THE

SHEET-METAL WORKER'S INSTRUCTOR

PRACTICAL RULES FOR DESCRIBING THE VARIOUS PATTERNS REQUIRED BY ZINC, SHEET-IRON COPPER, AND TIN-PLATE WORKERS

BY

REUBEN HENRY WARN

PRACTICAL TIN-PLATE WORKER

SEVENTH EDITION

BY

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PREFACE.

The art of delineating the developed forms required for the immense variety of shapes into which sheet metals—tin, copper, zinc, and steel—are bent is based on geometry and mensuration. These, therefore, have been treated in Chapters I.—III. and VII. as preliminary to the actual projection of patterns. The behaviour of metals and their properties occupy two chapters. Afterwards the numerous forms of joints are described and illustrated, with methods of union, and tools, and appliances.

The subject of all possible developments is far too extensive to be exhausted in one volume. But the essential principles are here, which have wider applications. Many forms which appear to be difficult when first approached are found to be extended applications and modifications of simpler problems.

The fact that seven editions of this work have been sold is to the Publishers a sufficient justification for its re-issue in their series of "Trade and Technical Manuals."

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THE SHEET-METAL WORKER'S INSTRUCTOR.

CHAPTER I.

DEFINITIONS OF TERMS USED IN GEOMETRY.

A KNOWLEDGE of the elementary problems of geometry is absolutely essential to the marking out of work in sheet metal. And in order to this it is essential to have a clear and correct knowledge of the nomenclature employed. Many terms are used loosely in the shops, and are frequently employed incorrectly. No great harm is perhaps done, because the meaning of terms, however loosely used, is generally understood by those who make use of them. But it is better to employ them in their true and only geometrical, or mathematical, sense. The grasp of clear definitions and relations is also a good mental exercise. I do not propose to write a treatise on geometrical problems, but simply to preface the practical work of this book with those problems, and the definitions which relate to the same, which are of cardinal value and of fundamental importance. I shall make these as precise and brief and clear as possible, so that their meaning may be at once grasped. I need scarcely point out that there are other methods of working some of the problems than those here given; but their introduction would not be consistent with the scope of the book, and, if desired, the larger treatises, treating specially of geometrical problems, must be consulted. I will, therefore, now treat, under their separate sections, of the meaning of the terms used in geometry, of the construction and measurement of angles, of the use of set squares, and of the construction of the principal elementary figures.

DEFINITIONS.

1. A line is length without breadth.

2. The extremities of a line are points.

3. A straight line is that which lies evenly between its extreme points.

4. A plane superficies, or surface, is that in which any two points, being taken, the straight line between them lies wholly in that superficies.

5. The extremities of a superficies are lines.

6. A plane rectilineal angle is the inclination of two straight lines to one another in a plane which meet together, but are not in the same straight line (Fig. 1).

7. When a straight line, A, standing on another straight line, B, makes the adjacent angles equal to one another, each of the angles is called a right angle, and the straight line which stands on the other is called a perpendicular to it (Fig. 2).

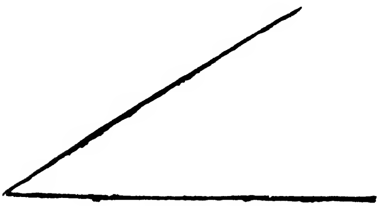


Fig. 1.

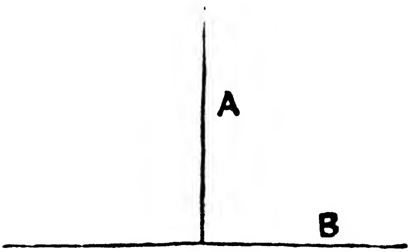


Fig. 2.

8. An obtuse angle is that which is greater than a right angle (Fig. 3).

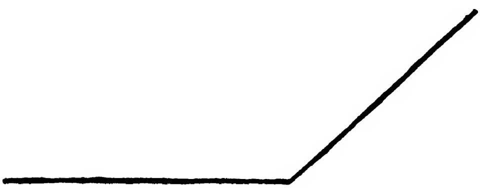


Fig. 3.

9. An acute angle is that which is less than a right angle (Fig. 1).

10. A term or boundary is the extremity of anything.

11. A figure is that which is enclosed by one or more boundaries.

12. A circle is a plane figure contained by one line, which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal (Fig. 4).

13. And this point is called the centre of the circle (Fig. 4, C).

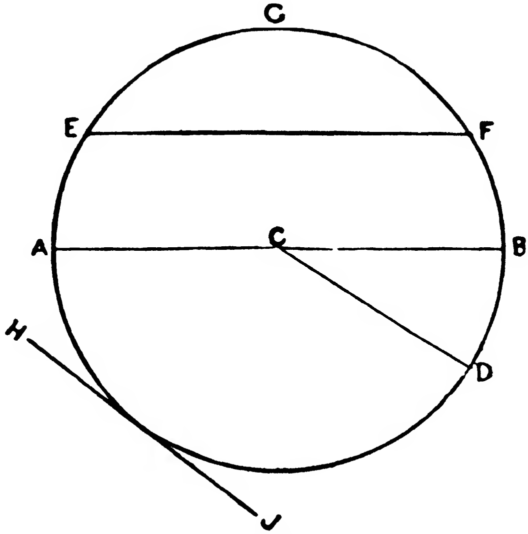


Fig. 4.

14. A diameter of a circle is a straight line drawn through the centre, and terminated both ways by the circumference (Fig. 4, AB).

15. A radius is a straight line drawn from the centre to the circumference (Fig. 4, CA).

16. A semicircle is the figure contained by a diameter, and the part of the circumference cut off by the diameter (Fig. 4, AGB).

17. A segment of a circle is the figure contained by a straight line, and the circumference which it cuts off (Fig. 4, EGF)

18. Rectilineal figures are those which are contained by straight lines.

19. Trilateral figures, or triangles, are those which are contained by three straight lines.

20. Quadrilateral figures by four straight lines.

21. Multilateral figures, or polygons, by more than four straight lines.

Triangles are classified according to (*a*) the equality or inequality of their sides, and according to (*b*) the magnitude of the angles.

According to sides :—

22. An equilateral triangle is that which has three equal sides (Fig. 5).

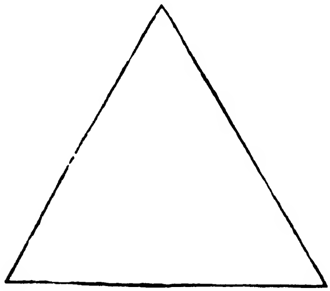


Fig. 5.

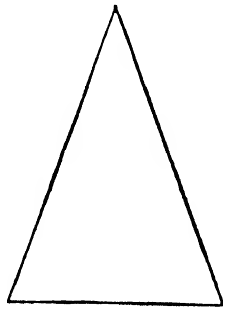


Fig. 6.

23. An isosceles triangle is that which has two sides equal (Fig. 6).

24. A scalene triangle is that which has three unequal sides (Fig. 7).

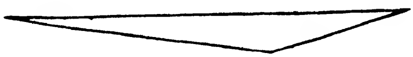


Fig. 7.

According to angles :—*

25. A right-angled triangle is that which has a right angle (Fig. 8).

26. An obtuse-angled triangle is that which has an obtuse angle (Fig. 7).

27. An acute-angled triangle is that which has three acute angles (Fig. 5).

28. The hypotenuse in a right-angled triangle is the side opposite the right angle (Fig. 8, BC).

* It is interesting to note, that since the three interior angles of any triangle are equal to two right angles (Euc. I. 32), only one of the three angles of a triangle can be a right angle or an obtuse angle, the remaining two being necessarily acute angles, since they make up the other right angle together.

29. A square is that which has all its sides equal, and all its angles right angles (Fig. 9).

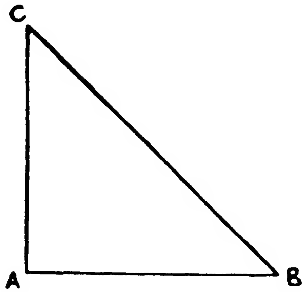


Fig. 8.

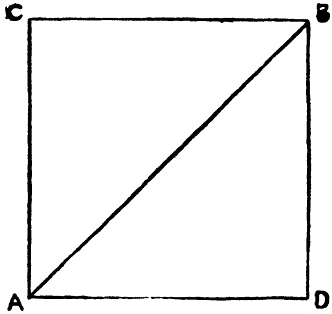


Fig. 9.

30. An oblong is that which has all its angles right angles, but only its opposite sides equal (Fig. 10).

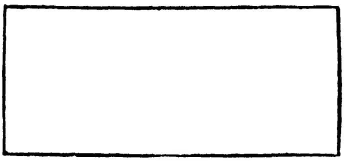


Fig. 10.

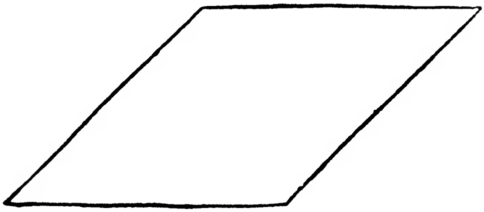


Fig. 11.

31. A rhombus is that which has all its sides equal, but its angles are not right angles (Fig. 11).

32. A rhomboid is that which has its opposite sides equal, but all its sides are not equal, nor its angles right angles (Fig. 12).

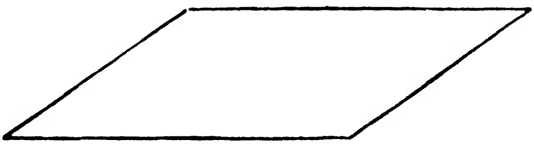


Fig. 12.

33. A line joining two opposite angles of a quadrilateral is called a diagonal (Fig. 9, AB).

34. A quadrilateral which has its opposite sides parallel is called a parallelogram (Figs. 9, 10, 11, 12).

35. A trapezoid is a quadrilateral having two, and two only, of its sides parallel (Fig. 13).

36. A trapezium is a quadrilateral which has none of its sides parallel (Fig. 14).

Fig. 13.

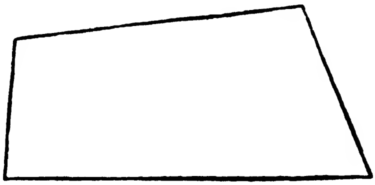


Fig. 14.

37. In a polygon, if its sides are all equal, it is called a regular polygon; if unequal, an irregular polygon.

38. A pentagon is a five-sided figure.

39. A hexagon is a six-sided figure.

40. A heptagon is a seven-sided figure.

41. An octagon is an eight-sided figure.

42. A nonagon is a nine-sided figure.

43. A decagon is a ten-sided figure.

44. An undecagon is an eleven-sided figure.

45. A duodecagon is a twelve-sided figure.

46. A sector of a circle is the figure contained by two straight lines drawn from the centre and the circumference between them (Fig. 4, CBD).

47. A chord is a straight line, shorter than the diameter, lying in the circle, and terminated at both ends by the circumference (Fig. 4, EF).

48. A tangent is a straight line which meets a circle, and being produced does not cut it, *i.e.* does not enter the circle (Fig. 4, HJ).

49. An ellipse is a plane figure bounded by one continuous curve described about two points (called foci), so that the sum of the distances from every point in the curve to the two foci may be always the same.

50. A solid is that which has length, breadth, and thickness.

51. The boundaries of a solid are surfaces.

52. A solid angle is that which is made by more than two

plane angles, which are not in the same plane, meeting at one point.

53. A pyramid is a solid figure contained by planes, one of which is the base, and the remainder are triangles whose vertices meet at a point above the base, called the vertex or apex of the pyramid (Fig. 15).

54. A prism is a solid figure contained by plane figures of which two that are opposite are equal, similar, and parallel to one another; and the others are parallelograms (Fig. 16).

NOTE.—The bases of prisms and pyramids may be regular or irregular figures, and the prism or pyramid is known as triangular, square, pentagonal, etc., according to the shape of its base.

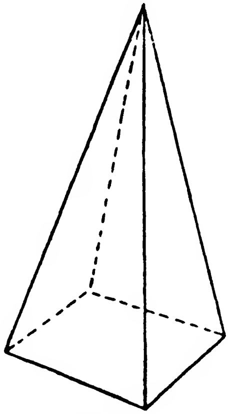


Fig. 15.

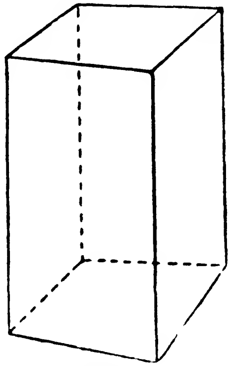


Fig. 16.

55. A cube is a solid figure contained by six equal squares (Fig. 17).

56. A sphere is a solid figure described by the revolution of a semicircle about its diameter, which remains fixed (Fig. 18).

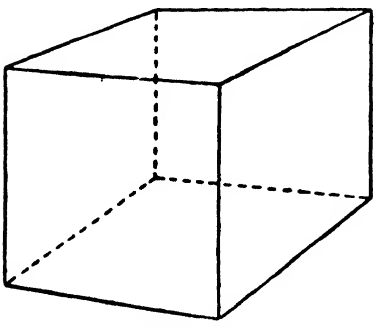


Fig. 17.

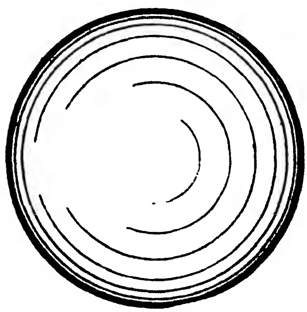


Fig. 18.

57. The axis of a sphere is the fixed straight line about which the semicircle revolves.

58. The centre of a sphere is the same as that of the semicircle.

59. The diameter of a sphere is any straight line which passes through the centre, and is terminated both ways by the superficies of the sphere.

60. A cone is a solid figure described by the revolution of a right-angled triangle about one of the sides containing the right angle, which side remains fixed (Fig. 19).

N.B.—If the fixed side be equal to the other side containing the right angle, if it be less than the other side, an obtuse-angled cone, and, if greater, an acute-angled cone.

61. The axis of a cone is the fixed straight line about which the triangle revolves.

62. The base of a cone is the circle described by that side containing the right angle which revolves.

63. A cylinder is a solid figure described by the revolution

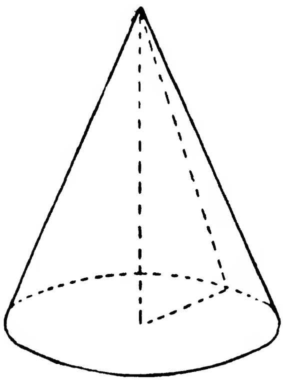


Fig. 19.

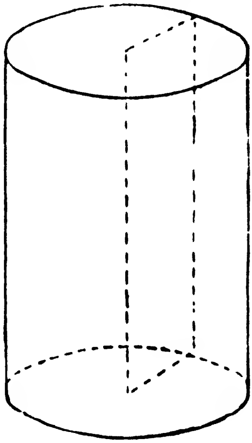


Fig. 20.

of a right-angled parallelogram about one of its sides, which remains fixed (Fig. 20).

64. The axis of a cylinder is the fixed straight line about which the parallelogram revolves.

65. The bases of a cylinder are the circles described by the two revolving opposite sides of the parallelogram.

66. A tetrahedron is a solid figure contained by four equal and equilateral triangles.

67. An octahedron is a solid figure contained by eight equal and equilateral triangles.

68. A parallelopiped is a solid figure contained by six quadrilateral figures of which every opposite two are parallel.

69. The sections of a cone form a remarkable series of curves, which were known and described by the ancient Greeks.

(*a*) If the cone be cut by a plane parallel to its base and the cut part removed, the exposed part is a circle.

(*b*) If the cone be cut obliquely so as to preserve the base entire, the section is an ellipse.

(*c*) When a cone is cut by a plane parallel to one of its sloping sides, the section is a parabola, and if cut parallel to its axis, the section is an hyperbola.

CHAPTER II.

THE CONSTRUCTION OF ANGLES.

A KNOWLEDGE of the readiest methods by which angles may be constructed is essential to the correct marking out of work. In the shops, angles are marked by the use of the common protractor, or for large work by a scale of chords, or by means of set squares. Without these aids, however, all the angles usually wanted can be constructed by the compasses alone. I will illustrate the latter method first, and afterwards that by means of the scale of chords, and of set squares.

I. TO CONSTRUCT ANGLES OF A GIVEN NUMBER OF DEGREES WITH COMPASSES.

70. To construct an angle of 60° (Fig. 21).

With centre A, and any radius, describe the arc BC cutting AB in B. With B as centre and the same radius, describe an arc intersecting the arc BC in C. Join AC. The angle CAB contains 60°.

71. To construct an angle of 30° (Fig. 21).

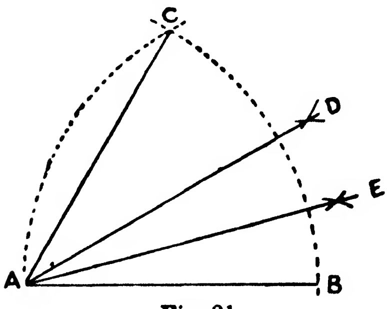


Fig. 21.

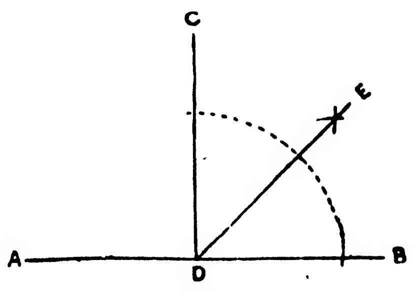


Fig. 22.

Construct an angle of 60° as above, and bisect it. The angle DAB contains 30°.

72. To construct an angle of 15° (Fig. 21)

Construct an angle of 30° as above, and bisect it. The angle EAB contains 15°.

73. To construct an angle of 45° (Fig. 22).

Erect a perpendicular on AB at D, and bisect either of the angles ADC, or CDB at E. The angle EDB contains 45°.

74. To construct an angle of 75° (Fig. 23).

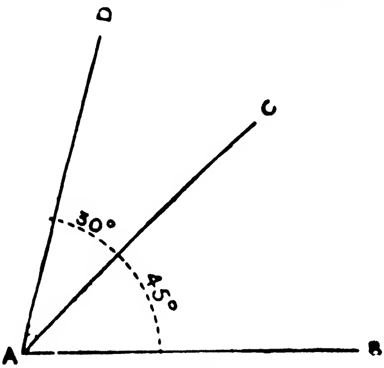


Fig. 23.

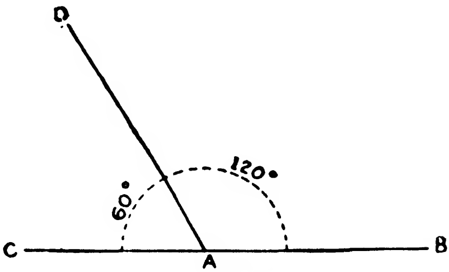


Fig. 24.

Construct an angle CAB, of 45°, as above. On AC construct an angle, CAD, of 30°. The angle BAD contains 75°.

75. To construct an angle of 120° (Fig. 24).

Produce BA to C, and on AC construct an angle, CAD, of 60°, by Prob. 79. The angle BAD contains 120°.

76. To construct an angle of 135° (Fig. 25).

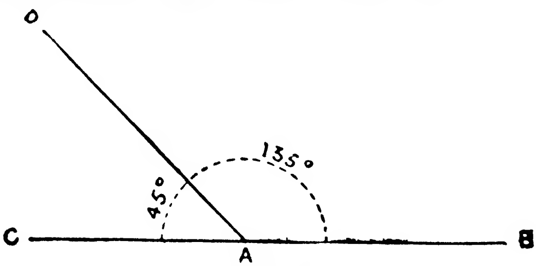


Fig. 25.

Produce BA to C, and on AC construct an angle, CAD, of 45°, by Prob. 73. The angle BAD contains 135°.

77. To construct an angle of 150° (Fig. 26).

Produce BA to C, and on AC construct an angle, CAD, of 30° by Prob. 71. The angle DAB contains 150°.

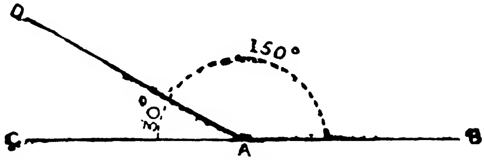


Fig. 26.

78. To construct an angle of 105° (Fig. 27).

Produce BA to C, and on AC construct an angle, CAD, of 75° by Prob. 74. Then the angle DAB contains 105°.

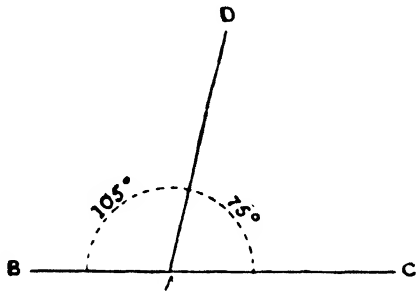


Fig. 27.

II. The Scale of Chords.

The scale of chords (Fig. 28) may be constructed of any size. A small scale is suitable for small work, but for large work the scale should be, say, from 18" to 24" in length. The reason is, that error in marking off the angles of lines of considerable length is lessened thereby. With the small protractors and small scales a little error soon becomes magnified on work of large magnitude.

79. To construct the scale of chords (Fig. 28).

With any radius describe the quadrant CE. Erect its perpendicular CD. Draw the chord, and trisect its arc. Divide each third into 6 equal parts by trial. With E as centre and the distances to each of the 18 divisions as radii, describe arcs cutting ED, the base of the quadrant, and the base produced. The divided line is the scale of chords, reading to 5°.

We will now proceed to investigate its use. Suppose it be

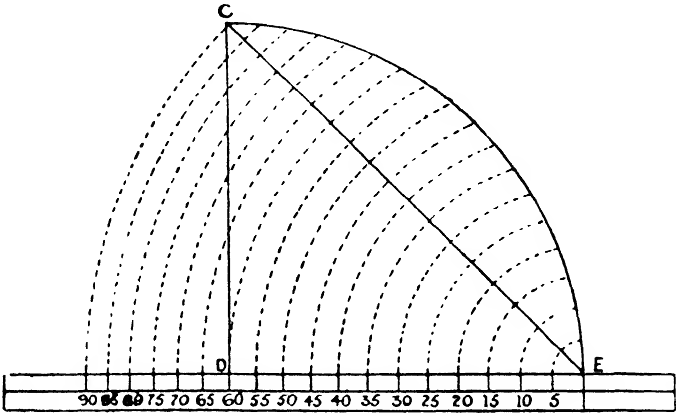


Fig 28.

required to construct an angle of, say 65°. It is spoken of as *plotting* an angle of 65°.

From the centre A (Fig. 29), and with the distance E, to 60° (Fig. 28), as radius, describe the arc BC. With B as centre, and the length E, to 65° (Fig. 28), as radius, cut the arc BC in C. Join CA. Then the angle CAB is an angle of 65°.

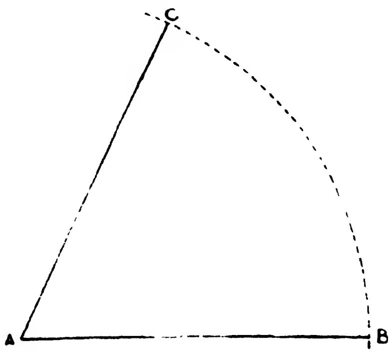


Fig. 29.

In this manner we may construct any angle we require from the scale of chords. In the scale we have taken, the angles differ by 5°. If we required them to differ by 10°, as 10°, 20°, 30°, 40°, etc., we should divide the quadrant into 9 equal parts ($90 \div 9 = 10$). If a difference of 1° were required, divide the quadrant into 90 parts ($90 \div 90 = 1$). It must be noted, too, that the length E, to 60°, must always be taken as radius because that is the radius used in constructing the scale.

We can not only construct angles by this scale, but measure unknown angles.

To measure the angles CAB (Fig. 30), by the scale of chords.

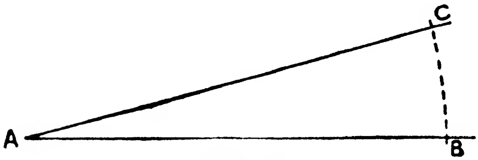


Fig. 30.

With centre A, and radius E, to 60° (Fig. 28), strike the arc BC. Measure the length of the arc BC with compasses, and apply the length to the scale (Fig. 28). The angle is thus found to contain 15°.

III. The Use of Set Squares.

Set squares are in constant use in the workshop. They are called "set" squares to distinguish them from the "try" squares, the latter checking external angles, the former internal. The set squares are made in two forms, the "mitre," in which the hypotenuse makes angles of 45° with the sides, and the 30° and 60° square, in which the hypotenuse makes those angles with the sides. By means of these angles, namely, 90° of the sides, and 45°, 30°, and 60°, many angles can be plotted at once

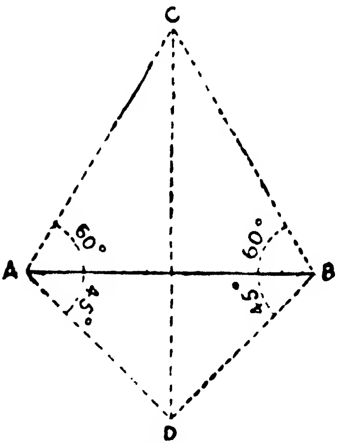


Fig. 31.

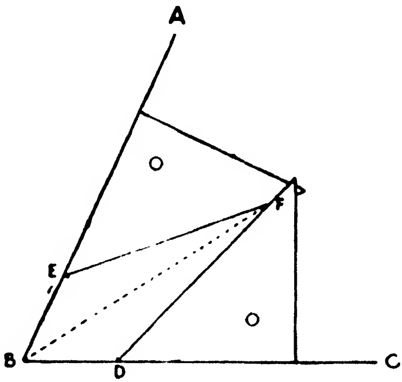


Fig. 32.

80. To bisect a straight line AB by the aid of a set square (Fig. 31).

At the extremities of the line AB, make equal angles of either

45° or 60° on one side, as shown in Fig. 31. On the other side, at the two extremities, make two equal angles of either 45° or 60°. Produce these lines which make the equal angles until they intersect. The line CD joining the points of intersection bisects AB.

81. To construct angles of given dimensions by means of set squares.

Figs. 33–38 show clearly how this may be done.

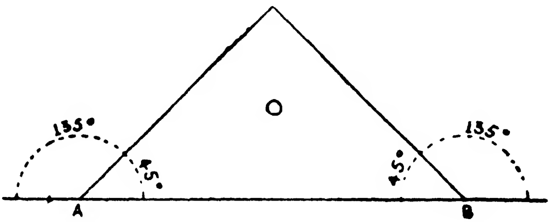


Fig. 33.

82. To bisect an angle by the aid of two set squares (Fig. 32).

Suppose the angle ABC be required to be bisected. From B

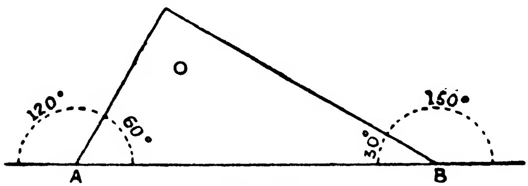


Fig. 34.

mark off distances BD, BE, equal to each other. Place set squares as shown, and the straight line joining B with the point of intersection F bisects the angle ABC.

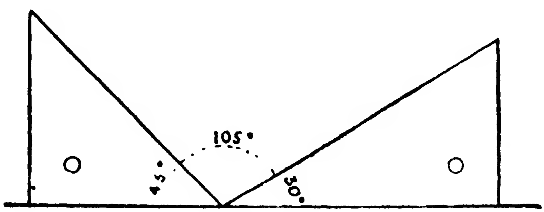


Fig. 35.

In Fig. 33 the square with angles of 45° is used for marking angles of 45° or 135°. In Fig. 34, the square with angles of 60° and 30° marks those angles, and also angles of 120° and

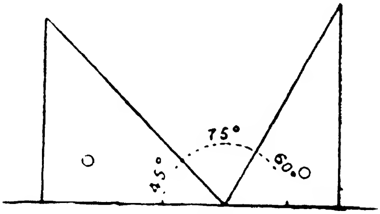


Fig. 36

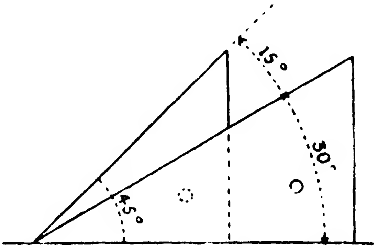


Fig. 37.

150°. In Fig. 35, the two squares in combination with 30° and 45° adjacent set off an angle of 105°. In Fig. 36, the same, but with 60° and 45° adjacent, set off an angle of 75°.

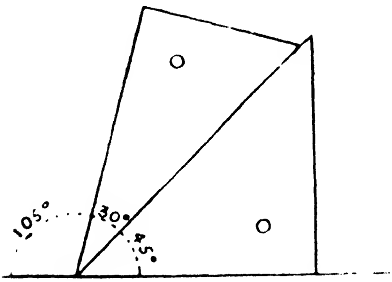


Fig. 38.

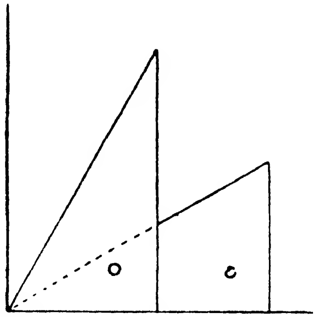


Fig. 39.

In Fig. 37 the same square with 45°, and 30° superimposed, sets off an angle of 15°. In Fig. 38 the same set makes angles of 75° and 105°.

83. To trisect a right angle.

Fig. 39 shows clearly how this may be done by means of two set squares, each of 60°.

84. Set squares and the straight edge are also invaluable for constructing right angles, perpendiculars, and parallels.

Fig. 40 shows their mode of use more plainly than any verbal explanation.

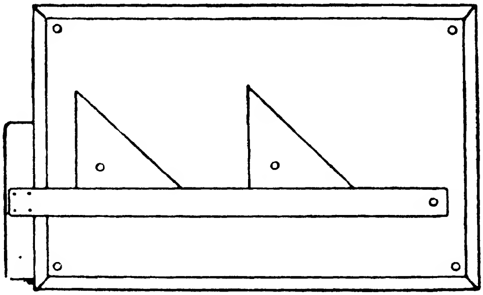


Fig. 40

CHAPTER III.

SOME USEFUL PROBLEMS IN GEOMETRY.

Some Important Geometric Truths.

(*a*) All straight lines drawn from the centre to the circumference of a circle are equal. (Euc. I. def. 30.)

(*b*) The angles at the base of an isosceles triangle are equal to one another. (Euc. I. 5.)

(*c*) The greater side of every triangle has the greater angle opposite to it. (Euc. I. 18.)

(*d*) If a straight line fall on two parallel straight lines, it makes the alternate angles equal to one another. (Euc. I. 29.)

(*e*) The opposite sides and angles of a parallelogram are equal to one another. (Euc. I. 34.)

(*f*) Triangles upon the same base, or on equal bases, and between the same parallels, are equal to one another. (Euc. I. 37, and I. 38.)

(*g*) Parallelograms upon the same base, or upon equal bases, and between the same parallels, are equal to one another. (Euc. I. 35, and I. 36.)

(*h*) If a parallelogram and a triangle be upon the same base, and between the same parallels, the parallelogram shall be double of the triangle. (Euc. I. 41.)

(*i*) The angle in a semicircle is a right angle. (Euc. III. 31.)

(*j*) If a straight line be parallel to one side of a triangle, it cuts the other sides, or those produced, proportionally. (Euc. VI. 2.)

(*k*) In equal circles, the angles at the centre standing on equal arcs, are equal to one another. (Euc. III. 27.)

(*l*) In equal circles, equal straight lines cut off equal arcs. (Euc. III. 28.)

(*m*) In any right-angled triangle, the square which is described on the side subtending the right angle is equal to the squares described on the sides which contain the right angle. (Euc. I. 47.)

Useful Geometrical Problems.

85. To construct an equilateral triangle on a given base. AB (Fig. 41).

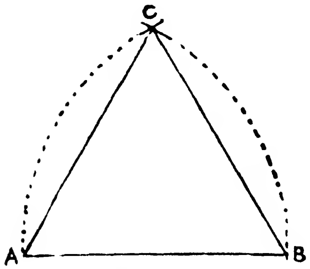


Fig. 41.

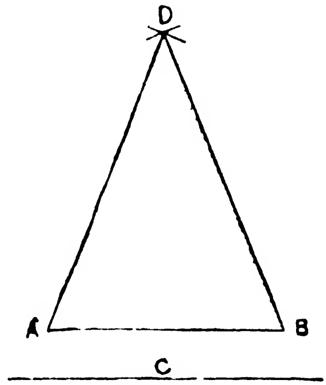


Fig. 42.

With centre A and radius AB describe the arc BC, and with B as centre and with the same radius describe the arc AC intersecting the arc BC at C. Join CA, CB.

86. To construct an isosceles triangle, having given the base AB and one side C (Fig. 42.)

With A as centre and length C as radius describe an arc. With B as centre and the same radius describe an arc, cutting the former at D. Join DA, DB.

87. To construct a rhombus, the diagonal AB and one side E being given (Fig. 43).

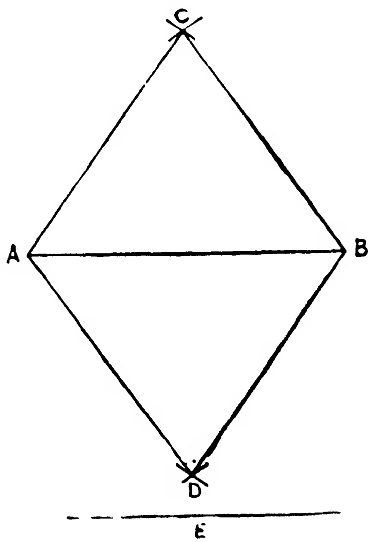


Fig. 43.

With A as centre and radius

equal in length to the line E describe an arc above and below the line AB. With B as centre and the same radius describe an arc above and below to cut the former arcs. Join CA, CB, DA, DB.

88. To construct a triangle having given the three sides. (Fig. 44.)

Draw AB equal to the length of D as base. With centre A and radius equal in length to the line F describe an arc. With centre B and radius equal to E describe an arc cutting the former in C. Join CA, CB.

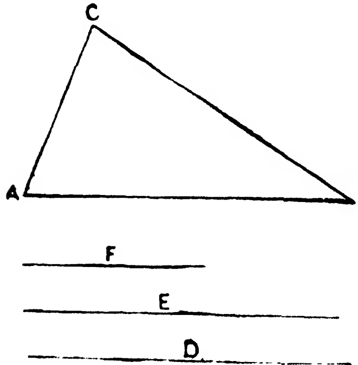


Fig. 44.

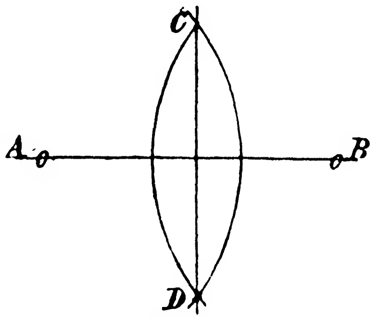


Fig 45.

89. To bisect a given straight line; that is, to divide it into two equal parts at right angles (Fig. 45).

Let AB be the given line. From any part, say *o o*, with radius greater than half the length *o o*, describe curves cutting each other in CD. Then a straight line drawn through the points of intersection will bisect the line AB.

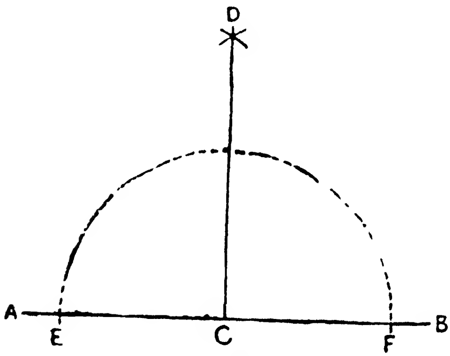


Fig. 46.

90. To draw a perpendicular to a given line AB from a given point C in it (Fig. 46).

With centre C and any radius describe an arc intersecting

AB at the points EF. With centres EF and any radius greater than half the arc EF describe arcs to intersect at D. Join DC.

91. To draw a perpendicular to a given line AB from a point C outside it (Fig. 47).

With C as centre and any radius describe an arc cutting AB

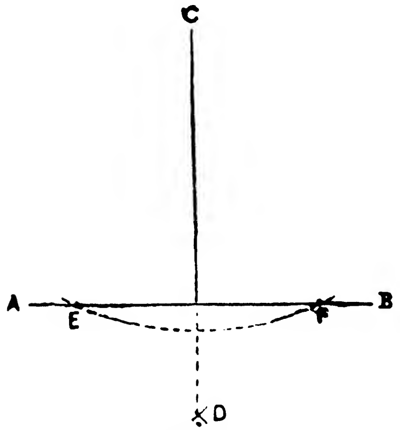


Fig. 47.

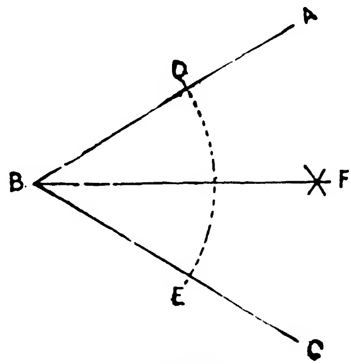


Fig. 48.

in EF. With EF as centres and any radius describe arcs to intersect at D. Join CD.

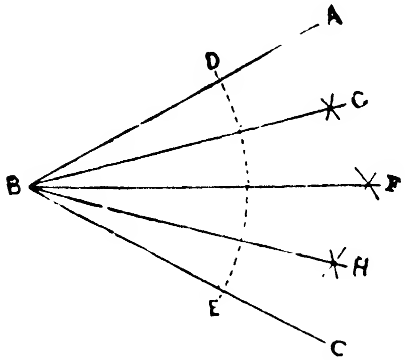


Fig. 49.

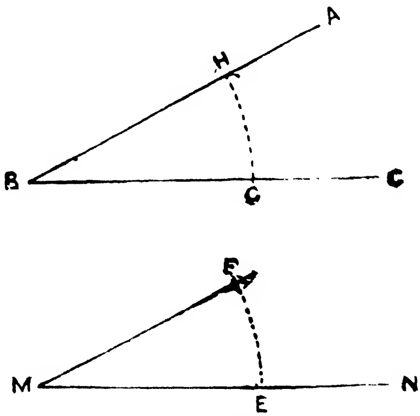


Fig. 50.

92. To bisect a given angle ABC (Fig. 48).

With centre B and any radius describe an arc cutting the lines in D and E. With centres D and E and any radius describe arcs intersecting in F. Join BF.

93. To divide a given angle into four equal parts (Fig. 49).

Bisect the angle at F as in the previous problem. Then bisect the angles ABF, FBC by lines BG, BH.

94. To make an angle equal to a given angle ABC (Fig. 50).

Draw any straight line MN. With centre B describe an arc

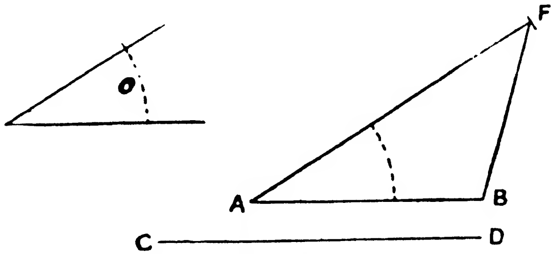


Fig. 51.

cutting the lines in GH. With centre M and the same radius describe another arc EF. With centre E and radius GH cut the arc EF in F and join MF. Then the angle FME is equal to the angle HBG.

95. To construct a triangle having given two sides AB, CD and the contained angle *o* (Fig. 51).

At the point A make with the line AB an angle equal to the

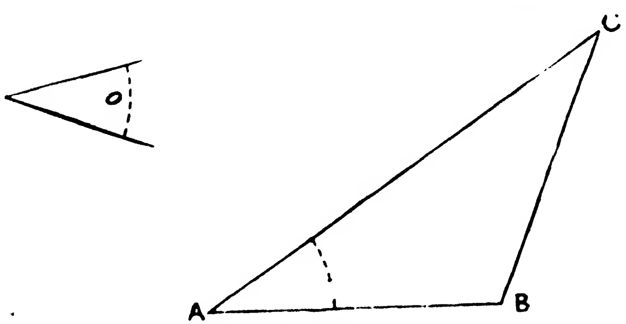


Fig. 52.

angle *o*, by the previous problem. From the line drawn cut off a part AF equal to CD and join BF.

96. To construct an isosceles triangle having given one of the equal sides AB and one of the equal angles *o* (Fig. 52).

At A make with the line AB an angle equal to *o*. With centre B and radius equal to AB describe an arc cutting the line AC in C. Join CB.

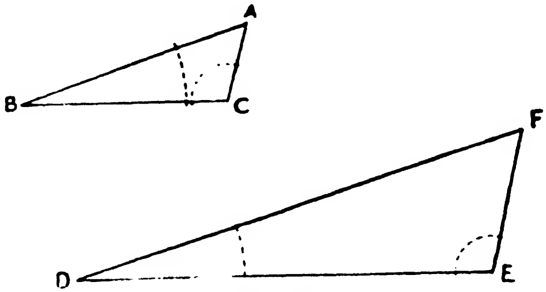


Fig. 53.

97. To construct a triangle similar to a given one ABC on a given line DE (Fig. 53)

At the point D, with the straight line DE make an angle equal to the angle ABC. At the point E with the line DE make an angle DEF equal to BCA. Produce the lines DF, EF, till they meet at F.

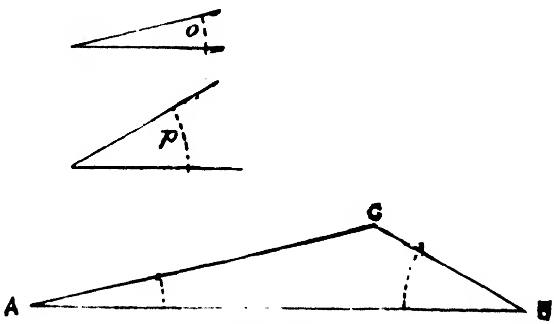


Fig. 54.

98. To construct a triangle having given the base AB and the angles *o* and *p* at the base (Fig. 54).

At the points A and B, with the line AB make angles equal to *o* and *p* respectively. Produce AC, BC to meet at C.

99. To construct an isosceles triangle, the base FG and the vertical angle A being given (Fig. 55).

With centre A and any radius describe an arc BC. Join the points B and C. At points F and G make angles equal to ABC. Produce FD and GD until they meet at D.

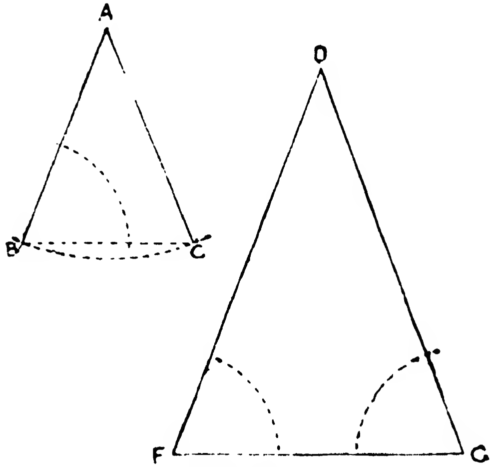


Fig. 55.

100. To construct a polygon on a given base AB similar to a given one MNOPQ (Fig. 56).

Draw the diagonals MO, MP. At the point B make an angle

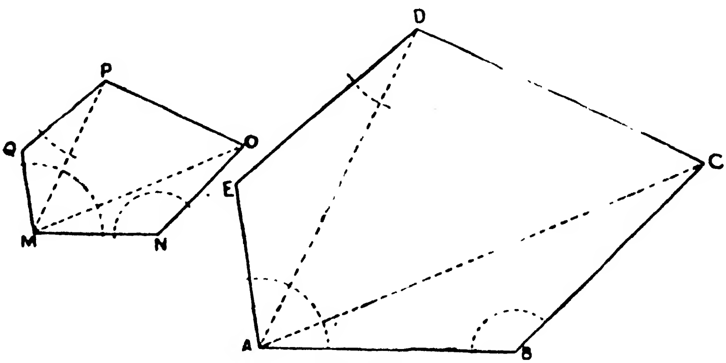


Fig. 56.

equal to MNO. At the point A make three angles BAC, CAD, DAE equal to the three angles NMO, OMP, PMQ respectively.

Produce AC, BC till they meet at C, and at C make an angle equal to MOP. Produce CD, AD till they meet at D, and at D make an angle equal to MPQ. Produce AE, DE till they meet at E.

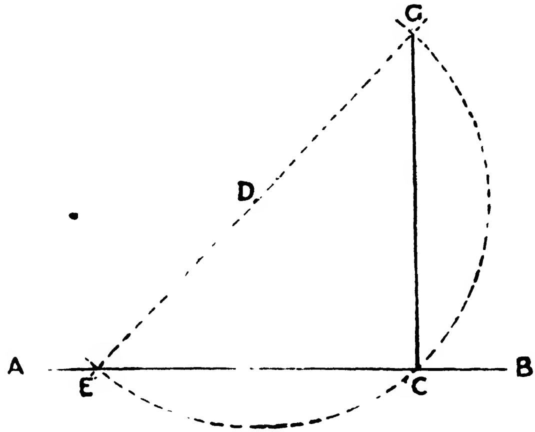


Fig. 57.

101. To draw a perpendicular to a given straight line AB from a given point C within it (Fig. 57).

Take any point D nearer to C than to A. With centre D and radius DC describe an arc ECG. Join ED and produce it to meet the arc in G.
Join CG.

102. To draw a perpendicular to a given line from a given point C outside it (Fig. 58).

Take any point A in AB not opposite C. Join AC and bisect AC in *d*. From *d* as centre and *d*A as radius describe an arc cutting AB in B. Join CB which will be the perpendicular required.

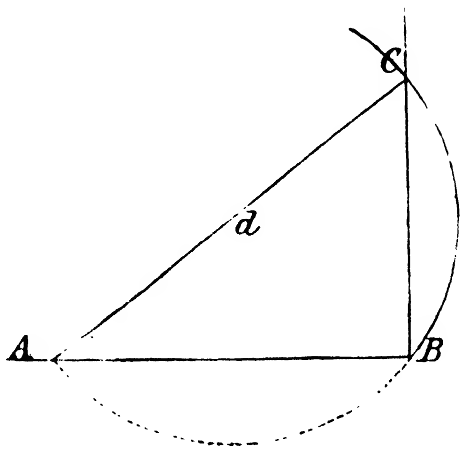


Fig. 58.

103. To construct a right-angled triangle, its hypotenuse AB, and one side CD being given (Fig. 59).

Bisect AB and describe a semicircle on it. With centre A and radius CD cut the semicircle in E. Join AE, BE (the angle in a semicircle is a right angle).

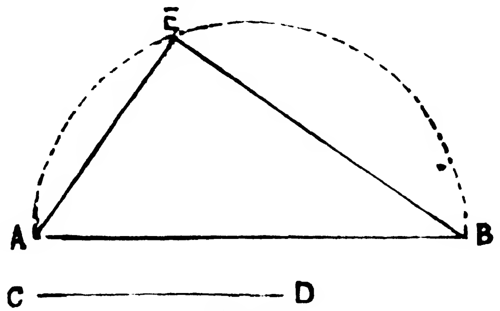


Fig. 59.

104. To construct a right-angled triangle having given the base AB and one of the acute angles C (Fig. 60).

Bisect AB and on it describe a semicircle. At the point A make an angle equal to C. Join BD.

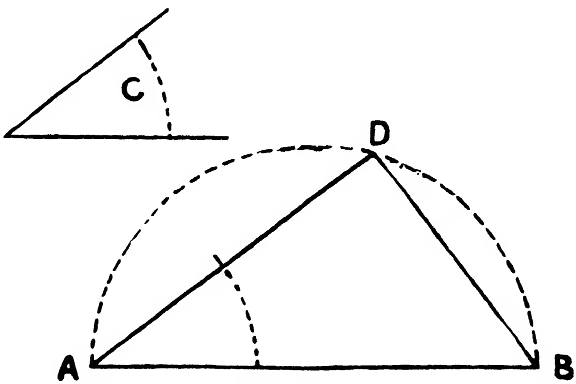


Fig. 60.

105. To construct a square having given the diagonal AB (Fig. 61).

Bisect AB at C. Erect the perpendicular DE. With centre C and radius AC describe a circle cutting the perpendicular in D and E. Join AD, DB, BE, EA.

106. To construct a rectangle having given one diagonal AB and one side C (Fig. 62).

Bisect AB at D. With centre D and radius AD describe a

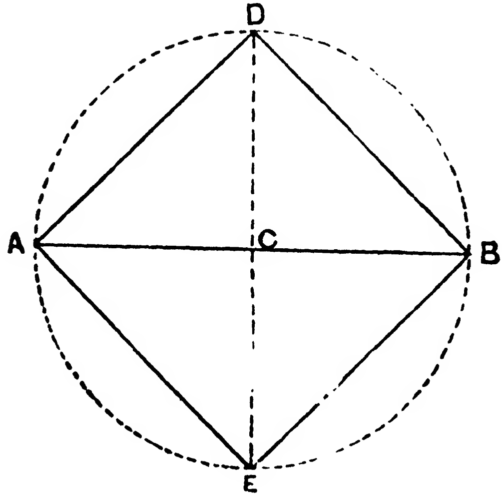


Fig. 61.

circle. With centres A and B, and radius equal in length to C, cut the circumference in E and F. Join AE, EB, BF, FA.

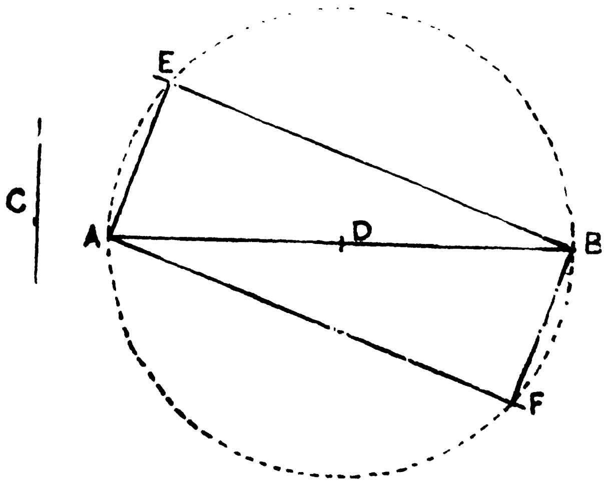


Fig. 62.

107. To draw a line parallel to a given straight line at a given distance from it (Fig. 63).

Let AB be the given straight line, and the line AC represent the distance between the parallels. Then with A as centre and

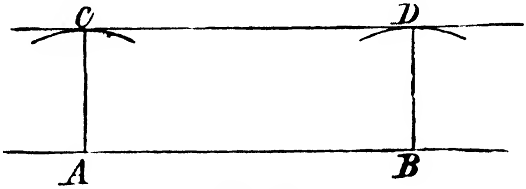


Fig. 63.

the radius AC describe the arcs C and D. Draw the line CD so as to touch these curves, and CD will be parallel to AB as required.

108. To draw a line parallel to a given line AB through a given point C (Fig. 64).

Take any point D in AB. With D as centre and DC as

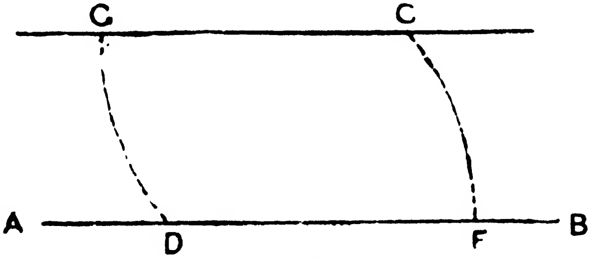


Fig. 64.

radius describe the arc CF. With C as centre and the same radius describe the arc DG. Cut off DG equal to FC. Draw GC the parallel line required.

109. To construct a triangle, given its perimeter or boundary length AB, and two of its angles CD (Fig. 65).

At A and B make angles BAE, ABE each equal to C and D respectively. Bisect the angles BAE, ABE. Through the point H where the lines intersect, draw HM, HN parallel to AE and BE.

110. To divide a line into any number of equal parts (in this case seven) (Fig. 66).

Let AB be the given line which is to be divided into seven

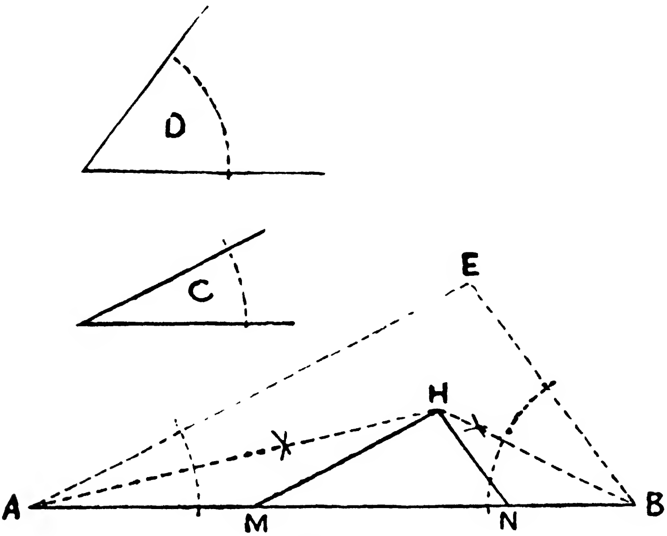


Fig. 65.

equal parts. From the point A draw another line (at any angle with AB), and with any convenient opening of the compasses set off seven equal parts, as 1, 2, 3, 4, 5, 6, 7. Join the points 7 and B, and draw parallel lines from 6, 5, 4, 3, 2, 1, to cut the line AB, which will be divided into seven equal parts as desired.

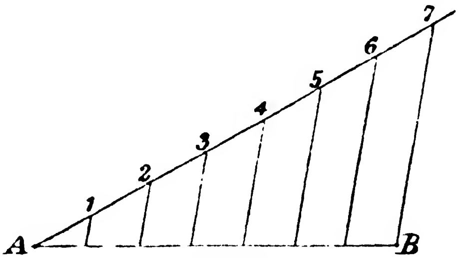


Fig. 66.

111. To divide a line AB proportionally to a given divided line CD (Fig. 67).

Draw a line AD equal to CD at any angle to AB. Join BD. From the points EFG, transferred from the points *e*, *f*, *g* on line CD, draw lines parallel to BD. Then AB is divided proportionally to CD.

112. To find a mean proportional to two given lines AB, BC (Fig. 68).

Place them in the same straight line ABC. Bisect ABC at D and describe a semicircle on it. At B erect a perpendicular

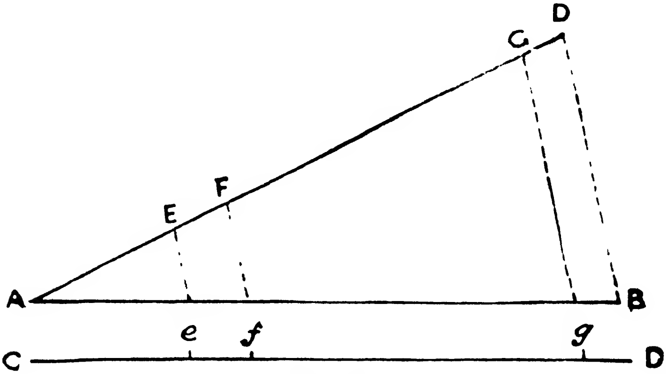


Fig. 67.

to touch the semicircle at F. Then AB : BF : : BF : BC. (Euc. VI. 13.)

113. To describe any regular polygon (Fig. 69).

Describe a circle and draw its diameter. Divide the diameter

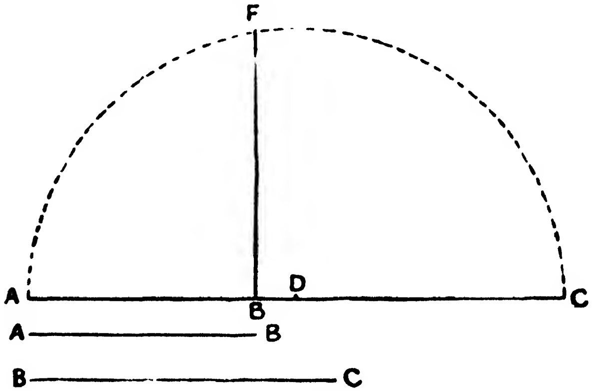


Fig. 68.

into the same number of equal parts as the polygon is to have sides (in this case five). With centres A and B and radius AB describe arcs cutting each other at C. From C draw through the second division (*in all cases*) and produce to D. Step off the arc AD four times and thus complete the pentagon. This method is not geometrically accurate, but for practical work it is accurate enough.

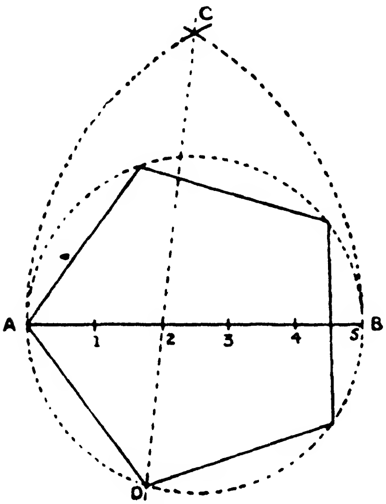


Fig. 69.

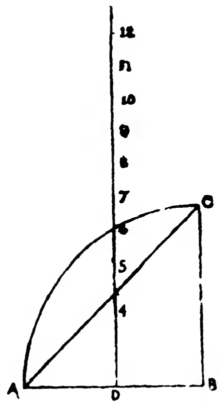


Fig. 70.

114. To describe any regular polygon on a given straight line AB (Figs. 70, 71).

At B in Fig. 70 erect a perpendicular BC, equal to AB. Join AC. Bisect AB in D, and erect a perpendicular, cutting AC in 4. *The point 4 is the centre of a square described on AB.* With B as centre and AB as radius describe the quadrant A6C. *The point 6 is the centre of a hexagon described on AB.* Bisect 6–4 at 5. *The point 5 is the centre of a pentagon described on AB.* Step off on the perpendicular, spaces equal to 4–5 or 5–6 as shown. Fig. 71 shows a pentagon and a heptagon constructed by this method.

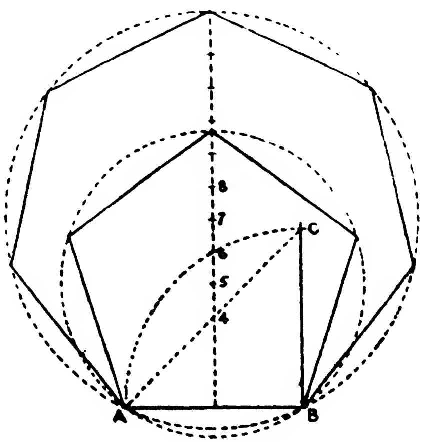


Fig. 71.

115. Upon a given straight line to describe any regular polygon (in this case a pentagon) (Fig. 72).

Produce *ab* indefinitely, from *b* as centre with a radius *ba*, describe the semicircle *ac*5, which divide into as many equal parts as there are to be sides in the polygon, in the present example five, through the second division from 5 draw the line *bc*, which will form another side, bisect these sides as shown at *f*, *g*, ***h***, *e*, the point of intersection at *o* is the centre of the circle of which *a*, *b*, *c*, are points in the circumference, then producing the dotted lines *b*1 to *c*, and *b*2 to *d*, will divide the circle into the number of parts required.

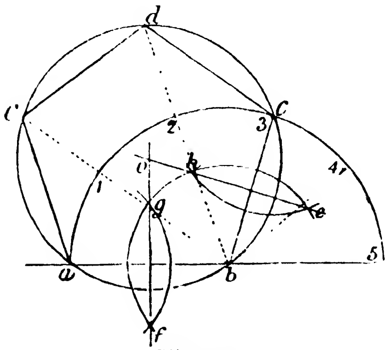


Fig. 72.

116. Upon a given side to draw a regular pentagon (Fig. 73).

Let AB be the given side, from its extremity B erect a perpendicular B*f* equal to half AB, join A*f* and produce it till *fb* be equal to B*f*, from A and B as centres with a radius equal to B*b*, draw arcs intersecting at E, which will be the centre of a circle containing five divisions equal to AB.

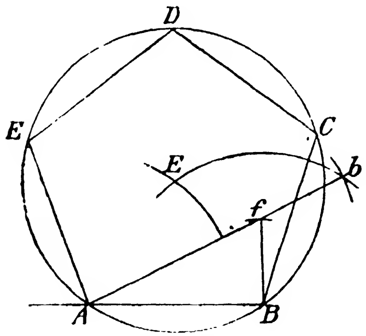


Fig. 73.

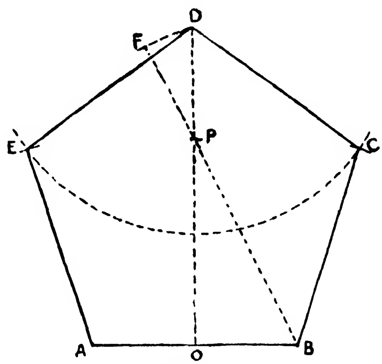


Fig. 74.

117. To construct a pentagon on a given line AB (Fig. 74).

Bisect AB in O, and draw OD perpendicular to AB. Make OP equal to AB. Join BP, and produce it to F. Make PF equal to AO. With B as centre and BF as radius describe an arc cutting OD in D. D is the apex of the pentagon. With D as centre and AB as radius describe an arc EC. With A, B as centres and AB as radius cut this arc in E and C. Join BC, CD, DE, EA.

118. To construct a hexagon on a given straight line AB (Fig. 75).

With A as centre and radius AB describe the arc BE. With B as centre and the same radius describe the arc AF cutting

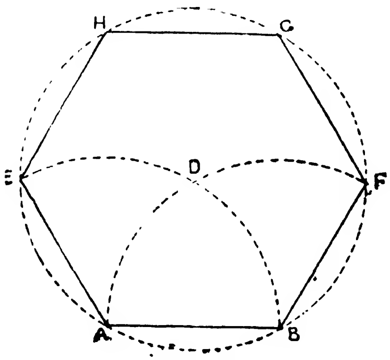


Fig. 75.

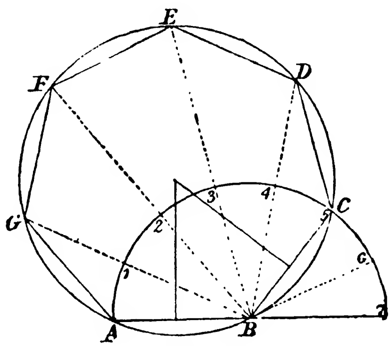


Fig. 76.

the other arc at D. With D as centre and the same radius describe the circle ABFGHE. With centres F and E and the same radius cut the circle in G and H. Join the points BFGHEA.

119. Fig. 76 is a heptagon drawn on the same principle as the pentagon (Fig 72), and which will give a sufficient explanation how this or any other polygon having a given number of equal sides is drawn.

120. To construct a regular octagon (Fig. 77).

Describe the circle ADBFCE and bisect its diameter by the perpendicular CD. Bisect the angles BMC, AMC, and produce these lines to cut the circle as shown. Join the points where the diameters cut the circle.

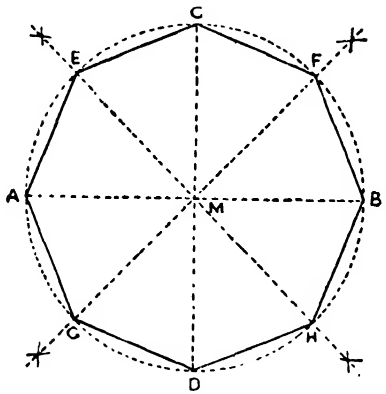


Fig. 77.

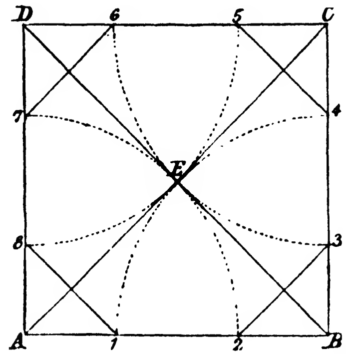


Fig. 78.

121. To describe an octagon in a given square (Fig. 78).

Let ABCD be the given square. Draw the diagonals ACDB, then from the angular points ABC and D, with a radius equal to AE, describe curves cutting the sides of the square in 1, 2, 3, 4, 5, 6, 7, 8, then by joining these points the polygon will be complete.

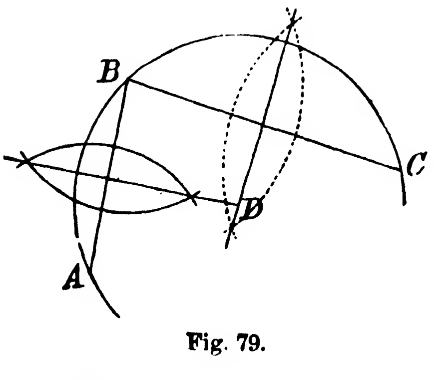


Fig. 79.

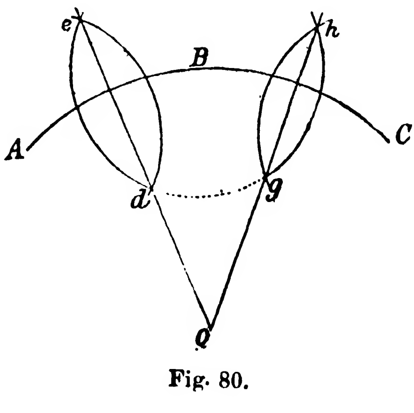


Fig. 80.

122. To draw a circle through any three given points (provided they are not in a direct line) (Fig. 79).

Let A, B, C be the three given points; join AB and BC;

bisect AB and BC, and produce the bisecting lines until they cut each other in the point D, then D will be equi-distant from each of the three points, and the centre of the circle required.

123. To find the centre of a circle or the radius of a curve (Fig. 80).

From the point B as a centre, with radius greater than half the distance to the other points, A and C, draw a portion of a circle, as *edgh*, and from A and C as centres, with the same radius, draw curves to intersect or cut the part of a circle first drawn at *ed* and *gh*. From these points of intersection draw lines *ed* and *hg* until they meet at *o*, which will be the centre of the curve required.

124. To draw a tangent to a circle or portion of a circle without having recourse to the centre (Fig. 81).

Let A be the point from which the tangent is to be drawn. Take any other point C in the circle AC, join AC, and bisect the curve AC at *f*, then from A as centre with the radius A*f*, equal to the chord of half the curve, describe the curve *ef*D, making *f*D equal to *ef*, then through the points A, D draw the line DAB, which will be the tangent required.

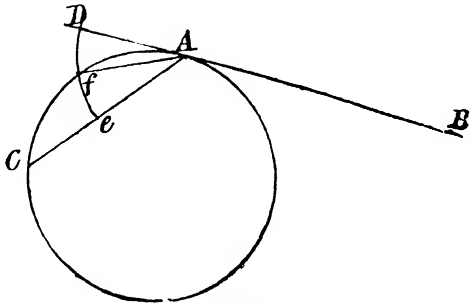


Fig. 81.

125. Having an arc of a circle given, to raise perpendiculars from any point or points without finding the centre (Fig. 82).

Let AB be the given curve or arc, and A 1, 2, 3, 4, 5, the points from which perpendiculars are to be erected. Make the dimension 5–6 equal to 4–5, from 4 and 6 as centres, with radius greater than half the distance between them describe arcs intersecting each other at 7; a line drawn through the point of intersection at 7–5 gives one of the perpendiculars required, the other points as far as 11 will be found in the same manner. If a perpendicular is to be raised at A, the extremity of the curve, a method somewhat different must be employed; suppose the perpendicular 1–11 to be erected, from

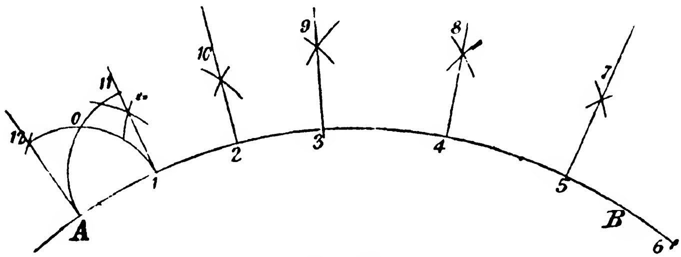


Fig. 82.

1 with the radius 1A describe the curve A-11, and from A with the same distance describe 12–1, make $o12$ equal to $o11$, and A-12 will give the perpendicular wanted.

126. To draw a straight line equal to the circumference of a given circle (Fig. 83).

Let A, D, B, c, be the given circle. Draw the diameter AB, and through its centre o draw the perpendicular cD. Draw a diagonal line Ac; set the compasses in c, and with a radius at any distance beyond its centre a describe the arc E; now with the compasses in A draw another arc intersecting at E, and draw the line oE, then three times the diameter, with the distance ab added, will be a close approximation to the length of the circumference.

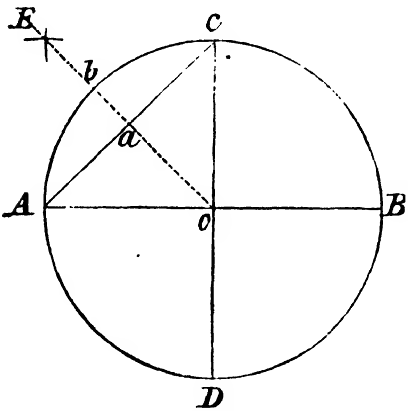


Fig. 83.

127. To draw a straight line equal in length to any given arc of a circle (Fig. 84).

Let A B or C D be the given arcs. From A with radius AB, and *vice versâ*, describe arcs intersecting at G. Draw EF parallel to AB, then from G draw lines through A and B cutting at E and F, then EF in the length of the curve from A to B.

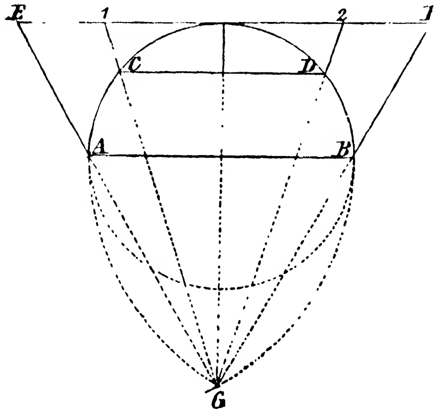


Fig. 84.

Again, lines drawn from G, through C and D, cutting at 1 and 2, will give the length from C to D. On the same principle the length may be found from any other point in the semicircle.

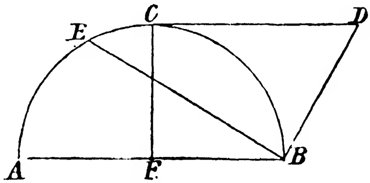


Fig. 85.

128. To find the length of a semicircle by another method (Fig. 85).

Let ACB be the semicircle. Make AE equal to AF, and draw EB, then draw BD at right angles to EB, and draw CD parallel to AB; CD is the length of the quadrant CB, and twice CD the length of the semicircle ACB.

129. To draw an ellipse with a string and pencil (Fig. 86).

Draw the given diameters AB and CD at right angles to each other at their centre, E. Take the distance from E to A with compasses, then using C as centre, draw an arc to cut the diameter AB in *o*, *o* (these two points *o*, *o* are called the *foci* of the ellipse). Place a pin at each point where the curve cuts the line AB, as at *o*, *o*, and another at C, pass a string round the three pins, and tie it securely, thus forming a triangle with the string, as *oo*C. Take

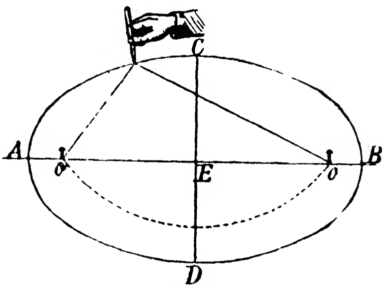


Fig. 86.

out the pin at C and substitute the point of a pencil, which may be drawn along, moving with the string, and the point will thus trace a perfect ellipse.

130. To draw an ellipse with the trammel (Fig. 87).

The trammel is an instrument consisting of a right-angled cross, ABCD, grooved on one side, and a tracer, E, with three movable studs 1, 2, B, two of which slide in the grooves just mentioned, the other at B is provided with a pencil to trace the curve of the ellipse. For the application, suppose AC to be the major axis, and BD the minor, lay the cross of the trammel on these lines; then adjust the sliders of the tracer so that 1B may be equal to *o*C, and EB equal *o*D; then by sliding the tracer in the grooves of the cross, the pencil at B will describe the ellipse.

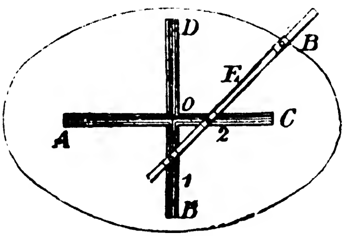


Fig. 87.

131. Fig. 88 illustrates precisely the same principle of drawing the ellipse as Fig. 87, and is inserted because the trammel (which is perhaps preferable to any other method of drawing this curve) is not always at hand; and this is a trammel easily constructed, and answers every purpose.

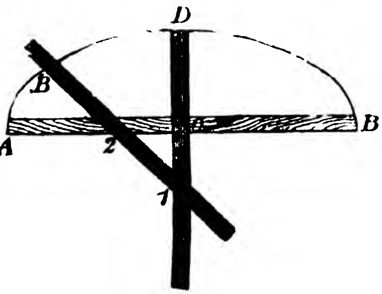


Fig. 88.

Take A*o* as the major axis, and D*o* as the minor, on which another straight edge is to be fastened and extended as shown, put a bradawl or nail through at 1 and 2, and apply the pencil at B, then by sliding the tracer round, keeping the bradawl against the axes of the ellipse, one quarter of the curve will be described; now move the tracer to another quarter, and describe it in the same manner, and continue in like manner until the ellipse is completed.

132. To strike an ellipse with the compasses, the length or major axis being given (Fig. 89).

Divide the given length 1–5 into five equal parts, then with 2 as the centre and a radius 2–4, and *vice versâ*, describe curves intersecting at A and B; then from the points A and B draw lines through 2 and 4 indefinitely. With 2 as centre and radius 2–1 describe the curve CD, and from 4 the curve EF; now with centre A and radius AD describe the curve DF, and from B the curve CE, which completes the ellipse.

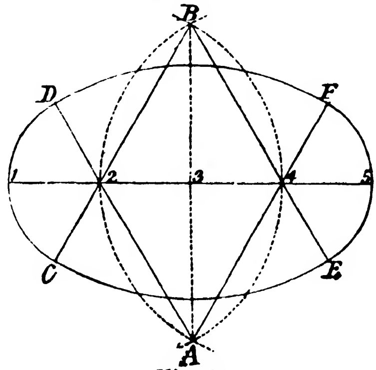


Fig. 89.

133. To describe an ellipse within a given square, or when the major and minor axes are given (Fig. 90).

Draw the major axis AB, and minor axis CD, and the diagonal BD, take the distance AE on the major axis, and transfer from B to 1 on the diagonal BD, also transfer the distance ED to the point 3; take half 1–3 in the point 2 as centre, and any distance towards B greater than its half, and *vice versâ;* from B and 2 describe arcs intersecting at 4 and 5, through these points draw a line until it cuts the minor axis at C, make E*g* equal to E*o*, and ED equal to EC, from C and D draw lines through *o* and *g* indefinitely, then with centre *g* and radius *g*A describe the curve 6A8, and from centre *o*, 5B7; then from centre C and radius CD draw the curve 8D5, and from centre D the curve 6C7, which will complete the ellipse.

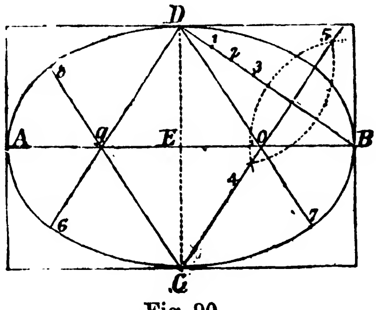


Fig. 90.

134. Fig. 91 represents another method of drawing an ellipse (this is recommended for general practical use).

Let AB be the given length, and *b*D the width. From B set off *g* equal to *b*D the given width. Divide *g*A into three equal parts. Set off two of these parts on each side of E, as *s* and *t*. From *s* and *t* as centres, with radius *st*, describe curves or arcs, cutting each other in *u*, *w*. From *u* and *w* draw lines through *s* and *t*, and produce them to *o*, *n*, *l*, *m*. Take *s* as centre, *s*A as radius, draw the curve *o*A*l*; and with *t* as centre draw the curve *m*B*n*. Then with *u* and *w* as centres, and radius *wl*, strike the curves *lbm* and *o*D*n*, which will complete the oval required.

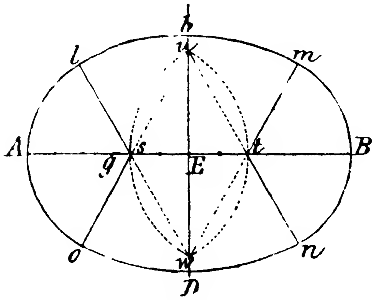


Fig. 91.

135. Fig. 92 shows a kind of ellipse very frequently used in the manufacture of various articles—a method of getting this shape is required so as to cut a "flue," or tapering body (which will be shown in a future example).

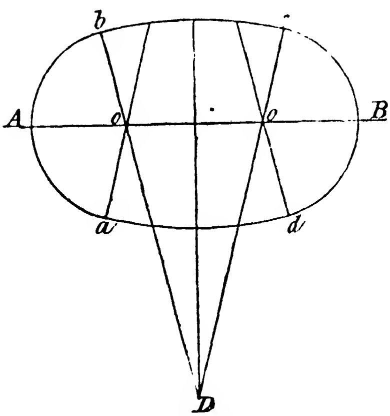


Fig. 92.

Take AB, the given length, set the compasses to nearly half the required width. From A and B mark off the points *o*, *o*, and strike semicircles *a*A*b* and *c*B*d*. Take any distance on this curve, as from A to *b*, or further if required, and mark off a corresponding distance from B to *c* and *d*. Produce lines from *bo* and *co* until they meet as at D. Then with radius D*b* strike the remainder of the curves *bc* and *ad*, which will give the ellipse required.

136. Fig. 93 illustrates another method. If two semicircles AEC, FBG, are described as shown in this figure, and both semicircles divided into the same number of equal parts, 1, 2, 3, 4, 5, and if through the points of division of the larger semicircle

lines are drawn perpendicular to AC, and through the corresponding points in the smaller one parallel to AC, the points of

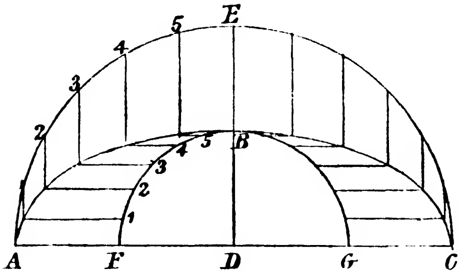


Fig. 93.

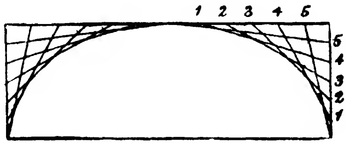


Fig. 94.

intersection will be points in the elliptic curve, giving a graphic illustration of what an ellipse really is.

Fig. 94 is another method of drawing an ellipse by intersecting lines, so simple in construction as to need no further explanation.

Fig. 95 shows another way of striking an ellipse of definite length and width.

Set off the length AB and width CD on lines intersecting at the centre *o*. Measure off *o*E equal to *o*C. Join AC, and on AC measure off CF equal to AE. Bisect AF in G, and from G draw a line perpendicular to AC, cutting AB at J and CD at H; then J will be the centre from which the small arc will be drawn, and H that from which the large one will be drawn.

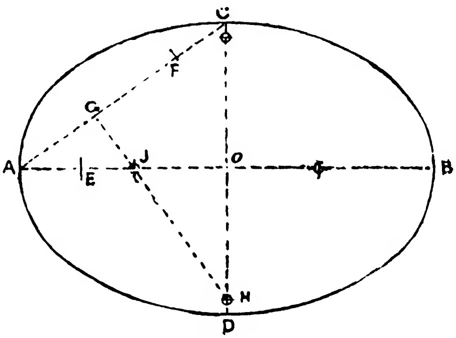


Fig. 95.

137. Fig. 96 shows a method of drawing an oval or egg-shaped body.

Draw the line AB, and bisect it in C, with centre C draw the circle A*o*B*d*, and draw the diagonals A*ob* and B*ob*; then with A as centre, and radius AB, draw the curve B*b*, and with centre B the curve A*b*; now with *o* for centre, and radius *ob*, describe the curve *bb*, which completes the oval required.

138. To draw a parabola by the intersection of lines, its axis, height, and base or ordinate, being given (Fig. 97).

Let AC be the base, and DE the axis, and E its vertex. Produce the axis to B and make EB equal to DE, join AB, CB, and divide them into the same number of equal parts, join the divisions by the lines 1–1, 2–2, etc., and their intersections will produce the curve required.

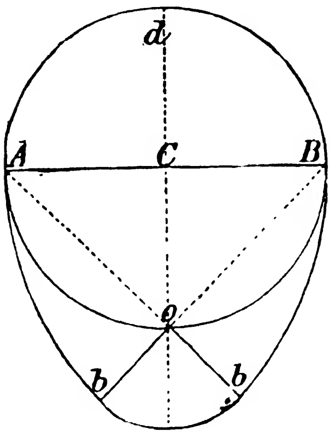


Fig 96.

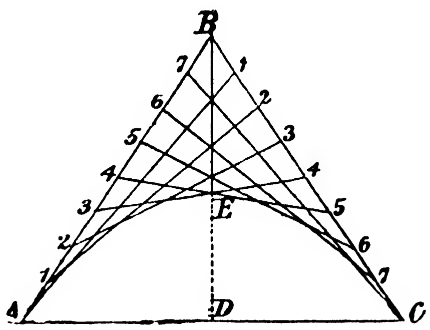


Fig. 97.

139. The chord and height of a segment of a circle of large radius being given, to find the curve without having recourse to the centre, which is supposed to be unattainable (Fig. 98).

Let AC be the chord line, and DB the height, through B

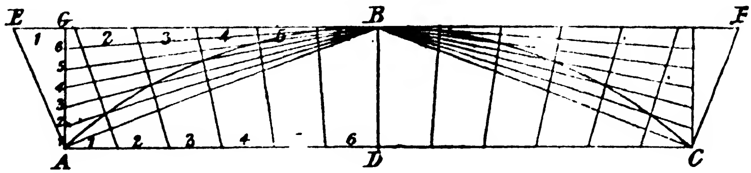


Fig. 98.

draw EF parallel to AC, join AB and BC, draw AE at right angles to AB, and CF at right angles to BC, divide AD and EB into any number of equal parts (say 6), join the corresponding numbers, 1–1, 2–2, 3-3, etc. Also divide AG into the same number of equal parts, and from each division draw lines to B, and the points of intersection will be points in the curve.

140. To strike a segment (or part of a circle) by a triangular guide, the chord and the height being given (Fig. 99).

Let AB be the chord of the segment, and DC the height (or

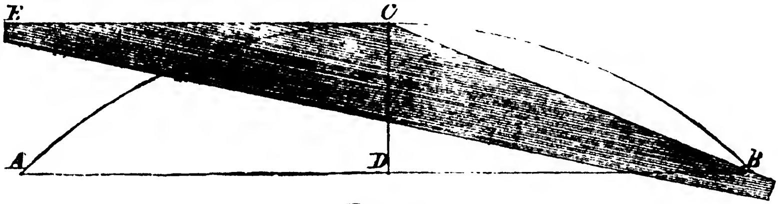


Fig. 99.

versed sine), join BC and cut CE parallel to AB, and make it equal to BC, fix a pin in B and another in C, and with the triangle ECB describe the curve CB, then remove the pin B to A, and by guiding the sides of the triangle against AC, strike the other part of the curve ACB.

141. To construct a plain scale, for drawing a small plan proportionate to a larger one (Fig. 100).

Draw a line AB, say 12 inches long, and mark off the inches as from 1 to 12; then draw BC, say 2 inches long. Draw a line from 12 to C, and draw parallel lines from the points 11, 10, 9, 8, etc., to cut the line BC. By using the distances on the line BC, as 1, 2, 3, 4, etc., as inches, we get a scale of 2 inches to the foot.

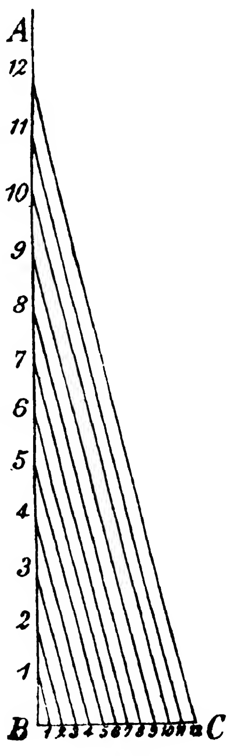


Fig. 100.

CHAPTER IV.

ENVELOPES OF SOLID BODIES.

THOSE who have read carefully the preceding sections on plane figures, and on solid bodies, will be prepared to follow me through the succeeding pages, in which I propose to treat of the forms of the envelopes of solid, or hollow bodies. The subject bears both on the principles of development of those enveloping forms, and upon their mensuration.

The basis of the art of developing the envelopes of solid forms is to obtain the outline of every portion of the enveloping surface as a *plane surface.* The simplest objects therefore to develop are those the surfaces of which are *plane* when in place. To this group belong all the rectangular forms. It is when we come to objects having various curvilinear surfaces, or surfaces having angles other than right angles, that the student begins to find difficulty:

Solid figures are parallelopipeds, prismatical, pyramidal, conical, cylindrical, spherical, or polygonal, the latter being either regular, or of irregular and mixed forms.

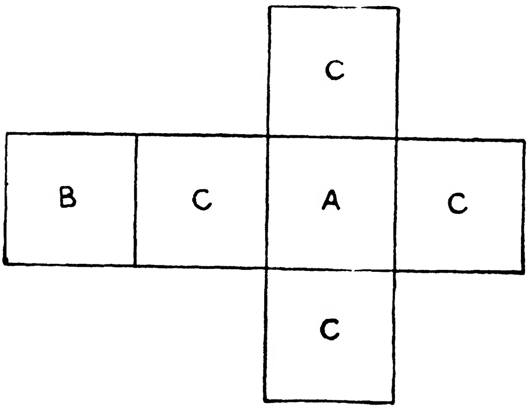


Fig. 101.

A rectangular parallelopiped is a solid bounded by six parallelograms, of which Figs. 16 and 17, p. 7, are examples. Fig. 101 shows the envelope of Fig. 16, developed. It simply consists of six equal squares. If we call A the bottom, then B will be the top, and C, C, C, C will be the four sides. All right prisms have two ends alike in shape and dimensions, and situated in parallel planes, and the other bounding figures are parallelograms. Fig. 102 is a hexagonal prism, and Figs. 103 and 104 illustrate the envelopes of the same. The only difference in these consists in the manner in which they are folded, the vertical seams in Fig. 103 being avoided in Fig. 104; the vertical lines between the CC sections in Fig. 104 simply indicating the folds of the sheet.

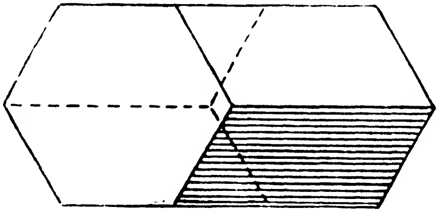


Fig. 102.

But since the main purpose of these, and the following figures in this section, is to show how the envelopes of solid bodies are built up of plane geometric figures, these envelopes are not to be taken in all cases as illustrative of the practical details of the methods in which work is built up. This information must be sought in the body of the book (Chap. VI.).

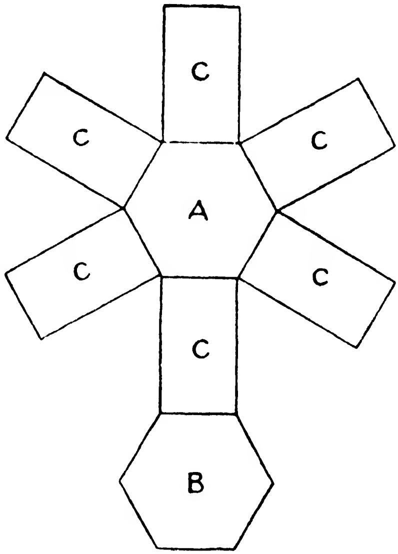


Fig. 103.

In Figs. 103 and 104, A and B correspond with the hexagonal ends of the prism, and C, C, C, C, C, C are the sides. The ends AB might be any other shape, depending on the class of prism, but in any case the sides connecting them would be parallelograms.

A term in frequent use in the succeeding pages is "frustum." It denotes a slice of a solid cut in a plane parallel with the base, and this figure has its application in many problems in this work.

A prismoid (Fig. 105) is a frustum of a wedge, or, more exactly, a solid which has two parallel plane faces, each having

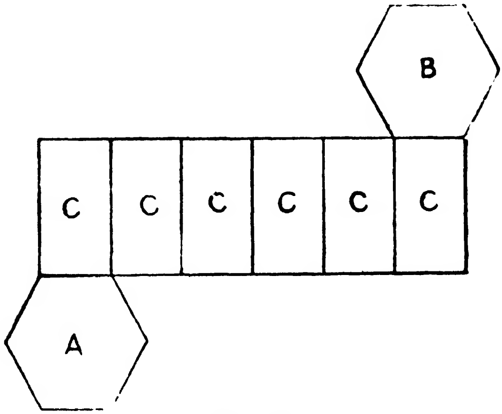


Fig. 104.

the same number of sides, but connected by faces the forms of which are trapezoids. Fig. 106 is the envelope of Fig. 105. A is the larger face, B the smaller face parallel with it, C, C the wider sides, D, D the narrower ones. Of course, the sides are not necessarily equal

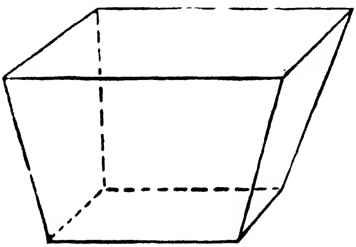


Fig. 105.

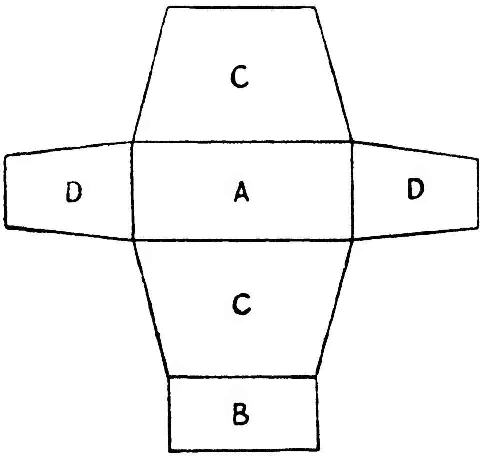


Fig. 106.

tapered or symmetrical, as shown. The shape of the sides will depend on the form of the prismoid.

The envelope of the wedge would be, for the base a rectangle, for the sides two trapezoids, and for the ends two wedges.

A pyramid is a solid of which the base is either a triangle, quadrilateral, or polygonal figure, and the sides triangles. The base may have any number of sides, but the sides are always triangles, and the point at which they meet is called the vertex of the pyramid. In Fig. 107 A is a plan view, and B an elevation of an octagonal pyramid, *a* being the base, and *b* the vertex. Fig. 108 is the envelope of Fig. 107, formed by eight isosceles triangles, the height *c* of each being equal to the slant height *c* of the pyramid in Fig. 107, and the width *d* of the base being equal to the width *d* of each side *d* of the base of the pyramid in Fig. 107. No matter how few, or how numerous, are the number of sides contained in the base of Fig. 107, the solid is still termed a pyramid; hence it is usual to define this figure concisely as a solid bounded by three or more triangles which meet at a point, and by another rectilineal figure (the base). The

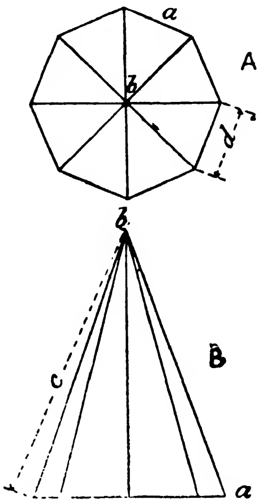


Fig. 107.

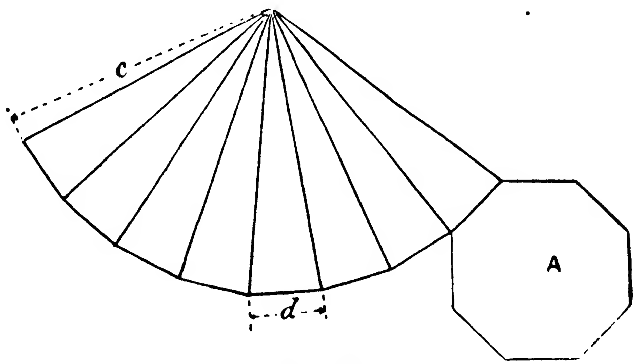


Fig 108.

base of the pyramid, Fig. 108, A, is that of the plan A in Fig. 107.

Fig. 109 is a frustum of the same pyramid, *a* being the base, and *b* the plane of the cut face or top of the frustum, *c* is the vertical height, *d* the slant height, *e* the width of the faces of the octagon at the base, and *f* their width on the top face. Fig. 110 shows the envelope of Fig. 109. The slant height *d* in Fig. 110 corresponds with *d* in Fig. 109, and the widths *e*, *f*, at bottom and top similarly. In this case the envelopes of the faces are not triangles, as in Fig. 8, but trapezoids, and the two sides of the trapezoids which are not parallel converge to the apex *g* of the frustum. Whatever shape the base of the frustum may be, and it may vary precisely as in the pyramid, the sides are always trapezoids. The base and crown of the frustum *a* and *b* in Fig. 110 correspond with *a* and *b* in Fig. 109.

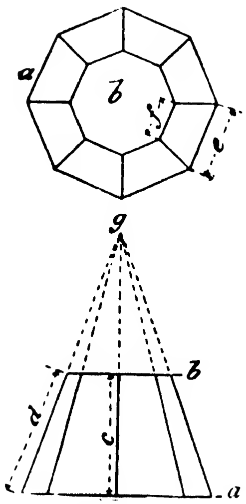


Fig. 109.

If we conceive all the angles in the pyramid (Fig. 107) to be obliterated, we obtain the cone (Fig. 111) in which the base is a circle. The cone is defined as being formed by the revolution of a right-angled triangle round one of the sides which contains the right angle, the side round

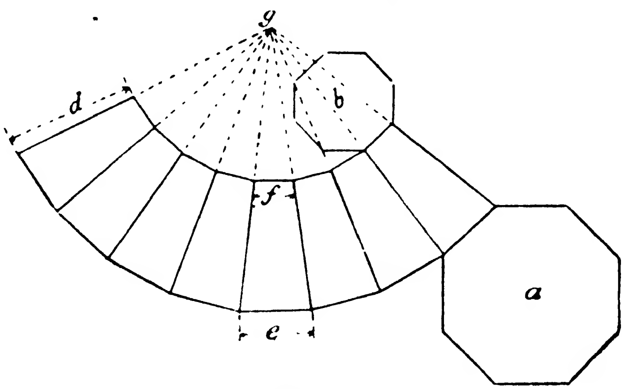


Fig. 110.

which the revolution takes place being fixed: *a* is the base, *b* the vertex, *c* the slant height; A is the envelope of the cone, and *c* the plan of the base *a*. The length of the curved edge of A is equal to the circumference of *c*. The envelope of a cone, therefore, is a sector A of a circle, and a circular base c.

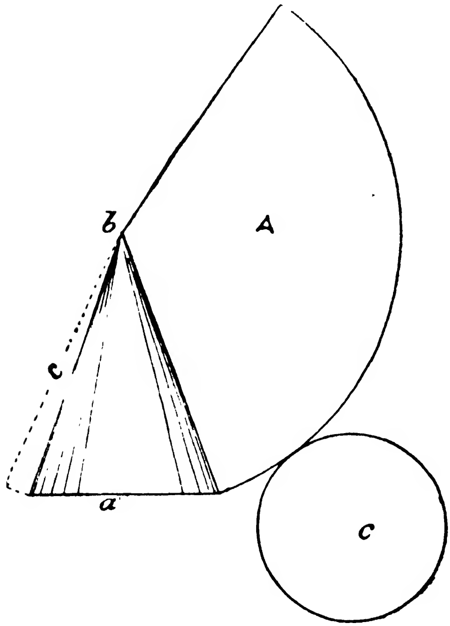


Fig. 111.

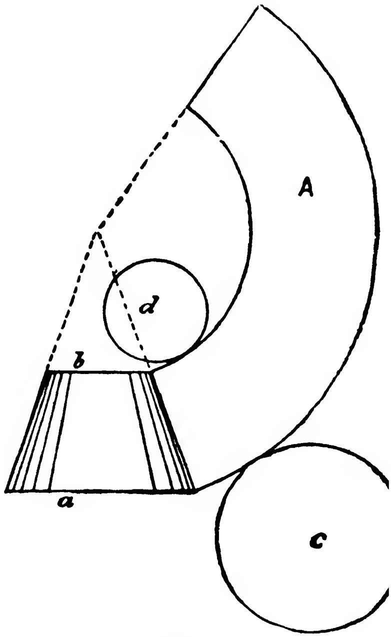


Fig. 112.

Fig. 112 is a frustum of a cone with its envelope. The envelope of the body is a portion of an annulus, or segment of a circle, and the base and crown are circles, *c* and *d* corresponding with the base *a*, and top, or crown *b*, respectively of the frustum.

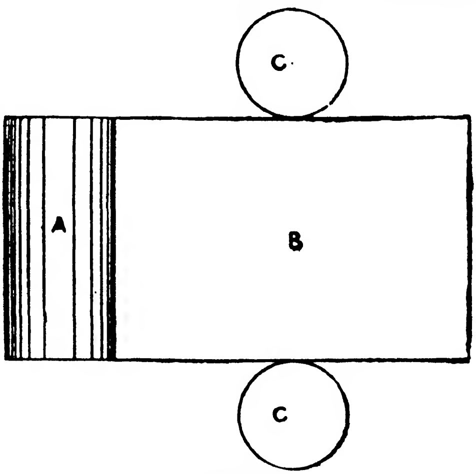


Fig. 113.

The envelope of the cylinder A in Fig. 113 consists of a rectangle B, and two circular ends, C, C. The length of the rectangle is equal to the circumference of the cylinder, and the breadth equal to its height.

The two trays in Figs. 114, 115 are examples of mixed forms. In each case the bottom of the tray is a rectangle with the corners taken off. In Fig. 114 the sides are trapezoids. In Fig. 115 the sides are rectangles, and the corners are quadrants of a truncated cone, so that if the four corners were cut out and placed edge to edge they would form a complete truncated cone.

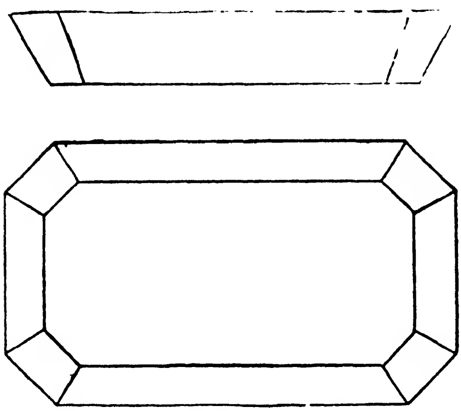


Fig. 114.

Regular polygonal solids such as the prisms, hexagonal, pyramidal, and other figures, and mixed figures which combine these forms, can, since their sides are bounded by right lines, be readily drawn directly by reference to their primitive forms. But such is not the case with figures having curved sides. A large number of figures of the latter class are shown in this work (see problems).

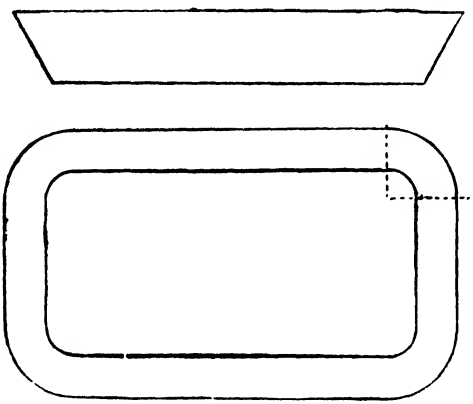


Fig. 115.

When the sides are curves, the curves may be considered as passing through an infinite number of points located at certain distances from a given centre line. The method then adopted to obtain these curves is to draw any convenient number of lines called ordinates, parallel with the base of the figure, and to measure off the correct distances corresponding with the outline of the figure upon these, and draw the required curves through these points. The greater the number of ordinates drawn, the more correct will be the curvature so obtained. The method of obtaining the lengths of the ordinates is usually that of direct transference of the lengths of the ordinates drawn across the elevation to the corresponding ordinates drawn on the developed plane of a given side.

When it is not practicable to obtain an envelope in the form, first of a true plane, as it has been in the foregoing examples, then various devices are resorted to in order to obtain the nearest approximations practicable. In the case of all forms of a spherical character these devices are necessary. Such cases occur in globes, or portions of globes, in the elbows of bend pipes, and in similar work which is curved in two or more directions. In the case of cylindrical and conical work in which the bending occurs in one direction only, simple bending or rolling suffices. But in the case of spherical work, in which bending occurs in all directions, the tendency always is to puckering. In very thin work done in malleable metals or alloys, this puckering is altogether taken out in the operation of raising (see Chap. IX.). The thicker the work, the more difficult is the task of removing the puckering, and then it becomes convenient to divide the work into narrow sections, each of which can be more readily bent than pieces of larger area. Examples of this occur in Chap. VI. A complete

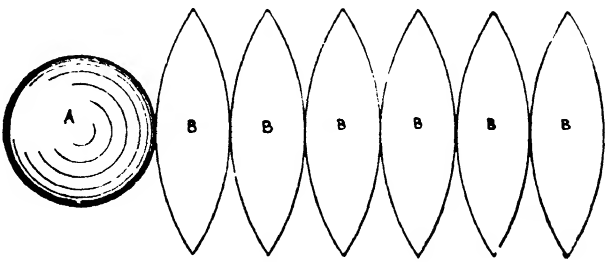


Fig. 116.

sphere can be raised from two plates, each equal in area to a hemisphere. But a sphere (Fig. 116), **A**, can be covered with the minimum of hammering by dividing the enveloping surface into several narrow sections called gores, B. Each piece is equal in length to the semi-circumference, and of width at the centre equal to that proportion of the circumference of which it forms a fraction. The widths between the centre and the ends are obtained by means of ordinates, the length of each ordinate having the same proportion to the circumference of the zone in which it occurs as the width at the centre has to the circumference there.

CHAPTER V.

PROJECTION.

THE principles of the art of projection are few and simple, their applications are practically infinite. If principles are understood, there is no difficulty with their applications. The problems in this work consist almost wholly of applications of the principles of projection, and all I propose, therefore, to do in this section is to explain the fundamental principles on which these and kindred constructions are based.

Perspective views are drawn in order to picture solid objects as they appear to the eye of an observer. In perspective views, the boundary lines pass towards a vanishing point, and the effect of reality is also further imparted by appropriate shading, imitating the effects of light, and one view suffices. But in projection of solid bodies, two views at least are always necessary, one in which the object is viewed from above, termed the plan, and the other in which it is viewed from one side, termed the elevation, and these two views are taken at right angles to each other, and perspective is not introduced in any degree, the views being those of plane surfaces only, in which the effects of point of view and of distance are absolutely disregarded.

The above explanation is given in an elementary form for the sake of obtaining a clear working basis. The projection of solids, however, involves very much more than this in practice. Plans and elevations occur under so many circumstances that they have to be distinguished by special terms. A body may be hollow, and the plan or elevation be taken in planes passing through the body, and then the views are termed a sectional plan and a sectional elevation respectively. Or there may be plans taken from below as well as from above figures. There will be end elevations as well as side elevations, and plans and elevations of faces which are at angles other than at right angles with each other. There will be developed plans, or plans of the boundaries, or envelopes of solid or hollow bodies, and the problems in this work largely consist of these.

I am not going to attempt anything like a set treatise in projection; but I propose to put, in as brief a space as possible, a few of the fundamental facts upon which the practice is based, illustrating the subject from my own point of view, which is always that of the workman. I learned my projection, in the first place, as I did other matters, in the workshop, and not from books. Book-work came later in life, and so, naturally, I look at most matters from the practical standpoint. Hence, I shall not talk about hinged boards, and waste a lot of time in describing points, which a glance at a drawing makes obvious, but lay down a few principles only. Taking now some simple illustrations, I will briefly indicate the essential methods of projection of solids to different planes.

If we look back at some of the perspective figures of the simpler geometrical solids shown in Chap. I., such as the cube, the sphere, the cone, the pyramid, the cylinder, the prism, we see at once what kind of figures are indicated even without the aid of shading. But we are quite unable to take any working dimensions from such drawings. We may gather some idea of relative proportion, as, for instance, between diameter and height; but we cannot put rule or compasses on, and measure dimensions for the purpose of working from. But the latter is absolutely essential in all practical construction. If, however, we take either of those figures, and lay down, or project two or more views to proportionate scale, we can measure anything off. The cube and sphere, of course, appear the same, and measure the same, from whatever face we regard them, but it is not so with other figures. The cone is projected as in Fig. 117, the pyramid as in Fig. 118, the cylinder as in Fig. 119,

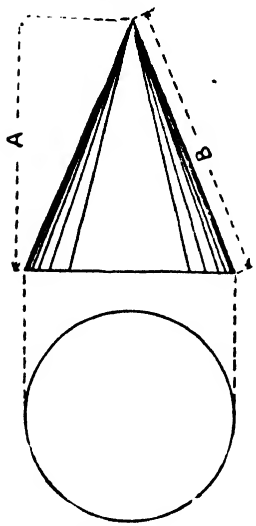


Fig. 117.

and the prism as in Fig. 120, A. In each case, horizontal and vertical dimensions can be taken.

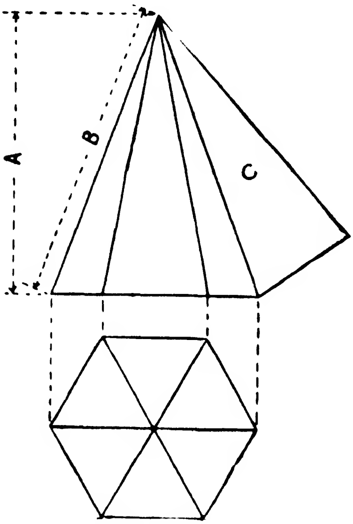


Fig. 118.

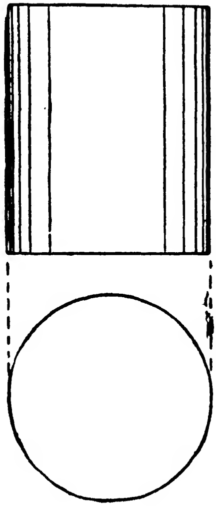


Fig. 119.

In order to obtain these projections, parallel lines at right angles to the faces are drawn from the boundaries in one view

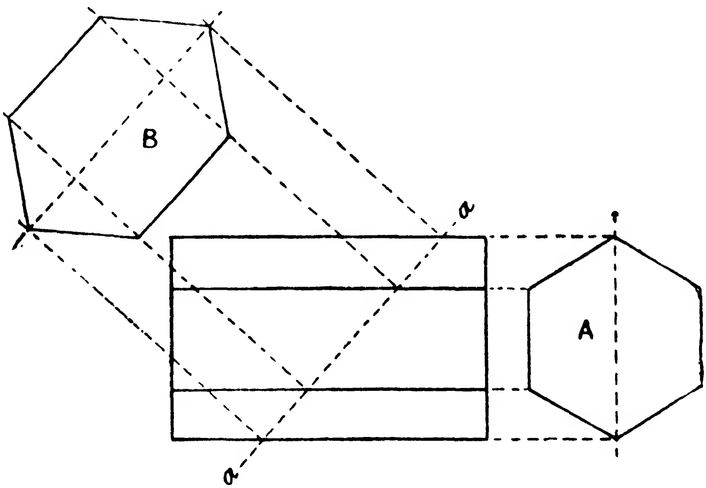


Fig. 120.

to obtain the boundaries in the other view, and by this simple method projections of the most apparently intricate character are obtained.

In the figures of the cylinder and the prism the cross-sections taken at right angles with the vertical axis are all alike, and then the plan view is identically the same in any horizontal plane. But in the case of many figures, as, for example, in those of the pyramid and cone, and others, the sides of which are not parallel with each other, the horizontal sections vary

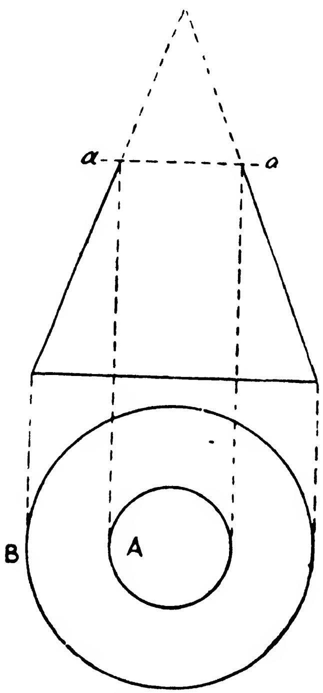


Fig. 121.

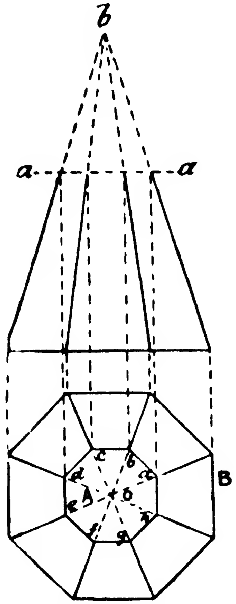


Fig 122

throughout, so that in the case of such figures cut by horizontal planes the shapes of the cut surfaces are projected to the plan by means of lines taken from the boundaries perpendicular to the cut faces, as shown in Figs. 121, 122, A being the plan of the cut face, and B the plan of the bottom face.

Again, in the case of solids having faces which are not parallel with each other, it is obvious that the elevation (Figs. 117, 118) gives the vertical height A measured on the vertical axis, or centre line, or plane. Such a dimension must usually be known. But it is also as often necessary to know the sloping or slant height B. Then measurement must be taken along B, or a projection made with lines perpendicular to B, as shown to the right of Fig. 118 at C.

In connection with these illustrations, there is an important distinction to be noticed. It is a very simple matter, and yet

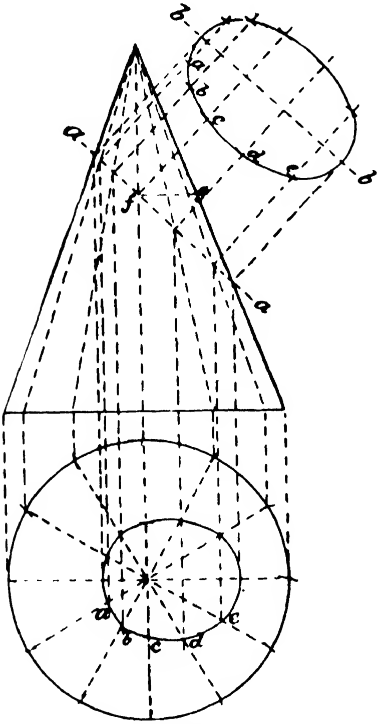


Fig. 123.

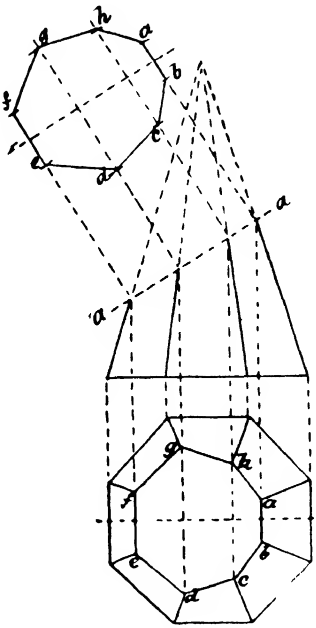


Fig. 124.

I have seen blunders made through not making the distinction between what I will call the actual plan and the developed plan of an object. Thus, if we cut a cone with an oblique line, *aa* (Fig. 123), the cut surface will be an ellipse. But the length of the ellipse will not be that obtained by projecting the boundary lines downward to the plan. That view would give the appearance of the cut face as seen from above. But the true dimensions of the ellipse must be measured upon a development obtained by projecting boundary lines perpendicularly to the cut face *a-a*.

Similarly the dimensions of the cut face of the pyramid (Fig. 124) would not be those which appear upon the actual plan, but those of the development obtained by perpendicular lines raised upon the cut face *a-a*.

Again, in Fig. 120 the projection of the faces of the prism from the plan view A will not give the correct widths of the face *a-a*, but the projection must be made perpendicular to

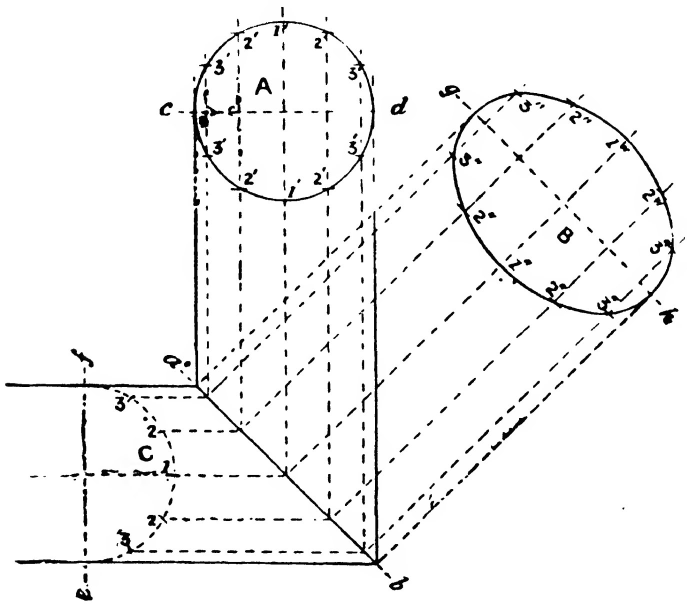


Fig. 125.

the face as at B. Likewise, in the cut cylinder (Fig. 125), the projection upwards will give the appearance of the cut face as it appears from above, but a view for working purposes must be obtained by a projection of perpendiculars from the cut face *a-b* as at B.

These illustrations lead us to the consideration of another fundamental matter, namely, the manner of obtaining the projections of the boundary lines of these views in detail. From the views given it is clear how the extreme lengths and widths of such boundary lines are obtained, but not how the intermediate curved portions are got. These are plotted by means of parallel lines of projection termed "ordinates," and the method has extensive illustration in the problems given in this work.

The principle is illustrated in the case of the rectangle seen in plan in Fig. 126 at A. Two projections are shown, one from the corners *a*, *b*, *c*, perpendicular to the diagonal *ab*, the other perpendicular to the face *c-b*. In each case the projection lines are carried perpendicularly from the plane which it is desired to represent. So, in the truncated cone (Fig. 121) cut on the plane *a-a*, lines are carried down perpendicularly to the cut face.

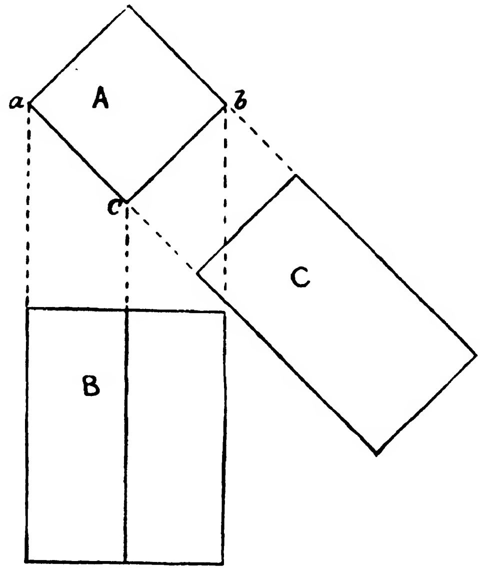


Fig. 126.

In Fig. 122 the method by which the projection of a pyramid and its truncated face *a-a* is obtained is shown. The base is drawn in plan below. Thence lines are carried upwards from the angles to the elevation above, and lines drawn from those angles to the apex *b*.

To obtain the form of the cut surface, first draw lines from the angles of the figure in plan to the centre *o*. Obviously these lines will be precisely the same as the corresponding lines drawn in the elevation. From the angles of the cut surface *a-a* carry perpendiculars down to the plan. The points where these cut the radial lines will be the terminating angles *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, of the cut surface.

In Fig. 124, where the cut surface is made diagonally, *a-a*, the same method of projection is adopted; but the shape of the cut surface is not that of a regular octagon, but is irregular.

And where the form of the diagonally cut surface is projected to show its real dimensions, as in the figure, the form is not that of a regular octagon. In this figure (Fig. 124) the widths across the angles are measured from the plan view below, and add the reference letters, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, represent similar dimensions, and these dimensions across the angles being connected with straight lines give the outline required.

To project the outline of an hexagonal prism cut obliquely (Fig. 120).

Draw the diagonal *a-a*, raise the perpendiculars upon it from the angles of the hexagon. Upon these lines set off the widths, and connect the points so obtained.

To obtain the projected form of the cut face of a cone, cut at an angle *a-a* (Fig. 123).

Divide the circumference of the base into any convenient number of equal parts, in this case twelve. Draw lines from each point of division to the centre in the plan, and to the apex in the elevation. Both sets of lines will then correspond—that is, the lines in the plan will correspond precisely with those in the elevation. Draw the diagonal *a-a*, which represents the plane of the cut surface of the cone, and from the points in which this line intersects the points of equal division project lines downwards to the plan. The points in which these perpendicular lines intersect the radial lines will be points in the development of the cut surface. Curved lines drawn through these points of intersection will represent the appearance of the cut surface viewed from above.

To obtain the development of the actual dimensions and form of the cut surface, raise perpendiculars on that surface from the lines of equal division.

Draw a centre line, *b-b*, parallel with that cut surface, and on each side of the same set off distances corresponding with those obtained on the lines in plan, *a*, *b*, *c*, *d*, *e*, both sides being alike. At the centre line the width will be obtained by the horizontal *f-g*. A curve traced through the points of intersection so obtained will give the dimensions and form required, and will become an ellipse.

To project the cut face formed by the meeting of two cylinders at right angles (Fig. 125), viewed first perpendicularly to the axis of the lower cylinder, and second perpendicularly to the cut face.

Strike a semicircle C equal in radius to the radius of the cylinder, and divide it into any number of equal parts, in this case six, 1, 2, 3, 1, 2, 3. Draw ordinates through the points of division parallel with the sides of the cylinder. At the points where these lines cut the diagonal face *a-b* raise parallel lines perpendicular with the axis of the lower cylinder. Draw a line, *c-d*, at right angles with these, and set off therefrom the points 1′1′, 2′2′, 3′3′ at the same distances from the centre line *c-d* as the points 1, 2, 3 are from the centre line *e-f* of the semicircle below. A curve drawn through these points of intersection will give the form of the cut face as seen from above, and this is seen to be a circle.

But to obtain in the second place the true dimensions of the cut face, project perpendiculars from the same ordinates 1, 2, 3 as before, but perpendicular to the joint line *a-b*, draw a centre line *g-h*, and set off thence points of division 1″, 2″, 3″, as before, from the divisions on the semicircle. A curve drawn through these points of intersection will give the correct outline of the cut face.

To project the appearance which the ends of a cylinder, A, present when laid at an angle *o* (Fig. 127).

Strike a semicircle, *a*, equal in radius to the radius of the cylinder and divide it into any convenient number of equal parts, and carry parallel lines from the points of division down the cylinder body. From the points where these lines of division intersect the faces of the cylinder carry parallel lines along indefinitely, perpendicular to the base line *b-b*, with which the cylinder makes an angle. Draw the diametral outline of the cylinder at B, and draw and divide out a similar semicircle *c* to that drawn on A, and carry parallels through these points of division. The points of intersection of these parallels with the lines projected from points of division in the faces of A will be those through which the curves must be drawn to show the appearance presented by the cylinder ends, corresponding with the angle at which A is laid.

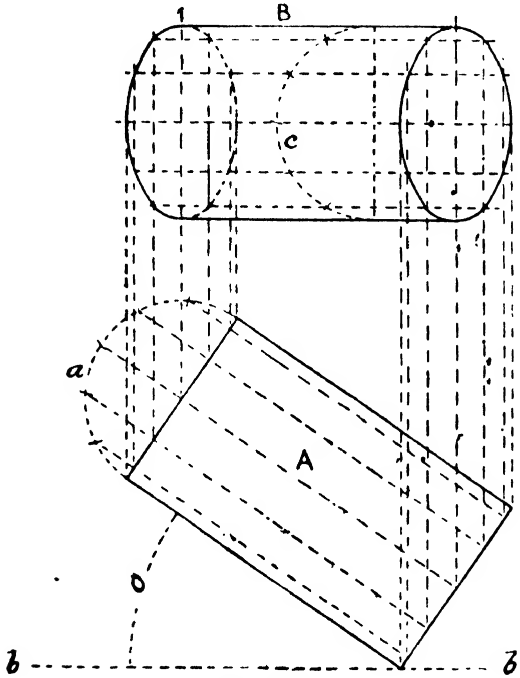


Fig. 127.

These illustrations will suffice to indicate the principles on which projections are constructed. Many applications of these principles occur in the practical problems.

CHAPTER VI.

PRACTICAL PROBLEMS.

To arrange practical problems on a rigid basis or system is hardly possible. I have endeavoured, as far as practicable, to adopt the natural basis of the geometrical forms to which the figures in the problems are referable. In the case of parallelopipeds, of prisms, and of prismoids no problems occur. The forms of the envelopes of such bodies require no illustration beyond those given in Figs. 101–106, in explanation of the essential forms of their envelopes. I have, therefore, placed pyramidal forms in the first section, after which conical figures follow naturally. Then I have introduced the problems relating to the cylinder, then the spherical forms. Elliptical and oval forms follow. Finally, I have arranged mixed figures in the last section, being combinations of ellipses with circles, and of parallel sides, and convex and other ends, of forms built up of polygons, and curved outlines. This, though necessarily involving some slight mixing up of forms, in some examples affords the only true methodical basis of classification. It facilitates reference, and permits of similar problems being studied in their relation with each other.

SECTION I.—PYRAMIDAL FIGURES.

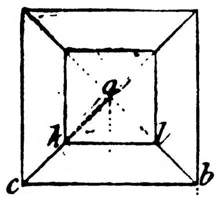
The pyramid and the cone (Figs. 15, 19, 118, 117) are right figures, and are termed right to distinguish them from oblique figures. In the right pyramid and the right cone, the apex is situated plumb over the centre of the base. In the oblique pyramid and in the oblique cone it is not situated over the centre of the base. In the right cone the generating or slant line is always of the same length; in the oblique cone, the generating lines are of unequal lengths, and may be considered as being infinite in number. In the right cone a plane taken in any direction through the axis of the cone is always perpendicular to the base. In the oblique cone only one such plane is perpendicular to the base, that, namely, which cuts the longest and the shortest generating lines. The amount of obliquity of a cone or pyramid is measured by the angle which its axis makes with the base.

Since conic frustra, or truncated cones and pyramids are figures constructed on the same principles as the entire cones and pyramids, the above general remarks apply substantially to them both. The problems immediately following have reference to right pyramids.

PROBLEM 1.—**To strike the pattern for a square tapering article (or frustum of a quadrilateral pyramid).**

Fig. 128 represents the plan, and Fig. 129 the vertical height or elevation. Draw the diagonals, and take the distance from the centre *a* to *b* (Fig. 128), and mark off the same in Fig. 129 from *g* to *d*. Also take the distance (Fig. 128) from

FIG. 128.



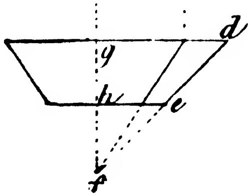


Fig 129.

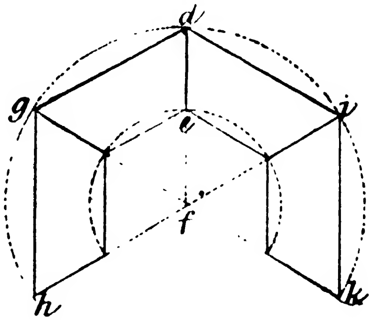


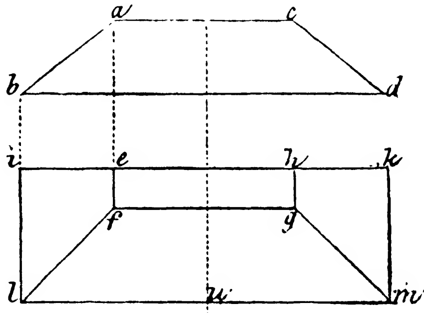
Fig. 130.

a to *l* or *k*, and mark off the same in Fig. 129 from *h* to *e*. Draw a line through the points *d*, *e*, to cut the perpendicular line at *f*. Then draw (Fig. 130) the perpendicular line *df*, and take the radius *fd* (Fig. 129), and with it in Fig. 130 describe the circle *hdk*; with radius *fe* in Fig. 129, still using *f* as centre in Fig. 130, draw the smaller circle *e*. Take the length of one side of the base from *c* to *b* (Fig. 128), and mark off the same four times on the larger circle (Fig. 130), at *h*, *g*, *d*, *i*, *k*. Draw lines through these points to the centre *f*; join these points by lines *hg*, *gd*, etc., and also join the points on the smaller circle in the same manner, which will complete the pattern.

PROBLEM. 2.—**To describe the pattern for a square or rectangular tapering top or tray, with sides and bottom, in one piece.**

Fig. 131 shows the vertical height and one-half of the plan. Draw (Fig. 132) the horizontal line *bd* and the perpendicular line *op*. Draw the rectangle *efgh*, the same size as *efgh* in Fig. 131.

Fig. 131.



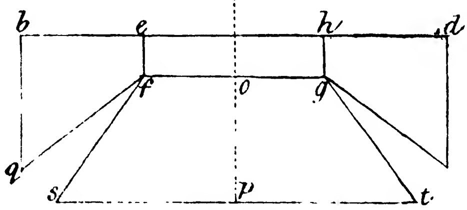


Fig. 132.

Take the length *ab* (Fig. 131) and mark off a corresponding distance from *e* to *b*, *h* to *d*, and *o* to *p* (Fig 132), and draw through the points *b*, *p*, and *d* the lines (at right angles) *bq*, *st*, and *dr*; and transfer the length *il* to *bq* and to *dr*; also the length *ul* from *p* to *s* and from *p* to *t*. Then draw the lines *qf*, *sf*, *tg*, and *rg*, which will complete one-half of the pattern.

PROBLEM 3.—**To strike the pattern for a diamond-shaped tapering body, in one piece.**

Fig. 133 shows in plan the size and shape required for top and bottom; in Fig. 134 from *i* to *f* is the vertical height. Transfer the lengths *ac* and *ae* (Fig. 133) to Fig. 134, from *f* to *g* and from *f* to *h* respectively, also the distances from *a* to *b* and *a* to *d*, to *il* and *ik*, and draw through *g* and *l* a line to cut the perpendicular at *m*, and another line through *h* and *k* to *m*.

With the lengths *mg*, *mh*, *ml*, and *mk* (Fig. 134) as radii, describe the curves *g*, *h*, *l*, *k*, in Fig. 135, from the centre *m*, and draw the line *gm*. Transfer the length *ec* (Fig. 133) from *g* to *r* and from *g* to *n* (Fig. 135), also from *n* to *o* and from *r* to *b*, and

Fig. 133.

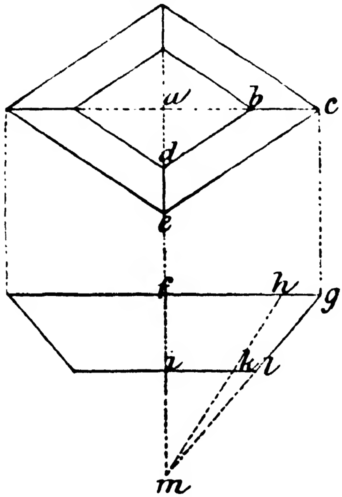


Fig. 134.

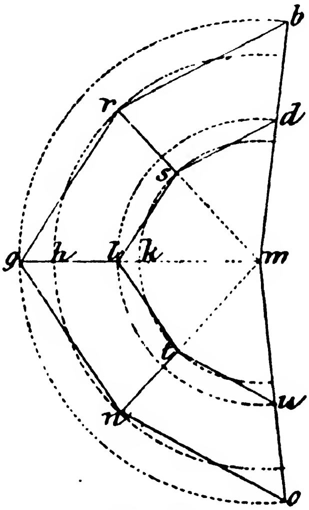


Fig. 135.

draw lines from *r*; *b*, *n*, and *o* to the centre *m*. Connecting these points by straight lines *br*, *rg*, *ds*, *sl*, etc., will complete the pattern.

Problem 4.—**To describe the pattern for a hexagon mould or tray, having the bottom and sides in one piece.**

Fig. 136 shows the elevation and half the plan. To obtain a development of the pattern, draw (Fig. 137) the perpendicular *bc*, and draw the half-hexagon *efghi*, of the same size as *efghi* (Fig. 136).

Divide the lines *hg* and *gf* into equal parts, and draw the lines *ak* and *am* through the points of bisection; then carry the length of *ab* (Fig. 136) from *l* to *k* (Fig. 137). Draw through *k* the line *no* parallel with *hg*. Take the length *kl* (Fig. 136) and mark off the same from *k* to *n* and from *k* to *o*, and draw the lines *hn*, *go*; *hgno* is the sixth part of the pattern. Proceed in the same manner to draw the remainder, one-half of the pattern (as well as of the plan) only being shown here.

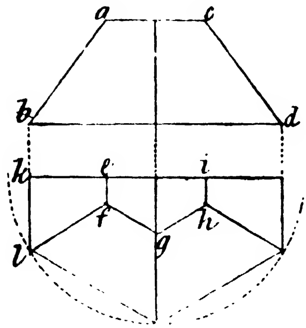


Fig. 136.

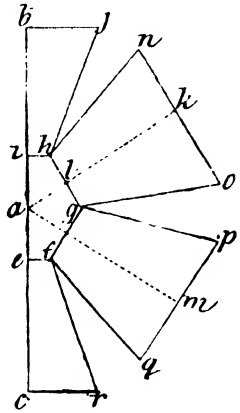


Fig. 137.

PROBLEM 5.—**To strike the pattern for a tapering octagon body (or frustum of an octagonal pyramid) in one piece.**

Fig. 138 represents the plan of the top and bottom (the

Fig. 138.

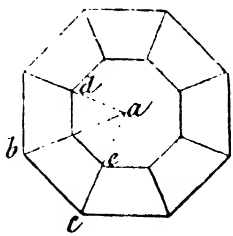
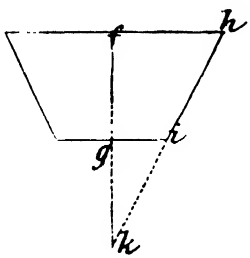


Fig. 139.



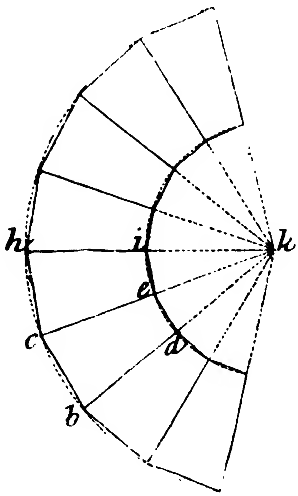


Fig. 140.

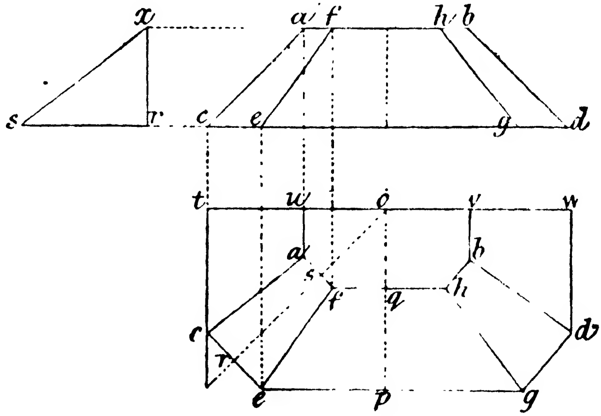
method of striking this figure is given in Figs. 77, 78, p. 34), Fig. 139 is the elevation, from g to f being the vertical height required. Take the distance from the centre *a* (Fig. 138), to one of the extreme points, as *c*, and from *f* (Fig. 139) mark off the same distance to *h*, and the distance *a* to *e* (Fig. 138), mark off from *g* to *i* (Fig. 139), draw the line *hi* to cut the perpendicular line *fg* prolonged, at *k*. With radius *kh* (Fig. 139), draw an arc of a circle as *hcb* from *k* (Fig. 140), and with radius *ki*, still using *k* as a centre, strike the curve *ied* (Fig. 140). Take the distance *bc* (Fig. 138) and mark off the same distance eight times on the larger curve (Fig. 140), as *bch*, etc. Draw lines from all these points to the centre *k*. Draw straight lines from these points as from *h* to *c*, and *c* to *b*, etc., and likewise from the intersecting points of the smaller curve *i* to *e*, *e* to *d*, etc., which will complete the pattern.

PROBLEM 6.—**To describe the pattern of an irregular octagon pan or tray, with the sides or bottom in one piece.**

Figs. 141 and 142 show the required elevation and plan, having drawn which, proceed with the development of the

Fig. 141.

Fig. 142.



pattern in Fig. 143. Draw the half-octagon *uafhbv* to the same dimensions as the corresponding letters in Fig. 142. Draw the horizontal line *tw* and the perpendicular line *oc*. Divide the sides *af* and *hb* with centre lines *or* and *op*, then carry the length of the line *ac* (Fig. 141)—being the slant height of the larger sides—from *q* to *c*, from *u* to *t*, and from *v* to *w* (Fig. 143).

Draw from *t* and *w* perpendicular lines, and through the point *c* draw the line *eg* parallel with *tw*.

Take the distance from *t* to *c* (Fig. 142), mark off the same in Fig. 143 from *t* to *c*, *w* to *d*, *c* to *e*, and *c* to *g*, and draw the lines *ca*, *ef*, *gh*, and *db*.

Then in Fig. 141 draw the perpendicular line *xr*, and from Fig. 142 take the horizontal projection of the smaller sides, equal to the distance *sr*, and transfer the same from *r* to *s* (Fig. 141), and draw the line *sx*, the length of which should now be transferred from *s* to *r* (Fig. 143). Draw *yz* parallel to *af*.

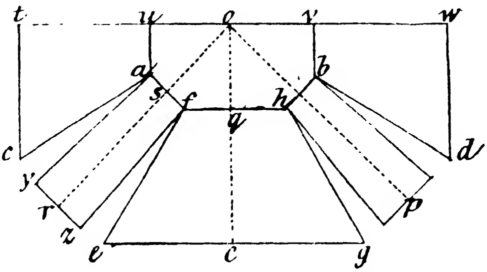


Fig. 143.

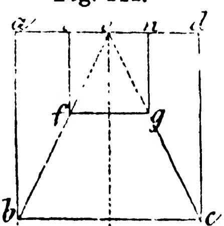
Take the distance *re* (Fig. 142), and mark off the same from *r* to *y* and *r* to *z* in Fig. 143, draw lines *ya* and *zf*. For the other side, *p*, proceed in like manner, which on being done will complete half the pattern.

These have been examples of right pyramid figures. We will now take those of oblique form.

Problem 7.—**To describe the pattern of a square funnel where one side is vertical.**

Fig. 144, *a*, *b*, *c*, *d* shows the plan of the top, *e*, *f*, *g*, *h* the plan of the hole or bottom, of the funnel. Fig. 145, from *i* to *k*, shows the elevation. Draw lines from the points *bf* and *cg* to cut each other on the centre line at *o*. Transfer the distances *ob* and *oa* (Fig. 144) to Fig. 145 from *i* to *m* and from *i* to *l* respectively, also the lengths *of* and *oe*, from *k* to *n*, and from *k* to *o*. Draw the line *mn* to cut the perpendicular line at *p*. Also the line

Fig. 144.



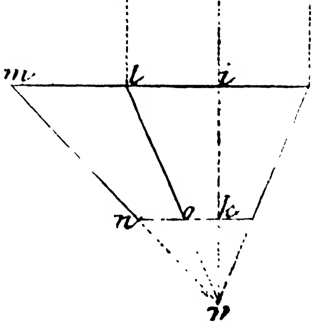


Fig. 145.

through the points *l* and *o*, which will cut the perpendicular line at the same point *p*, if the distances are taken correctly. Take the distances from *pm*, *pl*, *pn*, and *po* in Fig. 145 as radii, and describe the curves *m*, *l*, *n*, *o* from the centre *p* (Fig. 146). Take the length from *b* to *c* (Fig. 144) and mark off the same from *m* to *c* (Fig. 146), and draw the lines from *m* and *c* to the centre *p*; also take the distance *b* to *a*, and mark off the same from *m* to *g* and from *c* to *d* (Fig. 146), and the distance *a* to *d* (Fig. 144) mark off from *d* to *a* (Fig. 146). Draw lines from these points to the centre *p*, and connect these points with straight lines, as *ad*, *dc*, etc., and connect the corresponding points on the smaller curves *n* and *o*, which will complete the pattern required.

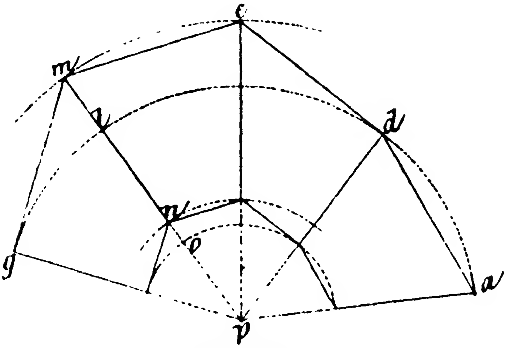


Fig. 146.

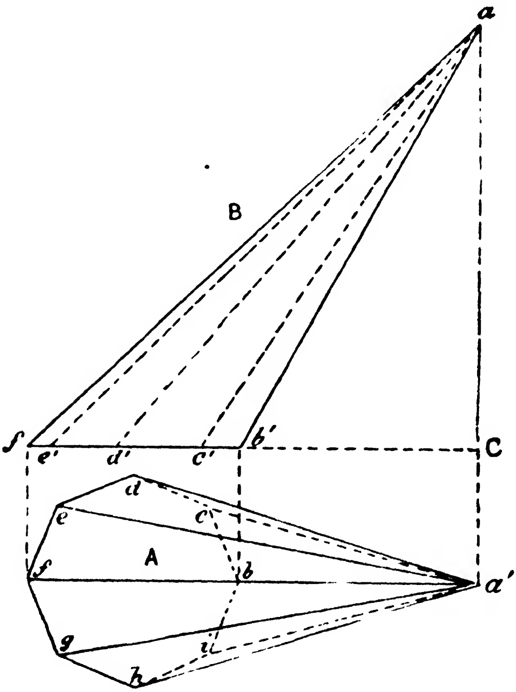


Fig. 147.

NOTE.—This will be found a very useful method for striking a square or rectangular tapering top or sides. Whether the tapering be proportionate or not, by drawing lines as *bf* and *cg* (Fig. 144) from the angles (which show the positions of the top and bottom of the article required) to cut the centre line wherever the point o may come, and by taking it as a working centre, one-half or a section of the pattern may be developed.

PROBLEM 8.—**To draw the envelope of an oblique pyramid, in this case one of octagonal form** (Fig. 147).

Draw the base A, in plan—b, c, d, e, f, g, h, i, and the outline only of the pyramid in elevation, B (Fig. 147), a being the apex in elevation, and a' the same in plan. There are certain lengths, those of the angular edges in plan, which have to be transferred to the elevation in order to obtain the correct lengths of those edges for the development. It would not do to

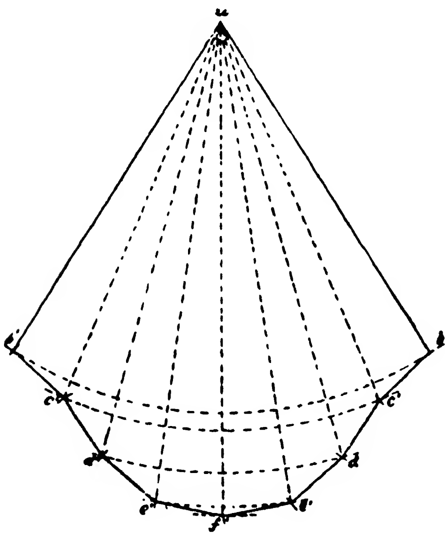


Fig. 148.

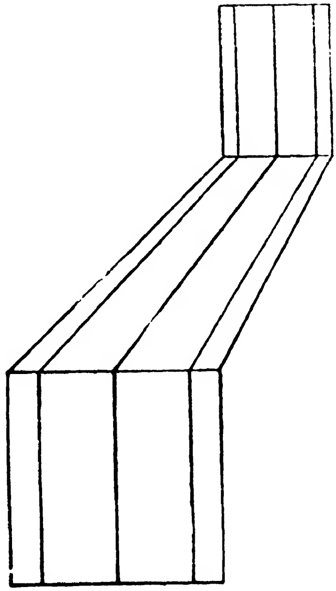


Fig. 149.

take the lengths obtained by the projection of the edges directly upwards, for the reasons given in connection with Figs. 117, 118, pp. 54, 55. But the lengths $a'c$, $a'd$, $a'e$, must be measured off with compasses and transferred from C on the base line to Cc', Cd', Ce'. Then the lengths ac', ad', ae' will be the correct lengths of those edges of the pyramid for the purpose of development. The lengths ab', af' will be correct in the elevation. To mark the outline of the pattern (Fig. 148), with radius ab (Fig. 147), mark off ab' in Fig. 148, with radius ac in Fig. 147, mark off ac' in Fig 148, and so on, as far as af'. Then take the length of one side of the hexagon, say from b to c, or c to d, in Fig. 147, step it off in Fig. 148 from b' to c', c' to d', and so on successively from arc to arc. Connect the points of intersection thus obtained with straight lines, which with the lines ab', ab', the outline being symmetrical on both sides of af', will complete the figure required.

The frustum of an oblique pyramid is sometimes required, as, for example, in the construction shown in Fig. 149.

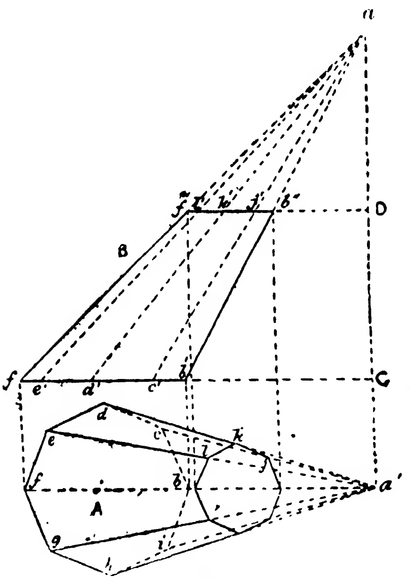


Fig. 150.

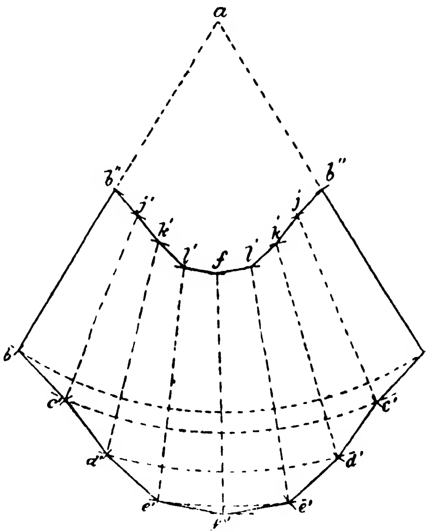


Fig. 151.

PROBLEM 9.—**To draw the envelope of a frustum of an oblique pyramid, in this case one of octagonal form, suitable for Fig. 149.**

Let Fig. 150 represent the frustum in plan A, and in outline elevation B. As in the previous example, the lengths $a'c$, $a'd$, $a'e$, will be transferred to Cc', Cd', Ce', on the base line, the points b', f' being obtained by simple projection. The points j', k', l' may be obtained by direct measurement by transferring the dimensions $a'j$, $a'k$, $a'l$, below from D to j', D to k', D to l', on the plane of the top of the frustum. But as these points occur where the lines $c'a$, $d'a$, $e'a$ cut the plane of the top of the frustum, there is no need to measure these points directly.

The developed pattern is obtained (Fig. 151) by first setting out the base by the method previously described. The outline of the top of the frustum is obtained by measuring off, and transferring the lengths of the edges on Fig. 150, as f', f'', $b'b''$, $c'j'$, $d'k'$, $e'l'$, to Fig. 151, the points so obtained being connected by the straight lines $b''j'$, $j'k'$, and so on.

PROBLEM 10.—**To draw the development of an oblique pyramid, shown in plan (A) and elevation (B) in Fig. 152.**

The correct lengths of the sides are shown projected, $a'b$ to Cb', and $a'c$ to Cc', giving for the true lengths of the angles ab', ac'.

Take the length ac', and with it strike the radius $a'c$ in Fig. 153. With the length ab' in Fig. 152, strike the radius $a'b$ in

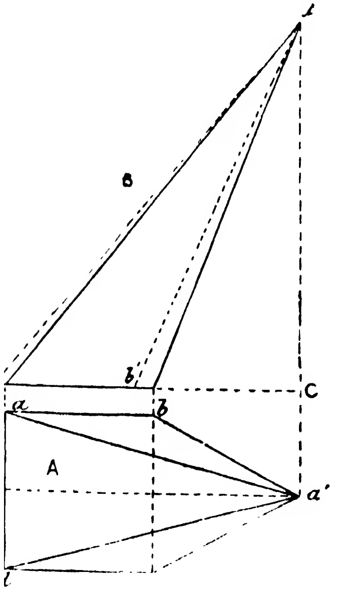


Fig. 152.

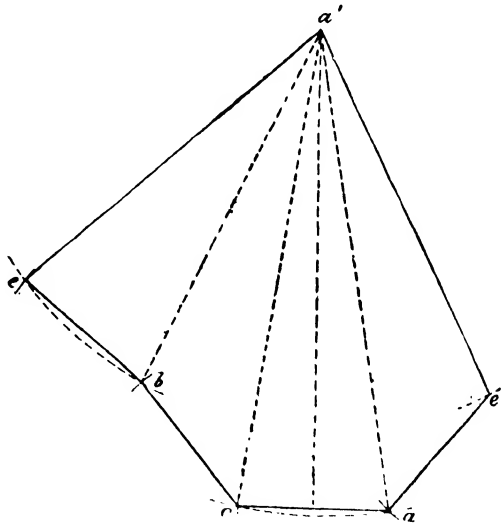


Fig. 153.

Fig. 153. Take the length of a side, as bc in Fig. 152, and set it off from e to a (Fig. 153), and from a to c, from c to b, and from b to e. Connect the points of intersection of these lengths with the arcs of circles already drawn by means of straight lines, and also a' with e and e, and the outline will be that required.

Problem 11.—To draw the development of an oblique truncated pyramid, shown in plan and elevation in Fig. 154.

The correct lengths of the sides are shown projected, as in the previous figure, the same reference letters being employed. The lengths *ab″*, *ac″*, on the plane D, are also the correct lengths for the small end of the pyramid. In Fig. 155 the outline of the base is developed precisely the same as in the previous

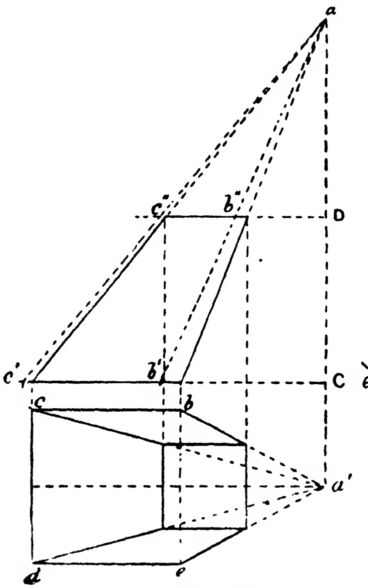


Fig. 154.

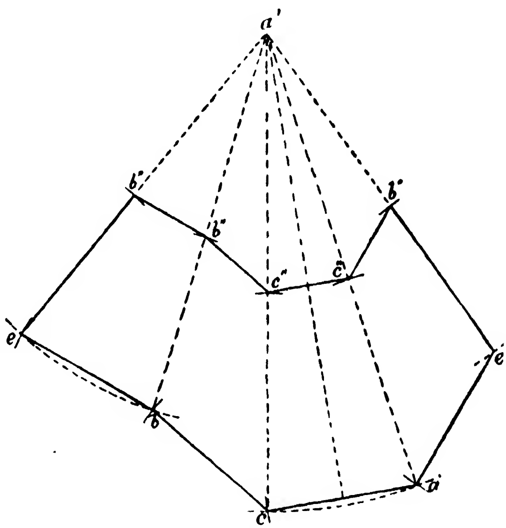


Fig. 155.

example. To develop the top edge, the lengths *c′c″* in Fig. 154 are transferred to *ac″*, *cc″* in Fig. 155, and the lengths *b′b″* in Fig. 154 to *bb″*, *eb″* in Fig. 155. Connecting these points with straight lines gives the outline required.

Since a right pyramid can be inscribed in a cone, and may be considered as a cone with its angles obliterated, it is convenient, when practicable, to inscribe its faces within a curve which coincides with the angles of the faces. But in some cases, as in the case of large figures, it may not be convenient to strike a radius so large, and then a method of triangulation is resorted to. This method is of wide application, not only for polygonal bodies, but for those bounded with curves of large radius.

Let Fig. 156 represent the plan and elevation of a hexagonal truncated pyramid of which the development is wanted without the use of a long radius. For purposes of triangulation two dimensions are required—the true length of the diagonal AE in elevation, corresponding with DB in plan; and the true length of the slant edge AB in plan. This last in the example can be obtained by direct measurement of the side *b* in elevation. But in the absence of an elevation, it can be obtained from the plan thus: In the plan view, raise a perpendicular BC on AB. Measure off the length BC equal to the vertical height *a* of the truncated pyramid, and draw a diagonal AC. Then AC will represent the true slant height of AB. To obtain the diagonal AE, which obviously cannot be got from an elevation because of the slope of the face, draw the diagonal BD. Upon it raise a perpendicular BE. Measure off the length BE, equal to the vertical height *a*, and draw a diagonal ED. ED will then represent the true length of BD in plan, and AC in elevation. From the lengths CA

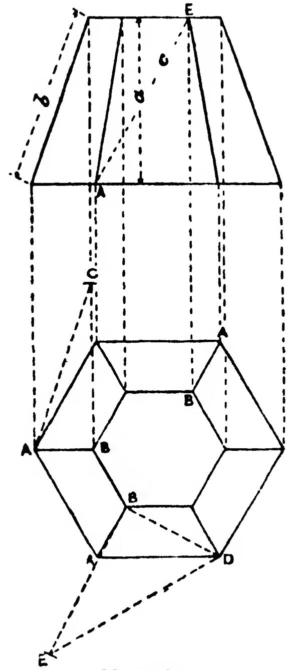


Fig. 156.

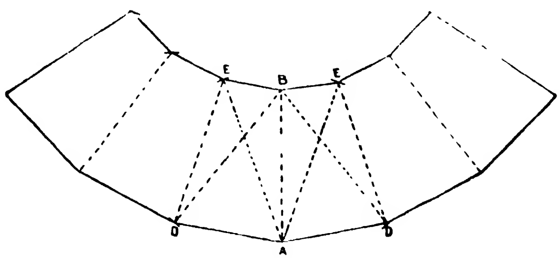


Fig. 157.

and ED points of intersection may be obtained for the construction of the developed envelope of the pyramid thus:—

Take a point A (Fig. 157), and set off the length AB equal to AC in Fig. 156, just obtained. Take the length of one side of the base AD in Fig. 156, and set it off from A to D, D (Fig. 157). Take the length DE of the diagonal face (Fig. 156), and set it off from B to D and D (Fig. 157), intersecting the lines just drawn. With the same length DE set off arcs from A at E, E. Take the length of the top face BB (Fig. 156), and set it off from B to E, E, intersecting the arcs just drawn. Draw the lines DE, DE through the points of intersection. A repetition of these operations will give the pattern of envelope required.

PROBLEM 12.—**To develop the pattern for the frustum of a pyramid by triangulation when it is too large to draw the entire development by problem 5 on p. 67** (Figs. 138–140).

In the example, let the pyramid be hexagonal. Then in Fig. 158 draw a line AB equal in length to one of the sides or faces of the base of the pyramid, and prolong it in each direction. From A as a centre, and radius AB, describe a semicircle, and divide it into half as many equal parts as the pyramid has faces; in this case three. Through one of the points of division C draw a line CA. Then CAB is the angle made by two adjacent faces of the pyramid, and AB, AC are the lengths of the faces measured on the base. Bisect the angle CAB in D. Draw a line CF, making the angle FCA equal to the angle CAD. To obtain the length of the faces on the smaller end of the frustum, set off that length from A to G, and draw the line GF parallel with AD. Draw FH parallel with AC; FH will then represent the length of one of the faces on the smaller end. Draw HJ parallel to AB, and equal in length to FH. Join JB. Then the plan of two faces of the pyramid is represented by the figure ABJHFC. To obtain the slant heights, first let fall a perpendicular HK from H upon AB. Measure off KL equal to the perpendicular height of the frustum. Join LH, which will

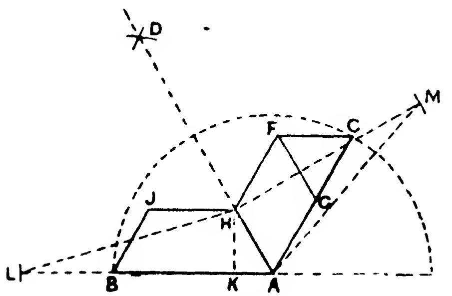


Fig. 158.

be the slant height of the face HK. Draw HM perpendicular to AH, and equal to the perpendicular height. Join AM, which will give the slant height of the edge AH.

To obtain the pattern (Fig. 159), draw a line AH equal in length to AM (Fig. 158), with A as centre, and radius AK (Fig. 158), describe arcs at KK (Fig. 159). With H as centre, and radius equal in length to the line LH (Fig. 158), draw arcs intersecting these at KK. Draw lines from A through K, K, and produce them to B and G. Measure off the lengths AB, AG equal to the length AB in Fig. 158. Draw lines HJ, HF parallel with AB, AG, and equal in length to HJ, HF in Fig. 158. Join BJ, GF. Then the figure BAGFJH will be the development for two adjacent sides of the frustum required.

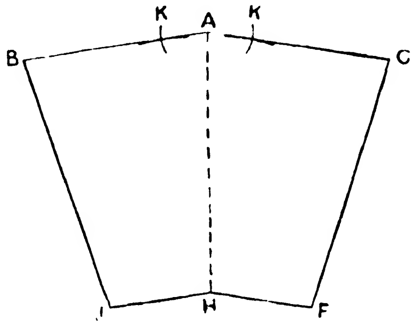


Fig. 159.

Figs. 160, 161 show the same kind of construction applied to

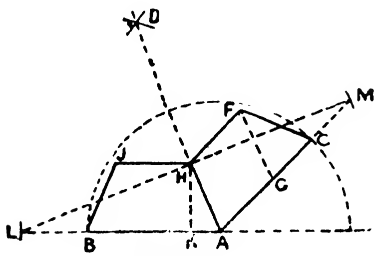


Fig. 160.

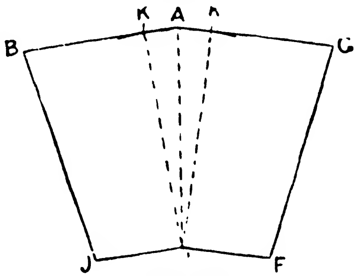


Fig. 161.

the frustum of an octagonal pyramid. As the same reference letters are used as in the preceding figures there is no need to repeat the description given.

PROBLEM 13.—**To draw the envelope of a truncated pyramid by triangulation, the perpendicular height, *a*, being given.**

Let Fig. 162 represent the pyramid in plan, the angles of the two faces being represented by the letters *a, b, c, d, e, f, g, h*. Draw a line *ij*, set off on it a distance *jk*, at right angles, equal in length to the given height of the frustum. The length of the diagonal *ik* then equals the slant height of the face *cdgh* of the frustum. For the slant height of the side *bcgf*, which is the same as *adhe*, draw the diagonal *gb*, make *gk* perpendicular to it, equal in length to the vertical height, and *bk* will be the length of the diagonal or slant face. For the similar face of *abfe*, draw *af*, make *fl* perpendicular and equal to the height of the frustum, and the diagonal *al* will represent the slant face.

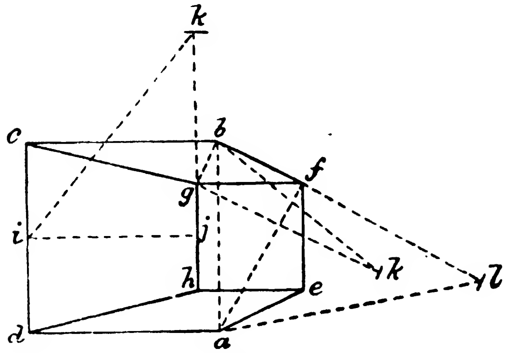


Fig. 162.

For the developed pattern (Fig. 163), draw *ij* equal in length to *ik* in Fig. 162, and draw two lines at right angles therewith. On these lines set off equi-distantly from *i* and *j*, *ic*, *id*, *jg*, *jh*, at the same distances as the same letters are from *i*, *j* in Fig. 162. Take the diagonal *bk* in Fig. 162, and set it off from *g* to *b*, and from *h* to *a* in Fig. 163. Take the length of the sides *cb* and *da* in Fig. 162, and set it off from *c* to *b*, and from *d* to *a* in Fig. 163. Join *cb*, *da*. Draw *gf* parallel with *cb*, and *he* parallel with *da*, and equal in length to *gf* and *he* in Fig. 162. Take the diagonal *al* (Fig. 162), and set it off from *f* to *l* in Fig. 163. Draw a straight line from the point of intersection to *b*. Draw *fe'* parallel with *bl*. The lines *le'*, *ae* will complete the pattern.

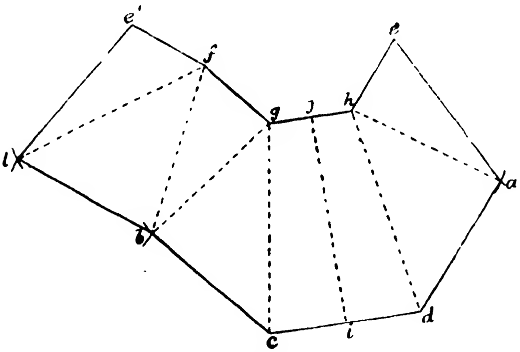


Fig. 16

SECTION II.—CONICAL FIGURES.

Envelopes of cones, or conic frustra, and portions of the same occur so frequently in sheet-metal work that a clear understanding of the principles of their development is essential. As stated on p. 49, the envelope of a cone is a sector of a circle; the envelope of a frustum of a cone is a segment of a circle. The following considerations will serve to fix these points in the mind, and to illustrate the basis of practical methods of development.

If a sheet of paper or card is bent round a solid cone (Fig. 164), its butting edges being cut to just meet down the cone, and its other edge being cut flush with the base of the cone, the sheet will, when unrolled, form the sector *abc*. The curved length *e* will exactly equal the circumference which corresponds with the diameter *d* of the base of the cone, and the width *f* with the slant height *g* of the cone.

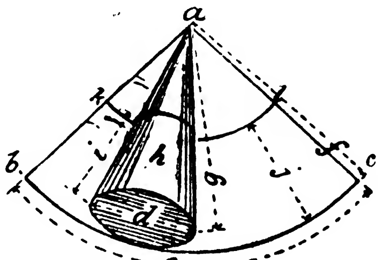


Fig. 164.

Similarly the envelope of a conic frustum or truncated cone *h*, of slant height *i*, would be a segment *bckl* of width *j*.

Obviously, too, in order to form such envelopes, the actual radius of the base of the cone, or of the top of the truncated cone, is not taken, but a radius taken along the slant height of the cone, that is, in the figure *ac* or *lc*. So that taking the truncated cone shown in plan and in elevation in Fig. 165, and its envelope in Fig. 166, the radius of the envelope corresponding

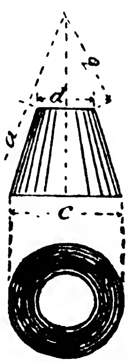


Fig. 165.

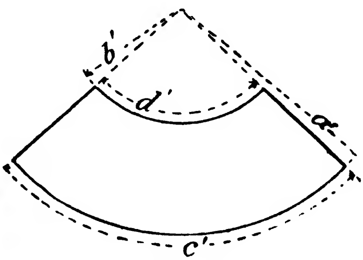


Fig. 166.

with the radius a of the base will be a', and that corresponding with the radius b will be b'. The measurement of the envelope round the curve c' will equal $c \times 3{\cdot}14159$, and that round the curve d will be $d \times 3{\cdot}14159$. These principles, though simple, are very important, and have their practical applications in many of the problems in this work.

The actual measurement of a distance like c',d' round curved lines, is not usually done by reckoning diameter $\times$ 3·14159. This would not be convenient, because of the practical inconvenience of measuring round a curve. To bend a rule round a curve is not an accurate method. Paper scales are used in drawing offices, and these are often bent round curves. But in the shops, when a measurement is taken round a curve, the method adopted is usually this: The compass or divider is set to a definite fractional length of the curve, as, say, 1 inch, ½-inch, ¼-inch, and stepped round as many times as is requisite. Of course the dimensions so stepped round are not arcs of circles, but chords, but the amount of inaccuracy is slight. With care, dimensions can be marked round curves in this way with all the practical accuracy needful.

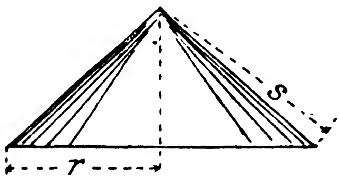


Fig. 167.

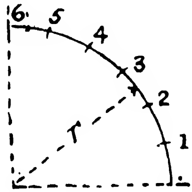


Fig. 168.

In the particular application in question, that of the circumferences corresponding with the base of the cone, or of the crown of a truncated cone, the following is a good method. Taking two cases, that of a cone (Fig. 167), and that of the frustum of a cone (Fig. 170): in the first case, let Fig. 167 represent a cone, the envelope of which is required. In Fig. 168 strike a quadrant of a circle of radius r, equal to the radius r of the base of the cone. Divide the quadrant into any number of equal parts—six in this case. Strike in Fig. 169 a circle of radius s, equal to the radius s of the slant height of the cone (Fig. 167). Prick out on this four sets of six divisions, each transferred from Fig. 168, and these will give the circular length of the envelope corresponding with the circumference of the base, and *abc* will represent the outline of the envelope.

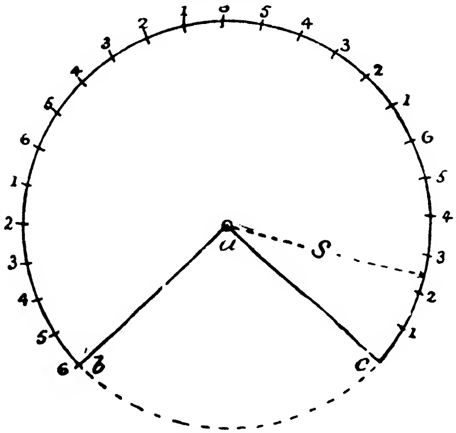


Fig. 169.

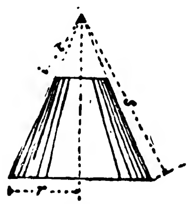


Fig. 170.

So, again, let Fig. 170 represent a truncated cone of which the envelope is required. Divide a quadrant (Fig. 171), of radius *r*, equal to radius *r* in Fig. 170, into any number of equal parts, and transfer these to a portion of a circle (Fig. 172), of radius *s*, equal to the length *s* of the slant height of the cone. Divide this into four sets of six divisions each, corresponding

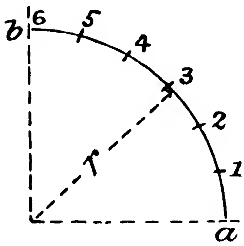


Fig. 171.

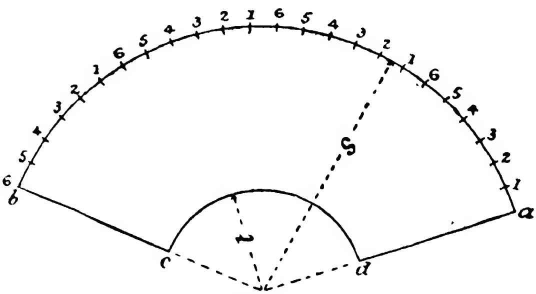


Fig. 172.

with four quadrants of Fig. 171. Then the circular length so divided out will correspond with the circumference of the base in Fig. 170. The slant height *t*, from the apex to the crown of the frustum, will be the radius *t* for striking the curve which will correspond with the circumference of the crown of the frustum. There is no need to divide this out, because if lines are drawn from the extremities of the larger curve to the centre, whence both curves are struck, they will cut off the precise arc *cd* required for the envelope. So that the segment *abcd* (Fig. 172) is the envelope required for the truncated cone in Fig. 170.

Note, by the way, how the shape of the envelope is affected by the relative heights and diameters of the cones. So that a practised eye can tell almost beforehand of what size and shape a piece of sheet will be wanted for any job.

As already remarked, the right cone, that is, a cone with

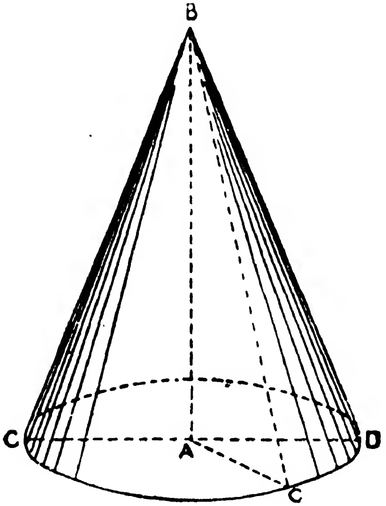


Fig. 173.

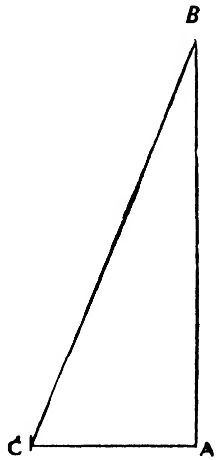


Fig. 174.

equal slant or inclination all round, is formed by the revolution of a right-angled triangle around one of the sides contained in the right angle, and the hypotenuse of the triangle develops in its revolution the surface of the cone. Thus the right-hand portion of Fig. 173 shows the surface of a cone formed by the revolution of the right-angled triangle ABC about the axis AB. AB is the vertical height of the cone, CB its slant height, and AC its radius, and the diameter of its base is CD; B is its apex.

The next figure (174) shows how one dimension can be obtained from others.

If the height AB (Fig. 174) and the radius AC are given, then, if these are laid down on vertical and horizontal lines, the length and inclination of the slant BC are obtained. Or if the radius AC and slant CB are given, then the height AB can be obtained therefrom. Slant, or slant height, signifies therefore not the measurement of an angle, but the length of a sloping side.

The relation between the cone and its envelope, and the frustum of a cone and its envelope, is shown in Fig. 175. ABC is a cone, BCDE is a frustum of the cone ABC. The envelope of the cone is struck with one radius AC; that of the frus-

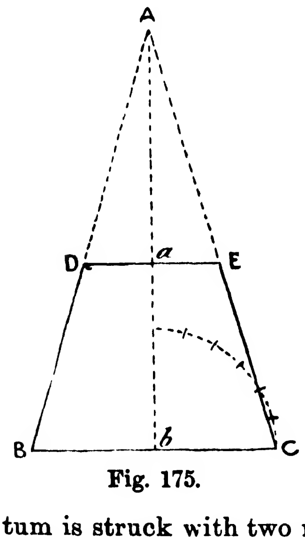


Fig. 175.

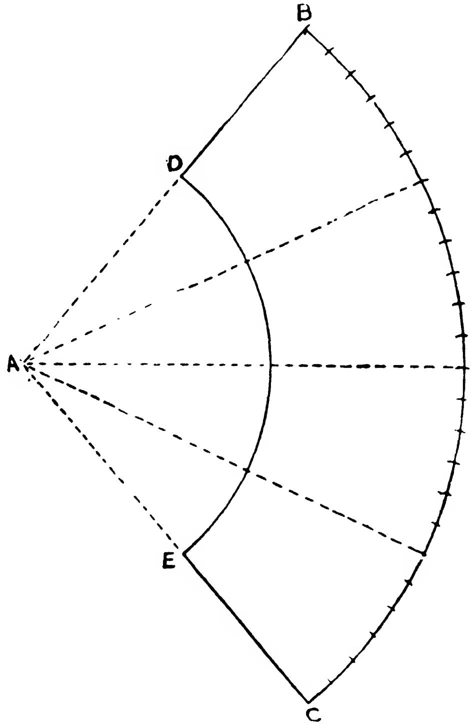


Fig. 176.

tum is struck with two radii AC and AE (Fig. 176). The length of the arc C cut off by the radial lines in Fig. 176 is equal to the circumference of BC in Fig. 175. This, as already shown, is obtained most conveniently by dividing a quadrant of BC (Fig. 175) into a given number of equal parts, and measuring off the length of four such quadrants on the arc BC in Fig. 176. The formation of the envelope of the frustum of the cone by the revolution of the radii *a*E and *b*C is shown in the perspective diagram in Fig. 177. The vertical height of the frustum is *ab* and the slant height is EC. Sometimes one may be given, sometimes another. Having one set of dimensions given, we can find the others. If the radius of the bottom or base *b*C, and that of the top *a*E, is given, and the height *ba*, on setting these out the slant height CE is obtained, and the line or edge CE being prolonged to A gives the centre A, from which the arcs for E and C are struck. Conversely, having the radius of the base *b*C, the vertical height *ba*, and the slant height CE given, the length CE is set off from C to cut the horizontal line *a*E, passing through *a*, and CE prolonged to A gives the centre for the radii E and C.

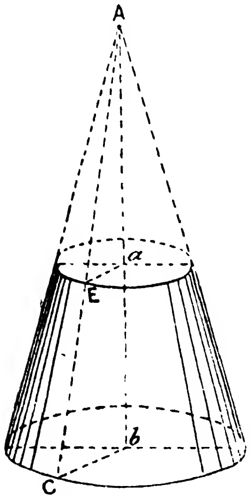


Fig. 177.

Alternatively by calculation—

PROBLEM 14.—**To find the slant height of a cone from the radius of the base and the perpendicular height.**

If the sides of a right-angled triangle are given, the hypotenuse is found thus:—

Add the squares of the sides and extract the square root of the sum.

Hence, for a cone. Take the *radius* and the *perpendicular height*, square each, add together, and extract the square root, which will be the *slant height.*

Articles are sometimes required, the whole or portions of which are the envelopes of conic frustra, but for which it is difficult or impossible to obtain radii direct with trammels or compasses set in a definite centre. In such cases approximate curves can be obtained by the method of triangulation.

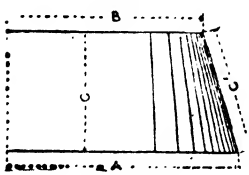


Fig. 178.

PROBLEM 15.—**To develope conic frustra by triangulation.**

Let Fig. 178 represent the envelope of a conic frustum, the slant height of which is so slight that the apex of the completed cone would be situated too far away to be utilized as a centre. There are four dimensions, the larger radius A, the smaller B, the vertical height C, and the slant height C′. But it is not even necessary to draw out Fig. 178 in order to get the slant height C, since the latter can be obtained by the construction shown in Fig. 179, which represents a part plan of the frustum, in this case a quarter.

In Fig. 179 draw the arcs AA and BB, representing respectively the radii A and B of Fig 178 in plan. These are now to be divided into any convenient number of equal parts, as *a*, *b*, *c*, *d*, *e* on the arc AA, which, prolonged to the centre *o*, divides the arc BB equally at *f*, *g*, *h*, *i*, *j*. Since the working plan of the plate ABBA (Fig. 179) has to be developed on the slant surface C′ in Fig. 178, the triangulation in Fig. 179 is adopted. The distances A*a*, B*f*, etc., are of course exactly the same in plan in Fig. 179 as they would be on the slant face C′ in Fig. 178.

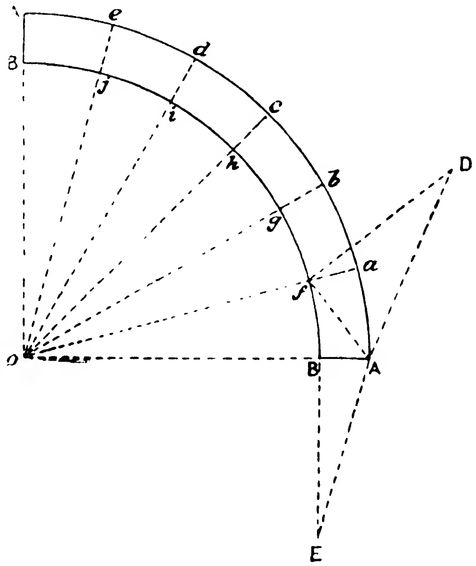


Fig. 179.

But the diagonal dimension A*f* in Fig. 179 has to be developed proportionally for the slant face C′. To obtain this, raise a line *f*D perpendicular to A*f*, and measure off on it the length *f*D, equal to the perpendicular C in Fig. 178. Join AD, and this length AD will be the correct length A*f* on the slant face C′. in Fig. 178.

If the view (Fig. 178) is not drawn, the slant height C′ for the width of the plate is obtained thus :—

Raise at B (Fig. 179) a perpendicular to A*o*, and measure off on it a length BE, equal to the vertical height C, in Fig. 178.

Draw the diagonal AE, which will be the width of the slant face C′ in Fig. 178.

The method of drawing the plate from these data is shown in Fig. 180.

Draw a line between two points, AB, at a distance apart equal to the slant height C′. Strike circle arcs A*a* and B*f*,

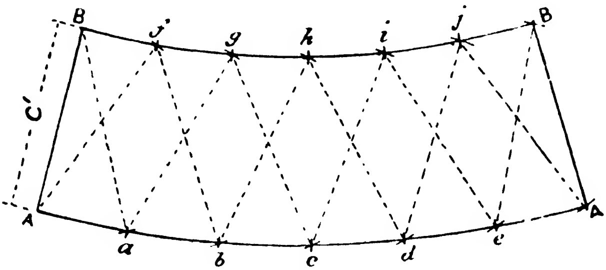


Fig. 180.

corresponding with the divisions in Fig. 179. Take the length AD in Fig. 179 as radius, and from points A and B in Fig. 180 strike circle arcs cutting *a* and *f*. Using the same dimensions, and taking the points of intersection *a* and *f* as centres, find new points of intersection *b* and *g*, and again use these as centres, and so on, as shown. Curves AA and BB drawn through these points will give the envelope required.

Fig. 181.

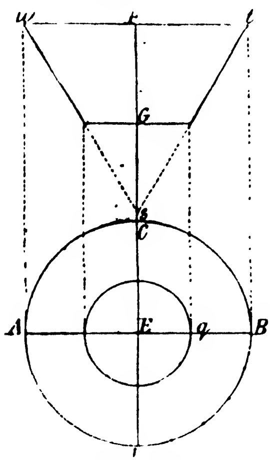


Fig. 182.

In the next example (Fig. 181) the allowance for seam or lap is introduced. This has been omitted previously in merely geometrical problems, but it necessarily comes in for practical work. The term "flue" is a shop synonym for taper or slant.

Problem 16.—**To strike a pattern for a round, tapering, or flue article (or a frustum of a cone).**

Fig. 182 represents in plan the diameter of both top and bottom, and Fig. 181 from G to F the vertical height, being an elevation of Fig. 182. Divide the circle with lines, as AB and CD, at right angles; then draw a line as *ab* in Fig. 183, and take the depth required, as from F to G; mark it off from *a* to *d*, and draw the lines *ac* and *de* at right angles with *ab*; take the radius of the larger circle EB with the compasses, and mark off the distance from *a* to *c*; take also the radius of the small circle E to *g*, and mark it off from *d* to *e*; then draw a line through the points *c*, *e* to cut the line *adb*; with *b* as centre and radius *be* strike the curve *eh*; open the compasses to *c*, still using *b* as centre, and strike the curve *cioqf*. The circle (Fig. 181) is divided into quarters; take one of them and divide it into any convenient number of equal parts, as D to B; from *c* (Fig. 183) measure off a corresponding number of distances to *i*; the curve *c* to *i* shows one quarter of the pattern required; by adding on a like distance, as from *i* to *o*, half the pattern is represented, and the distance *cioqf* is the whole pattern required in one piece; draw a line from *f* to the centre *b*, and the required lap to be added on as shown.

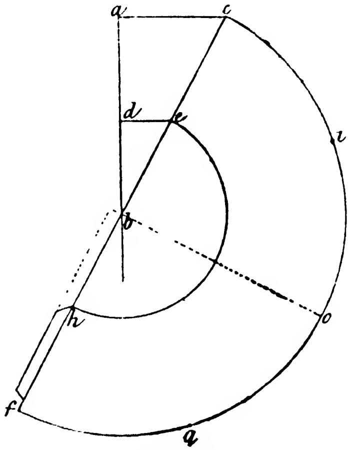
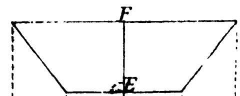


Fig. 183.

Fig. 184.



PROBLEM 17.—**To describe the plan of a round flue body to be cut in three pieces.**

Fig. 185 represents the top and bottom of the body in plan; Fig. 184 is its elevation; the plan is to be divided into three parts, which is done in a very simple manner. The radius by which a circle is struck equals one-sixth of the circumference, so that drawing a line from every alternate point set out by the radius on the circumference to the centre will divide the circle into three, as B, C, D (Fig. 185).

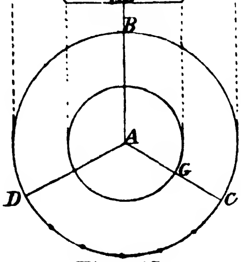


Fig. 185.

The pattern for this body is shown in Fig. 186. Draw *obe* at right angles; take the perpendicular height FE (Fig. 184) and mark it off from *b* to *d* (Fig. 186). Draw the line *df* at right angles with *bo*. Take the radius of the outer circle at AB (Fig. 184) and mark it off from *b* to *e* (Fig. 186). The radius A G in Fig. 185 is to be marked off from *d* to *f* in Fig. 186. Draw a line through the points *e*, *f* to cut the line *bo* at *o*. Take *o* as centre, and with radius *of*, strike the curve *fn*. Open the compasses to *e*, still using *o* as centre, and strike the curve *em*. Divide the circle arc DB in Fig. 185 into any convenient number of equal parts, and measure off a corresponding number of equal parts in Fig. 186 from *e* to *m*. Draw a line from *m* to the centre *o*, which gives the pattern of one-third of the body required, the perpendicular height of which will be equal to EF in Fig. 184.

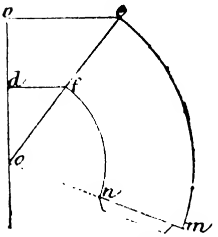


Fig. 186.

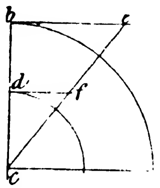


Fig. 187.

Fig. 187 gives the flue or the slanting height of the same articles; the only difference between this and Fig. 186 is, that the radius is taken from *d* and *b*, instead of *f* and *e*.

Problem 18.—**To obtain the radius required for striking the pattern of a slightly tapering article, without the necessity of producing lines to meet.**

Let the two circles in Fig. 188 represent the diameters *ef* and *gh* at the top and bottom, then the distance from *a* to *b* will show the flue on all sides. Take the distance from *a* to *b* with the compasses, and measure off, or find how many times that distance is contained between *a* and the centre *o* on the diameter line: in this case 9. Now let the vertical height, from *c* to *d*, be multiplied by 9, and the product will be the radius required, or the length of string or wire to strike the curve with.

To give an example:—Suppose the diameter of the larger circle is 18 inches, and that of the smaller one 16 inches, the distance from *a* to *b* would be 1 inch, and from *a* to *o* would be 9 times as much as from *a* to *b*. Now suppose the vertical height from d to c be 2 feet. Then 9 times 2 being 18, a radius of 18 feet would strike the required curve for the pattern.

Oblique conic frustra are sometimes required, as in the example given in Fig. 189, the joints running either horizontally or at an angle.

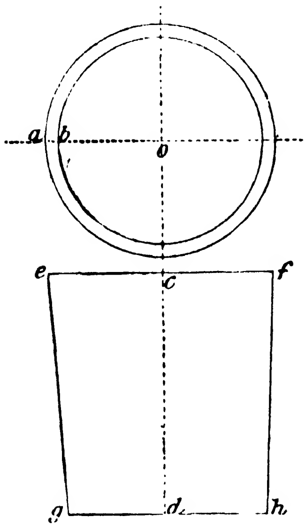


Fig. 188.

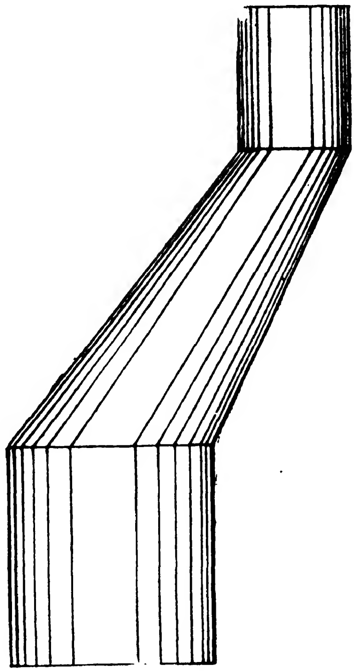


Fig. 189.

PROBLEM 19.—**To strike the pattern of a tapering piece of pipe to join two vertical cylinders, to form a double elbow.**

Draw ACEG and BDFH (Fig. 190) according to the plan required: produce the lines CE and DF until they meet at the point O. Draw the semicircle on AB, and divide into equal parts b, c, d, e, f, and draw the perpendiculars through these points to cut the section line CD at g, h, i, k, n; from these points draw lines to the point O. With O as centre, and radius Oi, draw the curve NAB (Fig. 191); with the compasses set to the divisions b, c, d, e, f on the semicircle AB (Fig. 190) mark off twelve points from N to B (Fig. 191), and draw lines from these points to the centre O. With radius OC (Fig. 190) strike the curve CP (Fig. 191), and with radius OD (Fig. 190) strike the curve DB (Fig. 191).

If curves are drawn with O as centre from the points *g*, *h*, *k*, and *n* (Fig. 190) (which are not shown here), the points of intersection with NPB (Fig. 191) will give the exact direction of the curve BPN; and if curves are drawn from the same centre from

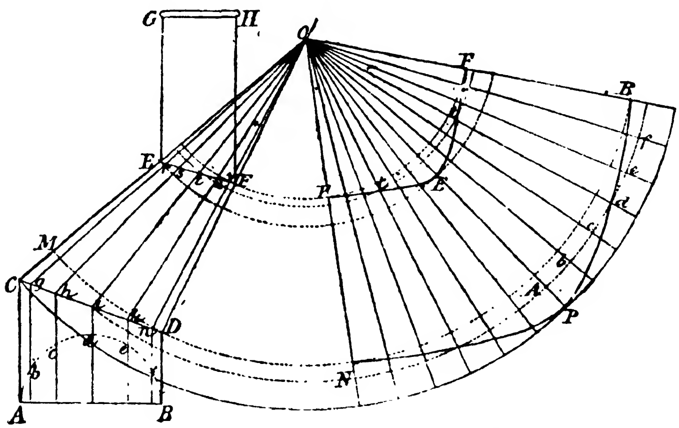


Fig. 190. Fig. 191.

the points of intersection on the line EF (Fig. 190), the points of intersection (Fig. 191) will give the direction of the curve F*t*E*t*F, which will give the pattern for the tapering part of the angular pipe. The other parts can be drawn as previously described.

Note.—The curves mentioned which give the pattern should be drawn from the various points with freehand. As there are many curves in geometry and in mechanical drawing which are drawn better by hand from given points than by instruments, the student is recommended to practise free-hand drawing at the same period that he studies other portions of mechanical art.

Problem 20.—**To describe the pattern of an oblique cone, or the frustum of a cone cut parallel with the base.**

The vertical position of the two diameters is shown by the two circles in Fig. 192. Now take the vertical height from F to *b* (Fig. 193), and draw FG and *ba* parallel to AB—the diameter line—(being drawn through the two centres from which the circles are struck), and draw AH perpendicular to AB; also draw perpendiculars from E to *a* and from B to G, and draw a line through the points G and *a* to cut the line AH at H. Now FH represents the vertical height of the cone, and FG the base, and the line *ba* shows the frustum or section required, being cut off parallel with the base.

Divide half the plan into equal parts, as 1, 2, 3, 4, 5, and draw lines from these points to A. Now, using A as a centre, draw arcs from these points, as 1D, 2C, etc., to cut the diameter AB, and draw perpendiculars from these points to the base line FG. Next, using the point H as a centre (Fig. 193), describe arcs from G, *hf*, *e*, *d*, *c*, F to A, also from *b*, *i*, *k*, etc. Now with the compass set to the divisions on the plan 1, 2, 3, 4, 5, B (Fig. 192), mark off the same distances in Fig. 193 from G to

Fig. 193.

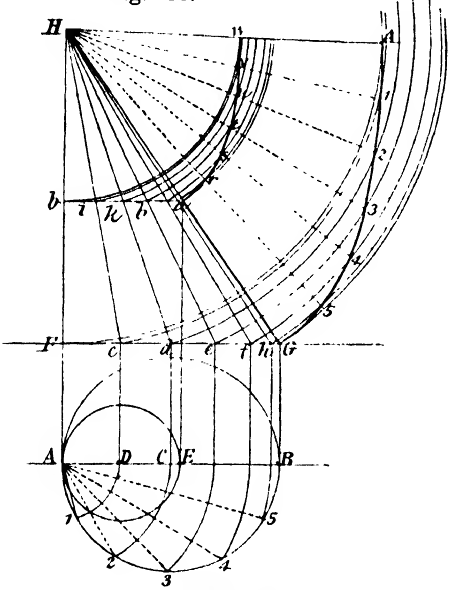


Fig. 192.

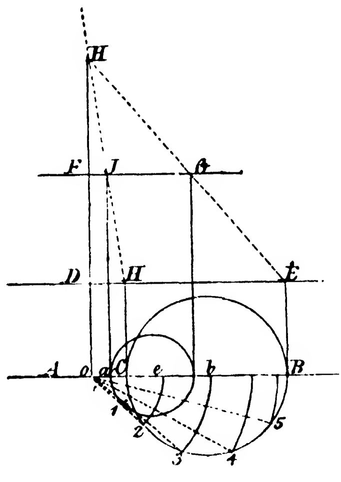


Fig. 194.

5, 4, 3, 2, 1, and A, stepping from the outer curve into the second and third, and so on, and draw lines from these points of intersection to the centre H. By drawing curves through these points of intersection as A, 1, 2, 3, 4, 5, G, and also through *w*, *v*, *u*, *t*, *s*, *r*, *a*, one-half of the development will be obtained.

Fig. 194 is a further illustration of the same principle. The two circles struck from centres on the line AB show the vertical positions of the sections of the cone required, and from D to F is the elevation. Draw perpendicular lines from C to H and from B to E, being the diameter of the base, also carry perpendiculars showing the diameter of the top of the cone to J and G, join HJ and EG, and produce them to meet at the point H. Draw a perpendicular line from H to cut the line AB at *o*, which will be used as a working centre, as the point A is in Fig. 192; the development will then be obtained in the same manner as in Fig. 192.

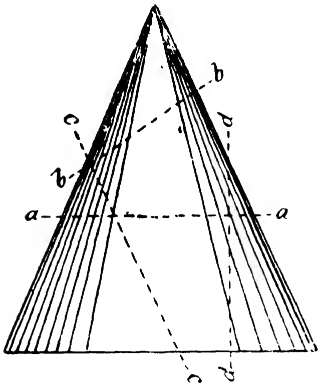


Fig. 195.

A cone cut in various sections is seen in Fig. 195. These are: a plane *a-a* cut parallel with the base of the cone, the section being a circle; a plane *b-b* cut obliquely to maintain the base entire, the section being an ellipse; a plane *c-c* cut parallel to one of the sloping sides, the section being a parabola; a plane *d-d* cut parallel with its axis, the section being a hyperbola. We will take the envelopes of cones cut at *b-b*, *c-c*, *d-d*.

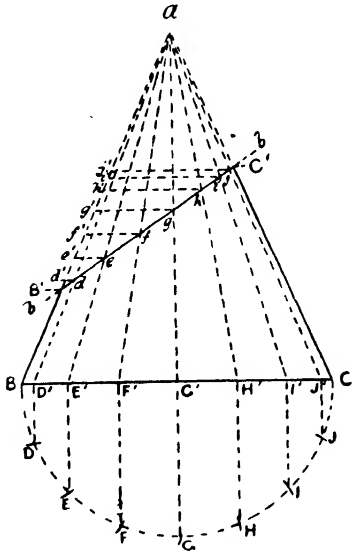


Fig. 196.

Problem 21.—To obtain the development of the envelopes of the cone cut in elliptical section (Figs. 196, 197).

Fig. 196 shows the cone cut at *b-b*, the cut section forming an ellipse; and Fig. 197 is the development of the lower part of the envelope of the cut cone. Obviously also the supplementary portion of the figure is the envelope of the upper portion of the cut cone.

In Fig. 196, let *a*BC represent the outline of the cone. Strike a semicircle BGC, equal in radius to half the length of the base BC, and divide it into any convenient number of equal parts, B, D, E, F, G, H, I, J, C. Carry perpendiculars up to cut the line BC in D′, E′, F′, G′, H′, I′, J′, and draw lines thence to the apex *a*. They will then cut the diagonal *b*-*b* at *d*, *e*, *f*, *g*, *h*, *i*, *j*. Carry horizontals along to meet the slant edge B*a* in *d*′, *e*′, *f*′, *g*′, *h*′, *i*′, *j*′. Then the lengths B*d*′, B*e*′, B*f*′, B*g*′, B*h*′, B*i*′, B*j*′ will be the actual lengths of the lines D′*d*, E′*e*, F′*f*, G′*g*, H′*h*, I′*i*, J′*j*. To obtain the envelope, take the length *a*B as radius,

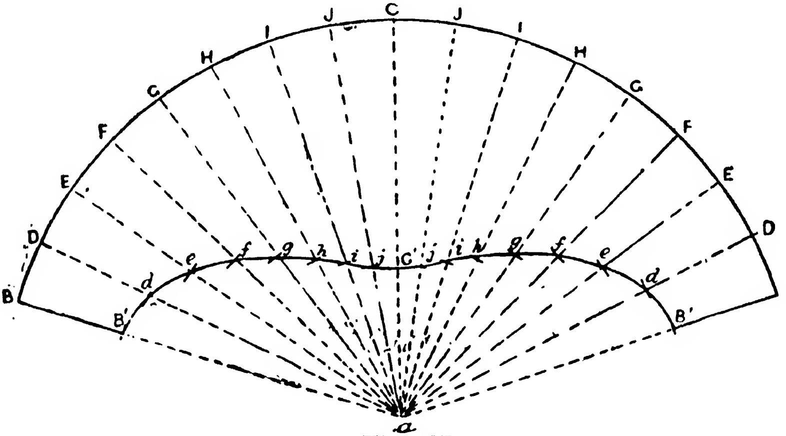


Fig. 197.

and strike a circle arc (Fig. 197) *a*BB. From the point C set off to right and left the points C, J, I, H, etc., to B, using the lengths of the points of division CJI, etc., in Fig. 196 above. Draw lines from the points of division to *a*. On these lines set off the projected lengths from the previous figure, thus: Take the length CC′, and set off from C to C′ in Fig. 197. Take the length J′*j* (Fig. 196), and set it off from J to *j* (Fig. 197). Take the length I′*i* (Fig. 196), and set it off from I to *i* (Fig. 197), and so on. A curve drawn through the points G′, *j*, *i*, *h*, *g*, *f*, *e*, *d*, B′ to right and left will give the form of the envelope of the cut surface. The construction lying to one side of this curved line will be that for the lower part of the cone; that lying to the other side, for the upper part of the cone.

Problem 22.—**To obtain the development of the envelopes of the cone cut in parabolic section** (Figs. 198, 199).

Draw a semicircle BGC on the base BC of the cone. From the point where the line *c*-*c* cuts the base BC of the cone, carry a perpendicular downwards, cutting the semicircle in H. Divide the arc BH into any convenient number of equal parts, B, D, E, F, G, H, and carry perpendiculars up to cut BC in D′, E′, F′, G′. Carry these to the apex *a*, cutting the parabolic section in *d*, *e*, *f*, *g*. To obtain the working lengths of these lines, carry the horizontals out to cut B*a* in *d*′, *e*′, *f*′, *g*′.

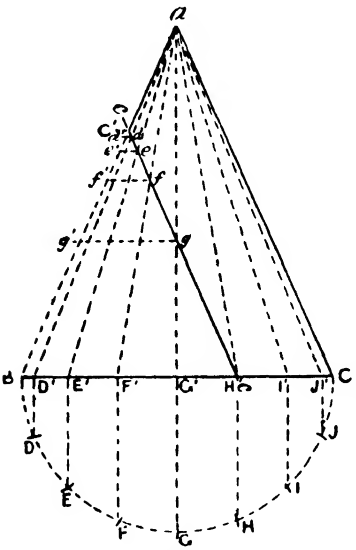


Fig. 198.

For the developed sheet, with the radius *a*C, strike a circle arc *a*CC (Fig. 199). Take any point B, and from it set off to right and left the points of division B, D, E, F, G, H, corresponding with those above in Fig. 198, and draw lines thence to *a*. From *a* set off *a*C′ equal to *a*C′, in the upper figure, *ad*′, *ae*′, *af*′, *ag*′ equal to the corresponding distances in Fig. 198. A line drawn through these points will give the outline of the cut surface required. The figure is

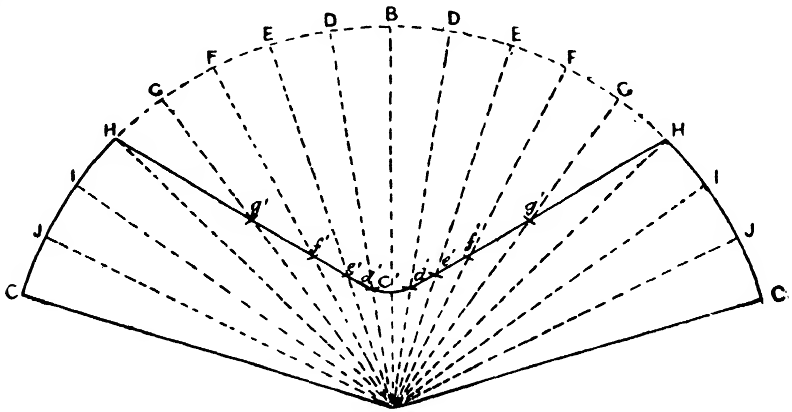


Fig. 199.

completed by dividing the arc HC into any convenient number of parts, H, I, J, C, and setting off corresponding divisions, H, I, J, C, in Fig. 199, and connecting C*a*. Then C*a*CHC′H is the envelope of that portion of the cone which includes the apex in Fig. 198, and HC′HB is the outline of the supplementary portion of the cone.

PROBLEM 23.—**To draw the outline of the cut face of a parabola** (Figs. 200, 201, *c-c* in Fig. 198).

Draw a base line AB (Fig. 200), and on it erect a perpendicular CD. From C as a centre measure off to right and left lengths CA, CB equal to the length HH′ in Fig. 198, which is the width of the base of the parabola at the section in that figure, and erect the perpendiculars. Measure off the height or length of the axis to D equal to the length of the line *c-c* in Fig. 198, and draw the horizontal. Divide half the base, and the entire length, each into the same number of equal parts, 1, 2, 3; 1′, 2′, 3′. Erect perpendiculars on 1, 2, 3. Draw diagonals from 1′, 2′, 3′ to the apex D. Make these lines symmetrical. A curve traced through the intersections of these lines will give the outline required.

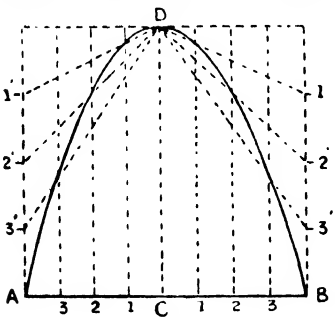


Fig. 200.

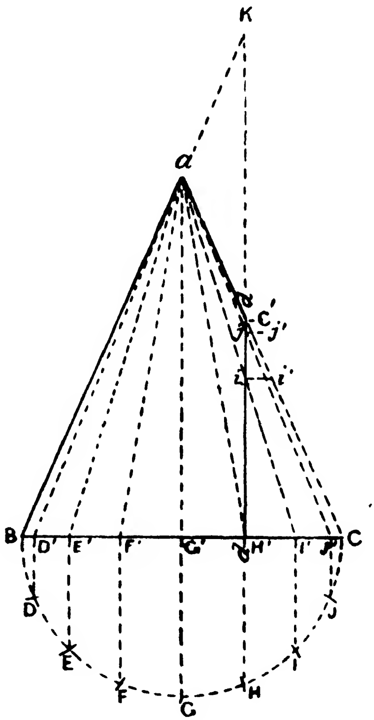


Fig. 201.

PROBLEM 24.—**To obtain the envelope of the cone cut in hyperbolic section** (*d-d* in Figs. 195 and 201).

Strike a semicircle, as before, on the base BC of the cone (Fig. 201), and carry a perpendicular from *d* to cut this semicircle in H. Divide the arc HC into any convenient number of equal parts, H, I, J, C. Make IJ cut BC in I′J′, and carry lines from I′J′ to the apex *a*, cutting *dd* in *i* and *j*. From these points of intersection carry horizontals to cut *a*C in *i′* and *j′*.

For the envelope: With radius *a*C (Fig. 201) strike a circle arc (Fig. 202) below. From a point C set off the distances C, J,

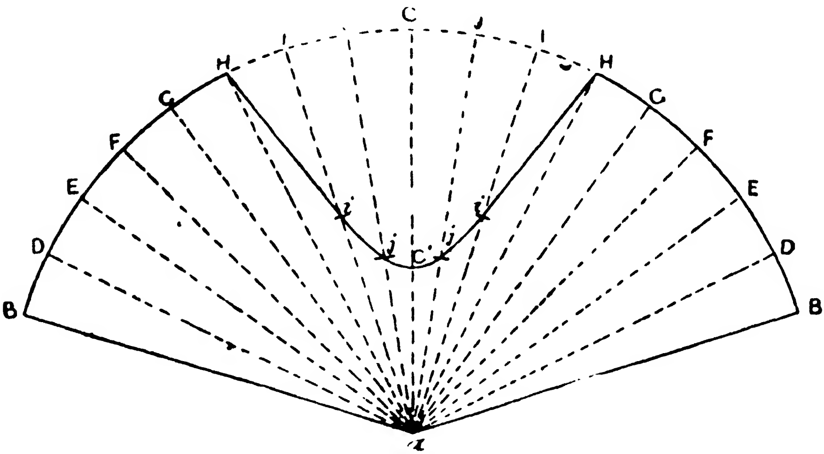


Fig. 202.

I, H to right and left, corresponding with C, J, I, H (Fig. 201), and draw lines thence to *a*. Set off CC′, J*j′*, I*i′*, corresponding in lengths with CC′, C*j′*, C*i′* (Fig. 201), and draw a line through the points of intersection. To obtain the envelope of the other portion of the cone, divide the arc BH (Fig. 201) into any convenient number of parts, B, D, E, F, G, H, and step these round on the circle arc (Fig. 202), from H to B, and connect B*a*. Then B*a*, BH, C′H will represent the envelope of that part of the cone which contains the apex, and HC′, HC the other portion.

PROBLEM 25.—**To draw the outline of the cut face of a hyperbola** (Fig. 203; *d-d* in Figs. 195 and 201).

Draw a base line AB (Fig. 203), and erect a perpendicular CD;

CA, CB, each equal in length to H′H in Fig. 201, giving the width of the base. Measure off CD equal to the height or axis of the hyperbola of the section. Beyond this measure off DK equal in length to the major axis, obtained by prolonging B*a*, H′*d* in Fig. 201 till they meet in K. Divide half the base and the entire sides into the same number of equal parts, 1, 2, 3, 1′, 2′, 3′. Draw lines from 1, 2, 3 to K, and from 1′, 2′, 3′ to D. A curve drawn through the points of intersection of these lines will give the outline required.

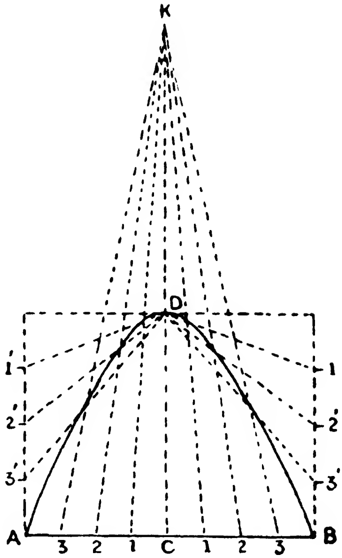


Fig. 203.

Section III.—Cylindrical Figures.

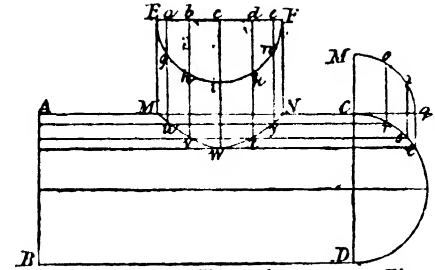
Problem 26.—**To strike the pattern for a T-piece, or two cylinders jointed at right angles.**

Strike a semicircle of the diameter of the smaller cylinder (Fig. 204) from E to F, also extend the lines DC and AC as shown at M and *q*, and describe a quadrant from M to *q* with the same radius with which the semicircle EF is struck. Now divide the semicircle into a convenient number of parts—in this case six, as *g*, *h*, *i*, *k*, *n*, F. Divide the quadrant into three equal parts, as *o*, *p*, *q*. Also strike a semicircle from C to D of the size of the larger cylinder, and draw perpendiculars from *op* and *q* to cut the curve in the larger cylinder at *r*, *s*, *t*. Now draw lines from the points *r*, *s*, *t* parallel to CA; and draw also perpendiculars from the points *g*, *h*, *i*, *k*, *n*, to intersect the horizontal lines *r*, *s*, *t*. Those points of intersection, *u*, *v*, *w*, *x*, *y*, N, will show the course of the curve generated by the smaller cylinder being fitted against the larger one at right angles.

Now draw twelve perpendiculars as shown in Fig. 205: F, *e*, *d*, etc., and take the lengths in Fig. 204 from FN, *ey*, *dx*, *cw*, *bv*, *au*, EM, and transfer the same to the corresponding letters in Fig. 205, and draw a curve through these points, which will give the pattern for the smaller cylinder.

Next, to obtain the hole to receive the smaller cylinder, proceed in Fig. 206 to draw the line AB, and bisect it at C. Take the distances C*r*, C*s*, and C*t* (Fig. 204), and mark off on

Fig. 204.



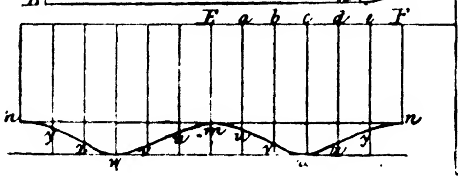


Fig. 205.

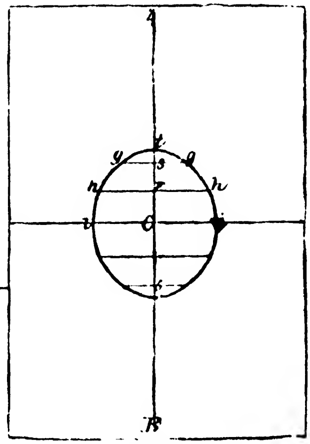


Fig. 206.

each side of C (Fig. 206) like distances, shown by the corresponding letters, and through these points draw lines *hh* and *gg*, etc., at right angles with AB. Take the distance from *c* to *i* (Fig. 204), and mark off a corresponding distance on each side of C to *i* (Fig. 206); also the lengths *bh* and *ag* (Fig. 204), and transfer the same to Fig. 206, on each side of *r*, to *h*, and each side of *s*, to *g*; a curve drawn through these points will give the required aperture to receive the smaller cylinder.

Problem 27.—**To strike the pattern of an elbow at right angles, in a round pipe.**

Draw ABCFED (Fig. 207), which shows the size of the elbow required. On the line CF strike a semicircle of the diameter of the cylinder. Divide the semicircle into any convenient number of equal parts, as *a*, *b*, *c*, *d*, *e*, F. Extend the line AD indefinitely, and set off twice the number of parts that there are on the semicircle (Fig. 207), from C to F, F (Fig. 208), on each side of the centre C, and draw perpendicular lines F*n*, *em*, *dl*, *ck*, *bi*, *ah*, C*g*, etc. Extend the line BC to cut the perpendicular C*g*, and draw lines from the points in the semicircle *a*, *b*, *c*, etc., to cut the perpendiculars at *h*, *i*, *k*, etc.

Draw a curved line through all the points of intersection, as

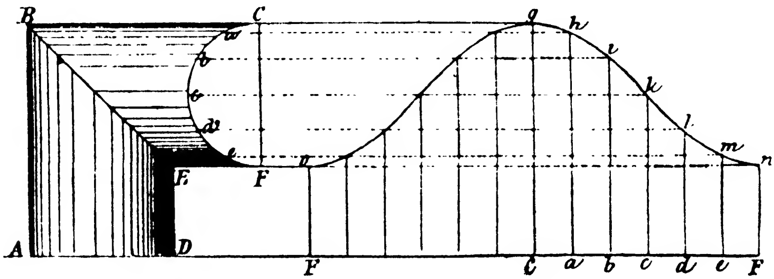


Fig. 207. Fig. 208.

n, *m*, *l*, etc., to *o*, which forms the curve required for the pattern. This curve should, from *n* to *m*, commence somewhat at right angles with the perpendicular *n*F, also from *g* to *h* to give a curve, and not to show a point at *g*.

PROBLEM 28.—**To strike the pattern of two cylinders for joining at an oblique angle.**

Let DAEC (Fig. 209) represent the larger cylinder, and let HFIG be drawn to the required size of the cylinder that has to be connected to it at any angle or position required. Draw the line FG at right angles with HF, describe the semicircle from F to G, and divide it into six equal parts, as 1, 2, 3, etc., draw lines through these points at right angles with FG. Now strike the semicircle ABC representing the diameter of the larger cylinder, and extend the lines DA to M, and CA to L, take the radius of the semicircle *o*F, and from A mark off the same distance to M, then take half the length of the base of the cylinder, from K to H or K to I, and mark off a like distance from A to L. A quarter of an ellipse is required, as shown from L to M, the radius of which may be obtained in the following manner. Draw a line from M to N, also one from L to N, at right angles, and draw the diagonal line LM, draw a line from the point N to cut through the diagonal LM at right angles, producing the points *c* and *b*: with *c* as centre, and radius *c*L, draw the curve from L to *a*; with *b* as centre, and radius *ba*, draw the remainder of the curve from *a* to M. Divide the curve from L to M into three equal parts, and draw perpendiculars from these points to meet the curve AB, as *df*,

Fig. 209.

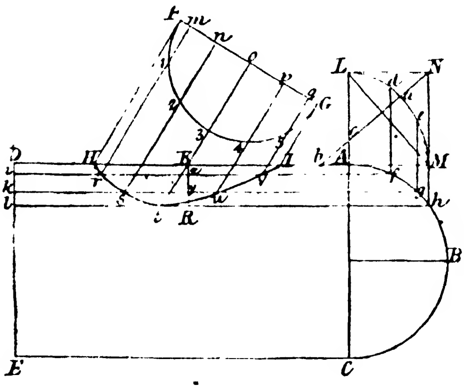
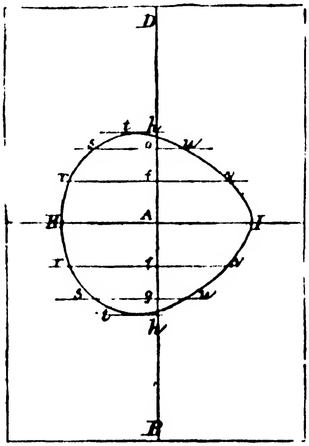


Fig. 211.



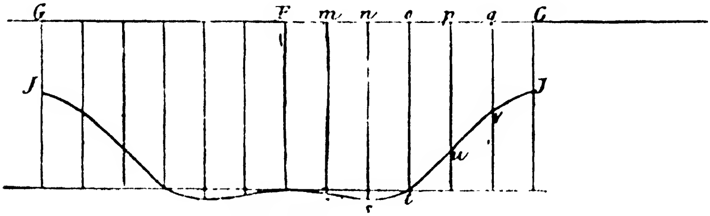


Fig. 210.

eg, M*h*, draw also lines parallel to AD from *f* to *i*, *g* to *k*, and *h* to *l*: the points where these lines are intersected by the lines drawn through the smaller cylinder will be the points through which to trace the curve, as *r*, *s*, *t*, *u*, etc. Draw twelve perpendicular lines in Fig. 210, as F, *m*, *n*, *o*, etc., at the same distances apart as the divisions in the semicircle FG (Fig. 209), and take the length of the lines in Fig. 209, as FH, *mr*, *ns*, etc., and transfer the same to the perpendiculars in Fig. 210 marked by the corresponding letters. Draw a curve through the points thus obtained, J, *v*, *u*, etc., which will give the pattern for the smaller cylinder.

To obtain the curves for the hole in the larger cylinder. Draw DB and HI (Fig. 211) at right angles, take the distances from A to *f*, *g*, and *h* (Fig. 209), and mark off like distances on each side of A on the line DB, as *f*, *g*, *h*, and draw lines from these points parallel to HI. Draw a perpendicular from the point K to R (Fig. 209), and transfer the lengths KH and KI (Fig. 209) from A to H and A to I (Fig. 211), also the distances *xr* and *xv* (Fig. 209) from *f* to *r* and *f* to *v* (Fig. 211), and the distances from *y* to *s* and *y* to *u* (Fig. 209) transfer to Fig. 211 from *g* to *s* and *g* to *u*; take the distance from R to *t* (Fig. 209), and mark off the same from *h* to *t* (Fig. 211); the curve drawn through the points thus obtained will give the shape of the aperture required to receive the smaller cylinder.

Problem 29.—**To strike the pattern of an elbow in a round pipe at any angle required (in this case an obtuse angle).**

Draw ACEG and BDFH (Fig. 212) according to the angle required in the pipe. Draw the section line CD from the two

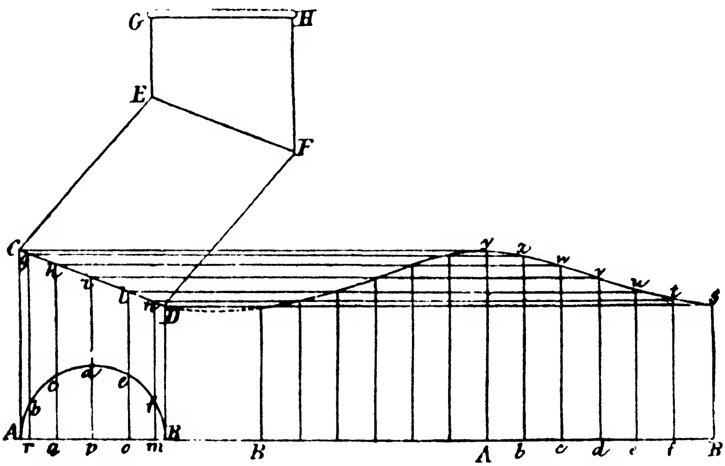


Fig. 212. Fig. 213.

points of the angle, showing where the joint is required. Extend the line AB indefinitely. Draw the semicircle on AB of the diameter of the cylinder, divide it into a convenient number of parts, as A, *b*, *c*, *d*, *e*, *f*, B, and take a corresponding number of points, *b*, *c*, *d*, *e*, *f*, from A to B, B (Fig. 213). Draw the perpendiculars B*s*, *ft*, *eu*, etc. Now through the points *b*, *c*, *d*, etc., on the semicircle (Fig. 212) draw the perpendiculars *gr*, *hq*, *ip*, *lo*, and *nm*. Either take the length of the lines as AC, *rg*, *qh*, etc., and mark off the same lengths from A to *y*, *b* to *x*, *c* to *w*, etc. (Fig. 213), or draw lines parallel to AB from the points on the section line CD (Fig. 212), as *g*, *h*, *i*, etc., to intersect the perpendiculars in Fig. 213 at *y*, *x*, *w*. A curve drawn through these points of intersection, *s*, *t*, *u*, *v*, etc., will give the pattern required. The perpendiculars on the left hand side of A*y*, in Fig. 213, though not lettered, correspond exactly with those on the right-hand side.

PROBLEM 30.—**To describe a cylindrical section through any given angle.**

Let AB (Fig. 214) be a transverse section of a right cylinder, and CD the line of the required section. Draw the circle at AB, and on the semicircle take any number of points, as 1, 2, 3, 4, etc., from which points draw lines perpendicular to AB, produced to cut the line CD in 1, 2, 3, 4, etc. From these points draw the lines 1*a*, 2*b*, 3*c*, 4*d*, etc., perpendicular to CD, and make the lengths of these ordinates respectively equal to the lengths of the corresponding perpendiculars on the transverse section of the cylinder below, that is to say, let *d*4, *c*3, etc., below, equal 4*d*, 3*c*, etc., on the line CD above; continue this process throughout, and through the points found in this way the required section will be drawn.

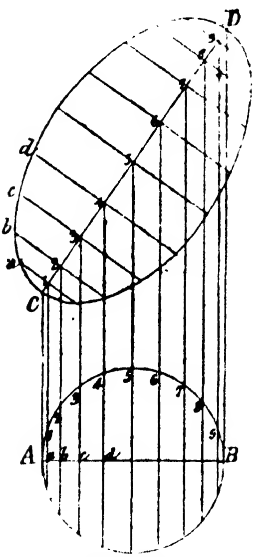


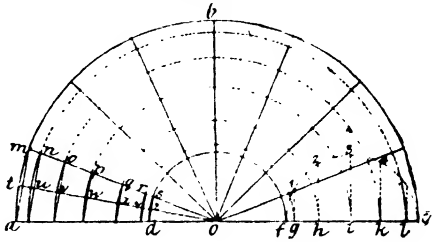
Fig. 214.

NOTE.—This problem will also show that a cylinder being cut obliquely to its axis, the surface so cut becomes an ellipse. See Fig. 125, p. 58.

Problem 31.—**To strike the pattern for a round pipe, to form a semicircle for connection to other pipes.**

Fig. 215 represents the pipe in plan, of outer radius *oa*, and inner radius *od*. Draw the semicircle from *f* to *c* (Fig. 215), which represents the diameter of the pipe, divide it into a convenient number of equal parts, as 1, 2, 3, 4, 5*c*, and draw perpendiculars from these points of division to meet the line *fc*, as from 1 to *g*, 2 to *h*, 3 to *i*, etc. From *g*, *h*, *i*, *k*, and *l*, and from *o* as a centre, draw the semicircles shown. Now divide the curve *abc* into any convenient number of parts, showing the number of transverse sections the pipe will be composed of; take one of those sections as from *a* to *m*, bisect it at *t*, and draw a line from *t* to the centre *o*. Next connect the points of intersection which the semicircles make with the lines *mo* and *ao*, with straight lines *mta*, *nu*, *ov*, etc.

Fig. 215.



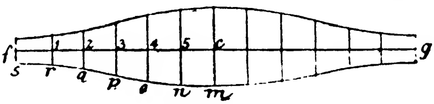


Fig. 216.

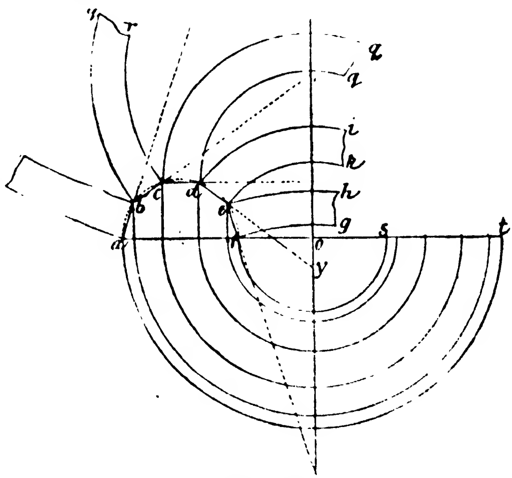


Fig. 217.

Draw the horizontal line *fg* (Fig. 216), and on this line set off as ordinates twice the number of distances that there are on the semicircle *fc* (Fig. 215), namely, 1, 2, 3, 4, 5, *c*, and take the distances *tm*, *un*, *vo*, etc. (Fig. 215), and measure off the same on each side of the centre line *fg* (Fig. 216) from *c* to *m*.

5 to *n*, 4 to *o*, etc. A curve drawn through the points thus obtained will give the pattern for one section.

Fig. 217 also shows a semicircular pipe of outer radius *ot*, and inner radius *of*, but the joints of the various pieces of which it is composed will run in the circumferential instead of in the transverse direction, as in the previous example.

Draw a semicircle from *a* to *f*, showing the diameter of the pipe, divide it into a convenient number of equal parts, as *a*, *b*, *c*, *d*, *e*, *f*, and draw a line through *o* perpendicular to the base of the semicircle. Through the points *ab*, *bc*, *cd*, etc., draw lines to cut this perpendicular. These points of intersection will be used as centres, and the distance between these and the corresponding points on the semicircle *af*, from which they are produced, will give the radii for the various curves of which the pipe is required to be made. Portions of the segments are shown developed, as *fg*, *eh*, for the inner segment; *ek*, *di*, for the second one; *dq*, *cq*, for the middle one; *cr*, *bq*, for the fourth; and the fifth is seen at the extreme left. The centre of *ek* is at *y*, and the location of the positions of the centres of the other radii is also apparent. The lengths of each segment will be obtained by stepping round a definite number of points of division on the plan view, and then transferring these to their corresponding segments.

Problem 32.—**To strike the pattern for a lobster-back cowl.**

Describe the semicircle from E to F (Fig. 218) to any given size, and divide it into a convenient number of parts, as 1, 2, 3, etc.; through those points draw perpendiculars to meet the line *o*B. Taking *o* as centre, describe the curve BHA, also let curves be drawn from all the perpendiculars, 1, 2, 3, etc. Divide the curve from B to A into as many parts as there are sections required, in this case four, as shown at KHGA, draw a line from *o* to *p* through the centre of one of these sections, and draw straight lines to connect the points where

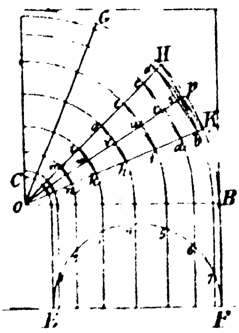


Fig. 218.

the curved lines are intersected by the lines H*o* and K*o*, as *ab*, *cd*, *ef*, *gh*, etc. Mark off twice the number of points that are contained in the semicircle, and at the same distance apart, as ordinates on the line EE (Fig. 219), 1, 2, 3, 4, etc. Measure off from *p* to H and from *p* to K (Fig. 218), and mark off the same distances from E to H and E to K (Fig. 219). Also measure off from *s* to *a* and from *s* to *b* (Fig. 218), and mark off the same

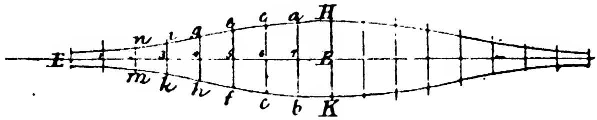


Fig. 219.

distance from 7 to *a* and 7 to *b* (Fig. 219), also the distance from *t* to *c* and *t* to *d* (Fig. 218), and mark off a like distance from 6 to *c* and *c* (Fig. 219), and so on with the remaining distances in Fig 218, which are marked with corresponding letters in Fig. 219. A curve drawn through the points thus obtained will give the development of one section, which, as the four sections are alike, will render further explanation unnecessary.

SECT. IV.—THE SPHERE.

PROBLEM 33.—**To describe the pattern for a globe, formed of twelve pieces joined together.**

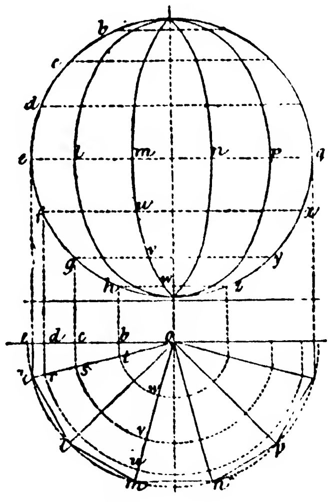


Fig. 220.

Fig. 221.

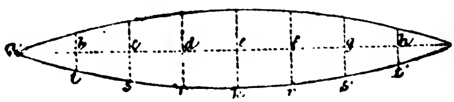


Fig. 222.

In Fig. 220 describe a circle of the required size, and draw the perpendicular *aw*, now divide the half-circle into any number of parts, as *a*, *b*, *c*, *d*, etc., and draw the horizontals *eq*, *fx*, *gy*, etc.; next draw in Fig. 221 half a circle, and divide it into six equal parts, having half a side from *e* to *k* perpendicular with the base; now in

Fig. 222 draw the line *ai*, and mark off the ordinates *a*, *b*, *c*, *d*, *e*, etc., at the same distance apart as the corresponding letters in Fig. 220, also draw perpendiculars in Fig. 220, from the points *h*, *g*, *f*, *e*, etc., to cut the line *ko* (Fig. 221), and let the distances from *b* to *t*, *c* to *s*, *d* to *r*, and so on, be transferred to the corresponding points in Fig. 222, which points *t*, *s*, *r*, etc will give the course of the curve required for the pattern.

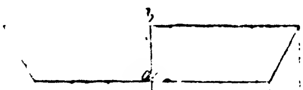
Section V.—Ellipses and Ovals.

Problem 34.—**To strike the pattern for a tapering elliptical article in four pieces or sections** (Fig. 223).

Draw the diameters AB and CD at right angles, then draw the smaller ellipse (as explained in Fig. 91) first to the size required for the bottom, then from the same centres, which in this instance are *a*, *b*, *c*, *d*, describe the outer oval as much larger as is required for the top of the article.

Fig. 223 shows the perpendicular height. Draw AB and DE (Fig. 225) parallel, at the same distance apart as *a*, *b* (Fig. 223)

Fig. 223.



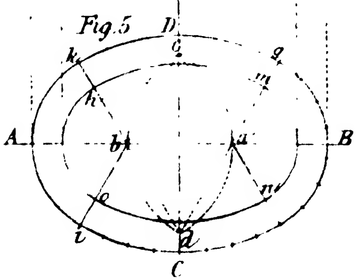


Fig. 224.

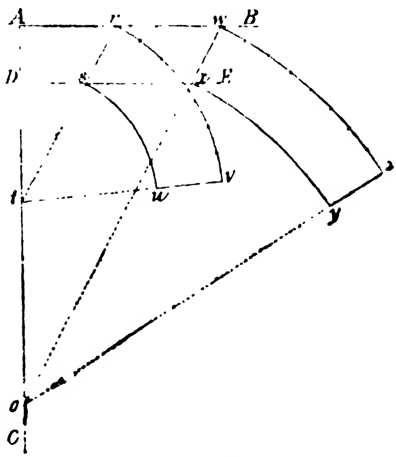


Fig. 225.

being the vertical height, draw AC at right angles with AB and DE, take the radii with which the end section of the oval is struck, and mark them off on Fig. 225, that is, take the radius *a*B (Fig. 224) and set it off from A to *r* (Fig. 225), and the radius *am*, and set it off from D to *s*. Draw a line through *rs* to cut the perpendicular AC at *t*, then with *t* for a centre and *ts* as radius, strike the curve *su;* then with *tr* as radius, draw the curve *rv*. Take the length of the larger end of the curve (Fig. 224) by marking off a convenient number of parts, then take a like distance by marking off a corresponding number of parts, from *r* to *v* (Fig. 225); now draw the line from *v* to the centre *t*, which will give the pattern of the end section *r*, *v*, *u*, *s*.

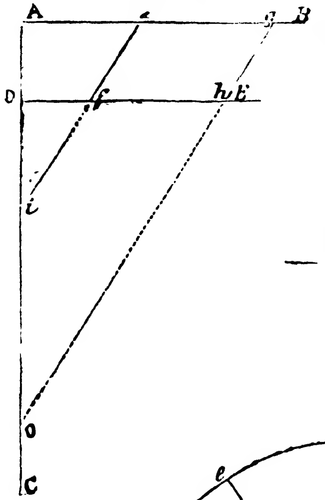
For the pattern of the side take the radius *dk* or *dg* (Fig. 224), and mark off an equal distance from A to *w* (Fig. 225), and take the radius *dh* (Fig. 224), and mark off an equal distance from D to *x* (Fig. 225); draw a line through the points *w*, *x* to cut the perpendicular at *o;* with *o* as a centre, and radius *ox*, strike the curve *xy;* with radius *ow*, strike the curve *wz*, take the length of the curve *kg* (Fig. 224), and take a corresponding length of curve from *w* to *z* (Fig. 225), then draw a line from *z* to the centre *o*, which by adding the usual laps will complete the side.

PROBLEM 35.—**To describe a tapering elliptical body in one piece.**

Draw the two ellipses (Fig. 228), and proceed with Fig. 226 (as in Fig. 225). Draw AB and AC at right angles, and draw DE parallel to AB at the required depth from A to D; from the centre *a* (Fig. 228) take the radius *am*, by which the curve BC is struck, and mark off the distance on Fig. 226 from A to *e*, also take the radius of the smaller curve *an* (Fig. 228), and mark off from D to *f* (Fig. 226); draw *ef* to cut the perpendicular line AC at *i*. Take the radius of the curve of the side from *b* to A (Fig. 228) and mark off that distance from A to *g* (Fig. 226), then take the radius from *b* to E (Fig. 228), being the radius by which the curve EF is struck, and mark it off on the line DE to *h* (Fig. 228), draw a line through the points *g*, *h* to cut the perpendicular line AC at *o*, and this will give the radii for describing the pattern, the development of which will be found in Fig. 229.

To commence, Fig. 229, draw the line *ab*, set the compasses from *i* to *e* (Fig. 226), and on the line *ab* (Fig. 229), taking *c* for a centre, strike the curve *ead*, make the length of the curve *ead* the same as B*m*C (Fig. 228). Draw lines from *d* and *e* through the centre *c*, and extend them indefinitely to *f* and *g*. Take the distance *if* (Fig. 226) for radius, still using *c* (Fig. 229) as a centre, and strike the curve *hi*. Now take the distance *o* to *g* (Fig. 226) with the compasses, and with it from

Fig. 226.



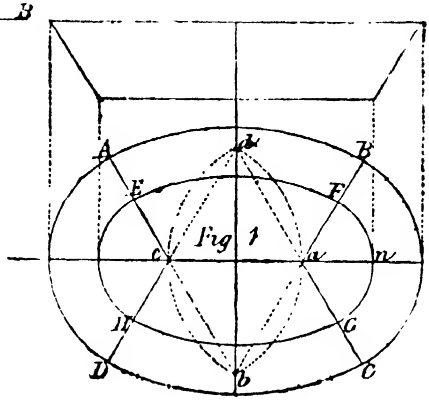


Fig. 227.

Fig. 228.

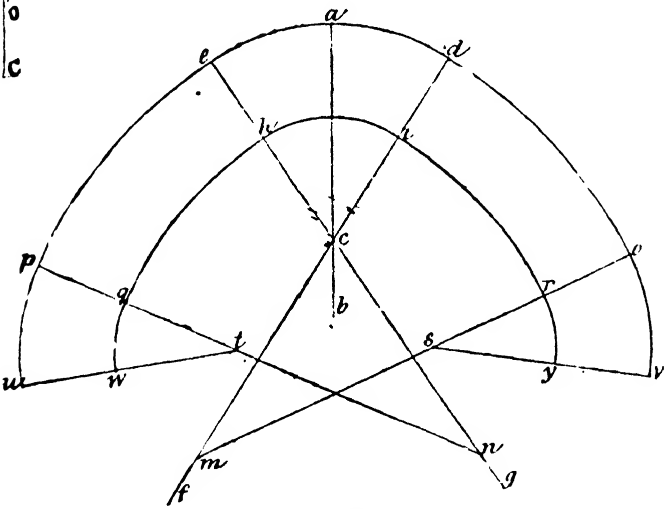


Fig. 229.

e (Fig. 229) mark off the point *n* on the line *eg*, likewise mark off a like distance *d* to *m* on the line *df*; using *n* and *m* as centres, strike the curves *do* and *ep*; then with radius equal to *oh* (Fig. 226), still using *m* and *n* as centres, strike the curves *ir* and *hq*.

Divide out the length of the curve C to D (Fig. 228), and take the same distance from *d* to *o* (Fig. 229), draw a line from *o* to the centre *m*, make the distance from *e* to *p* the same as from *d* to *o*, draw a line from *p* to the centre *n*, mark off the points *t* and *s* on the lines *pn* and *om*, equal to the distance from *a* to *c*; then using *t* and *s* as centres, with radius *tp*, strike the curves *pu* and *ov*; then with radius *tq* strike the curves *qw* and *ry*, take the length of the curve from *a* to *d*, mark off like distances from *o* to *v*, and from *p* to *u*, draw lines from *u* to the centre *t* and from *v* to *s*, which will complete the pattern.

PROBLEM 36.—**Another method of describing an elliptical tapering body.**

Fig. 230 shows the ellipse of the size required for the bottom. Draw the diameters AB and CD at right angles, and describe the ellipse.

Fig. 231 shows the vertical height and flue, or slant, required. To strike the pattern G, which is the end section, *d* being the centre from which the curve *mn* is struck, draw a line at *de* at right angles with AB. Produce or extend the curve *mn* to cut the line *de* at *e*, draw the line *eg* at right angles with *de*, make *eg* equal to the vertical height *ab* in Fig. 231. Draw *gh* at right angles with *ge*; *cb* in Fig. 231 shows the flue required, mark off that distance from *g* to *h*. Draw a line through the points *he* to cut the line AB at *i*, taking *i* for a centre on the line AB, with the radius *ih* draw the curve *hf*, and with radius *ie* draw the curve *ek*. Measure off the length of curve *mn* from *e* to *k*, and draw a line through *ik* to *f*.

Proceed with the side section H in the same manner. Extend the line CD indefinitely. D being the centre from which the curve *nl* is struck, draw a line D*o* at right angles with CD; extend the curve *nl* as dotted, until it cuts the line D*o*; draw a line *op* at right angles with D*o*, make *op* equal to the vertical height *a* to *b* (Fig. 231). Draw *pr* at right angles with *po*, make *pr* equal to the required flue *bc* (Fig. 231), draw a line through *ro* to cut the extended line CD at *s*; with *s* for centre, and radius *sr*, draw the curve *rt*, and with radius *so* strike the curve *ou*. Measure the length from *m* to *q*, taking a like distance from *o* to *u*; now draw a line through *su* to *t*, which completes the pattern. This pattern will, after being well studied, be found an excellent introduction to Fig. 232; it is a different method from that described in Figs. 223 to 225, and in other diagrams, but the result will on practice be found

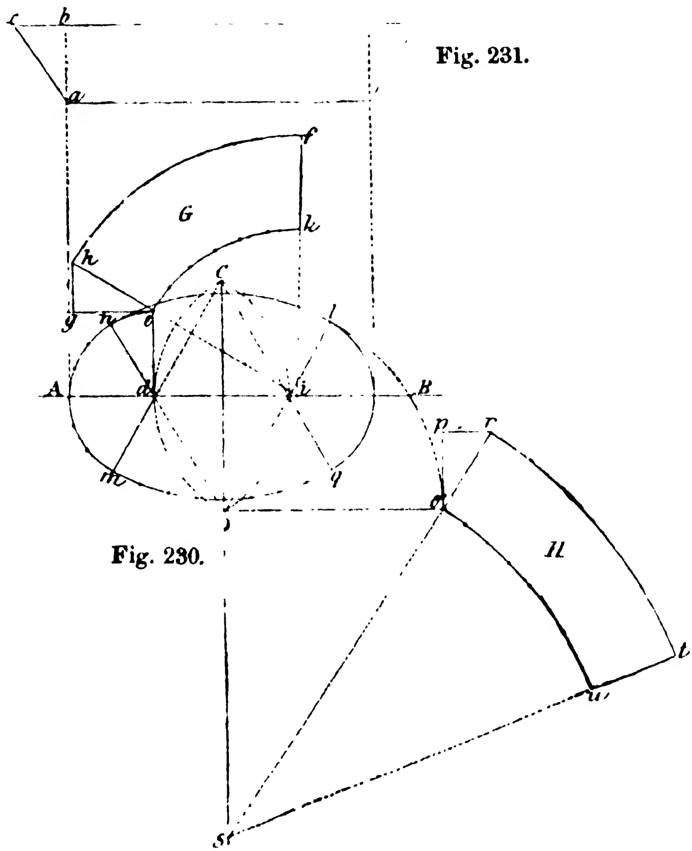


Fig. 231.

Fig. 230.

precisely the same. In these and the foregoing figures the tapering must be equal on all sides.

Problem 37.—**To describe a tapering elliptical body, where the tapering is not equal on all sides.**

(*In this case more tapering at the ends than at the sides.*)

Fig. 232 shows the diameter of two distinct ellipses, each one being described from a separate set of centres. To proceed with the larger or outer ellipse, which (as well as the smaller or inner one) is constructed in the same manner as described in Fig. 91, p. 40, take the required diameters, as AB and CD, the centres being EF and GH. The smaller ellipse will have to be constructed in the centre of the larger one, according to the given length and width required for the bottom of the article, the centres by which this ellipse is struck being *ab* and *cd*. Fig. 233, from *a* to *b*, shows the vertical height.

Fig. 234 shows a pattern of the side, which is obtained in the following manner. H being the centre from which the curve KL (Fig. 232) is struck, draw the line HR at right angles with the perpendicular line GH, and extend the curve KL as dotted, to meet the line HR. Draw the line RS perpendicular with the line HR, mark off the depth from R to S, equal to the vertical height *ab* (Fig. 233). Draw ST at right angles with RS, take the distance at D between the two ellipses on the line CD (the width between them being the flue of the sides), mark off the same distance from S to T, draw a line through the points TR, and extend it as shown by the dotted line to cut the extended perpendicular line CD at *u*; with radius *u* R, strike the curve *m*H*m* (Fig. 234) (the centre of this curve is not shown, lying outside our drawing). Next, *c* being the centre from which the curve *fg* (the side of the smaller ellipse) is struck, draw the line *cl* at right angles with the perpendicular line CD, extending the curve *fg* with the same radius to meet the line *cl*. Draw a line *lm* at right angles with *cl*, again marking the distance *ab* (Fig. 233) from *l* to *m* (Fig. 232). Draw a line *mn* at right angles with *lm*. Take the distance again between the two ellipses at the side C, and mark off that distance on the line *mn*, draw a line through the points *n* and *l* to cut the perpendicular line CD prolonged, at *o*. Take the distance from *l* to *n*, or from R to T, and measure off a like distance from H to *g* (Fig. 234), being the slant height of the body at the centre of the side. Take the distance from *o* to *l*, and from *g* with that dimension, *o* to *l*, mark off the point *w* below (Fig. 235); with *w* as centre, and radius *ol*, strike the curve *hgk*. Extend the line *cbg* (Fig. 232), which shows the division of the smaller ellipse, to *k* on the curve of the larger one; the point L is the right sectional line of the larger ellipse, while the extended line, *cbg* to *k*, is the sectional line of the inner one; let the distance between *k* and L be equally divided as at *i*, draw a line from *i*

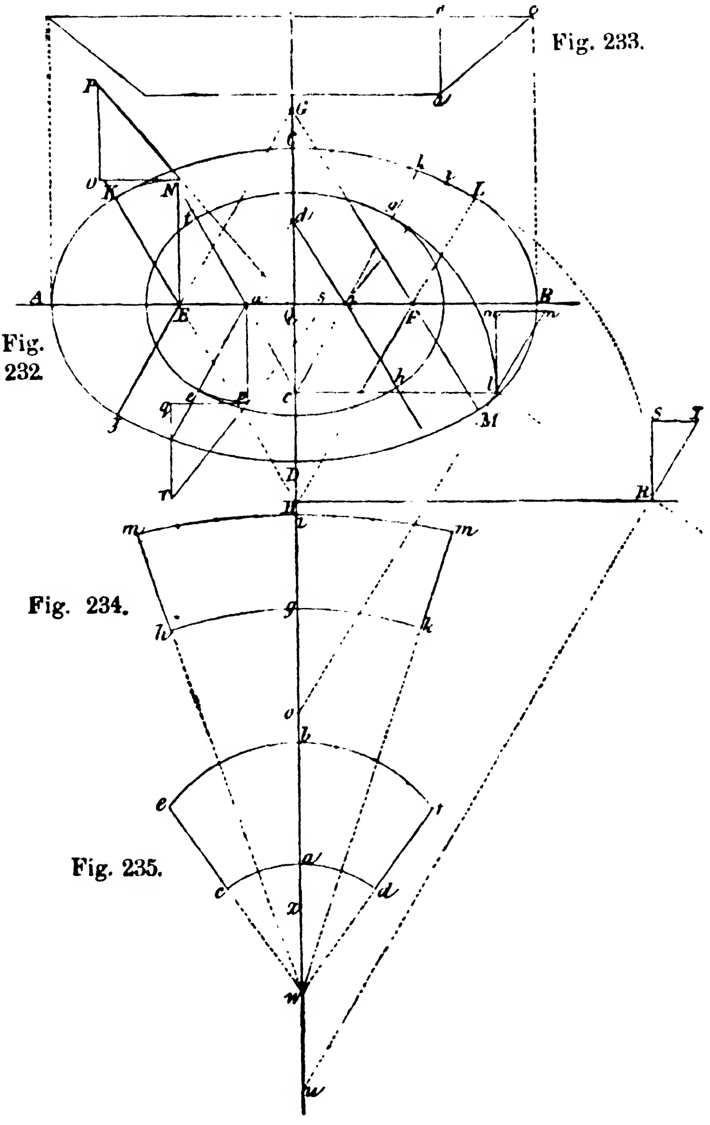


Fig. 233.

Fig. 232

Fig. 234.

Fig. 235.

to *b*. the latter point being the centre from which the end of the smaller ellipse is struck, take the length of the curve from *i* to

C, measure off a like distance from H to *m* on each side of the perpendicular line (Fig. 234), and draw lines from *m* to the point *w* (Fig. 235), being the centre from which the bottom curve *ghk* is struck; this will complete the pattern *mh*, *mk*, for the side.

Fig. 235 shows a pattern of the end, which is obtained as follows: E (Fig. 232) being the centre from which the end JAK of the large ellipse is struck, draw a line from E to N at right angles with the diameter AB, extend the curve JAK, to cut the line EN at N. Draw a line from N to *o* at right angles with NE, make N*o* equal to *ab* (Fig. 233), the vertical height. Draw *o*P at right angles with *o*N, take the distance between the two ellipses at the end on the line AB, being the flue of the end, and mark off a corresponding distance from O to P. Draw a line through PN to cut the diameter AB at Q. With radius QN, and *x* (Fig. 235) as a centre, describe the curve *ebf*. Take *a* (Fig. 232) (being the centre from which the end of the smaller ellipse, *fe*, is struck) and draw *ap* at right angles with the centre line AB, extending the curve *fe* to cut the line *ap* at *p*, draw *pq* at right angles with *ap*, again taking the vertical height *ab* (Fig. 233), and marking off a like distance from *p* to *q*. Draw *qr* at right angles with *gp*, take the distance from A to the end of the smaller ellipse, being the flue of the end, and mark off the same from *q* to *r*. Draw a line through the points *r*, *p* to cut the centre line at *s*, take the distance from *r* to *p*, or from P to N, which will give the slanting depth at the centre of the end, mark off the same distance from *b* to *a* (Fig. 235), take the radius from *s* to *p* (Fig. 232), and from *a* (Fig. 235), with that radius mark off the point *w*. With *w* as a centre, and with radius *sp*, strike the curve *cad*; take the length of the curve from B to *i* (Fig. 232) and mark off a corresponding distance on Fig. 235 from *b* to *f*, and from *b* to *e*, draw lines from *f* and *e* to the centre *w*, being the centre from which the curve of the bottom is struck, which will complete the pattern *ec*, *fd* for the end.

PROBLEM 38.—**To describe the pattern of an egg-shaped or oval tapering body** (Fig. 236).

Draw AB and CD at right angles, and from E, with radius

EH, draw a circle cutting the line CD at F; from G and H draw lines through F, and produce them indefinitely, and G, H, and F will be the centres from which to strike the remainder of the figure; then from the same centres draw the larger oval as much larger as the flue requires.

In Fig. 237 draw AB and ED of the required depth apart, and BC at right angles therewith; mark off B*a* and D*e* equal to EC and EJ (Fig. 236). Draw a line through *ae* to cut the perpendicular line at *h*. Take the distance HA (Fig. 236), and

Fig. 236.

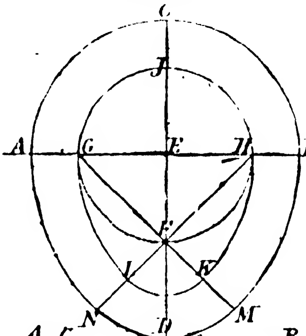
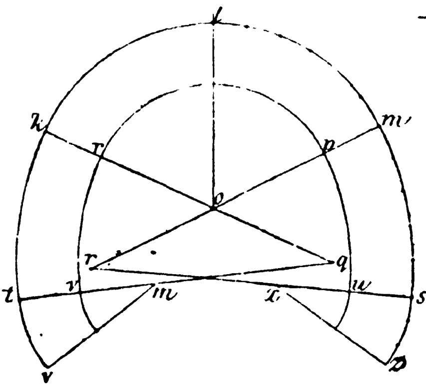


Fig. 238.



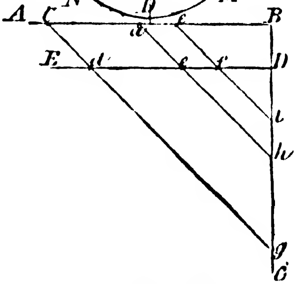


Fig. 237.

mark it off from B to C on the line BA in Fig. 237. Take HG (Fig. 236) and mark it off from D to *d* on the line DE (Fig. 237), and produce C*d* to *g*. Take the radii FM and FK, in Fig. 236, and mark them off from B to *e* and from D to *f* in Fig. 237, and produce *ef* to *i*

In Fig. 238 draw the line *lo*. With radii *ha* and *he* (Fig. 237), using *o* (Fig. 238) as a centre, strike the curves *klm* and *rp*. Take the length of curve from A to C (Fig. 236) and dot off a like distance from *l* to *m* and from *l* to *k* (Fig. 238). Draw lines from *m* and *k* through the centre *o*, and produce them indefinitely; take the radius g to C (Fig. 237), and with that radius from k mark off q (Fig. 238), and from m mark off r; take q and r as centres, and radius rm, and draw curves ms and kt; make ms and kt of the same length as AN and BM, in Fig 236; draw lines from t to the centre q, and also from s to r; with radius gd, in Fig. 237, draw the curves pu and rv in Fig. 238. Take the radius ie, in Fig. 237, and from t and s, in Fig. 238, mark off m and x for centres, and thence strike the curves ty and sz; make sz of the same length as ND in Fig. 236, and draw lines from z to the centre x, and from y to m. With radius if, in Fig. 237, describe the curves from u and v, which will complete the pattern.

PROBLEM 39.—**To describe the pattern for a travelling sitz bath.**

Fig. 239 represents the plan of the top and bottom required, and Fig. 240 the elevation. (It will be seen that the tapering is as much in the front as at the back, but the back being much higher than the front, the tapering is not equal in proportion to the depth; this, however, may be governed according to dimensions required.) Let the horizontal line gd (Fig. 241) be drawn to represent the required length of the bottom, as shown by the dotted lines brought down from the plan (Fig. 239), and let the line AB represent the slanting length of the top, as shown by its meeting the perpendiculars drawn from the top of the plan (Fig. 239); now draw lines Ag and Bd, and produce them to meet at the apex E, and draw a vertical line from E to cut the line AB (Fig. 239) at F, which will be used as a working centre.

Let one-half the ellipse be divided into any number of parts, as 1, 2, 3, 4, 5, 6, 7, A, and using F as a centre, describe the arcs therefrom cutting the diameter line AB, at f, e, d, c, b, a; and from these points draw perpendiculars, as shown by the dotted lines. It will now be observed that the radius from F to a and from F to 7 will be (in this case) the same as from F to 6, by which the arc from 6 to a is drawn, which will prevent separate arcs being described from points 7 and a, as occurs from all the other points; therefore a perpendicular from the point 6 must be drawn to cut the line AB (Fig. 241) at i, and from this point a line must be drawn to s, parallel with gd (to

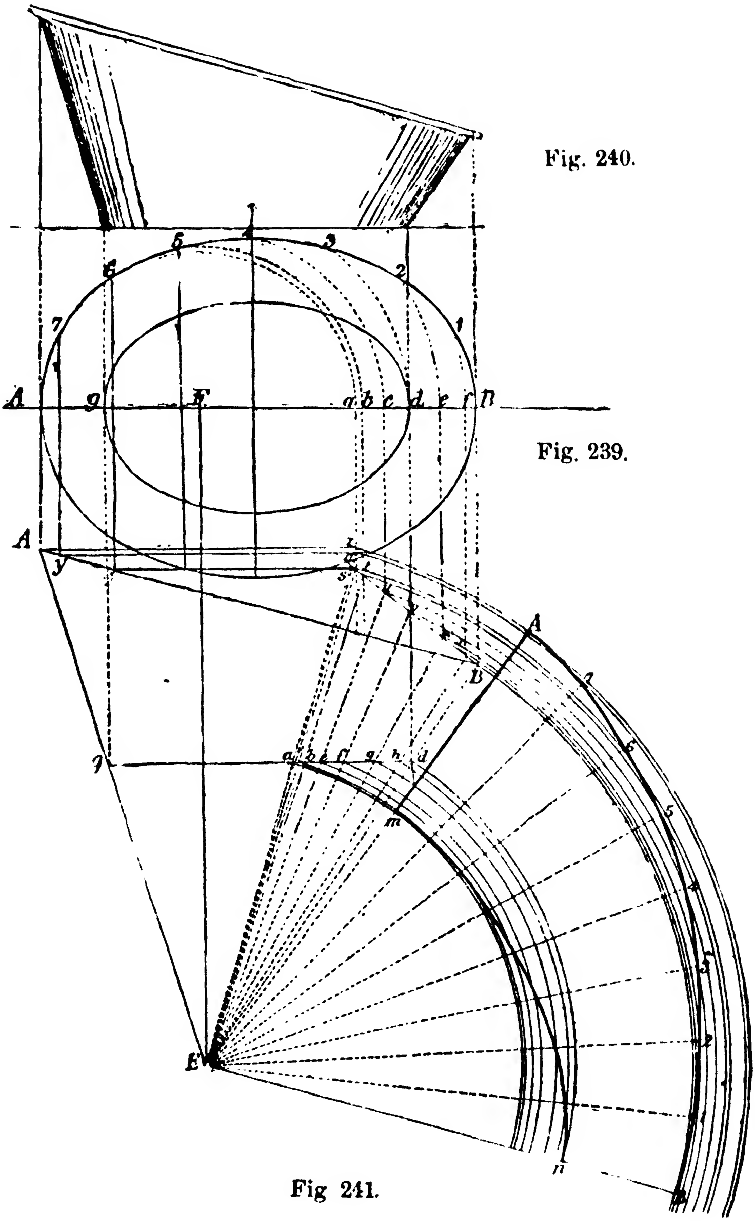


Fig. 240.

Fig. 239.

Fig 241.

cut the perpendicular from a), and from the point s, a line must be drawn to B, which will give the various heights (where intersected by the perpendiculars drawn from b, c, d, e, f) of the bath from B to the point 6 (Fig. 239). Now, from point 7 (Fig. 239), draw a perpendicular to cut the line AB (Fig. 241) at y, and a perpendicular from A (Fig. 239) to A (Fig. 241); then, from y and A draw lines parallel to gd, to cut the perpendicular dotted line from a (Fig. 239), at o and z, which will give the two remaining points which were wanting in the process of describing the arcs. From all the points, z, o, s, t, u, v, w, etc., describe arcs of indefinite length; then draw lines from all these points to the apex E, and the points where the horizontal line gd (Fig. 241) is cut by these radial lines, as at d, h, g, f, etc., will be the points from which another set of arcs concentric with the first will be drawn. Take the same compasses with which the half-ellipse in Fig. 239 was divided out into equal parts, and measure off the same number of distances B, 1, 2, 3, etc., in the development (Fig. 241), stepping from the first arc into the second arc, thence into the third, and so on; and draw lines from all those points to the apex E, which will give the points of intersection on the smaller set of arcs. Now drawing the curves through these points of intersection from m to n, and from the point A through 7, 6, 5, etc., to B, will give one-half of the pattern required.

PROBLEM 40.—**To describe the pattern for a hip bath.**

Fig. 242 shows the plan, illustrating the relative positions of the bottom and of the top. Let distance between the lines QR and ST below represent the perpendicular height from M to N (Fig. 243), the lines QR and ST being both drawn parallel to the diameter line AB (Fig. 242).

Draw perpendicular lines from the extreme points B and A in the plan, to k and g on the line QR, which will represent the top of the bath; also perpendiculars from a to h, and from n to i, on the line ST, showing the position and length of the bottom. The smaller oval need not necessarily be drawn, only the points a and n marked off, showing the position of the bottom or the required slant of the toe and back, as the shape of the bottom will be found in the development to come in proportion.

Draw lines through the points gh and ki, and produce them

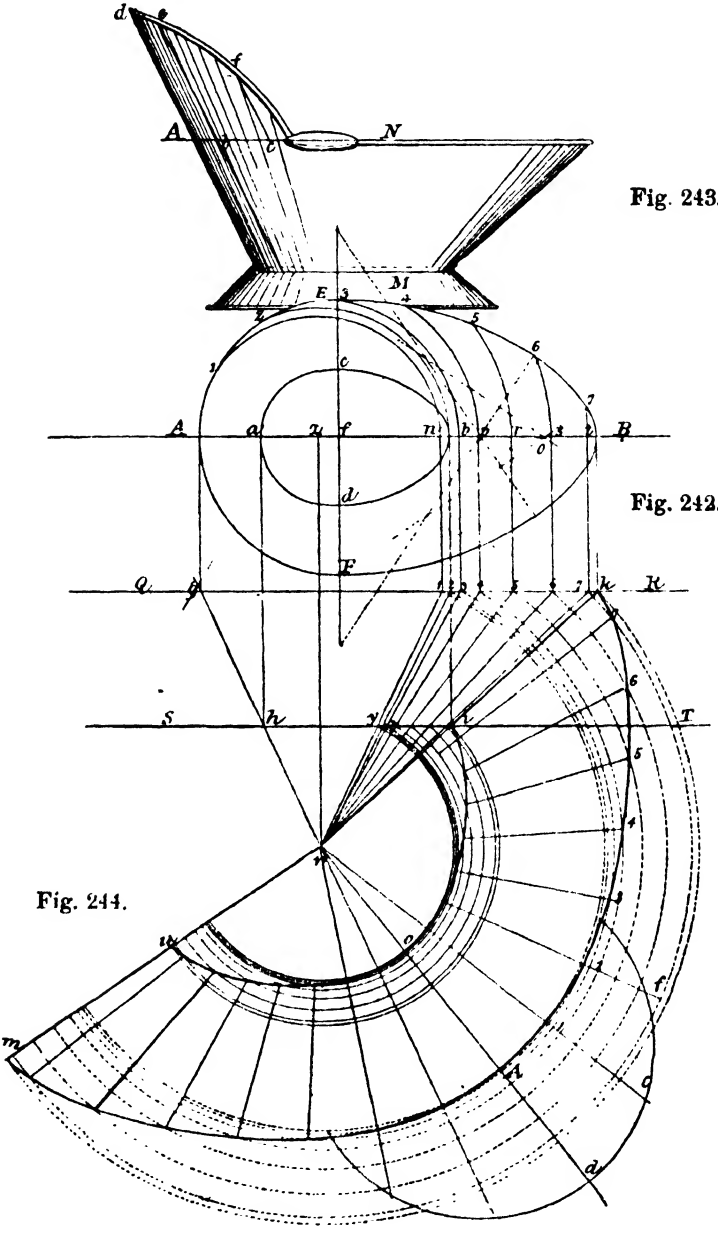


Fig. 243.

Fig. 242.

Fig. 244.

to meet at w (Fig. 244); now from w draw a perpendicular line to cut the diameter AB at x in the plan, which is to be used as a working centre; next let one-half the plan, from A to B, going round the oval, be divided into any number of parts as 1, 2, 3, 4, 5, 6, 7, and from these points, using x as a centre, describe arcs cutting the diameter line AB, at t, s, r, etc., and draw perpendiculars therefrom to cut the line QR, as indicated by figures corresponding with those marked round the curve. Now, by using w as centre, and describing the various portions of circles, as shown, starting from 1, 2, 3, 4, 5, 6, 7, and k, on the line QR, likewise drawing portions of circles starting from the points from y to i on the line ST, and taking the same distances from k to A, and from A to m (on the arcs k, 7, 6, 5, etc.) as those round the plan from B to A (in Fig. 242); and by drawing lines from 7, 6, 5, 4, etc., leading towards the centre w, all the points of intersection will be obtained, from which the curves may be drawn (by freehand) to give the development of the pattern in one piece.

To obtain the shape of the back, draw the curve *def*, in Fig. 243, as required, and mark off points b and c, perpendicular to 1 and 2 in Fig. 242, and draw *be* and *cf* parallel with A*d*; now transfer the lengths of A*d*, *be*, and *cf*, in Fig. 243 from A to d, 1 to c, and 2 to f, in Fig. 244, which will give the course of the curve of the back to correspond with the same in Fig. 243.

Section VI. Mixed Figures.

Problem 41.—**To describe the pattern for a cone with an elliptic base.**

In Fig. 245 let AB represent the major diameter, and DD the minor, and let E (the centre of the base) to F (the apex) represent the vertical height. Now draw half of the ellipse, from A to B, and divide it round into a convenient number of equal parts, as 1, 2, 3, 4, 5, 6; and from these points, using E as centre, describe arcs to cut the base line AB. Now taking F for a centre, with radius FB, draw the curve BA in Fig. 245, and so from all the points from B to D, in Fig. 245, describe the curves in Fig. 246 to A. With the same compasses set for the divisions 1, 2, 3, 4, 5, 6, in Fig. 245, mark off twice that number in Fig. 245, but in measuring off the distances with the compasses in Fig. 245, commencing on the outer curve, from each point step into the next curve, as shown by the figures 1, 2, 3, 4, 5, 6, and then from 6 step back in like manner from the inner to the outer curves to A; lines are drawn from the points thus obtained to the centre F. A curve drawn through the points B, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, A will complete the pattern; the lengths of the lines drawn from FA, F1, F2, F3, etc. (Fig. 246), will be equal to the lengths FA, Fb, Fc, Fd, etc. (Fig. 245).

Fig. 246.

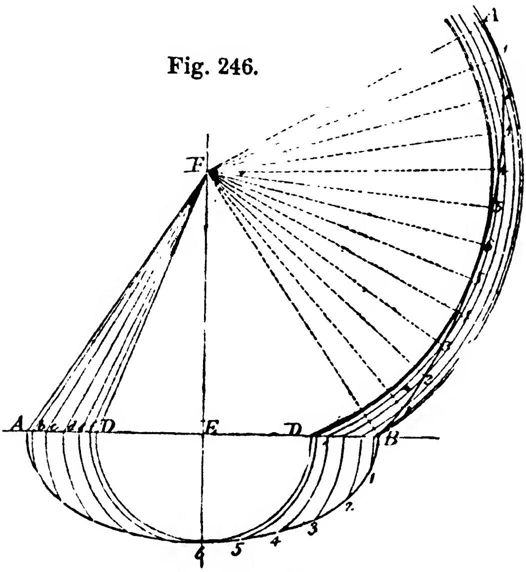


Fig. 245.

PROBLEM 42.—**To describe the pattern for a cone and cylinder, to intersect or meet at right angles with their axes.**

Fig. 251 is a perspective representation of a cone having a cylinder attached to it at right angles with its axis. The patterns which have to be described are first, the development of the cone, second the curve of the cylinder, to fit when placed against the cone, and third, the shape required for the hole or aperture in the cone to receive the cylinder (or to meet edge to edge for joining). Fig. 247, CB shows the base, and DA the elevation of the cone, AC and AB are the sides. From the centre D strike the semicircle CEB. Now to develop the pattern of the cone, take A as a centre, and with radius AB, strike the curve BCB (fig. 250), find the length of the curve BEC (Fig. 247), and set off twice that length on BCB in Fig. 250.

The next problem is to find the curve which will be generated by the intersection of the cylinder (which is a round body of parallel diameter) with the cone (which is a round body of ever-decreasing diameter). The lines *an* and *bu* (Fig. 248) represent the lengths of the cylinder at top and bottom. Draw the semicircle, and divide it into equal parts *d*, *e*, *c*, *f*, *g*, *b*, and from these points draw lines parallel to *an* to cut the line AB, and from these points of intersection with AB, *n*, *v*, *w*, *x*, *y*, *z*, *p* draw perpendicular lines as dotted to cut the base line BC. Next from C draw the perpendicular line C*b*, and take half the length of the line from *u* to *n* (Fig. 248), which will be where it is intersected by the line *cx*, and transfer the same from C to *a* (Fig. 247). Take the radius of the semicircle *k* to *c*, and mark off the same from C to *b*, and describe the curve *ab*. Divide it into half as many parts as the semicircle has been divided into (but the distances will not be the same), and from these points of division draw the lines *eg*, *fh*, and *bi* parallel to CD. Now, using D as a centre, strike the curves from the dotted lines drawn from the points *p*, *z*, *y*, etc., to meet the horizontal lines just drawn in rotation, as follows: join the perpendicular drawn from *p* on the line AB with a curve, and extend it to *p* on the line *a* CD, and join *p* to *u*. Next, from the perpendicular brought down from *z*, draw a curve to meet the line *eg* at *z*, and raise a perpendicular from *z* to *t* cutting the line *gz*, and from the line brought down from *y* strike the curve to meet the line *fh* at *y*, and raise a perpendicular to cut the line *fy* at *s*, and from the line *x* strike the curve *x* to meet the line *bi*, and raise the perpendicular to meet the line *cx* as shown at *r*. The other curves, *w*, *v*, and *n*, follow back in the same manner on the lines *fh*, *eg*, and CD, producing the points *q*, *o*, and *n* (Fig. 248). The points *u*, *t*, *s*, *r*, *q*, *o*, *n* will show the course of the curve sought.

By drawing the horizontal lines *b*, *g*, *f*, *c*, *e*, *d*, *a* (Fig. 249) at the same distance apart as the distances indicated by the corresponding letters on the semicircle (Fig. 248), and by producing the perpendiculars from *n*, *o*, *q*, *r*, *s*, *t*, *u*, prolonged from Fig. 248, to intersect the horizontal lines drawn in Fig. 249, the points of intersection will show the course of the curve for the pattern of the cylinder.

Now, to find the shape of the aperture in the pattern of the cone. First draw the line AC (Fig. 250), and taking A as a

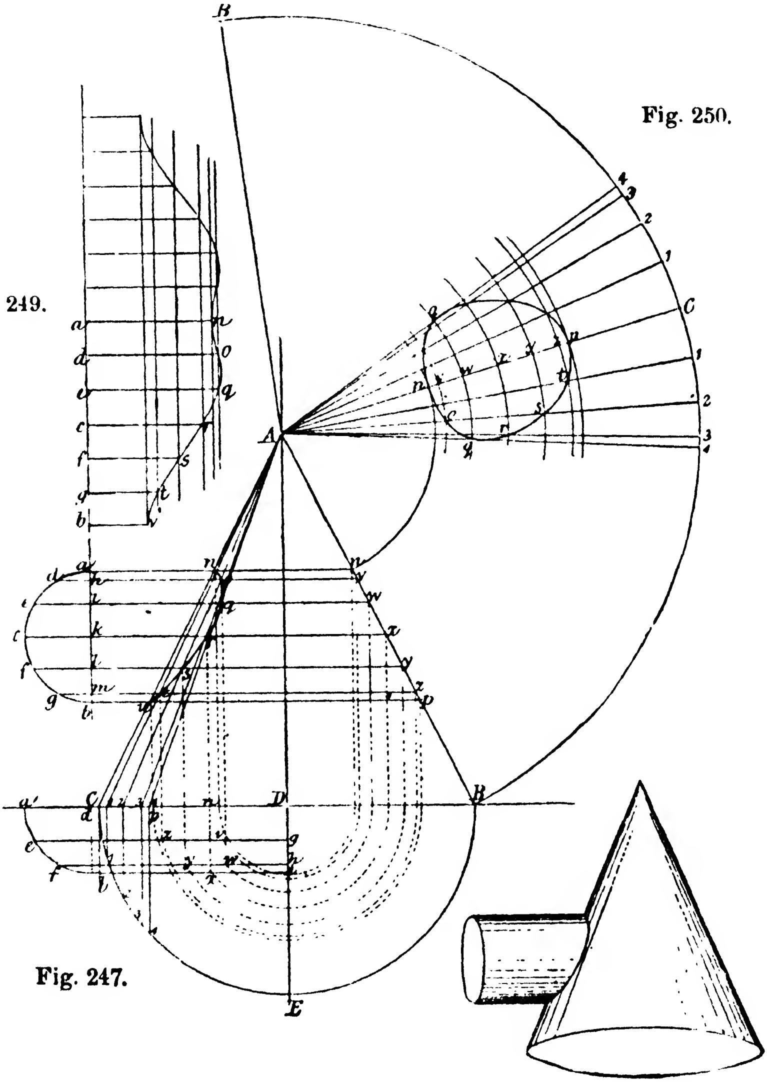


Fig. 249.

Fig. 248.

Fig. 250.

Fig. 247.

Fig. 251.

centre, with radii A*n*, A*v*, A*w*, etc., produced from Fig. 247, describe the arcs *n*, *v*, *w*, *x*, etc. Now, from A (Fig. 247) draw lines through the points *t*, *s*, *r*, *q*, *o*, to cut the line CD at 1, 2, 3, 4, and from these points draw the perpendiculars to cut the curve CE at 1, 2, 3, 4. Take the distances C1, C2, C3, and C4, on the curve CE, and from C (Fig. 250) mark off the same distances as shown by corresponding figures. Draw lines from these figures to A, the points of intersection with *n*, *v*, *w*, *x*, *y*, *z*, *p* will give the course of the curve. The line drawn from A through *o* also cuts through the point *s*, being line 2; so, observe that in Fig. 250 the points *o* and *s* are the required points for the curve on the same line 248.

PROBLEM 43.—**To describe the pattern for a tapering article, elliptical at the base and round at the top.**

(*Such as an elliptical canister top, having a round hole for the neck and cover.*)

Fig. 252 represents the plan and elevation of the top required. Take AB and CD, being the given diameters. Draw a diagonal line from the points *a* and *b*, being the intersecting points of the curves; make the line *ab* (Fig. 253) equal to *ab* (Fig. 252). Now take the distance from F to E (Fig. 252) being the vertical height of the article, and mark off a like distance from F to E (Fig. 253), draw *cd* parallel to *ab*, make *cd* (Fig. 253) equal in length to the diameter of the circle from *c* to *d* (Fig. 252). Draw lines through *bd* and *ac*, and prolong to cut the perpendicular line at *g*. With radius *gb*, from centre *i* strike the circle ABc (Fig. 255), or boundary line. In Fig. 252, *v* being the centre from which the curve *a*C*f* is struck, draw the line *vw* at right angles with CD, and extend the curve *a*C*f* as dotted to cut the line *vw*. Draw *wx* at right angles with *vw*, take the vertical height EF and mark off an equal distance from *w* to *x*, draw *xy* at right angles with *wx*, take the distance from D to *r*, being the flue of the side, and mark off a like distance from *x* to *y*, draw a line through *y* and *w*, and prolong to cut the perpendicular CD at *z*. Draw a perpendicular line AB (Fig. 254); taking B as centre, with radius *zw* (Fig. 252), strike the curve *f*A*g* (Fig. 254). Take the length of the curve from C to *f* (Fig. 252), and set off a like distance from A to *g* and from A to *f* (Fig. 254). Draw lines from *f* and *g* to the centre B, and draw the line *fg*, which will give one section of the base of the pattern.

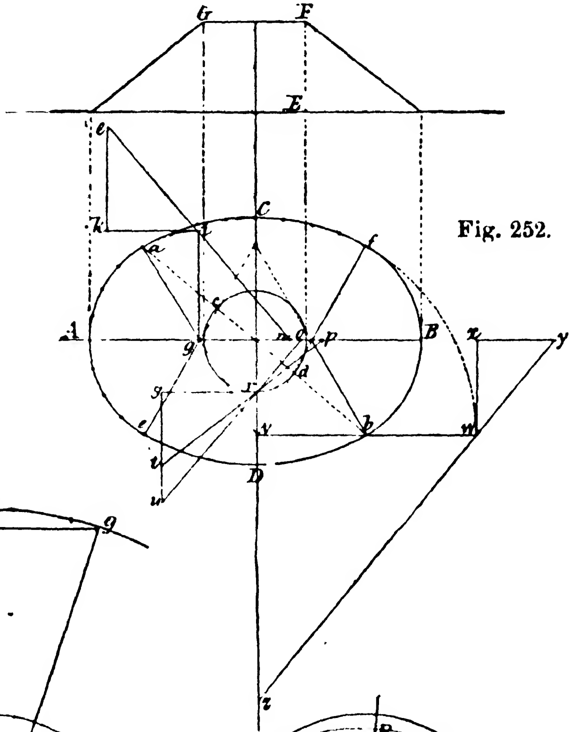


Fig. 252.

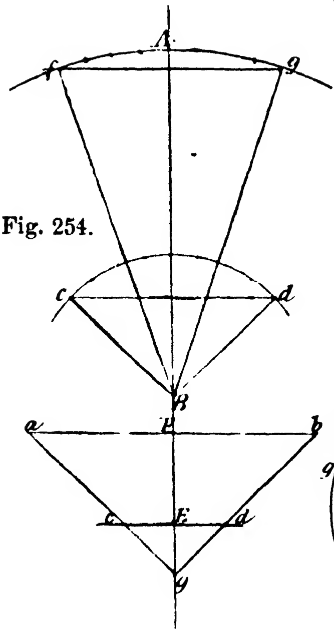


Fig. 254.

Fig. 253.

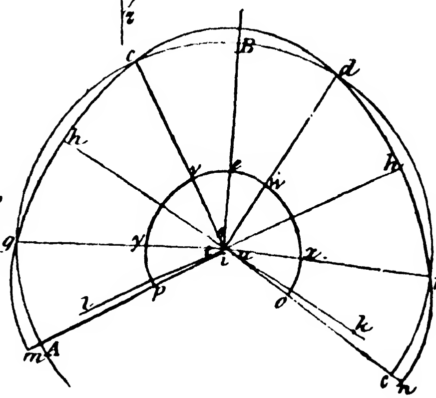


Fig. 255.

The curve of the end of the base being struck from *g* (Fig. 252) as centre, draw a line from *g* at right angles with

AB, and extend the curve *ea* to cut the line produced from *g* to *i*. Draw the line *ik* at right angles with *ig*, making *ik* equal to the vertical height EF. Draw *ke* at right angles with *ik*, make *k* to *e* equal the flue of the end, that is from A to *g*, being the distance from the extreme curve of the ellipse to the edge of the circle on the line AB.

Draw a line through the points *e* and *i* to cut the centre line AB at *n*; with radius *ni* from B (Fig. 254) as centre, strike the curve *cd*, take the length of the end curve from *e* to *a* (Fig. 252), and mark off a like distance from *c* to *d* (Fig. 254), draw the chord line *cd*, which will give the end section of the base.

Take the distance from *c* to *d* (Fig. 254) and mark off an equal distance on the circle AB*c* (Fig. 255), that is, from *c* to *d*, and draw lines from the points *c* and *d* to the centre *i*, bisect these lines by the perpendicular B*i*, take the distance from either B to *d* (Fig. 254) or *n* to *i* (Fig. 252) as radius, and from *c* or *d* (Fig. 255) mark off point *e* on the line B*i* with *e* as centre, strike the curve *cd*. Take the distance from *f* to *g* (Fig. 254) and mark off a like distance on the circle or boundary line from *c* to *g* (Fig. 255) and *d* to *f*. Draw lines from *g* to the centre i, and also from *f* to *i*; bisect the distances from *g* to *c*, and from *d* to *f* at *hh*, extend these lines to *k* and *l*. Take the distance from *z* to *w* as radius (Fig. 252) and from *d* (Fig. 255) mark the point *l* on the line *hl*, and from *g* mark off the point *k*. Using *l* and *k* alternately as centres, strike the curves *df* and *cg*. Take the distance from *d* to B, and mark off from *g* to A and *f* to *c*, and draw lines from the centre *i* to A and *c*. Again, using *ni* (Fig. 252) as radius, strike the curve *fn* (Fig. 255) from a centre on the line *ni*, also from a centre found on the line *mi* strike the curve *gm*, which will complete the curve for the base.

Now, to describe the curve for the circular hole in the top—the line CD (Fig. 252) being drawn through the centre by which the circle is struck, from point *r* draw *rs* at right angles with the diameter line CD, take the distance EF, being the vertical height required, and mark off the same distance from *r* to *s*, draw the line *s* to *u* at right angles with *rs*, take the distance from D to *r*, and mark off from *s* to *t*, being the slant of the side. Draw a line through the points *tr* to cut the diameter line AB at *p*, then take the distance from the ellipse to the circle, that is, from A to *g*, being the slant of the end, and mark off a like distance from *s* to *u*, draw a line through *u* and *r* to *o* on the line AB, take the distance from *u* to *r* or from *e* to *i*, which should be the same, being the slanting depth of the end, and mark off like distances from *n* to *o* (Fig. 255), from *m* to *p*, and from B to *e* (the outer curve). Take the distance *or* (Fig. 252) as radius, making *s* (Fig. 255) the centre on the line B*e* prolonged, and strike the curve *v*, *e*, *w*, through the point *e* : with the same radius strike the curve *ox*, with *u* as centre on the line *ci*; also with *t* as centre on the line *mi*, strike the curve *py*; with radius *pr* (Fig. 252) strike the curves *yv* (Fig. 255) and *wx*, with centres found on the lines *hk* and *hl*, which will complete the pattern required.

Problem 44.—**To strike the pattern of an oblong tapering pan in two parts or sections.**

In Fig. 256 draw lines EC and FD, at the distance apart required for the straight portions of the ends. Draw HJ and G*k*, at the distance apart required for the straight portions of the sides. Take the points *a*, *b*, *c*, *d*, for centres, and then draw curves for the corners, as at GC and *ef*, etc., at each of the corners, then draw straight lines to meet the curves as at HG, CD, *fg*, and so on, which will give the size of that article both top and bottom. The line AB shows where the two halves meet for joining together. In Fig. 257, draw *ab* and *ac* at right angles, take the required depth from *a* to *d*, equal to the vertical depth of the pan, draw *de* parallel to *ab*, then take the radius from *a* to G, in Fig. 256, and mark off the same distance from *a* to *b* in Fig. 257; also take the radius of the small curve from *a* to *e*, in Fig. 256, and mark off the same distance from *d* to *e* (Fig. 257), draw the line through *b* and *e* to cut the line *ac*; from *e* to *b* will give the slanting depth.

Fig. 258 is the development of the pattern. Make the straight part from *i* to *l* equal GH, in Fig. 256, the depth from *i* to *n* and *l* to *o* to equal *be* (Fig. 257), take the distance *bc* (Fig. 257) and mark it off from *i* to *k* and *l* to *m* (Fig. 258), take *k* and *m* as centres, and with radius *cb* (Fig. 257) strike the curves *iq* and *lp*, divide out the length of the curve from C to G

(Fig. 256), and dot off the same distance from *l* to *p* (Fig. 258) make *iq* equal *lp*, draw the lines *pm* and *qk*; with radius *ce* (Fig. 257), still using *m* and *k* as centres, strike the curves *os* and *nr*. Draw lines *pu* and *st* at right angles with *pm*, making *pu* equal to CB (Fig. 256), draw *ut* parallel to *ps*, finish the opposite end N in like manner, adding the lap *w* in both instances as required. Where wiring or edging is required, add on accordingly.

Fig. 259 shows two circles divided into four equal parts, A,B,C,D, equal to the four corners of Fig. 256; with a little calculation the pattern may be obtained without going through

Fig. 259. Fig. 258.

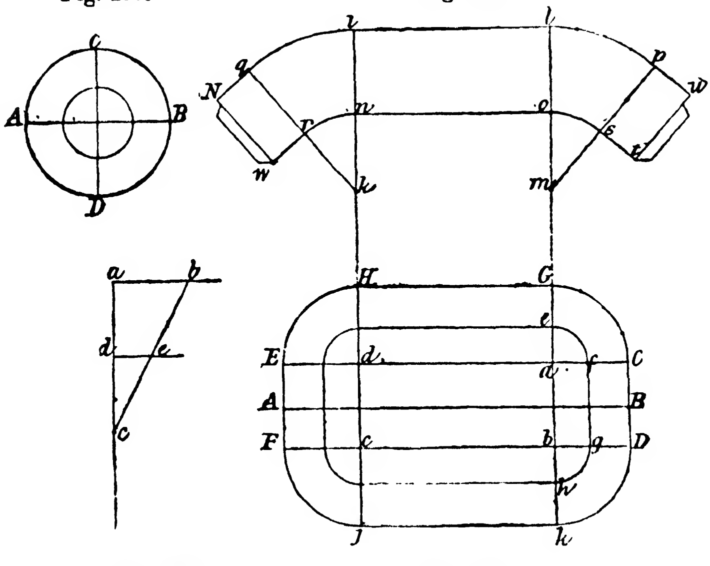


Fig. 257. Fig. 256.

the process of again constructing Fig. 256. To illustrate this, take an article, say ten inches from A to B (Fig. 256) and seven inches from H to *j*, the corner EH to be the quadrant of a 4-inch circle, as from A to C (Fig. 259). The diameter AB being four inches, subtracting four inches from seven inches would leave three inches straight at end, as from F to E.

Again, subtracting the 4-inch circle from ten inches, the given length, will leave six inches straight in the sides, as from H to G; then drawing lines *no* and *il* (Fig. 258), at the required depth as previously described, draw *lo* and *in*, the perpendiculars, seven inches apart; then drawing the corners as previously described, adding on an inch and a half, from *p* to *u*, and *q* to N at right angles with *pm* and *qk*, will give the required pattern.

PROBLEM 45.—**To strike the pattern of an oblong pan, with round corners, but struck from different centres, and tapering more at the ends than at the sides.**

To construct the plan, Fig. 260, first draw the larger rectangle and the diameter lines, also the diagonals, and from the diagonals draw the four lines showing the width and length for the bottom. Draw the quadrants (or quarter circles) for the corners, as *gf*, from the centre *c* to any size required, and from the points *g* and *f* draw lines to the centre *a*, which will give a proportionate size for the corners of the bottom, as shown in the curve *de*, struck from *b* as centre.

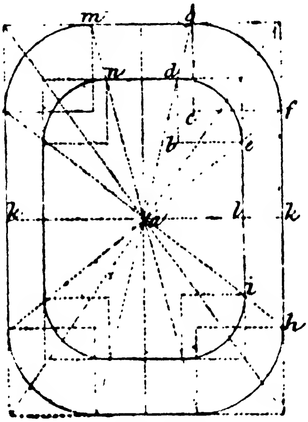


Fig. 260.

Having drawn the plan, proceed now with Fig. 261. To obtain the radii required, draw *ab* and *ad* at right angles. From *a* to *c*, take the vertical depth of the pan required, and draw *ce* parallel with *ad*, then transfer the lengths of *ag*, *af*, and *ak* (Fig. 260), to *ad*, *ak*, and *al* respectively (Fig. 261), also the distances *ad*, and *ae*, and *al* (Fig. 260), to *ce*, *cf*, and *cg* (Fig. 261), and draw lines from the points *de*, *kf*, and *lg*, to cut the perpendicular *ab* at *b* (all cutting at one point), then take the lengths from *c* to *g* and *b* to *d* (Fig. 260) (being the radii of the corners) and carry them from *a* to *m* and *a* to *n* (Fig. 261), and draw the lines *mp* and *no* parallel with *kf*.

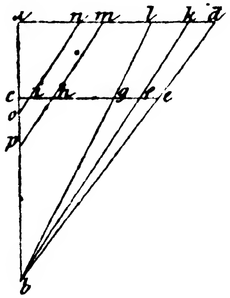


Fig. 261.

To proceed with the pattern drawn in Fig. 262, draw the perpendicular line *ab*, take the lengths of *bd*, *bk*, and *bl* in Fig. 261 as radii, and describe from the point *a* (Fig. 262) as a centre, the curves, *cd*, *ef*, and *gw*, also take the radii *be* and *bg*, and from the same centre *a* (Fig. 262) strike the curves *ik* and *uo*. Then transfer the length *mg* (Fig. 260) to the curve *cd* (Fig. 262), and draw the lines *ca*, *da*, and *cd*. From the points *c*, *d*, *i*, and *k*, draw perpendicular lines, *cp*, *dq*, *ir*, and *ks*. Take the length *pm* (Fig. 261), and carry the same from *c* to *p* and *d* to *q* (Fig. 262); take also the length *on* (Fig. 261), and mark off the same from *i* to *r* and *k* to *s* (Fig. 262). From *p* and *q* as centres (Fig. 262), with radius *pm* (Fig. 261) strike the curves *ct* and *dx* to meet the curve *ef* (Fig. 262), draw the lines *tp* and *xq*, also the lines *ta* and *xa*; then from *r* and *s* as centres, with radius *on* (Fig. 261) describe the curves *il* and *km* (Fig. 262). Now take the distance from *f* to *k* (Fig. 260), and mark off the same distance from *t* to *v* on the curve *g* (Fig. 262), also the same distance from *x* to *w*, and draw lines from *v* to *a* and from *w* to *a*. Draw the lines from *t* to *v* and from *x* to *w*, also *lu* and *mo*, parallel with *tv* and with *xw*, which completes the development of one half of the pattern.

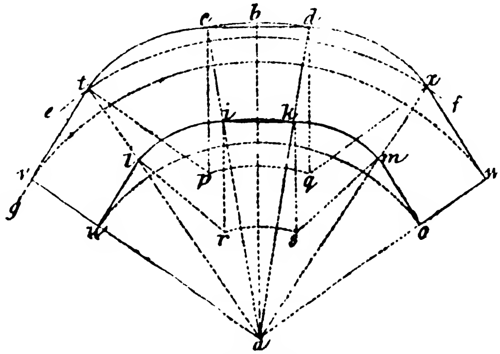
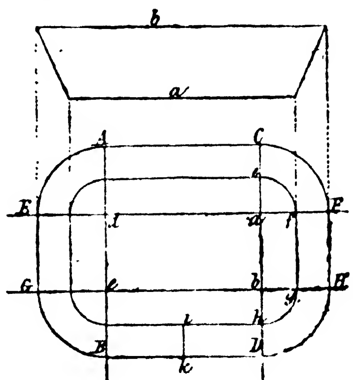


Fig. 262.

Problem 46.—**To strike the pattern for a tapering oblong article in one piece, such as a flue oblong candlestick.**

Fig. 263 shows the plan outline of both top and bottom, and Fig. 264 the vertical height and elevation. Take the perpendicular height *ab* (Fig. 264) and mark it off from *b* to *d* (Fig. 265). Take the radius for the corners *a*C (Fig. 263) and mark it off from *b* to *c* (Fig. 265); also take the radius *ae*, and mark off from *d* to *e*; draw a line from *c* through *e* to cut the perpendicular line *ba*, which gives the slanting height and the radii required for striking the corners. Draw the lines *ef* and *ac* (Fig. 266) at the same distance apart as *e* to *c* in Fig. 265. Draw the perpendiculars *ae* and *cf* (Fig. 266) at the same distance apart as A C (Fig. 263), giving the straight part of the side required. With radius *ac* (Fig. 265), using *b* and *d* (Fig. 266) as centres, strike the curves *ap* and *cg*, and with radius *ae* (Fig. 265), still using the same centres in Fig. 266, strike the curves *eq* and *fh*. Take the length of the curve DH (Fig. 263) and dot off the same distance from *c* to *g* (Fig. 266), making *ap* equal to *cg*, draw lines from *p* and *g* to the

Fig. 264.



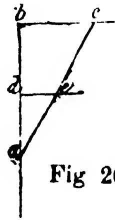


Fig 265

Fig. 263.

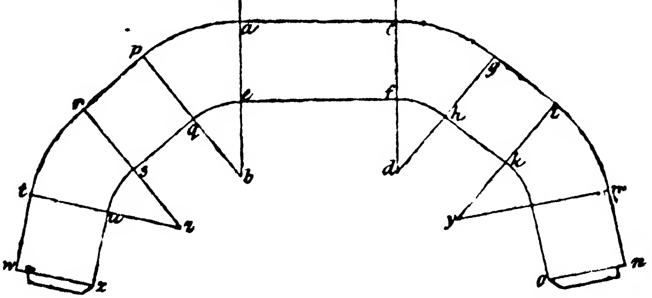


Fig. 266.

centres *b* and *d*, draw *pr* and *qs* at right angles with *pb*. Take the distance from E to G (Fig. 263) and mark the same distance from *p* to *r* and *q* to *s* (Fig. 266). Draw *rz* parallel with *pb*, from *r* mark off point *z*, the same length as *p* to *b*; then, using *z* as centre, strike the curves *rt* and *su*, making the curve *rt* equal *pa*; draw a line from *t* to the centre *z*, draw *tw* and *ux* at right angles to *tz*; taking the distance from B to *k* (Fig. 263), mark off the same distance from *t* to *w* and *u to x*;

draw *wx* parallel with *tu*. Proceed in the same manner with the other end, in which *ik* corresponds with *rs*, *my* with *tz*, and *no* with *wx*. Add on the lap, as shown, which will make the pattern complete in one piece, being joined together at *ki* (Fig. 263).

PROBLEM 47.—**To strike a pattern for the tapering sides of a tray having various curves.**

Fig. 267 shows the plan and elevation of the article, for which a pattern for the tapering sides is required. Having drawn the plan, it is required to show the points or centres from which the various curves are struck, as shown here by *m*, *b*, *h*, and *l*. The tapering being equal on all sides, the curves for the bottom and top are struck from the same centre, that is, the curves *ie* and *gc* are both struck from one centre, viz. *h*.

To prepare for the development of the pattern, construct Fig 268, making the distance from A to *b* the required vertical height (Fig. 267), and take the radius by which the curves *ac* and *de* are struck, that is, the distance from *b* to *a* and *b* to *d*, and mark off the same on Fig. 268 from A to *c* and from *b* to *d*; draw the line through the points *c* and *d* to cut the perpendicular at *e*. Take also the distance *he* or *hi*, and mark off the same from A to *i* (Fig. 268), and the distance from *h* to *t* mark off from *b* to *t* (Fig. 268). Draw a line through the points *i* and *t* to *k*. (The radius *mr* and *mv*, in this case, being the same as from *h* to *i* and *h* to *t*, does not require to be transferred to Fig. 268.)

To commence describing the pattern, take *ec* (Fig. 268) as radius, and from *b* as centre describe the curve *ac* (Fig. 269), take the length of the curve from *a* to *c* (Fig. 267) and mark off a corresponding distance from *a* to *c* (Fig. 269), and draw lines from *a* and *c* to the centre *b*. Now take the radius from *e* to *d* (Fig. 268), and again using *b* as centre (Fig. 269), strike the curve from *e* as far as the line *ab*. Take the distance from *a* to *o* in Fig. 267, and mark off the same from *c* to *i* (Fig. 269), likewise the distance from *c* to *s* (Fig. 269); take a like distance from *a* to *s* (Fig. 269), and draw lines through the points thus obtained from the points where the curve *e* intersects the lines *ab* and *cb*, and produce them indefinitely as *sg* and *ik*.

Take the radius *ki* (Fig. 268), and with it, from the curve *e* (Fig. 269), mark off the point *k* on the line *ei* prolonged, also from *d* the point *g* on the line *ds* prolonged; using *k* as centre, strike the curve *ex*, and from *g* as centre, strike the curve *df*.

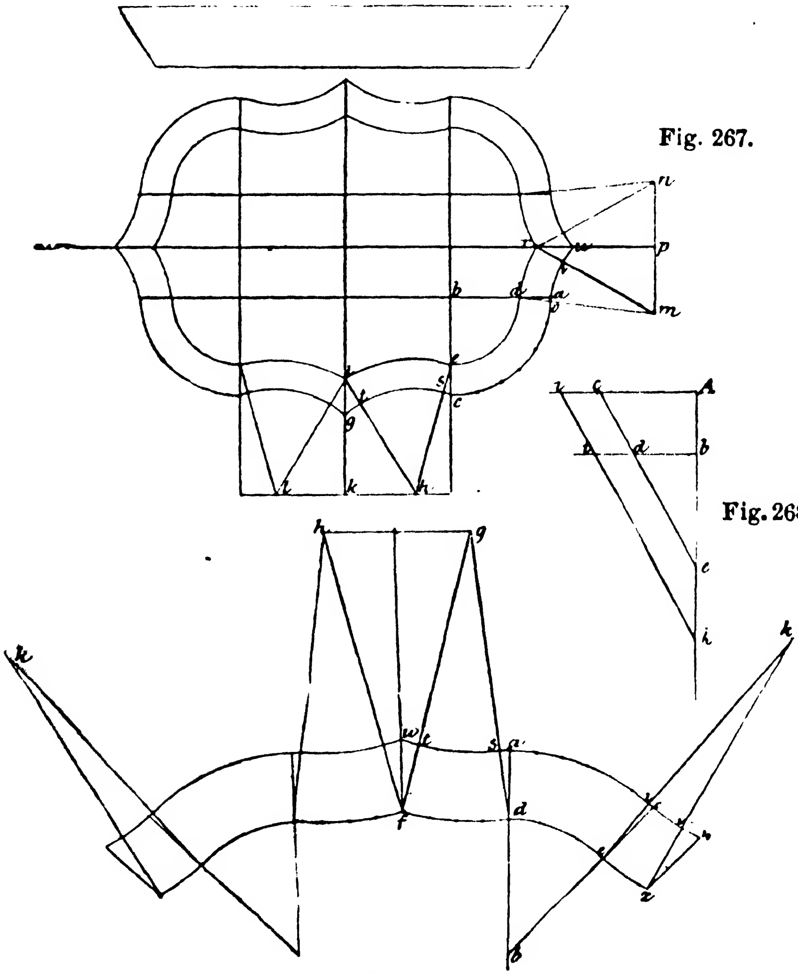


Fig. 267.

Fig. 268.

Fig. 269.

Again, from *k* and *g* as centres, and radius *kt* (Fig. 268), strike the curves *au* and *iw*; take the length of curve from *d* to *r* (Fig. 267), and mark off a corresponding distance from *e* to *x* (Fig. 269), and draw the line *xk*; the distance *vu* in Fig. 267 will show what is required to be added on the curve from *v* to *w* in Fig. 269; draw the line *xw*, which will give one end of the pattern to meet for joining at *ru* (Fig. 267). Now take the length of the curve from *e* to *i* (Fig. 267) and mark off a corresponding distance from *d* to *f* (Fig. 269). Draw a line from *f* to the point *g*, mark off the distance from *t* to *u* equal to that from *t* to *g* (Fig. 267), and draw a line through the points *f* and *u* (Fig. 269), and produce it indefinitely as far as *g*. Draw a line from *g* to *h* at right angles with the line prolonged from *f* and *u*, make the distance from the centre line to *h* equal to the distance from the centre line to *g*, which will be the centre for the next curve, and proceed in like manner to complete the pattern symmetrically.

Problem 48.—**To describe a pattern for a tapering top, the base being straight and parallel at the sides, and with circular ends, the hole in the top to be circular, and parallel with the base.**

(*Similar to a tea-bottle top.*)

Fig. 270 shows the plan and elevation required. Draw the lines *ad* and *cb*, at a distance apart equal to the required width of the top, and draw *ac* and *db* at right angles with them, and through the centres from which the circular ends are struck. Then draw the diagonal lines *ab* and *cd*, which will give the centre *o*, and draw the diameters AB and CD at right angles through the centre *o*. Take the distance from E to F, being the vertical height, and mark off a like distance on the vertical line H to G (Fig. 271), draw the lines *ab* and *cd* also at right angles with the line HG. Take the length of the diagonal line *ab* (Fig. 270) and make the length *a*H*b* (Fig. 271) equal to *ab*, take the distances from the centre *o* to *t*, and *o* to *e* (Fig. 270) (being the diameter of the circle), and mark off corresponding distances on Fig. 271 from G to *c* and G to *d*. Draw lines from points *bd* and *ac* to cut each other at *g*; with radius *ga* or *gb*, and with *i* (Fig. 272) as centre, strike the curve ABC (which will give a boundary line to describe the pattern on).

The curve of the ends (Fig. 270) being semicircles, extend the line *bc* to *g*, which will be at right angles with *ac*, take the distance EF (being the vertical height) and mark off a like distance from *c* to *g*, draw *gh* at right angles with *cg*, take the

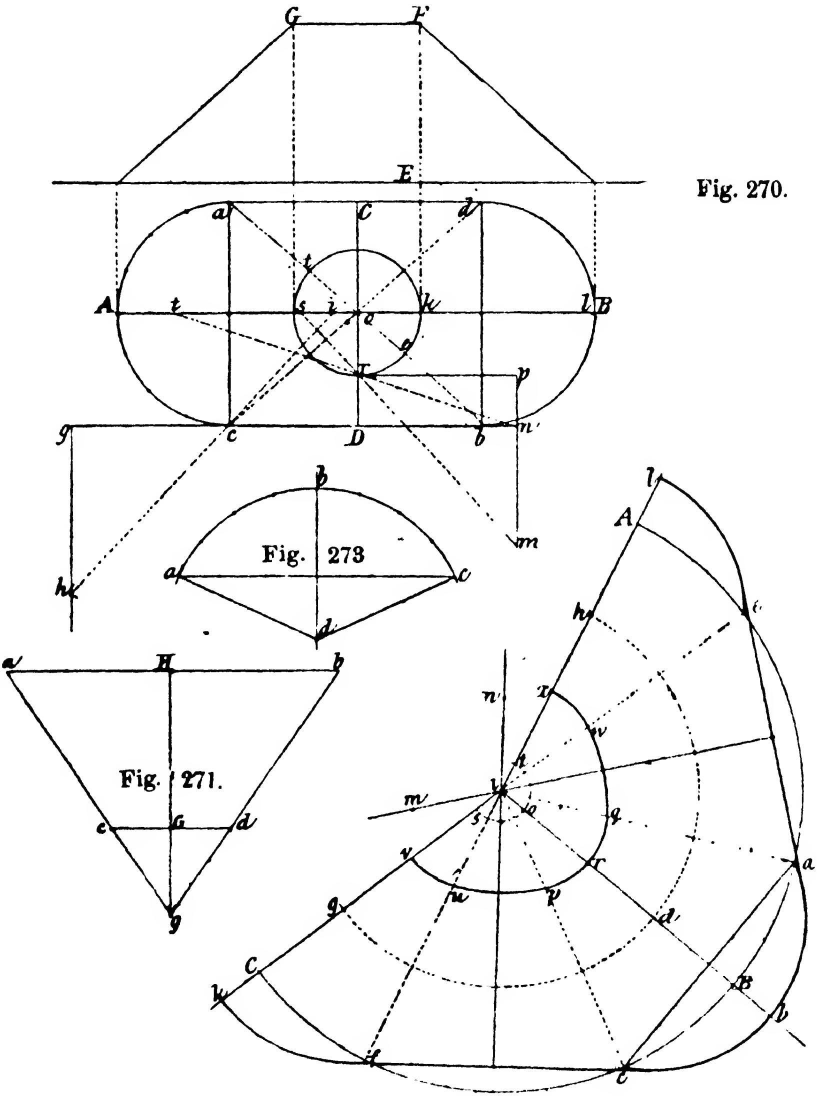


Fig. 270.

Fig. 273

Fig. 271.

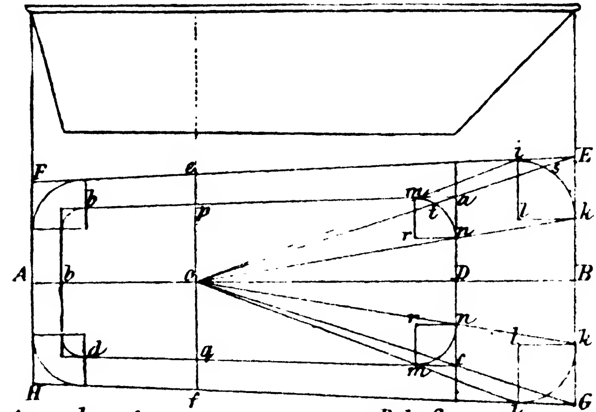
Fig. 272.

distance from *k* to *l*, being the taper of the end, and mark off a like distance from *g* to the point *h*, draw a line from *hc* to cut the line AB at *i*; with radius *ic*, taking *d* as centre, strike the curve *abc* (Fig. 273), now take the length of the curve *aAc* (Fig. 270), and mark off a corresponding distance on *abc* (Fig. 273), draw the chord from *a* to *c* (Fig. 273). In Fig. 272 draw a line from the centre *i* to B on the curve ABC, and take the length of the chord *ac* in Fig. 273, marking off an equal distance from *a* to *c* (Fig. 272) on the curve ABC. Take the distance from *a* to *d* (Fig. 273), and from *a* (Fig. 272) mark off the point *d* on the line *i*B; with *d* as centre (with the same radius as the curve *abc* is struck by, in Fig. 273), strike the curve *abc* (Fig. 272), take the distance *cb* (Fig. 270), which is the straight part of the side, and mark it off on Fig. 272 from *a* to *e* and *c* to *f* on the circle ABC, draw lines from *e*, *a*, *c*, and *f* to the centre *i*; take the distances from B to *c* or B to *a*, marking off the same distance from *e* to A and *f* to C, draw lines from the centre *i* to A, and *i* to C, and produce them, as at *k* and *l*; with radius *id* draw the circle as dotted *gdh*, take *g* and *h* as centres, with radius *db*, and strike the curves from *f* to *k*, and *e* to *l*, draw lines *ae* and *cf* from the highest parts of the curves, and the base of the pattern will be finished.

To get the curve for the hole in the top, bisect the lines *ae* and *cf* (Fig. 272) through the centre *i*, and produce them indefinitely as *n* and *m*. From *r* on the circle (Fig. 270) draw a line *rp* at right angles with CD, take the vertical depth EF, and mark a like distance from *r* to *p*. Draw *pm* at right angles with *rp*, take the distance from *kl* (which shows the slant of the end), and mark off a corresponding distance from *p* to *m*; take the distance from *r* to D (being the slant of the side), and mark off a corresponding distance from *p* to *n* on the line *pm*, draw a line through the points *mr* to cut the centre line AB at *s*, also draw a line through the points *nr* to cut the centre line at *t*; take the distances from *m* to *r* or *h* to *c*, and mark off like distances from *b* to *r*, *l* to *x*, and *k* to *v* (Fig. 272), being the slanting depth of the end of the pattern. Take the distance from *s* to *r* (Fig. 270), and from *r* (Fig. 272) mark off the point *o*, and from *v* the point *s*, and from *x* the point *t*; using *t*, *o*, and *s* as centres, strike the curves *x*, *w*, *v*, *u*, and *p*, *r*, *q*, with the radius *sr* (Fig. 270). Now take the radius *tr* (Fig. 270), and using *m* and *n* as centres, strike the curves *wq* and *pu*, which will complete the pattern required.

PROBLEM 49.—**To strike the pattern for an oblong tapering bath.**

Fig. 274.



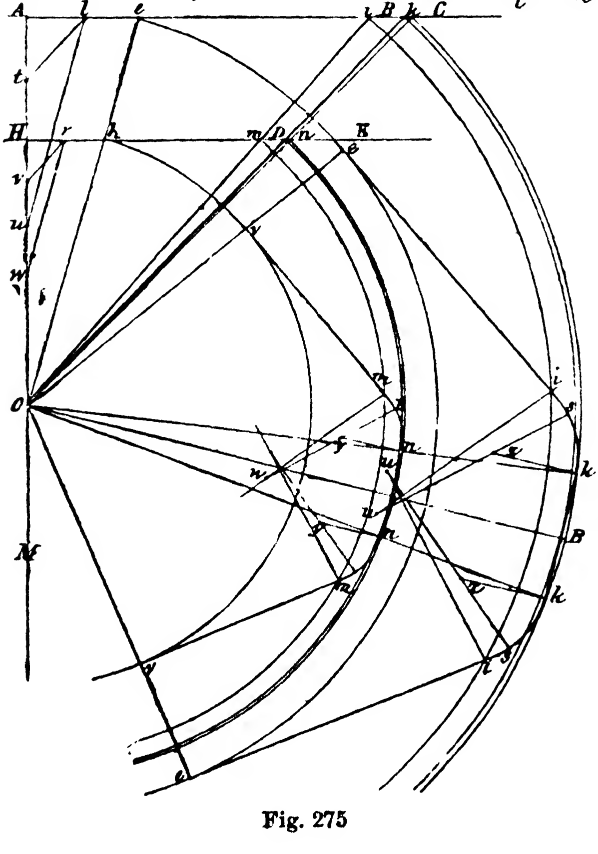


Fig. 275

Fig. 274 represents the elevation and plan for the bottom and top, showing a much greater slant or fall at the head than at any other part. Having drawn the lines FE, HG, FH, and EG, which represent the size and outline of the article at the top, draw also four similar lines, which represent the size and the position required for the bottom. Draw the diameter line AB, and draw lines from the angles in the top and bottom as, from E and *a*, G and *c*, and produce them to meet the centre line

AB, at *o*. Draw the corners as *ik*, and draw lines from *i* and *k* to the centre *o*; where these cut on the lines *ab* and *ac* will show proportionate corners for the bottom (these corners being quarter-circles, are struck from *r* and *l* as centres). Draw a line *ef* through the point *o* at right angles with AB.

Fig. 275 illustrates the way in which the pattern has to be developed. Taking first the upper part of the figure:—Draw the perpendicular AM, take the vertical height from A to H, equal to the depth of the bath in Fig. 274, and draw AC and HE at right angles with AM. Take the length of the lines from O to B, O to *k*, O to *i*, and O to *e* (Fig. 274), and from A (Fig. 275) mark off the points B, *k*, *i*, and *e*. Again in Fig. 274 take the distances OD, O*n*, O*m*, and O*p*, and from H (Fig. 275) mark off corresponding distances at D, *n*, *m*, and *h*. Draw the line from points B and D to cut the perpendicular line AM at O, also the lines from *k* and *n*, *i* and *m*, *e* and *h*, which will also cut the perpendicular AM at O.

Take the radius of the large corner, *li* (Fig. 274), and mark off the same from A to *l* (Fig. 275), also the radius of the small corner *rn*, and mark off the same from H to *r*. Draw the lines from *l* to *t* and *r* to *v* parallel with the line BDO, also the lines from *l* to *u* and *r* to *w* parallel with the line *eh*.

With O as a centre draw curves from the points set out on the line AC, that is, from *k*, B, *i*, and *e*, also on the line HE from the points *n*, D, *m*, and *h*. On the lower part of the figure draw the line BO, and from B mark off on each side to *kk* the same distance indicated by the corresponding letters in Fig. 274, draw lines from *kk* to O, also a line from *k* to *k*. Now from the points where the lines *kk*O intersect the curve *n*, draw a line from *n* to *n*, and draw perpendicular lines from the points *nn* to *yy*, likewise from *kk* to *xx*. Take the length of the line *tl*, and from *k* mark off the point *x*; from *x* as centre, with radius *tl*, strike the curve *ks* on each side. Taking the length of the curve from *k* to *s* (Fig. 274), mark off the same distance from *k* to *s* (Fig. 275), draw a line from *s* through the point *x* indefinitely, take the length of the line *u* to *l*, and from *s* mark off the point *u*; using *u* as centre, with radius *ul* strike the remainder of the curves from *s* to *i*, to right and left of B.*

* The tapering at the end being so much more than that of the sides, it

Taking the distance from *i* to *e* (Fig. 274), mark off a like distance from *i* to *e* (on the curve *e*, Fig. 275), draw lines from *i* to *e*, also from *el* to centre O. With radius *vr*, from *n* mark off the point *y* for a centre, and strike the curve *nt*, making it the same length as *nt* (Fig. 274); draw a line from *t* through the centre *y*, and produce it to *w*. Taking the distance from *w* to *r* as a radius, from *t* mark off the point *w* for a centre, and strike the remainder of the curve from *t* to *m* to meet the curve drawn from *m* on the line HE. Draw a line from *m* to cut the line *e* at *v* on the curve *h*. This will complete the pattern of the head, and the part of the body shown at the line *ef* in Fig. 274. The remaining portion (or toe), being about equal tapering, can be obtained in the same manner as an ordinary oblong article which is fully described in Figs. 263–266.

PROBLEM 50.—**To strike the pattern of an article where the sides are straight and the ends semicircular.**

Fig. 276 shows in plan the size and shape of the required article at top and bottom. Fig. 277 is the elevation. Having drawn Figs. 276 and 277, showing the plan and elevation, proceed with Fig. 278. Draw *ab* and *ac* at right angles, take the depth ED (Fig. 277), and mark it off from *a* to *d*, draw *de* parallel to *ac*; take the radius AB (Fig. 276), and mark off the same distance from *a* to *c* in Fig. 278; take the radius AG (Fig. 276), and mark off the same distance from *d* to *e*, in Fig. 278; then draw a line through the points *c* and *e* to cut the line *ab* at *b*, which will give the slanting height and the radius. This pattern is to be made in halves, joined at each end, as at B (Fig. 276). To strike the pattern, shown in Fig. 279, make the straight part for the side *fh* equal to DE (Fig. 276). Extend the lines *fm* and *hn* indefinitely, take the distance from *c* to *b* (Fig. 278), and mark off that distance from *f* to *g* and *h* to *i*; then with radius *bc* (Fig. 278), taking *g* and *i* as centres, strike the curves *fo* and *hp*; with radius *be* (Fig. 278), still using *g* and *i* as centres, strike the curves *mr* and *ns*. Divide the curve from D to B (Fig. 276)

becomes necessary to strike the corners in the pattern in two sections as divided by the line *Eao*, Fig. 274, the end section being struck in proportion to the slant of the end, and the remainder in proportion to the slant of the side.

into any convenient number of parts with the compasses, and mark a similar number of parts on the curve *fo* (Fig. 279); make *hp* equal *fo*, and draw lines from *p* to *i* and *o* to *g* to the centres, completing one-half of the pattern required.

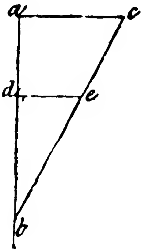


Fig. 278.

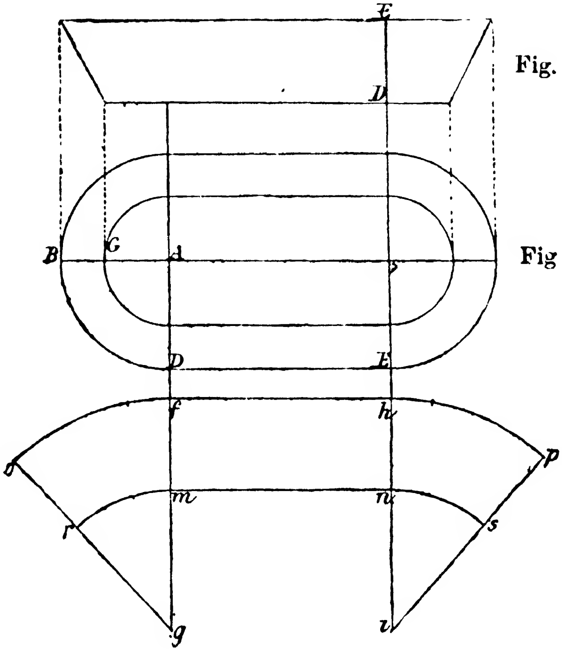


Fig. 277.

Fig 276.

Fig. 279.

Problem 51.—**To describe the pattern of a round-end bath, tapering more at the ends than at the sides.**

Fig. 280 shows the bath in elevation and in plan. It will readily be seen that the pattern required for the end of this bath is a section of the oblique cone in Figs. 192–194, p. 91. The semicircles CBD and EFG, for the larger and smaller portions of the ends respectively, being struck from N and M as centres, extend lines perpendicularly from these centres to N and M on the line AB (Fig. 281), and draw two circles thereon, corresponding with the semicircles in Fig. 280. One quarter of the circumference is all that is required to be divided here, viz. from B to C. Lines *df* and *gh* are drawn at the same distance apart as *t* and *u*, the vertical height in Fig. 280. The vertical line from m to o will be obtained as explained in Figs.

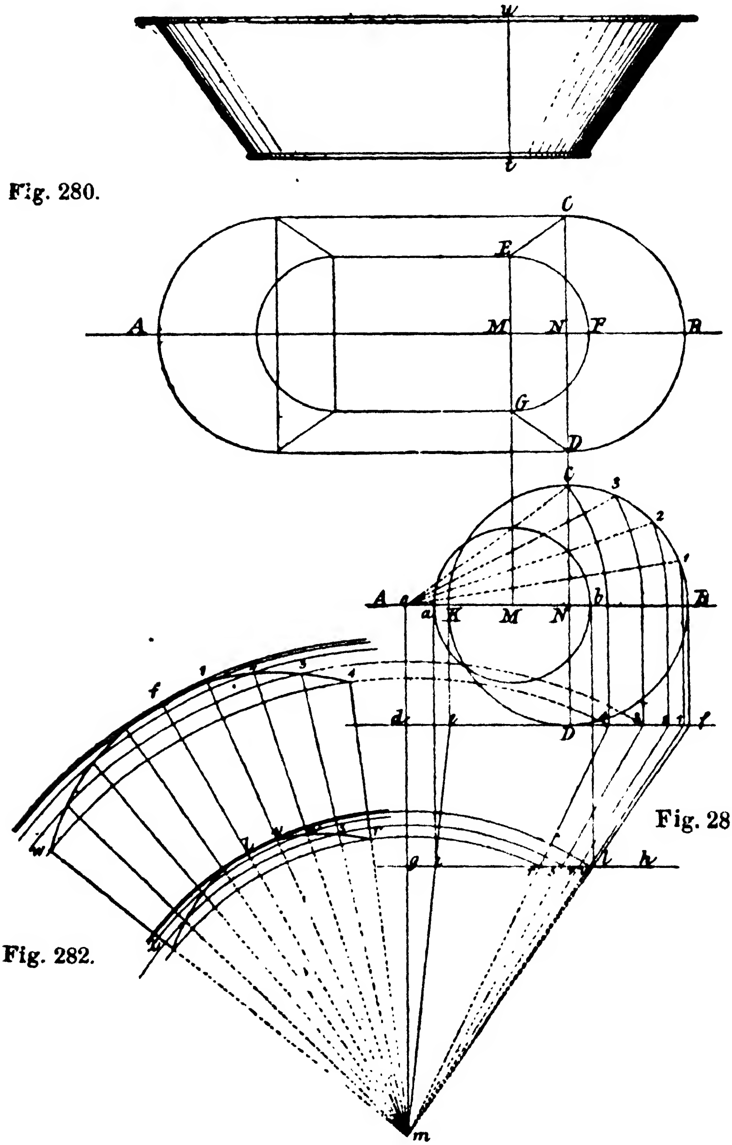


Fig. 280.

Fig. 281

Fig. 282.

193, 194. It will be observed that lines drawn from 1, 2, 3, and C to o, the working centre, will also divide the same section of the smaller circle into a like number of equal parts (a line drawn from C to *o* also cuts the perpendicular from M, the centre of the smaller circle). Now, using *o* as a centre, describe the arcs from C, 3, 2, 1 to cut the diameter AB, and draw the perpendiculars from AB to meet the line *df* at 4, 3, 2, 1, *f*; from the points thus obtained, strike the various curves in Fig. 282, as previously explained in Figs. 246, p. 120, and 193, p. 91. From the centre *m* draw the line *lf*, and with the compass set to the divisions in the quadrant BC (Fig. 281), mark off the same distances from *f* to 1, 2, 3, 4 (Fig. 282), from the outer to the inner curve, also from *f* to *w*. Now, draw lines from these points to the centre *m*, and draw a curve through the points of intersection, viz. through 4, 3, 2, 1, *f*, etc., also through the points *r*, *s*, *u*, *v*, etc., which will give the development of the pattern for the end, so far as shown by the semicircles CBD and EFG in the plan.

Problem 52.—**To describe the pattern for the top of a jack screen.**

[It will be seen that, in striking this pattern, sufficient allowance must be made for hollowing the top into the required concave form, in addition to the leading points obtained. This must be left to the judgment of the workman, as there are no known rules to describe it.]

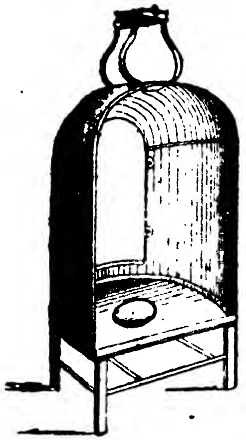
Fig. 287 represents in perspective the article of which the development of the top is required.

Fig. 283 shows the elevation or the shape of the front of the screen, and Fig. 284 gives the shape of the top in plan, the top being intended to be made in three pieces.

Draw in the plan the shape of the hole for the jack to work through, as shown by *mqn*, and draw lines from *u* and *r*, from any part of the outer curve that will make the back and side pieces look proportionate or convenient for material, to the centre *p*. Now, divide one side of the elevation into any number of equal parts, 1, 2, 3, etc., and draw perpendiculars therefrom to cut the line *wv* in the plan, as indicated by corresponding figures; and from these points describe circle arcs, cutting the lines *rp* and *up*.

Now, the height of the top, as far as the back piece will come, is shown by the arc drawn from 6, intersecting the line of jointure of the back piece at *s*. Therefore from B (Fig. 285) mark off 1, 2, 3, 4, 5, 6, at the same distance apart as the same figures in Fig. 283, and draw the horizontal lines from B to 6. Now, the curve *aeb* in Fig. 283 represents the opening in the

Fig. 287.



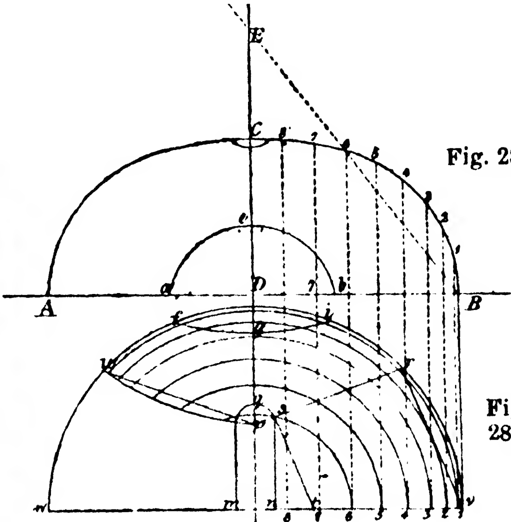


Fig. 283.

Fig. 284.

Fig. 288.

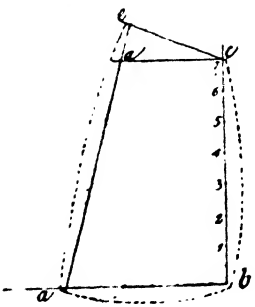


Fig. 286.

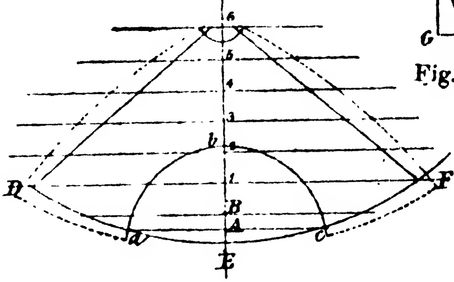


Fig. 285.

top for the doorway, and the curve *fgi* (Fig. 284) gives its course in the plan, shown by the point *g* coming between the second and third arc, as the point *b* (Fig. 283) comes between the second and third perpendicular; draw FGH at right angles (Fig. 288), and take the distance from D to *e* (Fig. 283), and mark off the same from G to F, also the distance from *g* to the outer curve in the plan, and mark off from G to H, drawing the line FH; now let the difference between the distances from FG to FH be added on from B to A (Fig. 285) and draw the line *ac;* draw the curve *abc* about one-fourth wider than *ab* in Fig. 283, as it will draw in much closer by hollowing. Draw line from B to 6 (being the height of the back piece, as previously stated), and extend it to cut the perpendicular at E (Fig. 283); now with the radius EB in that figure, describe the curve for the bottom of the pattern through the points *a* and *c* (Fig. 285), and take the distance from *f* to *u* (Fig. 284), and an allowance for the seam, and mark off from *a* to D, and *c* to F, and draw lines from points so obtained to 6; now draw the dotted curves for the hollowing as shown, according to judgment.

To obtain the pattern for the side, draw the line *rv* at right angles with *rs* (Fig. 284), and also the line *st* in the same manner; now draw *cba* (Fig. 286) at right angles, and by taking the distance from *r* to *s* in Fig. 284, and by measuring off the same distance on the line AB (Fig. 283) from B, it will just reach the perpendicular 7; now take the distance from B to 7 on the curve, and mark off the same from *b* to 7 on the perpendicular in Fig. 286, and draw *ca* at right angles with *cb*. Take the distance from *s* to *t* (Fig. 284) and mark off the same from *c* to *a* (Fig. 286), also take the distance from *r* to *v* (Fig. 284) and mark off the same from *b* to *a* (at the base, Fig. 286), and draw a line through the points *aa*, and extend it; now take the distance *n* to *t* (Fig. 284) and let the same be added on from *a* to *e* (Fig. 286) and draw the line *ce*, this will give the main points, and the size required for the pattern of the side; curves for hollowing and wiring to be added on as shown by dotted lines.

PROBLEM 53.—**To draw an ogee arch.**

In Fig. 289 divide the width AB into four equal parts, A, *c*, *g*, *d*, B, and on *cd* erect the square *cedf:* the points *cdef* are the centres of the four quadrants composing the arch.

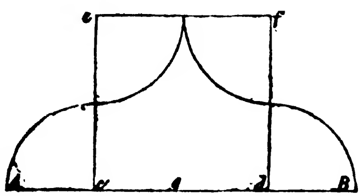


Fig. 289.

PROBLEM 54.—**Another method.**

In Fig. 290, let AB be the width, and CD the height of the arch, join AD, BD, and bisect them in *g* and *h* respectively; then from the centres A, *g*, D, *h*, B describe arcs intersecting at *e*, *f*, C, which are the centres of the four arcs composing the arch.

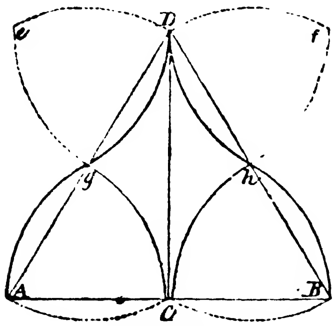


Fig. 290.

PROBLEM 55.—**To find the covering of an ogee dome, the plan of which is hexagonal.**

In Fig. 291, let ABCDEF be the plan, and HIJ the elevation, Divide HJ into any number of equal parts, as 1, 2, 3, 4, 5, 6. and through these points draw perpendiculars to FG; through the points in FG draw lines parallel to FE (the side of the hexagon) to EG, bisect EF in *a*, and draw *a*G, which is the seat of one side of the dome. Now to find the development of one section, set off the lines 1, 2, 3, 4, 5, 6, K at the same

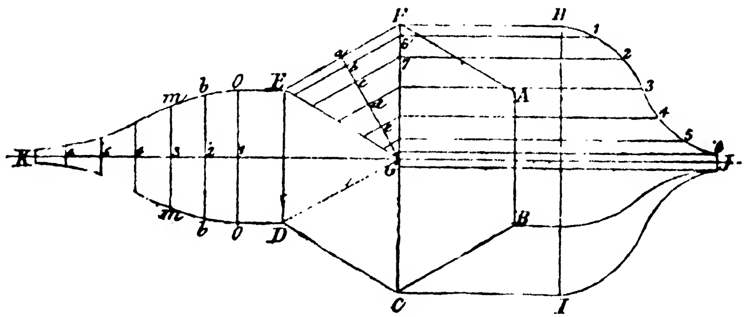


Fig. 291.

distance apart as 1, 2, 3, 4, 5, 6 on the elevation from H to J. Then take *a*E or *a*F on the plan, and transfer it to 1*o* on each side of 1 on the pattern; now take *b*6 on the plan, and transfer it from 2 to *b* on each side; then *c*7 on the plan transfer to 3*m*, and so on to K, and through the points *o*, *b*, *m*, etc., trace the curve as shown, and it will form the covering for one side of the dome. All the sides being equal, of course, the pattern of one side is all that is required.

PROBLEM 56.—**To describe the pattern for a rectangular base and bottom in one piece, where the flue or curve is equal on all sides.**

(Such as may be used as a base for either an Aquarium or a Fern-case.)

Draw Fig. 292, which represents half the plan; Fig. 293 shows the elevation and profile of the base. Next, in Fig. 294, draw the rectangle *Ckta*, of the same size as ACDB in Fig. 292. Take the vertical distance between line *b* and line *h* (Fig. 293), and divide the curve into any number of equal parts, as *c*, *d*, *e*,

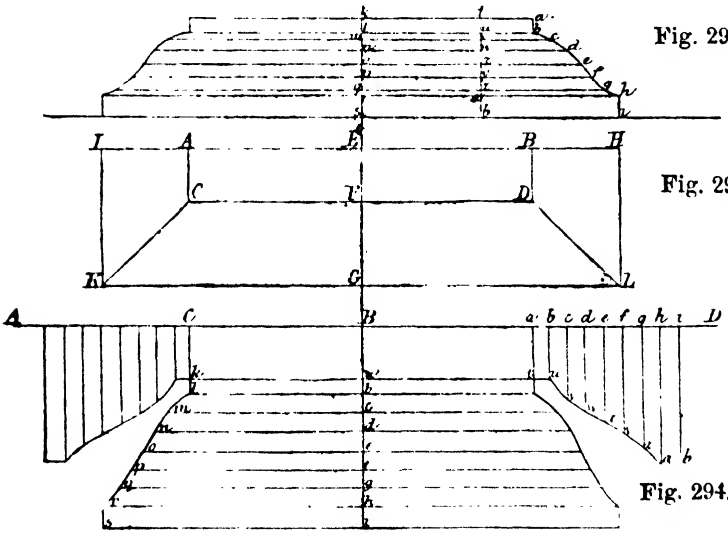


Fig. 293.

Fig. 292.

Fig. 294.

f, *g*, and mark off corresponding distances on the perpendicular line from *a* to *i* (Fig. 294), also from *a* to *i* on the line BD in the same figure, likewise from C to A on the line BA, and draw parallel lines from these points. The distances from *k* to *a*, *m* to *c*, and *n* to *d*, etc., in Fig. 293, will show the required distances, namely, from *b* to *l*, *c* to *m*, and *d* to *n*, to be taken on each side of the centre line, in Fig. 294. Then, by taking the distance from B to D in Fig. 292, and marking off the same from *a* to *t*, Fig. 293, and drawing the perpendicular *tb*, the required lengths of the lines at the ends will be obtained, as from *b* to *u*, *cv*, *dw*, *ex*, etc., for transference to Fig. 293, shown similarly lettered (Fig. 294). A curve drawn through the points thus obtained will give one-half the required pattern.

PROBLEM 57.—**To describe the pattern for a cover and neck of an irregular octagon article, such as a tureen.**

Fig. 295 represents the elevation and the required curves, and Fig. 296 shows one half the plan.

To obtain the pattern for the cover, first draw the half octagon *z*, *x*, *u*, *b*, *i*, *v*, *n* (Fig. 297) to the same size as *z*, *x*, *a*, *b*, *i*, *d*, *n* (Fig. 296), and draw through the centre the perpendicular line *bh*.

Draw the line (Fig. 297) from *x* to *f* at right angles to *xu*; also from *v* draw the line *vk*, at right angles with *vi*; then take the length of the line *xw* (Fig. 295) and transfer the same from *z* to *y* and from *n* to *m* (Fig. 297), and draw the perpendiculars *yw* and *ml*; then take the length of *yw* (Fig. 296) and mark off the same from *y* to *w* and from *m* to *l* (Fig. 297), and draw the lines *wx* and *lv*. Now, in the plan (Fig. 296), draw the line *xf* from *x*, at right angles with *xa*, transfer the length *xf* from *y* to *f* (Fig. 295), raise the perpendicular *yx*, and draw line *a* from *x* to *f*. Now, take the length of *xf*, and mark off the same length from *x* to *f*, and from *x* to *k* (Fig. 297), and draw the line *wg* through the point *f*, parallel to *xu*, and the line *in* through *k* similarly (Fig. 295); take the distances from *f* to *g* and from *f* to *w* (Fig. 296), and mark off the same from *f* to *g* and *f* to *w* (Fig. 297), and draw the lines *wx* and *gu*.

Now take the distances from *o* to *b* and from *o* to *h*, in the plan (Fig. 296), and mark off the same from *o* to *b* and from *i* to *h* in the elevation (Fig. 295), and draw the line *bh*. Take the length of *bh* (Fig. 295) and mark off the same from *b* to *h* (Fig. 297), and through the point *h* draw *gn* parallel to *ui*; take the length of *hg* (Fig. 296) and mark off the same from *h* to *g* and from *h* to *n* (Fig. 297), and draw the lines *gu* and *ni*, which completes half the pattern for the top.

When describing the patterns for the sides or neck, note that, the octagon being irregular, and the angles leading towards

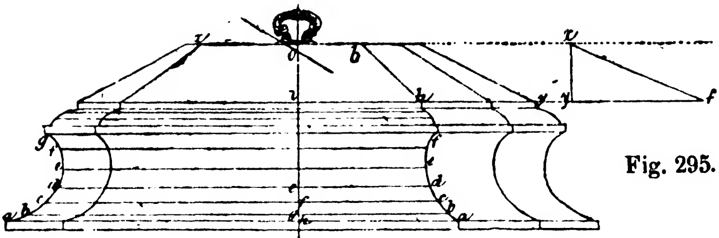


Fig. 295.

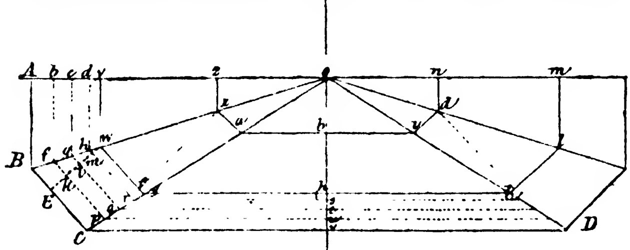


Fig. 296.

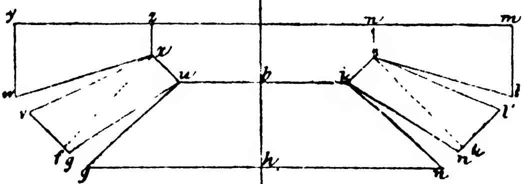


Fig. 297.

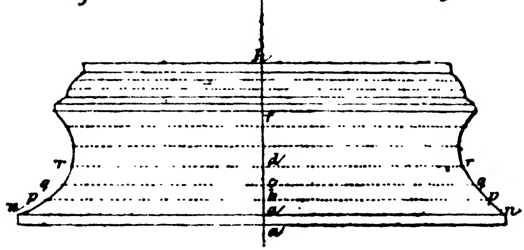
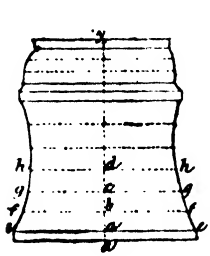


Fig. 298.

Fig. 299.



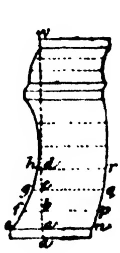


Fig. 300.

the centre, the several sides will take different curves; there fore each section will vary.

To obtain the pattern for the end, of which BA (Fig. 296) represents one-half, the process is simply to divide the curve through *a*, *b*, *c*, etc., to *g* and *w* (Fig. 295), draw the horizontal lines from these points, and also draw perpendiculars from the same points, and extend them to cut the lines A*o*, B*o*, in the plan (Fig. 296). On the perpendicular line *ay* (Fig. 299), take a number of distances corresponding with those in Fig. 295, and draw the horizontal lines *a*, *b*, *c*, *d*, etc., and on each side of these points mark off the points *ee*, *ff*, *gg*, etc., of the same length as AB, *bf*, *cg*, *dh*, etc. (Fig. 296). A curve drawn through the points so obtained will give the pattern for the end.

NOTE.—The lines in Fig. 2, AB, *bf*, *cg*, etc., are not all that could be obtained from the points in the curve (Fig. 295), but they will sufficiently illustrate the principle, and would be sufficient in most cases.

To obtain the pattern for the side from B to C, in Fig. 296, draw the lines from *f* to *p*, from *g* to *q*, and from *h* to *r*, etc., parallel to BC, and draw a line from *w* to E, at right angles with *wf* or BC, precisely the same as the line *xf* was drawn at right angles with *xa*. Then draw the perpendicular line *aw* (Fig. 300), also the horizontals, *a*, *b*, *c*, *d*, etc., at the same distance apart as in Fig. 299; and take the distances from E to B and from E to C, and mark off the same from *a* to *e* and from *a* to *n* (Fig. 300), also the distances from *k* to *f* and from *k* to *p*, and mark off the same from *b* to *f* and from *b* to *p*, and so on until all the points are obtained, which will give the direction for the curve to be drawn to give the pattern for the small side.

Now, draw lines from *p*, *q*, *r*, etc., parallel to CD (Fig. 296), and take the distance *ov*, *ou*, *ot*, *os*, and *oh*, and transfer the same, on Fig. 295, from *h* to *a*, *g* to *b*, *f* to *c*, and *e* to *d*, and so on, to *ih*; and through the points thus obtained draw the curve from *h* to *a*. Take the distances between the horizontals (previously drawn from the curve at the extreme end), and on the perpendicular *ah* (Fig. 298) mark off corresponding distances, and draw the horizontal lines. Take the distances from *v* to C, *u* to *p*, *t* to *q*, and *s* to *r*, etc. (Fig. 296), and transfer the same to each side of the perpendicular line (Fig. 298), from *a* to *n*, *b* to *p*, *c* to *q*, etc. A curve drawn through these points will complete the necessary patterns.

Problem 58.—**To describe the pattern for a triangular pedestal or pyramid, with all three sides alike (an equilateral triangle).**

Fig. 301 shows the pedestal in elevation, and Fig. 302 in plan. To draw the plan for this figure, let the point *x*, and the centre point *o*, be located in a horizontal line at right angles with the perpendicular line *uo*, which represents the real centre of the article, not the centre of either of the sides. In order that the curves may be equal on all sides, divide the circle into three equal parts, *xgh*, and draw lines therefrom to the centre *o*. Now, draw the required shape for one side in Fig. 301, and let this side be divided into any number of parts, *a*, *b*, *c*, *d*, *e*, *f*, etc., and draw the horizontal lines therefrom across the profile, likewise perpendiculars from the same points to intersect the lines *go* and *ho* in Fig. 302, and from these last points of intersection draw lines parallel to *hx* and *gx*. By raising perpendiculars from all the points on the line *ox*, two of which are shown from *x* and *r*, the direction of the curve will be obtained, showing the point or angle of the article, which at first may appear to have more curve than the view on the left hand. But by referring to the plan (Fig. 302) it will be seen

Fig. 301.

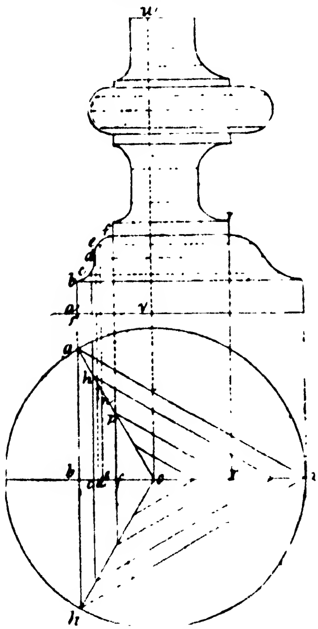


Fig. 302

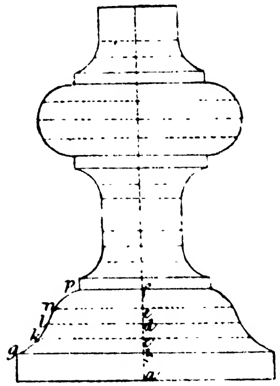


Fig. 303

that the one (left hand) gives a view of the side, and the other (the right hand) that of a point or angle. To develop the pattern (Fig. 303) the distances between the lines *a*, *b*, *c*, *d*, etc., are transferred from those indicated by the corresponding letters in Fig. 301, and the lengths from *b* to *g*, *c* to *k*, etc., are equal to those similarly marked in Fig. 302, which will give the course of the curves to complete the pattern.

Problem 59.—**To describe the patterns for the sides of an irregular octagon pan.**

Figs. 304 and 305 show the elevation and half plan of the article required, having the flue or curve equal on all sides, as shown by the distances from *c* to *l*, and from *y* to *x* being alike; therefore all that is needed is to divide the curve (Fig. 304) into equal parts, as *a*, 1, 2, 3, *b*, and take the same from *a*, 1,

Fig. 304.

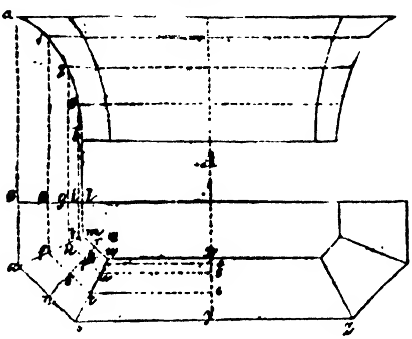
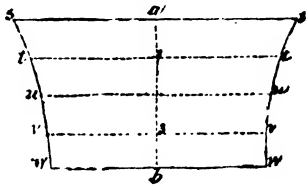


Fig. 305.

Fig. 306.



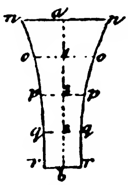


Fig. 307.

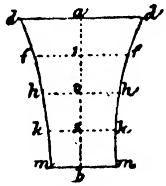


Fig. 308.

2, 3, *b* on the perpendicular lines in Figs. 306, 307, and 308; and the widths of these patterns will be obtained from the plan as indicated by similar reference letters (Fig. 305) as previously described in connection with Figs. 292–294, p. 145.

Figs. 309 and 310 show also the elevation and half plan of an irregular octagon article, in which the curve will not be alike on all sides, but proportionate, and its angles or section lines all leading to the centre.

Draw the elevation (Fig. 309); also draw half the plan (Fig. 310) AGFEDC, as required, and draw the sectional lines, from G, F, E, D, to the centre B. Divide the curve from *c* to *f* into equal parts, as 1, 2, 3, 4, 5, and draw horizontal lines across the elevation from these points; also perpendiculars from the same points to cut the line DB, and carry the lines from D*e* to E*g* and F*g* parallel with DE and EF.

To obtain the pattern for the end, draw the perpendicular (Fig. 313), and mark off distances equal to 1, 2, 3, 4, 5 (Fig. 309),

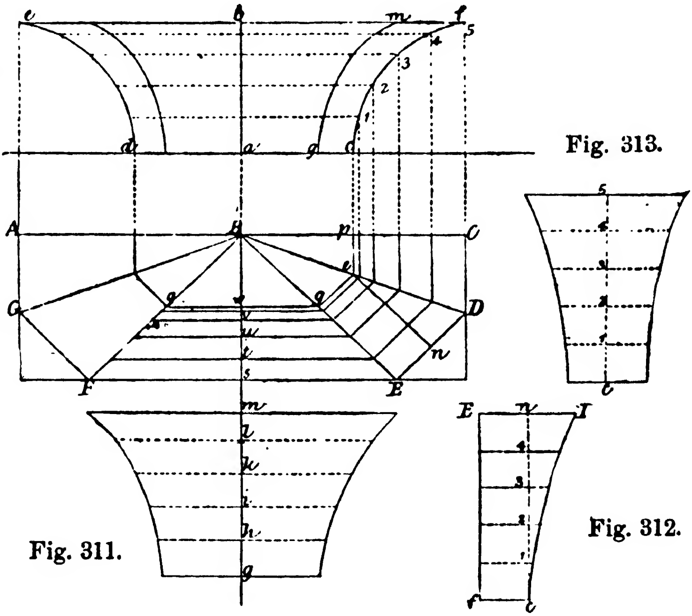


Fig. 309. Fig. 310. Fig. 311. Fig. 312. Fig. 313.

and set off the distances from CD to *pe* in Fig. 310 on each side, which will give the widths of the pattern through which its curves will be drawn.

Next draw the line *en* (Fig. 310) at right angles with *ge*; and as this line *en* is of the same length as from *p* to C, draw the perpendicular *cn* (Fig. 312), and draw the parallel lines 1, 2, 3, 4, *n*, at the same distance apart as in Fig. 313. Now mark off the distance from *n* to E and from *n* to D, the same as from *n* to E and from *n* to D (Fig. 310); likewise transfer all the distances on each side of the line *ne* in the same order to Fig. 312, as shown; and lines drawn through the points thus obtained will give the pattern for the small side.

The projection of the side from *w* to *s* (Fig. 310) being much less than that of the end from *p* to C, proceed as follows:—take the distances (Fig. 310) from B, the centre, to *w*, *v*, *u*, *t*, and *s*, and mark off the same (Fig. 309) from the perpendicular *ab*, to *g*, *h*, *i*, *k*, *l*, and *m*, and draw a curve through these points, which shows the fall of the side before mentioned; now draw the perpendicular (Fig. 311) and draw the parallel lines *g*, *h*, *i*, *k*, *l*, *m*, at the same distance apart as the corresponding letters in the elevation (these will not be equal distances apart, as in Figs. 312), and make them the same length as the lines *s*, *t*, *u*, *v*, *w* (Fig. 310); this being done will give the course of the curve required to complete the pattern.

PROBLEM 60.—**To describe the pattern for a vase of octagonal shape.**

Draw the profile in elevation (Fig. 314) to the required design, and the half octagon in plan (Fig. 315) of the size corresponding with the extreme points in Fig. 314 (as described in Fig. 78, p. 34), and to be placed so that one half of the side, as from A to E, may be perpendicular with the base. Draw section lines from points A, B, C, and D, to the centre G, divide the curve in the elevation (Fig. 314) into any convenient number of parts, as *a*, *b*, *c*, *d*, etc., and from these points draw horizontal lines across the elevation; also draw perpendiculars from the same points and extend them to cut the line AG in the plan (Fig. 315) at *a*, *b*, *c*, etc. Now mark off the same distances as shown by corresponding letters in Fig. 316, and through these points draw parallel lines *mm*, *nn*, *oo*, etc. Take the length of the line (produced from *a*), from *c* to *m* (Fig. 315), and mark off the same from *a* to *m* on each side of the perpendicular line in Fig. 316, also the distance from *b* to *n* (being the line produced from *b*, Fig. 314), and mark off the same from *b* to *n* (Fig. 316), also let the distance from A to *o* (Fig. 315) be carried from *c* to *o* (Fig. 316). Now take the distance from *d* to *p* (Fig. 315) and transfer the same from *d* to *p* (Fig. 316), and so on; transferring all the distances between the lines EG and AG, to

Fig. 316, until all the points in the development of the pattern are obtained. Draw a curve through those points, which will complete the pattern. The lower portion, similarly developed, as shown below.

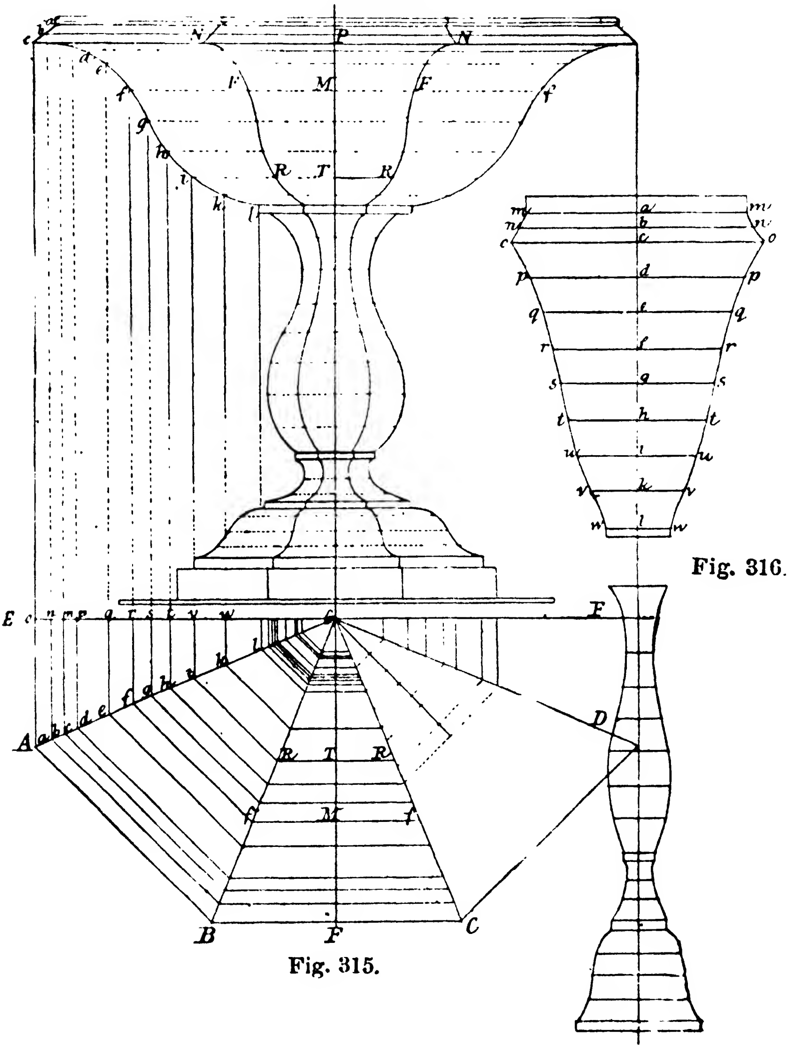


Fig. 314.

Fig. 316.

Fig. 315.

To make the pattern look more complete, the course of the curves N, F, R may be obtained in the following manner (although not necessary for the development of the pattern). From the points *a*, *b*, *c*, *d*, *e*, *f*, etc., on the line AG (Fig. 315) draw lines parallel to AB, to cut the section line BG, and again produce them parallel with BC, to cut the line CG. Now if vertical lines are raised from B and C in the plan (Fig. 315) they will cut the horizontal line *c* in the elevation at N and N; also by following the line from *f* in the elevation to *f* M *f* in the plan and raising perpendiculars as before, these will give the points FF in the elevation; likewise by following the perpendicular from *i*, produced at RR in Fig. 315, perpendiculars raised from these points will give the points RR in Fig. 314. By tracing all the points in like manner the course of the curves NFR, etc., will be found. And it will be observed that these curves will show the same width as the pattern (Fig. 316) all the way through, while the length of the pattern will correspond with the length measured round the curve of the side, as at *a*, *b*, *c*, *d*, *e*, *f*, etc.

PROBLEM 61.—**To describe the pattern for a Hexagon Base.**

Draw Fig. 317, the half hexagon, so placed that half of a side,

Fig. 318.

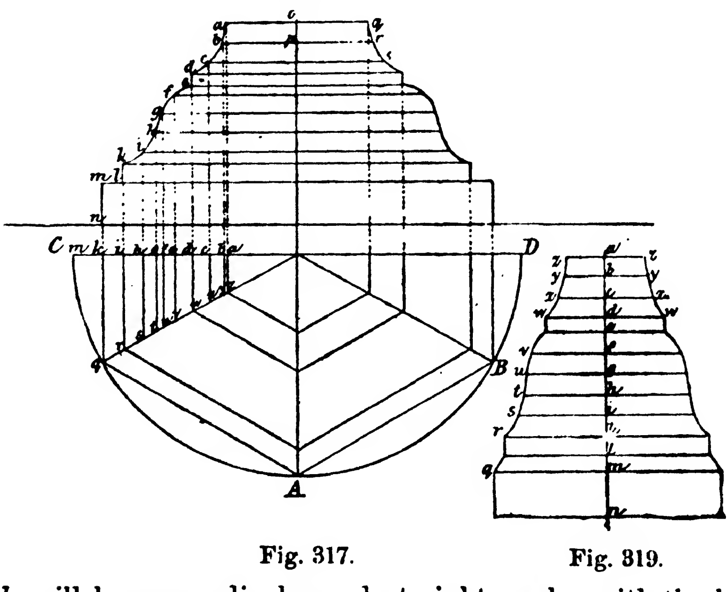


Fig. 317. Fig. 319.

as *qk*, will be perpendicular and at right angles with the base. Draw a perpendicular line *o* A, and lines from B and *q* to the centre. In Fig. 318 (the elevation) divide the curve into a convenient number of parts, and draw the horizontal lines, and also the perpendiculars therefrom, and extend them to cut the line drawn from *q* in Fig. 317 to the centre at *q*, *r*, *s*, *t*, etc. Draw the perpendicular *an* in Fig. 319, and set out on this line all the distances of the straight lines and angles, likewise the points of division on the curve of the elevation (Fig. 318), as *n*, *m*, *l*, *k*, *i*, etc., to the points marked by corresponding letters in Fig. 319, and through the points thus obtained draw parallel lines at right angles with the perpendicular *an*, and on each side mark off the points, *az*, *by*, *cx*, *dw*, etc., at the same distances as indicated by the corresponding letters in Fig. 317. By connecting these points with curves and right lines (according to the plan in Fig. 318) the required pattern will be obtained.

PROBLEM 62.—**To describe the pattern for a vase having twelve sides (duodecagon).**

Draw the profile in elevation, Fig. 320 (the two outer curves only), also draw the perpendicular line CD through the centre, and draw the horizontal line AB (Fig. 321), on which construct half the plan by methods similar to those in Fig. 315, p. 153, and draw the sectional lines from the points F, G, H, I, K, L, to the centre E, having half of one side from F to A at right angles with the base. Now divide the outer curve in the elevation (Fig. 320) into any convenient number of parts, as *a*, *b*, *c*, *d*, *e*, *f*, etc., and from these points draw horizontal lines across the elevation, also draw perpendiculars from the same points and extend them to cut the line FE, in Fig. 321. Observing the points from which they are produced, take the distances between the lines AE and FE, in Fig. 321, and transfer them to the lines marked by corresponding letters in Fig. 322. Now connect the points FG, GH, and HI, etc. (Fig. 322), and draw lines parallel therewith from all the points produced on the line FE; observing that where there are straight parts in the elevation, as from *m* to *n*, and from *t* to *u*, the same distance is to be taken, as marked by the corresponding letters in the development of the pattern (Fig. 322), and the two lines, as *t* and *u*, for example, will be connected by lines at right angles (as they are both of the same length).

Now, having carried parallel lines from all the points on the line FE to all the other sectional lines in the plan, as from F to

Fig. 320.

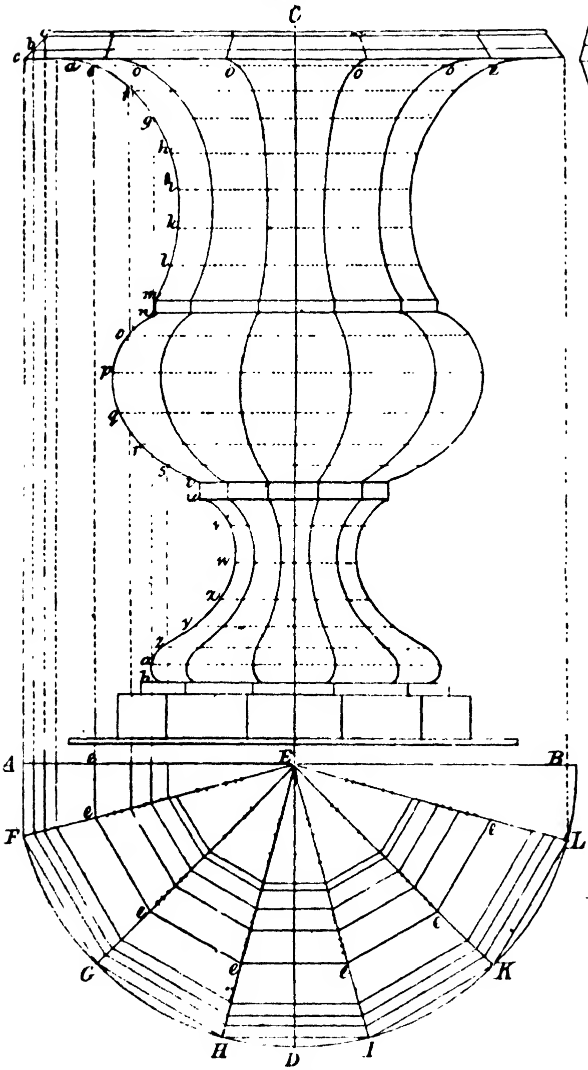


Fig. 321.

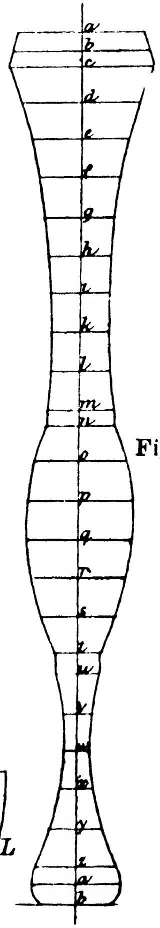


Fig. 322

G, G to H, and H to I, etc., by raising perpendiculars from these points, or by marking off points perpendicular to them on the corresponding horizontal lines in Fig. 320, the course of all the curves may be obtained, showing all the joints and angles. For example, by following the perpendicular line drawn from *e* (Fig. 320), to *e* on the line FE (Fig. 321), and all the other sectional lines marked by *e*, perpendiculars raised from these points, on the lines G, H, I, and K, will give the direction of the various curves on the horizontal line *e*, *e* (Fig. 320), *o*, *o*, *o*, *o*.

CHAPTER VII.

MENSURATION.

A KNOWLEDGE of mensuration is required by the sheet-metal worker for calculating the areas of surfaces, both plane and curved, in order to estimate quantities of material; and of volumes or capacities, in order to judge of the cubic contents of vessels of various forms. I will run through the most useful rules under these heads :—

I. The areas of plane surfaces.

II. The areas of the envelopes of solid or hollow bodies.

III. The capacities or volumes of hollow bodies.

I. First, then, as to the areas of plane surfaces. Quadrilaterals are rectilineal figures with four sides. The simplest cases that occur among quadrilaterals are those of *parallelograms*, which include the square, the rectangle, the rhombus, and rhomboid. In each of these the sides are parallel. In the square and rhombus, the four sides are of equal length, and in the rectangle and rhomboid the opposite sides are of equal length. In the rhombus, however, the angles are not right angles. In each case the rule is—

Rule 1.—Multiply the length by the breadth to get the area. Also, knowing the area of a parallelogram, and one dimension, the other can be calculated.

Rule 2.—In the case of a square, if the area is known, the length of one side can be found by extracting the square root of the area. Thus the area of a square measuring 10 inches along each side is—$10 \times 10 = 100$, and the square root of 100 is, $\sqrt{100} = 10$.

To find the area of a triangle.

Rule 3.—Multiply the base by the perpendicular height, and take half the product.

Conversely, knowing the area, and one dimension—either the base, or height—it is easy to find the other dimension. The rule is—

Rule 4.—Divide *twice* the area by the base or height, as the case may be, and the quotient will be the other dimension required.

The area of a trapezoid, which has only two of its sides parallel, is obtained by the rule—

Rule 5.—Multiply the sum of the two parallel sides by the perpendicular distance between them, and half the product will be the area.

A quadrilateral is a rectilineal figure with four sides. But it is convenient to separate the quadrilaterals with parallel sides —the square, rectangle, rhombus, and rhomboid from those in which the sides are not parallel, and to consider the latter after

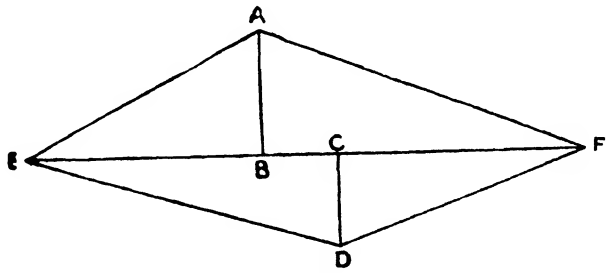


Fig. 323

the triangles, because the method of measuring triangles finds application in these. Hence the rule to find the area of a quadrilateral (Fig. 323).

Rule 6.—Draw a diagonal EF, and find the area of each triangle, of which the diagonal forms the base, and add together their sum for the area.

Or—

Rule 7.—Multiply the sum of the vertical heights or perpendiculars AB, CD by the diagonal EF, and half the product will be the area.

The method of estimating the area of any polygonal rectilineal figure, whether regular or irregular, is based upon the foregoing rules. Any such figure can be divided up into triangles, trapezoids or rectangles, and the areas of each calculated separately. Hence the rule—

Rule 8.—Divide the polygonal figure up into any convenient figures, and the sum of the areas of these will be the area of the entire figure.

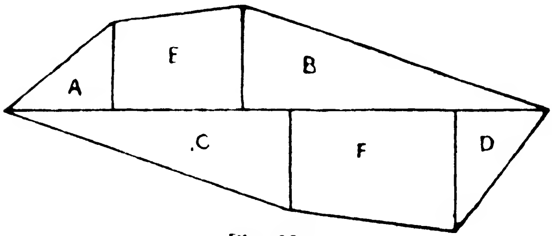


Fig. 324.

Thus rectilinear figures like Figs. 324, 325, are divided up into triangles. Fig. 324 contains four triangles, A, B, C, D, and two trapezoids E, F; and Fig. 325 is divided into two triangles A, B, and one rectangle C

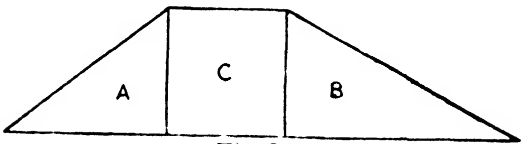


Fig. 325.

To find the area of a circle.

Rule 9.—Square the diameter, and multiply the product by 0·7854.

Or—

Rule 10.—Square the radius, and multiply the product by 3·14159.

Conversely, having the area of a circle given, to find the diameter or the radius.

Rule 11.—Divide the area by 0·7854, and extract the square root of the quotient for the diameter.

Or—

Rule 12.—Divide the area by 3·14159, and extract the square root of the quotient for the radius.

To find the area of an annulus, or circular ring.

Rule 13.—Find the areas of both the inner and the outer circles. Subtract the area of the inner circle from that of the outer circle, and the remainder will be the area of the ring.

To find the area of a sector of a circle.

Rule 14.—As 360° is to the number of degrees in the angle made by the radii of the sector, so is the area of the complete circle to the area of the sector of which it forms a part. Thus if the angle included between the radii B, A, D, in Fig. 326, were 60°, then the area of the sector would be one-sixth of that of the complete circle.

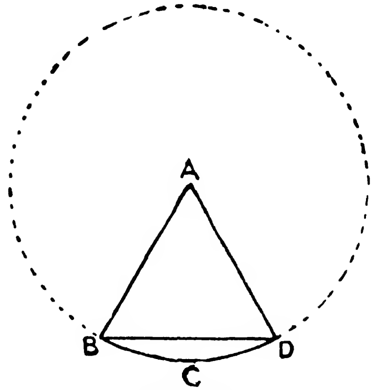


Fig. 326.

The rule for finding the area of a segment of a circle is usually based on the rule just given. Thus—

Rule 15.—Find the area of the sector A, B, C, D, which has the same arc, and then subtract from it the area of the triangle A, B, D, included between the radii of the sector and the chord of the segment.

This is the most convenient way of putting the problem, but it is applicable only when the arc of the segment measures less than 180 degrees.

When the area of a segment greater than the semi-circle is required, then the area of the smaller or supplementary segment is obtained first, and subtracted from the area of the complete circle.

II. The areas of the envelopes of solid, or of hollow bodies, are calculated mainly by the obvious application of rules already given in the preceding section, and of principles laid down in the chapter on the development of surfaces. We will take the principal figures in order.

Any cubical figure or vessel contains six faces (see Fig. 101, p. 44). If the faces are equal, obtain the area of one by multiplying the length by the breadth, and multiply the product by six. If the figure is rectangular only, so that the dimensions of the sides are different from those of the ends, reckon the areas of the sides separately from the areas of the ends, and add the products together.

The area of the enveloping surface of a cylinder comprises a rectangle and two circles. The rectangle Fig. 113, p. 49, is as long as the circumference of the cylinder, and as wide as its height, and the circles are of the same area as the ends of the cylinder. The area of the enveloping surfaces of a segment of a cylinder, Fig. 327, and of a sector of a cylinder, Fig. 328, can be found by the application of previous rules.

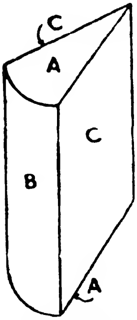


Fig. 327.

The ends A, A of Fig. 327 are segments, the areas of which are found by Rule 14, given in connection with Fig. 326. The curved face B is a rectangle of length equal to the height, and of width equal to the width measured round the arc, the two sides C, C are rectangles also. In Fig. 328, A, A are sectors, the areas of which are found by Rule 15, given in connection with Fig. 326. B is a rectangle, the width of which is obtained by measuring round the arc or curve, and C is a rectangle.

The areas of the enveloping surfaces of a prism are found by the application of foregoing rules. If the ends are triangles the areas are obtained by Rule 3, if rectangles by Rule 1, if polygonal by Rule 8. The sides of prisms are rectangles.

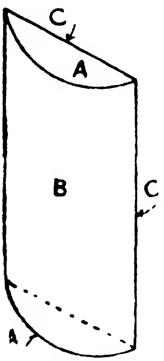


Fig. 328.

The areas of the envelopes of prismoids are found by the application of the rules for rectangles, Rule 1, and for trapezoids, Rule 4.

The areas of the envelopes of pyramids embrace the areas of the bases, which are triangles (see Rule 3), rectangles, or polygons (see Rules 1, 8), and the sides which are triangles, Rule 3.

The envelopes of a frustum of a pyramid embrace the top and base, either triangles, rectangles, or polygons, and the sides which are trapezoids, for the areas of which see Rule 8.

The areas of the envelopes of cones include the base, which is a circle, and the curved surface, which is a sector of a circle (see Fig. 111, p. 49), the area of which is found by Rule 14. Or—

Rule 16.—**Multiply the circumference of the base by the slant height of the cone, and take half the product for the area.**

It is also capable of demonstration that the slant height of a right circular cone bears the same relation to the radius of the base as the area of the curved surface or envelope bears to the area of the base.

The area of the envelope of a frustum of a cone includes that of the base and top, or cut surface, and that of the curved surface. The base and top are circles, the areas of which are obtained by Rule 9. The area of the curved surface is that of a portion of an annulus or circular ring, the area of which can be calculated by Rule 13.

Or by the following method—

Rule 17.—Multiply the sum of the circumferences of the two ends of the frustum by the slant height of the frustum, and take half the product for the area of the curved surface or envelope of the body.

It is also demonstrable that the slant height of a frustum of a right circular cone bears the same proportion to the difference of the radii of the ends as the area of the curved surface bears to the difference of the areas of the ends.

To find the area of the surface of a sphere.

Rule 18.—Square the diameter, and multiply the product by 3·14159.*

To obtain the area of the curved surface of a zone of a sphere, or of a segment of a sphere.

Rule 19.—Multiply the circumference of the sphere by the height of the zone, or segment.

III. The volumes or cubic capacities of hollow or solid bodies are referred to a standard called the cube, as a cubic inch, foot, yard, etc. Hence every dimension in a body must, for convenience of calculation, be brought to the same standard. This

* The area of the surface of a sphere is equal to the area of the surface of a cylinder, the diameter and the height of which are each equal to the diameter of the sphere. Also, the area of the surface of a sphere is equal to four times the area of its diameter.

The latter definition is easily remembered, and is useful in calculating the areas of the hemispheres, because the area of the sheet or disc of metal required for raising a hemisphere must be equal in area to the combined areas of two discs, each equal to the diameter of the hemisphere.

is usually inches in the class of work which forms the subject-matter of the present volume.

To find the volume or contents of any rectangular body or vessel.

Rule 20.—Multiply together the three dimensions, length, breadth, and depth, or height. This is just the same as saying, Multiply the area of the base by the height.

And conversely, knowing the area and contents, the height can be deduced by dividing the contents by the area. Or, knowing the contents and the height, the area of the base can be obtained. The contents divided by the height will give the area.

Rule 21.—The cubic contents of a parallelopiped, of a prism, and of a cylinder, can be obtained by multiplying the area of the base by the height. So, too, the contents of bodies like Figs. 327, 328 can be obtained by the above rule. The area of the base of Fig. 327 is found by Rule 14, and that of Fig. 328 by Rule 15. Having found these areas, multiply them into the height or distance between the ends.

To find the solid contents of a circular ring (Fig. 329).

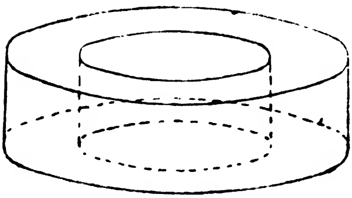


Fig. 329.

Rule 22.—Multiply the area of the ring, as found by Rule 13, by the depth.

To find the contents of any pyramid, or of a cone.

Rule 23.—Multiply the area of the base by the height, and one-third of the product will be the contents.

Conversely, the area of the base, and the height of a pyramid, or of a cone, can be found from the other dimensions.

Knowing the volume, and the area of the base, the height can be found by dividing three times the volume by the area. The area of the base can be found by dividing three times the volume by the height.

The contents of the frustum of a pyramid or of a cone can be found by the following rule.

Rule 24.—Multiply together the areas of the two ends, and take the square root of the product, add the square root so obtained to the two areas. Multiply the sum by the height of the frustum. One-third of the product will be the contents required.

The contents of a prismoid may be found by the following rule.

Rule 25.—Add together the areas of the two ends, and four times the area of a section parallel to the two ends and mid-way between them; multiply the sum by the height. One-sixth of the product will be the volume.

To find the contents of a sphere.

Rule 26.—Multiply the cube of the diameter by 0·5236.

To find the contents of a spherical shell.

Rule 27.—Subtract the cube of the inner diameter from the cube of the outer diameter, and multiply the result by 0·5236.

The contents of the zone of a sphere is found thus:—

Rule 28.—To three times the sum of the squares of the radii of the two ends add the square of the height; multiply the sum by the height, and the product by 0·5236. The result is the contents.

To find the contents of a segment of a sphere.

Rule 29.—To three times the square of the radius of the base add the square of the height, multiply the sum by the height, and the product by 0·5236. The result is the contents.

SIMILAR SOLIDS.

By similar solids is meant solid bodies which, though alike in form, differ in dimensions. An important property of these solids is that their volumes are as the cubes of corresponding lengths, in the case of spheres as of the cubes of their diameters. Hence it is that an apparently slight increase in lineal dimensions increases enormously the volumes or contents of bodies.

The proportions of all vessels are to one another as the cubes of their diameters. We give two examples illustrating this law, and the method in which dimensions are obtained of any sized vessel larger or smaller than one gallon. This applies, however, only to those cases where the utensil is made of a straight strip. For a gallon saucepan the straight strip is 24 inches long by 7 inches wide. Now suppose we require a saucepan to hold 5 quarts. First find the diameter of a gallon saucepan, which equals $24 \div 3{\cdot}1416 = 7{\cdot}639$. Then, because the proportions of all vessels are to one another as the cubes of their diameters, we cube this diameter $= 7{\cdot}639^3$, and this equals 445·768. Now, since the larger vessel is to hold 5 quarts, we multiply 445·768 by $\frac{5}{4}$, this fraction representing the relative proportions of the quantities held by the 5 quart and 1 gallon saucepan. The product obtained is 557·210; and on taking the cube root of this last quantity, we obtain the diameter of the 5-quart vessel. This is found to be 8·22. Multiplying this by 3·1416, we get the length of a strip for a 5-quart saucepan, $8{\cdot}22 \times 3{\cdot}1416 = 25{\cdot}824$, or $25\frac{7}{8}$ inches. The breadth of the strip is obtained by the following proportion :—$7{\cdot}639 : 7 :: 8{\cdot}22 : x$, and x is found to equal 7·53, or $7\frac{1}{2}$ inches. It will be seen that the first and third terms of this proportion are the diameters of the two vessels, while the second (and the fourth when found) are the breadths of the strips. The next example, calculated in precisely the same manner, shows how to obtain the length and breadth of a strip for a 3-quart saucepan. Taking again, as the standard, a strip 24 inches by 7 for a utensil with a capacity of 4 quarts, we obtain the diameter: $24 \div 3{\cdot}1416 = 7{\cdot}639$. Cubing this according to the law, we get 445·768. Now, since the new vessel is to hold 3 quarts, we multiply 445·768 by $\frac{3}{4}$, and the product is 334·326. Extract the cube root as before. $\sqrt[3]{334{\cdot}326} = 6{\cdot}99$, and this is the diameter for the smaller saucepan. 6·99 multiplied by $3{\cdot}1416 = 21{\cdot}959$, and this is the length of the strip. Roughly we may say it will be 22 inches long. For the breadth: $7{\cdot}639 : 7 :: 6{\cdot}99 : x = 6{\cdot}40$, or $6\frac{3}{8}$. 22 inches by $6\frac{3}{8}$ will, therefore, be the required dimensions.

CHAPTER VIII.

METALS AND ALLOYS.

THE properties of the common metals and alloys are well marked; and the different degrees in which these qualities are possessed by the different metals and alloys render each better adapted for certain purposes than the others. These properties are:—

(1) **Metallic lustre.**
(2) **Tenacity.**
(3) **Ductility.**
(4) **Malleability.**
(5) **Conductivity.**
(6) **Fusibility.**
(7) **Specific gravity.**

Each of these qualities is of special value in its place. The capacity for taking a polish in brightening, and planishing, and finishing copper and tinned goods; tenacity, or the strength, of a metal or alloy to resist stress, pressure, pulling, bending in vessels, bars, rods, wires; ductility, or capacity for drawing out, upon which properly the art of wire-drawing is based. Without malleability it would be impossible to roll thin sheets, or to flatten or raise them into curved forms. The good conducting power of metals for heat renders them suitable for warming and domestic purposes, while their power of conducting electricity is a property of equal value, as bearing on wires and plates. Fusibility lies at the basis of all casting; but though the sheet-metal worker is but slighty interested in this branch, a knowledge of the fusibility of alloys is essential to the practice of brazing and soldering. The specific gravities or relative weights of the metals is an important property, even from the point of view of the worker in sheet metals, since all sheets of tin, lead, copper, and zinc are sold by pounds weight to the foot. A few remarks, therefore, by way of comment and explanation of these several qualities, possessed in common by metals and alloys, may, therefore, fitly preface the descriptions of the metals and alloys to follow. Taking :—

(1) **Metallic lustre.**—This is, in fact, nothing more than the power of reflecting light rays. If a surface absorbs light rays largely, the reflection is broken, and the appearance of the surface will not be bright, but dull. A broken or rough surface absorbs and scatters the light rays; a smooth surface, in the sense of being polished, reflects them. A porous substance cannot be polished. For a surface to be capable of taking a polish and becoming lustrous it must be dense, close—or, as we say, hard. Thus no amount of polishing would make the surface of wood lustrous like that of iron, and no amount of polishing would make the surface of iron as lustrous as that of the harder steel. Metals not hard enough in themselves to take a high polish can be rendered harder and more lustrous by the admixture of another metal. Thus, tin and copper in various proportions form speculum metal and bell metal, each extremely hard and lustrous; and so of alloys of other metals.

(2) **Tenacity** is equivalent to strength, or the resistance offered by a body to forces tending to pull its particles asunder. It is measured in pounds or tons per square inch. That is, if the tensile strength of a bar of iron is 20 tons per square inch, that means that a load of 20 tons, suspended at the end of a short bar 1 inch square, in cross section, would just suffice to tear the bar asunder. Tenacity, in this sense of breaking strength, is not of so much relative interest to the sheet-metal worker as it is to the engineer. Still, there are some matters cognate thereto which it is well to be aware of, such as the effect of the presence of impurities, the effect of temperature, the effect of drawing out, etc. In brief, the presence of foreign matters varies, in some cases and in certain proportions tending to increase, in others to diminution of strength. The effect of increase of temperature is to lessen the tenacity of metals, the effect of excessive drawing out is to lessen the tenacity by overcoming the cohesive strength, and replacing the fibrous condition by the crystalline; on the other hand, the tenacity is raised by

moderate drawing out. Steel and iron possess the highest tenacity, while zinc, tin, and lead possess the least.

(3) **Ductility.**—In proportion to the ductility of metals and alloys they are adapted for the purpose of wire-drawing: hence, steel, iron, and copper, being highly ductile, are used for this purpose. Gold, silver, and platinum stand highest in the range of ductility; but their cost precludes their use for any but some special purposes. Tenacity is closely related to ductility, inasmuch as a weak metal will break before it can be reduced to a fine wire. Hence, zinc, tin, and lead, though soft, will not stand drawing down, because their tenacity is so low. Ductile metals become hardened and crystallized during the process of wire-drawing, until they reach the limit of the coherence of their particles. Then annealing becomes necessary. This is effected by heating the metal, and allowing it to cool slowly, the effect of heat being to produce a natural rearrangement of the molecular particles.

(4) **Malleability** is not identical with ductility, though in some respects akin to it. The effect of hammering or rolling is to destroy the cohesion of the particles of metal, to restore which annealing is necessary. The softest metals are not the most malleable, neither are the most tenacious metals the most readily rolled and hammered. Lead and tin are soft; iron and steel are strong, or tenacious; but neither are malleable, as are gold, silver, and copper. Copper is the only really malleable substance used by sheet-metal workers, and that can be hammered into almost any form. Sheet-iron and steel can be bent and rolled, but cannot be raised under the hammer or in dies to anything like the same extent as copper. The malleability of thick metals is generally increased by heat, that of thin metals is not practically affected by it. The malleability of metal lies at the basis of the formation of works in sheet metal. There is an essential difference between the operations of the plater and boilermaker and those of the sheet-metal worker. The materials are largely the same—steel, iron, and copper—but the differences in thickness render the methods of working different. The first-named class of artisans do much of their work by the aid of heat; the second, in the cold. The difference is due to the relative thicknesses of the plates used by the first, and of the

sheets used by the second. A thick plate cannot be bent to a quick curvature unless it is heated; a thin sheet can be bent, or hammered, or stamped, in the cold to almost any outline. The reason of this is readily apparent on a little consideration.

Take a plate of thick metal, a sheet of thin metal, and a sheet of indiarubber, and note the effect of bending in each case. The thick plate can only be bent by the application of much force, assisted, if the curvature be quick, by heat; the thin steel can be bent most readily to the same curvature; the indiarubber also with extreme ease. In each case the effect of bending is to extend the outer layers, and compress the inner layers. The layers in the centre of the plate, or sheet, are neither extended nor compressed, and this central plane of bending is called the neutral axis. The difference in bending thick and thin metal plates is due to the fact that in the first the layers which are in compression and extension are at a considerable distance from the neutral axis, while in thin plates these layers are practically coincident therewith, so that in a thin plate there is no appreciable amount of compression or extension, hence the ease with which they can be bent.

But if the metal in the plates were highly elastic and mobile, like indiarubber, then, even though thick, extension and compression would take place in thick plates as in thin. The effect of heating thick plates is to cause the molecules to move over one another, and to become rearranged permanently, and this not necessarily in a state of high extension or compression, such as would result if the plates had been bent cold, but in a safe and natural way, provided the amount of bending does not exceed the limit which the nature of the material will permit it to sustain. The same kind of thing occurs in thin sheet-metals which are subjected to severe rolling, hammering, or stamping. Some movement and rearrangement of the particles of metal takes place, and the greater the amount of curvature or distortion of form produced, the more severe will be the stresses produced in the substance of the material. If a flat plate is puckered up or raised by hammering, or if it is deeply beaded or dished, or set out, it will be brought into so high a state of tension that it will probably crack, unless heating is resorted to for the purpose of rearranging the particles of metal. It is therefore obvious that the result of hammering, rolling, and stamping is to cause the particles of metal to glide over one another, extending some parts and compressing others, with the frequent coincidence also of thinning down some of the portions which have been subjected to the most severe treatment. If, therefore, the metals did not possess this property of malleability and of ductility, but were such that their particles could not be made to glide one over the other, no irregular metallic forms could be produced by hammering or stamping, but casting would be the only method available for obtaining these forms. We shall see the practical issues of this valuable property possessed by the metals.

(5) **Conductivity** for heat is a property which renders the metals so valuable for heating purposes. The conducting power of metals varies, but it so happens that copper, which is the best conductor among the metals in common use, is also the most malleable. Iron is also an excellent conductor. The thinner the sheets, too, the more rapidly is heat transmitted through their substance. And, moreover, heat is transmitted so quickly through thin malleable sheets that there is no risk of fracture occurring, due to unequal contraction, as there is in many metallic substances.

(6) **Fusibility.**—The melting of iron, steel, copper, and brass does not concern the worker in sheet metal, but the relative fusibilities of the numerous brass, lead, and tin solders are matters of much practical importance to him. These all melt at comparatively low temperatures, and it is essential to know at what temperatures certain solders melt, in order to employ on any given job a solder, the melting point of which is well below that of the material which has to be united. Coke or charcoal fires, jets of gas, and copper bits are used to fuse the various solders employed.

(7) **Specific Gravity.**—The specific gravity of a metal is estimated relatively to that of a given equal bulk of pure water at a given temperature (62°). Beyond the commercial classification of sheets by weight, the relative weights of metals do not concern the sheet-metal worker much.

The manner in which the physical properties of the alloys is affected by small variations in the proportions of their constituents is often remarkable. Malleability, ductility, fusing points, even appearances are often radically modified. Some metals are more readily influenced in this way than others. Among familiar examples we may note the effect which very minute percentages of carbon, phosphorus, and silicon exercise on steel.

Taking very common examples, it is remarkable that the union of two soft and malleable metals, as copper and tin, results in alloys ranging from the tough yellow gun metal to the brittle bell and speculum metals of silvery whiteness. So, too, copper alloyed with the very brittle and crystalline zinc forms the soft, yellow brass, which is bent and cut with so great ease. Or, copper with lead forms an alloy so soft as to be hardly workable. Again, tin and lead alloyed together fuse at a temperature lower than that of either of the constituents—a fact which renders them valuable as solders. And by adopting different proportions, various fusing points higher and lower are obtained, suitable for soldering different qualities of metal or alloy.

Copper Alloys.

Copper is not only highly valuable in the pure state, but its value is even perhaps greater when alloyed in various proportions with tin, lead, zinc, or other metals. It is only necessary to instance gun metal, brass, bell metal, and the solders. The subject of alloys is one of so great interest and value that volumes might be devoted to them. But, in strictness, the subject is of greater interest to the founder than to the metal plate worker. Still, there is very much of interest in it to the latter, since all brass sheets and wires are alloys. All tinning of copper vessels is effected by a union of the surfaces of dissimilar metals; the differences in qualities of sheets and wires depend mainly on the proportions in which certain elements occur. All solders, whether hard or soft, are alloys. So that for these and for kindred reasons a knowledge of the principles which underlie the union of dissimilar metals to form alloys is desirable, and this will indicate the way in which I propose to treat the subject here.

Whether alloys are true chemical compounds has been doubted. At least, they are not recognized as such in science. The reason is, that there is no fixed and definite proportion in which, and in which alone, combination of the metallic elements occurs. In a true chemical compound such is the case. They invariably combine in definite proportions known as their combining weights, or in multiples of those combining weights. But true alloys are formed apart from any such definite combinations, so that one or other of the elements in one alloy shall be in excess by comparison with another alloy of the same metals. It seems, however, as though true chemical combination must take place, but that the compound is mechanically associated with an excess of one or more of the elements. The reason for assuming the existence of a true compound is, that an alloy usually possesses physical characteristics very different from those possessed by its separate elements—a feature in which it closely resembles most true chemical compounds. The strength, tenacity, hardness, and fusing points of alloys are generally higher than those of their constituent elements, in some cases very much higher—effects which do not seem possible by a mere mechanical mixture of elements.

Copper is alloyed with tin, lead, and zinc in various proportions. When alloyed with tin alone it forms the gun metals, bronzes, bell metals, and speculum metal. When alloyed with zinc only it forms various brasses, muntz metal, and spelter solders. Alloyed with lead only, it forms the very common pot metals. Alloyed with tin, zinc, and lead, it forms various gun metals and bronzes.

Alloys of copper with zinc alone are used chiefly to form spelter solder, muntz metal, and some brasses. Copper and zinc mix in all proportions; but exact proportions are difficult to determine, because zinc volatilizes readily. The fusibility of copper-zinc alloys increases with the proportion of zinc. The colour ranges, with the successive additions of zinc, from the red of copper to silvery white, and the malleability decreases until a crystalline character prevails. The table in the appendix may be consulted for proportions, but the following remarks are generally applicable to the copper-zinc alloys.

An alloy of about 1 oz. of zinc to 16 ozs. of copper is used for cheap jewellery; one of 3 ozs. to 4 ozs. of zinc to 16 ozs. of copper for sundry alloys once known as Bath metal or pinchbeck; alloys of from about 6 ozs. to 8 ozs. of zinc to 16 ozs. of copper form common brass, the latter being slightly more fusible than the former. Muntz metal is formed of 40 parts of zinc to 60 of copper, but other proportions mix well. Equal parts of zinc and copper form soft spelter solder; or, 12 ozs. or 14 ozs. of zinc to 16 ozs. of copper would probably be the ultimate proportions after volatilization.

Copper and tin also mix in all proportions, successive additions of tin increase the fusibility of the alloy, the malleability diminishes, and the colour gradually changes from red to white.

Copper-tin or gun metal alloys range from about 1 oz. of tin to 16 ozs. of copper in the softest; to 2 ozs. or $2\frac{1}{2}$ ozs. of tin to 16 ozs. of copper in the hardest. Beyond the last proportion, up to 5 ozs. of tin to 16 ozs. of copper, range the bell metal alloys; from $7\frac{1}{4}$ ozs. to $8\frac{1}{4}$ ozs. of tin to 16 ozs. of copper form speculum metal.

The alloys of copper with lead alone are used in the cheap pot metals. The fusibility is increased with successive additions of lead, the malleability is soon lost, and the red colour of copper gives place to a leaden hue. About 6 ozs. of lead to 16 ozs. of copper is the limit at which a true alloy can be formed; with an increase in the proportion of lead the latter separates in cooling.

Alloys of copper with zinc, tin, and lead are largely used under the names of brasses, bronzes, gun metals, and pot metal. There is practically no limit to the range of these alloys. For some few proportions the table in the appendix may be consulted.

Speaking generally, those alloys are not proportioned separately, but the copper is added to a brass alloy. In many mixtures lead is not used at all, but copper, tin, and zinc only. Antimony is also sometimes used. A little iron added to yellow brass hardens it. Lead, on the contrary, makes it more malleable. Zinc added to a pure mixture of copper and tin (gun metal) makes it mix better, and increases the malleability. Pot metal is improved by the addition of a little tin, and also of antimony.

Pewter.

Pewter is chiefly tin alloyed with one-fifth to one-sixth part of lead. Sometimes, however, tin and copper are used, in which case the copper may be less than one per cent. To make such a mixture a preparation called "temper" is first obtained by melting two parts of tin to one of copper, and by adding a small quantity of this "temper" to a large quantity of tin, a perfect mixture of the metals is secured. Pewter should not contain a greater proportion of lead than one part to four of tin, with specific gravity of 7·8. If the quantity of lead exceeds this amount, the specific gravity will be higher. With an excess of lead, the latter is apt to be dissolved out by the action of acid, and contaminate the liquids drank out of the pewter vessels.

Britannia Metal.

Britannia metal, sometimes called "white metal," is an alloy of variable composition, of which tin forms the principal element; used for teapots and spoons chiefly. Its manufacture was introduced about 1770. It is cast into ingots, and rolled into sheets. Its typical composition is given as 3½ cwt. of block tin, 28 lbs. of antimony, 8 lbs. of copper, and 8 lbs. of brass. The effect of the antimony is to give hardness.

CHAPTER IX.

FLATTENING, RAISING, STAMPING, SPINNING, BENDING.

THE operations of flattening, raising, stamping, spinning, and bending sheet metals depend for their success upon the malleability of the metals so heated. There are few vessels the figures of which are so complicated and intricate that they cannot be worked into shape. But a high degree of skill is required in some of this class of work, and the best work must necessarily be done by hand.

The flattening of thin sheet metals is effected as in principle the other operations are performed, by causing certain parts of the metal to glide or spread over other parts immediately adjacent, so that parts which are tense by comparison with parts adjacent become loosened and extended to approximate to the condition of the parts adjacent, until all parts are in an equal condition of tension. This is done by hammering the parts in tension, by means of what are termed "solid blows," so spreading or extending the metal laterally, and allowing the bulged or loose parts room to expand or spread out. It would not do to hammer the loose or bulged portions, because that would increase the bulge by spreading or enlarging the area or curvature of the bulged portion. The metal there is bulged because it is prevented from expanding by reason of the tight parts adjacent, and the only way in which the bulge can be removed is by just giving it room to spread out, and this can only be effected by removing the excess of tension from the parts adjacent, so allowing them to spread out away from the bulge, leaving the latter free to expand into a true plane. A very minute amount of bulging is sufficient to cause buckle in a plate, the term "buckle" being commonly used to signify the condition of local tension and bulging; but however minute the degree of difference, nothing save the removal of the local tensions by hammering or by rolling will produce a true and level plate. The condition of buckle, even when slight, is easily recognized. When of considerable extent, it is recognized by the straight-edge, or by casting the eye across the surface. Even when slight in extent, it can be recognized by the bending of the plate backwards and forwards, when if buckled there is a lack of elasticity evident to the practised hand and ear. There is the feeling that the continuity of the plate is broken, and a whip-like sound as of crackling or flapping.

The term "solid," or "opposed blows," signifies this—that the blows are delivered upon the plate between two hard, unyielding, and strictly opposed surfaces, as the face of the hammer and the face of the anvil. These blows invariably compress, and thin, and extend, or spread, and also harden the metal. The term is used in opposition to "hollow blows," in which the metal is struck upon a yielding body, or with no body beneath it; the effect in this case being the reverse of the previous, the metal being bent, thrown up, or thickened. In much raised work the blows are of a dual character, partly solid, partly hollow, as when it is desired to produce curvatures without altering the thickness of the metal.

The formation of curved, dished, or hollow vessels by the process of raising depends for results upon the malleability of metals. If a metal or alloy were rigid in the same sense or degree that a piece of cast-iron or tempered steel is rigid, then the formation of curved surfaces by hammering would be impracticable. Cast-iron and tempered steel, we say, are not malleable, that is, not from the practical point of view. In other words, the particles of which they are composed are so hard, rigid, and crystalline that they possess no faculty of relative movement over one another, no property of gliding, or viscosity. Therefore they do not yield under the hammer, but fracture only. The property of malleability possessed eminently by copper, and in a lesser degree by some other metals and alloys, is one to which there appears to be no limit in practice. I am not now alluding to the extreme tenuity to which sheets can be brought by hammering, but to the alterations in form which are practicable by the simple process of hammering, provided due annealing is resorted to, sufficient in amount to counteract the brittleness induced by hammering, and restore the original ductility. In hammering, the metal is thinned and thickened alternately—now spread out, now thrown up again at the will of the workman, and almost as if by instinct. The first operation in raising a dish, a flaring rim, a hemisphere, is the creation of a series of puckers or wrinkles, as they are termed. This is the result of trying to bring a larger circle into the circumference of a smaller one, the metal possesses a sufficient rigidity to resist, and becomes waved in consequence round the edges. But these wrinkles are obliterated by the subsequent process of razing, in which the projecting flutes are set down in detail, and made to glide into the adjoining depressions, with the result that a general curve, more or less regular, is formed.

In doing such work, the blows delivered partake of the solid or dead blow, and the hollow or elastic character, the result being that the metal is not permanently thinned or thickened in different localities, but its thickness is averaged about equal all over alike. It is difficult to describe the process clearly; it is a matter of intelligent dexterity, which the workman is better able to perform than a writer to describe. The principles, however, are those just laid down.

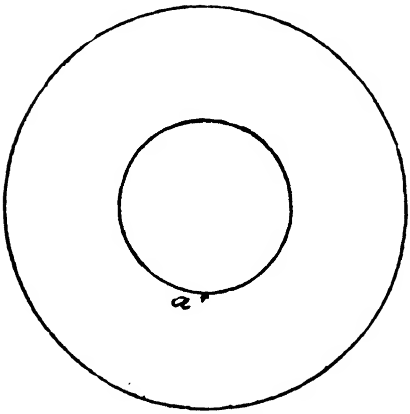


Fig. 330.

The details of raising hollow works are very numerous. In the case of a vessel with flaring sides, it is usual to mark a circle to indicate where the dishing is to commence. Fig. 330 shows such a circle (*a*) marked on a disc for a pan; Fig. 331, one marked and partly worked on a cylinder for a cylindrical bell-mouthed vessel. The dishing or hollowing for a sheet like Fig. 330 is often commenced in a hollowing block (Fig. 332), as in the case of a hemispherical or dished object. The metal is first beaten roughly down, into the hollow most suitable to the form required, which wrinkles or flutes the edges, after which the razing down commences. In many cases the wrinkling is done over the edge of a suitable

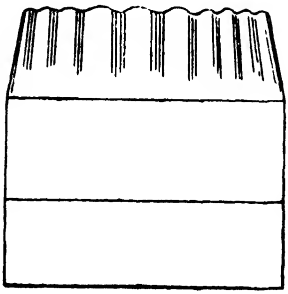


Fig. 331.

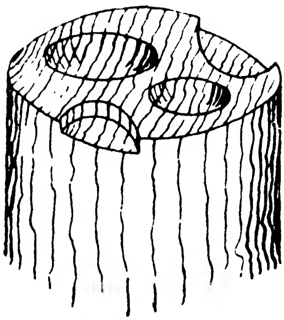


Fig. 332.

stake, the metal being bent with hammer or mallet. In all but the shallowest work, the bending is done in courses, or narrow circles, or curves, the metal being annealed after the formation of each course. Frequently in repetitive work several similar pieces are hollowed at once, so saving time. The pieces are secured together by the outer sheets being prolonged to form clips for the temporary embracement of the inner sheets. The accuracy of the work is facilitated by turning the sheets round upon one another from time to time, in order to correct inequalities.

The raising is done in courses or circles, and the term "raising a course" is applied to each successive operation of this kind. The effect is the same as when metal is spun in the lathe, only that the hammer takes the place of the burnisher. In each case the metal is stretched, and thinned, or thrown up, and thickened—becoming accommodated to the new form imposed upon it by burnisher or hammer. Thus, to take a familiar example, that of a water-ball 9 ins. in diameter. The area of the surface of a sphere 9 ins. diameter is—

$$9 \times 9 \times 3{\cdot}14159 = 254 \text{ ins.},$$

half of which is $\frac{254}{2} = 127$ ins.; 127 ins. is therefore the area of the disc required, neglecting the allowance for the seam, to form one-half the ball. Hence—

$$\sqrt{\frac{127}{0{\cdot}7854}} = 12\tfrac{3}{4}$$

the diameter of the disc required. The circumference of the 9-in. ball is 2 ft. $4\frac{1}{4}$ ins.; the circumference of the disc of $12\frac{3}{4}$ ins. diameter is 3 ft. 4 ins. Now, although the superficial amount of metal in the disc is just equal to that in the hemisphere to be formed from it, yet it is evident that the disposition of the metal must be altered. For the disc circumference of 3 ft. 4 ins. will be contracted to the hemispherical circumference of 2 ft. $4\frac{1}{4}$ ins., and the diameter of the disc of $12\frac{3}{4}$ ins. will be extended to the hemispherical curve of 1 ft. $2\frac{1}{8}$ ins. This is shown graphically in Fig. 333, in which A represents the size of the disc, B that of the hemisphere in elevation, and C the hemisphere in plan. The circumference of A has to be contracted to that of C; but, at the same time, the distance across A has to be extended to form the curved surface B′ of B. Obviously, therefore, in raised metal work, a rearrangement of the particles of metal must take place, the relative disposition of its molecules must be changed, notwithstanding that the total area remains the same, or nearly the same, and the thicknesses should also be practically the same all over. In all work of this character, therefore, the piece of metal required

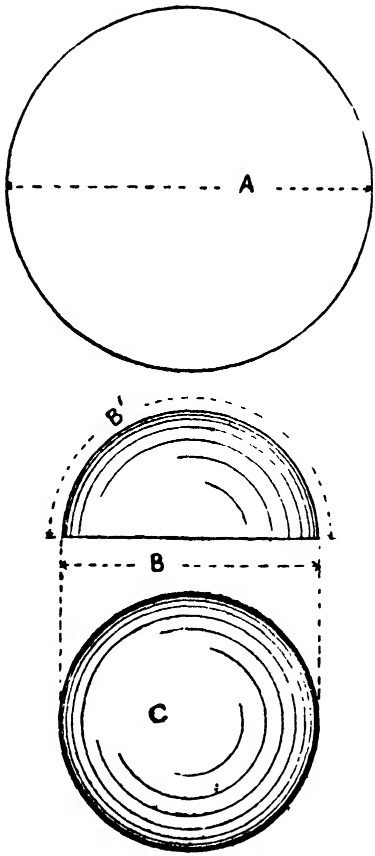


Fig. 333.

to produce a given form should be cut to approximately the same area as that of the finished work, so that there shall be no excess and consequent waste or shortness of material, or inequality of thickness. The rules given for mensuration in Chapter VII. will be found of value for calculations of this kind. Fig. 334 shows an early stage of the process of raising such a disc as Fig. 330, or Fig. 333; and Fig. 335 illustrates on an enlarged scale another stage in which a few of the flutes are shown being razed out. In raising work in successive courses thus, the wrinkles or flutes which are outside in one course are made inside in the next course, in order that the hammer shall work equally on the inside and outside of the vessel. After a vessel has been raised, the marks left by the hammers are obliterated, either with a smooth-faced hammer or with a wooden mallet.

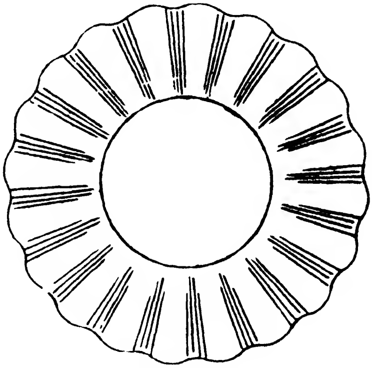


Fig. 334.

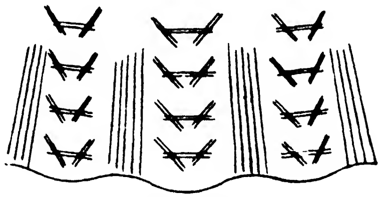


Fig. 335.

In the art of metal spinning, the thin sheet of malleable metal or alloy, cut to the form of a flat disc at first, is bent into concentric curves by means of gentle and continuous pressure applied with a blunt but perfectly smooth burnisher held against the disc during its revolution in the lathe. The burnisher thins and spreads and thickens according as it is moved from centre to circumference of the spinning sheet, or the reverse way, both movements being made to alternate, at will, according as the metal requires spreading, or thickening. The pressure is necessarily heavy. There are only three or four types of burnishers used; one, a plain round rod with a rounding end; another having its end shaped of a more or less globular form; another curved something like an engineer's curved burnisher; another flattened and rounded at the end. These are made of hardened steel, perfectly polished in order not to scratch the work. The lathe is of special construction (see Fig. 465, p. 256). Wooden chucks or forms are turned of suitable shapes for jobs in hand, and mounted on the face-plate of the headstock, and the thin sheet metal is gradually forced to take the outlines of the forms by continuous pressure exerted by the burnisher. Much of the work is done by a double-action of burnishers, and sticks or rubbers acting on opposite sides of the sheet, so bending it to double curvatures, and lessening the tendency of the work to chatter or wobble. The lathe-rest is fitted with pins, which afford fulcra for the different positions of the burnisher and rubber, and the work is held by pressure on the centre between the chuck and a holding-block, cupped holder, or other suitable means thrust against it by the poppet mandrel. As the work is advanced the burnisher is moved along, taking its leverage from successive pins as suitable fulcra, from which to operate the sweep of the burnisher forward and backward.

In the work of spinning, the metal is stretched and bent as in bending rolls, the bending and stretching taking place, however, in concentric circles instead of in a cylindrical form. Very intricate shapes can be produced in this way, including the turning over and wiring of edges. It is, however, practicable only with very thin sheets. When comparatively thick sheets have to be operated on, then quick curvatures can only be imparted by hammering or by stamping. Spinning is frequently resorted to in order to finish work already stamped roughly to shape in dies. Thus the common hand-bowls used for washing are made by first stamping in a press, which brings them nearly into shape, but leaves a lot of pucker marks from the stamps. The bowls are then put in the spinning lathe, and the puckers all removed with steel burnishers. The concentric rings seen on these bowls when new are the marks left by the burnishing tool.

Spun work of awkward outlines, in which there are considerable differences in diameter, is usually made in two or more pieces, spun separately and soldered together. The easiest material to work is Britannia metal. Copper and brass require to be annealed, if the work done on them is considerable. The surface of the metal has to be greased or oiled previous to applying the burnisher, and the work runs at high speeds—dependent on diameter. The forms on which the work is spun are conveniently made of wood, but sometimes also of metal.

The bending of thin sheets is done by hand, and by machine. Sheets are stamped to almost any outlines in dies by machine. Hand operations are performed by means of stakes, beak irons, creasers, formers, and in rolls of various kinds.

Thin sheet metals are bent to sharp angles over the head of the hatchet stake shown in Fig. 378, p. 203, the hammer or mallet being used to set the metal first, after which the sheets would be bent down with the hand. For some jobs the sheet would be pinched in the vice and bent over. For bending large strips into curves of moderate radius, a former is used. This is of wood about two feet in length, and from two to four inches in diameter. It is shouldered to be held against the chest of the workman, and the edge of the bench, in a horizontal position. The strip of metal is bent by hand over this, first in diagonal directions, and then square across until the required curve is imparted to it. Small tubes and beadings, when made by hand, say for special jobs, are bent in the crease shown in Fig. 392B, p. 206. The metal is laid over a suitable groove, beaten down into it with a wire and mallet, and the top edges folded and hammered down over the wire with a hollow punch. Large tubes are worked upon the beak irons shown in Fig. 384, p. 204. The larger tubes are bent in the bending rolls shown in Fig. 418, p. 215. There are two general types of these rolls, depending on the position of the adjustable roll. Fig. 336 shows the arrangement generally adopted for thin sheet metals, Fig. 337 that for thick, or engineer's work.

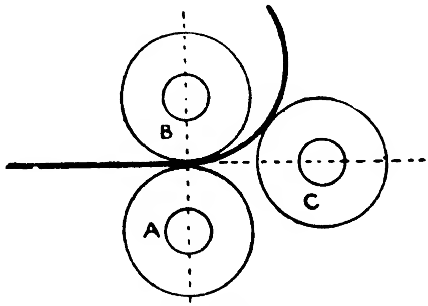


Fig. 336.

In both the principle of operation

is the same, only the position of the adjustable roll is different.

Fig. 337 shows a section through the common engineer's rolls. AA are fixed rolls, that is, their centres are unalterable. B is an adjustable roll, being capable of movement upwards and downwards. C is the plate which is being bent. It is clear that by depressing roll B, the curvature imparted to C will be greater, and that by raising B the curvature imparted to C will be less; so that by the simple raising and depressing of the roll B, any curvatures greater than the diameter of B can be imparted to C. So, likewise, in Fig. 336, A and B are the fixed rolls, corresponding with AA, in Fig. 337, only B is capable of a very minute amount of adjustment vertically, in order to enable the sheets to be gripped between A and B, but C is the adjustable roll, corresponding with B in Fig. 337. It is adjustable vertically, and the effect is the same as in Fig. 337, namely, the diminution or increase in the radius of the curve. Raising roll C produces a curve of less radius, depressing it produces a curve of greater radius. There is a narrow portion of the sheet at the edge which does not become curved, and this has to be set down with a hammer or mallet subsequently.

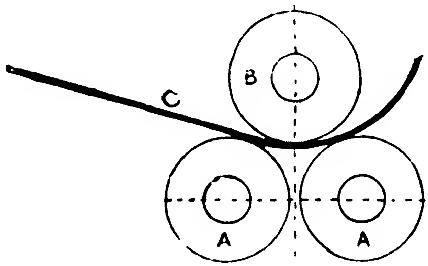


Fig. 337.

Sheets are straightened also in bending rolls unless they are very thin. It is clear that the adjustable roll can in each case be so placed in relation to the others that a sheet can be passed through in a curve of very large radius, or in a straight line. In Fig. 337 this can be done by raising roll B until its lower edge is in the same plane as the top edges of AA; in Fig. 336, by lowering roll C until its top edge is in line with the top edge of A. And this is how sheets are straightened in the bending rolls. It is not done at one pass, but the sheets are first bent to moderately quick curvatures, first in one direction, then in the reverse direction, and after each pass the position of the adjustable roll is altered to diminish the curvature until at length, when the final passes are taken, the sheets are practically flat, kinks and buckles having been removed by the processes of rolling. For engineer's heavy plates there is a special flattening machine made with a series of rolls between which plates are straightened, but such are not used by sheet-metal workers.

The principles and methods of rolling involved in Figs. 336, 337 are extended to include other rollers besides those which are of plain cylindrical form devised for the bending of sheets. Rolls having suitable contours are used to a very large extent for producing corrugations, beadings, mouldings, and such like.

Small copper pipes, tubes, and spouts, are readily bent into any curves without wrinkling, if they are first filled with lead. One end of the pipe is closed with thick brown paper, and the pipe laid in a box of damp sand, while the lead is being poured in. The lead must be soft. A bit of iron rod is cast in with the lead, its end standing out at a distance of a few inches to afford the necessary leverage for bending the pipe. This, of course, is melted out after the bending is done. The bending is variously effected with a mallet, or with leverage, or with both in combination. Before running the lead out the work should be covered with a solution of whiting in water. Copper pipe is also filled with resin before bending. Lead is better for quick bends, resin for long ones. Only the part to be bent and that immediately beyond need be filled, a bullet, or wad of paper, or cotton waste being inserted at the locality beyond which the filling material is not required. The part which has to be bent must be annealed first to a cherry red, in daylight. Portions which have to be left straight must be left unannealed, or hard.

There are many methods and rigs-up adopted for bending copper pipes. Much depends on the size of the pipe. Up to about five inches diameter the leverage of manual labour is sufficient, but above that hydraulic power is generally employed. The bending is always done by leverage or pressure, never by hammering. In all coppersmiths' shops there is a strong bending-block sunk in the floor for the purpose of pipe-bending.

It is of cast-iron about twelve inches square, and standing up to about the ordinary height of a workbench. It is made (Fig. 338) to receive the various attachments required for pipe-bending. The top of the block is shouldered down to receive a strap which confines a bending-block, called a lead piece. The latter is a stout plate of lead with a hole or holes in it for the insertion of pipes. The lead being soft does not bruise the pipe which is being bent. It is secured with the strap. On one side of the block a back plate is inserted with pins fitting into the holes cast in the block, which affords an essential point of leverage in the bending of pipes. Holes are cast in the top of the block to receive pins which also form suitable points of leverage.

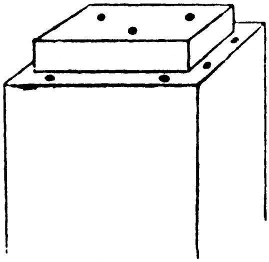


Fig. 338.

Most work in sheet copper is planished at some stage or other. The object of planishing proper is to close and harden the grain of the metal, taking the limpness out of it, and to make it more elastic and rigid so that it will retain its shape. Often this operation is performed before any work is done upon the sheets, in order to make them stiff enough to work upon. Often it is done at a later stage. The art consists in hammering over the whole surface in detail until every portion has been subjected to the hardening effect of the hammer blows. The hammering is done in straight lines, or in concentric curves, depending on the nature of the work. The planishing is done in a bottom stake, fixed in the floor-block, or on a level block of metal. Various hammers are used for different work. Copper goods are polished with a file first, followed by emery cloth applied on a stick, then by fine emery, rubbed on with hempen rope, wrapped round with a single hitch, and drawn to and fro, and finally with a metal burnisher and sweet oil. This is usually made like the engineer's similar tool, from an old file.

Pitching signifies the setting out or raising of a portion of a vessel, as the depth of a cover, or a flange, or an end of a tube. A belge is a bellied portion of a piece of work. A lag is a sharp bend in a vessel, as at the bottom of a kettle or gluepot, or similar article. To bend the lag is termed to stag it. Poppling denotes the imparting of a very slight concavity to a disc of sheet metal, as in the bottoms of some vessels, so drawing them in or raising them to a small degree. It is done by a few hammer blows. "Flaring" is used in the same sense as "flue," to signify the conical or tapered forms of vessels.

CHAPTER X.

JOINTS.

The commoner joints used by sheet-metal workers are shown in the figures adjoining. Some are used much more frequently than others. Some are suited for thin sheets only, some for thicker ones, some are weak, others are strong.

Fig. 339, A, is a lap joint suitable for thin sheets soldered or brazed together. The edges are thinned down or scarfed to a feather edge and overlapped, and the solder or spelter is run in, and the edges may be smoothed subsequently. It is used for flat work, and for tubes. Fig. 339, B, is a lap joint in which the edges are not thinned down, but left from the shears, and the angles filled up with solder, a method adopted in the thinnest sheets.

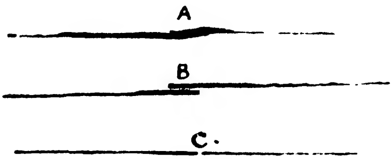


Fig 339.

Fig. 339, C, shows a butt joint, in which the edges are simply brought together, and soldered, or brazed. It is stronger if made with spelter than with solder, but is not strong unless the sheets are of moderate thickness.

Forms of butt joints used when the sheets to be joined are

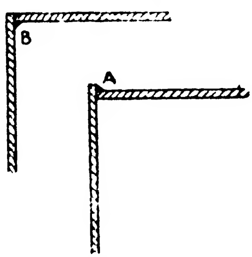


Fig. 340.

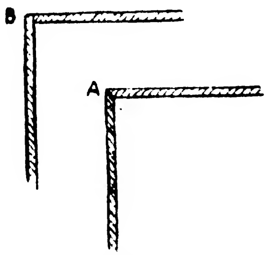


Fig. 341.

not in line, but at right or other angles, are shown in succeeding figures. The joints in Figs. 340, 341, are all made by butting and soldering. In Fig. 340, A, the solder is outside, an edge of one of the sheets being prolonged sufficiently to make an angle for the solder. In Fig. 340, B, the solder is along the inner angle. This is a common joint. In Fig. 341, A, the sheets butt with a mitre joint, and the solder is between the faces only, as in hard soldering, or along the inner angle, if soft soldered. In Fig. 341, B, the joint is brazed, and there is no spelter along the inner angle, as there is in the angle of the soldered joints in Fig. 340, B.

Fig. 342 shows lap joints of various types. Thus, in Fig. 342,

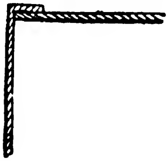


Fig. 342.

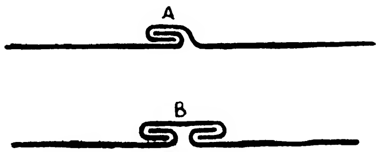


Fig. 343.

one sheet is bent for the other to lie against, and solder is run in, and a ridge of solder is left down the angle.

Very many of the strongest lap joints are made by folding over the edges one upon another, and closing with the hammer, with or without the addition of solder. Examples are given in Fig. 343. Fig. 343, A, is a lap joint. It is not soldered, not being required to be water-tight, but the folded edges are closed by hammering flat with a seam set. Fig. 343, B, is another form of lap joint, also closed by flattening. Figs. 344 are used for sheets at right or other angles, the first, A, being soldered; the second, B, not soldered. The first would slide asunder without solder, the second could only slide endwise. C is also soldered. Fig. 345, A, is a roll joint used for lead-roofing sheets. Fig. 345, B, is that used for zinc, which will not bend so readily as lead. Neither is soldered, and the water will not get through unless it rises above the top of the wood roll. The form of seam

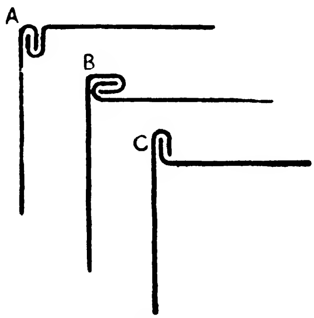


Fig. 344.

shown in Fig. 346 is a locked joint, formed by the folding over of seams upon one another, and closing by hammering, without solder. It is used for sheet-metal tubing, for stoves, etc.

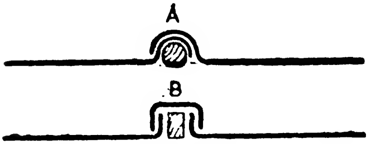


Fig. 345.

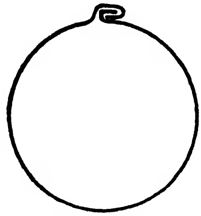


Fig. 346.

Such seams are formed by machine now, but years ago they were all done by hand, as they are still in some little shops and for some special jobs. The edges were first bent over a hatchet stake, and then folded down over a straight edge laid across the bench (Fig. 347). The pipes were bent over a bending bar secured parallel with the edge of the workbench, the sheets being bent between the bar and the edge of the bench, the bar being the fulcrum over which the pipes were bent in detail. But such work is done in rolls now more expeditiously and accurately.

The strongest joint possible in thin sheet-metal work is that which is cramped and brazed. It is employed for the longitudinal joints of many cylindrical vessels in which great strength is wanted,

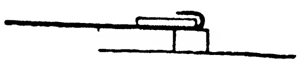


Fig. 347.

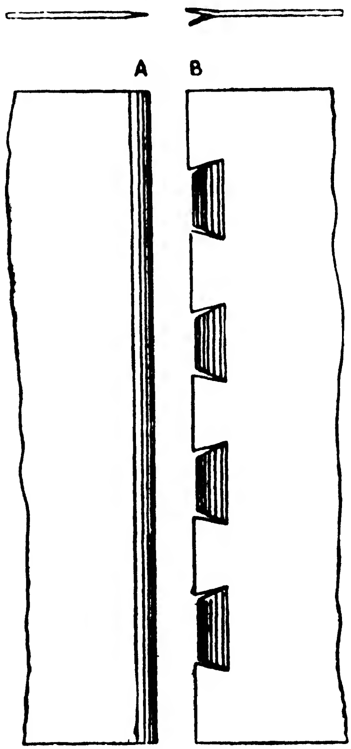


Fig. 348.

or which are exposed to great heat. It is employed for straight and bend pipes, for the circular bottoms of many hollow vessels, and much work besides.

In Fig. 348 the method of qreparing a straight cramped joint is shown; in Figs. 349, 351, one for a circular bottom. The method of making these cramps is as follows:—

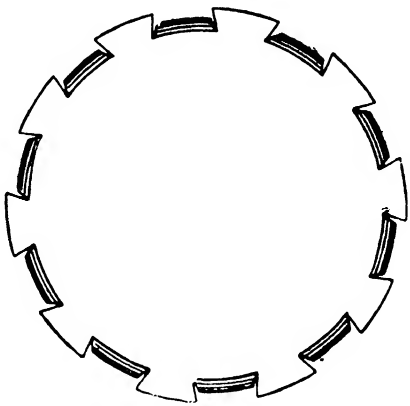


Fig. 349.

One edge is simply thinned down with the hammer (Fig. 348), and is left plain, A. The other, B, is thinned, and notched dove-tail fashion, or obliquely, and the cramps thus separated are turned upwards and downwards alternately, and the thinned plain edge A is inserted between the vees formed by the cramp B. The cramps are then hammered down upon the plain edge, and brazed, and the joint is subsequently dressed off, and finished neatly.

The notching of the cramps is done with shears or snips, and the depth will vary from about $\frac{3}{16}$ to $\frac{3}{8}$ of an inch, dependent on the substance of the plate. The insertion of the plain edge within the cramped edge is as readily effected in the case of a circular disc (Figs. 349—351) as in that of the longitudinal joint of a cylindrical body. Fig. 349 shows such a bottom in plan, Fig. 350 in perspective, ready to fit into the bottom of the vessel in Fig. 351. Half the cramps go within and half without the vessel, those which go without being turned upwards, as shown in Fig. 350, to pass through the opening. After insertion, they are hammered down on the outside and inside of the opening.

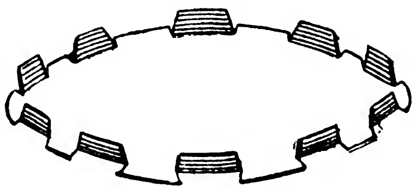


Fig. 350.

After the cramps are hammered down upon the plain edge, the joint is wetted with borax dissolved in water, and spelter is run around the course of the cramps, and melted into the joint.

The cleaning is done with a file, and any inequalities in the seam are knocked down on a suitable stake until the seam is of the same thickness as the sheet metal, after which the work requires annealing.

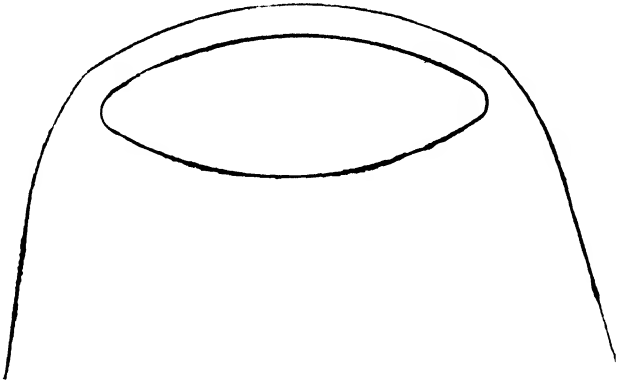


Fig. 351.

Fig. 352 shows a straight piece of pipe cramped and secured with binding wire in readiness for brazing.

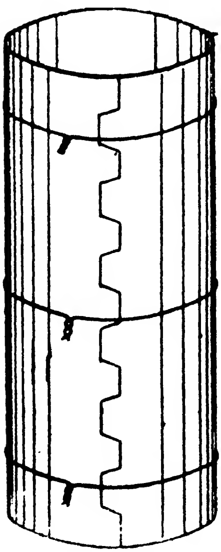


Fig. 352.

Bend pipes in copper are often made up, not from pipes bent, but from sheets bent, and cramped. The seams may be on the sides, or on the throat and back. The cramped joint is used in both cases.

Pipes made of heavy sheet are thinned only and lapped, not cramped, but light sheets must be cramped. If thinned and lapped, the thinning must be done on opposite sides of the strip.

To chatter a cramped joint means to shake and loosen it slightly by hammering, so that the seams shall be slightly opened, or just sufficiently to permit of the spelter running between the joints of the cramps.

The same effect is produced in the cramped joints of circular discs by springing the disc inwards and outwards several times.

Copper steam-pipes when straight are made with one longitudinal seam, scarfed or thinned down at the edges and brazed. Bends are made with two seams, one on each side. As the pipes are heated from the outside, the difficulty of brazing increases with the thickness, due to risk of overheating the outside before the inside is raised to a temperature sufficiently high. It is usual to make bends of material one gauge thicker than straight pipes, in order to compensate for the thinning of the metal which occurs during the extension which is caused by bending. The only part of the bend which remains of its original thickness is the inner curve; the sides, and the metal at and around the larger curve, becoming thinned down by the bending.

Brazed joints in copper pipes are made as shown in Figs. 353, 356. One type is called a flush-joint (Figs. 353, 355), used when the pipe is required parallel outside; and the other the socket

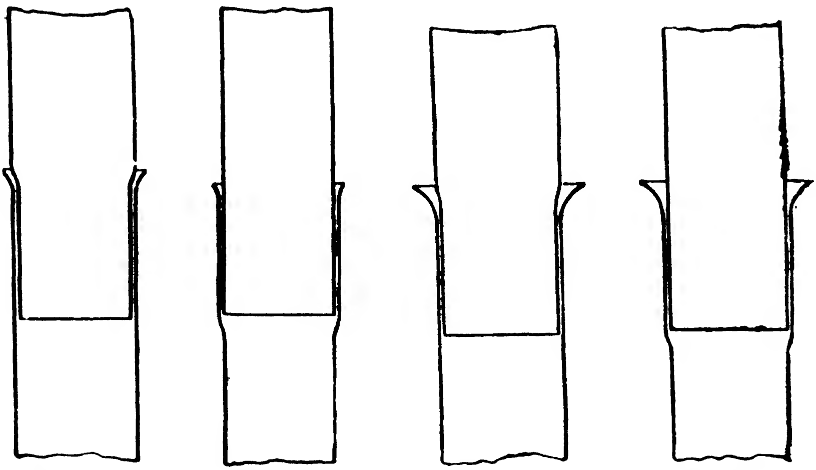


Fig. 353. Fig. 354. Fig. 355. Fig. 356.

joint (Figs. 354, 356), used when the pipe is required parallel inside. In the flush joint for soft soldering (Fig. 353), one pipe is reduced in diameter, forming a male end to fit the outer pipe. The latter is slightly bell-mouthed in order to provide a groove to prevent the solder from running over and down the outside. In the socket joint (Figs. 354, 356), the inner pipe is left parallel, and the outer pipe enlarged to embrace it, and also slightly bell-mouthed to hold the solder. The flush joint

(Fig. 355) for brazing is similar to that for soldering in Fig. 353; but the bell-mouthed end is larger, in order that it may hold sufficient spelter to fill the joint. The same remark applies to the socket joint (Fig. 356) for brazing. The fitting of the ends must not be too tight, but there must be sufficient freedom of fitting to permit of the spelter or solder forming a thin film between the two. The joints are cleaned by insertion in a strong solution of salt and water. They are then heated to a cherry-red and plunged into water. Afterwards they are scoured with sand and water and dried. The ends which have to be soft soldered are tinned first.

Fig. 357 shows a type of joint used when brazing a branch or outlet on a copper pipe. The outlet is slightly flanged to fit the main pipe, and the opening in the latter is slightly burred or bell-mouthed to fit a little way within the outlet. The two are then wired together and brazed. Fig. 358 shows a form of joint between a funnel and its spout. The end of the spout is flanged slightly, and the end of the funnel collared to match. The spout is then put in, and the joint closed with

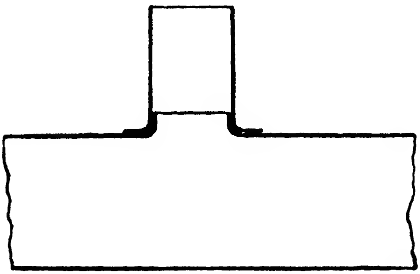


Fig. 357.

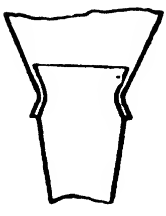


Fig. 358.

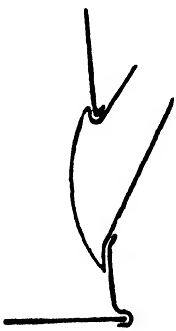


Fig. 359.

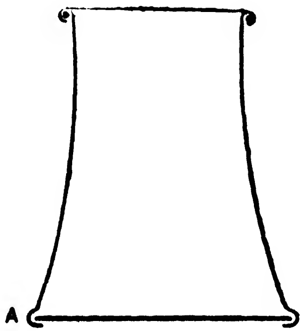


Fig. 360.

the hammer, and soldered. A similar type of joint is made use of between the spout of a coffee pot and the body of the pot (Fig. 359).

Fig. 360 shows a method of insertion of a bottom in a cylindrical vessel, and also the wiring of the edge. A groove or crease is partly formed, A, with a creasing-hammer (Fig. 407, p. 209) on a creasing-iron (Fig. 392B, p. 206), and the bottom is laid in this, and the crease closed over and soldered in, the solder being run either from the inside or outside.

Riveting as done by sheet-metal workers differs in some respects from that done by engineers' boiler-makers. The difference is due mainly to the difference in the thicknesses of sheets and plates. When the boiler-maker's plates are to be riveted, the closing-up of the rivet is often sufficient to ensure a close joint between the plates. Still, in good work, it is also usual to ensure the closing of the plates first, either by hydraulic pressure, or by hammering. Then, again, riveted joints in thick plates are caulked along the edges. But with thin sheet metals, caulking is not practicable, and the use of cement, or of insertion layers, is not desirable, nor usual in good work. Hence the joints of the sheets are carefully closed before riveting up, by a process called scrubbing. This consists in hammering the sheets all around the rivet tail in lines tangential to the rivet (Fig. 361), so closing the plates down around the rivet, and drawing the rivet tail up through the plate, as far as practicable. The cross pane or scrubbing hammer (Fig. 402) is used for the purpose. The tails of the rivets are generally finished of an octagonal conical form (Fig. 362), instead of cup-headed like engineers' rivets are usually done.

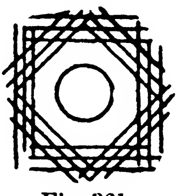


Fig. 361.

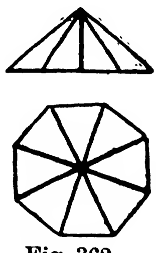


Fig. 362.

With the methods of union adopted by the engineer, the sheet-metal worker has, as a rule, little to do. It occasionally happens, however, that a heavy piece of work has to be done in sheet-iron, or perhaps thick copper, for which soldering and brazing are unsuitable, and in which riveting is the only practicable method. The methods of union then adopted are shown in Figs. 363, 365.

Fig. 363, A, shows a lap joint, single riveted, for uniting sheets approximately in one plane for flat surfaces, or for large tubes. No thinning down of the edges is ever done, but the square edges stand up in the finished work. Fig. 363, B, is a butt joint, ringle riveted. In this the faces of the sheets are continuous. Sometimes two strips are used, one inside, one outside; but this is seldom if ever done by sheet-metal workers, being only necessary in the case of vessels which have to stand very high pressure, as steam boilers. So, too, the method of double riveting—that is, the adoption of two parallel rows of rivets in a seam instead of one—is only resorted to in the case of vessels subject to high pressures.

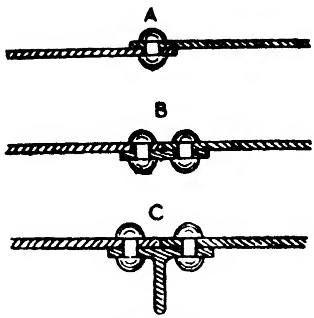


Fig. 363.

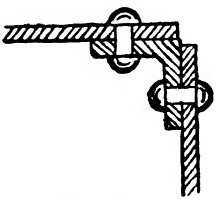


Fig. 364.

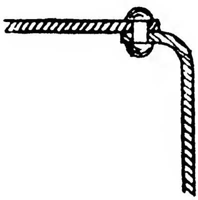


Fig. 365.

Fig. 363, C, shows a method of union adopted in which the plate which has to stand pressure is reinforced by the angle iron riveted across the joint. This is adopted in some tubes of large diameter and length, and in large flat surfaces otherwise unstayed. For jointing stout plate at right or at other angles, the two methods shown in Figs. 364 and 365 are employed, one (Fig. 364) is by means of angle iron, being the older method, and Fig. 365 is by flanging or bending, the better method for most work.

CHAPTER XI

SOLDERING.

To describe in minute detail the various operations of hard and soft soldering as adapted to various classes of work would demand much more space than could possibly be spared in a work of this character. I will, however, try to give a clear idea of the general principles and practice involved therein. Notwithstanding there is much both in principle and practice which is common to both methods, I think it well for the sake of clearness to keep the treatment of the two distinct, and will therefore do so.

Soldering in its generally understood sense signifies the close union of metallic surfaces by means of a thin film of molten metal or alloy run between the surfaces. In general also there are two main classes of soldering, conveniently distinguished by the terms hard and soft, the first-named being likewise known as brazing. Hard soldering is of the strongest character, requiring a high temperature, namely, a good red heat, visible in daylight; soft soldering is usually done at a temperature below that of melting lead. The hard solders are chiefly alloys of copper, and are known as spelter solders. Silver solders, used by the jewellers, also come under this class. The soft solders are mainly alloys of tin and lead. The proportion in which the metals are mixed to form soldering alloys are very various; such variations being rendered necessary in order that a soldering alloy shall always have a lower melting point than that of the metal or alloy which it has to unite, and also in some cases to obtain as nearly as possible uniformity of colour and strength between the two. Many a job has been spoiled by the use of an unsuitable solder, the work melting at the same time, or before the solder; and many a job strong in itself is rendered weak by the use of a solder too weak for it.

So that in all cases the solder should be selected in order that it may melt at a temperature considerably below that of the materials which it has to unite; and the necessary knowledge may in some cases be obtained either directly from tables, or by experiment, in the manner to be noted presently. Moreover, the harder metals and alloys should always be united with hard solders, and the softer ones with soft solders, and then the seams will be of about equal strength with the other parts.

It is practicable to obtain a most extensive range of solders with melting points from below that of boiling water, up to those suited for copper and iron goods The lowest melting points occur in what are termed the bismuth solders, the highest in the spelter and silver solders. An alloy of three of lead, five of tin, and three of bismuth, melts at 202° Fah.; one containing equal parts of lead, tin, and bismuth, at 254° Fah.; one containing four of lead, four of tin, and one of bismuth, at 320° Fah. Such solders, and others with various melting points, are used chiefly for pewter work. Tin and lead mixed in various proportions form the most useful range of soft solders, their melting points ranging from about 380° Fah. to 558° Fah. The commonest alloy is two of tin and one of lead, melting at 340° Fah.; one of tin and two of lead is the plumber's sealed solder, melting at 441° Fah.

It is absolutely necessary to the intimate and perfect union of all soldered works, that metal shall unite to metal. That is, the presence of any dirt or oxide in the joint will effectually prevent perfect union of the surfaces in contact. During the brief period of the raising of the temperature of the work to the melting point of the solder, some film of oxide will almost invariably form unless means be taken to prevent it. So that before a perfect joint can be made two precautions are necessary. One is to clean the surfaces first, the next is to keep them clean, until the solder is fused and run in. To effect the latter, a flux is used both to protect the cleaned surfaces from the action of the air, and also to dissolve any oxide which may form on the already cleaned surface. The selection of suitable fluxes is necessary to the making of a perfect joint.

The metals and alloys which are united by soft soldering are chiefly copper and its alloys, as brass, and gun metal, lead and tin, and alloys of the same, as the Britannia metals, and the pewters, tinned iron and zinc. Copper, brass, and their alloys, are, however, more frequently united by hard soldering or brazing. When united with soft solder, the alloy used is about two parts of tin to one of lead, the ordinary tinmen's solder. Tinned iron, or what is commonly called sheet tin, is united with the same solder, two of tin, one of lead, and chloride of zinc for flux. Lead is united with a similar solder, or with others containing larger proportions of lead, but the flux used is tallow. Britannia metal is united with solder containing two of tin and one of lead, and with chloride of zinc, or resin as flux; zinc with the same solder, and flux.

The bulk of soft soldering is done with the copper bit, or by means of sweating on, or by wiping. The copper bit, sometimes erroneously termed a soldering iron, is a convenient reservoir of heat for melting the solder while in contact with the work. Before using it is tinned, that is, heated to a low red, filed bright, and rubbed first with sal ammoniac, and then upon a piece of tin or of solder, to coat the surface, after which it is wiped with rag, or tow.

The edges to be soldered are scraped clean, and brought together, and protected with powdered resin, or with chloride of zinc, or other flux. Then the strip of solder being held in the left hand, and the copper bit in the right, the two are brought into contact, and drawn along the edges of the work. The solder is thereby melted in small quantity, and is worked and spread and smoothed along the joint with the copper bit. The bit must not be made too hot, or it will render the solder too fluid, and repel it.

The sweated joint is generally adopted for broad surfaces, and not for narrow edges. The surfaces are cleaned, and a thin layer of tin or of solder is spread over each, with the copper bit. The two tinned surfaces are then brought together, and raised to a temperature sufficiently high to melt the films of metal, and cause them to unite.

The wiped joints are used for lead pipe. The melted solder is poured round the jointed pipes in quantity, and is smoothed to a rounding form with special irons (Fig. 366), and with a well-greased pad of thick cloth.

The tinning of copper cooking utensils is akin to soldering, and is done to prevent the formation of oxide, which is poisonous. Stew pans, tea-kettles, and similar articles, are treated in this way. Previous to tinning, the inside of the vessel is washed with hydrochloric acid to remove all dirt. All trace of the acid is then removed by scouring with clean, sharp sand, and common salt, and washed in clean water. The inside is rubbed over with soldering fluid, or with sal ammoniac, and the outside is coated with a solution of whiting to protect the vessel from the fire, over which it is now heated. Melted block tin is poured

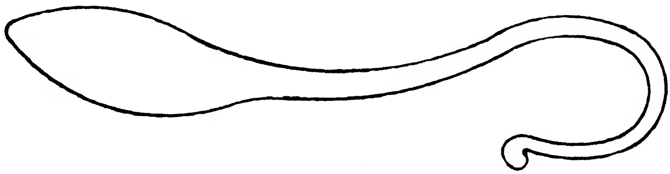


Fig. 366.

in, and rubbed over the inside of the vessel with a wad of sal ammoniac. When all parts are coated the superfluous tin is poured off, and the inside wiped smoothly with a wisp of tow held in a gloved hand in the case of an open vessel, or wrapped round a wire if the vessel has a narrow mouth. Washing in clean water follows to remove any sal ammoniac remaining, and the article is dried in sawdust. Finally, the inside is polished with a rag and whiting, and the outside with a rag and crocus.

In hard soldering, or brazing, the parts to be united have to be raised to so high a temperature that the copper bit is of no use, but the heat of a coke or charcoal fire, or in some cases of the blowpipe, is employed. Moreover, since the work takes some considerable time to execute, and because it is raised to so high a temperature, it is usual to secure the parts with fine iron wire, called binding wire. The flux used is borax, a compound which dissolves almost all oxides and earthy impurities that are likely to form on the joint. The hard or spelter solders mostly contain zinc, and the eye partly judges of the completion of the joint by the blue flame which accompanies the volatilization of the zinc. The spelter is granulated, and the borax is pounded fine in water. The two are mixed, and applied together, or separately, and sprinkled or spread along the joint. The heat is applied very gradually in order that the ebullition or boiling up of the borax, due to driving off of its water of crystallization by the heat, shall not displace the spelter from the joint. Afterwards the heat is increased, and at a low red the borax fuses, and at a bright red the solder fuses, and runs quickly into the joint. After covering or charging a joint with borax first and spelter afterwards, the water in the borax is slowly dried off, and if any borax has spread beyond the joint, this should be wiped off to prevent the spelter spreading farther than is necessary.

Fig. 367 shows a charger or spatula used for spreading borax and spelter over joints which have to be brazed. It is made from a bit of $\frac{1}{4}$-in. rod, flattened at one end, and turned into a

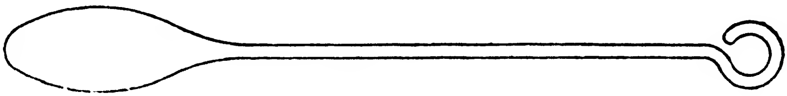


Fig. 367

handle at the other. To clean a joint in copper pipe, and at the same time to soften or anneal it for brazing, cover it with a strong solution of salt and water, and heat to a cherry red in ordinary daylight. Quench in cold water, and scour with tow and clean sand and water.

CHAPTER XII.

TOOLS AND APPLIANCES.

The tools used by tinmen, coppersmiths, and sheet-metal workers in general are of unique forms. In many respects they resemble those employed by smiths, but beyond some general outlines the resemblance ceases. Smith's tools are heavy, those of sheet-metal workers are light; they are usually also longer and much more slender than those of the smith. Instead, too, of being set or fixed in an anvil, they are set in a bench, or block of wood or iron on the ground, or on the workbench.

The anvil group includes the anvil (Fig. 368) and anvil stakes. The generic term "stake" is applied to many forms

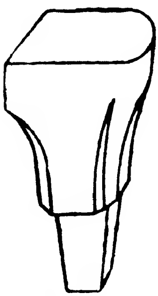


Fig. 368.

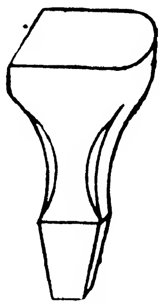


Fig. 369.

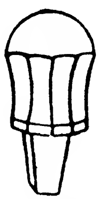


Fig. 370.

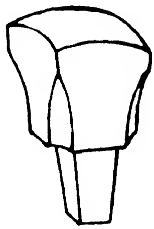


Fig. 371.

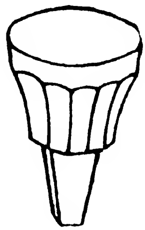


Fig. 372.

of tinmen's tools of the anvil type. The anvil proper (Fig. 368) is about 6 ins. or 8 ins. square; the anvil stakes are of very similar outline, but range down to about a half-inch square. The faces may be flat, as in Fig. 368, and as in the anvil stake (Fig. 369); or convex, as in Fig. 371. Many are designated by special names, as "roundhead" (Fig. 370), "longhead" (Fig. 371), "ovalhead" (Fig. 372), the forms of which are clearly shown in the drawings. There are also names derived from special functions, as the "tea-kettle bottom" stake (Fig. 373), the "round-bottom" stake (374), the "half-moon" stake (Fig. 375). There is also the "pepper-box head" stake (Fig. 376), and the "bullet" stake (Fig. 377).

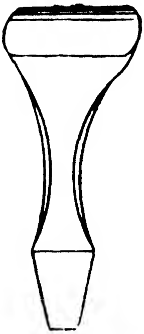


Fig. 373.


Fig. 374.

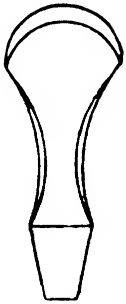


Fig. 375.


Fig. 376

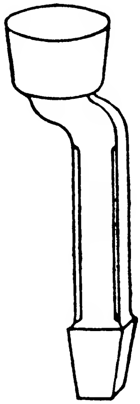


Fig. 377.

These are all special variations of the anvil form; but in some other stakes the resemblance to the anvil type is scarcely discernible. Many of these have been named because of their special functions or resemblances. Thus the origin of the term "hatchet" stake (Fig. 378) is sufficiently obvious. Here the

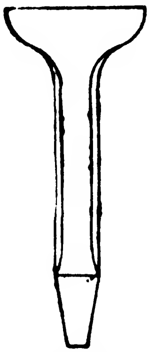


Fig. 378.

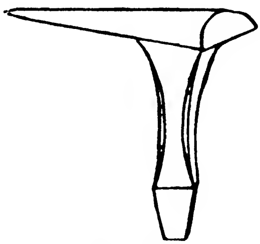


Fig. 379.

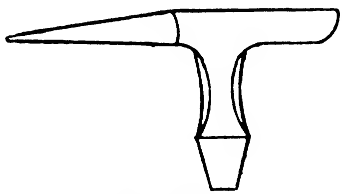


Fig. 380.

working face, or, rather, edge, is sharp, and its function is the bending of sheet metal to sharp angles over the keen edge. The "funnel" stake (Fig. 379) and the "extinguisher" stake (Fig. 380) derive their names from their original or special uses; the "saucepan belly" stake (Fig. 381) is a special tool. The "pipe" stake (Fig. 382) and the "side" stake (Fig. 383) are used for bending tubular forms over. Fig. 384 is called a "bick" iron, or "beak" iron. It is like an elongated anvil, and its function is the bending of long pipes or tubes. Those

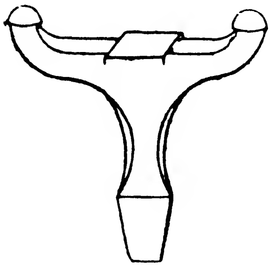


Fig. 381.

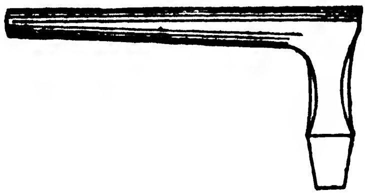


Fig. 382.

of circular form are bent on the circular end, those of rectangular form on the square end. A "tee" stake is shown in Fig. 385. It is used for hammering the backs of bend pipes into

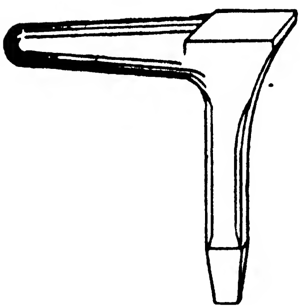


Fig. 383.

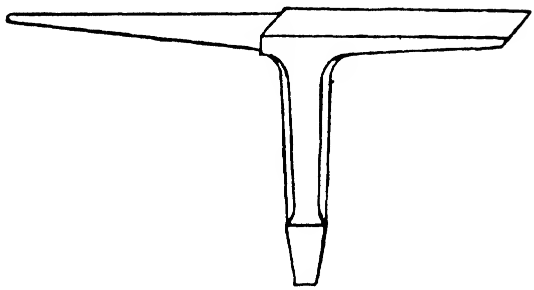


Fig. 384.

shape and finish, when such bend pipes are made in halves, jointed with seams at the sides. Fig. 386 is a teapot neck tool, Fig. 387 a bottom stake. These do not exhaust all the forms, but they include the principal ones.

These tools, it will be observed, are provided with square tapered necks. These are for insertion in sockets, horses, or in the floor-block. Fig. 388 illustrates a socket which fits into the floor-block and receives a stake. The horse (Fig. 389) is an appliance in the ends of which the small stakes are fixed, the horse itself being set in the block on the floor or bench. Fig. 390 shows another type of horse, with a stake inserted in it. Fig. 391 is the floor-block, which receives the sockets and stakes.

The term "stake" is often applied to the appliance in Fig. 392A, which corresponds in

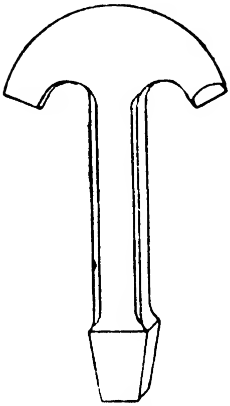


Fig. 385.

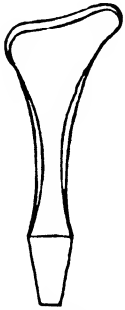


Fig. 386.

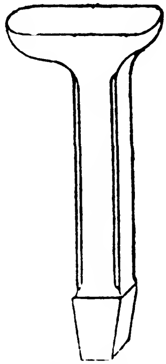


Fig. 387.

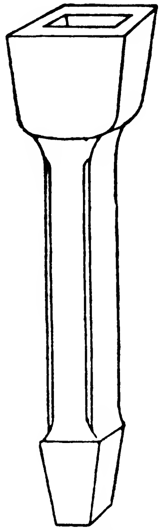


Fig. 388.

appearance with the bottom swage of the smith. It is termed a "grooving stake," and thin sheet metal is bent by being hammered down into an appropriate groove with a mallet, driving a rod of metal into the concave portion. The creasing iron (Fig. 392B) is an appliance of

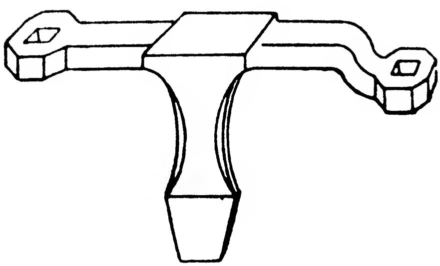


Fig. 389.

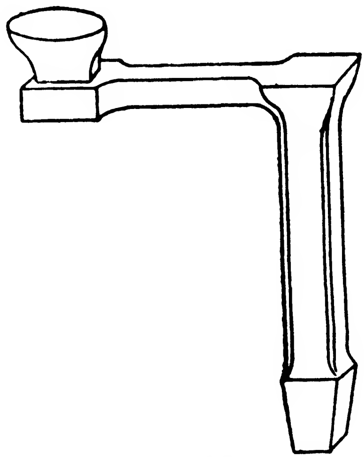


Fig. 390.

the same class, but narrower. Fig. 393 is a tool having somewhat similar function, called a seam set, or grooving tool, being used for hammering seams into.

Mandrels are used for bending, hammering, raising, and finishing various works upon. They are generally slid over

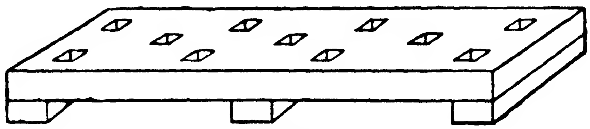


Fig. 391.

square bars inserted in the mandrel block. They are of cast-iron and of all sizes. They are circular or segmental in form, like those used by boiler-makers. Thus a single mandrel will have two or four faces, cast to different curvatures (Fig. 394). They will roughly average 5 ins. or 6 ins. in diameter, and may be from 2 ft. to 4 ft. in length; but all depends, of course, on the nature of the work done in the shop.

A mandrel block (Fig. 395) is a massive structure of cast-

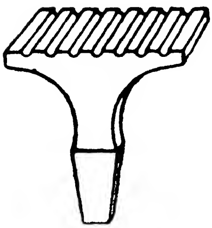


Fig. 392A.

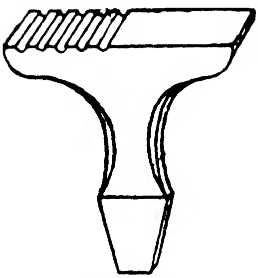


Fig. 392B.

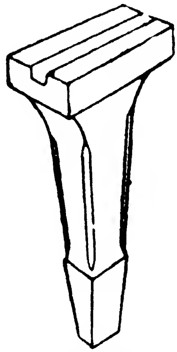


Fig. 393.

iron, used for the insertion of mandrels of various kinds, so that they may stand out in a horizontal direction. It is formed of two cast iron plates set on end and bolted together about 2 ft. apart. The plates stand about 5 ft. high, by about 4 ft. wide, by about 2 ins. thick. Various round and square holes are cast in them for the insertion of mandrels in positions required. Fig. 396 shows a round mandrel on the bar, which is inserted in one of the holes in the block.

A cod (Fig. 397) is an elliptical iron casting having a square hole in its longitudinal direction, to fit over a square bar set in the mandrel block. Its use is for insertion in bend pipes, to serve the purpose of an anvil during the hammering down of brazed seams.

The principal hammers used in the various operations of the sheet-metal worker are here illustrated. Figs. 398 to 400 are planishing

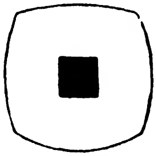


Fig. 394.

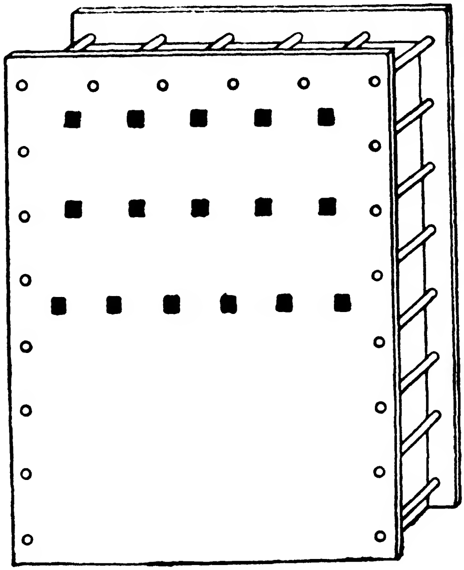


Fig. 395.

hammers, having broad faces. These are ordinarily used, but sometimes they are provided with spring faces. These spring-faced planishing hammers are used for finishing surfaces more highly than is practic-

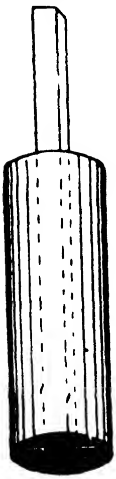


Fig. 396.

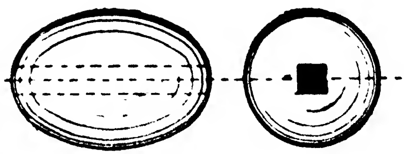


Fig. 397.

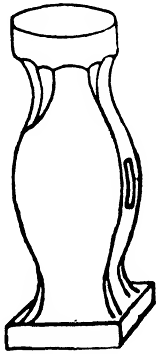


Fig. 398.

able with the ordinary bright-faced hammers. These last, when used in opposition to a bright stake or head, produce a circular mark, which has to be subsequently obliterated. When work has to be highly polished, and in plated work, such marks

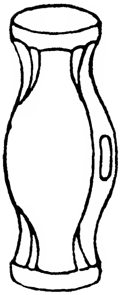


Fig. 399.

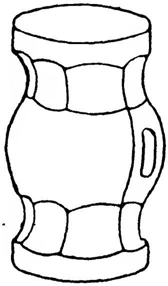


Fig. 400.

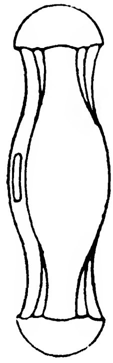


Fig. 401.

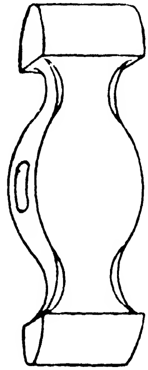


Fig. 402.

are objectionable; and in these cases the hammer face is muffled, in order to leave no marks. Often, too, the head or stake is muffled, as when the side of the work next the stake has to be perfectly smooth. To make a spring face, the face of an ordinary planishing hammer is covered with a piece of thin steel, bent up and secured with binding wire, a strip of cloth or similar elastic substance being interposed between the hammer face

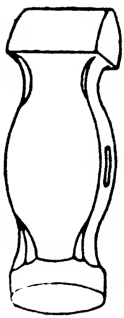


Fig. 403.

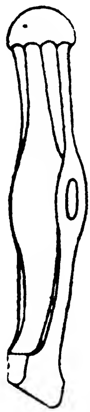


Fig. 404.

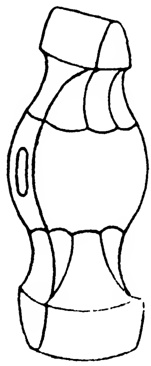


Fig. 405.

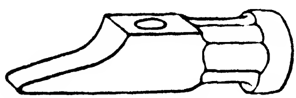


Fig. 406.

and the steel, and secured with binding wire, or a ring; or instead of sheet steel, cloth or parchment is used for covering the face of the hammer, or of the stake. In each case the blows are less elastic than those delivered by a wooden mallet, and less decisive than those delivered by naked metallic faces. The face which is required to be the smoother is brought opposite the muffled tool or stake. Or when both faces are wanted smooth, then both tool and stake are muffled.

Fig. 401 is a hollowing hammer, used for dishing work. Fig. 402 is a cross-pane hammer. Fig. 403 is a cross-pane and

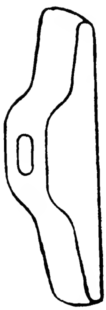


Fig. 407.

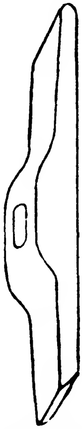


Fig. 408.

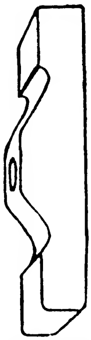


Fig. 409.

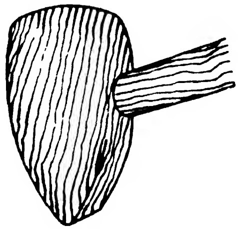


Fig. 410.

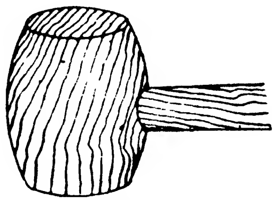


Fig. 411.

face hammer. Fig. 404 is a bullet-head hammer. Fig. 405 is a scrubbing hammer for hammering round rivets to close the sheets together. Fig. 406 is a riveting hammer. Fig. 407 is a creasing hammer for forming creases or grooves for the

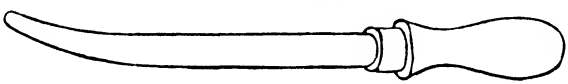


Fig. 412.

insertion of sheets or wires. Figs. 408, 409 are paning hammers for closing up seams. Figs. 410, 411 are wooden mallets for use where the employment of metal hammers would be objectionable. Fig. 412 is a steel burnisher for polishing surfaces, for which hammers could not be used.

APPENDIX.

TABLE I.

Tin Plates—Dimensions and Weights.

Mark.	Dimensions of Sheets.	Number of Sheets in a Box.	Weight of each Box.
	Inches.	Sheets.	Pounds.
IC	14 × 10	225	108
IX	14 × 10	225	136
IXX	14 × 10	225	157
IXXX	14 × 10	225	178
IXXXX	14 × 10	225	199
IC	14 × 20	112	108
IX	14 × 20	112	136
IXX	14 × 20	112	157
IXXX	14 × 20	112	178
IXXXX	14 × 20	112	199
IC	28 × 20	56	108
IX	28 × 20	56	136
IXX	28 × 20	56	157
IXXX	28 × 20	56	178
IXXXX	28 × 20	56	199
IC	12 × 12	225	108
IX	12 × 12	225	136
IXX	12 × 12	225	157
IXXX	12 × 12	225	178
IXXXX	12 × 12	225	199
DC	17 × 12½	100	94
DX	17 × 12½	100	122
DXX	17 × 12½	100	143
DXXX	17 × 12½	100	164
DXXXX	17 × 12½	100	185
DC	17 × 25	50	94
DX	17 × 25	50	122
DXX	17 × 25	50	143
DXXX	17 × 25	50	164
DXXXX	17 × 25	50	185
DC	34 × 25	25	94
DX	34 × 25	25	122
DXX	34 × 25	25	143
DXXX	34 × 25	25	164
DXXXX	34 × 25	25	185

TABLE II.

WEIGHT OF SHEET ZINC PER SQUARE FOOT; ALSO PLATES 8 FT. BY 3 FT. THICKNESS, MEASURED BY THE B. W. GAUGE IN LBS. AND OZS.

Thickness by B. W. Gauge.	Weight per Sq. Ft.		Weight of a Sheet 8′ × 3′.		Thickness by B. W. Gauge.	Weight per Sq. Ft.		Weight of a Sheet 8′ × 3′.	
	lbs.	ozs.	lbs.	ozs.		lbs.	ozs.	lbs.	ozs.
10	5	0	120	0	22	1	1	25	8
11	4	9	109	8	23	0	15	22	8
12	4	0	96	0	24	0	13¼	19	14
13	3	7	82	8	25	0	12⅛	18	3
14	3	0	72	0	26	0	10¾	16	2
15	2	10	63	0	27	0	9½	14	4
16	2	6	57	0	28	0	8⅞	13	5
17	2	1	49	8	29	0	8	12	0
18	1	12	42	0	30	0	7⅜	11	1
19	1	9	37	8	31	0	6⅞	10	5
20	1	5	31	8	32	0	6⅜	9	0
21	1	3	28	8	33	0	6	9	0

TABLE III.

WEIGHT AND THICKNESS OF SHEET LEAD.

Weight in lbs. per sup. foot.	Thickness in inches.	Weight in lbs. per sup. foot.	Thickness in inches.
1	0·017	7	0·118
2	0·034	8	0·135
3	0·051	9	0·152
4	0·068	10	0·169
5	0·085	11	0·186
6	0·101	12	0·203

TABLE IV.

Weight of Twelve Inches Square of Various Metals.

Thick-ness.	Wrought Iron.	Cast Iron.	Steel.	Gun Metal.	Brass.	Copper.	Tin.	Zinc.	Lead.
$\frac{1}{16}$	2·50	2·34	2 56	2·75	2·69	2·87	2 37	2 25	3·68
$\frac{1}{8}$	5·00	4 69	5·12	5·50	5 38	5·75	4·75	4·50	7·37
$\frac{3}{16}$	7·50	7·03	7·68	8·25	8·07	8·62	7·12	6·75	11·05
$\frac{1}{4}$	10·00	9·38	10·25	11·00	10 75	11·50	9·50	9·00	14·75
$\frac{5}{16}$	12·50	11·72	12·81	13 75	13·45	14·37	11·87	11·25	18·42
$\frac{3}{8}$	15·00	14·06	15·36	16·50	16·14	17·24	14·24	13·50	22 10
$\frac{7}{16}$	17·50	16·41	17·93	19·25	18·82	20·12	16 17	15·75	25·80
$\frac{1}{2}$	20·90	18·75	20·50	22·00	21·50	23 00	19 00	18·00	29·50
$\frac{9}{16}$	22·50	21·10	23·06	24·75	24·20	25·87	21·37	20·25	33·17
$\frac{5}{8}$	25·00	23·44	25·62	27·50	26·90	28·74	23·74	22·50	36·84
$\frac{11}{16}$	27·50	25·79	28·18	30 25	29·58	31·62	26·12	24·75	40·54
$\frac{3}{4}$	30·00	28·12	30·72	33·00	32·28	34·48	28·48	27·00	44 20
$\frac{13}{16}$	32·50	30·48	33·28	35·75	34·95	37·37	30 87	29 25	47·92
$\frac{7}{8}$	35·00	32·82	35·86	38·50	37·64	40 24	32·34	31·50	51·60
$\frac{15}{16}$	37·50	35·16	38·43	41 25	40·32	43·12	35·61	33·75	55·36
1	40·00	37·50	41·00	44·00	43·00	46·00	38·00	36·00	59·00

TABLE V.

Weight of Metals. To Find Weight in Lbs.

Aluminium	cub. in.	× 0·094
Brass	„	× 0·31
Copper	„	× 0 32
Cast-Iron	„	× 0·26
Wrought-Iron	„	× 0·28
Lead	„	× 0·41
Mercury	„	× 0·49
Nickel	„	× 0·31
Tin	„	× 0·26
Zinc	„	× 0·26

TABLE VI.

Weight of Copper Pipes per foot run.

Bore in Inches.	Thickness of Metal in Parts of an Inch.					
	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$
	lb.	lb.	lb.	lb.	lb.	lb.
$\frac{1}{2}$	0·426	0·946	1·561	2·270	3·075	3·973
$\frac{5}{8}$	0·520	1·185	1·845	2·649	3·547	4·540
$\frac{3}{4}$	0·615	1·324	2·129	3·027	4·020	5·108
$\frac{7}{8}$	0·709	1·514	2·412	3·425	4·493	5·676
1	0·804	1·703	2·696	3·784	4·966	6·243
$1\frac{1}{4}$	0·993	2·081	3·263	4·540	5·712	7·378
$1\frac{1}{2}$	1·182	2·459	3·831	5·297	6·857	8·514
$1\frac{3}{4}$	1·372	2·838	4·388	6·055	7·805	9·646
2	1·560	3·217	4·967	6·808	8·748	10·783
$2\frac{1}{4}$	1·750	3·591	5·531	7·566	9·694	11·918
$2\frac{1}{2}$	1·940	3·975	6·103	8·327	10·643	13·066
$2\frac{3}{4}$	2·128	4·352	6·668	6·081	11·590	14·190
3	2·316	4·729	7·238	6·737	12·534	15·325

TABLE VII.

Weight of Brass Pipes, per foot run.

Bore.	Thickness in Parts of an Inch.						
Inch.	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
$\frac{1}{4}$	0·22	0·53	0·94	1·43	2·01	2·68	3·44
$\frac{1}{2}$	0·40	0·89	1·47	2·15	2·91	3·75	4·70
$\frac{3}{4}$	0·58	1·25	2·01	2·86	3·80	4·83	5·95
1	0·76	1·61	2·55	3·58	4·70	5·92	7·25
$1\frac{1}{4}$	0·94	1·96	3·09	4·31	5·64	6·98	9·46
$1\frac{1}{2}$	1·12	2·34	3·67	5·01	6·49	8·05	9·71
$1\frac{3}{4}$	1·33	2·66	4·14	5·70	7·36	9·11	10·94
2	1·48	3.04	4·69	6·44	8·27	10·20	12·21
$2\frac{1}{4}$	1·65	3·40	5·23	7·16	9·17	11·27	13·46
$2\frac{1}{2}$	1·83	3·75	5·77	7·87	10·06	12·35	14·72
$2\frac{3}{4}$	2·01	4·11	6·31	8·59	10·96	13·42	15·97
3	2·19	4·47	6·84	9·31	11·85	14·69	17·42

TABLE VIII.

Weight of Lead Pipes.

Bore.	Length.	Weight of each Length in Lbs.					
		Common.		Middling.		Strong.	
Inches.	Feet.	Thickness in Inches.	Lbs.	Thickness in Inches	Lbs.	Thickness in Inches.	Lbs.
½	15	0·2	16	0·29	22	...	26
¾	15	...	24	...	28	...	36
1	15	0·11	30	0·15	40	0·17	46
1¼	12	...	36	...	44	...	53
1½	12	...	48	...	56	...	70
2	10	0·17	56	0·21	70	0·34	83
2½	10	...	70	...	86	...	100

TABLE IX.

Weight of Wrought-Iron Pipes per foot run.

Bore in Inches.	Thickness of Metal in Parts of an Inch.							
	1/16	⅛	3/16	¼	5/16	⅜	7/16	½
	lb.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
¼	0·21	0·5	0·9	1·3	1·9	2·5	3·1	3·9
⅜	0·3	0·7	1·1	1·6	2·3	2·9	3·8	4·6
½	0·4	0·83	1·4	2·0	2·7	3·5	4·3	5·3
⅝	0·46	1·1	1·6	2·3	3·1	3·9	4·9	5·9
¾	0·54	1·2	1·9	2·6	3·5	4·5	5·5	6·6
⅞	0·6	1·3	2·1	2·9	3·9	4·9	6·1	7·3
1	0·7	1·5	2·4	3·3	4·3	5·5	6·7	7·9
1¼	0·87	1·8	2·9	3·9	5·2	6·4	7·8	9·3
1½	1·0	2·1	3·3	4·7	5·9	7·4	8·9	10·6
1¾	1·2	2·5	3·8	5·3	6·8	8·4	10·1	11·9
2	1·4	2·8	4·3	5·9	7·6	9·5	11·3	13·2
2¼	1·53	3·1	4·8	6·6	8·5	10·4	12·4	14·5
2½	1·7	3·5	5·3	7·3	9·3	11·4	13·6	15·9
2¾	1·9	3·8	5·8	7·9	10·1	12·4	14·7	17·2
3	2·03	4·1	6·3	8·6	10·9	13·3	15·9	18·5

TABLE X.

Diameter of Whitworth Gas Taps.

Size. Inches.	Diameter.	Diameter at Bottom of thread.	Number of threads per inch.
1/8	0·3825	0·3367	28
1/4	0·518	0·4506	19
3/8	0·6563	0·5889	19
1/2	0·8257	0·7342	14
5/8	0·9022	0·8107	14
3/4	1·041	0·9495	14
7/8	1·189	1·0975	14
1	1·309	1·1925	11
1 1/8	1 419	1·3755	11
1 1/4	1·65	1·5335	11
1 3/8	1·745	1·6285	11
1 1/2	1·8825	1·766	11
1 5/8	2·021	1 9045	11
1 3/4	2·047	1·9305	11
1 7/8	2·245	2·1285	11
2	2·347	2·2305	11
2 1/4	2·5875	2·471	11
2 1/2	3 0013	2·8848	11
2 3/4	3·247	3·1305	11
3	3·485	3·3685	11
3 1/4	3·6985	3 582	11
3 1/2	3·912	3·7955	11
3 3/4	4·1255	4·009	11
4	4·439	4·225	11

TABLE XI.

Soft Solders. Composition of, and Melting Points.

Solder.	Tin.	Lead.	Bismuth.	Melts at Fahr.
Bismuth solder	3	5	3	202°
Ditto...	2	2	1	229°
Ditto...	1	1	1	254°
Ditto...	4	4	1	320°
Tinman's coarse solder ...	3	2		334°
Tinman's fine solder ...	2	1		340°
Plumber's fine solder ...	1	2		441°
Plumber's coarse solder ...	1	3		482°
Solder for soldering lead ...	1	1 1/2		
Solder for soldering tin ...	1	2		
Solder for soldering pewter	2	1		

TABLE XII.

Hard Solders or Brazing Mixtures.

Solder.	Copper.	Tin.	Zinc.	Brass.	Silver.
Hard solder for copper and brass	1		1		
Ditto			1	5	
Dito harder	2		1		
Ditto	3		1		
Silver solder	8		8		1
Ditto				1	1
Ditto				1	2

TABLE XIII.

Old B.W.G.

Descriptive number B.W.G.	Equivalent in parts of an inch	Descriptive number B.W.G.	Equivalent in parts of an inch.	Descriptive number B.W.G.	Equivalent in parts of an inch.
0000	0·454	11	0 120	24	0·022
000	0·425	12	0 109	25	0·020
00	0·380	13	0·095	26	0·018
0	0·340	14	0·083	27	0·016
1	0·300	15	0 072	28	0·014
2	0 284	16	0·065	29	0·013
3	0 259	17	0 058	30	0·012
4	0·238	18	0·049	31	0·010
5	0·220	19	0·042	32	0·009
6	0·203	20	0·035	33	0·008
7	0·180	21	0·032	34	0·007
8	0 165	22	0·028	35	0·005
9	0·148	23	0·025	36	0·004
10	0·134				

TABLE XIV.

Standard Sheet and Hoop Iron Gauge (B.G.),

Issued in March, 1884, by the South Staffordshire Ironmasters' Association, for the use of Sheet and Hoop Iron Makers.

No. of Gauge.	Thickness in			Approximate Weight per superficial foot of Sheet Iron in lbs.
	Ordinary fractions of an inch.	Decimals of an inch.	Millimetres.	
3°	$\frac{1}{2}$	0·5000	12·700	20·000
2°	...	0·4452	11·288	17·808
1°	...	0·3964	10·068	15·856
1	...	0·3532	8·971	14·128
2	...	0·3147	7·993	12·588
3	...	0·2804	7·122	11·216
4	$\frac{1}{4}$	0·2500	6·350	10·000
5	...	0·2225	5·651	8·900
6	...	0·1981	5·032	7·924
7	...	0·1764	4·480	7·056
8	...	0·1570	3·988	6·280
9	...	0·1398	3·551	5·592
10	$\frac{1}{8}$	0·1250	3·175	5·000
11	...	0·1113	2·827	4·452
12	...	0·0991	2·517	3·964
13	...	0·0882	2·240	3·528
14	...	0·0785	1·994	3·140
15	...	0·0699	1·775	2·796
16	$\frac{1}{16}$	0·0625	1·587	2·500
17	...	0·0556	1·412	2·224
18	...	0·0495	1·257	1·980
19	...	0·0440	1·118	1·760
20	...	0·0392	0·996	1·568
21	...	0·0349	0·886	1·396
22	$\frac{1}{32}$	0·03125	0·794	1·250
23	...	0·02782	0·707	1·1128
24	...	0·02476	0·629	0·9904
25	...	0·02204	0·560	0·8816
26	...	0·01961	0·498	0·7844
27	...	0·01745	0·4432	0·698
28	$\frac{1}{64}$	0·015625	0·3969	0·625
29	...	0·01390	0·3531	0·556
30	...	0·0123	0·3124	0·492
31	...	0·0110	0·2794	0·440
32	...	0·0098	0·2489	0·392
33	...	0·0087	0·2210	0·348
34	...	0·0077	0·1956	0·300
35	...	0·0069	0·1753	0·276
36	...	0·0061	0·1549	0·244
37	...	0·0054	0·1371	0·216
38	...	0·0048	0·1219	0·192
39	...	0·0043	0·1092	0·172
40	...	0·00386	0·0980	0·1544

Note.—The weight in Steel can be found by adding 2 per cent., or $\frac{1}{50}$th, to the weight in Iron.

TABLE XV.

New Legal Standard Wire Gauge.

Descriptive number S.W.G.	Equivalent in parts of an inch.	Descriptive number S.W.G.	Equivalent in parts of an inch.	Descriptive number S.W.G.	Equivalent in parts of an inch.
No.		No.		No.	
7/0	0·500	13	0·092	32	0·0108
6/0	0·464	14	0·080	33	0·0100
5/0	0·432	15	0·072	34	0·0092
4/0	0·400	16	0·064	35	0·0084
3/0	0 372	17	0·056	36	0·0076
2/0	0·348	18	0·048	37	0·0068
0	0·324	19	0·040	38	0·0060
1	0·300	20	0·036	39	0·0052
2	0·276	21	0·032	40	0·0048
3	0·252	22	0·028	41	0·0044
4	0·232	23	0·024	42	0·0040
5	0·212	24	0·022	43	0·0036
6	0·192	25	0·020	44	0·0032
7	0 176	26	0·018	45	0·0028
8	0·160	27	0·0164	46	0·0024
9	0·144	28	0 0148	47	0·0020
10	0·128	29	0 0136	48	0·0016
11	0·116	30	0·0124	49	0·0012
12	0·104	31	0·0116	50	0·0010

TABLE XVI.

Table of the Decimal Equivalents of an Inch.

$\frac{1}{32}$	=	0·03125	$\frac{3}{8}$	=	0·375	$\frac{23}{32}$	=	0·71875
$\frac{1}{16}$	=	0·0625	$\frac{13}{32}$	=	0·40625	$\frac{3}{4}$	=	0·75
$\frac{3}{32}$	=	0·09375	$\frac{7}{16}$	=	0·4375	$\frac{25}{32}$	=	0·78125
$\frac{1}{8}$	=	0·125	$\frac{15}{32}$	=	0·46875	$\frac{13}{16}$	=	0·8125
$\frac{5}{32}$	=	0·15625	$\frac{1}{2}$	=	0·5	$\frac{27}{32}$	=	0·84375
$\frac{3}{16}$	=	0·1875	$\frac{17}{32}$	=	0·53125	$\frac{7}{8}$	=	0·875
$\frac{7}{32}$	=	0·21875	$\frac{9}{16}$	=	0·5625	$\frac{29}{32}$	=	0·90625
$\frac{1}{4}$	=	0·250	$\frac{19}{32}$	=	0·59375	$\frac{15}{16}$	=	0·9375
$\frac{9}{32}$	=	0·28125	$\frac{5}{8}$	=	0·625	$\frac{31}{32}$	=	0·96875
$\frac{5}{16}$	=	0·3125	$\frac{21}{32}$	=	0·65625	1	=	1·0
$\frac{11}{32}$	=	0·34375	$\frac{11}{16}$	=	0·6875			

TABLE XVII.

Inches and their equivalent Decimals of a Foot.

Inches.	Equivalents.	Inches.	Equivalents.	Inches.	Equivalents.
$\frac{1}{16}$	0·0052083	$2\frac{1}{2}$	0·2083	$5\frac{7}{8}$	0·489583
$\frac{1}{8}$	0·010416	$2\frac{5}{8}$	0·21875	6	0·5
$\frac{3}{16}$	0·015625	$2\frac{3}{4}$	0·22916	$6\frac{1}{4}$	0·52083
$\frac{1}{4}$	0·02083	$2\frac{7}{8}$	0·239583	$6\frac{1}{2}$	0·5416
$\frac{5}{16}$	0·0260416	3	0·25	$6\frac{3}{4}$	0·5625
$\frac{3}{8}$	0·03125	$3\frac{1}{8}$	0·260416	7	0·583
$\frac{7}{16}$	0·0364583	$3\frac{1}{4}$	0·27083	$7\frac{1}{4}$	0·60416
$\frac{1}{2}$	0·0416	$3\frac{3}{8}$	0·28125	$7\frac{1}{2}$	0·625
$\frac{9}{16}$	0·046875	$3\frac{1}{2}$	0·2916	$7\frac{3}{4}$	0·64583
$\frac{5}{8}$	0·052083	$3\frac{5}{8}$	0·302083	8	0·6666
$\frac{11}{16}$	0·0572916	$3\frac{3}{4}$	0·3125	$8\frac{1}{4}$	0·6875
$\frac{3}{4}$	0·0625	$3\frac{7}{8}$	0·322916	$8\frac{1}{2}$	0·7083
$\frac{13}{16}$	0·0677083	4	0·3333	$8\frac{3}{4}$	0·72916
$\frac{7}{8}$	0·072916	$4\frac{1}{8}$	0·34375	9	0·75
$\frac{15}{16}$	0·078125	$4\frac{1}{4}$	0·35416	$9\frac{1}{4}$	0·77083
1	0·083	$4\frac{3}{8}$	0·364583	$9\frac{1}{2}$	0·7916
$1\frac{1}{8}$	0·09375	$4\frac{1}{2}$	0·375	$9\frac{3}{4}$	0·8125
$1\frac{1}{4}$	0·10416	$4\frac{5}{8}$	0·385416	10	0·8333
$1\frac{3}{8}$	0·114583	$4\frac{3}{4}$	0·39583	$10\frac{1}{4}$	0·85416
$1\frac{1}{2}$	0·125	$4\frac{7}{8}$	0·40625	$10\frac{1}{2}$	0·875
$1\frac{5}{8}$	0·135416	5	0·416	$10\frac{3}{4}$	0·89583
$1\frac{3}{4}$	0·14583	$5\frac{1}{8}$	0·427083	11	0·916
$1\frac{7}{8}$	0·15625	$5\frac{1}{4}$	0·4375	$11\frac{1}{4}$	0·9375
2	0·16	$5\frac{3}{8}$	0·447916	$11\frac{1}{2}$	0·9583
$2\frac{1}{8}$	0·177083	$5\frac{1}{2}$	0·4583	$11\frac{3}{4}$	0·97916
$2\frac{1}{4}$	0·1875	$5\frac{5}{8}$	0·46875	12	1.00
$2\frac{3}{8}$	0·197916	$5\frac{3}{4}$	0·47916		

TABLE XVIII.

Fractional Parts of a Pound Avoirdupois and their Decimal Equivalents.

Ounces.	Lbs.	Ounces.	Lbs.	Ounces	Lbs.
$\frac{1}{4}$	0·015625	$5\frac{1}{2}$	0·34375	11	0·6875
$\frac{1}{2}$	0·03125	6	0·375	$11\frac{1}{2}$	0·71875
1	0·0625	$6\frac{1}{2}$	0·40625	12	0·75
$1\frac{1}{2}$	0·09375	7	0·4375	$12\frac{1}{2}$	0·78125
2	0·125	$7\frac{1}{2}$	0·46875	13	0·8125
$2\frac{1}{2}$	0·15625	8	0·5	$13\frac{1}{2}$	0·84375
3	0·1875	$8\frac{1}{2}$	0·53125	14	0·875
$3\frac{1}{2}$	0·21875	9	0·5625	$14\frac{1}{2}$	0·90625
4	0·25	$9\frac{1}{2}$	0·59375	15	0·9375
$4\frac{1}{2}$	0·28125	10	0·625	$15\frac{1}{2}$	0·96875
5	0·3125	$10\frac{1}{2}$	0·65625	16	1·000

TABLE XIX

DIAMETERS, CIRCUMFERENCES, AREAS, SIDES OF EQUAL SQUARES, SQUARES, AND CUBES.

Diameter in inches.	Circumference in inches.	Area in Sq. inches.	Sides of Equal Squares, inches.	Area in Sq. feet.	Square, in inches.	Cube, in inches.
$\frac{1}{8}$	0·3927	0 0122	0·1107	...	0·0156	0·00195
$\frac{1}{4}$	0·7854	0 0490	0·2215	...	0·0625	0·01563
$\frac{3}{8}$	1·1781	0·1104	0·3323	...	0·1406	0·05273
$\frac{1}{2}$	1 5708	0 1963	0·4431	...	0·25	0·125
$\frac{5}{8}$	1·9635	0·3068	0·5438	...	0·3906	0 24414
$\frac{3}{4}$	2·3562	0·4417	0·6646	...	0·5625	0·42138
$\frac{7}{8}$	2·7489	0·6013	0 7754	...	0·7656	0·66992
1	3·1416	0·7854	0·8862	...	1·0	1·0
$1\frac{1}{8}$	3·5343	0·9940	0·9969	0·0069	1·2656	1·42383
$1\frac{1}{4}$	3·9270	1·2271	1·1017	0·0084	1·5625	1·95313
$1\frac{3}{8}$	4·3197	1·4848	1·2185	0·0102	1·8906	2 59961
$1\frac{1}{2}$	4·7124	1·7671	1·3293	0·0122	2·25	3·375
$1\frac{5}{8}$	5·1051	2·0739	1·4401	0·0143	2·6406	4 2910
$1\frac{3}{4}$	5 4978	2·4052	1·5'08	0·0166	3·0265	5·3593
$1\frac{7}{8}$	5·8905	2·7611	1·6616	0·0191	3·5156	6 5918
2	6·2832	3·1416	1·7724	0·0225	4·0	8·0
$2\frac{1}{8}$	6 6759	3·5465	1·8831	0·0245	4·5156	9·5957
$2\frac{1}{4}$	7·06*6	3·9760	1·9939	0·0275	5·0625	11·3906
$2\frac{3}{8}$	7·4613	4·4302	2·1047	0·0307	5·6406	13·3965
$2\frac{1}{2}$	7·8540	4·9087	2·2.55	0·0340	6·25	15·625
$2\frac{5}{8}$	8·2467	5·4119	2·3262	0·0375	6·8906	18·0879
$2\frac{3}{4}$	8·6394	5·9395	2·4370	0·0411	7·5625	20·7969
$2\frac{7}{8}$	9·0321	6·4918	2·5478	0·0450	8·2656	23·7637
3	9·4248	7·0686	2·6586	0·0490	9·0	27·0
$3\frac{1}{8}$	9·8175	7·6699	2·7694	0·0531	9·7656	30 5176
$3\frac{1}{4}$	10·210	8·2957	2·8801	0·0575	10·5625	34·3281
$3\frac{3}{8}$	10·602	8·9462	2·9909	0·0620	11·3906	38·4434
$3\frac{1}{2}$	10·995	9·6211	3 1017	0·0668	12·25	42·875
$3\frac{5}{8}$	11·388	10·320	3·2124	0·0730	13·1406	47·634
$3\frac{3}{4}$	11·781	11·044	3·3232	0·0767	14·0625	52·734
$3\frac{7}{8}$	12·173	11·793	3·4310	0·0818	15·0156	58·185
4	12·566	12·566	3·5448	0·0879	16·0	64·0
$4\frac{1}{8}$	12·959	13·364	3·6555	0·0935	17·0156	70·1895
$4\frac{1}{4}$	13·351	14·186	3·7663	0·0993	18·0625	76·7656
$4\frac{3}{8}$	13·744	15·033	3·8771	0·1052	19·1406	83·7402
$4\frac{1}{2}$	14·137	15 904	3·9880	0·1113	20·25	91·125
$4\frac{5}{8}$	14·529	16·800	4·0987	0·1176	21·3906	98·9316
$4\frac{3}{4}$	14·922	17·720	4·2095	0·1240	22·5625	107·1719
$4\frac{7}{8}$	15·315	18 665	4·3202	0·1306	23·7656	115·8574
5	15·708	19·635	4·4310	0·1374	25·0	125 0
$5\frac{1}{8}$	16·100	20 629	4·5417	0·1444	26·2656	134·6113
$5\frac{1}{4}$	16·493	21·647	4·6525	0·1515	27·5625	144·7031
$5\frac{3}{8}$	16·886	22·690	4·7633	0·1588	28·8906	155·2871
$5\frac{1}{2}$	17·278	23·758	4·8741	0·1663	30·25	166·375
$5\frac{5}{8}$	17·671	24 850	4·9848	0·1739	31·6406	177·9785
$5\frac{3}{4}$	18·064	25·967	5·0956	0·1817	33·0625	190·1094

Diameter in inches.	Circumference in inches.	Area in Sq. inches.	Sides of Equal Squares, inches.	Area in Sq. feet.	Square, in inches.	Cube, in inches.
5⅞	18·457	27 108	5·2064	0·1897	34·5186	202·7793
6	18·849	28·274	5·3172	0·1979	36·0	216 0
6⅛	19·242	29·464	5·4280	0·2062	37·5156	229·7832
6¼	19·635	30·679	5·5388	0·2147	39·0625	244·1406
6⅜	20·027	31·919	5·6495	0·2234	40 6406	259·0840
6½	20·420	33·183	5·7603	0·2322	42·25	274·625
6⅝	20·813	34·471	5·8711	0·2412	43·8906	290·7754
6¾	21·205	35·784	5·9819	0·2504	45·5625	307·5469
6⅞	21·598	37·122	6·0927	0·2598	47·2656	324·9512
7	21·991	38·484	6·2034	0·2693	49·0	343·0
7⅛	22·383	39·871	6·3142	0·2791	50·7656	361·7051
7¼	22·776	41·282	6·4350	0·2889	52·5625	381·0781
7⅜	23·169	42·718	6·5358	0·2990	54·3906	401·1309
7½	23·562	44·178	6·6465	0·3092	56·25	421·879
7⅝	23·954	45·663	6·7573	0·3196	58·1406	443·3223
7¾	24·347	47·173	6·8681	0·3299	60·0625	465·4844
7⅞	24·740	48·707	6·9789	0·3409	62·0156	488·3730
8	25·132	50·265	7·0897	0·3518	64·0	512·0
8⅛	25·515	51·848	7·2005	0·3629	66·0156	536·3770
8¼	25·918	53·456	7·3112	0·3741	68·0625	561·5156
8⅜	26·310	55·088	7·4220	0·3856	70·1406	587·4277
8½	26·703	56·745	7·5328	0·3972	72·25	614·125
8⅝	27·096	58·426	7·6436	0·4089	74·3906	641·6191
8¾	27·489	60·132	7·7544	0·4209	76·5625	669·9219
8⅞	27·881	61·862	7·8651	0·4330	78·7656	699·0449
9	28·274	63·617	7·9760	0·4453	81·0	729·0
9⅛	28·667	65·396	8·0866	0·4577	83·2656	759·7988
9¼	29·059	67·200	8·1974	0·4704	85·5625	791·4531
9⅜	29·452	69·029	8·3081	0·4832	87·8906	823·9746
9½	29·845	70·882	8·4190	0·4961	90·25	857·375
9⅝	30·237	72·759	8·5297	0·5093	92·6406	891·6660
9¾	30·630	74·662	8·6403	0·5226	95·0625	926·8594
9⅞	31·023	76·588	8·7513	0·5361	97 5156	962·0968
10	31·416	78·540	8 8620	0·5497	100·0	1000·0
10⅛	31·808	80·515	8·9728	0·5636	102·5156	1037 9707
10¼	32·201	82·516	9·0836	0·5776	105·0625	1076·8906
10⅜	32 594	84·540	9·1943	0·5917	107·6406	1116·7715
10½	32·986	86·590	9·3051	0·6061	110·25	1157·625
10⅝	33·379	88·664	9·4159	0·6206	112·8906	1199·4629
10¾	33·772	90·762	9·5267	0·6358	115·5625	1242·2969
10⅞	34·164	92·885	9·6375	0·6499	118·2656	1286·1387
11	34·557	95·033	9·7482	0·6652	121·0	1331·0
11⅛	34·950	97·205	9·8590	0 6804	123·7656	1376·8926
11¼	35·343	99·402	9·9698	0·6958	126·5625	1423·8281
11⅜	35·735	101·623	10·080	0·7143	129·3906	1471·8184
11½	36·128	103·869	10·191	0·7270	132·25	1520·875
11⅝	36·521	106·139	10·302	0·7429	135·1406	1571·0098
11¾	36 918	108·434	10·413	0·7590	138·0625	1622·234
11⅞	37·306	110·753	10·523	0·7752	141·0155	1674·5605
12	37·699	113·097	10 634	0·7916	144·0	1728·0

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