EXPERIMENTAL
ELECTRICAL ENGINEERING

AND

MANUAL FOR ELECTRICAL TESTING

FOR ENGINEERS AND FOR STUDENTS IN
ENGINEERING LABORATORIES

VOL. I.

BY

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PREFACE TO THE FOURTH EDITION

While a member of the electrical engineering staff at Cornell University it was my privilege to proof-read and, in part, edit the first edition of Professor Karapetoff's "Experimental Electrical Engineering" published in 1909. Since that time, in teaching in the laboratories of another institution, the successive editions of this text, supplemented by mimeographed sheets applying to local equipment and conditions, have been used as the foundation of experimental procedure. When it came time to revise the Third Edition of Volume I, Professor Karapetoff, who for some years has not been actively engaged in undergraduate laboratory instruction, asked to be relieved of the task of the revision. He did the writer the honor of inviting him to undertake this work in his stead. This I have been very happy to do, endeavoring throughout the revision to maintain the same standards of technical excellence and the simplicity of literary style which characterize the other editions.

Some material formerly included in the text has been omitted, notably the chapters on telephone practice, and the short chapter on primary cells. The decision to omit this material came largely as the result of answers to a questionnaire sent to more than one hundred electrical engineering teachers throughout the country. Among other information requested was a statement of the extent to which they were giving the various experiments listed in Volume I, and their opinions as to the relative importance of each. It was found that very few were giving any telephone experiments, and almost none gave work on primary cells. Moreover, there are excellent texts on communication and telephone practice, and books of physics usually treat the primary cell.

No new chapters have been added in the revision, but all have been revised and most have been supplemented. Many new cuts have been made, a large number *replacing general characteristics by curves to scale taken on actual equipment.* Diagrams have been modernized and cuts of commercial apparatus brought up to date. *Complex notation has not been avoided,* rather its use is encouraged, since it is felt that practically all engineering schools have now adopted this highly efficient method of vector representation, even in the work given to non-electrical engineering students.

A few literature references have been made, chiefly to articles in those technical periodicals to which the student is most sure to have access, with a view to encourage him in the reading of such engineering literature.

Some subjects have been made more nearly independent of Volume II. Thus, the material on the polyphase induction motor is carried through the circle diagram and the analytical solution of the simpler equivalent circuit. The material on the alternator has been expanded to include the simpler phases of alternator regulation. Such additions have been made largely as the result of suggestions received from the electrical engineering teachers, who sent in the questionnaire above referred to.

The writer wishes particularly to acknowledge the very valuable replies received from these teachers, many of whom sent fairly complete files of their laboratory instruction sheets. Very practical comments were also received from engineers engaged in the manufacture of instruments and other electrical equipment. Thanks are due Professor G. R. Patterson for his careful work in the preparation of new drawings and to others of the staff for criticisms and suggestions.

B. C. Dennison

Carnegie Institute of Technology,
Pittsburgh, Pa.
April 24, 1933.

CONTENTS OF VOLUME I

CHAPTER I

MEASUREMENT OF RESISTANCE

Drop-of-Potential Method

Substitution Method

Wheatstone Bridge

CHAPTER II

AMMETERS AND VOLTMETERS — CONSTRUCTION

MOVABLE-IRON INSTRUMENTS

PERMANENT-MAGNET AND D-C-ELECTROMAGNET TYPE INSTRUMENTS

ELECTRO-DYNAMOMETER-TYPE INSTRUMENTS

OTHER TYPES OF INDICATING AMMETERS AND VOLTMETERS

RECORDING OF GRAPHIC INSTRUMENTS

CHAPTER III

AMMETERS AND VOLTMETERS — CALIBRATION

DIRECT–CURRENT POTENTIOMETERS

ALTERNATING–CURRENT POTENTIOMETERS

CHAPTER IV

WATTMETERS AND POWER-FACTOR METERS

ELECTRO–DYNAMOMETER–TYPE WATTMETER

INDUCTION-TYPE WATTMETERS

POWER-FACTOR METERS

CHAPTER V

WATTHOUR METERS

COMMUTATOR-TYPE WATTHOUR METER

INDUCTION-TYPE WATTHOUR METERS

CHAPTER VI

REACTANCE AND RESISTANCE IN A-C CIRCUITS

MUTUAL INDUCTION

CHAPTER VII

THE MAGNETIC CIRCUIT

LIFTING AND TRACTIVE ELECTROMAGNETS

CHAPTER VIII

PERMEABILITY AND HYSTERESIS LOOP

TRACTIONAL METHOD

BALLISTIC METHOD

STEADY DEFLECTION METHODS

TESTING OF PERMANENT MAGNETS

CHAPTER IX

MEASUREMENT OF CORE LOSS

THE MECHANICAL-TORQUE METHOD

THE WATTMETER METHOD

CHAPTER X

DIRECT-CURRENT GENERATOR

NO-LOAD CHARACTERISTICS

LOAD CHARACTERISTICS OF SHUNT-WOUND GENERATOR

SERIES-WOUND GENERATOR

CHAPTER XI

DIRECT-CURRENT GENERATOR (*Continued*)

AUTOMATIC VOLTAGE REGULATORS

OPERATION OF GENERATORS IN PARALLEL

THE THREE-WIRE SYSTEM

AUTOMOBILE LIGHTING GENERATOR

CHAPTER XII

DIRECT-CURRENT MOTOR

THE SHUNT-WOUND MOTOR

THE SERIES-WOUND MOTOR

THE COMPOUND-WOUND MOTOR

BRAKES AND DYNAMOMETERS

TROUBLES IN DIRECT-CURRENT MACHINES

CHAPTER XIII

LOSSES IN DIRECT-CURRENT MACHINERY

EXPERIMENTAL SEPARATION OF LOSSES

RETARDATION METHOD

SEPARATION OF LOSSES BY COMPUTATION

CHAPTER XIV

DIRECT-CURRENT MACHINERY — OPPOSITION RUNS

CHAPTER XV

ARMATURE REACTION AND COMMUTATION

MAGNETIC FLUX AT NO LOAD

ARMATURE REACTION

COMMUTATION AND COMMUTATION POLES

BRUSHES AND THEIR TESTING

CHAPTER XVI

THE TRANSFORMER

CHAPTER XVII

THE TRANSFORMER (*Continued*)

CHAPTER XVIII

INSTRUMENT TRANSFORMERS

POTENTIAL TRANSFORMERS

CURRENT TRANSFORMERS

CHAPTER XIX

THE ALTERNATOR — OPERATING FEATURES

VOLTAGE REGULATION

FREQUENCY METERS

CHAPTER XX

THE ALTERNATOR — VOLTAGE REGULATION

CHAPTER XXI

ALTERNATORS IN PARALLEL

CHAPTER XXII

THE POLYPHASE INDUCTION MOTOR

ELEMENTARY THEORY

STARTING CONDITIONS

PERFORMANCE TESTS

CHAPTER XXIII

THE SINGLE-PHASE INDUCTION MOTOR

CHAPTER XXIV

STORAGE BATTERIES

The Lead-Acid Cell

The Nickel–Iron Cell

OPERATION AND CONTROL OF STORAGE BATTERIES

BATTERY BOOSTERS

CHAPTER XXV

D-C MOTOR STARTERS AND CONTROLLERS

FACE PLATE D-C MOTOR STARTERS AND SPEED REGULATORS

THE DRUM-TYPE CONTROLLER

LIST OF EXPERIMENTS IN VOLUME I

CHAPTER I

MEASUREMENT OF RESISTANCES

CHAPTER II

AMMETERS AND VOLTMETERS — CONSTRUCTION

CHAPTER III

AMMETERS AND VOLTMETERS — CALIBRATION

CHAPTER IV

WATTMETERS AND POWER-FACTOR METERS

CHAPTER V

WATTHOUR METERS

CHAPTER VI

REACTANCE AND RESISTANCE IN A-C CIRCUITS

CHAPTER VII

THE MAGNETIC CIRCUIT

CHAPTER VIII

PERMEABILITY AND HYSTERESIS LOOP

CHAPTER IX

MEASUREMENT OF CORE LOSS

CHAPTER X

DIRECT-CURRENT GENERATOR

CHAPTER XI

DIRECT-CURRENT GENERATOR (*Continued*)

CHAPTER XII

DIRECT-CURRENT MOTOR

CHAPTER XIII

LOSSES IN DIRECT-CURRENT MACHINERY

CHAPTER XIV

DIRECT-CURRENT MACHINERY OPPOSITION RUNS

CHAPTER XV

ARMATURE REACTION AND COMMUTATION

CHAPTER XVI

THE TRANSFORMER

CHAPTER XVII

THE TRANSFORMER (*Continued*)

CHAPTER XVIII

INSTRUMENT TRANSFORMERS

CHAPTER XIX

THE ALTERNATOR — OPERATING FEATURES

CHAPTER XX

THE ALTERNATOR — ARMATURE REACTION

CHAPTER XXI

ALTERNATORS IN PARALLEL

CHAPTER XXII

THE POLYPHASE INDUCTION MOTOR

CHAPTER XXIII

THE SINGLE-PHASE INDUCTION MOTOR

CHAPTER XXIV

STORAGE BATTERIES

CHAPTER XXV

D-C MOTOR STARTERS AND CONTROLLERS

EXPERIMENTAL ELECTRICAL ENGINEERING

CHAPTER I

MEASUREMENT OF RESISTANCE

1. Fundamental Relations. — Experiment shows that electrical conductors offer a certain opposition to the passage of an electric current. It is necessary, therefore, to apply a difference of potential (electrical pressure) at the terminals of the conductor to produce a flow of current through it. This is analogous to the difference of pressure necessary at the ends of a pipe in order that water may flow through it. It is also found that the value of a steady current I produced in a conductor is directly proportional to the electrical pressure E applied. In other words

$$I = k \times E, \quad \text{or} \quad E = RI \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where R is the coefficient of proportionality, called the *resistance* of the conductor. This experimentally determined relation between electromotive force (abbreviated emf) and current is called *Ohm's* law.

The resistance of a conductor is its most important electrical feature, and many methods are available for accurately measuring and comparing resistances. The practical unit of resistance is called *the ohm* and is such a value that a conductor is said to possess a resistance of one ohm when an applied emf of one volt causes a current of one ampere to flow. According to the definition of this unit established by international agreement it is represented by the resistance, at 0° C, of a column of pure mercury 106.3 cm in length, weighing 14.452 grams, and of uniform cross-section.

In the measurement of resistances two methods are in most common use:

(a) Drop-of-potential method.
(b) Wheatstone bridge methods.

The first method is based directly on Ohm's law given above; the second is a null or balance method, the unknown resistance being compared with a known or standard resistance. These and some other methods are described below and illustrated by applications to certain practical problems.

DROP-OF-POTENTIAL METHODS

2. The Voltmeter-Ammeter Method. — The connections for determining the resistance of a conductor by the drop-of-potential method are shown in Fig. 1. In this figure, *Ba* is a storage battery or other steady source of direct current, X is the unknown resistance, and *Am* is an ammeter for measuring the current. *Vm* is a voltmeter connected to measure the drop of potential across the resistance X.

If E is the voltmeter reading in volts and a steady current of I amperes is read simultaneously on the ammeter, the resistance of the conductor is $X = E/I$ (in ohms), according to Ohm's law mentioned above.

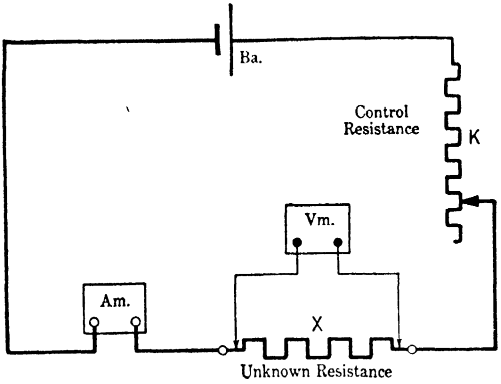

FIG. 1. Measurement of resistance by the drop-of-potential method.

Usually several readings are taken with different values of current and an average value of X calculated. It is important not to use a large enough current to heat the conductor appreciably, since the resistance of most conductors varies with temperature. It is therefore necessary to note the temperature at which resistance measurements are taken.

To insure accuracy in the results it is important that the current flowing through the voltmeter shall be negligible as compared with that through the resistance, or else that a suitable correction be made for the voltmeter current. Suppose that in Fig. 1, the voltmeter reads 100 volts and the ammeter 0.5 ampere. Assume that the voltmeter resistance is 10,000 ohms. The current through the voltmeter is therefore $100/10{,}000 = 0.01$ ampere. The true current through X is therefore $0.5 - 0.01 = 0.49$ ampere, and the unknown resistance X is $100/0.49 = 204$ ohms, rather than $100/0.5 = 200$ ohms, obtained by neglecting the voltmeter current. This latter value is seen to be in error some 2 per cent. In general terms, if E_x be the voltage drop across the unknown resistance X when a total current I flows in the circuit, and if r is the voltmeter resistance, from eq. (1) may be written

$$E_x = (I - E_x/r)X, \quad \text{or} \quad X = E_x/(I - E_x/r) \quad . \ . \ . \ (2)$$

When the drop-of-potential method is to be used for very accurate measurements a sensitive galvanometer or a potentiometer (§73) is used. In the latter no current is shunted around the unknown resistance; in the former it can be made of negligible amount.

A modification of the drop-of-potential method known as the *comparison-of-drops method* is shown in Fig. 2. Here a standard resistance R is used in series with the unknown resistance X, and the voltage drop measured in succession across X and across R. Let the readings be E_x and E_r. If the current I has not changed during the measurement we have, neglecting the current through the voltmeter,

$$I = E_r/R = E_x/X \quad . \ . \ (3)$$

or

$$X = RE_x/E_r \quad . \ . \ . \ (4)$$

The voltmeter is conveniently transferred from one resistance to the other by means of a double-pole, double-throw switch (Fig. 2), or by using flexible leads which may be connected in turn across R and then across X.

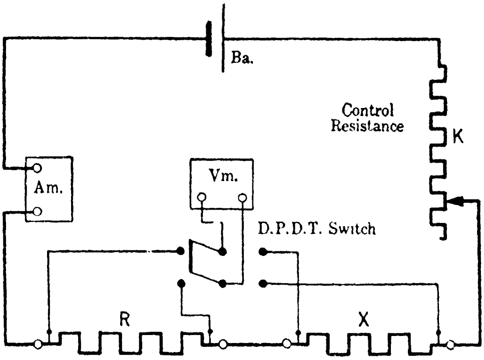

FIG. 2. Measurement of resistance by the comparison-of-drops method.

The advantage of the comparison method is that only one instrument, the voltmeter (or galvanometer), needs to be calibrated, and even for this it is not necessary to know the actual value of divisions on its scale, since only the ratio of the readings enters the result. When the unknown resistance X is comparatively high and a voltmeter of fairly low resistance is used an appreciable fraction of the current may be shunted through the voltmeter, so that eq. (4) is no longer exact. Where allowance for the voltmeter resistance must be made this may be done as follows. From eq. (2), $I = E_x/X + E_x/r$; likewise $I = E_r/R + E_r/r$, so that $E_x/X + E_x/r = E_r/R + E_r/r$. Solving for X gives

$$X = E_x/(E_r/R + [E_r - E_x]/r) \quad . \ . \ . \ . \ . \ . \ . \ (5)$$

It is clear that if either r is very great or $E_r = E_x$, eq. (5) reduces to eq. (4). It is evident that best results are obtained when the unknown and standard resistances are practically equal.

In a modification of this method suggested by Professor Springer variable resistances are connected in the leads to the voltmeter switch and these are varied until the two voltmeter deflections are equal. Under this condition the same current flows through the voltmeter in each case, therefore the same currents pass through X and R, giving drops E_x and E_r truly proportional to X and R. If R_r and R_x are the resistances added in the voltmeter leads, and r is the voltmeter resistance

$$IR/(R_r + r) = IX/(R_x + r) \quad \text{or} \quad X = R\frac{R_x + r}{R_r + r} \quad . \quad . \quad . \quad (6)$$

3. EXPERIMENT 1-A. — Resistance Measurement by the Drop-of-Potential and the Comparison Methods. — The purpose of the experiment is to determine the values of certain unknown resistances by the methods explained in the preceding section.

(1) *Ammeter-and-voltmeter method.* Connect the apparatus as in Fig. 1. Close the circuit and adjust the regulating rheostat K to give the highest current which does not appreciably heat the resistance to be measured. Read volts and amperes simultaneously. Reduce the current in steps, and take similar readings at each step. In order to see the error due to the current through the voltmeter, perform the foregoing test with a high resistance, using a small current, then with a low resistance, using a large current. Read the values of current with the voltmeter on and off, if there is an appreciable difference.

(2) *Comparison-of-drops method.* Connect as in Fig. 2 and read the voltage drops across X and R, keeping the current constant for the two readings. Repeat for several settings of the rheostat K. Perform the preceding measurements for several values of the unknown resistance, preferably using the same unknowns that were measured in (1) so that a comparison of results by the two methods will be possible. Try the effect of using standard resistances, R, which give practically the same voltage drop as that across X, also standards which give drops rather widely different. Select a comparatively high-resistance X and connect a millivoltmeter or a sensitive galvanometer in series with a high resistance across X. Note any changes in the galvanometer indications when the voltmeter is connected across R, across X, and when removed. Do the same with the resistance X quite low.

(3) Introduce variable known resistances into the leads to the voltmeter switch and take readings on one or more of the resistances already measured, using the method of Professor Springer. Note the values of resistance necessary to give equal voltmeter deflections across X and R. Before leaving the laboratory find the resistances

of the galvanometer circuit, of the voltmeter, and of the standard resistances R.

Report. For each unknown resistance in part (1) plot volts against amperes as abscissas, and draw the straight line through the origin which comes nearest to passing through all the points. Compute the unknown resistances from the values on the straight lines. Calculate the corrected values of X using formula (2) and compare. Show that the error due to the voltmeter current is greater for high values of X and explain. For part (2) plot volts E_r against E_x as abscissas and draw straight lines through these points. Compute the unknown resistances from the ratios of these voltages. Describe the experiment in which the galvanometer was used and estimate theoretically the error due to the current through the electromagnetic voltmeter with different values of resistance which were measured. For part (3) state results obtained and give your opinion relative to the accuracy and convenience of this method.

4. EXPERIMENT 1-B. — Measurement of High Resistance by Means of a Voltmeter. — The purpose of the experiment is to determine, by means of a high-resistance d-c voltmeter (or galvanometer), and constant-potential supply, the values of certain high resistances. The method is applicable to the measurement of insulation resistance (below one megohm), of high-resistance "grounds" on electric circuits, of the unknown values of resistances to be used as multipliers for instruments, etc.

Suppose that a voltmeter of unknown resistance r, when connected across the supply voltage of value E, gives a deflection D'. Then, if connected across the same potential but in series with the unknown resistance X, the reading will be reduced to some value D'' such that

$$D'' = D'r/(r + X) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

This follows from the fact that the reading of a voltmeter is proportional to the current through it. When the meter alone is connected across the supply the current flowing in the meter circuit is $I' = E/r$. When the unknown resistance X is connected in series with the voltmeter across the same supply the current in the meter circuit is $I'' = E/(r+X)$, and from these two expressions eq. (7) follows.

If eq. (7) is solved for X it is found that

$$X = r(D' - D'')/D'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Proceed in the performance of the experiment as follows:

(a) Using the method outlined above, determine the resistance between the terminals of an open slate or composition-base switch which has first been immersed in water and then shaken free of all excess moisture. Take readings over a period of 30 minutes or more while the base is drying out. Use a voltmeter having a full-scale deflection of 150 volts or higher and a 110-volt d-c supply of fairly constant voltage. If higher values of voltage and corresponding voltmeters are available more accurate values of resistance may be obtained. Alternate the readings of the voltmeter with those taken with a sensitive galvanometer of very high resistance.

(b) Repeat the determinations of (a), using as the unknown resistance a coil of cotton-covered wire wound on a pipe or other conducting rod, having first saturated the coil with shellac. Measure the resistance between the copper wire and the rod. Again note the variation of resistance with time as the shellac dries.

(c) Measure the resistance to ground (frame) of the field and armature windings of a d-c generator or motor or a transformer. Try to determine to what extent this varies with the temperature of the machine, and with the voltage used in the measurements.

(d) Using a voltmeter multiplier of unknown resistance determine its value and calculate the new multiplying factor of the voltmeter with it in series.

Report. (1) Plot curves of insulation resistance of the composition-base switch as a function of time. (2) Do the same for the freshly shellacked winding. (3) Give the results of parts (c) and (d) of the experiment. (4) Calculate the upper and lower limits of the resistance which it is possible to measure using the voltmeter; using the galvanometer. Explain why more accurate results may be obtained using a high-resistance voltmeter, rather than one of low resistance.

5. Influence of Length, Cross-Section, and Material, of a Cylindrical or Prismatic Conductor upon its Resistance. — At a constant temperature the resistance of a conductor of constant cross-section (a) increases in direct proportion to its length, and (b) varies in inverse ratio to its cross-section. Hence, the resistance may be expressed by the formula

$$R = \rho l/A \qquad (9)$$

where l is the length of the conductor, A is its cross-sectional area, and ρ is a physical constant which characterizes the material of the conductor and is known as its *resistivity* or *specific resistance.* The above relation is somewhat analogous to that governing the flow of water in a pipe: the longer the pipe the greater is its frictional resistance; the larger its cross-section of opening the easier it is to force through it a given stream of water.

It will be seen from formula (9) that ρ is the resistance of a prismatic conductor of a certain material, of unit length and of unit cross-section. For the practical units in use and for the numerical values of ρ for copper see §37. For values applying to other metals the reader is referred to the elementary textbooks on electricity or to one of the numerous tables of physical constants in engineering handbooks

6. EXPERIMENT 1-C. — Determination of the Influence of Length, Cross-Section, and Material, of a Conductor on its Resistance. — The purpose of the experiment is to verify the relation stated in the preceding section and to determine the specific resistance ρ for a few commonly used conducting materials such as copper, aluminum, iron, German silver, and carbon. The connections shown either in Fig. 1 or in Fig. 2 may be used. Insert the conductor to be tested in place of X and adjust the current by means of the rheostat K. Read the current in the ammeter and measure the voltage drop across a certain length of the conductor. Use knife-edge contacts at the points of contact of the voltmeter leads, in order both to insure good contact and to have an exact length of conductor between contacts. Record this length. Reduce the current in steps, taking similar readings at each step.

Repeat the same determinations on conductors of (a) different length, (b) of different cross-section, (c) of different materials. In each case vary but one factor entering into formula (9). Do not use currents of great enough value to appreciably heat the conductor under test as this will, in general, change its resistance. When accurate results are required the conductor should be immersed in an oil-bath maintained at a particular temperature. Before leaving the laboratory measure accurately the cross-sections of the several conductors tested.

Report. (1) Show from the readings that the resistance of a conductor of given cross-section is directly proportional to its length. To do this, plot the observed volts against amperes for each length of conductor used and draw the corresponding straight lines through the origin. The ratio of ordinates at a given abscissa must then be equal to the ratio of lengths. (2) Show from the readings that the resistance is inversely proportional to the cross-section. For this purpose plot the observed volts to amperes as abscissas for each cross-section used, and draw the corresponding straight lines through the origin. The ratio of two abscissas for any chosen ordinate must be equal to the inverse ratio of the corresponding cross-sections of the samples. (3) Calculate the resistivity ρ of each material tested, and compare with the values given in an available reference book.

7. Influence of Temperature upon the Resistance of a Conductor. — The electrical resistance of a metal increases with its temperature, and within practical limits the increase in resistance is usually assumed to be proportional to the temperature rise. Thus, if α is the temperature coefficient (proportional increase in resistance per degree temperature rise) at 0° C, and R_0 is the resistance of the conductor at 0° C, the resistance R' at a temperature T' is

$$R' = R_0(1 + \alpha T') \quad \text{. . . . .} \quad (10)$$

For commercial copper the resistance increases 0.427 per cent for each degree centigrade, or $\alpha = 0.00427$, *the resistance at* 0° *C being taken as* 100 *per cent.* Thus, if the resistance of a copper conductor at 0° C is R_0 ohms, the resistance R_{15} at 15° C is determined by eq. (10), or $R_{15} = R_0(1 + 0.00427 \times 15) = R_0(234 + 15)/234$. (Note that $234 = 1/0.00427$.) Similarly the resistance at, say, 25° C is $R_{25} = R_0(234 + 25)/234$. From these two expressions it follows that $R_{25} = R_{15}(234 + 25)/(234 + 15)$. In more general terms, if R' is the resistance at the temperature T' and R'', the resistance at T''

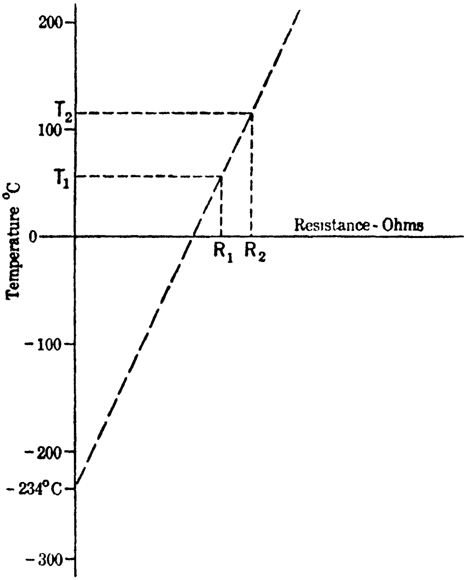

FIG. 3. The variation of resistance of a copper conductor with temperature.

$$R''/R' = (234 + T'')/(234 + T') \quad \text{. . . . . .} \quad (11)$$

Equation (10) suggests that the curve of resistance of metal conductors is a straight line intersecting the axis of ordinates at $-1/\alpha°$ C, and for copper at −234° C. This is illustrated in Fig. 3.

The temperature coefficient of materials has widely different values for different substances, as shown in the following table. For additional data see any electrical handbook. Carbon and electrolytes have a negative temperature coefficient, i.e., their resistance decreases with increase in temperature.

Table I

TEMPERATURE COEFFICIENTS OF COMMONER CONDUCTORS

(American Handbook)

Material	Temperature Coefficient at 0° C
Copper	0.00427
Aluminum	0.00423
Carbon	−0.0003
German silver	0.00031
Iron — pure	0.00625
Tungsten	0.0051
Manganin	0.00001 to 0.00004

The following are some of the problems in which it is essential that the temperature coefficient of the materials be known:

(1) A resistance used as a precision standard for measurements, or one used for the purpose of accurate regulation, in both of which cases it is desirable that the resistance coefficient be negligible. Thus a manganin shunt or resistance used as voltmeter multiplier increases only from 1/100 to 4/100 of 1 per cent for a temperature rise of 10 degrees.

(2) Some protective resistors must be made of materials in which the resistivity increases rapidly with the temperature. This protects the apparatus in series with the (ballast) resistor in event the applied voltage for some reason increases. An iron wire, sealed in an atmosphere of hydrogen to prevent oxidation, is often used for this purpose.

(3) The change in resistance of a conductor with temperature is often used for temperature measurements in the so-called resistance thermometers. These are especially convenient for use in inaccessible places, as when built into an electrical winding. They are also used when a continuous graphic record of temperature is required.

(4) The carbon-filament lamp has a negative temperature coefficient. The tungsten lamp which has replaced it has a high positive coefficient (see Table I). For this reason the tungsten lamp is less sensitive to changes in voltage.

The temperature coefficient of a solid conductor may be determined in a manner similar to that in which its resistance is measured (§3). The material tested is placed in an oil-bath maintained at a definite temperature. The bath is heated gradually while, every few minutes, readings of volts and amperes are taken as in Fig. 1. The corresponding temperatures are read on a thermometer placed in the oil, which must be continually stirred so that all parts of the bath shall be at the same temperature. For more accurate measurements of resistance than those given by the drop-of-potential method a Wheatstone bridge (§17) or a Kelvin double bridge (§34) may be used. For the conductivity measurement of an electrolyte see §§15 and 23.

8. EXPERIMENT 1-D. — Determination of Temperature Coefficients of Solid Conductors. — The experiment is performed in a manner similar to Experiment 1-A; the theory is given in the preceding section.

(a) Perform this experiment with materials such as copper or iron which have an appreciable temperature coefficient, then with manganin and German silver, which have a very low coefficient. First measure the resistances of the samples at room temperature, being careful not to heat them appreciably while the readings are being taken. Place the materials in an oil-bath and raise the temperature of the bath, say, 50° C, being sure to bring all parts of the bath to the same temperature by constant stirring. The bath may be heated by sending a relatively large current through the resistance to be measured and then taking resistance readings at currents which will not increase the temperature further.

(b) Demonstrate that the resistance of carbon decreases with increase in temperature by connecting a carbon-filament lamp as in Fig. 1 and, having first taken its resistance at room temperature, using a small current, vary the current over a wide range by means of the variable resistance *K*. Measure resistance and endeavor to get an approximate value of the coefficient by using the following rough scale (due to Poillet) of temperatures as functions of the color of the filament in the lamp being used for the measurement.

(c) Repeat the test using a tungsten-filament lamp.

Table II

COLOR TEMPERATURE SCALE

(Poillet)

Temperature °C	Color	Temperature °C	Color
525	Nascent red	1200	Bright orange
700	Sombre red	1300	White
800	Nascent cherry	1400	Brilliant white
900	Cherry	1500	Dazzling white
1000	Bright cherry	1600	
1100	Dull orange		

(d) Take resistance measurements on a length of iron wire suspended in air, first at room temperature using low values of current, and then increasing the current to a value which brings the wire up to a color recognized as one of those in the color-temperature scale of Table II.

Report. (1) Calculate the values of temperature coefficient of the metals tested in the oil-bath, referring α to 0° C. Thus, from eq. (10), $R' = R_0(1 + \alpha T')$ while $R'' = R_0(1 + \alpha T'')$, so that $R'/R'' = (1 + \alpha T')/(1 + \alpha T'')$. Solving for α gives

$$\alpha = (R'' - R')/(R'T'' - R''T') \quad \ldots\ldots \quad (12)$$

Compare the values so determined with those given in an electrical handbook.

(2) Give the results of the tests of the carbon-filament and the tungsten-filament lamps, tabulating resistances and estimated temperatures and plotting the corresponding curves. From the curves (extended) estimate the temperature coefficients of carbon and tungsten (at 0° C).

(3) Give the results obtained on the iron wire and calculate the temperature coefficient at a dull red heat.

(4) Discuss the current-time curves of carbon-filament and tungsten-filament lamps when suddenly connected across a supply of normal voltage. Which type of lamp is more liable to burn out on intermittent service?

9. Determination of the Temperature Rise in a Winding, from Increase in Resistance. — This problem is the inverse of that in Experiment 1-D. The temperature coefficient is now assumed to be known, and the temperature rise of a coil of wire is calculated from the increase in its resistance.

From eq. (11) one may write

$$R''/R' - 1 = \frac{234 + T''}{234 + T'} - \frac{234 + T'}{234 + T'}$$

or

$$T'' - T' = \frac{R'' - R'}{R'}(234 + T') \quad \ldots\ldots \quad (13)$$

This is the value of temperature rise determined from the increase of resistance above R', at an initial temperature T'. Somewhat more simply one might write from eq. (11) the final temperature T'' as

$$T'' = \frac{R''}{R'}(234 + T') - 234 \quad \ldots\ldots \quad (14)$$

Suppose, for example, that the resistance of a coil of copper wire is 4.573 ohms at 15° C, while at some unknown temperature it has been found to be 5.468 ohms. Applying eq. (13) to the example gives the rise in temperature

$$T'' - T' = \frac{5.468 - 4.573}{4.573}(234 + 15) = 48.7° \text{ C}$$

or the final temperature $T'' = 48.7 + 15 = 63.7°$ C, a value confirmed by eq. (14). The temperature so determined is the mean temperature of the coil.

This method is commonly used for the determination of the temperature rise of electrical machines. It is open to the objection that it gives only the average temperature rise of the winding. A measurement by means of thermometers is even less satisfactory since the thermometer usually can be placed only on the outside surface of the winding where the temperature is materially less than that inside the winding. An excessive temperature within a coil carbonizes the cotton or paper insulation and may finally lead to a short circuit. As the temperature rise of a coil depends upon the conditions of cooling, as well as upon the loss to be dissipated, an improvement in cooling is obtained by immersing the coil in oil or subjecting it to a draft of air (oil-cooled and air-blast transformers).

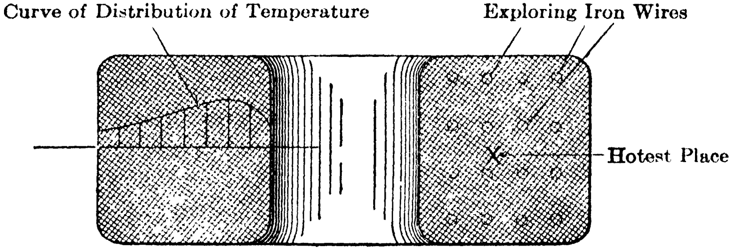

FIG. 4. Section through a coil, showing location of exploring wires, and the distribution of temperature.

The distribution of temperature within a coil (Fig. 4) may be investigated by placing exploring wires of known temperature coefficient in various positions within the coil as indicated by the small circles. As the coil heats, the temperature rise in the various spots is determined from the increase in temperature of the exploring wires. Copper wire is generally used for these resistance thermometers, the terminals of each resistance coil being brought out separately and its resistance being measured from time to time as the large coil heats. In place of the resistance coils, thermocouples are often embedded in the winding and temperatures found from the emf's of these thermocouples as read by means of a special potentiometer. If resistance coils are used their resistances are measured by the Wheatstone bridge rather than by the drop-of-potential method to insure greater accuracy in results.

10. EXPERIMENT 1-E. — Temperature Rise in a Winding determined by Rise of Resistance. — The purpose of the experiment is to determine the rise in temperature of a coil both from the increase in its own resistance and from the increase in resistance of resistance elements embedded in the winding; also to compare the values of temperature rise determined by these methods with that from thermometer readings. Provide a suitable coil, such as that shown in Fig. 4, having, if possible, resistance elements built in at known points in the coil. Place thermometers on the surface of the coil, covering the bulbs with cotton waste or putty to keep off air currents. Before heating the main coil and while it is at room temperature make a careful measurement of its resistance. Also, connect each of the exploring coils in turn as the unknown X arm of a Wheatstone bridge (Fig. 11) and determine its resistance. Now heat the main coil by means of an electric current, and take readings of current and voltage drop and periodic readings of the test resistances, noting also the thermometer readings and the time of each set of readings. Continue the test until the thermometers show a substantial rise in temperature. Repeat the test for at least one other condition of cooling, as (a) enclosed in a box, (b) cooled by an electric fan, etc., in each case maintaining the same current through the coil for the same length of time.

Report. Calculate the rise in temperature of the various parts of the main coil from the increase in resistance of the test coils, and plot temperature from inside to outside as a curve (see Fig. 4), or mark the temperatures on a sketch of the cross-section of the coil. Compare the average and maximum temperature rise shown by the exploring coils with the mean temperature shown by the increase in main coil resistance, and with the values given by the thermometers. What is the ratio of maximum rise to mean rise? to minimum rise? Indicate the effects of the different methods of cooling the coil upon its temperature rise. State advantages and disadvantages of oil cooling as compared to air-blast.

11. Resistances in Series and in Parallel. — Resistances are connected in electric circuits in various combinations; it is sometimes required to determine the value of a single resistance which would produce the same effect as that of two or more given resistances. A few simple cases are here considered.

(1) *Resistances in series.* From the very concept of resistance it follows that two resistances, R_1 and R_2 (Fig. 5), connected in series, are equivalent to a single resistance of value $R_1 + R_2$. The same is true

for any number of such resistances, so that, for resistances in series,

$$R_{\text{equiv.}} = \sum_1^n R \qquad (15)$$

(2) *Resistances in parallel* (Fig. 6). The equivalent resistance in this case is less than either of the component resistances, because the addition of each new resistance means a new path for the current. Let the common voltage across the resistances be e, and the currents i_1, i_2, etc. Then

$$i_1 = e/R_1;\ i_2 = e/R_2;\ \text{etc.}$$

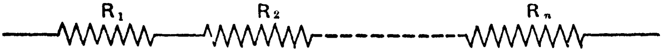

FIG. 5. Resistances in series.

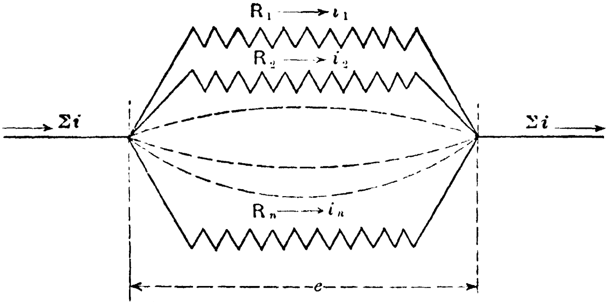

FIG. 6. Resistances in parallel

The equivalent resistance must, by definition, have such a value that the total current $\sum i$ would flow through it with the same voltage e; hence

$$\sum_1^n i = e/R_{\text{equiv.}} \qquad (16)$$

Substituting the values i_1, i_2, etc., we have

$$\sum_1^n e/R = e/R_{\text{equiv.}}$$

or, dividing both sides of the equation by e,

$$\frac{1}{R_{\text{equiv.}}} = \sum_1^n 1/R = 1/R_1 + 1/R_2 + 1/R_3 + \cdots \qquad (17)$$

Special cases. For two resistances in parallel, formula (17) becomes

$$1/R_{\text{equiv.}} = 1/R_1 + 1/R_2 = \frac{R_1 + R_2}{R_1 R_2}$$

so that

$$R_{\text{equiv.}} = R_1R_2/(R_1 + R_2) \quad \ldots \ldots \ldots \quad (17a)$$

or, stated verbally, when two resistances are in parallel the equivalent resistance equals the product of the resistances divided by their sum.

By a similar line of reasoning, with three resistances in parallel,

$$R_{\text{equiv.}} = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \quad \ldots \ldots \quad (17b)$$

Stated in words, with three resistances in parallel, the equivalent resistance is equal to the product of the three resistances divided by the sum of the products of the several pairs of resistances.

Formula (17) gives the value of the equivalent resistance in terms of the component resistances. The reciprocal value of resistance is called *conductance*, and the result (17) indicates that the equivalent conductance is equal to the sum of the conductances of the branches connected in parallel. Denoting the conductances by g, we have

$$g_{\text{equiv.}} = \sum_{1}^{n} g \quad \ldots \ldots \ldots \ldots \quad (18)$$

which is analogous to the expression (15).

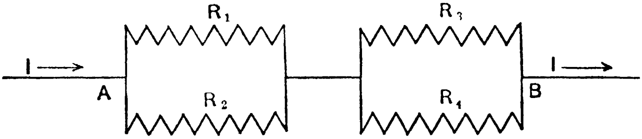

FIG. 7. Combination of resistances in series and in parallel.

(3) *Series-parallel combination of resistances.* Let it be required to find the resistance equivalent to the combination shown in Fig. 7. First find a resistance r' equivalent to the resistance offered by R_1 and R_2, by using formula (17) or (17a); in the same way find the resistance r'' equivalent to R_3 and R_4. The equivalent resistance of the whole combination is $(r' + r'')$.

A more complicated combination of resistances in series and in parallel is shown in Fig. 8. It corresponds to the practical case of a transmission line $OeO'e'$ with current-consuming devices R_1, R_2, . . . R_5 connected at different places. The problem is to find one single resistance which would allow the same total current to flow, with the same supply voltage between O and O'.

The problem is solved in steps: The resistances R_4 and $(2r_5 + R_5)$ are connected in parallel between the points d and d'. Their equivalent resistance R_4', according to formula (17a), may be calculated from the expression

$$R_4' = \frac{R_4R_5 + 2R_4r_5}{R_4 + R_5 + 2r_5}$$

In a similar manner we find

$$R_3' = \frac{R_3R_4' + 2R_3r_4}{R_3 + R_4' + 2r_4}$$

$$R_2' = \frac{R_2R_3' + 2R_2r_3}{R_2 + R_3' + 2r_3}$$

$$R_1' = \frac{R_1R_2' + 2R_1r_2}{R_1 + R_2' + 2r_2}$$

and finally

$$R_{\text{equiv.}} = R_1' + 2r_1$$

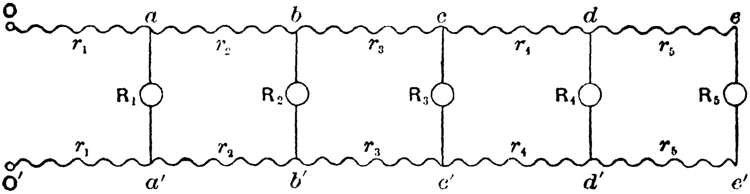

Fig. 8. Current-consuming devices R_1, R_1, etc., connected across a transmission line of appreciable resistance.

The problem is solved by calculating R_4' from the first equation and substituting its value in the second equation; then R_3' is calculated from this equation and its value substituted in the next equation, etc.

In more complicated networks, such, for example, as an unbalanced Wheatstone bridge, the component resistances cannot be reduced to series and parallel combinations, and the unknown currents are found by applying Kirchhoff's two laws.

12. EXPERIMENT 1-F. — Exercises with Resistances in Series and in Parallel. — Take several resistances of suitable value and measure them separately by the drop-of-potential method or with a Wheatstone bridge. Connect the resistances as shown in Figs. 5 to 8, or in any other desired series-parallel combination, and measure the resultant (equivalent) resistance. Compare the results with the formulas given in the preceding article.

13. EXPERIMENT 1-G. — Measurement of Resistance of a D-C Armature. — A direct current is passed through the armature from an external source, and the voltage drop at the terminals is measured with a low-reading voltmeter. The ratio of this voltage to the current gives the resistance of the armature. In performing the measurement it is advisable to hold the voltmeter leads first on the terminals of the machine, and then on the commutator bars under the brushes. This will make it possible to separate the voltage drop due to the armature resistance from the drop due to contact emf, and the resistance of the brushes themselves (§384).

This contact drop is not quite the same when the machine is at rest as it is when the machine is running. To determine the difference, the machine may be run at a low speed with the field circuit open, and the measurement repeated. It must be remembered, however, that even when the field circuit is open, there is some residual magnetism in the field, and quite possibly some flux due to armature reaction. This magnetism induces a voltage in the armature and may considerably affect the results. In order to eliminate its influence, the measurements must be repeated with the machine running in the opposite direction at exactly the same speed. The average of the two resistances will give the true resistance of the armature circuit.

In stating the results it is necessary to note to which temperature they refer. The resistance of the armature, and therefore the voltage drop, will be somewhat higher when the machine is hot. In contracts the efficiency is usually guaranteed at a certain temperature of the machine, say 75° C. If the resistance was measured when the machine was cold, for example at 25° C, its resistance at 75° C can be computed from the formula given in §7. Or else, if the resistance of the armature was measured when cold and again after a continued run at some load (temperature test), the temperature rise in the armature winding may be calculated from the increase in resistance, as explained in §7.

Report. Plot volts to amperes as abscissas and draw the straight line that passes through these points. The equation of the line is

$$E = E_b + IR_a$$

where E_b is the intercept of the line with the axis of volts, and R_a, the armature resistance, is the value determined from the slope of the line. When the contact drop at the brushes is not included, the straight line must pass through the origin. With the contact drop it will lie above the origin, because the contact counter emf has an appreciable value even with the smallest current. Give separately the values of the resistance of the armature proper and the voltage drop due to brush contact. Similarly, determine the contact emf at the brushes with the armature revolving. Draw a curve showing the voltage drop in the armature at higher temperatures; indicate the voltage drop due to the brush contact.

SUBSTITUTION METHOD

14. Theory of the Substitution Method. — With this method (Fig. 9) a certain current, supplied from a constant voltage source, is made to flow in the circuit containing the unknown resistance x; then an adjustable calibrated resistance r is substituted in its place and set at such a value as to give the same current as before. If the battery voltage remains the same, then the known resistance is equal to the unknown resistance. In Fig. 9, *Ba* is the battery, *Ga* is an ammeter or a galvanometer, which need not be calibrated, but the galvanometer, if used, should be protected by a shunt, as indicated in Fig. 9. This shunt may well be of the Ayrton (universal) type; it should be connected first with a very low resistance in the main circuit and this value increased to that giving the desired deflection (or sensitivity in null-methods). *Rh* is a regulating resistor. The double-throw switch is first thrown down and the current adjusted so as to obtain a desired deflection of the galvanometer. After this, the switch is thrown up, and r adjusted until the galvanometer gives the same deflection; then the resistance r is equal to x.

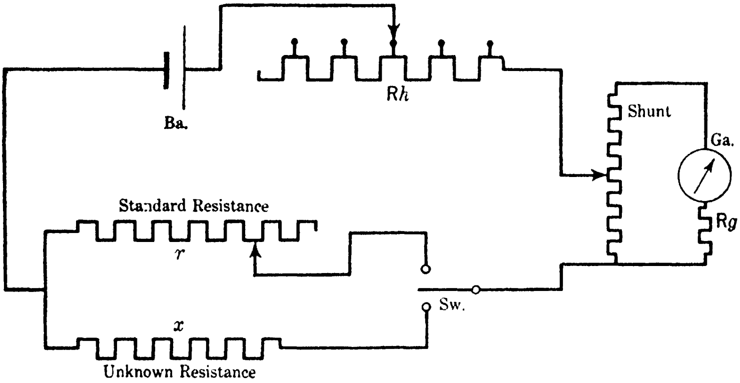

FIG. 9. Substitution method of measuring resistances.

When using this method, care should be taken that the battery emf does not change (on account of polarization) between the two readings, and also that the resistance *Rh* remains constant. Instead of being connected in parallel, r and x may be connected in series, as shown in Fig. 10, and short-circuited in succession.

The substitution method is sometimes used for measuring high resistances, such as insulation resistances: also for measuring the resistance of liquid conductors, as explained in §15.

15. Measurement of the Resistance of Electrolytes by the Substitution Method. — The difficulty in measuring the electrical resistance of most liquids is that they are decomposed by electric current. Gases are deposited on the electrodes, the resistance being thereby increased, and a counter emf of polarization set up. One way out of this difficulty is to measure the resistance of liquids with an alternating current (see §23). Another way is to use the substitution method shown in Fig. 10.

The liquid under test is placed in a glass tube *V*, provided with plugs at both ends, and with the electrodes *pp*, the upper electrode being adjustable. *R* is a calibrated resistance box. A certain current is sent through the circuit, until the conditions in the liquid become constant; then the deflection of the galvanometer is noted. After this, the upper electrode *p* is moved slowly by an amount *x*, as shown by the dotted lines. The current is brought back to its former value by increasing the resistance *R* by an amount *r* which is evidently equal to the resistance of the column *x* of the liquid. The effect of the polarization is thus eliminated, as it is present to some extent with both positions of *p*.

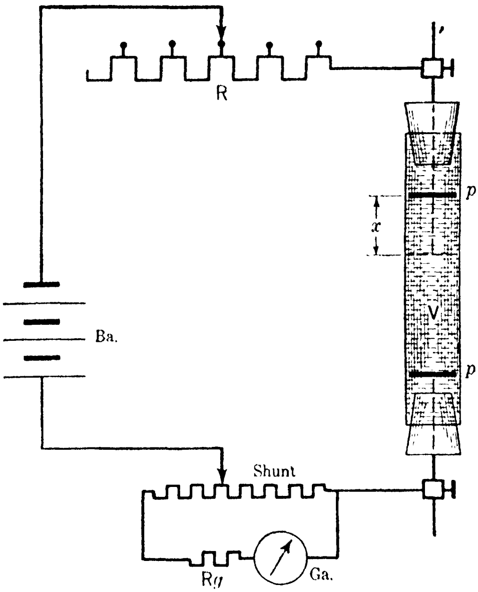

Fig. 10. Measuring the resistance of a liquid by the substitution method.

The resistance of a liquid is usually expressed as that of a column 1 cm high and having a cross-section of 1 sq cm. This is called the resistivity, or the specific resistance of the liquid. If the cross-section inside the glass tube is *A* square centimeters, the specific resistance of the liquid under test is rA/x. This is because the resistance is proportional to the length and inversely proportional to the cross-section of the conductor (see eq. (8), §5).

16. EXPERIMENT 1-H. — Measurement of Resistance by the Substitution Method. — The theory of the method and its application to the measurement of resistance of solid conductors are given in §14; the application to liquid conductors is explained in §15. The experiment should be so planned as to give the student some practice with the arrangements illustrated in Figs. 9 and 10. One problem of interest is that of the resistance of a given length of copper wire of a known B. & S. gauge number. Using the circuit of Fig. 10, first measure the resistance of a coil of copper wire and then, decreasing the length by, say, 50 ft, again measure the resistance. In the report discuss the advantages and disadvantages of the substitution method as compared with the drop-of-potential method and the comparison method. What are the possible sources of error in this method? Compare the resistances of the copper wire with that to be expected from values in the wire tables.

WHEATSTONE BRIDGE

17. *The combination of conductors known as the Wheatstone bridge* has many applications and is used in many modified forms. In its simplest form, Fig. 11, it is used for measuring resistances, and has the following features:

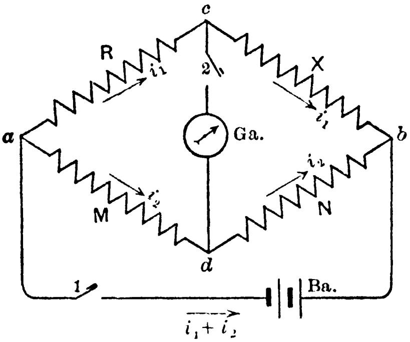

FIG. 11. The diagram of connections known as the Wheatstone bridge.

(1) The unknown resistance X is compared directly with a standard resistance R.

(2) No calibrated measuring instrument is required, the adjustment being made by bringing the galvanometer *Ga* to zero (null method).

(3) Only a very small current need be employed so that a single dry cell will often serve as the battery, although a 4-volt storage battery with series resistance is to be preferred.

(4) Minor variations in the voltage of the supply do not affect the readings of the bridge.

The unknown resistance X and the standard known resistance R are connected in series with each other and with the battery *Ba.* The combination is shunted by two other resistances M and N, the ratio of which is known. The galvanometer is bridged between the two sets of resistances, hence the name " bridge." Contact keys 1 and 2 are provided in the battery and galvanometer circuits.

The battery key 1 is closed first, and then the galvanometer key 2 is closed. If the galvanometer gives a deflection, the " balancing " resistance R and the "ratio" resistances M and N are varied until the galvanometer comes back to zero. Then

$$X/R = N/M \quad \text{. . . . .} \quad (19)$$

Proof: Suppose that the four resistances satisfy the condition that no current flows through the galvanometer; in other words, the difference of potential between the points c and d is zero and the bridge is "balanced." From this condition it follows that the voltage drop in the branch ac is equal to that in the branch ad; or, with the notation in the sketch,

$$Ri_1 = Mi_2$$

Similarly

$$Xi_1 = Ni_2$$

Dividing the second equation by the first, we obtain equation (19).

Instead of using R for the standard resistance, we can use N for a comparison with X, while M and R may serve as ratio coils. From eq. (19) we have identically

$$X/N = R/M \quad \text{. . . . .} \quad (20)$$

The greatest accuracy of measurement is obtained when X is nearly equal to R, and consequently the ratio of N to M is nearly equal to unity. The bridge is normally used for resistances between one and ten thousand ohms, although it may be used with decreased accuracy beyond these limits. For lower values of resistance, the Kelvin double-bridge (§34) is to be preferred.

18. Slide-Wire Wheatstone Bridge. — A simple Wheatstone bridge is shown in Fig. 12; the ratio resistances M and N are formed by two parts of a straight wire; the ratio is varied by moving the sliding contact d. The slide-wire is made of a non-corrodible metal of high specific resistance and of a small temperature coefficient. The brass blocks a, b, c to which R and X are connected have a negligible resistance.

To measure the resistance X, key 1 is closed first to allow the current to become steady,[1] then key 2, and the contact d is moved along the wire until the galvanometer remains on zero when key 2 is closed or opened. Let the standard resistance R equal 10 ohms; with the position of the contact shown in the sketch, $M = 27$ div., $N = 73$ div. According to

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[1] In case R and X are low (below 1 ohm) in value it may be best to close the galvanometer circuit first and the battery circuit later to avoid errors due to heating R and X. It may even be desirable to interchange battery and galvanometer terminals when there is danger that $R + X$ will pass too much current. With this change of connections the equation for X is unchanged.

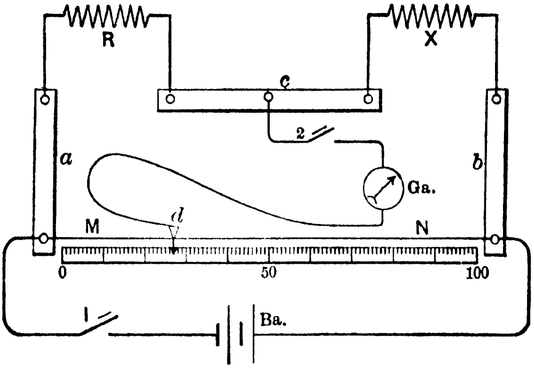

Fig. 12. Diagram of the (meter) slide-wire Wheatstone bridge.

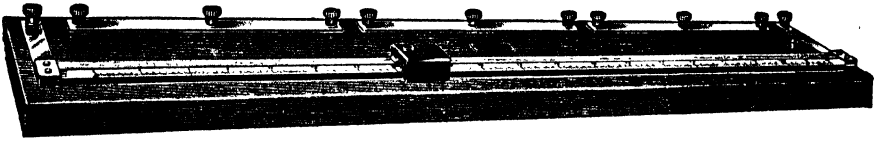

Fig. 13. Commercial form of the meter slide-wire bridge.

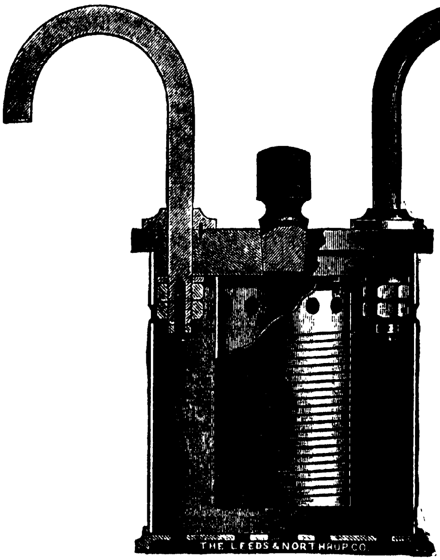

Fig. 14. A standard resistor.

formula (19),

$$X = \frac{10 \times 73}{27} = 27.04 \text{ ohms}$$

The details of the slide-wire, as constructed by one manufacturer, are shown in Fig. 13. Extra binding posts and straps make it possible to use the bridge for the Carey-Foster method, described in §25.

The balancing resistance R may have various forms; usually it consists of a coil of wire (Fig. 14) wound on a metal spool and protected by a metal case. The heavy horn-shaped terminals are used for suspending the device from mercury cups, in order to minimize the error due to contact resistances. The resistor is designed for immersion in a constant-temperature oil-bath. In a later design special potential terminals are provided to prevent error due to contact drop.

19. EXPERIMENT 1-I. — Resistance Measurements with a Slide-Wire Bridge.

— This experiment may well be combined with Experiment 1-N. The theory and the use of the bridge are explained in the two preceding sections. Four kinds of problems may be solved with the bridge:

(a) Checking a standard resistance against a primary standard.

(b) Determining the values of unknown resistances, preferably certain of those which have already been measured by other methods.

(c) Adjusting a resistance to a desired value.

(d) Determining the upper and lower limits of resistance which can be accurately measured with this equipment.

The student is expected to make himself familiar with the use of the bridge, and to practice on these four problems. Although great accuracy is not expected in this exercise, since it is only a preparatory one for more advanced work with the Wheatstone bridge, the student should try to get consistent and reliable results.

20. Bridge Measurements with Alternating Currents.

— A chemical decomposition takes place in some conductors when a direct current is passing through them. This results in the so-called polarization or formation of gases and a counter emf at the electrodes; the apparent resistance of the conductor may be considerably increased thereby. Such an action is observed with many liquids known as electrolytes, and in ground connections; in the latter case an electrolytic action takes place between the conductor and the earth. If polarization is known to be present, or is suspected, it is not advisable to determine the resistance with a Wheatstone bridge using a direct current. Alternating current is then used, and the d-c galvanometer is replaced either by an ordinary telephone receiver or by a vibration galvanometer (§45) responsive to small alternating currents. An a-c galvanometer similar to the d-c instrument is also available. The field magnet is made of soft laminated steel and is excited with the same current as the coil; this gives a steady deflection with a sensitiveness somewhat less than that of the d-c galvanometer. (One manufacturer offers a sensitivity expressed as from 5 to 0.5 microampere per millimeter of deflection in the pointer type, or from 0.05 to 0.025 microampere per millimeter deflection in the lamp-and-scale type.) If a telephone receiver is used, a sound is heard as long as the bridge is out of balance. This sound is reduced to a minimum when the bridge is exactly balanced. It is hardly possible to reduce it completely to zero if there are any harmonics in the supply current, so the source of alternating current should be as pure as possible.

Where the resistance to be measured requires an alternating current, the telephone receiver, properly tuned to the frequency of the supply, is one of the best devices available. The frequency of the supply has to be kept as nearly constant as possible to be within the most effective range of the receiver, otherwise there is a loss in sensitivity.

The necessary alternating current may be obtained from an ordinary lighting circuit with a high resistance in series, or through a step-down transformer. The measurement of the resistance of electrolytes at low frequency is subject to error due to polarization, and only in special cases can 60-cycle current be used for accurate measurements. A frequency of 1000 cycles is very satisfactory, as with it there is generally little or no trouble from polarization. An induction coil with a vibrating interrupter may be used and can be supplied with a direct current from a small battery, but it gives a very poor wave form for bridge measurements. Special forms of 1000-cycle generators and oscillators (such as the Vreeland oscillator and the microphone hummer) are also on the market for use as a power supply where very accurate measurements are desired.[1] These produce a current of the desired frequency, practically free of all harmonics, and the sound in the telephone receiver is reduced substantially to zero at the position of balance.

21. Portable Direct-Reading Slide-Wire Bridge. — A convenient portable Wheatstone bridge of the slide-wire type is shown in Fig. 15*a*; it is intended for all-round practical work, such as locating faults in the insulation of wiring installations. The slide-wire makes one turn around a drum. This reduces the length of the case, and yet the wire is long enough to give reasonable accuracy. The contact which corresponds to point *d* in Fig. 12 is formed by a " toucher " consisting of a metal roller which is moved along the slide-wire by the rotation of the knurled handle (Fig. 15*b*). The instrument is entirely self-contained, the galvanometer

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[1] Leeds & Northrup Catalog 48, p. 21.

g and a few dry cells *Ba* being mounted in the case. Five standard resistances *R*, of 1, 10, 100, 1000, and 10,000 ohms, are supplied with the bridge. They are mounted inside the box, and the leads are taken to the five sockets shown. Any one of these may be used, according to the value of the unknown resistance; the desired value is selected by inserting a plug in the corresponding socket.

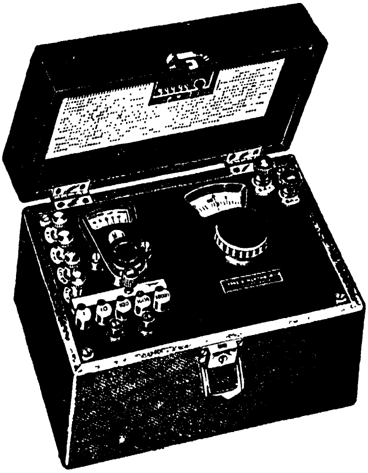

FIG. 15*a*. A portable slide-wire bridge, or "Ohmmeter."

In the operation of this "ohmmeter" the unknown resistance is first connected across the *X* posts, the galvanometer pointer clamp is then released, a standard resistance having a value near the estimated resistance of the unknown is plugged in, and, with battery and galvanometer keys closed, the slide-wire knob is rotated until the galvanometer deflection comes to zero. The value of the unknown resistance is equal to the slide-wire scale reading multiplied by the plug setting. For example, with a plug setting of 100 and a scale reading of 0.62 the required resistance is 0.62 × 100 = 62 ohms.

A similar instrument is available which uses alternating current from a commercial 60-cycle or 25-cycle source through a small transformer within the case. It is recommended in the measurement of resistances in which there may be danger of error due to polarization as discussed above. A galvanometer of the a-c type is used to denote balance of the bridge, and operation is substantially the same as with the d-c bridge having self-contained battery.

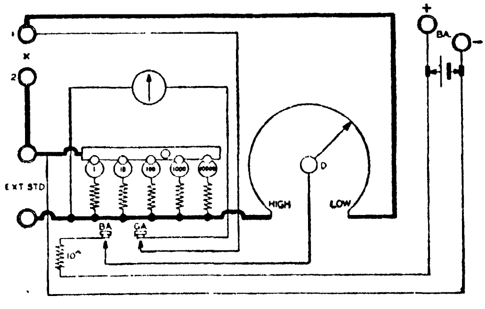

FIG. 15*b*. Diagram of connections with the portable ohmmeter.

22. EXPERIMENT 1-J. — Practice with Portable Slide-Wire Bridges. — For the theory of operation and the description of these instruments see §§20 and 21.

(a) Inspect the instrument, read the accompanying instructions, and obtain a clear understanding of its connections and operation.

(b) Use the device for measuring a set of calibrated resistances, within as wide a range as possible, paying particular attention to the sensitiveness of the instruments at different values of the balancing resistances. (1) Perform the measurements first with direct current, using the galvanometer; then (2) repeat some of the measurements with alternating current and with the telephone receiver in place of (or in parallel with) the a-c galvanometer.

(c) Determine the resistance of an electrolyte with direct and with alternating current, and note the difference.

(d) Measure the resistance between a line and the ground (resistance of a fault).

Report. Give the numerical results. State your opinion about the instruments, their advantages and shortcomings, the limits of accuracy, the best range, the relative advantages of the use of the telephone receiver over that of the galvanometer, and of direct current over alternating current, and vice versa.

23. Kohlrausch Bridge. — A disadvantage of a stretched slide-wire (Fig. 13) or single turn of wire is that it cannot be made long enough for accurate measurements without making the instrument itself too large. Kohlrausch solved this difficulty by winding the slide-wire in a number of turns (usually 10) on an insulating cylinder, as shown in Fig. 16. The contact *d* (see Fig. 12) is mounted inside a hood and moves along the spiral of resistance wire when the hood is rotated upon its threaded shaft (Fig. 16). The circular scale on the hood, together with the glass index, make it possible to read to 1/1000 of a turn, or 1/10 000 of the length of the slide-wire. Two additional plug controlled end coils, each of 4.5 times the resistance of the slide-wire, may be inserted in circuit. A standard resistance of the dial type, variable in steps of 1 ohm from 0 to 1000 ohms, is shown on the left. The electrolyte to be measured is seen in a special glass flask, while a thermometer is provided to give the temperature of the liquid.

Alternating current is used for the measurement of the resistance of electrolytes and is supplied by some suitable source of high (audio) frequency current, such as a Vreeland oscillator. A telephone receiver serves as the detector to indicate balance of the bridge. A vibration galvanometer (§45) may also be used.

The form of the vessel is too complicated to allow the specific resistance of the liquid to be deduced from the dimensions. This is no serious objection, however; the constant of the vessel may be determined once for all by measuring in it the resistance of a standard liquid whose specific resistance is known. For example, it is known that the specific resistance of a 25 per cent solution of zinc sulphate is 21.4 ohms per cc at 18° C. Let the resistance of such a solution, as measured in a Kohlrausch vessel, be 152 ohms. Then the constant of the vessel is 21.4 ÷ 152 = 0.141. The resistance of any other liquid measured in this vessel must be multiplied by this coefficient to get its specific resistance.

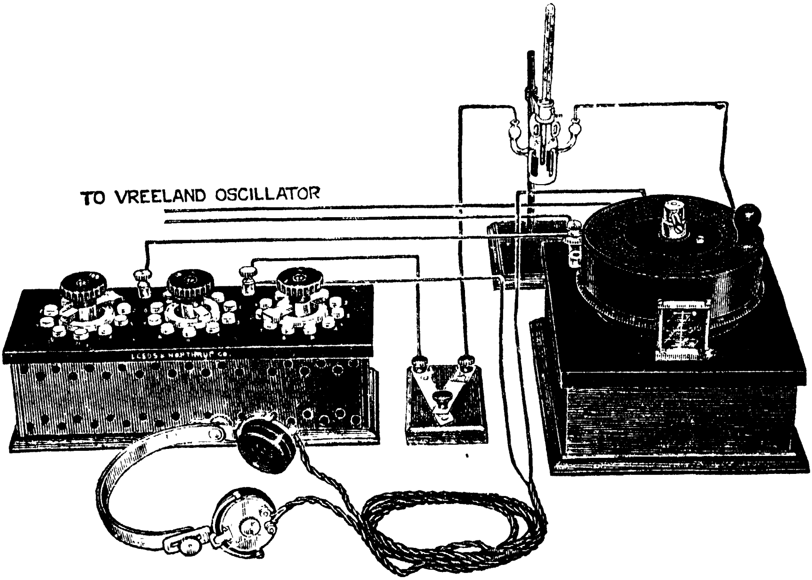

FIG. 16. Resistance measurement of an electrolyte with the Kohlrausch bridge, using alternating current.

Different types and sizes of cells are suited to different electrolytes and various concentrations. For example, polarization may occur in some cases unless the electrodes are platinized, and in other cases platinized electrodes appear to act catalytically and assist chemical action. Then again, platinized electrodes may, because of their spongy nature, absorb so much of the electrolyte as to cause errors in measurement when later used on solutions of different nature or concentration. Care must be taken to secure a cell giving a correct current density for the electrolyte employed, and the cell design should take into account other electrical characteristics, such as plate capacity and stem inductance and resistance. The rate of heating and cooling of the cell is of importance, and cells of the pipette type are to be recommended since they not only permit filling with minimum exposure of the electrolyte to the action of air, but also allow the cell to be completely immersed in a bath of constant temperature. An accurate determination of the temperature is necessary, since electrolytes have comparatively high temperature coefficients of conductivity. For more accurate work with high resistances, an oil-bath is therefore employed. The Kohlrausch bridge is better adapted for an accurate determination of the resistance of an electrolyte than the substitution method described in §15.

24. EXPERIMENT 1-K. — Measurement of Resistances with the Kohlrausch Bridge. — For the description of the bridge see the preceding section. The measurements to be performed are similar to those specified in Experiments 1-H, 1-I, and 1-J.

25. Carey-Foster Method of Comparing Resistances. — This is a modification of the ordinary Wheatstone bridge and is used for an accurate comparison of two nearly equal resistances, as in the determination of the percentage of error of secondary standards. The connections are shown in Fig. 17; the bridge illustrated in Fig. 13 may be used with this method. *P* and *Q* are two resistances to be compared to each other; *R* and *X* are two other resistances, the numerical values of which it is not necessary to know. It is advisable, however, to have *R* and *X* not very different from each other, in order to obtain a balance near the center of the slide-wire.

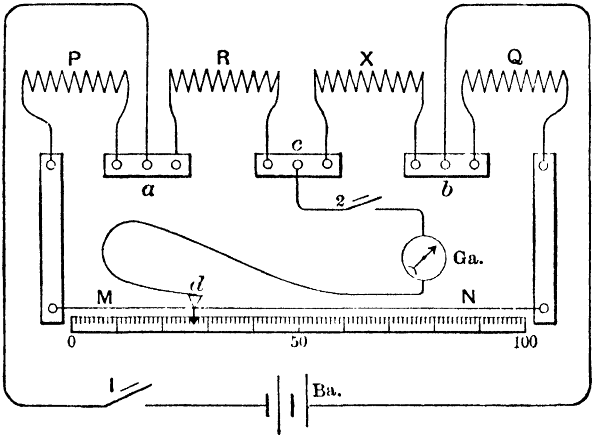

FIG. 17. Carey-Foster bridge.

The bridge is balanced as usual; then the positions of *P* and *Q* are interchanged by means of a special switch; a new balance is obtained by slightly moving the contact *d*. The resistance of the slide-wire between the two positions of the contact is equal to the difference between the resistances *P* and *Q*.

The reason for this is that, the branches R and X being the same in the two measurements, the total resistance of each of the two other branches must also be the same. If P is smaller than Q, the difference must be compensated by the corresponding amount of resistance in the slide-wire.

The method presupposes that the resistance of the slide-wire per division of the scale is known. The slide-wire is calibrated by measuring with the bridge the difference in the resistance of two standards P and Q, which difference has been determined before in some other way. This is best done by taking two equal resistances P and Q, say of 1 ohm each, and then shunting P by a comparatively high resistance, for example 100 ohms. This reduces the resistance of P to a certain value P'. According to the formula (17) in §11 we have:

$$\frac{1}{P'} = \frac{1}{1} + \frac{1}{100}; \qquad \text{or} \qquad P' = 0.9901 \text{ ohm}$$

The difference between Q and P' being known, the slide-wire may be calibrated, at least in the middle portion used for measurements. A balance may be obtained at any desired point of the slide-wire by changing the value of R.

26. EXPERIMENT 1-L. — Comparison of Resistances by Means of the Carey-Foster Bridge. — The method is explained in the preceding section. Special types of Carey-Foster bridge are on the market, with which an accuracy of comparison of over one one-hundredth of a per cent is possible. For ordinary practice with the method, the bridge shown in Fig. 13 is sufficient.

(a) Connect four resistances as indicated in Fig. 17 and practice in comparing their values.

(b) Calibrate the slide-wire as explained above.

(c) Determine the limits of accuracy of the method by varying the difference between P and Q, the ratio $R : X$, and the absolute values of the four resistances.

27. Testing Rail Bonds. — On most electric roads the current for operating cars returns to the power house or substation through the track rails; this saves one conductor. In order to make the use of the rails for this purpose possible they must be either welded or "bonded" together, that is, connected by more or less flexible copper conductors as shown in Fig. 18. Rail bonds must be periodically inspected, as they get loose or broken, causing an excessive resistance between the rails. Poor bonding brings with it an excessive drop of voltage and loss of power, electrolytic corrosion of gas and water pipes in the vicinity (due to current passing from rails to pipes or the opposite), telephone troubles, etc.

If rail bonds could be tested while the road was not in operation, the simplest method would be to send a current from the station and to measure the drop across each bond with a low-reading voltmeter. Usually, however, testing must be done with cars running on the same line, so that the current in the rails is widely fluctuating. In order to utilize this current, special devices are used as described below. Where no station current is available a portable storage battery is used, with the leads clamped to the rails adjacent to the bond under test. The resistance of the bond is expressed in feet of solid rail; thus a resistance of 2 ft means that the voltage drop caused by the bond contacts is the same as would be caused by two additional feet of solid rail. Convenient methods of testing bonds are as follows:

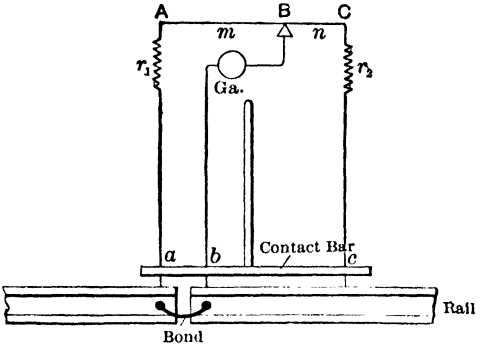

FIG. 18. Diagram of a bond tester.

(a) If no special bond tester is available, *two ordinary millivoltmeters* may be used, one connected between *a* and *b* (Fig. 18), the other between *b* and *c*. The millivoltmeters are read simultaneously, so that the fluctuations in the rail current do not affect the ratio of the readings. This ratio is evidently equal to the ratio of the resistances of the bond *a-b*, and the rail *b-c*. It is advisable to have some protective resistance in series with the millivoltmeter connected across the bond, since the voltage drop across a loose or broken bond may be many times in excess of the range of the instrument. The protective resistance may be short-circuited with a push button. Good contacts with the rail are obtained by means of file edges or hack-saw blades.

(b) A convenient device for testing rail bonds — the so-called *Roller bond tester* — is shown in Fig. 19; a diagrammatic view of its connections is represented in Fig. 18. It operates on the principle of a slide-wire Wheatstone bridge. The unknown resistance *a-b* of the bond is compared to a definite length *b-c* of the solid rail by means of the slide-wire *A-C* and the galvanometer connected between *b* and *B*. The slider *B* is moved back and forth until the galvanometer shows zero. Then the ratio of the resistance of the bond to that of the rail is equal to the ratio

$(r_1 + m) \div (r_2 + n)$. It is not necessary to calculate this ratio every time; the scale of the instrument (Fig. 19) gives the resistance directly in feet of solid rail.

The resistances r_1 and r_2 act as an extension of the slide-wire, making the useful part of it longer and therefore more accurate. Without these resistances, the short slide-wire would contain all the values from zero to infinity. Fluctuating currents in the rail do not affect the balance, because fluctuations occur in all the branches of the bridge simultaneously.

One observer is sufficient with this instrument; he holds the contact bar by the upright in one hand, and operates the knob of the slider with the other hand. The instrument itself is suspended from his shoulders. The large scale indicates the resistance; the small scale is for galvanometer deflections. In many cases, it is sufficient to know that the resistance of the bond is not beyond a certain limit; then the pointer of the slider is simply set at this limit. On all bonds having resistance below this limit, the galvanometer deflects one way; on defective bonds it deflects the other way, provided, of course, that the direction of the rail current remains the same. This makes the instrument very convenient for rapid testing, even with an unskilled observer.

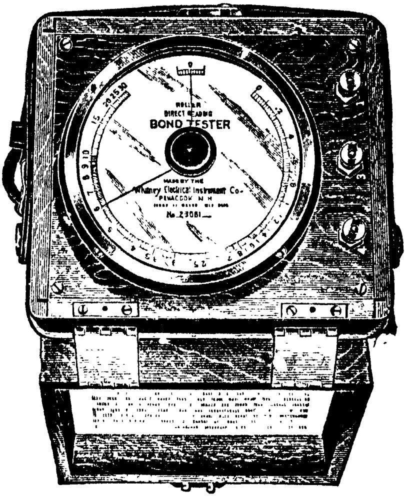

FIG. 19. The Roller bond tester.

28. EXPERIMENT 1-M. — Testing Rail Bonds. — The method and the apparatus are described in the preceding article. Become familiar with the device in the laboratory before trying it on an actual track. Two rails should be provided in the laboratory, connected by a variable resistance; this resistance is to represent various states of a bond, from a perfect contact to a broken bond.

(a) Connect the rails to a d-c circuit, placing a regulating resistance in series with them, to produce fluctuating currents as in practice.

(b) Having set the resistance of the bond very low, measure it first with a steady current, then with fluctuating currents.

(c) Repeat the same test with higher resistances of the bond.

(d) Having become thoroughly familiar with the instrument, measure a number of rail bonds on a street-car line in actual operation.

In performing this experiment, particular attention should be paid to the accuracy of the instrument throughout its range, and to the influence of current fluctuations. Determine the limits of error in the readings due to a poor contact between the rail and the saw blade or the file surface used for making the contact. How would the accuracy of results be affected by current passing from rail to rail through the ground?

29. Plug-Type Wheatstone Bridge. — Series and Decade Arrangement of Coils. — For accurate work, bridges are used in which the three "known" resistance arms *R*, *M*, and *N* (Fig. 11) consist of carefully adjusted resistance coils (Fig. 20). These coils are mounted inside of a box and leads are taken to contacts on its top. Resistances are varied either by inserting plugs, as in Fig. 20, or by dial switches, which allow a somewhat quicker adjustment.

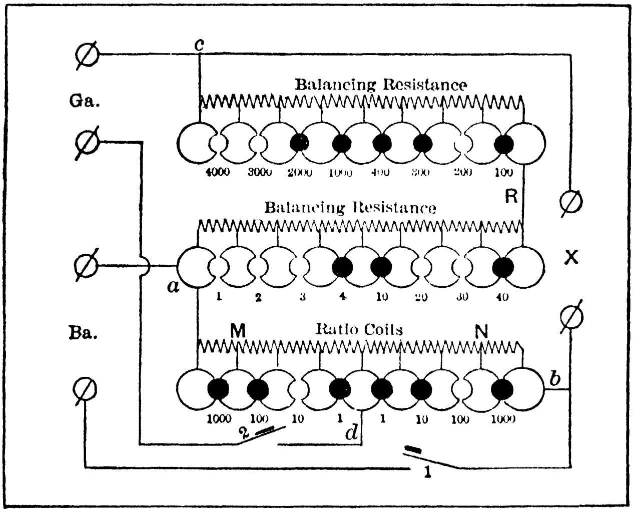

Fig. 20. Diagram of a plug-type Wheatstone bridge.

The older method of connecting coils is that shown in Fig. 20; the coils are connected in series, and each plug short-circuits a coil. Thus, in the upper row all the coils are short-circuited, except 4000, 3000, and 200 ohm, and the reading is 7200 ohms.

The newer or the "decade" method is shown in Fig. 21*a*. Here the resistances which are not in use are simply left out of the circuit. Therefore only one plug is necessary for each decade or row of ten coils. A more recent type of decade resistance requiring but four resistance elements to vary the resistance in steps of 1 ohm from 0 to 9 ohms is shown in Fig. 21*b*.

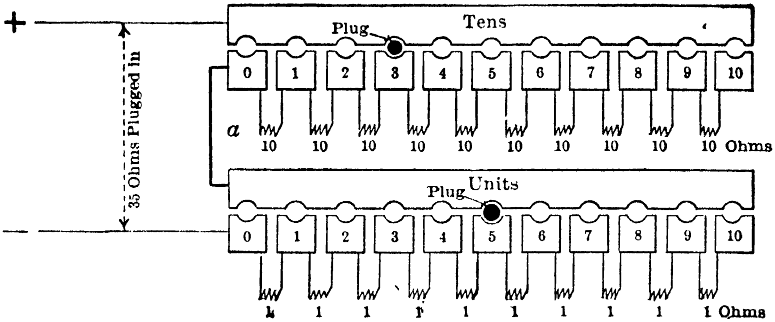

FIG. 21*a*. Decade arrangement of resistances.

The advantages of the decade arrangement are:

(1) A smaller number of plugs is required.

(2) Additional resistance between the plugs and the contact blocks is less.

(3) The result is more easily read, no summation being necessary.

(4) There is less possibility of making a mistake or losing a contact plug.

30. Example of a Plug-Type Bridge. — A simple bridge is shown in Fig. 20; the lettering is the same as in Fig. 11. The bridge has three arms: two ratio arms M and N in the lower row, and the balancing resistance R in the two upper rows of coils. The box is provided with three pairs of terminals — for the battery, for the galvanometer, and for the unknown resistance X. The battery key 1 and the galvanometer key 2 are also mounted on the box.

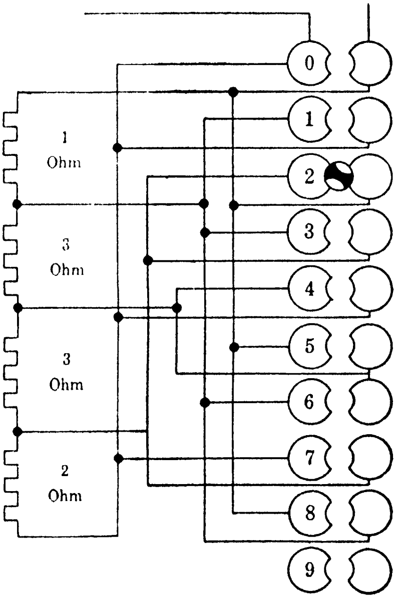

FIG. 21*b*. Modified decade arrangement of resistances.

Unlike the slide-wire bridge (Fig. 12), plug-type bridges have only a limited number of ratios M/N — usually in powers of 10. The balancing is done by the third arm R. In the instrument under consideration, the resistance of this arm may be varied from 0 to 10,000 ohms in steps of 1 ohm. Thus, with the ratio $N/M = 1000/1$, resistance settings can be made up to 10,000,000 ohms. On the other hand, by selecting $R = 1$ ohm and $N/M = 1/1000$, resistance settings down to 0.001 ohm are possible. The practical range of the bridge is within considerably narrower limits, because it is not sufficiently accurate near the theoretical limits of its range.

To measure a resistance, connect it to the terminals marked X; select a ratio N/M, say 100/10. Press the battery key and then the galvanometer key; vary the resistance R until the galvanometer remains at zero when its key is pressed. *Best results are obtained when the ratio is selected so that the balancing can be done with all four places of the resistance R — thousands, hundreds, tens, and units.* Thus, in measuring a resistance, say, between 25 and 26 ohms, if a ratio $N/M = 1$ is chosen the values of R which most nearly give a balance are $R = 25$ ohms and $R = 26$ ohms. This gives the unknown resistance within 4 per cent. On the other hand, if a ratio $N/M = 1/100$ is chosen the balance can be obtained to the nearest ohm or two in over 2500, say 2532, giving the result within 1/10 per cent. The right ratio N/M can always be selected if the approximate value of X is known in advance; otherwise, the best ratio is found by trials. With the setting shown in Fig. 20 the unknown resistance

$$X = 7256\frac{100}{10} = 72{,}560 \text{ ohms}$$

In some bridges used for precision work, certain of the coils may be joined in series or in multiple or in any combination of series and multiple. The coils may thus be checked against each other in many combinations. For example, all the 10-ohm coils taken in parallel may be compared with any 1-ohm coil. In such a high-grade bridge the precision of adjustment is 1/20 of 1 per cent for the coils of the 1/10-ohm series, and 1/50 of 1 per cent for the other coils of the rheostat. The ratio coils may be certified to be like each other to within 1/100 of 1 per cent.

31. Portable Testing Sets. — The above-described bridges are primarily intended for *stationary* work. There is a demand for self-contained *portable* bridges, for locating faults and for other emergency purposes in the operation of electric plants. The slide-wire bridge described in §21 is one of this kind. A more accurate and convenient instrument, shown in Fig. 22, consists of a portable testing set (Wheatstone bridge) with a rheostat comprising four decade resistors each having ten steps of resistance change and a zero position. This gives a total range of 11,110 ohms, adjustable in steps of 1 ohm. The ratio arms are controlled by a rotary switch (marked "multiply by") between the limits of 1/1000 and 1000. The resistors are wound on nonmetal spools. A built-in galvanometer of sensitivity 1 megohm[1] is provided, also a three-position shunt to vary the galvanometer sensitivity during the balancing of the bridge. Provision is made for connection to a more sensitive external galvanometer if desired. The set may be changed over from the Wheatstone bridge to the Murray or Varley loop by throwing the switch shown at the rear of the panel, and three resistors for the Murray loop tests are provided in the ratio switch. It is possible, by means of an external binding post, to use the rheostat independently of the rest of the bridge.

FIG. 22*a*. Portable testing set (Leeds & Northrup Company).

Explicit directions are supplied with the instrument, so that any one with a very elementary knowledge of electricity can use it.

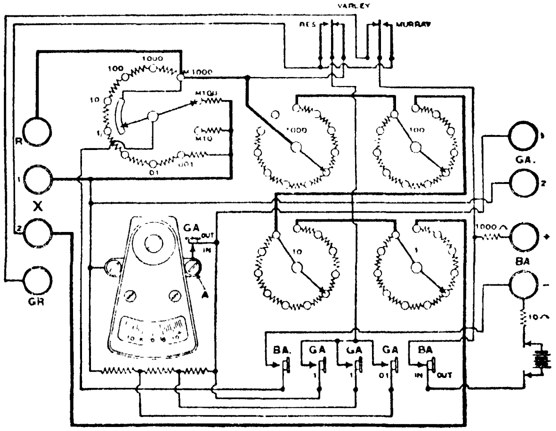

FIG. 22*b*. Diagram of the portable testing set.

32. EXPERIMENT 1-N. — Measurement of Resistances with a Plug- or Dial-Type Wheatstone Bridge. — A few types of bridges are

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[1] A sensitivity of 1 megohm indicates that the galvanometer will have a deflection of one division when an emf of 1 volt is applied to the galvanometer through a resistance of 1,000,000 ohms.

described in §§29 to 31. The problems which may be solved with these bridges are the same as are enumerated in the preceding experiments with simpler bridges. If possible, the student should connect the coils of the bridge itself in various combinations, so as to check them against each other. The purpose of the exercise is not only to learn how to perform measurements, but also to study the bridge itself, and to ascertain the limits of its accuracy, the best range, the needed sensitiveness of the galvanometer, the best battery voltage, the influence of temperature, the disturbing effect of thermoelectric force, the best galvanometer resistance, and the effect of the relative position of the galvanometer and the battery on the accuracy of the measurements.

33. EXPERIMENT 1-O. — Measurement of Low Resistance by Means of a Modified Wheatstone Bridge. — The accuracy of the Wheatstone bridge in the measurement of resistances below, say, 0.1 ohm may be materially improved by the following expedient, the purpose of which is to avoid error due to lead and contact resistances. Referring to Fig. 23, the standard resistance R and the unknown low resistance X are first equipped with suitable potential taps *inside* the main terminals. M and N are resistances of several hundred ohms and are connected to the potential taps at t' and t''. A represents the resistance of leads and contacts between R and X. The battery Ba should be a storage battery capable of supplying a considerable current but below that at which R and X show an appreciable temperature rise. In the operation of the bridge the galvanometer is connected first at a and a balance obtained by adjusting the arm N to some value N_a; then to the point b and a balance obtained with N at N_b.

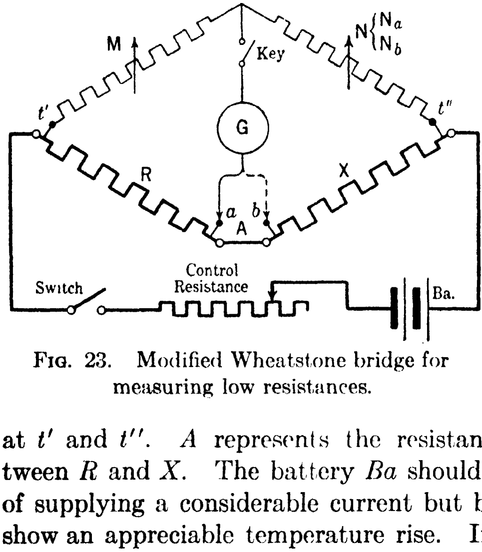


FIG. 23. Modified Wheatstone bridge for measuring low resistances.

For condition (a), $R/(X + A) = M/N_a$; for (b), $(R + A)/X = M/N_b$.

From these equations it follows that $N_aR = MX + MA$, and $N_bR + N_bA = MX$. Eliminating A from these equations and solving for X gives

$$X = R\frac{N_b(M + N_a)}{M(M + N_b)} \quad \ldots\ldots\ldots \quad (21)$$

In this method the principal sources of error, the effects of lead and contact resistance in the ordinary bridge, are largely eliminated, with a corresponding improvement in accuracy of results.

The purpose of this experiment is to become acquainted with this method of measuring moderately low resistances and to determine its limits of accuracy. Measure again some of the low resistances, such as ammeter shunts, series fields, and short sections of conductor, which were measured by the ammeter-voltmeter method and compare results. Measure also certain standard resistances and determine the limits of resistance and degree of accuracy obtainable with this equipment. Report findings in the form of tables and discuss results.

34. Measurement of Very Small Resistances (the Kelvin Bridge). — The Wheatstone bridge in its ordinary form (see §33) is not suitable for measuring small resistances, say below 0.1 ohm, because the resistance of the contacts at *a*, *b*, *c*, and *d* (Fig. 11) may constitute an appreciable part of the resistances *X* and *R* themselves. The drop-of-potential and comparison-of-drops methods (§2) give satisfactory results with resistances considerably below 0.1 ohm, if care is taken to have the potential leads properly applied, so as to eliminate the contact resistance, and if, in the latter method, a constant current supply is available. For an accurate comparison of very low resistances the potentiometer is used (§75).

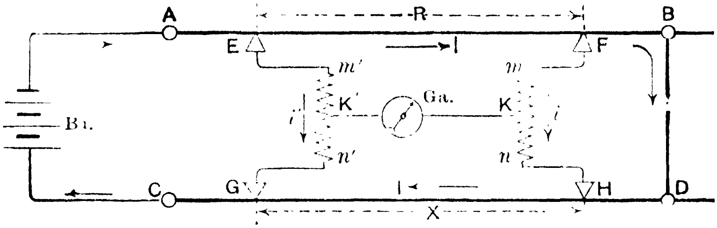


Fig. 24. Schematic diagram of the Kelvin double bridge.

Lord Kelvin combined the Wheatstone bridge and the drop-of-potential method into a scheme which is exceedingly accurate for measuring low resistances, say down to 0.0001 ohm. The diagram of connections of this so-called Kelvin double bridge is shown in Fig. 24.

The resistances *R* and *X* to be compared are connected in series with a low-voltage battery *Ba*, and some regulating resistance not shown in the figure. The connections to the galvanometer are taken from the points *E*, *F*, *G*, and *H* separate from the main terminals. This is a matter of paramount importance, because the contact resistances at *A*, *B*, *C*, and *D* should not enter into the result. Four ratio coils,

m, n, m', and n', are used instead of two ratio coils, M and N, of the ordinary Wheatstone bridge (Fig. 11).

The unknown resistance X is measured by adjusting either R or the ratio coils until the galvanometer gives no deflection. The two pairs of ratio coils are adjusted simultaneously, so that the relation

$$m/n = m'/n' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (22)$$

is preserved. Generally this is best done by keeping $m = m'$ and $n = n'$. When a balance is obtained the following relation holds true

$$R/X = m/n = m'/n' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

from which X can be calculated.

Proof: When no current flows through the galvanometer, the points K' and K are at the same potential. Therefore the voltage drop from E to K' is the same as the voltage drop EFK. With the notation in the sketch we have

$$i'm' = IR + im$$

The same holds true with respect to the point G; hence,

$$i'n' = IX + in$$

From these two equations we get

$$R/X = (i'm - im)/(i'n' - in)$$

Substituting for m' its value from eq. (21), we obtain

$$R/X = \frac{i'mn'/n - im}{i'n' - in} = \frac{i'n'm/n - inm/n}{i'n' - in} = m/n$$

which proves the relation (23).

There are two types of Kelvin double bridge: In one type a set of a few standard resistances R is supplied with the bridge, and the adjustment is made by the ratio coils. In the other type the ratio coils are made to give only a limited number of combinations, but the standard resistance R is made adjustable, so as to give practically any value of resistance within certain limits. Such a variable standard low resistance is shown diagrammatically in Fig. 25.

AB is a heavy piece of resistance metal of uniform cross-section and uniform resistance per unit of length; CD is another piece of resistance metal of smaller cross-section, and the two are joined by a heavy copper bar AC into which both are silver-soldered; LL are the current terminals and PP are the potential terminals. The resistance of AB between the marks 0 and 100 on the scale S is 0.001 ohm. From the point 1 on CD to 0 on AB the resistance is also 0.001 ohm. The slider M moves along the resistance AB, and its position is read on the scale S which is divided into 100 equal parts and can be read by means of a vernier to thousandths. Subdivided this way, the resistance between the tap-off points *PP* may have any value from 0.001 to 0.01 ohm in steps of 0.000001 ohm. Using a ratio of 1 to 10 in the bridge, resistances from 0.1 ohm to 0.01 ohm may be measured in steps of 0.00001 ohm, and by using the inverse ratio, all values from 0.001 ohm to 0.0001 ohm in steps of 0.000 0001 ohm.

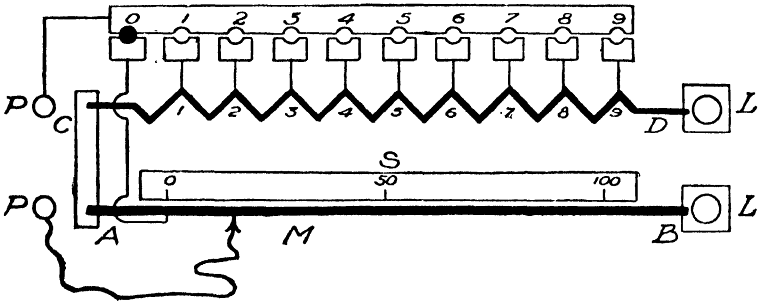


FIG. 25. Variable low resistance, used as a standard in the Kelvin double bridge.

It will be noted that there are no contacts in the main circuit between the terminals *LL*. The only contacts are the tap-off points to the potential terminals *PP*. The resistance of these contacts is negligible, as they are in series with the ratio coils, which have a resistance of several hundred ohms.

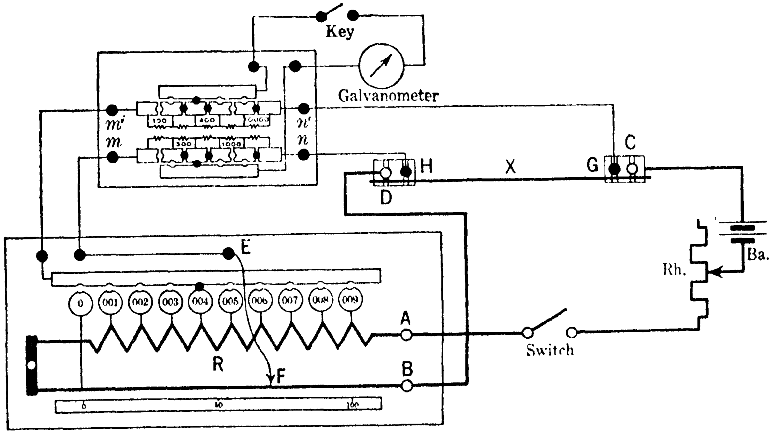


FIG. 26. Complete diagram of connections of the Kelvin double bridge.

35. EXPERIMENT 1-P. — Measurement of Low Resistances with the Kelvin Double Bridge. — (See §34.) Connect the variable low resistance, ratio coils, unknown resistance *X*, galvanometer, control resistance, storage battery, and switch as in Fig. 26. First use the Kelvin double bridge to check such standard low resistances (ammeter shunts) as may be available; then measure certain of the low resistances measured by the ammeter-voltmeter method, and in Experiment 1-O.

Compare the values by the different methods and discuss the comparative accuracy obtained. What are the possible sources of error in this method? What are its advantages and disadvantages as compared with the comparison-of-drops and the ammeter-voltmeter methods?

36. Hoopes Conductivity Bridge. — One of the important practical applications of the Kelvin double bridge is the determining of the conductivity of bars of copper and aluminum. This is done regularly in factories manufacturing wire for electrical purposes; wire is also tested by large consumers before a consignment is accepted. It is desirable in such cases to modify the standard Kelvin bridge described in the preceding articles, so as to fulfil the following requirements:

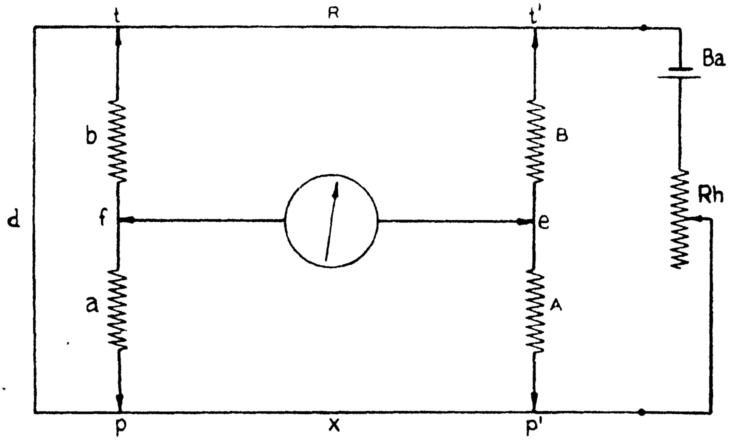


FIG. 27. Schematic diagram of the Hoopes conductivity bridge.

(1) The apparatus must be direct reading in percentage of conductivity of some agreed pure metal, instead of giving results in ohms.

(2) The apparatus must be adapted for testing a large number of similar samples in a short time.

(3) It must be easily handled by a person who has but little electrical knowledge.

A bridge which satisfies these conditions was designed by Mr. Wm. Hoopes (Figs. 27, 28, and 29). When Figs. 24 and 27 or 28 are compared it will be seen that the electrical connections in the Hoopes bridge are identical with those in the regular Kelvin bridge. *DE* (Fig. 28) is a standard wire made of the same material as the samples to be tested, in order to have the same temperature coefficient. It has resistances between point *F* and points *G*, *H*, and *I* corresponding to the standard resistances for three B & S gauge wires, for example, No. 10, No. 11, and No. 12.

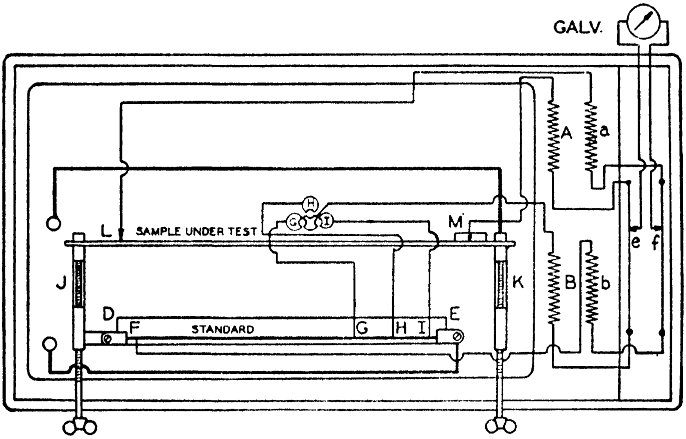
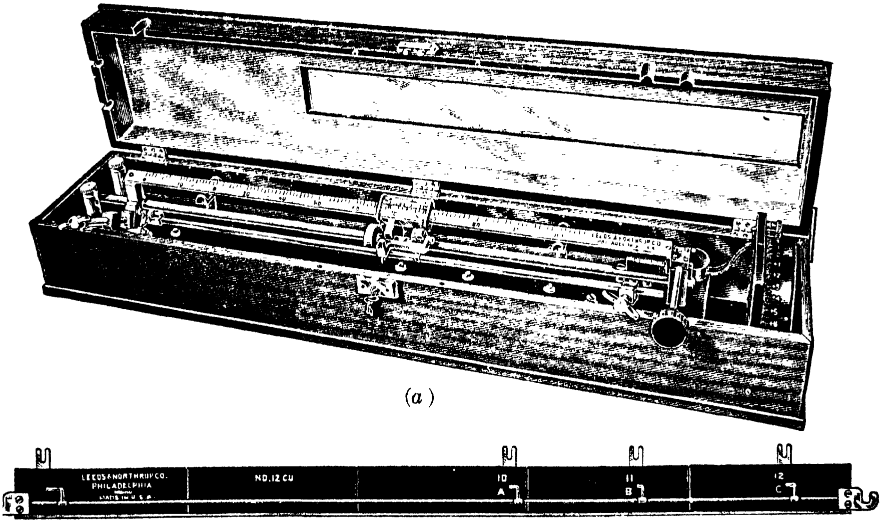


Fig. 28. Complete diagram of the Hoopes conductivity bridge.



(*a*)

(*b*)

Fig. 29. (*a*) Hoopes conductivity bridge, (*b*) Standard for the Hoopes bridge.

J K is a sample under test, cut to a standard length of 38 in. in a special cutting-off machine. The potential contact *L* on the sample is fixed, and contact *M* is movable. The ratio arms consist of equal resistances, *A* and *B*, also *a* and *b*. *A* is joined to *B* by a circular slide-wire resistor with a movable contact point, *e*. Similarly *f* moves on the slide-wire joining *a* and *b*, the two slide-wires being mounted inside the drum shown at the right in Fig. 29. *e* and *f* move together so as to maintain the two ratios equal. The drum is set at a point on its scale corresponding to the percentage by which the weight of the sample differs from that of a standard wire of the same gauge number and length, said difference being preferably determined by weighing the sample on a special balance reading this per cent difference directly.

Current for the bridge is supplied by a 2-volt storage cell and should have approximately the following values for the several gauge numbers:

No. 18	3 amp	No. 6	11 to 12 amp
No. 15	4 amp	No. 3	15 amp
No. 12	4 to 5 amp	No. 0	18 to 20 amp
No. 9	8 to 10 amp	No. 000	50 to 100 amp

The manufacturers of the Hoopes bridge have available a special rheostat on which the resistances at the several contact buttons are correct for the 2-volt supply and the several sizes of wire. They are marked in wire sizes.

The operation of the bridge is as follows: The standard wire (see Figs. 28 and 29*b*) is clamped in the front of the bridge by means of lugs at either end, whereupon potential points *F*, *G*, *H*, and *I* (Fig. 28) are connected in the bridge by special forked terminals (Fig. 29*b*). The ratio arm drum is now set at the scale reading corresponding to the per cent excess or deficit of weight of the test sample. Point *M* is then moved (by finger lever and knurled knob) until the bridge shows a balance. The vernier reading of this scale gives the per cent conductivity of the sample directly to an accuracy of better than 0.2 per cent. Obviously in such a bridge a large number of samples of the same size of wire can be tested in a very short time.

The Hoopes bridge illustrated in Fig. 29*a* is enclosed in a metal-lined box having a glass window in the top. This makes it possible to balance the bridge and read the scale settings without opening the box, thus the sample and standard resistance are kept at practically a constant temperature, being shielded from draughts.

37. EXPERIMENT 1-Q.—Determination of Conductivity of Wires with Kelvin or Hoopes Double Bridge.—(See §§34 and 36.) Consider this an experiment in which the utmost of precision is to be obtained. Secure samples of straight copper wire or rod and, using either the Kelvin or Hoopes double bridge, determine the conductivity of each sample.

(a) *Kelvin bridge.* Connect the equipment as in Fig. 26, mounting the unknown resistance between proper clamps held, say, 36 in. apart. The clamps must be provided with the proper current and potential terminals (*D*, *C*, and *G*, *H*), the latter connected to knife-edges resting upon the sample. Use a heavy short lead *DB* between standard and unknown resistances. Set $m = m'$, and $n = n'$ in the ratio arms. Be sure that the polarity of connections is correct so that a balance will be possible. Pass a relatively large current through the bridge and sample and vary the standard low resistance until a balance is obtained. Record the value of *R*, of the ratio m/n, and the temperature at which the readings were taken.

Measure the exact distance between knife-edges and the total length of the sample. Weigh the sample accurately and take the diameter at several points along its length, using micrometer calipers, in order to get an accurate average. Repeat the above procedure for other samples of wire.

(b) *Hoopes conductivity bridge.* If this equipment is available, use the Hoopes bridge to determine the conductivity of the several samples of wire, following the instructions of §36. Vary the length of one sample by small amounts and remeasure the conductivity.

Report. For each sample calculate the following resistivity figures: (a) ohms per mil-foot, (b) ohms per centimeter cube, (c) ohms per meter-gram (all referred to 20° C), (d) per cent conductivity (referred to the International standard for annealed copper).

Discuss the limits of resistance which can be measured by the double bridge with error not exceeding 0.04 per cent.

Consider the following as the International standards for annealed copper, all at 20° C.

Mass Resistivity	0.15328 ohm per meter-gram.
	875.2 ohms per mile-pound.
Volume Resistivity	1.7241 microhms per centimeter cube.
	0.017241 ohm per meter-milligram-square.
	0.67879 microhm per inch-cube.
	10.371 ohms per mil-foot.
Density	8.89 grams per cubic centimeter.

The equations for resistivity, as calculated from measured resistance and dimensions of the conductor, as may be derived from eq. (8), are:

(1) *Resistivity in ohm-centimeters*, or ohms per centimeter cube is

$$\rho_v = RA/l \quad \ldots\ldots\ldots\ldots \quad (24)$$

where *R* is the resistance of the wire in ohms, *A* is the area in square centimeters, and *l* is the length in centimeters.

(2) ***Resistivity in ohms per mil-foot*** **is**

$$\rho_v' = Rd^2/l \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

where R **is in ohms,** d **is in mils (1 mil = 1/1000 in.) and** l **is in feet.**

(3) *Resistivity in gram-centimeters* is

$$\rho_m = RM/l^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (26)$$

with R in ohms, M = the mass of the sample in grams, and l = its length in meters.

38. Insulation Measurements and Localization of Faults. — For more complete practical measurements of insulation resistance of electrical apparatus and of lines, for localization of faults, and for disruptive tests on insulation see Vol. II. Faults in d-c machinery are described in §§314 to 319 of this volume.

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CHAPTER II

AMMETERS AND VOLTMETERS — CONSTRUCTION

39. In dealing with electric power, two of the most important quantities to be measured are *current,* and *difference of potential.* Technical instruments for measuring electric current are commonly called *ammeters,* because the practical unit of electric current is the " ampere." Instruments for measuring differences of potential, or electric pressures, are called *voltmeters,* since the practical unit of difference of potential is the " volt."

An ammeter is connected in series with the circuit in which the current is to be measured (Fig. 30); a voltmeter is connected at two points of the circuit between which it is desired to determine the difference of potential. There is no fundamental difference between the construction of ammeters and that of voltmeters; with the exception of electrostatic voltmeters all usual voltmeters are in reality ammeters calibrated in volts. In other words, a voltmeter measures the current passing through it, which current is proportional to the voltage at the terminals of the instrument, and the scale is marked directly in volts.

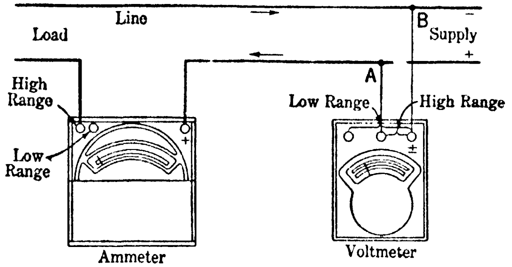

Fig. 30. Standard connections for a voltmeter and an ammeter.

To illustrate, let an ammeter of high resistance, say 1000 ohms, be connected *across* a line, and let the indication be 0.1 ampere. According to Ohm's law, it takes 100 volts to cause a current of 0.1 ampere to flow through a resistance of 1000 ohms; therefore the voltage of the line is 100 volts. To use the instrument as a voltmeter, the scale may be conveniently changed so as to have the division " 100 v " correspond to what was formerly " 0.1 amp."

The resistance of a voltmeter must be high in order to make its power consumption small. It will be seen from Fig. 30 that the current flowing in a voltmeter returns to the generator and is lost so far as the load is concerned. The resistance of the instrument windings is usually insufficient, and for this reason a "multiplier" or high resistance R (Fig. 31) is connected in series with the instrument. The multiplier[1] may be mounted within the instrument case or may be external to it. By varying the resistance of the multiplier the range of a voltmeter may be varied at will. Let, for example, a current of 0.01 ampere be required for the full-scale deflection of a certain voltmeter movement. Then, if the range of the instrument is to be 150 volts, the total resistance of the voltmeter circuit, including the winding and the multiplier, must be $150/0.01 = 15{,}000$ ohms. In order to use the same instrument up to 300 volts, an additional 15,000-ohm multiplier must be provided in series, and the scale readings multiplied by 2.

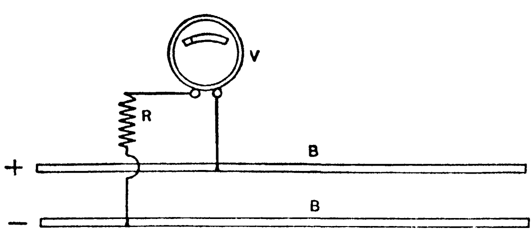

FIG. 31. A series resistance used as a voltmeter multiplier.

On a-c circuits a potential transformer (§56) is frequently used in place of a multiplier. With an electrostatic voltmeter (§55) a series resistance-type multiplier cannot be used, and the total voltage is reduced by means of condensers in series.

40. Classification of Ammeters and Voltmeters. — Ammeters and voltmeters are built on various principles, enumerated below; each system has its advantages and shortcomings, and a field of application of its own. Some instruments may be used with both direct and alternating currents; others are suitable for one kind of current only. The types in practical use at present are:

(1) Movable-iron-type instruments in which the exciting coil is stationary and a piece of iron tends to move into a certain position in the magnetic field due to this coil.

(2) Permanent-magnet movable-coil instruments characterized by a stationary permanent magnet, or an electromagnet excited with a constant direct current. The current to be measured flows through a light movable coil (d'Arsonval type) or through stretched wires placed in the magnetic field.

(3) Electro-dynamometer-type instruments based upon the mechanical torque or force between two coils through which the current to be measured is passed.

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[1] Strictly speaking the term "multiplier" should be limited to a series resistor, external to the meter, and used to extend the range of an instrument beyond some particular value for which the instrument is already complete.

(4) Hot-wire instruments in which the current to be measured heats a wire and thus changes its length.

(5) Thermoelectric-type instruments in which the current to be measured heats the thermojunction of a secondary circuit. The secondary current is measured with a galvanometer.

(6) Induction-type instruments based upon the principle of split-phase alternating field.

(7) Electrostatic voltmeters based on the mechanical attraction between two metal bodies at different electric potential.

These types are described in more detail below.

MOVABLE-IRON INSTRUMENTS

41. Description of the Three Types. — (A) One of the oldest instruments of this type is shown in Fig. 32. *C* is a stationary coil which carries the current to be measured. *P* is a soft-iron plunger so suspended that it can move freely up and down. The shaft which supports the plunger also carries the counterweight *Q* and the pointer *N*. When a current flows through the coil, the plunger is drawn in, against the increasing leverage of the weight *Q*. The corresponding deflection is shown by the pointer on the scale. When the circuit is opened, the counterweight brings the moving system back to zero.

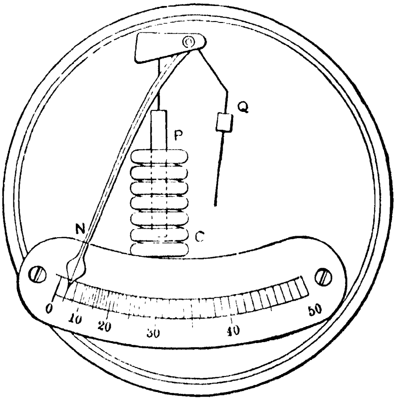


FIG. 32. Plunger-type movable-iron type instrument (ammeter).

In the form shown in Fig. 32, the instrument is used only as a current indicator where no particular accuracy is expected. The disadvantages of this type of construction are as follows: (a) The gravity control is inconvenient because it requires a careful setting of the instrument. (b) The moving part is heavy and causes considerable pivot friction. (c) The scale is short and very much suppressed at lower divisions. (d) The energy consumption in the instrument itself is considerable.

Such an instrument may be used on alternating as well as on direct current, because the force of attraction between the plunger and the coil does not depend on the direction of the current. In instruments intended for a-c circuits the plunger is laminated, or made of iron wires, to prevent the formation of eddy currents and subsequent heating (§217).

The spool on which the coil is wound must be made of an insulating material, or, if of metal, must be subdivided so that no secondary currents can be induced in it. The calibration of the same instrument is somewhat different on alternating and on direct current, because of the influence of hysteresis, eddy currents, and saturation in the plunger. The calibration also depends to some extent on the frequency and on the wave-form of the alternating current.

(B) A much better instrument of the same type, the so-called *Thomson inclined-coil ammeter*, is shown in Fig. 33. The coil is placed at about 45 degrees to the direction of the shaft; a piece of soft iron (iron vane) is mounted on the shaft in an inclined position. When the coil is energized, it produces a magnetic flux, as indicated by the arrows; the vane tends to move so as to become parallel to the lines of force. In doing so it turns the shaft, and the deflection is shown on the scale. The motion is opposed by a spiral spring, the counterweight serving to balance the moving part.

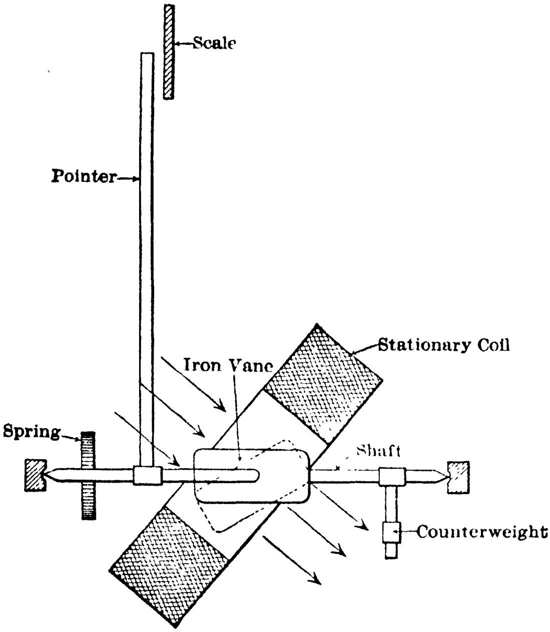


Fig. 33. A cross-section of the Thomson inclined-coil ammeter.

This instrument is more compact than the above-described plunger-type instrument; the moving part is lighter, and the friction is much reduced, making the instrument more sensitive. Moreover, the spring control is more positive than the gravity control used in the plunger instrument. Thomson inclined-coil instruments are made partially "dead-beat" by means of an aluminum vane fastened to the moving part. Frictional resistance of the air to movements of this vane absorbs a considerable portion of the energy of oscillations, and the pointer assumes its final deflection after swinging to and fro.

(C) In the instrument shown in Figs. 34 and 35 a movable iron piece and a fixed iron piece are used. When a current flows through the field coil these pieces are magnetized in the same direction and therefore repel each other. Spiral springs are used for the controlling force, and the moving system is effectively damped by means of an aluminum vane attached to the moving part and swinging in a practically enclosed box. Because of the use of two iron pieces the mechanical force is increased and it is possible to reduce the amount of iron and consequently the core loss. The indication of the instrument is said to be practically independent of the wave-form of the alternating current and of its frequency within wide limits.

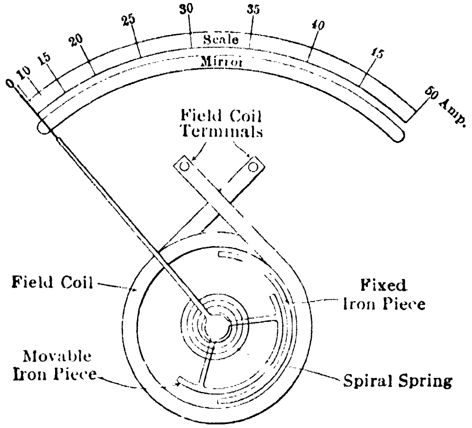


FIG. 34. The Weston iron-vane ammeter.

42. Theory of Iron-Core Instruments. — (a) *Direct currents.* The relationship between the position of the iron plunger or vane and the mechanical pull or torque exerted by it is rather a complicated one, as may be seen from the curves shown in Fig. 156. The skill of the designer consists in choosing such dimensions and proportions as will produce a scale of desired characteristics. For some purposes a uniform scale is desired; for others an extended scale on part of the range and a suppressed scale for the rest of the range is preferable.

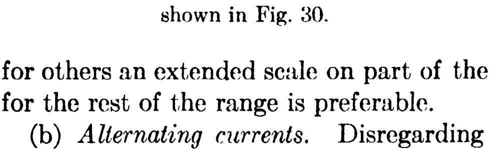


FIG. 35. The movable part of the ammeter shown in Fig. 30.

(b) *Alternating currents.* Disregarding hysteresis, the indication of a movable-iron instrument is independent of the direction of the current through the exciting coil, the iron core being affected equally by the magnetomotive force of either direction. Therefore, when an alternating current is sent through the coil, the indication of the instrument is nearly the same as if every second half-wave were reversed and the alternating current converted into a pulsating unidirectional current. During each alternation the mechanical force upon the moving part of the instrument varies between zero and a maximum, but with usual frequencies the inertia of the moving part is sufficiently large to prevent it from following the instantaneous variations of the current. The pointer therefore assumes the position corresponding to the average value of the mechanical force.

In any given position of the iron plunger or vane, the flux density is approximately proportional to the exciting current, since in an instrument of this kind the flux density is always well below the knee of the saturation curve (Fig. 148). The density of the lines of force in the air is also proportional to the exciting current. But the mechanical attraction or repulsion between the external flux and that in the iron part is proportional to the product of the corresponding flux densities; hence, in this case it is proportional to the *square* of the exciting current. Thus we have:

$$\text{Instantaneous pull} = ki^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

where i is the instantaneous value of the alternating current and k is a coefficient of proportionality. Since the moving element assumes the position corresponding to the average pull we have

$$\text{Average pull} = k\frac{1}{T}\int_0^T i^2\,dt = kI^2_{\text{eff.}} \quad . \; . \; . \; . \; . \; . \quad (2)$$

where dt is an infinitesimal element of time and T is the duration of one cycle of alternating current. In this expression

$$I_{\text{eff.}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

is called the *effective* value of the alternating current, and is defined as the square root of the mean of the squares of the instantaneous values. For direct current the effective value is the same as the steady value. Thus, theoretically, a soft-iron instrument calibrated with direct current should indicate the effective values of alternating currents. In reality this is not quite true because of the disturbing effect of hysteresis and of eddy currents (§§216 and 217). A good instrument calibrated with direct current is correct within a few per cent with alternating currents of moderate frequency and of reasonably good wave-form.

With a given position of the plunger, the pull is proportional to the square of the current; hence, if the coefficient of proportionality were approximately constant, the deflections of the pointer would be proportional to the square of the current. In a properly designed instrument the coefficient of proportionality, k, may be made smaller for the positions of the moving part at higher currents, and the scale thus made approximately uniform for a considerable part of the range.

43. Polarized-Vane Ammeter. — A simple d-c ammeter which comprises a permanent magnet in its construction is shown in Fig. 36. It consists of a movable soft-iron vane *V* which is polarized by a stationary permanent magnet *NS*, and of a stationary current coil *C* which causes a deflection of the polarized vane when a current flows through the coil. The coil surrounds a stationary iron core *I*, one extremity of which faces the soft-iron vane. The vane is fastened to a pivoted staff mounted between bearings. Just above the vane and on the same staff is also mounted an aluminum cylinder *a*, which rotates between the poles of the permanent magnet and acts as a damping device.

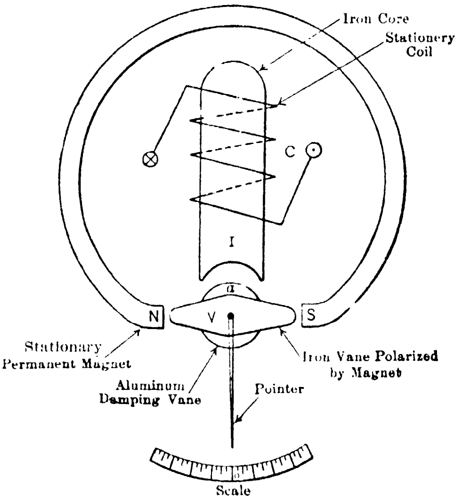


FIG. 36. A polarized-vane ammeter.

When no current flows through the coil, the permanent magnet causes the vane to take a definite position which corresponds to the zero reading on the scale. When a current flows through the coil, the iron core which it surrounds becomes magnetized. The end facing the polarized vane will attract one end of the latter and repel the other, thus causing a deflection of the vane. The polarity of the iron core depends upon the direction of the current, and the magnetic force depends upon the magnitude of the current and thus determines the amount by which the polarized vane is deflected. The permanent magnet serves as the restoring force. This type of instrument is widely used for indicating battery charge and discharge on automobiles, and for this reason the zero is usually in the center of the scale.

PERMANENT-MAGNET AND D-C-ELECTROMAGNET TYPE INSTRUMENTS

44. Permanent-Magnet Movable-Coil Type Instruments. — The principle upon which these instruments are constructed will be seen from Figs. 37 and 38. The magnetic circuit of the instrument consists of a well-aged permanent magnet M with pole-pieces N and S made of soft iron. The soft-iron cylinder I completes the magnetic circuit, leaving two narrow air-gaps. A light coil C of fine wire is pivoted or suspended in such a way that it can swing freely in these air-gaps. When a direct current is passed through the coil, the latter tends to turn, owing to the force which acts upon any conductor carrying a current, when placed in a magnetic field. The deflection is shown on the scale by the pointer P. The purpose of the soft-iron pole-pieces and the cylinder is to provide short air-gaps of constant length, in which a uniform radial flux distribution will be produced. This makes it possible to obtain a uniform scale, which is one of the advantages of this type of instrument. The mechanical torque between the coil and the magnetic flux is proportional to the current and to the flux density at the place where the coil is located. If this density is uniform, the torque is proportional to the current, so that with a proper restoring force it is possible to have a uniform scale.

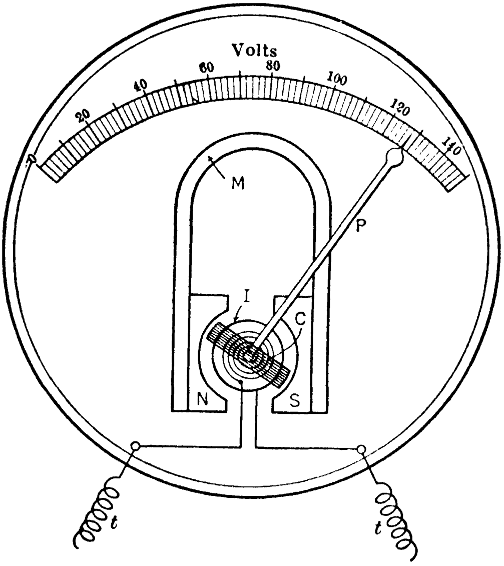


Fig. 37. Active parts of a permanent magnet, movable-coil instrument.

Instruments of this type are constructed for various purposes, from a delicate *galvanometer*, carrying currents of the order of microamperes, to a rugged recording ammeter in which the coil is designed, say, for 0.1 ampere. Such sensitive galvanometers were specified in Chapter I to denote balance in the Wheatstone and Kelvin bridges, etc. In a galvanometer the coil is usually supported between two stretched vertical wires, the torsion of which acts as the restoring force. The same wires serve also as the leads for the current. The *bifilar* suspension used in some galvanometers consists in supporting the coil by means of two wires from the top. When the coil is deflected the wires are forced out of their vertical position, slightly lifting the coil. Gravity then acts as the restoring force.

In ordinary ammeters and voltmeters, and in less delicate galvanometers, the movement of the coil is opposed by two spiral springs, one on top and the other on the bottom (Fig. 38). The springs are coiled in opposite directions so that one of them is twisted while the other is untwisted during the movement of the coil. This compensates for a possible non-uniformity in the material of the springs. In most instruments, the same springs are used for leading the current into and out of the coil. In some cases separate leads are used, independent of the springs; such leads must be very soft so as to offer practically no opposition to the movement of the coil. Without the controlling force of the springs, the needle would be deflected to the end of the scale with any current.

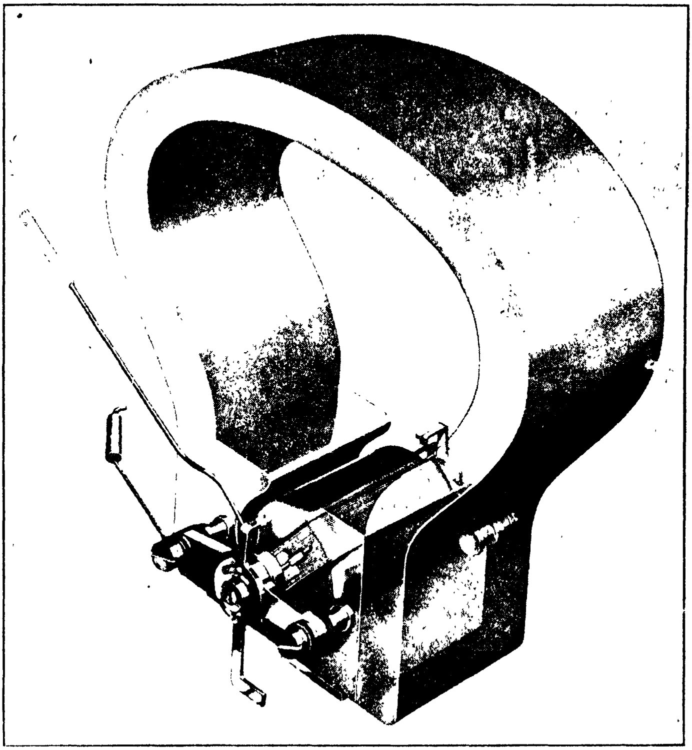


FIG. 38. Arrangement of parts in the permanent-magnet, movable-coil instrument.

It is easy to see that only direct or pulsating currents will give an indication when passed through the movable coil of a permanent-magnet instrument. When alternating current is passed through an instrument of this type, the needle merely trembles at its zero position without giving any deflection. This is because it receives opposite impulses in such rapid succession that it has no time to move in either direction. For the use of this principle in the " vibration galvanometer " see §45.

Moving-coil instruments are easily made " dead-beat " by winding the moving coil on an aluminum frame. Eddy currents induced in the frame during the movement of the coil effectively check any tendency to swing about the point of equilibrium. In some makes of instruments of this type, it is claimed that the damping is so adjusted that the pointer passes a little beyond the final position and immediately returns to it. The purpose of this arrangement is to make sure that the movement is not impeded by friction.

The mechanical support of the coil deserves particular attention. The pivots are made of the best-grade hardened steel and are mounted in sapphire jewels to reduce friction. The instrument must be carefully handled, since shocks dull the pivots and may damage the jewels, thus increasing the friction and reducing the accuracy of calibration. In one make of instruments, resilient jewel mountings are used, so that when the instrument is dropped or subjected to a blow, the moving coil and the jewels move together until the coil strikes the iron cylinder I, where the shock is absorbed.

The moving coil carries only very small currents, usually not above 0.02 ampere; the resistance of the coil is comparatively low. Therefore, an instrument used as a voltmeter is provided with a series resistor (Fig. 31). When used as an ammeter, a moving-coil instrument is provided with a so-called *shunt* of low resistance which carries all but a small fraction of the current to be measured (§49).

Single Air-gap Construction. The magnetic circuit in Fig. 37 has two air-gaps. The parts may also be so arranged as to have but a single air-gap. In this case the pole-pieces N and S are made to overlap each other, and the lines of force in the air-gap are parallel to the axis of rotation of the coil. The central cylinder is omitted and the coil embraces one of the pole-pieces. Only one side of the coil is active in producing the torque, instead of two, but the construction is somewhat simplified. Moreover, the air-gap is between two flat surfaces, instead of between two cylindrical surfaces as in the double air-gap construction. The single-gap type is used but little.

Unipivot Construction. The movement shown in Fig. 39 differs from that shown in Fig. 38 in that the movable coil is supported on only one pivot, which is located at the center of the soft-iron sphere. The counterweight shown in the figure is so adjusted as to make this point the actual center of gravity. The coil therefore swings freely in all directions, and the instrument need not be kept perfectly level. The spiral spring shown on top furnishes the restoring force and also serves as one of the leads to the coil. The other lead is a very fine volute spiral attached to the bottom of the coil and marked " flexible connection." Instruments of this type are available in ranges from very delicate galvanometers responsive to a few microamperes to ordinary portable ammeters and voltmeters of usual commercial rating.

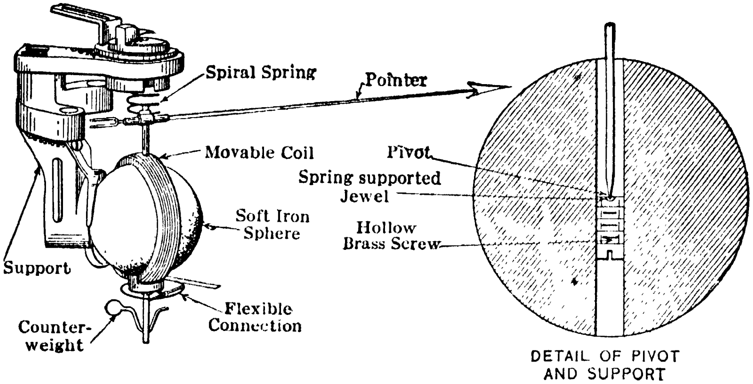
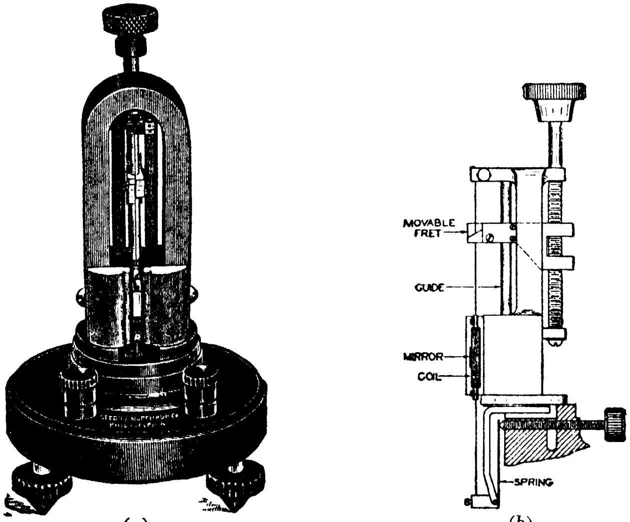


Fig. 39. Unipivot type instrument.



(a) (b)

Fig. 40. Vibration galvanometer (Leeds & Northrup Co.)

45. Vibration Galvanometer.—The vibration galvanometer (Fig. 40*a*) is an instrument for the detection of small sinusoidal alternating currents of a particular frequency, and is used in many cases in which the telephone receiver was formerly employed. The instrument is particularly useful in a-c " null " methods, for example, in various arrangements of the Wheatstone bridge for measuring resistances with alternating currents (§20) and for comparing inductances and capacitances.

The usual vibration galvanometer is of the permanent-magnet movable-coil type (§44). The coil is quite narrow and is held in position between two flat suspension strips. These strips are stretched taut so that the *natural frequency* of vibration of the moving system is of the order of magnitude of, say, 60 complete oscillations per second. The mechanical tuning to a particular frequency is usually accomplished in two ways. For rough adjustment a fret can be moved along one of the suspensions, thus changing its effective length. To secure the fine adjustment the tension of the suspension strips is altered. (See Fig. 40*b*.)

When an alternating current is passing through the coil it is alternately deflected to the right and to the left with each half-wave; in other words, the coil is set in vibration. When the moving system is tuned to the particular frequency of the alternating current, that is, is in *resonance* with it, the sensitiveness of the instrument is at a maximum, and the amplitude of vibration with a given current is the greatest. The sensitivity drops very rapidly when the instrument and the current are " out of tune " even by a fraction of 1 per cent. For this reason it is of importance to keep the frequency of the supply very nearly constant, and it is often necessary to have a separate generator driven by a motor which is provided with a particularly sensitive speed governor.

A small mirror is mounted on the coil, and a beam of light is reflected by this mirror to a scale. When no current is passing through the coil a sharply defined straight image of the lamp filament is seen on the screen. As the coil vibrates, the image becomes extended into a band of light. The width of the band is practically proportional to the magnitude of the current. To give an idea of the current range of the instrument, it may be stated that a well-known standard vibration galvanometer has a sensitivity of 40 mm per microampere at 60 cycles; it can be tuned to frequencies of from 50 to 80 cycles.

A higher harmonic in the current affects the deflection to an imperceptible extent, so that the instrument may be said to possess a highly selective sensitivity. For this reason it is superior to a telephone receiver as a detector. In Wheatstone-bridge work the sound in the telephone receiver under balanced conditions can be reduced to a minimum but not to zero, unless the current is purely sinusoidal; on the other hand, the vibrations of the galvanometer coil can be practically stopped, giving a much more sharply defined reading.

The advantages of a d-c galvanometer over a " tuned " instrument are quite apparent, and for this reason various methods have been proposed and used whereby an ordinary d'Arsonval galvanometer may be employed with small alternating currents. This usually involves a rectification of the galvanometer current. A synchronous commutator has been used successfully for such rectification, and also crystal and copper-oxide rectifiers, in conjunction with a vacuum-tube amplifier (Vol. II).

46. Peak-Reading Voltmeter. — In some cases the wave-form of an alternating voltage is badly distorted and only the instantaneous maximum value is of interest. Such a case arises, for example, in testing high-voltage cables where the voltage becomes distorted because of the high capacitance of the cables and where at the same time it is important not to subject the insulation, even for an instant, to a voltage in excess of that specified.

An instrument which indicates only the recurring maximum value of an alternating current or voltage consists of two small parallel wires or strips upon which a tiny mirror is mounted. These wires are placed in the field of a strong permanent magnet or an electromagnet excited with direct current. When a current flows through the wires, since it moves upward in one wire and downward in the other, they move in opposite directions within the magnetic field, thus tilting the mirror. With alternating current they oscillate, and the amplitude of the oscillation is proportional to the instantaneous maximum of the current. A beam of light from a straight-filament lamp is reflected from the mirror of the oscillating system to a ground-glass scale. When the wires are at rest the image of the filament appears on the scale as a narrow vertical line. When the system is oscillating the line of light spreads out into a band with the maximum point sharply defined, and the width of the band is proportional to the amplitude of the current through the strips. The scale may be calibrated directly in volts or in amperes.

The instrument is practically a simplified oscillograph (Vol. II) in which the motion along the time axis is omitted. In another type of peak or crest voltmeter, a part of the high voltage is rectified and measured with a d-c voltmeter.

47. Thomson D-C Astatic Instruments. — The magnetic field in an instrument of this type is produced by an electromagnet which must be excited from some source of direct current. A permanent magnet may also be used.

Two moving coils are mounted diametrically opposite each other upon an aluminum disk and are placed in the magnetic field parallel to the shaft. Two small pieces of soft iron are rigidly mounted on the shaft, and are held in the zero position by the components of the magnetic field perpendicular to the shaft. This magnetic attraction thus acts as the restraining force in place of the usual springs. The moving torque is produced by the interaction of the direct current through the coils of the moving element with the lines of force of the stationary electromagnet.

The current is conducted into the moving coils by two small silver-alloy spirals, which exert very little force as springs. The actuating and restraining forces are both proportional to the field strength of the electromagnet, so that the exciting current need not be kept exactly constant. This is of importance in switchboard service where the stationary coils are excited from the bus-bars, the voltage between which may vary at times by several per cent.

To compensate for the effect of external magnetic fields, two sets of magnetic poles are used, arranged astatically; that is, a stray field which increases the torque on one side of the armature decreases it by the same amount on the other side. In addition, the mechanism is magnetically shielded by an iron case. The damping of the moving part is produced by eddy currents induced in the aluminum disk on which the movable coils are mounted.

48. Accurate Measurement of a Small Emf. — There are cases in which a small emf, say below 1 volt, down to a few millionths of a volt, must be measured without drawing any current from the source of the emf to be measured. With such a requirement an ordinary type millivoltmeter or galvanometer is not applicable, and a potentiometer, which is described in Chapter III, may be used. However, a potentiometer and standard cell may not always be available. Where extreme accuracy is not required, the following "microvolter" method of measurement may be used (Fig. 41). It is based on the potentiometer principle but requires no standard cell. The main circuit consists of a battery *Ba* and a rheostat *R*. An accurately adjusted low resistance *r* of negligible temperature coefficient is shunted around part of *R*, in

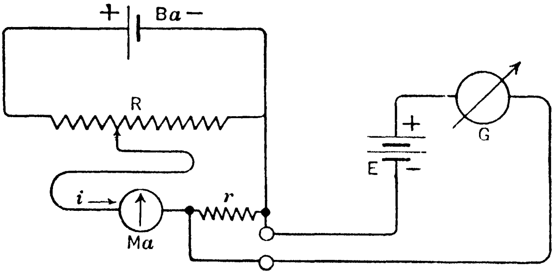


FIG. 41. The microvolter method of measurement of small emf's.

series with a milliammeter *Ma*. The unknown voltage *E* is connected across *r* in series with a sensitive galvanometer *G*. The current through *r* is varied by means of the sliding contact on *R* until the galvanometer shows zero deflection. Let the milliammeter current under these conditions be *i* amperes. Then we have $E = ir$, and from this expression the unknown emf, *E*, may be computed. For a given *r* the milliammeter may be calibrated directly in microvolts, making the above computation unnecessary.

For voltages of the order of 50 millivolts and higher, the foregoing scheme may be simplified by the use of the same indicating instrument as galvanometer and as milliammeter. The diagram of connections, which Dr. Northrup calls the "millivolter" principle, is shown in Fig. 42. The main circuit is through the battery *Ba*, a variable resistance *R*, a standard resistance *r*, and the resistance marked *Cu*. The latter is made of copper and is equal to that of the moving coil of the galvanometer *G*. In position 1, the unknown emf, *E*, is connected across the standard resistance *r* in series with the galvanometer. The resistance *R* is adjusted until the galvanometer deflection is zero, so that the voltage drop across *r* is equal to *E*. The connections are then quickly changed to position 2, in which the galvanometer is used as a direct-reading ammeter. If the current and the resistance *r* are known, the emf, *E*, may be computed. For a given *r* the galvanometer may be calibrated directly in millivolts. In the second position the resistance *Cu* is eliminated, since the movable coil of the galvanometer takes its place, making the total resistance of the main circuit equal to that in position 1, provided the contact resistances are equal, or are negligible.

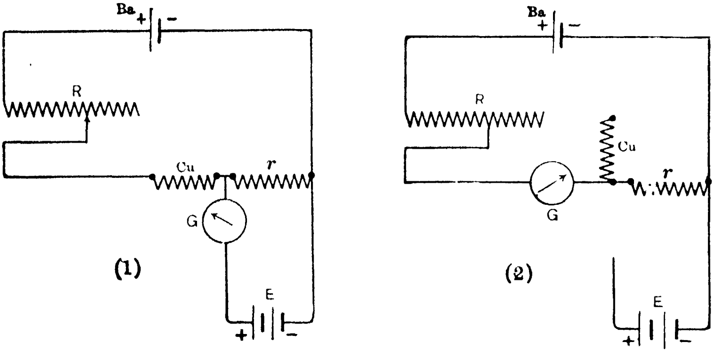


FIG. 42. The millivolter method of measurement of small emf's.

The instruments built on these two principles have been used mainly for temperature measurements by means of thermocouples, but they are also suitable for the calibration of ammeters and shunts and for accurate measurements of small emf's.

49. Ammeter Shunts. — A movable-coil permanent magnet instrument, when used as an ammeter, usually has to be provided with a so-called *shunt*, of low resistance (Figs. 43 and 44), which carries the main current to be measured. The instrument itself is connected across the terminals of the shunt and measures the millivolt drop across it. In an instrument of this type it is impracticable to build a coil which would carry more than a small fraction of 1 ampere. Thus, an ammeter of the moving-coil type, when used to measure even a moderate current, is simply a millivoltmeter connected across a shunt.

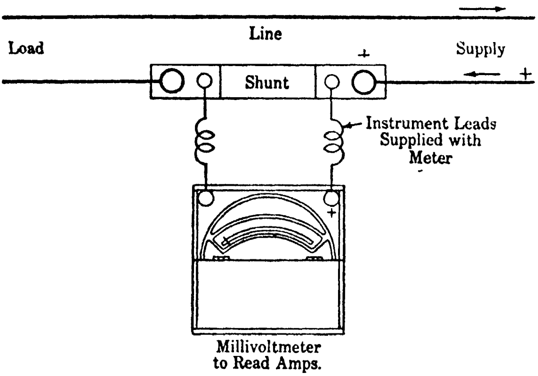


FIG. 43. Current measurement with a millivoltmeter across a shunt.

Let the resistance of a shunt be 0.001 ohm, for example, and let the full-scale deflection of the instrument be 50 millivolts, or 0.050 volt. According to Ohm's law, the line current is

$$i = \frac{0.050}{0.001} = 50 \text{ amperes.}$$

The current through the instrument itself is but a few milliamperes and can be neglected.

In commercial instruments the shunt and the millivoltmeter are usually calibrated together, and the scale reads directly in amperes instead of in millivolts. In ammeters of moderate capacity the shunt may be placed inside the case, so as to make the instrument self-contained. In larger instruments, the shunt is separate (Fig. 44) and is connected to the millivoltmeter by flexible leads.

Ammeter shunts are made of strips of manganin, German silver, or other material of high specific resistance and low temperature coefficient. These strips are sweated into heavy brass blocks which serve as the current terminals. In addition to these heavy terminals there are small potential terminals for connecting the shunt to the millivoltmeter. The latter terminals are placed so as to measure the drop across the body of the shunt, independent of the distribution of current in the blocks.

This distribution may vary with an uncertain contact at the main terminals.

One common mistake which the beginner is apt to make when using a moving-coil instrument is to forget to connect a multiplier or a shunt. This invariably results in the burning out either of the moving coil (Fig. 38) or of the spiral springs. Sometimes the pointer is bent or broken.

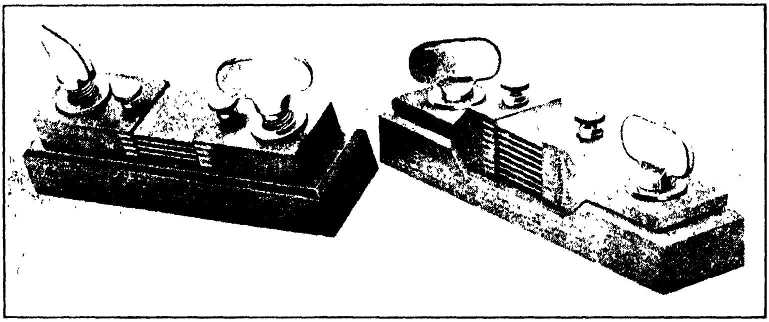


Fig. 44. Commercial shunts (Weston Co.).

ELECTRO-DYNAMOMETER-TYPE INSTRUMENTS

50. Electro-Dynamometer Voltmeter. — An instrument of this type (Fig. 45) consists of two stationary coils *SS* and a movable coil *M*. All the coils are connected electrically in series. Spiral springs are used as the controlling force for the movable coil and hold it at a certain angle with the stationary coils when no current is flowing. When a current is passing through the coils, the moving coil tends to place itself in a plane parallel to that of the stationary coils, and the resulting deflection is read on the scale. The general make-up of the instrument is the same as in Fig. 38, except that stationary coils are substituted for permanent magnets.

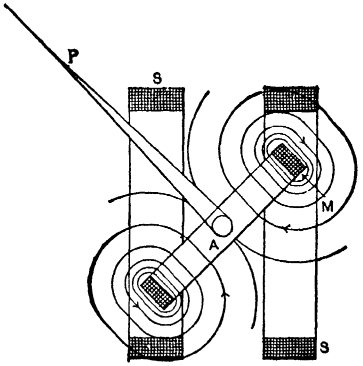


Fig. 45. Arrangement of coils in a dynamometer-type voltmeter.

The instrument can be used with either direct or alternating current. The coils being connected in series, the current reverses simultaneously in both, so that the force of attraction between the two does not change its sign. In fact electro-dynamometer-type instruments may be calibrated with direct current and used on a-c circuits. They may be constructed as voltmeters, ammeters, wattmeters, power factor meters, phase meters, etc.

Damping is accomplished by means of an air damper consisting of an aluminum vane. The entire set of coils is mounted in a double closed iron shield, which protects it from external magnetic fields and electrostatic influences. For very accurate measurements, reverse readings with direct current should be made, to eliminate the effect of any slight residual magnetism in the shield. In a double or astatic electrodynamometer (Fig. 65) the directions of the currents in the two elements may be so chosen as to neutralize the effect of a *uniform* external field, such as that of terrestrial magnetism. There is no assurance, however, that the influence of a non-uniform local field will also be neutralized.

Voltmeters of this type usually have such a large non-inductive resistance (multiplier, §39) in series with the coils that at a given voltage the instrument takes practically the same effective current within a wide range of commercial frequencies. However, when using the instrument at high frequencies or for low voltages at which the resistance of the multiplier is reduced, one must be careful to calibrate the instrument accordingly. The unavoidable inductance of the coils may in such cases play an appreciable part and make the calibration depend upon the frequency of the voltage to be measured.

Let a certain alternating current flow through the voltmeter and let the moving part take a position of equilibrium, at which the electromagnetic torque between the coils balances the spring torsion. If an instantaneous value of the current is i, then the electromagnetic torque at that instant is equal to ki^2, where k is an instrument constant *for that particular relative position of the coils.* The torque varies periodically between zero and a maximum, and the average value is equal to the spring torque. In other words

$$\text{Spring torsion} = k \cdot \text{ave}\,(i^2) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

But, by definition, *the square of the effective value I of an alternating current is equal to the mean of the squares of the instantaneous values over a cycle.* Hence, eq. (4) may also be written as

$$\text{Spring torsion} = kI^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

Now let a direct current be sent through the instrument, of such a value, i_{dc}, that the deflection is the same as before. We then have

$$\text{Spring torsion} = ki_{dc}^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

Comparing eqs. (5) and (6), we see that $I = i_{dc}$. In other words, a dynamometer-type instrument calibrated with direct current indicates true effective values of alternating current, irrespective of waveform. In application to a voltmeter this conclusion presupposes that the ohmic resistance of the instrument is so high as compared to its reactance (§137) that the current taken by the instrument is practically independent of the frequency and is simply proportional to the applied voltage.

The force of torsion of the spring is nearly proportional to the angle of deflection. Hence, if k were constant for different angles, the deflection would be proportional to the square of the current flowing through the instrument, that is, proportional to the square of the voltage. This follows directly from eq. (5). The corresponding scale would be very much suppressed at the lower range and open at the upper range. Sometimes such a scale is desirable; but if a more uniform scale is preferred, the instrument may be so designed as to make about two-thirds of the scale in the upper range more nearly uniform.

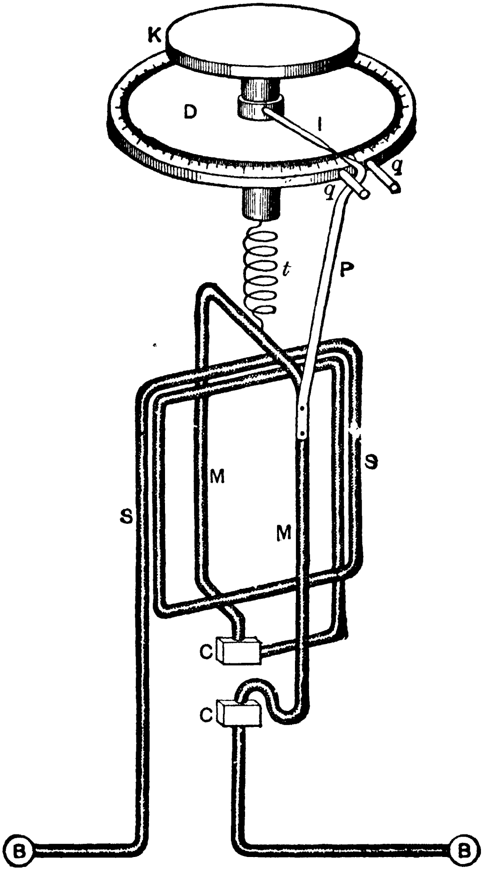


Fig. 46. Arrangement of parts in the Siemens electro-dynamometer.

51. Electro-Dynamometer Ammeters.—The principal difficulty in constructing an ammeter on the same principle as the voltmeter described in the preceding section, consists in devising a satisfactory method of conducting the current to be measured, or a fraction of it, to and from the moving coil. This problem has been only partly solved, and the two instruments described below, though quite accurate and valuable in standardization work, are not used to any extent for other purposes.

(a) *Siemens electro-dynamometer.* This is the original ammeter of this type and is illustrated in Figs. 46 and 47. *BB* are the terminals of the instrument; the current flows from the left terminal through the stationary coil *SS* to the upper mercury cup *C*; thence through the movable coil *MM* and the lower cup *C* to the right terminal *B*. The coil *MM* is suspended from the top of the instrument by a cocoon thread (not shown in Fig. 46), and is controlled by the spiral spring *t* operated by the torsion knob *K*. The zero of the scale is between the stops *qq*. When a current flows through the instrument, the pointer *P* strikes against the right stop *q*. The knob *K* is then turned to the left, until *P* comes back to zero. The index *I* shows the angle of torsion on the dial *D*.

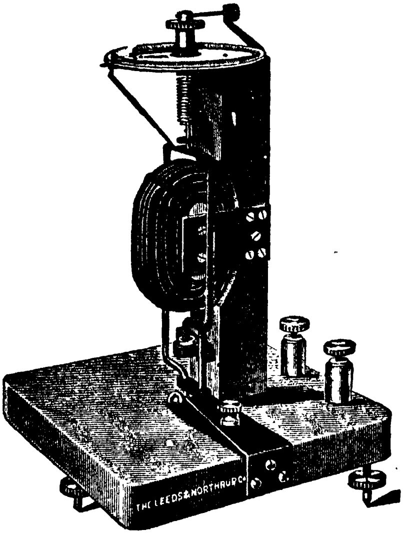


Fig. 47. An electro-dynamometer ammeter.

This instrument is not direct-reading, and the presence of mercury necessitates leveling and makes the instrument inconvenient to handle except for special purposes. The moving coil is practically undamped, so that readings are slow and require some skill, especially with fluctuating currents. When calibrating or using it with direct current, the operator must neutralize the influence of the terrestrial magnetism upon the moving coil by placing the instrument in such a position that the plane of the moving coil is perpendicular to the magnetic meridian. To compensate for the influence of stray fields, readings should be taken with the current flowing through the instrument first in one and then in the reverse direction, and the results averaged.

The theory of the electro-dynamometer voltmeter given in the preceding section is applicable here. Since the torsion-head reading is always taken with the same relative position of the stationary and moving coils, the coefficient k is an absolute constant of the instrument. If the angle of torsion is α, we have

$$\text{Spring torsion} = C\alpha \quad \ldots\ldots\ldots\ldots \quad (7)$$

where C is the spring constant. Equating formulas (5), (6), and (7), we get

$$kI^2 = ki_{dc}^2 = C\alpha$$

or

$$I = i_{dc} = \sqrt{\alpha} \cdot \sqrt{C/k} = D\sqrt{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where $D = \sqrt{C/k}$ is the calibration constant of the instrument. We see from eq. (8) that in the Siemens dynamometer the current is proportional to the square root of the angle of torsion. The constant D can be determined by comparison with a standard d-c instrument, and the same constant used when measuring effective values of alternating currents.

The definition of the effective value of current or voltage, given in §50, holds true whether the quantity varies according to the sine law, or in a more complicated manner. *For sinusoidal quantities the effective value is equal to the amplitude divided by the square root of* 2. A proof of this proposition will be found in almost any elementary textbook on alternating currents. See also the note on p. 90.

(b) *Weston direct-reading ammeter.* This type of electro-dynamometer ammeter is similar in its construction to the voltmeter shown in Fig. 45. The stationary coils carry the whole current to be measured, while the movable coil is connected across a shunt (Figs. 43 and 44), and the current passing through the coil is proportional to the voltage drop in the shunt. In this manner the necessity for carrying heavy currents through mercury cups is obviated. In order to be absolutely accurate at all frequencies within a certain range, both the shunt and the moving coil should be altogether non-inductive. In practice this is not possible, but a very close approximation of this condition is obtained by making the shunt practically non-inductive and keeping the resistance of the moving-coil circuit high as compared to its reactance. In standard laboratory instruments of this type, the self-inductance of the movable coil circuit is practically compensated for by the use of condensers. For an instrument without condensers, indications are guaranteed to be correct within 1/4 of 1 per cent of full-scale value, both for direct current and for alternating currents of any frequency up to 133 cycles per second. For a compensated instrument the error can be made less than 1/10 of 1 per cent. Instruments of this type may also be compensated for frequencies of 500 cycles and above.

Because of the necessity of making the resistance of the moving-coil circuit comparatively high, the voltage drop across the shunt and the power consumption of the instrument must also be much higher than in a permanent-magnet movable-coil instrument. For this reason these instruments are more suitable for calibration and testing purposes than for switchboard service. Damping, and freedom from the effects of external magnetic fields, are obtained as in the direct-reading voltmeter described above. These instruments are also made as milliammeters for ranges lower than 1 ampere. The field and movable coils are then connected in series, and the total current is made to flow through both.

(c) *Transformer-type instrument.* Soft flexible leads can be designed to conduct currents up to about 1 ampere into the moving coil, by making the instrument somewhat larger and not very accurate. Beyond this value a current transformer (§56) may be used, and the range of the ammeter thus increased indefinitely. The recording instrument shown in Fig. 65, when designed as an ammeter, is used with a current transformer.

OTHER TYPES OF INDICATING AMMETERS AND VOLTMETERS

52. Hot-Wire Instruments.—An expansion type hot-wire instrument is shown in Fig. 48. The current to be measured, or some part of it, passes through the leads *tt* and the stretched platinum-iridium or platinum-silver wire *AB* and heats the latter so that it sags appreciably. The resulting downward motion of the point *C* is transmitted to the pointer *P*. The deflection is magnified by means of the wire *CD*, the point *F* of which is under the tension of the spring *KH*, through a silk thread wound on the pulley *G*. When *C* moves downward, *F* and *H* move to the left, and the silk thread turns the pulley. The pointer *P*, mounted on the same shaft with *G*, shows the deflection on the scale.

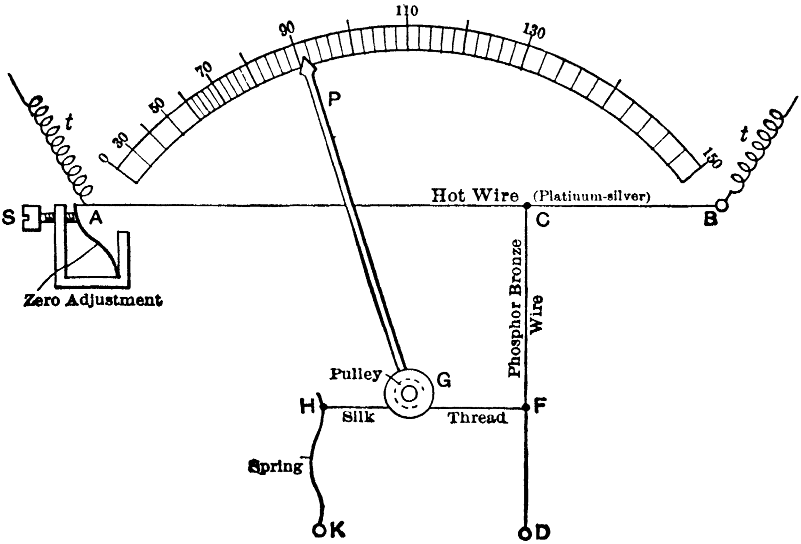


FIG. 48. Elements of the expansion-type hot-wire instrument.

An instrument of this construction can be used equally well with direct or alternating current. A hot-wire ammeter for large currents is used with a shunt (Fig. 43); the maximum current through the hot wire itself is usually limited to a few amperes or, in special instruments for radio work, to a few hundredths of an ampere. The scale of a hot-wire instrument is not uniform because the heat energy put into the wire increases as the square of the current. In some hot-wire instruments used for high-frequency measurements the scale is calibrated in squares of the current; this gives nearly uniform scale divisions.

The instrument shown in Fig. 48 is made " dead-beat " as follows: An aluminum disk is mounted on the same shaft with the pulley G and placed between the poles of a permanent magnet. Eddy currents induced in the disk during its motion effectually damp the vibrations of the needle.

The base of the instrument, between points A and B, is made of an alloy whose coefficient of thermal expansion is equal to that of the hot wire itself. This is done in order to compensate for changes in room temperature which should not deflect the pointer from zero. Nevertheless, a hot-wire instrument easily gets " off zero," and for this reason a zero adjustment screw S is provided at A.

Let r be the resistance of the hot wire and i an instantaneous value of a variable periodic current flowing through it. The total amount of energy converted into heat in the wire during a cycle, corresponding to an interval of time T, is

$$W = \int_0^T (i^2 r)\, dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

With a rapidly fluctuating current, the final temperature of the wire corresponds to the *average* heat input per second. In other words, the temperature of the wire, with alternating or fluctuating current, corresponds to the average amount of power

$$P_{\text{ave.}} = \frac{1}{T}\int_0^T (i^2 r)\, dt = r \cdot \frac{1}{T}\int_0^T i^2 \cdot dt = rI^2 \quad . \; . \; . \quad (10)$$

where I is the effective value of the alternating current (§30). A constant direct current, i_{dc}, in order to give the same scale deflection, must develop the same amount of Joulean heat in the hot wire. In other words

$$P_{\text{ave.}} = ri_{dc}^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

Hence, $I = i_{dc}$, and for this reason a hot-wire instrument, calibrated with direct current, indicates true effective values of an alternating current of any wave shape.

53. Thermoelectric Ammeter. — The original instrument based on this principle, and known as the Duddell thermogalvanometer, is shown in Fig. 49. The direct or alternating current to be measured is sent through the resistor or "heater" shown at the bottom of the figure and causes its temperature to rise, thus warming a thermojunction or thermocouple attached to the end of a movable coil suspended in the air-gap of a permanent magnet *NS*. A small direct current flows in the coil and causes it to be deflected by the permanent magnet against the torsion of the quartz fiber *Q*. The sensibility of the instrument depends upon the resistance of the heater and upon its distance from the thermal junction. The instrument can be calibrated with direct current and used with alternating currents, for the same reason as the hot-wire meter described in the preceding section. This thermogalvanometer has become of some importance in measuring high-frequency a-c currents, notably in radio work, telephone transmission, X-ray tubes, etc. The instrument is also suitable for use, in some cases, as an intermediate d-c to a-c standard.

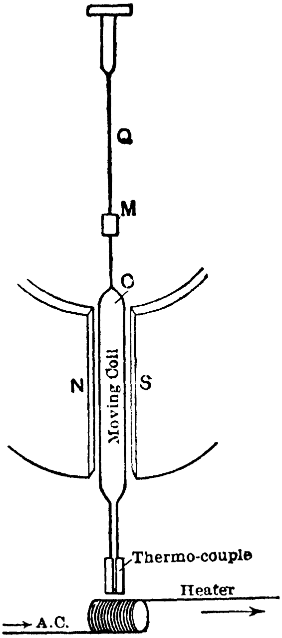


FIG. 49. The Duddell thermogalvanometer.

54. Induction-Type Ammeters and Voltmeters. — These instruments (Fig. 50) are suitable for alternating current only, and are based upon the principle of the revolving magnetic field produced by two alternating currents out of phase with each other; the principle is the same as in the split-phase induction motor (§508). The torque in an ammeter or voltmeter shown in Fig. 50 may be explained in exactly the same manner as the starting torque in a single-phase induction motor with an auxiliary phase winding. The laminated iron core *CC* is shaped like the magnetic circuit of a bipolar dynamo, and a hollow aluminum drum *D* serves as an armature. This drum is pivoted on jewels between the poles, and an indicating pointer is mounted on the same shaft. The restraining force is furnished by phosphor-bronze spiral springs as in most other types of instruments (Fig. 39). The current I_1 to be measured flows through the primary winding *PP*. A secondary winding, *SS*, is placed on the same core and furnishes an induced secondary current I_2 to a pole winding *AA'*. This arrangement of the windings

PP and *SS* is similar to that in a current transformer (§56). The magnetic fluxes are shown by the lines ϕ_m and ϕ_s and are also indicated vectorially in Fig. 51. The main flux ϕ_m is due to the combined magnetomotive forces of both the primary and the secondary windings, and its direction in the aluminum armature is substantially horizontal, that is, parallel to *ss*. The secondary flux ϕ_s is due to the pole winding *AA′*,

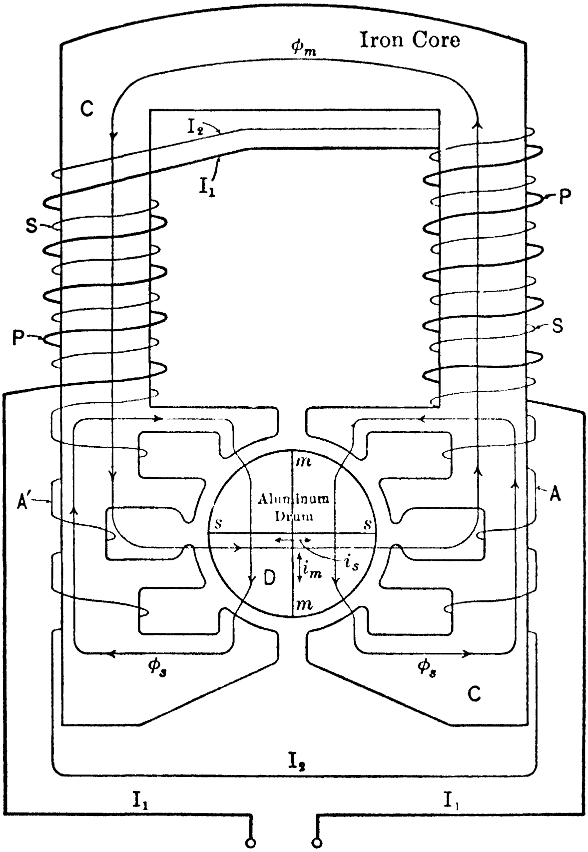


FIG. 50. Schematic diagram of the induction-type ammeter or voltmeter.

and its lines of force run in the drum substantially in the vertical direction, parallel to *mm*. According to Faraday's law, these two fluxes induce in the aluminum drum secondary currents in such a direction as to oppose the fluxes. Because of the unavoidable leakage reactance of the drum, these currents lag in time phase behind the corresponding induced voltages.

The drum currents, i_m, due to the main flux, flow substantially in the planes parallel to *mm*, while the currents, i_s, due to the secondary flux, flow in planes parallel to *ss*. The torque of the instrument is due to the algebraic sum of the torques produced by the interaction of $\phi_m \cdot i_s$ and $\phi_s \cdot i_m$, taking into account the phase angles (both in time and in space) between the currents and the fluxes. The flux ϕ_m produces no torque with the current i_m which it induces. The action is simply a repulsion along the lines of force. On the other hand, the currents i_s tend to cut across the lines of force of the flux ϕ_m and give a torque. Similarly, ϕ_s gives no torque with its own secondary currents i_s, but only with the currents i_m.

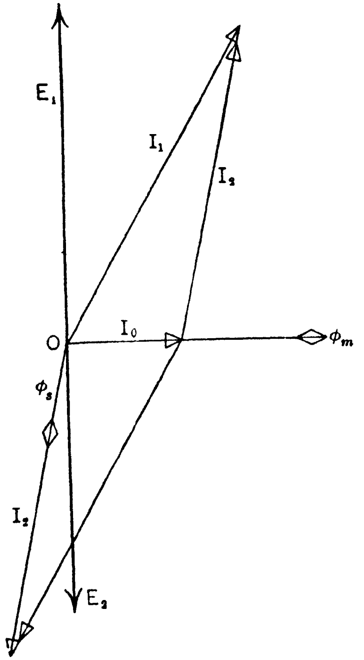


FIG. 51. A vector diagram of the currents and fluxes in the instrument shown in Fig. 50.

For those who do not care to follow the theory in detail, it is sufficient to understand that the magnetic circuit and the windings of the instrument produce two fluxes, differing from each other both in *time* phase and in *space* phase. This is a necessary and sufficient condition for the production of a revolving or gliding magnetic field (§484). The gliding field, though imperfect in comparison with that in a polyphase machine, is sufficient to induce in the aluminum drum secondary currents as in the squirrel-cage rotor of an induction motor. Because of these currents the drum tends to follow the field. The resisting springs limit the motion of the drum to an angle at which their torsion balances the electromagnetic torque.

Figure 51 is drawn under the assumption of an equal number of turns in the *P* and *S* windings, so that the currents are added directly instead of the magnetomotive forces. The main flux, ϕ_m, is due to the geometric sum of the currents I_1 and I_2; in other words, it corresponds to the magnetizing component I_0 of the primary current. The secondary flux ϕ_s is due to the current I_2 only, and is displaced in time phase with respect to ϕ_m by nearly 90 degrees. The construction of the instrument is such that ϕ_m and ϕ_s are displaced in space by 90 degrees, so that the above-mentioned condition for the production of a revolving field is fulfilled.

An instrument of this type may be so designed as to be practically independent of the effect of the frequency within rather wide limits. It is claimed that the maximum difference in readings for any two frequencies between 25 and 60 cycles is 1/2 of 1 per cent of the full-scale deflection. To correct for the changing resistance of the drum with temperature, the secondary coil circuit is wound partly with copper and partly with wire of low temperature coefficient. When the temperature of the drum increases it is necessary to have a somewhat larger flux for the same torque. The flux actually becomes larger because the increased resistance of the secondary circuit necessitates a larger induced emf, and consequently a larger ϕ_m.

In a voltmeter of this type the primary coil is wound with many turns of fine wire, and an external non-inductive multiplier is used, of such high resistance that the indications are practically independent of the frequency, as in the case of the ammeter. Instruments of this type need to be heavily damped electromagnetically, so that the pointer will not overswing and cause unnecessary wear on the bearings. Advantages of these instruments are: a long open scale, freedom from error due to external fields, a high torque, the absence of moving connections, and ruggedness.

55. Electrostatic Voltmeters. — The operation of this type of instruments is based on electrostatic attraction between two metal bodies carrying opposite electric charges. One of these bodies is usually stationary and the other movable (Figs. 52 and 53). They are insulated from each other and are connected to the opposite sides of the line, so that they are charged with equal and opposite quantities of electricity. The movable and stationary parts are mutually attracted, and the deflection of the movable part is shown on the scale. In general, the scale is suppressed at lower voltages since the attraction increases as the square of the voltage. An instrument may be so designed as to give a practically uniform scale throughout the upper range.

The voltmeters shown in Figs. 52 and 53 have a gravity control; that is, the electrostatic force of attraction must overcome the unbalanced part of the weight of the moving part. For this reason these instruments must be carefully leveled before use. Protective resistance rods, *bb*, are provided in series with the instrument to prevent damage in case of a high-voltage arc-over. The resistance of these rods is of the order of magnitude of 1/10 megohm per kilovolt rating of the instrument.

The vane type (Fig. 52) is suitable for potentials of 3000 to 10,000 volts. The pan type (Fig. 53) is made for potentials up to 50,000 volts. In still another type, the movable plate is in the form of a round flat cup and is enclosed in a cylindrical glass case filled with oil. The restoring force in this type is furnished by a spring instead of by gravity, the instrument being built for potentials as high as 150,000 volts.

In the voltmeter shown in Fig. 54, the stationary part consists of two curved metal plates *BB* connected to the line, either directly or through the condensers *CC*. The moving element consists of two suspended hollow cylinders *MM*. When an electric pressure is applied between the plates *BB*, the moving element becomes charged by induction, and its cylinders tend to approach the stationary plates. They can do this by moving counter-clockwise, the shape of the stationary plates being such that this movement reduces the distance between them and the moving element. A spiral spring is used as the controlling force, and the deflection is read on the scale. The action of this voltmeter may also be explained on the basis of a general law of electrostatics that a charged system tends to assume a position of maximum capacitance (permittance) or minimum elastance.

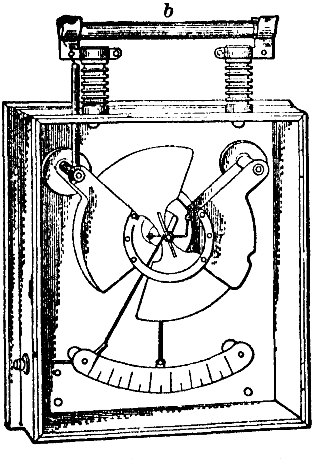


FIG. 52. The vane-type electrostatic voltmeter.

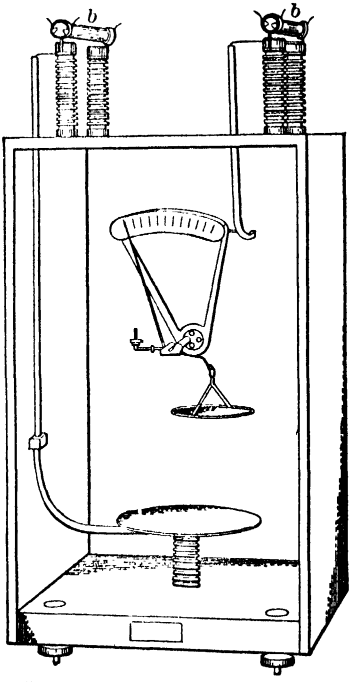


FIG. 53. The pan-type electrostatic voltmeter.

The plates *BB* and the cylinders *MM* are enclosed in an iron tank filled with oil. The tank shields the voltmeter from the influence of external electrostatic fields. The minimum safe distance between the stationary and the moving parts is greatly reduced by the use of oil, and the actuating forces are greatly increased, not only because of the smaller distance between the parts, but also because of a greater specific inductive capacity of oil over air. Moreover, the oil acts as a damper and makes the instrument nearly "dead-beat." The oil also buoys up the moving element, removing practically all weight from the bearings. This greatly reduces the friction and makes the instrument much more sensitive.

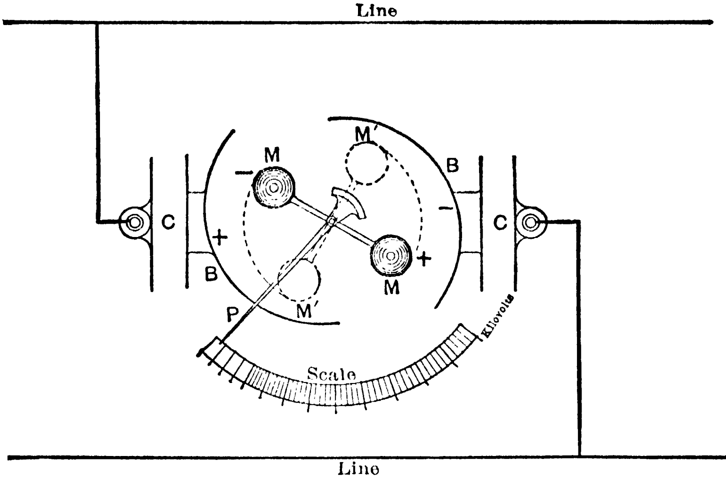


Fig. 54. An electrostatic voltmeter for high voltages.

With low voltages, the necessary area of the plates, in order to have an appreciable attraction, becomes quite large. For this reason, electrostatic voltmeters are mostly used for high-tension work. In a low-voltage static voltmeter designed by Lord Kelvin the required surface is obtained by using several small plates in parallel; this instrument is known as the *multicellular* voltmeter.

The voltage range of an electrostatic voltmeter may be greatly increased by the use of condensers in series with it. It is known from electrostatic theory that when two or more condensers are connected in series the total voltage is divided in the ratio of the reciprocals of the capacitances (Vol. II). The reciprocal of a capacitance is sometimes called the *elastance* of the condenser, and it may be said that the voltage is divided in proportion to the elastances of the individual condensers in series. Thus, condensers play the same part for an electrostatic voltmeter as multiplier resistances used with other types of voltmeters.

The smaller the capacitance of a condenser (that is, the higher its elastance), the smaller the voltage to which the voltmeter itself is subjected.

In the voltmeter shown in Fig. 54 the terminals are of the condenser type. They consist of concentric cylindrical layers, alternately insulating and conducting, which form in effect a large number of condensers in series. Some of these layers of the condenser terminal may be short-circuited in order to obtain a full-scale deflection at one-half or at one-quarter voltage. This voltmeter is made for potentials up to 200,000 volts, and may have one insulated terminal only, the other curved plate *B* being connected directly to the tank and the ground.

56. Instrument Transformers. — On an a-c circuit an ammeter is often connected to the line indirectly, through a so-called *current transformer* which serves somewhat the same purpose in the a-c line that the shunt does in the d-c system (Fig. 55). Such a transformer consists of a laminated iron core provided with two windings. The primary winding is connected in series with the line and the secondary to the ammeter. The ratio of the currents in these two windings is very nearly as the inverse ratio of the numbers of turns. Let the primary winding have 4 turns and carry a current of 100 amperes, and let it be desired to use a 5-ampere ammeter. The secondary winding must then have 80 turns, since 100 × 4 = 5 × 80. The advantages of using a current transformer are as follows: (1) All ammeters may be made for one range, say 5 amperes, and used for any current measurement when provided with a transformer of suitable ratio (the ammeter scale is usually marked

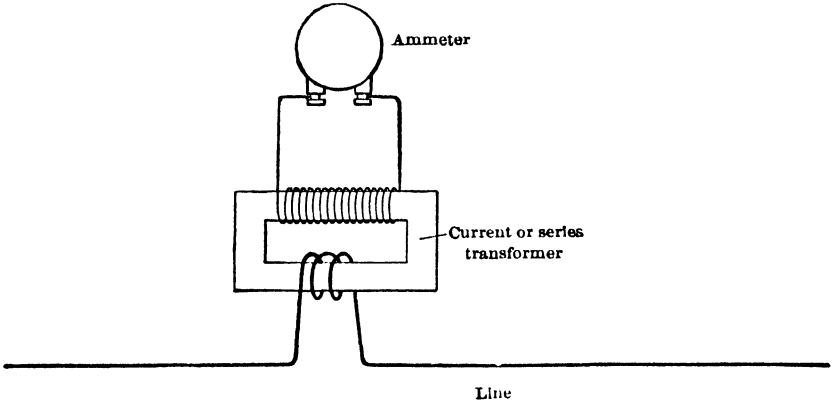


FIG. 55. An ammeter connected to an alternating-current circuit through a current transformer.

to read the true line amperes). (2) On a high-tension circuit it is much safer and more convenient to have the instrument itself insulated from the line. For further details regarding current transformers see Chapter XVIII.

A voltmeter is practically always connected to a high-tension a-c line through a so-called *potential* (or shunt) *transformer* (Fig. 56). Such a transformer also consists of a laminated iron core and two windings. The ratio of the voltages is very nearly equal to that of the numbers of turns. For example, if a 110-volt voltmeter is to be used on a 2200-volt line, the ratio of the numbers of turns must equal 20 to 1. The advantages of a shunt transformer are: (1) All voltmeters may be made for one range only, say 150 volts, and may be used for any desired voltage by choosing a suitable transformer. The voltmeter scale may be marked so as to read the line voltage directly. (2) It is more convenient and much safer to have the measuring instruments insulated from the high-tension line. (3) The instrument need not be near the line but may be in any convenient location. For details of the theory and methods of calibration of potential transformers see Chapter XVIII.

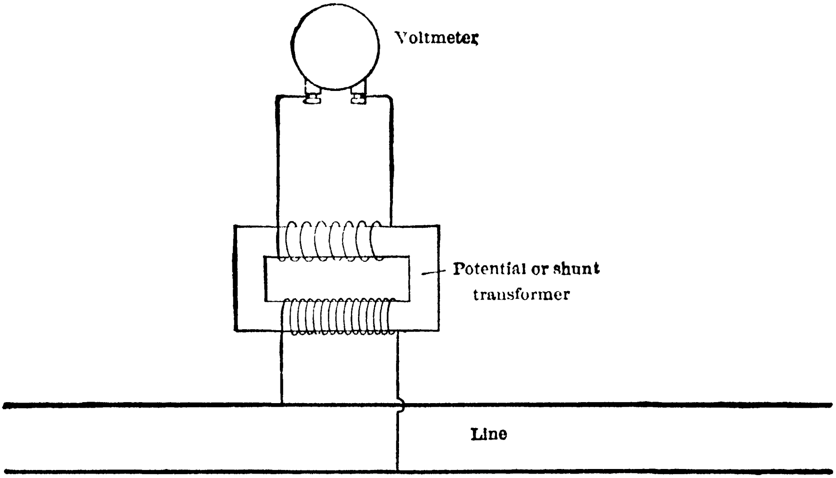


FIG. 56. A voltmeter connected to an alternating-current circuit through a potential transformer.

A separate low-tension winding, called the *tertiary* winding, is sometimes provided in a high-tension power transformer for the measurement of voltage, and a voltmeter is connected directly to this winding. This does away with a separate shunt transformer and its errors. If the numbers of turns in the primary and the tertiary windings are known, it is easy to compute the factor by which the voltmeter reading must be multiplied in order to obtain the primary voltage.

57. Requirements for Good Ammeters and Voltmeters. — The principal requirements for a good measuring instrument are:

(1) Permanency of calibration.

(2) Insensibility to stray magnetic fields.

(3) " Dead-beat " quality, i.e., the instrument should not swing too long before giving a definite indication.

(4) Suitable character and legibility of the scale.

(5) A high ratio of the deflecting torque to that of friction.

All these requirements are met to a larger or smaller degree in modern ammeters and voltmeters. However, the stricter the requirements, the more expensive the instrument becomes; it is therefore advisable not to demand greater accuracy than a specific case may require. The following remarks refer to the foregoing requirements:

(1) *The calibration* may be affected by a change of form or relative position of parts inside the instrument; by an increased friction in pivots; by aging of springs or of iron; by permanent magnets losing part of their magnetism.

(2) Some of the above-described types of instruments are inherently less affected by *external stray fields* than others, and all of them can be sufficiently protected by being inclosed in a steel case. Hot-wire instruments are practically unaffected by stray fields, since their action is not based on a magnetic effect; induction instruments are affected very little, since their air-gap is small and their own field quite strong. Electromagnetic instruments are affected the most, and should never be placed near dynamos, or on a switchboard within a loop of bus-bars and cables carrying strong currents. Electro-dynamometer-type instruments are affected by terrestrial magnetism when used on direct current; therefore in accurate measurements it is essential to reverse the current in the instrument and to take the average of two readings.

(3) *The " dead-beat " quality* is always desirable in an instrument, though usually it can be attained only by the addition of an extra damping device. An aluminum vane is quite frequently used for this purpose, the damping being attained either by the resistance of the air to the movement of the vane, or by placing the vane between the poles of a permanent magnet, so that any movement will induce eddy currents in it. Oil damping is used in a few instruments. Moving-coil permanent-magnet instruments are easily made " dead-beat " by making the frame of the movable coil of aluminum. Eddy currents induced in it by the permanent magnet are sufficient to make the instrument dead-

beat. (The word " aperiodic " is sometimes used instead of the expression " dead-beat.")

(4) Different types of instruments have a somewhat *different character of scale.* The scale of moving-coil instruments is usually perfectly uniform, all divisions from zero to full scale being equal (Fig. 37). On the other hand, hot-wire instruments have a non-uniform scale, divisions being crowded on the lower part of the scale (Fig. 48), so that the instrument can be read with fair accuracy at not less than 40 per cent of its full range. Sometimes the scale is suppressed on purpose in its lower part so as to have larger divisions in the useful range of the instrument. In instruments based on the hot-wire or dynamometer principle, the scale is of necessity non-uniform, since the deflecting force is proportional to the square of the current. A *uniform scale is by no means always desirable in switchboard service,* though it is very convenient in testing and for experimental purposes, since it increases the useful range of the instrument.

(5) The friction torque in a measuring instrument must be kept as low as possible by using high-grade sapphire jewels and highly polished pivots of hard steel. The absolute value of the friction itself is not a reliable guide to the sensitiveness of an instrument, but in a good instrument the friction torque must be a very small fraction of the actuating torque. In other words, a high-torque instrument may have a somewhat higher friction. The useful torque may be measured by holding the instrument with the axis of the moving element in a horizontal position and placing a small weight upon the otherwise counterbalanced needle. The weight is shifted until the desired deflection is obtained when the needle is horizontal. The torque is then equal to the product of the weight and its distance from the pivot. See also §120. Two competitive instruments may be compared on the basis of gram-centimeters of torque for full-scale deflection, although the instrument with a higher torque is not necessarily superior in all other respects.

58. EXPERIMENT 2-A. — Study of Ammeters. — The purpose of the experiment is to learn the construction, good qualities, and limitations of the principal types of instruments described in §§41 to 55. This should help the student to handle instruments properly in his future work, and to select intelligently the right instruments for a given purpose. The instrument to be investigated must be connected into a circuit; if necessary, another instrument, used as a standard, must be also connected into the same circuit, in order to observe the comparative behavior of the two. There are many points in regard to which two instruments of the same range and of different type may be compared to each other; the following are among the most important:

(1) Are there any such defects in the construction that the instrument cannot possibly have a permanent calibration?

(2) Is the instrument "dead-beat," and if not, how can it be made dead-beat in the most convenient way? The degree to which this quality is present is expressed through the "damping factor," defined in the Standardization Rules of the A.I.E.E. as the "ratio of the angular deviations of the pointer in two successive swings from the position of equilibrium." These deviations may be observed by suddenly applying to the meter a current which gives, say, 60 or 70 per cent of the full-scale deflection.

(3) If the instrument is absolutely dead-beat (aperiodic), note how long it takes the pointer to assume its final deflection. Also see to what degree the moving part is capable of following rapid fluctuations of current. This point is particularly important in hot-wire instruments; they are somewhat sluggish on fluctuating loads, because it takes a certain length of time for a wire to change its temperature. Characterize this sluggishness numerically, by varying the current up and down at a certain speed and by a certain percentage; observe the behavior of the instrument under test, as compared to that of the standard instrument.

(4) See if the pointer always returns to zero, or at least to the same point, when the circuit is opened; if not, investigate the cause. In some instruments a special screw, accessible from the outside, is provided for setting the pointer at zero; it is denoted by *S* in Fig. 48.

(5) Can the instrument be used indifferently in a vertical and horizontal position? Is the moving part sufficiently well balanced, so that small differences in the position of the instrument do not affect the calibration?

(6) How is the force produced which deflects the pointer from zero position, and what is the resisting force?

(7) Explain the character of the scale (uniform, crowded on one side, etc.) by the character of the moving and resisting forces. What changes should be made in the instrument to get a more uniform scale, or one that is still more suppressed on one side?

(8) Is the calibration affected by the temperature, and if so, how much? Is there in the instrument any compensation for the influence of temperature?

(9) If the instrument has a shunt, determine if the same has a negligible temperature coefficient, in other words, if the calibration of the instrument changes because of the heating of the shunt.

(10) Determine how far the instrument is affected by stray magnetic fields and a proximity of large iron masses. Does the iron case shield the instrument entirely from these influences, and does it not introduce an error by itself?

(11) Does the instrument show an appreciable hysteresis effect, so that indications on increasing and decreasing currents are different?

(12) Is the calibration the same on direct current as on alternating current, and if not, what is the percentage difference and the cause of the difference?

(13) Is the calibration affected by the wave-form of alternating current?

(14) What is the energy consumption in the instrument itself, and how does it compare with that of other competitive instruments of the same range?

(15) What is the absolute value of the resisting torque (in gram-centimeters) exerted by the spring or by some other restraining device at full-scale deflection? What is the friction torque of the instrument? Inspect the jewels and the pivots with a magnifying glass and report their condition.

Report. Describe the general arrangement of the test and the construction of the instruments used, and give the calibration curves. Also answer, for each instrument studied, the above 15 questions. Do not repeat the questions; simply refer to them by number.

59. EXPERIMENT 2-B. — Study of Voltmeters. — The experiment is performed in the same way as Experiment 2-A.

60. Polyphase Boards. — In many practical cases, especially in polyphase work, it is necessary to measure currents and voltages in more than one part of the circuit. At the same time it is not always possible or convenient to have a sufficient number of ammeters and voltmeters. In such cases a so-called polyphase board or table may be used, by means of which one ammeter and one voltmeter may be connected in succession to several independent circuits or parts of a circuit. The polyphase boards described below may be constructed for any number of circuits.

(a) *Plug-type board.* The polyphase table shown in Fig. 57 is very convenient for laboratory purposes. The line wires, in which it is desired to measure current (and power), are connected by plugging the leads into the sockets beneath the table. When the ammeter plug is out the circuit is closed from the upper socket through the spring contact to the lower socket. If it is desired to measure current, say, in line 1, the plug is inserted in the corresponding receptacles as far as it will go. The pin, shown at the bottom of the plug, enters the small hole and opens the spring contact, as shown by the dotted lines. This does not open the circuit for it is shunted between the sockets by the ammeter. When the plug is removed the spring closes the circuit before the meter circuit is opened. Thus the circuit is always closed.

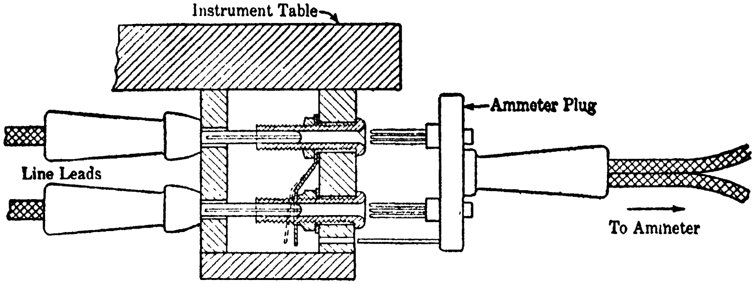


FIG. 57. A plugging arrangement for using one ammeter, one voltmeter, and one wattmeter in several circuits, without interrupting the main current.

By attaching voltmeter leads, one to the ammeter terminal and one, say, to the neutral wire, inserting the polyphase plug into any line automatically connects the voltmeter between that line and neutral. In power measurements the current winding of the wattmeter is connected in series with the ammeter and the potential winding in parallel with the voltmeter. Inserting the plug then automatically connects ammeter, wattmeter, and voltmeter in the required phases.

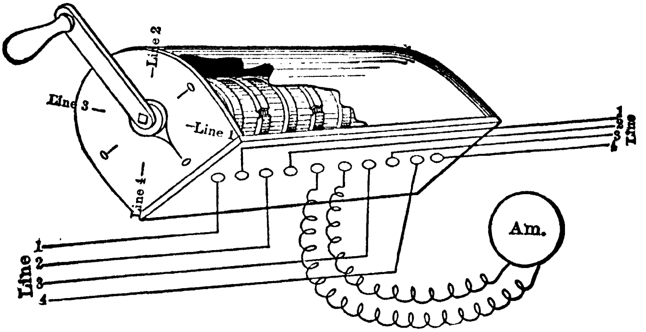


FIG. 58. A controller for connecting an ammeter and a wattmeter into several lines in succession, without opening the main circuit.

(b) *Controller-type board.* Another convenient device for the same purpose is shown in Fig. 58. It is intended for transferring an ammeter and the current winding of a wattmeter from one circuit to another. A separate switch must be used for transferring the voltmeter, as with the polyphase boards shown in Figs. 59 and 60. The device is an ordinary drum-type controller; the lines and the ammeter leads are connected to stationary fingers. The revolving drum is provided with

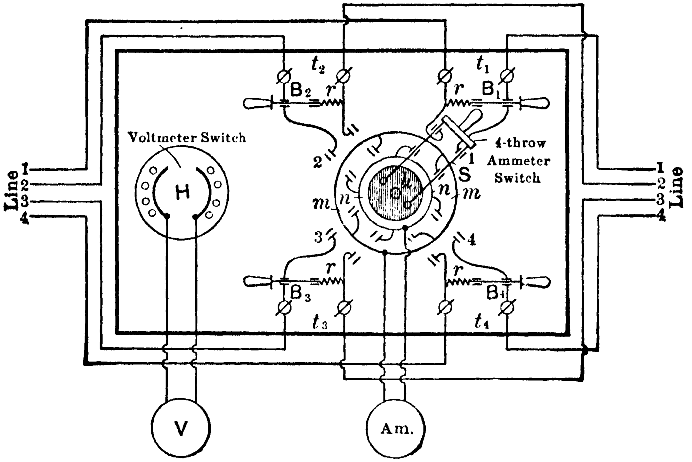


FIG. 59. A polyphase board with a multi-throw switch for metering currents.

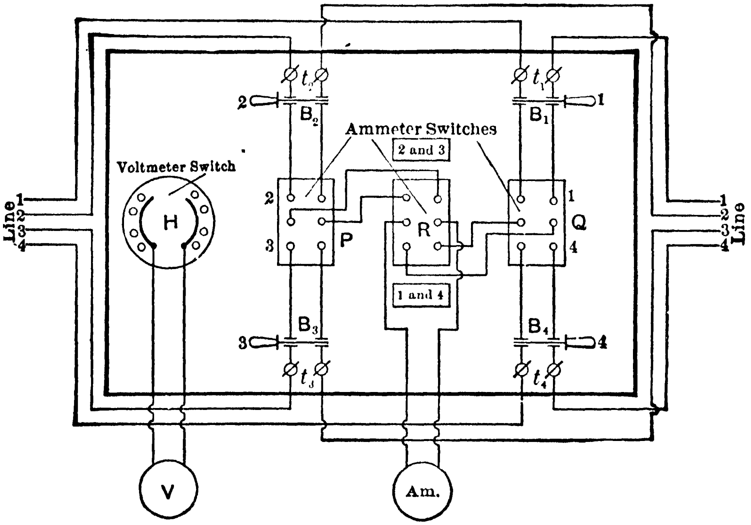


FIG. 60. A polyphase board with three double-pole double-throw switches for metering currents.

copper contacts which establish the required connections between any of the lines and the ammeter. When the pointer on the handle shows "line 1," the ammeter measures the current in this line; when the handle is in one of the " O " positions, all the lines are connected direct, and the ammeter is out of the circuit. This type of polyphase board is very convenient to handle, especially when readings must be taken in rapid succession.

(c) *Boards with knife-switches.* The boards shown in Figs. 59 and 60 are not so convenient in operation as the two devices described above, but they can be built with less difficulty, by using ordinary knife-switches. The ammeter and the voltmeter switches are entirely separate. The lines are connected to four pairs of terminals t_1, t_2, t_3, and t_4. When the switches B_1, B_2, B_3, and B_4 are closed, the currents flow directly through the lines, outside the ammeter. To read amperes in phase 1, throw the central radial switch S (Fig. 59) to position 1, and open the switch B_1. The current from the left-hand terminal, t_1, flows through the left-hand blade of S, connection n, ammeter Am, connection m, and the right-hand blade S to the right-hand terminal t_1, and to the line. Resistances r in series with the switches B are intended to compensate for the resistance of the ammeter and the leads, so that the total resistance of the line remains the same, whether the switch B is open or closed.

The polyphase board shown in Fig. 60 differs from that in Fig. 59 in that the connections are made by means of three ordinary double-pole double-throw switches P, Q, and R, instead of a special radial switch S.

RECORDING OR GRAPHIC INSTRUMENTS

61. It is often of importance to have a continuous record of the performance of electrical machinery, partly as data for economic and engineering calculations, partly as a check on station attendants, machine-tool operators, etc. Recording or curve-drawing ammeters, voltmeters, wattmeters, etc., are used for this purpose. The record is traced automatically on a strip of coördinate paper by a pen fastened to the end of the pointer of the instrument. The paper is moved at a constant speed by a clock mechanism.

The principle of action of almost any of the indicating instruments described in §§41 to 55 may be used in the construction of a graphic instrument, but much skill is required in providing the necessary force for overcoming the pen friction, without impairing the accuracy of the instrument. There are three methods of accomplishing this end:

(a) The instrument is so designed as to have a high torque (§62).

(b) The instrument acts merely as a relay for an independent source of power which moves the pen (§63).

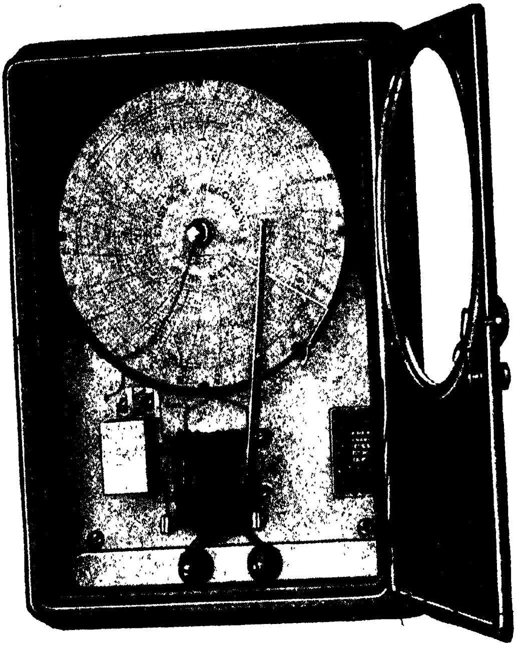


FIG. 61a. A Bristol round-chart recording ammeter.

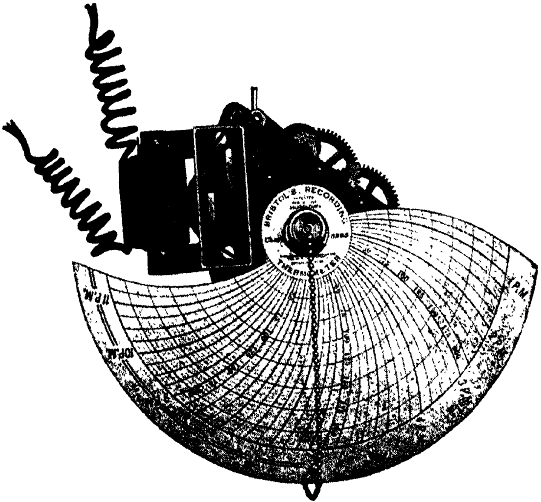


FIG. 61b. Telechron motor driving the movement of a recording meter.

(c) The record is made by means of an electric spark or beam of light, involving no friction (§64).

62. High-Torque Recording Instruments. — A Bristol recording ammeter is shown in Fig. 61*a*. The current to be recorded flows through the stationary coil. The magnetic field produced by the coil attracts the iron disk which rests on knife-edge supports. The supports have a lateral spring action, which opposes the movement of the disk. The motion of the disk is transmitted to the recording pen-arm, which traces a record on a sheet of coördinate paper, ruled as shown in Fig. 62. The paper is driven by the clock mechanism in the upper part of the instrument or by a small "telechron" motor (Fig. 61*b*). One revolution of the paper may be had in twenty-four hours, six hours or even one hour, as desired. A radius-averaging instrument, similar to a planimeter, is available for records of the type shown in Fig. 62. With such a device the average value of the current, voltage, etc., over a desired period of time may be readily obtained.

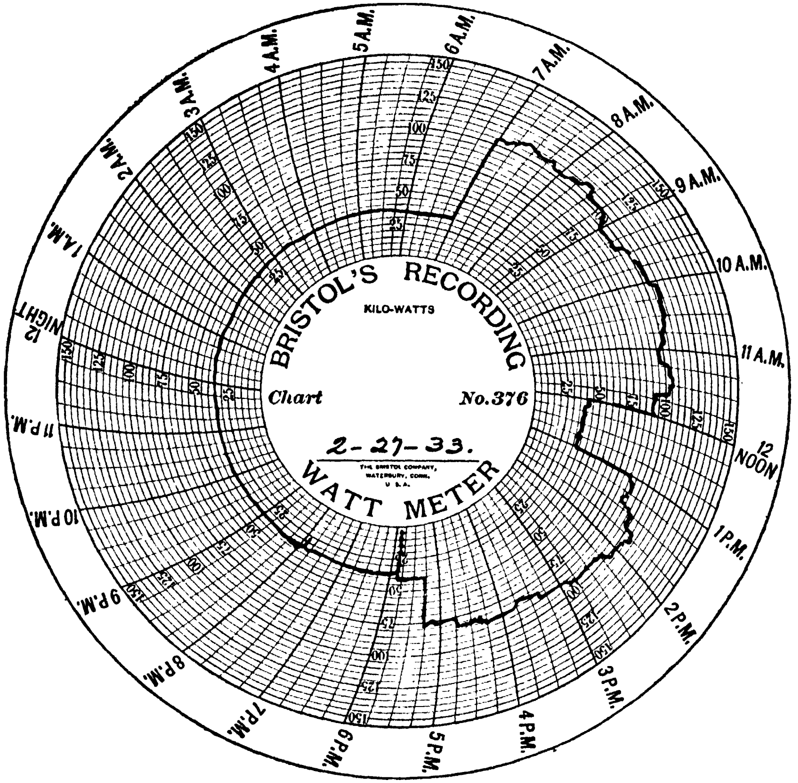


Fig. 62. A 24-hour record taken on a round-chart recording wattmeter.

In the recording instruments of the type shown in Fig. 63 (Esterline), a continuous record is obtained on a roll of paper driven by the clock shown to the left. The coördinate paper has straight abscissas but curved ordinates. The latter feature is not objectionable for most purposes and greatly simplifies the construction of the instrument. Curve-drawing instruments are available, however, in which the pen is made to move on a straight line, so that the record is obtained in rectangular coördinates (Fig. 66). A scale is provided on top so that the meter may be also used as an indicating instrument.

Direct-current ammeters and voltmeters of this type have the usual permanent-magnet movable coil movement (§44) shown in Fig. 64, except that it has to be designed for a much higher torque on account of pen friction. A stationary inkwell is attached to the meter element; the pointer consists of an alloy tube, terminating in a glass writing pen at the outer end. The inner end of the tube is bent downward and submerged in the ink. The pointer is flexibly supported and fitted with an adjustable counterweight, by means of which the pressure of the pen on the paper may be adjusted. The flow of ink through the pointer is due to capillary action, and is adjusted to the rate at which the ink is used by the pen. It is impossible for the ink to feed too rapidly and blot the record.

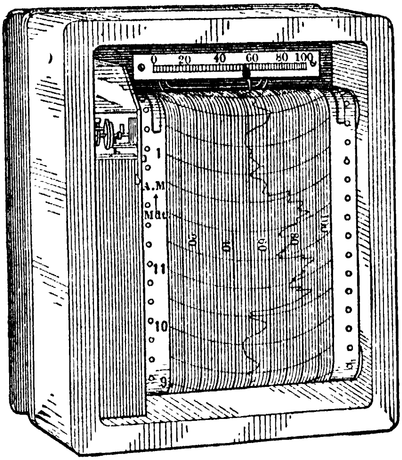


FIG. 63. An Esterline recording instrument.

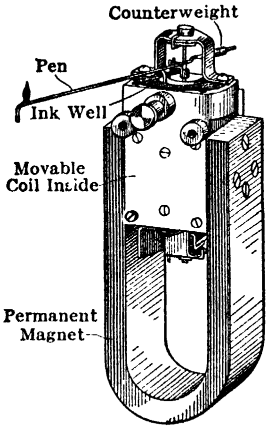


FIG. 64. A permanent-magnet, movable-coil type recording instrument.

For a-c ammeters and voltmeters (also for wattmeters, see Chapter IV), the electro-dynamometer-type movement is used (Fig. 65). The double element makes the instrument astatic, that is, insensitive to external fields, and the useful torque is doubled. No pivots or jewels are used, and the moving element is carried on a steel suspension wire. For damping, an aluminum vane is attached at the bottom of the movable part. This vane is immersed in the oil chamber shown at the bottom of Fig. 65.

The usual rate of paper feed in Esterline instruments is from 0.75 to 12 in. per hour; for recording rapid fluctuations a special clock drive may be provided, with speeds as high as 12 in. per minute. On account of the considerable friction pull on the pen, no great accuracy can be expected at such a high rate of feed. A dot record, instead of a continuous record, may also be obtained, when extremely high sensitivity and the smallest possible power consumption are required, as in recording temperatures, electrolytic voltages, etc. In this case the pen is counterweighted so as to rest normally above the chart, and an electromagnet, closed intermittently by the clock, draws the pen down, making a succession of dots (from 2 to 120 dots per minute).

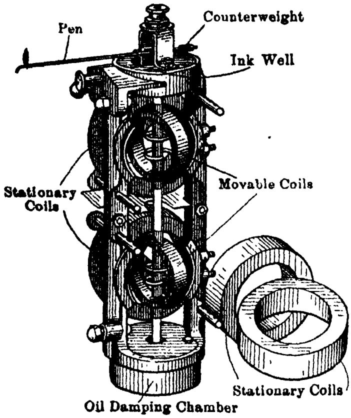


FIG. 65. An astatic electro-dynamometer used as a recording instrument.

63. Curve-Drawing Instruments on the Relay Principle. — The above-described recording instruments must be designed with a comparatively large torque, so that the pen friction will not impair the accuracy of indications. This necessitates considerable power consumption, and even then leaves the accuracy at small deflections somewhat indefinite. Therefore, in some graphic instruments the energy necessary to move the pen and to overcome its friction against the paper is supplied from an independent source. The energy of the indicated quantity merely closes one of the two relay contacts, which contact energizes a solenoid or a small motor and makes it move the pen in one direction or the other.

A relay-type curve-drawing voltmeter with the movement based on the Kelvin-balance principle (§85) is shown in Fig. 66. The details of connections are shown in Fig. 67. The instrument has four stationary coils and two movable coils between them, arranged exactly as in the Kelvin balance. The movable part is provided on the left side with a contact arm which closes one or the other of the two contacts between which it swings. The auxiliary source of direct current is represented schematically by a battery. The two solenoids, *CC*, which operate the pen, are connected in parallel, and each is connected to one of the abovementioned contacts. The condensers *KK* and the resistance *R* are for the purpose of reducing sparking at the contacts. The leverage *MM* of the pen is connected by the spiral spring *S* to the movable part of the voltmeter.

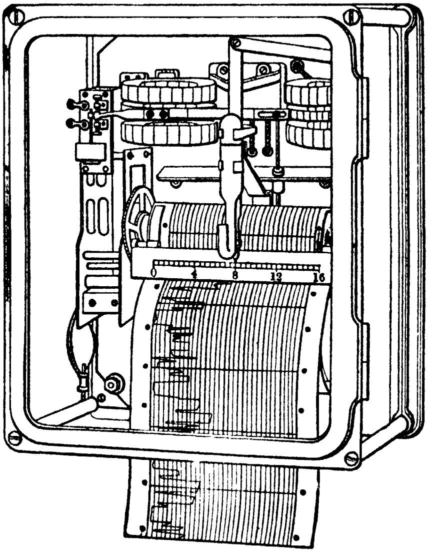


Fig. 66. A Kelvin-balance relay-type graphic voltmeter.

The six coils constituting the voltmeter itself are all connected in series across the circuit in which it is desired to record the voltage; *VR* is the series resistor. Let the position of the pen at a certain moment correspond exactly to the voltage of the line, and let both contacts be open. If now the line voltage drops, the spring *S* will overcome the attraction between the movable and the stationary coils and will close the upper contact. A current then flows from the battery into the left solenoid *C*, a pull is exerted on the plunger *P*, and the pen begins to move to the left, tracing a record. The movement of the pen reduces the tension of the spring *S*; when the pen arrives at the correct point, the electromagnetic pull between the stationary and the moving coils of the voltmeter becomes just sufficient to overcome the tension of the spring, and the contact is broken. The pen remains in this position until a new change in voltage occurs.

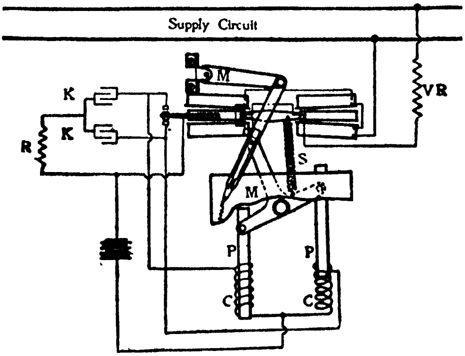


Fig. 67. A diagram of connections in the recording voltmeter shown in Fig. 66.

In reality the pen follows small variations in voltage almost instantaneously. The pen is made of glass and is provided with a reservoir of a sufficient size to hold ink for a continuous record of two months. The quickness of motion of the pen is regulated by two dash-pots connected to the moving coils. The sensitiveness of the record may also be varied by adjusting the distance between the contacts. The rate of feed of paper may be varied within certain limits.

In some more recent instruments of the relay type, the pen carriage is mounted on a nut which can move on a threaded shaft. The shaft is driven by a small electric motor which takes the place of the solenoid magnet, so that the straight-line motion is obtained directly. A complete line of such instruments is available, comprising ammeters, voltmeters, wattmeters, power-factor meters, frequency meters, etc. In all these instruments the clock mechanism which drives the paper roll is made electromagnetically self-winding and requires no winding by hand whatever, or else the clock is driven by a small synchronous motor.

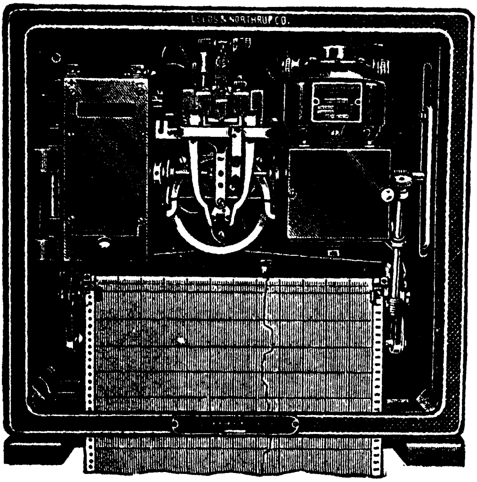


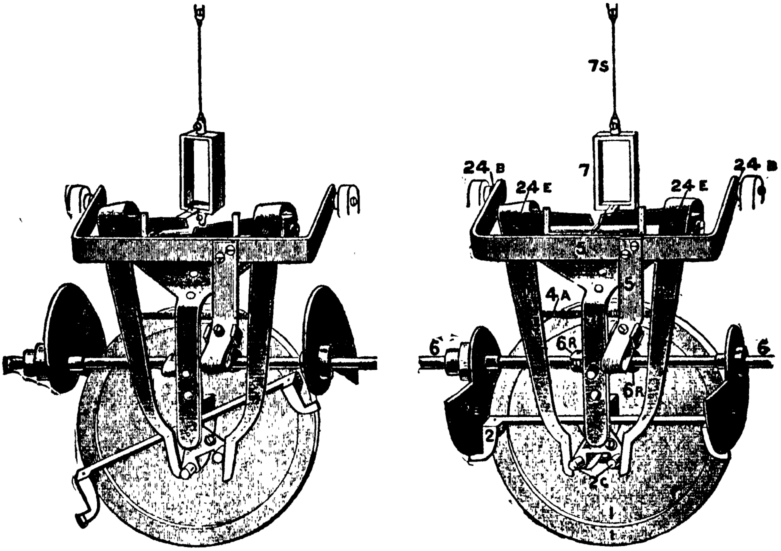
Fig. 68. Self-balancing recording Wheatstone bridge. (Leeds & Northrup Co.)

Among the most ingenious of the relay types of recording meter are the self-balancing Wheatstone and Kelvin bridges, also potentiometers, as manufactured by the Leeds & Northrup Company. In these instruments, when the galvanometer needle swings either way from zero (bridge balanced) this position of the needle causes the automatic balancing device to turn a disk bearing the slide-wire balancing resistance, thus bringing the bridge back to balance. The recording mechanism — a pen bearing upon a ruled chart — records the position of the contact on the slide-wire and so the value being measured (resistance, temperature, etc.). Figure 68 shows such a self-balancing Wheatstone bridge which records the temperature of a winding as obtained from its change in resistance (§7). Figure 69, *a* and *b* exhibits the self-balancing mechanism of the recorder.

64. Recording Instruments for Rapidly Fluctuating Currents or Voltages. — The recording instruments described above are not suitable for accurately recording rapidly fluctuating currents or voltages which sometimes occur in practice or in research work. Special instruments have been built for such cases; among these the following may be mentioned:

(a) A dotted record is obtained with an electromagnet energized with an interrupted current (see the end of §62).

(b) An ordinary indicating instrument is used and its indications are followed as accurately as possible by the observer with a second indicating pointer. This pointer is connected mechanically to the recording pen or stylus, which traces a curve on a moving paper roll or on a smoked cylinder.



(a) (b)

FIG. 69. The balancing mechanism of the meter in Fig. 68. (*a*) Meter in process of rebalancing, (*b*) Meter balanced.

(c) A continuous electric spark is made to play between the indicating pointer and a metal support over which the paper passes. Such a discharge may be easily produced by means of a small induction coil and an interrupter. The spark makes a fine burned line on the paper, without the pointer touching it.

(d) A small mirror may be attached to the movable part of an indicating instrument, as in a reflecting galvanometer, and a record of its movements obtained on a roll of sensitized paper by a beam of light. Such an arrangement resembles an oscillograph (Vol. II).

65. EXPERIMENT 2-C. — Study of Recording Instruments. — Different types of recording instruments are described in §§61 to 64.

The points to be investigated are as follows:

(1) Construction of the indicating and recording parts.

(2) Promptness of response to sudden fluctuations of current or voltage.

(3) Pen friction and its influence on the sensitiveness of the instrument.

(4) Character of the scale of the record.

Report. State your findings and give your opinion of the good points and shortcomings of the instrument. Explain how you would compute the average and the effective values of the current or voltage, if the record is plotted to curved coördinates as in Fig. 62.

Note. The ratio between the amplitude and the effective value of a sinusoidal alternating current or voltage may be deduced as follows: The square of the effective value is equal to the average of the squares of the instantaneous values over a cycle. Let a quantity, a, vary with the time, or with some other independent variable, x, according to the sine law Then

$$a = A_m \sin x \quad \text{. . . . .} \quad (12)$$

where A_m is the amplitude (maximum value) of a. According to the foregoing definition, we have for the effective value, A, of the variable a:

$$A^2 = A_m{}^2 \operatorname{ave} (\sin^2 x)\Big|_0^{2\pi} \quad \text{. . . . .} \quad (13)$$

But

$$\operatorname{ave} (\sin^2 x)\Big|_0^{2\pi} = \operatorname{ave} (\cos^2 x)\Big|_0^{2\pi} \quad \text{. . . . . . .} \quad (14)$$

because sin x and cos x pass through the same range of values over a cycle. Hence

$$A^2 = \tfrac{1}{2}A_m{}^2\Big[\operatorname{ave} (\sin^2 x + \cos^2 x)\Big]_0^{2\pi} = \tfrac{1}{2}A_m{}^2 \quad \text{. . . .} \quad (15)$$

and consequently

$$A = A_m/\sqrt{2} \quad \text{. . . . .} \quad (16)$$

CHAPTER III

AMMETERS AND VOLTMETERS — CALIBRATION

66. Primary and Secondary Standards. — The absolute values of the ampere, volt, and ohm are established by an international agreement, and primary standards of these quantities are maintained by the respective governments — in this country by the Bureau of Standards, Washington, D. C.

It is not necessary to have primary standards of all the three quantities — the volt, the ampere, and the ohm. According to Ohm's law, when two of these are known, the third is represented by their product or ratio. The unit of resistance, or the international ohm, is officially defined as being represented by the resistance of a column of mercury of a certain length and mass, and at a specified temperature. (See §1.) The international ampere is defined as the unvarying current which deposits in 1 second of time 0.001118 gram of silver out of a solution of nitrate of silver, under certain definite conditions. As a consequence, the international volt is the emf which, being applied at the terminals of a standard ohm, produces in it a current equal to 1 ampere. For details regarding these units see the publications of the Bureau of Standards or an electrical handbook.

The mercury ohm and the electrochemical ampere are hardly suitable for practical purposes. Secondary standards are therefore used in practice, calibrated by means of these. Such secondary standards are: Resistances made of wire or strip, and a standard electrochemical cell (§74) of a known emf. Current is determined either as the ratio of volts to ohms, or by means of a standard ampere balance (§92).

The calibration of ammeters and voltmeters with a standard cell requires rather delicate and complicated arrangements involving the use of a potentiometer and allows of an accuracy not needed for many practical purposes. In places where many instruments must be calibrated in a comparatively short time, and where extreme accuracy is not required, it is customary to use good ammeters and voltmeters as intermediate standards. They are checked from time to time with the primary standards kept in the laboratory. We shall first describe simple calibrations by means of intermediate standards, and then give the methods of checking the standard instruments themselves.

67. Calibration of D-C Ammeters with a Standard Millivoltmeter and a Shunt. — Standard d-c instruments are invariably of the permanent-magnet moving-coil type, because of the accuracy and sensitiveness of this construction. As is explained in §49, an ammeter based on this principle is in reality a millivoltmeter which measures the voltage drop across a shunt (Fig. 43). Therefore, a standard millivoltmeter and a set of standardized shunts covering the required range of currents are all that is necessary for calibrating a d-c ammeter (Fig. 70). The ammeter or ammeters to be calibrated are connected to a steady source of direct current (usually a storage battery) in series with a standard shunt and the regulating resistance *R*. A standard millivoltmeter is connected across the terminals of the shunt. The current is adjusted to a desired value, and the ammeters and the millivoltmeter are read simultaneously. The current is then changed, new readings are taken, etc.

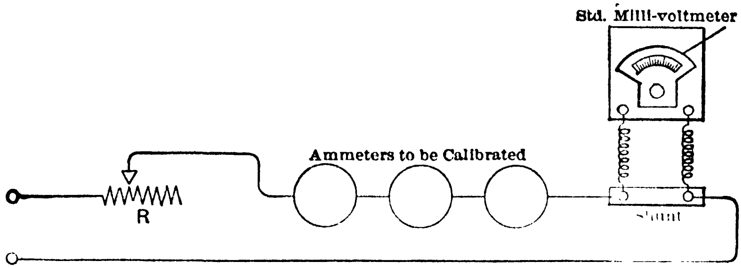


FIG. 70. Calibration of ammeters with a standard shunt and a standard millivoltmeter.

A standard millivoltmeter for accurate calibration resembles the standard voltmeter shown in Fig. 71. It is similar to ordinary portable instruments based on the d'Arsonval principle (§44), but is several times larger and has a scale shown in Fig. 72. With this scale, parts of divisions may be read much more accurately than with an ordinary scale, such, for example, as in Fig. 37. The scale is provided with a mirror. When readings are being taken, the observer's eye must be in such a position that the end of the pointer is seen to cover its image in the mirror, to avoid error of parallax. The calibration of the instrument depends to a slight degree upon its temperature; a thermometer is provided in the base of the instrument so that the necessary correction may be taken into account.

The results of the calibration are given in the form of a table; or a curve is plotted giving true amperes vs. actual readings as abscissas. Some prefer to plot calibration curves in the form shown in Fig. 73; instead of true readings, corrections only are given, in scale divisions or in percentages.

If a wide range of amperes is to be covered, one shunt is not sufficient. Suppose, for example, that the millivoltmeter, when connected

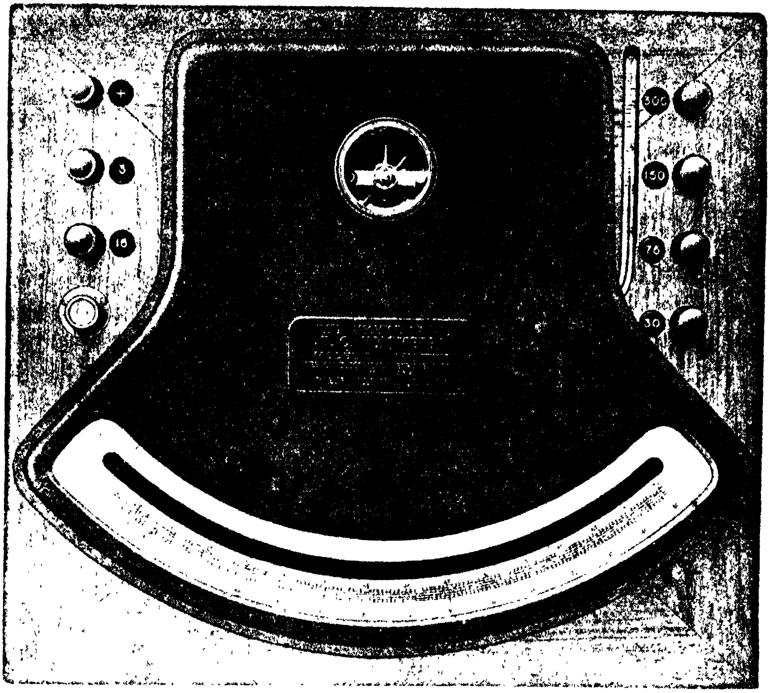


Fig. 71. A standard semi-portable voltmeter.

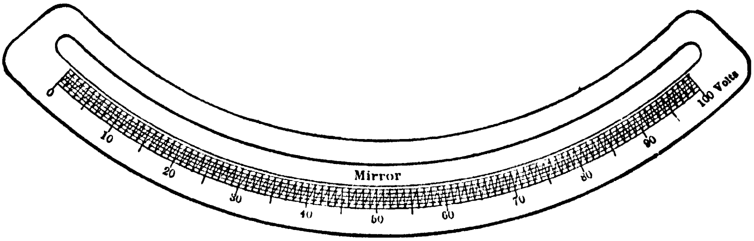


Fig. 72. The precision scale used in standard instruments.

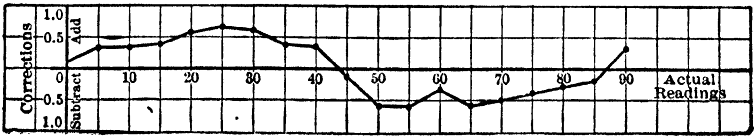


Fig. 73. Calibration curve of an instrument.

across a certain shunt, gives a full-scale deflection with a current of 1000 amperes flowing through the shunt. With 100 amperes the deflection will be only 10 per cent of the scale; it is hardly advisable to go below this, as the accuracy of the readings is impaired. Thus, below 100 amperes another shunt of ten times the resistance is required; it will give a full-scale deflection at 100 amperes and may be used down to 10 amperes. The next shunt will serve from 10 to 1 amperes, etc.

Not only ammeters, but also millivoltmeters and shunts, may be calibrated in a similar way. (See also §83.) To calibrate a shunt, connect it in series with a standard shunt (Fig. 70) and measure the voltage drop with a calibrated millivoltmeter, first across the standard shunt, and then across the shunt to be calibrated. If the line current is constant, the ratio of the readings will give the ratio of the resistances of the two shunts. An ammeter should be kept in the circuit in order that the observer may be sure that the current remains unchanged between readings. It is well to use a double-throw switch with mercury contacts when changing the millivoltmeter from one shunt to the other; ordinary knife-switches have an appreciable contact resistance which may vitiate the results.

To calibrate a millivoltmeter, connect it across a shunt in parallel with a standard millivoltmeter and take simultaneous readings on both instruments.

68. EXPERIMENT 3-A. — Calibration of D-C Ammeters with a Millivoltmeter and a Shunt. — The method is explained in the preceding section. Connect one or more ammeters as in Fig. 70 and calibrate them throughout their range, starting the readings at about 1/10 of full scale and increasing by equal steps. Always bring the needle of the test meter exactly to the scale division and read the standard. Use a reading glass with each meter. Now calibrate a shunt; find its temperature correction, if any. Calibrate a millivoltmeter; see if the length of the leads and the presence of a switch in the circuit influence the indications.

Report some of the results in the form of curves showing true amperes to actual readings as abscissas; other results in the form of correction curves, as in Fig. 73. From which form of curve is the corrected value obtained more quickly? more accurately?

69. Calibration of D-C Voltmeters with a Standard Voltmeter. — The arrangement of the instruments is shown in Fig. 74; the voltmeters V_1, V_2, V_3, etc., are connected in parallel between two bus-bars *A-m* and *b-n*. The terminals *a* and *c* of a high-resistance potentiometer-type rheostat are connected across the source of supply; by means of the sliding arm, H, connected to b, any part of the total line voltage may be applied at the terminals of the voltmeters. When H is in position a, the pressure between m and n is 0. When H is in position c, the total line voltage is applied between m and n. One of the voltmeters is a standard instrument (Fig. 71) with which the others are to be compared. An ordinary embedded-type field rheostat is convenient for use as a voltage regulator. It usually has only two terminals, a and b; it is easy to solder a third terminal c.

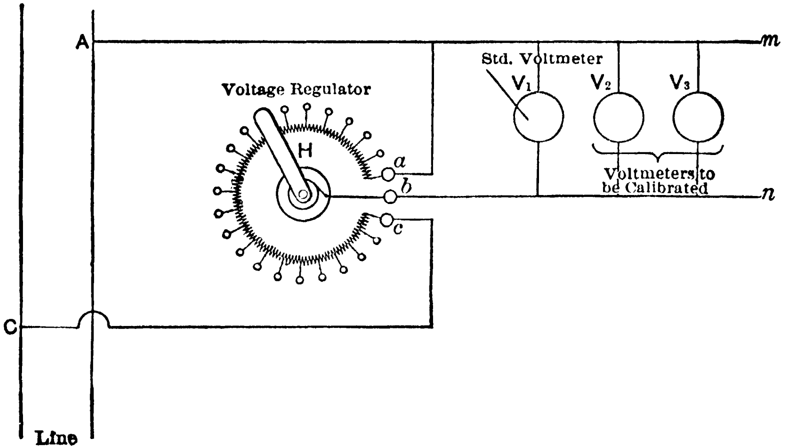


FIG. 74. Calibration of voltmeters.

70. EXPERIMENT 3-B. — Calibration of D-C Voltmeters with a Standard Voltmeter. — The general procedure is explained in §69; the diagram of connections is shown in Fig. 74. Calibrate a voltmeter with one or more different multipliers. Find out if the instrument itself or the multipliers have an appreciable temperature coefficient. Plot calibration curves as specified in §68.

71. Calibration of an Alternating-Current Ammeter with an Intermediate D-C-A-C Standard. — A hot-wire instrument (§52) may be calibrated with direct-current (Fig. 70) and used with alternating currents. The same is true of an electro-dynamometer-type ammeter (§51) provided that the precaution is observed in regard to terrestrial magnetism and stray fields.

A movable-iron ammeter (§41) or an induction-type instrument (§54) must be calibrated with a current of the frequency for which the instrument is intended to be used, and of approximately the same wave-form. A precision electro-dynamometer-type instrument or a Kelvin balance (§92) may be used as such a standard. If neither is available, the instrument may be calibrated with a d-c standard, using an intermediate or transfer instrument (Fig. 75). A hot-wire or a thermoelectric instrument (§§52 and 53) is especially well adapted for such service, or an electro-dynamometer can be employed which may not be good enough as a primary standard. The double-pole double-throw switch S is first thrown to the right and the intermediate standard C is read simultaneously with the instrument B to be calibrated. Then the switch is thrown to the left and the current is adjusted by means of the rheostat R until the transfer instrument C reads the same as before. Then the indication of the standard ammeter A is the correct current for the previously noted reading of B.

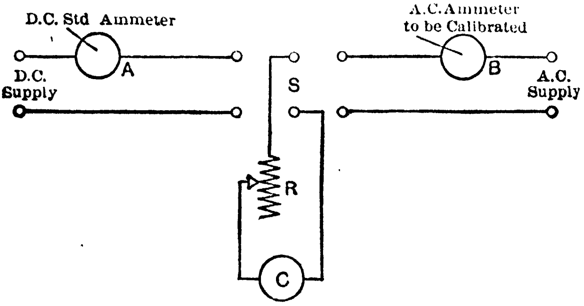


FIG. 75. Calibration of an alternating-current ammeter by means of an intermediate A-C–D-C instrument.

72. EXPERIMENT 3-C. — Calibration of an A-C Ammeter with an Intermediate D-C–A-C Standard. — The method is described in the preceding section, and the connections are shown in Fig. 75. Begin the readings at about 1/10 of full-scale and increase in equal steps to the highest current which all three instruments can carry. Plot the results as shown in Fig. 73.

73. EXPERIMENT 3-D. — Calibration of an A-C Voltmeter with an Intermediate D-C–A-C Standard. — The arrangement of the test is similar to that described in §71, except that the three instruments are in this case connected in parallel, as in Fig. 74. It is left to the student to devise a convenient diagram of connections, taking Fig. 75 as a guide. Plot the results as shown in Fig. 73.

74. Standard Cell. — Standard voltmeters and millivoltmeters used for calibration, as described in §§67 and 69, are sufficiently accurate for most practical purposes, provided they are periodically compared to a primary standard of emf. At the present writing, the Weston cadmium cell, which is a primary cell of very accurately known emf, is used as such a standard. In calibrating an instrument by means of a standard cell, a potentiometer (§75) is used, because the standard cell must only balance the voltage across a part of the main circuit, and must not be allowed to deliver anything but a very minute current. The construction and chemical composition of the cadmium cell are shown in Fig. 76.

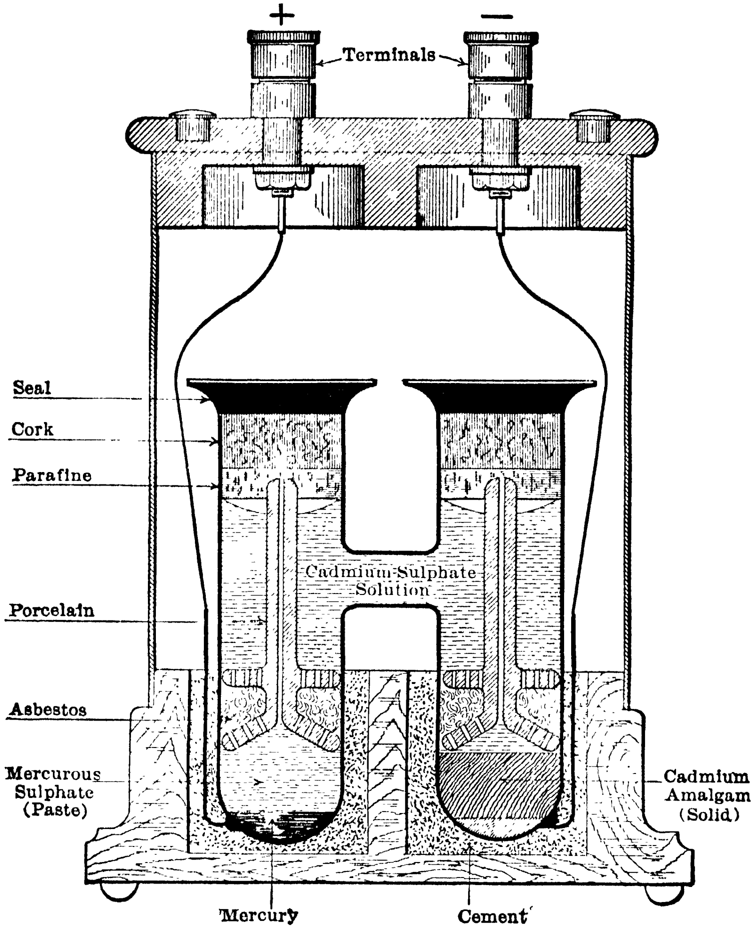


Fig. 76. Cross-section of a Weston standard cell.

There are two types of cadmium cell. The so-called *saturated form* (or Weston normal cell) contains solid crystals of cadmium sulphate in the solution. The *unsaturated form* (or Weston standard cell) contains cadmium sulphate solution saturated at 4° C. The saturated cell is used as the international standard the world over, and its emf is accepted as 1.01830 volts at 20° C. Unfortunately, it has an appreciable temperature coefficient and for this reason is hardly ever used for general purposes. The unsaturated form has a practically constant emf within the usual range of temperature met with in practice, and for this reason is generally employed, even in accurate scientific research.

Each cell is provided by its maker with a certificate giving its exact emf, and it should be sent periodically to the Bureau of Standards for certification. Minor improvements have been introduced in the construction of the cell from time to time. For leading-in wires, tungsten has been found to be an acceptable substitute for platinum; a harder and less soluble glass has been substituted as the material for the container, etc. For these reasons it is important to use, for each individual cell, the particular value of the emf found by comparison with a certified cell.

The two important precautions to be observed with a standard cell are as follows: (1) In preliminary measurements, never close its circuit on a resistance of less than 10,000 ohms, preferably more. Even a minute current of 1/10 milliampere polarizes the cell, and it is doubtful if it ever sufficiently recovers to be used again as a reliable standard. (2) Never subject the cell to a temperature below 4° C. or above 40° C. It may take days and even weeks before its emf at the usual room temperature again reaches the normal constant value.

DIRECT-CURRENT POTENTIOMETERS

75. Principle of the Potentiometer. — In a circuit involving the potentiometer principle a known fractional part of some steady supply voltage is balanced against an unknown emf, with a view to determining the value of the latter. Thus, in Fig. 77 a steady source of emf *W* sup-

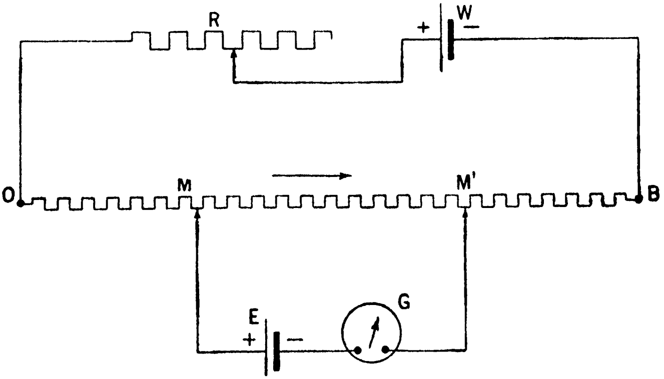


FIG. 77. Circuit illustrating the potentiometer principle.

plies an unvarying current through the slide-wire resistor OB having a graduated scale. The unknown emf E is applied across a variable portion MM' of OB, care being taken that the polarity is such that the voltage drop between M and M' opposes E. Zero deflection of the galvanometer in series with E shows when the drop $MM' = E$. If now E is replaced by a standard cell of known emf and a new balance obtained, the total drop OB will be determined and the fractional part MM' can be calculated. From the above it follows that a potentiometer is a piece of electrical apparatus by means of which direct currents and voltages can be measured to a high degree of precision, by means of a standard cell (§74) as the known emf. The name potentiometer has been extended to devices for measuring alternating currents and voltages (§91) on a similar principle, even though no standard cell is used there. The potentiometer has a wide application for calibration of ammeters, voltmeters, resistances, etc., used as secondary standards. With a potentiometer, voltages from one-millionth of a volt to several thousand volts can be accurately compared to the emf of a standard cell, and currents measured from a small fraction of an ampere to many thousand amperes.

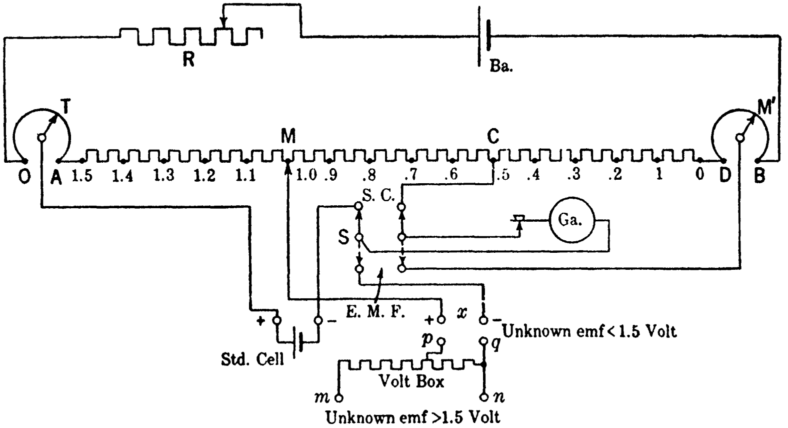


FIG. 78. Typical diagram of potentiometer circuits.

The ordinary potentiometer connections are shown in Fig. 78; for a clear understanding of the principle involved the student should keep in mind two points:

(a) The standard cell balances the unknown voltage, *but it is not used as a source of current.*

(b) The " zero " method is used, so that it is not necessary to know the galvanometer constant.

An adjustable resistor *OB* of the dial type is connected in series with one or two storage cells *Ba* and a regulating rheostat *R*; a constant current is thus established in *OB*. Any desired fraction of the total voltage drop may be had between the contacts *M* and *M'* by moving them along the resistor, or by regulating the rheostat *R*. The contact *M* is for coarse regulation, *M'* for fine regulation; contact *C* is fixed. The drop between *C* and *T* is balanced against the standard cell and that across *M* and *M'* against the unknown emf. A balance is recognized when the galvanometer needle remains at zero. The ratio of the resistance from *M* to *M'* to that from *C* to *T* is equal to the ratio of the emf's under comparison, provided the current in *OB* remains constant, which can be checked by throwing the switch *S* upward and again checking the drop between *C* and *T* against the emf of the standard cell. In case these emf's do not balance, *R* is varied to restore balance, and the measurements are repeated. By means of the double-throw switch *S* the connections are conveniently changed from the standard cell to the unknown emf and back.

In order to make the arrangement direct-reading, the contacts *C* and *T* are set beforehand so as to balance the voltage of the standard cell at the proper divisions of the scale — in case of the Weston cadmium cell at about 1.0183 volts. (See §74.) The exact setting depends upon the value of the emf of the cell used, and this emf must be known. The switch *S* is thrown upward, and the rheostat *R* adjusted until the galvanometer returns to zero. Then the switch is thrown downward, and the galvanometer balance again obtained by shifting *M* and *M'*.

The potentiometer scale usually has a range of about 1.5 volts so that only voltages below this limit can be directly compared to that of a standard cell. For voltages above 1.5 volts, a multiplier, or the so-called "volt-box," is used, shown at the bottom of the sketch. It consists of a high resistance with one or more taps at a known part of it, say 1/10, 1/100, etc. The unknown emf is connected across the terminals *m* and *n*; the terminals *p* and *q* are connected to the "unknown emf" terminals of the potentiometer. For example, let the unknown voltage be about 110 volts and the resistance between *p* and *q* be 1/100 of the total resistance of the volt-box. Then the voltage across *pq* is only about 1.1 volts, and it can be directly compared to the emf of the standard cell. Multiplying the result by 100, we get the actual unknown voltage.

76. EXPERIMENT 3-E. — Study of a Simple Potentiometer. — Before attempting to use the very sensitive and somewhat delicate potentiometer described in principle in §75 it is desirable for the student to make clear to himself the working principles of the potentiometer by means of a somewhat simpler equipment. Replace the graduated resistances and slide-wires of Fig. 77 by a simple slide-wire on a graduated scale (Fig. 13), preferably 1 meter long. Contacts *M*, *M'*, *C*, and *T* may be replaced by voltmeter leads having knife-edge terminals. In place of the standard cell a dry cell of known emf may be used.

(a) Connect the circuits as shown in Fig. 77 and practice measuring an emf small enough to be within the range of the slide-wire.

(b) Arrange the connections for measuring amperes by means of a standard shunt (Fig. 70); use the potentiometer in place of a millivoltmeter.

(c) Compare two resistances by measuring the drop at their terminals (Fig. 2).

(d) Finally, improvise a volt-box and make clear to yourself the method of measuring voltages beyond the range of the slide-wire.

77. Leeds & Northrup Type-K Potentiometer. — A convenient form of potentiometer is the Leeds & Northrup Type K shown in Fig. 79. The actual connections are given in Fig. 80, the same notation being employed as in Fig. 78. The larger portion of the resistance, *AD*, is replaced by fifteen 5-ohm coils in series, with the terminals each representing a drop of 0.1 volt, and connected to a series of contact buttons. The dial and the handle corresponding to contact *M* are visible in Fig. 79 near the center of the panel. The slide-wire resistor between *D* and *B* is wound on a Bakelite cylinder and has eleven turns totaling some 16 ft in length with a resistance which is made exactly 1.1 times that of one coil between *A* and *D*. Thus the total voltage drop across the slide-wire is 0.11 volt, and each turn corresponds to 0.01 volt. The turns of the drum are read on the glass scale shown in the figure, and fractions of a turn by the 200 divisions on the periphery of the drum. Thus the divisions of this scale each represent 0.000 05 volt, which can be further estimated to 0.000 01 volt.

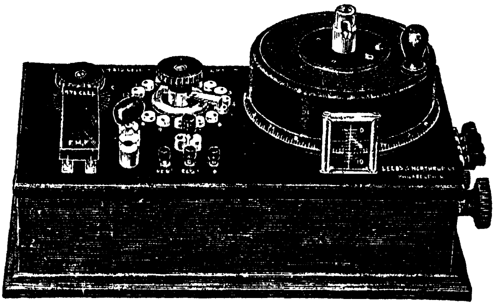


FIG. 79. Leeds & Northrup Type-K potentiometer.

The regulating rheostat *P* is mounted within the potentiometer case; the current control handles and binding posts are shown at the right end of the case. Note that no contacts are interposed in the path of the current, in the calibrated resistance between *O* and *B*.

The normal range of the potentiometer is 1.61 volts, but this may be changed to 1/10 this value for the measurement of very small emf's by introducing the shunt marked 10 : 1; this is done by changing the plug shown at the left, from hole 1 to that marked 0.1. The resistance of the shunt *S* (Fig. 80) and that of *OB* bear such a ratio that when the shunt is applied only 1/10 the former current flows through *OB*, hence the voltage drop across *OB* is but 1/10 of its previous value. The resistance *K* which is automatically inserted when the ratio plug is moved has such a value that the working current of the potentiometer remains unchanged. This gives the instrument a range of 0.161 volt to zero, in steps of 0.000 005 volt, or 1/200 millivolt, by actual scale readings, or to 1/1000 millivolt (1 microvolt) by estimation.

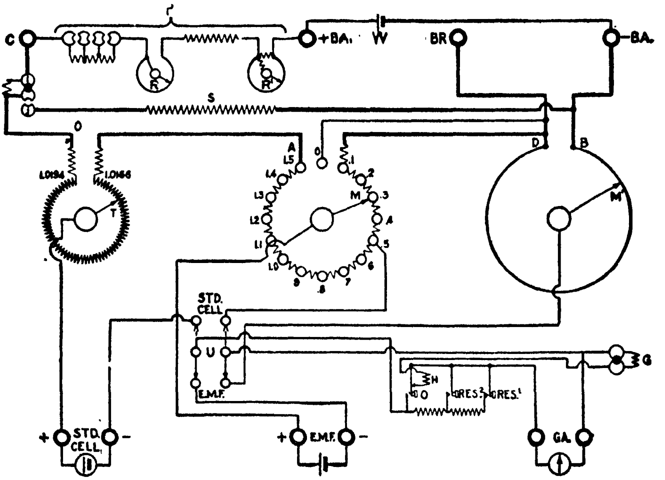


FIG. 80. The circuit diagram of the Type-K potentiometer.

The connections to the standard cell and to the unknown emf are similar to those shown in Fig. 78. An external galvanometer is connected to the binding posts marked *GA*. There are three keys marked "Res¹," "Res²," and "*O*" in the galvanometer circuit; with the first two keys a protective resistance is connected in series with the galvanometer. This is to prevent violent deflections of the galvanometer before a balance is obtained, and also to protect the standard cell against excessive currents which might polarize it. The resistances marked *G* and *H* are for the purpose of damping the galvanometer when key *O* is released.

Contacts M and M' are used for obtaining balance with the unknown emf only, and not when adjusting the current — this being done by means of the rheostat P. It will be seen from the diagram of Fig. 80 that the standard cell is connected between the point 0.5 of the potentiometer resistance and the sliding contact on the dial to the left, marked T (temperature). The emf of the Weston cell being about 1.018 volts, the resistances connected to this dial permit the operator to set the potentiometer exactly at the certified emf of the cell, taking into account small variations due to change in temperature, or any other cause (§74).

When switch U is thrown *up* the balance is obtained by regulating P, *with any position of M and M'.* To evaluate the unknown emf the switch U is thrown *down* and a new balance obtained by moving M and M'. In the meantime the battery current may have changed, a condition which may be checked by throwing switch U *up* and pressing the galvanometer button. If the bridge is still in balance the setting of M and M' gives the value of the unknown emf.

78. Checking Potentiometer Resistances. — Provision is made in the potentiometer shown in Fig. 79 for checking the resistances of the dial and of the slide-wire. The coils in the series AD are 5 ohms each and are connected to brass blocks with reamed holes (Fig. 79). A pair of flexible cords is furnished, with taper plug terminals to fit the reamed holes. The resistances of these coils can be measured with an ordinary Wheatstone bridge and thus compared with each other to a high degree of accuracy, even if the bridge is not very accurate. For potentiometer work the essential point is that they should be like each other, not that they should be accurately of any particular value.

The calibration of the extended wire DB on the cylinder may be checked against the dial resistance AD by converting the potentiometer itself into a Wheatstone bridge. To do this, short-circuit the binding posts marked $-Ba$ and C (Fig. 80) with a heavy copper connector, not smaller than No. 10 B & S; also short-circuit the terminals provided for the unknown emf and throw switch U down. Connect the battery, plus a protective resistance, between the terminals marked $-Ba$ and "Bridge." The contacts M and M' form junctions of the two arms on each side of the bridge with the galvanometer between. On one side there are 16 equal resistances of 5 ohms each, the resistance of the temperature dial being supplemented with an extra resistance, so as to make it exactly 5 ohms. If the extended wire be read in its 1000th parts, each of the 16 coils of the potentiometer circuit will be equal to 1/16 of 1000, or 62.5 divisions when balanced against the wire.

79. Directions for Using Leeds & Northrup Type-K Potentiometer. —Because of the extended use of the above-described potentiometer, it has been deemed advisable to reprint here the explicit directions for its use, as given by the makers.

Setting up. — All connections must be made as indicated by the stamping on the potentiometer. Particular attention must be given to the polarity of the standard cell, battery, and emf, the corresponding plus and minus signs being marked.

If used with a wall-type galvanometer having a telescope and scale, it will be found convenient to place the potentiometer so that the telescope is directly over the glass index of the extended wire, thus permitting the observer to read the galvanometer deflections and potentiometer settings without change of position.

Potentiometer current. — A medium-sized storage cell will be found advantageous, producing a steady current. Errors of measurement are frequently due to an unsteady source of emf.

Setting for standard cell. — Set the standard cell switch to correspond with the known emf of the standard cell, as given in its certificate. *Place plug in hole 1, and see that it is always in this position when checking against the standard cell.* Place the double-throw switch at **STD. CELL.** Only the switch-key at the left of the three contacts marked "**RES,**"^{1} "**RES,**"^{2} and "**O,**" should be used in the preliminary balances so as to include the greatest extra resistance in series with the galvanometer. This precaution is always to be taken before the final balance is obtained, to prevent violent deflections of the galvanometer.

Adjust the regulating rheostat *P* until the galvanometer shows no deflection. The final galvanometer reading should be taken with the switch-key at *zero* so as to remove the series resistance and thus increase the sensibility. The key "**RES**"^{2} contact can be used as an intermediate point of contact as the galvanometer balance is approached.

Measurement of unknown emf. — The potentiometer gives direct readings for voltages up to and including 1.61 volts. For voltages higher than the direct range of the instrument, a volt-box or multiplier must be used, except as described at the end of this section.

The standard cell balance having been obtained as described under "Setting for Standard Cell," proceed as follows:

Place the double-throw switch in position "**E.M.F.**" and use the galvanometer key "**RES**"^{1} so as to include again the highest series resistance. The balance for the unknown emf is now obtained by manipulating the tenths switch and rotating the contact on the extended potentiometer wire. The final position of the two contacts in conjunction with the position of the plug at left of instrument indicates the voltage under test, as described later.

Plug at **0.1.** — Plug at **0.1** shunts the potentiometer circuit so that the readings taken from the settings of the tenths switch and slide-wire contact must be divided by 10. Having first set the standard cell switch and balanced with plug in position **1** change the ratio plug to **0.1** and proceed to balance for unknown emf as before by rotating the tenths switch (using the galvanometer key marked "**RES**"[1]) until a condition of balance is obtained exactly or approximately. To secure an exact balance, rotate the contact on the extended wire. The unknown emf can now be read from the position of the tenths switch and the extended wire contact by dividing by 10.

Examples. A balance was obtained with the tenths switch at *1.3*, the extended wire contact at *176*, and the plug at **1**. The voltage under test, therefore, is *1.3176*. If the plug at **0.1** had been used, the same reading would have indicated 0.13176 volt.

To ascertain if current in the potentiometer circuit has altered during a measurement, it is only necessary to plug in again at **1**, place the double-throw switch on **STD. CELL** and close the galvanometer key. No deflection indicates that current is constant. If previous balance does not exist, the regulating rheostat must again be adjusted until the galvanometer shows no deflection.

To measure voltages from 1.6 to 16. — Voltages up to 16 volts may be measured by using a greater voltage across the **Ba** posts. For this purpose a battery of about 20 volts should be used. Insert the ratio plug at **0.1** and throw the switch to **STD. CELL.** Then balance the galvanometer by means of the regulating rheostat. When the rheostat has been set to secure a balance, insert the ratio plug at **1**, set the switch on **E.M.F.**, and read the voltage in the usual manner. Multiply the instrument reading by 10.

The manufacturers of the Type-K potentiometer also offer the Type-K2 potentiometer having a third range of emf from 0 to 0.016 volt especially adapted to measuring the emf of thermocouples at low temperatures.

80. The Queen-Gray Standard Potentiometer. — This potentiometer, shown in Fig. 81, differs from the Type K just described chiefly in the following particulars:

(a) An intermediate switch of ten 0.01-volt steps and a single-turn slide-wire of 0.01-volt range replace the 11 turns of slide-wire covering the 0.11-volt range.

(b) The resistance standards of the potentiometer working circuit are 50 ohms each, instead of 5 ohms, thus requiring a working current of 0.002 ampere or 1/10 the value in the Type-K instrument.

(c) As a result of the introduction of the intermediate switch two

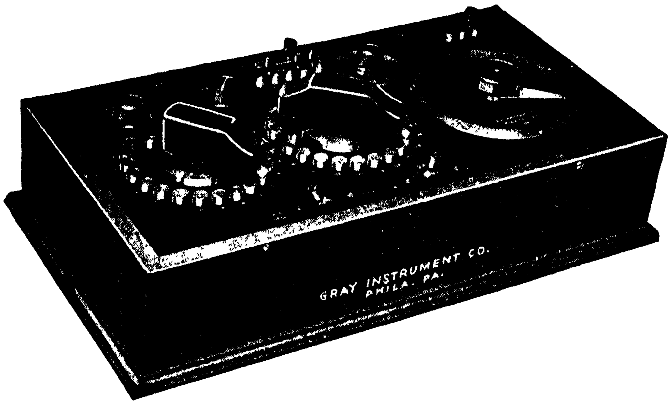


Fig. 81. The Queen-Gray standard potentiometer.

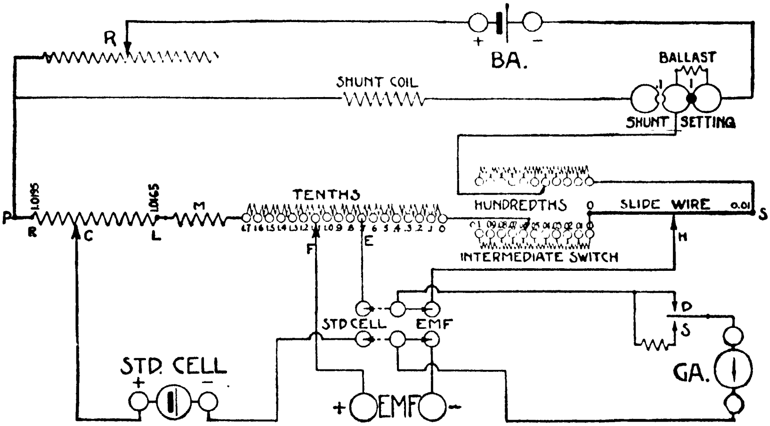


Fig. 82. Circuits of the Queen-Gray standard potentiometer.

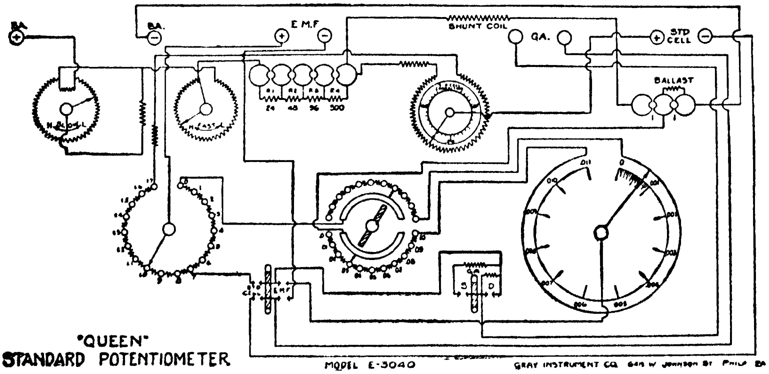


Fig. 83. Complete diagram of the Queen-Gray standard potentiometer.

contacts are inserted into the potentiometer circuit of the Gray instrument.

Figure 82 shows the simplified diagram, and Fig. 83 the complete diagram of circuits and connections of the Queen-Gray potentiometer. By throwing the ratio switch marked 1 and 0.1 the upper range of the instrument may be changed from 1.7 volt to 0.17 volt; and, with the ratio switch in position 1 but using a battery voltage of about 20 volts the range of the instrument is extended to 17 volts without the use of a volt-box. The lowest scale division on the one-turn slide-wire is 0.000 002 volt when the 0.1 ratio is used. This reading may be estimated to 0.000 001 volt, or 1 microvolt.

81. The General Electric Precision Potentiometer. — Figure 84 is of the General Electric precision potentiometer for which the diagram

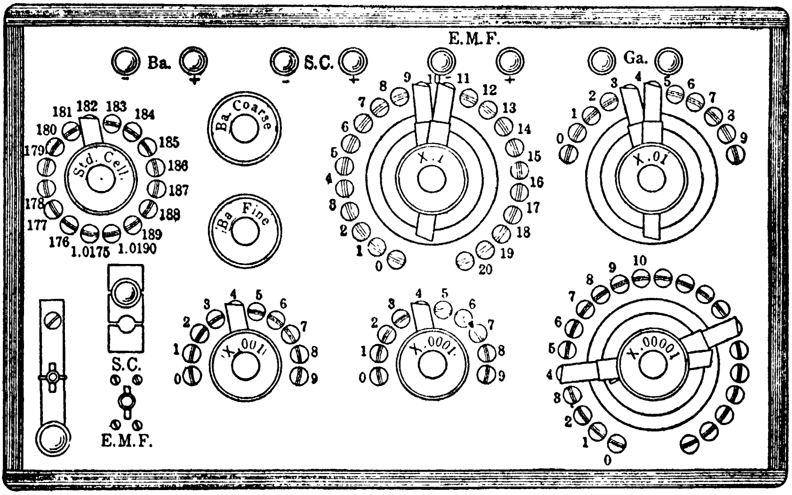


FIG. 84. The General Electric precision potentiometer.

of circuits and connections is Fig. 85. The manufacturer describes the circuit of this potentiometer as follows: "The current flows through the adjusting rheostats, having fixed resistances in parallel and in series with them, through the resistance coils of the two main dials, past the ratio plug, through the standard cell coil, and through the small resistances for adjusting the standard cell setting. Branch circuits flow from these dials carrying 1 per cent and 0.1 per cent of the current, respectively, to circuits which include the other three dials. None of the brush contacts carries any appreciable amount of the potentiometer current so that the error due to contact resistance is negligible."

In this potentiometer the balance against the standard cell is made without changing the position of the ratio plug. Voltages from 1 microvolt to 0.21 volt, using the ×0.1 ratio, of 10 microvolts to 2.1 volts using the ×1 ratio, both at normal working current of 0.01 ampere, are read directly from the dial settings and no slide-wire is used. It will be noted, however, that through the use of branch circuits contacts are introduced into the paths of small fractions of the working current.

By employing a battery of sufficient voltage, values of unknown emf up to 21 volts may be measured directly, but a volt-box (or multiplier) is recommended for anything over 2.1 volts, the multiplier being normally furnished with ratios of 10, 50, and 500, and having a resistance of 100 ohms per volt. With regard to the accuracy of measurement with this potentiometer the manufacturer states that the error will be less than 1/100 of 1 per cent of the dial readings. For the measurement of current, standard shunts are available for currents from 0.1 ampere to 500 amperes.

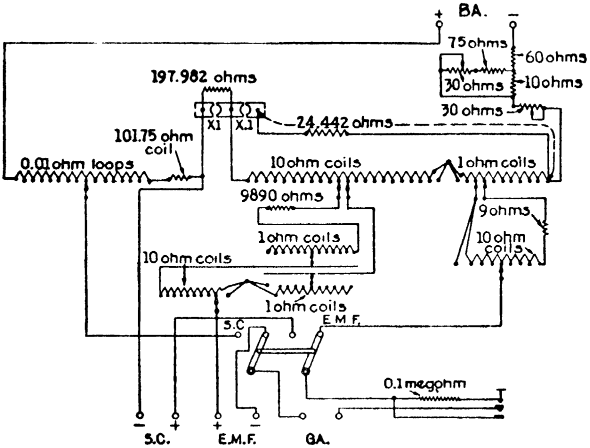


Fig. 85. Circuit diagram of the General Electric precision potentiometer.

82. EXPERIMENT 3-F. — Calibration of a Voltmeter with a Potentiometer and Standard Cell. — The theory and the general method of measurements are given in §§75 and 76. For a description of a practical potentiometer and the instructions for its use, see §§77 to 81. Study carefully the connections of the available potentiometer; read the instructions furnished by the maker. By means of the potentiometer calibrate a d-c voltmeter or measure an unknown voltage below 1.6 volts. Measure one or two voltages in the range between 1.6 and 16 volts, and finally a voltage above 16, using a volt-box. Pay particular attention to the sensitiveness and accuracy obtained, and to the factors which influence these. If possible, check the resistances of the potentiometer itself, as explained in §78; or devise another method more suitable for the apparatus at hand.

83. EXPERIMENT 3-G. — Calibration of an Ammeter with a Potentiometer and a Standard Resistance. — The experiment is similar to that described in §68, except that the voltage drop across the shunt (Fig. 70) is measured with a potentiometer instead of with a millivoltmeter. For the use of the potentiometer see §§75 to 81. In case the potentiometer is to be used without volt-box to measure emf's above 1.6 volts, how is the setting for the standard cell to be made?

84. EXPERIMENT 3-H. — Comparison of Two Resistances by Means of a Potentiometer. — The method is the same as in Fig. 2 except that a potentiometer is used instead of an ordinary voltmeter or millivoltmeter. For the theory and a description of the potentiometer see §§75 to 81. Compare the accuracy of this method with that obtained by the drop-of-potential and the comparison methods.

85. Deflection Potentiometer. — The so-called null-type potentiometer described above is suitable only for the measurement of steady currents and voltages, because it takes some time to obtain a balance. Moreover, the accuracy obtainable is often much greater than is required for many practical purposes. A need has been felt for some time for a potentiometer-type instrument of an accuracy intermediate between that of a null-type potentiometer and of the best indicating voltmeters and millivoltmeters of the direct-deflection laboratory type. This problem has been solved in the so-called *deflection potentiometer*[1] developed by Dr. H. B. Brooks of the Bureau of Standards, and, in the form manufactured by the General Electric Company, shown in Figs. 86 to 88. In this instrument an approximate balance is obtained as in the ordinary potentiometer (§75); the last one or two significant figures in the value of the unknown voltage or current are given by the deflection of the same galvanometer which indicates the approximate balance. This instrument has the following advantages over the null-type potentiometer: its setting requires much less time, and small fluctuations in the unknown current or emf can be directly followed on the galvanometer pointer without requiring new settings. At the same time it is more accurate than the best standard indicating instruments, and the unknown quantities are determined directly in terms of a standard cell and accurately adjusted resistances.

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[1] See Bulletin of the Bureau of Standards, Vol. 8, No. 2.

The particular instrument shown in Fig. 86 can be used (a) to measure voltages within the direct range from zero to 1.5 volts to within 0.0001 volt without the use of a multiplier, (b) in connection with a volt-multiplier for measurements of emf's higher than 1.5 volts, and (c) to measure current by means of shunts, the instrument when so connected reading directly in amperes.

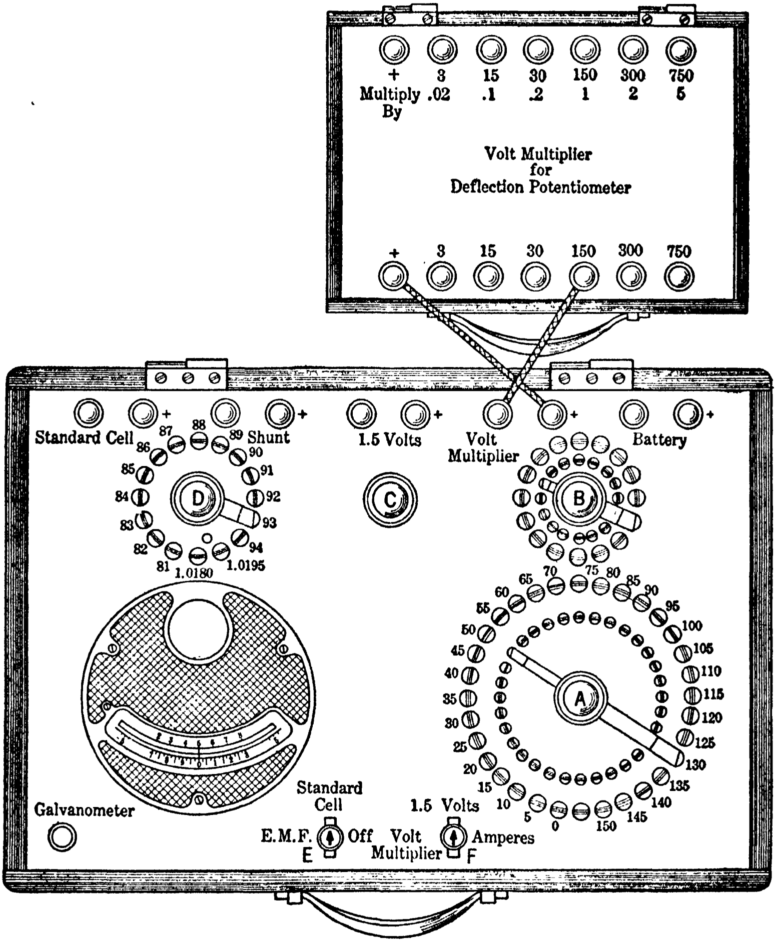

FIG. 86. The Brook's deflection potentiometer, as built by the General Electric Company.

The volt-ranges obtained by means of the multiplier, viz., 3, 15, 30, 150, 300, and 750, together with that of the potentiometer direct, 1.5 volts, have been chosen with a view to facilitating the checking of indicating voltmeters having scales usually found in commercial use.

RANGES OF EMF MEASUREMENTS

Maximum EMF in Volts	Multiplying Factor*	Resistance in Ohms across Which the Unknown EMF is Connected	EMF per Small Division of Galvanometer Scale
1.5	0.01	40	0.001†
3.0	0.02	80	0.002
15.0	0.1	400	0.01
30.0	0.2	800	0.02
150.0	1.0	4000	0.1
300.0	2.0	8000	0.2
750.0	5.0	20000	0.5

* The multiplying factor is that number by which the potentiometer readings as given on the instrument should be multiplied to reduce to volts.

† By estimating parts of a division, the values in this column can be read to about one-tenth of that given.

Current is measured by reading the drop across special current-carrying shunts of rather high resistance. There must be a total resistance amounting to 40 ohms in the potential circuit from each shunt; these ballast resistances are made an integral part of the shunt and have the values shown in the table below. An allowance of 0.025 ohm is made for the resistance of the leads from the shunt to the potentiometer. With these special shunts the reading is either some direct decimal of the current value, or is modified by the factor 5. At rated current for the shunt there is a fairly high reading of the potentiometer. There are five sizes normally offered for use with the instrument. The current capacity and multiplying factor of each are given in the following table.

DATA REGARDING SHUNTS FOR USE WITH POTENTIOMETER

Maximum Rating of Shunt, Amperes	Resistance of Shunt, Ohms	Ballast Resistance, Ohms	Maximum Potentiometer Reading	Multiplying Factor	Amperes per Small Division of Galvanometer
1.5	1.00	38.975	150	0.01	0.001
6.0	0.20	39.775	120	0.05	0.005
15.0	0.10	39.875	150	0.10	0.01
60.0	0.02	39.955	120	0.50	0.05
150.0	0.01	39.965	150	1.00	0.10

Shunts for higher ratings will be supplied upon request.

The potentiometer resistance is composed of thirty coils, the outer ring of switch contact points around the main dial *A* forming the 30 drop points. These are numbered 0, 5, 10, 15, etc., up to 150. The figures on the galvanometer scale (Fig. 87) are in the same units, and indicate up to three more or less than the number on which the main dial switch rests. In this way any value between the main dial points can be read by inspection of the galvanometer deflection, one small division on the galvanometer scale being equal to 1/10 of a main dial unit.

Two scales are marked on the galvanometer. In the lower one, zero is located at the center. At the right of zero each consecutive ten small divisions are marked 1, 2, and 3 respectively, the corresponding locations at the left being marked 9, 8, and 7. On the upper scale the figure 5 is located at the center, the figures at the right and left being respectively 6, 7, 8 and 4, 3, 2. If the point on which the main dial switch stops ends in zero, the upper scale is read; if with a figure 5, the lower. It will be seen that if the needle is deflected toward the right, the total reading is always more than that indicated by the main dial switch; if to the left, it is less.

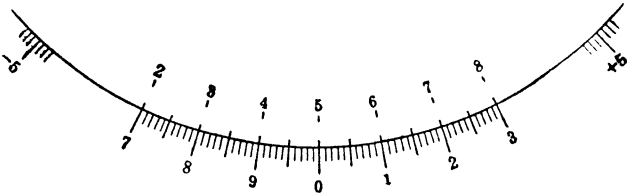

FIG. 87. The galvanometer scale of the deflection potentiometer.

For example, if the reading on the main dial is 55 and the galvanometer reads on the upper scale 4.71 (the last figure estimated as 1/10 of a small division), the total reading would be 54.71. But if the main dial read 60 and the needle were deflected to the same position, the upper scale would be read, viz., 9.71, the total reading being 59.71. If in the two examples cited the galvanometer had been deflected to the right the total reading would have been more than indicated by the main dial switch.

86. Operating Instructions for the General Electric Type of Deflection Potentiometer. — The following specific instructions are given by the makers of this type of instrument and are here reproduced in order to give a better idea of the possibilities of the deflection potentiometer.

Storage cell. Connect a storage cell to posts marked **BATTERY**, observing the polarity signs. The emf of this cell must not exceed 2.1 or be less than 1.9 volts. If the potential is not within this range, it will be impossible to adjust the current to the correct value. The use of a lead cell between these limits assures a practically steady current. If the emf of the cell is too high, discharge it through a suitable resistance until the voltage is within the limits given above. Do not insert outside resistance in the circuit as this will introduce an error in the instrument readings.

Resistance of leads. The total resistance of any two leads connecting the storage cell, volt-multiplier, or current-carrying shunts should not exceed 0.05 ohm. (Five feet of No. 16 B & S lamp cord for each lead will be well within this limit.)

Standard cell. Connect a standard cell to posts so marked. A Weston standard cell is used to determine the current flowing through the potentiometer coils. The emf of this cell, when in good condition, is always between 1.0180 and 1.0195 volts. The sixteen points of the standard cell dial " D " are numbered consecutively from 1.0180 to 1.0195.

Galvanometer zero adjustment. Place the switch of the standard-cell dial on the point corresponding to the emf of the cell. Make sure that the galvanometer needle is at zero; if it is not, adjust by means of the small screw at the left of the scale opening. Turn the switch-key E to the point marked " Standard Cell." Press down the galvanometer key and adjust the current by means of the coarse and fine rheostats in the upper right and upper center of the panel until the needle is brought exactly to the zero mark. This adjustment of current through the potentiometer coils should be made frequently in order to compensate for slight changes in the emf of the storage cell.

Measurement of emf not exceeding 1.5 volts. Attach the terminals of the circuit to be tested to posts marked " 1.5 VOLTS," turn switch-key E to E.M.F., and switch-key F to the point marked **1.5 VOLTS**. Now connect the galvanometer through the high protective resistance by pressing the galvanometer button partway down.

Galvanometer locking device. Lock in this position by turning the button either to the right or to the left. Turn keys E and F to " E.M.F." and to " **1.5 VOLTS** " respectively, and rotate the main dial switch until the needle shows the least deflection; press the galvanometer key down as far as possible, thus cutting the protective resistance out of the galvanometer circuit, and read the value of deflection on the galvanometer scale. Multiply the reading by 0.01 in order to reduce it to volts. If no balance can be obtained it is probable that the unknown emf leads are reversed.

Measurement of emf higher than 1.5 volts. The volt-multiplier must be used if an emf higher than 1.5 volts is to be measured. Connect the unknown emf to the line side of the volt-box, with the positive side of the line at the plus binding post and the negative side at the post marked somewhat above the expected value of the emf. Connect the corresponding binding posts on the instrument side of the volt-box to the terminals marked " **VOLT MULTIPLIER** " on the potentiometer. Having checked the adjustment of the potentiometer against the standard cell as before described, turn key **E** to " **E.M.F.**" and key **F** to " **VOLT MULTIPLIER.**" Proceed to get a reading as previously outlined, multiplying it by the factor given on the line side of the volt-multiplier.

Current measurements. Select a shunt having the smallest rated capacity which will include the current to be measured, and connect the drop terminals of the shunt to the posts on the potentiometer marked " **SHUNT.**" Connect the current terminals of the shunt in series with the line carrying the current to be measured. Turn the key **E** to " **E.M.F.,**" and the key **F** to " **AMPERES.**" Proceed to obtain a balance as already described. Multiply the readings by the factor from the table which corresponds to the particular shunt. The result gives the value of current in amperes.

Checking galvanometer coils. To check the accuracy of the galvanometer, connect in the way before described for measuring voltages within the range of the instrument, viz., 1.5 volts. Supply the emf to the posts marked **1.5 volts** by connecting to the drop terminals of a shunt having not more than 0.1 ohm resistance. Adjust the current through the potentiometer coils as before; turn keys **E** and **F** to " **E.M.F.**" and **1.5 volts** and regulate the current in the shunt until the galvanometer shows no deflection when the main dial switch is on any convenient point. When it is accurately balanced, move the main dial switch to the next point in either direction and note the deflection. If the galvanometer is reading correctly, the needle should be over either the point marked " -5 " or " $+5$ " depending upon which way the switch was moved. A deviation of two-tenths of a small division from the points -5 or $+5$ means a maximum error, when making measurements in the regular way, of only one-tenth of a small division at full-scale deflection (2.5), or 1/100 of 1 per cent when the main dial switch reads 100.

Checking potentiometer coils. Proceed as in checking the galvanometer, except that a balance should be secured on each of the 30 points of the main dial, beginning with point 5, by adjusting the current through the shunt, moving the switch at each balance one point back and noting the deflection of the needle. The deflection should be the same on all the coils if they are in good condition. These coils are adjusted to within 1 part in 10,000 before the instrument leaves the factory. The check above described is only approximate, not better than to within 1 part in 500, but serves to show if any coil has suffered serious damage.

87. Theory of the Deflection Potentiometer. — The principle of the Brooks deflection potentiometer described above is shown in Fig. 88. *E* is the unknown voltage to be measured, and is above 1.5 volts. *CD* is the resistance of the volt-box, the galvanometer circuit being tapped off at points *C* and *F*. The total resistance of the volt-box is *R*, and the point *F* corresponds to one *p*th part of it from the left. Therefore the resistances *CF* and *FD* are marked R/p and $R(p - 1)/p$ respectively. *AB* is the main potentiometer resistance fed from the storage battery e_1; it corresponds to the resistance *OB* in Fig. 78. The current in the main potentiometer circuit is regulated by means of the resistances r_3 and r_6, corresponding to the dials *B* and *C* in Fig. 86. When the contact points *N* and *P* are moved to the right the current i_1 in *AB* is increased, and vice versa. The galvanometer tap-off point, *H*, corresponds to point *M* in Fig. 73. It will be seen further that simultaneously with the change in the position of point *H* the resistance r_4 of the galvanometer circuit is varied at point *L*. The variable points *H* and *L* are obtained in Fig. 86 by means of the two rows of buttons controlled by the handle *A*.

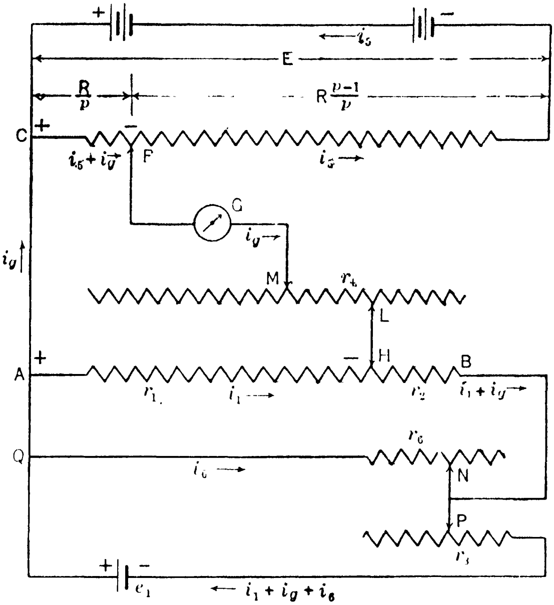

Fig. 88. The principle of the deflection potentiometer.

The purpose of this seemingly complicated arrangement is to keep the total resistance of the galvanometer circuit constant, independent of the positions of the sliders *HL* and *NP*, as is required by the theory indicated below. *The galvanometer circuit is understood here to comprise all the resistances in Fig. 88, connected as shown, and considering the galvanometer itself as if it were a source of voltage, with the emf's e_1 and E removed and a short circuit put in place of each.* Beginning with the galvanometer, the part *GMLH* of the galvanometer circuit has no branches; between *H* and *A* the galvanometer circuit consists of the branch r_1 in parallel with the series-parallel combination *HBNPQ*. The latter consists of r_2 in series with the parallel combination of r_3 and r_6. Between *A* and *C* there are no branches, and from *C* to *F* there are two branches in parallel, namely $CF = R/p$ and $DF = R(p - 1)/p$. The circuit is completed from *F* to *G* by a single conductor. The resistances r_3 and r_6 are in parallel in the galvanometer circuit as defined above. They are so adjusted that, when the slide *NP* is moved to the right or to the left, the combined resistance of r_3 and r_6 in parallel remains constant. Similarly, when *HL* is moved to the right or to the left, the resistance r_4 compensates for any change in the combined resistance of r_1, r_2, r_3, and r_6, considering the latter combination from the point of view of the galvanometer circuit.

The galvanometer deflections must be correct in the sense of giving the last decimals of the unknown voltage or current (a) when a different tap *F* is taken on the volt-box; (b) when the volt-box is dispensed with for emf measurements below 1.5 volts, and (c) when an ammeter shunt is inserted between *C* and *F* for current measurements. A different ballast resistance is introduced in each case into the galvanometer circuit to keep its total resistance constant. This is obtained automatically on the dial switch *E* (Fig. 86) so that no attention on the part of the operator is required beyond setting the switch to correspond with the particular ammeter shunt or voltage ratio used.

(1) *An emf less than* 1.5 *volts.* Consider the simplest case of measuring an emf, *E*, below 1.5 volts, and assume this emf to be due to a cell of negligible resistance, connected directly between the points *C* and *F*, with the volt-box omitted. Let an approximate balance be obtained at *H*, as explained in the preceding section, and let there be a small current i_g in the galvanometer circuit. The problem is to find *E* from the potentiometer setting and from the value of the galvanometer current i_g. A current i_g in the direction indicated in the sketch means that the difference of potential *C-F* is less than *A-H*, the points *A* and *C* being practically at the same potential. Consider the unknown voltage *E* to consist of two component parts, E_1 and ϵ, and let E_1 acting alone give a perfect balance ($i_g = 0$) with the setting at the same point *H*. We can now apply the principle of superposition of currents in a network of conductors in which two or more emf's are applied, viz., that *the actual*

current in a conductor is equal to the sum of the currents produced in the same conductor by the individual emf's or *their partial combinations acting separately.* Let the emf's e_1 and E_1 be applied simultaneously as the first partial combination. The emf E_1 by supposition is such that in combination with e_1 it gives $i_g = 0$, and the setting at H reads the value of E_1 directly. As the other partial emf take ϵ, acting separately, with e_1 replaced by a short circuit. If the equivalent resistance of the galvanometer circuit, as described above, is R_g, then

$$\epsilon = i_g R_g \quad \text{. . . . .} \quad (1)$$

so that the galvanometer current is a measure of the unbalanced part of the unknown voltage E, and with the assumed direction of i_g the correction ϵ must be subtracted from E. The resistance R_g must be a constant for all settings of HL and NP, in order that the galvanometer may be calibrated directly in decimal fractions of a volt, as is the main potentiometer dial. Then only can a galvanometer reading be added directly to a dial reading, as is stated in the directions for the use of the deflection potentiometer. Thus the potentiometer setting measures E_1, and the galvanometer current indicates the remainder of E.

If the source of emf, E, has a considerable resistance, ρ, so that the voltage drop ρi_g has an appreciable value, this resistance must be considered as a part of the resistance of the galvanometer circuit, and the ballast resistance reduced accordingly. Since ρ itself may be variable and may affect the value of E, it is safer not to use the direct deflection principle for such measurements but to apply a null potentiometer. A good galvanometer requires a current of only 10 microamperes per scale division, so that the internal resistance of the source E must be quite high indeed, or E itself very small, in order to affect appreciably the accuracy of the direct-deflection principle.

(2) *An emf greater than* 1.5 *volts.* Now, let the unknown voltage E be applied through a volt-box, as shown in Fig. 88, and let it be again required to determine E in terms of the setting at H, of the volt-box ratio p, and of the galvanometer current i_g. In order to do this, imagine a small emf, ϵ, to be inserted in the galvanometer circuit, either in the part AC or in the part FM, in such a direction as to oppose the current i_g. Adjust ϵ to such a value that it satisfies eq. (1), that is, produces a current $-i_g$, and apply again the principle of superposition, assuming the voltages E, e_1, and ϵ to act simultaneously. Since the first two together produce the current i_g and the third one the current $-i_g$, the combination will give a zero galvanometer current. Thus, with this combination the voltage drops CF and AH are exactly equal to each other and the potentiometer setting at H reads the value which is lower than the

true voltage CF by the known amount ϵ. But in the actual measurement ϵ is read on the galvanometer directly, in the form of the current i_g, so that the true voltage CF is again equal to the algebraic sum of the setting at H and the galvanometer reading.

It must be clearly understood that the deflection potentiometer does not read the actual voltage CF, but that which would exist if the galvanometer current were equal to zero. Only under such conditions does the volt-box subdivide the applied voltage in the correct ratio p of the resistances. In other words, with the galvanometer properly calibrated as a voltmeter and with its resistance kept constant, the deflection gives the true electrostatic difference of potential between the points F and H, as if the galvanometer circuit were broken between these points, and as if the circuits CDE and ABe_1Q had only one common connection AC with zero current flowing through it. The potentiometer thus measures that value of the voltage drop CF which when multiplied by p gives the correct value of the voltage E across the volt-box CD. Any internal resistance in E or any resistance in series with the volt-box has nothing to do with this result. The applied emf, E, may be increased to $E + E'$, and the part E' consumed in some resistance, leaving again part E across the volt-box. Thus, the given voltage source $E + E'$ of appreciable resistance may be replaced by an emf, E, of zero resistance, and the reasoning given above will apply. The resistance of the galvanometer circuit, external to the potentiometer, must comprise only R/p and $R(p - 1)/p$ in parallel, but no external or internal resistance of the unknown emf, E.

(3) *Voltage drop across an ammeter shunt.* Now, let the resistance CF be that of an ammeter shunt. Assume a counter emf, $\epsilon_1 = R_s i_g$, to be inserted between C and F in series with the ammeter shunt of resistance R_s, and a counter emf, $\epsilon = R_g' i_g$, to be inserted between F and M. Here R_g' is the resistance of the galvanometer circuit from C through the potentiometer to F; that is, R_g' is different from the former value R_g in that it does not include any of the resistances of the circuit CDE. With these two additional emf's in the circuit, the current in CF becomes equal to i_5 and the current in FD remains equal to i_5. The galvanometer current i_g is reduced to zero. Thus, the voltage drop iR_s in the shunt is smaller than the potentiometer voltage V between A and H by the amount $\epsilon_1 + \epsilon_g$; in other words

$$iR_s = V - i_g(R_g' + R_s) \qquad \qquad (2)$$

Thus, in order that the galvanometer may indicate as a voltmeter, the resistance of its circuit must be equal to $R_g' + R_s$, and the resistance of the part FD or the internal resistance of E does not enter into the problem. With the ballast resistance properly adjusted, galvanometer deflections represent the true corrections to the approximate settings at H, and with a simple conversion coefficient the instrument reads directly in amperes.

Another way of explaining the meaning of the galvanometer deflection when measuring a current with a shunt is as follows: Let the applied emf, E, be considered as made up of a part E_1 which will give $i_g = 0$, and of a residual ϵ_3. If E_1 act alone, it will send a current V/R_s through R_s, where V is the difference of potential corresponding to the null setting on AB. If ϵ_3 act alone, with e_1 replaced by a short circuit, it will send the current i_g through the branch $CAHF$, and also the current $(R_g'/R_s)i_g$ through R_s. The sum of these two currents resulting from ϵ_3, plus the current caused by E_1, gives the actual current i_5 at the time of observation; that is,

$$
\begin{aligned}
i_5 &= \frac{V}{R_s} + i_g + \frac{R_g'}{R_s} \cdot i_g \\
&= \frac{1}{R_s}[V + (R_s + R_g')i_g] \\
&= \frac{1}{R_s}(V + R_g i_g) \qquad \qquad (3)
\end{aligned}
$$

Here R_g consists of R_g' plus the resistance of the shunt. This expression is identical with eq. (2) and is not limited in any way by the resistance in the source of current.

88. Temperature Measurement by Potentiometer. — One of the most important industrial uses of the potentiometer is in the measurement and recording of temperatures. By means of thermocouples inserted in various parts of generators and transformers, in furnaces and ovens, etc., the potentiometer, by measuring the emf developed in the heated thermocouples, gives the temperatures sought. Special potentiometers are available with self-contained battery, standard cell, galvanometer, and cold-junction compensation, while the scale reads directly the temperature of the hot junction. Still other potentiometer pyrometers are available which trace a continuous record of the temperature or several temperatures and are self-balancing. They may be used for the purpose of automatic control of temperatures, to operate signals, etc. (See Leeds & Northrup Catalogue 87; also Bureau of Standards Technological Paper 170.)

ALTERNATING-CURRENT POTENTIOMETERS

89. The difficulties in the way of measuring alternating emf's on the potentiometer principle (Fig. 78) are as follows: two continuous emf's differ from each other in magnitude only, whereas two alternating emf's may differ in magnitude, phase, frequency, and wave-form. Moreover, there is no standard of alternating emf corresponding to a standard cell for continuous voltages. If the problem be limited to that of comparing two strictly sinusoidal emf's of the same frequency, then the known emf must be adjusted not only in magnitude but in phase as well. There are at least two practical methods of accomplishing this end, viz.:

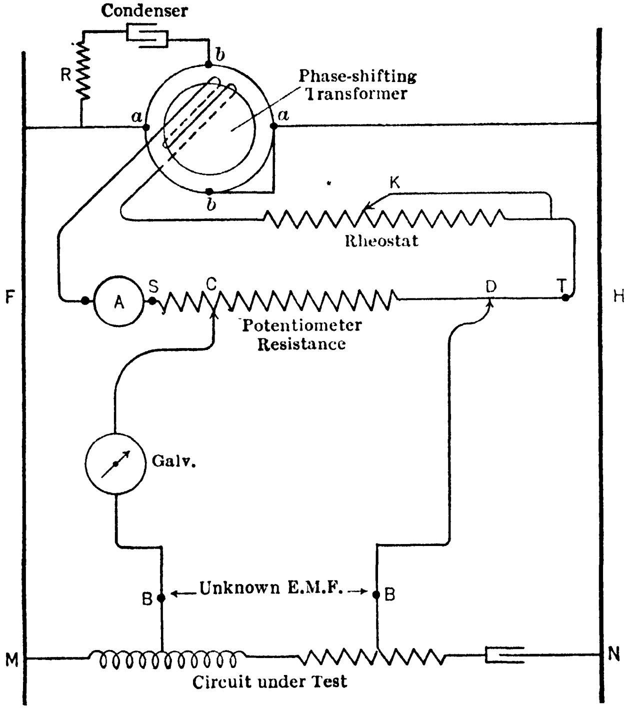

FIG. 89. The principle of the Drysdale a-c potentiometer.

(a) By means of a phase-shifter (Drysdale, Fig. 89).

(b) By resolving the known emf into two components in quadrature with each other (Larsen, Fig. 90).

An a-c potentiometer is quite useful for the calibration of ammeters, voltmeters, and other a-c instruments, for the determination of the ratio and of the phase angle of instrument transformers, for capacity and inductance measurements, for the study of alternating leakage fluxes, for research with artificial transmission or telephone lines, and generally for accurate measurements with alternating currents.

90. Drysdale Potentiometer. — The principle of the Drysdale potentiometer is shown in Fig. 89. The unknown voltage drop to be measured is that between the terminals *BB*, across part of the circuit *MN*, fed from the same a-c line *FH* to which the phase-shifting transformer of the potentiometer is connected. The stationary part of this transformer is wound like the stator of a two-phase motor (§484); the movable part has one diametral winding. One phase of the stator, *aa*, is connected directly across the line; the other, *bb*, is connected to the same line in series with a resistance *R* and a condenser. The connections are similar to those in a split-phase motor (§508) so that a revolving magnetic field is produced in the air-gap. The phase of the secondary induced emf depends upon the position of the secondary winding with respect to the two primary windings. The secondary part may be turned into any desired position by means of a worm gear, and the angle of shift from the zero position is indicated on a scale. See also the potential-shifter in §132b.

No standard cell can be used directly, and for this reason the current in the "slide-wire," or in the potentiometer circuit proper, *ST*, is adjusted by means of an a-c ammeter *A*. Since this current is but a fraction of an ampere, a dynamometer-type, hot-wire, or thermoelectric instrument should be used. Any of these instruments may be accurately calibrated with direct current (§71). The use of this milliammeter limits the accuracy of the whole potentiometer because the accuracy of the result cannot be greater than that with which the current in *ST* can be adjusted to a prescribed value. The galvanometer which indicates the equality of the voltages *CD* and *BB* is a null instrument of the vibration type, tuned accurately to the frequency of the supply (§45). A telephone receiver may be used instead, although it is difficult to obtain perfect silence in it on account of unavoidable higher harmonics. A vacuum-tube amplifier and oxide rectifier with a d-c galvanometer is used successfully.

To measure an alternating voltage, the current in the ammeter *A* is adjusted by means of the rheostat *K* to such a value that the indications on the dials corresponding to the slide-wire *ST* read directly in volts. Then the contact points *C* and *D* and the phase-shifter are adjusted until the galvanometer indication is zero. The unknown voltage is then read directly on the dials and the corresponding phase angle on the phase-shifter. This angle has no particular meaning unless two emf's are measured. Then the difference of the two angles represents the electrical phase displacement between the two emf's. Non-inductive ammeter shunts are available, by means of which alternating currents can be measured to the same degree of accuracy as alternating voltages. Since it is possible to measure a current, a voltage, and the phase angle between the two, the instrument may also be used for power measurements.

91. Larsen Potentiometer. — This potentiometer, in the form further developed by Dr. A. E. Kennelly and his co-workers, is shown in

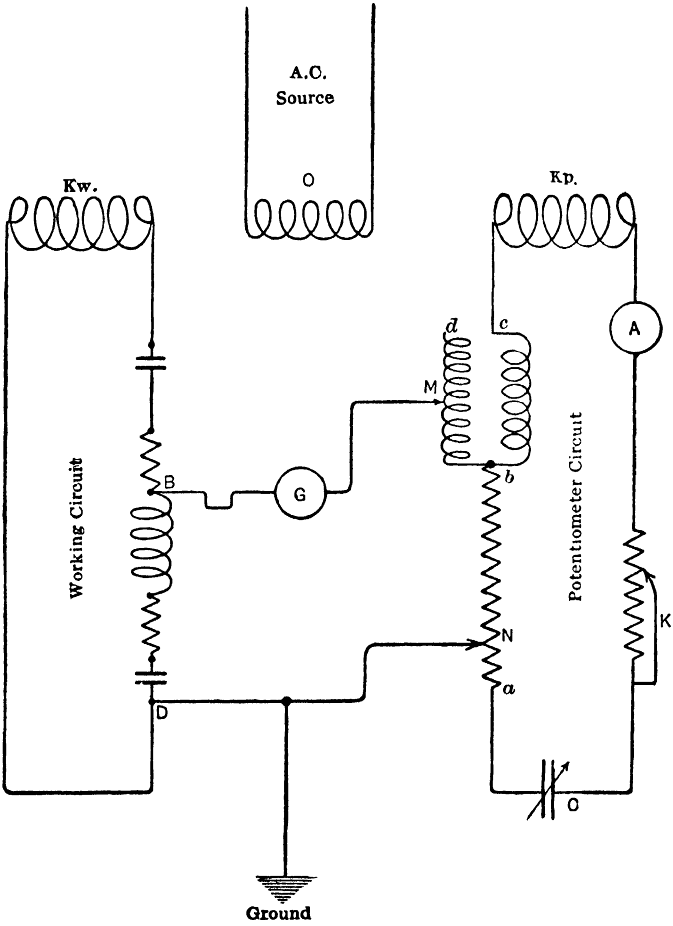

FIG. 90. The Larsen a-c potentiometer.

Fig. 90. It is particularly useful at higher frequencies, say above 1000 cycles per second, when the source of testing current is an oscillator (Vreeland oscillator, pliotron, etc.). In such a case the amount of power necessary to operate the phase-shifting transformer in Drys-

dale's potentiometer is prohibitive, and it becomes necessary to vary separately the magnitude of each of the two quadrature components of the known emf. In Fig. 90, *O* is the source of alternating power, connected inductively through the coil K_w to the working circuit (circuit under test) and through the coil K_p to the potentiometer circuit. The two important parts of this circuit are a non-inductive resistance *ab* and the reactive coil *bc*, without iron core. The winding *bd* is placed in an inductive relation to *bc*. When the current in the galvanometer *G* and in the winding *bM* is zero, the flux linking with *bM* is due to the current in the winding *bc*; that is, this flux is in phase with the current in the potentiometer circuit. Hence, the voltage induced in *bd*, or in any part of it, is in quadrature with the flux or with the potentiometer current. The voltage drop across *ab*, or across any part of it, is in phase with the potentiometer current. Thus, by means of the sliding contacts *M* and *N* it is possible to impress between points *B* and *D* an emf of any desired magnitude and phase within the limits of the device. When the galvanometer current is zero, the known and the unknown emf's are equal and opposed to each other. The known value is read on the dials, as in any potentiometer.

Because of the absence of a standard cell, the potentiometer current is adjusted to a certain value on the ammeter *A* by means of the rheostat *K* and condenser *C*. The latter serves to change the phase of the current. The ground connection shown in the sketch is advisable at higher frequencies, in order to minimize parasitic currents due to the distributed capacitance of the apparatus. Stray alternating magnetic fields must also be guarded against.

92. The Kelvin Ampere Balance. — When using a potentiometer and a standard cell for the measurement of a current, the latter has to be obtained each time indirectly as the ratio of an emf to a resistance. A direct secondary standard of current is available in the so-called Kelvin balance (Figs. 91 and 92). It is based on the electro-dynamometer principle (§51), but differs from the usual instruments in that the coils lie in parallel planes and the electromagnetic attraction is balanced by means of a weight instead of by a spring. Such an instrument is calibrated either by means of a silver voltameter, or with a potentiometer, using a standard resistance and a standard cell.

A Kelvin balance consists of four stationary coils *A* (Fig. 91), and two movable coils *B* suspended by metal strips *C*; these strips serve also as current leads for the coils *B*. When the instrument is used as an ammeter, the six coils are connected in series; a current flowing through the coils produces attractions and repulsions, as marked in Fig. 92. The influence of terrestrial magnetism is neutralized by making the current in the two movable coils flow in opposite directions.

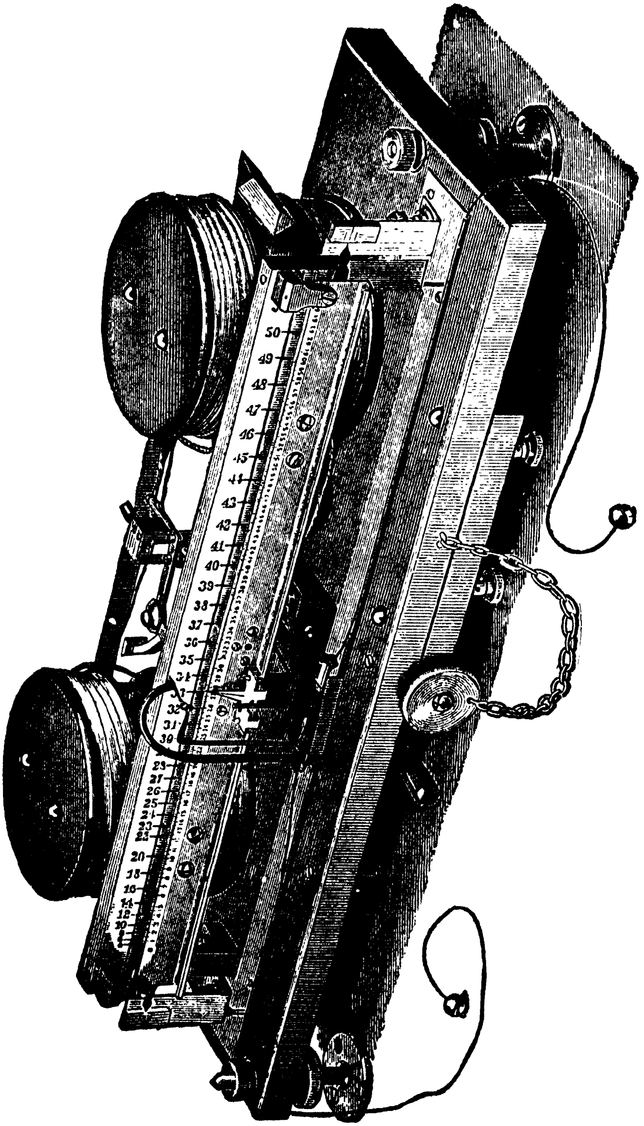

Fig. 91. The Kelvin balance for measuring currents.

A weight is placed in the trough *D*; with this weight the movable coils are in balance without current when the sliding weight *w* is in its extreme left position; this position corresponds to the zero of the scale.

When a current is flowing through the instrument, it is necessary to move *w* to the right in order to keep the movable coils in balance. The value of the current is indicated on the scale by the position of the weight *w*. It will be seen in Fig. 91 that the instrument is provided with two scales: one is the accurate uniform scale; the other, the so-called inspectional scale, is calibrated directly in amperes. The ampere divisions are proportional to the square roots of the distances from the left end of the scale, because the attraction between the coils is proportional to the square of the current (§42). The inspectional scale is accurate enough for ordinary purposes. When a greater accuracy is required the uniform scale is used; the current is then calculated by extracting the square root of the reading and multiplying it by the constant of the instrument.

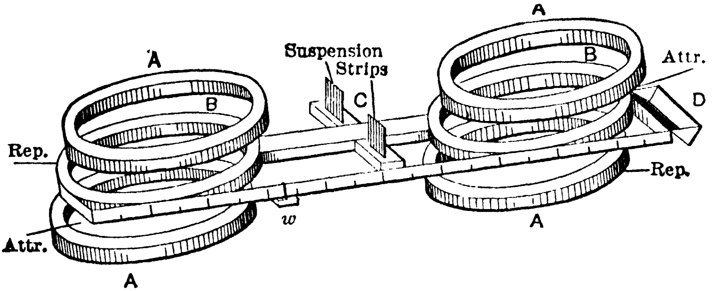

FIG. 92. The principle of the Kelvin balance.

The weight *w* is slipped into its proper position by means of a self-releasing pendant, hanging from a hook carried by a sliding platform, which is pulled in the two directions by two silk threads passing through holes to the outside of the glass case (Fig. 91). For the fine adjustment a small rider is provided, as in an ordinary chemical balance. The rider is actuated by a fork, the handle of which is visible below the base. The round knob in the base, fastened to the chain, lifts the moving coils off the suspension when the instrument is not in use. Four pairs of weights (sliding and counterpoise) are supplied with each instrument; of these the carriage of the moving weights and its counterpoise constitute the first pair. These weights are adjusted in the ratios of 1 : 4 : 16 : 64, so that each pair gives a round number of amperes, or half-amperes, or quarter-amperes, or of decimal subdivisions or multiples of these magnitudes of current, on the inspectional scale.

These instruments can be used equally well with direct or alternating currents, and are made in seven sizes covering the range from 0.01 ampere to 2500 amperes. The smallest size may also be used as a voltmeter, when provided with a suitable high resistance in series (multiplier). Balances are built on the same principle for measuring watts; also composite balances for measuring amperes, volts, and watts with one and the same instrument. Detailed instructions for using the balance always accompany the instrument.

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CHAPTER IV

WATTMETERS AND POWER-FACTOR METERS

93. A wattmeter is an instrument for measuring electric power delivered to a circuit. The name " wattmeter " comes from the name " watt," which is a practical unit of power, in the volt-ampere-ohm system. There are three types of wattmeters: (a) *indicating wattmeters,* which show the average value of power or the rate at which energy is consumed over a short interval of time; (b) *recording,* or graphic, *wattmeters* which trace a curve of such average values on a record sheet; (c) *watthour meters,* sometimes also called integrating wattmeters, which sum up and record the *total energy* delivered during an interval of time. The latter are treated in the next chapter.

To make this distinction clearer, consider a 100-volt d-c circuit, in which the current regularly fluctuates between 50 and 70 amperes every 10 seconds. Imagine a wattmeter of each of the three above types connected to such a circuit, and observe their behavior — say for 5 minutes. The needle of the indicating wattmeter will fluctuate regularly between the divisions of the scale, corresponding to 5000 and 7000 watts. The recording wattmeter will trace a wavy line between the same values (Fig. 66). The dial of the watthour meter will show at the end of 5 minutes

$$\frac{(7000 + 5000)}{2} \times 5 = 30{,}000 \text{ watt-minutes,}$$

or

$$\frac{30{,}000}{60} = 500 \text{ watthours.}$$

94. Power in A-C Circuits. — The power expended in a d-c circuit (Fig. 30) is equal to the product of the current delivered and the voltage at which it is supplied. In practical units

$$\text{Watts} = \text{amperes} \times \text{volts}$$

or, with the customary notation,

$$p = i \times e \quad \ldots\ldots\ldots\ldots \quad (1)$$

Thus, in a d-c circuit, power may be measured with an ammeter and a voltmeter, unless the load is rapidly fluctuating. In an a-c circuit the presence of self-induction or capacitance so modifies the relations that the " effective volts " times " effective amperes " (§42) is not equal to the true average watts expended in the circuit. The relation (1) is, however, always true for the corresponding instantaneous values of current and voltage. In other words,

$$dw = p \cdot dt = i \cdot e \cdot dt \quad \ldots\ldots\ldots\ldots \quad (2)$$

represents the true electrical energy delivered to an a-c circuit during an infinitesimal element of time dt; i and e are instantaneous values of current and voltage. The foregoing expression is true for any wave-shape whatsoever. The average energy per second, that is the average rate at which energy is delivered, or the *power in watts*, is

$$P = (1/T) \int_0^T i \cdot e \cdot dt \quad \ldots\ldots\ldots\ldots \quad (3)$$

where T is the time of one cycle of the alternating current.

Assume first that both the current and the voltage vary according to the sine law. We have then

$$i = I_m \sin 2\pi ft \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$e = E_m \sin (2\pi ft \pm \phi) \quad \ldots\ldots\ldots\ldots \quad (5)$$

Here ϕ is the *phase angle* (§145) by which the emf leads or lags behind the current, and $f = 1/T$ is the frequency, or the number of cycles per second (Fig. 123). Substituting the values of i and e from eqs. (4) and (5) in eq. (3) we obtain

$$P = (I_m E_m / T) \int_0^T \sin 2\pi ft \cdot \sin (2\pi ft \pm \phi) \cdot dt$$

or, after integration,

$$P = \tfrac{1}{2} E_m I_m \cos \phi.$$

Substituting the *effective* values, E and I, of the voltage and current, as shown by the ammeter and voltmeter (see note on p. 90), we find that the *average real power* is

$$P = EI \cos \phi \quad \ldots\ldots\ldots\ldots \quad (6)$$

Equation (6) shows that the true power delivered to an a-c current circuit depends not only upon the effective values of the current and voltage, but also upon the phase angle between the two. The expression EI is sometimes called the *apparent power* or " volt-amperes "; the ratio between the true power and the apparent power, or $\cos \phi$, is called the *power factor* of the part of the circuit under consideration (see also §145).

Now, let both the current and the voltage be *non-sinusoidal*, although of the same frequency. Equation (3) still holds true, and P represents the true average power. But in order to define the power factor a further assumption is necessary. For example, let the effective values of the current and voltage, as measured, say with hot-wire instruments, be again denoted by I and E, and let the average true power, measured with a *wattmeter*, be P. Then by analogy with eq. (6), the power factor may be defined as

$$\cos \phi = P/(EI). \qquad (7)$$

Equations (6) and (7) are identical in form, but the angle ϕ in eq. (6) is an actual phase angle, whereas in eq. (7) it is a fictitious angle which refers to the *equivalent* sine values of current and voltage. The foregoing definition of the phase displacement ϕ with non-sinusoidal currents is not the only possible one. For another definition see the discussion on the measurement of reactive power (§107).

It will be seen from formula (3) that, in order to measure the true average power in an a-c circuit, an indicating instrument is required which automatically integrates the instantaneous products of current and voltage. Such an instrument is called an *indicating wattmeter*. Two types of such wattmeters are described below the electro-dynamometer type and the induction type.

ELECTRO-DYNAMOMETER-TYPE WATTMETER

95. An indicating wattmeter based on the electro-dynamometer principle (§§50 and 51) is shown in Fig. 93. The internal electrical connections are shown in Fig. 94. The instrument has two stationary coils and a moving coil between them, as in the voltmeter shown in Fig. 45. The movable coil is controlled by spiral springs, as in Fig. 38.

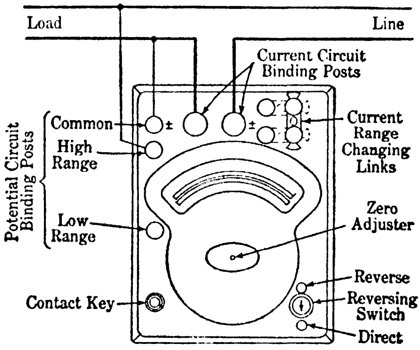

FIG. 93. External connections and general appearance of an indicating wattmeter.

The stationary coils consist of a few turns of heavy wire or strip connected in series with the circuit, as in an ammeter. The movable coil consists of many turns of fine wire, and is similar to the movable coil of a permanent-magnet moving-coil voltmeter. It is connected across the circuit, in series with a high non-inductive resistance. The current through the movable coil is practically proportional to, and in phase with, the voltage. The instantaneous torque between the stationary and the movable coil is proportional to the product of the instantaneous currents in them, and is thus proportional to the product of the current and the terminal voltage. According to eqs. (1) and (2), the *mechanical impulse* due to this torque (torque times an infinitesimal interval of time *dt*) is proportional to the instantaneous energy delivered to the circuit. If the movable part of the instrument had no inertia, it would vibrate in accordance with the fluctuations of instantaneous power. But with usual frequencies of alternating currents, these fluctuations follow in such quick succession that the movable element assumes the position corresponding to the *average* impulse; thus it automatically integrates the power over a cycle, according to expression (3).

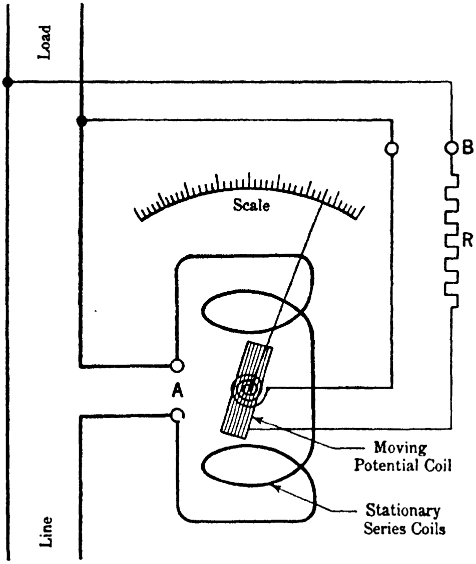

FIG. 94. The arrangement of parts in an indicating wattmeter.

With a non-inductive load, that is, when the current and the voltage are in phase, the torque is positive at all instants. On a partly inductive load the product of the instantaneous volts and amperes is negative over a part of the cycle (Fig. 123). Since the average torque in the instrument is counterbalanced by a spiral spring whose restraining torque is proportional to the deflection, *the indication corresponds to the true average power regardless of the power factor.* The same is true for non-sinusoidal currents and voltages of any wave-form.

The scale is divided directly in watts or in kilowatts (1 kilowatt = 1000 watts). The instrument may be calibrated with a standard wattmeter or with a standard d-c ammeter and voltmeter, and may be used on either direct or alternating current. The current terminals (Fig. 93) are connected in series with the circuit in which it is desired to measure the power. The potential terminals are connected across the line. In some portable precision instruments, the potential circuit should be kept open when no reading is being taken, so as not to overheat the movable coil and the multiplier.

A slight error is introduced by the increase in the resistance of the moving coil with the temperature, since the current through the potential circuit then becomes less with the same applied voltage. In some high-grade wattmeters, this error is compensated by means of a non-inductive shunt around the movable coil. This shunt is made of a material possessing a comparatively high resistance-temperature coefficient, so that with the increase in temperature a larger fraction of the total current flows through the moving coil. At high frequencies the temperature compensating shunt introduces a phase angle of sufficient magnitude to cause a slight error (§98). For further details of construction of dynamometer-type wattmeters see §101.

96. Correction for Power Consumption in the Wattmeter Itself. — An indicating wattmeter, such as is shown in Fig. 94, consumes some power in its series and potential circuits. The inaccuracy introduced thereby is in some cases large enough to require a correction. It will be easily seen from Fig. 94 that the current in the series winding of the wattmeter is the sum of the load current and the current consumed in the potential circuit of the wattmeter itself. Therefore, the instrument indicates more power than is actually consumed in the load. The corresponding correction is

$$i^2R = E^2/R \quad \ldots\ldots\ldots\ldots \quad (7a)$$

where i is the current in the potential circuit, R the ohmic resistance of this circuit, and E the load voltage. In accurate measurements, the resistance of the potential circuit and the load voltage may be ascertained and the above correction computed and subtracted from the wattmeter reading.

Sometimes the potential leads are connected to the "line" side of the current coils, as, for example, in the watthour meter shown in Fig. 108. In this case, the current coils carry the true load current, but the voltage across the potential terminals of the instrument is the line voltage and not the load voltage. If the resistance of the series winding is r and the load current is I, the correction for the power consumed in the instrument itself is equal to I^2r.

Before connecting a wattmeter in a circuit, it is advisable to estimate the possible error due to the power consumption in the instrument itself, and to decide whether the potential terminals should be connected across the load or across the line. Consider, for example, a 110-volt supply; at this voltage the current through the potential circuit of a wattmeter is of the order of 21 milliamperes, and consequently the power consumption is about 2.3 watts. Consider two wattmeters, one rated at 100 amperes and loaded at 100 amperes, the other rated at

1 ampere and loaded at 1 ampere. Assuming the resistance of the series circuit of the first wattmeter to be 0.5 milliohm and that of the second 0.7 ohm, we get the results shown in the following table. The first reading in each case refers to the wattmeter connected as in Fig. 93, and the second reading corresponds to the case with the potential coil across the line. A careful study of this table will show the difference in the conditions in the two cases. One should consider the corrections both in actual watts, and in per cent of meter readings.

POWER CONSUMPTION IN TWO WATTMETERS

Connection	(1) Load Current, Amperes	(2) Amperes through Current Coil	(3) Load Voltage	(4) Voltage across the Potential Circuit	(5) True Power, Watts	(6) Instrument Indication, Watts	(7) Correction	
							E^2/R Watts	I^2r Watts
Voltage coil across the load.........	100	100.021	110	110	11,000	11002.3	2.3	None
Voltage coil across the line..........	100	100	110	110.05	11,000	11005	None	5.0
Voltage coil across the load.........	1	1.021	110	110	110	112.3	2.3	None
Voltage coil across the line..........	1	1	110	110.70	110	110.70	None	0.70

It is usually preferred to have the wattmeter connected as in Fig. 94, and to correct for the loss of power in the potential winding. The reason is that it is more difficult to measure the resistance r of the series winding, and, moreover, it depends on the contact resistance at the current terminals.

97. Compensation for Power Consumed in the Instrument.—Some wattmeters are provided with a compensating winding which automatically corrects for the power consumption in the instrument itself, thus simplifying its use. Such a compensated wattmeter is shown in Fig. 95, the compensating coil being connected in series with the potential coil. Without the compensating coil, a small current flows through the series and the potential windings of the wattmeter (Fig. 94) even when the load is zero, and causes the pointer to indicate some power, although in reality the load power consumption is zero.

The number of ampere-turns in the compensating winding is equal and opposite to that created by the series coils, so that the pointer indicates zero at no load. The compensation is good at all voltages, since the current in the compensating coil is always the same as in the series winding (at no load). The potential circuit, between the terminals *C* and *B*, consists of the compensating coil, the movable potential coil, and a high resistance *R*.

In cases in which the series winding and the potential winding are supplied from two independent sources, the compensating coil should be left out of the circuit, and the terminal *D* used, instead of *C*. The "uncompensated," or "independent," terminal *D* is connected to the potential coil through a low resistance *r* equal to the resistance of the compensating coil. This is done in order to have the same total resistance between *B* and *D*, as between *B* and *C*. In this way the instrument calibrated between one pair of terminals reads correctly between the other pair of terminals.

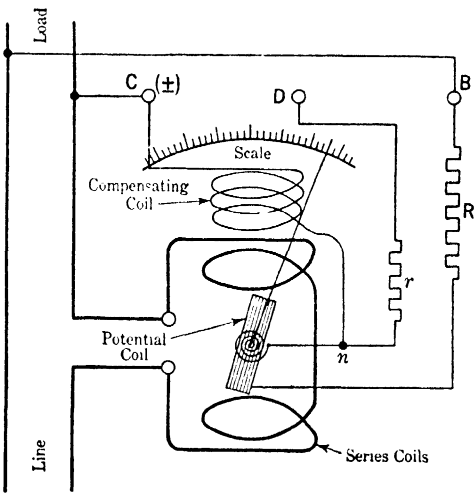

Fig. 95. Compensation for the power consumption in the wattmeter itself.

The uncompensated terminal *D* should be used, instead of *C*, when the wattmeter is being calibrated with a separate source of current for the series winding. Such is, for example, the case when a large storage cell is supplying the current, while a battery of small cells gives the required voltage. With an arrangement of this kind, a large wattmeter can be calibrated with a comparatively small expenditure of power. See also §133.

98. Inaccuracy at Low Power Factors. — Theoretically there should be no inductance in the potential circuit of a dynamometer-type wattmeter, in order that the current in the moving coil be in phase with the line voltage. In reality, the potential coil and the multiplier have a small amount of inductance, so that, strictly speaking, the current in the potential circuit lags slightly behind the applied emf. The inaccuracy thus introduced is usually negligible, especially at high values of power factor, but with very low power factors it may become quite noticeable.

The influence of the power factor is shown in Fig. 96. I is the vector of the current in the series coil. E_1 is the vector of the applied emf at a low value of the power factor of the load (large phase displacement ϕ_1). If the potential circuit had a resistance r but no inductance, the current in the moving coil of the wattmeter would have the value E_1/r and would be in phase with E_1. Under these conditions the indications of the instrument at a given current I would be proportional to the working component $On_1 = E_1 \cos \phi_1$ of the voltage. In reality, owing to the inductive reactance x of the potential circuit, the current in that circuit is lagging behind E_1 by a small angle α and is reduced to a value E_1/z, where $z = \sqrt{x^2 + r^2}$. The result is the same as if the applied voltage were reduced to the value $Oa_1 = E_1 \cos \alpha = E_1 r/z$ (Fig. 96), and had the working component Om_1. When the same reasoning is applied to conditions in the wattmeter at a high power factor, indicated by the small angle ϕ_2 between I and E_2, the points n_2 and m_2 are obtained. It is easy to see that, with the same current I, the same voltage E, and the same angle of lag α in the potential circuit, the proportional error, n_1m_1/On_1 at the low power factor is much larger than the error, n_2m_2/On_2, at the high power factor. When the angle α for a given instrument is known, readings can be corrected by eq. (9) or (10) below.

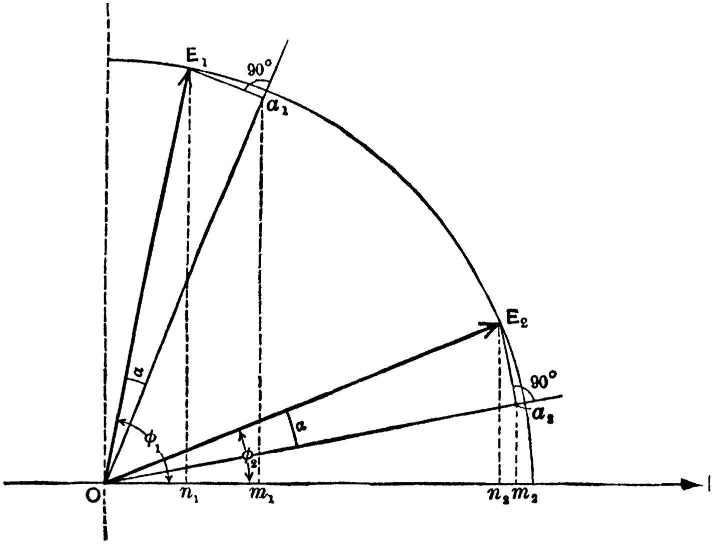

FIG. 96. Vector diagram, showing the influence of inductance in the potential circuit of a wattmeter.

To express the error analytically we can write, for any angle ϕ in Fig. 96:

$$Om = E \cos \alpha \cos (\phi - \alpha)$$
$$On = E \cos \phi$$

Hence the error in the ratio is

$$e = (Om - On)/On = \frac{\cos \alpha \cos (\phi - \alpha)}{\cos \phi} - 1 \quad . \; . \; . \quad (8)$$

Expanding, we find

$$e = \tfrac{1}{2} \sin 2\alpha \tan \phi - \sin^2 \alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

When α is small, say one or two degrees, and ϕ is of considerable magnitude, we can approximate $\sin 2\alpha = 2 \sin \alpha$ and $\sin^2 \alpha = 0$. The preceding expression is then simplified to the following *approximate* one

$$e = \alpha \tan \phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

where α is in radians. For example, when $\cos \phi = 0.6$, $\sin \phi = 0.8$, and $\tan \phi = 4/3$, the error per degree of α is

$$e = (\pi/180)(4/3) = 2.3 \text{ per cent}$$

Equation (10) shows the error of the wattmeter as a fraction of On (which, with the current I, gives the true power to the load) and of the power factor angle ϕ, which latter is not known until the error e is determined. It is quite sufficient ordinarily to calculate e on the basis of ϕ determined from the actual wattmeter reading divided by $E \times I$. A method preferred by some[1] for calculating the error in wattmeter readings is as follows:

Referring to Fig. 96 let $\phi_1 - \alpha = \delta$. The actual load is

$$P = E_1 I \cos \phi_1 = E_1 I \cos (\alpha + \delta) = E_1 I (\cos \alpha \cos \delta - \sin \alpha \sin \delta) \quad (11)$$

Since the current in the potential coil is reduced by the presence of the reactance x to r/z of the correct value, and lags E by the angle $\alpha = \cos^{-1} r/z$, the actual wattmeter reading is

$$W = E \frac{r}{z} I \cos (\phi - \alpha) = E \cos \alpha I \cos \delta \quad . \; . \; . \; . \; . \quad (12)$$

So that

$$\cos \delta = W/(EI \cos \alpha) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

From (13)

$$\sin \delta = \sqrt{1 - \cos^2 \delta} = \sqrt{(EI \cos \alpha)^2 - W^2}/(EI \cos \alpha) \quad (14)$$

---

[1] See "Measurement of a single-phase load at a low power factor," by A. Dovjikov, *Elec. Jour.*, July, 1923.

Substituting from eqs. (13) and (14) into (11), and remembering that $\sin \alpha = x/z$ and that $\cos \alpha = r/z$

$$P - W = -\frac{x}{r}\sqrt{(EIr/z)^2 - W^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or, since r/z is nearly unity,

$$P - W = -\frac{x}{r}\sqrt{(EI)^2 - W^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Equation (16) gives the error in watts between the true power, P, and the wattmeter reading, W, while the proportional error is

$$(P - W)/W = -\frac{x}{r}\sqrt{\frac{(EI)^2 - W^2}{W^2}} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Or

$$(P - W)/W = -\sin \alpha \sqrt{\frac{(EI)^2 - W^2}{W^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$= -\sin \alpha \sqrt{(EI - W)/W \times (EI + W)/W} \quad (19)$$

Under ordinary circumstances, and with a good instrument, the inaccuracy due to inductance is negligible, but under extreme conditions a correction, as in Fig. 96, may be necessary. When the dielectric loss in a high-grade condenser is being measured, and in similar cases in which the voltage and the current are practically in quadrature with each other and the measured power is extremely small for the values of current and voltage involved, special methods of power measurement may be necessary. The reader is advised to consult the Bureau of Standards or makers of accurate measuring instruments. He will also find considerable information in the publications of the principal electrotechnical and physical societies.

For corrections due to errors in instrument transformers see §100.

99. EXPERIMENT 4-A. — Calibration and Study of an Indicating Wattmeter of Electro-Dynamometer Type. — The instrument is described in §§95 to 98; it is calibrated with direct current, and may be used on either direct or alternating current.

(1) Connect the wattmeter under test to a d-c line and provide a suitable load; into the same circuit connect an ammeter and a voltmeter which have been previously standardized. Begin the calibration with the lowest load; read volts, amperes, and watts. Gradually increase the load by approximately equal steps and take similar readings at each step. Now reverse both potential and current leads and repeat the calibration. In connecting the wattmeter keep in mind the explanations in §§96 and 97 and take properly into account the energy consumption in the instrument itself. To see the influence of this factor, use in turn the uncompensated and the compensated potential terminals (Figs. 93 and 95) and note the difference in the indications with the same load. See if the i^2R correction accounts for the discrepancy; try this with a large and with a small load.

(2) Connect the wattmeter in such a way as to have the error for the power consumption in the instrument itself first in one and then in the other form mentioned in §96. Make clear to yourself the order of magnitude of this error and the best way of connecting the wattmeter with a large and with a small current, and at a high and at a low voltage.

(3) Measure an a-c load at a constant current and voltage, and at a variable power factor, beginning with the highest. Devise a method for ascertaining whether the current is leading or lagging. Show how to determine whether power is flowing into the load, or the load is pumping power back into the line. Measure a large load at a low power factor by overloading the current and the potential coils of the wattmeter for a short time. Show that the error is much greater than at a high power factor. Devise some other method of measuring the power input into a load of very low power factor.

(4) Make a study of the influence of terrestrial magnetism and of stray fields, both direct and alternating current, upon the accuracy of the instrument. Try various combinations, such as a stray d-c field and a-c currents in the instrument, etc., and make clear to yourself, theoretically, which combinations should affect the indications and which should not.

Report. Give a calibration curve in the form shown in Fig. 73. State in detail your findings in regard to the energy consumption in the instrument itself and in regard to the influence of the terrestrial field and stray fields. Give data showing that the true power is less than the apparent power when the load is partly inductive. Figure out the power factor of the load according to eq. (7) in §94. State the difficulty experienced in measuring a load at a very low power factor and the method used for overcoming this difficulty.

100. Effect of Inaccuracies in Instrument Transformers upon a Wattmeter. — When a correctly calibrated indicating wattmeter or watthour meter is connected to a line through very accurate current and potential transformers (§56), its indications are simply multiplied by the nominal ratios of these transformers. When greater accuracy is required and when the transformer errors cannot be neglected, the wattmeter should be calibrated and its potential circuit adjusted in combination with the transformers. This is not always possible on account of heavy currents or high voltages involved, and for this reason it may be necessary to correct the wattmeter readings for the effect of errors in the instrument transformers. These errors are discussed in detail in Chapter XVIII. The errors in the ratio are taken directly from the calibration curves of the transformers, and the resultant error in the wattmeter due to this cause is very nearly equal to the sum of these errors. Thus, if the current ratio is 0.2 per cent high and the voltage ratio is 0.5 per cent high, the per cent error in the volt-amperes will be

$$(1 + 0.002)\,(1 + 0.005) - 1 = 0.002 + 0.005 + 0.00001$$

or very nearly 0.7 per cent, the last term being negligible.

The wattmeter error due to the phase angle of the transformers depends upon the algebraic difference of the phase angles of the current and voltage transformers respectively. For example, if the angular error due to the current transformer is 1° 40′ leading, and that due to the potential transformer is 30′ leading, the effect is the same as if the phase displacement between the current and the voltage were increased by 1° 10′. Let this *resultant* phase angle error be denoted by α, and let the correct phase angle between the line current and the line voltage in the primary circuit be ϕ, with the current lagging. The secondary phase angle, that is the one in the wattmeter circuit, is $\phi - \alpha$, and the wattmeter reads high. The correction ratio, to be subtracted from unity, is

$$\begin{aligned}\epsilon &= [\cos(\phi - \alpha) - \cos\phi]/\cos\phi \\ &= 2\sin(\phi - \tfrac{1}{2}\alpha)\sin\tfrac{1}{2}\alpha/\cos\phi \quad \ldots\ldots\ldots \quad (20)\end{aligned}$$

When α is small, say one or two degrees, and ϕ is a much larger angle, the foregoing expression is simplified *approximately* to

$$\epsilon = \sin\alpha\tan\phi \quad \ldots\ldots\ldots \quad (21)$$

or even to

$$\epsilon = \alpha\tan\phi \quad \ldots\ldots\ldots \quad (22)$$

where α must be in radians. See also §98.

The error in a current transformer is a function of the current itself, and the error due to the resultant phase angle is a function of the power factor ϕ of the load. Thus, the total error due to the inaccuracies in the current and potential transformers is variable and cannot be allowed for in the construction or adjustment of the meter itself. For this reason it is essential to keep the errors in instrument transformers at a minimum.

101. Construction Details of Dynamometer-Type Wattmeters. — Most indicating wattmeters used for practical measurements are of the direct-deflection type described above. For very accurate work, or for calibration purposes, dynamometer-type wattmeters of the torsion type are also used, similar to the Siemens dynamometer shown in Fig. 47 except that the movable coil is in the potential circuit and consists of many turns of fine wire, so that no mercury cups are necessary. The current through the moving coil is so small that it can be easily conveyed by soft metal leads or by the suspension wires.

As a primary watt standard, a Kelvin-type balance instrument is sometimes used, similar to the ampere balance described in §92, except that the movable coils are wound of many turns of fine wire and used as potential coils of a wattmeter. The attraction and repulsion between two sets of coils, caused by the currents in the coils, is measured by balancing it against a weight or the torsion of a spiral spring. If a weight is used, it is arranged to be moved along a scale, and its distance from the fulcrum, when equilibrium is established, is a measure of the torque between the sets of coils. If a spring is used, it is put in tension by means of a knob carrying a pointer that moves over a circular scale. The scale in either case is marked in watts. The weight or the spring is adjusted to bring a pointer (attached to the moving coils) to the zero position. The motion of the moving coils is very slight, making the bearing wear negligible; the coils are always in the same position when in balance, eliminating possible errors due to varying magnetic conditions in different positions; the coils are close together, eliminating errors due to external magnetic fields; no iron is used in the magnetic circuit, precluding frequency errors.

For measurement of very small amounts of power, a direct-deflection electro-dynamometer is sometimes used in which the moving coil is provided with a mirror, and the deflection is read by means of a telescope on an illuminated scale, as in a mirror-type galvanometer. In very accurate instruments of this type, no metal is used in the supporting parts so as to avoid eddy currents. The current coils are made of stranded copper to minimize eddy currents.

Accurate wattmeters for calibration purposes are sometimes of the astatic type, that is, with a double movement similar to that shown in Fig. 65. The effect of the external fields is thereby minimized. High-grade magnetic shielding is a more positive protection against irregular external fields and should be used whenever possible. This is of particular importance in measurements at a low power factor at which the moving torque is comparatively small. To increase the deflection a weak controlling spring or metal strip is used for suspension.

When using an indicating wattmeter at a low power factor, it is sometimes necessary to overload its potential or current circuit or both in order to obtain a sufficient deflection. For example, a 10-ampere, 150-volt wattmeter will give a full-scale deflection of 1500 watts at these values of current and voltage on a non-inductive load. At a power factor of 10 per cent the deflection with the same values of current and voltage will be only 150 watts, which may not be sufficiently accurate. For such a measurement a 5-ampere, 75-volt wattmeter is preferable, provided that its coils can stand 100 per cent overload for a short time. The full scale of the latter wattmeter corresponds to 375 watts, so that 150 watts can be read satisfactorily. For this reason the coils of many indicating wattmeters on the market are guaranteed to stand a double current for a short time. Special wattmeters are also made for measurements at low power factor and with a correspondingly higher current consumption in the potential circuit. For the necessary precautions and errors in measuring power at low values of power factor, see §§98 and 100 above.

The range of a given a-c wattmeter may be increased indefinitely by the use of current and potential transformers (§56 and Chapter XVIII). As a matter of fact, most industrial wattmeters are of the 5-ampere, 110-volt type, even though the scale may read thousands of kilowatts. For most practical purposes the errors due to these transformers are negligible, but in special cases and for accurate measurements they must be carefully considered (§100).

For power measurements in a polyphase circuit, two or more single-phase wattmeters may be used, or one wattmeter transferred from phase to phase, as shown in §60. A more convenient way is to use a so-called polyphase wattmeter which consists of two movements, such as are described above, mounted on the same shaft so that their indications are automatically added and the sum is read on the scale. It is also possible to mount the two movements side by side and have them connected by a light aluminum rod. The so-called " two-wattmeter " method of polyphase power measurement is explained in §105.

The stationary coils of wattmeters used for the measurement of very large currents are sometimes made hollow and cooled by circulating water, thereby increasing their current-carrying capacity as much as ten times. This method is resorted to for very accurate measurements, especially at low values of power factor when the errors due to a current transformer are inadmissible.

In high-grade indicating wattmeters, the entire coil system is mounted in a closed iron case which effectually shields it from external magnetic fields and electrostatic influences. In relatively high-tension measurements it is of importance to connect one end of the potential circuit to the line in which the current coils are connected, in order to avoid a large difference of potential within the instrument and an electrostatic action between the coils. A large difference of potential occurs then only in the multiplier resistance outside the instrument.

In most instruments of the electro-dynamometer type the natural damping is insufficient and an additional damping is usually provided by means of a light aluminum vane connected to the moving system and placed in an enclosed box. Graphic or curve-tracing wattmeters are usually of the electro-dynamometer type, and are similar in their general construction to the graphic ammeters and voltmeters described in §§61 to 63.

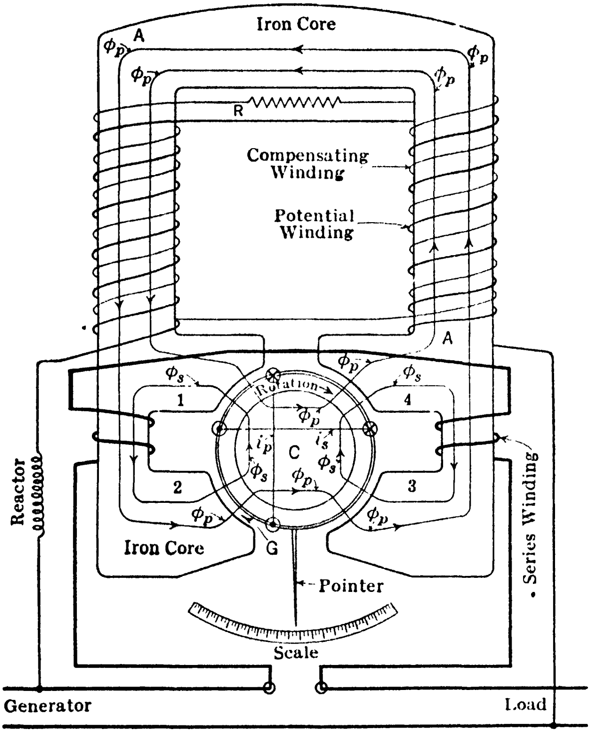

Fig. 97. An induction-type indicating wattmeter.

INDUCTION WATTMETERS

102. For a-c switchboard service, where no extreme accuracy is required, wattmeters based upon the principle of a revolving magnetic field are sometimes used. The magnetic and electric circuits of such an *induction-type wattmeter* are shown in Fig. 97. The iron core AA consists of thin laminations to reduce eddy currents, and the magnetic circuit is completed through a cylindrical iron core *C*. In the annular air-gap *G* between these two cores an aluminum drum is mounted in which secondary currents are induced by the alternating fluxes in the pole-pieces. Because of the reaction of these currents upon the inducing fluxes (§103) the drum tends to rotate. Its deflection is controlled by a spiral spring and is indicated on the scale. There are three windings: the series winding through which the main line current flows; the potential winding connected across the line in series with a high reactance; a compensating winding closed upon itself through an adjustable resistance *R*. Very often these instruments are not connected to the line directly, but through potential and current transformers (Chapter XVIII).

With a non-inductive load, the flux due to the shunt winding must be in time quadrature with that due to the series winding, because under those conditions the torque exerted upon the aluminum drum is a maximum. The current in the potential winding cannot be made to lag behind the terminal voltage by exactly 90 degrees, because of the unavoidable resistance both in the winding itself and in the reactor in series with it. For this reason the compensating winding, short-circuited through the resistance *R*, is used. By varying this resistance, the secondary current is adjusted to such a value that the flux in the core, due to the combined magnetomotive force of this current and of that in the potential winding, lags exactly 90 degrees behind the terminal voltage. If a two-phase supply is available, a non-inductive load may be connected to one phase through the series winding of the wattmeter, and the potential circuit to the quadrature phase. Resistance *R* is then adjusted until the pointer shows zero. Under these conditions the wattmeter will give a maximum indication when the potential circuit is connected to the same phase as the load. A wattmeter so adjusted at zero power factor should give readings proportional to the value of power factor, because only the components of the two fluxes in time quadrature with each other produce a torque upon the aluminum drum.

The action of the secondary winding in preventing temperature errors is the same as in the ammeter and voltmeter based on the induction principle (§54). The voltage circuit consists partly of copper and partly of an alloy having zero temperature coefficient. The amount of each is so proportioned that the resistance drop varies at the same rate as the reactive drop, thus maintaining a constant angle of lag at all temperatures.

The instrument is somewhat affected by the frequency and by the wave-shape of the current and voltage, but is accurate enough for switchboard service. Its long scale, extending over nearly 300 degrees, and a rugged construction with no leads going into the moving part, constitute additional advantages. It has to be calibrated by means of a dynamometer-type wattmeter (§95), which in turn is standardized on direct current.

For polyphase circuits a similar wattmeter consists of two single-phase movements such as those described above, electrically and mechanically independent, but with the two aluminum drums mounted on the same shaft. The total torque on the shaft is equal to the sum of the torques of the two movements, and the meter automatically adds and indicates the total power in the two phases. An iron shield between the two elements prevents stray fields set up by one element from affecting the torque of the other, and the meter is not affected by any unbalancing of a polyphase circuit. For further details see §105 below and also Vol. II.

103. Theory of Induction-type Wattmeter. — Consider first the case of a non-inductive load, with the compensating resistance *R* properly adjusted so that the potential flux ϕ_p and the series flux ϕ_s (Figs. 97 and 98) are in time quadrature with each other. Each of these fluxes induces secondary currents in the aluminum armature, and the torque is due to the interaction of the flux ϕ_p with the current i_s, and of ϕ_s with i_p.

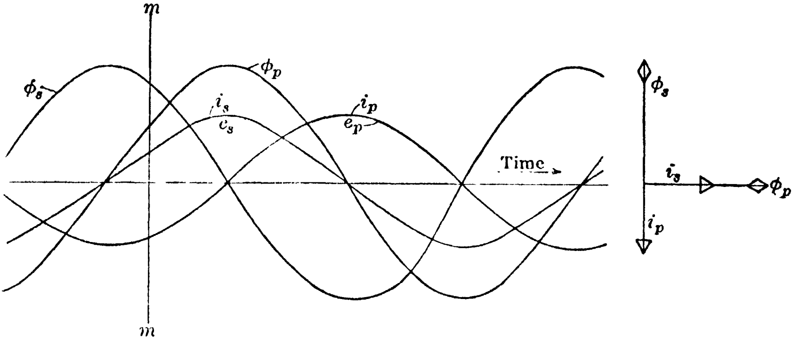

Fig. 98. The currents and fluxes in the wattmeter shown in Fig. 97.

Consider first the action of the flux ϕ_s, assuming it to be sinusoidal in time. It induces in the drum an electromotive force $e_s = -d\phi_s/dt$ in a lagging time quadrature with ϕ_s, as shown by the e_s curve in Fig. 98. The current due to this emf is lagging behind it by a certain angle, which depends upon the relative resistance and reactance of the paths in the drum. This current may be resolved into a component i_s in phase with e_s, and another in quadrature with e_s. The latter is not shown in the figure, because it does not contribute to the useful torque. Similarly, the flux ϕ_p induces a voltage e_p, and of the resulting current only the in-phase component, i_p, is shown as a sine-wave. The same relations are shown vectorially to the right.

We shall assume that at the instant of time marked *mm* the fluxes ϕ_p and ϕ_s have the directions indicated in Fig. 97. These directions are arbitrary and depend upon the direction in which the corresponding coils are wound and connected to the circuit. At the instant under consideration, the flux ϕ_s is decreasing, and, therefore, the corresponding secondary current i_s is shown linking with it and so directed as to strengthen it. On the other hand, the flux ϕ_p is increasing, and the induced current i_p is shown linked with it and opposing the flux. By applying the familiar rule for the mechanical force between a flux and a current, it will be found that the action of i_s and ϕ_p is such as to tend to rotate the drum clockwise, and the action of ϕ_s and i_p is in the same direction. It will be seen from Fig. 98 that ϕ_p and i_s pass through zero at the same instant, so that the torque due to these two merely fluctuates between zero and a maximum, but never reverses. The same is true of ϕ_s and i_p. Thus, the direction of the torque deduced for a particular instant is true for any instant; that is, the tangential mechanical effort is cumulative. The drum assumes the position at which the counter torque of the controlling spring is equal to the average electromagnetic torque. The disregarded quadrature components of the two currents, with the corresponding fluxes, give alternating torques, the average value of each being zero.

Now let the load be partly inductive. The main current may be considered as consisting of the component $I \cos \phi$ in phase with the voltage, and $I \sin \phi$ in quadrature with it. The component $I \cos \phi$ acts in the wattmeter as a non-inductive current, and therefore produces upon the moving element a torque proportional to the true power $EI \cos \phi$. It remains to be shown that the quadrature component $I \sin \phi$ contributes no torque. For this purpose the sine-waves and the vectors of ϕ_s, e_s, and i_s in Fig. 98 must be shifted by 90 degrees in one or the other direction, according to whether the current is leading or lagging. Then by repeating the reasoning given above it may be shown that the individual torques either give the average value equal to zero or else the two torques oppose each other. Thus, theoretically at least, the instrument calibrated at unity power factor reads the true power at any other phase angle.

Should the impressed frequency be increased, the current and the flux in the potential circuit would be reduced and the compensation would no longer be correct. The currents induced by both fluxes in the aluminum drum would also be different. Thus, generally speaking, the calibration depends upon the frequency. It is possible, however, so to proportion the instrument that the calibration would remain sensibly constant within the range of frequency variations encountered in power-plant operation. A non-sinusoidal applied voltage or current may be considered as a superposition of a fundamental sinusoidal quantity and of various harmonics of higher frequencies. Inasmuch as the calibration of the instrument is affected by the frequency, it cannot give correct readings on a circuit of any arbitrary wave-form, as is approximated by the dynamometer wattmeter. However, by proper design the instrument may be made to read with a sufficient degree of accuracy within the usual range of wave-form, or it may be specially calibrated for a particular wave-form.

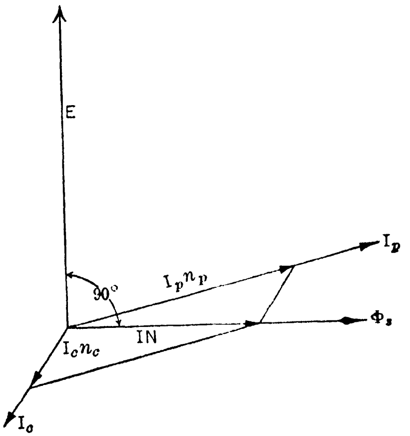

Fig. 99. The corrective action of the compensating winding in the wattmeter shown in Fig. 97.

The diagram in Fig. 99 explains the corrective action of the compensating winding. E is the vector of the applied voltage and I_p is the current in the potential winding which lags behind the voltage by less than 90 degrees. The current in the compensating winding is I_c. The corresponding numbers of turns are n_p and n_c. The combined magnetomotive force, IN, of the two windings is obtained as the diagonal of the parallelogram constructed on $I_p n_p$ and $I_c n_c$. The flux ϕ_s is due to IN. It will be seen that, by properly selecting the proportions, the flux ϕ_s can be made to lag behind E by exactly 90 degrees, even though I_p is not in quadrature with E. It is of advantage to have the secondary load R consist of a non-inductive resistance so as to have the current I_c as nearly as possible in phase opposition with E and thus to affect the primary excitation $I_p n_p$ by as large an angle as possible. The diagram in Fig. 99 is an ordinary transformer diagram (Fig. 283).

104. EXPERIMENT 4-B. — Calibration and Study of an Induction-Type Wattmeter. — The instrument is described in §§102 and 103. Provide a suitable load in which magnitude and power factor may be varied at will. A phantom load (§142) and a phase-shifting transformer (§141) will be found convenient. Have an ammeter, a voltmeter, and a standardized dynamometer-type wattmeter in the circuit with the induction-type wattmeter being studied. For greater accuracy it may be desirable to have two standard wattmeters, one for the higher and one for the lower range of the instrument to be calibrated. A power-factor meter (§118) would save time in making load adjustments. The source of power must be nearly sinusoidal, and its frequency must be kept constant for each value at which a test is made. Arrange the instruments in the circuit in such a way that their own power consumption will have the least influence upon the results and that a correction can be made for this power consumption (§§96 and 97). (1) If a two-phase circuit is available check the phase compensation and set it right. Otherwise adjust the instrument to read correctly at the lowest possible power factor and also at unity power factor. Show experimentally that the influence of this adjustment is much greater at low values of power factor. (2) Adjust the load so as to have the highest safe value of current and the lowest possible power factor. Read all the instruments. Reduce the current in steps, keeping the phase angle nearly constant, and take similar readings at each step. (3) Take similar runs at one or two higher values of power factor and finally at non-inductive load. (4) Investigate the effect of frequency and of wave-form upon the calibration of the instrument. (5) Note discrepancies in the construction, if any, between the instrument tested and the one described in the text above.

Report. (1) Give the diagram of connections used and the form of the data sheet. (2) Plot curves of percentage of error against true watts as abscissas. (3) Describe the adjustment of the compensating resistance and give data showing its influence upon the accuracy of the meter under different load conditions. (4) Give data showing the influence of the frequency and wave-form upon the instrument error. (5) Referring to Figs. 97 and 98, explain in detail why reversing either the current or the potential leads makes the armature deflect in the opposite direction. (6) Describe differences, if any, between the instrument tested and the one shown in the text.

105. The Two-Wattmeter Method. — Let a source of electric power (Fig. 100), direct or alternating, be connected to a steady load by means of three conductors, and let it be required to measure the average power delivered to the load. This can be done by means of two wattmeters, *A* and *B*, connected as shown in the figure. Conductor 2 is taken to be the "common return wire" for the other two. Wattmeter *A* measures the power between the conductors 1 and 2, and wattmeter *B* that between 3 and 2. The total power delivered to the load is equal to the algebraic sum of the two readings. If the source of power delivers direct current and the load is constant, the wattmeter readings represent the actual energy delivered to the load per unit time. With alternating or variable currents and voltages, the wattmeter readings give the *average* power or the average rate at which the energy is delivered.

The method of connections shown in Fig. 100 is usually referred to as the *two-wattmeter method,* and it gives the correct result for the total power in a three-wire system with any kind of currents and voltages, balanced or unbalanced. The only evident limitation is that each wattmeter in itself must be sufficiently accurate for that particular kind of current and frequency.

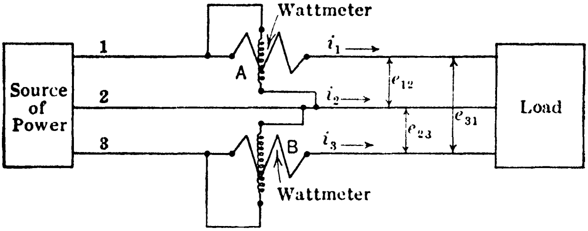

Fig. 100. The two-wattmeter method.

It is immaterial which of the three conductors is chosen as the return conductor. The individual readings will be different, but the algebraic sum is the same. If the load consists of pure non-inductive resistances, and if both wattmeters, *A* and *B*, are so connected as to read in the positive direction, the total power is equal to the arithmetical sum of the readings. However, when the load is partly inductive, or contains a counter emf, it is possible for one of the readings to become negative. Then it is necessary to reverse the terminals of that particular wattmeter and to take the difference of the two readings as the total average power delivered to the load. If the negative reading is greater than the positive, then the power is being " pumped back " to the assumed source of supply. This is possible, for example, with two interconnected power stations.

For this reason, when using the two-wattmeter method on unbalanced or inductive loads, or between two sources of power, it is of importance to determine in advance the polarity of the terminals of the instruments. This can be done by connecting each wattmeter to a resistance load in such a way as to obtain a positive deflection of the pointer. If with the same terminal sequence the meter reads positive on an unknown load, its reading is to be taken as positive, indicating that power is actually delivered to the load, and not returned to the source. Otherwise the terminals are reversed to read positive, and the reading itself is considered negative.

In practice, two separate single-phase wattmeters are seldom used. A polyphase wattmeter which contains two wattmeter movements on the same shaft is preferable, because the reading gives the total power directly, provided that the terminals are properly chosen, as explained above. If the torque of one movement is positive and that of the other is negative, the instrument automatically registers the difference of the two, provided that the positive indication is greater. If a polyphase wattmeter is not available, a single-phase instrument is used with a polyphase board (§60) so arranged as to read the power in the leads 1–2 and 3–2 in succession.

The two-wattmeter method is largely used for measuring power in three-phase circuits, where, when the load is partly inductive, the two wattmeter readings are different from each other, *even though the currents and the voltages are perfectly balanced.* With sinusoidal currents and voltages, when the power factor of the load is 50 per cent, one of the readings becomes zero, and, at lower values of power factor, becomes negative. This is of importance in measuring the input of an induction motor at light loads, when the power factor may be quite low. The proper sign of the two readings must be carefully determined. The following check upon the sign of the smaller wattmeter reading applies upon balanced loads: when the ratio $W/(EI)$ for the meter having the greater reading is less than 0.866 the sign of the smaller wattmeter reading is negative.[1] For further details regarding the measurement of power in polyphase circuits see Vol. II. The information given above and the theory given in the next section are sufficient for the power measurements in tests on alternators and induction motors treated in this volume, and for the connections of watthour meters considered in the next chapter.

106. Theory of the Two-Wattmeter Method. — The systems of connections used within the source of power (Fig. 100) and within the load are unimportant to this discussion. Since electricity behaves like an incompressible fluid, the sum of the line currents must be zero, that is

$$i_1 + i_2 + i_3 = 0 \quad \ldots\ldots\ldots\ldots \quad (23)$$

provided that all the currents are considered positive in the same direction, as indicated by the arrows (Fig. 100). Equation (23) applies

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[1] H. K. Humphrey, A method for determining the sign of the smaller wattmeter reading in balanced three-phase power measurements, *Jour. A. I. E. E.*, May, 1926.

equally well to the instantaneous values of the currents or to the vectors of currents in an a-c system.

The voltages between the wires are denoted by two subscripts, the former denoting the wire *assumed* to be at the higher potential. The current i_1, in reaching conductor 2, performs work corresponding to the difference of potential e_{12}. The wattmeter A automatically indicates the *average* power between conductors 1 and 2, or $P_A = \frac{1}{T}\int i_1e_{12}\,dt$. Similarly, wattmeter B gives the average power $P_B = \frac{1}{T}\int i_3e_{32}\,dt$, so that the total power P delivered to the load is

$$P = \frac{1}{T}\int (i_1e_{12} + i_3e_{32})\,dt \qquad (24)$$

It is of importance to note that the subscripts of the voltage between conductors 3 and 2 are marked in that order, whereas in the sketch the same voltage is marked with the subscripts 2–3. The latter marking corresponds to the *cyclic* order and leads to the equation

$$e_{12} + e_{23} + e_{31} = 0 \qquad (25)$$

or to the vector sum

$$E_{12} + E_{23} + E_{31} = 0 \qquad (25a)$$

If now a new pair of wattmeter readings are taken with the wire 3 as the common return, the series coils of the wattmeters in wires 1 and 2, the expressions under the integral sign in eq. (24) must be changed to $(i_2e_{23} + i_1e_{13})$. It is proposed to show that the two expressions are identical.

Using eqs. (23) and (25),

$$\begin{aligned} i_2e_{23} + i_1e_{13} &= -(i_1 + i_3)e_{23} + i_1(e_{12} + e_{23}) \\ &= i_1e_{12} - i_3e_{23} = i_1e_{12} + i_2e_{32} \end{aligned}$$

The two expressions are thus identical, and the same total power is obtained no matter which of the three conductors is selected for the common potential terminal.

It may be shown from the vector diagram for a three-wire a-c system that the sum of the two wattmeter readings equals the total power under all conditions of load. Consider first three load devices connected between the three lines and a common neutral O. Figure 101 shows the vectors of the currents and voltages in an unsymmetrical three-wire system with unbalanced inductive loads. Wattmeter 1 is connected in line 1 and between lines 1 and 2, while wattmeter 2 is connected in line 3 and across 3–2. The vectors E_{01}, E_{02}, and E_{03} represent the voltages between the three lines and the neutral of the load. The readings of the wattmeters are

$$P_1 = I_1E_{21}\cos\phi_1, \qquad \text{and} \qquad P_2 = I_3E_{23}\cos\phi_2$$

But $E_{21} = E_{01} - E_{02}$, and $E_{21}\cos\phi_1$, the projection of E_{21} upon I_1, equals the sum of the projections of E_{01} and $-E_{02}$ upon I_1, or $E_{21}\cos\phi_1 = E_{01}\cos\theta_1 - E_{02}\cos(\alpha - \theta_1)$.

Similarly, $E_{23} = E_{03} - E_{02}$, so that $E_{23}\cos\phi_2$, the projection of E_{23} upon I_3, equals the sum of the projections of E_{03} and $-E_{02}$ upon I_3, or $E_{23}\cos\phi_2 = E_{03}\cos\theta_3 - E_{02}\cos(\delta - \theta_3)$.

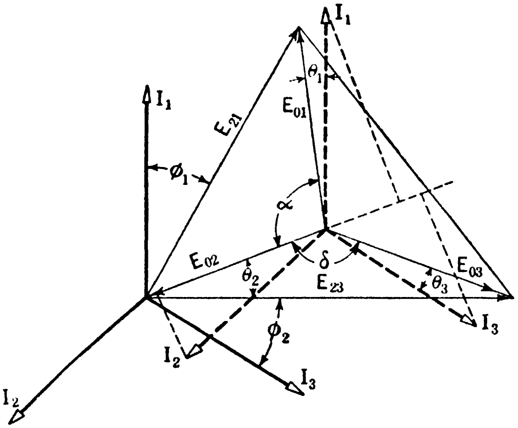

FIG. 101. Vector relations in the measurement of power by the two-wattmeter method.

Substituting these values in P_1 and P_2 and adding the readings

$$P_1 + P_2 = I_1E_{01}\cos\theta_1 + I_3E_{03}\cos\theta_3 - E_{02}[I_1\cos(\alpha - \theta_1) + I_3\cos(\delta - \theta_3)] \quad (26)$$

But $I_1\cos(\alpha - \theta_1) + I_3\cos(\delta - \theta_3)$ represents the sum of the projections of I_1 and I_3 upon E_{02}, as may be seen from the figure. Now, since $I_1 + I_3 = -I_2$, the sum of these projections upon E_{02} must equal the projection of $-I_2$ upon $E_{02} = -I_2\cos\theta_2$.

Substituting this value in eq. (26), one gets

$$P_1 + P_2 = I_1E_{01}\cos\theta_1 + I_2E_{02}\cos\theta_2 + I_3E_{03}\cos\theta_3 \quad . \quad . \quad (27)$$

which is the total power in the three branches of the load under all conditions. Therefore, the sum of the readings in the two-wattmeter method gives the total power in any three-phase (or three-wire) system. For a further study of methods of power measurement in polyphase systems see Vol. II, Chapter XL.

The foregoing theory and method of connection may be extended to any number n of conductors, instead of three. The required number of wattmeters is $n - 1$; one of the conductors is considered as the common return and is used for the potential terminal of all the meters. A three-phase system with an additional return conductor, or with the neutral points grounded, is a four-wire system, and three wattmeters are normally required to measure its total power. It is possible, by the use of three current transformers, to measure power in a four-wire system by means of two wattmeters. (See Handbook for Electrical Metermen, Fig. 77.)

107. Reactive-Component Wattmeter. — Let a dynamometer-type or induction-type wattmeter (§§95 and 102) be connected in the usual way to measure the true power in a partly inductive single-phase circuit, with the current and voltage strictly sinusoidal. Let the instrument reading in the usual notation be

$$P = IE \cos \phi. \qquad (28)$$

See Fig. 102 and eq. (6) in §94. Now let, the same wattmeter be connected in the following way: Its current coils are to be in series with the main circuit, but its potential circuit is to be connected to a source of alternating voltage identical with the former voltage but displaced with respect to it in time by 90 electrical degrees. The current through the potential winding is thereby displaced by 90 degrees, and is now in phase with the component $I \sin \phi$ of the load current. Thus the reading of the instrument is

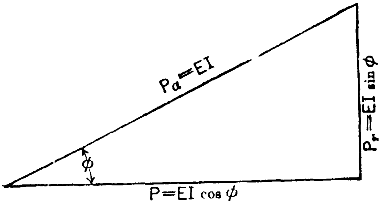

FIG. 102. True, reactive, and apparent power.

$$P_r = IE \sin \phi \qquad (29)$$

The quantity P_r is known as the *reactive power*, and is a measure of the rate at which the magnetic or electrostatic energy is surging between the load and the source of power. The value of $\sin \phi$ is called the *reactive factor*.

With P and P_r represented as the two sides of a right-angle triangle (Fig. 102), the hypotenuse gives the *apparent power* or volt-amperes

$$P_a = EI \qquad (30)$$

Also

$$P_a = \sqrt{P^2 + P_r^2} \qquad (30a)$$

The value of P_a can be determined directly as a product of the ammeter and voltmeter readings.

From the triangle (Fig. 102), we have

$$\tan\phi = P_r/P \qquad (31)$$

and

$$\cos\phi = P/P_a \qquad (31a)$$

Therefore, the phase angle ϕ and the power factor, $\cos\phi$, may be determined from the two wattmeter readings, P and P_r. The apparent power may also be computed as

$$P_a = P/\cos\phi = P_r/\sin\phi \qquad (32)$$

With a *non-sinusoidal* current and voltage, a wattmeter connected in the usual way still gives the true average power. If, however, we again connect it as before, with its potential winding quadratured, the new reading has to be *defined* as the reactive power. Thus we again get the same right-angle triangle, from which the values of ϕ and P_a may be readily computed. It must be clearly understood, however, that these are merely *defined* values and not unique physical values, as is the true power.

With non-sinusoidal currents at least two other definitions of the quantities P_r, P_a, and ϕ are possible: (a) to take P_a as the product of the actual ammeter and voltmeter readings, and to construct a right-angle triangle upon the values of P and P_a; (b) to take $\cos\phi$ from the indication of a reliable power-factor meter (§109) and to construct a right-angle triangle using the experimental values of P and ϕ. These two procedures give different values for the remaining parts of the triangle. Thus, with non-sinusoidal currents, the exact values of the reactive power, power factor, and apparent power depend upon the adopted definitions and upon the measuring instruments used. With sinusoidal currents there is but one set of values, and with currents which depart but moderately from a sine law, the discrepancies are not great enough to impair the usefulness of the concept of reactive power for practical purposes. See also §114 below.

A wattmeter connected to measure the reactive component of power gives a switchboard attendant useful information about the condition of his circuits and about the amount of idle circulating current. Sometimes he is in a position to reduce the reactive lagging current or even to change it to a leading component. For such cases the reactive wattmeter is provided with a scale having its zero in the center, the two sides being marked "leading" and "lagging." For polyphase connections of reactive power meters, see Vol. II.[1]

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[1] See, also, "Measuring the reactive component," by John Auchincloss, *G. E. Rev.*, April, 1924.

108. EXPERIMENT 4-C. — Study of an Indicating Reactive-Power Meter. — The theory is explained in the preceding section. Provide a source of two-phase power, nearly sinusoidal, from an alternator, or a rotary convertor, or by transforming to two-phase from a three-phase supply (see phase transformation in Vol. II). Take an ordinary indicating wattmeter and provide a load such that the current can be kept constant while the power factor is varied at will. Connect a double-throw switch in the potential circuit of the wattmeter, to enable it to be connected at will to either phase. The provisions must be such that the voltage in the auxiliary phase is kept strictly equal to and in exact quadrature with that in the loaded phase. Have an ammeter and a voltmeter in the circuit, and if possible a power-factor or phase-angle meter (§109). (a) Begin with a nearly non-inductive load; read volts, amperes, power factor, watts, and reactive watts. Reduce the power factor of the load in steps, keeping the voltage and the total current constant, and take similar readings at each step. (b) If possible, repeat the run, using a badly distorted source of voltage, or a load that distorts the current wave, such as an arc lamp or a rectifier. (c) Show that the sign of deflection of the reactive-power meter is reversed when changing from a leading to a lagging current.

Report. (1) For a few sets of readings, construct the triangle shown in Fig. 102, using different combinations of observed quantities, for example, EI and P, ϕ and P_r, P and P_r, etc. With a sine-wave voltage and current, the result should be nearly the same. (2) Give a few triangles or computed values to show the discrepancies with non-sinusoidal waves. (3) Explain theoretically, by drawing a-c vectors, that the indication of a reactive-power meter is reversed when the current changes from a lagging to a leading one.

POWER-FACTOR METERS

109. Some operating features of a-c power plants and transmission lines depend essentially upon the *power factor* of the load (§94). If the switchboard were provided with all the necessary ammeters, voltmeters, and wattmeters the power factor could be computed, but it would be a tedious process to figure this out periodically. With the usual fluctuating load it is especially convenient to have an indicating or graphic instrument which shows the power factor *directly* without computation. Two types of power-factor meters are used in practice, viz., the ***iron-vane*** type and the ***electro-dynamometer*** type. These instruments are built both for single-phase and for polyphase circuits, but in a polyphase circuit their indications have a definite meaning only when the voltages and the load are perfectly balanced. At this writing there is no universally accepted definition of power factor for an unbalanced load.

110. Iron-Vane-Type Power-Factor Meter. — The principle of this power-factor meter is shown in Figs. 103 and 104. In a three-phase meter the stationary part consists of a laminated iron core A, provided with a regular three-phase winding (Fig. 354) connected to the line through the current transformers T. The movable part consists of a soft-iron vane V, of the shape shown in Fig. 104, and magnetized by a stationary coil P, connected across one of the phases of the circuit, either directly or through a potential transformer. The currents in the polyphase winding of the instrument, being induced through the current transformers, are practically in phase with the line currents. The current in the coil P bears a definite phase relation to the line voltage. Therefore the position of the vane will depend *on the phase relation* between the line current and the line voltage, or, which is the same, *on the power factor* of the load. The instrument is calibrated directly in percentage power factor, the calibration extending over all the four quadrants of the scale. This is necessary because currents may be leading or lagging, and the phase angle between them and the voltage will be less or more than 90 degrees, according to whether the power is being transmitted in one or the other direction.

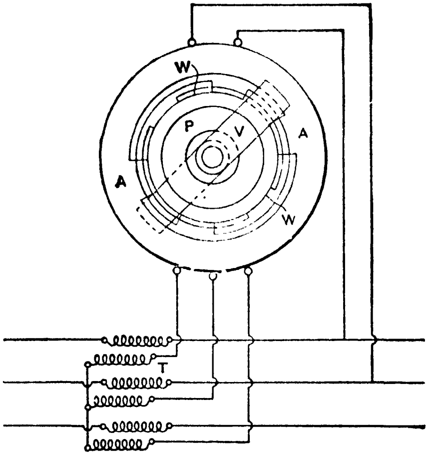


FIG. 103. An iron-vane power-factor meter.

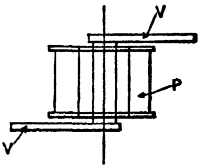


FIG. 104. The potential coil and the movable vane of the power-factor meter shown in Fig. 103.

In order to understand the action of this meter, the reader should make clear to himself that the stationary winding produces a rotating magnetic field (§484). This revolving field may be thought of as being due to two fictitious alternating fields, at right angles in space and in time quadrature with each other (§507). For one of the fictitious exciting coils the space position, as well as the corresponding time phase, is arbitrary, and we choose it so that the current will be in phase with that in the potential coil *P*. The iron vane will then orient itself in the direction of this field, since the force of attraction between two alternating fields in phase with each other is a maximum. The average force of attraction between the iron vane and the second alternating field is nil, the two fields being in time quadrature with each other; this force alternately becomes an attraction and an equal repulsion. If the time phase between the line current and the voltage should change, the position of the two fictitious coils in space would also change, and the iron vane would assume a new position.

For two-phase circuits the instrument is provided with two windings in space quadrature, instead of three. For a single-phase circuit an instrument is used with two windings, also in quadrature, connected as in a split-phase induction motor (§484). One of the windings is connected across the line in series with a high inductance and the other in series with a high resistance, so that a time-phase displacement between the two currents is obtained, as is necessary for the production of a revolving magnetic field. The above-described power-factor meter is by no means an accurate measuring instrument; its indications are affected by the magnitude of the line current and by the line voltage. But it is useful for a comparative judgment of the variations of power factor during the day, and for regulating the machines so as to maintain approximately a desired power factor.

111. EXPERIMENT 4-D. — Study and Calibration of an Iron-Vane Power-Factor Meter. — The instrument is described in the preceding section. Provide a three-phase load (see §442), and have an ammeter, a voltmeter, and a wattmeter connected in the circuit by means of a polyphase board (§60). Connect the power-factor meter as in Fig. 103 and apply the largest safe inductive load. Read the four instruments, and gradually increase the power factor of the load, keeping the same total current, until the load becomes non-inductive. Repeat the same test with two or three smaller values of current. If an over-excited synchronous motor is available, the calibration may be extended to the quadrants corresponding to leading current.

Before or after the calibration, investigate a few features of the construction of the instrument, as follows:

(1) Open the potential circuit and observe the vane rotate under the influence of the field produced by the series winding alone. Reverse two of the series leads, whereupon the vane will revolve in the opposite direction.

(2) Reverse the potential leads; the pointer will turn by 180 degrees.

This corresponds to the change from output to input, in other words, from alternator to synchronous motor.

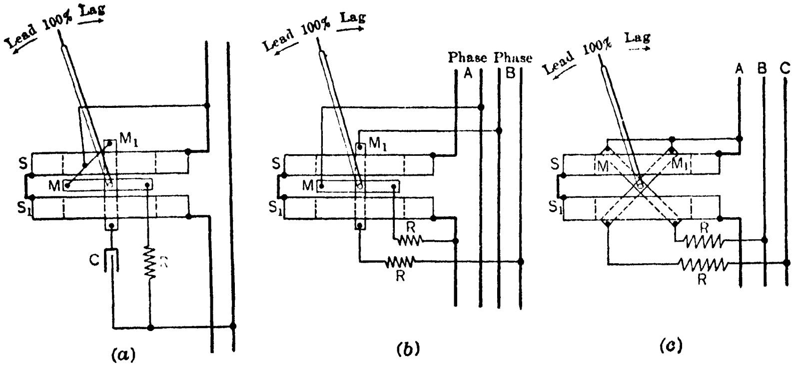
(3) The instrument is intended to be read with balanced load only; unbalance the load and see what effect this has on indications of the power-factor meter.

(4) Connect the meter to a single-phase supply circuit through a split-phase arrangement, like that shown in Fig. 378. Take a few calibration points at different values of power factor and of line current.

(5) Investigate the effect of the frequency, of the line voltage, and of the load current upon the accuracy of the instrument.

Report. (a) Give the calibration curve of the instrument at the rated current, voltage, and frequency. (b) Give the findings in regard to the errors due to each of the three factors. Explain the cause of these errors. (c) Give the findings with the unbalanced load. (d) Report the operation of the instrument on the single-phase system.

112. Electro-Dynamometer-Type Power-Factor Meter.[1] — One of the dynamometer-type power-factor meters is shown in Fig. 105, in application to the single-phase, two-phase, and three-phase circuits



(a) (b) (c)

FIG. 105. An electro-dynamometer-type power-factor meter.

respectively. It consists of two stationary current coils SS_1 and of two movable potential coils MM_1, the latter being fastened together at right angles to each other. The movable coils are pivoted to rotate freely and have no control springs.

In the *single-phase meter*, Fig. 105(*a*), the movable voltage-coil M is connected in series with a resistor R across the circuit, and its current is therefore practically in phase with the voltage. The other voltage

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[1] See "A new development in alternating-current instruments," by Paul MacGahan, *Elec. Jour.*, July, 1923.

coil, M_1, is connected in series with a condenser C across the circuit, and its current is practically 90 degrees out of phase with the voltage. At 100 per cent power factor, the current in M is, therefore, in phase with the current in the current coils S and S_1, and the torque between M and the fixed coils is a maximum. This torque tends to make M parallel to the fixed coils. Under the same condition the current in M_1 is 90 degrees out of phase with the current in S and S_1, and its average torque is zero, because a maximum current in M_1 coincides with zero current in S and S_1, and vice versa.

As the power factor changes from 100 per cent to some other value, the current in M, being in phase with the voltage, becomes more and more out of phase with the current in SS_1, and its torque decreases. The current in M_1 comes more and more into phase with the main current, and its torque increases. The direction of the torque depends on whether the current is leading or lagging. The coils are so connected that the two torques oppose each other. The moving system therefore assumes a definite position for every value of the power factor of the circuit, the position being such that the two torques are equal and opposite. Instead of the condenser C, the current phase in one of the movable coils may be shifted by means of an inductance. Whether inductance or capacitance is used, the instrument is affected by frequency changes and must be recalibrated for a different frequency. Otherwise the values of R and C (or L) must be properly adjusted.

The *two-phase meter* is shown at Fig. 105(*b*). In this case the 90-degree phase relationship between the currents in the two voltage coils is obtained by connecting the two coils to the two phases of the circuit. In other respects the principle of operation is the same as that of the single-phase meter.

Figure 105(*c*) shows a diagram of the *three-phase meter*. In this case one voltage coil is connected across the phase AB and the other across AC, the current coils being in the phase A. At 100 per cent power factor the voltage (or the current) in one of the potential phases leads the main current by 30 degrees, and in the other phase it lags by 30 degrees, so that the torques of the two coils balance each other. A change in power factor increases the lead of one phase voltage and decreases the lag of the other, or vice versa, causing a relative change in the torques of the two coils and making the movable system assume a new position of equilibrium at which the two torques again balance each other. A dynamometer-type power-factor meter possesses a much higher accuracy than the one with an iron vane (§110), and if carefully constructed, should give an accuracy of at least 1 per cent, from about one-fifth of its current rating up.

113. EXPERIMENT 4-E. — Study of Electro-Dynamometer-Type Power-Factor Meter. — The purpose of the experiment is to make the student familiar with the power-factor meter described in the preceding section. (1) Provide a single-phase load in which power factor may be varied at will by the addition of inductance or capacity, and connect into the circuit an ammeter, a voltmeter, a wattmeter, and the power-factor meter under test. Set the load at nearly 100 per cent power factor and change it in steps to as low a value as possible, keeping the current and the voltage nearly constant. Also produce a partly leading current, if feasible. At each setting read the four instruments. Take a few readings at a slightly lower and at a slightly higher frequency, and also at frequencies much above and below the rated value for the power-factor meter. Be sure that the other instruments give reliable readings at such frequencies. Connect an adjustable resistance in each potential circuit, and by adjusting these try to bring the calibration back to that corresponding to the scale of the instrument.

(2) Take a few similar readings on a two-phase load and on a three-phase load, both balanced and unbalanced.

(3) Adjust the single-phase load to be exactly of unity power factor, and disconnect the moving coil in series with the condenser. The indication of the instrument should remain unchanged, thus proving that the current in M_1 is in phase quadrature with the main current. Show similarly that at any other power factor, the resultant torque on the moving system is equal to the differential action of the two coils.

(4) Show that, in the three-phase meter, the moving coils exert equal and opposite torques at unity power factor when the planes of the movable coils are at an angle of 45 degrees with the stationary coils. Disconnect one of the movable coils and read the indication on the scale. Do the same with the other coil, having the first coil connected. Repeat this experiment at one or two other values of the load power factor.

Report. (1) Give the calibration curve of the single-phase instrument at the rated frequency. Explain how and why the frequency of the supply affects the calibration. Explain the effect of the additional resistance, and whether this resistance should be added to the capacitive or resistance circuit, for a higher and for a lower frequency than the rated frequency. (2) Give the results of the calibration of the meter on a two-phase and on a three-phase circuit with a balanced load. Explain the meaning of the readings with the unbalanced load. (3) Give the results of the tests with one of the potential coils disconnected, on both the single-phase and the three-phase circuit. Show that the observed behavior of the instrument could be predicted theoretically.

114. Reactive-Factor Meter. — Besides the power factor (§94), which for sinusoidal quantities is defined as the cosine of the angle between the current and the voltage, the so-called *reactive factor*, which is the sine of the same angle (§107), is sometimes used. A reactive-factor meter can be made out of the above-described power-factor meter by simply changing the scale to read the sines of the corresponding angles instead of the cosines. In some reactive-factor meters the functions of the outer and the inner windings (Fig. 103) are reversed, the inner coil being connected through a current transformer to one of the line conductors, and the outer polyphase winding connected across the line through two V-connected potential transformers.

The advantage of a reactive-factor meter over a power-factor meter is that the former shows the idle component of the current most emphatically. Thus, with a power factor of 95 per cent, the reactive factor is 31.2 per cent (and not 5 per cent), because

$$\overline{0.950}^2 + \overline{0.312}^2 = 1.000$$

Similarly, with a power factor of 98 per cent, the reactive factor is 19.7 per cent (and not 2 per cent). This large indication will induce the attendant to correct the waste more quickly, and therefore makes the meter particularly useful in securing the proper operation of rotary converters, where a slight drop in power factor may cause considerable heating of the armature. In this respect the meter plays a part similar to that of a wattmeter connected to measure the reactive power (§107).

With non-sinusoidal currents and voltages the equivalent phase-displacement angle ϕ has a different value according to whether it is computed from eq. (28) or eq. (29), in §107. Let these values be denoted by ϕ_1 and ϕ_2 respectively; the theory shows that $\phi_1 > \phi_2$ if the reactive power P_r be measured with a dynamometer-type wattmeter. Thus, while with sinusoidal currents and voltages we have

$$\cos^2 \phi + \sin^2 \phi = 1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

here we must write

$$\cos^2 \phi_1 + \sin^2 \phi_2 < 1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34)$$

Assume that we have an accurate power-factor meter and an accurate reactive-component meter, both carefully calibrated with sinusoidal currents. According to eq. (33), the sum of the squares of their indications on a circuit with sinusoidal currents and voltages should be 100 per cent squared. If the same instruments be used on a circuit with an irregular wave-form, the sum of the squares of their indications, according to eq. (34), will be less than 100 per cent squared.

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CHAPTER V

WATTHOUR METERS

115. The Watthour as a Unit of Consumption of Electrical Energy. — A certain amount of coal is required to generate a definite quantity of electrical energy. The amount of energy used by the consumer depends upon the average load in watts, and upon the duration of time through which the power is used. For these reasons it is proper to charge for electrical energy on the basis of the watthour or the kilowatthour consumption (§93). A 50-watt incandescent lamp uses, in 1 hour, 50 whr (watthours) of energy. Two lamps, used half an hour each, or 4 lamps used for a quarter of an hour each, also consume together the same amount of energy. Within certain limits it does not make much difference to the company supplying the power during what period of time a certain amount of energy is consumed. If the rate is 10 cents per kwhr (kilowatthour), the cost to the consumer is, in either case,

$$10 \times 50/1000 = 0.5 \text{ cent}$$

When larger amounts of energy are consumed, the kilowatthour consumption may be only one of the factors in the total charge for the electric service. The character of the demand, that is, steady, fluctuating, with high peaks, long hours, short hours, at off-peak times for other loads, etc., determines the cost of service, and in many cases is taken into consideration in the monthly bill (§134). Still other possible items in the bill for the electrical service may be the rating of the installed machinery, a charge for the installation itself (lines, transformers, etc.), a penalty for low power factor, a penalty for phase unbalancing, etc. All these factors are outside the scope of this chapter, which is concerned only with the measurement of the actual watthours consumed.

The reason for measuring electrical energy in watthours, instead of simply in watts, is that 1 watt represents a definite amount of energy liberated or absorbed *in 1 second*; in other words, it is only a *rate* of expenditure of energy. This follows from the definition of the watt as "volt times ampere," where the ampere is a quantity of electricity *per second.* On the other hand, the energy contained in 1 lb. of coal is measured in Btu, calories, joules, etc., all of which are units independent of time. Therefore, watts must be multiplied by time in order to reduce them to the same physical meaning and dimensions as the heat units.

An instrument which measures the total electrical energy consumed in a circuit during a certain period of time is called a *watthour meter*. Sometimes it is referred to simply as an " electric meter." The monthly bill of a small consumer is usually computed on the basis of the reading of the watthour meter on his premises.

The various watthour meters on the market are inherently small motors, so constructed that the instantaneous speed is proportional to the rate of consumption of energy in the circuit to which they are connected. Therefore, the *totalized* number of revolutions over a certain interval of time is proportional to the total energy consumed during that time. For this purpose the meter shaft is connected by means of a worm gear to a counting or totalizing gear train with dials, shown in Fig. 106. The speed of the meter and the gear ratio are usually so adjusted as to make the meter direct-reading, or at least to have a simple multiplier, such as 10 or 100. With the position of the hands shown in the figure, the reading is 18,255 kwhr. If the reading one month later is, for example, 18,490 kwhr, the consumption during the month was 235 kwhr, and at a rate of, say, 10 cents per kwhr, the monthly bill will be $23.50.

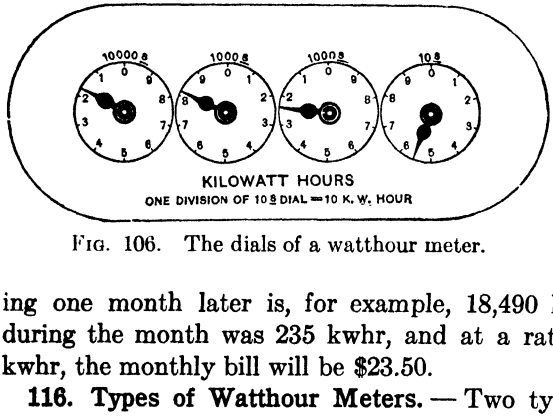

Fig. 106. The dials of a watthour meter.

116. Types of Watthour Meters. — Two types of *indicating* wattmeters are described in the preceding chapter, viz., the dynamometer type and the induction type. Corresponding to these two types are two types of watthour meters — *commutator meters*, used on d-c circuits, and *induction-type* meters, used on a-c circuits. A third type is the mercury-motor meter described in §122, which was produced for the purpose of eliminating the commutator. On a d-c circuit in which the voltage is kept practically constant, watts are proportional to amperes, so that it is sometimes sufficient to measure ampere-hours instead of watthours (§124).

COMMUTATOR-TYPE WATTHOUR METER

117. The Thomson commutator meter (Figs. 107 and 108) consists of a small d-c vertical-shaft motor, without iron in its magnetic circuit. The armature A is connected across the line, in series with a high resistance R, the value of which depends upon the voltage of the supply.

In the regular operation of the meter the armature runs so slowly that its counter emf is negligible in comparison with the line voltage. Consequently, the current through the armature is practically constant and is equal to the line voltage divided by the resistance *R*, plus that of the armature itself.

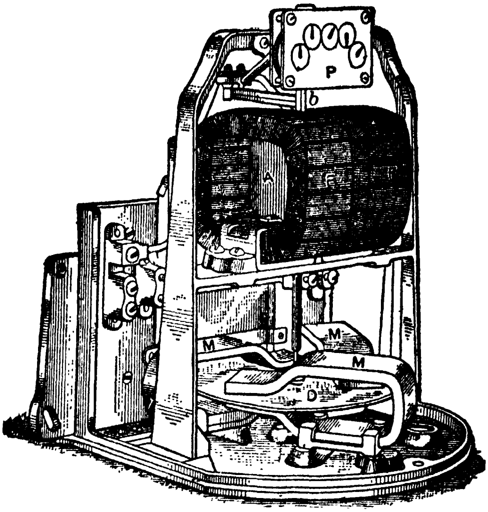

Fig. 107. A general view of the Thomson watthour meter.

The stationary field coils *FF* are in series with the line, and the current flowing through them is equal to the line current. Thus, the torque between the field and the armature is proportional to "line current times line voltage," or to the power delivered. In this respect the device is similar to the indicating wattmeter shown in Fig. 94.

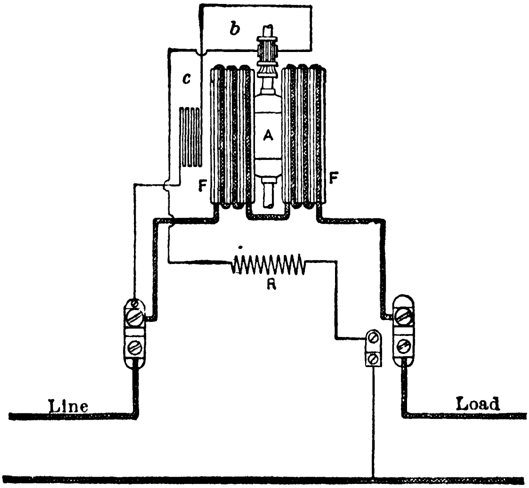

Fig. 108. The electrical connections in a commutator-type watthour meter.

A motor connected in this manner would run away unless it were loaded. To make its speed proportional to the power to be measured, the meter is provided with a Foucault current brake (Fig. 107). This brake consists of an aluminum disk *D* mounted on the armature shaft, and placed between the poles of the permanent magnets *MM*. The rotation of the disk in the field of the magnets causes eddy currents to be induced in it. The counter torque of these currents balances the driving torque of the motor element. The speed of rotation is usually adjusted by moving the brake magnets in and out. In some meters an iron disk, mounted near the pole-pieces, shunts some of the flux around the air-gap. By shifting this disk the useful flux is adjusted so that the meter runs at the correct speed.

To understand the operation of the meter and the action of this brake, assume first that the meter has no friction, and that, at a certain load current I and line voltage E, the meter runs at a certain speed s, determined by the counter torque of the eddy currents in the brake. At low speeds the eddy currents are proportional to the speed. The retarding torque between these currents and the permanent magnets is proportional to the value of the currents themselves, that is, proportional to the speed. Thus, the torque due to the eddy currents is Bs, where B is a coefficient of proportionality. The driving torque is proportional to EI, and we have the relationship

$$kEI = Bs \qquad (1)$$

In this expression k is the constant of the instrument, and kEI is the driving electromagnetic torque, equal to the brake torque. It will thus be seen that the instantaneous speed of the meter is proportional to the power EI to be measured. The unavoidable bearing and brush friction would somewhat modify this simple relationship, were it not for the fact that all modern meters are provided with a friction compensating device which is described in the next section. With this compensation the speed of the meter is practically proportional to the load, down to a small per cent of its rating. This means that the total number of revolutions of the meter during a given interval of time, as shown on the dial P, is proportional to the total electrical energy that has passed through the meter. This is true irrespective of whether the load is constant or fluctuating, since the armature adapts itself quickly to changes in load, and even at a rapidly fluctuating load, rotates at a speed corresponding to the average load.

In some more recent meters the revolving armature is made spherical. This gives a more advantageous utilization of the materials and secures a higher torque with a lower watt loss in the meter itself. Some large-capacity meters are provided with two armatures on the same shaft, connected astatically. Such an arrangement minimizes the effect of stray magnetic fields, since any stray field which tends to weaken the torque of one armature strengthens that of the other.

A magnetic shield is provided between the stationary field coils and the permanent magnets M. In the earlier unprotected meters it was found that a strong electromagnetic field due to a severe short circuit was sufficient to affect the strength of the permanent magnets and thus to throw the meter out of calibration.

Most d-c meters of the above-described type cannot be used with ammeter shunts, and the whole line current must flow through the meter. This presents serious difficulties with currents ranging up into several thousand amperes. By a proper modification of the design, meters are produced in which the stationary element is designed for 5 amperes at full load, and the meter is connected across a shunt, like an ammeter.

118. Compensation for Friction. — The unavoidable friction in the bearings and between the commutator and the brushes produces a retarding torque, which would cause the meter to run slow on light loads. This would represent an appreciable loss to the company supplying the power because a great majority of meters run on light load most of the time. For example, an uncompensated 10-ampere meter might not start at all, or might run very slowly with only one 0.25-ampere lamp in the circuit. Thus a customer could leave such a lamp in the circuit permanently without an adequate return to the company.

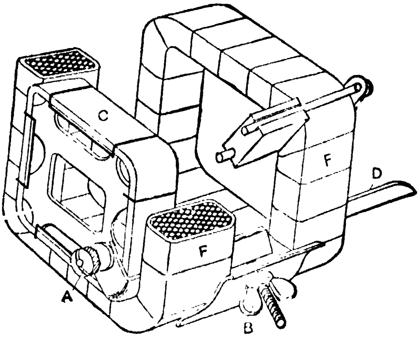

FIG. 109. Compensation for friction in a Thomson watthour meter.

To remedy this, a compensating coil c (Figs. 108 and 109) is provided in the armature circuit; the field produced by this coil gives an additional torque just sufficient to balance the friction of the meter. As it is impossible to predetermine the exact amount of friction, especially since it varies with the pressure of the brushes on the commutator, with the wear of the jewel supporting the shaft, etc., this compensating coil is made adjustable. A possible arrangement is shown in detail in Fig. 109. *FF* are the main field coils; *C* is the compensating coil. Its distance from the armature can be varied by means of the screws *A* and *B* and the guide *D*.

In some meters the compensating coil is stationary, but is provided with several taps connected to the buttons of a dial. By means of a small lever more or less turns of the compensating coil may be introduced into the circuit, and in this way the amount of compensation may be varied at will. Another method of adjusting the amount of compensation is by means of a variable shunt connected in parallel with the compensating coil. In some meters the compensating coil shown in Fig. 109 is made to move up and down, instead of to and from the armature.

A meter which is *over-compensated* is liable to creep at no load, registering energy when none is consumed. The well-established policy on the part of all reputable operating companies is to furnish each customer with a meter accurately calibrated and adjusted against creeping at no load. In spite of this, a meter is occasionally found which creeps at no load, from one or more of the following causes: (a) vibration which reduces the friction, (b) a stray magnetic field, and (c) too high a voltage. Sometimes the potential circuit is connected by mistake on the load side of the meter, when the friction compensation has been adjusted with the potential circuit connected on the service side. With the former connection the compensating current flows through the main or series coil, and the resultant ampere-turns increase the compensating torque; this also may cause the meter to creep.

Even with the most careful adjustment in the shop, there is a possibility of creeping, and for this reason a simple anti-creep device is often used. The most common method is to force a small U-shaped piece of iron wire or " clip " over the edge of the brake disk. This clip is attracted by the drag magnets and tends to remain stationary in a position where it is closest to one of the poles. When the meter is rotating under load, the accelerating force, as the wire approaches a magnet, is equal to the retarding force as it leaves the magnet, so that the effect on the accuracy of the meter is negligible. In placing the clip, care should be taken not to injure the disk, and to locate the clip so that it is not likely to hit the magnets.

In some meters a piece of iron wire is attached to the disk near the shaft, giving the same effect. The effect of the clip is varied by changing its length and position. To make this adjustment properly requires good judgment. The tester must take account of the character of the installation and of the location, and set the clip so that it will not prevent the meter from starting on a light load, and yet so that the meter is not likely to creep under the worst conditions which may exist at some later time. The minimum starting current should be determined after adjusting the clip.

The *starting current* of a meter is defined as the smallest current required at normal voltage to start the moving element from rest, and to cause it to rotate continuously through one or more revolutions past the anti-creeping device. When the meter has an anti-creeping clip, the starting current should be taken when the clip is nearest to a magnet pole. If a low-reading ammeter is not available, the starting current may be estimated in terms of the total watt rating of the test lamps required to start the meter.

119. Calibration of Commutator-Type Watthour Meters. — The calibration of a watthour meter comprises (a) an adjustment of the brushes and the bearings to give the best operating conditions; (b) an adjustment of the compensating device so that the meter does not creep at no-load and yet starts with a very small load-current; (c) an adjustment of the brake magnets to make the meter run at the proper speed corresponding to the gear ratio of the recording dial. In some cases, parts (a) and (b) are omitted in order to determine the percentage of error of the meter "as found." Thus, in the case of a complaint on the part of a customer, the percentage of "slow" or "fast" of the meter is determined before any adjustments whatever are made. Should the meter be found incorrect, it is carefully inspected and adjusted, and then calibrated to read correctly. The three practical methods of calibration are described below.

(1) *Comparison with a good indicating ammeter and voltmeter* (*or indicating wattmeter*). The d-c watthour meter under test is connected to a steady load, and the input into this load is measured in the usual way with a moving-coil ammeter and voltmeter (§41). A calibrated indicating wattmeter (§95) may be used in place of these two instruments. The number of revolutions of the meter shaft during a certain interval of time is determined with a stopwatch. From these data and the constant of the meter, that is the number of watt-seconds per revolution, the percentage of error may be readily computed. The connections must be such that the watthour meter does not measure the potential loss in the standard instruments, or the instruments measure the potential loss of the watthour meter.

Let, for example, a watthour meter be tested at 200 amperes and 120 volts, and let the constant marked on the meter be 10 whr per revolution of the disk. Let the disk be found to make 30 revolutions in 46 seconds. Then the true energy that has passed through the meter is

$$200 \times 120 \times 46/3600 = 306.7 \text{ whr}$$

The energy indicated by the watthour meter is

$$10 \times 30 = 300 \text{ whr}$$

Thus, the meter ran *slow*

$$(306.7 - 300)/306.7 = 2.2 \text{ per cent}$$

and this represents its percentage of error.

(2) *Comparison with a standard watthour meter* (*portable standard*). The disadvantage of the preceding method is the necessity of keeping

the load nearly constant, and of using one or two indicating instruments and a stopwatch. For this reason a standard portable watthour meter is being used to an increasing extent. It is connected to the same circuit as the meter under test, and its number of revolutions (and fractions thereof) is determined for an interval of time, during which the meter under test makes a predetermined number of revolutions. If the constants of the two meters are known, the percentage of error of the meter under test can be readily determined.

Let the meter to be tested have a constant of 0.4 whr per revolution of the disk, and let the corresponding constant of the standard meter be 0.5. Let the meter to be tested make 45 revolutions while the standard meter makes 35.2. The corresponding amounts of energy indicated by the two meters are $0.4 \times 45 = 18.0$ and $0.5 \times 35.2 = 17.6$ whr. Thus, the meter under test runs *fast* by

$$(18.0 - 17.6)/17.6 = 2.3 \text{ per cent}$$

The portable standard meter usually has a number of stationary coils for different ranges of current, and it has multiplier resistances for at least two different voltages. This permits the use of the standard meter at a large percentage of its rated current, for which the meter is particularly accurate. The temperature of the potential circuit affects somewhat the speed of rotation, and for this reason a meter should not be tested until its potential circuit has been connected for at least twenty minutes. The standard meter is sometimes provided with "heating connections," so as to bring the potential circuit quickly to the working temperature.

The rotating standard should be set in a place free from vibration and stray magnetic fields. It should be kept away from iron masses, motors, electrical apparatus, and conductors carrying heavy currents. The stationary coils should be placed in the plane of the magnetic meridian, and several comparative tests should be made with the current and potential connections reversed, so as to determine the possible effects of a stray field.

Rotating standards are not made much in excess of 100-ampere current capacity. To calibrate a meter of larger capacity, a known fraction of the total current is passed through the standard meter (Fig. 110). The arrangement is based on the Wheatstone-bridge principle. A and B are two manganin shunts, the drop across A being about 0.1 volt and that across B about 0.4 volt. The third branch of the bridge is formed by a small shunt C of the capacity of the standard meter, and the fourth branch consists of the standard meter in series with an adjustable resistance D which is set so as to reduce the current in the galvanometer G to zero. Let, for example, the resistances A and C be such that the voltage drop across either is the same when there is a current of 2000 amperes in A and 40 amperes in C. In other words, the galvanometer G will show no deflection when the current through C is 2 per cent of that in A; the total line current is then 51 times that passing through the standard meter.

(3) *Calibration with a standardized load resistance.* A resistance made of some metal of zero temperature coefficient is carefully calibrated in the laboratory by accurately determining the wattage consumption for all voltages within the working range of the meters to be tested; the results are tabulated or plotted as a curve. Such a resistance is then used as an artificial load for the meter under test, and the voltage at its terminals is measured with a voltmeter. The table or the curve will then give the true watts. The number of revolutions of the meter disk per second, multiplied by the meter constant, will give the meter watts. The percentage of error, if any, may be computed from these data.

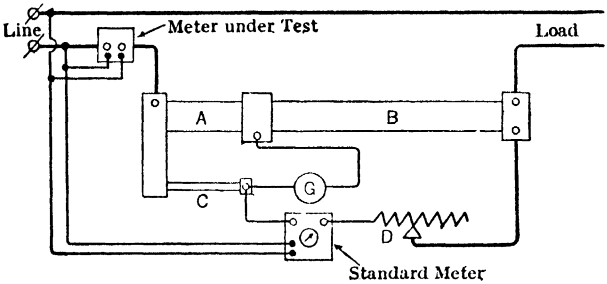

FIG. 110. A shunt for calibrating large-capacity meters.

Care must be exercised in regard to the connections to the voltmeter, which must indicate the voltage applied not only to the meter, but also to the cable terminals of the resistance. As the internal resistance of a d-c voltmeter is very high, the current taken by the voltmeter may be ignored for all loads in excess of 1 ampere; for loads less than 1 ampere the voltmeter should either be connected to the load side of the watthour meter or a correction made for the current taken by the voltmeter. For loads in excess of 1 ampere, the drop in the field coils of the watthour meter also enters as a factor, depending upon the point of connection of the potential circuit of the meter. If the potential circuit is connected to the load side of the watthour meter, the voltmeter should also be connected at this point, and then no error will be introduced. If the potential circuit is connected to the service side of the watthour meter, and the voltmeter is also connected at that point, allowance should be made for the drop in the watthour-meter field coils when computing the wattage consumed by the resistances.

120. Torque and Friction. — A high torque per unit weight of the moving element is considered to be a desirable feature in almost any electrical measuring instrument, because the disturbing influence of the pivot friction is less and the meter registers more accurately at light loads. Special instruments have been devised by means of which the torque on the shaft of a meter can be directly measured in gram-centimeters, and two or more meters of different makes may be compared. The two torque-measuring devices described below may be used for testing either indicating instruments or watthour meters.

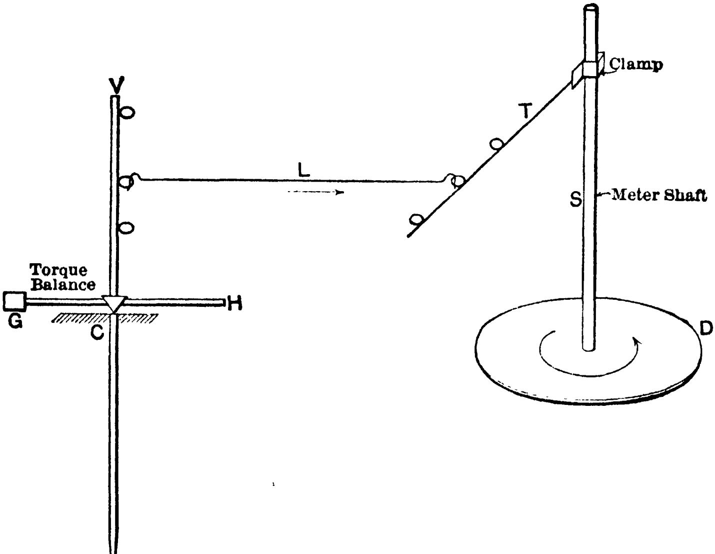

FIG. 111. A torque balance for testing watthour meters.

(a) *Torque balance.* The instrument is shown diagrammatically in Fig. 111. A light arm *T* is clamped to the shaft *S* of the meter under test, and is connected by a link *L* to a sensitive balance *GH*. The balance has a knife-edge support at *C*; the position of equilibrium is indicated by the pointer *CO*. A definite weight *G* is attached to the balance, causing it to tip toward the left. Then the meter is loaded electrically until the pull on the link *L* brings the pointer *O* back to zero. The critical load is measured on an indicating wattmeter; the mechanical torque is calculated from the weight *G* and the leverage *TLVC*. From these data the torque is calculated per watt of load and per ounce weight of the revolving part of the meter, supported by the lower bearing. This gives a basis for the comparison of competitive meters of similar construction. The rods V and T are provided with several loops, so as to vary the torque in steps; two or more weights are supplied with the instrument, to cover a wide range of meters to be tested.

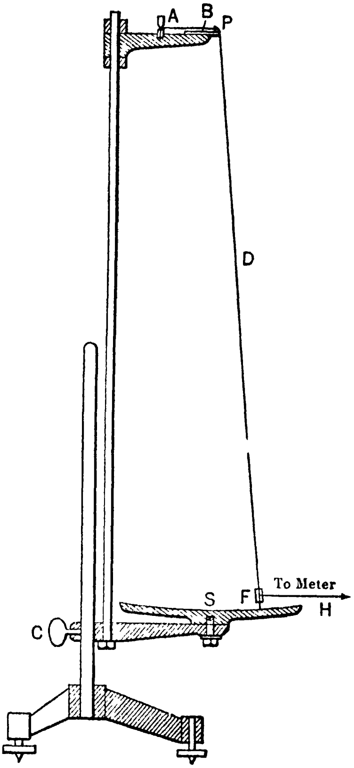

FIG. 112. A pendulum device for measuring the torque of a meter.

(b) *Pendulum device.* The essential feature of this device (Fig. 112) is a scale S on a concave spherical surface turned from a brass casting to a radius of curvature of 1 meter. The bob is supported from an adjustable arm, and is so arranged that the point of support P is at the center of curvature of the spherical scale. A silk fiber D is used as the suspension and is wound on a friction pin A, for convenience in adjusting its length. The fiber passes through a V-shaped notch in the end of a brass strip B, which is capable of a small horizontal adjustment to aid in the initial centering. The whole is mounted on an ordinary clamp stand, the tripod being fitted with leveling screws. The clamp C allows the scale and bob F to be lowered as a unit, for convenience in applying to an instrument.

The bob consists of a small, hollow brass cylinder with a fine sewing-needle passed through it perpendicular to the axis. The silk fiber H, by which the horizontal force to be measured is transmitted to the bob, is attached to the needle and passes out along the axis of the hollow cylinder. The point of attachment is made at the center of mass of the bob, which, for convenience, is marked by a file scratch. The mass of the bob alone is adjusted to 0.5 gram, but in order to change the range, concentric cylinders, each cut in halves, are made to fit snugly over the inner cylinder. These extra cylinders make the total mass 1, 2, 5, 10, and 20 grams respectively, giving a range of 40 to 1 for full-scale deflection. A radial saw-cut in each half-cylinder allows these extra weights to be slipped into place without disconnecting either of the silk fibers.

The graduations on the scale consist of concentric circles, the radii of which are proportional to the trigonometric tangents of the angles of deflection of the pendulum, so that

Horizontal force = weight of bob × 0.001 × reading in divisions

In measuring the torque of a deflection instrument — for example, a voltmeter — the horizontal thread is fastened to the pointer at a convenient distance from the pivot, the whole pendulum system adjusted to the proper height, and the voltmeter moved horizontally until the desired deflection is produced. We then have

Torque = arm × weight × 0.001 × reading in divisions

In the case of a watthour meter it is only necessary to attach the thread to the edge of the disk, apply current and voltage to the meter, and allow the thread to wind upon the disk as far as it will.

(c) Meters can also be compared by determining their so-called *friction-torque ratio.* For this purpose the friction compensator is adjusted at no load so that the meter is just balanced, i.e., so that it will creep under the slightest vibration. When this is done, a load is applied equivalent to the full capacity of the meter, and the speed of the revolving part is measured. Then the friction compensator is disconnected and the speed measured again at the same load. The percentage of decrease in speed is proportional to the friction-torque ratio; thus, 2 per cent decrease in speed would mean a friction-torque ratio of 1 : 50; 1 per cent, 1 : 100, etc. The lower this ratio, the better is the meter, at least, so far as the disturbing influence of its friction is concerned.

121. EXPERIMENT 5-A. — Calibration of a Commutator-Type Watthour Meter. — The purpose of the experiment is to study the performance of a commutator-type meter described in §§117 and 118, and to obtain experience in the methods of calibration indicated in §119.

(1) Adjust the bearings and the brush tension on the meter, disconnect the compensating winding, and make the anti-creeping device ineffective, if present. Connect the meter to a load at the normal voltage, and determine the minimum starting current at which the disk positively rotates. Be sure that the current required for the potential circuit does not flow through the field coils. Connect the compensating winding and adjust it as accurately as possible, to prevent the meter from creeping at no load. Again determine the starting current.

(2) With the final adjustment so obtained, try to make the meter

creep at no load by subjecting it to vibration or stray fields, or by raising the supply voltage. Adjust the anti-creep device to make the meter reasonably immune against creeping due to the foregoing causes. Determine the starting current with the meter finally adjusted and ready for service.

(3) Calibrate the meter by one or more of the methods described in §119. Begin with an overload of about 25 per cent and reduce the load in steps to zero, taking readings at each step. Adjust the permanent magnets so as to make the meter read correctly at full load. Check at a few other loads.

(4) Determine the torque per watt per unit weight of the moving element, and the friction-torque ratio, as explained in §120.

Report. (a) Describe the findings in regard to the light load and anti-creeping adjustments, and give the value of the starting current. (b) State the influence of jarring, external magnetic fields, brush tension, etc. (c) Give the calibration curve of the meter using percentage of load as abscissas and percentage of " slow " or " fast " as ordinates. (d) Give the value of the specific torque and the friction-torque ratio of the meter. (e) Mention the features of construction, the observed behavior of the meter, a method of testing, etc., different from those described in the text.

122. Mercury Watthour Meter. — The above described commutator meter is widely used on d-c circuits with satisfactory results, but it has the following disadvantages: (1) The commutator and the brushes are liable to get out of order. (2) Because there is no iron in the magnetic circuit, a large number of turns is required on both windings to produce a sufficient torque. (3) The moving part being comparatively heavy, the friction is considerable and varies somewhat during the life of the meter. (4) A commutator meter is not well adapted for operation with a shunt, and this limitation makes it inconvenient and bulky with heavy currents. (5) The meter is to some extent affected by external fields, even with a double astatic armature. (6) The pivot and the jewels are likely to give trouble, especially in locations subjected to vibration.

It is claimed that these objectionable features are largely eliminated in the d-c watthour meter of the mercury-motor type, shown in Figs. 113, 114, and 115. It is essentially a homopolar electric motor with mercury contacts. The armature consists of the copper disk *A* floated in the mercury chamber *F* (Fig. 115) between the poles of an electromagnet *Y*. *EE* are the contact ears to the chamber, and the line current flows through the mercury and through the copper disk. This disk is slotted, as shown in Fig. 114, so as to guide the current at right angles across the field, thus rendering it more effective in producing the torque. The exciting winding *S* (Fig. 115) of the electromagnet is connected across the line, and a few series turns *C* are provided to compensate for the armature reaction (§365).

According to the fundamental law of electromagnetism, a conductor carrying an electric current in a magnetic field tends to move at right angles to both the current and the field. The direction of the current in the disk is radial, and the lines of force cross the disk essentially in a direction parallel to its shaft because of the magnetic return plate above the disk (marked " Laminated Steel " in Fig. 113). Thus, the mechanical force upon the disk is tangential and is proportional to the load current. The field strength varies only with the applied voltage and is therefore nearly constant. The brake or damping disk *D* is placed in the field of permanent magnets and performs the same function as in any other watthour meter; that is, it acts as a load and causes the meter to run at a speed proportional to the electromagnetic torque (§117).

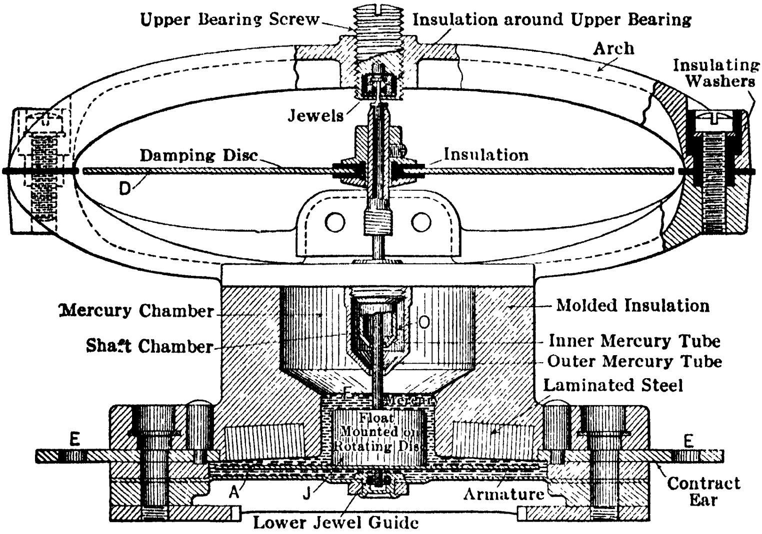

FIG. 113. A cross-section of a mercury watthour meter.

The armature disk *A* is provided with a float *J* (Fig. 113) which gives the moving system a slight buoyancy upwards, so that the main bearing of the meter is on top, and the lower jewel acts merely as a guide. The double sleeve or tube *O* prevents mercury from escaping in shipment. Any spray of mercury that may enter the outer tube is caught in the inner tube, and as the meter is again put in an upright position this mercury falls immediately back into the regular chamber.

On account of the presence of iron with variable permeability, the accuracy of the mercury meter is somewhat affected by fluctuations in the line voltage. A reduction in the line voltage causes the meter to run slightly fast and and an increase in voltage above normal causes it to run slow. This factor must be considered in an installation with a poor voltage regulation. The meter is not affected appreciably by changes in temperature. An increase in temperature will increase the resistance of the damping disk, thus reducing the eddy currents for a given speed, and, therefore, reducing the counter torque; consequently the meter will tend to speed up. However, a similar change occurs in the resistance of the potential winding, resulting in a decrease in the motor field and a corresponding decrease in the motor torque. The combined effect is that the speed remains practically unchanged.

On account of the inherently low starting torque of mercury meters, there has been a slight tendency to creep on over-voltage. To offset this, an anti-creeping device is used which consists of a very small irregular-shaped piece of iron placed between the float J and the armature disk A. The effect is to stop the meter when the position of the iron is such that the greatest field flux passes through it.

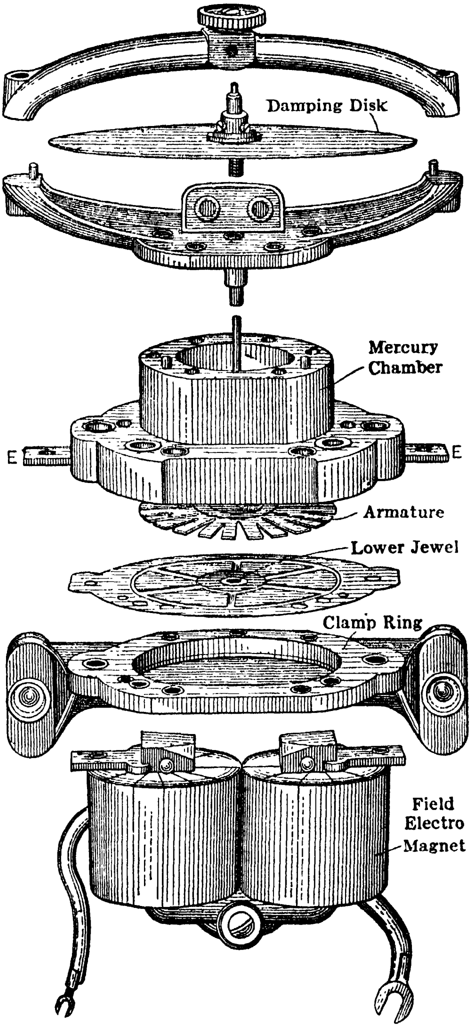

Fig. 114. The principal parts of the mercury meter shown in Fig. 113.

A small amount of dross is sometimes found in the mercury chamber; it is due to oxidation of the copper and the formation of copper-mercury amalgam. This trouble has not been found to be serious, although some foreign companies platinize the surface of the armature and then bake it in japan in order to reduce the quantity of copper-mercury amalgam. It is also of importance to eliminate all ammonia in the manufacture of the molding materials from which the mercury chambers are usually made. Any trace of ammonia readily forms an ammonia-mercury amalgam which is a heavy plastic substance and soon affects the accuracy of the meter. A method has been found of chemically treating condensite or Bakelite mercury chambers so as to greatly reduce the formation of dross.

The friction compensation is accomplished by means of the thermocouple *ab* (Fig. 115) which is heated by the coil *H* in the potential circuit. The local circuit of the thermocouple is closed through a resistance wire *B*, the adjustable clamp *K*, and the copper wire *R*. The leads *c* and *d* from this circuit are connected to the contact ears *EE* of the mercury chamber. At no load, or at light loads, the thermocouple circuit furnishes an armature current sufficient to overcome the friction.

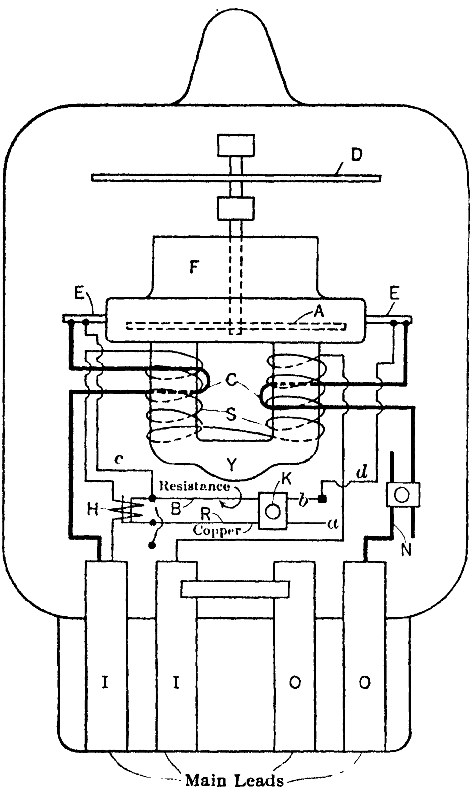

Fig. 115. The electric circuits of the meter shown in Figs. 113 and 114.

123. EXPERIMENT 5-B. — Calibration of a Mercury Watthour Meter. — The procedure is similar to that of Experiment 5-A. Vary the line voltage to a considerable extent, and study the effect of poor voltage regulation on the accuracy of the meter. Report results as in Experiment 5-A.

124. Ampere-Hour Meters. — In a d-c circuit in which the voltage is kept practically constant, watts are proportional to amperes, and it is sufficient to measure ampere-hours, instead of watthours. Multiplying the former by the actual or agreed voltage of the supply gives the energy consumption in watthours. Electrolytic meters are to some extent used in such cases, especially abroad. The current flowing through the meter decomposes a certain chemical contained in it. The amount of material decomposed is proportional to coulombs or to ampere-hours which pass through the meter; the scale can be made direct-reading.

An ampere-hour meter may also be built on the same principle as the commutator meter shown in Fig. 108. In this case the stationary field is supplied by a permanent magnet, and the current through the armature is made proportional to the line current by means of an ammeter shunt. The mercury-motor meter shown in Fig. 115 is used extensively as an ampere-hour meter, the shunt electromagnet being replaced by a permanent magnet. In this case the torque is proportional only to the current through the armature and does not depend upon the line voltage.

There is also a small commutator-type d-c ampere-hour meter of the general form well known in Europe for a number of years, in which three flat coils, suitably disposed between two aluminum disks, rotate in the field of two powerful permanent magnets. The armature is connected across a suitable shunt of 35 to 40 millivolts drop. The demand for such meters has been principally in foreign countries and for very small consumers, in rated capacities of 1, 2, and 5 amperes, sometimes larger. The meter reads in ampere-hours, and, if desired, the dial can be arranged to read in kilowatthours at a given voltage.

Ampere-hour meters are used to a considerable extent in storage-battery operation, in determining the economy of electric railway cars, in electroplating, etc.

INDUCTION-TYPE WATTHOUR METERS

125. Watthour meters used on a-c circuits operate on the principle of the split-phase induction motor (§508), and in this respect are similar to the corresponding type of indicating wattmeters (§102). A representative magnetic circuit of an induction-type watthour meter is shown in Figs. 116 and 117. Other watthour meters on the market differ from it in minor details only. The magnetic circuit is built up of soft-steel laminations to suppress eddy currents. The series or current winding is placed on the pole-pieces *a*, *b*, and the potential winding (shunt coil) on the pole-piece *c*. The revolving part. or armature, of the meter consists of a corrugated aluminum disk placed partly in the air-gap between the poles. The shaft which supports the disk is geared to the recording train, as in the d-c meter shown in Fig. 107. The alternating magnetic fluxes excited by the windings on the pole-pieces induce secondary eddy currents in the disk, and the reaction between these currents and the fluxes sets the disk in rotation.

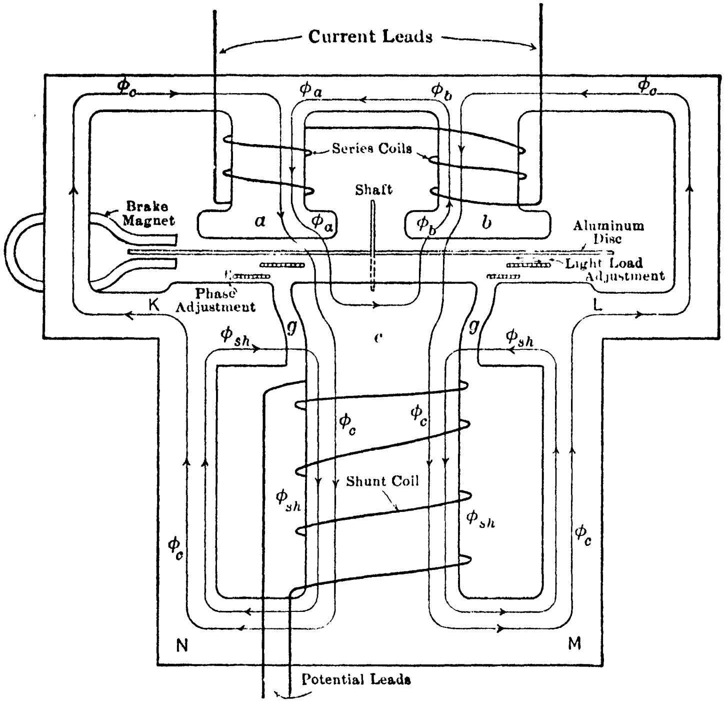

FIG. 116. An induction-type watthour meter.

It is shown in §127 that the torque is a maximum when the flux excited by the shunt winding lags by 90 degrees in time behind that in the series winding. This condition must obtain at unity power factor, because the meter must then revolve at its highest speed. But at unity power factor the voltage across the potential winding is in phase with the line current; therefore the flux in the pole c must be *lagged* behind the applied voltage by exactly 90 degrees. For this purpose the lower part, *KLMN*, of the magnetic circuit is shaped like the core of a choke coil, with small air-gaps gg on top. Most of the flux excited by the shunt coil takes the path marked ϕ_{sh}, and only a small part, ϕ_c, acts upon the aluminum disk. The portion ϕ_{sh} serves to reduce the shunt current by its reactive or choking action and to keep it as nearly as possible in quadrature with the voltage.

The resistance of the shunt coil is kept low, so that the potential circuit is highly inductive and the current lags by nearly 90 degrees behind the applied voltage. However, it is impossible to displace the current by fully 90 degrees because of unavoidable resistance and core loss. The exact quadrature lagging is accomplished by means of a short-circuited secondary coil the action of which is similar to that described in §§102 and 103. Such a " lagging " or " phase-adjustment " coil is shown schematically in Fig. 116 above the top of pole *c*. In some meters it consists of a few turns of wire short-circuited through an adjustable resistance. In others it consists of a single turn punched out of copper plate and mounted so that its position may be varied up and down by means of an adjusting screw.

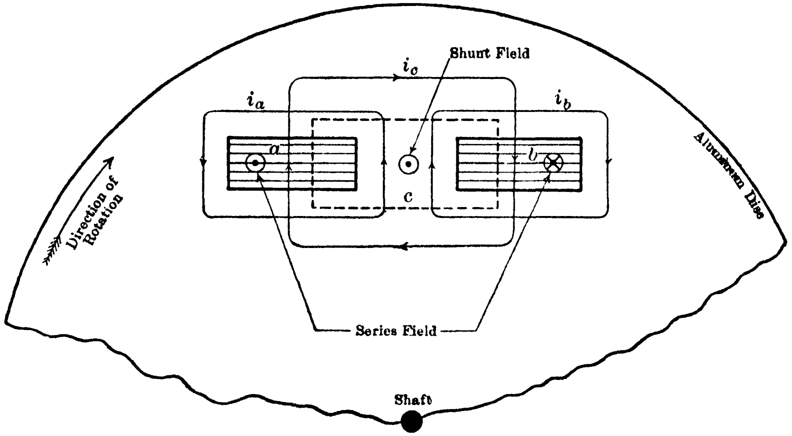

Fig. 117. Currents induced in the aluminum disk by the several fields.

The speed of the meter at full non-inductive load depends upon the position of the brake magnets (§117), one of which is shown schematically to the left in Fig. 116. These magnets act on the same aluminum disk as the a-c field, and retard the motion of the disk by induced eddy currents. The speed of the meter is adjusted either by moving the magnets in and out, or by shunting part of their flux through a piece of iron.

On account of the unavoidable friction, the meter requires a light-load adjustment (§118). Otherwise it would run slow at light loads and would not start until the load torque was high enough to overcome the friction. The friction compensating device usually consists of a short-circuited turn of flat copper shown in the main air-gap in Fig.

116 just below the aluminum disk. When this turn is concentric with the pole *c*, the secondary currents induced in it only reduce the magnitude of the flux ϕ_c. But when this turn is shifted to one side, it acts as a "shading coil," and its action becomes somewhat similar to that of the poles *a* and *b*. The currents induced in the aluminum disk are shifted both in time and in space position so as to exert a small electromagnetic effort upon the disk. This effort may be adjusted to compensate the friction and to make the meter start on a very small fraction of the rated load. Creeping at no load is sometimes prevented by punching two small slots on the periphery of the aluminum disk. When one of these slots comes under the shunt pole, the induced currents are so modified that the meter stops. These slots have no appreciable effect when the series coils are also excited. See also §118 in regard to light-load adjustment and creeping.

It is well to remember that the shaft of an a-c meter is constantly vibrating because of the alternating action upon the aluminum disk. This subjects the supporting jewel to a type of wear absent in a d-c meter. A light moving element is therefore more important in an a-c meter. In some meters a steel ball is used in place of a jewel for the lower bearing, thus substituting rolling friction for the ordinary jewel friction. A flexible or yielding top bearing is usually necessary to prevent the noise due to vibration.

Reactive-energy meter. A watthour meter may be used to measure the total reactive energy instead of the true energy. For this purpose, its potential circuit is connected to a source of voltage displaced by 90 degrees with respect to the load voltage. This subject is treated in §107 in application to indicating wattmeters, and the theory and the connections are directly applicable to watthour meters.

126. Polyphase Watthour Meters. — A two-phase or three-phase watthour meter is obtained by combining two single-phase meters described above. The two armatures are mounted on the same shaft and act on the same recording train, thus automatically adding together the indications of the two component meters. In some polyphase meters the two field elements act on the same aluminum disk. The brake magnet also acts on the same disk, thus making the whole construction light and compact.

Two methods of connecting a polyphase watthour meter to measure the energy consumed by a three-phase system are shown in Fig. 118(*a*) and (*b*). In (*a*) the meter may be said to be "self-contained," being connected directly into the system. This is possible only provided the voltage and current of the system are relatively low in value. Figure 118(*b*) shows both potential and current transformers used to connect the meter to the lines (see §56, and Chapter XVIII) of a high-voltage system. (Induction-type watthour meters are built only for moderate currents and voltages.) There are polarity marks at the terminals of the transformers. Unless the transformers are connected with due regard to polarities it is quite possible that one element of the meter will oppose the other, or both might be reversed, running the meter backward. In both the schemes of connections shown the two-wattmeter method (§§105 and 106) of measurement is used.

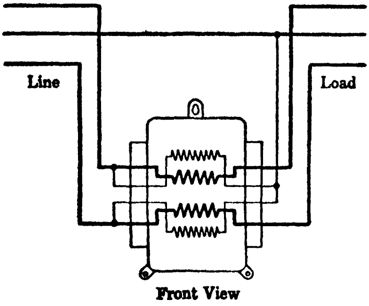

FIG. 118a. Installation diagram of a self-contained three-phase watthour meter.

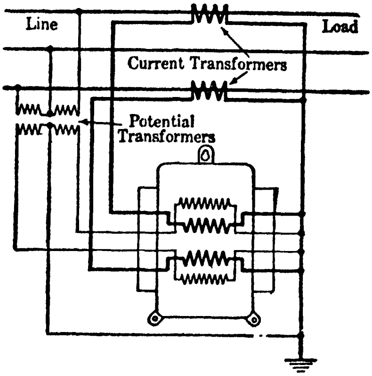

FIG. 118b. Installation of a three-phase watthour meter using instrument transformers.

127. Theory of Induction Watthour Meter. — It is customary to dismiss the theory of this type of instruments (Figs. 116 and 117) by stating that a gliding magnetic flux is produced in the air-gap due to the combined action of the series and shunt fields, as in a split-phase induction motor. The aluminum disk is said to follow this gliding flux because of induced eddy currents, as in the familiar Arago experiment. The difficulty with this explanation is that the consecutive poles do not follow alternately around a cylindrical air-gap, as in a motor. There are only two series poles and one shunt pole, active in the production of the gliding field, and it is not clear what becomes of this assumed gliding flux after it has traveled, say, from *a* to *b*. It cannot go any farther; and, should it return, a counter torque would be exerted upon the aluminum disk. Another explanation is therefore offered below, based upon a consideration of the actual currents and fluxes in the air-gap.

In Fig. 119 the sine-waves ϕ_a and ϕ_b represent instantaneous values of the fluxes produced by the windings on the poles *a* and *b*. The vectors of these fluxes are shown in Fig. 120. An instantaneous flux is considered positive when it is directed upward in the air-gap. Hence ϕ_a is positive when ϕ_b is negative, and vice versa. In a properly adjusted meter, with a non-inductive load, the flux ϕ_c, due to the winding on pole c, lags 90 degrees behind ϕ_a, or behind ϕ_b, depending upon the desired direction of rotation of the armature. It is shown in the figures lagging behind ϕ_a. The transformer action causes these three fluxes to induce secondary currents in the disk. These currents surround the projections of the corresponding poles and are denoted by i_a, i_b, and i_c respectively, in Fig. 117. Current i_a is so located with respect to the flux ϕ_a that their mutual mechanical action has no component in the plane of the disk. However, current i_a flows partly in the field ϕ_c, and there is a tangential mechanical force between the two. Similarly, there is an interaction between the current i_b and the field ϕ_c, and between the current i_c and the fields ϕ_a and ϕ_b. All these torques are in the plane of the disk, and it remains only to be shown that the actions are cumulative and therefore must result in the rotation of the disk.

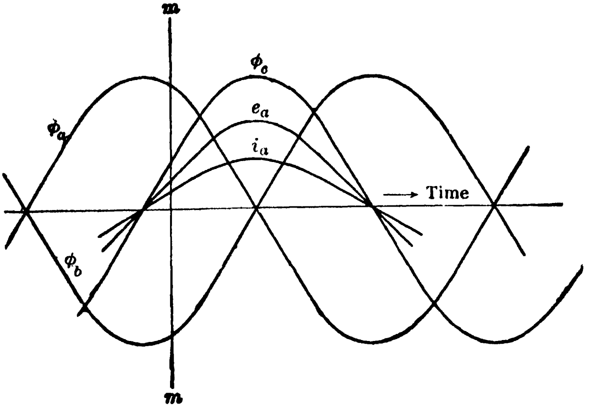

FIG. 119. The fluxes and the currents in the meter shown in Figs. 116 and 117.

The voltage e_a, induced by ϕ_a, lags behind ϕ_a by 90 degrees, and is therefore shown in phase with ϕ_c. The total current I_a (Fig. 120A), due to e_a, lags behind the latter by a certain angle β, depending upon the ratio of the reactance of the path to its resistance (§145). This current, I_a, may be resolved into the component i_a in phase with e_a and i_a' in quadrature with it. Since ϕ_c and i_a are in time phase with one another, the mechanical force between the two is always in the same direction, varying from a positive maximum to zero. The current component i_a' is in quadrature with ϕ_c, and therefore the instantaneous force is alternately positive and negative, the average value being equal to zero. Thus, for our purposes it is only necessary to consider the components i_a, i_b, and i_c of the currents which are in phase with the corresponding induced voltages.

To show that the electromagnetic forces are cumulative, consider an instant of time such as is represented by *mm* (Fig. 119). The three fluxes have the directions shown in Fig. 117 by two dots and a cross within small circles. The absolute values of the fluxes ϕ_a and ϕ_b are decreasing; that of ϕ_c is increasing. Hence, according to Faraday's law of induction, the three currents have the instantaneous directions shown by the arrowheads in Fig. 117. Applying the well-known rule for the direction of the mechanical force between a flux and a current, it will be found that all four combinations cause the disk to rotate from left to right. Since we have shown before that the direction of each force keeps the same sign throughout a complete cycle, the cumulative action is thus proved for any instant.

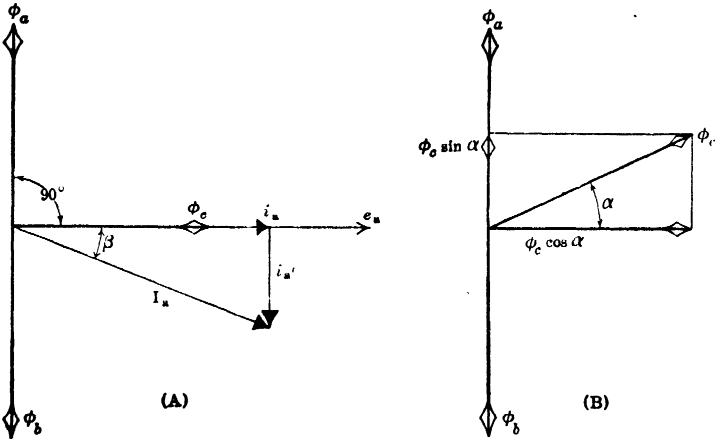

FIG. 120. The vectors of the quantities shown in Fig. 119.

Now let the line current lag behind the applied voltage by an angle α (Fig. 120B). This is the same as if the applied voltage, and consequently the flux ϕ_c, were advanced by the same angle α with respect to their former phase position, and the fluxes ϕ_a and ϕ_b remained intact. The flux ϕ_c may be resolved into two components in time: $\phi_c \cos \alpha$ in quadrature with ϕ_a, and $\phi_c \sin \alpha$ in phase with ϕ_a. The discussion given above for the flux ϕ_c may be applied to the component $\phi_c \cos \alpha$, because it occupies the same space and time position as the total flux ϕ_c occupied at unity power factor. Thus the torque due to this component of the flux is reduced in the ratio of $\cos \alpha : 1$ as compared to its former value. The component $\phi_c \sin \alpha$ simply increases the flux ϕ_a and reduces ϕ_b by the same amount. This affects the values of the corresponding currents i_a and i_b (Fig. 117). But since one is reduced by the same amount by which the other is increased, their total torque with $\phi_c \cos \alpha$ remains the same as if the component $\phi_c \sin \alpha$ did not exist. Thus, the net result is that the tractive effort upon the disk is reduced in the ratio of $\cos \alpha : 1$. An ideal meter should read correctly at any value of power factor, if it is correctly adjusted at unity power factor.

Two assumptions are made in the foregoing theory which do not hold true in an ordinary single-phase motor designed for delivering power: (1) The reaction of the disk currents upon the fields is neglected. (2) The rotational emf's and the corresponding currents in the disk are neglected. These assumptions are justified in a meter, because the disk currents are small and the speed of rotation is very low.

128. Calibration of Induction-Type Watthour Meters. — The points to be covered and the procedure are in general the same as in the case of a commutator-type meter (§119). The following instructions cover only those points which have not been taken up before, or where the procedure differs from that followed with a d-c meter. The preliminary adjustments include the bearings, the friction compensation, the full-load speed, and the phase lag (§125). An inaccurate lagging of the potential phase is of little effect at a large non-inductive load, but becomes of increasing importance as the load decreases and becomes more inductive, that is, as the meter speed is reduced. After the brake magnets are adjusted so that the meter runs correctly near unity power factor, the lagging coil is adjusted so that the meter reads nearly correct at a power factor of about 50 per cent. Then the reading at 100 per cent power factor should be checked again, and the two adjustments set by several trials until the meter has as nearly as possible the same constant at both values of power factor. If a two-phase source of supply is available, with the phases exactly in quadrature, a further check on the phase adjustment is possible at zero power factor, by connecting the current leads to a non-inductive load on one phase, and using the other phase for the potential circuit. See also §§132 and 133 regarding phase-shifters and a phantom load.

Of the three methods of calibration described in §119, a direct comparison with a standard watthour meter (rotating standard) is the most convenient. The standard watthour meter is usually also of the induction type and its number of watthours per revolution is known. Accurate current and potential transformers may be used to bring the rotating standard and the meter under test to within approximately the same range, although a standard wattmeter usually has several ranges, which are obtained by properly interconnecting its coils.

The indicating-wattmeter method, with a stopwatch, is also used to a considerable extent, especially when the voltage and the frequency are steady. When a standardized load resistance is used, care must be taken to have it wound non-inductively, or else its true power consumption and the power factor must be determined by separate tests.

Rotating standards are now available with a flickering neon-lamp attachment in the light of which the disk of the test meter is viewed. Special notches on the edge of the disk appear to stand still when the two meters are brought to the same speed. This "Stroboscopic" method greatly shortens the time required for a calibration and increases its accuracy. (See literature references at the end of this chapter.)

The constant of an induction-type watthour meter is affected by the wave-form and frequency of the supply. Care must be taken, therefore, to see that these are standard, or else the deviations must be known. A good meter is affected but little by reasonably small departures from the sine-wave and from the rated frequency, but a meter of a new or unknown type should be carefully tested in this respect, especially at low values of power factor.

129. EXPERIMENT 5-C. — Calibration of an Induction-Type Watthour Meter. — The meter is described in §§125 to 127. The experiment is performed in the same order as Experiment 5-A, keeping in mind the difference in the construction of an a-c meter and the necessity of considering not only the magnitude of the load but its power factor and frequency as well. These points are covered in §128. If a phase-shifter is available (§132), make use of it in part of the calibration, and make clear to yourself its advantages over an inductive load. A phase-shifter may be improvised in the laboratory to show its mode of operation, even though its accuracy may not be sufficient for actual use. Provide a phantom load (§133), and compare the ease of manipulation, the accuracy, and the energy consumption with those when an actual load is used.

130. Some Causes of Inaccuracy. — In the 1916 edition of the Electrical Meterman's Handbook, published by the National Electric Light Association, the following table was given on page 414 in regard to possible troubles in an induction-type watthour meter and their effect upon the operating characteristics. This table will be found useful in the practical management of meters. Moreover, it will give a thoughtful student an opportunity to verify his understanding of the theory. Given a trouble, say a short circuit in the potential coil, he should be able to predict theoretically its effect upon the performance of the meter.

Condition Found	Possible Causes
Slow. Full load and light load	Short circuit in current or potential coils; loose magnets.
Slow. Principally at light load	Defective bearing; other excessive friction; loose light-load adjustment.
Fast. Full load and light load	Weakened or loose magnets; short circuit in series reactance of potential circuit (non-inductive load).
Fast. Principally at light load	Loose or defective light-load adjustment; vibration; short circuit in current coils.

A frequent cause of trouble with polyphase meters, especially when used with instrument transformers (Chapter XVIII), is wrong connections. A clear understanding of polyphase connections (Vol. II) is the only safe guide in cases not covered by standard diagrams furnished by the maker.

131. EXPERIMENT 5-D. — Troubles in Induction-Type Watthour Meters. — The principal troubles are enumerated in the preceding article. A meter should be provided in which the coils may be partially short-circuited at will, bearings, magnets, and compensating coils put out of adjustment, etc. The student can then check the points covered in the foregoing table, as well as provide other combinations of troubles. If a polyphase meter is available, the effect of wrong connections should be studied, with and without instrument transformers, especially the effect of reversed leads.

Report. Give the findings in each case of trouble, and explain theoretically that the observed behavior of the meter could be predicted from its construction and mode of operation.

132. Potential Phase-Shifters. — A watthour meter which may be used on partly inductive loads must be tested for its accuracy of calibration at different values of power factor. It is rather inconvenient to provide such an adjustable load in the laboratory, and somewhat tedious to set it at a desired value of current and power factor. It is much more convenient always to load the meter on a lamp bank or other resistor at practically unity power factor, and to supply the potential windings with a current properly shifted in phase. For example, let it be required to test a meter at a phase displacement of 30 degrees between the voltage and the current, the latter lagging. It is much simpler to advance the voltage by 30 degrees than to lag the current by the same amount. There are two types of devices used for shifting the potential phase in instrument testing, particularly in the calibration of watthour meters. Either of them may be conveniently combined with the artificial load described in the next section.

(a) *The two-component potential shifter.* The principle upon which this device is built may be understood from Fig. 121. Two transformers or auto-transformers (§§386 and 402) *A* and *B* are connected to phases I and II, respectively, of a two- or three-phase supply. Each transformer is provided with a number of taps. By properly cross-connecting these taps, a voltage of any desired magnitude and time phase may be obtained. In the sketch the taps *n* and *p* are shown connected together, and the potential leads to the meter are taken from the taps *m* and *q*. The theory of this method may be explained by reference to the vector diagram. OV_1 and OV_2 are the vectors of the applied voltages I and II, assumed to be at a known phase angle to each other, such as 60°, 90°, or 120°.

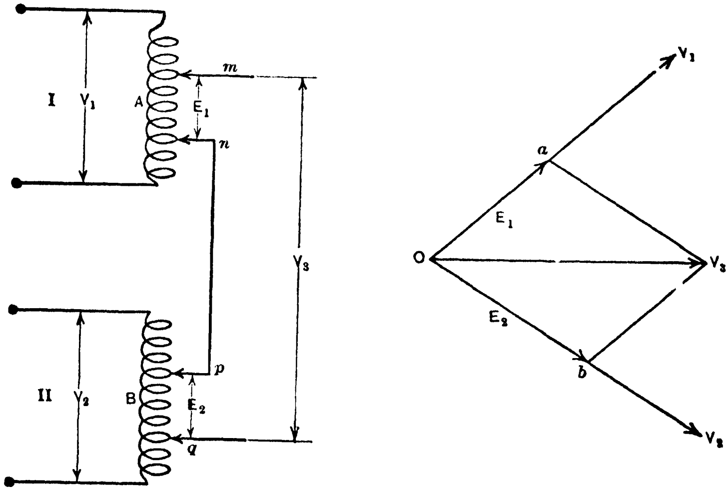

FIG. 121. A two-component potential phase shifter.

The voltage between the taps *m* and *n* is a certain fraction of the total voltage OV_1 and is shown as the vector $E_1 = Oa$ in phase with OV_1. Similarly, the voltage $Ob = E_2$ between *p* and *q* is a fraction of the voltage OV_2. The secondary potential difference V_3 between *m* and *q* is a geometric sum of the voltages E_1 and E_2, and is therefore equal to the diagonal OV_3 of the parallelogram constructed on *Oa* and *Ob*. By giving different positive and negative values to E_1 and E_2, the resultant secondary voltage V_3 may be adjusted to any desired magnitude or phase position with respect to V_1 and V_2.

In a convenient practical device of this kind the connections are such that the phase of the secondary voltage is shifted by simply turning a handle on a dial or inserting a plug in one of a number of holes. Moreover, the magnitude of the voltage remains constant and only its time-phase is shifted. A scale is provided which indicates either angles or the corresponding values of power factor. If the meter load is slightly inductive, the scale itself is shifted so as still to have the correct 100 per cent point when the voltage and the current are in phase with each other. When the phase displacement between the current and the voltage is exactly 90 degrees, an indicating wattmeter connected in the circuit shows zero. It must be remembered that this device requires a two-phase or three-phase source of supply even for testing single-phase meters, but it does away with the necessity of providing an adjustable inductive load.

(b) *The revolving-field potential shifter.* This device is described in §90 in application to an a-c potentiometer. The potential shifter is shown there with a split-phase stator, because only a single-phase supply is supposed to be available. With a two-phase supply, the windings *aa* and *bb* are connected directly to the two phases, so that the condenser and resistance *R* are unnecessary. With a three-phase supply, the stator must be wound accordingly, or else the three-phase system must be converted into a two-phase system, by one of the known methods (see Phase Transformation, Vol. II). The point on the dial corresponding to zero power factor is determined by an indicating wattmeter which shows zero at this point.

133. Artificial or Phantom Load. — The regular testing of many watthour meters under full-load conditions results in the wasting of considerable amounts of energy, requires bulky resistances, and causes excessive heat losses which sometimes make the meter room rather uncomfortable. For this reason it is preferable to supply the load current for the meter through a separate transformer, at a considerably reduced voltage, say at about 10 volts for a 110-volt meter. The potential leads of the meter are, of course, connected to a source of the rated voltage. With such an arrangement it is possible to have the same current and voltage relations in the meter as if it were measuring an actual load, at the same time saving about 90 per cent of the energy which otherwise would be required for calibration. Such an artificial or phantom load may be conveniently used in combination with one of the potential phase-shifters described in the preceding section.

A phantom load usually consists of a constant-potential transformer with low-voltage secondary, resistors connected to the secondary, and switches for varying the load. The primary of the transformer is connected to the same source of supply as the potential winding of the watthour meter. The secondary circuit of the load also contains the series winding of the watthour meter, an ammeter, a wattmeter, etc. With a 10 to 1 transformer ratio only 1 ampere is drawn from the supply for every 10 amperes that flow through the meter. The phase relationship between the current and the voltage may be adjusted by means of a potential phase-shifter. Phantom load combinations are also available in portable form for testing meters on consumers' premises.

134. Maximum-Demand Indicators (Demand Meters). — The actual cost of supplying a certain number of kilowatthours of energy to a consumer generally depends on the rate at which he consumes this energy (§115). A consumer who uses 1 kw regularly for eight hours a day is more desirable to the operating company than another who uses 4 kw for two hours. The amount of energy per month may be the same in both cases, but the first consumer requires less capacity of the generating apparatus and a smaller transmission line than the second one; he also causes smaller fluctuations of the load. Therefore the company may give him a better rate per kilowatthour. This principle of "discrimination" in charging for electrical energy is used by many operating companies. The consumer is charged so much per kilowatthour of actually consumed energy, and then an extra charge is made for each kilowatt of his *maximum demand.*

This demand or peak is usually agreed to be the maximum amount of energy consumed in any consecutive thirty minutes during the month, or in any consecutive fifteen minutes, etc., according to the local conditions. This amount of energy, divided by the time, is taken as the maximum demand in kilowatts, although it only gives the average rate of energy consumption during the peak period. For example, if the total energy consumption during a half hour of heaviest load is 1250 kwhr, the demand is taken to be 2500 kw. This method of reckoning does not penalize the customer for very short peaks.

Instruments for measuring the maximum demand during a month, or other agreed period of time, are called maximum-demand indicators, or demand meters. Some of them are attachments to watthour meters; some are modified watthour meters; and some are based on the thermal principle. It would lead too far to describe these numerous devices in this book. The following description of one of the oldest demand indicators will suffice to illustrate the general principle. Much information on the subject will be found in various publications of the National Electric Light Association.

135. The Wright Maximum-Demand Indicator. — This device is shown schematically in Fig. 122*a*. It records the maximum current of certain duration which has passed through it at any time since it was last set. A liquid is hermetically sealed in a glass tube having a bulb at each end. A band of resistance metal is placed around the left-hand bulb *A*. The current to be measured passes through this band, heats the air in the bulb, and the expansion of the air forces the liquid up into the right-hand side of the tube, causing it to overflow into the middle or indicating tube. The liquid deposited in the indicating tube remains there until the indicator is reset. The glass tube is carried on a backing, so hinged that the meter can be reset by tipping the tube and allowing the liquid to run out of the indicating tube into the side tubes. It is the difference in temperature of the air in the two bulbs that causes the indicator to register. Any change in the temperature of the external air causes equal air expansion in both bulbs, and hence does not affect the reading.

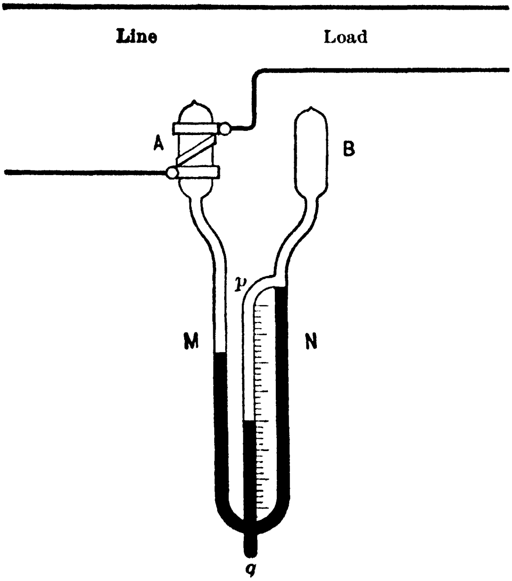

FIG. 122*a*. Wright maximum-demand indicator.

The instrument is purposely made slow-acting; standard indicators are so designed that if the maximum load lasts only five minutes, the meter will register about 80 per cent; if ten minutes, 95 per cent, and the full 100 per cent is registered when the load has continued about forty minutes. In this way the customer is not penalized for short overloads which do not inconvenience the power supply station.

135A. Other Forms of Demand Meters. — A common form of demand meter is obtained by including in the watthour meter some form of demand indicator. This may consist of a pen and chart, the pen moving forward with the kilowatthours consumed, for a period of fifteen minutes, and then being atomatically returned to zero by a clock mechanism. Each time the hand is set back to zero the chart is advanced a small amount. Thus the record of the chart gives both the demand during each fifteen-minute period, and the time in the day, or day in the month, when this demand occurred.

Another form of demand meter is obtained by means of a demand attachment placed (Figs. 122*b* and 122*c*) in the ordinary watthour meter. In place of the standard register is substituted one in which, to the usual dials, are added a scale of kilowatt demand, and a hand which moves over the scale just as the pointer moved over the chart in the type of meter previously described. This hand pushes forward a loose pointer and leaves it at the maximum reading, when the clock mechanism trips and returns the moving hand to zero. Thus, the loose pointer remains at its maximum indication until reset by hand. This type of demand meter shows at a glance the maximum demand during the period, but it does not show the time at which this demand occurred. The records of the switchboard attendant, if the meter is in a power house or substation, should show the approximate time that the maximum indiaction was produced.

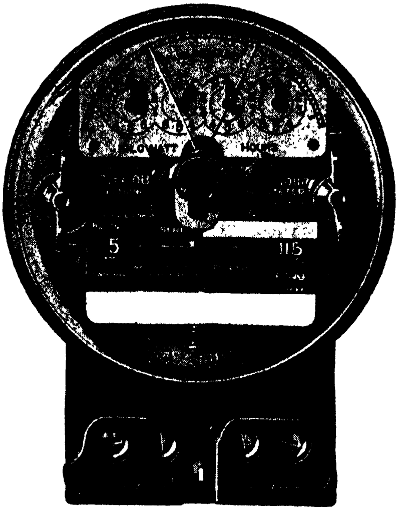

Fig. 122*b*. Westinghouse induction-type watthour meter with demand attachment.

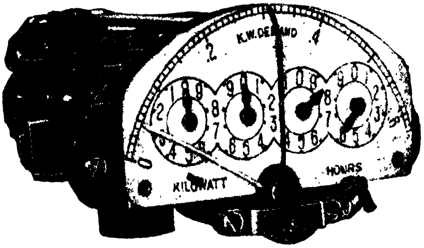

Fig. 122*c*. The demand attachment for the Westinghouse induction watthour meter.

Other types of kilowatthour demand meters are built, as well as of kilovolt-ampere demand meters. In one of the latter, two watthour-meter elements are enclosed in the same case, one connected to register kilowatthours, and the other to give reactive kilovolt-amperes (§107). Both elements operate to move the pen of a recording mechanism over a ruled chart, the values of kilowatthours, and of reactive kilovolt-amperes being combined by the mechanism of the meter into kilovolt-amperes.[1]

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[1] See "Metering of Alternating Current," published by the Educational Department of the Westinghouse Electric and Manufacturing Co.

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CHAPTER VI

REACTANCE AND RESISTANCE IN A-C CIRCUITS

136. Physical Concept of Inductance. — When an electric current exists in a conductor, a magnetic field is created around it. As long as the current remains constant, this field does not react in any way upon the electric circuit. But if the current varies, the flux will also change; in doing so it induces in the conductor an emf. In other words, the magnetic field reacts upon the circuit and either affects the rate of change of current, or else requires an additional applied voltage to counteract its effect. This induced emf, in accordance with the fundamental law of electromagnetic induction, has such a direction as to oppose the change in current, and consequently to retard the change in flux. The action is very much the same as if the magnetic field had some kind of inherent *inertia.* The value, ϵ, of this counter emf is expressed by the relationship

$$\epsilon = -L\, di/dt \qquad (1)$$

where i is the instantaneous current, di/dt is its rate of change with time, and L is a coefficient of proportionality, called the *inductance,* or the coefficient of self-induction, of the part of the circuit under consideration. The minus sign is necessary since, when the current increases, the reaction of the magnetic flux induces an emf which opposes this increase. Thus, ϵ is negative when di/dt is positive, and vice versa. The practical unit of inductance in the ampere-ohm system is called the henry. *An electrical device or circuit is said to have an inductance of one henry if one volt of counter emf is induced in it when the current changes at a rate of one ampere per second.*

137. Difference between Inductance and Ohmic Resistance. — The presence of inductance in an a-c circuit usually necessitates an increase in the applied emf in order to maintain the same current as without the inductance. An increase in resistance has a similar effect. There is a very fundamental difference between the two, however, as a consideration of the following points will show:

(1) In an ohmic resistance the drop caused by it is at any moment proportional to the *instantaneous value* of the current. An inductance becomes apparent only when the current is varying; the induced emf, or resulting voltage drop, being proportional to the *rate of change* of the current, and not to its absolute value; see eq. (1) above.

(2) The ohmic resistance and the accompanying heating of a metallic conductor are said to be due to collisions of streaming electrons, which constitute the electric current, against the molecules of the metal. Inductance, on the other hand, is supposed to be caused by the inertia of the magnetic flux surrounding the conductor.

(3) The energy spent in an ohmic resistance is converted into heat, and is lost electrically. The electric energy spent in an inductance, when the current increases, is stored in the form of electromagnetic energy of the field, and is given back to the circuit, when the current decreases, through the medium of induced emf.

(4) A wire, whether straight or wound into a coil, has the same ohmic resistance, while its inductance in the second case is increased many times because of the concentration of the magnetic flux. Thus, generally speaking, the ohmic resistance of a cylindrical conductor does not depend upon the shape of its geometric axis, while its inductance is essentially determined by the shape into which the conductor is bent.

138. Inductance and Reactance. — In many practical problems inductance is treated in connection with sinusoidal alternating currents of a constant frequency. It is convenient, therefore, to combine the frequency factor with the value of inductance; this simply means a change in the unit and in the physical dimension of the quantity which expresses the inertia of the magnetic field. The new physical quantity is called the *reactance* of a circuit or of a device. It will be shown that reactance, like ordinary resistance, may be expressed in ohms.

The following considerations lead to the concept of reactance. According to the foregoing definition of inductance, the applied voltage necessary for overcoming the effect of an inductance of L henrys is

$$e = L\,di/dt \quad \ldots\ldots\ldots\ldots \quad (1a)$$

where the voltage $e = -\epsilon$ may be properly called the instantaneous inductive drop of potential in the conductor or circuit of inductance L. The voltage e is at any instant equal and opposite to the counter emf ϵ induced in the conductor by the varying magnetic flux. It is more convenient in the following discussion to consider the voltage e rather than ϵ.

Let the current vary according to the sine law, Fig. 123, at a frequency of f cycles per second. The instantaneous values of the current may be expressed by the formula

$$i = I_m \sin 2\pi f t \quad \ldots\ldots\ldots\ldots \quad (2)$$

where I_m is the amplitude of the wave, and t is time in seconds. Substituting the value of i from eq. (2) in eq. (1a), we obtain the following value for the inductive drop:

$$e = 2\pi f L I_m \cos 2\pi f t \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

This result interpreted means that a sinusoidal current through a pure inductance sets up a sinusoidal counter emf of the same frequency and requires an applied voltage of the amplitude $E_m = 2\pi f L I_m$.

Denoting $$2\pi f L = x \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

we get $E_m = xI_m$. The same relation holds true for the effective values, I and E, of current and voltage (see note on page 90). Thus we obtain

$$E = xI \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

The quantity x is called the *inductive reactance* of the circuit or electrical device under consideration. Equation (5) shows that a reactance, being the ratio of a voltage to a current, may be expressed in ohms like an ohmic resistance. It will be seen from eq. (4) that inductive reactance is a composite concept; its value depends on the inductance of the device and on the frequency of the supply. In practice the frequency usually remains constant, and hence the reactance also remains constant (save when the magnetic conditions surrounding the conductor are changed).

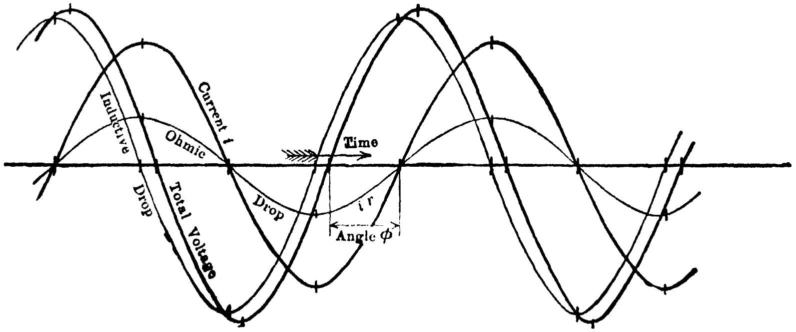

Fig. 123. Instantaneous values of current and voltage in an alternating-current circuit containing resistance and inductance.

The current i and the voltage e (marked "Inductive Drop" in Fig. 123) do not reach their maxima simultaneously; the current passes through zero when the emf reaches its maximum, and vice versa. This is in accord with eqs. (2) and (3), for the current is a sine function and the voltage varies according to a cosine function. Since $\sin 2\pi f t = \cos (90° - 2\pi f t)$, the two waves are said to be displaced in time phase by 90 electrical degrees with respect to each other. The current lags behind the voltage, because the latter reaches its maxima at the instants of time corresponding to $2\pi ft = 0, 2\pi, 4\pi$, etc., while the current reaches its highest positive values when $2\pi ft = 0.25\pi, 2.25\pi, 4.25\pi$, etc. The curves shown in Fig. 123 may be obtained experimentally with an oscillograph (see Vol. II).

139. Experimental Determination of Reactance. — An experimental study of reactance is made somewhat complicated by the presence of unavoidable ohmic resistance. The influence of the resistance may be eliminated as is shown in §143 below, or it may be reduced to a negligible amount. The latter conditions may sometimes be obtained by using a coil of comparatively low resistance and of high inductance and testing it at a high frequency and low current. The ohmic drop is then small while the reactive drop may be relatively quite high.

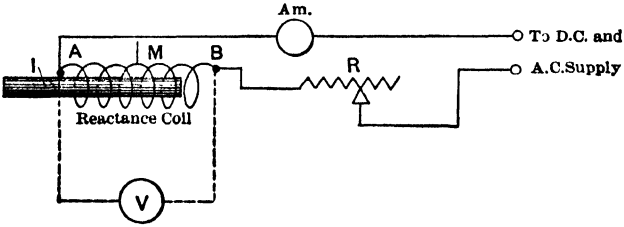

FIG. 124. Resistance and reactance in series.

The reactance of such a coil of low resistance, AB (Fig. 124), may be determined experimentally, by measuring the current flowing through the coil and the a-c voltage at its terminals. According to eq. (5) the reactance is equal to the ratio of the voltage to the current. The current is measured on the ammeter *Am*, the voltage on the voltmeter V connected across the terminals of the coil. The rheostat R is for the purpose of regulating the current.

An iron core I is shown within the coil; the presence of iron intensifies the flux produced by the coil, and thus greatly increases its reactance. By moving the core in and out the reactance may be varied within wide limits. This method is often used for regulating current in an a-c circuit.

Four factors influence the accuracy of determination of reactance by this method: The ohmic resistance of the coil, the iron loss in the core (hysteresis and eddy currents), the saturation of the iron core, and higher harmonics in the supply voltage. These factors either may be kept down by a suitable choice of conditions, or their influence may be determined and corrected for, as is explained below.

140. Factors Which Affect the Value of Reactance. — The reactance of a coil with an iron core depends on the following factors:

(a) Frequency of the supply (Fig. 125).
(b) Position of the core (Fig. 126).
(c) Intensity of current (Fig. 127).
(d) Number of turns in the coil.

In these diagrams the student is asked to pay attention, for the time being, only to the curves marked "Reactance x." It will be seen from these curves that for a given coil:

(a) The reactance is directly proportional to the frequency of the supply (Fig. 125), in accordance with the definition of reactance; see eq. (4).

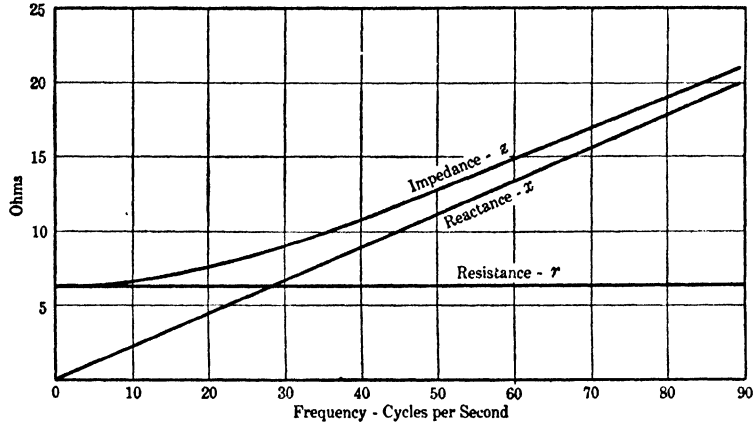

FIG. 125. Influence of frequency on reactance and impedance (resistance and inductance are kept constant).

(b) The reactance depends essentially upon the presence and the position of the iron core, or plunger. The reactance decreases as the plunger is *drawn out* of the coil; the lower limiting value of the reactance is that of the coil without iron. In Fig. 126 the curve marked "Impedance" (see §143) also represents reactance since in this test the two were nearly equal.

(c) With iron, the value of the reactance, determined as the ratio of reactive volts to amperes, depends upon the intensity of the current flowing through the coil. This is because the magnetic flux is *not* proportional to the current, the iron approaching its saturation limit (§165). In this case the current wave is not sinusoidal, even when the voltage varies according to the sine law. Strictly speaking, the theory given above does not hold true when eq. (2) ceases to be applicable, but it is possible to speak of the reactance of a coil as defined by eq. (5). In this case the reactance reaches its maximum at some point within the range of maximum permeability of iron, in other words, where the flux per ampere of current is a maximum. Without iron, there is no reason why the reactance should depend on the value of the current, as both the flux in the coil and the applied voltage are proportional to the current.

(d) The inductance, as well as the reactance, of a given coil is proportional to the square of the number of turns in series. Consider two coils of the same outside dimensions and with the same space available for winding. Let one of the coils be wound with a finer wire, so as to accommodate m times as many turns in the same space; assume that the currents in the two coils are adjusted so as to produce equal magnetic fluxes. Let the induced voltage and the current in the first coil be E and I respectively — those in the second coil E' and I'. Since the flux in the second coil is the same as in the first, I' must be $= I \div m$, in order to have the same number of exciting ampere-turns. The flux in the second coil is interlinked with m times as many turns as that in the first coil; consequently $E' = mE$. Thus, the reactance of the second coil

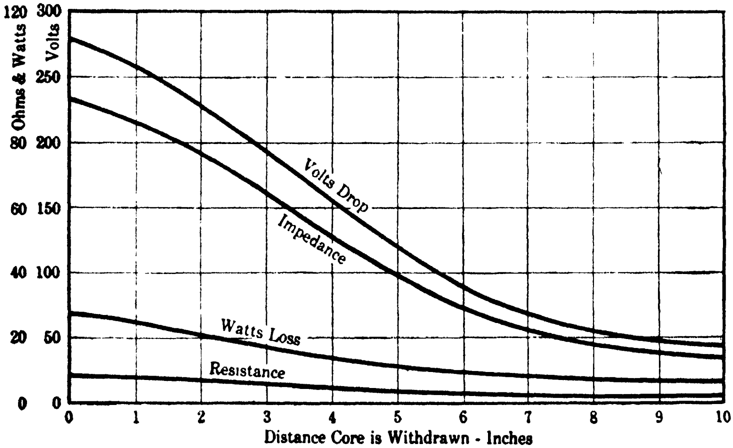

FIG. 126. Influence of the position of the plunger (Fig. 124) on the reactance and impedance of the circuit.

$$x' = \frac{E'}{I'} = \frac{mE}{I \div m} = \frac{m^2E}{I} = m^2x$$

where x is the reactance of the first coil. This proves the above proposition.

141. EXPERIMENT 6-A. — Study of a Reactance Coil without Iron. — The purpose of this experiment is to investigate the influence of the frequency and of the number of turns in a coil upon its reactance, as discussed in the preceding article. The connections are shown in Fig. 124; a coil should be used with a considerable number of turns of heavy wire in order to have an appreciable inductance with a negligible resistance. (a) First investigate the influence of the frequency.

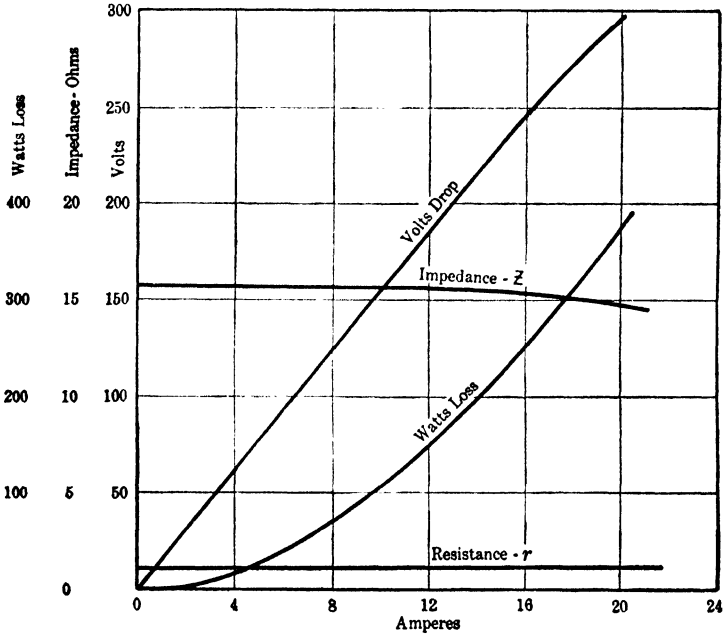

FIG. 127. Influence of the magnitude of current on the impedance of a coil (effect of saturation in iron).

Connect the circuit to an a-c generator the speed of which may be varied at will. Bring the speed up to the highest safe value and keep the current down to a relatively low value by means of rheostat *R* and by cutting down the field excitation of the alternator. Read the voltage across the coil, the current in it, and the speed of the alternator. Increase the current in three or four steps and take similar readings at each step, keeping the speed constant. Reduce the speed in steps to the lowest practicable limit, and at each step take a few readings with different values of current and voltage.

(b) To investigate the influence of the number of turns, it is convenient to have a coil wound with two or more wires in parallel. By using separate sections of the winding, or combining them in series and in parallel, different numbers of active turns are obtained. The experiment may be performed at a constant high frequency; the reactance should be determined at low, medium, and high values of current.

(c) In cases where inductance is harmful, for example in high-grade resistance units, coils are wound *non-inductively*, that is, so as to produce practically no magnetic flux. This is accomplished by connecting the two halves of a winding in such a way that one tends to produce a magnetic flux in the direction opposite to that of the other. Try a non-inductive arrangement by sending a current through the coil with two sections of the winding connected in opposition. If the coil has only one winding, take the middle tap M (Fig. 124) as one terminal, and use the points A and B connected together as the other terminal. Before closing the circuit, insert enough resistance R to prevent an inrush of current. The voltage drop across the coil will be found to be small, and consequently its reactance will be low.

Report. Compute the values of reactance from eq. (5) and plot them to frequency as abscissas; figure out the inductance of the coil (in henrys) according to formula (4). Give the data and the results, showing that the inductance increases as the square of the number of turns. Describe the experiment with the coil wound non-inductively. Point out and explain any inaccuracies in the results due to the influence of the ohmic resistance.

142. EXPERIMENT 6-B. — Study of a Reactance Coil with an Iron Core. — The factors to be investigated are enumerated in §140. (a) The influence of the frequency and of the number of turns may be investigated as in Experiment 6-A. However, the current must be kept constant for each set of readings, since the value of the reactance depends somewhat upon the current, due to the presence of the iron core. This part of the experiment may be omitted, if Experiment 6-A has been previously performed.

(b) To investigate the influence of the position of the plunger, first remove it (Fig. 124), and adjust the current in the coil to some nominal value. Then gradually move the plunger in, at the same time cutting the resistance R out of the circuit, so as to keep the current constant (Fig. 126).

At each position read the volts across the coil, the current, the position of the plunger, and the frequency. Take similar sets of readings at two or three smaller values of current. Take another set of curves, keeping the voltage across the coil constant, and allowing the current to drop as the plunger is inserted into the coil.

(c) To investigate the influence of the intensity of current upon the reactance (Fig. 127), set the plunger at the position of maximum flux and take readings of volts and amperes, increasing the current in steps. Repeat the test with two or three different positions of the plunger.

Report. Plot curves showing variation in the reactance of the coil with the frequency, with the position of the plunger, and with the intensity of the current, as in Figs. 125, 126, and 127. Explain the character of the curves theoretically, and point out the inaccuracies due to the ohmic resistance.

143. Impedance, or Combination of Reactance and Resistance. — Reactance coils have been considered so far as devoid of ohmic resistance, or at least as having a negligible resistance. In many practical cases, however, the ohmic resistance of a circuit is of equal if not greater importance than its reactance. It is necessary, therefore, to investigate the combined effect of a reactance and a resistance in series.

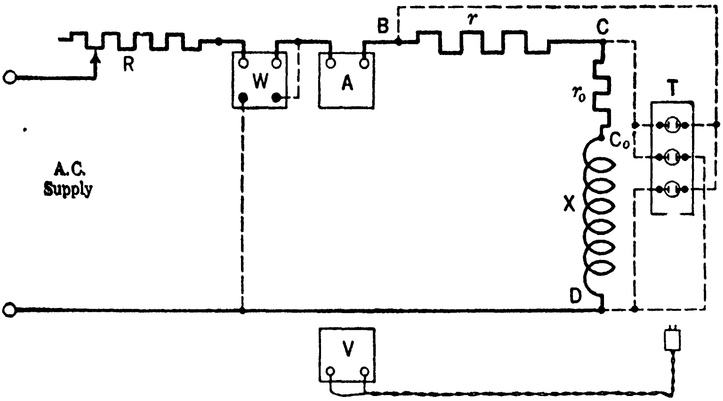

FIG. 128. Diagram of connections for investigating electrical relations in a circuit containing resistance and reactance in series.

Experience and theory show that a reactance x and a resistance r, connected in series, as in Fig. 128, cannot be added arithmetically, even though both are expressed in ohms. They must be added *geometrically*, at right angles, as in Fig. 129. For example, let a reactance of 4 ohms be connected in series with a resistance of 3 ohms. To force a current of 10 amperes through the reactance alone, an emf of 40 volts is necessary; to force the same current through the resistance alone, 30 volts are required. But when the two are connected in series, only 50 volts are necessary, instead of 40 + 30 = 70 volts. In other words, the combined effect of 4 ohms and 3 ohms is in this case equivalent to 5 ohms. and not to 7 ohms.

This peculiar relation can be explained by means of the equations established in §138. When a coil has some ohmic resistance r, in addition to an inductance L, the emf which must be applied at its terminals in order to produce a current i, according to eq. (1a), is

$$e = L\frac{di}{dt} + ir \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

If the current wave is sinusoidal, then, substituting the value of i from eq. (2) and remembering the relation (4), we obtain

$$e = xI_m \cos 2\pi ft + rI_m \sin 2\pi ft \quad . \; . \; . \; . \; . \; . \; . \quad (7)$$

This shows that the applied voltage at any instant is a sum of two voltages, each varying as a sine-wave (Fig. 123). The wave of the inductive drop is the same as before; the wave of the ohmic drop, ir, is in phase with the current, both being represented by the sine function. The total voltage is also represented by a sine-wave, for the sum of two sine-waves of the same frequency is also a sine-wave.

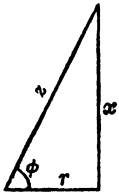

FIG. 129. Geometrical addition of a resistance and a reactance, the result being an impedance.

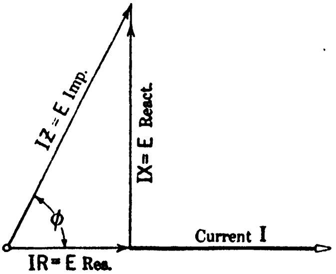

FIG. 130. The triangle of voltage drop, corresponding to Fig. 129.

It will be seen from Fig. 123 that the amplitude of the wave marked "total voltage" is less than the sum of the amplitudes of the two component waves. Its phase position is also intermediate between the two. We may express this voltage as

$$e = zI_m \sin (2\pi ft + \phi) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

where z and ϕ are two new quantities which determine the magnitude and the position of the resultant e. Equating (7) and (8) we obtain

$$z \sin (2\pi ft + \phi) = x \cos 2\pi ft + r \sin 2\pi ft \quad . \; . \; . \; . \quad (9)$$

This relation holds true at any instant t; applying it first for $ft = 0$, and then for $ft = 1/4$, we obtain

$$\left.\begin{aligned} z \sin \phi &= x \\ z \cos \phi &= r \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (10)$$

whence

$$\left.\begin{aligned} z &= \sqrt{x^2 + r^2} \\ \tan \phi &= x/r \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (11)$$

For given values of x and r, the values of z and ϕ may either be calculated from eq. (11) or constructed as in Fig. 129. The quantity z is called the *impedance* of a piece of apparatus or of a circuit, and is expressed in ohms. It is equal to the quadrature geometrical sum, and not to the arithmetical sum, of the component resistance and reactance in series.

This addition of quantities in quadrature is commonly represented by the so-called "complex" or "symbolic" system of notation, in which a quantity in quadrature with another, taken as the reference, is expressed by the letter j preceding the quantity. Thus $r + jx$ means that x is to be considered as 90 degrees ahead (in a counter-clockwise direction) of r, and the two are to be added geometrically (Fig. 129).

Returning now to the above-stated numerical example, we see that a 4-ohm reactance and a 3-ohm resistance act together as an impedance $z = \sqrt{4^2 + 3^2} = 5$ ohms. Equation (8) may be represented symbolically thus:

$$E = I(r + jx) = zI \angle\phi \quad \ldots\ldots\ldots \quad (12)$$

where $\angle\phi$ means that the applied voltage wave leads that of the current by the angle ϕ (Fig. 123). This equation, compared with eq. (5), clearly indicates the influence of an ohmic resistance in series with a reactance.

The analytical relationship represented by eqs. (7) and (8) is shown graphically in Fig. 130, where I is the effective value of the current, $I = I_m/\sqrt{2}$. The two legs of the right triangle represent the vectors Ir and Ix of the ohmic and reactive drop respectively. The resultant vector, or the hypotenuse Iz, represents the total applied voltage. The current vector I is in phase with the ohmic drop. The triangles in Figs. 129 and 130 are similar, and the latter is obtained from the former by multiplying all its sides by the effective value, I, of the current. It is well to keep in mind, however, that *the triangle in Fig.* 130 *is a vector diagram of quantities which vary according to the sine law with the time, whereas the triangle in Fig.* 129 *is a combination of constant scalar quan-*

tities, r, x, and z, and the impedance $z = r + jx$ is not a vector but an "operator," i.e., it operates to change the length of the current vector by which it is multiplied, and rotates it through an angle. Thus Iz is a voltage which is z times the current in value and at the angle ϕ ahead of it.

When a coil has an appreciable ohmic resistance, its reactance is determined as follows: Let an alternating voltage E be applied to the terminals of the coil and let I be the current in the coil. Then the ratio E/I gives the impedance z of the coil. The ohmic resistance r is measured with direct current. In the first eq. (11), r and z are now known, and we have

$$x = \sqrt{z^2 - r^2} \quad \text{. . . . .} \quad (11a)$$

At high frequencies the effective ohmic resistance (§154) may be considerably higher than the resistance measured with direct current. In this case a wattmeter should also be used when measuring E and I, and the effective resistance computed from the relationship $P = I^2r$, where P is the wattmeter reading.

144. EXPERIMENT 6-C. — Study of a Reactance Coil with an Appreciable Ohmic Resistance. — The purpose of the experiment is: (1) to prove that a resistance and a reactance in series must be added geometrically; (2) to investigate the influence of the factors enumerated in §140, as modified by the presence of resistance. The diagram of connections is shown in Fig. 128, except that the wattmeter W is not needed. The non-inductive resistance r is connected in series with a reactive coil x. The latter must be so chosen as to have as small an ohmic resistance as possible; this unavoidable resistance is denoted by r_0. If possible, use a high-frequency source, say a 500-cycle alternator. *Note*: The usual 60-cycle alternator, when operated with "open delta" connection, will produce a considerable 180-cycle voltage. By means of the switch or plug-board T, the voltmeter V may be made to measure the voltage either across the non-inductive resistance or across the reactive coil alone, or across the whole impedance.

(1) Remove the iron core from the reactive coil, send a considerable alternating current through x and r, and read the current and the three voltages. Take similar readings at smaller values of current, keeping the frequency constant. (2) Take a similar set of readings at a constant current and resistance, varying the frequency. (3) Take a similar set of readings at a constant current and frequency and with a variable resistance r. (4) Repeat a few readings with the iron core in the coil. (5) Take a few readings with different numbers of effective turns. (6) Measure the resistances r and r_0 with direct current.

Report. (a) From the first set of readings plot the straight lines between total volts and amperes, and partial volts and amperes. Compute z as the ratio of the total volts to amperes. Compute $r + r_0$ from the d-c measurements, and determine x as in Fig. 129. Draw a triangle of voltages and show that the angle opposite the total voltage is obtuse because of the presence of resistance r_0. (*b*) Show that the values of the true reactance of the coil are proportional to the frequency, while the actual readings of volts across the coil, without being corrected for r_0, increase more slowly than the frequency; give a theoretical reason. (c) From the third set of readings plot a curve as in Fig. 131, and the diagram shown in Fig. 132. (d) Explain theoretically the reason for the observed difference in the voltage readings across the coil with the iron core in and out, at the same current and frequency. Also explain the observed effect of the change in the number of turns.

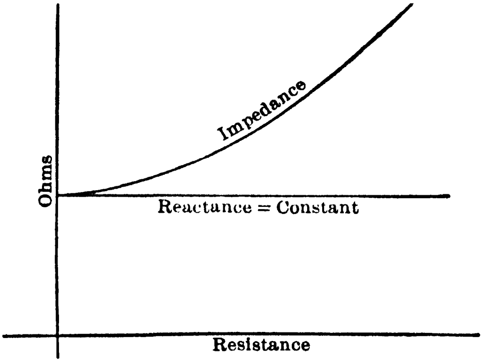

Fig. 131. Influence of variable resistance on the value of an impedance.

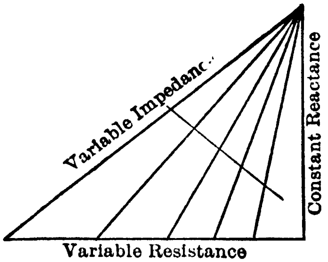

Fig. 132. Explanatory diagram to the curves shown in Fig. 131.

145. Phase Angle and Power Factor. — The phase angle ϕ, Fig. 130, is of great importance in determining the average power spent in a circuit (§94). The actual power varies from moment to moment with the instantaneous values of current and voltage. When inductance is present, the power supplied by the source becomes at times negative; this happens during the parts of the cycle when the inductance gives back the stored electromagnetic energy (see §§136 and 137). For many practical purposes, only the average value of power, during one complete cycle of alternating current, is of importance.

Let e be an instantaneous value of the voltage and i the corresponding value of the current, as in §143. Then the average power during an interval of time T corresponding to one cycle is

$$P = (1/T)\int_0^T ei\,dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (13)$$

Substituting the value e from eq. (6), we get

$$P = \frac{L}{T}\int_0^T i\,di + \frac{r}{T}\int_0^T i^2\,dt,$$

or

$$P = \frac{L}{T}\left(\frac{i^2}{2}\right)_0^T + rI^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

where I is the effective value of the current (§50). The first term on the right side of eq. (14) is equal to zero, because i is the same for any two moments differing by T. This shows that, with periodically fluctuating currents, no power is permanently lost or gained in an inductance; it is merely stored during a part of each cycle and returned to the circuit during another part of the cycle. The power lost in the resistance is represented by the familiar expression rI^2 and is not returned to the source of supply in the form of electrical energy.

Now let the current and the voltage vary according to the sine law, the current being expressed by eq. (2), or $i = I_m \sin 2\pi ft$. If the voltage leads the current by an angle ϕ (Fig. 123), its equation is

$$e = E_m \sin(2\pi ft + \phi) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

Substituting these values in eq. (13) and integrating, we obtain

$$P = (1/T)\int_0^T E_m I_m \sin 2\pi ft \sin(2\pi ft + \phi)\,dt$$

but $\sin(2\pi ft - \phi) = \sin 2\pi ft \cos\phi - \cos 2\pi ft \sin\phi$, so that

$$P = \frac{E_m I_m}{T}\left[\int_0^T (\sin^2 2\pi ft \cos\phi)\,dt + \int_0^T (\sin 2\pi ft \cos 2\pi ft \sin\phi)\,dt\right]$$

Substitute for $\sin^2 2\pi ft$ its equal $(1 - \cos 2 \times 2\pi ft)/2$ and for $\sin 2\pi ft \cos 2\pi ft$ the value $(\sin 2 \times 2\pi ft)/2$. After performing the integrations and substituting limits, one obtains

$$P = \tfrac{1}{2}E_m I_m \cos\phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

or, using effective values in place of amplitudes,

$$P = EI\cos\phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16a)$$

Referring to Fig. 130, this means that the average power is equal to the product of the voltage and the " in-phase " component, $I \cos\phi$, of the current; or else the power is equal to the current times the " in-phase " component, $E \cos\phi$, of the voltage.

The value $\cos\phi$ is called the *power factor* of the circuit. The component of the current in phase with the voltage is known as the power or energy component, that in quadrature with it as the reactive or wattless component. Similar names are sometimes applied to the components of the voltage in phase and in quadrature with the current.

The average power given by eq. (16a) is also known as the *real power* (true watts). The expression EI is called the *apparent power*, or total volt-amperes; the expression $EI \sin \phi$ is called the *reactive power*, or the *reactive volt-amperes*.

146. EXPERIMENT 6-D. — Power Relations with Resistance and Reactance in Series. — The purpose of the experiment is to make clear the relations deduced in §145. The connections are shown in Fig. 128. A voltmeter switch or plug-board T is used, by means of which the voltmeter may be made to indicate at will either the total voltage, the voltage across the inductance, or that across the resistance. The resistance r_0 is meant to represent the small ohmic resistance unavoidable in the reactance coil x.

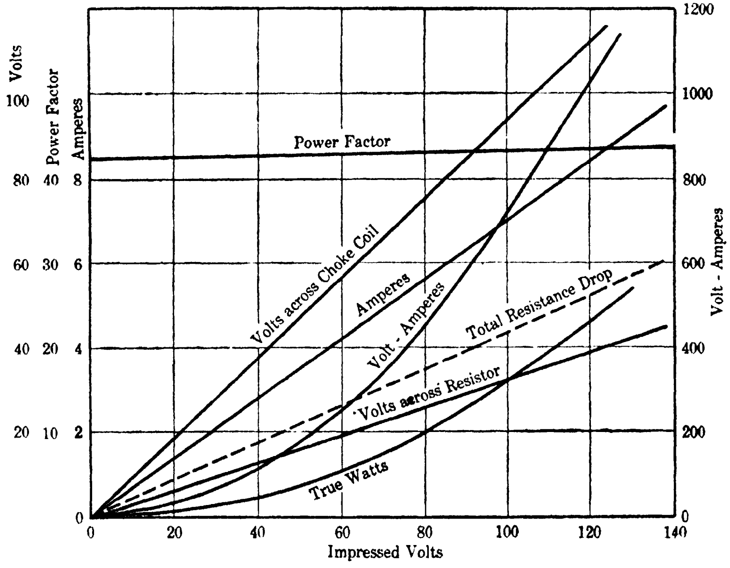

Fig. 133. Variation of the values of current, power, etc., in the circuit shown in Fig. 128, while the impressed voltage varies.

Select by trials such values of r and x as will give about the same drop across BC as across CD; read amperes, volts, and watts. Take several sets of readings, varying the applied voltage by means of the rheostat R. Repeat the same test with different values of r and x.

Report. (1) Plot amperes, watts, and component volts, to total volts as abscissas (Fig. 133). Explain the shape of the curves from theoretical considerations.

(2) Plot corrected values of pure resistance drop and pure reactance drop, as shown by dotted lines. This is done by adding the Ir_0 drop to Ir and subtracting the total ohmic drop geometrically from the applied voltage, according to Fig. 130.

(3) For a few points selected from the curves, show that the power read on the wattmeter checks with the calculated expression $(r + r_0) \cdot I^2$.

(4) Check the values of $\cos \phi$ calculated as the ratio of true watts to apparent watts with those determined from the triangle of voltages (Fig. 130).

147. Impedances in Series. — The relations deduced in §§143 to 146 may now be extended to the case of two or more resistances and reactances connected in series (Fig. 134). Such conditions arise in practice, for example, when both the line and the current-consuming devices possess resistance and reactance, and it is desired to calculate the generator emf and the power factor at the generator bus-bars.

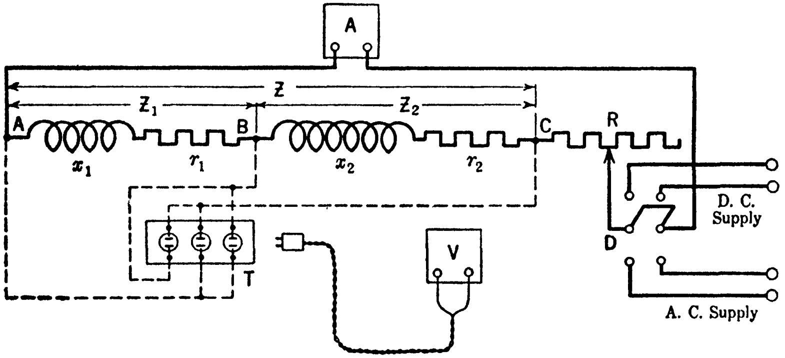

FIG. 134. Diagram of a circuit for testing impedances in series.

The relations are shown in Fig. 135*b*, the diagram being constructed for a given current I (horizontal vector). It is understood that in a vector diagram a voltage or current notation always means the effective value. The total voltage drop across AC (Fig. 134) does not depend on the order in which the resistances and reactances are connected. Therefore, we may substitute an equivalent resistance, $R = r_1 + r_2$, for the two separate resistances r_1 and r_2. In a similar way an equivalent reactance, $X = x_1 + x_2$, may be introduced.

Figure 135*a* shows the graphical addition of z_1 and z_2. AP, the equivalent resistance, $= r_1 + r_2 = R$; PC, the equivalent reactance, $= x_1 + x_2 = X$; and so the combined or equivalent impedance $= AC = (r_1 + r_2) + j(x_1 + x_2) = Z$. Thus AC is the geometric sum of AB and $BC = z_1 + z_2$. The sum of two impedances is obtained, not by the arithmetic addition of z_1 and z_2, but as the geometric sum. Numerically the combined impedance is

$$Z = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2} = \sqrt{R^2 + X^2}$$

When the impedances of Fig. 135*a* are multiplied by the current I the component voltage drops are obtained. These are shown in the triangle $AP'C'$ of Fig. 135*b*, which is similar to Fig. 135*a* except for a change of scale and the inclusion of the vector of the current. AM' is the vector of the voltage across r_1; $M'P'$, that across r_2. $M'B' = P'N'$ is the voltage across x_1 and $N'C'$ that across x_2. The voltage across AB (Fig. 134) is represented by the vector AB'; the voltage across BC by $B'C'$. It may be seen that AC' is the geometric sum of AB' and $B'C'$. This same method is applicable when three or more impedances are connected in series. On such a vector diagram the angles of phase displacement between the current and the various voltages may be measured on the diagram.

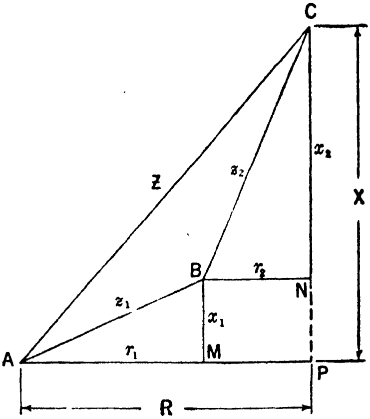

FIG. 135*a*. Graphical addition of two impedances.

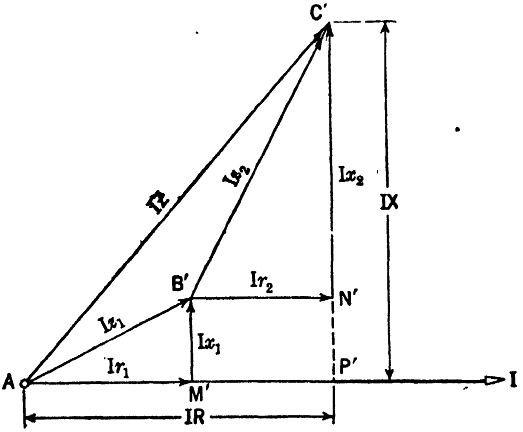

FIG. 135*b*. Vector diagram for two impedances in series.

148. The Three-Voltmeter Method of Measuring Power. — A special case of the diagram shown in Fig. 135*b* is represented in Fig. 136. The reactance $x_1 = 0$, so that one of the impedances is reduced to a non-inductive resistance r_1; in other respects the two diagrams are identical. This case is of some practical importance, as it leads to a method of determining power in an inductive circuit, without the use of a wattmeter. Let BC (Fig. 134) be any apparatus, such as a single-phase motor, and let it be necessary to determine the power consumed in it, without using a wattmeter. According to eq. (16*a*), the power depends upon the current, the voltage, and the phase displacement between the two. The current I and voltage across BC are measured directly. In order to determine the phase angle between the two, a non-inductive resistance r_1 is connected in series with the apparatus BC, and the voltages AB and AC are also measured. The resistance r_1 being non-inductive, the voltage drop AB is in phase with the current, so that the triangle ABC (Fig. 136) may be constructed in its true phase relation to the current I. This gives the desired phase angle ϕ at B. Projecting BC on I gives $E \cos \phi$; this value, multiplied by the current, gives the required power. If the value of r_1 is known, it is not even necessary to have an ammeter, since the current can be calculated as the ratio of the voltage to the resistance.

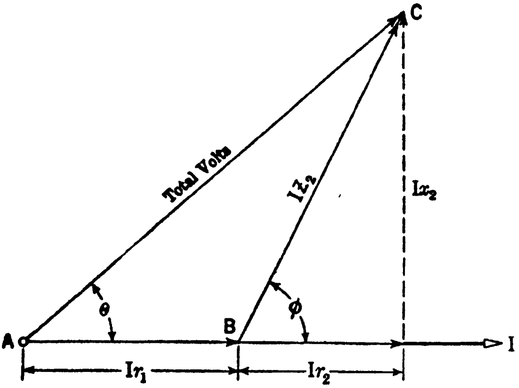

FIG. 136. A special case of the diagram shown in Fig. 135*b*, when the reactance x_1 is zero.

This method is called *the three-voltmeter method,* because the phase angle is determined from three voltmeter readings. It has been used to some extent in former years, but is now applied in exceptional cases only, since indicating wattmeters have come into universal use.

149. EXPERIMENT 6-E. — Voltage and Power Relations with Impedances in Series. — The connections are shown in Fig. 134; a wattmeter should be added, as in Fig. 128. (a) Adjust the desired values of resistances and reactances, and take readings of volts, amperes, and watts. Read total watts, and watts within each impedance separately. Gradually increase the applied voltage by regulating the resistance R; take similar readings with each setting of the rheostat. Measure the ohmic resistances with direct current. (b) Repeat the experiment with different values of resistances and reactances. (c) Determine the power consumed in a given impedance, using first a wattmeter, and then the three-voltmeter method.

Report. Plot the results to total volts as abscissas. Draw the dia-

gram of voltages $AM'B'N'C'$ (Fig. 135b), and check the angles with those calculated from the wattmeter readings. Check the power determined by the three-voltmeter method with that read on the wattmeter, and with the calculated I^2r loss.

150. Impedances in Parallel.—Two impedances in parallel are shown in Fig. 137; it is required to find the total line current I and its phase relation to the alternating voltage E applied between M and N.

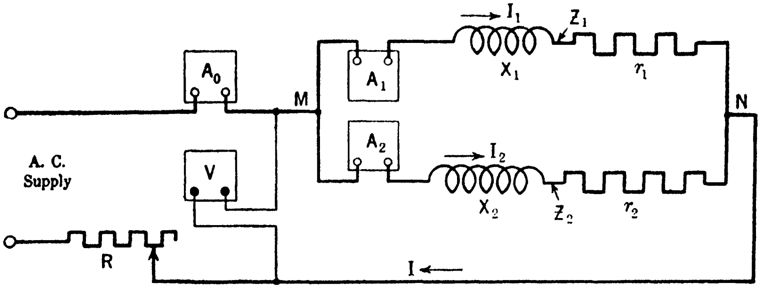

FIG. 137. Two impedances in parallel.

Consider first the simplest case, when one branch has a pure reactance x only; the other an ohmic resistance r (Fig. 138). The current in the first branch is $I_1 = E/x$, and lags 90 degrees behind E; the current in the second branch is $I_2 = E/r$, and is in phase with E. The total current I is the geometric sum of the two and is represented by the diagonal of the rectangle. Let the *equivalent* impedance of the circuit be defined as one which will give the same current I and the same phase angle ϕ as the given parallel combination, at the same voltage E. Denoting this impedance by z, we have, from Fig. 138

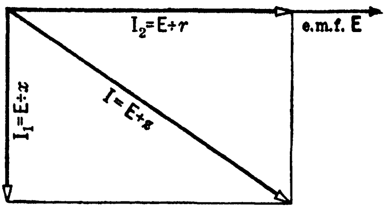

FIG. 138. Vector diagram of currents, with resistance and reactance connected in parallel.

$$(E/z)^2 = (E/r)^2 + (E/x)^2$$

or

$$\frac{1}{z} = \sqrt{\left(\frac{1}{x}\right)^2 + \left(\frac{1}{r}\right)^2} \quad \dots\dots\dots \quad (17)$$

This is different from expression (11), deduced for the case of a resistance and a reactance in series.

When resistance and inductive reactance are present in both parallel branches, the two component currents, I_1 and I_2, are lagging behind the

impressed emf, OA (Fig. 139), by certain angles ϕ_1 and ϕ_2. The total current $I = OC$ is the geometric sum of the two and lags behind OA by the angle ϕ.

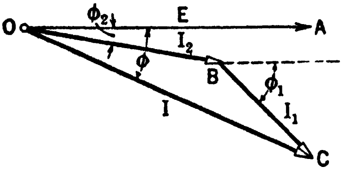

FIG. 139. Component currents and total current in the case of two impedances in parallel.

The same relations are shown in greater detail in Fig. 140. The applied voltage between the points M and N (Fig. 137) is represented by the vector OE. The voltages across the resistance and across the reactance in branch 1 are represented by the triangle OP_1E, as follows: OP_1 is the drop in the resistance r_1; P_1E is that in the reactance x_1. The current I_1, being in phase with the ohmic drop, is represented by the vector OC_1. The triangle OP_2E gives similar relations for the branch 2. If there are more branches in parallel, a triangle may be constructed for each branch. The apexes P_1, P_2, . . . of all such triangles lie on a semicircle drawn on OE as a diameter. This is because the triangles are right triangles, and have OE for hypotenuse. The total current I in the line is the geometric sum of I_1 and I_2 and is represented by the vector OC. If there are more than two branches in parallel, the vectors of the component currents are added together as if they were mechanical forces radiating from the point O. These relationships may be checked experimentally with an oscillograph (Vol. II).

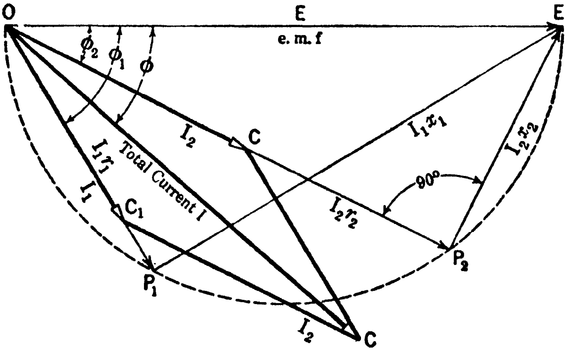

FIG. 140. Vector diagram of currents and voltages, corresponding to the connections of Fig. 137.

151. Admittance, Conductance, and Susceptance. — The reciprocal of a *pure resistance* (without inductance or capacitance) is called *conductance*, is denoted by g, and is measured in units written ohm^{-1} or mho. Thus, if the resistance of a wire is $r = 2$ ohms, its conductance is $g =$

0.5 mho (§11). When conductors are connected in parallel, their conductances are added arithmetically.

Similarly, the reciprocal of a *pure reactance* (without resistance) is called *susceptance*, is denoted by b, and is measured in mhos. The reciprocal of an *impedance* is called *admittance*, is denoted by y, and is measured in mhos. These concepts are useful in the study of circuits containing impedances in parallel.

Thus, in the simplest case of pure resistance and pure reactance (Fig. 138), we have a conductance g in parallel with a susceptance b. The currents through the two are $I_1 = Eb$ and $I_2 = Eg$, respectively. The total current is $I = Ey$, and eq. (17) becomes

$$y = \sqrt{b^2 + g^2} \quad . . . . \quad (18)$$

which is analogous to eq. (11).

Expressed as a complex number (see §143), $y = g - jb$, the sign of the susceptance term being negative for inductive reactance, and positive for capacitive reactance (see Vol. II).

In the more general case, shown in Figs. 137 and 140, the series connection of a resistance, r, and a reactance, x, in any one branch is first replaced by its equivalent parallel combination of a conductance, g, and a susceptance, b. The reciprocal of impedance = admittance, or $1/z = y$, and in complex numbers, $y = 1/(r + jx)$. This division is performed by rationalizing the expression through multiplication of numerator and denominator by $(r - jx)$. This requires that the value of j^2 be known. Since multiplying a quantity by j means that the quantity must be considered 90 degrees ahead of the reference, it follows that j^2 represents a rotation of 180 degrees, or a minus sign replaces j^2. Therefore,

$$y = 1/(r + jx) = (r - jx)/(r - jx)(r + jx) = r/(r^2 + x^2) - jx/(r^2+x^2)$$

Thus,

$$y = r/z^2 - jx/z^2 \quad . . . . \quad (19)$$

so that

$$g = r/z^2 \quad . . . . \quad (20)$$

and

$$b = x/z^2 \quad . . . . \quad (21)$$

Since the conductance of the circuit carries the *in-phase* component of the current through the admittance,

$$Eg = I \cos \phi = Ey \cos \phi \quad . . . . \quad (22)$$

so that

$$\cos \phi = g/y \quad . . . . \quad (23)$$

Similarly, b carries the *quadrature* (wattless) component of the current and

$$\sin \phi = b/y \quad \ldots \quad (24)$$

Thus, by applying eqs. (20) and (21), each branch is reduced to a conductance and a susceptance in parallel. All the conductances can now be added to form the equivalent conductance

$$g_{\text{equiv.}} = \sum g \quad \ldots \quad (25)$$

and all the susceptances to form the equivalent susceptance

$$b_{\text{equiv.}} = \sum b \quad \ldots \quad (26)$$

The total current I in the circuit may now be found.

$$I = E\, g_{\text{equiv.}} - jEb_{\text{equiv.}} = I \cos \phi - jI \sin \phi$$

The total power consumed by the circuit is then

$$W = EI \cos \phi = E^2 g_{\text{equiv.}} \quad \ldots \quad (27)$$

Equivalent admittances and impedances. When an admittance y is in series with an impedance it is necessary to convert the admittance to an *equivalent impedance* which can then be added to the series impedance to give the combined impedance of the circuit. The impedance $z_{\text{equiv.}}$, equivalent to an admittance y, is found by the method employed above, namely, that of rationalization of the complex expression. Thus

$$\begin{aligned} z_{\text{equiv.}} &= 1/y = 1/(g - jb) = (g + jb)/(g + jb)(g - jb) \\ &= g/(g^2 + b^2) + jb/(g^2 + b^2) = g/y^2 + jb/y^2 \quad \ldots \quad (28) \end{aligned}$$

In this expression the equivalent resistance is

$$r_{\text{equiv.}} = g/y^2 \quad \ldots \quad (29)$$

and the effective reactance is

$$x_{\text{equiv.}} = b/y^2 \quad \ldots \quad (30)$$

In practical computations of a-c problems it is often convenient to use such equivalent combinations of the series or parallel type, so as to be able to add conductances or susceptances in parallel and resistances or reactances in series. The method is applicable to any number of impedances and makes it possible to dispense with graphical constructions, such as are shown in Figs. 135*a* and 140. For an application of this method see §154 below.

152. EXPERIMENT 6-F. — Study of Impedances in Parallel. — The purpose of the experiment is to illustrate the relations described in §§150 and 151. The connections are identical with those in Fig. 137, except that one ammeter may be used for all branches, being connected through a polyphase board (see §60), provided the impedances being measured are sufficiently great, compared with the ammeter impedance, so that the insertion of the meter in one branch will not change the distribution of current between the branches. Otherwise, separate ammeters and wattmeters should be used, the wattmeter readings being required to determine phase relations. Before starting the test, review the precautions and suggestions of §139. (a) Begin with the simplest case, in which one branch contains only resistance and the other only reactance. Measure the three currents, volts, and watts. Gradually increase the voltage and take similar readings. Finally measure the resistance r with direct current. (b) Repeat the experiment with other values of resistance and reactance. (c) Take the more general case of r and x in both branches, as in Fig. 137. Read amperes, watts, terminal volts, and the component voltages in each branch. (d) Illustrate experimentally the existence of equivalent series and parallel combinations explained in §151. Show that the total current and the power factor obtained in one of the readings under (a) can be also obtained by means of a series combination of a resistance and a choke coil. Show that two impedances in parallel may be replaced by one. Should difficulty be experienced in adjusting the required values empirically, by trials, an estimate according to eqs. (20) and (21) may be helpful.

Report. (1) Show that the three currents form a right triangle when a pure resistance is connected in parallel with a pure inductance; check the phase angle ϕ with that determined by the wattmeter. Also show that the total power corresponds to the I^2r loss in the resistance, and that no power is lost in the reactance.

(2) Draw one or more vector diagrams, as in Fig. 139, and show that the relations obtained from the volt and ampere readings check with those calculated from the wattmeter readings.

(3) For at least one set of readings construct a complete diagram, as in Fig. 140.

(4) Apply the theory given in §151 to the readings obtained under (d).

153. EXPERIMENT 6-G. — Motors and Lamps Connected in Parallel to the Same A-C Supply. — This experiment is a practical illustration of the case discussed in §§150 to 152. The load in many industrial plants consists partly of incandescent lamps, which are practically a non-inductive load, and of induction motors operating at a comparatively low power factor. The resultant load in the power house is the sum of the power taken by the lamps and that taken by the motors;

the resultant power factor lies somewhere between the average power factor of the motors and 100 per cent.

A laboratory experiment may be considered as sufficiently representative of commercial conditions when performed with a load consisting, say, of a 0.5- to 1.0-hp 110-volt induction motor and 10 to 20 incandescent lamps of 50 watts each, connected in parallel with the motor (Fig. 141).

In actual service the load and power factor vary in innumerable combinations of which the extremes may well be the following: (1) Motors operating at no load and therefore at minimum power factor, while the lamps are turned on one by one. This will show the effect of an increasing non-inductive load in improving an otherwise unsatisfactory power factor load; (2) the motor operating at or near full load and therefore at a high power factor, while the lamps are turned on one at a time. This will show a less marked improvement in power factor as the non-inductive load is added to one of already fairly satisfactory value. The relations in intermediate cases can be readily understood from the results of these two.

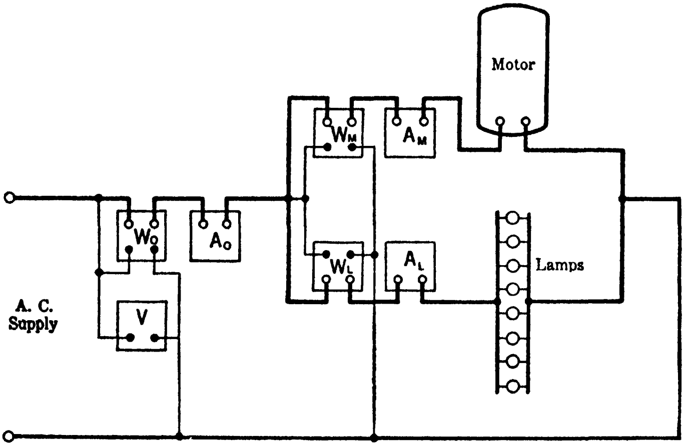

FIG. 141. A diagram of connections for studying two impedances in parallel.

Connect the lamps and the motor in parallel to the power supply, as in Fig. 141, and arrange switches and instruments to read watts and amperes supplied to the lamps, to the motor, and to both. The voltage must be kept as nearly constant as possible during the whole experiment. Take two comprehensive sets of readings, under the conditions described in (1) and (2) above.

Report. To lamp load in watts as abscissas plot watts combined load, watts motor load, lamp current, motor current, total current, resultant power factor, and motor power factor; also calculate and plot on the same curve sheet the reactive amperes (in quadrature with the voltage), and the energy component of the current (in phase with the voltage). Two separate sets of curves should be plotted: for tests (1) and (2). The diagram, Fig. 139, is simplified in this case to that in Fig. 142 because the load in one of the branches is non-inductive. Check a few points on the above curves by means of this diagram. For instance, take the observed component currents, find from the diagram the total current and the resultant power factor, and compare them with the values directly observed. Or else take the observed total current and one of its components and determine from the diagram the other component current.

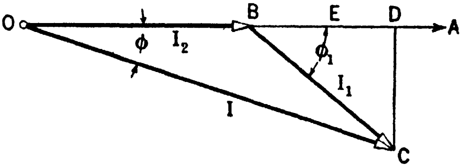

Fig. 142. Current relations with an inductive load and a non-inductive load in parallel.

154. The Effective Resistance and Reactance of a Coil with an Iron Core. — When hysteresis and eddy currents in a core (§§216 and 217) are of considerable magnitude, it is sometimes convenient to express their effect upon the circuit by a suitable change in the ohmic resistance of the coil. For example, let the power input into such a coil be 400 watts at a current of 10 amperes and at a terminal voltage of 50. Let the resistance of the winding measured with direct current be 3 ohms. Then the copper loss alone is $(10)^2 \times 3 = 300$ watts, and the core loss is $400 - 300 = 100$ watts. The power factor of the coil is $400/(50 \times 10) = 0.8$ or $\phi = 36.9$ degrees.

The *effective resistance* of a coil or circuit is defined as the total power converted into heat (copper loss, eddy-current loss, core loss, skin effect, etc.) divided by the square of the effective value of the current. Accordingly, the effective resistance of the coil under consideration is 4 ohms. This value depends both upon the true I^2r loss and upon the magnitude of the core loss. Consequently it varies with the current and with the frequency.

Sometimes an additional power loss is caused by eddy currents induced in surrounding metal objects by the magnetic flux of the circuit. Moreover, with high frequencies the flux within the conductor causes a non-uniform distribution of current throughout the cross-section of the conductor (skin effect). In all such cases the so-called effective resistance of the coil is taken as the actual watt input divided by the square of the current. The effective impedance is found by dividing the terminal volts by the current. From these two quantities the effective reactance is computed by taking the square root of the difference of the two squares.

Figure 143*a* is the circuit diagram of the impedance coil considered as an equivalent resistance, r_e, made up of the actual (ohmic) resistance of the copper, r_0, and the apparent resistance, r_c, equivalent in effect to the core loss, together with the effective reactance x_e. Figure 143*b* is the vector diagram for this circuit. The applied voltage *OC* of 50 volts is taken as the reference. The current, *I*, of 10 amperes lags behind *OC* by the angle $\phi = \cos^{-1} 0.8$. *OB* and *BC* are the effective resistance and effective reactance drops, respectively, obtained as the components of *E* in phase and in quadrature with *I*. *OA* is the 30-volt drop due to the actual (ohmic) resistance of the winding, and *AB* that equivalent to the core loss (10 volts). Ix_e is then found to be 30 volts, and $x_e = 3$ ohms.

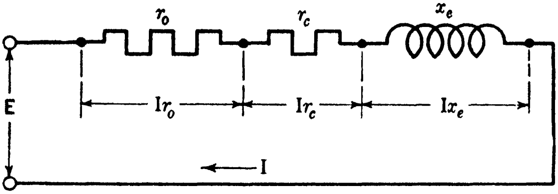

FIG. 143*a*. Circuit of an impedance coil considered as a true resistance, r_0, a resistance r_c, due to the core loss, and a reactance, x_e, all in series.

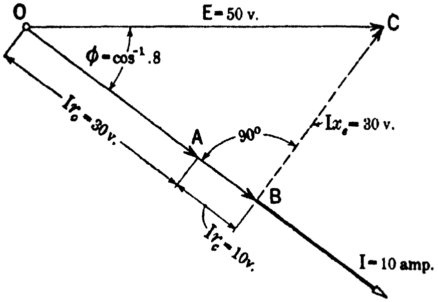

FIG. 143*b*. Vector diagram of an impedance coil considered as in Fig. 143*a*.

In some cases it is preferable to represent the impedance coil as a resistance, r_0 (Fig. 144*a*), in series with two branches, g_c and b_c in parallel, expressing the additional losses in the coil by means of a fictitious resistance (or conductance, g_c) shunted across the reactance of the coil. This resistance, or leakage conductance, is of such a value as to cause a power loss equal to that in the core. Thus, in the foregoing numerical example, r_0, the actual resistance of the coil carries the total current *I* of 10 amperes at the angle $\phi = \cos^{-1} 0.8$ as before (Fig. 144*b*) with a drop of $OA = 30$ volts. The drop across g_c and b_c must then be $E_2 = E - Ir_0 = 50 - (30 \times 0.8 - j\,30 \times 0.6) = 26 + j\,18$, or 31.6 volts.

Since the core loss $= E_2^2 g_c$, $g_c = 100/(31.6)^2 = 0.1$ mho, $I_c = E_2 g_c = (26 + j\,18) \times 0.1 = 2.6 + j\,1.8$ or 3.16 amperes.

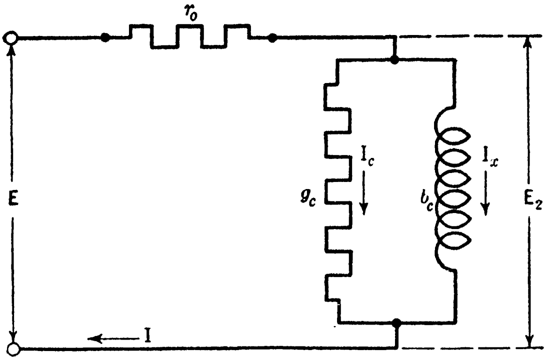

FIG. 144*a*. Circuit diagram of an impedance coil treated as a resistance, r_0, in series with a conductance, g_c and a susceptance b_c, in parallel.

The two components of current through g_c and b_c are in quadrature and combine to make up $I = 10 \cos^{-1} 0.8 = 8 - j\,6$. Therefore $I_x = I - I_c = (8 - j\,6) - (2.6 + j\,1.8) = 5.4 - j\,7.8$ or, numerically, $I_x = 9.5$ amperes. Therefore, $b_c = 9.5/31.6 = 0.3$ mho, or $X_c = 1/b_c = 3.33$ ohm.

From the results obtained by the two methods of treatment of the impedance coil it is seen that the value of the reactance depends upon the manner in which the core loss is assumed to affect the equivalent electric circuit of the coil.

Resistance and reactance at high frequencies are treated in the chapter on High-Frequency Measurements in Vol. II.

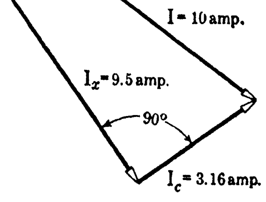

FIG. 144*b*. The vector diagram for the impedance coil when represented by the circuit of Fig. 144*a*.

155. EXPERIMENT 6-H. — Effective Resistance and Reactance. — The purpose of the experiment is to determine the effect of core loss and of eddy currents in adjacent metal objects upon the true resistance and reactance of a coil, as discussed in the preceding section. To make the conditions more striking, use a solid core or one consisting of thick laminations or wires. The connections are the same as in Fig. 128

except that the resistance r is not necessary. Be sure that the constants of the measuring instruments are known within the range of frequencies used in the following tests. (1) Run the source of alternating current at the highest possible frequency, and send as large an alternating current through the coil as it can stand without damage; read volts, amperes, and watts. Reduce the current in steps, keeping the frequency constant, and take similar readings at each step. (2) Repeat the preceding run at a few lower values of frequency. (3) Keep the current constant, and take readings at different frequencies. (4) Repeat several characteristic settings of the preceding three runs with metal masses placed near the coil in such positions that eddy currents are induced in them by the magnetic flux. (5) Measure the resistance of the coil with direct current.

Report. (a) Plot the readings against the variable quantities as abscissas, and draw smooth curves. (b) From these curves compute the values of the effective resistance by dividing the power input by the square of the current. (c) Compute the values of impedance by dividing volts by amperes. (d) Knowing the impedance and the resistance, compute and plot the corresponding values of reactance. (e) For different values of current and frequency, plot the effective values of resistance, reactance, and impedance as affected by the proximity of metal masses, and compare with the values of true ohmic resistance and true reactance. (f) Explain theoretically the general character of the variation of these quantities. (g) Show by one or two numerical examples, using data taken from the curves, how the effect of the core loss and of the adjacent metal bodies may be taken into account (a) by an added series reactance, and (b) by a conductance shunted around the coil. Draw vector diagrams similar to Figs. 143*b* and 144*b*, and explain them.

MUTUAL INDUCTION

156. The reactance of a coil, such as the coil C_1 shown in Fig. 145, is reduced by the proximity of another coil, C_2, if the circuit of the latter is closed and contains no external source of emf, such as an alternator. The reason for this reduction in the primary reactance is that the flux excited by the first coil induces a current in the secondary coil, and this current opposes the action of the primary current. As a result, the total magnetomotive force, the flux, and consequently the counter emf induced in the first coil, are reduced.

The effect of the secondary coil depends, among other factors, upon its distance from the primary coil and on the character of the circuit on which it is closed (resistance r_2 and reactance x_2). The action is greatly intensified if both coils are mounted on the same iron core, *I*. This interaction of two coils, without direct metallic connection, is called *mutual induction.*

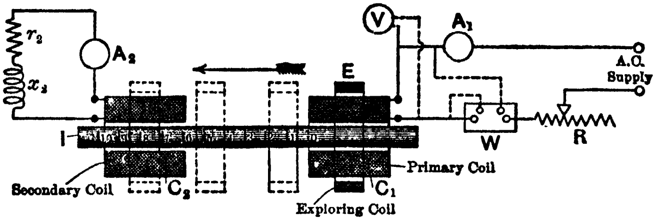

Fig. 145. Apparatus to study the effect of mutual induction in a circuit.

In some practical cases, mutual induction is highly desirable; thus the action of a transformer (Chapter XVI) is based entirely upon it. The so-called "coupled circuits" in radio work are other examples of application of mutual inductance. In some cases mutual induction has a harmful effect; for example, when a power-transmission line parallels a telephone line, currents are induced in the latter which cause noises in the telephone receivers and make the understanding of speech difficult or impossible (inductive interference).

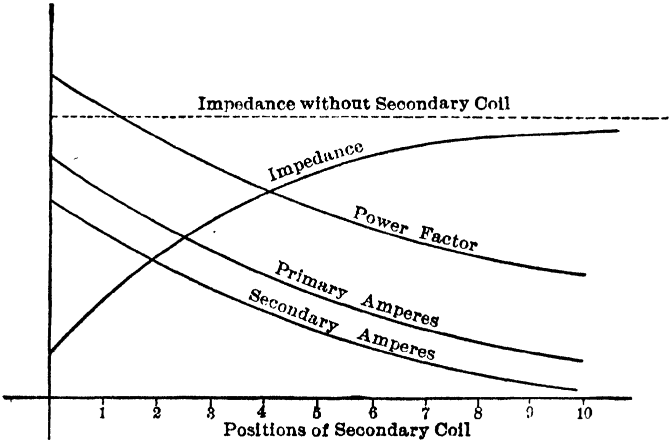

Fig. 146. Curves showing the effect of mutual induction, with the connections as in Fig. 145.

The electrical relations between two circuits subjected to a mutual induction are usually too complicated to be expressed by practical formulas or diagrams. All that is desired here is to give the student at least a qualitative understanding of the physical phenomenon and the influence of the factors which enter into it (Fig. 146). This is explained in the following experiment.

157. EXPERIMENT 6-I. — Effect of Mutual Induction on the Reactance of a Circuit. — The phenomenon described in the preceding paragraph may be studied experimentally with the apparatus shown in Fig. 145. The coil C_1 under test is denoted the "Primary Coil." It is connected to an a-c supply, with an ammeter A_1, a voltmeter V, and a wattmeter W in the circuit, as usual. Another coil, C_2, marked "Secondary Coil," is closed on a circuit consisting of a resistance r_2, reactance x_2 and an ammeter A_2. The secondary coil may be placed at any desired distance from the primary coil. In order to increase the inductive action between the two coils, they may both be put on a common iron core I. To investigate the influence of the position of the secondary coil, short-circuit it upon itself through the ammeter A_2, and bring it as close as possible to the primary coil (position "O"). Adjust the primary current by the resistance R to the maximum safe limit; read volts, amperes, and watts. Gradually move C_2 away, keeping the primary amperes constant. Take readings until the secondary coil is removed so far that its influence on the primary circuit is hardly noticeable. Finally open its circuit and again read the primary values.

As the coils are separated, the flux between them varies. The values of the flux can be ascertained by an exploring coil E connected to a separate voltmeter. The coil E is moved from one position to another, as shown by the dotted lines, and the variations in the flux are measured by the variations in the induced emf. The voltmeter current is so small, especially if a high-resistance voltmeter is used, that the presence of the exploring coil does not appreciably affect the electrical relations between the primary and secondary coils. Repeat the test without the iron core.

Next investigate the influence of the character of the secondary circuit upon the mutual inductance. Place the secondary coil at a short distance from the primary coil, and take several readings of volts, amperes, and watts, varying the values of x_2 and r_2. Keep either the primary volts or primary amperes constant, in order to have a basis for a comparison of the results.

Report. Plot the results to positions of the secondary coil as abscissas (Fig. 146). Plot the effect of varying secondary inductance and resistance; use as abscissas the quantity which was varied. Give a physical explanation of the observed relations and construct a few vector diagrams.

158. Coefficient of Mutual Inductance. — By analogy with the definition of the coefficient of self-induction in §136, the coefficient of mutual induction, M, is defined by either of the following equations:

$$e_2 = -M \frac{di_1}{dt} \qquad \dots\dots\dots\dots \qquad (31)$$

or

$$e_1 = -M\frac{di_2}{dt} \quad \ldots\ldots\ldots\ldots \quad (32)$$

In these equations the subscripts 1 and 2 refer to two electric circuits linked by a common magnetic flux (Fig. 145). It is shown in works on theoretical electricity and magnetism that the coefficient M is the same in both equations. In other words, when the current in circuit 1 varies at a rate of 1 ampere per second, an emf of M volts is induced in circuit 2, and vice versa. The coefficient of mutual induction M is measured in henrys, as is the coefficient of self-induction L.

The coefficient of self-induction, L, is always positive. The coefficient of mutual induction, M, may be either positive or negative. Let, for example, M equal +8 millihenrys between a stationary and a movable coil, in a certain position of the latter in which M is a maximum. Now, if the movable coil be gradually turned about its axis, the value of M would decrease, become zero, and then become negative. When the coil has been turned by 180 degrees the coefficient of mutual induction is −8 millihenrys.

Let the current in circuit 1 vary according to the sine law, eq. (2) in §138. Then the instantaneous secondary induced voltage is

$$e_2 = -2\pi fMI_m \cos 2\pi ft$$

that is, the secondary induced voltage lags behind the primary current by 90 electrical degrees. Using the effective values, we get

$$E_2 = x_m I_1 \quad \ldots\ldots\ldots\ldots \quad (33)$$

where

$$x_m = 2\pi fM \quad \ldots\ldots\ldots\ldots \quad (34)$$

The quantity x_m is called the *mutual reactance* of the two circuits and is measured in ohms. Equations (33) and (34) correspond to eqs. (5) and (4) in §138. For a definition of *mutual impedance,* in application to telephone and telegraph circuits, see the Standards of the American Institute of Electrical Engineers.

The mutual inductance or reactance of two circuits may be measured by sending a pure alternating current I_1 through one of them and measuring the voltage E_2 induced in the second one, on open circuit. The ratio of the two, according to eq. (33), will give the mutual reactance x_m. The frequency being known, M can then be computed from eq. (34). See also §160.

159. EXPERIMENT 6-J. — Determination of the Mutual Inductance of two Circuits. — Provide two coils without iron cores and place them in such a relative position that an alternating magnetic flux produced by one induces an appreciable voltage in the other. (1) Send a large alternating current through coil 1; read this current and the voltage induced in coil 2. Reduce the current in steps and read the secondary volts at each step, keeping the frequency constant. (2) Repeat the readings at one or two other frequencies. (3) Keeping the primary current constant, vary the position of the secondary coil in a systematic manner and read the secondary volts in each position. Investigate separately the influence of the distance between the centers of the coils and of the angle between their axes. Find the positions of maximum, minimum, and zero M. Note positions for which M is positive and those for which it is negative. (4) For two or three positions of the coils send an alternating current through 1 and measure the voltage induced in 2; then send a current through 2 and measure the voltage in 1. The two currents need not be equal, but the frequency must be the same in both cases. (5) Try the effect of an iron core in each of the two coils in succession, and then in the two simultaneously.

Report. (a) Compute the coefficient of mutual induction for the positions tested. (b) Give theoretical reasons for the observed positions of maximum and minimum M. (c) From the readings under (4) show that the two voltages are proportional to the currents and that this result proves the statement following eq. (32). (d) Give the results of the test with the iron cores, and explain theoretically.

160. Measurement of Inductance with the Wheatstone Bridge.— Small inductances, such as are used in telephone and telegraph work and in scientific research, are sometimes measured by means of a Wheatstone bridge, with interrupted or alternating currents. Some of these methods, which are particularly convenient for the measurement of self- and mutual induction and of capacitance, will be found in Vol. II.

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CHAPTER VII

THE MAGNETIC CIRCUIT

161. The Magnetic Circuit. — The electric circuit normally consists of materials which are good conductors of electric current, separated by non-conductors or insulators. The magnetic circuit, likewise, is made up of materials, usually iron in some form, which offer paths in which magnetic effects may be readily produced, separated by materials which are not easily magnetized. Unlike the electric circuit there are no insulating materials for magnetism, although air, electrical insulation, copper, and all except the ferro-magnetic metals are very inferior conductors of magnetism. In most electrical apparatus the magnetic circuit consists of iron or steel parts on either side of short air-gaps, varying from a few ten-thousandths of an inch in the transformer, to an inch or more in the large high-speed alternator.

In the electric circuit the relation between the current, resistance, and emf is expressed by the familiar Ohm's law, or $I = E/R$, and $E = RI$ (eq. 1, §1). In the magnetic circuit practically the same relationship exists between the *magnetic flux*, ϕ, the *magnetomotive force*, **mmf**, and the *reluctance*, $\mathcal{R}$. Thus,

$$\phi = \mathbf{mmf}/\mathcal{R} \qquad (1)$$

may be said to express Ohm's law of the magnetic circuit. The common source of magnetomotive force in electrical apparatus is a coil of wire carrying an electric current. Familiar examples of this are found in measuring instruments (Fig. 33), the exciting windings of generators and motors (Fig. 198), and the windings of transformers (Fig. 271), etc. It is natural, therefore, to express mmf in terms of current and turns, or in *ampere-turns.*

In the cgs system the unit of mmf is the *gilbert,* which is such a value of mmf that acting through a distance of 1 cm in air it will produce a flux of unity density (see §163). The ampere-turn compares with the gilbert as 0.4π to 1, so that, expressed in gilberts, the mmf of a coil of wire of N turns carrying a current of I amperes is

$$\mathbf{mmf} = 0.4\pi NI \text{ gilberts} \qquad (2)$$

162. Equipment for Investigating Magnetic Relations. — The fundamental magnetic relations, discussed above, may be conveniently studied on such apparatus as is shown in Fig. 147. It consists of several sets of

U-shaped pieces, m, m', of steel or iron, within the magnetizing coils pp, which are connected to a source of direct current. K is a reversing switch by means of which the exciting current may be reversed. When K is closed, a current flows through the coils pp and produces a magnetic flux, as shown by the dotted lines. This flux may be varied by the following means:

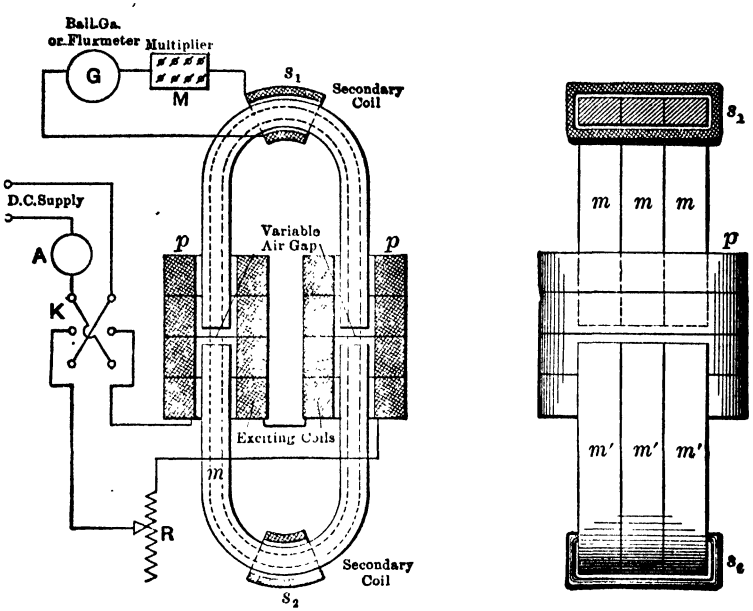

FIG. 147. An apparatus for the study of relations in magnetic circuits.

(a) By regulating the exciting current with the rheostat **R**.

(b) By changing the number of coils in the circuit.

(c) By increasing or decreasing the air-gap between the upper and the lower cores.

(d) By changing the number of sections m, m'.

(e) By replacing the cores by others of different magnetic material.

The exciting current is read on the ammeter **A**. The magnetic flux in the core is measured by means of the exploring coils s_1 and s_2, connected to a ballistic galvanometer (§177) or to a fluxmeter (§180).

163. Magnetic Flux and Flux Density. — The magnetic state in a medium has a definite direction at each point, and may be considered as some kind of tension or attraction along certain paths. The lines which follow these directions are called " magnetic lines " or " lines of force." They are shown by dotted lines in Fig. 147. The familiar experiment in which iron filings arrange themselves around a magnet in definite patterns illustrates the directions of the lines of force. Apart from their directions these lines are purely imaginary, but it is convenient to measure the strength of a magnetic field by assigning a *numerical* value to each line of force. With this assumption, lines of force are said to be closer to one another in places where magnetization is more intense, and vice versa. This leads to the concept of *density of lines of force,* or *flux density,* which plays an important part in the theory of magnetism.

The total number of lines of force within an area is called the magnetic flux. The flux within a coil, such as s_1 (Fig. 147), is usually measured in terms of the total or integrated emf which can be induced in the coil by the flux. It is an experimental fact that, when the magnetic flux linking with a coil increases or decreases, an emf is induced in the coil proportional to the rate of change of the flux with time. This is the familiar Faraday's law of induction, expressed by the formula

$$e = -n d\phi/dt \qquad \text{. . . . . (3)}$$

The rational unit of magnetic flux in the volt-ampere-second system is therefore defined by the following statement: *A magnetic flux varies at the rate of one unit per second when the emf thereby induced in the coil which surrounds it is one volt per turn.* Such a unit of flux is called the *weber,* or the *volt-second;* it is too large for general use. The practical unit used, called the *line of force,* is based on a similar definition except that the cgs unit of emf is taken instead of the volt. This emf unit is 10^{-8} times as large as the volt, which factor, therefore, enters into all formulas which relate to induced emf in volts, due to rate of change of flux linkages expressed in lines of force.

As an example of the above, suppose that an exploring coil of 100 turns surrounds the cores m, m' in Fig. 147, and that, while the current in the exciting winding is brought to zero at a uniform rate by means of the rheostat R, during a period of 20 seconds, a low-reading voltmeter shows 0.05 volt. From these data the original flux is figured out as follows: The emf induced in each turn of the secondary coil is $0.05 \times 10^8/100 = 50{,}000$ cgs units. According to the above definition of flux, this result shows that the flux was decreasing at the rate of 50,000 lines of force per second. As it took 30 seconds for the flux to be reduced to zero, the original flux was $50{,}000 \times 30 = 1{,}500{,}000$ lines of force.

The unit of flux, or one line of force, as defined above, is called *the maxwell,* so that fluxes are expressed in maxwells. The weber, or the unit of flux, defined on the basis of 1 volt per second, is too large, and the maxwell is too small a unit for many practical purposes. A compromise is reached by using the following intermediate units:

1 kilomaxwell (or kiloline) = 1000 maxwells (= 1/100,000 weber).
1 megamaxwell (or megaline) = 1,000,000 maxwells (= 1/100 weber).

It would be almost impossible to measure a magnetic flux accurately by gradually reducing it to zero, so as to get a constant voltmeter deflection as described above. The ballistic method described below (§166), though based on the same principle, is much more convenient and is largely used in practice.

164. Influence of Length and Cross-Section of a Magnetic Circuit upon Its Reluctance. — Just as the resistance of an electrical conductor depends upon its length, its cross-section, and its resistivity (or conductivity), so is the *reluctance* of a magnetic circuit of constant cross-section and uniform material equal to its length times the reluctance of a unit cube, divided by area. (*Note*: This is strictly true only for non-magnetic materials like air or for magnetic materials in which the flux density is constant throughout the circuit). The relation is expressed (see also eq. 4, §4) as

$$\mathcal{R} = \rho l/A = l/\mu A \qquad (4)$$

where $\mu = l/\rho$ is the *permeability* of the material.

The resistance of an electrical conductor is normally considered to be constant, except for minor changes due to temperature variation; but there are conductors, such as the electrical arc, in which the resistivity (or conductivity) is a function of the current carried. Similarly, but to a far greater extent, the permeability of magnetic materials varies over a range of several thousand times as the density of magnetic flux in the material is changed. As the higher flux densities are approached in iron a given increase in the exciting ampere-turns results in smaller and smaller increments of flux density, thus the iron becomes more and more nearly " saturated " (§165). This is discussed in greater detail in Chapter VIII.

Because of this variable permeability the concept of reluctance is not often used in application to steel and iron. The properties of the magnetic circuit are in this case better expressed in the form of a saturation curve for the whole circuit (Fig. 148), which gives the relation between the flux density (or total flux), in some part of the circuit, and the total magnetizing force. Thus the use of eq. (4) is practically confined to short air-gaps in which the lines of force are nearly parallel to one another and to carefully mapped fields of flux in the air spaces between the field poles and armatures of electrical machines, etc. (The student is referred to a comprehensive literature on " Field Mapping " appearing recently in the technical journals.) While the value of permeability for air is constant and equal to unity in the gilbert-maxwell system its value naturally depends upon the units used for mmf and flux. For further details in regard to this question see the table in §165, also the author's "Magnetic Circuit," Chapter I.

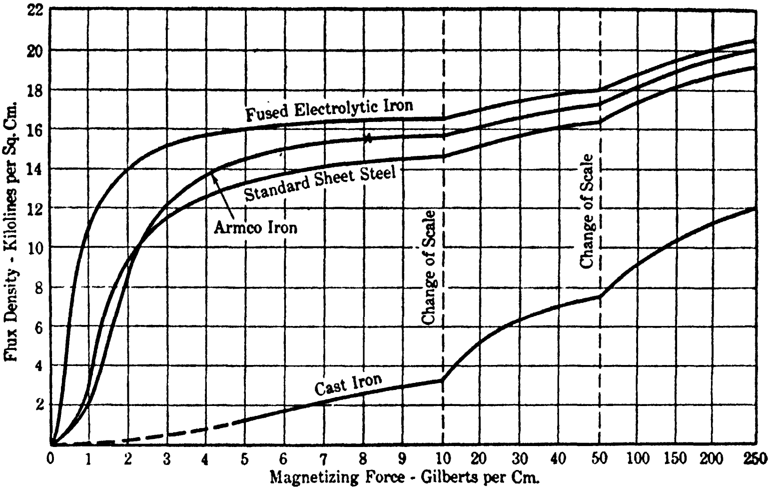

Fig. 148. Some magnetization (B–H) curves for several ferromagnetic materials (G. E. Co.).

165. Flux Density and Ampere-Turns per Unit Length (B-H Curve). —The magnetic properties of given samples of steel or iron may be conveniently represented in the form of curves, such as are shown in Fig. 148. These curves give the relationship between the mmf and the corresponding flux for several materials. For these samples the mmf is expressed as the *magnetizing force*, or magnetic intensity, H, *per unit length of the material.* The flux is given in terms of flux density B, which by definition is equal to *the flux per unit cross-section of the material.* The curves shown in Fig. 148 are known as the saturation, magnetization, or B-H curves. Another term used for flux density is *induction*; it is somewhat misleading because of the use of the same word in connection with induced emf.

As an example of the determination of data from which to plot such B-H curves, suppose that the equipment of Fig. 147 is being used to test a sample of Armco Iron. Let the total number of turns in the exciting coils be 320, let the measured flux (see §166) be 154,000 maxwells, and the length of the magnetic circuit be 100 cm. Then, with an exciting current of 2 amperes, the magnetizing force is $H = 320 \times 2/100 = 6.4$ ampere-turns per linear cm, or $6.4 \times 1.257 = 8.04$ gilberts per cm. Let the cross-section of the iron be 10 sq cm, then the flux density is

B = 15.4 kilolines per sq cm. The value $H = 8.04$ characterizes the material at the density B = 15.4 kilolines per sq cm, without regard to the length or the cross-section of the whole circuit. Thus the point marked X on the B-H curve for Armco Iron is obtained.

With the aid of such a B-H curve, the number of ampere-turns for a uniform magnetic circuit of any dimensions, and made of the same kind of magnetic material, may be readily computed, when the total flux is given. It is only necessary to find the flux density B, to obtain the corresponding value of H and to multiply H by the length of the magnetic circuit.

For *air*, the theoretical relationship between B and H *is a straight line.*

$$B_a = \mu_{aa} H_a \quad \dots\dots\dots\dots \quad (5)$$

where μ_{aa} is the absolute permeability of the air for the particular units in which B_a and H_a are expressed. The symbols B and H are provided with the subscript a to indicate the air. The numerical value of μ_{aa} depends upon the units used, and is not always equal to unity as is sometimes erroneously assumed. The following table gives the values of μ_{aa} for the principal units in practical use.

Flux Density B_a	Magnetizing Force H_a	Absolute Permeability of air μ_{aa}
Maxwells per sq in.	Amp-turns per in.	3.193
Maxwells per sq cm.	Amp-turns per cm.	1.257
Maxwells per sq cm.	Gilberts per cm.	1.000

It will be seen from the table that with the units for H_a and B_a in common use, μ_{aa} is different from unity. In this connection it may be noted that the unit *gauss* stands primarily for the flux density of "1 maxwell per sq cm," and is simply an abbreviation for this expression. The name gauss is also sometimes applied to the expression "1 gilbert per cm length." It will be seen from the third line in the above table that numerically the two expressions for B_a and H_a are equal, because $\mu_{aa} = 1$, but from a physical point of view they have different dimensions, since μ_{aa} is *not* a numeric. Hence, the name gauss can be properly applied only to one of them and will be used only to indicate the maxwell (or line) per square centimeter. There is a growing tendency to replace the unit "gilbert per centimeter" by "oersted."

As an example of application of eq. (5), let the magnetic circuit shown in Fig. 147 have two air-gaps of 1 mm each. With a flux density B_a = 15.4 kilolines per sq cm, the corresponding $H_a = B_a/\mu_{aa} = 15{,}400/1.257 = 12{,}250$ ampere-turns per cm. Therefore, the mmf required for the two gaps is $2 \times 0.1 \times 12{,}250 = 2450$ ampere-turns.

The foregoing method of computation of the ampere-turns required for a flux in the air is applicable whenever the flux density is fairly constant, so that H_a may be directly multiplied by l. Such is the case of a uniformly wound torus ring, and of the middle portion of a very long straight solenoid where the lines of force are parallel to one another. In the latter case let n_1 be the number of turns per unit length, and i the exciting current. Then $H_a = n_1 i$, and B_a may be found from eq. (5). If the cross-section of the solenoid is A, the total flux is $\Phi = B_a A$.

The total number of ampere-turns required for a composite magnetic circuit, consisting of iron parts and air-gaps in series, is obtained by adding together the mmf's required for the separate parts. Thus, in the foregoing example $2 \times 320 = 640$ ampere-turns are necessary for the iron parts and 2450 ampere-turns for the air-gaps. Hence, the whole required excitation or mmf is $640 + 2450 = 3090$ ampere-turns. Since the number of turns in the exciting coils is 320, the required magnetizing current is $3090/320 = 9.65$ amperes.

166. EXPERIMENT 7-A. — Fundamental Relationship between Magnetic Flux and MMF. — The purpose of the experiment is to illustrate the fundamental relationship between a magnetic flux and its exciting ampere-turns. The connections are shown in Fig. 147. The constant of the ballistic galvanometer (§177) or of the fluxmeter (§180) is supposed to be known;[1] otherwise the instrument is calibrated as described in §179. Before beginning the experiment, thoroughly demagnetize the cores from the residual flux which they may contain after a previous experiment. To do this, excite the cores to a considerable degree and then gradually reduce the current to zero by the rheostat R, at the same time continually reversing the current with the switch K. Instead of reversing the current an alternating current may be used. Eliminate the air-gap as thoroughly as possible by bringing the cores tightly together, and excite the circuit with a small current, keeping the galvanometer circuit open.

(1) Close the galvanometer circuit, and with its coil at rest, read the exciting current. Then suddenly reverse switch K and observe the galvanometer deflection. It is preferable in practice to reverse the exciting current, instead of opening the circuit, on account of the indefinite residual magnetism which remains when the circuit is opened. By reversing the current the flux is changed by a definite amount $\Phi - (-\Phi) = 2\Phi$. Again open the galvanometer circuit, increase the exciting

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[1] By the constant of the ballistic galvanometer is meant the total flux change (through the exploring coil) corresponding to a certain (unit) deflection of the galvanometer.

current by a definite amount, and produce a second discharge through the galvanometer by reversing the switch K. Proceed in this way as far as the exciting coils will stand the increasing current, being careful not to allow the exciting current to decrease even momentarily. To keep galvanometer deflections within reasonable limits, vary either the resistance M, or the number of turns in the coil s_1. Note each time the value of this resistance and the number of turns in the secondary coil, since the galvanometer constant depends upon these two quantities.

(2) Again carefully demagnetize the cores and repeat the experiment with a small air-gap, say 0.010 in. Such an air-gap may be easily obtained by interposing a thin piece of fiber or paper between the cores; these materials, being non-magnetic, have the same effect as air. Take a magnetization curve as before. Make similar tests with larger air-gaps.

(3) Substitute cores made of other kinds of magnetic materials, say wrought iron, steel, and cast iron, and repeat the experiment.

(4) Check the statement previously assumed, namely that the mmf depends on the number of ampere-turns, and not on the number of turns or on the current separately.

If a fluxmeter (§180) is used instead of a ballistic galvanometer, it is not necessary to reverse the exciting current. After the cores have been demagnetized, connect the fluxmeter to the secondary coil s_1, and increase the magnetizing current in steps. Read the exciting current and the fluxmeter deflection at each step.

Report. (1) Plot curves as in Fig. 149. (2) Explain the characteristic shape of these curves. (3) Plot curves of ampere-turns necessary for the air-gaps alone, by subtracting from the total ampere-turns the excitation required for the iron parts of the circuit. (4) From these latter curves show that the air-gap ampere-turns are proportional to the length of the gap and to the flux density. Verify eq. (4).

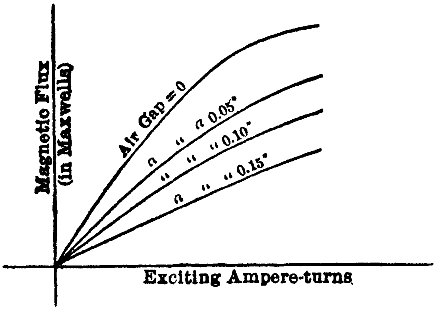


Fig. 149. — Curves showing the influence of small air-gaps in an iron circuit.

167. EXPERIMENT 7-B. — Influence of the Length and Cross-Section of a Magnetic Circuit on its Reluctance. — The arrangement of apparatus and the method of measuring the flux are the same as in §166.

(a) Take one section of the cores, m, m' (Fig. 147), and press the ends together as accurately as possible, so as to reduce the air-gap to a minimum. Take a magnetization curve, as in Fig. 149. (b) Take the cores apart and fasten between them two straight pieces of iron, of the same cross-section and same quality of material as the rest, so as to increase the length of the magnetic circuit, and take a similar curve. More ampere-turns are necessary to produce the same flux, because of the greater length of the magnetic circuit. (c) Take out the straight pieces, bring the U-shaped pieces together as before, and add more sections m, m' in parallel, so as to increase the cross-section of the magnetic circuit. Again take a magnetization curve. It will be found that the same flux is obtained with a considerably smaller number of ampere-turns. (d) Repeat the same experiment with cores made of a different magnetic material. (e) Before leaving the laboratory, measure all the lengths and cross-sections of the samples.

Report. (1) Plot all the magnetization curves for which data were taken in the laboratory. (2) Show that the number of ampere-turns necessary to produce a given flux is proportional to the length of the magnetic circuit. (3) Show that with a given number of ampere-turns, the total flux is proportional to the cross-section of the circuit, in other words, that the reluctance at a given flux density is inversely proportional to the cross-section. (4) Show that, with a given length and cross-section of the magnetic circuit, the reluctance increases when the flux density increases, and note whether or not this is a linear relation.

168. Magnetic Leakage. When two or more paths in parallel are offered to an electric current, it is divided into parts inversely proportional to the resistances (or proportional to the conductances) of the paths. In a similar way a magnetic flux is divided when two paths are offered to it, as in Fig. 150. The apparatus shown there is the same as in Fig. 147 except that the exciting coils *pp* are placed unsymmetrically. The whole flux goes through the lower core because the exciting coils are placed on it, but only part of this flux goes through the upper core. The remainder finds its way back through the air as shown by the dotted lines. This latter portion is called the *leakage flux*, because in most practical cases it is not utilized for the purpose for which the magnetic circuit is formed. Thus, in the electric generator or motor,

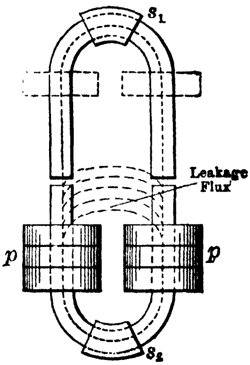


Fig. 150. Arrangement of coils for studying magnetic leakage with the apparatus illustrated in Fig. 147.

part of the flux, instead of going through the armature of the machine, is shunted around it and passes directly from pole to pole through the air. The lower core in Fig. 150 corresponds to the field frame of the machine, the upper one to its armature.

Magnetic leakage makes it necessary to produce a larger flux than is actually utilized. This means more exciting ampere-turns, and consequently more copper, more expenditure of energy for excitation. Magnetic leakage is particularly objectionable when the flux density in the iron is high — approaching saturation — since an increase in flux then means a disproportionately larger increase in the number of exciting ampere-turns. For example, let the leakage in a certain case amount to 20 per cent of the useful flux (this is not an uncommon figure in practice). The increase in the exciting ampere-turns necessary for producing these 20 per cent additional lines of force may be 50 per cent or more, as compared to the ideal case of no leakage, owing to the saturation in the iron caused by the additional flux.

For this reason it is of importance to know how to measure magnetic leakage, and to learn the principal factors upon which it depends, so as to minimize it. In the apparatus shown in Fig. 150, the leakage flux, or the difference between the total and the useful fluxes, may be measured with a ballistic galvanometer or a fluxmeter as before, by taking discharges first through the coil s_2 (total flux), then through s_1 (useful flux). It is more difficult to study accurately the actual distribution of the leakage flux in space, because this requires measuring the flux density in its magnitude and direction from point to point. At the same time, a study of the map of magnetic leakage is of considerable practical importance, because it enables the designer to modify the form and the dimensions of the frame of a machine so as to reduce the leakage to a minimum.

A preliminary study of the distribution of a leakage flux in air may be made with a small magnetic needle freely suspended from a silk thread. The direction of the needle in different places of the stray field gives an idea as to the directions of the leakage lines of force. After this, the flux may be investigated either with a ballistic galvanometer, a fluxmeter, or a bismuth spiral (§210). When a ballistic galvanometer is used, a small exploring coil, connected to it, is brought into the place in which it is desired to measure the leakage. The coil is held on a pivot by a wound spring locked by a catch. With the galvanometer coil at rest, the catch is released and the test coil quickly turns by 180 degrees. The flux cut during this movement produces in the galvanometer a discharge proportional to the flux. If turning a test coil is impracticable, for instance because of a narrow space, the coil is made flat, and after having been brought into the desired place, is quickly withdrawn from the field. In some other cases the exciting current must be broken or reversed in order to produce the desired galvanometer deflection.

169. Factors on which Magnetic Leakage Depends. — Generally speaking, the magnitude of a leakage flux depends on the relative values of the reluctances of the useful path and of the stray paths. Any factor which decreases the reluctance of a stray path or increases the reluctance of the useful path tends to increase the leakage. Thus, for example, making the U-shaped cores (Fig. 150) narrower, that is, bringing the two legs of each core closer together, will increase the leakage, because the path for the leakage flux through the air becomes shorter. Increasing the air-gap between the two U's also increases the leakage flux (Fig. 150), because the reluctance of the useful path is thereby increased. For the same reason the percentage of leakage increases with the increase in the saturation. Moving the exciting coils lower increases the cross-section of the leakage flux (in the vertical plane perpendicular to the paper), and consequently increases the leakage flux itself.

An important case of increase in leakage is that due to some counteracting ampere-turns (coils shown by dotted lines on the upper core). This corresponds in practice to armature currents which counteract the excitation produced by the field coils. The phenomenon is known in generators and motors as *armature reaction* (§§365 to 370). That armature reaction increases magnetic leakage in certain cases may be understood from the following example. Let it be required to maintain a certain flux of Φ lines of force in the upper core (Fig. 150), whether or not a current flows through the upper coils shown by dotted lines. Suppose that each section of the coils has 40 turns, and that at no load the necessary flux Φ is produced with 10 amperes exciting current, or $10 \times 6 \times 40 = 2400$ ampere-turns. Let the leakage flux at no load be 20 per cent of Φ. Now let an opposing current of 15 amperes be sent through the upper coils, creating an armature reaction of $15 \times 2 \times 40 = 1200$ ampere-turns. Without the leakage, the lower exciting coils would have to furnish $2400 + 1200 = 3600$ ampere-turns, in order to give the same flux as before. With the leakage, an increase of 1200 ampere-turns is not sufficient, as shown by the following simple calculation.

First assume the reluctance of the iron paths to be negligible as compared to that of the two air-gaps. Then the mmf across the leakage paths is proportional to the excitation on the lower cores. When this excitation is increased from 2400 to 3600 ampere-turns, the leakage flux is increased from 0.2Φ to 0.3Φ. The lower core must now carry a flux of 1.3Φ instead of 1.2Φ, and the leakage constitutes 30 per cent instead of 20 per cent of the useful flux Φ. In reality the reluctance of the lower cores is not negligible, so that more than 3600 ampere-turns must be placed on it in order to leave 3600 between the lower pole tips. Thus, on account of the armature reaction, the magnetic leakage is increased, and an additional mmf is required on the exciting poles which is greater than the corresponding armature ampere-turns. This shows the importance of keeping the magnetic leakage as low as possible.

The magnitude of the leakage flux, with the same excitation, depends upon the reluctance of the leakage paths, and is therefore affected by the shape of the pole-pieces, and particularly by that of the pole tips. This matter is of extreme importance in the design of the magnetic circuit of electrical machinery. In Fig. 150 the amount of the leakage flux may be increased by adding pieces of iron on the inner side of the pole tips so as to reduce the reluctance of the air paths.

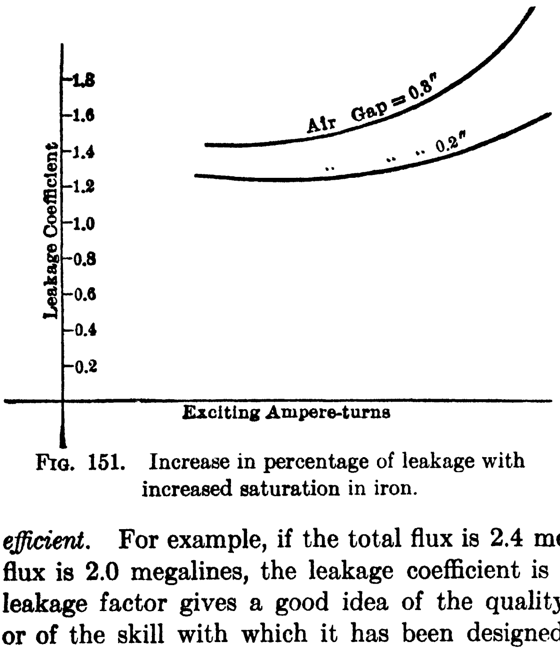


Fig. 151. Increase in percentage of leakage with increased saturation in iron.

The ratio of the total flux to the useful flux is called the *leakage factor* or *leakage coefficient*. For example, if the total flux is 2.4 megalines, and the useful flux is 2.0 megalines, the leakage coefficient is 1.2. The value of the leakage factor gives a good idea of the quality of a magnetic circuit or of the skill with which it has been designed. In Fig. 151 the influence of the saturation in iron and of the length of the air-gap is shown by plotting the corresponding values of leakage coefficient as ordinates.

170. EXPERIMENT 7-C. — A Study of Magnetic Leakage. — The purpose of the experiment is to illustrate the relations explained in the two preceding articles. The apparatus used is shown in Figs. 147 and 150. Magnetic leakage depends upon several factors, and the influence of each factor should be investigated separately. The total flux is measured by discharges through the coil s_2 (Fig. 150) and the useful flux by those through the coil s_1.

(a) *Influence of the air-gap and saturation.* Set the exciting coils *pp* as shown in Fig. 150, and place the upper core as close to the lower core as possible (no air-gap). Take an accurate curve of fluxes in s_1 and s_2 with various values of the exciting current (Fig. 149). Read the fluxes as in Experiment 7-A (§166). Make similar runs with two or three different values of air-gap.

(b) *Influence of the position of the magnetizing coils.* Select a value of air-gaps, place the exciting coil symmetrically with respect to the two cores, as in Fig. 147, and measure the fluxes in both cores. Then gradually move the coils lower, as in Fig. 150, and take similar readings. Repeat the experiment with different values of air-gap and of the exciting current.

(c) *Influence of the shape of pole-pieces.* Put pieces of iron on the inner side of the ends of the lower core, so as to assist the leakage through the air. Do the same with the pole tips of the upper core. Investigate the influence of the shape of these extra pole-pieces with different degrees of saturation in the iron and with different lengths of the air-gap.

(d) *Effect of the armature reaction.* Excite the coils on the upper core to oppose the lower coils, as shown in Fig. 150. Determine the value of the flux in s_2 when the flux in s_1 has a given value, with and without the opposing coils. Reverse the current in the upper coils, so as to make them assist the lower exciting coils; observe the change in the leakage. Make similar tests with different positions of the coils, different degrees of saturation in the iron, and various lengths of air-gap.

Report. Tabulate or plot the results in such a form as to bring out clearly the influence of each factor. Make use of the leakage coefficient in order to reduce the results of the experiment to a more general form (Fig. 151).

171. EXPERIMENT 7-D. — Maps of Stray Flux. — This experiment supplements the preceding one, the purpose being to determine the actual paths of leakage lines of force. The method of measurement is described in §168. Establish certain magnetic conditions, as in Fig. 150, and investigate as closely as possible the direction and the magnitude of the stray flux density at various places. Change the conditions, as indicated in the preceding experiment, and find the effect upon the distribution of the stray flux. Preferably select the same conditions for which values of the leakage coefficient have been determined in the preceding experiment.

Report. Plot the principal directions of the stray flux in several vertical and horizontal planes; mark the magnetic densities per square inch or per square centimeter. Make one or two free-hand sketches, showing actual lines of force, their distribution, and relative density.

State whether the densities at all points increase proportionately with the increase in the exciting current. Discuss the influence of the factors enumerated in §170, and show how they should theoretically affect the distribution and the density of the stray flux.

Note. For a further study of magnetic flux and leakage in d-c machines see Chapter XV.

LIFTING AND TRACTIVE ELECTROMAGNETS

172. Electromagnets are used in practice for various duties, particularly for exerting mechanical pull and for lifting weights. They are especially convenient for intermittent work, where a short stroke and quick action are required. They are used for releasing crane and elevator brakes, for operating switches and motor starters, for supporting weights carried by a crane (Fig. 152), etc. Electromagnets are also widely used as relays for closing all kinds of auxiliary or operating circuits.

Electromagnets used for heavy mechanical duty may be subdivided into two classes: *lifting magnets* and *tractive magnets.* The first kind (Fig. 152) is employed for supporting weights; the second kind (Figs. 153, 154, and 155), for performing mechanical work by the movement of the plunger. The characteristics of each class will be considered separately.

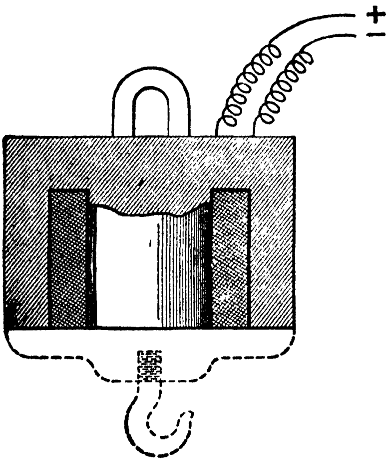


FIG. 152. A lifting electromagnet, with special armature.

173. Lifting Electromagnets. — Electromagnets of this type, Fig. 152, are used in shops and in rolling mills, where large pieces of steel are regularly carried by cranes. An electromagnet is suspended from the crane in place of the usual hook, and is lowered to the piece of steel to be carried. When the electromagnet is energized by closing the circuit of its exciting coil, the magnetic object to be lifted is attracted to it and may be safely carried by the crane. To release the piece, it is sufficient to open the exciting circuit of the electromagnet. The advantage of this method over the ordinary hook is that the work is performed more quickly, as no time is lost in fastening the hook and in unhooking the piece at the end of the travel; also a large number of small and irregular pieces, such as scrap- and pig-iron, can be instantly picked up or dropped. The lifting electromagnet shown in Fig. 152 consists of a steel casting with an annular cavity for the exciting coil. Lines of force pass through the inside core, then through the object to be lifted, and the path is completed through the outside part of the electromagnet frame. The armature and the hook, shown in the sketch by dotted lines, are not used in regular operation; they are shown merely to indicate the method by which the electromagnet may be tested for tractive effort.

Lifting electromagnets are made in different shapes, according to the purpose for which they are intended to be used. Thus, in rolling mills, where large sheets are carried, they are often made with several short, projecting poles, each provided with an exciting coil. This is done in order to support the sheet in several places simultaneously. In carrying pig-iron of irregular shape, the ring-form electromagnet is commonly used. It need only be lowered onto the pile of pig-iron and the coil energized, whereupon the pieces will cling to the poles on all sides.

The lifting power of an electromagnet depends not only on its construction and the exciting current, but also on the form of the piece to be lifted and the quality of iron in it. This is because the path of the magnetic flux is closed through the piece to be lifted; hence the reluctance of this piece determines in part the value of the flux. To make the conditions definite, when testing such an electromagnet, an armature should be used with the smallest possible reluctance; such an armature is shown by dotted lines in Fig. 152. Under such conditions the electromagnet develops its maximum lifting power. If, however, the electromagnet is intended for a definite service, such as for lifting certain kinds of plates in a rolling mill, the test may be performed with such plates.

Holding power of an electromagnet. If the flux density existing between the electromagnet and the piece to be lifted is uniform the holding effort may be expressed by Maxwell's formula

$$F = B^2A/8\pi \text{ dynes} \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$= B^2A/24.7 \times 10^6 \text{ kg} \quad \ldots\ldots\ldots \quad (7)$$

where B is in lines per square centimeter, A is in square centimeters and is the area of flux path. When the more usual units of flux density in kilolines per square inch and area in square inches are employed the pull in pounds is

$$F' = B_0{}^2A_0/72.3 \text{ lb.} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Example. A lifting magnet similar to that of Fig. 152 in contact with a thick steel plate has an area of flux path of 300 sq in. and establishes a flux density of 60 kilolines per sq in. The holding power is therefore $F' = 60^2 \times 300/72.3 = 14{,}950$ lb.

174. EXPERIMENT 7-E. — Test of a Lifting Electromagnet. — Suspend the electromagnet (Fig. 152) from a secure place, and provide an iron armature with a hook, as shown by dotted lines; weigh the armature before using it. Place the armature under the electromagnet, and starting with an excitation such that the armature is supported by the magnetic attraction, gradually reduce the current until the armature drops. Note this critical value of the current. Place some weight on the hook, and repeat the determination. Proceed in this way until the saturation limit is reached, or to the safe limit of heating. In performing this test, have under the armature a suitable support, preferably on springs, to limit the motion and to reduce the noise when the armature is released and strikes the support. Repeat the same test, varying the individual factors, viz., (a) use a different armature; (b) interpose an air-gap, by placing a piece of paper, fiber, etc., between the magnet and armature; (c) lift irregular objects. Before leaving the laboratory, measure the dimensions of the magnet and determine the number of turns in the exciting coil. In the absence of a regular lifting magnet, the apparatus shown in Fig. 147 may be used for the experiment, and the influence of various factors investigated.

Report. To exciting current as abscissas, plot curves of maximum supported weight. From formula (7) or (8) compute the corresponding values of magnetic flux density in the air-gap, and plot them on the same curve sheet. Show by a few examples that this density checks within reasonable limits with that computed from the exciting ampere-turns and the dimensions of the magnet. Use a standard *B-H* curve (Fig. 148) such as is found in various textbooks and handbooks. Indicate the influence of the various factors on the lifting force of the electromagnet.

175. Tractive Electromagnets. — Three common types of tractive electromagnets are shown in Figs. 153 to 155; the difference in their characteristics may be judged from the curves of pull shown in Fig. 156. The electromagnet shown in Fig. 153 consists of a coil surrounding a plunger; the curve of pull shows that, as the plunger is drawn into the coil, the pull first increases and then decreases. The addition of an iron frame on the outside (Fig. 154) increases the pull at the end of the stroke, as shown by the curve marked "Iron Clad." A further increase in pull is obtained by adding an inside core *C* (Fig. 155). The plunger is tapered and the core is countersunk, in order to increase the pull by reducing the reluctance of the air-gap. In many cases, however, a flat-faced core and stop are sufficient. Because an iron-clad electromagnet with a core gives the greatest pull, it does not follow that this type should be used in all cases. The selection of one or another type depends upon the character of the work for which the magnet is intended. The requirements in a particular case being known, the best type may be selected with reference to the curves of pull.

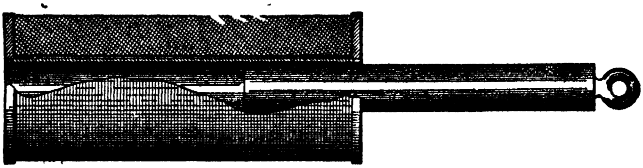


Fig. 153. A coil-and-plunger type tractive electromagnet.

Alternating-current electromagnets are built along similar lines, except that the magnetic circuit usually has to be laminated to reduce eddy currents (§217). If the plunger consists of iron strips, a square cross-section is more convenient. The pull, being proportional to the square of the flux density, varies from zero to a maximum and back twice during each cycle of the alternating current. This causes the plunger to vibrate, and if it touches a stop, produces chattering. This is sometimes remedied by the use of a *shading coil*, or solid metal sleeve at the end of the plunger (Fig. 377). Secondary currents induced in this sleeve are considerably out of phase with the primary current, so that the resultant mmf in the air-gap is never equal to zero. Consequently, the variations in the pull are not so great as without the shading coil.

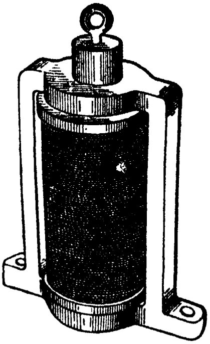


Fig. 154. An iron-clad tractive electromagnet.

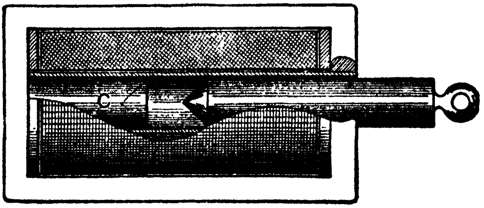


Fig. 155. An iron-clad tractive electromagnet, provided with a core *C* for increasing the pull at the end of the stroke.

A convenient apparatus for testing tractive electromagnets is shown in Fig. 157. The coil C of the electromagnet under test is connected to a source of d-c or a-c supply, through the switch S, regulating rheostat R, and the ammeter A. The plunger **P** is suspended from the beam of the balance. The balance has fulcrums at F_1 and F_2, and two sliding weights, w_1 and w_2. With this construction a large pull is counter-balanced with small weights

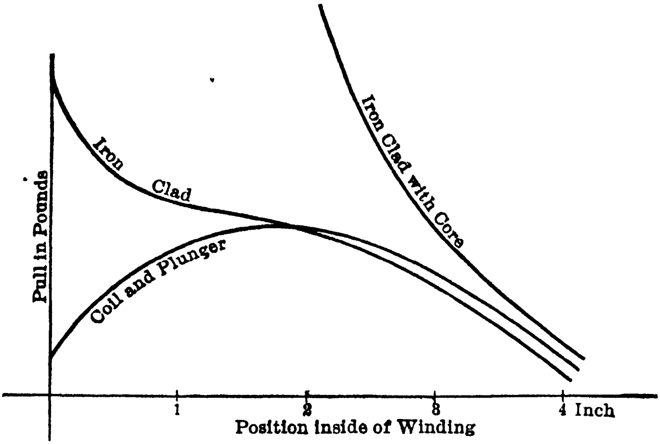


FIG. 156. Curves of comparative pull of the electromagnets shown in Figs. 153 to 155.

so that the apparatus is sensitive and convenient to handle. The curves shown in Fig. 156 were taken by means of such a device. A simpler apparatus involving the use of a spring balance, and similar to the Thompson permeameter of Fig. 162, is often used for laboratory testing.

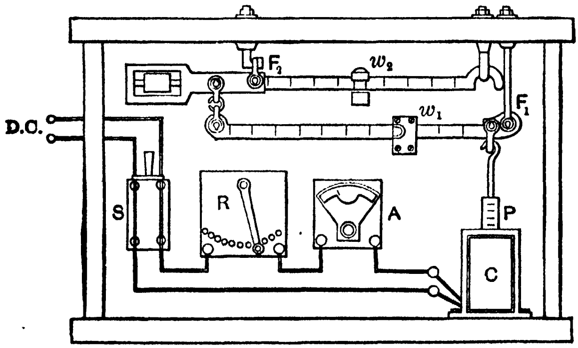


FIG. 157. An apparatus for testing the tractive effort of electromagnets.

176. EXPERIMENT 7-F. — Test of a D-C Tractive Electromagnet. — The purpose of the experiment is to determine curves of pull (Fig. 156) as functions of both plunger position and current, and to study different types of electromagnets. An apparatus similar to that of Fig. 157 or

Fig. 162 should be provided. Start the test using a simple solenoid and plunger, similar to Fig. 153. First determine the neutral position of the plunger within the coil such that the scale indication is the same with current on and current off. This scale reading is the " tare " indication. Proceed as follows: (a) Adjust the current to the highest value permitted by heating of the equipment and hold this current constant. Take readings of the scale and plunger location for successive positions of the plunger from neutral to the most remote possible with the equipment. (b) For a fixed position of the plunger, vary the current in equal steps from a maximum to the lowest at which definite scale readings are obtainable. (c) With the current as in (a), move the plunger suddenly into the coil, noting any momentary effect upon the ammeter reading. Withdraw the core suddenly and again note. (d) Provide a stop or fixed core in the electromagnet similar to Fig. 155 and take readings as in (a). (e) If possible, provide both yoke and stop and take readings as in (a). Obtain the dimensions of the electromagnet and core and number of turns in the exciting winding.

Report. (1) Plot curves of net pull (total pull less tare) against plunger position for tests (a), (d), and (e). (2) Plot pull against current for fixed position of plunger. (3) Explain any effect upon the current of sudden movements of the plunger. (4) Justify the shapes of the curves from the general theory of the tractive electromagnet.

177. The Ballistic Galvanometer. — A ballistic galvanometer is a measuring instrument in which deflections are proportional to very brief electric discharges through it. An electric discharge through such a galvanometer may be made proportional to a sudden change in a magnetic flux. For this reason the ballistic galvanometer is widely used for measuring magnetic fluxes. Like most galvanometers used in physics, a ballistic galvanometer consists of a coil of fine wire suspended in the field of a permanent magnet. When a current passes through the coil, the latter is deflected according to the strength of the current. The torsion of the suspension wires, or spiral springs, returns the coil to zero when no current flows. The construction is similar to that shown in Fig. 38, except that the coil is suspended by means of straight vertical wires which also provide the restoring force. The coil is made wider, that is, its vertical parts are farther from the axis of rotation, to increase the moment of inertia, and frequently adjustable weights are attached to the movement.

This greater moment of inertia is the only essential difference between the ballistic galvanometer and the usual type of galvanometer used for steady deflections. A greater inertia is necessary in order to increase the time of natural swing of the moving system, and thus to prevent an appreciable movement of the coil before the total electric discharge has passed through it. Only under this condition does the instrument give deflections which are proportional to total discharges, and are independent of their duration or of the instantaneous values of the current. If, for example, the movable part is made so heavy that its period of swing is 10 seconds, the coil will not move appreciably before the discharge is over, with any discharge that lasts, say, less than 0.1 second.

For measuring a magnetic flux (Fig. 147), the ballistic galvanometer is connected to the secondary coil s_1, if necessary, through a resistance M, to reduce deflections. As long as the flux remains constant, the galvanometer reads zero; but if the flux is suddenly changed, a transient electric current is sent through the galvanometer circuit, and the coil is thrown by a definite amount. It then continues to swing to and fro, in a manner determined by its inertia, the restoring force of the suspension wires, and the dissipative forces of induced currents and friction. The first deflection, multiplied by a constant of the instrument, is a measure of the change in flux. For example, let the galvanometer constant be 50,000 maxwells per 1 cm scale deflection. Then, should the deflection be 15 cm, the change in flux is $50{,}000 \times 15 = 750{,}000$ maxwells, or 750 kilolines.

A change in flux is brought about by opening or closing the switch K (Fig. 147), by reversing the current with the same switch, or by changing the setting of the rheostat R.

The theory and calibration of the ballistic galvanometer are explained in §§178 and 179.

178. Theory of the Ballistic Galvanometer. — The general features of the instrument are described in the preceding section, and it will now be shown that deflections of the instrument are proportional to variations in the flux through the secondary coil. It will be proved: (a) that an electrical discharge through the galvanometer is proportional to the corresponding change in flux linking with the test coil; (b) that the total impulse communicated to the movable coil of the galvanometer is proportional to the total quantity of electricity that has passed through it; and (c) that the deflection of the galvanometer coil is proportional to the impulse communicated to it. From these three facts it will follow that deflections are proportional to variations in flux.

(a) *An electric discharge through a ballistic galvanometer is proportional to the change in flux.* Let Φ be an instantaneous value, in webers, of a variable magnetic flux linked with an exploring coil, for example the flux within the test coil s_1 in Fig. 147. The instantaneous emf, e, induced in the coil by a variation of this flux, according to §163, is

$$e = -n\,d\Phi/dt$$

where n is the number of turns in the coil. The minus sign is necessary because of the usual convention as to the positive direction of the emf. Namely, when the flux increases, so that its derivative is positive, the induced emf tends to produce a current which would reduce the flux. Hence, if the magnetizing current of the flux itself be considered positive, the induced emf and the current which it would induce are negative.

Let r be the total resistance of the galvanometer circuit and L its inductance. According to eq. (6) in §143, the instantaneous current i in the galvanometer is determined by the relation

$$e = ir + L\,di/dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

Substituting the value of e from above gives

$$-n\,d\Phi/dt = ir + L\,di/dt$$

or

$$-n\,d\Phi = ir\,dt + L\,di \quad . \; . \; . \; . \; . \; . \; . \quad (10)$$

Let Φ_1 and Φ_2 be the initial and the final values of the flux during the short interval of time t of the change. Integrating the preceding equation over the time t, we obtain

$$n(\Phi_1 - \Phi_2) = r\int_0^t i\,dt + Li\Big|_0^t \quad . \; . \; . \; . \; . \; . \; . \quad (11)$$

The integral on the right-hand side represents the total quantity of electricity, in coulombs or ampere-seconds, which is discharged through the galvanometer; let this quantity of electricity be denoted by Q. The second term on the right-hand side of eq. (11) is equal to zero, because the galvanometer current is zero both at the beginning and at the end of the discharge period. Thus we have

$$n(\Phi_1 - \Phi_2) = rQ \quad . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

and

$$Q = \frac{n}{r}(\Phi_1 - \Phi_2) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12a)$$

This proves the above statement that an electrical discharge through the galvanometer is proportional to the change in flux.

(b) *The total impulse communicated to the movable coil is proportional to the total quantity of electricity* Q. The instantaneous mechanical torque between the movable coil and the magnetic flux of the galvanometer itself, due to its permanent magnet, is proportional to the instantaneous current i and to the flux density in the air-gap. The latter density being constant, the torque impulse, J, may be written in the form

$$J = C\int_0^t i\,dt = CQ \qquad \text{. . . . . (13)}$$

where C is a coefficient of proportionality which depends upon the construction data of the instrument. The coil begins its motion as a result of this impulse.

(c) *The deflection is proportional to the impulse communicated to the movable coil.* The galvanometer coil moves against the following forces:

(1) The torsion of the suspension wire or springs.
(2) The currents induced in the coil and in its aluminum frame.
(3) The air resistance.

Let α be the final deflection with a given discharge of Q coulombs, and let x be an instantaneous intermediate deflection. The values of α and x may be in degrees, or in length units on the scale. The torque due to torsion is proportional to the angle of torsion. Hence, the total resisting impulse of the suspension wire may be written as

$$J_1 = C_1'\int_0^\alpha x\,dt \qquad \text{. . . . . (14)}$$

where C_1' is a coefficient of proportionality. As in a pendulum, so in the galvanometer coil, if the current impulse is over before the coil has begun to move, the motion of the coil is harmonic, and with small angles the time of one swing is independent of its amplitude. For a double α, all intermediate angles x are doubled, while the time elements dt remain the same. Hence, the value of the integral in eq. (14) is doubled. Thus, generally, J_1 may be said to be proportional to α, or

$$J_1 = C_1\alpha \qquad \text{. . . . . (15)}$$

where C_1 is another coefficient of proportionality.

The currents induced in the circuit of the coil and in its aluminum frame, during the motion of the coil, are proportional to the instantaneous values of the angular velocity dx/dt. Their impulse is

$$J_2 = C_2\int_0^\alpha (dx/dt)\,dt = C_2\alpha \qquad \text{. . . . . . . (16)}$$

where C_2 is a coefficient of proportionality.

The air resistance, or windage, may also be assumed to be proportional to the instantaneous angular velocity, so that its impulse is

$$J_3 = C_3\alpha \qquad \text{. . . . . (17)}$$

Now, equating the communicated impulse, eq. (13), to the sum of the resisting impulses, we obtain

$$CQ = (C_1 + C_2 + C_3)\alpha \quad . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

This proves that the final deflection, α, is proportional to Q.

(d) *The final formula.* Denoting the "ballistic" constant of the galvanometer by b, we have from eq. (18)

$$Q = b\alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

Consequently, eq. (12) becomes

$$n(\Phi_1 - \Phi_2) = br\alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

Let, for example, the galvanometer constant be $b = 15 \times 10^{-6}$ coulomb (ampere-second) per cm of scale deflection. Let the resistance of the galvanometer circuit, r, be equal to 2000 ohms, and the number of turns in the exploring coil be $n = 100$. Let a certain flux be established through the test coil, and then the exciting circuit suddenly opened, causing a galvanometer deflection $\alpha = 30$ cm. The final $\Phi_2 = 0$, and we have, according to eq. (20),

$$\Phi_1 = 15 \times 10^{-6} \times 2000 \times 30/100 = 9 \times 10^{-3} \text{ weber} = 900{,}000 \text{ maxwells}$$

179. Calibration of a Ballistic Galvanometer. — A ballistic galvanometer is usually calibrated with a standard mutual inductance, although it may also be calibrated with a standard condenser or with a standard inductance coil.

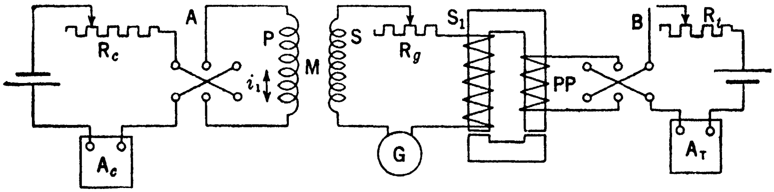


FIG. 158. Connections for the calibration of a fluxmeter, using a standard mutual inductance.

(a) The secondary winding, S, of a *standard mutual inductance M* of a known fixed value (§156) is left permanently in the galvanometer circuit, in series with the secondary coil S_1 (Fig. 158) used for measuring the unknown flux. The primary coil, P, of the mutual inductance is connected separately to a source of direct current, and provisions are made for quickly reversing this current using the double-throw switch **A**. The switch **B** similarly permits the reversal of current through the exciting coils PP of the magnetic tester. A galvanometer deflection may thus be produced by reversing the current either in the magnetizing coils PP of the magnetic circuit under test, or in the primary winding of the mutual inductance. In the first case the instantaneous induced secondary voltage in the galvanometer circuit is $e = -N\,d\phi/dt$; in the second, it is $e = -M\,di_1/dt$, where i_1 is the instantaneous primary current in the mutual inductance. A comparison of these two expressions shows that the product Mi_1 takes the place of the product $n\phi$ in the theory given in §178, and in the final formula (20). Thus, from eq. (20), reversing the current through the coils pp gives the galvanometer deflection

$$\alpha_1 = 2n\phi/br \quad \ldots\ldots\ldots\ldots \quad (21)$$

Similarly, reversing the current I through the primary of the mutual inductance M gives a deflection

$$\alpha_2 = 2MI/br \quad \ldots\ldots\ldots\ldots \quad (22)$$

Therefore

$$n\phi/MI = \alpha_1/\alpha_2 \quad \ldots\ldots\ldots\ldots \quad (23)$$

or

$$\phi = \alpha_1 MI/\alpha_2 n \quad \ldots\ldots\ldots\ldots \quad (24)$$

Substituting the value of ϕ so obtained in eq. (21) gives the value of b, the constant of the galvanometer, or of ϕ/α, the flux per centimeter scale deflection of the galvanometer in the given circuit. In much experimental work it is not necessary that the value of b be determined but only that ϕ, as given by eq. (24), be known. If M is in henrys and I is in amperes, ϕ is in webers (§163).

(b) *Calibration by means of a standard condenser.* Let a *condenser* of a known capacitance, say C microfarads (Vol. II), be charged at a known voltage e, and then discharged through the ballistic galvanometer; let the deflection be α. According to the definition of capacitance, the charge $Q = Ce \times 10^{-6}$ coulomb, so that eq. (19) gives $Ce \times 10^{-6} = b\alpha$. In this equation all the quantities except b are known, so that the latter can be determined.

(c) *Calibration using a standard inductance coil. A standard inductance coil* is usually a long straight solenoid without iron core, with a short concentric exploring coil placed about its central part. The flux density B in the middle part of the coil can be computed as explained in §165. When the cross-section A of the coil is known, the flux $\phi = BA$ becomes known, and formula (20) may be used for the determination of the constant b. The standard inductance coil is excited with a known primary current I, and the exploring coil is connected in the galvanometer circuit. The primary circuit is then opened or the current is reversed, and the deflection is noted. In the first case the change in flux is equal ϕ; in the second, 2ϕ. This is a modification of the mutual-inductance method described under (a) above.

180. Grassot Fluxmeter. — The disadvantages of the ballistic galvanometer described in the preceding sections are:

(1) It is necessary to bring about a change in the flux in order to measure it.

(2) The change must be sudden, in order that the electric discharge be completed before the galvanometer coil begins to move.

(3) The indications are only instantaneous, since the moving coil immediately begins to return to zero.

These disadvantages are largely eliminated in the Grassot fluxmeter, shown in Figs. 159 and 160. Its construction is essentially that of a ballistic galvanometer, except that the moving coil is suspended from a single cocoon fiber of almost negligible torsional stiffness. In an ordinary ballistic galvanometer the coil is returned to zero under the influence of the torsion of the suspension wire; in the fluxmeter the coil remains substantially at rest in any position, and creeps back but very slowly. The upper end of the fiber is attached to the flat spiral spring *R* to minimize the effect of shocks. The stiff wire frame *E*, fastened to the coil *B*, allows of a central attachment of the very thin silver strips *s* and *s'*,

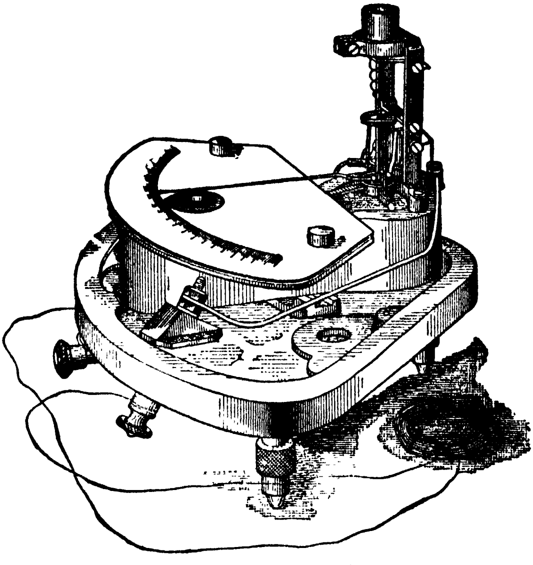


Fig. 159. The Grassot fluxmeter.

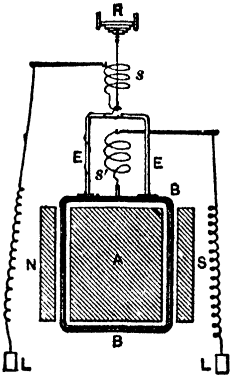


Fig. 160. Arrangement of parts in the Grassot fluxmeter.

CHAPTER VIII

PERMEABILITY AND HYSTERESIS LOOP

182. Steel and iron intended for use in the magnetic circuit of electrical machinery and apparatus is usually tested for its magnetic properties. There are two standard tests of this kind, viz., one for permeability and one for core loss. The permeability test is considered in this chapter and the core-loss test in the next.

183. Magnetization or Saturation Curve. — The properties of a sample of magnetic material are conveniently represented by means of magnetization or *B*-*H* curves, such as are shown in Fig. 148 in §165. From such a curve the ampere-turns necessary for producing a desired flux density can be computed, and one can also judge as to whether or not the shipment from which a sample was taken comes up to the standard established for the particular application.

By definition, the ratio of B to H is called the *absolute permeability* of the material; we shall denote it by μ_a. Thus we have

$$\mu_a = B/H \quad \ldots\ldots\ldots\ldots \quad (1)$$

Since both B and H may be expressed in different units the value of the absolute permeability is not definite unless the units are stated. For example, let a piece of steel require $H = 25$ ampere-turns per cm at a flux density of $B = 15$ kilolines per sq cm. According to the foregoing definition, $\mu_a = 15/25 = 0.6$. A flux density of 15 kilogausses is identically the same as one of $15{,}000 \times 2.54^2 = 96{,}800$ maxwells per square inch. The corresponding H in the English measure is $25 \times 2.54 = 63.3$ ampere-turns per inch. The new value of μ_a is $86{,}800/63.3 = 1525$.

To avoid this ambiguity, the *relative permeability*, μ_r, is often used; it is defined as the ratio of the flux density produced in the sample under test to that in the air, with the same magnetizing force H. In other words,

$$\mu_r = B_{\text{iron}}/B_{\text{air}} \quad \ldots\ldots\ldots\ldots \quad (2)$$

The kind of units used for B is immaterial, as long as it is the same for both the iron and the air. In the foregoing example the flux density in the air for $H = 25$ is $1.257 \times 25 = 31.4$ maxwells per sq cm (§165). Hence, $\mu_r = 15{,}000/31.4 = 477$. This number means that at the flux density of 15 kilogausses the sample of steel under test is magnetically 477 times " better " than the air.

It is not always required to figure out the values of permeability from a *B-H* curve. Often the *B-H* curve itself is the desired characteristic of a sample. The *B-H* curve is also called the *saturation curve* or the *magnetization curve*, and the three names are used interchangeably. The measuring devices described below are intended for obtaining the *B-H* relationship and are called permeameters.

184. Hysteresis Loop. — Such a magnetization curve as is shown in Fig. 148 for cast iron is also shown in Fig. 161 as the curve *OAM*. It is assumed that at *O* the sample is completely demagnetized. The saturation curve *OAM* is therefore called the *virgin curve*. Let this curve be obtained experimentally, beginning at point *O*. If now, after point *M* has been reached, the excitation is reduced again, the flux density does not decrease as rapidly as it increased, but varies according to the curve *MR*. When the exciting circuit is opened, the iron still possesses a *residual* magnetic density *OR*. When the exciting current is reversed this density is gradually decreased. The value *OK* of the magnetizing force which reduces the residual magnetism to zero is called the *coercive force* (*OR* = *retentivity*) H_c. When the exciting current is further increased (in the negative direction), the magnetic flux in the iron is reversed and varies according to the part KM_1 of the curve. Decreasing the

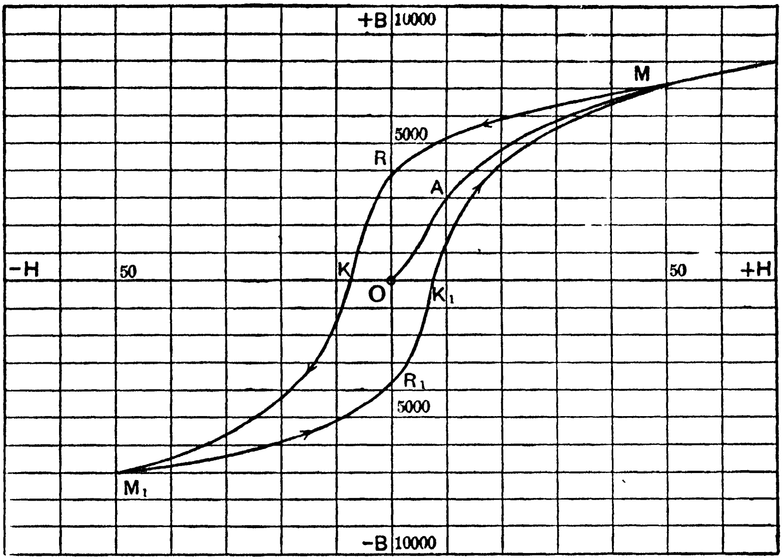


FIG. 161. Magnetization curve and hysteresis loop of a sample of cast iron.

current again and reversing it causes the densities to vary according to the branch M_1K_1M of the curve.

This peculiar phenomenon, viz., that two different flux densities are possible with the same value of exciting current, is called *hysteresis.* It is attributed to some kind of molecular friction in iron; the root of the word signifies " to lag behind." The curve MKM_1K_1 is called the *hysteresis loop.* In accurate permeability tests, it is advisable to take both a virgin curve and one or more hysteresis loops.

When a hysteresis cycle is performed for the first time on a piece of virgin iron the loop is not closed, but the final point M lies somewhat above the initial point M. After the cycle has been repeated for a number of times the loop becomes closed and the iron is said to be in a *cyclic* state. The locus of points, such as M and M_1, for different final values of H in the cyclic state, is called the *normal* or *commutation curve,* and may be slightly different from the true virgin curve.

A large value of residual magnetism $OR = B_r$ is objectionable in cases where the flux is supposed to follow closely any variations in exciting current, as for example in the field poles of a generator or motor. On the other hand, large residual magnetism is desirable in steel intended for permanent magnets, such as are used in certain types of measuring instruments, in telephone receivers, and in magneto generators. For a further discussion of hysteresis see Chapter IX. The testing of permanent magnets for residual magnetism and for coercive force is described in §§211 to 214.

185. Magnetic Flux in Air. — In some of the tests described below, it is required to compute the ampere-turns necessary to produce a certain flux density in air, instead of iron (see §161). This can be done with sufficient accuracy only when the lines of force are parallel to each other, as for example, in a small air-gap between two large pieces of iron, or near the center of a long solenoid. Theory and experience show that under these conditions *one ampere-turn per centimeter of length of path in the air produces a flux density B_a = 1.257 lines of force* (maxwells) *per sq cm* (theoretically $4\pi/10$ lines per sq cm). The magnetic density in air is denoted by B_a to distinguish it from that in iron, which is designated by B. Thus, to produce a flux density of B_a lines per square centimeter (gausses) in an air-gap l centimeters long (in the direction of lines of force) the required number of ampere-turns is

$$ni = B_a l/1.257 = 0.796 B_a l \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

If l is in inches, and B_a in lines per square inch, the formula becomes

$$ni = 0.796(B_a/2.54^2) \times (2.54l) = 0.313 B_a l \quad . \; . \; . \; . \quad (4)$$

Let it be necessary, for example, to produce a flux of 3,000,000 lines in an air-gap 0.3 in. long, and of a cross-section of 50 sq in. The flux density $B_a = 3{,}000{,}000/50 = 60{,}000$ maxwells per sq in. The required mmf is therefore (eq. 4) $ni = 0.313 \times 60{,}000 \times 0.3 = 5634$ ampere-turns.

186. Types of Testing Devices. — The methods and the devices for experimentally obtaining the *B-H* curve and a hysteresis loop (Fig. 161) of a given specimen of steel or iron are described below under the following three headings: (1) tractional method, (2) ballistic method, (3) steady-deflection methods. The *B-H* curve can also be determined with alternating current, by the wattmeter method, when measuring the core loss (§231).

With the tractional method the magnetic flux density is measured in terms of the mechanical attraction between two parts of a magnetic circuit. This is one of the oldest methods and is gradually giving way to the more accurate and convenient ballistic method. In the latter, the flux within the specimen is suddenly reversed by reversing the current in the magnetizing winding (§177). A transient current is thereby induced in a secondary winding, and the total electric discharge is read on a ballistic galvanometer. From this deflection the change in flux may be computed. In the past, steady-deflection methods have not been much used, except for testing permanent magnets; but with the development of methods for balancing the restoring torque of the fluxmeter (see §181) they are becoming of greater importance. In one of the older steady-deflection methods the piece of steel under test is used as part of the magnetic system of a moving-coil galvanometer, and the deflection is a measure of the flux in the specimen. In another arrangement the flux in the specimen is made equal to that in a standard sample, and the mmf's in the two are directly compared. The property of bismuth to increase its electrical resistance in a magnetic field has also been utilized. When steel laminations are to be used with a-c excitation exclusively, as for example in a transformer core, it is sometimes preferred to determine their magnetization curve under the same conditions (§231), that is, with alternating current.

The selection of this or that piece of apparatus or method of taking a *B-H* curve depends upon the nature of the test, for example, a scientific investigation, a special industrial test, routine commercial tests, etc. The desired rapidity of individual determinations, the required accuracy of the results, and the form of available samples also influence the choice of equipment and procedure.

Samples of solid iron are usually tested in the form of straight rods of circular or rectangular cross-section and of convenient length, as these are easily prepared. Sometimes it is desired to test a specimen in ring form, so as to avoid an air-gap; in exceptional cases it may be necessary to test a casting or a forging in bulk (§197). Laminations are tested whenever possible in rectangular strips assembled into packages. Sometimes it is required to test them in ring form, or in more complicated stampings of the same shape in which they are used in the assembly of a piece of electrical apparatus. Devices and methods are available for all such cases, and the selection of the best device for the purpose in hand is a matter of judgment based essentially upon first-hand experience with various devices and methods of testing.

187. Procedure in Testing. — The following remarks are taken almost verbatim from Circular 17 of the Bureau of Standards. Although these instructions were written specifically for the ballistic method, they will be found useful with any other method of determination of permeability and hysteresis loop. Some of the recommended precautions are necessary only in extremely accurate testing, and in each particular practical case one has to decide which of them may be omitted without detriment to the results.

The induction (flux density) which a bar of iron or steel will assume under a given magnetizing force depends upon the previous magnetic condition of the specimen and upon the rate of change from one magnetic state to another. It is modified by the presence of mechanical vibration and depends to some extent on temperature. It is therefore desirable to state the conditions under which the test is made.

All the ordinary tests on magnetic material except aging should be made at a constant temperature of 25° C. This is in view of the fact that all the magnetic quantities, including the normal induction, the hysteresis loop, and the losses due to hysteresis and eddy currents, depend upon the temperature of the specimen. The relation is not a simple one and is not the same for all materials. In view of these facts it seems desirable to make all such measurements at a fixed temperature.

It is important that there be no mechanical vibration of the specimen during the test. Such vibrations tend to give an induction greater than normal for increasing magnetizing forces, and too small values for decreasing forces. Hence, the test specimen should always be protected from mechanical vibrations in ballistic measurements.

The results found for rolled sheets usually depend upon whether the material is magnetized parallel to the direction of rolling or at right angles to this direction. When not otherwise specified and the dimensions of sheets submitted permit it, the test pieces should be so cut, that the flux traverses half of them parallel to the direction of rolling, and half normal thereto.

The measurement of flux density requires a knowledge of the cross-sectional area of the specimen. For rods and bars the cross-section is determined from the dimensions. In sheet metal, however, it should not be determined by direct measurement, but from values of mass, length, and density. The density of each specimen should be determined experimentally, as experience shows that the assumption of any specified value introduces an uncertainty in the result which is greater than the inaccuracy of the magnetic measurements.

Normal induction. If a bar of thoroughly demagnetized iron is subjected to a magnetizing force, it experiences a certain induction (flux density). This induction will be greater if the magnetizing force is applied suddenly than for a slower growth of magnetizing current. If the magnetizing force is repeatedly applied and removed, the values of the induction obtained differ somewhat. If the magnetizing force is reversed, a change of induction approximately twice the preceding values is obtained. For the first few reversals the change of induction is not constant, but becomes so after a large number of reversals. One-half this constant value of the change in induction on reversal of the magnetizing force is the normal induction, and the locus of such points is the curve of normal induction.

The magnetic properties of a piece of iron or steel may be considered as defined by the curves of normal induction and hysteresis. Before determining the normal induction data, it is necessary that the specimen be freed from its previous magnetization. This is accomplished by subjecting it to a cyclic magnetizing force of one period per second, which is gradually reduced from an initial value, and carries the induction (flux density) well beyond the point of maximum permeability to a final value somewhat lower than the lowest induction to be studied.

After thorough demagnetization, the lowest magnetizing force to be used is applied and reversed many times, until the iron is brought to a cyclic magnetic state. The induction is then measured and the next higher value of the magnetizing force applied in the same manner. This process is repeated until the required number of points is determined. This is a somewhat laborious operation, but it has been found necessary in order to obtain reliable results.

Hysteresis loop. Before determining the hysteresis loop, the iron is demagnetized as above, and the magnetizing force is applied and increased until the iron is brought up to the maximum induction for which the loop is required. This magnetizing force is repeatedly reversed until the iron is in a normal condition. The magnetizing force is now reduced from its maximum value to a lower one, and the change in magnetic induction corresponding to the change in force is noted. After this pair of values has been determined, the *maximum* magnetizing force is again applied and the iron once more brought back to a normal magnetic condition. It is not necessary, however, to repeat the process of demagnetization. Another point is then determined in the same manner as the first but for a greater decrease in mmf, and so on. Points corresponding to negative values of the magnetizing force are obtained by simultaneously reversing and reducing the magnetizing force. Before each determination of a point on the loop the iron is brought back to its normal condition at maximum induction.

This method of measuring the magnetic constants differs somewhat from the old " step by step " method, which is still employed in many of the modern commercial permeameters. It has the advantage of making the measurement under conditions more nearly approximating those of commercial practice, and is practically free from the effects of magnetic viscosity. Further, it is possible to get more consistent results by this method than by the older one, as the effects of imperfect initial demagnetization are not so serious. The numerical data obtained by these two methods are not identical, and in publishing results of work of the highest precision it is desirable to specify the method of measurement.

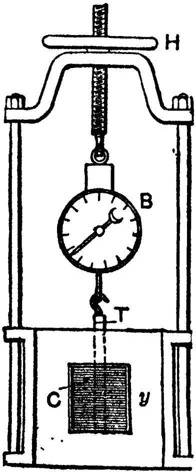


FIG. 162. Thompson permeameter.

TRACTIONAL METHOD

188. Thompson Permeameter. — One of the simplest pieces of apparatus for testing iron and steel, the so-called *Thompson permeameter*, is shown in Fig. 162. It is based on the measurement of the mechanical force necessary to separate two parts of a magnetic circuit. The sample *T* to be tested is made in the form of a rod; it is placed within an iron yoke *y*, of negligible magnetic resistance (reluctance). The closed iron circuit thus formed is magnetized by the coil *C*. Then the sample *T* is pulled out of the yoke by turning the handwheel *H*; the pull is read on the spring balance *B*.

The apparatus is intended primarily for approximate relative measurements; the better the sample (the higher its permeability), the more force is required to pull it out of the yoke, with the same magnetizing current. The device has been used for such approximate tests by some electric manufacturing companies. Whenever a new lot of steel is received, samples made out of it are tested in the permeameter. The force necessary to pull a specimen out of the yoke must be at least equal to that previously found for the lowest acceptable grade of material; otherwise the entire consignment is rejected. The right number of ampere-turns is obtained by regulating the field rheostat.

The permeameter may also be used for the determination of the actual magnetization curve (Fig. 161) of a sample, by comparing the mechanical force necessary to pull out the sample under test, with that required with a standard sample whose magnetization curve has been previously determined by some other method. The comparison is made on the basis of the fact that the mechanical force of attraction (see §173) between two parts of a magnetic circuit is proportional to the square of the magnetic density at the separating surface and to the area of this surface (Maxwell's law). For a proof of this law and of eq. (6) below see, among other works, the author's " Magnetic Circuit," the last chapter.

Thus, with a given magnetizing current, if F_1 is the spring balance reading with the sample under test, and F_2 that with the standard sample, we have

$$\left(\frac{B_1}{B_2}\right)^2 = \frac{F_1}{F_2} \qquad (5)$$

If the magnetic density B_2 is known from the magnetization curve of the standard sample, B_1 may be calculated, and the magnetization curve plotted for the sample under test.

The magnetization curve of a sample can also be obtained with the Thompson permeameter even without having a standard sample. Let B be the flux density in the sample, in kilolines per square centimeter; let B_a be the density in the air after the sample has been withdrawn; let A be the cross-section of the sample in square centimeters, and F the mechanical pull in kilograms. The theoretical formula for the total mechanical force at the separating surface is (see §173)

$$F = AB^2/24.7 \qquad (6)$$

When the sample is pulled out, not all the flux disappears, since B_a kilolines of force per square centimeter still remain. Thus, only $(B - B_a)$ kilolines per square centimeter vanish, and the indication of the spring balance corresponds to this number. The preceding equation in our case becomes

$$F = A(B - B_a)^2/24.7 \qquad (7)$$

Solving it for B, we get

$$B = 4.97\sqrt{F/A} + B_a \qquad (8)$$

In this expression B_a may be approximated from eq. (3) in §185, if the number of exciting ampere-turns ni and the length l of the coil, that is, the height of the window in the iron yoke, are known. It must be remembered, however, that B_a in formula (3) is in gausses, whereas in eq. (8) it must be in kilogausses.

If B and B_a are expressed in kilomaxwells per square inch, A is in inches, and F in pounds, the preceding formula becomes

$$B = 8.5\sqrt{F/A} + B_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In this case B_a must be computed from eq. (4) and the result divided by 1000, to convert it into kilolines.

The Thompson permeameter is adapted for making quick comparative tests rather than for taking exact magnetization curves. A spring balance is not reliable enough to allow of accurate measurements, and moreover, the results are vitiated by the two air-gaps between the yoke and the sample. To reduce the error due to this cause the sample must fit nicely into the hole in the upper part of the yoke, and the two surfaces which are pulled apart must be carefully machined.

The *Ewing Magnetic Balance* is another device based on the same principle. It has a beam with a sliding weight instead of a spring balance. The specimen rests on the yoke with its cylindrical surface, instead of touching the yoke with an end, as in the Thompson permeameter. This feature does away with the surfacing of one end of the sample, and makes it unnecessary to have a hook on the other end; moreover, a better contact is obtained. The scale is calibrated directly in flux densities by means of a standard sample, previously tested by some other method.

189. EXPERIMENT 8-A. — Magnetization Curves with Thompson Permeameter. — The purpose of the experiment is to obtain, for a given sample, a B-H curve and a hysteresis loop, such as are shown in Fig. 161. It is desired that the student test samples of forged, rolled, and cast steel, wrought iron, and cast iron, to see the difference in the magnetic properties of these materials. For the general procedure in testing see §187, although this particular permeameter is not an accurate device and many refinements recommended there are hardly necessary. (a) To demagnetize the sample, place it in the apparatus and magnetize it well above the maximum density to be used in the test. A regulating rheostat and a reversing switch should be provided in the circuit of the exciting winding (Fig. 147). By operating the double-throw switch, reverse the magnetism of the sample repeatedly, and at the same time gradually reduce the exciting current by means of the regulating rheostat. A frequency of about one reversal per second is recommended. (b) First take a virgin curve OAM (Fig. 161);

read the exciting current and the pull necessary to tear the sample from the yoke. Carry the magnetizing current as high as the coil will permit. Then take a complete hysteresis loop. (c) Repeat the same test with other samples. Before leaving the laboratory, measure the cross-section of the samples tested and the axial length of the coil; also ascertain the number of turns in the coil. Each point should be obtained as an average of several readings, because of the inaccuracies in each observation. The sample and the yoke must be nicely surfaced and the spring balance must exert an axial pull. Another precaution, important in all permeability tests, is that once the current has been raised to a certain value it should not be reduced again until that part of the magnetization curve is completed. Otherwise, confusing hysteresis effects will be introduced.

Report. Plot the spring-balance readings to exciting amperes as abscissas. This should be done before making any calculations, in order to obtain smooth curves and to eliminate possible errors of observation. For one of the samples change the curve into a magnetization curve, such as is shown in Fig. 161, by using formula (8) or (9). If the sample has been previously tested by another method, check the results. With this curve as a standard, plot a similar curve for another sample, using formula (5).

190. Du Bois Magnetic Balance. — This is also a tractional device (Figs. 163 and 164), more sensitive and accurate than either of the two

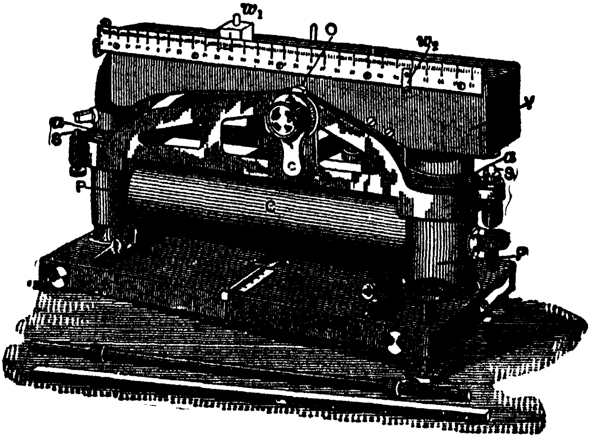


FIG. 163. Du Bois magnetic balance.

described in §188. The sample *T* under test is clamped between two heavy pole-pieces *PP* and magnetized by the coil *C*. Clamps are provided for accommodating either a round or square sample, or a bundle of laminations. In Fig. 163 two specimens are shown lying on the table. The magnetic circuit is closed through the iron yoke YY; the yoke is supported at O by knife-edges, like the beam of a balance. The shorter arm of the yoke is made of the same weight as the longer one, by weighting it with lead; the yoke is in balance when the coil C is not energized and the weights W_1 and W_2 are on zero.

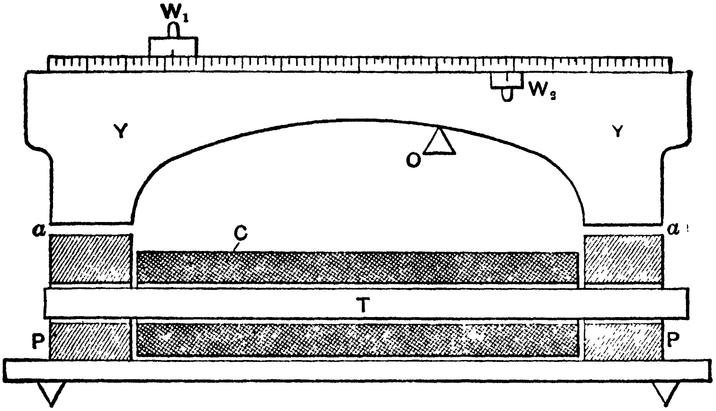


FIG. 164. Schematic representation of the Du Bois magnetic balance.

When the sample is magnetized, the yoke tips over to the left, because the attraction in the left-hand air-gap acts on a longer lever. To balance the yoke, the weights are moved to the right. When the balance is obtained, the flux density in the sample is read directly on the scale, in maxwells per square centimeter. The movement of the yoke is limited by adjustable stops, SS (Fig. 163). To simplify the balancing, one of the stops and the yoke may be provided with platinum tips, connected to a local circuit consisting of a dry cell and a bell or a galvanometer so that a signal is given when the yoke touches the stop.

A certain flux in the sample and in the pole-pieces gives a definite force of attraction at the air-gaps aa. Therefore the scale of the instrument may be calibrated directly in fluxes. But as all samples are of the same cross-section, a definite density corresponds to each flux. Therefore the scale is calibrated directly in magnetic densities. The calibration is done by means of a specimen whose magnetization curve has been determined by the ballistic method described below. A good feature of the instrument is that the reluctance of the air-gaps is large as compared to that of the sample. Therefore small inaccuracies in the contacts between the sample and the pole-pieces are of no consequence.

191. EXPERIMENT 8-B. — Magnetization Curves of Iron with Du Bois Balance. — The experiment is performed in the same order as Experiment 8-A (§189). Before leaving the laboratory, weigh the sliders and measure the length and the cross-section of the air-gaps and their horizontal distance from the knife-edge supports. Make a copy of the scale; measure the cross-section of the samples; note the number of turns in the magnetizing coil.

Report. (1) Plot curves of flux density to ampere-turns per centimeter (or inch) as abscissas (Fig. 161). If possible, use the same sheet of cross-section paper for all the samples, so as to have a direct comparison for the quality of the materials. (2) Show on a numerical example how to use these curves for determining the number of ampere-turns required to produce a given flux in a magnetic circuit. Take a circuit like the one shown in Fig. 147. The dimensions of the cores and the length of the air-gap are supposed to be given. (3) Compute the actual pull on the yoke of the balance (§188) and compare the calculated magnetic torque with that of the weights.

BALLISTIC METHOD

192. The Development of the Ballistic Method. — When a magnetization curve is taken, two quantities are measured, the exciting ampere-turns and the flux produced by them. The chief difficulty lies in measuring the flux. It can be measured more accurately by a discharge through a ballistic galvanometer than by the tractive method described in §§188 and 190. The use of the ballistic galvanometer for measuring fluxes is described in §§177 to 179. Although this is the most accurate method of measuring fluxes, it is only recently that devices have been perfected which make the method convenient for technical purposes. It may not be amiss to give here a brief history of the development of this method.

The original arrangement (Fig. 165) required samples made in ring form, each provided separately with an exciting and a secondary winding. This, of course, was a serious objection from a practical point of view. Hopkinson introduced the "divided-bar" apparatus (Fig. 166) in which the samples *TT* are made in the form of rods; these are much easier to prepare than rings. Permanent coils are used; the magnetic circuit is closed through a heavy yoke *YY*. An objection to this method is that, because of the imperfect contact between the yoke and the sample, the exciting current is greater than the real quality of the sample requires.

Ewing remedied this defect by eliminating the reluctance of the contacts and the yoke (Fig. 167). Two measurements are taken with different useful lengths of the magnetic circuit, and from these data the disturbing reluctance of the joints is eliminated. An objection to this method is that it requires two identical samples, which are sometimes difficult to obtain. Picou, on the other hand, eliminated the influence of the imperfect contacts (Fig. 168) by introducing additional windings which compensate for the reluctance of the air-gaps. This compensation was further developed in the early or duplex type Fahy permeameter (Fig. 174), which permits of a rapid and convenient testing of a large number of similar samples with a considerable degree of accuracy. This instrument has been largely replaced by the Fahy simplex permeameter of Figs. 175 and 176.

The Burrows permeameter (Fig. 172) offers a still greater refinement in the compensation for imperfect magnetic joints and is used in standardization work or where a great accuracy is required, values correct within 1 per cent being obtainable. The J-permeameter was developed by the Western Electric Company particularly for the testing of cobalt steel used for permanent magnets and requiring a higher magnetizing force than is possible with the Burrows and similar permeameters. The Gokhale permeameter of the General Electric Company is designed to give the saturation values of the magnetic samples tested. Drysdale's device (Fig. 170) has been developed to meet a need for approximate magnetic testing of large castings or forgings, without the preparation of special test samples.

Another objection to the ballistic method has been found in the ballistic galvanometer itself; in its original form, with a telescope and with undamped oscillations, it was not a convenient device for practical work. Ballistic galvanometers of sufficient sensitiveness have been more recently developed, provided with a pointer and a scale like ordinary voltmeters and ammeters. They are perfectly damped electromagnetically, so that readings may be taken in quick succession. The fluxmeter (§180), where it can be used, makes it possible to obtain continuous indications of fluxes instead of instantaneous deflections of a ballistic galvanometer.

The above-mentioned devices, based on the ballistic method, will now be described more in detail.

193. Ring Method. — A sample of iron or steel to be tested is turned in the form of a ring *I* (Fig. 165) of a comparatively small radial width.

If sheet steel is to be tested, separate rings are punched and assembled. The ring is uniformly wound with a known number of turns of wire, connected to the terminals *P* (primary). A secondary or exploring coil *E* is wound on the first coil and connected to a ballistic galvanometer.

The core is excited from a source of d-c supply, through a regulating rheostat R and a double-throw switch D. An ammeter is connected in the primary circuit. This is essentially the same arrangement as that shown in Fig. 147 and described in §162, except that the magnetic circuit consists of iron only, and no inaccuracy is introduced because of an air-gap. The circuit may well be made identical with that of Fig. 158, where means are provided for the calibration of the galvanometer during the progress of the test.

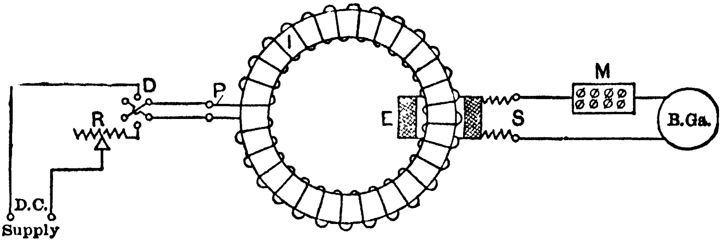


FIG. 165. Testing permeability of a sample in ring form.

The first step is to demagnetize the sample thoroughly by exciting it to as high saturation as possible, and then gradually reducing the excitation to zero, at the same time continually reversing the current. After this, the sample is magnetized again in steps, and the corresponding deflections of the galvanometer are noted. This gives the virgin curve (Fig. 161); a complete hysteresis loop may also be taken in a similar way, if desired. The curves shown in Fig. 161 may then be plotted if the dimensions of the core, the number of turns on both windings, and the constant of the ballistic galvanometer are known. For the exact procedure in testing see §187.

In some cases it is possible to have specimens in the form of very long rods. Experience shows that a rod whose length is at least 500 times its diameter may be considered magnetically as infinitely long. This means that the same number of ampere-turns *per unit length of its middle part* are required in order to produce a certain flux, whether the rod is straight or made into a closed ring without air-gap. This is not true for a shorter rod because of the leakage and of the demagnetizing action of the ends. With a very long rod the reluctance of the path of the lines of force back through the air is practically zero, because of the very large cross-section of this path. For this reason the required number of ampere-turns for the middle part is nearly the same as it would be if the rod were bent into a ring.

194. Divided-Bar Method. — An objection to the practical use of the ring method is that the preparation of samples is both expensive and slow. Hopkinson found a way out of this difficulty by making samples in the form of two straight bars, *TT* (Fig. 166). The magnetic circuit is closed through a heavy iron yoke *YY* of negligible reluctance, so that the reluctance of the circuit is practically equal to that of the samples and of the joints. The left-hand bar is securely clamped to the yoke by the screw *n*. The right-hand bar is provided with a handle *H*, by means of which it may be pulled out of the yoke. The exciting or primary coil *C* is wound in two sections and is connected to the terminals *P*. The exploring coil *E* is inserted between the two sections and is connected to the terminals *S*. The electrical connections are the same as in Fig. 165 or Fig. 158.

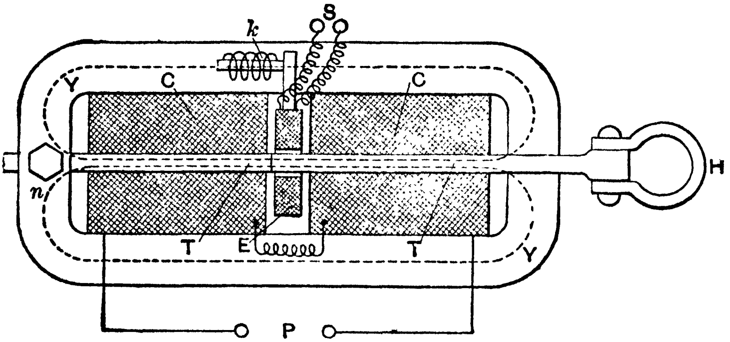


FIG. 166. Hopkinson divided-bar method of testing iron.

To measure the flux produced in the samples by the coils *CC*, the right-hand sample is suddenly withdrawn from the yoke. This releases the coil *E*, and the spring *k* instantly snaps it out of the magnetic circuit. The change in flux embraced by this coil produces a discharge in the ballistic galvanometer, as in the ring method. In calculating the value of *H*, the length of the magnetic circuit is taken as that between the inside faces of the yoke, because of the high permeability of the rest of the circuit. There is also a slight leakage error due to the flux through the coil that does not pass through the bar, but this may be predetermined by a test with a non-magnetic bar.

Instead of using two samples and pulling out one of them, one long sample may be used, and the ballistic discharge obtained by changing or reversing the current in *CC*, as in the ring method. The apparatus is fairly accurate for some practical purposes, but great care must be exercised to have good magnetic contacts between the yoke and the sample.

195. Ewing Double-Bar Method. — To eliminate the influence of the contacts and the reluctance of the yoke, Ewing proposed the apparatus shown in Fig. 167. The magnetic circuit is formed by two identical test bars *AA′*, *BB′*, and two heavy yokes *YY′* made of wrought iron. Each bar is surrounded by a magnetizing coil, as in Fig. 183. A secondary coil, connected to a ballistic galvanometer, is wound on one of the exciting coils. The coils are mounted in the wooden case *C*; the terminals *PP′* are connected to a source of d-c supply, the terminals *SS′* to a ballistic galvanometer. The number of turns in the secondary coil may be varied at will by the switch shown on top of the box. At the same time a resistance is automatically inserted into the galvanometer circuit, equal to that of the sections of the coil that are cut out. In this way the total resistance of the galvanometer circuit is kept constant, and its deflections are proportional to the number of secondary turns; see eq. (20) in §178. The reason for varying the number of turns is to have the galvanometer deflections within the best range of the instrument with widely different values of flux.

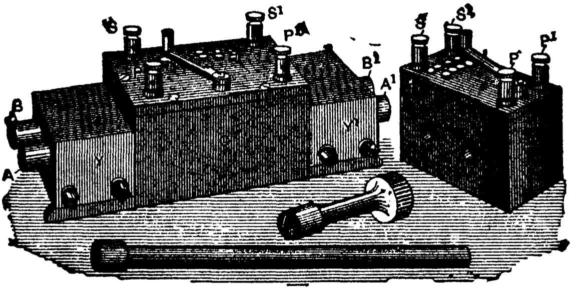
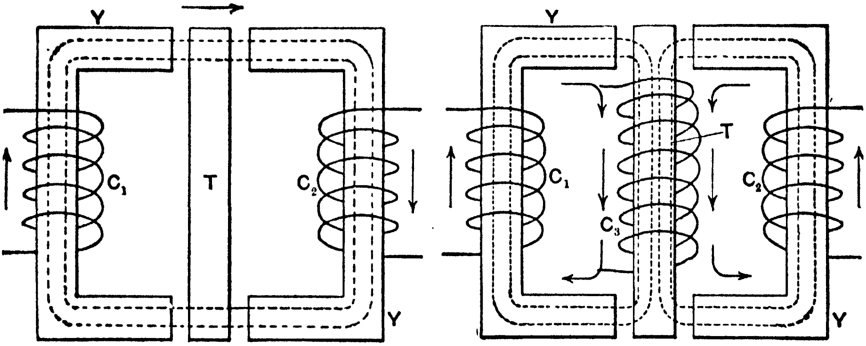


FIG. 167: Ewing double-bar method of testing iron.

The test is performed similarly to that with a ring sample. After a magnetization curve has been obtained, one of the yokes and the box *C* are removed by loosening the screws. A shorter box *D* is slipped in place, and the magnetic circuit completed again, the yokes being brought closer together. A magnetization curve is taken as before. It differs from that previously obtained because the circuit is shorter and the number of magnetizing turns in the box *D* is smaller. The reluctance of the yokes and that of the contacts is practically the same in both cases, and may be eliminated by comparing the two curves. For example, let 850 ampere-turns be necessary in order to produce a certain flux with the box *C*, 10 in. long. Let 450 ampere-turns give the same flux with the box *D*, 5 in. long. This shows that 400 ampere-turns were used in the first case for $5 \times 2 = 10$ in. of the length of the samples, so that the true ampere-turns per inch of the sample are $400/10 = 40$. If the shorter box were not used, the reluctance of the contacts and the yokes would have to be neglected; the number of ampere-turns per inch of the sample would have to be taken as 850/20 = 42.5, which is about 6 per cent too high. The readings with the shorter box enable one to separate the 50 ampere-turns lost in overcoming the reluctance of the contacts and the yokes.

The details of the apparatus shown in Fig. 167 are due to Mr. J. W. Esterline. In particular he deserves credit for the boxes, for the method of clamping, and for the scheme of using variable secondary turns with constant resistance.

196. Picou Permeameter. — In this apparatus the disturbing influence of the magnetic contacts is compensated by additional magnetizing windings. The principle of compensation is shown in Figs. 168 and 169. The sample T under test is of rectangular cross-section, and may consist of a bunch of steel laminations. It is clamped within the magnetizing coil C_3 between two U-shaped yokes YY, provided with compensating windings C_1, C_2. The winding C_3 supplies just enough ampere-turns to maintain the flux in the test sample. The coils C_1 and C_2 provide the magnetomotive force necessary to overcome the reluctance of the yokes and the air-gaps. A secondary coil is wound on each of the three magnetizing coils, and may be connected at will to a ballistic galvanometer.



FIGS. 168 AND 169. Principle of compensation for the reluctance of air-gaps in the Picou permeameter.

The first step in testing a sample is to compensate for the reluctance of the yokes and the air-gaps. For this purpose the circuit of the main coil C_3 is left open; the coils C_1 and C_2 are energized so as to produce a flux through the yokes only, as shown by the dotted lines in Fig. 168. Each coil supplies the ampere-turns necessary for overcoming the reluctance of one yoke and of two air-gaps. The desired value of the flux is established by trial discharges through a ballistic galvanometer. The current in one of the coils is then reversed, so that the flux is made to pass through the sample under test, as shown by the dotted lines in Fig. 169. Each of the coils, C_1 and C_2, has now to overcome the reluctance of its yoke, of two air-gaps, and in addition the reluctance of one longitudinal half of the sample under test. The flux is naturally reduced; to bring it to its former value, the middle coil C_3 is excited so as to assist the two other coils.

When the former value of the flux has been reëstablished in the yokes (as recognized by ballistic deflections), it is evident that the two outside coils again supply the ampere-turns necessary for the yokes and air-gaps, while the middle coil carries the flux through the sample under test. Knowing the number of ampere-turns per unit length of the coil C_3 and taking a ballistic discharge through the secondary coil which surrounds it, a point on the magnetization curve is obtained. The same test is repeated with other values of flux to obtain a complete set of data for a *B-H* curve.

197. Drysdale Permeameter. — This instrument, shown in Figs. 170 and 171, has been used for testing frame castings of electric generators and motors, without preparing special samples. The magnetizing and the exploring coils (Fig. 170) are wound on a plug (Fig. 171); the plug is inserted into a hole drilled at any desired place in the casting to be

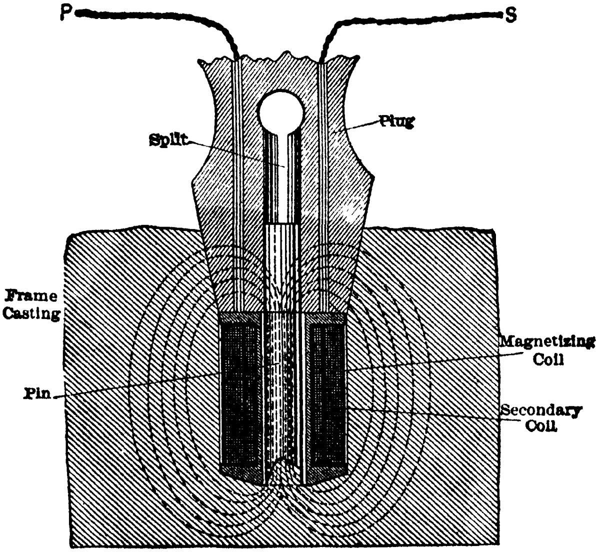


FIG. 170. Cross-section of the Drysdale permeameter, showing the test coils and plug in the casting under test.

tested. The magnetic circuit is excited as shown by dotted lines, and a discharge taken through a ballistic galvanometer or a fluxmeter. The hole drilled into the casting has the shape shown in Fig. 170. The pin or core left in the center constitutes the sample under test. A special hollow drill is used in preparing the sample so that it will fit the testing plug. A casting should be tested in several places, in order that its average permeability be determined. The instrument is calibrated by testing with it a sample of known magnetic properties.

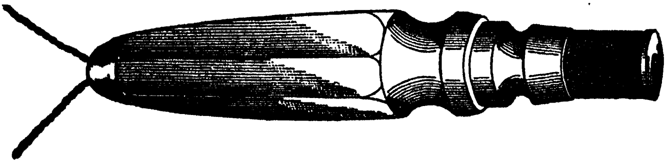


FIG. 171. Plug and exploring coil of the Drysdale permeameter.

198. Burrows Permeameter. — This permeameter, complete with control box and rheostats, but without galvanometer and scale, is shown in Fig. 172. The magnetic circuit and windings of the Burrows permeameter are shown in Fig. 173. This permeameter makes use of the compensated double-yoke method and is a modification of the ring method (Fig. 165). It is considered by the Bureau of Standards[1] and by the American Society for Testing Materials[2] as the standard device for

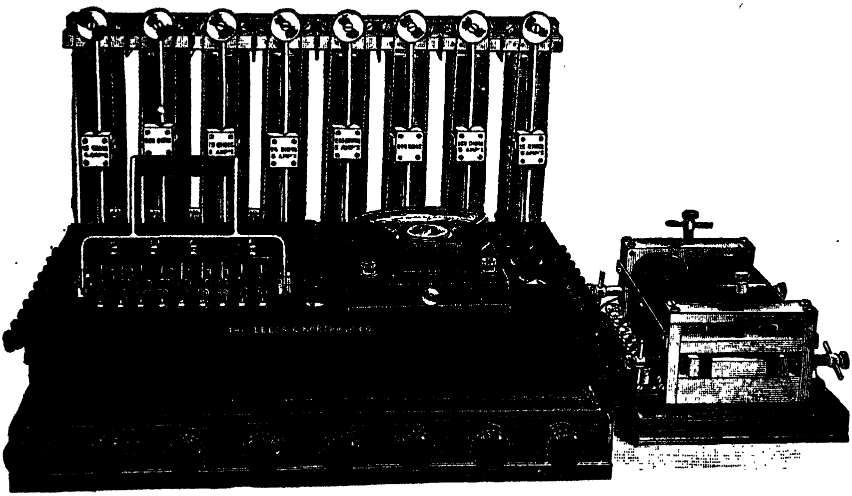


FIG. 172. Burrows permeameter and control equipment.

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[1] See Scientific Paper 117 and Circular 17, Bureau of Standards.

[2] See A.S.T.M. Standard Test Specifications Serial A 34–18 and A 34–24.

obtaining normal induction (§187), residual induction, and coercive force data on straight samples of maximum permeability not exceeding 10,000. Measurements are reported to be accurate within 1 per cent for inductions between 1000 and 20,000 gausses, on samples of maximum permeability of 5000, provided they are uniform in magnetic characteristics throughout the length of the sample. Normally a maximum magnetizing force of 300 gilberts per centimeter is obtainable. The magnetic circuit consists of two soft-iron yokes, *B* and *C*, joined by the bar under test and by an auxiliary bar which should have nearly the same properties as the test bar. The two bars should be approximately of the same size, although small differences in area may be compensated for. The test pieces should be at least 10 in. long and of such dimensions that they will pass through openings $\frac{3}{4}$ by $1\frac{3}{4}$ in. Round rods may be used, but they require a special clamping device.

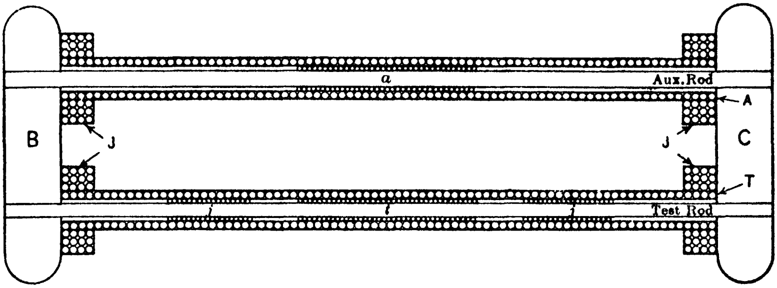

FIG. 173. Arrangement of coils and magnetic circuit in the Burrows permeameter.

The principle of compensation employed may be understood from the following: The magnetic leakage over the central portion of the test bar is reduced to zero by a proper distribution of the magnetizing force. As a result, the true magnetizing force *H* may be accurately computed from the exciting current, the concentration of the magnetizing winding over the center of the bar being known. The magnetizing winding over the test bar is denoted by *T*; that of the auxiliary bar by *A*. The four coils marked *j* are for the purpose of providing an adjustable magnetizing force for the joints and the yokes, so as to reduce the leakage to zero. The secondary or test coils, which may be connected at will to the ballistic galvanometer, are denoted by the corresponding small letters, *t*, *a*, and *j*. In the form recommended by the A.S.T.M. there are two additional test coils, similar to *j*, *j*, around the auxiliary bar.

The adjustment and compensation described below consist in distributing the mmf's *A*, *T*, and *J* in such proportion that (a) the fluxes in the middle parts of the two bars are equal, and (b) the flux over a considerable portion of the test bars remains constant. The purpose of this adjustment is to cut down as much as possible "free mmf's," that is, mmf's produced at one point of the magnetic circuit and used up at some other point. Such free mmf's, or differences of magnetic potential, cause leakage fluxes through the surrounding air and therefore impair the accuracy of measurement. The mmf must be consumed as much as possible in the reluctance of the iron path bit by bit, where produced, leaving no free ampere-turns. If the permeability of the two bars and the contact reluctances were exactly the same, the mmf of each exciting winding (*A* or *T*) with the same number of ampere-turns would be used up completely upon the corresponding half of the magnetic circuit. Because of unavoidable differences in the material and clamping, such is not the case, and one of the mmf's may partly exert its influence upon the other half of the circuit, thus shunting part of the flux through the air. Therefore, after the current in the main exciting winding **A** is set at normal, that in the winding surrounding the auxiliary bar is changed until the flux in the middle portions of the two bars is the same. This is determined by connecting the two secondary coils, *a* and *t*, in series opposition and discharging the secondary current through a ballistic galvanometer when the primary currents are reversed. With the proper adjustment of the two primary currents the galvanometer deflection should be zero.

Because of the reluctance of the joints and the yokes, a free mmf, and consequently a leakage flux, would exist along the test bar if it were not for the compensating coils *JJ*. The current in these coils must be so adjusted that the main winding **T** furnishes the mmf only for the bar itself, while the ampere-turns in *JJ* are just sufficient for maintaining the flux through the joints and the yokes. In this case the mmf in **T** is consumed bit by bit where produced and there is no leakage flux, so that the flux along the bar remains constant, as in a ring sample. The secondary coils *jj* have only 50 turns each, while *t* has 100 turns. The two *j* coils are therefore connected in series with the ballistic galvanometer and in opposition with *t*. The current in *JJ* is then adjusted until there is no galvanometer deflection when the primary currents are reversed. In the A.S.T.M. form the four secondary coils *j* of 50 turns each are connected in series to oppose the emf induced in the two test coils *a* and *t*, of 100 turns each, also connected in series. A zero deflection of the galvanometer connected to these coils indicates that there is no leakage flux and that the reluctance of the joints has been compensated for. Therefore an approximate uniformity of flux is secured through the greater portion of test material.[1]

The induction or *B-H* curve is taken by a step-by-step process begin-

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[1] See Leeds & Northrup Bulletin 533, p. 4.

ning at the point of lowest magnetization. The sample is previously demagnetized, and at each step the magnetic circuit is compensated as described above. The flux in the sample is measured by reversing all the magnetizing currents simultaneously and noting the deflection on a calibrated ballistic galvanometer. The values of H are determined from the number of turns and the exciting current in *T*. The various auxiliary switches and rheostats shown in Fig. 172 have been developed in order to simplify the manipulation of the instrument. However, considerable practice is required before the operator is sufficiently expert to obtain results with speed and accuracy. Detailed instructions accompany the permeameter and should be followed closely in its operation. Publications of the Bureau of Standards are available covering the use and the theory of the instrument.

199. Fahy Duplex Permeameter.[1] — This device is also of the ballistic type. An H-shaped magnet core is employed, a standard specimen **A** (Fig. 174) bridging one end of the H and the test specimen **X** bridging the other. The crossbar **J** of the H-shaped core has an adjustable airgap for the purpose of reducing flux in the specimens due to the residual flux of the main core. The sample *X* and auxiliary bar *A* are magnetized in part by the winding **M**, placed on core **J**, and in part by four coils **C** so arranged that they augment **M** in magnetizing one bar and oppose **M** with regard to the other. Test coils **D**, **D′**, **S**, and **T** are each of the same number of turns and are so connected that upon reversal of current

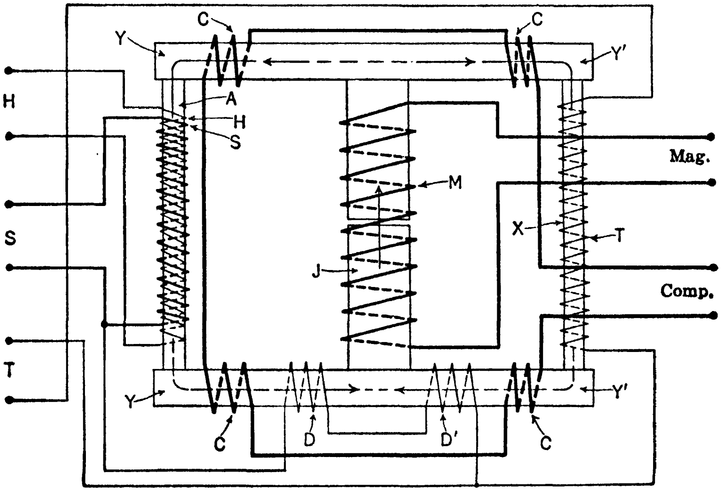

Fig. 174. The Fahy duplex permeameter.

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[1] See Bureau of Standards Scientific Paper 306.

in the magnetizing coil *M* the sum of the integrated emf's in coils S and D′ opposes the sum of the emf's in coils T and D. Between the adjacent binding posts of S and T will appear the difference of the emf's of coils D and D′. This is adjusted to zero value with the specimens removed, by varying the current in coil M and coils C. When this difference is zero the fluxes through the two yokes are equal. Consequently, when the combined emf in the series circuit of coils S, D, D′, and T is also zero it follows that the leakage fluxes by the two paths are equal (since any difference between fluxes through coils D and S is leakage flux). Coil H consists of a large number of turns of fine wire wound on the same form as coil S. The indication of the ballistic galvanometer when connected to coil H, upon reversal of currents in coil M and coils C, will, if the specimens have been withdrawn, give the intensity *H* of the magnetizing field between yokes *YY*

The flux through bar A is measured by reversing the current in M and measuring the ballistic discharge through coil S. Similarly the flux in X is determined by the discharge of coil T. The *B*-*H* curve of bar A is supposed to be already known, since it is a standard test bar, so that the magnetization curve of the test bar is obtained by comparison. The compensation for equal leakage must be made for each point on the saturation curve.

The Fahy duplex permeameter may be used to obtain absolute measurements of flux density without the use of a standard bar, which is then removed from the permeameter, but for this purpose the Fahy Simplex Permeameter is recommended.

200. Fahy Simplex Permeameter. — Figure 175 shows this much-simplified permeameter, and Fig. 176 is an assembly of the permeameter with its control equipment. The permeameter consists of a U-shaped laminated core on which is an exciting winding. Between the poles of the core are two test coils. One coil is wound on a split brass tube within which the test bar is placed and then clamped tightly against the poles of the electromagnet. This coil is brought out to binding posts marked B and measures the flux in the sample. The other test coil, brought to terminals marked H, is wound on a non-magnetic core and covers uniformly the space between the pole-pieces of the clamping device. This coil measures the mean flux density in the air due to the mmf between the ends of the test piece, and, consequently, the mean value of magnetizing force *H* acting on the specimen. Both coils B and H give readings on the ballistic galvanometer when the current in the magnetizing coil M is reversed. An empirical constant, supplied by the manufacturer, is needed to correct the readings of H for the slight difference in magnetic potential between the ends of the coil H and the terminals of the test piece. In a publication[1] of the Bureau of Standards it is stated that this apparatus gives consistent and reproducible results within allowable experimental error. It checks the values obtained with the Burrows permeameter within about 2 per cent and may be used with laminated specimens having permeabilities as high as 6000. On samples which are not uniform in magnetic properties throughout the length of the bar

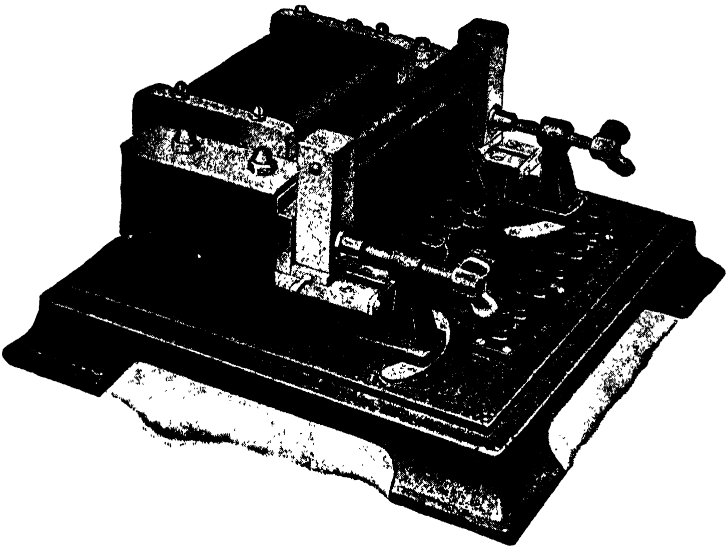

Fig. 175. The Fahy simplex permeameter.

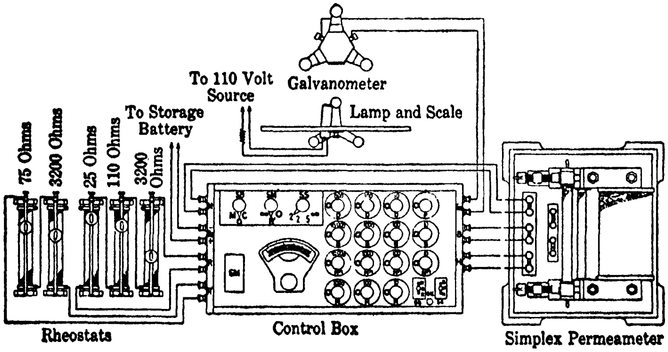

Fig. 176. The Fahy simplex permeameter and complete control equipment for magnetic testing.

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[1] "Performance of the Fahy Simplex Permeameter," by Raymond L. Sanford, Bureau of Standards Research Paper 174.

the results by this permeameter are superior to those obtained with the Burrows permeameter. Care must be taken that the test pieces make good contact with the iron blocks at the end of the **H** coil, otherwise the measured values of *H* are too low. In general, this permeameter can be operated with fair results by a relatively inexperienced tester and with little loss of time.

201. The J-Permeameter.[1] — There is a lack of uniformity in the magnetizing force *H* along a specimen when it is made part of the return path for the flux, for either an H-shaped or a U-shaped electromagnet (as in the Fahy duplex and Fahy simplex permeameters). Also, there is a lack of uniformity of magnetizing force within a solenoid, diminishing toward the ends. The J-permeameter is an attempt, by combining these two defects in such a way that they tend to compensate, to establish a uniform magnetic field over a given length of test bar and accurately measure both the field strength and the resulting induction, over a wide range in values of magnetizing force (20 to 1000 gilberts per centimeter).

Since at high values of magnetizing force there may be considerable flux outside the sample but within the test coil surrounding it, the indications of the galvanometer connected to the test coil and intended to give the value *B* in the sample may be greatly in error. This error is prevented in the J-permeameter by a compensating coil **C** (Fig. 177) wound over the **H** coil which parallels the test specimen much as in the Fahy simplex permeameter. Coil **C** is connected in opposition to the **B** coil which has the same area and number of turns and surrounds the sample. The combined effect of these two coils is to give a galvanometer reading proportional, not to the total induction *B* in the iron under test, but to *B-H* (ferric induction). To obtain the total induction *B*, the value of *H* given by the **H** coil is added. This permeameter is especially valuable for the

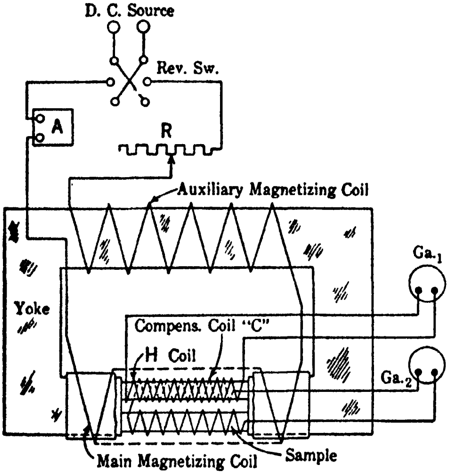

FIG. 177. The magnetic and electric circuits of the J-permeameter.

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[1] See "An improved permeameter for testing magnet steel," by B. J. Babbitt, *Jour. Optical Soc. Amer. and Rev. Sci. Instruments*, July, 1928.

testing of cobalt magnet steel and similar materials used for permanent magnets.

202. The Gokhale Saturation Permeameter.[1] — The saturation permeameter, Fig. 178, has a heavy U-shaped yoke between the poles of which the specimen is clamped. An exciting winding consisting of a large number of turns of rather heavy wire surrounds the sample. There is no winding on the U-shaped core. This permeameter is intended primarily for obtaining the saturation value of the ferric induction (B-H) of specimens of iron and steel up to magnetizing forces of 4500 gilberts per cm (9100 ampere-turns per in.), and may be used for approximate determinations of B vs. H down to $H = 200$ gilberts per cm. Like the J-permeameter it has means for automatically compensating for the " spatial " flux within the potential coil but outside the sample.

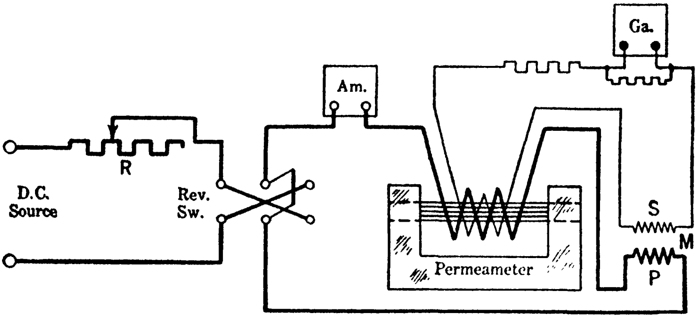

FIG. 178. Circuit diagram of the saturation permeameter.

Compensation for spatial induction is accomplished by a variable mutual inductance M (Fig. 178) of which the primary coil **P** is in series with the exciting winding of the permeameter and the secondary coil **S** is connected in series opposition with the potential coil. To obtain the H vs. (B-H) curve of a specimen it is clamped in the permeameter after an approximate setting of M has been made. Readings of the ballistic galvanometer are now taken by bringing the exciting current to various desired values and reversing. When, at the higher values of H, the curve of (B-H) vs. H is found to be a straight line the setting of M is varied until the values of (B-H) become constant, whereupon it is known that the iron is fully saturated and that M compensates for the spatial induction.

In Fig. 179 (*a*) is the curve taken by this permeameter for ferric induction vs. H on a sample of 2.5 per cent silicon steel.[1] Curve (*b*), which gives B vs. H, is derived from curve (*a*) by adding H to the ferric induction.

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[1] See "Saturation permeameter," by S. L. Gokhale, *Jour. A.I.E.E.*, March, 1928.

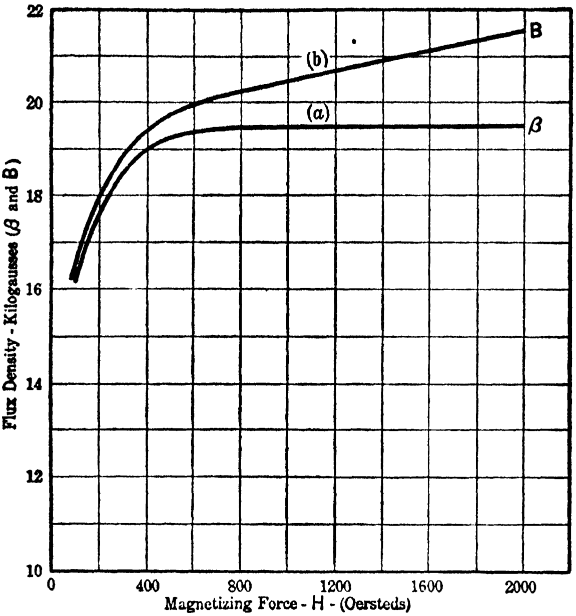


FIG. 179. Curves of ferric induction β and total induction B, as obtained with the saturation permeameter on $2\frac{1}{2}$ per cent silicon steel.

203. EXPERIMENT 8-C. — Magnetization Curves of Iron by the Ballistic Method. — The experiment comprises the use of any of the instruments described in §§192 to 202. The sample is thoroughly demagnetized (§187) and a virgin curve *OAM* (Fig. 161), taken step by step. The test should be performed on samples of silicon steel, mild steel, wrought iron, and cast iron. If the constant of the ballistic galvanometer is not known, the instrument must be calibrated, as explained in §179. Aside from learning the method itself, and the operation of a permeameter, it is expected that the student will make clear to himself: (a) the details of its construction, (b) the sources of error, and (c) the limits of accuracy.

Report. (1) Plot the magnetization curves of the samples, using ampere-turns per centimeter as abscissas and flux densities in kilolines per square centimeter as ordinates, both total induction, B and ferric induction, B-H. Also plot values of absolute and relative permeability (§183) to values of B as abscissas. (2) Illustrate on a numerical example the use of these curves for determining the number of ampere-turns in a given case — for instance, to produce a given flux in a magnetic circuit like the one shown in Fig. 147. The dimensions of the cores and the length of the air-gap are supposed to be given. (3) Discuss the advantages and disadvantages of the apparatus used, sources of error, limits of accuracy, etc.

204. EXPERIMENT 8-D. — Hysteresis Loop of Iron by the Ballistic Method. — The meaning of the hysteresis loop is explained in §184, and the steps in the experimental procedure in §187. Any of the devices described in §§192 to 202 may be used. See also the instructions for the preceding experiment. For each sample tested obtain data for at least two different hysteresis loops corresponding to different values of maximum H.

Report. (1) Plot the hysteresis loops of the samples tested, using ampere-turns per centimeter as abscissas and kilolines per square centimeter as ordinates. Mark in particular the values of coercive force and of residual flux density and compare the materials tested with respect to these points. (2) Compute the hysteresis loss per cycle, in joules, by integrating one of the curves, as explained in §232. (3) Discuss the advantages and disadvantages of the apparatus used, sources of error, limits of accuracy, etc.

STEADY DEFLECTION METHODS

205. Koepsel Permeameter. — In its principle the device is a movable-coil millivoltmeter (Figs. 180 and 181), in which the permanent magnet is replaced by an electromagnet; the sample P under test is a part of this electromagnet. The instrument consists of two heavy pole-pieces J, J (Fig. 180) in which is fastened the rod P to be tested; S is the magnetizing coil. The flux produced in the sample and in the

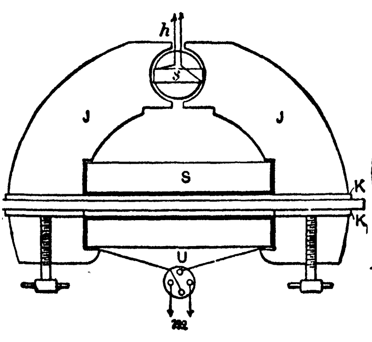


Fig. 180. A cross-section of the Koepsel permeameter.

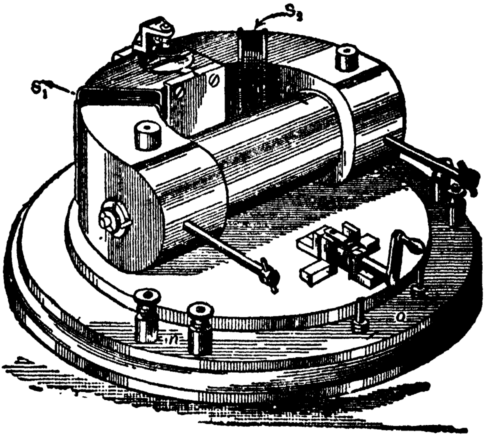


Fig. 181. The Koepsel permeameter, with the cover off.

pole-pieces is measured by means of a movable coil s, similar to those used in d-c ammeters and voltmeters (Fig. 38). The coil is supplied with current from a small auxiliary dry battery; deflections are indicated by a pointer on the dial (Fig. 182). The scale is calibrated directly in magnetic densities B per square centimeter.

U in Figs. 180 to 182 is a double-throw switch for reversing the current in the magnetizing coil S; W_m in Fig. 182 is a rheostat for regulating the current in the same coil. W_h is a rheostat for regulating the current in the moving coil; it has three handles, for coarse, medium, and fine adjustment. S_1 and S_2 in Fig. 181 are compensating coils connected in series with S and opposing it magnetically. These coils are adjusted so that the instrument shows zero when the sample is taken out.

The apparatus is intended for taking complete magnetization curves, similar to that shown in Fig. 161. The values of B are read off directly on the scale (Fig. 182) when the current i in the moving coil has been adjusted to a value given on the instrument. This current $i = k \div q$, where k is an instrument constant, and q is the cross-section (in square centimeters) of the sample under test. The smaller this cross-section the smaller the flux, and the larger must be the current in the moving coil in order to produce the same deflection. The exciting ampere-turns are obtained by multiplying the current I in the coil S by its number of turns per unit length.

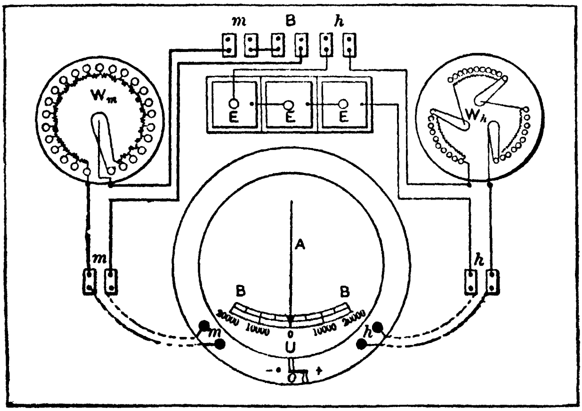


FIG. 182. The electrical connections in the Koepsel permeameter.

Care must be taken to protect the apparatus from external magnetic fields; strong magnets or large pieces of iron should not be allowed near it when readings are taken. The sample itself must have no ends protruding out of the yoke, as they produce a stray field. The apparatus is sensitive even to terrestrial magnetism; in order to eliminate this error the device must be placed so that the line marked on the dial is in the direction "north-south." When the instrument is in this position and both currents are "on," the pointer must remain on zero when there is no test sample in the apparatus.

In this permeameter, as well as in some others, there is a source of error which is difficult to eliminate, namely, imperfect contacts between the yoke and the rod under test. As a result, it takes more ampere-turns to produce a certain flux than it would without these unavoidable air-gaps, and the apparent permeability comes out lower than it is in reality. An air-gap only 1/1000 in. long is equivalent in its magnetic resistance to nearly ½ in. of a good iron rod; therefore, when accurate results are required, the experimentally observed *B-H* curve must be properly corrected. Such correction curves are usually supplied with the Koepsel apparatus.

206. EXPERIMENT 8-E. — Magnetization Curves of Iron with Koepsel Permeameter. — Test a steel specimen, a cast-iron specimen, and a bunch of iron laminations. Special clamps are provided with the instrument for accommodating round and rectangular samples. Set the instrument in the *N-S* direction; see that the needle shows no deflection, without the sample, and with both currents on. Insert a sample and demagnetize it (§187); use the commutator shown in front in Fig. 181 for reversing the current. Take the curves shown in Fig. 161. Correct the obtained data in accordance with the standardization certificate of the instrument. Investigate the sensitiveness of the apparatus to stray magnetic fields, to terrestrial magnetism, and to large iron masses in its proximity.

Report. Similar to that in §203.

207. Esterline Permeameter. — This device is essentially a small d-c generator; the sample bar to be tested completes the magnetic circuit of the machine. The working principle of the device may be understood with reference to Fig. 180, if the moving coil *s* be replaced by an ordinary d-c armature (§233) driven by a motor. The magnetizing coil *S*, the sample *P* and the pole-pieces *J*, *J* are essentially the same as in the Koepsel permeameter. The flux in the sample, instead of being measured by a deflection of the moving coil *s*, is measured by the voltage induced in the revolving armature. The magnetic density is computed from the formula

$$\text{Density } B = \text{constant} \times \text{volts/speed}$$

This follows from the fact that the voltage induced in the revolving armature is proportional to the flux and to the speed of rotation. Compensating coils are used to correct for the reluctance of the air-gap and of the parts of the circuit other than the bar under test.

The exciting ampere-turns are varied by changing the number of turns in the magnetizing coil, or by regulating the current in the coil by a suitable resistance, as in the Koepsel permeameter. The Esterline apparatus can be substantially built, is not affected by terrestrial field, and is suitable for ordinary commercial work. It is claimed that an inexperienced operator can obtain accurate results after very little practice. The device is described in detail in the *Proceedings of the American Society for Testing Materials*, VI, 1906.

208. Ewing Permeability Bridge. — An ingenious method of comparing specimens of steel and iron to a standard sample was devised by Professor Ewing (Fig. 183). *S* is a standard rod whose magnetization curve is known; *T* is the sample under test. Both are surrounded by magnetizing coils and are clamped in heavy iron yokes *PP*. The magnetic circuit is closed as shown by the arrows. The standard sample is excited with a certain number of ampere-turns; the number of ampere-turns on the sample under test is varied until the fluxes in both samples become equal. This condition obtains when the pivoted magnetic needle *m* returns to zero. When the fluxes in the two samples are different, some lines of force must find their path through the yokes *H H* and the air-gap between them, deflecting the needle. When the fluxes become equal, no stray flux passes through *H H*, and the needle returns to zero under the influence of the permanent magnet *g*.

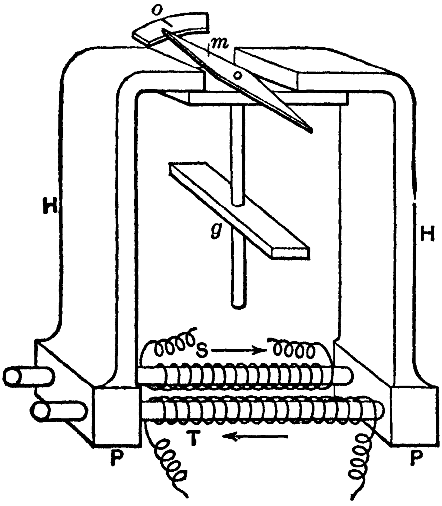


FIG. 183. The Ewing magnetic bridge.

In the apparatus, as it is actually made, the number of turns around the standard sample *S* is fixed; that around the specimen *T* may be varied by a radial arm and contacts, somewhat as in Fig. 167. A double row of contacts is provided, so that when a section of the coil is cut out of the circuit, an equivalent resistance is inserted in its place. This is done in order to maintain a constant current. The two magnetizing coils are connected in series, so that the ratio of active turns gives directly the ratio of ampere-turns. With this arrangement one ammeter is used instead of two.

The magnetization curve of the sample under test is plotted by changing the abscissas of the magnetization curve of the standard sample. For example, let the standard sample require 80 ampere-turns per in. at a density of 100 kilomaxwells per sq in. Send through the magnetizing coils a current, such as to produce 80 ampere-turns per in. in the standard sample; adjust the number of turns of the specimen under test, until the needle *m* returns to zero. Let the number of turns in the standard coil be 100, that in the other coil 150. This shows that in order to obtain a density of 100 kilomaxwells in the sample under test, $80 \times 150/100 = 120$ ampere-turns per in. are required. A calibration curve is usually furnished with the instrument to correct for unavoidable air-gaps.

The apparatus is called a " bridge " because of its resemblance to the Wheatstone bridge; the two horns *H H* and the magnetic needle correspond to the galvanometer circuit of the bridge (§17).

209. EXPERIMENT 8-F. — Magnetization Curves of Iron with Ewing Permeability Bridge. — The apparatus is described in §208. The conduct of the experiment and the requirements for the report are the same as in §203.

210. Bismuth Spiral. — The metal bismuth has the peculiar property of increasing its electrical resistance appreciably when placed in a magnetic field. Per cent increase in resistance is nearly proportional to the magnetic density of the field. The curve shown in Fig. 184 gives an idea of this increase for an average specimen of bismuth wire. This property of bismuth has been utilized to some extent for measuring magnetic densities. Fine bismuth wire is made into a flat spiral wound non-

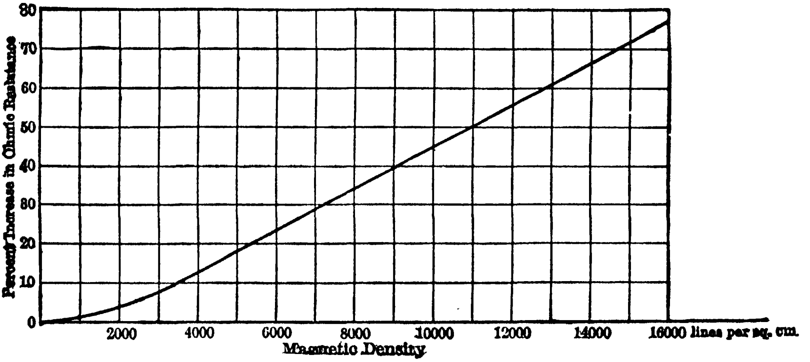


FIG. 184. The resistance of a bismuth spiral as a function of magnetic flux density.

inductively, the wire being doubled back on itself. The winding is held rigidly between two thin pieces of mica. Such a spiral is placed in a magnetic field the flux density of which it is desired to measure. The ends of the spiral are connected to a Wheatstone bridge (§17). From the increase in the electrical resistance of the bismuth wire, the flux density is determined by using the curve shown in Fig. 184. Bismuth spirals used for measuring flux density in air-gaps of electrical machines, stray fluxes, etc., are provided with handles and with suitable terminals. For testing permeability of iron with a bismuth spiral, an apparatus similar to that shown in Fig. 166 has been used. The exploring coil *E* is omitted, and the spiral is inserted between the ends of the two samples *T*, *T* although the presence of the rather large air-gap makes the calculation of the *B*-*H* curve for the iron difficult. Bismuth has an appreciable resistance-temperature coefficient, so that an observed change in resistance may in part be due to this cause. A carefully computed correction for temperature must be applied when using this method.

TESTING OF PERMANENT MAGNETS

211. Permanent magnets are used in a number of electrical devices, such as magneto generators, d'Arsonval-type measuring instruments (§44), watthour meters (Fig. 107), and telephone receivers. The magnetic flux of a permanent magnet is due to its residual magnetism (§184), and before using a magnet in a piece of apparatus it is often necessary to measure its residual flux, or at least to see that it is not below a certain required value. The magnitude of the residual flux *OR* (Fig. 161) in itself is not always a sufficient indication of its " permanency." It is also necessary to determine the width of the hysteresis loop, in other words the value *OK* of the coercive force. The greater the coercive force, the more constant will the strength of the magnet remain under various adverse conditions of service, such as vibration, temperature changes, external magnetic fields, etc.

In the regular manufacture of permanent magnets, sample bars are taken from each lot of magnet steel, as received from the steel manufacturer, and several bars are heat-treated or " quenched " at each of four different temperatures near 800° C. After the bars have been carefully heat-treated, hysteresis curves are taken on each sample to determine that the coercive and residual values for the steel are above permissible minimum values. After being magnetized and heat-treated, all magnets are carefully aged, being demagnetized by a definite percentage in the course of the aging process. After aging, the magnets for indicating instruments are not tested, but are brought through the processes of manufacture in such a way as to require a small amount of demagnetizing to give the proper strength after complete assembly in the meter.

When it is necessary to protect a permanent magnet against losing a considerable portion of its original magnetism, the following precaution may be used. After the magnet has been properly energized, it is lifted through the exciting coils, a U-shaped keeper coming up with it. A flexible keeper is then laid over the exterior of the magnet, remaining there until the magnet has been placed on the magneto frame, or in whatever piece of apparatus it is designed for.

The method of testing depends on the purpose for which a magnet is to be used. Narrow-gap magnets used in watthour meters are simply " weighted " as described in §212. Wide-gap magnets used in ammeters and voltmeters are usually tested in a d'Arsonval-type device described in §213. Magnets for magneto generators may be tested in the same kind of device, or in a hand-operated magneto described in §214.

212. Testing of Narrow-Gap Magnets. — A magnet intended to be used in a watthour meter is usually tested with an aluminum disk. The latter is mounted in a vertical plane on a horizontal shaft delicately pivoted and balanced. The magnet is mounted in the same position with respect to the disk as it would occupy in a meter, so that its lines of force thread through the disk. The disk receives a sudden impulse from a weight arm moving through a definite angle. The disk then begins to move, and is finally stopped by the eddy currents induced in it by the magnet under test. The angle of deflection of the disk is a measure of the strength of the magnet. The stronger the magnet the smaller the angle of deflection.

The test is a purely relative one, giving an indication of the strength of the magnet under test as compared to a standard magnet the residual flux of which has been determined ballistically (§177), by withdrawing a test coil from one of its legs. A watthour meter usually has two permanent magnets. After having been " weighted " as described above, these magnets are paired to give the various strengths required by meters of different torques. The strength of a pair of magnets after assembling is determined by placing them on a special test meter having a definite torque, and noting the angle of rotation of the test meter when a like test meter with standard magnets has completed one revolution.

213. Testing of U-shaped Permanent Magnets. — (a) *With a d'Arsonval meter.* The principle is shown in Fig. 185. *A* is a permanent-magnet movable-coil type instrument (§44) from which the permanent magnet has been removed and replaced by a pair of soft-iron poles which project through the case of the instrument. A small current of known and fixed value from a cell *C* is put through the movable coil of the instrument.

B is a d-c milliammeter on which the value of this current may be read and kept constant by adjusting the rheostat R_1. If desired, the winding of the movable coil and the current through it may be so adjusted that, when a permanent magnet under test is placed across the soft-iron pole-pieces, the scale of the instrument A will indicate the flux directly in maxwells, or the density in gausses. The principle is similar to that used in the Koepsel permeameter (§205).

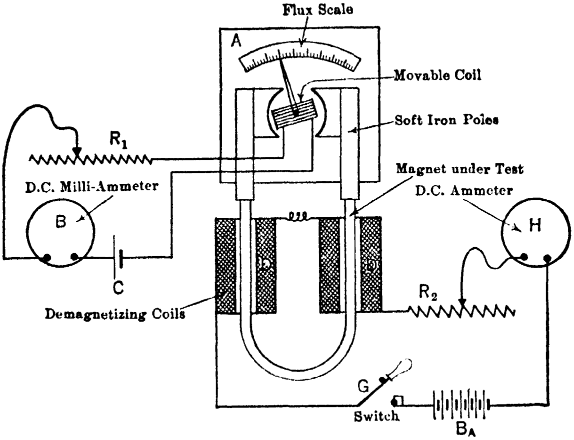


FIG. 185. A movable-coil device for testing permanent magnets.

Surrounding the magnet are two coils DD, wound and connected in such a way as to weaken the magnet. H is a d-c ammeter which measures the demagnetizing current through these coils. G is a switch and R_2 is an adjustable rheostat for regulating the demagnetizing current. The scale of the instrument H can be so drawn as to read directly in demagnetizing ampere-turns, or in ampere-turns per unit length of the magnet. B_A is a source of d-c current.

The procedure in using this instrument is as follows: After the current through the instrument B has been adjusted to the desired constant value for which it is calibrated, the magnet under test is placed upon the pole-pieces, the switch G being open. Because of the reaction between the magnetic flux of the magnet and the current in the movable coil, the latter is deflected and the instrument reads the residual magnetism of the magnet.

When this has been done, and with the resistance R_2 at a maximum, the switch G is closed. The resistance R_2 is then gradually reduced until the instrument A reads zero, showing that the coercive force of the magnet is exactly balanced by the demagnetizing effect of the coils

DD. While in this condition, the indication of the instrument *H* gives the current necessary to demagnetize the magnet, or directly the coercive force of the magnet.

(b) *With a magnet-testing generator.* A modification of the preceding method, sometimes used for testing large permanent magnets for magnetos, consists in replacing the movable coil (Fig. 185) by an ordinary d-c armature with a commutator and brushes (§233). The principle is similar to that used in the Esterline permeameter (§207). The armature is connected to a low-reading voltmeter and is driven by hand through a suitable gearing. The driving mechanism includes a centrifugal clutch which allows the armature to slip at or above a critical speed. Thus, as long as the handle is rotated above the operating speed of the clutch, the armature is driven at a fixed speed. Under these conditions the indication of the voltmeter is proportional to the flux of the magnet under test, and the scale may be calibrated directly in flux units.

The generator used for this test is similar to the familiar " megger " employed in insulation measurements (Vol. II). An electromagnetic tachometer driven at a constant speed can also be used. It is only necessary to replace the regular permanent magnet of the instrument by the magnet under test, and to provide suitable pole-shoes made of soft iron.

If the number of armature turns and the induced emf are known, the useful flux through the armature, at a given speed, may be computed from the usual formula for a d-c machine, eq. (1) in §234. Or else, in addition to the usual winding on the armature, a coil of a known number of turns is so wound that it can be made to embrace the whole magnetic flux through the armature. To carry out the calibration by means of this additional coil, a permanent magnet is slipped upon the pole-pieces to provide the necessary magnetic flux, and the search-coil terminals are connected to either a ballistic galvanometer (§177) or a fluxmeter (§180) by flexible wires. The armature is now turned through 180 degrees, starting and finishing through the search-coil. By comparing the throw on the ballistic galvanometer with that produced by reversing the current in a standard solenoid provided with a secondary winding (§179), the total flux through the search-coil, and therefore through the armature, may be readily calculated. It must be remembered, however, that with either method it is the useful flux of the magnet that is determined, and not the total flux. The coercive force is measured in the same manner as in the arrangement shown in Fig. 185.

214. EXPERIMENT 8-G. — Testing of Permanent Magnets. — The principal methods in use are described in §§211 to 213. The student should make clear to himself the following points: (a) the heat-treatment and magnetization of steel bars; (b) the effect of vibration, high and low temperatures, external fields, etc., upon the residual flux; (c) different methods of testing, their advantages, disadvantages, and accuracy.

CHAPTER IX

MEASUREMENT OF CORE LOSS

215. A laminated steel core, used in the armature of an electrical machine or in a transformer, is subjected to an alternating magnetization. Each particle of the core is regularly magnetized in one and then in the opposite direction many times a second, according to the frequency of the supply or the speed of the machine. Under such conditions the core may become appreciably heated. This heating is objectionable for two reasons:

(1) The temperature may reach a limit dangerous to the insulation of the windings.

(2) The efficiency of the machine is lowered, because of the energy thus converted into heat.

This heating of a steel core, or the corresponding loss of energy, is due to two distinct and independent causes; viz., (1) molecular friction, or hysteresis; (2) induced eddy currents.

216. Physical Nature of Hysteresis. — The phenomenon of hysteresis is described in §184. A definite amount of energy is necessary to subject a unit mass of iron to one complete cycle of magnetization (Fig. 161). This energy depends upon the limits of magnetic flux density at points M and M_1, and upon the chemical and physical properties of the sample, its heat treatment, etc. It is proved in §232 that this amount of energy is proportional to the area of the hysteresis loop. Therefore, the hysteresis loss per cycle increases with the flux density and is greater for kinds of steel which possess greater residual magnetism and higher coercive force. Mr. T. Spooner (*Elec. Jour.*, March, 1925) states that hysteresis loss may be expressed in terms of coercive force, H_c, and maximum density, B_m, with an accuracy of about 15 per cent, by the formula

$$W_h = B_m H_c/\pi \text{ ergs per cc per cycle} \quad \ldots \ldots \quad (1)$$

where B_m is in gausses and H_c in gilberts per centimeter.

Anderson and Lance (*Engineer*, Sept. 22, 1922) state that, using the same units as in eq. (1),

$$W_h = aB_m H_c \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2)$$

where

$$a = 0.2133 + 0.0108 \times 10^{-3} \times B_m \quad \ldots \ldots \quad (3)$$

with fair accuracy for densities below 15 kilogausses.

In explanation of the nature of hysteresis loss it may be assumed that the magnetons (small elements of iron in which its magnetism resides; see Ref. 11) in the iron do not follow the gradual increase in the magnetizing force continuously but in small steps, and every time a new configuration of equilibrium or orientation of molecules takes place, they oscillate about their new stable positions, and dissipate some energy in these vibrations.

The power P_h lost in hysteresis in a mass M of magnetic material may be represented by the empirical formula:

$$P_h = \eta f M B^n \text{ watts} \quad \dots\dots\dots\dots \quad (4)$$

where f is the number of complete cycles of magnetization per second, B is the maximum flux density reached during each cycle, and η is a physical constant which characterizes a given material. The exponent n usually has a value of between 1.5 and 1.8, and in some computations is assumed to be equal to 1.6, although for densities in excess of 12 kilogausses it becomes variable and in excess of 2.0. Another empirical formula which sometimes can be made to represent experimental results more closely is:

$$P_h = fM(\eta_1 B + \eta_2 B^2) \quad \dots\dots\dots\dots \quad (5)$$

This formula has the advantage of two empirical coefficients, η_1 and η_2, and has no fractional exponent. It obviously fails at densities so high that the loss increases faster than the square of B.

In stationary machinery, for example in a transformer, the energy necessary for overcoming hysteresis loss is supplied electrically in the form of an additional component of the exciting current. In a generator, the hysteresis causes an opposing torque between the armature and the field, and the extra energy for overcoming this torque is supplied mechanically by the prime mover. In a motor the loss is supplied from the line, and it reduces the useful torque available on the shaft.

It is of interest to note that the counter torque caused by hysteresis is independent of the speed of the machine. Let this torque be T and the speed of the machine N rpm. The power necessary for overcoming this torque is proportional to " torque times speed," so that

$$P_h = K'NT \text{ watts}$$

where K' is a numerical constant which depends on the units used. Equating this expression to eq. (4), we have

$$K'NT = \eta M f B^n$$

The number of cycles of magnetization f is proportional to the speed N of the machine, that is, $f = K''N$, where K'' is another constant. Substituting and dividing both sides of the equation by $K'N$, we finally get

$$T = K\eta MB^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where K is another constant. Equation (6) shows that the hysteresis torque does not depend on the speed of the machine, but only on the flux density and on the quality and quantity of magnetic material.

217. Physical Nature of Eddy Currents. — Steel and iron not only possess high permeability for magnetic flux but also have a comparatively good conductivity for electric currents. Therefore, when a magnetic flux in iron varies with time, causing emf's to be induced, currents are produced in it as in any other conductor subjected to an alternating flux. These currents are called *eddy currents,* or Foucault currents, after a French physicist by that name. According to Lenz's law, these currents assume such a direction that they tend to oppose the changes in the magnetic flux, and therefore form closed paths linking with the flux in planes perpendicular to it. By laminating the core in the proper direction, the paths of eddy currents are restricted and the resistance of their paths is increased. In order to prevent these currents more effectively from flowing from one sheet to another, the laminations are sometimes insulated from each other by japan, varnish, tissue paper, etc. However, for many practical purposes the iron oxide, formed on the surface of laminations during the process of annealing, constitutes a sufficient insulating against eddy currents.

The power loss due to eddy currents may be represented theoretically by the formula

$$P_e = \epsilon M(tfB)^2 \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where ϵ is a factor, approximately constant, which depends upon the electrical resistivity of the iron, its temperature, the distribution of the flux, the wave-form of the flux, and the units used. M is the mass of the laminations, t the thickness of each; B is the maximum flux density during a cycle, and f is the frequency, or the number of cycles of magnetization per second.

The following comment is made by Mr. Spooner (*loc. cit.*) regarding the accuracy of such formulas as eq. (7) and the effect of resistivity of the material: " while high resistance means decreased eddy losses, nevertheless, due to other factors which accompany high resistance, the eddy losses are not inversely proportional to the resistivity. Also the losses are not proportional to the square of the thickness of the sheets but decrease less rapidly than the square law with decreasing thickness." Nevertheless, eq. (7) does indicate in general the manner in which eddy-current loss varies with different factors.

By a reasoning similar to that used in the preceding section, it can be proved that in a revolving machine the resisting torque due to eddy currents is proportional to the speed of rotation. Since the hysteresis torque is the same at all speeds, this difference in the character of the two losses is utilized in practice for separating the hysteresis loss from that due to eddy currents (§230).

218. Methods of Measuring Hysteresis and Eddy Currents. — The core loss due to hysteresis and eddy currents lowers the efficiency and increases the temperature rise in machinery. It is important, therefore, to know how to measure this loss on samples of steel intended for use in the magnetic circuit of electrical apparatus. Results of core-loss tests are usually plotted in the form of curves, such as are shown in Fig. 186.

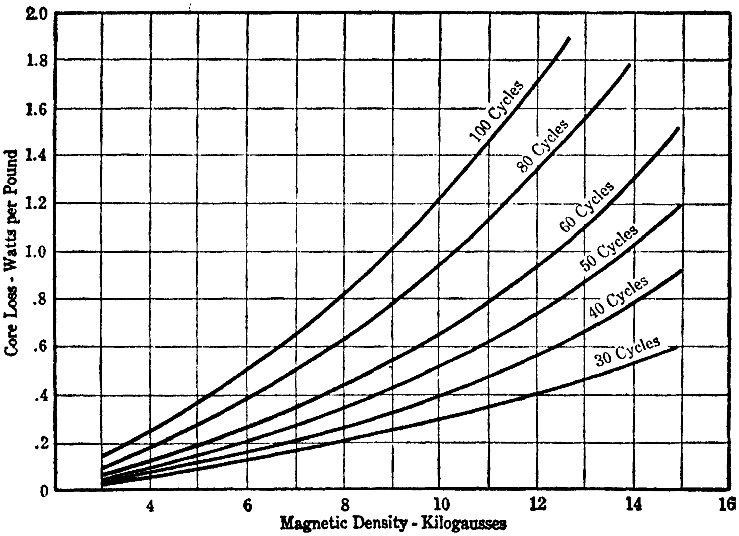


FIG. 186. Core loss at different magnetic densities and at different frequencies of magnetization. (Allegheny Steel Co.)

The curves give the total loss which comprises that due to both hysteresis and eddy currents. In most practical cases it is not required to separate the two; but should this be necessary, it can be done as is explained in §230.

There are two practical methods of determining core loss, viz., (1) the wattmeter method and (2) the mechanical-torque method. With the wattmeter method, steel laminations under test are assembled into packages so as to form the core of a choke coil which is energized with an alternating current. The power input into the coil read on a wattmeter, less the copper loss, is equal to the core loss. With the mechanical-torque method, the laminations under test and an external magnetic field are rotated relatively to each other, as if the core were that of the armature of an electrical machine. The torque due to the core loss is measured by means of a torsion spring or counterweight.

The torque method should give more accurate results for sheet steel intended for armature cores of generators and motors, because they are subjected to a " rotating " hysteresis. The wattmeter method is more suitable for transformer iron subjected to alternating magnetization. In practice, however, no such strict distinction is made between the two methods, because in most cases it is sufficient to know that a new lot of steel laminations has a core loss which is not above a certain limit. It makes little difference in which way this limit is ascertained, provided that the same method is used in all cases. The wattmeter method has important practical advantages and is being used more and more, in preference to the mechanical-torque method.

A third possible method consists in integrating the area of a hysteresis loop of the sample (Fig. 161). This method is seldom used because it is rather lengthy; besides, it gives the hysteresis loss only, and not the total core loss.

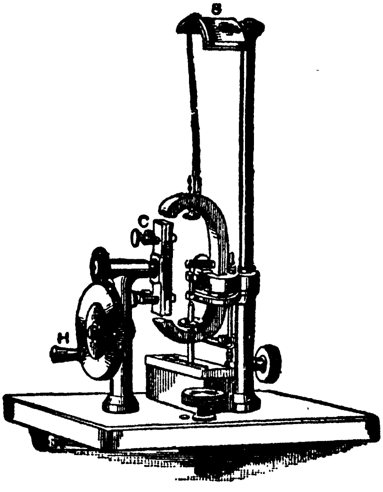


FIG. 187. The Ewing hysteresis tester.

THE MECHANICAL-TORQUE METHOD

219. Ewing's Magnetic Tester.—This is the simplest and the oldest device (Fig. 187) based on the principle of measuring hysteresis loss by its mechanical torque. A few strips of sheet iron under test are clamped in the carrier *C* which is made to revolve by turning the handle *H*. The carrier turns between the poles of a permanent magnet which is suspended on a knife-edge. The hysteresis in the specimen causes the magnet to be deflected, and the deflection is observed by means of a pointer on the scale *S*.

The same test is repeated with a standard sample whose hysteresis loss has been previously determined by some other method. The ratio of the deflections gives the ratio of the values of hysteresis loss in the two samples.

The deflections of the pointer are practically independent of the speed of rotation; it is shown in §216, eq. (6), that hysteresis torque is independent of the frequency of magnetization. When the speed of rotation is quite high, the influence of eddy currents becomes noticeable, and the deflection is somewhat increased. The loss determined with this tester refers to a certain flux density only, and no provision is made in the apparatus for varying the density. The density within the samples is practically independent of the quality of iron tested, the value of the flux being determined by the reluctance of the two air-gaps. The instrument is adapted only for rough relative tests.

220. EXPERIMENT 9-A. — Hysteresis Loss in Iron with the Ewing Magnetic Tester. — Test a few samples of sheet steel and iron as explained in the preceding article. Investigate the influence of the speed of rotation. Make a sketch of the mechanical details of the device. Test one of the samples with laminations thoroughly insulated from each other, and then without insulation; see if the influence of eddy currents makes itself perceptible in the second case. Form an opinion in regard to the accuracy of the instrument and its applicability for practical work.

221. Blondel's Hysteresimeter. — This device (Figs. 188 and 189) is based on the same principle as the above-described Ewing magnetic tester, but is mechanically superior to it. It has a U-shaped permanent magnet *MM* which can be rotated about a vertical axis by means of the handle *H*. The sample sheets under test are made in the form of a ring *R* and are fastened on the support *S*. This support, with its pivoted vertical shaft *P*, tends to revolve, but is restrained by an opposing spiral spring shown in the figure. When the magnet *M* revolves, the support with the sample turns until the hysteresis torque just balances the torsion of the spring. The deflection is practically independent of the speed of rotation, as in the Ewing apparatus described above, and is directly proportional to the hysteresis constant η of the sample (§216). A standard sample, for which the hysteresis constant has been determined by another method, is used with this instrument. The ratio of the deflections obtained with the standard sample and with the sample under test, is equal to the ratio of their hysteresis constants.

The magnet *M* is selected of such a strength as to give a magnetic density *B* of about 10,000 lines per sq cm within the sample. The reluctance of the air-gap is so large, as compared to that of the sample itself, that differences in permeability of different samples hardly affect the above value. Thus all samples are compared at a standard density of 10,000 maxwells per sq cm. To insert a sample into the apparatus, first remove the glass which covers the scale, then take off the pointer and finally the support *S*. The sample is then clamped to the support and the parts returned to their respective places. In testing samples with this hysteresimeter it is always necessary to take readings with rotation both ways; the average of the two indications is the true zero position. This eliminates the error due to an uncertain previous magnetization of the sample. It is advisable to rotate the handle at a speed of 2 to 3 rps.

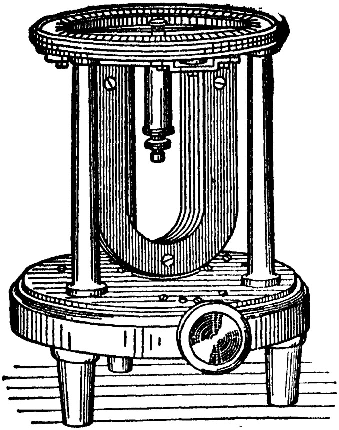


Fig. 188. The Blondel hysteresimeter.

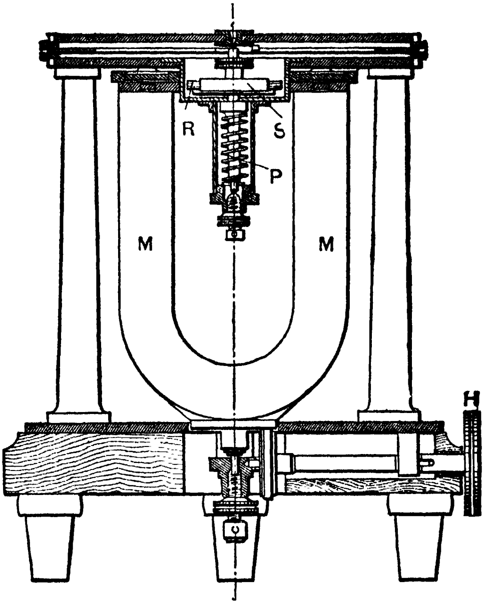


Fig. 189. A cross-section of the Blondel hysteresimeter.

222. EXPERIMENT 9-B. — Hysteresis Loss with the Blondel Hysteresimeter. — See the directions for the preceding experiment.

223. Holden-Esterline Core-Loss Meter. — The two above-described hysteresis testers are very simple and convenient to use, but have the following disadvantages:

(a) Since they are rotated by hand, the frequency is so low that samples are tested essentially for hysteresis loss only, while usually the total core loss is of importance.

(b) Hysteresis loss is determined at one density only.

(c) Readings are merely relative, so that a standard sample is required.

These objections are remedied in the device shown in Fig. 190. In this instrument the magnetizing field is revolved by a motor at any required frequency. A six-pole electromagnet is used in place of a permanent magnet, so that it is possible to test samples within a wide range of magnetic densities, and the instrument can be made to read the core loss directly in watts. The exciting field, with its six inwardly projecting poles, is made of stamped laminations and mounted on a hub which can be rotated by means of a belted motor. The poles of this field are wound with exciting coils. For a-c work the frame is left stationary, the coils being connected to a two-phase or three-phase circuit so as to produce a rotating magnetic field (§484).

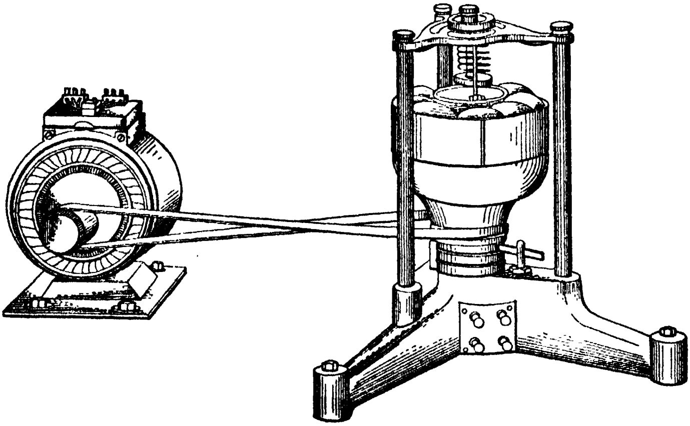


FIG. 190. The Holden-Esterline core-loss meter.

The sample to be tested consists of concentric rings mounted on a brass spider. The shaft has a conical pivot below and rests on a jeweled bearing. The upper end of the shaft is guided by a bearing in the dial plate. When the poles are rotated, or a rotating field is produced by a-c currents, the eddy currents and hysteresis in the sample tend to cause it to rotate also. This rotation is prevented by the coiled spring attached to the spider at one end and to the milled head on the dial plate at the other end. The pointer attached to the milled head is set at zero when there is no twist in the spring. The angle of torsion by which it is necessary to turn the core back to zero, when the field is rotating, is a measure of the loss.

Instead of calibrating the dial in torque units, such as foot-pounds or gram-centimeters, it can be calibrated directly in watts loss at a certain standard frequency, for instance, at 60 cycles per second. At lower frequencies the readings must be reduced in proportion, and at higher frequencies increased. The air-gap is of sufficient length to make the flux density in the specimen practically proportional to the exciting current. Moreover, the permeability of the sample at the usual densities has very little effect on the flux in the instrument, so that the dial can be calibrated once for all. The apparatus is well adapted for commercial work; it is strong mechanically, and not much skill or electrical knowledge is required to operate it. A somewhat similar device has also been constructed in which the punchings are made to revolve and the field is stationary.

224. EXPERIMENT 9-C. — Testing Iron with the Holden-Esterline Core-Loss Meter. — For a description of the device see the preceding section. Test a few samples over a wide range of densities and frequencies, driving the field with a motor. Take a few readings with a revolving field produced by polyphase currents. Study the mechanical details of the apparatus. If possible, check the calibration of the dynamometer spring. See also §220.

Report. Plot curves of core loss in watts per unit weight of laminations at various densities and frequencies (Fig. 186). Separate the hysteresis loss from eddy currents, as is explained in §230. Show the difference in the values of the loss with the field produced by direct current and by polyphase currents; explain the discrepancy.

THE WATTMETER METHOD

225. In the above-described core-loss testing devices, the sample is subjected to the so-called " rotating hysteresis " of the same general character as in the armature of a generator or a motor. In a transformer core the iron is subjected to " alternating hysteresis," and the power loss may be considerably smaller.

A package of laminations to be tested under the latter conditions is placed within a magnetizing coil excited with an alternating current of the desired frequency and magnitude, and the core loss is read on a properly connected wattmeter. The general arrangement of the magnetic and electric circuits described below is due to Dr. J. Epstein and has been used for a number of years by the *German Association of Electrical Engineers.* Later it was adopted with some modifications by the *American Society for Testing Materials,* and another modification has been developed for accurate work by M. G. Lloyd of the Bureau of Standards.

Where a complete knowledge of the core loss in a particular sample is required, curves of such loss are taken at different values of flux density and frequency. Where only a check measurement is desired on a shipment, it is sufficient to determine the loss at one or two standard flux densities and at a standard frequency. Such a value is sometimes known as the *loss value* (Verlustziffer). In Europe the loss value is usually given in watts per kilogram of steel laminations at 50 cycles per second, and at the flux density of 10 or 15 kilolines per sq cm. In this country the loss value is most commonly given for a density of 10 kilolines per sq cm, at 60 cycles, and per pound of laminations (see Fig. 186).

226. The Epstein Apparatus. — This device, in the form recommended by the *American Society for Testing Materials,* is shown in Fig. 191. The arrangement of the magnetizing coils and of the test specimens with butt joints is shown in Fig. 192*a*. The joints are held tight by the clamps indicated in Fig. 191. The specifications for the samples are as follows: " The magnetic circuit shall consist of 10 kg (22 lb) of the test material, cut with a sharp shear into strips 50 cm (19 11/16 in.) long and 3 cm (1 3/16 in.) wide, half parallel and half at right angles to the direction of rolling, made up into four equal bundles, two containing material parallel and two containing material at right angles to the direction of rolling, and finally built into the four sides of a square with butt joints and opposite sides consisting of material cut in the same manner. No insulation other than the natural scale of the material (except in the case of scale-free material) shall be used between laminations, but the corner joints shall be separated by tough paper 0.01 cm (0.004 in.) thick."

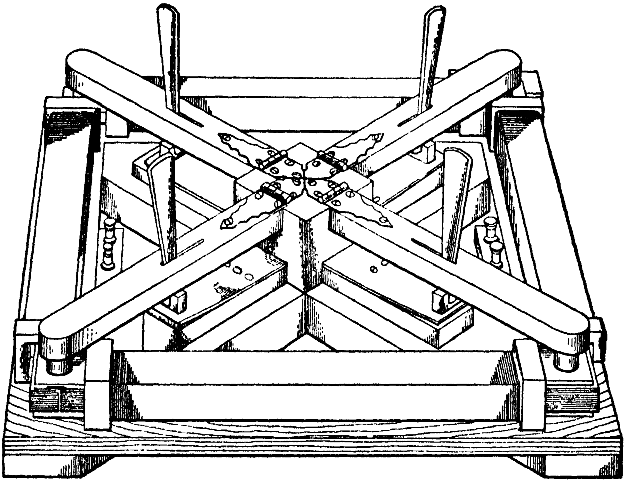
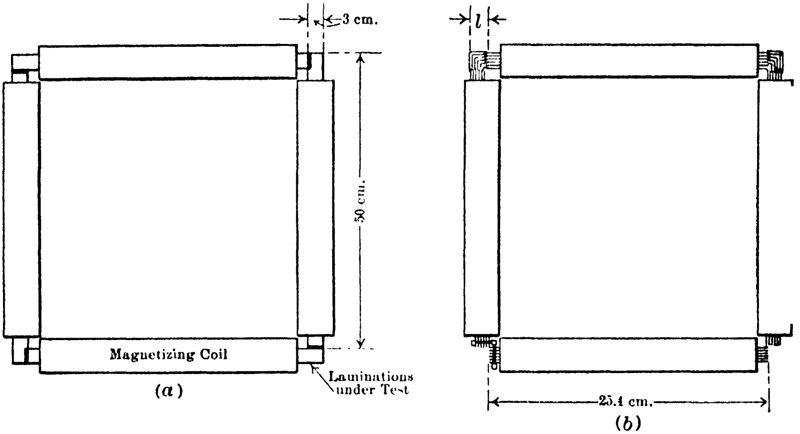


FIG. 191. The Epstein apparatus for measuring core loss.

The coil specifications are as follows: The inside cross-section of the spool on which a coil is wound is 4 by 4 cm; the thickness of the wall not over 0.3 cm, the winding length 42 cm. The primary winding of each of the four coils to consist of 150 turns of copper wire, the total resistance per coil to be between 0.3 and 0.5 ohm. The secondary winding on each coil to consist of 150 turns of copper wire, and the resistance to be not over 1 ohm; shall be wound beneath each primary winding and used to excite the voltmeter and wattmeter potential coils.

The arrangement shown in Fig. 192*b* is used by the *Bureau of Standards.* It requires less than 2 kg of iron for a sample, as against the 10 kg required for the standard Epstein device. The sheets are placed on edge and are connected at the ends by corner pieces of the same material or of a material of known core-loss constants. This arrangement does away with the air-gaps unavoidable with butt joints. The specifications are as follows: " The material to be tested is usually submitted in the form of 10 sheets 30 cm (12 in.) square. This material is cut into strips 25.4 by 5 cm (10 by 2 in.). The final cutting to size is done at the Bureau. These are assembled into four bundles, in each of which adjacent strips are separated by strips of pressboard of equal width and thickness, but 2 cm shorter. Each bundle is wrapped with gummed paper and is inserted in a solenoid, and the four bundles are then arranged in a square so that the plan view shows the edges of the strips. The solenoids are wound upon fiber forms, which are 22.7 cm long, and have inside dimensions 5 by 1 cm. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of these corner pieces as there are test pieces, and they are graduated in



(*a*) (*b*)

Fig. 192. The arrangement of coils and test pieces for core loss; (a) 10-Kg sample with butt joints, (b) 2-Kg sample with corner pieces.

length so as to give a uniform lap of about 2 mm. A clamp is tightened over these laps so as to give a good magnetic joint.

"Each solenoid has two windings. The secondary consists of 121 turns of No. 14 double-cotton-covered wire. Over the secondary are wound 242 turns of No. 14 double-cotton-covered wire. The four solenoids are connected in series, making a total of 968 magnetizing turns and 484 turns in the secondary."

The *electrical connections* for use with either Fig. 192*a* or Fig. 192*b* are shown in Fig. 193. Ordinary indicating instruments are used in commercial testing; the *Bureau of Standards*, in its scientific work, has used an astatic electro-dynamometer-type instrument with a mirror and scale.

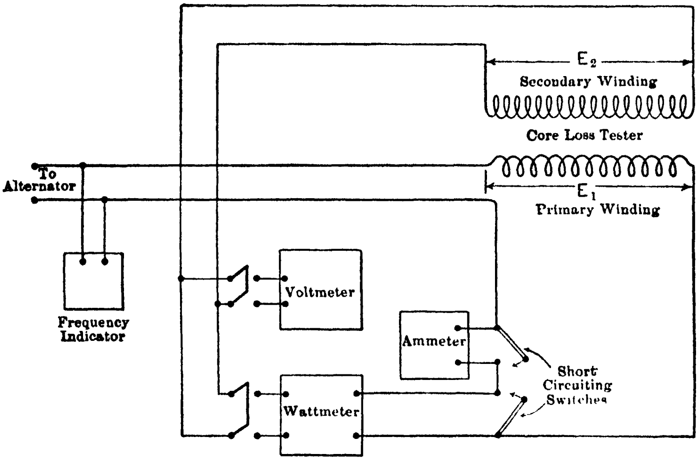


FIG. 193. Electrical connections for measuring core loss (wattmeter method).

The current winding of the wattmeter is in the primary circuit, and its potential winding is connected across the secondary terminals of the core-loss tester. When so connected, the wattmeter measures the core loss in the iron samples plus the I^2R loss in the whole secondary circuit, consisting of the potential windings of the testing device and the wattmeter, and a multiplier resistance. When the total *secondary* resistance R is known and the induced voltage E_2 has been read on the voltmeter, the secondary copper loss E_2^2/R is computed and subtracted from the wattmeter reading, which then gives the true iron loss. The corresponding flux density is computed from formula (11) given below. With the potential winding of the wattmeter connected in the secondary circuit, an error will arise when there is some flux threading the core and linked with the primary but not with the secondary (leakage flux). This is avoided by winding the secondary under the primary and making the two coextensive in length.

After a number of values of core loss have been read on the wattmeter, at different values of induced voltage and frequency, curves of core loss may be plotted (Fig. 186). If it is desired to separate the hysteresis loss from the eddy currents, readings are preferably taken at 60 and at 30 cycles, or at some other two frequencies in the ratio of 2 to 1, because computations are thus simplified (§230).

The hysteresis loss depends upon the amplitude of the exciting current and is independent of its wave-form, unless the current has more than one maximum for each alternation (saddle-form wave). The eddy-current loss depends upon the wave-form of the primary current. In all core-loss tests it is therefore essential either to have an alternator which gives a nearly sinusoidal voltage, or to test samples with a voltage of the same wave-form as that on which the corresponding lot of laminations is to be used (see, however, §229). The applied voltage should be adjusted either by means of the field rheostat of the alternator or by using an auto-transformer (§402) with numerous taps. As little resistance as possible should be used in the supply circuit, since, owing to the distortion of the wave-form of the exciting current, a resistance drop caused by this current would also be distorted and would modify the wave-form of the voltage across the coil. The alternator used to supply the core-loss tester should be of sufficient rating to be but lightly loaded at the current drawn during the test. Its wave-form will then not be altered by armature reaction, regardless of the form or power factor of the test current.

With the growing use of high frequencies for various purposes, there is some need for determining core loss at such frequencies. A dynamometer-type wattmeter and voltmeter are no longer suitable on account of their inductance. One method that has given satisfactory results at high frequencies consists in measuring the core loss with a calorimeter while the induced secondary voltage is determined with an electrostatic voltmeter. A tungsten arc or vacuum-tube oscillator can be made to furnish the required high-frequency current.

227. Theory of the Wattmeter Method. -- By varying the voltage (or frequency) applied to the primary of the testing device the maximum flux density in the core can be made of any desired value, as the following relations will show.

The emf induced in the secondary winding of N_2 turns by a flux, ϕ, kilolines, is

$$e = -N_2\, d\phi/dt \times 10^{-5} \text{ volts} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (8)$$

With a sine-form applied voltage the flux ϕ will be practically sine-form and may be written

$$\phi = \phi_m \sin 2\pi ft \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

Therefore, from (8) and (9)

$$\begin{aligned} e &= -2\pi f N_2 \phi_m \cos 2\pi f t \times 10^{-5} \text{ volts} \\ &= -2\pi f N_2 A B \cos 2\pi f t \times 10^{-5} \text{ volts} \quad . \; . \; . \; . \; . \; (10) \end{aligned}$$

where A = core area, and B = maximum flux density in the core, in kilolines per unit of area. Hence the amplitude of the secondary voltage will be

$$e_{\text{max.}} = 2\pi f N_2 A B \times 10^{-5} \text{ volts}$$

so that, in effective value

$$E_2 = 4.44 f N_2 A B \times 10^{-5} \text{ volts} \quad . \; . \; . \; . \; . \; . \; (11)$$

This is the voltage across the voltmeter and wattmeter in Fig. 193 — neglecting the small drop due to the instrument current in the low-resistance secondary winding.

For the standard Epstein sample and apparatus, letting M = weight of sample in kilograms and δ = specific gravity of the steel, eq. (11) may be reduced as follows:

$$\begin{aligned} E &= 4.44 f \frac{600 \times M \times 1000 \times B \times 10^{-5}}{200\delta} \\ &= \frac{13.32 f \times M \times B}{\delta} \times 10^{-2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (12) \end{aligned}$$

For example, if $f = 60$ cycles, B = 10 kilolines per sq cm, $\delta = 7.5$, and M = 10 kg, solving eq. (12) gives E = 106.5 volts.

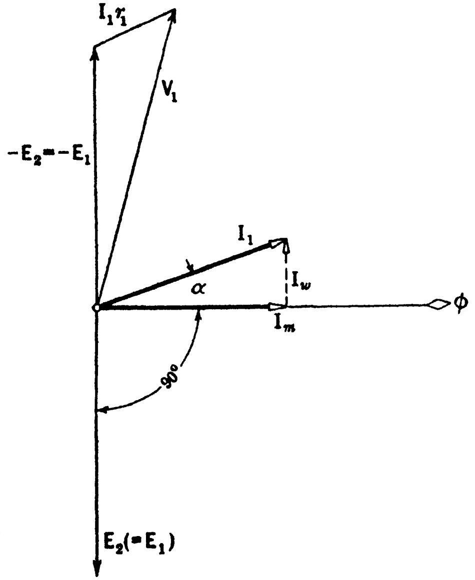


FIG. 194. Vector relations involved in the Epstein test.

That the wattmeter, energized by the primary current, I_1, of the tester and the secondary voltage, E_2, will read the core loss, may be seen by reference to the vector diagram.

Figure 194 is drawn for a tester having a ratio of 1 : 1 between primary and secondary turns, and for the secondary winding on an open circuit: I_1 is the current drawn by the tester. ϕ, the flux in the core, lags behind I_1 by the angle α. The component, I_m, of the total current in phase with ϕ is called the magnetizing component, while I_w in quadrature with ϕ is called the core-loss component, and is in phase with the induced emf E_1 ($=E_2$), reversed. Therefore the wattmeter (Fig. 193) reads the product $-E_2 \times I_1 \times \cos (90° - \alpha) = -E_1 \times I_w$. That this is the core loss of the tester is evident from the following:

The total power input is $W_1 = V_1 I_1 \cos \theta$. But V_1 is the vector sum of $-E_1$ and $I_1 r_1$, so that the projection of V_1 upon I_1 must equal the sum of the projections of $-E_1$ and $I_1 r_1$ upon I_1; i.e.,

$$V_1 \cos \theta = -E_1 \cos (90° - \alpha) + I_1 r_1 \cos 0$$

or

$$W_1 = -E_1 I_1 \cos (90° - \alpha) + I_1 r_1 \times I_1 \cos 0 \quad . \; . \; . \quad (13)$$

$$= -E_1 I_w + I_1^2 r_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

(Note from the diagram that the product $-E_1 I_w$ is positive.)

Equation (13) shows that the total input to the tester consists of the I^2R loss in the primary winding plus the product $-E_1 I_w$ ($= -E_2 I_w$), where $-E_2 I_w$ is the reading of the wattmeter. But the total input to the tester is made up of the I^2R loss of the winding plus the core loss, so that $-E_1 I_w (= -E_2 I_w)$ must equal the core loss.

Of course, when the secondary coil supplies the voltmeter and wattmeter potential, there must be an additional input to the tester of E_2^2/R where R is the equivalent resistance of the two meters in parallel, it being assumed that the loss due to the instrument current in the resistance of the secondary coil is negligible.

The wattmeter connected as in Fig. 193 indicates the average power due to the current I_1 times the induced voltage E_2. If the number of primary turns is the same as in the secondary this reading gives the true power input into the core and into the secondary circuit. Subtracting the secondary loss E_2^2/R, the result gives the core loss. If the number of secondary turns is different from that in the primary winding the wattmeter reading must be reduced or increased in the corresponding ratio to find the true power input.

In the early forms of Epstein apparatus there was no secondary winding, and the potential coil of the wattmeter (as well as the voltmeter) was connected directly across the primary terminals. In this case the wattmeter reading included the I^2R loss in the primary winding, and, moreover, the voltmeter reading had to be corrected for the primary voltage drop in order to obtain the true induced emf from which the flux should be computed. For these reasons a secondary winding has been added in the later types of apparatus and the potential taken from this winding.

In eq. (11), A, the net cross-section of the core, is usually computed from the weight of the laminations, their length, and the known specific gravity of iron. A specific gravity of 7.5 may be assumed for high-resistance steel, that is, for all steels having a resistance of over 2 ohms per metergram, and 7.7 for low-resistance steel. The flux in the air spaces between the laminations is neglected.

The following example is from the test of a sample of transformer steel, using the Epstein apparatus. The steel consists of 10 kg of No. 27 gauge ($t = 0.0172$ in.) low-silicon steel cut as specified in §226. The loss value at 60 cycles, 10 kilogausses, is to be determined. From eq. (12) the voltage E_2 necessary to give the required flux density is

$$E_2 = (13.32 \times 60 \times 10 \times 10)/(7.7 \times 10^2) = 103.8 \text{ volts}$$

From the test at this voltage the wattmeter reading is 40 watts. The resistances of the voltmeter and potential circuit of the wattmeter are 3540 ohms and 2000 ohms, respectively.

The E^2/R correction for instrument losses is therefore $E^2/R = (103.8)^2/3540 + (103.8)^2/2000 = 3.05 + 5.4 = 8.45$ watts.

Therefore, the core loss $= 40 - 8.45 = 31.55$ watts.

Loss per kilogram $= 31.55/10 = 3.155$ watts.

Loss per pound $= 31.55/(10 \times 2.2) = 1.43$ watts.

This value is satisfactory for this grade of steel as shown by curves of commercial loss furnished by the manufacturers (Fig. 195).

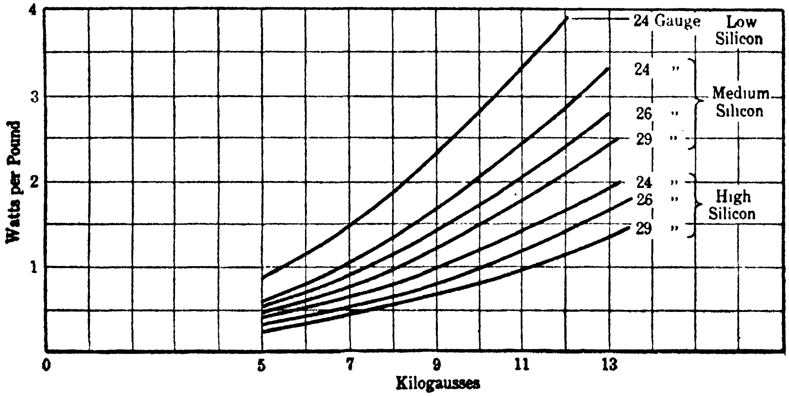


FIG. 195. Loss curves (at 60 cycles) of electrical steels of various qualities.

In the apparatus built according to the specifications of the *American Society for Testing Materials* (Fig. 192*a*) the flux density is practically uniform throughout, and therefore the wattmeter reading of loss is divided by the total weight of laminations. In the apparatus designed by the *Bureau of Standards* (Fig. 192*b*) a small correction is necessary for the overlap of the corner pieces where the flux density and the loss are lower. Assume the corner pieces to be made of the same material and to be of the same thickness as the samples under test. Let M be the mass of the test pieces and m that of the corner pieces. The wattmeter reading is divided, not by $(M + m)$ but by $(M + m)(1 - 1.4c)$, where $1.4c$ is an empirical correction for the lower flux density in the overlaps. The factor c is equal to the mass of the corner pieces which lap, expressed in terms of the mass of the test pieces. In other words

$$c = (m/M) - (l/25.4) \qquad \ldots\ldots\ldots \quad (15)$$

where l is the dimension shown in Fig. 192*b*, in centimeters, and 25.4 is the length of the test pieces, in centimeters. When the corner pieces are of different material the correction is somewhat more involved. See the Circular of the *Bureau of Standards*, No. 17.

228. EXPERIMENT 9-D. — Determination of Iron Loss by the Wattmeter Method. — The method and the apparatus are described in §§220 to 222. The following range meters should be available for this test, using the standard Epstein tester and samples:

One low power factor wattmeter (§95) of current ranges 1 and 2 amperes; voltage ranges, 75 and 150 volts.
One dynamometer-type voltmeter, 75 and 150 volt ranges.
One low-range ammeter, for 1 and 2 amperes.
One accurate frequency meter.

These instruments should be calibrated in advance of the test in order that the test may be performed at voltages corresponding to the desired flux densities (see eq. (12)). The diagram of connections is shown in Fig. 193. (1) Weigh the samples, measure their length, and assemble as instructed in §226. Before clamping the joints, tap lightly together until, with a given voltage applied to the primary coil, the exciting current is a minimum. Compute the coefficient of proportionality between voltmeter readings and flux densities, according to formula (12). (2) Begin readings at a standard frequency and at the highest feasible flux density. Read watts, volts, amperes, and the frequency. Be sure that the resistance of the secondary circuit is known for each set of readings. Read volts and watts simultaneously and keep the voltmeter and wattmeter potential circuits open when reading amperes. Reduce the voltage in steps and take readings at each step, keeping the frequency constant. Between the steps reduce the current gradually so as to demagnetize the material properly. Whenever the magnetizing circuit has been broken, it must be closed through a considerable resistance which is continuously reduced to zero, in order to prevent a large first surge and consequent high magnetization which would require subsequent demagnetization. (3) Repeat the preceding test at several other frequencies, above and below standard, including the frequency exactly equal to one-half standard. (4) Make a comparative test of iron loss with and without insulation between the laminations, taking care that in one set of readings bright metal surfaces touch each other. This test is preferably made at a high frequency at which the effect of eddy currents is more noticeable. (5) Before leaving the laboratory, record all the data concerning the apparatus that are needed in computations.

Report. (1) Correct the wattmeter readings for the secondary copper loss and plot values of core loss per kilogram or per pound against flux densities as abscissas, for each frequency investigated. (2) Separate hysteresis from eddy currents by the straight-line method indicated in §230, and compare one or two results with those obtained by the two-point method. (3) Compute the exponent according to which the hysteresis loss varies with the flux density. (4) Write down the final numerical formula from which the loss per kilogram of material may be computed at any flux density and frequency within the range investigated experimentally. (5) Give numerical data to illustrate the effect of imperfect insulation upon the increase in eddy-current loss. (6) Plot curves of magnetizing volt-amperes per kilogram of iron (§231) for different frequencies and show that for a given flux density the values are proportional to the frequency. (7) For one or two flux densities compute the actual instantaneous ampere-turns per centimeter, corresponding to maximum B (§231), and compare with an available B-H curve for similar grade of steel. Indicate how the air-gaps in the magnetic circuit were allowed for.

229. Iron-Loss Voltmeter. — This meter is for use in connection with a wattmeter for determining the iron loss in distributing and power transformers on the basis of sine-wave voltage and normal frequency, when the testing is done on a circuit of any wave-shape and approximate frequency. The iron loss in a transformer varies with the wave-shape and frequency. Guarantees are, therefore, based on sine-wave and a normal frequency; but the standard conditions are difficult to obtain in practice. However, for any other wave-shape and frequency there is some equivalent voltage that will produce the same iron loss as would the sine-wave of normal frequency at normal voltage. The iron-loss voltmeter enables the tester to adjust to this equivalent voltage for the circuit at hand.

The iron-loss voltmeter consists of a wattmeter movement in series with the winding on an iron core (Fig. 196). With this connection the meter element measures the loss in the iron core. The scale is calibrated on a circuit of sine-wave voltage and is marked in " volts." The reading therefore indicates the sine-wave voltage of normal frequency equivalent to the actual voltage, frequency, and wave-shape of the circuit to which the instrument is connected. If the voltage or frequency of the circuit on which a transformer test is being made is adjusted until the iron-loss voltmeter indicates the rated sine-wave voltage of the transformer being tested, the iron losses in the transformer will be the same as they would be on a circuit of that voltage and sine-wave shape. There is also a watt scale marked on the meter which indicates the total watts consumed in the meter. This is for the correction of wattmeter readings taken when the voltmeter is in circuit.

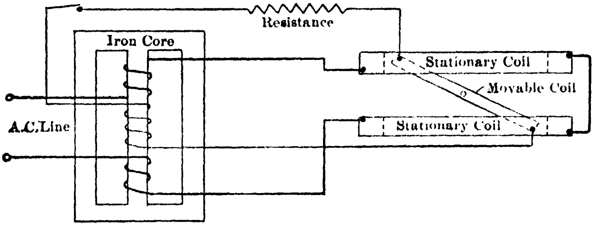


FIG. 196. Details of the iron-loss voltmeter.

In application the iron-loss voltmeter is connected across the terminals of the transformer under test in the same manner as an ordinary voltmeter. A wattmeter is also connected in the circuit in such a way as to measure the total input of both the transformer and the iron-loss voltmeter. The voltage of the circuit is then adjusted by any convenient means until the voltmeter reads the normal voltage of the transformer. The total power input is read on the wattmeter and the watts input of the instrument is read on the watt scale of the iron-loss voltmeter, the difference being the normal iron loss of the transformer. The standard iron-loss voltmeter is usually calibrated for 60 cycles and for 125- and 250-volt circuits.

230. Analysis of Loss Curves. — At a given flux density, the hysteresis loss is proportional to the frequency, while the eddy-current loss is proportional to the square of the frequency (§§216 and 217). Thus, the total core loss P per unit weight of laminations may be represented by the formula

$$P = Hf + Ff^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where f is the frequency, and H and F are functions of the flux density B. The letter H stands for hysteresis and F for Foucault or eddy currents. Dividing both sides of eq. (16) by f, we get

$$P/f = H + Ff \qquad (17)$$

This relationship is utilized in the separation of hysteresis from eddy currents in two ways, according to whether the total loss curves (Fig. 186) are available at two frequencies only, or at three or more frequencies.

(a) *The two-point method.* Let values of total loss P be available at a certain flux density, at two frequencies, f_1 and f_2. Writing eq. (17) for these two frequencies, we have

$$\left.\begin{aligned} P_1/f_1 &= H + Ff_1 \\ P_2/f_2 &= H + Ff_2 \end{aligned}\right\} \qquad (18)$$

Solving these as simultaneous equations for H and F, we get

$$F = [(P_1/f_1) - (P_2/f_2)]/(f_1 - f_2) \qquad (19)$$

$$H = [f_1(P_2/f_2) - f_2(P_1/f_1)]/(f_1 - f_2) \qquad (20)$$

F and H being known, the individual losses $P_h = Hf$ and $P_e = Ff^2$ may be computed at any frequency.

The computations are particularly simple when the values of the losses are required only at a standard frequency f_1, and when the value of the total loss is known at this frequency and also at half the frequency. Substituting in the preceding equations $f_2 = \frac{1}{2}f_1$, we get after reduction

$$\text{Hysteresis loss } P_h = Hf_1 = 4P_2 - P_1 \qquad (21)$$

$$\text{Eddy-current loss } P_e = Ff_1^2 = 2P_1 - 4P_2 \qquad (22)$$

Example. In the test of a standard sample of steel in the Epstein tester the following data are obtained.

At 60 cycles, 103.8 volts total core loss = P_1 = 31.55 watts.

At 30 cycles, 51.9 volts total core loss = P_2 = 14.2 watts.

From eq. (21)

$$P_h = 4 \times 14.2 - 31.55 = 25.25 \text{ watts}$$

From eq. (22)

$$P_e = 2 \times 31.55 - 4 \times 14.2 = 6.3 \text{ watts}$$

These are the separate values of hysteresis loss and eddy-current loss at 60 cycles and 103.8 volts, i.e., at 10 kilogausses.

(b) *The straight-line method.* Let some value, B, represent the flux density at which the losses are to be separated, and let the total loss be known for at least three frequencies (see Fig. 186). The preceding method is then inconvenient, since the values of H and F determined from each pair of data would be somewhat different on account of unavoidable inaccuracies in the experimental data and in the computations. In such a case it is both simpler and more accurate to draw graphically the *most probable straight line* which covers all the data. Equation (17) represents a straight line between f and P/f, and the corresponding points may be easily plotted on a sheet of cross-section paper. A straightedge is placed over these points and a straight line is drawn which passes as nearly as possible through these points. The intercept on the axis of ordinates gives the value of H, and the remainder of the ordinate at any frequency represents the value of Ff for that frequency. After this, the values of $P_h = Hf$ and $P_e = Ff^2$ may be readily computed.

The exponent according to which the hysteresis loss varies with the flux density is determined in the following way: Let the exponent of B in the formula (1), §216, be unknown. The formula may be written in the form

$$P_h = aB^n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (23)$$

where a is a constant with which we are not at present concerned. Taking logarithms of both sides of this equation, we get

$$\log P_h = \log a + n \log B \quad . \; . \; . \; . \; . \; . \; . \; . \quad (24)$$

This is the equation of a straight line between $\log P_h$ and $\log B$; the unknown exponent n is numerically equal to the trigonometric tangent which this line makes with the axis of abscissas (Fig. 197). Thus, in order to determine n at a certain frequency, the eddy-current loss must first be separated from the total loss, as is explained above. The rest is then replotted to a logarithmic scale, as in Fig. 197. The points usually lie on a nearly straight line; the slope of this line gives the exponent n.

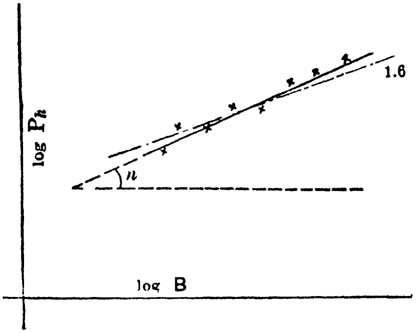


FIG. 197. Determination of the exponent according to which hysteresis loss varies with the magnetic density.

It is convenient to plot this curve on a sheet of logarithmic paper. In this case the abscissas and the ordinates are plotted directly without looking up logarithms. Do not attempt to mark the origin; with the logarithmic scale it is at minus infinity. All that is needed is the slope n of the curve; this slope is independent of the position of the origin.

In some cases it may be desired to accept the usual (Steinmetz) exponent $n = 1.6$ as being correct, and it is merely required to find the value of a in eq. (23); in other words, the constant η in formula (4). In such a case a line having the slope of 1.6 is drawn as closely as possible through the experimental points. The coefficient a is then calculated for this line from eq. (24). In a similar manner one may check the fact that the eddy-current loss increases as the square of the magnetic density.

The total core loss, P, also increases as some power of the flux density; the exponent has a value at ordinary densities between 1.6 and 2. This value can be found by plotting the total loss curve to a logarithmic scale, as is explained for the hysteresis loss above.

If the exponent $n = 1.6$ is assumed, the equation for the total core loss, from eq. (4) and eq. (7) is, for any given core,

$$P_{h+e} = k_1 f B^{1.6} + k_2 f^2 B^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

But, from eq. (11), for the given core and number of turns,

$$B = k_3 E_1 / f \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (26)$$

From eqs. (25) and (26), combining constants, the equation for core loss becomes

$$P_{h+e} = K'\left(\frac{E_1}{f}\right)^{1.6} \times f + K''E_1^2 \quad . \; . \; . \; . \; . \; . \quad (27)$$

Equation (27) states that in a given transformer, or similar device, the hysteresis loss in the core varies with the 1.6 power of the induced voltage and inversely with the 0.6 power of the frequency; the eddy-current loss varies as the square of the induced voltage and is independent of the frequency.

Equation (27) is frequently of value in the separation of the total core loss into its two components when two or more readings of loss are obtained at different voltages and frequencies.

Example. The core loss in a certain Epstein sample at 60 cycles and 103.8 volts = 31.55 watts. At 100 volts and 40 cycles the core loss is 36.1 watts. It is required to determine the individual losses at the former condition.

Applying eq. (27), one has the following:

$$\text{At 60 cycles:} \quad 31.55 = K'\left(\frac{103.8}{60}\right)^{1.6} \times 60 + K'' \times (103.8)^2$$

$$= K' \times 144.6 + K'' \times 10{,}775$$

$$\text{At 40 cycles:} \quad 36.1 = K'\left(\frac{100}{40}\right)^{1.6} \times 40 + K'' \times (100)^2$$

$$= K' \times 173.2 + K'' \times 10{,}000$$

solving gives $K'' = 5.84 \times 10^{-4}$. This value, substituted in the second term of eq. (27), gives the eddy-current loss, at 60 cycles, as

$$P_e = 5.84 \times 10^{-4} \times (103.8)^2 = 6.3 \text{ watts}$$

Therefore, by subtraction, the hysteresis loss at 60 cycles is

$$P_h = 31.55 - 6.3 = 25.25 \text{ watts}$$

The above solution is performed quickly, using the log-log slide rule.

In some investigations it is inconvenient to use a fractional exponent in the expression for the hysteresis loss or for the core loss. Moreover, for some grades of iron, these exponents are not constant, but are higher at higher flux densities than at lower. It is sometimes more convenient to represent either the total loss or the hysteresis loss (as the case may be) at a constant frequency by the expression

$$P = cB + gB^2 \quad \ldots\ldots\ldots\ldots \quad (28)$$

where c and g are certain constants. In this formula the two terms do not represent the hysteresis loss and eddy-current loss respectively, but the total loss is represented empirically by a parabolic expression. Dividing both sides of this equation by B, we get

$$P/B = c + gB \quad \ldots\ldots\ldots\ldots \quad (29)$$

The values of c and g may be determined by either method described above for the computation of H and F.

231. Magnetizing Current from Wattmeter Test. — When performing a core-loss test with the Epstein apparatus (§226) it is possible at the same time to determine the magnetization curve of the sample (OAM in Fig. 161). For this purpose it is only necessary to compute the magnetizing or reactive component I_m of the total primary current I_1 (Fig. 194). The latter is read on the ammeter shown in Fig. 193. Let the wattmeter reading be P watts, and let the induced primary voltage be E_1. The value of E_1 is computed from the voltmeter reading E_2, by multiplying it by the ratio of the numbers of turns, N_1/N_2. Thus, the component of the primary current in phase with E_1 is

$$I_w = P/V_1 \quad \ldots\ldots\ldots\ldots \quad (30)$$

and the magnetizing or reactive component of the primary current is

$$I_m = \sqrt{I_1^2 - I_w^2} \quad \ldots\ldots\ldots\ldots \quad (31)$$

In some cases I_m differs but a little from the total I_1, so that the correction for I_w may not be necessary.

The amplitude of the magnetizing force, or the ampere-turns per unit length of iron, is

$$H = N_1 I_m \sqrt{2}/l \quad \text{. . . . .} \quad (32)$$

where l is the total length of the magnetic circuit. In this formula the amplitude of the current, $I_m \sqrt{2}$, is used because the corresponding value of B from eq. (11) also refers to the maximum flux density. In this way the data for the B-H curve of the laminations may be computed. Care must be taken to make the butt joints as perfect as possible, since even a small air-gap will appreciably increase the magnetizing current. When paper of known thickness (§226) is used at the four joints the ampere-turns there consumed may be calculated by eq. (3) or eq. (4), §184, and subtracted from the total to get the net mmf for the iron, before substituting in eq. (32). No such allowance is needed in the arrangement with corner pieces shown in Fig. 192.

The excitation data per kilogram of material of an a-c core, at a constant frequency, may also be given as reactive volt-amperes plotted or tabulated against values of flux density B. This may be shown as follows: By analogy with eq. (11), the voltage induced in the primary winding is

$$E_1 = 4.44 f N_1 A B 10^{-5} \quad \text{. . . . .} \quad (33)$$

Multiplying both sides of this equation by the magnetizing current I_1, dividing by the volume Al of the laminations, and denoting the result by Q, we obtain

$$Q = \frac{\text{magnetizing volt-amperes}}{\text{volume}} = \frac{E_1 I_1}{Al}$$

$$= \pi f \left[\frac{N_1 I_1 \sqrt{2}}{l} \right] B 10^{-5} \quad \text{. . . . .} \quad (34)$$

According to eq. (32), the expression in the brackets on the right-hand side (eq. 34) represents the value of magnetizing force H. This magnetizing force is a function of B, so that the whole right-hand side of the equation is a function of B, say $F(B)$, and we have

$$Q = fF(B) \quad \text{. . . . .} \quad (35)$$

These equations show that it is possible to plot, from the data taken on the Epstein test, an *experimental* curve which will give magnetizing volt-amperes per pound or kilogram of material at a standard frequency, as a function of magnetic density B, as called for in Experiment 9-D. Furthermore, eq. (35) shows that for a given B the values of Q are proportional to the frequency, so that it is sufficient to plot the curve for one frequency only.

This method of representing the magnetizing current is very convenient in designing transformers, because the current can be computed immediately from the volume of the core and the assumed density B.

232. Hysteresis Loss from Area of Hysteresis Loop. — The hysteresis loss, in joules or watt-seconds per cycle, per cubic unit of iron, is proportional to the area of the hysteresis loop (Fig. 161). The magnitude of the loss varies with the maximum magnetic density reached during the cycle, corresponding to the points M and M_1.

The proof of this proposition is as follows: When a piece of well-laminated iron is undergoing a cycle of magnetization, emf's are induced in the exciting winding by the varying magnetic flux. The instantaneous energy supplied by the exciting circuit, or returned to it, is equal to *eidt*, where i and e are instantaneous values of the exciting current and of that part of the applied voltage which is equal and opposite to the induced voltage; dt is an infinitesimal element of time. The total net electric energy, in joules, supplied to the exciting coil by the source of a-c power, during one complete cycle of duration T, is

$$W = \int_0^T ie\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

According to the fundamental law of induction, eq. (8), we have

$$e = n\,d\phi/dt = nA\,dB/dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where n is the number of exciting turns, ϕ is the instantaneous flux, B the corresponding flux density (in webers per square centimeter or per square inch), and A is the cross-section of the iron core. Substituting this value of e in eq. (36) and dividing both sides of the equation by the volume Al of the laminations, we obtain

$$W/(Al) = \int_{\bigcirc} (ni/l)\,dB = \int_{\bigcirc} H\,dB \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The circle as a subscript to the integral signifies that the integration is to be extended over a closed figure. Thus, eq. (38) represents the area of the complete hysteresis loop, and the above-stated proposition is thereby proved. If the sample undergoes f cycles of magnetization per second, the hysteresis loss in watts per cubic unit is proportional to $f \times$ (area of hysteresis loop). Hence

Hysteresis loss in watts = constant $\times f \times$ loop area $\times$ volume (39)

This method is seldom used in practice for determining the hysteresis loss, because:

(1) It is tedious, since it requires a complete magnetization curve to be taken and integrated for each point on the hysteresis loss curve.

(2) In ballistic determinations of hysteresis, the iron is undergoing the magnetizing process slowly, whereas in actual machines it is magnetized and demagnetized many times per second. There are indications that the hysteresis loss is somewhat different in the two cases.

(3) The eddy-current loss is not taken into account, while in most practical cases the engineer is interested in the total core loss.

Nevertheless the method is of interest from a theoretical point of view, and it is recommended that the student integrate at least one hysteresis loop in connection with the experiments in Chapter VIII. The scale to which the loop is to be plotted has to be carefully considered in order to obtain the result in the desired units.

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tact with the commutator segments and conduct direct current to the line.

The stationary frame is made of magnetic material, either cast iron, cast steel, or rolled steel. The pole-pieces are of steel laminations and may be cast into the frame or bolted to it. The poles are provided with coils through which a direct current is sent, necessary for the excitation of the magnetic field. In practice, the exciting current is usually generated by the machine itself, but for certain purposes the generator is excited from a separate source.

A d-c machine is shown more in detail in Fig. 199. The view to the left is a plane perpendicular to the shaft. The upper left-hand quadrant is a cross-section through one of the field poles and through the armature and commutator. In the lower left-hand corner the radial connections are shown between the armature winding and the commutator. The brush rigging and the leads are shown on the right-hand side of the same view. The upper part of the sketch to the right is a longitudinal cross-section of the machine, in the vertical plane passing through the center line of the shaft. The lower part is a side view of the machine. The armature laminations are shown assembled in five stacks with ventilating spaces between. The radial lines shown in the armature core, in the upper left-hand quadrant of the other sketch, represent the ribs of a spacing plate which forms a ventilating duct.

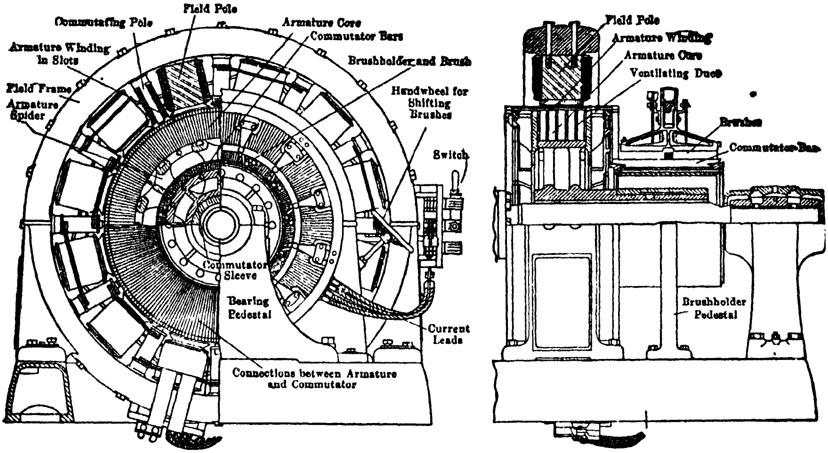

FIG. 199. A multipolar d-c machine.

In addition to the main or field poles, the so-called commutating poles, or interpoles, are shown midway between the main poles. The purpose of the interpoles is to provide a strong magnetic field over those armature coils in which the current is undergoing commutation due to their temporary connection to segments under the brushes. This field helps the reversal of the current and makes the commutation practically sparkless. For further details regarding interpoles (see §276).

NO-LOAD CHARACTERISTICS

234. Separate Excitation. — It is advisable to begin the study of the d-c generator with the simplest possible case, namely, that of the machine excited from a separate source and running at no load (Fig. 200). A voltmeter is connected across the armature terminals to measure the induced voltage. The field winding is connected to a source of d-c supply, in series with an ammeter and to a regulating rheostat. A wider adjustment of the field current is possible if a potentiometer-type rheostat (Fig. 200) is used rather than a series rheostat. A double-throw switch will permit reversing the direction of the exciting current, and therefore the polarity of the induced emf, without changing the direction of rotation of the machine.

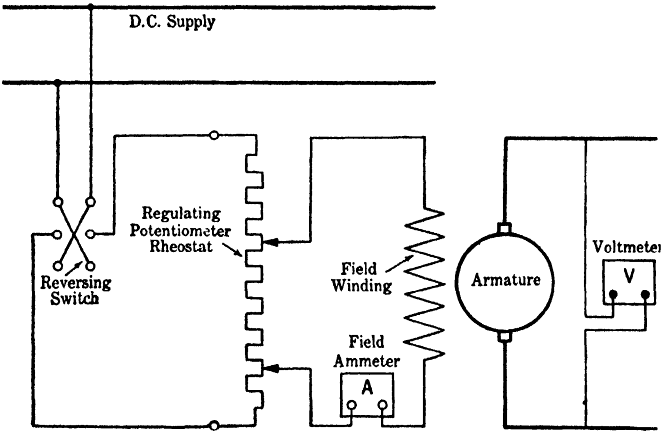

FIG. 200. Circuit diagram of a separately excited generator.

The voltage induced in the armature of a d-c machine is proportional to the following three factors: the magnetic flux per pole; the speed of the machine; and the number of armature conductors in series. The exact formula is

$$E = (p/a)(\text{rpm}/60)Z\phi \times 10^{-2} \quad \ldots\ldots\ldots \quad (1)$$

where E is the induced emf in volts, p is the number of poles, a is the number of armature circuits in parallel, Z is the total number of armature conductors, and ϕ is the flux per pole, in megalines.

Formula (1) may be proved as follows: From the law of induction

$$e = -N\, d\phi/dt$$

where N is the number of turns in series, and $d\phi/dt$ is the rate of change of flux through the turns. Likewise, ignoring signs,

$$E_{\text{ave.}} = N \times \text{ave}\, d\phi/dt$$

In the d-c machine, while a coil of wire moves from the center of a north to the center of a south pole the flux through a turn changes from $+\phi$ to $-\phi$, or the total change of flux $= 2\phi$. In one revolution the total change of flux through the turn $= 2p\phi$, and the total change per second $= 2p\phi$ (rpm/60), or $2p\phi(\text{rpm}/60) \times 10^{-2}$ is the average voltage induced in the turn (ϕ being in megalines). The voltage between terminals equals average volts per turn times turns in series, or,

$$E = 2p\phi\,(\text{rpm}/60) \times (Z/2a) \times 10^{-2}$$

which gives eq. (1).

(a) *Effect of varying field current.* Let us assume first that the speed of the machine is kept constant, and that the flux ϕ is varied by regulating the exciting current with the field rheostat. Let us assume also that the magnetic circuit of the machine has been thoroughly demagnetized so that it has no appreciable residual magnetism (§184). Then the relationship between the field current and the flux is found to be like the curve Odb in Fig. 201. (The curve of Fig. 201 and those which follow were taken in the laboratory on a 3.75-kw, 125-volt shunt generator, without commutating poles.) According to the foregoing formula, the induced voltage is proportional to the flux, so that while the curve is plotted between the field current and the induced voltage, it also gives a relationship between the exciting current and the flux which it produces. This curve is known as *the no-load saturation curve* of the machine, and its curvature is caused by the increasing magnetic saturation in the iron parts (Chapter VIII).

If the residual magnetism has not been completely removed, some voltage is induced by it even when the field circuit is open. For this reason the saturation curve may begin somewhat above the origin, and the initial voltage Of depends upon the amount of residual magnetism present. If such a residual magnetism is present, and the exciting current is gradually increased, a voltage curve fgb is obtained, which lies somewhat above Odb. When the magnetizing current is again reduced from its maximum value Oa, the voltage curve bhf lies still higher in its middle part, forming a hysteresis loop (§184). In practice, the difference between these curves on their upper " working part " is unimportant, but the student should see clearly the reason for discrepancies at low values of the exciting current.

(b) *Effect of varying speed.* Assume that the exciting current, and consequently the flux, is kept constant, and that the speed of the machine is varied from zero to the highest safe value. According to the fundamental law of induction, the induced voltage is proportional to the rate at which the armature conductors cut the flux. Therefore, with constant excitation, the relationship between the induced voltage and the speed of rotation is a straight line through the origin (Fig. 202). Five such straight lines are shown, each corresponding to a different value of the exciting current. At any given speed, it will be noted that increasing the field current from 1.6 to 2.0 amperes does not raise the voltage as much as increasing the current from 1.2 to 1.6 amperes. This is due to the saturation in the magnetic circuit of the machine,

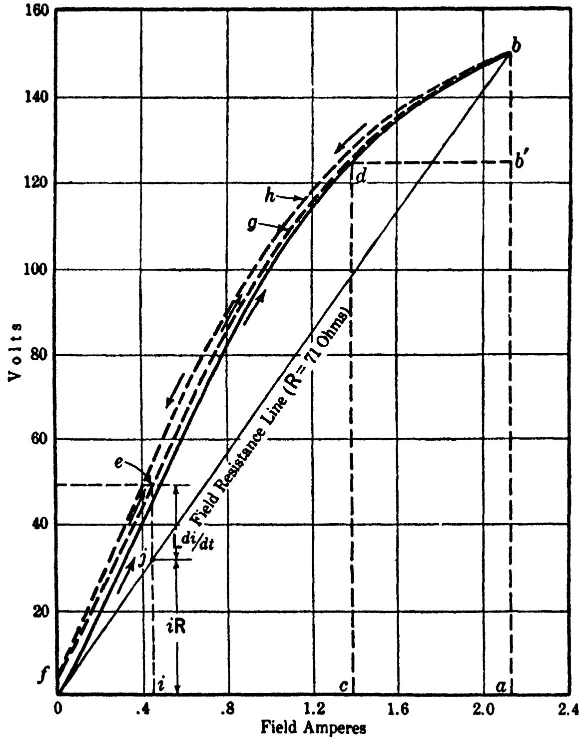

FIG. 201. No-load saturation curve of a direct-current generator.

(c) *Effect of changing the winding.* It is not so easy to investigate in the laboratory the effect of the number of armature conductors Z upon the induced emf. Manufacturers of electrical machinery often use this relationship in changing a generator from one rated voltage to another. For example, a standard 100-kw, 220-volt generator may have 400 armature conductors of a certain size, placed in 100 armature slots. Should a 110-volt generator of the same speed and output be desired, the same frame and armature may be used, but the required number of conductors would be 200, and each conductor would be of about double the cross-section, thus occupying practically the same space in the slot as two conductors of the standard machine. The current density and the I^2R loss in the two machines will be nearly the same.

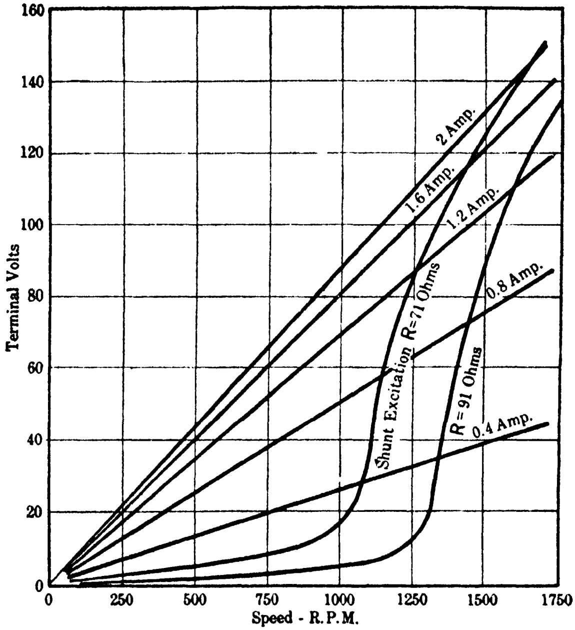

Fig. 202. Induced voltage at no load, at different values of exciting current as a function of speed.

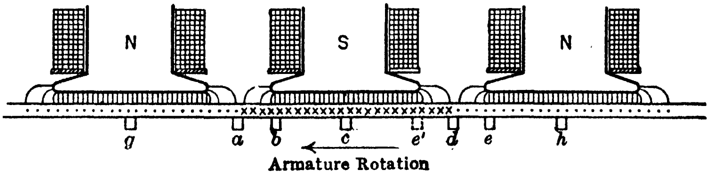

Fig. 203. Diagram to show the effect of brush shift upon the induced emf.

235. Brush Shift. — Another factor which affects the induced emf is the position of the brushes with respect to the field poles. Part of the armature and field of a d-c machine is shown in Fig. 203. The direction of the emf's induced in the individual conductors is shown by the usual dots and crosses. At no load, and with the brushes midway between the poles (that is, in the position *ad*), the action of all the conductors under one pole is cumulative, and the induced voltage is a maximum. The brushes are said to be placed on *the geometric neutral.*

Let the brushes be shifted into position *be*, and let *e'* be a position (*not* an actual brush) such that *de* = *de'*. Then the voltages induced in the groups of conductors *de* and *de'* neutralize each other, and the active group of conductors is reduced to *be'*. The emf's induced in the belt *ee'* are comparatively small, because the conductors are subjected to a reduced flux density between the main poles. For this reason the total induced voltage is not appreciably affected with small angles of brush shift. When, however, the brushes are shifted beyond the edges of the poles, the induced voltage is rapidly reduced and becomes zero when the brushes are opposite the centers of the poles, i.e., in the position *ch.*

Sparking may occur under the brushes when they are shifted into such a position that the corresponding armature conductors are in a strong magnetic field. This sparking is due to the emf and the resulting circulating currents induced in the coils short-circuited by the brushes through adjacent commutator segments (Fig. 267). When the contact with a commutator segment is broken, the inductance of the coil causes a spark to be maintained across the widening gap between the edge of the brush and the segment. For details see Chapter XV.

In recording the quality of commutation it is convenient to use some such comparative terms as perfect commutation, slight spark, pin spark, spark in spots, hot spark, vicious sparking, near flash, etc., or other terms descriptive of the observed phenomena.

When plotting results of tests or computations relating to induced emf., commutation, etc., it is convenient to use the so-called *electrical degrees* for abscissas. The distance between the centers of two adjacent pole-pieces, or between two sets of brushes of opposite polarity, is always taken as 180 electrical degrees. Curves so plotted are independent of the actual number of poles of the machine.

With a given polarity of the field poles, the polarity of the brushes is reversed when the armature is driven in the opposite direction. This follows from the fundamental law of induction. The polarity is also reversed when the exciting current is made to flow in the opposite direction, because the flux polarity of each pole becomes reversed.

236. EXPERIMENT 10-A. — No-Load Characteristics of a Separately Excited D-C Generator. — The purpose of this experiment is to study the performance of a separately excited d-c generator at no load, and to illustrate its theory, as explained in the preceding articles. Connect the machine for separate excitation (Fig. 200), and arrange to drive it by an adjustable-speed motor. For the first two parts of the experiment the brushes should be set on the geometric neutral. Use a data sheet similar to the one shown below.

	Field Amps.	Volts	Speed	Remarks
Instr. No.				
Constant. . .				

(1) *No-load saturation curve.* With the generator running at its rated speed, bring the field current up to as high a value as possible, by varying the rheostat setting. Record this maximum field current and the corresponding voltage and speed. Throughout this run the speed should be kept as nearly constant as possible, and small deviations recorded, in order to correct the observed curve accordingly. Reduce the field current slightly and again read the voltage and the speed. Be careful not to change the residual magnetism by reducing the field current too much and then raising it again. (Every time that is done the saturation curve will appear somewhat irregular.) Proceed in this way for a large number of values of the field current, from maximum to zero. This will give data for the descending branch *bhf* (Fig. 201) of the saturation curve. To obtain the corresponding data for the ascending branch *fgb*, begin with the field circuit open and read the residual voltage. Then, with the two contacts of the potentiometer resistance brought together, or, when a series resistance is used, with all the field resistance in the circuit, close the field switch and read the current, the voltage, and the speed, as before. Increase the field current in steps, recording the same data at each step, and use as nearly as possible the same values of field currents as in the descending branch. The warning about overrunning the values of the field current applies here also. If desired, the field poles may be demagnetized (§187, under "normal induction") and the virgin saturation curve *Odb* taken.

(2) *Speed characteristic.* The purpose of this run is to obtain the relationship shown in Fig. 202. Bring the speed of the machine up to the highest permissible value and excite the field with the highest possible

current. Record the current, the voltage, and the speed. Now, holding the field current constant, reduce the speed in small steps to zero, taking similar readings at each step. Repeat this run with two or three lower values of field current, always beginning with the maximum speed, and keeping the field current constant.

(3) *Brush shift.* Provide a template by means of which the position of the brushes may be determined either in degrees or in terms of commutator segments, with reference to the geometric neutral of the machine. With the generator running at a constant speed, shift the brushes backward and forward to satisfy yourself that the voltage is a maximum when they are in the neutral position. Shift the brushes in steps between the neutral and the quadrature position and at each step record the angle of shift and the voltage; also note the quality of commutation, that is, the amount of sparking, if any. The speed and the field current should be kept constant, or small deviations recorded. The same data sheet may be used by adding another column for the angle of brush shift.

Before leaving the laboratory, note the number of poles of the machine and count the total number of commutator segments.

(4) *Direction of induced emf.* Prove by experiment that reversing either the field current or the direction of rotation reverses the polarity of the machine, and that reversing both leaves the polarity unchanged. In recording the results of this experiment the direction of rotation is considered clockwise or counter-clockwise, looking at the commutator end of the machine. Make a rough sketch of the generator and denote the armature and field terminals by letters or numerals. Record the field current as entering the winding, say through terminal *C* and leaving through terminal *D*. Similarly indicate the direction of the induced emf by marking the plus and minus armature terminals.

Report. (a) Using the data obtained in part (1) of the experiment, plot the descending and ascending saturation curves. Explain why different values of voltage were obtained for the same field current in these two curves. (b) Plot the results of run (2) as in Fig. 202, and explain why the curves are straight lines. Find the value of the induced voltage for normal speed on each of these curves and check with the saturation curves obtained in part (1). (c) Plot the results obtained in part (3), using the values of induced voltage as ordinates and the angles of brush shift (in electrical degrees) as abscissas. Explain why the curve has only a slight slope for small angles of shift, and why the slope becomes considerable as the angle becomes larger. (d) Tabulate the results of part (4), giving the polarity of the field and armature terminals and the direction of rotation. Show from these results that the polarity of the brushes is reversed when either the field current or the direction of rotation is reversed.

237. Self-Excitation. — In the introductory treatment, the generator has been assumed to be excited from a separate source. In practice the machine is usually made to excite itself; in other words, the field current is generated in the armature of the machine. One of the usual connections between the field winding and the armature of a self-excited d-c generator is shown in Fig. 204. The field winding consists of many turns of comparatively small wire, and is connected across the brushes of the machine in series with a regulating rheostat, so that the exciting circuit is in parallel with the external circuit. Such a generator is said to be *shunt-wound*, or *shunt-excited*, the field winding being " shunted " across the armature terminals.

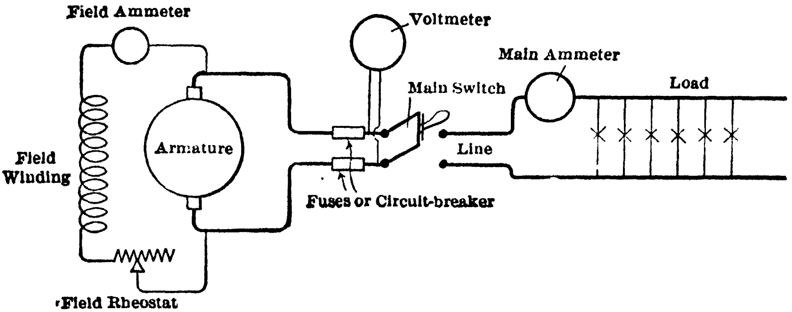

FIG. 204. Diagram of connections of a shunt-wound generator and load.

The no-load saturation curve (Fig. 201), which represents the relationship between the field current and the induced voltage at a constant speed, is practically the same whether the machine is self-excited, or separately excited. This is because the exciting current amounts to but a few per cent of the rated armature current, and hence its voltage drop in the armature and its armature reaction are very small. Similar curves for other than normal speeds are of interest in studying the initial self-excitation of the shunt-wound generator (Fig. 205). The ordinates of these curves, for any given field current, are proportional to the speed, since the emf generated in the armature winding is proportional to the rate of cutting the magnetic flux.

With the field connections shown in Fig. 204, the self-excitation is made possible because of the *residual magnetism* in the iron parts, which, after being magnetized, retain a small part of their magnetism (§184). Let the machine be running somewhere near its rated speed, and let both the load circuit and the field circuit be open. The residual magnetism induces in the armature a small voltage, and this voltage is available at the brushes. Now, if the field circuit be closed, a small current will flow through the field winding. With the proper polarity, this current strengthens the original flux, so that a higher voltage is induced in the armature. This higher voltage causes a larger current to flow in the field winding, which current in turn increases the flux, etc., until the flux, the induced emf, and the exciting current reach their full stable values.

Thus, suppose that during this building-up process the emf is passing through the point marked e in Fig. 201, at which point the field current has the value i. This induced voltage, impressed across the field circuit, is consumed in two ways, namely; as iR drop in the total field circuit, and by the counter emf of self-induction, $L\,di/dt$, of the field winding, or

$$e = iR + L\,di/dt$$

The straight line Ob from the origin shows the relation between resistance drop iR and the field current i. It is called the " field resistance line "; numerically the slope of this line equals the resistance in the field circuit. A line from e perpendicular to the axis of abscissas cuts this field resistance line at j giving ij equal to the iR drop and ej, the remainder of the induced voltage, consumed by the changing current in the self-inductance of the field, or $ej = L\,di/dt$. Obviously the current continues to rise until iR equals the generated voltage $e = ab$ when $L\,di/dt$ is zero and the current has the value $i = Oa$ (Fig. 201). Evidently also, where $ej(= L\,di/dt)$ is great the voltage builds up rapidly, and vice versa.

238. Limits and Direction of Self-Excitation. — Let the line Of (Fig. 205) be the field resistance line with all resistance out of the rheostat. The limiting voltage of self-excitation at normal speed is determined by the point b of intersection of Of and the saturation curve Og as explained in §237.

Now let the resistance of the field circuit be increased so that the resistance line becomes Of'. The machine will build up its voltage only to b', which is less than b. Finally, let the resistance of the field circuit be so increased that the resistance line assumes the position Of''. In this case there can be no great building up since the straight line is practically tangent to the saturation curve, and the current produced in the field circuit by the residual magnetism is not sufficiently large to produce a large increase in the flux. As indicated, the saturation curve does not start from the origin, because of the residual magnetism, while Of'' does pass through the origin. Consequently, Of'' must cut the saturation curve at a point above that represented by the residual magnetism and raise the voltage somewhat. Practically, however, there is a definite limit to the field resistance, above which there is no appreciable building up of induced voltage. This is called the *critical resistance.*

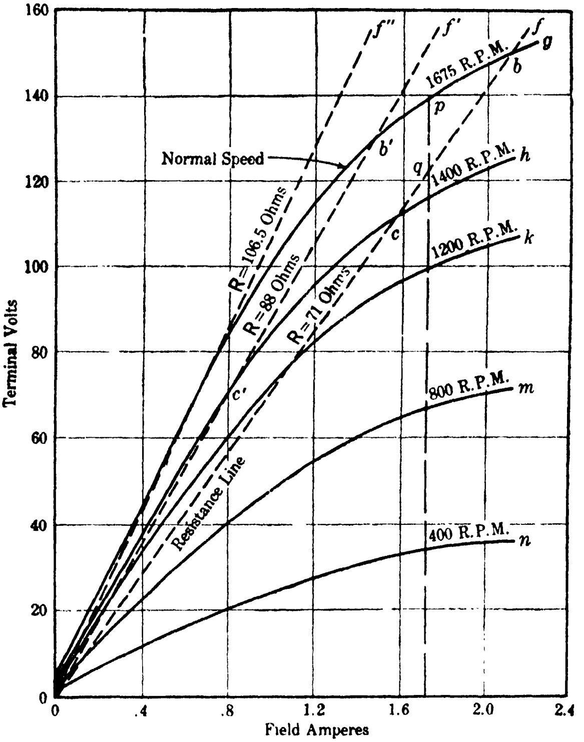

FIG. 205. No-load saturation curves of a 3.75 kw, 125-volt shunt generator, at different speeds.

So far, we have considered the process of self-excitation at the rated speed. At a lower speed, for example, that represented by the curve *Oh*, the same resistance lines *Of* and *Of'* intersect the saturation curve at lower voltages corresponding to points *c* and *c'*. The resistance line *Of'* is almost tangent to *Oh*, so that at this speed the machine will lose its voltage when the field resistance is slightly increased beyond that corresponding to *Of'*. We thus see that at a lower speed the generator has a lower critical value of field resistance. There is a speed, sometimes called the *critical speed,* below which no appreciable building up of voltage will occur even with all the external resistance cut out of the field circuit. The resistance line of the field winding itself is in this case practically tangent to the saturation curve at that speed.

With separate excitation (§234), the polarity of the induced voltage is reversed with the reversal of either the field current or the direction of rotation, but remains the same when both are reversed simultaneously. Now, with shunt excitation, let either the field connections or the direction of rotation be reversed. The induced voltage due to the residual magnetism will force a current through the field winding in the direction in which this current will tend to destroy the original residual magnetism, and the generator will not build up. When both the field connections and the direction of rotation are reversed the machine will build up as at first.

239. EXPERIMENT 10-B. — No-Load Characteristics of a Self-Excited Shunt-Wound Generator. — The purpose of this experiment is to determine the relation between the exciting current, the speed, and the terminal voltage at no-load, and to investigate the ability of the machine to build up its voltage. Connect up the generator as shown in Fig. 204, except that no load is necessary, and arrange to drive it by an adjustable-speed motor. If the voltage does not build up, try reversing the field connections. If the field still fails to build up, excite it from a separate source (Fig. 200), so as to restore the residual magnetism in the right direction. Use a data sheet similar to the one given for the preceding experiment.

(1) *Saturation curves.* Take the data necessary for plotting the descending branch of the no-load saturation curve at normal speed, as explained in the experiment on separate excitation (§236). Take similar data at two other speeds, for example, at 85 and at 70 per cent of the normal.

(2) *Maximum field resistance for building up.* Drive the generator at its rated speed and find the rheostat setting at which the field just begins decidedly to build up. Read the field current, the final voltage, and determine the time required for the voltage to reach its final value. Repeat for one or two other speeds. From the terminal voltage and final field current calculate the total resistance of the field circuit in each case.

(3) *Effect of speed upon generated voltage.* (a) With the external field resistance cut out, gradually increase the speed in small steps, reading volts, field amperes, and rpm, until the allowable range of speed has been covered. Record the speed at which the voltage just starts definitely to build up. (b) Increase the field resistance, say 20 per cent, and take similar readings of voltage and field current as speed is increased. See Fig. 202. (c) Repeat the preceding runs with one or two higher values of external field resistance.

(5) Explain the comparative shapes of the internal characteristics taken at different speeds. (6) For one or two characteristic curves, calculate the value of the critical resistance. Is this resistance a function of the speed of the machine? (7) Explain the comparative shapes of the external characteristics with the field winding shunted. Show that by this means the machine can be made to deliver a constant current with a varying terminal voltage and load resistance. (8) Describe and explain the results on the building up of the generator.

CHAPTER XI

DIRECT-CURRENT GENERATOR (*Continued*)

248. Compound Winding. — The preceding study of the shunt-wound generator shows that when the field rheostat is left unchanged the terminal voltage decreases appreciably as the load current increases. For many purposes the terminal voltage must be kept at least approximately constant, and this can be accomplished in three ways:

(1) The field rheostat is adjusted from time to time by hand.

(2) The field current is adjusted continuously by an automatic regulator (§§252 to 254).

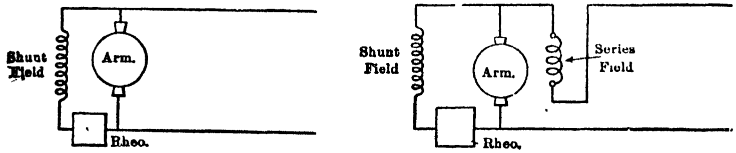

FIG. 212. Shunt-wound generator to the left, compound-wound generator to the right.

(3) A second field winding is provided (Figs. 212 to 214) through which the load current (or the armature current) of the machine is made to flow. Thus, as the load of the machine increases, the number of exciting ampere-turns is automatically made larger, and the voltage is correspondingly raised (Fig. 215).

When a generator, in addition to a shunt field winding, is provided with a series winding, it is said to be *compound-wound* (Fig. 212). The shunt winding in this case furnishes the excitation necessary to produce the rated voltage at no load. The series winding increases the generated voltage by an amount approximately proportional to the load, as required to compensate for the armature reaction and for the voltage drop in the series circuit of the machine itself. In the most general case this includes the voltage drop in the armature, the brushes, the series winding, the interpole winding, and the compensating winding, if present. If the machine is situated at a considerable distance from the place where the voltage must be maintained constant, the number of turns in the series winding may be increased so as to cover the line drop as well. In this case the voltage of the machine is greater at full load than at no load, and the generator is said to be *over-compounded*. When the number of series ampere-turns is such that the terminal voltage at full load is practically equal to that at no load the generator is said to be *flat-compounded.*

In a compound-wound machine the shunt winding may be connected either across the armature alone (Fig. 213), or across both the armature and the series winding (Fig. 214). The first connection is called the *short shunt* and the second, the *long shunt.* Since the voltage drop across the series winding hardly amounts to 1 per cent of the terminal voltage, the electrical characteristics of the two machines are approximately the same. One or the other type of connections is used, according to the local conditions.

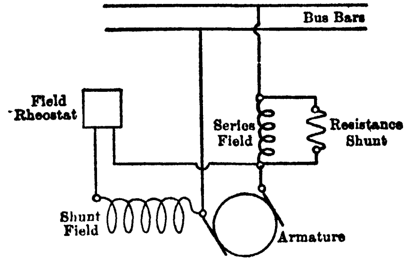

FIG. 213. Diagram of connections in a compound-wound generator, with the shunt-field winding connected across the armature (short shunt).

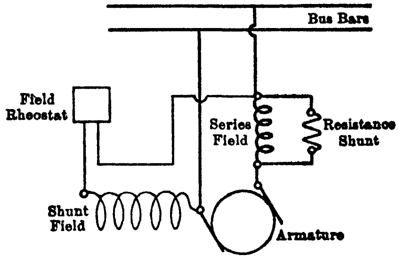

FIG. 214. Diagram of connections in a compound-wound generator, with the shunt-field winding connected across the terminals of the machine (long shunt).

249. Deviations from Constant Voltage. — When the series winding has been so adjusted that the voltage at full load is the same as at no load (curve $fc'e'$ in Fig. 215), the voltage at the intermediate load is somewhat higher than would be required by an ideal flat compounding. The ideal straight line is marked $fb'e'$. To keep the voltage absolutely constant, small adjustments of the shunt-field rheostat must be made by hand, unless an automatic voltage regulator is provided in addition to the series winding. The existence of this " hump " on the voltage curve may be explained as follows: Let the required number of field ampere-turns at no load be 8000; these must be provided by the shunt winding (see table below). Let the total required number of exciting ampere-turns at full load for flat compounding be 11,000. Of this number the shunt winding furnishes 8000 and the series winding 3000. At half load the machine is not so highly saturated by armature reaction (see §363) as at full load, so that the number of ampere-turns is less than 1500 and may be only 1000. Thus the total excitation actually required at half load for flat compounding is 9000. But in the machine the series-field winding furnishes 1500 ampere-turns at half load, so that the total excitation at this load is at least 9500. In reality it is more, because the current in the shunt-field winding is increased by the higher armature voltage. The resulting excitation is therefore higher than that required to produce the normal voltage, and the "hump" in the curve results.

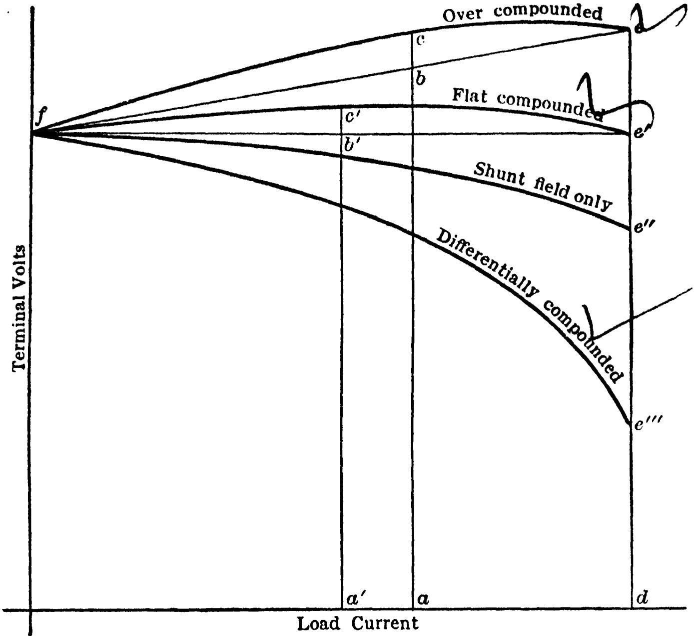

FIG. 215. Effect of series-field winding on voltage characteristics of a generator.

Per cent Load	Excitation Required	Excitation actually Obtained
0	8000	8000
50	9000	over 9500
100	11,000	11,000

It is difficult for a manufacturer to design a machine with just the right number of series turns for a desired percentage of voltage rise. The permeability of iron may be somewhat different from that assumed in the calculations, and the effect of armature reaction cannot be predetermined with very great accuracy. Moreover, different users may desire the same type of machine with different degrees of over compounding. Therefore generators are usually provided with a number of series turns sufficient for a reasonably high degree of over compounding. The *series winding is then shunted* or weakened, as desired, by resistance strips or grids (Figs. 213 and 214), so that only a part of the total current flows through the series field. By varying the resistance of the shunt, a desired characteristic may be obtained without changing the design of the machine.

If the terminals of the series-field winding be reversed so that the series excitation opposes that of the shunt field, the machine is said to be *differentially compounded,* that is, the series excitation increases the voltage drop for any given load. Such machines are used for special purposes only, for example, when it is necessary to limit the load current to a safe maximum value as the load resistance is decreased. The characteristics of the machine approach those of a constant-current generator rather than those of a constant-voltage generator.

In view of the deviation of the external characteristic (Fig. 215) of a compound-wound generator from the straight line, it is convenient to characterize this departure as the ratio (or percentage) of the maximum departure from the straight line to the straight-line voltage at the same point. The straight line is drawn between the points of no-load and full-load voltage on the actual curve. For the upper curve the departure ratio is bc/ab; for the flat-compound curve the departure ratio is $b'c'/a'b'$. In an ideal machine, without saturation, the voltage curve becomes a straight line and the percentage departure becomes zero.

250. Number of Turns in the Series Winding. — The necessary number of turns in the series winding for flat compounding may be determined by successive trials, or by the following experiment. Let the machine under test be a 110-volt, 200-ampere generator, and let it give its rated voltage at no load with a field current of 5.5 amperes. With the series winding disconnected, and the field rheostat setting unchanged, the voltage will drop as the machine is loaded. Suppose further that the field current must be increased to 6.1 amperes to maintain the rated terminal voltage at full load. If the total number of turns in the shunt winding is 2400 per pole, then $(6.1 - 5.5) \times 2400 = 1440$ ampere-turns represents the additional excitation to be furnished by the series winding. At full load the current is 200 amperes; consequently there must be about 8 turns in the series winding $(200 \times 8 = 1600)$. This is somewhat larger than is necessary, but an allowance must be made to compensate for the ohmic drop in the series winding itself, and the surplus may be reduced by a resistance shunt around the series turns.

The number of turns required for a certain degree of over compounding may be determined in a similar manner. With the same data as before, let it be required to provide a sufficient number of turns in the series winding to increase the full-load voltage to 116 volts. To determine experimentally the required number of series turns, load the machine, and increase the shunt-field current until it gives 116 volts at 200 amperes. Let the necessary field current be 6.8 amperes, or the total number of exciting ampere-turns per pole 6.8 × 2400 = 16,320. With the series winding in the circuit, the shunt current at full load will be 5.5 × 116/110 = 5.8 amperes, and the corresponding number of ampere-turns will be 5.8 × 2400 = 13,920. Thus the series winding will have to supply 2400 ampere-turns, which means 12 turns at 200 amperes.

If the number of turns in the shunt winding is not known it can be determined experimentally. Excite the machine separately by an auxiliary winding of a known number of ampere-turns and measure the terminal voltage at no load. Determine the current in the shunt-field winding necessary to induce the same voltage at the same speed. Let it take 72 turns of the auxiliary winding at a current of 100 amperes to produce 50 volts at the terminals, and let it take 3 amperes to induce the same voltage by using the shunt winding. The voltage in the two cases being the same, the flux per pole is also the same, and consequently the numbers of exciting ampere-turns must be equal. Thus the unknown number of turns in the shunt winding is 100 × 72/3 = 2400.

The number of turns in the shunt-field winding may also be determined ballistically (§177) by providing one of the poles with an auxiliary exciting winding of a known number of turns and with a tertiary winding connected to a ballistic galvanometer. The armature is kept stationary and the shunt-field winding on the pole under test is disconnected from the rest of the pole windings. The pole under test is then excited once by means of the regular field coil and then by means of the auxiliary winding. The two currents are so adjusted that the deflections of the ballistic galvanometer upon a reversal of the current are the same in the two cases. This means that the flux, and consequently the number of exciting ampere-turns, is also the same. Thus we have the condition $n_1 i_1 = n_2 i_2$, from which the number of turns in the shunt-field winding may be computed.

251. EXPERIMENT 11-A. — Exercise in Compounding a D-C Generator. — The purpose of this experiment is to investigate the action of the compound winding explained in §§248 to 250. The machine used for this experiment should be a shunt-wound generator, and the student himself is expected to predetermine and provide a suitable series winding for different amounts of voltage rise.

(1) Take an external characteristic of the machine as a shunt-wound generator, without the series winding (§242), having first set the field rheostat so as to get the rated voltage at full-load current. Keeping the rheostat in this position and beginning at no load, increase the load in steps until the machine is considerably overloaded. At each step record the field current, terminal volts, and the load current. This will give the data necessary for calculating the voltage regulation of the machine without the series winding.

(2) To obtain the number of turns that must be supplied by the series winding for flat compounding, measure the field current necessary to produce the normal voltage at no load and subtract this current from that required to produce the normal voltage at full load. Determine the number of turns in the shunt winding by the method given in §250 and from these data compute the required number of turns in the series winding to make the generator flat-compounded (§250), and actually place a slightly greater number of turns on the machine.

(3) Adjust a shunt around the series winding so as to have the same voltage at full load as at no load, and take data necessary for plotting an external characteristic of this compound-wound generator at normal speed and normal full-load voltage.

(4) Determine the number of series turns necessary to over-compound the machine, say by 10 per cent, and check the results experimentally. Shunt the new series winding by a variable resistance and adjust it so as to make the machine flat-compounded as before.

(5) Reverse the connections to the series-field winding and take the data necessary for plotting an external load characteristic, beginning with the rated voltage at no load (differential-compound winding).

(6) Disconnect the shunt-field winding and take the necessary data for plotting a curve of the voltage produced by the series field alone (series-wound generator, §247).

Report. (1) Plot the external characteristics of the machine without the series winding, and for all the cases where it was used. (2) Plot the saturation curves obtained by using the shunt winding alone and the auxiliary turns alone. Explain how the number of turns in the shunt winding was obtained from these two curves and give the numerical results. (3) Explain how the number of turns in the series winding was predetermined to make the generator flat-compounded, and over-compounded. (4) Give the results of applying the resistance shunt to the series winding. (5) Calculate the regulation of the machine as a shunt, flat-compounded. over-compounded and differentially compounded generator.

AUTOMATIC VOLTAGE REGULATORS

252. Vibrating-Type Regulator. — The compound winding described above does not give a perfect voltage regulation, nor does it correct for speed fluctuations of the prime mover. Moreover, the machine must be made somewhat larger and heavier, in order to accommodate the series winding. Various attempts have been made to do away with the series winding and to regulate the voltage of the machine automatically — for example, by solenoids acting on the shunt-field rheostat. An objection to such regulators is that they are rather sluggish and do not follow variations in voltage quickly enough.

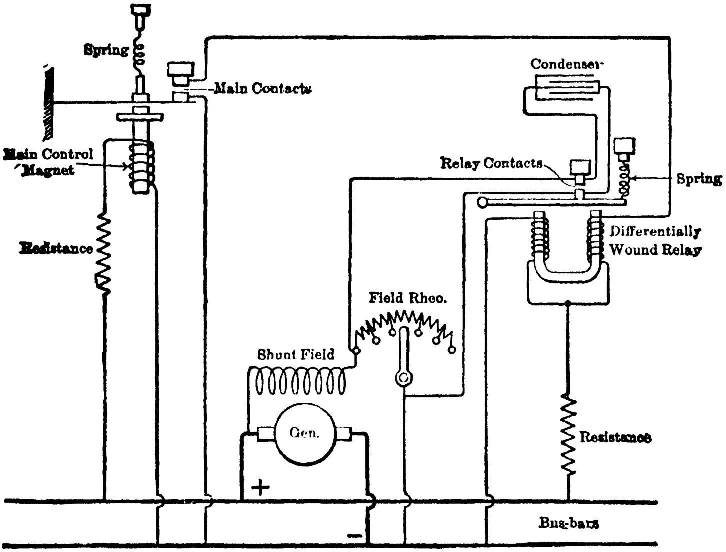

FIG. 216. Diagram of connections of a vibrating-type voltage regulator for use with a direct-current generator.

Mr. Tirrill, inventor of the vibrating-type voltage regulator which bears his name, made use of the idea of periodically short-circuiting the field rheostat, or a portion of it (Fig. 216), whenever the voltage was below normal, thus allowing the field current to rise and to increase the voltage generated. This is done by means of light vibrating contacts built to have a minimum of inertia. The relative lengths of time during which the rheostat is short-circuited or active determine the average value of the field current and consequently the voltage of the machine. The leads from the field rheostat are short-circuited between contact tips marked " relay contacts "; a condenser is placed across the contacts to reduce sparking. These contacts are closed and opened by the differentially wound electromagnet, or relay, controlled by the "main contacts." The main contacts are closed and opened by the control electromagnet connected across the main bus-bars.

The action of the regulator is as follows: Let the main contacts be open; then only the left-hand side of the differential electromagnet (see Fig. 216) is energized, whereupon it opens the relay contacts and introduces the field rheostat into the circuit. The terminal voltage of the machine immediately drops below normal. This reduces the current in the control electromagnet, and the spring above it overcomes the pull exerted upon the small iron armature and closes the main contacts. Now a current flows through the right-hand branch of the differential relay and destroys its previous magnetization. The spring above it closes the relay contacts and short-circuits the field rheostat, causing the field current to rise. The line voltage rises slightly above normal, the main contacts are again opened, and the operations of the two electromagnets are repeated in the same order. In reality, both contacts are continually vibrating; the periods during which the relay contacts are closed adjust themselves automatically, so as to maintain a constant voltage at the terminals of the machine.

An effect similar to over compounding is produced by providing an additional differential series winding on the main control electromagnet. The voltage then rises as the load increases. It is not necessary for the whole main current to flow through this winding, but only for a small current proportional to it. To accomplish this, a low-resistance shunt is inserted in the main circuit; the series winding of the electromagnet is connected across the terminals of this shunt, as in a d-c ammeter (Fig. 43). The series winding opposes the action of the shunt winding; therefore, with a heavy load on the line the bus-bar voltage must be higher before the control magnet can overcome the tension of its spring. In other words, the spring keeps the main contacts closed for a comparatively longer time.

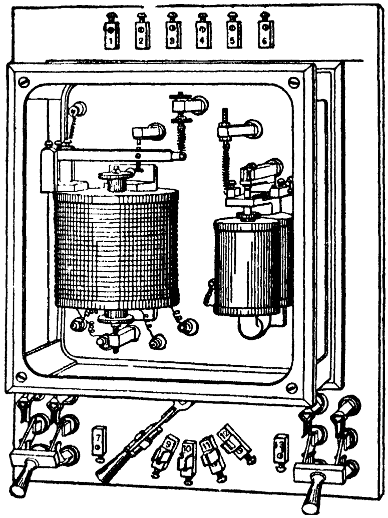

Fig. 217. Tirrill voltage regulator for direct-current generators.

Figure 217 shows the external view of a Tirrill regulator; the main contacts are seen to the left and the differential electromagnet to the right. The radial switch below is for connecting the regulator to the field of any of the machines working in parallel. The double-throw switches are for reversing the polarity of the contacts so as to wear them equally.

253. EXPERIMENT 11-B. — Study of the Vibrating-Type D-C Voltage Regulator. — The regulator described in the preceding section is connected as shown in Fig. 216, and its performance studied under various conditions met with in practice. Each factor which affects the performance of the regulator should be studied separately, keeping all other factors constant. A very explicit and detailed book of instructions, furnished by the makers of the device, gives all the necessary directions for adjustments and operation. Vary the current output of the machine, the terminal voltage, the setting of the rheostats, the adjustment of the springs, etc. Vary the load gradually, and then suddenly. Observe the promptness of action of the regulator under the extreme conditions. Devise some means for finding out if there is any flicker in the terminal voltage of a machine controlled by a Tirrill regulator. If an oscillograph or a recording voltmeter is available take actual curves of the terminal voltage under a few extreme conditions.

Report the results systematically, giving a theoretical explanation of the principal features of the observed performance.

254. Counter-EMF Voltage Regulator. — From the above description of the vibrating-type voltage regulator it will be seen that the main contacts are closed or opened only when the line voltage is slightly below or above normal. This sometimes causes an objectionable flicker, noticeable in lamps. For this reason another regulator has been developed in which a small " counter-emf motor " is connected in series with the shunt field of the generator, and the current through the latter is regulated by changes made automatically in the counter emf of the motor. The motor field is energized from the bus-bars through a resistance. This device is also provided with a main control magnet and contacts corresponding to the main contacts of the Tirrill regulator. These contacts are connected across the motor field so that they short-circuit it when they are closed.

The counter-emf motor is equipped with an eddy-current brake consisting of a low-resistance metal disk which rotates in a strong magnetic field produced by one or more magnets energized by the generator field current. This brake prevents the motor from overspeeding and gives it greater uniformity of rotation.

The generator field rheostat is so adjusted that without the regulator it holds the voltage somewhat above normal. Consequently, the control contacts are originally open, putting full field on the motor. The counter emf induced in the motor opposes the generator field current and at once lowers the generator voltage. When the voltage is lowered sufficiently, the main contacts close, short-circuiting the motor field and thus lowering the counter emf. This again causes the generator voltage to rise slightly, and the same cycle is repeated. The contact action is so rapid that an average field is held on the counter-emf motor, and its action varies from time to time with changes in the load and in the speed of the main generator.

Sometimes an additional winding is provided over the main winding on the control magnet, to prevent hunting and to insure a speedy contact action. The leads to this " speed " winding, when used, should be connected to give the most rapid regulation. This anti-hunting winding opposes the main winding when the contacts are open, thus causing them to close rapidly again, preventing the building up of the counter emf.

A detailed instruction book and an exact diagram of connections are furnished by the makers of this device. For an experimental study of it, see the preceding experiment on the vibrating-type regulator.

In addition to the types of regulator described above, mention should be made of the " quick acting " regulator of the Brown-Boveri type in which the resistance in the field of the generator is varied by a rocking contact; also of the carbon-pile regulator (see §560) in which the pressure between carbon disks placed in the field circuit is varied by a coil placed across the generator voltage. A rise in voltage causes the coil to diminish the pressure on the disks, thus increasing the resistance in the generator field, which lowers the voltage. See "The Carbon-pile regulator," by E. H. Wolff, in *Elec. Jour.*, for Feb., 1930.

OPERATION OF GENERATORS IN PARALLEL

255. It is common practice in power houses to have more than one generator connected to the same bus-bars and operated simultaneously (Fig. 218). This gives a more flexible and economical arrangement than one large machine of a size sufficient to supply the heaviest load of the day. The chief advantages of having more than one machine are:

(1) The number of machines actually used during certain hours can be made to correspond more closely to the demand; this permits the operation of the machines near the point of their maximum efficiency.

(2) An accident to one machine does not shut down the whole station.

(3) In large power houses it is necessary to use several machines, simply because the total output is considerably above the capacity of the largest generator built.

With the system of distribution of electrical energy used at present (constant-potential system), machines which operate simultaneously are connected *in parallel*, so that each runs at full voltage and supplies a part of the current demand.

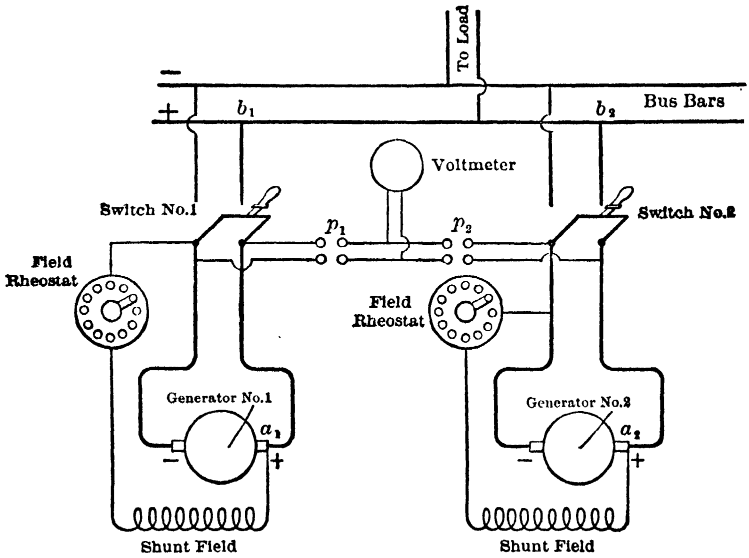

FIG. 218. Two shunt-wound generators in parallel.

Certain conditions must be fulfilled in order that two or more machines may supply power to the same bus-bars without interfering with each other, and that each may take a proportionate share of the total load. When but one machine is connected to the line, the power generated can only flow into the load. When, however, two generators are connected to the same line, the current from the first machine may flow both to the line, and also into the second machine, driving it as a motor. In this case, the second machine not only does not help the first one, but constitutes an additional useless load for it. Even if this is prevented, and both machines are working as generators sending power into the line, it does not necessarily follow that each machine is taking its proper share of the load. The conditions necessary for satisfactory parallel operation of two or more direct-current machines are discussed in the following articles. Shunt-wound machines are considered first, and then compound-wound machines.

256. Shunt-Wound Generators in Parallel. — Two identical shunt-wound machines driven by identical prime movers and with the governors properly set, usually run satisfactorily in parallel, and automatically divide the load equally, for there is no reason why one machine should carry a larger share of the load than the other. Should one machine tend to take more load, its voltage would drop, and the other machine with is higher voltage would automatically assume a greater share of this load.

A diagram of connections for two shunt-wound generators in parallel is shown in Fig. 218. Let machine 1 be supplying the whole load at a certain time, and then let the load increase to such a value that it becomes necessary to add the other machine. Generator 2 is first brought up to speed and excited so as to give the same terminal voltage as 1 (and the right polarity, of course); then its main switch is closed. The first machine still continues to supply the total load, unless the excitation of the second machine is somewhat *increased* and that of the first machine *reduced* (in order to keep the bus voltage constant). By regulating the field rheostats of both machines the load can be divided in any desired proportion. One voltmeter only is used with any number of machines in parallel. Each machine is provided with voltmeter terminals, such as p_1 and p_2; a plug is inserted in one of the receptacles, and thus connects the voltmeter to the desired machine.

If it is desired to disconnect machine 1 and to transfer the whole load to the other machine, this should *not* be done by simply opening the main switch of the first machine. This would cause an arc at the switch, the line voltage would suddenly drop, and the second machine might be damaged by the extra load suddenly thrown upon it. This may not be noticeable on small laboratory generators, but is of consequence with large machines. Therefore, before the main switch of one of the machines is opened, its field excitation must be *lowered* and that of the other machine or machines *increased,* until the machines which are to continue in operation carry practically the whole load. Then the main switch of the first machine can be opened without disturbing the established electrical relations. In some cases the throttle on the prime mover also has to be adjusted in transferring the load from one machine to the other.

By reducing the field excitation of the first generator below the no-load point, its armature current is reversed, and the machine begins to run as *a shunt motor,* driving its prime mover. This, of course, must be avoided in practice, but should it accidentally occur, no particular harm would result, as the machine continues to run in the same direction. In some installations, reverse-current relays are provided in connection with circuit-breakers to open the circuit of the machine in case the current becomes reversed.

If the two machines have different voltage characteristics (Fig. 219), that is to say, if the curve of voltage drop in one differs from that in the other, it may easily occur that although the machines divide the current properly at a certain load, the machine which has the smaller internal drop becomes relatively overloaded on heavier loads, and vice versa. Thus, in Fig. 219 the load currents *ab* and *ab'* are equal to each other, but the load current *cd* is considerably higher than *cd'*. At this latter load the machine whose characteristic is shown to the left will carry less than its proportionate share of the load. Under such circumstances it may be necessary to connect some resistance in the armature circuit of the better machine, until it gives the same per cent drop as the other. Although it is not always possible to make the two external characteristics identical by this means, the two machines can usually be made to work satisfactorily under all load conditions unless the saturation of their magnetic circuits and the percentages of armature reaction differ too greatly.

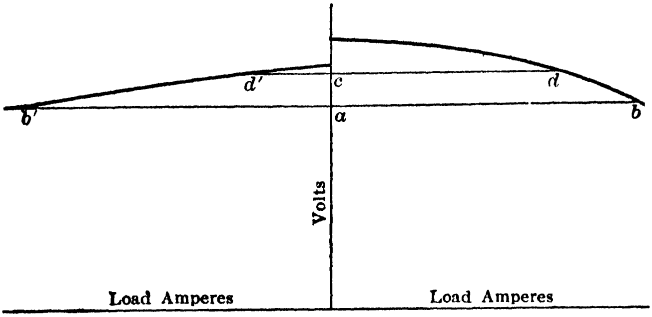

FIG. 219. Division of load between two generators of different characteristics.

257. EXPERIMENT 11-C. — Operation of Shunt-Wound Generators in Parallel. — The student group is given two shunt-wound generators, preferably of different types and with different characteristics. The machines are to be connected in parallel, as shown in Fig. 218, and loaded on a rheostat. Ammeters should be used to measure the total load and that between each machine and the bus-bars. Ammeters should also be provided in the field circuits.

(a) Bring first one machine and then the other to proper speed and voltage and take several points on the external characteristic. With the load of one machine set at normal value adjust the voltage of the other machine to equal that of the loaded machine. Test the polarity of the machines by connecting them on one side only, and read the voltage between the remaining terminals. It should be zero. (Unbalance the voltages and note that there is now a reading of the voltmeter.) With polarity correct and voltages equal, connect the machines in parallel.

(b) Now practice transferring the load from one machine to the other by regulating the field rheostats. Start and stop one of the machines while the other is running, without disturbing the load. Weaken the field of one of the machines until the current in it is reversed and it is driven as a motor by the other. Try to run one or both machines in parallel with the laboratory d-c supply.

(c) Investigate whether the machines divide the current properly at all loads. Take a load nearly equal to the total capacity of the two machines, and adjust the field rheostats so that each machine takes its share of the load. Read the load current and voltage; also the field current, the load current, and the speed, of each machine. Gradually reduce the load, and at each step read the same quantities.

(d) If the machines do not divide the load approximately in the correct proportion, introduce some resistance into the armature circuit of the machine that has the tendency to take a larger share of the load. Adjust the resistance to the best value, and then take a complete curve from full load to no load, showing the division of current between the two machines.

Report. (1) Explain the results obtained in (b), and discuss any operating difficulties encountered.

(2) Plot the external characteristics of the two generators, and from them state the manner in which you would expect the machines to perform in parallel.

(3) Plot the combined external characteristic of the two machines from run (c), and on the same sheet show the percentage of total current delivered by each machine. Discuss these curves with reference to (2) above.

(4) Explain the effect of added resistance in the leads of one generator with reference to the curves obtained.

(5) How would a forward shift of the brushes of one machine affect the parallel operation? Explain in detail.

258. Compound-Wound Generators in Parallel. — Two compound-wound generators connected in parallel to the same bus-bars are shown in Fig. 220. The general scheme of connections is the same as in Fig. 218, except for the addition of the series windings and the equalizer connection explained below. Frequently, the series windings are provided with resistance shunts to insure approximately the same degree of compounding in each machine (Figs. 213 and 214).

The equalizer connection, $a_1E_1E_2a_2$, is provided between the brushes a_1 and a_2, and adjacent to the series windings, since two compound-wound generators in parallel do not operate in a stable manner without it.

The reason for this is as follows: Let the two machines be identical and let the switch E_1 be open at first, so that there is no equalizing connection; let machine 2 for some reason have a slightly higher induced emf. This machine will then take a slightly larger share of the load than machine 1. Therefore, a larger current will flow through the series winding of 2 than through that of 1. The emf of 2 will thereby be *still more increased,* since voltage rises with load in a compound generator, and the share of the load of the other machine will be decreased. This effect will continue until machine 2 not only carries the whole load, but also, unless the machines are provided with reverse-current circuit-breakers, reverses the current in 1 and drives it as a motor. Such an incident would do no harm to a shunt-wound machine, but with a compound-wound machine the reversed current flowing through the series field weakens the total field, and the machine begins to run faster and faster, driving its prime mover. This causes the other machine to become heavily overloaded, until finally one of the circuit-breakers disconnects the two machines. Unless the circuit-breakers act promptly,

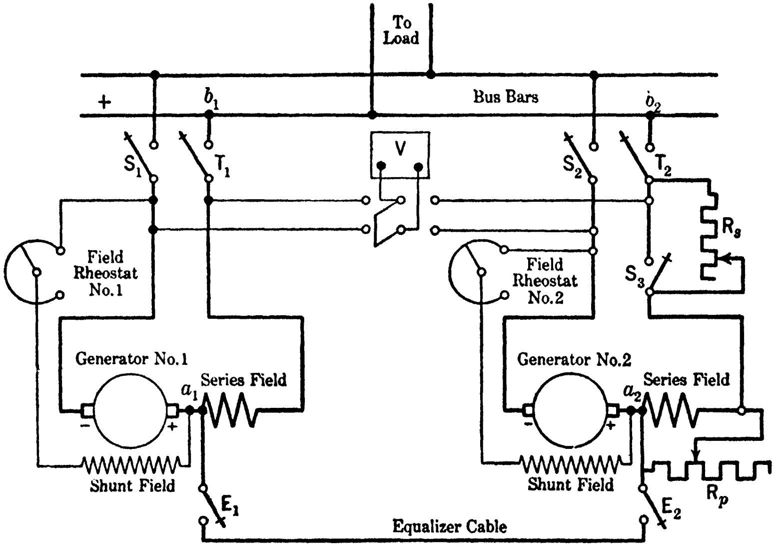

FIG. 220. Diagram of connections for testing two compound-wound generators in parallel.

machine 1 may run away and be destroyed, or a serious "flash-over" may occur at the commutators, owing to the abnormal currents carried.

In a machine provided with interpoles, the interpole winding must be considered a part of the armature and not a part of the series field. The equalizer connection must be made between the main series winding and the interpole winding, and not directly to the brush terminal of the machine. Otherwise the equalizing current will flow not only through the series coils but also through the interpole coils, thus making the commutating field either too strong or too weak for the actual armature current.

259. Action of the Equalizer. — An equalizer connection between the points such as a_1 and a_2 (Fig. 220) remedies the foregoing unstable operation. Its action is as follows: Let, as before, the two machines be identical, but suppose that for some reason the induced emf of machine 2 is higher, and therefore the current supplied by it is larger, than that of machine 1. This tends to make the potential of a_2 above the positive bus larger than that of a_1. Thus part of the current of machine 2 flows from a_2 to a_1 and to the positive bus-bar through the equalizer and the series winding of machine 1. Therefore, the field of machine 2 is weakened and that of machine 1 is strengthened by the excess current; machine 1 is thus helped to keep up its voltage. In short, *the equalizing connection prevents the currents in the series fields of two or more identical machines from differing widely from each other, however different their armature currents may be.*

Therefore, when connected by a suitable equalizing bus-bar or cable, two or more similar machines cannot have widely different induced emf's, nor can their load be disproportionately distributed. Even if one machine should send power back into the other, the series field current cannot become reversed. The equalizer is mainly useful on the rising part of the external characteristic (Fig. 215). Beyond the hump in the curve, if one machine tends to take a larger load its voltage will drop and the other machine will then be made to take more load. The operation on this part of the curve is thus seen to be similar to that of two shunt-wound machines in parallel.

A difficulty arises when the machines have *widely different characteristics,* as is explained in the case of the shunt-wound generators. The two conditions necessary for satisfactory parallel operation of two compound-wound machines of different types are as follows:

(1) The two machines must have similar external characteristics when running separately. If such is not the case, the machine which has a higher degree of compounding will take a larger share of the total output as the load on the two increases, because its induced emf will be

higher than that of the other machine. The equalizing connection cannot remedy this fault.

(2) When each machine is delivering its proper share of load, no current should flow through the equalizer. This means that the resistances of a_1b_1 and a_2b_2 must be inversely as the rated currents of the respective machines. As a general rule, the resistances a_1b_1 and a_2b_2 do not satisfy this requirement, and it becomes necessary to introduce additional resistance in one of the circuits. Then the machines will divide the current properly at all loads.

In operating two *identical* machines in parallel the system of " crossed series fields " is sometimes used, as illustrated in Fig. 221. Here the armature of generator 1 is connected in the circuit with the series field of generator 2, and vice versa. This serves to equalize the voltages more perfectly than the usual equalizer connection, for now any tendency of 1 to take more than its share of the total current is met by an increase of the current through the series field of 2, causing a rise in its induced voltage, while at the same time the increased current from the armature of 1 means a decreased current in its own field and a reduced value of induced voltage.[1]

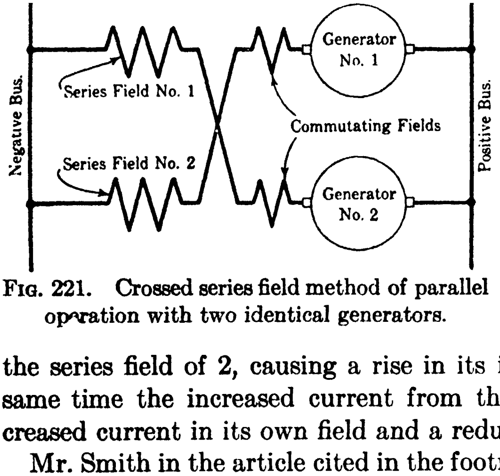

FIG. 221. Crossed series field method of parallel operation with two identical generators.

Mr. Smith in the article cited in the footnote states that in the parallel operation of compensated machines, in which the effect of armature reaction is eliminated, some difficulty may be encountered. This may be overcome either by giving the brushes a slight shift in the direction of rotation, thus restoring part of the armature reaction, or by using differential series windings in conjunction with stronger cumulative series field windings, and connecting the equalizer to the points between the cumulative and differential windings. With this connection an excess of current from machine 1 increases the effect of its differential field and lowers its induced voltage, while an excess current through the cumulative field of 2 raises its induced voltage. Thus a redistribution of load current is effected in the desired direction.

260. Directions for Paralleling Two Compound Generators.—*Method* 1. Let machine 1 be supplying the load alone, and let it be

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[1] See "Some Problems of Parallel Operation of Direct Current Generators," I. C. Smith, *Elec. Jour.*, August, 1926.

required to connect generator 2 in parallel with it. The main switches S_2 and T_2 (Fig. 220) are open, as well as the equalizer switch E_2, the equalizer switch E_1 being considered closed. R_p and R_s are either absent or out of circuit, and S_3 is closed. As a first step, machine 2 is brought up to speed and excited so as to give a voltage slightly above that across the bus-bars, then equalizer switch E_2 is closed. No current yet flows in the equalizer, and the voltage of machine 1 is not affected. Next switches S_2 and T_2, preferably combined in one double-pole switch, are closed. Initially, since the generated voltage of 2 about equals that of 1, a_1 is at approximately the potential of a_2, and little or none of the current from machine 1 will flow through the equalizer. But as 2 begins to supply current to the load, this current flows through the series field of machine 2, raising its voltage and causing it to assume some part of the total load. Redivision of the load and correct bus voltage are attained by adjustment of the field rheostats of both machines, after which control of voltage and division of load are automatic.

In disconnecting one of the machines its load should be reduced to zero by reducing its shunt field and at the same time adjusting that of the other machine if necessary to keep the bus voltage constant. Next, the double-pole switch is opened, then the equalizer switch. The fundamental rule to be followed in either connecting or disconnecting a machine is to do it with as little electrical disturbance as possible, which has been the aim in the procedure outlined above.

With large machines it is often considered desirable to have three separate switches as shown in Fig. 220. Then either of the following methods of paralleling the machines may be used.

Method 2. After having brought the incoming machine up to speed and to bus voltage, first close switch E_2 and then T_2. At once adjust the field rheostat of 1 to keep the bus voltage at its correct value, otherwise the current which 1 supplies through the series field of 2 weakens the series field of 1 and lowers the bus voltage. Now having corrected the voltage of 2 to equal bus voltage, close S_2, and by adjustment of the two field rheostats secure the correct load distribution and bus voltage.

Method 3. Having brought the incoming machine up in speed, close the equalizer switch E_2 and adjust the voltage of 2 to be slightly higher than bus voltage. Now close S_2, which puts the two machines in parallel as shunt machines. Readjust the field of machine 2 so that the line current of 2 is zero. This means that a_1 and a_2 are at the same potential. Now close T_2, and as current begins to flow in the series field of 2 it will supply some of the load. Now adjust both field rheostats to secure the correct load distribution and bus voltage.

It is possible that, when the two switches connecting a machine to the bus-bars are united into one double-pole switch, the machines cannot be thrown in parallel as smoothly, since, with this arrangement, the main circuit is closed before the field is properly adjusted. However, Method 1 is used to a considerable extent. The double-pole connecting switches can be used to better advantage if a second equalizer connection is established between the other extremities of the series-field coil, so that merely closing the equalizer switch will connect the two series windings in parallel. In this case the equalizer switch is placed in the second equalizer connection and switches E_1 and E_2 are omitted. In inexpensive isolated plants, three-pole switches are sometimes used, combining all the three switches into one; this is done when a momentary rush of current is not considered harmful.

261. EXPERIMENT 11-D. — Operation of Compound-Wound Generators in Parallel. — The student group is given two d-c generators of somewhat different characteristics. The machines may be provided with series windings beforehand, or the student himself may be expected to put them on.

(a) First take several points on the external characteristics of each machine in order to determine their fitness to operate in parallel.

(b) As explained in §§255 to 260, three conditions are necessary in order that the machines operate properly in parallel:

(1) An equalizing connection must be provided.
(2) The machines must have similar external characteristics when running alone.
(3) The voltage drop at normal load in the generator lead a_1b_1 (Fig. 220) should equal that in lead a_2b_2.

Provide circuit-breaker protection for the machines, then try to operate them in parallel without any of the above conditions being fulfilled and see what happens. Have ammeters in the equalizing lead (at first open), and in each generator lead and shunt field circuit. Record the observed data (all currents and the voltage of the common bus and across each armature), provided the machines are sufficiently stable to permit readings.

(c) Now introduce the equalizer, making sure that it has a very low resistance. Try paralleling the two machines by each of the three methods described above. Vary the total load from combined rated value to zero, and read all currents and voltages.

(d) Arrange as in Fig. 220 to connect suitable variable resistances, R_s and R_p in series and in parallel, respectively, with the series field of the machine having the greater over-compounding. To determine the effect of the values of lead resistances adjust the total load to equal the combined rating of the machines, first with R_s and R_p out of the circuit. Vary the shunt fields until the machines divide the load in the desired ratio. Maintaining conditions otherwise constant, diminish the load to one-third normal value and note its division between the machines. Now insert R_s, and attempt by varying both R_s and the shunt field settings to obtain conditions such that the machines share the load in the desired ratio at both full load and one-third load. When this is accomplished take a complete run and record all readings.

(e) To determine the effect of changing the characteristic of one machine cut R_s out of circuit and connect R_p as indicated. Again adjust the shunt field settings and R_p until the machines properly share both rated load and one-third load; then take a load run.

(f) To determine the effect of resistance in the equalizer take several readings at a constant load but with different resistances in the equalizer circuit.

Report. (1) Plot the external characteristics of the two machines and explain the adjustments which would seem to be necessary for best operation in parallel. (2) Describe what happened when you tried to operate the set without the equalizing connection. (3) Explain the operation before the resistance R_s was inserted, and show how nearly the resistances of a_1b_1 and a_2b_2 were inversely proportional to the rated currents of the machines after the best performance was secured. (4) Plot a curve showing the division of the current between the two machines at various loads under the best conditions you were able to obtain (*a*) with R_s in the circuit; (b) with R_p connected. (5) Report your findings regarding the effect of the resistance in the equalizer connection upon the operation in parallel. (6) Discuss the operation to be expected at a speed or at a voltage different from the rated values. (7) Discuss the three methods of paralleling compound generators as to time required, system disturbance, and practice needed to secure good results.

THE THREE-WIRE SYSTEM

262. To secure economy in the transmission of large blocks of power or to make it possible to operate motors, etc., on either of two voltages, the three-wire system of d-c distribution is frequently used. Thus, in Fig. 222 there are between the wires *a* and *b* 250 volts, and between *a* and *c* or *b* and *c* only 125 volts. The two outside wires are the principal (positive and negative) conductors; the middle, or neutral, wire carries the current corresponding to the unbalance in the loads connected between outer wires and neutral.

The present tendency is away from the three-wire d-c system because of the change to alternating currents for lighting and power and also because it is no longer necessary to apply different voltages to a d-c motor in order to get a wide range of speed. A speed range as high as 3 to 1 or even greater can be obtained with modern commutating-pole motors (§276) by field control on an ordinary two-wire system. Nevertheless, there are a great many three-wire d-c installations in actual operation, and it is essential that the student become familiar with their fundamental electrical relations.

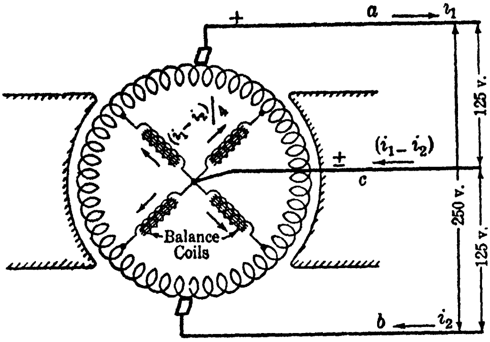

FIG. 222. Three-wire generator in which the neutral point is obtained by means of balance coils within the rotor.

In a three-wire lighting circuit the lamps are connected between the middle wire and either of the outside wires (Fig. 225). The voltage across the outside mains is usually about 220 volts, and the copper economy (not counting the middle wire) corresponds to this pressure; at the same time, standard 110-volt lamps are used. The weight of copper with the same power output and with the same percentage of line loss is inversely proportional to the square of the line voltage, i.e., at 220 volts the wires need be only 1/4 as large as for 110 volts. The cross-section of the middle conductor generally need not be more than 25 per cent of that of either of the outside wires but for safety is usually made the same, so that the addition of a third wire on a 220-volt system still leaves a 25 per cent margin of economy, as compared to an ordinary 110-volt two-wire distribution.

263. Methods of Dividing the Voltage. — The two outside wires in the three-wire system are connected to the terminals of the main generator (Figs. 222 to 225); special devices are used for dividing the voltage so as to obtain a point for connecting the middle conductor. The methods in common use are:

(1) Balance coils (Fig. 222).

(2) Motor-generator or balancer set (Figs. 223 and 225).

(3) Storage battery (Fig. 224).

The original method of obtaining a three-wire system was to use two d-c machines in series and to connect the outside terminals to the main conductors, while the common terminal was connected to the neutral wire. This method is hardly ever used, except perhaps in the case of high-voltage d-c generators for traction purposes.

(1) The balance coils (Fig. 222) are either built into the rotor and their junction point brought out by means of a single slip-ring, or they are ordinary reactance coils or auto-transformers (§402) provided with taps at the center and joined. The generator *G* then has four slip-rings connected to four equidistant points of the armature winding, as in a two-phase rotary converter (Vol. II). The balance coils form a star-connected quarter-phase system, and the middle wire *c* is connected to the neutral point of this polyphase system. This neutral point being symmetrically situated with respect to the outside wires *a* and *b* on the d-c side, its potential is always midway between the potentials of these wires, and thus the voltage is divided into two equal parts. The energy consumption in the balance coils is small, their inductance being large as compared to their resistance. This method of dividing the voltage is particularly well adapted for use with rotary converters, because they are already provided with slip-rings, and it is usually possible to obtain a neutral point on the main transformers which supply the converter. For a description of rotary converters and polyphase systems see Vol. II.

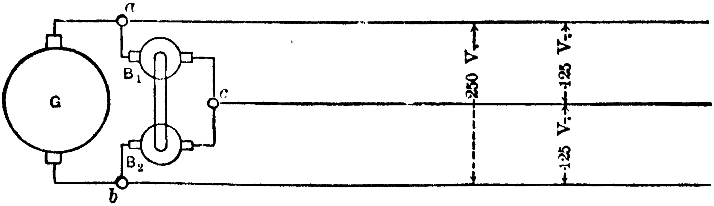

Fig. 223. Dividing line voltage in two by means of a balancer set.

(2) The balancer set[1] (Figs. 223 and 225) consists of two identical shunt-wound machines, B_1 and B_2, of comparatively small current capacity. The armatures are directly connected mechanically and are connected in series electrically, between the two outside wires. The middle wire is connected at *c*, between the two armatures. The two machines of the balancer set being identical, excited to the same voltage, and run at the same speed, the voltage is divided at *c* into two equal parts. When the load becomes sufficiently unbalanced, one of the machines runs as a generator and the other as a motor; the voltages still remain practically the same, the induced emf's being approximately equal. The details of the distribution of currents are explained in the next article.

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[1] See "Direct-Current Balancer Sets," by A. C. Lanier, *Elec. Jour.*, Vol. IX, p. 1036.

(3) A storage battery may also be used for dividing the voltage (Fig. 224), but the comparatively high cost and depreciation of storage cells prevent their extensive use for this service.

An advantage of balance coils over a motor-generator set is that the former are less expensive and require but slight attention. On the other hand, a three-wire generator with slip-rings is more expensive than an ordinary two-wire machine. Here, as in most practical problems, preference is given to one or the other system according to the local conditions in each particular case.

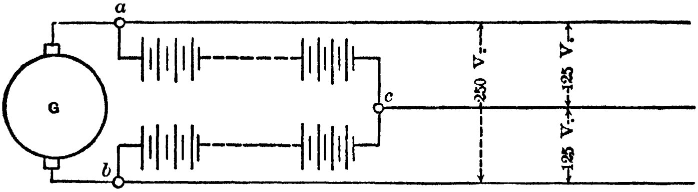

FIG. 224. Dividing line voltage in two by means of a storage battery.

264. Effect of Unbalancing. — The distribution of currents in a three-wire system with an unbalanced load is illustrated in Fig. 225, the total voltage of 220 being divided in two by a motor-generator set. It is assumed that the load is 500 amperes on one side and 400 amperes on the other. The difference, equal to 100 amperes, returns through the middle wire to the balancer set. The balancer set, not being connected to a prime mover, cannot generate additional electric energy; it merely equalizes or transfers power from the less loaded side to the more heavily loaded side. If the machines were ideal, 100 amperes would be divided into 50 and 50 in each machine, flowing in the opposite directions, the lower machine running as a motor, the upper as a generator. The main generator would supply the average of the two currents, i.e., 450 amperes at 220 volts. With this distribution of currents, Kirchhoff's first law is fulfilled at points a and b, namely $500 = 450 + 50$, and $400 = 450 - 50$.

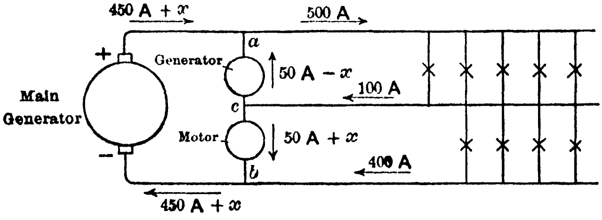

FIG. 225. Distribution of currents in a three-wire system with an unbalanced load.

In reality, the current distribution is somewhat different, because of the losses and the voltage variations of the balancer set. A certain no-load current x flows through the set even when the load is perfectly balanced; this current is necessary for overcoming the iron loss and friction in each machine. Moreover, the main generator has to supply some current for the shunt fields of both machines. Superimposing this current x upon the useful currents mentioned above, we get the distribution of currents shown in Fig. 199. The current in the balancer machine working as a generator is reduced and that in the motor increased by the amount x. The condition $\sum i = 0$ at the points a and b is again fulfilled.

With an unbalancing of load currents, the voltages between the middle wire and each of the two outside conductors become somewhat unequal. With a properly designed balancer set, the difference should not be more than a very few per cent; moreover, it may be corrected to some extent by regulating the field rheostats of the balancer machines. An investigation of the current relations in the balance coils (Fig. 222) leads to similar results.

265. Efficiency of a Balancer Set. — Referring to Fig. 225, the efficiency of the balancer set is equal to the ratio of the output of the machine working as a generator, to the input into the machine working as a motor; the excitation losses of both machines should be added to the input. This is according to the standard definition of efficiency of any motor-generator set. It will be easily seen, however, that in the case of a balancer set the efficiency, according to this definition, becomes negative when the load is nearly balanced, because then both machines operate as motors, each overcoming part of the total losses. As unbalancing increases, the efficiency gradually rises to zero and then becomes positive. There is no physical contradiction in this result, provided that the actual conditions of operation of the set are clearly kept in mind.

There is another way of looking at the efficiency of a balancer set, namely, from the standpoint of the three-wire system as a whole. The losses in the balancer set increase the output of the main generator above the useful demand for power; the ratio of this demand to the output of the main generator may be called the efficiency of the three-wire system, with reference to the balancer set.

To illustrate the above definitions of efficiency, assume that the no-load current of the balancer set, x, is 10 amperes (Fig. 225), and that the field circuit of each machine consumes 2 amperes, both fields being connected in parallel across the outside conductors.

The efficiency of the motor-generator set itself, according to the first definition, is

$$\frac{(50 - 10)110}{(50 + 10)110 + (2 + 2)220} \quad 70.6 \text{ per cent}$$

The efficiency of the three-wire system, according to the second definition, is

$$\frac{500 \times 110 + 400 \times 110}{(450 + 10 + 2 + 2)220} = 97 \text{ per cent}$$

Each expression has a definite physical meaning, and either is used, according to the conditions of the problem. If a greater accuracy is required, the copper loss in the armatures of the balancer set, and the resulting unbalancing of the voltages, must be taken into account in the above expressions for efficiency.

266. EXPERIMENT 11-E. — Study of a Three-Wire System. — Wire up the generator and the balancer set as in Fig. 223, and provide variable resistances to be used as a load. Load one side of the line until the balancer set runs as nearly as possible at its full rated capacity. Note the input and the output of the two machines of the balancer set, the voltages on the loaded and unloaded sides of the line, the speed of the set, the currents in the three line wires, and the field currents of the balancer machines. Then gradually load the other side of the line, keeping the load on the first side and the total voltage constant. As the unbalancing decreases, the balancer load decreases, though the total generator load increases. Take readings as indicated above with different degrees of unbalancing, until the load on both sides is the same, and the balancer runs idle. Increasing the load further will merely reverse the conditions in the balancer set: the machine which has been running as a motor will run as a generator, and vice versa.

If a generator with slip-rings is available, or a rotary converter, a three-wire system with balance coils (Fig. 222) may be tested. It is not absolutely necessary to have three or four slip-rings; if only two slip-rings are available, the neutral point may be obtained with one balance coil. In fact, it is desired that the student investigate the effect of using only one balance coil instead of two. With a three-phase rotary converter, the three balance coils are connected in Y, and the middle wire connected to the neutral point. The same load experiment should be performed with balance coils as with the motor-generator set. Note that a balance coil consumes a magnetizing current, which is an alternating current, and cannot be detected by a permanent-magnet moving-coil ammeter (§44).

Report. Draw a diagram of actual connections. To amperes in the main generator as abscissas plot the currents in the two outside wires and in the neutral, the voltages, the generator output, and the currents in the balancer armatures and fields; also the speed of the set. Show that the current readings satisfy Kirchhoff's first law at points a, b, and c

(Fig. 225). Figure out the efficiency of the balancer set and of the three-wire system, as in §265. Explain theoretically the observed departures in speed and voltage of the balancer set from the no-load values. Give similar results for the test in which balance coils were used.

AUTOMOBILE LIGHTING GENERATOR

267. The electric generator of an automobile is always connected in parallel with a storage battery, Fig. 226. The main purpose of the generator is to keep the battery charged, although at times the generator also furnishes current to the lamps, the horn, the ignition coil, etc. An ordinary shunt-wound machine is not suitable for the purpose, because the generator is geared or connected by a chain or belt to the engine, and thus has to run at widely different speeds. If it were adjusted to furnish the proper charging current at a certain moderate speed, it would overcharge the battery at higher speeds, and also overload itself. Of the various arrangements tried for limiting the generator current and voltage at high speeds, the third-brush excitation shown in Fig. 226 has proved to be the simplest and the most reliable.

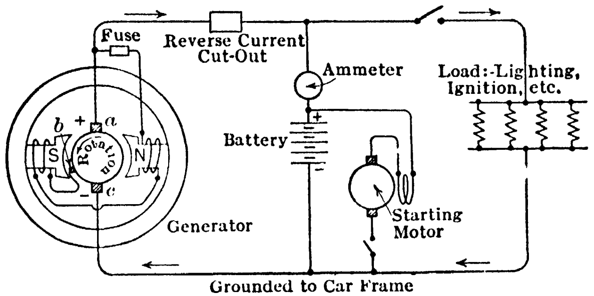

FIG. 226. The electric circuit of an automobile, showing a third-brush generator.

The third brush bears upon the commutator somewhere between the main brushes, and the shunt-field winding is connected between this brush and one of the terminals of the machine. The operation of such a generator at no load is not much different from that of an ordinary shunt-wound machine, in that the induced voltage is proportional to the speed. When, however, the generator is connected in parallel with a storage battery, the generator current affects the field and the induced voltage of the machine in such a way as to produce current-speed characteristics shown in Fig. 227. Each curve refers to a particular position of the third brush. Beginning with a certain low speed, the generator current increases and reaches a maximum, and when the speed is further increased the current drops again, never exceeding the desired safe limit. At low generator speeds, when its emf is below that of the battery, the two are automatically separated by either a reverse-current cut-out shown in Fig. 226 or a potential relay. This prevents the battery from discharging into the generator, and at these speeds the load is supplied by the battery alone.

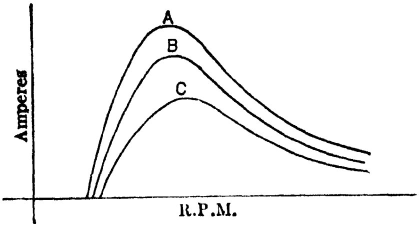

FIG. 227. Current characteristics of a third-brush generator.

The automatic regulation of current in this machine is due partly to the armature reaction, and partly to the currents induced in the armature coils short-circuited by the third brush. Each of these factors will now be considered separately.

268. Armature Reaction and the Effect of Short-Circuited Coils. — The difference between the flux density distribution in the air-gap of a d-c generator at no load and with the armature loaded is shown in Fig. 263. The whole flux is shifted in the direction of the rotation of the armature. Thus, with the connections shown in Fig. 226, as the armature current increases, the air-gap flux between the brushes *a* and *b* decreases. Therefore the voltage induced in the armature between *a* and *b* is reduced, and consequently the exciting current also decreases in the same proportion, still further reducing the flux and the field current. By making the armature ampere-turns relatively large and the air-gap small, the flux is made unstable and easily shifted by the armature current. In this manner any appreciable increase in the armature current is effectively checked.

The armature coils short-circuited by the third brush are in a comparatively strong field, so that large currents are induced in them. By actually determining the direction of these currents it will be found that their mmf opposes that of the field coils between *a* and *b*. Thus, these currents further reduce the flux between *a* and *b*, and the field current. The magnitude of these currents is nearly proportional to the speed of rotation, and it is for this reason that the current output of the machine decreases at high speeds (Fig. 227). The magnitude of the induced currents at a given speed depends somewhat on the brush contact resistance, and the currents may be increased by using a third brush *b* made of soft graphite.

269. Operating Characteristics of a Third-Brush Generator. — The three curves shown in Fig. 227 refer to different settings of the third brush. By shifting it in the direction of the rotation of the armature, the field current is increased and the curve marked *A* is obtained. The generator becomes connected to the battery at a lower speed, the charging current reaches a higher absolute maximum, and it reaches it at a lower speed. Such a setting would be used in a car for city service at comparatively low speeds, and with frequent starting and use of the lights. For summer touring when there is a tendency to overcharge the battery, the third brush should be shifted back to obtain a characteristic corresponding to curve *C*.

The maximum generator current depends on the counter emf of the battery. When this counter emf is high the generator voltage is also high, and so is the voltage across the field circuit. The exciting current is consequently somewhat higher than normal, the main field is comparatively stiff, and it takes a stronger armature current to shift it. The machine is thus able to maintain a higher charging rate. On the contrary, when the battery is nearly discharged, the maximum charging current that the generator is capable of delivering is correspondingly lower.

An important precaution to be observed in the operation of the third-brush generator is that the machine must not be run without the battery, unless the field circuit be opened by removal of the fuse, or the armature short-circuited. Otherwise the voltage will rise to a value many times that reached in regular operation, and the insulation of the machine may be injured or lamps, if they are turned on while this condition remains, will be burned out at once.

On some cars the so-called single unit is used; this acts as a motor for starting the engine, and as a generator afterwards. It is provided with a series-field winding for starting purposes and with the usual shunt-field winding connected to the third brush. In starting, the shunt field assists the series field, thus increasing the torque. In generator operation the action of the series field is differential, still further opposing the tendency of the armature current to rise at high speeds.

In at least one make of third-brush generator a thermostatic device is mounted in the machine housing for a further regulation of its output. The device has four contacts which in turn short-circuit four sections of a resistance in series with the field winding. In cold weather, or at the beginning of a run when the generator is comparatively cool, all the field resistance is short-circuited, and the generator delivers its full output, charging the battery at a high rate. As soon, however, as the generator begins to heat up, one section of the field resistance after another is introduced into the circuit, and the generator current is materially reduced. For a theoretical discussion of the third-brush generator see Langsdorf's " Principles of Direct-Current Machines," McGraw-Hill Book Co., New York City.

270. EXPERIMENT 11-F. — Characteristics of a Third-Brush Generator for Automobile Service. — Connect the machine to an adjustable-speed motor and provide some arrangement for conveniently regulating the speed within as wide limits as would be met in automobile service. A suitable storage battery must be provided, also a lamp load or a resistance load, and the connections made as in Fig. 226, omitting the starting motor. In addition to the zero-center ammeter in series with the battery, an ammeter should be inserted in the load circuit and one in series with the field winding. A voltmeter should be connected through a multi-point switch to measure at will the voltage at the generator or battery terminals, as well as the field voltage.

(1) Adjust the reverse-current cut-out or potential relay to close the generator circuit at a voltage slightly above that of the battery. Keep the load switch open, run the generator at various speeds and adjust the position of the third brush for the maximum possible output. (2) Get the data necessary for the upper curve shown in Fig. 227 and also read the terminal voltage, the field voltage, and the field current. (3) Close the load switch and investigate the influence of a considerable resistance load upon the generator characteristics, and upon the charging current. Investigate also the effect of a reasonable amount of resistance in the field circuit. Shift the third brush into two or three other positions and repeat the preceding run at each position. (4) Set the third brush at the point of maximum output and investigate the influence of the battery counter emf upon the charging rate. Take a battery in a different state of charge from the first one, or add a nearly discharged cell to act first cumulatively and then against the main battery. Finally run the generator with a very high counter emf at its terminals and again with the armature short-circuited. (5) Run the machine carefully on open circuit at a low speed to prove that the voltage would rise to a dangerous value at higher speeds. (6) Try a hard carbon brush and a soft graphite one for the third brush, and also brushes of different width. Investigate the effect of these factors upon the performance characteristics of the generator.

Report. Plot the various characteristics to speed as abscissas. State briefly the effect of the variable factors observed. Show that the observed changes in the characteristics could be expected theoretically, at least qualitatively.

271. Special Types of D-C Generators. — Several other special types of d-c generators have found application in special fields of work.

The Rosenberg generator, in which the direction of emf is independent of the direction of rotation, and the current which it will supply to the battery with which it is usually connected in parallel is essentially constant, is especially adapted to train lighting. Also, there are several types of the arc-welding generator which is of increasing importance with the growth of welding practice in the fabrication of buildings and machines. They will be found described in the technical literature and in standard textbooks of direct-current machinery.[1]

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[1] See "Principles of Direct-Current Machines," by A. S. Langsdorf, McGraw-Hill Book Co., New York City.

CHAPTER XII

DIRECT-CURRENT MOTORS

272. A d-c machine, such as is shown in Figs. 198 and 199 and described in §233, is *convertible*; that is to say, it may be operated either as a generator or as a motor. When such a machine is driven by a source of mechanical power (prime mover) it delivers electrical energy and operates as a *generator*. When electrical energy is supplied to the machine, it is capable of developing mechanical energy available at its shaft and operates as a *motor*.

This convertibility is a direct result of two fundamental laws of electromagnetism; viz.: (1) When an electric conductor, forming a part of a closed circuit, is moved across a magnetic field, a current is induced in the conductor such as to oppose the motion; the mmf of this current tends to strengthen the field in front of the conductor and to weaken the field behind it. (2) When a current is sent through a conductor located in a magnetic field, the conductor tends to move across the lines of force and the mmf of the current strengthens the field behind the conductor and weakens it in front; the emf induced in the conductor by its motion through the field tends to oppose the passage of the original current, and is known as the back emf or *counter emf* of the motor. In the first case the conductor acts as an element of a generator; in the second, as that of a motor.

273. Types of Motors. — Three types of d-c motors may be distinguished according to the method of field excitation, viz.: the shunt-wound motor (Fig. 228), the series-wound motor (Fig. 229), and the compound-wound motor (Fig. 230). In the shunt-wound motor the field winding is connected directly across the supply circuit, so that (for a constant voltage supply) the field current of the motor is practically constant at all loads. In the series-wound motor, the field winding is in series with the armature circuit, and as the load increases the field current is also strengthened automatically. In a compound-wound motor (§§306 to 310) the series field may be connected either so as to assist the shunt field (cumulatively compounded motor) or to oppose the shunt field (differentially compounded motor).

There is a marked difference in the performance characteristics of these types of motors, and each type has a field of application of its own. The shunt-wound motor is essentially an adjustable-speed motor, the

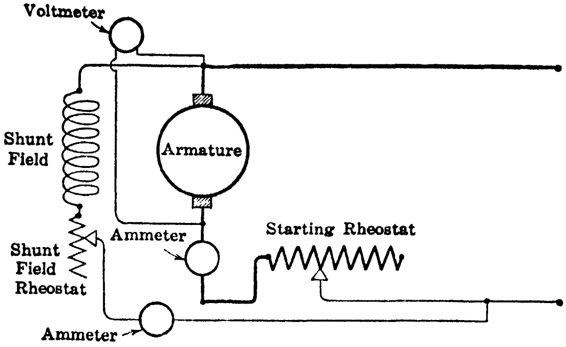

Fig. 228. Diagram of connections for starting and speed control of a shunt-wound motor.

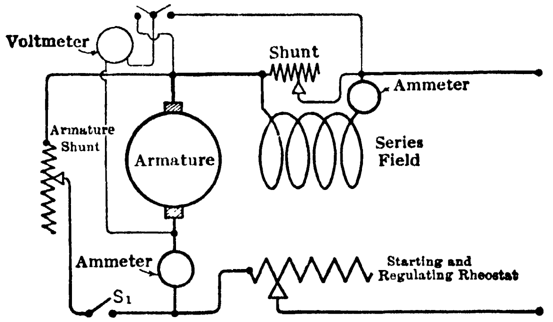

Fig. 229. Diagram of connections for starting and speed control of a series-wound motor.

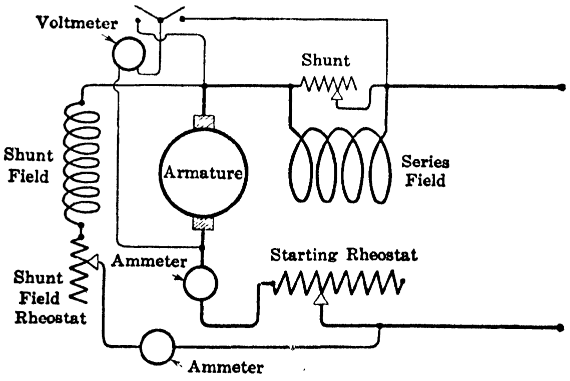

Fig. 230. Diagram of connections for controlling a compound-wound motor.

speed of the series-wound motor increases automatically as the load decreases. A theoretical explanation of these characteristics is given below, in the treatment of these types of motors. Shunt-wound motors are used for machine-tool drive, and for driving various machines in textile and other industries, where a constant or nearly constant speed is required independent of the load. Series-wound motors are used in railway, crane, and hoisting work, where a heavy starting torque is necessary, and where the motors have to be frequently started and stopped. In these cases the load is usually fluctuating, and it is desirable to have a motor which automatically slows down as the load increases, in order not to produce too large fluctuations in power demand.

In some special cases a motor is desired, with characteristics intermediate between those of the two above-mentioned types, and for this purpose both series- and shunt-field windings are provided. Such a cumulatively compounded motor possesses a considerable starting torque and slows down with the load, but does not run away when the load is taken off. This type of motor is used with elevators, also for driving planers, punches, etc. In some exceptional cases an absolutely constant speed is required, independent of the load. A differentially compounded motor may be made to satisfy this requirement quite closely, although its speed is somewhat affected by fluctuations in the terminal voltage. A speed regulator (§279) will keep the speed of a motor constant even with a fluctuating terminal voltage. By suitably increasing the number of turns in the series winding, a differentially compounded motor can even be made to run faster at full load than at no load, should a particular service require such a characteristic.

A d-c motor usually requires a starting rheostat, and sometimes a field-regulating rheostat. In addition to these rheostats, switches may be required for reversing the direction of rotation, for disconnecting from the line, etc. Standard auxiliary devices of this kind are described in Chapter XXV, and the reader is advised to familiarize himself at least with the simplest of these while studying the performance of d-c motors.

THE SHUNT-WOUND MOTOR

274. The diagram of connections of a shunt-wound or separately excited d-c motor is shown in Fig. 228. It is an adjustable-speed machine in the sense that the field strength at constant applied voltage remains practically constant and independent of the load, but may be adjusted at will by means of the field rheostat. The interaction between the armature current and the magnetic field of the poles produces a tangential force (electromagnetic torque). By means of the commutator and the brushes, the relative position of the groups of armature conductors which carry current in a given direction remains unchanged in space during the rotation of the armature, so that the action persists indefinitely.

The rotation of the armature in the field causes an emf to be induced in the armature conductors, as in the d-c generator (§234). According to the principle of action and reaction, the induced emf opposes the applied voltage and is therefore called the counter emf. Under normal operation this counter emf is but a few per cent lower than the applied voltage, and the small difference between the two allows a current, just sufficient for the required load, to flow through the armature. An electric motor must develop a counter emf in order to deliver mechanical power, because any relative motion between the armature conductors and the magnetic field results in a counter emf. On the other hand, no counter emf is required for the starting torque, which is due to a static action between the armature conductors and the main poles.

275. Performance Characteristics. — The relationship between the applied voltage E and the counter emf e is

$$E = e + IR \quad \ldots \ldots \ldots \quad (1)$$

where I is the armature current and R is the resistance of the armature circuit including the brushes and the interpole winding, if any. At the rated armature current, the term IR amounts to but a few per cent of E in order to keep the efficiency of the motor sufficiently high. Thus, for many practical computations, the counter emf, e, is assumed to be essentially equal to the applied voltage, especially when the value of e is not accurately known. Multiplying both sides of the foregoing equation by the armature current I we get

$$EI = eI + I^2R \quad \ldots \ldots \ldots \quad (2)$$

In this expression, EI is the input into the armature and I^2R is the heat loss in the armature circuit. Hence, according to the law of the conservation of energy, the amount eI must represent the power developed by rotation, i.e., useful output of the machine plus its iron losses and friction. This relationship makes it possible to write down immediately an expression for the electromagnetic torque between the armature and the field. The expression given in §234 for the induced emf may be applied directly to the counter emf, e, that is,

$$e = (p/a)(\text{rpm}/60)Z\phi \times 10^{-2} \quad \ldots \ldots \quad (3)$$

Multiplying both sides of this equation by I, we get the following expression for the motor output, including the iron loss and friction:

$$eI = I(p/a)(\text{rpm}/60)Z\phi \times 10^{-2} \quad \ldots \ldots \quad (4)$$

Let P be an output expressed in kilowatts, and let it be required to compute the corresponding torque, T, in meter-kilograms. We have then

$$P \times 10^3 = 981 \times 10^5 \times T \times 2\pi(\text{rpm}/60) \times 10^{-7} \quad . \; . \; . \quad (5)$$

In this expression, the factor 10^3 converts kilowatts into watts; 981×10^5 converts kilogram-meters into ergs; and 10^{-7} converts ergs into watt-seconds (joules). Solving for T, we obtain

$$T = 16.2P/(\text{rpm}/60) = 974P/(\text{rpm}) \quad . \; . \; . \; . \; . \quad (6)$$

If T is desired in foot-pounds, we have

$$T = 974 \times 3.281 \times 2.205P/(\text{rpm}) = 7040P/(\text{rpm}) \; . \; . \; . \quad (7)$$

The use of horsepower units for measuring the input or the output of electrical machinery is being generally discontinued in favor of the kilowatt. Should P, in the foregoing formulas, be expressed in horsepower, the following conversion ratios would be used: 1 British horsepower = 746 watts; 1 metric horsepower = 736 watts.

Applying eq. (6) to the output eI in eq. (4), we obtain the following expression for the torque, T, in meter-kilograms:

$$T = 0.0162I(p/a)Z\phi \times 10^{-2} \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

or in foot-pounds

$$T = 0.117I(p/a)Z\phi \times 10^{-2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

It will thus be seen that the torque is independent of the speed of the machine and is entirely determined by the current, the flux, and the armature winding. It must be clearly understood that eqs. (5) and (6) do not give the net available torque on the shaft, but the gross electromagnetic torque, which includes that for overcoming the iron loss and the friction and windage losses.

The performance characteristics of a shunt-wound motor at a constant field current (Fig. 231) may be readily understood in the light of the foregoing theory. The field flux remains essentially constant throughout the entire operation range, except for a comparatively small effect of the armature reaction (§365) which somewhat reduces the net flux at high values of armature current.

When the terminal voltage E is kept constant, then, according to eq. (1), the counter emf, e, decreases somewhat as the load current increases. At a constant flux, the speed of the machine decreases in the same proportion as the required counter emf, that is, but a few per cent between no load and full load; see eq. (3). Because of the armature reaction which reduces the flux, a somewhat higher speed is necessary to induce a given counter emf, while the armature current for a given torque must be larger. This increases the IR drop so that the counter emf need not be as great as it would have to be without the armature reaction. We thus see that the speed curve is shown drooping while the current curve is shown rising faster than the torque. The other characteristic curves shown in the figure require no explanation.

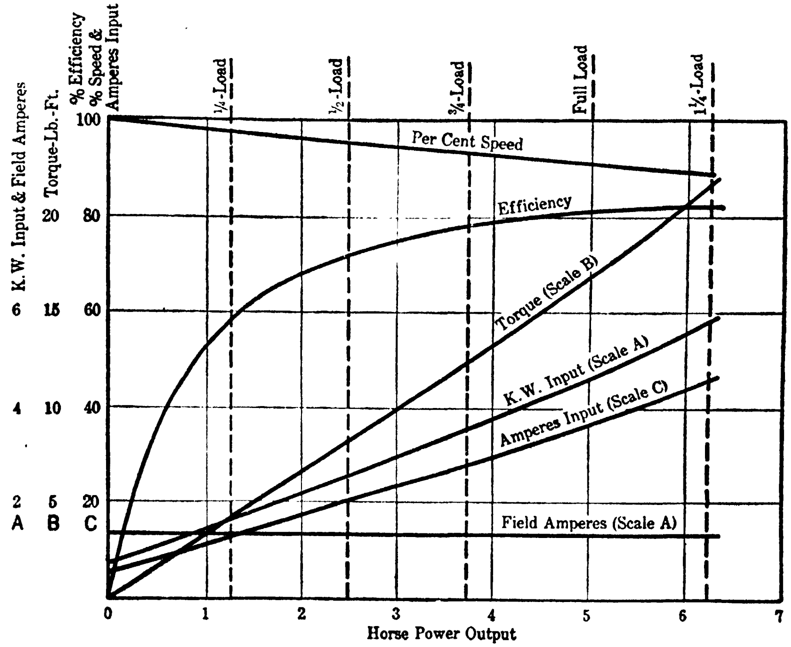

Fig. 231. Performance curves of a shunt-wound motor rated 5 hp, 125 volts, 1675 rpm.

At another value of field current, a new set of performance curves must be used. The smaller the field current the higher the speed of the machine and the more noticeable the effect of the armature reaction. The machine takes a higher armature current for the same torque, but a given torque corresponds to a greater output. As in the case of the generator (§244), a complete picture of the operating characteristics may be obtained by using a three-dimensional surface with the armature current, field current, and speed plotted along the axes of coördinates.

276. Commutating Poles or Interpoles.—The modern motor, in order that it may be suitable for heavy duty, frequent overloads, or a wide range of speeds by field control, is usually provided with so-called *commutating poles* or *interpoles* (Figs. 232 and 199). The interpoles are placed midway between the main poles and are much narrower than these. The magnetic field produced in the air-gap by these auxiliary poles is concentrated over the conductors undergoing commutation.

The induced emf's due to these poles are of such magnitude and direction as to assist and to accelerate the reversal of the current in the armature conductors. The emf of self-induction in the coils under commutation is proportional to the armature current to be reversed, so that the commutating flux must also be nearly proportional to the armature current. For this reason the winding on the commutating poles is connected in series with the armature (Fig. 232), and the magnetic flux density in the interpoles is kept below the knee of the saturation curve.

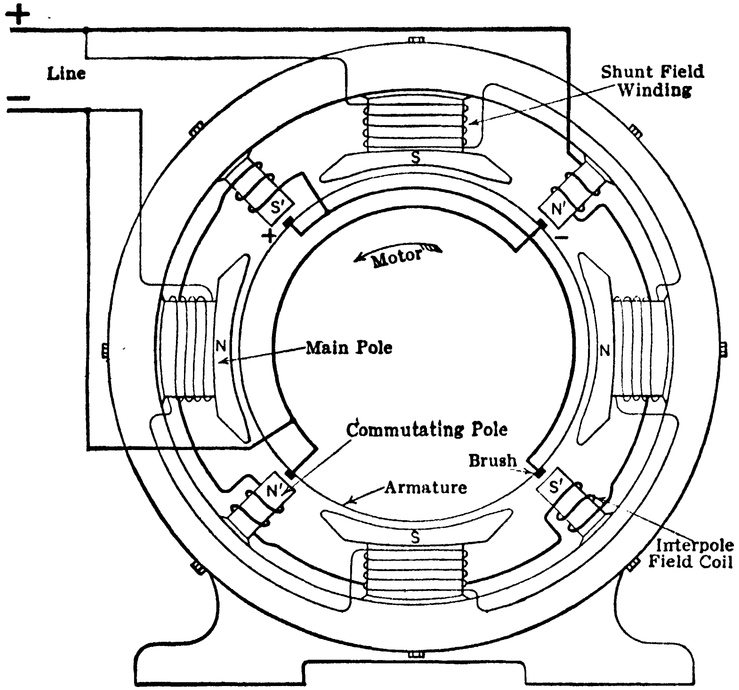

FIG. 232. A four-pole motor with commutating poles (interpoles).

In a motor without commutating poles the main magnetic field is designed to be of sufficient strength and stiffness to make satisfactory commutation possible at the rated speed and for values somewhat above and below this. With the motor operating in one direction only the brushes may have a permanent shift from the geometric neutral. However, when the motor is intended to run in either direction, the brushes must remain in the neutral position. Although some increase in speed may be obtained by weakening the field, a limit is soon reached at which the armature reaction so distorts the field that serious sparking occurs.

In a motor provided with interpoles the design constants are different, and the magnetic field need not be as stiff. A correct commutating field is maintained independent of the main field, at all reasonable values of the armature current, so that the magnetic field can be weakened much more, and a far wider range of speeds can be obtained. Modern shunt-wound motors with commutating poles may have a speed range as high as four-to-one or higher. At any speed within this range the motor is supposed to furnish its rated output without excessive sparking or heating.

The polarity of a commutating pole in a motor is that of the trailing main pole; this corresponds to shifting the brushes against the direction of rotation in a motor without interpoles. In a generator, with the same direction of rotation, the commutating poles are of the opposite polarity.

With very difficult conditions of operation, such as sudden overloads and current reversals met with in some applications, as for example in steel-mill drive, it becomes necessary to remove the effect of the armature reaction more completely. *Compensating windings* are then provided on the main pole faces to prevent the shifting of the magnetic field. The interpole winding in this case consists of fewer ampere-turns. For further details regarding armature reaction, commutating poles, compensating windings, and commutation see Chapter XV.

277. Speed Control. — An inspection of eq. (3) shows that the speed of the motor may be varied at will by varying one of the following factors: field flux, armature voltage, or the number of armature conductors in series. Of these three methods, the first one, or the so-called field control, is used most, because of the convenience of regulating the comparatively small exciting current. The weaker the field the higher the motor speed with the same applied voltage. The lower speed limit is reached when the field rheostat is short-circuited and the field winding is connected directly across the line. Sometimes the motor cannot be operated at this speed for any length of time without overheating the field winding.

The upper speed limit is determined by several considerations, namely (a) commutation, (b) heating of the armature winding, and (c) centrifugal stresses. At higher speeds, that is with a weakened field, the effect of the armature reaction is more pronounced, and the frequency of commutation is higher. For these two reasons the commutation is less satisfactory at higher speeds.

In a motor with interpoles (Fig. 232) the brushes are normally left at the geometrical neutral, and this is especially important when the motor must run in both directions. In a motor without commutating poles, the brushes may be shifted to some extent to obtain the best commutation, or to adjust the speed. The latter possibility is based on the fact that the direct armature reaction is determined by the brush shift (§235), so that by shifting the brushes the value of the flux per pole is varied. No considerable speed variation can be obtained by this means, because sparkling results as soon as the brushes have been shifted by any appreciable angle from the best position. However, this method is sometimes convenient for adjusting the speed to an exact value, after an approximately correct speed has been obtained with the field rheostat.

278. Special Motors of Adjustable-Speed Type. — Instead of varying the field strength by regulating the field current, the magnetic flux may be varied by changing the reluctance of the magnetic circuit. An adjustable-speed motor of this type is shown in Fig. 233 (Stow motor). The pole-pieces are hollow and have center cores which consist of plungers actuated by a handwheel. As these cores are withdrawn the field flux is decreased without changing the field current. In four-pole motors of this type all the pole-pieces are moved simultaneously by a beveled-gear transmission. The reason why the effect of the armature reaction in this motor is reduced at higher speeds may be seen by a comparison of Fig. 233 with Fig. 234; the latter represents the field of an ordinary motor. In the field of the Stow motor (Fig. 233) the reluctance of the magnetic path of the distorting lines of force, due to the armature current, increases with the effective length of the air-gap. The ratio of the field ampere-turns to the armature distorting mmf is not decreased with increase of speed, as it is in the case of the ordinary motor where the field flux is varied by decreasing the field current. Therefore, the field distortion does not increase with the speed. In the

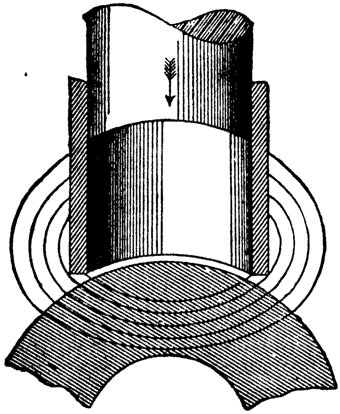

FIG. 233. Pole-piece in a Stow motor; cross-magnetization is reduced by an air column.

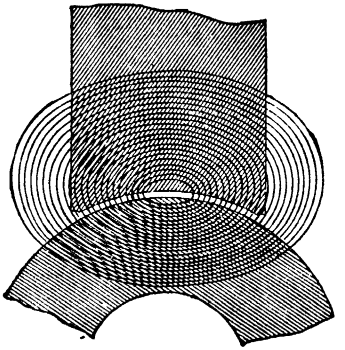

FIG. 234. Pole-piece in an ordinary motor; field distortion is the greatest when the field is the weakest.

ordinary motor the field distortion is greater at higher speeds, since the ratio of the field ampere-turns to the armature distorting ampere-turns is smaller. Another advantage of the Stow motor is that the speed adjustment is gradual, and not in steps.

In the Lincoln adjustable-speed motor the armature is made slightly larger at one end than at the other, and can be shifted along the shaft by a handwheel. As the armature is withdrawn from the pole-pieces, the useful magnetic area decreases and the length of the air-gap increases because of the conical shape of the armature. The result is that the useful flux decreases, and the speed of the motor is thereby increased. Since in this motor the useful flux is reduced for speed control by increasing the reluctance of the air-gap, the magnetic distortion is said to be actually less at high speeds than at low speeds. This is the opposite to that which takes place in a motor with field-current control. Motors of this type have been built with a speed range as high as ten-to-one.

279. Centrifugal Speed Regulator. — In some drives it is required to hold the motor speed absolutely constant, independent of fluctuations in the load and in the voltage supply. An automatic centrifugal governor is then mounted on the motor shaft and arranged to control two contacts. One of these contacts is stationary and the other movable, being mounted on the end of a governor arm. The action of the governor arm is controlled by a spring which can be adjusted for any desired speed. The contacts are connected through two collector rings, brushes and brush-holders, to close a shunt circuit across the motor field rheostat. This shunt circuit is thus rapidly opened and closed, resulting in an average field excitation of the correct value to give the required motor speed. There is no danger of breaking the motor field, because the device operates in shunt with the field rheostat.

In a large motor the field current may be too strong for the control contacts. One or more relays are then used; they are mounted separately, and are similar to the voltage regulator described in §252. The motor contacts control the circuits of these relays which in turn close and open the shunted by-pass across the field rheostat.

280. EXPERIMENT 12-A. — No-Load Test of a Shunt-Wound Motor — Field Current Control. — The purpose of this experiment is to determine the relation between the field current and the speed of a shunt-wound d-c motor at no load, with a constant applied voltage. The connections are shown in Fig. 228, and the theory is explained in §§274 to 279. The data sheet shown in §289 may be used, the last column being omitted. In performing this and other experiments on motors it is advisable to use a carefully calibrated, reliable speed indicator, such as the vibration tachometer, the centrifugal tachometer, or one of the electromagnetic type. The ordinary revolution-counter takes too much time and gives only the average speed over a comparatively long period.

(1) Set the brushes exactly in the geometric neutral, close the field circuit with the resistance all cut out, so as to have the strongest possible field, and have all the starting rheostat " in," so as to cut down the starting current. Start the motor by closing the armature circuit, and cut out the starting resistance in steps as the armature speeds up. Bring the machine up to the highest safe speed by reducing the field current and let it run for several minutes to warm up the bearings. The voltage at the armature terminals must be kept constant throughout the test. If the supply voltage is fluctuating, connect an adjustable resistance or a booster (see §284) in the armature circuit, and always obtain the correct voltage at the armature terminals before measuring the speed. Read the armature current, the field current, the speed, and the armature voltage.

(2) Increase the field current in steps, and at each step repeat the preceding readings. Having obtained the speed with the highest field current, reduce the current again to the same minimum value as in the first set of readings and check the speed reading to observe the effect of residual magnetism.

(3) Take a few readings with the brushes shifted from the neutral position by different amounts in each direction. Note the effect upon the speed, and record the degree of sparking, if any (§235).

(4) Drive the machine as a generator at a convenient constant speed and take a no-load saturation curve (§§236 and 239). See also §291.

(5) Measure the armature resistance, including the brushes, and determine the resistance of the commutating pole winding, if present.

Report. To the values of field current as abscissas plot the corresponding speeds; mark the effect of residual magnetism, if any. To the same abscissas plot the armature current and the power input into the armature, corrected for the I^2R losses. These corrected values represent the watts lost in supplying the iron loss and friction at no load. To the same abscissas plot the values of the induced emf from the no-load saturation run, correcting these readings, where necessary, for speed departures. By means of this saturation curve show that in each run as a motor the product of the speed and the field flux was nearly constant, as it should be, according to eq. (3). Show that the relationship between the field current and the speed (as a motor) could be predicted from the constant-speed saturation curve as generator. Give the results of the test with the brushes shifted.

281. Methods of Controlling Speed by Varying the Armature Voltage. — With a constant field excitation, the speed of a shunt motor may be varied in several ways, the principal ones being:

(1) Armature rheostatic control.
(2) A source of variable voltage.
(3) Separate connections to mains of different voltages.

A variable voltage supply is usually provided in one of the following three ways:

(1) By means of a separate generator (Ward-Leonard System) or motor-generator set.
(2) By means of a counter emf, or booster set.
(3) By means of a three-wire system.

These methods are described below more in detail.

282. Rheostatic Control. — If the starting rheostat of Fig. 228 is made rugged enough to carry full-load current without overheating, it may be used to vary the speed of the shunt motor. This is sometimes useful when it is desired to run the motor for limited periods of time at speeds below that obtained with full field (shunt field rheostat all cut out). Rheostatic speed control is a convenient but wasteful method of varying the voltage across a motor armature. From eq. (1), $e = E - IR$, where R now includes armature plus control resistance. From eq. (3) one may write

$$\text{rpm} = \frac{ea \times 60 \times 10^2}{pZ\phi} \quad \ldots\ldots\ldots\ldots (3a)$$

or, for a constant field, ϕ

$$\text{rpm} = ke = k(E - IR) \quad \ldots\ldots\ldots (10)$$

Thus, speed at constant field is proportional to the counter emf, or to the difference between applied voltage and IR drop. This means that to reduce the speed 50 per cent the IR drop should equal 1/2 the applied volts, and for a given current $I^2R = 1/2\ EI(= 1/2$ the total input to the motor). The efficiency of operation under these conditions, including the losses of the motor itself, immediately becomes below 50 per cent.

In addition to the wastefulness of this method, the presence of the resistance in series with the armature results in wide variations of speed with changes in load, as indicated by eq. (10), where, for constant field, the current I varies directly with load torque. A further objection to this method of speed control is that it takes a long time to adjust the speed of the motor to a desired value, owing to the interrelation between speed, counter emf, voltage drop, and current. The student should analyze this problem in detail.

With small motors the so-called *potentiometer method of rheostatic control* (Fig. 235) gives a closer speed regulation, although this method is still more wasteful of energy than the straight rheostatic control. A resistance *R* is connected across the supply mains; its value is such that the current through it is several times larger than that through the motor armature. The armature circuit is shunted off at any two desired points of this resistor by means of the connections *A* and *B*, one of which may be for larger steps and the other for fine adjustments. The whole arrangement resembles the potentiometer connections shown in Fig. 77, whence its name.

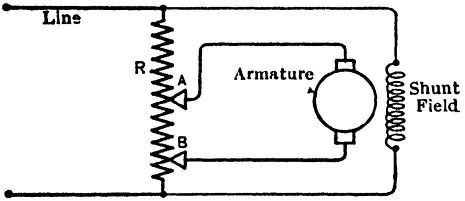

Fig. 235. Speed regulation of a small motor by means of a potentiometer-type resistance.

283. The Ward-Leonard (Motor-Generator) Method of Speed Control. — Here the armature voltage of a motor *M′* (Fig. 236) is varied by using a separate motor-generator set. The primary source of supply may be either direct or alternating current, because either source will serve to drive the approximately constant-speed motor *M* which in turn drives the d-c generator *G*. A three-wire power line is shown in the diagram.

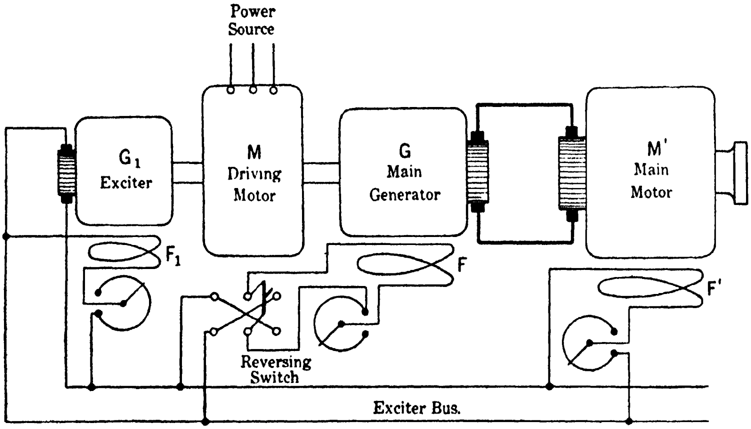

Fig. 236. An adjustable-speed motor driven by a motor generator set (Ward-Leonard system).

The fields of both the generator G and the variable-speed motor M' are preferably excited from a separate source, such as a small motor-generator set, to permit an independent adjustment of the exciting current, and the reversal of the direction of rotation of the main motor M'. Instead of the small auxiliary motor-generator, an exciter may be mounted on the shaft of the main set MG, or the exciting current may be drawn from an independent source of direct current, such as a storage battery. The Ward-Leonard control, though quite expensive, is extensively used for elevator and hoist control and for the main drive in steel mills where the motors are rated at several thousand horsepower. The speed control is almost unlimited in either direction of rotation, and the resultant economics in steel production outweigh the extra investment in the motor-generator sets.

In steel-mill drive the large motor-generator set is usually provided with a heavy flywheel, and the speed of the driving motor M is regulated by a current or power relay, or, in the case of induction motors, by a " slip regulator " (Chapter XXII, and Chapter LX, Vol. II). By these means the fluctuations in the power taken from the supply circuit are greatly reduced. When a sudden overload comes on the main motor M', the driving motor M of the motor-generator set, because of its drooping speed characteristic, slows down and allows the inertia of the flywheel to supply a part of the overload. When the load is suddenly thrown off the main motor, the motor-generator set is speeded up, thereby again storing energy in the flywheel. The method of driving an adjustable-speed motor at a variable voltage from a motor-generator set is generally referred to as the Ward-Leonard system. The use of a flywheel on the motor-generator set to reduce fluctuations in the power demand is usually ascribed to Ilgner.

284. Booster Control. — A more efficient, but less flexible, method than the one just described for speed control of d-c motors is that in which a small generator is connected in series with the armature of the motor to be regulated. By varying the field of this " booster " its voltage adds or subtracts any desired value from zero to the maximum of which it is capable. The booster armature must of course be able to carry the full current of the motor, but its voltage range (and rating) can usually be much lower than that of the main motor. Thus, for a speed range of 40 per cent, i.e., 20 per cent above and 20 per cent below normal, the voltage and rating of the booster generator need be but 20 per cent that of the motor to be controlled. Figure 237 shows in a schematic way the connections of motor and booster. Note that the field of the booster is regulated by means of a potentiometer resistance across the d-c supply, and that a reversing switch is also provided in this field circuit. The booster may be driven by a d-c motor running from the main d-c supply or it may be driven by an a-c motor, etc. This method of control is especially useful in the testing laboratory in the loss tests of motors and generators where it is very necessary that the speed be held at particular values.

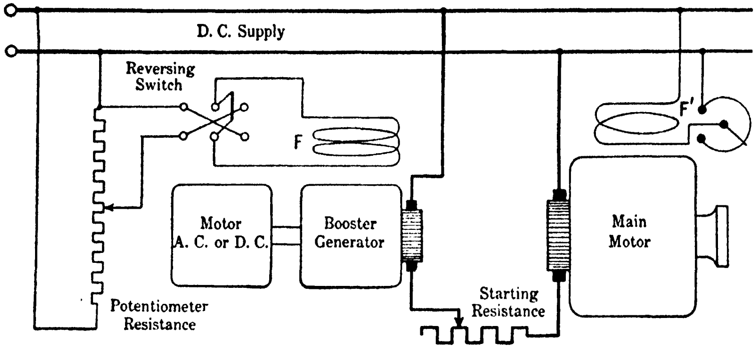

FIG. 237. Series booster method of speed control.

285. The Three-Wire System. — A shunt-wound motor connected to a three-wire system (§262) is shown in Fig. 238. The field winding is connected between the outside conductors, and the field current can be varied within certain limits by the rheostat in its circuit. One armature terminal is connected to the negative line conductor at *c*. The other terminal, *b*, may be connected at will either to the neutral conductor *n*, or to the positive conductor at *d*. To obtain the lowest speed, say 400 rpm, the motor field is fully excited, and the armature is connected across 110 volts. By gradually weakening the field, the speed (assuming a non-commutating-pole motor) can usually be at least doubled before difficulties in commutation begin; this gives any desired speed between 400 and 800 rpm. The armature may then be switched on to the full voltage, 220 volts, and the field again fully excited. This will again give a speed of 800 rpm; by weakening the

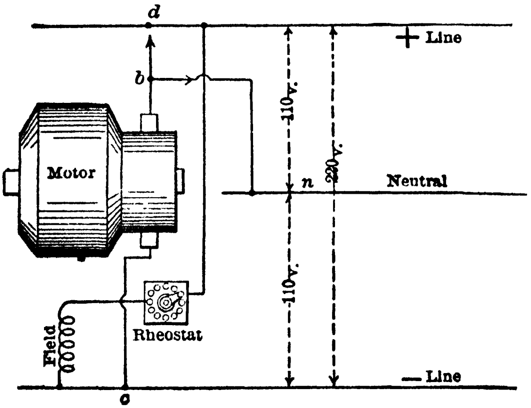

FIG. 238. Motor connected to a three-wire supply.

field as before, the speed can be raised to 1600 rpm. This gives at least the same speed range of four-to-one as can be obtained with a commutating-pole motor on a two-wire supply.

A still wider speed range may be had by using the so-called *unsymmetrical* three-wire system. The total line voltage, say 250 volts, may be divided by the middle wire into 160 and 90 volts. This gives three motor speeds in the ratio of 25 : 16 : 9, without the field control. By weakening the field this range can be at least doubled. This system has not been used much. probably on account of additional complication in maintenance.

Instead of a three-wire system, two motors on a two-wire system can be used for driving the same load. By connecting the armatures in series and in parallel (as in an electric car), half speed and full speed can be obtained. It is even unnecessary to have two separate motors; the two armature windings can be put on the same core and made to revolve in the same field. This is called the " two-commutator motor " (Fig. 239). When *a* is connected to *b* and *b'*, and the connections at *c* and *d* are open, the two armature windings are in series and the motor runs at half speed. When *a* is connected to *c*, and *b'* to *d*, the connection between *b* and *b'* being open, the two windings are in parallel and the motor runs at full speed. The field circuit is provided with a regulating rheostat, by means of which the range of speed is further increased and intermediate speeds are made possible.

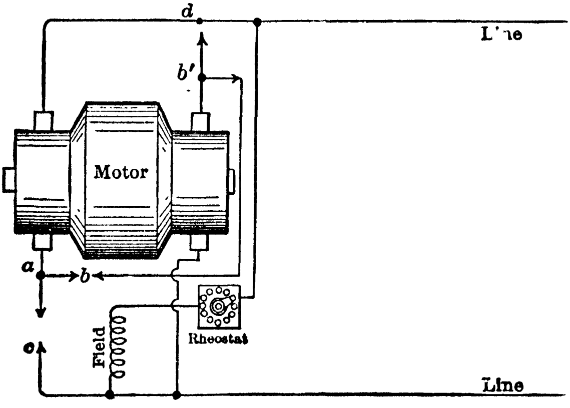

FIG. 239. A two-commutator motor; the armature windings may be connected either in series or in parallel.

286. EXPERIMENT 12-B. — Speed Characteristics of the Shunt-Wound Motor with Armature Rheostat Control. — Connect the motor under test to a generator, a blower, or some such steady load, and provide an adjustable resistor in series with the armature circuit of the motor. This resistor must be able to carry the full rated current of the motor for a considerable time without overheating. Excite the motor field to the highest possible value and start the armature in the usual way by cutting out part of the resistance in the armature circuit. Adjust the load and the armature resistance in such a way as to have the rated current through the motor at about 90 per cent of the line voltage across the armature terminals. Read armature volts and amperes, the field current, the motor speed, and the voltage drop across the regulating resistance. Throw off the load completely and take similar readings. Perform similar runs with the voltage at the armature terminals reduced to 80, 70, 60, etc., per cent of the line voltage and with the same rated armature current. Also take a few runs at a different value of the field current. Make runs with the motor loaded and unloaded when the armature is connected directly across the line, all of the regulating resistance being short-circuited. Make a similar investigation with the regulating rheostat connected across the line, as in Fig. 235. Use a very small motor for such a test and keep the current through the resistor at a value equal to several times the motor current.

Report. (1) Use the full-load volts across the armature as abscissas, and plot the following curves: (a) volts at the armature terminals at full load (45-degree line); (b) same at no load; (c) volts lost in the regulating resistance at full load and at no load; (d) motor speed at full load and at no load. (2) Devise a method for plotting the results of the test with the potentiometer arrangement, so as to bring out its good and its objectionable features. (3) Compare these two methods of speed regulation in regard to convenience, expense, efficiency, and speed fluctuation between no load and full load.

287. EXPERIMENT 12-C. — No-Load Tests of a Shunt-Wound Motor Using Armature Voltage Control. — The purpose of this experiment is to investigate the limits and the relative advantages of the various methods of speed control described in §§281 to 285. This study may be conveniently performed at no load because, with a given field excitation and at a constant applied voltage, the speed of a shunt-wound motor varies but a few per cent between no load and full load. This statement does not hold true for the rheostatic control (§282), with which the voltage across the armature is necessarily a function of the armature current.

(1) *The Booster System.* Connect the booster generator in series with the laboratory supply, as in Fig. 237. Excite the motor under test to a convenient value and keep the field current constant throughout the

run. Bring the motor-generator set to its rated speed, with the generator field circuit open. Start the motor under test and bring it up to speed by gradually short-circuiting the starting rheostat. Read the armature volts and the ampere input into the motor armature, and measure the speed. Excite the booster generator field in steps, so as to increase the voltage across the motor armature, and at each step repeat the above readings. Again open the generator field, and, after the motor under test has slowed down to its normal speed, take check readings at this speed. Reverse the field connections of the generator and again increase the excitation in steps, taking at each step the same readings as above. Repeat the run with two or three other values of field current in the motor under test.

(2) *Ward-Leonard system (motor-generator set).* Connect the motor under test to an available generator forming part of a motor-generator set, as in Fig. 236, and have both machines separately excited from the supply circuit. Set the field excitation of the motor under test at some convenient value and keep it constant throughout the first run. Run the generator at its normal speed *with the field circuit open* and close the switch between the two machines. Start the motor and bring it up to speed by gradually exciting the generator. Vary the generator excitation in steps, and at each step read the amperes and volts at the motor armature terminals, and its speed. Reduce the excitation of the generator again to zero and reverse the direction of the exciting current of the motor. Gradually cut out the field rheostat, causing the motor to start and to run in the opposite direction. Take one or two check readings at the same values of armature voltage as before. Investigate the effect of brush shift in each direction upon the speed and the armature current of the motor. Note the degree of sparking, if any occurs. Repeat the run with one or two other values of field current in the motor under test.

(3) *Three-wire system.* If a three-wire system is not available, it can be improvised by one of the means described in §263. Connect the field circuit across the outside mains and keep the field current constant throughout the test. Run the motor with the armature connected at full voltage and at half voltage (Fig. 238). With an unsymmetrical three-wire system, make a run with the armature across each of the three voltages. Read the field current, armature current, volts, and speed. Perform similar runs with two or three other values of field current.

(4) Measure the resistances of the motor and booster armatures.

Report. Describe and draw the various connections used during the experiment, and shows by mean of curves or tables that the motor speed at no load and with a given field current is nearly proportional to the voltage at the armature terminals. Give a theoretical reason for this fact. Compare the range of speeds obtainable with the various systems tried, and point out the relative complication and expense of each. Give a few examples of industrial applications in which each of these systems might be preferred. Since the armature volt-amperes represent the input necessary for overcoming the iron loss and friction in the motor (§275), show from the readings that with a given field current the losses increase with the speed, and that with a given speed they increase with the field excitation. Explain separately the reasons for the increase in hysteresis loss and in eddy currents with the speed and with the field excitation.

288. The Ampere-Speed Curve of a Shunt-Wound Motor. — A direct measurement of the mechanical output of a motor is avoided in practice whenever possible, because a Prony brake (§311) can be used only with comparatively small motors, and a transmission dynamometer (§§312 and 313) or an electrodynamometer is expensive and not always available. For these reasons it is often preferred to compute the output from a given or assumed input, and the losses (Chapter XIII). Such computations are simplified and made more accurate if the relationship between the electrical input and the speed is determined experimentally.

This relationship, or the ampere-speed curve, is obtained by connecting the motor under test to a generator, a blower, a centrifugal pump, or some such steady load. The motor is then run within the desired limits of input, voltage, field current, etc., and the corresponding values of speed are carefully measured. Because of a steady load, the speed can be kept constant and can be accurately measured. The electrical input can also be read accurately. The losses within the motor being known from another test, the performance curves (Fig. 231), including the output, may be computed quite accurately, without actually measuring the output.

For example, let a set of readings in an ampere-speed test of a small shunt-wound motor be as follows: armature current 15 amperes, speed 880 rpm, terminal voltage 220 volts, and the field current 1 ampere. Let the resistance of the armature (including the brush contacts) be 0.50 ohm, and let the iron loss and friction, found from another test, be 200 watts. We then have

Input into the armature	15 × 220	= 3300	watts
Input into the field	1 × 220	= 220	"
Total input	3300 + 220	= 3520	"
Armature copper loss	$15^2 \times 0.5$	= 113	"
Core loss and friction		= 200	"
Output	3300 − (113 + 200)	= 2987	"
Efficiency	2987/3520	= 84.8	per cent
Torque, §275, eq. (6)	974 × 2.987/880	= 3.63	m-kg

The foregoing results could not have been obtained so readily without the ampere-speed curve because the speed itself is an important characteristic, and if the exact speed at a given input is known, the proper value of the core loss and friction can be obtained from the no-load loss. Moreover, it would not be possible to compute the useful torque from the output, without knowing the speed. It is true that the full-load speed can be estimated from the no-load speed, by allowing for the armature voltage drop; but the armature reaction and the voltage drop at the brushes make this computation somewhat uncertain.

An ampere-speed test, in place of a brake test, is useful not only in the case of shunt-wound motors, but with all kinds of motors, a-c as well as d-c.

289. EXPERIMENT 12-D. — Ampere-Speed Curves of a Shunt-Wound Motor. — The purpose of this experiment is to obtain the relationship between the ampere input into the motor armature and its speed, with the different values of field current (Fig. 231). This test should preferably be performed on the same motor as Experiment 12-A in order that the performance characteristics may be computed. If the supply voltage is unsteady, provide a regulating rheostat or a booster by means of which the voltage at the motor terminals may be kept constant when taking readings. Use the same data sheet as in §290, omitting the last column.

(1) Connect the motor under test to a suitable generator, blower, centrifugal pump, or some such steady load. If an electric generator is used it may be convenient to feed the power back into a supply circuit rather than to dissipate it in resistances. Set the brushes initially in the neutral position, although in a motor without interpoles it may be necessary to shift the brushes to a position that will give the best average commutation under all load conditions. Excite the motor under test with the largest field current possible, and bring the motor gradually up to speed on a light load. Increase the load until the armature current in the motor under test reaches its highest safe value. Run the set for a sufficient time to make sure that the friction and lubrication conditions are constant. Read armature amperes, field amperes, terminal volts, and motor speed. Reduce the load in steps to zero, keeping the field current of the motor and the voltage across the armature terminals constant. Take the same reading at each step. For some moderate value of load take readings with the brushes shifted into different positions in each direction from neutral.

(2) Repeat the same run at several other values of the field current, in each case going to the limit imposed by speed commutation, or heating.

(3) Measure the resistance of the armature winding including the brushes, using a wide range of currents. Measure also that of the interpole winding, if present.

Report. Against armature amperes as abscissas, plot the speed curves at the different values of field excitation. On the same curves indicate the effect of brush shift. For several points on these curves compute the losses, the output, and the efficiency, as in §288, and plot the performance curves shown in Fig. 231. A separate set of curves must be obtained for each value of field current, and two or three sets may be plotted on the same curve sheet with ink of different colors.

290. EXPERIMENT 12-E. — Brake Test of a Shunt Motor. — The purpose of the experiment is to obtain directly the performance curves shown in Fig. 231. The motor should be connected as shown in Fig. 228, and a suitable brake or dynamometer provided (§§311 to 313). (Frequently the most convenient and accurate load device is a separately excited shunt generator, direct-connected or belted to the motor which is to be tested. The various losses of the generator being known [see Chapter XIII] from previous tests and measurements, they need only be added to the measured output [volts times amperes] of the generator to give the generator input, which is, of course, the motor output.) In Fig. 228, one circuit includes the field winding with its regulating rheostat and an ammeter; the other circuit is formed through the armature of the motor, with an ammeter and a starting and regulating rheostat. The latter is needed if the supply voltage is not steady. A voltmeter is connected across the armature terminals. The readings may be conveniently recorded on a data sheet similar to the one shown below.

	Armature Amperes	Field Amperes	Volts	RPM	Torque
Instr. No.......					
Constant.......					

Begin the test with the highest load, say 25 per cent overload, and at the highest field current. Read the armature amperes, field amperes, terminal voltage, speed, and brake load. Shift the brushes a few degrees in each direction and take the same readings; note sparking, if any.

With the brushes correctly set reduce the load in steps to zero, and take a sufficient number of readings for plotting the curves. The field current and the terminal voltage must be kept constant throughout the test. Make similar runs at a few other values of field current. Also make runs with other values of line voltage, say 10 per cent above and a like amount below normal, in order to see the influence of this factor on the performance of the motor.

Report. For each value of field current, plot the performance curves shown in Fig. 231. On the same curve sheet, indicate the effect of varying the supply voltage and of shifting the brushes.

291. No-Load Saturation Curve of a Shunt-Wound Motor. — When it is desired to obtain experimentally a no-load saturation curve (§236) of a d-c machine, and no driving motor is available, the machine under test may be driven as a motor at no load, the speed being held constant by varying the armature volts as the field is changed, and a saturation curve obtained in this way. The machine is connected as in Fig. 228, the field circuit being regulated independently of the armature circuit. In performing this test it is of importance to select the proper speed — one that can be maintained within wide limits of field current.

The best way is to begin with the highest possible value of the field current, and at the highest available voltage at the armature terminals. The value of the speed obtained under these conditions is then maintained at lower values of the field current, by lowering the armature voltage accordingly. The values of the voltage read at the armature terminals are practically equal to the induced emf, but if greater accuracy is desired the ohmic drop through the armature may be subtracted from the readings.

It is not important to obtain experimentally a saturation curve at exactly the speed for which it is to be used. The induced voltage at no load is strictly proportional to the speed, so that the observed values may be easily recomputed for any other desired speed.

292. EXPERIMENT 12-F. — No-Load Saturation of a Shunt-Wound Motor. — The purpose of the experiment and the connections for it are explained in the preceding section. Begin with the highest possible value of field current, and with the full line voltage, or even over-voltage, applied at the armature terminals. Read the field current, the armature current, the armature volts, and the speed. Reduce the field current in steps, and at each step introduce more resistance into the armature circuit; or vary the armature voltage in some other way (§283), so as to keep the speed nearly constant. Read the instruments at each step. Before leaving the laboratory, measure the resistance of the armature, including the brushes. For this measurement use approximately the same values of current as were read during the test.

Report. Correct the observed voltage readings for the *ir* drop. Correct the induced emf's so obtained for the effect of incorrect speed, if any. Plot the corrected voltages against field current as abscissas, at a constant speed. Show by a few readings that the induced voltage is proportional to speed.

293. Dynamic Braking. — Dynamic braking (Fig. 240) is used for stopping or slowing down a motor, and consists in reconnecting it so as to convert it into a generator and make it dissipate its stored kinetic energy in the form of I^2R heat in an external resistance. This change is accomplished by means of two switches, *a* and *b*, which in this case are shown as contactors, operated by individual electromagnets. When the switch *a* is closed and *b* is open, the machine runs in the usual way as a motor. With the switch *a* opened and *b* closed, and with the field circuit connected to the line as before, the motor armature will generate an emf and force a current through the resistance *R*.

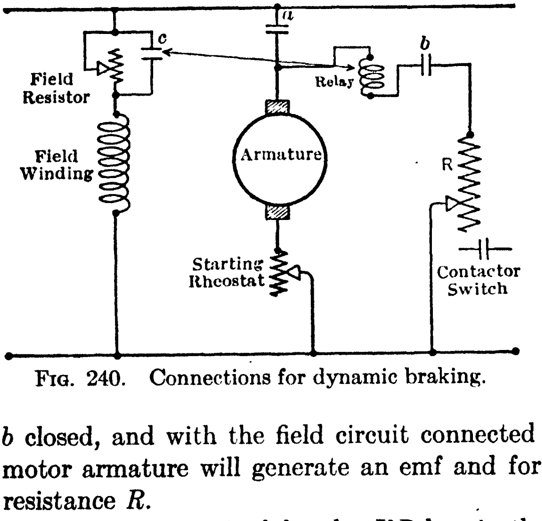

Fig. 240. Connections for dynamic braking.

The energy required for the I^2R loss in the resistance is supplied at the expense of the stored mechanical energy in the armature, flywheel, gears, lifted load, etc. In many cases dynamic braking is utilized only for slowing down, as it is most effective at high speeds, and a mechanical brake is applied to stop the machine.

An adjustable-speed motor may be running at a high speed with its field weakened, when the operator desires to stop the motor. The braking effect is strongest with the full field, and therefore a contactor switch *c* is sometimes provided to short-circuit the field rheostat. This switch may be made to operate automatically by a relay in series with the brake resistance, so that *c* is closed as soon as a current begins to flow through *R*.

The advantages of dynamic braking over a mechanical brake are: (a) The stored energy is largely dissipated in an external resistance which may be remotely located, while a friction brake develops heat close to the motor. (b) A quicker reduction of speed is obtained, because a time element is involved in the setting of the friction brake. (c) The motor shaft is relieved of a considerable strain to which a mechanical brake subjects it. (d) The wear and tear and the upkeep expense of the mechanical brake are reduced when it is used for the final stop only. On the other hand, dynamic braking involves certain additional complications and expense; and if it is used frequently, the motor capacity must be somewhat greater, to take care of the additional heating of the armature.

Dynamic braking is also used to bring the motor quickly from full speed to a desired lower speed. In this case, during braking the resistance *R* (Fig. 240) is connected in parallel with the armature, and the two are connected to the line in series with the starting rheostat. The voltage drop in the latter reduces the voltage at the motor terminals to a value below that of its counter emf, so that the armature generates a current which circulates through the resistance in parallel with it. The armature quickly comes to a lower speed, at which the regular motor connections may be reëstablished, or the circuit opened and a mechanical brake applied.

In the case of a series motor, dynamic braking is often used on unbalanced hoists for lowering the load, but it is necessary to excite the fields separately. The series-field winding is connected across the line in series with a resistance, thus converting the machine into a separately excited generator (Fig. 200).

The so-called regenerative control, or pumping-back of power into the line, is somewhat similar in principle to dynamic braking. The power is not wasted, however, but is recovered for use in other devices connected to the same line. A notable example is the regeneration of power by an electric train going down grade. The series fields are excited from a separate low-voltage generator, and the speed can be nicely controlled within wide limits. Air brakes need be used for stops only. See also Chapter XIV.

294. EXPERIMENT 12-G. — Dynamic Braking and Regenerative Control of a Shunt-Wound Motor. — For the purpose and the method of dynamic braking see the preceding section. The experiment can be better performed on a motor connected to a heavy flywheel or belted to another machine, as otherwise the time of stoppage may be too short for reliable readings. Connect the motor as in Fig. 240, using a double-throw switch in place of the contactors *a* and *b*. Provide an ammeter in the brake resistor circuit and one in the field circuit. Connect a voltmeter across the armature, and have a good tachometer for reading instantaneous speeds. Special recording instruments (§64) should be used for accurate results. (1) Bring the motor up to speed at no load with the full field current. Record the field current and the speed, and change the armature circuit quickly from the line to the brake resistor. Read load amperes, armature volts, and the instantaneous speed as accurately as possible, every few seconds until the motor stops, or at least until it slows down to a low speed at which it can be readily stopped by a mechanical brake. Note the position of the regulating handle on the brake resistor. (2) Repeat the test with a few different values of brake resistance. (3) Perform similar runs with different values of field resistance. (4) Make a few runs in which the field resistance is " in " when the armature is connected to the line, and is short-circuited at the same time that the armature is switched over to the brake resistor. This may be done by hand or with an automatic magnet switch *c* shown in Fig. 240. (5) Vary the brake resistance during the process of retardation in such a way as to keep the armature current as nearly constant as possible, thus obtaining the maximum braking action. (6) Try the arrangement described in the preceding section for bringing the motor quickly to a lower speed, with the starting resistance in the circuit. Adjust the values of the resistances for the best results. (7) Bring the machine to a speed somewhat above normal and then, without disconnecting the armature from the line, quickly increase the field current by a moderate amount (too great a change will cause an excessive current). Observe the reversal of the armature current due to the generator action. A zero-center ammeter in the armature circuit is convenient for such a test. (8) Measure the resistance of the armature and of the brake resistor for the points used in the test.

Report. (1) Plot the readings against time as abscissas, or tabulate them. (2) Give the best value or values of brake resistance for use with this motor in regular operation, and prove these values from your test data. If the brake resistance should be short-circuited in steps during the process of retardation, give the values of step resistances and show how this may be done automatically by magnet switches, actuated either by the armature voltage or by the load current. (3) Give the diagram of connections and the results of the test specified under (6) above. (4) Consider theoretically the case of a compound-wound motor (§306) and explain what should be done with the series field during dynamic braking. (5) Give the general theory and the equations from which the time of stoppage or of speed reduction for a given shunt-wound motor could be predetermined by calculation. The reduction in the kinetic energy during an infinitesimal interval of time is equal to the I^2R loss in the brake resistance plus the losses in the motor itself (§339).

THE SERIES-WOUND MOTOR

295. The diagram of connections of a series-wound motor is shown in Fig. 229, and the performance characteristics in Fig. 241. The fundamental property of such a motor, at a constant line voltage, is that its torque varies greatly and inversely with the speed. The motor is capable of exerting quite a high starting torque without an excessive current, but as the load torque is reduced the motor speeds up practically without limit. Unless precautions are taken, a series-wound motor without load will run away, and the armature is likely to burst because of the excessive centrifugal force.

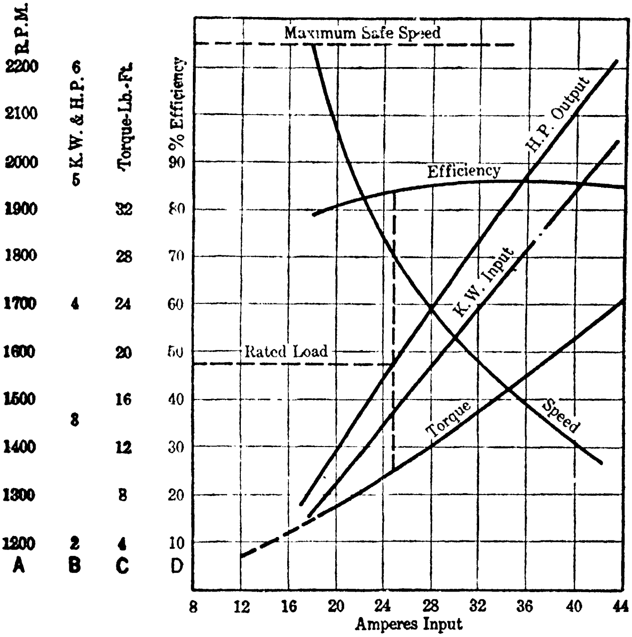

Fig. 241. Performance curves of a series-wound motor.

The reason for this behavior of the series motor lies in the fact that the armature current also excites the fields. At no load, the power input being only that to supply the losses, the motor current is small. This small current produces but a weak magnetic field, while the counter emf is practically equal to the applied voltage, the *IR* drop in the machine being very small. Hence, the motor armature speeds up until the counter emf induced by the weak field reaches the required value. This relationship may also be seen from eq. (3) in §275. At no load, e is practically equal to the constant applied voltage E. Consequently, the product of rpm and the flux Φ must be constant. But Φ is small, being excited by a small no-load current. Hence the speed must be high.

The speed of a series-wound motor thus varies automatically with its load torque, and this characteristic makes the motor particularly well adapted for railway, crane, and hoist service. Constant speed is not essential for such service; on the contrary, it is desired that the speed should automatically decrease as the load torque increases. At constant speed the heavy demands upon the power supply and the excessive overloads of the motor itself would make its operation unsatisfactory.

296. Torque Characteristics. — The electromagnetic torque of an electrical machine is proportional to the products of the armature current and the air-gap flux, other factors remaining unchanged, and is independent of the relative speed between the conductor and the field. This is a fundamental law of interaction between an electric current and a magnetic field (§272). The same relationship is also represented by eq. (8) in §275.

In a series-wound motor the field is excited by the same current that flows through the armature. Hence, without saturation in the iron parts, the torque would be proportional to the square of the current. Such is nearly the case at high speeds and light loads when the current is small. At start, when the machine takes a heavy current, the magnetic circuit is highly saturated, and the torque varies more nearly as the first power of the current.

Thus, in a series-wound motor both the speed and the torque are definite functions of the current, and therefore there is a definite relationship between the speed and the torque, the so-called *torque-speed* characteristic. If values of speed be plotted as abscissas and the corresponding values of torque as ordinates, the maximum ordinate is at zero speed, and the torque drops rapidly as the speed increases. The torque approaches the axis of abscissas at high speeds. This curve is used in plotting time-speed curves of electric trains and in other acceleration problems.

297. Speed Control of a Series Motor. — Since the series-wound motor is inherently a variable-speed machine (Fig. 241), the speed control in this case means an external change (made by a resistance or otherwise) in order to obtain a different speed characteristic. The two methods most commonly employed for this purpose are: a resistance in series with the motor circuit, and a resistance shunted around its field winding (Fig. 229). When two or more motors are connected to the same load, as for example in an electric car or locomotive, two ranges of speed are obtained by connecting the motors in series and in parallel. In special cases, such as in testing, the armature may also be shunted by closing switch S_1. All these methods will now be considered more in detail.

(1) *Series resistance.* This method is wasteful of power, requires large, well-ventilated rheostats, and is used only with comparatively small motors for intermittent service, such as crane and hoist work. The torque of a series-wound motor is a function of the current only. The speed of the motor depends upon the voltage at the armature terminals, so that a resistance in series with the motor makes it possible to obtain a desired torque at a lower speed. The power loss is in the same proportion as the voltage drop in the resistance. As the load torque varies, the current taken by the motor also varies, so that the voltage drop across the series resistance fluctuates with the load. Thus, the motor operates at a variable terminal voltage, and its inherent variable-speed characteristics are accentuated. A counter emf motor-generator set or booster (Fig. 237), when applied to a series motor, accomplishes the same purpose without so much power waste. Moreover, it can be used both to cut down and to boost the line voltage.

(2) *Shunt around the field winding.* The effect of shunting the field winding by a resistance (Fig. 229) is to weaken the field flux for a given armature current and consequently to speed up the motor. The counter emf being practically constant and equal to the applied voltage, a weaker flux means a higher speed. With a given load torque the motor draws more current from the line with the field shunted, because the torque is proportional to the product of the armature current and the field flux. The shunting of the series field is used in railway motors for running at a high speed on level stretches. Such motors are provided with interpoles for better commutation.

(3) *Series-parallel connection of two motors.* When two series-wound motors are connected in series across a constant-voltage line, each is working at one-half of the applied voltage, so that with a given current the counter emf is only about one-half that at full voltage. Therefore, with a given torque, the motor speed is reduced to one-half. This connection is used on electric cars for starting and for running at lower speeds.

(4) *Shunted armature.* Just as shunting the field winding speeds the motor up, so shunting the armature by a resistance slows it down. This is because the field flux becomes relatively stronger. This method is wasteful of power, because the shunting resistance is practically across the line voltage. Thus, for example, in a 500-volt motor the voltage drop across the field may be of the order of 20 volts, leaving 480 volts across the armature. One ampere shunted around the field winding represents a loss of only 20 watts, whereas 1 ampere shunted around the

armature means a loss of 480 watts. For this reason, this method is hardly ever used in actual service. For its use in testing see §299.

298. Performance Characteristics from the Ampere-Speed Curve and Losses. — The direct method of obtaining the performance characteristics of a series-wound motor (Fig. 241) is by loading the motor on a Prony brake (§311), on a dynamometer (§§312, 313), or on a calibrated d-c generator. With large motors the brake becomes bulky, inaccurate, and difficult to handle, and a transmission dynamometer of suitable characteristics is seldom available. In such a case the performance curves shown in Fig. 241 may be quite accurately computed from the so-called ampere-speed curve and the losses, both of which are determined experimentally.

(a) *The ampere-speed curve of a series-wound motor* is obtained by belting or otherwise connecting the motor mechanically to a steady load, and running the motor under the desired conditions of current, voltage, and field strength. The results of such a test are plotted with current as abscissas against speed as ordinate. Each such curve refers to a particular value of terminal voltage, with the field winding shunted or not, as the case may be (Fig. 229). Such a test can be performed quite accurately, the load being steady and there being no brake to keep adjusted while the speed is being measured.

There is a definite relationship between two ampere-speed curves taken at two different terminal voltages, but with the same current (that is, at the same field excitation), since the speeds vary as the counter emf's. For example, let the speed of a motor be 1200 rpm at 150 amperes and with 220 volts at its terminals. Let the ohmic drop in the machine at this current be 15 volts. The speed at the same current and at the terminal potential difference of 180 volts can be computed as follows: The counter emf is 205 volts in the first case and 165 volts in the second. With the same field strength in both cases, the speeds are proportional to the counter emf's. Thus, the unknown speed is $1200 \times 165/205 = 966$ rpm.

Similarly, there is a definite relationship between two speeds with the field shunted and unshunted. For example, the current in the foregoing illustration is 150 amperes, the resistance of the armature 0.06 ohm, and that of the field winding 0.04 ohm. Let the field winding be shunted by such a resistance that the armature current is 200 amperes when the field current is 150 amperes. The ohmic drop in the machine is $200 \times 0.06 + 150 \times 0.04 = 18$ volts, and the counter emf is 202 volts. Hence the speed under these conditions will be $1200 \times 202/205 = 1183$ rpm. This computation is not quite exact, because at 200 amperes the armature reaction is greater than at 150 amperes, and consequently the net field flux is somewhat weaker.

(b) *The losses of the series motor* consist of the I^2R loss in the armature and field windings, the iron loss, and friction. The copper loss is usually computed from the resistance of the windings, and the iron loss plus friction (or the total no-load loss) may be determined from a simple test based upon the following considerations:

The friction and windage loss depends upon the speed only. The iron loss, or the armature core loss, consists of hysteresis and eddy currents. It depends upon the speed and upon the field excitation, and increases with both. Therefore, to measure the rotation loss, the motor must be run at no load, at the same speed, and with the same field current as at a certain desired load. The power input into the armature is then almost entirely used for overcoming the no-load losses and is a measure of these losses.

For this purpose the motor is run with the armature shunted by a variable resistance (Fig. 229), as is mentioned in §297 under (4). By regulating the line rheostat and that in parallel with the armature, it is possible to adjust the speed and the field current to correspond to one of the points on the ampere-speed curve. The power input into the armature, corrected for the I^2R loss, will give the sum of the no-load losses. The subject is treated more in detail in §§329 and 330, but the above information is sufficient for calculating the performance characteristics.

When the power input into the motor is known from the ampere-speed curve, the output is obtained by subtracting from the input the computed copper loss and the measured no-load losses. The efficiency is equal to the ratio of the output to the input. The useful torque developed by the motor on the shaft is computed from the output, by means of eq. (6) or (7) in §275.

299. EXPERIMENT 12-H. — Ampere-Speed Characteristics of a Series-Wound Motor. — The purpose of the experiment and the use of ampere-speed curves are explained in the preceding section. This experiment and the one following should be performed on the same motor.

(1) Wire up the motor, as in Fig. 229, and connect it to a suitable steady load. Start the motor in the usual way with the starting rheostat, and adjust the load so as to make the motor draw the heaviest safe current. The terminal voltage must be kept constant throughout each run. Read amperes, armature volts, and speed. Reduce the load in steps and take similar readings at each step. (2) Make similar runs at one or two other values of terminal voltage. (3) Make a few runs at the same terminal voltage as in the first run, but with the field wind-

ing so shunted as to reduce the field current by a definite percentage in each run. (4) Measure the resistances of the armature and field.

Report. (1) Plot the observed values of current and armature voltage to speed as abscissas. (2) Show that the curves taken at different terminal voltages agree among themselves. (3) Show that the curves with the field shunted and unshunted also agree among themselves. (4) Compute an ampere-speed curve for a different assumed terminal voltage and with a different amount of shunting.

300. EXPERIMENT 12-I. — Efficiency of a Series-Wound Motor from its Ampere-Speed Curve and No-Load Losses. — The purpose and the method of the experiment are described in §298. The experiment should be performed on the same motor as the preceding one, and the ampere-speed curves should be drawn in advance. All the runs specified below are to be performed at no load.

(1) Connect the motor as in Fig. 229. Select a point on an ampere-speed curve corresponding to the lowest speed and the highest current. Adjust the resistance in the main circuit and that in parallel with the armature until the motor runs at the same speed and with the same field current as on the selected point on the ampere-speed curve. When the field winding is not shunted, the field current is the same as the armature current, but with the shunted field it is important to have the correct field current, because the no-load losses depend on it. Having adjusted the speed and the field excitation to the exact values they have at the selected point on the ampere-speed curve, read the armature amperes and the voltage between the armature terminals. Take similar readings for a number of points on the same ampere-speed curve. In the same manner duplicate some points on the other ampere-speed curves, if performance characteristics are desired at other values of terminal voltage, and with shunted fields.

Report. For each set of readings, compute watts input into the armature and subtract the armature I^2R losses, if appreciable. Plot curves of no-load losses and of the total copper loss to correspond with each ampere-speed curve. From these data compute the performance characteristics shown in Fig. 241 Plot them either to amperes, to torque, or to speed as abscissas.

301. EXPERIMENT 12-J. — Brake Test on a Series-Wound Motor. — The purpose of the experiment is to obtain directly the performance curves of the series motor, as shown in Fig. 241. Precautions should be taken to keep the motor from running away. With a shunt motor the brake can be safely released, since the speed of the motor is practically the same at no load as when loaded. In a series motor the speed increases enormously as soon as the load is taken off, and either the armature, the commutator, or the bearings may be damaged if the motor be allowed to run at this speed. For this reason, always *open the electric circuit before releasing the brake;* or at least insert a sufficient resistance to keep the speed down.

As an additional precaution against running away, an underload circuit-breaker may be connected in the circuit; when the load is taken off and the current falls below a certain limit, this device automatically opens the circuit. However, the student should not rely absolutely on such a circuit-breaker, since it may " stick " just when it is expected to act. It is best to have one man of the section stand near the main switch, and open the circuit if the motor should reach a dangerous speed. The terminal voltage should be kept as nearly constant as possible, and if necessary a regulating rheostat should be used for this purpose. The same data sheet may be used as in §290.

Begin the test with the highest load, take readings of amperes, volts, speed, and torque. Then reduce the load by approximately equal steps, until the safe upper limit of the motor speed is reached.

Take a few readings with the field weakened by about 10 and 20 per cent by means of a shunt resistance, and read both the armature and the field current. Take runs with the supply voltage about 10 per cent above and below its normal value. Measure the resistance of the armature, including the brushes, and also that of the field winding.

Report. Plot the performance curves shown in Fig. 241, using the output, the torque, or the speed as abscissas. Check theoretically the observed values of speed, with the field winding shunted and unshunted, by computing the corresponding counter emf's (§298).

302. EXPERIMENT 12-K. — No-Load Saturation Curve of a Series-Wound Motor. — This curve may be obtained in any one of the following three ways: (1) By driving the machine as a generator with the field excited from an independent source (§236). (2) By driving the machine as a motor at no load, with the field separately excited; the test is then identical with that described in §§280 and 291. (3) By driving the machine as a series motor at no load, with the armature shunted, as in Experiment 12-I, and its speed kept constant.

303. Starting Torque of Series-Wound Motor. — In some kinds of service for which the series motor is used, a knowledge of its starting torque is of particular importance. This information may be obtained by means of a Prony brake or transmission dynamometer, in the way in which values of torque are obtained with the motor running, except that at standstill, the friction is somewhat indefinite. Read torque while the brake arm is being slowly raised and again while it is slowly lowered then take the average value. Values of starting torque are often required at high currents, which the motor can stand for only a few seconds without overheating. Such readings must be taken promptly, and the motor must be allowed to run light between the readings, to cool the windings.

The torque at a high value of current may also be estimated by computation from the measured torque at a moderate current, provided that the no-load saturation curve of the motor is known (§302). According to eq. (8), §275, the electromagnetic torque is proportional to the product of the current and the flux. Let a torque T_1 be exerted between the armature and the field with a current of 50 amperes flowing through the machine, and let it be required to determine the torque at 100 amperes. Let the following data also be known from the no-load saturation curve, taken at a constant speed:

Field Current (Amperes)	*Armature emf* (Volts)
50	72
100	90

The torque T_2 at 100 amperes is then determined from the expression

$$T_2/T_1 = (100/50) \times (90/72)$$

More generally

$$T_2/T_1 = (I_2\Phi_2)/(I_1\Phi_1) = (I_2e_2)/(I_1e_1) \quad . \ . \ . \ . \ (11)$$

where e_2 and e_1 are the induced voltages taken from a no-load saturation curve of the machine at the exciting currents I_2 and I_1, respectively.

It must be clearly understood that T_1 and T_2 are the values of the total electromagnetic effort between the field and the armature, and not the values of the useful torque available on the shaft. At standstill the latter values are smaller by the amount of static friction which must be measured or estimated separately.

A source of inaccuracy in formula (11) lies in the armature reaction (§365) which makes the value of the flux, when a torque is exerted, different from that with the same field excitation and with no armature current. It would therefore be more accurate to use the values of the induced voltages from an ampere-speed curve (§298) where the armature reaction is present at its full value.

304. EXPERIMENT 12-L. — Static Torque Test on a Series-Wound Motor. — The purpose of the experiment is a direct study of the starting torque of a series motor with a brake. Begin the test with the heaviest possible current and take readings promptly so as not to overheat the machine. Estimate the static friction from the difference in scale readings while the brake arm is being raised and lowered. Drive the motor as a generator by means of another available machine, and take its no-load saturation curve. If preferred, the no-load saturation curve may be obtained by one of the other two methods mentioned in §302. Measure the resistances of the windings.

Report. Plot the values of starting torque against amperes as abscissas. Give the data on the estimated static friction of the machine. Plot the no-load saturation curve and show how closely the points on the observed torque curve check experimentally with eq. (11). Explain the effect of the armature reaction on the torque at low and at high saturation and at the knee of the saturation curve. Assuming the torque to vary as the nth power of the current, give the values of n for small, medium, and high values of the current.

305. Motor for Starting Automobile Engine. — Motors used for cranking automobile engines are usually of the series type designed to give a particularly heavy starting torque, although their rating for continuous service is very small. So far as the motor itself is concerned, its tests are not different from those performed on any other series motor and described above. It is customary to plot the performance characteristics of such a motor to speed as abscissas rather than to its amperes, as shown in Fig. 241.

There is one feature which distinguishes the actual service performance of an automobile starting motor from that of a series motor connected to a large constant-potential source of supply. The automobile motor is operated from a storage battery of limited capacity, and without any starting or regulating resistance in series. Before the armature acquires any considerable speed and counter emf, the motor draws a very large current which is limited only by the low internal resistance of the battery (§531) and of the motor and leads. This current, and the voltage at the motor terminals, depend on the kind and size of battery used, on the state of its charge, the temperature, etc.

Thus, performance curves of an automobile starting motor, obtained with a constant voltage supply, will not give a true picture of its performance with a battery. On the other hand, the performance obtained with a battery may be misleading, unless the condition of the battery is known. In practice, such tests are made with a battery of a certain agreed type and size, which is well charged. Motors which under such conditions do not develop a certain specified torque are rejected.

THE COMPOUND-WOUND MOTOR

306. An ordinary shunt-wound motor is entirely satisfactory in most cases where an approximately constant or adjustable speed is required with variable load, as for example in machine-tool drive. However, there are cases of considerable practical importance where the characteristics of the shunt-wound motor can be improved by providing it with an additional series winding, similar to that of a compound-wound generator. Such a motor is called a compound-wound motor; the series winding can be connected either to strengthen or to weaken the field produced by the shunt winding. In the first case the motor is said to be *cumulatively* compounded; in the second, *differentially* compounded.

307. Cumulative Compounding. — Cumulative compounding of a shunt motor is used when the motor is frequently subjected to heavy short overloads, for example, when driving a punch and shears or a planer, or where a high starting torque, together with a moderate top speed are required, as on an elevator. With a shunt-wound motor, when a punch touches a sheet of metal, or when a planer tool begins a new cutting stroke, the motor instantly slows down and there is a rush of current into the motor armature to provide the necessary torque. With a cumulatively compound-wound motor this rush of current is reduced because the field is automatically strengthened when an overload occurs, so that the same torque is obtained with a smaller current than in the shunt motor.

The speed of a cumulatively compounded motor fluctuates more than than in a shunt-wound motor, because of the above-mentioned automatic strengthening of the field on overloads. This is an advantage rather than a drawback, in that the output is automatically reduced on overloads as compared to that which it would be at a constant speed.

A compound-wound motor possesses operating characteristics intermediate between those of a shunt-wound and a series-wound motor. By properly selecting the relative number of ampere-turns in the two windings, the characteristics may be made to approach those of either of the two simpler types of motors. It is even possible to adjust the characteristics during the operation. Thus, a compound-wound elevator motor may be started with a large number of series turns to get a high torque, and then part, say alternate poles, of the series winding may be short-circuited, or the winding may be shunted, to give a higher speed and more of the constant-speed characteristics of a shunt-wound motor. For further details regarding field windings see §250.

308. EXPERIMENT 12-M. — Effect of Cumulative Compounding on a Motor. — The experiment is conducted essentially as an ordinary load test on a shunt motor (§§289 and 290). In order to see more clearly the influence of the series winding, it is advisable to provide different degrees of compounding, either by dividing the series winding into sections, or by connecting a variable shunt around it (Fig. 230).

(a) Begin with the heaviest load and no compounding, that is, use the shunt winding only; adjust the load so as to have the armature current, say, 25 per cent above the normal rating. Read amperes, volts, speed, and field current; also the torque if a brake or dynamometer is used. Reduce the load in steps, reading as before, until no load is reached.

(b) Introduce, say, one-half of the series winding into the circuit, and take readings over the same range of input or brake torque as before. Insert all the series ampere-turns, vary the load, and read as before. The results of these tests will show the influence of compounding on the current taken by the motor, on its efficiency, and on speed variations as compared to an ordinary shunt motor. If it is possible to keep the brushes stationary they should not be shifted during the whole test, as this would change the armature reaction, and consequently the speed of the motor. Should it be necessary to move the brushes because of an excessive sparking, this must be noted in the results, and a record made of the number of commutator bars by which the brushes were shifted.

(c) Should time and equipment permit, the behavior of the motor under actual fluctuating load conditions should be observed, both with and without the series winding. A lathe is convenient for this purpose; a piece of work is put in it, such that the tool cuts during a part of the revolution, thus giving the motor a fluctuation. No *exact* measurements can be attempted under these conditions unless a special recording ammeter and a tachograph are available. In the absence of such, the current is read every few seconds, and speed observed as closely as possible with an ordinary tachometer. The results when plotted to time as abscissas will give an approximate idea of the general performance of the motor under sudden overloads, with and without the series winding.

Report. The results of the tests should be plotted to either torque, output, or armature amperes as abscissas, all on the same sheet, so as to show the influence of compounding on speed, efficiency, input, and output of the motor.

309. Differential Compounding. — There are cases in which a nearly *constant speed at all loads* is very essential. This is the requirement, for instance, in some textile mills, where one motor operates a large number of spinning or weaving machines. The speed of the motor should not depend on the number of machines in actual operation, for every change in speed affects the quality or the design of the product. Motors used for such purposes are provided with a shunt winding and also with a demagnetizing or *differential* series winding, for the following reason:

A shunt-wound motor usually slows down a few per cent as the load increases, and in order to bring up the speed to its former value, it is necessary to weaken the field of the motor by the field rheostat. In a differentially compounded motor this is done automatically. When the load increases, the armature current also increases; thus a larger current flows through the series winding. Since this winding is so connected as to oppose the shunt-field winding, the field strength is reduced, and the motor speeds up. By a proper adjustment of the number of turns of the series winding, a fairly constant speed can be secured throughout the working range of the motor. By a further increase in the number of turns the speed of the motor may be made to increase with the load.

It is not possible to obtain an absolutely constant speed at all loads because of the saturation of the iron parts. The speed depends upon the *flux*, whereas the compounding action is proportional to the number of *ampere-turns.* When the iron is far from saturation the flux is proportional to the ampere-turns, so that theoretically a constant speed may be maintained at all loads. With the usual degrees of saturation this condition does not hold true, and a motor compounded so as to give the same speed at full load as at no load runs at somewhat lower speeds at partial loads. This result of saturation is observed in a generator compounded so as to give the same voltage at full load as at no load; it then gives too high a voltage at intermediate loads (§249).

A differentially compounded motor does not start so easily as an ordinary shunt motor because the starting current weakens the field and thereby reduces the starting torque. If an excessive starting current is allowed to flow through the series field, the action of the latter may even become stronger than that of the shunt field, and the motor will have a tendency to start in the wrong direction. For this reason differentially wound motors are usually provided with a switch for short-circuiting the series winding during the starting period. After a considerable speed has been attained, and the current has dropped to a lower value, this switch is opened, and the motor runs up to its rated speed.

With the development of satisfactory centrifugal speed regulators, the field of application of differentially compounded motors has been narrowed down to special cases.

310. EXPERIMENT 12-N. — Effect of Differential Compounding on a D-C Motor. — The purpose and the theory of differential compounding are explained in the preceding section. The motor under test is connected to a steady load, such as a generator or a blower, either directly or through a dynamometer (§§312 and 313). A Prony brake is not so suitable for this experiment because of the difficulty of keeping the load constant while the speed is being read. If the output cannot be measured by any of the above-mentioned means, the speed readings may be referred to the values of armature current plotted as abscissas (the ampere-speed curve, §288).

(1) Disconnect the series winding altogether. Begin the runs with the largest armature current and the highest shunt-field current practicable. Read both currents and the line voltage, and measure the speed as accurately as possible. Keep the shunt-field current constant and reduce the load in steps to zero, taking similar readings at each step.

(2) Introduce part of the series windings so as to oppose the shunt field, or if it is not practicable to subdivide this winding, connect all the series turns in the circuit and arrange to shunt them by a variable resistance. Repeat the runs performed in (1) with several different values of effective series turns.

(3) Run the motor as in (1) and (2) above, but at a much lower value of the shunt-field current. Make such runs at two or three different values of exciting current to determine the effect of the saturation upon speed regulation.

(4) Determine the number of turns in the series winding (§250).

Report. (1) Plot the observed values as ordinates against the armature amperes or torque as abscissas. For each value of the shunt-field current state the number of series turns which gives the best speed regulation between no load and full load. Calculate the percentage of speed deviation for some of the runs, especially the maximum percentage of deviation from the desired speed with the high and low values of the shunt-field current. Explain the difference theoretically.

BRAKES AND DYNAMOMETERS

311. The Prony Brake. — The performance curves shown in Figs. 231 and 241 can be obtained directly by loading the motor and measuring the input, output, and speed. The simplest device for loading a motor is a Prony brake (Fig. 242); it is extensively used with small and medium-size motors. In the form shown in the sketch, it consists of an iron band *ab* lined with soft wood, heavy canvas, or regular brake lining. The band embraces the pulley *P* of the motor, and is fastened to the beam *k*, the end of which rests on the scale *S*.

When the motor armature revolves, friction is developed between the brake lining and the pulley; the power of the motor is thus converted into heat. The brake pressure is regulated by the handwheel *h*, and in this way various loads may be obtained. The turning moment, or the *torque*, as it is called, is proportional to the pressure exerted and is measured on the scale. In most cases it is necessary to carry away the heat developed by friction, in order to prevent burning the brake lining. The pulley shown in the sketch is cooled by a stream of water from the pipe *w*; the water is kept against the inner surface of the face of the pulley by centrifugal force and the flange prevents it from spilling.

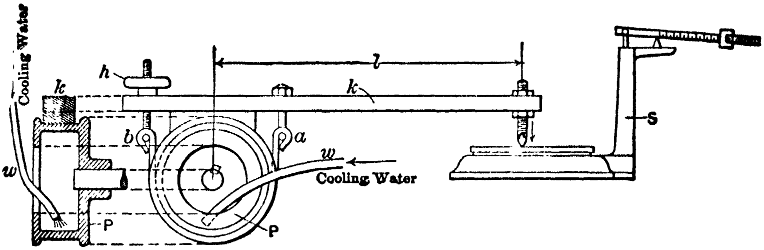

FIG. 242. A Prony brake with water-cooled pulley.

The brake arm exerts a certain pressure on the scale even with the loose band, because of its weight. This unbalanced weight, or tare, must be subtracted from the scale reading to obtain the net pressure. Let F be the *net* pressure on the scale at the end of a lever of length l. Then the torque $T = Fl$, in meter-kilograms, or in foot-pounds, according to the units used. The corresponding output, P, is computed from formula (6) or (7), §275.

For loading very large motors the so-called water-brake is used; power is absorbed in it by driving water through paths of high resistance. The resultant friction heats up and even evaporates the water, while cold water is being continually supplied from the mains.

Eddy-current brakes are used to some extent with small motors. Such a brake usually consists of a copper disk mounted on the motor shaft, and a set of electromagnets placed near the disk and supported on knife-edges or ball bearings. When the disk revolves, eddy currents are induced in it as it cuts through the field produced by the electromagnets. The electromagnets tend to follow the disk, as in the classical Arago experiment, but are prevented by a lever which rests on a scale, as in Fig. 242; the pressure on the scale measures the torque. The load is varied by regulating the exciting current of the electromagnets; artificial cooling of the disk is sometimes necessary. An eddy-current brake gives a steadier load than is possible with a mechanical brake and closely approximates the electro-dynamometer described in §313.

312. Transmission Dynamometer. — One objection to the Prony brake is that the friction between the brake lining and the pulley continually varies, so that the load is not quite steady while the speed is being measured; this limits the accuracy of the test. Another disadvantage is that a considerable amount of energy must be dissipated in a limited space. It is much more convenient to load the motor on a steady output, such as an electric generator, a pump, a blower, etc., provided that this output can be readily measured. One method is to connect the load to the motor under test through a transmission dynamometer, such as is shown in Figs. 243 and 244. This dynamometer does not absorb any power, but merely measures the torque between the driving and the driven shafts.

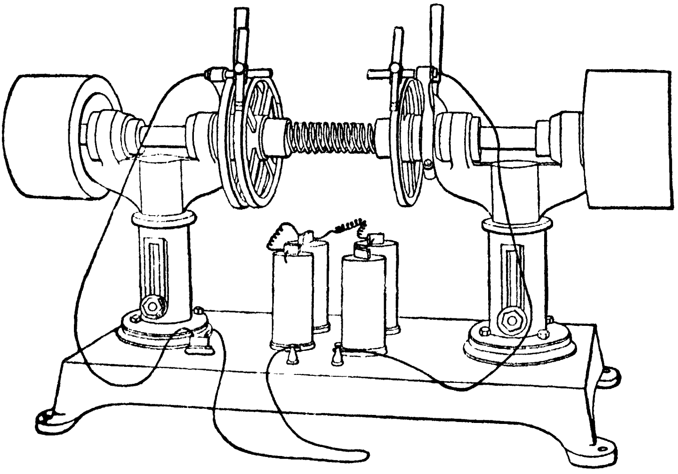

FIG. 243. A transmission dynamometer (see Fig. 244).

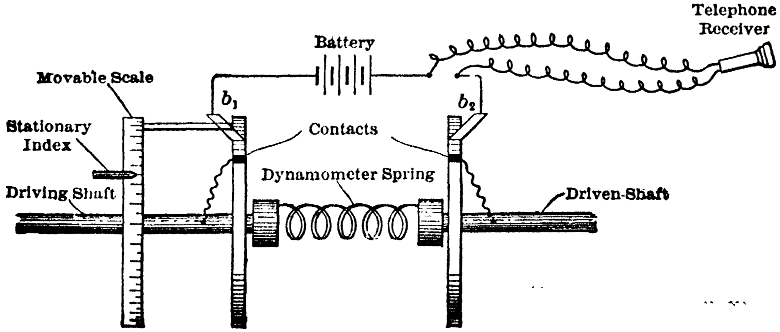

FIG. 244. Electrical connections in the dynamometer shown in Fig. 243.

The torque is measured by the angle of torsion of the spiral spring previously calibrated in pounds or in kilograms. This angle is measured electrically, as shown in the sketches. With the machine stationary, an electric circuit is established through a few dry cells, a telephone receiver, the dynamometer spring, brushes b_1 and b_2, and the corresponding contacts on two insulating rings. The brushes are set in such a position that they come under the contacts simultaneously when the spring is not under torsion. The circuit is then closed once per revolution, and this fact is recognized by a click in the telephone receiver. When the dynamometer is revolving, carrying a load, the spring is twisted and the contacts are no longer closed simultaneously; the click in the telephone disappears. The left-hand brush is then shifted along the contact ring until the click is heard again. The angle of shift is read on the stationary index, and is evidently equal to the angle by which the spring has been twisted. The transmitted power is calculated from the formula:

$$\text{Power output} = \text{constant} \times \text{angle} \times (\text{rpm}) \quad . \ . \ . \ . \quad (12)$$

The spring is previously calibrated by weights, and the torsion constant calculated. Several springs of different rigidity may be used with the same dynamometer to increase its useful range. The dynamometer may be direct-connected to the two machines or it may be provided with one or two pulleys, and one or both machines may be belted to it.

313. Electromagnetic Dynamometer. — This dynamometer (Fig. 245) is essentially a d-c machine which may act either as a generator or as a motor. Its revolving part is coupled or otherwise connected to the machine under test, while the stationary part, instead of being fastened to the bed-plate, is supported from the shaft on separate bearings. The stator is carefully balanced so that its center of gravity lies as nearly as possible in the vertical plane passing through the center line of the shaft.

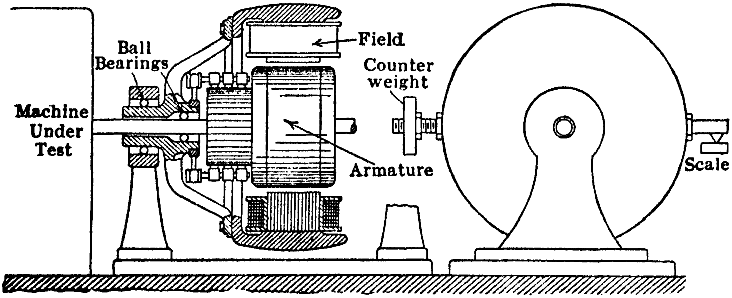

FIG. 245. An electromagnetic dynamometer.

When the dynamometer field is excited, and a current is flowing through its armature, a tangential electromagnetic effort is exerted on the stator, tending to set it in rotation. This torque is opposed by a lever arm pressing on a scale, as in Fig. 242. This pressure is read on the scale and the torque is computed as with the Prony brake.

The total torque exerted between the stationary and the revolving parts of the dynamometer includes that due to the regular armature currents, eddy currents, and hysteresis, as well as that corresponding to the inner bearing and brush friction. The windage torque is not recorded on the scale and is either neglected or estimated from a separate test.

When the machine under test is an electric motor or a prime mover, the dynamometer is made to act as a generator, either dissipating the electrical energy in a resistance or pumping it back into a line. When the machine under test is an electric generator, a pump, or any machine requiring mechanical energy to drive it, the dynamometer is made to act as a motor receiving electrical energy from the line.

An earlier form of this dynamometer, the so-called *cradle* dynamometer, consists of an ordinary electrical machine whose frame is mounted on a " cradle " supported on trunnions. The deflection of the frame is counterbalanced by weights moved along an arm. Such a device can be easily rigged up in a laboratory.

A calibrated machine. As already stated (§290), an ordinary d-c machine may be used for the same purpose as the above-described dynamometer, without having its stator frame supported on bearings, and without the scale. It is only necessary to calibrate the machine carefully as to its efficiency and losses at different loads and speeds, by one of the methods described in Chapter XIII. Then, when the input is known, the output may be found from a curve, or vice versa. Of course, there will be inaccuracies due to the variations in the core loss with the temperature, aging of the iron, and variable friction, and additional load losses due to the armature reaction and commutation that are difficult to estimate. For this reason the dynamometer arrangement described above is more accurate, since all the factors just enumerated are taken care of automatically in the mechanical force measured on the scale.

TROUBLES IN D-C MACHINES

314. Troubles in an electrical machine may be either of mechanical or of electrical nature. Mechanical troubles, such as unbalancing, defective bearings, etc., can be detected with comparative ease; they are remedied in the same way as in any non-electrical machine. Purely electrical troubles require more special knowledge for their detection; some of the symptoms and the detection of the more important electrical troubles are described below.

The current-carrying parts of a d-c machine are liable to troubles of electrical nature, namely, the armature, the commutator, and the field windings; these will be considered separately. It is hardly possible to give detailed and complete instructions for locating all kinds of troubles, since they often occur in a manner and place in which they are least expected. Sound judgment and an instinct bred of experience enable a trained man to locate a trouble in a few minutes, where a beginner may spend several days. For insulation measurements see Vol. II.

315. Troubles in Field. — The principal troubles in the field circuit are as follows: The winding may be partly or totally short-circuited, or it may be grounded to the frame. The coils may be connected in such a way as to produce two adjacent poles of the same polarity. The winding may be broken (open circuit). The machine may have lost its residual magnetism, or it may have it in the wrong direction.

All these troubles can be detected by comparatively simple means. A short circuit of low resistance may be detected by measuring the resistance of the winding with a Wheatstone bridge, or by the drop-of-potential method. A short circuit of comparatively high resistance is best detected with alternating current. The faulty coil is removed from the pole-piece and placed in a core-type transformer (Fig. 271) with the upper part of the core removable (Fig. 247). The primary coil of the transformer is connected to an available source of a-c supply, while the field coil under test serves as a secondary. When this coil is free from short circuit, the primary current is not affected by its presence. Any short circuit will cause an induced secondary current, and thus will increase the primary line current, read on an ammeter. A partial short circuit may be burned out by this method and thus made easier to locate.

If the winding is grounded to the frame, an ordinary incandescent lamp connected to a source of power will light up between the winding and the frame; or else the ground can be detected by a Wheatstone bridge, a galvanoscope, a megger (see Vol. II), etc. If the ground resistance is high, it is measured with a voltmeter as described in Chapter I, §4. One ground connection may not interfere with the operation as long as the rest of the circuit is thoroughly insulated from the ground. But should another point touch the ground a short circuit is produced, with all its harmful effects.

An open circuit, or a loose connection, is easily traced out with a test lamp or voltmeter. Apply voltage to the field circuit. Start with both test terminals at one end of the winding and move one terminal to points successively farther from the starting point. When the indication suddenly appears the open will have been passed.

A loss of residual magnetism is usually discovered when the generator fails to excite itself, although a broken or wrong connection has the same effect. Wrong polarity can be detected by bringing a magnetic needle near consecutive poles.

A portable incandescent lamp, connected to a d-c source, is sometimes used in testing the polarity of the field coils in a multipolar machine. This is done by placing a lighted carbon-filament lamp between the pole tips. According to the direction of the magnetic flux the loops of the filament will either draw closer together or become separated. In this way a dead pole can also be detected. Another test of the polarity of field coils consists in passing a current through the field winding and placing two ordinary iron nails or bolts on two adjacent poles and bringing their free ends close to one another. If the polarity is right the nails will be attracted, and if wrong they will be repelled.

316. Troubles in the Armature. — There are four principal faults to look for in an armature, viz., a short circuit, an open coil, grounded winding, and a reversed coil. The usual method of locating a short circuit (Fig. 246) consists in passing a current through the stationary armature and measuring the voltage drop between each two adjacent commutator segments by means of a millivoltmeter, or a low-reading voltmeter, *V*. At the place of short circuit, as between *a* and *b*, the drop is almost zero, or at least lower than that between the other segments. After the trouble has been thus located, it remains to determine whether the short circuit is in the coil *c* or between the two commutator bars. This is done by disconnecting the coil from the commutator and trying the voltage drop again between the same two commutator bars and between the ends of the coil. An open circuit is also tested by means of a low-reading voltmeter. In this case the deflection suddenly rises when passing over the fault. A ground is detected by the same means as in a field coil (§315).

A modification of the foregoing test consists in using alternating current, or an interrupted current from a battery and buzzer, instead of direct current, and applying a telephone receiver instead of a millivoltmeter. Where the winding and the commutator are clear of short circuit, a distinct humming is heard in the receiver; the humming ceases when passing over the short-circuited bars. A clear understanding of the armature connections is necessary in order to apply this test intelligently. Under certain conditions, in a multipolar armature, there may be other " silent " coils besides the damaged one, owing to the use of so-called " dead coils."

A core-type transformer is often used for locating short-circuited armature coils. The principle is the same as that described above for field coils. Armature coils may either be tested individually or as assembled on the armature core. In the latter case the arrangement is such (Fig. 247) that the total voltage induced in a perfect winding is equal to zero. The total width *a* between the transformer poles is smaller than the coil pitch, so that no voltage is induced in the coils in position 1, while the voltages induced in pairs of coils, such as 2 and 3, neutralize each other. However, should there be a local short circuit between the turns in coil 2, a current would flow through it; and if the transformer is large enough, the heat in the coil may be sufficient to be detected, or may partly burn the insulation. Also there will be a decided noise or " growl " when the short-circuited coil is energized by the transformer.

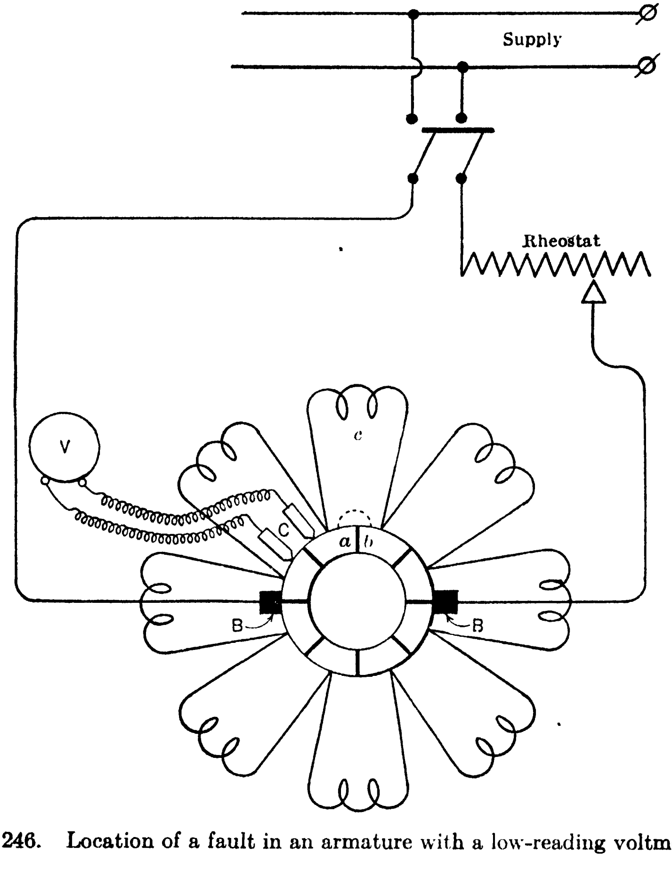

FIG. 246. Location of a fault in an armature with a low-reading voltmeter.

The armature is slowly turned on its support so as to bring all the coils in succession under the influence of the transformer. If the short-circuit current is not large enough to be detected by heating, noise, or by the increase in the transformer current, a piece of metal, or even a screwdriver, is passed around the commutator so as to short-circuit different coils in succession. A spark will be drawn from good coils, but the short-circuited coil will show no spark. However, the absence of sparking may also indicate an open-circuited coil. To decide the question, a light piece of sheet iron, shown at *d*, is held near the coil under test. The short-circuit current will create a local flux and the piece of iron will vibrate rapidly. No such vibration occurs with an open-circuited coil. A telephone receiver may also be applied to successive commutator bars to detect a faulty coil.

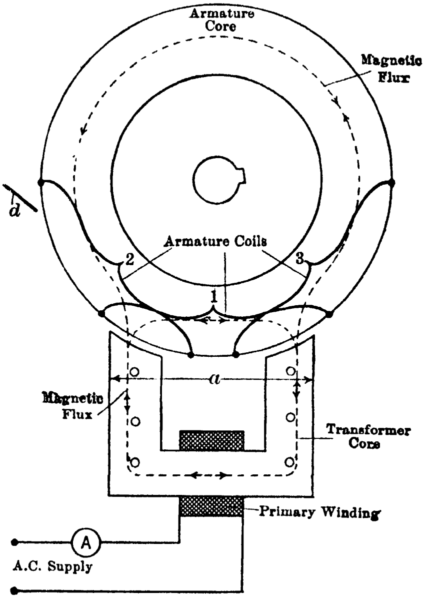

FIG. 247. A transformer for locating a short-circuited armature coil.

An *open-circuited* or short-circuited coil can sometimes be detected by inspection, especially when the machine is started as a motor. With a short-circuited coil the armature will not start on the first few rheostat points, and, when it does start, will take an excessive current. The defective coil will be warmer than the rest of the winding. With an open-circuited coil a vicious spark is visible under the brushes as the coil undergoes commutation, that is, passes from one group of coils to another. The corresponding commutator bars will be found burned and roughened. An open circuit may also be detected with a test lamp, or by a bar-to-bar test described above for locating a short-circuited coil. With a clear understanding of the armature connections the reader will be able to predict changes in the distribution of current or voltage due to an open circuit, and will plan his measurements accordingly.

The *presence of a ground,* or an electrical connection between an armature coil and the iron core, can often be determined with a test lamp by connecting the lamp between the frame of the running machine and each terminal in turn. Or a separate source of emf may be used, one terminal being connected to the frame and the other through the lamp to either terminal of the idle machine. The *location of the ground* may be determined by sending a moderate current through the armature and, one terminal of a low voltage lamp or voltmeter having been connected to the frame, moving the other terminal over the commutator. The minimum indication will occur when the contact touches the segment of the grounded coil. A telephone receiver may be used as the detector with alternating current. With a low-resistance armature and a dead ground it may be necessary to short-circuit the commutator with a bare copper wire and to send a heavy current between it and the shaft of the machine through the grounded coil. This will create around the faulty coil a local magnetic field which can be detected with a small piece of sheet iron.

A *reversed armature coil* is one in which the leads to the commutator are reversed. By passing a large current through the armature, this coil is made to create in the corresponding slots a magnetic flux of wrong polarity. This polarity can be determined with a compass needle or with a small bar of magnetized steel, placed in succession over different slots.

A more accurate test for a reversed coil is as follows: The armature is placed on two horses, and a field coil is mounted over it. The field coil is excited with direct current, a millivoltmeter is connected to two adjacent commutator bars, and then the field circuit is quickly opened. This will induce a voltage in the armature coils, and the millivoltmeter will show a transient deflection or "inductive kick." By connecting the millivoltmeter in succession to different coils, a coil will be found for which this deflection is a maximum. After this, each coil is investigated in this position as the armature is being rotated on the horses. A reversed coil will show a "kick" in the opposite direction. A faulty coil, for example one with a wrong number of turns, will cause a deflection of wrong magnitude. If the armature is large or has single-turn coils, or if the millivoltmeter is not sensitive enough, a greater deflection can be obtained by reversing the foregoing method. A heavy current is passed through the armature and this circuit is opened to give the kick. The millivoltmeter is connected across the field coil, which is held in the same position as before. This coil is made of a large number of turns of fine wire, to give a deflection of sufficient magnitude.

317. Commutator Troubles. — These are usually manifested in excessive heating and sparking; sometimes the cause lies in the commutator itself, and sometimes in the armature winding, as explained above. The principal commutator troubles proper are: rough surface, a high bar, high mica or other insulation between bars, a wrong setting of the brushes, and a short circuit between the bars. A rough surface or a high bar is usually detected by an inspection of the commutator as soon as the trouble has been noticed. A wrong setting of the brushes is discovered by shifting the brushes until the position of minimum sparking is found. Old machines usually require shifting of the brushes with various loads because of a large armature reaction. More modern machines generally have a permanent setting from no load to full load. In a machine with commutating poles (§276) the brushes must be set on the geometric neutral.

A short circuit between the bars is the most serious trouble and requires immediate attention; if the bars *a* and *b* (Fig. 246) are short-circuited they short-circuit the coil *c*. As this coil revolves in a strong magnetic field, heavy currents are produced in it, and the coil is usually burned out in a short time. The methods of locating this trouble are the same as those of locating a short-circuited armature coil and are described in the preceding section.

318. Some Causes of Sparking. — To give an idea of the variety of causes, both electrical and mechanical, that lead to poor commutation and sparking, a list prepared by a prominent manufacturer of carbon brushes is quoted below. See also §§371 to 385 on commutation and brushes.

1. Brushes off electrical neutral. — Shift to neutral by trial, or set on neutral by means of voltmeter.
2. Brushes spanning too many bars. — Trim down faces of brushes for short distance back from end or, if holders are clamp type, order thinner brushes.
3. Brush studs not parallel with the commutator bars. — Bend the brush studs or grind or shim under the bolts which fasten the studs to the yoke.
4. Incorrect brush spacing. — Check the spacing by counting the number of bars between studs or by placing a strip of paper around the commutator with divisions marked off equal to the number of studs, and correct the spacing by rotating the brush studs or the brush holders on the studs.
5. Brushes tight in brush holders. — Clean the holders with gasoline, and if brushes are still tight, sandpaper them down or file out the holders carefully.
6. Brush pressure too low. — Pressure should be 1¾ to 2¼ pounds per square inch cross-section for stationary motors and generators, 2½ to 4 pounds for elevator and mill motors, 3 to 5 pounds for crane motors, 4 to 7 pounds for railway motors.
7. Too low contact drop of brush. — Consult a brush manufacturer.
8. Insufficient abrasive action of brushes. — Use a commutator stone or more abrasive brushes.

9. **High mica.** — Use abrasive brushes or a commutator stone, or undercut the mica.

10. **Chattering.** — Due to rough and dirty commutator, high mica, high bars, and flat spots.

11. **Poor adjustment of interpoles.** — Consult the manufacturer of the machine.

12. Overloads. — Undercut the mica, use low-friction brushes and check up all causes for short-circuit currents, such as: (a) Brush off neutral. (b) Faulty brush spacing. (c) Too thick brushes. (d) Unequal air-gaps. (e) Crooked brush studs. (f) Too low contact drop of brushes. (g) Unbalanced armature winding.

13. Open circuit in armature coil. — Rewind that part of the armature.

14. Loose end connection. — Scrape and resolder all defective connections.

15. Worn bearings. — Shim or renew the bearings.

16. Unequal air-gaps. — Shim the short poles or grind off the faces of the long poles, or if from worn bearings, see paragraph above.

17. Short-circuit currents between brush studs caused by unbalanced armature winding. — Consult the manufacturer of the machine.

18. Eccentric commutator on high-speed machine. — Turn or grind.

19. Poor belt lacing. — Relace or, still better, use a continuous belt.

20. Pound of reciprocating engine driving the machine.

21. Unstable foundation.

22. Cross currents between generators operated in parallel, driven by reciprocating engines, due to variation in angular speed of engines. — Use heavier flywheel.

319. EXPERIMENT 12-O. — Locating Troubles in a D-C Machine. — The purpose of the experiment is to become familiar with the various methods described in §§315 to 318. Since an inexperienced person may spend a long time before finding a short circuit or a ground, it is advisable to let the student himself provide various faults in the machine, observe its behavior, and then perform the measurements described above for locating that particular trouble. In this way more can be accomplished within a given time, and various faults can be studied systematically. A fault sometimes causes different symptoms, according to whether the machine is running as a generator or as a motor; therefore, the machine should be tested as both whenever practicable.

(a) Begin the experiment by short-circuiting part of the field winding and operate the machine as a generator and as a motor; note the abnormal symptoms observed. Then do the same with the other troubles enumerated in §315: make wrong connections, produce an opposite residual magnetism, etc. Record all the symptoms observed, and make clear to yourself how the real cause could be located by gradually eliminating all other possible causes.

(b) Produce successively in the armature the faults described in §316 and observe the symptoms accompanying each. Care should be taken not to damage the machine, as these faults are usually accompanied by vicious sparking and heavy currents. The necks of two commutator bars may be provided with small screws for placing an artificial short circuit between them. For short-circuting, use a fuse wire which would melt before the coil could be damaged. It is interesting to observe the effect of two or more coils short-circuited simultaneously. Show how the coils can be located in this case by the drop-of-potential method.

(c) Produce some of the commutation faults described in §317. Be careful not to damage the machine, and in each case make clear to yourself the exact appearance of the trouble, as distinct from sparking due to some other cause.

(d) After having had sufficient practice with the above faults, the student should feel enough confidence in himself to locate some prearranged troubles, without knowing their place or nature. One man of the section, or the instructor, may be asked to arrange one or more faults in the machine, for the others to locate. This will make the experiment more profitable and interesting.

Report. Describe the observed faults and the methods used in testing and locating them. Where more than one method is possible, state the advantages and the disadvantages of each, and the scope of its application. Write brief instructions for an electrical repair shop as to what to do and in what order, when a small d-c machine is sent in as defective and when no other information is available as to the exact nature of the trouble.

CHAPTER XIII

LOSSES IN D-C MACHINERY

320. A d-c machine can be operated either as a generator or as a motor. In the former case it transforms the mechanical energy of the prime mover into electrical energy available at its terminals; in the second, it converts the electrical input from the line into mechanical energy available on its shaft. In both cases this transformation of energy is necessarily accompanied by losses in the machine itself.

These losses cause the output of the machine to be less than the input; the ratio of the two is called the *efficiency* of the machine. Efficiency is thus indirectly a measure of the losses. By definition

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

This expression holds true for either a generator or a motor. The efficiency may also be expressed in terms of the output and the losses thus:

$$\text{Efficiency of generator} = \frac{\text{electrical output}}{\text{electrical output} + \text{losses}} \quad . \ . \ . \quad (2)$$

$$\text{Efficiency of motor} = \frac{\text{mechanical output}}{\text{mechanical output} + \text{losses}} \quad . \ . \quad (3)$$

The efficiency of a motor may also be expressed in the following form:

$$\text{Efficiency of motor} = \frac{\text{electrical input} - \text{losses}}{\text{electrical input}} \quad . \ . \ . \quad (4)$$

321. Direct and Indirect Methods of Determining Efficiency.— The experimental methods used for determining efficiency differ according to whether it is expressed explicitly in terms of both the input and the output, as in formula (1), or only in terms of one of these and the losses, as in formulas (2), (3), and (4). The first method is sometimes called *the direct method* of determining efficiency, and the second, *the indirect.* Both are used in practice, and there are cases in which one or the other is preferable.

When both the input and the output are to be measured directly, an actual load test is necessary, and this immediately introduces difficulties. Whereas the *electrical* input or output can usually be measured very accurately with comparatively simple means, the measurement of *mechanical* power requires a brake, a transmission dynamometer, or similar devices which, with the exception of the electro-dynamometer (§313), are neither easily operated nor very accurate even with a small machine, and none are applicable to large machines. Moreover, the waste of power unavoidable with such a test is objectionable, and in many cases even prohibitive, because it may be impossible to get the necessary source of power. Besides, the direct method gives only the sum total of the losses and not the separate losses. The results obtained cannot be very accurate, because the losses are determined as a difference between two large and not very different quantities — the input and the output. An error of 1 per cent in either of these may result in an error of 10 per cent in the computed magnitude of the losses. These objections do not apply to the opposition or "loading-back" tests described in the next chapter.

The *indirect* method is therefore usually employed, especially with large machines, and by its use the segregation of the losses into the separate components is made possible. From the measured resistances the various I^2R losses are calculated. In addition, the machine under test is run at *no load,* and the power necessary for driving it is measured. This power is used entirely for overcoming the losses, and from it the core loss and the friction in the machine can be computed. There are three ways in which a machine may be run at no load, viz.:

(1) The machine may be driven electrically, as a motor.

(2) The machine may be driven mechanically, by a motor.

(3) The stored kinetic energy of the revolving part of the machine may be utilized (the retardation method).

All these methods, with suitable modifications, are applicable to a-c machines.

322. Losses in a D-C Machine. — The losses in an electric machine, running as a motor or as a generator, may be subdivided into the following three classes:

(a) The I^2R loss in the armature, commutator, brushes, field windings, rheostats, shunts, etc. Most of these losses can be computed from the measured ohmic resistances of the parts. The additional copper loss due to commutation and to eddy currents in the armature conductors is either neglected or estimated by means of known empirical coefficients for the type of machine under test.

(b) Iron loss, or core loss, consisting of hysteresis and eddy currents in the armature core, teeth, and adjacent parts, and a small additional loss due to eddy currents in the pole faces. These losses depend upon the magnetic flux and the speed of the machine. If the efficiency is desired for one particular speed and field current only, the core loss may be combined with the friction loss and the sum obtained from the data taken as the machine is run at no load under the desired conditions. Otherwise, these losses are preferably separated, as is explained below.

(c) Mechanical losses, viz., bearing friction, brush friction, and windage or air resistance. These losses depend only upon the speed of the machine. With a belt drive the friction depends on the tension of the belt. This increase in friction can hardly be taken into account and ordinarily is not charged to the electric machine.

323. Example of Computation of Efficiency from Losses. — Let it be required to calculate the efficiency curve of a 50 horsepower 220-volt shunt-wound motor. Suppose, then, that at no load the motor takes 10.6 amperes (armature current), that the field current which the motor requires is equal to 5.8 amperes, and that the resistance of the armature plus brushes, found by measurement, is 0.0377 ohm. Neglecting the armature copper loss, the iron loss and friction equals 220×10.6 watts $= 2.33$ kw. The excitation loss is 220×5.8 watts $= 1.275$ kw. The total loss independent of load is $2.33 + 1.275 = 3.605$ kw. The copper loss in the armature depends upon the load; since we do not know what current the motor takes at an output of 50 hp, we may find it by trials. A still better method is to construct the whole efficiency curve from nc load to one and a quarter load and then take from this curve the point corresponding to the rated load.

An ideal 50-hp motor would take, at full load, $50 \times 746/220 = 170$ amperes. The real motor will probably take 190 or 200 amperes. We therefore construct the efficiency curve for points, say, between 150 amperes and 250 amperes. The full-load point will surely lie on this curve. Take, for example, the 150-ampere point. The total electrical input is $(150 + 5.8) \times 220 = 34.28$ kw; the copper loss in the armature is $150^2 \times 0.0377 = 0.848$ kw. The total loss is $3.605 + 0.848 = 4.453$ kw.

$$\text{The efficiency,} = \frac{34.28 - 4.453}{34.28} = 87 \text{ per cent}$$

$$\text{The output} = \frac{34.28 - 4.453}{0.746} = 40.1 \text{ hp.}$$

In this manner the efficiency can be calculated for various values of output. The efficiency at an output of 50 hp is then found from the curve plotted. The efficiency of a generator is computed by a similar method.

For accurate results it is necessary to drive the machine at no load under conditions as nearly identical as possible with those under load. In the case of a generator, the speed must be equal to the expected speed of the prime mover, which may be slightly different at different loads. The field current must have the same value as is expected at the load for which the efficiency is to be computed, and the armature induced voltage must be equal to the desired terminal voltage of the machine under load, plus the ohmic drop in the series circuit (minus this drop for a motor). Because of the armature reaction (§365), the value of field current is difficult to estimate theoretically. Therefore, it is advisable, where possible, to precede the no-load test by a load test under the desired conditions of terminal voltage (§§242 and 243). From these tests the required field current can be determined and the no-load test performed under more nearly correct conditions. For additional load losses, see §324 below.

In the case of a motor, the difficulty lies in estimating the actual speed under different loads and with different values of field current. To obtain more accurate results, the no-load test should be preceded by taking an ampere-speed curve (§288). Then, for any desired current input, the counter emf at the proper speed can be computed and the motor run at no load under the same conditions.

In the testing of both generators and motors it is often desirable to take a series of no-load runs each at a fixed value of field current and for a range of speed obtained by varying the impressed voltage. From these curves the rotation loss is known for any condition which may arise. For the use of these curves in the separation of the losses into their component parts see §§341 to 344.

324. Conventional Efficiency and Directly Measured Efficiency.— The American Institute of Electrical Engineers in its Standards recognizes a *conventional* efficiency computed from the no-load losses, and the *directly measured* efficiency determined from an input-output test. The opposition or " circulating-power " tests described in Chapter XIV are particularly well adapted to determining the efficiency under the actual load conditions. At the same time, it is recognized that the conventional efficiency can be determined much more readily, and so long as competitive machines are always tested and compared by the same method, there is seldom an occasion for determining the efficiency from an actual load test.

In the calculation of conventional efficiency the following assumptions are made: (1) With given values of exciting current and speed, the core loss is the same at full load as it is at no load. (2) The copper loss in the loaded armature is equal to the computed I^2R loss, where R is the ohmic resistance measured with the armature stationary. (3) The voltage drop between the commutator and the brushes is the same in the machine when running as when standing still. (4) There are no commutation losses.

In reality all these assumptions are only approximately correct. In the loaded machine, armature reaction (§365) weakens the flux under one half of the pole, and strengthens it under the other half. As a result all armature teeth undergo cycles of magnetization corresponding to the increased flux density, so that the core loss in the teeth increases appreciably, core loss increasing nearly as the square of the maximum flux density to which a particular part of the laminations is cyclically subjected. On account of this non-uniform flux distribution, eddy currents are induced in the armature conductors, causing an additional copper loss not taken into account in the computed value of I^2R. There is also some additional loss due to a non-uniform distribution of current under the brushes, commutation spark, copper loss in the short-circuited armature coils (§373), and eddy currents in the pole shoes.

These *additional load losses*, or *stray load losses*, cannot be either computed or measured directly, but their existence has been proved experimentally by a comparison of the efficiencies of the same machine determined by the direct and indirect methods (§321). The efficiency computed from the losses comes out appreciably higher than that determined from the ratio of the output to the input.

Eliminating from the total losses as determined from the load test the friction, the computed I^2R loss, and the core loss at no load, the remainder will represent the stray load losses enumerated above. Certain numerical coefficients have been suggested, varying from 1.1 to 1.3, by which the measured no-load losses should be multiplied to obtain the corresponding losses under full-load conditions. Any such coefficient is but a crude empirical correction which should be applied with great caution and only to types of machines for which it has been determined experimentally. Some idea of commutation losses may be obtained by running the machine with the armature short-circuited and noting the increase in power necessary to drive it, over and above the calculated I^2R loss plus that required on open circuit.

325. Determination of the Total No-Load Loss in a D-C Machine Driven Electrically. — This method is applicable to a machine with either a series or a shunt winding, or with both. The machine is connected for running as a shunt-wound motor (Fig. 228) and the shunt-field winding is excited in the usual way. The series winding, if needed for excitation, should not be left in series with the armature circuit, but separately excited like the shunt winding, preferably from an independent low-voltage source. In a compound-wound machine the series winding may be left out entirely if the required field strength can be obtained with the shunt winding alone. For a modification of this method especially adapted to the series motor, see §329.

The machine is run at any desired values of speed and armature voltage by regulating the field rheostat and the rheostat or booster in the main circuit. The power input into the armature is measured as the product of the armature current and the voltage at the armature terminals. This input goes to overcome the core loss and friction, plus a small copper loss due to the current itself. The armature resistance being known, the latter loss may be easily corrected for. The efficiency is computed from these readings as is explained on the example in §323.

326. EXPERIMENT 13-A. — Efficiency from Losses, Machine Driven Electrically. — The purpose of the experiment and the method are explained in §325. Connect the machine under test as in Fig. 228, and run it at no load and at a high speed for a sufficient time to warm up the bearings. If the efficiency is desired for the machine acting as a generator, take a load characteristic either at a constant voltage (§243) or at a constant field current (§242). If the machine is to be used as a motor, take one or more ampere-speed curves (§289) at constant impressed voltage and constant field current (if a shunt machine). In either case, measure the resistance of the armature, including the brushes; also that of the series-field winding and any other resistance that is in series with the armature, and is an integral part of the machine, such as the interpole winding and the compensating winding (§276). From these data, compute in the laboratory the values of the emf induced in the armature for the points for which the losses are to be measured.

Start the machine in the usual way as a motor, and bring it up to the desired speed at the required value of the armature voltage. Measure accurately the armature current at no load, and note the values of speed, voltage, and field current. Take such readings for several points on the desired characteristic of the machine.

If it is not practicable to take a load characteristic or an ampere-speed curve, estimate the induced voltage in the case of a generator, and the speed under load for a motor, from the terminal voltage and the ohmic drop in the machine, disregarding the armature reaction.

Report. Plot the load characteristics or the ampere-speed curves, if any were taken. If not, show how the necessary data for these load runs were estimated. Explain the reason for determining the losses at the particular values of field current or voltage and speed selected. Correct the armature input at no load for the I^2R loss, or show that it was negligible. Compute the total losses for a number of values of armature load current and plot the corresponding efficiency curves against the output as abscissas.

327. Determination of the Total No-Load Loss in a D-C Machine Driven Mechanically. — There are cases in which the machine under test cannot be run as a motor, so that the test described in §§ 325 and 326 is not practicable. For example, the machine to be tested may be wound for 500 volts and only a 110-volt source of supply may be available. In this test (Fig. 248) the machine is belted to a small auxiliary motor and driven at the right speed and excitation, at no load. The input into the driving motor is equal to the losses in the machine under test, plus a correction for the losses in the driving motor itself.

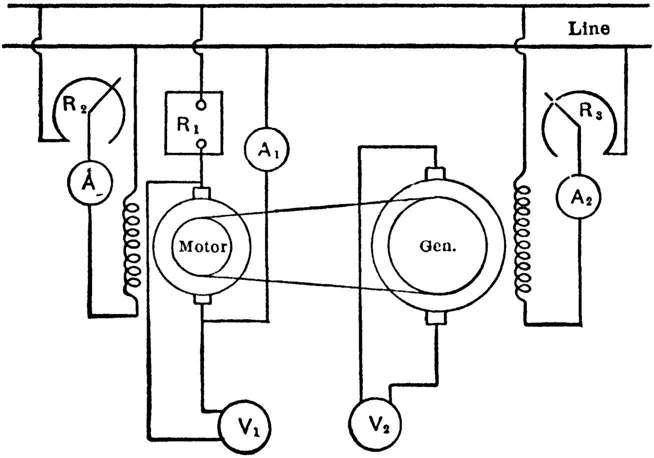

FIG. 248. Separation of losses in a direct-current machine. The machine is driven mechanically as a generator.

If the supply voltage is insufficient to excite the field of the driven machine it may be connected for self-excitation and a correction made for the small current in the armature.

The losses in the driving motor are taken into account as in the following example: Let the input into the armature of the auxiliary motor, in one of the runs, be 5 amperes at 104 volts, or 520 watts. Suppose that when the same motor was run alone (with the belt off) it took an armature current of 1.3 amperes at 101 volts, with the same excitation and at the same speed in both cases. Let the resistance of the motor armature, as found by measurement, be 0.16 ohm. Then the iron loss and friction in the motor amount to $131.3 - 1.3^2 \times 0.16 = 131$ watts. Therefore the iron loss and friction in the machine under test are

$$520 - 131 - 5^2 \times 0.16 = 385 \text{ watts}$$

It will be seen from these figures that the correction for the I^2R loss is small when the driving motor is running idle, and in many cases may be neglected altogether. The rating of the driving motor must be small as compared to that of the machine under test, in order that the driving motor may be run at a point near its maximum efficiency. In this case the correction for its own losses is comparatively small, and the result will be more accurate. The correct values of speed and of field excitation at which the machine under test must be driven are determined or estimated in the same manner as in the case of a machine driven electrically.

Instead of estimating the losses in the driving motor, its output may be determined directly by either of the following two methods:

(a) *The Prony brake.* After having taken the necessary readings with the machine under test, disconnect it, and apply a Prony brake (§§290 and 311) to the driving motor. Adjust the load in such a way as to duplicate the former input readings, that is, use the same field current, the same armature current, and speed. From the brake reading, the output can be directly computed, and the losses in the motor need not be known. The driving motor may be of either d-c or a-c type, and the theory of its operation has no bearing on the test. It is necessary only to duplicate the input readings with the machine under test and with the brake. The brake output is equal to the input into the machine under test.

(b) *A transmission dynamometer.* The machine under test is connected to the driving motor through a transmission dynamometer (§312). The input into the machine under test is read directly, and it is not even necessary to know the electrical input into the driving motor.

328. EXPERIMENT 13-B. — Efficiency from Losses, Machine Driven Mechanically. — The experiment is performed as explained in §327. The diagram of connections is shown in Fig. 248; the driving motor is at the left, the machine under test at the right. The fields of both machines are separately excited, if possible, so that the power loss in the fields does not enter into the calculations. Be sure that the meters are correct or that the calibrations are known. Mark carefully on the data sheet meter numbers and constants. (1) Drive the machine under test over the range of speeds at which it is to operate under load, and at the required values of field excitation, or armature voltage, as the case may be. During each run hold the field of the driving motor constant and vary speed by changing applied volts. For each point read amperes and volts input into the armature of the driving motor, both field currents and generator speed. Measure the pulley diameters at the center of the belt so as to know the ratio of speeds. (2) At the end of the experiment take off the belt and run the driving motor alone for the same values of speed and field current. (3) Measure the resistances of both armatures over a wide range of current.

Report. (1) Give the exact diagram of connections showing the location of each meter. (2) Tabulate, for each value of generator field, the values of input into the driving motor, also, at the same speeds, the input to the driving motor running alone. From these, figure out the core loss plus friction and windage losses in the machine under test. (3) For the speed at which the generator is to be operated plot the rotation losses against no-load volts, and from this curve read the loss at the values of load for which efficiency is to be calculated. (4) Plot these losses and the values of computed copper loss against values of output as abscissas, and plot values of efficiency against the same abscissas. (5) Explain why the efficiency passes through a maximum. At what load would it be most advantageous for this to occur? State how you arrived at the particular values of speed and field current to use in the various runs. What are the principal sources of inaccuracy in the test and calculations?

329. Efficiency of a Series-Wound Machine from Losses. — The machine may be driven either electrically or mechanically (§§325 and 327), and if the field winding be excited independently of the armature, the tests do not differ in any respect from those described above for a shunt-wound machine. It is possible, however, to drive the machine electrically as a motor at no load, with the field winding in series with the armature (Fig. 229), provided that a resistance is connected in *parallel with the armature,* as shown. By shunting the major part of the field current through this resistance it is possible to run the motor with a low field flux and at a moderate speed. The armature current is just sufficient to supply the core loss, plus the friction and windage losses. Thus, these losses can be measured at different speeds and at different values of field excitation. The efficiency of the machine may then be computed under any desired load conditions.

In §325, where a similar method is described, the operating characteristics of the machine are supposed to have been determined experimentally, in the form of an ampere-speed curve. This ampere-speed relationship may also be obtained by a simple computation from a test with the shunted armature, as is illustrated in the following numerical example.

Let a 220-volt series motor have an armature resistance of 0.4 ohm and a series field resistance of 0.3 ohm. Let it be required to obtain its performance characteristics at a load current of 30 amperes. At this current, the counter emf is $220 - 30(0.4 + 0.3) = 199$ volts. The motor is run at no load with the armature shunted, and the two rheostats are so adjusted that the field current is 30 amperes when the voltage between the brushes is slightly over 199 volts, say 200 volts. Let the armature current under these conditions be 3 amperes. Then the counter emf is $200 - 3 \times 0.4 = 198.8$ volts, which is close enough to the desired value, 199. Let the speed be 1250 rpm. Neglecting the effect of the armature reaction, we now have the same speed and the same core loss as with the machine loaded at 30 amperes. The no-load losses are equal to $3 \times 198.8 = 596.4$ watts. The useful output at this current is $220 \times 30 - 596.4 - 30^2 \times 0.7 = 5374$ watts. The efficiency is $5374/6600 = 81.4$ per cent, and the torque (§275) is $974 \times 5.37/1250 = 4.18$ meter-kilograms.

By performing similar computations and measurements at other assumed values of current, complete performance curves of the motor can be plotted.

330. EXPERIMENT 13-C. — Efficiency of a Series-Wound Machine from the Losses.

— The purpose and the method are explained in the preceding section. (1) Measure the resistances of the armature and the field, and compute the values of the counter emf for different values of load current to be used. (2) Connect the motor as in Fig. 229 and run it at no load and at the corresponding values of field current and counter emf. At each step measure the field amperes, armature amperes, armature volts, and speed. Be sure to measure the actual field current and to compute the voltage drop in the armature for the corresponding load current. As a check on the method, connect the motor to a suitable load and take an ampere-speed curve, as in §299.

Report. Give the values of the measured resistances and the computed values of counter emf. Plot field current, no-load losses, and copper losses against speed as abscissas. For the same abscissas compute and plot the input, the output, efficiency, and torque. Plot the observed ampere-speed curve, and explain the discrepancy, if any, with the computed one. Using the available test data, explain, by means of an example, how an ampere-speed characteristic could be computed for operation with the field winding shunted in a given ratio. See §297 under (2). Explain how the test and the computations would be modified if the machine under test were intended for use as a generator, and the characteristics were desired under those conditions.

EXPERIMENTAL SEPARATION OF LOSSES

331. The designer of electrical machinery — and sometimes the user — is not satisfied to know the sum total of the losses, but desires to know the values of the separate component losses described in §322 and shown, with the exception of those that are indeterminable, in Fig. 249. The same losses are classified in the Standardization Rules of the A.I.E.E. by the following table:

TABLE III

CLASSIFICATION OF LOSSES

(Standard. Rules of A.I.E.E.)

Accurately Measurable	Approximately Measurable or Determinable	Indeterminable
No-load core losses including eddy current losses in conductors at no load	Brush friction loss	Iron loss due to flux distortion
		Eddy-current losses in conductors due to transverse fluxes occasioned by the load currents
Load I^2R losses in windings	Brush contact loss	
No-load I^2R losses in windings	Losses due to windage and to bearing friction	Eddy-current losses in conductors due to tooth saturation resulting from distortion of the main flux
		Tooth-frequency losses due to flux distortion under load
		Short-circuit losses of commutation

Only those losses which are determinable or accurately measurable will be considered.

A knowledge of the separate losses is of importance for the following reasons:

(a) Knowing the losses as functions of the load, speed, etc., it is possible to determine the rating and performance of the machine for different classes of service (intermittent load, variable load, constant load, operation at other than normal speed, etc.).

(b) The magnitude of the separate losses is a check on the quality of materials, workmanship, and design.

(c) Knowing the values of the principal losses, the designer is enabled to vary the dimensions of the machine, still keeping its efficiency as high as competition may require, and preserving reasonable limits of temperature rise.

Of the several kinds of losses, only the core loss and mechanical losses need be discussed here. The copper loss, I^2R, is readily calculated from the measured resistances of the windings. However, see §324 in regard to additional load losses.

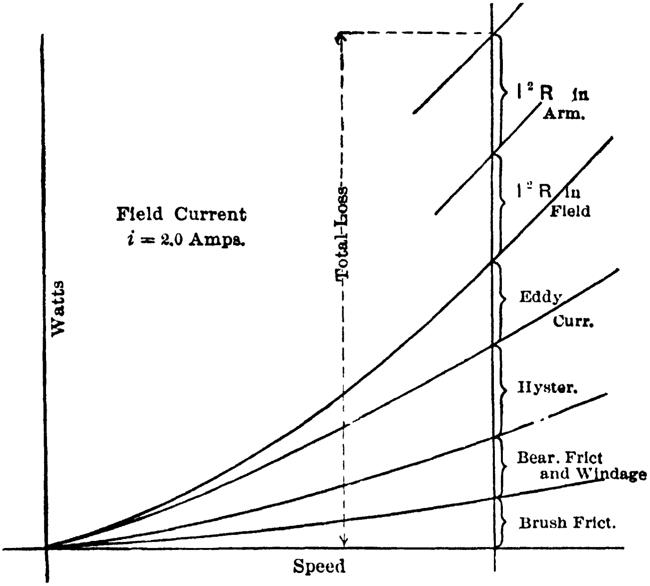

FIG. 249. Separate losses in a direct-current machine.

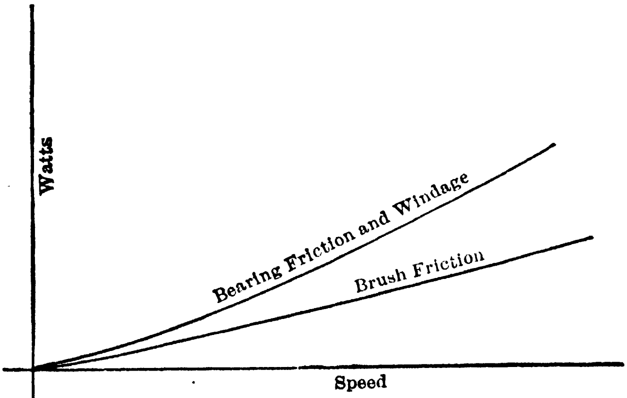

FIG. 250. Brush friction separated from bearing friction and windage.

332. Core Loss and Mechanical Losses. — Frictional losses vary with the speed of a machine, and may be plotted as in Figs. 249 and 250.

In ordinary testing, the bearing friction is seldom separated from the windage.

The core loss varies with the speed and the field current, and is usually represented by a set of curves, as shown in Fig. 251. The components of the core loss — hysteresis and eddy currents — are shown separately in Figs. 252 and 253. It is important for the manufacturer to have eddy currents separated from the hysteresis loss. A high hysteresis loss means that the iron used is of inferior quality, or the flux density is too high. A large eddy-current loss may indicate that the laminations are too thick or not sufficiently insulated from each other, or that the filing of the slots has been excessive, etc. Thus the remedy is quite different in the two cases.

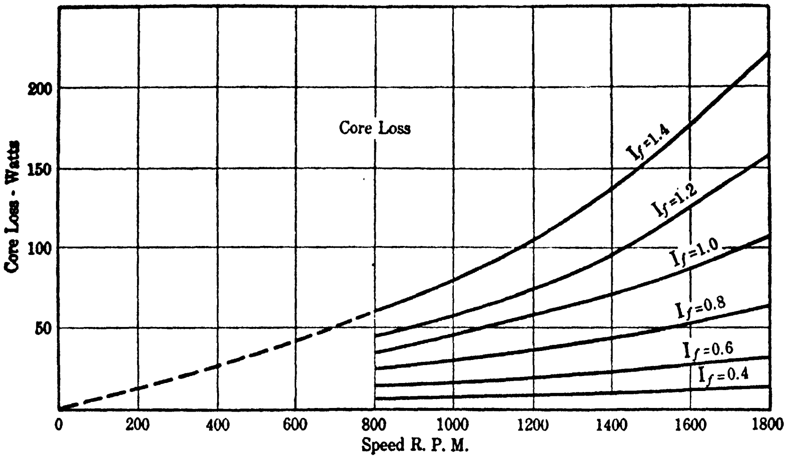

FIG. 251. Curves of iron loss as a function of speed and of exciting current (5-hp, 125-volt, 1675 rpm shunt motor).

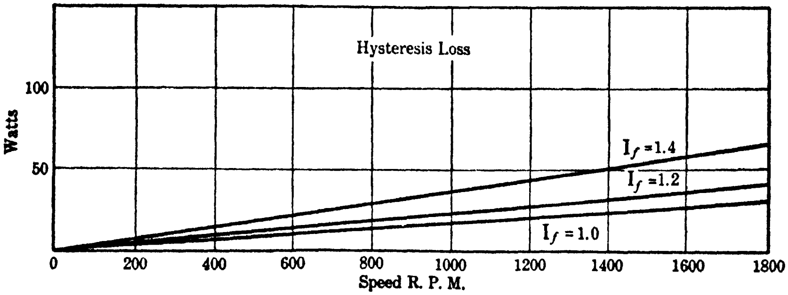

FIG. 252. Curves of hysteresis loss as a function of speed and of exciting current.

The sum of the core and mechanical losses, or, as it is sometimes called, the total no-load loss (Fig. 254), is determined by measuring the power necessary for driving the machine at no load. The three possible methods of doing this are the same as are enumerated in §321 under the indirect methods of measuring efficiency. In particular, the reader is advised to become thoroughly familiar with §§325 to 328 before proceeding any further. The total loss having been obtained under different conditions of speed, excitation, friction, etc., the individual losses are computed mostly by subtraction. For example, if it takes

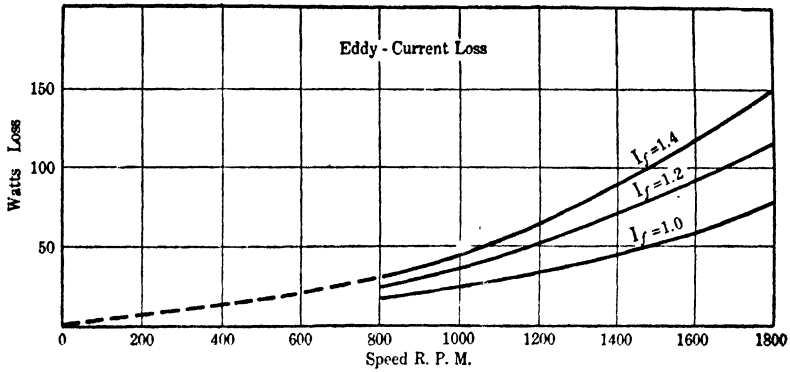

Fig. 253. Curves of eddy-current loss as a function of speed and of exciting current.

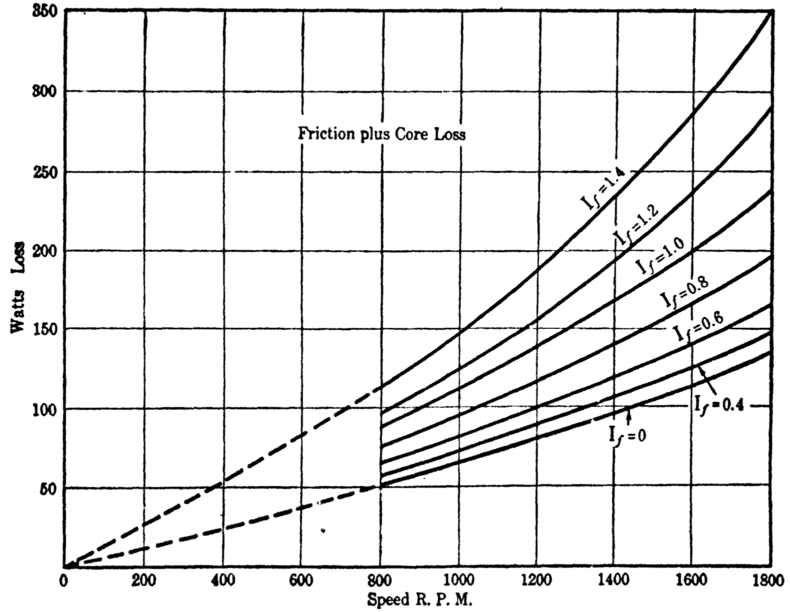

Fig. 254. Total no-load losses as a function of speed and of exciting current (5 hp motor).

P_1 watts to drive a machine with the brushes pressing on the commutator and P_2 watts with the brushes lifted, the brush friction evidently is equal to $(P_1 - P_2)$ watts.

The tester's skill consists in running the machine under such conditions that from the data obtained the individual losses may be separated with the least amount of labor and with the greatest possible accuracy. Before performing any of the following experiments, §§341 to 344 should be carefully studied and each experiment planned with the view to the subsequent separation of the losses by computation.

333. EXPERIMENT 13-D. — Separation of Losses, Machine Driven Electrically. — The purpose of the experiment is to separate the total losses in a d-c generator or motor into the components shown in Fig. 249. For general instructions see §§331 and 332. The machine under test is connected as shown in Fig. 228 and driven as a motor at no load over a series of curves, each at a constant field current, and over as wide a range of speed as is feasible. The machine should first be run for at least half an hour at a high speed in order to attain constant friction conditions. Lubrication must, of course, be kept the same during the whole test.

(a) Begin the test with the strongest field current possible and at the maximum speed that can be obtained with this field current, i.e., at a reasonable over-voltage. Read armature volts and amperes, field amperes, and speed. Then, *keeping the field current constant at this maximum value,* reduce the speed in 8 to 10 approximately equal steps to its lowest possible limit, by introducing resistance in the armature circuit or, better yet, by reducing the applied voltage by means of a reversed booster; take similar readings at each step. The values of "armature volts times armature amperes" (with a small correction for the armature I^2R loss) will give directly the data for the upper curve in Fig. 254.

(b) Make similar runs with smaller values of field current, to correspond to the other curves on the same sheet. If a separation of losses is to be made, as in Fig. 258, use at least six different values of field current and take the last value as small as possible, so as to obtain the required range of speed.

(c) The two lower curves in Fig. 254 cannot be taken experimentally since the machine requires an appreciable field current to run as a motor. The friction can be determined from the other curves by calculation (§341). The separation of the brush friction from the bearing friction and windage cannot be performed very accurately although one may raise all but one brush on each arm and still run the motor, and from the decrease in the input figure out what the reduction in power would be if *all* brush friction were eliminated. This test should be performed at a low field current (high speed), because then the friction constitutes a larger percentage of the total losses.

(d) Measure the resistances of the windings.

Report. Plot watts input into the armature (less the armature I^2R loss) against speed as abscissas, as in Fig. 254. Separate the friction from the iron loss as explained in §341; separate the hysteresis loss from the eddy-current loss as shown in Fig. 258. Do this at least for the normal excitation of the machine, although preferably for such a range of curves as is shown in Fig. 251, so as to show the influence of the value of field current on the iron losses. Plot all the losses at a chosen field current to speed as abscissas (Fig. 249). Show how the efficiency at full load and at the rated speed may be computed from these data, for the machine running as a generator and as a motor.

334. EXPERIMENT 13-E. — Separation of Losses, Machine Driven Mechanically. — The purpose of the experiment is to separate the total losses in a d-c generator or motor into the components shown in Fig. 249. For general instructions, see §§331 and 332. The machine under test is belted or direct-connected to a small auxiliary motor and driven at no load, at various speeds and with varying field excitation. The connections are shown in Fig. 248. It is preferable to excite the machine under test from a separate source. The input into the armature of the driving motor represents the sum of iron loss and friction in both machines, plus a small I^2R loss in the motor armature itself (§327).

(a) The machines should first be run for at least thirty minutes at a high speed in order to warm up the bearings and make the friction constant. Then excite the machine under test to its highest possible value, and raise the speed to the highest safe limit. Read all the instruments, as shown in Fig. 248, and also the speed of the set. Reduce the speed in steps, preferably keeping the field current of the driving motor constant and varying its armature volts, either by a booster or by inserting resistance in its armature circuit. It is desirable to keep the field current constant, because it is then easier to correct the data for the core loss in the driving motor. The field current of the machine under test must be kept constant during each run, readings being taken at each speed. This test corresponds to the upper curve in Fig. 254, after the losses in the driving motor have been subtracted from the volt-ampere input into its armature, as is explained at the end of §323.

(b) The same run should be repeated at several lower values of field current in the machine under test, until a sufficient number of curves has been obtained (five or six). The last curve is taken with the field circuit opened and corresponds to the frictional loss alone.

(c) Lift the brushes from the commutator, so as to eliminate their friction; the difference in the input with the brushes " down " and " up " gives the value of the brush friction.

(d) Take off the belt and run the driving motor alone at the same speeds and with the same values of exciting current as before, in order to determine its own losses. The loss in the belt can be only roughly estimated. The arrangement should be such as to make this loss as small as possible. Under normal conditions it is not over 2 per cent of the power transmitted.

(e) Measure the armature resistances of both machines.

Report. The requirements are the same as in §333, except that the iron loss is separated from the total loss directly, by using the experimentally determined friction curve. The method shown in Fig. 257 may be used as a check.

335. Separation of Losses in a Series-Wound Motor. — The reader is supposed to be familiar with the tests and computations described in §§329 and 330, and only additional information is given below.

The no-load losses in a series motor depend on two independent variables: the field current and the speed. With a given field strength, the speed is proportional to the induced emf, or very nearly to the voltage at the armature terminals. Thus, it may be said that the *iron loss and friction depend on the field current and on the voltage at the armature terminals.* For the purpose of plotting curves, one of the variables must be kept constant during each run. It is convenient to keep the armature voltage constant for each curve, and to vary the field current by means of the rheostats shown in Fig. 229. Data for several loss curves should be taken, within as wide limits of armature voltage and field current as possible, and plotted against speed as abscissas.

In order to separate the friction from the iron loss, a second set of runs is made, without the shunting resistance around the armature, and with the armature and the field simply connected in series. The terminal voltage must be kept sufficiently low; otherwise the motor will run away. When running at no load under such conditions, the motor takes very little current, and its field is so weak that the iron loss can be neglected and the whole input into the armature assumed to be equal to the friction loss. Note that this is not true with a shunt motor, since there the field has its full value at no load as well as at any other load, and the iron loss is not negligible under any circumstances. The values of friction given by this curve are subtracted from the ordinates of the curves previously taken, and new curves are plotted giving the iron loss separately as a function of speed and field excitation. The hysteresis loss is then separated from the eddy-current loss, as is explained in §§342 to 344. Incidentally, the first series of runs gives not only the total losses, but also data for ampere-speed curves corresponding to various terminal voltages.

336. EXPERIMENT 13-F. — Separation of Losses in a Series-Wound Motor. — The purpose and the method of conducting the experiment are explained in the preceding section. Connect the machine as in Fig. 229, with meters as shown. Run the motor at a high speed for at least thirty minutes to warm up the bearings.

(a) For the first series of runs (iron loss + friction) shunt the armature so that a greater part of the current passes around it. Run the machine at no load, and adjust the conditions so as to have full rated voltage or more across the armature terminals, and a field current at least 25 per cent above the rated current of the motor. Read field amperes, armature amperes, volts across the armature, and speed. Keeping the field current constant, reduce the voltage at the motor terminals in steps until the motor almost stops, and take readings at each step. Make similar runs with lower values of field current.

(b) For the second series of runs (friction alone) remove the shunting resistance, and run the motor light throughout the possible range of speeds. A considerable resistance must be inserted in the main circuit at all times to keep the motor from running away. Take numerous readings of amperes, volts, and speed.

(c) Measure the resistances of the armature and of the field windings.

(d) The ampere-speed curve of the machine may be taken experimentally, or it may be computed from the runs with the armature shunted. With these data the performance characteristics of the machine can be predetermined under any desired conditions of speed, terminal voltage, armature current, or field current.

Report. Plot the power input into the armature to speed as abscissas, correcting the voltage readings, if necessary, for the *IR* drop in the armature; each curve must refer to a constant field current. The friction curve may be plotted on the same curve sheet. Subtract the friction from the total loss. Separate the hysteresis from the eddy currents, as in §§342 to 344. Compute an ampere-speed curve, as in §329, and show how the efficiency can be determined for any desired input or speed.

RETARDATION METHOD

337. Instead of driving a machine electrically or mechanically for separating the losses, it is possible to bring it up to the highest safe speed and shut off the power; the machine will then gradually slow down to a standstill, converting its stored kinetic energy into the losses. While the machine is slowing down, instantaneous values of speed are read every few seconds or are recorded continuously. Such runs are made with various values of field current, and the results plotted as speed-time or *retardation* curves (Fig. 255).

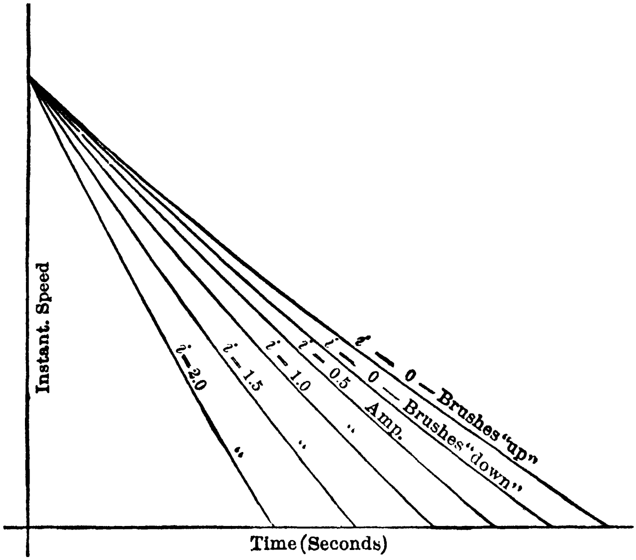

FIG. 255. Retardation or speed-time curves at various values of field current.

The form of these curves depends upon the relative magnitude of the iron loss and the friction, and on their variation with speed. A method is given in §339 for converting these curves, by calculation, into those shown in Fig. 254. Subsequently the losses can be separated, as explained in §§341 to 344.

This method is more applicable to large machines in which the stored energy of the armature is sufficient to maintain the rotation during a comparatively long time (several minutes), so that instantaneous speeds can be taken with considerable accuracy. If the machine has been brought up to speed mechanically, by an auxiliary motor, the belt is thrown off. If set in rotation electrically, the armature circuit is broken.

338. Measurement of Instantaneous Speeds. — The principal difficulty in performing a retardation test lies in an accurate measurement of speed; otherwise the method is simple and reliable, especially with large machines. Several methods have been used or proposed for measuring instantaneous speeds, and the principal ones are enumerated or described below.

(1) A small magneto-generator, belted to the machine and connected to a sensitive voltmeter, is a convenient device for the purpose.

(2) A centrifugal speed-indicator or a vibrating-reed tachometer may be used, provided that the rate of deceleration is not too great to cause an appreciable time lag in the scale indications.

(3) A recording tachometer or an oscillograph may be used to draw a retardation curve directly.

(4) An ordinary speed-counter may be used to record the total number of revolutions from the moment at which the connection to the external power is broken. This speed-counter is read every five or ten seconds without stopping it; the difference of each two consecutive readings gives the total number of revolutions during these two or three seconds. From these readings the average speed corresponding to the middle instant of each time period can be determined.

(5) A voltmeter is left across the armature terminals, and read every two or three seconds, or a recording voltmeter with high-speed chart is used. If the field current is kept absolutely constant, the induced voltage is exactly proportional to the speed. Before breaking the connections to the source of external power, the coefficient between the speed and the voltage must be determined, so as to be able to convert the observed voltage-time curve into a speed-time curve. This method is not reliable with the field winding opened, because the voltage induced by the residual magnetism is quite low and the brush contact drop is relatively large even at no load.

(6) It is important to measure the speed accurately during the first part of the retardation curves, just after the power is shut off, because the values of the losses are usually desired at these speeds. Sumpner's differential voltmeter method is convenient for the purpose. The machine is speeded up as a motor, and the armature circuit opened on one side of the line only, by a single-pole switch. A low-scale direct-reading voltmeter connected across the switch terminals shows practically zero as long as the switch is closed. When the switch is opened and the motor slows down, the voltmeter measures the difference between the line voltage and the emf induced in the machine under test. The line voltage being constant, and the induced emf proportional to the instantaneous speeds, the voltmeter readings are proportional to the drop in speed.

(7) The speed may also be read stroboscopically. For this purpose several concentric circles are drawn on a cardboard or metal disk and on the circumference of each circle a number of uniformly spaced projections are marked, the number being different for each circle. The disk is mounted on one end of the shaft of the machine under test.

An electrically driven tuning-fork is provided with an aluminum wing on each prong, and each wing has a slit in it. When the fork is stationary the slits overlap, so that the rotating disk can be observed. When the tuning-fork is set in vibration, the slit is alternately closed and opened, permitting periodic glimpses of the rotating disk. At a definite speed of the motor, the projections on one of the circles seem stationary when viewed through the tuning-fork. Knowing the frequency of the fork and the number of projections, the observer may compute the speed. An alternative method is to determine empirically the speeds at which different figures on the disk appear stationary. After the machine has been brought to a high speed, the observer views the disk through the tuning-fork, and marks the instants at which various figures become stationary. These instants may be marked electrically on a chronograph drum. For details see D. Robertson, " Separation of the No-Load Stray Losses in a C-C Machine by Stroboscopic Running-Down Methods," Institution of Electrical Engineers (British), Journal, Vol. 53, 1915, pp. 308 to 328.

(8) In addition to the speed, the rate of decrease in speed, or the deceleration, may also be measured, since it enters into the final formulas discussed in the next section. A simple method consists in connecting an electrostatic condenser in series with a galvanometer, across the armature terminals of the machine under test. As long as the speed and the voltage are constant, the charge on the condenser also remains constant, and the galvanometer deflection is zero. As the speed of the machine becomes lower, the voltage induced in the armature decreases and consequently the condenser charge is reduced in the same proportion. The current which now flows through the galvanometer is proportional to the rate of decrease of the charge or voltage with the time. But the voltage being proportional to the speed, the galvanometer current is proportional to the rate of change in speed with time, or to the deceleration of the machine. A voltmeter connected to the secondary winding of a potential transformer, the primary of which is connected to a d-c magneto driven by the test machine, reads proportionally to the rate of change of voltage and therefore to deceleration of the machine.

339. Theory of the Retardation Method. — The kinetic energy stored in the armature at a certain speed is equal to $1/2K\omega^2$, where K is the moment of inertia of the revolving part, and ω is its angular velocity. The latter is proportional to the speed n expressed in rpm, so that the stored energy may also be represented in the form $1/2Cn^2$, where C is a constant proportional to the moment of inertia of the revolving part.

Let P, expressed in watts, be the value of total loss (iron plus friction) corresponding to the speed n. Then, since the rate of change of energy with time equals power,

$$-d(1/2Cn^2)/dt = P \quad \dots\dots\dots\dots \quad (5)$$

Differentiating, we obtain

$$P = -Cn\,dn/dt \quad \dots\dots\dots\dots \quad (6)$$

or, in words, the total loss at any speed, n, equals the constant, C, involving the moment of inertia, times the speed, n, times the rate of change of speed, dn/dt (slope of the retardation curve).

In eq. (6), C is evaluated as follows: Run the machine light as a motor at a certain value of field current, I_{f0}, and at the rated speed, n_0. Let the power input under this condition be P_0. Now bring the speed above the value n_0 and, disconnecting it, allow it to slow down with the field current kept constant at I_{f0}, taking readings of n at frequent intervals until n_0 is passed. Plot this curve (rpm vs. time) and determine the value of dn/dt at n_0. Then, since the loss P_0 is known, eq. (6) may be solved for C. Other retardation curves are then run for various values of field current, at zero field, and at zero field, brushes raised. These curves are illustrated in Fig. 255.

To calculate from the curves the loss P at any value of field current and speed, take the appropriate retardation curve and from the curve read dn/dt at the desired value of n. This may best be done as follows: Referring to Fig. 256, take points n_1 and n_2 at each side of the desired speed n, and as close together as it is possible to read the difference $(n_1 - n_2)$ accurately. Then n may be said to be the average of n_1 and n_2, or $n = (n_1 + n_2)/2$. Writing eq. (6) in terms of average values,

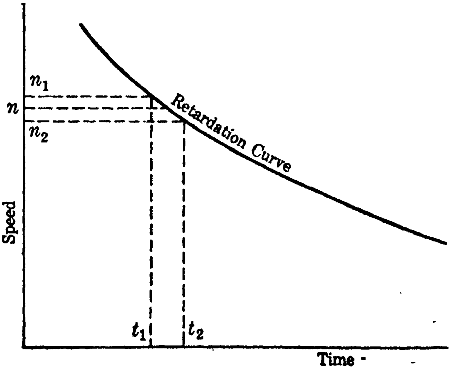

FIG. 256. Use of the retardation curve in the determination of losses.

$$P = -Cn_{\text{ave}}\frac{\Delta n}{\Delta t} = C \cdot (n_1 + n_2)/2 \cdot (n_1 - n_2)/(t_2 - t_1) \quad . \quad (7)$$

340. EXPERIMENT 13-G. — Efficiency and Separation of Losses by the Retardation Method. — The purpose and the method of conducting the experiment are explained in §§337 to 339.

(a) Have the machine wired so that it can be brought up to speed electrically as a motor (Fig. 228). Run the machine for about thirty minutes to attain steady conditions of lubrication. Bring it up to the highest safe speed, measure this speed, and then suddenly open the circuit (both armature and field), allowing the machine to slow down to a standstill. Measure instantaneous retardation speeds by one of the methods described in §338. Repeat the run several times until you have learned to read speeds quickly; a metronome may be of assistance.

The armature circuit can be safely opened by tripping a circuit-breaker, but when opening the shunt-field circuit special precautions must be taken in view of the high inductance of the field windings. It is well to provide a special *field-discharge* switch, which in opening the line circuit simultaneously closes the field winding upon itself, through a suitable resistance. In this way an easy path is offered for the inductive discharge, and the danger of breaking down the insulation is minimized.

(b) Bring the machine up to speed again; open the circuit and lift the brushes at the same moment. Take a retardation curve without the brush friction. Repeat this run several times until you get reliable results.

(c) Make retardation runs with five or six values of exciting current. It is advisable to keep the value of the field current while the machine is slowing down the same as in speeding up, because otherwise it takes too long for the current to settle to the desired value, on account of the high inductance of the field windings. In order to obtain high speeds with the stronger fields, apply voltages to the armature higher than the rated voltage of the machine.

(d) Determine the constant C of the revolving part, by the method described in §339.

(e) Before leaving the laboratory, measure the resistance of the armature winding.

(f) Sometimes the conditions are such that the machine under test cannot be brought up to the required high initial velocity electrically, as a motor, and has to be speeded up by an auxiliary motor. To obtain some experience and skill in this method, belt the machine under test to a suitable small motor, bring the set up to a high speed, and throw off the belt. Take a few retardation curves by this method.

Report. Plot the observed retardation curves (Fig. 255), and calculate C as in §339. Convert the retardation curves into the loss curves shown in Fig. 254, according to the directions given in §339. The remaining requirements are the same as in Experiments 13-D and 13-E. Compare the results obtained in parts (*a*) and (*f*) of the experiment and state the advantages and the disadvantages of the two methods of bringing the machine up to speed, especially in the case of a hypothetical very large machine — not the one tested.

SEPARATION OF LOSSES BY COMPUTATION

341. Separation of Friction from Iron Loss. — It is mentioned in §333 that, when the curves shown in Fig. 254 are obtained by driving the machine electrically as a motor, it is impossible to obtain data on the friction alone by test. The friction in this case is computed by applying the self-evident principle that *the friction loss at any speed is the limit towards which the total loss is tending* when the excitation and the applied voltage are reduced to zero, the speed being kept constant. In order to make use of this principle, the curves of total loss

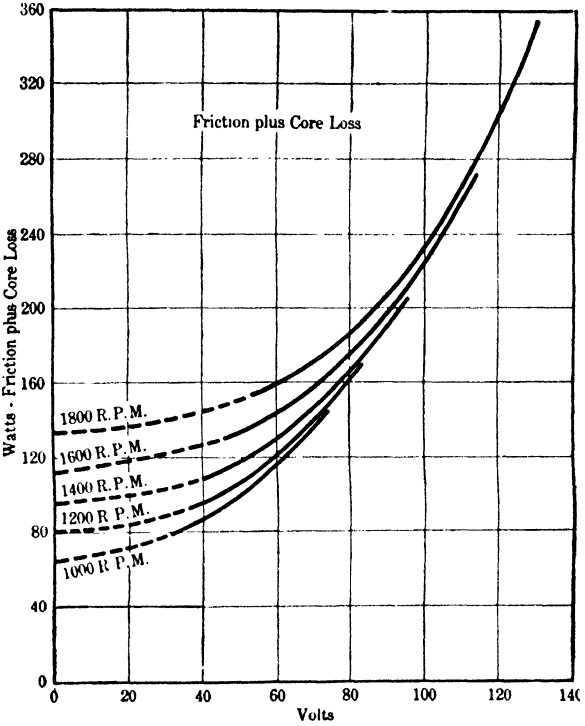

FIG. 257. Curves of total no-load loss as a function of voltage and speed (5-hp motor).

in Fig. 254 are replotted to induced emf's as abscissas (Fig. 257), each curve referring to a certain constant speed. With the machine running at no load, the voltage at the armature terminals is practically equal to the induced counter emf of the machine, but a correction for the IR drop in the armature can be readily made if so desired. These curves are then produced to the axis of ordinates as shown by the dotted lines; the intercepts give the values of friction at the corresponding speeds. Plotting these values of friction against speed as abscissas (as in Figs. 250 and 254) the required friction curve is obtained.

This method is based upon the fact that at a constant speed and at no load the field flux is proportional to the induced armature voltage. Thus, for each curve in Fig. 257, the points, going from the higher to the lower voltage, represent values of the losses at decreasing values of the field flux. In the limit, at the intercept with the axis of ordinates, the applied voltage, and hence the flux, is zero, so that no iron loss is present and the loss is due to the friction alone. Here, and in the rest of the chapter, " friction " is meant to include the air friction, or windage, as well as bearing and brush friction (Fig. 250).

The dotted parts of the curves in Fig. 257 are obtained either according to the judgment of the eye, or analytically, by assuming them to be parabolas. In the later case the general equation of the curves may be written in the form

$$P = Q + kE^2, \qquad (8)$$

where P is the total loss, Q is the friction loss, and E the induced voltage; k is a constant whose numerical value does not enter into the calculations. In this equation the core loss at a constant frequency is assumed to vary as the square of the induced emf, that is, as the square of the field flux. Writing this equation for some two points on one of the curves, we have:

$$\left.\begin{aligned} P_1 &= Q + kE_1^2 \\ P_2 &= Q + kE_2^2 \end{aligned}\right\} \qquad (9)$$

Eliminating k and solving for Q, we get

$$Q = \frac{P_1 - P_2(E_1/E_2)^2}{1 - (E_1/E_2)^2} \qquad (10)$$

This value of Q gives the point at which the curve should meet the axis of ordinates.

This equation is applied to the upper curve of Fig. 257 (1800 rpm) with the following results:

From the curve, $E_1 = 60$, $P_1 = 159$; $E_2 = 120$, $P_2 = 307$.

Then

$$Q = \frac{159 - 307 \cdot (60/120)^2}{1 - (60/120)^2} = 109 \text{ watts}$$

It happens that from another test the value of Q for this condition is known to be 131 watts, so that it is seen that this equation does not fit the curve as closely as is desired.

As a second approximation it would seem reasonable to assume as the equation of the curve

$$P = Q + kE^n \qquad \text{(11)}$$

Before the curve can be extrapolated to the origin to estimate Q it is necessary in any given case to solve for n and eliminate k, which can be done as follows:

Take the derivative of P with respect to E in eq. (11), and multiply by E. Then, calling $dP/dE = P'$

$$P'E = nkE^n \qquad \text{(12)}$$

or

$$k = P'E/nE^n$$

which, substituted in eq. (11), gives

$$P = Q + P'E/n \quad \text{or} \quad Q = P - P'E/n \qquad \text{(13)}$$

It remains to solve for n. From eq. (11)

$$nP = nQ + nkE^n \qquad \text{(14)}$$

Writing eq. (14) for two points on the curve, and subtracting,

$$n(P_1 - P_2) = P_1'E_1 - P_2'E_2$$

or

$$n = (P_1'E_1 - P_2'E_2)/(P_1 - P_2) \qquad \text{(15)}$$

In the above expressions, P' is found at a point on the curve by laying a straight-edge tangent to the curve and so getting the tangent to the curve (watts per volt). This should be done with considerable care.

Again using values from the upper curve of Fig. 257 to solve for Q, one finds the following values:

$$E_1 = 120, \quad P_1 = 307, \quad P_1' = 4.75;$$
$$E_2 = 60, \quad P_2 = 159, \quad P_2' = 1.02$$

Then

$$n = \frac{4.75 \times 120 - 1.02 \times 60}{148} = 3.44$$

Therefore, from eq. (13)

$$Q = 307 - \frac{4.75 \times 120}{3.44} = 141$$

This value for Q is but 7.5 per cent higher than the test value of 131 which might be considered satisfactory in most cases.

In the above numerical example the power lost is found to be proportional to the 3.44 power of E, rather than the square.

A more accurate solution. For some machines neither eq. (8) nor eq. (11) adequately represents the loss curve. These equations may then be replaced by a more general expression

$$P = Q + kE^2 + aE^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

This formula contains four parameters, Q, k, a, and n. The curve which it represents can therefore be made to pass through four points on the given experimental curve. Taking a derivative of both sides of eq. (16) with respect to E and multiplying the result by E, we get

$$P'E = 2kE^2 + naE^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where, as before,

$$P' = dP/dE$$

Eliminating aE^n between eqs. (16) and (17) gives

$$nP + (2 - n)kE^2 = nQ + P'E \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

To eliminate nQ, this equation is written for two points on the given curve, say points 1 and 2. Subtracting the results from each other we get

$$n(P_1 - P_2) + (2 - n)k(E_1^2 - E_2^2) = P_1'E_1 - P_2'E_2 \quad . \quad . \quad (19)$$

A similar equation can be written for two other points on the curve, say 3 and 4, thus

$$n(P_3 - P_4) + (2 - n)k(E_3^2 - E_4^2) = P_3'E_3 - P_4'E_4 \quad . \quad . \quad (20)$$

These are two simultaneous linear equations which can be readily solved for the unknown quantities n and $(2 - n)k$; the friction loss, Q, is then computed from eq. (18) applied to one of the four points on the curve. The values of the derivative P' for the four points (watts per volt) are again estimated by laying a straight-edge tangent to the curve at the desired point.

Applying these equations to four points on the upper curve of Fig. 257, Q is found to have the value 133, which is almost exactly the test value. Upon substitution of the values of n, k, and a, in eq. (20), for various values of E, the original curve is found to be duplicated with considerable accuracy.

342. Separation of Hysteresis and Eddy-Current Losses. — The curves shown in Fig. 254 give the total losses and the friction loss for different values of speed and field current. Subtracting the friction loss from the total losses gives the core losses (hysteresis and eddy currents) for various values of speed and exciting current (Fig. 251). The next step is to separate the hysteresis loss from the eddy currents. This is done on the basis of the fact that, *with a given excitation, the power loss due to hysteresis is proportional to the speed, and that due to the eddy currents increases as the square of the speed* (§§216 and 217). Thus, at a certain speed of n rpm, and at a constant excitation, the total iron loss

$$p = Hn + Fn^2 \qquad (21)$$

where H and F are two constants characterizing hysteresis and eddy-current (Foucault) losses respectively at the given field current. A small letter p is used here to distinguish the core loss from the total loss previously denoted by P.

This equation can be used for the separation of the losses in two ways, purely analytical and graphico-analytical. Both solutions are given below, and the reader is also referred to §230 for a somewhat similar treatment.

343. Analytical Separation of Iron Losses. — Select one of the curves in Fig. 251 and apply eq. (21) to two points taken sufficiently far apart and corresponding to two values of spped, n_1 and n_2:

$$\left.\begin{aligned} p_1 &= Hn_1 + Fn_1^2 \\ p_2 &= Hn_2 + Fn_2^2 \end{aligned}\right\} \qquad (22)$$

These equations can be readily solved for H and F. After this, the straight line Hn representing the hysteresis loss (Fig. 252) can be drawn, and the parabola Fn^2 may be plotted, giving the eddy-current loss (Fig. 253). The same procedure is then repeated for the other curves in Fig. 251, and in this way the hysteresis may be separated from eddy currents, for various desired values of speed and field current.

The disadvantage of this purely analytical method is that in selecting two points on a curve the assumption is made that these points are absolutely correct; in other words, that any two other points on the same curve would give exactly the same values of H and F. In reality, the values of H and F vary somewhat for different pairs of points selected on a curve. This means a tedious process of calculating H and F for several combinations of points and taking average values, unless the method of least squares is used, with its involved computations.

344. Graphico-Analytical Separation of Iron Losses. — The graphico-analytical method described below permits of taking simultaneously into account as many points on a curve as desired; in this way the most probable values of the separated losses are obtained in a comparatively simple manner. Dividing both sides of eq. (21) by n, we get

$$p/n = H + Fn \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (23)$$

This is the equation of a straight line between p/n and the speed n. A line DC of this kind is shown in Fig. 258. In order to draw this line, one of the curves in Fig. 251 is chosen and several of its ordinates are divided by the corresponding abscissas. The values of (p/n) so obtained are marked with crosses in Fig. 258, as ordinates for the corresponding values of speed. The most probable straight line, DC, is then drawn through these points according to the judgment of the eye. Producing this line to the axis of ordinates, a horizontal line AB is obtained, which subdivides the ordinates of DC into the constant part MN equal to the hysteresis coefficient H, and the variable part NP corresponding to the eddy currents, and equal to Fn.

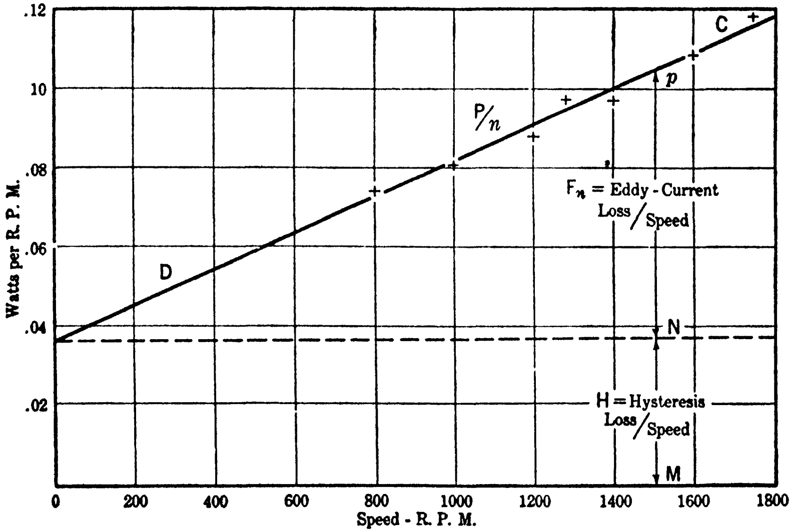

FIG. 258. Graphico-analytical method of separation of hysteresis from eddy-current loss.

H being known, the hysteresis loss is obtained by multiplying H by the desired values of n. Fn being known, the eddy-current loss is found by multiplying values of *Fn* by the corresponding values of *n*. The separation of the core losses for one particular value of field current is thus completed.

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CHAPTER XIV

DIRECT-CURRENT MACHINERY — OPPOSITION RUNS

345. Tests for determining the efficiency, voltage regulation, and temperature rise in electric generators and motors may be conveniently performed without the expenditure of much energy, when two machines of about the same size are available. The machines are then loaded to their full capacity *on each other*, no power being wasted in outside resistances or in a Prony brake. The machines are connected together, both mechanically and electrically (Figs. 260 and 261), and driven so that one machine acts as a generator, the other as a motor. The motor drives the generator, while the latter supplies electric power back to the motor. This is called the opposition or the " pumping-back " method of testing machines.

If the machines were ideal — without losses — the set would be self-contained and would require no power from the outside to drive it. In reality, the losses in both machines have to be supplied either by an auxiliary motor or a booster, or by connecting the set to a source of electrical supply. The power supplied to the set from outside covers only the losses in both machines, and herein lies the economy of the method. Let it be required, for example, to test two 500-kw generators at full load. Assuming the efficiency of each machine to be 90 per cent, the source of power would have to supply $500 \div (0.90)^2 = 618$ kw if one machine were to drive the other in the ordinary way, the second machine being loaded on resistances. When running the same two machines in opposition, the line has to supply only approximately 10 per cent of the $500 + 500$ kw, or only 100 kw. If a small driving motor or a booster is used (Fig. 260), the losses in these machines must also be supplied from the line; but even then the power demand is much smaller than with an ordinary load test.

In a generator or a motor driven in opposition with another machine, the electrical conditions may be so adjusted as to have full-load current and the rated voltage. Therefore the opposition or the pumping-back method is used for determining the efficiency, regulation, and temperature rise of a machine by direct measurement, since these three tests require the machine to be run under full-load conditions.

346. A Mechanical Analogue of Opposition Runs. — The following analogue (Fig. 259) may make clearer the underlying principle of all the opposition methods described below. Let it be required to determine the efficiency of a large crane or hoist under full-load conditions, when there is not enough power to drive it at full load, but when another identical hoist is available. Let the two machines be belted, as shown in the sketch, so that when hoist 1 lifts its weight P_1, hoist 2 lowers its weight P_2. The weights are equal and correspond to the rated capacity of the hoists; the whole system is mechanically balanced. Without friction, a very small force would cause a movement in either direction. In reality quite a considerable force is necessary to overcome the friction in both machines, and to start the movement.

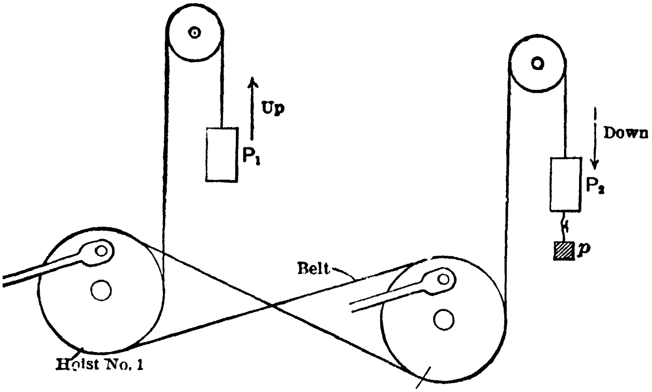

FIG. 259. A mechanical analogue illustrating the electrical opposition methods.

This extra power may be supplied either by driving one of the shafts or by adding an extra weight p to the load of one of the hoists. The first solution is analogous to the auxiliary motor shown in Fig. 260; the second solution corresponds to the case of electric power being supplied from the line, as in Fig. 261. Let each of the large weights on the hoists be equal to 10 tons, and let an additional weight of 2 tons be required to move the system at its rated speed. This corresponds to a friction loss of 1 ton per hoist; the average load is 11 tons, consequently the efficiency of either hoist is $(11 - 1)/11 = 91$ per cent.

347. Classification of Opposition Methods. — The methods by which two electrical machines may be run in opposition are usually classified according to the way in which the losses are supplied. The copper loss may be supplied either electrically or mechanically, and the iron loss + friction may also be supplied either electrically or mechanically. This gives the four combinations shown in the following table:

	Iron loss and friction supplied mechanically	Iron loss and friction supplied electrically
Copper loss supplied electrically	(1) Blondel	(3) Kapp — Parallel Potier — Series Hutchinson — Series and Parallel
Copper loss supplied mechanically	(2) Hopkinson	(4) Impracticable (?)

Blondel's method (1) is theoretically the most rational, because the electrical loss is supplied electrically and the mechanical losses are supplied mechanically; its disadvantage is that it requires a booster set and an auxiliary motor (Fig. 260). Omitting the booster set and supplying all the losses by the auxiliary motor gives Hopkinson's method (2); this method is as good as Blondel's for the determination of temperature rise and of voltage or speed regulation, but gives only approximate results for efficiency.

On the other hand, omitting the auxiliary motor in the Blondel scheme gives the three methods marked (3) in the table (Fig. 261). All the losses are supplied electrically, either by a booster or directly from the line, or by both methods. The correct method among these three is that of Hutchinson, in which the copper loss is supplied by the booster and the iron loss + friction from the line.

Kapp omits the booster and supplies all the losses from the line; Potier does not use the line and supplies all the losses by the booster. These two methods are simpler than Hutchinson's and just as good for the measurement of temperature rise and of voltage or speed regulation, but give only approximate results for efficiency. The method (4) has never been worked out in practice, for it does not seem rational to supply electrical losses mechanically, and vice versa. It may be possible, however, that a simple and satisfactory arrangement of this kind could be devised. All the above methods will now be described more in detail.

348. Losses Supplied Electrically and Mechanically. — We shall begin with Blondel's method, because it is more correct theoretically, and because all the other methods may be deduced from it. The connections are shown in Fig. 260. Two identical machines under test are belted or coupled together; their armatures are connected electrically in opposition. The shunt fields should be excited separately, from the line, whenever possible. The series fields and the interpole windings, if any, are left connected as in the regular operation. The combination of the armature, series field, and interpole winding is referred to below as the *series circuit* of the machine. A small auxiliary motor, belted to one of the machines, supplies the power for overcoming the iron loss and friction of the set. A booster is connected in series with the armatures to generate the voltage necessary for overcoming the resistance drop in the series circuits of the two machines. The booster is shown in the diagram driven by a separate motor.

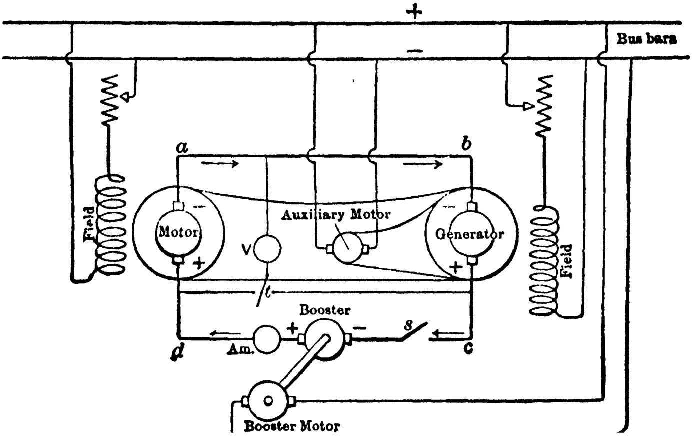

FIG. 260. Blondel's opposition method of testing two direct-current machines.

The set is started and brought up to speed by the auxiliary motor with the switch *s* open. Then the fields of both machines are excited to the required value, and when the voltmeter *V* gives the same reading on both sides, the switch *s* is closed. Very little current flows between the two machines, the armatures being in opposition. By exciting the booster so as to add its voltage to that of the generator, any desired current may be made to circulate between the two machines. The adjustment of the brushes and other similar precautions are the same as in a regular load test.

By regulating the field rheostats of both machines, the speed of the auxiliary motor, and the booster voltage, any desired conditions of current, voltage, and speed of the set under test can be established. When the two machines are equally excited, since the induced emf's are equal and opposite, the current is made to circulate only by the voltage supplied by the booster. Therefore, the copper loss is supplied entirely by the booster; consequently the iron loss and friction must be supplied by the auxiliary motor.

349. Details of Blondel's Method. — The test may be performed for the purpose of determining (a) the efficiency of the machine, (b) its voltage regulation, and (c) its temperature rise. The machine may be intended for use as a generator or as a motor. All these cases will be considered separately.

(a) *Efficiency.* Let the machine under test (Fig. 260) be intended to be used in regular service as a generator. Then its speed and voltage during the test are adjusted to the rated values, and the load current is taken larger than the specified value by the amount of the field current. During the test, the latter is supplied separately, but in regular operation it must be supplied by the armature itself. The other machine must in this case be identical with the machine under test, or else its efficiency must be known. Assuming it to be identical, its fields must be excited to the same value in order to have the same iron loss in both machines. The armature current is adjusted to a proper value by varying the speed and the field current of the booster.

Let P be the power output, in watts, supplied by the auxiliary motor, I the current circulating between the machines, E the rated voltage of the generator, i the exciting current during the test, and e the voltage between the positive terminals of the two machines under test. Voltage e is practically equal to that between the armature terminals of the booster, except for a small drop in the connection cd, the switch, and the ammeter. The resistance of the connection ab must be kept as low as possible. We have then:

Useful output of the generator $= E(I - i)$
Iron loss + friction in each machine $= \frac{1}{2}P$
Copper loss in the series circuit of each machine $= \frac{1}{2}eI$
Shunt excitation loss in each machine $= Ei$

the latter includes the loss in the field rheostat.

$$\text{Efficiency of the generator} = \frac{E(I - i)}{EI + \frac{1}{2}P + \frac{1}{2}eI} \quad . \; . \; . \; (1)$$

The output of the auxiliary motor is determined from its input and losses (§327); all the other quantities which enter into this expression are measured directly during the test.

Now let the machine under test be intended for service as a motor. With the same notation as before, we have:

$$\text{Efficiency of the motor} = \frac{EI - \frac{1}{2}P - \frac{1}{2}eI}{E(I + i)} \quad . \; . \; . \; . \; (2)$$

The denominator of this expression represents the total input into the armature and shunt field. The numerator is equal to the input into the armature less the core loss, friction, and the copper loss in the series circuit. If the motor is rated in terms of current or power input, the circulating current I can be adjusted directly to its proper value. If, however, the rating is given in terms of output, it may be necessary to make several trials before the value of I is found for which the numerator of formula (2) has the desired value.

(b) *Voltage or speed regulation.* The adjustment is the same as before, but the machines need not be alike. Having obtained the desired load conditions on the generator under test, reduce the booster voltage, or increase that of the other machine until the circulating current is reduced to zero, or, more correctly, to the value of the field current. This corresponds to throwing the load off the machine, as is required for the test on voltage regulation (§240). Measure the terminal voltage under these conditions. The percentage of voltage rise is by definition equal to the regulation of the generator. Another way of determining the voltage at no load is to open the switch between the generator and the motor and to drive the generator at no load by the auxiliary motor.

When the machine under test is intended to be used as a motor, the percentage regulation refers to the speed drop between no load and full load. In this case, the voltage across the motor terminals is kept constant by means of the booster field. After the full-load readings have been taken, the speed of the auxiliary motor is adjusted until the circulating current is reduced to zero, or, more correctly, to that which the motor under test takes at no load. Another way of accomplishing the same result is to throw off the belt between the two machines and to run the motor at no load.

(c) *Temperature rise.* The adjustments are the same as before; the machine under test must be run at the same current which its armature carries at the specified load, at the same field current, and at the rated speed. The other machine may have any characteristics, provided that it can carry the load required by the first machine. If the two machines are identical, the temperature run can be made on them simultaneously.

Before the run is begun, the resistances of the armature and the field, while cold, must be measured. The machines are then run the required number of hours at the specified load, and hot resistances are measured in order to calculate the temperature rise (§9). The temperatures of the armature iron and of the commutator are measured by thermometers, thermocouples, etc. Sometimes the temperature of the windings is also measured by thermometers, as a check on resistance measurements. See also §358.

350. EXPERIMENT 14-A. — Efficiency, Regulation, and Temperature Rise by the Blondel Opposition Method. — The machines are connected as in Fig. 260; instructions for starting the set are given in the preceding article. The runs for efficiency, regulation, and temperature rise may all be performed without changing the connections. Perform these tests, assuming first that the machine under test is running as a generator and then as a motor.

(a) Begin the experiment by measuring the cold resistances of the generator and of the auxiliary motor. Bring the set up to speed and adjust the current at about 25 per cent overload. Take the necessary readings, and then gradually reduce the armature current to zero, so as to obtain complete curves of efficiency and voltage regulation. Take a similar set of readings for speed regulation as a motor. Read the terminal volts E, the differential voltage e, the field current, the circulating amperes, the speed, and the input into the small auxiliary motor. The latter reading should comprise armature amperes, volts, field amperes, and speed. At the end of the test take off the belt and run the auxiliary motor light, in order to determine its own losses.

(b) Belt the motor again, bring the set up to speed, and adjust the conditions for the temperature run, so as to obtain an appreciable temperature rise in an hour or less. An average machine ought to be able to stand at least 50 per cent overload in current and 25 per cent overpotential for this period of time. At the end of the run, measure hot resistances, and determine the temperature rise both by thermometer readings and by increase in resistance.

Report. Plot curves of efficiency, voltage regulation, and speed regulation, using the circulating current as abscissas. On the same curve sheet, plot curves of output, for the machine working as a generator and as a motor, using the same abscissas. Give the data on temperature rise and compute the final results.

351. Losses Supplied Mechanically — Hopkinson's Method. — The above-described Blondel method of running two machines in opposition requires an auxiliary motor and a booster set. If it is desired to do away with the booster, the current between the two machines has to be circulated by weakening the field of one, or by strengthening the field of the other. In this case the auxiliary motor (Fig. 260) supplies the copper loss as well as the iron loss and friction of the set. This method is simpler than Blondel's, but the iron loss is not the same in the two machines, the fields being differently excited. This is of no consequence for regulation and temperature tests, as in this case the two machines need not even be identical. The method gives only approximate results for efficiency, but efficiency is usually determined from the losses and not from an opposition test.

When a temperature run is required on both machines, each may be run as a motor half of the time with the other acting as a generator. The set should be run at such a terminal voltage, slightly different from the normal, that the *average* core loss in the two machines will be equal to the core loss for each in actual operation. The voltage must be above the rated voltage if the machines are intended for operation as generators, and below the rated value for motors.

If P is the output of the auxiliary motor, the total losses in each machine may be assumed to be approximately equal to $\frac{1}{2}P$, and expressions (1) and (2) for the efficiency become·

$$\text{Efficiency of the generator} = \frac{(I - i)E}{IE + \frac{1}{2}P} \quad . \; . \; . \; . \; . \quad (3)$$

$$\text{Efficiency of the motor} = \frac{EI - \frac{1}{2}P}{E(I + i)} \quad . \; . \; . \; . \; . \quad (4)$$

352. EXPERIMENT 14-B. — Efficiency, Regulation, and Temperature Rise by the Hopkinson Opposition Method. - The method is explained in §351; the directions for performing the experiment are similar to those in §350.

353. Losses Supplied Electrically — Hutchinson's Method. — The losses may be supplied electrically by the three methods enumerated under (3) in the table above (§347). The theoretically correct method for determining efficiency is that of Hutchinson, where the losses are supplied partly in series, partly in parallel. The connections are shown in Fig. 261. The method is similar to that of Blondel (Fig. 260), except that the auxiliary motor is dispensed with, and the iron loss + friction is supplied from an external source of electric power, through the leads *pa* and *qd*. The set is started from the line with the rheostat *R*, using the left-hand machine as a motor, the switch *s* being open. When the machines are up to speed the generator field is excited, and when the voltages of the two machines are approximately equal the switch *s* is closed. After this, the required current is established between the two machines by suitably exciting the booster.

With two identical machines, the field current must be the same in both, at least for the efficiency test, in order to have the same iron loss. The copper loss is supplied mainly by the booster, as in the Blondel method. The line *pq* supplies the iron loss and friction and also a small part of the copper loss. The correction is easily applied, as is shown below, and does not impair the accuracy of the method.

Kapp's method is obtained from that of Hutchinson by omitting the booster; all the losses are supplied directly from the line. A current is made to circulate between the two machines by weakening the motor field. This method has the advantage of greater simplicity, and is just as satisfactory for regulation and temperature runs. It does not give quite correct values for efficiency, both the iron loss and the copper loss in the two machines being different.

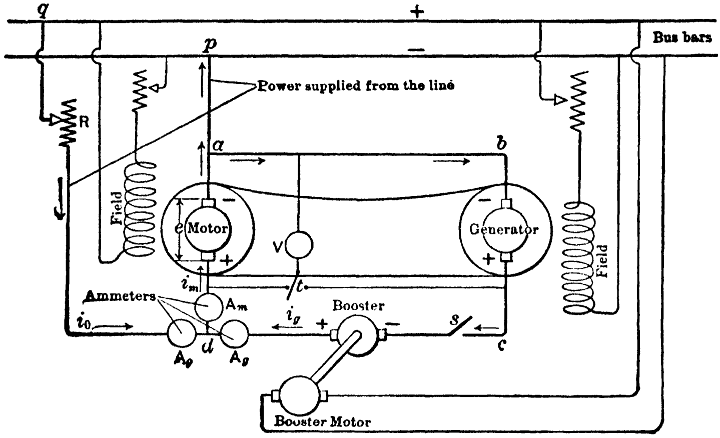

FIG. 261. Hutchinson's opposition method of testing two direct-current machines.

Potier's method is obtained from that of Hutchinson by omitting the connections to the line and supplying all the losses from the booster. The field of the motor must in this case be stronger than that of the generator, in order to obtain an additional torque for overcoming the iron loss + friction of the set. With this method the copper loss in both machines is the same, but the iron loss is somewhat different; the same remark applies to it as to Kapp's method.

354. Efficiency by Hutchinson's Method. — Let Ei_o be the power delivered to the set from the line (Fig. 261), and ei_g that delivered by the booster, where E and e have the same meaning as in §349. Then the expression $(Ei_o + ei_g)$ represents the total losses of the set, except the shunt excitation. Assuming these losses to be the same in both machines, the loss in each is equal to $\frac{1}{2}(Ei_o + ei_g)$ and the efficiency may be computed. The assumption of equal losses is not quite correct because the currents i_g and i_m in the two machines are somewhat different. If greater accuracy is required, the copper loss should be figured out separately; that is to say, with the notation shown in the diagram and in §349, we have

$$Ei_o + ei_g = i_g^2 r + i_m^2 r + P \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

This equation expresses the fact that the power Ei_0 supplied from the line, plus the power ei_g delivered by the booster, is equal to the copper loss in the series circuits of the two machines, plus their iron loss and friction P. The resistance r is that of the armature of either machine, including the brushes and the series field, if any. The fields being equally excited, the voltages induced in the two armatures are equal and opposite. Therefore, the differential voltage e between the points c and d must be equal to the voltage drop in the series circuits of the two machines. In other words,

$$e = i_g r + i_m r \qquad \text{. . . . . (6)}$$

The current i_0 supplied from the line is the difference between the motor current i_m and the generator current i_g, that is,

$$i_0 = i_m - i_g \qquad \text{. . . . . (7)}$$

Substituting the values of e and i_g from eqs. (6) and (7) in eq. (5), we get

$$P = Ei_0 - i_m i_0 r \qquad \text{. . . . . (8)}$$

In other words, the iron and friction loss, P, of the set is equal to the power Ei_0 supplied from the line, less the correction term $i_m i_0 r$. The iron and friction loss in each machine is $\frac{1}{2}P$. r being known, the copper loss in the series circuit of either machine may easily be calculated from the ammeter readings. The efficiency can thus be determined, as in the Blondel method, taking the excitation loss into account.

355. EXPERIMENT 14-C. — Efficiency, Regulation, and Temperature Rise by the Hutchinson Opposition Method. — The theory and directions are given in §§353 and 354. The order in which the experiment is performed and the requirements for the report are about the same as in §350. At the end of the test disconnect the line at p and q and run the set by supplying all the losses from the booster; this is Potier's method, mentioned in §353. Then disconnect the booster and supply all the losses from the line; this is Kapp's method mentioned in the same article.

356. Opposition Test on Series Motors. — The methods given in the table (§347) may be used with corresponding changes for testing series motors, more particularly railway motors. Such motors are tested in large quantities, and it pays to provide special arrangements in order to facilitate the handling of machines. In order to understand the details of these methods, and especially the necessary computations, the student should be familiar with the treatment of the series motor given in Chapters XII and XIII.

One of the opposition methods sometimes used for such motors is similar to that shown in Fig. 260, except that the motors, instead of being belted together, are geared to a countershaft. This shaft is driven by an auxiliary motor, which supplies the mechanical losses. The fields are connected in series with the armatures, and a booster supplies the copper loss.

A simpler method, which is a modification of Kapp's method, is shown in Fig. 262. The motors are coupled together and are connected electrically in opposition. The motor to the left is started from the line and drives the other machine, which acts as a generator and supplies current to the water rheostat; the switch S_1 is open. When a desired speed has been reached, and the voltmeter V_1 shows nearly zero, the switch S_1 is closed. By increasing the resistance of the water rheostat, the generator is made to send part of the current back into the motor, reducing the current drawn from the line. Finally the circuit-breaker S_2 is opened, and the machines run in opposition.

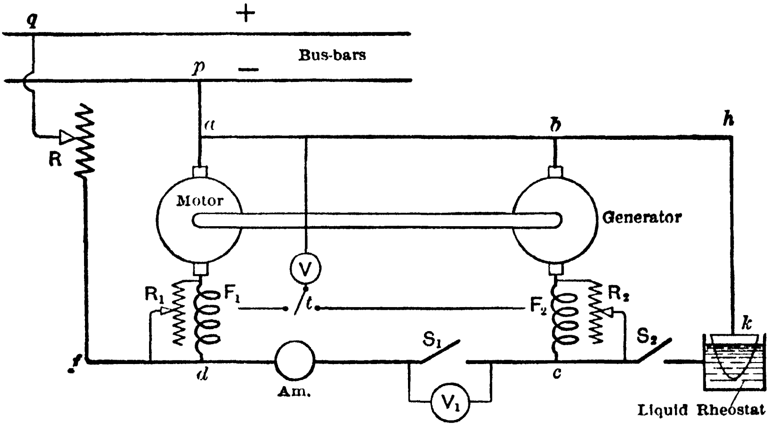

FIG. 262. An opposition test on two series-wound motors.

Shunt rheostats R_1 and R_2 are provided around the field windings F_1 and F_2, in order to regulate the speed of the set and the current circulating between the machines. The machine with a weaker field acts as a motor, the other as a generator. To equalize the conditions during the temperature run, each machine is made to act half of the time as a generator, and half of the time as a motor. The computations for efficiency are similar to those given above for shunt- and compound-wound machines.

357. EXPERIMENT 14-D. — Opposition Test on Two Series Motors. — Two identical railway motors are connected according to either scheme described in §356 and run at several speeds within their operating range, the purpose of the test being to obtain ampere-speed and efficiency curves. Make clear to yourself, in advance, the details of the proposed method of computing the results in order not to omit taking any necessary data during the test. After the preceding test, a heavy overload should be put on the motors for about an hour (temperature run). Measure cold and hot resistances, and also determine the temperature rise by thermometers or by some other approved method. See also the instructions for the other experiments in this chapter.

Report. Plot ampere-speed and efficiency curves of the motor; give the values of temperature rise by resistances and by thermometers.

358. Heat Tests and Temperature Measurements. — An accurate heat run and a careful determination of the hottest spots in the core, in the windings, and on the commutator, are of the greatest practical importance in acceptance tests of electrical machinery and apparatus, both revolving and stationary. For this reason, very detailed instructions as to loading, temperature measurements, and the highest permissible temperature limits are specified in the Standards of the American Institute of Electrical Engineers. These standards and rules are frequently revised with the progress of the art, such as better quality of insulation, more accurate apparatus for measuring temperatures, and new industrial conditions. For this reason it is preferred in this work not to give any detailed instructions for heat tests and temperature measurements. The reader is advised to consult the latest edition of the Institute Standards, the Rules of the National Electrical Manufacturers' Association (N.E.M.A.), or the corresponding rules issued by other national bodies in this country and abroad.

CHAPTER XV

ARMATURE REACTION AND COMMUTATION

359. The purpose of the experiments described in this chapter is to illustrate to the student (a) the distortion of the magnetic field in a d-c machine due to the armature current, (b) the influence of this distortion upon the commutation, and (c) the production of a correct field for commutation by means of commutating poles (interpoles) and a compensating winding. An outline of the elements of commutation is added for a more advanced study. Much of this subject is usually treated in the theory of electrical machinery, without experiments, but it is thought that the first-hand information which the student obtains in the laboratory will help him to appreciate more thoroughly the importance of these factors in the design and operation of d-c machinery. When undertaking a study of this chapter the student is supposed to be familiar with the first part of Chapter VII (The Magnetic Circuit) and with a few simple tests on generators and motors, described in the preceding chapters on d-c machinery.

MAGNETIC FLUX AT NO LOAD

360. Total Flux in Armature and in Field Structure. — Most of the magnetic flux excited by the field winding of a d-c machine finds its way between any two adjacent poles through the armature iron, and constitutes the useful flux of the machine. Some of the flux passes from pole to pole directly through the air, without entering the armature iron (Fig. 264). This part of the flux induces no emf in the armature conductors and is known as the *stray* or *leakage flux.* Both the total flux in the field poles and the useful flux through the armature may be measured *with the machine stationary,* by a modification of the ballistic method shown in Figs. 147 and 150.

An exploring or secondary winding of a known number of turns is placed on one of the pole-pieces near the yoke, to include the leakage flux, and another exploring coil is placed on the armature in such a way as to be linked with the total useful flux. The first exploring coil may be placed on the yoke instead of the pole-piece, in which case it will measure but one-half of the flux per pole (except in a bipolar machine with undivided magnetic circuit, where it will measure the total flux per pole). The machine is excited with a current through the regular field winding, and a ballistic galvanometer (§177) is connected to one of the secondary test coils, for example, to that on the field pole. When the primary circuit is opened, or the current in it is reversed (to eliminate the effects of the residual magnetism), a current impulse is sent through the ballistic galvanometer and the deflection is read on the scale. Knowing the galvanometer constant, the number of turns of the secondary winding, and the resistance of the secondary circuit, the observer can compute the change in the flux. The field is then excited again to exactly the same value, and the ballistic galvanometer is connected to the exploring winding on the armature. Its deflection is again noted when the field is destroyed or reversed. From this experiment the useful armature flux may be calculated as above, and the difference between the two will give the leakage flux.

With a large machine it is hardly practicable to open the field circuit suddenly, or to reverse the current in it, on account of the high self-induction of the winding. The induced emf may reach several thousand volts, which is dangerous to the operator and may damage the insulation of the winding. In such a case the flux should be varied in steps. For this purpose an adjustable resistance is connected in series with the field winding, and is short-circuited by a switch. By suddenly opening the switch the flux is changed by the desired amount, through the decrease in the field current, and the ballistic impulse noted. In this way the flux may be reduced to zero in several steps, or reversed, and the sum of the galvanometer readings used to compute the total flux. If a fluxmeter is available (§180) the field may be varied slowly, and a total magnetization curve taken.

The armature flux may also be computed from eq. (1), §234, by driving the machine at no load and at a constant speed, and measuring the induced voltage. This method is practicable only when the number of armature conductors and the number of armature circuits in parallel are known.

361. Leakage Coefficient. — The leakage coefficient, by definition, is the ratio of the total flux per pole to the useful flux in the armature. This ratio varies in modern machines from 1.15 to 1.40, according to the type and proportions of the machine and the saturation in the iron. A leakage coefficient of 1.30 means that out of every 130 lines of force in the field frame, only 100 actually pass through the armature; the rest find their path directly from pole to pole through the air.

It is important to keep the leakage flux as low as possible, since any increase in the total flux means a disproportionate increase in field copper, on account of saturation of the iron (§169). Moreover, a machine with a large leakage gives a poor voltage regulation because the armature reaction deflects the flux more easily into the leakage paths. The leakage coefficient increases with the saturation, because the reluctance of the useful path in the armature increases while that of the leakage paths in the air remains constant. With the same useful flux, the leakage is larger when the machine is loaded than at no load, because the mmf between the poles is larger.

The leakage coefficient may be determined as explained in §360 by measuring the fluxes in the field and the armature ballistically and taking their ratio. The galvanometer need not be calibrated; with the same number of turns on the armature and field poles, the ratio of the deflections gives directly the value of the leakage coefficient. If the number of turns is different, then both readings must be reduced to the same number of turns.

A compensation (or zero) method may also be used for measuring the leakage coefficient; with this method no ballistic galvanometer is required. A smaller number of exploring turns is placed on the pole-piece than on the armature; the two exploring coils are connected in opposition, and in series with an ordinary low-reading voltmeter of the moving-coil type (in place of a ballistic galvanometer). When the flux is suddenly changed, the emf's induced in the two coils are in opposite directions, and the voltmeter " kick " records the difference of the two. The ratio of the numbers of turns in the two coils is varied until the voltmeter needle remains at zero when the flux is suddenly increased or decreased. Then the inverse ratio of the numbers of turns is equal to the ratio of the fluxes, or to the leakage coefficient.

For example, let an exploring coil of 20 turns be put on the armature. Let us assume that, with 15 turns on the field, the voltmeter gave a small deflection in one direction; with 16 turns, a small deflection in the opposite direction. The ratio of the fluxes, or the leakage coefficient, is between $20/15 = 1.33$ and $20/16 = 1.25$. The true value may be estimated by noting the relative values of the deflections. This method is not so accurate as the ballistic method, but is more convenient and accurate enough for some practical purposes.

When a low-reading voltmeter is used for flux measurements, its moving system must be slightly weighted in order to increase its moment of inertia. Otherwise the pointer may move back and forth while the flux is being changed. This is because the flux often varies at different rates in the field and in the armature.

Commutating poles (§276) somewhat increase the leakage coefficient of the main poles by providing a low-reluctance path between adjacent pole tips. Therefore, in studying magnetic leakage experimentally, it is advisable to determine its value with and without the commutating poles, and also with the latter unexcited and excited with different numbers of ampere-turns.

362. EXPERIMENT 15-A. — Ballistic Measurement of Magnetic Flux in a D-C Machine. — The purpose of the experiment is to obtain a magnetization curve of a d-c machine at standstill, and with no armature current; also to determine its magnetic leakage under these conditions. The theory is explained in the two preceding sections.

(1) Place an exploring winding on one of the field poles and another on the armature along the center lines between two adjacent poles; preferably use fine wire. Demagnetize the machine thoroughly by repeated reversals of the field current, while reducing its value to zero; or use an alternating current and decrease it gradually to zero. Connect a ballistic galvanometer to the exploring winding on the field pole and increase the exciting current in steps from zero to the maximum practicable value. At each step read the field current and the ballistic throw. Be sure that the total resistance of the galvanometer circuit is known at each step, as it may be necessary to vary this resistance for some steps to obtain the best sensitivity of the galvanometer. Reduce the maximum current in steps, and if possible go over the whole hysteresis loop (§184).

(2) Repeat the experiment with the galvanometer connected to the exploring coil on the armature, using as nearly as possible the same values of exciting current. If a fluxmeter is available (§180) perform a similar run, using this instrument in place of the ballistic galvanometer.

(3) For a few values of the field current determine the leakage coefficient directly, using the zero method and a low-reading voltmeter. Use a very high, a medium, and a low exciting current in order to observe the effect of saturation upon the value of the leakage coefficient.

(4) Increase the pole arcs by adding iron bars or strips to the pole tips. It is not necessary to fasten them securely, as they will be held by magnetic attraction. Observe the increase in magnetic leakage due to the closer proximity of the pole tips. Similarly investigate the effect of excited and unexcited interpoles upon the leakage between the main poles.

(5) Before leaving the laboratory make a sketch of the magnetic circuit of the machine, especially of the form of the pole-pieces, as nearly as possible to scale. Count the number of turns on both auxiliary windings, and ask for the galvanometer constant.

Report. Give the diagram of connections used, the arrangement of the exploring coils on the machine, and an approximate shape of the magnetic circuit. Against values of exciting current as abscissas, plot values of the total flux in the poles and in the armature. To the same abscissas plot the ratio of the two, or the leakage coefficient. Check a few values of the latter with the data obtained with the zero method. Show the effect of bringing the pole tips closer together upon the value of the total flux, the armature flux, and the leakage coefficient. Analyze the data regarding the effect of the commutating poles upon the leakage.

363. Distribution of Flux in the Air-Gap. — For some purposes it is of interest to know not only the total flux in the air-gap of a machine, but also the space distribution of flux density (Fig. 263). In this sketch the armature circumference is developed into a straight line which is used as the axis of abscissas. The two adjacent brushes of opposite polarity are shown midway between the poles, and the distance between them is divided into 180 electrical degrees. Another scale is also shown, marked in commutator segments, of which there are 30 per pole in this particular machine. Against these abscissas, the corresponding values of flux density, found by experiment or computation, are plotted as ordinates.

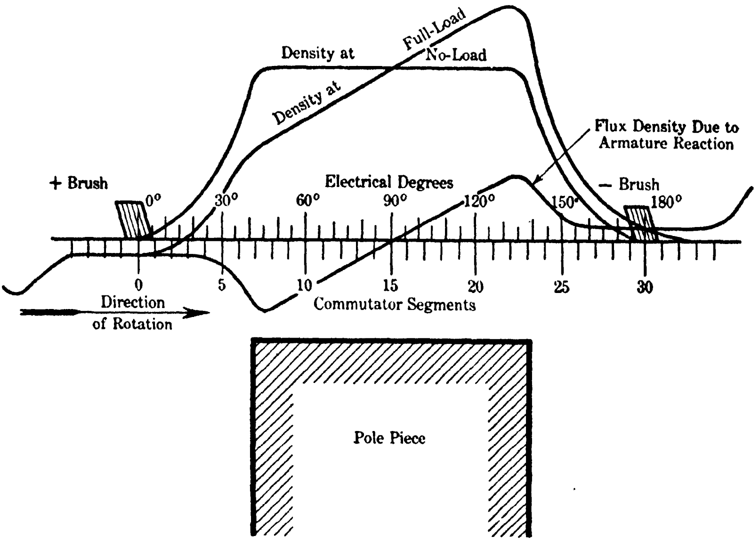

FIG. 263. Flux-density distribution under a pole, with the generator at no load and when loaded.

At no load the flux density under the pole is practically uniform, and gradually decreases to zero in the neutral zone midway between the poles, where the brushes are placed. When a current is flowing through the armature, it becomes a source of mmf, or *armature reaction*, which, if the field windings happened not to be excited, would produce such a distribution of flux as was indicated by Fig. 234, and is shown in Fig. 263 by the curve " Flux density due to armature reaction." This flux, due to the " cross-magnetizing effect " of the armature, may be thought to combine with the flux due to the field at no load and it is this combination of fluxes (or mmf's) which distorts the original flux distribution. The full-load flux distribution shown in Fig. 263 corresponds to that of a generator, the flux being shifted in the direction of rotation of the armature. With a motor, since the armature currents flow in the opposite direction, the field is shifted against the direction of rotation. The armature reaction is treated more fully in §§365 to 367 below.

Curves, such as are shown in Fig. 263, are of importance in the study of armature reaction, magnetic leakage, commutation, etc., and it is therefore desirable to become acquainted with experimental means for obtaining the necessary data. The principal methods are as follows:

(1) *Narrow test coil.* A long, narrow test coil, connected to a ballistic galvanometer or fluxmeter, is inserted in the desired place in the air-gap and then is withdrawn. Similar results may be obtained by leaving the coil in position and varying or reversing the field. In either case, the galvanometer deflection is a measure for the total flux through the coil, and consequently for the average flux density within its area. The coil is moved from place to place and the deflections plotted against its positions.

(2) *Wide test coil.* A test coil of a width equal to the pole pitch may be used instead of the narrow one. The wide coil is placed on the armature in such a position that its sides are opposite the centers of two adjacent poles. In this position as much flux is entering the coil from the *N* pole as from the adjacent *S* pole, and the total flux through it is zero. Therefore, if this coil be connected to the ballistic galvanometer and the field current changed or reversed, there will be no current impulse through the galvanometer. Let the test coil be shifted by a small electrical angle α, so that more flux enters it from one pole than from the other. Now if the flux be reversed, the galvanometer deflection will be a measure of the excess flux over the angle 2α. Consequently, the galvanometer deflection is proportional to the flux density near the center of the poles. Thus, by moving the coil in small steps, the *increase* in the galvanometer throw each time is a measure for the corresponding flux density. One of the armature coils may be used as such a test coil, by being disconnected from the rest of the winding and connected to a galvanometer or fluxmeter. Instead of revers-

ing the flux, the armature, in small machines, may be held in a position against a strong spring and then allowed to turn quickly by a small angle. The throw of the ballistic galvanometer due to the cutting of the lines of force within this small angle is again a measure of the average flux density over the angle of rotation.

(3) *A bismuth spiral* (§210) could be used for exploring the field, although this method is hardly ever applied in practice.

(4) *Double pilot brush.* The method mostly used in practice is that of measuring the emf induced in the armature coils as they pass a certain point in the air-gap. With the preceding methods the armature is stationary; here, the machine must be driven as a generator or as a motor. Two small brushes are mounted on a temporary brush-holder, and the distance between them is adjusted to be equal to the distance between the centers of two adjacent commutator segments. The brushes are insulated from each other and are connected to a low-reading voltmeter. When the armature is revolving, the voltage between these two " pilot " brushes is equal to that at the terminals of any armature coil moving in a certain position with respect to the field. In other words, the pilot brushes measure the voltage induced in a certain place of the field; this voltage is proportional to the average flux density over the width of one armature coil and within the arc corresponding to the width of one commutator segment.

By gradually moving the pilot brushes around the commutator, the flux-density distribution is measured over the whole pole pitch, and the results plotted as in Fig. 263. The same experiment, repeated with the armature loaded, gives a different distribution, because of the weakening and distorting action of the armature ampere-turns. In this case the observed voltmeter readings must be corrected for the voltage drop in the coil, in order to obtain the true induced emf's. This correction is added to the readings in the case of the generator and subtracted in the case of the motor. Further refinements of this method are possible whereby the interval of time during which the voltmeter (or a galvanometer) is in the circuit is considerably shortened, and the flux density averaged over a smaller angle.

The double-brush method requires an extra brush-holder on the machine, and a divided sector for measuring angles. A simpler arrangement, sufficiently accurate for practical purposes, consists of a template made of cardboard, with holes for voltmeter points. The holes are spaced so as to correspond to a certain number of commutator bars. The template is laid around the commutator, as closely as possible, but without touching it, and is securely fastened to the brush-holder. When the armature is revolving, voltmeter points are inserted in each two adjacent holes and the voltage measured. Ordinary metal points may scratch the commutator; it is better to use hard drawing pencils.

(5) *Single pilot brush.* Instead of measuring the voltage induced in an armature coil by means of two pilot brushes, the same voltage may be measured with a single pilot brush, by taking the difference of the two voltmeter readings between this pilot brush and one of the regular brushes of the machine. For example, let the voltage measured between the negative brush of the machine and the pilot brush, in a certain position, be 72 volts. Shift the pilot brush by one commutator segment towards the positive brush. If the new reading is 75 volts, then the voltage induced in this particular coil is 3 volts. This method is more suitable for obtaining a curve of voltage distribution along the commutator than for the flux-density curve. The ordinates of the latter curve are proportional to the values of the slope of the first curve ($B = \text{constant} \times de/d\alpha$). Conversely, the voltage-distribution curve is the integral, or the area curve, of the flux-density curve.

(6) *Oscillograph.* If an oscillograph (Vol. II) is available, the curve of flux-density distribution may be readily obtained by taking a curve of the voltage induced in an armature coil. For this purpose two small slip-rings are mounted on the armature shaft, with soft brushes bearing on each. These slip-rings are connected to an armature coil or to two adjacent commutator segments, while the brushes are connected to the oscillograph. The curve traced by the latter gives the voltage induced in that particular armature coil. The instantaneous values of voltages are proportional to the elementary fluxes cut by the conductors, so that the same curve gives the flux-density distribution. A correction for the IR drop is necessary when a load current flows through the coil. This method is very convenient when many distribution curves have to be taken for a special study.

364. EXPERIMENT 15-B. — Magnetic Flux Distribution in the Air-Gap of a D-C Machine at No Load. — The purpose of the experiment is to obtain the flat-top curve of flux-density distribution at no load, shown in Fig. 263. For a similar test on a loaded machine, see Experiment 15-C. The various available methods of measuring flux densities are described in the preceding section. Either two movable pilot brushes or a template and voltmeter points would ordinarily be used. Bring the machine to full speed, run it at no load as a generator or as a motor at normal voltage, with the regular brushes in the geometric neutral.

(1) Read volts between consecutive commutator segments and note the corresponding angular positions of the pilot brushes. Read also

the voltage between one of the regular brushes and a pilot brush. Keep the speed, the terminal voltage, and the field current constant, and note their values. Take similar sets of readings with the field highly saturated, and then with a very low saturation.

(2) Add strips of iron to the pole tips and again take readings for one of the field-distribution curves, at the same speed and voltage.

(3) Before leaving the laboratory, count the commutator segments, and note the number of poles. Make a sketch of the pole-shoe; measure its axial length and peripheral width; also the radial length and the average diameter of the air-gap. If possible, find the total number of armature conductors and the number of parallel circuits in the armature.

Report. Check the sum of voltmeter readings between the pilot brushes against the terminal voltage of the machine. Also check single-pilot and double-pilot readings, and make readjustments and corrections where necessary. Plot curves of flux-density distribution. Sketch the position of the pole-shoe over the curves. Explain the effect of saturation and of the added iron strips upon the shape of the curves. For a few points, show that the slope of the single-pilot brush curve is proportional to the ordinates of the double-pilot brush curve, and explain the reason. If the total number of armature conductors is known, compute the flux per pole, using eq. (1) in §234. Dividing this flux by the pole-pitch area will give the average flux density under the pole, and consequently the scale for the plotted curves.

ARMATURE REACTION

365. A current flowing through the armature winding of a d-c machine (Fig. 264) is a source of mmf which influences the value and the space distribution of the flux due to the field winding. In this sketch the surface of the armature is developed into a plane, and the relations hold true for any number of poles. The directions of the armature and field currents are shown in the usual way by dots and crosses, assuming the machine to be acting as a generator. The " belts " of armature currents between consecutive sets of brushes are in opposite directions.

In studying the armature reaction of a d-c machine, it is not necessary to consider the mmf of the individual armature coils, but only that of groups of coils between adjacent brushes. The function of the commutator consists in maintaining *stationary groups* of coils between adjacent sets of brushes while in reality *individual* coils are revolving (§371). The armature reaction, or the magnetic action of the armature currents upon the field, is due to these *quasi-stationary* groups of coils, and therefore is practically independent of the velocity of rotation. There is a small effect of the currents in the coils undergoing commutation, but this effect is neglected in the present discussion.

The armature reaction depends upon the position of the brushes. In Fig. 264 the brushes are shown in the geometric neutral, and the armature reaction is known as *transverse*. In Fig. 265 the brushes are shifted, and in addition to the transverse reaction there is also a *direct* armature reaction. These two components will now be considered separately.

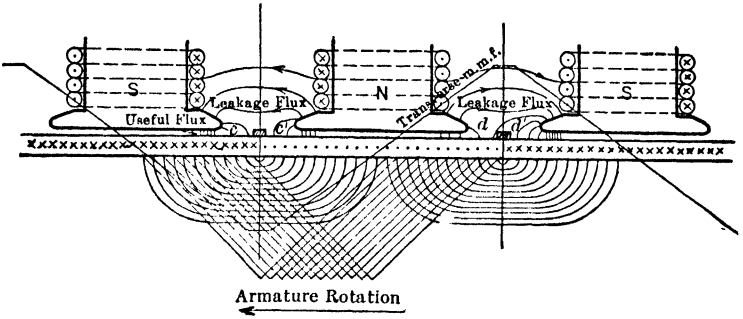

FIG. 264. Armature reaction in a d-c machine; brushes in the geometric neutral zone.

366. Transverse Armature Reaction. — Let the brushes be placed on the geometric neutral. In a real machine the armature conductors are connected at the ends, as is shown by the slanted straight lines. This is necessary in order to form a continuous symmetrical winding closed on itself, and one in which the same relationship with respect to the poles and the brushes is preserved as the armature revolves. However, it is easier to understand the mmf of the armature conductors, if we imagine these conductors grouped at each instant into " concentric " coils, also shown in the figure. Neither the electric nor the magnetic relations are thereby affected so far as the armature reaction is concerned, except that this arrangement holds true for an instant only.

We now have a wide fictitious armature coil with a mmf acting *up* along the axis of the brush *dd'*, and a similar one acting *down* along the axis of the brush *cc'*. This kind of armature reaction, with its poles midway between the main poles, is known as the *transverse reaction*.

Comparing the directions of the mmf's of the armature currents with those of the exciting field winding, we shall see that the flux is strengthened under the leading pole tips (those which the coils pass first) and weakened under the trailing tips. The armature currents in a d-c *generator* thus tend to shift the axis of magnetic flux of the machine in the direction of rotation of the armature. In a *motor*, with the same field polarity and with the same direction of rotation, the armature currents flow in the opposite directions, and the flux is shifted against the direction of rotation.

Theoretically, neglecting the effect of saturation, the flux is strengthened on one side of a pole as much as it is weakened on the other. This is illustrated in Fig. 263 by the curve of " flux density due to armature reaction," which curve represents the flux which the armature would set up if the field were not excited. But, on account of the saturation of the pole tips and of the armature teeth, the mmf required to produce a given increase in flux will produce a much greater decrease when applied in the opposite direction, especially in the region of the " knee " of the saturation curve. Therefore one result of field distortion is the weakening of the field. In a d-c machine the induced emf is proportional to the total flux, but is independent of its space distribution; formula (1) in §234. Therefore, the transverse armature reaction reduces the induced emf of the machine only indirectly, owing to the saturation of the magnetic circuit.

367. Direct Armature Reaction. — Consider the brushes to be shifted in the direction of rotation (Fig. 265) in order to improve the commutation (§235). The armature reaction is partly " transverse," that is, along the center lines of the spaces E and W, and partly *direct*, along the axes of the main poles. In order to separate these two components, we divide the total belts of armature currents into fictitious coils a and a' which exert a purely transverse or cross-magnetizing action, and b, b', and b'', which oppose the magnetomotive force of the field coils and constitute the so-called *direct armature reaction.* It will be seen that the direction of the mmf due to the coils b, b', b'' is such that shifting the brushes in the direction of rotation weakens the field flux, and shifting them against the direction of rotation strengthens it.

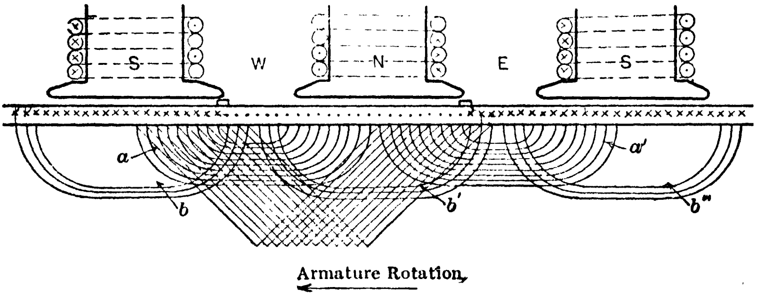

Fig. 265. Armature reaction in a direct-current machine; brushes shifted from the geometric neutral zone.

The foregoing conclusions have been reached by considering the machine to be operating as a generator. When it is working as a motor, with the same field polarity and with the same direction of rotation, the direction of the armature currents is opposite to that shown in Fig. 265, because the electromagnetic torque is now exerted in the direction of rotation. Thus, for a motor we have the opposite conclusions from those reached for the generator. All these results are tabulated below.

	Generator	Motor
Transverse reaction *shifts* the field	in the direction of rotation	against the direction of rotation
Direct reaction *weakens* the field when the brushes are shifted	in the direction of rotation	against the direction of rotation
Direct reaction *strengthens* the field when the brushes are shifted	against the direction of rotation	in the direction of rotation

The requirements for sparkless commutation are such that the brushes have to be shifted in the same direction in which the transverse reaction shifts the main field, unless the transverse mmf has been reversed by commutating poles (§276), by a compensating winding (§377), or by both.

The armature reaction influences the performance of a machine in various ways: It modifies the magnitude and the space distribution of the flux; it affects the terminal voltage of a generator, or the speed of a motor; and it affects commutation. Although these consequences are but different aspects of the same fundamental phenomenon, they are taken up separately in the experiments that follow, in order to present to the student the individual effects of each more clearly. The commutation problem is treated separately at the end of the chapter.

368. EXPERIMENT 15-C. — Magnetic Flux Distribution in the Air-Gap of a Loaded D-C Machine. — This experiment should be performed on the same machine and with the same pilot brushes as the preceding experiment, in order that the results may be compared. Plan all the runs keeping in mind the following two requirements: first, the results must be comparable to those in Experiment 15-B; second, only one factor must be varied at a time, the others being left unchanged.

(1) With the brushes in the neutral position run the machine as a generator and as heavily loaded as is possible without excessive sparking, taking readings as in Experiment 15-B. Use the same field current

and speed as in the first run of the preceding experiment. (2) If possible, also run the machine loaded, using first a high value and then a low value of field current; these values should be the same as at no load. (3) If the machine has no commutating poles, repeat one of the runs with the brushes shifted as much as commutation will permit. (4) Add iron strips to the pole tips and repeat one of the runs with the brushes in the neutral, and one with the brushes shifted. (5) Make some of the characteristic runs with the machine operating as a motor, loaded heavily on a generator, a blower, or some such steady load.

Report. Plot the data in such a form as to bring out the influence of each factor, viz., flux-density distribution with an armature current vs. that at no load; distribution with a brush shift vs. no brush shift; effect of low saturation vs. high saturation; generator vs. motor, etc. In each case show that the observed change from the shape of the no-load distribution curve could have been foreseen theoretically.

369. EXPERIMENT 15-D. — Effect of Armature Reaction on the Terminal Voltage of a Shunt-Wound Generator without Interpoles. — The purpose of this experiment is to investigate the influence of armature reaction and of brush shift upon the terminal voltage of a generator. The computations for the report require some of the data taken in Experiment 15-C, so that this experiment, the preceding two, and the one following should be performed on the same machine.

(1) With the generator running at its rated speed, excite the field as strongly as possible. Set the brushes on the geometric neutral and load the machine to the limit imposed by commutation. Read the field current, armature current, speed, and terminal volts. With a low-reading voltmeter determine the voltage between the heel and the toe of one of the brushes and check on a few other brushes; note whether the toe has a higher or a lower potential than the heel.

(2) Shift the brushes in steps by small known angles, both forward and backward, and at each step adjust the field current and the load current to the same values as at first. Take readings similar to those above. (3) Repeat the entire test with as low a value of field current as commutation will permit.

(4) Before leaving the laboratory, be sure to have accurate data for the no-load saturation curve of the machine and the armature resistance.

Report. (a) Plot curves of terminal volts vs. angles of brush shift, in electrical degrees. (b) For any set of readings, estimate the value of the armature reaction as follows: Add the *ir* drop in the armature to the terminal voltage, in order to obtain the voltage actually induced in the armature. Let the corrected voltage correspond to the ordinate *ab'* in Fig. 201, and let the field current be *Oa*. The corresponding voltage at no load is *ab*. Thus the armature reaction in terms of field current is equal to *ca*. In other words, if *Oa* is the value of field ampere-turns, then *ca* is that of the *equivalent* opposing armature ampere-turns. Perform such computations for a few points and plot or tabulate the results in such a way as to bring out clearly the effects of saturation and of brush shift. (c) Show that the general character of voltage changes could have been predicted from the theory of armature reaction. (d) Explain the nature of the voltage between the toe and the heel of the brush, and give the observed data of the value and the polarity of this difference of potential.

370. EXPERIMENT 15-E. — Effect of Armature Reaction on the Speed of a Shunt-Wound Motor Without Interpoles. — The purpose of this experiment is to investigate the influence of armature reaction and of brush shift upon the speed of the motor. For this experiment it is preferable to connect the motor to a generator, a blower, a centrifugal pump, or some such steady and adjustable load. The runs and the requirements for the report are essentially the same as in the preceding experiment, except that there the speed is kept constant and the voltage is allowed to fluctuate, while with the motor the terminal voltage is kept constant, and variations in speed are noted.

COMMUTATION AND COMMUTATING POLES

371. Stationary Groups of Armature Coils. — The function of the commutator in a d-c machine may be briefly explained as follows (Fig. 264): Consider a group of armature coils in series between two sets of brushes of opposite polarity. All these coils are under the influence of a magnetic flux of the same polarity so that the emf's induced in them are added. Consider a short interval of time during which the armature has moved by one commutator segment. For the whole machine the electrical conditions at the end of the interval are precisely the same as at the beginning of it, and hence the induced emf is the same. During the interval, the induced emf remains practically the same, except that one out of the many coils leaves the group under consideration, and another takes its place. Thus, the induced voltage between the brushes is slightly fluctuating, the frequency being equal to that at which the commutator segments pass under the brushes.

It may thus be said that, by means of the commutator and stationary brushes, *groups of coils* under the individual poles are kept practically stationary, in spite of the fact that in reality *individual coils* are revolving. Therefore, a commutator is an indispensable part of a d-c generator or motor in which conductors are connected in series within the machine itself, forming a closed winding. The latter limitation is necessary on account of the so-called homopolar machine which is not considered in this discussion.

372. Current Reversal in a Coil Undergoing Commutation. — In order to understand the cause of sparking under the brushes, the necessity for brush shift, and the purpose of commutating poles (interpoles), the student should familiarize himself with the elementary processes which take place in the coils undergoing commutation. In Figs. 266 and 267 the wavy lines marked 1, 2, 3 represent part of the armature

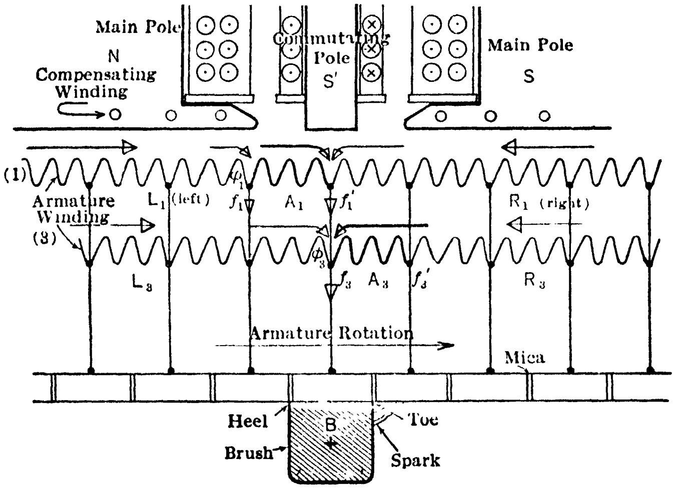

FIG. 266. Armature currents at an instant when the brush is in contact with one commutator segment.

winding at three different instants of time. The coil A, which is shown heavier, is the one in which the reversal of current is to be studied. The peripheral width of the brush B is assumed to be less than the width of one commutator segment or at most equal to it. The groups of coils marked L and R are respectively those to the left and to the right of the coil under commutation.

At the instant (1) the coil A, denoted in this position as A_1, belongs to the L group of coils, and the current is flowing through it to the right, as is shown by the arrowheads on top. This current reaches the brush through the commutator lead f_1' and the commutator segment. At the instant (3) the same coil, now marked A_3, belongs to the R group, and the current flows through it to the left, as shown by the lower arrowheads. Thus, while the armature has moved by one commutator segment to the right, the current in the coil under consideration has been completely reversed. The time interval (1-3) is exceedingly small; the student can take as an example a machine with which he is familiar and, knowing the number of commutator segments and the rated speed, can compute the fraction of a second that it takes for the armature to move by one commutator segment.

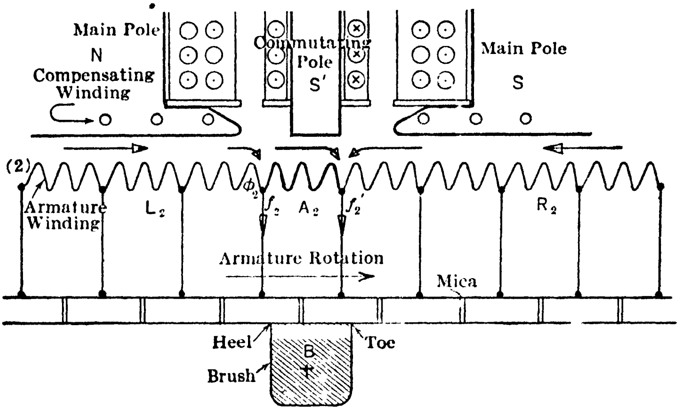

FIG. 267. Armature currents at an instant when the brush bridges two commutator segments.

When an electric current is forced to reverse rapidly, the emf of self-induction is brought into play, that is, the electrical inertia of the magnetic flux linked with the current in the coil under consideration (§136). This magnetic flux is denoted in Fig. 266 by ϕ_1 and ϕ_3 respectively, and being produced by the changing current, must be reversed with the reversal of the current. In so doing, the flux induces a counter emf which opposes the reversal of the current in A_3, and impedes the flow of current from the part R_3 of the armature winding through A_3, f_3, and the commutator segment to the brush.

Should this inductive emf be sufficiently high, the current from R_3, which up to the time that the segment and riser f_3' left the brush has been flowing in part by that path, will continue through this shunt path after segment and brush have separated, thus flowing across the air space and into the toe of the brush in the form of a spark. Thus the fundamental cause of sparking is a high counter emf induced in the coil under commutation, which forces part of the armature current to reach the brush through an air path and the adjacent commutator segment which is no longer in contact with the brush.

The voltage causing the spark, or the so-called *sparking voltage,* is not equal to the emf induced in the coil, on account of the voltage drop in the local circuit through the coil and the brushes. In a mathematical theory of commutation it is necessary to consider the variable resistance of this local circuit and the law of change of the current in the coil A during the interval of time (1-3). For the major part of this interval the coil is short-circuited by the brush, as shown in the position A_2 (Fig. 267). In this position the armature current is conducted to the brush through two leads, f_2 and f_2', and the coil A_2 forms a local circuit through these leads, the two commutator segments, and the brush. Thus the law of reversal of the current in A_2 depends not only upon the emf induced in it but also upon the contact resistance of the two parts of the brush. The contact area under the toe decreases, that under the heel increases with the time. The phenomenon is further complicated by the fact that the brush is often made wider than one commutator segment in order to reduce the current density. In this case two or more coils undergo commutation at the same time, so that their mutual inductance and the division of current among them must be taken into account.

373. Factors which Affect Commutation. — The simplified description of the phenomenon of commutation given above should be sufficient for a clear understanding of the following additional statements concerning it:

(1) *Coil inductance.* Any arrangement which reduces the inductance of the coils undergoing commutation improves the commutation. As such may be mentioned: smaller number of turns per commutator segment, shorter armature, properly designed end connections, fractional-pitch winding, etc.

(2) *Direction of brush shift.* The emf of inductance in the coil may be neutralized by the inductive action of an external field of proper magnitude and direction. Since the purpose of such a field is to facilitate the reversal of current, the direction of the "reversing" field, *in a generator,* must be the same as that to which the R (or the leading) group of coils is subjected. For this reason in a generator the brushes, if shifted at all, are shifted in the direction of rotation of the armature, anticipating the reversal. *In a motor,* the current flows against the induced emf, so that the field to which the coil has been subjected previous to the commutation period helps the reversal. For this reason, in a motor without interpoles, the brushes are shifted against the direction of rotation, if a shift is necessary to improve the commutation.

(3) *Armature reaction.* In a generator the effect of the transverse armature reaction (§366) is to strengthen the field under the *trailing*

pole tip and to weaken it under the *leading* tip. In Fig. 264 the symmetrical fringing field without the armature reaction is shown at *cc'* and one distorted by the armature reaction at *dd'*. This distortion of the field makes it more difficult to bring the coil under commutation within the influence of the proper reversing flux density. Moreover, when the brushes are shifted in the direction of rotation, a demagnetizing action of the armature upon the field is produced (§367), and the whole field flux is further shifted in the direction of rotation. Thus, in a machine without interpoles, and with a strong armature reaction, it sometimes happens that no sparkless position of the brushes can be found at a heavy load. The same is true for the motor, considering the opposite direction of brush shift.

(4) *Contact resistance.* The effect of the reactive emf in the coil (Fig. 267) is to make the lead f_2' take more than its share of current as compared to f_2, resulting in a higher current density under the toe of the brush, and, with a sufficiently high local current density, likewise in a spark. A brush of high contact resistance remedies this tendency to some extent, and makes it easier to obtain sparkless commutation. Theoretically, with a very high contact resistance the parts of the current conducted through f_2 and f_2' respectively should be nearly in the ratio of the corresponding contact areas under the brush. Thus, the current density under the heel and toe portions of the brush is more nearly equalized and a better commutation results. In practice other factors limit the range of application of this remedy.

(5) *Resistance leads.* An effect somewhat similar to that of increasing the contact resistance of the brushes is obtained by inserting resistances into the commutator leads, such as f_1, f_2, etc. Because of the resultant increase in the resistance of the local circuit of the short-circuited coil, the local current is cut down, and this tends to equalize the current density under the brush. This remedy has been applied mainly in a-c commutator motors.

374. Commutating Poles. — As is stated in the preceding section, the reactive emf in the coil undergoing commutation may be neutralized by subjecting the coil to the influence of an external field. Shifting the brushes for this purpose has practical limitations, including the inconvenience of changing their position with changes in the load. *It is much simpler to bring the necessary reversing field to a coil undergoing commutation than to try to bring the coil to the field.*

The commutating pole (or interpole) S' (Figs. 266 and 267), placed midway between the two main poles, is such a solution of the commutation problem. In a generator its polarity is the same as that of the leading pole S. In a motor the brushes are shifted backwards and consequently the commutating pole in the same position should be excited with the N polarity. Since the required commutating field should be proportional to the current to be reversed in coil A, the commutating poles are excited by a winding connected in series with the armature. This winding is marked " auxiliary field winding " in Fig. 268. The magnetic circuit of the commutating poles is kept at a low saturation, in order to make the required flux density as nearly as possible proportional to the load current.

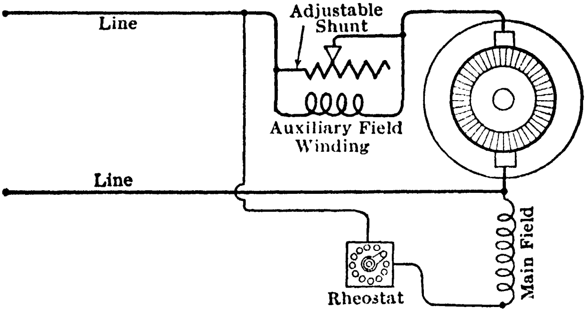

FIG. 268. Diagram of connections of a shunt-wound machine with interpoles.

The necessary number of ampere-turns on a commutating pole is much higher than that required to produce the same flux density in the air-gap of an unloaded machine. It must be remembered that the transverse armature reaction (§366) has its maximum mmf midway between the main poles, that is, under the center of the commutating pole, so that it is necessary first to compensate for these ampere-turns and then to set up a commutating field in the right direction.

In a machine provided with interpoles, there is no occasion for shifting the brushes from the geometric neutral. Should the brushes, for some reason, be shifted, the effect will be similar to a cumulative or differential compounding, because the flux of each commutating pole will then be added to or subtracted from that of the adjacent main pole acting upon the same group of armature conductors.

Inductive shunt. In a newly designed machine it is rather difficult to predetermine the exact number of turns for the commutating pole. For this reason a larger number of turns is sometimes provided than is necessary (Fig. 268) and an adjustable shunt is connected across the winding. When the machine is being tested this shunt is adjusted to obtain the best possible commutation at all loads. In a machine designed for rapid fluctuations or reversals of the armature current (as in some drives in steel mills), care must be taken to divide the variable current properly between the interpole winding and its shunt, under all conditions. The winding being much more inductive than an ordinary resistance shunt, a rising current would flow in preference through the shunt, while a decreasing current would tend to continue through the inductive path. This would provide a commutating field of wrong magnitude and might result in sparking, or even in flashing over, between the brushes. For this reason, shunts are sometimes made inductive in the form of coils, and their ratio of resistance to inductance is adjusted to agree with that of the interpole winding.

It is hardly fair to test the same generator with and without commutating poles. A machine designed with commutating poles has a relatively much stronger armature (measured in ampere-turns), and its other constants are different from what they would be were the machine designed without interpoles. The following experiment, however, will help the student to see clearly the influence of the commutating poles.

375. EXPERIMENT 15-F. — A Study of Commutating Poles in a D-C Generator. — The purpose of this experiment is to investigate the effect of commutating poles (Fig. 266) upon the commutation and voltage regulation of the machine. The generator should be connected as for an ordinary load test (Fig. 204) and driven by an adjustable-speed motor.

(1) *Load test.* For each speed in the following runs the field rheostat should be set to give normal voltage at the maximum load and the brushes set exactly on the geometric neutral. (a) With the commutating poles properly connected, run the machine at the maximum feasible speed, and obtain the necessary data for plotting the external characteristic at that speed (§242), that is, read terminal volts, load amperes, and the field current. Repeat the run at the lowest speed at which the desired voltage can be obtained, and at rated speed. (b) Cut out the interpoles and, using the same values of speed as before, repeat the preceding experiment in so far as sparking at the brushes will permit. Note on your data sheet the quality of commutation as indicated in §235. (c) Take a few readings with the reversed polarity of the winding on the commutating poles, and note the voltage regulation and the quality of commutation.

(2) *Brush shift.* With the commutating-pole windings in the circuit, load the machine to normal current at rated speed. Shift the brushes by steps, both forward and backward, as far as sparking will permit. Keep the load current and the terminal voltage constant, and at each step record the field current, the angle of brush shift in electrical degrees, the voltage between the toe and the heel of a brush, and the quality of

commutation. (b) Take similar readings at no load and at one or two intermediate loads. Take one or two sets of readings with the brushes shifted, and in the neutral position, without readjusting either the field rheostat or the load rheostat, so as to find the effect of the brush shift in either direction upon the terminal voltage and the load current. (c) Cut out the commutating-pole windings and repeat the preceding test.

(3) *Effect of inductive shunt.* Arrange some connections to shunt the interpole winding at will by a non-inductive or by an inductive shunt, preferably through a double-throw switch. Observe the effectiveness of each in taking care of commutation with sudden fluctuations of the load and with slow changes.

(4) *Limits of commutating flux density.* Provide a variable non-inductive resistance as a shunt to the interpole winding. Find by experiment the best value of such resistance to take care of commutation from no load to a reasonable overload. Find what fraction of the armature current flows through the interpole winding at this resistance. Investigate the limits within which the shunted resistance and the armature current may be varied (a) without appreciably affecting the quality of commutation and (b) still permitting commutation without excessive sparking. Measure the voltage between the heel and the toe of a brush with each of the various arrangements tried. Excite the interpoles from a separate source so as to have a fairly strong commutating field at no load. Find out the effect of such a field in either direction upon the quality of commutation at light loads. Observe whether or not the spark changes from the heel of the brush to the toe when the commutating field is reversed.

Report. (1) For each speed plot the external characteristics of the machine with the commutating-pole winding in and out. Indicate the results with the commutating winding reversed. Mark on the curves the quality of commutation. (2) For each load used in part (2) of the experiment tabulate or represent graphically the effect of each angle of brush shift upon the voltage drop and sparking. (3) Show theoretically that according to the direction of the brush shift the commutating-pole winding has a cumulative or differential compounding effect, and check this statement with the observed readings. (4) Explain in detail why an inductive shunt is more effective than a non-inductive one in taking care of sudden fluctuation in the load current; substantiate this explanation with your experimental data. What is the ideal proportion between the resistance and inductance of a shunt for the interpole winding and why? Would you use highly saturated commutating poles or only slightly saturated? Should there be any relationship between the degree of saturation in the interpoles and that in the iron core of the inductive shunt? (5) Tabulate the results of shunting the commutating-pole winding by various resistances, and explain why the commutation remains apparently perfect with quite a wide range of commutating flux. Give the data on the test with separately excited interpoles, and from the theory of commutation explain the observed findings as to sparking.

376. EXPERIMENT 15-G. — A Study of Commutating Poles in a D-C Shunt-Wound Motor. — This experiment is similar to the preceding one, except that in each run the applied voltage is kept constant, instead of the speed, and fluctuations in speed are noted, whereas in the generator variations in the terminal voltage are observed. A steady load, such as an electric generator, is preferable for this experiment. It is not necessary to refer the characteristics of the motor to the outputs as abscissas, unless a transmission dynamometer or a calibrated generator is available. It is sufficiently accurate to refer the speed and other readings to the power input into the armature as abscissas, correcting for the I^2R loss. Some results are preferably plotted to the armature current as abscissas.

377. Compensating Winding. — In a machine subjected to violent overloads, rapid reversals of current, etc., it is very important to remove the transverse armature reaction (§366) as completely as possible, in order to be assured of satisfactory commutation under all conditions. In many machines there is not enough space between the main poles to provide an interpole winding sufficient for this purpose. In such a case the transverse armature reaction is *compensated* by a distributed winding placed in slots in the main pole-shoes (Fig. 267). This distributed compensating winding is connected in series with the armature, and its ampere-turns are so computed as to be equal and opposite to those of the armature. Thus, most of the armature mmf is removed, and the commutating poles need be provided with comparatively few turns to create the proper commutating field. By the use of the compensating winding it becomes possible to design d-c machinery for higher speeds, shorter air-gaps and less material, so that the cost compares favorably with that of the ordinary commutating-pole machines. In addition the voltage regulation of generators and speed regulation of motors is greatly improved so that in these particulars, as well as in commutation, a superior machine results.

378. EXPERIMENT 15-H. — A Study of Compensating Winding in a D-C Machine. — The theory and purpose of the compensating winding are explained in the preceding section. In practice a machine with a compensating winding would always be provided with interpoles, so that it is difficult to separate the study of one from the other. The directions given for the two preceding experiments should therefore be carefully studied in planning this experiment. Pilot brushes (§363) will be found useful in ascertaining the extent of compensation of the transverse armature reaction. Curves shown in Fig. 263 should be taken under various load conditions (a) with both the interpoles and the compensating winding in the circuit, (b) with only one of them in the circuit and (c) with both cut out. A curve of flux-density distribution at no load should be plotted on the same sheet for direct comparison. In connection with this study, the quality of commutation should be investigated, and also the voltage regulation in the case of the generator and speed regulation for the motor. The effect of sudden fluctuations in the load and even of sudden reversals of the armature current may be made a subject for further study. Finally the relative proportion of ampere-turns upon the interpoles and in the compensating winding may be varied and the effect of these variations upon the characteristics of the machine and the quality of commutation determined.

379. Experimental Study of Commutation. — A complete theory of commutation includes a great many factors, each of which has been a subject of extensive experimental and analytical researches. Most of that work is outside the scope of this book. Only the principal factors and methods of testing are mentioned below. See also §382 below on testing of brushes, and the end of Chapter XII under " Troubles in D-C Machines." Some of the factors to be studied are:

(a) *The contact voltage drop* of various grades of carbon and graphite brushes as a function of mechanical pressure, speed, temperature, direction of current, its density, and frequency; the critical current density for each grade of brush. This subject is conveniently studied on separate collector rings, although the results may not be directly applicable to a commutator subdivided into segments and with a rapidly varying current density. A direct study on an actual machine or on a special revolving commutator is also possible but much more difficult.

(b) *The physical operating condition of brushes.* Under this heading the following topics may be studied: temperature of the heel and of the toe, in the positive and in the negative brushes; distribution of energy losses on the face of the brush; the relation between these losses, the width of the brush, and the temperature rise on the commutator; carrying of copper by the current from the commutator to the brush; life of brushes with various degrees of sparking; effect of undercutting the mica,

(c) *Inductance*, self and mutual, of the coils undergoing commutation may be studied by sending a high-frequency alternating current through a stationary armature and measuring the impedance of the coils in different combinations. Eliminating the ohmic drop, the reactance, and hence the inductance, may be computed. It must be remembered that a closed circuit containing the field winding will act as the secondary of a transformer, and that considerable eddy currents may be induced in the pole-pieces; both of these will reduce the true inductance that would be obtained by testing the armature entirely removed from the field structure.

(d) *Magnetic fields in the commutation zone.* A coil of very fine wire is placed in the same slots with an armature coil, so as to form a very close magnetic coupling with it. It is subjected to the same magnetic fields as the armature coil, but has no current flowing through it. The ends of the test coil may be connected to an oscillograph through two small slip-rings, and the effect of the fields studied directly from the shape of the curve of induced emf. Stationary test coils may also be mounted in the commutating zone for a study of the oscillating field produced by the currents in the short-circuited coils, as distinct from the field due to the transverse armature reaction. This has an important bearing upon the question of the effect of the transverse armature reaction on current reversal. Pulsations of current and voltage in the external circuit, pulsations of field current, eddy currents in pole faces, and such similar problems may also be studied in this connection. The effect of commutating poles, their flux density, width, possibility of over-commutation, etc., are also interesting topics for study.

(e) *The spark voltage.* The distribution of potential along the contact surface of the brush, and the voltage at different points between the brush and the commutator segment, are important factors in determining the quality of commutation. With a perfect reversal of current, the current density at all points under the brush is the same, and consequently the voltage drop across the contact surface is constant. Any departure from these two conditions characterizes a non-uniform distribution of current density under the brush. From six to ten points may be marked on the lateral surface of the brush, and the voltage measured between these points and the corresponding points on the commutator, sharp copper contacts being used. A low-reading voltmeter will give average values of these voltages; an oscillograph will indicate their actual variation with the time. When using an oscillograph it is preferable to mount an auxiliary slip-ring connected to one of the commutator segments and to take the voltage curve between the slip-ring and one of the brushes.

(f) *Time of reversal.* Theoretically, the time t of current reversal in an armature coil is determined by the duration of the short circuit, that is, by the relation $vt = b - m$, where v is the linear velocity of the commutator surface, b is the peripheral width of the brush, and m the thickness of one mica insulation. Actual oscillograms sometimes show a considerably shorter period of time, probably on account of imperfections in brush contact. An experimental study of this subject is of considerable practical interest, especially the effect of careful grinding of the brushes to fit the commutator surface.

(g) *Oscillograms of current.* With perfect commutation the current in a short-circuited coil should vary with the time from the full value in one direction to that in the opposite direction, according to the straight-line law. The degree of departure from this law is one of the criteria of commutation, showing the effect of inductance, external fields, and other disturbing factors. To obtain a curve of current reversal, a low-resistance ammeter shunt is connected into the circuit of one of the armature coils, and potential leads are taken from it through two slip-rings to an oscillograph. It is also advisable to have a similar ammeter shunt in one of the commutator leads, to get the distribution of the total current along the commutator segments, as a check on the first curve.

(h) *Commutation on short circuit.* With a very large machine it is sometimes difficult to provide a load of sufficient magnitude to investigate the quality of commutation on a heavy overload. The same overload current may be easily produced by running the machine with the armature short-circuited and the field excited from a separate source. The physical conditions of commutation are essentially the same as at a higher terminal voltage, except that the armature reaction has relatively a much greater influence because of the weakened external field. Nevertheless, such a test is of great practical importance, especially if one finds out the amount of allowance for more difficult conditions of commutation under short circuit, and the percentage of reduction in the criterion or in the limiting value of the current. In other words, if under the actual load conditions serious sparking begins at 500 amperes, at what current would the same degree of sparking take place on a short circuit?

380. Study of Commutation on an Equivalent Circuit. — The principal difficulties in studying commutation on an actual machine are due to the rotation of the armature. Moreover, with the coils embedded in slots it is not easy to eliminate or to modify the individual factors and thus to estimate their effect. For this reason some of the best investigations upon commutation have been made on an equivalent circuit (Fig. 269) in which only the commutator C revolves, while two identical stationary storage batteries, E_1 and E_2, in parallel, replace the emf's induced in the L and R groups of coils (Fig. 266). The alternate commutator segments are connected to two slip-rings, s_1 and s_2, which in turn are connected to the batteries through adjustable resistances, R_1 and R_2, and " ballast " inductances, L_1 and L_2. The purpose of the adjustable resistances is to enable one to balance the two currents in case the batteries give slightly different emf's; the inductances are intended to keep the currents in the two battery circuits practically constant, and unaffected by the rapid fluctuations of the current in the coil undergoing commutation.

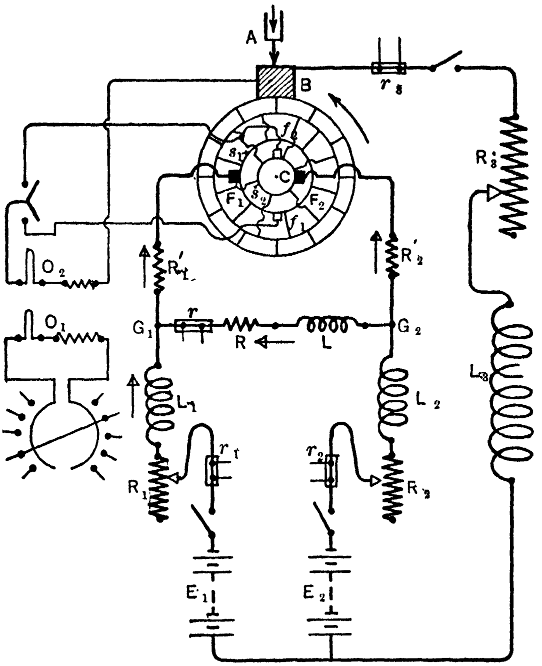

FIG. 269. An equivalent circuit with adjustable constants for studying commutation.

Only one brush, B, is used, and it is held against the commutator by means of a special brush-holder at A with an adjustable pressure. The peripheral width of the brush must be less than, or at the most equal to, the width of one commutator segment. The external circuit, or the load, is imitated by the resistance R_3 and inductance L_3.

The coil undergoing commutation is represented by a shunted combination of resistance R and inductance L between points G_1 and G_2.

As the brush B touches alternate commutator segments, the current through this shunted circuit is forced to flow in the alternate directions, thus imitating the action of an armature coil which is being transferred from one group of coils to the other.

The oscillograph shunts are marked r, r_1, r_2, r_3 and are used for taking curves of currents in the four main branches of the circuit. By means of a radial switch the oscillograph element O_1 may be connected at will to any of these shunts. The voltage curves between the brush and the commutator segments are taken with the second oscillograph element O_2, using separate brushes f_1 and f_2 so as to avoid the voltage drop in the main current brushes F_1 and F_2.

With this device, some of the factors and problems in commutation, enumerated above, can be studied much more conveniently and accurately than on an actual machine; the magnitude of the individual factors may be readily varied or their influence even eliminated.

381. EXPERIMENT 15-I. — Study of Commutation in a D-C Machine. — The theoretical elements of commutation are explained in §§371 to 375; the various factors to be studied experimentally are enumerated in §379. Most of this study is rather advanced and requires the use of an oscillograph and of special apparatus (§380), and more time than an average student can devote to the subject. Nevertheless, a thoughtful experimenter can derive considerable benefit even from a superficial study of commutation with limited means. In particular, some of the simpler problems enumerated in §379, under (a), (b), (d), (e), and (h), may well be attempted, in order to make clear to oneself the physical nature of the problem and the experimental difficulties involved.

BRUSHES AND THEIR TESTING

382. The two most common processes of manufacturing carbon brushes are termed the *squirting*, or extruding, process and the *molding*, or machining, process. In the former of these the material in the form of pulp is forced under pressure through a metal die and then cut off to the desired length and baked at a high temperature to carbonize the bond and permanently set the material. This method is used in making the cheaper grade of carbons, which do not have the strength to resist the breaking and chipping in service. The other process consists in molding the material into blocks under heavy pressure and then baking it. Carbons are cut from these blocks and machined to exact size.

The information on carbon and graphite brushes, given below, has been compiled mostly from the data furnished by the engineers of the National Carbon Company, to whom the author wishes to express his sincere appreciation.

383. Brush Composition. — According to their composition, brushes may be divided into the following classes:

Amorphous carbon brushes. The amorphous carbon brush is hard and has the lowest current-carrying capacity of any of the various grades. It is used in a few cases where commutating conditions are very severe, demanding a tough brush, but its field of application is not large.

Carbon-and-graphite brushes. The addition of graphite makes the brush softer, increases the current-carrying capacity, decreases the coefficient of friction, and reduces the contact drop. A great variety of characteristics is found in the various grades of brushes falling in this class, so that its field of application is quite large.

Electro-graphitic brushes are somewhat harder than brushes of the preceding class but as a rule are non-abrasive or practically so. They have better lubricating properties and higher current-carrying capacities than brushes of the first two classes, and on the whole, better commutating characteristics.

Pure graphite brushes. Brushes of this composition are the softest and have the highest current-carrying capacity of any of the brushes, with the exception of the metal-graphite type. On account of the softness of these brushes, as low a spring tension should be used as is consistent with satisfactory operation. Brushes of this composition have good lubricating qualities, making them suitable for high commutator speeds, and it is in this field that they find their most extensive application.

Brushes impregnated with a lubricating material. The pore space in a carbon brush is about 25 per cent by volume, and in this type of brush the carbon is impregnated with a lubricant. The coefficient of friction and some other characteristics of the brushes are improved by this treatment.

Metal-graphite composition brushes. Brushes composed of a combination of metal and graphite have the highest carrying capacity that it is possible to obtain, except in all-metal brushes. These brushes are used on plating generators, on d-c machines of very low voltage, and on collector rings where the carrying capacity is too high for carbon or graphite brushes.

384. Electrical Characteristics of Brushes. — The following tests and electrical characteristics of brushes are of practical importance:

(1) *Specific resistance.* This is the resistance in ohms of a cube whose edges are 1 cm or 1 in. long. This value is determined by meas-

uring the resistance of a long rod before it is cut into brushes of the regular size. Any of the methods mentioned in Chapter I for low-resistance measurements may be used.

(2) *Contact drop.* This, sometimes improperly called contact resistance, is the value in volts of the difference in potential between the contact surface of the brush and the commutator segment. This voltage seems to be more of the nature of counter emf and varies but slowly with the current density within wide limits of the latter. The apparatus for determining the contact drop consists of a copper ring driven by a variable-speed motor, the brushes being held firmly against the ring by holders which permit accurate measurements of the brush pressure. The current used flows from the brush to the ring and from the ring to the brush in such a way as to represent the conditions of positive and negative brushes of a d-c machine. Curves of current and voltage are plotted at different speeds, brush pressures, and current densities. A pyrometer is used to give the necessary temperature data on both the brushes and the ring.

There is a definite relation between friction and contact drop. Contact drop decreases with an increase of brush pressure; the friction increases with the pressure. There is a definite point at which the decrease in contact drop is overbalanced by the loss due to the increase in friction. Therefore, in the determination of brush tension both these losses should be considered, and the value chosen must effect a compromise between them. The pressure should seldom be less than 1¼ lb per sq in. For crane motors, mining locomotives, railway motors, and similar rough and intermittent service, the pressure ranges from 4 to 7 lb per sq in. (§318).

Although the specific resistance of a brush has some effect on the commutation of a machine, its effect is not so marked as that of the contact drop. A brush of high specific resistance tends to reduce the short-circuit currents on the brush face to some extent, but this effect is not very important. A more important advantage is the tendency to reduce short-circuit currents between the individual brushes of a stud, which currents may be caused by improper brush spacing. The chief disadvantage of high specific resistance is the high heat-loss due to the passage of current. This increases the temperature of the commutator and consequently decreases the rating of the machine. On the whole, the disadvantages of a brush of high specific resistance would greatly outweigh the advantages, were it not for the fact that high specific resistance and high contact drop usually go hand in hand.

In making tests on both the specific resistance and the contact drop it is well to investigate their values up to considerably higher temperatures than would be expected in normal operation. In fact, it is advisable to go up to the point of "glowing," because it is under such abnormal conditions that a good brush may save a machine, or at least tide over an emergency.

(3) *Current-carrying capacity.* The carrying capacity is the current density, in amperes per square centimeter or square inch of contact surface, that a brush can carry continuously without serious heating. In other words, the current-carrying capacity is fundamentally limited by the temperature reached on the contact face. For intermittent service, such as cranes or elevators, the brush may sometimes be rated at from 50 per cent to 75 per cent higher. When giving a figure for carrying capacity, the brush engineer considers not only the actual load current, but also the short-circuit currents in the coils undergoing commutation (§372), and the heating produced by friction and contact drop. Carrying capacity, though primarily an electrical characteristic, is subject to pronounced influence by all the mechanical or physical characteristics of brushes having any influence on the temperature. These mechanical or physical characteristics are: coefficient of friction, abrasiveness, density, and thermal conductivity. All of these are considered below.

(4) *Arcing and burning tests.* In selecting brush stock it is well to test it for its ability to withstand disintegration due to severe arcing and burning. Almost any arrangement that will sustain an arc is suitable for such a test, provided that the physical quantities which enter into the test can be measured and that the conditions can be repeatedly reproduced with a fair degree of accuracy.

385. Mechanical Characteristics of Brushes. — The following mechanical characteristics of brushes and tests are of practical importance:

(1) *Coefficient of friction.* The apparatus for determining brush friction consists of a copper slip-ring very accurately balanced and driven by an adjustable-speed motor. The brush-holders and studs are also accurately balanced on a pin bearing, the friction of which is so slight as to be negligible, so that only the actual brush friction is recorded on the fine spring balances used. Thermometers, or thermocouples, are so attached as to measure the temperature of the brush as nearly as possible to the brush face. Friction tests are run at different speeds and at different values of brush tension for each grade of brush under comparison.

Atmospheric conditions, composition of the brush, brush tension, current density, peripheral speed, composition of commutator or slip-ring, condition of commutator, i.e., whether slotted or flush, and temperature, are factors which have more or less effect upon the coefficient of friction. Therefore, the results obtained on a smooth slip-ring must be used judiciously and not generalized to other conditions of operation.

The hardness of a brush does not indicate its coefficient of friction. Hardness depends upon the hardness of the particles composing the brush and upon the strength of the binder holding these particles together. Friction depends on the structure of the particles making up the brush and upon the impurities in the carbon or graphite which give the brush an abrasive or cutting quality. It is, therefore, not true to say that a hard brush will wear a commutator faster than a soft brush. In fact, a soft brush will wear a commutator more rapidly than a hard brush with the same percentage of abrasive material, because of the fact that the soft brush wears away more rapidly, thus permitting the abrasive material to feed down on to the commutator at a more rapid rate. It is possible to get a high friction without abrasive action, but it is impossible to get abrasive action without a high friction. The most common impurities which cause abrasive action are mica, quartz, silica, silicon carbide (" Carborundum "), and iron oxide.

A film of oil between the brush and the commutator, besides acting as a lubricant, has the property of increasing the contact drop. However, the porosity of the brush and the high temperature at the brush face usually cause the oil film to disappear very quickly, and the injurious effect of oil on mica makes it dangerous practice to lubricate a commutator. On undercut commutators the practice is even more dangerous, because of the collection of dust and dirt in the slots which may result in flash-overs or short circuits.

(2) *Hardness.* The mechanical hardness of a brush is determined with an instrument called a scleroscope, in which a steel weight falls from a constant height upon the article whose hardness is to be measured, and then rebounds up a graduated scale, the height of the rebound being measured. This, of course, does not give the actual hardness, but a relative figure which is useful for purposes of comparison.

(3) *Mechanical strength or toughness.* The usual strength test on brush stock is really a flexure test. The test piece is supported on two knife-edges and a third knife-edge presses on the middle. A weight is moved along a graduated lever arm and the pressure is increased until the piece breaks. This force, properly correlated with the dimensions of the prism under test, gives the unit strength or toughness of the material.

(4) *Abrasiveness.* Resistance of the carbon brush to abrasion, or scouring action, is the ability to withstand the wear due to friction. This

term should not be confused with hardness. Hardness relates to compactness of the mass as a whole; abrasiveness relates to the cutting action given a brush by the more or less gritty particles in the brush mixture. The abrasive action of a brush depends largely upon the composition of the ash which forms a part of the brush. The amount of ash can be determined by heating the brush in a special furnace to a temperature which will burn out all the carbon. Silica, mica, and silicon carbide form the major part of the abrasive material found in the ash of carbon brushes. In general, it may be said that an abrasive brush should be used where the mica between the commutator segments is very hard and is not undercut, or where excessive reactance voltage creates a serious tendency toward high mica. An abrasive brush will keep the mica level with the commutator and will prevent the sparking and chattering due to high mica. Except in cases where there is a tendency toward high mica or heavy sparking which roughens the commutator, pronounced abrasive qualities in a brush are undesirable.

There seems to be no reliable test for abrasiveness. Efforts have been made to develop an apparatus to test this characteristic, but these have failed to produce results that check with actual experience with the different grades of carbons under service conditions. This is attributed to the influence of the commutating currents on disintegration at the brush face, which cannot be reproduced in a testing machine. Brushes which have shown decided abrasive characteristics in operation will sometimes glaze over on the test machine and run for a long period of time without wear.

(5) *Maximum peripheral speed.* Most types of brushes operate satisfactorily only as long as the linear speed of the commutator surface does not exceed a certain value. Therefore, when a brush is to be used on a high-speed surface, it is advisable to determine the speed limit or to ascertain it from the maker of the brush.

(6) *Vibrator test.* Railway motor brushes are subjected to a special test in a vibrator to determine their ability to resist breakage due to chattering. This test must be made at least as severe as in the actual service, and only the stock that will stand this test should be specified or used.

(7) *Density.* The real density of the material of a brush is the weight of the carbon exclusive of the pores compared to the weight of an equal volume of water. The average real density of brushes is approximately 2.00. The apparent density is the weight of the carbon with the pores compared to the weight of an equal volume of water. The average apparent density of brushes is approximately 1.5. The average pore space in brushes amounts to about 25 per cent. A low-density brush,

because of its low inertia, will more readily follow the irregularities of the rotating element and for this reason is particularly adapted to very high peripheral speeds.

(8) *Thermal conductivity.* This characteristic is of importance because it determines the amount of heat that can be conveyed away from the brush face to the holder and to the frame of the machine; consequently it determines in part the temperature of the commutator. Graphite has a higher thermal conductivity than most of the forms of carbon, and if it is possible to replace a carbon brush with a suitable graphite one, the temperature of the commutator may be reduced by several degrees. Thermal conductivity may be expressed either in the absolute or in the relative measure. The absolute thermal conductivity of a brush material is the number of watts which will flow through 1 cm cube at the temperature difference of 1° C between the opposite sides. Relative thermal conductivity is expressed in percentage of the thermal conductivity of some standard material.

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CHAPTER XVI

THE TRANSFORMER

386. A stationary transformer, in the usual sense of the word, is a device based on electromagnetic induction, for converting a-c power from one voltage to another. For example, a lamp load in a house may be 10 amperes at 110 volts. By means of a transformer (Fig. 270) of suitable design, this power can be obtained from a 2200-volt supply by drawing only 0.5 ampere. The product is 1100 volt-amperes in either circuit, and the power lost in the transformation is only 2 or 3 per cent.

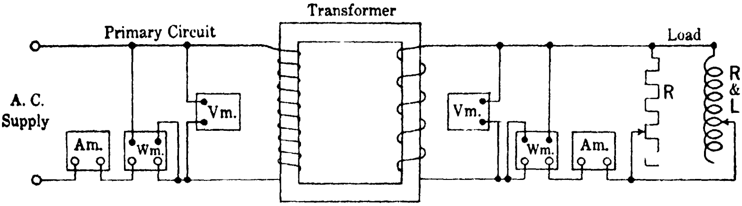

Fig. 270. Diagram of connections for a load test on a transformer.

A transformer (Figs. 271 to 273) consists of a laminated iron core and, usually, of two windings, primary and secondary, in an inductive relation to each other. Modifications of this are the single-winding, or auto-transformer, and the three-winding transformer used on special power applications (see Vol. II, §§796 and 819). In the two-winding transformer one of the windings is connected to the source of power, the other to the load. The winding connected to the supply should always be called the primary, the other the secondary. Sometimes it is preferable to refer to the high-tension winding and the low-tension winding respectively, and a transformer can usually be connected so as either to raise the voltage or to lower it. For an exception, in the case of instrument transformers, see §§422 and 430.

387. Core-Type and Shell-Type Transformers. — There are three ways of arranging the windings and the core relatively to each other. The so-called *core-type transformer* (Fig. 271) consists of one iron core and of two sets of coils, one set on each of its legs. The *shell-type transformer* (Fig. 272) consists of one set of coils, and each side is surrounded by an iron core. The combination type or the *distributed-core type transformer*, shown in Fig. 273, permits of a somewhat better utilization of the copper and iron with consequent improvement in efficiency and a reduction of cost. Electrically the three types are equivalent, the difference being mainly mechanical and determined by the convenience and cost of manufacture.

Transformers are usually immersed in oil, which keeps the windings and the core cooler in operation and also protects the insulation from moisture. In smaller sizes, the oil is cooled naturally through the containing steel case. In larger sizes it is usually necessary to increase the heat-radiating surface of the case by making it corrugated or by adding circulating pipes or radiators, and in some instances to provide blowers to cool the radiating surfaces. In many large transformers the oil is cooled by circulating water in pipes placed within the transformer tank, although it is sometimes preferred to circulate the oil in pipes in a separate water tank or under a spray. Another method of cooling is by forced circulation of air. No oil is used, the transformer case being left open at the top and bottom; air is blown through the core and windings by a suitable blower.

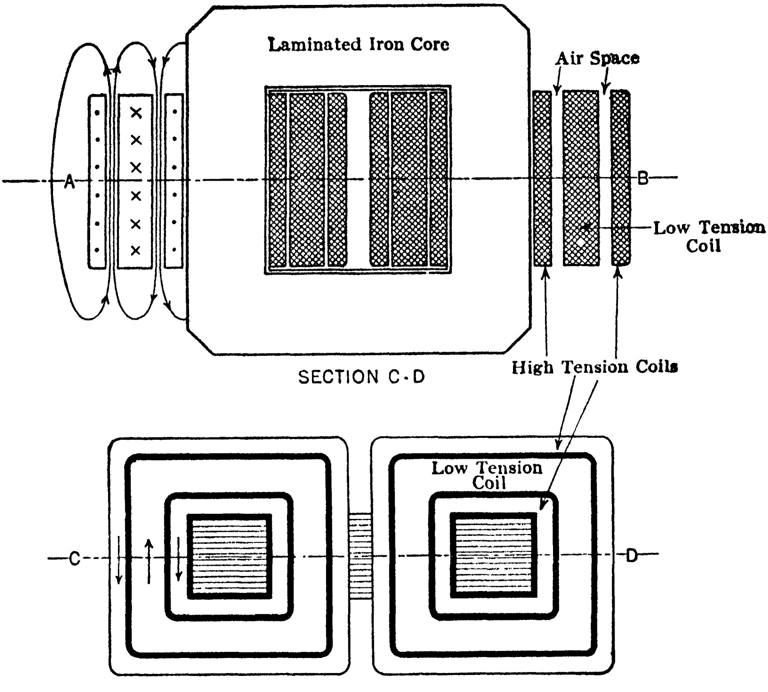

FIG. 271. A core-type transformer.

388. Introduction to Transformer Testing. — A student working with even a 400-volt transformer must remember that *it is dangerous* to come in contact with the high voltage; great care should be exercised in regard to touching any switches, wires, or instruments connected to the high-tension side. At the higher voltages, rubber matting should be put on the floor for protection so as to insulate the tester from the ground, and while handling the instruments he should not touch either the walls, pipes, or any objects, including other students, that might be connected to the ground. In addition, if instruments are connected directly in the high-voltage circuit, rubber gloves should be provided for the person who reads these instruments (Fig. 270) so that he will not touch them by mistake. *Directions of the instructor should be followed exactly,* and no changes attempted before consulting him. An electrical engineer has to deal with high-tension circuits in his practical work, and it is advisable for him to get some experience with such circuits before leaving college. He should know, however, that a mistake or negligence may prove fatal, and should proceed with extreme caution.

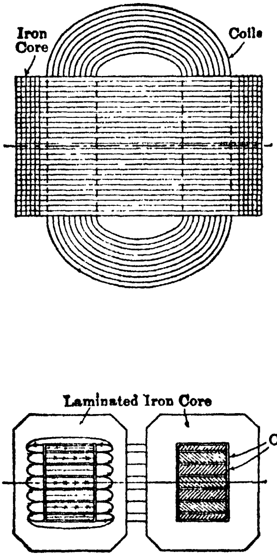

FIG. 272. A shell-type transformer; above, a single core, below, two separate cores.

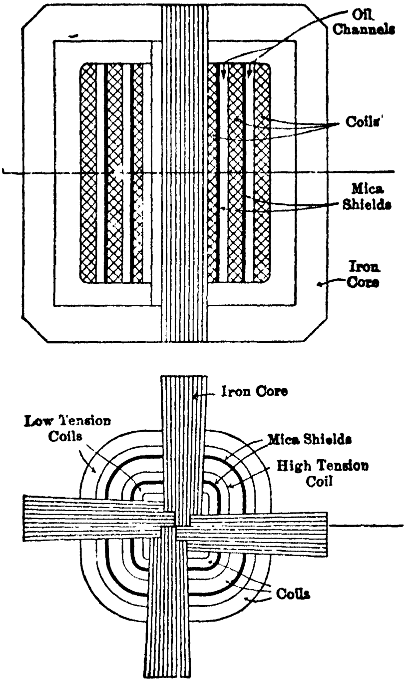

FIG. 273. A distributed-core transformer.

The following are the principal tests performed on standard transformers, either by the manufacturer or by the customer. The purpose of these tests is mainly to check the guaranteed performance characteristics, the quality of materials, and the correctness of winding and assembly.

(1) *Voltage-ratio test* at no load, to determine whether the transformer has been wound correctly, and the leads and taps properly brought out. Part of the checking of the connections is the so-called *polarity test* (§414) to determine which terminals are to be connected together for parallel operation.

(2) *Resistance of the windings.* This measurement is usually done by the drop-of-potential method (§2), although the Kelvin double bridge (§34) or the Wheatstone bridge (§29) may be used. This measurement serves several purposes, viz.: (a) It is a check on the winding and its assembly. (b) The results may be used in computing the efficiency and the voltage regulation of the transformer. (c) If performed before and after the heat run it gives the data for the computation of the temperature rise. (d) A comparison of the I^2R loss computed from the resistance measurements with that determined from the wattmeter test (§392) gives an idea of the extent of eddy currents in the conductors.

(3) *Core loss and magnetizing current.* This test is performed at no load to determine the amount of power lost in hysteresis and in eddy currents in the iron core, and the current which the transformer takes at no load. The results are used to check the quality of the iron and to compute the efficiency of the transformer. The temperature rise may also be estimated from this test and the preceding one. The upper limit of the no-load or magnetizing current is often prescribed by the contract, and is also a check on the design, the quality of the iron, and its assembly.

(4) *Short-circuit test,* to determine the impedance drop and the total copper loss. The results are used in computations of the voltage regulation and efficiency. This test also serves as a check on the winding, its connections, and soldered joints.

(5) *Heat run.* The purpose of this test is to determine the maximum temperature which the principal parts of the transformer reach after a certain number of hours of operation under prescribed conditions. This test is of great practical importance.

(6) *Test of insulation, to determine its strength and condition.* This includes a high-potential test between each winding and the core, between the windings, and sometimes between the sections or turns of the high-tension winding. The insulation resistance between the primary and the secondary winding is also sometimes measured. These

tests are described in Vol. II, but they may be performed as described in §4.

(7) *Load test.* This test is avoided as much as possible because of the expense and difficulties of providing a suitable artificial load, especially for a large transformer. If required for the heat run mentioned under (5) above, it is preferable to use the so-called opposition or loading-back method (§398) in which power is circulated between two transformers.

389. Voltage Ratio. — Consider a transformer (Fig. 270) connected on its primary side to a suitable source of a-c supply, and with its secondary winding open, that is, disconnected from the load. The alternating current in the primary winding excites in the core an alternating flux, which induces alternating emf's in both the primary and the secondary windings. The two windings being placed on the same iron core and subjected to the same alternating flux, the voltages induced *per turn* will be the same in both. Since the turns in each coil are connected in series, the total induced voltage per coil is proportional to its number of turns. We thus have the following fundamental relationship: *In a transformer the voltages e_1 and e_2 induced by the useful flux are in the same ratio as the numbers of turns, n_1 and n_2, in the corresponding windings.* Mathematically expressed,

$$e_1/e_2 = n_1/n_2 \qquad \ldots\ldots\ldots\ldots \quad (1)$$

This expression holds true either for the instantaneous or for the effective values of the induced emf's, e_1 and e_2. For a relationship between the induced voltage, the flux, and the number of turns, see eq. (6) in §227.

In the usual transformer, the primary voltage drop is very small, so that the primary applied voltage, E_1, is practically equal and opposite to the primary induced emf, e_1, even at full load. Similarly, the voltage drop in the secondary winding being small, the secondary terminal voltage is nearly equal to the secondary induced emf, e_2. Therefore, we have *approximately*

$$E_1/E_2 = n_1/n_2 \qquad \ldots\ldots\ldots\ldots \quad (2)$$

Equations (1) and (2) become practically identical at no load.

390. EXPERIMENT 16-A. — No-Load Voltage Ratio and Polarity of a Transformer. — The purpose of this experiment is not only to determine the polarity and voltage ratio but also to prove that the ratio of the induced voltages is equal to the ratio of the numbers of turns. It is advisable to have a transformer in which the number of turns is known by actual count or otherwise, and may be varied at will, either by interconnecting individual coils or by using different taps on the same coil.

Where only moderate voltages are involved one voltmeter is connected directly on the primary side and another on the secondary side, unless the voltages are so nearly equal that the same voltmeter may be used with a double-throw switch. An electrostatic voltmeter (§55) is sometimes suitable for the high-tension side, but when the high-tension voltage exceeds 220 volts a potential transformer (§56) of known ratio is commonly used with the voltmeter, connections for the test being as in Fig. 274. It is also advisable to have an ammeter in the primary circuit in order to get an idea of the order of magnitude of the no-load current taken by the transformer under different conditions. (For a special study of magnetizing current, see §405.) Proceed as follows:

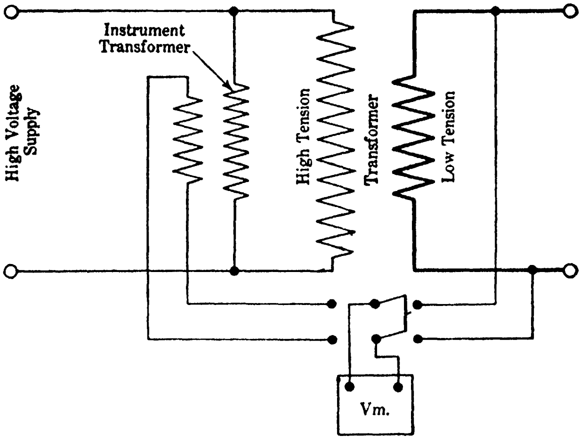

FIG. 274. Voltage ratio test of a transformer, using a voltmeter and a potential transformer.

(1) Select an arrangement of windings for which the number of turns is known and apply a voltage of known frequency to the primary terminals. Read the primary and secondary voltages and the primary current. Vary the applied voltage in steps from 20 per cent above to 20 per cent below normal, to see if the ratio of voltages remains independent of the applied voltage.

(2) Keep the applied voltage constant and vary its frequency. This can best be done by having an alternator driven by an adjustable-speed motor. Take readings as before of primary and secondary volts, as well as of frequency, to determine whether frequency affects voltage ratio. Note the increase in the magnetizing current as the frequency is decreased.

(3) Make a few check runs with a different arrangement of windings or with different taps on the same coils. Observe that the magnetizing

current increases as the number of primary turns is reduced, the voltage and the frequency being kept constant.

(4) Select two separate secondary (low-tension) coils and designate the terminals of one *a*, *b*, and those of the other *c*, *d*. Connect *a* to *c* by a jumper and, with normal voltage applied across the primary, measure the three voltages *ab*, *bd*, and *cd*. Keeping the same primary voltage, disconnect the jumper at *c* and put it to *d*. Measure the voltages *ab*, *bc*, and *cd*. Show from the readings that in one case the coils were connected cumulatively, and in the other case in opposition (§414).

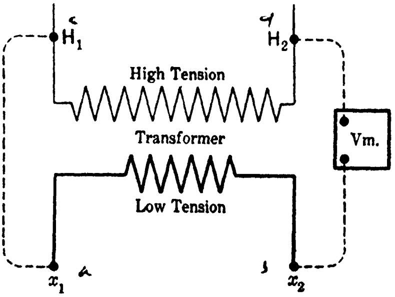

FIG. 275. Test for polarity.

(5) Test the transformer for polarity as follows: Referring to Fig. 275, join H_1 and X_1, thus connecting the primary and secondary windings together on one side. Apply the normal low-tension voltage to the high-tension winding and read the voltage between the remaining points, H_2 and X_2. If this voltage is less than the applied voltage, the transformer has *subtractive polarity*. If the resultant voltage is more than the voltage H_1H_2 the polarity is *additive*

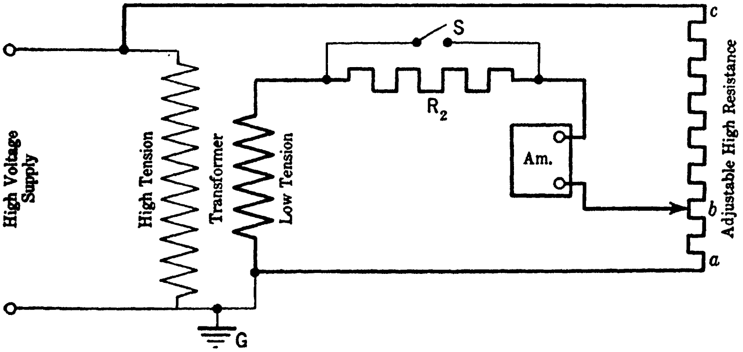

FIG. 276. Voltage ratio using a sensitive ammeter (or voltmeter) and balancing resistance.

(6) Determine the voltage ratio of a high-voltage transformer by the method of Fig. 276. The resistance *ac* must be adjustable in fine increments and be capable of withstanding full high-tension voltage without overheating. Note that the windings of the transformer must be connected in the subtractive sense. Choose the resistance R_2 such that if

subject to full low-tension voltage the resultant current will be within the range of the ammeter. In place of the ammeter and protective resistance a voltmeter with external multiplier may be used. Apply normal voltage and frequency to either winding. Start with the movable contact b at point a and switch S open. Then move b toward c until the ammeter reading approaches zero. Close switch S and readjust b if need be to bring the meter reading to a minimum again. The ratio of transformation will then be

$$N_1/N_2 = \frac{R_{ac}}{R_{ab}}$$

Report. (a) By means of curves or a table show the effect of the applied voltage and of the frequency upon the ratio of transformation. (b) Show that eq. (2) is nearly correct for most of the readings, and discuss the departures, if any. (c) Plot curves showing how the magnetizing current varies with the applied voltage, the frequency, and the number of primary turns. Explain these variations by the theory of an iron core excited with alternating current (§231). (d) From the experiment with two secondary coils, describe what is meant by their " polarity," and explain how transformer coils may be connected in series and in parallel. (e) Give the results of test (6) above and comment upon the accuracy possible by this method. State what other devices may be used in place of the ammeter. Should it be possible to bring a sensitive indicator to zero with these conditions? Why?

391. Ratio of Currents on Short Circuit. — Consider again the same transformer (Fig. 270) connected on the primary side to a source of comparatively low alternating voltage (not over 5 per cent of the rated voltage), and on the secondary side short-circuited through an ammeter of very low resistance. It is found experimentally that, under such conditions, with a core of low reluctance and with a winding of low-leakage reactance (§412), the currents, I_1 and I_2, in the primary and secondary circuits very closely satisfy the condition

$$I_1 n_1 = I_2 n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

both for the instantaneous and for the effective values of the currents; n_1 and n_2 are the corresponding numbers of turns. In other words: *In a short-circuited iron-core transformer, with low leakage, the currents are inversely as the numbers of turns in the two windings.*

This simple relationship follows directly from the fact that under such conditions the flux in the core is quite small. Thus, the applied voltage being low, the counter emf has to be low, and since this emf is induced by the flux in the core, the flux must be small. The secondary winding being short-circuited, even a small induced voltage causes a considerable current flow in it. This current would produce a large flux, were it not for the primary current which automatically adjusts itself to a value such that its mmf is almost equal and opposite to that in the secondary, because the resultant number of ampere-turns needed for a small flux is almost equal to zero; eq. (3) follows directly. The applied voltage must be kept low, so that the currents will not be excessive. This voltage is used to circulate the two currents, overcoming the ohmic resistance and the leakage reactance of the two windings.

One of the practical uses of the short-circuit test is to determine the equivalent resistance and reactance of the windings (§392). It is also used to check the design and the soundness of manufacture and assembly. Sometimes it is used to determine the temperature rise in the windings, although the results should be applied judiciously, because practically no heat is generated in the core on account of a low flux density. Therefore the core helps to conduct the heat away from the windings, and with rated currents in the windings the total loss in the transformer is below normal.

392. Equivalent Impedance, Resistance, and Reactance from Short-Circuit Test. — The secondary output on short circuit being equal to zero, and the core loss negligible, practically the whole primary input is expended in the I^2R loss in the windings. This loss could be computed from the resistances of the windings, but it is better measured directly on a wattmeter connected in the primary circuit. With transformer coils made of small-size conductors, the results by the two methods check within a few per cent, but with very large copper bars used for heavy currents, the actual copper loss is considerably higher than the computed copper loss. This is due to eddy currents and to a non-uniform distribution of alternating currents through the cross-section of the conductor (skin effect). This increase in the copper loss must be taken into account in computing the efficiency of a transformer.

A short-circuited transformer behaves like an impedance coil consisting of a resistance and a reactance in series (§143). While the energy is transmitted into the secondary circuit by induction and not by conduction, the result is the same as if the secondary winding were done away with and the impedance of the primary winding increased by a certain amount. This subject is considered theoretically in §409, but for the present the student may regard the foregoing statement and the following conclusion as experimental facts:

In a loaded transformer with a core of low reluctance, resistances and reactances can be transferred from one circuit to the other upon being multiplied by the square of the ratio of the numbers of turns. In other words, a resistance, r_1, in the primary circuit is equivalent to a resistance, r_2, in the secondary, if the two satisfy the condition

$$r_1/r_2 = (n_1/n_2)^2 \quad \ldots\ldots\ldots\ldots \quad (4)$$

A similar relationship holds for reactances, viz.,

$$x_1/x_2 = (n_1/n_2)^2 \quad \ldots\ldots\ldots\ldots \quad (5)$$

These equations are deduced in §409.

Because a short-circuited transformer behaves like an impedance coil with a very low flux density in the core, the current is proportional to the applied voltage. Moreover, one may speak of the *equivalent* impedance, reactance, and resistance of the transformer, in the sense in which these terms are applied to a choke coil. In practice, these three quantities are determined from a short-circuit test as follows:

(a) *The equivalent impedance,* $z_{\text{equiv.}}$ The effective value of the applied voltage E_1 on short circuit is divided by the corresponding current I_1, that is,

$$z_{\text{equiv.}} = E_1/I_1 \quad \ldots\ldots\ldots\ldots \quad (6)$$

(b) *The equivalent resistance,* $r_{\text{equiv.}}$ Let the wattmeter reading in the primary circuit be P watts, with the secondary short-circuited. The equivalent resistance is computed from the expression

$$P = I_1^2 r_{\text{equiv.}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

If a wattmeter test is not feasible, the resistances R_1 and R_2 of the windings are measured with direct current, and the equivalent resistance computed by means of formula (4), namely

$$r_{\text{equiv.}} = R_1 + R_2(n_1/n_2)^2 \quad \ldots\ldots\ldots \quad (8)$$

(c) *The equivalent reactance,* x_{equiv}. Knowing the equivalent impedance and resistance, one may determine the equivalent reactance either graphically or analytically, as for any choke coil (§143), that is, from the relationship

$$x_{\text{equiv.}}^2 = z_{\text{equiv.}}^2 - r_{\text{equiv.}}^2 \quad \ldots\ldots\ldots \quad (9)$$

393. EXPERIMENT 16-B. — Short-Circuit Test on a Transformer. —The purpose of the experiment is to check the ratio of the currents, and to determine the equivalent impedance, the true I^2r loss, and equivalent resistance of the transformer, as is explained in the preceding two sections. Connect a low-resistance ammeter across the secondary terminals, using as short and heavy leads as possible, and an ammeter in series with the primary winding. Use a low-range voltmeter across the primary terminals and connect a wattmeter to measure the input into the primary circuit (Fig. 270). Provide an arrangement for regulating the primary applied voltage, such as a rheostat or an auto-transformer (§402), in addition to which a separate alternator whose excitation may be varied at will is quite convenient.

(a) Apply a primary voltage sufficient to produce a maximum safe current in both windings of the transformer. Read both ammeters, the wattmeter, and the voltmeter. Reduce the voltage in steps, and take similar readings at each step. (b) Keeping the applied voltage constant vary the frequency of the supply from a value 20 per cent below to one 20 per cent above normal and read as in (a). (c) Insert in the secondary circuit a small non-inductive resistance, sufficient to affect the primary current and arrange to measure voltage drop across it. Take a set of readings as above. Cut out the resistance and insert an adjustable resistance in the primary circuit. Find the value of this resistance that will make the instrument readings the same as before with the same *total* voltage applied across primary plus resistance. Repeat with a few different values of resistance. (d) Perform a similar test with reactances. (e) If the ratio of turns is not known from the preceding experiment it may be estimated from the ratio of currents or determined by reading a few sets of no-load voltages. (f) Measure the resistances of both windings with direct current.

Report. (1) Plot the currents against the values of applied voltage as abscissas, and explain the reason why both curves are straight lines. (2) Show that the ratio of the currents is inversely as the ratio of the numbers of turns. (3) To the same abscissas plot the wattmeter readings, and also the computed total I^2R loss. Explain any considerable discrepancy between the two and show the effects of varying frequency. (4) Compute the values of the equivalent impedance, resistance, and reactance of the transformer. (5) Show from the data on the inserted resistances and reactances that eqs. (4) and (5) hold true.

394. Vector Diagram of an Ideal Transformer. — An ideal transformer is defined as one in which the resistances and the leakage reactances of both windings are equal to zero, and the reluctance of the magnetic circuit is so low that it takes a negligible mmf to excite the magnetic flux for normal operation. The vector diagram of such a transformer is shown in Fig. 277 and will be readily understood on the basis of its behavior on open circuit (§389) and on short circuit (§391). The particular ratio of turns selected in the figure is $n_1/n_2 = 2$. The diagram shows the following properties of an ideal transformer:

(a) The primary terminal voltage V_1 is equal and opposite to the primary induced emf, E_1, because of the assumption of zero impedance drop in the primary winding.

(b) The secondary terminal voltage V_2 is equal to the secondary induced voltage E_2, because by assumption the impedance drop in the secondary winding is zero.

(c) The induced voltages E_1 and E_2 are in the ratio of the respective numbers of turns and are in time phase with each other, being induced by the same flux.

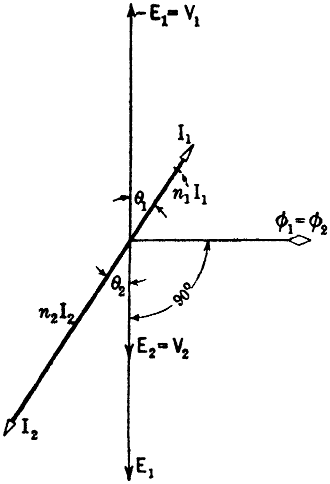

FIG. 277. The vector diagram of an ideal transformer.

(d) The primary and secondary mmf's, n_1I_1 and n_2I_2, are equal and opposite because the magnetic core by supposition requires a negligible magnetizing current. Hence, the currents are inversely as the numbers of turns, and are in phase opposition with each other.

(e) The induced voltages, E_1 and E_2, lag by 90 degrees behind the common flux Φ. This follows directly from the fundamental law of induction (§163). Let the flux vary with the time t according to the sine law, $\phi = \Phi \sin 2\pi ft$, where f is the frequency of the supply. The induced emf, e_0, per turn of either the primary or the secondary winding is equal to $-d\phi/dt$. Differentiating, we find $e_0 = -2\pi f\Phi \cos 2\pi ft$. Hence, the induced emf varies according to the cosine law, and passes through zero and maximum values one-quarter of a cycle later than the flux.

(f) The primary and the secondary power factors are equal to each other, because the two terminal voltages are in phase opposition, and so are the two currents. The corresponding phase angles are denoted in the figure by ϕ_1 and ϕ_2.

(g) The power input into the primary circuit is equal to the secondary output, no power being lost in an ideal transformer. For the currents we have $I_1n_1 = I_2n_2$, and for the voltages $E_1/n_1 = E_2/n_2$; besides, $\cos \phi_1 = \cos \phi_2$. Multiplying these three expressions, term by term, we get $E_1I_1 \cos \phi_1 = E_2I_2 \cos \phi_2$.

Figure 277 contains vectors of four different kinds of physical quantities, viz., voltages, currents, flux, and mmf's. Four different kinds of arrowheads are used to distinguish these quantities. This notation is more or less standard and facilitates the reading of a vector diagram.

In a good modern transformer, the impedance drop in either winding at full load amounts to but a small part of the induced voltage, usually 2 to 4 per cent. Also, the magnetizing ampere-turns constitute but a small proportion of the full-load ampere-turns of either winding, possibly 5 per cent. Hence, the diagram shown in Fig. 277 may be said to represent *approximately* the relations in an actual transformer, except at light loads. An exact vector diagram of an actual transformer is deduced in §407, but the foregoing diagram of an ideal transformer is sufficiently accurate for the solution of many practical problems, except those involving voltage drop and regulation.

395. Voltage Drop and Regulation. — The transformer windings possess a certain amount of resistance and of leakage reactance, both of which cause an internal voltage drop. Thus, with a constant applied primary voltage, the secondary terminal voltage depends upon the value of the load current and upon the power factor of the load. As the load fluctuates, the secondary voltage also varies. To maintain satisfactory service these fluctuations must not exceed a certain limit, and for this reason the determination of the voltage regulation of a transformer is of great practical importance.

For example, let a transformer have a ratio of turns of twenty to one. When the primary winding is connected to a 2200-volt supply the secondary voltage at no load is 110. Suppose that, at normal load, the voltage drop in the primary winding due to its impedance is such that when subtracted vectorially from 2200 volts it leaves 2160 volts to be transmitted into the secondary. With the ratio of transformation of twenty to one, the secondary induced voltage will be 108 volts. From this value the secondary voltage drop has to be subtracted vectorially, leaving, say, 106 volts at the secondary terminals. Thus, as the load varies between zero and its rated value, the secondary terminal voltage fluctuates between 110 and 106 volts. This is probably as wide a range as is permissible, and a wider range would not give satisfactory service to users of incandescent lamps.

According to methods of calculation adopted by the American Institute of Electrical Engineers, the *percentage of voltage regulation* of the foregoing transformer is

$$(110 - 106)/110 = 3.6 \text{ per cent}$$

This is the percentage ratio of the drop in voltage occurring between no load and the given load, to the no-load voltage.

The voltage variation on the secondary side, with a constant primary voltage, either may be determined by the *direct method,* i.e., by actually loading the transformer, or it may be calculated from the measured resistances and reactances of the windings. The second, or *indirect method,* is always preferred in practice, because it gives more accurate results and involves the expenditure of but a small amount of energy. This method is described in §§409 and 410. However, in order that the student may have experience in the actual operation of the transformer under normal conditions and so be able to judge regarding the magnitude of voltage variations, he should be given the opportunity of determining regulation by actually loading a transformer on non-inductive and inductive loads, as described in the following experiments. It is advisable that he determine the regulation of the same transformer by both direct and indirect methods in order that he may judge of their comparative merits.

396. EXPERIMENT 16-C. — Load Tests of a Single Transformer. — The purpose of this experiment is to determine the efficiency and regulation of a given transformer by actual measurement, and to note how these quantities are affected by load and power factor. This experiment is intended to illustrate the general properties of a transformer, whereas, for more accurate values, the relatively simple and more accurate short-circuit test is used to obtain needed values so that efficiency and regulation may be calculated (§409).

Connections for the test are shown in Fig. 270, except that potential and current transformers should be used in the high-tension supply if the voltage is much in excess of 220 volts (see §§421 and 428). The difference between the values of secondary voltage at full load and at no load is small, so that a transformer of comparatively poor regulation should be chosen for this experiment, and, in any case, the primary and secondary voltages and power should be read with great care. The determination of efficiency by a comparison of output and input is subject to considerable error since any inaccuracy in either quantity causes a relatively greater error in the difference which represents the loss.

Arrange to hold impressed voltage and frequency constant, by adjusting the excitation and speed of the alternator which is used as the supply. Record readings somewhat as suggested in the following table.

Tests should be made as follows:

A. *Unity power factor, varying load.* Begin with the highest safe current, say 50 per cent above rated value, and with the reactor of Fig. 270 disconnected. Vary the load current in 8 or 10 steps from maximum value to zero. Take all primary and secondary readings. See that values are consistent before proceeding, i.e., that the load power factor is 100 per cent by actual instrument readings, that efficiency values are reasonable, and that voltage drop increases normally with increase in load.

B. *Constant current, varying power factor.* Hold the load current constant at, say, rated value, and with the impressed voltage and frequency constant as before, vary the power factor of the load from a

minimum, using the reactor only, to unity, in steps of approximately 10 per cent.

TRANSFORMER LOAD TEST

	Conditions: ______ Constant, __________ Variable									
	Primary				Secondary			Calculations		
	Amp.	Volts	Watts	Freq.	Amp.	Volts	Watts	Effic.	Regul.	P.F. (Load)
Instr. No....										
Constant....										
Remarks.....										

This gradation may be secured closely enough by each time disconnecting the reactor and for, say, 40 per cent power factor, adjusting the value of resistance so that the output current is 40 per cent of the desired total, and then connecting the reactor and adjusting it so that the total current has the required value. A power-factor meter will aid materially in arriving at the desired values but must be checked by the readings of the other instruments.

Take all instrument readings as before and again scrutinize them for irregular or inconsistent values before proceeding. Carry the test into the region of leading currents, if possible.

C. *Constant power factor, varying current.* Holding power factor constant, at, say, 80 per cent, vary the load current from the maximum safe value to near zero, maintaining impressed voltage and frequency constant. Note whether the readings of the power-factor meter are reliable over the whole current range, as indicated by comparing with the values calculated from amperes, volts, and watts.

Report. 1. For Run A plot primary and secondary volts and watts, also calculated efficiency, voltage ratio and regulation, against load amperes. Compare the values of regulation and efficiency at rated load with the expected values. Comment upon the reliability and accuracy of the results. Explain the shape of the efficiency curve.

2. For Run B plot primary volts, secondary volts, regulation, efficiency, primary watts, W_1, secondary watts, W_2, and $(W_1 - W_2)$ against power factor.

Explain the shape of the regulation vs. power-factor curve. Why is not the highest value of regulation at the lowest power factor? Discuss variations in $(W_1 - W_2)$. Should this value vary?

3. For Run C proceed as for A and compare the results obtained on the two runs, especially as to regulation and efficiency. How do you explain the apparent errors in the readings of the power-factor meter?

397. EXPERIMENT 16-D. — Load Tests of Two Transformers in Series. — This experiment is intended to replace Experiment 16-C when the required apparatus is available, namely, two identical transformers, connected as in Fig. 278. The method is especially desirable when the ratio of transformation of the transformers is high. Thus, the power is supplied to transformer 1 at a low voltage and is stepped up in that and down in the other transformer. Therefore the load devices and all meters are in the low-voltage circuits. By using double-throw switches or a polyphase board the same meters are used in the supply and the load circuits, practically eliminating instrument errors. Moreover, when two transformers are used in series the difference between input to No. 1 and output from No. 2 is twice what it is in one transformer, at the same load, and is more accurately determined. Efficiency may be calculated as (output of No. 2/input to No. 1)$^{1/2}$; regulation = (supply volts − load volts)/(2 × supply volts). In general the same runs should be made as in Experiment 16-C and the same report requirements met.

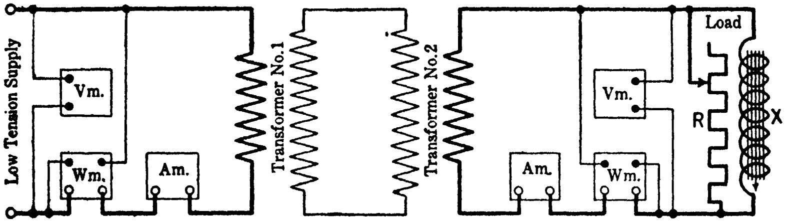

FIG. 278. Load test of two identical transformers connected in series.

398. Two Transformers Connected in Opposition. — Sometimes a transformer must be tested under full-load or overload conditions, for example in order to determine its temperature rise. A small transformer may be loaded on resistances or reactances without much difficulty, but with a large transformer, or with one wound for an extremely high voltage, such a test may be difficult and expensive. When two similar transformers are available, one transformer can be loaded on the other (Fig. 279) so that the load energy circulates between the two. This is the so-called " loading-back " method, also known as the " stray-power " or " opposition " method. It is used in testing generators and motors (Chapter XIV), as well as transformers. By its use no power is wasted except that necessary for supplying the losses in both transformers and in such auxiliary equipment as may be required.

In Fig. 279, *A* and *B* are two transformers under test. Their low-tension windings are connected in parallel to a power supply, and the high-tension windings are connected *in opposition* to each other. As the transformers are identical, the induced voltages will balance so that no current will flow through the secondaries, and but a small magnetizing current through the low-tension windings. At the same time, full iron loss takes place in the cores of the transformers since they are subjected to the full rated voltage. Now, if full-load current be made to circulate in the windings, the transformers will be under the same conditions in regard to the losses and voltage regulation as under actual full load. A convenient method of circulating a load current is to open one of the circuits, say the low-tension, and to introduce a source of alternating current at a comparatively low voltage. This auxiliary source is shown in the figure as a transformer *C*. By regulating the voltage of this source, full-load current can be produced in the primaries of the transformers under test; this current causes full-load current to flow in the secondaries, which therefore need not be opened. This is particularly convenient when the secondary coils are wound for a high voltage.

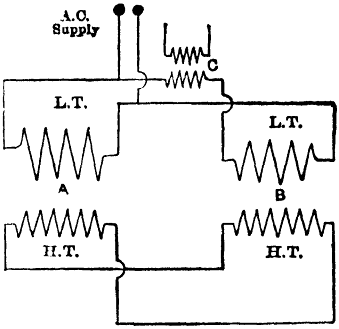

FIG. 279. Two transformers connected in opposition.

In this test the transformers are under actual load conditions as far as the losses and heating are concerned; at the same time, the energy supplied from outside is merely sufficient to cover the losses. For example, with two 1000-kw transformers of a full-load efficiency of about 98 per cent, the total expenditure of power is only about 40 kw. When but one transformer is available, the same test can sometimes be performed by connecting the two halves of each winding in opposition.

When an opposition test is performed for the determination of the temperature rise only (§358), the frequency (within reasonable limits) of the circulating current as well as its phase relation with reference to the primary voltage will not affect the result; therefore they may have any convenient values. In fact, if no suitable alternating current is available, both the high-tension and the low-tension circuits may be entered and sources of direct current, sufficient to produce the desired currents in the windings, may be introduced. They must, however, be of such values and flow in such directions in the primary and secondary windings that the magnetizing effects are equal and opposite, otherwise there is danger that the cores will be saturated and abnormal exciting current be drawn from the a-c supply. As long as the I^2R and core losses have the required values the final temperature of the transformers will be the same as under actual load conditions. See §392, however, in regard to the difference between the true ohmic and the effective resistance, and the corresponding difference between the computed and measured copper loss.

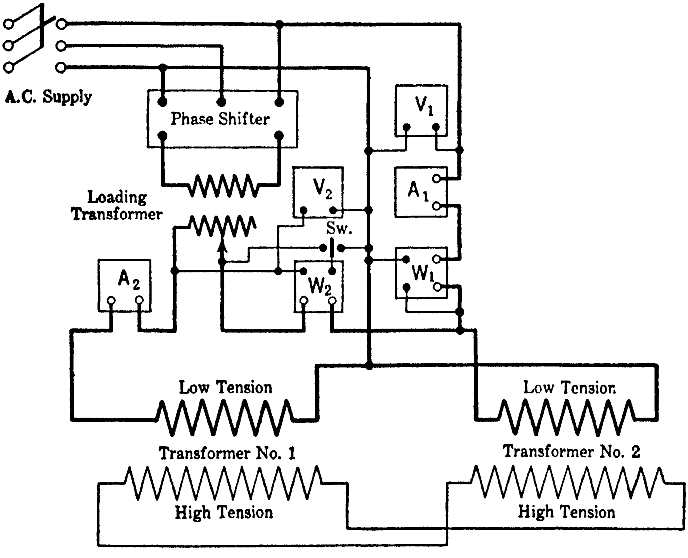

FIG. 280. Detailed connections for an opposition test on two transformers.

When the opposition test is used for a determination of the voltage regulation, the current furnished by the auxiliary source C not only must be of the correct frequency, but also must have the proper phase displacement with respect to the terminal voltage of one of the transformers. In this case, either the transformer C should be in the form of a phase-shifter or some form of phase-shifting device (resistance to raise, or reactance to lower, the power factor) should be inserted in the line to C. The proper phase position of the current is found by means of an ammeter a voltmeter, and a wattmeter (or a power-factor meter) connected with its current coil in the circuit of the two transformers and its potential coil across one primary winding (see Fig. 280). One transformer delivers power to the other, so that in one of them the phase angle between the secondary terminal voltage and the secondary current is less than 90 degrees, and in the other it is greater than 90 degrees.

399. EXPERIMENT 16-E. — Opposition Test on Two Transformers. — The purpose of this experiment is to perform a load test upon two comparatively large transformers with no expenditure of energy except that required by the losses which may be supplied from a limited source (§398). This method of loading may be used for any purpose for which a transformer has to be tested under full-load conditions. In practice it is widely used for heat runs. The connections are shown, simplified, in Fig. 279, and in more detail in Fig. 280. In this particular experiment it it desired to determine the voltage regulation and the efficiency, as well as the temperature rise. A voltmeter, using a potential transformer if necessary, may be connected across the secondary circuit, or regulation may be determined from the difference between the terminal voltages of the two primary (low-tension) windings. Ammeters, voltmeters, and wattmeters should be used in the primary circuit to read the following quantities: the voltage at the terminals of the two primaries, that furnished by the booster transformer, magnetizing and circulating currents, the power input from the source *C*, the core loss, and the wattmeter reading due to the circulating current and the voltage across the primary of one transformer.

(1) *Voltage regulation.* The circulating current must be of the same frequency as that of the applied voltage; the phase of the circulating current must be adjusted to correspond to the desired power factors, in order that it may be possible to determine the regulation at these power factors. Read the two primary voltages when the desired current circulates between the transformers, and then read the same voltages at no load. The percentage of regulation is computed according to the definition in §395, assuming that the total difference in the primary voltages is twice the voltage drop in one transformer. Take readings with the current lagging, leading, and in phase with the voltage.

(2) *Efficiency.* The data taken in (1) above permit also the calculation of the efficiency of the transformers, the wattmeter W_1 reading the combined core loss, and W_2, with switch *Sw* thrown to the left, reading the copper loss of the two transformers, the power input to one transformer having also been noted. The approximate value of per cent loss of one transformer is then found from the sum of the loss readings divided by twice the input to one transformer.

(3) *Heat run.* Measure very carefully the resistance of the windings while the transformer is cool, determining its temperature by properly placed thermometers. *Observe the precautions* detailed in §401 when measuring the resistance of these and other highly inductive circuits. Then put a safe overload on the transformers, both in current and in voltage, to produce an appreciable temperature rise, say, within an hour. This short run is permissible when the purpose of the run is to obtain experience with the method rather than to determine the ultimate temperature rise (which requires several hours). If time permits, in addition to circulating a current from the same source as the applied voltage, try a current from a different source and of a different frequency, and also a direct current. In the case of direct current, make sure that the mmf's of the primary and secondary windings are equal and opposite. Try a reversed or a wrong direct current in one of the windings and observe the effect on the core loss and on the magnetizing current. At the end of the run, measure the resistances of the windings again, and determine the temperature rise from the increase in resistance. Thus,

$$T'' - T' = \frac{R'' - R'}{R'} (234.5 + T')$$

where R' is the resistance at the beginning of the test when the temperature T' was noted. R'' and T'' are the final values (see §9, eq. 13).

Report. (1) Give the results of the voltage regulation run, and show that they check qualitatively with the theory given in §§394 and 395. Compare the accuracy of this method with that of determining the voltage regulation from the load tests.

(2) Describe the arrangement whereby the phase of the circulating current was altered and measured. Give the efficiency computations, and compare the accuracy of this method with that to be expected from computing the efficiency from the losses (§400).

(3) Give the results of the heat run, and explain how the current was circulated between the transformers. Describe and explain what happened when a wrong direct current was circulated through one of the windings.

400. Efficiency from Losses. — By definition, the efficiency of a transformer is the percentage ratio of its net power output to its power input. Let the output of a transformer be 1200 kw and the corresponding input 1223 kw. Then the efficiency at this particular load is 1200/1223 = 0.981 or 98.1 per cent. In this particular example the losses amount to 23/1223 = 0.0187 or 1.87 per cent of the input. Generally speaking, a greater accuracy of computation will result if the per cent loss is first calculated and then subtracted from 100 to get the efficiency.

It may seem at first that the simplest method of determining the efficiency of a transformer would be to have it actually loaded and to read the output and the input by means of wattmeters in the primary and in the secondary circuits; their ratio would give directly the efficiency of the transformer. However, this direct method has serious drawbacks and is hardly ever used in practice. In the test of a single transformer it involves the use of a wattmeter on the high-tension side, and requires the utmost accuracy in calibrating and in reading the wattmeters, because the values to be read differ by but a few per cent. When two identical transformers are loaded in series the method is wasteful of energy and is of doubtful accuracy. Moreover, with large transformers, the amount of power required for a full-load test is not always available. Therefore, it is customary to measure the losses in a transformer separately (at no load and on short circuit), and then to *calculate* the efficiency. This can be done by much simpler means and with a considerably greater accuracy than is possible with the direct method.

Transformer losses. The losses in a transformer consists of the I^2R or copper losses in both windings, and of the iron loss or *core loss* (hysteresis and eddy currents). The copper loss depends only on the load, and can be easily calculated for any load if the effective resistances of both windings are known. Thus, from eqs. (7) and (8) in §392

$$P = I_1^2 r_{\text{equiv.}} = I_1^2\left[R_1 + R_2\left(\frac{N_1}{N_2}\right)^2\right] \quad . \ . \ . \ . \ . \quad (10)$$

These resistances are usually measured with direct current, by means of a Wheatstone bridge or by the drop-of-potential method. With conductors of considerable cross-section, the eddy currents and a non-uniform distribution of current over the cross-section may cause the actual I^2R loss to be considerably higher than that figured out from the d-c resistance measurement. In this case it is preferable to measure the actual copper loss with a wattmeter on short circuit (§393). The iron loss depends on the magnitude of the magnetic flux and is practically independent of the load. This follows from the fact that with a constant impressed voltage the flux is also nearly constant (neglecting a small primary drop). Therefore the iron loss can be determined at no load and assumed to be the same at all loads.

As an illustration of the calculations involved, let it be required to determine the efficiency of a 2300- to 115-volt, 5-kva transformer, at three-quarters load, unity power factor. Let the resistances of the windings, corrected to an operating temperature of, say, 75° C (see §7), be 12.5 ohms and 0.032 ohm respectively, and let the iron loss, determined by a wattmeter at no load, equal 60 watts. At three-quarters of the rated load the output of the transformer is 3.75 kva, or 32.6 amperes at 115 volts. The primary current is equal to 1/20th of this, or 1.63 amperes; thus we have (see eq. 10)

$$\begin{array}{lcl} \text{Total copper loss} = \overline{1.63}^2 \times (12.5 + 0.032 \times \overline{20}^2) & = & 67.2 \text{ watts} \\ \text{Iron loss} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & = & 60 \quad \text{watts} \\ \hline \text{Total loss at } 3/4 \text{ load} \ldots\ldots\ldots\ldots\ldots\ldots & = & 127.2 \text{ watts} \end{array}$$

$$\text{The per cent loss} = \frac{127.2 \times 10^2}{3750 + 127.2} = 3.28 \text{ per cent}$$

The efficiency, therefore, = 100 − 3.28 = 96.7 per cent. In this way, values may be obtained for any desired load, and an efficiency curve plotted from no load up to the maximum feasible overload

401. EXPERIMENT 16-F. — Efficiency of a Transformer from Its Losses. — The experiment comprises three distinct measurements:

(1) Ratio of transformation.
(2) Resistances of the windings.
(3) Core loss.

The ratio of transformation is determined as in §390. The resistances of the windings are measured by direct current, usually by the drop-of-potential method. As the resistance of the low-tension winding is much smaller than that of the high-tension winding, it may be necessary to use different ammeters and voltmeters. *Always remove the voltmeter leads before opening or closing the d-c circuit,* especially on the high-tension side, and keep clear of the terminals, since the inductive " kick " which accompanies the sudden rise or fall of the current *produces a high voltage* which would damage the voltmeter and *might endanger the life of the operator.* As a check on the resistance measurements determine the I^2R

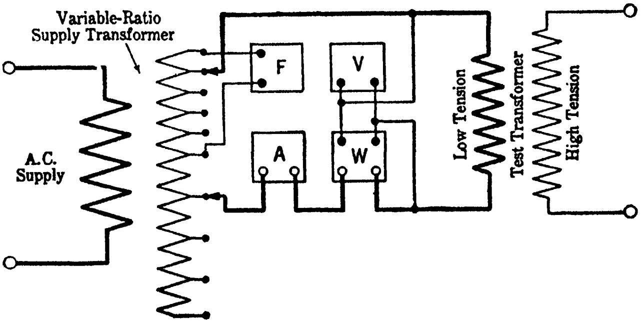

FIG. 281. Connections for making a core-loss test on a transformer.

loss with a wattmeter, as in §393. In each case determine the temperature at which the measurements were taken.

Secure a source of alternating emf, substantially sine-form. Arrange for convenient adjustment of voltage and frequency. Connect instruments and equipment as in Fig. 281, where a transformer is shown with a large number of taps so that a wide range of voltage will be possible. Note that power is applied to the low-tension winding of the transformer. The core loss is measured by a wattmeter exactly as in the Epstein test, §226. Measurements in the low-tension circuit are quite sufficient since 110 volts applied to the low-tension side of a 10 : 1 transformer will produce the same flux and give the same core loss as 1100 volts applied to the high-tension side.

Remember that high-tension terminals become alive the moment an a-c voltage is applied to the low-tension winding; in order to avoid the possibility of a fatal mistake, tape up the high-tension terminals before starting the test. Note that with connections as in Fig. 281 the readings of the wattmeter will include the losses in the voltmeter and in the potential circuit of the wattmeter itself, unless the latter is compensated for this loss. To determine whether this is so, disconnect both the voltmeter and the low-tension winding of the transformer and, with a moderate voltage applied across the low-tension leads, note whether there is a reading of the wattmeter. If there is the meter is not compensated and correction must be made as stated, i.e., by subtracting the potential circuit loss = E^2/R, where E is the voltage across the potential circuit of the meter, and R is its resistance. Similar correction must be made for the voltmeter if it is in circuit when the wattmeter readings are taken. Proceed with the conduct of the test as follows:

A. Holding the supply frequency constant at normal value vary the applied voltage at the terminals of the transformer in 10 or more nearly equal steps, from the minimum at which dependable readings are obtainable to a value at which the exciting current of the transformer approaches the rated load current, thus indicating saturation of the core. Read volts, amperes, and watts at each point.

B. Starting with normal volts and frequency vary the voltage and frequency always in the same proportion, in order to study the effect of frequency while the flux density is constant. Cover at least the range from one-half normal frequency to 25 per cent above normal. Read as in A.

C. Holding the applied voltage constant in value, vary the frequency over as wide a range as possible and take all readings.

Report. (1) State the ratio of transformation and the resistances of the windings.

(2) Check the calculated I^2R loss with that measured with the wattmeter, and use the higher value in subsequent calculations. Plot curves of core loss, corrected if necessary for the I^2R loss due to the exciting current, as follows: (a) Core loss and exciting current against voltage, frequency constant; (b) core loss and exciting current against frequency, voltage constant; and (c) core loss and exciting current against frequency, flux density constant. From the first of these curves pick the core loss of the transformer at normal voltage and frequency. What per cent is this of the rating of the transformer? What per cent is the exciting current of the rated load current?

(3) Following the method of §230 separate the core loss at normal voltage and frequency into its components.

(4) To kilowatts output at unity power factor as abscissas, plot I^2R loss, core loss, total loss, and efficiency, all on the same graph sheet. Show that the efficiency is a maximum, or the per cent loss a minimum, when the two losses are equal. On the same sheet indicate by crosses or circles values of efficiency at several loads, when the supply voltage is 10 per cent above normal; below normal. Select these values in the range of maximum efficiency under normal conditions. Similarly indicate the general effects of variations in frequency of the supply upon the losses, efficiency, and rating of the transformer. At constant applied voltage does the core loss increase or decrease with decrease of frequency? Why?

402. The Auto-Transformer. — An auto-transformer, or single-winding transformer, is shown in Fig. 282. The primary, or line, voltage is applied between the terminals A and B, and power is taken to the load off the terminals B and C of the same winding. The two parts of the winding are mounted on the same iron core, and are in the closest possible magnetic interlinkage with one another. The currents in the parts AC and BC of the winding may be quite different from each other, and for this reason, the corresponding coils are made of conductors of different cross-section, in order to save weight and space.

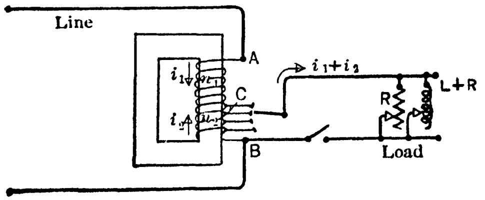

FIG. 282. An auto-transformer connected to a load.

Consider first the conditions at no load. The applied voltage causes a magnetizing current to flow through the winding, thus producing an alternating flux in the core. This flux induces, between the terminals A and B, a counter emf practically equal and opposite to the applied voltage. The drop of potential is uniformly distributed over the winding so that any desired fraction of the total voltage may be taken between one of the terminals and an intermediate point, or between two intermediate points. Thus, for example, if the total number of turns is 100 and the applied voltage is 1000, the voltage per turn is 10 volts. Singling out 40 consecutive turns anywhere in the winding will give a secondary emf of 400 volts. It saves an extra terminal to use one of the primary terminals, such as B, for the secondary load also. Different secondary voltages are obtained by using leads to different turns, or different taps, as they are sometimes called.

Now let the load switch be closed and let a considerable current flow in the secondary circuit. With a closed magnetic core and moderate saturation in iron, the necessary magnetizing ampere-turns are negligible as compared to the primary or secondary ampere-turns. The latter two are therefore practically equal to each other, and modifying the notation in Fig. 282 to represent effective, rather than instantaneous values of currents, we have

$$n_1 I_1 = n_2 I_2 \qquad \text{. . . . . (11)}$$

Here n_1 and n_2 are the numbers of turns in the parts AC and CB of the winding respectively, and the difference in the meaning of n_1 here and in a two-winding transformer (§391) should be carefully noted.

According to the first Kirchhoff law, the load current is equal to the sum of the currents in the transformer itself, or

$$I_l = I_1 + I_2 \qquad \text{. . . . . (12)}$$

The fundamental voltage relationship expressed by eq. (2) becomes

$$E_1/E_2 = (n_1 + n_2)/n_2 \qquad \text{. . . . . (13)}$$

where E_1 is the line voltage and E_2 is the load voltage. Equations (11), (12), and (13) comprise the principal relationships in an auto-transformer when the magnetizing current and the internal voltage drop within the transformer itself can be neglected. Under these simplified conditions, the vector diagram is identical with Fig. 277, although I_2 and n_1 have a different meaning. The load current may be added to the diagram by drawing a vector equal to $I_1 + I_2$ in the direction of I_2.

In the foregoing example, the part AC consists of 60 turns and BC of 40 turns. If the load current is, for example, 50 amperes, then 20 amperes are supplied from the line and 30 amperes by the secondary winding. Under these conditions the primary and the secondary mmf's are equal and opposite, since 20 × 60 = 30 × 40. The power furnished from the line is equal to that consumed in the load because 1000 × 20 =

400 × 50, the assumption being that no energy is lost in the transformer itself.

Theoretically, an auto-transformer requires less copper than an ordinary two-winding transformer, as may be seen from the following estimate. Let the amount of copper required per ampere-turn in the foregoing auto-transformer be A. The total amount of copper then is

$$(20 \times 60 + 30 \times 40)A = 2400\ A$$

With an ordinary transformer the corresponding amount of copper would be

$$(20 \times 100 + 50 \times 40)A = 4000\ A$$

Thus, in this particular case, over one-third of the amount of copper can be saved by using windings connected *conductively*, instead of *inductively*.

Auto-transformers are widely used for starting induction motors (§487), for voltage regulation, and for other purposes where the difference between the primary and the secondary voltage is not too great. A disadvantage of the auto-transformer is that it is impossible to insulate the high-tension coils from the low-tension circuit. This is inadmissible in many cases on account of the danger to life or property. For an application of the theory of the ordinary transformer to the auto-transformer, see the last paragraph in §412.

403. EXPERIMENT 16-G. — Load Test on an Auto-Transformer. — The purpose of the experiment is to illustrate the relations stated in the preceding article. The experiment is performed in a manner similar to Experiment 16-C above. Use an auto-transformer with regulating taps (Fig. 282), or one of the windings of an ordinary transformer, if it is provided with taps. An auto-transformer may also be improvised by connecting the two windings of an ordinary transformer in series.

(a) Determine the ratio of the voltages at no load. Connect the two outside points of the windings to a source of supply and measure the voltages between the various taps. Also apply a voltage across a part of the winding, and read the resulting higher voltage across the total winding. In so doing, be careful to have enough turns in the part of the winding connected across the line, so as not to draw an excessive current.

(b) Determine the ratio of the currents on short circuit (§393). Read the currents I_1, I_2, and $I_1 + I_2$ shown in Fig. 282, also the corresponding line voltage and the power input.

(c) Choose a suitable tap and load the transformer as in Fig. 282. Have ammeters connected so as to read the primary, the secondary, and the load current. There should also be a voltmeter and a wattmeter in each circuit. One voltmeter, one ammeter, and one wattmeter, in connection with a polyphase board (§60), are usually sufficient for the purpose. Begin with a maximum permissible load, and reduce it in steps to zero. At each setting read all the currents, voltages, and watts.

(d) Repeat the same experiment with a few other taps, taken nearer to A or to B. In the latter case, be careful not to overload the part BC of the winding, since the current I_2 increases inversely as the number of turns.

(e) Before leaving the laboratory, ascertain the number of turns between the various taps. Measure the resistances of the windings and the core loss.

Report. (1) Draw a diagram of connections in the auto-transformer and mark on it per cent voltage at different taps. (2) Plot I_1 and I_2 to load amperes as abscissas, and show that the conditions (11) and (12) are approximately fulfilled. (3) Plot curves of secondary voltage against load as abscissas, to bring out the influence of the power factor. Explain the observed differences in per cent regulation with different taps. (4) Compute the efficiency from the losses and check with that obtained from the wattmeter readings. (5) Analyze the results of the short-circuit test; check the secondary voltage measured in the load test with that computed from the transformer impedance (§411)

CHAPTER XVII

THE TRANSFORMER (*Continued*)

404. Magnetizing Current. — When the secondary or load circuit of a transformer is open, the primary current serves merely to maintain the transformer flux and is therefore called the magnetizing or exciting current. In usual power transformers it amounts to but 3 to 5 per cent of the full-load current, and is therefore neglected in the approximate diagram shown in Fig. 277. However, the value of the magnetizing current is of importance for some purposes. For example, the designer checks his computations by comparing the actual and the computed values, and indirectly checks the quality of iron and assembly. The operating engineer desires to keep the magnetizing current to a minimum, because during the periods of small power demand the magnetizing currents of the large number of transformers on the system constitute an appreciable load of very low power factor.

With the secondary circuit open, a transformer becomes identical with the Epstein apparatus (§226), and the theory given there applies here also. The following experiment on magnetizing current is intended to be an application of the theory, and should be performed accurately and intelligently so as to enable the student to compare his results with theoretical conclusions.

The experiment may be performed using either winding, whichever gives more convenient values of currents and voltages. The reader is again warned that as soon as a comparatively low voltage is applied to the low-tension winding a dangerous difference of potential may exist between the high-tension terminals. The initial rush of magnetizing current depends upon the state in which the iron was left by the preceding application of voltage, and upon the part of the voltage wave at which the circuit is closed. It is quite possible to obtain an initial inrush of magnetizing current of the order of magnitude of full-load current or greater, which requires that meters be protected during this period.

The vectorial relations at no load may be understood by reference to Figs. 277 and 283. The primary impedance drop at no load is usually so small that it can be neglected altogether. Therefore $V_1 = -E_1$, and the flux vector Φ is perpendicular to both. Assuming low saturation in iron, no hysteresis, and no eddy currents, the magnetizing ampere-turns, I_0n_1, are in phase with Φ. Since the magnetizing current, I_0, flows through the primary winding, its mmf corresponds to n_1 turns, and, with an ideal core, is in phase with the flux and in a lagging phase quadrature with the primary applied voltage V_1.

Because of saturation and hysteresis in iron, the magnetizing current is not sinusoidal even when the flux and the applied voltage vary according to the sine law. In this case the actual magnetizing current is *approximately* replaced by an *equivalent sine-wave current* so that a vector diagram may be used. Because of the power loss in hysteresis and eddy currents the vector I_0 of the equivalent magnetizing current is not in phase with the flux but has an energy component in phase with the applied voltage V_1, and leads the flux by the angle α.

Should the applied voltage be non-sinusoidal, the wave-form of the magnetizing current may still further depart from the sine law. The flux and the voltage are connected by the relationship $e = -N\frac{d\phi}{dt}$; the flux and the magnetizing current are connected by the saturation curve of iron. A peaked flux wave means higher flux densities, a larger magnetizing current, and a greater core loss, and should therefore be avoided.

405. EXPERIMENT 17-A. — Magnetizing Current in a Transformer. — The purpose of the experiment is to make a further study of the magnetizing current of the transformer as a function of the number of turns, applied voltage, its frequency, and wave-form. The transformer is wired up as in Fig. 270 but with the wattmeter omitted. The secondary terminals are left open and should be taped up. (1) To observe the transient current, i.e., the current rush upon connecting the transformer to the supply, replace the low-reading ammeter by one having a full-scale deflection of at least twice the full-load current of the transformer, and then try closing and opening the switch which controls the supply, several times. A better idea of the nature of the transient current may be had from oscillograms taken during this operation, provided the required equipment is available. Try the above first at rated voltage and frequency, and then at about 25 per cent above normal voltage. Study the effect of direct current, used in measuring the resistance of the windings, upon this current rush.

(2) To study factors affecting the exciting current of the transformer replace the larger ammeter used in (1) by a low-reading meter, protected by a short-circuiting switch when the transformer is thrown onto the supply. Begin with the highest voltage to which the transformer may be safely subjected, and read amperes, volts, and frequency, which

latter should be held constant. Reduce the applied voltage in steps, and take similar readings at each step.

(3) If an alternator driven by a variable-speed motor is available make similar runs at different frequencies.

(4) To investigate the effect of the number of turns, take a few readings at a constant voltage and frequency, using different taps. The two halves of the winding can also be used in series and singly.

(5) Before leaving the laboratory, determine the number of turns in the winding with which the experiment was performed, the dimensions of the iron core, and the resistance of the winding.

(6) If an oscillograph is available, a few characteristic curves of magnetizing current should be taken to show the steady as well as the transient conditions.

(7) If an alternator is available, in which the wave-form of the induced voltage may be varied at will, investigate the effect of a flat-topped and of a peaked voltage-wave upon the shape and magnitude of magnetizing current. An ammeter will show the difference in the effective values and an oscillograph may be used to ascertain the actual wave-form.

Report. (1) Correct the voltmeter readings for the effect of the ohmic drop and copper loss, as in §227, for the readings for which this correction is appreciable. (2) Plot amperes against the corrected voltages as abscissas, at different frequencies. (3) From the data taken with different numbers of turns, show that the general character of variation of the magnetizing current with the frequency, number of turns, and voltage, is in accordance with the theory of the transformer. Pick out the data for which the ratio of the voltage to the frequency is the same. Show that for such data the magnetizing current is approximately equal, and give a theoretical reason. (4) From the known dimensions of the core and number of turns, compute the actual flux densities and magnetizing volt-amperes per unit volume of iron, for the extreme conditions obtained during the test (§231). (5) Give the results of the test on the initial rush of current, and show theoretically that the initial value of magnetizing current may be large when the residual magnetism is in the wrong direction. (6) Give the results of the oscillograph test and of the runs in which the wave-form of the applied voltage was varied. Explain your findings and give theoretical reasons for the results.

406. Magnetic Leakage in the Transformer. — In the first vector diagram of the transformer (§394) a perfect magnetic coupling between the primary and the secondary windings was assumed. In reality there is a space between the two (Figs. 271, 272). Neglecting the magnetizing current, the primary and secondary currents are in phase opposition, and the instantaneous directions of the currents are shown in the usual way by dots and crosses. It will thus be seen that each half of the primary coil, with the adjoining half of the secondary coil, forms a fictitious coil whose mmf is in the opposite direction from that of the next fictitious coil. These combined mmf's produce the leakage fluxes, as shown in the figures, linked either with the primary winding alone or with the secondary winding alone. The currents in these windings vary with time approximately according to the sine law, hence the leakage fluxes vary according to the same law.

In accordance with the fundamental law of induction, the fluxes induce, in the corresponding windings, emf's in lagging time quadrature with the currents themselves. The effect is the same as if the coupling between the coils were perfect but a reactive coil were connected in series with the primary circuit of the transformer and one in series with its secondary circuit. The amount of inductance in these equivalent coils is called the primary and the secondary leakage inductance of the transformer, respectively.

An imperfect transformer with magnetic leakage may thus be replaced by a perfect one, with the imperfections taken outside and represented by fictitious impedance coils of proper magnitude. The effect of the primary inductance is to consume part of the applied primary voltage. The remainder supplies the resistance drop and emf induced by the useful flux. Consequently, the primary leakage affects the value of the useful flux and hence the value of the secondary induced emf. With a lagging load current, this emf is smaller than that which would be induced in an ideal transformer, with the same applied voltage. The effect of the voltage drop in the secondary inductance is to reduce the secondary terminal voltage still further.

Strictly speaking, the leakage inductance reduces the applied primary voltage and the induced secondary emf only with a lagging secondary current. When the secondary current leads the voltage, the leakage reactance may boost the voltage, as is explained below. It is only because in practice the lagging current predominates that the leakage fluxes in a transformer have come to be considered as causing an additional drop and not a rise in voltage.

407. The Complete Vector Diagram of the Transformer. — The vector diagram of the ideal transformer, shown in Fig. 277, needs corrections for the effects of exciting current; resistance, and the leakage reactances described in the preceding section. The complete vectorial relations are shown in Fig. 283 for a lagging secondary current. In this diagram the secondary quantities, with the exception of E_2, are shown in primary terms. Voltages are multiplied by the ratio of transformation, n_1/n_2, and the current is divided by this ratio. Thus primary and secondary vectors become comparable in size. The vectors I_1x_1 and $I_2x_2(n_1/n_2)$ are drawn in leading quadrature with the corresponding currents and are equal and opposite to the emf's induced by the leakage fluxes. They represent the components of the applied primary and of the induced secondary voltage which are consumed in overcoming the effect of the leakage fluxes.

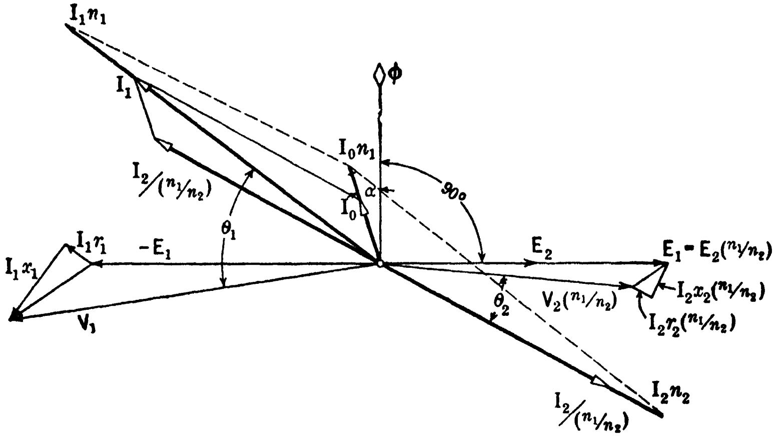

FIG. 283. The complete vector diagram of a transformer.

It will be seen from the geometry of the figure that with a lagging secondary current the reactance drop increases the difference between the induced and the terminal voltages. It is said, therefore, that on inductive load the internal magnetic leakage of the transformer affects the voltage regulation adversely. With a constant applied voltage and a constant secondary current, the secondary voltage reaches its minimum when the power factor of the load is such that $\tan \theta = x/r$.

It is instructive to draw a similar diagram for a leading load current. It will be found that the internal reactance of the transformer raises the secondary voltage, and $V_2(n_1/n_2)$ reaches its maximum when the load current is purely capacitive. Such a case arises, for example, when a transformer feeds a long transmission line at no load, supplying its charging current. Under these conditions there may be a partial resonance and a dangerous rise in voltage (Vol. II).

Sometimes the leakage reactance of a transformer is purposely made large to protect it against violent short circuits on the load side.

408. Simplified Transformer Vector Diagram. — In the commercial transformer the no-load current is such a small fraction of the load current that it may be neglected in the calculation of transformer regulation. This leads to a greatly simplified vector diagram. Thus, if the exciting current is neglected in Fig. 283, I_1 and $I_2/(n_1/n_2)$ become equal in value but opposite in phase. When the primary vectors are rotated through 180°, I_1 and $I_2/(n_1/n_2)$ coincide, therefore the Ir drops of primary and secondary become in phase and their sum, $I_1r_1 + I_2r_2\ (n_1/n_2)$, may be combined in an *equivalent* Ir drop shown as $I_1r_{\text{equiv.}}$ in Fig. 284. Since, from eq. (3), §391, $I_2 = I_1(n_1/n_2)$, it follows that $I_1r_{\text{equiv.}} = I_1r_1 + I_1r_2(n_1/n_2)^2$, or

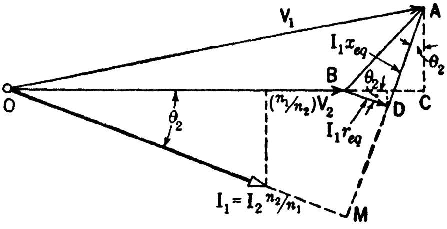

FIG. 284. Simplified vector diagram of the transformer.

$$r_{\text{equiv.}} = r_1 + r_2(n_1/n_2)^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1)$$

The Ix drops combine similarly into $I_1x_{\text{equiv.}}$, where, by a line of reasoning similar to that which gave eq. (1),

$$x_{\text{equiv.}} = x_1 + x_2(n_1/n_2)^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (2)$$

Thus, in Fig. 284, the Ir and Ix drops combine into a single triangle BDA.

When, to the secondary volts (in primary terms) or to $V_2(n_1/n_2)$, are added the equivalent Ir and Ix drops, the value of primary impressed volts is obtained. This is shown in the simplified diagram of Fig. 284. Note that the induced emf's are omitted in this diagram, as they are of no importance in the calculation of regulation.

Analytical solution of the diagram may be made by inspection of Fig. 284 in either of two forms:

Method (*a*). *I_1 as reference vector.* In the triangle OMA

$$V_1^2 = \overline{OM}^2 + \overline{MA}^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

and

$$V_1 = \sqrt{\overline{OM}^2 + \overline{MA}^2} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (3a)$$

But $OM = OB \cos\theta_2 + I_1r_{\text{equiv.}}$, and $\overline{MA} = \overline{OB} \sin\theta_2 + I_1x_{\text{equiv.}}$ Hence,

$$V_1^2 = [(V_2(n_1/n_2)\cos\theta_2 + I_1r_{\text{equiv.}}]^2 + [V_2(n_1/n_2)\sin\theta_2 + I_1x_{\text{equiv.}}]^2 \quad (4)$$

Method (*b*). *V_2 as reference vector.* In the triangle OCA

$$V_1^2 = \overline{OA}^2 = \overline{OC}^2 + \overline{CA}^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (5)$$

But
$$\overline{OC} = \overline{OB} + \overline{BD}\cos\theta_2 + \overline{DA}\sin\theta_2 \quad . \; . \; . \; . \; . \; . \; (6)$$

and
$$\overline{CA} = \overline{DA}\cos\theta_2 - \overline{BD}\sin\theta_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

Substituting in terms of electrical quantities

$$V_1^2 = [V_2(n_1/n_2) + I_1r_{\text{equiv.}}\cos\theta_2 + I_1x_{\text{equiv.}}\sin\theta_2]^2 + [I_1x_{\text{equiv.}}\cos\theta_2 - I_1r_{\text{equiv.}}\sin\theta_2]^2 \quad (8)$$

Since, in any practical case, $\overline{CA}$ is small compared with $\overline{OC}$, it is allowable to write

$$\overline{OA} = \sqrt{\overline{OC}^2 + \overline{CA}^2} = \overline{OC} + \overline{CA}^2/(2\overline{OC}) \quad . \; . \; . \; . \; . \; (9)$$

Equation (9) will give the numerical value of $\overline{OA} = V_1$, when slide-rule calculations are used, more accurately and more quickly than eq. (5). Similarly, from eq. (8) may be written

$$V_1 = [V_2(n_1/n_2) + I_1r_{\text{equiv.}}\cos\theta_2 + I_1x_{\text{equiv.}}\sin\theta_2] + \frac{1}{2}\left[\frac{(I_1x_{\text{equiv.}}\cos\theta_2 - I_1r_{\text{equiv.}}\sin\theta_2)^2}{V_2(n_1/n_2) + I_1r_{\text{equiv.}}\cos\theta_2 + I_1x_{\text{equiv.}}\sin\theta_2}\right] \quad . \; . \; (10)$$

Since regulation $= \dfrac{V_1 - V_2(n_1/n_2)}{V_1}$, and since V_1 is numerically almost equal to $\overline{OC}$,

$$\text{Regulation} = \frac{I_1r_{\text{equiv.}}\cos\theta_2 + I_1x_{\text{equiv.}}\sin\theta_2}{V_1} + \frac{(I_1x_{\text{equiv.}}\cos\theta_2 - I_1r_{\text{equiv.}}\sin\theta_2)^2}{2V_1^2} \quad . \; (11)$$

or, in per cent, and dropping subscripts,

$$\textit{Per cent regulation} = (\%\, Ir\cos\theta + \%\, Ix\sin\theta) + (\%\, Ix\cos\theta - \%\, Ir\sin\theta)^2/200 \quad . (12)$$

This is identical with the method of calculation of transformer regulation specified in the Standardization Rules of the A.I.E.E. No. 13-351 (May, 1930) except as to nomenclature. Quoting from the Institute Rules:

Let q_r = per cent resistance drop, q_x = per cent reactance drop, m = power factor, and n = reactive factor, of the load; then,

$$\textit{Per cent regulation} = mq_r + nq_x + (mq_x - nq_r)^2/200 \quad . \; . \; . (13)$$

The equations for regulation so far developed apply specifically to inductive loads. If it happens that the load is capacitive, i.e., that the load current leads the secondary voltage, $\sin\theta_2$ becomes negative in the above equations, with the corresponding change in signs in eq. (13).

The relations represented in Fig. 284 and by the equations which have been developed correspond to the approximate equivalent circuit shown in Fig. 285, except that in this equivalent circuit the exciting current is shown in the " exciting circuit " in parallel with the impedances of the windings, here shown in primary terms. In this exciting circuit the conductance g_o allows an in-phase or energy current to flow, to account for the loss due to hysteresis and eddy currents, while the susceptance b_o permits the magnetizing component of the no-load current to pass. In the calculation of regulation any effect of this exciting current was ignored altogether. Strictly speaking, this exciting circuit should be considered to be between the primary impedance and the secondary equivalent impedance, since the exciting current actually flows in the primary winding. This would add considerably to the complication of the solution and is unnecessary.

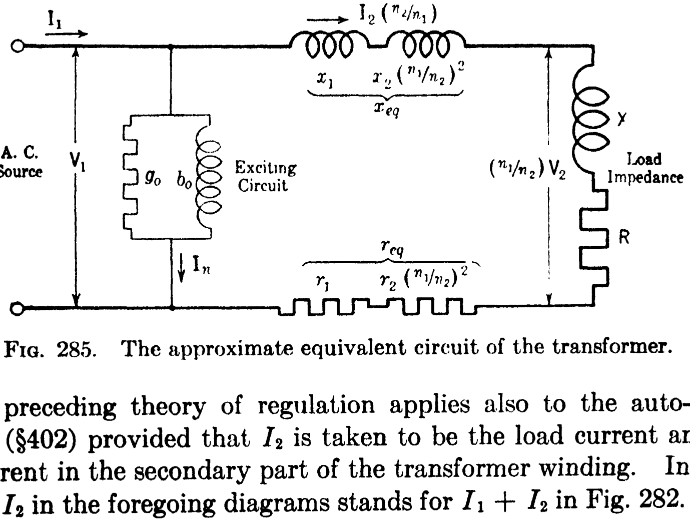

FIG. 285. The approximate equivalent circuit of the transformer.

The preceding theory of regulation applies also to the auto-transformer (§402) provided that I_2 is taken to be the load current and not the current in the secondary part of the transformer winding. In other words, I_2 in the foregoing diagrams stands for $I_1 + I_2$ in Fig. 282. The equivalent resistance is computed from the wattmeter reading on short circuit, and the equivalent impedance is the ratio of the primary volts to the primary amperes on short circuit. As a matter of fact, when predetermining the regulation of a transformer from the short-circuit test it is not necessary to know whether the transformer has two separate windings or only one. All tests and computations can be performed on an auto-transformer as if it had two separate windings.

409. Example of Predetermination of Regulation. — From the short-circuit test of the transformer, performed as outlined in §393, the equivalent resistance and reactance are calculated as in §392. Suppose that in a transformer rated 2200/110 volts, an applied emf of 120 volts, applied to the high-tension winding, causes a current of 25 amperes, high tension, and 500 amperes, low tension, to flow in the windings, with a power loss, as measured by the wattmeter, of 1406 watts. Then, since $P = I^2 r_{\text{equiv.}}$, $r_{\text{equiv.}} = 1406/(25)^2 = 2.25$ ohms (in high-tension terms). The leakage impedance of the transformer, $z_{\text{equiv.}} = E/I = 120/25 = 4.8$ ohms. The leakage reactance then has the value $x_{\text{equiv.}} = \sqrt{4.8^2 - 2.25^2} = 4.23$ ohms. It is found that the resistance of the ammeter and leads in the low-tension circuit has a value of 0.001 ohm, equivalent in high-tension terms to $0.001 \times (500/25)^2 = 0.4$ ohm. Therefore, the actual equivalent resistance of the transformer windings is $2.25 - 0.4 = 1.85$ ohm.

Let it be required to predetermine the voltage regulation of this transformer at the secondary terminal voltage of 110 volts and at an output of 70 kw, the power factor of the load being 80 per cent (lagging). The secondary current is

$$\frac{70{,}000}{110 \times 0.80} = 796 \text{ amperes,}$$

or, reduced to the primary circuit, $796/20 = 39.8$ amperes. The voltage at the primary terminals of the ideal transformer (Fig. 277) is 2200 volts. The equivalent ohmic drop is $1.85 \times 39.8 = 73.6$ volts; the equivalent reactive drop is $4.23 \times 39.8 = 169$ volts. We now construct the diagram, as in Fig. 284, and find that the voltage V_1 equals 2370 volts. The same result follows from formula (4), namely,

$$V_1^2 = (2200 \times 0.80 + 73.6)^2 + (2200 \times 0.60 + 169)^2$$

from which $V_1 = 2365$ volts. At no load this voltage produces a secondary emf of $2365/20 = 118.2$ volts; therefore, the regulation of the transformer, in per cent of the constant secondary voltage, at this particular load is

$$\frac{118.2 - 110}{110} = 7.45 \text{ per cent}$$

If it is preferred that regulation be calculated by eq. (12), first find the voltage drops in per cent. Then $\%Ir = 73.6/22 = 3.34$, while $\%Ix = 169/22 = 7.68$. Substituting in eq. (12):

$$\begin{aligned}\text{Per cent regulation} &= 3.34 \times 0.8 + 7.68 \times 0.6 + (7.68 \times 0.8 - 3.34 \times 0.6)^2/200 \\ &= 2.67 + 4.61 + 0.086 = 7.37\end{aligned}$$

410. EXPERIMENT 17-B. — Voltage Regulation of a Transformer from Short-Circuit Test. — The experiment is performed as explained in the directions for Experiment 16-B. It is expected, however, that a more advanced student will obtain better data and will apply the results to a predetermination of voltage regulation, as explained in the preceding section. If possible, at least one set of very accurate readings of terminal voltages should be obtained on actual load, preferably on an overload at low power factor (most unfavorable conditions of operation), so as to check the predetermined regulation with that observed experimentally.

Report. (a) Calculate the regulation of the transformer, using eq. (12), at rated current and at 100 per cent power factor, at 80 per cent power factor, both lagging and leading, and at zero power factor, lagging and leading.

(b) Construct Kapp's diagram as explained in the following section, and using the values taken from this diagram plot per cent regulation at full-load current against power factor as abscissas, covering the range from zero power factor, lagging, to zero power factor, leading. Show on this curve the values calculated in (a). Show also how the measured regulation checks values calculated from the short-circuit data.

411. Kapp's Diagram. — This diagram is of value primarily when it is desired to predetermine the regulation of the transformer at constant current but at a number of different power factors. Its usefulness for this purpose is increased if the following changes are made from the simpler diagram usually described.

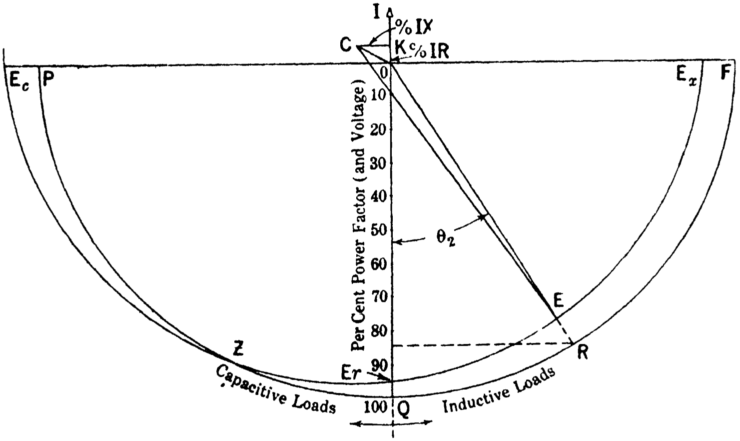

FIG. 286. Kapp's diagram for determining transformer regulation.

1. Express the Ir and Ix drops in per cent of V_1, and draw the triangle of drops (Fig. 286).

2. Draw two semicircles, each with a radius = 100 per cent, and with centers at O and **C** on the triangle of drops.

3. Divide the line OQ ($=$ 100 per cent) into ten equal parts and number them as on the diagram. This is the scale of power factor.

Now, to determine the regulation at any desired power factor draw a horizontal line from the point of desired power factor on the scale of that quantity to intersect the semicircle PQF, say, at R. Draw OR. The line OR will be at the desired angle θ_2 from OQ, the line of the current vector. Now OE is the secondary terminal voltage, in per cent, or, since OR = 100 per cent, $RE = V_1 - V_2(n_1/n_2)$, expressed in per cent, and is the regulation, in terms of the constant V_1.

From this it is seen that to determine the regulation at desired power factors it is only necessary to project from the power factor scale to the circle PQF, and laying a straight-edge along OR, read ER in per cent. This can be done for a large number of values in a few minutes, after the diagram is once drawn. Note that for lagging power factors the projection line is taken to the right, and for leading values it is drawn to the left. At the point Z the regulation of the transformer is perfect (zero), and beyond that point the values of regulation are negative, i.e., the secondary voltage rises with load, instead of falling.

If the secondary voltage, rather than the primary, is constant, OR will represent the secondary voltage and CR that of the primary. RE will still closely represent the difference $V_1 - V_2(n_1/n_2)$, but in per cent of the constant V_2.

To determine the regulation at other values of the load current, new drop triangles, OKC, must be drawn and new semicircles scribed from C as center. The principal difficulty in applying Kapp's diagram to an actual transformer is that the voltage drops are very low and the triangle OKC is comparatively small, so that the distance RE is difficult to read accurately, primarily because of the difficulty of locating the centers O and C exactly and of drawing accurate semicircles. The diagram does show, better than any other, the way in which power factor affects regulation, and the points of worst and best regulation.

A series of analytical calculations, using eq. (12), can be carried through rather quickly, especially if a table of calculated values is made up to facilitate the work. Values may also be read from Mershon's Chart, to be found in most textbooks dealing with short transmission lines. In this chart only per cent drops have to be laid off on an enlarged scale and the results are obtained with satisfactory accuracy.

412. The Effect of Subdivision of Windings upon the Leakage Inductance. — Consider two transformer windings (Fig. 287) of the same total number of turns. In the arrangement (a) shown to the left, one primary coil P is used and one secondary coil S of approximately the same thickness. Since at any considerable load the primary and the secondary ampere-turns are practically equal, the required amount of copper in P and in S is approximately the same, assuming the same current density and the same space utilization in both. The leakage flux is shown by curved lines, and the instantaneous currents by dots and crosses. With such "concentrated" or "bunched" coils the magnetic leakage is a maximum and the voltage regulation is poor.

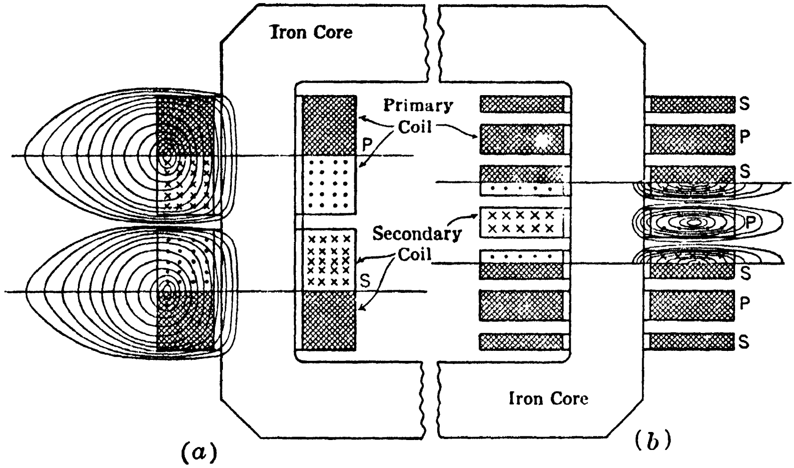

FIG. 287. Effect of subdivision of windings upon transformer leakage fluxes.

To reduce the leakage mmf's, the windings are subdivided into sections, as shown at (b) to the right. The primary and the secondary coils are placed alternately, with " half-coils " at the ends. With such an arrangement, each half of a primary coil with an adjacent half of the secondary coil forms a leakage mmf which excites a leakage flux. These fluxes are smaller than at (a) because their exciting mmf's are smaller, and moreover each flux is linked with fewer turns and therefore induces a lower emf. On the other hand, there are more emf's connected in series, so that the reactive drop decreases nearly inversely as the number of sections and not as its square.

The inductance of a coil occupying a certain space is equal to $\mathcal{P}n^2$, where $\mathcal{P}$ is the equivalent permeance of the magnetic circuit of the coil and n is its number of turns. Let the leakage inductance of the primary coil to the left be

$$L = \mathcal{P}n^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (14)$$

Assuming, for the moment, that the permeance of the leakage space between each two coils in the arrangements (a) and (b) will be the same, the inductance of each primary coil to the right is

$$L' = \mathcal{P}(n/q)^2 . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

where q is the number of sections into which the primary winding is subdivided. There are q sections in series, so that the total inductance of the subdivided primary winding is

$$qL' = q\mathcal{P}(n/q)^2 = \mathcal{P}n^2/q = L/q \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

In reality $\mathcal{P}$ is not quite the same with the subdivided winding, but the above deduction shows roughly the effect of subdivision of a transformer winding upon its leakage inductance. At no load, with only a small magnetizing current flowing through one of the windings, the leakage fluxes are negligible and the induced voltages are the same in both cases, so that in this respect the subdivided and the bunched windings are equivalent.

For the so-called interlacing of transformer coils in the three-wire and T-connection, see §§416 and 417.

The reader is warned against a hasty conclusion that the voltage regulation of a transformer with q sections is q times better than that of one with bunched windings. With reference to Fig. 284 it will be readily seen that reducing the length of the vector $I_1x_{\text{equiv.}}$ to one-half by no means reduces the arithmetical difference between OA and OB to one-half. Especially at high values of load power factor, the influence of the reactive drop on the voltage regulation is quite small, as may be seen from an examination of the figure. At a very low power factor the reactive voltage drop is nearly in phase with the terminal voltage, so that the influence upon the regulation exerted by it is nearly in proportion to its value. On short circuit the leakage reactance practically determines the current, and therefore the best method of studying the leakage reactance of a transformer is on short circuit (§409).

For the purpose of this study it is best to have a special transformer consisting of a comparatively large number of identical coils, with two half-coils at each end. Although in practice only one half-coil would be used at each end, it is better to have two, in order that the student may determine how much the leakage flux is increased when full coils are used at the ends. For the principal readings it is convenient to use a one-to-one ratio of transformation, in order that the secondary quantities may be transferred directly into the primary circuit.

One of the windings is short-circuited through a low-resistance ammeter, and an alternating voltage is applied to the other, just sufficient to circulate a safe large current in both windings. An ammeter, a voltmeter, and a wattmeter are connected on the primary side.

The ratio of primary volts to amperes will give the total equivalent impedance of the transformer. The wattmeter indication represents practically the I^2R loss only, because the useful flux on short circuit is quite small and the core loss is negligible. Dividing the wattmeter reading by the square of the supply current gives the equivalent effective resistance of the transformer (§392).

The hypotenuse and one of the sides of the impedance triangle (Fig. 129) being known, the other side is easily found either by computation or graphically, and thus the total equivalent leakage reactance of the transformer is ascertained. Its effect upon the voltage drop and regulation is determined as is explained in §408.

413. EXPERIMENT 17-C. — Study of Transformer Leakage. — The purpose of this experiment is to study the influence of the arrangement and subdivision of the transformer coils upon its leakage inductance. The method is explained in the preceding section. The frequency of the supply circuit must be accurately known for each reading. See §98 regarding wattmeter readings at low power factor.

(1) Begin with the most unfavorable conditions, that is, with all the primary coils concentrated on one side of the core, and all the secondary coils on the other. Bring the voltage up so as to give the maximum safe value of the current, and read the instruments; note also the frequency of the supply. Take one or two check readings at reduced voltages.

(2) Now " sandwich in " one of the windings between the two halves of the other, and repeat the readings. If the same values of current or voltage are used as before, the effect of the rearrangement will be brought out more clearly.

(3) Subdivide the windings still more, always keeping in mind the general arrangement indicated above, that is, there must be as small an mmf as possible in the space between two coils, and this mmf must be balanced in such a way as to make the primary and the secondary ampere-turns equal. This will necessitate the use of half-coils at the ends. Try also one or two combinations with the full coils at the ends, and note the increase in the leakage reactance.

(4) For a few characteristic arrangements of the coils, determine the actual voltage drop in the transformer under load, so as to be able to compare it with that computed from the short-circuit test. For this purpose load the transformer on reactances, on an unloaded induction motor, etc., so as to have as low a power factor as possible. This is desirable because on an inductive load the reactive voltage drop is much more nearly in phase with the line voltage, and therefore affects it much more than at a high power factor. Since the voltage drop constitutes but a small percentage of the terminal voltage, this test and especially the voltage readings must be taken quite accurately to avoid disappointment in working up the data.

(5) To increase the accuracy of the preceding run, a differential

method of measuring voltages may be used. For example, a primary and a secondary terminal may be connected together and the difference between the potentials of the two other terminals measured with a low-reading voltmeter. With a one-to-one ratio of transformation this reading ought to be nearly zero at no load.

(6) Before leaving the laboratory, measure carefully, with direct current, the resistances of all the coils used, and if possible determine the number of turns in each, at least relatively.

Report. (1) Show schematically the various arrangements of the coils used during the experiment, and refer to them in the report by number only. (2) Show that the primary and the secondary amperes were in the inverse ratio of the number of turns, and check or reject the readings in which this condition was not fairly closely fulfilled. (3) From the primary volt and ampere readings compute the corresponding impedances. Calculate the effective resistances from the wattmeter readings and check with the values obtained with direct current; see §408, eq. (1). (4) Determine the corresponding leakage reactances and correct where necessary for the frequency. Show that the relative values of these reactances are such as might be expected theoretically. (5) For the arrangement of the coils for which a load test was performed, compute the per cent voltage regulation and compare with that obtained from the test.

414. Transformers in Parallel. — When two or more transformers (Fig. 288) are connected in parallel on the primary side only, each secondary supplying its own load, the individual transformers may have entirely different characteristics and even different ratios of transformation. If, however, the secondaries are also to be connected in

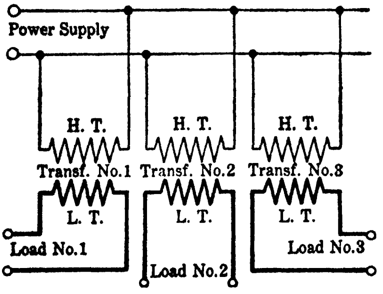

FIG. 288. Transformers with primaries in parallel, secondaries loaded independently.

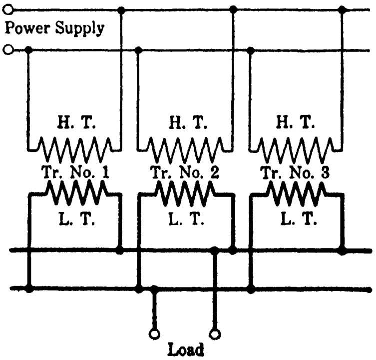

FIG. 289. Transformers with both primary and secondary windings in parallel.

parallel (Fig. 289), to supply a common load, the characteristics of the individual transformers must be such that the total load output is divided among them in a fairly correct proportion from no load to a reasonable overload. See in this connection the discussion on the parallel operation of d-c machines (§255).

When the ratio of the no-load voltages is the same for all the transformers, the load will divide between the transformers in inverse proportion to their equivalent impedances (Fig. 290). It follows that if the output is divided properly, say at full load, it will also be divided in the right proportion at any other load. The required condition is that the vectors of voltage drop in all the transformers, at rated load, must constitute the same fraction of the applied voltage and form the same phase angle with it. This means that all the transformers must have the same per cent ohmic drop and the same per cent reactive drop at their respective rated currents.

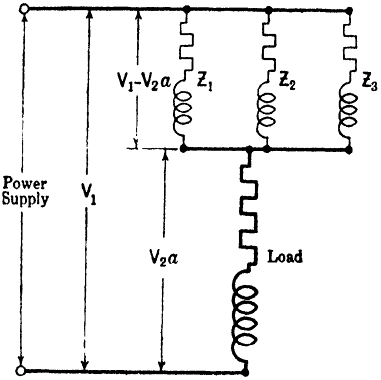

Fig. 290. Equivalent circuit of three transformers in parallel.

If one of the transformers has a smaller normal internal drop than the others it will tend to take a greater share of the total load. The division of the load is improved by adding a proper amount of external resistance or reactance, or both, in series with that particular transformer. When the transformation ratios of the individual transformers are slightly different, an exact correction requires an auto-transformer which acts as a booster for the transformer with the lower ratio.

Polarity test. When the secondaries of two or more transformers are to be connected in parallel, proper leads or terminals must be used to avoid a short circuit. Therefore, before connecting in parallel, the transformers must be tested and marked as to their polarity (see §390). When the polarity of all the transformers has been determined and marked in this manner, the terminals bearing the same letter are connected to the same side of the line, to insure an agreement in the polarity on the secondary side.

Another method of testing a transformer polarity consists in testing the secondary of a transformer against that of another of known polarity and of the same ratio as that to be tested. The primaries are connected in parallel to the same source of supply. On the secondary side one of the terminals of the standard transformer, say x_2, is connected to one of the terminals of the transformer under test. If this latter terminal happens to be also x_2, then the voltage between the two remaining free terminals will be zero; otherwise it will be equal to twice the secondary voltage of each transformer. This can be ascertained either with a voltmeter or with an incandescent lamp in series with a protective resistance or reactance.

415. EXPERIMENT 17-D. — Transformers in Parallel. — The purpose of this experiment is to study the connections, operation, and division of load between two transformers in parallel. The transformers should be of nearly the same ratio of numbers of turns, but preferably of different rating. The ratio of active turns in one of the transformers should be made adjustable by a small amount, by providing a tap on one of its coils. The percentage ohmic and reactive drop should also be different for the two.

(1) Test the polarity of the transformers, as is explained in the preceding section, so as to know which secondary terminals are to be put together.

(2) Connect the transformers in parallel, both on the primary and on the secondary side, as in Fig. 289, and provide a suitable load. Have an ammeter in the secondary circuit of each transformer and one in the load circuit. Adjust the load to such a value that one of the transformers is carrying its rated current while the other is underloaded. Read the three ammeters. Select a few different loads, at different values of power factor, and take similar readings.

(3) Perform a short-circuit test on each transformer, as in §409, and add a sufficient amount of external resistance and reactance in the circuit of one of the transformers or both until the two transformers have the same per cent ohmic drop and the same per cent reactive drop.

(4) Connect the two transformers again in parallel, as in (2) above, and show that they divide the load much more nearly in proportion to their ratings.

(5) Change slightly the ratio of one of the transformers by using a tap on one of its windings. Show that with unequal voltage ratios the transformers may be made to divide the load properly at some one value of the current, but that they will not divide it in the right ratio at any other current. Connect an auto-transformer to act as a booster for one of the transformers, and adjust it so that the transformers will divide the total current properly at all loads.

Report. (1) Give data on the transformers used, and describe the polarity test. (2) Give the numerical data showing which transformer took a disproportionately greater share of the load. Draw a few vector diagrams of the three currents to show how much the two transformer currents were out of phase with each other, and also how the ratio of the component currents varied at different loads. (3) Give the test data and the computations to show how the external resistances and reactances were adjusted to get the same per cent internal drop in both transformers. (4) Give the data to prove that, with such an adjustment, the transformers divided the load properly. (5) Show that with the different ratio of transformation the division of the load was different at different values of the total current, and explain this fact theoretically. Give data to indicate how much an additional auto-transformer improved the distribution of the load.

416. Three-Wire A-C Distribution. — The three-wire distribution shown in Fig. 291 has the advantage of considerable copper economy over the ordinary two-wire system. It permits the use of lamps and other current-consuming devices of voltage E_2, say 110 volts, with the copper cross-section in the two external conductors corresponding to that required by the voltage $2E_2$. This cross-section is but one-quarter of that required at the voltage E_2 for the same voltage drop and with a given power consumption. When the load on both sides is balanced, no current flows through the middle wire. This wire carries only the current corresponding to the difference of the loads on the two outside conductors, and can therefore be of comparatively small size. The d-c three-wire system is described in §262.

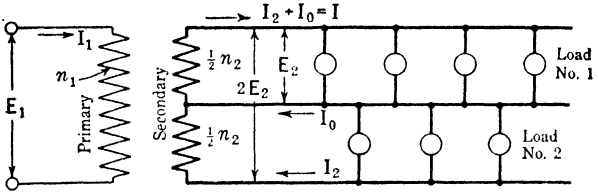

FIG. 291. Three-wire distribution from a single transformer.

In an a-c three-wire distribution, the total secondary voltage is obtained from an ordinary transformer which steps down the transmission voltage E_1 to the desired value $2E_2$. The middle or neutral wire is connected to the middle point N of the secondary winding, and thus is kept at all times at a potential midway between the potentials of the two outside conductors.

With the notation shown in the figure, the current through one-half of the secondary winding is I_2, that through the other half is $I_2 + I_0$, where I_0 is the current in the neutral. Neglecting the magnetizing current of the transformer, the primary and the secondary ampere-turns must be equal (§408), so that the currents are connected by the relationship

$$I_1 n_1 = \tfrac{1}{2} n_2 I_2 + \tfrac{1}{2} n_2 (I_2 + I_0) \quad . \; . \; . \; . \; . \; . \; . \; . \; (17)$$

or

$$I_1 n_1 = (I_2 + \tfrac{1}{2} I_0) n_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (18)$$

This relationship holds true algebraically for instantaneous values of the currents, and geometrically for their effective values. When the power factor of the load is different on the two sides of the neutral, the three mmf vectors in the foregoing equation form a triangle.

Figure 292 is the vector diagram of the transformer with two independently loaded secondary windings, the nomenclature being changed somewhat from that of Fig. 291 in order to distinguish between terminal and induced voltages. In Fig. 292, E_2 is the induced voltage common to each secondary. The load currents I_2 and $I_2 + I_0$ are shown equal in value but at different power factors. As a consequence the terminal voltages of the two secondary windings, namely, V_2' and V_2'', differ in value and in phase.

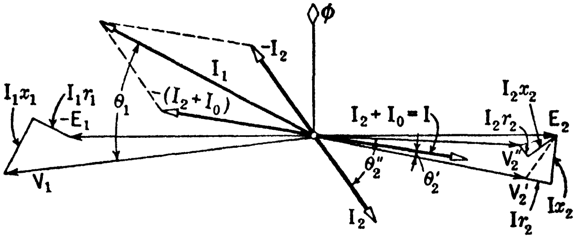

FIG. 292. Vector diagram of a transformer with two independently loaded secondary windings.

The effect of these load currents appears in the primary as currents $-I_2$ and $-(I_2 + I_0)$, assuming a one-to-one ratio of transformation, or $n_1 = n_2/2$.

The total primary current, neglecting exciting current, is I_1, the vector sum of $-I_2$ and $-(I_2 + I_0)$. $-E_1$ is the component of applied voltage required to balance the induced voltage, while $I_1 r_1$ and $I_1 x_1$ are the impedance drops in the primary winding. Adding them to $-E_1$ gives the applied voltage V_1.

Because of the internal drop in the transformer itself, the secondary voltage with an unbalanced load is somewhat lower on the side which is more heavily loaded or which has the lower power factor. Care must therefore be taken to subdivide or to " interlace " the transformer windings on the two sides, so as to minimize this effect, as is shown below. Sometimes an automatic voltage regulator is used on each of the two external conductors to keep the voltage constant and independent of the load.

Interlacing of coils. Owing to the difference in the currents in the two halves of the secondary winding, the impedance drop is also different. The resultant unbalancing of the two secondary voltages is remedied in part by using adjacent secondary coils on the opposite sides of the neutral. Let there be, for example, eight secondary coils, numbered consecutively from 1 to 8. Instead of connecting the coils 1, 2, 3, 4 in series on one side and the coils 5, 6, 7, 8 on the other side of the neutral, the coils 1, 3, 5, 7 are used for one circuit and 2, 4, 6, 8 for the other. In this way the leakage fluxes due to those coils are distributed symmetrically and the reactive drop in the two circuits is equalized. A similar interlacing of coils is used in one of the transformers in the T-connection (Vol. II), §783.

417. EXPERIMENT 17-E. — Currents and Voltages in a Transformer on a Three-Wire System. — The diagram of connections is shown in Fig. 291. The primary voltage must be kept constant, and if necessary a regulating rheostat must be provided. Insert ammeters to read all three secondary currents, wattmeters, and a voltmeter to read the voltages. Connect an ammeter, a voltmeter, and a wattmeter on the primary side.

(1) Provide a suitable non-inductive load and distribute it as evenly as possible between the two sides of the secondary circuit, so as to have as nearly as possible a balanced load. The current in the neutral should then be almost equal to zero. Read all the voltages, currents, and watts.

(2) Reduce the load on one side in steps to zero, keeping the current on the other side constant. At each step read all the voltages, currents, and watts as before.

(3) Connect a wattmeter on one side of the secondary circuit, bring the load again to the same value as at the beginning of the preceding run, and leave the load on the side without the wattmeter constant. On the other side make the load more and more inductive, keeping the current itself constant. In other words, keep the currents in the two outside wires constant and equal to each other, but vary the phase relation of one with respect to the other. . At each step read carefully the watts, the voltages, and the currents, including that in the middle conductor.

(4) If the transformer coils may be connected in different combina-

tions (§412) investigate the effect of the interlacing of the secondary coils upon the voltage balance, as is explained in the preceding section. Take readings of the two secondary voltages under the two extreme conditions of load unbalancing, namely, (a) with a maximum load on one side and a zero load on the other side, and (b) with the two currents equal in magnitude but of widely different power factor. For each of these combinations use the two extreme arrangements of the secondary coils, namely, the one in which the consecutive transformer coils are all on the same side of the circuit, and the other in which every second coil belongs on the opposite side of the circuit.

(5) Before leaving the laboratory, measure accurately the ratio of transformation at no load.

Report. (a) Plot the results obtained in (2) against the current in the neutral as abscissas. Check eq. (18) by using the ratio of the no-load voltages for the ratio of the numbers of turns. Show that the current in the neutral is equal to the arithmetical difference of the currents in the two outside conductors. (b) Against the current in the neutral as abscissas, plot all ampere, volt, and watt readings, and the values of power factor obtained from test (3) above. Draw as complete a vector diagram as data will permit for conditions similar to those in Fig. 292 and compare it with that diagram. Show from the results that eq. (18) now holds true geometrically. (c) Show in detail the arrangements of coils used in experimenting with the effect of interlacing. Give the data to indicate the amount of unbalancing of the two secondary voltages in two comparative runs. Explain the observed facts theoretically by drawing a sketch of the transformer with its leakage fluxes, as in Fig. 287.

418. Constant-Current Transformer. — Electric power is normally generated and transmitted at substantially *constant potential.* There are a few cases in which a *constant current* is required with a varying load. Such is the case of street lighting with series-type incandescent or arc lamps. A transformer which converts constant-voltage a-c energy into constant-current a-c energy, and thus allows a series circuit to be fed from a constant potential source, is called a constant-current transformer.

A constant-current transformer (Fig. 293), like any ordinary transformer, consists of two windings placed on an iron core. The essential feature of the transformer is that its secondary coil is movable, whereas in an ordinary constant-potential transformer both coils are stationary. The stationary primary winding is connected to the a-c supply circuit; the movable secondary coil is connected to the load circuit in which the current is to be maintained constant and independent of the number of lamps in operation. When the load circuit is opened, the secondary coil rests on the primary, since only part of its weight is balanced by the counterweight. Closing the load circuit causes secondary currents to flow in the movable coil, and it is repelled upward, away from the fixed coil. The farther upward it moves, the lower becomes the emf induced in it, because the magnetic leakage between the two coils increases with the distance between the coils (Fig. 287). The counterweight is so adjusted that the unbalanced weight of the coil is just equaled by the repulsion between the windings, at a particular value of load current. When the number of lamps, or the resistance of the circuit, decreases, the current at first increases; this increases the repulsion between the coils, and the movable coil is repelled farther upward. The useful flux is thereby reduced and the induced voltage again decreases until the required current is reached and the coil again "floats." A good transformer of this type will hold the current constant within 1 per cent from full load to short circuit.

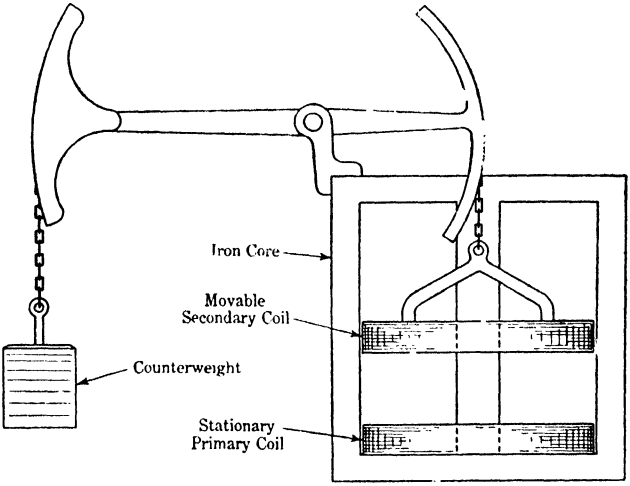

FIG. 293. Arrangement of parts in the constant-current transformer.

In an experimental study of the current-transformer it is sometimes desirable to find the phase angle between the primary and secondary voltages or currents. The phase angle between the voltages can be found as follows: Let the primary terminals of the transformer be denoted A_1 and B_1; the secondary terminals A_2 and B_2. Connect B_1 and B_2 together and measure the voltages A_1–B_1, A_2–B_2, and A_1–A_2. The vectors of these voltages form a triangle from which the phase relation may be determined. Since constant-current transformers are

usually built for high voltages, it will be necessary to employ potential transformers in making these measurements. The connection between primary and secondary vectors can be made on the secondaries of these potential transformers (see Fig. 294).

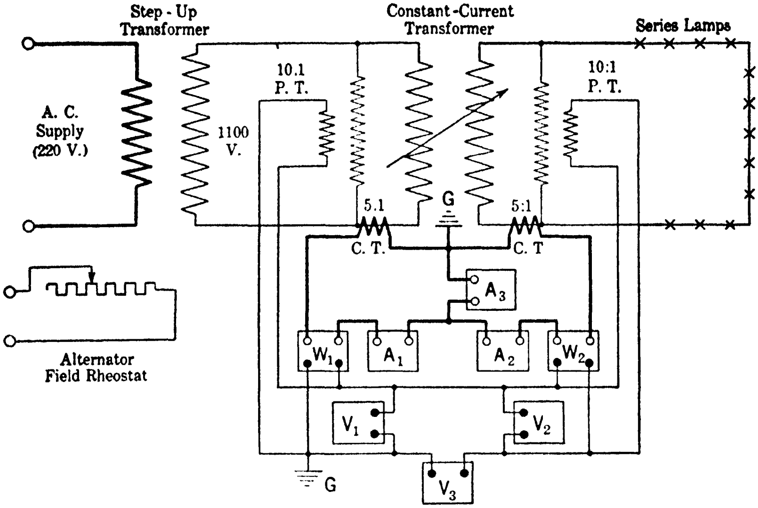

FIG. 294. Detailed connection diagram for testing a 1100/1100 volt constant-current transformer, using instrument transformers.

The phase relation between the currents can be determined by using, temporarily, a common conductor for one side of the primary and secondary circuits, with an ammeter in this line, although the arrangement using instrument transformers as shown in Fig. 294 is to be preferred.

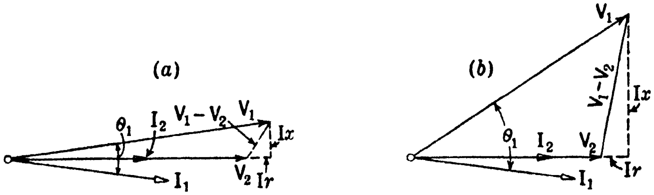

FIG. 295. Simplified vector diagrams of a 1:1 ratio constant-current transformer; (a) Coils together, (b) Coils separated. (Load non-inductive.)

With either arrangement the primary amperes, the secondary amperes, and the common amperes are read simultaneously. This again gives a triangle of vectors from which the desired phase relation may be read. The phase angle between the primary voltage and current is obtained from the wattmeter, ammeter, and voltmeter readings; the secondary

phase angle similarly. As a check on the relations thus obtained a wattmeter may be connected to read watts corresponding to primary volts and secondary amperes, and then vice versa. Figure 295 illustrates the phase relations in such a constant-current transformer with unity power factor in the secondary (load) circuit, (a) coils close together, and (b) coils separated. Note the increased reactive drop with the coils separated.

Large constant-current transformers are sometimes provided with two stationary and two movable coils, mounted on the same core. One set of coils is independent of the other, and each secondary is connected to a different load circuit. Each movable coil is provided with a separate counterweight.

419. EXPERIMENT 17-F. — Test of a Constant-Current Transformer. — The theory and connections are described in the preceding section. Both primary and secondary voltages are usually high in such a transformer, and great care must be taken to protect the experimenter and the equipment. It is advisable that this be made an example of metering in high-voltage circuits, by the introduction of both current and potential transformers. Figure 295 suggests the arrangement of meters and equipment for this experiment, it being assumed that the primary and secondary circuits operate at approximately 1100 volts. This may require a service transformer to step up the laboratory a-c supply from, say, 220 volts to 1100 volts. Note that, as suggested in §418, combined primary and secondary amperes, as well as combined primary and secondary voltages, are read on the secondary side of the instrument transformers. Wattmeters W_1 and W_2 give input to and output from the transformer. Obviously, with 10 : 1 potential transformers, and 5 : 1 current transformers there is a multiplying factor of 50 to apply to the wattmeter readings, of 10 for voltmeter, and of 5 for ammeter readings. The instrument transformers should be so connected that A_3 reads the difference (vectorial) rather than the sum of A_1 and A_2. However, ammeters A_1, A_2, and A_3 should have approximately the same impedance as otherwise the relations will be affected by meter impedances. V_3 should be the difference between V_1 and V_2.

The load may consist of a series of lamps so arranged that they may be short-circuited one at a time, or of ordinary 110-volt laboratory resistors in series. If the latter, care must be exercised in the operation of the switches, using an insulated " hook-switch " or similar device to make it less possible for the operator to come in contact with live parts of the circuit. The circuit should never be opened in the secondary circuit as the full 1100 volts will appear across the switch which opens this circuit.

With normal voltage applied to the transformer primary (and maintained at this value throughout the test), place the moving coil midway of its travel and adjust the load impedance so that rated current flows in the secondary. Vary the counterweight until the coil just floats. The transformer is now ready for the test. Proceed as follows:

(a) Starting the run with the load impedance such that the moving coil is at the lower limit of its travel, gradually decrease the load impedance until the coil moves to the top, taking readings of all meters for positions of the coil, say, 2 in. apart, using a scale placed on the core for this purpose.

(b) Determine the limits of current within which the transformer (with necessary changes in the counterweight) can regulate for constant current. For example, with a transformer rated at 6 amperes, try the regulation at 5 amperes and at 8 amperes.

(c) Investigate the effect of the power factor of the load upon the performance of the transformer.

(d) Determine how promptly the transformer acts with change in load. For example, with a change in load impedance which causes the coil to move over about one-half its travel, in how many seconds does the coil cease to move and the current become steady?

(e) Measure the voltage induced in the secondary winding on open circuit, and with the coil at the bottom, middle, and top of its travel. Do the same with a constant resistance in the load circuit, moving the coil into these several positions by hand. Measure all voltages, currents, and powers for this condition.

(f) Look up the methods used in practice to keep the secondary circuit closed when a series incandescent or a series arc lamp burns out.

Report. (1) Compute the values of the non-inductive resistance in the load during the first run, and plot curves of primary and secondary volts, amperes, and watts, coil position, efficiency (W_2/W_1), combined volts, primary and secondary power factors, all against load resistance.

(2) Give data showing the current regulation at the different values of the secondary current. If the regulation is not satisfactory give theoretical reasons for the observed values.

(3) Discuss variations of the secondary voltage with coil position when the secondary was on open circuit; also current variations when the coil was moved into various positions while load impedance remained constant.

(4) Explain theoretically the effect of power factor of load upon the performance of the transformer, and give the observed data in support of the explanation.

(5) Explain the action of the transformer with sudden changes in the load.

(6) Construct vector diagrams showing the phase relations of primary and secondary quantities for the moving coil at the bottom, middle, and top of its travel (see Fig. 295).

(7) Report on item (f) above, and state how the transformer is operated in commercial installations; also what happens if a lamp burns out.

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19. J. B. Gibbs, *Elec. Jour.*, July, 1926, p. 341, The Taylor-connection; three-phase to two-phase (transformation).

20. E. H. Waldo, *Trans. A.I.E.E.*, Vol. 48 (1929), p. 821, Some notes on transformer design.

CHAPTER XVIII

INSTRUMENT TRANSFORMERS

420. Instrument transformers are shown in the circuits of Figs. 55, 56 and 294 in connection with a-c ammeters, voltmeters, and wattmeters. They are also used to energize watthour meters, relays, power factor meters, trip coils, regulators, etc. By the use of instrument transformers it is possible to measure alternating currents or potentials of practically any values using 5-ampere ammeters or 150-volt voltmeters. The transformers not only serve as multipliers for the instruments but also insulate them from the high-voltage and make it possible to locate the meters at a distance from the high-tension lines.

When instrument transformers are used the instrument readings must of course be multiplied by the transformer ratio to obtain the actual value of current or voltage. Therefore any error in the ratio of the transformer will cause a corresponding error in the metered values, while any phase angle introduced by the transformer between line and meter values will introduce a further error when, as in wattmeters, power-factor meters, etc., the phase relations affect the readings. Thus it is essential that methods be available for the calibration of instrument transformers as to both ratio and phase angle.

Potential transformers and current transformers will be considered separately, although in theory and methods of calibration they are essentially similar. A general knowledge of the construction, operating characteristics, and vector diagrams of the transformer is presupposed, and is covered in the two chapters immediately preceding.

POTENTIAL TRANSFORMERS

421. When an alternating voltage materially exceeds 220 volts it is usually safer, more convenient, and more economical to connect the voltmeter or other potentially operated device to the line through a small transformer (Fig. 56) called a *potential transformer*. For high voltages such a transformer is usually of the oil-immersed type and in its essential construction details does not differ from a power transformer (Chapter XVI), except for its small size and nominal rating (15 volt-amperes being not uncommon).

In order that a voltmeter, connected through a potential transformer, may indicate the true primary voltage, either it must be calibrated together with the transformer or else the ratio of the transformer itself, when carrying its instrument load, must be accurately known. In a good potential transformer the voltage drop is very small and the ratio of terminal voltages remains sensibly constant and nearly equal to the ratio of the numbers of turns, so long as the transformer is not overloaded. A transformer designed to give a ten-to-one ratio of terminal voltages when excited from the high-tension side will not give a one-to-ten ratio when excited from the low-tension side. This is because one of the windings is usually provided with additional (compensating) turns to correct for the drop in the transformer.

A transformer used with a watthour meter, wattmeter, or similar device, must be connected with the proper *polarity* (§414). Failure to do this will result in the meter running backwards, or in a polyphase meter giving the difference, rather than the sum, of the power consumed in two phases. See Vol. II, Chapter XL, for a discussion of polyphase measurements.

Besides an error in ratio, a potential transformer also introduces a small error in the phase angle of the measured voltage. Figure 296 is the simplified vector diagram of a loaded potential transformer, showing separately the drops due to the load current I_2 and the exciting current I_0. Note the effects of the latter drop upon the ratio E'/E'' and the phase angle α between E' and E''. In a well-designed transformer this error in angle amounts to but a fraction of a degree (10 to 20 minutes) and is of no consequence when the transformer is used with a voltmeter or relay. But when it is used in connection with a wattmeter or watthour meter this error affects the apparent phase angle between the voltage and the current, and consequently the indication of the meter. At high values of load power factor the error is negligible. For example, $\cos 3° = 0.9986$, and $\cos 4° = 0.9975$, so that a phase error of one degree affects the indication of a wattmeter only about 0.1 per cent. On the other hand, at lower values of power factor an error of one degree leads to a more considerable error in the measured power. For example, $\cos 59° = 0.515$, while $\cos 60° = 0.5$, the error due to one degree being 3 per cent. This is more than is usually permissible in metering energy (see also §§98 and 100).

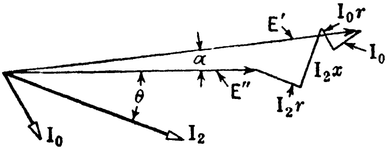

FIG. 296. Vector diagram of a potential transformer, including effect of exciting current.

Since the errors in voltage ratio and phase angle in a potential trans former are very small their determination requires accurate instruments and a skilled observer. The principal methods used and the theory of each will be given below.

422. Determination of Errors in Ratio and Phase Angle. — The various methods used for the measurement of ratio and phase angle in a potential transformer may best be explained with reference to Fig. 297. $OA = E$ is the vector of the unknown secondary voltage of the transformer under test, and $OB = E'$ is a reference voltage of known magnitude. It is required to determine the value of E in terms of E', and the phase angle α between them. Let AC be the component of OA perpendicular to OB. Since angle α is always small, normally less than one degree, OC is nearly equal to OA ($\cos 1° = 0.99985$). Thus the algebraic difference between OA and OB is practically equal to $CB = m$, or numerically,

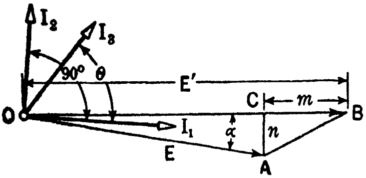

FIG. 297. Relations in the determination of ratio and phase angle error.

$$E' - m = E \quad \ldots\ldots\ldots\ldots \quad (1)$$

CB should not be confused with the *vector* difference AB between the two vectors.

Also, if $AC = n$ is known, the angle α may be found from the relation,

$$\sin \alpha = n/E \quad \ldots\ldots\ldots\ldots \quad (2)$$

Thus, when it is possible to determine m and n, the unknown voltage E and its phase displacement α from the reference voltage E' may be computed from eqs. (1) and (2).

In practice E' is usually the secondary voltage of a standard transformer of the same nominal voltage ratio as the transformer under test. This reference transformer must have been carefully calibrated, both as to ratio and phase angle, in a specially equipped laboratory. Methods in which the ratio and phase angle of the test transformer are determined with reference to a standard transformer are known as *relative* or *comparison* methods (§§ 423 to 425). On the other hand, the *absolute* methods (§426) are those by means of which the ratio and phase angle are determined directly by comparison of the primary and secondary voltages of the transformer under test.

As an example of the results obtained by the comparison method, suppose that the ratio of the standard transformer is 0.3 per cent above and that the ratio of the transformer under test is 0.1 per cent below that of the standard transformer. Then the absolute error of the test transformer is 0.2 per cent high. Similarly, if the phase error of the standard transformer is 20 minutes, lagging, and that of the test transformer is 30 minutes (lagging with respect to the standard transformer), then the absolute error of the test transformer is 50 minutes (lagging).

423. Opposition Form of the Comparison Method. — In this method of testing, the potential transformer under test and a standard transformer of the same nominal ratio, each carrying a suitable meter load, are connected in parallel on the high-tension or primary side. On the low-tension side they are connected in opposition. Any meter which will read accurately the small voltage difference between the two transformers will serve to make the comparison. This meter might be a sensitive low-reading voltmeter, although a-c voltmeters are not accurate at fractional scale readings. It might be a vacuum-tube voltmeter, but this is not sufficiently portable and direct reading. It might be a d-c voltmeter or galvanometer with a synchronous rectifier or oxide rectifier, etc. The type of meter preferred for the purpose, however, is the comparator voltmeter, although a suitable separately excited wattmeter of the electro-dynamometer type is sometimes used (§424).

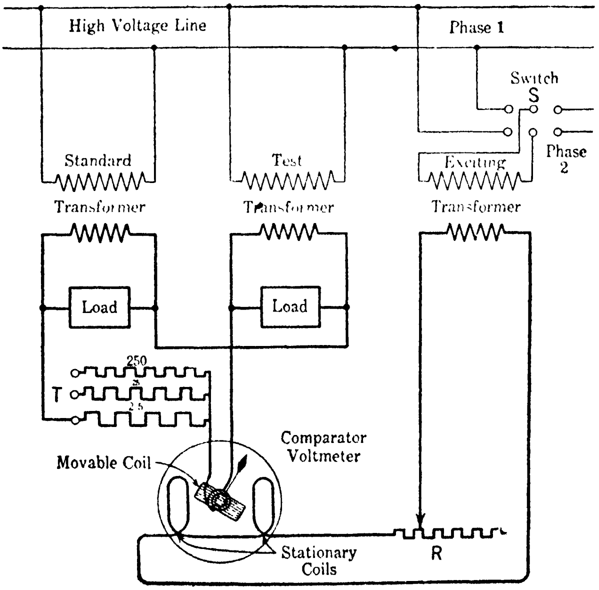

FIG. 298. The comparator voltmeter used in the calibration of potential transformers.

The comparator voltmeter is of the electro-dynamometer type. The stationary coils are excited from a separate or "exciting" transformer (Fig. 298), which may be connected to the same phase (1) as the test transformer, or to phase 2 in which the voltage wave is at a known angle, preferably 90 degrees, from phase 1.

The three terminals of the comparator voltmeter, denoted by T, are connected to the movable coil through different series resistances, and the markings 2.5, 25, and 250 refer to the values of full-scale deflection of the instrument, in volts. For transformers of the same nominal ratio the 2.5-volt scale is usually sufficient; if not, the 25-volt scale is used. The 250-volt scale is used in preliminary trials only, to protect the instrument. The steps of the resistance R are marked for different secondary voltages, from 95 to 125, so that the specified constant current (about 0.2 ampere) can always be made to flow through the field winding. Because of the separate field excitation, the scale divisions are practically uniform, and not crowded at the lower part of the scale as in the ordinary dynamometer-type voltmeter.

First the switch S is thrown to the left and the voltmeter deflection read. Since the exciting transformer is connected to the same phase as the other two transformers, and the resistance R is non-inductive, the field current is practically in phase with the secondary voltages of the two transformers under comparison. This current is marked I_1 in Fig. 297. A dynamometer-type instrument measures the average torque between the current in the stationary winding and the in-phase component of the current in the movable coil (§95). The current in the movable coil is due to the differential voltage AB, and because of the large resistance in series with this coil, the current is practically in phase with AB. Thus, with a constant field current, the deflection of the instrument is proportional to the component of AB in phase with I_1. Because of the small magnitude of the angle, between I_1 and E this component of AB is practically equal to m. If m and E' are known, the unknown voltage E may be computed from eq. (1).

Then the switch S is thrown to the right, bringing the field current practically in quadrature with E and E'. The new current is denoted in Fig. 297 by I_2. By the same reasoning as before, we find that the voltmeter deflection is practically equal to n; n being known, the angle α is computed from eq. (2).

The auxiliary phase not in quadrature with the main phase. Even if a two-phase supply is not available, a quadrature emf may be improvised by using a T-connection (Vol. II). However, should it be impossible to get a quadrature voltage, the field current for the second reading may be displaced with respect to I_1 by a known angle θ (Fig. 297). For example, if one of the remaining phases of a three-phase supply is used, the angle θ can be made practically equal to 60°. In this case the voltmeter measures the component of the voltage AB in phase with I_3.

The projection of AB on I_3 is equal to the sum of the projections of m and n on I_3. Remembering that the directions of OB and I_1 practically coincide, we get

$$AB \cos (AB, I_3) = m \cos \theta + n \sin \theta \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this expression the left-hand side represents the actual voltmeter reading; m is known from the first reading and the angle θ is known. Therefore, n may be computed, and then α determined from eq. (2). For $\theta = 60$ degrees, the foregoing expression becomes

$$n = 1.16 \, [AB \cos (AB, I_3) - 0.5m] \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since m is small as compared to E or E', a considerable error in its value will affect the value of the transformer ratio but very little. For this reason the angle θ need not be known very accurately. Thus, the differential method has the advantage that only moderate accuracy in measured quantities is required.

Signs of errors. The scale of the comparator voltmeter is marked to indicate whether the voltage of the transformer under test is higher or lower than that of the standard transformer. As a check, it may be noted that adding a secondary load on the transformer (especially an inductive load) lowers its secondary terminal voltage. If the addition of such a load increases the voltmeter reading with current I_1, then the standard transformer has a higher voltage, and vice versa. As to the phase angle, the reversed secondary voltage *at no load* usually leads the primary voltage. The addition of a non-inductive load always tends to cause the secondary voltage to lag. Such an increase in load should be tried with the current I_2, and if the reading n increases, then E was originally lagging behind E', and vice versa.

424. Wattmeter Used as a Comparator. — The comparator voltmeter described above is a special instrument and may not always be available. The preceding method can be used with an ordinary dynamometer-type indicating wattmeter, provided that its deflections can be made of sufficient magnitude and also provided that certain precautions are observed. The instrument selected should be of low current-range, say 2 or 5 amperes, and its stationary coils may have to be overloaded to obtain a magnetic field of sufficient strength. The potential coil always possesses some unavoidable inductance, so that sufficient resistance must be used in series with it to bring the current approximately in phase with the differential voltage to be measured. The wattmeter must be calibrated in volts with a given field strength. The use of a wattmeter for the calibration of potential transformers antecedes that of the comparator voltmeter, and the latter has been developed in order to overcome the limitations and inaccuracies involved in the use of an ordinary wattmeter.

425. Comparison Method Using Two Watthour Meters. — This method is a modification of that shown in Fig. 298; the connections are shown in Fig. 299. The two potential transformers to be compared are connected to the potential circuits of two induction-type watthour meters (§125) **A** and **B**. The series coils of these meters are supplied with current from an auxiliary transformer connected to a non-inductive load.

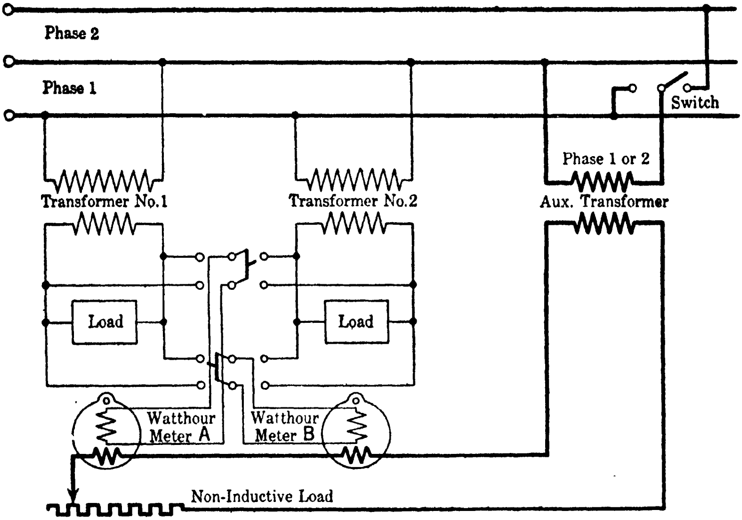

FIG. 299. Two watthour meters used in calibration of a potential transformer.

Consider first the general case illustrated in the vector diagram of Fig. 300, drawn for transformer 1 and showing the current in the series coils of the watthour meters at an angle θ from the primary voltage E'. Note that the (reversed) secondary voltage E'' of the transformer may either lead E', when the current I_2 of the burden is of very low power factor, or lag behind E' when the burden is of higher power factor. Call angle α positive when E'' leads E'.

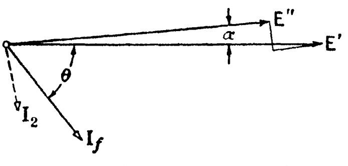

FIG. 300. Vector relations during test of potential transformer.

Let A_1 be the number of revolutions of meter **A** during a chosen interval of time t_1, when connected to transformer 1. Let B_2 be the corresponding number of revolutions of meter **B** running from transformer 2. Let p and q be the constants of the meters. Watthour meter **A** on transformer 1 reads

$$pA_1 = E''I_ft_1 \cos (\theta + \alpha_1)$$

Correcting this for the ratio R_1 of transformer 1 gives

$$R_1pA_1 = R_1,I_ft_1 \cos(\theta + \alpha_1) = E't_1I_f \cos(\theta + \alpha_1)$$

To be exact, however, the meter, when corrected for the ratio of the transformer, should give the energy,

$$P = E'I_ft_1 \cos\theta$$

which requires that the meter reading be multiplied by the ratio $\cos\theta/\cos(\theta + \alpha_1)$. Therefore, the corrected indication would be

$$R_1pA_1 \cos\theta/\cos(\theta + \alpha_1) = E'I_ft_1 \cos\theta \quad \ldots\ldots \quad (5)$$

Similarly, for meter **B** on transformer 2, corrected for ratio R_2,

$$R_2qB_2 \cos\theta/\cos(\theta + \alpha_2) = E'I_ft_1 \cos\theta \quad \ldots\ldots \quad (6)$$

From eqs. (5) and (6) one may write

$$R_1pA_1 \cos\theta/\cos(\theta + \alpha_1) = R_2qB_2 \cos\theta/\cos(\theta + \alpha_2) \quad \ldots \quad (7)$$

Now throwing the switches shown in Fig. 299 so as to put meter **A** on transformer 2 and meter **B** on transformer 1, gives readings A_2 and B_1 so that one may write, reasoning as above, that

$$R_2pA_2 \cos\theta/\cos(\theta + \alpha_2) = R_1qB_1 \cos\theta/\cos(\theta + \alpha_1) \quad \ldots \quad (8)$$

Dividing (8) by (7) gives

$$\frac{R_2A_2 \cos(\theta + \alpha_1)}{R_1A_1 \cos(\theta + \alpha_2)} = \frac{R_1B_1 \cos(\theta + \alpha_2)}{R_2B_2 \cos(\theta + \alpha_1)} \quad \ldots\ldots \quad (9)$$

Therefore,

$$(R_2/R_1)^2 = \frac{A_1B_1 \cos^2(\theta + \alpha_2)}{A_2B_2 \cos^2(\theta + \alpha_1)}$$

or

$$R_2/R_1 = \frac{\sqrt{A_1B_1} \cos(\theta + \alpha_2)}{\sqrt{A_2B_2} \cos(\theta + \alpha_1)} \quad \ldots\ldots \quad (10)$$

CASE I. *Determination of ratio.* In eq. (10), R_2 and α_2 are known for the standard transformer (2); R_1 and α_1 for the test transformer are to be determined. R_1 may be calculated if the data are substituted in eq. (10) provided the tests are performed with $\theta = 0$. This condition will result if the auxiliary circuit is supplied from phase 1 and the load on the circuit is non-inductive. Assume $\theta = 0$ in eq. (10), then

$$R_2/R_1 = \sqrt{A_1B_1/A_2B_2} \cos\alpha_2/\cos\alpha_1 \quad \ldots\ldots \quad (11)$$

Since α_2 and α_1 are normally less than one degree, the ratio $\cos\alpha_2/\cos\alpha_1$ will be essentially unity, so that one may write

$$R_2/R_1 = \sqrt{A_1B_1/A_2B_2} \quad \ldots\ldots\ldots \quad (12)$$

This may be put in a form for accurate calculation with the slide-rule by writing

$$\sqrt{\frac{A_1B_1}{A_2B_2}} = \sqrt{\left(1+\frac{A_1-A_2}{A_2}\right)\left(1+\frac{B_1-B_2}{B_2}\right)} = \sqrt{1+\frac{A_1-A_2}{A_2}+\frac{B_1-B_2}{B_2}}$$

$$= 1+\frac{A_1-A_2}{2\mathrm{A}_2}+\frac{B_1+B_2}{2B_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

or

$$\frac{R_2-R_1}{R_1} = \frac{1}{2}\left[\frac{A_1-A_2}{A_2}+\frac{B_1-B_2}{B_2}\right] \quad \ldots\ldots \quad (13a)$$

The calculations may be further simplified if the test is so conducted that $A_2 = A_1$. In the derivation of eq. (13a), use is made of the fact that normally A_2 and A_1 are nearly equal so that $(A_1 - A_2)/A_2$ is a very small quantity. So also is $(B_1 - B_2)/B_2$. Thus, where products or squares of these very small quantities occur they may be neglected.

From eq. (13) the ratio R_1 of the test transformer is found, since R_2 for the standard transformer is already known.

CASE II. *Determination of phase angle.* If the auxiliary transformer is connected to phase 2 of a three-phase system, the angle θ will be very closely 60 degrees. Rewriting eq. (10)

$$\frac{\cos(\theta+\alpha_1)}{\cos(\theta+\alpha_2)} = \frac{R_1}{R_2}\sqrt{\frac{A_1B_1}{A_2B_2}} \quad \ldots\ldots\ldots \quad (14)$$

Expanding the first term and dividing by cos θ

$$\frac{\cos(\theta+\alpha_1)}{\cos(\theta+\alpha_2)} = \frac{1-\tan\theta\tan\alpha_1}{1-\tan\theta\tan\alpha_2}$$

Replacing the quantity $\sqrt{A_1B_1/A_2B_2}$ by its equivalent in eq. (13) one may now write eq. (14) as

$$\frac{1-\tan\theta\tan\alpha_1}{1-\tan\theta\tan\alpha_2} = \frac{R_1}{R_2}\left[1+\frac{A_1-A_2}{2A_2}+\frac{B_1-B_2}{2B_2}\right]$$

Multiplying both sides of this equation by $1 - \tan\theta\tan\alpha_2$, and dropping terms such as $\tan\alpha_1(A_1 - A_2)/2A_2$, then collecting terms

$$\tan\alpha_2 - \tan\alpha_1 = \frac{1}{\tan\theta}\left[\frac{R_1-R_2}{R_2}+\frac{A_1-A_2}{2A_2}+\frac{B_1-B_2}{2B_2}\right] = \alpha_2 - \alpha_1 \quad (15)$$

Or, expressing α_2 and α_1 in minutes,

$$\alpha_2(\text{in minutes}) = \alpha_1 + \frac{3438}{\tan\theta}\left[\frac{R_1-R_2}{R_2}+\frac{A_1-A_2}{2A_2}+\frac{B_1-B_2}{2B_2}\right] \quad (16)$$

Since R_1 is known from the previous test and R_2 and α_2 are known for the standard transformer, α_1 may be calculated from eq. (16). Some typical ratio and phase angle values at loads of various values and power factors are shown in Fig. 301 (Westinghouse Company).

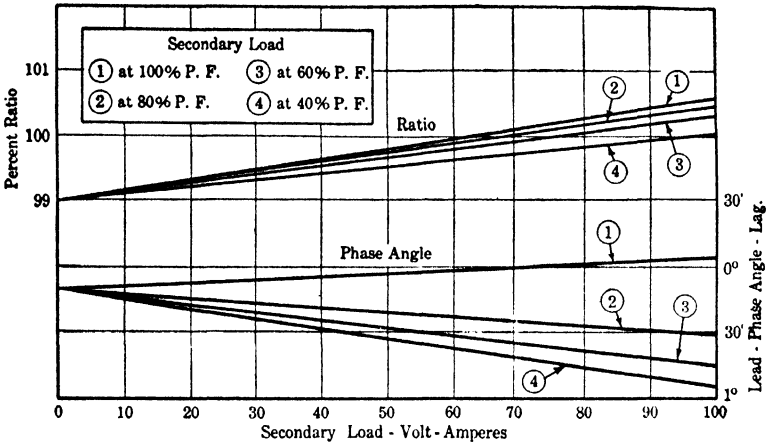

FIG. 301. Typical ratio and phase angle curves for commercial voltage transformers, ratio 20 to 1, 60 cycles.

426. Absolute Methods of Calibrating Potential Transformers.— Such methods require special electrical measuring devices of high accuracy, and for this reason the work of standardization of potential transformers is usually undertaken only in especially well-equipped laboratories. Standardized transformers are then used for the determination of errors of commercial transformers as outlined above. In what follows, only the *potentiometer methods* will be given. For other methods the reader is referred to special papers on the subject in the *Trans. A.I.E.E.*, Bulletins of the Bureau of Standards, etc. (see bibliography at close of chapter).

The potentiometer method. The principle of the potentiometer method for determining transformer ratio was illustrated in Fig. 276 as one of the methods for determining the ratio of power transformers. A greater accuracy is necessary in the calibration of instrument transformers, and it is also necessary to modify the method to include the determination of the phase angle.

In Fig. 302 the voltage is stepped up in any suitable transformer to the rated high-tension value of the transformer X under test. Primary and secondary windings of X are connected in a subtractive sense. Sliding contacts c and d are arranged to move along the taps of a high non-inductive resistance placed across the high-tension winding, until the voltage E_2 is nearly balanced by the drop E_d between c and d. The ratio of the resistance between c and d to the total resistance bd gives E_d

in per cent of the high-tension voltage E_1. The balance between E_2 and E_d may be revealed by any of several detectors.

(a) *Galvanometer and synchronous rectifier.* One of the most sensitive detectors consists of a d-c galvanometer, with suitable multiplying resistance or shunt, connected in the circuit with a synchronous rectifying device (commutator or reversing switch) driven either by a small synchronous motor or from the alternator of the supply. The galvanometer and multiplier may be calibrated directly in volts or millivolts.

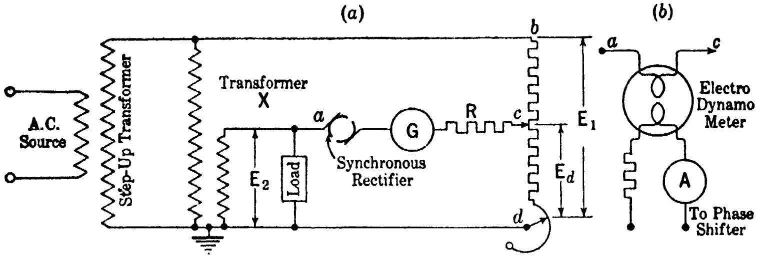

FIG. 302. Potentiometer method of calibrating potential transformers,
(a) Using synchronous commutator and galvanometer as detector,
(b) Using a dynamometer with field excited through a phase shifter.

A particular advantage of the galvanometer and synchronous rectifier is that the galvanometer may be made to read two components of a voltage by shifting the brushes through a corresponding angle, say, 90 degrees. Referring to Fig. 303, let AB represent the vector difference between E_2 and E_d. If the two components, AC and CB, can be determined, this will permit the calculation both of the phase angle α and the value of E_2 in terms of E_d (known as a certain percentage of the high-tension voltage) so that the ratio R_1 of the transformer will be known.

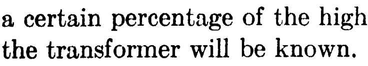

FIG. 303. Voltage relations in the calibration of a potential transformer, potentiometer method.

In the application of this method the galvanometer and rectifier may be connected between c and d with these contacts brought practically together, thus giving a small voltage drop in phase with the high tension. If the rectifier brushes are now moved into the position giving the maximum galvanometer deflection, the detector is evidently sensitive to voltages in line with E_d. Now connecting as in Fig. 302, and moving contacts c and d until E_d as nearly as possible balances E_2 and at the same time adjusting the brush setting slightly until the galvanometer reads a minimum, it is certain that the galvanometer is insensitive to voltage drop in the direction BC and of maximum sensitivity to voltage in the direction of E_d. The slight reading of the galvanometer under this condition is the component $AC = m$. Shifting the brushes 90 degrees and reading the galvanometer the new reading is the component $CB = n$. Then, practically, $E_2 = E_d + m$, and $\tan \alpha = n/E_2$.

(b) *Electro-dynamometer and phase shifter.* In place of the synchronous rectifier and galvanometer a separately excited electro-dynamometer (or wattmeter) may be used, the phase of the exciting current being adjusted by a phase shifter (see Fig. 302*b*). Shifting the phase of the exciting current is equivalent to shifting the brushes of the synchronous rectifier, and the calibration is carried on in the manner described above.

(c) *Vibration galvanometer as detector.* In some laboratories a vibration galvanometer is preferred as the detector. In order to get both ratio and phase angle of the transformer it is necessary to bring E_d into phase with E_2, which may be done by inserting a variable inductance L in the circuit of the resistances R_1 and R_2 of Fig. 304. The effect of varying L may be obtained by shunting a variable portion of R_2 by a condenser C. To secure the required data for the calibration the resistance R_1 is varied, as also the shunted portion of R_2 until the vibration galvanometer shows a balance. Then, the ratio of transformation is[1]

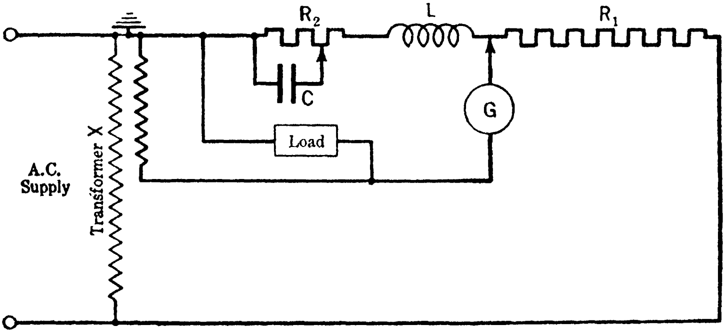

FIG. 304. Calibration of a potential transformer, using a vibration galvanometer as detector.

$$E_1/E_2 = \frac{R_1 + R_2}{R_2} \cos\theta \quad \ldots \ldots \quad (17)$$

and

$$\theta = \tan\theta = \omega L\left(\frac{1}{R_2} - \frac{1}{R_1 + R_2}\right) \text{radians}$$

$$= 3438\,\omega L\left(\frac{1}{R_2} - \frac{1}{R_1 + R_2}\right) \text{minutes} \quad \ldots \quad (18)$$

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[1] See "The Testing of Instrument Transformers," by Agnew and Silsbee, *Trans. A.I.E.E.*, Vol. 31, p. 1636.

427. EXPERIMENT 18-A. — Calibration of a Potential Transformer. —The purpose of the experiment is to become familiar with one or more of the methods of calibration described in the preceding sections. The diagrams of connections and general procedure are sufficiently explained there. The ratio and the phase angle of the transformer under test should be determined for light load, for rated load, and for an overload; also for a practically non-inductive load and for one at low power factor. In view of the small magnitude of the quantities to be measured, it is particularly important to make the connections and to take the readings with the utmost care. Before the test and during the experiment, it is desirable to investigate the principal sources of inaccuracy and errors, and to estimate their possible effect upon the results. The same potential transformer should be calibrated by one of the comparison methods, and again by an absolute method, and the results compared.

Report. Give the detailed diagrams of connections used and the original readings. Compute the ratio error and the phase angle at different loads, and compare their relative magnitudes with what might be expected from published data and curves on commercial transformers (Fig. 301). Indicate the sources of inaccuracy in the methods employed and in the apparatus used. In each case give the order of magnitude of the error.

CURRENT TRANSFORMERS

428. On a high-tension a-c circuit, or one in which the current exceeds a certain value, it is safer, more convenient, and more economical to connect ammeters, current relays, or the series windings of wattmeters, through a *current transformer* (Fig. 55). Such a transformer consists of a closed iron core, and two windings, one of which (the primary) is connected in series with the line and the other (the secondary) to the measuring instrument, relay, etc. For high voltages, such a transformer is usually immersed in oil, and in its essential details it is similar to a power transformer (Chapter XVI), except for its small size and a nominal rating (sometimes 10 volt-amperes or less).

In order that an ammeter connected through a current transformer may indicate the correct line current, either the instrument must be calibrated with the transformer, or else the current ratio of the transformer must be accurately known. In a good current transformer the magnetizing current and the core loss are very small, so that the ratio of the currents is very nearly equal to the inverse ratio of the turns (§408), at least within a certain range. As with the potential transformer there are cases in which an accurate knowledge of the ratio of transformation is of importance, so that some acquaintance with methods of calibration is essential.

Besides an error in the current ratio, a current transformer introduces a small error in the phase angle between the primary and the secondary currents (Fig. 283). This angle modifies the reading of a watthour meter or a wattmeter, and therefore must be kept low. Its influence is much more pronounced on an inductive load, as is shown by a numerical example in §421. The principal methods of experimental determination of the ratio error and of the phase angle of a current are given below. Such measurements usually require quite accurate instruments and a skilled observer, unless only a rough check is required.

429. Types of Current Transformers and Some Precautions in Use. — A current transformer which is provided with two windings, a primary and a secondary, may be called " self-contained." Each winding sometimes consists of sections which may be connected in series or in parallel, thus considerably increasing the range of the device. This is particularly convenient in portable transformers used for calibration and testing purposes. For heavy primary currents the primary winding is sometimes omitted altogether, and the line conductor or the bus-bar is put through the hole in the iron core (Fig. 305). Such a transformer is variously called the " through " type, the " loop-through " type, the hole or loop type, the stud or bus-bar type, the open primary type, etc. With the same transformer the primary conductor may be passed through the hole once for the high current range, twice for the intermediate range, and three or four times for the low range.

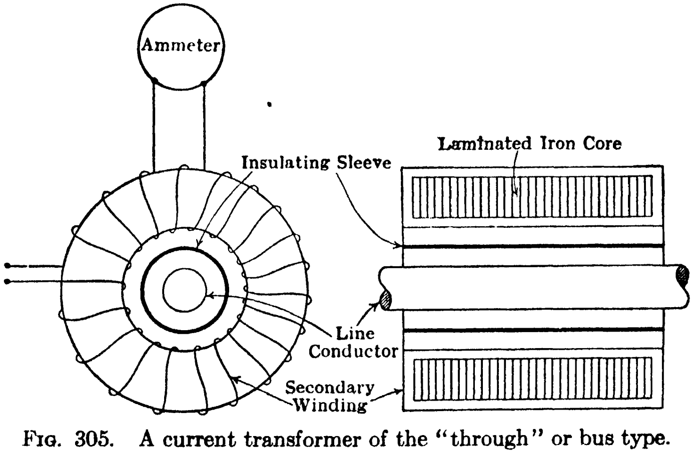

FIG. 305. A current transformer of the "through" or bus type.

There is also a " split-type " current transformer in which one side of the core is detachable, or hinged to the other so that the transformer can be slipped over a line conductor and then tightly clamped in place. Such transformers are used for current measurements in circuits which it would be inconvenient or impossible to open. They are apt to lead to quite a considerable and variable error, especially in the phase angle, and should not be used with a wattmeter if accurate values are required. They are principally useful for an approximate measurement of current, for finding a fault in a cable, etc.

A low exciting current is essential in a current transformer, since it insures a constant ratio and a small phase angle. For this reason the core must be of extremely low reluctance and the winding so proportioned as to require but a low flux density. An accurate current transformer should have no joints in the magnetic circuit. In a split-type transformer the two variable air-gaps increase the exciting current and therefore affect the accuracy of the device. If such a transformer is used for permanent work, the joints must be carefully overlapped and the laminations drawn tightly together with insulated bolts.

Because of the unavoidable magnetizing current and internal losses, current transformers are properly *compensated* within their range with extra turns, to reduce the influence of these errors. For this reason, a transformer which gives a correct ten-to-one ratio will not give a correct one-to-ten ratio when the functions of its primary and secondary windings are reversed.

When using a current transformer with a wattmeter or watthour meter, it is of importance to observe the proper *polarity* of connections. Otherwise the meter may indicate backwards, or a polyphase meter (§126) may indicate the difference instead of the sum of the readings of the two elements.

The primary of a current transformer should never be left connected to the line with the secondary circuit open, as this will set up a heavy flux through the core, saturating the iron, and may induce a dangerous voltage in the secondary winding. Moreover, considerable residual magnetism is liable to remain in the core after the line circuit is opened, and the transformer ratio will be impaired. If for any reason it becomes necessary to disconnect the meter from the secondary winding, the latter should first be short-circuited. Some portable transformers are provided with a special short-circuiting switch for this purpose. Should an open circuit occur accidentally while the transformer is connected to the line, the residual magnetism can be removed by inserting a resistance between the secondary terminals and gradually reducing it to zero while a considerable alternating current is passing through the primary. If this is not done, the transformer may show quite an appreciable error. Such demagnetization should always precede a calibration.

Care must be taken to avoid placing two current transformers close together or having a heavy current lead adjacent to a transformer under test, as stray fields affect the ratio of transformation.

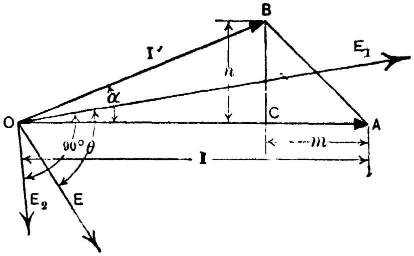

Fig. 306. The general principle of measurement of the ratio error and of the phase angle in a current transformer.

430. Errors in Ratio and in Phase Angle. — The various methods used for the determination of the ratio and of the phase angle in a current transformer may be best explained by reference to Fig. 306. $OA = I$ is the vector of the unknown secondary current in the transformer under test, and $OB = I'$ is a reference current. It is required to determine the magnitude of I and the phase angle α. Let BC be perpendicular to OA. Since angle α is always small, not over 2 degrees in the extreme, OC is nearly equal to OB, $\cos 2°$ being equal to 0.9994. Thus, the *algebraic* difference between OA and OB is practically equal to $AC = m$, or

$$I - I' = m \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (19)$$

AC may be considerably smaller than the *geometric* difference AB between the two vectors. If $BC = n$ is known then

$$\sin \alpha = n/I' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (20)$$

Thus, if m and n are known, the unknown current I and its phase displacement α with respect to the reference current can be computed from eqs. (19) and (20).

In practice, I' is usually the secondary current of another current transformer of the same nominal ratio as the transformer under test, the two transformers being connected in series on the primary side. This reference transformer must be carefully calibrated in an especially equipped laboratory, both as to its ratio and as to its phase angle. The ratio and the phase angle of the transformer under test can then be determined in their relation to those of the standard transformer, by comparatively simple means. Such methods are known as the *relative* methods of calibration (§§432 and 433). On the other hand, the *absolute* methods (§434) are those by means of which the ratio and the phase angle are determined directly by comparison of the primary with the secondary current in the transformer under test. See the last paragraph in §422.

In the methods described below it is necessary to supply the potential circuits of the wattmeter or of the watthour meters with a source of alternating voltage differing by a given phase angle from that supplied to the current coils. A potential phase-shifter (§132) may be conveniently used for this purpose.

THE CALIBRATION OF CURRENT TRANSFORMERS

431. Methods of Testing Current Transformers. — The following are methods of testing current transformers:[1]

A. *Relative (or Comparison) Methods.*
  1. *Deflection.*
     - Interchanged ammeter method.
     - Interchanged wattmeter method.
     - Interchanged watthour meter method.
  2. *Balanced.*
     - Differential wattmeter method.
     - Bridge circuit method.
     - Null bridge method.

B. *Absolute Methods.*
  1. *Deflection.*
     - Two-ammeter method.
     - Two-wattmeter method.
  2. *Balanced.*
     - Mutual inductance method.
     - Resistance method.
     - Baker test ring.

Of the methods enumerated above, the comparison methods are in general simpler and require less sensitive and delicate detecting instruments. However, the absolute methods are capable of greater accuracy and require no standardized transformer for purposes of comparison. Some brief description will be given below of two of the comparison and two of the absolute methods.

432. Interchanged Watthour Meter (Agnew) Method. — In Fig. 307, which should be compared with Fig. 299 for potential transformers, connections are shown for calibrating a current transformer by comparison with a standard transformer, using two watthour meters. This method

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[1] Dr. F. B. Silsbee, "Methods of Testing Current Transformers" (*Trans. A.I.E.E.*, Vol. 53, p. 282).

is exactly analogous to the two-watthour-meter method of calibrating a potential transformer and is carried out in the same manner. The same equations for ratio and phase angle apply. One of the watthour meters should be of the rotating standard type and have a graduated disk. This method yields satisfactory accuracy but is slow because of the time required for the meters to register sufficient readings.

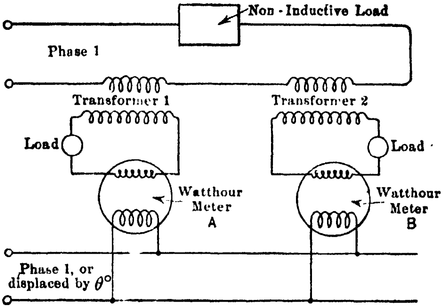

FIG. 307. The use of two watthour meters for calibration of current transformers.

433. The Null Bridge Method. — The connections for the null bridge, or Silsbee, method are given in Fig. 308. The primary windings of the standard and test transformers are connected in series to a suitable source of current. The secondary windings are so connected that the currents flow around the circuit in the same direction. A bridge circuit, connected between the points f and b, will carry the difference of the secondary currents, or $I_s - I_x$. By suitable adjustments of r_2, M, and r_1 the deflection of detector G may be brought to zero, showing that the points f and b are at the same potential. Writing the sum of the potential drops from b to f for the condition of balance,

$$(I_s - I_x)r_1 + (I_s - I_x)j\omega L + I_s j\omega M - I_x r_2 = 0$$

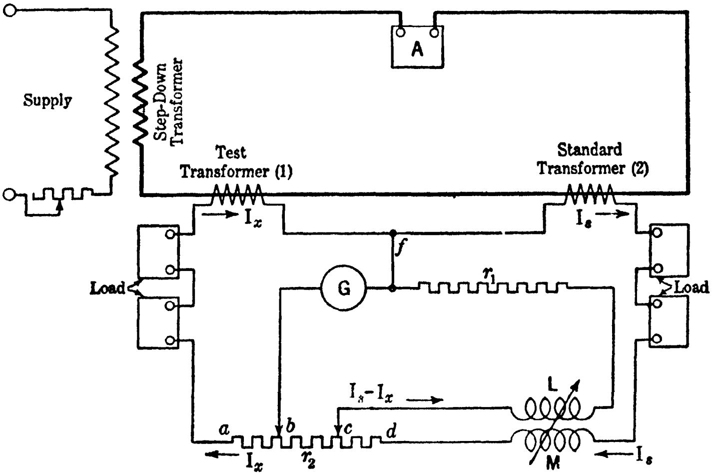

FIG. 308. Null bridge method of testing current transformers.

or $$I_s r_1 + I_s j\omega L + I_s j\omega M - I_x r_1 - I_x r_2 - I_x j\omega L = 0 \quad . \; . \; (21)$$

so that $$\frac{I_s}{I_x} = \frac{r_1 + r_2 + j\omega L}{r_1 + j\omega(L + M)} = \frac{1 + r_2/r_1 + j\omega L/r_1}{1 + j\omega L/r_1 + j\omega M/r_1}$$

$$= \frac{1 + a + jc}{1 + j(b + c)} \quad . \; . \; . \; . \; . \; . \; . \; . \; (22)$$

Where $a = r_2/r_1$, $b = \omega M/r_1$, and $c = \omega L/r_1$, all of which will be small compared with unity in any practical case.

Let $R_x = I'/I_x$ be the ratio of transformer **x**, and $R_s = I'/I_s$ be the ratio of the standard transformer. Then

$$R_x/R_s = I_s/I_x = \sqrt{(1 + a)^2 + c^2}/\sqrt{1 + (b + c)^2} \quad . \; . \; . \; . \; (23)$$

$$= \frac{1 + a + c^2/2}{1 + b^2/2 + bc + c^2/2} \text{ (approximately)} \quad . \; (24)$$

Dividing, and dropping higher powers of the very small quantities,

$$R_x/R_s = 1 + a - bc - b^2/2 = 1 + a \text{ (approximately)} \quad . \; (25)$$

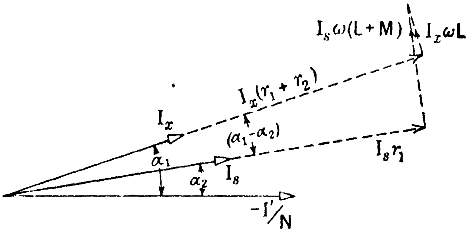

FIG. 309. Vector relations in bridge circuit at balance.

Phase angle. It may be seen from Fig. 309 drawn to represent the vectors of eq. (21)

$$\tan(\alpha_1 - \alpha_2) = \frac{I_s\omega(L + M) - I_x\omega L/\cos(\alpha_1 - \alpha_2)}{I_s r_1}$$

$$\cong \frac{I_s\omega(L + M) - I_x\omega L}{I_s r_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (26)$$

or, dividing by $I_s r_1$,

$$\tan(\alpha_1 - \alpha_2) = \frac{\omega L}{r_1} + \frac{\omega M}{r_1} - \frac{I_x}{I_s} \times \frac{\omega L}{r_1} \quad . \; . \; . \; . \; (27)$$

$$= c + b - \frac{I_x}{I_s}c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (28)$$

Substituting from eq. (25) the approximate expression for $I_x/I_s = 1/(1 + a) = 1 - a$, where a is a small quantity, then

$$\tan(\alpha_1 - \alpha_2) = b + ac \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (29)$$

ABSOLUTE METHODS

434. Absolute Methods. — The most obvious of the absolute methods is that in which two ammeters of proper range are connected into the primary and secondary circuit of the current transformer and simultaneous readings taken for various values of current. The accuracy of this method is limited by the precision of the meters, and this falls off as the square of the current in a-c instruments. It is not permissible to increase the accuracy of the meters by substituting lower-reading instruments at the lower values of current as the burden imposed by the ammeters varies inversely as the square of the meter rating. The two-ammeter method also fails to give the phase angle of the transformer. Two more important methods — the two-wattmeter method and the resistance method — will be described with more detail.

435. The Two-Wattmeter Method. — Two wattmeters of appropriate range are connected as in Fig. 310, with the current coils of the two meters in the primary and secondary circuits, respectively, and the potential coils energized from a common source of emf. By taking readings with the potential first substantially in phase with the current, and then displaced by an angle of 60 or 90 degrees, both ratio and phase angle of the transformer may be calculated. This method is also limited in range because wattmeters are not available for extremely high values of current. The accuracy of the method falls off directly as the percentage of rated current decreases.

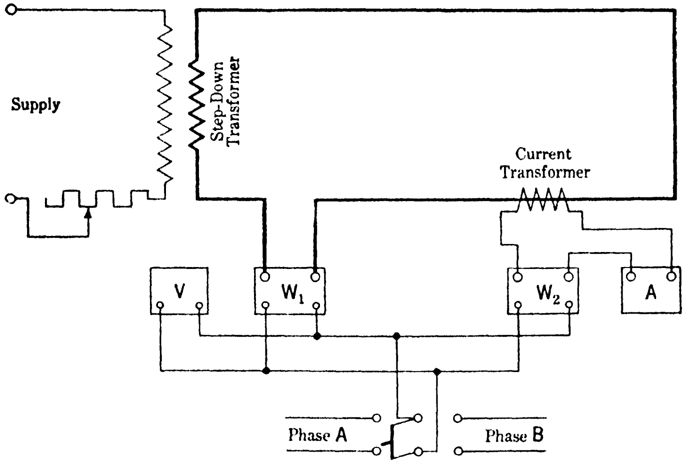

FIG. 310. Two-wattmeter method of current-transformer calibration.

In the application of the method, simultaneous readings are taken of the two wattmeters, the ammeter and voltmeter, at the desired current.

Changing the potential to the other phase, and with the same value of current as before, similar readings are taken. Let A_1 and A_2 be the "watts per volt" readings of wattmeters W_1 and W_2 when the potential circuits are excited from phase A. B_1 and B_2 are similar values when the excitation is from phase B. Then (see Fig. 311)

$$A_1 = I_1 \cos(\theta + \alpha) \quad \text{(30)}$$

and

$$A_2 = I_2 \cos\theta \quad \text{(31)}$$

Therefore

$$I_1/I_2 = A_1 \cos\theta / A_2 \cos(\theta + \alpha) = R \quad \text{(32)}$$

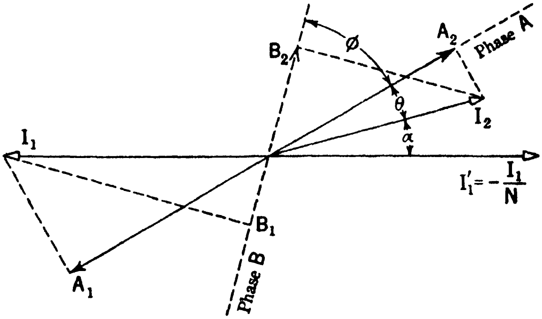

FIG. 311. Relations in the two-wattmeter method.

Expanding, and dividing by $\cos\theta$,

$$R(\cos\alpha - \tan\theta \sin\alpha) = A_1/A_2 \quad \text{(33)}$$

From the readings with phase B, one may write

$$R(\cos\alpha - \tan(\theta + \phi)\sin\alpha) = B_1/B_2 \quad \text{(34)}$$

But $I_1 = A_1/\cos(\theta + \alpha)$; also $I_1 = B_1/\cos(\phi + \theta + \alpha)$

Therefore

$$A_1 \cos(\phi + \theta + \alpha) = B_1 \cos(\theta + \alpha) \quad \text{(35)}$$

Expanding eq. (35) and dividing by $\cos\alpha$, one may solve for $\tan\alpha$, or

$$\tan\alpha = [B_1 \cos\theta - A_1 \cos(\phi + \theta)]/[B_1 \sin\theta - A_1 \sin(\phi + \theta)]$$

$$= \frac{\cos\theta - \frac{A_1}{B_1}\cos(\phi + \theta)}{\sin\theta - \frac{A_1}{B_1}\sin(\phi + \theta)} \quad \text{(36)}$$

The readings of amperes, volts, and watts permit the calculation of the values of $\cos\theta$ and $\cos(\phi + \theta)$. Thus, $\cos\theta = A_2/I_2$ and $\cos(\phi + \theta) = B_2/I_2$. The values of θ and ϕ substituted in (36) will give $\tan\alpha$.

For special cases where $\theta = 0$ and $\phi = 90$ degrees (two-phase system with $\theta = 0$)

$$\tan\alpha = -B_1/A_1, \quad \text{and} \quad R = A_1/A_2 \cos\alpha \cong A_1/A_2 \quad . \; . \; (37)$$

If $\theta = 0$ and $\phi = 60$ degrees (possible with three-phase system)

$$\tan\alpha = \frac{A_1 - 2B_1}{\sqrt{3}A_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (38)$$

436. Resistance Method. — Probably the most generally used of the absolute methods is one in which non-inductive resistances (shunts) of such values that they give practically equal drops (about 1 volt) are connected in the primary and secondary circuits of the current transformer under test. One resistance, normally that in the secondary circuit, and a mutual inductance, are varied until the drops in the primary and secondary circuits are equal and in phase, as shown by a sensitive galvanometer. A vibration galvanometer is an excellent detector. Incidentally it is not affected by the phase of the difference in the drops.

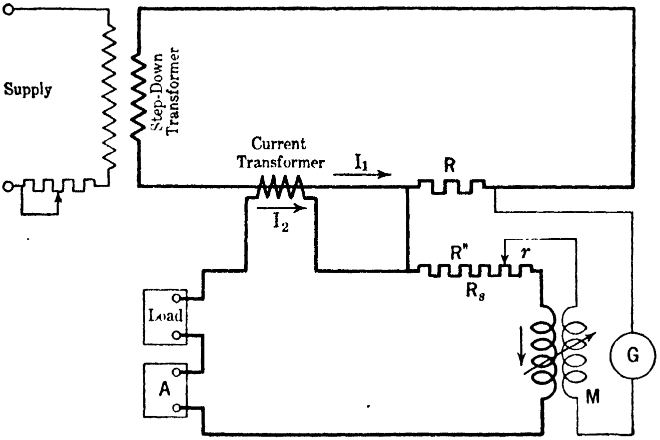

FIG. 312. Connections for the resistance method.

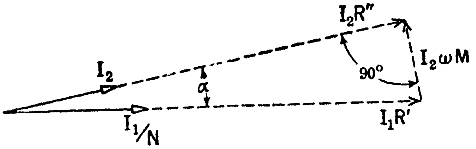

FIG. 313. Relations in the resistance method.

Figure 312 shows the connections for the test, and Fig. 313 the vector relations of the currents and voltage drops. For the case of balance

$$I_1R' + j\omega MI_2 = I_2R'' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (39)$$

From this equation the ratio, R, of the transformer becomes

$$R = I_1/I_2 = R''/\sqrt{R'^2 + \omega^2 M^2} \quad \ldots\ldots \quad (40)$$

The phase angle is given by the relation,

$$\tan \alpha = \omega M/R'' \quad \ldots\ldots\ldots\ldots \quad (41)$$

Since a suitable variable resistance is not always available to use as the non-inductive shunt in the secondary circuit a modification of the resistance method is often made as shown in Fig. 314, where both the primary resistance, R', and the secondary resistance, R'', are shunts of fixed value, with the drop in R'' greater than that in R'.

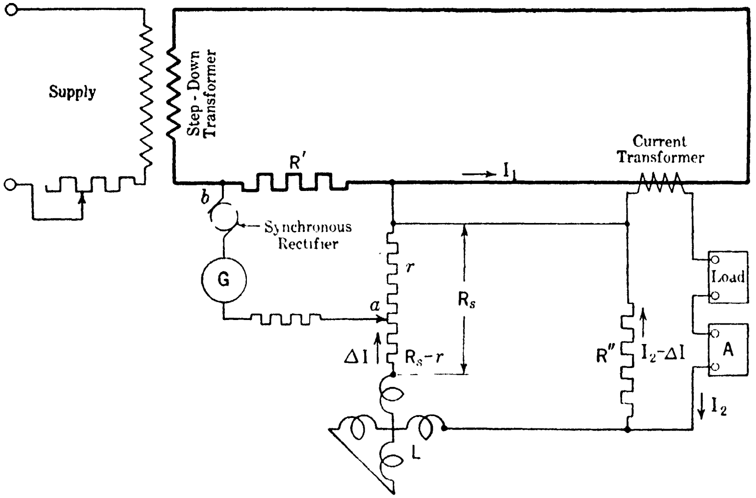

FIG. 314. A modification of the resistance method.

The voltage drop in the resistance R' is balanced by that in a variable portion of the resistance R_s, which, with the variable inductance L, is shunted around the secondary resistance R''. The resistances must be strictly non-inductive. R' and R'' should be in approximately the ratio of the primary and secondary currents, except that the drop in R'' must be somewhat greater than that in R', to obtain a balance. These drops may well be of the order of 1 volt each, at normal current. The galvanometer G may be a vibration galvanometer, a separately excited electro-dynamometer, or a d-c galvanometer with a synchronous rectifier, or operated through a vacuum-tube amplifier and oxide rectifier.

Assuming the use of a vibration galvanometer, the procedure is as follows: The secondary current I_2 is adjusted to the desired value as read by one of the meters which constitute the normal burden on the current transformer. Then L is varied and point a moved along R_s until a balance is obtained, as shown by zero deflection by the galvanometer.

When the synchronous rectifier and d-c galvanometer are used a position of the rectifier brushes is first found which makes the detector sensitive both to IR and IX drops, as shown by the galvanometer when changes are made in either drop. Be sure that the galvanometer circuit is free from inductive effects due to stray fields, as may be determined by connecting together points b and a, which should be arranged to be close to one another. There should now be no deflection of the galvanometer for any position of the brushes. The variable inductance, L, and the shunted portion of R_s should now be varied until a balance is obtained. Then (see Figs. 314 and 315)

$$\Delta Ir = I_1R' \quad \text{or} \quad \Delta I = I_1R'/r \qquad (42)$$

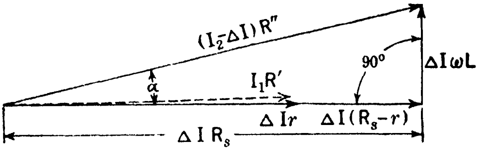

FIG. 315. Relations in the modified resistance method.

From the vector diagram it is seen that

$$\frac{\Delta IR_s}{\cos\alpha} = (I_2 - \Delta I)R'' \quad \text{or} \quad \frac{\Delta IR_s}{\cos\alpha} + \Delta IR'' = I_2R'' \qquad (43)$$

From (42) and (43)

$$\frac{I_1R'}{r}\left(\frac{R_s}{\cos\alpha} + R''\right) = I_2R''$$

or

$$\frac{I_1}{I_2} = \frac{R''r}{\left(\frac{R_s}{\cos\alpha} + R''\right)R'} \cong \frac{R''}{R'}\,\frac{r}{R_s + R''} \qquad (44)$$

The phase angle is given by the relation

$$\tan\alpha = \frac{\Delta I\omega L}{\Delta IR_s} = \frac{\omega L}{R_s} \qquad (45)$$

437. EXPERIMENT 18-B. — Calibration of a Current Transformer. — Following in general the outline given for Experiment 18-A for the potential transformer, carry through the calibration of a current transformer by one comparison and one absolute method, comparing the results obtained by one method with those obtained by the other, and also with those expected from published results on commercial transformers. Figure 316 shows typical ratio and phase angle curves of a commercial transformer at several usual values of burden.

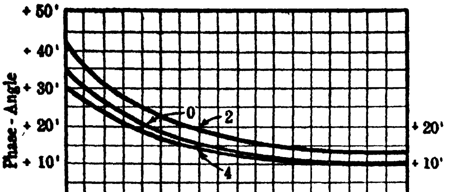

FIG. 316. Typical phase angle and ratio curves of a commercial 25-cycle current transformer.

438. The Two-Stage Current Transformer. — The "two-stage" current transformer devised by Mr. H. B. Brooks (Fig. 317) gives greater accuracy of ratio and freedom from phase-angle errors than can be attained by current transformers of ordinary construction. It consists of two iron cores linked with three secondary windings. The larger or principal core is wound with a principal secondary winding, *s*, of 5-ampere rating. The smaller core is wound with a winding *a'* of very much smaller rating, called the auxiliary secondary. This smaller core is also encircled by a 5-ampere winding, *s'*, of the exact number of turns which the principal secondary winding, *s*, on the left-hand core would have if the principal core had infinite permeability and zero core loss. The winding *s'* is in series with the principal secondary winding, and in opposition to the primary winding *p'*. The terminals of *s* and *s'* are connected to the current winding of a wattmeter, watthour meter, etc. The auxiliary secondary coil is connected in the meter to an auxiliary winding of the same number of

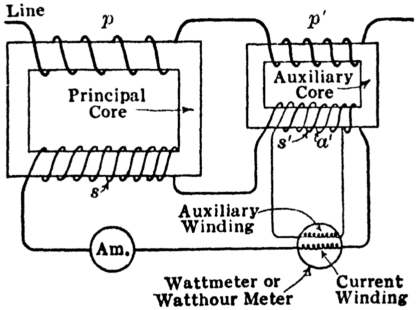

FIG. 317. A two-stage current transformer which corrects errors in ratio and phase angle.

turns as the regular current winding. A modification of the two-stage transformer, making it possible to use a standard-type wattmeter, has been developed.[1]

In operation, there will be no flux in the auxiliary core if the conditions are such that the principal secondary current is correct in ratio and phase, because then the mmf's of p' and s' are exactly equal and opposite. Any departure from the correct value sets up a mmf in the auxiliary core, and a current will flow in the auxiliary windings of both the transformer and the meter and give resultant ampere-turns of nearly correct value in the meter. If the principal secondary current is too low, the auxiliary secondary current is automatically additive, and vice versa. If the principal secondary current has the right value but is out of phase, the auxiliary current is automatically of such phase as to correct it. In other words, the auxiliary transformer is so constructed and connected that the difference of the mmf's of p' and s' acts as its primary excitation, and the winding a' as its secondary. By a suitable modification in the secondary circuit, the auxiliary winding in the meter may be made unnecessary.

Instead of the two physically distinct transformers shown diagrammatically in Fig. 317, it is more convenient to use a single primary winding and a single secondary winding encircling both cores, with the auxiliary secondary winding and a few turns of the main secondary winding surrounding the auxiliary core only. This method of construction produces a two-stage transformer which is physically a single compact unit.

439. Standardization of a Through-Type Transformer. — The comparison methods of calibration of current transformers necessitate the use of a standardized transformer of the same nominal ratio as the one under test. On a large electric supply system, current transformers of many different ratios are usually required, and the problem of reducing the number of standard transformers to a minimum is of considerable practical importance. Some portable transformers have the primary winding divided into sections so that by varying the connections between the sections, different current ratios may be obtained. Even then the number of ratios is usually limited to four.

A " loop-through " transformer (§429) may be conveniently provided with a primary winding of a variable number of turns and used as a universal standard. For example, let the highest desired primary current be 800 amperes and let the secondary winding consist of 160 turns designed to carry 5 amperes, this being the standard instrument

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[1] Bushing-Type Current Transformers for Metering, Boyagian and Skeats, *Trans. A.I.E.E.*, March, 1929.

current. The primary winding is then conveniently made of 16 turns of 10-strand cable, giving 160 turns in series for the one-to-one ratio. The same turns may also be used in several series-parallel combinations and as one straight parallel combination. One or two taps on the secondary winding will still further increase the possible number of current ratios.

Care must be taken to have both windings well distributed around the iron core, and always to use the same turns for a given ratio, because the calibration depends appreciably upon the position of the two windings on the core. Every lack of symmetry affects the ratio as well as the phase angle of the transformer. With a well-distributed secondary winding the difference in the ratio caused by changing from a distributed to a bunched primary winding may be less than 0.1 per cent; but if the secondary winding is bunched, then changes in the primary winding affect the ratio so much as to make such a transformer unsuitable for calibration purposes.

With proper precautions in the arrangement of the two windings, such a transformer may be calibrated at the one-to-one nominal ratio, and the same phase angle used with any other primary number of turns. Thus, a convenient universal standard is obtained with only one set of constants. If there are taps on the secondary winding, a separate calibration must be made for each.

Such a transformer may be calibrated and standardized without outside assistance, by means of two watthour meters (§432) and then used for checking other current transformers. With the one-to-one nominal ratio, one of the watthour meters is connected in series with the primary winding of the transformer and the other in series with its secondary winding. With a perfect transformer the two readings should be identical, and any discrepancy is due to an error in the ratio or to a phase angle. Referring to Fig. 307 and to the theory in §432, the standard one-to-one transformer is here assumed to be perfect. Therefore it is omitted, and the corresponding watthour meter is connected directly in the primary circuit. Thus, the foregoing formulas are directly applicable, if this particular condition is kept in mind.

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CHAPTER XIX

THE ALTERNATOR — OPERATING FEATURES

440. An alternator is an electric generator for producing alternating current or currents by a relative motion of conductors and a magnetic field. More specifically, a synchronous alternator is one in which the magnetic field is excited with direct current and in which there results a reversal in direction of the induced emf each time a conductor passes from a pole of one polarity to that of the other. In other words, the "frequency" (see §441) of the induced voltage is proportional to the rpm of the machine. The essential parts of a synchronous machine are shown in Figs. 318 and 319. The stationary armature core consists of steel laminations held together in a suitable iron or steel frame. Slots are punched in these laminations, and the armature coils placed in the slots are held in place by suitable wedges. In low-speed machines the revolving spider is made of cast iron, cast steel, or welded steel plate; and in high-speed machines, of steel forgings of great tensile strength.

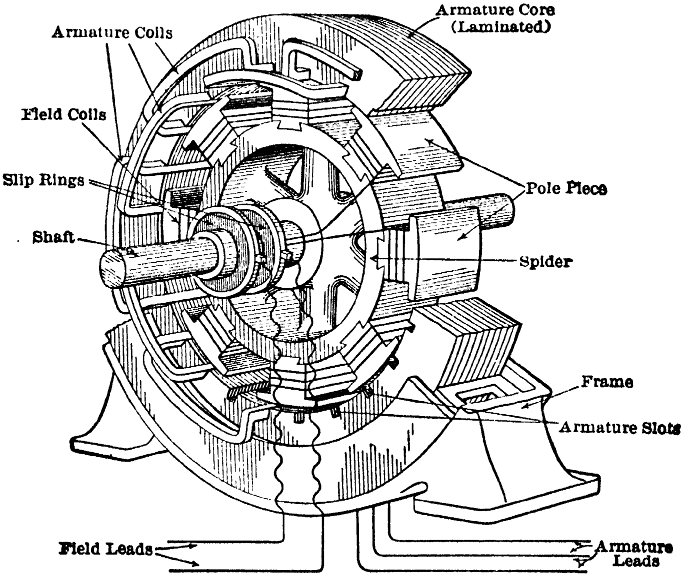

FIG. 318. The principal features of the revolving field alternator or synchronous motor.

Individual pole-pieces are fastened to this spider, and each is provided with a field coil. The field winding is excited with direct current and produces opposite magnetic polarity in adjacent pole-pieces. The exciting current is conducted into the field coils through two slip-rings shown in the sketch.

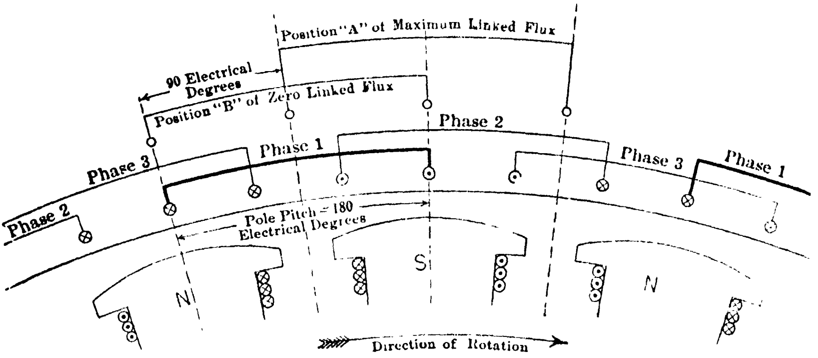

FIG. 319. A three-phase armature winding, one slot per pole per phase.

The particular alternator shown in Fig. 318 is an eight-pole, three-phase machine; it has four armature coils per phase, or one coil side per pole. The armature winding and the field poles are shown partly developed in Fig. 319. All the armature conductors which at the instant under consideration are under the centers of the poles belong to phase 1 and are electrically connected in series. At this instant the voltage induced in these conductors is a maximum, because they are cut by the magnetic flux at its maximum density. The directions of the exciting current and of the induced voltage are shown in the conventional manner by dots and crosses. As the poles move on, the voltage induced in this phase decreases to zero, then increases in the opposite direction, and reaches its negative maximum after the poles have moved by one *pole-pitch*, i.e., when the north poles come into the position previously occupied by the south poles, and vice versa. Thus, one alternation of induced voltage occurs during the interval of time within which the poles move over one pole-pitch.

The coils marked " phase 2 " are identical with those in phase 1, but are displaced with respect to them by two-thirds of the pole-pitch, in the direction of rotation of the poles. The pole-pitch is said to correspond to 180 electrical degrees, and therefore phase 2 is displaced with respect to phase 1 by 120 electrical degrees. The same voltage which is induced in phase 1 is also induced in phase 2, one-third of a cycle later. In other words, the wave or the vector of the voltage induced in phase 2 lags behind that in phase 1 by 120 electrical degrees. Similarly the coils which belong to phase 3 are so placed that the voltage wave induced in them leads that in phase 1 by 120 degrees.

Such an arrangement of armature coils is called a *three-phase* winding, and the three induced voltages may be used either singly or in combination. Most alternators used in power and lighting systems are of the three-phase type. For the general properties and advantages of three-phase over single-phase generation and transmission see Vol. II, under Polyphase Systems. For the elementary experiments described in this volume it is sufficient to become familiar with the simplest method of connecting the three phases, the so-called Y-connection described in §442 below.

All the tests described in this chapter may be performed on either a polyphase or a single-phase machine. For the sake of experience the student should perform them on a single-phase machine first, because the connections and the computations are somewhat simpler. Later, the same experiments should be performed on a three-phase alternator, partly as an exercise in polyphase relations and measurements, and partly because the theory of voltage regulation, explained in Chapter XX, applies more correctly to polyphase machines.

The machine shown in Fig. 318 has a revolving field and a stationary armature. Most machines used for lighting and power are of this type, because the armature is usually wound for fairly high voltage, and a stationary structure can be better insulated and more easily supervised in operation. Moreover, in a three-phase machine a revolving armature would necessitate three slip-rings instead of the two needed for the exciting current. There is, however, a type of alternator with a revolving armature and stationary poles, similar in its construction to the d-c machine (Fig. 199). It is used mostly in small sizes for special purposes. There is also a so-called inductor type of alternator used mostly in high-frequency work, with both the exciting and the armature windings stationary. The pole projections alone are rotated, and the emf is induced in the armature coils by flux variations due to periodic changes in the reluctance of the magnetic circuit.

An alternator, like a d-c machine, not only is capable of converting mechanical energy into electrical, but also may be operated as a motor, converting an electrical input into mechanical energy available on its shaft. It is then called a synchronous motor. In this chapter and in the two following chapters, simple experiments are described on the synchronous generator only. For more advanced and special tests on the synchronous generator, and for the theory of and experiments on the synchronous motor and rotary converter, see Vol. II.

441. Induced Voltage and Frequency. — According to the fundamental law of induction (§163), the emf induced at any instant in a single turn of an armature coil is

$$e = -d\phi/dt \quad \ldots\ldots\ldots\ldots \quad (1)$$

where ϕ is the instantaneous value of the field flux linking with the turn under consideration, and t is time. Multiplying both sides of this expression by dt and integrating within an arbitrary finite interval of time Δt, we get the following expression for the *average voltage* induced during this interval:

$$e_{\text{ave.}}\,\Delta t = \int_t^{t+\Delta t} e\,dt = -\Delta\phi \quad \ldots\ldots\ldots \quad (2)$$

or

$$e_{\text{ave.}} = -\Delta\phi/\Delta t \quad \ldots\ldots\ldots\ldots \quad (3)$$

Here $\Delta\phi$ is the change in the flux linked with the turn during the interval of time Δt.

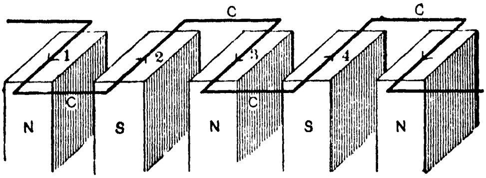

FIG. 320. Relative position of the poles and armature winding when the induced emf is a maximum.

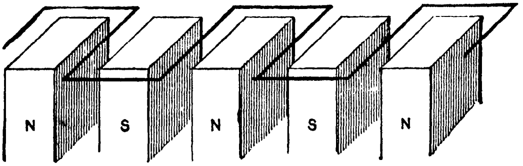

FIG. 321. Relative position of the poles and armature winding when the induced emf is zero.

In a certain position A of an armature coil (Fig. 319), it embraces the whole flux per pole. In another position, B, distant from the first by 90 electrical degrees, the total flux linking with the turn is zero, equal fluxes entering and leaving it due to its position over the poles of opposite polarity. These two characteristic relative positions of the field poles and armature winding are also shown in Figs. 320 and 321. The interval of time within which the flux linked with a coil changes from its full value Φ to zero is one-quarter of a cycle or $\frac{1}{4}T$, where T is the duration of one cycle. Substituting in eq. (3) $\frac{1}{4}T$ for Δt, we get, apart from the minus sign,

$$e_{\text{ave.}} = 4\Phi/T \quad . \quad (4)$$

where Φ is the total useful flux per pole. In practice it is customary to characterize an alternating voltage or current not by the duration of a cycle, but by the frequency f which is defined as the number of cycles per second. For example, if the duration of one cycle $T = \frac{1}{60}$ of a second, the frequency $f = 60$ cycles per second. Thus, in general terms

$$f = 1/T \quad \dots \dots \dots \dots \quad (5)$$

Substituting this value in eq. (4) we get

$$e_{\text{ave.}} = 4f\Phi \quad \dots \dots \dots \dots \quad (6)$$

The emf wave of most alternators closely resembles a sine wave (Fig. 123). This wave-form is usually required in practice, and the pole-pieces and winding are designed accordingly. For a sine wave the ratio between the maximum and the average value is equal to $\pi/2$; the effective value (§§50 and 52) is obtained by dividing the maximum value by the square root of two ($\sqrt{2}$). Since all ordinary voltmeters and ammeters read effective values, and most practical computations, data, and ratings refer to effective values, it is convenient to change eq. (6) accordingly. We then get

$$e_{\text{eff.}} = \tfrac{1}{2}\pi 4f\Phi/\sqrt{2} \quad \dots \dots \dots \dots \quad (7)$$

or

$$e_{\text{eff.}} = 4.44f\Phi \quad \dots \dots \dots \dots \quad (8)$$

Let the armature winding consist of N turns in series per phase, and let the voltages induced in all the turns be equal and in phase with one another. Then the preceding expression for the induced emf per turn must be multiplied by N to obtain the total induced voltage per phase. In reality the requirement for the best utilization of the winding space and other mechanical and electrical considerations make it necessary to distribute the coils of a phase group into several adjacent slots so that their induced emf's are somewhat out of phase with one another. Sometimes it is also desirable to use coils of a width, or pitch, smaller than the pole-pitch. Therefore, the total induced emf is somewhat reduced, and this reduction is taken into consideration by the so-called *breadth factor* k_b, due to the spreading out of the winding, and the *pitch factor*, k_p, due to the use of a fractional pitch coil. Both these factors are smaller than unity. The foregoing equation then becomes

$$E_{\text{eff.}} = 4.44k_bk_pfN\Phi 10^{-2} \quad \dots \dots \dots \quad (9)$$

In this expression $E_{\text{eff.}}$ is in volts and the flux Φ per pole is in megalines; this necessitates the factor 10^{-2}. For further details see the author's book entitled " The Magnetic Circuit." under " breadth factor."

The relation between the number of poles p of the machine, its speed of rotation n, in rpm, and the frequency f of the induced voltage, in cycles per second, is as follows: At f cycles per second the induced voltage has a frequency of $60 \times 2f = 120f$ alternations per minute, because each cycle corresponds to two alternations. During one revolution of the machine, p poles pass under each group of conductors, thus inducing p alternations. Consequently the number of alternations per minute is pn, and we have

$$120f = pn \qquad (10)$$

or the generated frequency, in terms of rpm and number of poles, is

$$f = pn/120 \qquad (10a)$$

When the number of poles or the speed of the machine is not known, the frequency can be measured directly with a *frequency meter.* For a description of this meter see §§454 to 456 below.

442. Three-Phase Y-Connection. — The three separate armature windings shown in Figs. 318 and 319 are usually connected according to Fig. 322. The vectorial relations of currents and voltages are shown in Fig. 323. Each winding is connected on one side to a common point O, and on the other side to a line conductor. Consider that the load is similarly connected to the line and to the neutral point O'. The two neutral points may or may not be connected together. The neutral connection, or the fourth conductor, is shown by the dotted line and is often omitted. Sometimes one or both neutral points are simply connected to ground.

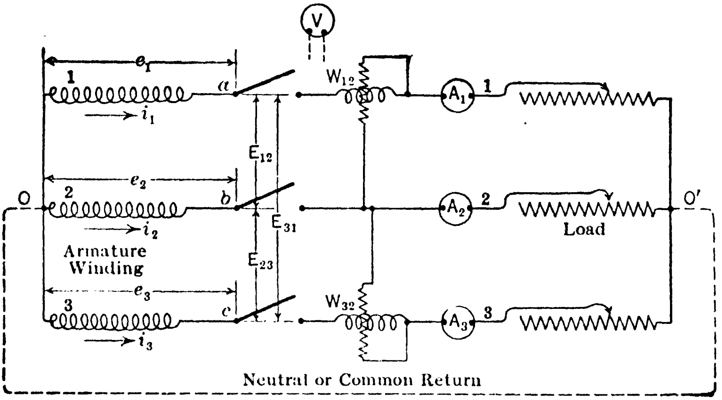

FIG. 322. A three-phase Y-connected system.

An ammeter is shown in each phase, and the load can be so balanced that the three instruments read exactly the same. In this case, considering the three branches of the load to have the same power factor, the three currents differ from each other by 120 degrees in time phase, as shown in Fig. 323. According to the first Kirchhoff law, the current in the neutral connection is equal to the sum of the three line currents. But the geometric sum of three equal vectors displaced by 120 degrees is zero. Therefore, *in a balanced three-phase system with strictly sinusoidal currents, the common return current is equal to zero.* For this reason the return conductor is often omitted when the load can be kept fairly well balanced. With non-sinusoidal currents there is some current in the neutral, even on balanced load. This current consists of third, ninth, fifteenth, etc., harmonics ("triple-*n*," or multiples of the third, harmonics).

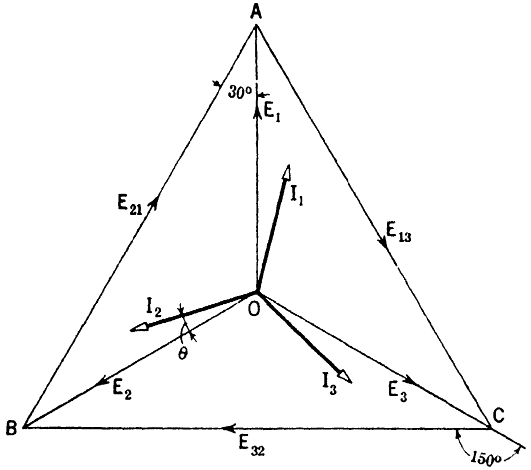

FIG. 323. Current and voltage relations in a Y-connected three-phase system, at a balanced partly inductive load.

Two kinds of voltages are distinguished in a Y-connection, viz., the *phase voltages*, E_1, E_2, E_3, and the *line voltages*, E_{12}, E_{23}, and E_{31}. At any particular instant, e_1 (instantaneous value of the voltage E_1) indicates the amount by which the potential of the point *a* (Fig. 322) is above that of point *O*. Similarly e_2 gives the potential of point *b* above *O*. Therefore, the instantaneous value of the voltage between *a* and *b* is equal to the difference between the instantaneous values e_1 and e_2. But a relationship which holds algebraically for instantaneous values of currents or voltages holds true geometrically for their vectors. For this reason in Fig. 323 each vector of line voltage is shown as the geometric difference between the vectors of phase voltage. Thus, $E_{12} = E_2 - E_1$, etc. The three phase voltages are displaced in phase by 120 degrees because of the arrangement of the windings of the alternator. As a consequence the three line voltages are also displaced 120 degrees with respect to one another, but are 30 degrees displaced with reference to the phase voltages.

From the geometry of the figure it will be found that *in a balanced system with sinusoidal emf's the ratio between the line voltages and the phase voltages is equal to the square root of three* ($\sqrt{3}$); in other words,

$$E_{12} = E_1\sqrt{3} \quad \ldots\ldots\ldots\ldots \quad (11)$$

With a balanced system the points *O* and *O′* are at the same potential, and the phase voltages may be measured with respect to either. When the induced voltages are not sinusoidal the ratio of line voltage to phase voltage differs from $\sqrt{3}$.

With a non-inductive load the three currents are in phase with the corresponding voltages aO', bO', and cO'. With a balanced load these voltages are equal to the phase voltages E_1, E_2, and E_3, so that the currents I are in phase with the corresponding phase voltages. When the load is partly inductive, the currents I lag behind the phase voltages by the power-factor angle θ, as shown in Fig. 323.

The power delivered to the load can be measured either in each phase separately, or by the two-wattmeter method (§105), using the wattmeters indicated as W_{12} and W_{32} in Fig. 322. See also Figs. 100 and 118. A more detailed treatment of polyphase systems will be found in Vol. II, Chapters XXXVIII to XLIV.

443. Corresponding Experiments on A-C and D-C Generators. — The fundamental experiments on alternators, such as the no-load test, voltage regulation under load, efficiency from losses, heat run, etc., are similar in their purpose and general method to those described for d-c machinery in Chapters X to XV. Since the reader is expected to have performed at least the most elementary experiments on a d-c generator before he begins a study of the alternator, it would be an unnecessary repetition to give here all the minute instructions for similar fundamental experiments. The student will do well to consult the corresponding sections on d-c machinery and also his own laboratory reports, partly to get a clear idea of the points of similarity and the differences, and partly so as to be on his guard against the difficulties that may be encountered and the mistakes of judgment committed when performing an experiment for the first time.

The principal additional factors in testing an alternator, as compared to a d-c generator, are as follows:

(1) A separate source of d-c excitation is always required.

(2) The induced voltage is characterized not only by its magnitude but also by its frequency (§§440 and 441).

(3) The load current is characterized not only by its magnitude, but also by its phase angle with respect to the terminal voltage of the machine. Therefore a wattmeter or a power-factor meter is usually required in the circuit.

(4) An alternator is often a three-phase or a two-phase machine, and some knowledge of polyphase relations is necessary (§442).

444. EXPERIMENT 19-A. — No-Load Characteristics of an Alternator. — The purpose of the experiment is to investigate the influence of the exciting current and speed upon the induced emf at no load. The theoretical formula is given in §441. The corresponding d-c experiment is 10-A. It is preferable to perform this experiment on a single-phase machine, or to use one phase only of a polyphase winding, because the relationship is the same in each phase. The diagram of connections is shown in Fig. 324. A frequency meter (§454) may also be connected across the terminals.

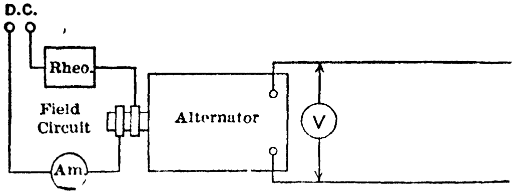

FIG. 324. Single-phase alternator at no load.

(1) Have the alternator connected to an adjustable-speed motor of preferably not more than 10 per cent the rating of the alternator, in order that the losses of the latter will constitute a fair load on the motor, thereby permitting their more accurate determination. Wire up the driving motor so as to be able to read the power input, and later to compute the corresponding values of output (see Experiment 13-B). It is best to hold the field current of the motor constant and to adjust the speed of the set by varying the armature impressed volts, since the use of a constant field simplifies the calculation of the motor rotation losses.

(2) Run the set for 15 minutes or until the friction of the bearings becomes constant.

(3) With no field current on the alternator drive it at several speeds from, say, 20 per cent above normal to 20 per cent below normal. Read speed and all motor values (armature volts, field current, armature current, rpm if different from that of the alternator). Repeat for a fixed value of alternator field.

(4) Driving the alternator at rated speed take its no-load saturation curve, beginning at a very low value of field current and raising the voltage in about 10 approximately equal steps to the maximum value obtainable. Approach each voltage point from below to avoid hysteresis effect. For each point read speed (constant), terminal volts, frequency, field amperes, and drop across the alternator field. Read also all motor values. Starting from the highest point take a few readings with the field current of the alternator reduced in steps toward zero, to determine the effect of residual magnetism (Fig. 201). If the machine is a three-phase alternator take several simultaneous readings of armature phase volts and line volts (§442).

(5) Disconnect the driving motor from the alternator and drive the motor alone at the value of field current previously used. Take all readings. Vary the speed over the range covered during the test, by varying

the armature voltage. For this purpose a series resistance may be used, but this requires some time for motor speeds to become constant. An adjustable booster, connected to boost or buck the shop voltage, gives a better means of speed control.

(6) Measure the resistances of motor and alternator armature windings.

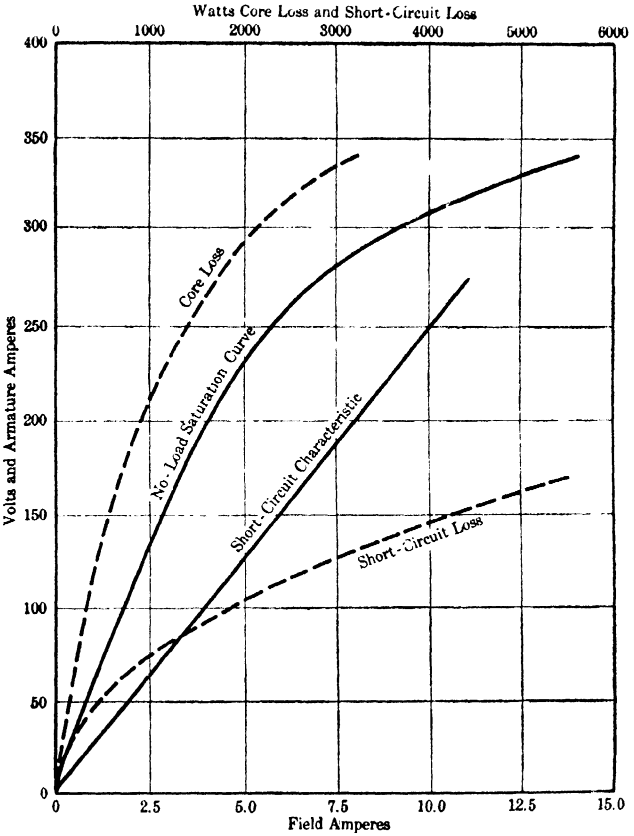

FIG. 325. No-load and short-circuit characteristics of a 37.5-kva alternator.

Report. (1) Plot the observed saturation curves of the alternator, as in Fig. 325. (2) Show from the data that with a given field current the induced voltage is proportional to the speed. Compute the speed at which the machine has to be run for, say, frequencies of 25 cycles, 50 cycles, and 60 cycles. (3) From the input and losses of the driving motor compute its output, which is equal to the core loss, friction and windage of the alternator, for each point taken. Subtracting the friction and windage will give the core loss alone. Plot its values against alternator voltage (see Fig. 325) as abscissas. (4) Check the ratio between the phase and line voltages and explain the departure from the theoretical ratio of $\sqrt{3}$, if any. (5) Calculate the alternator armature I^2R loss at rated kilovolt-amperes.

445. Short-Circuit Test. — This test consists in running the alternator at a constant speed with the armature winding short-circuited through a low-resistance ammeter. The field excitation is varied but is kept quite low so as not to produce an unsafe armature current. The results are plotted as armature amperes vs. field amperes (Fig. 325). On the straight part of the saturation curve the induced voltage is proportional to the field current. Since the short-circuit armature current is proportional to the induced voltage and since the latter is low on short circuit, the armature current is also proportional to the field current.

The short-circuit test is used in practice for various purposes, the principal applications being as follows:

(a) *To check the assembly and the connections of the machine.* For a standard-type machine, of which several have been built and thoroughly tested before, it is often unnecessary to perform a load test; in such cases a no-load run (§444) to check the magnetic circuit, and a short-circuit test to check the armature winding, will be sufficient. If the short-circuit curve checks closely with those taken on the preceding machines, it shows that the machine has been assembled and connected correctly, and that the soldered connections are good.

(b) *Heat run and copper loss.* Since the output on short circuit is zero, only a small motor is required to drive the alternator and to furnish the losses which consist mostly of friction and copper loss. The iron loss is quite small because the field flux is small. Thus, a heat run for the armature winding alone may be conveniently and economically performed, although its results must be used judiciously, because the iron core, being cooler, is helping to conduct the heat away. The input into the driving motor, corrected for the losses in the motor itself, may be taken for the value of the copper loss, friction and windage of the alternator, and used in efficiency computations (§447).

(c) *Voltage regulation.* Knowing the exciting current necessary to produce the rated current on short circuit, one may determine by computation the voltage regulation of the loaded machine at zero power factor (or operating as a synchronous condenser). With certain assumptions, the performance at other values of power factor can also be estimated. This subject is treated more in detail in Chapter XX and in Vol. II.

The technique of the short-circuit test is quite simple, and it is thus desirable for the beginner to perform it, aside from the practical uses to which the results may be put.

446. EXPERIMENT 19-B. — Short-Circuit Test on an Alternator. — The purpose of the experiment is to obtain the relationship between the exciting current and the armature current when the armature winding is short-circuited, and to determine the armature I^2R loss when loaded. The practical uses of such a test are explained in the preceding section. The experiment should be performed on the machine used in Experiment 19-A, in order that the efficiency may be computed from the losses, as explained in §447. The alternator is connected as in Fig. 324, except that an ammeter is placed across the terminals instead of a voltmeter. On a three-phase machine an ammeter should be used in each phase, or else the same ammeter should be transferred from phase to phase (§60). In the latter case, impedances equal to that of the ammeter should be used in the other two phases to balance the circuit.

(1) Connect up the driving motor as in Experiment 19-A, so as to be able to read its input and to compute the corresponding output (§327). Run the machine under test short-circuited and at the rated speed. Starting with very low values, bring the field current up until the armature current reaches a maximum safe value. Read the a-c armature current, the corresponding field current, the input into the driving motor, and its field current and speed. Reduce the alternator excitation in steps to zero and take similar readings at each step. The last readings should be taken with the alternator field circuit and armature circuit open. Disconnect the driving motor and run it alone to determine its own losses. Measure its armature resistance. The corresponding d-c experiment is 13-B.

(2) Increase the alternator speed to a safe upper limit and adjust the field current so as to maintain an overload armature current that the machine can carry for some time without dangerous overheating. Read the speed, the field current, and the armature current. Keep the field current constant and reduce the speed in steps to as low a value as possible. At each step read the speed and the armature current.

(3) Measure the resistance of the armature winding with and without the ammeter and connections.

Report. (1) Plot the short-circuit curve at the rated speed, as in Fig. 325. (2) Plot the readings taken at a constant field excitation and variable speed, and give a theoretical explanation for the general shape of the curve (see the end of §463). (3) From the values of input into the driving motor compute its output, which represents the sum of the losses in the alternator. Subtract the I^2R loss in the ammeter and its leads, and plot the remainder against field current as abscissas. The value at zero excitation gives the friction and windage loss, the remainder represents the armature copper loss and a small iron loss. For compari-

son, plot on the same sheet the computed I^2R loss in the armature. (4) Compute the synchronous reactance according to the definition given in §463.

447. Efficiency from Losses. — The efficiency of an alternator or of a synchronous motor may be computed from its losses in a manner similar to that described in Chapter XIII for d-c machines. The losses in a synchronous machine consist of the following: (a) copper loss in the armature winding; (b) the amount of power necessary for excitation, including the loss in the field rheostat; (c) hysteresis and eddy currents in the armature core; (d) windage and friction; (e) eddy currents in the pole faces, end-shields, frame, etc.

The copper loss in the armature may be computed from the resistance of the winding, or it may be measured directly on short circuit (§445). The latter method is used when the winding consists of heavy conductors, and there is reason to suspect an appreciable eddy-current loss and skin effect, which will make the effective resistance higher than that measured with direct current.

The core loss and the friction and windage can be determined by one of the three methods described in Chapter XIII in application to d-c machines, viz.: (a) The machine is driven mechanically at no load by a small auxiliary motor. (b) The machine is driven electrically as a synchronous motor. (c) The retardation method is used. The reader should have no difficulty in devising similar experiments for a synchronous machine.

The excitation loss may or may not be taken into account in computing the efficiency. This depends upon the agreement between the parties concerned and upon local conditions, such as the source of the exciting current. When specifying the efficiency of a synchronous machine it is therefore necessary to state whether or not the excitation loss has been included, and, if included, how it was computed.

The losses in the pole faces cannot be measured in any simple way, and are usually disregarded, although under some circumstances they may reach considerable proportions. This is especially true in single-phase machines and in polyphase machines with an induced emf of poor wave-shape, that is, one departing considerably from a pure sine wave. The pole-face losses are of the nature of additional load losses, which are discussed in §324; the statements made there in regard to conventional efficiency apply to synchronous machines as well.

When the losses are known, the efficiency at a desired load is computed by means of one of the formulas in §320. An element of uncertainty is introduced by the difficulty of predetermining the exact value of the field current required for the desired load current at a given power factor and terminal voltage. The most accurate way is to make an actual load run under these conditions. If this is not feasible, the required field excitation is determined as explained in §468, or by a more accurate method described in Vol. II. Another element of uncertainty is caused by the difference in the core losses at no load and in the loaded machine, at the same value of field current. Because of the impedance drop in the armature and of the armature reaction, the true field flux under load is different from that at no load, both in its magnitude and in its space distribution; hence the core loss is also different. This factor is difficult to take into account by computation; if the value of core loss used has been determined from a no-load test, the efficiency should be plainly marked "conventional."

The data obtained in Experiments 19-A and 19-B are sufficient for computing the conventional efficiency from the losses. The former experiment (run 4) gives the core loss and friction; the latter experiment (run 1) gives the same friction and the armature copper loss. The latter is augmented, it is true, by some unavoidable core loss which has to be estimated. For a determination of the efficiency of a synchronous machine from an opposition run, see similar experiments on d-c machines and on transformers (§§347 and 398).

448. EXPERIMENT 19-C. — Efficiency of a Synchronous Machine from its Losses. — The theory is explained in the preceding section, and the data available from the preceding two experiments may be used. The field current required for a given load and voltage is best determined by a direct test, or is computed according to §468.

VOLTAGE REGULATION

449. When a synchronous alternator is loaded at a constant speed and constant field current, the terminal voltage is a function not only of the load current but also of its phase displacement with respect to the terminal voltage. In other words, the operating characteristics of an alternator at a constant speed comprise four factors:

A — terminal voltage.
B — field current.
C — armature current.
D — power factor of the load.

It is impossible to represent the influence of all these factors by a single curve, or even by a single set of curves or a surface (§244). Several sets of curves are necessary. For example, in Fig. 326, *A* is plotted against

D for various values of C, keeping B constant and equal say to B_1. Similar sets of curves may be obtained for $B = B_2$, $B = B_3$, etc., covering the useful range of field current.

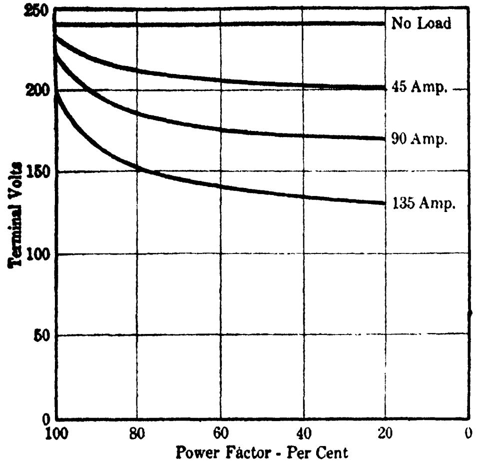

FIG. 326. Voltage characteristics of an alternator as a function of the power factor of the load.

In Fig. 327, A is plotted against C for different values of D, keeping B constant. There are several different combinations of sets of performance curves, depending on which two quantities are used as coördinates, and which quantities are maintained constant. Of these combinations the following four are of particular practical importance:

(1) Voltage characteristics at a constant armature current and excitation (Fig. 326), but with varying power factor.

(2) Voltage characteristics at a constant power factor and excitation (Fig. 327), but with varying load current.

(3) Excitation characteristics at a constant load current and constant terminal voltage (Fig. 328), but varying power factor.

(4) Load saturation curves at a constant armature current and power factor (Fig. 336), but varying excitation.

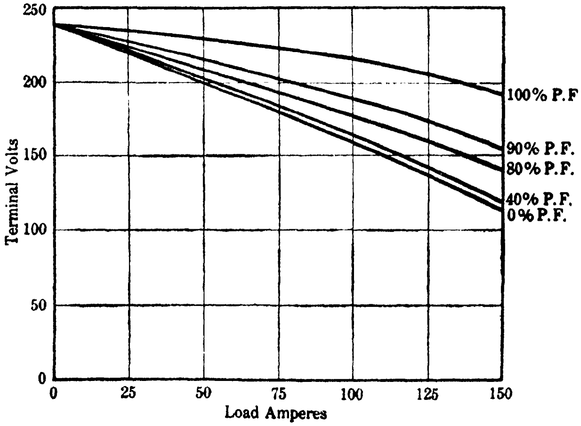

FIG. 327. Voltage characteristics of an alternator as a function of the load current.

One complete set of curves gives a sufficient number of points for determining the operating conditions at any desired values of A, B, C, and D. Therefore, from any one complete set, any of the remaining sets can be plotted by recombining the proper points on the curves of the first set.

More combinations may be obtained by imposing additional conditions upon the four variables. For example, curves may be plotted between B and C when the product CD is constant and A is constant. Each curve gives a relationship between the armature current and the field current at a constant terminal voltage, when the output of the machine remains constant. Such curves are known as *phase characteristics*, and correspond to the well-known V-curves of a synchronous motor. These phase characteristics are of interest in the operation of a synchronous generator when the prime mover (usually a water-wheel) has no governor and is made to deliver a definite amount of power. The input into the generator is constant, and consequently the output is also approximately constant.

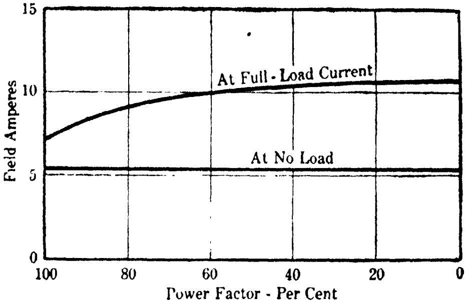

FIG. 328. Excitation characteristics of an alternator.

An inspection of the curves in Figs. 326 and 328 shows that with a given lagging current the voltage drop increases as the power factor becomes lower, that is, as the phase displacement between the current and the voltage increases. When the armature current is leading, the opposite is the case. A leading current causes an internal voltage rise instead of a drop. With a sufficiently large leading current, the field excitation necessary for maintaining a given terminal voltage may be less than at no load.

A theoretical explanation of the fact that the terminal voltage is a function not only of the armature current, but of the power factor as well, is given in Chapter XX. It is sufficient here to take this statement as an experimentally observed fact.

450. Load Test on an Alternator. — In order to obtain experimentally the curves shown in Figs. 326 to 328, the alternator is loaded as is shown in Fig. 329. The diagram of connections is shown in application to a single-phase machine, or to one phase of a polyphase machine, although the great majority of the alternators used for lighting and power purposes are three-phase machines. However, the single-phase test is here preferred since, in the polyphase machine, the readings, as well as the connections, are somewhat more involved. Moreover, it is difficult to keep the load in the three phases accurately balanced. For these reasons the beginner should first get his experience in loading a single-phase alternator.

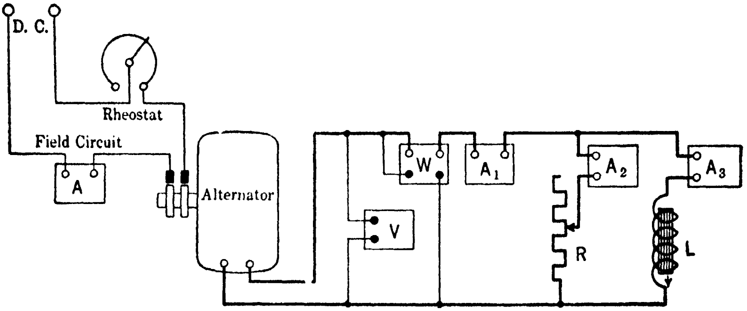

FIG. 329. Connections for loading a single-phase alternator.

The load is shown as consisting of a resistance and an inductance in parallel. This will be found to be a convenient arrangement for regulating both the current and the power factor, although the same result may be accomplished with a resistance and a reactance in series. When the resistance R and the inductance L are used in parallel, it is advisable, although not strictly necessary, to have an ammeter in each branch, as this facilitates current adjustment and is of assistance in working up the data.[1] When R and L are in series, voltage readings across each will make it easier to adjust the power factor to the desired value.

W is a wattmeter, and the power factor is computed as the ratio of watts to volt-amperes. The precautions mentioned in §96 regarding the power consumption in the instrument itself should be considered, although in most cases this will be negligible. An indicating power-factor meter (§109) is convenient for an approximate load adjustment, even though it may not be accurate enough for final readings.

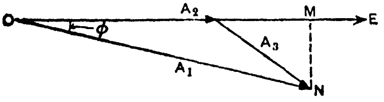

FIG. 330. Diagram of currents read on the three ammeters in Fig. 329.

When working up the data, it is useful, at least for a few points, to draw the three ammeter readings as vectors (Fig. 330). OE is an arbitrary direction of the vector of ter-

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[1] When only one ammeter is used one may adjust for a given total load current at approximately a desired power factor, say 80 per cent, by opening the circuit through L and varying R until the line current has a value of 80 per cent of the desired total current. If then the circuit of L is closed and the value of L varied until the ammeter reads the required total current this current will be at approximately the desired power factor (80 per cent).

minal voltage, and the current A_2, through the non-inductive branch of the load, is in phase with it. On the base A_2, the triangle $A_2A_3A_1$ is constructed, the other two readings being used as the remaining sides. This is because the geometric sum of the branch currents A_2 and A_3 must equal the total line current A_1. The angle ϕ at O is equal to the phase displacement between the total current and the voltage, and the power factor is equal to the ratio of OM to ON. This furnishes a check on the reading of the power-factor meter and on the value of the power factor obtained as the ratio of watts to volt-amperes.

If the main purpose of the experiment is to become familiar with the technique of testing, it may be sufficient and more instructive to take a few curves of each kind rather than a complete set. In performing the experiment always begin with the most difficult conditions, i.e., the highest field excitation, the lowest power factor, and the largest possible armature current.

However, when the particular purpose of the load test is to analyze the armature reaction (Chapter XX), the load saturation curves (Fig. 336) are of particular importance. Several of these should be taken, both with the current lagging and with it leading. To complete the set a curve should be taken at a power factor as near zero as possible. A synchronous motor constitutes a convenient load at very low values of power factor. For a leading current it is over-excited; for a lagging current under-excited, or even run on residual field only.

Sometimes it is impossible to run a synchronous motor at abnormally high or low values of field current, because the machine will not stay in step, especially when operated single phase. It can then be driven by a small d-c motor, to enable it to operate in parallel with the machine under test. When the power factor of the load is below 20 per cent, the terminal voltage of an alternator, at a given armature current, remains practically constant and independent of the power factor (Fig. 326). This performance feature of the alternator makes it unnecessary to adjust the power factor of the load exactly to zero.

451. EXPERIMENT 19-D. — Load Test on an Alternator. — The purpose of the experiment is to determine the influence of the armature current, load power factor, and field current upon the terminal voltage of the machine. The general procedure is discussed in §450, and sample curves are shown in Figs. 326 to 328. According to the specific purpose of the test and the available experimental facilities any two factors may be kept constant and the other two varied. The readings may be conveniently recorded on a data sheet such as is shown below, where it is assumed that three ammeters were used.

	Amperes			Volts	Watts	Power Factor	Field Amps.	Speed
	A_2 Non-ind.	A_3 Ind.	A_1 Total					
Instr. No..								
Constant..								

If the no-load and short-circuit curves of the machine have not been obtained before (§§444 and 446), they should be taken in connection with this experiment. The resistance of the armature should also be measured.

Report. (1) From the wattmeter readings compute the values of the power factor and check them with those read on the power-factor meter and with those obtained from the three ammeter readings (Fig. 330). (2) Plot the test data in some systematic way, so as to bring out clearly the characteristics of the machine and the influence of the desired factor. Show on one or two examples how to obtain " cross-curves," that is, curves for which some other quantity is kept constant. For example, if the terminal voltage is variable for all the curves that have been taken, show how to combine certain points on different curves so as to obtain a relationship between the field current and the armature current for a constant terminal voltage and some constant power factor. (3) If familiar with portions of the theory given in Chapter XX, use the data obtained to illustrate and to check the effect of the armature reaction and reactance upon the voltage regulation of the machine.

452. Voltage Regulator. — Because of the armature reaction and leakage reactance, the terminal voltage of an alternator varies within wide limits with changes in load; this characteristic makes an automatic voltage regulation highly desirable. Numerous attempts at compounding alternators have so far proved to be too complicated. Another solution, namely, the use of an automatic regulator *outside* the machine, is quite satisfactory. The vibrating-type regulator, described in §252 in application to d-c generators, is used with some modifications for maintaining constant terminal voltage of alternators.

The regulator is shown diagrammatically in Fig. 331. Its function consists in periodically short-circuiting the field rheostat of the exciter. The relative duration of short circuit and open circuit of the contacts is made to depend on the terminal voltage of the alternator. The exciter rheostat is short-circuited between two relay contacts, when the differential relay under the contact arm is deënergized. This relay is controlled by the main contacts, which are closed and opened by the d-c electromagnet shown at the left. The duration of the contact is determined by the position of the right-hand lever actuated by the a-c control electromagnet.

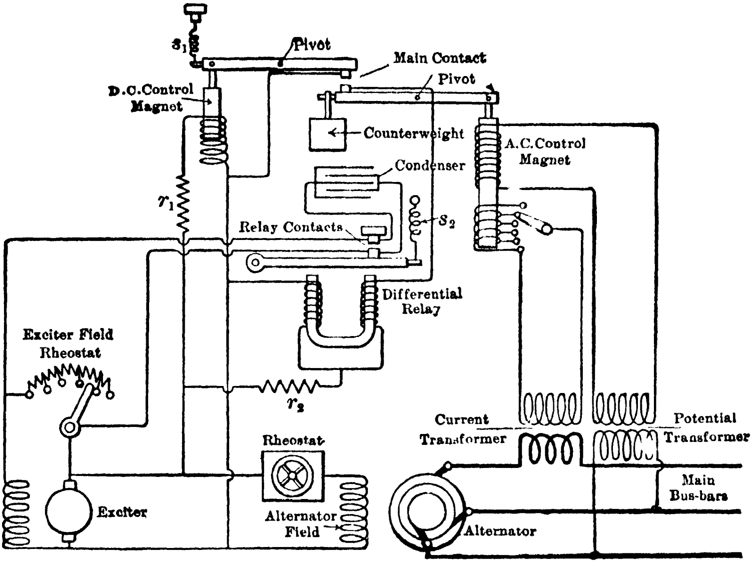


FIG. 331. Diagram of connections of the Tirrill voltage-regulator used with an alternator.

Assume the main contacts to be open, so that only the left-hand side of the differential relay is energized and the relay contacts are opened, thus inserting resistance in the exciter field. This brings the exciter voltage down, and the spring s_1 closes the main contacts. A current flows through the right-hand leg of the differential relay, destroying its magnetism. Thereupon the relay contacts are closed by the spring s_2, short-circuiting the rheostat, and the exciter voltage is increased. The d-c control magnet now overcomes the action of the spring s_1 and again opens the main contacts, etc. Thus, both contacts are vibrating all the time, maintaining a certain exciter voltage. A condenser is placed across the relay contacts to reduce sparking.

Since the lower main contact is attached to a movable arm, the interval of time during which this contact is closed depends upon the position of the arm. Should the terminal voltage of the alternator rise above normal, the core of the a-c control magnet is pulled upward. This moves the main contacts farther apart, and the time of short circuit of the exciter rheostat is reduced. The alternator voltage is thus brought back to normal. Should the a-c voltage be low, the same solenoid brings the main contacts closer together.

Sometimes it is desired to have the alternator over-compounded, that is, to give a higher voltage with increasing load. This is sometimes advisable in order to compensate for the impedance drop in a long transmission line. For this purpose the a-c control electromagnet is provided with a second winding connected to the main line through a current transformer, as shown in the sketch. When the line current increases, the series winding pulls the core of the a-c control magnet down, thus forcing the main contacts to operate on a higher plane. This increases the tension of the spring s_1 and a higher exciter voltage is required to balance it. The increased exciter voltage gives the correct a-c voltage under the changed load conditions.

The current and potential transformers are so connected to the busbars that at unity power factor the current in the series winding is practically in quadrature with that in the potential winding. Referring to Fig. 323, it will be seen that the current I_1 is in quadrature with the voltage E_{32} when $\theta = 0$. At zero power factor of the load, the currents in the two windings are in phase opposition with each other. Thus the controlling action of the series winding is a minimum at 100 per cent power factor and reaches its maximum at zero power factor when more excitation is needed. The number of turns in the compensating winding is adjustable for different degrees of over compounding. More accurate voltage regulation at the distant end of the line is obtained by means of a line-drop compensator such as is used with voltmeters.

When two or more alternators in parallel (Chapter XXI) are controlled by voltage regulators, it is necessary to check reactive circulating currents between the individual machines by automatically adjusting their excitation. In this case the a-c control electromagnet of each regulator, in addition to a potential winding, is provided with a compensating winding of the type already described. When the power factor is unity and the load current is properly balanced between the different machines, the field produced by the current coil is 90 degrees out of phase with the field produced by the potential coil, and has but a slight effect. Should a circulating current tend to flow between the generators, the regulator will be instantly affected and will raise or lower the generator excitation, as required, to eliminate the circulating current. This is because the latter is nearly 90 degrees out of phase with the load current, and therefore in phase with, but directly opposed to, the current in the potential coil, thereby changing the pull of the potential winding until balanced conditions have been restored. Should it be necessary to compensate also for the voltage drop in the line, additional current transformers and line-drop compensators are required.

453. EXPERIMENT 19-E. — Study of the A-C Voltage Regulator. —The device is described in the preceding section. An experimental investigation of a similar d-c regulator is outlined in Experiment 11-B. An additional study should be made of the effect of the power factor of the load and of the adjustment of the counterweight placed on the arm controlled by the a-c electromagnet.

FREQUENCY METERS

454. The frequency, or the number of cycles per second of an alternating-power supply, can be measured directly by means of instruments called frequency meters. Three types of such instruments in general use may be designated as

(1) The vibrating-reed type.
(2) The differential-voltmeter type.
(3) The iron-needle type.

These three types are described below in more detail. For frequency measurements by means of a-c bridges see Vol. II.

455. The Vibrating-Reed Frequency Meter. — This meter, also known as the Frahm frequency meter, is based upon the principle of mechanical resonance, the property by virtue of which an elastic body vibrates vigorously when subjected to rhythmic impulses of the same frequency as the natural period of vibration of the body itself.

How this well-known law has been utilized for the measurement of frequency may be gathered from Fig. 332. A number of reeds *R* (usually 3 mm wide) consisting of special spring steel, carefully tempered, are fastened in a row to a bridge-piece *B*, to which is attached the armature *A* of a small electromagnet *M* mounted close to it. When the instrument is connected across the circuit whose frequency is to be measured, the current, after passing through a series resistance *G*, excites the magnet, which thus imparts to the armature *A* an impulse for each alternation of the current. The vibration set up in *A* is transmitted to the reeds *R*, and just as one tuning-fork in operation will excite another of the same period of vibration, so will that reed which is in tune with the frequency of the current at once respond vigorously. In order to give a clear indication, the reeds have a small portion of their upper ends bent over at right angles and enameled white, so as to make them conspicuous against the black interior of the instrument.

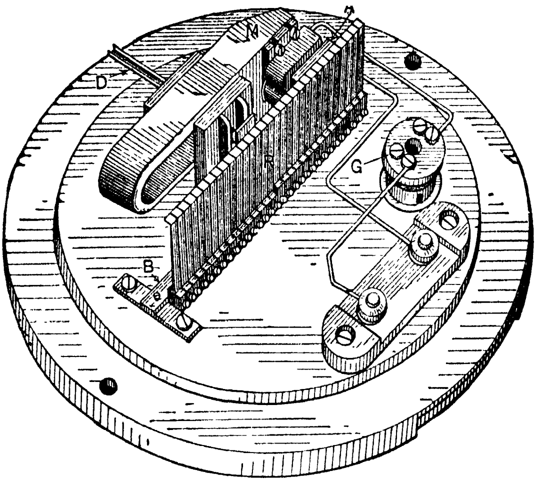


FIG. 332. Vibrating-reed frequency meter.

On first consideration it may seem that this method of indication is only capable of giving results in an intermittent, step-by-step fashion. The number of reeds in one meter is naturally limited, and it would therefore appear that indications would depend on the chance of the frequency to be measured corresponding exactly to that of one of the reeds in the instrument. In practice this is not the case. The vibrations of each reed are not confined to one mathematically exact value, but extend over a certain range of frequencies. They commence about 2 per cent below full value, reach their greatest amplitude at the exact frequency for which they are tuned, and at 2 per cent above this still show clearly. Actually, therefore, more than one reed is usually in motion, and from the relative lengths of the vertical bands formed by them the exact value of the frequency may be estimated to a fraction of a unit.

The instrument is fitted with an adjusting screw (shown at *D*) by means of which the distance between the armature and the magnet may be varied. This enables the user to secure full amplitude of the reeds for voltages varying within 20 per cent of that for which the instrument was calibrated. This is particularly desirable with portable meters, where the voltage to be employed is frequently not known beforehand.

The fact that in Frahm instruments the reeds are excited mechanically through the bridge-piece affords means for providing any meter with a second scale range (double that of the first) by a very simple device. For this purpose a second electromagnet, either excited by direct current or having a permanent magnet for its core instead of one of soft iron, is connected in. The current then merely weakens and strengthens the flux of the permanent magnet, thus giving to the armature but one impulse for each complete cycle (2 alternations). Consequently, double the frequency is required with this magnet to cause any particular reed to vibrate in tune. A special felt-lined damping bar is fixed behind the scale, just out of reach of the swinging reeds under normal conditions. In cases of sudden overload, the reeds touch this bar and are thereby prevented from swinging through so great an arc as to suffer permanent deflection or to become liable to fracture. These instruments have no pivoted moving parts or jeweled bearings and are accurate in any position of the meter; their indications are independent of the voltage or the wave-form of the supply They are not affected by external magnetic fields, and their energy consumption is very small.

456. The Differential-Voltmeter Type Frequency Meter. — The frequency meter shown in Fig. 333 is essentially a combination of two induction voltmeters described in §54. Two split-phase electromagnets, M_1 and M_2, act in opposite directions on the aluminum disk D, thus constituting a differential voltmeter. To make the instrument respond to changes in frequency, the winding of one of the electromagnets is connected in series with an inductance L, and the other winding with a resistance R. The current in the branch M_1, containing the resistance, is practically independent of the frequency; the current in the branch M_2 decreases as the frequency increases, thus giving preponderance to the electromagnet M_1. For each frequency there is a definite stable position of the aluminum disk, and the instrument is calibrated in cycles per second by connecting it to an alternator whose speed is measured directly with a speed counter.

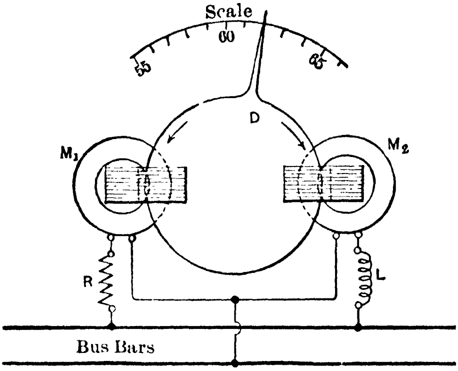


FIG. 333. A frequency meter of the differential-voltmeter type.

The aluminum disk, acted upon by the magnets, is so shaped that when the shaft turns in one direction the torque of the magnet tending to rotate it decreases, while the torque of the other magnet increases. The pointer therefore comes to rest where the torques of the two magnets are equal. This arrangement insures freedom from error due to varying voltages.

457. The Iron-Needle Type Frequency Meter. — This meter (Fig. 334) has two sets of stationary field coils, 1 and 2, mounted with their axes perpendicular to each other. An iron needle is pivoted and free to rotate within these coils. A reactance, x_1, is connected in series with coil 1, and a resistance, r_2, in series with coil 2. The first combination is shunted by a resistance r_1, the second by a reactance x_2. The whole is connected across the source of supply, in series with the reactance x.

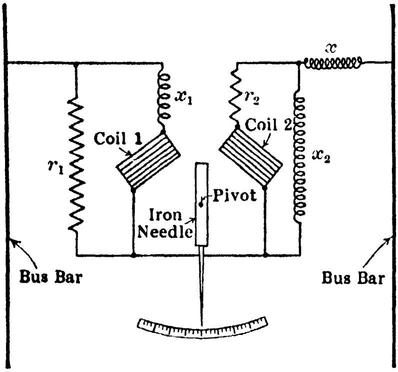


FIG. 334. The iron-needle type frequency meter.

At a low frequency, a larger current flows through coil 1 and a smaller one through coil 2. The magnetic field of coil 1 predominates, and the iron needle takes the corresponding position of the resultant field, indicating the frequency on the scale. At a high frequency, the field due to coil 2 predominates. Thus, for every frequency within the limits of the instrument range there is a definite position of the iron needle.

The reactor x serves to cut down higher harmonics in the supply voltage. By a proper design of the reactors, the instrument has been made practically independent of fluctuations in the applied voltage.

CHAPTER XX

THE ALTERNATOR — VOLTAGE REGULATION

458. The brief outline of theory given below is intended as a supplement to the preceding chapter, in order that the reader may account at least qualitatively for the observed changes in the terminal voltage of a loaded synchronous alternator. Some statements are given without a rigid proof; for such proofs the reader is referred to special works on the subject, among others to the author's "Magnetic Circuit," and to Vol. II.

459. Causes of Voltage Drop in an Alternator. — There are four component causes of the difference between the no-load voltage and the terminal voltage under load, with the same field current and speed, viz.:

(1) *Armature resistance.* The resistance drop is in phase with the armature current I and is equal to IR. When there are considerable eddy currents in armature conductors, the effective resistance (§§154 and 392) should be used, and not the value measured with direct current.

(2) *Leakage reactance of armature winding.* Alternating currents in the armature winding excite magnetic leakage fluxes, linking partly with one phase only, partly with two or more phases. These fluxes act like those of self- and mutual inductance (Chapter VI). With a symmetrical polyphase winding the effect of the mutual inductance can be combined with that of the self-inductance of each phase, giving the so-called equivalent inductance per phase. Multiplying this inductance by $2\pi f$, where f is the frequency of the armature currents, we get the so-called equivalent reactance x per phase. The voltage drop Ix, due to this reactance, is in leading quadrature with the current I.

(3) *Armature reaction.* When a current is flowing through the armature winding, it becomes a source of mmf which is combined with that of the field winding, weakening (or strengthening) and distorting the field flux. The flux having thus been modified, the emf induced by it in the armature winding is different from that induced by the original flux at no load. The actual effect of the armature reaction is quite complicated; but for practical purposes the mmf of the armature may be resolved into two components:

(a) Direct armature reaction, which ampere-turns can be directly subtracted from (or added to) those of the field winding.

(b) Transverse armature reaction, of which the effect in a generator is to shift the field flux against the direction of rotation of the poles, and in a synchronous motor in the direction of rotation of the poles.

The armature reaction is simpler in a polyphase than in a single-phase machine, because in a polyphase machine the pulsating mmf's of the individual phases are combined into one resultant mmf of constant amplitude which glides along the air-gap at the same angular velocity as the field poles (§484). Therefore the relative position of the field and armature mmf's remains unchanged with the time (at a fixed power factor).

In a single-phase machine the armature reaction is pulsating while the field mmf is rotating. A pulsating mmf may be resolved into two mmf's gliding in opposite directions, each component being of one-half the amplitude of the original mmf. The mmf gliding in the direction of the poles is combined with that of the field winding, as in a polyphase machine. The other component excites a flux which is partly wiped out by eddy currents which it induces in the pole faces, and partly induces an additional triple-frequency voltage in the armature conductors. This subject is outside the scope of this work.

(4) *Field leakage flux.* This is flux established by the field but, since it does not enter the armature, it can generate no voltage in the armature winding. The leakage flux is proportional to the mmf acting between adjacent poles. In the unloaded alternator the magnetic potential between the poles is that required to force the flux through the air-gap and armature path. In the loaded machine, to this magnetizing force must be added that required to balance the armature reaction. As a consequence the flux leakage between the poles is altered, as is the value of the emf generated.

460. Armature Reactance. — Three kinds of paths for armature leakage fluxes are usually distinguished, viz., those in the slots, those between the tooth tips, and those around the end connections. The voltage induced by these fluxes, divided by the corresponding armature current, is called the armature reactance, which acts as a reactive coil interposed between the machine and the external circuit. In order to be able to predetermine the performance characteristics of an alternator or synchronous motor, its armature reactance must be known.

The leakage field linking with the armature conductors depends not only upon the currents in these conductors, but upon the field excitation as well, because the latter limits the leakage fluxes to certain paths. It depends also upon the position, relative to the poles, in which the conductors of a phase are found at the time of maximum current. Therefore, it would be wrong, although it is possible, to measure the armature reactance on a machine standing still, by simply sending an alternating current (or currents, in a polyphase machine) through the armature winding from an external source, as though it were an ordinary impedance coil (§143). (The proper interpretation of such data is complex and will not be considered here. See Vol. II and the bibliography.) Obviously, the fluxes excited by such external currents are entirely different from the leakage fluxes produced by identical currents with the machine excited and running at synchronous speed.

No simple direct method of measurement of armature reactance being available, the following indirect methods are used:

(a) From a short-circuit test (§445). In this case the direct armature reaction is practically the only one present, and it can be computed provided the number of turns and the arrangement of the armature winding are known. The effect of the armature reaction having been eliminated, the remaining voltage drop is due to the armature impedance. For details see §462.

(b) From the no-load saturation curve and a load saturation curve at zero power factor (Fig. 336). In this case also the direct armature reaction can be eliminated, as explained in §464, and the reactive voltage drop in the armature computed.

(c) From estimated permeances of leakage paths per unit length of coils. The values of unit permeances are based upon previous tests on machines with similar windings. For details see the author's "Magnetic Circuit," p. 229.

(d) From an air characteristic, as described below.

The air characteristic. It has been found by numerous experiments that the true leakage reactance of the armature of a polyphase synchronous machine may be computed from an impedance test on the same armature supplied with alternating currents from an external source, *the field structure being completely removed.* The straight line plotted between volts and amperes from such a test is known as the air characteristic of the machine. From the total measured voltage drop, the resistance drop is subtracted vectorially, and the remainder is equal to the voltage induced in the armature by the magnetic fluxes.

These fluxes may be roughly divided into (a) the true leakage fluxes, the same that exist in normal operation, and (b) those which extend over a pole-pitch and which correspond to the useful fluxes in a loaded machine. Subtracting the induced voltage due to the latter fluxes gives the voltage induced by the true leakage fluxes. This voltage, divided by the corresponding current, gives the desired armature reactance.

In evaluating the voltage due to the (b) fluxes we shall use the following notation and units:

E the emf per phase, induced by the (b) fluxes, in volts.
f the frequency, in cycles per second.
I the effective value of the armature current, in amperes.
k_d the combined breadth and pitch factor of the armature winding for the fundamental of the revolving mmf.
L the gross axial length of the stator.
m the number of phases.
M the amplitude of the fundamental of the revolving mmf of the armature, in ampere-turns.
n the number of armature turns, per pole per phase.
p the number of poles of the machine.
Φ the flux described under (b) above, in maxwells per pole.

It has been proved by M. Schenkel (*Elektrotechnik und Maschinenbau*, Vol. 27, 1909, p. 207) that

$$\Phi = 0.8\pi ML \quad \ldots\ldots\ldots\ldots \quad (1)$$

With the approximate assumptions made in the deduction of this formula, the flux Φ comes out independent of the number of poles and of the bore of the machine. In this expression

$$M = 0.9k_d mnI \quad \ldots\ldots\ldots\ldots \quad (2)$$

(see the author's "Magnetic Circuit," p. 130). The flux Φ induces in the armature winding the following voltage per phase:

$$E = 4.44k_d fnp\Phi 10^{-8} \quad \ldots\ldots\ldots\ldots \quad (3)$$

Substituting the value of Φ from eq. (1) and using in it expression (2) for M, we get

$$E = f(k_d n)^2 mILp10^{-7} \quad \ldots\ldots\ldots\ldots \quad (4)$$

By means of this formula the voltage due to the fluxes extending from pole to pole may be evaluated, provided that n and k_d can be ascertained. Subtracting E arithmetically from the total reactive voltage computed from the air characteristic, the armature reactance can be determined as is explained above.

This method involves several assumptions, but it gives results which are sufficiently close for practical purposes. In good synchronous machines the purely reactive drop at the rated current is less than 20 per cent of the rated voltage; moreover, this drop is mostly added to the induced or terminal voltage geometrically, at a rather large angle. Consequently, even a considerable error in the determination of the armature reactance will affect the resultant calculated voltage comparatively little. On the other hand, with modern highly saturated machines even a small difference in the induced voltage means a noticeable increase in the exciting ampere-turns, and for this reason the armature reaction should be known as accurately as possible.

461. Armature Reaction and Reactance from Short-Circuit Test. — The voltage induced by the field flux in a short-circuited armature (§445) is used up in overcoming its impedance drop. In large and medium-sized alternators of the usual proportions, the reactive drop is several times larger than the resistance drop, so that for many practical purposes the latter may be neglected. Therefore, *in a short-circuited armature the current is approximately in quadrature with the induced voltage.* This means that while the induced voltage in an armature conductor reaches its maximum when the conductor is opposite the center of a pole, the current in the same conductor reaches its maximum when the conductor is midway between two poles. By actually considering instantaneous values of currents and positions of the poles, the following general proposition may be proved:

In a polyphase synchronous machine in which armature currents reach their maximum values when the corresponding conductors are midway between the poles, the direct armature reaction is the only one present. It opposes the field mmf when the armature current is lagging behind the induced emf, and strengthens it when the current is leading. By direct armature reaction is meant an mmf with its maxima opposite the center lines of the field poles and moving synchronously with the poles.

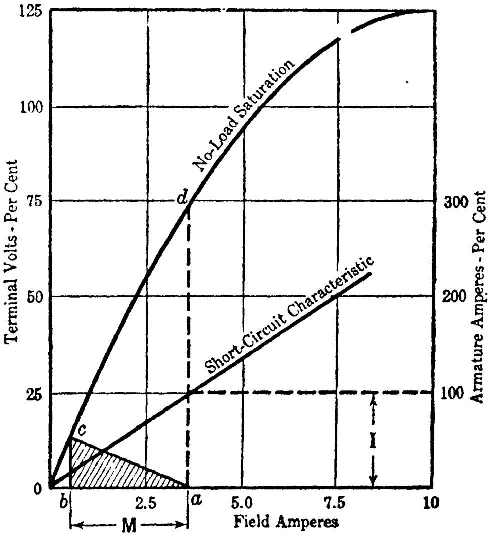


FIG. 335. The no-load and short-circuit characteristics used in the determination of armature reactance, armature reaction, and synchronous reactance.

Neglecting the armature resistance, the short-circuit current satisfies the foregoing condition, and hence on a short circuit the direct armature reaction is practically the only one present. This makes the short-circuit test and the corresponding curve (Fig. 325) particularly valuable in the study of alternator characteristics.

Let the field ampere-turns (Fig. 335) be Oa at rated armature current I on short circuit, and let the demagnetizing action of the armature be equivalent to $ab = M$ ampere-turns (see eq. 2, §460), calculated from the current I and the number of *effective* armature turns per pole. Therefore, the *net* excitation is equal to Ob ampere-turns, and the induced voltage as shown by the no-load saturation curve is equal to bc. This voltage causes the current I to circulate through the armature winding, and therefore bc is equal to Ix, since the ohmic drop IR is neglected.

In the absence of the short-circuit curve, it is possible, in a machine in which the design constants are known, to calculate the field excitation necessary for a given short-circuit current I as follows: Compute the value Ix from the design constants and find the corresponding abscissa Ob on the no-load saturation curve. To this value add the calculated ampere-turns M of the direct armature reaction. This will give the required field ampere-turns Oa, necessary to circulate the current, I, on short circuit.

The problem is different when the machine constants are not known, because the short-circuit test gives only the value of Oa, in amperes, for a given value of armature current I. However, if, in addition to the no-load and short-circuit characteristics, the load saturation curve at zero power factor is also available, it becomes possible to determine both armature reactance and direct armature reaction with reasonable accuracy. This method is explained in §465.

462. EXPERIMENT 20-A. — Armature Reactance from the Short-Circuit Test. — Using the data taken in Experiments 19-A and 19-B draw the no-load and short-circuit characteristic curves as in Fig. 335. Having obtained complete winding data, calculate the armature reaction M at a certain current I (see eq. 2, §460). Subtract the armature reaction M from the total excitation Oa required to circulate the current I on short circuit, thus finding the net excitation Ob. Note the induced armature volts bc corresponding to the excitation Ob. Consider the voltage $bc = Ix$ drop, since, at the low power factor in the alternator on short circuit, the Ir drop may be neglected. From the Ix drop calculate the reactance x of the armature winding.

463. Synchronous Reactance from Short-Circuit Test. — The difficulties connected with an experimental separation of the armature reaction from the armature reactance, especially when a zero power-factor load saturation curve (§464) is not available, have led to the adoption of a fictitious variable quantity known as the *synchronous reactance* of the machine. The author is not in favor of this term or of the quantity itself, but it has been used in the past and is occasionally mentioned in the literature of the subject. The student should therefore become familiar with the definition, uses, and limitations of this quantity.

By definition, the synchronous reactance, x_s, of an alternator armature is a fictitious reactance which would cause the same voltage drop as the combined action of the true armature reactance x and of the armature reaction M. In other words, the drop $Ix_s = ad$ (Fig. 335) is taken to correspond to the induced voltage at no load for the field excitation Oa, as if the armature reaction did not exist. Actually, both the true armature reaction M and the true reactive drop Ix are always present in the loaded alternator, and though for certain practical purposes they may be combined into one fictitious quantity, the synchronous reactance, it must be used with great discretion and is liable to give results of doubtful accuracy.

Synchronous reactance is a safe concept to use as long as the operation is considered at very low values of power factor and on the straight part of the saturation curve. When these two conditions are fulfilled simultaneously, x_s remains constant and equal to its value as determined from a short-circuit test. As examples of the use of the term synchronous reactance, the following two practical problems will be considered. Later its use in the calculation of alternator regulation will be discussed (§467).

(a) *Transient short-circuit current.* The value of the steady armature current obtained from the short-circuit curve (Fig. 335) is considerably below that which flows at the first instant after a *sudden* short circuit. Let the machine be running at no load and then be suddenly short-circuited. The main magnetic flux, which was excited by the field winding alone, cannot be changed instantly by the armature reaction, because the exciting current rises, on account of the transformer action or mutual induction between the armature and field windings, and opposes the armature reaction. Thus, for the first few cycles the short-circuit current is practically limited by the true armature reactance x alone (the effect of resistance being almost negligible). Consequently, the transient short-circuit current is greater than the sustained short-circuit current in the ratio of x_s to x. In large low-frequency turbo-alternators this ratio may be quite large, and the momentary short-circuit current is sometimes destructive to the machine.

(b) *Variations of short-circuit current with speed.* Within wide limits of speed, and at a given field excitation, the short-circuit current increases very slowly with the speed. This is because both the synchronous reactance and the induced voltage are proportional to the speed. Neglecting the resistance of the armature winding, the short-circuit current is equal to the ratio of the induced voltage to the synchronous reactance, and is therefore independent of the speed. At low frequencies the effect of the resistance becomes appreciable, and the short-circuit current rapidly decreases toward zero.

464. Synchronous Reactance from Zero Power-Factor Test. — In Fig. 336 the curve *abw* represents the terminal voltage with a purely inductive load, when the armature current is kept at a constant value *I*, and the field excitation varies from the value *Oa* on short circuit to increasingly higher values.

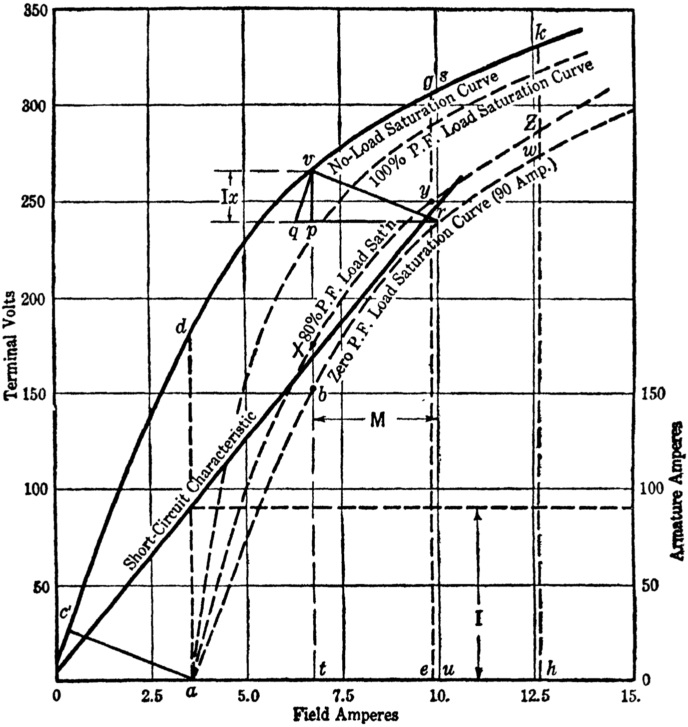


FIG. 336. Saturation curves of a 37.5 kva alternator, showing Potier triangle.

The synchronous reactance at point *r* can be computed from the equation $sr = Ix_s'$. It will be seen from the figure that $sr < vb < ad$, so that the synchronous reactance decreases considerably as the magnetic circuit of the machines becomes saturated. It would be inaccurate, therefore, to use the value of x_s, computed from the short-circuit test, for a predetermination of voltage drop and regulation within the normal operating range of the machine, even at zero power factor. It is better to get this value from the distance *sr* between the zero power-factor load and the no-load saturation curves at normal voltage on the former. This value will be used later (§468) in the determination of alternator regulation by the A.I.E.E. method.

The relationship between the terminal voltage E and the induced emf, E_0, at zero power factor, with the current lagging, is also shown vectorially in Fig. 337. The armature reaction exerts a direct demagnetizing action, and the induced voltage is reduced from OD_0 at no load to the value OD. Now, taking OD as the true induced emf and neglecting the ohmic drop, we obtain the terminal voltage by subtracting the true reactive drop Ix. The same difference between E_0 and E is also shown to be due to a fictitious reactance drop Ix_s. With the current I in leading quadrature, both BD and DD_0 are added to OD_0, making the terminal voltage E higher than the induced emf, E_0.

The diagram shown in Fig. 337 also explains why an over-excited synchronous motor at no load draws a leading current from the line. The machine is shown over-excited because the induced voltage E_0 is greater than the terminal voltage E. The current I is lagging with respect to E, but is leading the line voltage, $-E$ (equal and opposite to E), by 90 degrees. In this case both the armature reaction and the reactive drop tend to reduce the induced voltage, and the current I automatically assumes such a magnitude that the reduced voltage is equal and opposite to the given line voltage.

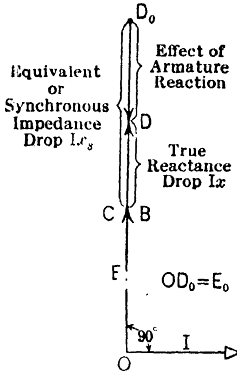


FIG. 337. Voltage drop in a synchronous machine at zero power factor, with a lagging current.

465. Armature Reaction and Reactance from the Potier Triangle.— The zero power factor load saturation curve is drawn between armature terminal volts and field amperes, the armature current being held constant at normal value and the power factor maintained at approximately zero. The load of near zero power factor is obtained either by using adjustable reactors or by connecting the alternator to an under-excited synchronous motor. Such a test does not involve the expenditure of much energy and can usually be performed without great expense even on the largest machines. It is found that if the zero power-factor load saturation curve (Fig. 336) is shifted parallel to itself until its initial point a coincides with the point O, and then upward with the point a moving along the no-load saturation curve to some point c', the zero power-factor curve can be made to coincide closely with the no-load saturation curve, since the two curves are of nearly the same shape.

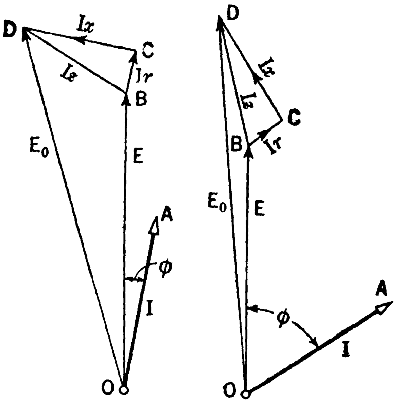
This fact indicates that the zero power-factor curve is essentially parallel to the no-load curve at a distance and in the direction of *c'a*. The oblique line *c'a* should then fit anywhere between the two curves, as its equal, *vr*, at rated voltage shows. Then, since, for the same generated voltage, the excitation for zero power factor exceeds that for the no-load curve by the amount of the direct armature ampere-turns at current I, *the horizontal component pr* of this oblique line equals the value of direct armature ampere-turns $AT_a = M$. Likewise, for the same *net excitation* (as *Ot*) the terminal voltage at zero power factor is less than that at no load by the amount of the Ix drop (ignoring the Ir drop), which value is given by the vertical component *pv* of the slanting line *vr*. This is the Potier triangle method of determining the values of armature reactance and armature reaction. The use of these values in predetermining the regulation of the alternator at any desired load and power factor will be explained in §469.

466. EXPERIMENT 20-B. — Zero Power-Factor Load Test. — The purpose of the experiment is to obtain the zero power-factor load saturation curve *arw* shown in Fig. 336, and to apply to it the theory given in the preceding sections. The experiment should be performed on a three-phase or two-phase machine on which the no-load and short-circuit characteristics have been previously taken (§§444 and 446), or else these curves must be included as part of this experiment. A load is provided, and the readings at zero power factor are taken, as explained in §451. As a matter of fact, the load saturation curve at zero power factor is included in Experiment 19-C, but it is desired here to obtain this curve and the other two shown in Fig. 336 with particular accuracy, for the purpose of applying the theory. The analysis of the data can be carried much further if the results of Experiment 20-A on the same machine are available.

Report. (1) Plot the curves shown in Fig. 336. (2) Cut out a piece of tracing paper to correspond to the shape of the no-load saturation curve and find by trials the direction in which it must be shifted to fit the lower part and the knee of the other curve. (3) Check the length and direction of the line $c'a = vr$ as follows: From a point r on the zero power-factor load saturation curve at about rated voltage draw the line $rq = Oa$, horizontally to the left (Fig. 336). From the point q draw a straight line parallel to the initial part *oc'* of the no-load saturation curve to intersect the latter at v. Draw *vr* and compare with *c'a*. The two should be essentially equal and parallel. Either (2) or (3) will give *c'a* in magnitude and direction, and consequently the values of $M = AT_a$ and Ix. (4) Check the value of x thus obtained with that found by calculation from the winding data (§460). (5) Plot the theoretical zero power-factor load saturation curve as the locus of points r. (6) On the same curve sheet, plot a curve of values of synchronous reactance against terminal volts at zero power factor. (7) From the points r on the zero power-factor curve and s on the no-load saturation curve determine the rise in voltage when zero power-factor load is removed from the alternator, and calculate the regulation at zero power factor, lagging.

467. Factors Affecting Voltage Regulation at Values of Power Factor other than Zero. -- This subject cannot be fully treated here because of the complexity of the numerical relations. The brief description given below is intended merely to impart to the reader a correct qualitative idea of the factors involved. Although the armature reaction and reactance act simultaneously, it is advisable to consider their effects separately.

(a) *Effect of armature reactance.* The vectorial relations at a high and at a low value of power factor are shown in Figs. 338*a* and 338*b* respectively. The terminal voltage is denoted by E and the phase angle between it and the load current by ϕ. The net induced voltage, E_0, is obtained by adding to E the ohmic drop Ir in the armature, in phase with the current, and the reactive drop Ix in leading quadrature with I.



(a) (b)

FIG. 338. Voltage drop in an alternator; (a) at high power factor, (b) at low power factor, lagging.

It will be seen from the geometry of the figures that, with the same value of the induced voltage E_0, the higher the power factor of the load, the higher will be the terminal voltage E, and for a leading armature current the terminal voltage may be even higher than the induced emf. In other words, the armature reactance may cause a rise instead of a drop in voltage. In other words, the voltage regulation of the machine is improved as the current vector moves farther and farther in the counter-clockwise direction. This is in accordance with the actual performance curves shown in Figs. 326 to 328.

(b) *Direct armature reaction.* It has been stated in §461 that a lagging armature current, which reaches its maximum when the corresponding conductors are midway between the poles, exerts a demagnetizing action upon the field flux. Such a current is displaced in time phase by 90 degrees with respect to the no-load voltage, because the latter reaches its maximum when the groups of conductors are opposite the centers of the poles. This current is practically a reactive current, although its phase displacement with respect to the terminal voltage is not quite equal to 90 degrees.

When the power factor of the load is not zero, the total armature current, I, may be resolved into two components, one of which reaches its maximum midway between the poles and the other when the conductors are opposite the centers of the poles. The statements made in §§461 and 462 in regard to the direct armature reaction apply to the first component of the current. The practical difficulty consists in finding the value of this component.

The larger the phase angle ϕ of the load current, the greater is this component of the current and the more pronounced is the demagnetizing or direct armature reaction. But the influence of the armature reactance is also more pronounced at higher values of ϕ (Fig. 338*b*), so that the two factors are cumulative in lowering the terminal voltage of the machine.

(c) *Transverse armature reaction.* The mmf of the other component of the current, viz., that which reaches its maximum when the conductors are opposite the centers of the poles, must also be considered. It can be shown that its action consists in shifting the main flux towards the trailing pole tip. The effect is qualitatively the same as if fictitious exciting coils were placed between the main poles in lagging space quadrature with the real exciting coils. For this reason the armature reaction due to this component of the current is called the transverse armature reaction. It does not directly weaken the flux, but indirectly reduces its value by saturating the trailing pole tip. Moreover, it modifies the distribution of flux density on the pole face, and thereby also distorts the wave shape of the induced emf. The actual influence of the transverse armature reaction upon the induced voltage of the machine is rather difficult to take into account quantitatively.

Thus, in estimating the full-load voltage of an alternator, the correct procedure (for which credit is due to A. Blondel) requires that the armature current be resolved into two components, one of which is in quadrature and the other in phase with the induced emf at no load. The first component (if lagging) exerts a demagnetizing action on the field poles; the other, a cross-magnetizing or transverse action, as described above. After each action has been taken properly into account, and the field flux has been corrected for the effect of both, the net induced emf E_0 may be found from the no-load saturation curve of the machine.

After this, the diagram shown in Figs. 338*a* and 338*b* may be constructed, and the terminal voltage E determined. It is also possible to begin with a given terminal voltage and current, and to determine the necessary field excitation.

Although this correct method of predetermination of the voltage characteristics of an alternator is used to some extent in designing a new machine or in checking its performance, it can hardly be applied to testing a machine whose constants are not known, because of the difficulty of obtaining correctly the two components of the armature reaction. For this reason, various approximate and empirical methods for predetermining the voltage regulation of an alternator have been proposed and used. Of these methods the one recommended by the American Institute of Electrical Engineers is described below. For a more detailed treatment of the subject see Vol. II and the author's book entitled " The Magnetic Circuit."

468. The A.I.E.E. (Synchronous Impedance) Method of Predetermining Alternator Regulation. — In Fig. 336, $sr = Ix_s$ is the synchronous reactance drop of the alternator at the current I. In the A.I.E.E. method x_s is assumed to be constant at the value obtained from the point r at normal voltage on the zero power-factor characteristic. In the calculations which follow, it is treated like any reactance. The determination of regulation by the A.I.E.E. method is illustrated by the vector diagrams of Figs. 338*a* and 338*b* in which the reactive drop, $Ix = DC$, becomes the synchronous reactance drop, Ix_s. Then E_0 is the no-load voltage and regulation $= (E_0 - E)/E$.

Analytical solution. Inspection of the vector diagram (Fig. 338*a*) yields the following equation:

$$\begin{aligned} E_0 &= (E + Ir_e \cos\phi + Ix_s \sin\phi) + j(Ix_s \cos\phi - Ir_e \sin\phi) \\ &= A + jB \quad \ldots \quad (5) \end{aligned}$$

or numerically

$$E_0 = \sqrt{A^2 + B^2} \quad \ldots \quad (6)$$

Since B is normally small compared with A, one may write

$$E_0 = A + B^2/2A \quad \ldots \quad (7)$$

Graphical solution. To obtain the regulation of the alternator at constant kilovolt-amperes and over a wide range of power factor, Kapp's diagram of Fig. 286 may well be used. The diagram for the alternator will differ from that of the transformer in the greater values of the voltage drops and, therefore, of resultant regulation. A variation of Kapp's diagram[1] is shown in Fig. 339.

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[1] Suggested by Professor Forman, of West Virginia University.

The diagram is first drawn for 100 per cent power factor with E and Ir_e horizontal. IZ_s then assumes the position shown. Continue the line of IZ_s to any length easily divided into 10 parts and draw the power-factor quadrants, marking the power-factor scale. Draw also the semicircular locus of IZ_s. The broken-line construction shows the procedure to obtain the no-load voltage, E_0, and resultant regulation at about 80 per cent lagging power factor. It is an easy matter, if voltages are drawn in percentage of normal voltage E, to draw a series of arcs in the vicinity of A about O as center and with radii differing, say, 5 per cent, and thus read the regulation values directly from the position of C with reference to the arcs. (See Mershon's diagram in any electrical handbook.)

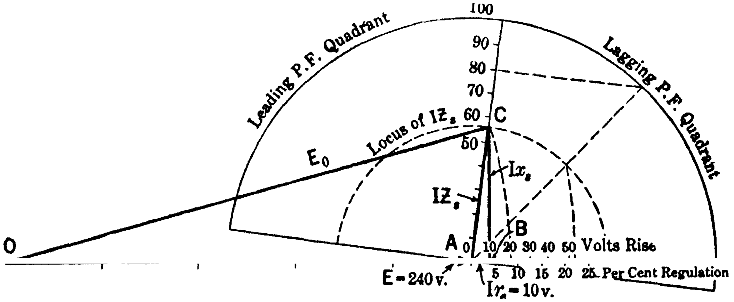

FIG. 339. Graphical solution of synchronous impedance (A.I.E.E.) diagram.

469. Potier Method of Calculating Alternator Regulation. — In §465 a method was described for determining the armature leakage reactance drop Ix and the armature reaction M from the Potier triangle. These values are employed in the general, or Potier, method of estimating alternator regulation as illustrated in Fig. 340 (see Fig. 512, Vol. II). This method is one in which *actual* rather than fictitious voltage drops are added to terminal volts to give the voltage generated. Then, to the mmf required to produce this voltage the armature reaction is added vectorially to find the total excitation. From this the no-load voltage and regulation may be obtained.

Referring to Fig. 340, note that to the terminal voltage, E, are added the drops Ir_e and Ix to give the generated voltage, E_g. The value of the latter is carried across to the no-load saturation curve to read the net field excitation, ON. Now, to ON the vector of armature reaction M is added at the angle ϕ (*internal* power-factor angle) from the vertical, giving AT *total* which is swung down to F to permit taking the corresponding no-load voltage E_0 from the no-load curve.

The Potier method is theoretically correct only for non-salient pole

machines since only there may the reactance x be considered independent of power factor. Also only there may field mmf and armature mmf be considered strictly sine-form quantities and so be properly combined by vector addition. However, the Potier method is generally applied to salient pole alternators and gives reasonably satisfactory results. A method which is theoretically correct for the salient-pole machine is that of Blondel, referred to in §467. This method is described in Vol. II, Chapter XLVIII.

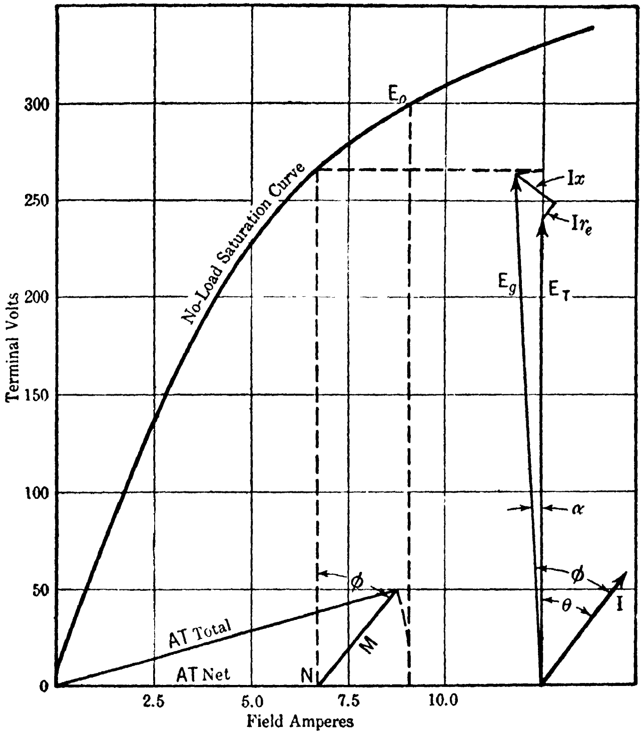

FIG. 340. Potier method of calculating alternator regulation.

470. Load Saturation Curves at Various Power Factors.

— One defect of the A.I.E.E. method was that the value of synchronous impedance was considered constant regardless of the saturation of the magnetic circuit. A method of determining regulation at a desired power factor, say 80 per cent, and one in which the variation of IZ_s with saturation is recognized, is illustrated in Fig. 341. It is a graphical method of determining points on the load saturation curve at the desired power factor and for the stipulated current.

The values used in the construction of Fig. 341 are taken from Fig.

336, in which the synchronous impedance drop is the vertical distance between the zero power-factor load saturation curve and the no-load saturation curve, expressed in volts. To determine the terminal volts at 80 per cent power factor, rated current, corresponding to a field excitation of value *oh*, lay off the drop Ir_e at the angle ϕ from the line *DA*.

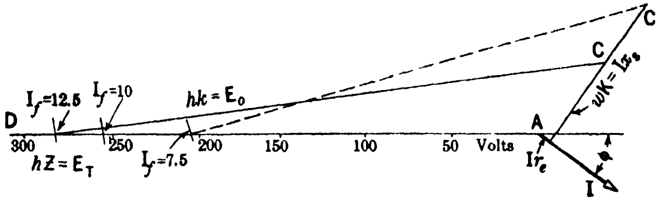

FIG. 341. Graphical determination of points on load saturation curve.

Leading the Ir_e vector by 90 degrees lay off the value of synchronous impedance drop *wk*. From the point *C* at the end of *wk* as center, strike an arc with radius *hk* to intersect line *DA* and thus determine the terminal voltage *hz*. Repeating this construction for the excitation *ou* gives the terminal voltage, *uy*, etc. Thus the 80 per cent load saturation curve *axyz* (Fig. 336) is determined. In all except the smaller alternators the effect of resistance drop may be neglected.

To obtain the regulation at 80 per cent power factor, rated current, the excitation *oe* of Fig. 336 which gives rated terminal voltage on the 80 per cent power-factor load saturation curve is noted and the corresponding no-load voltage *eg* follows, giving regulation. This seems a tedious method of calculating the regulation at one value of power factor and load current. However, it has the advantage that the varying value of synchronous impedance is recognized.

471. EXPERIMENT 20-C. — Load Saturation Curves of an Alternator at Different Values of Power Factor. — The purpose of the experiment is to check the actual test curves shown in Fig. 336 with those computed according to the method given in the preceding section. The results of Experiments 19-A, 19-B, and 20-B may be used, if the data are sufficiently accurate. In such a case, no actual new experiment is necessary, but only additional computations for the report. It is advisable, however, to take a special set of curves with very careful settings and with all the readings at exactly the values of armature current for which the computations are to be performed. The method of loading the machine and the data sheet are the same as in §451.

Report. (1) Plot the experimental curves as in Fig. 336.

(2) For different values of field current, measure and plot the values

of the synchronous reactance drop, using the load saturation curves at zero power factor.

(3) Construct diagrams like those in Figs. 341 and 339 for the values of power factor for which experimental curves are available; choose points of low, medium, and high saturation.

(4) Compare the computed values of terminal voltage with those measured on test, and find out whether the method of synchronous reactance gives *optimistic* or *pessimistic* curves, that is, whether the computed values lie above or below those observed on test.

(5) In case of a serious discrepancy between the observed and computed values, suggest a further empirical correction in order to bring the computed values nearer the test data.

CHAPTER XXI

ALTERNATORS IN PARALLEL

472. Two d-c machines can be put in parallel as soon as their voltages are approximately equal, provided that the correct polarity is observed in connecting them (§255). But before two alternators are switched in parallel, three conditions must be fulfilled: their terminal voltages must be (1) of the same magnitude; (2) of the same frequency; (3) in phase with each other (considered with reference to the parallel circuit of the two machines). Unless all these requirements are met, there will be a disturbing circulating current between the machines. Only when the terminal voltages of the alternators, considered with reference to the series circuit of the two machines, are equal and opposite *at all moments* — which requires the fulfilment of the above conditions — will circulating current be prevented and each machine send its share of the total current into the line.

The process of adjusting the magnitude, frequency, and phase of the voltage of an alternator or synchronous motor to those of another machine or source of supply, is called *synchronizing* the machine. The word means " bringing into the same rhythm."

473. The Three Adjustments Involved in Synchronizing. — Let us investigate what would happen if two alternators should be connected to the same bus-bars without having all the above-stated conditions fulfilled.

(1) If the two machines are so excited as to give *different voltages,* the other two conditions being fulfilled, a reactive current will circulate between the two machines, leading in the machine excited lower, and lagging in the machine excited higher. This has the effect of strengthening the field of the first machine and weakening the field of the second machine; the resultant voltage at the bus-bars will be somewhere between the voltages of the two machines. The set may still work satisfactorily, provided that this reactive current is not too heavy. However, such a circulating current does not help the operation and gives an unnecessary I^2R loss in the armatures of both machines. Should this current become sufficiently large, it will cause a dangerous temperature rise in the armature windings or possibly open one of the circuit-breakers, even though the external load current may be quite small.

(2) If the voltages of the two machines are *not in phase* with each other, in other words, if the maxima of their emf waves do not occur at the same moments, even though the two emf's are of the same frequency and magnitude, an energy current will circulate between the machines. This current, sometimes called the synchronizing current, tends to bring the machines into phase, and it may be quite large, if the difference in phase is considerable. If the torque due to this synchronizing current is large enough to pull the lagging machine into phase promptly, the current itself will be of short duration and will not damage the machine. Otherwise, the armature winding of the machine may be overheated and the insulation damaged.

(3) If paralleling without synchronizing is attempted, when the two frequencies differ from each other by a few per cent, the conditions are approximately those described under (2) above, with the phase angle continually changing. To show this clearly, two sine-waves of nearly the same frequency should be drawn. It will be found that at some instants the waves are in phase with each other, then they are in phase quadrature, in phase opposition, and, after a number of cycles, in phase again. Unless the energy component of the circulating current is sufficient to pull the lagging machine into synchronism promptly, and to hold it there, the machines may be overheated or otherwise injured by the excessive current. The theoretical side of the foregoing statements is elaborated somewhat more fully in §§478 and 479 below.

Although modern alternators are built to withstand short-circuit currents there is considerable disturbance to the system when they occur, and the opening of the circuit-breaker of course prevents the alternator from carrying any part of the system load.

After the correct speed has been approximately obtained, the field current of the alternator is so adjusted as to give about the same voltage as that across the bus-bars to which the machine is to be connected. It only remains then to bring the machine *into phase* with the bus-bar voltage. This is done either by means of properly connected synchronizing lamps, or by special instruments, called synchroscopes or synchronism indicators.

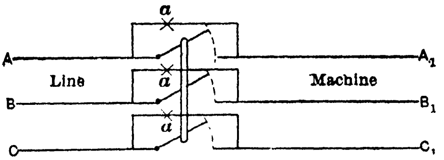

Fig. 342. Synchronizing lamps.

474. Synchronizing Lamps.

— Synchronizing lamps are usually connected as shown in Fig. 342, around the main switch which connects the machine to the bus-bars. As long as the alternator voltage is not of the same frequency as that of the line, the lamps will be alternately lighted and extinguished with a frequency equal to the difference of the frequencies of the two voltages, that is, with the frequency of the beats of the circulating current. By varying the alternator speed it is possible to extinguish the lamps; this will show that the machine is in perfect synchronism, and the switch can be closed. The sketch illustrates the case of a three-phase machine; with a single-phase machine one of the lines, say CC_1, is omitted.

The reason for current beats in the synchronizing lamps is as follows: Let the two alternators be running at slightly different frequencies, e.g., 60 and 59.5 cycles per second. The terminal voltages cannot be made equal and opposite at all instants. Let the voltages be approximately equal and opposite at a certain instant, so that there will be no circulating current due to voltage difference when the machines are connected in parallel. A second later the machines will be out of phase by one-half of a cycle, so that their induced emf's will be added together, and a circulating current will flow through the lamps. Another second later, the voltages will be again in opposition and the lamps will be extinguished. The student should plot two such sine-waves of voltage and add the ordinates point by point in order to get a better idea of the shape and the frequency of the resultant voltage. He will find that the frequency of the so-called "beats," that is, the frequency at which the synchronizing lamps are lighting up, is not the true frequency of the resultant current but that of its "envelope." Each flash of the lamps corresponds to several cycles of the circulating current, but the eye receives only one continuous impression.

Some operators prefer to have synchronizing lamps crossed, as shown in Fig. 343. The machine is in synchronism when the lamps glow the brightest. As an advantage of this arrangement, it is claimed that should a lamp burn out during the process of synchronizing, the operator would immediately notice it, while with the first arrangement he might assume the machines to be in perfect synchronism and close the switch with disastrous results. However, the possibility of a synchronizing lamp burning out is rather remote, and with two- or three-phase machines the burning out of one set of lamps would not affect the others. Also it is generally considered less easy to determine moments of maximum darkness than moments of maximum brilliancy of the lamps.

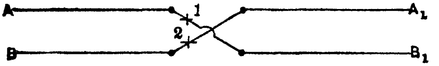

FIG. 343. Crossed synchronizing lamps.

With three-phase machines, crossed synchronizing lamps in two phases are very convenient, especially when the lamps are arranged in a circle, as in Fig. 344. In this case, maximum brightness occurs in the three sets of lamps in rotation, so that the light appears to be traveling along the circle. The direction in which the light rotates depends on whether the speed of the incoming machine is low or high. When the machine is in synchronism, the rotation of the light ceases; the lamps marked " 3 " are dark, while the lamps " 2 " and " 1 " glow with equal brightness.

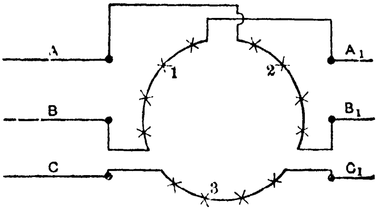

Fig. 344. Synchronizing lamps arranged in a circle.

When two 220-volt single-phase machines are to be put in parallel, at least four ordinary 110-volt synchronizing lamps should be used in series, because at certain moments during the process of synchronizing, the emf's of the machines may be acting in the same direction instead of in opposition, thus giving 440 volts. It is even better under these conditions to have 5 lamps in series, so as not to let them glow too brightly; it is then easier to observe the periods of maximum brightness. With two three-phase machines of the same voltage as above, three 110-volt lamps in series in each phase are sufficient, although four lamps may give better service. For higher voltages synchronizing lamps are usually connected through small transformers.

An ordinary indicating *voltmeter can be used for synchronizing*, instead of lamps. It is connected between the machines, in parallel with or in place of one string of lamps, either directly or through a potential transformer. When the machines are in synchronism, the voltmeter pointer comes to zero unless the crossed connection is used. When the frequencies are unequal it swings to and fro.

When providing a synchronizing connection between two polyphase machines, it is necessary to find out their *phase sequence*. Otherwise, when A is connected to A_1 (Fig. 342), B may be connected to C_1, and C to B_1. If synchronizing lamps were placed in only two lines and the switch were here closed in accordance with their indication, there would be a short circuit between the machines if the phase sequence were incorrect. Once the proper phases have been found, however, it is safe to synchronize afterwards in one phase only, provided that neither the direction of rotation nor the connections are changed.

475. Synchronism Indicators. — In larger installations, synchronizing lamps have gradually given place to the more accurate instruments, so-called synchroscopes or synchronism indicators. Such a device has the appearance of an ordinary switchboard instrument (Fig. 345), except that the pointer has no retaining spring or weight, and is free to revolve through 360 degrees. When the speed of the alternator to be synchronized is low, the pointer revolves in one direction; when it is high, the pointer rotates in the opposite direction. When the speed is correct, the pointer stands still; and when the machine is " in phase," the pointer shows zero, indicating that the main switch may be closed.

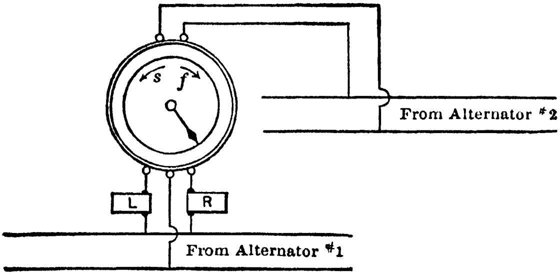

FIG. 345. Connections to a single-phase synchronism indicator.

The usual synchronism indicator resembles a small a-c motor. The field winding is connected to one of the machines, or to the line, and the armature to the other machine (Fig. 346). When the frequencies of the two machines are different, the resultant field in the synchronism indicator constantly changes its position, making the armature revolve in one or the other direction. When the frequencies are the same,

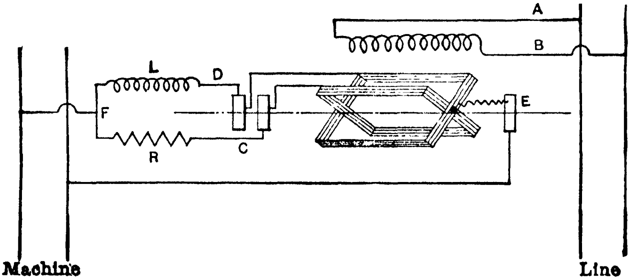

FIG. 346. Circuits of the single-phase synchroscope.

the field is stationary in space; when finally the machines are in phase with each other, the field occupies such a position that the pointer, connected to the armature, shows zero.

The synchronism indicator, illustrated in Figs. 345, 346, and 347, is a single-phase device. The stationary field winding, *AB*, is connected to the bus-bars (line); the revolving armature is connected to the machine to be " thrown in," in series with an inductance *L* and resistance *R*, usually an incandescent lamp. The armature is of the drum type; it has two coils rigidly fastened at right angles to each other and connected in series. Their junction is connected through the collector ring *E* to the binding post marked *E* in Fig. 347. The other two terminals are brought out through collector rings *C* and *D* to the binding posts marked with the same letters in Fig. 347.

R and *L* are used for splitting the phase of the armature current, so that the current in the coil which is in series with *L* lags behind that in the other coil. Each armature coil is subjected to a torque from the stationary field. When the frequencies of the two machines are equal, the armature of the synchronism indicator assumes the position in which the two torques are equal and opposite. When the frequencies are slightly different, the position of equilibrium varies with the time and the armature revolves at a speed equal to the difference of the two frequencies.

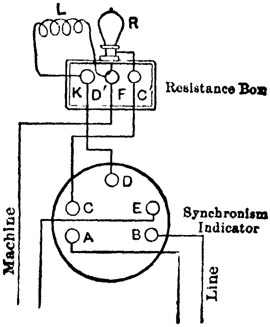

Fig. 347. Connections between the phase-splitter and the synchroscope.

In another make of synchronism indicator the split-phase arrangement shown in Fig. 346 is stationary, so that no slip-rings are necessary. A rotating magnetic field is produced around this winding by currents from the bus-bars passing through it. In this rotating field is placed a movable iron vane (Fig. 104) or armature magnetized by a stationary coil connected across the terminals of the incoming machine and corresponding to the winding *AB* in Fig. 346. The iron vane takes a position where zero of the rotating field occurs at the same instant as zero of the stationary field. Thus its position at every instant indicates the phase angle between the voltage of the incoming machine and that of the bus-bars. As this angle changes, on account of the varying frequency, the iron vane with the pointer attached to it rotates, and when synchronism has been reached it remains stationary.

For other types of synchronism indicators, see a handbook for electrical engineers. When using a single-phase synchroscope with polyphase machines, the same precaution should be observed in regard to the phase sequence as in the case of synchronizing lamps (§473).

With the development of automatic power plants and substations, operated without attendants, a need arose for an automatic synchronizer. Such a device is essentially a relay which closes the operating circuit of the main switch of the incoming machine when the latter is in synchronism. A dash-pot on one of the parts prevents the contact from being closed too soon and requires the existence of a condition of synchronism for a certain period of time before the circuit is closed.

476. EXPERIMENT 21-A. — Exercises in Synchronizing an Alternator. — The purpose of the experiment is to acquire skill in connecting a synchronous machine in parallel with a source of supply. This source should preferably be a regular a-c line used in the laboratory, so as to concentrate the student's attention on one machine only, with its driving motor. If such an outside source is not available, another alternator, driven by a separate motor, may be used. The alternator under test should be driven by a shunt-wound d-c motor, with a fine speed adjustment. The wiring diagram is the same as in Fig. 329, except that the load may be omitted if the machine is to be synchronized with an a-c system which carries a sufficient load.

(1) Connect synchronizing lamps " dark " or " straight," as in Fig. 342; at first use the machine single-phase and synchronize two leads only. After this has been successfully accomplished, find the phase sequence and synchronize all three phases. Do not close the switch until the machines are so nearly at the same frequency that the lamps remain extinguished for several seconds, thus insuring that the incoming machine is in synchronism and in step.

(2) Practice synchronizing with the lamps crossed (Figs. 343 and 344), and make clear to yourself the advantages and disadvantages of both methods. Vary the number of lamps in series and form an opinion as to whether a bright or a dull glow gives the best results.

(3) Try synchronizing with d-c and a-c voltmeters built on different principles (Chapter II), and select the one that gives the best results.

(4) Connect a synchronism indicator, and practice synchronizing with it. Find out if the use of lamps at the same time is of assistance.

Repeat each experiment several times, and then make a final trial to find how many seconds it should take under ordinary practical conditions to start the set, to connect it in parallel, and to make the final adjustments until it is carrying its correct share of the load. The ammeter and the wattmeter need not be read accurately but approximate readings will help in writing the report.

(5) Try connecting the machine in parallel without having one of the three necessary conditions fulfilled (§§472 and 473). Since this may cause an excessive transient current, such a test must be performed with great caution. Nevertheless it is desired that the student form an idea as to the ability of the machine " to pull itself into step."

(6) Connect a choke coil without iron core in each lead between the alternator and the line. This choke coil will limit the transient current

when the machine is switched in without accurate synchronizing. Bring the machine to a speed slightly above synchronism and close the switch. In slowing down, the machine will synchronize itself. Then short-circuit the choke coils. Experiment with different values of reactance and find the best amount.

(7) Repeat the preceding test with series resistance in place of reactance.

Report. (1) State your findings as to the amount of time that it should take an experienced operator to synchronize the alternator with the different methods tried. (2) Describe the phenomena observed when the main switch was closed without having the machine properly synchronized. (3) Give the data and your opinion as to the usefulness and practicability of using choke coils or resistors in the main leads for the first closing of the switch. (4) Show that the results obtained with the choke coils and resistors are in accord with the theory given in §§478 and 479.

477. Operation of Synchronous Machines in Parallel. — Let it be assumed for the sake of simplicity that a comparatively small alternator under test is operated in parallel with a source of power supply of practically unlimited capacity, so that fluctuations in the power output or the circulating current of the small machine do not appreciably affect the operation of the other machines. The bus-bar voltage and its frequency are therefore supposed to be absolutely constant under all possible conditions of operation of the machine under test. The magnitude and the phase angle of the current which the alternator under test is sending into the network, with its very large connected load, are functions of two independent variables, viz., (1) of the speed setting of the prime mover or electric motor which drives the alternator, and (2) of the field excitation of the alternator. The influence of these two factors will be considered separately.

478. Field Excitation. — Let the prime mover or the driving motor of the machine under test be so adjusted that the alternator sends practically no power into the network, and furthermore let the excitation of the alternator be so adjusted that the induced emf is equal to the bus-bar voltage, so that there is no appreciable reactive circulating current. Now, let the field current of the alternator be somewhat reduced, the new conditions being shown in Fig. 348*a*. In this vector diagram E_b is the bus-bar voltage and E_m is the machine voltage, or more exactly, the induced emf corresponding to the new excitation. E_r is the difference of the two or the resultant voltage acting upon the armature of the machine. In most alternators the armature resistance may be neglected as compared to its reactance, so that E_r causes a lagging quadrature current, I, between the machine and the rest of the system.

With respect to the induced voltage, E_m, of the machine this current I is in leading quadrature; hence it exerts a direct armature reaction which strengthens the field excitation and increases the induced emf (§464). Thus, the induced emf E_m may be thought of as consisting of two parts: E_o, which is the emf induced with the same excitation at no load, and E_a, due to the effect of the armature reaction. The conditions with the field over-excited are shown in Fig. 348*b*. Here the armature current lags behind the induced emf, the armature reaction demagnetizes the field, and the reactive voltage drop E_r is again equal to the difference between the induced voltage and that at the bus-bars.

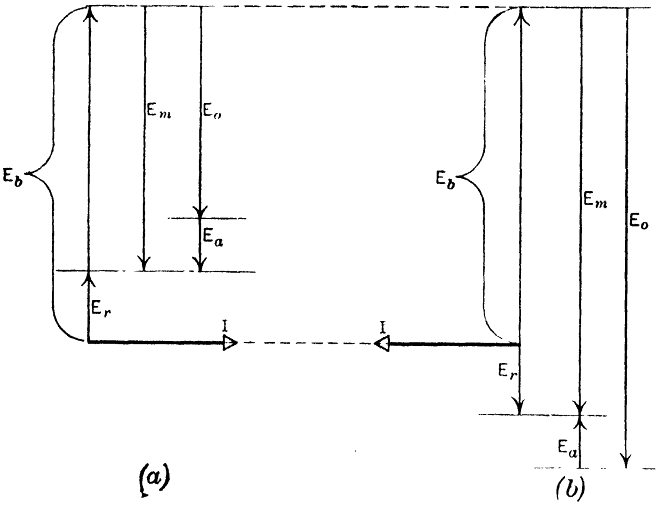

FIG. 348. Reactive circulating current between a large source of power supply and a synchronous machine; (*a*) machine under-excited, (*b*) machine over-excited.

Thus, by varying the field excitation it is possible to cause an alternator operating in parallel with others to deliver a lagging or leading current of any safe desired magnitude, without appreciably changing the working component of the current. The total reactive current in the connected system depends upon the character and magnitude of the load, and is fixed for a given load. Thus, when the reactive current delivered by one of the alternators is reduced, the total reactive component supplied by the other machines is increased by the same amount. With several interconnected power stations or substations, it is of advantage to furnish as much reactive current as possible from the station nearest to the load, so as to reduce the needless I^2R loss in the transmission lines

479. Speed Setting. — With two or more d-c generators operating in parallel (§255) the amount of power delivered by one of them can be increased by increasing its field excitation. Even though the prime mover driving that particular machine should slow down somewhat, the increased induced emf would cause the machine to deliver a greater current. A d-c generator can also be made to deliver a higher current by increasing the speed of its prime mover or driving motor. On the other hand, the power output of an alternator working in parallel with others can be increased by the second expedient only, that is, by changing the speed setting of its prime mover. As is shown above, an increase in the field excitation merely produces a reactive current which demagnetizes the field.

Let an alternator working in parallel with a large system be so excited that its induced emf is practically equal to the bus-bar voltage, and let the governor of the prime mover be adjusted for a very small output, so that the alternator delivers practically no current to the network. Now let the setting of the governor be changed to that for a slightly higher speed. The new conditions are shown in the vector diagram in Fig. 349. E_b is again the bus-bar voltage and E_m is the induced emf of the machine. Because of the higher setting of the governor, the reversed vector E_m leads E_b slightly. The difference between the two, E_r, will cause the machine to deliver a current. Again neglecting the armature resistance as compared to its reactance, we may say that this current I is in lagging phase quadrature with E_r. This current is almost in phase with E_m and therefore is an energy current which produces a useful counter torque.

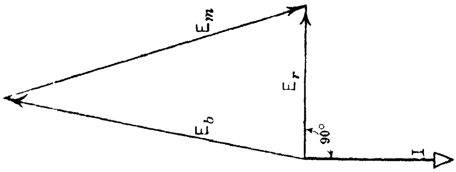

FIG. 349. Diagram to show that the phase of the induced emf has to be advanced to make the alternator deliver an energy current.

The alternator under consideration experiences but a momentary acceleration, after which it continues to run in synchronism, delivering an energy current to the line and assuming an increased amount of the load because of a higher setting of the governor. The same consideration applies to the case in which an alternator is driven by a shunt-wound d-c motor. Weakening the field of the latter acts in the same way as setting a centrifugal governor for a higher speed.

Let two identical alternators be driven side by side unloaded and connected in parallel to the same bus-bars. Let the center lines of the shafts of the two machines lie on the same straight line. It is assumed that other alternators are furnishing all the required power. Now let the governor setting of machine 1 be changed to make the machine deliver some power to the line, while machine 2 still continues to run idle. Originally the revolving parts of the two machines were running in agreement, that is, for an observer viewing along the shaft line, the field poles of one machine would seem to cover the corresponding poles of the other. When machine 1 is loaded, its revolving part is shifted forward with respect to that of machine 2. This phenomenon may be made visible by drawing a radial line in an identical position on the revolving part of each machine and illuminating both lines with an arc or neon lamp fed from the bus-bars. The stroboscopic effect will cause these two lines to appear stationary even when the machines are running; the shifting forward of the line on the loaded machine can be readily observed. Two contacts and a telephone receiver can also be used, as in Fig. 244.

As the prime mover is set for higher and higher input, the revolving structure of the alternator is shifted forward accordingly, thus generating a larger energy current and developing a higher opposing torque. When the angle of shift has reached a certain limit (about 90 electrical degrees) a further increase in the angle will give a smaller torque. Thus, if the governor is set for an output higher than this maximum output of the alternator, the latter pulls out of step, that is, becomes electrically " uncoupled " from the rest of the system, and the set, having lost its load, will tend to run away.

480. Operation as a Synchronous Motor. — Like a d-c generator, an alternator is convertible in its action and may be made to operate as a so-called synchronous motor. To show this operation experimentally, it is convenient to drive the machine by a d-c shunt-wound motor whose field may be adjusted within wide limits. The alternator is synchronized in the usual way, and the field currents of both machines are so adjusted that the alternator delivers practically no current to the line. If the field current of the driving motor be reduced, so that the set tends to speed up, the synchronous machine will operate as a generator delivering power to the a-c line. However, if the field excitation of the driving motor be increased, so that the set tends to slow down, an energy current will flow into the alternator armature from the bus-bars, driving the a-c machine as a motor. This will convert the d-c motor into a generator and will cause it to send electric power back into its source of d-c supply.

In this case there is also a definite overload limit for the synchronous motor similar to that which exists when the machine is acting as a generator. With motor operation, the field structure is shifted back as the load increases; when the angle of shift has exceeded that corresponding to the maximum torque, the machine drops out of step and comes to rest.

For further details regarding the synchronous motor and hunting of synchronous machines when operated in parallel, see Vol. II. Hunting is also discussed more in detail in the author's article " Hunting and Parallel Operation of Synchronous Machines," *Sibley Journal of Engineering*, March, 1920.

481. EXPERIMENT 21-B. — Parallel Operation of Alternators. — The purpose of the experiment is to study the division of the load between two or more synchronous alternators in parallel, and the influence of the field excitation upon the reactive component of the current. The theory is given in §§477 to 480 above. The machine under test should be driven by a shunt-wound d-c motor and connected in parallel with the laboratory power supply or with another alternator. The speed of the driving motor should be adjustable in fine steps near the synchronous speed of the alternator. Instructions for synchronizing are given in §§474 to 476. A suitable load should be provided, consisting of a combination of adjustable resistances and reactances (Fig. 329), unless the power from the alternator under test can be conveniently used in the system with which it is working in parallel. The following instruments should be provided for both the alternator and the driving motor: a voltmeter, an ammeter in the armature circuit, and a field ammeter. A wattmeter and polyphase board should be connected for measuring the a-c output.

(1) Having synchronized and " switched in " the machine under test, adjust the field currents of both the a-c and the d-c machine to such values that the alternator sends practically no current into the line. Keep the motor excitation constant and increase the excitation of the alternator in steps until the machine delivers its maximum safe current. At each step read all the instruments. Devise some simple arrangement to show that the current delivered by the machine is of low power factor and is lagging and not leading. A power-factor meter (§109) may be used for this purpose. If the machine under test is working in parallel with another machine of limited capacity and not with a large network, it may be necessary to adjust the load and the field excitation of the other machine in order to keep the terminal voltage of the machine under test constant.

(2) Bring the excitation of the alternator again to the value that it had at the beginning of the preceding run. Reduce the field current

in steps and take similar readings. Show that the current in this case is leading.

(3) Reëstablish the normal conditions as at the beginning of the first run. Now keep the alternator field current constant and decrease the motor excitation in small steps to the lowest permissible value. At each step read all the instruments. Devise some simple method by which the forward shift of the alternator field with the load may be demonstrated, and if possible measured. Determine the limits within which the alternator excitation has to be varied in order to keep the current strictly non-inductive between no load and full load.

(4) Having brought the conditions back to normal, open the main switch on the d-c side to show that the alternator can operate as a *synchronous motor.* Take readings of the instruments on the a-c side, with different values of the alternator field, from the maximum obtainable to the minimum. This will give data for the so-called V-curve of the synchronous motor at nearly no load, that is, the curve of armature amperes against field current with the synchronous motor driving only the unloaded d-c machine. For each point taken be sure to determine whether the armature current is leading or lagging.

(5) With the conditions as in (1) above, increase the field current of the driving motor above normal, by small steps, and observe the reversed flow of power in both machines, reading all meters in both d-c and a-c circuits. During this run the d-c motor becomes a generator and returns power to the d-c supply.

(6) With conditions as in (5) hold the d-c output constant and vary the field of the synchronous motor over the available range and get data for the V-curve at a constant, and appreciable, load. Note that varying the field of the synchronous motor merely changes the reactive component of the armature current, as witnessed by the almost constant reading of the wattmeter.

Report. (1) Show from your readings that varying the field excitation of a synchronous motor or generator mainly influences the reactive component of the armature current. Describe how the sign of the reactive component (leading or lagging) was determined experimentally and check the results with the theory of armature reaction. (2) Explain how, by varying the field of the driving motor, the alternator is changed into a synchronous motor. Substantiate the explanation by the data taken in the laboratory. (3) Plot the V-curves (armature amperes against field amperes) for the unloaded and for the loaded synchronous motor; also the inverted V-curves, i.e., power factor against field current, at constant load. (4) Compute a few values of the efficiency of the set, with the power flow in each direction. (5) Describe the demonstration of the mechanical shifting of the field structure with the load, both forward and backward. (6) Predict theoretically the combined effect of field current regulation of both the driving motor and the synchronous machine. Assume the set to constitute one of the several plants on a large interconnected system, with a central load dispatcher to whom the local conditions as to the load and power factor are reported from each generating station at frequent intervals. Under what conditions would he direct the operator to regulate the prime mover (in this case the driving motor), and under what conditions would he direct him to adjust the alternator field? How could this regulation be accomplished automatically? (7) Expand the vector diagram of Fig. 349 to apply to two identical machines in parallel, each delivering one-half of the total load current I_0 when $-E_b$ and E_m are in phase. Show how a phase displacement between $-E_b$ and E_m, causing the corrective current I, will cause an increased load on the leading machine and a decreased load on the lagging machine.

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CHAPTER XXII

THE POLYPHASE INDUCTION MOTOR

482. The construction and principle of operation of the polyphase induction motor are shown in Figs. 350 to 352. The stationary part, or the *stator*, consists of steel laminations with slots along the inner edge. A winding is placed in these slots and connected to a two-phase or three-phase source of supply (§§440 and 442). The winding is similar to that shown in Figs. 353 and 354, and the combination of alternating currents in it produces a *revolving magnetic field* in the air-gap, as is explained below. An instantaneous distribution of this flux in a six-pole machine is shown in Fig. 352. The poles are alternately marked *N* and *S*, and the flux density is assumed to be distributed in space approximately according to the sine law. This distribution of flux density moves uniformly along the air-gap, so that at any other instant the distribution is the same as in the sketch, except that the points of maximum and zero flux density have been shifted. The motion of the

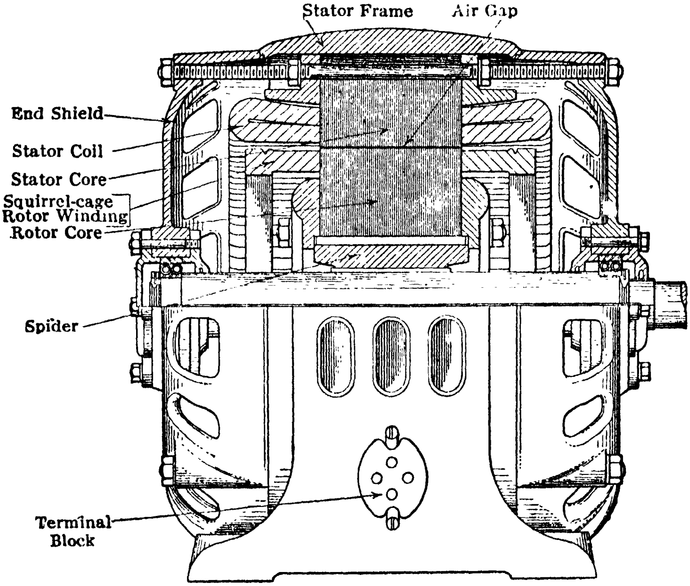


FIG. 350. An induction motor.

air-gap flux takes place at the *synchronous speed*; that is, the flux glides a distance equal to that occupied by a pair of poles during the interval of time corresponding to one complete cycle of the exciting currents. The flux revolves to the left or to the right, depending upon the connections between the stator winding and the source of power, i.e., upon the phase sequence.

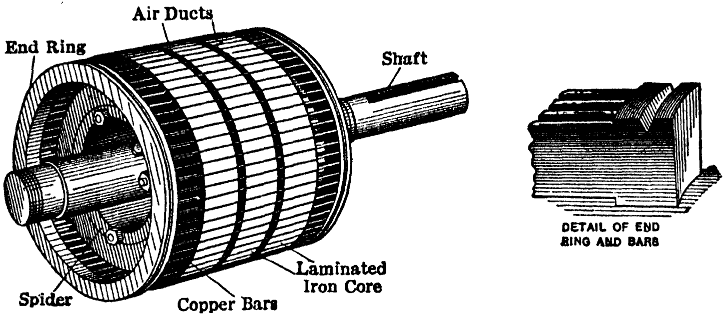


Fig. 351. A squirrel-cage rotor.

The *rotor* also consists of steel laminations assembled on a spider or directly on the shaft, and provided with slots on the periphery. A winding is placed in these slots and is short-circuited upon itself. This winding usually consists of copper bars (Fig. 351) connected at both ends to rings made either of copper or of an alloy of comparatively high resistivity. Such a winding is known as a *squirrel-cage winding*. It is the simplest and most reliable type of winding known.

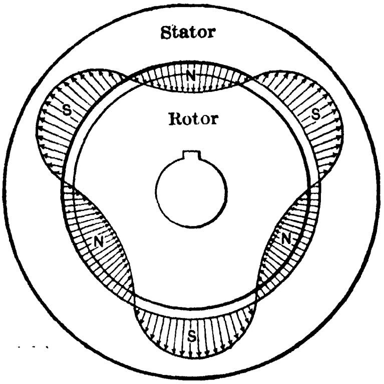


Fig. 352. The magnetic field in the air gap of a six-pole induction motor.

Just as a d-c motor requires a starting resistance in series with the armature, so an induction motor should have a starting resistance in series with its rotor winding, in order to reduce the starting current. This also increases the starting torque per ampere of line current. The resistance should be cut out gradually as the rotor speeds up, and under the normal operating conditions the rotor resistance should be as low as possible, in order to reduce the secondary copper loss to a minimum and to increase the efficiency of the motor. Unfortunately, there are no simple means for connecting a starting resistance in series with a squirrel-cage winding, and therefore such a winding is usually designed as a compromise between the best starting and operating conditions. The resistance of the end-rings, as a rule, is not high enough for the best starting torque and current and is too high for the best possible efficiency at full speed. The double squirrel-cage motor does have both a good starting torque and a satisfactory performance at normal load. (See bibliography.) These characteristics are obtained at an increase in cost over the simple squirrel-cage motor.

Although squirrel-cage rotors are largely used because of their simplicity and lower cost, rotors which must start at a heavy torque, as in crane service, or on systems in which the starting current must be limited on account of its effect upon the line, are usually provided with a *three-phase winding* similar to the stator winding. This winding has three terminals to which resistances may be connected temporarily while the motor is being started. These resistances are usually stationary, and the connection with the rotor is established through three slip-rings and brushes. Sometimes the resistances are mounted within the rotor and are connected to the winding either manually or automatically, by a centrifugal device. Such resistances are normally used in the rotor only during the starting period, and then gradually cut out as the motor speeds up. Adequate resistances, external to the rotor, may be used to control the speed of the loaded motor (see §488).

ELEMENTARY THEORY

483. The Principle of Operation. — The principle upon which a polyphase induction motor operates is similar to that in the familiar experiment in which a permanent magnet is rotated in front of a copper disk and sets the disk in rotation by means of eddy currents induced in it. If the disk carries no mechanical load, its tendency is to revolve at the same speed as the magnet, because then the eddy currents are reduced to zero and there is no electromagnetic torque, since none is needed. When a reasonable mechanical load is applied to the disk it slows down to such a speed that the induced currents form with the magnet a torque equal and opposite to the load torque.

The difference between this arrangement and the induction motor is that in the latter the revolving field is produced by means of stationary windings, and the power for driving the rotor is furnished to these windings from the line electrically. The secondary winding consists of parallel bars, instead of a solid copper disk, in order to force the secondary currents to flow in directions parallel to the shaft, and to suppress harmful eddy currents in other directions. The emf is perpendicular to the directions of both the flux and the current. (Thus, with the magnetic flux perpendicular to the surface of the rotor and the currents parallel to the shaft, the driving force is tangential to the rotor surface in planes perpendicular to the shaft.

484. The Rotating Magnetic Field. — The key to an understanding of the performance of the polyphase induction motor lies in a clear conception of its revolving field. A theory of this field is given in Vol. II, and the practical results are as follows:

(1) When two stator windings (Fig. 353) are displaced *in space* by 90 electrical degrees and are supplied with two-phase currents, that is, with currents displaced by 90 electrical degrees *in time*, a magnetic field is produced which moves synchronously along the air-gap, covering a space equal to two pole-pitches, or 360 electrical degrees, during each cycle of the alternating currents.

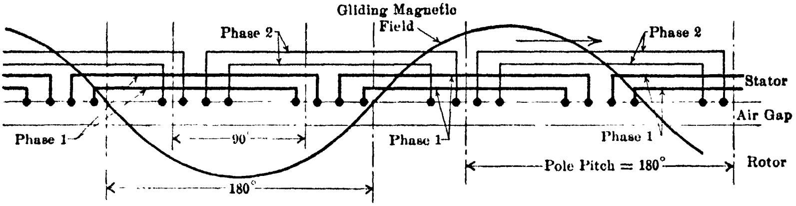


FIG. 353. A two-phase winding, and the gliding magnetic flux produced by it.

(2) When three stator windings (Fig. 354) are displaced *in space* by 120 electrical degrees and are supplied with three-phase currents, that is, with currents displaced by 120 electrical degrees *in time*, a magnetic field is produced which moves synchronously along the air-gap, covering a space equal to two pole-pitches, or 360 electrical degrees, during each cycle of the alternating currents.

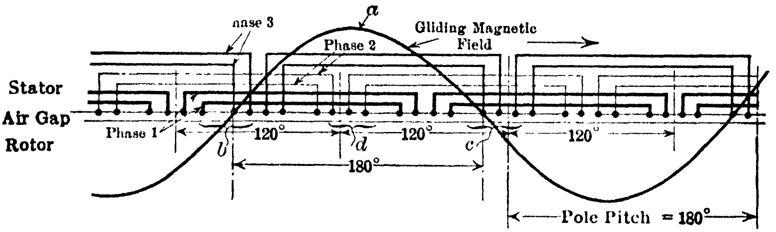


FIG. 354. A three-phase winding and the gliding magnetic flux produced by it.

(3) (The flux glides in the direction from the phase in which the current is leading to the one in which it is lagging.) To reverse the direction of rotation of the flux in a two-phase machine, the terminals of one of the

phases are reversed. To reverse the flux in a three-phase machine, two out of three terminals are interchanged.

Figure 355 is the vector diagram of the induction motor.

At no load the revolving flux is excited by the stator winding only, and the current, I_n, drawn from the line consists of the magnetizing current, I_ϕ, in quadrature with the terminal voltage, and of a small energy component, I_w, which supplies the core loss, the copper loss, and the friction and windage.

When the machine is loaded, the air-gap flux is due to the combined action of the primary and secondary mmf's, as in a transformer (§394). Since the secondary currents oppose the primary, larger currents are drawn from the line to compensate for the secondary mmf, and to leave a sufficient mmf for the production of the air-gap flux. Thus, the stator current, I_1, consists of the no-load current, I_n, plus the component, $-I_2$, which balances the rotor current $+I_2$ (Fig. 355).

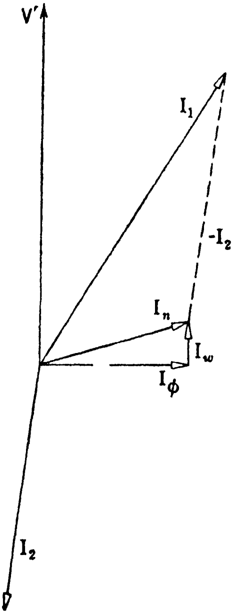


FIG. 355. Vector diagram of a three-phase induction motor.

An induction motor is usually called upon to operate at a constant primary voltage. Under these conditions the air-gap flux is nearly constant and independent of the load. This property of the polyphase induction motor greatly simplifies its theory. The applied voltage is distributed in the stator windings approximately in the same manner as in the primary winding of a transformer, viz.:

(1) The greater part of it is balanced by the useful counter emf induced in the windings by the air-gap (mutual) flux.

(2) A small part is balanced by the emf's induced in the stator windings by the leakage fluxes (in the slots and around the end connections). This is called the primary reactance drop of the machine.

(3) A small part goes to overcome the ohmic drop of the primary current in the resistance of the stator windings.

In a well-designed machine, parts (2) and (3) are small, so that the counter emf (1) is nearly equal to the applied voltage. If the latter is constant, then the counter emf is also nearly constant. But this counter emf is induced by a flux revolving at a constant speed independent of the load. Consequently, the air-gap flux is also nearly constant, that is, independent of the load. In reality the flux decreases by a small amount as the load increases, on account of the voltage lost in (2) and (3).

In a machine designed for a special use, or in a very small motor, the primary impedance drop may be quite large, and the air-gap flux may decrease considerably with the increase in the load. Also when a motor is just starting, at several times its rated current, the air-gap flux may drop to about one-half its normal value, because of a large primary impedance drop. Therefore, the statement that the air-gap flux is nearly constant applies only to the working range of a standard induction motor of moderate or large size, designed for continuous operation.

485. Slip and Torque. — The speed of the revolving magnetic field depends upon the frequency of the line currents and upon the number of poles of the stator. During one alternation of the supply current the field travels from one pole to the next pole; therefore the number of revolutions per minute of the rotating field is equal to the number of alternations per minute divided by the number of poles of the stator winding. For example, on an ordinary 7200-alternation lighting circuit the revolving field of a 6-pole induction motor makes 7200/6 = 1200 revolutions per minute (§441).

The speed of the rotor is lower than that of the revolving field, because the secondary conductors must cut the lines of force in such a direction that the induced currents tend to pull the rotor into synchronism. In practice, the speed of the armature at full load is a few per cent less than that of the revolving field. For example, the actual speed of the above-mentioned motor is somewhere near 1140 rpm. This difference between the speed of the revolving field and the actual speed of the rotor is called *the slip* of the induction motor, and is usually measured in per cent of the speed of the revolving field. (This latter speed is also called the *synchronous speed*, and is the speed of a synchronous alternator with the same number of poles and generating the same frequency.) It is also the actual speed of an ideal rotor which has no ohmic resistance. The synchronous speed of the above motor is 1200 rpm, and the slip at full load is

$$\frac{1200 - 1140}{1200} = 5 \text{ per cent}$$

The slip depends on the load and increases with it. At no load the motor runs almost synchronously, because a very small difference in speed between the revolving field and the rotor is sufficient to induce secondary currents which, with the air-gap flux, form a small driving torque equal to that of the friction and windage. As the load increases, the rotor slows down somewhat, while the speed of the revolving field remains the same. Therefore the rotor conductors cut the air-gap flux at a higher and higher relative speed. As a result, larger and larger secondary currents are induced which, with a nearly constant magnetic field, exert a higher and higher tangential force upon the rotor. For each load there is a speed at which the driving electromagnetic torque is equal to the load torque plus that due to friction and windage.

Low-resistance rotor. Experiment and theory show the general character of the speed-torque curves of an induction motor to be that shown in Fig. 356. Consider first the case of a low-resistance rotor. The working part of the torque curve is hf. At no load and zero torque the machine runs at practically the synchronous speed Oh, where the origin O is at the stand-still point. As the load increases the operating point moves from h towards f. At the rated torque, corresponding to the ordinate kg, the machine runs at the speed Ok, the slip being equal to hk. As the torque is further increased, the point f is reached at which the motor " pulls out " and quickly comes to a stop. The part ff_0 of the curve corresponds to an unstable operation, and Of_0 is the starting torque. If the actual torque applied to the motor at start is less than Of_0, the machine will start, run up the part f_0f of the curve and will operate in a stable manner somewhere between f and h. If the applied load torque at start is greater than Of_0 the motor will not start at all.

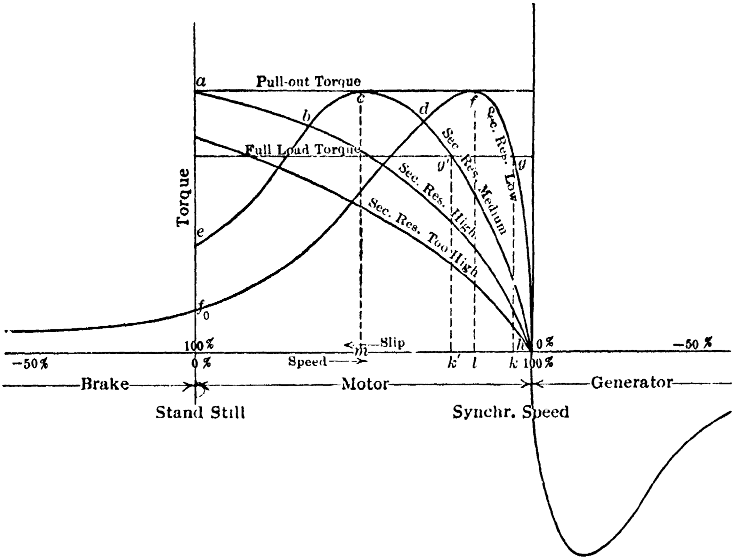


FIG. 356. Speed-torque curves of a polyphase induction motor, at several values of rotor resistance.

The existence of the critical point f is due to the leakage inductance, L_2, of the rotor winding. When currents flow through the secondary winding they cause magnetic fluxes around the conductors, in slots and at the end connections. These fluxes cause the currents to lag behind the induced voltages. The tangent of the angle of lag, ϕ_2, is equal to the ratio of the reactance $2\pi f_2 L_2$ to the resistance r_2 (see Chapter VI) or $\tan \phi_2 = 2\pi f_2 L_2 / r_2$. Both L_2 and r_2 are approximately constant, but the frequency of the secondary currents increases with the slip, so that as the load increases these currents get more and more out of phase with the induced voltages. The electromagnetic torque is proportional not to the whole secondary current I_2, but only to that part of it which is in phase with the flux. This is illustrated in Fig. 357. At a small load the secondary current is I_2', and as the load increases the current becomes equal to I_2''. The component Oa'' of I_2'', in phase with the flux Φ, is greater than the corresponding component Oa' of I_2', and the motor is capable of carrying the increased load. This corresponds to the stable portion hf of the curve. As the load further increases, the slip increases, and while the secondary current also increases, its vector assumes such a phase position, I_2''', that its component, Oa''', in phase with Φ is smaller than before, thus representing a decreased rather than an increased torque, and the motor pulls out and stops. These results may be briefly summarized as follows:

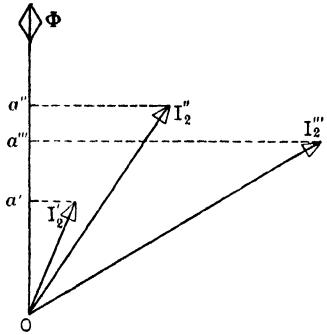


Fig. 357. A vector diagram showing that a larger secondary current may have a lower torque component than that of a smaller current.

At low values of slip, a low-resistance rotor of a polyphase induction motor behaves like a friction coupling, transmitting mechanical power and drawing an energy current from the line. At very high values of slip, the rotor behaves like the secondary of a transformer carrying a purely inductive load. At intermediate values of slip both characteristics are present to a varying degree.

The effect of increased rotor resistance. Should the rotor resistance of an induction motor be increased, without any other change in its dimensions, the speed-torque curve assumes the shape hce (Fig. 356), instead of hff_0. Theoretically the value of the pull-out torque, cm, remains equal to fl, but it takes place at a lower speed Om, or at a higher per cent slip hm. For the same useful torque as before, say $k'g' = kg$, the slip hk' is greater, because a higher induced emf is required in the rotor to establish the same current through a higher impedance. The starting torque is now equal to *Oe* and is greater than before. By suitably increasing the rotor resistance still further, the speed-torque curve can be made to assume the shape *hba*, with the maximum torque at start. Such a motor would have a very low efficiency at running speeds and would show great variations in speed with the load.

The advantages of the curves *abh* and f_0fh can be readily combined by using an additional resistance in the rotor, for starting only. The machine is started on the curve *ab* if the highest obtainable torque is desirable, then part of the resistance is cut out and the performance continues on the curve *bcd*. Finally, when the machine has reached a considerable speed, all the external resistance is cut out, and the rotor reaches its normal speed on the curve *dfg*. The resistance may be cut out in any number of steps. A liquid rheostat, in which the number of steps is infinite, is sometimes used.

STARTING CONDITIONS

486. The Starting Torque. — The starting torque of a polyphase induction motor is approximately proportional to the square of the applied voltage, for the following reason: When standing still, the machine is identical with a transformer (§§407 and 408) and may be replaced by a combination of resistances and reactances. But in such a combination all the currents are proportional to the applied voltage, the equivalent impedance being constant. Therefore the net mmf and the air-gap flux are proportional to the applied voltage. But the torque is proportional to the product of the air-gap flux and the secondary current, and as each of these is proportional to the applied voltage, their product, or the torque, is proportional to the square of the voltage.

At a given voltage the starting torque is a function of the line frequency. Although a motor designed for a certain frequency could not be expected to operate satisfactorily at a widely different frequency, small variations in frequency are unavoidable. Sometimes a high starting current of the motor may pull down the alternator speed a few per cent. Theory shows that the starting torque at a given voltage is inversely proportional to the expression $[(r_{\text{equiv.}}^2 + (2\pi f L_{\text{equiv.}})^2]f$, where $r_{\text{equiv.}}$ is the total or equivalent resistance of the motor, $L_{\text{equiv.}}$ is the equivalent leakage inductance, and f is the frequency (see the author's "Electric Circuit," p. 129). Thus, the starting torque increases when the frequency decreases, and the increase is more rapid than the corresponding decrease in the frequency.

487. Starting of Induction Motors. — Small induction motors, or those provided with a high-resistance rotor, can be started at full voltage by simply connecting the stator to the line. This is usually permissible only when the machine starts at a light load, as in a motor connected to a centrifugal blower, centrifugal pump, etc., and when the first rush of current is not too objectionable. A starter with primary resistances is also sometimes used (Fig. 358).

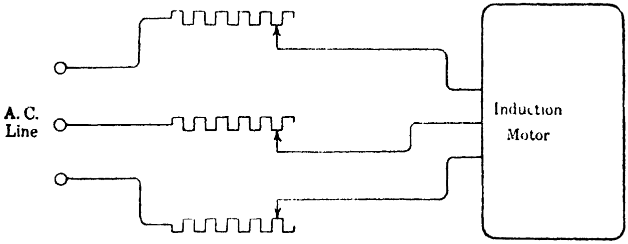


FIG. 358. Starting a three-phase induction motor using resistances in the stator circuit.

A medium-sized motor, provided with a squirrel-cage rotor, is usually started at a reduced voltage (Fig. 359) with two V-connected auto-transformers (Vol. II). Taps are provided in the transformer windings at different fractions of the total number of turns, so that a starting voltage may be chosen to suit the required starting torque. In order to reduce the starting current, as low a starting voltage as will give satisfactory acceleration of the motor should be selected, because the starting current is proportional to this voltage. On the other hand, the starting torque decreases as the square of the applied voltage (§485), and this fact determines the lowest limit of the starting voltage. As the rotor speeds up the current reduces so that the stator may now be connected directly to the line and the auto-transformers be cut out. For details of induction motor starters see Vol. II.

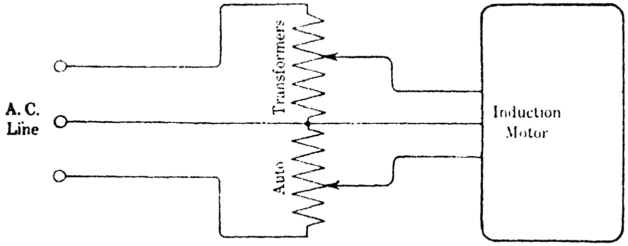


FIG. 359. Starting an induction motor by means of two auto-transformers connected in "V."

The foregoing method is not well suited to a large induction motor, a motor connected to a line on which the starting current is strictly limited, or one which has to be started and stopped at frequent intervals. In such cases a three-phase rotor is provided with three slip-rings (Fig. 360) which at the start are connected to three external resistances, and later short-circuited upon themselves. Internal resistances which revolve with the rotor are but seldom used because an arrangement whereby such resistances can be regulated and short-circuited, automatically or manually, is usually more complicated and less reliable than slip-rings with stationary resistances. The relationship between the starting torque and the resistance of the rotor circuit is explained in §485 and illustrated in Fig. 356. It is important to connect all the resistance " in " again as soon as the motor is stopped, to be sure that the machine is ready for the next start.

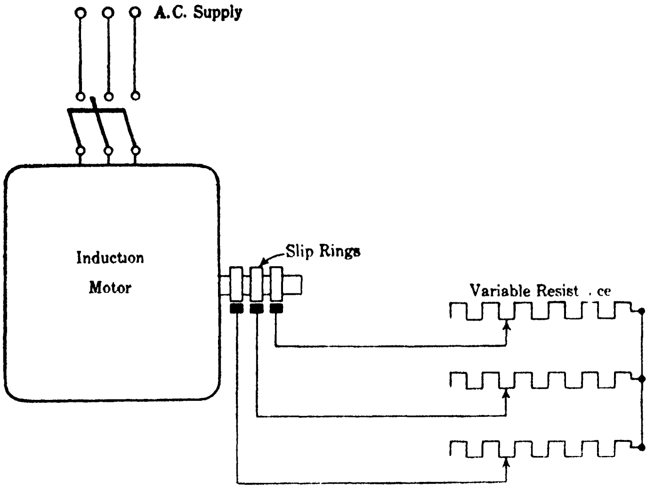


FIG. 360. Starting an induction motor by means of resistances in the rotor circuit.

There are some special methods of starting induction motors, devised to do away with the complication of the two standard methods described above. Such special arrangements are seldom used and need be mentioned here only briefly.

(1) Starting with the stator windings Y-connected and running with the same windings delta-connected. This is equivalent to applying a starting voltage equal to $1/\sqrt{3}$ = 57.7 per cent of the line voltage.

(2) Starting at no load, with a centrifugal clutch between the motor and its load.

(3) Starting simultaneously with the generator which supplies the motor with power.

(4) In a plant equipped with several motors, using one or more auxiliary lines from sources of lower voltage or frequency, for starting only.

(5) The rotor is provided with two separate windings, one of high resistance, active at start only, the other of low resistance which automatically comes into play as the motor speeds up, and then carries the bulk of the secondary currents. Several arrangements are in existence for accomplishing this result. The double squirrel-cage motor (see §482) operates on this principle; see also (8) below. Two squirrel-cage windings, one of high resistance but low reactance, the other of low resistance but high reactance, are placed on the rotor. At start, when the frequency of the rotor currents is high (equal to that of the stator) the high-reactance winding carries very little current, while the high-resistance, low-reactance winding carries sufficient current, of high power factor, to produce a good starting torque. As the motor speeds up the reactance of the low-resistance winding diminishes, owing to decreased rotor frequency with the result that this winding carries most of the rotor current and the motor operates like any motor with a low-resistance rotor. Thus slip and efficiency at full load are quite satisfactory. At the same time the motor is capable of being thrown directly upon the supply voltage and draws a moderate current but produces a high starting torque. A section through a typical double squirrel-cage rotor slot is shown in Fig. 361.

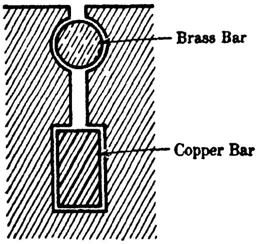


FIG. 361. Section through the rotor slot of a typical double-squirrel cage motor.

(6) Rotor windings are divided into two unequal parts which are connected in opposition for starting and are cumulative in their action in normal operation.

(7) The use of inductances has been suggested in place of resistances in the secondary circuit (Fig. 360) to limit the secondary currents at the start. The disadvantage of this method is that the starting torque is greatly reduced, and the power factor of the machine is lowered.

(8) Choke coils of high reactance, but of low ohmic resistance, are connected across the slip-rings in parallel with comparatively high non-inductive resistances. The frequency of secondary currents at start being high, a large portion of them flows through the resistances. As the motor speeds up the secondary frequency decreases and a larger and larger fraction of the currents flows through the reactances. At full

speed the whole starting arrangement may be short-circuited, or left in, if so desired for the sake of simplicity.

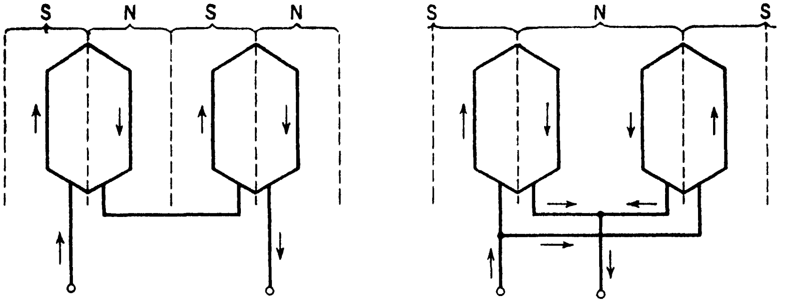
488. Speed Control of Induction Motors. — An induction motor is not well adapted for speed control because of the constant speed of its revolving flux. The electrical energy input from the line is transmitted through the air-gap into the rotor, with the exception of a few per cent lost in the primary copper loss and stator core loss. Let the tangential effort between the stator and the rotor be F kg, the linear speed of the revolving field v_1 meters per second, and that of the rotor conductors v_2 meters per second. Then the electrical input into the rotor is Fv_1 kg-meters, while the mechanical power developed by the rotor is only Fv_2 kg-meters. The remainder, $Fv_1 - Fv_2$, is used up in the rotor, mainly as the secondary copper loss. Thus, of the power transmitted into the secondary, the fraction v_2/v_1 is converted into mechanical power (including friction and windage), and the remainder, $(v_1 - v_2)/v_1$, is the fraction of the electric power lost in the rotor winding. This relationship is usually expressed in the words: *Per cent secondary copper loss is equal to per cent slip.* Let, for example, 100 kw be transmitted inductively from the stator into the rotor, and let the machine be operating at 5 per cent slip. Then the Joulean loss in the rotor is 5 kw, and 95 kw is used to drive the mechanical load and to overcome the torque due to the friction, windage, and hysteresis in the rotor core.

A clear understanding of this relationship is necessary for the study of different methods of speed control. If all but a small percentage of the input into the secondary is to be converted into mechanical power, then the difference between the speed of the air-gap flux and that of the rotor must be small. If the rotor is to run efficiently at various operating speeds, the speed of the stator flux must be adjustable. But this speed is a function (a) of the number of poles, and (b) of the frequency of the supply, so that at least one of these factors has to be varied.

On the other hand, if the speed of the rotor is to be varied without modifying the speed of the revolving flux, there are the following two possibilities: (c) a large secondary i^2r loss is permitted to occur; or (d) the excess electrical power delivered to the rotor is returned into a network or converted into useful mechanical power in another machine. Of these four possibilities (a) and (c) are used regularly, while (b) and (d) are used in some large drives. The practical means for accomplishing these results are as follows:

(a) *Change in the number of poles.* Either the same winding is used and properly reconnected, or two separate windings are placed in the same slots and used one at a time. This method allows only two or three speeds and is used, among other applications, in marine propulsion and in some electric locomotives. Figure 362 shows how the same winding may be connected for either two poles or four poles.

(b) *The frequency of the supply* can be varied at will if the generating set furnishes power only to the motors whose speed is to be adjusted. Thus, in an electrically propelled ship, the speed of the main turbo-alternators can be varied within certain limits for the purpose of regulating the speed of the induction motors which drive the propeller shafts. Recently methods have been devised for controlling motor speeds through the use of variable frequencies obtained from vacuum tubes of the "Thyratron" type.



(a) Simplified Diagram of One Phase of a 4-Pole Stator

(b) Same Winding Reconnected for Two Poles

FIG. 362. Illustrating the use of the same winding for both a four-pole and a two-pole stator.

(c) *The use of secondary resistances* for speed control is the standard method for all intermittent work, on cranes, hoists, switching locomotives, etc. For lack of a better and simpler method, a considerable loss of power in external resistances is accepted as being inevitable. For details of rheostats and controllers see Vol. II. Speed-torque curves for different values of secondary resistance are shown in Fig. 356, and the diagram of connections in Fig. 360.

(d) *The regenerative control* in the secondary circuit may be of two kinds: (1) The excess secondary electric power is converted into electric power of the primary frequency and is returned to the primary circuit, or it is converted into direct current and used in another circuit. (2) The excess secondary electric power is sent through another motor and converted into mechanical power which is used on the same shaft with the main motor, or separately. All such methods of regenerative control require at least one and often two or three additional machines, and are therefore used only in very large drives, such as rolling mills and mine hoists, where the advantages of speed control within wide limits outweigh the additional complications and expense. It is in such drives that polyphase commutator motors find their place as adjuncts to the induction motor. Several descriptions of such systems will be found in the volumes of *General Electric Review* and *Electric Journal.* See also Vol. II, Chapter LV.

Figure 363 shows one arrangement of apparatus in the Kraemer system of speed control. Here a rotary converter takes a-c power from the slip-rings of a large induction motor and delivers d-c power to a d-c motor which is coupled to an induction generator electrically connected to the a-c supply. Weakening the field of the d-c motor causes it to drive the induction generator above synchronous speed and deliver power to the a-c supply. Thus power is drawn from the secondary of the main motor and it is caused to slow down. In a similar manner, if the field of the d-c machine is increased it slows down; the induction machine now operates as an induction motor drawing power from the a-c source. This power is fed to the commutator of the rotary converter from the d-c motor, acting as a generator. Thus power is delivered to the rotor of the main motor and it is caused to speed up, even to values above synchronism. For a range of speed control of the main motor of, say, 20 per cent above and below synchronism, the auxiliary machines must each be of about one-fifth the full-load rating of the main motor. Note that control of the field of the rotary converter alters the phase position of the current it draws from the secondary of the motor so that the power factor, as well as the speed, of the main motor may be regulated.

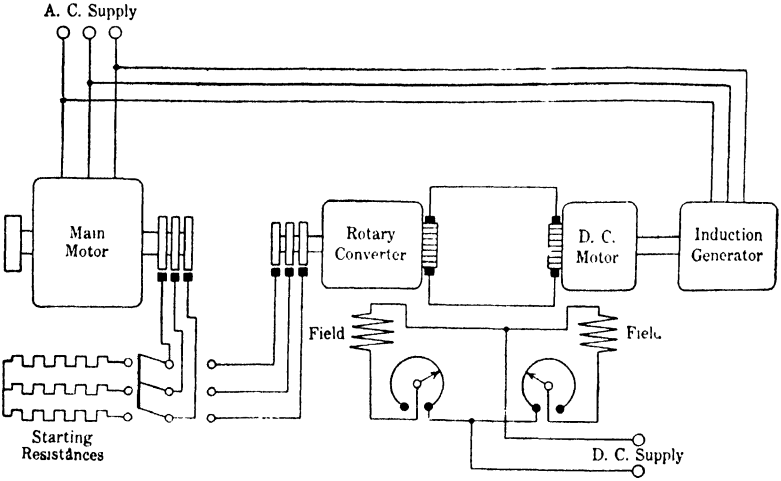


FIG. 363. One form of the Kraemer system of speed control of a large induction motor.

(e) *Concatenation.* Two similar wound-rotor motors, coupled mechanically and connected electrically in series, i.e., the rotor of motor 1 to the stator (or rotor, depending upon the ratio of stator to rotor turns) of motor 2, tend to run at one-half synchronous speed. At this speed the slip of motor 1 is 50 per cent so that the frequency of its rotor currents is one-half rated frequency which is correct for motor 2, considering the speed at which it is compelled to run. (Whichever winding acts as the secondary on motor 2 is either short-circuited or connected to a variable resistor bank.) Two motors operated in this manner develop twice the normal torque of one motor at one-half synchronous speed (ignoring slip). This arrangement has been used to some extent in railway service where the reduced speed and heavy torque are of advantage on steep grades. It is shown in Fig. 560, Vol. II.

489. EXPERIMENT 22-A. — The Starting Torque and Current of a Squirrel-Cage Induction Motor. — The purpose of the experiment is to investigate the value of the starting current and starting torque as functions of the applied voltage, frequency, and rotor resistance. The necessary theory is explained in §§486 and 487. A suitable Prony brake must be provided (§311), preferably with a spring balance, so arranged — for instance with a handwheel and worm — that the lever can be moved up and down while the motor is starting, in order to eliminate the bearing friction of the machine and detect high and low spots in the motor torque. Auto-transformers with numerous taps can be used for applying different voltages (Fig. 359), although it may be simpler to use as power supply a separate alternator in which voltage and frequency can be varied at will. It is sufficient to have an ammeter, wattmeter, and voltmeter in one phase only, if a preliminary experiment shows that the three phases are fairly well balanced. This will save time and so reduce heating of the motor. An indicating tachometer should be connected to the motor shaft. The frequency of the supply must be kept constant except where its influence upon the starting torque is to be determined. Copy the name-plate data of the motor under test.

(1) Find the lowest voltage at which the motor will start at no load. Read volts, watts, and amperes at the instant of starting, find the number of seconds which it takes to attain the final speed, and read the speed, the final volts, watts, and amperes.

(2) Repeat the test at successively higher values of the voltage, taking care not to overheat the motor. Should the windings become dangerously hot, run the motor for a time at no load and at the rated voltage, to allow the windings to cool off through the fanning action of the rotor.

(3) Make a few runs similar to (1) and (2) at frequencies to 10 per cent higher and lower than the rated frequency of the motor.

(4) Apply a brake to the motor and, locking the rotor, gradually raise the terminal voltage until the current reaches the highest safe value (it may be twice the rated current or even higher). Read the current, the voltage, power, and the maximum torque as the brake arm is raised and lowered. Release the brake and allow the motor to come up to speed. Read the final volts, amperes, watts, speed, and starting time. The test should be made as quickly as possible so as not to overheat the machine.

(5) Repeat (4) at successively lower values of voltage and starting current.

(6) If another squirrel-cage rotor of different resistance is available, take a few readings with it so as to have a basis for comparison with the first machine.

Report. (1) Tabulate the data of runs (1) and (2) in such a way as to bring out clearly the effect of the starting voltage upon the current inrush and upon the time of running up to speed. (2) Show that the effect of the frequency upon the starting torque and current is such as would be expected theoretically. (3) Show from the readings how nearly the starting current is proportional to the terminal voltage; extrapolate the straight-line relationship to the full rated voltage. On the same

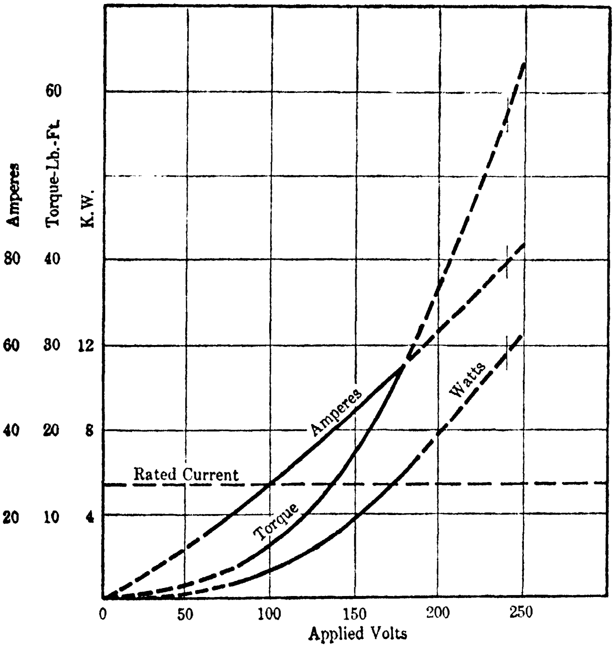


FIG. 364. Blocked rotor curves on a 10-hp, 3-phase, 240-volt induction motor.

curve sheet, mark the rated current of the machine. (4) Show from the readings that the maximum starting torque is nearly proportional to the square of the terminal voltage. Extrapolate the parabolic curve to the rated voltage. From the rated output and speed of the motor, compute the full-load torque (§275) and mark it on the same curve sheet. (5) From the curves of current and torque, suggest a value of starting voltage suitable for a kind of service in which the motor starts practically at no load. Also suggest a starting voltage for another kind of service where the motor has to overcome a starting torque of the same order of magnitude as the full-load torque. (6) If tests have been made with a rotor of different resistance, show that with the same starting voltage the starting torque is higher for the rotor of higher resistance.

The curves of Fig. 364 were taken on a 10-hp, 240-volt motor having a wound rotor but with the rotor short-circuited so that it has essentially the characteristics of a squirrel-cage motor.

490. EXPERIMENT 22-B. — The Starting Torque and Current of an Induction Motor with a Phase-Wound Secondary. — The purpose of the experiment is to investigate the effect of a variable rotor resistance upon the starting torque and current. The necessary theory is explained in §§486 and 487. It is advisable to use the same stator as in the preceding experiment. A suitable Prony brake must be provided (§311), preferably of the self-recording type, because the starting torque of a phase-wound rotor varies considerably in its different positions with respect to the stator. It is essential therefore to note several values of the torque with the same setting of the secondary rheostat, moving the rotor by a small angle at a time. A self-recording brake, or "stationary torque recorder," can be improvised out of an ordinary Prony brake, a spring balance, and a steam-engine indicator. An ammeter should be provided in each stator phase and in each rotor phase so that the resistances in the three phases can be balanced, unless a special balanced three-phase resistor is available, when readings in one phase are sufficient. Stator watts should be measured, and voltmeters should be provided for the line voltage and that between the slip-rings. If it is necessary to balance the resistances it may be convenient to connect the voltmeter in succession between each of the slip-rings and the neutral point. A reliable indicating tachometer should be connected to the shaft. All the tests specified below are to be performed at the rated stator voltage and supply frequency.

(1) Start the test with a value of the secondary resistance such that approximately rated current flows in the stator. Read the maximum

starting torque on the brake; also the primary and the secondary volts and amperes and the primary watts.

(2) Repeat the test at successively lower values of the starting resistance, until the stator current is at its highest allowable value (say $2\frac{1}{2}$ times normal). Be sure to go beyond the value at which the starting torque reaches its maximum (see Fig. 365). If necessary to prevent excessive heating the applied voltage may be dropped to about three-fourths normal and readings taken at the lower resistances. Current and torque readings so taken can be extrapolated to correspond to rated voltage as suggested under report requirements.

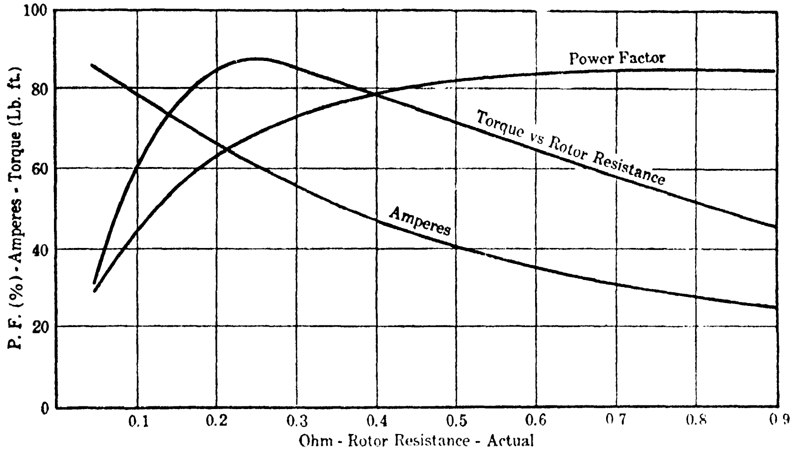


FIG. 365. Effect of varying the rotor resistance of a 10-hp wound-rotor induction motor with blocked rotor.

(3) Try starting the motor on a single-phase line (Fig. 378) and make clear to yourself the limitations of this method.

(4) Try some special methods of starting described in §487.

Report. (1) Describe the preliminary experiments on fluctuations of the starting torque with different positions of the stator, and explain whether the average, the minimum, or the maximum torque has been used in the computations and curves specified below. (2) Against rotor resistance as abscissas, plot the observed values of starting torque, primary current, primary watts, and power factor, also secondary current and secondary voltage. Rotor resistance, per phase of Y, is equal to the ratio of the secondary Y voltage to the secondary current. If the primary applied voltage was other than normal for certain points, change the currents and the secondary voltage readings directly as the voltage, and the torque and watts as the square of the voltage. (3) From the preceding curves note the value of the starting resistance for which the starting torque is a maximum. (4) Specify the resistance values to be used in a starting rheostat to give the following values of starting torque, expressed in terms of full-load value: (a) 100 per cent, (b) 150 per cent, (c) 200 per cent. If there are two possible values, which would you use, and why? (5) Make a comparison between the starting characteristics of a squirrel-cage motor and one with a phase-wound secondary. (6) Describe the tests with special starting arrangements and state the advantages and disadvantages of each.

PERFORMANCE TESTS

491. The Performance Curves. — The operating characteristics of a certain polyphase induction motor are shown in Fig. 366, plotted to horsepower output as abscissas, although the characteristics may also be plotted against kilowatts input, against primary amperes, against torque, slip, etc., depending upon the purpose for which the curves are to be used.

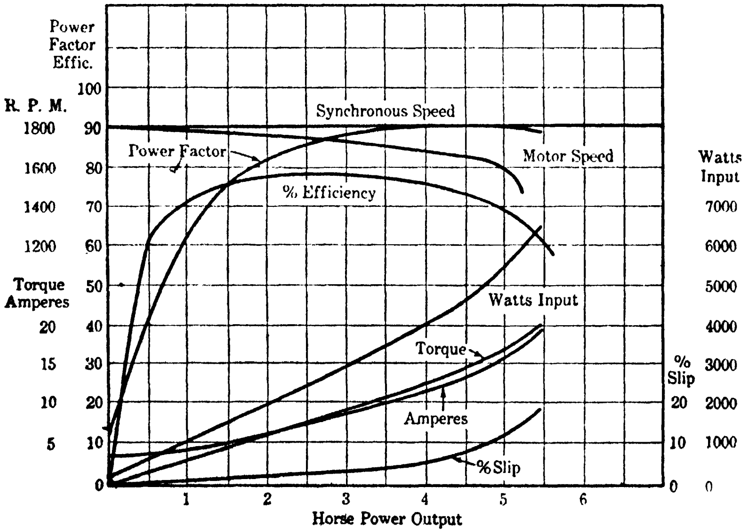


FIG. 366. Performance curves of a 3-hp, 220-volt, 1750-rpm, 3-phase induction motor.

The curves are self-explanatory and should be studied critically to note how the various performance items — power-factor efficiency, current, etc. — vary with the load. A few words in regard to "apparent input" and the calculation of power factor may not be amiss. If the effective value of the primary current per phase is I_1 and the primary Y

voltage is E_1, then the apparent input per phase, in volt-amperes, is defined as $E_1 I_1$. The true or real power input per phase is less, and is equal to $E_1 I_1 \cos \phi_1$ watts, where ϕ_1 is the phase-displacement angle between E_1 and I_1. The total apparent input is equal to 3 $E_1 I_1$, or, using the line voltage (§442) and kilovolt-amperes,

$$P_{\text{app.}} = E_\Delta I_1 \sqrt{3}/1000 \quad \ldots\ldots\ldots \quad (1)$$

where $E_\Delta = E_1\sqrt{3}$.

Similarly, the total true input, in kilowatts, as measured by a wattmeter, is

$$P_{\text{true}} = E_\Delta I_1 \cos \phi_1 \sqrt{3}/1000 \quad \ldots\ldots\ldots \quad (2)$$

The general shape of the curves shown in Fig. 366 follows from the theory of the motor given in §§483 and 485, and from the analytical solution given in §503. At no load, or at zero output, the true input is small and covers only the losses in the motor. The power factor is low because the greater part of the primary current is a reactive component required to excite the main flux. The efficiency is zero (since there is no output), and the rotor runs practically at synchronous speed because the opposing torque is almost zero. As the load is applied, the primary current increases at first slowly, because in addition to a magnetizing component it contains a power component corresponding to the increasing rotor current, and these two components are nearly in quadrature. Thus the power factor and the efficiency are increased, and the slip also becomes greater, to enable larger currents to be induced in the secondary. When the motor is overloaded, ~~the slip and the~~ frequency of the secondary currents increase so much that the effect of the secondary leakage inductance causes a low secondary power factor. This lowers the primary power factor, the effect being similar to that of an inductive load in a transformer (§396). For this reason the power-factor curve is shown slightly drooping at heavy loads. The copper loss increases rapidly with the current, and therefore the efficiency also passes through a maximum and decreases on an overload.

492. The Load Test. — The foregoing curves can be computed either from an actual load test, or from a no-load and short-circuit test, as in the case of a transformer. The first or direct method is described below, and the second or indirect one is treated in §503 and in Vol. II.

A load test may be performed in two different ways, according to whether the mechanical output is measured directly or is computed from the input and the losses. The output may be measured directly by means of a Prony brake (§311), by means of a dynamometer (§§312 and 313), or by using a calibrated generator as a load. The general arrangement is nearly the same as in the load test of a d-c motor (Chapter XII).

The principal difficulty in performing an accurate brake test on an induction motor is to hold the load constant while the speed and the slip are measured. Moreover, the load must be exactly the same when watts and amperes are read in the three phases. Time may be saved and more nearly simultaneous data taken if readings are made in one phase only, provided a check shows the values in the several phases to be substantially equal. In any case, it is preferable to use a steadier load than is possible with a Prony brake. Although a blower or centrifugal pump may be used to supply the steady load, it will usually be found more convenient to employ an electric generator for the purpose. Provision is thus made for varying the load at will. The test is conducted similarly to a brake test, except that the output of the motor is not read directly, but is calculated afterwards from the output of the generator plus its losses. The latter can be partly measured and partly computed, as is explained below.

Experience shows that with this method more accurate results are obtained than with an ordinary brake test. One reason is that the speed and the load are easily maintained constant for any length of time, so that the input, rpm, and slip can be read quite accurately. Another reason is that all the readings and calculations are purely electrical, the mechanical torque being calculated from the input to the generator instead of being read on the brake.

With any load test on an induction motor it is of importance to measure the speed and the slip accurately. Although the speed may differ by only a small percentage from its synchronous value, there is no provision in a squirrel-cage motor for adjusting the speed to a desired value, as one can do in a d-c motor by means of a field rheostat. For this reason the exact speed and its variations with the load must be accurately known in order to provide a proper gear ratio or pulley for a particular drive. When the output is computed from the losses, the slip must be determined with particular accuracy, because the secondary copper loss is directly proportional to it (§488). The speed and slip measurements are described in §§493 to 496.

The electrical input into the stator can best be measured with a polyphase wattmeter (§102) so that the whole power is obtained in one reading. If a polyphase wattmeter is not available and the phases are not closely enough balanced so that a reading of watts per phase is sufficient, the two-wattmeter method is used (§105), a single-phase wattmeter being read in succession in two current phases. It would hardly be practicable to provide a separate ammeter and voltmeter for each phase; a special " polyphase board " is used, by means of which the same instruments may be connected in succession in the three phases. Any of the polyphase boards described in §60 can be used for the purpose.

493. Measurement of Frequency and Speed. — The speed measurement of an induction motor deserves particular attention. The speed depends on the load and on the frequency of the supply since the latter determines the speed of the revolving field. If the power for testing is taken from an industrial circuit, the frequency will normally be held constant because of the time service now associated with most power systems. However, if the supply is from a laboratory generator, the frequency will usually vary, and unless the exact frequency is known at the time when the speed is taken, the speed determination is of little value.

When the generator is accessible, its speed may be measured simultaneously with that of the motor, so that the two speeds refer to the same frequency. If the generator is not accessible, a small synchronous motor may be run from the same line to which the induction motor is connected. As the speed of a synchronous motor is always equal to that of a generator with the same number of poles, and automatically follows speed variations of the latter, the synchronous motor will at any moment give the actual speed of the generator. A calibrated frequency meter (§454) may also be used for measuring the frequency of the supply.

Instead of measuring the speed of the motor and the frequency of the supply, it is sometimes preferable to measure simultaneously the speed and the slip of the induction motor. Their sum gives the synchronous speed, from which the frequency of the supplied currents can be computed. If, for example, the motor speed is 702 rpm, and the slip is 22 rpm, the synchronous speed is 724 rpm. If the motor is a 10-pole machine, the frequency of the supply at that particular moment is 7240 alternations per minute, instead of the standard 7200. When plotting a speed curve, the corresponding correction must be made.

The slip usually constitutes but a small percentage of the speed; at the same time an accurate knowledge of its value is of considerable importance to the designer as well as to the user of the motor. When the slip is determined as a difference between the synchronous speed and the actual speed of the motor, a small error in the determination of either may lead to a considerable error in the value of the slip figured out as the difference of the two. For this reason it is preferable to measure the slip and the speed directly and independently of each other, and to check the frequency of the supply as the sum of the two.

494. Stroboscopic Slipmeters. — The term "stroboscopic" is usually applied to devices and methods based upon the fact that the eye receives a continuous impression when the frequency of flickering of the light exceeds a certain limit, a moving body being visible only at certain points on its path.

(a) *Sectored-disk slipmeter.* A simple slipmeter based on this principle and involving the use of a sectored disk is shown in Fig. 367. A pasteboard or sheet-metal disk, with white sectors painted on it, is mounted on the shaft of the motor. The number of sectors should be equal to the number of poles of the motor. An a-c arc lamp, a neon lamp, or even a small incandescent lamp, fed from the same supply as the motor, is placed before the disk. If the motor were revolving synchronously the white sectors would appear stationary; since, however, the speed of the rotor is a few per cent below synchronism, the sectors appear to the eye to be slowly rotating in the direction opposite to that of the shaft.

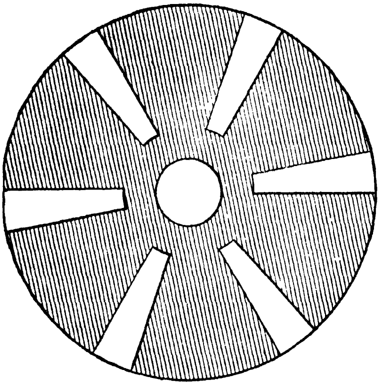


FIG. 367. A sectored disk for measuring the slip of an induction motor by the stroboscopic method.

The reason for this phenomenon is that the light from an a-c lamp varies from a maximum to zero, or nearly zero, during each alternation of the current. With a synchronous rotation of the disk, each sector has just enough time during one alternation to move into the position of the preceding sector. Thus, when the light is renewed, the observer's eye finds the sectors in seemingly the same position as before. Since the flickerings of the light occur several thousand times per minute, the phenomenon appears continuous. When the rotor lags behind synchronism, the sectors do not have enough time to move through one polar division during one alternation of the current, so that, each time the lamp relights, the eye finds the sectors in a slightly different position. This lagging behind goes on continually, and the sectors appear to the eye to be slowly moving backward, while in reality they are revolving at a high speed with the shaft.

The higher the slip of the motor, the faster are the sectors moving backward through the field of vision; the number of *apparent* revolutions of the disk is a direct measure of the slip. In order to count the sectors more conveniently, it is advisable to limit the field of vision so as to see only one sector at a time. Let us assume, for example, that 120 sectors have passed through the field of vision in one minute, while the speed of the motor, as measured by an ordinary speed-counter, was

1765 rpm; let the motor be a 4-pole, 60-cycle, machine. With these data the slip of the motor and the actual frequency of the supply are figured out as follows:

With 120 sectors passing through the field of vision during one minute, the rotor has slipped 120 alternations of the supply. As the motor has four poles it took 120/4 = 30 slip revolutions to lose 120 alternations. Hence, the synchronous speed of the motor was 1765 + 30 = 1795 rpm, and the slip amounts to

$$\frac{30}{1795} = 1.67 \text{ per cent}$$

When the speed was measured, the frequency of the supply was equal to 1795 × 4 = 7180 alternations per minute, instead of the standard frequency of 7200.

It will be seen from the above example that the stroboscopic slipmeter is more suitable for 25-cycle motors and for light loads than for high-frequency motors and for heavy loads, because in the latter case the sectors pass quite rapidly through the field of vision, and it is difficult to count them. In some testing departments this defect is overcome by the use of gears of ratio 1.02, 1.04, etc., which may be changed as the slip of the motor increases. When the sectored disk is driven through gears of ratio 1.02 the disk appears to stand still at an actual slip of 2 per cent; an apparent slip of 2 per cent is an actual slip of 4 per cent, etc. The same result may be attained if the sectored disk rides on a cone pulley along which it may be moved by an adjusting screw until it appears to stand still. The ratio of diameters of cone and disk is indicated on a scale which reads directly the slip.

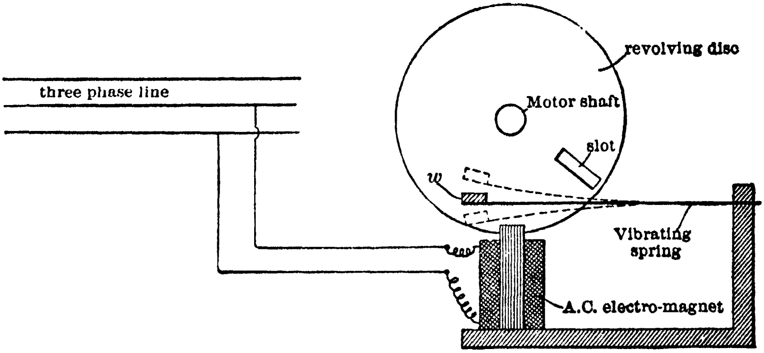


FIG. 368. A vibrating-reed slipmeter.

(b) *Vibrating-reed slipmeter.* Another stroboscopic slipmeter, which does not require a lamp, consists of an a-c electromagnet (Fig. 368) connected to the source of supply and provided with a steel reed clamped near one end. Alternating current flowing through the electromagnet sets the reed into synchronous vibrations. The reed is loaded at its extremity with the weight *w*, so as to make its natural period of vibration correspond to that of the supply. A disk with a slot in it is mounted on the shaft of the motor, and the vibrating reed is viewed through this slot. If the rotor were revolving synchronously, the reed would appear to the eye to be stationary, because it would always be viewed by the observer during the same part of its vibration. Since the rotor lags behind synchronism the reed appears slowly moving up and down. The number of strokes per minute is proportional to the slip, as in the neon lamp slipmeter.

The reed may vibrate at a frequency which is a multiple of that of the supply, and it is safer to calibrate this instrument experimentally, by measuring the slip of a motor by some other method. This will give a constant by which to multiply the number of strokes in order to find the number of alternations slipped per minute.

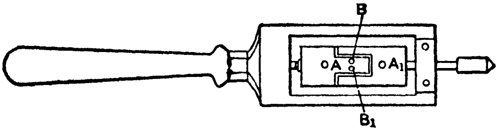


FIG. 369. A commutator-type slipmeter.

495. Commutator Slipmeters.— In places where many induction motors are tested regularly, it is desirable to have a more convenient device for measuring slip than the above-described stroboscopic slipmeters. A device of this kind is shown in Figs. 369 and 370. It is a commutator with as many segments as the motor has poles, and is pressed by hand against the end of the motor shaft, as an ordinary speed-counter. The device is connected through a resistance to the power supply and to a sensitive d-c ammeter. If the speed of the motor were exactly synchronous, the impulses of the current sent through the commutator into the ammeter would be always at the same point on the alternating emf wave, and its indication would be steady. However, since the rotor *lags* behind the revolving field, these impulses occur at different intermediate values of the current, and the ammeter needle swings at a frequency equal to the *difference* of the speeds of the revolving field and the rotor, and hence proportional to the slip of the motor. If the slip is not too high, the

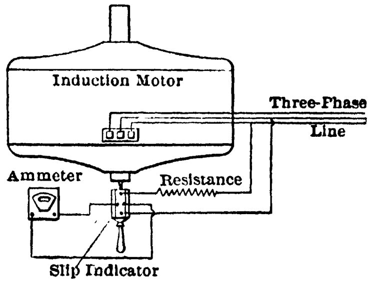


FIG. 370. The electrical connections of the commutator-type slipmeter.

number of swings per minute can be easily counted and the slip calculated. A polarized bell can be used instead of an ammeter, and the number of strokes per minute counted.

When the motor has a wound rotor the commutator device is unnecessary as it is then possible to get slip from the frequency of the rotor currents. This may be done by connecting a d-c millivoltmeter with suitable protective resistance and switch across one of the leads which short-circuit the rotor. The meter will make one complete swing (right and left) for each cycle of rotor current. Since rotor frequency = stator frequency × slip,

$$\text{Slip} = \frac{\text{rotor frequency}}{\text{stator frequency}}$$

As with the stroboscopic slipmeter, it is difficult to count the number of impulses when the frequency is high and the slip is considerable. The Bianchi slipmeter (Fig. 371), similar in its principle to that in Fig. 369, has an attachment for automatically registering the number of revolutions of slip. The impulses produced by the revolving commutator D, instead of being counted by the observer, are sent through the electromagnet M which actuates a ratchet-and-pawl recording mechanism q, through the permanent magnet c. The number of revolutions of slip is thus recorded on the dial Z_2; at the same time the number of revolutions of the rotor is shown on the dial Z_1, connected to an ordinary speed-counter. The operation of this device is entirely automatic; it is pressed against the shaft of the motor at p, as an ordinary speed-counter, and is held for, say, one minute. The reading on one scale Z_1 gives the actual speed of the motor; the reading on the other scale gives the num-

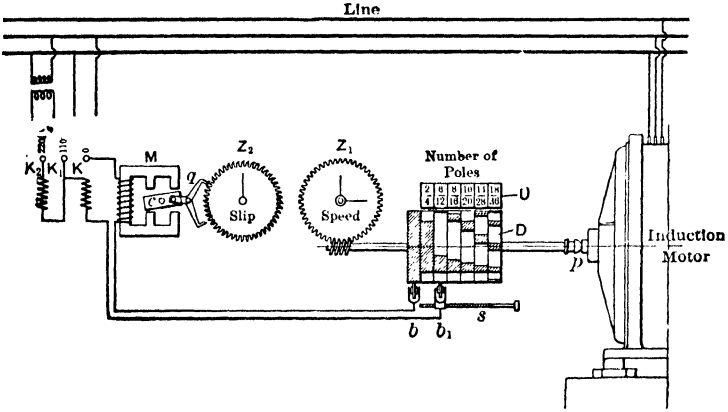


FIG. 371. The Bianchi automatic slipmeter.

ber of revolutions of slip; adding the two gives the synchronous speed of the motor, and consequently the frequency of the supply.

The slipmeter is connected to the line between either the terminals K and K_1, or K and K_2, according to the voltage of the motor. For higher voltages a potential transformer is used. The drum D is provided with different combinations of contacts, for motors with various numbers of poles. The setting is done with the screw s which moves the roller brush b_1. The number of poles is indicated on the scale U.

496. A Differential-Gear Slipmeter. — The slipmeter shown in Fig. 372 is a convenient arrangement for mechanically comparing the angular velocities of two shafts. One shaft is driven at a constant speed by the synchronous motor of the slip indicator and the other at the speed of the motor under test by means of a flexible coupling between them. The synchronous motor shaft carries a flywheel on one end and on the other a gear wheel which may be made to mesh with any one of a nest of gears mounted on a parallel shaft. The various gears adapt the instrument for testing motors with various numbers of poles.

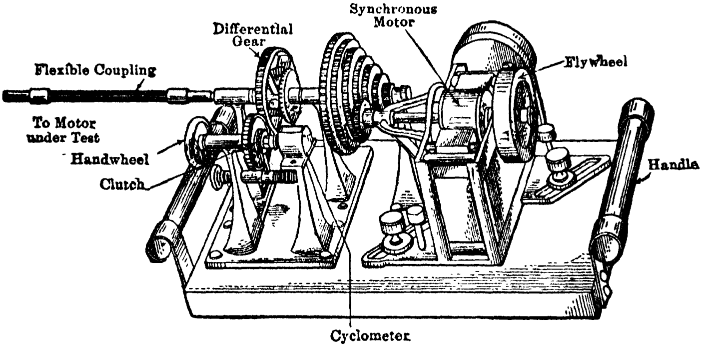


FIG. 372. A differential-gear slipmeter.

The parallel shaft is equipped at its other end with a bevel-gear wheel meshing with two pinions of a differential gear. Meshing with these pinions on the other side is another bevel gear, carried on the shaft connected to the machine under test. A short auxiliary shaft shown in front of the differential gear has on one end a gear wheel meshing with the large wheel of the differential gear, and on the other end a small handwheel by which to hold this shaft. A clutch, operated by a lever, connects the auxiliary shaft to a cyclometer which registers the number of revolutions of the auxiliary shaft. The gear ratio is such that one revolution of the auxiliary shaft corresponds to one-half revolution of the differential gear.

The indicator is used as follows: With the speed of the alternator held constant at the normal frequency of the motor, the indicator is connected to the induction motor shaft by means of a flexible coupling. The bevel gear on the shaft carrying the coupling is then driven at the speed of the induction motor. By grasping the handwheel on the auxiliary shaft, and thus holding the large gear of the differential stationary, the synchronous motor is mechanically brought up to the speed of the induction motor by power transmitted through the other side of the differential and the nest of gears. When the line switch of the synchronous motor is closed it will fall into step with the alternator and run at synchronous speed. With the synchronous motor running and driving one bevel gear of the differential at synchronous speed, and the induction motor driving the other bevel gear at the speed of the induction motor, the larger gear of the differential will rotate and drive the auxiliary shaft and cyclometer at a speed proportional to the difference in the other two speeds. In this way, the slip of the induction motor is mechanically recorded.

497. EXPERIMENT 22-C. — Load Test of an Induction Motor. — The purpose of the experiment is to obtain the performance curves shown in Fig. 366 by actually loading the motor, using for the purpose a Prony brake, an electro-dynamometer, or a calibrated d-c shunt generator. Proceed as follows:

(a) Arrange for an a-c source of constant voltage and frequency, and for a suitable load.

(b) Insert the necessary meters, using a polyphase board if desirable. Bring the motor up to speed and see that all meters read properly.

(c) Holding supply voltage and frequency constant vary the load on the motor in approximately equal steps from zero to the maximum of which the motor is capable. At each load measure (see §492) input volts, amperes, and watts, also frequency, rpm, and slip, using one of the methods already described. Take readings of torque if the Prony brake or electro-dynamometer is used, or of armature volts and amperes if a d-c generator supplies the load. In the latter case hold the current constant in the shunt field, which should be separately excited.

(d) Record the data as in the table which follows, substituting for torque the electrical items listed above if the d-c generator is used. In this event it is necessary also to calibrate the generator by disconnecting it from the motor and running it at various speeds over the range covered in the test, holding the field current constant at the value previously used and varying the speed by changing the impressed voltage. The power input to the armature under this condition, when corrected for

Date.................................Section.....

Observers:

....

....

....

....

Length of Brake Arm.....................................

Tare or Zero Brake Reading..................

Make of Motor...

Type — Open, semi-enclosed, enclosed, ventilated frame { Yes No

Volts.................................Amps. per terminal...........

HP.................................Mfrs' No.................No. of Poles.....

Speed { At full load. .... / At no load................ Alts............Cycles ....

Phase...................................Connections Y or Δ

Type of Rotor { Phase wound Squirrel cage Fans { Yes No

Method of Starting...

	Amperes			Volts			Watts		Speed		Slip Indic.	Torque
	A	B	C	AB	BC	CA	A — AB	C — CB	Motor	Synchr.		Lb
Instr. No...												
Const.......												

the small armature plus brush I^2R loss, equals the core loss plus friction and windage of the generator.

(e) Measure the resistance of the generator armature, including that of the brushes, using the range of current employed as load. Plot resistance drop, IRa, against current.

(f) Repeat the load run on the motor at, say, 80 per cent of normal volts, proceeding otherwise as in (c).

Report. Give the actual diagram of electrical connections and describe the apparatus used, viz., the kind of brake or dynamometer, frequency meter, slipmeter, etc. Compute the data for the curves at both values of impressed volts and give sample computations. Plot the curves and mark the points of full load, $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ load, as well as 25 per cent overload. Discuss the effect of voltage upon power factor and efficiency at $\frac{1}{2}$ load; at full load. How does voltage affect the overload capacity of the motor?

498. Losses in the Induction Motor. — As is explained in §492, it is difficult to hold a brake load constant while readings are being taken. A convenient modification of the brake test consists in connecting the motor to a steady load, such as a blower, centrifugal pump, or electric generator, and omitting the output readings from those specified in the preceding experiment. The output is computed from the input and the losses in the machine. These losses are as follows:

(a) The primary copper loss.
(b) The secondary copper loss.
(c) The core loss in the stator.
(d) The core loss in the rotor.
(e) Bearing friction, slip-ring friction, and windage.

The primary copper loss may be computed from the resistance of the winding at the proper operating temperature. The actual loss may be somewhat higher (usually 10 to 20 per cent) because of eddy currents and an unequal distribution of the current in the conductors. In the rotor, at ordinary loads, the frequency is so low that the d-c or "ohmic" resistance may be used. The "effective" a-c resistances are sometimes determined from the wattmeter readings on the so-called short-circuit test similar to that on a transformer (§393). It is difficult, however, to eliminate the core loss from such readings.

The stator core loss depends upon the stator flux. With a constant applied voltage, the stator flux is practically constant over the ordinary range of loads (§484). The rotor core loss is zero at synchronism, and is quite small at usual values of slip, because the frequency of the secondary magnetization is low. As the load increases, the secondary core loss increases while the primary core loss slightly decreases. For most practical purposes it is sufficient to disregard the secondary core loss altogether and to consider the primary core loss as having its maximum value (the no-load value) at all loads.

When the core loss and the mechanical losses are known separately, the *correct* procedure in computing the output of an induction motor, for a given input and slip, is as follows:

(1) From the measured primary current and resistance, the primary copper loss is computed and added to the primary core loss. The sum of these losses is subtracted from the input, and the result represents the stator output, or the input into the rotor.

(2) According to the theory given in §488, the percentage of secondary copper loss is equal to the per cent slip. Therefore, the input into the secondary, after being reduced by the percentage equal to that of slip, gives the gross mechanical output.

(3) This total mechanical output is further reduced by the amount of friction and windage, and the result represents the useful net output available on the shaft.

In practice this procedure is sometimes difficult to follow, because there is no simple method of measuring the core loss and the mechanical losses separately. With the motor running at no load, the total input is equal to the core loss, plus the friction and windage, plus the primary copper loss. The secondary copper loss is negligible because the slip is practically zero. Correcting the wattmeter reading for the computed primary copper loss gives the total no-load loss, that is, friction, windage, and core loss. The experimental separation of the core loss is explained in §501 below, and if it is performed on the motor under test, the correct procedure outlined above can be followed. If, however, only the total no-load loss is known, an *approximate* procedure is followed, viz.:

(1) The primary copper loss and the total no-load losses are subtracted from the input, and the result is taken to be equal to the input into the rotor.

(2) The input into the secondary is reduced by the percentage of slip and the result is taken to represent the useful output available on the shaft.

This approximate procedure is accurate enough for large and medium-sized machines in which the losses and the slip are comparatively small. But with small motors and with special low-efficiency motors, the accurate procedure should be followed.

499. An Example of Computation of Output from Input and Losses. — Let a 10-kw, 440-volt, three-phase induction motor be found to take 800 watts at no load; let the no-load current be 5.5 amperes per phase and the primary resistance 0.75 ohm per phase. The iron loss and friction are equal to 800 watts less a correction for the copper losses in the primary and the secondary windings. But the secondary copper loss is negligible at no load, because the slip is very small. Thus the correction amounts to $3 \times \overline{5.5}^2 \times 0.75 = 68$ watts, and the losses in question are equal to $800 - 68 = 732$ watts.

Now take a point on the current curve, for example that corresponding to an input of 15 amperes per phase. Let the measured power input at this current be 9920 watts and the slip 5.4 per cent. The primary copper loss is

$$3 \times \overline{15}^2 \times 0.75 = 506 \text{ watts}$$

therefore $9920 - 506 = 9414$ watts represents the output plus all the losses except the primary copper loss.

From now on, the procedure differs according to whether the correct or the approximate method is followed, as outlined in the preceding section.

(a) *The approximate method.* The input into the secondary is $9414 - 732 = 8682$ watts. The secondary copper loss is $0.054 \times 8682 = 469$ watts. The net output is $8682 - 469 = 8213$ watts. After the output corresponding to a given input has thus been determined, the efficiency, the torque, etc., may be computed as in an actual brake test.

(b) *The exact method.* Let the friction and windage be determined separately (§501), and let them be equal to 415 watts at the synchronous speed. Then the iron loss is $732 - 415 = 317$ watts. The input into the secondary is $9414 - 317 = 9097$ watts, the secondary copper loss is $0.054 \times 9097 = 491$ watts, and the total power output, including that converted into friction and windage, is $9097 - 491 = 8606$ watts. At 5.4 per cent slip, the friction and windage are less than at the synchronous speed. Let their estimated value, or one taken from an available curve, be 370 watts. Then the net useful output is $8606 - 370 = 8236$ watts, which is only 23 watts more than the value determined above by the approximate method.

500. EXPERIMENT 22-D. — Performance of an Induction Motor from Input and Losses. — The purpose of the experiment is to obtain the performance curves shown in Fig. 366 without actually measuring the output with a brake or dynamometer (§§498 and 499). The experiment should preferably be performed on the same motor as Experiment 22-C, in order to check the results. The motor is belted or direct-connected to a generator or a blower, serving as a load, and readings are taken as explained in §497, except that the output readings are omitted.

At the end of the load test, careful readings should be taken of amperes and watts at no load, preferably in the form of a complete no-load test as described in §501, and the resistance of the primary winding determined at the proper operating temperature. When a resistance is measured between two terminals of a Y-connected winding, the result represents the resistance of two phases in series. Therefore, in order to obtain the *average* resistance per phase, take the resistances between the terminals A–B, A–C, and B–C, add them together, and divide the result by 6. If possible, separate the core loss from the windage and friction by one of the methods described in the following section.

Report. The same curves should be plotted as in §497, the results to be derived by the method described in §§498 and 499. In figuring out the primary copper loss, use the value of the primary resistance at the highest temperature specified for that particular class of insulation in the Standards of the American Institute of Electrical Engineers. Compare the performance curves with those obtained on the same motor by a brake test, and explain the discrepancies, if any.

On a separate sheet plot curves of total loss, primary and secondary I^2R losses, iron loss, and friction, against the output as abscissas, so as to get a clear idea of the relative importance of these losses at various loads.

501. Separation of Iron Loss from Friction. — The same three methods may be used for the separation of the core loss from the mechanical losses in a polyphase induction motor as in a d-c machine (Chapter XIII), viz.: (a) the machine may be driven mechanically, (b) the machine may be driven electrically, (c) the retardation method may be used.

(a) *The machine driven mechanically.* The induction motor under test is belted or otherwise mechanically connected to a small shunt-wound d-c motor and is driven at different speeds near synchronism, both below and above the synchronous speed. The stator circuit is left open so that there are no losses except the friction and windage. The input into the driving motor and its losses being known, the friction and windage loss of the motor under test can be determined. The driving motor should be comparatively small in order to run at a point of high efficiency; then a small error in the estimation of its losses will not affect the results appreciably. From the wattmeter readings taken on a no-load run on the same induction motor, the sum of iron loss and friction can be determined. Subtracting the friction loss from this sum, the value of the iron loss alone becomes known.

It is also of interest to take a run with the stator connected to a source of a-c supply while the machine is being driven by the auxiliary motor. An ammeter and a polyphase wattmeter should be connected in the a-c circuit. When the field of the d-c motor is strong, its armature tends to run rather slowly and the induction motor drives it as a generator. As the d-c field is weakened and the motor speeds up, it begins to drive the induction motor, overcoming its friction and windage and the secondary core loss. This is shown by a reduction in the wattmeter indication. When the synchronous speed is reached, the a-c circuit furnishes only the primary core loss and the primary copper loss.[1] The driving motor supplies the power for overcoming the rotor friction and windage.

If the field of the d-c motor be further reduced, the motor will drive the induction machine as a generator, " pumping " power back into the a-c line. The wattmeter indication will be reversed, and the readings of both the a-c ammeter and wattmeter will increase with the speed. Just at synchronism, on account of the hysteresis torque, there may be a noticeable decrease in the current input into the driving motor. For the operation of the induction machine as a generator, see Vol. II, Chapter LIV.

(b) *The machine driven electrically.* The induction motor under test is connected in the usual way to a source of a-c power, with a voltmeter, an ammeter, and a wattmeter in the primary circuit. A provision should be made for varying the applied voltage within wide limits. The first run is made at a reasonable over-voltage, and then the voltage is reduced in numerous steps to the lowest value at which the motor can run at a nearly synchronous speed. The current, the voltage, the watts, and the speed are read at each step. The watt readings are corrected for the primary copper loss, and a curve is plotted between the watts and the volts. This curve gives the core loss plus friction, as a function of the applied voltage. Being extrapolated to zero voltage, the curve gives the friction plus windage only, because the core loss would be zero if the motor could be run at zero voltage. Figure 373 gives actual test curves of this form.

In the extrapolation mentioned above a small inaccuracy is involved, because the speed of the machine goes down somewhat as the applied voltage is reduced. This can be corrected during the test by increasing the generator speed to keep the motor speed constant. The value of friction plus windage obtained by the extrapolation described above may be used in computing the output of the motor from its input (§498). For an accurate extrapolation of such a curve see §341.

(c) *Retardation method.* The machine is brought up to a speed well above the synchronous value, either by an auxiliary motor or by raising

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[1] In some motors there is an appreciable "hysteresis torque" at synchronous speed, tending to hold the motor in synchronism and supplying part of the rotation loss.

the speed of the supply generator. The source of power is then cut off and the machine is allowed to slow down. The friction plus windage may be calculated from the slope of the retardation curve. For details see §§337 to 340.

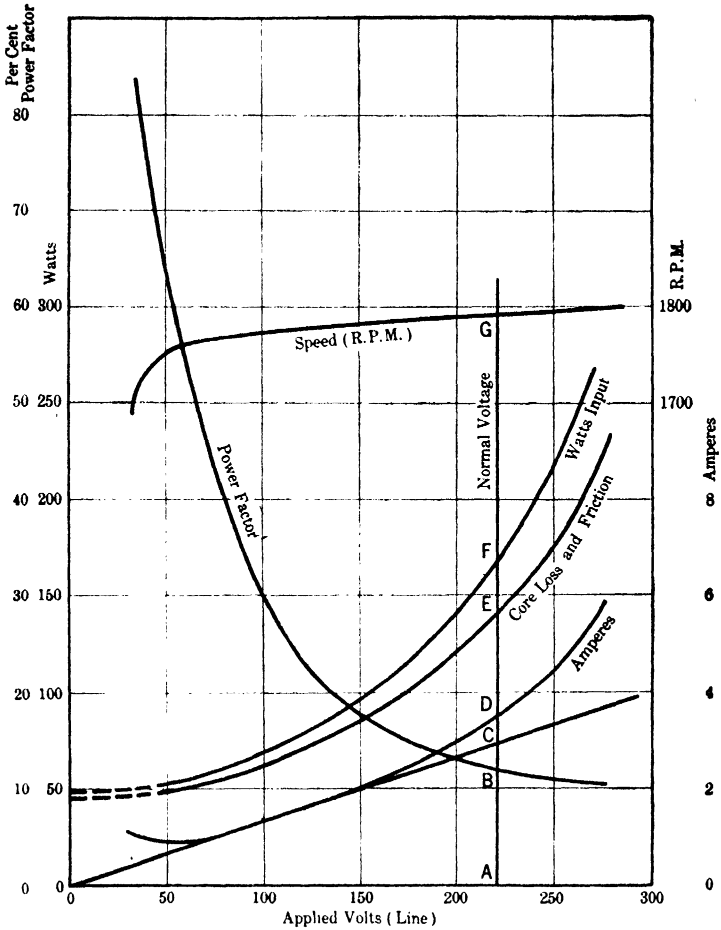

FIG. 373. Running-light curves of a three-phase induction motor, rated 3 hp., 220 volts, 1750 rpm., 60 cycles.

502. EXPERIMENT 22-E. — Separation of Friction from Core Loss in an Induction Motor. — The three available methods and the necessary computations are explained in the preceding section. This is a somewhat advanced experiment which requires considerable skill and precision if it is to give satisfactory results. The friction loss should be determined by all three methods on the same motor, so as to obtain a mutual check and to form an opinion as to the relative convenience and accuracy of the methods.

503. Calculation of Performance from the Equivalent Circuit. — The induction motor is electrically equivalent to a transformer operating from a circuit of constant potential and serving a variable non-inductive load. Therefore, it may be represented by the same equivalent circuit as the transformer, a varying load on the motor being equivalent to a varying resistance R, in the secondary circuit of a transformer. (Compare Figs. 285 and 374.)

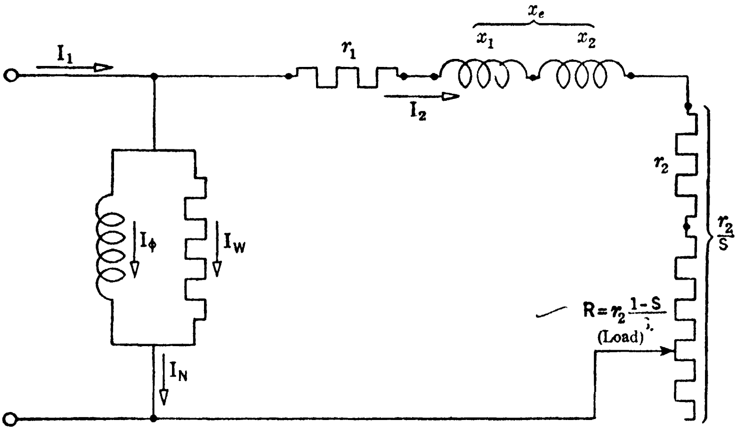

FIG. 374. The approximate equivalent circuit of the induction motor.

The performance of the motor may be determined with an accuracy sufficient for most purposes by solving this approximate equivalent circuit. Such a solution involves:

(a) The determination of the constants of the circuit from the no-load and blocked-rotor data of the motor.

(b) The calculation, at several assumed values of slip, of the various items of performance from the circuit equations.

In the treatment which follows, *values will be per phase* unless otherwise stated. A three-phase motor is assumed in the table of calculations.

Determination of circuit constants. From the running-light data of the motor, at normal voltage and frequency, the current, I_N, watts input, W_n, and voltage, V', are known (all values are per phase). Let $\cos \theta_n$ be the no-load power factor. Then the power component, I_W of the no-load current $= W_n/V'$, and the magnetizing component, $I_\phi = I_N\sqrt{1 - \cos^2 \theta_n}$.

Let W_{1b} and I_{1b} be the power input and stator current with the rotor blocked and rated voltage, V', applied; and $\cos \theta_b$ = rotor-blocked power factor. Then, assuming that the equivalent circuit represents the motor under these conditions also, and remembering that the slip, s, is unity, it follows that the load resistance $R = 0$. The current, I_{2b}, through the right-hand branch of the circuit is: $I_{2b} = I_{1b} - I_n$, the subtraction being performed graphically (vectorially). The power in the right-hand branch $W_{2b} = W_{1b} - W_n = I_{2b}^2\,(r_1 + r_2)$.

Therefore,

$$r_{\text{equiv.}} = r_1 + r_2 = (W_{1b} - W_n)/I_{2b}^2$$

being the sum of the *effective* resistances. By subtracting the stator resistance, increased 10 to 20 per cent above the value measured with direct current to get its effective value, the effective rotor resistance (in stator terms) is found. This may be decreased, say, 20 per cent, to approximate its ohmic value which should be used in calculations involving low values of slip. The combined stator and rotor reactance may be found as follows:

$$Z_{2b} = \sqrt{(r_1 + r_2)^2 + (x_1 + x_2)^2} = V'/I_{2b}$$

so that

$$(x_1 + x_2) = \sqrt{Z_{2b}^2 - (r_1 + r_2)^2}$$

All constants of the circuit are now known.

504. Example of Calculation of Performance. — The procedure re-

TABLE OF PERFORMANCE CALCULATIONS OF A THREE-PHASE INDUCTION MOTOR

slip, s, (assumed)	0.005	0.01	0.015	0.03	0.05	0.10
r_2/s	8.88	4.44	2.96	1.48	0.888	0.444
$R = (r_2/s) - r_2$	8.836	4.396	2.916	1.436	0.844	0.40
$r_1 + r_2 + R = r_1 + r_2/s$	8.941	4.50	3.02	1.54	0.949	0.505
$g_2 = \dfrac{r_1 + r_2/s}{(r_1 + r_2/s)^2 + (x_1 + x_2)^2}$	0.112	0.221	0.325	0.60	0.876	1.15
$b_2 = \dfrac{(x_1 + x_2)}{(r_1 + r_2/s)^2 + (x_1 + x_2)^2}$	0.0053	0.021	0.046	0.167	0.395	0.978
$I_{2w} = V'g_2$ (amperes)	28.35	56.2	82.5	153.3	222.5	293
$I_{2n} = V'b_2$	1.36	5.33	11.72	42.5	100.5	248.5
$I_2^2 = I_{2w}^2 + I_{2n}^2$	802	3178	6937	25,405	59,600	147,600
$I_{1w} = I_{2w} + I_W$	38.06	65.9	92.2	163	232	303
$I_{1n} = I_{2n} + I_\phi$	86.8	90.8	97.2	128	186	334
$I_1 = \sqrt{I_{1n}^2 + I_{1w}^2}$	94.8	112	133.7	207	298	450
$\cos\theta_1 = I_{1w}/I_1$	0.40	0.59	0.69	0.79	0.78	0.67
$W_2 = (I_2^2R) \times 3/10^3$ = (power output, in kw)	21.25	41.9	60.6	109.6	150.9	177.1
Hp = output/.746	28.5	56.2	81.2	147	202	237.5
Torque = $\dfrac{7.04 \times W_2 \times 10^3}{\text{syn. speed } (1 - s)}$	334	662	962	1765	2490	3080
Input = $3 \times I_{1w}V'/10^3$ (kw)	29.0	50.3	70.3	124.2	177.0	230.0
Effic. = $\dfrac{\text{output}}{\text{input}}$	0.73	0.83	0.86	0.883	0.853	0.77
Speed = syn. speed $(1 - s)$	448	445.5	443	436.5	427.5	405

quired to obtain the performance of the motor at various slips is illustrated in the preceding table in which the data apply to a 150-hp, 440-volt, three-phase, 450 (synchronous) rpm motor. The constants of the approximate equivalent circuit are: $r_1 = 0.061$, $r_2 = 0.0444$, $x_1 + x_2 = 0.4285$, $I_N = 9.71 - j85.5$. Friction plus windage = 1000 watts (total).

For a more detailed and exact treatment of the equivalent circuit of the induction motor see Vol. II, Chapters 52 and 53. There the circle diagram solution of both the approximate and the exact equivalent circuits of the motor are given. For the benefit of those who wish to use the circle diagram but do not have access to Vol. II it will be outlined, without proof, in the article which follows.

505. Performance from the Circle Diagram. — In the equivalent circuit discussed in §503 it will be noted that, as the slip varies, the reactance of the circuit remains constant but the resistance changes. The locus of the current vectors in such a circuit is a circle (see §945, Vol. II). The complete circle diagram which corresponds to the approximate equivalent circuit of Fig. 374 is shown in Fig. 375. Note that the point P_s on the circle corresponds to the condition of rotor blocked, and to effective rather than ohmic resistance in the rotor.

Construction and use of the circle diagram. Lay off the no-load current, $I_N = OA$ at the no-load power factor, having first constructed the power-factor quadrant *prq*. From A draw a horizontal line AK to the right, its final length to be determined later. From O lay off the vector OP_s

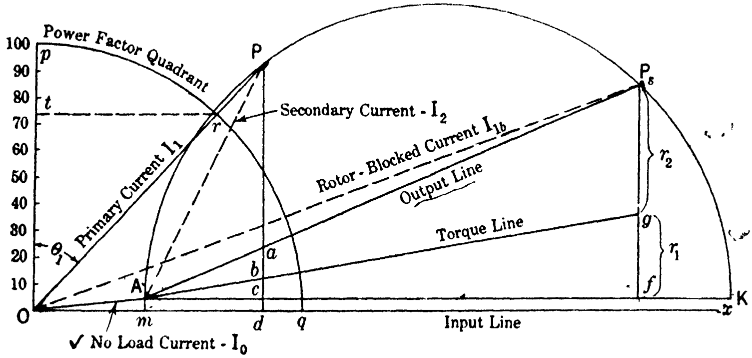

FIG. 375. The circle diagram corresponding to the approximate circuit of the induction motor. (Fig. 547 of Vol. II.)

equal to the rotor-blocked current I_{1b} at rated voltage and at blocked power factor. Draw AP_s. This is the *output line* in the diagram. Erect a perpendicular bisector to the line AP_s to intersect the line AK at the center of the circle, and draw in the semicircle APP_sK. Draw the horizontal line Omx. This is the *input line*.

Divide the height P_sf into two parts, P_sg and gf proportional to the rotor and stator resistances respectively, and draw the line Abg. This is the *torque line.*

To read the performance of the motor for any point P on the diagram proceed as follows:

Current input, $I_1 = OP \times k$, where k = amperes per inch of diagram
Watts input $= Pd \times k \times V' \times$ (number of phases).
Cos $\theta_1 = Ot$ on scale of power factor.
Output $= Pa \times k \times$ (number of phases) $\times V'$; horsepower = output/746.
Efficiency $= Pa/Pd$.
Torque $= (Pb \times k \times V' \times$ number of phases $\times 7.04)/$syn. rpm.
Slip $= ab/Pb$.
Rpm = syn. rpm $\times (1 - s) =$ syn. rpm $\times (Pa/Pb)$.

Many of these items may be read directly from special scales on the diagram, for which see p. 327, Vol. II.

506. EXPERIMENT 22-F. — Performance from Calculations based on the Equivalent Circuit. — Having obtained the running-light and rotor-blocked data of the motor, also the stator resistance, either from previous tests or from special tests for this purpose, calculate the various performance items of the motor at several values of slip, using either the analytical or the circle diagram method. Plot these curves as in Fig. 366 and compare with the actual test values. Explain what simplifying assumptions have been made in these solutions and the general effect of these assumptions upon the results obtained.

Explain any advantages which the circle diagram method of calculating performance would seem to have over the analytical method.

For advanced tests on the induction motor, see Vol. II.

CHAPTER XXIII

THE SINGLE-PHASE INDUCTION MOTOR

507. A single-phase induction motor differs from the polyphase motor shown in Figs. 350 and 351 in that its stator is provided with a single-phase winding similar to that shown in Fig. 320, for connection to a single-phase supply. The rotor is usually provided with a squirrel-cage winding, although it may also have a three-phase winding and slip-rings. It may even have a rotor similar to the armature of a d-c motor, except that the brushes are short-circuited. At standstill such a motor has only an alternating and not a gliding magnetic field, and will not start without some auxiliary means, since there is no reason why there should be more torque in one direction than in the other. Electromagnetically, such a motor at standstill is similar to a stationary transformer on short circuit.

To make the motor self-starting, an auxiliary winding is usually provided on the stator, in space quadrature with the main winding (§508). Another method consists in starting the machine, if it have a commutator-type rotor, as a repulsion motor (§512) and then short-circuiting all the individual rotor coils.

Once brought up to a considerable speed, the simple single-phase motor develops a torque and if without load runs almost up to synchronism. After this, when loaded, it behaves essentially as the polyphase induction motor treated in the preceding chapter. Its performance characteristics, though inferior to those of a polyphase motor, are essentially those shown in Fig. 366, and its speed-torque curve (Fig. 356) has the shape *hfd*; beyond point *f* the torque rapidly drops to zero.

There are two different ways of explaining the fact that, once brought up to speed, a single-phase induction motor possesses a torque and can be operated under load near the synchronous speed. Both theories are given in Vol. II, and the fact itself may be accepted here as a result of electromagnetic reactions between the stator and the rotor, and may be shown by a simple experiment as follows: bring a two-phase or three-phase induction motor up to speed, apply a moderate load, and then break the connection between one of the phases and the line. The motor will continue to operate single-phase, although at a lower power factor and at a much lower overload capacity.

An important characteristic of the single-phase induction motor is that at synchronous speed and within a few per cent slip below and above synchronism it has a revolving air-gap flux of practically constant amplitude. The net magnetizing mmf of the stator (stator ampere-turns minus the opposing rotor ampere-turns along the same axis) furnishes only an alternating flux. Hence, in order that the rotating field exist, the rotor, when in motion, must develop an alternating mmf stationary in space, and in time and space quadrature with the net stator mmf. This is because two such mmf's are necessary to produce a revolving flux (§484).[1] At lower speeds this quadrature mmf is smaller than the net magnetizing mmf along the stator axis, and the revolving field becomes irregular, or, as it is called, elliptical. At standstill the quadrature field disappears, and for starting purposes it must be excited by means of an auxiliary winding on the stator.

Single-phase induction motors are used in small sizes almost entirely, for the following reasons: (a) Their weight and cost per kilowatt of rating are considerably higher than those of two-phase or three-phase motors. (b) They either have to be started and brought up to speed at no load, or else the starting current reaches several times the rated current, even for a moderate torque. This current is objectionable from the point of view of the service to other users of electric power. (c) The power factor and the overload capacity are lower than those of polyphase motors. In small sizes, and for certain classes of service, single-phase induction motors are very useful, because they can be connected to an ordinary single-phase lighting circuit and do not require a special three-phase wiring with more expensive transformers, switches, etc.

508. Starting with an Auxiliary Phase (Split-Phase Principle). — A revolving magnetic field, however imperfect, is necessary in order to start an induction motor as such. In a single-phase motor this field is usually produced by adding an auxiliary stator winding in space quadrature with the main winding (Fig. 376). If the current in this auxiliary phase could be made to lag behind or to lead the current in the main winding by exactly 90 degrees and be equal to it, a perfect revolving field and a high starting torque could be obtained.

If the phase displacement between the two currents is produced by series resistance and inductance, it is always considerably less than 90 degrees. Thus, a fluctuating or elliptical revolving field is produced, one that reaches higher values opposite the centers of the main coils than opposite the centers of the auxiliary coils. Nevertheless, such a field can be made of sufficient magnitude to start the rotor at no load and

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[1] See Vol. II, Chapter LVI. Also "Principles of A. C. Machinery" by R. R. Lawrence (Chapter 43).

to bring it up to a considerable speed, beyond which the rotor itself furnishes the quadrature field necessary for running up to full speed. Figure 376 illustrates the split-phase system of starting. Here the main phase is connected directly across the line and the auxiliary phase across the line in series with a resistance. By winding the auxiliary phase of high-resistance wire an external resistance may be omitted.

The auxiliary phase is opened at a proper speed, either by hand or automatically by a centrifugal device, and the motor then runs on the main phase only. The rotor is sometimes connected to its load through a friction clutch operated by a centrifugal device, so that the rotor comes up to speed light, and is not engaged until it is capable of developing the required torque. To reverse the direction of rotation, the electrical connections either to the main phase or to the auxiliary winding are reversed.

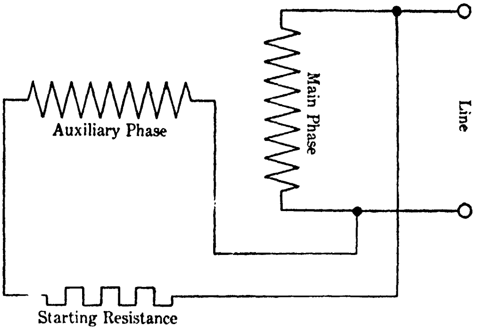


FIG. 376. Connections for starting a single-phase induction motor by means of an auxiliary phase.

Further applications of the split-phase principle. Watthour meters, indicating instruments, a-c magnetic contactors, phase-shifting transformers, etc., operating on the principle of the single-phase induction motor (Chapters II to V) are constructed. In them the auxiliary phase winding has to be left in the circuit permanently. In all such devices the part corresponding to the rotor either is stationary or moves at such a low speed that the induced secondary currents do not excite a sufficient quadrature flux, and the latter has to be produced by an auxiliary primary current.

The shading coil. A special case of an auxiliary phase is the so-called shading coil S (Fig. 377) placed unsymmetrically upon a stator pole. It has been used in ammeters, fan motors, etc., and usually consists of a strip of copper short-circuited upon itself. Without this coil, the main a-c winding C would produce in the iron core I and in its air-gap a uniform alternating flux, causing symmetrical eddy currents in the pivoted aluminum disk D. There would be no tangential force tending to turn the disk. The shading coil acts as a short-circuited secondary of a transformer, weakening the flux within itself and causing it to lag with respect to the flux outside the coil. Thus, two sets of unsymmetrical eddy currents are produced in the aluminum disk; each set forms a torque with the other flux and tends to rotate the disk. The electromagnetic reactions are similar to those shown in Fig. 117.

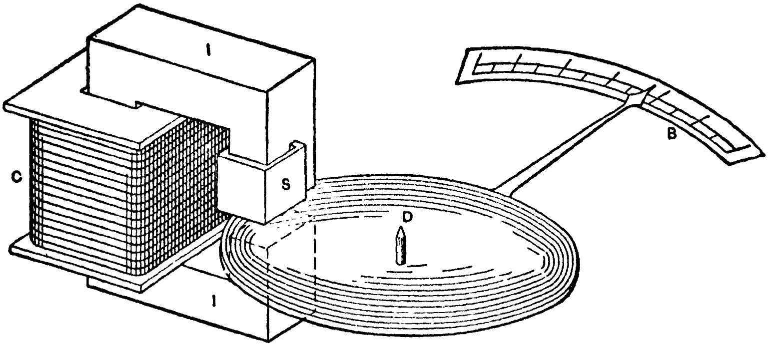


FIG. 377. The use of the shading coil in the older type induction meter.

509. The Condenser Motor. — In the so-called " condenser motor " a static condenser of proper value, connected in series with one of two stator windings, will cause the currents in the two windings to be actually in quadrature, one leading and one lagging the applied voltage. This condition results in a satisfactory starting torque and a high power factor of the motor. The condenser may be kept in circuit after the motor is up to speed and carrying load. If the condenser has been properly chosen the motor will operate essentially as a two-phase motor, except that it will have a higher power factor. Professor B. F. Bailey (*Trans. A.I.E.E.*, April, 1929, p. 596) gives diagrams similar to those shown in Fig. 378 for single-phase motors starting on the split-phase

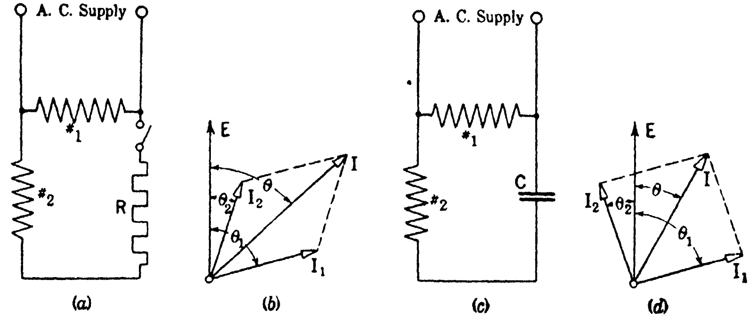


FIG. 378. Diagrams of split-phase motors at start.
(a) Circuits of the resistance split-phase motor.
(b) Vector diagram of the motor of (a) at start.
(c) Circuits of the capacitance split-phase motor.
(d) Vector diagram of (c) at start.

principle, one with resistance, and one with capacitance in series with the auxiliary winding.

It will be noted that in the resistance split-phase motor at start the currents I_1 and I_2 in the two windings differ in phase by an angle much less than 90 degrees, indicating an unsatisfactory starting torque. The total current, I, is large and at a fairly low power factor ($\cos \theta$).

In the condenser motor (diagrams [c] and [d]) the currents I_1 and I_2 are practically 90 degrees apart, indicating an excellent starting torque. The total current I is less in value and at a much higher power factor than that in the resistance split-phase motor. Professor Bailey makes the following interesting comparison of the performance of a certain motor as two-phase, resistance split-phase, and condenser motor. Possibly the starting torque of the resistance split-phase motor could have been increased at the expense of a higher starting current and a lower power factor.

	Starting Torque	Starting Current	Power Factor	Power Demand
Two-phase	1	1	0.696	1
Condenser split-phase	1.245	0.973	0.99	1.39
Resistance split-phase	0.213	0.76	0.795	0.855

510. EXPERIMENT 23-A. — Adjustment of the Starting Phase in a Single-Phase Induction Motor. — The purpose of the experiment is to obtain the best starting conditions for a single-phase motor in which an auxiliary stator phase is used for starting (§§508 and 509). It is desired to produce a torque sufficient for a positive start at no load, with as small an initial current as possible. Wire up the machine as in Fig. 376, with an ammeter and a wattmeter in the line and in both phases. A voltmeter should be connected to measure at will the line voltage, that across the starting phase, and that across the resistance or impedance in series with it.

(1) Find out by preliminary trials whether a resistance, an inductance, or a capacitance in series with the auxiliary phase gives best starting conditions. Increase and decrease this resistance or reactance until the smallest initial line current has been obtained at which the motor is sure to start. Since numerous trials at a full voltage would overheat the motor, conduct these tests and adjustments at a reduced voltage, with the rotor stationary. The aim should be to obtain, in the current that flows through the auxiliary winding, as high a component in time quad-

rature with the main current as possible. This can be determined by a properly connected wattmeter or power-factor meter.

(2) Having obtained the best conditions, check the results by actually letting the rotor start and come up to speed. Measure the initial line current and watts, the currents and the watts in the two phases, and the voltages across both phase windings and across the starting resistance or reactance. Determine whether the current in the auxiliary phase lags behind or leads that in the main phase. Estimate the number of seconds which it takes to reach the full speed, and find the best intermediate value of speed at which the auxiliary phase should be disconnected unless you find it should be left in circuit. Measure the line current when the motor is running at full speed, with the auxiliary phase " in " and " out."

(3) Adjust the starting resistance or reactance to bring the motor up to speed within the shortest possible time, even though at the expense of a much larger starting current. Make the same measurements as before.

(4) Put a Prony brake on the motor, and by adjusting the starting resistance determine the largest practicable starting torque that the motor can develop without an excessive current. Make the same measurements as before.

(5) If a satisfactory capacitance is available, operate the motor as a capacitor motor (condenser motor) over a wide range of load, using, preferably, a calibrated d-c generator as the load device. Experiment with the capacitance to determine the best value for the loaded motor. Take all generator and motor readings so as to be able to determine both the complete performance of the motor and the phase relations of the several vectors of voltage and current.

(6) To investigate the air-gap flux, place in the air-gap a flat exploring coil of a width equal to the pole-pitch and connect the coil to a voltmeter. Block the rotor, set the starting resistance or reactance for the best conditions found under (1) above, and connect the stator to a reduced line voltage so as not to overheat the motor. Move the exploring coil in steps over a pole-pitch, and at each step measure the induced voltage in magnitude and in phase position. The latter can be determined with a wattmeter, or by balancing the induced voltage against that of a potential phase-shifter (§132) or an a-c potentiometer (§89). Repeat the same measurements with the main phase disconnected, and then with the auxiliary phase disconnected. For comparison take a few readings with a wrong value of starting resistance or reactance, that is, with one which gives a poor starting torque.

Report. (1) Draw vector diagrams of starting currents and voltages in their proper magnitude and relative phase position, under the different conditions investigated. (2) Tabulate the test data to show how different values of starting resistance or reactance affect the total starting current, the time of coming up to speed, and the starting torque. (3) With the assistance of the test data explain the effect of disconnecting the auxiliary phase too early or leaving it " in " too long. (4) Plot the complete performance curves of the condenser motor and discuss them in comparison with those of the polyphase motor tested in Experiment 22-C. (5) Give the results of the test with the exploring coil. From these results deduce the space and time distribution of the actual flux in the air-gap, especially the magnitude and the time phase of the quadrature flux with respect to the main flux.

511. A Three-Phase Motor on a Single-Phase Circuit. — There are cases in which it becomes desirable or necessary to operate a three-phase induction motor on a single-phase circuit, for example when one of the line wires or a transformer is out of order. A satisfactory starting arrangement is shown in Fig. 379*a*. Two of the phase windings, *OA* and *OC*, are connected in series across the line, as the " running " or main phase. The third winding, *OB*, is used for starting only, and is connected at *b*, between a resistance *r* and an impedance *z* (as nearly as possible a pure inductance). After the motor has reached a consider-

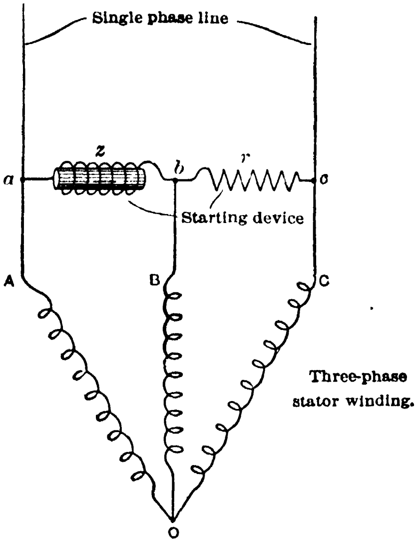


Fig. 379*a*. Starting a three-phase induction motor on a single-phase circuit.

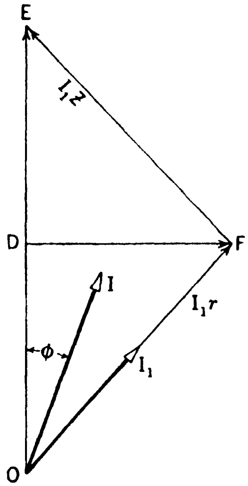


Fig. 379*b*. Current and voltage relations with the connections according to Fig. 378.

able speed, the circuits of the starting device and of the phase *OB* are opened, and the machine operates as a single-phase motor.

The voltages, with *OB* open, are shown in Fig. 379*b*. *OE* is the line-voltage, *OI* is the current in the main phase *AC*, lagging behind *OE* because of the inductance of the winding. The voltages across *r* and *z* are represented by *OF* and *FE* respectively. The current I_1 through *r* and *z* is in phase with *OF* and lags considerably behind *FE*. The voltage across the starting phase, *OB*, is represented by the vector *DF*, since this phase is connected between the middle point of *AC* and the junction of *r* and *z*. It will be seen from the sketch that the voltage *DF* may be made to lag considerably behind *OE*, so as to imitate a two-phase combination. However, when a current flows through *OB*, the phase relations become less favorable and the phase angle between *DF* and *OE* decreases. Nevertheless, by suitably selecting *r* and *z*, a torque can be produced sufficient for starting the motor.

512. EXPERIMENT 23-B. — Exercises in Starting a Three-Phase Induction Motor on a Single-Phase Circuit. — The method is explained in the preceding section. (1) First of all satisfy yourself that a three-phase motor can be operated on a single-phase circuit. For this purpose connect it to a three-phase circuit in the usual way, bring it up to speed at no load, and read volts, amperes, watts, speed, and slip (Chapter XXII). Disconnect one of the stator phases and take similar readings. (2) Stop the motor and adjust the resistance *r* and the impedance *z* (Fig. 379*a*) so as to get any starting torque whatever. Then by varying their values improve the starting conditions as far as possible. After this, either *r* or *z* should be kept constant, and curves taken showing the influence of the other factor. Read the currents, voltages, and watts in the several branches of the circuit, so as to permit the construction of diagrams, like the one shown in Fig. 379*b*. Readings should also be taken with the phase *OB* closed and open, in order to see the influence of the current in this phase upon the current and voltage relations in the circuit *abc*. If a starting torque can be obtained, its value should also be investigated with a Prony brake. (3) For the rest of the experiment follow the instructions for the preceding experiment, with suitable modifications. For a companion experiment see §516.

Report. (1) Give the no-load data for the motor operated single-phase and three-phase. Explain theoretically the relative values of exciting current and watts input under these two conditions. (2) Draw vector diagrams of starting currents and voltages under the best obtainable conditions, with the phase *OB* open and closed. (3) See the report requirements of the preceding experiment for the remainder of the work on the data taken.

513. Starting as a Repulsion Motor. — There is a class of single-phase motors (Fig. 380) which possess quite a considerable starting torque; this torque is obtained, however, at the expense of a commutator and brushes. The particular motor, shown in the sketch, has a stationary field winding, *FF*, connected across the line and producing an alternating field indicated by the arrows. The armature *A* is similar to that of a d-c machine, and is provided with a commutator. The brushes *bb* are placed at an angle with respect to the direction of the field, and are short-circuited upon themselves, thus forming a closed armature circuit. The operation of the motor may be explained as follows:

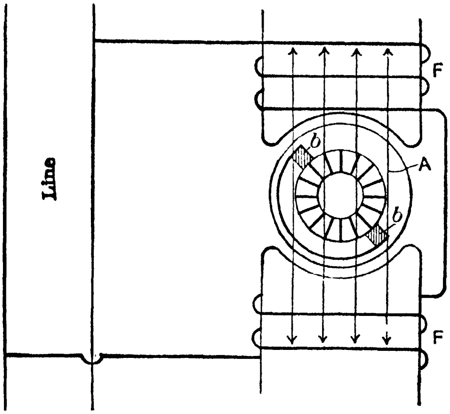


FIG. 380. Schematic diagram of a repulsion motor.

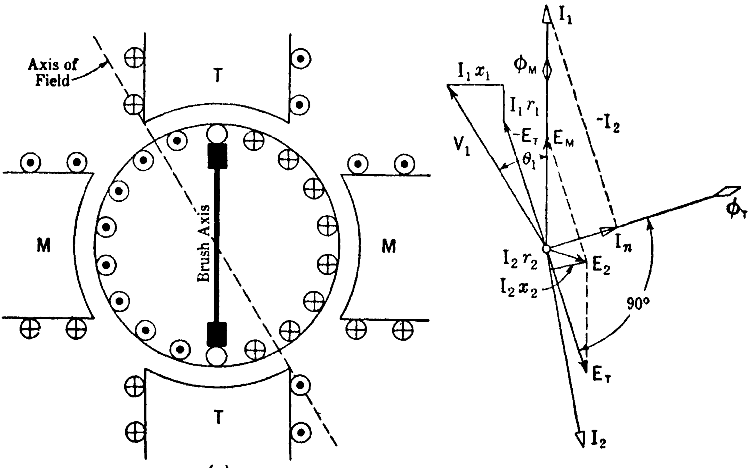
On account of the shift of the brushes from the axis of the field the flux set up by the stator will be composed of two components: one along the axis of the brushes and one at right angles, quite as though there were two sets of poles or separate field windings along these two axes. This is illustrated in Fig. 381(*a*) where *T* is the component field winding along the brush axis exerting transformer action resulting in the armature current, and *M* is the component at right angles and, with the armature current, producing motor action, or torque.

Referring to Fig. 381(*b*), I_1 is the stator current. The flux ϕ_m will be essentially in phase with I_1 because this current produces the only mmf along the *M*–*M* axis and the reluctance of the circuit is practically constant, owing to the air-gap.

Along the *T*–*T* axis the motor behaves like any transformer. The primary current, I_1, is practically balanced by a secondary current I_2, which, assuming a one-to-one ratio of turns, is equal and opposite to I_1 except for an exciting component I_n to which is due the net mmf, along this axis, causing the flux ϕ_T. The current I_2 flowing in the armature causes resistive and reactive drops, I_2r_2 and I_2x_2, the sum being the total drop E_2. This latter voltage is not due to transformer action alone but to a combination of transformer action, producing the component voltage

E_T, and to generator action due to the conductors moving through the field ϕ_m causing the voltage E_m. Motor power is measured by the product of the counter emf E_m with the component of the current I_2 in phase with E_m reversed.

Returning now to Fig. 381(*a*), it is seen that the axis of rotor current, I_2, is such that it can produce no net torque with the field ϕ_T even if they were not practically in time-phase quadrature as shown in (*b*). The rotor current, however, is so distributed that it will produce torque with the field M provided I_2 and ϕ_m are not in time-phase quadrature. Figure 381(*b*) shows that they are practically in phase opposition so that motor torque is assured.



(*a*) (*b*)

FIG. 381. Diagrams of the repulsion motor.
(a) Resolution of the field into components. (b) Vector relations.

In Fig. 381(*b*), the stator vector, $-E_T$, which is the drop due to the emf induced in the stator winding by the field ϕ_T, and I_1r_1 plus I_1x_1, the impedance drop of the entire stator winding, have been added to give the impressed stator voltage, V_1. Stator power factor is shown by angle θ_1. At the moment of starting the motor the counter emf E_m is absent, with the result that current I_2 is larger and lags somewhat behind the position shown in Fig. 381(*b*). Conditions are thus favorable for the production of a good starting torque. The speed of the motor greatly increases as the load decreases, the repulsion motor being in this respect similar to a d-c series-wound motor (Fig. 241).

The desirable starting characteristics of a repulsion motor and the desirable running characteristics of an induction motor can be combined by starting the machine as a repulsion motor and then short-circuiting all the individual commutator segments when a sufficiently high speed has been reached. The machine then operates as an ordinary squirrel-cage single-phase induction motor. In practice the commutator is automatically short-circuited by a centrifugal device which at the same time lifts the brushes in order to do away with unnecessary friction. Such motors are useful for drives in which the load cannot be conveniently disconnected at start and a large starting current is permissible. For a more detailed theory of the repulsion motor at start see Vol. II.

Combined repulsion and induction motors are also built which have both the drum winding with commutator, and a squirrel-cage winding. In the operation of this repulsion-induction motor the brushes remain on the commutator, there being no centrifugal device. The motor starts as a repulsion motor, and as the speed increases, with resultant decrease in the repulsion motor torque, the squirrel-cage winding exerts a supplementary torque which reverses above synchronous speed. As a result the motor starts with a good torque but does not overspeed with reduction of load. Instead of the dropping speed-torque curve of the usual repulsion motor it has a nearly constant speed. Its principal defect is that it is expensive and there is necessarily some wear of the brushes and commutator.

514. EXPERIMENT 23-C. — Study of a Repulsion-Start Single-Phase Induction Motor. — The purpose of the experiment is to investigate the effect of various factors which affect the starting characteristics of the motor described in the preceding section. The factors to be varied are the brush shift, the ratio of the primary and secondary turns, and a resistance or reactance in place of the short-circuiting conductor between the brushes. The desired characteristics to be investigated are the starting torque and current, the time of coming up to speed, sparking at the brushes, etc. On the basis of the instructions for Experiment 23-A, the student should write similar instructions for this test, never varying more than one factor at a time. It is of particular importance to bring the experimental results regarding the starting torque and current in accord with the theory of the brush shift given in Vol. II.

515. EXPERIMENT 23-D. — Brake Test on a Single-Phase Induction Motor. — The purpose and the method of the experiment, the curves to be obtained, etc., are similar to those described in §§491 to 497, except that, with but one stator phase, the wiring, the readings, and the computations are much simpler. It is of importance to load the motor, if possible, up to its pull-out torque, because a comparatively low ratio of the pull-out torque to the full-load torque is a weak point of the single-phase induction motor, and should be known when choosing such a motor.

If the stator has an auxiliary phase winding, in space quadrature with the main winding, it should be connected to an a-c voltmeter and the induced emf measured at each load. The quadrature flux at no load is practically equal to the main flux, so that the voltage induced in the auxiliary winding at no load is a measure of both the main flux and the quadrature flux. The observed decrease in the quadrature flux with load should be shown to be at least approximately in accord with the theory given in Vol. II.

516. EXPERIMENT 23-E. — Comparison of Characteristics of an Induction Motor Operating Three-Phase and Single-Phase. — The method of operation of a three-phase motor on a single-phase circuit is explained in §§511 and 512. The purpose of this experiment is to obtain complete performance characteristics (Figs. 356 and 366) of a three-phase motor from a brake test, and then to take similar characteristics of the same motor operated on a single-phase circuit. All the necessary information in regard to the test itself will be found in §§492 to 497.

Report. Plot on the same curve sheet the performance curves of the motor operated three-phase and single-phase, so that a direct comparison may be made. Where possible, explain the observed differences theoretically.

517. EXPERIMENT 23-F. — Performance of a Single-Phase Induction Motor from Losses. — The reasons for determining the performance of a single-phase motor from the losses, instead of by direct brake test, are the same as those given in §498 for polyphase induction motors. The experiment is conducted as explained in §500. See also §515 in regard to the pull-out torque and the measurement of the quadrature flux. The only difference in figuring out the results is in the expression for the secondary copper loss. It is shown in §488 that in a polyphase induction motor the per cent secondary copper loss is equal to per cent slip (the input into the secondary being taken as 100 per cent). In a single-phase motor, at small values of slip, the per cent secondary copper loss is very nearly *twice* as large as per cent slip. For example, if the slip of the motor at a certain load is 4 per cent, the secondary copper loss amounts to 8 per cent. In other words, 8 per cent of the input into the secondary is converted into heat in the rotor windings. This added copper loss is due to a much more unfavorable distribution of the secondary currents, as a result of the rotor providing an mmf for the quadrature flux (§507). The exact relationship between the slip and the secondary copper loss is much more involved, and the reader is referred to special treatises on the subject.

518. The Phase Converter. — A single-phase induction motor can be used to convert single-phase energy into two-phase or three-phase energy, and is then called a phase converter. When a single-phase motor is running at no load, the air-gap flux is practically a uniformly rotating one, as in a polyphase machine. Hence, the emf induced in the auxiliary winding (Fig. 376) is in time quadrature with the counter emf induced in the main winding, and therefore is approximately in time quadrature with the voltage applied to the main phase. Thus the single-phase source of supply and the auxiliary phase, together, form a source of two-phase energy. By properly choosing the number of turns in the auxiliary winding, the two voltages can be made of nearly the same magnitude.

If the auxiliary phase is electrically loaded, the phase relations become more complicated; but as long as the load does not exceed a certain moderate limit, the machine continues to run at nearly synchronous speed, taking energy from the main phase and delivering it to the auxiliary phase. Some of the transferred energy is stored, for a fraction of a cycle, in the mechanical inertia of the revolving part and in the magnetic fluxes, since the instantaneous amounts taken from the main phase and delivered to the auxiliary phase are not equal to each other.

Such a phase converter can be used for supplying energy not only to a two-phase circuit, but to a three-phase circuit as well. In this case the auxiliary winding has fewer turns than the main phase, and the two windings are T-connected (Vol. II), forming a source of three-phase supply. Such converters have been successfully used on electric locomotives driven by three-phase induction motors but operated from a single-phase trolley circuit. They can also be used for other purposes where polyphase currents are needed and only a single-phase source of supply is available.

519. EXPERIMENT 23-G. — Load Test on a Phase Converter. — The machine is described in the preceding section. An ammeter, a voltmeter, and a wattmeter should be provided in each circuit and in the line, or else the same instruments transferred from circuit to circuit by means of a suitable polyphase board (§60).

(1) Start the motor in the usual way, with an auxiliary phase (§§508 and 510), and then disconnect one of the terminals of the auxiliary winding from the source of supply. With the machine running at full speed and at no load, measure the voltages across the main phase, across the auxiliary phase, and across the two together. This will give a triangle of vectors so that the voltages can be drawn in their proper relative phase positions.

(2) Connect the auxiliary phase to a small non-inductive load and measure the currents, the voltages, and the watts in both phases; also the speed, the slip, and the frequency of the supply. One point of the two windings should be left in common as before, so that the three voltages may be read in their proper phase position. Increase the load in steps until the temperature limit is reached or until the machine stops. Read the instruments at each step, as before.

(3) Perform similar runs with inductive loads of different power factor. If possible, also try a leading load current.

(4) Provide a balanced two-phase load, one phase connected to the auxiliary winding, the other across the main line. In this case, currents and watts should be read in four places, viz., in both load phases, in the main winding, and in the line. Make a regular load run as before, and observe the unbalancing in the voltages and currents as the load is increased.

(5) Adjust the number of turns in the auxiliary winding so as to have a no-load voltage equal to 0.866 of that in the main winding. If this is not feasible, use a transformer across the auxiliary phase to bring the voltage to the desired value. Connect the middle point of the main winding to one of the terminals of the auxiliary phase (or of the secondary winding of the transformer), to form a T-connection (Vol. II). Provide a balanced three-phase load, for example a three-phase motor, and make a regular load test as above. Be sure that measuring instruments are read in the proper places so that vector diagrams can be constructed later.

(6) Invert the machine by connecting it to a source of three-phase power and load it single-phase across the main winding. Show that with such an arrangement there is less unbalancing in the polyphase supply than if the same single-phase load were taken directly across one of the phases, without the converter.

Report. (1) Plot all the observed quantities against suitable abscissas, such as the load current, load watts, or the input, whichever quantity seems to be the most suitable from an inspection of the data. (2) On the same curve sheet plot efficiency and power-factor curves. (3) Draw a few characteristic vector diagrams showing all the currents and voltages, to illustrate the conditions on a single-phase, two-phase, and three-phase load, at a high and at a low power factor. (4) Show that the observed variations in the terminal voltages are at least qualitatively in accord with the theory of the machine. (5) Give the data on the operation of the inverted machine; compute the percentages of power delivered to the load directly and through the auxiliary phase. Connect the observed facts with the theory.

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CHAPTER XXIV

STORAGE BATTERIES

520. A storage cell or electric accumulator is a device in which electric energy can be stored in chemical form and then delivered again to an external circuit as electric energy. Two or more cells connected together form an *electric storage battery.*

521. The Storage Battery as a Supplementary Source of Power. — A storage battery (Fig. 382) is frequently connected in parallel with one or more d-c generators (or rotary converters) for the purpose of steadying the load and voltage of the latter and of acting as a reserve source of power. When the load is light (Fig. 383) some current is supplied to the battery, charging it. During peaks of the load the battery discharges in parallel with the generators, relieving them of the major portion of

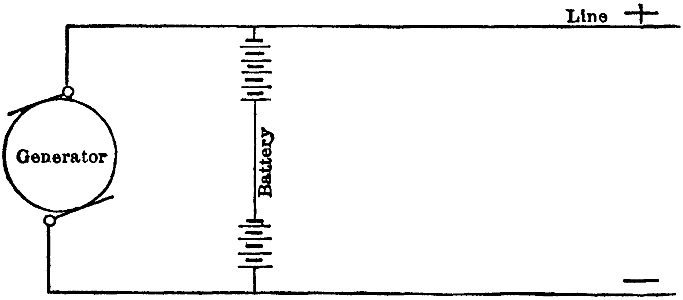

FIG. 382. A storage battery used as a supplementary source of power.

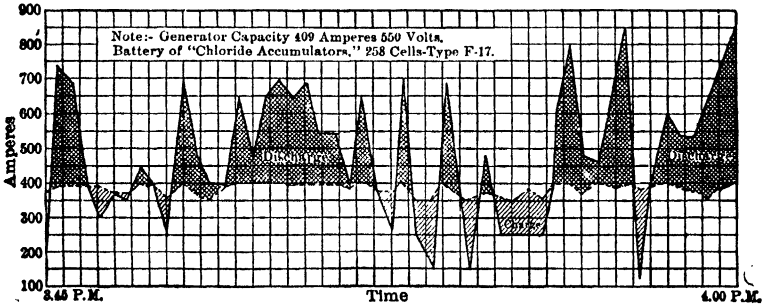

FIG. 383. Effect of a storage battery in removing peaks and depressions in a system load.

the overload. In an emergency or during the hours of very light load, say at night, the generators can be shut down, and the battery made to supply the entire load. The advantages of this use of the battery are as follows:

(a) The size of the prime movers and generators can be somewhat reduced, since the battery carries the peaks of the load over and above the average value.

(b) The prime movers and the generators can be made to operate at a more nearly constant load, within the range of their best efficiency.

(c) Mechanical stresses in rotating machinery, due to sudden changes in the load, are reduced.

(d) The power-generating machinery may be shut down during certain hours of the day, with resulting economies in operation; this also provides time for minor repairs.

(e) The battery constitutes an emergency source of power which is available to tide over short interruptions in the regular sources of supply (stand-by service).

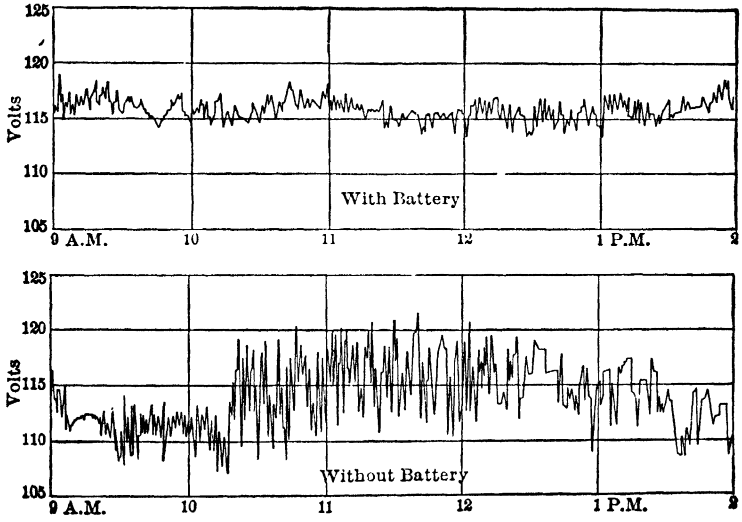

FIG. 384. Effect of a battery in steadying the line voltage.

On the other hand, the first cost of a storage battery and of its accessories is quite high, and there is a considerable additional expense for its upkeep and depreciation. Nevertheless, stand-by storage batteries are used in some large electric power and railway substations, especially in cities where it is of particular importance to avoid interruptions in service. A battery placed at the receiver end of a long feeder helps to maintain a more nearly constant load voltage (Fig. 384) by reducing fluctuations in the line current and therefore causing a more uniform line voltage drop.

522. A Hydraulic Analogue. — The part which a storage battery plays in the distribution of electrical energy is somewhat the same as that of a storage tank in a water-supply system (Fig. 385). Without the tank, the pumps would have to supply a variable demand and their capacity would have to be sufficient for the *maximum* demand. Moreover they would have to be operated twenty-four hours a day; the power consumption would be much increased and the efficiency reduced, to say nothing of the severe mechanical strains imposed by sudden variations of the load. Besides, there would be no stored water in reserve for short interruptions in the pumping plant.

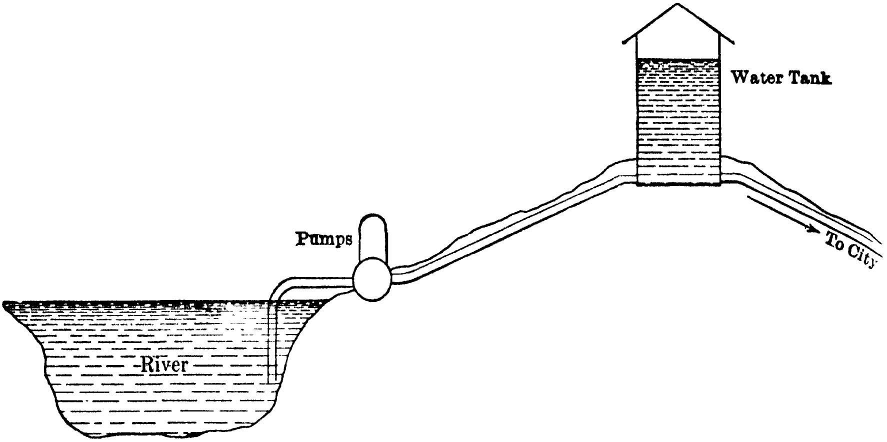

Fig. 385. A water-supply plant, as an analogue to an electric plant with a storage battery.

A water tank of sufficient capacity remedies all these drawbacks. The capacity of the pumps needs to be sufficient for the *average* demand only, and they may be operated at practically full load. When the demand is below the average, the excess water is pumped to raise the level in the tank. When the demand is above the average, the tank supplies the necessary excess of water into the city mains. With a variable load, the tank allows a more constant pressure to be maintained in the mains and makes it possible to shut down the pumps during the hours of light demand.

523. Storage Batteries for Starting and Lighting Automobiles and for Electric Vehicles. — Perhaps the commonest use of storage batteries is for starting internal-combustion engines on automobiles, trucks, tractors, motor boats, farm-lighting generators, etc. The same battery also operates the lights, the signal horn, ignition system, special heating appliances, gauges, etc. The general arrangement is shown in Fig. 226. The battery must be reasonably light and durable, must repeatedly stand an excessive rate of discharge during the period of starting, and must not be seriously damaged by long overcharging with a moderate current. Batteries of similar type are used for the propulsion of electric passenger vehicles, trucks, some street cars, mining and industrial locomotives, boats, etc.

Among other uses of storage batteries may be mentioned house and farm lighting, radio, miscellaneous uses on board ship, telephone, telegraph and signal field, operation of large electric switches, relays, etc., in power plants, portable mine lamps, laboratory service, etc.

524. Types of Storage Cells. — Two types of storage cells are in practical use at the present time, although some other chemical combinations are theoretically possible. The most generally used type is the so-called lead-acid cell. A more recent cell, which has a rather limited field of application, is the nickel-iron cell, also known as the alkaline or Edison cell. A brief description of these cells is given below; for a more detailed description and for instructions as to their care and management, troubles and their remedies, the reader is referred to numerous special works on the subject.

THE LEAD-ACID CELL

525. Chemical and Physical Features. — The positive active material is peroxide of lead (PbO_2), and the negative active material is sponge lead (Pb). Dilute sulphuric acid (H_2SO_4) is used as electrolyte. When the cell discharges, both active materials are partially converted into lead sulphate ($PbSO_4$), and the acid thus becomes more dilute. On charging, a reverse action takes place, the plates being again restored to lead peroxide (positive plate) and sponge lead (negative plate). The specific gravity of the electrolyte increases to its maximum value, and the cell is again ready for discharge. These chemical changes may be represented by the following formula:

$$\underset{\xrightarrow{\textit{discharge}}}{PbO_2 + Pb + 2H_2SO_4} = \overset{\xleftarrow{\textit{charge}}}{2PbSO_4 + 2H_2O} \quad \ldots \quad (1)$$

In reality the chemical reactions are more complicated and are not established beyond dispute in all their details. However, the above fundamental equation is sufficient for a general understanding of the operation of lead storage batteries.

Certain impurities in the electrolyte, or in the water added to it, are injurious to the plates and may ruin them in a short time. Therefore, pure sulphuric acid and distilled water should be used whenever possible. If there is any doubt, the proposed acid and water should be submitted to the manufacturer of the battery for analysis and approval.

The density of the electrolyte in a cell, both in the fully charged condition and at normal discharge, varies over quite a wide range, depending on the design of the cell and the service for which it is adapted. Where cells are designed to go into a very limited space and where minimum weight is essential, there is very little space for electrolyte, and in order to provide sufficient quantity of sulphuric acid to give full capacity, it is necessary to make the electrolyte somewhat more dense. In such cases the specific gravity on full charge runs as high as 1.275 to 1.300. However, where the space is not so restricted, a lower density is used, down to about 1.200 to 1.210. From the standpoint of the life of the plates and wood separators, the lower-density electrolyte is preferable.

The foregoing values of density refer to 25° C. Tables and charts are available which give the correct density at other temperatures and also proportions of water and concentrated acid by weight and by volume for an electrolyte of a desired density.

As the cell is being discharged, the specific gravity of the electrolyte decreases by a definite amount, on account of the formation of water and the abstraction of some of the SO_4 radical which forms lead sulphate on the plates. Thus, if the gravity of a fully charged cell is 1.275, at the end of discharge it will usually be between 1.175 and 1.150. The battery should not be discharged below this point. The limits of density variations are usually within 10 per cent, and therefore an accurate hydrometer should be used for tests, especially when the state of charge or discharge is to be judged by the specific gravity of the acid.

When a cell is fully charged, the specific gravity of the electrolyte reaches its maximum and remains constant if the battery receives an overcharge. With reference to the fundamental chemical equation given above, this means that all sulphate has been converted into lead and lead peroxide. This is the best indication of a complete charge of a cell. Similarly, other states of charge and discharge can be judged by the specific gravity of the electrolyte, since changes in its density are approximately proportional to the ampere-hours discharged. This procedure is fairly safe with a familiar battery whose performance has been observed during some preceding cycles of charge and discharge.

Acid should never be added to the electrolyte except when all the plates in the cell are fully charged and the density is still below a prescribed minimum. If there is any doubt about this point, it is safer to operate the cell with a weak electrolyte than to impair its life by one that is too strong.

Sulphation. Whenever a battery is discharging, lead sulphate ($PbSO_4$) is formed on both the positive and the negative plates, as a part of the process of producing current. After a normal discharge the sulphate is finely crystalline and of such a nature that it is easily reduced by the current flowing through the battery on charge. If charging is neglected and the battery allowed to stand in a discharged state, the sulphate deposit becomes thicker and more dense; it fills the pores of the plates and eventually makes the active material dense and hard, thus prolonging the time required for a charge. In this latter condition the battery is spoken of as being "sulphated."

If sulphation is allowed to go too far, it will cause buckling of plates, loss of active material, a much decreased capacity and efficiency of the cell, and an increased internal resistance. A prolonged charging at a low rate of current will restore a moderately sulphated cell or battery. In more aggravated cases, several cycles of charge and discharge, or even a special treatment, may be necessary. For such special treatments see some practical book of instructions for the care and handling of storage batteries. By keeping the battery well charged and filled with proper electrolyte, and by avoiding over-discharge, serious sulphation can be avoided.

526. Mechanical Construction of Plates. — Both active materials, viz., sponge lead and peroxide of lead, are mechanically weak and must be properly supported. According to the method of support and manufacture, two types of plates are distinguished, the Planté plate, and the Faure or pasted plate.

A *Planté plate* consists originally of pure lead with its surface area greatly increased by cutting a large number of grooves. The plate is then used as an anode in an electrolyte of dilute sulphuric acid containing a small amount of some other acid, such as nitric, which will attack lead. When a direct current is passed from an outside source, an external layer of lead is oxidized into lead peroxide, and the thickness of this layer is increased by repeated charge and discharge. The plate is then ready to be used as a positive plate in a storage cell. If it is to be used as a negative plate the active layer is "reversed," or reduced, into sponge lead. This is done by using the plate as a negative in a cell and charging it.

A widely used positive plate of Planté type is shown in Figs. 386 and

387. The grid consists of cast lead-antimony alloy, designed to resist the " forming " effect of electrolytic action of the cell, and of sufficient rigidity to prevent troublesome growth and buckling. The grid is provided with circular openings slightly tapering toward the center. Into these openings are forced by hydraulic pressure rosettes or buttons of soft lead which constitute the active portion of the plate. These rosettes are formed of strips of pure lead, corrugated transversely and rolled into a spiral. After being forced into place in the grid they are subjected to the " forming " process, whereby the active material or lead peroxide is developed electrochemically on the transverse surfaces. The expansive action of this forming process, combined with the " hour-glass " shape of the openings, securely locks the buttons in place.

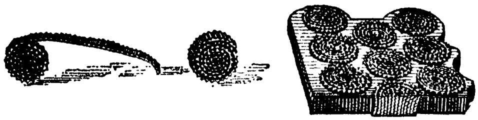

FIG. 386. Lead buttons used in the positive plates shown in Fig. 387.

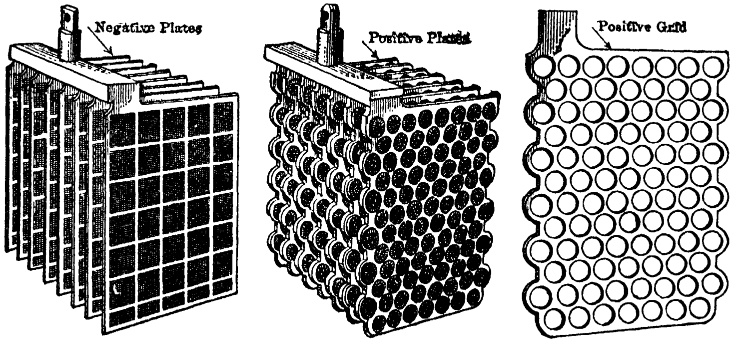

FIG. 387. An example of Planté positive and negative plates.

A *Faure* or *pasted plate* generally consists of a flat frame or grid of lead-antimony alloy into which the active materials, in the form of a paste, are pressed. The structure of the grid is such that the active materials are held securely in place. There are many forms of grids; the one shown in Fig. 387 for the positive plate of the Planté type gives a general idea of their structure. In the " box type " negative pasted plate, somewhat like that shown in the same sketch to the left, the grid is made of horizontal and vertical ribs spaced about 1 in. apart. These ribs form pockets closed on both sides with perforated sheet lead, and in these pockets the active material is permanently held in place. Since this active material need not be self-supporting, it may be of such composition as is best suited to maintain the desired porosity and permanent capacity in service.

A positive plate with lead peroxide held in compartments is shown in Fig. 388. It has a grid composed of a number of parallel vertical lead-alloy rods united integrally to horizontal top and bottom frames, the former being provided with the usual conducting lug. Each vertical rod forms a core which is surrounded by a cylindrical pencil of peroxide of lead, the active material. This, in turn, is enclosed by a hard-rubber tube having a large number of horizontal slits. These slits serve to provide access for the electrolyte to the active material, and yet are so fine as practically to eliminate the washing out of the material. The outside tubes are reinforced by leaving the exposed edge solid, that is, without slits.

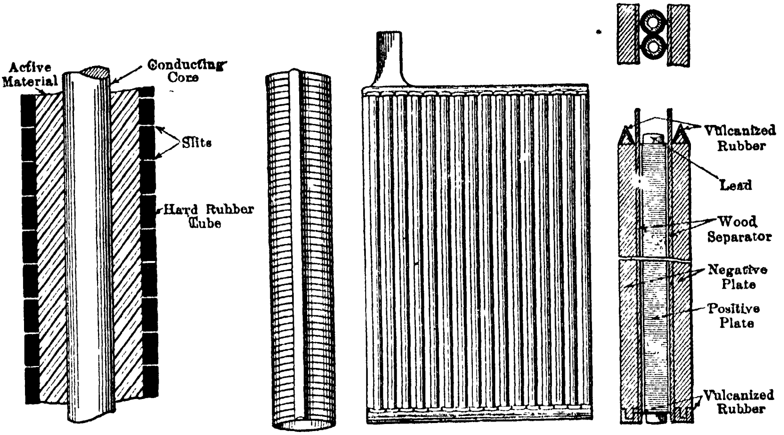

Fig. 388. A positive plate with the active material in hard-rubber tubes.

Each tube is provided with two parallel vertical ribs projecting on opposite sides, at right angles to the face of the plate. These ribs not only serve to stiffen the tubes, but also act as insulating spacers, allowing the use of plain wood-veneer separators. The rubber tubes have a certain amount of elasticity, allowing them to compensate for changes in volume of the active material, due to expansion and contraction during charge and discharge.

A complete cell embodying this plate is shown in Fig. 389, with part of the jar cut away to show the construction. There is one more negative plate than there are positive plates, the two outside plates being negative. The same excess of negative plates is shown in Fig. 387.

Separators. The active plates are generally separated from each other by thin sheets of wood veneer known as separators (Figs. 388 and

389). The separators are porous to permit electrolytic conduction, but they prevent metallic conduction which would short-circuit the cell. The separators are usually corrugated on one side and smooth on the other. The wood must be either naturally free from injurious substances or treated to remove them. The wood separators in tractor batteries are generally kept away from the positive plates by means of thin sheets of perforated hard rubber.

In one make of storage cell, threaded-rubber separators are used in place of wood. Such a separator consists of a sheet of soft-rubber compound containing a great many cotton threads running transversely through the sheet. Each thread acts as a wick, and the threads take the place of pores in a wooden separator.

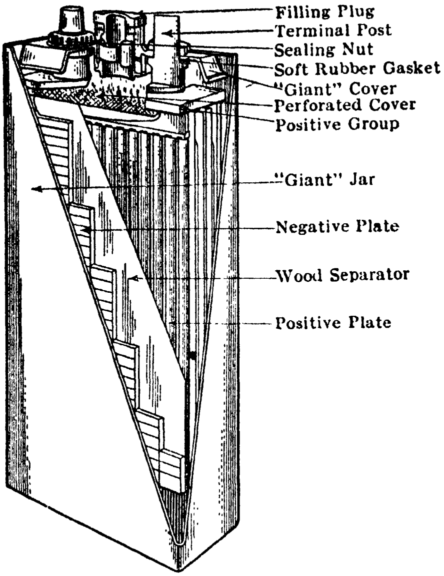

FIG. 389. A cut-away view of a complete lead-acid storage cell.

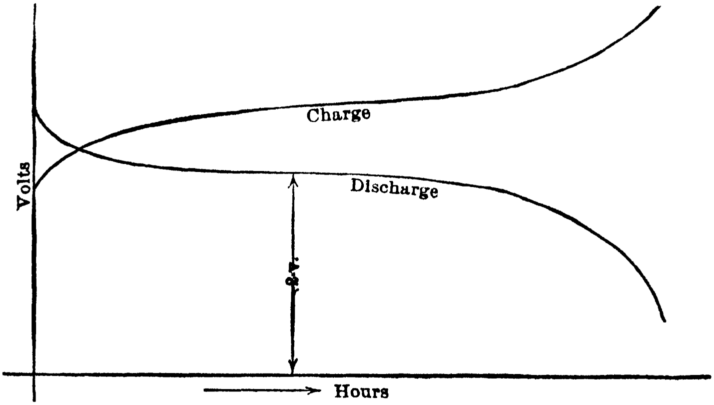

FIG. 390. Curves of charge and discharge of a lead-acid storage battery.

527. Voltages during Charge and Discharge. — A fully charged storage cell has an emf of a little over 2 volts on open circuit. If it is allowed to be discharged indefinitely, the voltage will at first remain practically constant at about 2 volts, then will gradually fall off, at first slowly, then more and more rapidly, down to zero (Fig. 390). The voltages given in the curve are supposed to be measured while a normal discharge current is passing through the cell. The voltage drop in the cell is caused by the internal resistance of the cell (§531) and by some polarization due to a weakening of the acid in the pores of the active material.

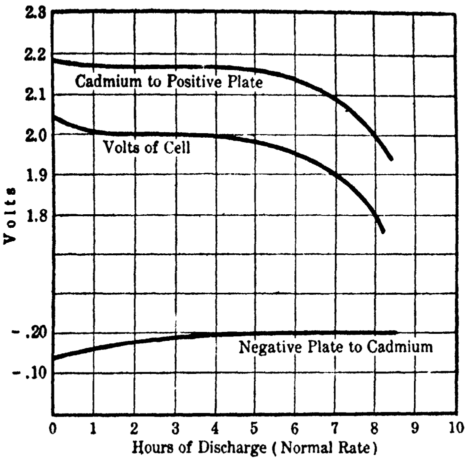

FIG. 391*a*. Typical curves on cadmium test, during discharge of lead-acid cell.

A complete discharge down to zero voltage would be impracticable, because for all ordinary purposes the terminal voltage of the battery must be constant within rather narrow limits. Moreover, such a complete discharge, if frequently repeated, would in time sulphate the plates (§525) and shorten their life. In practice, a cell is considered to require a new charge when the voltage has dropped to 1.75 volts (or better, 1.8 volts), although this value varies with the rate of discharge, being lower for the higher rates, and vice versa. With stationary batteries this voltage is measured while the cells are supplying a current which corresponds to the eight-hour rate of discharge.

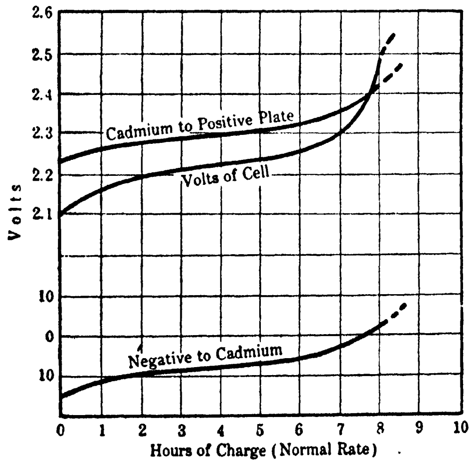

FIG. 391*b*. Curves on test with cadmium electrode during charge of lead-acid cell.

When a cell is being charged, the external voltage applied at its terminals must be high enough to overcome the counter emf of the cell and to force the charging current through its ohmic resistance. At the beginning of a charge the charging voltage is a little above 2 volts per cell; see Fig. 390, also Fig. 391*b*. As the battery becomes recuperated, this voltage must be gradually increased; until at the end of the charge it is necessary to apply about 2.6 volts in order to get full charging current through the cell. The end of the charge is also recognized by an excessive liberation of gases (gassing) due to decomposition of water in the solution.

One of the best indications of a complete charge is given by the specific gravity of the electrolyte, measured with a hydrometer (§525). The open-circuit voltage of a cell does not indicate its state of charge, although it is affected by the specific gravity of the electrolyte, short-circuited plates, etc. The voltage during charge or discharge will vary with temperature and age of the cell. For a cell in normal condition the open-circuit voltage[1] is given by the formula

$$E = 1.85 + 0.917(G - g)$$

where G = specific gravity of the acid, and g = the specific gravity of water at the cell temperature.

528. Cadmium Tester. — In order to ascertain separately the state of charge on each plate, a cadmium tester or electrode is sometimes used. It consists of a stick of pure cadmium placed in the acid of the cell under test. It is well to have the cadmium protected by a hard-rubber tube with perforations for the circulation of the acid. At the end of the charge (Fig. 391*b*) the voltmeter shows about 2.43 volts between lead peroxide and cadmium, and about 0.05 volt between the lead plate and cadmium. This is more or less positive indication of the end of the charge. The voltage between the two plates is equal to the sum of the two readings:

$$2.43 + 0.05 = 2.48 \text{ volts}$$

The same tester can be used to ascertain the end of discharge. In this case the voltages are approximately +2.00 and −0.20 volts respectively; the cell voltage is

$$2.00 - 0.20 = 1.80 \text{ volts}$$

Typical curves of voltage between battery plates and the cadmium electrode during charge and discharge are shown in Fig. 391*a* and *b*.[1]

Voltages on charge and on discharge should be read while a normal current is passing through the cell. On charge the cadmium tester is first electronegative and later electropositive to the sponge lead plate and is negative to the peroxide plate. On discharge the cadmium is negative to both plates. This explains the plus and minus signs in the

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[1] Standard Handbook, 5th ed., McGraw-Hill Book Co.

foregoing equations. Different authorities give different numerical data for the potentials to cadmium, and these values depend somewhat upon the conditions of the test and upon the cadmium electrode itself.

A high-resistance voltmeter, or a potentiometer, is essential in this test, as otherwise there will be an appreciable polarization about the cadmium electrode that will vitiate the results. For details of the use of the cadmium electrode and for the necessary precautions, see Technological Paper 146, of the Bureau of Standards.

If it is found that one of the plates is not fully charged, the charge must be continued until the cadmium electrode shows the required voltage. If it is feared that an excessive charge may damage the other plate, the plate which requires additional charging may be charged in a separate cell. A cadmium tester gives reliable indications in the hands of an experienced observer, especially when many tests are made on batteries of the same type. Otherwise, it is safer to judge about the state of the charge from the acid density and the voltage.

529. Capacity of Storage Cells. — The *capacity* of a cell, or the amount of electricity which it can give on discharge, is measured in *ampere-hours.* A cell which can supply 25 amperes for 8 hours, before the lower limit of the emf, say 1.8 volt, is reached, is said to have a capacity of $25 \times 8 = 200$ ampere-hours. Experience shows that the capacity of a lead cell depends essentially on the rate of discharge. The more rapid the discharge the less is the capacity; thus the above cell, if discharged at a rate of 100 amperes, would be completely discharged in one hour instead of two hours. Therefore, *when speaking of the capacity of a storage battery it is always necessary to mention the number of hours in which the battery is supposed to be discharged.* It is customary to rate stationary batteries on the basis of an eight-hour discharge and batteries used for the propulsion of electric automobiles on the basis of a four-hour to six-hour discharge. Storage batteries used in electric railway substations, for taking up fluctuations of the load, are usually rated on the arbitrary basis of one-hour discharge, or even less, and batteries used to supply the control-bus in a power station are often rated on a one-minute discharge basis.

If a battery is intended to be discharged within a shorter period of time than normal, its rated capacity must be reduced in a ratio usually given by the manufacturer. In the table below the data refer to a particular type of cell only, but they give a general idea of the order of magnitude of changes in capacity. The second column contains the terminal voltages at which the discharge should be stopped. In each case the voltage is supposed to be measured with a discharge current flowing at the corresponding rate.

Hours Discharge	Final Voltage	Relative Value of Current	Relative Capacity in Ampere Hours
8	1.75	1	8 (100%)
3	1.72	2	6 (75%)
1	1.65	4	4 (50%)
⅓	1 52	8	2⅔ (33⅓%)

When a battery is discharged at the one-hour rate for one hour and is completely discharged *at that rate*, it still has additional capacity which is available at lower rates of discharge. Additional capacity may also be taken out at the one-hour rate if the battery is allowed to stand and recuperate. In fact, the full eight-hour capacity of a battery may be taken out at the one-hour rate, provided the discharge is intermittent in character and is distributed over a total elapsed time of eight hours. Higher rates than the one-hour rate are utilized in many applications, such as momentary load regulation and standby service. In the latter service, rates as high as four times the one-hour rate are occasionally used.

The above-described behavior on discharge and the reduction in capacity at higher rates is principally due to a reduced density of the electrolyte in the pores of the plates. When the cell is allowed to recuperate, fresh acid gradually finds its way into the pores, an additional quantity of lead sulphate can be formed, and more ampere-hours taken out of the cell.

For starting and lighting batteries of gasoline automobiles, the American Society of Automotive Engineers prescribes a double rating in the following rule:

" Batteries for combined lighting and starting service shall have two ratings. The first shall indicate the lighting ability and shall be the capacity in ampere-hours of the battery when it is discharged continuously at the 20-hour rate to a final voltage of not less than 1.75 per cell, the temperature of the battery at the beginning of such discharge being 80° F. The second rating shall indicate starting ability of the battery, and shall be the minimum amperes when the battery is discharged continuously at the 20-minute rate to a final voltage of not less than 1.5 per cell, the temperature of the battery at the beginning of such discharge being 80° F (27° C)."

Effect of Temperature. The capacity of a lead-acid storage cell decreases as its temperature is lowered. This is particularly unfortunate for starting batteries of automobile engines and of electric vehicles in extremely cold weather, for then the maximum of energy is usually required but the available capacity of the battery may be only 50 per cent of what it is in the summer.

One would naturally suppose from the high rates of current demanded from the automobile storage battery that failures in service would be due principally to this reason. As a matter of fact, the deterioration of the battery plate is due largely to overheating in the summer. This overheating acts in two ways. It tends to loosen the active material in the battery plates and also carbonizes the wood insulation between the plates. Therefore, the charging rate should not be excessive, especially during warm weather.

530. Electrical Efficiency. — Two kinds of efficiency of a storage cell or battery are distinguished, viz., the ampere-hour efficiency and the watthour efficiency. Let an ampere-hour meter (§124) and a watthour meter (§116) be connected in series with a cell which has been fully charged, and let the cell be discharged to a low point at a predetermined constant or variable rate. Then let the cell again be fully charged at a desired rate. Let the two instruments be read both on charge and on discharge. The ratio of the total ampere-hours delivered by the cell to those put into it gives its ampere-hour efficiency. A similar ratio of the readings of the watthour meter gives the watthour efficiency of the cell. The second efficiency is lower than the first because the average voltage on discharge is lower than the average charging voltage. The numerical values of efficiency refer to the particular rates of charge and discharge used and to the chosen end-voltage.

531. Internal Resistance. — The resistance of a storage cell is due to a number of factors, including the resistance of the electrolyte and separators, the resistance of the active material which varies during charge and discharge, and that of the grids and terminals. The resistance of a storage cell rises toward the end of discharge to more than double its resistance when fully charged. When a discharged cell is charged, the internal resistance falls again to its original value, corrected for temperature.

Although there is a considerable change in the density of the electrolyte, the change in its conductivity is small. The change in resistance which takes place at the plates of a lead-acid cell is largely explained by the variable amount of diluted electrolyte in the pores of the active materials. The more the acid in these pores is diluted, through an excessive discharge, the higher is the internal resistance.

Another cause of variations in the internal resistance is the presence of lead sulphate (§525). During discharge the lead and peroxide particles become more or less densely covered with a layer of non-conducting lead sulphate which is reduced on the subsequent charge. If this layer of sulphate is allowed to become hard and dense, the internal resistance of the cell may rise to a high value.

The determination of the *true* ohmic resistance of a storage cell is rather difficult, because this resistance is very small, variable, and masked to some extent by the effect of polarization. Moreover, it is not the true resistance but rather the *virtual* resistance of the cell that is of interest to the user. This virtual or equivalent resistance represents the total drop of voltage in the cell, due to whatever causes. If it were not for a variable amount of polarization, the simplest method of determining the internal resistance R of a cell would be to observe its voltage E_0 on open circuit, and then immediately note the voltage E with a certain charging current I flowing through the battery. Then, evidently, we should have

$$R = \frac{E - E_0}{I} \quad \text{. . . . .} \quad (2)$$

Because of polarization, a better method is to measure two terminal voltages, E_1 and E_2, corresponding to two different values, I_1 and I_2, of charging current. We then have

$$\left.\begin{aligned} E_1 - p - E_0 &= RI_1 \\ E_2 - p - E_0 &= RI_2 \end{aligned}\right\} \quad \text{. . . . .} \quad (3)$$

where p is the counter emf of polarization. Eliminating p and solving for R, we obtain

$$R = \frac{E_1 - E_2}{I_1 - I_2} \quad \text{. . . . .} \quad (4)$$

An objection to this method is that the emf, p, of polarization is not quite constant with various rates of charge. Another objection is that the difference $E_1 - E_2$ is rather small, and this impairs the accuracy of the result. It is advisable to perform a large number of tests with various values of I_1 and I_2 and to take an average of the calculated values of R. Experience shows that more consistent results are obtained on discharge than on charge; the same formulas (2) and (4) are used as given above.

With the so-called " break " method of measuring the resistance of a cell on discharge, the external load resistance is adjusted to a certain value, and when the conditions become steady, the circuit is suddenly opened. The emf, read on a voltmeter, rises instantly by a certain amount and then continues to rise *gradually* as stronger electrolyte is diffused into the pores of the plates, taking the place of the electrolyte weakened by the discharge. From this sudden rise in voltage and a knowledge of the current that was flowing through the cell, the internal resistance can be calculated. For example, let the voltage rise from 1.8 volts to 1.9 volts when the switch is opened, and let the current be 100 amperes. The resistance of the cell is 0.1 ÷ 100 = 0.001 ohm.

Instead of opening the circuit, the observer may suddenly change the current by a definite amount and read the corresponding instantaneous rise in voltage. The virtual resistance can be computed from eq. (4). See also §23 on the resistance of electrolytes.

532. Charge and Discharge Characteristics. — Supplementing the curves shown in Fig. 390, a more complete set of characteristic curves for a lead-acid storage cell is shown in Fig. 392.

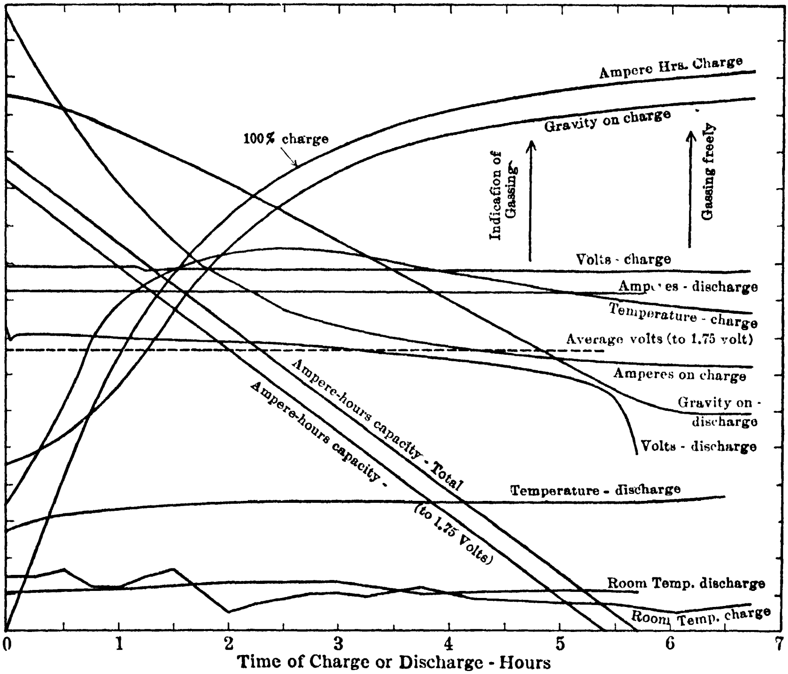

FIG. 392. Charge characteristics of a lead-acid cell at a constant voltage, also its discharge characteristics.

In Fig. 390 the cell is supposed to be charged at a nearly *constant current*, except that at the end of the charge the current is reduced to the "finishing rate," to avoid violent gassing which loosens the active material and shortens the life of the cell. This gassing is due to decomposition of the water in the electrolyte into hydrogen and oxygen. At the end of the charge, when the remaining amount of lead sulphate is small, the chemical reaction in the pores can go on only at a moderate speed, and if there is an excess current it is wasted in decomposing water.

When charging at a constant current, the required voltage increases as the charge proceeds.

The curves shown in Fig. 392 refer to charging at a *constant voltage* of about 2.3 volts per cell. At the beginning of the charge the difference between 2.3 volts and the counter emf of the cell is rather large, and therefore the initial charging current is also large. With the cell nearly discharged, a large charging current causes no gassing and simply reduces the total time required for charging. As the charge process advances and the counter emf of the cell increases, the current is automatically reduced; and with a terminal voltage of 2.3 volts the final or finishing rate of current is nearly correct, that is, it permits the final reduction of lead sulphate without excessive gassing.

Thus, the advantages of the constant-potential system of charging, over the older constant-current system, are (a) that no current regulation is required, and (b) that the total time of charging is considerably reduced. In practice, a modified constant-potential system of charging is often used (§540). When only one cell is being tested, the current usually has to be adjusted to a desired value by means of a rheostat as the charge proceeds.

It has been found that if the charging rate is kept below a value equal to the number of ampere-hours out of the cell, excessive gassing and high temperature will ordinarily be avoided. As an illustration, if a cell having a capacity of 200 ampere-hours is completely discharged, the initial rate of charge may be 200 amperes, but must be continually reduced, so that when the cell is half charged the rate will be 100 amperes, and when it is three-quarters full, the rate will be 50 amperes. Toward the end of the charge, when the rate has been finally reduced to the normal or finishing rate, it need not be further reduced, and the charge may be completed at this rate. Some gassing will result at the end, but the amount will not be excessive.

533. EXPERIMENT 24-A. — Test of a Lead Storage Cell. — A complete test on an unknown cell should comprise the following points: (a) performance curves, such as shown in Figs. 390 to 392, at various rates of discharge; (b) variations of voltage and capacity with temperature; (c) charge and discharge rates and capacity per unit weight of the cell and per unit area of both plates; (d) the efficiency of the cell under different conditions; (e) the internal resistance of the cell and its variations; (f) the relationship between the maximum density of the electrolyte and the discharge characteristics; (g) loss of charge by leakage and local action; (h) durability of the cell and its adaptability for a proposed service.

All these tests are either described in the preceding sections or follow directly from their purpose and from the general characteristics of the lead storage cell. Although each test in itself is rather simple, a complete series, such as outlined above, would take many weeks to perform. If it is desired merely to become acquainted with the technique of the principal tests, one partial discharge and a following charge will be sufficient. If it is not convenient to charge and to discharge a single cell, two or more cells in series may be tested, but all voltage and density readings should be taken on one and the same cell.

The cell or cells (Fig. 393) are connected to a double-throw switch, by means of which they can be connected at will either to a source of direct current for charging or to a load resistance for discharge. An ammeter and an ampere-hour meter are provided in the battery circuit. A watthour meter may also be used as a check, if the battery voltage is sufficiently high for its potential circuit.

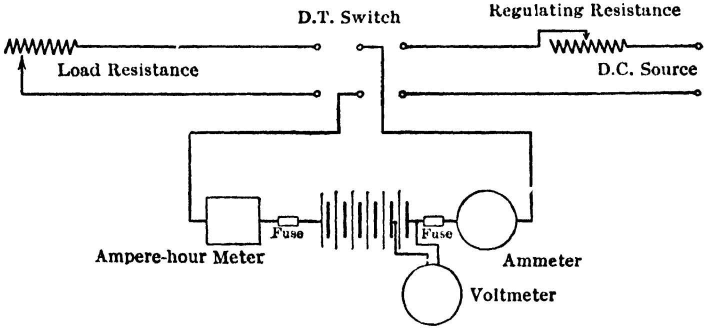

FIG. 393. Connections for testing a storage battery.

Assuming the cell to be fully charged, take the initial readings of voltage and specific gravity on open circuit, and measure the internal resistance and the temperature of the cell and of the room. Connect the cell to the load rheostat, and adjust the current to a desired value and keep it constant throughout the test.

At first read all the instruments and thermometers at frequent intervals; later, when the conditions become steady, less frequent readings are sufficient. Use a cadmium electrode as a check on voltage readings. Use first a high-resistance voltmeter and then one of lower resistance. Note the difference in results.[1] From time to time measure the internal resistance. Always stir the liquid before reading the hydrometer so as to measure the true average density.

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[1] See "Cadmium Electrode for Storage Battery Testing," by Holler and Braham, Bureau of Standards Technological Paper 146.

To save time, two or more independent cells or batteries can be tested simultaneously, some charging, others discharging. One voltmeter with a good multipoint switch is sufficient for all. An ammeter shunt may be used in each circuit, with a common millivoltmeter for reading the currents. In a student laboratory, tests at low rates of discharge, sulphation tests, investigation of local action, durability, service tests, etc., may be continued day after day by different observers, and the results entered in a common log to be analyzed at the end of the complete test.

At the end of the experiment, measure all the dimensions of the cell and of its elements, so as to be able to make a drawing to scale. Determine the weight of the plates, of the electrolyte, and of the complete cell. Do not keep the negative plates out of the liquid longer than is necessary; they may be damaged by the action of the atmosphere.

Report. Plot the observed performance curves and give answers to the items (a) to (h) enumerated at the beginning of the experiment. Where the laboratory data are insufficient, specify in detail the necessary additional tests and measurements.

THE NICKEL-IRON CELL

534. Chemical and Physical Features. — The secondary cell, known as the Edison or alkaline cell, is shown in Fig. 394. In a fully charged state the active material of the positive plates is nickelic hydroxide, $Ni(OH)_3$, and that of the negative plates is metallic iron (Fe). The electrolyte is a 21 per cent solution of caustic potash (potassium hydroxide, KOH). When the circuit is closed and the cell allowed to discharge, iron is oxidized to ferrous hydroxide, $Fe(OH)_2$, while the high-nickel oxide is reduced to lower oxides, and finally to nickelous hydroxide, $Ni(OH)_2$. On charging, the iron oxide is again reduced to iron and the nickelous oxide is oxidized to the higher oxide, $Ni(OH)_3$. These reactions can be expressed by the following reversible equation:

$$\underset{\xrightarrow{\textit{discharge}}}{Fe + 2Ni(OH)_3} = \overset{\xleftarrow{\textit{charge}}}{Fe(OH)_2 + 2Ni(OH)_2} \quad . \; . \; . \; . \; . \quad (5)$$

Because of this reaction the cell may be termed a " hydroxyl lift " cell.

The function of the electrolyte is to furnish hydroxyl ions (HO) at the anode and to absorb them at the cathode. During the discharge, potassium ions tend to migrate from iron to nickel, leaving hydroxyl ions to combine with the iron. Potassium ions recombine with hydroxyl ions moving from nickel. Thus the electrolyte becomes concentrated in the pores of the positive plate and weakened in the pores of the negative plate. On charge, an opposite migration takes place. Diffusion ultimately destroys these differences in concentration and leaves the electrolyte in the original condition, since at any instant the total quan-

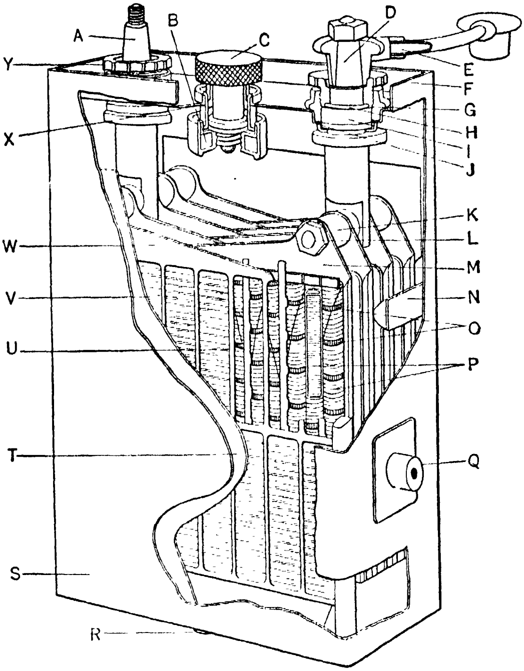

Fig. 394. Details of construction of Edison's alkaline or nickel-iron secondary cell.

A — Negative pole.
B — Valve.
C — Filler cap.
D — Positive pole.
E — Copper wire swedged into steel lug.
F — Cell cover welded to container.
G — Stuffing box.
H — Gland ring.
I — Stuffing box gasket.
J — Weld to cover.
K — Spacing washer.
L — Connecting rod.
M — Positive grid.
N — Grid separator.
O — Seamless steel ring.
P — Positive tubes (nickel hydrate and nickel in layers).
Q — Suspension boss.
R — Cell bottom (welded to sides).
S — Solid steel container.
T — Side insulator.
U — Pin insulator.
V — Negative pocket (iron oxide).
W — Negative grid.
X — Cell cover.
Y — Hard rubber gland cap.

tity of water and potassium hydroxide (including that in the pores of both plates) remains constant.

For this reason, the specific gravity of the electrolyte, at a given temperature, remains practically constant both on charge and on discharge, and a hydrometer cannot be used to ascertain the state of charge of a cell, as in a lead-acid accumulator (§525). Distilled water has to be added somewhat more frequently to the alkaline electrolyte than to the lead-type cell in order to make up for the losses due to gassing on charge, and the whole electrolyte has to be completely renewed about once in three or four years of service, although some alkaline batteries on light-duty service have operated for twenty years without renewal of electrolyte.

The fundamental principle of the cell is, in short, the oxidation and reduction of two metals in an electrolyte which does not dissolve, and will not combine with, either the metals or their oxides. Sodium hydroxide may be used in place of potassium hydroxide but is not recommended.

Under normal conditions the gravity of electrolyte should be from 1.200 to 1.220. When potassium electrolyte has fallen to 1.160 it should be renewed. Sodium electrolyte may be used down to 1.130.

A small amount of lithium hydroxide (LiOH) is always added to the electrolyte. It is said that lithium causes the active material within the nickel electrode to swell and to make a better contact with the containing tubes and with the nickel flakes added to nickelic hydroxide for better electric conductivity. Whatever the explanation may be, the fact has been established that lithium hydroxide appreciably increases the capacity of the cell.

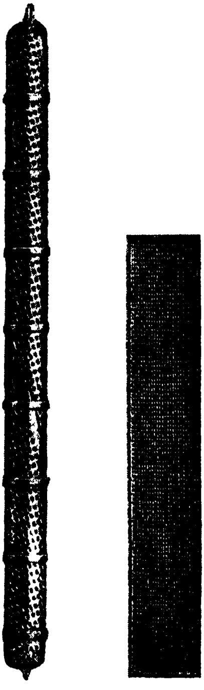
(a) (b)

FIG. 395a. The positive tube of the nickel-iron cell; Fig. 395b, the negative pocket.

535. Mechanical Construction of the Plates. — The active material of the positive plates (nickelic hydroxide) is packed under high pressure in small, reinforced steel tubes (Fig. 395a) with alternate layers of nickel flakes. This is necessary because nickel hydroxide itself is a poor conductor of electricity. The tubes are provided with numerous lateral perforations to allow the electrolyte to permeate the active material, and are solidly mounted in a steel grid (Fig. 396), as may be seen at *M* and *P*, Fig. 394.

Negative plates consist of thin rectangular pockets (Fig. 395*b*) of perforated sheet steel mounted in steel frames or grids (Fig. 396). The active materials, iron oxide and metallic iron, are contained in the pockets. The grids, tubes, and pockets are nickel plated. Mercuric oxide, subsequently reduced to mercury, is added to increase the electric conductivity of the active material.

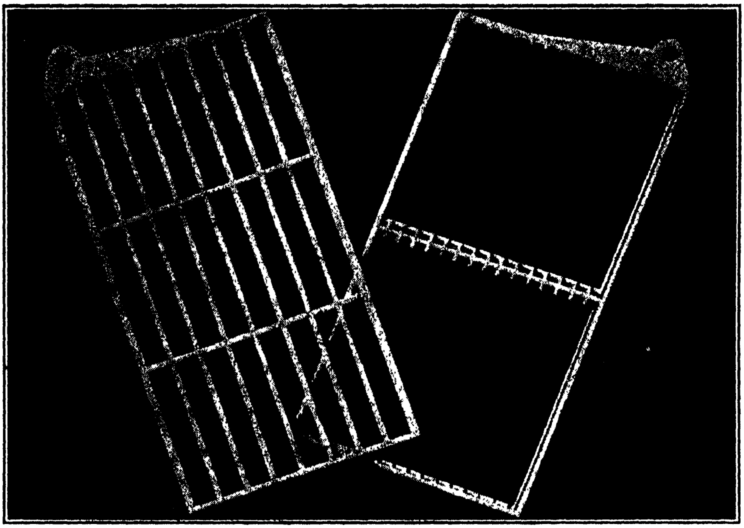

FIG. 396. The grids of the nickel-iron battery, negative on the left, positive on the right.

There is always one more negative plate in the cell than there are positive plates. All like plates are connected together by nickeled steel rods, and the whole assembly is placed in a container of nickeled steel. Hard-rubber separators are placed between adjacent plates of opposite polarity, and hard-rubber bushings, rods, and supports are used throughout to insulate the plates from each other and from the container. The whole mechanical construction has been devised so as to secure the rigidity, strength, and durability necessary in traction work and for uses as portable batteries.

536. Electrical Characteristics. — The normal open-circuit emf of an alkaline cell is about 1.34 volts. When the cell is fully charged, the emf rises (see Fig. 397) to a somewhat higher value for a short time. The open-circuit voltage should never be used to determine the state of charge or the extent of discharge.

The proper method of determining the state of charge is to read the cell voltage while the battery is on charge or discharge. On charge, if the voltage remains constant at 1.8 volts per cell, or above this, for thirty minutes, with normal charge current flowing through the battery, then the battery can be considered fully charged. A very good check on this is to observe the extent of gassing. If a cell is gassing freely with normal current flowing, it is a pretty good indication that a condition of full charge has been reached. In this connection, a distinction must be made between gassing and frothing.

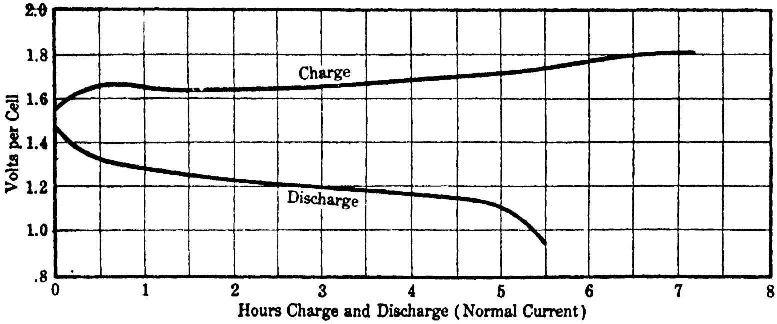

FIG. 397. Charge and discharge curves of the nickel alkali cell. (Edison Storage Battery Co.)

Frothing at the filler openings of a battery indicates either too rapid charging, too high a level of electrolyte, or impurities in the cells. If frothing takes place while charging at normal rate with the level of the electrolyte at the proper height, it is a sure indication that some form of animal fat or oil has entered the cell. If the frothing continues for any length of time, the cell should be emptied and rinsed out, and new electrolyte put in.

If a cell is discharged immediately after charging, the initial voltage, with the rated current, is 1.45. The voltage rapidly falls to 1.3 volts and then drops slowly to 1.15 volts (Fig. 397). At this point it begins to fall more rapidly, and the discharge is usually stopped at about 1.0 volt. The average voltage while discharging at the normal five-hour rate to the foregoing point is about 1.2 volts. When discharging at five times the normal rate, the final voltage can be as low as 0.5 volt (see Fig. 398). The advisable final voltage between these limits may be taken to vary according to a straight-line law.

The normal charging and discharging rates for an Edison battery are seven and five hours, respectively. Gas is evolved throughout the charge, and as this gas contains considerable hydrogen, an open flame should not be brought near, and an electric spark at a battery terminal may cause an explosion. The temperature rises constantly when the battery is discharging or charging.

The capacity and efficiency of an Edison cell increase with the temperature, but temperatures above 50° C will shorten the life of the cell. At normal current a reduction in temperature from 115° F to 50° F lowers the capacity about 10 per cent; below 50° F the capacity and efficiency decrease rapidly. The solution crystallizes into a mass resembling snow at $-17°$ F. The ampere-hour efficiency of the alkaline cell is about 82 per cent, and the watthour efficiency about 60 per cent if the period of discharge begins immediately after the completion of the charge. About 10 per cent loss occurs if the battery stands idle for two days and some 13 per cent in ten days (at 75° F). At lower temperatures the loss of charge is less. For a general discussion of the voltage, capacity, efficiency, and internal resistance of storage cells see §§527 and 532.

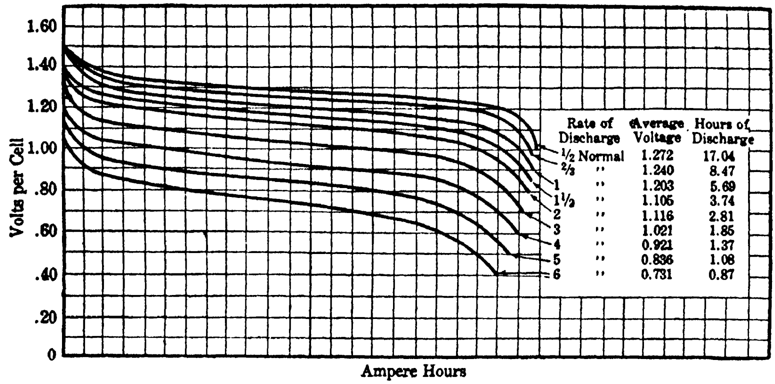

FIG. 398. Discharge curves of nickel alkali cells at various rates. (Edison Storage Battery Co.)

537. A Comparison of the Lead-Acid and Nickel-Iron Cells. — The Edison cell may be considered to be competitive with the lead-sulphuric acid cell, except in some classes of service, such as submarine, central station standby, automobile and airplane starting, and large telephone installations. A comparatively high internal resistance and low emf of the nickel-iron cell are the principal inherent obstacles to its universal use in power work. On the other hand, an alkaline battery is generally somewhat lighter and takes less space than a lead battery of the same capacity and voltage. Thus, the alkaline battery weighs from 87 to 105 lb per kwhr of capacity as compared with 105 to 130 lb for the lead battery. The alkaline battery is also much more robust and foolproof, it has a longer life, and can stand an amount of hard usage, abuse, and neglect that would ruin a lead battery, so that it is especially adapted to motive-power service.

The following disadvantages of the nickel-iron cell may be mentioned: (a) Its first cost is higher. (b) The energy and current efficiencies are lower than those of a lead cell. (c) The voltage varies considerably during a discharge. (d) The cell is very inefficient at low temperatures. (e) After a full charge it loses 10 per cent of its energy in forty-eight hours.

On the other hand, the nickel-iron cell has the following advantages over the lead-acid cell:

(a) *On charge*: An Edison battery may be forced to full charge at almost any rate of current, without injury, provided the electrolyte temperature does not become excessive. A short boosting charge at a high rate is particularly desirable in electric-vehicle service. The battery may be charged in the wrong direction without permanent injury. It emits no corrosive fumes, and there is no acid to handle.

(b) *On discharge*: A repeated excessive discharge will not result in a permanent injury. Even an occasional short circuit is said to have a beneficial effect, if any. The ampere-hour capacity is more independent of the rate of discharge than that of a lead cell.

(c) *Mechanical strength*: A nickel-iron cell is not subject to buckling or sulphation, and there is no shedding of active material which, as sediment, ultimately may cause a short circuit. Continued vibration has practically no effect on the cell.

(d) *When standing idle*: An Edison battery may stand for a long time in any state of discharge, without damage. The electrolyte may be drained or partially removed without serious injury to the plates. The cell will retain over half its charge at the end of six months or more of idleness. Cases are quoted of a prompt recovery of an alkali battery after remaining unused for three years. The manufacturers recommend that the cells be stored in the discharged condition, and that the plates be kept covered with the electrolyte.

538. EXPERIMENT 24-B. — Test of a Nickel-Iron Storage Cell. — The general scope of the test is the same as in §533, but the differences in the chemical reactions and in the physical characteristics of the two cells must be kept in mind. Readings with the hydrometer and the cadmium electrode are, of course, to be omitted.

OPERATION AND CONTROL OF STORAGE BATTERIES

539. Desirable Tapering of Charging Current with Time. — In order to charge a battery fully after discharge it is necessary to pass through the cells in the proper direction (opposite to that of discharge) an amount of electricity equal, in ampere-hours, to that taken out on discharge plus some excess to make up for the losses.

If the charging rate in amperes is not too high, practically all the current is useful for charging the plates. If the charging rate is increased, a point will be reached where " gassing " begins, due to bubbles of oxygen and hydrogen formed at the surface of the plates by decomposition of the water in the electrolyte; this gassing increases with further increase in the charging rate. This decomposition absorbs an amount of electricity proportional to the amount of gas produced, and this portion of the current is wasted, as it produces absolutely no useful effect in charging the plates. Charging rates sufficiently high to produce violent gassing not only are wasteful of electric energy, but also tend to dislodge the active material from the plates and produce excessive temperature rise, thus very materially shortening the life of the plates; and it cannot be too strongly emphasized that *the excess which produces gassing does not charge the plates.*

In general, any charging rate is permissible which does not produce excessive gassing or a cell temperature exceeding the safe value, usually given as 43° C with lead-acid cells, and 50° C with nickel-iron cells.

The value of the charging current at which appreciable gassing begins depends upon several factors, such as state of charge, temperature, specific gravity of electrolyte, type of plate, etc., but the principal factor is the state of charge of the battery. When a battery is fully charged, any rate of charge, however small, will produce gassing, but the rate may be reduced to such a low value that, unless abnormally prolonged, the small amount of gassing which results is practically harmless. This safe rate is called the " finishing " rate (§532). During the earlier stages of the charge, the charging rate may be several times the finishing rate, without producing violent gassing; and the more completely the battery is discharged, the higher the charging rate may be without causing excessive gassing.

From the above it will be observed that there is a wide range of satisfactory charging methods, from that employing a constant current at the finishing rate (the " Constant Current " method) requiring with a lead battery about sixteen hours for a full charge, to that involving a comparatively high rate at the beginning, tapering off to the finishing rate at the end, under which conditions a battery may be charged in five hours or even less.

For each stage of charge there is a maximum value of current which should not be exceeded, and when the charging current can be automatically tapered in accordance with this rate, the charging is completed within the least possible time. It is often of importance to accomplish this aim at least approximately, for example in batteries used on locomotives, trucks, etc., where the time taken for charging may reduce the amount of service of which the vehicle is capable, unless spare batteries are available.

An *ampere-hour meter* (§124) can be used for terminating the charge automatically. The ampere-hour meter must be connected in the battery circuit at all times during discharge as well as charge, in order that it may register the state of charge of the battery when the battery goes on charge. It should run slower on charge than on discharge, in order to take into account the losses in the battery and to give it a proper overcharge before the charging circuit is opened. Under these conditions an ampere-hour meter designed to open the charging circuit when the indicator hand returns to zero will give satisfactory results, provided a suitable instrument is employed and is maintained in operating condition.

In important installations of lead batteries a *recording and signaling hydrometer* is sometimes used. As is stated in §523, the specific gravity of acid electrolyte is the best indicator of the state of charge. However, it is often troublesome or impossible for an attendant to take a sufficient number of readings and to determine accurately when the gravity has reached a predetermined point, either on charge or discharge. Moreover, the readings taken with an ordinary hydrometer require a correction for temperature in order to be directly comparable, and this the attendant frequently has not the opportunity to compute.

The recording hydrometer is placed on a pilot cell, that is, a cell whose condition is representative of the normal cells of the battery. The instrument traces a continuous record of density on a sheet of coördinate paper, the ordinates being automatically corrected for the temperature. When the density reaches its maximum, a contact is automatically closed and a visual or audible signal is given to warn the operator that the charge should be ended. Similarly, a signal is given to end the discharge when the specific gravity has reached its permissible minimum.

540. The Modified Constant-Potential System of Charging. — A simple method of obtaining an automatically tapering rate of charge consists in connecting the battery, in series with a comparatively low resistance, across a constant-potential source of direct current. This is known as the *modified constant-potential system,* to distinguish it from the older " straight " constant-potential system with which no current-limiting resistance is used (§532).

Let the finishing value of current be i_f, that is, the value usually prescribed by the manufacturer at which the charge should be completed when gassing begins. Let an applied voltage of e_f volts per cell (about

2.5 volts for a lead cell) be required to maintain this finishing rate of current at the end of a charge. If the bus voltage is E and the number of cells in series is n, the excess voltage is $E - ne_f$, and the required protective resistance R is determined by the condition

$$R = (E - ne_f)/i_f \qquad (6)$$

At the beginning of a charge, the current, i_b, will be

$$i_b = (E - ne_b)/R \qquad (7)$$

where e_b is the applied voltage per cell necessary to cause the charging current i_b to flow at the beginning of a charge. For a given type of cell the relationship between i_b and e_b is approximately known.

The protective resistance R, computed from eq. (6), must give a value of current i_b not to exceed a safe limit for the cells and for the charging equipment. If the initial current is excessive, several remedies are possible; for example:

(a) A higher resistance R may be used, thus lengthening the total duration of charge.

(b) A generator, a rectifier, or some other source of current with a distinctly drooping voltage characteristic may be used, to limit the initial charging current.

(c) The protective resistance R may be reduced in one or two steps, after the initial period of charging is over.

The foregoing difficulty arises when the available source of voltage is not high enough for the battery, leaving no margin at the end of the charge. On the contrary, when the available source is too high, the protective resistance, adjusted for the proper finishing rate, will unnecessarily cut down the initial rate of current, thus lengthening the total time of charging. Moreover, the efficiency of the whole arrangement is reduced when a considerable part of the applied voltage is absorbed in a resistance. In such a case the following remedies are possible:

(d) A motor-generator set is installed to reduce the bus voltage to the best value.

(e) Two or more batteries are charged in series.

(f) The protective resistance R is increased in one or two steps after the initial period of charging.

(g) Counter emf cells may be used (§546), if the required reduction in voltage is not too great.

By the use of counter emf cells the characteristics of the modified constant-voltage method may be secured with a bus voltage somewhat higher than normal, and without manual adjustment during charge, whereas, if an ordinary rheostat is used, it must be adjusted from time to time to secure the same results.

If the line voltage varies from time to time, the number of counter emf cells in the circuit may be adjusted by means of a cell switch (§548) or a series of knife-switches, to maintain a constant voltage on the charging bus. These cells may also be used to provide a second constant-voltage bus at a lower voltage than the main bus, for charging batteries of fewer cells.

The value of the protective resistance may be changed automatically during the charge by using an additional relay contact on the ampere-hour meter. When a certain prescribed number of ampere-hours has passed through the battery this contact is closed, and a circuit-breaker is tripped or an electrically operated switch is closed in the resistor circuit, thus increasing or reducing the resistance by the desired amount.

541. EXPERIMENT 24-C. — Constant-Potential System of Charging a Storage Battery. — The method is described in §§532 and 540. The purpose of the experiment is to study the effect of different values of bus voltage and protective resistance upon the magnitude of charging current at different stages of charge. To save time, it is advisable to have three or more identical batteries, one fully charged, one discharged to the lowest allowable limit, and the others at known intermediate stages of discharge. By closing the charging circuit for a short time only, their state of charge is not materially altered during the experiment. In this way it is possible to duplicate the conditions which would obtain with one and the same battery at different stages of charge, without waiting for a battery to become charged to each particular stage.

A suitable source of direct current should be provided, such as a generator whose voltage can be varied within wide limits. An ammeter, a voltmeter, and an adjustable series rheostat are connected in the generator circuit. Each battery is provided with a separate switch by means of which it can be connected to the generator.

(1) Measure the voltage of the fully charged battery, adjust the generator voltage to be slightly above it, and connect the battery so that it is charging, with the series rheostat short-circuited. Raise the generator voltage to charge the battery at the finishing rate with the electrolyte gassing freely. Read volts and amperes. Substitute the other batteries in turn, keeping the generator voltage constant, and read the charging current. For a fully discharged battery it may be necessary to reduce the generator voltage or to introduce some protective resistance. This test is intended to illustrate the straight constant-potential method of charging.

(2) Measure the voltage of the fully discharged battery, adjust the generator voltage to be slightly above it, short-circuit the rheostat, and

connect the battery on charge. Raise the generator voltage to produce as high a charging current as the equipment will stand. Keeping the voltage at this value, substitute the fully charged battery and read the current. This test will show that with the straight constant-potential system of charging, the end of the charge is needlessly prolonged, because of a low final current.

(3) Adjust the generator voltage and the series resistance by trials until the charging current has the correct "finishing" value for the fully charged battery and does not exceed a safe value for the discharged battery. Read the currents, the voltages across the different batteries, and the voltage drop in the rheostat. This illustrates the modified constant-potential system.

(4) Select a generator voltage somewhat below that in the preceding test and take readings with the different batteries, using first a value of resistance which gives the correct finishing rate and then one which gives a proper initial current. Try the remedies (a), (b), and (c) mentioned in the preceding section, or some other remedy that the facilities on hand will permit.

Select a generator voltage somewhat above that found in test (3) and repeat test (4). Use remedies (e), (f), and (g). Take a generator voltage much above that required for the last period of charge, and repeat the test. Try a remedy that seems to be the most suitable for such a case.

Report. (1) Give the data on the straight-potential charging and explain its disadvantages and limitations. (2) Give the data on the modified constant-potential system, and estimate the saving in time as compared with charging the same battery at a constant current. (3) Apply eqs. (6) and (7) to the values of currents, voltages, and resistances found by experiment. (4) Give the data on the tests with the generator voltage below and above normal and explain the remedies which have been applied in order to reduce the total time of charging to a minimum.

542. Control of Battery Charge and Discharge. — The exact method by which the discharge rate of a battery is controlled, or by which the total output is divided between the battery and the generators, depends upon the function of the battery, its size, and the local conditions. The methods of controlling the charge also differ widely, from a simple knife-switch to a sensitive automatic booster. In the following experiments the student is given an opportunity to become familiar with a few typical systems of battery control. With these as guide he will be able to understand and to choose among many other intermediate methods of battery control met with in practice.

The various cases are treated more or less in the order of their complexity, and boosters are described separately at the end. The automatic control of an automobile battery used for starting and lighting is described in §267.

543. A Low-Voltage Battery Charged from a High-Voltage Circuit. — There are small batteries which have to be charged from sources of much higher voltage. For example, a 125-volt battery used for operating switches in an electric railway substation may have to be charged from an available 600-volt trolley circuit. A 40-volt battery used for some special purpose may have to be charged from an available 110-volt supply, etc. If power economy is not important, and a constant voltage between the low-voltage buses is not essential, the battery is simply charged in series with a resistance which absorbs the excess voltage.

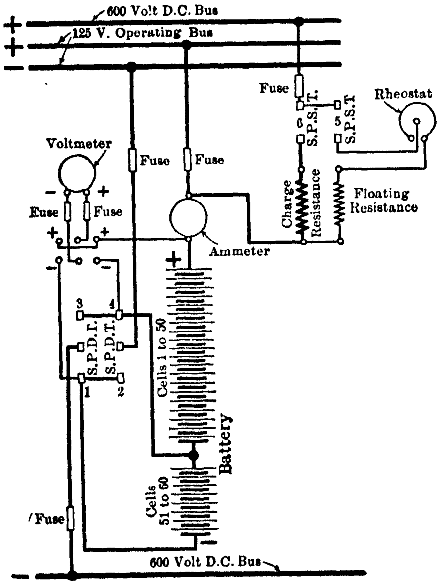

FIG. 399. A low-voltage battery connected to a high-voltage circuit through a resistance.

A typical installation is shown in Fig. 399. Two resistances, a "floating" resistance and a "charging" resistance, are shown between the 600-volt bus and the 125-volt operating bus on the positive side, with a knife-switch in circuit with each. Ordinarily the charging resistance is disconnected, while the floating resistance allows an amount of current to pass from the 600-volt bus equal to that required by the 125-volt load, plus a small "trickling" charge into the battery. It is important that this "floating" resistance be of suitable value to effect these results, and for convenient adjustment a small variable rheostat is shown in series with the fixed resistance.

When the battery is to be charged, the charging resistance is connected in parallel with the floating resistance. A group of 10 extra cells is provided, which may be used at will or cut out when fully charged. Normally the switches 1, 2, and 5 are closed. At the beginning of the charge, switches 1, 4, 5, and 6 are closed, and at the end of the charge the switches 3, 4, 5, and 6. The voltmeter reads the total battery voltage when its switch is closed to the left, and that of the cells 1 to 50 when the switch is closed to the right.

544. EXPERIMENT 24-D. — Charging a Storage Battery Through a Resistance. — The purpose of the experiment is to learn the characteristics of the circuit shown in Fig. 399. The voltages need not be the same as shown in the sketch. A 110-volt laboratory circuit can be used for the higher voltage, and a battery of comparatively few cells charged from it. A suitable load rheostat should be provided, adjustable within wide limits. It is advisable to have at least three identical batteries, one fully discharged, one fully charged, and one about half discharged. In this manner, the current and the voltage relations may be studied under different operating conditions, without waiting for a battery to undergo a complete charge and discharge. If the number of cells is sufficiently large, the same result can be accomplished by varying the number of cells in the circuit. For example, 20 nearly discharged cells in series may approximately represent about 16 charged cells, so far as the total voltage is concerned. An increase in the internal resistance of nearly discharged cells may be imitated by adding a corresponding resistance in series with the battery.

(a) Select and adjust the values of the three resistances shown in Fig. 399, to cover operation under the extreme conditions of load and charge. (b) Measure the operating bus voltage under various conditions of charge and discharge that may arise. (c) Try different numbers of extra cells (marked cells 51 to 60 in the diagram) and make clear to yourself their function in steadying the bus voltage, and the amount of charge which they receive as compared to the main cells. (d) Measure voltage fluctuations between the low-voltage buses when the voltage between the high-tension buses varies within reasonable limits.

Report. (1) Explain how the values of the three resistances were selected, and why an adjustable rheostat is needed in series with the floating resistance. (2) Give the limits of unavoidable fluctuations of the voltage between the battery buses, and explain the conditions under which each value occurs. (3) Explain the amount of charging current which the battery receives under different load conditions and at different stages of charge. (4) Give data to show the function of the extra cells, and explain which number should be recommended in the particular installation tested. (5) Give data on the effect of fluctuating voltage supply, and explain the results theoretically.

545. Charging Battery Sections in Parallel. — A simple method of charging a storage battery when the generator voltage is not sufficiently high is shown in Fig. 400. The battery is divided into two halves which are connected in series when discharging, and in parallel for charging. This is done in order to obtain a sufficient voltage for charging, without raising the line voltage maintained by the generator. An example will make this clearer. Consider a battery intended for an ordinary 110-volt lighting circuit. The voltage of each cell at the end of discharge is about 1.8 volts; therefore the required number of cells is $110 \div 1.8 = 62$. But the voltage necessary with this number of cells at the end of a charge is $2.6 \times 62 = 161$ volts, which is far above the line voltage. With the battery divided into two halves in parallel, only 80.5 volts are required for charge; the excess line voltage is absorbed in the rheostat *R*. The battery output on discharge is also regulated by the same rheostat. This method, although very simple, is used only in small installations, where the loss of power in the rheostat is not objectionable.

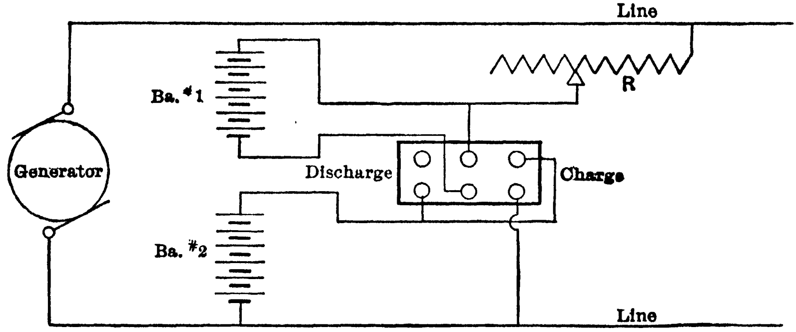

FIG. 400. Charging two halves of a storage battery in parallel.

A more economical method is to divide the battery into three equal parts; let them be denoted by the letters *A*, *B*, and *C*. First the parts *A* and *B* are charged in series, up to one-half of the total required ampere-hours; then the parts *B* and *C* are charged with an equal amount of electricity. Finally the parts *A* and *C* are connected in series and the charge is completed. Less energy is wasted in the resistances with this arrangement, although it may take longer to charge the battery. The voltage at the end of the charge is $\frac{2}{3} \times 161 = 107$ volts.

Other combinations are also possible; for example, *A* and *B* may be connected in parallel with each other and in series with *C*. The battery is charged at as high a rate as possible until *C* is completely charged. Then *C* is disconnected, *A* and *B* are connected in series, and the charge is completed.

546. Counter-Emf Cells. — In place of, or in addition to, a rheostat in series with a storage battery, counter-emf cells may be used on both charge and discharge. These cells consist of unformed lead plates in dilute sulphuric acid. Because they have no active materials, they possess no capacity for discharge. When current is passed through such a cell it offers a counter-emf of about 2.3 volts. The difference between a resistance and a counter-emf cell is that in the former the voltage drop is proportional to the current, whereas in the latter it is nearly independent of the current.

Counter-emf cells must be so connected that current always passes through them in the same direction. Otherwise, plates would be gradually "formed," like Planté plates (§526), and the cells would attain a considerable ampere-hour capacity.

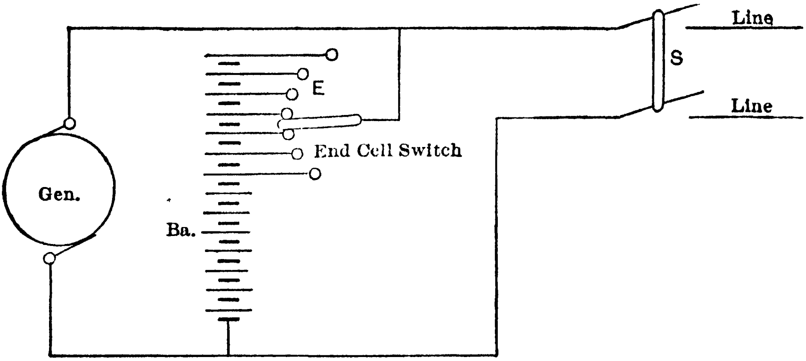

FIG. 401. Use of a single end-cell switch.

On charge, the counter-emf cells absorb the excess generator voltage. At the beginning of the discharge the same cells cut down the battery voltage while it is too high. The counter-emf cells can be connected and disconnected as a unit, or, for a closer regulation, one by one, by means of an end-cell switch (Fig. 401).

547. EXPERIMENT 24-E. — Charging a Battery in Sections. — Wire up the two halves of a battery as in Fig. 400, and make connections to a suitable generator. Ascertain the state of charge of the battery with a hydrometer, or by some other method. Provide the necessary ammeters and voltmeters, and a load in the form of adjustable resistances. Operate the installation under the following conditions:

(a) The battery and the generator supplying power to the line in parallel.

(b) The battery being charged, the generator at the same time supplying power to the line.

(c) The battery alone supplying power, the generator being shut down.

One other operating condition would be with the generator working alone, the battery being disconnected for inspection or repairs, but this need not be considered in this experiment.

For each of conditions (a), (b), and (c) select a few characteristic loads (light load, medium load, full load, and overload) and take all the necessary ammeter, voltmeter, and hydrometer readings, so as to have a complete record of the electrical relations in the circuit, with special reference to the performance of the battery. Observe voltage and current fluctuations when the load is varied, first gradually and then suddenly. Perform all these tests both with a resistance and with counter-emf cells, and observe the difference in the behavior of the two.

Devise a convenient arrangement of switches for charging the battery in three parts, as explained in §545. Connect the battery accordingly and observe the process of charging. Try a division of the battery into different sections which in your judgment will permit charging it with the least loss in the rheostat, or in a minimum time, or will have some other advantage. With the various arrangements tried, first use a regulating rheostat and then counter-emf cells; make clear to yourself the advantages and the disadvantages of both.

Report. (1) Draw the exact diagram of connections used, with all the instruments, fuses, etc. (2) Give the data on the division of the load between the battery and the generator. Estimate what this division would have been, were the battery fully charged or almost completely discharged. (3) Describe the effect of the battery when the load is changed suddenly. (4) Give the voltage drop in the rheostat when charging the battery in two and three sections, and estimate the total loss of energy in the rheostat, or in the counter-emf cells, as compared to the energy actually put into the battery. (5) Describe a better division of the battery into sections if one has been tried or suggested itself. (6) Give your opinion, substantiated by test data, as to the relative advantages of a resistance and counter-emf cells. (7) Assume the battery and the generator to represent an isolated plant; select a type of prime mover and a variable load curve of reasonable character. Write instructions as to the hours of day and night when the generator should be running, when the battery should be charging, and when it should be supplying the load alone. Also give brief instructions as to the care which the battery should receive.

548. End-Cell Switches. — In many small installations there is no demand for current during certain hours of the day. In such cases battery connections shown in Fig. 401 may be used. The battery is charged during the hours when the main switch *S* can be left open, the generator voltage being raised to the required value (say 161 volts) for charging the battery. During a discharge the battery voltage and output are regulated by the so-called *end-cell switch E*, by means of which more cells may be connected in the circuit as the voltage of each cell becomes lower.

End-cell switches are also sometimes used in installations where charging is done by means of special machines, so-called " boosters " (Fig. 405). Thus, storage batteries in stations and substations of electric lighting companies in large cities are often regulated by end-cell switches and charged by boosters. Large end-cell switches are sometimes operated by small auxiliary electric motors, which are started and stopped either by the switchboard attendant, or automatically by a contact voltmeter.

The movable contact piece of an end-cell switch must be wide enough not to open the battery circuit while the arm is moved from one stationary segment of the switch to the next. On the other hand, if the contact were allowed to bridge two adjacent segments, it would short-circuit the cell connected to these two segments, which is not permissible. Therefore, the contact piece is made in two parts, with a *protective* resistance between, this resistance limiting the current in the short-circuited cell during the instant when the arm is moved from one contact to the next.

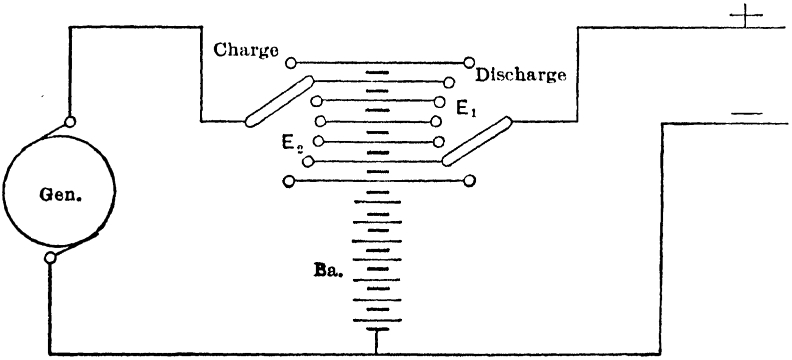

FIG. 402. Use of a double end-cell switch.

In some cases it is not practicable to have the main switch opened while the generator voltage is being raised for the charge; at the same time the size of the installation may not warrant the complication of a booster set. Two end-cell switches may be used in such cases, as shown in Fig. 402. By means of the end-cell switch E_1 the required voltage is maintained on the line, while the charging current and voltage are regulated by the generator field rheostat and the end-cell switch E_2.

With this arrangement the end cells are carrying the sum of the charging current and the line current, and are therefore charged faster than the rest of the battery. However, actual practice does not show any serious disadvantage of such an arrangement, provided that charging is done judiciously and during the hours of small demand. By actually computing the limits of voltages it will be found that more contact points are necessary with a double end-cell switch than with a single end-cell switch.

Counter-emf cells (§546) may be used as a modification of, or in addition to, the connections shown in Figs. 401 and 402. When the battery is placed on charge, the generator current is divided; part of it flows through the main battery and the rest passes through the counter-emf cells to the line. In this way the generator voltage may be raised sufficiently to charge the whole battery, and at the same time part of the generator current supplies the load at the normal voltage. As the charge progresses and the generator voltage is increased, more counter-emf cells are connected in series with the line, by means of an end-cell switch.

Figure 403 shows a complete diagram of connections for the arrangement according to Fig. 402. The switch S_1 controls the generator circuit, S_2 the battery circuit; S_3 is a double-throw switch. For charging the battery, S_3 is thrown to the right, and the switches S_1 and S_2 are closed. The charging current flows from the positive terminal of the generator, through S_3, to the charging end-cell switch E_2; thence through the battery, ammeter A_2, overload circuit breaker CB_2, underload circuit breaker CB_1, and switch S_2 to the negative terminal of the generator. At the same time, the line is supplied with current through the discharge end-cell switch E_1 and the ammeter A_1. The underload circuit breaker prevents the battery from sending current back into the generator; it can be used only in battery plants in which the load is varying slowly, so that there are distinct periods of charge and discharge.

For discharging the battery into the line, in parallel with the generator, the switch S_3 is thrown to the left; this cuts the charging end-cell switch E_2 out of the circuit, while the underload circuit breaker CB_1 is short-circuited by the lower blade of the switch S_3. The discharge is regulated by the end-cell switch E_1 and by the field rheostat of the generator. On the other hand, if it is desired to shut down the generator, the switch S_1 is opened, and the battery continues to supply current alone. This is usually done when the demand for current is quite small. If the switch S_2 is opened, instead of S_1, the battery is disconnected from the line, while the generator continues to supply power to the load.

V is a voltmeter which, by means of the switch *V Sw.*, can be connected at will to show the generator voltage, the battery voltage, or the line voltage.

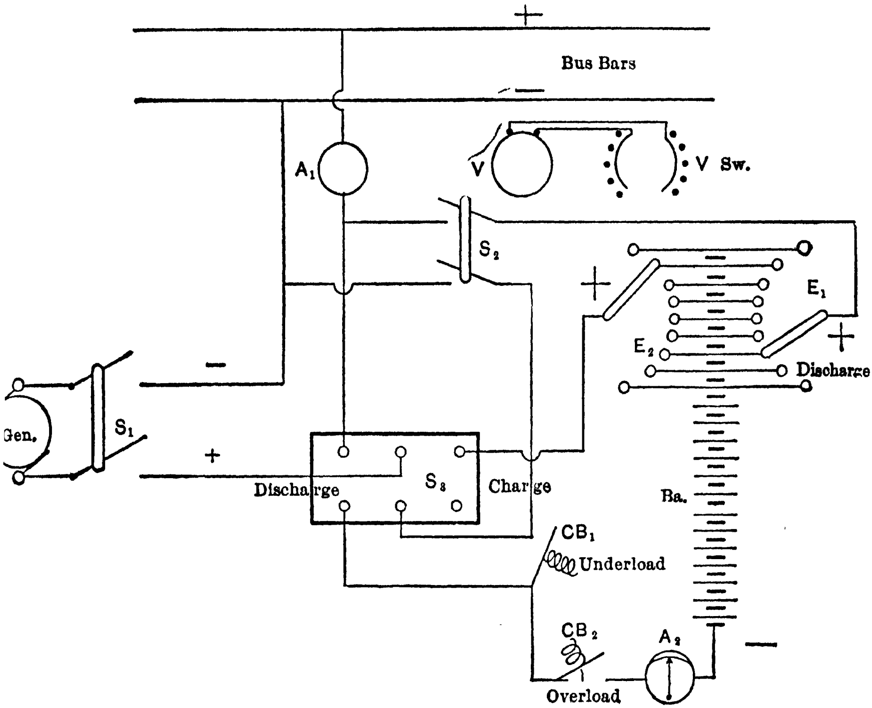

FIG. 403. Details of connections of a battery controlled by a double end-cell switch, as in Fig. 402.

549. EXPERIMENT 24-F. — Control of a Storage Battery by an End-Cell Switch. — The connections with a single end-cell switch are shown in Fig. 401 and with a double end-cell switch in Fig. 402, or more in detail in Fig. 403. Try both systems in actual operation, under the conditions specified in §547. Devise a diagram of connections with which counter-emf cells can be used, and the generator made to supply the load while charging the battery at an increased voltage.

Report the actual diagrams of connections and the numerical results of the test. State voltage fluctuations with sudden variations of the load, and give the relative fluctuations of the current in the generator and in the battery. Figure out the number of points actually necessary on the single end-cell switch and on the double end-cell switch in a practical installation.

550. Battery Control when a Variable Bus Voltage Is Permissible. — In special cases in which a generator and a storage battery are connected in parallel to a load consisting of devices which can be operated satisfactorily within rather wide limits of voltage, end cells can be used without an end-cell switch. A typical installation of this kind is shown in Fig. 404, in the form in which it is used in power stations and substations for operating remote-control oil switches, disconnecting switches, etc. The difference between this case and that shown in Fig. 399 is that here a motor-generator set is available for charging the battery.

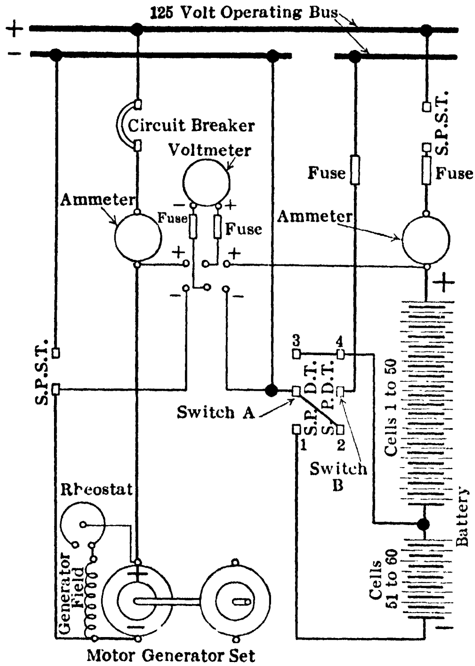

FIG. 404. Use of end cells to limit voltage variations.

In order to permit giving the battery a charge to the maximum voltage by raising the voltage of the generator, and without subjecting the connected apparatus to this high voltage, a tap is taken from the battery to the switchboard by means of which a group of 10 cells may be cut out. At the beginning of charge, the entire 60 cells are connected to the generator, whose voltage is raised sufficiently to deliver the charging current, while 50 cells are connected across the load circuit. Under these conditions the load current passes through the end-cell group, in addition to the charging current of the main battery, and the charging of the end cells is completed, therefore, before that of the main battery. The end-cell group is then cut out and the charging of the remaining 50 cells is completed. The maximum voltage of these 50 cells at the end of charge will be nearly 140 volts. The signal lamps used with oil switches are designed to stand this voltage for a short time, and the standard remote-control apparatus will operate satisfactorily at this voltage.

The negative bus is divided into two sections, and two single-pole, double-throw knife-switches, *A* and *B*, are provided, one connected to each section of the negative bus. When *A* is thrown down, the 60 cells of battery are connected across the generator terminals; and when *B* is thrown down, the two sections of the bus are connected together. Current is then furnished to the load circuit by the dynamo with the battery floating in parallel. This is the normal position of these switches.

At the beginning of the charge, switch *B* is thrown up, connecting the load circuit across 50 cells, and the voltage of the generator, which is still connected across 60 cells, is raised until the desired charging current is obtained. When the end group is fully charged, as indicated by free gassing and maximum specific gravity, switch *A* is thrown up, cutting out the end-cell group, and the charging of the main battery is completed.

551. EXPERIMENT 24-G. — Storage Battery with End Cells, Operating in Parallel with a Generator. — The connections and the method of operation are described in the preceding section. The order of the experiment and the requirements for the report are similar to those in §§544, 547, and 549, with such modifications as follow directly from the differences in the electrical connections.

552. Floating Batteries. — In power plants in which considerable voltage and load fluctuations on generators are not objectionable, or are unavoidable, storage batteries have been often used without any means for adjusting instant charge and discharge, simply as shown in Fig. 401. This allows the battery to be freely charged or discharged with the fluctuations of the bus voltage. Such " floating " batteries are used in some electric-railway substations, and also in plants containing cranes and elevators. The number of cells is so selected that when the generators give approximately their full output, the bus-bar voltage is equal to the battery emf. Under such conditions no current flows into or out of the battery. The end-cell switch is adjusted from time to time to correct for the state of charge of the battery.

The battery current at different loads depends upon the voltage characteristics of the generators. Shunt-wound generators with drooping characteristics should be used in this case. Then, with a load below the rated load, the bus voltage is higher than the battery emf, and a charging current flows into the battery. When the generators are carrying an overload the bus voltage is reduced and the battery discharges into the line, helping the generators (or rotary converters). With such an arrangement, the generator load is more nearly constant than without the battery. The battery is never entirely discharged or fully charged, but is maintained in a medium condition. Once every week or two it is necessary to give the battery a thorough charging, and even an overcharge, in order to prevent sulphation (§525).

Such a floating battery is more effective the wider the voltage fluctuations. The voltage at the end of a feeder varies much more than in the substation, on account of the ohmic drop in the feeders; therefore, it is better to have a floating battery at the end of the line. Further advantages are as follows: The average voltage being lower than in the substation, fewer cells are required; voltage fluctuations being more pronounced, the battery is charged and discharged within wider limits; the load on the generators or rotary converters is steadier; there is a considerable saving in line copper, since the feeders have to carry more nearly an average current, instead of the maximum current. The chief disadvantage of placing the battery at the farther end of a feeder is the additional space and attention required outside the substation.

Standby service. Owing to the desire always to have the battery fully charged for an emergency, and because of a comparatively rapid depreciation of the cells on load-peak service, the foregoing arrangement of allowing the battery to discharge freely during overloads has been gradually abandoned. It has been a standard practice for the lighting companies in the larger cities to provide storage batteries which are kept fully charged and connected to the bus-bars at all times, in order to insure against an interruption of service in the congested business districts. The batteries were used for no other purpose than insurance of continuity of service. In case of failure of the customary sources of energy, the energy of such a battery is instantly available to supply the deficiency, without even the necessity of throwing any switches either by hand or automatically. The batteries are capable of discharging at very high rates and therefore are not protected by any form of overload circuit-breaker, nor is any device provided for breaking the battery circuit under load. In case of any interruption of power the batteries must meet the full demand until power is restored, or until the charge in the batteries is exhausted.

Such a "standby" battery is usually provided with an end-cell switch, and is charged at regular intervals with a plain non-automatic booster (§554).

553. EXPERIMENT 24-H. — Performance of a Floating Battery. — Connect the battery in parallel with a shunt-wound generator, as in Fig. 401, and select such a number of cells that at a desired load the battery neither charges nor discharges. Provide an artificial feeder of considerable resistance so as to represent the actual conditions of railway service; the resistance being such as to give from 10 to 15 per cent voltage drop at full load. The load, representing street cars, is connected at the further end of the line. Connect an ammeter in the generator circuit, one in series with the battery, and one in the main line. Connect a voltmeter so as to measure, at will, the voltage across either end of the line.

(a) First disconnect the battery and use the generator alone, increasing the load from zero to a heavy overload, without regulating the field rheostat. Measure the line amperes and the volts at each end of the line.

(b) Apply the same values of load with the battery and the generator working in parallel. Observe the proportion of the load taken by the battery, and the improved conditions of voltage regulation. If the results show that the battery is charged more than it is discharged, add one or more cells, and vice versa. Repeat the experiment until the battery gives the best performance; in other words, keeps the generator load as steady as possible.

(c) Add a few turns of series winding on the generator field, but not enough to make the machine flat-compounded. Observe the difference in the performance of the battery, and the new voltage regulation.

(d) Connect the battery at the farther end of the line and repeat the same runs. Investigate the influence of the resistance of the line, of the position of the load on the line, and of compounding the generator.

Report. Give the results in such a form as to show the relative usefulness of the floating battery under the various conditions investigated. Point out the limitations in its use.

BATTERY BOOSTERS

554. Non-Automatic Shunt-Excited Booster. — With the exception of smaller plants in which storage batteries are charged as described in the preceding sections, most of the important battery plants are provided with so-called *boosters*, or special generators for charging batteries. The same boosters are sometimes used for regulating the discharge.

A simple non-automatic booster is shown in Fig. 405. Its armature *B* is in series with the battery and is direct-driven by a motor *M*. The fields *F* are separately excited across the bus-bars; *E* is an end-cell switch (§548).

When the battery is discharging, the switch *S* is thrown up so that the booster is cut out of the circuit; the discharge is regulated by the end-cell switch *E*. For charging the battery, the switch *S* is thrown down and the booster emf raised by means of the field rheostat *FR*,

thus giving, together with the generator pressure, a voltage sufficient for charging. As the charge progresses, this voltage is regulated by means of the same rheostat *FR*.

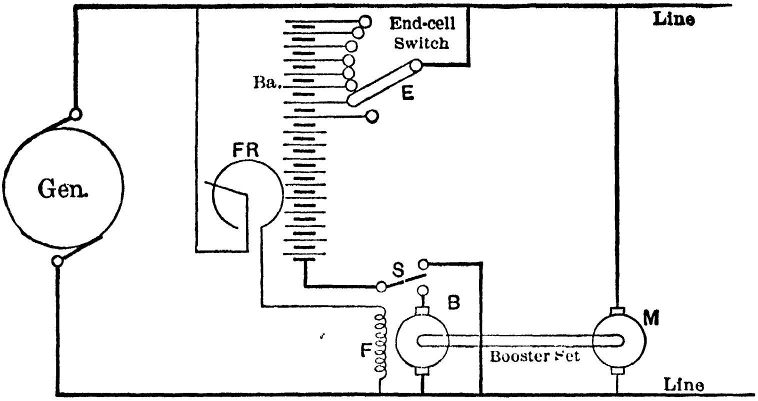

FIG. 405. Battery charged by a non-automatic booster and discharged through an end-cell switch.

Instead of using an end-cell switch, the same booster may also be used for regulating the discharge of the battery. In this case a reversing switch is connected in its field circuit, so that the direction of the induced emf may be changed. The booster is then called *reversible.* Sometimes the booster field is connected across the battery and not across the line; there is not much difference between the two methods.

555. EXPERIMENT 24-I. — Storage Battery Control by a Shunt-Wound Non-Automatic Booster. — The connections are shown in Fig. 405. Ammeters should be inserted to read the generator current, the line current, the battery current, and the booster field current. A voltmeter with a suitable switch should be provided to read the line voltage, the battery voltage, and the booster voltage. To represent changes in the battery emf that depend on the state of its charge, one or more cells may be disconnected, or more cells added to the battery. Some resistance may also be used in series with the battery, to imitate an increase in its internal resistance with discharge (§531).

(1) Assume the battery to be fully charged and the booster disconnected. Connect the battery to discharge in parallel with the generator, and set the end-cell switch to give a normal division of the load between the two. Read all the instruments and note the position of the end-cell switch. Repeat the readings under conditions corresponding to the battery half discharged and fully discharged.

(2) Connect the booster to charge the battery and again take a few sets of readings to correspond to the various stages of charge. See §§532 and 539 in regard to the desirable tapering of the charging current. Decide for yourself what amount of attention and manual regulation of the booster will be necessary during a charge under ordinary operating conditions.

(3) Add a reversing switch in the booster field, disconnect the end-cell switch, and investigate the limits, the advantages, and the disadvantages of regulating the battery discharge by means of the booster field, especially with a rapidly fluctuating load.

Report. (1) Give numerical data to illustrate the performance of the plant at different stages of battery charge and discharge. (2) Give data on the performance of the booster, both on charge and on discharge. (3) Describe the advantages and the disadvantages of using the booster, in place of the end-cell switch, for regulating the battery discharge.

556. Automatic Boosters. — The above-described manually controlled booster is used in plants in which there is a distinct period of time during which the battery can be charged. The rest of the time the battery is either discharging or is kept in reserve (standby service, §552). Sometimes, however, a battery is used for taking up instantaneous load fluctuations, with intervals of charge and discharge rapidly and irregularly following each other. Such is the case in some railway installations, in elevator service, in buildings with large printing presses, etc. In such plants it would be out of the question to regulate the booster field by hand in order to keep the generator output even approximately constant. It then becomes necessary to vary the field strength of the booster automatically, by means of a series winding in the load circuit (Fig. 406), or in the generator circuit (Figs. 409 and 410). Such boosters are generally known as automatic boosters. By properly proportioning and connecting the field windings, a great variety of performance characteristics of a battery plant can be obtained.

Of late years the tendency has been away from using storage batteries with automatic boosters for load regulation, batteries being used more as a reserve, and being charged by plain non-automatic boosters at regular intervals (§554). Nevertheless, automatic boosters deserve the student's attention in view of the fundamental principles involved and because of some of their ingenious features. Moreover, there may be a return to automatic boosters in proportion as storage cells are improved and made to stand numerous charges and discharges with less deterioration.

The following typical automatic boosters are described below:

(a) *Differentially compounded booster* (Fig. 406) in which one or two series windings are placed on the booster field poles.

(b) *Constant-current booster* (Fig. 407) which differs from the preceding one in that its armature is in series with the main generators and not with the battery.

(c) *Shunt-wound booster* with a separate series coil actuating a *carbon-pile regulator* (Fig. 409).

(d) *Shunt-wound booster* with its field controlled by a separate *counter-emf machine* (Fig. 410) excited by the main generator current.

A series winding on the booster itself is objectionable because it necessitates a somewhat larger machine, and because, in some instances, the booster field flux does not follow rapid fluctuations of the current in the series coils promptly enough. For these reasons, *externally controlled* or *relay boosters* have been developed, of which those mentioned under (c) and (d) above are the best-known examples.

With a clear understanding of the four above-mentioned typical kinds of connections, the reader should have no difficulty in comprehending other booster systems not covered in this book.

557. Differential Booster. — A booster of this type is shown in Fig. 406 and differs from the non-automatic booster (Fig. 405) in being provided with an additional series-field winding, F_2, which opposes the action of the shunt winding F_1. At a certain average load, F_1 and F_2 neutralize each other, and the booster emf is = 0. At this load the generator voltage must be made equal to that of the battery, so that the battery neither charges nor discharges. At a heavier load the action of F_2 is stronger than that of F_1, and the booster emf is added to that of the battery, causing it to discharge. On light loads the action of

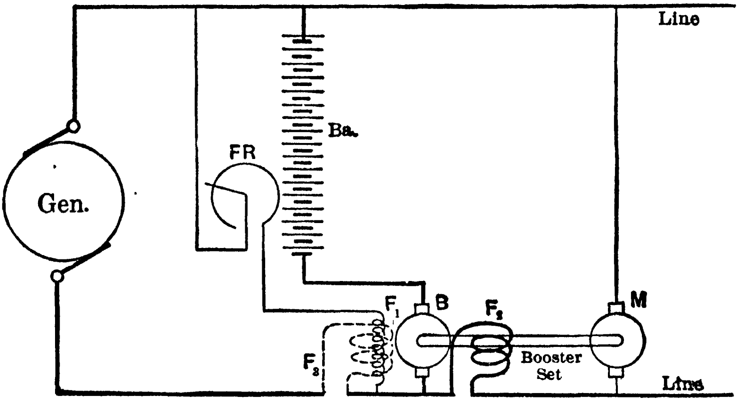

FIG. 406. Automatic differential booster.

F_1 is stronger than that of F_2, and the booster sends a charging current into the battery This action is entirely automatic, and the booster tends to maintain a nearly constant load on the generators, the battery taking up load fluctuations.

Experience shows that the action of the booster is improved and a closer regulation is obtained by adding a third field winding F_3, shown by the dotted line and acting in the same direction as F_2. This "inside" series field is particularly useful in correcting for the effect of polarization which occurs during short charges and discharges of the battery. It also corrects for variations in internal resistance due to changes of temperature as well as to changes in the state of charge of the battery.

In order to understand the action of this additional winding, first assume the battery to be very little polarized, and the current in the shunt-field winding to be so adjusted that the battery takes its proper share of the load. This means that, when the generator is delivering its assigned current, the mmf's of the three field windings of the booster are such that the battery current is zero. Now let the battery be badly polarized by a preceding heavy discharge. Then, with the same external load as before, but without F_3, the battery would supply less than its share of current, thereby overloading the generator. With the winding F_3, a heavier current delivered by the generator automatically raises the booster emf and thus causes the battery to deliver a larger current, relieving the generator. Similarly the winding F_3 allows the booster voltage to be raised in the direction of charge when the generator output is below normal.

558. Constant-Current Booster. — A booster connected as shown in Fig. 407 is used in plants in which it is desired to protect a constant-voltage load, such as the lamps L, from elevator motors X and Y, electric welders Z, etc., whose current demand is subject to violent rapid fluctuations. The voltage across the lighting load must be practically constant; a considerable drop in line voltage when a large motor is being started is not only harmless but may even be desirable.

The booster and battery are connected between the lamp load and the motor load. The booster is provided with a shunt-field winding and a series winding. The shunt field causes an emf to be induced in the same direction as that of the generator. The series field is connected to oppose it. At first, assume the load XYZ to be steady, and let the field rheostat FR of the booster be so adjusted that the generator furnishes a desired current while the battery supplies the rest. Now let the load be suddenly increased and let the first tendency in the generator be to deliver a larger current. This current, acting in the winding F_2, will lower the booster emf in the positive direction or increase it in the negative direction. The net result will be that the difference of potential between the points *p* and *q* will be reduced, and the battery will therefore be forced to take a larger share of the load.

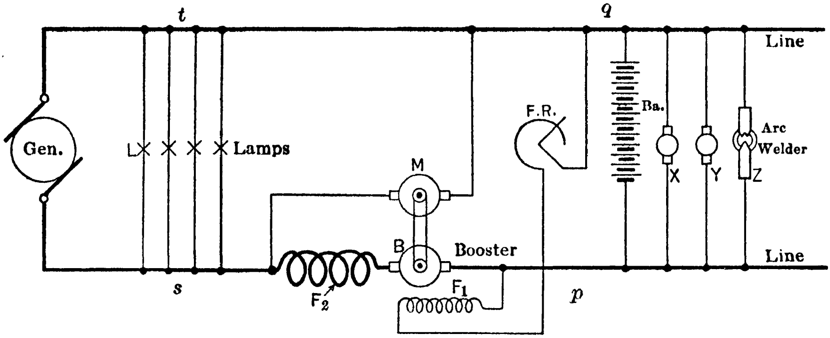

FIG. 407. A constant-current booster.

Should the load *XYZ* be reduced or disconnected, the booster voltage would rise and the voltage between the points *p* and *q* would be increased. The battery would be made to deliver a smaller current or even to charge. Thus, the generator is made to deliver a substantially constant current to a fluctuating load, and therefore it is possible to maintain a constant voltage at the lamps, between the points *s* and *t*. An end-cell switch may be used with this booster, if desired, for manual adjustment of the battery from time to time.

559. EXPERIMENT 24-J. — Operation of a Compound-Wound Booster. — The purpose of the experiment is to study the performance characteristics of the automatic boosters connected as shown in Figs. 406 and 407.

(1) Study the performance of the installation with a steady load. The booster voltage, the generator voltage, and the battery voltage are first adjusted to give the desired division of the current at some one particular load. Then this load is varied in steps from zero to a maximum, and all the currents and voltages are read at each step. This gives an idea of the closeness of the automatic regulation of the system.

(2) Change the characteristics of the battery to correspond to a different state of charge, to a different polarization, different internal resistance, etc. Take similar readings with different load currents.

(3) Change the load suddenly and measure the promptness or slowness of adjustment to the new conditions. Note the flickering, if any, of the lamps.

(4) Adjust the field strength of the booster, the number of cells of the battery, and any other variables, to obtain the best all-round average performance of the system within some assumed limits of the load and its fluctuations. Take performance data within as wide limits as possible.

Report. Give the data on the best possible performance of the booster with a variable load, and describe whatever additional adjustment by hand is necessary or desirable. State the advantages and the disadvantages of such a booster as compared to a plain non-automatic shunt-wound booster (§553), and give concrete examples of practical installations in which one or the other kind is preferable.

560. Carbon-Pile Booster Regulator. — In order to avoid placing a bulky series winding on the field poles of a booster, several external regulators have been developed, one of which is shown in Figs. 408 and 409.

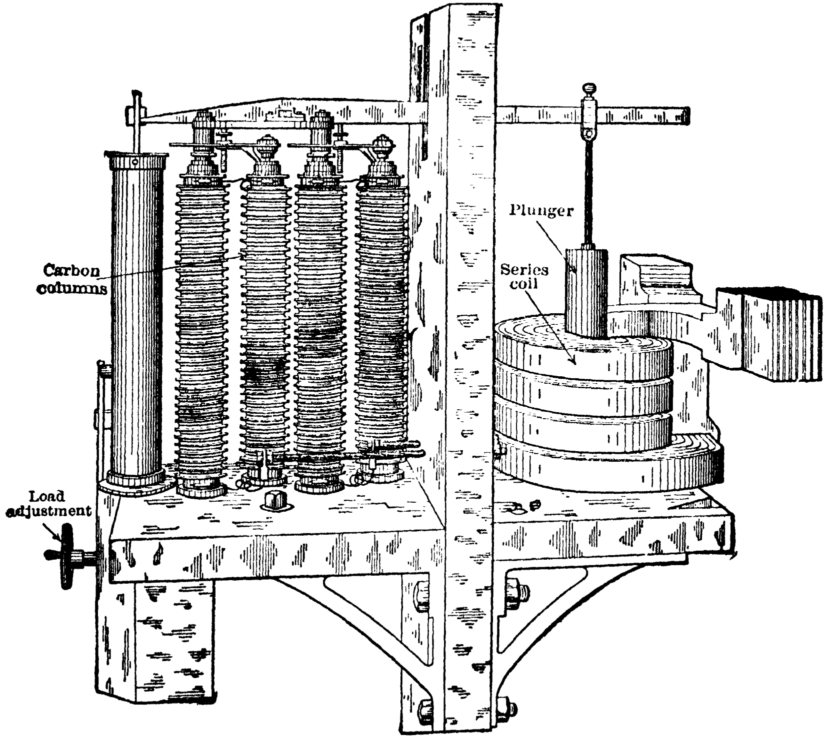

Fig. 408. Carbon-pile battery regulator.

The generator current, instead of passing through a series winding placed on the booster, passes through the solenoid *A* of the regulator.

An iron core, actuated by this solenoid, compresses through a suitable leverage two sets of columns, *CC*, consisting of carbon disks. These carbon piles are connected to the shunt field of the booster and to the battery, as shown in Fig. 409. The electrical resistance of the piles consists chiefly of the contact resistance between the disks, and therefore varies within wide limits depending upon the pressure exerted by the core of the solenoid *A*.

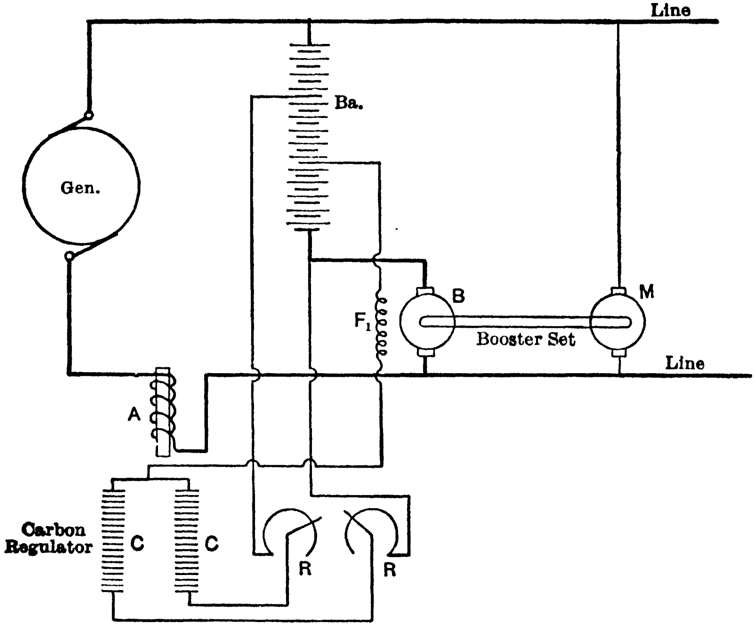

Fig. 409. Electrical connections of a booster controlled by a carbon-pile regulator.

With the normal value of the generator current, the pressure on both piles is adjusted to be the same. Their resistances being then equal, no current flows through the booster field F_1. The whole arrangement somewhat resembles the familiar Wheatstone bridge. When the generator current is below or above normal, one column is compressed more than the other, the bridge is not balanced, and a current flows through the booster field in one or the other direction, causing the battery either to charge or to discharge. The device is entirely automatic in its action, and takes the place of one or more series-field windings (§557).

In reality the arrangement is more complicated than that shown in Fig. 409. It would be too wasteful to circulate through the carbon columns a current sufficient to energize the booster field through a mere unbalancing of the resistances. Therefore, except in very small installations, *the carbon regulator actuates the field of a separate exciter, which in turn supplies current to the booster field.* The exciter being a much smaller machine than the booster, considerably less energy is lost in the regulator. The booster and the exciter are usually driven by the same motor.

To adjust the generator output. The handwheel shown in Fig. 408 serves to regulate the spring tension and thus to control the relative output of the generator and the battery. For best results the spring should be so adjusted that at any time the average load is thrown on the generator and the battery is allowed to take up only very short fluctuations and momentary peaks. Since the average load itself varies with the time, the spring tension should be adjusted at frequent intervals. In most plants it is not feasible to make such an adjustment by hand, and therefore a small electric motor is provided which automatically varies the tension of the spring. This motor is electrically interconnected with the booster armature in such a way that the generator current is made to correspond to the load current averaged over comparatively long intervals of time, while the battery is allowed to absorb momentary fluctuations. A load-limiting device is also available which prevents more load from being transferred to the generator after its safe output has been reached. Above this limit the battery is made to carry sustained peaks as well as rapid fluctuations of the load.

561. Booster Regulation by Counter Emf. — In the arrangement shown in Fig. 410 the booster shunt field F_1 is automatically regulated by a small counter-emf machine C. The field F_2 of this machine is excited by the main generator current. At a certain desired value of this current, the counter emf of the machine C can be made equal to the bus voltage so that no current will flow through the booster field F_1. When the generator current is below this value, the booster field is excited in such a direction that the battery is charged, and vice versa. CM is the motor which drives the counter-emf machine C.

The size of the counter-emf set can be considerably reduced and the sensitiveness of regulation increased by introducing a relay machine between the machine C and the booster field winding (Fig. 411). This additional machine is known as the exciter. The counter-emf machine C, instead of acting directly on the booster field, as in Fig. 410, modifies the field of the exciter *Ex.*, which in turn controls the booster field current. The armature of the counter-emf machine is connected in series with the exciter field, across the main bus-bars. When a normal current flows through the main generator, and consequently through the field F_2, the voltage induced in the exciter armature just balances that across the bus-bars; the booster excitation is then equal to zero.

When the generator current is above normal, the voltage in C is higher, and the exciter field is energized in such a direction as to assist the battery

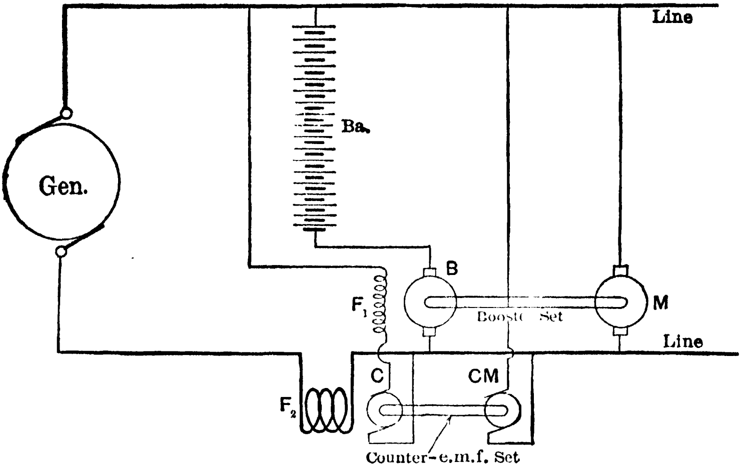

FIG. 410. Automatic booster controlled directly by a counter-emf machine.

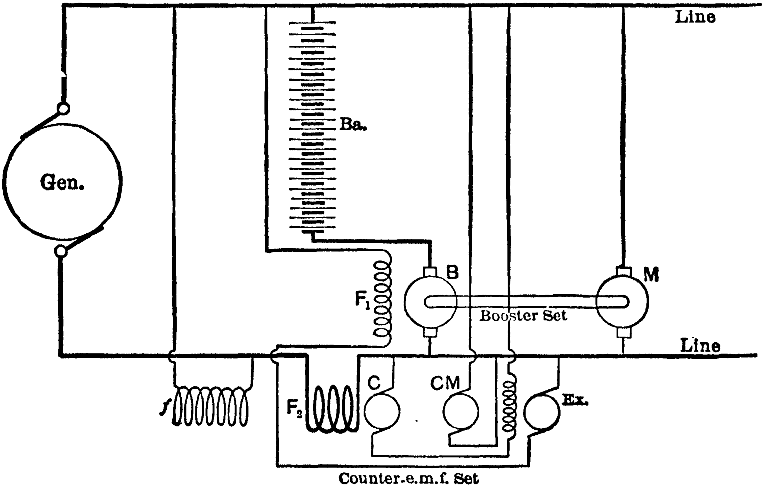

FIG. 411. Automatic booster controlled by a counter-emf machine through an exciter.

to discharge. The opposite takes place when the generator current is below normal. With proper relations, the system keeps the generator load practically constant.

The counter-emf machine may be provided with an additional field winding, f, which may be connected either directly across the bus-bars, as shown in Fig. 411, or in series with the armature C and the exciter field. The addition of this winding makes the system more flexible and permits of an adjustment for any desired performance of the battery. Further adjustment is made possible by a rheostat in series or in parallel with the field winding F_1, and also by shunting the main series-winding F_2 by adjustable resistances. The machine C is driven by the motor CM. The exciter may be driven by the same motor, or it may be a high-speed machine driven by a separate small motor.

562. EXPERIMENT 24-K. — Performance of a Relay-Operated Battery Booster. — The experiment refers to the systems of booster control such as are described in §§560 and 561. The general order of tests is the same as in §559, except that currents and voltages have to be read in more places, and there are more numerous initial adjustments to be made in order to obtain the best operating results.

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CHAPTER XXV

DIRECT-CURRENT MOTOR STARTERS AND CONTROLLERS

563. Classification of Controllers. — A motor controller is a device, or group of devices, which governs, in some predetermined manner, the electrical conditions under which the motor operates. The usual basic functions of various controllers are as follows:[1]

(a) To provide safe and proper conditions for starting a motor.
(b) To vary the motor speed.
(c) To reverse the direction of rotation.
(d) To stop the motor.

A non-reversing controller, used only for accelerating a motor to full speed, is often called a " starter." A controller used to regulate the speed of a motor is often spoken of as a " speed regulator."

According to their type of construction, controllers may be divided into the following classes:

Face-plate controllers (Figs. 412, 413, and 421).
Drum controllers of either the cylinder or the cam type (Figs. 423 and 424).
Lever-type controllers (Fig. 416).
Magnetic-contactor controllers (Figs. 418 to 420).
Liquid controllers, or liquid rheostats (Vol. II).

Controllers may also be classified according to the type of motor which they govern, such as controllers for shunt motors, for series-wound motors, for squirrel-cage induction motors, for phase-wound induction motors, etc. They may also be more definitely described by naming the principle of regulation, for example, rheostatic control, voltage control, etc.

In some controllers all the basic functions are performed by hand; such devices are called manual controllers. In other controllers all the basic functions are performed by electromagnets; such controllers are called " full-magnetic." There is also an intermediate type known as semi-magnetic. When governing functions are performed by electromagnets, an additional device is necessary whereby the operator (or a

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[1] **"Control for Direct-Current Industrial Motors," by R. T. Kintzing, *Elec. Jour.*, January, 1927.**

change in the physical conditions, such as pressure, temperature, etc.) closes a control circuit which in turn operates the electromagnets. Such a device, called a " master switch," may be in the form of a small drum controller, a float switch, a set of push-buttons, etc. Controllers can be further characterized by the adjectives automatic, semi-automatic, non-automatic, reversing, non-reversing, with or without dynamic braking, etc.

Each type of controller has its field of application. The selection of the proper controller depends on the size and type of the motor and the work to be performed by it. For reasons of safety it is necessary to enclose the live parts of all manually operated controllers. Remote controllers are usually of the magnetic-contactor type actuated by master switches which are entirely enclosed thus preventing contact with live parts. The controller itself is located at a distance and for many applications need not be enclosed.

The purpose of this chapter is to acquaint the student with the simplest and most common types of starters and controllers used with d-c motors (Chapter XII). Magnetic-contactor controllers, those for polyphase induction motors, and railway controllers are described in Vol. II. The diagrams of connections are subject to innumerable minor variations depending upon the purpose in view and the progress of the art. It is intended here merely to give a general idea of the typical connections. An exact diagram usually accompanies a controller or may be procured from the maker.

FACE-PLATE D-C MOTOR STARTERS AND SPEED REGULATORS

564. Low-Voltage Protection. — A typical open-type starter for small d-c shunt or compound motors is shown in Fig. 412, and the more modern safety-type enclosed starter in Fig. 413. It consists primarily of a resistor having a number of taps brought to buttons on the face-plate and intended to be placed in series with the motor armature during starting. The resistor is mounted in a metal box the slate or composition cover of which forms the face-plate. The starter is provided with a low-voltage protection consisting of an electromagnet which holds the starting arm in the running position as long as the coil is energized. Should the power supply be interrupted for any reason the starting arm will remain in the running position until the speed of the motor, and consequently its generated voltage, have fallen materially below normal, when the arm will be released and will return to the " off " position, under the action of the spiral spring at its pivot. The holding coil may be connected either across the line in series with a resistance, or in series with the shunt field of the motor. With the latter connection, the starting arm will be released if the field circuit becomes open, thus protecting the motor from drawing an excessive current or even running away in case of loss of field. Figures 412 and 414 show the holding coil in the field circuit and across the line, respectively. The holding coil is wound on an iron core and has two pole-pieces. The starting arm either is of iron or is provided with an iron keeper which is held by the pole-pieces, thus maintaining the arm in the running position. Before the motor is started, the starting arm is in the position shown in the sketch, and the main switch is open. To start the motor, the main switch must first be closed and then the starting arm moved slowly to the right. On the first notch all the starting resistance is connected in series with the armature, and the field circuit is connected directly across the line; as the arm is moved farther the resistance is gradually cut out until on the last notch the armature is

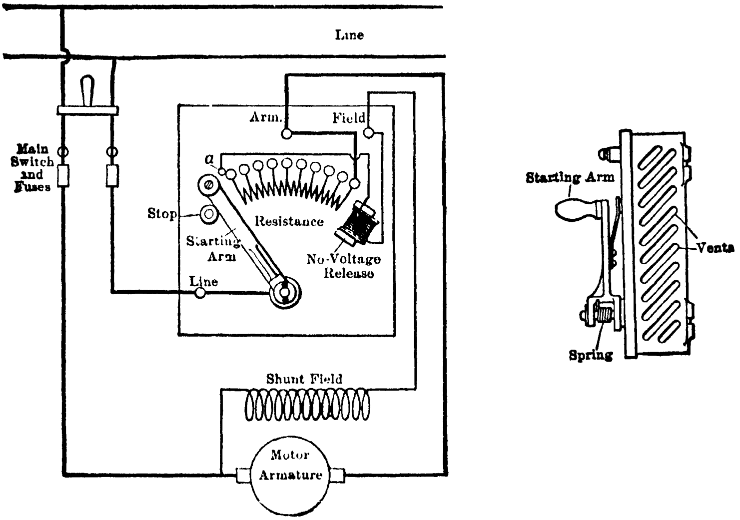

FIG. 412. Motor starter with no-voltage protection (three-point starter).

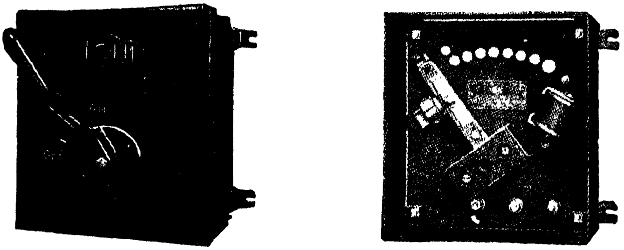

FIG. 413. Safety-type (enclosed) motor starter (Cutler-Hammer Co.).

connected directly across the line and the low resistance of the starter is now in the field circuit. In this position the starting arm is held by the release electromagnet. In starters of larger size, the starting arm is provided with a laminated copper brush which, in the running position, completely short-circuits the rheostat between two large contact pieces; these offer a much better contact than a face-plate segment.

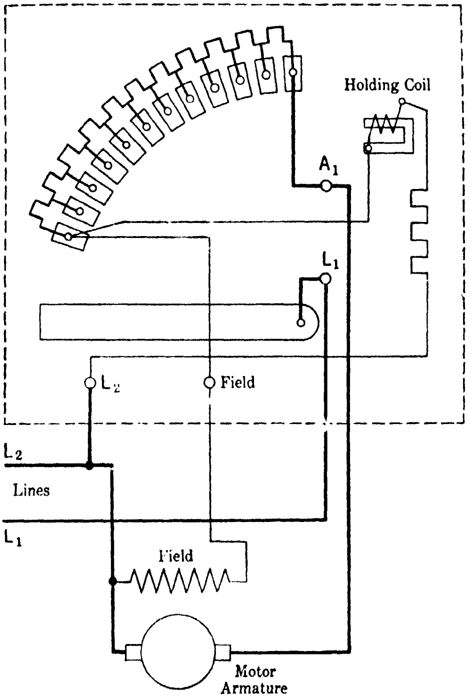

FIG. 414. Motor starter with holding coil across the line (four-point starter)

To stop the motor the main switch is opened or the circuit-breaker is tripped. This cuts off the power supply of the motor and it begins to slow down. This deënergizes the release coil, and the starting arm flies back to its zero position, under the action of the spiral spring seen in the side view to the right (Fig. 412). Whenever the motor is stopped, the whole starting resistance is *automatically* introduced in the armature circuit, protecting the motor for the next starting.

The small auxiliary contact *a*, connected to the first starting button, serves two purposes: (1) It permits the motor field to be energized a moment before the armature circuit is closed, thus allowing for the time lag due to a considerable inductance of the field winding. (2) It takes up the spark when the starting arm flies back to zero position. After the contact has been considerably burned by this spark, the button *a* can be easily replaced.

By following the electrical connections it will be seen that even with the starting arm in the " off " position the shunt field remains connected across the motor armature. This is in order to avoid sudden opening of the field and thus prevent a rise of voltage across the field coils when the circuit is ruptured at *a*. Thus, the spark is not nearly so destructive as it would be if the field circuit were also open.

In some cases, the motor field winding and the under-voltage release coils are connected independently across the line (Fig. 414), instead of being in series with each other. This is done, for example, when it would be difficult to provide the required number of ampere-turns on the coil in any other way, especially at a high voltage.

565. Overload Protection. — The motor and the starting rheostat are protected from an overload or short circuit either by fuses or by a circuit breaker. Formerly, some starting rheostats were equipped with the so-called overload coil connected in series with the line. With such a construction (Fig. 415), when the current exceeds a certain limit, the overload coil *c* attracts the iron armature *i* which strikes and closes the auxiliary contact *p*. This short-circuits the no-voltage release coil, and the starting arm flies back to "zero" position. The armature *i* may be set by means of the screw *r* to open the circuit at a desired value of the current. This form of protection is becoming obsolete, since a small coil with contacts cannot be made either as accurate or as reliable as a separate properly designed circuit breaker with a well-made time-element attachment. A separate magnetic contactor with an overload relay can also be used. This type of protection is described under magnetic contactors in Vol. II.

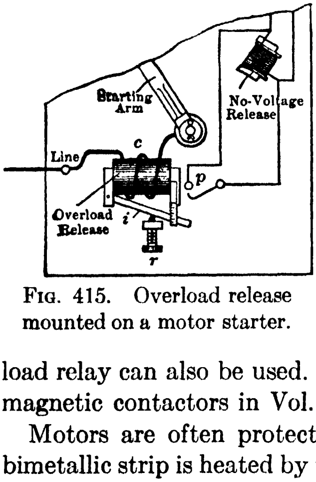

FIG. 415. Overload release mounted on a motor starter.

Motors are often protected by thermal overload relays in which a bimetallic strip is heated by the motor current at the same rate, as nearly as possible, as that at which the motor heats. Above a certain temperature this relay trips and opens the line contactor in the motor supply circuit.

566. Multiple-Switch Starters. — Face-plate starters are commonly used for motors up to about 50 hp at 230 volts. Above this the currents are of such a magnitude that it is difficult to insure a sufficiently good contact between the segments and the arm, using the type of construction shown in Figs. 412 to 414. If a manually operated starter is desired a separate switch may be used for each step, giving the multiple-switch starter of Fig. 416.

The connections are similar to those shown in Fig. 412, including automatic under-voltage protection. The switches are mutually interlocked, so that they can be closed in a certain order only. The first switch to the left, when closed, is held in position by the no-voltage coil, provided the latter is energized. This first switch pushes out of the way a stop, or interlock, which otherwise prevents the second switch from being closed. Now the second switch may be closed; this being done, the stop of the third switch is lifted up, etc. Should the power be cut off, the no-voltage coil releases the first switch, and then all the switches open at once.

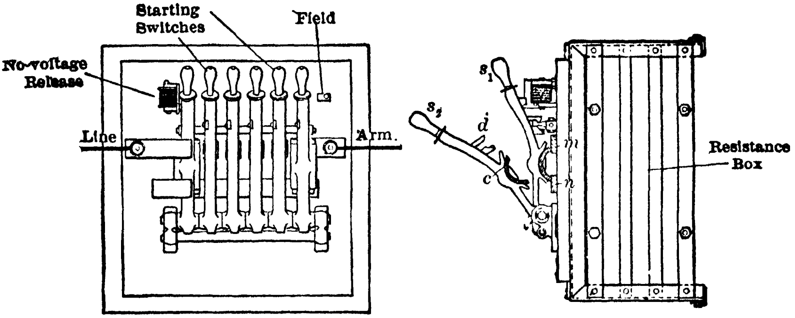

FIG. 416. Unit-switch motor starter, with no-voltage protection.

Some additional details of construction may be seen in the side view to the right. The switch s_1 is shown closed; the switch s_2, open. The switches are provided with curved copper brushes *c* which close the circuit between a bus-bar *n* and contacts *m*, connected to the sections of the starting resistance. The retaining hooks *d* and their locks are also shown in the sketch.

567. Simple Magnetic Starters. — (a) *Solenoid type.* Instead of operating a face-plate motor starter by hand, the starting arm may be acted upon by an electromagnet energized from an auxiliary circuit (Fig. 417). This is convenient when the motor and the starter are located at an inaccessible place, when it is desired to secure a rate of acceleration independent of the operator's judgment, or when the motor is started and stopped automatically by a float in a water tank, by a pressure gauge in an air reservoir. etc.

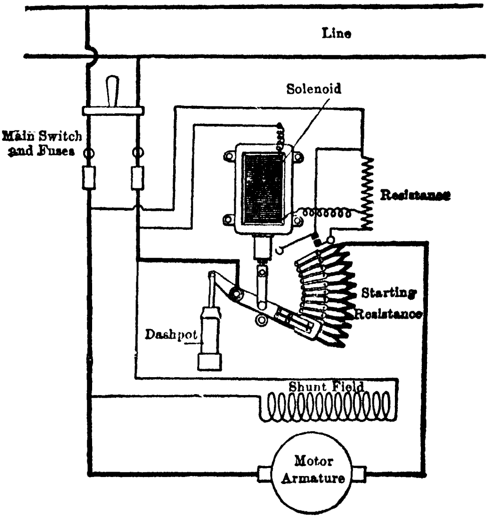

FIG. 417. A solenoid-operated motor starter.

With the arrangement shown in Fig. 417, the solenoid is energized when the main switch is closed. The iron plunger then moves the starting arm upward, cutting out the resistance. The rate at which the arm is moved is regulated by the dash-pot. At the end of the travel an auxiliary contact, shown in the sketch, is opened by the arm, and some additional resistance is introduced in the solenoid circuit. The current in the coil is thereby reduced to a value just sufficient to hold the starting arm in the upper running position. When the power is off, or the main switch is opened, the solenoid is deënergized, and the starting arm falls by gravity to its zero position.

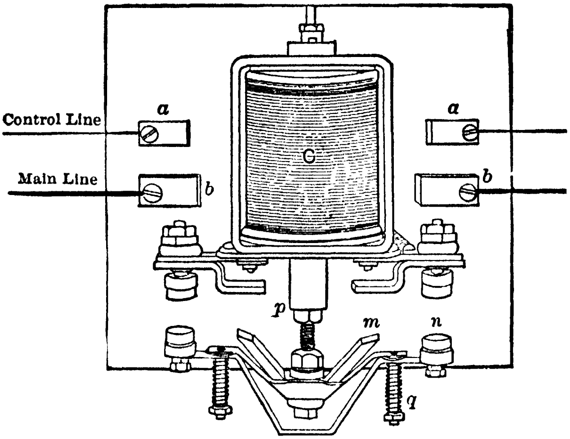

FIG. 418. An electrically operated switch for remote control.

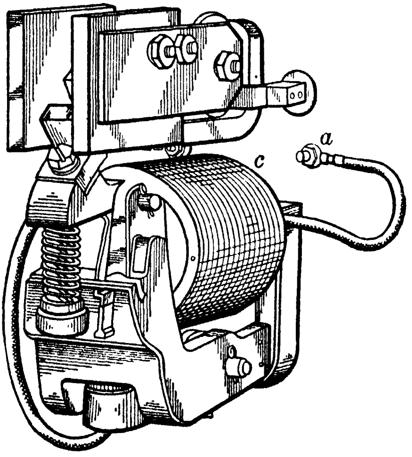

FIG. 419. A clapper-type contactor with magnetic blow-out.

It is not necessary to run the main line to the place where the operator is located. An electrically operated main switch, Figs. 418 and 419, may be placed near the starter, and only small control wires run to the operator's place. By closing a small auxiliary switch the solenoid *C* of the main switch is energized, and in turn it closes the main circuit. This energizes the starter solenoid, and the starting arm begins to move. This arrangement is sometimes used with motor-driven pumps and air compressors. The circuit *a* energizing the solenoid of the main switch is then closed or opened automatically by a float switch, or by a pressure indicator, when the water lever in the tank, or the air pressure, is beyond certain limits. This automatically starts and stops the motor, as needed.

The rate at which the rheostat steps are cut out is determined by the friction resistance of the dash-pot. The rate which is satisfactory at a light load may be too high when the motor starts on a heavy overload. To remedy this, a current-limiting relay may be inserted in the main circuit, with its contacts in the solenoid circuit. When the starting current exceeds a certain limit this relay shunts part of the operating current around the solenoid and thus weakens the magnetic pull. The motion of the lever is retarded or stopped until the main current drops sufficiently, and the relay again allows the solenoid to be energized to its full strength.

(b) *Magnetic contactor type.* A simple magnetic starter operating in a manner similar to that of the unit-switch starter (Fig. 416), except that it closes the individual switches automatically, at definite time intervals, is shown in Fig. 420. This starter consists of a multi-finger starting and accelerating contactor with a timing device, a temperature overload relay, and a starting resistor, mounted on an insulating panel and enclosed in a metal case. The operation is as follows: When a push-button or other control switch is closed, the solenoid, shown in the lower right corner of the starter, is energized and closes the line contactor (shown in the upper left of the figure). This connects the motor across the line in series with the total starting resistance, and the motor starts. At the same time tension is exerted on the remaining fingers of the starter, tending to close them. This is prevented by the timing device in which a pendulum escapement is set in motion, operating a pawl and ratchet mechanism which, depending upon its adjustment, allows the remaining fingers of the starter to close one after the other and cut out the starting resistance, thus bringing the motor up to speed, usually in 2 to 8 seconds.

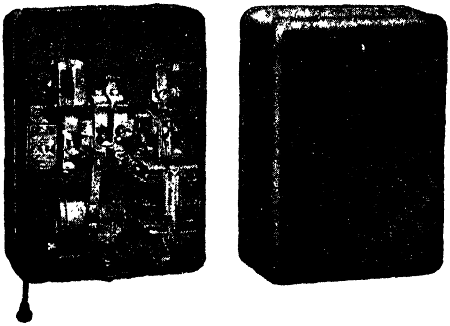

FIG. 420. A three-point automatic motor starter. (G. E. Co.)

Overload protection in this starter is provided by a hand-reset temperature overload relay, the reset handle being at the lower left corner of the starter (see §565). In case of overload this relay opens the circuit of the solenoid and allows the switches to drop out, the relay acting after a time depending upon the amount of overload on the motor.

Undervoltage protection is provided by the contactor and accelerating fingers dropping open when the line voltage drops below a certain value.

Such a starter as the one just described is suitable for motors of $\frac{3}{4}$- to 20-hp rating and, like the solenoid type, may be actuated from a distance by a push-button, pressure switch, float switch, or other automatic device.

With large motors, or when the connections are more complicated, several unit switches, such as the one in Figs. 418 or 419, are used, instead of a starting arm or the automatic unit-switch. Such magnetic controllers are described in Vol. II, Chapters LIX and LX.

568. The Determination of Proper Steps in a Starting Rheostat.—When a shunt motor of which the normal armature current is I_a is to be started by means of a series resistance the required steps in the starting resistance may be calculated as follows: Suppose that the motor must start a load amounting to normal torque with current peaks on the several starting points of pI_a, where p is the ratio of peak current to normal current. Let r_a be armature resistance and R_1, R_2, etc., be the resistance on the various points.

On the first point $pI_a(R_1 + r_a) = E$, the applied voltage, or

$$R_1 = (E/pI_a) - r_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

On this point of the starter the motor speeds up until the current falls to I_a and the counter emf reaches the value

$$E_a' = E - I_a(R_1 + r_a) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

On the second point of the starter the current again rises to pI_a, thus

$$(R_2 + r_a)pI_a = E - E_a'$$

Substituting the value of E_a' from eq. (2)

$$R_2 + r_a = I_a(R_1 + r_a)/pI_a$$

or

$$R_2 = (R_1 + r_a)/p - r_a \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Continuing this line of reasoning it may be shown that on the nth point of the starter

$$\begin{aligned} R_n &= (R_1 + r_a)/(p^{n-1}) - r_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4) \\ &= (E/pI_a)/(p^{n-1}) - r_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5) \\ &= E/(I_ap^n) - r_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6) \end{aligned}$$

The values of the resistance steps follow as the differences between the rheostat resistances on successive points. This gives, as the nth step of resistance

$$n\text{th step} = R_n - R_{n+1} = (E/I_a)(1/p^n - 1/p^{n+1})$$
$$= E/I_a\left(\frac{p-1}{p^{n+1}}\right) \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

Example: It is required to determine both the total rheostat resistance and the successive steps of resistance in a starter to be used with a 110-volt shunt motor in which $r_a = 0.06$ ohm, $I_a = 30$ amperes, and p is to be 1.5. Then, from eq. (1)

$$R_1 = (110/1.5 \times 30) - 0.06 = 2.38 \text{ ohm}$$

The first step in the starter, from eq. (7), is

$$\text{Step } 1 = 110/30(0.5/1.5^2) = 0.815 \text{ ohm}$$

The other values of starter resistance R, and of the successive steps follow in the accompanying table.

TABLE OF CALCULATED VALUES

Point No.	$(E/pI_a)/p^{n-1}$	R_n	Resistance Steps
1	2.44	2.38	0.815
2	1.625	1.565	0.542
3	1.083	1.023	0.361
4	0.722	0.662	0.241
5	0.481	0.421	0.161
6	0.320	0.260	0.107
7	0.213	0.153	0.071
8	0.142	0.082	0.047
9	0.095	0.035	0.035
10	0.063	0	. .

569. EXPERIMENT 25-A. — Study of a D-C Motor Starter with Under-Voltage Protection. — A starting box should be provided of the type shown in Fig. 412, and a suitable shunt-wound motor to be operated in connection with it; also an ammeter, a voltmeter, and means for loading the motor.

(1) Wire up the motor and the starter, and practice starting and stopping. Make clear to yourself the order in which the main switch (or an overload circuit breaker) and the rheostat handle should be operated, and to what extent the arrangement is "fool-proof"; also what would happen if the operations were performed in a wrong order.

(2) Explain why the handle does not fly back immediately after the

main switch is opened; prove the explanation by an experiment. Determine the minimum line voltage at which the coil can hold the arm. Interpose pieces of thin paper between the coil and its armature, and observe the effect on the magnetic attraction.

(3) Apply a certain brake load and start the motor by moving the rheostat arm at a certain definite speed. Every few seconds read instantaneous values of line amperes and volts across the armature. With three observers, after some practice, it is possible to take readings on an instrument every two seconds. A much better way is to use a high-speed recording ammeter and voltmeter (§64). Repeat the experiment with different rates of starting, and with loads of different magnitude.

(4) Measure the total starting resistance and the resistances of separate steps. This may be done by applying a steady current and measuring the voltage drop between adjacent segments.

(5) If an electrically operated starter is available, such as shown in Fig. 417 or 420, connect it up and operate, in order to observe its action. Take a few readings as before, to characterize the performance of the device quantitatively. Study in particular the dash-pot adjustment, the influence of the kind of dash-pot oil and its temperature, the effect of an abnormal line voltage, of the motor load, special timing device, etc.

Report. Draw diagrams of the actual connections used during the experiment. Explain your findings concerning the operation of the under-voltage release coil. Give curves showing the variation of motor volts and amperes during the period of starting; show the influence of the load and of the rate at which resistance steps were cut out. Plot the resistance of the steps and the total resistance of the rheostat, to steps as abscissas. Explain the theoretical reason for which the resistances of various steps are made different. Following the method of §568, work out proper resistances for the steps in a new starting rheostat to be designed for a given motor. Give your findings on the operation of the solenoid starter.

570. Speed Reduction by Armature Resistance. — With small motors, when economy of operation is not essential, the speed may be reduced by inserting some resistance in series with the armature (§281). This is also the usual method of control in intermittent service, for example, on cranes, hoists, electric cars, etc.

With small motors, a face-plate controller is used similar to the one shown in Fig. 412, except that the resistor must be designed for continuous service, and a notched sector holds the starting arm indefinitely in any position. The release coil has a pivoted armature carrying a hook on one end. This hook engages in any of the slots of the sector, and holds the arm in a desired position as long as the coil is energized.

When the power is off the armature is released and the spiral spring returns the arm to the "off" position. If no such interlock is provided, a separate under-voltage circuit breaker should be used to protect the motor should the power be off.

A reversing controller for a series motor is shown in Fig. 421; it is of a type used with small hoists. No interlocking or automatic features are required in this case beyond a separate overload circuit breaker, since the operator watches the performance constantly.

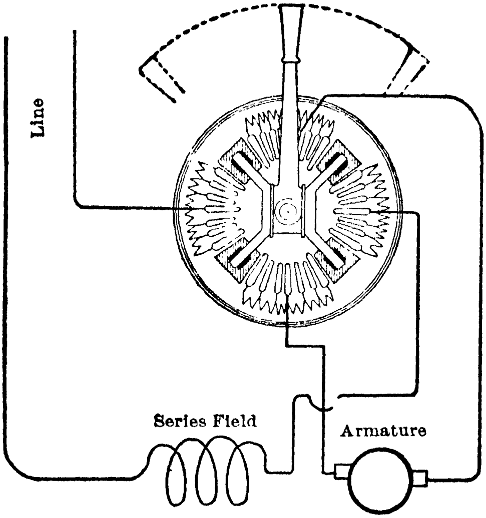

FIG. 421. A face-plate controller for a series-wound motor.

The regulating lever is made in three pieces separated by strips of insulation. The two outside parts form electrical connections; the middle part is insulated. The function of the controller consists in cutting the resistances in and out. Reversing is done by moving the handle in the opposite direction; this reverses the armature leads.

Magnetic blow-out coils (§578) are placed either on the contact arms or on the controller shaft. In the latter case, the iron of the framework is so designed as to complete the magnetic circuit and to give a blow-out action at the four points at which the circuit is broken.

Only one half of the regulating resistances are used at any one time; therefore one half of them may be omitted and the other half connected to both "forward" and "reverse" contacts. On the other hand, it is of some advantage to use separate resistances, because then they are in the circuit only half of the time and have more time to cool off.

In a rheostat intended for speed control by resistance in the armature circuit, the absolute and the relative values of the resistances of the individual steps depend essentially upon the character of the service which the motor has to perform. Two typical kinds of service are distinguished in industrial practice, viz., the *fan service* and the *machine service*, better called the increasing torque-speed service and the constant torque service. In a fan, the required torque decreases rapidly as the speed is reduced, and vice versa, on account of the nature of air resistance. In a lathe, on the other hand, the torque with a given cut remains essentially constant within a wide range of speeds. The ampere-speed curves in the two cases are entirely different, and so are the values of voltage drop in the series rheostat necessary to produce the same variation in speed.

571. EXPERIMENT 25-B. — Rheostatic Speed Control of a D-C Motor. — The underlying principles are explained in the preceding section. Particular attention should be paid to the proportioning of the individual resistance steps for different types of motors, shunt, series, and compound-wound, and for different classes of service. The student should clearly see that with any particular setting of the rheostat the motor speed does not remain constant as the load varies, because the voltage drop in the rheostat is a function of the armature current. When experimenting with a series motor it is advisable to have an underload circuit breaker in the line so as to protect the motor against running away on a light load.

572. Field Control. — In the above-described starting rheostats, no provision is made for regulating the field current of the motor, to vary its speed. If speed control is required, an additional rheostat must be connected in the field circuit (Figs. 228 and 422). The motor should always be started with as strong a field as possible, in order to get a

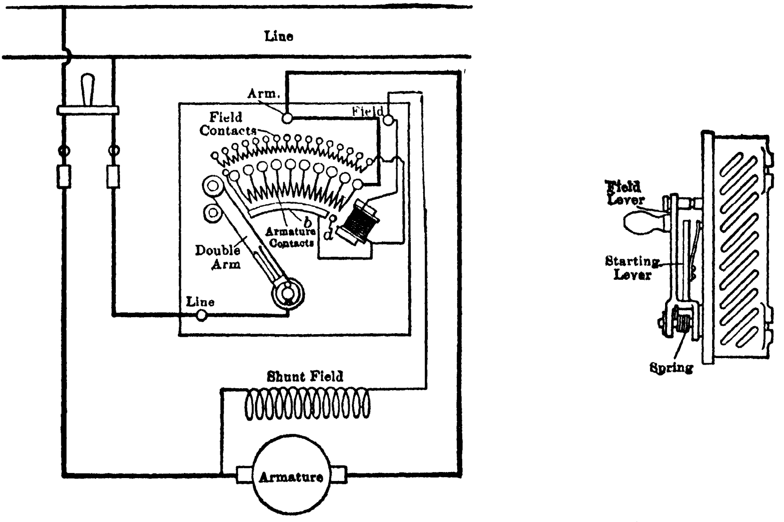

FIG. 422. A combination motor starter and speed regulator.

good starting torque, without an excessive rush of armature current. Therefore, the starting and the field rheostats must be suitably interlocked, either mechanically or electrically.

In the controller shown in Fig. 422 the lower row of contacts is connected to the starting resistance and the upper row to the field rheostat. A double lever is provided, the outside arm being for the field contacts and the inside one for the starting contacts (see side view to the right). Only the outside arm is provided with an operating handle. In starting, the two arms are moved together, but the field arm is electrically inoperative, because the field current flows directly through the starting level, the bar *b*, the under-voltage protection coil, and the motor field winding. At the end of the starting period, the starting lever is attracted and held by the under-voltage protection magnet, while the field lever may be moved back to increase the speed of the motor. The upper row of contacts is now operative, since the starting lever no longer touches the short-circuiting bar *b*, but rests on the blind segment *d*.

When the main switch is opened, the starting lever is released, and on its way back it strikes the field lever, so that both arms are returned simultaneously to the " zero " position. It will be clearly seen from the above description that it is impossible to start the motor with a weakened field.

In some cases it is preferred to have the field rheostat separate from the starter. In one device of this kind the field rheostat is kept normally short-circuited by the armature of an electromagnet; this insures the starting of the motor with full field. The short circuit is not removed by energizing this electromagnet, since the armature is too far from it to be attracted. But when the field-regulating arm is brought to its " zero " position, corresponding to the strongest field, it pushes the armature against the electromagnet, and removes the short circuit. After this, any desired amount of resistance may be added to the field circuit, and the motor speed increased. As soon as the main switch is opened, the armature of the field rheostat is again released and the field rheostat short-circuited, independent of the position of its operating arm.

573. Field-Accelerating and Decelerating Relays. — Besides the two methods for insuring the starting of the motor with a full field, described in the preceding section, an independent relay may be used in the main circuit with its contacts across the field rheostat. As long as the current through the motor armature exceeds the limit for which the relay is set, the field rheostat is automatically short-circuited, no matter in what position its regulating lever may be. When the current drops below that limit the relay again becomes inoperative. An advantage of this so-called *field-accelerating relay* is that it protects the motor not only during the starting period, but also in regular operation, should the motor become overloaded. The relay causes the motor to operate with the full field, that is, at the lowest possible speed and highest torque, until the overload has been removed.

The field-accelerating relay, if used alone, without the so-called *field-decelerating relay*, may under certain circumstances cause the following difficulty: Let the motor be running at its maximum speed and let the rheostat be turned rapidly to an intermediate value. The motor field will increase, reversing the current in the armature and causing it to act as a generator, until the speed of the motor has decreased to a balanced value. A relatively small increase in the field strength will usually cause a heavy current to flow, as there is very little resistance in circuit. This excessive current causes the armature of the field-accelerating relay to lift and short-circuit the balance of the field rheostat, thus materially increasing the regenerative effect. The action of the relay under these circumstances is undesirable. In such cases a decelerating relay should also be used for reversing the action of the accelerating relay on regeneration.

The decelerating relay limits the rate of retardation of the motor by preventing too sudden a strengthening of the field due to manipulation of the field rheostat or by other means. It has a compound winding; the series winding is connected in the armature circuit, and the shunt winding across the line or armature terminals. During accelerating periods of the motor the decelerating relay will not operate, as the directions of current through the series and shunt windings oppose each other. However, when the field strength is increased to a sufficient extent to produce regenerative action and the armature current is reversed, the series and shunt coils aid each other, producing a resultant flux large enough to separate the relay contacts, and reinsert the field resistance momentarily. This results in a fluttering action which may occur several times until the motor is operating at normal speed with the rheostat again short-circuited.

574. EXPERIMENT 25-C. — Study of a Field-Control Speed Regulator for a D-C Motor. — The purpose of the experiment is to investigate the performance of a face-plate controller, such as described in the preceding two sections. The characteristics of the motor itself are covered in §275.

(1) Connect the speed regulator to a motor and practice operating it; make clear to yourself the automatic features of the device.

(2) Measure the speed and the field current of the motor with several positions of the regulating handle.

(3) Measure the total resistance of the field rheostat, and the resistances of the separate steps; also the resistance of the motor field.

(4) Connect an ordinary motor starter (Fig. 412) and a separate field rheostat in place of the combination starter and regulator. Devise an electrical or mechanical interlocking arrangement which would prevent starting the motor with weakened field.

(5) Provide an overload relay to short-circuit the field rheostat; experiment with its operation and with the adjustment of current setting and the time element. Investigate the reversal of the armature current on regeneration and try the corrective action of a field-decelerating relay.

Report the connections used, and the automatic features of the devices tested. Plot curves of motor speed, field current, and total resistance in the field circuit, to rheostat steps as abscissas. Describe a method of proportioning the individual resistances of a proposed field rheostat for a given motor, to give desired speed steps at different loads.

575. EXPERIMENT 25-D. — Overload Protection of a D-C Shunt or Compound-Wound Motor. — The purpose of the experiment is to investigate the relative advantages and disadvantages of the various means for protecting a motor against sudden or sustained overloads. The principal methods in use are enumerated in §565. Test the principal types of controllers described in §§564 to 573. For each device tested ascertain the following features: (a) accuracy of setting in amperes; (b) the time element as a function of current with sudden and with sustained overloads; (c) constancy and reliability in operation under severe industrial conditions of dirt, heat, vibration, rust, etc.; (d) delay in restoring the service; (e) relative expense; (f) immunity against ignorance or malicious tampering. In some of these tests the actual motor may be dispensed with and replaced by an ordinary load rheostat, after the overload characteristics of the motor have been ascertained.

Report. Describe the devices tested; give the numerical data obtained and the characteristic features (a) to (f). Indicate the advantages and the disadvantages of each method and its particular field of application.

THE DRUM-TYPE CONTROLLER

576. One of the most common types of manual controllers is the drum-type controller; it is quite widely used with individually driven machine-tools, on small cranes and hoists, on electric cars, and in many other industrial applications. It is used either for governing the main currents, or as a master switch for control currents, when the main controller is of the magnetic-contactor type. There are two types of drum controllers, the cam type (Fig. 423) and the cylinder type (Fig. 424).

In a cam controller the individual contactors are mounted on stationary vertical posts, and the corresponding cams are fastened to a vertical shaft operated by a handle. When the shaft is turned, individual cams force the contactors to close in the desired sequence. The shape of the cams is such that the contacts are made and broken quickly, to prevent burning of the contactor tips. Blow-out coils and arc shields (§578) are provided at the contactors at which the circuit is finally broken.

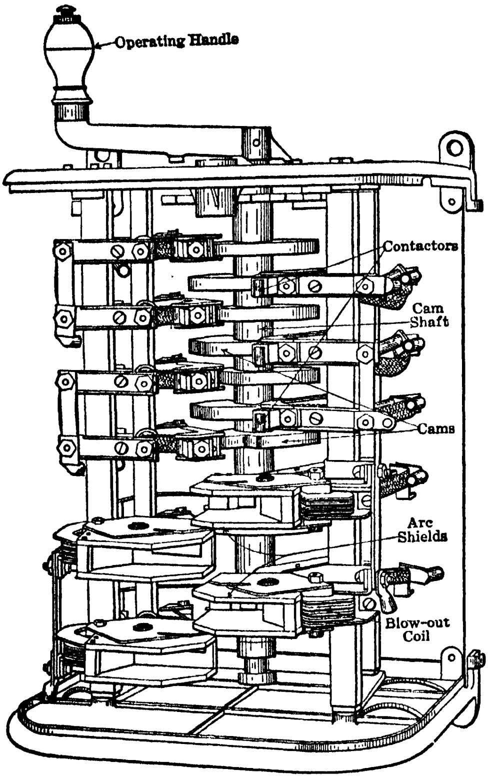

FIG. 423. A cam-type drum controller.

In a cylinder-type drum controller (Figs. 424 and 58), the connections are made between stationary fingers and movable copper strips or segments mounted on the shaft operated by a handle. A controller of this kind is described in detail in the next sections.

577. Mechanical Features of a Drum Controller.—The drum-type controller shown in Fig. 424 is one that has been quite widely used on street cars, and though intended for two series-motors, is representative of the general construction of drum controllers for other types of motors.

The wires coming from the line, from the motors, and from the starting and regulating resistances, are all connected to stationary "fingers," and these are interconnected in various combinations by the copper segments mounted on the revolving drum. The drum is operated by a handle; in each position of the handle various fingers are connected in a different way, so as to vary the speed of the motor, the direction of rotation, etc. However complicated the connections inside the controller may be, the operator does not need to know about them; he simply turns the handle in one or the other direction, guided by the indications marked on the cover. A notched wheel is shown on top of the drum, under the cover. A roller is forced into one of these notches by a spring when proper contacts are made inside, thus indicating to the operator the correct running positions.

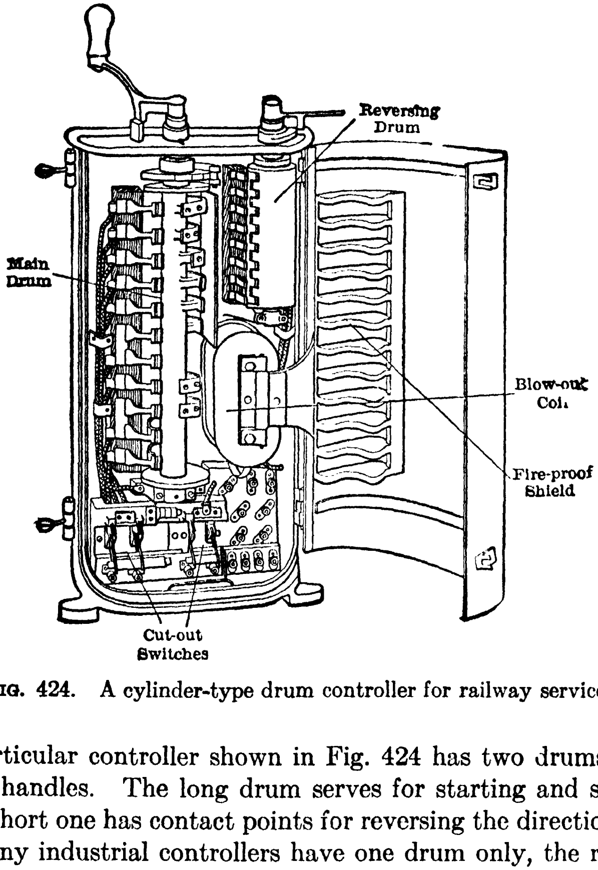

FIG. 424. A cylinder-type drum controller for railway service.

The particular controller shown in Fig. 424 has two drums and two operating handles. The long drum serves for starting and speed control; the short one has contact points for reversing the direction of rotation. Many industrial controllers have one drum only, the reversal of rotation of the motor being obtained by turning the handle from the " off " point in the opposite direction.

The two handles are mechanically interlocked in such a way that the reversing handle can be operated only when the regulating handle is in the " off " position. Moreover, the regulating handle can be operated only when the reverse handle is either in the " forward " or in the " reverse " position, but not in the " off " position. The purpose of this interlocking is to insure that the last contact is opened and the arc drawn in a proper place on the regulating drum, where a magnetic blow-out field is provided for extinguishing it.

578. The Magnetic Blow-Out. — In a controller which is operated many times a day, some fingers and the corresponding contact strips would soon become burned and pitted by the action of the electric arc when the circuit is ruptured, if means were not provided to reduce sparking.

The usual remedy is a blow-out coil, such as is shown in Figs. 423 and 424. Its action is based upon a fundamental fact of electromagnetism (Fig. 425) that in a magnetic field an electric current tends to move across the lines of force, or, as it is sometimes expressed, to cut them.

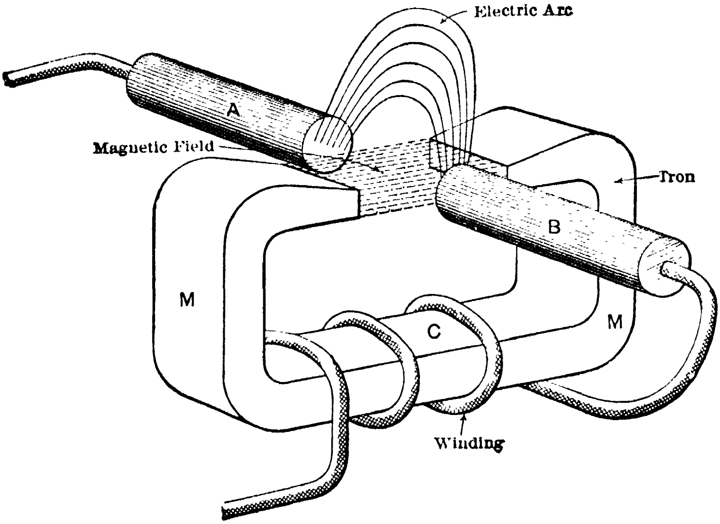

FIG. 425. The action of a magnetic blow-out.

The pole-pieces, *MM*, are so placed that the general direction of the lines of force is at right angles to the movement of ions in the arc between the electrodes *A* and *B*. The electromagnetic action tends to move the arc outside the field; the length of the arc is thereby increased and it is promptly ruptured. The coil of a blow-out magnet is usually energized in series with the line, so that the arc-expelling action is stronger the larger the current to be ruptured. For small shunted arcs, permanent magnets are sometimes used.

The proper design of the blow-out is very important. The magnetic field must have the correct strength and distribution to reduce the wear on the contacts and to prevent the arc from flashing over to other parts of the device. The arcing parts are separated from the rest by refractory fireproof partitions, shields, boxes, etc., whose shape depends upon the general design of the controller.

In some types of controllers so-called *arc-splitters* are used in connection with the blow-out magnet. The function of these splitters is to increase the length of the arc, without having it projected much beyond the edge of the arc box. These splitters also cool the arc, which materially assists in rupturing it. These arc boxes are used with magnetic-contactor control and also with drum controllers.

579. Elementary Controller Connections. — Before attempting to read actual diagrams of connections of drum controllers, such as those shown in Figs. 430 and 431, it is advisable for the beginner to become acquainted with the general principles underlying such diagrams. For this purpose, four elementary diagrams are shown below, each illustrating one kind of control only, viz.:

(1) Cutting out the starting resistance in steps (Fig. 426).

(2) Speed control by the field rheostat (Fig. 427).

(3) Reversing the armature terminals for a change in the direction of rotation (Fig. 428).

(4) Change from one terminal voltage to another (Fig. 429).

In all these diagrams the surface of the controller drum is assumed to be developed in a plane. Different fingers are marked *a*, *b*, *c*, etc.; the drum segments are denoted by the letters *x*, *y*, *z*, *u*, and *v*. It will be seen that in most cases the segments are electrically interconnected and their combination acts in the circuit as one piece of metal. Different positions of the controller fingers on the segments are indicated by the dotted vertical lines numbered 1, 2, 3, etc.

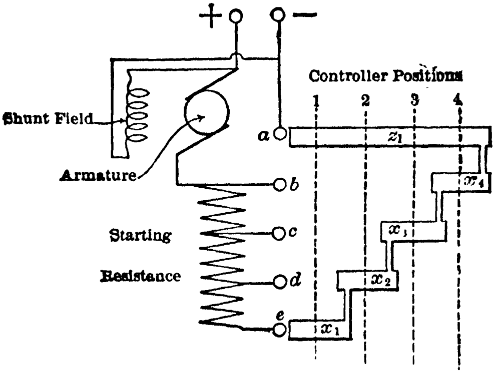

Fig. 426. Connections to the starting resistance.

In Fig. 426 the starting connections only are shown. On the first notch the fingers *a*, *b*, *c*, *d*, *e* must be imagined as placed on the dotted line 1. It will be seen that the line current from the positive terminal passes through the armature and the whole starting resistance to finger *e*. From there it passes through all the drum segments to the finger *a* and out. On the second notch the finger d touches the strip x_2, and the section de of the starting resistance is cut out. On the third notch still more resistance is cut out, and finally on the fourth notch the current flows through the armature without any starting resistance in series; this is the running position of the drum.

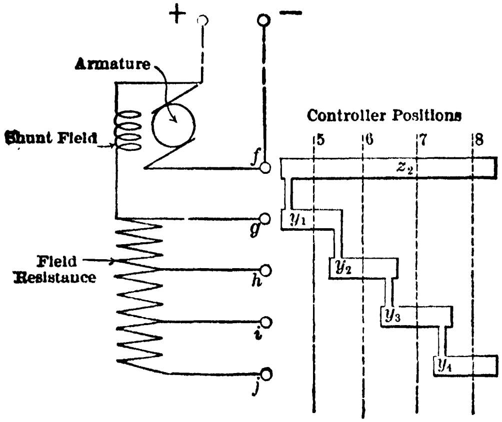

FIG. 427. Connections to the field resistance.

Figure 427 represents controller connections for speed control by means of a variable resistance in the field circuit; for the sake of clearness the starting connections are omitted. On the fifth notch the field is excited directly across the line, without any resistance in series with it. This gives the strongest field, and therefore the lowest speed. On the sixth notch the resistance between the fingers g and h is inserted into the circuit, on the next notch that between g and i etc., until on the last notch the whole field resistance is in the circuit, and the motor runs at its highest speed.

Connections for reversing the motor are shown in Fig. 428. When the drum is in the " Forward " position, the current from the positive

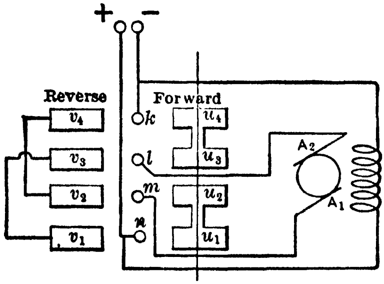

FIG. 428. Connections for reversing the armature.

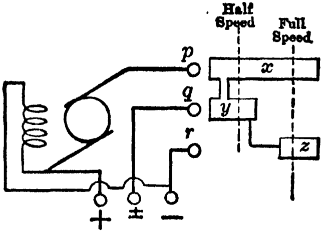

FIG. 429. Three-wire connections.

terminal flows through n, u_1, u_2, and m to the armature terminal A_1, and thence returns to the line through the terminal A_2. When the controller handle is in the " Reverse " position, the current passes through n, v_1, v_3, and l to the armature terminal A_2, thus flowing through the armature in the opposite direction. Therefore, the motor will run in the opposite direction, since the field connections are not reversed.

Controller connections for operating a motor on a three-wire system (§§262 and 285) are shown in Fig. 429. In the position marked " Half speed " the armature is connected between the positive and the neutral (±) wires; at full speed it is connected between the positive and the negative terminals.

580. Experimental Controller. — In studying the connections and experimenting with an actual drum controller, the student is handicapped by the fact that the controller is all wired up, and some of the wiring is not accessible. Moreover, the controller is usually intended for a specific duty only, and cannot very well be used for various purposes.

It is therefore advisable to have in the laboratory an *experimental* controller, especially adapted for exercises in wiring. No permanent connections should be made between the strips on the drum, but each strip should be provided with one or more binding posts so that the student himself may establish any desired connections. Some of the segments must be long, others short; and they must be arranged stepwise, for gradually cutting in or out of resistances. Such a controller, if properly designed, is very useful for a study of the operations explained in §579.

The controller should be mounted horizontally in order to be more accessible, and should have no cover, but only a board on which the fingers are mounted. It is not advisable to have a blow-out coil in connection with it, as the device should be kept as simple as possible. The action of a magnetic blow-out should be studied on a separate electromagnet.

With such a controller the student can realize separately the connections shown in Figs. 426 to 429, and check them by actually operating a suitable motor. Then he can wire the same controller for a combination of starting and field control, starting and reversing, dynamic braking (§293), or for any desired combination of standard and special operations.

581. EXPERIMENT 25-E. — Wiring of an Experimental Drum-Type Controller. — The purpose of the experiment is explained in the preceding section. The fundamental connections are described in §579. At the end of the experiment all the connections should be removed in order that the next group of students may have the benefit of designing their own connections.

Observe separately the action of a blow-out coil in a circuit containing considerable inductance, by interrupting the circuit at a suitable place. At first use a separately excited electromagnet, or a permanent magnet, and observe its action on the arc. Reverse the direction of the main current, also the polarity of the magnet, and in each case notice the effect. Then take a regular blow-out coil, connected in series with the main circuit, and investigate its action. When experimenting with a blow-out coil the student must be careful to have the wires well insulated. The emf of self-induction, when the circuit is opened, may give quite a dangerous shock.

Report. Give diagrams of the actual connections used. Describe the tests with the blow-out coil, and show on a sketch where and how a magnetic blow-out could be applied to the controller used in the experiment.

582. Examples of Industrial Controllers. — With a clear understanding of the above simple diagrams, the connections in Figs. 430 and 431 can be followed through with little or no difficulty. Figure 430 represents the connections in a reversing controller. The motor shown is compound-wound, but the series field has nothing to do with the operation of the controller, which can be also used with a shunt-wound motor. The controller has two starting positions, 1 and 2, in which the armature resistance is cut out in steps; the other positions are for field control.

The line wires, the armature leads, and one of the field terminals are connected to the fingers corresponding to the long strips, and are in the circuit in all positions. The " Forward " strips and the " Reverse " strips are so connected that the current flows through the armature in the opposite directions. The shorter upper strips are for cutting out the starting resistance in steps. The lower diagonal strips are for gradually introducing resistance into the field. Fewer steps are provided on the " Reverse " to illustrate the possibility of so doing, although in a standard controller identical steps would usually be provided for both directions of rotation.

The controller shown in Fig. 431 performs essentially the same functions as the preceding one, but has a somewhat different arrangement of contacts. It can also be used for dynamic braking (§293). In the lower right-hand corner a protective panel is shown consisting of a single-pole overload circuit breaker and a clapper-type contactor, automatically operated by an electromagnet.

Assume both the circuit breaker and the contactor to be open and the controller in the " off " position shown in the diagram. If the operator closes the circuit breaker by hand, a current will flow through the path $L_1M_1N_1zqQC$, and through the contactor coil to L_2. The contactor armature is attracted and the main circuit is closed between L_2 and M_2. Simultaneously with this motion, the relay contacts are closed, so that the contactor coil is energized directly between M_1 and L_2.

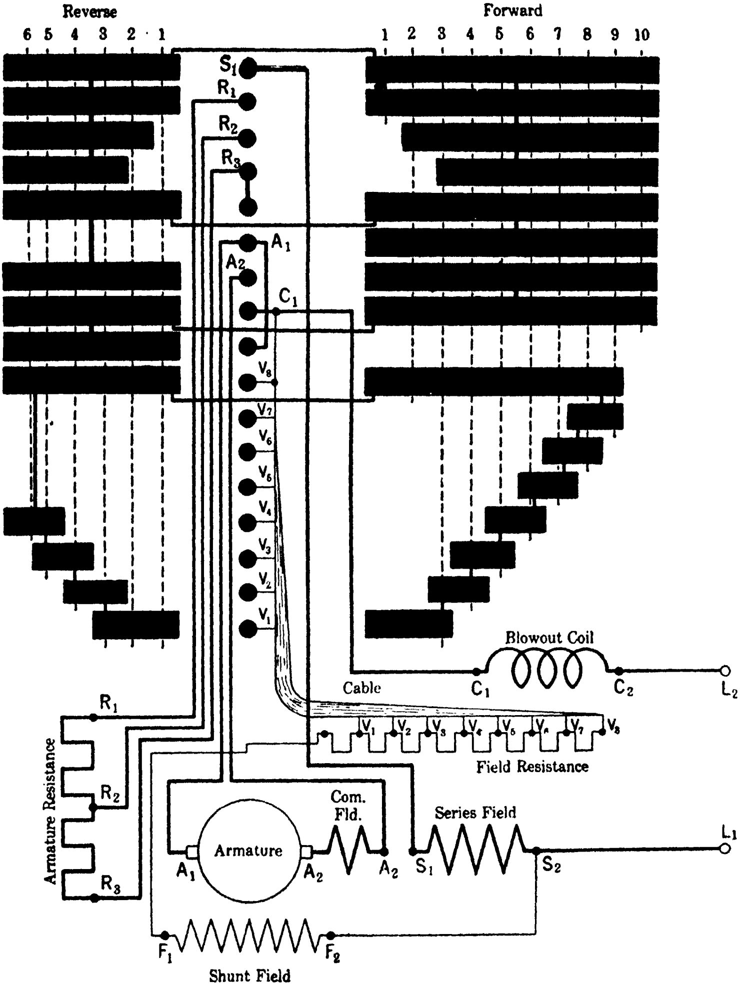

FIG. 430. Connections of a reversing industrial controller.

This is necessary, because the finger Q does not touch the segment q in some of the operating positions of the controller.

Should the circuit breaker be tripped, either by hand or because of an overload, the contactor coil is deënergized and the main line is disconnected between L_2 and M_2, as well as between L_1 and M_1. If, instead of the protective panel, an ordinary line switch and fuses are used, the line conductor L_1 is connected directly to the controller finger N_1, and the conductor L_2 is continued as $P_1P_2P_3$. The lead CQ is omitted altogether.

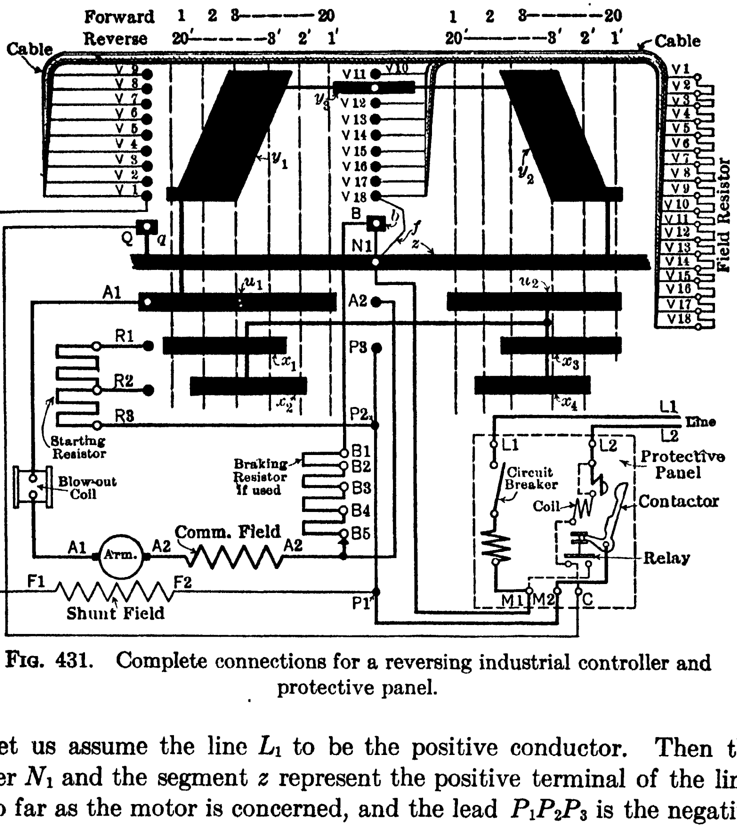

FIG. 431. Complete connections for a reversing industrial controller and protective panel.

Let us assume the line L_1 to be the positive conductor. Then the finger N_1 and the segment z represent the positive terminal of the line, in so far as the motor is concerned, and the lead $P_1P_2P_3$ is the negative terminal. It is only necessary to follow the armature and the field connections between these terminals.

On the first notch forward the two rows of controller fingers occupy the positions indicated by the two vertical lines marked 1, 1. The armature current follows the path $zA_1A_2u_2x_1R_1R_2R_3P_2$, with the whole starting resistor in the circuit On the second notch the finger R_2 touches the segment x_2 and the part R_1R_2 of the starting resistance is cut out. On the third notch the terminal A_2 is directly connected to P_3 through u_2 and x_3, and all of the starting resistance is eliminated. After this, the armature connections are not changed, and the field resistance is introduced in steps.

Should the controller drum be turned in the opposite direction, the first step would correspond to the vertical lines marked 1′, 1′. In this case A_1 is in contact with u_2, and A_2 touches u_1, causing the armature current to flow in the opposite direction. The rest of the connections are the same, as for forward rotation.

On the first notch forward the field current follows the path $zy_1V_1F_1F_2P_1$, with no resistance "in." The same field strength is maintained on the next few notches while the armature resistance is being cut out. On the last operating notch (the twentieth), the field is energized from N_1 through the jumper f, contact finger V_{18}, the whole field resistor, and the finger V_1. The motor is then running at its highest speed. In the intermediate positions part of the field resistance is short-circuited by the slanted segments y_2 and y_1, and the controller handle may be left in any of these positions indefinitely.

When the controller drum is quickly returned to the "off" position, the motor is converted into a generator. By following the connections in the "off" position, and assuming the motor to be still running, it will be found that the armature circuit is closed upon itself through the braking resistor B_1 to B_5, while the field winding is excited from the line but is weakened by the resistance between V_1 and V_{11}. The value of the braking resistance can be so adjusted as to reduce the initial generated current to a safe magnitude and to stop the motor within a reasonable interval of time. If dynamic braking is not desired, the braking resistor and the connection to B in the controller are omitted.

583. EXPERIMENT 25-F. — Operation of a Drum-Type Controller with a D-C Motor. — The purpose of the experiment is to make a study of the connections in a given drum controller. The experiment supplements that described in §581. The most instructive method of performing the experiment is for the student to trace the actual connections without the use of a diagram. Having made his own diagram, he can then determine what the controller can do, and check his conclusions by actually operating a motor with it. The connections can be traced with a lamp, a voltmeter, or a buzzer, and the resistances measured by the drop-of-potential method or with a simple Wheatstone bridge (Chapter I). If such a problem seems too advanced, or would take too much time, the student may be given a diagram of connections and only asked to check it on the controller and to decide what operations the controller is designed to perform. Then he can check his conclusions by actually operating a motor.

Sketches should be made of the important mechanical features of the controller, of the blow-out coil, interlocking if any, adjustment of fingers, removable parts, arc-resisting partitions, etc. Tests should be made to determine to what extent the controller is fool-proof, and what special instructions should be given the operator. The student should make clear to himself what protective apparatus, such as fuses, a switch, circuit breaker, an overload relay, etc., should be installed with the controller in order to safeguard, as much as possible, the controller itself, the motor, the operator, and the line.

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INDEX

Numbers refer to pages