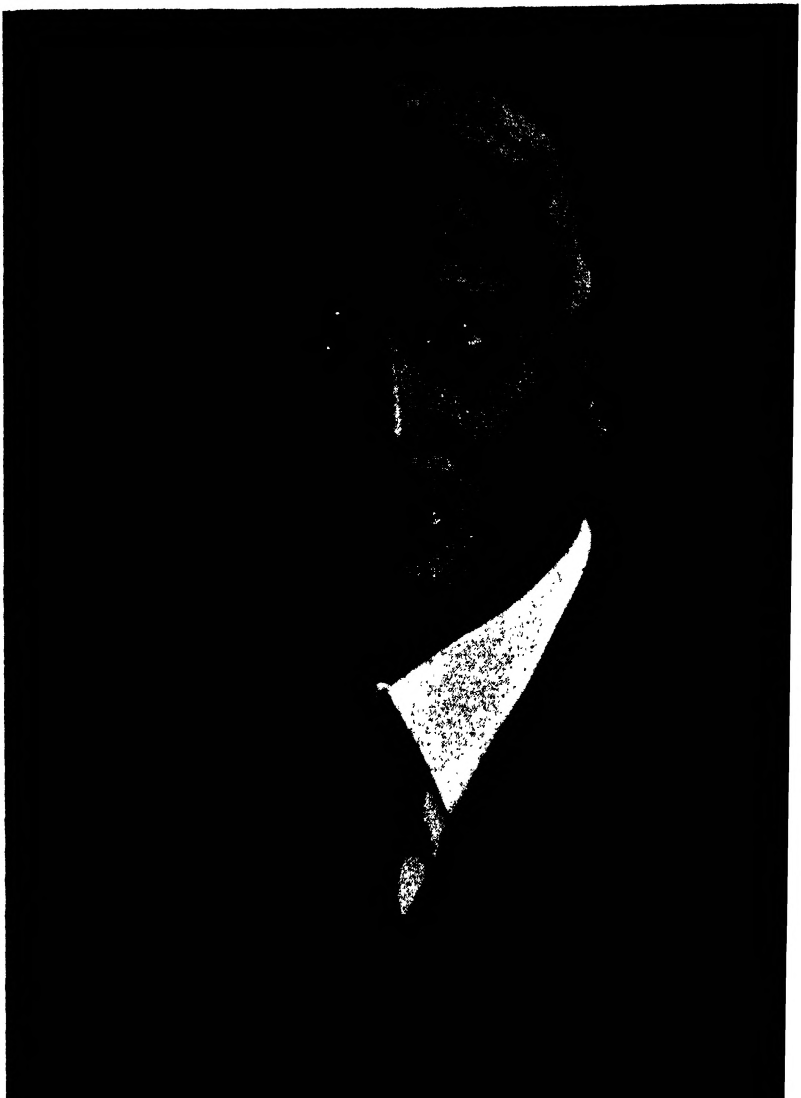
INDUSTRIAL HIGH FREQUENCY ELECTRIC POWER



(Photo: Orren Jack Ti

THE LATE DR. E. F. NORTHRUP

INDUSTRIAL HIGH FREQUENCY ELECTRIC POWER

by

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PREFACE

In 1944 Mr. C. F. Partridge of Birmingham Central Technical College requested me to give a course of lectures on "Industrial High-frequency Technology." This book has been written around the notes prepared for those lectures. Many acknowledgments and references to sources of information are made, but not, may I say, in order to shift the responsibility for shortcomings, which is entirely mine.

I wish to record my gratitude to Mr. J. P. Reed and Mr. W. B. Jones of Research Department, Tube Investments Ltd., for translations of papers by Losinsky and Vologdin; to Mr. L. E. Newnham and Mr. L. C. Ludbrook, for valuable help; to colleagues at Birlec Ltd., and in particular to Mr. P. F. Hancock and Mr. Alan Shew, for advice on numerous points.

E. MAY.

DECEMBER 1948.

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CHAPTER 1

A Summary of the Basic Circuit-theory

THE ATOMS of the elements which make up material substances appear to consist of two kinds of electricity, the electron and the proton. The number and arrangement of these electrical "particles" determine the kind of element and its properties. The quantity of electricity, or electric charge, of an electron is the same as that of a proton, but is of opposite kind, so they are referred to as negative and positive charges respectively. The practical unit of quantity of electricity, the coulomb, is equivalent to $6{\cdot}28 \times 10^{18}$ electrons, and the practical unit of current, the ampère, is equivalent to this number of electrons flowing past every second.

All the protons, and some of the electrons, are tightly bound up in the nucleus or core of the atom, and the remainder of the electrons ("planetary electrons") encircle this nucleus. Normal atoms have equal numbers of electrons and protons, so that their total effective electric charge is zero.

Substances from the atoms of which electrons can easily be detached are good conductors of electricity, the conduction-current consisting, in the case of solids, of the slow drift of these freed electrons under the influence of an applied voltage ("electron-motive-force"). Good insulators are those substances from the atoms of which electrons are not easily removed, and the break-down-voltage is high.

In gases and vapours, the conduction-current consists of the movement in opposite directions of both electrons ($-ve$) and ionised atoms ($+ve$), the latter being atoms from which electrons have been removed, for example, by collision with high-speed electrons.

In liquids the conduction-current consists of the slow movement of ion-groups bearing charges of opposite sign. The "ions" in this case are often quite complex groups of molecules.

In high-vacuum tubes, such as are used for the generation of high-frequency power, the space-current consists of high-speed electrons travelling through empty space from cathode to anode; a third electrode placed between them serves to control the current, and is called the control-grid.

In all these cases, then, the term "electric current" implies the movement of electric charges, either in the simplest form of moving electrons, or as charges borne by atoms, molecules or groups of molecules. The force causing the charges to move is that which they experience when immersed in an electric field of force. The electric field is usually provided by applying or inducing an E.M.F. in the circuit, and in practical units the field-strength is measured in volts per centimetre length of circuit path.

The effects produced by electric currents—heating, magnetic, and chemical—are applied by the engineer to a wide range of work, and this book is largely concerned with the application of magnetic and heating effects, and especially to the rapid production of heat in the place where it is required.

In electrical engineering work where the power required is appreciable, the current is established in one of two ways. The necessary electric field for driving the electrons is either applied directly, or it may be induced by the presence of a changing magnetic field. In the former case this means applying a voltage directly to the part in which the current is required. In the latter case the voltage will be induced in the part, without direct electrical connection to the source of power.

Electrical resistance

The essential characteristic of electrical resistance is the absorption of electrical energy when a current exists in the resistive part. In a true resistance this energy is converted into heat-energy, as for example in an "electric fire" or a furnace heating-element. It is often convenient to refer to the "equivalent resistance" of a device which absorbs electrical energy and converts it into some form other than heat-energy. For example, a 200-v electric motor driving a lathe is drawing, say, 5 ampères from the 200-v supply. Energy is being absorbed from the supply at the rate of 1000 watts, and most of this energy is being dissipated at the tool-tip and in the mechanical drive. The motor is not an electric fire (or ought not to be!), but so far as the supply is concerned it behaves exactly as would a 1 KW "fire"; i.e. the "equivalent resistance" of the motor load is $\frac{200}{5}=40$ ohms. It is interesting to note that the

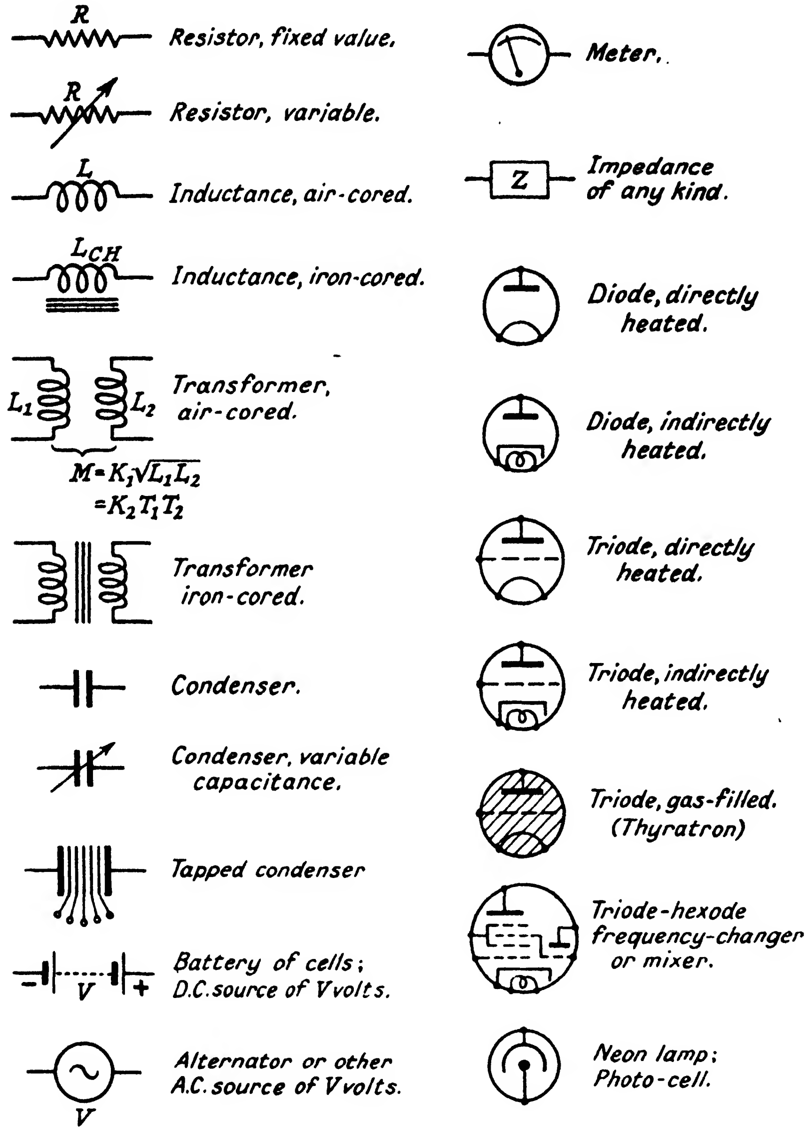


Fig. 1·1. Circuit Diagram Symbols.

whole of the energy is ultimately turned into heat in this case, and it is sometimes said that "heat is the ultimate form of all energy". The industrial H.F. engineer is concerned with the generation of heat in sufficient quantity in the right place, and makes use of the electric loss characteristics of the work materials to this end.

The resistance or opposition offered to a current is measured in ohms, and can be defined as the ratio of voltage applied across the part to the resultant current through the part. Where the current is a changing one (e.g. alternating) this ratio gives total opposition or impedance, including inductive and capacitive effects. Hence a stricter definition of resistance is the ratio,

$$\frac{P}{I^2}=R \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}1$$

where P=average power developed, in watts

I=effective or root-mean-square value of current, in ampères.

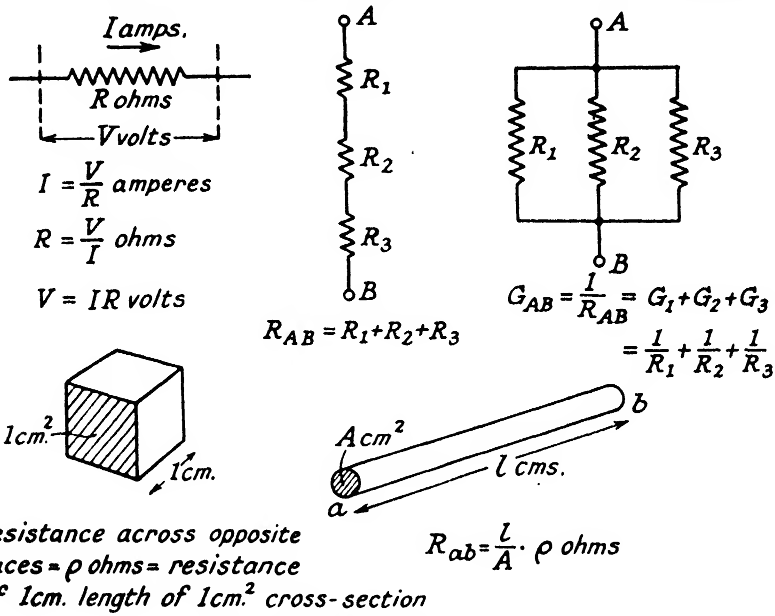


Fig. 1·2. Electrical Resistance.

For a pure resistance (no inductance or capacitance) we also have

$$\frac{V}{I}=R \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}2$$

where V=voltage across R

I=current through R, ampères.

Fig. 1·2 summarises the important circuit properties of resistance elements.

Specific resistance or resistivity of a material

This is defined as the resistance offered to current by a 1 cm length of the material, of 1 cm² cross-sectional area, and is given the symbol ρ. It is measured in ohms/cm/cm².* Typical values of ρ are quoted in Table I (Appendix A). As indicated in Fig. 1·2, resistances in series add, while conductances in parallel add (conductance is the reciprocal of resistance, i.e. $G=\frac{1}{R}$ mhos). From this it follows that a length l cms of a material, of A cm² cross-sectional area, has a resistance given by:

$$R=\frac{l}{A}.\rho \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}3$$

Temperature coefficient

The electrical resistance of most materials changes appreciably with temperature. In particular the resistance of pure metals increases at a constant rate with increase of temperature. The amount by which a resistance of 1 ohm (at t_1° C) increases when the temperature is raised by 1° C is called the "temperature coefficient of increase of resistance" based on the initial temperature t_1. It may be written α_{t_1}. Values of α for various materials, and the corresponding reference temperatures, are given in Table I (Appendix A).

For any resistance R_1 at t_1° C, the corresponding resistance R_2 for some new temperature t_2° C is given by:

$$R_2=R_1\,[1+\alpha_{t_1}\,(t_2-t_1)] \text{ ohms} \quad . \quad . \quad . \quad 1{\cdot}4$$

The increase of resistance due to temperature-rise is very considerable in the case of pure metals; for example, copper increases in resistance by 41% for a temperature rise from 15° C to 115° C, while the filament resistance of a tungsten lamp increases from its "cold" resistance to about twelve times this value when at running temperature. On the other hand, some alloys have very small temperature coefficients, and may exhibit negative coefficients over certain temperature ranges. Some forms of carbon also have a negative coefficient.

H.F. resistance

Equation 1·3 assumes the current uniformly distributed over the whole cross-section of the conductor. This is not so when current, and hence magnetic flux, are changing, and the current flows mainly in the outer skin of the conductor, where the magnetic linkages, and hence the current-path reactance, have minimum values. The effect is very pronounced at high frequencies. The

* *i.e.* ohm-cms.

ratio of conductor-resistance at a frequency f to its resistance to an unchanging current is approximately:

$$\frac{R_f}{R_o} \propto d\sqrt{\frac{\mu f}{\rho}}, \text{ where } R_o \propto \frac{\rho}{d^2} \ (d = \text{diameter}),$$

μ being the magnetic permeability of the conductor material.

Because of skin effect, hollow metal tubing is widely used for H.F. coils, which may be cooled if necessary by the passage of water through them. The effective penetration depth of the current is usually taken as being:

$$p = \frac{1}{2\pi}\sqrt{\frac{\rho}{\mu f}} \text{cms}; \quad \rho \text{ is in E.M. units,}$$

i.e. ohm-cms $\times 10^9$. The wall thickness of a hollow conductor should preferably be at least twice this penetration depth. A graph giving values of p for a wide range of values of ρ and f is given in Fig. 6·32, in which μ is taken as unity.

Ferromagnetic materials are not, of course, used industrially as primary conductors of high-frequency currents; their high effective resistance would result in heavy power losses. For such primary conductors high-conductivity copper, silver, and sometimes aluminium are used, and as much *useful surface* is provided as conditions permit. Current penetration depths in H.C. copper at a working temperature around 75° C and at frequencies of 1, 3, 10, and 400 KC/S (common "standard" frequencies) are respectively 0·23, 0·13, 0·07, and 0·011 cm approximately. The effective H.F. resistance of coils and of work-pieces for induction heating can be calculated in terms of the "penetration-depth" with sufficient accuracy for most practical purposes (see Chapter 6).

Work-pieces to be heated by H.F. induction are frequently of steel—e.g. mild steel billets and bars to be through-heated to 1100-1250° C for forging, and medium-carbon steels to be surface or locally heated for hardening. Such steels are highly magnetic at temperatures below some 700-750° C, and suddenly lose their high permeability, i.e. $\mu \rightarrow$Unity, in this temperature range. This abrupt reduction in μ at the Curie temperature results in an increase in current penetration depth and gives rise to some interesting operating problems which are discussed elsewhere (e.g. Chapter 9). The effective value of μ depends mainly upon the work-coil ampère-turns (see Chapter 6); it may be mentioned here that for most ordinary steels μ_{eff} may be as high as 200 in through-heating applications on heavy sections, and tends to approach unity in the intense fields used in some surface-hardening applications.

The distribution of alternating current in a conductor is further modified by the proximity of other current-carrying conductors. The effect is important in heavy-current bus-bars,* coils, "concentrators", and in induction heating circuits generally.

Electrical inductance—mutual and self

Whenever the magnetic flux linking a circuit changes, an E.M.F. is induced in the circuit, given by:

$$e = -\frac{d(\Phi T)}{dt} \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad 1{\cdot}5$$

i.e. e is proportional to the rate of change of linkages or "line-turns", $\Phi \times T$. The flux Φ may be produced by a current i_1, flowing in a circuit A, as shown in Fig. 1·3, and some part of this flux may link with another circuit B. Suppose Φ is the total number of lines of force (i.e. total magnetic flux) produced by the T_1 turns of circuit A when carrying a certain value of current i_1 ampères. If all this flux linked with all the turns T_2 of B, the total linkages of B due to i_1 would be ΦT_2. In practice this never holds, and the linkages of B will be $K\Phi T_2$. Thus K is a factor always less than unity, and is called the "coupling factor" between A and B. Its value may be very nearly 1·0 when both coils are mounted close together on a common laminated-steel core, as in low-frequency transformers, or it may be as low as ·01 in the case of certain air-cored high-frequency transformers.

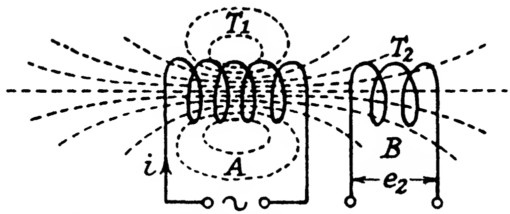


Fig. 1·3. Self and Mutual Inductance.

The E.M.F. induced in circuit B will be:

$$e_2 = -\frac{d(K\Phi T_2)}{dt} \times 10^{-8} \text{ volts}$$

$$= -KT_2\frac{d\Phi}{dt} \cdot 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}6$$

e_2 is the E.M.F. of mutual induction between circuits A and B.

* See Chapter 9, Fig. 9·24.

Suppose, as is true for air-cored coils, that Φ is proportional to i_1. Then we can re-write equation 1·6:

$$e_2 = -M\frac{di_1}{dt} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}7$$

where $\frac{di}{dt}$=rate of change of current i_1 in ampères/sec and M is a constant incorporating K,T_2, and the constant of proportionality between Φ and i_1. M is called the coefficient of mutual induction (or simply the mutual inductance) between A and B, and is measured in henrys. The circuits A and B would possess a mutual inductance of 1 henry if e_2 were 1 volt when $\frac{di_1}{dt}$ is 1 ampère/sec. The value of M obviously depends upon coil turns, dimensions and relative positions.

Again, if i_1 is changing and consequently Φ is changing, an E.M.F. is induced in circuit A by its own changing flux. This follows from the fundamental law expressed in equation 1·5. This is an E.M.F. of self-induction and is given by:

$$e_1 = -L_1\frac{di_1}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}8$$

where L_1 is the coefficient of self-inductance of circuit A (or simply the inductance of A) expressed in henrys. Thus L_1 would be 1 henry if $e_1 = 1$ volt when $\frac{di_1}{dt} = 1$ ampère/sec. The millihenry (1 mH$=10^{-3}$ H) and the microhenry (1 μH$=10^{-6}$ H) are also used The minus sign in equation 1·8 denotes that e_1 is in opposition to the voltage applied to circuit A, i.e. an inductance reacts against any change of current in it. It offers no opposition to a steady unchanging current.

Similarly, if circuit B were supplied with a changing current i_2, an E.M.F. of self-induction

$$e_2 = -L_2\frac{di_2}{dt} \text{ volts}$$

would oppose the applied voltage.

Neglecting circuit resistance, the self-induced voltage e will be exactly equal and opposite to the applied voltage v, so that the applied voltage will be:

$$v = +L\frac{di}{dt} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}9$$

In practical cases a circuit must possess some resistance, since ρ is not zero. Then:

$$v = v_R + v_L$$
$$= iR + L\frac{di}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}10$$

The power being taken from the supply in such a circuit (Fig. 1·4) is obtained by multiplying the voltage-equation 1·10 by the instantaneous current i;

i.e. $$vi = i^2R + Li\frac{di}{dt} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}11$$

= power being taken when current is i amps and changing at $\frac{di}{dt}$ ampères/sec.

The i^2R term represents the rate of conversion from electric to heat-energy in the circuit, while $Li\frac{di}{dt}$ represents the rate at which energy is being stored in the magnetic field of the circuit. Suppose the final value of i is I ampères, then the total energy stored in the magnetic field is:

Magnetic energy =

$$\int_0^I Li\,di = \tfrac{1}{2}LI^2 \text{ joules} \quad . \quad . \quad 1{\cdot}12$$

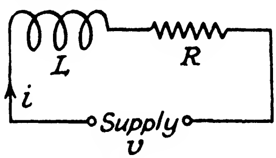


Fig. 1·4.
Circuit having Inductance and Resistance.

This energy is released back into the circuit when the current is reduced to zero.

Self and mutual inductance effects being essentially associated with changing currents, they are of great importance in alternating current circuits, and especially so at high frequencies.

The calculation of self-inductance

This is a matter of considerable difficulty if high accuracy is required. There are, however, certain useful generalisations and approximate relations which are given below:

(1) $L_{\mu H} = T^2RF$ for a coil of T turns of (mean) radius R inches. F is a form factor, $F = \frac{\cdot 8R}{6R + 9l + 10d}$, l being winding length in inches, and d winding depth (inches) (see Fig. 1·5 (a)).

(2) For fixed proportions:

$$L \propto \text{linear dimensions},$$
$$L \propto T^2.$$

(3) Wheeler's Formula for cylindrical coils, single layer:

$$L_{\mu H}=\frac{R^2T^2}{9R+10l} \quad \text{(ref. Fig. 5 (a))}$$

is accurate within 5 % if $l \geq R$. R and l are in inches.

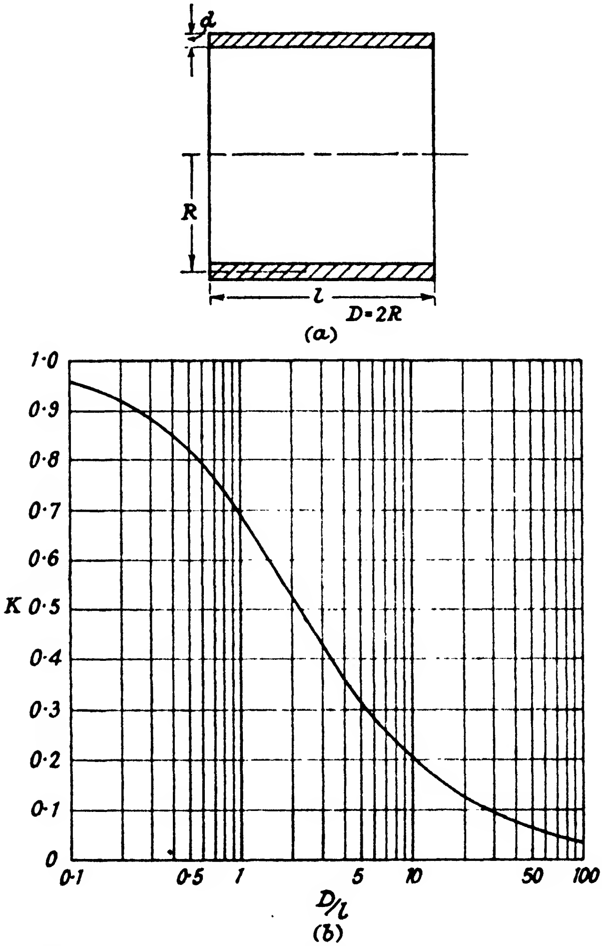


Fig. 1·5. Nagaoka and Lorenz Constant for Calculation of Inductance.

(4) Single turn of radius R, of wire radius r (cms):

$$L_{\mu H}=4\pi R\left[\log_e \frac{8R}{r}-\frac{7}{4}\right].10^{-3}.$$

(5) Closely-wound single-layer coil, of n turns per cm. length:

$$L_{\mu H}=K\pi^2D^2n^2l\times 10^{-3}.$$

D and l in cms. K is plotted in Fig. 1·5 (b). $D=2R$.

(6) Twin feeder consisting of two conductors of radius r, separation D between centres, $D \gg r$,

$$L_{\mu H} \doteqdot \cdot 4 \log_e \frac{D}{r} \text{ per metre.}$$

(7) Concentric feeder, consisting of two thin-walled tubes, inner of radius R_i, outer of radius R_o,

$$L_{\mu H} \doteqdot \cdot 2 \log_e \frac{R_o}{R_i} \text{ per metre.}$$

The inductance of twin flat bus-bars is plotted in Fig. 9·24.

Electrical capacitance

Fig. 1·6 shows an arrangement of two metal (i.e. conducting) plates which may be connected to the D.C. source of V volts when the switch S is closed. The plates each have an area of A cm² and are separated by a distance d cms, with only air between them. The arrangement is called a condenser or capacitor.

When the switch is closed for the first time, the pointer of the ammeter kicks and then returns to zero, indicating that a brief current-pulse occurs. During the time of this pulse electrons are sucked from plate a and pumped round the circuit into plate b; it is convenient to regard the energy-source as an electron-pump. For given values of A, d, and V, a definite number of electrons is transferred, the total quantity of electricity involved being called the "condenser charge". The result is that plate a is now "positively charged" (since negative charges, electrons, have been removed) and plate b is "negatively charged". Charges of opposite sign attract each other, and it becomes increasingly difficult for electrons to leave plate a because of this attraction. Thus it is that the condenser charge is a fixed amount for a given applied voltage, and this is expressed as:

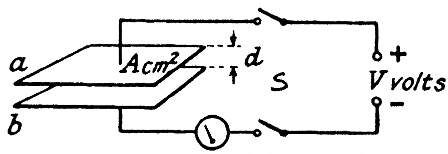


Fig. 1·6. Electric Condenser.

$$Q = CV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}13$$

where Q = total charge pumped round from a to b, in coulombs,

V = applied voltage,

C = a constant, called the capacitance of the condenser, and measured in "farads".

A capacitance of 1 farad implies that 1 coulomb of electricity

($6{\cdot}28\times10^{18}$ electrons) is transferred from a to b when $V=1$ volt; i.e. $C=\frac{Q}{V}=1$ coulomb/volt, or 1 farad.

For two parallel plates as shown, the capacitance is given by:

$$C=\frac{KA}{4\pi d}\cdot\frac{1}{9\times10^5}\ \mu\text{F}.$$

K is a constant, the value of which depends upon the kind of insulating material between the plates. It is called the dielectric constant, or permittivity, and is practically unity for air. Values of K for various materials are given in Table III, Appendix A.

Practical condensers, even when constructed to have very large capacitance, do not approach the farad. Instead, practical values are measured in microfarads, 1 microfarad being one-millionth of 1 farad.

$$1\ \mu\text{F}=10^{-6}\ \text{farad}.$$

A still smaller unit, the micromicrofarad, is also used.

$$1\ \mu\mu\text{F}=10^{-12}\ \text{farad}.$$

From 1·13 it follows that one-millionth of a coulomb is transferred from one plate to the other when 1 volt is applied across them, if the capacitance is 1 μF.

Condenser current

The rate at which the charge is transferred from a to b is obviously the same thing as the rate of flow of electricity round the circuit, that is, the current in the wires connecting the two plates. Hence:

$$\text{Circuit current}=i=\frac{dQ}{dt}\ \text{ampères} \quad . \quad . \quad . \quad 1{\cdot}14$$

or, since $Q=VC$, and C is constant,

$$i=C\frac{dV}{dt}\ \text{ampères} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}15$$

where C is expressed in farads, and $\frac{dV}{dt}$ in volts/second, i.e. the condenser circuit current is proportional to capacitance and to rate of change of condenser voltage. It follows that when an alternating voltage is applied to C an alternating current will flow continuously in the condenser-circuit, whereas when a steady uni-directional voltage is applied a current flows only while the condenser voltage is building up to that of the supply.

Practical circuits always possess resistance, inductance, and capacitance, though any one or two of these three fundamental properties may be so small as to be negligible under certain conditions. Consider next the particular case of a circuit consisting of capacitance and resistance in series (Fig. 1·7). The total voltage across C and R is V,

i.e. $$V=v_C+v_R \quad . \quad . \quad 1{\cdot}16$$

$$=\frac{q}{C}+iR \quad . \quad . \quad 1{\cdot}17$$

where q=condenser charge at any chosen instant, and i=circuit current at that instant.

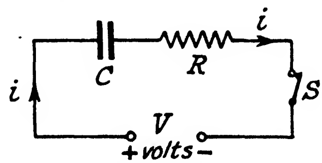


Fig. 1·7.
Circuit having Capacitance and Resistance.

Equation 1·17 expresses the facts:

(i) When S is first closed, and while C is as yet uncharged,

$$V=iR \text{ (since } v_C=0\text{)},$$

i.e. $$i=\frac{V}{R} \text{ initially} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}18$$

(ii) $q=Q=VC$ only when $i=0$, that is, after current has ceased.

CR time-constant

Fig. 1 8 shows how v_C (and hence q) and v_R (and hence i) change with time, after closing the switch for the first time:

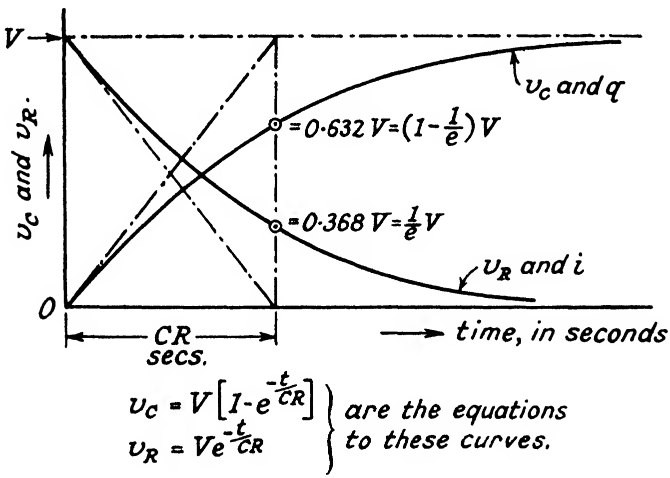


Fig. 1·8. Charging and Discharging Curves.

This graph shows that i never quite falls to zero, nor, consequently, does q ever quite reach the value $Q=VC$, but these values are approached very rapidly if the product CR is small. As indicated in the figure, at a time CR secs (C in farads, R in ohms; or C in μF, R in MΩ) after commencement of charging, v_C and q have built up to ·632 of the limiting values V and VC while v_R and i have fallen to ·368 of their initial values V and $\frac{V}{R}$.

It is interesting to note that if the initial rate of growth of v_C were maintained, v_C would achieve the value V in CR secs. Similarly, v_R and i would fall to zero in this time. The product CR is called the time-constant. That the product has the units of seconds can be seen thus:

$$CR=\frac{Q}{V}\times\frac{V}{I}=\frac{Q}{I};\ \text{i.e.}\ \frac{\text{Coulombs}}{\text{Coulombs/sec}},\ \text{i.e. seconds.}$$

When a condenser, initially charged to a voltage V, is discharged through a resistance of R ohms connected across its terminals, the discharge current again dies away exponentially from an initial peak value of $\frac{V}{R}$ ampères. The decay curve representing v_R and i in Fig. 1·8 represents, with appropriate scales, all the quantities v_R, v_C, i, and q for the discharge, since now $v_R=v_C$.

The condenser-resistance circuit is of very great importance, and applications of it are discussed later. It is worth noting here that one important application is to the energising of circuits for definite periods of time. (See Chapter 8.)

Energy stored in condenser

Multiplying 1·17 by i, we get:

$$Vi=\frac{q}{C}i+i^2R \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}19$$

=rate at which energy is being taken from supply.

If q builds up to a final value Q, the total energy stored in C is, since $i=\frac{dq}{dt}$,

$$\text{Energy stored}=\frac{1}{C}\int_0^Q q\,dq=\tfrac{1}{2}\frac{Q^2}{C}=\tfrac{1}{2}CV^2 \text{ joules} \quad . \quad 1{\cdot}20$$

This energy is stored as potential energy of the electric field in the dielectric between the plates. It is given back to the circuit when the condenser is discharged by placing a conducting path across the plates.

Effect of solid dielectric between the condenser plates

It is found that when the space between the condenser plates is occupied by a solid insulating material or "dielectric" instead of air, the dimensions remaining unchanged, a larger capacitance is obtained. The factor by which the capacitance is increased is called the dielectric constant (symbol K) of the dielectric; its value commonly ranges between 2 and 6, though there are notable exceptions (see Table III, Appendix A).

Mica is much used, especially for high-voltage condensers, for capacitances below 1 μF. Oil-impregnated and wax-impregnated papers are very commonly used for relatively large capacitances,(1) and are suitable for voltages up to 1 KV. For higher voltages a number of units are connected in series. Ceramic condensers are now being widely used for H.F. circuits (see Fig. 8·10).

The break-down voltage of various dielectrics is given in Table III, Appendix A. It is given under "Dielectric Strength", that is, maximum volts per millimetre of thickness.

The behaviour of solid dielectrics under applied voltage has been the subject of much investigation and discussion.(2) It is convenient to imagine molecular "dipoles" in the material, which are caused to turn in a kind of viscous resisting medium when an electric field is established by the application of voltage (see Fig. 1·9). This is an attempt to account for a very important property of most solid dielectrics—namely, the absorption of some electrical energy which is converted into heat in the dielectric, and which cannot be accounted for by consideration of the normal finite "insulation resistance" alone. It is seen that such dielectrics are not perfect, that is, not loss-free, and a condenser with such a dielectric can be represented as shown in Fig. 1·10 (a) or (b), in which C represents a loss-free condenser and R a resistance in which the loss is supposed to occur.

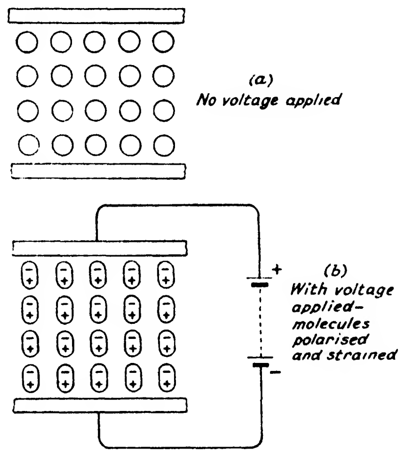


Fig. 1·9. Condenser with Solid Dielectric.

The value of the equivalent series resistance R_S or parallel resistance R_P is discussed in Chapter 7.

Sinusoidal current in a pure resistance

Fig. 1·11 shows the circuit and the time-graph of the current in the circuit. The current time-graph is similar to that obtained by plotting the *vertical* height of the free end of a rotating stick above or below a datum line, as shown in Fig. 1·12.

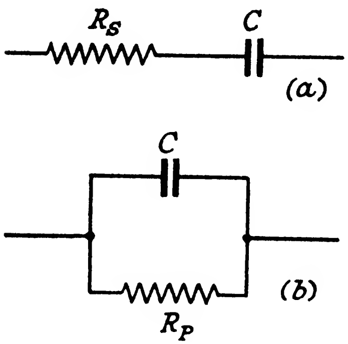


Fig. 1·10.
Equivalent Circuits for Fig. 1.9.

It is therefore convenient to represent the alternating current of Fig. 1·11 by a "rotating vector" such as OA in Fig. 1·12. One revolution of this representing vector is equivalent to one cycle of the current, so that:

$$\omega T = 2\pi \text{ radians}$$

if T is the periodic time; and since $T=\frac{1}{f}$, f being the frequency in cycles/second, we have:

$$\omega = \frac{2\pi}{T} = 2\pi f \text{ radians/second}$$

=angular velocity of vector representing a sinusoidal current of frequency f cycles/second.

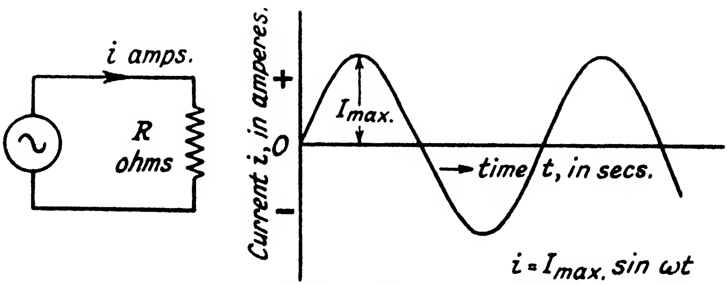


Fig. 1·11. Sinusoidal Alternating Current in Pure Resistance.

Average value of sinusoidal current

This is found as the average height of the current curve, and, over a whole cycle, is zero. The practical significance of this is that the average value of any effect depending directly upon the value of the current, such as electro-plating, deflection of simple moving-coil meter, etc., is zero over a whole cycle.

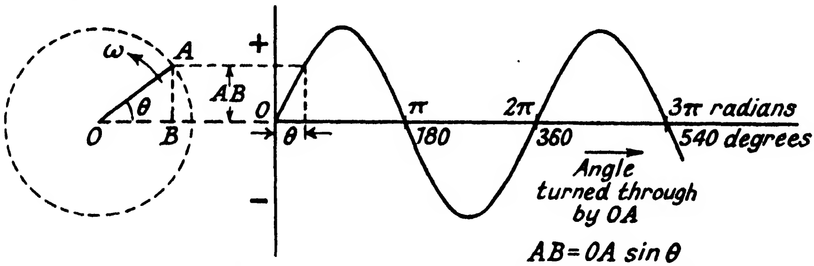


Fig. 1·12. Vector Representation of Sinusoidal Quantities.

The average value over a positive or a negative *half*-cycle is important, and is found thus:

$$I_{av.}=\frac{1}{\pi}\int_{\theta=0}^{\theta=\pi} I_{max}\sin\theta.d\theta=\frac{I}{\pi}\Big[-\cos\theta\Big]_0^{\pi}$$

$$=\frac{2}{\pi}I_{max} \qquad =\cdot 637 I_{max} \quad . \quad . \quad . \quad . \quad 1\cdot 21$$

Similarly, for a current having the waveform shown in Fig. 1·13, consisting of half-cycle pulses only, the average value is:

$$I_{av.}=\frac{I_{max}}{\pi}=\cdot 318\ I_{max} \quad . \quad . \quad . \quad . \quad 1\cdot 22$$

This is the value which would be read by a moving-coil meter carrying such a current.

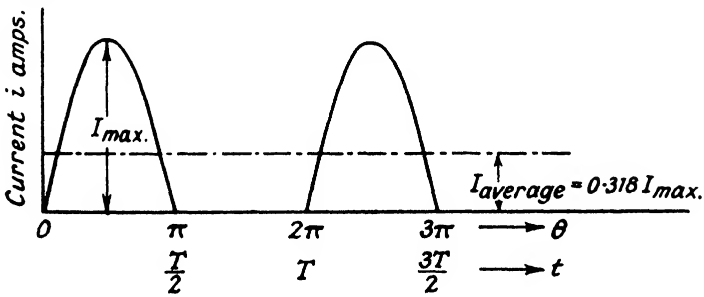


Fig. 1·13. Average Value of Repeated Half Sine-wave Pulses.

Root-mean-square value of sinusoidal current (RMS value)

The power developed in a circuit of constant resistance R is given at any instant by:

$$P=i^2R \text{ watts,}$$

i being the current in R at that instant.

In single-phase alternating current circuits the power obviously varies from instant to instant, and the average power is therefore important. Then:

$$P \text{ average} = (\text{Average square of current}) \times R \text{ watts.}$$

In Fig. 1·14 it is seen that the (current)^2 curve is also sinusoidal, but of twice the frequency of the sinusoidal current; i.e. there are two power pulses, both positive, per cycle of current. The practical significance of this is that the heating effect is independent of direction of current.

The average value of (current)^2 is seen to be half the peak value,

i.e. $$(i^2)_{av.} = \frac{I_{max}{}^2}{2} \quad . \quad . \quad . \quad . \quad 1·23$$

and the average power is therefore:

$$P_{av.} = \frac{I_{max}{}^2}{2} R \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 1·24$$

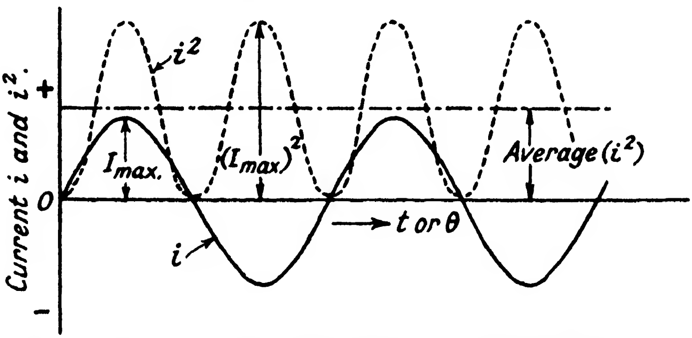


Fig. 1·14. Current and Power Time-graphs for Resistive Circuit-element.

Comparing this with the D.C. case, in which $P = I^2R$, it is seen that the sinusoidal alternating current is equivalent to a steady current of $\frac{I_{max}}{\sqrt{2}}$ ampères, from the power aspect.

This value, the square-root of the average or mean square of the alternating current, is usually written I_{RMS} or simply I.

Thus average power in an A.C. circuit is given by:

$$P_{av.} = I_{RMS}{}^2 R \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1·25$$

and for a sine-waveform current:

$$I_{RMS} = \frac{I_{max}}{\sqrt{2}} = ·707 I_{max}.$$

Similarly, since for a resistance:

$$v = iR$$

it follows that $$\frac{V_{max}}{\sqrt{2}} = \frac{I_{max}}{\sqrt{2}} R, \text{ or } V_{RMS} = I_{RMS}R \quad . \quad . \quad . \quad 1·26$$

The majority of commercial alternating current meters are for convenience calibrated to read RMS values, since it is these values which quickly lead to a value for the average circuit-power.

Finally, since voltage across R is proportional to current in R, it follows that the voltage rises and falls in step with the current, i.e. voltage and current are in phase. The product of voltage and current, instant by instant, would of course yield the same power curve as would be obtained from i^2R; the waveform would be that shown for i^2 in Fig. 1·14. In-phase current and voltage are represented vectorially by two superimposed vectors. The behaviour of a resistive circuit carrying an alternating sinusoidal current is summarised in Fig. 1·15.

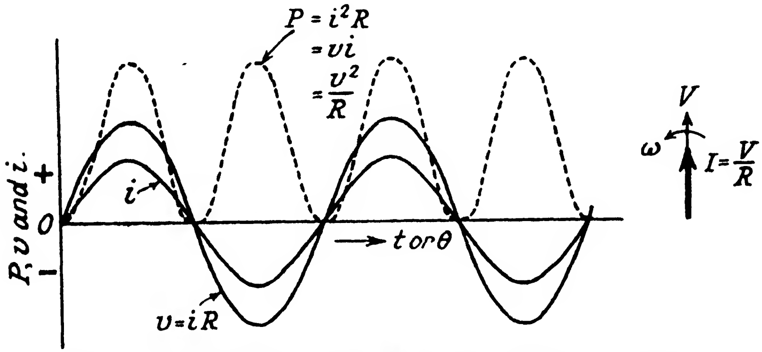


Fig. 1·15. Waveforms and Vectors for Resistive Element.

Inductive circuit carrying sinusoidal current

Equation 1·9 gave the applied voltage to such a circuit as that of Fig. 1·16 as:

$$v = L\frac{di}{dt} \text{ volts}$$

L being the circuit inductance in henries, and $\frac{di}{dt}$ the rate of change of circuit current in ampères/second. Current and applied voltage are plotted in Fig. 1·16, and it is seen that i lags one quarter-cycle (90° or $\frac{\pi}{2}$ radians) behind. This is also shown in the vector diagram.

For a sinusoidal current we have:

$$i = I_{max} \sin \omega t$$

so that $$\frac{di}{dt} = \omega I_{max} \cos \omega t = \omega I_{max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad . \quad . \quad 1\cdot27$$

i.e. the rate of change of any sine-waveform has also a sine-waveform of amplitude ω times the original amplitude, and leads it in time by one quarter-cycle, which is equivalent to $\frac{\pi}{2}$ radians in a vector diagram.

It follows from equation 1·9 that the voltage across L is sinusoidal, of same frequency as the current in it, leads the current by 90° or $\frac{1}{4}$-cycle, and has an amplitude given by:

$$V_{max}=\omega L I_{max} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}28$$

Inspection of Fig. 1·16 shows that the average power in the case of a "pure" inductance is zero. Power is taken from the supply while the current is building up, this power being the rate of energy

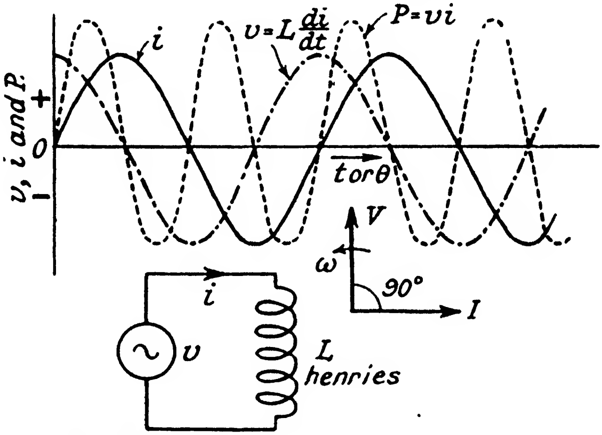


Fig. 1·16. Time-graphs and Vectors for Inductive Circuit-element.

supply to the magnetic field which builds up in phase with the current. Then when the current is falling again to zero, power is given back to the supply.

The power circulating between the supply and the inductance is called Reactive Power. Referring again to equation 1·28 it is seen that the quantity ωL is such that when multiplied by current (ampères) a voltage results; hence ωL is measured in ohms, and is the opposition set up by the inductance L to a current having a frequency $f=\frac{\omega}{2\pi}$.

Re-writing 1·28 in RMS values:

$$V=I\omega L \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}29$$

$$\text{or } I=\frac{V}{\omega L}; \text{ or } \omega L=\frac{V}{I}$$

V and I being respectively RMS voltage across and current in the inductance of L henrys.

Also, $\omega L = 2\pi f L = X_L$ ohms . . . . . 1·30

the "reactance" offered by L at frequency f. Note that X_L is proportional to frequency.

Capacitive circuit, sinusoidal current

Equation 1·15 gave the current in a condenser as:

$$i = C\frac{dv}{dt} \text{ ampères}$$

C being the condenser capacitance in farads, and $\frac{dv}{dt}$ the rate of change of voltage across C, in volts/sec. Applied voltage and condenser current are plotted in Fig. 1·17, and it is seen that

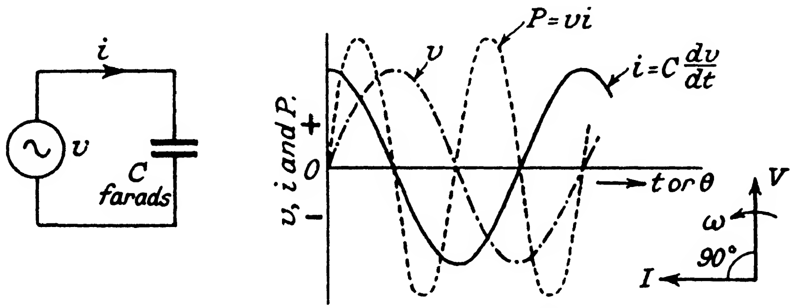


Fig. 1·17. Time-graphs and Vectors for Capacitive Circuit-element.

i leads v by $\frac{1}{4}$-cycle; this is also shown as an angle of lead of $\frac{\pi}{2}$ radians in the vector diagram.

For a sinusoidal supply voltage we have:

$$v = V_{\max} \sin \omega t.$$

So that

$$\frac{dv}{dt} = \omega V_{\max} \cos \omega t = \omega V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad . \quad . \quad 1·31$$

whence the condenser current is:

$$i = C\frac{dv}{dt} = \omega C . V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right) \quad . \quad . \quad . \quad . \quad 1·32$$

i.e. the current waveform is also sinusoidal, leads the voltage waveform by $\frac{\pi}{2}$ radians or $\frac{1}{4}$-cycle, and its amplitude is:

$$I_{\max} = \omega C . V_{\max} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1·33$$

so that $$I = \omega C.V \quad \text{(RMS values)} \quad . \quad 1{\cdot}34$$

and $$V = I \times \frac{1}{\omega C}.$$

The quantity $\frac{1}{\omega C}$ is called the reactance of the condenser, and is the opposition offered to an alternating current of frequency $f=\frac{\omega}{2\pi}$ cycles/sec. It is usually written X_C, so that

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ ohms} \quad . \quad . \quad . \quad . \quad 1{\cdot}35$$

Note that X_C decreases as frequency increases. As in the case of inductance, the power is reactive, its average value being zero. For the condenser, however, power is taken from the supply when v is rising, and given back when v falls again.

Summarising thus far, we see that for resistance or inductance or capacitance carrying sinusoidal current, this current is limited to a definite value for a given applied voltage, the current value being such as to set up a back-voltage equal and opposite to the applied voltage. In the resistance case, this can only occur as a result of a continuous expenditure of energy from the supply at the rate of I^2R watts. In the case of both inductance and capacitance, there is no such continuous expenditure, but instead only a surging to and fro of energy between supply and "load", the load being purely "reactive".

For resistance only we have: $I=\frac{V}{R}$,

for inductance only: $I=\frac{V}{X_L}$,

for capacitance only: $I=\frac{V}{X_C}$.

V in each case being the RMS voltage across the element (R or L or C) and I the RMS current in the element.

Combinations of R, L, and C

Consider first the case of a resistance R ohms and an inductance L henrys connected in series and carrying a sinusoidal current of RMS value I ampères.

Fig. 1·18 shows the circuit and vector diagrams. The vector diagram is conveniently drawn to a scale of RMS values, since these bear a fixed relation to the peak values or amplitudes. It is constructed thus:

First a current vector is drawn, of any suitable length. The voltage vector V_R representing the voltage across R is then drawn, along the current vector since current and voltage are in phase for a resistive element. The length of the V_R vector is determined by the scale chosen (e.g. 10 volts/inch, or 100 volts/inch), the magnitude of V_R being IR volts.

The voltage vector V_L is then drawn, leading the current vector by 90°. Its length is again proportional to its magnitude, which is $I\omega L$ or IX_L volts.

The supply voltage V_S required to maintain the current I in this circuit is then given by finding the vector sum of V_R and V_L as indicated in the diagram. It is seen that there is a phase-angle

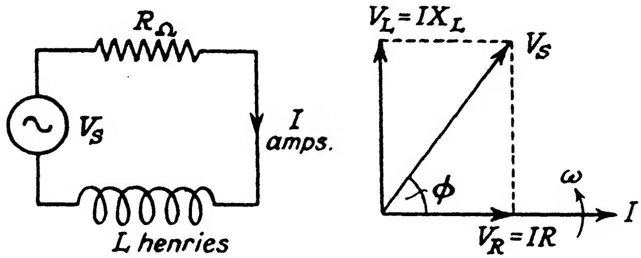


Fig. 1.18. Series Combination of L and R.

ϕ between V_S and I, the current lagging. V_S and I are related by $I=\frac{V_S}{Z}$, where Z is the total opposition or impedance of the circuit.

Circuit impedance

The triangle of voltages shown in Fig. 1·19 is taken from Fig. 1·18. From this we see that:

$$V_S=\sqrt{V_R^2+V_L^2} \quad \ldots \quad 1\cdot36$$

i.e.

$$IZ=I\sqrt{R^2+X_L^2} \quad \ldots \quad 1\cdot37$$

so that

$$Z=\sqrt{R^2+X_L^2} \quad \ldots \quad 1\cdot38$$

This quantity Z is called the impedance of the circuit consisting of R and L in series. It is the total opposition set up by the circuit to an alternating current. It is measured in ohms, and varies with frequency since X_L is proportional to f.

Again, from Fig. 1·19 we see that:

$$V_R=V_S\cos\phi \quad \ldots \quad 1\cdot39$$

The average power taken from the supply will be that dissipated in R, i.e. I^2R watts.

This may also be written, since $V_R = IR$, as:

$$P = I^2R = V_RI \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}40$$

The "apparent power" obtained from the product of supply voltage and circuit current is referred to as the circuit volt-ampères; this is:

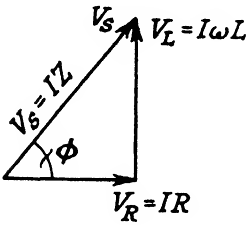


Fig. 1·19.
Triangle of Voltage-vectors from Fig. 1·18.

$$\text{Volt-ampères} = V_SI \quad . \quad 1{\cdot}41$$

Substituting in 1·40 the value of V_R given by 1·39, we have:

$$\text{Power} = V_SI \cos\phi \text{ watts} \quad . \quad 1{\cdot}42$$

So that the ratio

$$\frac{\text{Circuit-power}}{\text{Circuit volt-ampères}} = \cos\phi \quad . \quad . \quad . \quad 1{\cdot}43$$

This ratio is called the power factor, and it can be seen from Fig. 1·19 that:

$$\text{Cos } \phi = \text{power factor of circuit} = \frac{R}{Z} \quad . \quad . \quad . \quad 1{\cdot}44$$

i.e. it is the ratio of circuit-resistance to total impedance. The power factor of an inductive circuit is referred to as a lagging power factor, since the current lags behind the applied voltage. These results also hold for the practical case of an inductive coil possessing distributed resistance, and for such a coil with added

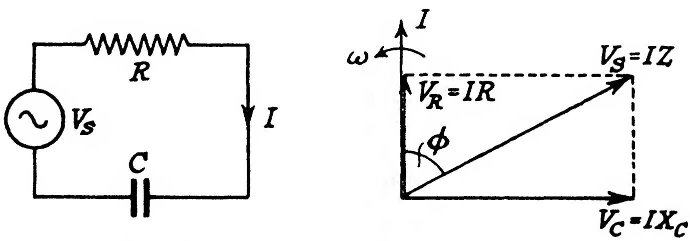


Fig. 1·20. Series Combination of C and R.

external resistance. R is in every case the total circuit-resistance.

Similar results can be obtained for the circuit shown in Fig. 1·20.

In this case we have:

$$V_S = \sqrt{V_R^2 + V_C^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}45$$

i.e.

$$IZ = I\sqrt{R^2 + X_C^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}46$$

so that $$Z=\sqrt{R^2+X_C^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}47$$
=circuit impedance in ohms.

Power factor $=\cos\phi=\frac{R}{Z}$, and is a "leading power factor".

The j notation

The calculation of A.C. circuit problems is often greatly simplified by using the j notation. Fig. 1·21 (a) shows the "impedance triangle" for the circuit shown in Fig. 1·17 (L and R in series, carrying a sinusoidal current). The impedance triangle is obtained by dividing each side of the voltage triangle of Fig. 1·19 by the common current I.

As we have seen, the circuit impedance Z is given by:

$$Z=\sqrt{R^2+X_L^2} \text{ ohms} \quad . \quad . \quad . \quad . \quad 1{\cdot}38$$

i.e Z is the sum of R and X_L *added at right angles.*

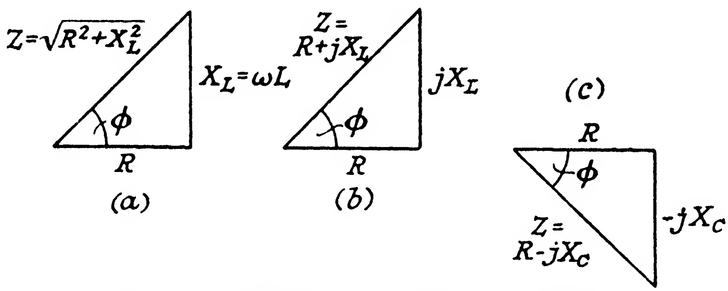


Fig. 1·21. Impedance Triangles, and j Notation.

In the j notation, this is written simply as:

$$Z=R+jX_L \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}48$$

the prefix j indicating that X_L is to be added *at right angles* to R (see Fig. 1·21 (b)).

Thus any quantity, when multiplied by this operator j, is turned through 90° from the reference axis—in this case the axis along which R is set out. Conventionally, the rotation is counter-clockwise.

If this quantity is then turned through a further 90° in the same direction, it will lie along the reference axis and can be added (with due account paid to its sign) directly to any other quantity measured along this axis. Thus if a length R is marked off from O to the right, and a length $+X$ is turned twice through 90° counter-clockwise about the point O, and starting along the direction of

R, it will again be along this axis, but must be regarded now as being equal to $-X$, i.e. turning X through 180° is equivalent to multiplying $+X$ by -1. It is also equivalent to multiplying $+X$ twice by j, so that $j^2=-1$.

Finally, note that swinging X through a further 90° counter-clockwise, i.e. multiplying again by j, we have $j \times j^2 X$, i.e. $-jX$, since $j^2=-1$. This, of course, is the same result as if X had merely been turned through 90° clockwise from the original positive direction along the axis of R.

To sum up: Multiplying a quantity by j means turning it counter-clockwise through 90° from the reference axis; multiplying it by $-j$ means turning it 90° clockwise from the reference axis; multiplying it by j^2 means reversing its original direction along the reference axis, when it can be added or subtracted directly to or from other quantities along this axis.

It follows that, for the circuit of Fig. 1·20 (C and R in series) the circuit impedance will, in this notation, be written as:

$$Z=R+(-j)X_C=R-jX_C \quad . \quad . \quad . \quad . \quad 1{\cdot}49$$

The significance of this is indicated in Fig. 1·21 (c).

It can be seen from, e.g., Fig. 1·21 (a) that not only is the magnitude of Z fixed absolutely by the magnitudes of R and X_L, but so also is the phase-angle ϕ (i.e. the angle by which the voltage applied to the circuit leads the circuit current), for $\tan\phi=\frac{X_L}{R}$. Thus there is yet another way in which we can refer to the circuit impedance, i.e. by writing $Z\angle\phi$, meaning an impedance of magnitude Z ohms, where $Z=\sqrt{R^2+X_L{}^2}=R+jX_L$, and having a ratio of reactance to resistance $\frac{X_L}{R}$ equal to $\tan\phi$.

This notation is particularly useful in simplifying the procedure for obtaining products and quotients of impedances.

We next consider two circuit arrangements in which there is both dissipated and reactive power, but the reactive power may be confined to the "load" circuit and need not continuously surge between this circuit and supply. Such circuits are called "resonant circuits".

L, C, and R in series; Series resonance (Fig. 1·22).

The essential feature of such a series circuit is that the same magnitude and direction of current exists in all parts of the circuit simultaneously. Consequently, the voltages applied across the three components are:

$$V_R = IR, \qquad V_L = IX_L, \qquad V_C = IX_C.$$

These are shown, with their respective phasing, in the vector diagram. The supply voltage is the vector-sum of these components, and although the diagram shows the case of V_L greater than V_C, it is clear that if $V_L = V_C$, the supply voltage V_S is equal simply to V_R and is in phase with it. For any other condition, V_S is greater than V_R and leads or lags relative to current according as V_L or V_C is the greater.

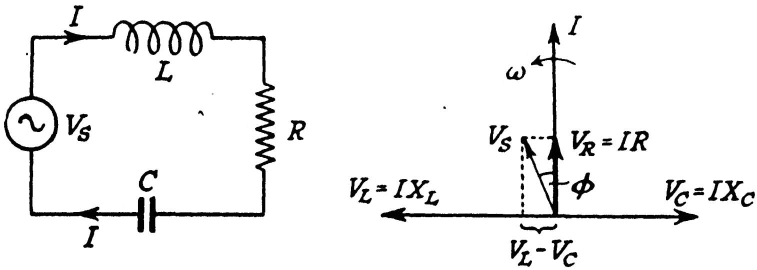


Fig. 1·22. Series Combination of L, R, and C.

The particular case for V_L and V_C equal (but of opposite sign) corresponds to a particular frequency—the "resonance frequency" f_O of the circuit. Thus:

Let $$V_L = V_C,$$

i.e. $$I\omega_O L = I \times \frac{1}{\omega_O C},$$

or $$\omega_O^2 = \frac{1}{LC}; \text{ but } \omega_O = 2\pi f_O,$$

so that $$f_O = \frac{1}{2\pi\sqrt{LC}} \text{ cycles/sec, for } V_L = V_C \qquad 1{\cdot}50$$

L is in henrys, and C in farads.

For the general case (V_L not necessarily equal to V_C), it is seen from Fig. 1·22 that

$$V_S = \sqrt{V_R^2 + (V_L \sim V_C)^2} \text{ volts},$$

the sign $\sim$ indicating "difference between",

i.e. $$V_S = IZ = I\sqrt{R^2 + (X_L \sim X_C)^2}$$

or $$Z = \sqrt{R^2 + (X_L \sim X_C)^2} \text{ ohms}$$
$$= \text{total impedance of circuit.}$$

This may be written, using j notation, as:

$$Z=R+jX_L-jX_C=R+j(X_L-X_C) \text{ ohms} \quad . \quad . \quad 1{\cdot}51$$

Z obviously varies with frequency, the manner in which it does so being shown in Fig. 1·23. For frequencies less than f_O the effective impedance is capacitive, while for frequencies above f_O it is inductive.

The circuit-current for constant applied voltage is also shown in Fig. 1·23. The current is largest at f_O, the resonant frequency, for then the circuit impedance is a minimum, and equal to R.

At f_O therefore:

$$I_{f_O}=\frac{V_S}{R} \text{ ampères} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}52$$

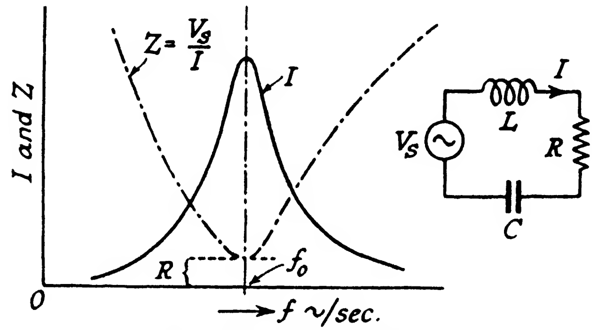


Fig. 1·23. Series Resonance.

Now the reactance of L at f_O is $2\pi f_O L$ ohms, i.e. $\omega_O L$ ohms, so that the voltage across L at resonance is

$$V_L \text{ at } f_O=I_{f_O}\omega_O L$$

$$=\frac{V_S}{R}\cdot\omega_O L \text{ (from 1·52)} \quad . \quad . \quad . \quad . \quad 1{\cdot}53$$

i.e. the ratio $\quad \dfrac{V_L}{V_S}=\dfrac{\omega_O L}{R}$ at resonance.

This ratio may be very large, especially if R is small, and it represents a voltage-magnification. It is an important quantity in tuned circuits and is given the symbol Q;

i.e. $$Q=\frac{\omega_O L}{R}=\frac{\text{Inductive reactance at } f_O}{\text{Circuit-resistance at } f_O} \quad . \quad . \quad 1{\cdot}54$$

The qualification "at f_O" is necessary in connection with R as well as the obvious case of L, because resistance does vary with frequency.

This large voltage across L (Q times the supply voltage, at resonance) is, of course, balanced out by an equal but opposite voltage across C.

The power dissipation *at resonance* is:

$$P = I^2R \text{ watts}$$
$$= V_S I \text{ watts, since } V_S = V_R = IR,$$

so that all the power being drawn from the supply is dissipated, i.e. there is no continuous surging of reactive power between supply and external circuit. Even so, it is clear that large amounts of reactive power exist in the external circuit, since the products $V_L I$ and $V_C I$ are Q times larger than $V_S I$, the dissipated power. The explanation is that the reactive power oscillates between L and C, although it must be obtained from the supply in the first case. This happens during the first few cycles after connection to the supply, the initial conditions being termed "transient" to distinguish them from the "steady state" that we have considered here.

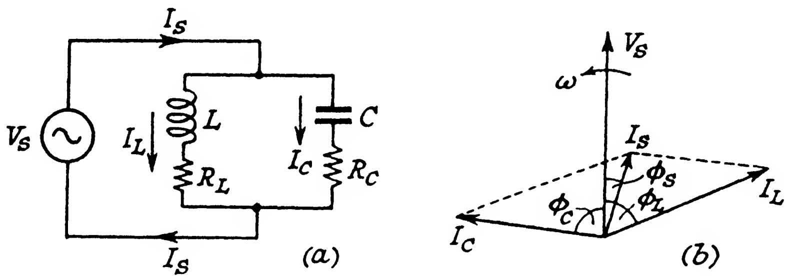


Fig. 1·24. Parallel Combination of L, R, and C.

Parallel-resonant circuit

The general case of this circuit is shown in Fig. 1·24, in which both the inductive and the capacitive branches contain resistance. Each branch has therefore a complex impedance, denoted by Z_L and Z_C respectively.

$$Z_L = \sqrt{R_L^2 + X_L^2} = R_L + jX_L.$$

$$Z_C = \sqrt{R_C^2 + X_C^2} = R_C - jX_C.$$

The rule given in Fig. 1·2 for parallel resistances can be extended to complex impedances. Thus the total impedance presented to the supply points AB is given from:

$$\frac{1}{Z_{AB}} = \frac{1}{Z_L} + \frac{1}{Z_C},$$

i.e.

$$Z_{AB} = \frac{Z_L Z_C}{Z_L + Z_C} \quad \ldots\ldots \quad 1{\cdot}55$$

The denominator of equation 1·55 is the total impedance of the closed circuit consisting of L, R_L, C, and R_C, and will vary with frequency in the manner shown in Fig. 1·23 for a simple series circuit LCR, R being in this case R_L+R_C.

The numerator may be written as:

$$(R_L+jX_L)(R_C-jX_C)$$

which becomes:

$$R_LR_C+jR_CX_L-jR_LX_C+X_LX_C.$$

In the circuits we shall need to consider, the reactance terms X_L and X_C will generally be several times larger than the resistance terms R_L and R_C. Ignoring the smaller terms in the numerator, equation 1·55 may be written as:

$$Z_{AB} \doteqdot \frac{X_LX_C}{Z_L+Z_C} \text{ ohms} \quad . \quad . \quad . \quad . \quad 1{\cdot}56$$

Now $$X_L=2\pi fL=\omega L$$

and $$X_C=\frac{1}{2\pi fC}=\frac{1}{\omega C},$$

so that $X_LX_C=\frac{L}{C}$ and is constant.

In deriving equation 1·50 we had $\omega_O{}^2=\frac{1}{LC}$, ω_O being $2\pi f_O$, where f_O is the resonant frequency of the series LRC arrangement.

Multiplying through by L^2 gives:

$$\omega_O{}^2L^2=\frac{L}{C},$$

so that $$X_LX_C=\frac{L}{C}=(\omega_OL)^2 \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}57$$

That is, the numerator of 1·56 is a constant, and equal to the square of the reactance of L at the resonant frequency of the closed L, R_L, C, R_C system, while the denominator of 1·56 is the series impedance of this closed system. Thus:

$$Z_{AB} \doteqdot \frac{(\omega_OL)^2}{Z_{series}} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}58$$

Z_{AB} therefore varies with frequency in the same manner as the *reciprocal* of the impedance of a series LCR circuit, and it will therefore have a similar frequency characteristic to that of the *current* in a series circuit, since for the latter:

$$I_S=\frac{V_S}{Z_{series}}.$$

Fig. 1·25 shows the impedance-frequency relation for the parallel arrangement of Fig. 1·24. It is seen to be similar to the current-frequency curve of Fig. 1·23.

The supply current I_S is also shown, for constant supply voltage V_S.

The parallel circuit therefore exhibits a rapid rise of impedance presented to the supply, round about a certain frequency. This resonance frequency f_O' will therefore correspond to a minimum supply current (for constant supply voltage), and corresponds with an inductive load corrected to unity power factor, familiar to industrial power engineers. For fixed values of L and C, the frequency of resonance is given from the approximate relation of equation 1·58 as the frequency for which Z series is a minimum, i.e. $f_O' \doteqdot \dfrac{1}{2\pi\sqrt{LC}}$ cycles/sec, the frequency for series resonance f_O.

This approximation is sufficiently accurate for most purposes

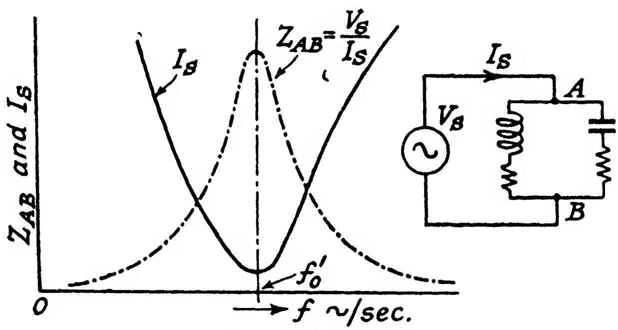


Fig. 1·25. Parallel Resonance.

though, taking as the criterion of resonance that V_S and I_S shall be in phase, the accurate expression for f_O' is $f_O' = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}\left(\dfrac{L-CR_L^2}{L-CR_C^2}\right)}$ cycles/sec.

$$\doteqdot f_O \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 1{\cdot}59$$

At frequencies below f_O, the inductive branch offers the lower impedance and therefore carries the larger current, so that the effective load is inductive. Similarly, at frequencies above f_O the effective load is capacitive. At f_O, the total reactance is zero, i.e. $Z_L = Z_C$, and the circuit as a whole behaves like a resistive load of high ohmic value, its magnitude being:

$$Z_{AB} \text{ at } f_O = \frac{(\omega_O L)^2}{R} = \frac{L}{CR} \text{ ohms} \quad . \quad . \quad . \qquad 1{\cdot}60$$

Now $\dfrac{\omega_O L}{R} = Q$, the magnification factor referred to on p. 28.

The parallel impedance at resonance may therefore be written:

$$Z_{AB} \text{ at } f_O = \frac{\omega_O L}{R} . \omega_O L = Q . \omega_O L \text{ ohms} \quad . \quad . \quad 1{\cdot}61$$

i.e. the resonance-impedance is Q times as large as the reactance of the coil (or the condenser) branch alone at that frequency.

Further, it follows from this that the current I_L is Q times as large as the supply current, at f_O, as also is the condenser current I_C;

i.e. $$I_L \doteqdot I_C \doteqdot QI_S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}62$$

The explanation is that I_L and I_C are (approximately) in opposite phase, I_L lagging and I_C leading by (nearly) ¼-cycle relative to the common supply voltage. Therefore a large *circulating* current exists in the closed *LCR* system, and only a small current is drawn from the supply, in phase with the supply voltage, the power thus drawn supplying the loss in the resistive part of the closed system (see Figs. 1·24 and 1·26).

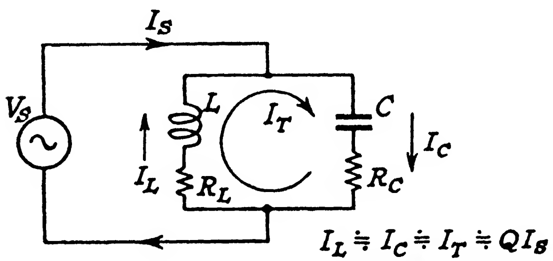


Fig. 1·26.
Current Relations at Resonance, for Fig. 1·25.

It is thus apparent that a large amount of reactive energy may be circulating in the closed *LCR* circuit, and this circuit is therefore often termed a "tank" circuit, implying that it is an A.C. energy-reservoir from which energy can be drawn. The circulating "tank" current is:

$$I_T \doteqdot QI_S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}63$$

H.F. power generators employing a valve oscillator commonly use a tank circuit of this kind. Also, the furnace coil and power-factor-correction condensers form a similar tank circuit in the case of the H.F. alternator plant. In both cases it would be hopelessly uneconomic to have the whole of the required coil-current flowing through the power source.

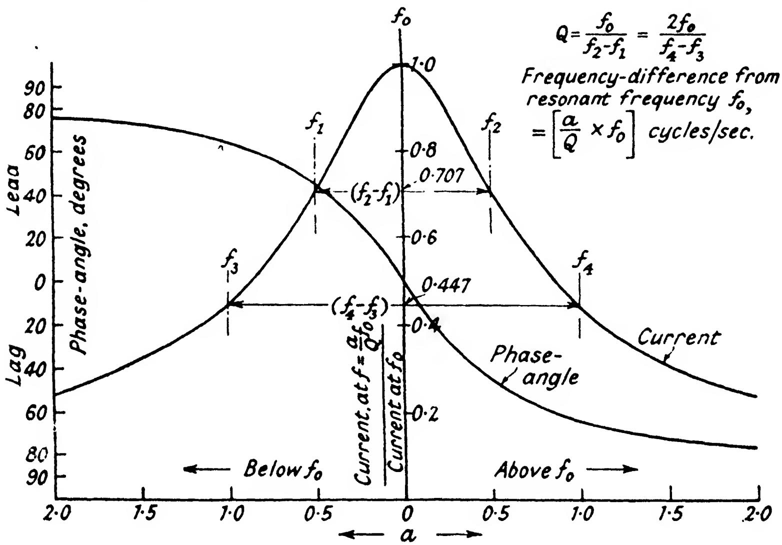
Generalised resonance relations

It is possible to show the behaviour of both series and parallel resonant circuits by means of a single pair of curves, provided that the Q value is reasonably high, say not less than 10.[3]

The current-frequency characteristic for the series circuit and the phase angle of circuit current in relation to the supply voltage are plotted in Fig. 1·27. These curves also represent circuit impedance and phase angle for the parallel circuit, except that "lead" and "lag" must be reversed for this.

Natural oscillations in an LCR system

A circuit arrangement employed in arc and spark H.F. generating plants, and also of importance in the theory of the valve oscillator, is shown in Fig. 1·28 (a).



[Courtesy of Messrs. Iliffe.

Fig. 1·27. Generalised Resonance Curves.

With switch S in position 1, the condenser C charges up to the supply voltage V, and thus receives and stores an amount of energy $\frac{1}{2}CV^2$ joules or watt-seconds (C in farads).

When S is thrown to position 2, C will discharge through L and R, and provided that R is less than a certain critical value, the discharge current is oscillatory, as shown in Fig. 1·28 (b). The oscillation is termed a "natural oscillation" as distinct from forced oscillations under the influence of a voltage applied from an external source.

The frequency of natural oscillation(4) is:

$$f_N = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \text{ cycles/sec} \quad . \quad . \quad . \quad . \quad 1{\cdot}64$$

but the second term under the root sign, $\frac{R^2}{4L^2}$, does not affect the value of f_N very greatly unless R is so large as to make the oscillation very highly damped. Hence the frequency is given approximately by:

$$f_N \doteqdot \frac{1}{2\pi\sqrt{LC}} \text{ cycles/sec} \quad . \quad . \quad . \quad . \quad 1{\cdot}65$$

i.e. f_N is very nearly the same as both the series and parallel resonance frequencies of this circuit, although all three differ slightly.

The value of R which just makes the circuit aperiodic (non-

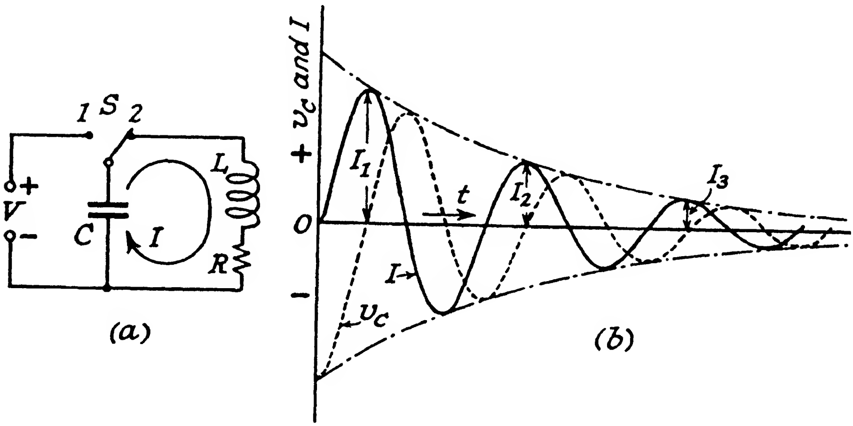


Fig. 1·28. Damped Oscillatory Discharge.

oscillatory) is the value which makes the whole term under the root sign zero, i.e.

$$\frac{R^2}{4L^2} = \frac{1}{LC}, \text{ or } R_{\text{crit}} = 2\sqrt{\frac{L}{C}} \text{ ohms} \quad . \quad . \quad . \quad 1{\cdot}66$$

The oscillation is damped, i.e. successive amplitudes become smaller, as the energy is converted into heat in the circuit resistance R. Neglecting radiation of energy away from the circuit, the whole of the energy is ultimately used up in this way. Therefore, if t secs be the time taken for such an oscillation to die away to a negligible amplitude, the RMS current in the resistance is given from:

$$I^2Rt = \tfrac{1}{2}CV^2, \text{ i.e. } I = V\sqrt{\frac{C}{2Rt}} \text{ ampères} \quad . \quad . \quad 1{\cdot}67$$

If N such "trains" of oscillations occur every second (e.g. by rocking S to and fro N times/sec) the total energy supplied to R per second is $\frac{NCV^2}{2}$ joules/sec, i.e. the average power is:

$$P=\frac{NCV^2}{2} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}68$$

The first current maximum of the discharge has a value of $V\sqrt{\frac{C}{L}}$ ampères, assuming that the energy-loss in R up to this point has been negligible. For then the voltage of C will be zero at the moment of current maximum, so that the whole energy of the system is at this moment stored in the magnetic field of the inductance L; then:

$$\tfrac{1}{2}CV^2=\tfrac{1}{2}LI^2 \text{ joules} \quad . \quad . \quad . \quad . \quad 1{\cdot}69$$

or

$$I=V\sqrt{\frac{C}{L}} \text{ ampères.}$$

The ratio of successive amplitudes in the same direction is given by:

$$\frac{I_1}{I_2}=\frac{I_2}{I_3}, \text{ etc.}=e^{\frac{R}{2Lf_N}}$$

and is called the decrement of the circuit.

The natural logarithm of this is more convenient, and is:

$$\log.\text{dec}=\log_e\frac{I_1}{I_2}=\frac{R}{2Lf_N}.$$

Other equivalent expressions are:

$$\log.\text{dec}=\pi R\sqrt{\frac{C}{L}}=\pi R\omega C=\frac{\pi}{Q} \quad . \quad . \quad . \quad 1{\cdot}70$$

Q being, as before, the ratio $\frac{\omega_0 L}{R}$.

Magnetically-coupled circuits

Energy can be transferred from one circuit to another without the necessity for a conductive connection between the two. An example was seen in mutual inductance coupling. There are two kinds of such magnetically-coupled circuits which are of importance—"iron-cored" transformers and "air-cored" transformers.

The theory of the iron-cored transformer is fully dealt with in many text-books, and only a summary of the elementary treatment is given here.

The primary winding is energised from an A.C. power source, and an alternating flux is produced in the core. If this flux is sinusoidal, and links completely with the whole of both primary and secondary windings (Fig. 1·29), these having T_1 and T_2 turns respectively, then the RMS terminal voltages on no-load will be:

$$V_1 = 4{\cdot}44 f T_1 \Phi_{max}\, 10^{-8} \text{ volts} \quad . \quad . \quad 1{\cdot}71 \text{ (a)}$$

$$V_2 = 4{\cdot}44 f T_2 \Phi_{max}\, 10^{-8} \text{ volts} \quad . \quad . \quad 1{\cdot}71 \text{ (b)}$$

The maximum value of the total flux, Φ_{max}, is the product of flux-density B_{max} and effective core cross-section area A cm². In transformers for 50 cycles/sec, B_{max} may be as high as 14,000 lines/cm², while peak densities of a few hundred lines/cm² only are permissible at frequencies of several Kilocycles/sec, otherwise excessive "iron loss" results. The iron loss is made up from two components:

(1) Eddy-current loss, proportional to $f^2 B_{max}^2$, for "thin" laminations. If the laminations are so thick, or the frequency so high, that "skin effect" is appreciable, then the eddy-current loss is proportional to $\sqrt{f}$ for a given B_{max}.

(2) Hysteresis loss, proportional to $f B_{max}^{\alpha}$, and is independent of lamination thickness. ($\alpha \doteqdot 1{\cdot}6$ to $1{\cdot}8$ for most steels.)

For high-frequency operation the core laminations must be extremely thin, in order that the eddy-current loss shall be not excessive and that the flux density shall be uniform—i.e. the iron utilisation factor shall be high.* Thus for 10 KC/S, the lamination thickness should not be greater than ·005″, and preferably less (e.g. ·002″). There is, however, considerable difficulty in manufacturing high-silicon steels thinner than ·005″.

The no-load primary current required to produce the flux is given approximately from:

$I_o = \sqrt{I_m^2 + I_e^2}$ ampères (see Fig. 1·30 (a))

I_m = "magnetising component" of I_o

$$\doteqdot \frac{\text{(Ampère-turns per cm length of flux path)} \times \text{(path length, cms)}}{\sqrt{2}\; T_1}$$

I_e = "energy component" of I_o

$$\doteqdot \frac{\text{(Watts loss per lb. weight of core, for given } B_{max} \text{ and } f) \times \text{(core weight)}}{V_1}$$

* See, e.g., (1) "Medium Frequency Magnetization of Sheet Steel," R. Pohl *J.I.E.E.*, Vol. 94, Part II, No. 38.

(2) "Iron-Loss Measurements by A.C. Bridge, and Calorimeter," J. Greig and H. Kayser, *J.I.E.E.*, Vol. 95, Part II, No. 44.

(3) Fig. 1·31, with the exception of the curves for ·005″ stalloy and ·002″ nickel-iron alloy, is reproduced from the paper by Greig and Kayser.

The ampère-turns required per cm length of flux path, to produce B_{max}, and the watts loss per lb weight of core material, are read from curves supplied by the steel manufacturer. See Fig. 1·31 for losses at medium frequencies.

On load, the secondary terminal voltage V_2 will have nearly the value given by equation 1·72 if leakage fluxes are small, and the secondary load current will be, approximately:

$$I_2 = \frac{V_2}{Z_{load}} = \frac{V_2}{\sqrt{R^2+X^2}} \text{ ampères} \quad . \quad . \quad . \quad 1{\cdot}72$$

Again, for "tight coupling" corresponding to very small leakage fluxes:

$$V_1 I_1 \doteqdot V_2 I_2$$

I_1 being the on-load primary current; and

$$I_1 T_1 \doteqdot I_2 T_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}73$$

Now the ratio $\frac{V_2}{I_2} = Z$, the load impedance, and since the voltage ratio is equal to the turns ratio:

$$I_2 = \frac{V_2}{Z} = \frac{T_2}{T_1} \cdot \frac{V_1}{Z}.$$

Also, from 1·73, $I_1 = \frac{T_2}{T_1} I_2$, so that the ratio $\frac{V_1}{I_1}$ = effective impedance presented to supply is

$$Z_{eff} = \frac{V_1}{I_1} = \frac{V_1}{\left(\frac{T_2}{T_1}\right)^2 V_1} . Z$$

$$= \frac{Z}{K^2} \text{ ohms, where } K = \frac{T_2}{T_1} \quad . \quad . \quad 1{\cdot}74$$

Thus the transformer is an impedance-matching device. That is, a load having an actual impedance of Z ohms is made to appear as if it were an impedance of $\frac{Z}{K^2}$ ohms to the source of supply, by using a transformer with a turns-ratio $\frac{T_2}{T_1} = K$. A vector diagram of conditions in an ideal transformer (zero resistance and leakage) is given in Fig. 1·30 (b).

The two orthodox types of core construction are illustrated in Fig. 1·29. Recently a new kind of construction has been described, which uses a continuous rolled strip of silicon steel or high-permeability nickel-iron alloy, annealed, impregnated and baked after "winding" to form a core.(5) The core is subsequently cut into two parts, the coils slipped on, and the core and coils assembly held together in a suitable frame, or with bands. The surfaces forming the butt-joints in the core are carefully ground, and each joint is said to be the equivalent of an air-gap of only ·0005″. This construction

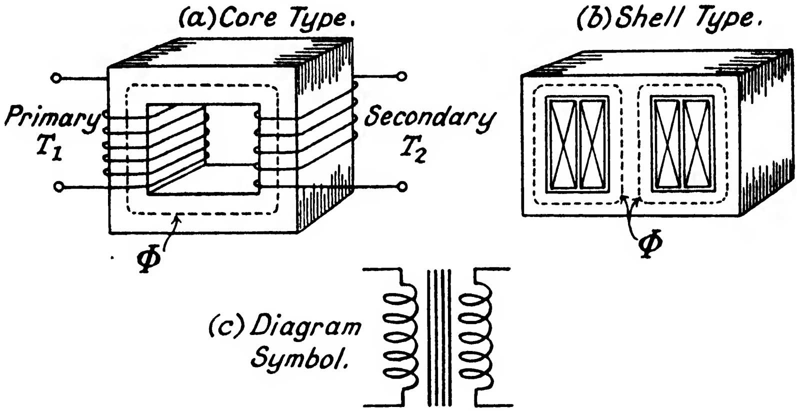


Fig. 1·29. Iron-core Transformers.

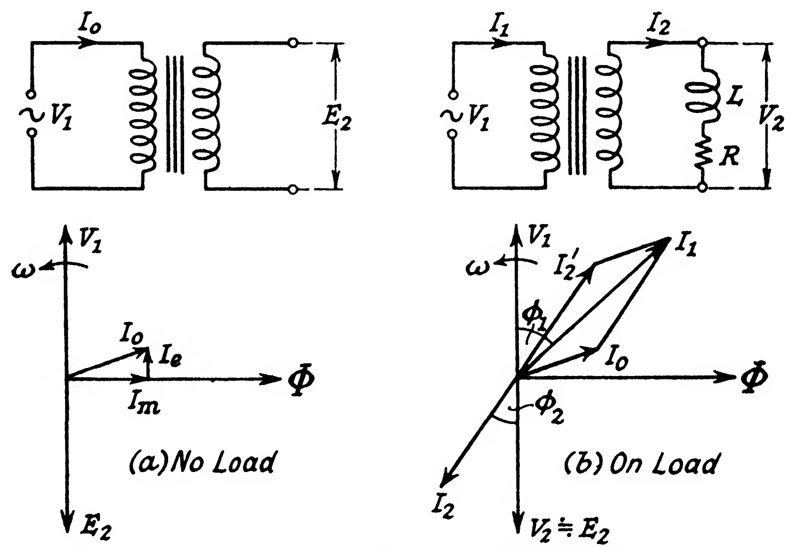


Fig. 1·30. Vector Diagrams f Ideal Transformer.

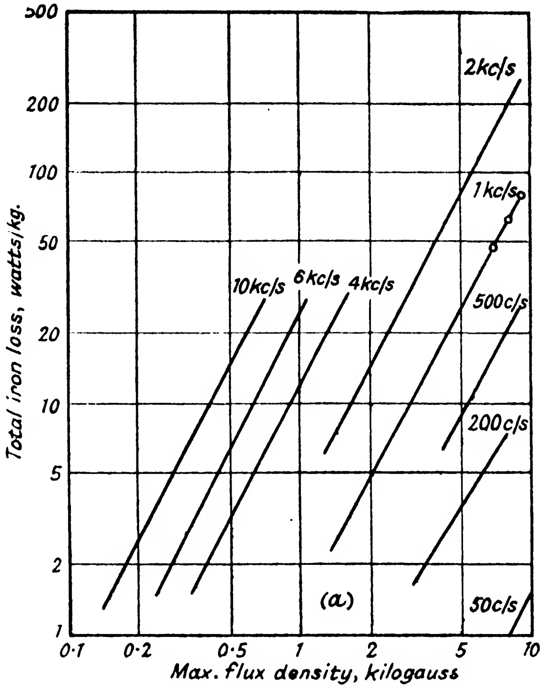
seems particularly well suited to transformers working at relatively high frequencies, as the very thin steel required is easier to handle in this form. There is also a saving of about 30% in volume and weight as compared with ordinary steels, owing to the higher

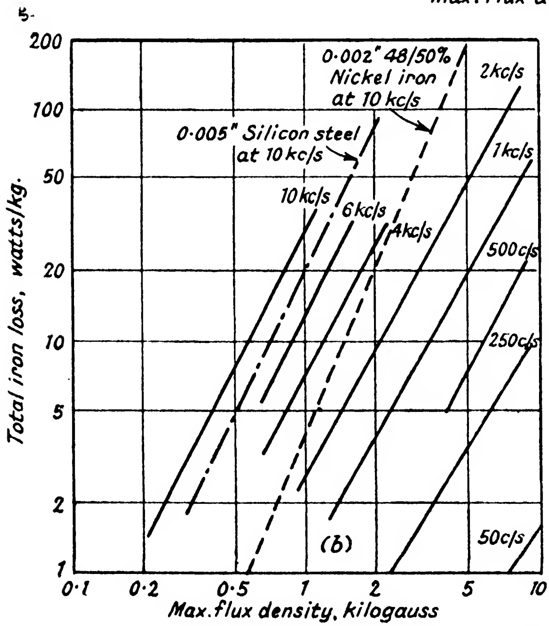
(a) ·014″ Silicon steel.

(b) ·007″ Silicon steel.

— - — ·005″ Silicon steel at 10 KC/S.

- - - - ·002″ 48/50% Nickel-iron at 10 KC/S.





(a) and (b) are reproduced from "Iron-Loss Measurements by A.C. Bridge and Calorimeter," Prof. J. Greig and H. Kayser, *J.I.E.E.*, Vol. 95, Part II, No. 44, by courtesy of the Institution of Electrical Engineers. The other two curves are included by courtesy of G. K. and N. Research Laboratories and Birlec Ltd.

Fig. 1·31. Total Iron Loss.

permissible flux-densities in these special alloys (e.g. Hipersil, Hypernik).

Equation 1·71 shows that, for a given frequency and voltage, the product $T\Phi_{max}$ is fixed, so that a very large core and few turns, or many turns and a small amount of iron, may be used. Obviously, a happy medium is generally chosen, in which the total normal-load losses are approximately equally divided between the iron and the copper.* Since the core flux is very nearly constant at all loads, the iron-loss is constant, while the copper-loss, being proportional to I^2, increases as the square of the load-power, if load voltage and power factor remain constant.

Copper current densities range from 1500 ampères/sq. in. to 3000 ampères/sq. in., depending upon the method of cooling, and for high frequencies skin effect must be allowed for. Some high-power high-frequency transformers utilise water-cooled copper tubing instead of solid windings, and current densities may then be about 10,000 ampères/sq. in.

Equation 1·71 also indicates that a reduction in size accompanies an increase in frequency for a given KVA rating, since both T and Φ_{max} can be reduced for a given voltage. On the other hand, B_{max} must also be reduced as the frequency is increased.

At a given frequency the power output of transformers increases as the fourth power, and losses increase as the cube, of the linear dimensions. The available cooling surface area, however, increases only as the square of the linear dimensions, so that artificial cooling is resorted to for ratings above 5 KVA or so, at 50 cycles/sec. The rating depends upon the permissible temperature rise due to the total losses. These losses are most readily determined from open-circuit and short-circuit tests at normal frequency, with a wattmeter measuring input power. The O/C test is made at normal primary volts, and the total power input may be taken as iron-loss. The S/C test is made at a low primary voltage, just sufficient to give normal full-load current in the short-circuited secondary, and the total power input is taken as being entirely copper-loss.

For some purposes, as, for example, in the arc-oscillators described

* This equal division is not, however, practicable in high-power H.F. transformers with water-cooled windings. In these it is easy to extract the copper-loss heat, but much more difficult to keep the iron cool; a high ratio of copper to iron loss is therefore usual. Efficiently water-cooled cores, made, e.g., from solid blocks of compressed "dust" and drilled for water passages, would, by permitting higher flux-densities to be used, reduce the copper loss and improve the efficiency. The new material "Ferroxcube" (see *Philips Technical Review*, Vol. 8, No. 12) has interesting possibilities.

in Chapter 2, transformers having very large leakage reactance are used. That is, a considerable fraction of the flux produced by each winding does not link the other winding. The transformer then behaves as if separate inductances were added in the primary and secondary circuits, and the quantity ($\omega \times$ "leakage inductance") is called the leakage reactance. This transformer reactance is usually quoted in terms of the reactance voltage, developed at normal full-load, expressed as a percentage of the rated voltage. When used to supply an inductance load the effect is to reduce the load voltage below the rated open-circuit value, but when the load is capacitive the terminal voltage is *increased*.

On the other hand, iron-cored "work-head" transformers used for induction-heating, at frequencies up to 10 KC/S, commonly work into a circuit having a lagging power factor of the order of ·1, and the leakage reactance of such transformers is made as low as possible.

Outline of the design procedure for iron-cored H.F. work-head transformers

Work-head transformers for induction heating usually have a single-turn secondary winding to supply a large current, often several thousands of ampères to a single-turn work-loop. The power factor of such a load is invariably low, e.g. ·25 lagging or less, and since it is impracticable to use low-voltage heavy current capacitors across the work-loop, the transformer must be rated for the full KVA rating and not merely the KW rating, i.e. the rating is $\frac{\text{KW}}{\text{P.F.}}$ Kilovolt-ampères. Correction capacitors are connected across the primary winding to bring up the power factor presented to the supply to approximately unity.

Knowing the power input required to the work, the work-loop current to develop this power is found from:

$$P = I_2^2 R_C \text{ watts,}$$

where R_C is the "coupled resistance", due to the work, referred to the work-loop (see Chapter 6).

The magnitude of I_2 is practically determined by the reactance of the work-loop and its leads or feeders, and the leakage reactance of the transformer. The total reactance, referred to the primary (supply) side, is:

$$X_{\text{eff}} = T_1^2[\Delta + X_w] \text{ ohms,}$$

where X_w = reactance of loaded work coil and leads,
Δ = reactance of transformer, referred to single-turn secondary.

For a simple concentric arrangement of primary and secondary windings:

$$\Delta \doteqdot \frac{8\pi f D d . 10^{-8}}{L} \quad . \quad . \quad . \quad . \quad . \qquad 1{\cdot}75^*$$

where D = mean diameter of the two windings, cms,
d = radial air gap between windings, cms,
L = axial length of windings, cms.

It will generally be possible to design the transformer so as to have a leakage reactance less than 10% of that of the load. Clearance between primary and secondary must be kept to the minimum required for sufficient electrical insulation, and work-loop leads must be as short and as close together as possible. A convenient low-reactance core-type arrangement is to have concentric primary and secondary cylindrical windings on each vertical limb, with the two secondary shells in parallel, the output leads consisting of three interleaved flat straps of considerable depth and minimum spacing (see Fig. 9·23 (b)). These notes refer, however, to a single concentric arrangement on one limb.

Assuming a leakage-reactance 10% of that of the work-loop:

$$X_{eff} = 1{\cdot}1\ T_1{}^2 X_w \text{ ohms.}$$

Then, since

$$I_1 \doteqdot \frac{V_1}{X_{eff}} \doteqdot \frac{I_2}{T_1},$$

we have

$$T_1 = \frac{V_1}{1{\cdot}1 I_2 X_w} \text{ turns} \quad . \quad . \quad . \quad . \qquad 1{\cdot}76$$

The core cross-sectional area then follows from:

$$A = \frac{V_1 \times 10^{-8}}{4{\cdot}44 f T_1 B_{max}} \text{ cm}^2.$$

The value of D can now be chosen.

Axial winding length L is determined by the permissible copper loss; each winding is a cylindrical current sheet, the currents penetrating to a depth of p cms (see Fig. 6·32). The total copper loss for both windings, referred to the secondary current, is therefore:

$$P_{Cu} = I_2{}^2 R_{Cu} = I_2{}^2 \left[\frac{4\pi^2 D \sqrt{\rho f} . 10^{\ 9}}{L}\right] \text{ watts,}$$

* Only the axial component of leakage flux is considered here. For a treatment of factors influencing transformer leakage reactance, see A. Langley Morris, *J.I.E.E.*, Vol. 86, No. 521.

ρ being the copper resistivity in E.M. units, about 2000 E.M.U. at 70° C,

i.e. $$L \doteq \frac{I_2^2 \times 2D\sqrt{f}.10^{-6}}{P_{\text{Cu}}} \text{ cms} \quad . \quad . \quad . \quad . \quad 1{\cdot}77$$

It is now possible to check that the assumed transformer reactance of 10% that of the load can be realised or, preferably, reduced. From equation 1·75 we obtain an expression for the maximum permissible value of d, the gap between windings:

$$d_{\max} = \frac{\cdot 1 X_w L . 10^8}{8\pi f D} \text{ cms.}$$

If it is practicable to make d less than this the leakage will be less than 10%.

EXAMPLE

A work-head transformer is required to supply 75 KW to a load having a resistance of ·003 ohm and a reactance of ·015 ohm (i.e. P.F. = ·2 lagging). Primary supply is 500 volts, 10,000 cycles/sec. Transformer copper-loss not to exceed 5 KW.

Secondary current required:

$$I_2 = \sqrt{\frac{75{,}000}{\cdot 003}} = 5000 \text{ ampères.}$$

Primary turns, assuming 10% leakage reactance:

$$T_1 = \frac{500}{1{\cdot}1 \times 5000 \times \cdot 015} = 6 \text{ turns.}$$

Core cross-sectional area, for a core material for which the permissible $B_{\max}$ is 1250 lines/cm² at 10,000 cycles/sec, is:

$$A = \frac{500 \times 10^8}{4{\cdot}44 \times 10{,}000 \times 6 \times 1250} = 150 \text{ cm}^2.$$

The mean winding diameter D is therefore about 20 cms. Axial winding length L is given from:

$$L = \frac{5000^2[2D\sqrt{f}.10^{-6}]}{5000} = 20 \text{ cms.}$$

The value of d, the gap between windings, for 10% reactance is:

$$d_{\max} = \frac{\cdot 0015 \times 20 \times 10^8}{8\pi \times 10{,}000 \times 20},$$

i.e. $$d_{\max} = \cdot 596 \text{ cm.}$$

This value is easily realisable, and could in fact be somewhat reduced.

The total effective reactance presented to the supply is:

$$X_{\text{eff}} = 1{\cdot}1 T_1^2 \times \cdot 015 = \cdot 594 \Omega,$$

and since this is large compared to the resistance:

$$I_1 \doteqdot \frac{500}{\cdot 594} = 842 \text{ ampères},$$

so that Input KVA $= 500 \times 842 \times 10^{-3} = 421$ KVA.

$$\text{Supply power factor} = \frac{75+5}{421} = \cdot 19 \text{ lagging},$$

and capacitors of about 420 KVA are required to bring this to unity.

Core loss has been neglected here. Magnetising current may be calculated from the induction curve of the material, or, if the the magnetising volt-ampères per pound weight is known, from

$$I_m = \frac{\text{Mag. VA/lb} \times \text{core weight}}{\text{Supply voltage}}.$$

It may be asked, why use a transformer at all; i.e. why not use a multi-turn work-coil operating directly from the high-voltage supply? In many induction-heating applications it is necessary to arrange for accurately localised heating; especially is this so in surface-hardening applications, where very high power-concentrations are required.* In such cases the work-coil should have small clearance from the work, and its axial length is also small, so that it is impracticable to use a high-voltage multi-turn work-coil from both insulating and cooling considerations. A transformer makes it possible to use a single-turn low-voltage heavy-current work-loop which can be easily water-cooled. Another important point is that the single-turn inductor may be machined from a solid block to a high degree of precision; it is in fact a machine-part and not simply an electrical winding.

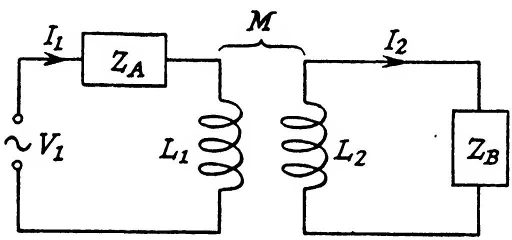


Fig. 1·32. Magnetically-coupled Circuits.

Magnetically-coupled circuits with low coupling factor

Where the magnetic coupling between primary and secondary circuits is loose, and where, consequently, "leakage" fluxes are large, it is convenient to refer to the self and mutual inductances of the circuits for analysis of behaviour, rather than to try to extend the simple treatment already given for the iron-cored transformer with approximately unity coupling-factor. Of course, the

* See, for example, Fig. 6·21.

analysis which follows can be applied to the iron-cored transformer, but it is not usually necessary to do this.

The general case of magnetically-coupled circuits is shown in Fig. 1·32.

L_1 represents that part of the total inductance of the primary circuit which is magnetically linked with L_2, the corresponding part of the total inductance of the secondary circuit.

M is the *mutual* inductance between L_1 and L_2, and is equal to $K\sqrt{L_1L_2}$, K being the coupling factor.

When an alternating current of RMS value I_1 and frequency f flows in L_1, an E.M.F. is induced in L_2, of RMS value

$$E_2 = -j\omega M I_1 \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}78$$

($\omega = 2\pi f$); $-j$ indicates that E_2 lags $\frac{1}{4}$-cycle behind the current I_1.

Z_A and Z_B are the impedances of primary and secondary circuits respectively, other than the reactances of L_1 and L_2; the total series primary impedance alone is therefore:

$$Z_1 = Z_A + j\omega L_1 \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}79$$

and the series impedance of the secondary circuit is:

$$Z_2 = Z_B + j\omega L_2 \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}80$$

The induced secondary-voltage E_2 will drive a current I_2 round the secondary circuit, given by:

$$I_2 = \frac{E_2}{Z_2} = \frac{-j\omega M I_1}{Z_2} \text{ ampères} \quad . \quad . \quad . \quad 1{\cdot}81$$

This alternating current will induce a voltage in the primary of $-j\omega M I_2$ volts, and this must be opposed by an equal but opposite component in the supply voltage V_1. The total supply voltage is therefore:

$$\begin{aligned} V_1 &= I_1 Z_1 + j\omega M I_2 \\ &= I_1\left[Z_1 + \frac{(j\omega M)(-j\omega M)}{Z_2}\right] \\ &= I_1\left[Z_1 + \frac{\omega^2 M^2}{Z_2}\right]. \end{aligned}$$

Then $\dfrac{V_1}{I_1}$ = effective impedance presented to supply $= Z_{\text{eff}}$,

i.e. $$Z_{\text{eff}} = \frac{V_1}{I_1} = Z_1 + \frac{\omega^2 M^2}{Z_2} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}82$$

The coupled impedance

$\dfrac{\omega^2 M^2}{Z_2}$ is called the "coupled impedance" and represents the equivalent impedance, due to the secondary circuit, which is in effect put in *series* with the primary self-impedance Z_1.

Since Z_2 is complex (except at resonance, when it behaves like a pure resistance) the coupled impedance will be complex:

$$Z\text{ coupled}=\frac{\omega^2M^2}{Z_2}=\frac{\omega^2M^2}{R_2+jX_2}.$$

This expression is rationalised by multiplying numerator and denominator by (R_2-jX_2), yielding:

$$Z\text{ coupled}=\frac{\omega^2M^2(R_2-jX_2)}{R_2{}^2+X_2{}^2}=\frac{\omega^2M^2(R_2-jX_2)}{Z_2{}^2} \quad . \quad 1{\cdot}83$$

The series-coupled resistance in the primary circuit is therefore:

$$R\text{ coupled}=\frac{\omega^2M^2}{Z_2{}^2}R_2\text{ ohms} \quad . \quad . \quad . \quad . \quad 1{\cdot}84$$

and the series-coupled reactance is:

$$X\text{ coupled}=\frac{-\omega^2M^2}{Z_2{}^2}.X_2\text{ ohms} \quad . \quad . \quad . \quad 1{\cdot}85$$

Note the change of sign in the reactance case. This means that an inductive secondary circuit behaves like series capacitance in the primary circuit, while a capacitive secondary behaves like a series inductance in the primary.

It may also be noted that if the secondary circuit is a tuned circuit operating at its series-resonance frequency, then the coupled impedance is a pure resistance, of high ohmic value if M is large and R_2 small.

Degree of coupling for maximum energy transfer to secondary circuit

The secondary circuit power is:

$$P_2=I_2{}^2R_2\text{ watts}$$

and for a given value of R_2 this will be a maximum when I_2 is a maximum.

Now I_2 is given in terms of the mutual inductance M, the impedances Z_1 and Z_2, and the primary supply voltage, by combining equations 1·81 and 1·82:

$$I_2=\frac{\omega MI_1}{Z_2}\text{ in magnitude, and }I_1=\frac{V_1}{Z_{\text{eff}}}$$

i.e.
$$I_2=\frac{\omega M}{Z_2}\cdot\frac{V_1}{Z_1+\dfrac{\omega^2M^2}{Z_2}}=V_1\cdot\frac{\omega M}{Z_1Z_2+\omega^2M^2} \quad . \quad . \quad 1{\cdot}86$$

This expression has a maximum value when

$$Z_1Z_2=\omega^2M^2,$$

i.e.
$$M=\frac{\sqrt{Z_1Z_2}}{} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}87$$

gives the coupling for maximum energy transfer to the secondary circuit.

Equation 1·86 indicates also that for a given degree of coupling there is a particular frequency for maximum energy transfer, given by:

$$\omega=\frac{\sqrt{Z_1Z_2}}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}88$$

Equation 1·87 expresses a particular case of a general principle, that maximum power is developed in a load when its impedance, as seen by the source, is equal to that of the source. Thus we have, from 1·87,

$$Z_1Z_2=\omega^2M^2$$

for maximum power, and the total effective primary impedance is

$$Z_{\text{eff}}=Z_1+\frac{\omega^2M^2}{Z_2}$$

$$=Z_1+\frac{Z_1Z_2}{Z_2}=2Z_1.$$

The coupled impedance must therefore be equal to Z_1 for maximum load power. This, however, although a necessary condition, is not a sufficient one, and is amplified in Appendix B.

The efficiency of power transfer to the load depends only upon the coupled resistance R_C and the primary ("source") resistance R_1;

ie. $$\eta=\frac{R_C}{R_1+R_C}.$$

It does not follow that the optimum coupling in any particular case is that given by equation 1·87; many transformers and other pieces of electrical equipment would be working at considerable overload and relatively low efficiency under such conditions, i.e. they are usually designed to supply power to a load impedance much higher than their internal impedance. An induction-heating transformer with an untuned secondary, for example, should be designed to have as low internal impedance as possible, to minimise the ratio of input to output KVA.

The air-cored auto-transformer

A simple matching device, sometimes used in induction-heating practice, consists of a coil, to the ends of which the high-frequency source is connected; the actual heating coil, usually a single turn, is then tapped across one or two turns only of the "primary" coil. The equations derived for two separate magnetically-coupled circuits can be applied to the calculation of this auto-transformer, the appropriate expression for the mutual inductance between primary and secondary circuits being found thus:

Neglecting losses, equation 1·82 becomes

$$X_{\text{eff}} = X_1 - \frac{\omega^2 M^2}{X_2} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}89$$

X_1 is the reactance of the whole ("primary") coil without secondary connections; X_2 is the reactance of the secondary circuit (i.e. $X_2 = X_S + X_{\text{wc}}$; see Fig. 1·33 (d)).

Then $$\omega M = \sqrt{X_2(X_1 - X_{\text{eff}})}$$

or $$M = \sqrt{L_2(L_1 - L_{\text{eff}})} \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}90$$

M is, therefore, easily determined from open- and short-circuit tests; that is:

$$\omega M = \sqrt{X_S(X_1 - X_{\text{s/c}})} \quad . \quad . \quad . \quad . \quad 1{\cdot}91$$

X_S being the reactance of the secondary with primary open, and $X_{\text{s/c}}$ being the effective primary reactance with secondary short-circuited (see Fig. 1·33 (a), (b), and (c)).

The power factor, even on-load, is always low (e.g. ·1), so that little error is involved in this, or in the following expression for work-coil current, by using the approximation that reactances are numerically equal to impedances.

From 1·78 $$E_2 = \omega M I_1,$$

and from 1·81 $$I_2 = \frac{E_2}{X_2} = \frac{E_2}{\omega(L_S + L_{\text{wc}})}.$$

The ratio of secondary to primary current is, therefore:

$$\frac{I_2}{I_1} = \frac{M}{(L_S + L_{\text{wc}})} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}92$$

It is important to keep the effective inductance L_{wc} of the work coil as small as possible, in order that a large current shall circulate in it. This means tight coupling between the work-coil and the work, and short closely-spaced leads from the transformer to the coil. A current step-up of about 5 is practicable; there is not usually any point in tapping across more than one or two turns of the primary, and for this condition the coupling factor K is about ·6 to ·7, where:

$$K = \frac{M}{\sqrt{L_1 L_S}}.$$

When used as a matching device between, say, a valve oscillator and a heating coil, fine control of power-loading can be obtained by shifting the primary connections, or by altering the position along the primary at which the "secondary" is brought out.

It is permissible to have two or more secondary circuits on one such auto-transformer; if two similar work-coils are used, they should be connected at points symmetrically either side of the centre of the coil.

Like the "focus inductor" or "concentrator" transformer

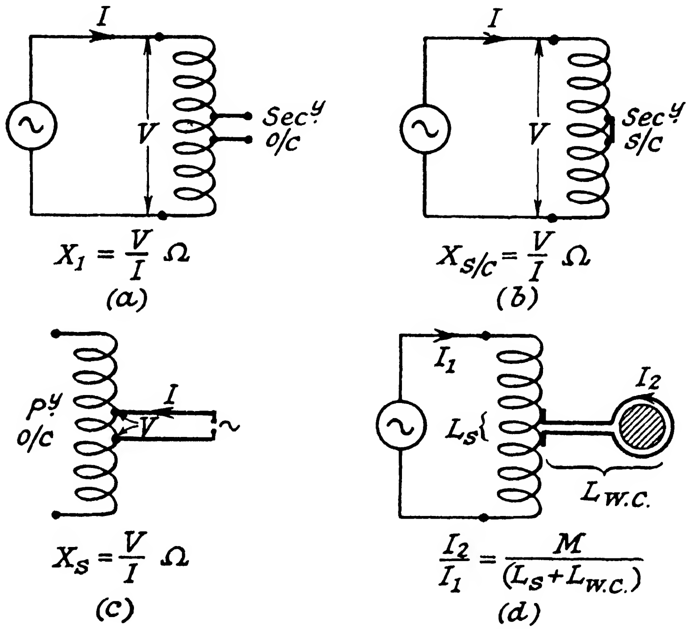


Fig. 1·33. Air-core Auto-transformer.

(see Chapter 6) an auto-transformer is used where conditions do not permit a long work-coil, of several turns, to be used. In general, it is only satisfactory provided that the work-coil diameter is small relative to that of the primary coil (say $\frac{1}{3}$ or $\frac{1}{2}$ maximum). Its use permits (and requires) close coupling to the work-piece as the work-coil voltage, for a given rating, is proportional to the number of turns; the auto-transformer makes it possible to use a low-voltage heavy-current work-coil.

Performance figures for a 16-turn auto, 11″ long, $7\frac{3}{4}$″ inside diameter, wound from 17/32″ copper tubing were:

Primary inductance, $L_1 = 26\ \mu H$.

Number of secondary turns	1	2	3
Secondary inductance L_s	·3	·95	2·3 μH
Mutual inductance M	1·8	3·6	5·4 μH
Coupling factor K	·596	·675	·72
Ratio $\frac{M}{L_s}$	6	3·8	2·35
*Ratio $\frac{I_2}{I_1}$	5·14	3·6	2·3

The secondary tappings were chosen so as to locate the secondary as nearly as possible at the centre of the coil.

Skin and proximity effects at high frequencies

An unvarying current distributes itself among the available parallel paths of a conducting system so as to dissipate minimum energy, i.e. the current divides in inverse proportion to the resistances of the paths. In a homogeneous conductor, for example, the current density is uniform throughout the cross-section, all filamentary paths offering equal resistance.

At very high frequencies the current distribution is such that the stored (magnetic) energy is a minimum, even though this may mean a great increase in the dissipated energy.† Thus currents flowing in the same direction in adjacent conductors follow paths which give minimum mutual induction, i.e. the current density is highest in the more remote parts of the conductors. Conversely, oppositely directed H.F. currents crowd into adjacent parts of the conductors. The skin effect in a single isolated conductor is a good example of the first case; the H.F. current flows in "filaments" which are as widely separated as possible, i.e. on a thin surface skin. The effective depth of this skin is only about ·07 mm for copper at 1 MC/S. An interesting example of the second case (oppositely-directed currents) is seen in Fig. 6·12, where a high-frequency current of several hundred ampères is confined to the edges of a slot cut in a copper disc.

The current distribution in a co-axial or concentric H.F. feeder is another example of the combined proximity and skin effects; current flows on the outer surface of the inner conductor, and on the inner surface of the outer conductor.

Both skin and proximity effects are corollaries of the "law" of minimum magnetic energy, and the distribution of high-frequency

* Used in conjunction with a work-coil of ·05 μH effective inductance.

† See, for example, *The Theory of Electricity*, Chapter V, G. H. Livens (Cambridge University Press), 1926.

currents will in general be such as to make the effective reactance of the whole system a minimum. Thus, in the induction heating of metals, the current induced in the charge concentrates in those parts of its surface which are closely coupled to the work-coil, for then the magnetic field within the charge is a minimum, and the effective reactance of the work-coil is, therefore, a minimum also. Incidentally, the induction of a current in the charge is accompanied by a mechanical force of mutual repulsion between charge and coil (the "motor" effect), the force on the charge being radially inwards if the charge is symmetrical within the coil. In induction-melting furnaces this repulsion gives rise to a stirring motion in the molten charge.

CHAPTER 2

Arc and Spark Oscillators

IN CHAPTER 1 (p. 34) it was stated that when a charged condenser C is allowed to discharge through an inductive circuit LR the discharge current is oscillatory, i.e. alternating (provided that R is less than $2\sqrt{\frac{L}{C}}$ ohms), and that the frequency of this current is approximately $\frac{1}{2\pi\sqrt{LC}}$ cycles/sec.

Fig. 2·1 shows the basic circuit of an arc-oscillator which uses this principle to produce trains of damped oscillations of the required frequency.

T is a mains-frequency transformer (50 cycles/sec) having a secondary voltage of about 6600 volts. C_1 and C_2 are similar capacitors, each made up from a number of units with an effective capacitance of about 2 μF. L is the "work" coil (e.g. the furnace coil in the case of an H.F. furnace) consisting of some 30 turns of copper tubing and having a negligible reactance at 50 cycles/sec, so that C_1 and C_2 are in effect in series across the supply, each being charged to one-half of the total voltage. They are thus equivalent to a single capacitance of one-half the value of either (i.e. about 1 μF effective capacitance) charged to the full voltage.

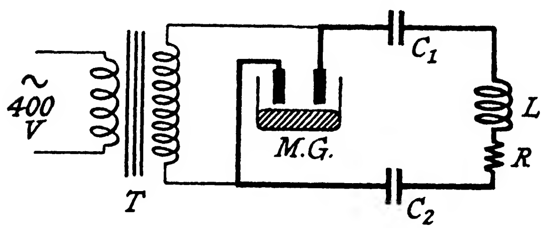


Fig. 2·1. Mercury Arc Oscillator.

MG is a "mercury gap" consisting of a pair of carbon, iron, or tungsten-faced-copper rods, each about $1\frac{1}{2}''$ diameter, suspended above a mercury-pool, and immersed in an atmosphere of hydrogen and mercury-vapour. The hydrogen pressure is kept slightly above atmospheric, while the mercury-vapour pressure depends upon the pool temperature, and is a minute fraction of one atmosphere over the normal working range of temperature. The high ion-mobility of hydrogen is an important factor in the cooling of the arc.

This double gap between the two carbons breaks down at a critical voltage, which depends upon the gap lengths, and the gap ceases to be an insulator and becomes a very good conductor in a matter of a few microseconds. The condensers then discharge round the circuit comprising the coil and the double gap. The coil current then has the damped oscillatory waveform shown in Fig. 1·28 (b).

The mains transformer has a high leakage-reactance to prevent it from passing a heavy 50 cycles/sec current through the gap during the time the latter is conducting, so that the major part of the gap current is that of the high-frequency discharge. This current will have an initial peak value of several hundred ampères and the voltage drop across the arc is of the order of 20 volts, so that an appreciable amount of power is dissipated in the arc-chamber, which is therefore water-cooled. A break in the water circuit is provided for visual inspection, and a water-flow switch may be incorporated for closing down the plant if the water supply fails.

The voltage to which the condensers charge before the formation of the arc across the gap depends upon the gap length, and this is controllable by means of a device for raising or lowering the mercury level in the chamber. This adjustment provides a simple and very effective control of H.F. output power, since the total energy per discharge is $\frac{1}{2}CV^2$ joules. For a 50 cycles/sec supply there will be not less than 100 discharges per second, each discharge consisting of several cycles of a damped H.F. oscillation, as shown in Fig. 1·28 (b).

The H.F. power developed is therefore at least $50CV^2$ joules/sec (i.e. watts), but probably only one-half of this is usefully dissipated in the charge within the work-coil, most of the loss occurring in the arc-chamber, and the remainder in other circuit-resistance, particularly coil H.F. resistance. A small amount of H.F. power is radiated, and may cause severe interference with telephone and wireless communications, the more so because the waves are damped, and so really consist not of a single frequency, but of a wide band of frequencies ranging around the nominal oscillation frequency. It is therefore essential that such equipment be well screened to reduce radiation to a negligible amount. Notes on screening are given in Chapter 8. Apparatus of this kind, working on a nominal frequency of, say, 20 KC/S (15,000 metres), may cause radio interference on short and ultra-short wavebands, if metal screens and mains-filters are not fitted.

Until the gap breaks down and the arc is formed, the mains transformer is working into a purely capacitive load. Consequently, the terminal voltage of the secondary (i.e. the total condenser voltage) is greater than the open-circuit value, as the transformer leakage-reactance voltage due to the leading current is in phase with the induced secondary voltage. The leakage reactance is usually about 100%, that is, the transformer-reactance voltage is equal to the induced voltage if the condenser current is the rated full-load current, which is the usual case. Under these typical conditions the condensers are being charged from a source the voltage of which may be twice that given on the maker's name-plate.

Some idea of the magnitudes of current, voltage, and power in generators of this type may be got from the following example:

Supply frequency, $f=50$ cycles/sec.

Secondary voltage (rating) $=6600$.

$C_1=C_2=2{\cdot}4\ \mu F$, i.e. $C_{eff}=1{\cdot}2\ \mu F$.

$V=$ voltage at which gap-breakdown occurs

$=14{,}000$, say (adjustable).

$L=50\ \mu H$.

$$\text{Energy per discharge} = \tfrac{1}{2}CV^2 \text{ joules}$$
$$= \tfrac{1}{2}\times 1{\cdot}2\times 14{,}000^2\times 10^{-6}$$
$$= 118 \text{ joules.}$$

Total H.F. power, assuming one oscillatory discharge per half-cycle of mains supply:

$$= 2f\times\tfrac{1}{2}CV^2$$
$$= 100\times 118 \text{ watts}$$
$$= 11{\cdot}8 \text{ KW.}$$

Initial peak current:

$$I_1 \doteqdot V\sqrt{\frac{C}{L}}$$
$$= 14{,}000\sqrt{\frac{1{\cdot}2}{50}}$$
$$= 2170 \text{ amperes.}$$

Oscillation frequency:

$$f_{\mathrm{HF}} \doteqdot \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{10^6}{2\pi\sqrt{50 \times 1{\cdot}2}}$$

$$= 20{,}500 \text{ cycles/sec.}$$

Critical value of circuit-resistance:

$$R_{\mathrm{crit}} = 2\sqrt{\frac{L}{C}} = 2\sqrt{\frac{50}{1{\cdot}2}} = 12{\cdot}9 \text{ ohms.}$$

The total circuit-resistance, including that of the arc and the work-coil, will normally be considerably less than this.

Mercury-arc oscillators normally operate at frequencies between 10 KC/S and 80 KC/S. Power considerations necessitate fairly large capacitance values (of the order of 1 μF), and the inductance values of typical loaded coils are such as to bring the oscillation-frequency within this range.

It is quite usual for several oscillation-trains to occur (e.g. 6 or 8) in each mains half-cycle, the breakdown occurring at a correspondingly lower voltage; nevertheless, the H.F. power is of the order indicated in the simple treatment given above.

Determination of efficiency of energy-conversion

The obvious basis for calculation of efficiency is the ratio:

$$\frac{\text{Useful energy supplied to load}}{\text{Energy input from 50 cycles/sec mains}}.$$

The load-energy is conveniently measured calorimetrically, and input energy can be obtained from watt-hour meter readings, or watt-meter and time readings. Alternatively, the loss-energy can be calculated from readings of inlet and outlet temperatures of cooling-water supplied to the arc-chamber and work-coil, together with measurements of rate of flow in each cooling system.

The useful energy supplied to the load is then very nearly =(Total electrical input − total energy to coolant).

The efficiency of the oscillatory circuit (in which most of the losses occur) can be determined with the aid of an oscillograph. A ring-coil of a few turns placed near to one end of the work-coil will have an E.M.F. induced in it of similar waveform to that in the work-coil, and this may be traced on the screen of a cathode-ray tube having its *Y*-deflection plates connected to this "search-coil". The sort of trace which is obtained is shown in Fig. 2·2.

If one train of damped oscillations occurs every $\frac{1}{2f_s}$ secs (f_s=mains supply frequency) it may be possible to measure fairly accurately the oscillation frequency from the oscillogram; it is better, however, to use even a simple form of calibrated "frequency meter" for this (see Chapter 8). Now it can be shown that the number of cycles which occur before the oscillation-amplitude is $\frac{1}{P}$ of its first peak-value is given by:

$$n=1+\frac{\log_e P}{\delta} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 2\cdot1$$

δ being the logarithmic decrement of the circuit (p. 35) and equal to $\pi R\omega C$;

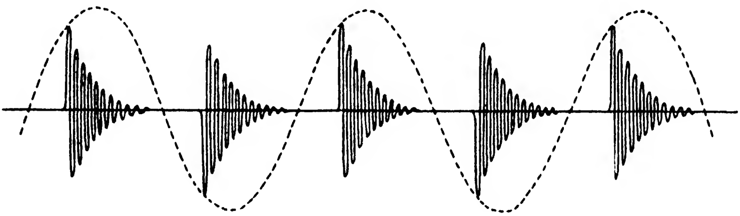


Fig. 2·2. Oscillatory Waveform of Arc Oscillator (for simplicity, only one discharge per mains half-cycle is shown).

i.e.
$$R=\frac{\delta}{2\pi^2 f_N C}\text{ ohms} \qquad . \quad . \quad . \quad . \quad . \quad . \quad 2\cdot2$$

and
$$\delta=\frac{\log_e P}{n-1} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 2\cdot3$$

If, therefore, oscillograms are taken for no-load and loaded conditions, and δ and f_N determined as indicated, the effective value of R can be calculated for each condition.

Thus, if R_O=effective R on no-load,

and R_E=effective R on load,

then the oscillatory-circuit efficiency will be:

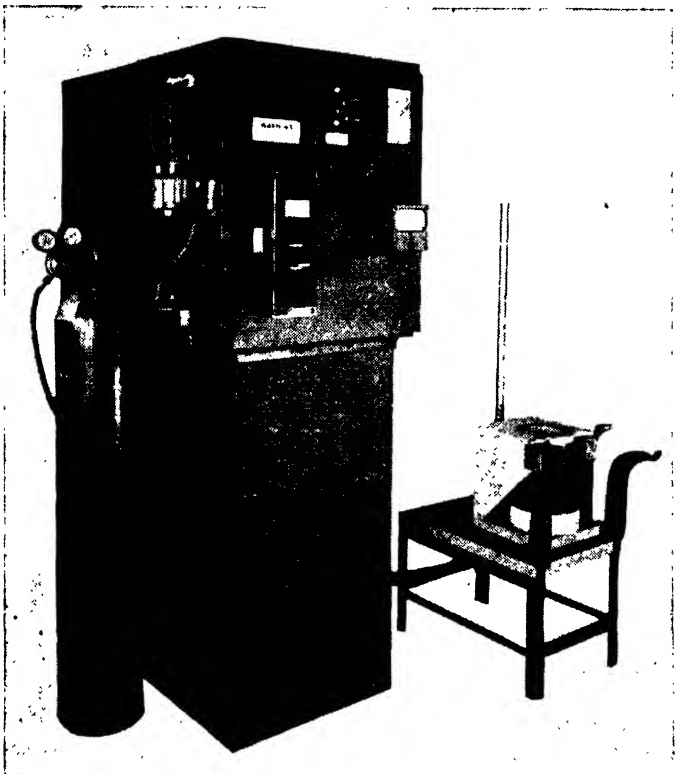
$$\eta=\frac{R_E-R_O}{R_E}\times 100\,\% \qquad . \quad . \quad . \quad . \quad . \quad . \quad 2\cdot4$$

In calculating δ, a value of 2, 5, or 10 may be found convenient for P.

Fig. 2·3 illustrates a 20-KW mercury-arc type H.F. generator with a small melting furnace, made by the Ajax Electrothermic Corporation.

The spark oscillator

A similar circuit to that shown in Fig. 2·4 is used in conjunction

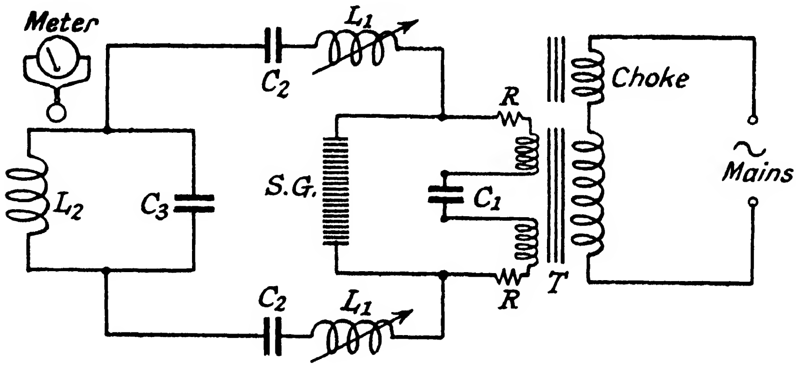


[*Courtesy of Birlec Ltd.*

Fig. 2·3. Ajax 20-KW Mercury-arc High-frequency Generator.

with a spark-gap instead of an arc. Actually several gaps are arranged in series to facilitate cooling. Each gap is about four-thousandths of an inch long, between tungsten discs of about 1″ diameter, the discs forming the end-faces of brass cylinders. Some of the cylinders are water-cooled, the remainder air-cooled. The "Lepel" unit, for example, has 48 water-cooled and 12 air-cooled gaps.[6]

The condenser C_1 is made up from a number of capacities in series, with tappings brought out to a stud switch, to provide a means of controlling the output power, maximum power corresponding to minimum number of condensers in circuit. Inductances L_1 are air-cored coils arranged for smooth variation, and are tuned to give maximum tank current as indicated by a thermocouple meter in a loop circuit loosely coupled to the work-coil leads. Safety devices include interlock switches on doors and inspection plates; water-flow switch for opening the mains



[*Courtesy of Lepel H.F. Laboratories and Gaston Marbaix.*

Fig. 2·4. Lepel 35-KVA Spark Oscillator.

R: Damping resistors ·0125 Ω.
L_1: Tuning inductances.
C_1: Power-control condensers (several series units; variable from ·1 μF to ·4 μF).
C_2: Isolating condensers; ·02 μF effective, each.
C_3: Tank condenser ·15 μF effective.
L_2: Work-coil.
SG: Multiple spark-gaps.
T: High-voltage transformer: Primary 440 V, 75 A, Secondary 14,000 V, 2·3 A.

supply circuit if insufficient water is fed through the spark-gap cylinders and work-coil; and a relay, also in the mains supply to the high-voltage transformer, which closes only when the air-fans are energised. Units of this type have been constructed having a rating of 25-35 KW. The frequency range is about 100 to 300 KC/S, and the units are particularly suitable for localised heat-treatment work (see Chapter 6). Screening of the power unit and work-coil is necessary. The multiple-gap arrangement gives very rapid quenching of the spark and consequently a very large number of oscillation-trains occur during each half-cycle of the supply. Thus it is possible to use small capacities and so generate relatively high frequencies.

The inverter[7]

Mercury-arc valves can be used for the inverse process to that of rectification, i.e. they can be made to produce A.C. power from a D.C. source, at a high conversion efficiency. Unfortunately, the frequencies obtainable are relatively low, owing to the time required for de-ionisation of the arc-path. Thus, using a normal grid-controlled mercury-arc rectifier, the upper frequency limit is about 1000 cycles/sec for reliable operation; frequencies up to about 15,000 cycles/sec have been achieved using thyratrons on low power. Ignitrons, in which a mercury-pool cathode is used without the disadvantage of permanent ionisation due to a "keep-alive" circuit, might perhaps enable the inverter to challenge the H.F. alternator in the future. It is unlikely that it will ever be suitable for frequencies exceeding 10,000 cycles/sec.

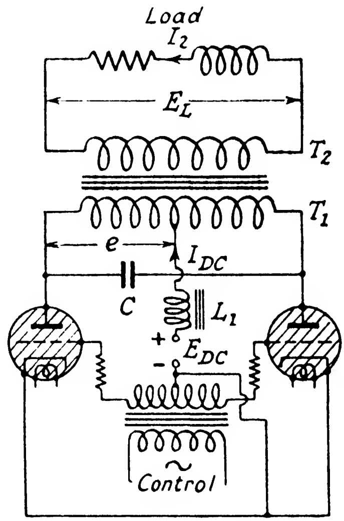


Fig. 2·5. Inverter.

There are several circuit arrangements. One of the simplest, and probably the most suitable for industrial H.F. power generation, is the type shown in Fig. 2·5, in which two valves are operated in parallel, each valve conducting alternately and so producing an alternating flux in the transformer core, and A.C. power in the load. Separate excitation from an external source is shown; it is possible to derive a suitable control-voltage for the grid-circuits from the output transformer.

It is a characteristic of the mercury-arc valve that, once the arc is formed between plate (anode) and cathode, it can be extinguished only by reducing the plate voltage to a low value; in practice, the plate voltage is reduced to zero or, more often, driven negative (relative to cathode potential) in order to extinguish the arc. In the parallel-type inverter, the initiation of an arc in one valve (say V_1) is caused to drive the plate voltage of the other valve, V_2, considerably negative for a short time, and so extinguish the arc to that plate. This is achieved by virtue of the charge accumulated in the commutating condenser C during the period of current flow through V_2; for when V_1 conducts, the volt-drop across it is very small (e.g. 15 volts) so that the left-hand terminal of C is in effect connected to the common cathode-point, and the polarity of C at this moment is such as to make the plate voltage of V_2 negative. During the time that V_1 is conducting, C is re-charged with the reverse polarity, and in due course, when the grid voltage of V_2 initiates a new arc in V_2, the plate of V_1 is driven negative.

The waveform of the output voltage varies considerably according to the circuit constants, but the fundamental frequency of the output is that of the exciting source. The magnitude of the output voltage also has a wide range of possible values, and may even rise (when referred to the whole primary winding) to 20 or more times the D.C. supply voltage if the load power-factor is low and lagging.

Wagner(7) has published a complete analysis of the operation of the parallel-type inverter, and has set out a basis for design. In this he uses a design constant K defined thus:

$$K=\frac{\text{Condenser KVA}}{\text{Load KVA}}$$
$$=4YZ$$

for the particular case of $T_2=\frac{1}{2}T_1$ (T_1 and T_2 are total primary and secondary turns).

In general, for any transformer ratio:

$$K=\left(\frac{T_1}{T_2}\right)^2 YZ,$$

where Z=load impedance at the nominal (i.e. control source) frequency f

$=\sqrt{R^2+\omega^2L^2}$ ohms,

$Y=2\pi fC$ reciprocal ohms.

The load power-factor $\cos\phi=\frac{R}{Z}$.

Curves are given in the article referred to showing primary and secondary waveforms for a wide range of values of K and load power-factor.

Fig. 2·6 relates relevant design quantities with K and power factor, and enables the necessary design calculations to be made.* The procedure is:

(1) Determine the load impedance and power factor.

(2) Determine the effective (i.e. R.M.S.) value of load voltage E_L required for given power $(P=\frac{E_L^2}{Z}\cos\phi)$.

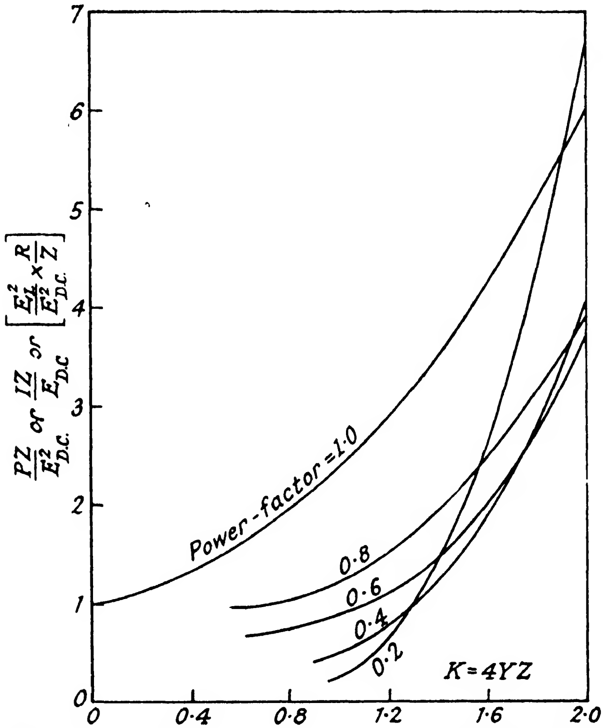


Fig. 2·6. Inverter Design Curves. (*After Wagner*)[7b].

(3) Select a value for K, and hence calculate the appropriate value for C. K should be made approximately unity. Larger values for K give longer available de-ionising time, but much poorer voltage regulation with change of load power-factor. When $K=1$, the condenser KVA equals the load KVA.

(4) From the appropriate curve in Fig. 2·6 read off the value of $\frac{PZ}{E^2}$, and hence calculate the required D.C. supply voltage E.

* In this figure, the current I is the D.C. input, i.e. I_{DC} in Fig. 2·5. It is assumed that the inductance L_1 is infinitely large, so that this current is constant.

The vertical scale of the graph is correct for the particular case of $T_2=T'$, where $T'=\frac{1}{2}T_1$ (i.e. secondary winding identical with each *half* of the primary). For the general case of $\frac{T'}{T_2}=n$, vertical scale readings must be divided by n^2. For given load-power, the required D.C. voltage decreases and current increases as K increases; from this point of view it is desirable to keep K small since the heavier current increases de-ionisation difficulties.

In connection with item (3), Wagner suggests, as an empirical rule for K:

$$K=\sqrt{1-(\text{power factor})^2}.$$

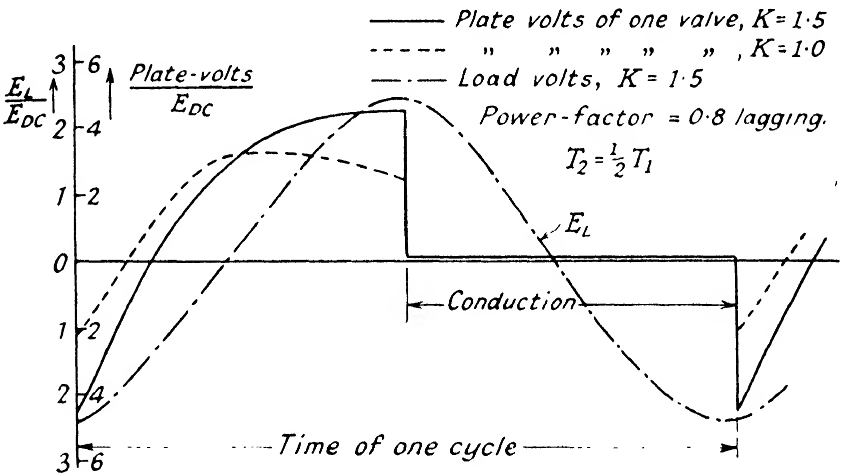


Fig. 2·7. Inverter Waveforms. (*After Wagner*)[7b].

This will give a de-ionisation time greater than one-tenth of the cycle time, except for power factors higher than ·99.

For a chosen capacitance of commutating condenser, an increase in load resistance (i.e. a decrease of load-power) increases the value of K. Hence if the inverter is stable at full load, it will operate on all lighter loads since the available de-ionising time will be greater.

The output voltage and the corresponding plate-cathode voltage of one valve are shown in Fig. 2·7 for the particular case of $K=1{\cdot}5$, power factor $=\cdot 8$ lagging. The effect of the value of K on the time available for de-ionisation can be seen by comparing the full-line graph of plate voltage ($K=1{\cdot}5$) with the dotted-line graph

($K = 1{\cdot}0$). It will be seen that the choice of a value of K requires the careful consideration of several conflicting factors—voltage regulation, de-ionising time available, D.C. volts required, and magnitude of current in the arc to be extinguished.

It is possible to compensate for the bad voltage regulation which results in excessive rise in output volts as load is removed. One method is to reduce the commutating capacitance as the load

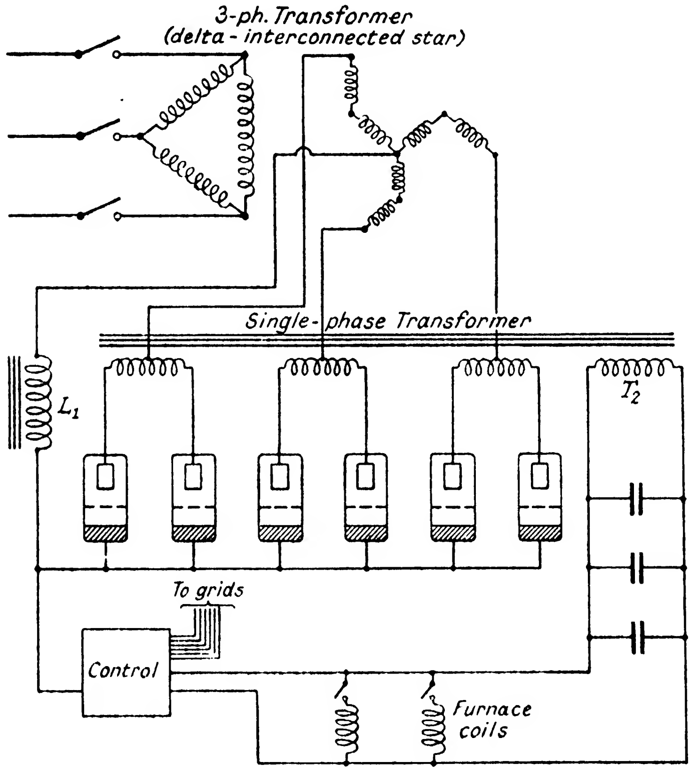


Fig. 2·8. Inverter with Three-phase Input.

becomes lighter. Alternatively, if the D.C. input power is obtained from a grid-controlled rectifier, the output voltage of this rectifier can be automatically reduced as the inverter load is removed (see "Power Supplies", Chapter 8).

The series choke L_1 in the D.C. line should have a high reactance to currents at the operating frequency f, preferably at least twice the load impedance.

Inverters with A.C. input

If the D.C. source (Fig. 2·5) were replaced by an A.C. source having a frequency which is low relative to that of the output, the inverter would function during input half-cycles over which the supply voltage has the polarity indicated in Fig. 2·5. An arrangement is shown in Fig. 2·8 for a three-phase mains-frequency input, giving single-phase high-frequency output power to an induction-furnace load. The mercury-arc converter may contain all six anodes in a single vessel, or six separate units may be used. The operation is similar to that of a normal three-phase half-wave rectifier (see Chapter 8), in that the current in the reactor L_1 is unidirectional and each phase in turn supplies current, but differs in that two anodes are associated with each phase. Each pair of anodes carries current in turn, normally for one-third of each supply period; during this time the current is carried alternately by each of the two anodes, exactly as in the arrangement shown in Fig. 2·5, due to the combined effects of grid-excitation and capacitance-commutation.

The transformer windings associated with the three pairs of anodes are arranged on a common core to supply single-phase H.F. power to the winding T_2, as they do not transform any three-phase current. The commutating action of a capacitor is transmitted to the inverter through this transformer, the load-capacitor bank being made large enough to produce commutation in addition to neutralising the reactance of the furnace coil and the transformer.

Inverters for induction-furnace work are made self-exciting, i.e. they are oscillators. Consequently, the output frequency varies somewhat with load conditions; this is not objectionable, and is preferable to capacitor-switching which would be required if the frequency were held constant. Units operating from a three-phase input have been built having ratings up to 300 KW, and units of 1500 KW are planned; their static nature and high power-conversion efficiency (over 90%) are valuable features, and their increasing application to induction melting, and to through-heating of billets for forging, seems certain.*

* See (i) British Patent No. 510,005.

(ii) "Mercury Arc Frequency-Changing Equipment for Induction Heating," S. R. Durand, *Iron and Steel*, 1947, Vol. 24, No. 14.

(iii) "Electronic Frequency Converters for Induction Melting Furnaces," S. R. Durand, *Iron Age*, Sept. 25th, 1947, Vol. 160, No. 13.

(iv) Article by H. F. Storm, *The Allis-Chalmers Electrical Review*, March, 1945.

CHAPTER 3

High-frequency Alternators

The wound-rotor alternator

FIG. 3·1 ILLUSTRATES the principle of this type of alternator. The machine consists of two essential parts: The rotor or field system, and the stator, the armature or energy system. The stator is built up from thin steel laminations; a "solid" construction (e.g. a forging) may be used for the rotor, though ring-laminations, keyed to a forged or fabricated spider, are usual in large machines.

An 8-pole rotor is shown for simplicity; a much larger number of poles is usual in H.F. alternators. Each pole is excited by its own field-coil, supplied from a D.C. source; sometimes alternate poles only are wound.

An armature-coil is wound around each stator-projection corresponding to a rotor-pole. For the rotor position shown in Fig. 3·1, it is clear that a large part of the total flux of each pole passes through, and therefore links with, the corresponding coil. If the rotor now be imagined to have moved through $\frac{1}{8}$ revolution, again each stator-coil is linked by a large flux, but the direction of this flux through the coil is reversed, north and south rotor-poles having been interchanged for each coil.

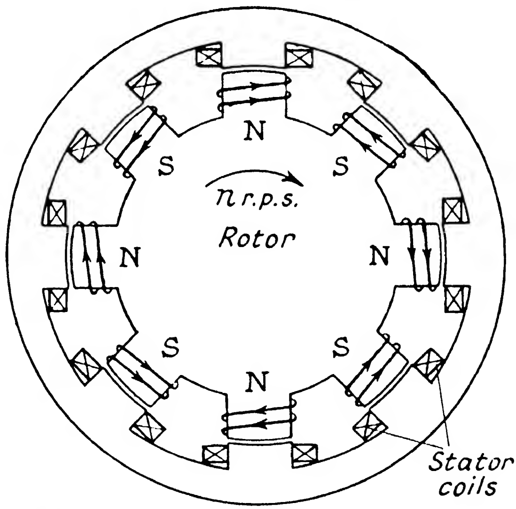


Fig. 3·1. Principle of Wound-rotor High-frequency Alternator.

Thus in $\frac{1}{2p}$ revolution (p=*pairs* of rotor poles) the flux through each coil has changed from Φ lines in one direction to Φ lines in the reverse direction, i.e. the total flux-change for each coil is 2Φ lines.

A similar reversal of flux takes place every $\frac{1}{2p}$ revolution. For a rotor speed of n revolutions/sec, the time of one such flux reversal is $\frac{1}{2pn}$ seconds.

The average rate of change of flux through a coil is therefore:

$$\left(\frac{d\Phi}{dt}\right)_{\text{av.}}=\frac{2\Phi}{\frac{1}{2pn}}=4pn\Phi \text{ lines/sec.}$$

The average voltage thus generated in each coil is:

$$E_{\text{av.}}=T.\left(\frac{d\Phi}{dt}\right)_{\text{av.}}\times 10^{-8} \text{ volts}$$
$$=4pn\Phi T.\times 10^{-8} \text{ volts,}$$

where T=number of coil turns.

If the flux-reversals through a coil are sinusoidal, a sinusoidal voltage is generated having an RMS value of $1{\cdot}11\times E_{\text{av.}}$, so that

$$E_{\text{coil}}=4{\cdot}44pn\Phi T.10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad 3{\cdot}1$$

It will be clear that a complete cycle of flux-change takes place every $\frac{1}{p}$ revolution, so that the frequency, i.e. the number of complete cycles per second, is:

$$f=pn \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}2$$

A speed of about 1500 or 3000 r.p.m., i.e. $n \doteqdot 25$ or 50 r.p.s., is usual, the alternator being driven by an induction motor or a synchronous motor from a 50 cycles/sec supply.

Similar voltages are generated in all the coils, which may be connected in series or series-parallel to give the required output voltage, and current-carrying capacity. Relatively high voltages (e.g. 1000 volts) are usual. The alternator output voltage is controlled by the D.C. rotor-field excitation. The D.C. supply is obtained from a generator coupled to the alternator, or, alternatively, from a rectifier unit supplied directly from the 50 cycles/sec mains.

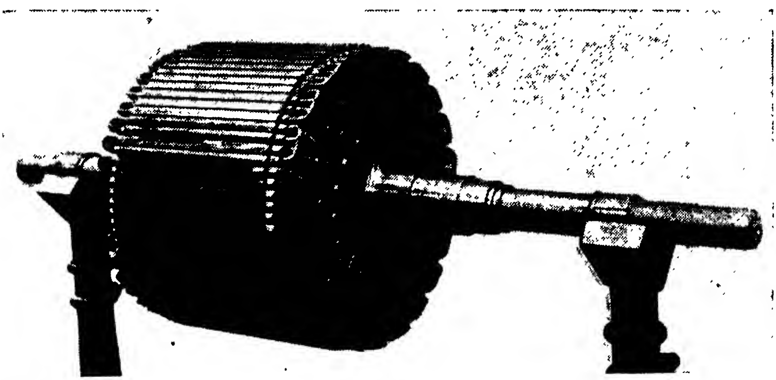
Wound-rotor machines are used for frequencies up to about 2000 cycles/sec (80 poles, 3000 r.p.m.), and with power ratings up to at least 500 KW. Practical constructions differ from the elementary type shown in Fig. 3·1 in three respects:

(1) The number of poles is large, $p=\frac{f}{n}$ pairs.

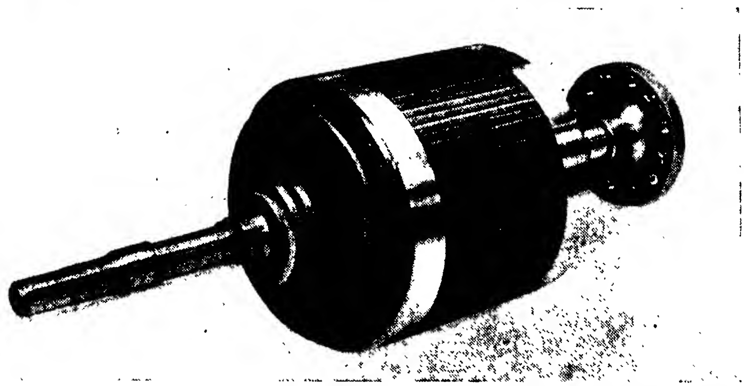
(2) The stator may be wound for a two- or three-phase output.

(3) The stator windings are located in slots, rather than around polar-projections.

The rotor of a high-frequency alternator of this type is, therefore, a toothed cylinder, with the exciting winding arranged in the



(*a*)



[*Courtesy of English Electric Co. Ltd.*

(*b*)

Fig. 3·2 (a) and (b). Wound Rotor for 2200 cycles/sec Alternator.

slots in a similar fashion to that shown for the stator winding of Fig. 3·10. (This figure refers to an inductor alternator, but the stator-winding diagram is applicable to both rotor and stator of the wound-rotor type of machine.)

The rotor is given a smooth finish by enclosing the ends of the field-coils in brass end-bells. Fig. 3·2 (a) shows such a rotor just after winding; the finished rotor, complete with end-bells, slip-rings and fan, is seen in Fig. 3·2 (b).

The rating of the actual machine was 150 KW, single phase, 1200 volts, 2200 cycles/sec at 3000 r.p.m.

Under load conditions the armature-winding ampère-turns set up a reaction magneto-motive force which may either oppose or assist the rotor M.M.F., depending upon the power factor of the load being a lagging or a leading one respectively. Hence the generated E.M.F. on load may be smaller or larger than the no-load value for the same field-current. Also the terminal voltage V of the alternator on load will be less than the generated E.M.F. E_g by an amount $I_A Z_A$, with due account taken of phase, where I_A = armature current in ampères, and Z_A = armature impedance in ohms. Z_A is a complex impedance, $R_A + jX_A$, R_A being the armature resistance and very small compared to X_A, the armature reactance.

The percentage change in terminal voltage when load is switched off is called the Regulation, i.e.

$$\% \text{ Regulation} = \frac{E_g - V}{V} \times 100,$$

for that particular load and power factor. The effects of armature reaction and reactance are illustrated in Fig. 3·3.

The regulation of high-frequency alternators is inherently high, because of the large internal reactance (though this is sometimes partially compensated by means of a series condenser), and the terminal voltage variations are further accentuated, in induction-heating work, by the changes which occur in load power-factor during a heating cycle. It is not always possible to correct these changes fully, and some form of automatic voltage-control is nearly always used, e.g. Thyratron or Amplidyne regulator.

The voltage available at the load is further reduced by the (mainly reactive) impedance of the bus-bars or cable between alternator and load. Information on bus-bar reactances will be found in Chapter 9.

Load power-factor correction

The normal applications of H.F. alternators involve induction effects associated with a magnetic field produced by the load current. (Examples are induction-heating, and magnetic operation of vibrating apparatus.) Consequently, the effective load on the H.F. alternator is nearly always a combination of inductance and resistance, and the alternator works into a load having a lagging power factor. The heating of the armature coils sets a limit to the current which may safely be allowed to flow, and this heating will be the same, for a given current, irrespective of load power-factor. The power developed in the load, on the other hand, is directly proportional to load power-factor, cos ϕ.

$$P_{\text{load}} = VI_L \cos\phi \text{ watts}$$

where $\cos\phi = \dfrac{R_L}{Z_L}$; $\left(\cos\phi \doteqdot \dfrac{R_L}{X_L} = \dfrac{1}{Q}, \text{ when } \phi \to 90^\circ\right)$.

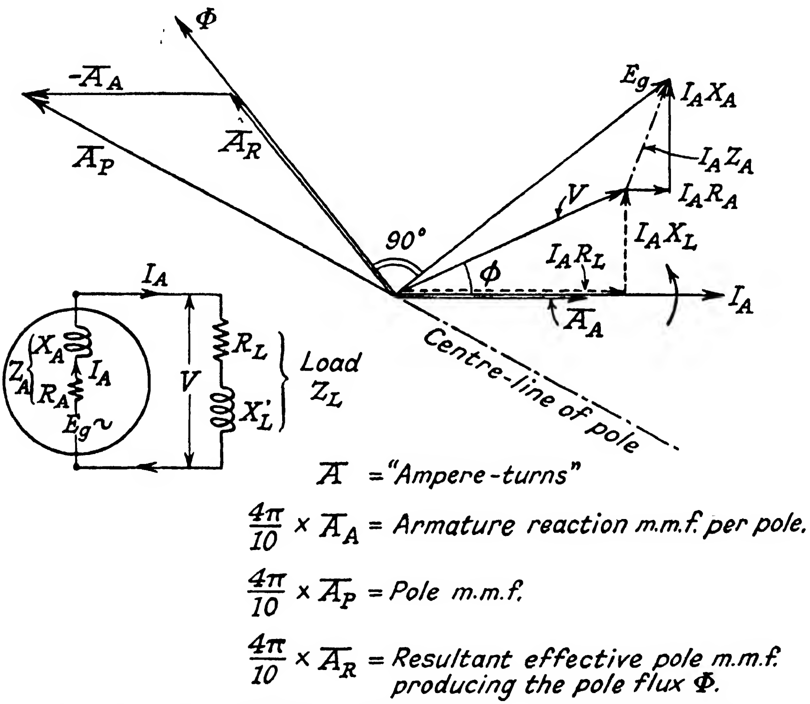


Fig. 3·3. On-load Vector Diagram for High-frequency Alternator.

It is essential, therefore, to compensate for the inherent low lagging power-factor of the induction-load, in order that the required amount of load-power shall be obtained for a reasonably small power dissipation in the alternator windings and the leads. This is achieved by using correction-condensers (Fig. 3·4), which take a leading-current from the alternator. If the condenser current is made equal to the reactive component $I_L \sin\phi$ of the load-current, then the alternator current will be in phase with its terminal-voltage, and will be a minimum for that particular value of load-power (see also Fig. 1·24 (a)).

A bank of condensers is always used with switchgear for selecting the amount of capacitance required for minimum alternator current at a fixed terminal voltage, i.e. the capacitance for correction to unity power-factor.

In the inductor-type alternator, operating at frequencies above 1000 cycles/sec, it is also necessary to compensate for the high internal reactance of the machine itself, due to its armature-inductance; for this a series condenser is used.

The motor-alternator set, Fig. 3·5, is a G.E.C. unit installed at Brockhouse Castings Ltd, Wednesfield, for steel melting. The alternator rating is 400 KW, single phase, at 1000 or 2000 volts, 1000 cycles/sec. It has 80 poles, alternate poles only being wound,

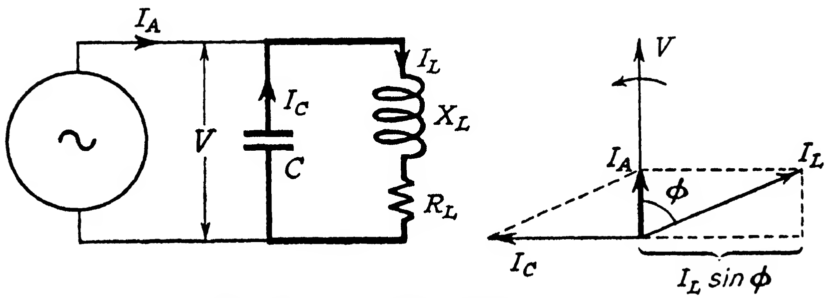


Fig. 3·4. Load Power-Factor Correction.

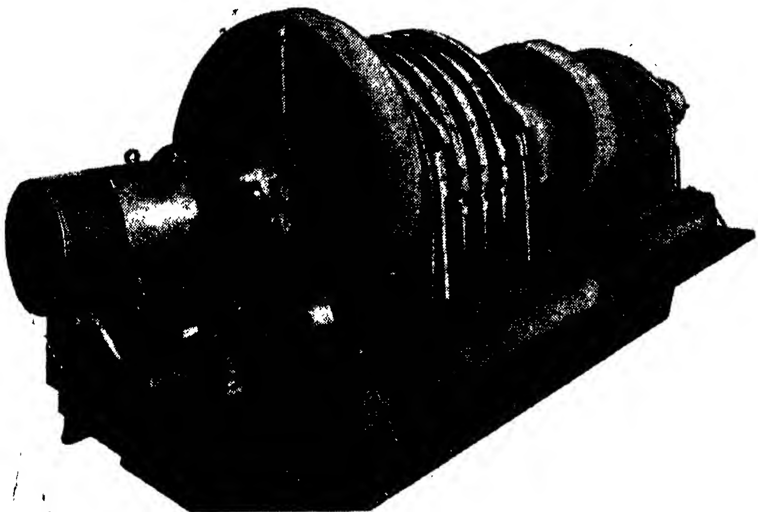
and the speed is 1500 r.p.m. To minimise windage losses and noise, the surface of the rotor is given a smooth cylindrical finish. The rotor shaft is of non-magnetic steel.

Stranded conductors are used for the stator windings to reduce skin-effect copper losses. The stator core is built up of thin silicon steel laminations varnished on both sides, and the core clamping rings are made of bronze to minimise stray load losses.

The main exciter for the alternator field is a 13·3-KW 140-v compound-wound D.C. generator directly coupled to the alternator shaft. There is also a pilot exciter, belt-driven from the main shaft, rated at 320 volts, 2·25 ampères. Two exciters are used to obtain very rapid regulation of the H.F. output voltage.

The induction motor is a standard 650-h.p. slip-ring machine, 400 volts 50 cycles/sec., three-phase, with brush lifting and short-circuiting arrangements, and is hand-started by means of a liquid starter. Both motor and alternator are cooled by filtered air blown axially through them.

The plant supplies power to three steel-melting furnaces of 3-cwt, 5-cwt, and 10-cwt capacity respectively. The two smaller furnaces operate at 1000 volts and may be used together or independently; the larger furnace is used alone, at 2000 volts. The alternator voltage is changed by appropriate interconnection of stator leads. Each furnace is contained in a steel casing which houses a laminated magnetic circuit,(30) the inductor coil, and the refractory lining. The inductor is made up from several sections

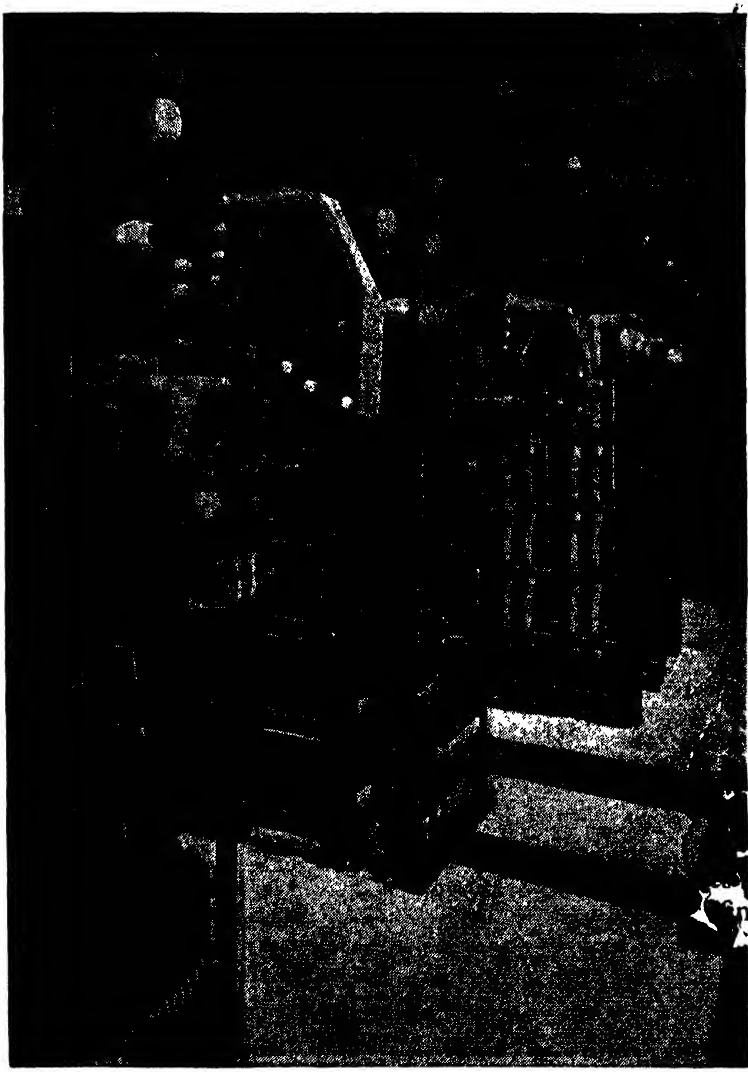


[*Courtesy of G.E.C. Ltd.*

Fig. 3·5. G.E.C. 400-KW 1000 cycles/sec Motor-alternator Set.

in series, each section consisting of a number of turns without joints and being independently supplied with cooling water.

The H.F. correction condensers and contactor gear used in conjunction with the 1000-cycle alternator and furnaces described are illustrated in Fig. 3·6. A bank of fixed condensers is permanently connected in circuit; four banks of switched condensers are available for maintaining the power factor as near to unity as possible under changing load conditions during melting. Each bank is divided into two equal parts suitable for 1000-v working, and each part can be operated independently when using the smaller (1000-v) furnaces. The two parts are connected in series



[*Courtesy of G.E.C. Ltd.*

Fig. 3·6. Banks of Fixed and Variable High-frequency Condensers, and Contactor Gear.

for use at 2000 volts with the large furnace. The condensers consist of a number of naturally-cooled unit boxes, containing oil-immersed condenser spools of special design for H.F. service. A fan supplies filtered air to the condenser room for cooling, and an interlock on the fan starter prevents power from being supplied to the furnaces until the fan is running.

Inductor alternators

It seems probable that, for industrial applications requiring frequencies higher than 400 cycles/sec, the wound-rotor high-frequency alternator will be entirely superseded in time by the inductor alternator, which is both cheaper and more efficient.

The high peripheral speeds, up to 100 metres/sec, and large

(*a*)

A: Stator laminations.

B: Stator frames or yokes.

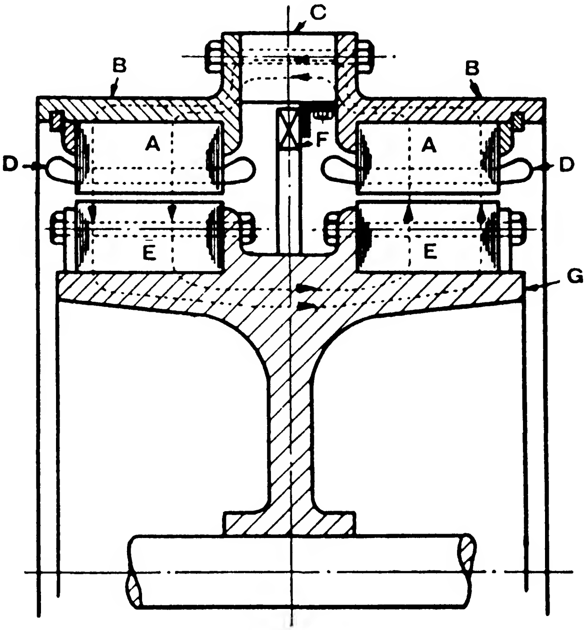
C: Magnetic distance-piece.

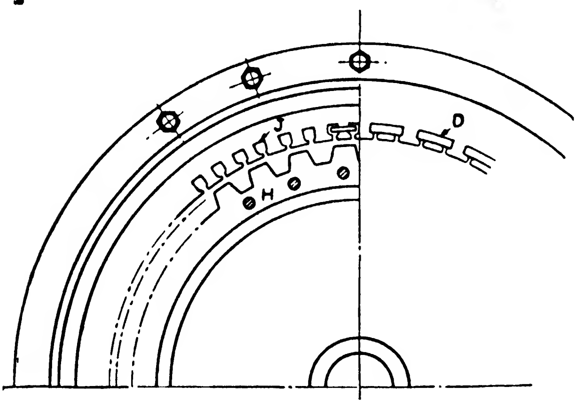
D: Armature winding in semi-closed slots, *J*.

E: Rotor laminations.

F: D.C. field coil.

G: Steel spider.





H: Rotor profile.

J: Stator-slots.

(*b*)

[*Courtesy of B.T.-H. Ltd.*

Fig. 3·7. Homopolar Alternator with Split Stator Frame.

number of rotor teeth which are permissible in the inductor type make frequencies up to 50,000 cycles/sec possible.

Two basic types of inductor alternator are in common use. Both have toothed cylindrical rotors, usually laminated, and differ essentially only in the method of producing the working flux. Figs. 3·7 and 3·8 show the principal features of the two types.

Fig. 3·7 shows the homopolar type of inductor alternator, which, in its present-day form, differs only in constructional details from the machine designed and constructed by B. G. Lamme in 1902.[10] There are two laminated stator-rings fitted tightly into a steel yoke; the rings are slotted for the stator (A.C.) windings. The rotor consists of a pair of toothed cylinders, the number of rotor teeth or poles being half the number of stator-teeth. The magnetic circuit consists of both stator-rings, both rotor-cylinders, the yoke and the cast-steel rotor-centre, and is excited by means of an annular coil supplied with D.C.

Each stator-coil embraces one stator-tooth, the flux in which depends upon the rotor position, being large when opposite a rotor-tooth, or small when opposite a rotor-slot. A complete cycle of flux variation in a stator-tooth (and therefore through a coil) occurs for a rotor movement of one rotor-tooth-pitch, and the frequency of the induced voltage is therefore:

$$f = p_r n \text{ cycles/sec} \qquad 3{\cdot}3$$

where p_r = number of rotor poles or teeth,

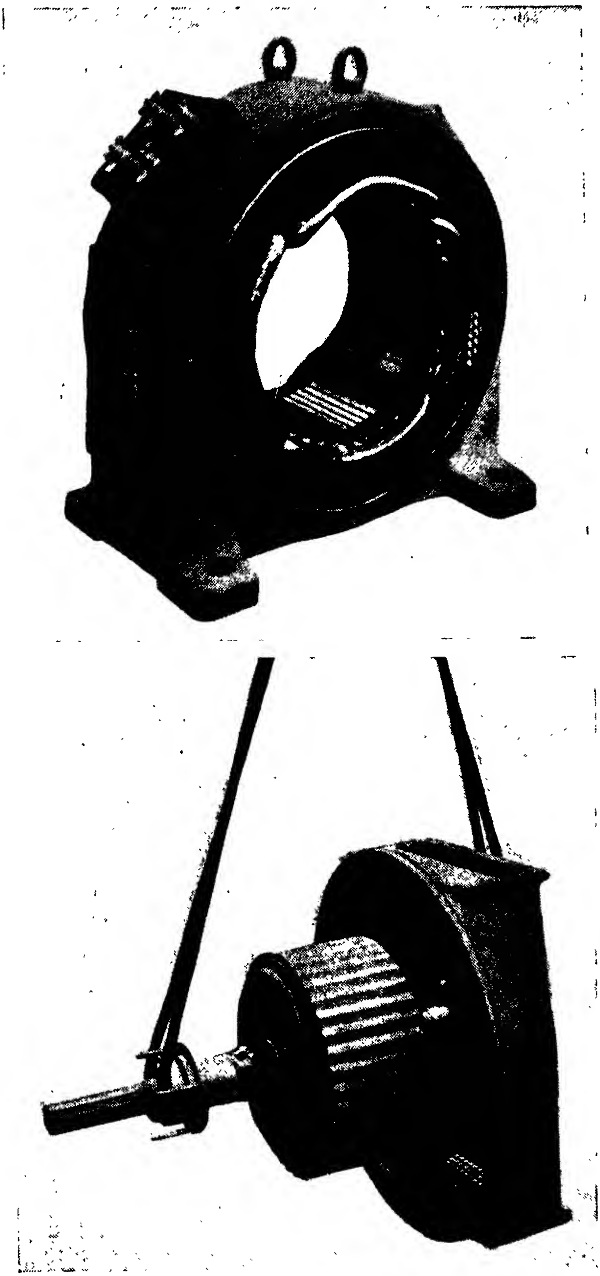
n = rotor speed in revs/sec.

Machines of this type have been constructed for frequencies up to 15,000 cycles/sec, though the majority are for frequencies between 1000 and 10,000 cycles/sec. See also Fig. 3·15.

The second type of inductor generator (Fig. 3·8) is a heteropolar machine; that is to say, whereas in the homopolar type the whole of one stator-ring behaves as one pole (either N. or S.), in this case the stator consists of 2 or 4 slotted poles of alternate polarity (N., S., N., S.), each pole being excited by its own D.C. winding as in a D.C. machine, which it resembles closely in this respect.

The slots in the pole-faces contain the A.C. coils, and the relation between stator- and rotor-teeth is the same as for the homopolar type. The principle of operation is the same for both types, and the details of calculation given later apply equally well to either.

There are certain advantages in the heteropolar arrangement. For example, the time-constant of the field system is much lower



(*a*)

(*b*)

[*Courtesy of B.T.-H. Ltd.*

Fig. 3·8. B.T.H. 10-KVA 2000 cycles/sec 4-pole Heteropolar Inductor Alternator. (a) Stator. (b) Rotor.

than in the homopolar machine, and this makes possible rapid changes in power output, such as are required in multi-stage heat-treatment processes. Also, only one stator-core is used, and the yoke may be made of a light alloy as it does not carry the flux. It will be noticed that there is an alternating flux in the rotor of this machine of frequency $f_r = p_{DC}n$, where p_{DC} is the number of pairs of D.C. stator-poles.

H.F. alternators inherently possess a large armature reactance ωL_a, and this would, if uncorrected, result in a large reduction in terminal voltage when put on load. It may, in fact, be impossible to develop appreciable power in an external load. It is usual, therefore, to employ series capacitance (Fig. 3·9) to "neutralise" the armature reactance partially; if the series-condenser reactance X_c is made equal to the armature (inductive) reactance X_a, and the

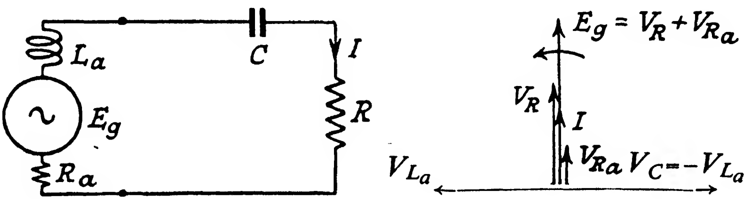


Fig. 3·9. Vector Diagram for Loaded Alternator showing effect of Series Condenser.

external load is purely resistive, the circuit behaves like the series-resonance circuit described in Chapter 1.

In Fig. 3·9 L_a represents the armature (i.e. stator) leakage-inductance which gives rise to the reactance $X_a = \omega L_a$ ohms. R_a is the armature resistance, C the series capacitance, and R the load resistance. For resonance conditions, i.e. $X_c = X_a$, the circuit current is:

$$I = \frac{E_g}{R_a + R}; \quad E_g = \text{generated voltage.}$$

$$\text{Terminal voltage} = \sqrt{V_R^2 + V_C^2} \text{ volts.}$$

$$\text{Load voltage} = V_R = IR \text{ volts.}$$

$$\text{Load-power} = V_R I = I^2 R \text{ watts.}$$

Note that if the load-resistance were zero (i.e. machine connected directly across C) the circuit current would be limited only by R_a, while armature and condenser voltages would rise to very high values. These points are illustrated in Fig. 3·14, which shows performance curves of an inductor generator. True resonance is avoided, in practice, by only partially compensating for the armature reactance.

The arrangement of rotor and stator teeth and slots, and the method of winding the stator-slots, is shown in Fig. 3·10. The flux-distribution in the gap is also shown.

An armature-coil consists essentially of the two coil-sides embracing a tooth. The E.M.F. induced in a coil can be calculated

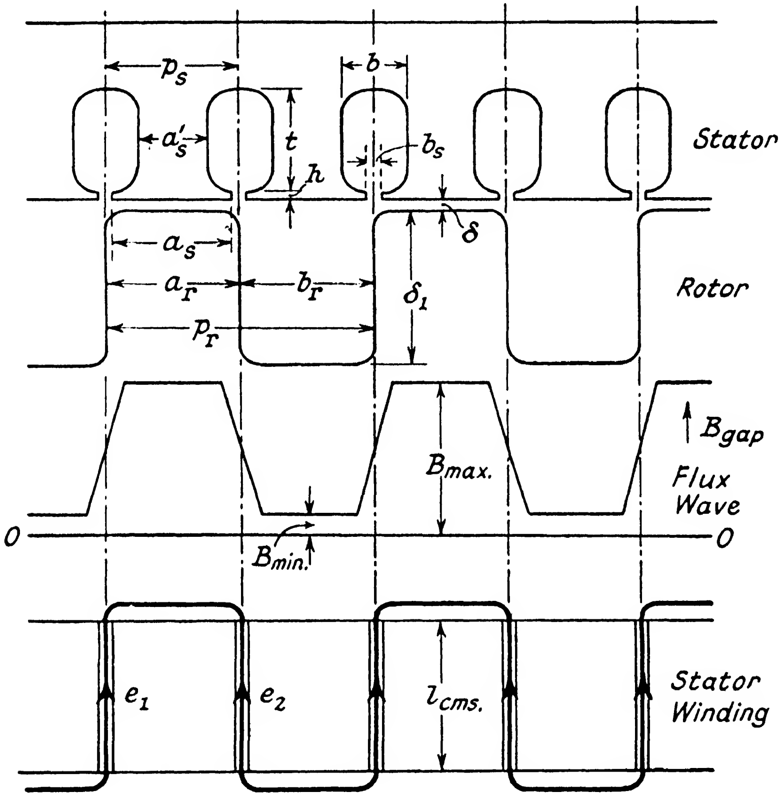


Fig. 3·10. Arrangement of Stator- and Rotor-teeth, Stator Winding, and Gap-flux Waveform for Conventional Inductor Alternator.

either from a consideration of flux-linkages, or, more conveniently, from the conception of flux-cutting. From this latter point of view, the E.M.F. in a conductor of length l cms being "cut" by a magnetic field of density B lines/cm², having a relative velocity of v cms/sec, conductor, field, and motion being all at 90° to one another, is:

$$e = Blv \times 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}4$$

It is seen from this expression that the voltage generated in the conductor varies in exactly the same way as the flux-density varies. Thus in Fig. 3·10 the flux-distribution round the gap of the inductor alternator is represented by a trapezium-shaped wave, its density varying from B_{min} to B_{max}. When the rotor moves, the effect is as if this flux-wave moved with it, though the wave-shape will vary somewhat with different rotor positions; this variation will not be large in the case of semi-closed stator-slots.

The flux cutting any conductor in one stator-ring varies between B_{max} and B_{min}, but does not reverse. Thus for a single-turn coil, with its coil-sides located in adjacent slots, the total generated voltage is the *difference* between the voltages in the coil-sides. The maximum E.M.F. in coil-side No. 1 is:

$$e_1 = B_{max} lv \,.\, 10^{-8} \text{ volts}$$

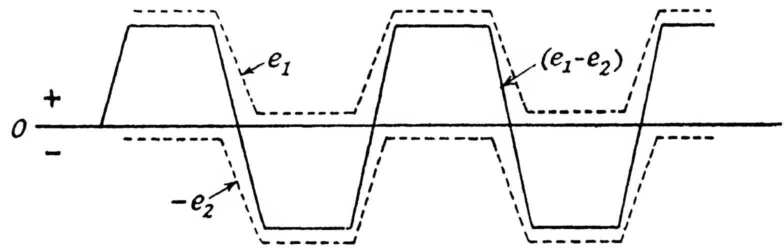


Fig. 3·11. Coil-side E.M.Fs and Resultant Coil E.M.F. for Fig. 3·10.

and at this instant the E.M.F. in coil-side No. 2 is:

$$e_2 = B_{min} lv \,.\, 10^{-8} \text{ volts.}$$

The maximum coil-E.M.F. is therefore:

$$e_{coil} = e_1 - e_2 = (B_{max} - B_{min}) lv \,.\, 10^{-8} \text{ volts} \quad . \quad . \quad 3{\cdot}5$$

One half-cycle later, when the rotor has moved a stator-slot pitch, the coil-E.M.F. is again a maximum, but has reversed, since e_2 is now the greater and e_1 the smaller voltage. The E.M.F. wave $(e_1 - e_2)$ is shown in Fig. 3·11.

For a winding consisting of S coils in series, with Z conductors in series per stator-slot, the maximum generated voltage is:

$$E_{O max} = (B_{max} - B_{min}) ZlSv \,.\, 10^{-8} \text{ volts} \quad . \quad . \quad 3{\cdot}6$$

where Z = number of conductors in series per stator-slot
l = axial iron-length of stator, in cms
S = number of coils in series, in one complete circuit of the stator windings
v = peripheral speed of the rotor, in cms/sec.

The arrows on the armature-conductors in Fig. 3·10 represent directions of E.M.Fs at, for example, the moment of maximum voltage. The rotor position shown in the diagram corresponds to the moment of zero voltage.

The author is indebted to Prof. N. M. Oboukhoff for permission to include the following notes on a design method for these machines. For a detailed treatment, reference should be made to Prof. Oboukhoff's papers on the subject.(14)

Putting $\alpha = \frac{B_{\text{min}}}{B_{\text{max}}}$.

E_{max} = amplitude of fundamental component of E.M.F. wave

$E = \frac{E_{\text{max}}}{\sqrt{2}}$ = RMS value of fundamental component of E.M.F. wave

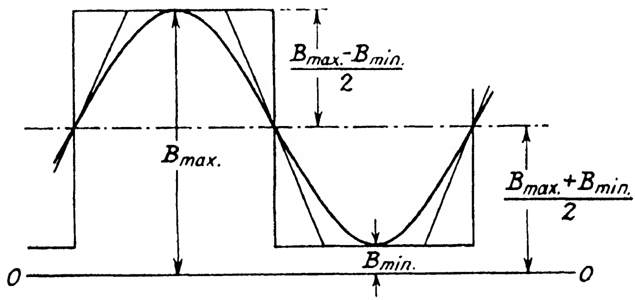


Fig. 3·12. Rectangular, Trapezoidal and Sinusoidal Flux Waveforms.

$$\beta_1 = \frac{E_{\text{max}}}{E_{O\text{max}}}$$

$$\beta = \frac{E}{E_{O\text{max}}} = \frac{\beta_1}{\sqrt{2}}$$

then $$E = \beta(1-\alpha)B_{\text{max}}ZlSv \,.\, 10^{-8} \text{ volts} \quad . \quad . \quad . \quad 3{\cdot}7$$

Now the actual distribution of gap-flux can be represented, in any particular case and with sufficient accuracy, by one of the several wave-shapes shown in Fig. 3·12, i.e. rectangular, trapezoidal, or sinusoidal. These wave-shapes also represent the armature E.M.F. time-graph, since the windings are concentrated into one effective slot per pole-pitch.

Rectangular and sinusoidal waveforms of flux-distribution (and therefore of E.M.F.) are limiting cases, for which:

Rectangular waveform: $\beta_1 = \frac{4}{\pi} = 1{\cdot}27$; $\beta = \frac{4}{\pi\sqrt{2}} = {\cdot}905$, sine curve: $\beta_1 = 1;\ \beta = \frac{1}{\sqrt{2}} = \cdot 707,$

and in practice, therefore: $1 < \beta_1 < 1\cdot 27,$

and $\cdot 707 < \beta < \cdot 905.$

For trapezoidal waveforms a value of $\beta = \cdot 8$ to $\cdot 9$, i.e. average value $= \cdot 85$, can be taken.

For high-frequency machines, especially with rotor-teeth slightly rounded, the E.M.F. waveform is approximately sinusoidal, so that β may be taken as ·707 to ·8, average value ·75.

The effect of armature reaction is much less in these basic inductor-generators than in normal machines, so that the generated E.M.F. on-load is practically the same as on no-load; the armature-flux behaves practically entirely as leakage-flux, i.e. it results in the armature reactance already mentioned.* Thus the armature reactance can be measured reasonably accurately at standstill, the armature being supplied from another source with current at the appropriate frequency. The reactance value corresponding to the rotor position shown in Fig. 3·10 is then used for calculation purposes. Alternatively it can be determined from a resonance test, when its magnitude will be equal to the reactance $\frac{1}{2\pi f C}$ of the series condenser which gives maximum current in a resistive load.

In design the armature reactance may be estimated as follows:

Armature reactance $= \omega L_a = 2\pi f L_a$ ohms.

$$L_a = \frac{L}{M},$$

where L_a = inductance of complete armature-winding

L = inductance of 1 circuit

M = number of parallel circuits in armature-winding.

$$L = \Delta Z^2 l S . 10^{-8} \text{ henries} \quad . \quad . \quad . \quad . \quad 3\cdot 8$$

where Δ = flux linked with an armature-coil, per unit length of stator-core, and per ampère-turn of stator-slot.

Combining equations 3·6, 3·7, and 3·8, the following design formulae are obtained:

$$Z = \beta \frac{L}{\Delta} \cdot \frac{B_{max}(1-\alpha)v}{E} \quad . \quad . \quad . \quad . \quad 3\cdot 9$$

* Armature reaction is not negligible in certain types with special notchings, e.g. Fig. 3·18, in which armature-windings are concentrated in a few slots.

$$Sl=\frac{L}{\Delta}.\frac{10^8}{Z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}10$$

$$Sl=\frac{\Delta}{L}.\frac{E^2}{\beta^2}.\frac{10^8}{v^2B_{max}{}^2(1-\alpha)^2} \quad . \quad . \quad . \quad . \quad 3{\cdot}11$$

α can be calculated, or estimated graphically, or determined experimentally. Prof. Oboukhoff states that Δ has an almost constant value, being about 14 for semi-enclosed stator-slots and about 10 for open slots. It can be calculated for any particular magnetic circuit, or experimentally determined.

The procedure for design of the inductor type of machine may now be outlined:

Rotation-speed n and peripheral-speed are chosen; $v=100$ to 140 metres/sec for H.F. machines with reasonably "smooth" rotors. The rotor diameter, D, is thus fixed from:

$$D=\frac{v}{\pi n}.$$

The pole pitch: $P_r=\frac{v}{f}.$

Number of poles: $p=\frac{f}{n}.$

The number of parallel paths in the armature-winding is selected; the number of coils in series in each circuit is then calculated. For a chosen air-gap (·030″ is a typical value), α can be estimated or determined experimentally.

Using the appropriate value for Δ as quoted previously, equations 3·9, 3·10, and 3·11 yield Z and l.

Armature-conductors can be loaded to a greater extent than in normal machines (even up to 6000 ampères/sq. in.).

Slot-shape is defined, and then Δ, α, Z, and l checked back. The leading dimensions D, l, Z, P_r, S, v, Δ, α, and slot measurements are thus fixed. The method is applicable to alternate-pole machines if equations 3·9, 3·10, and 3·11 are replaced by:

$$Z=2\beta\frac{L}{\Delta}.\frac{vB'_{max}}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}12$$

$$Sl=\frac{L}{\Delta}.\frac{10^8}{Z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}13$$

$$Sl=\frac{\Delta}{4L}.\frac{E^2.10^8}{\beta^2B'_{max}{}^2v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}14$$

where B'_{max} is the maximum air-gap flux.

In the foregoing the ratio $\alpha=\frac{B_{min}}{B_{max}}$ is required to be known. Prof. Oboukhoff describes an alternative procedure based upon the ratio $\gamma=\frac{\Phi_{max}}{\Phi_{min}}$, Φ_{max} being the total flux in a rotor-tooth, and Φ_{min} the flux in a rotor-slot. These flux-values are more readily determined than the flux-densities.

From Fig. 3·12 it may be shown that

$$1-\alpha=K\left(1-\frac{1}{\gamma}\right) \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}15$$

where $$K=K_s=\frac{\pi}{2}\left(\frac{1}{1+{\cdot}285\left(1-\frac{1}{\gamma}\right)}\right) \quad . \quad . \quad . \quad 3{\cdot}16$$

for a sinusoidal flux-wave;

and $$K=K_r=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}17$$

for rectangular flux-wave.

For medium-frequency machines (500 to 2000 cycles/sec) a average value for the flux-variation coefficient γ is 3·5 to 4·0. For high-frequency machines (say, 10,000 cycles/sec) $\gamma=1{\cdot}75$ to $2{\cdot}25$; the reason for the difference lies in the ratio $\frac{P_r}{\delta}$ $\left(\text{i.e. } \frac{\text{Pole-pitch}}{\text{Air-gap}}\right)$.

Taking	$\gamma=4$ for medium frequencies
and	$\gamma=1{\cdot}75$ for high frequencies,
we get	$K_s=1{\cdot}29$ (medium frequencies)
and	$K_s=1{\cdot}40$ (high frequencies).
So that	$K_s \doteqdot 1{\cdot}345$ for machines between medium and high frequency, for sinusoidal flux variation.

Similarly, for trapezoidal wave-shape (Fig. 3·12):

$$1-\alpha=K\left(1-\frac{1}{\gamma}\right) \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}18$$

where $$K=K_t=\frac{2a_p}{a_p+a'_p}\cdot\frac{1}{1+{\cdot}5\left(1-\frac{1}{\gamma}\right)\left(\frac{2a_p}{a_p+a'_p}-1\right)} \quad . \quad 3{\cdot}19$$

Substituting $K\left(1-\frac{1}{\gamma}\right)$ for $(1-\alpha)$ in 3·9, 3·10, and 3·11, and putting $K\beta=\backsim$, we get:

$$Z=\backsim\left(1-\frac{1}{\gamma}\right)\frac{L}{\Delta}\cdot\frac{v.B_{max}}{E} \quad . \quad . \quad . \quad . \quad 3{\cdot}20$$

$$Sl=\frac{L}{\Delta}\cdot\frac{10^8}{Z^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 3\cdot21$$

$$Sl=\frac{1}{\sim^2\left(1-\frac{1}{\gamma}\right)^2}\cdot\frac{\Delta}{L}\cdot\frac{E^2.10^8}{v^2\,B_{\max}{}^2} \quad . \quad . \quad . \quad . \qquad 3\cdot22$$

making possible the calculation of leading dimensions of an alternator in terms of γ.

An empirical expression for γ developed by Prof. Oboukhoff is:

$$\gamma=1+\left[\frac{\left\{P_r(1+y)-y\,\frac{2a_r}{P_r}\right\}\frac{2a_r}{P_r}\left\{(1+y)-y\,\frac{a_s}{P_s}\right\}\frac{a_s}{P_s}}{A\delta+CB_{\max}{}^q}\right] \qquad 3\cdot23$$

a_r, P_r, a_s, P_s are in millimetres (see Fig. 3·10). A, C, q, and y are numerics which can be regarded as constants for a given type of generator. δ is the simple air-gap length. For the case of the cylindrical-rotor inductor-alternator:

$$C=10^{-7} \text{ and } q=2$$

provided that the point of maximum voltage is not passed in the no-load characteristic.

Coefficients A and y depend upon $\frac{\delta}{P_r}$; thus, for a cylindrical rotor:

$$\begin{aligned}
A&=12 &&\text{for } \frac{\delta}{P_r}\lesseqgtr\cdot01\\
&=11 &&\text{for } \cdot01<\frac{\delta}{P_r}\lesseqgtr\cdot025\\
&=10 &&\text{for } \frac{\delta}{P_r}>\cdot025
\end{aligned}$$

and

$$\begin{aligned}
y&=3 &&\text{for } \frac{\delta}{P_r}\geqq\cdot08\\
&=2\cdot25 &&\text{for } \cdot05<\frac{\delta}{P_r}\lesseqgtr\cdot08\\
&=1\cdot5 &&\text{for } \cdot01<\frac{\delta}{P_r}\lesseqgtr\cdot05\\
&=1\cdot0 &&\text{for } \frac{\delta}{P_r}\lesseqgtr\cdot01.
\end{aligned}$$

Finally, from equation 3·23, the optimum ratio $\frac{a_r}{P_r}$ is shown to be equal to $\frac{y+1}{4y}$.

Tables of values for β, K, and $\backsim$ are given in the papers referred to. It is sufficient here to note that an average value of ·955 can be taken for $\backsim$, with a maximum error of 5 % or 6 %, for all practically-occurring wave-shapes. With specially-shaped teeth, giving an approximately sinusoidal flux-wave, $\backsim \doteqdot 1{\cdot}0$ for H.F. machines and ·913 for medium-frequency machines.

The no-load characteristic (generated voltage plotted against field excitation current) exhibits a property peculiar to these machines; that is, with increasing excitation, the generated voltage increases to a definite *maximum* and then begins to fall again. This is because, with increasing magnetic-reluctance in the rotor-teeth, the slot flux increases at a greater rate than the tooth-flux, so that the ratio $\gamma = \frac{\Phi_{\text{tooth}}}{\Phi_{\text{slot}}}$ falls, and (see equations 3·9 and 3·20) the generated voltage falls when $B_{\max}$, is increased beyond a particular value corresponding to the beginning of the knee of the magnetisation curve of the rotor material. The "negative voltage" $-e_2$ of one coil-side becomes large relative to the useful voltage e_1 of the other coil-side.

It is usual to arrange that the normal operating point lies close to this maximum-voltage point; i.e. the on-load exciting flux should be as near as possible to that which gives maximum no-load voltage. This means that $B_{\max}$ is of the order of 8000 to 11,500 lines/cm², depending upon the rotor material. The relations between maximum flux-densities in stator- and rotor-teeth, and corresponding air-gap flux-density $B_{\max}$, are:

Rotor: $$B_r = \frac{B_{\max}}{\psi t}\left[2 - K\left(1 - \frac{1}{\gamma}\right)\right] \quad . \quad . \quad . \quad . \quad 3{\cdot}24$$

Stator: $$B_s = \frac{K'}{\psi} \cdot \frac{P_s}{a'_s} . B_{\max} \quad . \quad . \quad . \quad . \quad . \quad . \quad 3{\cdot}25$$

where $t = \frac{2a'_r}{P_r}$; a'_r=pole-breadth at the root

$$K' = \frac{1}{1 + \left(1 - \frac{1}{\gamma}\right)\left(\frac{a_p}{a_p + a'_p} - {\cdot}5\right)} \text{ for trapezoidal flux-wave}$$

$K' = \frac{2}{\pi} K$ for sinusoidal flux-wave

K=(see equations 3·16 and 3·17)

ψ=iron "stacking factor".

Saturation usually occurs in the rotor-teeth first, though the densities in rotor- and stator-teeth do not differ greatly. Adequate cooling of the rotor-teeth is an important problem.

The performance curves of a small inductor-type generator are shown in Figs. 3·13 and 3·14. In curve (*a*) the open-circuit characteristic exhibits the maximum-voltage point already referred to; further increase of field-excitation results in a fall in generated

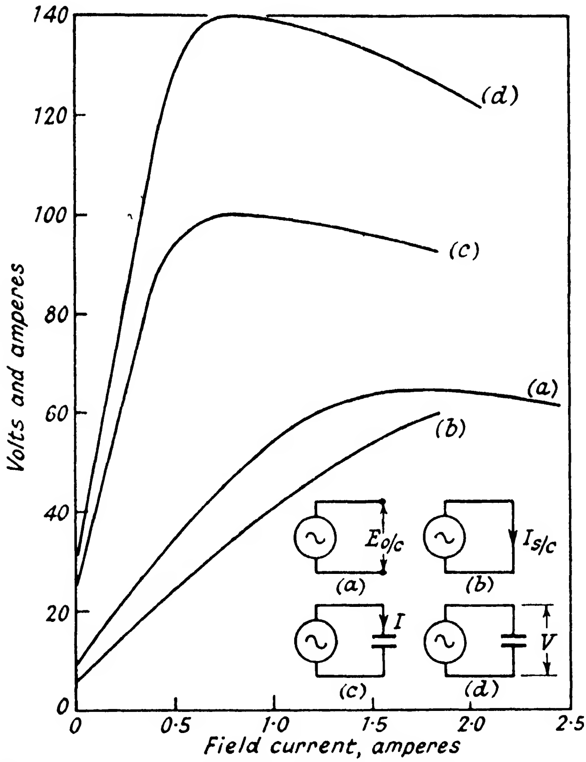


Fig. 3·13. Characteristics of Small Inductor Alternator.

voltage. The short-circuit characteristic is shown as curve (*b*). The rating of this machine was 46 volts, 15 ampères, 1400 cycles/sec, 2100 r.p.m. The relatively large short-circuit current at zero field-current is in agreement with the previous statement that armature-reaction effects are small. Curve (*c*) shows "short-circuit" current against excitation, with a condenser across the alternator, and negligible external resistance; the maximum current is now 100 ampères, and occurs at a field-current for which true short-circuit current (curve (*b*)) was only 35 ampères, and for which the open-circuit voltage (curve (*a*)) was the nominal voltage of the machine. Curve (*d*) shows the terminal voltage of the machine under the same conditions as (*c*), i.e. terminals shunted

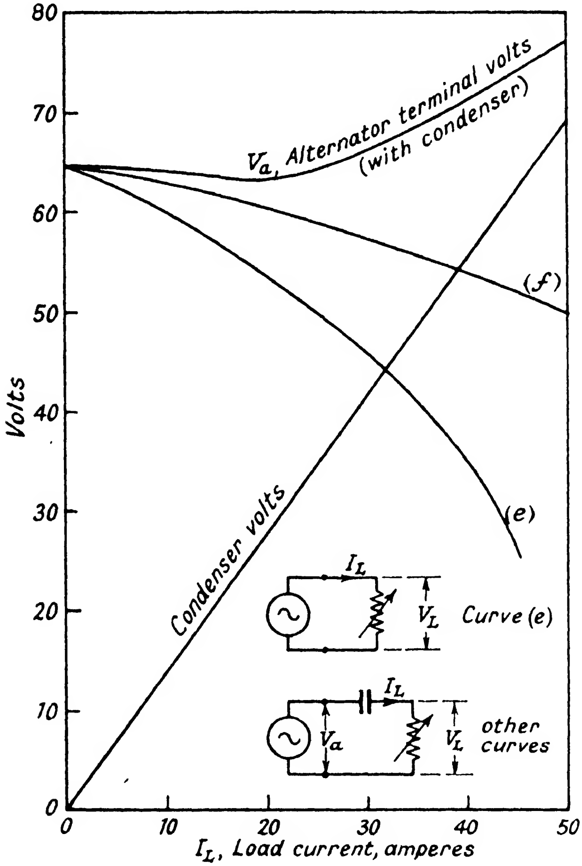


Fig. 3·14. Load Characteristics of Small Inductor Alternator.

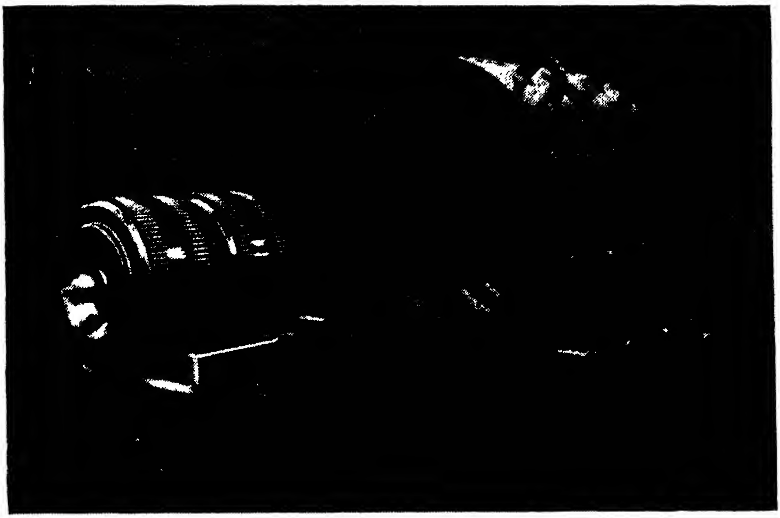
by a condenser; maximum voltage now occurs, of course, at the same point as maximum current, and is 140 volts.

The ratio of voltage to current (from (*c*) and (*d*)) shows that the condenser reactance was 1·4 ohms at 1400 cycles/sec $\left(X_c=\frac{140}{100}=1{\cdot}4\Omega\right)$. The actual capacitance was adjusted to resonate with the generator leakage-reactance, which was therefore about

$1{\cdot}4\Omega$. Now the ratio of voltage to current (from (*a*) and (*b*)) at the same excitation (0·8 ampère) gives:

$$Z=\frac{47}{34}=1{\cdot}38\Omega.$$

That is, assuming that the internal E.M.F. of the armature on true short-circuit (*b*) is the same as on open-circuit, then the armature behaves as an impedance of $1{\cdot}38\Omega$; the armature and circuit-resistance in this case (·47 ohm) contributes but little to this impedance, so that the armature-reactance value given approximately by $\dfrac{\text{Open-circuit volts}}{\text{Short-circuit current}}$ is seen to be practically identical



[*Courtesy of E.C.C. Ltd.*

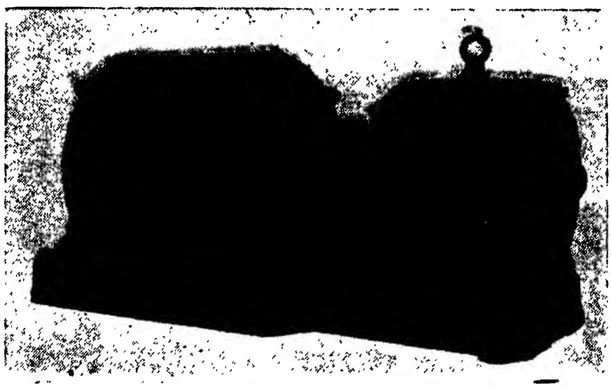
Fig. 3·15. Double 12·5-KVA 5000 cycles/sec Homopolar Inductor Alternator.

with the armature leakage-reactance. This provides more proof that armature reaction is small and that only reactance need be considered. Actually, for $Z=1{\cdot}38\Omega$ and $R={\cdot}47$, the reactance is $1{\cdot}3\Omega$.

Curve (*e*), Fig. 3·14, shows the load-characteristic without the series-condenser; in curve (*f*) the effect of the series-condenser is seen. A pure-resistance load was employed in each case, and the generator excited for maximum no-load volts (1·8 ampères field-current, 64·5 volts O/C). The condenser voltage rises in direct proportion to the load current (for constant frequency), and the maximum voltage which can be developed across it is 140 volts, corresponding to zero load resistance.

The KVA rating of the series-condenser will be the product of maximum voltage (i.e. zero load-resistance condition) and normal full-load current, since the condenser must be able to withstand the maximum voltage which the circuit is capable of developing, but it will not be required to pass currents in excess of the normal full load for any appreciable time.

Figs. 3·15 and 3·16 illustrate an interesting example of two homopolar inductor-alternators mounted in one frame. Each alternator is rated at 12·5 KVA, 5000 cycles at 3000 r.p.m. The stator-cores, of low-loss iron, are fitted into a fabricated steel



[*Courtesy of E.C.C. Ltd.*

Fig. 3·16. Double 12·5-KVA 5000 cycles/sec Motor-alternator Set.

yoke; the stator-coils are hand-wound, in semi-closed slots, and comprise five effective conductors per slot, of two parallel ·042″ D.C.C. The armature reactance of one alternator is approximately 20 ohms, requiring a series-condenser of about 1·6 microfarads to neutralise it. Former-wound exciting coils are located between the stator-rings. The laminated rotor is mounted on a cast-steel centre, held by phosphor-bronze clamp-rings. In Fig. 3·16 the double-alternator is shown coupled to a standard squirrel-cage induction motor by means of a Bibby coupling. Ball- and roller-bearings are used throughout.

The heteropolar alternator shown in Fig. 3·8 (a) and (b) has a 4-pole field system wound in four specially-notched slots. It is rated at 420 volts, 2000 cycles/sec, 10 KVA, 0·8 power factor, single phase, at 3000 r.p.m., and operates satisfactorily at this lagging power factor without a series-condenser.

Special types of inductor-alternator

Fig. 3·10 shows 2 stator-slots per rotor-pole, which is the usual case for a single-phase machine. To extend the same scheme for 2- or 3-phase output, the number of stator-slots may be doubled or trebled, or two or three sets of stator-rings may be mounted axially. The first alternative requires an increased diameter, the second an increased length.

An ingenious third alternative is described in British Patent No. 483660 (B.T.-H. Co. Ltd., and J. H. Walker).[13] The metho is applicable to homopolar and heteropolar types. The armature-windings are exactly as for a single-phase machine, but the number of stator-slots is:

$$S = 2p \pm 2 = 2(p \pm 1)$$

p being the number of rotor-poles. It will be remembered that in the normal single-phase stator, $S = 2p$. The E.M.F. in each successive armature-conductor is thus displaced in phase by $\frac{360°}{S}$ relative to the previous one, the phase displacement totalling 180° between any two conductors separated by one-half the total winding. The armature-winding is opened at appropriate points to give a polyphase output, e.g. for a 3-phase supply it is opened at 3 or 6 points equidistant in space, the 3 or 6 segments of the winding being suitably combined to give star or delta connections. For the heteropolar construction the number of pairs of exciting poles must be a multiple of the number of phases, as also must S, the number of stator-slots. For homopolar constructions, S must be a multiple of the number of phases.

This special construction also gives an improved waveform compared to that of the normal inductor type, being almost perfectly sinusoidal.

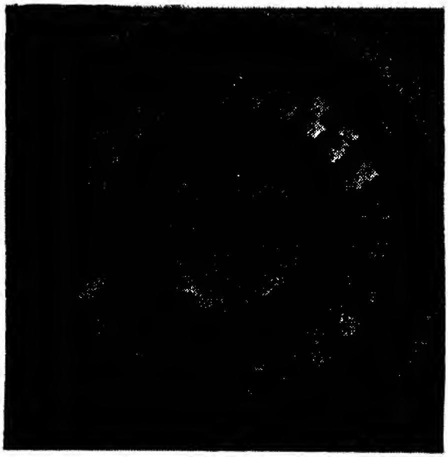
Other interesting features mentioned in this specification are:

(1) The possible use of low-frequency A.C. excitation instead of D.C., with heteropolar construction, when, by the addition of a squirrel-cage winding to the rotor, no separate driving motor is required, i.e. the machine operates as a frequency changer.

(2) The possible excitation by means of D.C. coils located at the bottom of the armature slots, or by coils wound on the rotor.

(3) The possible provision of non-magnetic wedges to fill the rotor-slots, to reduce noise and windage.

A very interesting recent development is the "vernier"-type H.F. inductor-alternator,[15] *in which the number of rotor-teeth exceeds that of the stator-teeth. Fig. 3·17 shows a homopolar arrangement in which there are 26 rotor- and 24 stator-teeth, giving 2 zones of* strong flux, in those parts of the gap where stator-teeth are approximately opposite rotor-teeth, and 2 zones of weak flux where stator-teeth are approximately opposite rotor-slots. In general, the number of such "cycles" of flux change round the gap is equal to the difference in number of stator- and rotor-teeth. Open slots are employed on the stator.



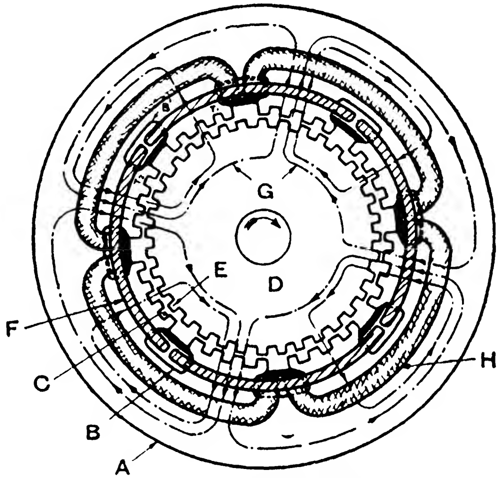
[*Courtesy of Metropolitan-Vickers ElectricalCo. Ltd.*

Fig. 3·17. Stator and Rotor Core Plates for Vernier Alternator.

A distributed stator-winding arrangement is indicated in the figure. It can be seen that a rotor movement of half a tooth-pitch would shift the strong and weak flux zones through a quarter-revolution, while a movement of one tooth-pitch results in a complete cycle of flux change through each coil as in the normal inductor generator. The effect is as if a 4-pole rotor were running at $\frac{f}{2}$ revolutions/sec; the generator frequency f being $P_R n$ cycles/sec; n is the actual rotor speed, and P_R the number of rotor-teeth or -poles. The average number of stator-teeth embraced by a coil is 3, and the teeth are wider than in a normal type since they are fewer in number; consequently, the average voltage per turn is much higher. For the case shown, the open-circuit voltage is about 60% higher than in a normal type having 52 stator-slots and the same number of conductors per slot.

Another factor which increases the generated voltage is the larger ratio $\frac{\Phi_{max}}{\Phi_{min}}$ for each stator-tooth, which is realisable at maximum excitation. Finally, the smaller number of stator-slots allows of more copper area per slot for a given stator diameter, so that the output from a vernier machine is considerably greater than that from the basic inductor type of the same size.

Another recent development in inductor-alternator design is that described in British Patent application 14710/41* (B.T.-H. and J. H. Walker). This new type of heteropolar construction requires a much smaller number of stator-slots and A.C. coils than do the conventional types (homo- or heteropolar) for a given frequency. Also there is a true flux reversal through each A.C. coil, and each coil is twice as effective in generating an E.M.F. as those in conventional types (in which the flux changes, but does not reverse); this feature, together with the increased winding-space factor and greater flux per pole made possible by the small number of stator-slots, results in a machine of less weight than conventional types for the same power output.



[Courtesy of B.T.-H. Co. Ltd.

Fig. 3·18.

Principle of B.T.-H. Lightweight Alternator.

A: Stator laminations. *E*: Rotor-teeth.
B: Large stator-slots. *F*: Field-coils.
C: Small stator-teeth. *G*: Flux paths.
D: Rotor laminations. *H*: Armature-coils.

The arrangement of stator, rotor, D.C., and A.C. coils employed is shown in Fig. 3·18, representing a machine with a 4-pole D.C. field system, and a 36-tooth rotor. The stator is divided by means of 8 large slots, to have 8 large teeth, and each large tooth is subdivided to have 4 small teeth and 3 small slots. Each large stator-slot is twice the width of a small slot or tooth, and small stator-teeth have the same angular pitch as the rotor-teeth. Thus alternate large stator-teeth have their small teeth opposite rotor-teeth at the instant when the remaining large teeth have their

* See also British Patents Nos. 554827, 570107, 570125, 574325, 589039.

small teeth opposite rotor-slots, so that there are alternate zones of strong and weak flux round the gap. These zones change places for a rotor movement of a half rotor-slot pitch, and a complete cycle of flux change occurs for every slot-pitch of rotation. An E.M.F. would therefore be generated in a coil wound round a large stator tooth, and its frequency would be:

$$f=pn \text{ cycles/sec,}$$

p being the number of rotor-poles or -teeth, and n the speed in revolutions per second.

An increase in flux through one such coil would be accompanied by a decrease of flux through a similar coil on an adjacent large stator-tooth. With the field-coil arrangement shown, these two fluxes are of opposite polarity, so that, in practice, A.C. coils are wound to embrace adjacent pairs of large stator-teeth, as shown, the E.M.F. per turn in the double-pitch coil being equal to the sum of the E.M.Fs per turn in two single-pitch coils.

There is no significant flux change through the D.C. coils, only a cyclic redistribution of flux within each coil, so that no alternating E.M.F. is induced in these coils.

Only 4 D.C. and 4 A.C. coils, accommodated in 8 slots, are required for this machine. A conventional heteropolar machine with a 36-tooth rotor would require 4 D.C. and 36 A.C. coils, and a total of 76 stator-slots; the corresponding figures for the homopolar machine are 1 D.C. coil, 36 stator-coils, and 72 stator-slots.

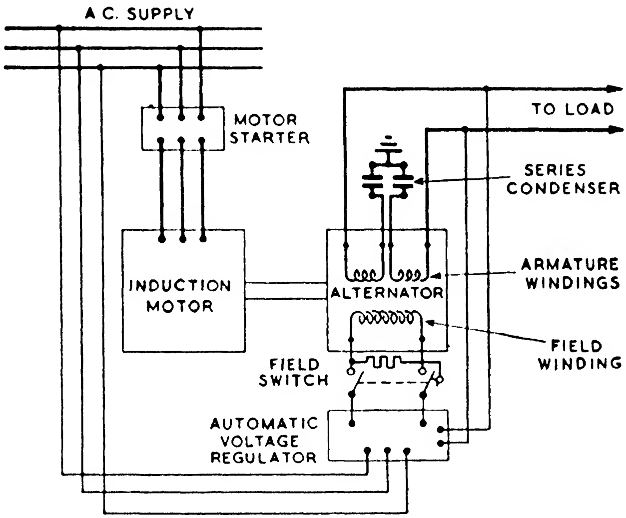
The waveform of the new-type alternator is very good, and the machines are relatively quiet. Frequencies up to 50 KC/S have already been obtained. By using a pair of machines, with the field of one excited from the A.C. output of the other, it is possible to obtain a double-frequency output. The two alternators must have the same nominal frequency.

Voltage control of H.F. alternators

The high armature-reactance inherent in H.F. inductor-alternators causes rather large changes of terminal voltage with load. It is to minimise this large voltage-regulation that series-condensers are used with most machines. Some form of automatic voltage-regulator is desirable, however, which will maintain almost constant terminal voltage in spite of variations in loading; a few cases occur in which manual control is sufficient, in particular, where only one load, requiring relatively long heating-time, is being supplied. All forms of controller, whether manual or automatic, operate on the alternator field-current.

Several kinds of automatic voltage-control gear are available. There are three main categories: electro-mechanical, such as the carbon-pile, Tirrill, and Isenthal types, special generators such as the Amplidyne, and electronic. *The latter type is desirable if very high speed response is required;* the B.T.H. electronic voltage-regulator, for example, maintains terminal voltage within 1 % of a selected* value over the full range of load.

The speed of response depends upon the time-constant of the alternator field-circuit; the advantage of heteropolar construction in this respect has been referred to. A schematic diagram of



[*Courtesy of B.T.-H. Co. Ltd.*

Fig. 3·19. Schematic Diagram—High-frequency Alternator Set for Induction Heating.

a high-frequency voltage-controlled alternator set for induction heating is given in Fig. 3·19.

Parallel operation of alternators

It is probable that H.F. substations of 1000 KW rating and upwards will be installed in many works during the next few years. In most cases the demand for H.F. power will grow over a period of years, and the most economical layout will be that which permits additional plant to be laid down as the load increases. For

* The Amplidyne generator is particularly useful for forcing a rapid change of field-current in a homopolar alternator, since reversal of polarity of field supply is easily obtained from it.

example, a 500-KW set may be installed for a forging shop, with the expectation that the load will be increased to 1000 KW in two years' time. It is obviously desirable that the two sets shall be suitable for parallel running on common bus-bars, as this makes for flexibility in operation.

H.F. alternators for parallel operation should have identical voltage regulation from no-load to full load, and their driving motors should also have similar characteristics. Dr. J. H. Walker recommends that the rating of a combination of alternators should be 10% less than the sum of their individual ratings, to allow for possible variations in percentage slip between induction-motors.[(13b)] To bring an inductor machine on to live bus-bars, it is run up to approximate synchronism and then connected, with its field unexcited. It is pulled into synchronism, and its field-current is then adjusted so that the alternator takes its proper share of the reactive load (see Chapter 9).

Industrial applications of H.F. alternators

Metal melting, heating for forging, and heat-treatment form the principal demand for H.F. alternator power, and these are considered in some detail in Chapters 6 and 9.

Signalling and remote control over mains-frequency power transmission and distribution systems, by means of superimposed "high-frequency" currents, are developments in which H.F. alternator equipment is employed. The control-frequency component is easily filtered, and rectified to operate a relay; alternatively, the relay itself can be arranged to resonate to the control frequency. Any one of many relays can be operated remotely, from a fixed-frequency alternator, by making the relay resonate to the frequency at which *trains* of control-oscillations are fed to the system; that is, the H.F. alternator output is modulated to a selected rhythm, and this rhythm causes one relay, or one group of relays, to operate.[(76)]

Portable electric tools such as drillers and grinders can be made lighter in weight by using high-speed motors. The maximum speed obtainable with an induction motor at 50 cycles/sec is 3000 r.p.m.; commutator motors must be used for higher speeds at this frequency. Portable tools are now being made which incorporate H.F. induction motors, power being supplied from a special motor-alternator. Typical frequencies are 150 and 200 cycles/sec, corresponding to maximum motor speeds of 9000 and

12,000 r.p.m. Reduction-gears between motor and output spindle give the correct tool or grinding-wheel speed.

For a given volt-ampère rating, the size and cost of a transformer or choke for operation at a "high" frequency will be less than of one for 50 cycles/sec, and this has been considered sufficient justification for employing frequencies of the order of 500 cycles/sec in some large installations of gas-discharge lighting.

The vibration testing of structures, particularly aircraft structures, to determine resonance frequencies has been carried out, using suitable electro-magnets, powered by a variable-frequency motor-alternator, to force the structure into vibration. Alternators for this purpose are usually required to cover a frequency range up to about 2000 cycles/sec.

(Where it is required merely to vibrate a component or structure at its own principal resonance-frequency, a simple method is to feed the pick-up voltage, generated in a vibration detector, to a thermionic valve power-amplifier, using the output of this amplifier to maintain the mechanical vibration, i.e. the component or structure behaves as the "tank-circuit" of a power-oscillator (see Chapter 5). Fatigue-testing can conveniently be done in this way.)

Motor-alternator losses and efficiencies

The declared efficiency of these machines is covered by B.S.S. 269: 1927, and is the ratio of power output to output plus losses. The losses comprise fixed loss (friction, windage,* and core loss at no-load), exciting circuit losses, direct load loss (armature I^2R loss, R being the D.C. resistance at 75° C), and stray load loss (being the additional losses in core, armature windings, and other metal parts due to load current). Losses in one 400 KW 1000 cycles/sec inductor machine were: fixed loss 22 KW, sum of field, direct and stray load losses 12 KW, total 34 KW; the full-load efficiency of the alternator was therefore 92%. Motor efficiency was 96%, giving 88·4% overall. Core and stray losses increase with frequency, and alternator efficiency falls to 75–80% at 10,000 cycles/sec.

Cooling air requirements average 100 cu. ft./min per KW of losses, with an air temperature rise of about 20° C above an inlet temperature of 25° C.

* See also p. 320.

CHAPTER 4

The Triode Valve

FOR FREQUENCIES higher than about 50,000 cycles/sec, dynamo-electric machines become increasingly impracticable, mainly because of inertia-effects which set a limit to the permissible peripheral speed of the rotor. Instead, high-vacuum thermionic valves are used; there is no appreciable inertia of the "moving parts" (electrons) except at very high frequencies indeed; and, for the same reason, there is no reactance within the valve, which therefore behaves like a purely resistive device.

We are here concerned especially with the valve as a power-oscillator, or oscillation-generator, or power-converter, and a brief consideration of Fig. 4·1 will be helpful at this stage. (Refer also to Fig. 1·28.)

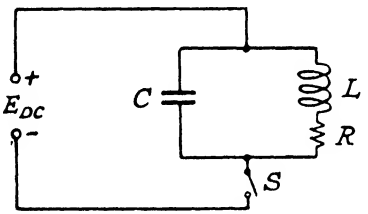


Fig. 4·1. Principle of Valve Oscillation-generator.

If switch S is closed for a moment, C is charged to the voltage E_{DC} (this neglects the effects of possible inductance and resistance in the supply leads and the supply), and if S is then opened again, C will commence a damped oscillatory discharge through LR. After one cycle of this oscillation, C is again charged with the polarity it had initially, but to a somewhat lower voltage, depending upon the damping, i.e. the ratio $\frac{R}{2L}$. If the switch S again be closed for a moment, the voltage of C will be raised once more to E_{DC}. In this way the losses in the LCR oscillatory circuit are made good every cycle by the momentary closing of S, and a *maintained* oscillation, of constant amplitude, exists in the closed LCR circuit. Alternating-current power, having a frequency of approximately $\frac{1}{2\pi\sqrt{LC}}$ cycles/sec, is obtained from the D.C. source, and the device is an oscillation-generator or power-converter. The actual load may be coupled in any convenient manner to the oscillatory circuit, and its effect will be to increase the effective value of R, and probably to alter the effective values of either L or C, or both, thus modifying the frequency.

In the valve-oscillator, the triode valve behaves, in effect, like the switch S, and by its action the oscillatory or "tank"-circuit is supplied, during a part of each cycle, with energy from the D.C. source. The valve is not, however, a simple switch, and its characteristics must be carefully studied in order that it may be properly applied, when the efficiency of power conversion from D.C. to any frequency up to at least several megacycles/sec may be as high as 75 or 80%.

The triode high-vacuum thermionic valve

As implied in its name, this valve has three electrodes:

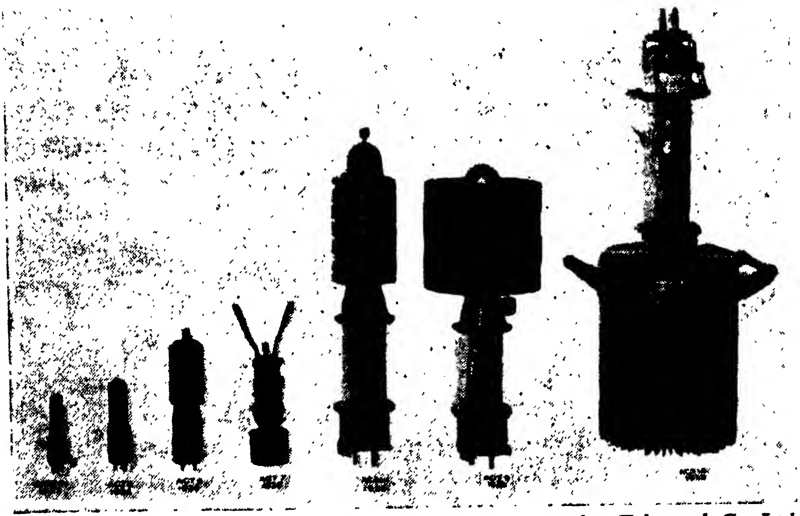
(1) A thermionic electron-emitting cathode, heated for the purpose either (a) by the direct passage of a current through it, in which case it is made up in the form of a wire or ribbon "filament", or (b) by conduction and radiation from a separate heater. In this case the cathode is commonly in the form of a hollow tube, with the heater inside and insulated from it electrically.

(2) An electron-collecting electrode, the anode or plate, which, when kept at a higher potential than the cathode, attracts and collects electrons emitted from the cathode. It is usually either cylindrical or box-shaped. The plate becomes heated by the bombardment of these arriving electrons, as well as by radiation from the cathode, and in the larger valves steps are taken to cool it artificially with either water or air-blast.

(3) A control electrode, the "grid", usually, though not essentially, in the form of a wire wound on supports so that it encloses the cathode, and stands between the cathode and plate. Electrons may pass from the cathode to the plate through the spaces between adjacent turns of the grid winding, but the attraction of the plate is considerably modified by the p.d. between grid and cathode. The grid-voltage thus controls the current through the valve.

Almost any valve is capable of satisfactory operation at all frequencies up to 2 MC/S, and within this limit the most important factors are the maximum permissible plate-power dissipation and the cathode emission. If, for example, a high-frequency power output of 5 KW is required, using a single valve, and the efficiency with which the valve converts from D.C. to A.C. power is 75%, then a D.C. power input of 6·67 KW is necessary, and of this 1·67 KW is dissipated at the plate of the valve. The valve must be of a size and type capable of dissipating this power, as well as the cathode-power and grid-power, without overheating. Most power valves at present available, when used as Class C oscillators, run well within their dissipation ratings, and the limiting factor on output power is the cathode emission.

The H.F. grid-current which can be carried safely through the



[*Courtesy of Marconi's Wireless Telegraph Co. Ltd.*

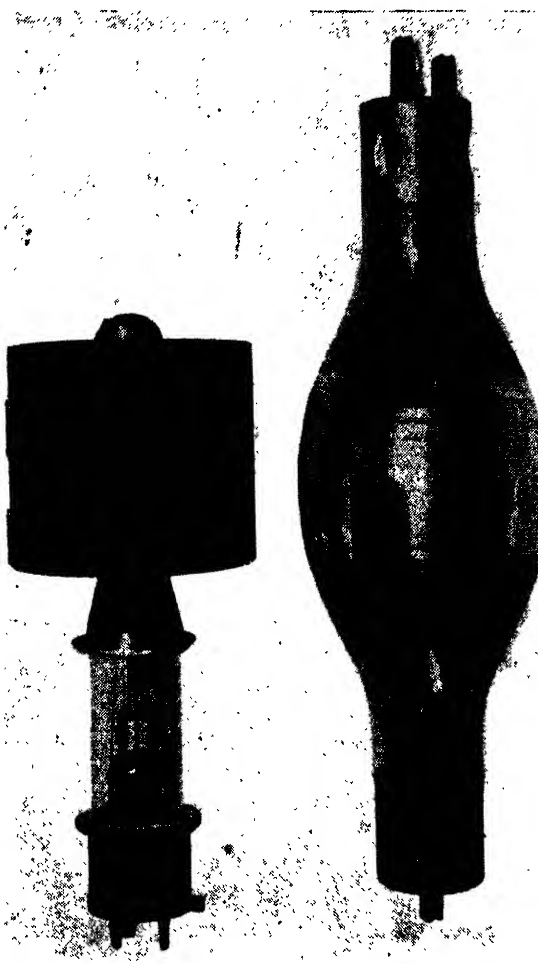
Fig. 4·2. Range of Air-cooled Valves. Left to right: 15 W, 75 W, 300 W, 300 W, 1·5 KW, 1·3 KW, 10 KW.

seals, particularly the grid seal, is a factor which limits the oscillation frequency of a valve (see Chapter 5).

Special characteristics of high-power valves[18]

The glass-bulb construction is used for plate-dissipations up to about 1·0 KW. The glass bulb is an envelope enclosing the whole electrode structure, and the valve is cooled by radiation, and by air-cooling, either natural or blown.

Valves which have to dissipate power of the order of 8 KW or more are usually water-cooled. In such cases the plate or anode is in the form of a long cylinder, sealed at one end, and joined to a glass portion at the other, the glass portion serving to insulate and seal the various leads. The anode is immersed in a water-cooling



[*Courtesy of Marconi's Wireless Telegraph Co. Ltd.*

Fig. 4·3. Two Triodes with Similar Ratings: 2·5 KW output, Class C, at medium frequencies.

system. For the same overall size, water-cooling increases the permissible power-dissipation some 20 times compared with natural cooling. Since the cooling water is in electrical contact with the anode, a long length of rubber hose must be used for inlet and outlet, about 1 KV per metre being the maximum permissible volt-drop along the hose.

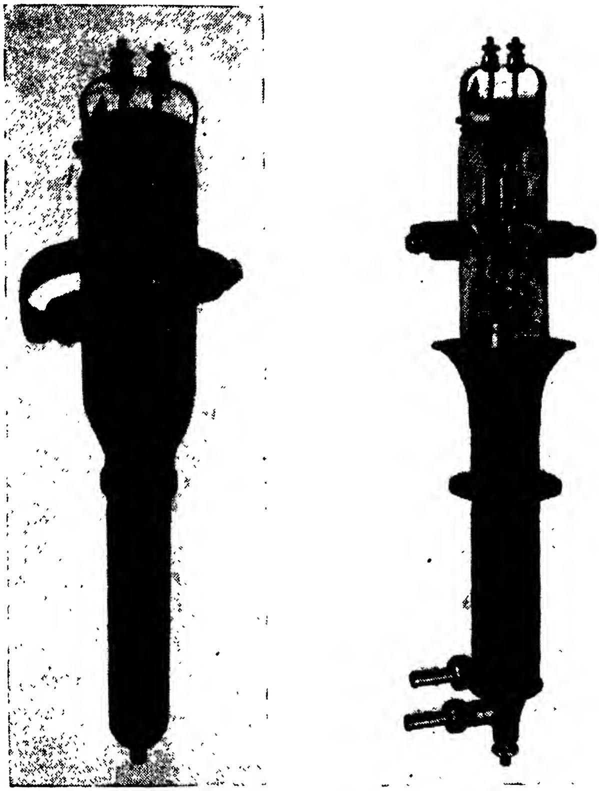
Valves which dissipate powers between 1 KW and 10 KW sometimes employ forced air cooling.(16) The simplicity and convenience of the system are important factors. The structure of

[*Courtesy of Mullard Wireless Service Co. Ltd.*

Fig. 4·4. Silica-envelope Triode, Mullard TX10-4000 7·5 KW output, Class C, at 12 KV, 2 MC/S.

the valve is similar to that for water-cooling, but the anode is surrounded by a ribbed cooler, through which air is blown at a pressure slightly greater than atmospheric. The temperature rise of the air in passing through the cooler is of the order of 40° C.

Yet another solution is to employ a valve with a silica envelope, which can withstand much higher temperatures than glass. Such valves are radiation-cooled, additional fan-cooling being sometimes provided; the maximum dissipation-power for which this construction is used is about 20 KW. Recent developments indicate that the silica-envelope valve, with a thoriated-tungsten



[*Courtesy of Mullard Wireless Service Co. Ltd.*

(*a*) (*b*)

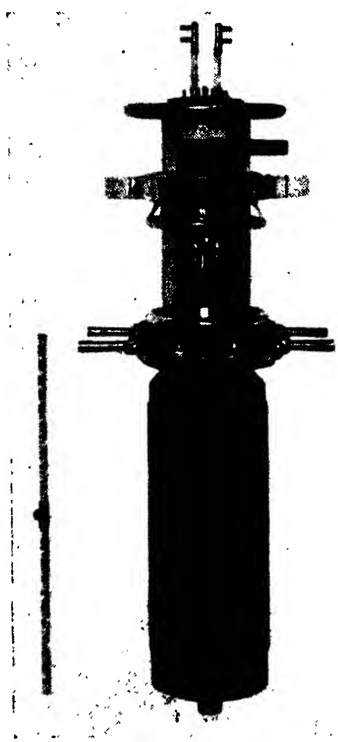
Fig. 4·5 (a) and (b). Water-cooled Triode, Mullard TX 12-20. 24 KW output, Class C, at 12 KV, 2 MC/S.

cathode, will probably become widely used for industrial power-oscillators.

Oil-cooling of cooled-plate valves has been used,[17] and will perhaps become more common in the future. It has been found that the plates of standard valves can be operated without detriment to life at temperatures considerably higher than those normally encountered in water-cooled valves. It is important, with water-cooling, that the outlet temperature of the water should not exceed 70° C., to avoid any tendency towards the formation of steam bubbles on the plate at local "hot spots".

Examples of several types of valve are illustrated in Figs. 4·2 to 4·8.

Fig. 4·2 shows a range of air-cooled valves with plate-dissipation ratings (left to right) of 15, 75, 300, 300, 1500, 1300, and 10,000 watts respectively. The largest (10 KW) is the ACT14, the blown-air cooled version of the Marconi CAT6, two of which are used in water-jackets in the 20 KW plant illustrated in Fig. 5·30.



[*Courtesy of Marconi's Wireless Telegraph Co. Ltd.*

Fig. 4·6. Water-cooled Triode, 200 KW output, Class C, at 1·5 MC/S. 120 KW output at 22 MC/S.

Fig. 4·3 shows a comparison of two valves of different types but having similar ratings. The radiation-cooled glass valve is the Ediswan ES1500A, one of the largest of its type. The other valve has a radiator fixed around its plate. Both valves will develop 2·5 KW output (Class C).

Fig. 4·4 shows a silica valve, the Mullard TX10-4000, radiation-cooled, with a plate-dissipation of 4 KW. This valve is employed in the 5 KW plant illustrated in Fig. 5·29.

A water-cooled valve is shown in Fig. 4·5 (a), and is seen in its water-jacket at (b). This is the Mullard TX12-20.

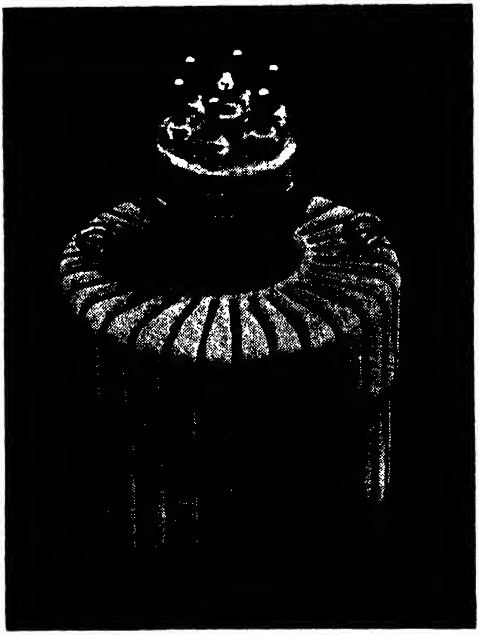
A very large water-cooled triode (Marconi) is illustrated in Fig. 4·6. This valve will develop 200 KW output at 1·5 MC/S, or 120 KW at 22 MC/S.

A forced-air cooled triode for very high frequencies is shown in Fig. 4·7. This valve has a rating of 1·8 KW output (Class C) at 150 MC/S. The radiator is of milled copper, and is arranged to fit on to a blower. The valve is made by G.E. (U.S.A.).

Fig. 4·8 illustrates an interesting modern high-frequency triode with a Class C output rating of 1 KW at 30 MC/S (R.C.A. type 833A). These relatively small high-frequency valves are particularly suitable for low-power dielectric heating plant.

Triode characteristics

The performance of the valve in any particular circuit can be determined from the static-characteristic curves of the valve. These show how plae current and grid current vary with the voltages applied to these electrodes. It may be mentioned that always, in all valves, electrode voltages are referred to cathode as zero-potential. Although triodes for different purposes have different characteristics, all follow the general form of those shown in Fig. 4·9.



[*Courtesy of Marconi's Wireless Telegraph Co. Ltd.*

Fig. 4·7. G.E. Air-cooled Triode for High Frequencies. 1·8 KW output, Class C, at 150 MC/S.

These are the "static characteristics", i.e. they are for the valve alone, and not for a valve in conjunction with a series "load" impedance.

For calculation purposes it is convenient to assume that the curves of Fig. 4·9 (a) are straight lines, all of the same slope, and uniformly spaced. This implies that although the total "D.C. resistance" of the valve $\left(\text{i.e. that the ratio } \dfrac{\text{Voltage across valve}}{\text{Current through valve}}\right)$ may be very low, as at point P, or very high, as at point Q, yet the resistance the valve offers to a *change* of plate-voltage is always the same, whatever the grid-voltage, provided this is constant during the change in plate-volts. Using the symbols of Fig. 4·9 (a), we see that

$\frac{\delta v_p}{\delta i_p}$ for any part of any characteristic

$=$a constant, called the plate A.C. resistance

$=R_p$ ohms.

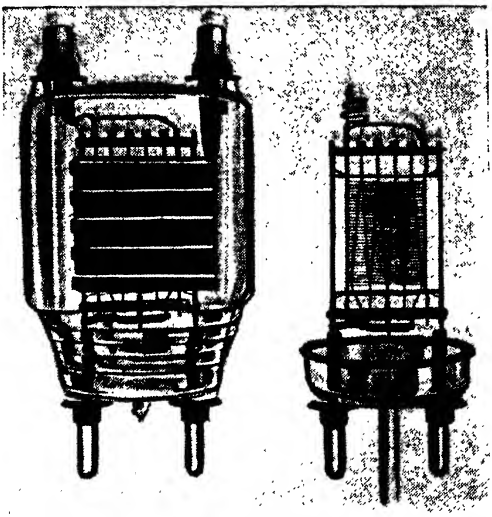
Thus R_p is the reciprocal of the slope of the plate-volts—plate-current curves, the slope being the plate A.C. conductance.

Again, uniform spacing of the v_p-i_p curves implies that the *change* in plate-current for a given change in grid-voltage is always the same, whatever the plate-voltage, provided the latter is constant during the grid-volts change. Referring to Fig. 4·9 (b),

$\frac{\delta i_p}{\delta v_g}$ for any part of any characteristic

$=$a constant, called the A.C. mutual conductance

$=g_m$ mA/volt.



[*Courtesy of Marconi's Wireless Telegraph Co. Ltd.*

Fig. 4·8. H.F. Triode 1 KW output, Class C, at 30 MC/S (R.C.A. 833A).

The justification for assuming that R_p and g_m are quite constant, for all electrode voltages, is that the actual characteristics of triodes do not depart very greatly from the linear relations of the theoretical characteristics, and that calculations based on the latter agree very well with performances actually obtained. This agreement is partly due to the fact that calculations generally involve the product R_pg_m, and this tends to remain constant even when R_p and g_m change, since an increase of one commonly accompanies a decrease of the other.

Fig. 4·10 shows the linear characteristics implied in the use of the valve "constants" R_p and g_m.

For such characteristics, the plate-current for any plate-voltage v_p and grid-voltage v_g is given by:

$$i_p=\frac{1000v_p}{R_p}+g_mv_g \text{ mA} \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}1$$

It is helpful to use numerical values, and Fig. 4·10 represents the theoretical $v_p - i_p$ curves of a small triode valve having an A.C. resistance R_p of 8700 ohms, and mutual conductance g_m of 2·8 mA/volt.

From these curves we see that an increase of 56 mA in plate-current (e.g. from 50 mA to 106 mA) may be obtained either by a grid-volts change of +20 volts, or a plate-volts change of approximately 490 volts. Thus 1 volt change at the grid is equivalent to

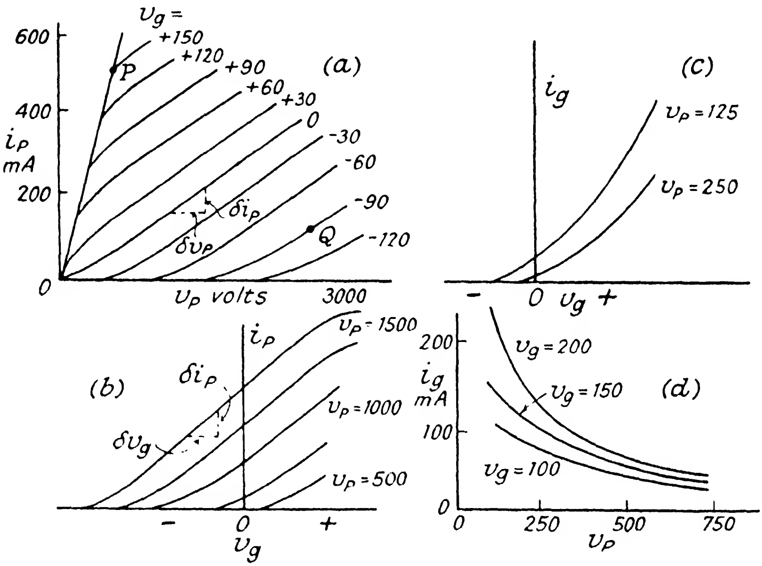


Fig. 4·9. Triode Characteristics.

(a) Plate-volts, plate-current curves
(b) Grid-volts, plate-current curves
(c) Grid-volts, grid-current curves
(d) Plate-volts, grid-current curves

of triode valve.

24·5 volts change at the anode, and the valve is said to have an amplification factor of 24·5.

Thus:

$$\text{Amplification factor } \mu = \frac{v_p \text{ change for given change in } i_p}{v_g \text{ change for same change in } i_p}.$$

Let the "given change" in i_p be 1 mA. Then the change in v_p required to produce it will be $\delta v_p = \frac{R_p}{1000}$ volts, since $\delta i_p = \frac{\delta v_p}{R_p}$ ampères.

Alternatively, 1 mA change in i_p could be obtained by a change in v_g given by $\delta v_g = \frac{1}{g_m}$, since 1 volt change in v_g gives g_m mA change in i_p.

Therefore the amplification factor

$$\mu = \frac{R_p g_m}{1000} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}2$$

The factor $\frac{1}{1000}$ is, of course, due to the fact that g_m is commonly measured in *milliampères* per volt.

For the valve with the curves of Fig. 4·10 we have seen that $\mu = 24{\cdot}5$. This means that if the grid-voltage is changed by 1 volt,

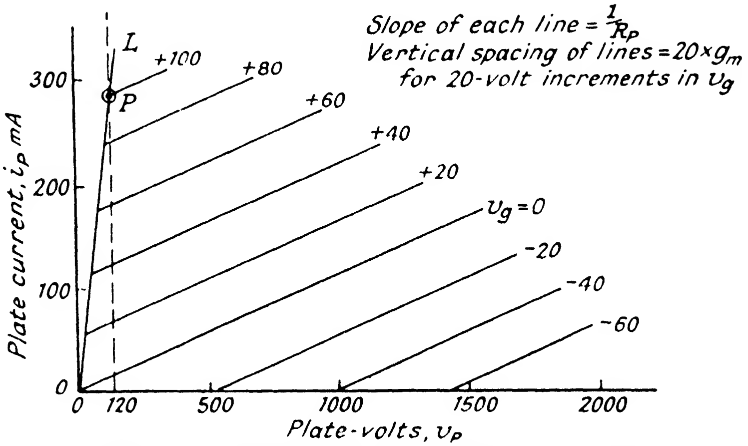


Fig. 4·10. Idealised Plate Characteristics for Triode.

the plate circuit-current change will be that which would result from 24·5 volts change in the plate-circuit.

The "limiting-edge", OL

Referring to Fig. 4·10 it will be obvious that no plate-current can exist when the plate-voltage is zero, even though the grid-voltage be considerably positive. Actually, what happens is that anode-current falls sharply at a critical point, for any given positive grid-voltage, and there is a corresponding rise in grid-current. These critical points lie on a line such as *OL*, which is therefore called the "limiting edge". This line is of great importance in connection with the design and operation of power-oscillators and amplifiers such as are used for the generation of H.F. power. It can be represented reasonably accurately by the equation $v_p = 1{\cdot}2v_g$; for example, in Fig. 4·10 the point P is the intersection of OL with the $v_g = 100$ curve, and lies vertically over $v_p = 120$ volts. To limit grid-current to a safe value it is necessary to arrange that, during cyclic variations of v_p and v_g, the former never falls below 1·2 times the peak positive value of the latter; in power-oscillators (Chapter 5) the minimum value of v_p is generally made between 1·2 and 2·0 times $v_{g\max}$.

Load-current controlled by triode valve

The usefulness of the triode lies in two properties:

(1) A change in grid-voltage is accompanied by a practically instantaneous corresponding change in plate-current. The effect is so nearly instantaneous as to make the valve satisfactory for operation up to at least several megacycles/sec.

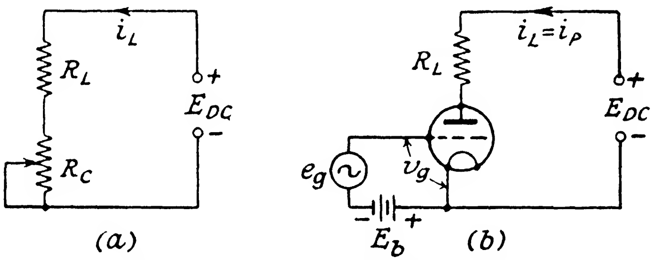


Fig. 4·11. Control of Power in Load R_L.

(2) A given grid-volts change is equivalent to a several times larger plate-volts change. The equivalent plate-volts change is μ times the actual grid-volts change; i.e. the valve is an amplifier. If the valve is such as to permit relatively large plate-current changes corresponding to these voltage changes, then it is a power-amplifier.

The triode valve may therefore be used to control the current and power in a load.

Consider the circuits shown in Fig. 4·11.

It is required to vary the power supplied to a load-resistance R_L. This could be achieved by the method shown at (a), the control being the slider of resistance R_c, which must be operated by mechanical means and therefore would be quite unsuitable for high-frequency variation of load current. The alternative shown at (b) is to use a high-vacuum valve in place of R_c, and to control the circuit current by varying the grid-voltage. The valve in effect is then a variable resistance like R_c, but it will be able to follow extremely rapid changes of grid-voltage. The valve is, however, much more than this, for though an alternator is shown in Fig. 4·11 (b) as the source of the control or "grid-drive" voltage, the valve can be arranged to supply its own "drive", in which case it becomes a power-oscillator. This is really a special case of the power-amplifier, and the general behaviour of amplifiers will be considered first.

The behaviour of the circuit of Fig. 4·11 (b) is shown graphically in Fig. 4·12.

CDE represents the $v_g - i_p$ curve of the valve when working with

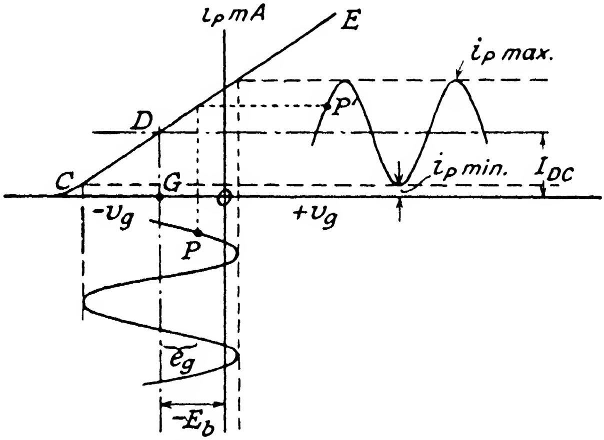


Fig. 4·12. Graphical Presentation of Triode Amplifier with Resistance Load.

the particular load-resistance R_L in series with it, and with a fixed plate-circuit supply-voltage E_{DC}. The slope of *CDE* is not g_m mA/volt, as for the static curves, but is less than this value, because of the effect of the load-resistance R_L upon the plate-voltage when the current i_p changes. Thus when e_g is positive, causing i_p to increase, there is an increased volt drop across R_L $(=\delta i_p R_L)$, and a consequent fall in plate-volts, v_p; this fall would, acting by itself, produce a fall in i_p. The result is therefore that a given change in grid-voltage yields a smaller change in plate-current, when the valve has a load-resistance in series with it, than is shown by the static curves. For the "static" $v_g - i_p$ curves, the slope is:

$$g_m = \frac{1000\mu}{R_p} \text{ mA/volt.}$$

For the "dynamic" curve *CDE*, the slope is $\frac{1000\mu}{R_p+R_L}$ mA/volt.

A negative "bias" voltage $-E_b$ is shown applied to the grid of the valve, and the alternating "drive" voltage e_g is superimposed on this bias. It will be seen later that the magnitude of this bias voltage may have a profound effect upon the behaviour of the valve; for the simple case considered here the bias can be regarded as a means of maintaining the effective grid-to-cathode voltage negative during at least the greater part of the drive-cycle, and thus minimising grid-current. It also serves to reduce the standing current drawn in the absence of a drive-voltage, while allowing the same *variation* in i_p to take place, when e_g is applied, as for the case of zero-bias. This latter point assumes, of course, that the amplitude of e_g is not so great as to make the effective grid-voltage, v_g, more negative than the value *OC* at the instant when e_g is a negative maximum. If this occurs, plate-current ceases during the time that v_g exceeds the "cut-off" voltage *OC*. The magnitude of *OC*, the "cut-off" voltage for the particular value of E_{DC}, is $\frac{E_{DC}}{\mu}$, since a negative grid-voltage of this amount will just neutralise the effect of a plate-voltage of $+E_{DC}$. The cut-off point is, of course, independent of load-resistance.

Fig. 4·12 shows the varying total grid-voltage waveform $v_g=(-E_b+e_g)$ stretched out along a convenient time-axis provided by the chain-dotted line, drawn vertically down through the point *G* corresponding to the bias-voltage $-E_b$. Similarly, the waveform of the varying plate-current i_p is conveniently shown on an axis provided by the horizontal line drawn through *D*. The i_p curve is obtained by "reflection" of the grid-volts wave form from the line *CDE*. A typical pair of points *PP'* illustrates this.

Voltage and power-amplification

We are concerned only with the alternating components of load current, voltage, and power. Fig. 4·13 shows a simplified "equivalent circuit" for the arrangement of Fig. 4·11 (b), and involves A.C. components only.

The valve behaves like a generator, of internal E.M.F. μE_g, internal resistance R_p, and external load-resistance R_L.

Circuit current (Fig. 4·13) will be:

$$I=\frac{\mu E_g}{R_p+R_L} \text{ ampères} \quad . \quad . \quad . \quad . \qquad 4{\cdot}3$$

The A.C. component of load voltage will be:

$$V=IR_L=\frac{\mu E_g R_L}{R_p+R_L} \text{ volts} \quad . \quad . \quad . \quad . \quad 4{\cdot}4$$

Drive voltage is E_g RMS in the actual circuit, so that the voltage gain will be:

$$G=\frac{V}{E_g}=\frac{\mu R_L}{R_p+R_L} \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}5$$

The voltage gain obviously rises with increasing values of R_L, and approaches the limiting value of μ as R_L is made large compared with R_p. The A.C. power developed in the load R_L will be:

$$P_L=VI \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}6$$

$$=\frac{\text{Max. plate-volts swing}}{2\sqrt{2}}\times\frac{\text{Max. plate-current swing}}{2\sqrt{2}}$$

$$=\frac{(v_{p\max}-v_{p\min})(i_{p\max}-i_{p\min})}{8} \text{ watts} \quad . \quad . \quad . \quad . \quad 4{\cdot}7$$

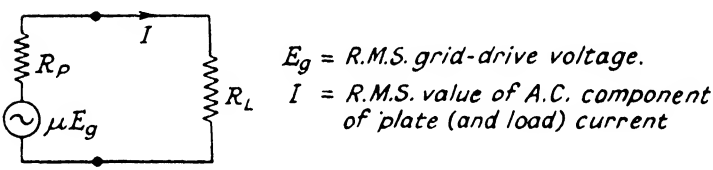


Fig. 4·13. Equivalent "A.C." Circuit for Triode Amplifier.

since for the case of sinusoidal variations in i_p and v_p the maximum "swing" of either is equal to the "double amplitude" of its A.C. component.

Efficiency of power conversion

Now it will be clear from the i_p time-graph in Fig. 4·12 that the average current drawn from the supply is the same whether there is any grid-drive or not (provided E_b, the bias voltage, remains unchanged). This current is labelled I_{DC} in the diagram. The input power to the circuit, from the D.C. source, is:

$$P_I=E_{\mathrm{DC}}I_{\mathrm{DC}} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}8$$

The efficiency of power conversion from D.C. to A.C. in the load R_L is therefore:

$$\eta=\frac{P_L}{P_I}=\frac{P_L}{E_{\mathrm{DC}}I_{\mathrm{DC}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}9$$

Condition for maximum A.C. power in the load

Equation 4·6 may be expanded by substituting for I and V the values given in 4·3 and 4·4. Then:

$$P_L = VI = \frac{\mu^2 E_g{}^2 R_L}{(R_p + R_L)^2} \text{ watts} \quad . \quad . \quad . \quad . \quad 4\cdot10$$

and the value of R_L which gives this expression its maximum value is:

$$R_L = R_p \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad 4\cdot11$$

as can be shown by plotting P_L for various ratios of $\frac{R_L}{R_p}$, or by differentiating 4·9 with respect to R_L and equating $\frac{dP_L}{dR_L}$ to zero. The graphical method is instructive, as it brings out the manner in which P_L varies with R_L; a typical curve is shown in Fig. 4·14.

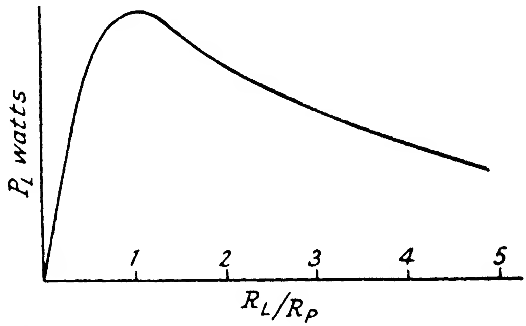


Fig. 4.14. Effect of Load Resistance Value on Power Output.

The condition for maximum load-power expressed in equation 4·10 is only true for sinusoidal current variation, and assumes the theoretical linear characteristics of Fig. 4·10. In practical valves some distortion of the plate-current waveform, relative to the drive-voltage waveform, is inevitable. If such distortion is very undesirable, as in speech-amplifiers and the like, the actual value of R_L chosen must be such as to give a reasonably large power-output together with a minimum of distortion. We are not concerned with such cases here, but readers who wish for further information on this point will find a list of books dealing with it under reference number [19].

The load-line

Finally, in connection with circuit (b) of Fig. 4·11, it will be seen that the action may very conveniently be illustrated by a construction superimposed on the *static* $v_p - i_p$ characteristics. This is shown in Fig. 4·15 which uses the assumed linear characteristics, based on R_p and g_m, of a small power triode.

PQ is a "load-line" located by the 2 points *R* and *Q*. *R* represents the value of the current i_p which would flow in R_L if the whole supply voltage E_{DC} were across it, i.e. if v_p were zero. *Q* represents the plate-voltage v_p when $i_p=0$, i.e. E_{DC}, since there is no voltage-drop across R_L when $i_p=0$. Points along the line *PQ* then give corresponding plate-volts, plate-current, and grid-volts throughout a complete cycle of signal voltage of amplitude $E_{g\max}$. The sinusoidal variation of these three quantities is indicated.*

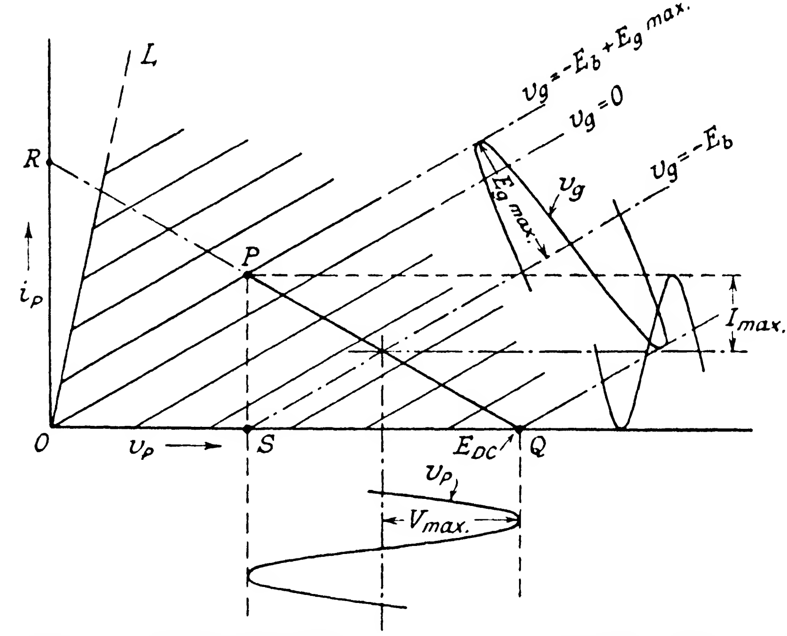


Fig. 4·15. Graphical Analysis of Triode Class A Amplifier, based on Plate Characteristics.

The average A.C. power in the load is:

$$P_L = \frac{V_{\max}}{\sqrt{2}} \cdot \frac{I_{\max}}{\sqrt{2}}$$

$$= \frac{SQ}{2\sqrt{2}} \cdot \frac{SP}{2\sqrt{2}}$$

$$= \frac{SQ.SP}{8} \text{ watts} \quad . \quad . \quad . \quad . \quad 4{\cdot}12$$

as already given in equation 4·7.

If load-lines of various slopes are tried, it would be found that the maximum value of the product *SQ.SP*, and hence maximum

* For maximum output with Class A operation, the mid-point of *PQ* (the "operating point") is chosen to correspond to the maximum permissible plate dissipation.

output power, for a given grid-volts swing, occurs when the slope is the same as that of the $v_p - i_p$ characteristics, i.e. when $R_L = R_p$, which agrees with equation 4·10. It is this particular value of R_L for which the load-line in Fig. 4·15 has been drawn.

Grid-current will flow when the grid-voltage is actually positive; i.e. over that part of the v_g curve (Fig. 4·12) between $v_g = 0$ and $v_g = E_{g\max} - E_b$. Thus a small amount of power is required in the grid-drive circuit.

The transformer-coupled load

For three reasons the load resistance R_L is not usually connected directly in series with the valve as shown in Fig. 4·11 (b), but is instead coupled to it indirectly, e.g. by means of a transformer.

(1) Since only A.C. power is required in the load, the power-dissipation due to the "standing current" component is eliminated by transformer coupling. A lower value of E_{DC} for the same A.C. conditions is then required.

(2) The load resistance is hardly ever of the correct value to yield sufficient power or a satisfactory efficiency. We have seen (Chapter 1) that a transformer is an impedance-matching device; it can therefore be used to match the actual load impedance to that of the valve, presenting to the valve the correct impedance for high load-power and conversion efficiency.

(3) The use of a transformer isolates the load electrically from the H.T. D.C. supply to the valve.

The simple case of an iron-cored coupling-transformer and pure-resistance load will be considered (Fig. 4·16), using the same valve yielding the same load-power as in Fig. 4·15.

The primary-winding resistance will generally be so small that any volt-drop due to the standing current I_{DC} is negligible; the supply voltage E_{DC} can therefore be reduced from the value corresponding to point Q in Fig. 4·15 to the value shown in Fig. 4·16. The plate-voltage is therefore E_{DC} in the absence of grid-drive e_g.

When plate-current decreases (e_g negative) the voltage induced in the primary is such as to raise the plate-volts; conversely, when the plate-current rises, the primary induced-voltage is such as to reduce the plate-volts. The plate-volts "swing" would be identical with that for Fig. 4·15 if the loaded transformer behaved like the original series-connected R_L. Thus, if the transformer magnetising-current is negligible compared with normal load-current, the two methods of load-connection yield the same A.C. power to the load if $\frac{R_2}{K^2}=R_L$, K being the ratio $\frac{T_2}{T_1}$.

The power input is, however, considerably smaller owing to the reduction in E_{DC}, so that a higher efficiency results from transformer-coupling to the load. Transformer losses occur, of course, which have not been taken into account here.

So far we have considered only cases in which plate-current is flowing for the whole of the drive-cycle; this is referred to as Class A operation. It is used only where rather high fidelity is required between output and input waveforms, the efficiency usually realisable being low. The theoretical maximum for the

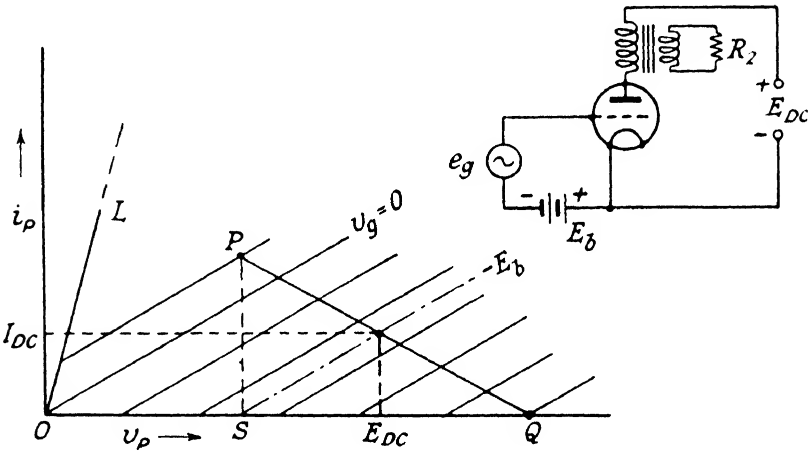


Fig. 4·16. Transformer-coupled Load.

transformer-coupled case is less than 50%. Practical efficiencies range from 15% to 30%, depending upon the permissible amount of waveform distortion.

The chief objection to such distortion, e.g. in speech-amplifiers, is that it results in "cross-modulation" and the introduction of new frequencies in the output not necessarily in harmonic relation to the input frequencies, when *two or more* frequencies are present in the input voltage e_g.

For the purpose of H.F. power generation a great simplification is possible, since only one frequency is required to be generated at any one time. This enables a resonant plate-load to be used and high power-conversion efficiencies to be realised, since the tuned load permits Class B or C operation of the valve, while yielding almost perfectly sinusoidal load-current.

These two classes or modes of operation are each further subdivided, according as grid-current is permitted or not. This provides a further simplification here, since the type of amplifier with which we are concerned always involves grid-current. Only this case, therefore, will be considered.

CHAPTER 5

Class B and Class C Operation of Power Amplifiers with Tuned Loads

The tuned plate-load

IT WAS shown in Chapter 1, p. 31, that a parallel combination of L and C behaves at resonance ($f \doteqdot \frac{1}{2\pi\sqrt{LC}}$ cycles/sec) like a resistance of $\frac{L}{CR}$ ohms; R is the series-resistance round the closed combination. Such an arrangement could therefore replace the load resistance R_L of Fig. 4·11 (b), and (*if the drive-voltage* e_g *had the frequency to which this load is tuned*, i.e. $f \doteqdot \frac{1}{2\pi\sqrt{LC}}$ cycles/sec) it would have the same effect, provided that $\frac{L}{CR} = R_L$.

It has also been seen that such a closed LCR system will oscillate if supplied with energy and then left to itself (p. 34). It is not essential, therefore, that the tuned load should be continuously supplied with an alternating current of sine wave-form, in order that the *circulating* current in it, and hence the voltage across it, shall be sinusoidal and shall persist. It is sufficient that pulses of energy shall be supplied to it periodically to make good the "damping losses" due to the dissipation in R. Such pulses can be supplied by virtue of correctly-timed current-pulses drawn from the D.C. supply and passed through the load-circuit, the magnitude and duration of the pulse being controlled by the valve.

Fig. 5·1 (a) shows such an arrangement, while (b) and (c) indicate the nature of the plate-current pulse for two values of bias. Case (b) shows E_b=cut-off voltage $\frac{-E_{DC}}{\mu}$; the current-pulses in this case have the form of half sine-waves, similar to those shown on p. 17. This is known as Class B operation, and the definition for it is that plate-current flows for one-half of the drive-cycle.

Case (c) shows E_b more negative than the cut-off voltage $\frac{-E_{DC}}{\mu}$. This represents Class C operation; current flows for less than half of each drive-cycle, the pulse waveform being the top portion only

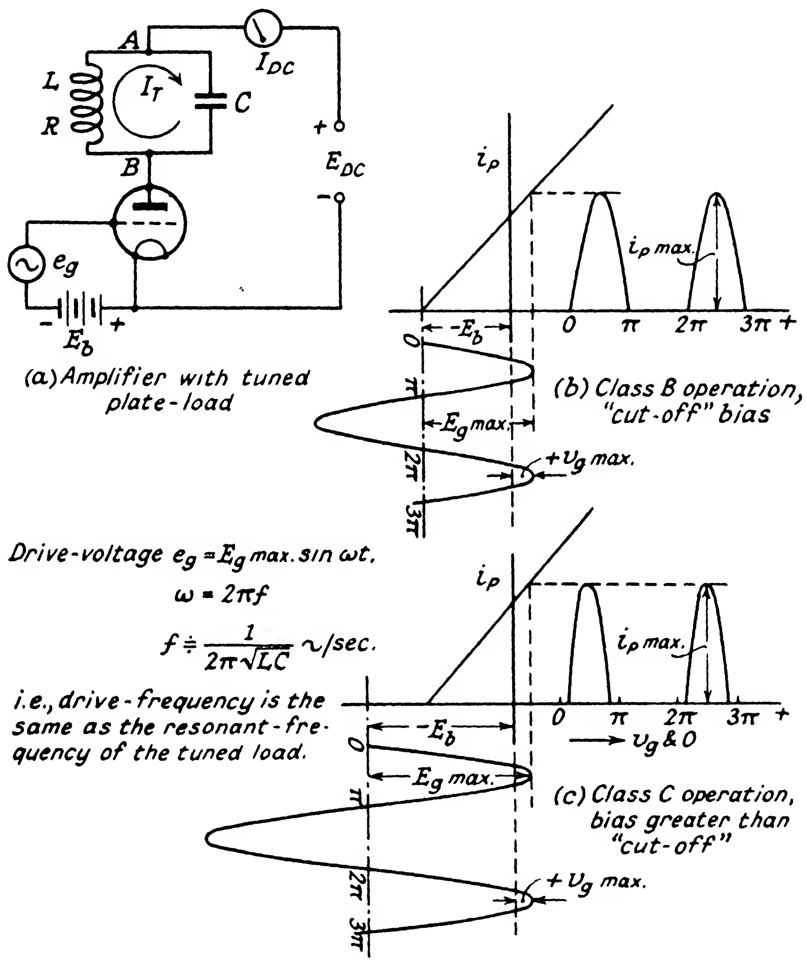


Fig. 5·1. (a) Amplifier with Tuned Load. (b) Class B operation. (c) Class C operation.

of a true sine-wave, provided that the dynamic characteristic of the valve is linear. This qualification was also implied in stating that the i_p waveform is a true half sine-wave in the Class B case.

In this amplifier the *circulating* current I_T in the tuned "tank"-circuit *LCR* persists and is practically sinusoidal and of constant amplitude, despite the pulse-like nature of the supply current i_p. This is due to the energy-storage of L and C. It will be shown that the i_p pulse contains a sinusoidal component of the same frequency as the drive, and it is this A.C. component which supplies the power to the tuned load, the resonant frequency of which is adjusted to be that of the drive.

Since the tank-circuit presents a high impedance (between points A and B, Fig. 5·1 (a)) only to a current having its resonance

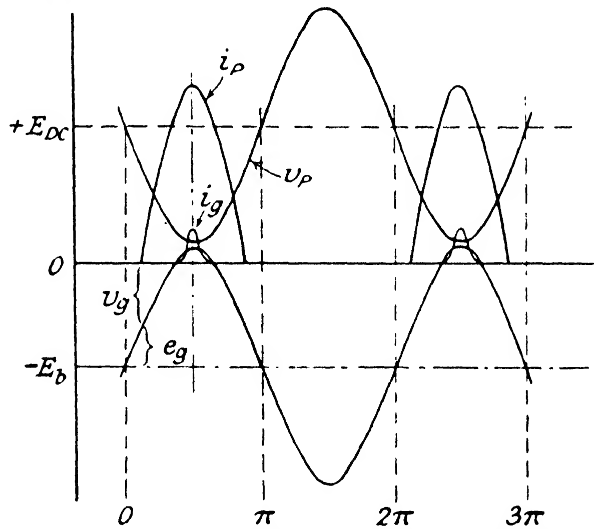


Fig. 5·2. Time-graphs of Plate- and Grid-voltages and Currents in a Class C Amplifier.

frequency, it follows that only this sinusoidal component of i_p develops an appreciable voltage between A and B. Consequently the voltage across AB (i.e. the tank voltage v_T) is almost perfectly sinusoidal, and the plate-voltage v_p must likewise rise and fall sinusoidally, being a minimum when v_T is a "positive" maximum ($A+ve$ to B), and a maximum when v_T is a "negative" maximum ($A-ve$ to B). The phase relations of v_T, v_p, and e_g are shown in Fig. 5·2.*

It will be observed that the standing current is zero for Class B and C amplifiers, and this is the reason for their high efficiency.

* In Fig. 5·2, plate-voltage is represented as the vertical distance of the v_p curve from the zero-voltage line; the vertical distance of the v_p curve from the E_{DC} line represents the tank voltage v_T, since at every instant $v_p+v_T=E_{DC}$.

To determine the D.C. input and A.C. output power, and hence the conversion efficiency, it is necessary now to consider the current-pulse in some detail. Class B is, of course, merely a limiting case of Class C, and is the case for which the duration of the current-pulse is exactly one half-cycle.

Analysis of the plate-current pulse

In Fig. 5·3 the pulse is shown in heavy outline as a part of a complete sine-wave. It is convenient to treat it as a cosine-wave, so that the zero for the horizontal angle-axis is located at the point of peak pulse-current, $i_{p\max}$.

The duration of the pulse, expressed as an angle, is 2θ, i.e. from $-\theta$ to $+\theta$, where θ may be less than 90° or $\frac{\pi}{2}$ radians, as for Class C, or equal to $\frac{\pi}{2}$ radians, as for Class B.

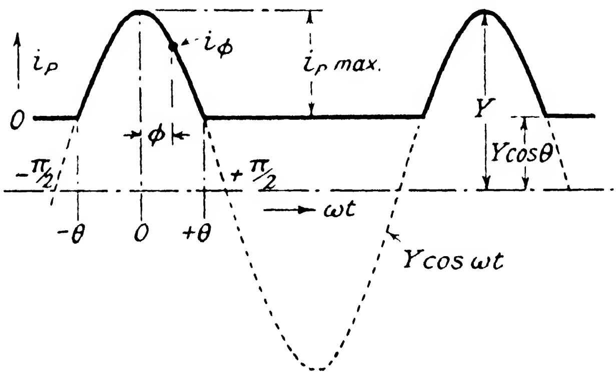


Fig. 5·3. Graphical Treatment of Plate-current Pulse.

Average value of pulse current over one or more whole cycles

This will be the reading obtained on a moving-coil meter carrying the pulse current, i.e. I_{DC}.

From Fig. 5·3 we see that $i_{p\max}=Y-Y\cos\theta=Y(1-\cos\theta)$.

So that $$Y=\frac{i_{p\max}}{1-\cos\theta}.$$

At an instant corresponding to any angle ϕ we have:

$$i_\phi=Y\cos\phi-Y\cos\theta=Y(\cos\phi-\cos\theta)$$

$$=\frac{i_{p\max}}{1-\cos\theta}(\cos\phi-\cos\theta) \quad . \quad . \quad . \quad . \quad 5\cdot1$$

The average value of the current pulse over a whole cycle is then given by finding the average height of the i_p curve; the same result is obtained by finding the average height of the half-pulse (from 0 to $+\theta$) over a half-cycle, i.e. over π radians:

$$i_{pav.}=I_{DC}=\frac{i_{pmax}}{\pi(1-\cos\theta)}\left[\int_{\phi=0}^{\phi=\theta}\cos\phi d\phi-\cos\theta\int_{\phi=0}^{\phi=\theta}d\phi\right]$$

$$=\frac{i_{pmax}}{\pi(1-\cos\theta)}[\sin\theta-\theta\cos\theta] \quad . \quad . \quad . \quad 5{\cdot}2$$

$$=i_{pmax}f_1(\theta).$$

A table of values of $f_1(\theta)$ is appended, and the function is also plotted in Fig. 5·5, p. 122.

θ=half-conduction-angle, degrees:	0	10	20	30	40	50	60	70	80	90
$f_1(\theta)=\frac{I_{DC}}{i_{pmax}}$:	0	·034	·072	·109	·147	·183	·218	·252	·286	·318

The average plate-current, I_{DC}, is measured with a moving-coil ammeter as shown in Fig. 5·1 (a).

Amplitude of fundamental alternating component of current pulse

The significance of the A.C. components is shown in Fig. 5·4 in which the fundamental (frequency f) and second-harmonic ($2f$) components have been plotted. The peaky curve represents the sum of these two and a D.C. component (I_{DC}), and it is seen that this sum represents the true current-pulse fairly accurately. The graphs were drawn for the special case of the half sine-wave pulse. The departure from the exact half sine-wave is not due to incorrect values of D.C. and A.C. components, but to the fact that only the first two A.C. components have been plotted. The amplitudes of the higher harmonics become progressively smaller.

For our simple amplifier in which the load is tuned to the frequency of the grid-drive, the fundamental A.C. component of the pulse determines the load-power. Its amplitude is found by employing the method of Fourier's analysis, using the construction shown in Fig. 5·3.

We have already seen that

$$i_\phi=\frac{i_{pmax}}{(1-\cos\theta)}(\cos\phi-\cos\theta) \quad . \quad . \quad . \quad . \quad 5{\cdot}1$$

$$=I_{DC}+A_1\sin\omega t+A_2\sin 2\omega t+\ldots$$

$$+B_1\cos\omega t+B_2\cos 2\omega t+\ldots$$

The curve is symmetrical about its peak value at 0, and so the coefficient A_1 is zero. The amplitude of the fundamental component is therefore simply B_1. Then:

$$B_1 = \frac{1}{\pi}\int_0^{2\pi} i_\phi \cos\phi d\phi$$

$$= \frac{2}{\pi}\,\frac{i_{p\max}}{1-\cos\theta}\int_0^{\theta}(\cos\phi - \cos\theta)\cos\phi d\phi$$

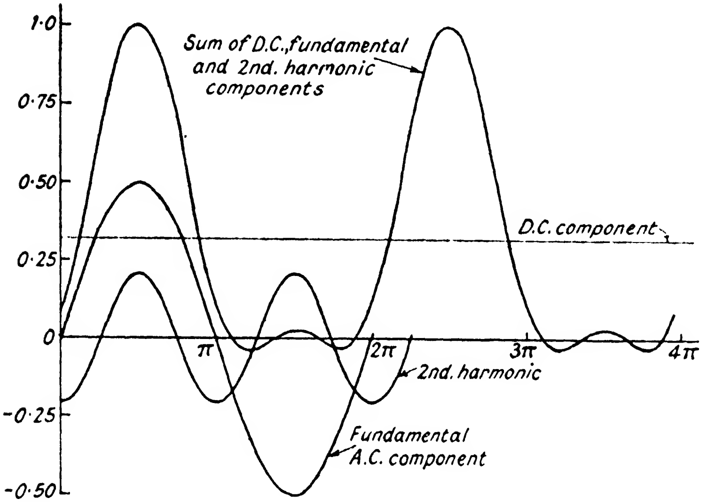


Fig. 5·4. Sum of D.C. and Fundamental and Second Harmonic A.C. Components of Class B Pulse.

$$= \frac{i_{p\max}}{\pi(1-\cos\theta)}\,[\theta - \sin\theta\cos\theta] \qquad . \quad . \quad . \quad 5{\cdot}3$$

$$= i_{p\max}\cdot f_2(\theta).$$

The ratio $\frac{I_{f\max}}{i_{p\max}} = f_2(\theta)$ is tabulated for values of θ from 0 to 90°, and is plotted in Fig. 5·5. $I_{f\max}$ is the amplitude of the fundamental component of the current pulse.

θ=half conduction-angle, degrees:	0	10	20	30	40	50	60	70	80	90
$\frac{I_{f\max}}{i_{p\max}} = f_2(\theta)$:	0	·072	·146	·213	·280	·339	·390	·435	·471	·500

Equivalent tank-circuit

Since only the fundamental A.C. component of the current pulse excites a circulating current in the tank-circuit, the equivalent circuit for the latter becomes as shown in Fig. 5·6.

The tank-circuit is in effect being supplied with power from an A.C. source of V_T volts RMS, where:

$$V_T=\frac{v_{T\max}}{\sqrt{2}}=\frac{\text{plate-volts swing}}{2\sqrt{2}} \quad . \quad . \quad . \quad . \quad 5{\cdot}4$$

The tank is tuned to resonate with the "supply" frequency (since it is tuned to the drive frequency) so that the tank-circuit presents to V_T a purely resistive load of $\frac{L}{CR}$ ohms. The current

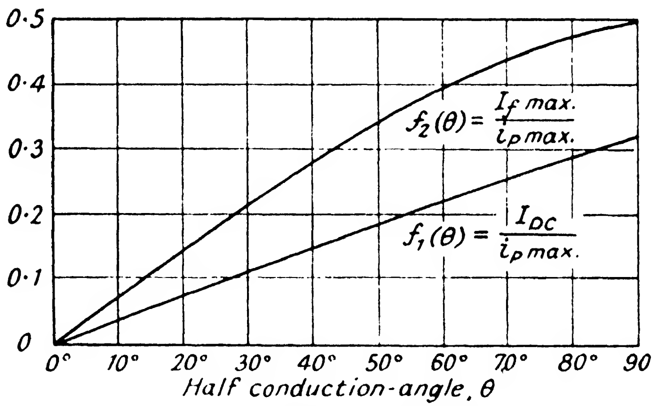


Fig. 5·5. Graphs of $f_1(\theta)$ and $f_2(\theta)$ for "1st Power" Law.

drawn from the "supply" is I_f, the RMS value of the fundamental A.C. component of i_p,

i.e.
$$I_f=\frac{V_T}{\frac{L}{CR}}=\frac{V_T}{Q\omega_O L} \text{ ampères} \quad . \quad . \quad . \quad . \quad 5{\cdot}5$$

$\omega_O L$ being the reactance of L at the resonance frequency. The circulating current in the tank, I_T, is Q times as large as the supply current, i.e.

$$I_T=QI_f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}6$$

The derivation of 5·5 and 5·6 was given on p. 32, Chapter 1. The power being supplied to the tank is:

$$P_T=V_T I_f=I_T^2 R \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}7$$

$$=\frac{V_T^2}{Q\omega_O L} \text{ watts (from 5·5)} \quad . \quad . \quad . \quad . \quad 5{\cdot}8$$

It is undesirable to make Q less than about 10 or 12, and it is clearly necessary that V_T should be as large as possible, for high power. With Class B and C amplifiers of this type, $V_{T\max}$ can be made about ·85 or ·9 E_{DC}; i.e. the plate-voltage v_p can be made to swing between ·1 or ·15 E_{DC} and 1·85 or 1·9 E_{DC} (see Fig. 5·2).

The resistance R of the tank-circuit would ideally be entirely due to the load. In practice it is made up from the series-coupled resistance of the true load, plus the inevitable series-resistance of the tank-circuit itself. For the common case of magnetic-coupling to the load (Fig. 5·7) the relations are:

$$\text{Coupled resistance } R_{\text{coupled}} = \frac{\omega^2 M^2 R_2}{Z_2^2} \text{ ohms} \quad . \quad 1{\cdot}84$$

$$\text{Coupled reactance } X_{\text{coupled}} = \frac{-\omega^2 M^2 X_2}{Z_2^2} \text{ ohms} \quad . \quad 1{\cdot}85$$

where
$$Z_2 = \sqrt{R_2^2 + X_2^2}$$
$$X_2 = \omega L_2.$$

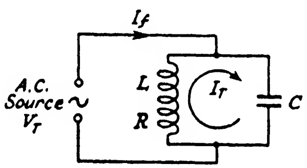


Fig. 5·6. Equivalent Tank-circuit.

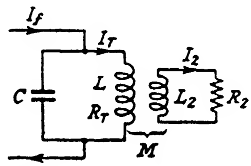


Fig. 5·7. Load Inductively Coupled to Tank.

The effective series resistance of the loaded tank-circuit is therefore:

$$R = R_T + R_{\text{coupled}} \text{ ohms} \quad . \quad . \quad . \quad 5{\cdot}9$$

R_T being the inherent tank-resistance, which must be kept as low as possible. Since the tank power $P_T = I_T^2 R$, the "tank efficiency", i.e. the ratio

$$\frac{\text{Useful power to load}}{\text{Total power to tank}} = \eta_T = \frac{R_{\text{coupled}}}{R} \quad . \quad . \quad 5{\cdot}10$$

The effective series reactance of the loaded tank is:

$$X = X_L - X_{\text{coupled}} - X_C \text{ ohms} \quad . \quad . \quad . \quad 5{\cdot}11$$

i.e. the coupled reactance behaves like a series *capacitive* reactance; this, of course, would alter the resonant frequency of the tank, which must be retuned whenever the loading is altered. It must be remembered that we are dealing here with an amplifier having a fixed drive-frequency. Retuning would be carried out by alteration of the value of L or C, and the tank is correctly tuned when the average value of the plate-supply current, I_{DC}, is a minimum for a given load; the effective series reactance X is then zero.

Graphical treatment of the complete amplifier (Class B)

It was seen in the case of the Class A amplifier of Fig. 4·11 (b) that a relatively large power output can only be obtained if the plate-volts and plate-current "swings" are large. With the pulse-operation inherent in B and C amplifiers, large swings can be obtained without the necessity for a large standing-current com-

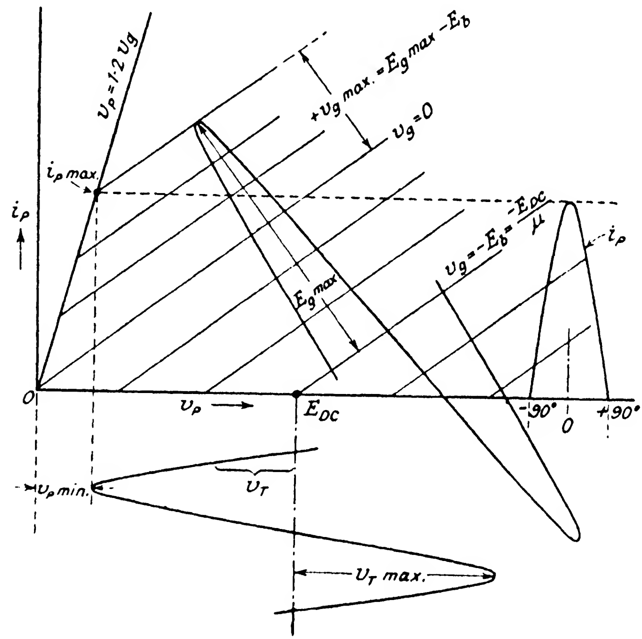


Fig. 5·8. Graphical Treatment of Class B Amplifier with Tuned Load.

ponent. It should be realised that this is due to the tuned load, the voltage across which must vary sinusoidally, because of the large circulating tank-current; it is for this reason that the Q value of the loaded tank must be kept reasonably high, e.g. not less than 12; values between 20 and 50 are common in industrial H.F. practice.

Fig. 5·8 shows graphically the relations of all the quantities involved for the Class B amplifier, and is based upon the static $v_p - i_p$ characteristics of the valve. For simplicity linear characteristics are assumed.

A complete cycle of drive-voltage $e_g = E_{g\max} \sin \omega t$, superimposed on a negative-bias voltage $-E_b$ results in a sinusoidal grid-to-cathode voltage variation between the limits

$$v_g = +(E_{g\max} - E_b) \text{ and } v_g = -(E_{g\max} + E_b).$$

For a Class B amplifier, $E_b = \frac{-E_{DC}}{\mu}$, and plate-current flows during the whole of the positive half-cycle of drive-voltage, rising to a peak value of $i_{p\max}$.

$i_{p\max}$ should lie on, or just to the right of, the limiting edge, and is made as large as the cathode-emission reasonably permits.

The corresponding plate-voltage variation is sinusoidal, the tank-voltage amplitude $v_{T\max}$ being fixed by the tank-impedance and the fundamental A.C. component of the current-pulse, i.e.

$$v_{T\max} = I_{f\max} . Z_T \text{ volts} \quad . \quad . \quad . \quad . \quad 5{\cdot}12$$

where Z_T=effective impedance of loaded tank.

The correct tank impedance therefore is:

$$Z_T = \frac{v_{T\max}}{I_{f\max}} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}13$$

$$= \frac{2v_{T\max}}{i_{p\max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}14$$

Since $I_{f\max} = {\cdot}5 i_{p\max}$ for Class B.

It can be seen that the whole design of a Class B amplifier depends upon the correct choice of the full-load peak current, $i_{p\max}$. The valve manufacturer usually quotes the permissible average plate-current I_{DC} for Class B operation, and if this is known, then:

$$i_{p\max} = \frac{I_{DC}}{{\cdot}318} = \pi I_{DC} \text{ ampères} \quad . \quad . \quad . \quad . \quad 5{\cdot}15$$

from equation 5·2 and graph.

Alternatively, as an approximation, we can insert assumed grid- and plate-voltage values in equation 4·1, i.e.

$$i_p = \frac{v_p}{R_p} + g_m v_g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4{\cdot}1$$

So that

$$i_{p\max} = \frac{v_{p\min}}{R_p} + g_m v_{g\max} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}16$$

Putting $v_{p\min} = 1{\cdot}2 v_{g\max}$ (see p. 107), and also $v_{p\min} = {\cdot}15 E_{DC}$, this being an average value realised in practice, we get:

$$i_{p\max} = \frac{v_{p\min}}{R_p}\left(1 + \frac{\mu}{1{\cdot}2}\right)$$

$$= \frac{{\cdot}15\, E_{DC}}{R_p}\left(1 + \frac{\mu}{1{\cdot}2}\right) \text{ ampères} \quad . \quad . \quad 5{\cdot}17$$

A third method of estimating the proper value for i_{pmax} is given on p. 131 in connection with the Class C amplifier, and is based upon the filament emission current.

Input power, output power, and efficiency of power-conversion

The input power from the D.C. supply is:

$$E_{DC}I_{DC}=P_I \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}18$$

The output power to the tank is:

$$P_O=V_TI_f$$

$$=V_T.\frac{\cdot 5\pi I_{DC}}{\sqrt{2}}, \left(\text{since } I_f=\frac{\cdot 5 i_{pmax}}{\sqrt{2}}\right) \quad . \quad . \quad . \quad 5{\cdot}19$$

And if we assume, as for equation 5·17, that

$$v_{pmin}=\cdot 15E_{DC},$$

i.e.

$$v_{Tmax}=\cdot 85E_{DC},$$

$$P_O=\frac{\cdot 85E_{DC}}{\sqrt{2}}.\frac{\cdot 5\pi I_{DC}}{\sqrt{2}} \text{ watts}$$

$$=\cdot 667E_{DC}I_{DC}$$

$$=\cdot 667P_I \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}20$$

So that the efficiency of conversion from D.C. to A.C. power is:

$$\eta=66{\cdot}7\,\%.$$

Higher efficiencies are realisable for Class B, depending upon how small v_{pmin} can safely be made without excessive grid-current, or the appearance of a sharp dip in the centre of the plate-current pulse-form. The limiting value of η for the case where $v_{pmin}=0$, i.e. peak tank-volts$=E_{DC}$, is 78·5 %, but is, of course, not realisable.

Plate dissipation power

The difference between D.C. input power and A.C. output power to the tank and load represents power that must be dissipated by the valve,

i.e.

$$\underset{\text{plate-dissipation}}{P}=P_I-P_O$$

$$=P_P.$$

The whole of this power is converted to heat at the plate, and in the larger valves the plate is cooled artificially. Small valves, e.g. up to 1 KW rating, are "radiation cooled".

The plate-power at any instant is the product of plate-volts and plate-current, v_pi_p watts. In pulse-operated tuned amplifiers current flows only while the plate voltage is relatively low, so that fairly high efficiencies are obtained.

For a Class B amplifier to yield 5 KW of power to the tank, the valve must be capable of dissipating some 2·5 KW at its plate, since the realisable efficiency is about 2/3.

The grid circuit

The negative bias voltage is:

$$E_b = \frac{-E_{DC}}{\mu} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}21$$

since the valve is biased to cut off.

The amplitude of the drive-voltage e_g must be such that the chosen value of i_{pmax} can be realised,

i.e. $$E_{gmax} = \frac{E_{DC}}{\mu} + v_{gmax} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}22$$

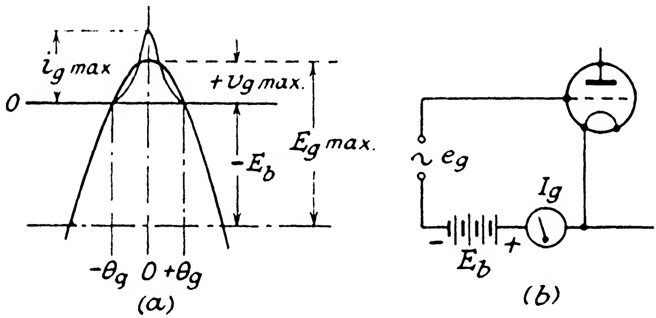


Fig. 5·9. The Grid-current Pulse.

For the average case of $v_{gmax} \doteqdot \frac{v_{pmin}}{1{\cdot}2} \doteqdot \frac{{\cdot}15E_{DC}}{1{\cdot}2}$,

we have $$E_{gmax} = E_{DC}\left(\frac{1}{\mu} + {\cdot}125\right) \text{ volts} \quad . \quad . \quad . \quad 5{\cdot}23$$

During the time that v_g is actually positive, a grid-current pulse occurs. This is illustrated in Fig. 5·9.

If the grid-current characteristics of the valve are available, as shown in Fig. 4·9 (d), it will be possible to read off the peak grid-current i_{gmax} for the assumed or calculated values of v_{pmin} and $+v_{gmax}$. The grid-current pulse does not follow a linear law, as does the plate-current nearly; rather it approximates to a square law. The average grid-current over a whole cycle can be found in terms of the peak value, from the curve of Fig. 5·10, which is the equivalent of Fig. 5·5 but for a squared-sine function.

The half-conduction-angle θ_g for grid-current is seen from Fig. 5·9 (a) to be related to E_b and $E_{g\max}$;

$$E_b = E_{g\max} \cos \theta_g,$$

i.e.

$$\cos \theta_g = \frac{E_b}{E_{g\max}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}24$$

Hence θ_g can be determined, and the average grid-current I_g obtained from Fig. 5·10.

If no information is available concerning grid-current, an estimate may be made, for preliminary design purposes, based upon:

$$i_{g\max} \doteqdot \cdot 25\, i_{p\max} \qquad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}25$$

whence

$$I_g = i_{g\max} f(\theta_g) \doteqdot \cdot 25 i_{p\max} f(\theta_g) \qquad . \quad . \quad . \quad 5{\cdot}26$$

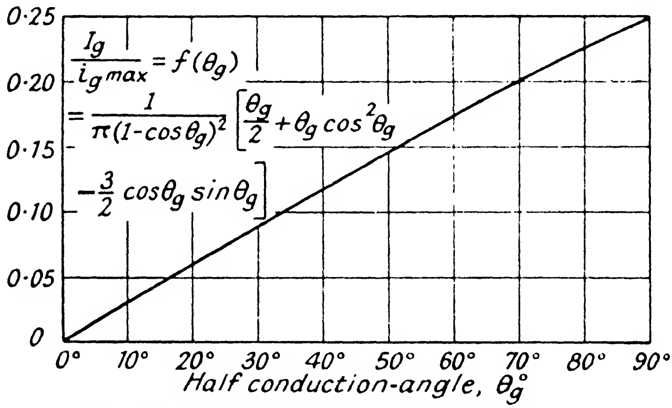


Fig. 5·10. Graph of $f(\theta_g)$ for "Square Law".

The power-dissipation in the grid-circuit can then be estimated. Grid-current flows only while e_g is in the neighbourhood of $E_{g\max}$, and a fairly accurate estimate of total grid-circuit power is obtained by assuming that e_g is constant at $E_{g\max}$ during the time of the grid-pulse. The result is usually between 5 and 10% high: .

$$P_{\text{drive}} \doteqdot E_{g\max} I_g \text{ watts} \qquad . \quad . \quad . \quad . \quad 5{\cdot}27$$

Part of this power is dissipated at the grid, the instantaneous grid-power being:

$$P_g = v_g i_g \text{ watts.}$$

The majority of the grid-drive power, however, is dissipated in the bias source; if batteries are used to provide the bias-voltage E_b, these will be charged as a result of grid-current, the power thus converted being:

$$P_B = E_b I_g \text{ watts} \qquad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}28$$

The average power dissipated in heat *at the grid* is therefore:

$$P_g \doteqdot (E_{g\max} I_g - E_b I_g)$$
$$= I_g (E_{g\max} - E_b)$$
$$= I_g v_{g\max} \text{ watts, approximately} \quad . \quad 5{\cdot}29$$

$v_{g\max}$ is the maximum *positive* grid-voltage, and can be measured with a peak valve-voltmeter (see Chapter 8). Average grid-current is measured with a moving-coil milliammeter, as shown in Fig. 5·15.

The usual method of providing grid-bias is by means of a grid-resistor, across which a bias-voltage $E_b = I_g R_g$ is developed. This is described on p. 138.

Worked example of Class B amplifier design

Valve: R.C.A. type 207, water-cooled. Maximum permissible plate dissipation 10 KW; $\mu = 20$.

Plate supply volts	$E_{DC} = 12{,}000$
Maximum D.C. plate-current, ampères .	$I_{DC} = 2{\cdot}0$
Maximum D.C. grid-current, ampères .	$I_g = 0{\cdot}2$
Required operating frequency, MC/S .	$f = 1{\cdot}0$
Assumed tank-circuit Q on full load .	$Q = 20$

Valve plate-circuit relations

Peak plate-current, $i_{p\max} = \pi I_{DC} = 6{\cdot}28$ ampères.

Amplitude of fundamental component of plate-current, $I_{f\max} = 0{\cdot}5 i_{p\max} = 3{\cdot}14$ ampères.

Putting $v_{p\min} = {\cdot}15 E_{DC} = 1800$ volts, gives $v_{T\max} = (12{,}000 - 1800) = 10{,}200$ volts.

Tank H.F. power $= \dfrac{10{,}200}{\sqrt{2}} \times \dfrac{3{\cdot}14}{\sqrt{2}} \times 10^{-3}$ KW $= 16$ KW.

D.C. input power $P_{DC} = 12{,}000 \times 2{\cdot}0 \times 10^{-3} = 24$ KW.

Plate dissipation power $P_D = 24 - 16 = 8$ KW.

Plate conversion efficiency $= \dfrac{16}{24} \times 100 = 66{\cdot}6\,\%$.

Valve grid-circuit relations

From manufacturers' curves, $i_{p\max} = 6{\cdot}28$ ampères at $v_p = 1800$ volts, when $+v_{g\max} = 900$ volts. The grid current for these conditions is $i_{g\max} = 0{\cdot}85$ ampère.

Grid-bias voltage, $E_b = \dfrac{-E_{DC}}{\mu} = -600$ volts.

Peak drive volts, $E_{g\max} = 600 + 900 = 1500$ volts.

Half-conduction-angle for grid-current, $\theta_g = \cos^{-1} \frac{600}{1500} = 67°$,

giving $f(\theta_g) = \frac{I_g}{i_{g\max}} = \cdot 185$.

Average (D.C.) grid-current, $I_g = \cdot 185 \times \cdot 85 = \cdot 157$ ampères.

Grid drive-power $\doteqdot E_{g\max} I_g = 1500 \times \cdot 157 = 235$ watts.

Grid-resistor required, $R_g = \frac{E_b}{I_g} = \frac{600}{\cdot 157} = 3820$ohms, used in conjunction with a series H.F. choke, as described on p. 138.

Tank-circuit relations (on full load)

RMS tank-voltage, $V_T = \frac{10,200}{\sqrt{2}} = 7220$ volts.

RMS supply current to tank, $I_f = \frac{3\cdot 14}{\sqrt{2}} = 2\cdot 22$ ampères.

Tank impedance, $Z_T = \frac{7220}{2\cdot 22} = 3250$ ohms.

Tank circulating current $I_T = QI_f = 20 \times 2\cdot 22 = 44\cdot 4$ ampères.

Tank inductance (effective value when loaded) from equation 5·8:

$$L = \frac{V_T{}^2}{\omega_O Q P_T} \times 10^6 \ \mu\text{H}$$
$$= 72 \ \mu\text{H}.$$

Tank tuning capacitance $\left(\text{from } f \doteqdot \frac{1}{2\pi\sqrt{LC}}\right)$.

$$C = \frac{10^{12}}{4\pi^2 f^2 L} = \cdot 00035 \ \mu\text{F}.$$

The Class C amplifier[20]

As already stated, Class B operation is a particular case of Class C, i.e. the limiting case, in which the plate-current pulse occupies a half-cycle. The general case is now considered, in which the plate-current flows for any period up to half a cycle.

The effect upon the plate-current pulse of increasing the negative grid-bias voltage beyond the "cut-off" value of $\frac{E_{DC}}{\mu}$ was seen in Fig. 5·1 (c). On the assumption that the pulse-waveform is identical with the top portion of a sine-wave, this pulse was analysed (p. 119) and the average and fundamental-component values obtained in terms of the peak pulse current.

Fig. 5·11 shows graphically the relations of all the quantities involved for the Class C amplifier, and is based on the static $v_p - i_p$

characteristics of the valve. For simplicity, linear characteristics are assumed. Reference should also be made to Figs. 5·1 and 5·2.

Plate-circuit relations

The safe peak plate-current, $i_{p\max}$, may be determined from equations 5·15 or 5·17, in the absence of complete valve characteristics, or it can be estimated thus:

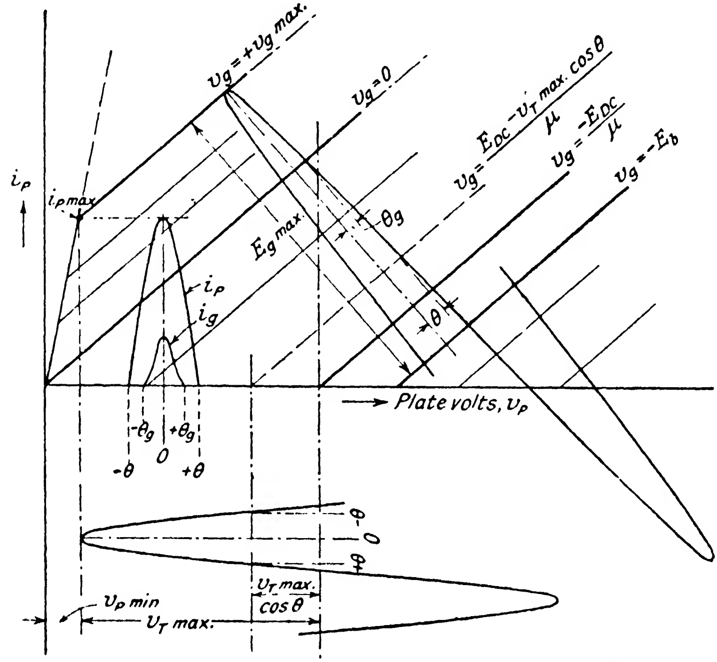


Fig. 5·11. Graphical Treatment of Class C Amplifier with Tuned Load.

For a given type of filament or cathode the permissible peak pulse current per watt of cathode-heating power is given below.

Type of emitter:	Pure tungsten	Thoriated tungsten	Oxide-coated
Peak mA per watt of heating power*:	4·5-7·5	15-25	20-50

For a filament or heater power of P_f watts, the value of $i_{p\max}$ is then given from

$$i_{p\max} = P_f \times \text{peak mA/watt} \quad . \quad . \quad . \quad . \quad 5{\cdot}30$$

* Peak grid-current is usually about one-quarter of the peak plate-current, and these tabulated values are thus approximately 80% of the corresponding total permissible space-current values. For pure tungsten filaments, the peak space-current may be made equal to the total emission, but for thoriated-tungsten and oxide-coated filaments large safety-factors are applied.

For example, the Marconi-Osram CAT6 valve-filament takes 75 ampères at 19 volts, and the peak permissible plate-current is 8 ampères; a tungsten filament is used. This corresponds to 5·6 mA/watt.

Having determined a suitable value for $i_{p\max}$, the amplitude of the fundamental-component of plate-current is obtained from Fig. 5·5 for any chosen value of θ, the "half conduction-angle"; let this be $I_{f\max}$ ampères.

As before, the minimum plate-voltage is about $\cdot 15E_{DC}$, so that the tank-voltage amplitude is $\cdot 85E_{DC}$.

The power supplied to the tank-circuit is then:

$$P_O=\frac{v_{T\max}}{\sqrt{2}}\times\frac{I_{f\max}}{\sqrt{2}}\text{ watts} \quad . \quad . \quad . \quad . \quad 5{\cdot}31$$

(See also equation 5·7.)

$$\doteqdot \cdot 425E_{DC}\, i_{p\max} f_2(\theta)\text{ watts} \quad . \quad . \quad . \quad 5{\cdot}32$$

$$=[\text{a constant}]\times f_2(\theta)\text{ for a fixed value of } i_{p\max}.$$

This power increases with the conduction angle 2θ, after the manner of the curve in Fig. 5·5.

The D.C. power input to the amplifier is:

$$P_I=E_{DC}I_{DC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}18$$

$$=E_{DC}i_{p\max} f_1(\theta)\text{ watts} \quad . \quad . \quad . \quad . \quad 5{\cdot}33$$

$$=[\text{a constant}]\times f_1(\theta)$$

for a fixed value of $i_{p\max}$, $f_1(\theta)$ being the ratio $\frac{I_{DC}}{i_{p\max}}$ plotted in Fig. 5·5.

The input power increases with the conduction angle 2θ after the manner of the $f_1(\theta)$ curve in Fig. 5·5.

The efficiency of power-conversion from D.C. to A.C. in the plate-circuit is therefore:

$$\eta=\frac{P_O}{P_I}\doteqdot \cdot 425\frac{f_2(\theta)}{f_1(\theta)} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}34$$

The efficiency falls steadily from about ·90 for a 20° pulse to ·667 for a 180° pulse (i.e. Class B). On the other hand, output power increases steadily as the pulse-angle increases. A useful criterion for selecting the value of θ is the product ηP_O. This has a maximum value at $\theta=90°$ (i.e. a 180° pulse), but does not diminish sharply until θ is reduced to about 60°. Since a compromise must be made between large power-output from a given valve, and efficiency of power-conversion, θ is usually chosen in the range 60° to 75°, i.e. a current-pulse of 120° to 150° duration. The valve-efficiency then ranges from 76% at $\theta=60°$, to 72% at $\theta=75°$, so that about three-quarters of the D.C. input power is converted to A.C. power in the tank-circuit. Not all of this is ultimately transferred to the load, as losses occur in both tank- and work-coils, and in the tank condenser.

The importance of having as low a value as possible for $v_{p\min}$ is obvious, but the plate-voltage must not be allowed to fall below the peak positive grid-voltage, $+v_{g\max}$. The values already quoted, i.e. $v_{p\min}=\cdot 15E_{\mathrm{DC}}=1\cdot 2v_{g\max}$ are typical, though $v_{p\min}$ may even be twice $+v_{g\max}$.

Tank circulating current I_T will be:

$$I_{T\ \mathrm{RMS}}=\frac{QI_{f\max}}{\sqrt{2}}\ \text{ampères (see also equation 5·6).}$$

In induction-heating applications it is usually necessary to make I_T very large, and Q is usually between 20 and 50, but tank $I_T{}^2R_T$ losses increase rapidly as Q is increased. It is clear that the tank KVA (i.e. product of tank-voltage and circulating current) will be large:

$$\text{Tank KVA}=V_TI_T=QP_O \quad . \quad . \quad . \quad . \quad 5\cdot 35$$

Thus, if tank power is 10 KW and $Q=50$, the tank KVA is 500.

In equation 5·9, R_T represented the effective inherent resistance of the tank-circuit itself, the tank-circuit loss being $I_T{}^2R_T$ watts. It is necessary to know how this power-loss is divided among the several components of the tank, i.e. tank-coil, work-coil, and tank condenser.

$$\text{Power-loss in tank-coil}=I_T{}^2R_{\mathrm{TC}}\ \text{watts} \quad . \quad . \quad 5\cdot 36$$

R_{TC} being the H.F. resistance of the coil.

Similarly:

$$\text{Power-loss in work-coil}=I_T{}^2R_{\mathrm{wc}}\ \text{watts} \quad . \quad . \quad 5\cdot 37$$

when, as is the usual case, the work-coil is in series with the tank-coil.

The tank-coil is a more or less permanent component, being never, or only infrequently, changed. It may therefore be made from large diameter copper tubing, and will generally be water-cooled. The work-coil, on the other hand, is usually changed for each different job, and nearly always is of necessity made from relatively small tubing, e.g. $\frac{1}{4}''$ to $\frac{1}{2}''$ outside diameter, and water-cooled. The power-loss in these coils is best determined calorimetrically, from measurements of temperature-rise and rate of flow of cooling water, together with measured tank-current.

The power-loss in the tank-condenser depends upon the condenser power-factor, tan δ (see Ch. 7). It is given by:

$$P_C = \text{Condenser KVA} \times \tan\delta \quad . \quad . \quad . \quad 5{\cdot}38$$

Dealing here only with the simple case shown in Fig. 5·1 in which a single condenser is in parallel with the tank-coil, the condenser KVA is the tank KVA, so that

$$P_C = QP_O \tan\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}39$$

A power-factor of ·0005 is reasonable for mica condensers, so that for the 500 KVA tank quoted above, the condenser power-loss would be 250 watts. It may be necessary to cool the condenser artificially in some cases, e.g. by fan or by immersion in oil.

The selection of suitable values for tank inductance L and capacitance C follows from equation 5·8:

$$P_T = \frac{V_T{}^2}{Q\omega_O L} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}8$$

So that

$$L = \frac{V_T{}^2 . 10^6}{2\pi f Q P_T} \text{ microhenries} \quad . \quad . \quad . \quad 5{\cdot}8\,(a)$$

and

$$C = \frac{10^{12}}{4\pi^2 f^2 L} \text{ microfarads } (L \text{ in } \mu\text{H}) \quad . \quad . \quad 5{\cdot}8\,(b)$$

It will be seen later (p. 145) that in many cases either the coil or the condenser is tapped to provide the grid-drive voltage from the tank; the valve then requires no *external* driving-source, and becomes a self-oscillator. Special requirements of the tank-circuit for these cases are discussed in the section dealing with oscillators.

Grid-circuit relations

For selected values of $i_{p\max}$ and θ, it is necessary to determine the correct values of grid-bias voltage E_b and drive-voltage amplitude $E_{g\max}$.

Drive-voltage and tank-voltage are in phase, and plate-current commences when the instantaneous plate-voltage v_p is μ-times the corresponding grid-voltage v_g, but of opposite sign. Referring to Fig. 5·11, plate-current commences when the tank-voltage $v_T = v_{T\max} \cos\theta$, i.e. when

$$v_p = E_{DC} - v_{T\max} \cos\theta \text{ volts.}$$

At this instant, therefore, the grid-voltage is:

$$v_g = -\left[\frac{E_{DC} - v_{T\max} \cos\theta}{\mu}\right] \text{ volts.}$$

This voltage is, however, $E_{g\max} \cos\theta$ above the bias voltage, $-E_b$. The drive-voltage amplitude is therefore:

$$E_{g\max}=E_{g\max}\cos\theta+\left[\frac{E_{DC}-v_{T\max}\cos\theta}{\mu}\right]+v_{g\max},$$

$v_{g\max}$ being the peak positive grid-voltage,

i.e.
$$E_{g\max}=\frac{\left[\frac{E_{DC}-v_{T\max}\cos\theta}{\mu}\right]+v_{g\max}}{1-\cos\theta}\text{ volts} \quad . \quad . \quad 5{\cdot}40$$

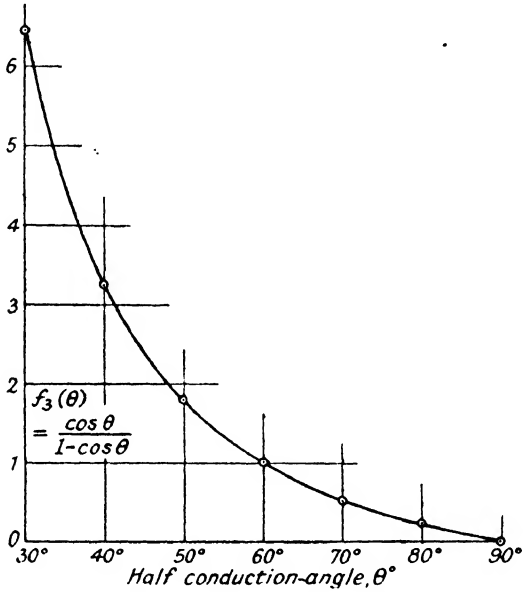


Fig. 5·12. Graph of $f_3(\theta)$.

The negative bias voltage E_b is, from Fig. 5·11:

$$E_b=-\left[\frac{E_{DC}-v_{T\max}\cos\theta}{\mu}+E_{g\max}\cos\theta\right].$$

Substituting for $E_{g\max}$ from equation 5·40, and putting $(E_{DC}-v_{p\min})$ for $v_{T\max}$, this expression becomes:

$$E_b=-\left[\frac{E_{DC}}{\mu}+\left(\frac{v_{p\min}}{\mu}+v_{g\max}\right)\frac{\cos\theta}{1-\cos\theta}\right]$$

$$=-\left[\frac{E_{DC}}{\mu}+\left(\frac{v_{p\min}}{\mu}+v_{g\max}\right)f_3(\theta)\right]\text{volts} \quad . \quad . \quad 5{\cdot}41$$

The function $f_3(\theta)=\frac{\cos\theta}{1-\cos\theta}$ is plotted in Fig. 5·12.

θ:	10°	20°	30°	40°	50°	60°	70°	80°	90°
$f_3(\theta)$:	65·7	15·65	6·47	3·275	1·80	1·00	0·52	0·22	0

Equation 5·41 involves no approximations, except, of course, that it is based upon linear valve-characteristics. For the particular case of $v_{p\min} = \cdot 15E_{DC} = 1\cdot 2v_{g\max}$, it simplifies to:

$$E_b = -E_{DC}\left[\frac{1}{\mu} + \left(\frac{\cdot 15}{\mu} + \cdot 125\right) f_3(\theta)\right] \text{ volts} \quad . \quad . \quad 5\cdot 42$$

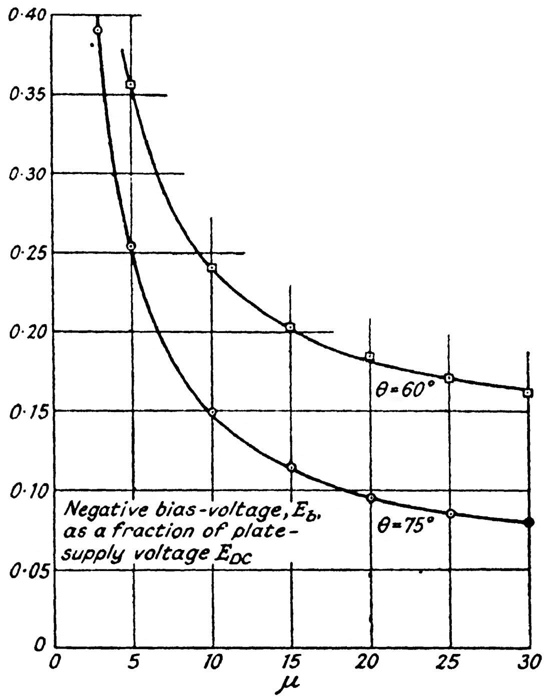


Fig. 5·13. Effect of Amplification Factor μ on Bias Voltage Required.

It is of interest to note the dependence of E_b upon the amplification-factor μ. This varies, for different triodes, between 3 and 100. Fig. 5·13 shows the negative bias voltage as a fraction of E_{DC}, for various values of μ, and for two values of θ, i.e. $\theta = 60°$ and $\theta = 75°$. The curves are calculated for the particular case of equation 5·42.

Grid-current flows during the time that the grid-potential is positive, i.e. from $-\theta_g$ to $+\theta_g$ in Fig. 5·9 (a), where

$$\cos \theta_g = \frac{E_b}{E_{g\max}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 5\cdot 24$$

The half conduction-angle θ_g can thus be found. The peak grid-current can be determined accurately only from the grid-current characteristic-curves; these are not always available, so that $i_{g\max}$ is usually assumed to be $\cdot 25 i_{p\max}$. Then the average D.C. grid-current is:

$$I_g = i_{g\max} f(\theta_g),$$

the value of $f(\theta_g)$ being read from Fig. 5·10.

Total grid-drive power is then given approximately as:

$$P_{\text{drive}} \doteqdot E_{g\max} I_g \text{ watts} \quad . \quad . \quad . \quad . \quad 5{\cdot}27$$

Methods of obtaining a negative bias voltage

Three methods are available for providing the bias voltage E_b. These are:

(1) Battery bias, arranged as shown in Figs. 5·1 (a) and 5·9 (b). Alternatively, a rectified supply from an A.C. source, e.g. the 50 cycles/sec mains.

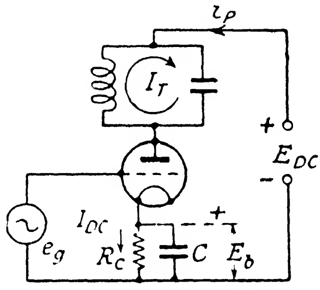


Fig. 5·14. Cathode Resistor Bias.
$E_b = I_{DC} R_c$ volts.

(2) Cathode-resistor bias (Fig. 5·14). A resistance R_C is connected between cathode (or filament) and the negative terminal of the D.C. supply E_{DC}, and is shunted by a H.F. by-pass condenser which carries the A.C. components of the plate-current, the current in R_C being the D.C. component I_{DC}. The cathode is thus raised to a steady potential of $+E_b = I_{DC} R_C$ volts above the negative D.C. line, and since the *average* grid-voltage is that of this line, the grid is held at an average potential of $-E_b$ relative to the cathode. It should be noted that the effective plate-supply voltage E'_{DC} is now less than the total D.C. supply voltage by the amount E_b.

Obviously, Class B or Class C bias is impossible by this method *in the absence of any drive* (unless a second resistor is connected between cathode and H.T. positive, when initial bias can be obtained). However, if the bias-condenser has a large capacitance it can store a sufficient charge, received from the pulses of plate-current, to give Class C bias when drive is applied. The power dissipated in R_C is $E_b I_{DC} = I_{DC}^2 R_C$ watts.

(3) Grid-resistor bias (Fig. 5·15).

Grid and cathode form a diode rectifier, of which the grid is the "anode", and which permits current only when the grid potential is positive relative to the cathode. When grid-current flows the grid condenser C_g is charged, with the polarity shown, thus introducing a D.C. component into the grid-cathode voltage, the grid-voltage being negative. In the absence of any possible discharge-path such as R_g, C_g would eventually become charged to the peak-voltage of the drive, i.e. to $E_{g\max}$, and the grid-potential could not then rise above zero (see Fig. 5·16). This would, of course, severely limit the power-output. With R_g connected as shown, the voltage across C_g will always be less than $E_{g\max}$, so that v_g can become positive during the peak of the drive-voltage. Variation of R_g thus provides a means of adjusting the peak positive grid-volts, $+v_{g\max}$. The negative-bias voltage then is:

$$E_b = I_g R_g \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad 5{\cdot}43$$

I_g being the *average* grid-current, as measured with a moving-coil meter. A grid-current meter is usually incorporated, as shown in Fig. 5·15.

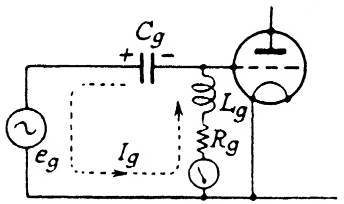


Fig. 5·15. Grid Resistor Bias. $E_b = I_g R_g$ volts.

Grid-resistor bias is nearly always used in self-oscillators, and then not only is a finite value of R_g essential for the reason already given, but also to allow for quick automatic adjustment of the bias to any change in drive-voltage. This requirement is important, and restricts the choice of values for C_g and R_g.

The grid-resistor will be required to dissipate the greater part of the total drive-power:

$$P_{Rg} = E_b I_g = I_g^2 R_g \text{ watts} \quad . \quad . \quad . \quad . \quad 5{\cdot}44$$

This power will be of the order of 1 to 5% of the D.C. plate input power, for triodes, the smaller percentages corresponding to the larger valves. A mat-type of resistance is convenient where the grid-resistor power is considerable; tappings are brought out to a stud-switch for variation of R_g.

The H.F. choke L_g shown in Fig. 5·15 is necessary to suppress the A.C. component of current which would otherwise flow in R_g due to the drive-voltage e_g across it. The reactance of L_g should preferably be not less than $5R_g$ ohms.

A combination of grid-resistor and cathode-resistor bias is sometimes used, the latter being in the nature of a safety precaution to protect the valve in the event of removal of drive-voltage e_g.

H.F. grid-current

In H.F. power amplifiers and oscillators, it is important that the H.F. current flowing in the grid-circuit shall not exceed a specified value. For example, in the R.C.A. type 207 triode, a water-cooled type with a plate dissipation of 10 KW max, D.C plate-current of 2·0 ampères, D.C. grid-current of 0·2 ampère, the specified maximum H.F. grid-current is 30 ampères. This current is almost entirely due to the capacitance-component (i.e. susceptance, $B_C=\frac{1}{X_C}$) of the input admittance, Y.

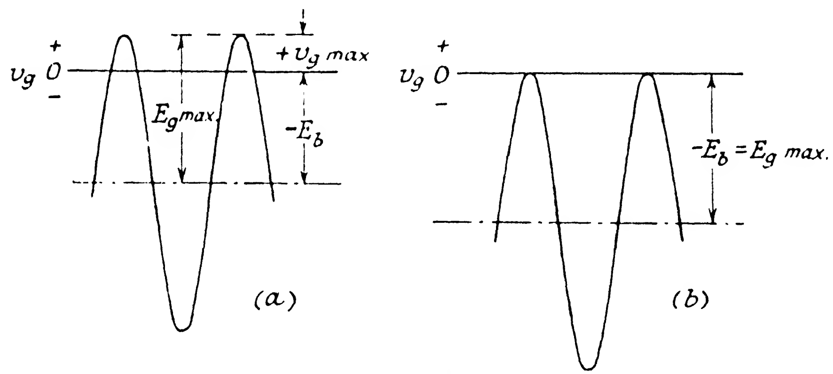


Fig. 5·16. Effect of Value of R_g, showing v_g when (*a*) $-E_b$ is less than $E_{g\max}$; (*b*) $-E_b = E_{g\max}$, i.e. when $R_g = \infty$.

The grid-cathode capacitance of this valve is 18 $\mu\mu$F, and grid-plate capacitance 27 $\mu\mu$F. When used with a tuned load, the effective load impedance is resistive, the voltage developed across the load being, say, n times the grid-drive voltage. This voltage ne_g is opposite in phase to the drive voltage (see Fig. 5·17) so that the effective voltage across the grid-plate capacitance is $(n+1)e_g$. The magnitude of the total H.F. grid-current $I_{g\mathrm{HF}}$ is therefore:

$$\begin{aligned} I_{g\mathrm{HF}} &= e_g\omega C_{gf} + (n+1)e_g\omega C_{pg} \\ &= \omega e_g[(C_{gf}+C_{pg})+nC_{pg}] \text{ ampères} \qquad 5\text{·}45 \end{aligned}$$

So that the effective input capacitance is:

$$\begin{aligned} C_{\mathrm{eff}} &= C_{gf}+C_{pg}+nC_{pg} \\ &= C_{gf}+(n+1)C_{pg} \qquad 5\text{·}46 \end{aligned}$$

In the particular case of the 207, for example, n will be about 4 for typical Class C conditions (peak drive-voltage $\doteqdot$ 2000, peak load-voltage $\doteqdot$ 8500), so that the effective input capacitance is:

$$C_{\text{eff}} \doteqdot 18+(5\times 27)=153 \ \mu\mu\text{F}.$$

Thus, e.g., the R.M.S. H.F. grid-current, for a peak drive-voltage of 2000 volts, at 10 MC/S, would be:

$$I_{g\text{HF}}=\frac{2000}{\sqrt{2}}\times 2\pi\times 10^{7}\times 153\times 10^{-12} \text{ ampères}$$
$$=13{\cdot}6 \text{ ampères.}$$

The R.F. "by-pass" condenser

It is the normal practice to shunt the D.C. plate-input terminals with a condenser having negligible reactance to current at the operating-frequency of the amplifier. This eliminates the A.C.

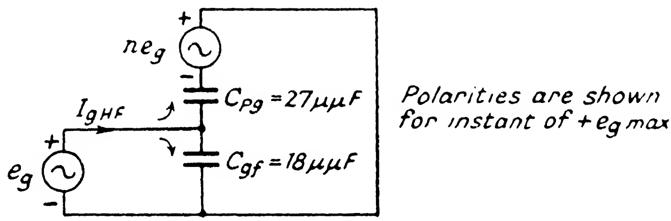


Fig. 5·17. Equivalent Circuit for Input Admittance of Triode with Tuned Load.

components of the plate-current from the D.C. source, which then supplies a practically steady current I_{DC} to this condenser and the valve-circuit. The "smoothing" of this supply current may be yet further assisted by the incorporation of a H.F. choke-coil between the D.C. positive and this by-pass condenser. An alternative point of view of the action of this condenser is to regard it as the energy-reservoir for the valve-circuit. Energy is drawn from it in pulses by the valve-circuit, and supplied to it at an almost steady rate from the D.C. source. Fig. 5·18 shows the two cases referred to.

The rating of the by-pass condenser C_b must be such that it is capable of withstanding the full working voltage E_{DC} and of carrying the fundamental component I_f of the plate pulse-current, together with small-amplitude components of higher frequency, e.g. the maximum amplitude of the second-harmonic component is about $\cdot 4 I_{f\text{max}}$.

Shunt-fed tank-circuits

The point A in Fig. 5·18 (a) and (b) is effectively at cathode-potential to H.F. currents, since the reactance of C_b is negligible at high frequencies. The tank-circuit is therefore in effect across the valve, which may conveniently be regarded as a H.F. generator

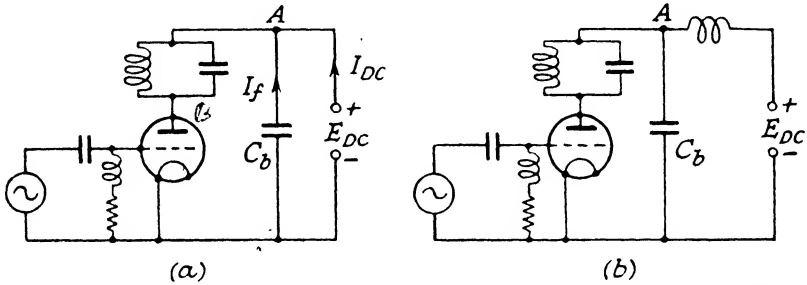


Fig. 5·18. (a) By-pass Condenser. (b) Choke and By-pass Condenser.

supplying the tank, as shown in Fig. 5·19 (a), which is concerned only with alternating quantities.

Fig. 5·19 (b) shows the usual arrangement which permits the tank-circuit to be grounded instead of operating at an average potential of E_{DC} volts above "ground". The coupling-condenser C_C must have negligible reactance at the operating frequency, while the "decoupling" choke L_{CH} should have a reactance at

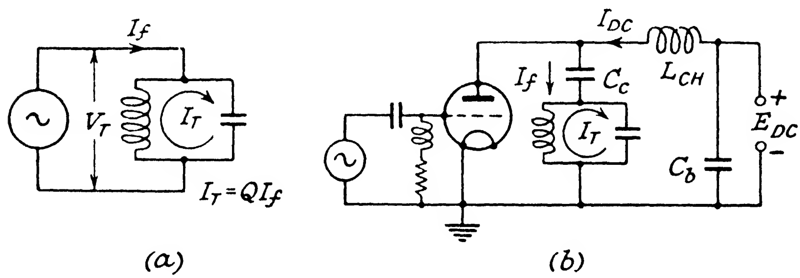


Fig. 5·19. Shunt-fed Amplifier with Grounded Tank.

least ten times the impedance presented by the tank. This choke is usually of the basket-winding type.

In many industrial applications, direct conductive connection is made between the tank and the work-circuit, so that a grounded tank is essential. This method of supplying the tank with H.F. energy is referred to as "shunt" or "parallel" feed, and is characterised by the fact that the D.C. component of plate-current does not flow through the tank-coil.

The arrangement previously considered, and illustrated in Fig. 5·18 (a) and (b), is referred to as "series feed", in which the D.C. component of plate-current flows through the tank-coil, which is at high-potential.

Worked example of Class C amplifier design

In the following example the valve used is the Marconi CAT6, a water-cooled valve with a maximum permissible plate dissipation of 12 KW. The operating conditions arrived at are approximately those realised in the H.F. induction heating plant shown in Fig. 5·30, early models of which employed two of these valves connected in parallel. The calculations below are for one valve:

Plate supply voltage	$E_{DC}=10{,}000$
Peak plate-current	$i_{p\max}=8{\cdot}0$ ampères
Maximum permissible D.C. plate-current	$I_{DC}=2{\cdot}0$ ampères
Total emission current	$I_E=10{\cdot}0$ ampères
Plate resistance	$R_P=5000$ ohms
Amplification factor	$\mu=45$

This valve is also available as a forced-air-cooled type (see Fig. 4·2), when it is known as the ACT14; the maximum plate dissipation is reduced to 8 KW.

Plate-circuit

Let the half-conduction-angle for plate-current, $\theta=60°$. The corresponding amplitude of the fundamental component of plate current, for $i_{p\max}=8{\cdot}0$ ampères, is (from Fig. 5·5):

$$I_{f\max}=0{\cdot}39\times 8=3{\cdot}12 \text{ ampères.}$$

Minimum plate volts:

$$v_{p\min}={\cdot}15E_{DC} \text{ approx.}$$
$$=1500 \text{ volts;}$$

∴ Amplitude of tank voltage:

$$v_{T\max}=8500 \text{ volts.}$$

H.F. power in tank:

$$P_O=\frac{8500}{\sqrt{2}}\times\frac{3{\cdot}12}{\sqrt{2}}\times 10^{-3}=13{\cdot}25 \text{ KW.}$$

Supply current (D.C.) $=0{\cdot}22\times 8{\cdot}0=1{\cdot}76$ ampères (from Fig. 5·5).

D.C. power input $=10{,}000\times 1{\cdot}76\times 10^{-3}=17{\cdot}6$ KW.
Plate dissipation power $=17{\cdot}6-13{\cdot}25=4{\cdot}35$ KW.

Plate conversion efficiency $\eta=\frac{13{\cdot}25}{17{\cdot}6}\times 100=75\,\%$.

Grid-circuit

The peak plate-current of 8·0 ampères is obtained with v_{pmin} $=1500$ volts, for a grid-voltage, $+v_{gmax}=850$ (from manufacturers' curves).

The required bias voltage is therefore:

$$E_b=-\left[\frac{E_{DC}}{\mu}+\left(\frac{v_{pmin}}{\mu}+v_{gmax}\right)f_3(\theta)\right] \quad . \quad . \quad 5{\cdot}41$$

$$=-1100 \text{ volts approximately.}$$

In the application referred to, the bias voltage may rise to -2000. The above method of calculation based upon linear valve characteristics predicts that such a change in bias voltage would reduce the output power to about 11·6 KW; actually it falls to 12·5 KW. The curve in Fig. 5·12 shows how rapidly the required bias voltage increases as θ is reduced below 60°.

The peak grid-current, corresponding to $+v_{gmax}=850$ volts, $v_{pmin}=1500$ volts, is approximately 1·75 ampères, and for the calculated bias of 1100 volts we have:

$$\cos\theta_g=\frac{E_b}{E_{gmax}}=\frac{1100}{1950}=\cdot 564,$$

i.e. $$\theta_g=55{\cdot}5^\circ \text{ and } f(\theta_g)=\cdot 16.$$

Then average grid-current, $I_g=\cdot 16 i_{gmax}$

$$=\cdot 16\times 1{\cdot}75=0{\cdot}28 \text{ ampère.}$$

This is reduced to 0·23 ampère, approximately, when $E_b=-2000$ volts. The grid-resistor required for this voltage is:

$$R_g=\frac{E_b}{I_g}=\frac{2000}{\cdot 23}=8700 \text{ ohms,}$$

and the grid drive power:

$$P_{drive}\doteqdot E_{gmax}\,I_g=2850\times\cdot 23=656 \text{ watts.}$$

Tank-circuit (loaded)

RMS tank-volts, $V_T=\frac{8500}{\sqrt{2}}=6000.$

Tank supply current, $I_f=\frac{3{\cdot}12}{\sqrt{2}}=2{\cdot}2$ ampères;

∴ Required impedance of loaded tank$=\frac{6000}{2 \cdot 2}=2730$ ohms.

Q may be as high as 50 in many induction-heating applications; then tank circulating current $I_T = QI_f = 50 \times 2 \cdot 2 = 110$ ampères.

Tank inductance and capacitance values can be calculated from equations 5·8 (*a*) and (*b*), p. 134, for any given frequency and Q value.

The triode-valve oscillator

In the power amplifiers discussed in the previous section, the tank-voltage developed across the tuned plate-load is for all practical purposes sinusoidal, and a large circulating current exists in the tank. It is quite easy, therefore, to obtain a suitable drive-voltage $e_g = E_{g\max} \sin \omega t$ from the tank, i.e. the A.C. drive is obtained from the A.C. output. This may be done in several ways; Fig. 5·20 shows the principle, while Fig. 5·21 shows three methods of providing a coupling between output and input circuits.

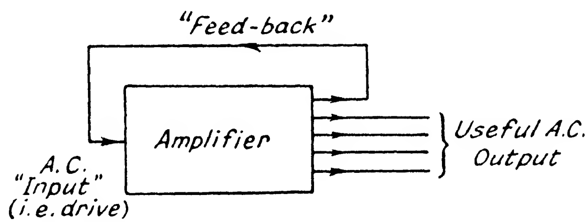


Fig. 5·20. Principle of Self-driven Amplifier—"Feed-back Oscillator".

At the present time practically all industrial H.F. power derived from valve-systems is generated by such "self-oscillators" as these, as distinct from power-amplifiers driven from entirely independent sources. In addition to simplicity, the power-oscillator has the important property of automatically adjusting itself to changes in the constants of the tank-circuit consequent upon changes in the nature of the load coupled to it. It has been pointed out that the load-constants will in general modify the resonance-frequency of the tank, but with the drive derived from the tank-circuit, the latter is of necessity tuned to the drive-frequency automatically. Small changes in frequency are unimportant in industrial applications of H.F. power, so that the power-oscillator is quite satisfactory. Class C operation is always used, with a current-pulse angle of 120°-150°.

Apart from the provision of a suitable drive-voltage and drive-power from the output-tank circuit, the arrangement is a power-amplifier as already described. The problems peculiar to the oscillator are therefore those concerning the feed-back coupling. Fig. 5·21 (a) shows the drive-voltage obtained by mutual induction between tank and grid coils. For any given value of tank-current I_T the induced voltage in the grid coil is:

$$E_g = \omega M I_T \text{ volts, RMS} \quad . \quad . \quad . \qquad 5{\cdot}47$$

and the drive-voltage can be varied by adjustment of the coupling between the two coils. Equation 5·47 shows that the drive-voltage increases with tank-current, so that any small oscillatory tank-current will be reinforced and will build up, provided that the coupling is sufficient, and the drive-voltage has the correct

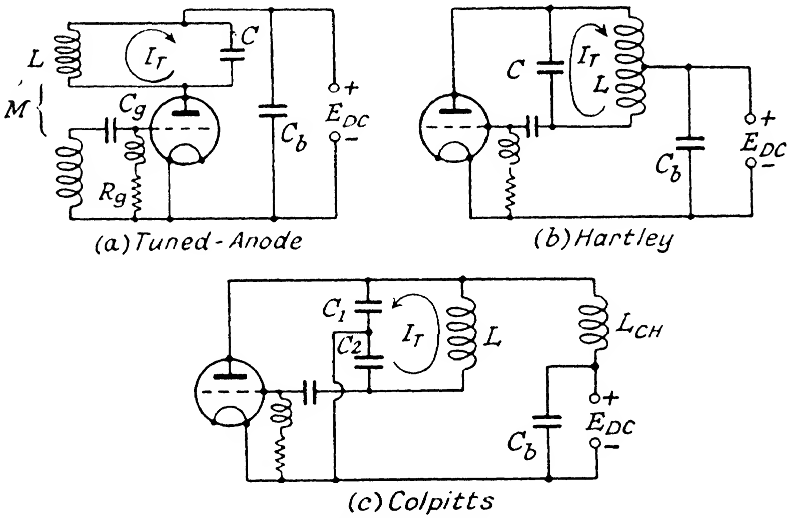


Fig. 5·21. Series-feed Oscillator Circuits.

phase-relation to the tank-current, the ultimate limit to the tank-current being set by the "limiting edge" (see Fig. 4·10) which, for a given drive-voltage, determines the value of $v_{p\min}$ and hence $v_{T\max}$. The coupling will normally be adjusted so that $+v_{g\max}$ does not exceed $v_{p\min}$; an increase in coupling beyond this point would be accompanied by a reduction in tank-current (since a trough appears in the centre of the plate-current pulse form); the resultant reduction in drive-voltage tends to make the required coupling less critical.

Self-bias by means of grid-resistor and grid-condenser is invariably used in these triode feed-back oscillators, the negative bias voltage building up automatically as the oscillation-amplitude builds up. Any change in drive amplitude is thus followed by a counteracting change in bias, e.g. if the amplitude is reduced, the bias is reduced, tending to neutralise the change in drive-voltage. It is important that the bias change occurs reasonably quickly, otherwise a state of intermittent oscillation may occur, known as "squegging". The rate at which the bias alters, consequent upon an alteration in drive-volts, is determined by the time-constant $C_g R_g$, which should therefore be not too large; on the other hand, $C_g R_g$ should have a value corresponding to not less than, say, two cycles of the working frequency in order that the bias voltage shall be reasonably "smooth".

In both the Hartley (tapped-coil) and Colpitts (tapped-condenser) oscillators, the magnitude of the grid drive-voltage is adjusted by tapping the appropriate branch of the tank-circuit. Fig. 5·22 (a) shows the tank-circuit arrangement for the Hartley, in which the simplicity of drive-adjustment by coil tapping is seen. The oscillation frequency is not directly altered by alteration of the grid tapping-point; frequency variation is obtained from the tank-condenser tapping-point, while alteration of effective load-impedance on the valve is obtained by means of the plate tapping-point. Dealing with each adjustment in turn, the grid-drive voltage is, of course, that developed between the "earthy" point of the coil, and the grid tap. This voltage will be roughly $\frac{T_g}{T_t} \times$ tank voltage, where T_g=turns between cathode and grid taps, and T_t =total effective turns on the tank-coil, i.e. the turns across which the tank condenser is connected. The peak tank-voltage will be approximately given by:

$$v_{T\max} \doteqdot \frac{T_t}{T_p} \times \text{Peak plate-volts swing}$$

$$\doteqdot \frac{T_t}{T_p} \times \cdot 85 E_{DC}$$

under normal working conditions.

These voltage relations based upon turns ratios are only approximate, since the volts per turn are not constant throughout the coil, because the flux linking any turn depends upon the location of that turn, and is greatest at the centre of the coil. Fig. 5·23 shows the measured voltage-distribution along a Hartley tank-coil two diameters long, and having 14 turns.

Secondly, the frequency of oscillation is given approximately by $f=\frac{10^6}{2\pi\sqrt{LC}}$ cycles/sec, where C is the tank-condenser capacitance in microfarads, and L the effective tank inductance in microhenries; i.e. the inductance of that part of the coil which is spanned by the condenser.

Thirdly, the effective load impedance on the valve is determined by the ratio of the turns T_P between the cathode and anode taps to the total effective turns.

Referring to Fig. 5·22 (a), assuming that the voltage is uniformly distributed along the coil, and that the peak voltage across T_P,

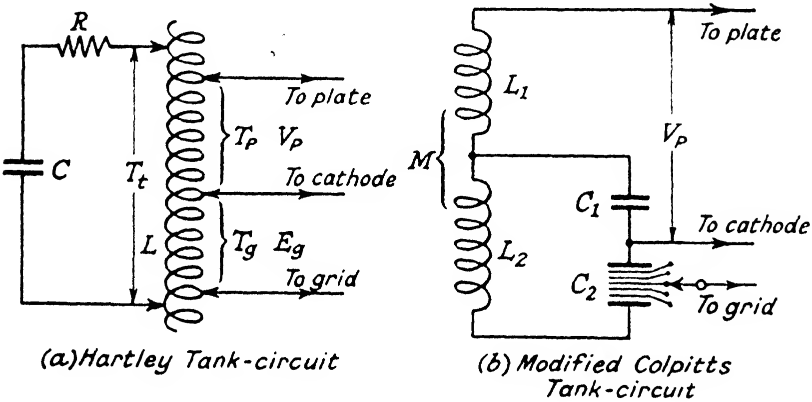


Fig. 5·22. (a) Hartley Tank-circuit. (b) Modified Colpitts Tank-circuit with Provision for Load Matching.

i.e. the peak plate-volts swing, is constant and equal to, say, $\cdot 85E_{DC}$, we have:

$$\text{RMS voltage across } T_t=\frac{T_t}{T_P}\times\frac{\cdot 85E_{DC}}{\sqrt{2}}$$
$$=\frac{T_t}{T_P}.V_P=V_T.$$

Circulating tank-current is then:

$$I_T=V_T\omega C \text{ ampères.}$$

$$\text{Tank power}=I_T{}^2R \text{ watts}$$
$$=V_T{}^2\omega^2C^2R$$
$$=\left(\frac{T_t}{T_P}\right)^2V_P{}^2\frac{C}{L}R \text{ watts,}$$

since $$\omega^2C^2=\frac{C}{L} \text{ at resonance.}$$

This power is supplied from the source, which is connected across T_P and which is working into a resistive load R_{eff}, so that

$$\text{Tank power}=\frac{V_P^2}{R_{eff}},$$

i.e.
$$R_{eff}=\left(\frac{T_P}{T_t}\right)^2 R_O,$$

where R_{eff}=actual load impedance into which valve works.

$R_O=\dfrac{L}{CR}=$ (resistive) impedance of tank-circuit at resonance when supplied across the total turns T_t.

Thus the load-impedance presented to the valve can be varied by adjustment of the plate tap. The usual procedure is to make the coupling to the load as tight as possible, i.e. in effect to make R as large as possible and hence $\dfrac{L}{CR}$ small, and then to adjust the plate tap for maximum load power. It is the usual case that the coupling to the load is inherently low, values of $R_{coupled}$ (i.e. that part of R which is due to the actual load) normally ranging from a fraction of 1 ohm up to about 5 ohms, for induction heating.

Fig. 5·22 (b) shows a modified form of the Colpitts tank-circuit, in which the grid-drive voltage is varied by means of the tapped-condenser C_2, which may be of the bare pressed-mica type. Variation of effective load-impedance presented to the valve is obtained by adjustment of the mutual inductance between the coil L_1 and the tank-coil L_2. L_1 carries only the plate-current, and may be solid copper wire of, say, 10 S.W.G., arranged to move axially over or through the tank-coil. The latter carries a large circulating current (e.g. 250 ampères for about 20 KW load-power) and so is made from 1″ copper tubing, water-cooled.

The coupling of the load to the tank-circuit is dealt with in Chapters 6 and 7, but it may be noted here that, for induction heating, it is usual to split the tank-coil into two halves, the work-coil being connected across the break so that it carries the tank circulating current. Frequency variation is then obtained by means of tappings on the two sections of the tank-coil, by condenser-switching, or a combination of both.

It is seen from Fig. 5·22 (b) that the voltage V_P, which is approximately constant for a given H.T. D.C. voltage, is divided between the plate coupling coil L_1 and tank-condenser C_1. Now the voltage drop across L_1 due to its own current and self-inductance is negligible, but the voltage induced in it due to the tank-current, i.e. ωMI_T, is controllable and can be large. The direction of winding and the connections of L_1 are such that when ωMI_T is large, the voltage across C_1, and hence across the whole tank-circuit, is small, resulting in low tank-current and power-output. Maximum tank-current corresponds to minimum coupling between L_1 and the tank-coil L_2. It will be seen that this arrangement has a partial compensating effect on any change of tank-current, at a given setting.

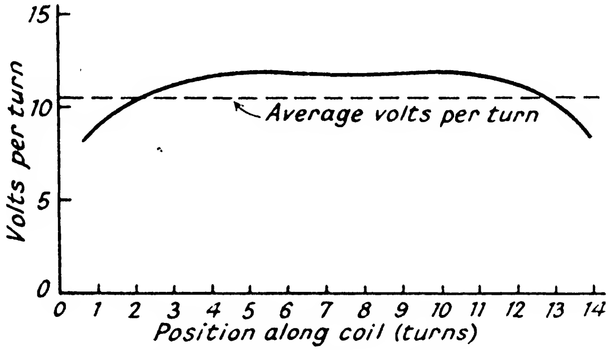


Fig. 5·23. Voltage Distribution along a 14-turn Coil.
$L = 17{\cdot}5$ cms. $D = 8{\cdot}75$ cms.

The effect of load-variation on oscillator performance

The design of the power-oscillator is based upon full-load power output. Variations of loading occur due to:

(*a*) Changes in the electrical characteristics of the work during heating.

(*b*) Introduction of work into, or its removal from, the inductor while the oscillator is switched on. A special case is automatic feeding of work into the inductor.

Consider an oscillator correctly adjusted for, and delivering, its rated power output. A reduction in loading will be accompanied by:

(*a*) A rise in tank-circuit Q, voltage, current, and KVA.

(*b*) An increased H.F. drive-voltage between grid and cathode, since it is obtained from the tank.

(*c*) A decrease in peak and average values of plate-current, consequent upon the lower value of $v_{p\min}$.

(*d*) An increase in grid-current.

(*e*) An increase in grid-bias voltage consequent upon (*d*).

With the drive correctly adjusted for full-load conditions, the removal of load might result in excessive grid-current. This in turn might lead to "blocking"[21] of the oscillator, and perhaps even melting of the grid-wires. A grid-current relay used in conjunction with a high-vacuum diode gives protection against serious over-driving (see Fig. 8·5).

It is best, where possible, to arrange that the oscillator is switched off before removing the load, or, in automatic plant, that new work enters the inductor field as treated work leaves. Violent changes of load should be avoided.

Although, in general, low coupling-factors are the rule (a tank-circuit Q as high as 50 is unfortunately often necessary), it does sometimes happen that, with a load in position, the coupling is so tight that the valve will not commence to oscillate; this involves manual readjustment of controls. With continuous repetition processes, initial adjustments should then compromise between satisfactory starting and subsequent heating.

The changes in loading due to physical changes in the work itself consist, in the case of eddy-current heating of steel parts (Chapter 6), of a steady increase in power to the work as the work temperature rises, followed by a rapid decrease in power as the magnetic change-point is reached.

In the heating of dielectric materials (see Chapter 7) the load-power tends to increase as the heating proceeds, due to changes in both power-factor and dielectric constant of the work, for the case of plastics. On the other hand, where the heating results in drying out (i.e. dehydration), the power-factor and dielectric constant fall as the moisture is removed, and the oscillator load is reduced.

It is desirable that the ratio of average to peak load-power shall be as high as possible, so keeping down the capital cost of generators to a minimum. Several alternative methods are discussed in Chapter 9. Automatic high-speed impedance-matching is the logical development for valve generators.

The impedance-matching device shown in Fig. 5·22 (b) makes it possible to transform a wide range of work-coil impedance values to that required to load the oscillator fully. If this or any other suitable matching device were motorised, and the motor controlled so as to maintain constant oscillator plate-current, then the oscillator would deliver full load-power over the whole heating cycle, provided that the work-coil impedance variations were within the scope of the matching device.

Although not sharply critical, the value of the coupling condenser used with this arrangement (Figs. 5·22 (b) and 5·24) is chosen so as to have approximately the same reactance, numerically, as that of the plate coupling-coil L_1. The tank impedance is resistive (to the fundamental frequency), but the plate looks into an inductive resistance because of the presence of L_1. The inductive reactance depends upon the magnitudes of M and tank-circuit Q, but it approximates to ωL_1 at full load (i.e., M and Q *relatively* small). Making the reactances of coupling coil and coupling condenser approximately equal may be likened to using a series-condenser with a H.F. inductor-alternator for the purpose of approximately neutralising the internal reactance of the alternator. An inductive component in the plate load broadens the plate-current pulse and also makes it lopsided; if L_1 and C_c are badly mismatched, power output and conversion efficiency will be reduced.

Valves connected in parallel

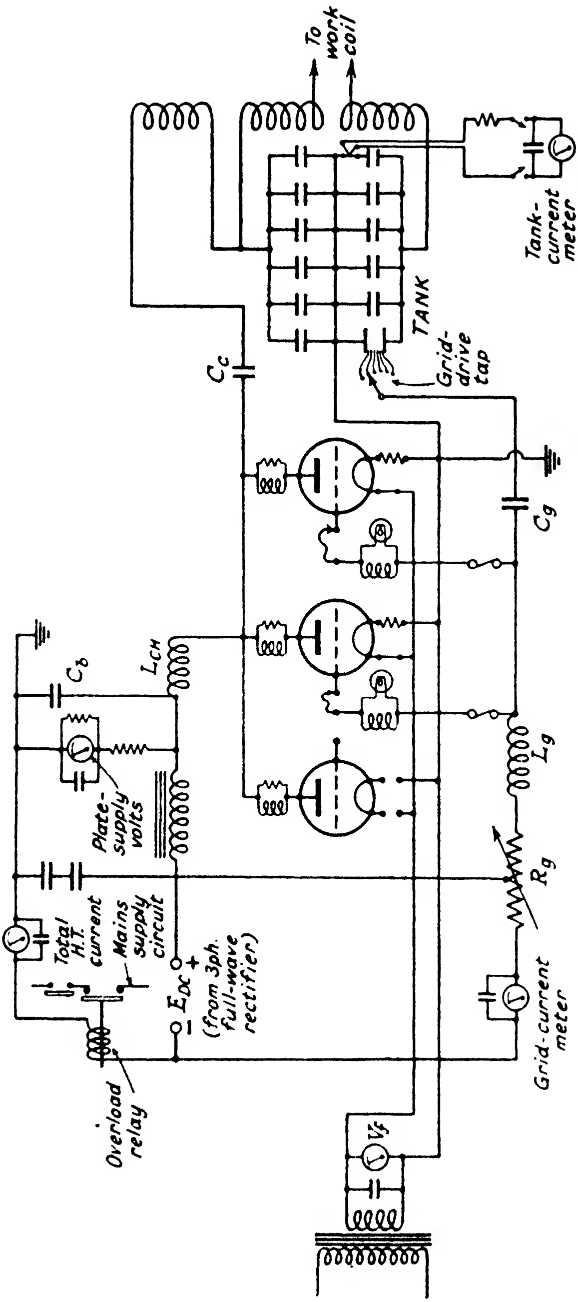
The power output from an oscillator can be increased by using two or more similar valves in parallel, as an alternative to using a single valve of bigger rating. Parallel operation affords an easy method of conversion of an existing oscillator to a higher power, provided that the power-supply unit is suitable. The plates of the several valves are strapped together and connected to a common point; similarly the grids* and cathodes are each strapped to common terminals, and these connected to the oscillatory and supply circuits in the same manner as for a single-valve oscillator; valves for parallel operation should be of similar types, with as nearly as possible exactly similar characteristics. Points which should be observed when using valves in parallel are:

(1) The effective plate resistance of n valves will be $\frac{R_p}{n}$, R_p being the plate resistance of a single valve.

(2) The power output will be nP_O, P_O being the power output of one valve.

(3) The fundamental component of the total plate-current will be nI_f; the corresponding component of plate-voltage will be the same as for a single valve.

* A non-inductive resistance of 10 to 100 ohms should be connected in each grid-lead, to suppress parasitic oscillations; alternatively, a parallel combination of V.H.F. choke and damping resistor, as shown in Fig. 5·24.



[Courtesy of R.T.E. Ltd.

Fig. 5·24. 20 KW Oscillator; Two CAT6 Valves in Parallel (Simplified Circuit).

(4) The correct load impedance will be $\frac{1}{n}$ that for a single valve working under the same conditions as each of the n valves in parallel.

(5) The drive voltage is the same as for one valve; the drive power must be increased n times.

In order to realise point (4), it is necessary to reduce the inductance between the plate tapping-point and the earthy point on the tank-coil. In practice this limits the number of valves which may be operated in parallel. It is not usual to parallel more than four valves. Fig. 5·24 shows the circuit diagram of a 20 KW 500 KC/S oscillator intended for induction heating. This employs two CAT6 valves in parallel; the third valve shown is a spare.

Valves in push-pull

An alternative method of increasing the output power by using two valves is to operate them in push-pull (see Fig. 5·25). The valves are in parallel across the D.C. input, but their drive-voltages are in phase-opposition, each valve in turn drawing its pulse-current. The pulses flow alternately in the two halves of the centre-tapped tank-coil.

Features of push-pull operation are:

(1) The effective plate resistance is twice that of a single valve.

(2) The power output is twice that of a single valve operating under the same conditions as either of the push-pull pair.

(3) The fundamental component of plate-current is that of a single valve; the corresponding tank-voltage is twice that for a single valve.

(4) The correct load impedance (plate to plate) is twice that for a single valve.

(5) The total drive voltage (grid to grid) is twice that for a single valve; drive power is also doubled.

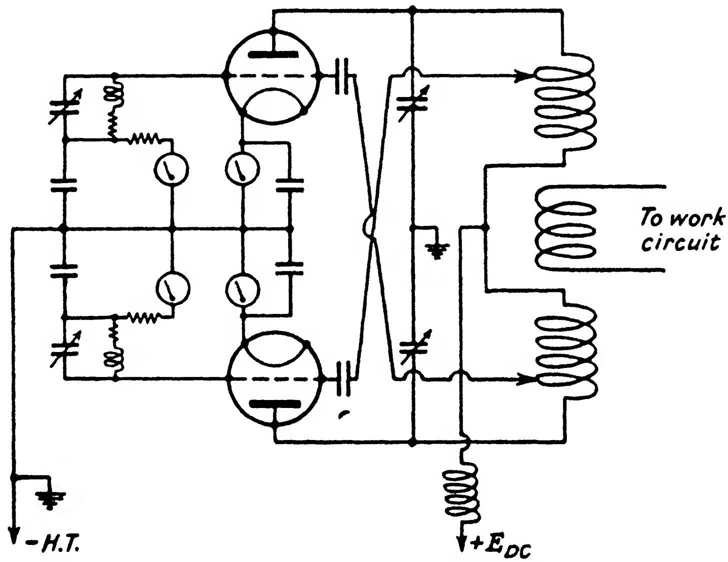
The advantages of push-pull operation are:

(1) Increased power and frequency stability.

(2) All oscillatory voltages and currents, with the exception of even harmonics, are symmetrical with respect to "earth" and to the D.C. supply. There are no A.C. components in the D.C. supply, other than even harmonics. With exactly similar valves, there are no even-harmonic components in the tank-circuit.

(3) Valve inter-electrode capacities (grid to cathode, and plate to cathode) are effectively halved, since they are in series. This is a useful property when working at very high frequencies, since valve capacities set a limit to the frequency at which satisfactory operation is obtained.

The impossibility of obtaining exactly similar characteristics for two valves, and the difficulty of determining the exact electrical centre of the tank-coil both make it desirable to include a H.F. choke in the plate-supply lead. This choke allows the H.F.



[*Courtesy of "Electronics"*.

Fig. 5·25. Push-pull Oscillator.

potential of the feed-point to vary somewhat, according to the degree of asymmetry of the valves and circuit. Inductive coupling to the load is convenient, as indicated in Fig. 5·25.

A comparison of single-valve, two-valve parallel, and two-valve push-pull oscillators is made in Fig. 5·26, the valves being represented merely as sources of A.C. power. The generated voltage V is the RMS value of the alternating component of plate-voltage, and I_f the RMS fundamental component of plate-current.

The impedances Z_1, etc., represent the plate-circuit "load" impedance into which each arrangement works.

There are advantages in using valves in parallel in oscillators for induction-heating; in particular, heavy-current low-voltage work-circuits are possible with parallel operation. On the other hand, there is a bias towards push-pull operation in dielectric-heating equipment, which normally employs much higher frequencies.

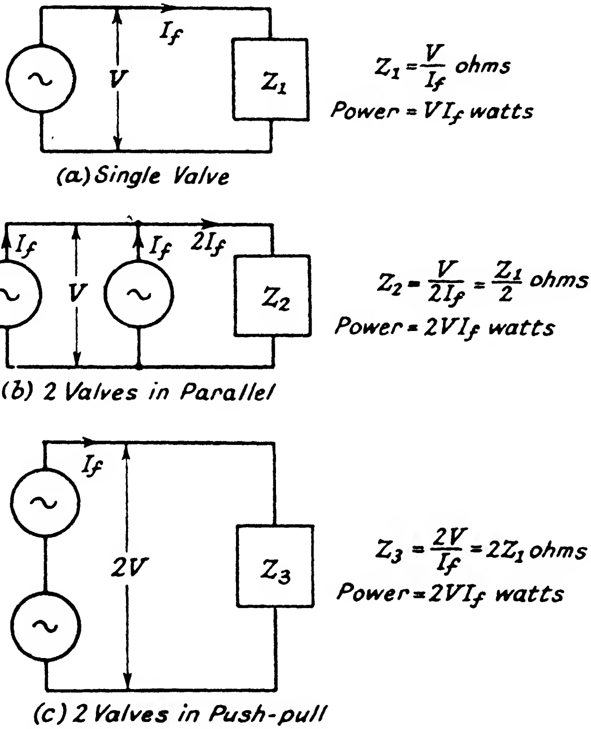


Fig. 5·26. Comparison of Single Valve, Two Valves in Parallel, and Two Valves in Push-pull.

The rating of valve-type H.F. power generators

In general, the conversion of mains-frequency power to high-frequency power by means of a valve oscillator involves four stages:

(1) Transforming to high-voltage A.C. power from the normal mains voltage.

(2) Rectifying and smoothing to give high-voltage D.C. power (see Chapter 8).

(3) Converting to high-frequency A.C. power.

(4) Supplying high-frequency power to the article to be heated.

These four stages are illustrated in Fig. 5·27 for the case of H.F. induction-heating. There are four corresponding ways in which the generator can be rated:

(1) On the mains input KVA.

(2) On the D.C. input power to the oscillator valve or valves.

(3) On the total H.F. power output from the valves.

(4) On the H.F. power supplied to the work-circuit connected to the output terminals of the generator.

The rating by the first method will be larger, for a given H.F. power in the work, the lower the transformer efficiency and power factor, and rectifier efficiency. The second method of rating is a reasonable guide to the probable useful output power only if the plate-conversion efficiency and tank-circuit efficiency are known. Similarly, method (3) is vague unless the efficiency of the tank is

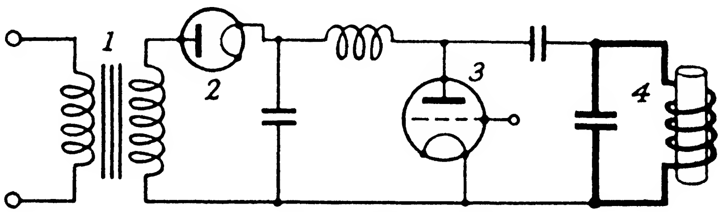


Fig. 5·27. Power Oscillator Rating.

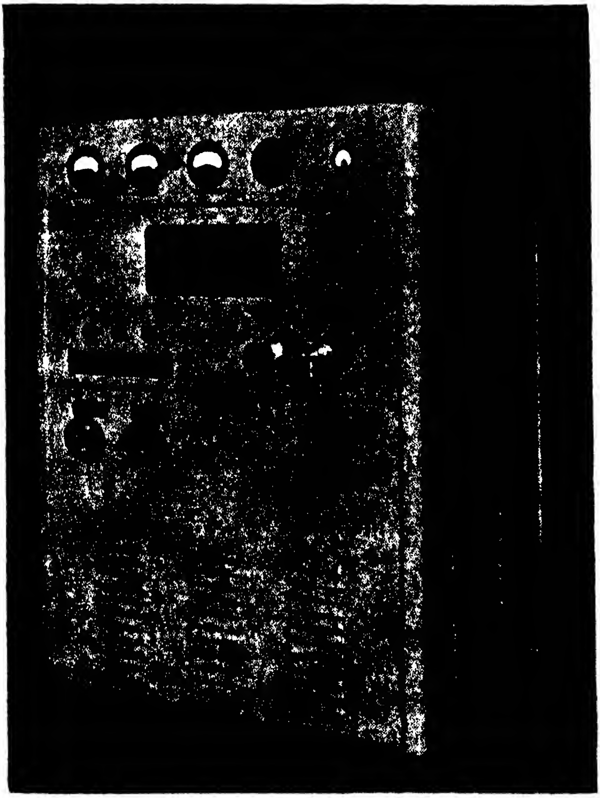
known. Tank-circuits for induction-heating will, in general, have rather greater losses than their counterparts in radio work, since a relatively high Q value is maintained for the sake of flexibility in handling a wide range of work. That is, the tank-current is made very large, and this necessarily involves a greater power-loss in the tank. The ability to cope with widely differing types of load makes this extra power-loss worth while, in the case of general-purpose plants; where the generator is intended for use on one particular job, a lower Q value and higher efficiency can be realised.

Method (4) is clearly the rating which is of first importance to the process engineer, while the mains engineer will require to know the input KVA and the power factor of the plant.

Hence, in order to compare different types of H.F. equipment, since the capital and running costs depend upon the useful H.F. power output to the work-circuit and the mains input KVA and power factor, both of these ratings should be specified precisely.

Complete valve-generator equipments for induction-heating

Figs. 5·28 and 5·29 show general exterior and interior views of a small H.F. heating unit by Radio Transmission Equipment Ltd. A Colpitts oscillator is used, with a tank-circuit like that illustrated in Fig. 5·22 (b). The mechanical arrangement for altering the coupling between the plate-coil and the tank-coil, by raising or

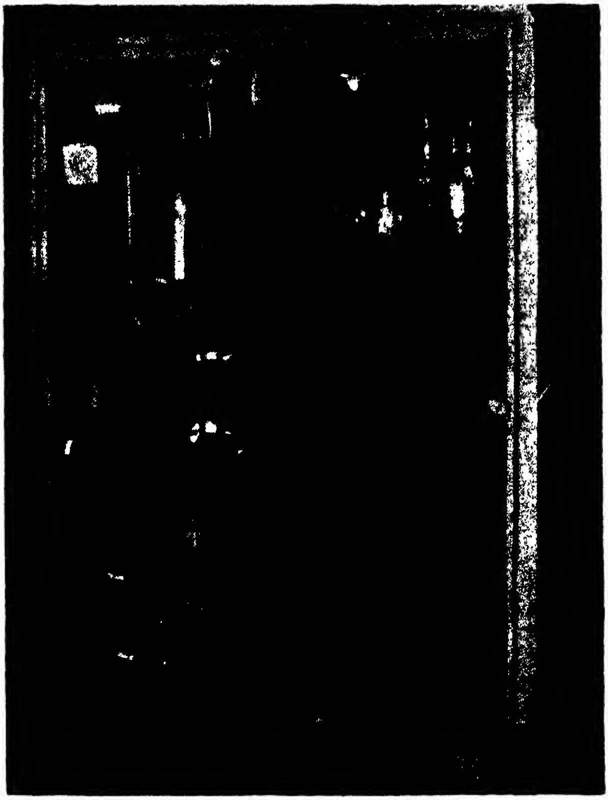


[*Courtesy of R.T.E. Ltd.*

Fig. 5·28. Induction-heating Unit, 800 KC/S 5 KW output, Exterior View.

lowering the former over the latter, can be seen. The tank-coil is split into two halves, and the inner ends brought out to the front panel for connection to the work-coil. The output power to the load is 4 to 5 KW, obtained from a single silica valve of the type illustrated in Fig. 4·4. This valve, the Mullard TX 10-4000, has tungsten seals, obviating forced air-cooling; it has a maximum permissible plate dissipation of 4 KW, and is radiation-cooled. The tank- and inductor-current is of the order of 80 ampères, and the latter coil is water-cooled. The frequency is about 800 KC/S, the precise value depending somewhat on the type of inductor used, and the characteristics of the work being heated.

The H.T. D.C. power to the oscillator is supplied from a three-



[*Courtesy of R.T.E. Ltd.*

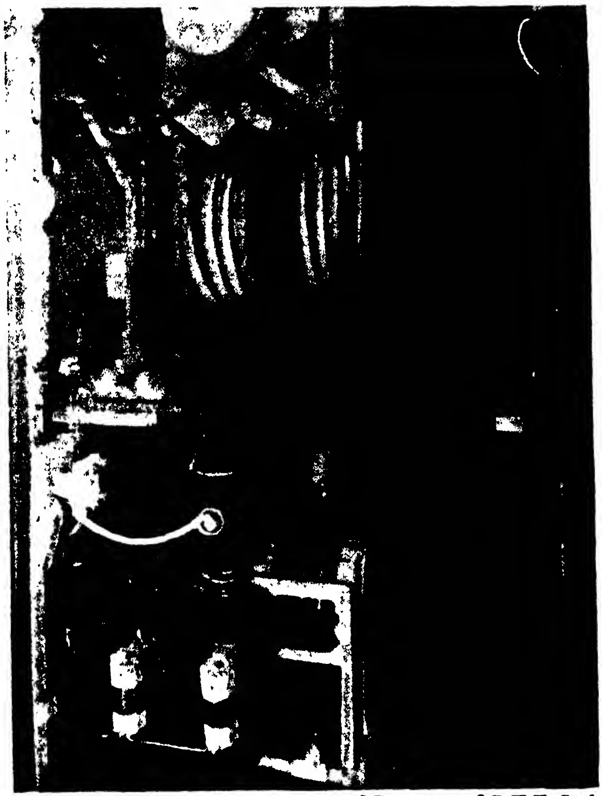
Fig. 5·29. Induction-heating Unit, 800 KC/S 5 KW output, Interior View.

phase full-wave rectifier employing hot cathode mercury vapour valves. Taps are provided on the H.T. transformer so that half, three-quarters, or full voltage may be used. A number of safety devices for protection, both of the plant and operating personnel, are incorporated (see Chapter 8). The front-panel meters, left to right, indicate:

Filament volts; plate-current; grid-current; H.F. (tank) current.

In addition, an hour-meter is fitted inside the cubicle, to record valve service hours.

The selector switch on the meter panel is for adjustment of grid-drive, using the method indicated in Fig. 5·22 (b). The push-



[*Courtesy of R.T.E. Ltd.*

Fig. 5·30. Induction-heating Unit, 400 KC/S 20 KW output. View showing Valves and Tank-circuit.

buttons on the central panel control valve filaments (ON and OFF), and, similarly, plate-voltage. The lower panel carries hand-wheels for the adjustment of oscillator-valve filament voltage, and the variable coupling to the tank; the latter adjustment provides a control of load power.

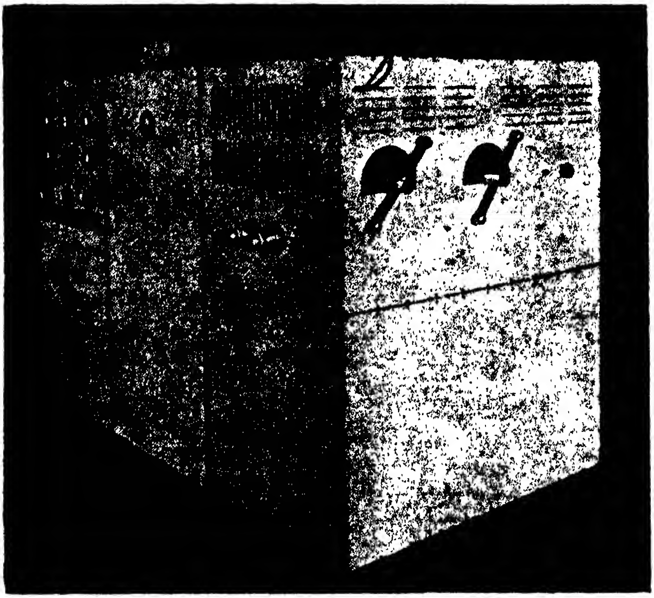
Power consumption at full load is approximately 12 KVA at 400 volts, 50 cycles/sec (3-phase), at a power factor of 0·9. A water supply of 30 gallons per hour at a pressure of 20 to 40 lbs/square inch, and a maximum incoming temperature of 80° F. is required for the inductor.

The overall dimensions are approximately:

Length 4′ 0″; Width 3′ 6″; Height 5′ 6″;

and the weight, complete, is 13 cwts.

Fig. 5·30 shows a view of the valves and oscillatory circuit of a 20 KW fixed frequency unit. Two water-cooled valves, of the CAT6 type, are used in parallel. The relatively long hoses

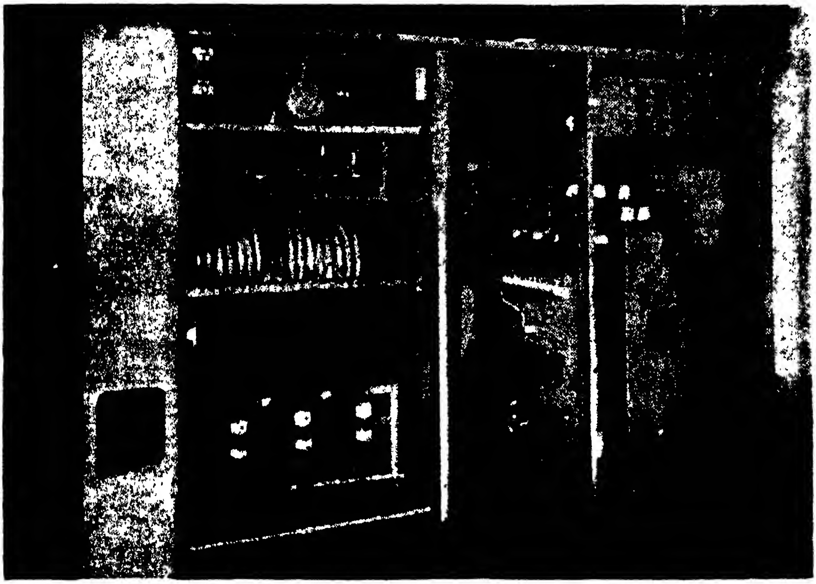


[*Courtesy of R.T.E. Ltd.*

Fig. 5·31. Variable-frequency Induction-heating Unit, 175 to 500 KC/S 20 KW output. Exterior View.

required because of the high plate-voltage (10 KV D.C. input), are coiled up on a drum. The third water jacket normally holds a spare valve. The split tank-coil is made from 1″ diameter copper tubing; the water feeds can be seen. The mica tank-condenser units are mounted above the tank-coil, and are fan-cooled. The leads from the tapped condenser to the grid-drive selector switch are visible in the top left corner of the photograph.

Figs. 5·31 and 5·32 show exterior and interior views of a variable frequency 20 KW unit manufactured by R.T.E. Ltd. The nominal frequencies are 175, 220, 320, 400, and 500 KC/S, any one being selected by means of the levers shown on the right-hand side-panel (Fig. 5·31). These levers operate switches connected to tappings on each half of the condenser-bank and split tank-coil (Fig. 5·32). The tank-coil current is about 200-250 ampères, and both it and the inductor are water-cooled, this requiring 30 to 60 gallons per hour at not less than 40 lbs per square inch. The two active water-cooled valves require a total of 300 gallons per



[*Courtesy of R.T.E. Ltd.*

Fig. 5·32. Variable-frequency Induction-heating Unit, 20 KW output, Interior View.

hour at not less than 20 lbs per square inch. The power consumption at full-load (20 KW in the load) is about 45 KVA at a power factor of ·9. The overall dimensions are approximately:

Length 9′ 0″; Width 5′ 0″; Height 6′ 0″;

and the weight, complete, is 26 cwts.

In these sets the work-coil, or the primary of a work-head transformer, is connected to the output terminals by means of screwed unions such as the "Enots" union or the Lockheed "Conelock". These unions make both electrical and water connections.

The number of coil-turns and the coupling to the work are chosen so as to load the generator fully, as indicated by the plate-current meter; for example, the maximum permissible plate-current (I_{DC}) is about ·90 ampère for the 5 KW unit, and about 3·8 ampères for the 20 KW unit. Increasing the number of work-coil turns (or primary-coil turns, if a concentrator or other type of transformer is used) usually increases the loading.

Overloading results in excessive plate-current, too little (or even reversed) grid-current, and low or zero tank-current. The fault is automatically interrupted by the operation of the plate-current "instantaneous overload" relay which trips the contactor in the H.T. transformer mains-supply circuit. The relay has then to be re-set manually.

Such induction-heating oscillators are designed for general purpose work, i.e. it should be possible to develop the rated output KW in a wide range of work-circuit impedances, from the low impedance of a typical surface-hardening transformer with a small work-loop (see, e.g., Fig. 9·33 and text), to the high impedance of a small melting furnace with its inherently loose coupling and low power-factor. The maximum output KVA at which the rated output KW can be developed is thus a measure of the flexibility of an oscillator.

CHAPTER 6

Induction Heating

PROBABLY THE most important industrial application of H.F. power at the present time is that of heating metallic objects by eddy currents, induced in them by high-frequency magnetic fields. This method of heating can be applied to melting-furnaces, heat-treatment (hardening and annealing), sintering, brazing, welding, hot-machining, forging, and surface cementation and alloying.

For melting purposes, the advantages of the method over non-electric methods are:

(1) The heat is generated within the charge itself, which is therefore the hottest part of the furnace. The thermal efficiency is high, and very rapid melting is possible.

(2) Melting can be carried out *in vacuo* or in a reducing atmosphere, if required.*

(3) The charge can be kept clean and free from impurities, and the inherent stirring action gives uniformity of composition.

For other purposes, such as localised heat-treatment, advantages of induction-heating include:

(4) Accurate control of the temperature and the location of the heating is possible.

(5) "Surface" hardening to any required depth, with immunity from damage to the steel due to overheating.

(6) Very short heat-treatment times, usually of the order of a few seconds for hardening, and a few minutes for billet-heating.

(7) Cleanliness in operation.

(8) Low power costs.

The principle of induction heating is illustrated in Fig. 6·1, which represents the heating-coil (work-coil, or inductor) L_1, magnetically coupled to a "load", which will behave electrically as an inductance L_2 and resistance R_2 forming a conducting path. The inductor also possesses resistance R_1.

* See footnote to p. 221.

The useful power developed in the load, on which the heating depends, is $I_2{}^2R_2$ watts, and since this power is obtained from the source energising the inductor, it can be expressed as

$$P = I_1{}^2R_c \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}1$$

where I_1 is the inductor current in ampères, and R_c is the "coupled-resistance" effectively in series with L_1 and R_1.

It was shown in Chapter 1 that:

$$R_c = \frac{\omega^2M^2R_2}{R_2{}^2+(\omega L_2)^2} \text{ ohms}$$

The power wasted in heating the inductor, due to its own resistance R_1, is $I_1{}^2R_1$ watts and, for this to be relatively small, the coupled resistance should be as large as possible.

If we assume for the moment that R_1, R_2, L_1, L_2, and M are

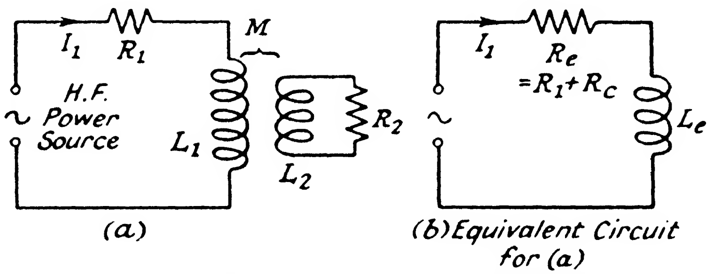


Fig. 6·1. Basic Circuit for Induction Heating.

independent of frequency, then the coupled resistance tends towards a maximum value, as the frequency is increased; thus:

$$R_{\text{coupled}} \xrightarrow[\omega L_2 \gg R_2]{} \frac{M^2R_2}{L_2{}^2}$$

i.e. in the limit when $\omega L_2 \gg R_2$, we have:

$$\frac{R_c . L_2{}^2}{M^2R_2} \longrightarrow 1{\cdot}0.$$

Fig. 6·2 shows how the ratio $\dfrac{R_cL_2{}^2}{M^2R_2}$ rises towards this limiting value as ω is increased. It is seen that R_{coupled} has a value of ($0{\cdot}9 \times$ limiting value) when $\dfrac{\omega L_2}{R_2} = 3$, and that further increase in frequency does not greatly increase this value.[29]

Actually, of course, R_2 and L_2 do alter with change of frequency, but the classical theory developed by Burch and Davis leads to a corresponding result, as will be seen.

Cylindrical charge

Fig. 6·3 represents a solid cylinder of metal, immersed in an alternating magnetic field parallel to its polar axis and of uniform density B lines/cm².

In any cylindrical shell of thickness t, and radius r, an E.M.F. is induced, given by:

$$e=\pi r^2 \times \frac{dB}{dt} \times 10^{-8} \text{ volts.}$$

$\frac{dB}{dt}$ = time rate of change of B.

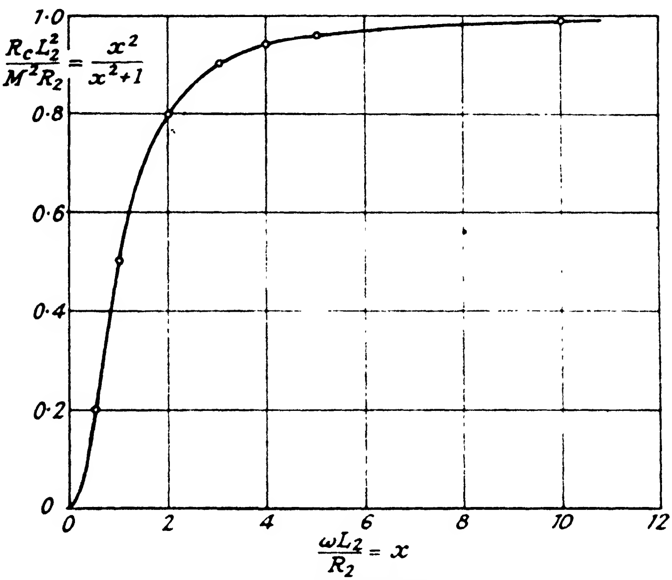


Fig. 6·2. Relation between R_c and frequency, for Fig. 6·1 (b).

For an axial length of 1 cm, the resistance of such a shell is:

$$R=\frac{l\rho}{a}=\frac{2\pi r\rho}{t} \text{ ohms.}$$

At *very low* frequencies the current in the shell would be limited only by this resistance, so that

$$\underset{f\to 0}{i} = \frac{e}{R} = \frac{rt\frac{dB}{dt}.10^{-8}}{2\rho} \text{ ampères.}$$

The current flowing in such "shells" of the solid cylinder is thus proportional to the radius of the shell; because the frequency is low, the current will in any case be relatively small, since e is small. It is not possible to use very high flux-densities because of the resulting large mechanical forces on the inductor. At the very low frequencies for which the foregoing holds, the power developed in the cylinder is proportional to the square of the frequency, since $P \propto e^2$; furthermore, the power-loss in the inductor producing the magnetic field would be constant, since its current and resistance would be constant, so that the power transference efficiency would be proportional to the square of the frequency, at the very low frequencies considered.

As the frequency of the magnetic field is increased, the reactances of the current paths in the cylinder modify the flux and current distributions, and at very high frequencies the current is practically localised in a thin surface skin on the cylinder. Fig. 6·4 (a) shows typical current distributions at various frequencies.(29)

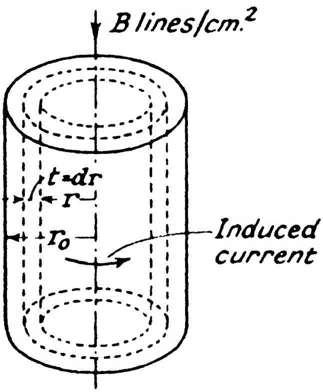


Fig. 6·3. Solid Cylinder.

High-frequency magnetic flux and current distributions in a cylindrical charge (22, 67, 68)

Consider the solid cylinder as being made up from a large number of thin concentric tubes. The E.M.F. induced in any tube is proportional to the alternating flux passing through it; the resultant current round the tube lags somewhat behind this E.M.F. because of the inductance of the current path. The effect of this current is, therefore, to attenuate the flux inside the cylinder and also to retard its phase relative to the flux outside the cylinder.

Consequently the flux density suffers a progressive decay and lag from surface to centre of the charge. At sufficiently high frequencies it is attenuated to a negligible value at a depth which is small compared to the charge radius; Fig. 6·4 (c) illustrates this condition. The summation of the flux vectors Φ_1, Φ_2, etc., gives a logarithmic spiral, and the resultant flux Φ_R lags 45° behind the surface flux Φ_O, and therefore also 45° behind the coil-current producing the flux.(68)

Steinmetz(22) showed that the total flux Φ_R in the charge is

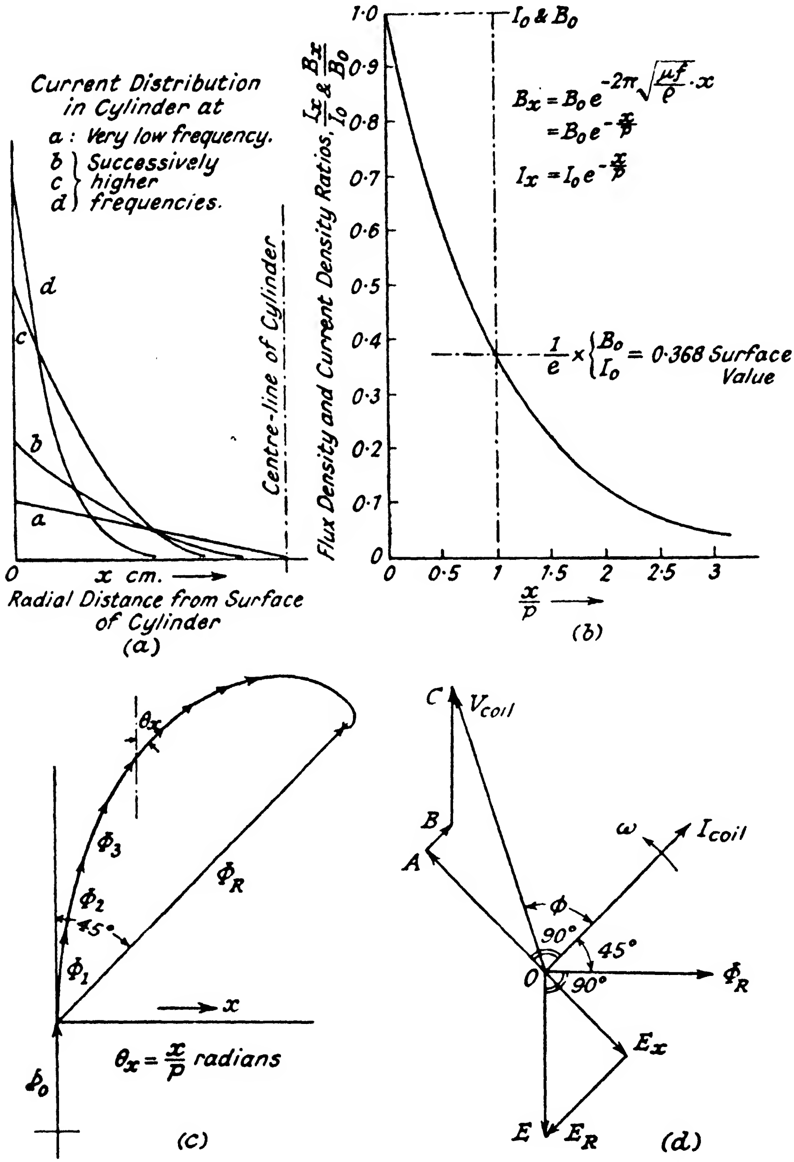


Fig. 6·4.

(a) Current Distribution in Solid Cylinder at Various Frequencies.
(b) High-frequency Current and Flux-densities in Solid Cylinder.
(c) Vector Diagram for Flux Distribution at High Frequencies.
(d) Vector Diagram for Loaded Coil.

equivalent to a flux of uniform density, equal to the surface density, penetrating to a depth given by:

$$\beta=\sqrt{\frac{\rho}{4\pi\omega\mu}}\text{ cms} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}2$$

$$=\cdot113\sqrt{\frac{\rho}{\mu f}}\text{ cms} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}3$$

where ρ = resistivity of charge in E.M.U.
$=10^9\times$(resistivity in ohm-cms)
$\omega=2\pi f$
μ = effective permeability of charge.

The charge current density is similarly attenuated and retarded; from the point of view of the heating effect, the current distribution is conveniently represented by the exponential decay curve in Fig. 6·4 (b), and its equivalent penetration depth taken as the depth at which the current-density has fallen to $\frac{1}{e}$ of the surface value; i.e.

$$\text{Current penetration depth } p=\frac{1}{2\pi}\sqrt{\frac{\rho}{\mu f}}=\sqrt{2}\beta\text{ cms} \quad . \quad . \quad 6{\cdot}4$$

$$=\cdot16\sqrt{\frac{\rho}{\mu f}}\text{ cms} \quad . \quad . \quad . \quad . \quad 6{\cdot}5$$

Approximately 90% of the total heat is generated in this layer of depth p cms. We shall see that for efficient through-heating, p should be less than one-half the charge radius, while for surface-hardening the frequency must be high enough to make p equal to, or less than, the hardened layer depth required.

A vector diagram for the high frequency conditions is given in Fig. 6·4 (d). This shows the resultant flux Φ_R in the charge lagging 45° behind the coil-current I. The resultant induced E.M.F. in the charge is E, lagging 90° behind Φ_R. E_X and E_R are the reactive and resistive components of E (relative to the coil-current I), and it is seen that the power factor of the charge is cos 45°, i.e. 0·707 lagging. This is also the theoretical maximum power factor of the loaded coil with perfect coupling; practicable arrangements yield load power factors between ·02 and ·4.

The voltage applied to the coil terminals (V in Fig. 6·4 (d)), is the vector sum of the components OA, AB, and BC, where

OA = Reactive voltage due to flux in the air-gap between coil and charge.

AB = Resistive voltage due to the resistance of the coil-winding.

BC = Resultant charge voltage E referred to the coil.

The power factor of the loaded coil is cos ϕ.

According to Babat and Losinsky,[24] equation 6·5 gives:

$$p=\frac{20}{\sqrt{f}}\text{ mm} \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}6$$

for carbon steel at 20° C and

$$p=\frac{500}{\sqrt{f}}\text{ mm} \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}7$$

for carbon steel at 800° C, i.e. above the Curie point.

On sharp convex surfaces, current penetrates rather more deeply; on concave surfaces rather less. Equation 6·6 suggests the effective value of μ is about 100; actually μ_{eff} depends upon the magnetic field strength and hence upon the power concentration, and this, in turn, depends upon the kind of application concerned.*

The power dissipated as a result of the induction of the current in the surface skin is given by:

$$P=\frac{H_t^2\sqrt{\rho\mu f}}{8\pi}\times 10^{-7} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}8$$

$$=\frac{H_t^2\rho}{16\pi^2 p}\times 10^{-7}\text{ watts/cm}^2\text{ of surface area} \quad . \quad . \quad 6{\cdot}9$$

H_t being the magnetic field-strength along the surface of the cylinder wall, i.e. the tangential component of the field. As before, ρ is in E.M. units. See equations 6·14 and 6·16 for the derivation of 6·8.

The heat is developed in this surface skin, the interior of the metal receiving heat from the skin by thermal conduction. In the induction melting-furnace, this conduction is obviously an essential part of the process. In surface-hardening it is necessary to heat the skin to the required temperature so quickly that a minimum of heat is transferred to the interior of the work; this necessitates the use of high-power H.F. generators, usually of inductor-alternator or valve type.

We see, therefore, that for a cylindrical charge, the current-path at high frequencies becomes a thin surface shell; the interior of the cylinder then has no effect upon the electrical behaviour of the system. The effective resistance and reactance of the charge are therefore those for the surface shell only. The magnetic field linked with the shell will be less than that which would be associated with the solid cylinder in an unchanging field (i.e. zero frequency), so that the charge inductance decreases somewhat with increasing frequency, though the reactance continues to increase. The charge-resistance obviously increases as the shell thickness decreases, i.e.

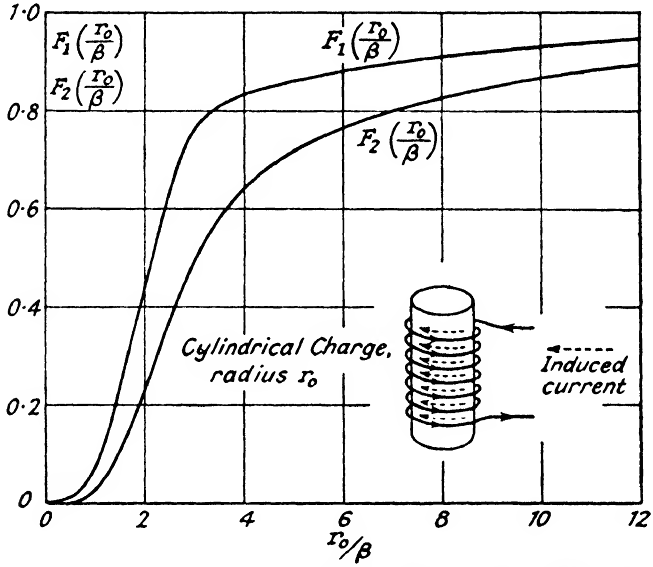
* See p. 189.

as frequency increases, and can be calculated from the current-path length and the skin thickness.

Coupled resistance and reactance

Burch and Davis have developed the mathematical theory[23] for various kinds of load or charge, and shown that the coupled resistance, in the case of a long coil and a long cylindrical charge, is:

$$R_{\text{coupled}}=2\pi\sqrt{2\pi}\,r_o\sqrt{\omega\rho\mu}\,n^2l.F_1\left(\frac{r_o}{\beta}\right)\times10^{-9}\text{ ohms} \quad 6{\cdot}10$$



[*Courtesy of Messrs. Ernest Benn Ltd.*

Fig. 6·5. Coupled Resistance and Reactance Functions for Long Coil and Charge.

where r_o = outer radius of cylindrical charge, cms,
ρ = resistivity of charge in E.M.U.,
$= 10^9\times(\rho$ in ohm-cms),
n = turns/cm axial length of inductor,
l = axial length of charge in cms,

$F_1\left(\frac{r_o}{\beta}\right)$ = a function of $\frac{r_o}{\beta}$, represented by curve 1 in Fig. 6·5.

β, ω, and μ are as for equation 6·2.

The corresponding coupled reactance for a non-magnetic charge is:

$$X_C=[-4\pi^2r_o^2\omega n^2l.10^{-9}]\times F_2\left(\frac{r_o}{\beta}\right)\text{ ohms} \quad . \quad 6{\cdot}11$$

i.e. it reduces the effective inductive-reactance of the inductor or work-coil (see also equation 1·85).

It is seen from Fig. 6·5 that the coupled resistance rises rapidly with increasing values of $\frac{r_o}{\beta}$, up to $\frac{r_o}{\beta} \doteqdot 3$; beyond this point the increase is not great. This is analogous to $\frac{\omega L_2}{R_2}=3$ in Fig. 6·2.

For values of $\frac{r_o}{\beta} \gg 3$, the coupled resistance may be taken as being approximately equal to the limiting value, i.e.

$$\left.\begin{aligned} R_C &\doteqdot 2\pi\sqrt{2\pi}\, r_o\sqrt{\omega\rho\mu}\,.\,n^2l\times 10^{-9} \text{ ohms} \\ &= 4\pi^2 r_o\sqrt{f\rho\mu}\, n^2l\,.\,10^{-9} \text{ ohms} \end{aligned}\right\} \quad . \quad 6{\cdot}12$$

These values are for a long coil and charge. In melting practice it is usual for the coil length to be made approximately equal to its diameter, while some heat-treatment coils are very short indeed, consisting of one or two turns of large diameter. Correction factors have then to be applied to equations 6·10, 6·11, 6·12. The corrections for R_C are dealt with on page 187; the correction for X_C is to multiply equation 6·11 by the value of K (Fig. 1·5), corresponding to the ratio of charge diameter to length (within the coil).

The power developed in the charge

For the cylinder of Fig. 6·3, at very low frequencies, the power developed in a thin shell is:

$$dP = i^2R \text{ watts}$$

$$= \frac{r^2t^2\left(\frac{dB}{dt}\right)^2.10^{-16}}{4\rho^2}\cdot\frac{2\pi r\rho}{t}$$

i.e. putting $t=dr$,

$$dP = \frac{\pi r^3 dr\left(\frac{dB}{dt}\right)^2.10^{-16}}{2\rho} \text{ watts.}$$

Integrating with respect to r between the limits $r=0$ and $r=r_o$, we have for the total power developed in the cylinder, per cm length:

$$P = \frac{\pi r_o^4\left(\frac{dB}{dt}\right)^2.10^{-16}}{8\rho} \text{ watts.}$$

If the magnetic field is produced by a long coil of n turns/cm, carrying a uniform current I ampères RMS, then $B=\mu H=\frac{4\pi\mu nI}{10}$, and $\frac{dB}{dt}=\omega B=\frac{4\pi\omega\mu nI}{10}$, whence

$$\underset{\omega\to 0}{P}=\frac{2\pi^3\omega^2\mu^2n^2I^2r_o^4 . 10^{-18}}{\rho} \text{ watts/cm. length} \quad . \quad 6{\cdot}13$$

Thus *at very low frequencies* the power developed in the charge is proportional to r_o^4, and dividing the charge up into a number of thin rods would reduce the heating. This is, of course, the reason for laminating the cores of transformers, etc. If the cylinder were replaced by x thinner rods, each of radius $\frac{r_o}{\sqrt{x}}$, the total power developed in the rods would be proportional to $\frac{r_o^4}{x^2}\times x$, i.e. to $\frac{r_o^4}{x}$; the heating would be reduced to $\frac{1}{x}$ of that for the original cylinder of radius r_o. The reduction in eddy-current loss in sheet laminations as their thickness is reduced is even more marked, the loss being proportional to the square of the thickness.*

We consider next a simple approximate treatment for the power developed in a long cylindrical charge within a long coil, the supply frequency being so high that the current penetration depth p into the charge is 1/10th the radius of the charge, or less.

The field-strength at the centre of the long coil, unloaded, is:

$$H=\frac{4\pi I_1T_1}{10l} \ (l=\text{coil length, cms});$$

and is independent of the coil radius.

When loaded with the charge, the field-strength at the centre is zero, since the magnetic field penetrates only the outer skin of the charge. Thus the charge ampère-turns neutralise the coil ampère-turns, and for a charge of about the same length as the coil we have:

$$I_2=I_1T_1.$$

I_1 and I_2 being coil and charge currents respectively.

The power developed in the charge is:

$$P=I_2^2R_2 \text{ watts}$$

where

$$R_2=\frac{2\pi r_o}{lp}\times\rho$$

$$p=\frac{1}{2\pi}\sqrt{\frac{\rho}{\mu f}},$$

* i.e. in a given total cross-sectional area,

(a) for n rods, rod radius $\propto \frac{1}{\sqrt{n}}$, and total loss $\propto \frac{1}{n}$.

(b) for n sheets, sheet thickness $\propto \frac{1}{n}$, and total loss $\propto \frac{1}{n^2}$.

and $\qquad I_2 = I_1 T_1\,;\; T_1 = nl.$

So that $\qquad P = I_1{}^2[4\pi^2 n^2 l r_o \sqrt{\rho\mu f}\,.\,10^{-9}]$ watts . . . 6·14

$\qquad = I_1{}^2 R_C$ watts.

This value for R_C is the same as that in equation 6·12 for values of $\frac{r_o}{\beta} \gg 3$.

Dividing equation 6·14 by the cylindrical surface area $2\pi r_o l$ and substituting $H = \cdot 4\pi n I$, the power generated per unit surface area of the cylinder is

$$\Delta P = \frac{H^2\sqrt{\rho\mu f}}{8\pi}\,.\,10^{-7}\ \text{watts/cm}^2,$$

as given in equation 6·8.

A generalised treatment giving the power developed in the cylindrical charge at any frequency involves the use of Bessel functions.(25) McLachlan has shown that the average power developed *per cubic centimetre* is approximately

$$\Delta P = 4\pi n^2 I^2 \mu\omega\,.\,F_3\left(\frac{r_o}{\beta}\right) \times 10^{-9}\ \text{watts/c.c.} \quad . \quad . \quad 6{\cdot}15$$

This expression is reasonably accurate for a long coil, or, alternatively, for a coil in which the internal clearance between it and the charge is small compared to the coil length. As before, n is the number of turns per centimetre length of the coil, and I is the RMS coil-current in ampères. The power function $F_3\left(\frac{r_o}{\beta}\right)$ is plotted in Fig. 6·6, from which it is seen that, for a given frequency, there is an optimum size of charge corresponding to maximum power per unit volume. This optimum occurs at $\frac{r_o}{\beta} \doteqdot 2{\cdot}5$. Alternatively, for a given size of charge there exists an optimum frequency,

$$f_{\text{opt}} \doteqdot \frac{6{\cdot}25\rho}{8\pi^2\mu r_o{}^2}\ \text{cycles/sec.}$$

For other than very small pieces constituting the charge, the "optimum frequency" given by this expression is often too low from another point of view, e.g. the inductor current required for a given power might be excessive. In such cases, frequencies higher than the "optimum" are used. An optimum frequency in the sense of maximum power per unit volume with given inductor ampère-turns is of interest mainly in those cases where the charge is to be uniformly heated throughout, e.g. in melting, and in heating of billets for forging.

The power factor of a loaded work-coil or inductor is low, usually not greater than ·3. Consequently the power factor is given sufficiently accurately from $\frac{R}{X}$, R being the total effective inductor resistance (mainly the coupled resistance R_C), and X the effective reactance, including the "negative reactance" coupled in due to the charge. Now R is proportional to $\sqrt{\omega}$ over the useful range of $\frac{r_o}{\beta}$, while X is proportional to ω; hence power factor is

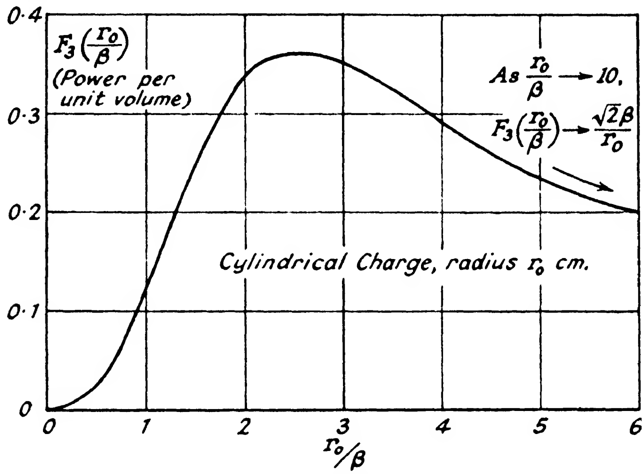


Fig. 6·6. Power Function for Cylindrical Charge. (*After McLachlan.*)[25]

proportional to $\frac{1}{\sqrt{\omega}}$. For economy in condenser correction KVA it is therefore desirable to use as low a frequency as is consistent with an efficient $\frac{r_o}{\beta}$ value, though condenser size and cost per KVA both increase as frequency is reduced. For surface-hardening and similar applications, however, the thickness of the heated skin is the important factor, and this determines the frequency to be used.*

Hollow cylindrical charge

Provided that the wall-thickness exceeds the nominal penetration-depth p, equation 6·14 is applicable. The total power developed

* There is an important distinction between alternator and valve-oscillator operation in respect of power-factor, for it is necessary to keep the oscillator Q fairly high—e.g. 20 or more, so that the power-factor of the inductive branch should be ·05 or less. There is no such restriction with an alternator supply, and the loaded-coil power-factor is made as high as is practicable.

in the hollow cylinder will then be the same as that in a solid cylinder of the same radius and material, under similar conditions, so that average power per cubic cm is correspondingly greater.

Burch and Davis have shown that an optimum frequency exists for through-heating a thin-walled cylinder(23):

$$f_{\text{opt}}=\frac{\sqrt{3}\rho}{4\pi^2R_it}\ \text{cycles/sec},$$

ρ=charge resistivity in E.M.U.,

R_i=inside radius of cylinder, cms,

t=wall-thickness of cylinder, cms.

This expression holds only for the condition $t \ll p$.

Vaughan and Williamson(66) give the following expression for optimum frequency for heating a hollow cylinder with wall-thickness not greater than $\cdot 13\times$outside diameter:

$$f_{\text{opt}}=\frac{\cdot 088\rho}{d_mt}\ \text{cycles/sec},$$

ρ=charge resistivity in E.M.U.,

d_m=mean diameter of cylinder, cms, $=(d-t)$,

t=wall-thickness, cms.

For thin-walled cylinders (and also for thick-walled and solid cylinders which are to be surface-heated only) it is convenient to use the conception of power developed per unit of surface area rather than power per unit volume. Such cases will, in general, correspond to values of $\frac{r_o}{\beta}>10$, for which $F_3\left(\frac{r_o}{\beta}\right)$ has approximately the value $\frac{\sqrt{2}\beta}{r_o}$.

Substituting this value in equation 6·14 and multiplying by the ratio $\frac{\pi r_o^2 l}{2\pi r_o l}\left(\text{i.e.}\frac{\text{volume of charge}}{\text{surface area of charge}}\right)$, we have for the power developed per unit area:

$$\Delta P=2\pi n^2I^2\sqrt{\rho\mu f}.10^{-9}\ \text{watts/cm}^2 \quad . \quad . \quad 6{\cdot}16$$

approximately. This assumes small clearance between coil and charge, i.e. close coupling. Corrections for larger clearances can be applied by multiplying equations 6·14, 6·15 and 6·16 by the factor $\left(\frac{H}{H_o}\right)^2$, the appropriate value of $\frac{H}{H_o}$ being read from the curves in Fig. 6·11. The result expressed in equation 6·16 is a particular case of the power relation given in equation 6·8, making the substitution

$$H = \frac{4\pi n I}{10}$$

which is approximately correct for a long coil and close coupling to the cylindrical charge. For surface-hardening of steel parts, ΔP must be of the order of 500-5000 watts/cm² (see Fig. 6·13).

Flat strip or plate charges[(69)]

The corresponding volume and surface power equations for flat strip or plate (Fig. 6·7) are:

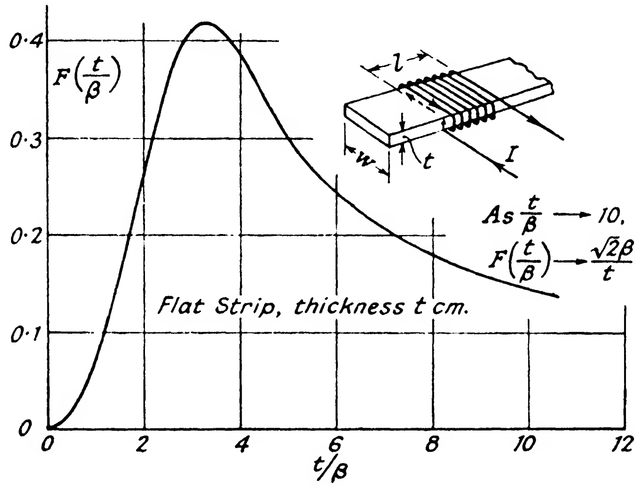


Fig. 6·7. Power Function for Flat Strip. (*After Baker.*)[(69)]

Average power developed per unit volume,

$$\Delta P/\text{c.c.} = 4\pi n^2 I^2 \omega \mu . F\left(\frac{t}{\beta}\right) \times 10^{-9} \text{ watts/c.c.} \quad . \quad 6{\cdot}17$$

where t=plate thickness in cms. The function $F\left(\frac{t}{\beta}\right)$ is plotted in Fig. 6·7, and is seen to exhibit a maximum in the region of $\frac{t}{\beta} = 3{\cdot}2$.

When $\frac{t}{\beta} > 10$, $F\left(\frac{t}{\beta}\right) = \frac{\sqrt{2}\beta}{t}$ approximately.

Power developed per unit of surface area:

$$\Delta P/\text{cm}^2 = \Delta P/\text{c.c.} \times \frac{\text{Plate volume}}{\text{Plate surface area}}$$

$$= \Delta P/\text{c.c.} \times \frac{lwt}{2(w+t)l}$$

$$\doteqdot \Delta P/\text{c.c.} \times \frac{t}{2}, \text{ when } t \ll w,$$

so that $$\Delta P/\text{cm}^2 = 2\pi n^2 I^2 \sqrt{\rho\mu f} \times 10^{-9} \text{ watts/cm}^2 \quad . \quad . \quad 6{\cdot}18$$

when the penetration depth $p \ll t$, i.e. $\frac{t}{\beta} \geqq 10$, and $t \ll w$.

Plane surface heated from one side by single straight inductor (Fig. 6·8)

An approximate expression for the total power developed at the surface, for the case of penetration-depth small compared to height of inductor above surface, is:

$$P \doteqdot \frac{l}{h} . I^2 \sqrt{\rho\mu f} \times 10^{-9} \text{ watts} \quad . \quad . \quad . \quad 6{\cdot}19$$

l=active length of inductor.

Most of this power is developed in the region immediately beneath the inductor (Fig. 6·8).*

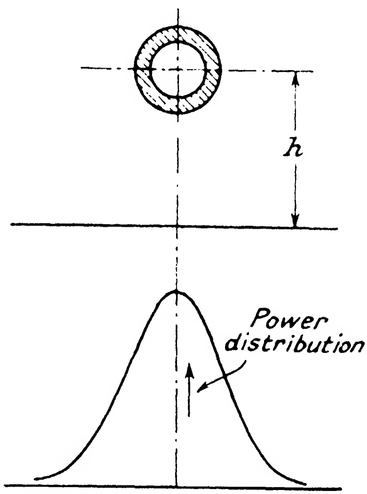


Fig. 6·8. Proximity Heating.

Work-coils or inductors[41, 42]*

The heating of many articles can best be carried out with a conventional type of coil of circular form; in particular, when that part of the article to be heated is itself cylindrical or near-cylindrical. Examples of such work are: shaft bearing-surfaces, circular cutters, gear-wheels, cams, screw-cutting dies, taps, reamers, tubes (internal or external surface treatment), and through-heating of round stock.

Other cases arise in which the shape of the article is so irregular that it is impossible to use a true coil at all; the inductor then becomes simply a current-carrying element of any convenient shape, placed as closely as possible to those parts of the article which are

* See also "Design Data for Induction Heating Coils", G. H. Brown, *Electronics*, August 1944.

to be heated. For a uniform hardened layer over such irregular surfaces a very high frequency is necessary. A typical example is the intricate internal surface of the trigger-mechanism housing of a rifle. Although the current penetration depth is greater at salient points on an irregular surface, such points actually heat more quickly than other parts of the surface, partly because of lower conduction heat-loss, and, in certain cases, closer proximity to the inductor.

Water-cooling of the inductor is essential in practically every application; this may involve the use of very small bore copper-tubing in the case of internal surface-treatment in confined spaces; more often it will be possible to use reasonably large tubing, e.g. $\frac{1}{4}''$ or more outside diameter, when water-cooling presents less difficulty.

Where cylindrical helix coils can be used, Babat and Losinsky[24] recommend an effective coil-length of 1/10 to 1/20 of the length of the conductor forming the coil, so that single-turn coils are preferable for hardening cylindrical parts over an axial length less than the diameter of the part. Multi-turn coils are better if the length of the heated part is 2 to 3 times the diameter. Fig. 6·9 illustrates how the proportions of the work-coil and the article affect the distribution of the heating, and hence the location of the hardened layer. The diagrams show examples of both internal and external coils, which may be of single- or multi-turn types. It will be observed that the clearance between the coil and the work may have a pronounced effect upon the heat pattern.

C. B. Kirkpatrick[41] has shown that where it is required that the magnetic field-strength produced by a circular helical coil at a point along the axis but beyond the end of the coil is to be a maximum, the radius of the coil for this condition is:

$$r_{opt}=(Z_O Z_1)^{\frac{1}{3}}\,(Z_O^{\frac{2}{3}}+Z_1^{\frac{2}{3}})^{\frac{1}{2}} \text{ cms} \qquad . \quad . \quad . \quad 6{\cdot}20$$

where Z_O=axial distance from the point to the near end of the coil, cms.

Z_1=axial distance from the point to the far end of the coil=Z_O+length of coil, cms.

For field distributions within the coil, see Fig. 6·14.

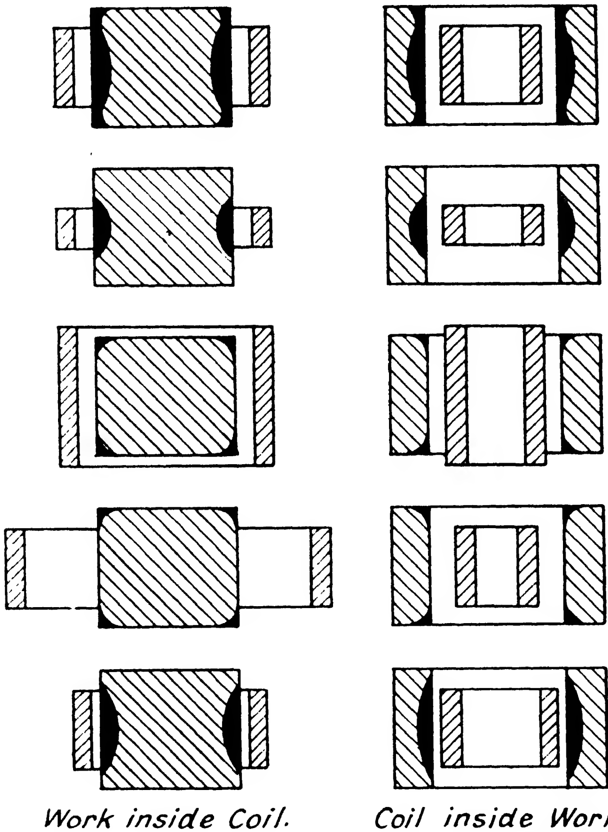
Similarly, for maximum field-strength at a point Z cms along the polar axis of a plane circular coil,

$$r_{opt}=\sqrt{2}\,Z \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}21$$

For heating large or small flat surfaces, plane spiral coils are used, generally of not more than 4 turns. Kirkpatrick points out that there is negligible difference in performance between equidistant and equiangular spirals. The approximate length of tubing required for winding either is:

$$S \doteqdot \pi DN + \frac{D}{4N} \text{ cms} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}22$$

N being the number of turns in the spiral, and D the overall



[*Courtesy of I.E.E.*

Fig. 6·9. Effect of Relative Proportions of Charge and Coil on Location of Heated Zones.

diameter in cms. The field-strength along the polar axis falls relatively slowly, as compared with that of a helical coil, at short distances from the plane of the coil. Two such coils, spaced coaxially a distance of about twice the radius of the smallest turn, will produce a reasonably uniform field between them, when carrying equal co-phased currents.

H.F. work-head transformers

It is often preferable to make the work-coil the secondary circuit of a transformer in order to obtain very high current concentration over a small region of the work-piece. High power concentration is especially necessary for surface hardening, and for hardened depths of the order of 1 mm or less a transformer is essential. The continuous progressive hardening method, which is described later, in conjunction with a "concentrator" transformer provides a means of surface-hardening large surface areas (of components having a regular section) with very modest generator power ratings.

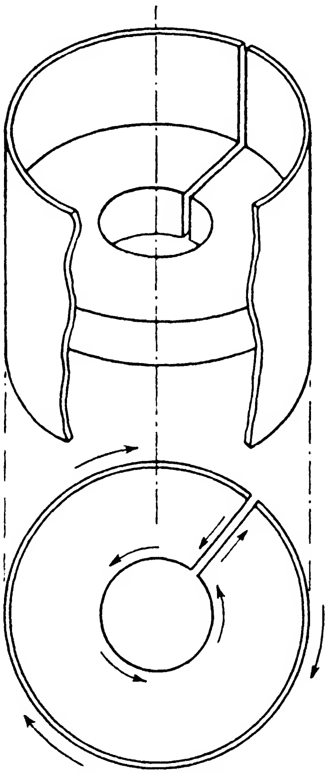


Fig. 6·10. Principle of "Current Concentrator" or "Focus Inductor" Transformer.

Laminated-steel-cored transformers are used to some extent, at motor-alternator frequencies, but the development of "air-cored" transformers, mainly for use with valve oscillators, has produced some remarkable and very ingenious "concentrators".

The principle is illustrated in Fig. 6·10.[43] A primary coil (not shown) surrounds a copper cylinder which is cut to form an open-circuit single-turn secondary. Internally across the centre of the cylinder (or at one end in many applications) is a disc, correspondingly cut. When the primary coil is energised the path of the skin current in the cylinder and disc is as indicated in the plan view of Fig. 6·10. The current, which is distributed more or less uniformly over the whole outer surface of the cylinder, is thus concentrated in the bore of the disc. A cylindrical load, such as a shaft, passed through the bore, has all the available generator-power concentrated over a small section of its length equal to the axial length of the disc. To realise maximum loading of the generator it is, of course, necessary to match the load correctly to it, and this is done by proper choice of primary coil turns.

Clearance between primary and cylinder should be as small as electrical insulation permits $\left(\text{e.g. } \frac{1}{16}'' \text{ to } \frac{1}{8}''\right)$. Clearance between the work and the disc is made as small as dimensional tolerances permit, and is commonly between ·020″ and ·040″.

It is normal practice to water-cool the cylinder, for example, by means of a cavity-wall construction, and also the disc, which is made hollow for the purpose. For some hardening applications it is convenient to make the quench-spray integral with the disc.

[*Courtesy of Birlec Ltd.*

Fig. 6·11. "Concentrator" Type of Work-head Transformer for Single-shot Local Hardening.

The "concentrator" is also used for many "single-shot" hardening jobs, where the stationary work-piece is heated and then either quenched *in situ* or drop-quenched. Fig. 6·11 shows a single-shot concentrator with its quench ring. This is one of several concentrators used in a 5 KW unit described and illustrated in Chapter 9 (Fig. 9·7).

Where a high current-concentration is required round a long, narrow slot, as, for example, in the hardening of knife-edge bearings, a cylinder of the form shown in Fig. 6·12 is effective.

Focus-inductors for brazing small parts are shown in Figs. 6·30 (a) and (b). Here an external secondary cylinder, with specially shaped water-cooled work loop, is slipped over an insulating sleeve covering the primary coil. Each of these units is rated at 3 KW. The rather complex-shaped work loop in Fig. 6·30 (b) is used for brazing the Bourdon gauge part shown.

Another application of the principle is seen in Fig. 6·31, and is used for a continuous soldering process employing a conveyor belt.[38] This is described in a later section.

Several types of focus-inductors are described and illustrated in U.S. Patent 1378187, filed by Dr. Edwin F. Northrup.[43] This remarkable patent, which was granted in 1921 (the original application date was 1918) should be familiar to all engineers concerned with induction heating, the development of which owes so much to this great pioneer.

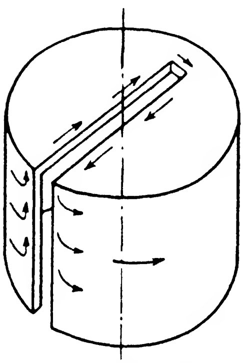


Fig. 6·12. Slot-type of Concentrator.

Shielding

It sometimes happens that the work-piece has one or two projecting parts which necessarily lie within the heating coil, and which must be protected against overheating. Such protection can be provided by placing a copper shield between the projection and the coil; alternatively it is sometimes possible to wind a reverse loop in one or more turns of the coil in the region of the projections to reduce the magnetic field-strength locally. A separate shield acts as a closed secondary circuit, in which the induced current locally weakens the field; a piece of copper tubing is often a convenient form for the shield, as it is easily water-cooled.

Inductor H.F. resistance and efficiency

The efficiency of the inductor is the ratio

$$\eta = \frac{\text{Power developed in charge}}{\text{Power input to inductor}}$$

$$= \frac{I_1^2 R_C}{I_1^2 (R_1 + R_C)} \quad \text{(see Fig. 6·1)}$$

$$= \frac{R_C}{R_1 + R_C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}23$$

It is most important that the inductor efficiency shall be as high as possible, for this influences the size of generator required. In addition, a highly-efficient inductor results in lower running costs, in both electric energy and cooling-water. H.F. energy is necessarily somewhat expensive, and yet it is not uncommon for much of the energy to be wasted in an inefficient inductor.

The only really satisfactory method of determining inductor efficiency is a calorimetric one, in which the total heat-energy developed in the work and the heat-energy given to the inductor cooling-water are measured. A simple treatment of the case for a long coil and long charge is given here, in order to indicate what factors influence the efficiency, and what order of efficiency it is reasonable to expect.

The high-frequency resistance of a coil may be calculated with reasonable accuracy by regarding the coil as a ribbon of thickness p, the current penetration depth given in equation 6·5. Assuming a close-wound ribbon-coil of N turns, inner radius r_{cms}, and length l $_{\text{cms}}$, the resistance is:

$$R_1 = \frac{L\rho_1}{pw} \text{ ohms.}$$

L = total length of "ribbon", $= 2\pi rN$, cms,

ρ_1 = resistivity of coil material, E.M.U.,

$p = \frac{1}{2\pi}\sqrt{\frac{\rho_1}{\mu f}}$; $\mu = 1$ for copper (see also Fig. 6·32),

w = "ribbon" width = diameter of tubing, or width of rectangular section,

$\doteqdot \frac{l_1}{N} = \frac{1}{n}$,

$N = nl$,

n = turns/cm axial length,

l_1 = axial length,

i.e.
$$R_1 = 4\pi^2 r n^2 l_1 \sqrt{\rho_1 f} \,.\, 10^{-9} \text{ ohms} \quad . \quad . \quad . \quad 6{\cdot}24$$

This expression agrees fairly well with values obtained in practice, provided that the actual wall-thickness of the inductor material exceeds the penetration depth (say $t > 1{\cdot}5p$). For high-conductivity copper at a mean temperature of 75° C, we can write:

$$R_1 \doteqdot 2\sqrt{f} r n^2 l_1 \,.\, 10^{-6} \text{ ohms} \quad . \quad . \quad . \quad . \quad 6{\cdot}25$$

The cooling-water temperature is, of course, less than the copper temperature; the latter can be ascertained by means of waxes of known melting-points, or temperature-indicating points.

For a turns space-factor F_S, the value for R_1 given in equation 6·25 must be multiplied by F_S, where

$$F_S = \frac{l_1}{Nw} = \frac{1}{nw}.$$

The coupled-resistance R_C due to the presence of the charge is, for the long coil and long charge, and for $\frac{r_o}{\beta} \gg 3$ (see equations 6·12 and 6·14),

$$R_C = 4\pi^2 n^2 l r_o \sqrt{\rho\mu f} . 10^{-9} \text{ ohms.}$$

The inductor efficiency is therefore:

$$\eta = \frac{r_o\sqrt{\rho\mu}}{r_o\sqrt{\rho\mu} + r\sqrt{\rho_1}} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}26$$

A simpler expression, from which the inductor efficiency is easily obtained, is the ratio:

$$\frac{\text{Power-loss in coil}}{\text{Power developed in charge}} = \alpha = \frac{r\sqrt{\rho_1}}{r_o\sqrt{\rho\mu}} \quad . \quad . \quad 6{\cdot}27$$

whence

$$\eta = \frac{1}{1+\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}28$$

Where charge and coil-lengths are unequal, but both may yet be considered "long" (e.g. length $\geqq 3 \times$ diameter) the above ratio will be approximately:

$$\alpha = \frac{r}{r_o}\sqrt{\frac{\rho_1}{\rho\mu}} . \frac{l}{l_1} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}29$$

For short coils and charges, the field-strength at the centre of the coil is not independent of coil radius, and the value of α is not simply proportional to the first power of the ratio r/r_o. Thus in the extreme case of a single-turn inductor of negligible axial length, the field-strength is inversely proportional to the radius of the turn, and so

$$I_2 = I_1 \frac{r_o}{r}$$

and

$$\alpha \longrightarrow \left(\frac{r}{r_o}\right)^3 \sqrt{\frac{\rho_1}{\rho\mu}} . \frac{l}{l_1} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}30$$

It is seen that close coupling between work and inductor is essential, with short inductors, if a high efficiency is to be realised.

The conductivity of copper is appreciably increased by annealing, and work-coils should preferably be annealed after winding. Annealing before winding is also necessary if the tubing used is initially in the hard-drawn state. High-conductivity copper is obviously desirable for all coils. Silver plating is an advantage at high radio frequencies, and is sometimes used for relatively low-frequency coils as a protection against the corrosion to which copper is so susceptible.

Approximate calculation of coil power factor, and the number of turns required—motor-generator equipments

The power factor of a loaded coil is always low, commonly of the order of ·05 to ·1, and rarely greater than ·3. Because of this certain approximations are permissible which simplify calculations without introducing serious error. These are:

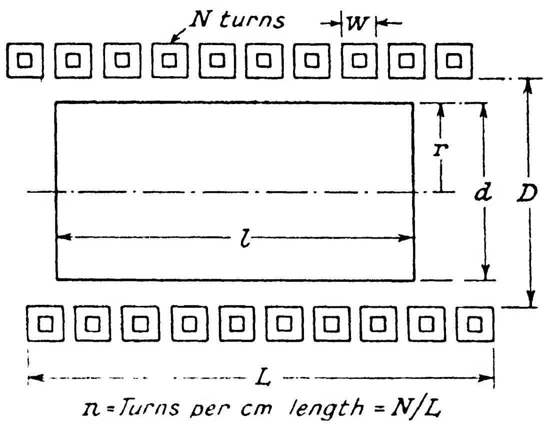


Fig. 6·13. Diagram for Equations 6·35 and 6·38 (Cylindrical Coil and Charge).

(*a*) In a parallel coil-condenser combination having unity overall power factor, the coil-current is, approximately:

$$I_{\text{coil}} = I_{\text{supply}} \times \frac{1}{\text{P.F. of coil}} \quad . \quad . \quad . \quad . \quad 6·31$$

(*b*) The power factor of the loaded coil is, approximately:

$$\text{P.F.} = \frac{R_e}{X_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6·32$$

(*c*) The condenser-current is approximately equal to the coil-current, and both are then given from:

$$I_{\text{coil}} = I_{\text{condenser}} = \frac{V}{X_e} \text{ ampères} \quad . \quad . \quad . \quad 6·33$$

R_e and X_e are the effective resistance and reactance, respectively, of the loaded coil; V is the coil supply voltage.

$R_e = R_1 + R_c$ (see equations 6·10 and 6·25).

$X_e = X_1 + X_c$ (X_c is negative for a non-magnetic charge; see equation 6·11).

X_1 is the reactance of the unloaded coil, and is ω times the coil inductance; i.e.

$$X_1 = \omega K_1 \pi^2 D^2 L n^2 . 10^{-9} \text{ ohms} \quad . \quad . \quad . \quad 6·34$$

(The symbols L, D are used here to denote coil length and inside diameter; l, d, and r denote charge length (within the coil), and outside diameter and radius of charge, see Fig. 6·13.)

Consider first non-magnetic charges, including steel at a temperature above its Curie point:

Substituting the appropriate expressions in equation 6·32, the power factor of the coil is seen to be:

$$\text{P.F.} = \frac{\left[F_a DL . 10^{-6} + 2\pi^2 d\sqrt{\rho} . l . F_1\left(\frac{r}{\beta}\right) . 10^{-9}\right]}{2\pi^3\sqrt{f} . 10^{-9}\left[K_1 D^2 L - K_2 d^2 l . F_2\left(\frac{r}{\beta}\right)\right]} \quad . \quad 6·35$$

$= \cos\phi$. K_1 and K_2 are found from Fig. 1·5 (*b*).

The coil efficiency, η, is conveniently found as the ratio of the second term of the numerator of 6·35 to the whole numerator. The total power-input to the coil is:

$$P = \frac{\text{Power to be developed in work}}{\eta} \text{ watts}$$

$$= VI_{\text{coil}} \cos\phi \text{ watts}$$

$$= \frac{V^2}{X_e}\cos\phi, \text{ (from equation 6·33)} \quad . \quad . \quad . \quad 6·36$$

So that

$$X_e \doteqdot \frac{V^2 \cos\phi}{P} \quad . \quad . \quad . \quad . \quad . \quad 6·37$$

$$= X_1 + X_c.$$

Substituting for X_1 and X_c the values given by equations 6·34 and 6·11, we obtain an expression for turns per centimetre length of coil:

$$n \doteqdot \frac{V}{\pi}\sqrt{\frac{\cos\phi \times 10^9}{\omega\left[K_1 D^2 L - K_2 d^2 l . F_2\left(\frac{r}{\beta}\right)\right]}} \quad . \quad . \quad 6·38$$

and total turns,

$$N = nL.$$

Coil-current, and condenser KVA required for unity overall P.F.

$$I_{\text{coil}}=\frac{P}{V\cos\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}39$$

$$\text{Condenser KVA} \doteqdot VI\times 10^{-3}=\frac{P}{\cos\phi}\times 10^{-3} \quad . \quad 6{\cdot}40$$

for unity overall P.F. (P is in watts).

The KVA rating of a given condenser is proportional to V^2f. The relation between condenser capacitance and KVA rating is:

$$C=\frac{I}{2\pi fV}\times 10^6 \text{ microfarads}$$

$$=\frac{\text{KVA}\times 10^9}{2\pi fV^2} \text{ microfarads.}$$

Corrections to R_c and X_c

(1) Correction for shortness of coil and radial clearance between coil and work(23):

Value of R_c given in equation 6·10 to be multiplied by $\frac{H^2}{H_o^2}$

where H^2=average square of magnetic field-strength over surface of charge.

H_o=field-strength within a long coil.

Graphs of the ratio $\frac{H}{H_o}$ are given in Fig. 6·14 for four ratios of coil diameter to length, and for various ratios of charge diameter to coil diameter.

(2) Corrections for shortness of charge:

(*a*) Value of R_c given in equation 6·10 to be multiplied by $\frac{1}{K_2}$

where K_2=Nagaoka's constant for the charge $\frac{d}{l}$ ratio (see Fig. 1·5 (b)). Burch and Davis(23) have suggested a factor $\frac{1}{K_2{}^2}$, but this, in the author's experience, tends to give values of R_c which are too high.

(*b*) Value of X_c given in equation 6·11 to be multiplied by K_2. This correction is included in equation 6·35.

(3) Corrections for magnetic charge:

(*a*) A figure for the *effective* μ to be included under the root sign in the numerator of equation 6·35.

(*b*) An additional $+ve$ term to be included in the expression for X_e, the effective loaded coil reactance, to take account of the flux in the permeable skin of the charge.

We have seen that this flux is equivalent to a flux of the same density as the surface value, distributed uniformly throughout a skin of depth β, and lagging by an angle of 45° behind the work-coil current. The "reactance component" of this flux, i.e. the component in phase with the coil-current, is therefore $\frac{1}{\sqrt{2}}$ of

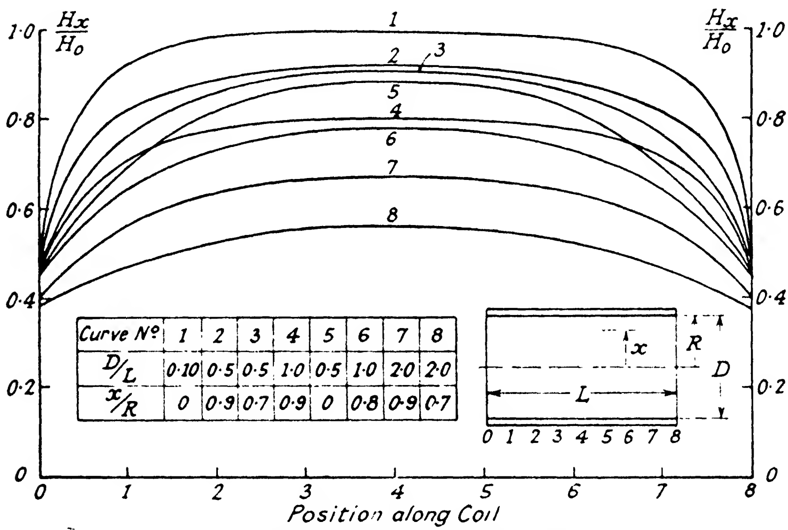


Fig. 6·14. Magnetic Field-strength in Short Coils.

the total flux in the skin of the charge, and if we replace β with $\frac{p}{\sqrt{2}}$ (equation 6·4) we have

$$\text{Reactive flux} = \frac{\mu H \times \pi d p}{2},$$

$$H = \cdot 4\pi n I.$$

Then the corresponding term to be *added* within the large brackets in the denominator of equations 6·35 and 6·38 is:

$$+\left(\frac{K_2 L d\sqrt{\rho\mu}}{\pi\sqrt{f}}\right)$$

The use of the factor K_2 is an attempt to account for the fact that this "skin flux" will not link with all the coil turns.

The coil power-factor, with a magnetic charge, is often quite high, e.g. ·3 to ·5, so that the approximation of equation 6·32 is not valid, and the relation

$$\text{P.F.}=\frac{R_e}{Z_e}=\frac{R_e}{\sqrt{R_e^2+X_e^2}}$$

must be used instead.

In nearly all applications of induction heating to magnetic materials, heating is carried beyond the Curie point, and it is usual in such cases to base coil calculations on the non-magnetic state, i.e. $\mu=1$.

The following empirical power-relations are useful in such cases:

(1) The initial loading (charge cold) is greater than the loading ultimately obtained beyond the Curie point, and rises rapidly to a peak of approximately twice this latter value.

(2) The loading beyond the Curie point is approximately 60% for surface hardening, or 80% for through-heating, of the *average* power throughout the heating cycle.

The work-coil voltage is assumed to remain constant.

The effective value of permeability μ can be estimated thus:

Most steels saturate at a flux density around 20,000 lines/cm², and for the saturated condition,

$$\mu_{\text{eff}} \doteqdot \frac{20{,}000}{\cdot 4\pi n I_{\max}} \doteqdot \frac{14{,}000}{\cdot 4\pi n I}.$$

The work-coil ampère-turns/cm, i.e. nI, are almost always great enough to saturate the steel during at least $\frac{4}{5}$ of the magnetisation cycle. If the approximate ampère-turns/cm can be estimated, and the magnetisation curve of the steel is available, it will be possible to estimate the effective μ fairly accurately; it usually lies between 20 and 40 for through-heating applications, and may approach unity at the very high field intensities used for surface-hardening.

Finally, it should be noted that for given coil and charge overall dimensions, and for a given frequency, the coil input power is proportional to $\frac{V^2}{N^2}$, so that, wherever practicable, a test should be carried out on an actual charge in conjunction with a roughly calculated coil. In this way an accurate determination of the correct number of coil-turns required can be made, and at the same time other essential design data can be collected. The coil voltage will be somewhat less than the rated generator (or output transformer) voltage,* because of the (mainly inductive) volt-drop in the leads, cables, and bus-bars. This drop should not normally exceed 10% of the rated voltage of the supply (see Chapter 9).

Cooling of work-coils and transformers

The heat generated in work-coils, or in the windings of work-head transformers, is taken away by cooling water fed through the tubular copper conductors forming the coils. "Focus" or "concentrator" type transformer secondaries, usually cylindrical, should have a cavity wall construction for water-cooling, with the possible exception of those used for continuous progressive hardening, where quench water can be used for cooling the cylinder.

Copper-loss power, and the rate of flow and temperature rise of cooling water are related thus:

$$P=\frac{G\theta}{3{\cdot}15}\text{ KW} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 6{\cdot}41$$

where P=power-loss in coil,
G=water flow gallons/minute,
θ=water temperature rise, ° C.

The maximum permissible temperature rise is usually reckoned to be about 44° C for an inlet temperature of 20° C; temperatures in excess of this may result in furring in the bore of the copper tubing.

In designing a coil it is necessary to check that sufficient water, at the available supply pressure, can be fed through it to carry away the coil-loss without excessive temperature rise. Two or more parallel water paths may be required; for example, water may be fed in at the centre of the coil, and taken out from each end.

The water flow which may be expected through a circular bore tube is given from:

$$G=12{\cdot}4d^2\sqrt{\frac{dP}{fL}}\text{ gallons/minute} \quad . \quad . \quad . \qquad 6{\cdot}42$$

since the flow is turbulent.[65]

d=inside diameter of tube, inches,
P=pressure drop along L inches of pipe, lb/sq. in,
f=friction coefficient, approximately ·011 for commercial copper tubing.

Where special sections are used (e.g. rectangular, or flattened from circular) it is a good plan to check the water flow through a given length at a known pressure (say, 40 lb/sq. in). The flow

* The phase-angle of the cable reactive volt-drop relative to the load voltage depends upon the load power-factor; a leading P.F. may cause the load voltage to exceed that of the generator.

through any length at any pressure can then be estimated, since it is proportional to $\sqrt{\frac{P}{L}}$. If the coil diameter is small, winding will probably reduce the flow considerably (perhaps even to one-half) and a flow-test on a wound coil is recommended in such cases.

The outlet water is sometimes taken to a cooler and recirculated instead of being run to waste. One advantage of this is the reduced risk of solid deposit within the coil tubing.

Coil insulation

There is no special problem in the electrical insulation of work-coils used with motor alternators, and such materials as hardwood, bakelite, Tufnol, mica, micanite, woven glass braid, tape, or sleeving, steatite, and alundum can be used according to the nature of the work. Normal precautions, such as total enclosure of live parts, must, of course, be taken.

The insulation problem is much more difficult in the case of coils used in conjunction with valve oscillators. Voltages from 1 KV to 6 or 8 KV at frequencies from 400 KC/S to 5 MC/S are normal. If bare coils are used, water from the atmosphere condenses on them, and flashover may occur. Water and steam from the quench system are also present in most hardening applications.

Complete embedding of the coil in wax, or even transformer oil, within a suitable non-metallic container, is possible in some cases. Alternatively, the coil may be wound from tubing which has a plastic covering extruded over it. Polythene is an excellent material for this purpose. Adequate water cooling is essential, to avoid damage to the insulation by heat. Insulating coatings which will withstand fairly high temperatures can be made with alundum cement, or a mix consisting of water-glass and magnesia, alumina or asbestos. Refractory coil-liners may be cast in a mix consisting of sillimanite or alumina powder and a silicon-ester (ethyl-silicate) binder.

Supporting structures for coils

The magnetic field outside the coil may be confined, either by means of packets of thin laminations of low-loss material such as nickel-iron or silicon-steel, or by using a closed cylindrical screen of copper* or aluminium. Where the field is thus confined the outer container or other structure may be of steel. Otherwise the amount of metal in the structure should be a minimum, and

* See, e.g., British Patent No. 392764.

preferably non-magnetic; coil boxes are commonly made with non-metallic top, bottom, and side plates (e.g. of wood, asbestos, or bakelite) contained by a skeleton framework of metal, the framework being arranged so that it does not form any closed loops. Where the $\frac{r}{\beta}$ or $\frac{t}{\beta}$ ratio of such metallic members can be kept small, i.e. less than 1, as in the framework of furnaces and heaters operating from motor-alternator sets, it is an advantage to use a high-resistance non-magnetic material such as austenitic stainless steel, nichrome, K-Monel, or No-Mag. But where $\frac{r}{\beta}$ is not small, as is almost inevitable at valve-oscillator frequencies, it is best to use a high-conductivity material such as copper, brass, or aluminium. These considerations also apply to any structural metal parts inside the coil, such as water-cooled skids and guides.

Melting furnaces arranged for pouring as well as melting *in vacuo* must be contained within a strong metal vessel; stainless steel is commonly used and if the furnace power is large, or the overall dimensions must be kept to a minimum, this casing may have to be water-cooled.*

In all cases, those metal parts of the structure which are accessible to an operator must be earthed.

The induction melting furnace (23, 28, 29, 30, 36)

The circuit arrangement is shown in Fig. 6·15 (a). H.F. alternators are generally used, although valve oscillators have been made for melting with power ratings up to 250 KW.(29)

The furnace coil is made from circular, oval, or rectangular section copper tubing, and water-cooled. Special solid drawn sections are sometimes used, with a cooling tube brazed on the outside (Fig. 6·15 (b)). Other interesting constructions are described in British Patent Specifications Nos. 283302, 347986, and 405913.

In large furnaces (e.g. 100 lb weight of charge and above) the coil is often tapped to provide control of power and of the electromagnetic stirring action during the melt (Fig. 6.15 (c)).

In small furnaces, and also for low melting-point alloys, a crucible is used to contain the charge. A rammed refractory lining, sintered in the furnace, is used in larger furnaces (e.g. above 100 lb). High-conductivity metals and alloys (copper, gold, aluminium, brass) are melted in conducting (graphitic) crucibles.

* See footnote to p. 221.

Any metal used in the supporting structure must be split up so as to avoid forming closed loops, or placed in positions where the magnetic field normal to the plane of the metal is weak. The field outside the inductor may be localised by means of packets of steel laminations, as in the Stobie furnace*; this construction makes it possible to use a closed metal container for the furnace structure.(30)

The furnace is mounted in trunnions for pouring, and is arranged

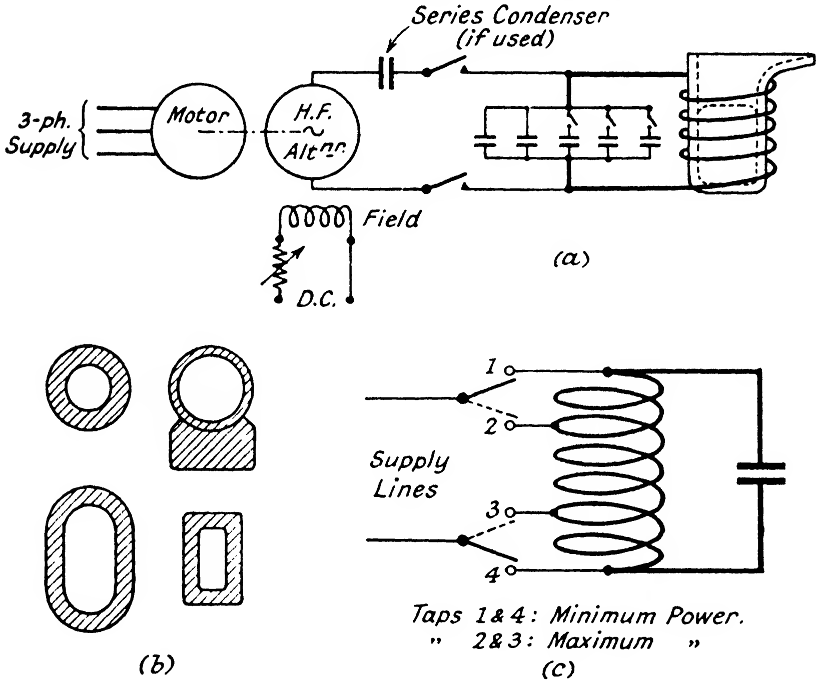


Fig. 6·15. (a) Circuit for Melting Furnace. (b) Typical Coil-tubing Sections. (c) Coil-tappings for Power Control.

to swing about an axis which passes through the lip, i.e. it is "lip-tilting"

Furnaces have been built with capacities up to 10 tons, and power ratings up to 1200 KW. The heat content of 1 ton of molten steel is approximately 325 KW-hours, and an overall efficiency of about 55% is realisable with a 1-ton furnace. The furnace efficiency is about 66%, the furnace losses in a particular case being: coil copper-loss, 13·5%; heat-loss through furnace lining, 6·8%;

* British Patent No. 405041; see also British Patents Nos. 376660 and 383058.

heat in lining, 5%; radiation and stray loss, 8·1%. In the case quoted the lining was already hot from a previous melt. The time to melt the charge of 1 ton was 75 minutes, during which the input power rose from 340 KW to 580 KW, and the furnace power factor averaged ·11. The cooling-water rate of flow was 16·8 gallons per minute, and the temperature rise increased steadily from 9° C to 22° C.[28]

The correction condensers used in conjunction with motor-alternator equipment are paper-dielectric, oil immersed, having a power factor of about ·002. The power-loss in these is:

$$P_c = (\text{Condenser KVA} \times \text{power factor}) \text{ KW}.$$

In the case quoted above the maximum condenser KVA was 4970; the maximum power-loss would thus be approximately 10 KW.

Part of the installed capacitance is arranged for switching in as required to maintain the overall power factor as close to unity as possible. This operation can be arranged for automatic correction, using a reactive-KVA relay, or alternatively condensers are manually switched to maintain approximately zero reading on a KVAR meter. Switching at reduced voltage is usual.

A 400-KW 1000 cycles/sec G.E.C. alternator set, and the condenser banks used in conjunction with the melting furnaces supplied from the set, are illustrated in Figs. 3·5 and 3·6.

Automatic control of alternator voltage, maintaining constant voltage under varying load conditions, is desirable. Where only one furnace is being supplied from an alternator, manual control of voltage can be used as a means of controlling furnace power.

Outline of the electrical design of a 200-lb steel-melting furnace

Power Considerations. The heat content of 200 lb of molten steel is approximately 30 KWH. Additional energy has to be supplied because of the coil-loss, the heat conducted through the lining from charge to coil cooling-water, heat given to the lining, and radiation and stray loss. The melting time will be assumed to be one hour.

It is usual to make the furnace proportions such that the ratio of height to diameter of the total molten charge is about 1·1 to 1·3; the dimensions chosen for this example are:

Height of molten charge	$11\frac{1}{2}''$ or 29·25 cms
Diameter of molten charge	$9\frac{1}{8}''$ or 23·2 cms.

The lining will be slightly tapered, the charge diameter being less than $9\frac{1}{8}''$ at the bottom and greater at the top. A lining thickness of $1\frac{3}{4}''$ (4·45 cms) will be assumed. The rate of heat loss through the lining, with the charge molten, will be:

$$H=\frac{KAt}{d}\ \text{KW},$$

where K=coefficient of conductivity, in KW/cm^2 of lining surface, per cm of thickness, per ° C temperature difference across lining.

$\doteqdot 1{\cdot}2\times10^{-5}$ for an average quartzite lining,

A=wetted area of lining, cm^2,

t=temperature difference across lining, ° C,

d=thickness of lining, cms.

In the case considered, when the charge is molten,

$$H=\frac{1{\cdot}2\times10^{-5}\times\pi\times23{\cdot}2\times29{\cdot}25\times1500}{4{\cdot}45}$$
$$=8{\cdot}65\ \text{KW}.$$

An average loss throughout the heat of about ·8 of this value, i.e. about 7 KW, will be taken, since the normal procedure is to commence a new melt with the lining already hot.

The weight of the heated lining is about 55 lb (density 100 lb/cubic foot). The mean temperature of the lining will rise from about 500° C to 800° C, and over this range its specific heat is ·28. This heat to the lining corresponds to about 8·7 KW-hours.

For a one-hour rating the power to the charge, the lining, and through the lining is therefore some 46 KW. If heat losses from the surface of the charge are minimised by using a slag and/or a lid, we can assume a stray loss of 4 KW, making a total of 50 KW; the power input to the furnace coil must then be $\frac{50}{\eta}$, η being the coil efficiency, which should average ·8 over the whole melt. A coil power input of 62·5 KW is therefore required to melt a charge of 200 lb of steel in one hour.

If this furnace is to be the sole load a 75 KW or 100 KW alternator (the nearest standard ratings) would be suitable.

A frequency of 3000 cycles/sec would enable charge pieces as small as $1\frac{1}{2}''$ to be heated efficiently; the coil design will be based upon this frequency and a power input of 60 KW to the molten charge. Higher power input can be obtained by using coil-tappings.

Coil Design. The values for the symbols in equation 6·35 are:

$$d=23{\cdot}2 \text{ cms}; \quad l=29{\cdot}25 \text{ cms}; \quad \frac{d}{l}={\cdot}793; \quad K_2={\cdot}742;$$

$$D=32{\cdot}1 \text{ cms}; \quad L=42{\cdot}0 \text{ cms}; \quad \frac{D}{L}={\cdot}765; \quad K_1={\cdot}75.$$

Coil space factor, $F_S=1{\cdot}2$ (assumed).
Molten charge resistivity, $\rho=200{,}000$ E.M.U.
Penetration depth, $p=1{\cdot}4$ cms, see Fig. 6·32.

$$\frac{r}{\beta}=\frac{\sqrt{2}r}{p}=11{\cdot}7; \quad F_1\left(\frac{r}{\beta}\right)={\cdot}95; \quad F_2\left(\frac{r}{\beta}\right)={\cdot}89.$$

Correction factors: Average $\left(\frac{H}{H_o}\right)^2={\cdot}53$; $\frac{1}{K_2{}^2}=1{\cdot}82$.

Substitution in equation 6·35* gives

$$\text{Coil power factor (charge molten)}=\frac{{\cdot}00162+{\cdot}0055}{3400\times 22{,}000\times 10^{-9}}$$

$$={\cdot}095 \text{ lagging.}$$

$$\text{Coil efficiency}=\frac{{\cdot}0055}{{\cdot}00712}\times 100=77{\cdot}3\%.$$

$$\text{Coil turns per cm length, } n=\frac{V}{\pi}\times\sqrt{\frac{{\cdot}095\times 10^9}{60{,}000\times 2\pi\times 3000\times 22{,}000}}$$

$$={\cdot}00062V,$$

where V=coil supply voltage
and total coil turns $=nL={\cdot}026V$.

The most convenient standard generator voltages are 500 and 800; the higher voltage makes possible rather finer adjustment of power by means of coil-tappings. Assuming an 800-volt supply, with 770 volts available at the coil terminals, the total coil turns required will be $770\times{\cdot}026\doteqdot 20$ turns. Tappings may be brought out from each of the first two turns at each end, and the change-over switch (Fig. 6·15 (c)) connected to suitable points as determined from tests. The maximum coil voltage will thus be in the region of $\frac{20}{16}\times 770=963$ volts, and consequently condensers should be rated for 1000 volts working.

Condensers. The condenser KVA required, based on 1000 volts rating, will be approximately:

$$\frac{60}{{\cdot}095}\times\left(\frac{1000}{770}\right)^2\doteqdot 1100 \text{ KVA};$$

* Including correction factors.

and the corresponding maximum furnace power will be:

$$60 \times \left(\frac{963}{770}\right)^2 = 94 \text{ KW}.$$

Tappings giving lower maximum power would, of course, be selected if the alternator rating were only 75 KW.

Cooling-water. The coil cooling-water is required to carry away about 16 KW of losses, at 60 KW input, or about 25 KW at the maximum rating; for this to result in a water temperature rise of not more than 30° C, the water-flow through the coil must be not less than

$$G = \frac{25 \times 3{\cdot}15}{30} = 2{\cdot}62 \text{ gallons/minute.}$$

A coil tubing section of $\frac{3}{4}''$ o/d, $\frac{1}{8}''$ wall, $\frac{1}{2}''$ bore can be accommodated, and the flow required can be obtained with a pressure drop of approximately 13 lb/sq. in., so that the coil need not be split into two sections for cooling.

Surface-hardening (24, 32, 33, 34, 63)

Remarkable progress has been made in the application of high-frequency power to surface-hardening during the past ten years, particularly in the U.S.S.R. and the U.S.A.

The surface of the metal is heated to the required temperature by induced eddy-currents to a depth δ, depending upon the nominal current-penetration depth p, the time for which power is supplied, and the thermal capacity of, and thermal conduction from, this hypothetical layer of depth p. Heating is in most cases accomplished in a few seconds, and is followed by a quenching operation; a water-hardening plain carbon steel (·35 to ·6% C) is normally used. A list of suitable steels is given in Chapter 9.

There are three principal ways of applying this method of heat treatment:

(1) The simultaneous treatment of the whole of the surface to be hardened, i.e. "single shot".

(2) The treatment of only one part of the surface at a time, e.g. "tooth-by-tooth" hardening of large cutters and gear-wheels.

(3) Continuous progressive treatment of a relatively small portion of the work, which moves at a constant speed through the effective zone of the inductor. It is, of course, immaterial whether the work or the inductor is arranged to move. The heated portion then passes into the quenching zone. Examples

are the surface-hardening of steel rails and shafts, and internal or external surface-hardening of tubes.

The non-uniform distribution of the induced eddy-currents was shown in Fig. 6·4. At any distance x within the metal from the surface (x being measured along the normal to the surface), the current has initially the value

$$I_x = I_o e^{-2\pi\sqrt{\frac{\mu f}{\rho}}.x} \text{ ampères, approximately} \quad . \quad 6{\cdot}43$$

I_o being the current on the surface.

The depth at which the current has fallen to $\frac{1}{e}$ of the surface value I_o (i.e. $\cdot 368 I_o$) is taken as the equivalent current-penetration depth p, and is

$$p = \frac{1}{2\pi}\sqrt{\frac{\rho}{\mu f}} \text{ cms} \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}44$$

Practically all the heat generated by the induced currents originates in this layer (average i^2 in layer$=(1-\cdot 368^2)I_o^2 = \cdot 86 I_o^2$). At temperatures above the magnetic change-point (recalescence or Curie point), $\mu = 1$, and an average value of ρ for steel is 100,000 E.M.U. at about 800° C. Hence at this temperature,

$$p \doteqdot \frac{500}{\sqrt{f}} \text{ mm.}$$

Before the change-point is reached, μ may have a high value*(27) in the case of many steels, so that the corresponding value of p is very much less than this. The increase in penetration-depth at the Curie temperature acts as a kind of safety-valve which minimises the risk of overheating the surface of the steel. It is, of course, unavoidable that the surface temperature slightly exceeds the correct hardening temperature in order to realise the latter at the required depth, but it is an interesting fact that, in this process, it is possible to over-heat the steel by some 250° C without causing a deterioration of its structure.† This is due to the fact that the heating process is of very short duration, a few seconds in most cases, and there is not time for appreciable grain-growth. The same amount of overheat during a conventional heat-treatment process would ruin the steel. Furthermore, this permissible overheat, in the H.F. process, may lead to greater hardness values being realised with certain types of steel.

* According to C. Dannatt, "μ is constant and independent of frequency up to at least 10^4 cycles/sec, and is definitely dependent upon frequency above 10^5 cycles/sec, and falls to a considerably lower value than the initial μ_o".(27)

† It is in fact necessary to exceed the "normal" hardening temperature in order to achieve the transformation to austenite in the very short heating-time; see footnote to p. 204.

The factors affecting the depth δ of the hardened layer have been mentioned; it is possible to achieve a selected value of δ in either of two ways:

(1) By making p approximately equal to, or slightly greater than, δ, and to arrange that the heating is so rapid that thermal conduction from the heated layer is negligible. This necessitates a high-power source.

(2) By making p very much less than δ, and allowing the heat to be conducted, from the layer in which it is generated, inwards to an effective depth δ. In this case the heating time is relatively long, and the power required correspondingly reduced.

(1) and (2) represent extremes; intermediate heating-cycles are, of course, available.

Losinsky recommends the following *minimum* frequencies for achieving a hardened depth δ mm[32]:

For smooth cylindrical shapes, $f_{min} = \dfrac{50{,}000}{\delta^2}$ cycles/sec . 6·45

For complex shapes, i.e. where the layer is to be developed along an irregular contour, $f_{min} = \dfrac{500{,}000}{\delta^2}$ cycles/sec . . . . 6·46

Higher frequencies can, of course, be used. The frequency does not appear to affect the metallurgical properties of the hardened layer. An objection to frequencies higher than strictly necessary is the inevitable increase in inductor-voltage. This should not exceed some 1000 to 1500 volts, because of the risk of break-down between the inductor and the work, especially when the latter is incandescent. It is important to note that the dielectric strength of air is considerably reduced at high frequencies, particularly if the "electrodes" have sharp points or edges.[60] The air-gap between the inductor and the work should be between 2 and 3 mm if possible, smaller clearances involve risk of breakdown (though strips of mica or other dielectric may be introduced where convenient), while larger clearances reduce the coupling unnecessarily. If a transformer is used, with a single-turn low-voltage heavy-current work loop, the clearance may be reduced to the minimum required by dimensional tolerances, e.g. ·01″ or ·02″.

Selection of the induction-heating cycle

Fig. 6·16 gives temperature-gradient diagrams[32] for the two ways referred to for achieving a hardened depth δ. Case (i) is for the rapid-heating method in which conduction plays little part, i.e. $p=\frac{500}{\sqrt{f}} \geqq \delta$. Case (ii) is for the "conduction-heating" cycle, in which $p=\frac{500}{\sqrt{f}} \ll \delta$.

The area enclosed by the temperature-gradient curves and the axes is a measure of the total heat-energy used. The amount of heat required for raising the layer δ to the hardening temperature T is proportional to the cross-hatched area.

The ratio:

$$\frac{\text{Total heat developed in work}}{\text{Heat required in layer } \delta}=K_T,$$

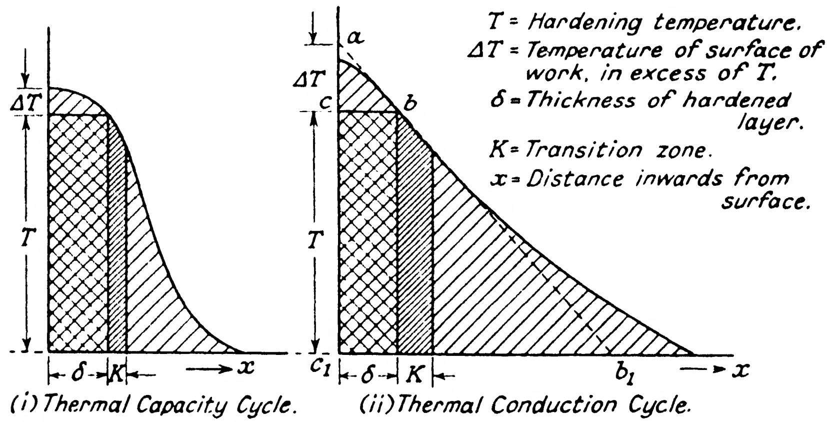


Fig. 6·16. Thermal Gradient for: (i) "Thermal Capacity (very rapid) Cycle" Surface-hardening; $p \doteqdot \delta$. (ii) "Thermal Conduction (very high frequency) Cycle" Surface-hardening; $p \ll \delta$.

the "coefficient of superfluous heat". When heating by the rapid cycle (Fig. 6·16 (i)),

$$K_T \doteqdot 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}47$$

The corresponding value of K_T for the slower (thermal conduction) cycle (Fig. 6·16 (ii)) is estimated by substituting the dotted line ab_1 for the actual gradient.

Then
$$K_T=\frac{\text{Area of triangle } ab_1c_1}{\text{Area of rectangle } T\delta}$$

$$=\frac{(\Delta T+T)X}{2T\delta}$$

$$X=\frac{T+\Delta T}{\Delta T}.\delta$$

from the similar triangles ab_1c_1 and abc,

whence
$$K_T = 1 + \frac{\Delta T}{2T} + \frac{T}{2\Delta T}$$
$$\doteqdot 1 + \frac{T}{2\Delta T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 6{\cdot}48$$

since $\frac{\Delta T}{2T}$ is very small.

Thus for a hardening temperature $T = 800°$ C, and an overheat ΔT of 200° C, $K_T = 3$, i.e. the total heat-energy used is three times that required for the layer δ. The broad transition zone is one consequence of this, and gives a more uniform distribution of the internal stresses set up by heating and hardening. It is seen from Fig. 6·16 (ii) that the temperature drop from the surface to the interior is more uniform than is the case with the first cycle.

Characteristics of the "thermal conduction" heating cycle, compared to the "thermal capacity" cycle are:

(1) Lower specific power in watts per square centimetre of surface to be treated. This power must, however, be applied for a longer time, and the total amount of energy used is greater.

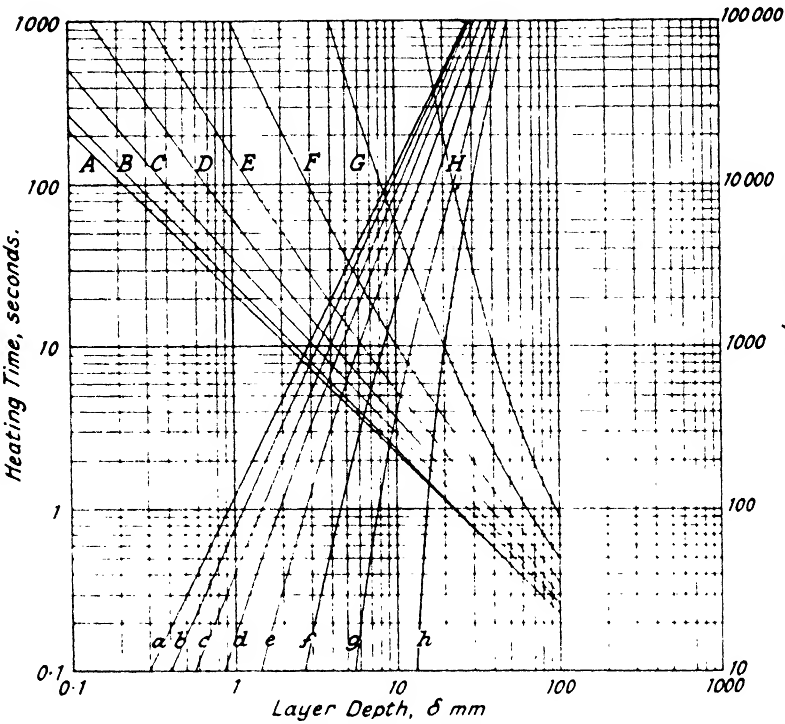
(2) Broader zone of transition from the hardened surface layer to the unchanged core.

(3) Easier and cheaper control of the lower generator power. For example, it is easier to control, accurately, 20 KW applied for 10 secs than to control, say, 120 KW for 1 sec.

The adoption of the first heating cycle, i.e. "thermal capacity" cycle, may, however, be imperative, e.g. for obtaining a very thin hardened skin (δ less than 1 mm) without using an excessive frequency. Thus, from equation 6·23, the minimum frequency for obtaining this depth in an irregular contour is 500 KC/S; a frequency considerably higher than this (at least 1000 KC/S) is necessary in order to apply conduction-heating from a thin surface layer. If such frequencies are not available, the only alternative is to use a high specific power, for a relatively short time, at the lower frequency.

Whichever heating cycle is adopted, or if some intermediate cycle is used, the energy required merely to heat the layer of depth δ to the appropriate temperature is always the same. For carbon steels this energy is about 4600 joules per cubic centimetre for an initial temperature of 20° C and a final temperature of 800° C. The corresponding energy-input to a steel article for a hardened depth of δ mm is, therefore, 460δ joules/cm² of surface heated. The total energy-input to the article, per cm², will be K_T times this value, i.e.

$$\Delta E = K_T \times 460\delta$$



Power Curves	A	B	C	D	E	F	G	H
Time Curves	a	b	c	d	e	f	g	h
Frequency, ∿/sec.	∞	10^6	10^5	5×10^4	10^4	5×10^3	10^3	10^2

Fig. 6·17. Relation between Hardened Layer-depth, Heating Time, and Specified Power Input to Work for various Frequencies. (*After Losinsky.*)[32]

and for a heating time of t secs, the specific power-input to the article is:

$$\Delta P = \frac{K_T \times 460\delta}{t} \text{ watts/cm}^2 \quad . \quad . \quad . \quad . \quad 6{\cdot}49$$

K_T will normally lie between 2 and 4, according to the heating cycle, being low for the "capacity" cycle, and high for the "conduction" cycle. The power-output from the H.F. generator must be somewhat greater, to allow for losses in the leads and inductor, and in heat radiation.

For a specified value of δ, the depth of the hardened layer, and a chosen frequency f, the heating time and the power required per cm² of surface can be obtained from the graphs given in Fig. 6·17, which are reproduced from Losinsky's article quoted under reference number (32). The curves are based on a hardening temperature of 850 to 900° C, and embrace the entire range of possible heating cycles.

Both horizontal and vertical scales are logarithmic, and the δ-time curves are very nearly straight lines for frequencies of 50,000 cycles/sec and higher. Hence, for such frequencies, the curves can be represented approximately by the equation

$$t = K\delta^n \text{ secs} \qquad (\delta \text{ in mm}).$$

The appropriate values of K and n are approximately:

f	K	n	$t=K\delta^n$ secs
$\gg 10^6$	1·0	2	$t=\delta^2$
10^6	·75	2·125	$t=\cdot 75\delta^{2\cdot 125}$
10^5	·4	2·3	$t=\cdot 4\delta^{2\cdot 3}$
5×10^4	·15	2·68	$t=\cdot 15\delta^{2\cdot 68}$

In the same way the "specific power" in watts per square centimetre of surface can be represented over the frequency range from 5×10^4 to ∞ by equations of the form:

$$\Delta P = h\delta^{-m} \text{ watts/cm}^2 \qquad (\delta \text{ in mm}).$$

f	h	m	$\Delta P=h\delta^{-m}$
$\gg 10^6$	2300	1·0	$\frac{2300}{\delta}$
10^6	2500	1·04	$\frac{2500}{\delta^{1\cdot 04}}$
10^5	3500	1·10	$\frac{3500}{\delta^{1\cdot 1}}$
5×10^4	6200	1·25	$\frac{6200}{\delta^{1\cdot 25}}$

The power-time graphs show that for a specified hardened depth more power must be supplied, but for a shorter time the lower the frequency used. For example, a hardened layer 2 mm deep requires 1250 watts/cm^2 for 3·4 secs at 10^6 cycles/sec, or 2500 watts/cm^2 for 1·2 secs at 5×10^4 cycles/sec. There is a small reduction in total energy dissipated in the work at the lower frequency.

Alternatively, for a fixed power-input, a smaller surface must be treated at the lower frequency, but for a correspondingly shorter time, to obtain a specified hardened depth. The area treated per second depends mainly upon the generator power, and to a lesser degree upon the heating cycle.

It seems reasonable to suppose that there will be a minimum permissible heating time for steel heat-treatment. Experience suggests that this is so, and that a heating time of at least ·1 sec is necessary; the previous heat-treatment of the steel is an important factor (see Chapter 9).*

The following example gives some indication of the values involved in a typical application, the surface-hardening of a small cam approximately circular in profile.

Cam diameter, ·75 in	$= 1{\cdot}9$ cms
Axial length, ·40 in	$= 1{\cdot}0$ cm
Surface area of cam	$= 1{\cdot}9 \times \pi \times 1{\cdot}0 = 6$ cm^2
Hardened layer depth	$= 1$ mm (·040 in)

The cam was hardened in a concentrator-type inductor, supplied from a 20-KW valve oscillator, with approximately 15 KW power-input to the work. The specific power was therefore 2·5 KW/cm^2, and the heating time was 1·0 sec. These values agree closely with those predicted from Losinsky's curves (Fig. 6·17).

The relation between work power, total surface area to be heated, and hardened depth, is summarised in Fig. 6·18. The four shaded zones correspond with hardened depths of 1, 3, 5, and 10 mm respectively. The upper limiting-line of each zone represents $f = 5 \times 10^4$, and the lower line $f = 10^6$ cycles/sec. The effect of frequency upon the power required becomes less as δ increases.

Continuous progressive surface-hardening(32)

Where very large surface areas are to be treated, as, for example, steel plates, rails, shafts, and tubes, simultaneous heating of the whole surface is impossible, and the obvious alternative is

* See also "Influence of Rate of Heating Steel on the Pearlite-Austenite Transformation, with special reference to Surface Hardening".—Eilender and Mintrop, *Stahl und Eisen*, Vol. 68, p, 83 (Feb. 26th, 1948).

continuous progressive heat-treatment. Losinsky has described applications of this method developed at the Svetlana Works, Leningrad, and the following notes are based upon his work, except where otherwise stated.

The set-up for surface-hardening a length of steel plate is shown in Fig. 6·19. The plate moves with a velocity v cms/sec in the direction shown, relative to the inductor. Beyond the inductor is

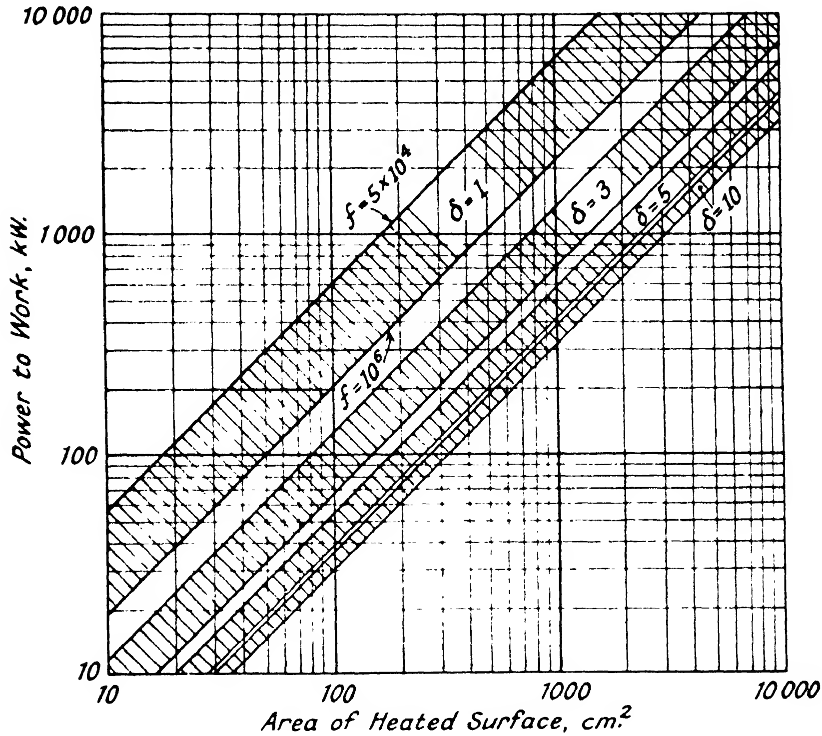


Fig. 6·18. Layer-depth, Surface Area, Total Power, and Frequency Relations. (*After Losinsky.*)(32)

the spraying-head for quenching the heated portion when this moves forward from the influence of the inductor. The depth δ of the heated layer at a given point gradually increases as the plate passes under the inductor. To obtain a uniform layer depth, the width of the inductor should be:

$$g \geqq (5 \text{ to } 10) \times \delta \quad . \quad . \quad . \quad . \quad . \quad 6{\cdot}50$$

To minimise the amount of "idle flux", the clearance between the inductor and the plate surface should be not greater than 2 to 5 mm, and may be as small as ·5 mm.

The velocity of travel of the plate, v, is given from

$$v = \frac{g}{t} \text{ cms/sec} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad 6{\cdot}51$$

t being the time, in seconds, taken to achieve the hardening temperature throughout the required layer-depth; t can be determined from the curves in Fig. 6·17. A range of ·2 to 10·0 cms/sec for v covers all requirements.

The distance m between the inductor and sprayer will depend upon the kind of sprayer used, but should not be less than 2 to 3 times the layer-depth, δ. It is sometimes possible to apply the quench-spray from the inductor itself (as is often done in the case

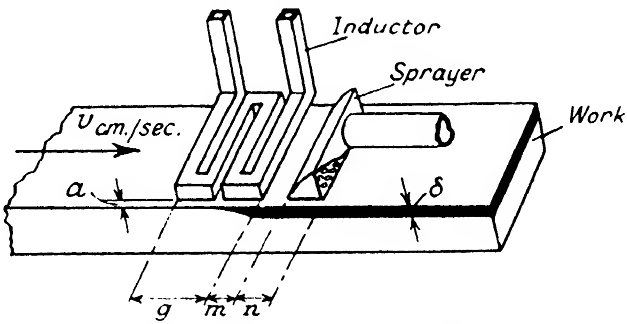


Fig. 6·19. Progressive Surface-hardening Set-up for Flat Plate. (*After Losinsky.*)(32)

of simultaneous total treatment of small objects), but in this case care must be taken that the spray cannot wet any part of the surface not as yet fully heated. This can be achieved by suitably focusing the spray, and by using shields.* The width of the sprayer, n, should be about 10 to 20 times δ, to cater for the fastest rate of travel of the work. The water pressure would normally be from 20 to 60 lb/sq. in.

The orifices are drilled in that surface of the sprayer nearest to the work-surface, and consist of 1·0- or 2·0-mm diameter holes, spaced at intervals of 4 to 5 mm and arranged in a series of intersecting diagonal lines. The water is preferably fed to the sprayer from more than one pipe, further to assist in obtaining a uniform hardened layer.

* An air-blast is sometimes useful for preventing water from flooding back into the heating zone.

The rise in work-surface temperature as the surface moves under the inductor is shown in Fig. 6·20 for three values of v, $v_1 < v_2 < v_3$, assuming the same specific power per cm^2 in each case.

The shaded zone represents the permissible degree of overheat, $\Delta T = T_2 - T_1$.

At the lowest velocity, v_1, the final surface temperature exceeds the permitted value; for v_2, conditions are satisfactory, while for the higher velocity, v_3, the hardening temperature is not reached before the surface leaves the inductor.

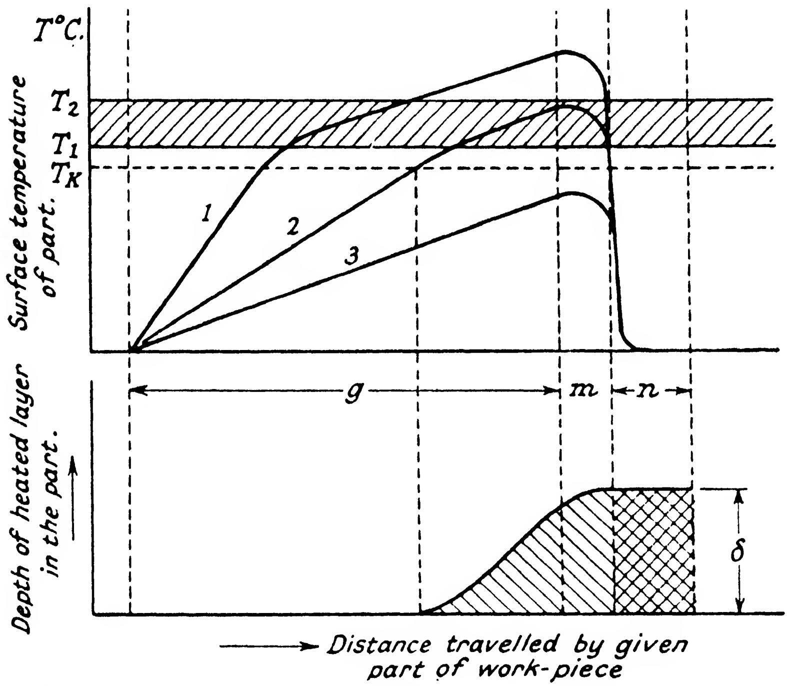


Fig. 6·20. Thermal Gradient along Flat Plate, and Hardened Depth. (*After Losinsky*.)(32)

The temperature T_K corresponds to the magnetic change point (Curie point); at this temperature the steel becomes non-magnetic, and the heating rate is then reduced, since the current-penetration depth is increased, and the current density thereby reduced. It has already been pointed out that this same effect reduces the risk of overheating the surface layer.

The lower curve in Fig. 6·20 shows the development of the heated and subsequently hardened layer, corresponding to the correct work-velocity v_2.

Similar curves would apply to the alternative condition of fixed velocity and variable specific power, curve 1 corresponding to maximum power, curve 3 to minimum power.

Warping of the plate, due to thermal expansion of the heated side, may occur where the plate thickness is less than ten times the hardened layer thickness, when only one side of the plate is heat-treated.

Prof. V. P. Vologdin has described apparatus developed in the V. I. Ulyanov (Lenin) Electrotechnical Institute in Leningrad, for the surface-hardening of steel rails by the continuous progressive system.(33) The depth of hardened layer required was 5 mm, for which a relatively low frequency (2000 cycles/sec) obtainable from

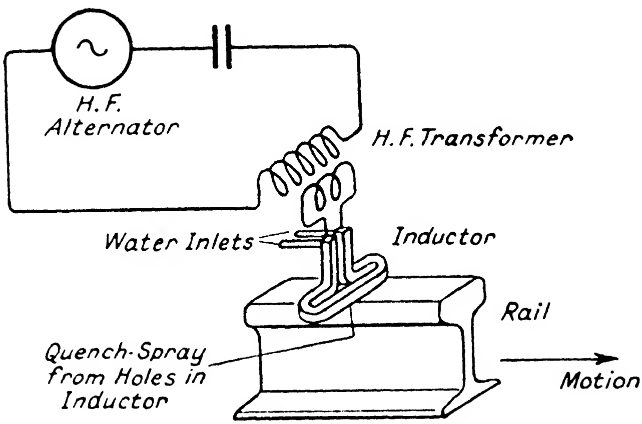


Fig. 6·21. Set-up for Surface-hardening of Rail. *(After Vologdin.)*(33)

an alternator was satisfactory. An efficiency of 80% was claimed for these alternators as compared to 50-60% for a valve generator; lower cost and simpler operation and maintenance were further factors in favour of the alternator. Fig. 6·21 shows the set-up for hardening the running-surface of a rail. A step-down transformer was used, since the inductor, consisting of only 1 or 2 turns, required a current of several thousand ampères at only 20 to 30 volts. This transformer was located as near as possible to the inductor; in the plant described, the hardening-head (inductor and transformer) travelled over the fixed rail. The ends of steel rails were also hardened by this method, a batch of ten or a dozen rails being treated together, and the two ends hardened simultaneously by means of two identical heads. These heads were mounted on carriers in lathe-beds, heating one rail-end at a time and then moving on to the next.

The development of steel-cored inductors, applicable to the heating of flat surfaces, is referred to by Prof. Vologdin; some of these laminated cores have been water-cooled, as have the cores of H.F. power transformers developed by the L.E.T.I. Using frequencies of a few kilocycles per second, the localisation of the inductor flux in this way seems a logical step. Further, recent developments in the manufacture of "iron dust" cores may prove to be of use in eddy-current heating inductors operating at the higher frequencies necessary for obtaining thin hardened layers

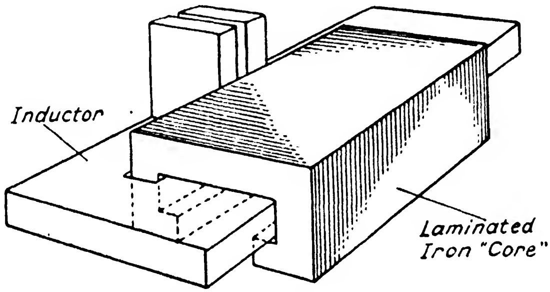


Fig. 6·22. Iron-cored Inductor.

(of the order of 1 mm). The principle of the laminated-steel core is illustrated in Fig. 6·22.*

Cylindrical work

For the continuous progressive hardening of the outer surface of a cylindrical article, e.g. a rod or a tube, the article is passed through a helical coil as indicated in Fig. 6·23. It is sometimes necessary to rotate the article at some 50 to 500 r.p.m. in order to avoid uneven heating due to possible eccentricity between inductor and work, and for uniformity in quenching.† The development of the hardened layer, and the relative power absorption at points along the length of the work, are shown in Fig. 6·23, for both stationary and moving work. As is also the case with flat plate, the power absorbed increases with temperature at first, due to the increased resistivity, but drops sharply at the Curie point. The hardened layer-depth will depend on work-velocity and specific power.

* See also British Patents Nos. 446111, 485966, 487365.

† Rotation speed should preferably be high enough for any point on the work-surface to make at least 2 or 3 revolutions during its passage through the inductor.

A peculiar effect has been observed[63] in the hardening of hollow pieces. If a number of pieces of similar material and given outside diameter are surface-hardened under identical conditions, the hardened depth is much the same as for a solid piece, until the wall thickness is reduced to about three times this depth, when there is a sudden increase in hardened depth, and it is, in fact, difficult to avoid through-hardening. The effect is presumably due to the absence of thermal conduction inwards, but the quite rapid increase in hardened depth is remarkable.

After passing through the inductor, the heated part is quenched

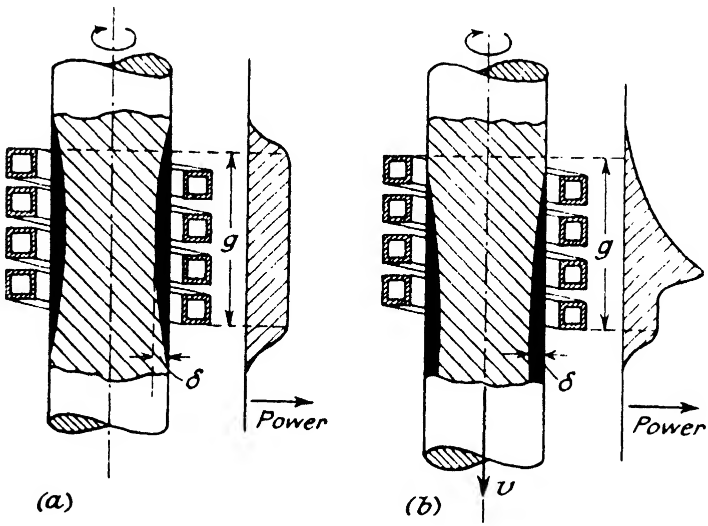


Fig. 6·23. Development of Hardened Layer and Relative Power Absorption for: (a) Stationary Work. (b) Moving Work. (*After Losinsky.*)[32]

by the sprayer (Fig. 6·24). Alternatively, if oil-quenching is to be used, the part moves down into a vat of oil. If necessary, the oil must be cooled by artificial means. Losinsky[32] describes the set-up for hardening in this way the surface of 0·5 % carbon-steel tubes, 30 mm outside diameter and 800 mm long. The inductor had 5 turns and an overall length of 70 mm, with internal diameter 38 mm, and was made from square-section copper tubing 10 mm × 10 mm × 1 mm wall thickness; it was water-cooled. The water sprayer, located 10 mm from the inductor, was fed at a pressure of 2 to 2·5 atmospheres through 3 pipes. The H.F. power frequency was 56,000 cycles/sec, the power being obtained from a valve generator, the electrical input to which was approximately 120 KVA. The hardened layer was 2 mm deep, its hardness being 60-61 Rockwell C. Transition zone width was 1·8 mm. The maximum deformation of the tube was only ·3 to ·4 mm.

A focus-inductor or "concentrator" is used for the progressive hardening of small parts such as gudgeon pins and plain shafts.

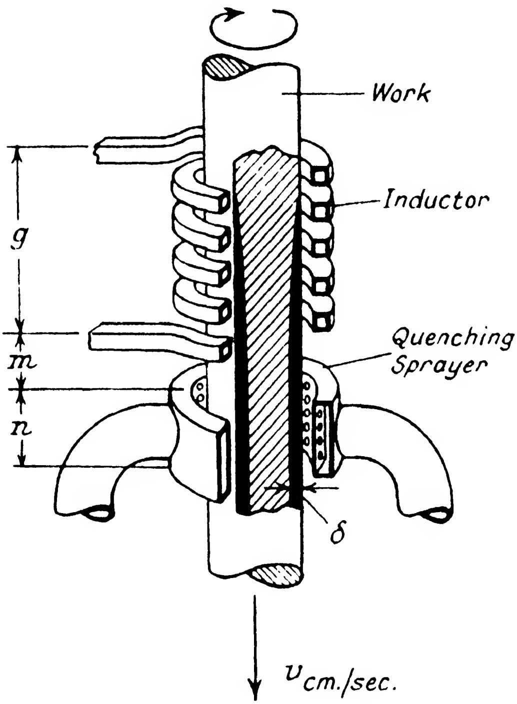


Fig. 6·24. Arrangement of Quench Spray Relative to Inductor. (*After Losinsky.*)(32)

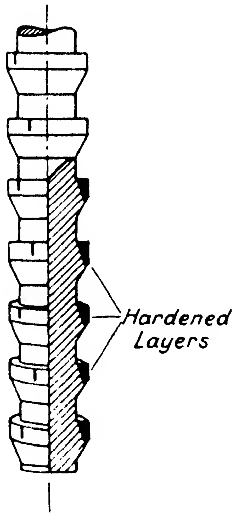


Fig. 6·25. Portion of Broach. (*After Losinsky.*)(32)

A typical installation is described and illustrated in Chapter 9 (see Fig. 9·4).

Grooved cylindrical work

The surface-hardening of steel bars and tubes having longitudinal grooves, e.g. splined shafts, can be carried out in the same way, but a steam-blanket formed in the groove may reduce the quenching rate and prevent proper hardening at the bottom of the groove. This can be countered by the use of a high-pressure spray.

Cylindrical articles having circumferential ridges, e.g. broaches, present an excellent application of this method of hardening. Maximum current, and hence maximum heat, are developed in the ridges (where the coupling to the inductor is greatest), which is precisely the condition required, since the cutting-edges of the broach are to be hardened while leaving the core of the shaft tough and strong (see Fig. 6·25).

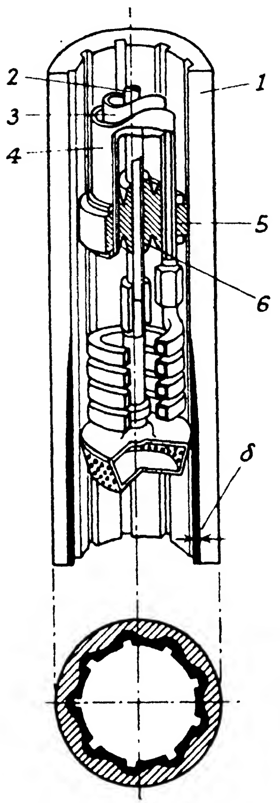


Fig. 6·26. Internal Hardening of Splined Tube, with Co-axial Feeder to the Inductor. (*After Losinsky.*)(32)

Hardening of inside surfaces of cylindrical work*

The internal surface of tubes and other hollow articles can be hardened by using a helical inductor and a quench-spray within the tube. The more convenient arrangement is to move the tube over the fixed coil and spraying-head, which are combined in a single unit, fed entirely from one end with electric energy, cooling water, and quenching water. It is necessary to employ a centring device to maintain clearance at all points between the inductor and the walls of the tube. Fig. 6·26 shows such a hardening unit, within an internally-splined tube, *1*. The long feeder consists of three concentric copper tubes, *2*, *3*, and *4*. Water is fed to the inductor and spraying-head through tube *2*, and the return water-path from the inductor is between tubes *3* and *4*, which are electrically connected. Tubes *2* and *3* are centred by means of the insulator, *6*; the centring of the unit within the work is accomplished by means of the collar, *5*. The clearance between inductor and work should not exceed 5 mm. The total inductor voltage will generally be not more than 1000 volts, and the inductor current several thousand ampères.

Losinsky's paper(32) illustrates an installation, housed in a tower, for the external and internal surface-hardening of cylindrical

* See also, e.g., British Patents Nos. 490514, 503130, 505073, 508027, 509895, 520479, 529927, 535202, 538241, 545122, 547579, 548551, 548970, 560189.

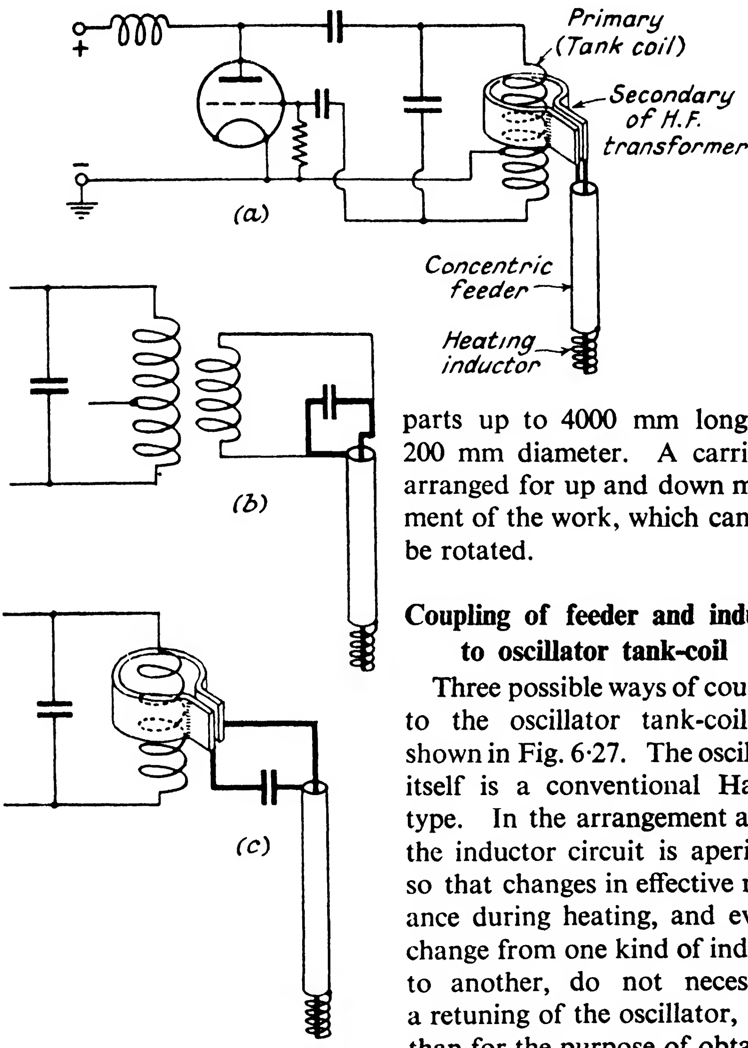


Fig. 6·27. Methods of Coupling Feeder and Inductor to Tank-circuit of Oscillator. (*After Losinsky.*)(32)

parts up to 4000 mm long and 200 mm diameter. A carrier is arranged for up and down movement of the work, which can also be rotated.

Coupling of feeder and inductor to oscillator tank-coil

Three possible ways of coupling to the oscillator tank-coil are shown in Fig. 6·27. The oscillator itself is a conventional Hartley type. In the arrangement at (a), the inductor circuit is aperiodic, so that changes in effective reactance during heating, and even a change from one kind of inductor to another, do not necessitate a retuning of the oscillator, other than for the purpose of obtaining a particular frequency. On the other hand, circuits (b) and (c) employ tuned inductor circuits, which necessitate careful tuning of the oscillator to give maximum inductor current. The advantage of arrangement (b) is that the coupling coil and the leads to the concentric feeder do not carry the full inductor current I, but only the power-component of it, which may be as little as 1/20 or even $1/50I$ (i.e. Q=20 to 50).

The inductor-circuit condensers are located as close as possible to the feeder input. The advantage of circuit (c) is that the voltage of the coupling loop is low, only $\frac{1}{Q}$ of the inductor voltage; at resonance, the condenser voltage is very nearly equal to the inductor voltage. All parts of this series-tuned inductor circuit carry the full inductor current. Circuit (a) is the most flexible, but (b) or (c) may be preferred for particular types of work involving only occasional changes in the set-up.

Through-heating

Through-heating by induction is being applied to forging work, continuous progressive through-hardening and tempering of bar stock, and continuous annealing of rods and tubes. The process is suitable for steel and for non-ferrous metals and alloys, although the overall efficiency of power conversion is necessarily lower for high-conductivity non-ferrous materials.

The application to forging has several advantages which offset the high capital cost of equipment; they are:

(1) Rapid heating, faster than by any other method.

(2) The relative absence of scale, due to the fast heating, results in longer life of forging dies, and gives a better finish.

(3) High electro-thermal conversion efficiency, the heat being generated in the metal itself.

(4) Greatly improved working conditions, since there are no fumes and only the workpiece is at high temperature.

(5) Small floor space required for heater units, again because of the rapid heating, and also because no thermal lagging is required.

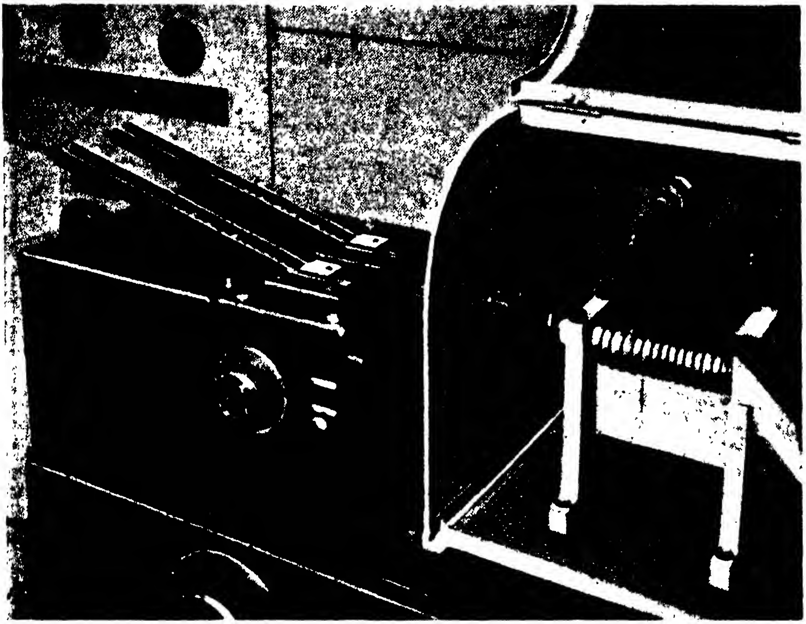
(6) The method is eminently suitable for automatic working.

An interesting small-scale application of heating for forging is illustrated in Fig. 6·28. This shows the feed mechanism and inductor for heating small steel slugs to 1050-1100° C. The slugs are fed automatically, at a selected rate, through two silica tubes surrounded by an oval helix coil. The generator is a 20-KW unit of the kind shown in Fig. 5·30. Each slug weighs approximately ·1 lb, and the unit delivers 20 slugs per minute to the forging press.

In 1943 the Ajax Electrothermic Corporation installed in the Youngstown Pressed Steel Division of the Mullins Manufacturing

Corporation what was claimed to be the largest plant of its kind in the U.S.A. The installation was for induction-heating bar stock, used for making 105-mm steel shells, and comprised eight 700-KW 960-cycles/sec motor-alternators, each supplying seven bar-heaters. Two of these heaters, and their correction condensers and control gear, are seen in Fig. 6·29.

Heating times and power ratings for through-heating are discussed in Chapter 9, and an automatic heater for bolt-heading is described. An outline of the electrical design of this heater is given below.



[*Courtesy of Birlec Ltd.*

Fig. 6·28. Feed Mechanism and Inductor for Heating Small Steel Slugs for Forging.

Specification. To heat $\frac{7}{8}''$ diameter steel bars, for a distance of $2\frac{5}{8}''$ from one end, to 1200° C, at a maximum rate of 800 per hour. Provision to be made for heating a longer length, $3\frac{1}{2}''$, at a corresponding lower rate. Power supply: 500-v 10-KC/S motor-alternator; assume 450 v available at the coil unit.

Consideration of heating rate and generator loading (see Chapter 9) indicate that a 6-coil unit will be required, giving a cycle time per coil of 27 secs, of which 25 secs will be usefully employed in heating the bar; automatic unloading and reloading occupies the remaining 2 secs. The 6 coils will be electrically in series.

The heat-content of the specified heated length of $2\frac{5}{8}''$ is 166 KW-secs; to this must be added conduction and radiation losses, estimated at 10%, making the total energy input to the work 183 KW-secs. Corresponding energy-input to coil, assuming 75% efficiency, is 244 KW-secs, so that the average power-input to each coil, for a heating time of 25 secs, is 9·75 KW.

[*Courtesy of Birlec Ltd.*

Fig. 6·29. Ajax Induction Heaters for 3″ diameter steel bars, 19″ heated length. Each 100 KW, 400 V, 960 cycles/sec. Heating time, 3 mins 52 secs. Note water-cooled condensers.

It is found that, to heat a length of $2\frac{5}{8}''$ of the bar uniformly, a total length of $2\frac{7}{8}''$ should be within the coil. Then the minimum length of coil to cope with a $3\frac{1}{2}''$ heated length is approximately 4″.

Other quantities required for calculation of coil turns and power factor are:

Coil. Inside diameter, $D=3{\cdot}34$ cms

Length, $L=10{\cdot}0$ cms

$$\frac{D}{L}=\cdot 334$$

$$K_1=\cdot 87.$$

Bar. Diameter, $d=2{\cdot}22$ cms

Length in coil, $l=7{\cdot}3$ cms

$$\frac{d}{l}=\cdot 304$$

$$K_2=\cdot 88$$

$$\rho=110{,}000 \text{ E.M.U.}$$

No correction factors are required for coil and charge shortness with these proportions.

Penetration depth $p=\cdot 5$ cm, whence

$$\frac{r}{\beta}=\frac{\sqrt{2}r}{p}=3{\cdot}14; \qquad F_1\left(\frac{r}{\beta}\right)=\cdot 77; \qquad F_2\left(\frac{r}{\beta}\right)=\cdot 5.$$

From equation 6·35 we have:

$$\text{Loaded coil P.F.}=\frac{\cdot 0000334+\cdot 000082}{6200\times 10^{-9}\times 81{\cdot}1}$$

$$\text{i.e. } \cos\phi \qquad =\cdot 228 \text{ lagging.}$$

$$\text{Coil efficiency} \qquad =\frac{\cdot 000082}{\cdot 0001154}\times 100=71\,\%.$$

And from equation 6·38, the number of turns/cm length of coil is:

$$n=\frac{450}{6\pi}\sqrt{\frac{\cdot 228\times 10^9}{9750\times 62{,}800\times 81{\cdot}1}}$$

$$=1{\cdot}62.$$

Total coil turns $N=nL=16$ approx.

The water-flow required for each coil, assuming 30% of total power-input will be taken away in the cooling-water, with 44° C rise, is:

$$G=\frac{\cdot 30\times 9{\cdot}75\times 3{\cdot}15}{44}=\cdot 21 \text{ gallons/min.}$$

The coil tubing section will be $\frac{1}{4}''$ square with $\frac{1}{16}''$ wall thickness; each coil will require approximately 7 ft of tubing. The pressure drop through each coil for the required water-flow will be approximately 10 to 15 lb./sq. in.

Total power-input to 6-coil unit $=6\times 9{\cdot}75=58{\cdot}5$ KW.

Coil-current $=\dfrac{P}{V\cos\phi}=\dfrac{58{,}500}{450\times{\cdot}228}=570$ ampères.

Total condenser KVA required, based on 500-v rating

$$=\left(\frac{500}{450}\right)^2\times 450\times 570\times 10^{-3}=317.$$

Test results obtained on the unit described in Chapter 9 were:

Coil efficiency (calorimetrically) $=72{\cdot}5\,\%$
Loaded power factor $={\cdot}218$
Installed condenser KVA $=315$
(Rated at 500 v, 9300 cycles/sec).

Examples of other applications of induction heating

Induction heating has been applied to brazing. An interesting recent example is in the manufacture of a type of steel tube.(37) Strip-steel is bevel-edged and then wound spirally on a mandrel, with copper strip interwound.* The spiral is passed through a high-frequency coil and heated to copper-brazing temperature (about 1120° C) in a reducing atmosphere. The brazed tube can then be drawn to any required diameter. The process is quick and continuous, and the tubes have high bursting-strength, since the grain runs spirally.

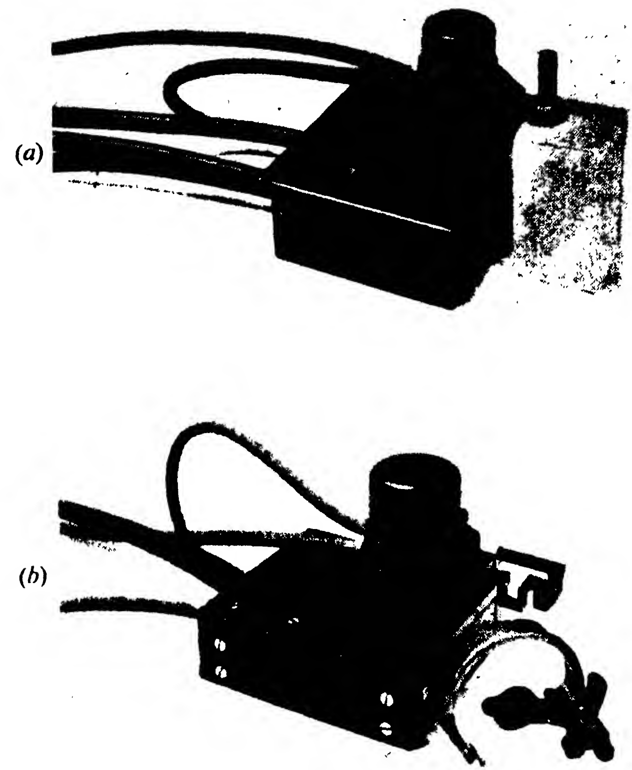
Another application is the brazing of tungsten-carbide tool-tips, reamer cutters, milling cutters, etc., to steel shanks or discs. The expensive carbide cutting-edge is placed in position with a thin shim of copper, brass, or silver-solder between it and the steel body of the tool. Heat is then generated in this region by induced eddy-currents. Normal silver-solder fluxes may be used, or the parts may be brazed in a reducing atmosphere such as pure hydrogen, cracked ammonia, or partially-burned town's gas from which water vapour and sulphur have been removed.(39) A controlled atmosphere is recommended for H.F. brazing applications where the higher melting-point brazing alloys are used.

Pancake or conical spiral coils are convenient for tipping lathe-tools; alternatively, a rectangular helix can be used. Copper tubing of about $\frac{3}{16}''$ or $\frac{1}{8}''$ outside diameter is usually necessary, in order to accommodate sufficient turns, unless a concentrator is

* See also British Patent No. 477526.

used. Cleanliness and accurate temperature control are the principal advantages of the H.F. method here.

Two set-ups for brazing small parts are shown in Fig. 6·30 (a) and (b); a description will be found in the section "H.F. Transformers". These concentrator units are designed to work with a small spark-gap converter.

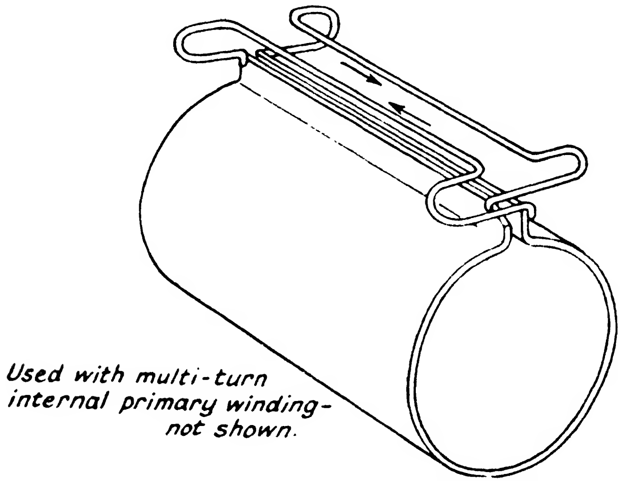


[*Courtesy of Birlec Ltd.*

Fig. 6·30. Ajax "Focus Inductors" for Brazing Small Parts.

Soft soldering with the aid of induction-heating[38] may offer real advantages over other methods, where large quantities of an article are to be mass-produced, despite the high first-cost of the equipment. The H.F. method makes possible the application of the correct amount of heat, correctly located, without risk of surface contamination, and without individual attention to each article during soldering. As a result, high production-speeds can be realised, and a uniformly high standard of product maintained.

An excellent example of the application of H.F. power to high-speed repetition soldering, recently described in the technical press,[38] is in the fixing and sealing of bases to rectangular cans containing condensers. The plant uses a conveyer belt which carries 2500 cans through the heating coil in one hour. The coil is ingenious, as in effect it embraces the can, while actually allowing the can to be moved through it, although the conveyer belt moves on a plane parallel to that of the "single-turn" coil. The principle is illustrated in Fig. 6·31 and is applicable to other conveyer belt



[*Courtesy of* "*Electronics.*"[38]

Fig. 6·31. Secondary of Work-head Transformer for Soldering Articles on a Moving Conveyor Belt.

H.F. operations, including rectangular and cylindrical work-pieces; rotation of the work-pieces may be necessary in the latter case.

Because of the low melting-temperatures of soft solders (180° C to 300° C), the specific power for such work is low. The equipment mentioned above is powered by a 2-KW oscillator operating at 400 KC/S: only one-half of this available output power is actually used for the quoted production rate.

The application of H.F. induction-heating to a process for separating magnesium, beryllium, calcium, and strontium from their ores and compounds is described in British Patents Nos.

466763, 469760, and 534811.(40) In this process the ore is mixed with a reducing agent and the mix heated, by induction, in a vacuum. After drawing off gases resulting from initial heating, the temperature of the mix is allowed to increase further, to promote reduction of the ore and distillation of the metal. When the reducing agent employed is not an electrical conductor, e.g. a carbide, the ore and reducing-agent in finely divided state are placed within or around a primer which can be heated by H.F. induction, and which then heats the mix by conduction. The primer is preferably a graphitic crucible in which the reduction process is carried on. Reducing-agents which are electrical conductors, and which may be used, are graphitic carbon, silicon, aluminium, calcium, or alloys of these elements.

In this process, H.F. heating results in a high thermal efficiency, because of the localisation of the heat, and also simplifies vacuum and distillation problems. The vacuum is maintained higher than that permitting a reverse reaction in which magnesium vapour recombines with oxygen from carbon monoxide generated in the process. A vacuum of ·01 mm is preferred, but must be kept below 2 mm in any case. Too low a vacuum (i.e. pressure greater than about 2 mm) is readily indicated by the appearance of myriads of luminous pin-points in the reaction-chamber, indicating recombination between magnesium and oxygen. Too high a vacuum, in relation to the induction frequency, produces a glow discharge in the metal vapour, and much of the electrical energy input is absorbed by this discharge. Typical reactions are*:

$$(1)\quad 2(MgO.CaO)+\tfrac{1}{6}FeSi_6 \xrightarrow[\cdot 005\text{ to }\cdot 5\text{ mm Hg}]{1100\text{ to }1175^\circ\text{ C}} 2Mg\nearrow+\tfrac{1}{6}Fe+2CaO.SiO_2.$$

$$(2)\quad 3(MgO.CaO)+2Al \xrightarrow[\cdot 005\text{ to }\cdot 5\text{ mm Hg}]{1050\text{ to }1175^\circ\text{ C}} 3Mg\nearrow+3CaO.Al_2O_3.$$

The reclamation of rubber and metal from scrap metal-to-rubber bonded articles is another recently developed process in which H.F. heating is employed. Eddy-current heating of the surface skin of the metal causes the rubber in contact with it to melt, and the rubber is then easily removed.

* For a review of vacuum techniques used in conjunction with induction heating see "Vacuum Metallurgy", R. A. Stauffer; paper published in the High Vacua Convention Supplement to *Chemistry and Industry*, Oct. 9th, 1948.

Localised annealing has also been carried out with the aid of H.F. induction-heating, a typical example being the treatment of the rim of a motor-car headlamp body pressing, prior to forming the rim to a rather complex section. Fig. 9·12 shows how the H.F. energy is supplied to the body pressing. The rim is annealed for a distance of about $1\frac{1}{2}''$ from the edge; the remainder of the

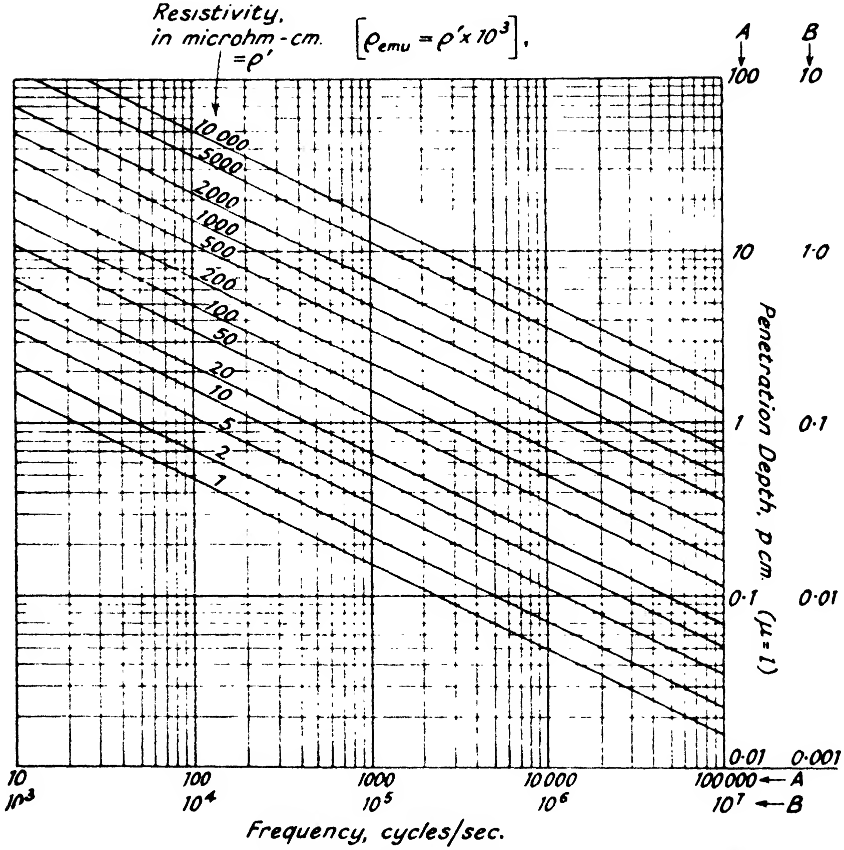


Fig. 6·32. Current Penetration Depths for Various Resistances and Frequencies.

body retains the work-hardness achieved during the drawing operation. A description of the unit built for this operation will be found in Chapter 9, in the section dealing with handling equipment.

The normalising and annealing of welded joints in steel pipes forms an excellent application of induction-heating. The coil is wound tightly over an asbestos cloth wrapped around the joint.

After the heat-treatment, the coil is unwound and, if of water-cooled tube, annealed before it is again used.*

Induction-heating of thin strip such as steel and aluminium for annealing, tube-forming, tin-plate flowing, etc., is attractive because the rapid heating obtainable allows the strip to be run at high speeds through "furnaces" of relatively short length. Strip materials in which the penetration depth is a small fraction of the strip thickness (say, $\frac{1}{3}$ or less), are conveniently heated in a flattened helical coil, as indicated in Fig. 6·7. The heating of aluminium and other high-conductivity strip, and of steel strip in the temperature range below the Curie point, can be done in this way; in particular, steel strip heating to 250° C, to cause electro-deposited tin to flow to give a bright finish, has been a very successful application in the United States (see, e.g., British Patent No. 568463).

Strip material in which the nominal penetration depth is approximately equal to, or greater than, the strip thickness, such as steel strip above the Curie-point temperature, can be heated efficiently by passing the strip between slotted laminated steel or nickel-iron pole systems wound with exciting coils, arranged so that the magnetic field is normal to the strip surface. The coils are supplied with energy from an A.C. source and circulating currents are generated in the plane of the strip instead of around the perimeter of the strip-section. The supply frequency required may be as high as 10,000 cycles/sec in the case of thin steel strip, while relatively thick aluminium strip may be heated at normal mains frequencies (see, e.g., British Patent No. 593195, and U.S. Patents Nos. 2448009 to 2448012 inclusive).

* Alternatively, heavy-current dry asbestos-covered flexible cable may be used. It is possible to use a mains-frequency supply for preheating for welding, and for stress-relieving after welding, if the required temperature is less than 600° C or so. See, e.g., U.S. Patent No. 2184534.

CHAPTER 7

Dielectric Heating

IT WAS pointed out in Chapter 1 (p. 15) that solid dielectric materials used in condensers were not loss free, and this is particularly so when a condenser forms part of an A.C. circuit, that is when the electric field in the dielectric is an alternating one. The effect of the power-loss, which results in heating of the dielectric, can be represented by either a series or a parallel resistance, as shown in Fig. 1·10 (a) and (b).

The supply current to either arrangement will lead the applied voltage by an angle less than 90°, and the component of this current which is in phase with the applied voltage is the power-loss component; the power-loss will be $VI \cos \phi$. In practice, $\cos \phi$ is

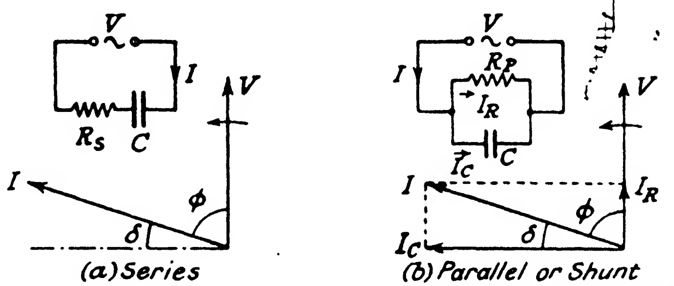


Fig. 7·1. Power-loss in a Condenser represented by: (a) Equivalent Series Resistance. (b) Equivalent Shunt Resistance.

small (rarely greater than ·1, and commonly between ·05 and ·0001), and it is convenient and usual to measure the small "loss angle" δ rather than the large angle ϕ (Fig. 7·1).

The power factor in either (a) or (b) is $\sin \delta$, and the power-loss is $VI \sin \delta$. When δ is very small, $\sin \delta \doteqdot \delta \doteqdot \tan \delta$, and the power factor is generally taken as $\tan \delta$. Strictly, in order that systems (a) and (b) shall both present exactly the same effective impedance to the supply, the two capacitances cannot have the same value, but provided that δ is small, they may be taken as equal.

The value of the equivalent series or parallel resistance is obtained thus:

(*a*) Series resistance:

$$\text{Power factor} \doteqdot \tan\delta = \frac{R_S}{X_C},$$

whence $$R_S = X_C \tan\delta = \frac{\tan\delta}{2\pi f C} \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}1$$

(*b*) Parallel resistance:

$$\tan\delta = \frac{I_R}{I_C} = \frac{X_C}{R_P},$$

whence $$R_P = \frac{1}{2\pi f C \tan\delta} \quad . \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}2$$

The equivalent series and parallel resistances can each be expressed in terms of the other by combining equations 7·1 and 7·2:

$$R_S = \frac{1}{R_P(\omega C)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}3$$

$$R_P = \frac{1}{R_S(\omega C)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}4$$

The heating of dielectrics in an alternating field is being used industrially in many ways; examples are:

(1) Pre-heating of bakelite powders prior to moulding.(44,45)

(2) Heating of laminated wood structures to set the glue; synthetic resin "glues" are being used in this process.(46,47,49)

(3) Pre-heating of rubber prior to moulding or curing.

(4) Dehydration of food-stuffs, textiles, pharmaceutical products, etc.

(5) Heating of thermo-plastic materials.

(6) Manufacture of bakelite laminated sheet.

(7) Softening of bakelite sheet prior to stamping, punching, etc.

(8) Cooking; for example, in the manufacture of cakes and biscuits.

Dielectric heating is now widely used for medical purposes, e.g. in the technique known as high-frequency diathermy.(52)

In addition, recent researches suggest the possible application of H.F. electric fields to the stimulation or the destruction of micro-organisms.(53)

Dielectric heating differs from eddy-current induction-heating in three important respects. Firstly, much higher frequencies are used—2 to 200 MC/S being the present range. 50 MC/S is a commonly used frequency for general purpose work, perhaps because satisfactory diathermy equipment has been available for low-power applications.

Secondly, the specific power per unit volume is much lower. This is because present-day applications are limited to the heating of materials to temperatures not usually exceeding 200° C.

Thirdly, the heat is generated in the body of the material, whereas induction-heating is essentially a surface effect.

These factors, frequency, specific power and heat distribution, are dealt with in the following paragraphs.

Specific power required for heating a dielectric

Consider a rectangular block of dielectric material, of volume Ad cm³ (Fig. 7·2) and of permittivity K.

The capacitance of a condenser consisting of this block sandwiched between two metal plates on the faces of area A, is:

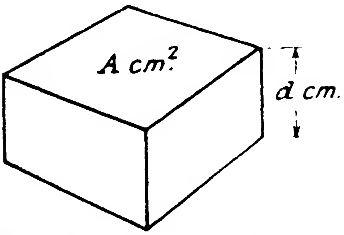


Fig. 7·2. Dielectric Block for Equation 7·5.

$$C=\frac{KA}{4\pi d}\cdot\frac{1}{9\times 10^{11}} \text{ farads.}$$

The condenser current, when a sinusoidal voltage V (RMS) is applied, is:

$$I=V\omega C \text{ ampères.}$$

So that the volt-ampères

$$VI=V^2\omega C.$$

The power dissipated in the dielectric is $VI \tan\delta$,

i.e. Power developed $=V^2\omega C\tan\delta$

$$=\frac{V^2\omega KA.\tan\delta}{4\pi d\times 9\times 10^{11}} \text{ watts} \quad . \quad . \quad . \quad . \quad 7{\cdot}5$$

The product $K\tan\delta$ is called the loss-factor of the material.*

The specific power developed, i.e. watts per unit volume, is:

$$\Delta P=\frac{V^2\omega K\tan\delta}{4\pi d^2\times 9\times 10^{11}} \text{ watts/cm}^3 \quad . \quad . \quad . \quad 7{\cdot}6$$

The power actually required to heat 1 cm³ of the material through a temperature range $T°$ C in t secs is:

$$P/\text{cm}^3=\frac{4{\cdot}2\,D.S.T}{t} \text{ watts} \quad . \quad . \quad . \quad . \quad 7{\cdot}7$$

D=density in gms/cm³,

S=specific heat in calories/gm/° C.

* The dielectric constant K varies with temperature, and it has been found, in the majority of cases, materials which have a high temperature coefficient of K also have a large loss-angle, and that the ratio of the two is practically constant. See M. Gevers and F. K. du Pré, *Philips Technical Review*, Vol. 9, No. 3, 1947.

Alternatively, the power required to raise the temperature of a mass of m gms weight of the material through $T°$ C is:

$$P=\frac{4{\cdot}2mST}{t} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}8$$

for a heating time of t secs.

For example, to heat 1 kilogramme of rubber from 20° C to 150° C in 1·5 mins, assuming a specific heat of 0·4, requires a power input to the rubber of approximately 2·5 KW. If the rubber were in the form of a block 12·6 × 12·6 × 6·3 cms (assuming a density of 1·0), i.e. $A=159$ cm^2, $d=6{\cdot}3$ cms, and taking $K=2{\cdot}2$, $\tan\delta={\cdot}02$, the voltage required to develop this power, from equation 7·5, is approximately 9000, for a supply frequency of 50 MC/S. The heating-circuit current would be approximately 1·4 ampères, and the KVA$=\frac{2{\cdot}5}{{\cdot}02}=125$. The power developed in the work is equal to that which would be dissipated in a series resistance R_S of $\frac{2500}{1{\cdot}4^2}=1275\Omega$, or a parallel (shunt) resistance $R_P=\frac{9000^2}{2500}=32{,}400\Omega$.

Reducing d, the volume remaining constant, enables V to be reduced in proportion, for the same load-power. The current would be increased, and the KVA would remain as before. Alternatively, for a constant voltage, the power is inversely proportional to d, and directly proportional to A and to frequency.

The upper limit for the applied voltage is set by electrical breakdown of the air between the plates or electrodes, by tracking across the surface of the material between the electrodes, and by corona discharge from the electrodes and leads. Breakdown within the dielectric material itself is generally unlikely to occur before one or other of these effects. The dielectric strength of air is lower at high frequencies than under conditions of steadily applied potentials.[(60)] Corona discharge can be avoided by rounding the edges of the electrodes and by using tubular leads rather than thin wire or strip.

Equation 7·8 gives the H.F. power required in the work; in practice the total H.F. power output from the oscillator will have to exceed this by about 20% to allow for losses in the tank-circuit and couplings.

An account of the mechanism of dielectric heating, in the light of modern theories, will be found in an article by Dr. L. Hartshorn, published in *Wireless World*, January 1945.

Distribution of heat in the work

The elementary treatment given for the dielectric block of Fig. 7·2 assumes uniform electric field distribution within the block. This is justifiable unless the dimensions of the block are of the order of $\frac{\lambda}{20}$ or more, λ being the wavelength of electromagnetic propagation in the material, *i.e.*

$$\lambda = \frac{30{,}000}{f_{\text{MC/S}}\sqrt{K\mu}} \text{ cms, approximately.}$$

Nearly all dielectrics are non-magnetic, so that, taking x equal to or less than $\frac{\lambda}{20}$ as the criterion, we have an empirical rule for the maximum frequency for reasonable field uniformity

$$f \doteqdot \frac{1500}{x\sqrt{K}} \text{ MC/S,}$$

x cms being the principal dimension of the block. Below this frequency the electric field distribution is substantially the same as for a steady field; the parallel-plate arrangement of Fig. 7·2 then gives a fairly uniform field throughout the dielectric. (See also Fig. 8·14.)

Equation 7·6 shows that the specific power is proportional to the square of the voltage-gradient, V/d; i.e. to the square of the electric field-strength. A uniform field ought, therefore, to result in uniform heating throughout the dielectric. Heat is usually lost by conduction from the outer skin, however, so that in most cases the interior is raised to a somewhat higher temperature than the surface. The internal temperature of rubber blocks and "preforms" can be checked by means of a thermo-couple encased in a small metal tube, rather like a bodkin, pushed into the material. Most dielectric materials are poor conductors of heat, and the readings obtained will usually be steady over a period of a minute or so; this is a useful property in one respect, since little heat is lost during the time of transferring the block to the mould.

In the older method of heating moulding material by conduction from the heated mould itself, it was not possible to obtain a uniform temperature throughout the material, those parts in contact with the mould becoming appreciably hotter than the interior. A combination of this method and H.F. pre-heating before placing the material in the mould can be made to produce practically perfect uniformity of temperature and hence of curing; this makes possible larger mouldings and reduced moulding pressures. A further advantage of H.F. pre-heating is that the greater part of the water content is driven off before the material is put into the mould, which need not then be opened for the purpose of "breathing" during the moulding operation.

In some applications of dielectric heating, heat is required to be generated locally and not uniformly throughout the article. Examples are found in the jointing of wood and of plastics. The electrodes are then shaped and positioned so as to concentrate the field as much as possible in the region where the heat is required. Also, where applicable, the introduction of materials having higher dielectric constants, or higher loss-factors, than those of the bulk of the article, will serve to localise the heat.

Power oscillators for dielectric heating

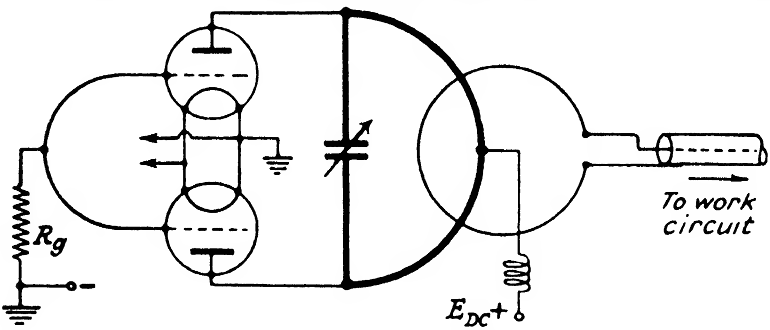
The frequency to be employed depends to some extent upon the size of the article to be heated.(70) The usual procedure is to determine the maximum permissible voltage without flashover, and then to calculate the appropriate frequency required to develop the necessary power in the work. For large articles, several feet long and a foot or so thick, "low" frequencies (2 to 5 MC/S) must be used. This is partly because of the difficulty of generating sufficient power at very high frequencies; partly because considerable power would be radiated from the leads and electrode system if higher frequencies were used; and partly because of the difficulty of obtaining uniform field distribution in the work when the electrodes are an appreciable fraction of a wavelength long. The latter point can, however, be compensated for to some extent by using matching stubs (see p. 247). Again, large articles will, in general, be associated with large capacitance, and then the inductance which would be required in conjunction with this would be absurdly small at very high frequencies.

Articles having dimensions of a few inches may be heated by sources operating at 10 to 50 MC/S, and at least one example has been described in the technical press in which a frequency of 200 MC/S was used, for the purpose of "spot-glueing" wood in a manner not unlike the spot-welding of metals, though the process was actually simpler, being performed from one side only of the parts being glued together.(50)

For frequencies up to 15 MC/S or so, power oscillators similar to those described in Chapter 5 are used, and most high-power valves are suitable. Special valves, and certain circuit modifications may be necessary for higher frequencies, and are essential at ultra-high frequencies (50 MC/S and above).

Push-pull oscillators are very suitable for the rather high frequencies required in dielectric heating, because of their inherent symmetry with respect to the cathode-point. Fig. 5·25, in Chapter 5, shows the circuit-diagram of a generator suitable for 5 MC/S or so. A push-pull oscillator circuit for 200 MC/S is illustrated in Fig. 7·3.(50)

Many applications of dielectric heating require only a low-power



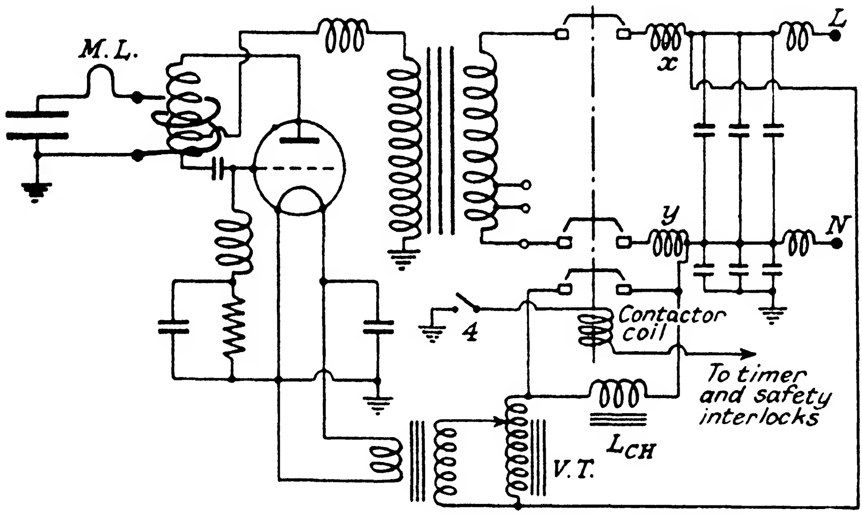
[*Courtesy of "Electronics"*.(50)

Fig. 7·3. Oscillator for 200 MC/S.

oscillator, e.g. ½ KW or 1 KW output. It is common practice in such cases to supply the oscillator with "raw A.C." ("self-rectifying oscillator"). Fig. 7·4 shows a circuit diagram of a system developed by the R.G.D. Company and B.I.C.C. Ltd., for making joints in polyvinyl-chloride cable covering.

The diagram illustrates several other interesting features, additional to the raw A.C. supply. The valve filament transformer is supplied from a Variac transformer, *V.T.*, giving fine control of filament-voltage. A series choke L_{CH} in the supply to the Variac reduces the filament voltage to roughly one-half its normal running value when plate-power is off; this choke is short-circuited by a pair of contacts on the main contactor during the operating period.

The work-circuit is shown inductively coupled to the plate-coil of the oscillator, which is basically a Colpitts type, with valve inter-electrode capacitances providing the divided condenser and cathode tap. The tuning capacitance consists of the valve inter-electrode capacitances and the capacitance reflected from the work-circuit. Thus no manual tuning of the work-circuit is required in this case. An alternative method of energising the work electrodes is to couple them directly to the plate-coil through condensers of relatively high capacitance (at least ten times the effective work capacitance; ceramic pot-condensers or mica condensers are suitable). Yet a third alternative is to earth the plate of the valve, and insulate the filament circuit, the D.C. potential of which is then below earth potential by the full amount of the H.T. supply



[Courtesy of R.G.D. Ltd. and B.I.C.C. Ltd.

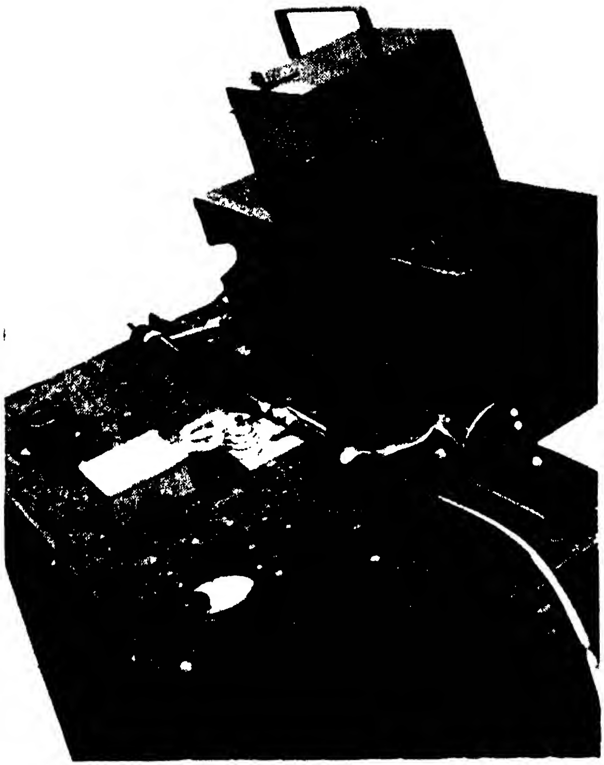
Fig. 7·4. Circuit Diagram of 450-Watt Self-rectifying Oscillator for Dielectric Heating.

voltage. The work electrodes may now be connected directly to the plate-coil, the earthed electrode being connected to the plate.

The meter-loop *M.L.* shown in the work-circuit is coupled to a meter-circuit comprising a screened pick-up coil, miniature diode rectifying valve, and moving-coil meter; this meter indicates the work-circuit current. Further details are given in Chapter 8.

An "all-wave" mains input filter is provided, to suppress the transmission of H.F. energy through the mains. Overload coils *x* and *y* are also included in the mains leads; excessive current in these coils causes the normally closed contacts *4* to open and so de-energise the contactor coil.

The work electrodes used with this oscillator for the jointing of P.V.C. cable covering are interesting. There are in effect three electrodes; a small flexible metal "finger" making contact with a strip of copper-foil wound round the joint; the metallic conductors of the cable itself; and the relatively long metal troughs in which lengths of cable either side the joint lie. Fig. 7·5 (a) shows the set-

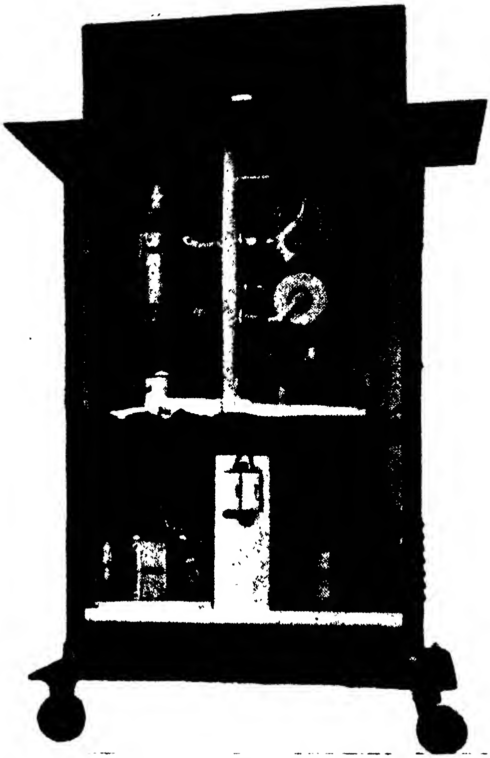


[*Courtesy of R.G.D. Ltd. and B.I.C.CC. Ltd.*

Fig. 7·5 (a). Dielectric-heating Unit for Making Joints in P.V.C. Cable Covering.

up, and the effective manner in which the electric field is concentrated at the joint. The technique employed is to leave a gap of about $\frac{1}{4}''$ between the two ends of the cable covering, and to fill this gap in with plastic P.V.C. This is prepared by dissolving a small amount of P.V.C.[61] in hot plasticiser and cooling off the solution to form a gel. The gel is mechanically mixed with a further quantity of powdered P.V.C., the resulting product being in a plastic putty-like state, convertible under heat and pressure to resilient P.V.C.

The region of the joint is covered with empire cloth and metal foil, and H.F. power at some 30 MC/S is applied for 30 secs. Under the action of the heat developed, a physical change takes place which converts the P.V.C. from the plastic to the resilient



[*Courtesy of R.G.D. Ltd. and B.I.C.C. Ltd.*

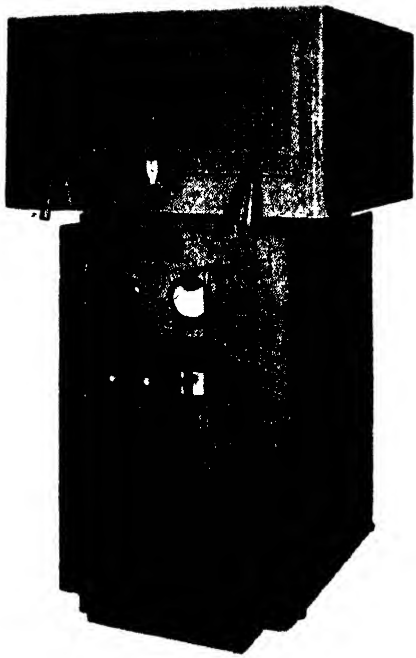
Fig. 7·5 (b).

state, the change consisting of the diffusion of plasticiser from the gel into the particles of unplasticised P.V.C. mixed with the gel.

The covering in the region of the joint is now homogeneous, and the metal foil and empire cloth are unwound and removed.

A rear view of the oscillator interior is seen in Fig. 7·5 (b). The timer (see also Fig. 8·7) and mains equipment are housed in the base of the unit; H.F. components are mounted on glass stand-off insulators on a cast aluminium frame alongside the oscillator valve.

A description of the ingenious dielectric-heating application, popularly known as the "Electronic Sewing Machine", will be found in British Patent Specification 573,518. The pieces to be joined, at least one of which must be thermo-plastic in nature, are placed together and passed between pressure-rollers which are also electrodes connected to the H.F. source.



[*Courtesy of Rediffusion Ltd.*

Fig. 7·6. Dielectric-heating Generator.

Fig. 7·6 shows a general view of a Redifon unit with a power-output of 2 KW maximum at 16 to 18 MC/S. This rating is sufficient to plasticise 1 lb of average wood-filled phenolic moulding powder preforms per minute, or to evaporate water from chemicals or other materials at some 6 lb per hour, or to heat about ½ lb of rubber per minute for moulding.

The unit illustrated is fitted with a large heating chamber which will accommodate multi-impression loading trays up to 21½″ × 19½″ × 4½″ maximum height. This chamber contains a pair of electrodes suitable for pre-heating moulding powders and preforms, rubber sheet or blocks, laminated board, and similar applications. Special electrode systems can be fitted for such applications as glueing and welding. The generator can be converted to eddy-current heating by replacing the electrodes with a conversion unit consisting of condensers and a pair of water-cooled terminals for the work-coil connection.

A process timer is incorporated in the oscillator unit. Thus for dielectric heating, the closing of the heating-chamber door, after reloading, automatically brings the top electrode down on to the work, switches power on, and starts the timer. The timer gives visual indication of the heating-cycle progress, and switches power off after a predetermined period.

The oscillator power supply is single-phase, with full-wave rectification providing D.C. to a pair of TYS 4·500 triodes. Cooling is by means of air circulated by a high-speed fan, the air being drawn in through glass-wool filters near the base of the unit. A thermostat control switches off the mains supply if the safe internal temperature is exceeded. A twin overload trip-circuit opens the main contactor if either valve is overloaded.

A control is provided for adjustment of output power. At full load (2 KW output) the input power is 3·6 KW; the consump-

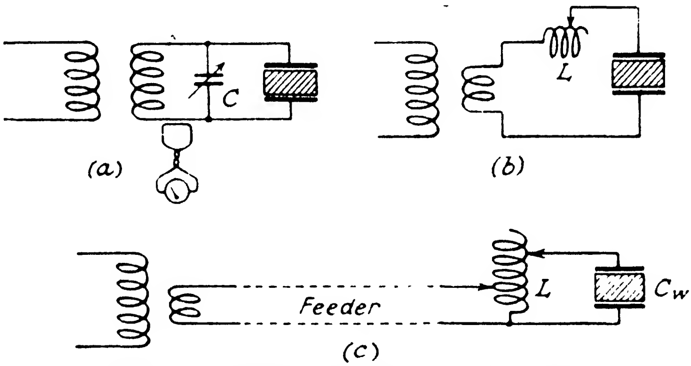


Fig. 7·7. Methods of Coupling Load to Oscillator.

tion at no-load is only 250 watts. The equipment is fully screened and mains filters are incorporated.

Dimensions of the oscillator unit are:

Width	20″ overall.
Depth	30½″ overall.
Height	38⅝″ overall, including terminals.
Weight	350 lb, approximately.

Coupling the load to the oscillator

The work electrodes and the work itself behave like a condenser in which power-loss occurs. It is usual to make this condenser a part of a resonant circuit, tuned to the oscillator frequency so that the maximum transference of power to the work is obtained. Three common coupling methods are illustrated in Fig. 7·7.

Methods (a) and (b) are suitable only for those applications in which the work is very close to the oscillator. The variable element (C in (a), L in (b)) is adjusted to give maximum current in the work-circuit, as indicated on either a thermal or rectifier meter in a small loop circuit placed near the coupling-coil.

When the work is some distance from the oscillator, as, for example, where a high-power unit, housed in a special room, feeds several heating jigs, a transmission line or feeder must be used. Fig. 7·7 (c) shows the principle involved.

The load-circuit would present an impedance, to a supply connected across the whole coil L, of:

$$Z=\frac{L}{C_w R}$$

assuming the supply frequency to be the resonance frequency of the load-circuit.

The actual impedance presented to the load-end of the feeder can be made any desired fraction of this value, by suitable selection of the tapping point along L; e.g. by tapping L half-way, the impedance is reduced to approximately one-quarter of the value given above. The higher the effective load impedance, i.e. the impedance presented to the feeder-line, the lower will be the line current, and hence the lower the line losses due to resistance, for a given load-power. The line voltage must, of course, be increased in proportion.

Tuning may be carried out by adjusting the oscillator frequency to that of the load-circuit, or preferably by tuning the work-circuit to a fixed oscillator frequency. For relatively low frequencies (up to, say, 3 MC/S, i.e. wavelength $\lambda=100$ metres), or which is the same thing from our point of view, for feeder lengths less than $\frac{1}{20}$ wavelength, any convenient kind of feeder with suitable voltage and current ratings may be used, and no special precautions are necessary. Where the feeder is relatively long $\left(\text{greater than } \frac{\lambda}{20}\right)$ its own impedance must be taken into account. Two cases will be considered: (*a*) Matched or un-tuned feeders; the tuned load is properly matched to the feeder, the combination behaving like a pure resistance at the feed-points. The current is uniform along the feeder. (*b*) Tuned feeders; feeder and load are tuned to resonance as a unit. Current distribution along the feeder is not uniform.

Properties of long feeders(48,52,54)

Transmission lines or feeders possess both inductance and capacitance distributed throughout their length, and energy is stored in the magnetic and electric fields associated with an energised feeder. A feeder may therefore be represented as in Fig. 7·8 (a). A mechanical model of such a feeder is shown at (b), in which the little masses represent the inductance, and the springs the capacitance, of the feeder. Series resistance and shunt leakage conductance, which account for the power-losses in the feeder, are not shown in (a).

Transmission-line theory is presented in a number of text-books. It is intended to show here how the results obtained can be imitated with the mechanical model, for in this way it is possible to develop

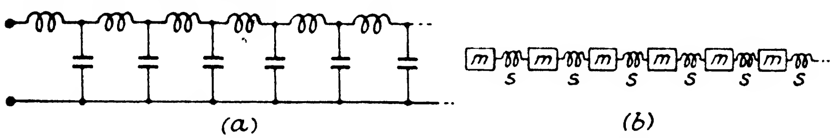


Fig. 7·8. (a) Feeder. (b) Mechanical Model of Feeder.

a simple and useful mental picture of the behaviour of the feeder. In using the model, the mechanical and electrical equivalents are:

Mechanical	*Electrical*
Force	Electromotive force (Voltage; E.M.F.)
Mass	Inductance
Displacement	Charge
Velocity	Current
Acceleration	Rate of change of current
Compliance or Resilience	Capacitance

By compliance is here meant the extension or compression of a spring per unit of applied force; this is analogous to the capacitance of a condenser, expressed as the charge stored per volt applied.

Fix one end of the model "line" so that it cannot move, e.g. to the floor, and hold the free end vertically above the fixed point with the line lightly stretched. The fixing of the far end of the line is equivalent to the feeder being open-circuit at its far end. It will be apparent immediately that this condition does not prohibit the flow of current into the feeder when a voltage is applied to the sending end, for a force applied to the model line results in a movement of its component parts, the movement or displacement being biggest at the sending end and zero at the far end. The current is a "charging" current, and the springs in the line are further extended or compressed according to the nature of the applied force.

Continuous slow oscillation of the sending end of the line, equivalent to the application of an alternating voltage to the open-circuit feeder, results in a progressively smaller movement (and therefore velocity) of all parts of the line, down to zero velocity at the far end. At very low frequencies, the open-circuit feeder thus behaves as a pure capacitance load on the supply, the maximum

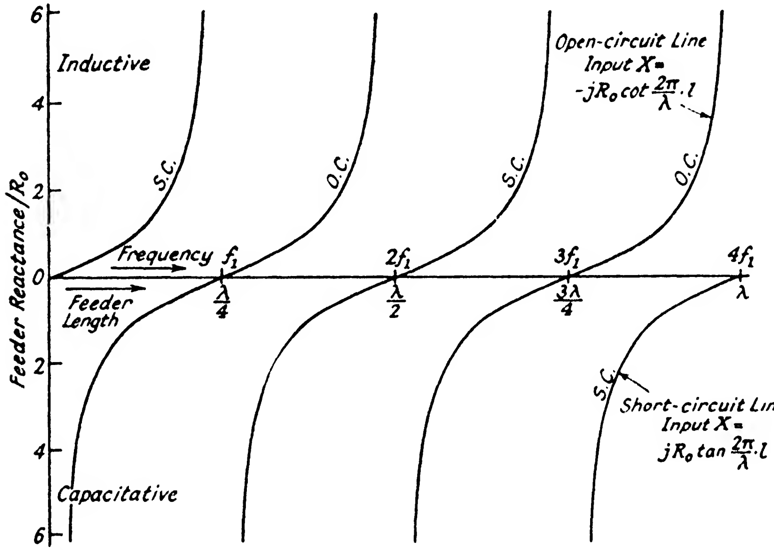


Fig. 7·9. Reactance of Open-and Short-circuited Feeders of Various Lengths.

voltage (force) occurring one-quarter cycle after the maximum current (velocity).

If the frequency of the force applied to the line is gradually increased, the amplitude of oscillation increases first to a maximum and then falls again (at the sending end, for constant applied force). At the maximum point, it will be observed that the instant of maximum applied force coincides with that for maximum velocity, i.e. the feeder now behaves like a pure resistance of low value. Thus the open-circuit feeder is in effect a series-resonant circuit at this particular frequency, f_1. Increasing the frequency to twice this value, it will be found that an oscillation of large amplitude builds up at the centre of the line, while the amplitude at the sending end is practically zero; moreover, the instants of maximum force and velocity once more coincide. At this double frequency, $2f_1$, therefore, the feeder presents a pure resistance load of very high ohmic value to the supply. At frequencies between f_1 and $2f_1$, it would be found that the effective impedance is inductive.

The open-circuit feeder thus presents to the supply an impedance which may be either capacitively reactive, or inductively reactive, or purely resistive, and the ohmic value for any of these conditions may be high or low. The reactance variation is shown by the appropriate "O.C." curves in Fig. 7·9. The resistive load corresponding to f_1 will be zero, and infinite at $2f_1$, if the line is loss-free.

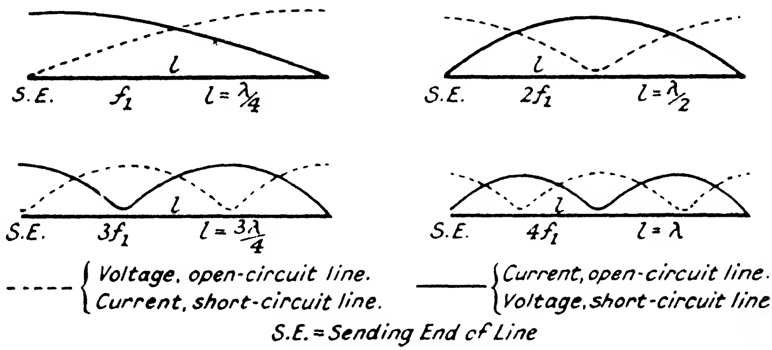


Fig. 7·10. Voltage and Current Distributions along O.C. and S.C. Feeders of Various Resonant Lengths.

The modifications to the curves, to take account of line-losses, are not significant for such lines as are likely to be used industrially.

Similar experiments performed with a model line having its far end free to move without any opposition, corresponding to a short-circuited feeder, will justify the "S.C." curves in Fig. 7·9. The corresponding voltage and current distributions along a feeder, at f_1, $2f_1$, etc., both for open-circuit and short-circuit conditions, are shown in Fig. 7·10. These can easily be verified with the model line. The horizontal line represents in each case the physical length of the feeder. The vertical heights, at any point, between this line and the sine curves represent the RMS value of the current in, and voltage across, the feeder at that point. The feeder is one wavelength long at the frequency $4f_1$, one-half wavelength at $2f_1$, three-quarter wavelength at $3f_1$, one-quarter at f_1.

The development of pronounced "standing waves" of voltage and current along the feeder at these critical frequencies (multiples of f_1) *is due to the reflection of energy from the far end of the* line (where there is no load to absorb it), *in such phase as to combine* with the *forward* travelling waves so as to reinforce them at some points and to neutralise them at others. The points of voltage or current minima are called voltage or current nodes; maximal points are called loops or antinodes.

Terminating the feeder in a pure resistance load can be represented in the model by fixing a light disc to the far end of the line, so that it acts as an air damper* when the line oscillates. If several sizes of damping disc are tried, it will be found that standing waves still persist, being most pronounced at the same critical frequencies as before; the standing-wave amplitudes are largest for very small discs (approximately short-circuit condition) and for very large discs (approximately open-circuit condition), but there will be one particular size of disc with which no standing waves can be developed, at any frequency; energy is now being dissipated by the disc without any reflection. The line is now properly "matched" to the load, and the line current and hence line-losses are a minimum for a given load-power. The equivalent electrical feeder condition is the termination in a load resistance of $R_O=\sqrt{\frac{L}{C}}$ ohms, L and C being the inductance and capacitance per unit of length of feeder. R_O is called the characteristic or surge impedance of the feeder. In relatively long feeders, or in feeders supplying a large amount of power to the load, it is desirable to match the load to the feeder by arranging that the effective load impedance is R_O ohms. Thus in Fig. 7·7 (c) the work-circuit can be matched to the feeder by suitable adjustment of the tapping on the coil L. Proper matching results in minimum line-losses, and the risk of voltage breakdown is reduced.

The magnitude of R_O depends upon the dimensions and spacing of the conductors, and upon the permeability and dielectric constant of the insulation; in H.F. feeders the permeability is always unity, and in the great majority of cases the dielectric constant is also unity, air being the usual dielectric. For μ and K both $=1$,

$$R_O=\sqrt{\frac{L}{C}}=276 \log_{10}\frac{D}{r} \text{ ohms} \quad . \quad . \quad . \quad 7{\cdot}9$$

for twin parallel-wire feeders (Fig. 7·11 (a)),

* Alternatively a liquid damping medium, e.g. oil or water, may be used in conjunction with smaller discs.

and $$R_O = 138 \log_{10} \frac{r_o}{r_i} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 7\cdot10$$

for coaxial feeders (Fig. 7·11 (b)).

With solid dielectric material having a dielectric constant K, C is proportional to K, so that

$$R_O \propto \frac{1}{\sqrt{K}},$$

i.e. R_O is reduced when a solid dielectric is used.

Solid insulation also reduces the physical length of a feeder relative to its electrical length in wavelengths. For air insulation, a wavelength of feeder is approximately

$$\lambda = \cdot95 \times \frac{300}{f} \text{ metres,}$$

f being in MC/S. This value must be multiplied by $\frac{1}{\sqrt{K}}$ for solid

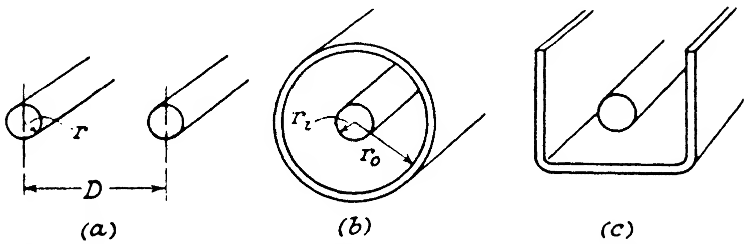


Fig. 7·11. Feeders: (a) Twin Line. (b) Coaxial. (c) Channel.

insulation; it is twice the distance between two adjacent maxima along the feeder.

Coupling the load to the source through a matched feeder

Twin parallel wire feeders ("open-wire" feeders), which may actually be constructed from tubes in some cases, normally have a characteristic impedance of 200 to 800 ohms, these limits being imposed by practicable physical dimensions. A feeder having $R_O = 600\Omega$, with its load properly matched to it, presents an impedance of 600 ohms to its source at the feed end, and this impedance has then to be transformed to the correct load impedance for the source. Referring to Fig. 7·12, three methods are illustrated for performing this second matching operation, which is necessary in order that the source shall deliver the maximum possible power to the load. In (a) the mutual coupling between the tank-coil L_1 and the secondary coil L_2 is adjusted to give maximum load-power, as indicated by an ammeter coupled to the work-circuit. This would, of course, coincide with maximum feeder current if the load and feeder are matched. A similar result is obtained by adjustment of the tapping points on the tank-coil, with the arrangement shown in Fig. 7·12 (b).

Another method of adjusting the coupling to the tank-coil employs a variable capacitance as shown in Fig. 7·12 (c), the arrangement forming what is known as a reactance transformer. If it be assumed that the feeder current is small compared to the

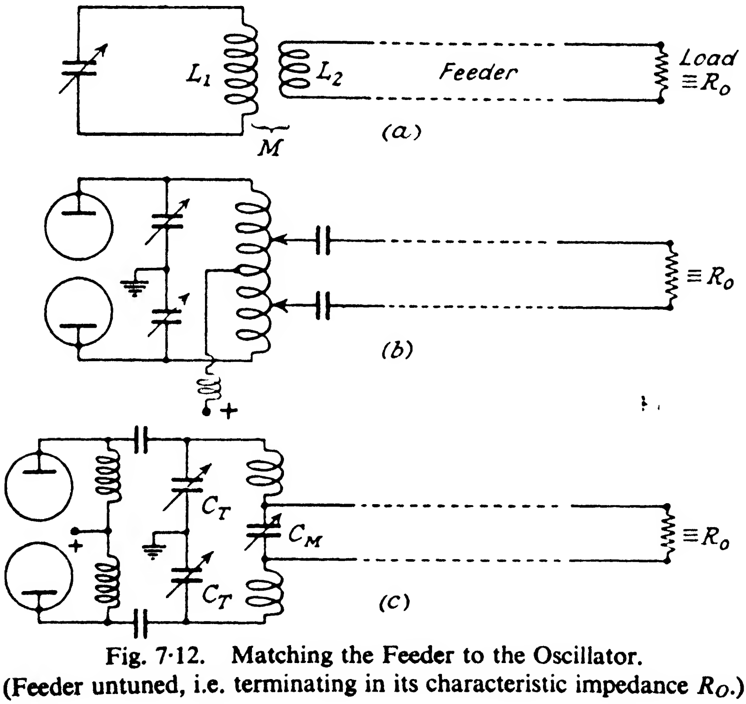


Fig. 7·12. Matching the Feeder to the Oscillator.
(Feeder untuned, i.e. terminating in its characteristic impedance R_O.)

tank-current, it is easily shown that the reactance of the matching condenser C^M is approximately

$$X=\frac{\sqrt{R_L R_T}}{Q} \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}11$$

where R_L=load resistance=feeder characteristic impedance R_O.

R_T=effective impedance of tank-circuit$=\frac{V_T}{I_f}$.

Q=ratio $\frac{\text{Tank volt-ampères}}{\text{Load-power}} \doteqdot 12$ to 20 when fully loaded.

Coaxial feeders may be used instead of open-wire types; the characteristic impedance is much lower, usually ranging between 50 and 200 ohms. No radiation takes place from these feeders. Fig. 7·11 (c) shows a form of feeder which combines the principal advantages of both the open-wire and coaxial types—simple and relatively cheap construction, with practically no radiation.

In all three systems of Fig. 7·12 it is assumed that the load is correctly matched to the feeder, which therefore presents a pure resistance R_O between its points of connection at the feed end. Three adjustments are therefore necessary in all:

(1) The load must be tuned to the oscillator frequency, for example, by one of the methods shown in Fig. 7·7.

(2) The load must be matched to the feeder, for example, by the method shown in Fig. 7·7 (c), in which the feeder tapping point is adjusted to give uniform voltage and current distribution along the whole length of the feeder.

(3) The coupling to the tank-circuit must be adjusted to provide the required amount of power in the load. This will usually correspond to the maximum possible power-output from the valves, in which case the coupling is adjusted until the D.C. feed-current to the valves rises to its permissible maximum value.

The correct settings for the three adjustments are arrived at by repeated trials; the initial adjustments should be made with reduced oscillator power, for example, by reducing the D.C. supply volts to the oscillator (see Chapter 8).

Coupling to the load through a tuned feeder

If the load impedance presented to the feeder at its far end were a resistance of a value other than R_O, the current in the feeder would not be the same at all points along its length, but would, in the case of a long feeder, exhibit a series of alternate maxima and minima, as shown in Fig. 7·13.

Curve (a) shows the current distribution when Z is a resistance less than R_O; if $\frac{R_O}{Z}=K$ (K greater than 1), then the current at the load end of the feeder will be K times as large as at $\frac{\lambda}{4}$ from the end. Similarly, curve (b) gives the current distribution when Z is a pure resistance K times greater than R_O; the current at the load is $\frac{1}{K}$ of the value at a point $\frac{\lambda}{4}$ from the load.

For any line, with any kind of terminating impedance, the ratio

$$\frac{V_x}{I_{x\pm\frac{\lambda}{4}}}=R_O \quad . \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}12$$

V_x being the line voltage at any point x, and $I_{x\pm\frac{\lambda}{4}}$ the line current one-quarter wavelength away from x. For the particular case of a pure resistance load, Z, unequal to R_O, the input impedance of a feeder of such a length that the feed end corresponds to a current minimum would be a pure resistance greater than R_O, say R_{max}. Again, the input impedance corresponding to a current maximum would be a pure resistance less than R_O, say R_{min}.

So that $$\frac{V_x}{I_x}=R_{max}; \quad \frac{V_{x+\frac{\lambda}{4}}}{I_{x+\frac{\lambda}{4}}}=R_{min},$$

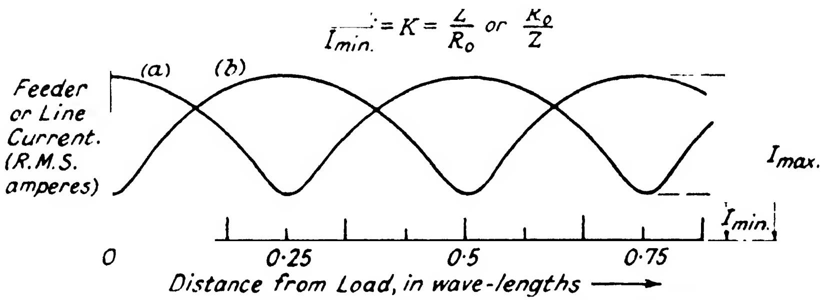


Fig. 7·13. Standing-wave Ratios on Incorrectly Terminated Feeder.

and $$\frac{V_x \times V_{x+\frac{\lambda}{4}}}{I_x \times I_{x+\frac{\lambda}{4}}}=R_{max}\,R_{min}=R_O{}^2,$$

i.e. $$\sqrt{R_{max}R_{min}}=R_O \quad . \quad . \quad . \quad . \quad . \quad . \quad 7{\cdot}13$$

If the load resistance Z is greater than R_O, a current maximum occurs at each point along the feeder an odd number of quarter-wavelengths from the load. When the input end of the feeder coincides with such a point, it behaves, in conjunction with the resonant load, as a low resistance, and the usual method of coupling to the power-source (i.e. the oscillator tank) is to make this low resistance a part of a series-tuned circuit, as shown in Fig. 7·14 (a). Two condensers, one in each line, are preferable to a single condenser, in order that the two lines shall be balanced and so produce a minimum of radiation. The tuning procedure is to adjust C_1 and C_2 simultaneously for maximum current at the feeder input, for a given coupling between L_1 and L_2. This coupling is then increased, C_1 and C_2 being readjusted to suit, until the maximum permissible plate-current to the oscillator is obtained; alternatively, until the required amount of power is being supplied to the load, if this is less than the maximum power the oscillator is capable of supplying.

If the load resistance Z is less than R_O, a current maximum occurs at each point along the feeder a whole number of half-wavelengths from the load, so that a feeder an integral number of half-wavelengths long will require to be series-tuned.

Similarly, load resistances greater or less than R_O, in conjunction with feeders respectively a whole number of half-wavelengths or an odd number of quarter-wavelengths long, present a high resistance at the input end, and are usually tuned by means of the

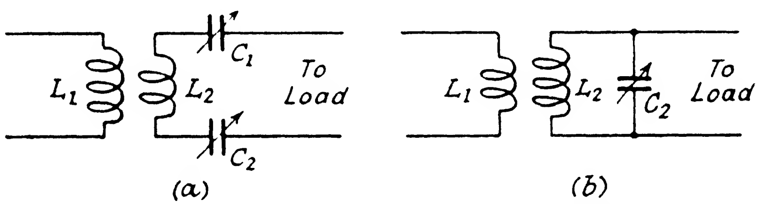


Fig. 7·14. Matching a Tuned Feeder to the Oscillator.

parallel-resonance system shown in Fig. 7·14 (b). Again, the coupling between L_1 and L_2 is adjusted to give maximum permissible plate-current to the oscillator, with the circuit L_2C_2 tuned to resonance, as indicated, for example, by maximum brightness in a gas-discharge tube placed near L_2.

Stub-matching[70]

It may be necessary to couple the load to the source through a feeder which is neither matched to the load nor of the correct length for resonant operation. It is assumed that the load-circuit itself is tuned to the oscillator frequency, since this condition is essential for maximum power in the load. The input impedance of the feeder plus load is now complex, i.e. there is a reactance component; the impedance is:

$$Z_{\text{input}} = \frac{R_O(Z + jR_O \tan \theta)}{R_O + jZ \tan \theta} \quad . \quad . \quad . \quad . \quad 7 \cdot 14$$

where R_O = characteristic impedance of feeder.

Z = effective resistance of the tuned load.

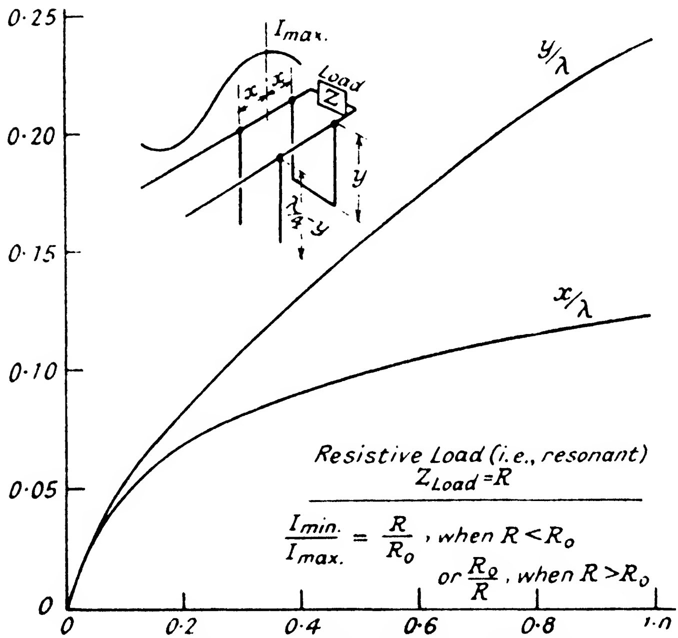
θ = electrical length of feeder, expressed as an angle;

$= \frac{l}{\lambda} \times 2\pi.$

l = feeder length, in metres.

λ = wavelength, $\doteqdot \frac{\cdot 95 \times 300}{f}$ metres.

f = frequency in megacycles per second.



[*Courtesy of Radio Society of Great Britain* ("*Amateur Radio Handbook*").

Fig. 7·15. Chart for Stub-matching.

It is possible to make the input impedance purely resistive, and equal to R_O ohms, by connecting a pure reactance, of a particular value, across the feeder at a suitable point. Referring to Fig. 7·13, the effect of the actual load resistance Z is similar to that of a resistance R_{min} terminating the feeder at a current maximum, or a resistance R_{max} placed at a current minimum. R_{min} is less than R_O, R_{max} greater than R_O. There is some point, between the current minimum and maximum points, at which the feeder could be terminated in a resistance R_O, *together with some reactance*, to give the same current distribution as for the actual load. If, therefore, the correct amount of reactance of opposite kind is connected at this point, the feeder is correctly matched between the input end and this point. The remainder of the feeder, from this point to the load, is unmatched, and should be as short as possible.

A short length (up to one-quarter wavelength) of open-circuited or short-circuited line is used as a matching reactance, offering respectively capacitive or inductive reactance, and is placed within one-eighth wavelength of a current maximum along the feeder, preferably that maximum which occurs nearest to the load. The length and type of line to use, and its correct position, for any particular standing-wave pattern along the initially unmatched feeder, are given in Fig. 7·15. Such short lengths of reactive line are called matching stubs.

An interesting application of matching stubs has been mentioned by J. P. Taylor, and concerns the glueing of long planks of wood to make laminated spars for aircraft.[46] The electrodes were about 25 ft long, and at a frequency of 8·5 MC/S, this represents nearly one-quarter wavelength. The voltage would, therefore, vary along the length of the electrodes, for with the feeder connected at the centre of the electrodes, these behave like approximately one-eighth wavelength lines, open-circuited at the far end; they present capacitive reactance to the feeder, together with the resistance corresponding to the power-dissipation in the work between them. The voltage distribution along the electrodes was made more uniform by connecting short-circuited (inductive) stubs across the electrodes at the two points corresponding to one-third intervals of the total electrode length. In this way the voltage variation was kept within 10%.

CHAPTER 8

Auxiliary Equipment and H.F. Measurements

Power supplies for valve oscillators

THE MOST convenient power-source, and the most common, is the alternating current mains. In Britain the standard frequency is 50 cycles/sec, and standard voltages are 230 single-phase, and 400 three-phase. Power at the appropriate voltages for filament and plate circuits of a valve-oscillator is easily obtained from suitable transformers supplied from the A.C. mains.

A.C. power is quite satisfactory for valve filaments. It is necessary, with high-power valves, to provide some means of increasing the filament voltage gradually to its normal working value, rather than to switch the full voltage on directly. This can be achieved by using a variable series resistance, or a filament transformer with high leakage-reactance, or a normal transformer supplied from a Variac, or other regulator-transformer. The maximum current during the initial heating-up period should not be allowed to exceed approximately 1½ times the normal running current, even momentarily. In calculating the series resistance required to do this, the cold resistance of the filament can be regarded as negligible. Similarly, if a "leaky" transformer be used, its short-circuit current should not exceed 1½ times the normal filament current. Some series resistance is desirable in any case, for accurate setting of the filament voltage; a reliable voltmeter should be included on the meter panel.

The plate-power supply system may be one of several kinds:

(1) "Raw" A.C. ("self-rectifying oscillator").

(2) D.C. from single-phase half-wave rectifier.

(3) D.C. from single-phase full-wave rectifier.

(4) D.C. from polyphase (e.g. 3-phase) half-wave rectifier.

(5) D.C. from polyphase full-wave rectifier.

(6) As for (2) to (5) inclusive, using grid-controlled rectifiers, giving smooth control of D.C. output voltage from zero to the rated maximum, and/or electronic power-switching.

"Raw" A.C. is used to some extent in low-power oscillators, notably for dielectric heating with power of the order of 1 KW or less. High-voltage A.C. from a transformer is applied directly

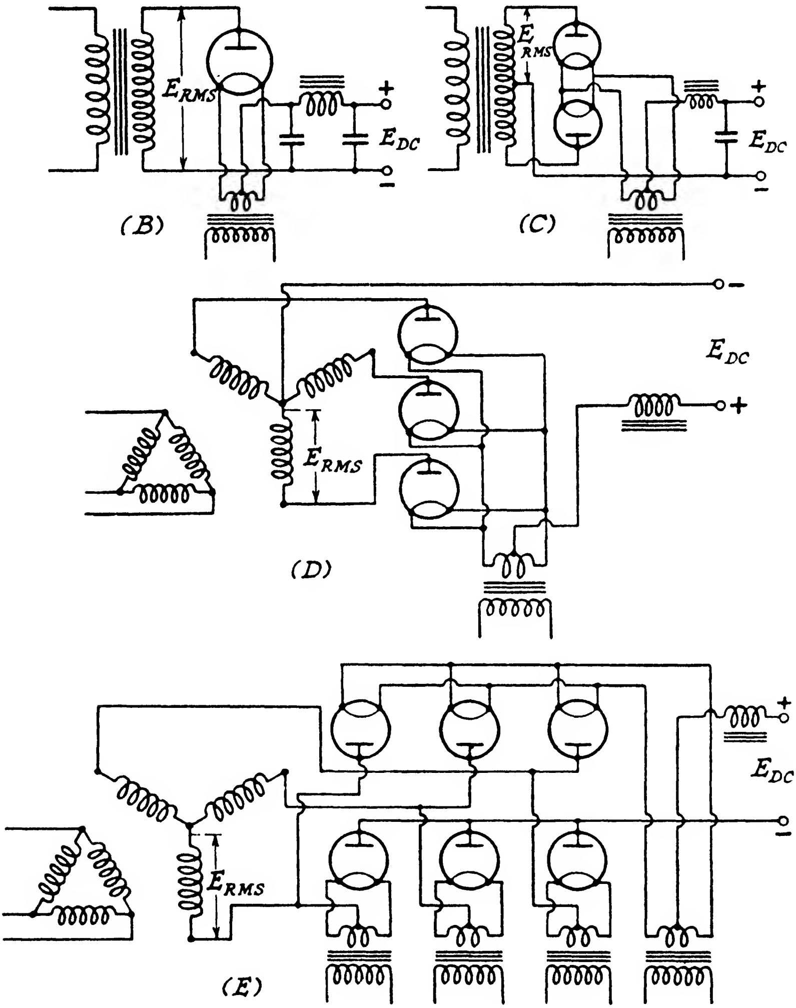


Fig. 8·1. Rectifier Circuits: (*B*) Half-wave, Single Phase. (*C*) Full Wave, Single Phase. (*D*) Half-wave, Three Phase. (*E*) Full Wave, Three Phase.

to the oscillator in lieu of the D.C. supply indicated in Fig. 5·19. An example of "raw" A.C. feed is seen in Fig. 7·4.

The oscillator functions only during alternate half-cycles of the mains supply, when the anode voltage is positive relative to the cathode; also the oscillation-amplitude is varying during the active half-cycle. The system has two serious disadvantages: the valve is necessarily generating less power than its rated maximum; and any radiation from the plant is amplitude-modulated and will cause interference with radio communications over a wide band of frequencies. Very thorough screening is therefore essential. On the other hand, the system possesses the merits of simplicity and low cost. Screening is necessary whatever the plate-power supply system used, so that screening costs need not be taken into account in assessing the relative merits of different systems. The "raw" A.C. system demands very complete screening, but it is not worth while making other than thoroughly efficient screens in any case.

The use of rectifiers giving practically constant D.C. output is usual in high-power oscillators. Mercury-vapour rectifying valves are almost universally employed because of their large current-handling capacity and high efficiency. Where the hot-cathode type of valve is used it is necessary to arrange for a time-delay between the switching on of the rectifier filaments and the application of the high voltage to the rectifier circuits. The length of this delay is quoted by the valve manufacturer; a greatly increased delay must be allowed when the valves are put into service for the first time, or after a long period without use.

Fig. 8·1 shows circuits for the systems (2) to (5) inclusive, and the table below gives the information necessary for selecting suitable rectifiers. Reference should be made to, e.g., *The J. and P. Transformer Book** for information on rectifier transformer design and ratings.

Circuit	$\frac{E_{DC}}{E_{RMS}}$	$\frac{E_{inv.}}{E_{DC}}$	$\frac{I_{DC}}{I_{av.}}$
Single-phase, half-wave	·45	3·14	1
Single-phase, full-wave	·9	3·14	2
Single-phase, bridge	·9	1·57	2
3-phase, half-wave	1·17	2·09	3
3-phase, full-wave	2·34	1·05	3

$E_{inv.}$ = peak inverse voltage across rectifier.
$I_{av.}$ = rectifier average anode current.
(Rectifier manufacturer specifies maximum safe values for these.)

A smoothing choke giving substantially constant load-current is assumed, and for this the maximum instantaneous rectifier current is equal to the load-current.

* Johnson and Phillips Ltd., Charlton, London, S.E.7.

The single-phase half-wave rectifier has no advantages other than apparent cheapness and simplicity, and is unsuitable for any but very low-power work. A reservoir condenser must be connected either across the rectifying valve or across the load, if it is desired to have reasonably constant load current; otherwise only discontinuous pulses are obtained. The output voltage is low, and the volt-ampère rating of the transformer is high. In common with other simple half-wave rectifiers, such as the 3-phase system of Fig. 8·1 (D), the flux in each transformer leg has a uni-directional component superimposed on the alternating component; this leads either to a bulky core or high magnetising current. In a 3-phase transformer this D.C. component can be avoided by using the inter-connected star secondary arrangement shown in Fig. 2·8.

Single-phase full-wave rectifiers are quite satisfactory for power up to about 2 KW. The bridge circuit has the merits of lower inverse voltage across the valves (as there are two valves in series), and the transformer secondary rating is lower; on the other hand, it requires more valves and a slightly more expensive filament-transformer system. In both, the fundamental frequency of the ripple in the output voltage is twice the supply frequency. Smoothing is achieved by means of one or more choke and capacitance filters.

The minimum inductance in henries for such a filter (series choke followed by shunt condenser) is numerically approximately equal to the load resistance in kilohms. The capacitance of the shunt condenser used with the choke must be large enough to make the filter resonant at a frequency considerably below the ripple frequency. The choke should be placed in the positive line rather than the negative.

Three-phase rectifiers are commonly used for high-power work. The full-wave system is desirable; the output voltage is high, the transformer KVA only slightly exceeds the D.C. output power, and the ripple voltage is small, and of high frequency (six times the supply frequency) so that smoothing is easy. A simple choke is sufficient for industrial power-oscillators.

Mention has been made of the need for means of reducing the D.C. H.T. voltage to a low value while preliminary adjustments are being made to an oscillator. A simple way of making this provision is to have tappings on the high voltage transformer to give, say, one-half, three-quarters, and full voltage as required. An alternative which permits smooth variation of H.T. volts from zero to maximum is the use of grid-controlled mercury-vapour rectifying valves. These valves remain non-conducting, even when the plate is at a high positive potential, provided that the grid potential is made sufficiently negative. The moment during the transformer voltage-cycle at which the valve commences to conduct is determined by control of the grid voltage. Fig. 8·2 shows the waveform of the unsmoothed rectified voltage for two different "cut-off" or conduction-delay periods. The smoothing choke in the D.C. line will have developed across it the greater part of all the alternating-voltage components of this complex rectified

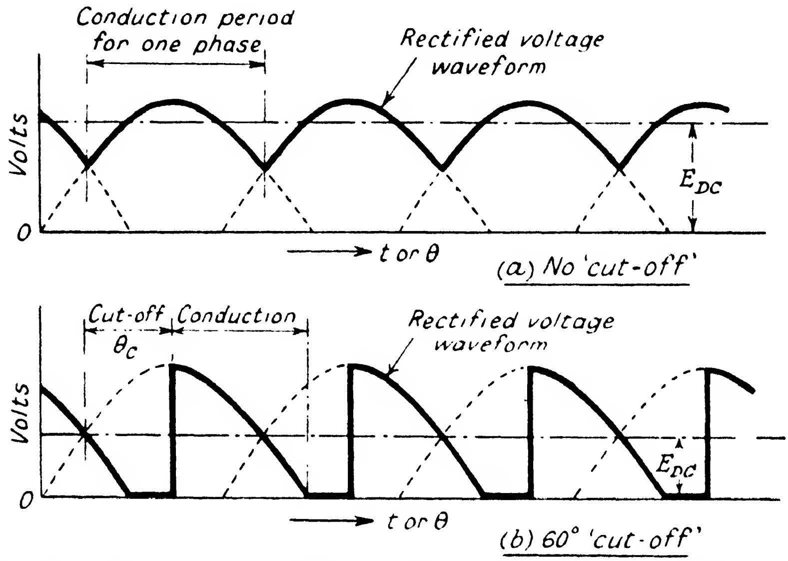


Fig. 8·2. Waveform of Rectified Voltage from 3-phase Half-wave Grid-controlled Rectifier: (a) No Cut-off. (b) 60° Cut-off.

voltage. The load-voltage is then practically unvarying and equal to the average value of the total rectified voltage. This average value obviously falls as the cut-off angle θ_C increases, ultimately becoming zero.(62)

The system provides an excellent means of accurate and fine control of oscillator output, with negligible waste of power. The grid-control voltage is usually obtained from the mains A.C. supply, and the cut-off angle is conveniently varied by phase-shifting the grid-voltage, relative to the anode-voltage, with a phase-shifter of the induction-regulator type. This is a form of induction "motor", the rotor of which forms the rotatable secondary of a transformer, and which supplies the control-grids voltages. The rotor is turned through any chosen angle by means of a hand-wheel and gearing; it is prevented from "motoring" in the ordinary sense. This controlled rotation of the rotor alters the phase-angle of the grid voltages of the rectifying valves, and so controls the D.C. output volts of the rectifier.

"Phase-shift control" of rectified voltage can also be achieved by means of a capacitance-resistance phase-shifting system, as indicated in Fig. 8·3 which illustrates the principle applied to a single-phase full-wave rectifier.

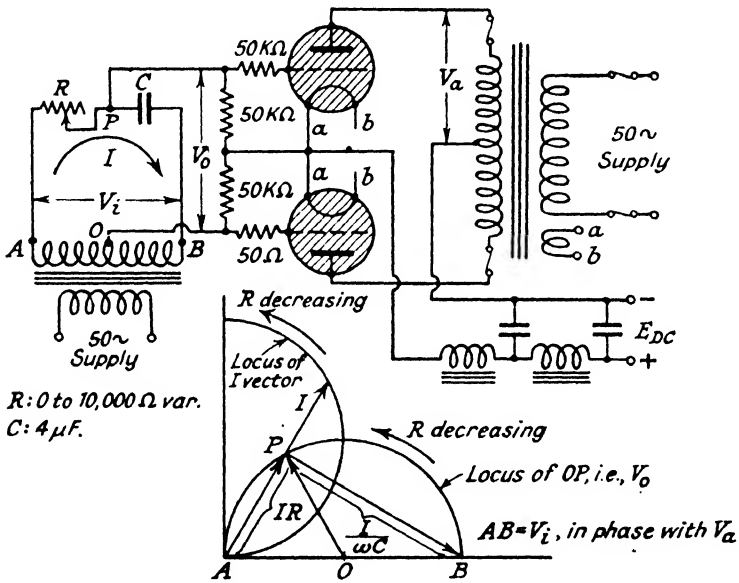


Fig. 8·3. Single-phase Full-wave Rectifier with Grid-control of D.C. Voltage.

The vector diagram, which ignores the small effect of the two 50 KΩ resistors forming the phase-splitter for the two grids, shows that a grid-voltage phase-shift from 0 to 180° can be obtained by varying *R* between zero and infinity. The rectified output voltage can therefore be adjusted to any value between zero and ·9 E_{RMS}.

Mercury-pool high-voltage rectifiers have been developed for power-oscillators, and it is probable that, with grid-control becoming virtually a necessity, this type will in time largely replace the hot-cathode rectifier, particularly for high-power ratings. The pool-rectifier can withstand occasional heavy overload without damage, and repetition high-speed power switching can be accomplished, by means of a blocking-voltage on the grids, without the mechanical difficulties associated with contactors.

Yet another advantage of grid-controlled rectifiers is that they provide a method of high-speed control of power during a heating cycle. They cannot compensate for changes in load impedance in such a way as to maintain full load on the oscillator, since full load must correspond with maximum supply voltage E_{DC}, but variation of power in a given load impedance can be achieved by varying the D.C. supply voltage to the oscillator.

Finally, supply mains voltage variation may be compensated by using a suitable rectifier grid-voltage phase-shift circuit. Summarising, grid-controlled rectifiers make possible any combination of the following functions:

High-speed switching of oscillator power supply.

Manual or automatic control of oscillator supply voltage, and hence of load-power.

Compensation for mains voltage variation.

Screening of H.F. equipment

All industrial H.F. equipment employing an arc or spark oscillator must be thoroughly screened to prevent electro-magnetic radiation beyond the immediate vicinity of the plant, whatever the nominal frequency of the oscillations. Similarly, apparatus operating at any frequency above 10,000 cycles/sec, even though the oscillation is of constant amplitude, must be screened. Screening is achieved by enclosing the apparatus within a metal box, cubicle, or room, the metal being connected to a low-resistance earth. When a screened cubicle or room is required, it is sufficient to use small-mesh wire gauze, the mesh being not larger than ¼″, and the gauge not thinner than 25 SWG. Galvanised iron wire, preferably copper-plated, is suitable. For a cubicle, the gauze is fastened to an angle-iron framework, all metal parts being adequately bonded to give a low resistance from all points to earth. Special care must be taken to ensure good electrical contact between the door and the remainder of the framework. A wooden floor is usually laid over the floor-gauze.

To screen an existing room, all surfaces of walls, ceiling, floor, and windows must be covered with wire-gauze, adequately bonded throughout, and earthed. Where the room can be screened during building, the gauze can be embedded in the plaster of walls and ceiling, and laid beneath the flooring. Metal door and window-frames should be incorporated.

In all cases, a 1-inch overlap must be allowed at joints in the wire-gauze, and soldered joints made at 12-inch intervals. Abutting surfaces of doorways and window-frames should be covered with sheet brass, copper, or zinc, to make contact with door and window screens. The sheet metal should be soldered to the wire-gauze at 12-inch intervals.

Other suitable materials for screening are sheet metal (aluminium, copper, galvanised iron), and metal-faced plywood; $\frac{3}{16}''$ mesh "expanded metal" is satisfactory in lieu of $\frac{1}{4}''$ G.I. wire mesh. Metallic paints are of no use for screening.

The connection between the screen and earth must be rigidly and permanently secured along its whole length, and protected from mechanical damage and corrosion. The earth electrode may be an earth-plate or a spike, though it is rarely that a sufficiently good earth can be obtained in this way. Connection to a main water-pipe is preferable, where convenient.

Full details of construction of screened cubicles and rooms, and a list of suppliers of materials and complete cubicles are contained in a Post Office Engineering publication, *Radio Interference*, C.1101.

It is not sufficient merely to suppress radiation from the plant by screening. Interference with communications systems can also be caused by H.F. power being fed into the supply mains from the plant, and transmitted through the mains to other apparatus. This is prevented by placing electric filters in the supply mains, at the point where the supply enters the screened room or cubicle. It is essential that all mains supplies entering the cubicle be filtered, whether they are actually connected to the H.F. plant or not, or alternatively, and preferably, that all electric supplies within the cubicle or room be obtained from one common filtered source.

Filter units recommended by the Post Office Engineering Department for single-phase and three-phase circuits are shown in Fig. 8·4. These are "all-wave" filters, designed for the suppression of interference over the whole of the radio frequency range at present in use, i.e. up to at least 50 MC/S.

The use, in the filters, of several values of capacitance in parallel, is necessary because of the inherent small self-inductance of any condenser. This self-inductance is larger the greater the capacitance. Every condenser thus has a resonance-frequency, which in the case of the larger capacitance values may lie in the relatively low radio-frequency band. At its resonance-frequency, a condenser behaves like a series-resonant system, of low resistance. The impedance-variation with frequency is as shown in Fig. 1·23; at frequencies higher than the resonance-frequency the condenser

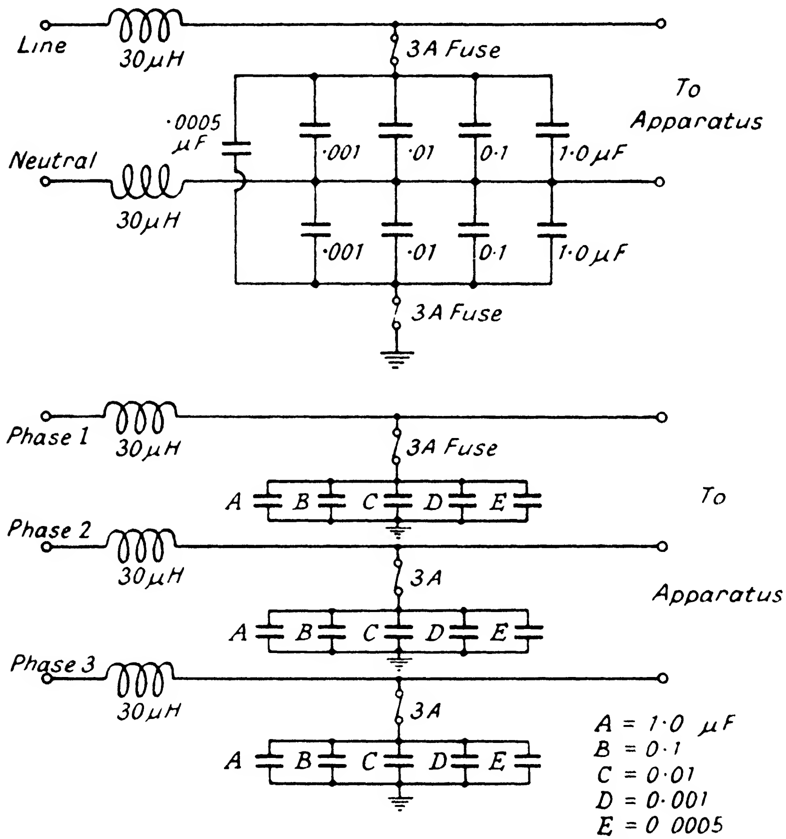


Fig. 8·4. Single-phase and 3-phase All-wave Mains Filters for Suppression of Mains-borne Interference from Power Oscillator.

offers *inductive* reactance, i.e. it no longer possesses the essential character of a capacitance. The inductive reactance is greater the higher the frequency, and the condenser is useless as a shunt-element of low impedance at frequencies much higher than the resonance-frequency.

The inductance of a suppression condenser lies usually between

·01 and 1·0 μH, and resonance-frequencies commonly lie within the frequency band 100 KC/S to 10 MC/S. The length and disposition of the condenser leads account for a relatively large part of its inductance, and must be kept as short as possible. The complete filter-unit must be enclosed in a metal case, the case being earthed and preferably fixed on to the screened wall.

Filters are rated in volts and ampères, corresponding to the supply voltage and the total current required by all the apparatus to be connected to the filtered source. The series inductances must be capable of carrying this total current; the condensers must comply with the specification laid down in B.S.S. No. 613. For most purposes, and for all supplies up to 500 v A.C., a test-voltage of 1500 A.C. between condenser-terminals is sufficient; the same test-voltage must be withstood between either terminal and the metal case of the condenser, where applicable.

The series-inductances are air-cored, and wound with insulated copper wire or strip. Every inductor has self-capacitance, and consequently a resonance-frequency. The inductor, therefore, behaves like the parallel inductance-capacitance system of Fig. 1·25, which offers a falling capacitive-reactance at frequencies higher than the resonance point. Any particular inductor is therefore suitable for only a limited frequency-range, and for complete filtering over a wide frequency-range (such as will be necessary with spark- and arc-oscillators), two or more inductors in series may be required, each suitable for a part of the whole range.

Safety devices and precautions

High-frequency industrial plant will require some or all of the following protective safety measures:

Protection of persons from electric shock and H.F. burns.

Protection of plant from damage consequent upon failure of components, cooling systems, and electric supplies.

Protection of plant from overload.

Protection of the thermionic valves from the premature application of H.T. supplies, and from too-rapid application of full filament-voltage from cold, in the case of tungsten filaments.

The whole of the electrical plant should be enclosed within an earthed metal case if possible. Inspection panels should be fitted with electro-mechanical interlock-switches which make it impossible for other than authorised persons to energise the plant unless all panels are in place. The contempt for danger bred of familiarity with high-voltage and high-frequency apparatus must be discouraged. An electric shock from the H.T. supply to a high-power valve-oscillator will almost certainly prove fatal; the peculiar type of flesh-burn due to contact of the skin with metallic parts in a high-frequency circuit is most unpleasant and distressing, as the flesh will not usually heal again, and a grafting operation may be necessary. It is essential, therefore, to arrange that the work-inductor in eddy-current heating plant, and the work-electrodes in dielectric heating plant are screened from the operator when the plant is in use. The screen may also serve for interference-suppression. Work-coil or electrode voltages of the order of several thousands are the rule. Inductor currents of several hundreds of ampères are common, and where these currents are in the form of successive trains of damped oscillations (arc- and spark-oscillators) the electro-mechanical forces have a low-frequency component; mechanical resonance of the coil at this frequency must be avoided. A fractured coil may have serious consequences.

Damage to the plant as a result of failure of components or electric supplies can be prevented by the adequate use of fuses, and a mains circuit-breaker fitted with magnetic or thermal overload and no-volt release mechanisms.

Protection against damage consequent upon failure of cooling systems is given by using electromagnetic interlock-switches energised from the fans-circuits, where fans are used, and water-flow switches placed in the cooling-water outlet pipe or pipes. These flow-switches open if the rate of flow of cooling-water falls below a predetermined value. Several kinds of flow-switches are available. It is worth noting that an apparent failure of the electric supply can actually be due to a reduction in the mains water pressure and resultant decreased flow; pilot lamps should be used in conjunction with the principal interlocks.

To prevent damage to mercury-vapour rectifier valves, a time-delay switch must be fitted, so that the H.T. supply cannot be switched on until the filaments of the valves have been energised for a certain time—seconds or minutes, depending upon the kind of rectifying valve.

Precautions to observe when using high-power valves

In valve-oscillator plant, additional high-speed overload protection must be incorporated, because of the low thermal-overload capacity of the plate and grid of the valve. To this end a relay is connected in the plate-supply circuit of the oscillator valve (usually in the negative H.T. return lead) to remove the plate-voltage if the specified full-load plate-current is exceeded. The relay should have a total operating time of about 1/10th second, and certainly less than 1/5th second. It is, of course, a D.C. relay, and its winding carries the "average plate-current" I_{DC}. A similar relay is sometimes placed in the grid circuit (between grid-resistor and filament) to disconnect the power-supply if excessive grid-current flows. A circuit arrangement is shown in Fig. 8·5, which provides automatic protection against excessive grid drive voltage consequent upon the removal of load from an oscillator (see also Chapter 5).

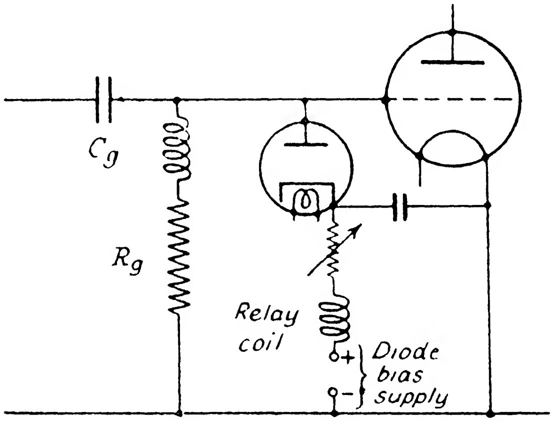


Fig. 8·5. Diode-limiter for Protection against Excessive Grid-current.

The diode is biased so that it passes no current unless the drive voltage exceeds a predetermined value; above this value, the diode load damps the drive circuit and minimises the increase in grid-current. The relay operates and interrupts the H.T. supply if the diode current exceeds a certain amount.

The necessity for using water-flow relays in conjunction with water-cooled valves has been stressed. In addition, an outlet-water thermometer and a water flow-meter are desirable; a combined interlock and flow-meter may be used. Outlet water temperature should not exceed 70° C, and under no circumstances should the water be allowed to boil. The flow must be sufficient to prevent the formation of steam-bubbles on the surface of the plate. A "stethoscope" consisting of about 6 ft of high-grade insulating tubing can be used to check this; one end is pressed against the jacket, at various points, while listening tests are made.

Distilled water is to be preferred for cooling, to reduce the probability of scale formation, which would reduce the cooling action. The R.C.A. recommend that, in any case, water having a hardness greater than 10 grains per gallon, or a resistivity less than 4000 ohm-cms should not be used. A 10% hydrochloric acid solution can be used for scale-removal, the plate being well rinsed after treatment.

The power dissipated at the plate can be checked from readings of inlet and outlet water temperatures (t_i and t_o° C respectively), and the water flow (G gallons per minute), using the expression

$$P_D = \frac{G(t_o - t_i)}{3{\cdot}15}\,\text{KW} \quad . \quad . \quad . \quad . \quad . \quad 8{\cdot}1$$

It may be possible to de-gasify a valve in which the vacuum has been impaired by overload, by operating it at half normal voltage for a period, the voltage then being raised in steps back to the normal value. At each step the valve should be operated long enough for stable conditions to be attained.

The high-speed plate-current-overload relay referred to should be supplemented with a series resistor to protect the valve during the time required for the relay to act. The minimum value of this resistor which will give adequate protection with minimum power-loss, for various power-supply ratings is:

Maximum power-output from rectifier:	16	40	100	250	640	1600	KW
Series resistor (R.C.A. rating):	25	50	200	250	275	300	ohms

A time-delay relay should also be used in the plate-circuit to prevent the application of plate-voltage before the filament has reached its normal operating temperature. The life of the tungsten filament can be extended by operating it at the lowest voltage consistent with sufficient emission. This can be determined by reducing the filament voltage, under normal operation on load, until a reduction in output is just detectable. The filament voltage must then be increased by the amount equivalent to the maximum expected supply-voltage variation, and then further increased by 2% to allow for minor variations in emission. Provision may be made, if desired, to reduce the voltage to about 80% of normal value during off-load periods.

Meters should be included to read filament voltage, plate supply-voltage E_{DC}, average plate-current I_{DC}, average grid-current I_g, and tank circulating current I_T (see H.F. Measurements section). An ammeter reading H.F. grid-current (in the grid lead) is desirable during tests with a new design.

Timing mechanisms

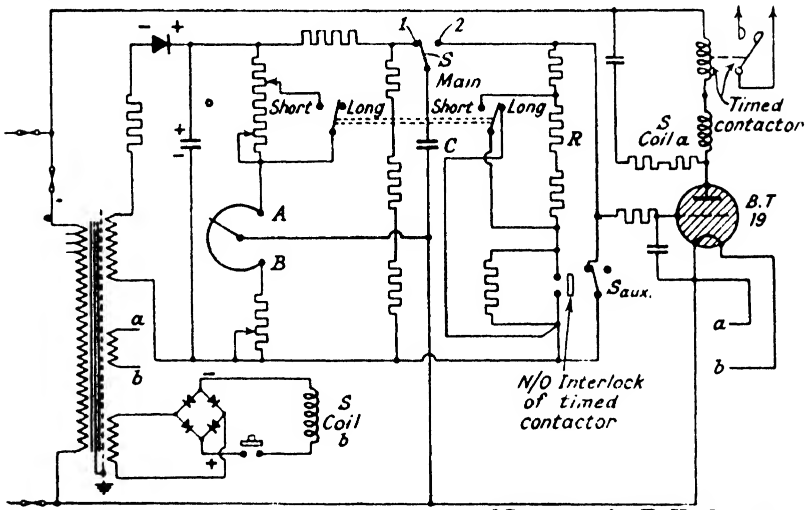
Many applications of H.F. power require the use of a timing mechanism, so that power is supplied to the work-circuit for a predetermined time, controlled within close limits. Process times range from a fraction of a second to several minutes, though it is not usually required that a single timer should compass this range. Where an extended range is required, as in experimental work, the electronic timer is best; for restricted ranges, other than extremely short times (1/10th second or less), several simple systems are available. These include pendulum timers, motor-driven-cam timers, synchronous-clock timers, and dashpot timers, as used, for example, in the control of resistance-welding machines. Some of these are described in J. L. Miller's book *Modern Assembly Processes.*[39]

Timers in which the minimum controlled-time interval is greater than 1/10th second are arranged to close a contactor for the required interval. This contactor controls the mains input power in the case of spark oscillators, the D.C. field or A.C. output in the case of H.F. alternators, and the plate-circuit supply transformer in the case of valve oscillators, though oscillator grid switching is sometimes used.

An alternative method of control of the valve-oscillator is available when grid-controlled mercury-vapour rectifying valves are used for the H.T. D.C. supply. The grid-voltages can be arranged so that the valves are normally non-conducting, and no D.C. power is available; the valves are then allowed to conduct, by changing the grid-voltages suitably, while the timing circuit is energised, so that D.C. power is supplied to the oscillator for this period. If time intervals shorter than 1 sec are required, this is the most satisfactory form of control. It can be combined with the phase-shift voltage-control described on p. 252.

A general purpose electronic timing circuit is shown in Fig. 8·6. The contactor is controlled by the mercury-vapour thyratron-valve, the grid-voltage of which is normally sufficiently negative, relative to the cathode, to prevent conduction through the valve.

The contactor coil is therefore normally not energised, and the contactor is open. The timing circuit *CR* drives the thyratron grid positive because of the charge in *C*, when relay *S* moves from position 1 to 2; the valve conducts and the contactor closes. *C* discharges through *R*, and the grid-voltage eventually resumes a negative value sufficient to suppress conduction. The contactor then opens again. An A.C. supply to the contactor coil and thyratron circuit is essential, as the negative grid-voltage cannot interrupt the valve current, but can only prevent it building up after its suppression during the negative half-cycle of anode-voltage.

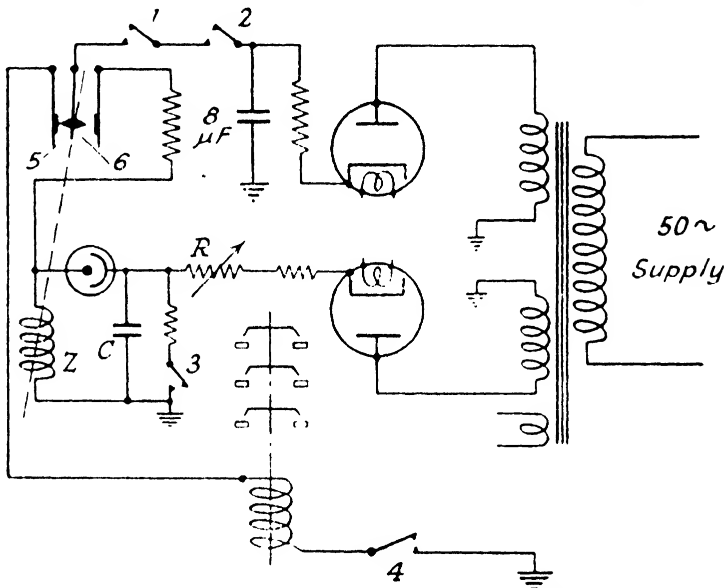


[*Courtesy of B.T.-H. Co. Ltd.*

Fig. 8·6. Electronic Timer-circuit with Thyratron Valve.

The time-interval for which the valve conducts and energises the contactor coil is, in this circuit, determined by the voltage to which *C* is initially charged; this voltage is selected by means of the potential-divider *AB*.

Fig. 8·7 gives the circuit diagram of a simple and effective time-switch developed by the B.I.C.C. Company, and employed by them in dielectric heating. Two separate half-wave rectifiers are used, providing D.C. for the time-circuit and the main contactor coil respectively. Switches *1*, *2*, and *4* are in the contactor-coil circuit. *1* and *2* close when the cage surrounding the work is closed, thus initiating the heating process; *4* is normally closed, but opens when overloading results in excessive current in coils x and y.* The closing of the work-cage also opens switch *3*, and the condenser C becomes charged, its voltage rising until sufficiently high to cause the neon tube to strike. C then discharges very quickly through the relay coil Z, so causing contacts *5* to break and *6* to make, the contactor-coil circuit is broken, and the heating process stops. This condition is maintained, so long as the cage remains closed, by virtue of a small "hold-on" current in Z, supplied from No. 1 rectifier circuit, through contacts *6*. On opening the cage, switches



[*Courtesy of British Insulated Callenders Cables Ltd.*

Fig. 8·7. Electronic Timer-circuit, with Neon Tube.

1, *2*, and *6* open, *3* and *5* close, and the system is ready for the next operation. The process time is the time required by C to become charged to the striking-voltage of the neon tube, and is varied by means of the resistance R.

Single-shot surface-hardening units often employ two timers, controlling heating and quenching times respectively. In such cases one may use two completely independent timers, usually arranged so that the quench commences at the end of the heating cycle. An alternative is a "double-timer" unit, which can be set to give any required heating and quenching times, with any desired degree

* See Fig. 7·4, p. 231.

of overlap. It is sometimes helpful to be able to start quenching before switching off the heat.

High frequency measurements

Measurement of current. Of the three important quantities—current, voltage, and power—the easiest of measurement in H.F. circuits is current, and many industrial H.F. power installations are equipped with means of current measurement only. The fact that the power developed in the work-piece is proportional to the square of the current in the work-circuit is justification enough for the inclusion of at least an H.F. ammeter.

Meters suitable for the measurement of high-frequency currents are the thermo-couple, hot-wire, and rectifier types. The latter includes metal-rectifier, crystal and thermionic-valve rectifier instruments. Small currents, of the order of a few ampères, are sometimes measured on a meter connected directly in the circuit; more often, either for convenience or because the current is large, a current-transformer is used, enabling the meter to be mounted on an instrument panel remote from the circuit. Iron-cored transformers are suitable for frequencies up to 20 KC/S; dust-cored or air-cored transformers are used for higher frequencies.

The air-cored current-transformer has various forms. For measuring very large currents the primary conductor is sometimes threaded once through a toroidally wound secondary to which the low-current meter is connected.(55) It can be shown that, if primary and secondary circuit resistances are negligible, the ratio of primary to secondary currents is proportional to the ratio of secondary to primary turns, and the calibration is nearly independent of frequency. A very common alternative form has a single-turn or multi-turn pick-up coil loosely coupled to the main circuit; a rectifier and high-resistance meter circuit then in effect measures the induced voltage across the pick-up coil This voltage is proportional to both primary-current and frequency, so that several calibration curves may be necessary.

Five arrangements for measuring H.F. currents are shown in Fig. 8·8.

In (a) the thermo-couple ammeter is connected directly in the line, while in (b) a thermal meter is connected to the toroidal secondary of a transformer, and in (c) the thermal meter is coupled to a loop in the line. If the meter is required merely as resonance indicator, or where its sensitivity is to be varied, the coupling arrangement shown in (c) can be made adjustable, the meter loop being mounted so that it can be swung.

(d) shows the usual full-wave bridge-circuit employing metal rectifiers. The standard meter-rectifier assemblies are for 1, 5, and 10 mA movements, and larger currents are read with the aid of a current-transformer. The upper frequency limit for accurate

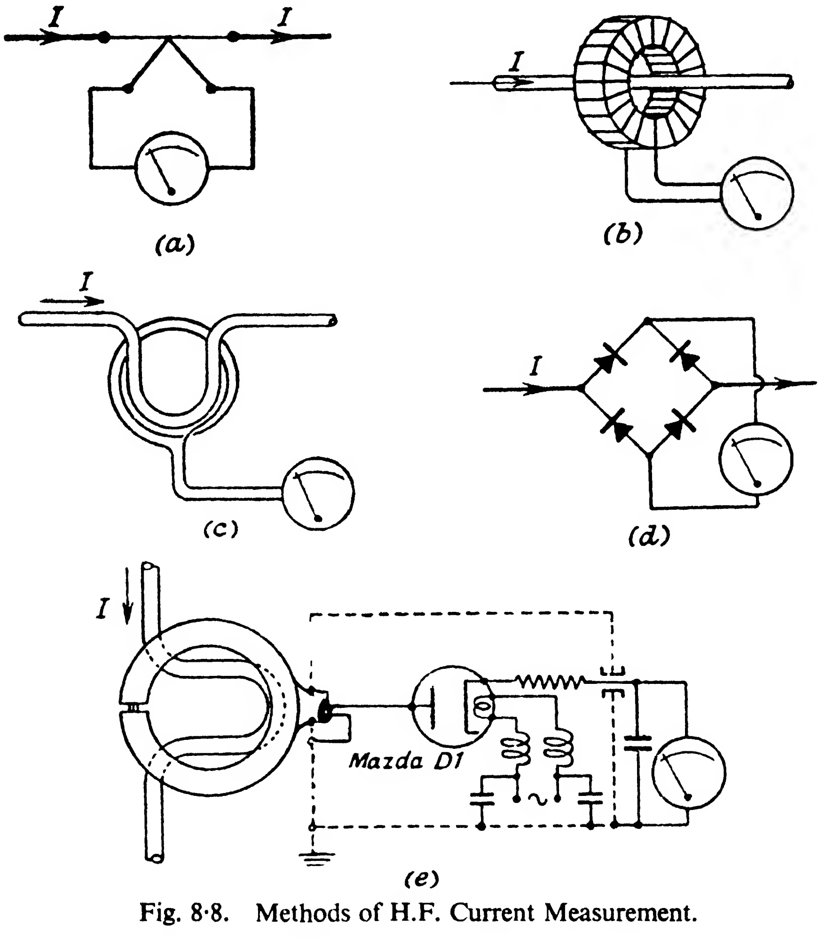


Fig. 8·8. Methods of H.F. Current Measurement.

readings with the standard units is about 100 KC/S, but satisfactory results can be obtained up to several megacyles/sec if "Westector" units are used in conjunction with a microammeter.

The arrangement shown at (e) is used in the B.I.C.C. equipment illustrated in Fig. 7·4, and is particularly suitable for the high frequencies commonly used in dielectric heating, where the other methods described are difficult to apply. The pick-up coil is screened, after the manner of a screened direction-finding loop, so as to eliminate stray pick-up and to make a robust construction. The rectifier unit is contained within an earthed metal screening box. The valve is a miniature thermionic diode such as the Mazda D1; the current rating of this valve is liberal, and it is not essential to use a microammeter in conjunction with it. A 5 mA movement is satisfactory.

Measurement of voltage

A simple voltmeter circuit for the measurement of peak voltages is given in Fig. 8·9 (a). Peak positive grid-voltage, peak grid-drive voltage, tank voltage, work-coil voltage, etc., are conveniently measured by this means.

The electrostatic voltmeter (circuit (a)) must be suitable for the direct reading of the peak voltage; for example, to measure the

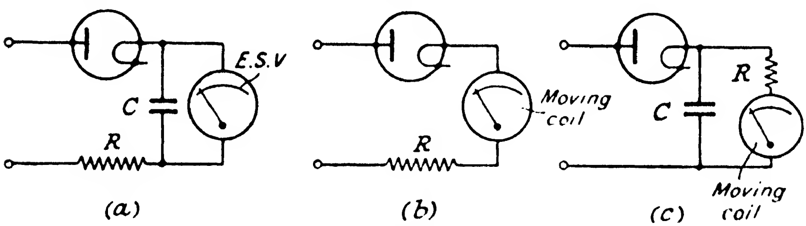


Fig. 8·9. Diode Voltmeters.

peak voltage of an input of 5000 v RMS, i.e. 7070 volts, a 7500-v or 10,000-v electrostatic meter would be required. The capacitance of *C* is not critical—about ·0005 μF is sufficient; it must be large enough to make the rectifying-valve capacitance negligible by comparison, and it must be capable of withstanding the peak voltage indefinitely. Fig. 8·10 shows suitable types. The valve must be able to withstand an inverse voltage of twice the peak voltage; its filament battery or transformer must be carefully insulated for this voltage. The resistance *R* limits the initial charging current when connection is first made to the voltage-source; its value is not critical, 10 to 50 KΩ is suitable.

When a moving-coil instrument is used (circuit (b)), it should be of a high-sensitivity type, e.g. 1·0 mA for full-scale deflection. The resistance *R* is made large enough to limit the current through the valve to something less than that required for full-scale deflection. The meter reading is proportional to the average value of the alternating voltage being measured, but where the input waveform is practically sinusoidal and of constant amplitude, the RMS and peak voltage values can be deduced from the average value.

Provided that R is very large compared to the valve resistance,

$$I_{av}=\frac{V_{av}}{2R}, \text{ very nearly,}$$

for this half-wave rectifier circuit. Then for sine-waveform,

$$I_{av}=\cdot318I_{max}=\cdot318\frac{V_{max}}{R}=\frac{\cdot318\times\sqrt{2}V_{RMS}}{R}$$

i.e. $$V_{RMS}=\frac{I_{av}\times R}{450} \text{ volts, when } I \text{ is in milliampères} \quad . \quad 8\cdot2$$

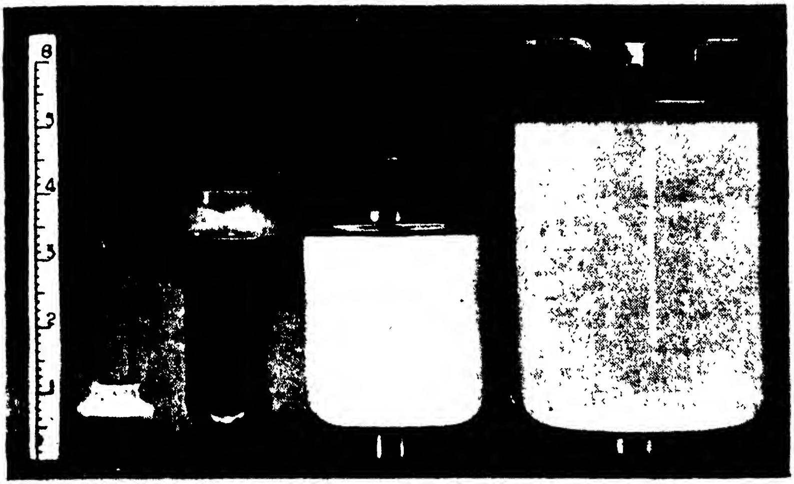


Fig. 8·10. High-voltage Condensers.

and $$V_{max}=\sqrt{2}V_{RMS}$$
$$=\frac{I_{av}\times R}{318} \text{ volts } (I \text{ in mA}) \quad . \quad . \quad . \quad . \quad 8\cdot3$$

Whence, using a 1-mA meter, the resistance R should be at least 318,000 ohms per kilovolt of the peak voltage to be measured. The power required to be dissipated in this resistance is:

$$P=\tfrac{1}{2}\,\frac{V_{RMS}^2}{R}$$
$$=\tfrac{1}{4}\,\frac{V_{max}^2}{R} \text{ watts} \quad . \quad . \quad . \quad . \quad 8\cdot4$$

i.e. about 1 watt per kilovolt of the peak voltage, using a 1-mA meter.

A third rectifier-voltmeter is shown in Fig. 8·9 (c). Provided that the product CR is large (at least equal to a period, in seconds, corresponding to 10 cycles of the voltage being measured), the voltage across C is steady and very nearly equal to V_{max}, the peak voltage of the source. The series resistance R must then have a value of 1 megohm per kilovolt, for a 1-mA meter. The meter reading will be proportional to V_{max}. For example, suppose a peak voltmeter is required, to read up to 10 KV peak, on frequencies from 100 KC/S upwards; the moving-coil meter has a 1-mA movement. The series resistance required is 10 MΩ, and the minimum value of C is about ·001 μF, since $CR = 10 \times 10^{-5}$ sec (C in μF, R in MΩ). The power to be dissipated in R would be $\frac{V_{max}^2}{R} = 10$ watts.

A high-frequency voltmeter for the measurement of small

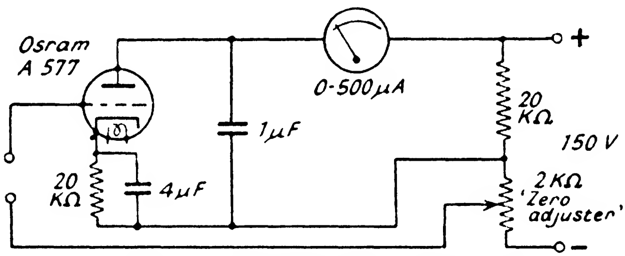


Fig. 8·11. Triode Voltmeter, 0–15 Volts.

voltages (e.g. of the order of 10 volts) is required for the determination of Q values by the method described on p. 280. Fig. 8·11 gives the circuit diagram of a suitable valve-voltmeter, of the auto-bias type. This has a nearly linear scale, and can be calibrated on a 50 cycles/sec supply.

The component values shown are approximately correct for a 0–10-V range, but it is a good plan to use a variable cathode-resistor which can then be set to the most suitable value as determined by trial.

If the meter is used for voltage measurements in circuits carrying D.C., it will usually be necessary to incorporate a series-condenser in the grid-lead. In that case, and wherever circuit conditions are such that there is no conductive path between grid and cathode, it is necessary to connect a resistor of 1 or 2 megohms between grid and the low potential end of the cathode-resistor. This does not arise in connection with Q measurement by the method shown in Fig. 8·18.

The new crystal-voltmeter, incorporating the "crystal valve" developed for radar, extends the field of easy voltage-measurement into the high radio-frequency range, e.g. to 50 MC/S. No auxiliary supplies are required, and a range from a few volts to kilovolts is covered by means of a set of capacitance voltage-dividers.

Power measurement

Accurate measurement of power at valve-oscillator frequencies is a matter of considerable difficulty. There is as yet no H.F. counterpart to the dynamometer-wattmeter which is used for D.C. and "low frequency" A.C. power-measurements; the dynamometer instrument is, however, perfectly satisfactory at present-day alternator frequencies, i.e. up to 15 KC/S or so.

In repetition work, where the coupling between the work-piece and the inductor is constant, giving a constant coupled-resistance, it is only necessary to maintain constant inductor-current in order to generate the same power in each successive work-piece. The total energy supplied to a work-piece is:

$$E = I^2 R_C t \text{ joules} \qquad 8{\cdot}5$$

where I = inductor current, ampères.

R_C = coupled resistance of work-piece, ohms.

t = heating time, seconds.

In many cases, therefore, it is not necessary to measure the power supplied to the work-piece; it is sufficient to measure the inductor current, and this is comparatively easy. If a satisfactory job can be repeated accurately with the aid of some indicator, such as a current indicator, precise measurements of load-power are not essential.

Research and development work, on the other hand, demand that the power from the generator and in the load be known with some accuracy.

The determination of maximum electrical power-output from an H.F. source can be made by measuring the current in a known resistance, the power developed being I^2R watts. The power developed in an article comprising the normal load on an H.F. source can best be determined calorimetrically. It is possible that satisfactory electronic wattmeters will be developed in the near future.

A method has been described by J. D. Kraus and R. W. Teed for measuring the high-frequency power-input to a load directly on a single meter.[57] Although developed as a diathermy "dose-meter", the apparatus described is suitable for dielectric heating and eddy-current heating measurements, and for all frequencies up to at least 50 megacycles/sec.

The underlying principle of the method is illustrated in Fig. 8·12 (a) and (b). Diagram (a) shows a work-circuit for dielectric heating, with a voltmeter across it. This "voltmeter" is conveniently a low-current thermo-couple or a rectifier type. At (b) is seen the equivalent circuit for (a); when there is no work between the electrodes, the no-load circuit-loss resistance is represented by

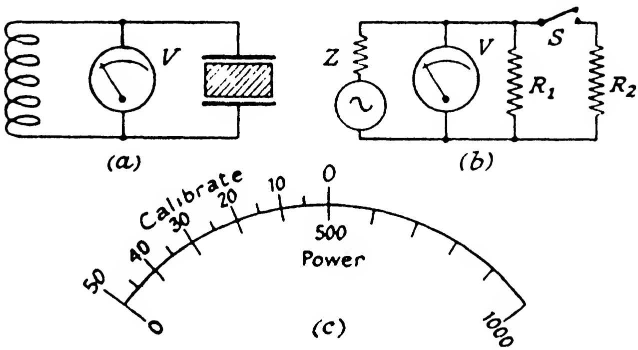


Fig. 8·12. Principle of Kraus and Teed Wattmeter.
(*See reference* (57).)

R_1, while closing switch S introduces the resistance R_2 representing the work.

The voltage V will fall somewhat when the circuit is loaded because of the source impedance Z. It can be shown that the useful load-power is equal to:

$$\frac{V_2\left(\frac{y_2}{y_1}V_1 - V_2\right)}{R_1} \text{ watts}$$

where

$$y_1 = \frac{V_1}{R_1}$$

$$y_2 = \frac{V_2}{\frac{R_1 R_2}{R_1 + R_2}}$$

i.e. the load-power can be measured in terms of meter-reading V_2

and the difference between meter readings with and without a load. Kraus and Teed refer to the quantity $\frac{\left(\frac{y_2}{y_1}V_1 - V_2\right)}{R_1}$ as the "calibration number". The meter, in the apparatus they describe, is provided with a variable shunt, so that its sensitivity can be varied; for large calibration numbers, the shunt is adjusted to make the meter more sensitive, and *vice versa.* The procedure is to tune the work-circuit without a load, and adjust the meter shunt to bring the pointer of the meter to the zero mark on the "calibrate" scale (Fig. 8·12); a load is then introduced, the circuit re-tuned, and the new reading noted on the upper ("calibrate") scale. The meter shunt is now re-set so that its dial reading corresponds to this "calibration number", and the new meter reading indicates the load-power on the lower (power) scale. The calibration of the instrument must be made with the aid of artificial loads in which the power developed can accurately be determined calorimetrically, or from current and resistance measurements, or photometrically.

The "calibrate" scale can be marked with any convenient divisions, e.g. 0 to 50. The shunt-dial is then marked correspondingly, and the shunt resistance so adjusted that deflection to the zero of the "calibrate" scale is normally obtained, on no-load, with the shunt-dial set to about half-way, i.e. to 25 on a 0-50 scale. The power-scale is then determined experimentally as indicated. The calibration holds only for one frequency or a small range of frequencies. An overall accuracy within 10% is claimed for the instrument.

Photometric power-measurement is useful for calibration purposes, and for determination of maximum power-output in cases where the frequency is so high that other methods become difficult. Incandescent lamps (with caps removed) are used as the load on the H.F. source under test, and the light-output from the lamps measured, preferably by means of a photo-cell and meter. The electrical power-input for a given light-output can be determined from a simple D.C. calibration, and a graph plotted relating photo-cell meter readings and "load" power.

Strength and distribution of electric fields

Methods of determining the strength and distribution of high-frequency electric fields can usefully be applied to dielectric heating, particularly in those cases where non-uniform heating is required.

Provided that the frequency is not so high as to result in non-uniform voltage distribution along the electrodes (i.e. provided that $\lambda \gg$ electrode length), the H.F. field distribution will be the same as that of a steady D.C. field or a low-frequency A.C. field. The distribution of these latter fields can be determined with precision by means of the electrolytic tank.[73] The principle of this method is illustrated in Fig. 8·13, in which a set-up for plane parallel electrodes is indicated.

The tank is of glass or other convenient insulating material, part-filled with slightly-conducting liquid, such as water from the

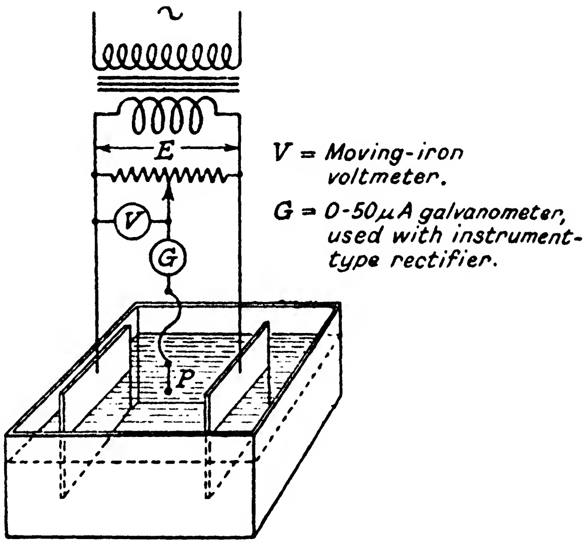


Fig. 8·13. Plotting Electric Field Distribution by Means of the Electrolytic Tank.

town mains. A scale model of the H.F. electrode system is placed in the liquid, and a small alternating voltage applied between the electrodes; the use of an A.C. supply eliminates polarisation difficulties. For any given position of the probe *P*, in the liquid between the electrodes, there will be a corresponding setting of the potentiometer slider *S* for which the current in the galvanometer is zero. The reading *V* of the voltmeter then gives the potential at the probe relative to that of the electrode to which the voltmeter is connected. The probe must be small, practically a needle-point, and of the same metal as the electrodes. It is convenient to link up the probe to a pencil through some form of pantograph, so that the pencil can indicate on a drawing-board the position of the probe in the tank. The usual procedure is to set the potentiometer slider so as to make V, say, 1/10 of the total electrode-voltage E, and, by moving the probe, to determine sufficient points at this potential to enable an equipotential line to be drawn. The process is then repeated for $V=\frac{2}{10}E$, and so on, until a complete set of equipotential lines is constructed. The lines representing the electric field are then drawn in so as to cut the equipotential lines at right

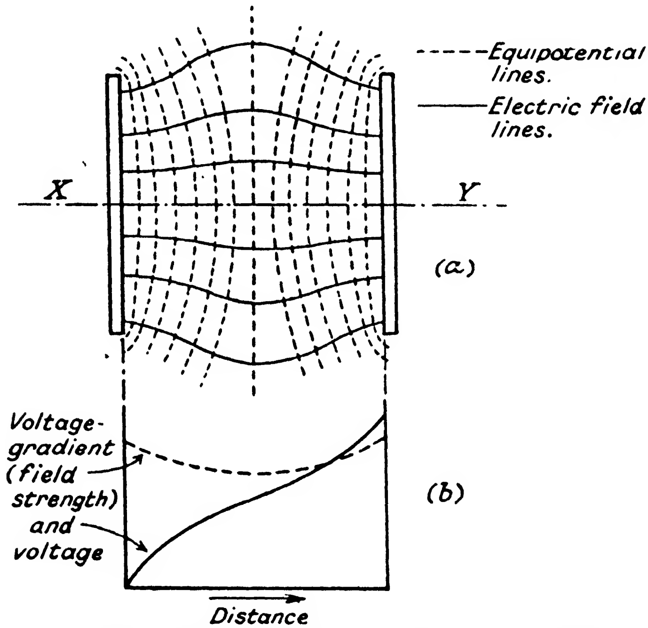


Fig. 8·14. Electric Field Distribution between Plane Parallel Plates.

angles, since equipotentials and lines of force are necessarily orthogonal. Fig. 8·14 (a) shows the kind of diagram obtained for plane parallel electrodes.

Fig. 8·14 (b) shows how the voltage and voltage-gradient (volts/cm, i.e. the field-strength), vary along the axis XY through the centre of the electrode system. It will be observed that the quantity actually measured in this method is potential, and that in order to determine the electric field-strength in a small region, at least two measurements must be made.

An instrument for measuring directly the electric field-strength in strong high-frequency fields has been described by K. S. Lion,[58] and is illustrated in Fig. 8·15. It makes use of the electrodeless luminous discharge which occurs in a low-pressure gas in a high-frequency electric field. A small spherical glass envelope, about 2 cms diameter, and containing a neon-argon mixture at about 0·1 mm mercury-column pressure, is used as a probe in the H.F. field being explored. The light-output from the discharge is proportional to the field-strength; a photo-electric cell, amplifier, and meter are used to measure the light-output, and the meter

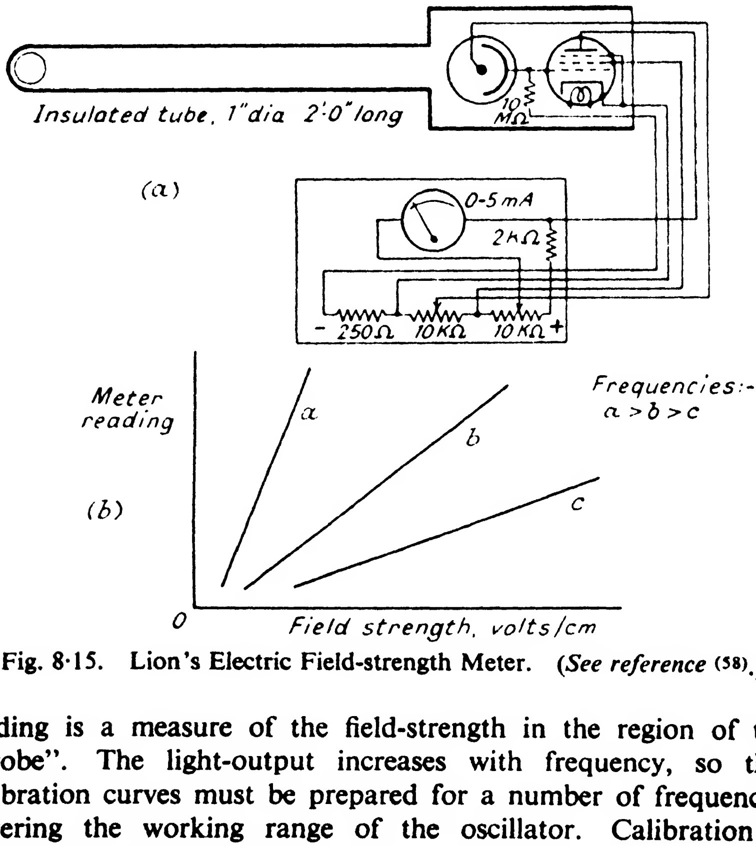


Fig. 8·15. Lion's Electric Field-strength Meter. (*See reference* (58).)

reading is a measure of the field-strength in the region of the "probe". The light-output increases with frequency, so that calibration curves must be prepared for a number of frequencies covering the working range of the oscillator. Calibration is performed by inserting the probe between the plates of an air-condenser with known applied voltages. The plates should be large relative to their spacing, to minimise the difference between actual and calculated field-strength; this can then be taken as being $\frac{V}{d}$ volts/cm (V=voltage between plates, d=spacing in cms).

Lion points out that in strong H.F. fields the discharge is not extinguished between half-cycles, provided that the oscillation-amplitude is not deeply modulated. Where the oscillator is operating from a "raw" A.C. supply, however, the discharge is extinguished at the end of each useful half-cycle of the mains-frequency. Special calibration will be required for such cases. Typical calibration curves are shown in Fig. 8·15 (b). In fields homogeneous through the volume of the probe, an error not exceeding 2% is claimed for measurements made with this instrument. In non-homogeneous fields the meter indicates the average value.

Where only the distribution of the field is required to be known, this can be determined with a model of the electrode-system immersed in carbon tetrachloride containing silk fibres, a D.C. field being produced by applying a steady high voltage to the electrodes. Alternatively, using an H.F. source, benzene containing powdered carbon in suspension can be used. The silk fibres or carbon particles align themselves to the field-pattern, in the same way as iron-filings in a magnetic field.

Measurement of frequency

A knowledge of the approximate frequency is essential in any industrial application of H.F. power. It is desirable that the frequency be *accurately* known, and it is possible that future legislation governing the use of H.F. equipment will make it necessary to operate each piece of equipment at a precise frequency, or, more probably, within a narrow frequency band. Thoroughly screened plant of the self-oscillator type is certainly preferable for industrial work to the much more complicated master-oscillator and driven power-amplifier which is standard radio practice.

Unless the frequency is required to be known very accurately indeed, an absorption-type frequency-meter of simple design is adequate for its measurement, and can be made to give values correct to within 1%. It is capable of indicating the relative magnetic field-strength in the neighbourhood of its pick-up coil, and the presence and relative amplitude of the stronger harmonics.

The essential feature of all absorption-type frequency-meters is the coil-condenser combination, tunable over the frequency range required. A range of plug-in coils makes even a very simple construction suitable for a wide band of frequencies (e.g. 0·1 to 100 MC/S).

Fig. 8·16 shows four arrangements of the absorption frequency-meter. In each case the *LC* circuit is tuned to resonate with the field of the tank-coil or work-coil of the power-oscillator. Resonance is indicated in (a) and (b) by the lighting of the flash-light lamp; as resonance is approached, the frequency-meter must be moved further away from the H.F. source, so that the indication is sharp, and, of course, to protect the lamp.

In circuit (c) the resonance-indicator is a neon lamp (without series resistance). Circuit (d) incorporates a rectifier (crystal, Westector, or valve) and a moving-coil meter. The shunt is

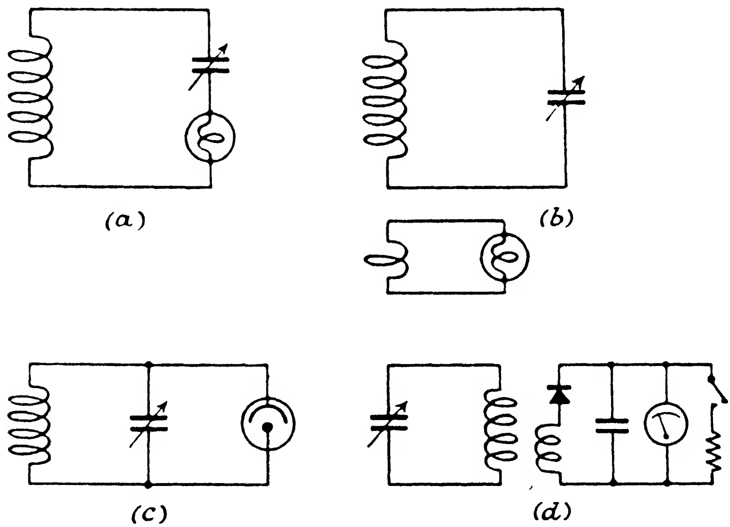


Fig. 8·16. Absorption-type Frequency-meters.

switched in during preliminary searching for resonance, to protect the meter.

In all of these arrangements the scale of the variable condenser is calibrated directly in frequency, or graphs are prepared giving dial reading-frequency relations for the various coils. If a linear scale is required, the tuning condenser must be of the "straight-line frequency" type. A convenient method of calibrating is to use a reaction-type wireless receiver, tuned in turn to each of several transmissions on known frequencies. At each frequency reaction is increased until the receiver just commences to oscillate, as indicated by a low-pitched growl in the 'phones. The frequency meter is now coupled loosely to the coil in the oscillating circuit, and the meter tuning adjusted to the point where the receiver ceases to oscillate. This is repeated for each known transmission frequency, and a calibration curve can then be drawn.

All the frequency-meter components, other than the coil, should be housed within a small metal box (aluminium or steel); the coil should plug into a socket mounted on a face of the box. A wireless receiver having its tuning dial calibrated in KC/S or metres is itself a sufficiently accurate frequency meter for some purposes. A characteristic signal is heard from the loud-speaker when the receiver is tuned to the H.F. generator frequency. It is necessary to check that the signal is due to the fundamental frequency of the generator, and not to a harmonic.

Where small changes in frequency are to be measured accurately, as in the determination of tank-circuit Q values (p. 278), a heterodyne frequency-meter is preferable to the absorption-type, being capable of giving a much sharper indication of resonance. The meter contains a low-power variable-frequency oscillator, of high frequency-stability.

A signal from this oscillator is combined with one picked up from the source under measurement, and a current having the difference-frequency of these two signals is produced and passed through headphones. The frequency of the local oscillator is varied until a zero-beat point is obtained between two sharply rising whistles. At the zero point, the local oscillator has the same frequency as the source being measured.

A simple and convenient application of the principle is shown in Fig. 8·17. The local oscillator is built around the triode portion of a triode-hexode valve of the type commonly used as the frequency changer in superheterodyne radio receivers. The signal from the test-source is applied between control-grid and cathode of the hexode portion, and the plate-current of the hexode is further controlled by the local oscillator signal to the injector-grid, which is internally connected to the oscillator-grid. The hexode plate-current then contains, among many components, one at the difference-frequency of the two signals, and this is the only component which may be within the audio range. All H.F. components are filtered out by means of the H.F. choke and the condenser C_F. Audio frequency currents, when present, flow through the 'phones. A range of standard 4- or 6-pin plug-in coils can be used in the oscillator circuit, to cover a wide range of frequencies. A calibration curve can be drawn for each coil from readings obtained with the aid of a radio receiver. A characteristic noise will be heard in the speaker when the receiver is tuned to the frequency of the local oscillator; if a communications-type receiver is available, suitable for receiving C.W. morse, the receiver can be accurately tuned by the zero-beat method. A good slow-motion dial on the frequency-meter is essential.

The measurement of *Q*

The measurement of the *Q* value of tank- and work-circuits enables one to determine the operating conditions of the power-oscillator, and the coupling between inductor (work-coil) and work.

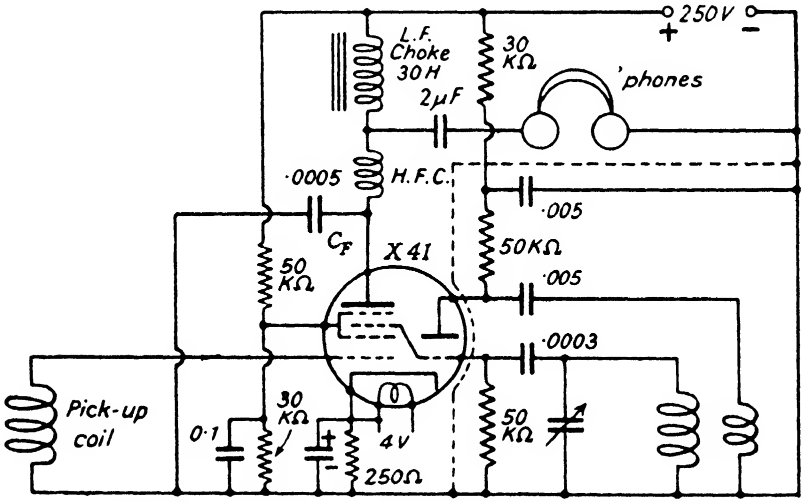


Fig. 8·17. Triode-hexode Heterodyne Frequency-meter.

It leads directly to the determination of coupled resistance and hence of load-power, power-losses, and electrical efficiency of the work-circuit.

The equipment required for measuring *Q* comprises:

Variable-frequency test oscillator, preferably with available power-output of the order of 50 watts.

Frequency-meter, preferably of the heterodyne type.

Valve voltmeter, range 0-10 volts.

During construction of an industrial power-oscillator, measurements can be made on the experimental tank-circuit itself; otherwise a "dummy" tank-circuit can be used, in which the tank-coil can be wound to give approximately the same inductance and

H.F. resistance as the tank-coil proper. The actual coil, or a replica, is preferable, however. The dummy tank-condenser can be any convenient low-loss air-dielectric variable type, calibrated, and set to the same capacitance value as that of the tank-condenser proper. Inductance and capacitance measurements can be carried out on any suitable bridge, e.g. the "General Radio" bridge, type 650-A.

Assuming, for simplicity, that the work-coil is a small coil in series with the main tank-coil, the procedure for determining the tank-circuit Q, unloaded and loaded, is as follows:

The test-oscillator coupling-coil is very loosely coupled to the unloaded tank-coil, and the oscillator frequency set to the resonance frequency f_o of the tank, as indicated by maximum reading on the voltmeter across the tank-condenser (Fig. 8·18). The oscillator output is then adjusted to give a conveniently large reading, say, V volts, on this meter. The oscillator frequencies above and below f_o at which the voltmeter reading falls to $\cdot 707V$ are then determined. Writing Δf for the difference between these two frequencies f_1 and f_2 (see Fig. 1·27), we have

$$Q=\frac{f_o}{\Delta f}=\frac{f_o}{f_2-f_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad 8\cdot 6$$

Alternatively, with the oscillator frequency constant at f_o, the tank-circuit can be off-tuned by means of a *calibrated* condenser, and the change in capacitance, ΔC, between the two settings giving $\cdot 707V$ noted. Then we have, approximately:

$$Q=\frac{2C}{\Delta C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 8\cdot 7$$

where C is the capacitance value at resonance.

This procedure is repeated with the tank-circuit loaded; the resonance-frequency f_o' will be somewhat higher than f_o, the effective inductance of the work-coil being reduced in the presence of the work, if this is non-magnetic.*

Let the unloaded and loaded circuit Q values be Q_1 and Q_2 respectively.

Then $$Q_1=\frac{1}{\omega_o C R_1} \quad [\omega_o=2\pi f_o] \quad . \quad . \quad . \quad . \quad 8\cdot 8$$

and $$Q_2=\frac{1}{\omega_o' C R_2} \quad [\omega_o'=2\pi f_o'] \quad . \quad . \quad . \quad . \quad 8\cdot 9$$

* Non-magnetic stainless steel (see Table I, Appendix) at room temperature has approximately the same resistivity as medium-carbon steel at hardening temperature, so that dummy loads of stainless steel may be used to represent the latter.

where R_1 and R_2 are the effective series resistance values in the two cases, and can be calculated.

The power-loss in the tank- and work-coil circuit on no-load when used with the power-oscillator will then be:

$$P_L = I_T^2 R_1 \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad 8{\cdot}10$$

I_T being the tank-current in ampères.

On load, the total power developed in the circuit is $I_T^2 R_2$ watts, and the useful power in the work itself is:

$$P_W = I_T^2(R_2 - R_1) \text{ watts} \quad . \quad . \quad . \quad . \quad 8{\cdot}11$$

$(R_2 - R_1)$ being the coupled-resistance of the work.

The electrical efficiency of the circuit is:

$$\eta = \frac{R_2 - R_1}{R_2} \times 100\% \quad . \quad . \quad . \quad . \quad 8{\cdot}12$$

Provided that the change in circuit inductance due to the intro-

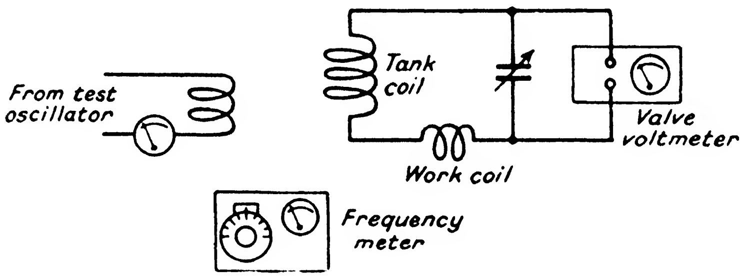


Fig. 8·18. Set-up for Q Measurement.

duction of the work is small (i.e. ω_o' differing only very slightly from ω_o) the electrical efficiency is approximately

$$\frac{Q_1 - Q_2}{Q_1} \times 100\% \quad . \quad . \quad . \quad . \quad . \quad 8{\cdot}13$$

The relative proportions of the total loss which occur in tank- and work-coils respectively can similarly be determined from Q measurements taken with and without the work-coil short-circuited, without any load in the coil.

When the test-circuit has a high Q value, as, for example, when it is not loaded, it will be found more convenient to determine the frequencies above and below f_o at which the voltmeter reading falls to ·447V. In this case we have (from Fig. 1·27):

$$Q = \frac{2f_o}{\Delta f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 8{\cdot}14$$

Δf being the difference between the upper and lower frequencies. The circuit diagram of a suitable voltmeter is given in Fig. 8·11.

It has been stated that the coupling between the test-oscillator and the circuit being examined must be very loose. This is necessary in order that the test-circuit shall have a negligible effect upon the oscillator. Only a very small amount of power will be required to be supplied to the test-circuit to develop, say, 10 volts across the tank-condenser, and the very loose coupling to give this should result in no detectable change in the reading of a thermal ammeter in series with the oscillator coupling-coil.

An alternative method of measuring Q is based upon the definition of Q given in equation 1·54.

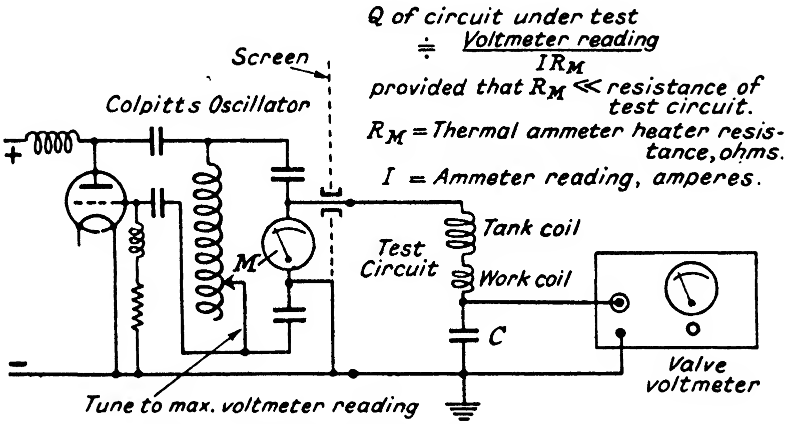


Fig. 8·19. Alternative Method of Q Measurement.

$$Q=\frac{V_L}{V_S}=\frac{\omega_o L}{R}$$

$$=\frac{V_C}{V_S}=\frac{1}{\omega_o CR} \quad . \quad . \quad . \quad . \quad . \quad . \quad 1{\cdot}54$$

When a voltage V_S is injected into a series circuit resonant to the source frequency, there is developed across the coil a voltage QV_S, and a similar voltage appears across the condenser. A convenient set-up is shown in Fig. 8·19; the injected voltage V_S is IR_M, R_M being the heater resistance of the thermal ammeter reading I ampères.

The procedure for measuring Q is therefore to inject a known voltage into the resonant circuit and to measure that developed across one of the elements, L or C. In practice, the voltage across a low-loss condenser is measured on a valve-voltmeter, and if the input voltage is carefully set to a particular value, the valve-voltmeter scale may be calibrated to read Q directly.

"Q meters" are available which incorporate a calibrated oscillator, valve-voltmeter, and calibrated variable condenser. The Simmonds Q meter, for example, has an oscillator with a range from 60 KC/S to 50 MC/S in seven bands, the calibration being accurate within 2%. The valve-voltmeter range is from 0·1 to 5 volts in 5 ranges; its Q scale is calibrated 0-110, and a switch gives additive ranges up to 500. The variable condenser has a maximum capacity of 650 $\mu\mu$F, and silvered mica low-loss condensers can be switched in to give a total capacitance of 2600 $\mu\mu$F; with the aid of a vernier dial, capacitance values within this range can be adjusted to 0·1 $\mu\mu$F.

A constant input of 10 millivolts to the circuit under test is provided from the volt-drop developed across a non-inductive resistance of 0·04 ohm carrying 0·25 ampère at the desired frequency. The resistance consists of a thin conducting film to which the H.F. current is fed radially, which results in a truly non-inductive element; in addition, the thin film is free from skin effects.

Such a measuring instrument can also be used for the determination of capacitance, power factor, and dielectric constant. It is therefore of great value for measurements in connection with dielectric heating as well as induction heating.

CHAPTER 9

Industrial Applications and Operating Problems

Induction melting equipment

CORELESS INDUCTION melting furnaces range in size from 1 or 2 lb weight of charge to 10 tons, with power ratings from 3 KW to 1200 KW. The power-source for a small furnace may be a spark-oscillator, valve-oscillator, or motor-alternator; the use of the latter is ruled out if the charge pieces are very small. The motor-alternator set is almost always used for the larger furnaces, say, 50 lb and over, the usual frequency-range being 500 to 3000 cycles/sec.

The chief electrical operating problems are power and power-factor control. It is usual for only one furnace to be supplied from either a spark- or valve-oscillator, and in such cases power control is effected by adjustment of the oscillator; there is no power-factor control problem in these self-oscillators. Where the power is supplied from an alternator, furnace power control can be obtained by means of an auto-transformer, or tappings on the furnace coil, in large furnaces; or in small furnaces by means of a tapped reactor, this being connected either in series with the furnace coil ("tank" reactor) or in the supply line to the parallel combination of furnace coil and condensers ("line" reactor). Tap-changing, which is carried out off-load, may be accomplished by using contactors or manually operated switches. It is not usual to supply more than one large furnace at once from the same alternator, and consequently furnace power is controlled from alternator excitation, but even so, furnace-coil taps are an advantage as they make it possible to use the full alternator power rating throughout the melt. A furnace with coil-taps may yield up to 25% more output than one without taps. Power-factor control is achieved by condenser switching, at reduced voltage, either manually with reference to an indicating power-factor meter or KVAR meter, or automatically through a KVAR relay. About 40% of the whole condenser bank is arranged for switching.

The furnace coil is supplied either through rigid bus-bars or through flexible cables. The latter avoid the need for some form of knife or mercury-pool contact, as the cables can flex when the furnace is tilted for pouring. Flexible cables may be water-cooled within rubber hose-pipes. It is usual to incorporate interlock-switches to prevent tilting with the furnace energised.

The Durville method of precision casting is sometimes used with small induction furnaces. The mould is clamped in position over the furnace, and the melt then takes place in a very limited amount of air. When the charge is molten and ready for pouring, the H.F. power is switched off, and the furnace is inverted. Sufficient time is allowed for the cast to solidify, and then the normal furnace position is resumed and the mould removed.

The choice of frequency for melting depends firstly upon the size of the pieces constituting the solid charge; the frequency must be high enough to give a satisfactory $\frac{d}{p}$ value (say $\frac{d}{p}$ not less than 4) where d=smallest dimension of piece.* Secondly, the stirring effect is greater the lower the frequency, being proportional to the square of the coil ampère-turns. The stirring must not be too violent, and a frequency 500 or 1000 cycles/sec is usual for large furnaces, 3000 cycles/sec for smaller foundry furnaces, and 10 KC/S or above for laboratory furnaces.

In certain cases, for example, decarburisation and refining free of sulphur and phosphorus, it is an advantage to be able to control the stirring action and the power-input independently. A solution to this problem has been provided by the Swedish ASEA "double frequency" furnace in which the furnace coil is supplied with poly-phase low-frequency stirring current superimposed on the single-phase high-frequency heating current (see also British Patents Nos. 423326, 433339, 444000, 508255).

The clay-graphite crucible referred to later in connection with non-ferrous foundry work is occasionally employed, in conjunction with a lift-coil type of furnace, for steel melting up to 200-lb capacity, as this kind of crucible is strong enough to be carried, with its molten charge, to the moulds. The tilting furnace is, however, employed for practically all steel melting, pouring directly into the moulds or alternatively into a ladle. Steel melting furnaces have also been built which can be lifted bodily, complete with their

* This is not an essential condition for clean small charge pieces which pack well, such as nickel shot, which can be melted down at quite low frequencies, e.g. 1000 cycles/sec.

water connections, but without electrical connections, and transported by overhead crane.

Crucibles and built-up linings may be acid, neutral, or basic according to metallurgical requirements. The most common applications of the "coreless" induction furnace are steel re-melting and alloying, for which an acid material such as quartzite, or an "acid-neutral" material such as sillimanite, is used. High manganese alloys require a basic lining such as magnesite or dolomite, as also do such operations as the removal of carbon, phosphorus, and sulphur.

The refractory crucible or lining should have low thermal expansion, at least over the normal working temperature range; this range can be restricted by continuous operation of the furnace. The surface in contact with the charge becomes sintered and the depth of the sintered layer increases steadily in use. The large thermal gradient between charge and coil exists mainly across the unsintered layer, which has the lower thermal conductivity, so long as this layer is not too thin. Ultimately, however, excessive thermal gradients exist across the sintered layer, which cracks up.

To achieve long life, the wall should be repaired by ramming new lining material against the sintered layer at regular intervals, e.g. after 15-20 melts. It is possible, with such treatment, to obtain some 200 melts from an acid lining such as quartzite. Basic linings have larger thermal expansion (and at temperatures of the order of 1700° C, violent shrinkage), and consequently have shorter life.*

The remarkable change in electrical resistivity of refractory materials with temperature can be used to provide a continuous and accurate indication of the state of the lining of a melting furnace. Measurements made on a number of materials by K. G. Quicke and the author showed that the resistivity could be represented with fair accuracy by the expression:

$$\rho_\theta = K\theta^{-n} \text{ ohm-cms} \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}1$$

n being in the region of 6 or 7. Thus at 1000° C resistivity falls, taking $n=6$, to one-millionth of its value at 100° C. Thermal conductivity, however, does not increase greatly with increasing temperature (below the sintering temperature), so that a nearly

* A detailed treatment of the metallurgy of the induction melting furnace, and of the forming and repairing of linings, is given in A. G. Robiette's book, *Electric Melting Practice*.[36] See also British Patents Nos. 303574, 310458, 347986, 422984, 521048.

uniform temperature-gradient exists through the lining. Assuming a linear gradient from 1500° C to 100° C through a lining t cms thick, and taking $n=6$, the resistance of the lining, between hot and cold faces, is:

$$R=\int_{o}^{t}\frac{K}{a}\theta_x{}^{-n}.dx \text{ ohms}$$

where $$\theta_x=\left(100+\frac{x}{t}1400\right) \text{°C},$$

a=wetted area of lining, cm²,

i.e. $$R\doteqdot\frac{Kt}{7000a\times 100^5}$$

or about 1/70th of the "cold" resistance of the lining (i.e. resistance at 100° C).

As the lining becomes progressively more eroded and the sintered layer moves outwards towards the furnace coil, the gradient increases and the lining resistance falls still further. Thus any circuit arrangement for measuring the lining resistance can be used to indicate its state. One such arrangement uses the molten charge as one electrode and the coil as the other (see British Patent No. 506665). Another method is to embed independent electrodes in the refractory; an advantage of this latter method is that it permits better electrical insulation to be used around the coil itself. It should be noted, however, that it is desirable to earth the molten charge, and it is therefore convenient to use the charge as one electrode in any case. From what has been said of lining resistivity at high temperatures, it will readily be understood that a sufficiently low-resistance connection to the molten charge can be obtained by burying an "earth mat" in the bottom of the lining, in the region which becomes sintered when the furnace is in use.

It may be necessary to hold a molten charge at temperature for some time before pouring. The furnace power-input required to do this is, of course, that which supplies the losses but contributes no additional heat to the charge. The rate of heat conduction through the lining, and the radiation and stray heat losses, form the "fixed loss" from the full molten charge, and $=P_F$ KW, say. The electrical loss (comprising coil, bus-bars, and capacitors losses) is directly proportional to the furnace load, i.e. $P_{Cu}=ax$ KW, where x is the actual load, and a is the ratio of copper-loss to total input at full load. Then the input power required to hold the molten charge is:

$$x_H = P_F + a x_H = \frac{P_F}{1-a} \text{ KW} \quad . \quad . \quad . \quad . \quad 9{\cdot}2$$

For example, the fixed loss P_F in a 2500-lb stainless-steel furnace was 40 KW, and the electrical loss was 81 KW at the full-load input of 350 KW. Then $a = \frac{81}{350} = 0{\cdot}231$,

and holding power $\quad x_H = 40 + {\cdot}231 x_H$

$$= \frac{40}{1 - {\cdot}231} = 52 \text{ KW.}$$

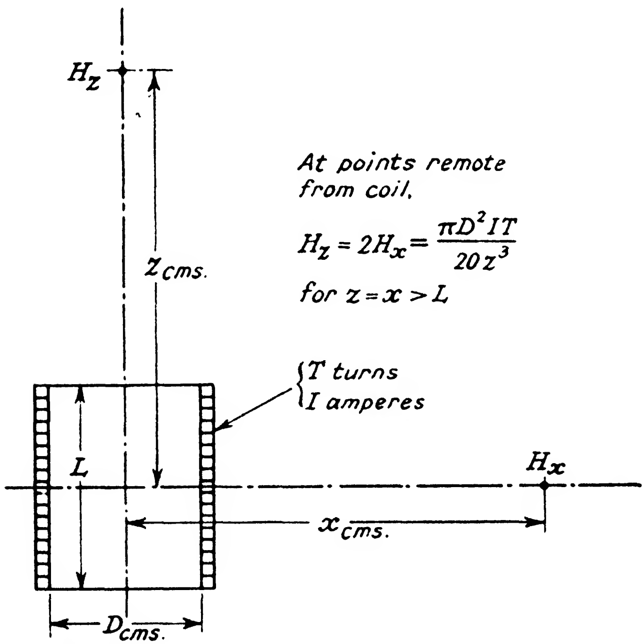


Fig. 9·1. Field-strengths Outside Furnace Coil.

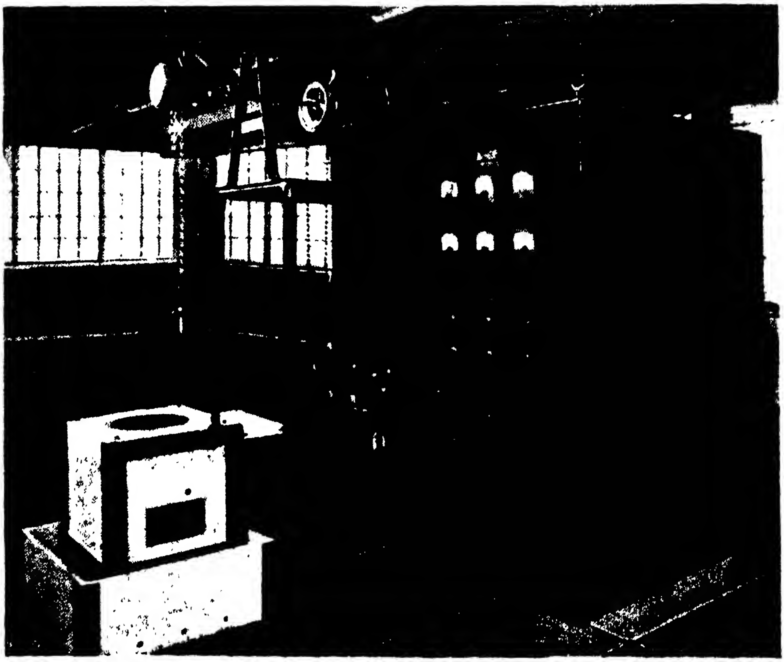
The fixed loss and electrical losses are characteristics of the furnace and can be calculated from readings of alternator output KW, and furnace cooling-water temperature rise and rate of flow with and without power on, all with a full molten charge.

The problem may arise in connection with melting furnaces of considerable power, but not of the type using lamination packets to confine the external field, of the minimum permissible distance from the coil to structural steel work. The external field-strength at points relatively remote from the coil is given approximately from

$$H_z = \frac{\pi D^2 IT}{20z^3} \quad . \quad . \quad . \quad . \quad . \quad . \qquad 9{\cdot}3$$

$$H_x = \frac{\pi D^2 IT}{40x^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 9{\cdot}4$$

where D = coil diameter, centimetres.
I = furnace coil-current, ampères.
T = furnace coil turns.



[*Courtesy of Birlec Ltd.*

Fig. 9·2. Ajax 50 KW 3000 cycles/sec., 400-volt Lift Coil Melter; 200 lb. Brass Charge.

x and z are distances, in centimetres, measured as shown in Fig. 9·1. Equations 9·3 and 9·4 are sufficiently accurate for typical melter coil proportions, provided that x or z is equal to or greater than coil-length L.

The corresponding heating effect due to eddy currents in a thick metal plate to which this stray field is tangential is:

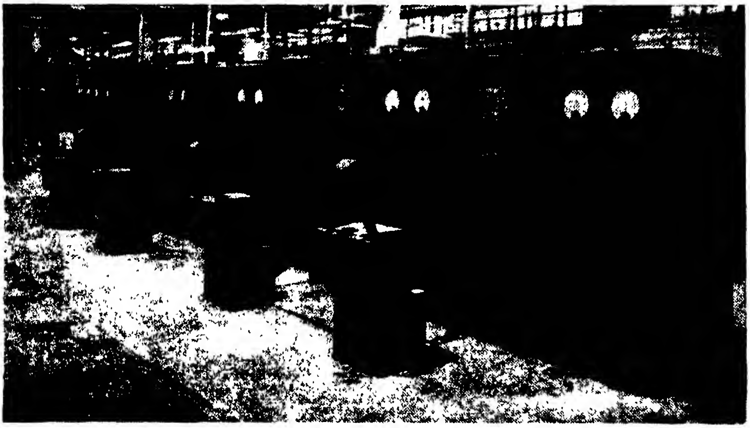
$$P = \frac{H^2\sqrt{\rho\mu f}.10^{-7}}{8\pi} \text{ watts/cm}^2 \quad . \quad . \quad . \quad . \qquad 9{\cdot}5$$

(see p. 173).

and should not usually exceed ·1 if the structure is to run reasonably cool.

Stray heating of structures more than $3D$ away from coil in the x direction, or $4D$ in the z direction, will not usually be severe, but depends, of course, upon the coil power-rating. So far as possible, structures should be arranged so that the larger surfaces are tangential rather than normal to the field.

The melting furnace seen in Fig. 9·2 is rated at 50 KW, 400 volts, 3000 cycles/sec. It is of the "lift-coil" type, i.e. the coil unit and furnace casing can be lifted bodily, leaving the crucible, containing a molten charge, free for pouring. A second crucible containing



[*Courtesy of Birlec Ltd.*

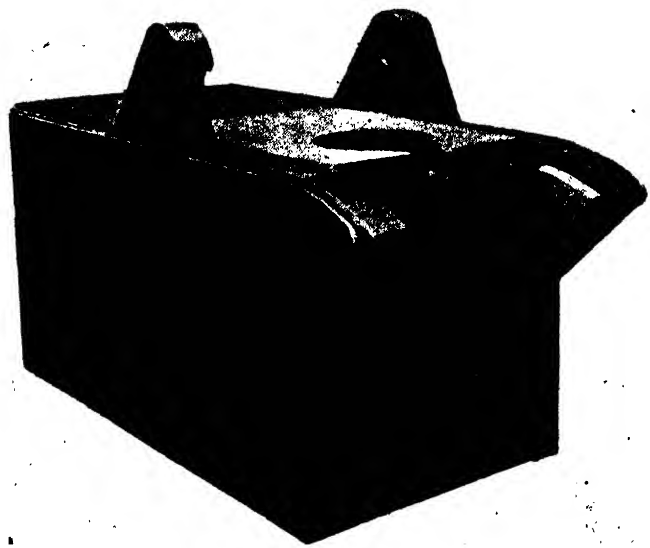
Fig. 9·3. Installation of Five Melting Units for Independent Simultaneous Operation from Common 500-v 10,000 cycles/sec Supply.

a cold charge takes its place, and another melt commenced with a minimum of delay. This kind of furnace is widely used in American non-ferrous foundries, and will melt 200-lb brass charge in 50 mins, the charge being contained in a conducting (graphite) crucible.

An installation consisting of a group of four 30-lb melters and one experimental station is seen in Fig. 9·3. The control cubicle for each furnace contains the line contactor, the correction capacitors, their contactors and selector switches, and meters reading furnace KW and reactive KVA.

An interesting feature is that all furnaces may be operated simultaneously, with independent power-control of each. This is obtained by having a tapped reactor in each line circuit. The rotary selectors can be seen in the middle of the right-hand side of the cubicles; they have six positions, and side-contacts interlock with the line contactors to ensure off-load operation. This method of power control is simple and effective, provided that care is taken to avoid series resonance, i.e. the furnace capacitors must be selected to give zero or a slightly lagging reactive-KVA reading.

The four 30-lb furnaces are for precision casting of high-temperature alloy steels, and are used in conjunction with moulds made by the disappearing-wax process. Flexible leads are used between the cubicles and the furnace-coils, and are water-cooled.



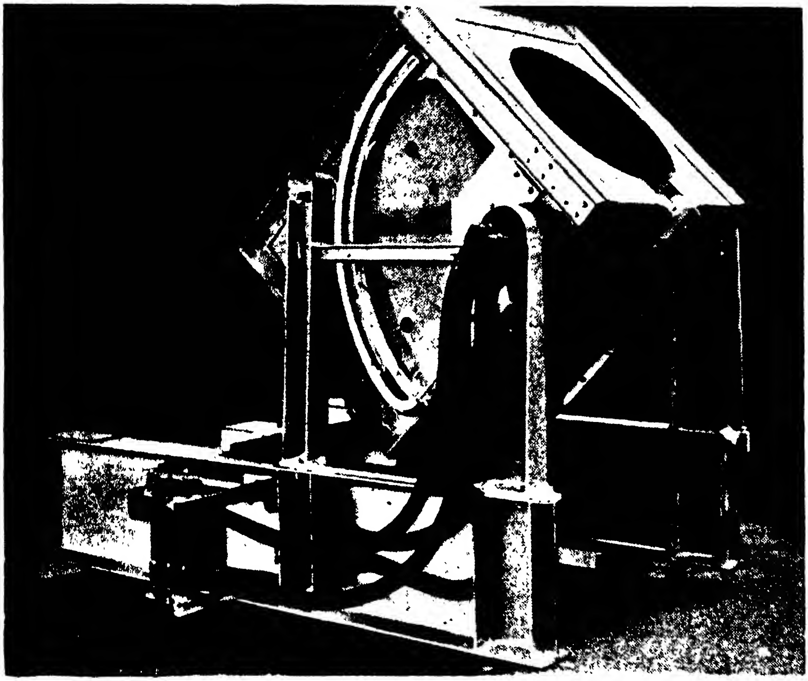
[*Courtesy of Birlec Ltd.*

Fig. 9·4 (a). 200-lb Melting Furnace with Motor-operated Tilting Gear.

Each furnace is rated at 17 KW, 500 volts, 10,000 cycles/sec. The 75-KW alternator supplying them is equipped with an automatic regulator which maintains constant voltage at all loads.

Fig. 9·4 (a) shows a 200-lb 75-KW melting furnace built for high temperature resistance-alloys research. The tilting mechanism is motor-driven, and is based on the double-pivot principle. Initial tilting occurs about an axis through the front base-line of the furnace box, until the top pivots have moved forward into the overhung bearings; tilting for pouring then occurs about the axis of the top pivots, which is arranged to pass through the spout. The advantage of the double-pivot scheme is that it allows the mould or ladle to be placed in position before tilting the furnace.

The Allis-Chalmers Manufacturing Company of Milwaukee has developed mercury-arc converter equipment for supplying induction melting furnaces. The "inverter" transforms 60 cycles/sec three-phase power to single-phase H.F. power at, e.g., 880 volts, 300 KW, 800 to 1000 cycles/sec, with a basic circuit arrangement similar to that shown in Fig. 2·8. A conversion efficiency in excess of 90%,



[*Courtesy of Messrs. Allis-Chalmers Manufacturing Co.*

Fig. 9·4 (b). 2000-lb. Induction Melting Furnace for Inverter Operating prior to Installation. Side cover removed to show Electrical and Water Connections.

even at only half-load, is claimed.* Melting furnaces from 750-lb capacity, and holding furnaces of 6000-lb capacity, have been built. The former will melt down alloy steels from a cold charge in about 45 minutes; the latter, supplied with molten steel from large arc furnaces by means of bottom-pouring ladles, can, by means of the phase-shift grid-control regulator, hold the charge temperature constant within $\pm 10°$ F over a prolonged continuous-pouring operation. (See Fig. 9·4 (b).)

* See footnote, p. 292.

Since these converters are self-oscillators, the frequency varies somewhat according to the amount and condition of the charge, but no capacitor-switching is required. The mercury-arc converters and the capacitor bank are both water-cooled; S. R. Durand* states that it has been found desirable to preheat the converters for a few minutes, using low-voltage 60 cycles/sec power, before starting up, to ensure reliable operation.

Through heating

The application of induction through heating of steel bars and billets for forging was referred to in Chapter 6, and three examples were quoted. This method of heating for forging is becoming very popular in Great Britain, particularly where full mechanisation is practicable, and several high-power plants are now in operation.

Other applications of through heating are:

Continuous hardening and tempering of bar stock.

Hot cropping.

Preheating for welding and during welding.

Heating after welding, for stress relief (400 to 650° C), or for normalising of weld material (e.g. 950° C).

Heating of aluminium alloy and brass billets for hot pressing (e.g. at 450-500° C and 650° C respectively).

Heating of steel or non-ferrous rings, e.g. gears for shrink-fitting on to wheels or bosses. (This operation often proves to be an excellent application for mains-frequency induction-heating, using an iron-cored transformer in which the ring forms the single-turn secondary circuit.)

The high heating speed possible with induction-heating is an important advantage of the method. This, together with the relatively small size of the furnace, and the remarkably cool and clean working conditions, result in a very different shop layout from that associated with conventional oil- or gas-fired soaking furnaces.

Heating times

The minimum heating time for a bar or billet is found approximately by assuming the whole surface suddenly brought up to and held at the final temperature required, and calculating the time taken for the centre temperature to rise to within 25 or 50° C of

* See "Electronic Frequency Converters for Induction Melting", S. R. Durand, *Iron Age*, Sept. 25th, 1947, Vol. 60, No. 13.

this, according to the degree of uniformity desired. Obviously, while energy is being supplied there must be some temperature gradient, but this disappears rapidly after the billet is removed from the coil. The heating time depends upon the thermal conductivity, density and specific heat of the material, and is much shorter for aluminium and copper, for example, than for steel.* Using the method of calculation developed by E. D. Williams and L. H. Adams,(72) and assuming a ratio of temperature-difference (surface to centre) to temperature rise of ·02, we obtain for round steel bars or billets: $t=15d^2$ secs, d being billet diameter in inches.

However, with induction-heating the surface is not instantaneously brought up to the final temperature; on the other hand, heat is generated in a layer of some thickness. It is found in practice that the minimum heating time for solid steel billets is approximately: $t=18d^2$ secs.

Although small billets can actually be through-heated in somewhat shorter times even than this, it is not always practicable to use such high rates, and under average conditions the time is:

$$t=25d^2 \text{ secs} \quad . \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}6$$

The corresponding average heating time for square-section solid steel billets is:

$$t=32b^2 \text{ secs} \quad . \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}7$$

where b'' is the length of the side of the square. In a round section all points on the surface are equidistant from the centre; in a square the RMS value of the distance from the centre is $1{\cdot}13b$. The heating time for the square is therefore $25\times(1{\cdot}13b)^2=32b^2$.

The heating times given from equations 9·6 and 9·7 hold for bars or billets enclosed within a conventional helical coil.

Power ratings for through heating for forging

Fig. 9·5 shows the through-heating time for cylindrical and square steel billets, according to equations 9·6 and 9·7.

Consider the round section, for which $t=Kd^2$ secs, K being 25. The heat content at 1250° C per inch length is:

$$\text{Heat content}=H\times\frac{\pi d^2}{4} \text{ KW secs,}$$

where $H=$ heat content per cubic inch
$=102$ KW secs.

* Heating time is inversely proportional to the thermal diffusivity α of the material; $\alpha=\dfrac{\text{Thermal conductivity}}{\text{Specific heat}\times\text{density}}$. Some average values of α, taking steel as unity, are: copper 10, aluminium 7, aluminium alloys 5, brass 3.

The average power-input to the billet over the duration of the heat is:

$$P=\left(\frac{\text{Heat content}}{\text{Heating time}}+\text{Average radiation loss}\right)\text{KW per inch length.}$$

At 1250° C, free radiation from the surface would be approximately 200 watts per square inch, and for typical conditions the average radiation loss may be taken as one-quarter of this figure.

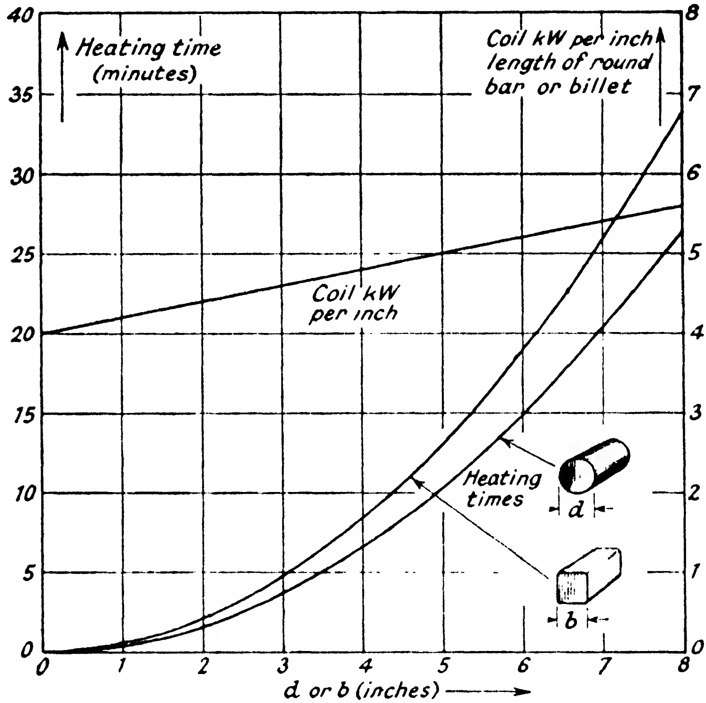


Fig. 9·5. Average Through-heating Times for Steel Billets, and corresponding KW per inch of Billet Length.

Thus for the heating time given from equation 9·6,

$$P=\left(\frac{H\pi d^2}{4Kd^2}+\cdot05\pi d\right)\text{KW per inch length}$$

$$=(3\cdot2+\cdot16d)\text{ KW per inch length.}$$

End-radiation loss is negligible for long billets, or a train of short billets passing through a coil. Assuming a coil efficiency of 80% we get as an empirical rule:

$$\text{Coil power-input}=\left(4+\frac{d}{5}\right)\text{ KW per inch length} \quad . \quad 9\cdot8$$

This expression is plotted in Fig. 9·5. It is an interesting point that coil power rating, for a given heating rate, is principally a function of heated length, and is influenced to only a minor degree by billet diameter, unless the diameter exceeds 3 inches or so. Through-heating induction furnaces for solid round sections, therefore, average 20 to 25 inches length per 100 KW of input power.

Heating times and power ratings for thick-walled hollow steel cylinders lie between limits corresponding to (i) solid cylinder $\left(\frac{d_i}{d_o}\to 0\right)$, and (ii) plane slab $\left(\frac{d_i}{d_o}\to 1\right)$.

Empirical expressions are

$$t \doteqdot m(d_o-d_i)^2 \text{ seconds} \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}9(a)$$

$$P_{\text{coil}} \doteqdot \left[\frac{n(d_o+d_i)}{(d_o-d_i)}+\frac{d_o}{5}\right] \text{KW per inch length} \quad 9{\cdot}9(b)$$

For (i), $m=25$, $n=4$; for (ii), $m=50$, $n=2$. d_o and d_i are outer and inner diameters, in inches.

No thermal-conduction time is required for thin-walled cylinders (wall thickness $\lesseqgtr p$); the heating rate for these is limited only by metallurgical considerations and available power.

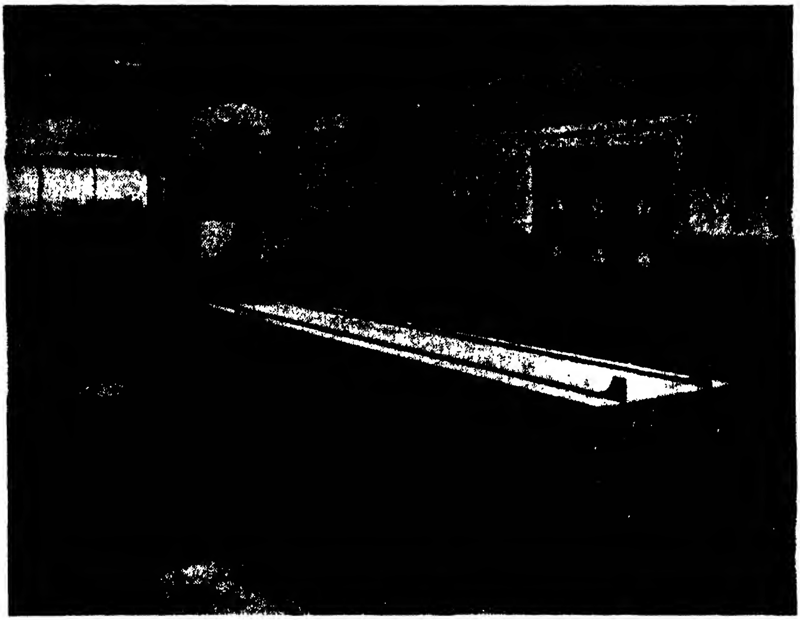
The overall efficiency obtainable, expressed in lb weight of steel heated to 1200-1250° C per Kilowatt-hour of mains input energy to the motor-alternator set is from 4·5 to 6·5 lb/KW hour; 5·5 lb/KW hour should be realised comfortably where it is practicable to use coils designed specifically for each billet size to be heated.

The bulk of through-heating work is done with motor-alternator sets, within a frequency range of 1000 to 10,000 cycles/sec. At the latter frequency, ½″ diameter bar is the smallest size (in steel) which can be heated economically.

Recent British developments in heating for forging employ a "conveyor" type of coil, through which billets are carried continuously on a moving refractory hearth. In one design, a rotary hearth is used in conjunction with an arcuate coil. The heating time per billet is slightly longer than the minimum obtainable with a simple helical coil, but there are great advantages in mechanical handling without the need for skids or guide-rails inside the coil. The "conveyor furnace" efficiency is about the same as that of the orthodox induction furnace (70 to 75 %, based on coil input power), because close coupling and good thermal insulation are possible. One such furnace, of the rotary type, rated at 250 KW, 2200 cycles/sec, heated 300 billets per hour (2¼″ square × 4½″ long) to 1225° C, equivalent to 7·5 lb of steel heated per KW hour input to the coil, or 6·2 lb per KW hour from the 50 cycles/sec mains.

Induction-heating for sintering and hot-pressing

The heating of metallic powders by direct induction in the powder is not usually satisfactory, as it is difficult to obtain uniformity of temperature. In some metal-powder operations, however, the powder can be heated in a graphite mould machined to give to the powder, under compression, a shape and size conforming closely to the required finish-dimensions. In such cases induction-heating of the mould is very attractive because of the cleanliness of the method, the ease with which the furnace can



[*Courtesy of Birlec Ltd* .

Fig. 9·6. Installation of Four Sintering Furnaces.

be built into the press, so that heat and pressure can be applied simultaneously, and the precise control of temperature.*

Fig. 9·6 shows an installation for the manufacture of tungsten-carbide die-rings. Four furnaces, each mounted on a bogie-truck, cater for mould sizes from 6″ to 24″ diameter; the largest furnace, rated at 100 KW, 800 volts, 3000 cycles/sec, is seen in the working position within the hydraulic press. The structure in the

* See, for example, British Patents Nos. 383387, 504803, 506728, 530995, 530996, 532174.

foreground houses the furnace power-factor-correction capacitors. It is an interesting fact that the electrical characteristics of graphite change relatively little during heating from cold to 1500° C or so, so that the furnace load (at any given voltage) and the power factor remain almost constant. A practical consequence of this is that no capacitor contactors are required; it is sufficient to select the appropriate capacitance before switching on.

The motor-alternator and its auxiliary equipment is in a sub-station behind the press. The control panel is built into the wall of this sub-station. A hand-operated isolator is used, not a line contactor, and this is interlocked with the field-switch so that the field-circuit is opened first and closed last. A second interlock prevents the field contactor from closing unless the hand-controlled field-regulator is in the position corresponding to minimum voltage. Meters on the control panel indicate alternator voltage and current, field current, and furnace KW and KVAR. A continuous temperature recorder can be seen to the left of the control panel.

Cooling-water is fed to the centre of the furnace coil and is discharged at the ends, whence it is led to two bucket-type water-flow relays mounted in a sump to the right of the control panel. These relays are arranged to open the alternator field-circuit in the event of a reduction in water-flow below a predetermined minimum.

Surface hardening

The great majority of applications call for a hardened layer depth of ·01″ to ·1″ (·25 to 2·5 mm). The lowest frequency with which a 2·5-mm layer can be realised is about 10,000 cycles/sec, while a ·25-mm layer requires a frequency of at least one million cycles/sec. At the present time, most induction surface-hardening applications employ frequencies from about 300 KC/S to 1500 KC/S, and there is a tendency towards the use of higher frequencies. Publicity has been given to the possibility of using extremely high frequencies (of the order of 20 MC/S) to heat rapidly a skin so thin that it may be "self-quenched" by conduction to the cool interior of the steel, so producing a quench-hardened skin without the use of a water spray or other externally applied quenching medium.(74)

400 KC/S is a useful and typical frequency, and the following

table shows the heating time and specific power for realising various case-depths at this frequency:

Case depth, inches	*Heating time, seconds*	*Specific power KW/sq. in*
·020	·15	36
·030	·40	23
·040	·70	17
·050	1·3	13
·080	3·0	9
·100	4·5	6

In most applications, using this frequency, the specific power required is from 10 to 20 KW per square inch.

Plain carbon steels are usually employed when hardening is to be done by induction-heating, though alloy steels are sometimes used because of the special properties required in the unhardened core. The carbon content should be not less than ·35%, nor greater than ·60% if core ductility is important.[77]

A list of suitable steels is given below. B.S.S. 970, 1942.

EN8	·40 C	Normalised
EN9	·55 C	Normalised or cold-drawn
EN10	·55 C	Oil-hardened and tempered
EN11	·60 C, ·5 Cr	" " "
EN12	·40 C	As EN8, but O.-H. and T.
EN15	·40 C, 1-1·3 Mn	O.-H. and T.
EN16	·25-·40 C, Mn, Mo	O.-H. and T.
EN18	·35-·45 C, 1·0 Cr, ·6-1·2 Mn	
EN19	·35-·45 C, 1·0 Cr, Mo, ·5-·8 Mn.	

The previous heat treatment of the steel is important. Because of the very short heating times for surface hardening there is little diffusion, and it is necessary to *start* with a homogeneous fine-grain structure (sorbitic), such as is obtained by previous hardening and tempering.*

Normalised or (worse) annealed structures are undesirable for surface hardening. The pearlite hardens, but the ferrite persists, and the resultant duplex structure has low hardness. It is because of this that parts which are to be copper-brazed prior to surface-hardening should be made from a relatively high-carbon steel,

* See also "Influence of Rate of Heating of Steel on the Pearlite-Austenite Transformation, with particular reference to Surface-Hardening"—Eilender and Mintrop, *Stahl und Eisen*, Feb. 26th, 1948, Vol. 68, p. 83.

such as EN9, in which the proportion of ferrite to pearlite will be low enough to permit appreciable diffusion of the ferrite during the short heating time of the hardening operation.

To illustrate the importance of the previous heat-treatment, the following example is quoted, showing the hardness figures realised on specimens of ·48 C ·41 Mn steel previously heat-treated in three ways:

	Surface hardness (VPN/30)	
Previous heat-treatment	·030″ case	·060″ case
Hardened and tempered (Sorbitic)	757	823
Normalised at 850° C (Fine pearlite, plus ferrite)	550	780
Annealed at 850° C and furnace-cooled (Coarse pearlite and ferrite)	530	622

The ·030″ case corresponds to a shorter, and the ·060″ case to a longer heating time. It is clear that it is desirable to start with a hardened and tempered part, although this will mean higher initial and machining costs.

If medium carbon steel bar is used for components which are to be surface hardened, the bar size must be large enough to require the removal of its surface layer for a depth of not less than ·015″-·030″ in the regions to be hardened. This is because of the surface decarburisation which occurs during manufacture of the bar.

The hardness obtainable is about the same as for a carburised surface, but the wear resistance is somewhat lower. There is not enough data yet concerning shock resistance and fatigue resistance*; tempering at 150 to 180° C improves shock resistance.

The chief advantages of induction surface hardening are speed and cleanliness, low power-costs, absence of scale, and minimised distortion, enabling finish-machining prior to hardening. The method is applicable mainly to regular shapes and small components, though long parts of regular section can be hardened by the continuous progressive method.

Induction-heating of carburised parts

A combination of carburising and induction hardening processes is often convenient, particularly where the shape or size of the component makes it difficult to avoid deep heating. The component is carburised to the required depth, and in the subsequent

* Information on fatigue resistance will be found in an article by R. J. Brown, *Sheet Metal Industries*, Vol. 24, No. 240, April, 1947. See also reference 77 (b).

induction-heating process it does not matter that the heat-pattern extends beyond or deeper than the zone to be hardened. In this way large components can be surface hardened, using an H.F. source of comparatively low power rating.

The advantages of induction-heating for final hardening of carburised parts are speed, cleanliness, consistent results without the need for skilled operators, and, in most cases, considerably less distortion than is obtained by quenching from a batch furnace. The localised heating inherent in the H.F. method often makes it unnecessary to copper-plate parts which are to be kept soft; the piece is carburised all over, but hardened only locally.

The manufacture of gear-wheels could be simplified and cheapened if distortion did not occur during hardening, for then all machining could be carried out prior to this. The localised heating obtainable by induction methods is very attractive in this connection, but despite much publicity to the contrary, true surface heating suitable for direct-hardening steels cannot yet be realised for other than a very narrow range of gear sizes, if the whole gear is to be treated simultaneously.*

The tooth contour and the correspondingly large surface demand high power-input at a high frequency. Only coarse-pitch teeth, in which a hardened skin of the order of ·10″ to ·125″ depth is permissible, can be treated in motor-generator equipments (10 KC/S maximum at present), and few industrial valve-generators have as yet been built with power ratings of more than 50 KW. "Tooth-by-tooth" hardening is only applicable to coarse teeth, and inevitably gives soft areas at regions of overlap. Until suitable generators become available, enabling direct-hardening steels to be used, a possible solution is to combine carburising with induction-heating for final hardening. In this way, although the teeth are heated right through, the skin is hardened only to the depth of the carburised layer, and the body of the gear, which remains relatively cool, does not distort severely. A carburised alloy-steel gear, 8½″ overall diameter, with 42 teeth 1¼″ wide, was heated in this way in 25 secs (60 KW coil input power, falling to 40 KW at the change-point, 10 KC/S) and subsequently oil-quenched; the hardness was uniform over the whole tooth surface.

* An induction method of obtaining contour-hardness in gears by preheating the teeth to 600° C or so has been described by R. J. Brown, *Sheet Metal Industries*, March, 1947; also *Automobile Engineer*, October, 1947, and British Provisional Patent No. 6204/46.

There is another possible method of "surface" hardening relatively large gears, and this is briefly described in the next section.

Surface hardening of through-heated parts by controlled quenching

The process of induction surface hardening dealt with in the preceding paragraphs and in Chapter 6 depends upon the rapid generation of heat in a thin surface-skin.

Paradoxically, a relatively thin hardened layer can be generated on the surface of a carbon steel article which has been through heated—or heated to a considerable depth—by using a suitable quenching technique.

A hardened structure is produced in a carbon-steel only if the *quenching rate* is above a certain critical value. This critical rate can be reduced, e.g. by the addition of nickel, resulting in "deep hardening" steels. There are also special steels in which the critical quenching rate has been somewhat increased, and these are called "shallow-case" steels.

If, then, quenching of a deeply-heated part can be controlled so as to produce a large thermal gradient only very near the surface of the part while the interior temperature falls (due to heat flow to the cooled surface) relatively slowly, only a superficial layer will harden. Induction-heating of the part, although obviously not essential, is an advantage because of the ease with which it can be combined with the quench system. It is emphasised, however, that the "surface" hardening obtained in this way is due to the precise control of quenching rate, and not to the use of induction-heating.

Handling equipment and feed mechanisms

The proper choice of handling equipment to be used in any application of induction-heating is of the greatest importance. The ideal mechanism is one which results in a steady load being continuously imposed upon the H.F. generator, a condition which it is possible to approach where mass-production is required. Otherwise, the KW rating of the generator must be greater than is strictly necessary, since the generator is idle or only lightly loaded when it might be doing useful work, and the peak power demand is correspondingly greater.

Steady loading is impossible with a single work-coil operating at constant voltage (except for continuous progressive heating), since the power developed in the work varies considerably during the heating cycle, and the useful power taken is zero during the intervals between unloading and reloading. Multiple work-coils with a staggered loading sequence result in an almost steady load, the amplitude of the power ripple depending upon the number of coils used. This is illustrated in Fig. 9·7, which shows the power-time curves for single and multiple coils heating steel bars for forging, the output of bars per hour being the same for the two arrangements. The same steady loading is, of course, obtained if, instead of several coils, a single coil is used to energise a

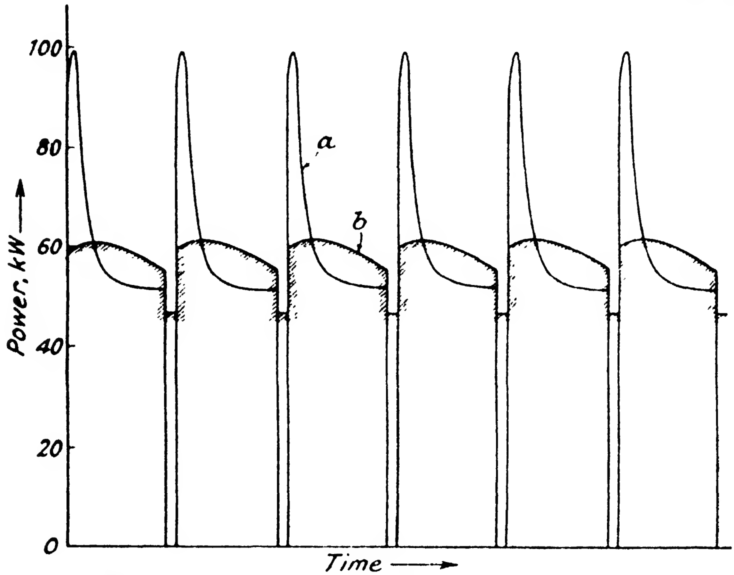


Fig. 9·7. Power-time Curves for (a) Single Coil, (b) Six Coils, Sequence-loaded, with same output of pieces per hour in each case.

secondary inductor having several work stations in it and loaded in sequence.

A second great advantage of multiple work stations with sequence loading is that the overall power factor is much more nearly constant than is the power factor of any one coil. Consequently, the amount of power factor correction capacitance need not be altered during the heating process, and the total condenser KVA to be installed is reduced. Alternatively, the frequency swing in self-oscillator generators is greatly reduced.

There is another factor which may necessitate the use of multiple coils. The required output of heated parts may be so great that it is impossible to heat them quickly enough in a single coil, even with unlimited available power. This is because of the time required for heat to be conducted inwards to the centre of the work from the heated surface layer. This time is, of course, determined by the dimensions of the work, the depth of the heated layer, and the degree of temperature uniformity aimed at.

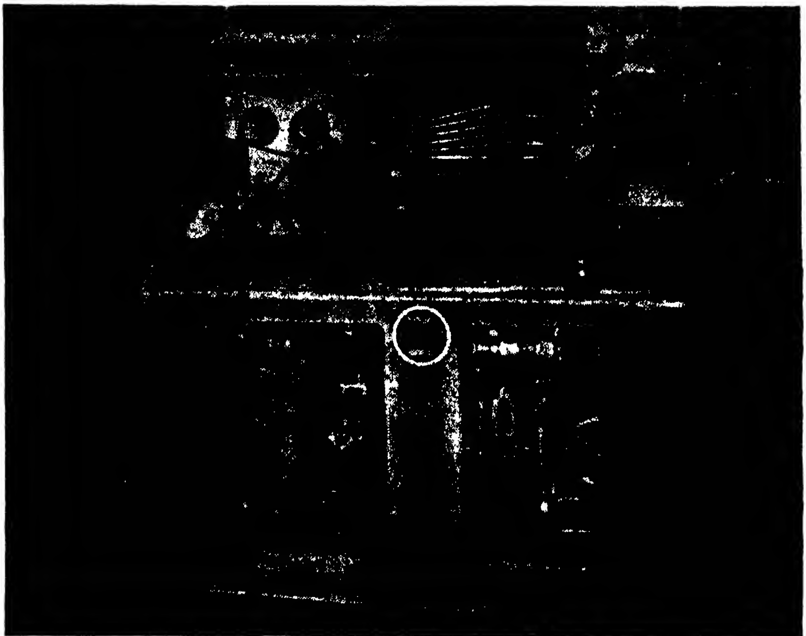
Continuous progressive heating of long bars, either for surface hardening or through heating, forms an ideal load, as conditions are quite steady and the full output of the generator can be maintained continuously. This applies also to the steady progression of a train of work-pieces, slugs, billets, etc., through a coil.

There are, however, many applications, particularly of surface hardening, where "single shot" heating must be used, and where, consequently, the load-power varies considerably during the heating cycle. There are two ways in which the resultant poor load-factor on the generator can be improved. The intervals between unloading and reloading can be made very short by using high-speed automatic handling gear; and, secondly, automatic adjustment of the work-coil power can be made during the heating-cycle. The latter requires high-speed operation if applied to surface hardening of small components, and a solution will perhaps be found in terms of automatic impedance-matching. The use of very high surface-power densities in surface hardening minimises power variations due to magnetic changes in steel parts.

In general, induction heat-treatment involves handling of the components singly, or a few at a time, and short-time treatment, in contrast with the long-time batch treatment of older methods. The induction processes are thus more akin to machine-tool practice than to normal heat-treatment furnace practice. The heating and handling equipment usually occupies only a small floor space, and fits naturally into a flow-production line.

Fig. 9·8 shows the automatic handling equipment for surface hardening steel shafts at the rate of 300 per hour. Two types of shafts are treated; in the one, the part to be hardened is a plain round portion $\frac{5}{8}''$ diameter, and in the other, the treated part is splined, $\frac{3}{4}''$ diameter over the splines. The treated length is about $3\frac{1}{2}''$ in both cases, and the case depth is ·030″. Interchangeable "concentrators" are used, one for each of the two types of shaft. The shafts are finish-machined before hardening, and only a small percentage require a subsequent straightening operation.

The driving motor, cam-shaft, oil pump, and water filter are housed in a fabricated steel base; the concentrator and electromagnetic quench-water valve are located in a brass screening box above the mechanical gear. The magazine, to the right of this brass box in the photograph, is loaded by hand. Each shaft in turn is automatically taken from the magazine, fed progressively through the concentrator and its integral quench-spray, withdrawn again and discharged on to a roller conveyer. The H.F. power supply to the primary coil of the concentrator is switched on only

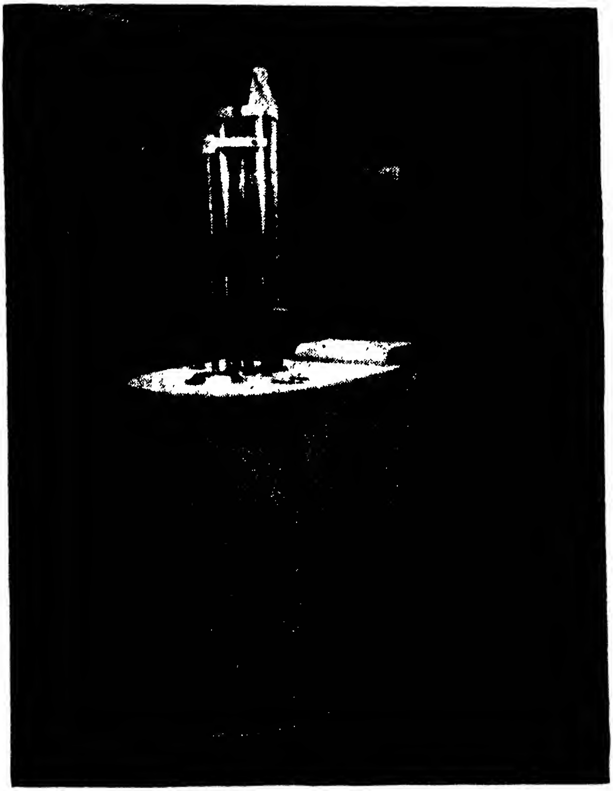


[*Courtesy of Messrs. Joseph Lucas Ltd. and Birlec Ltd.*

Fig. 9·8. Machine for Automatic Surface Hardening of Steel Shafts.

during the forward travel, through the inductor, of the part to be hardened. The quench-spray is maintained during the return stroke, so that the shaft is practically cold when discharged. A 20-KW, 400-KC/S valve-oscillator provides the H.F. power. This particular application of induction surface hardening showed remarkable saving compared to the lead-pot method, with three heat-treatments, which was formerly used, the initial cost of the equipment being recovered within a few months of installation.

Fig. 9·9 shows a machine for continuous progressive surface hardening of shafts up to 24" length. These are hollow, with an outside diameter of 13/16" and a bore of $\frac{7}{16}$", and have a line of small radial holes along one side. A shaft is mounted between centres, one of them spring-loaded, and fed vertically downwards at a controlled rate through a single-turn inductor and integral water

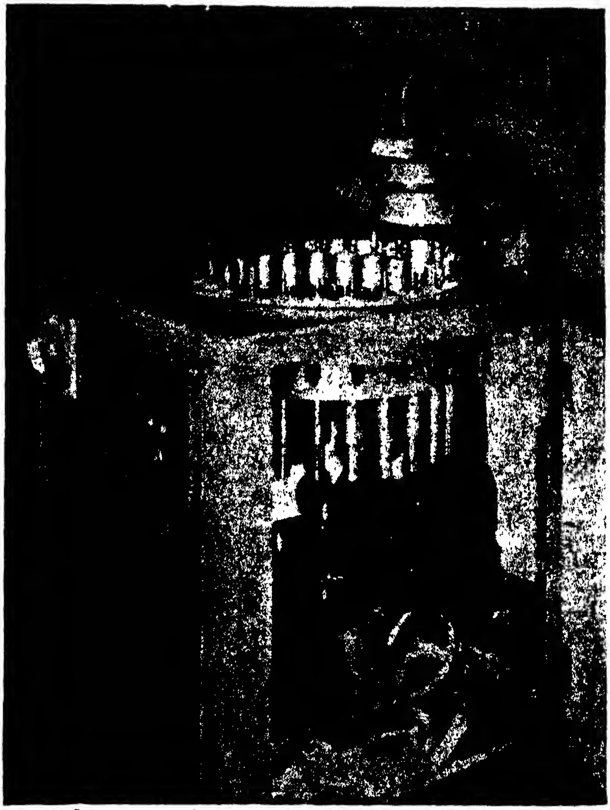


[*Courtesy of Birlec Ltd.*

Fig. 9·9. Vertical Machine for Selective Surface Hardening of Long Shafts.

quench-spray. At the end of the downward stroke, the drive is automatically disengaged and counterweights return the shaft quickly to the original position for unloading. H.F. power is switched on and off, to give the required hardened zones along the length of the shaft, by means of cams and limit-switches. The single-turn inductor is coupled to an 8-turn primary coil connected to a 20-KW valve-oscillator, the operating frequency being 375 KC/S. A hardened depth of ·040″ is realised, with a feed-rate of ·5″/sec.

An automatic machine for single-shot surface hardening of automobile ignition cams is shown in Fig. 9·10. Cams are placed by hand on stainless-steel spindles arranged in a circle round an



[*Courtesy of Messrs. Joseph Lucas Ltd. and Birlec Ltd.*

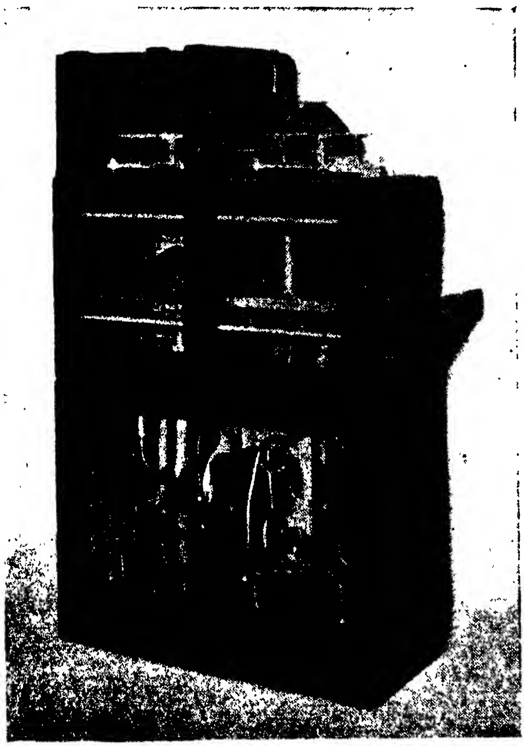
Fig. 9·10. Automatic Indexing Machine for Single-shot Surface Hardening of Cams.

indexing table; this table and parts of its mechanism are of aluminium, anodised to prevent corrosion.

Each cam, in turn, is elevated into the single-turn inductor of the word-head transformer. The heating time is 1·0 sec, and is followed by a quench-spray through holes drilled in the inductor; the cycle time is adjustable, but the normal production rate is

375 cams per hour. Several work-head transformers are interchangeable to suit different types of cam. To ensure good electrical insulation under arduous working conditions the primary coil of each transformer is embedded in wax; the rating of each is approximately 15 KW at 400 KC/S. A hardened layer depth of ·040″ is produced on cams having a surface area of about 6 cm^2.

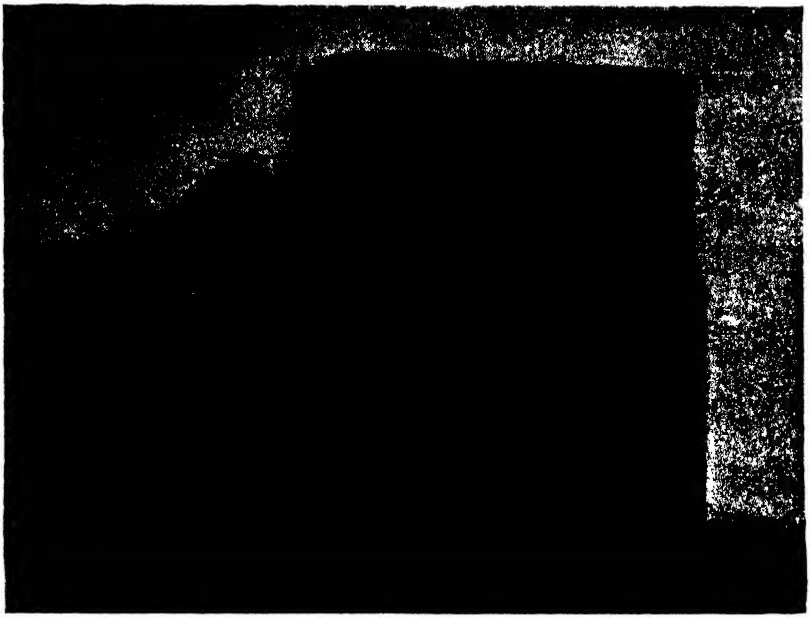
Another handling mechanism supplied with power from a



[*Courtesy of Messrs. Joseph Lucas Ltd. and Birlec Ltd.*

Fig. 9·11. Unit for Local Annealing of Headlamp Body Pressings, Rear View.

20-KW valve-oscillator is illustrated in Fig. 9·11. This shows a rear view, with guards removed, of a unit for local annealing of headlamp body pressings. There are two heating stations, and lamp-bodies are raised into the heating coils by means of compressed air rams, the two coils being loaded alternately. Unloading and reloading is taking place at one station while the other is heating. The coils are connected in series, and are continuously energised. A view of the loading table is seen in Fig. 9·12. Each safety door opens automatically when a heated pressing descends from its coil; the pressing is removed, another put in its place, and the door is then closed by hand. The cycle time is determined by a motor-driven camshaft, in conjunction with a continuously variable gearbox. The cams control air-valves in the supply to the rams, but additional valves controlled by the doors, prevent the rams lifting while the doors are open. Pneumatic door locks make it impossible to close a

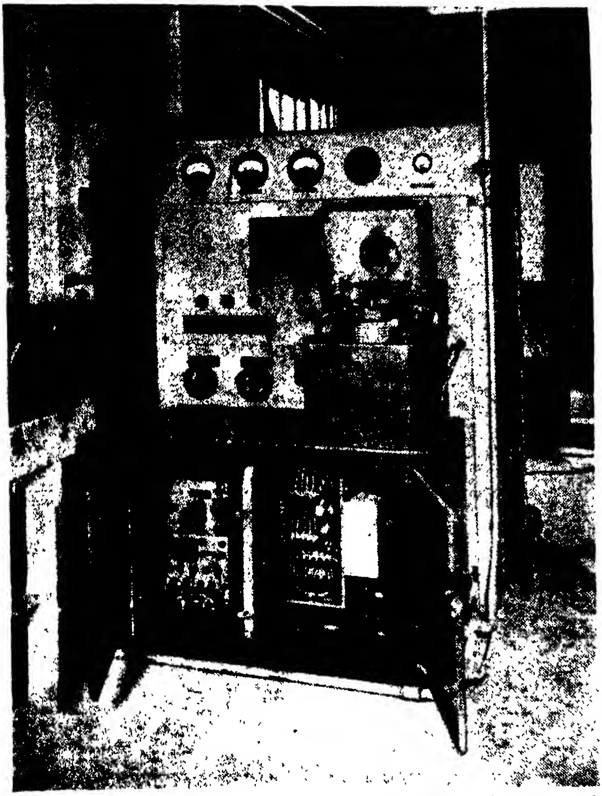


[*Courtesy of Messrs. Joseph Lucas Ltd. and Birlec Ltd.*

Fig. 9·12. Local Annealing Unit, Front View.

door if, through some delay, it has not been closed early enough for the pressing to be held up in the coil for the correct length of time. Those parts of the metal structure which are in proximity to the coils are made from brass to minimise stray loss. The handling equipment is designed to anneal steel bodies at any desired rate from 150 to 450 per hour.

Fig. 9·13 shows a simple form of handling equipment which is adaptable to a wide range of single-shot surface hardening and through hardening of small parts. The work inductor, of the "concentrator" type, is housed in a brass box mounted above a steel cabinet which contains the heat and quench timers. The large top cover over the furnace is opened only when it is necessary to change concentrators; to load a work piece, a small lid fitted in the top cover is opened. The lever on the right-hand side of the brass box raises or lowers a stainless-steel spindle, on which the work piece is located. After heating, the work is quenched in



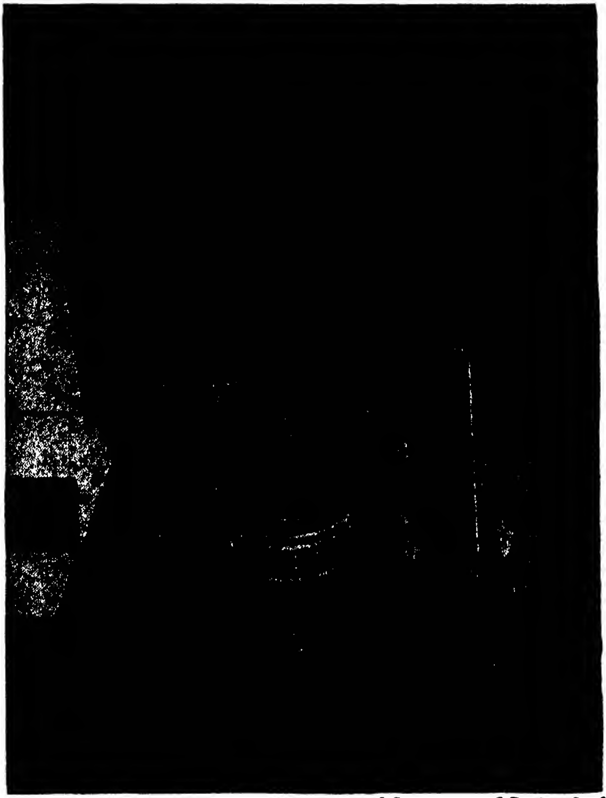
[*Courtesy of Birlec Ltd.*

Fig. 9·13. General View of 5-KW Local Hardening Unit.

position, the end of the heat-treatment cycle being indicated by lamp.

A close-up of the inductor box is seen in Fig. 9·14. This also shows a sprocket, located on the work-spindle, ready to be lowered into the concentrator, the primary coil of which can be seen. The operation here is through hardening of the sprocket teeth, the two rings of teeth being heated simultaneously. Other concentrators for use with this equipment treat small gears, spindles, and so on. The power supply is from a 5-KW valve-oscillator with a nominal frequency of 800 KC/S.

A recently developed induction-heater for bolt heading* is shown in Figs. 9·15, 9·16, and 9·17. This machine is automatic, and discharges steel bars, heated at one end, to a heading press, at any selected rate from 400 to 1000 per hour. The magazine in the top



[*Courtesy of Birlec Ltd.*

Fig. 9·14. Close-up of Work-head of Small Hardening Unit.

(Fig. 9·16) is loaded by hand with about a 30 minutes' supply of $\frac{3}{4}''$ or $\frac{7}{8}''$ diameter bars; the bars are fed to 6 loading mechanisms associated with a 6-coil heater unit (Fig. 9·17) and are heated for a selected length (e.g. equal to 3 or 4 diameters), and then discharged down the chute at uniform intervals. Adjustment is provided for various bar lengths and heated lengths.

* See also British Patent No. 593323.

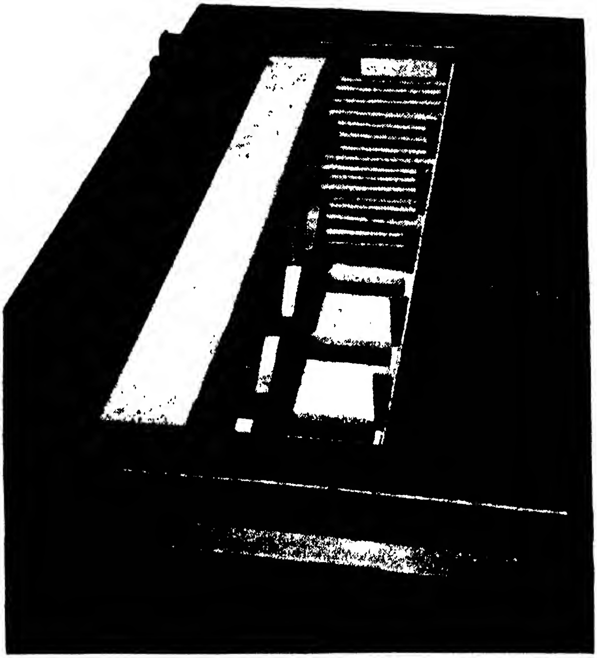
Electrically the 6 coils are in series across a 500-V 10-KC/S supply, but each is independently water-cooled. The sight-drains can be seen at the left-hand side of the front of the machine. The meter panel contains a supply voltmeter and ammeter, and sockets for plugging in a portable KVAR meter when making adjustments to the condenser bank to give unity overall power factor. The condensers are housed in the base of the machine, and the H.F. supply is through a lead-covered coaxial cable. The H.F. rating is 60 KW maximum; power control is by means of a tapped reactor, in the supply line, and also housed in the base.

[*Courtesy of Birlec Ltd.*

Fig. 9·15. Automatic Bar-heater for Bolt-heading.
60 KW, 10 KC/S.

A safety device is incorporated which protects the machine against possible damage in the event of a bar not being discharged properly, or of the magazine becoming empty. The ejection of a heated bar depends upon the presence of another bar to take its place, so that if the magazine were allowed to become empty, a heated bar would be fed into a coil a second time, and would certainly melt. To avoid this, a signal is sent to a timing unit each time a bar is due to be ejected. The bar, if properly ejected, momentarily closes a switch operated by a gate across the discharge chute, and a second signal causes the timer to relax. Failure to open the gate allows the timer to switch off the H.F. supply.

The handling mechanisms operate in sequence, resulting in an almost steady load on the H.F. alternator, the power ripple being only ± 2 or 3 KW. The overall power factor of the coil unit similarly is practically constant (variation about ± 5 KVA reactive), and is approximately ·22, requiring 315 KVA of correction con-



[*Courtesy of Birlec Ltd.*

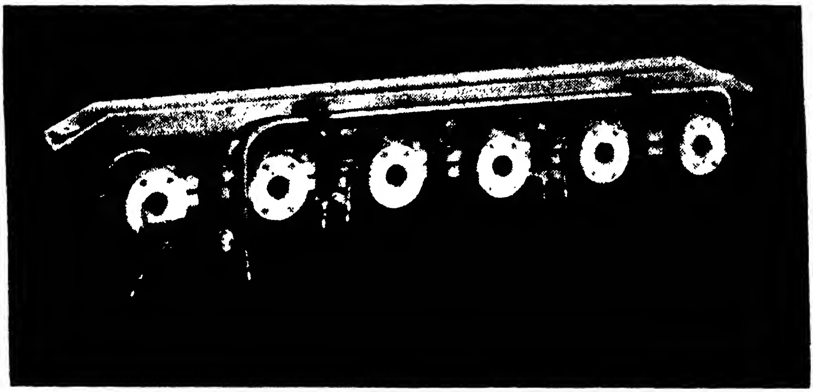
Fig. 9·16. Bird's-eye View of Bar-heater, showing Magazine partly loaded.

densers. A maximum of about 15 KW loss is required to be removed by the cooling-water, requiring a flow of about 1·5 gallons per minute. Electrical design data on this heater will be found in Chapter 6, section "Through Heating".

Radio frequency automatic soldering machine

Soldering by means of radio frequency heating has many advantages for quantity production. In some cases simple equipment is adequate for the purpose, but in others, in order to comply with specific requirements, machines built to very close limits of accuracy may be necessary. Fig. 9·18 shows a 3·5-KW radio frequency generator and continuous automatic soldering machine designed for sealing condensers of the wound-spool type into cylindrical brass bodies.

The requirements in this instance called for close tolerance in the finished overall length, a specified gap between the top of the condenser spool and the cap, and the ability to use the one machine for each of a considerable number of different types and sizes of condensers. The final product had to be air-tight, and during the course of the process no overheating of the insulation in the condenser was permissible.



[*Courtesy of Birlec Ltd.*

Fig. 9·17. 6-Coil Unit of 60-KW Bar-heater.

To comply with these requirements, the machine shown in Fig 9·18 was designed. A close-up of the unit with cover removed showing a condenser in the heating station, is reproduced in Fig. 9·19. The whole design follows machine tool practice; all the components being made to very small tolerances, and comprises a cast base to accommodate the driving gear, surmounted by a rotating head, divided for screening purposes into 6 compartments and containing pressure head, work seat, work-coil, and spring-loaded tongs in each compartment. Above this, under the top cover of the machine, are cam-operated plungers and forks which control the pressure heads.

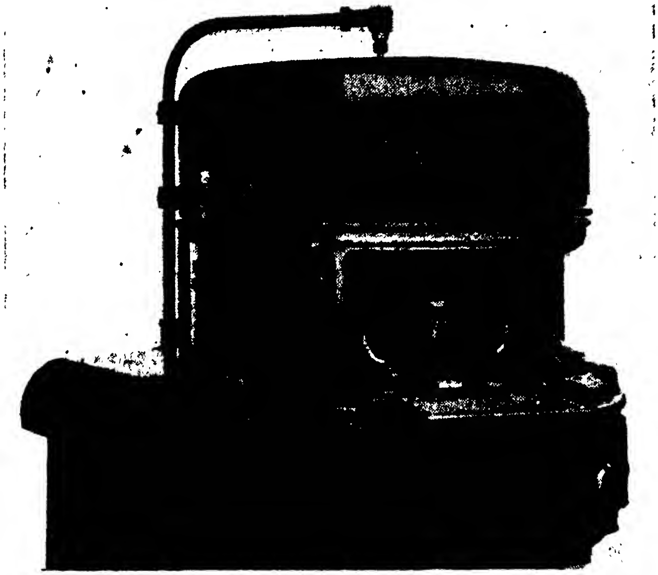
In order to make a satisfactory joint between the condenser body and cap, a load of 30 lbs is maintained on the component throughout the operation by means of the pressure heads. Reduction in this pressure causes an increase in the size of the gap round the body and excess solder can enter the condenser. Increase in pressure, on the other hand, may cause mechanical damage to the interior of the condenser. At the same time, the gap between pressure head and work seat must be adjustable to enable the machine to handle condensers of varying overall lengths. Provision is made, therefore, for such adjustment whilst the pressure exerted remains constant irrespective of this adjustment.

[*Courtesy of Messrs. Ferranti-Wild-Barfield Ltd.*

Fig. 9·18. H.F. Soldering Equipment for Brass Condenser-cans.

The pressure head in each compartment is retracted upwards when in the loading and ejecting stations, the work seats also being raised in these positions to facilitate operation. During the remainder of the cycle, however, the condenser is held between these two, being supported by the spring-loaded tongs until the pressure is applied after loading, and after the pressure is released prior to ejection. The two fingers of the tongs are automatically opened to release the component immediately before the ejecting plunger comes into operation, and are closed again before the loading station is reached.

Each compartment in the revolving head has its own work-coil, each work-coil being carried on copper fingers which engage with contacts supplying the radio frequency power in the heating station only. These fingers, together with the work-coil and the condenser, can be seen in Fig. 9·19. All work-coils, constructed of copper tubing, are water-cooled, the main feed being through a revolving coupling on the hollow central driving shaft, the discharge being through a similar coupling at the bottom of the shaft. The usual regulating valve and pressure switch are incorporated.



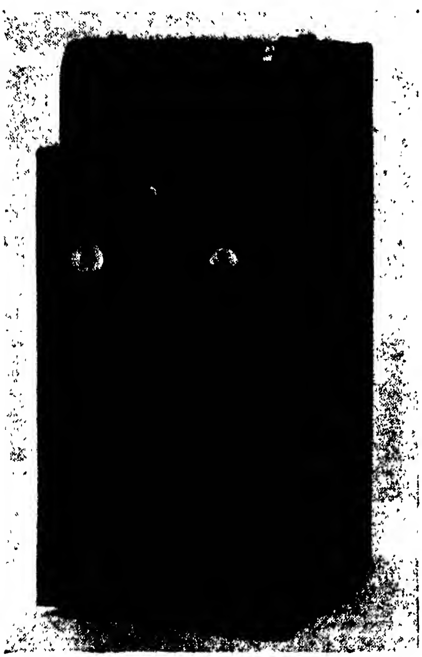
[*Courtesy of Messrs. Ferranti-Wild-Barfield Ltd.*

Fig. 9·19. Close-up of Work-head of Soldering Machine.

Driving gear is situated in the cast base, a motor driving through a fixed ratio worm gearbox. The central shaft carrying the rotating head is driven by a Geneva roller and star movement, the shaft rotating 60° at each step. Incorporated in the mechanism is a timing switch which starts the process timer controlling the period for which the radio frequency power is applied. Control of the heating time is, therefore, independent of speed of rotation, although interlocks are provided to prevent the application of the radio frequency power should the speed of the revolving head exceed that of the timer. The use of a timer provides more accurate control over the heating cycle and enables variations on this to be carried out without alteration to the rate of feed and delivery of components to and from the machine. The rate of delivery of components can be varied separately from 5 to 17 parts per minute.

Applications of dielectric heating

The use of radio frequency heating in the plastics moulding industry has brought numerous advantages over the older methods, and specially designed generators with built-in heating chambers have been developed for the purpose. These range from units capable of heating 4 ozs of material per minute, and typical examples are shown in Figs. 9·20 and 9·21. Fig. 9·20 shows an equipment rated at 750 watts output, for heating moulding materials for small mouldings, having a maximum capacity of about 12 ozs. The heating chamber is mounted on top of the cabinet, the lid of which carries one electrode, so that when it is opened, both electrodes are readily accessible for cleaning, whilst the clear space above the lower electrode makes it the more easy to load and unload.

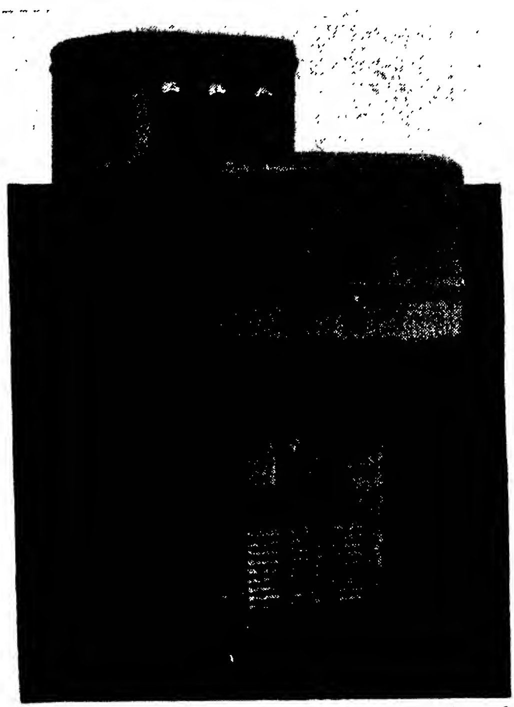


[*Courtesy of Messrs. Ferranti-Wild-Barfield Ltd.*

Fig. 9·20. 750-Watt Dielectric Heater.

Control of power-input in this unit is achieved by variation of the spacing of the electrodes. The lower electrode is capable of movement vertically, and its height, and hence the spacing between the two electrodes, can be adjusted by the control lever at the front of the equipment, which moves over a calibrated scale. An increase in the spacing reduces the input, and as the rate of heating is proportional to the power-input, the time cycle for heating can be altered to suit requirements. This electrode control, together with the starting push-button and the process timer, are the only controls needed, and consequently the operation of the equipment is well suited for the average moulding shop. Once the electrode spacing and process timer have been set for any particular charge, the operator need only load the unit, close the lid, and press the button.

Larger sets are generally similar in design, but some are equipped with automatic input control to compensate for the change in characteristics of the moulding materials as they rise in temperature. Normally, there is a tendency for the power-input to the charge to increase as the temperature rises, and allowance must be made for this in the initial setting of the electrode spacing so that, as the charge heats, the input does not rise above the maximum permitted for the particular generator. The starting input, therefore, must be somewhat lower than the maximum. With automatic control, the loading may be set initially to any figure not exceeding the maximum permissible and this input will be maintained automatically, irrespective of changes in characteristics of the charge during heating. In certain cases, this feature reduces slightly the heating cycle, and where maximum output is desired, offers some advantages over manual control.



[*Courtesy of Messrs. Ferranti-Wild-Barfield Ltd.*

Fig. 9·21. 3·5 KW Dielectric Heater.

Fig. 9·21 shows a 3·5-KW unit which takes electrodes up to 24" square. Under normal conditions an output of 18 ozs per minute of preheated materials is given, but in some cases this figure is very considerably exceeded. The general operation of this equipment is similar to that already described, and the production output is capable of providing preheated material for mouldings of considerable size.

Units such as those described may serve two or more adjacent presses provided that they are engaged on mouldings requiring the same quantity of materials and that the overall moulding cycles can be kept in step. Furthermore, in order to render the operation of the preheating equipment as simple and automatic as possible, remote control facilities may be provided whereby the commencement of the heating cycle may be started automatically by one of the movements on the press.

An interesting new dielectric-heating technique for thawing and heating frozen food uses a cavity magnetron to supply U.H.F. energy to a wave-guide in which the food is placed. The magnetron develops up to 5 KW of power at 1050 MC/S (i.e. $\lambda=28{\cdot}5$ cms, approximately) and heating times are extraordinarily rapid, being measured in seconds.* The very high frequency was chosen so that the voltage could be greatly reduced and so eliminate arcing in the load. In the article referred to the authors quote experimental findings which suggest that the heating of foods at this frequency is analogous to induction rather than to dielectric heating.

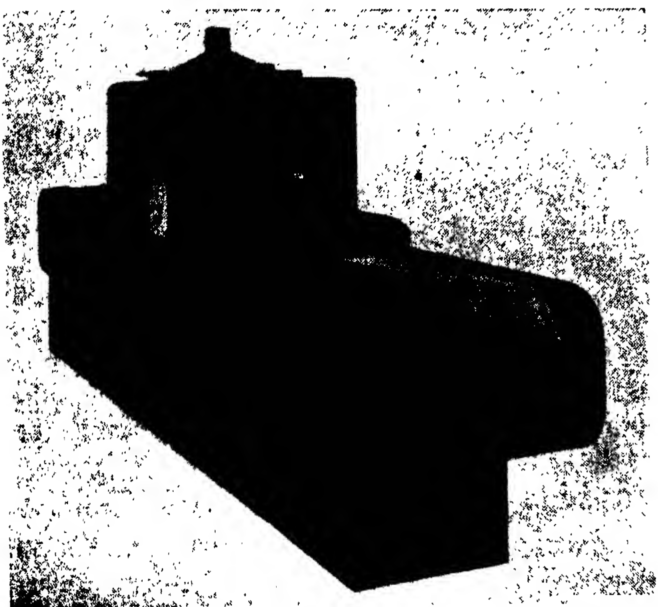
Radio frequency drying ovens

The use of radio frequency power for dielectric heating for drying has brought about the design of continuous units of which Fig. 9·22 shows an example. Textiles in various forms have been shown to be improved in qualities if advantage of the through heating by radio frequency is taken. Such method ensures that the material is heated to a uniform temperature for a uniform time, and the temperature gradient inevitable with other forms of heating is avoided. The conveyor equipment illustrated is similar in operation to any low temperature oven or furnace, but the heating is achieved by means of electrodes inside the oven proper. The lower electrode is below the conveyer belt and the upper electrode is suspended on insulated carriers. The height between electrodes can be varied and also the alignment can be altered to suit a desired rate of heating, both adjustments being effected by the hand-wheel controls on top of the oven. For some purposes a different input to the work is required initially from that required towards the end of the heating cycle, and the ability to have a different electrode gap at one end of the oven offers advantages.

* "UHF Heating of Frozen Foods," P. W. Morse and H. E. Revercomb, *Electronics*, October, 1947. (See also Chap. 24, *Radio Frequency Heating*, Brown, Hoyler, and Bierwirth, Van Nostrand, 1947.)

The work to be heated may take many forms, but in whatever form it is found, it is passed through the heating chamber by being placed on the endless belt conveyer. Speed of travel is variable, and a speed indicator is mounted on the oven to enable any desired rate of travel to be set.

The conveyor-belt tunnel-oven has also been applied to core-baking in foundries, the process time being thereby reduced from hours to minutes and a better product obtained.*



[*Courtesy of Messrs. Ferranti-Wild-Barfield Ltd.*

Fig. 9·22. Continuous Conveyor Dielectric Heater for Drying Textiles.

Installing and operating H.F. equipment

Motor-alternator equipment. Sets of 100 KVA rating and upwards are normally laid down in a brick sub-station, with its own fans and filters for supplying clean cooling air to the machines. The ventilating system should be arranged to maintain slightly greater than atmospheric pressure in the sub-station; this is especially important in foundries and other places where the dust-content of the unfiltered air is high. Dust and moisture should be excluded from the sub-station during and after erection of

* "Core Baking," *Automobile Engineer*, August, 1948.

machines. Hot outlet air should be ducted away from the machines to prevent recirculation.

There is a tendency towards the use of closed-circuit ventilation of H.F. motor-alternator sets, the cooling air being circulated through the machines and then through a fin-tube water-cooler, a system already in wide use for cooling large mains-frequency machines. Closed-circuit hydrogen cooling of H.F. alternator sets is already in use to some extent in the U.S.A.; its principal advantage is the great reduction of windage losses, but the somewhat better cooling properties of hydrogen (relative to those of air) permit an increase of up to 20% in rating for a given frame size. Windage losses usually form one-third to one-half of the total losses, and hydrogen cooling may reduce the windage loss to one-tenth of that for air.

Sets up to 250 KW rating may comprise either separate motor and alternator, usually direct-coupled, or a single two-bearing machine with both rotors on a common shaft. Larger ratings are built as separate machines. The set is laid on concrete foundations several feet thick; if the subsoil is light, this supporting concrete should be built up from a concrete raft. Great care must obviously be taken in lining up machines to avoid excessive vibration.

A routine inspection should be made periodically, say once a month, and the following items checked:

(1) Foundation bolts and coupling bolts tightened.

(2) Alternator air gap checked (special inspection plates are usually provided for this) with a set of long feeler gauges; eccentricity in excess of ·005″ should be regarded as serious.

(3) Electrical insulation of motor and alternator windings from the frames of the machines.

(4) Bearings and lubrication system.

(5) Cleanliness of air filters.

The frames of all electrical equipment should be efficiently earthed.

Main and auxiliary H.F. circuit-breakers are normally erected in the sub-station, control circuits being run back from the various H.F. units supplied from the set. The series-condenser unit, in the alternator armature circuit, is also part of the sub-station equipment, but the load condensers are located as close as possible to the loads with which they are associated.

H.F. power is supplied to the various loads (melting furnaces, billet-heaters, etc.) either through lead-covered coaxial cable, or interleaved flat copper bars. Cable is obtainable in long lengths (e.g. 100 yards), and in most cases the only joints required are those at the sub-station and the load. There is no external field from a coaxial cable, which may therefore be run in close proximity to steel structures. Characteristics of typical cables are given in the following table:

LEAD-COVERED COAXIAL CABLES FOR MEDIUM FREQUENCIES *

Cable No.	Overall diam. in inches	Frequency KC/S	Current Rating Amperes	Max. Working Voltage	Approx. KW Loss per 100 yds	Reactance per 100 yds, ohms	% Reactance drop per 100 yds †	Capacitance μF per 100 yds, inner to outer conductor
I.F. 1	·818	1 3 10	100 100 90	1000 „ „	1·5 „ „	·109 ·327 1·09	1·1 3·3 9·8	·0274
„ 2	1·10	1 3 10	180 170 150	„ „ „	2·0 „ „	·092 ·276 ·92	1·7 4·7 13·8	·0365
„ 5	1·736	1 3 10	370 350 260	„ „ „	3·5 „ „	·075 ·225 ·75	2·8 7·9 19·4	·0457
„ 6	2·202	1 3 10	520 470 330	„ „ „	4·3 „ „	·069 ·207 ·69	3·6 9·8 22·8	·0475
„ 7	1·09	1 3	190 180	2000 „	3·25 „	·12 ·36	1·2 3·25	·0228
„ 10	1·782	1 3	390 340	„ „	3·0 „	·075 ·225	1·5 3·8	·0412

* British Insulated Callender's Cables Ltd.

† Reactive voltage drop per 100 yards when carrying rated current, expressed as per cent of rated maximum working volts. If required, this drop may be compensated by suitable adjustment of the alternator series-condenser capacitance.

If flat bars are used,(64) they should be placed as close together as voltage considerations and their flexibility will allow. The inductive volt-drop may be estimated from

$$V = I\omega Ll \text{ volts,}$$

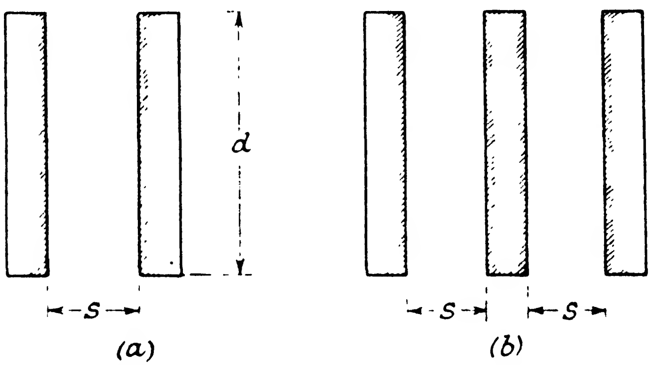
where I = line current, ampères,

L = line inductance per foot, henrys,

l = line length, feet.

The inductance per foot run of twin flat bars arranged as shown in Fig. 9·23 (a) is a function of the ratio of bar-spacing to bar depth, and is plotted in Fig. 9·24. The bar-spacing, S, rather than the distance between bar centres, is taken because current and flux penetration-depths in copper are small at frequencies of 1000 cycles/sec and higher.

Interleaving three bars as shown in Fig. 9·23 (b) reduces the inductance to approximately one-half that obtained with the simple arrangement shown at (a), for the same spacing, S. Similarly, for N interleaved bars, alternate bars being of opposite polarity, the inductance is reduced to $\left(\frac{1}{N-1}\right)$ of that of the twin bars of Fig. 9·23 (a). Interleaving also increases the permissible loading,



Shading indicates current distribution

Fig. 9·23. Twin and Interleaved Flat Copper Bars.

as better use is made of surfaces available for carrying current; this point is illustrated in the same figure.

Permissible current-loading of flat bars must be determined in relation to the current penetration depth p, and hence to the frequency of the installation. Experience suggests that a dissipation of ·20 watt per square inch of bar surface is reasonable and gives a temperature rise of 30 to 40° C. Taking the resistivity of copper bar at 60° C as ·8 microhm-inch, this dissipation corresponds to a current density of $1000\sqrt{\frac{\cdot 2}{\cdot 8p}}$ or $\frac{500}{\sqrt{p}}$ ampères per square inch (p in inches). That is, the bars may be loaded so that, assuming the current to be contained in a surface skin of depth p, the density in the skin does not exceed this value. The values of p

and current density for copper bars at 10, 3, and 1 KC/S are respectively ·028, ·052, and ·09 inch, and 3000, 2200, and 1670 ampères per square inch; corresponding current loadings per inch depth of bar, assuming both faces loaded (i.e. for inner interleaved bars) are 168, 230, and 300 ampères respectively. The current density for twin bars, in which only one face of each bar carries current, could be increased by about 40% for the same temperature rise. There is no electrical advantage in using bars thicker than $\frac{1}{8}''$ for frequencies of 3 KC/S and higher.

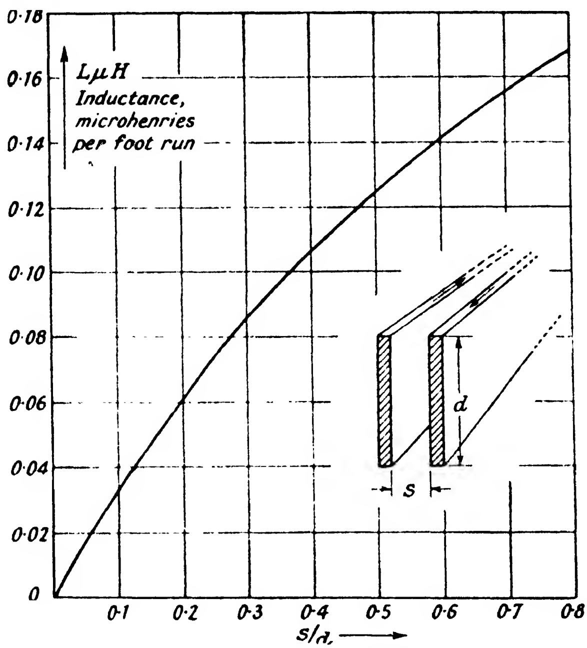


Fig. 9·24. Inductance Curve for Twin Flat Bars.

Current practice is to use coaxial cable for runs with few or no T-joints, such as from alternator to distribution "board", and from sub-station to individual work-heads; and to use flat bars, preferably interleaved, where many joints are to be made, as on the sub-station distribution "board". This consists of a bus system feeding the various output-contactors to which the outgoing coaxials are brought.

Interleaved flat bars are commonly used for the heavy-current connection between furnace-coil and condensers. The inductive reactance of this connection is made small by using thin deep bars, closely spaced as shown in Fig. 9·23; the condenser voltage will exceed the furnace-coil voltage by the amount of the reactive-voltage of this connection.

Contactors are similar to those used for 50 cycles/sec circuits, but suitably de-rated; for example, at 10 KC/S the current loading is reduced to about one-third of that for 50 cycles/sec. A high speed break, strong blow-out coils, and large arc-shields are required; the amount of steel in the construction is a minimum, and care is taken to keep inductive loops small.

Contactors controlling power-factor-correction condensers are usually operated off-load, and consequently may be of lighter construction, without blow-out coils or arcing tips. To ensure off-load switching, the selector switches are interlocked with the H.F. supply contactor, or in some cases with the alternator field switch.

Protection against damage due to overloads or short-circuits is provided by using thermal or magnetic relays supplied from iron-cored current transformers. These transformers have a multi-turn primary winding, for line currents less than 100 ampères; for larger currents, a toroidal secondary is slipped over the cable or bus-bar. Iron-cored current transformers must never be energised with the secondary winding open; if the relay or meter in the secondary circuit is removed, the winding should be short-circuited.

An insulated system is preferable to an earthed one, provided that sensitive earth-leakage detection equipment is used. Referring to Fig. 9·25, in which *S* represents the earthed metal structure of the furnace or heater, only a small H.F. fault-current will flow should a breakdown occur in the insulation between *S*, or the work piece or charge *W*, and the coil, because of the relatively high impedance of the choke L_{ch}. At the same time, the breakdown will allow a 50 cycles/sec current to flow through and operate the relay *R* which is arranged so that it then opens the alternator field contactor *1* and the line contactor *3*. Where required, as for example, in a melting furnace, a meter *M* can also be included in the 50 cycles/sec circuit to give a continuous indication of insulation resistance (see first section of this chapter).

The limiting of H.F. fault-current to a low value is important to the safety of the operator as well as to the protection of the alternator and series-condenser. However low the earth resistance, the reactance of the earth-strip may be of the order of one or several ohms, and a heavy fault-current, such as could occur in an earthed system, would then raise the potential of the heater framework to a dangerous value.

The heavy duty which would be imposed on the line contactor *3*, if called upon to break full-load current in a high-power installation, can obviously be reduced or avoided by opening the field contactor *1* first, but in homopolar machines the time-lag of the high-inductance field-circuit is a difficulty. A scheme developed in Germany* by G. W. Seulen, is to open the field contactor *1* and then close the alternator short-circuiting switch *2* before opening the line contactor *3*. The short-circuit current of the machine without its series-condenser is not excessive. The switch *2* is opened after the field-current has fallen to zero.

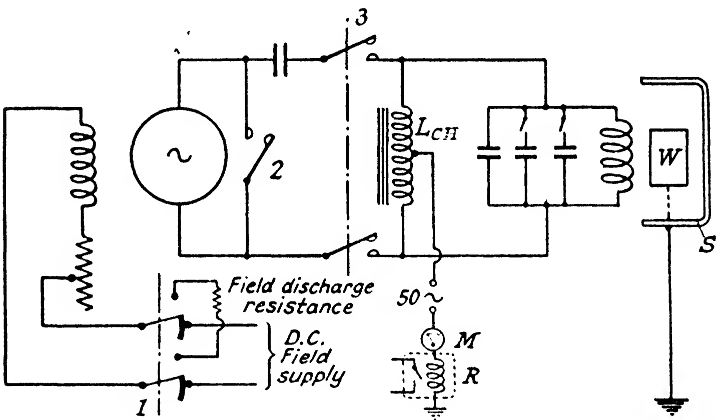


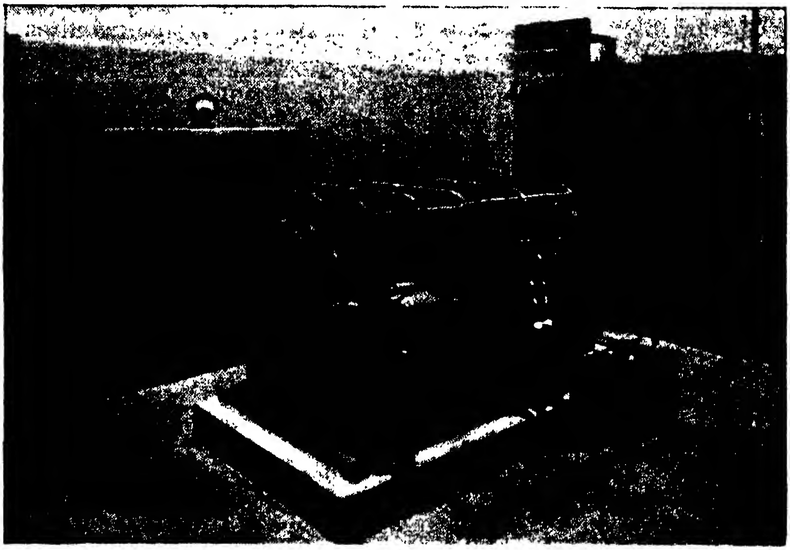
Fig. 9·25. Alternator and Induction Furnace, Insulated System, with Earth-leakage Indicator and Relay.

The equipment of a 75-KW 10,000-cycles/sec sub-station is illustrated in Fig. 9·26. The auto-transformer starter, arranged for remote push-button control, is seen against the far wall, and an automatic voltage-regulator of the thyratron type is mounted alongside the meter and control-cubicle. This cubicle faces outwards so that operation and supervision of the set do not require entrance to the sub-station, which is normally locked. The motor-alternator set is a two-bearing machine running at 3000 r.p.m. The small size of the alternator series-condenser is remarkable, considering that its KVA rating is approximately that of the alternator itself. A lead-covered coaxial cable carries the H.F.

* See German Patent No. 737192 (1943).

power to the load; the cable box in which this terminates is mounted near the series-condenser.

It may sometimes be necessary to parallel two or more alternators, for example, to make use of existing machines to supply a load too large for a single machine, or because it is not practicable to build a single machine of sufficiently high rating. Inductor alternators for parallel operation should have identical voltage-regulation from no-load to full-load, and their driving motors should have similar characteristics. If series-condensers are used with the alternators,



[*Courtesy of Birlec Ltd.*

Fig. 9·26. 75-KW 10,000 cycles/sec Sub-station, with Two-bearing Motor Alternator Set, Auto-transformer Starter, Alternator Series-condenser, Thyratron Voltage-regulator, and Control Panel.

these condensers must be regarded as part of the load-circuit rather than of the machines, as it is necessary to parallel the *alternator-windings*, as shown in Fig. 9·27. Without the tie *AB*, the two alternators tend to run 180° out of phase, the output voltage falls to zero and a heavy circulating current flows round the closed circuit comprising the windings and the series-condensers.

The procedure for parallelling is to bring up the speed of the unexcited incoming machine to approximate synchronism with the machine(s) already on the sub-station bus-bars, then to close the tie *AB*, and finally the switch *S2* (Fig. 9·27).* The incoming machine will pull into step and commence to supply its share of the power-component of load-current, but its excitation has then to be adjusted so that it also takes its share of the wattless component, i.e. so that there is ideally no circulating current. The correct excitation for two similar machines is that with which the machines supply equal currents to the load, the load-current then being equal to the sum of these two. J. H. Walker recommends that two similar machines operated in parallel should each be down-rated 10% to allow for losses due to residual circulating currents[13(b)].

Attention was drawn (Chapter 3) to the fact that the open-circuit characteristic of an inductor alternator exhibits a maximum; further increase in excitation beyond this point is accompanied by a *reduction* in generated voltage. This alternator characteristic, and the changes in work-coil impedance which occur as a load is fed into the coil and as the electrical and magnetic properties of the load change during heating, give rise to interesting induction-heater starting problems.

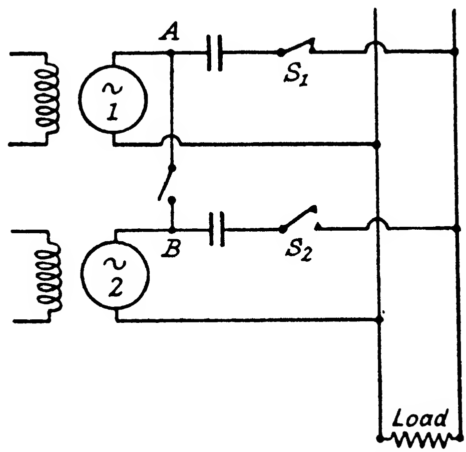


Fig. 9·27. H.F. Alternators in Parallel.

Consider first a heater having a full-load KW rating small compared to that of the alternator supplying it; e.g. one of several small billet-heaters supplied from a common alternator. In such a case the heater capacitor bank will be set to give approximately unity power factor under steady running conditions; the reactance of the capacitor bank is then approximately equal to that of the loaded coil. It will, however, be rather less than that of the empty coil, or of the coil with a cold (magnetic) charge. As a result, whether the heater is switched on with its coil empty, and the charge introduced subsequently, or else with the charge already in place, the initial power factor is less than unity, and leading.

* See, for example, "Short-Circuit Characteristics and Load Performance of Inductor-Type Alternators," A. Mandl, *J.I.E.E.*, Vol. 94, Part II, No. 38, April 1947.

Case (i): Coil already loaded with cold magnetic charge

The initial KW loading will be higher than (even double) the running KW, and the P.F. will be leading. As the charge heats up, the power and the leading KVAR will fall. Load and power factor will be substantially constant once the magnetic change-point has been passed in the outer skin of the charge.

Case (ii): Coil initially empty; charge introduced after switching on

The most common case of this kind is that of a continuous progressive heater, such as one in which billets are passing steadily through the coil, and eventually several, in different stages of heating, are in the coil at once. The initial KW loading will be low, being the coil, bus-bars and condensers losses—roughly one-fifth of the normal full-load KW. The P.F. will be low and leading. As billets move into the coil, the leading KVAR will first increase slightly, due to the permeability of the cold charge, and will then fall steadily to zero as the normal running condition is reached. In this case there is no power overload, the KW increasing steadily as the billets fill the coil.

In both cases, because the heater ratings are small, the alternator can supply the transient out-of-balance KVAR and, in case (i), the extra KW. Consider now the case of a continuous-flow billet-heater having a full-load rating up to the full capacity of the alternator.

It is impracticable to switch on with the coil already loaded with cold billets, partly because of the heavy KW demand, and also because billets already in the coil cannot have the correct cycle for proper discharge temperature. It may be possible to start up with the coil empty and with the capacitor bank already set to give unity P.F. under steady conditions, but this will depend upon the percentage change in coil reactance from starting to normal running, upon the *coil* power factor under normal running, and upon the alternator itself. If the alternator can deliver its nominal KVA rating, even with a leading power factor of about ·15, then such starting conditions are practicable provided that the ratio

$$\frac{[\text{Empty-coil reactance}]-[\text{Normal loaded-coil reactance}]}{\text{Normal loaded-coil reactance}}$$

is not greater than the normal-running power factor of the *coil.* If the reactance-ratio exceeds this figure, capacitor-switching must be performed, either manually or automatically, during the starting-up period, extra capacitors being switched in as required.

The reason why the alternator may not be capable of supplying its rated KVA at a low leading P.F., even for the short time required for starting up the heater, is the peculiar open-circuit (magnetisation) characteristic referred to. As with alternators of the wound-rotor

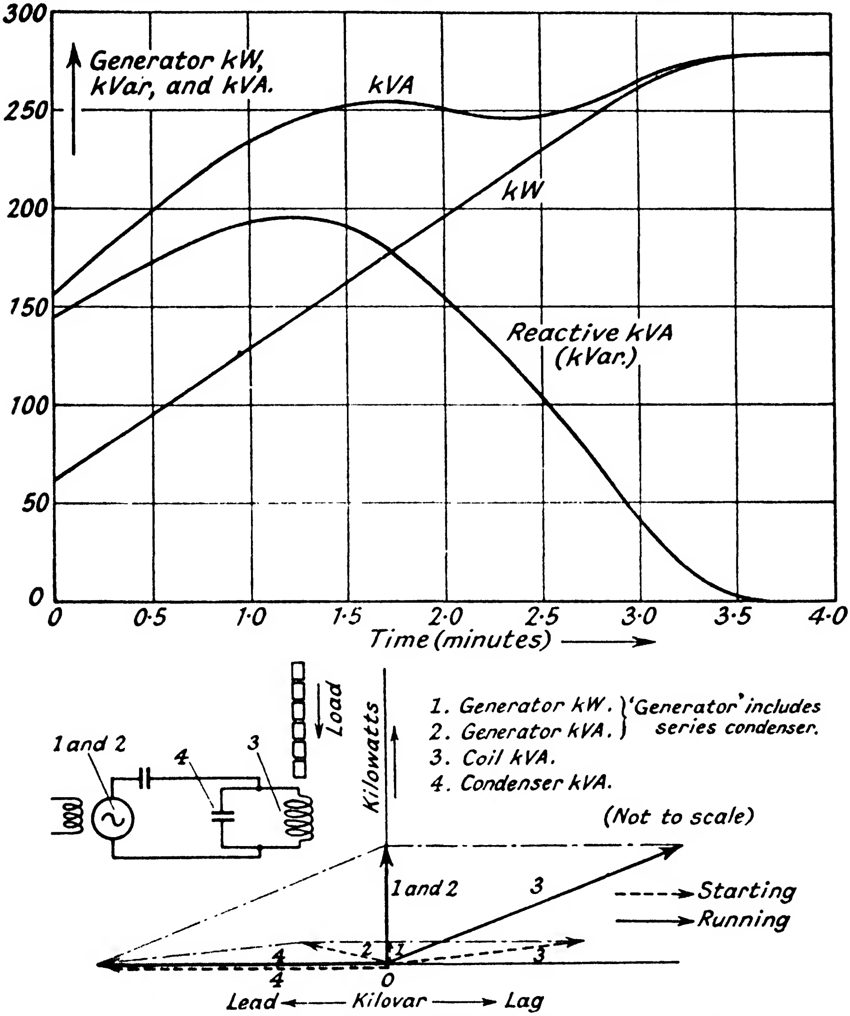


Fig. 9·28. Starting Conditions for 275/300-KW Continuous-flow Billet-heater.

type, a leading power factor has a self-exciting effect, but it is peculiar to the inductor type that excitation beyond a certain value actually reduces the generated voltage.

The graphs plotted in Fig. 9·28 show the starting conditions for a continuous-flow billet-heater which it was possible to start up without capacitor-switching. In this case the coil reactance-ratio was ·087, and the normal-running P.F. of the coil was ·148. It can be seen that the full-load KVA was approached, but not exceeded, during the starting-up period of 3 minutes.

The reactance-ratio can be reduced by using large coil-to-charge clearances, but the reduction in heater-efficiency entailed may rule this out in favour of capacitor-switching. Another factor which may have a considerable effect on the reactance-ratio is the increase in slip of the induction motor driving the alternator, as the alternator load increases. For example, a 3% increase in slip corresponds to a 3% reduction in alternator frequency; the heater-coil reactance

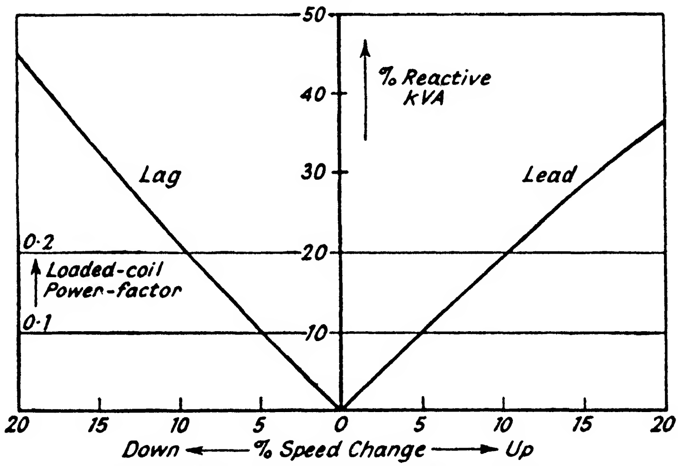


Fig. 9·29. Effect of Alternator Speed on Reactive KVA. Intersection of curve with appropriate coil power-factor line gives % slip at which reactive KVA=KW, numerically.

falls by 3%, and the capacitor-bank reactance increases by 3%. The effect, so far as starting conditions are concerned, is the same as if the coil reactance alone had decreased by 6%. It is obvious that a driving motor having a rather large full-load slip (in excess of 5%, say) may make capacitor-switching necessary, quite apart from the reduction in effective inductance of the heater-coil due to the presence of a hot charge. The slip effect is included in the curves of Fig. 9·28, which were plotted from KW and KVAR readings taken under actual starting conditions.

The effect of alternator speed on the reactive KVA of the load is plotted in Fig. 9·29. The speed change is expressed as a percentage of the speed for which the capacitor bank is correct for unity overall power factor; the reactive KVA of the load, which the alternator would be called upon to supply, is expressed as a percentage of the total capacitor KVA at this datum speed. This graph illustrates a particular case of the parallel resonant system of Figs. 1·24, 1·25, and 1·26. In general, where an induction motor is to be used to drive a H.F. alternator, it is advisable to specify that the motor slip at full-load shall be as small as possible, e.g. not more than 1·5 or 2·0%. The higher cost of a synchronous motor may be justifiable in some cases.

Valve-oscillator equipment

No special foundations are required for these static converters, though it is a good plan to place the equipment on 2″ steel channels, so that it is easy to move, and water on the floor does not get into the set. Also, water inside the set, due to a leaky hose or anode-jacket, is then easily drained away.

Preparatory work on the site consists in bringing up the water and electric supplies (including-earthing strip), and compressed air if required for any handling mechanism. Adequate drains must also be provided; sight-drains are preferable, but it is sometimes necessary to carry the water away overhead, and a closed pipe system has to be used.

Water-flow switches are always fitted to units which employ water-cooled valves, and it is important that these should be kept in good working order. A booster pump will be required to increase water supply pressure if the normal pressure falls below 20 lb/sq. in.; a working pressure of 30-50 lb/sq. in. is recommended. A closed recirculatory system, with a cooler, is desirable unless the purity of the mains water is found by analysis to be quite suitable.

Usually the oscillator is placed very near to the work-unit, but this is not essential; a transmission line can be used between the set and the work-head. This transmission line should terminate in a tuned tank-circuit at the work-head, if the distance is greater than 6 or 8 ft.; the line then carries only the power component of the work-head current, and the line-loss is kept small. The arrangement shown in Fig. 7·7 (c) is applicable to both induction and dielectric heating.

An arrangement which has already been used on a small scale, and which may become more common as power-ratings are increased, separates the oscillator section from the mains equipment and rectifiers. Oscillator valves and H.F. circuits only need then be located at the work-station, and are connected by high-tension D.C. cable to a remote sub-station in which the remainder of the equipment is housed.

Although it is obvious that heavy vibration on the site should be avoided if possible, vibration is not so serious as one might imagine; a number of high-power oscillators have been installed in heavy-press shops and there is no evidence to suggest that valve life is seriously reduced. "Shock-proof" mats should be used in doubtful cases.

A filter to prevent the transmission of H.F. energy along the supply mains is essential; it is commonly supplied as part of the oscillator equipment. A description of the mains filter is given in Chapter 8.

Before starting up a new equipment all interlocks should be checked in sequence—doors, water-flow switches, time-delay relays, interlocks on handling mechanisms, fan or blower interlocks—and hoses and anode-jackets inspected for water-leaks. Valve heaters should be run for several hours before switching on the H.T. transformer; 24 hours "baking" is not too long for hot-cathode mercury-vapour rectifiers, and it is most important that rectifier heater voltages are correctly set within the permitted limits of $\pm 2\frac{1}{2}\%$. The oscillator should then be allowed to operate continuously, without load, for an hour or so at a low plate-supply-voltage (e.g. 5 KV, where the maximum voltage is 10 KV). These precautions are taken to avoid flash-back in the rectifiers, which is most likely to occur when valves are newly installed or operated after a prolonged period of disuse. Flash-back (i.e. reverse current flow) will also occur if the temperature of the condensed mercury in the base of bulb exceeds 60° C or so, and a special blower is fitted for cooling the rectifiers in some high-power oscillators. The occurrence of a flash-back can be recorded by means of the reverse-current relay shown in Fig. 9·30. A voltage developed across R due to a reverse-current will cause the relay to operate; this relay may incorporate a counter or a telephone-type "eyeball" indicator; it can be arranged to shut-down the equipment (if suitable insulation precautions are taken), but this is not usually necessary because H.T. fuses or mains overload-relays will function.

Periodical inspections should be made to check the following items:

(1) Valve anodes (water-cooled) inspected for scaling, washed in a 10% hydrochloric acid solution if necessary, and afterwards well rinsed.

(2) Rubber-hose clips tightened, and hoses inspected for cracks which sometimes develop due to the high electric stress. Suspect hoses should be replaced, and not left until a leak or a burst occurs. In the event of a burst, the equipment ought not to be run again until wetted components have been thoroughly dried out.

(3) Anode-jacket rubber seals inspected for water-leaks, especially after installing or replacing a valve.

(4) Electrical joints in the tank-circuit inspected and tightened where necessary; brazed joints should be used where practicable. Intermittent failure to heat the work has occasionally been traced to a high-resistance joint.

(5) Contactor arcing-tips.

(6) Water-flow relays and filters.

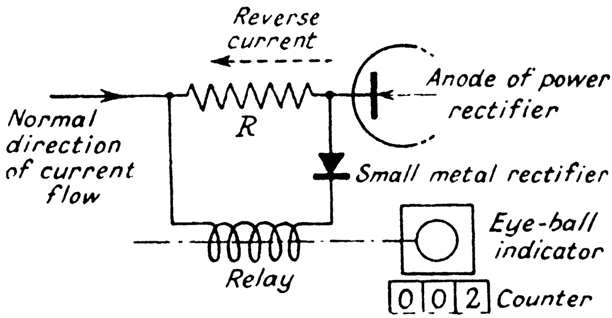


Fig. 9·30. Rectifier Flash-back Indicator.

It is a good plan, if forced-air-cooled oscillator valves are used, to coat a small area of the ribbed cooler with a temperature-indicating paint, so that a permanent record is made in the event of the normal running temperature being exceeded.

It is sometimes required to change over quickly from one coil set-up to another; this can be done with hand-operated switches or solenoid-operated H.F. contactors. In all cases, switching is carried out off-load, and auxiliary contacts on the switches or contactors are arranged to interlock with the contactor controlling the H.T. transformer which supplies power to the oscillator.

There is no objection to rapid switching of the oscillator itself; a duty of 1000 "ON" periods per hour is not unusual, and the only limitations are those imposed by the contactor. Electronic switching, e.g. by means of grid-controlled rectifiers or ignitrons, can still further reduce such limitations.

The emission current of an oscillator valve with a pure tungsten filament is extremely sensitive to filament voltage, while the useful life is greatly reduced by running the filament too hot. Accurate setting of filament voltage is thus very important, and in oscillators having two or more valves with a common filament supply and voltmeter it is usual to insert a low-value series resistor in each filament circuit. A set of these graded resistors is normally supplied, and values are selected to suit the marked filament characteristics of individual valves, so as to ensure equal emissions with a common supply voltage. It is therefore necessary to see that the correct resistance is used whenever a new valve is put into commission. This precaution is not required with thoriated-tungsten filaments; on the other hand, they should be run within $\pm 5\%$ of rated voltage even for light loads, whereas the voltage may be reduced, for pure tungsten, to the point at which the emission is just sufficient (i.e. at which the required output is just maintained). The filaments of high-power valves must be brought up to temperature gradually; time-delay interlocks are usually incorporated to ensure this. It is also good practice to reduce filament voltage slowly when the oscillator is being shut down.

Most troubles experienced with valve-oscillators are of a minor nature, and can be rectified quickly by the works electrician. Certain spares should be carried, e.g.,

1 oscillator valve,
2 rectifier valves,
1 grid resistance,
1 tank-condenser section,
Set arcing tips for contactor,
Set rubber seals for valve water-jackets.

The following list of faults and symptoms may be of use in locating causes of trouble:

Symptom	*Possible Faults*
(1) Mains C.B. trips out	Faulty C.B. No-volt-coil not energised or supply-voltage low. Faulty mains interference suppressor. Faulty transformer. Water supply failure. Plain overload.

Symptom	*Possible Faults*
(2) Blown H.T. fuses between H.T. transformer and rectifiers.	Rectifier flash-back (check supply voltage, ambient temperature, and rectifier heater voltage). Insulation breakdown (e.g. in rectifier valve-holder, or in H.T. leads). Faulty H.F. by-pass condenser or coupling condenser. Faulty oscillator tank condenser, or work-coil. Faulty oscillator valve. Plain overload.
(3) Blown grid fuses.	Grid-drive voltage too high on light load. Large variations in oscillator filament voltage (and hence emission), and in oscillator loading.
(4) Oscillator plate current-operated overload relay trips.	Plain overload. Faulty work-coil, or tank-circuit component. Faulty plate-coupling condenser.
(5) Erratic operation, output unstable.	Damaged grid resistance. Intermittent high-resistance contact, e.g. in tank circuit or grid circuit. Intermittent fault on work-coil.

The adjustments required for correct loading of the oscillator are described in Chapter 5. A brief summary is given here:

Excessive tank- and grid-currents, together with low D.C. plate-supply current, indicate insufficient loading; more work-coil turns (or more primary turns, if a work-head transformer is used), or closer coupling between inductor and work will correct these conditions. Conversely, the number of turns, or the coupling, should be reduced if the overload relay trips out, or if the tank- and grid-currents are low and plate-supply current is high. Some oscillator units have continuously variable control of loading; for example, that illustrated in Fig. 5·29, in which the coupling between plate-coil and tank-coil is continuously adjustable (see also Fig. 5·22 (b)). Such an arrangement makes it possible to match a wide range of work-coils to the oscillator.

It is a good plan to check the D.C. power-input (i.e. $E_{DC} \times I_{DC}$) without a work-piece in the inductor. This should correspond roughly to between $\frac{1}{6}$ and $\frac{1}{3}$ of the full-load D.C. power-input. An unusually high "no-load" reading might be due to shorted turns in the work-coil.

The calculation of work-coils and work-head transformers for valve-oscillators is more difficult than for alternator equipment, because the operating frequency and the work-coil voltage both depend partly upon the work-coil itself. The arrangement shown in Fig. 9·31 is that used in conjunction with oscillators of the kind illustrated in Figs. 5·24, 5·28, and 5·30, and approximates to a constant-current generator; i.e. the work-coil current does not change very greatly (for a given KW output, and D.C. input voltage) over a wide range of work-coil sizes. In such a case it is convenient to relate the power generated in the load to the ampère-turns per unit length; this enables a fairly accurate estimate to be made of the number of work-coil turns required and the voltage which will be developed across the coil. Note that whereas everything should be done to keep the coil-circuit power factor as high as possible in an alternator equipment, such is not the case with a valve-oscillator. Here it is necessary to make Q (i.e. the reciprocal of the power factor, approximately) high enough for the maintenance of oscillations of good waveform, and a Q value for the complete inductive branch of the tank-circuit lower than 15 to 20 is not desirable. Thus the "buffer inductance" of the

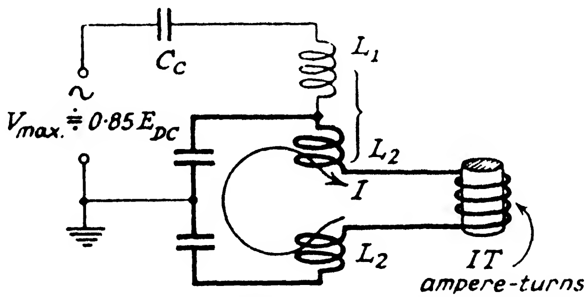


Fig. 9·31. Tank- and Work-circuit of Valve-oscillator.

split tank-coil $L2$ of Fig. 9·31 is essential—at least for a general purpose oscillator—as it increases the Q value. It also helps, in conjunction with the coupling coil L_1, to maintain approximately constant current, and it reduces the effect of the work-coil itsel upon the oscillation frequency.

It was shown in Chapter 6 that the power developed in a cylindrical charge of diameter d and length l cms with a close-fitting coil is:

$$P=(In)^2 \times 2\pi^2 dl\sqrt{\rho\mu f}.10^{-9} \text{ watts} \quad . \quad . \quad 9{\cdot}10$$

approximately, provided that the skin-effect is pronounced (see equation 6·14). In many cases, particularly for surface hardening, the specific power in KW/cm² of work-surface is the criterion; then

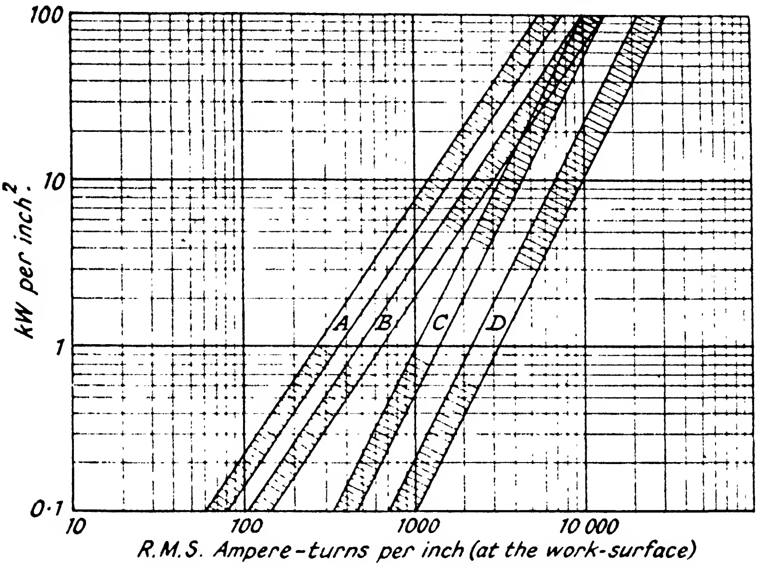
$$\Delta P=\frac{P}{\pi dl}=(In)^2 \times 2\pi\sqrt{\rho\mu f}.10^{-12} \text{ KW/cm}^2 \quad . \quad 9{\cdot}11$$

where In is the ampère-turns/cm length of charge.

It is seen from equation 9·10, substituting for $n = \frac{T}{l}$, that

$$P = (IT)^2 \times 2\pi^2 \frac{d}{l} \cdot \sqrt{\rho\mu f} \cdot 10^{-9} \text{ watts} \quad . \quad . \quad . \quad 9{\cdot}12$$

so that for constant current I the number of work-coil turns required to develop a load of P watts is the same for any size of load of given proportions.



[*Courtesy of B.T.-H. Co., Ltd.*

Fig. 9·32. Specific Power in Steel, and Loss in Copper Inductor.

A. Carbon Steel just below Curie point.
B. „ „ at 0° C.
C. „ „ in hardening and forging range.
D. Loss in close-fitting water-cooled work-coil.

Use R.H. edge of zone for short solenoid, $1 > \frac{l}{d} > {\cdot}5$.

Use centre for $3 > \frac{l}{d} > 1$.

Use L.H. edge for $10 > \frac{l}{d} > 3$.

Fig. 9·32 (B.T.-H. Co., Ltd.) gives KW per square inch and corresponding ampère-turns per inch for carbon steel in a solenoid at ·4 MC/S. (Note that in this figure dimensions are in inches.) Strictly, the KW/in² values are for the centre part only of the charge, the power falling off towards the ends (see Fig. 6·14), but since μ increases with decreasing field-strengths, the power does not fall off as rapidly as H^2 at the ends of the solenoid. R. M. Baker's approximation for μ (A.I.E.E. Technical Paper, 45-80, December 1944) was used in the calculations for Fig. 9·32. The difference in slope of the zones for steel below and above the Curie point is due to the change in effective μ with field-strength; at very high values of ampère-turns per inch, the effective permeability approaches unity. The change in oscillator loading which occurs as the Curie point is reached can be minimised by using high power-concentrations.

The inductor copper-loss values given in Fig. 9·32 are the minimum

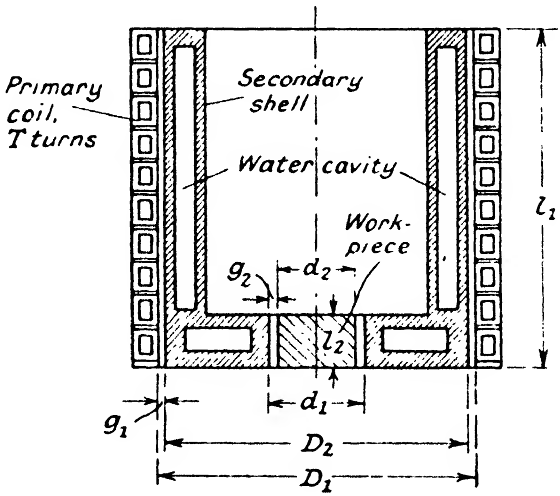


Fig. 9·33. Work-head Transformer for Surface Hardening.

theoretical values; losses obtained in practice may be as much as two or three times these.

A work-head transformer is almost always used for surface hardening in order to obtain sufficiently high power-concentration (Fig. 9·33). The radial clearance between the work and the low-voltage heavy-current inductor may be ·020″ to ·080″; it should be kept as small as mechanical tolerances permit. Similarly, the clearance between the primary winding and the secondary shell of the transformer can be made very small if a good insulating medium is used, e.g. an extruded coating of polythene on the primary-coil tubing, or a high-grade insulating tube slipped over the shell, and the primary coil embedded in wax. With such small clearances, the ratio of work-surface current to inductor current is approximately the ratio of their effective diameters (allowing for current penetration depth), and similarly the ratio of shell current to primary current is approximately the ratio of the corresponding effective diameters. This follows (see Fig. 9·34) from

$$I_2 = \frac{\omega M I_1}{Z_2} \text{ in magnitude (see Chapter 1, p. 46).}$$

Putting $Z_2 \doteqdot \omega L_2$ and $M = k\sqrt{L_1 L_2}$ we have

$$I_2 \doteqdot k\sqrt{\frac{L_1}{L_2}} . I_1 \text{ ampères,}$$

and for concentric arrangements of equal length and with small clearances,

$$k \doteqdot \frac{d_2^2}{d_1^2}, \text{ while } \sqrt{\frac{L_1}{L_2}} \doteqdot \frac{d_1}{d_2}$$

so that
$$I_2 \doteqdot \frac{d_2}{d_1} . I_1 .$$

Hence, to obtain the approximate number of work-coil ampère-turns which will produce the required number of work-surface ampère-turns, it is necessary to multiply the value obtained from Fig. 9·32 by the ratio of the effective diameters. A correction has to be applied twice if a transformer is used.

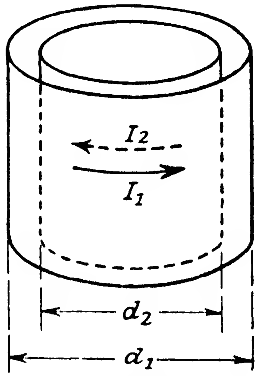


Fig. 9·34.
Detail of Transformer.

Example (Fig. 9·33)

Medium carbon-steel shaft, 2 cms diameter, for continuous-progressive surface-hardening to a depth of 1 mm in a "concentrator" type of transformer, at 400,000 cycles/sec, primary current 160 ampères.

$D_1 = 12$ cms	$D_2 = 11{\cdot}4$ cms	$l_1 = 12$ cms
$d_1 = 2{\cdot}2$ cms	$d_2 = 2{\cdot}0$ cms	$l_2 = 1$ cm or ·4 in.

Surface area of work within the inductor = 6·28 cm² or ·975 in².

Specific power required = 2·5 KW/cm² or 16·1 KW/in² (see Fig. 6·17).

For continuous progressive heating, the ampère-turns per inch required at the work-surface for this specific power may be taken as 3500 (i.e. between bands *A* and *C*, Fig. 9·32). Then total ampère-turns required at work surface

$$= 3500 \times {\cdot}4 = 1400$$

Ampère-turns required on primary coil

$$=1400\times\frac{D_1}{D_2}\times\frac{d_1}{d_2}=1620$$

whence primary turns required $=\frac{1620}{160}=10$ approx.

Allowance should be made for the reactance of the "slot" current-path between the secondary shell and the work-inductor. This is effectively part of Z_2 and will reduce the secondary current. The number of primary turns may require to be increased from 10 to 11 to allow for this.

The primary-coil voltage will be

$$V=IZ\doteqdot IX$$

where X is the effective reactance of the loaded transformer. The reactance is mainly due to the flux in the radial gaps between primary and secondary and between inductor and work, and is given approximately from

$$X\doteqdot 8\pi^3T^2f\left[\frac{D_1g_1}{l_1}+\frac{d_1g_2}{l_2}\right]\times 10^{-9}\text{ ohms}$$

With $T=11$ turns, this gives $X=6{\cdot}25$ ohms, so that the primary voltage on load $=160\times 6{\cdot}25=1000$ volts. The voltage would actually be somewhat higher than this, even as much as 25 % higher, due partly to the flux within the copper and steel, which has been neglected here. If such a transformer is made with detachable work loops, the primary should be insulated for a voltage corresponding to an open-circuit secondary, i.e. to that developed across the reactance of the primary-coil alone (see p. 10). In the example given, this would be roughly 4000 volts.

Copper-losses in the transformer primary winding and secondary shell may be estimated by using the equation given on p. 43, while that for the work loop is more conveniently found from curve D, Fig. 9·32. These losses are roughly 3 KW and 2 KW respectively. Total power input to the transformer is therefore some 21 KW, and the input KVA $\doteqdot 1250\times 160\times 10^{-3}=200$.

The efficiency of the transformer-inductor unit is therefore approximately 76 %, and the power-factor $\doteqdot{\cdot}105$.

The statements in the foregoing, that the tank Q should be not less than 20, and that the coupling between transformer primary and secondary and between work loop and work should be as tight as possible, may appear to be contradictory. The point is that very high current-densities are inevitable in the work loop and, to a lesser degree, in the transformer, whereas much lower densities, and therefore lower losses, can be achieved in the "buffer inductance" of the tank-coil *L2* (Fig. 9·31) within the oscillator cubicle, where there is less restriction on space. Transformer and loop losses are minimised by keeping the ampère-turns/KW down, and this can be done by using small clearances.

The choice of the dimensions D_1 and l_1 (Fig. 9·33) is not critical; primary and secondary copper-losses are determined by the ratio $\frac{D_1}{l_1}$, i.e. by the proportions rather than the actual dimensions. Since such transformers are used in conjunction with axially-short work loops, nothing is gained by increasing l_1 beyond a certain point, because the reduction in circumferential copper-losses is accompanied by an increased "axial" loss corresponding to the lines of flow towards the work loop. In this connection the centrally-placed work loops of Figs. 6·10 and 6·30 are electrically superior to the end-connected loop of Fig. 9·33, but the latter is often used because of mechanical convenience. For most purposes the ratio $\frac{D_1}{l_1}$ is chosen between ·66 and 1·0, both for internal and external secondaries; the actual value of D_1 is determined by such considerations as minimum bore of primary tubing for satisfactory water-flow, minimum permissible radial clearance between primary and secondary, and the inductance of the work loop; the larger these factors, the bigger D_1, and hence l_1, must be made.

Appendix A

TABLE I

Material	*Density*	*Melting Point ° C*	ρ *E.M.U.*	*Temp. Coefficient of Resistivity*
Aluminium	2·7	660	3200 (18° C)	·0038 (18° C)
Brass (65-35)	8·45	930	7500 (18° C)	·0010 (18° C)
Copper	8·89	1083	1780 (18° C) 22,000 (1083° C)	·0043 (18° C)
Chromium	6·93	1765	13,100 (20° C)	—
Gold	19·3	1063	2440 (20° C)	·0034 (20° C)
Graphite	1·56 (effective) 2·21 (neglecting porosity)	Volatilises above 3500° C	950,000 (average)	—
Iron	7·87	1535	9800 (18° C)	·0062 (18° C)
Lead	11·34	327	20,800 (18° C)	·0043 (18° C)
Magnesium	1·74	651	4460 (20° C)	·004 (20° C)
Molybdenum	10·2	2620	5000 (20° C)	·0047 (20° C)
Monel (cold drawn)	8·8	1350	44,000 (18° C)	·0019 (20° C)
Nickel	8·85	1440-1455	7800 (20° C)	·0054 (20° C)
Nichrome (80-20)	8·35	1375	103,000	·000098 (20-500° C)
Platinum	21·37	1773·5	11,000 (18° C)	·0038 (18° C)
Silver	10·5	960	1630 (18° C)	·0040 (18° C)
Steel (·1 % C)	7·85	1520	12,000 (18° C) 110,000 (1200° C) 200,000 (1520° C)	·0016 to ·0042 (18° C)
Steel (stainless) (18-8)	7·97	1430-1470	89,000 (18° C)	·0021 (20° C)
Tin	7·29	232	11,500 (20° C)	·0042 (20° C)
Tungsten	19·0	3382	5500 (20° C)	·0047 (20° C)
Zinc	7·14	419	6100 (18° C)	·0037 (18° C)

Magnetic change points ("Curie Points"):

* Iron	770° C
* Cobalt	1150° C
* Nickel	360° C
* Nickel-iron (30 Ni, 70 Fe)	70° C
* Permalloy (78 Ni, 22 Fe)	550° C
* Nickel-copper (Thermalloy, Thermoperm) .	10 to 70° C
Carbon steel (medium)	720° C

* From *Physical and Chemical Constants*, Kaye and Laby.

TABLE II

HEAT CONTENTS, ABOVE 0° C. KW SECS/LB.*

	°C 100 °F 212	200 392	300 572	400 752	600 1112	800 1472	1000 1832	1200 2192	1500 °C 2732 °F
Aluminium	40·0	83·2	129	178	281*S*	562*L*	661	—	—
Carbon	39·3	86·1	141	206	351	510	678	851	1120
Copper	17·6	35·7	54·6	73·2	113	154	196*S*	334*L*	401
Gold	5·8	11·8	17·9	24·2	37·5	48·8	64·6*S*	108*L*	128
Graphite	32·4	68·7	112	162	286	458	630	800	1142
Iron	20·7	43·1	68·5	98·0	159α	240β	314γ	372γ	470
Lead	5·9	12·0	18·3*S*	35·7*L*	48·2	61·0	73·4	—	—
Manganese	21·3	45·0	71·0	100	167	243α	315β	394γ	627*L*
Magnesium	46·7	96·0	148	202	314*S*	507*L*	—	—	—
Molybdenum	11·5	23·8	36·7	50·0	78·0	107	138	161	222
Nickel	20·7	43·2	68·0α	96·5β	146	198	249	302β	521*L*
Platinum	6·1	12·4	18·6	24·9	38·4	52·0	65·4	81·2	104
Silver	10·7	21·5	32·7	44·3	68·1	83·6*S*	168*L*	196	—
Steel (0·3% C)	24·1	48·2	72·0	96·5	158α	250β	314γ	364*S*	520*L* (1550° C)
Tin	10·7	22·0*S*	60·5*L*	71·0	92·3	114	135	—	—
Tungsten	6·3	12·4	18·8	25·1	38·2	51·5	65·5	79·5	102
Zinc	17·9	36·3	56·0	76·0*S*	168*L*	217	—	—	—

The heat-contents of most of the materials listed were computed from information given in *Metals Handbook*, 1939 Edition, page 85, published by the American Society for Metals.

* To convert to KW-hours per ton, multiply the values given above by $\frac{5}{8}$.

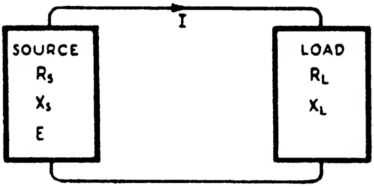
TABLE III

DIELECTRIC CONSTANT *K*, DIELECTRIC STRENGTH IN KV/MM AND POWER FACTOR TAN δ

Material	*K*	*KV/mm*	*tan δ at 1 MC/S*
Air	1·0	3·0	0
Amber (natural fossil resin)	2·9	—	·002
Cellulose acetate (sheet)	3·5 to 7·5	32 to 100	·03 to ·06
Cellulose nitrate	4 to 7	24 to 48	·07 to ·10
Di-Jell (various grades)	2 to 2·5	10 to 15	·0002 to ·001
Ebonite	2·5 to 2·5	30 to 110	·05
Glass—Crown	6·2	50 to 150	—
Glass—Flint	7·0	50 to 150	·004
Glass—Pyrex	4·5 to 4·9	30 to 150	·0028
Methacrylic resin	2·8 to 3·5	20	·02 to ·03
Mica—clear India	6 to 7·3	50 to 200	·0002 to ·003
Micanite	—	27 to 35	—
Mycalex	6 to 8	14	·003
Oil (transformer)	2·2 to 4·7	20 to 50	·005 to ·25
Perspex	2·8 to 3·44	10 to 15	·02 to ·046
Phenol, pure	5 to 6	16 to 20	015 to ·04
Phenol, black moulded	5·5	10 to 20	·035 to ·10
Phenol, paper base	5·5	16 to 52	·02 to ·05
Phenol, cloth base	5·6	6 to 24	·02 to ·08
Polythene (pure)	2·15 to 2·3	40	·00015 to ·0003
Porcelain (dry process)	6 to 7	10 to 25	·006 to ·01
Porcelain (wet process)	6 to 7	25 to 30	
Quartz, fused	3·75 to 4·1	13 to 14	·0002
Rubber	3 to 5	16 to 50	·006 to ·014
Steatite	6·1	12 to 27	·002 to ·003
Styrene (polymerised)	2·4 to 2·9	20 to 28	·0003
Shellac	2·9 to 3·7	8 to 20	·009
Titanium dioxide	90 to 170	16 to 20	·0006
Titanium dioxide—magnesium titanate	12 to 18	16 to 20	·0002 to ·001
Titanium dioxide—titanium-zirconium dioxides	40 to 60	16 to 20	·0005 to ·001
Urea-formaldehyde compounds	6 to 7	26 to 28	·03
Vaseline	2·2	9	
Vinyl resins (Vinylchloride acetate)—			
Unfilled	4	26	·017 to ·018
Filled	4	—	·02 to ·065
Water	81	—	—
Wood	2·5 to 7·7	3 to 4	·03 to ·08

Appendix B

POWER TRANSFERENCE FROM SOURCE TO LOAD



Current $$I=\frac{E}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}}.$$

Load power $$P_L=I^2R_L=\frac{E^2R_L}{(R_S+R_L)^2+(X_S+X_L)^2}.$$

The values of any of the quantities R_S, R_L, X_S, X_L, Z_S, Z_L, which yield maximum load power are found by differentiating the power equation with respect to the quantity concerned and equating the differential to zero. The results obtained are:

(1) A necessary condition for maximum load power is:

$$Z_L=Z_S \qquad (1)$$

(2) If there is complete freedom of choice of values for R_L and X_L, and X_L may be of either (a) the same kind as X_S, or (b) the opposite kind to X_S, R_S and X_S being fixed, then P_L is a maximum when

$$\left.\begin{array}{l}R_L=Z_S\\X_L=0\end{array}\right\} \qquad 2\text{ (a)}$$

or

$$\left.\begin{array}{l}R_L=R_S\\X_L=-X_S\end{array}\right\} \qquad 2\text{ (b)}$$

(3) The power is a maximum in a fixed load impedance Z_L when

either $$Z_S=0 \qquad 3\text{ (a)}$$

or when and $$\left.\begin{array}{l}R_S=0\\X_S=-X_L\end{array}\right\} \qquad 3\text{ (b)}$$

according as X_S and X_L are of the same or opposite kind.

(4) The power obtainable in cases 2 (b) and 3 (b) is greater than in 2 (a) and 3 (a) respectively; in case 2 (b) the efficiency is only 50 per cent.

(5) In all cases the power transfer efficiency is

$$\frac{R_L}{R_S+R_L}$$

(6) The ratio of output to input KVA is

$$\frac{Z_L}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}}$$

and is a maximum, i.e. unity, under conditions 3 (a) or 3 (b).

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