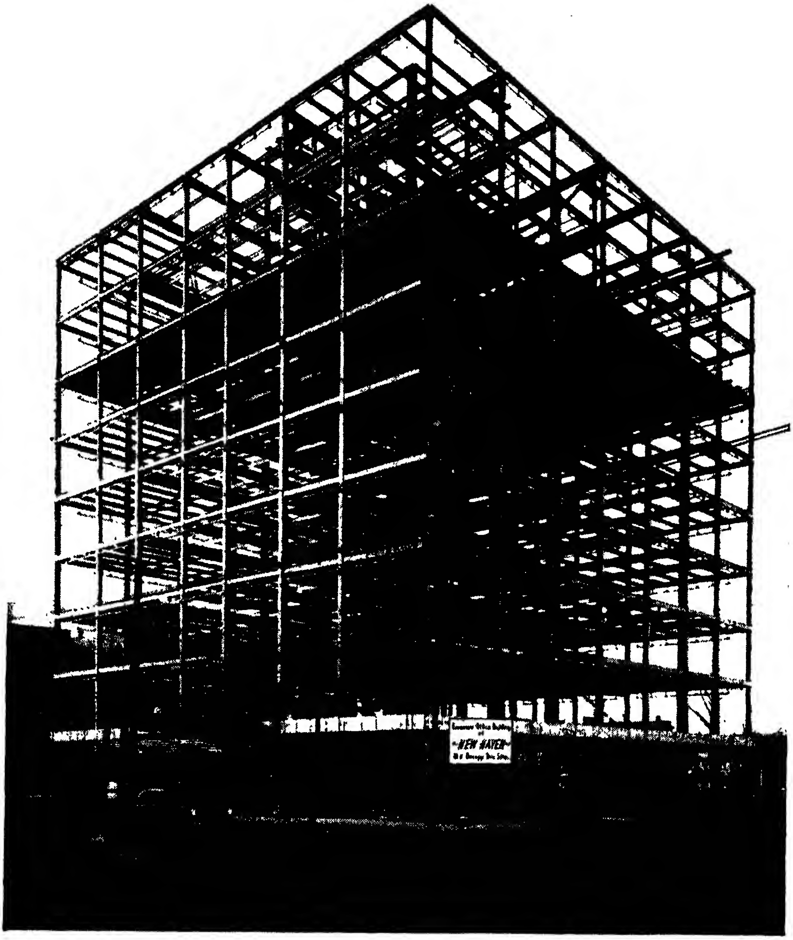
DESIGN OF
STEEL BUILDINGS



STRUCTURAL STEEL FRAMING FOR OFFICE BUILDING.

The New York, New Haven & Hartford Railroad Company
New Haven, Conn.

The floors of this building are one-way cinder concrete slabs with supporting beams spaced approximately eight feet on centers. All structural members were shop riveted. Field connections were welded at columns and bolted at intermediate points.

Architects	*Engineers*
The Office of Douglas Orr	Wilcox, Erickson, and Pfisterer

DESIGN OF STEEL BUILDINGS

by

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PREFACE TO THIRD EDITION

The purpose of this book, as stated in the preface to the first edition, is to present the general principles of structural design as applied to the more common types of buildings such as apartment houses, offices, and school and other institutional buildings. The general scope and method of presentation of the earlier editions have been retained in this revision, but the material on welded construction has been greatly expanded and now forms the subject matter of Chapter IX. In addition to a general discussion of welded framing connections, the application of welding to the design of plate girders and roof trusses is treated in detail.

All the examples in the text have been revised and made consistent with structural shapes now available in conformance with Simplified Practice Recommendation R-216-46 of the U. S. Department of Commerce, National Bureau of Standards. Text matter has been revised to take into account the 1946 revision of the American Institute of Steel Construction's Specification for the Design, Fabrication, and Erection of Structural Steel for Buildings, and the use of the Fifth Edition of the A.I.S.C. Manual is recommended in conjunction with this book. However, in accordance with the policy established in the earlier editions, the book is not predicated on a single specification, since it is believed desirable to point out the variation in specification requirements encountered in practice. To this end the A.I.S.C. Specification (1928) has been employed in many instances as illustrative of building codes predicated on an eighteen-thousand-pound-per-square-inch basic steel stress.

Chapter III on the design of beams has been rewritten extensively, and a more detailed treatment of the use of safe load tables included. As an aid to persons studying outside the classroom, answers to certain selected exercise problems have been given in Appendix E. All exercise problems are new.

This is the first edition of the book to appear under joint authorship. Although the authors collaborated closely, primary responsibility in preparing the chapters was divided as follows:

Revision of Chapters I through VIII, and X, by Hauf.

New material comprising Chapter IX, by Pfisterer.

It is assumed that the reader has some knowledge of the elementary principles of structural mechanics (statics and strength of materials) although much of this material is reviewed throughout the text in connection with design. The building design project, appearing as the last chapter of the first and second editions, has been brought up to date and placed in Chapter X. As in the earlier editions, the purpose of presenting the design of the structural framework and the set of working drawings for the small business building is to assist in bridging the gap between academic work and professional practice.

H.D.H.
H.A.P.

NEW HAVEN, CONNECTICUT
January, 1949

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CHAPTER I

GENERAL CONSIDERATIONS

1. Introduction. — Modern buildings may be classified structurally as either wall-bearing or skeleton construction. In the former type the floor loads are carried by the walls, their thickness being determined by the number of stories. In skeleton construction, the entire building, including the walls, is supported by a structural framework. The exterior walls in skeleton construction serve only as an enclosure and are carried on spandrel beams at each floor level; the wall thickness therefore is independent of the height of the building. A combination of the above types is used frequently for buildings not exceeding three or four stories in height, the exterior walls serving as bearing walls with an interior steel framework of columns, beams, and other structural members necessary to support the floor system.

The column locations and other points of support are usually determined by architectural considerations. The most economical column spacing for average loads is about twenty feet center to center although this will vary with the live load and the type of floor construction. In hotels, hospitals, school buildings, and similar structures the column spacing is generally governed by the partition arrangement. This may necessitate a less economical design, but it should be borne in mind that the steel frame is only a part of the entire building and that a sacrifice of economy in portions of the structural work is often offset by a saving in other operations. Just as there are several schemes that might be used to satisfy the architectural requirements of a building program, so there will usually be a number of different arrangements of the supporting framework. The choice of the most suitable structural scheme requires not only training in structural design but also a knowledge of the comparative costs of different types of work and a familiarity with field methods.

Although the principles developed in this book apply in general to all types of steel frame buildings, emphasis has been placed upon the design requirements of the more common structures such as business buildings, apartment houses, schools, and institutional buildings.

2. Design Procedure. — The general arrangement of the floor framing and especially the provision for columns should be kept in mind during the development of the architectural scheme. Preliminary framing plans should be made and the column dimensions approximated before a final scheme is adopted, inasmuch as the size of columns and the clearances required, especially in the lower stories, may affect the architectural layout. As soon as the arrangement of the floor framing is determined, the beams and girders are designed, followed by the final design of columns and foundations. In small buildings the structural framing is often shown directly on the architectural plans, but for operations of appreciable size a separate set of framing plans is prepared. The architectural and framing plans must be constantly checked against each other, as a change in position of a partition or window may necessitate the relocation of a beam.

3. Design Loads. — The loads for which a building is designed are classified as dead loads and live loads. High buildings and buildings of moderate height but extremely narrow width must also have special provision made to resist wind pressure. The subject of wind stress analysis and design of wind bracing is discussed in Chapter VIII. In some localities earthquake resistance is also a vital factor, but detailed consideration of seismic forces is beyond the scope of this book. The principal factors involved in earthquake-resistant construction are briefly discussed in Art. 136.

The dead load is the load due to the weight of the permanent parts of the building such as floors, including beams and girders, walls, roofs, columns, partitions, etc. The weights of different building materials to be used in determining the dead loads are usually specified in local building codes. In the absence of such specifications the values given in the accompanying table will be found satisfactory.

WEIGHTS OF BUILDING MATERIALS

Materials	Pounds per cubic foot	Pounds per square foot
Ashlar masonry, granite, gneiss	165	
Ashlar masonry, limestone, marble	160	
Ashlar masonry, sandstone, bluestone	140	
Brickwork, pressed brick	140	
Brickwork, common brick	120	
Brickwork, soft brick	100	
Concrete (stone)	144	
Concrete (cinder)	100	
Cinder fill	60	
Cement finish, 1 in. thick	...	12
Hollow tile, 4-in	...	16
Hollow tile, 6-in	...	22
Hollow tile, 8-in	...	30
Hollow tile, 2-in. split-furring	...	8
Plaster, lime or gypsum, $\frac{3}{4}$ in. thick	...	5
Plaster on metal lath, $\frac{3}{4}$ in. thick	...	8
Suspended ceiling (complete)	...	10
Hardwood flooring $\frac{7}{8}$ in. thick	...	4
Pine, spruce or hemlock sheathing	...	2.5
Creosoted wood-block flooring, 3 in. thick	...	15
Roofing felt, 3-ply, and gravel	...	5.5
Roofing felt, 4-ply, and gravel	...	6
Roofing felt, 5-ply, and gravel	...	6.5
Slate, laid in place, $\frac{1}{4}$-in	...	9.5
Slate, laid in place, $\frac{3}{8}$-in	...	14.5
Slate, laid in place, $\frac{1}{2}$-in	...	19.5
Roofing tile, laid in place, Spanish	...	8 to 10
Roofing tile, laid in place, shingle type, clay	...	12 to 14
Roofing tile, laid in place, flat, cement	...	15 to 20
Roofing tile, laid in place, book tile 2-in	...	12
Roofing tile, laid in place, book tile 3-in	...	20
Skylights with $\frac{3}{8}$-in. wire glass and frame	...	7.5
Windows (glass, frame and sash)	...	8
Glass block (4 in. thick)	...	20
Stair construction (steel)	...	50
Stair construction (concrete)	...	150
Stair construction (wood)	...	20

TYPICAL MINIMUM LIVE LOAD REQUIREMENTS

Classes of buildings	Minimum live loads per square foot of floor area					
	New York 1942	Phila-delphia 1941	Pacific Coast Bldg. Officials Confer-ence 1943	Chicago 1941	National Board of Fire Under-writers 1943	Dept.* of Com-merce 1945
Office space: Typical floors....	50	60	50	50	50	80
Hotels, Apartments, Lodging-houses, Hospitals	40	40	40	40	40	40
Dwellings.........	40	40	40	40	40	40
School classrooms .	60	50	40	50	50	40
Buildings or rooms for public assembly:						
With fixed seats..	75	60	50	75	75	60
Without fixed seats	100	100	100	100	100	100
Aisles and corridors	100	100	100	100	100	100
Warehouses.......	120	150	125–250	100–250	120	...
Garages: Public...	175	100	100	100	175	...
Private passenger cars only.......	75	100	100	50	175	...
Manufacturing:						
Heavy..........	120	200	125	100	120	...
Light...........	120	120	75	100	120	125
Stores: Wholesale..	120	110	100	100	100	125
Retail...........	75	110	75	100	100	125
Sidewalks.........	300	120	250	150	300	...

* The minimum live load requirements as recommended, in American Standard Building Code Requirements A 58.1-1945, sponsored by the National Bureau of Standards, are given in more detail in Appendix A.

The live load represents the probable load on the structure due to occupancy and is usually considered uniformly distributed over the floor area. The value, expressed in pounds per square foot, is taken large enough to cover the effect of ordinary concentrations which may occur. Buildings containing heavy machinery or similar large concentrations of load must be designed for such concentrations. Some building codes require that a square foot allowance be included in the live loads of office and loft buildings to cover the effect of partitions, the arrangement of which is usually left until after the building is erected in order to satisfy the particular requirements of the tenants. The effect of impact loads, such as those sustained by beams supporting elevator machinery, are usually provided for by increasing the actual moving loads by 100 per cent. This percentage may be reduced for the secondary framing and columns.

There is considerable lack of uniformity among different building codes as to the proper live load allowances for various types of occupancy. Several agencies, both governmental and professional, are working to bring about a higher degree of standardization in this matter. The recommendations of the U. S. Department of Commerce, the Pacific Coast Building Officials Conference, and the National Board of Fire Underwriters are included in the table on page 4. This table is only a digest of minimum live load requirements, and it must be strongly emphasized that the original codes should always be consulted for actual design work since they contain many restrictions and modifications which cannot be given in such a concise table.

4. Working Stresses. — Municipal building codes are not in complete agreement regarding the allowable values for working stresses to be used in the design of steel buildings. For many years it was general practice to use an allowable stress of 18,000 lb. per sq. in. for structural steel in tension, compression on short lengths, and on the extreme fibre in bending. This value for the basic working stress was formerly embodied in the Specification for the Design, Fabrication and Erection of Structural Steel for Buildings, promulgated by the American Institute of Steel Construction prior to 1936. In 1936 the Institute issued a new specification predicated on a basic working stress of 20,000 lb. per sq. in. The value of 20,000 lb. per sq. in. is also the basis of the 1946 revision of the A.I.S.C. Specification. This value for the basic working stress contemplates the use of structural steel meeting the requirements of the Standard Specification for Steel for Bridges and Buildings, promulgated by the American Society for Testing Materials and bearing the A.S.T.M. designation A 7-42. Steel meeting this specification must have an ultimate tensile strength of 60,000 to 72,000 per sq. in. and a yield point equal to at least half the tensile strength but in no case less than 33,000 lb. per sq. in. Although the basic stress of 20,000 lb. per sq. in. has been widely adopted, some codes still specify a basic stress of 18,000 lb. per sq. in. In all actual design work the local building code should be consulted. During the war years 1941–45, Federal Specifications permitted a basic stress of 22,000 lb. per sq. in. for certain classes of structures.

The ratio between the ultimate strength and the working stress has been defined as the *factor of safety*. This definition is likely to lead to a false sense of security, as failure of a structural member begins when the stress exceeds the elastic limit. This is due to the fact that deformations produced by stresses above this value are *permanent* and hence change the shape of the structure. Since, in design, stresses should always be kept within the elastic limit of a material, it is well to think of the factor of safety as the ratio between the elastic limit and the working stress. As the elastic limit of a structural grade steel meeting A.S.T.M. Specification A 7-42 is approximately half the value of the ultimate strength, the factor of safety when using a basic stress of 20,000 lb. per sq. in. would be 70,000/20,000 = 3.5 when based on the ultimate strength but only 35,000/20,000 = 1.75 when based on the elastic limit.

The relationship between ultimate strength, elastic limit, yield point, and deformation under stress is briefly reviewed in Appendix B.

CHAPTER II

REACTIONS, SHEAR AND BENDING MOMENT

5. Introduction. — A beam is a structural member subjected to transverse loads. Usually beams are in a horizontal position and the loads are applied vertically. There are three general types of beams: the simple beam, the cantilever, and the restrained beam.

A *simple* beam rests on two supports and may carry any system of loads between these supports. A simple beam carrying a concentrated load at the center is shown in Fig. 1*a*.

A *cantilever* beam is one which projects from a single support, such as a beam built into a masonry wall or pier (Fig. 1*b*). Another type of cantilever is the overhanging beam. This is merely a simple beam which projects beyond one or both of its supports (Fig. 1*c*).

A *restrained* beam is one that is rigidly connected at one or more supports or has more than one span. Beams having more than one span are usually called continuous beams (Fig. 1*d* and *e*). Beams of this type cannot be analyzed by the usual methods of statics and are said to be "statically indeterminate."

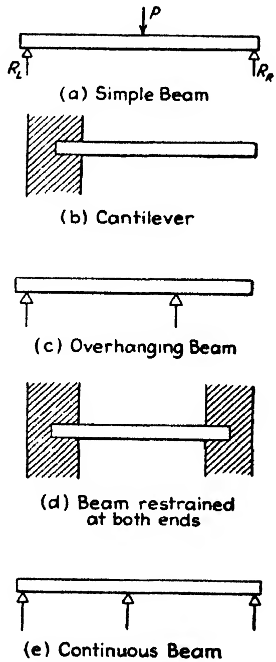


FIG. 1

6. Reactions. — The forces which support a beam are called its reactions. In Fig. 1*a* the left reaction has been designated as R_L and the right as R_R. Reactions are usually treated as concentrated forces. This, of course, is not absolutely true as can be seen from the case of a beam resting on a masonry wall. The beam extends past the face of the wall a certain distance and hence the reaction is distributed over the area of contact between the beam and the wall. This area is usually sufficiently small so that no appreciable error is introduced by considering the reaction to act at the center of the supporting area. The span of a simple beam is the distance between its reactions (Fig. 2).

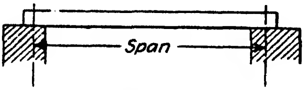


FIG. 2

7. Loads. — The loads on a beam are classified as concentrated and distributed. A concentrated load is one which extends over so small an area of the beam that it may be assumed to act at a point. A girder in a building receives concentrated loads at the points where the floor beams frame into it.

A distributed load is one which extends over a definite portion of a beam. If the load is of such a nature that it can be expressed as a number of pounds per foot of length of beam, it is said to be uniformly distributed. The beams supporting a floor slab in a building receive a uniformly distributed load throughout their length due to the weight of the slab.

8. Determination of Reactions. — After the system of loading has been determined, the next step in the design of a beam is the calculation of the reactions. This is accomplished by applying the three conditions of statics which state that for a body in equilibrium (1) the sum of the vertical components of all the forces acting on the body is zero; (2) the sum of the horizontal components of all the forces acting on the body is zero; and (3) the sum of the moments of all the forces acting on the body, using any point in the plane of the forces as a center of moments, is zero. These conditions may be expressed as follows:[1]

$$\Sigma V = 0 \qquad \Sigma H = 0 \qquad \Sigma M = 0$$

It is evident that in a beam supporting vertical loads only, there will be no horizontal components of the loads and hence no horizontal components of the reactions. The reactions will therefore be vertical and it is necessary to consider only $\Sigma V = 0$, and $\Sigma M = 0$.

Three equations may be written for a simple beam. (1) The summation of moments may be taken with the left reaction as the center of moments or (2) with the right reaction as the center of moments. (3) The algebraic sum of the loads and reactions is

[1] The Greek letter Σ (sigma) indicates a summation, i.e., an algebraic addition of all similar terms involved in the problem.

zero ($\Sigma V = 0$). In order to solve for the reactions, at least one moment equation must be used. This will give the value of one reaction. The other reaction may be found from the summation of the vertical forces, or by taking moments about the known reaction. Until one becomes accustomed to handling these equations it is best to solve for the reactions using the two moment equations and then check by $\Sigma V = 0$.

When computing reactions a distributed load may be considered as acting at its center of gravity.

EXAMPLE I

Compute the reactions of the beam shown in Fig. 3. Neglect the weight of the beam.

Solution: Taking moments about the left reaction ($\Sigma M_{RL} = 0$) and considering upward reactions as positive,

$$12R_R - 2400 \times 4 = 0$$
$$12R_R = 9600$$
$$R_R = 800 \text{ lb.}$$

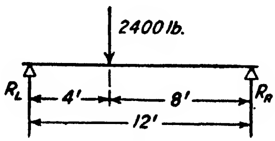


FIG. 3

Using $\Sigma M_{RR} = 0$ (taking moments about the right reaction)

$$12R_L - 2400 \times 8 = 0$$
$$12R_L = 19{,}200$$
$$R_L = 1600 \text{ lb.}$$

Using $\Sigma V = 0$ as a check, the sum of the loads must equal the sum of the reactions.

$$R_L + R_R = 1600 + 800 = 2400 \text{ lb.}$$

EXAMPLE II

Compute the reactions of the beam shown in Fig. 4. The beam weighs 40 lb. per lin. ft.

Solution: $\Sigma M_B = 0$ (taking B as the center of moments)

$$16R_A - (5000 \times 20) - (4000 \times 2) - (100 \times 14 \times \overline{7 + 2}) - (40 \times 20 \times 10) = 0$$
$$16R_A = 100{,}000 + 8000 + 12{,}600 + 8000$$
$$16R_A = 128{,}600$$
$$R_A = 8037.5 \text{ lb.}$$

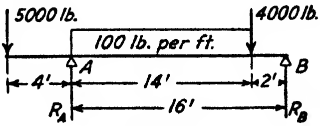


FIG. 4

$\Sigma M_A = 0$ (taking A as the center of moments)

$$16R_B + (5000 \times 4) + (40 \times 4 \times 2) - (4000 \times 14) - (100 \times 14 \times 7) - (40 \times 16 \times 8) = 0$$

$$16R_B = -20{,}000 - 320 + 56{,}000 + 9800 + 5120$$
$$16R_B = -20{,}320 + 70{,}920 = 50{,}600 \text{ lb.}$$
$$R_B = 3162.5$$

Note: In writing the ΣM_A equation, the moments produced by the loads to the left of R_A were given the same sign as the moment produced by R_B because they tend to rotate the beam in the same direction.

Using $\Sigma V = 0$ as a check, the sum of the loads must equal the sum of the reactions.[2]

Loads: $5000 + 4000 + 1400 + (40 \times 20) = 11{,}200$ lb.

Reactions: $R_A + R_B = 8037.5 + 3162.5 = 11{,}200$ lb.

In the analysis of an overhanging beam such as that shown in Fig. 5 it is often impossible to determine the direction of R_A by inspection. If the load P_2 is a great deal larger than P_1 it may be that R_A will act downward. A satisfactory method of solution is to assume that R_A acts upward and solve the problem in the usual manner. If the value for R_A comes out positive (+) the assumption was correct. If R_A comes out negative (−) the assumption was wrong and the reaction acts in the opposite direction. The numerical value, however, will be correct.

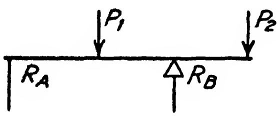


FIG. 5.

9. Shear. — Figure 6*a* represents a beam supporting a system of concentrated loads. It is evident from the sketch that the beam might fail by simply dropping between the supporting walls as shown in Fig. 6*b*. This type of failure is called a shearing failure. It is probable that the beam would fail in some other manner before being sheared off vertically, but the tendency to

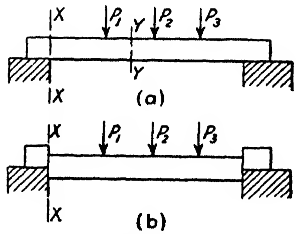


FIG. 6

[2] It is neither necessary nor desirable to carry the values of the reactions to the nearest pound. Three significant figures will give results, the accuracy of which is consistent with the assumptions of structural theory and conditions met with in practice. On this basis R_A and R_B of Example II would have values of 8040 lb. and 3160 lb. respectively. The figures have been extended here merely for the sake of completeness. Slide-rule computations have been used throughout this text, except in a few cases where extension of the numerical work seemed to clarify the explanation of a problem.

fail in this way would nevertheless be present. The force that measures this tendency is called the *vertical shear* or simply the *shear.*

From a study of Fig. 6 it becomes evident that just to the left of section $X-X$ (a right section[3] through the beam infinitely close to the support) the reaction of the wall on the beam is upward. Just to the right of the section the loads on the beam act downward. This combination tends to make the portion of the beam on the right of section $X-X$ slide past that on the left. At any other section such as section $Y-Y$ this same tendency exists, that is, the portion of the beam on the left of the section (called the left segment) tends to slip past the portion on the right of the section (called the right segment). The algebraic sums of all the forces acting on either segment are numerically equal. In order to evaluate the shear at any section, the following definition may be formulated:

The Shear at Any Right Section of a Beam is the Algebraic Sum of All the Transverse Forces on One Side of the Section.

Considering the shearing action at any section of a beam, the loads tend either to cause the portion to the right of the section to descend and the portion to the left of the section to ascend (Fig. 7) or vice versa (Fig. 8). The former is arbitrarily called positive (+) shear and the latter negative (−) shear. The letter V is frequently used as a symbol for shear.

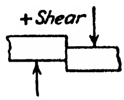


FIG. 7

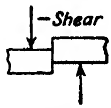


FIG. 8

10. Shear Diagrams — Concentrated Loads. — The variation in shear from section to section of a beam is clearly represented by means of a shear diagram. Figure 9 represents a beam with equal concentrated loads at the quarter points. The weight of the beam has been neglected. From the symmetry of the loading it is evident that the reactions are equal and have a value of $\frac{3P}{2}$. At some convenient distance below the loading diagram a base line representing zero shear has been laid off.

If a section of the beam adjacent to the left reaction is considered, the sum of all the forces acting upward on the left of the

[3] A right section is a section perpendicular to the longitudinal axis of the beam.

section (+ shear) is $\frac{3P}{2}$. The sum of all the forces acting downward on the left of the section (−shear) is zero. Hence from the definition of shear, the shear at the support equals $\frac{+3P}{2}$. This is termed the *end* shear and is equal to the reaction. This value is now plotted to some scale along the line representing the position of the left reaction.

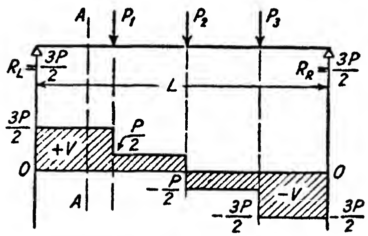


FIG. 9. Shear Diagram.

If another section is taken through the beam at A–A then, from the definition of shear, the sum of all the forces acting upward on the left of the section (+ shear) is $\frac{3P}{2}$. The sum of all the forces acting downward on the left of the section (−shear) is zero. Therefore the shear at section A–A is also $\frac{+3P}{2}$. (It should be kept in mind that in this problem the weight of the beam has been neglected.)

Section A–A might have been any section between the left reaction and the load P_1. Hence it is evident that the shear is constant between these two loads and therefore the shear line is horizontal.

If another section is taken just to the right of the load P_1, the shear at that section will be $\frac{+3P}{2} - P = \frac{P}{2}$. This value is plotted on the line extending down from P_1. The shear between P_1 and P_2 is constant and hence the shear line between these loads is also horizontal. This process is continued across the beam. The fact that the numerical value of the shear at the right end of the beam is equal to the right reaction will serve as a check.

11. Shear Diagrams — Uniform Loads. — Figure 10 is the shear diagram for a simple beam supporting a uniformly distributed load over the entire span. The total load on the beam is wL and each reaction is $\frac{wL}{2}$.

As in the preceding illustration the end shear is equal to the reaction. This is plotted to scale as before.

If the next section is taken at the center of the span, the shear at that section will be $\frac{wL}{2} - \frac{wL}{2} = 0$. If another section such as X–X is taken at the quarter point of the span, the shear there will be $\frac{wL}{2} - \frac{wL}{4} = \frac{+wL}{4}$. At a point three-quarters of the span length from the left reaction, the shear is $\frac{+wL}{2} - \frac{3wL}{4} = \frac{-wL}{4}$. It should be noted that under a uniform load the shear varies as a straight inclined line.

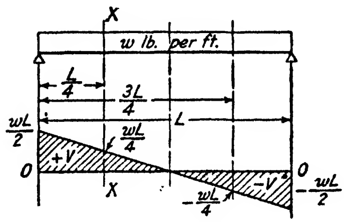


FIG. 10. Shear Diagram.

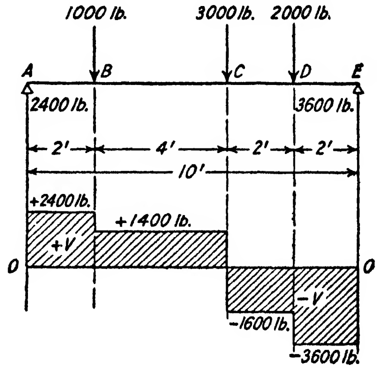


FIG. 11

It is evident from the foregoing discussion and from the examples which follow that the maximum shear for a simple beam or cantilever occurs adjacent to a reaction.

EXAMPLE I

Draw the shear diagram for the beam shown in Fig. 11. Neglect the weight of the beam.

Solution: Solving for reactions,

$$\Sigma M_E = 0 \text{ (taking } E \text{ as the center of moments)}$$
$$10R_A - (1000 \times 8) - (3000 \times 4) - (2000 \times 2) = 0$$
$$10R_A = 8000 + 12{,}000 + 4000 = 24{,}000$$
$$R_A = 2400 \text{ lb.}$$

$$\Sigma M_A = 0 \text{ (taking } A \text{ as the center of moments)}$$
$$10R_E - (2000 \times 8) - (3000 \times 6) - (1000 \times 2) = 0$$
$$10R_E = 16{,}000 + 18{,}000 + 2000 = 36{,}000$$
$$R_E = 3600 \text{ lb.}$$

Using $\Sigma V = 0$ as a check, the sum of the loads must equal the sum of the reactions.

Loads: $1000 + 3000 + 2000 = 6000$ lb.
Reactions: $2400 + 3600 = 6000$ lb.

These reactions have been recorded on the diagram.

The end shear at A is $+2400$ lb. There are no loads between A and B, therefore at any section between these points, the only force on the left of the section is 2400 lb. acting upward. Hence the shear for all sections between A and B is 2400 lb. and is positive. The shear for all sections between B and C is $+2400 - 1000 = 1400$ lb.; for all sections between C and D, $2400 - 1000 - 3000 = -1600$ lb.; and for all sections between D and E, $2400 - 6000 = -3600$ lb. With these values determined, the shear diagram may be drawn as shown.

EXAMPLE II

Draw the shear diagram for the beam shown in Fig. 12.

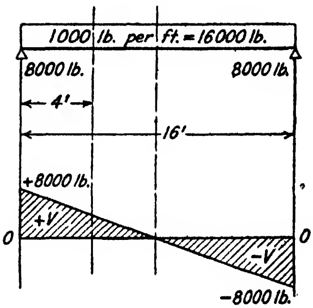


FIG. 12

Solution: The reactions and therefore the end shears each equal 8000 lb.

The shear at the center is

$$8000 - 1000 \times 8 = 0.$$

The shear four feet from the left reaction equals

$$8000 - 1000 \times 4 = 4000 \text{ lb.}$$

With these values the shear diagram may be constructed. It should be noted that for beams loaded as in Fig. 12 the shear line may be drawn by simply connecting the ordinates of the end shears. The shear line should cross the base line at the center of the span.

EXAMPLE III

Construct the shear diagram for the beam shown in Fig. 13. The weight of the beam may be neglected.

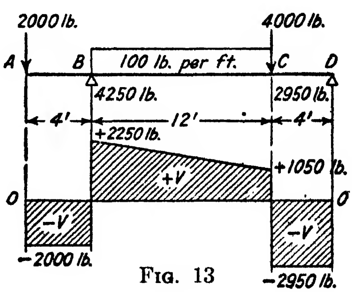


FIG. 13

Solution: Solving for the reactions,

$\Sigma M_D = 0$ (taking D as the center of moments)

$$16R_B - (2000 \times 20) - (4000 \times 4) - (1200 \times \overline{6 + 4}) = 0$$

$$16R_B = 40{,}000 + 16{,}000 + 12{,}000 = 68{,}000$$

$$R_B = 4250 \text{ lb.}$$

The sum of the loads is 2000 + 4000 + 1200 = 7200 lb. Therefore R_D = 7200 − 4250 = 2950 lb. This reaction should be checked, using *B* as the center of moments.

The end shear at *A* is −2000 lb. (see Fig. 8). As there are no loads between *A* and *B* the shear for any section in this panel is −2000 lb. At the section just to the right of *B* the shear is −2000 + 4250 = +2250 lb. At the section just to the left of *C* the shear is −2000 + 4250 − 1200 = +1050 lb. The load between *B* and *C* is a uniform load hence the shear line is a straight line sloping downward from left to right. At a section just to the right of *C* the shear is −2000 + 4250 − 1200 − 4000 = −2950 lb. This is also the shear for any section between *C* and *D*. Therefore the shear at *D* = −2950 lb. This value checks with the reaction.

EXAMPLE IV

A cantilever beam projects 14 ft. from the face of a wall. It supports a uniform load of 200 lb. per ft., extending from the fixed end to within 2 ft. of the free end, and a concentrated load of 1000 lb. applied at the free

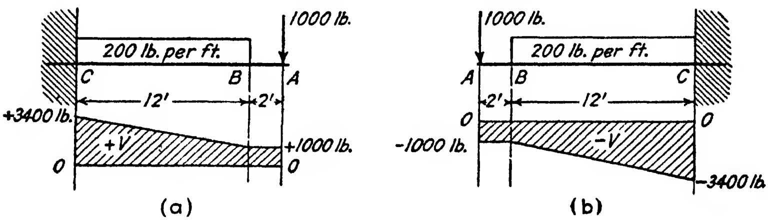


Fig. 14

end. Draw the shear diagram. The weight of the beam may be neglected.

The fixed end may be assumed at the left as shown in Fig. 14*a* or at the right as shown in Fig. 14*b*.

Solution 1: Assume the fixed end at the left as in Fig. 14*a*. The shear for all sections between *A* and *B* will be 1000 lb. If a section is taken just to the left of *A*, the *right* segment of the beam will tend to move downward relative to the *left* segment. Therefore the shear in panel *AB* is positive (see Fig. 7). Under the distributed load, the shear will increase uniformly making the shear at *C*, 1000 + (200 × 12) = 3400 lb. The diagram is constructed as shown.

Solution 2: Assume the fixed end at the right as in Fig. 14*b*.

The shear in panel *AB* will be 1000 lb. If a section is taken just to the right of *A*, the left segment will tend to move downward relative to the right segment. Therefore the shear in the panel is negative (see Fig. 8). Under the distributed load the shear increases uniformly making the shear at *C*, −3400 lb. The diagram is constructed as shown.

Although the sign of the shear at any section of the beam is (+) for Solution 1 and (−) for Solution 2, the numerical value is the same. Since in the design of beams it is the numerical value of the shear that is used, this ambiguity of signs presents no difficulties.

12. Bending Moment. — If the beam shown in Fig. 15*a* has a concentrated load P applied at the center, it will deflect as shown in Fig. 15*b*.[4] If the load were heavy enough it is conceivable

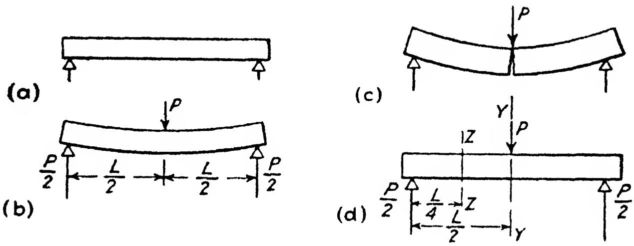


Fig. 15

that the bending would continue until the beam failed as indicated in Fig. 15*c*.

The tendency of the beam to fail in this manner is measured by the moment of the reaction about section $Y–Y$ taken through the point of application of the load (Fig. 15*d*). In Fig. 15*b* this moment is $\frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$ and is called the *bending moment* at section $Y–Y$.

The tendency of the beam to fail by bending is present at every point along its length and hence for any section that may be taken through the beam there exists a definite value for the bending moment. Thus at section $Z–Z$ shown in Fig. 15*d*, the bending moment is $\frac{P}{2} \times \frac{L}{4} = \frac{PL}{8}$ (using the left segment).

If the right segment were used the bending moment at $Z–Z$ would be

$$M = \left(\frac{P}{2} \times \frac{3L}{4}\right) - \left(P \times \frac{L}{4}\right) = \frac{3PL}{8} - \frac{PL}{4} = \frac{PL}{8}$$

[4] In ordinary steel or concrete beams this deflection may not be visible to the eye but it is present, nevertheless, and could be observed if sufficiently sensitive instruments were used.

where M = bending moment; $\left(\frac{P}{2} \times \frac{3L}{4}\right)$ is the product of the right reaction (R_R) multiplied by its distance from the section under consideration; $\left(P \times \frac{L}{4}\right)$ is the product of the load P by its distance from the section under consideration. The term $\left(P \times \frac{L}{4}\right)$ is given the opposite sign from the preceding term because the moment caused by it tends to counteract the effect of the right reaction.

The value for the bending moment at section Z–Z was the same regardless of whether it was computed using the left or right segment of the beam. In order to evaluate the moment at any section, the following definition may be formulated:

The Bending Moment at any Right Section of a Beam is the Algebraic Sum of the Moments of All the Forces on One Side of the Section.

This definition assumes that the beam is horizontal and that all the forces lie in a vertical plane.

If the beam tends to become concave *upward* at the section under consideration (Fig. 16), the bending moment at that section is positive (+). If the beam tends to become concave *downward* at the section (Fig. 17), the bending moment is negative (−).

FIG. 16

FIG. 17

One method which will always give the correct sign for the bending moment is to consider the moments of all upward forces as positive (+) and the moments of all the downward forces as negative (−). This system enables one to work from either end of the beam.

When computing the bending moment at any section of a beam the segment that will involve the simplest arithmetic should be used. In cantilever beams this will always be the segment between the free end and the section under consideration.

13. Bending Moment Diagrams — Concentrated Loads. — The variation of the bending moment from section to section of a beam may be represented by means of diagrams similar to those used for shear. Figure 18 represents a beam with equal concentrated loads at the quarter points. The weight of the beam has been neglected. From the symmetry of the loading it is evident that the reactions are equal and have a value of $\frac{3P}{2}$. At some convenient distance below the loading diagram a base line representing zero bending moment is laid off.

If a section is taken at A, the algebraic sum of the moments of all the forces to the left of the section is zero, since there are no forces to the left. Therefore, the bending moment at A equals zero.

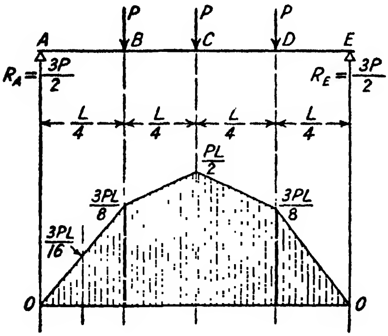


Fig. 18. Bending Moment Diagram.

If a section is taken at B, directly under the load, the algebraic sum of the moments of all the forces to the left of the section is $+\left(\frac{3P}{2} \times \frac{L}{4}\right)$ or $\frac{3PL}{8}$. This value is plotted to some scale on the line representing the section at B.

If the bending moment at any other section between A and B is plotted, it will be found to lie on a straight line connecting A and B. Thus for a section halfway between A and B the bending moment is $+\left(\frac{3P}{2} \times \frac{L}{8}\right) = \frac{3PL}{16}$. This value is just half the value at B. It may be noted, then, that for a system of concentrated loads, the bending moment between loads varies as a straight inclined line.

Referring again to the definition, the bending moment at C is $+\left(\frac{3P}{2} \times \frac{L}{2}\right) - \left(P \times \frac{L}{4}\right) = \frac{3PL}{4} - \frac{PL}{4} = \frac{2PL}{4}$ or $\frac{PL}{2}$. This value is plotted on the line representing the section at C.

In a similar manner the bending moments at D and E may be computed and plotted. The completed diagram is shown in the figure.

14. Bending Moment Diagrams — Uniform Loads. — Figure 19 represents a beam carrying a uniformly distributed load over the entire span. The total load on the beam is wL and each reaction is $\frac{wL}{2}$.

If a section is taken at the center of the span, the algebraic sum of the moments of all the forces to the left of the section is

$$M = +\left(\frac{wL}{2} \times \frac{L}{2}\right) - \left(w \times \frac{L}{2} \times \frac{L}{4}\right) = \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$$

in which M is the bending moment; $\left(\frac{wL}{2} \times \frac{L}{2}\right)$ is the left reaction times its distance from the section under consideration; $\left(w \times \frac{L}{2} \times \frac{L}{4}\right)$ is the load per foot, times the length of beam

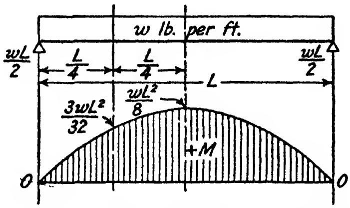


FIG. 19. Bending Moment Diagram.

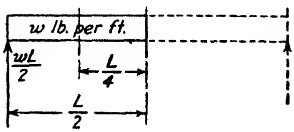


FIG. 20

covered, times the distance from the center of gravity of the load to the section. This is indicated more clearly by Fig. 20, where the left segment is shown by itself.

If the distances are measured in feet and the loads are given in pounds, the bending moment is expressed in foot-pounds.[5]

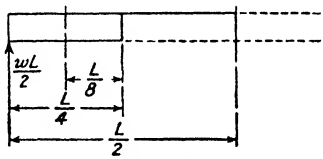


FIG. 21

In a similar manner the bending moment at any other section could be found. If a section is taken through the beam at a distance of one-quarter of the span length from the left reaction, the equation would be (see Fig. 21).

$$M = +\left(\frac{wL}{2} \times \frac{L}{4}\right) - \left(w \times \frac{L}{4} \times \frac{L}{8}\right)$$

$$M = \frac{wL^2}{8} - \frac{wL^2}{32} = \frac{4wL^2}{32} - \frac{wL^2}{32}$$

$$M = \frac{3wL^2}{32}$$

[5] In many textbooks on mechanics this unit of measure is designated as pound-feet. Although pound-feet is a more precise term from the standpoint of theoretical mechanics, foot-pounds has been used throughout this text because of its wide acceptance among practicing engineers.

If the bending moment at any other point were computed it would be found to lie on a parabolic curve, the vertex of which is at the center of the span. Therefore, it may be noted that under a uniform load the bending moment curve is a parabola. In order to plot the bending moment diagram for a beam supporting a uniform load it is necessary to compute the moment at even intervals of say one or two feet along the span length, or to construct the parabola by graphic methods after determining the maximum ordinate.

15. Relation Between Shear and Bending Moment. — In Art. 10 the shear diagram for a beam supporting equal concentrated loads at the quarter-points was constructed, and in Art. 13 the

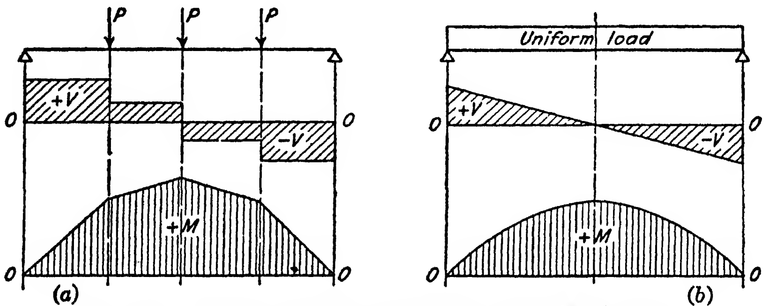


FIG. 22. Relation Between Shear and Bending Moment.

bending moment diagram for the same beam was drawn. These diagrams have been combined in Fig. 22*a*.

Figure 22*b* combines the shear diagram drawn in Art. 11 and the bending moment diagram of Art. 14 for a beam supporting a uniform load.

From Fig. 22 it is evident that

(1) On any portion of a beam where there are no loads, the shear may be represented by a straight horizontal line and the bending moment by a straight inclined line.

(2) On any portion of a beam supporting a uniformly distributed load, the shear may be represented by a straight inclined line and the moment by a parabolic curve, concave downward.

(3) At any point where the line representing the shear passes through zero, there is a maximum ordinate of the bending moment diagram.

A fourth relation which is brought out by Fig. 23 is

(4) If the shear is zero for any distance, the bending moment is constant for that distance and hence is represented by a straight, horizontal line.

It can be demonstrated mathematically that the above relations hold for any beam under any system of vertical loads. Table I of Appendix D shows the relation between the shear and moment curves and gives formulas for the maximum moment for the more common types of loading. The student should check these formulas by deriving them. More complete tables may be found in the handbooks published by the steel companies.

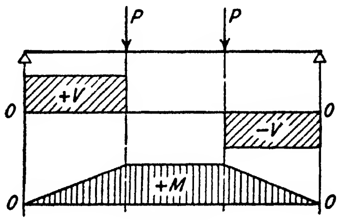


FIG. 23

In the examples which follow the loads and spans are the same as those used for the examples designated by corresponding numbers following Art. 11. The shear diagrams have been reproduced to show the relation between shear and bending moment and to aid in the construction of the bending moment diagrams.

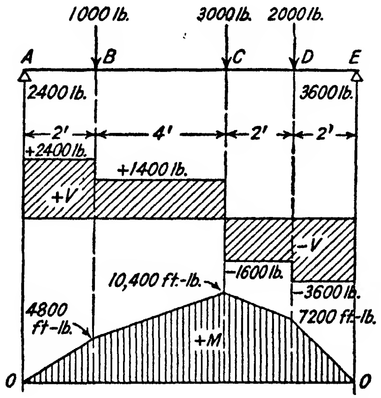


FIG. 24

EXAMPLE I

Draw the shear and bending moment diagrams for the beam shown in Fig. 24. Neglect the weight of the beam.

Solution: See Example I following Art. 11 for the computation of reactions and construction of the shear diagram.

Working from the left, the bending moments are

$M_A = 0$

$M_B = +2400 \times 2 = 4800$ ft.-lb.

$M_C = +(2400 \times 6) - (1000 \times 4) = 14{,}400 - 4000 = 10{,}400$ ft.-lb.

$M_D = +(2400 \times 8) - (1000 \times 6) - (3000 \times 2) = 7200$ ft.-lb.

$M_E = 0$

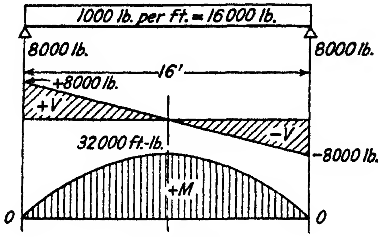


FIG. 25

EXAMPLE II

Draw the shear and bending moment diagrams for the beam shown in Fig. 25.

Solution: See Example II following Art. 11 for the computation of reactions and construction of the shear diagram.

The maximum bending moment occurs at the center and is

$$M = +(8000 \times 8) - (1000 \times 8 \times 4) = 64{,}000 - 32{,}000 = 32{,}000 \text{ ft.-lb.}$$

The moments at one or two-foot intervals along the span are computed in a similar manner and a curve drawn through the plotted points.

EXAMPLE III

Draw the shear and bending moment diagrams for the beam shown in Fig. 26. The weight of the beam is neglected.

Solution: See Example III following Art. 11 for the computation of reactions and construction of the shear diagram.

Working from the left, the bending moments are

$M_A = 0$

$M_B = -(2000 \times 4) = -8000$ ft.-lb. (See Figs. 16 and 17 for determination of sign.)

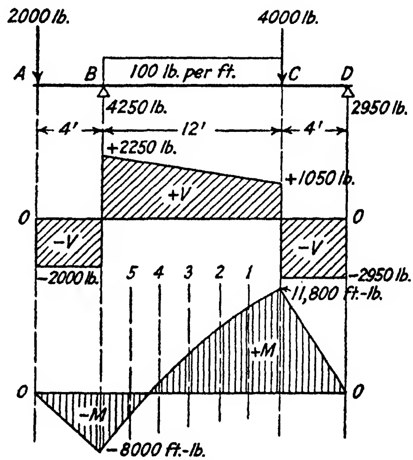


FIG. 26

Working from the right,

$M_C = +(2950 \times 4) = +11{,}800$ ft.-lb.

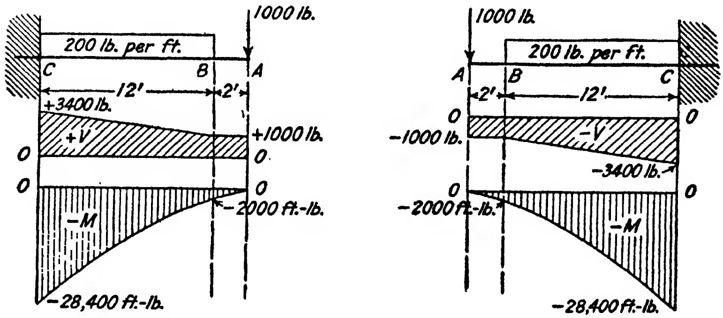
Solving for the bending moment at 2-ft. intervals between C and B,

$$M_1 = +(2950 \times 6) - (4000 \times 2) - (100 \times 2 \times 1) = 9500 \text{ ft.-lb.}$$
$$M_2 = +(2950 \times 8) - (4000 \times 4) - (100 \times 4 \times 2) = 6800 \text{ ft.-lb.}$$
$$M_3 = +(2950 \times 10) - (4000 \times 6) - (100 \times 6 \times 3) = 3700 \text{ ft.-lb.}$$
$$M_4 = +(2950 \times 12) - (4000 \times 8) - (100 \times 8 \times 4) = 200 \text{ ft.-lb.}$$
$$M_5 = +(2950 \times 14) - (4000 \times 10) - (100 \times 10 \times 5) = -3700 \text{ ft.-lb.}$$

EXAMPLE IV

A cantilever beam projects 14 ft. from the face of a wall. It supports a uniform load of 200 lb. per ft. extending from the fixed end to within 2 ft. of the free end, and a concentrated load of 1000 lb. applied at the free end. Draw the shear and bending moment diagrams. The weight of the beam may be neglected.

Solution 1: Assume the fixed end at the left as in Fig. 27*a*. For construction of the shear diagram see Example IV following Art. **11**.



(*a*) FIG. 27 (*b*)

Working from the free end, the bending moments are

$M_A = 0$

$M_B = -(1000 \times 2) = -2000$ ft.-lb. (See Figs. 16 and 17 for determination of sign.)

$M_C = -(1000 \times 14) - (200 \times 12 \times 6) = -28{,}400$ ft.-lb.

The moment at a point halfway between C and B is

$M = -(1000 \times 8) - (200 \times 6 \times 3) = -11{,}600$ ft.-lb.

The moments at other points between C and D are computed in a similar manner.

Solution 2: Assume the fixed end at the right as in Fig. 27*b*.

The curve will be as shown. The proof of this is left to the student. By reference to Figs. 16 and 17 it will be seen that the sign of the bending moment in a cantilever beam is the same whether the beam is assumed fixed at the right or left.

PROBLEMS

Compute the reactions of the beams in Problems 1 through 6 below, neglecting the beam weight.

1. Span = 18 ft. One concentrated load of 12,000 lb. is applied 6 ft. from the left reaction, and another, of 9000 lb., 4 ft. from the right reaction. (Answer given in Appendix E.)

2. Span = 16 ft. A concentrated load of 20,000 lb. is applied 5 ft. from the left reaction, and a uniform load of 900 lb. per ft. extends from the right reaction to mid-span. (Answer given in Appendix E.)

3. The beam is 21 ft. long over-all with the left reaction at the left end of the beam but with the right reaction located 15 ft. from the left end, thereby leaving an overhang at the right end of 6 ft. A concentrated load of 5000 lb. is applied at the end of the overhang, and a uniform load of 800 lb. per ft. also extends over the 6-ft. length of the overhang. There is also a concentrated load of 16,000 lb. located 5 ft. from the left end. (Answer given in Appendix E.)

4. Span = 21 ft. One concentrated load of 6300 lb. is applied 5 ft. from the left reaction, and another, of 18,000 lb., 7 ft. from the right reaction.

5. Span = 20 ft. One concentrated load of 15,000 lb. is located 6 ft. from the left reaction, and another of the same value is located 4 ft. from the right reaction. A uniform load of 1000 lb. per ft. extends over the 10-ft. length of beam between the concentrated loads.

6. The beam is 20 ft. long with the left reaction located 6 ft. from the left end and the right reaction located 4 ft. from the right end, making the distance between reactions equal 10 ft. A concentrated load of 9000 lb. is applied at the extreme left end and another of 7000 lb. at the extreme right end. A uniform load of 800 lb. per ft. extends over the entire beam.

7. Draw the shear and bending moment diagrams for the beam described in Problem 1. Use a length scale (L) of $\frac{1}{8}$ in. = 1 ft.; a shear scale (V) of 1 in. = 20,000 lb., and a bending moment scale (M) of 1 in. = 60,000 ft.-lb. (Answers for shear and bending moment values given in Appendix E.)

8. Draw the shear and bending moment diagrams for the beam described in Problem 2. Scales: L, $\frac{1}{8}$ in. = 1 ft.; V, 1 in. = 20,000 lb.; M, 1 in. = 60,000 ft.-lb. (Answers for shear and bending moment values given in Appendix E.)

9. Draw the shear and bending moment diagrams for the beam described in Problem 3. Scales: L, $\frac{1}{8}$ in. = 1 ft.; V, 1 in. = 20,000 lb.; M, 1 in. = 50,000 ft.-lb. (Answers for shear and bending moment values given in Appendix E.)

10. Draw the shear and bending moment diagrams for the beam described in Problem 4. Scales: L, $\frac{1}{8}$ in. = 1 ft.; V, 1 in. = 10,000 lb.; M, 1 in. = 60,000 ft.-lb.

11. Draw the shear and bending moment diagrams for the beam described in Problem 5. Scales: L, $\frac{1}{8}$ in. = 1 ft.; V, 1 in. = 30,000 lb.; M, 1 in. = 100,000 ft.-lb.

12. Draw the shear and bending moment diagram for the beam de-

scribed in Problem 6. Scales: L, $\frac{1}{4}$ in. = 1 ft.; V, 1 in. = 10,000 lb.; M, 1 in. = 30,000 ft.-lb.

13. Compute the reactions and draw the shear and bending moment diagrams for the beam and loading indicated. Beam length = 20 ft. The left end overhangs the left reaction 2 ft. and the right end overhangs the right reaction 4 ft., making the distance between reactions 14 ft. A uniform load of 1000 lb. per ft. extends over the entire 20-ft. length, and a concentrated load of 2000 lb. is applied at each end of the beam. Scales: L, $\frac{1}{8}$ in. = 1 ft.; V, 1 in. = 10,000 lb.; M, 1 in. = 10,000 ft.-lb.

14. A cantilever beam projects 9 ft. from the face of a wall. Three concentrated loads of 4000 lb. each are applied at 3-ft. intervals, starting with one load at the free end. Draw the shear and bending moment diagrams for these conditions, considering the fixed end to be at the right. Scales: L, $\frac{1}{4}$ in. = 1 ft.; V, 1 in. = 10,000 lb.; M, 1 in. = 60,000 ft.-lb.

15. A cantilever beam projects 8 ft. from the face of a wall. A uniform load of 500 lb. per ft. extends over the entire beam, and a concentrated load of 3000 lb. is applied 4 ft. from the free end. Draw the shear and bending moment diagrams for these conditions, considering the fixed end to be at the left. Scales: L, $\frac{1}{4}$ in. = 1 ft.; V, 1 in. = 6000 lb.; M, 1 in. = 20,000 ft.-lb.

16. A simple beam 12 ft. long carries a uniformly distributed load of 800 lb. per ft. over its entire length, and a concentrated load of 3000 lb. located 4 ft. from the left end. Draw the shear and bending moment diagrams for these conditions, checking analytically the distance from the right reaction to the point of zero shear. Scales: L, $\frac{1}{4}$ in. = 1 ft.; V, 1 in. = 6000 lb.; M, 1 in. = 20,000 ft.-lb.

CHAPTER III

THE DESIGN OF BEAMS

16. Resisting Moment. — In the preceding chapter, the external forces acting on a beam were discussed and a means of measuring their effect in terms of shear and bending moment was developed. It is now necessary to consider the forces acting within the beam that resist the tendency of the external forces to cause failure. The first of these to be investigated is the action within the beam that resists bending and is called the *resisting moment.*

Before proceeding with the derivation of an expression for the resisting moment, it will be well to recall the principle of mechanics which states that for every action there is an equal and opposite reaction. If a 10-lb. weight is placed on a table, the table reacts against the weight with a force of 10 lb. If the weight is increased to 20 lb., the table exerts a force of 20 lb. on the weight. The same principle applies to bending in a beam. If a beam is subjected to a system of loads the bending moment at every section is opposed by a resisting moment which is equal in magnitude to the bending moment and opposite in sense.

17. The Theory of Bending. — The derivation of the expression for resisting moment is based on two fundamental assumptions. The first of these is:

A PLANE SECTION OF A BEAM BEFORE BENDING REMAINS A PLANE SECTION AFTER BENDING.

The parallel plane sections *AB* and *CD* in Fig. 28*a* remain plane sections after the beam is bent as shown in Fig. 28*b*. They are, however, no longer parallel.

It may be observed from Fig. 28*b* that the portion of the beam between the two sections was rectangular before bending and became trapezoidal after bending. Therefore, the fibres in the upper portion of the beam must have become shorter than their original length and those in the lower portion longer. Furthermore, it is evident from Fig. 28*b* that the maximum shortening of the fibres occurred at the top surface and the amount of shortening decreased toward the center of the beam. Likewise, the maximum lengthening of the fibres occurred at the bottom surface and the amount of lengthening decreased toward the center. From this it follows that there must be some surface between the top and bottom of the beam where no change in length occurred. This surface is called the *neutral surface* and is designated in the figure by the line *NS*. The line in which this neutral surface cuts a

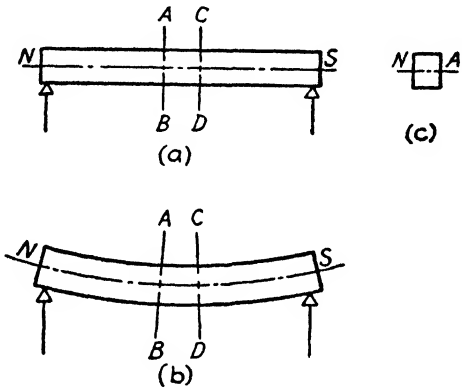


Fig. 28.

right section of the beam is called the *neutral axis* (Fig. 28*c*). It may be demonstrated mathematically that the neutral axis passes through the center of gravity of the section. In symmetrical sections such as rectangular beams and I-beams, the neutral axis is midway between the top and bottom surfaces.

The second assumption used in the derivation of the resisting moment is:

Within the elastic limit of the material, stress varies directly as the deformation and therefore as the distance from the neutral axis.

In other words, fibres that have the greater changes in length carry more stress than those that have smaller changes. This means that the maximum compressive stress in the beam shown in Fig. 28*b* occurs at the extreme top fibre and decreases uniformly to zero at the neutral axis. The maximum tensile stress occurs at the extreme bottom fibre and decreases uniformly to zero at the neutral axis. Most common structural materials have the same modulus of elasticity in tension as in compression. For materials of this class, the extreme fibre stress in tension is therefore equal to the extreme fibre stress in compression for homogeneous beams of symmetrical cross section.

18. The Beam Formula. — The expression for resisting moment that will now be established on the basis of the two foregoing assumptions is known as the *beam formula* or the *flexure formula*. It will be developed here by an elementary method in order to emphasize the physical significance of the various factors involved.

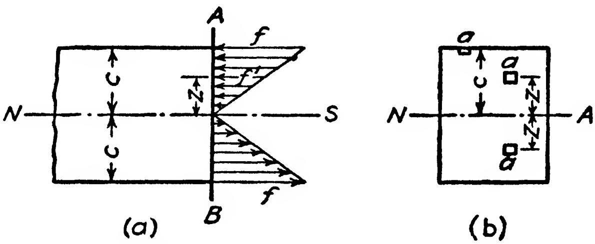


Fig. 29. Stress Distribution.

Figure 29*a* represents a portion of the beam shown in Fig. 28*b* cut at the section *AB*. The forces acting on the section are indicated by arrows. Figure 29*b* is an end view of this section on which is indicated the infinitely small end area of three fibres. Steel is not, of course, a fibrous material in the sense that wood is, but the concept of infinitely small fibres is very useful in studying stress relationships within any structural material.

Let f = the intensity of the stress at the extreme fibre in pounds per square inch;
a = the infinitely small area of a fibre in square inches;
c = the distance of the extreme fibre from the neutral axis in inches;
z = the distance of *any* fibre from the neutral axis in inches.

Using the above notation, the total stress in an extreme fibre is equal to the intensity of stress at that point multiplied by the area of the fibre, or fa.

It has been shown that the stress is proportional to the distance from the neutral axis. Referring to Fig. 29*a*, by similar triangles $\frac{f'}{f} = \frac{z}{c}$ or $f' = \frac{fz}{c}$. Therefore, the intensity of stress on a fibre at distance z from NA is $\frac{fz}{c}$ and the total stress on a fiber at distance z from NA is

$$f \times \frac{z}{c} \times a = \frac{faz}{c}$$

The moment of this stress about the neutral axis is

$$\frac{faz}{c} \times z \quad \text{or} \quad \frac{faz^2}{c}$$

It is clear that this process could be repeated for all the fibres in the beam. Furthermore, the sum of all such terms as $\frac{faz^2}{c}$ would be the sum of the moments of all the fibres in the section about NA or the resisting moment of the section. Expressed as an equation,

$$M_R = \Sigma \frac{faz^2}{c} = \frac{f}{c} \times (\Sigma az^2)$$

The term (Σaz^2) is called the moment of inertia[1] of the section with respect to the neutral axis and is usually designated by the letter I. The formula then becomes

$$M_R = \frac{fI}{c}$$

This expression for resisting moment holds for all shapes of beams, but the value of I will vary. In this demonstration a rectangular section has been used for the sake of simplicity, but the moment of inertia of any section may be computed by use of the calculus or other summation methods. Because the maximum value of the fibre stress occurs at the extreme top and bottom of the beam cross section, it is obviously desirable to place as much as possible of the material of which a beam is made near the top and bottom faces. This consideration is the basis for the familiar I and H shapes of structural-steel beams.

As previously stated, the bending moment and resisting moment

[1] Definition. — The moment of inertia of an area with respect to any axis is the sum of the products obtained by multiplying each element of area by the square of its distance from the given axis. If the elements of area are expressed in square inches and the distances in inches, the units for moment of inertia will be inches raised to the fourth power or inches^4.

at any section of a beam are equal. Therefore, we may write

$$M = M_R = \frac{fI}{c} \quad \text{or} \quad M = \frac{fI}{c}$$

This equation is known as the *beam formula* or the *flexure formula.*

19. Section Modulus. — The term $\frac{I}{c}$ is called the *section modulus* and is dependent upon the size and shape of the section. It is defined as the moment of inertia divided by the distance from the extreme outer fibre to the neutral axis and is sometimes expressed by the letter S. The American Institute of Steel Construction Manual and other steel handbooks give the section moduli of rolled shapes as well as their moments of inertia. These are usually listed under "Properties of Sections," "Technical Functions," or some similar title. The section moduli, moments of inertia, and other technical functions of American Standard I-beams are given in Table II of Appendix D. Since I-beams supporting vertical loads are normally placed with their webs vertical, the values of I and S to be used in the beam formula will be those for the x–x axis shown in the tables.

20. Use of the Beam Formula. — The beam formula may be stated in three different ways:

$$(1)\ M = \frac{fI}{c} \qquad (2)\ f = \frac{Mc}{I} \qquad (3)\ \frac{I}{c} = \frac{M}{f}$$

or, letting $\frac{I}{c} = S$,

$$(1)\ M = fS \qquad (2)\ f = \frac{M}{S} \qquad (3)\ S = \frac{M}{f}$$

In form (1) when the dimensions of the beam are known $\left(\text{represented by } \frac{I}{c}\right)$ and f is the maximum allowable fibre stress for the material, solving for M gives the maximum bending moment that the beam can resist, that is, the maximum resisting moment.

In form (2) when the maximum bending moment due to the loading is known as well as the dimensions of the beam, solving for f gives the stress in the extreme fibre.

In form (3) when the bending moment and allowable extreme fibre stress are known, solving for $\frac{I}{c}$ gives the section modulus required. This is the form used in design. When the required section modulus has been computed, a beam section is selected that has an $\frac{I}{c}$ equal to or greater than the one required.

It should be noted in using the formula that f is expressed in pounds per square inch, c in inches, and I in inches^4. Hence M must be in inch-pounds. M, as usually computed from the reactions and loads, is expressed in foot-pounds and must be converted to inch-pounds by multiplying its value by 12 before it is used in the formula.

EXAMPLE I[2]

A 10″ I 35 lb. has a span of 10 ft. How great a concentrated load at the center will the beam support if the maximum allowable fibre stress is 20,000 lb. per sq. in.?

Solution:

(1) Draw a sketch of the beam showing the loads (Fig. 30). In addition to the concentrated load, the beam must support its own weight.

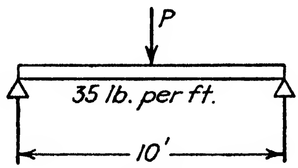


FIG. 30.

(2) From Art. 20, the maximum resisting moment of the beam is $M = fS$. Referring to Table II of Appendix D (American Standard Beams—Properties for Designing), the section modulus of a 10″I 35 lb. is found to be 29.2. Therefore,

$$M = fS = 20{,}000 \times 29.2 = 584{,}000 \text{ in.-lb. or } \frac{584{,}000}{12} = 48{,}700 \text{ ft.-lb.}$$

(3) From this resisting moment, the moment due to the beam weight must be deducted. Referring to Art. 14 and Fig. 19, the maximum moment due to the uniformly distributed load over the entire span occurs at the center and is equal to $\frac{wL^2}{8}$. Therefore,

$$M = \frac{wL^2}{8} = \frac{35 \times 10 \times 10}{8} = \frac{3500}{8} = 438 \text{ ft.-lb. (due to beam weight)}$$

[2] Steel I-beams and channels are designated by their nominal depths and their weight per foot of length. The designation 10″ I 35 lb. indicates that the I-beam mentioned has a nominal depth of 10 in. and weighs 35 lb. per ft. Simlar information regarding a channel section is indicated by 10″ [20 lb. The actual depths of many Wide Flange beams such as those rolled by the Bethlehem, Illinois, and Carnegie Steel Companies vary somewhat from the nominal depth. In this text the symbol I denotes an American Standard I-beam, and WF denotes a Wide Flange section.

The allowable bending moment due to the concentrated load is

$$M = 48{,}700 - 438 = 48{,}262 \text{ or } 48{,}300 \text{ ft.-lb.}$$

(4) Referring again to Table I, the maximum moment caused by a concentrated load at the center occurs under the load and is equal to $\frac{PL}{4}$.

$$\frac{PL}{4} = 48{,}300 \text{ ft.-lb.}$$

$$P = \frac{4 \times 48{,}300}{10} = 19{,}300 \text{ lb.}$$

EXAMPLE II

A 12″ I 31.8 lb. supports a total load of 30,000 lb. (including its own weight) uniformly distributed over a span of 14 ft. Find the maximum extreme fibre stress.

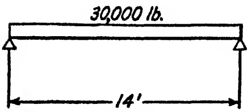


FIG. 31.

Solution:

(1) Draw a sketch showing the loads (Fig. 31).

(2) The maximum extreme fibre stress will occur at the section of maximum bending moment. From Table I the maximum moment is found to occur at the center of the span and is $\frac{WL}{8}$. Therefore,[3]

$$M = \frac{WL}{8} = \frac{30{,}000 \times 14}{8} = 52{,}500 \text{ ft.-lb.}$$

(3) From Art. 20, the extreme fibre stress is $f = \frac{M}{S}$. Referring to Table II, the value of S for a 12″ I 31.8 lb. is found to be 36.0. Therefore,

$$f = \frac{M}{S} = \frac{52{,}500 \times 12}{36} = 17{,}500 \text{ lb. per sq. in.}$$

[3] The difference between the two forms of the equation for bending moment at mid-span for beams carrying uniform loads should be clearly understood. The equation developed in Art. 14 was $M = \frac{wL^2}{8}$. In this expression w is the uniform load per foot of span and L is the span length in feet. It may be written $M = \frac{w \times L \times L}{8}$. The product of the first two terms of the numerator ($w \times L$) equals the total uniform load on the span. If the total uniform load is designated by W then $w \times L = W$ and the expression becomes $M = \frac{WL}{8}$. This second form is frequently more convenient to use than $M = \frac{wL^2}{8}$.

In the above equation the bending moment was multiplied by 12 to convert it to inch-pounds.

EXAMPLE III

Design an I-beam to carry a concentrated load of 27,000 lb. at the center of a 14-ft. span. The allowable extreme fibre stress is 18,000 lb. per sq. in.

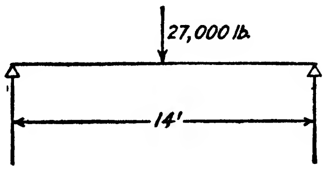


FIG. 32.

Solution:

(1) Draw a diagram showing the loads (Fig. 32).

(2) From Table I the bending moment due to the concentrated load occurs at mid-span and is

$$M = \frac{PL}{4} = \frac{27,000 \times 14}{4} = 94,500 \text{ ft.-lb.}$$

(3) From Art. 20, the section modulus required for this bending moment is

$$S = \frac{M}{f} = \frac{94,500 \times 12}{18,000} = 63.0$$

(4) Referring to Table II, a 15″ I 50 lb. is found to have a section modulus of 64.2. This beam is selected for the first trial.

(5) The bending moment at the center due to the weight of the beam is

$$M = \frac{WL}{8} = \frac{(50 \times 14) \times 14}{8} = 1230 \text{ ft.-lb.}$$

(6) The total bending moment at the center is

$$94,500 + 1230 = 95,730 \text{ ft.-lb.}$$

(7) The section modulus required for this moment is

$$S = \frac{M}{f} = \frac{95,730 \times 12}{18,000} = 63.8$$

Inasmuch as this required value is less than the section modulus of the assumed beam, the 15″ I 50 lb. is adopted.

21. Design Procedure. — Example III of the preceding article illustrates the steps necessary for the design of a simple beam to resist bending stresses. For beams with more complicated systems of loading, additional operations are often required. The following procedure gives the usual steps necessary for a complete design to resist bending.

(1) Make a sketch of the beam showing the loads.

(2) Compute the reactions.

(3) Find the points of maximum bending moment. These will occur where the shear line passes through zero. For the more complicated cases it may be necessary to plot the shear diagram.

(4) Compute the maximum bending moment. (The maximum numerical value regardless of sign is usually the one required.)[4] For complicated loadings it may be advantageous to plot the bending moment diagram.

(5) Find the required section modulus by the beam formula

$$S = \frac{M}{f}$$

(6) Select a beam with a section modulus somewhat *larger* than that required to carry the superimposed load. (The excess is to take care of the additional bending moment caused by the beam weight.)

(7) Compute the bending moment due to the beam weight at the point of maximum bending moment due to the superimposed loads.

(8) Add this moment to the moment previously computed for the superimposed load and compute the required section modulus for this total bending moment. This value should not be greater than the section modulus of the beam selected. If it is greater a new beam must be assumed and Steps (7) and (8) repeated.

When there are no other governing conditions the lightest-weight section that will carry the load should be used. In many instances, however, the required clear story height or other architectural considerations limit the depth, thereby necessitating the use of a heavier beam of equal strength but less depth. The above procedure may be abbreviated as one becomes proficient in design.

[4] It is evident from a comparison of Fig. 16 (Art. 12) and the discussion in the last paragraph of Art. 17 that positive bending moment induces tensile stress in the bottom fibres of a beam and compressive stress in the top fibres.

A study of Fig. 17 (Art. 12) will show that this condition is reversed in the case of negative bending moment, the top fibres being in tension and the bottom fibres in compression. Since steel is equally strong in tension and compression this consideration is seldom important in symmetrical sections such as I-beams except where overhanging or continuous beams are to be spliced.

22. Effect of Beam Weight. — In cases where the weight of the beam is very small compared with the applied load, it may be neglected. However, in fireproof construction, where the steel beams are surrounded by concrete, the weight of the beam plus the fireproofing is often an appreciable part of the load. Furthermore, the maximum bending moment with the beam weight included may not occur at exactly the same section as the maximum bending moment due to the superimposed loads alone. In cases where the weight of the beam is large compared to the superimposed load, the position and amount of the maximum bending moment should be checked after the weight of the beam is known.

A simple way to eliminate this difficulty and also Steps (7) and (8) is to assume a weight for the beam and show it as a load under Step (1). This method should not be employed, however, until enough beams have been designed so that the student's experience and judgment will enable him to make a reasonable assumption.

PROBLEMS

A handbook such as the American Institute of Steel Construction Manual should be used in the solution of the following problems. Although Table II of the Appendix contains all the necessary data for solving problems involving American Standard I-beams, it does not give the technical functions of the Wide Flange sections so generally used in present-day practice, or of other structural shapes such as channels and T-sections.

1. A 12″ I 35 lb. is used on a 15-ft. span. Find the total uniform load the beam will support in addition to its own weight if the allowable extreme fibre stress is 20,000 lb. per sq. in. (Answer given in Appendix E.)

2. An 18″ I 54.7 lb. supports a uniformly distributed load of 2000 lb. per lin. ft., including its own weight, over a span of 21 ft. In addition, there is a concentrated load of 6000 lb. applied 7 ft. from the left end. Find the position and value of the maximum extreme fibre stress developed by this loading. (Answer given in Appendix E.)

3. Design a beam for an 18-ft. span that will sustain a total uniform load (including an allowance for the beam weight) of 14,000 lb., plus concentrated loads of 18,000 lb. each, located 6 ft. from each end. The allowable extreme fibre stress is 20,000 lb. per sq. in. Select (*a*) the lightest-weight American Standard I-beam that will sustain the loading and (*b*) the lightest-weight Wide Flange beam. (Answer given in Appendix E.)

4. Find the maximum concentrated load that an 8″ [11.5 lb. can support at the center of a 10-ft. span if the allowable extreme fibre stress is 20,000 lb. per sq. in.

5. A 16″ WF 36 lb. is used as a beam 21 ft. long and supports equal concentrated loads at the third points of the span. Find the maximum value of the loads if the allowable extreme fibre stress is 18,000 lb. per sq. in.

6. A 5″ I 10 lb. is used as a cantilever beam which projects 4 ft. from the face of a wall. Find the maximum concentrated load it can support at the free end if the allowable extreme fibre stress is 20,000 lb. per sq. in.

7. A 21″ WF 82 lb. is used on a span of 24 ft. It supports a uniformly distributed load of 500 lb. per ft., including its own weight. In addition, a concentrated load of 32,000 lb. is applied 6 ft. from the left end, and another concentrated load of 21,000 lb. is applied 8 ft. from the right end. Find the maximum extreme fibre stress developed by this loading.

8. A 14″ WF 30 lb. rests on two supports spaced 18 ft. apart. The beam extends 6 ft. beyond the left support and 4 ft. beyond the right support. A concentrated load of 16,000 lb. is applied midway between the two supports, another concentrated load of 9000 lb. is applied at the left end of the beam, and a third concentrated load of 8000 lb. is applied at the right end of the beam. In addition, a uniform load of 400 lb. per ft., including the beam weight, extends over the entire length of the beam. Find the value of the extreme fibre stress:

(*a*) over the left support;
(*b*) under the 16,000-lb. load;
(*c*) over the right support.

In each case indicate the position of the extreme fibre stress in tension and compression, i.e., top or bottom of beam (see footnote to Art. 21, Step 4).

9. Design an I-beam to carry a single concentrated load of 21,000 lb. at the center of a 16-ft. span. Allowable extreme fibre stress = 20,000 lb. per sq. in.

10. Design an I-beam to carry a superimposed uniform load of 2000 lb. per ft. over a span of 20 ft. Allowable extreme fibre stress = 18,000 lb. per sq. in.

11. Design a Wide Flange section 21 ft. long to support concentrated loads of 15,000 lb. located at each third-point of the span. Allowable extreme fibre stress = 20,000 lb. per sq. in.

12. Design a Wide Flange section for the following conditions. The span is 18 ft. Four feet from the right reaction a concentrated load of 8000 lb. is applied. A uniform load of 900 lb. per ft. extends from the left reaction to the concentrated load. Allowable extreme fibre stress is 20,000 lb. per sq. in.

13. Design a Wide Flange section for the following conditions. The beam span is 18 ft. Six feet from the left reaction a concentrated load of 6000 lb. is applied. Another concentrated load of 3000 lb. is applied 6 ft. from the right reaction. A uniform load of 800 lb. per ft., including an allowance for beam weight, extends over the entire span. Allowable extreme fibre stress is 18,000 lb. per sq. in.

14. Design a beam for the following conditions. The over-all length is 24 ft. The beam overhangs the right reaction by 6 ft., making the distance between reactions 18 ft. A concentrated load of 8000 lb. is applied 6 ft. from the left reaction, and another concentrated load of 8000 lb. is applied 12 ft. from the left reaction. A concentrated load of 4000 lb. is applied at the extreme end of the overhang, and a uniform load of 1000 lb. per ft. extends over the 6-ft. distance between the right reaction and the end of the overhang.

23. Shearing Resistance. — After a beam has been designed for bending, its resistance to shear should be investigated. In most cases this investigation will not result in increasing the size of the beam except for those of short span, heavily loaded or subject to large, concentrated loads near the ends.

Two kinds of shear exist in a beam: *vertical* shear and *horizontal* shear. The action of vertical shear was discussed in Art. 9 of Chapter II. The existence of horizontal shear may be demonstrated as follows:

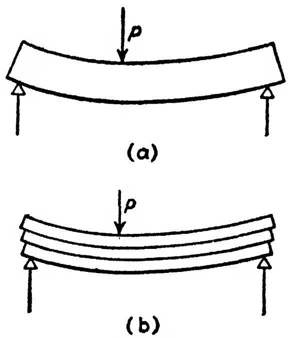


FIG. 33.

Figure 33*a* represents a beam which has deflected owing to the load *P*. Figure 33*b* represents a beam of the same dimensions as that shown in (*a*) but made up of three independent strips. The deflection in this case is more than that of the solid beam. Furthermore, it is evident that slipping has occurred along the surfaces of contact of the three independent pieces. This tendency of one layer of the beam to slip past another layer is also present in the solid beam of Fig. 33*a* but is prevented by the resistance of the beam to horizontal shear.

24. Relation between Horizontal and Vertical Shear. — Figure 34*a* represents a portion of the beam shown in Fig. 33*a*. The vertical lines represent two sections through the beam taken so close together that the shear on the sections may be assumed equal. Consider that a small particle of the beam between these sections has been removed. This particle is shown at a larger scale in Fig. 34*b* with the vertical shearing forces acting on it. It is evident from this figure that the particle is not in equilibrium and would rotate in a clockwise direction if no other forces were present. We know, however, that the particle is in equilibrium and therefore some other forces such as those represented by q in Fig. 34*c* must exist.

Since the particle is in equilibrium, moments taken about any point such as the corner B must equal zero. If the depth of the

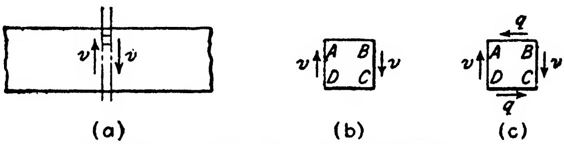


FIG. 34. Vertical and Horizontal Shear.

particle perpendicular to the paper is taken as unity, then the total stress on any one face such as AD is $AD \times 1 \times v$ or $v(AD)$. Therefore, taking moments about B,

$$v(AD) \times AB - q(DC) \times BC = 0$$

But

$$AD = BC \quad \text{and} \quad AB = DC$$

Therefore,

$$v = q$$

That is, at any point in a beam the intensity of the horizontal shearing stress is equal to the intensity of the vertical shearing stress.

25. Unit Shearing Stress. — The unit shearing stress at *any* fibre in a beam cross section is given by the general equation

$$v = \frac{VQ}{Ib}$$

in which v = unit shearing stress in pounds per square inch; V = total vertical shear at the cross section; Q = statical moment, taken about the neutral axis, of the area of the section outside the shear plane in question; I = moment of inertia of the beam section; b = width of beam at the shear area in question.[5]

[5] For the derivation of this equation, see any standard textbook on mechanics of materials.

The *average* unit shearing stress at a beam cross section may be expressed by the formula

$$v_a = \frac{V}{A}$$

in which v_a = average unit shearing stress in pounds per square inch and A is the total area of the beam cross section in square inches. For a rectangular beam cross section, it may be developed from the general equation given above that the *true* maximum shear occurs at the neutral axis and is equal to three halves of the *average* value. For I-beam and Wide Flange sections the determination of the maximum unit shear at the neutral axis from the general formula is more complicated. An approximation used in practice is to consider that the web of the steel beam resists all the shear and to compute the average unit shear on this basis. The allowable value for the average unit shearing stress is then taken low enough so that the actual maximum unit shear developed in the beam is within the limits of safety. For example, if the specified value for allowable *average* unit shearing stress were 13,000 lb. per sq. in. when figured on this basis, the actual maximum stress developed might be in the neighborhood of 15,000 lb. per sq. in. In practically all building codes, the value specified for allowable unit shearing stress in steel beams is based on this approximation. In conformance with this general practice, the term v as used in this textbook will denote *average unit shearing stress* rather than unit shearing stress, and the symbol A_w will indicate area of web (actual depth of beam times web thickness). The above formula therefore becomes

$$v = \frac{V}{A_w}$$

26. Use of the Formula for Unit Shearing Stress. — The formula given in Art. 25 is used in two ways. First, when a beam has been designed to resist bending, the shear in the web is checked by solving for v. This value should not exceed that laid down in the specifications. The unit shearing stress is, of course, greatest at the section of maximum total vertical shear. For simple beams this is adjacent to the support. V then becomes the shear at the support and A_w is the product of the depth of the beam by the web thickness. Should the value of v come out greater than the one specified, a beam with thicker web or greater depth must be selected and investigated.

When it is desired to find the maximum permissible total shear that a beam will carry, the specified unit shearing stress is multiplied by the area of the web, or

$$V = vA_w$$

Depths and web thicknesses of American Standard I-beams are given in Table II of Appendix D. Similar data for Wide Flange sections and other shapes will be found in the A.I.S.C. Manual and other steel handbooks. It should be noted that the actual depths of Wide Flange sections vary considerably, in many instances, from the *nominal* depths. The web area should always be computed by using the *actual* depth of the section.

Comparison of the web thicknesses of American Standard I-beams with those of Wide Flange sections will reveal that for beams of comparable nominal depths and weights the web thicknesses of American Standard I-beams are greater than those of Wide Flange sections. This variation results from the placement of more of the metal in the flanges of the Wide Flange section and, consequently, less in the web, in order to produce a more efficient beam shape from the standpoint of bending. Where heavy shears are involved, the greater thickness of the web of the American Standard I-beams frequently makes them advantageous. However, the majority of steel beams used in present-day practice are of the Wide Flange type, inasmuch as bending resistance controls the design of steel beams much more frequently than does shear. Shear becomes a controlling factor when heavy loads are carried on short spans or when large concentrated loads occur near the ends of a span.

It should be noted in passing that, because of the physical properties of the material, structural-steel beams subjected to excessive shearing stress do not fail by actual shearing of the metal along a plane section such as X–X in Fig. 6, Art. 9, or by the horizontal splitting, characteristic of wood beams, as indicated in Fig. 33*b*. When excessive shearing stresses produce failure, it occurs by diagonal buckling of the web, a phenomenon caused by the combined action of vertical and horizontal shear and the bending stresses. However, the web thicknesses and depths of rolled structural-steel beam sections have been so proportioned that danger of diagonal buckling is eliminated if the usual allowable shearing and bending stresses are not exceeded.

EXAMPLE I

If a 12″ I 40.8 lb. is used to carry the loads shown in Fig. 35, what is the maximum unit shearing stress developed within the beam? (The 10,000-lb. uniform load includes an allowance for the beam weight.)

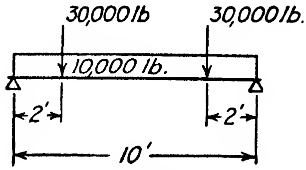


FIG. 35.

Solution:

(1) Compute the reactions. Since the loading is symmetrical, each reaction will be equal to half of the total load, or

$$R_L = R_R = \frac{10{,}000 + (2 \times 30{,}000)}{2} = 35{,}000 \text{ lb.}$$

(2) The maximum vertical shear occurs adjacent to, and is equal to, the reaction. Its value is therefore 35,000 lb.

(3) Referring to Table II of Appendix D for the depth and web thickness of a 12″ I 40.8 lb., the web area of the section is found to be 12 $\times$ 0.46 = 5.52 sq. in. Therefore,

$$v = \frac{V}{A_w} = \frac{35{,}000}{5.52} = 6340 \text{ lb. per sq. in.}$$

EXAMPLE II

What is the maximum permissible total shear on a 10″ I 25.4 lb., if the allowable unit shearing stress is 13,000 lb. per sq. in.?

Solution:

$$V = vA_w$$

$$v = 13{,}000 \text{ lb. per sq. in.}$$

Referring to Table II, the area of the web of a 10″ I 25.4 lb. is 10 $\times$ 0.31 = 3.1 sq. in. Therefore,

$$V = 13{,}000 \times 3.1 = 40{,}300 \text{ lb.}$$

27. Web Crippling. — Web crippling, sometimes called vertical buckling, is the term applied to the tendency of a beam web to fail by buckling due to excessive concentration of load. This tendency may exist at the end of a beam where it rests on top of its support, as indicated in Fig. 36*a*, or where a large concentrated load such as a column is applied at some interior point of the span, as shown by Fig. 36*d*. A common method of designing to prevent such failure assumes that the web acts as a column having as cross-sectional dimensions the web thickness t and having a distance along the beam equal to slightly more than the bearing length N. (See Fig. 36*a*.) The procedure permitting a value slightly greater than the bearing length N for one dimension of the assumed column is based on the fact that the portion of the web directly over the bearing does not act as a free strip but, because of its continuity with the rest of the web and the flanges,

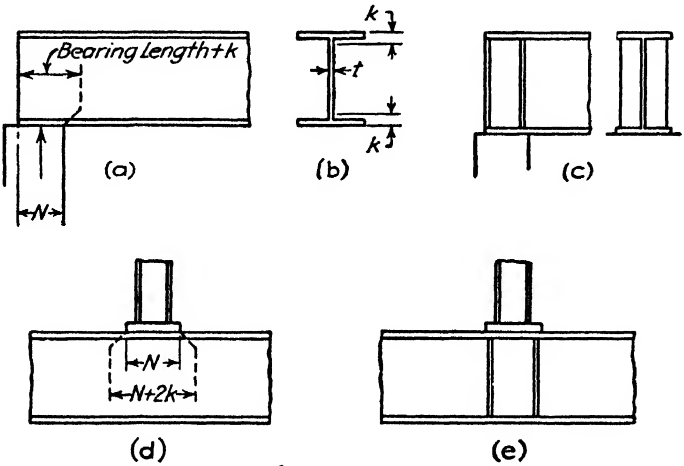


FIG. 36.

distributes part of the load to the adjacent portion of the web. For end reactions the A.I.S.C. Specification (1946) considers this effective column dimension to be the bearing length N plus the distance k, the latter value being equal to the distance from the outside face of the flange to the web toe of the fillet between flange and web. (See Fig. 36*b*.) Values of k for the various rolled sections are given in the steel handbooks.

The actual unit compressive stress f_b developed on the cross section of the assumed column is equal to the beam reaction R divided by the effective column area, or

$$f_b = \frac{R}{t(N + k)}$$

Substituting for f_b the allowable unit stress of 24,000 lb. per sq. in., as specified by the A.I.S.C. Specification, the required minimum length of bearing for any given reaction may be found by solving the above equation for N, or

$$f_b = \frac{R}{t(N+k)} = 24{,}000 \text{ (max.)}$$

from which

$$N \text{ (min.)} = \frac{R}{24{,}000t} - k$$

Where a heavy concentrated load P is applied to the top flange of a beam on the interior of the span, similar considerations apply except that the effective area of the assumed column is taken as equal to the bearing length N plus $2k$ (see Fig. 36*d*). Corresponding formulas for this condition are as follows:

$$f_b = \frac{P}{t(N+2k)} = 24{,}000 \text{ (max.)}$$

from which

$$N \text{ (min.)} = \frac{P}{24{,}000t} - 2k$$

Where the required length of bearing cannot be obtained, a beam with a thicker web must be used, or the thinner web must be reinforced by vertical stiffener plates riveted or welded on each side as indicated in Figs. 36*c* and 36*d*.

EXAMPLE

The end reaction of a 12″ WF 27 lb. is 28,000 lb. Determine the minimum required length of bearing at the support to prevent web crippling.

Solution:

(1) In order to apply the formula for minimum bearing length, it is necessary to know the values of t and k for the 12-in., 27-lb. Wide Flange section. Referring to the A.I.S.C. Manual or any other steel handbook, one obtains the following values: $t = 0.24$ in., $k = \frac{13}{16}$ or 0.81 in.

(2) Substituting and solving for N,

$$N = \frac{R}{24{,}000t} - k = \frac{28{,}000}{24{,}000 \times 0.24} - 0.81 = 4.05 \text{ in.}$$

28. Deflection. — In certain cases the deflection of a beam may be as important as its strength. The most common results of excessive deflection are cracks in plaster ceilings near the center of the span and cracks in marble, tile, and concrete floors over the tops of the girders supporting the beams. This latter condition is shown somewhat exaggerated by Fig. 37.

Most specifications limit the deflection of beams supporting plaster ceilings to $\frac{1}{360}$ of the span length. It should be noted, however, that beams usually receive a considerable portion of their load (the weight of the floor construction) before the plastering is started. This load, of course, causes an initial deflection, but it is only the deflection due to subsequent loading that may cause the plaster to crack. Although this would indicate that the total allowable deflection of a beam might be somewhat greater than $\frac{1}{360}$ of the span, it is considered better practice by many engineers to keep the total deflection within this limit.

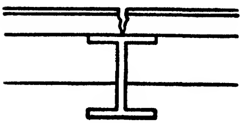


Fig. 37.

No definite limit for deflection has been determined that will entirely eliminate the cracking of masonry floors over girders. The cracking of concrete floors may often be prevented by placing additional steel reinforcement near the top surface of the slab where it crosses a girder. Tile, marble, and terrazzo floors should be designed so that joints occur over the girders.

29. Deflection Formulas. — The derivation of deflection formulas is outside the scope of this text. If the reader is not sufficiently familiar with the methods used in deriving deflection equations, as a result of previous work in structural mechanics, it is suggested that he consult any standard textbook on mechanics of materials.[6] The treatment which follows is primarily concerned with the application of deflection formulas. However, even a casual examination of the two equations below, together with the additional ones given in Table I of Appendix D, shows that the magnitude of deflection depends on the amount and distribution of the load, the span length, the stiffness of material comprising the beam as measured by (E), and the size and shape of the beam cross section indicated by the moment of inertia (I). More specifically, the deflection increases with an increase in load or

[6] The chapters on deflection in Laurson and Cox's "Mechanics of Materials" (John Wiley & Sons) and in Urquhart and O'Rourke's "Elementary Structural Engineering" (McGraw-Hill Book Co.) are among the several good references available.

span length, and decreases with an increase in E or I.

For a uniformly distributed load, $D = \dfrac{5Wl^3}{384EI}$

For a concentrated load at the center, $D = \dfrac{Pl^3}{48EI}$

In the above equations

D = the deflection at the center of span in inches;
W = the total uniformly distributed load in pounds;
P = the total concentrated load in pounds;
l = the span length in inches;
E = the modulus of elasticity (for structural steel, about 29,000,000 lb. per sq. in.);
I = the moment of inertia of the beam cross section about the axis of bending, normally the xx axis as indicated in the tables of technical functions.

If a beam is subjected to both a uniform load and a concentrated load at the center, the total deflection is the sum of the deflections due to each load figured separately.

EXAMPLE I

A 15″ I 42.9 lb. carries a load of 30,000 lb. uniformly distributed over a span of 20 ft. Find the deflection at mid-span. Neglect the deflection due to the beam weight.

Solution:

(1) From Table II the value of I for this beam is 441.8. The span in inches is $20 \times 12 = 240$ in.

(2) $D = \dfrac{5Wl^3}{384EI} = \dfrac{5 \times 30{,}000 \times 240 \times 240 \times 240}{384 \times 29{,}000{,}000 \times 441.8} = 0.422$ in.

EXAMPLE II

A 15″ I 42.9 lb. carries a load of 15,000 lb. concentrated at the center of a 20-ft. span. Find the deflection under the load. Neglect the deflection due to the beam weight.

Solution:

(1) From Table II the value of I for this beam is 441.8. The span in inches is $20 \times 12 = 240$ in.

(2) $D = \dfrac{Pl^3}{48EI} = \dfrac{15{,}000 \times 240 \times 240 \times 240}{48 \times 29{,}000{,}000 \times 441.8} = 0.337$ in.

30. Limiting Span Length. — In Art. 28 it was stated that many specifications limit the deflection of beams to $\frac{1}{360}$ of the span length. On the basis of the usual extreme fibre stresses allowed in design (20,000 or 18,000 lb. per sq. in.), this limit makes the permissible span for a steel beam of symmetrical section about 24 times its depth, when the beam is loaded uniformly to its full capacity. For unsymmetrical sections such as angles and tees, it errs on the side of safety. It is sometimes desirable that depths greater than the minimum allowable be used, especially where stiffness is a requirement.

An *approximate* rule for keeping the deflection due to a uniform load within the allowable limit is to have 1 in. of depth for every 2 ft. of span length, or, the depth of a beam in inches should not be less than half the span in feet. For example, when a beam is loaded to capacity, the minimum depth to use on a 24-ft. span is 12 in. (Consult the A.I.S.C. Specification (1946), Sec. 17*a*.)

The beam section required to resist bending will usually come within the limiting ratio of depth to length for deflection, but deflection may govern the depth of long-span beams carrying relatively light loads.

EXAMPLE

Design a standard I-beam to support a total load of 8500 lb., including its own weight, uniformly distributed over a span of 24 ft. The allowable extreme fibre stress is 20,000 lb. per sq. in. The deflection must not exceed $\frac{1}{360}$ of the span.

Solution:

(1) From Table I the bending moment is

$$M = \frac{WL}{8} = \frac{8500 \times 24}{8} = 25{,}500 \text{ ft.-lb. or } 306{,}000 \text{ in.-lb.}$$

(2) $S = \dfrac{M}{f} = \dfrac{306{,}000}{20{,}000} = 15.3.$

From Table II, an 8″ I 23 lb. has a section modulus of 16. This beam therefore satisfies the requirements for bending.

(3) According to the approximate rule of Art. 30 the depth in inches should equal half the span in feet in order to prevent excessive deflection. This apparently calls for a 12-in. depth and a 12″ I 31.8 lb. (the lightest-weight 12-in. section) could be used. This beam would deflect to the allowed limit (approximately) when loaded to its full capacity.

(4) However, a comparison of the section moduli of the 8″ I 23 lb. ($S = 16$) and the 12″ I 31.8 lb. ($S = 36.0$) shows that the 12-in. beam is more than twice as strong as the 8-in. section. Inasmuch as the deflection

is directly proportional to the load, the 12-in. I-beam would deflect less than half of the limit. An examination of the formula

$$D = \frac{5Wl^3}{384EI}$$

shows that the deflection is also inversely proportional to the moment of inertia of the section. With this in mind we may proceed as follows:

(5) The limit of deflection stated in the problem is $\frac{1}{360}$ of the span, or the allowable $D = \frac{\text{span}}{360} = \frac{24 \times 12}{360} = \frac{288}{360} = 0.8$ in.

(6) Solving the equation $D = \frac{5Wl^3}{384EI}$ for I,

$$I = \frac{5Wl^3}{384ED}$$

Substituting the allowable deflection for D,

$$I = \frac{5 \times 8500 \times 288 \times 288 \times 288}{384 \times 29{,}000{,}000 \times 0.8} = 113.8$$

(7) From Table II, a 10″ I 25.4 lb. has a moment of inertia of 122.1. This is the lightest-weight American Standard I-beam that will satisfy the requirements.

31. Procedure for Investigation of Deflection. — From a study of Art. 30 the following procedure for the investigation of the deflection of a uniformly loaded simple beam may be written.

(1) After a beam has been designed for bending, check it to find out whether the depth in inches is at least half of the span in feet. If it is not, compute the actual deflection by solving the equation

$$D = \frac{5Wl^3}{384EI}$$

(2) Find the allowable deflection. In the example of Art. 30 this was limited to $\frac{1}{360}$ of the span, but in some cases a different limit may be imposed. If the actual deflection exceeds the allowable, proceed as follows:

(3) Solve the deflection formula for the moment of inertia, substituting the allowable deflection for D,

$$I = \frac{5Wl^3}{384ED}$$

(4) Select a beam that has a moment of inertia equal to or greater than this value.

It is sometimes necessary to limit the deflection of beams carrying concentrated loads. The procedure in such cases is as follows:

(1) After the beam has been designed for bending, refer to Table I to find the correct deflection formula for the given load conditions.[7] Solve the formula for D.

(2) If D comes out greater than the allowable deflection, solve the same formula for I, substituting the allowable deflection for D.

(3) Select a beam that has a moment of inertia equal to or greater than this value.

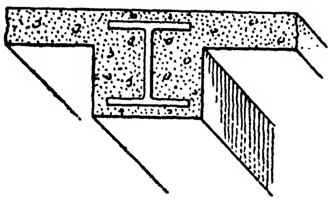


Fig. 38(a)

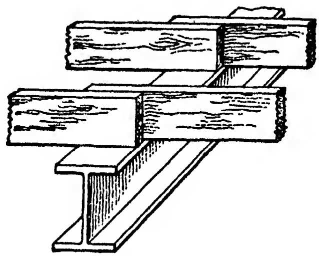


Fig. 38(b)

32. Lateral Support.—Another way in which beams may fail is by buckling of the top (compression) flange when lateral deflection is not prevented. The top flange, being in compression, is subject to the same buckling tendency that is characteristic of column action (see Fig. 75c), except that the web of the section prevents buckling in a vertical plane, thereby confining the distortion to a sidewise deflection. The bottom flange, being in tension, is not subject to buckling action.

In cases such as that shown in Fig. 38a, lateral support is supplied to beams by the floor construction. It is evident from the figure that lateral deflection of the top flange is prevented by the concrete slab. On the other hand, the type of floor system shown in Fig. 38b, where wood or steel joists simply rest on the top flange of an I-beam, offers no resistance to sidewise buckling. If the beam in question acts as a girder and supports other beams

[7] The A.I.S.C. Manual and the handbooks of the various steel companies contain more complete tables, covering a wider range of cases.

framed with connections like those shown in Fig. 56*b* and *c*, lateral support is also provided and the maximum unsupported length of the top flange of the girder is equal to the maximum distance between adjacent beams, or between the reaction of the girder and the nearest beam framing into it, whichever is greater.

Individual building codes vary in their provisions for controlling the stress in laterally unsupported beams. One widely used specification, however, requires that whenever the unsupported length L of the compression flange of a beam exceeds 15 times b, the width of the flange (both in inches), the basic allowable fibre stress of 20,000 lb. per sq. in. shall be reduced to the value given by the formula

$$f = \frac{22{,}500}{1 + \dfrac{L^2}{1800b^2}} \quad \text{or} \quad \frac{22{,}500}{1 + \dfrac{1}{1800}\left(\dfrac{L}{b}\right)^2}$$

To facilitate design work, the table of reduced allowable stresses on page 50 has been computed by substituting various values of L/b in the above formula. Values of L/b exceeding 40 are not permitted. (The column in the table headed "Ratio" should be disregarded for the time being. Its use is discussed later in Art. 33.)

EXAMPLE

A 10″ WF 21 lb. carries a uniformly distributed wall load of 28,000 lb. on a span of 9 ft. Conditions are such that no lateral support is furnished the beam. Determine whether the beam is overstressed according to the requirements discussed above.

Solution:

(1) Referring to the A.I.S.C. Manual or other steel handbook, the flange width of a 10″ WF 21 lb. is found to be 5.75 in. The value of L/b is therefore $(9 \times 12) \div 5.75$ or 18.8. Since this value exceeds 15, the allowable extreme fibre stress in the compression flange will be less than 20,000 lb. per sq. in.

(2) The allowable extreme fibre stress for this value of L/b may be found by solving the equation

$$f = \frac{22{,}500}{1 + \dfrac{1}{1800}\left(\dfrac{L}{b}\right)^2} = \frac{22{,}500}{1 + \dfrac{1}{1800} \times (18.8)^2}$$

or by referring to the table on page 50, and interpolating between the allowable stress of 18,910 lb. per sq. in. corresponding to $L/b = 18.5$,

TABLE OF ALLOWABLE UNIT STRESSES IN KIPS PER SQUARE INCH FOR BEAMS AND GIRDERS LATERALLY UNSUPPORTED

$$f = \frac{22{,}500}{1 + \dfrac{L^2}{1800b^2}}$$

L = unsupported length in inches.
b = width of flange in inches.

$\frac{L}{b}$	Unit Stress f	Ratio	$\frac{L}{b}$	Unit Stress f	Ratio
15.0	20.00	1.000	27.5	15.84	0.792
15.5	19.85	0.993	28.0	15.67	0.784
16.0	19.70	0.985	28.5	15.50	0.775
16.5	19.54	0.977	29.0	15.34	0.767
17.0	19.39	0.969	29.5	15.17	0.758
17.5	19.23	0.961	30.0	15.00	0.750
18.0	19.07	0.953	30.5	14.83	0.742
18.5	18.91	0.945	31.0	14.67	0.733
19.0	18.74	0.937	31.5	14.50	0.725
19.5	18.58	0.929	32.0	14.34	0.717
20.0	18.41	0.920	32.5	14.18	0.709
20.5	18.24	0.912	33.0	14.02	0.701
21.0	18.07	0.904	33.5	13.83	0.693
21.5	17.90	0.895	34.0	13.70	0.685
22.0	17.73	0.887	34.5	13.54	0.677
22.5	17.56	0.878	35.0	13.39	0.669
23.0	17.39	0.869	35.5	13.23	0.662
23.5	17.22	0.861	36.0	13.08	0.654
24.0	17.05	0.852	36.5	12.93	0.647
24.5	16.87	0.844	37.0	12.78	0.639
25.0	16.70	0.835	37.5	12.63	0.632
25.5	16.53	0.826	38.0	12.49	0.624
26.0	16.36	0.818	38.5	12.34	0.617
26.5	16.19	0.809	39.0	12.20	0.610
27.0	16.01	0.801	39.5	12.05	0.603
			40.0	11.91	0.596

Note: This table has been compiled from data contained in the fourth edition of the A.I.S.C. Manual, and the values given are based on the 1936 edition of the A.I.S.C. Specification. In the 1946 edition of the A.I.S.C. Specification a new formula for the design of laterally unsupported beams has replaced the one at the head of this table. The new formula, contained in Section 15 of the 1946 specification, takes into account the depth of the beam and the flange thickness, as well as the flange width and unsupported length, and results in higher permissible stresses for shallow beams and beams with thick flanges. The fifth edition of the A.I.S.C. Manual contains graphs and data to facilitate application of the new formula.

and 18,740 lb. per sq. in. corresponding to $L/b = 19$. The allowable extreme fibre stress is found to be 18,800 lb. per sq. in. (to three significant figures). In order to determine whether the beam is overstressed, this value must be compared with the actual extreme fibre stress developed by the loading.

(3) The stress developed in the beam is given by the beam formula $f = M/S$. For the stated loading condition, the maximum bending moment is

$$M = \frac{WL}{8} = \frac{28{,}000 \times 9}{8} = 31{,}500 \text{ ft.-lb.}$$

Referring again to a steel handbook, the section modulus of the 10″ WF 21 lb. is found to be 21.5. Therefore,

$$f = \frac{M}{S} = \frac{31{,}500 \times 12}{21.5} = 17{,}600 \text{ lb. per sq. in.}$$

(4) Since this value of 17,600 is less than the permissible value of 18,800 determined in Step (2) above, the beam is not overstressed.

PROBLEMS

1. Find the maximum permissible end shear for a 12″ WF 40 lb. if the allowable unit shearing stress is 12,000 lb. per sq. in. (Answer given in Appendix E.)

2. Find the maximum permissible end shear for a 10-in., 30-lb. American Standard channel if the allowable unit shearing stress is 13,000 lb. per sq. in.

3. A 14″ WF 30 lb. sustains an end reaction of 40,000 lb. Find the maximum unit shearing stress and determine the minimum length of bearing required at the support to prevent web crippling in accordance with the provisions of A.I.S.C. Specification (1946). (Answer given in Appendix E.)

4. A 16″ WF 36 lb. sustains an end reaction of 50,000 lb. Find the maximum unit shearing stress and determine the minimum length of bearing required at the support to prevent web crippling in accordance with the provisions of A.I.S.C. Specification (1946).

5. A 14″ WF 38 lb. sustains a concentrated load of 42,000 lb. applied 3 ft. from the left end of its 10-ft. span. Find the maximum unit shearing stress and determine the minimum length of bearing at the larger reaction necessary to prevent crippling of the web in accordance with the provisions of A.I.S.C. Specification (1946).

6. An 18″ WF 50 lb. supports a uniform load (including its own weight) of 24,000 lb. on a span of 24 ft. In addition, concentrated loads of 8000 lb. are applied 8 ft. from each reaction. Determine the deflection at the center of the span. (Answer given in Appendix E.)

7. A 21″ WF 62 lb. supports a uniform load (including its own weight) of 30,000 lb. on a span of 28 ft. In addition, three concentrated loads of 6500 lb. each are applied, one at the center and one at each quarter-point of the span. Compute the deflection at the center of the span.

8. A 10″ WF 29 lb. carries a uniformly distributed, superimposed load of 21,000 lb. on a span of 20 ft.

(*a*) Find the deflection at the center due to the superimposed load.

(*b*) What is the lightest-weight Wide Flange section that could be used to keep the deflection within the customary allowable $\frac{1}{360}$ of the span?

(*c*) What 10-in. and 8-in. Wide Flange sections may also be used, still keeping the deflection within $\frac{1}{360}$ of the span? (Answers to each part given in Appendix E.)

9. A 16″ WF 36 lb. has a span of 30 ft. and supports concentrated loads of 9300 lb. each at the third-points of the span.

(*a*) Find the deflection at the center due to the concentrated loads.

(*b*) What is the lightest-weight Wide Flange section that could be used to keep the deflection within the customary allowable $\frac{1}{360}$ of the span?

(*c*) What 14-in. and 12-in. Wide Flange sections may also be used, still keeping the deflection within $\frac{1}{360}$ of the span?

10. A 12″ WF 40 lb. is built into a concrete wall and projects from the face of the wall, forming a cantilever 10 ft. long. A load of 15,000 lb. is uniformly distributed over the entire 10 ft.

(*a*) Find the deflection at the free end of the cantilever due to this load.

(*b*) Assuming that it is desired to limit the deflection at the free end to $\frac{1}{2}$ in., select the lightest-weight 14-in., 12-in., and 10-in. Wide Flange sections that could be used.

11. A 21″ WF 62 lb. is used on a span of 20 ft. Conditions are such that no lateral support is provided. Find the total safe uniformly distributed load on the beam in accordance with the provisions of Art. 32.

12. A 16″ WF 40 lb. is to be used to support a single concentrated load at the center of a 22-ft. span. Conditions are such that lateral support will be provided at the point of application of the load but not elsewhere. Determine the maximum concentrated load that may be applied under these conditions if the provisions of Art. 32 govern.

33. Safe Load Tables. — The A.I.S.C. Manual and other steel handbooks give tables showing the total safe uniformly distributed load, as determined by the allowable extreme fibre stress in bending, that beams can carry on various spans. Such tables are invaluable where there is much designing to be done. Once the total uniformly distributed load to be carried has been determined, the beam is selected directly from the tables. Different handbooks have special arrangements of the tables, but a short study of any of them will reveal the method of selection.

The table on page 54 illustrates the make-up of a typical safe load table. It will be noted that the loads are given in *kips*, 1 kip being equal to 1000 lb. Loads tabulated above the solid cross lines near the top of the table produce maximum allowable shear on the beam web. Loads listed below the dotted cross lines near the bottom of the table produce deflections greater than $\frac{1}{360}$ of the span. When load values between these two lines are used, it is evident that shear and deflection requirements are within these limits without further checking.

A note at the head of the safe load table states that the allowable loads must be reduced if the beams are laterally unsupported. This is in accordance with the considerations discussed in Art. 32, where it was pointed out that the basic allowable extreme fibre stress of 20,000 lb. per sq. in. must be decreased whenever the unsupported length of the compression flange exceeds 15 times the flange width. The table on page 50, which gives the reduced values of extreme fibre stress for various values of L/b, also contains a column headed "Ratio." Figures in this column indicate the ratio of the safe load for different values of L/b to the values given in the safe load table, where full lateral support is assumed.

EXAMPLE

The safe load table indicates that a 12″ WF 27 lb. will sustain a total uniform load of 32 kips on a 14-ft. span if laterally supported. Find the allowable uniform load if conditions are such that no lateral support can be assumed and the provisions of Art. 32 govern.

Solution:

(1) From a table of technical functions, the actual flange width of the 12″ WF 27 lb. is found to be 6.5 in. (This should always be checked as the actual flange widths of many WF sections are different from the nominal widths shown at the top of the safe load tables.) The unsupported length in this case is equal to the span, or 14 ft. $\times$ 12 = 168 in. Therefore,

$$\frac{L}{b} = \frac{168}{6.5} = 25.8$$

(2) Reference to the table on page 50 shows that the ratio value for L/b of 25.5 = 0.826 and for L/b of 26 = 0.818. Interpolating between these values gives a value for the ratio of 0.821. The safe uniform

TYPICAL BEAM SAFE LOAD TABLE FOR SELECTED 12-IN. WIDE FLANGE SECTIONS

ALLOWABLE UNIFORMLY DISTRIBUTED LOADS IN KIPS FOR MAXIMUM BENDING STRESS OF 20 KIPS PER SQ. IN.

(1) For laterally unsupported beams allowable loads must be reduced in accordance with individual specification or building code requirements. (See Art. 32.)

(2) Loads above the solid cross lines produce maximum allowable shear on the beam web based on a maximum unit shearing stress of 13 kips per sq. in.

(3) Loads below the horizontal dotted lines produce deflections greater than $\frac{1}{360}$ of the span.

Span in Feet	Nominal Depth and Width — Weight per Foot					
	12 × 8			12 × 6½		
	50	45	40	36	31	27
6				98	82	74
7	118	106	92	87	75	65
8	108	97	87	77	65	57
9	96	86	77	68	58	50
10	86	78	69	61	52	45
11	78	71	63	56	48	41
12	72	65	58	51	44	38
13	66	60	53	47	40	35
14	62	55	49	44	37	32
15	58	52	46	41	35	30
16	54	49	43	38	33	28
17	51	46	41	36	31	27
18	48	43	38	34	29	25
19	45	41	36	32	27	24
20	43	39	35	31	26	23
21	41	37	33	29	25	22
22	39	35	32	28	24	21
23	38	34	30	27	23	20
24	36	32	29	26	22	19
25	35	31	28	25	21	18

load on the 12″ WF 25 lb. is therefore the tabular value multiplied by 0.821, or

$$W = 32 \times 0.821 = 26.3 \text{ kips}$$

(3) This is, of course, the total safe load including the weight of the beam, which should be deducted in order to obtain the net superimposed load the beam will support.

Although safe load tables are computed for uniformly distributed loads, they may be used also for other loading conditions. For example, a laterally supported beam will carry a single concentrated load at the center of its span equal to one-half the value tabulated for a uniform load. This relationship is derived by equating the formula for maximum bending moment caused by a concentrated load P at the center of a span to the corresponding formula for a uniform load W, as follows:

$$M \text{ (max.) concentrated load} = \frac{PL}{4}$$

$$M \text{ (max.) uniform load} = \frac{WL}{8}$$

$$\frac{PL}{4} = \frac{WL}{8}$$

from which

$$P = \frac{4WL}{8L} = \frac{W}{2}$$

If, therefore, it is desired to select a beam that will safely carry a concentrated load of 18 kips at the center of a 12-ft. span, the *load to be carried* should be multiplied by 2, thus being converted into an *equivalent uniform load*, or, as it is frequently called, an *equivalent tabular load* (abbreviated E.T.L.). In this case the E.T.L. would be $2 \times 18 = 36$ kips. Reference to the safe load table for 12-in. WF sections shows that a 12″ WF 27 lb. is good for 38 kips on this span. Inasmuch as the weight of this section is only $28 \times 12 = 336$ lb., the margin of 2 kips between the safe load in the table and the required E.T.L. is ample to take care of the beam weight. The 12″ WF 27 lb. section is therefore satisfactory.

Equivalent Tabular Load Factors

Expressions for equivalent tabular load may be set up for various loading conditions by equating $WL/8$ to the formula giving maximum bending moment for the particular loading concerned. The resulting expression for W is called the equivalent tabular load factor. The accompanying table gives E.T.L. factors for six conditions encountered frequently in practical work. Other factors may be developed, but for loadings much more complicated than those shown in the table, the solution of the expression for E.T.L. may entail as much work or more than application of the beam formula in the general design procedure of Art. 21.[8] The use of E.T.L. factors in connection with safe load tables is illustrated by the example which follows.

EXAMPLE

Using safe load tables based on a maximum bending stress of 20 kips per sq. in., design a Wide Flange section for a span of 18 ft. Concentrated loads of 7 kips are applied at each third-point, and a uniform load of 19 kips (including an allowance for beam weight) extends over the entire span.

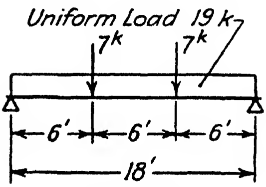


FIG. 39.

Solution:

(1) Draw a sketch of the beam showing the loads (Fig. 39).

(2) Reference to the table on page 57 shows the E.T.L. factor for equal concentrated loads at the third-points to be $2.67P$, making the E.T.L. $= 2.67 \times 7$ or 18.69 kips.

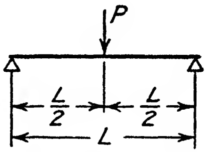
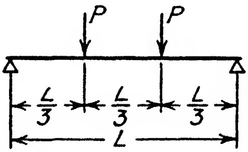
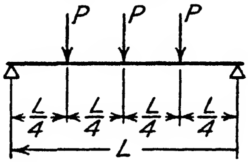
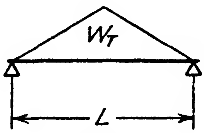
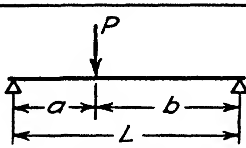
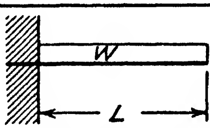
(3) The total equivalent uniform load is therefore

$$18.69 + 19 = 37.69 \text{ or } 37.7 \text{ kips}$$

(4) From the table on page 54 it will be observed that a 12″ WF 40 lb. will carry a safe load of 38 kips on an 18-ft. span. By consulting the complete safe load table in the A.I.S.C. Manual it will be found that a 14″ WF 38 lb. is good for 40 kips on this span. Either of these beams is therefore satisfactory.

(5) It should be noted particularly that the reactions of this beam are *not* equal to half the total equivalent uniform load ($37.7/2 = 18.85$ kips) but have values of $7 + 19/2 = 16.5$ kips each. This point is frequently overlooked by persons using safe load tables for the first time. In practical design work it is necessary to record reactions carefully so that accurate values may be used in the subsequent design of bearing plates,

[8] It is suggested that the student verify the E.T.L. factors given in the table. Bending moment equations for the loadings concerned may be obtained from Table I of Appendix B.

EQUIVALENT TABULAR LOAD FACTORS		
Simple Beam Concentrated Load at Center		E.T.L. = 2P
Simple Beam Equal Loads at Third-Points		E.T.L. = 2.67P
Simple Beam Equal Loads at Quarter-Points		E.T.L. = 4P
Simple Beam Triangular Load		E.T.L. = 1.33 W_T
Simple Beam Single Concentrated Load at any Point	 Where: $a = L/3$; $a = L/4$; $a = L/5$	E.T.L. = $\frac{8Pab}{L^2}$; E.T.L. = 1.78P; = 1.5P; = 1.28P
Cantilever Beam Uniform Load		E.T.L. = 4W

connection angles, or other structural members which may support the beam in question.

Attention is invited to the fifth case given in the table of E.T.L. factors on page 57, namely, that of a single concentrated load at any point on the span. Judgment must be exercised in the use of this E.T.L. factor when the beam also supports a uniform load. This is necessary because the maximum bending moment due to the concentrated load does not occur at the same point on the span as does that due to the uniform load, the former being under the load and the latter occurring at mid-span. However, if the maximum bending moments due to each load acting separately are added, the total will always be greater than the actual maximum moment developed in the beam. From this fact it follows that, if the E.T.L. for the concentrated load condition under discussion is added to the uniform load, the resulting total equivalent uniform load will be in error on the safe side. When the concentrated load is considerably larger than the uniform load, this discrepancy is negligible. However, when the uniform and concentrated loads are of the same order of magnitude, and the single concentrated load is located in the neighborhood of the third- or quarter-points of the span, the discrepancy may be large enough to result in the selection of a beam considerably stronger than necessary, consequently producing an uneconomical design. Under such conditions the general design procedure (Art. 21) should be employed in lieu of safe load tables.

Correction Factors

Before using a safe load table for design purposes, it must be checked to ascertain whether the allowable unit stresses upon which it is based are the same as those called for in the specification controlling the design of the particular project. It is obviously desirable to use a table based on the specified stresses if one is available. By means of correction factors, however, a safe load table based on an allowable extreme fibre stress of 20,000 lb. per sq. in. can be used in designing on 18,000 or 22,000 lb. per sq. in.

If the project specifications call for an 18,000 lb. per sq. in. maximum extreme fibre stress, it is evident that the safe loads shown in the A.I.S.C. table are *too large* and any particular safe load tabulated would have to be reduced by multiplying it by

$\frac{18}{20}$, the ratio between the different allowable unit stresses. Thus the table on page 54 indicates that a 12″ WF 31 lb. will sustain a safe uniform load of 33 kips on a 16-ft. span. At an allowable unit bending stress of 18,000 lb. per sq. in., the safe load for this same condition is $\frac{18}{20} \times 33 = 29.7$ kips. Consequently, when a beam to carry a given load is selected from the tables, *the load to be carried* must be multiplied by $\frac{20}{18}$ before entering the tables.[9]

If the project specifications permit a unit bending stress of 22,000 lb. per sq. in. (as was the case for some types of structures erected during the war years, 1941–45), the loads in the A.I.S.C. table based on 20,000 lb. per sq. in. are *too small.* Each safe load tabulated should, therefore, be increased by multiplying by $\frac{22}{20}$. When designing, however, it is simpler to apply a correction factor to the load to be carried. In this case, *the load to be carried* would be multiplied by $\frac{20}{22}$ before entering the tables.

When using correction factors as indicated above, the designer has to exercise his judgment as to the necessity for carrying out the checks for shear, deflection, and lateral support, particularly if the project specification requirements in these respects are very much different from those on which the safe load table is based. Obviously, the manipulations described above apply to the loads as controlled by bending requirements only. The necessary judgment in this connection can be developed only by practice and experience.

PROBLEMS

Note: The safe load table contained in the A.I.S.C. Manual or a similar table should be used in the solution of the following problems.

1. Design a Wide Flange section to carry a total uniform load of 20 kips on a span of 10 ft.,

(*a*) if full lateral support is provided,

[9] This operation is equivalent to reducing all the *listed tabular loads* by the ratio $\frac{18}{20}$. Care must be observed in subsequently recording the reactions of the beam, as these are, of course, determined by the *actual* loading and are not affected by the correction factor.

(*b*) if no lateral support is provided.
(Answer given in Appendix E.)

2. Design a Wide Flange section to carry a total uniform load of 80 kips on a span of 16 ft.,

(*a*) if full lateral support is provided,

(*b*) if no lateral support is provided.

3. Design a Wide Flange section to carry a total uniform load of 50 kips on a span of 24 ft. Lateral support is provided by a brace beam located at mid-span.

4. Design Wide Flange sections for the conditions listed below. Assume full lateral support.

(*a*) Span = 21 ft. Single concentrated load of 25 kips at center.

(*b*) Span = 30 ft. Concentrated loads of 28 kips at the third-points.

(*c*) Span = 24 ft. Concentrated loads of 10 kips at center and quarter-points.

5. Design beams for the conditions listed below. Assume full lateral support. Allowable extreme fibre stress = 18 kips per sq. in.

(*a*) Span = 22 ft. Total uniformly distributed load of 80 kips.

(*b*) Span = 24 ft. Concentrated loads of 15 kips at the third-points.

6. Design beams for the conditions listed below. Assume full lateral support. Allowable extreme fibre stress = 22 kips per sq. in.

(*a*) Span = 19 ft. Total uniformly distributed load of 85 kips.

(*b*) Span = 26 ft. Concentrated loads of 20 kips at center and quarter-points.

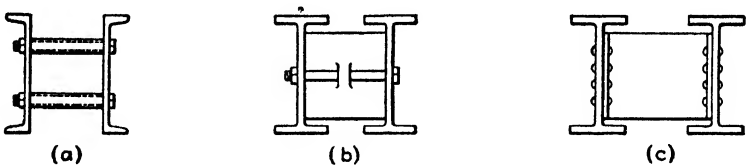


FIG. 40. Beam Separators.

34. Multiple Beam Girders. — Where the depth of a beam is restricted by head room or other considerations, it frequently happens that the beam section necessary to support the load is too deep. If the depth of the beam called for is not much greater than that allowed, a heavier section of less depth may be used. However, if the depth of the required beam is still greater than that which can be permitted, two beams of less depth may be used. These are usually fastened together in some manner and are called multiple beam girders. Figure 40*a* shows two channels held in place by gas pipe separators. This type of separator should be used only to hold the parts of the girder in position. It cannot transfer any load from one member to the other in case of unequal loading.

Cast-iron separators such as those shown in Fig. 40*b* give more rigidity to the girder than gas pipe separators. If cast-iron separators are made to fit tightly against the flanges of the members they will transfer some of the load from one to the other in the event of unequal loading. In beams less than 12 in. deep one bolt is used per separator and two bolts are required for beams 12 in. or more in depth.

Figure 40*c* shows a steel separator or diaphragm. This type makes the most rigid separator. Small sections of I-beams with the flanges placed vertically and riveted to the webs of the girders are often used as diaphragms. Many specifications limit the spacing of separators to 5 ft., which is a desirable maximum. They should also be placed at the ends of beams and under concentrated loads.

EXAMPLE

Design an I-beam to carry a uniformly distributed load of 40,000 lb. (including its own weight) over a span of 16 ft. The depth available for the beam is limited to 10 in., and the allowable extreme fibre stress is 18,000 lb. per sq. in.

Solution:

(1) The maximum bending moment is

$$M = \frac{WL}{8} = \frac{40{,}000 \times 16}{8} = 80{,}000 \text{ ft.-lb.} \quad \text{or} \quad 960{,}000 \text{ in.-lb.}$$

(2) The required section modulus is

$$S = \frac{M}{f} = \frac{960{,}000}{18{,}000} = 53.3$$

(3) From Table II, the lightest-weight section that will satisfy the requirements for bending is a 15″ I 42.9 lb. ($S = 58.9$). This beam exceeds the allowable depth, so two 10-in. beams will be tried.

(4) If two beams are used they will each take half of the load; hence, each beam must have a section modulus equal to at least half of the required total or 26.7. From Table II a 10″ I 35 lb. has a section modulus of 29.2. Therefore, two 10″ I 35 lb. are satisfactory.

(5) Separators of the type shown in Fig. 40*c* should be placed at each end and two more should be spaced equally across the span. These separators are sufficiently stiff so that each beam may be considered as furnishing lateral support for the other.

Multiple beam girders designed in this manner sometimes have a greater deflection than desirable. This may be investigated by the methods of Arts. 30 and 31, and heavier beams of the same depth may be selected having moments of inertia sufficient to prevent excessive deflection.

35. Unsymmetrical Sections. — It is frequently necessary to use unsymmetrical sections as beams. T-sections are well adapted for use as sub-purlins on roofs where some type of pre-cast concrete or gypsum slab forms the deck (see Fig. 111). Angles, either singly or in pairs, serve as lintels over openings in masonry walls. Where T-sections heavier than those ordinarily rolled are required, they may be made by splitting the webs of Wide Flange and Standard beam sections. Unsymmetrical sections are not economical for general use as beams, since the neutral axis is considerably closer to the flange than to the end of the stem (see Fig. 41).

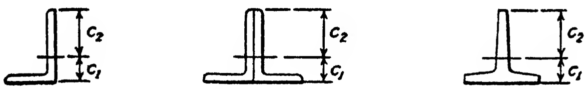


FIG. 41. Unsymmetrical Sections.

It will be recalled that the section modulus is equal to the moment of inertia divided by the distance from the neutral axis to the extreme outside fibre. For the sections shown in Fig. 41 this is $\frac{I}{c_2}$. If c_2 is assumed to be twice as great as c_1, it is evident that the extreme fibre stress at the top of the section will be twice that at the bottom, or, when the top fibre is stressed to the allowable limit, say 20,000 lb. per sq. in., the bottom fibre will carry a stress of only 10,000 lb. per sq. in. This matter is treated further in Arts. 108 and 109, under the discussion of bending and direct stress in the members of a roof truss.

36. Lintels. — Wherever an opening occurs through a brick, stone, or terra-cotta wall, some means must be provided for carrying the masonry above. Beams used for this purpose are called lintels. The most common examples occur over doors and windows. Figure 42 shows several types frequently used. Those shown at (*a*) and (*b*), made up of angles, are for relatively small openings in 8-in. and 12-in. brick walls. Sometimes the angles which are placed back to back are riveted together, but often they are independent. The I-beam and plate shown at (*c*) and the built-up member at (*d*) are for longer spans. When designing lintels similar to those shown at (*c*) and (*d*) the plates and the angles are not usually regarded as contributing to the bending strength, being used merely to provide bearing for the masonry supported.

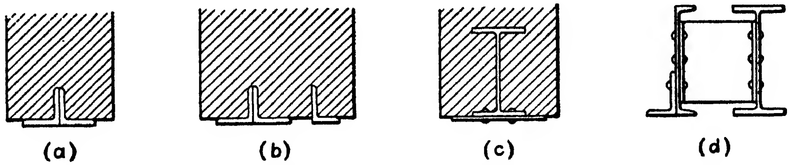


FIG. 42. Typical Lintels.

Just how much of the weight of the wall is borne by the lintel is uncertain. There is, of course, an arch action in the brick over the opening. This may be observed in many brick walls where cracking has occurred above a lintel, owing to partial failure or excessive sagging. Figure 43 illustrates such a condition. This would indicate that normally only a small triangular section of the wall is carried by the lintel. The height of this triangle is equal to about half the width of the opening.

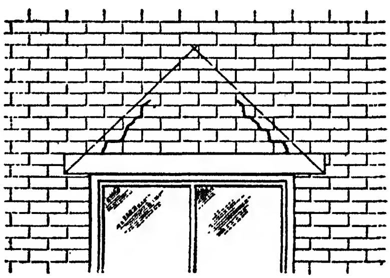


FIG. 43. Cracks over Lintel in Brick Wall.

When designing lintels care should be taken in determining the amount of load to be carried. The triangular loading may be assumed only when there is a height of unbroken masonry above the opening about equal to the span length and when there is a substantial pier between the opening under consideration and the next. Otherwise, the load should be figured as shown by the shaded area of Fig. 44*a*. Neither should the triangular loading be assumed for cases similar to that shown in Fig. 44*b*, as the openings above would destroy the arch action in the masonry. It should also be borne in mind that a lintel may receive some load from the floor above and this weight must be added to that of the supported wall.

Many lintels are made much larger than the size required by bending in order to provide enough bearing surface for the wall above. For instance, the outstanding legs of the angles shown in Fig. 42*a* would probably be at least $3\frac{1}{2}$ in. The height of the

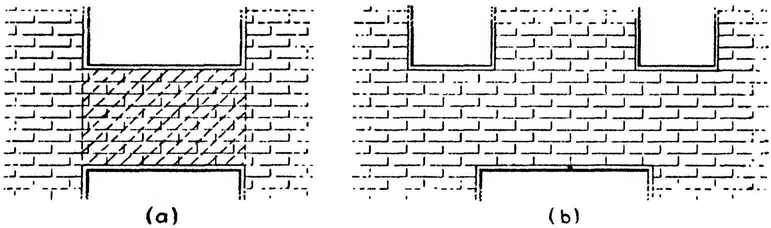


FIG. 44. Openings in Brick Walls.

vertical legs will, of course, vary with the span. Lintels over openings in brick walls should extend at least 4 in. beyond the face of the opening on each side. For wide openings a greater bearing length is usually necessary.

EXAMPLE

Design a lintel of two angles to carry an 8-in. brick wall over a 5-ft. 8-in. opening. Conditions are such that the triangular loading may be assumed. The weight of the brickwork is 130 lb. per cu. ft. The allowable extreme fibre stress in the steel is 18,000 lb. per sq. in. The lintel is to have 4-in. bearing. (See Fig. 45.)

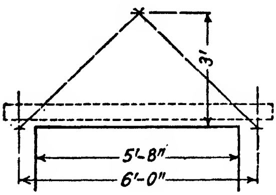


FIG. 45.

Solution:

Note: In the triangle of load the distances are based on the lintel span (i.e., center to center of bearing).

(1) If the brickwork weighs 130 lb. per cu. ft., the weight of a piece of wall 1 sq. ft. in area and 8 in. deep is $\frac{8}{12} \times 130$ or 86.7 lb.

(2) The area of the triangle is one half the base times the altitude, or Area $= 3 \times \frac{6}{2} = 9$ sq. ft., and the load supported $= 9 \times 86.7 = 780.3$ lb.

(3) From Table I the maximum bending moment for this type of loading (triangular) is

$$M = \frac{W_T L}{6} = \frac{780 \times 6}{6} = 780 \text{ ft.-lb.} \quad \text{or} \quad 9360 \text{ in.-lb.}$$

(4) The required section modulus is

$$S = \frac{M}{f} = \frac{9360}{18{,}000} = 0.52$$

As two angles are called for, the section modulus of each must be $\frac{0.52}{2}$ or 0.26.

(5) Reference to Table III shows that a $2\frac{1}{2} \times 2 \times \frac{5}{16}$-in. angle placed with the short leg vertical (axis $Y-Y$ in the tables) has a section modulus of 0.31. It is evident, however, that two such angles would not have enough bearing area to support the wall. Looking farther down the table we find that a $3 \times 2\frac{1}{2} \times \frac{1}{4}$-in. angle has the same weight as the $2\frac{1}{2} \times 2 \times \frac{5}{16}$-in. and a section modulus of 0.40. Two angles $3 \times 2\frac{1}{2} \times \frac{1}{4}$ in. might therefore be used.

Many architects and builders have a standard minimum angle which they use for lintels such as $3\frac{1}{2} \times 3 \times \frac{5}{16}$ or $3\frac{1}{2} \times 3 \times \frac{3}{8}$, placed with the $3\frac{1}{2}$-in. leg horizontal. In such cases the design is carried out to ascertain whether the standard lintel has sufficient strength. Many designers require the vertical legs of the angles to have $\frac{1}{2}$ in. of depth for every foot of span, regardless of the size called for, and specify a minimum thickness of $\frac{5}{16}$ in. for metal exposed to the weather.

When safe load tables for angles acting as beams are available, as in the A.I.S.C. Manual, they may be employed to advantage in the design of lintels. The triangular loading may be converted to an equivalent uniform load by means of the E.T.L. factor for this loading condition, shown in the table on page 57.

37. Bearing Plates. — When the end of a beam rests on a masonry wall, it is usually necessary to provide a steel bearing plate in order to distribute the reaction over an area sufficient to keep the average pressure on the masonry within the allowable limits. The required area of the plate is found by dividing the end reaction of the beam by the allowable unit bearing pressure on the masonry. In the absence of specific building code requirements, the values given in the table on page 66 may be used for safe bearing pressures on various kinds of masonry walls.

Referring to Fig. 46a, the dimension K of the plate, parallel to the beam length, is frequently limited by the thickness of the wall. Even on a 13-in. brick wall, K would be limited to 8 in. so as not to interfere with the outside course of the brickwork. The dimension N of the plate, parallel to the face of the wall, is determined by dividing the required area by the value established for K.

SAFE BEARING PRESSURE ON MASONRY WALLS

	Pounds per Square Inch
Portland cement concrete	600
Common brick, cement mortar	250
Common brick, lime and cement mortar	150
Hard brick, cement mortar	300
Rubble, portland cement mortar	140
Rubble, lime and cement mortar	100
Sandstone, cement mortar	300
Limestone, cement mortar	400
Granite, cement mortar	600
Hollow concrete masonry or clay tile units	80

The thickness of the plate is governed by bending considerations, which may be thought of as the tendency of the uniform bearing pressure on the bottom of the plate to curl the plate

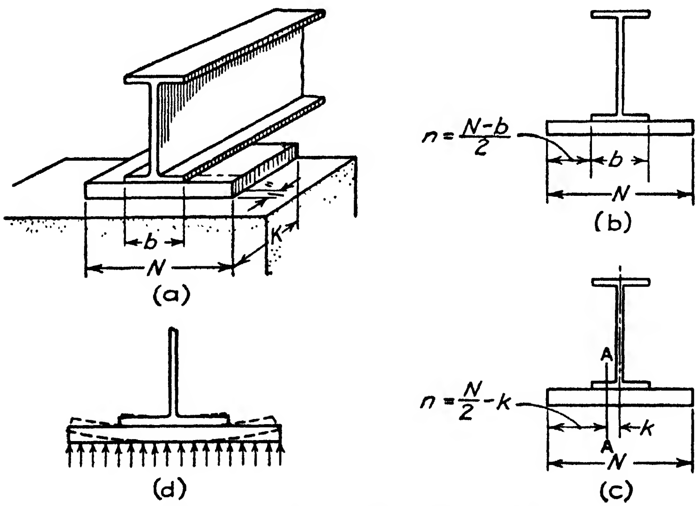


FIG. 46. Bearing Plate Design Data.

upward about the beam flange, as illustrated with great exaggeration in Fig. 46*d*. If the bearing plate is not thick enough to prevent distortion of this nature, the beam reaction will not be uniformly distributed over the area of contact between the plate and the masonry and there will be greater pressure directly under the beam than at the edges of the plate.

It will be observed from Figs. 46*a* and *d* that the projection of the plate on either side of the beam acts as an inverted cantilever with a uniformly distributed load. There is some uncertainty, however, as to just where the maximum bending moment will occur. If the beam flange is comparatively thick (and therefore stiff), it might be assumed that the flange would not tend to curl as shown in Fig. 46*d* but would remain flat. Under these conditions, the maximum bending moment in the plate would occur at the edge of the flange, and the cantilever projection n would have the value indicated in Fig. 46*b*. If the beam flange does not remain flat, however, the value to be used for the cantilever projection n will obviously have to be larger than indicated in Fig. 46*b*. The American Institute of Steel Construction recommends that the value of n be determined as shown in Fig. 46*c*, when designing bearing plates on the basis of an extreme fibre bending stress of 20,000 lb. per sq. in. Values of k (the distance from outer face of beam flange to web toe of fillet) for rolled beam sections are given in the A.I.S.C. Manual. The design of bearing plates is illustrated by the following example.

EXAMPLE

Design a bearing plate for a 10″ WF 21 lb. that delivers a reaction of 24,000 lb. to a wall built of common brick laid in cement mortar. The allowable unit bearing pressure on this type of wall is 250 lb. per sq. in. The allowable extreme fibre stress in the steel plate is 20,000 lb. per sq. in.

Solution:

(1) The required area of the plate is found by dividing the beam reaction R by the allowable unit pressure w.

$$A = \frac{R}{w} = \frac{24{,}000}{250} = 96 \text{ sq. in.}$$

(2) This area calls for an 8 × 12-in. plate. Referring to Fig. 46*a*, let K equal the 8-in. side and N the 12-in. side.

(3) From tables in the A.I.S.C. Manual or other steel handbook, the value of k is found to be $\frac{11}{16}$ in. or 0.69 in. Referring to Fig. 46*c*, the cantilever projection is

$$n = \frac{N}{2} - k = \frac{12}{2} - 0.69 = 5.31 \text{ in.}$$

(4) Considering a strip of the plate 1 in. wide (Fig. 46a), the maximum bending moment at section A–A, Fig. 46c, is

$$M = \frac{wn^2}{2} = \frac{250 \times 5.31 \times 5.31}{2} = 3530 \text{ in.-lb.}$$

(5) The required section modulus is

$$S = \frac{M}{f} = \frac{3530}{20{,}000} = 0.177$$

(6) The required thickness is $t = \sqrt{6S}$

This expression is derived from the definition of the section modulus $S = \frac{I}{c}$. The value of I for a rectangular section is $\frac{bt^3}{12}$, where b equals the width of the section and t the depth. Substituting this value,

$$S = \frac{I}{c} = \frac{bt^3}{12c}$$

But c equals half the depth of the section or $\frac{t}{2}$. Substituting this value,

$$S = \frac{2bt^3}{12t} = \frac{bt^2}{6}$$

The width of the section in this case has been taken as 1 in. Therefore,

$$S = \frac{t^2}{6} \quad \text{and} \quad t = \sqrt{6S}$$

Substituting the value of S found in Step (5),

$$t = \sqrt{6 \times S} = \sqrt{6 \times 0.177} = \sqrt{1.06} = 1.03 \text{ in.}$$

An 8 × 12 × 1-in. plate is adopted.

When heavy beams rest on relatively thin walls, the dimension K shown in Fig. 46a is often so limited that N becomes very large in proportion. This results in a large projection from the edge of the flange and a corresponding increase in thickness. Figure 47a shows one method of providing bearing for such a condition. The load is assumed to be equally divided between the beams and they are designed as inverted cantilevers. The shearing and bearing stresses in the webs of the supporting beams should also be investigated.

Beams bearing on masonry walls are usually provided with anchors as a means of tying the structure together. Figure 47b and c illustrates two common types. The bent rod shown at (b)

is usually made $\frac{3}{4}$ in. in diameter and the angles at (*c*) $\frac{3}{8}$ in. thick. Some steel companies have their own standard anchors, which are listed in their handbooks.

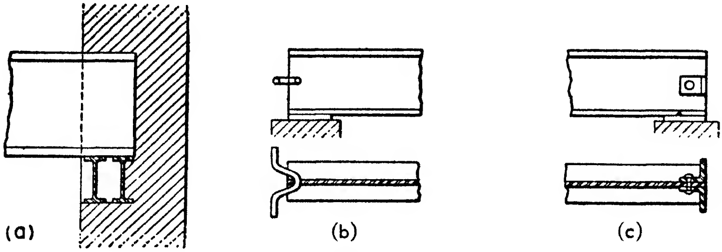


FIG. 47. Bearing Details and Wall Anchors.

38. Floor Framing. — Figure 48 illustrates two methods of framing commonly employed in ordinary office building construction. Third-point concentration is shown at (*a*) and center concentration at (*b*). When the span of a girder is much over 16 ft., third-point concentration is desirable. The area of floor supported by one beam is found by multiplying the span length by the sum of half the distances to the adjacent beams. Span

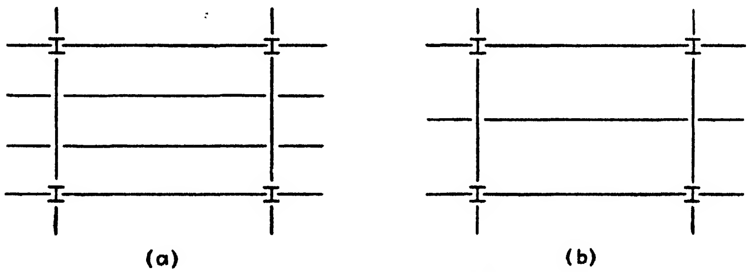
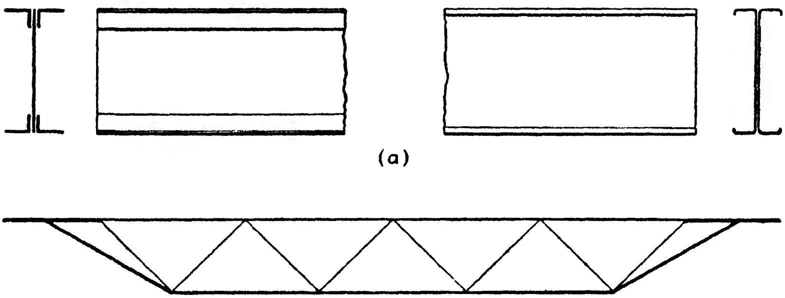


FIG. 48. Typical Floor Framing.

lengths are generally figured from center to center of supporting members, although this distance is sometimes reduced where beams frame against the flanges of large columns.

The arrangement of the framing is governed to a large extent by the type of floor system employed. In steel-frame office buildings, apartment houses, and similar structures, where concrete slabs reinforced with wire mesh are widely used, 7 to 10 ft. is a desirable distance between beams. This type of construction is discussed more fully in Chapter X, "Building Design Project," where methods of determining weights of floor construction, beam fireproofing, etc. are developed. Figure 132 in that chapter indicates the typical relation of steel beam, concrete floor slab, and fireproofing in this type of construction.

In addition to the standard types of reinforced concrete joist construction, several forms of steel joists are manufactured for supporting floors carrying light loads. The spacing varies from 12 to 30 in. depending on the size of joist, span length, and load. A thin concrete slab (2-in. minimum thickness) reinforced with ribbed metal lath is usually carried on top of the joists and a metal lath and plaster ceiling on the bottom. Galvanized wire or



(a)

(b)

Fig. 49. Types of Steel Joists.

light angle or channel bridging is used at intervals along the span to prevent twisting and buckling of the top flange. Figure 49*a* shows two types of joists constructed of thin metal sections welded together, and Fig. 49*b* illustrates the trussed or open-web steel joist. In the latter type the top and bottom chords are often made from Tee or channel sections or from two round bars placed side by side and electrically welded to a continuous web member at the points of contact.

Properties of joist sections, together with safe load tables and construction details, are given in the catalogues of the various manufacturers. (See Sweet's Architectural Catalogue File.)

PROBLEMS

1. A uniform load of 80,000 lb., including an allowance for beam weight, is to be carried on a span of 16 ft. The allowable extreme fibre stress is 20,000 lb. per sq. in., and the depth available for the beam is limited to

$10\frac{1}{2}$ in. Deflection shall not exceed 0.54 in. Design a beam (or multiple beam girder, if necessary) for this situation. (Answer given in Appendix E.)

2. A uniform load of 68,000 lb., including an allowance for beam weight, is to be carried on a span of 12 ft. The allowable extreme fibre stress is 20,000 lb. per sq. in., and the deflection shall not exceed 0.37 in. The depth available for the beam is limited to 9 in. Design a beam (or multiple beam girder, if necessary) for these conditions.

3. Design the necessary beam or beams for the following conditions: span = 25 ft., uniformly distributed superimposed load = 120,000 lb., allowable extreme fibre stress = 18,000 lb. per sq. in., deflection limited to $\frac{1}{360}$ of the span, and maximum depth available for beam = 17 in.

4. Two $4 \times 3 \times \frac{1}{2}$-in. angles are used as a beam on a span of 4 ft. The allowable extreme fibre stress is 18,000 lb. per sq. in. Find the total allowable uniformly distributed load,

(*a*) when the long legs are placed vertically,

(*b*) when the short legs are placed vertically.

(Answer given in Appendix E.)

5. Two $5 \times 3\frac{1}{2} \times \frac{3}{8}$ angles are used as a beam on a span of 5 ft. The allowable extreme fibre stress is 18,000 lb. per sq. in. Find the total allowable uniformly distributed load,

(*a*) when the long legs are placed vertically,

(*b*) when the short legs are placed vertically.

6. Requirements are the same as in Problem 5 except that the angles are $5 \times 3\frac{1}{2} \times \frac{5}{16}$ and the allowable extreme fibre stress is 20,000 lb. per sq. in.

7. Design a lintel composed of three angles to carry a 12-in. masonry wall over an opening of 6 ft. 2 in. Conditions are such that the triangular loading may be assumed. The weight of the masonry is 150 lb. per cu. ft. The allowable extreme fibre stress is 20,000 lb. per sq. in. The lintel is to have 4-in. bearing at each end.

8. Requirements are the same as in Problem 7 except that the wall is 8 in. thick, two angles shall be used, and the allowable extreme fibre stress is 18,000 lb. per sq. in.

9. Design a steel bearing plate for a 12″ WF 31 lb. that delivers a reaction of 25 kips to a wall built of portland cement concrete. The beam bears on the wall 6 in. Allowable extreme fibre stress = 20 kips per sq. in. (Answer given in Appendix E.)

10. Design a steel bearing plate for a 15″ I 42.9 lb. that has a reaction of 40 kips. The supporting wall is built of common brick laid in cement mortar, and the beam bears 10 in. on the wall. Allowable extreme fibre stress = 18 kips per sq. in.

11. Design a steel bearing plate for an 8″ WF 19 lb. that delivers a

reaction of 18 kips to a wall built of common brick laid in lime and cement mortar. The beam bears on the wall 8 in. Allowable extreme fibre stress = 20 kips per sq. in.

12. In Fig. 48*a* let the longer dimension of the typical bay shown be 24 ft. and the shorter 21 ft. The total load to be supported on the floor (including live load, weight of the concrete floor construction, and an allowance for the weight of beams and girders) is 160 lb. per sq. ft. The allowable extreme fibre stress in the steel framing is 20,000 lb. per sq. in., and deflection is limited in all cases to $\frac{1}{360}$ of the span. The relation of the concrete floor slab construction to the steel framing is as indicated in Fig. 38*a*. Adjacent bays are identical with the one under consideration, and hence corresponding beams will carry the same loads and have the same reactions.

(*a*) Design the members for the arrangement shown, using the lightest-weight Wide Flange sections that will carry the loads.

(*b*) Redesign the members placing the girders on the 24-ft. span with the beams on the 21-ft. span. Girders receive beams at third-points of span as in previous arrangement.

(*c*) Compare the weight of steel required by each arrangement, reducing your figures to pounds of steel per square foot of floor area.

CHAPTER IV

RIVETED CONNECTIONS

39. Introduction. — The component members of steel structures are held together by means of riveted, welded, or bolted connections. The use of welding in structural work is constantly increasing, but riveted connections are still widely employed in the fabrication and erection of steel buildings. Bolted connections, which are permissible under certain conditions, are discussed in Art. 49. Welded construction is treated in Chapter IX.

40. Riveting. — A rivet is composed of a cylindrical shank with a head on one end. The shank is made sufficiently long to extend through the parts to be connected leaving enough metal to form the second head when driven. Rivet holes are made $\frac{1}{16}$ in. larger than the nominal diameter of the shank, to facilitate placing the rivets, which expand on being heated, and to aid alignment. Before driving, the pieces are brought into position by means of bolts. The heated rivet is then inserted and driven until the second head is formed and the shank is "upset" to fill the hole completely. Driving is accomplished by means of a power-driven hammer with a die for upsetting the rivet and forming the head. Shrinking of the rivet as it cools draws the connected pieces more tightly together.

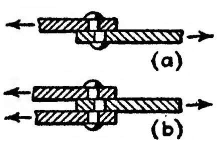


FIG. 50. Shear Failure.

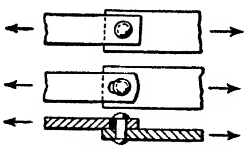


FIG. 51. Bearing Failure.

41. Failure of a Riveted Joint. — A riveted joint may fail by shearing the rivet shank as shown in Fig. 50 or by the rivets tearing through one or more of the connected members as illustrated by Fig. 51. The former is called a shear failure, and the latter a bearing failure. If the joint is so constructed that the rivets tend to shear on one plane (Fig. 50*a*) they are said to be in *single shear*. If the arrangement is similar to that shown in

Fig. 50*b*, where there are two shearing planes, the rivets are in *double shear*. In riveted joints designed to transmit tension, the net area of metal left at a section after deducting for the rivet holes must also be taken into account. This consideration is discussed in Art. 101, under the design of tension members in roof trusses.

42. Strength of a Rivet in Shear. — The resistance of a rivet to shear depends on the cross-sectional area of the shank and the allowable unit shearing stress. The A.I.S.C. Specification (1946) allows 15,000 lb. per sq. in. for the shearing stress on power-driven rivets, but some specifications call for a lower limit.

The strength of one rivet in single shear is

$$\text{R.V. (Single Shear)} = S_s \times A$$

in which R.V. = the value of one rivet;

S_s = the allowable unit shearing stress;

A = the cross-sectional area of the shank $\frac{\pi D^2}{4}$ based on the *nominal* diameter of the rivet.

The strength of a rivet in double shear is, of course, twice the single shear value.

EXAMPLE

Find the value of one $\frac{3}{4}$-in. rivet in single shear if the allowable unit shearing stress is 13,500 lb. per sq. in.

Solution:

(1) The area of the rivet $= \frac{\pi D^2}{4} = \frac{3.14 \times 0.75^2}{4} = 0.4418$ sq. in.

(This area may be obtained directly from the tables, found in nearly all handbooks, giving areas of circles.)

(2) R.V. (Single Shear) $= S_s \times A = 13{,}500 \times 0.4418 = 5964$ lb.

The shearing strength of $\frac{3}{4}$-in. and $\frac{7}{8}$-in. rivets for different allowable unit shearing stresses is given in Table IV in Appendix D. Other tables will be found in the steel handbooks.

43. Strength of a Rivet in Bearing. — The bearing on a rivet is the force exerted on it by the plates through which it passes. Although the bearing area is the cylindrical surface of contact between the rivet shank and a plate, the area used in computations is the projected area of this surface, that is, the area of a rectangle, the dimensions of which are the diameter of the rivet and the thickness of the plate. Here, again, the *nominal* diameter of the rivet is used. If two plates of different thicknesses are riveted together as shown in Fig. 52, it is evident that the rivet would tear through the thinner plate first. Hence, the value of the rivet in bearing is determined by the thickness of the thinner plate. The bearing area in this case is

$$\tfrac{3}{4} \times \tfrac{3}{8} \quad \text{or} \quad 0.75 \times 0.375 = 0.281 \text{ sq. in.}$$

In the joint shown in Fig. 53, the thickness of the thinner plate is $\frac{1}{4}$ in. but the two $\frac{1}{4}$-in. plates act in the same direction, the load P being divided between them. In the event of a bearing failure

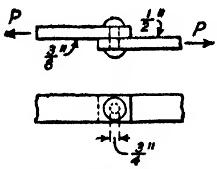


FIG. 52.

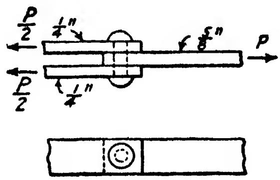


FIG. 53.

the rivet would have to tear through both plates acting to the left, not merely through one plate. The bearing area of the plates acting to the left of this joint is

$$(\tfrac{1}{4} + \tfrac{1}{4}) \times \tfrac{3}{4} \quad \text{or} \quad 0.5 \times 0.75 = 0.375 \text{ sq. in.}$$

The bearing area of the $\frac{5}{8}$-in. plate on the right is

$$\tfrac{5}{8} \times \tfrac{3}{4} \quad \text{or} \quad 0.625 \times 0.75 = 0.469 \text{ sq. in.}$$

Therefore, the bearing value of the rivet in this case is limited by the combined thickness of the two $\frac{1}{4}$-in. plates. It follows from this discussion that the bearing value of a rivet in a joint is determined by the thinner combined thickness of all the plates acting in one direction.

The strength of a rivet in bearing is equal to the unit bearing stress times the bearing area or

$$\text{R.V. (Bearing)} = S_B \times A$$

in which S_B = the allowable unit bearing stress in pounds per square inch;

A = the bearing area in square inches (i.e., the thickness of the metal times the nominal diameter of the rivet).

EXAMPLE

Find the bearing value of the ¾-in. rivet shown in Fig. 52 if the allowable unit bearing stress is 24,000 lb. per sq. in.

Solution:

(1) The bearing value of the ¾-in. rivet will be limited by the ⅜-in. plate. The bearing area is

$$\tfrac{3}{8} \times \tfrac{3}{4} \quad \text{or} \quad 0.375 \times 0.75 = 0.281 \text{ sq. in.}$$

(2) R.V. (⅜-in. Bearing) $= 24{,}000 \times 0.281 = 6750$ lb.

The bearing values of ¾-in. and ⅞-in. rivets in plates of various thicknesses and for different unit bearing stresses are given in Table IV. Other tables will be found in the steel handbooks.

Some specifications limit the allowable unit stress in bearing to twice the value used for single shear. The A.I.S.C. Specification differs in this respect, however, by allowing a higher unit bearing stress for rivets in double shear than for those in single shear. This higher allowable stress takes into account the friction in joints of the type shown in Fig. 53. Tests of riveted connections show that the bearing value of rivets in a plate tightly enclosed between two others is appreciably greater than that of rivets in plates merely lapped as in Fig. 52. The A.I.S.C. Specification (1946) allows 32,000 lb. per sq. in. for the bearing stress for rivets in single shear and 40,000 in double shear, when certain fabricating requirements are satisfied.

44. Design and Investigation of Riveted Connections. — The design of ordinary concentric joints is based upon the assumption that the stress is equally distributed among all the rivets of a connection. The number of rivets required in a joint is found by dividing the total stress to be transmitted by the limiting value of one rivet in shear or bearing, whichever is smaller. No piece should be connected by less than two rivets even though the computations indicate that one rivet is sufficient.

When investigating a connection to determine the safe load it will carry, the first step is to compute the strength of one rivet as governed by shear or bearing. When this value has been ascertained, the safe load is found by multiplying the strength of one rivet by the number of rivets in the connection.

EXAMPLE I

Determine the total stress P that the joint shown in Fig. 54 can transmit if ¾-in. rivets are used. The allowable unit shearing stress in the rivets is 15,000 lb. per sq. in., and the allowable unit bearing stress is 32,000 lb. per sq. in. for rivets subjected to single shear and 40,000 for rivets in double shear.

Solution 1:

(1) The rivets are in double shear. Therefore, the value of one rivet as governed by shear is

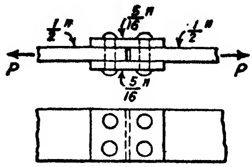


FIG. 54.

$$\text{R.V. (Double Shear)} = 2(S_s \times A)$$
$$= 2(15{,}000 \times 0.4418) = 13{,}254 \text{ lb.}$$

(2) Bearing is governed by the $\frac{1}{2}$-in. plate. Therefore, the value of the rivet in bearing is

$$\text{R.V. } (\tfrac{1}{2}\text{-in. Bearing}) = S_B \times A = 40{,}000 \times 0.5 \times 0.75 = 15{,}000 \text{ lb.}$$

(3) The limiting value of one rivet is 13,254 lb. There are two rivets in each half of the joint; therefore, the total stress P that the joint can transmit is

$$P = 2 \times 13{,}254 = 26{,}500 \text{ lb.}$$

Solution 2:

Note: Solution 1 demonstrates the general method for solving problems of this type. The following method involving the use of rivet tables is the one actually followed in practice.

(1) The rivets are in double shear. Referring to Table IV in the Appendix, the strength of a $\frac{3}{4}$-in. rivet in double shear, when the unit shearing stress is 15,000 lb. per sq. in., is

$$\text{R.V. (Double Shear)} = 13{,}254 \text{ lb.}$$

(2) Bearing is governed by the $\frac{1}{2}$-in. plate. Referring again to Table IV, the strength of a $\frac{3}{4}$-in. rivet in bearing in a $\frac{1}{2}$-in. plate, when the unit bearing stress is 40,000 lb. per sq. in., is found not to be listed. This means that the bearing value of the rivet exceeds the shear value. Hence tabulating it serves no useful purpose.

(3) Therefore, the total stress P that the joint will transmit is

$$P = 2 \times 13{,}254 = 26{,}500 \text{ lb.}$$

EXAMPLE II

How many $\frac{7}{8}$-in. rivets are required in the joint of Fig. 55 if the allowable unit stresses are 13,500 lb. per sq. in. in shear and 27,000 lb. per sq. in. in bearing? $P = 56{,}000$ lb.

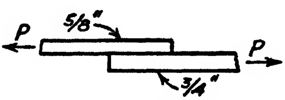


FIG. 55.

Solution:

(1) The rivets are in single shear. Reference to Table IV shows that

$$\text{R.V. (Single Shear)} = 8118 \text{ lb.}$$

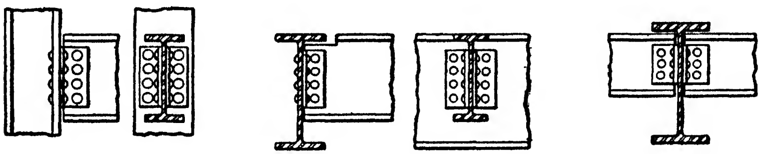
(2) The minimum bearing is furnished by the ⅝-in. plate acting to the left. Referring to Table IV,

$$\text{R. V. (⅝-in. Bearing)} = 14{,}766 \text{ lb.}$$

(3) The number of rivets necessary is

$$\frac{56{,}000}{8118} = 6.9 \quad \text{or} \quad 7 \text{ rivets}$$

45. Beam Connections. — The most common type of joint in steel frame buildings is that used to connect floor beams to other beams, girders, or columns. Such connections are made up of two angles riveted to the beam web. The outstanding legs are then riveted to the supporting members. Figure 56 shows three typical conditions.



(a) (b) (c)

FIG. 56. Typical Beam Connections.

The stresses to be considered in the types of joints shown in Fig. 56*a* and *b* are the shears in the rivets (double shear for those in the beam web and single shear for those in the outstanding legs) and bearing stresses in the beam web and in the flange or web of the supporting column or girder. In connections of the type shown in Fig. 56*c* the bearing in the web of the supporting girder frequently governs the design. It is customary to arrange the rivets in beam connections symmetrically even though this may necessitate one rivet more than actually required to resist the total stress.

EXAMPLE I

Find the allowable end reaction of a 15″ I 42.9 lb. if the beam is connected to the web of a 20″ I 65.4 lb. as shown in Fig. 56*b*. There are four ¾-in. rivets in the web of the 15-in. beam and eight ¾-in. rivets in the web of the 20-in. beam. The specifications allow a unit stress of 13,500 lb. per sq. in. in shear and 27,000 lb. per sq. in. in bearing. The connection angles are ⅜-in. thick.

Solution: Rivet values are taken from Table IV.

(1) Determine the strength of the four rivets in the 15-in. I-beam. These rivets are in double shear. The web thickness of the 15″ I 42.9 lb.

is 0.410 in. This offers less bearing surface than the combined thickness of the two angles (0.75 in.). Therefore,

R.V. (Double Shear) = 11,928 lb.
R.V. (0.410-in. Bearing) = 8222 lb. (using nearest tabulated value)

The strength of the four rivets is limited by bearing in the web of the I-beam and is 4 × 8222 or 32,890 lb.

(2) Determine the strength of the eight rivets in the 20-in. I-beam. These rivets are in single shear. The web thickness of the 20″ I 65.4 lb. is $\frac{1}{2}$ in. This provides more bearing surface than the $\frac{3}{8}$-in. thickness of the angle. Therefore,

R.V. (Single Shear) = 5964 lb.
R.V. ($\frac{3}{8}$-in. Bearing) = 7594 lb.

The strength of the eight rivets is limited by the single shear value and is 8 × 5964 or 47,700 lb.

(3) The maximum end reaction that can be transmitted from the 15-in. I-beam to the 20-in. beam is the smaller of the two values found in Steps (1) and (2) or 32,890 lb.

EXAMPLE II

Two 14″ WF 38 lb. beams frame into a girder as shown in Fig. 56*c*. The reaction of each beam is 36,000 lb., and the thickness of the girder web is $\frac{1}{2}$ in. The A.I.S.C. (1946) rivet specifications previously discussed control the design. Determine the number of $\frac{3}{4}$-in. rivets required in the web of each I-beam and in the girder web if the connection angles are $\frac{3}{8}$ in. thick.

Solution: Rivet values are taken from Table IV (last column).

Number of Rivets Required in Web of 14″ WF 38 lb.

(1) The rivets are in double shear. The web thickness of this section is 0.313 in. This offers less bearing surface than the combined thickness of the two angles (0.75 in.). Therefore,

R.V. (Double Shear) = 13,254 lb.
R.V. (0.313-in. Bear. D.S.) = 9376 lb. (using nearest tabulated value)

(2) The number of rivets required is found by dividing the beam reaction by the limiting value of one rivet, or

$$\frac{36{,}000}{9376} = 3.84 \quad \text{or} \quad 4 \text{ rivets}$$

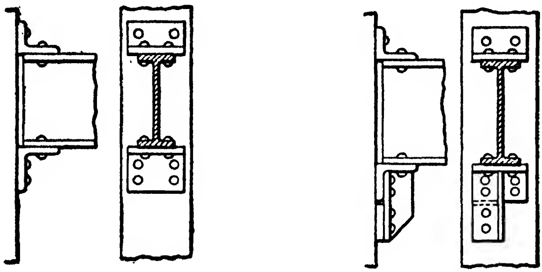
Number of Rivets Required in Girder Web

(3) Since the reactions of the two I-beams are equal, all the rivets through the girder web are in double shear. The thickness of the girder

web is ½ in. This offers less bearing surface than the combined thickness of the two angles (0.75 in.). Therefore,

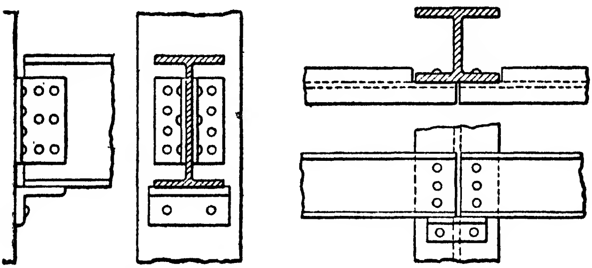
R.V. (Double Shear) = 13,254 lb.
R.V. (½-in. Bear. D.S.) = (greater than D.S. value)

(4) The number of rivets required is found by dividing the total reaction on the girder by the limiting value of one rivet, or 72,000/13,254 = 5.4 or 6 rivets, 3 on each side of the connection.



(a) (b)

FIG. 57. Typical Seat Connections to Columns.



(a) (b)

FIG. 58. Typical Beam and Column Connections.

46. Connection Details. — After the number of rivets required in a joint has been determined, it is necessary to detail the connection. This operation is usually left to the fabricating shop where the arrangement and spacing of the rivets is worked out according to certain standards for spacing, minimum distance of a rivet from the edge of an angle, etc. In order to standardize the shop work, many fabricating companies have adopted standard connection angles. Tables of standard connections giving safe loads, end clearances, and rivet layouts will be found in the A.I.S.C. Manual and other steel handbooks.

Seat connections such as those shown in (*a*) and (*b*) of Fig. 57 are sometimes used where beams frame into columns. In connections of this type the clip angle at the top of the beam carries no load but serves to hold the beam in position and stiffens the connection. When beams are connected to columns by means of web connections as shown in Fig. 58*a*, a small shelf angle is usually provided to facilitate erection but is not counted upon to carry any load. Figure 58*b* shows a common method of connecting spandrel beams to the wall columns of a building.

47. Rivets in Tension. — Several years ago most specifications relating to the use of structural steel were either silent on the subject of rivets in direct tension or prohibited their use where axial tension would be developed on the rivet. Bolts took the place of rivets under such conditions. However, in modern building construction rivets are frequently employed in situations where they must resist tension, and specifications have been set up prescribing working stresses. The Standard Specification for Structural Steel for Buildings of the American Institute of Steel Construction, as adopted in June 1936, allowed 15,000 lb. per sq. in. under certain conditions. The A.I.S.C. Specification (1946) permits 20,000 lb. per sq. in.

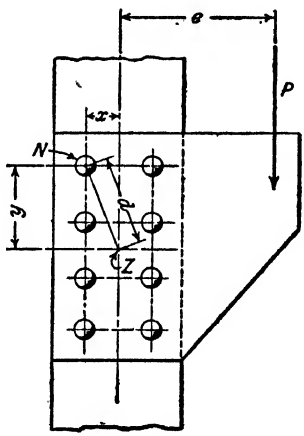


FIG. 59.
Eccentric Connection.

48. Eccentric Riveted Connections. — In all the connections previously considered, the rivets were arranged symmetrically about the line of action of the force. Under such conditions the stress in a joint is considered to be uniformly distributed among the rivets. It frequently happens, however, that beams are offset from column center lines to such an extent that they cannot be directly connected to the column flange. Where this occurs it is necessary to use some form of eccentric connection such as that shown in Fig. 59. The characteristic feature of this type of connection is that all the rivets are on one side of the force to be carried, thereby producing a turning moment at the joint. The stress in each rivet is made up of two parts: the ordinary uniform stress (load divided by number of rivets) and the moment-stress, which varies with the distance of the rivet from the center of gravity of the group. The joint should be so proportioned that the resultant of these two components on any one rivet does not exceed the allowable working stresses in shear or bearing.

The stresses in the rivets of an eccentric connection similar to the one shown in Fig. 59 may be determined as follows:

Let P = the load to be carried;
Z = center of gravity of rivet group;
e = the eccentricity (perpendicular distance from Z to the line of action of P);
x, y = coordinates of any rivet referred to Z as an origin;
d = distance of any rivet from the origin $= \sqrt{x^2 + y^2}$;
f_0 = moment-stress on a rivet at unit distance from Z;
f_m = moment-stress on any rivet.

The moment-stress on any rivet is equal to its distance from the center of gravity of the group, Z, multiplied by the moment-stress on a rivet at unit distance or

$$f_m = f_0 d = f_0 \sqrt{x^2 + y^2}$$

The moment of resistance of this stress for any rivet is

$$f_0 d^2 = f_0(\sqrt{x^2 + y^2})^2 = f_0(x^2 + y^2)$$

The total resisting moment of all the rivets equals the turning moment or

$$f_0(\Sigma x^2 + \Sigma y^2) = Pe$$

and

$$f_0 = \frac{Pe}{\Sigma x^2 + \Sigma y^2}$$

When f_0 has been determined from the above equation, the moment-stress on any rivet is found by multiplying this value by the distance of the rivet from the origin. This stress acts in a direction *perpendicular* to a line drawn from the rivet to the center of gravity of the group.

To determine the resultant stress on any rivet such as N in Fig. 59 it is first necessary to resolve the moment-stress into its vertical and horizontal components either analytically or graphically. The uniform stress on the rivet (vertical load divided by the number of rivets) is then added to the vertical component of the moment-stress to obtain the total vertical component. The resultant of this vertical component and the horizontal component of the moment-stress is found by use of the familiar equation

$$R = \sqrt{V^2 + H^2}$$

EXAMPLE

Determine the resultant stress on rivet B of the eccentric connection shown in Fig. 60. Compare this stress with the allowable rivet value if ¾-in. rivets are used with unit stresses of 13,500 lb. in shear and 24,000 lb. in bearing. The plate and column flange are each ⅜-in. thick.

Solution: The center of gravity of the rivet group is at Z.

(1) Compute the value of f_0

$$\Sigma x^2 = 8 \times (2\tfrac{3}{4})^2 = 60.5$$
$$\Sigma y^2 = (4 \times 6^2) + (4 \times 2^2) = 160$$
$$f_0 = \frac{Pe}{\Sigma x^2 + \Sigma y^2} = \frac{10{,}000 \times 9}{220.5} = 408 \text{ lb. per inch.}$$

(2) Compute the moment-stress (f_m) on rivet B. The distance from B to Z is

$$d = \sqrt{x^2 + y^2} = \sqrt{2.75^2 + 6^2} = 6.6 \text{ in.}$$

and the moment-stress is

$$f_m = f_0 d = 408 \times 6.6 = 2690 \text{ lb.}$$

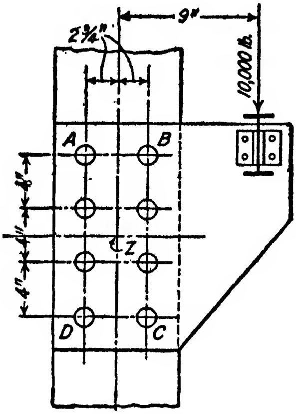


FIG. 60.

(3) Resolve the moment-stress into its vertical and horizontal components.[1] The horizontal component is $2690 \times \frac{6}{6.6} = 2450$ lb., and the vertical component is $2690 \times \frac{2.75}{6.6} = 1120$ lb.

(4) The uniform stress on the rivet is $\frac{10{,}000}{8} = 1250$ lb. The total vertical component is $1250 + 1120 = 2370$ lb. The resultant stress is

$$R = \sqrt{V^2 + H^2} = \sqrt{2370^2 + 2450^2} = 3400 \text{ lb.}$$

(5) Compare this stress with the rivet value From Table IV,

R.V. (Single Shear) = 5964 lb.
R.V. (⅜-in. Bearing) = (greater than S.S. value)

[1] It should be borne in mind that the direction of the moment-stress is at right angles to the line joining B and Z.

The resultant stress is obviously well within the rivet value as determined by single shear.

Bracket Connections

Figure 61 shows a bracket connection in which the rivets connecting the bracket to the column flange are subjected to direct tension in addition to being stressed in shear and bearing. Inasmuch as the shearing and tensile stresses in the rivets are independent of one another, sufficient rivets are provided to resist the shear or turning moment, whichever requires the larger number. For purposes of design the center of rotation is usually taken at the lowest rivet. Hence, the top rivet of the group will be most highly stressed in tension.

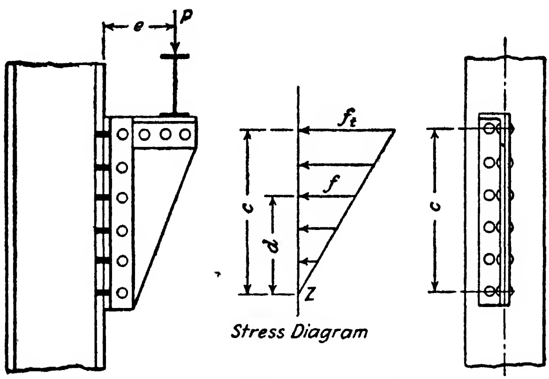


FIG. 61. Bracket Connection.

Let f_t in Fig. 61 be the tensile stress in the top rivet, c the distance of this rivet from the center of rotation Z, and d the distance of any other rivet from Z. The tensile stress f in any rivet at distance d from Z is expressed by the relation

$$\frac{f}{d} = \frac{f_t}{c}$$

or

$$f = \frac{f_t d}{c}$$

The moment of resistance of any rivet is

$$M' = \frac{f_t d}{c} \times d = \frac{f_t d^2}{c}$$

and the total moment developed by all the rivets is

$$M = \frac{f_t \Sigma d^2}{c}$$

This moment of resistance is equal to the turning moment Pe; hence,

$$\frac{f_t \Sigma d^2}{c} = Pe$$

EXAMPLE

Determine the maximum tensile stress on the top rivet of the connection shown in Fig. 61 if $c = 15$ in. with the intermediate rivets spaced 3 in. on centers. The load P is 10,000 lb. and the eccentricity is 9 in. Compare this stress with the tensile value of a $\frac{3}{4}$-in. rivet if the allowable stress for rivets in tension is limited to 15,000 lb. per sq. in.

Solution:

(1) Solving the above equation for the tensile stress in the top rivet,

$$f_t = \frac{Pec}{\Sigma d^2}$$

$$\Sigma d^2 = (3 \times 3) + (6 \times 6) + (9 \times 9) + (12 \times 12) + (15 \times 15) = 495$$

$$f_t = \frac{10{,}000 \times 9 \times 15}{495} = 2730 \text{ lb.}$$

(2) The area of a $\frac{3}{4}$-in. rivet is 0.4418 sq. in. Hence, the tensile value at 15,000 lb. per sq. in. is

$$0.4418 \times 15{,}000 = 6627 \text{ lb.}$$

The maximum tensile stress on the top rivet is well within the allowable limit for the loading shown.

Eccentric connections are generally designed by trial. No definite rule can be given as a guide to the first assumption since the strength of such a joint depends on the number and spacing of rivets as well as on the load and amount of eccentricity. Each trial, of course, serves as a guide for the next, the number required being largely dependent on the designer's judgment and experience. The A.I.S.C. Manual contains tables and other data to facilitate the design of eccentric beam connections.

49. Bolted Connections. — Bolts used in structural-steel erection are of two general types, *turned bolts* and *unfinished bolts.* The A.I.S.C. Specification (1946) considers as turned bolts all bolts, regardless of the manufacturing process, which have a tolerance of zero over the nominal diameter and 0.006 in. under. In addition, they must have " regular semi-finished " heads conforming to American Standard B 18.2-1941 of the American Institute of Bolt, Nut and Rivet Manufacturers. Turned bolts in close-fitting drilled or reamed holes are permitted in shop or field work where it is impracticable to drive satisfactory rivets. The diameter of the hole must not be more than $\frac{1}{50}$ in. larger than the external diameter of the bolt. When these conditions are fulfilled, the allowable unit shear and bearing stresses are the same as for power-driven rivets.

Unfinished bolts, as the name implies, are not finished to precise tolerances. They are manufactured by automatic machines from rod stock as received from the mill. Holes for unfinished bolts, as for rivets, are punched $\frac{1}{16}$ in. larger than the nominal diameter of the bolt. The A.I.S.C. Specification (1946) permits a unit shearing stress of 10,000 lb. per sq. in. on this type of bolt. The bearing stress is limited to 20,000 lb. per sq. in. when the bolts are in single shear and to 25,000 when in double shear. Unfinished bolts are commonly used to make field connections for certain classes of steel buildings less than 125 ft. high. The local building code should always be consulted, however, before specifications are drawn up prescribing the extent of bolted connections on any particular project. In the absence of specific local requirements, the provisions of Section 7, Paragraph (*e*) of the A.I.S.C. Specification (1946) may be followed.

A third type of bolt known as the *rib bolt* has come into use in recent years. The shank of this bolt is made with longitudinal ribs which project somewhat from the core of the shank and result in an over-all diameter slightly larger than the diameter of the hole. When driven into the hole through the pieces to be connected, the ribs are deformed, thereby wedging the bolt tightly.

Care must be taken in using bolts of any type to see that the threaded portion of the shank is of proper length. If the threads do not extend far enough from the end of the bolt to permit a full grip when the nut is tightened, washers must be used under the nut.

PROBLEMS

1. Compute the shearing and bearing values of a 1-in.-diameter rivet in $\frac{5}{16}$-, $\frac{1}{2}$-, and $\frac{3}{4}$-in. plates, if the allowable unit shearing stress is 13,500 lb. per sq. in. and the allowable unit bearing stress is 24,000 lb. per sq. in. for rivets in single shear, and 30,000 for those in double shear.

2. Find the maximum shear and bearing stresses that will be developed in the joint shown in Fig. 54 if $\frac{7}{8}$-in. rivets are used and $P = 30{,}000$ lb. (Answer given in Appendix E.)

3. Find the maximum shear and bearing stresses that will be developed in the joint shown in Fig. 62 if $\frac{3}{4}$-in. rivets are used and $P = 40{,}000$ lb.

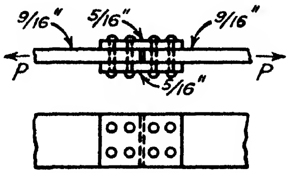


Fig. 62.

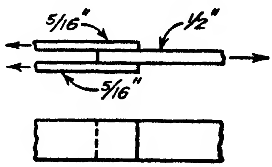


Fig. 63.

4. How many $\frac{3}{4}$-in. rivets are required in the joint shown in Fig. 63 to transmit a stress of 30,000 lb. across the joint? The allowable unit stress in shear is 13,500 lb. per sq. in., and in bearing 27,000 lb. per sq. in.

5. How many $\frac{3}{4}$-in. rivets are required on each side of the joint shown in Fig. 62 to transmit a stress of 35,000 lb. Allowable rivet stresses are controlled by A.I.S.C. Specification (1946).

6. Find the allowable end reaction of a 16″ WF 58 lb. if the beam is connected to the flange of a column as shown in Fig. 56*a*. There are 4 rivets through the beam web and 8 in the column flange. Rivet stresses are controlled by A.I.S.C. Specification (1946). The connection angles are $\frac{3}{8}$ in. thick and the column flange $\frac{1}{2}$ in. thick. Rivets are $\frac{3}{4}$ in. (Answer given in Appendix E.)

7. Find the allowable end reaction of a 16″ WF 36 lb. if the beam is connected to the web of a 18″ WF 50 lb. girder, as indicated in Fig. 56*b*. There are 4 rivets through the web of the 16-in. beam and 8 in the web of the 18-in. girder Allowable rivet stresses (pounds per square inch) are: shear = 13,500; bearing when in single shear = 24,000; bearing when in double shear = 30,000. Connection angles are $\frac{3}{8}$ in. thick, and $\frac{3}{4}$-in. rivets are used.

8. A 10″ WF 21 lb. frames into the web of a 14″ WF 34 lb. in a manner similar to that indicated in Fig. 56*b*. The reaction of the 10-in. beam is 24 kips. Allowable rivet stresses are controlled by A.I.S.C. Specification (1946). Connection angles are $\frac{3}{8}$ in. thick. Find the number of $\frac{3}{4}$-in. rivets required in

(*a*) the web of the 10-in. beam,

(*b*) the web of the 14-in. beam.

Make a sketch of the connection.

9. Find the number of rivets required in the web of the 14-in. beam of Problem 8 above if a similar 10-in. beam frames opposite the first one. Make a sketch of the connection.

10. A 14″ WF 30 lb. is connected to the flange of a column consisting of an 8″ WF 31, in a manner similar to that indicated by Fig. 56*a*. The reaction of the 14-in. beam is 38 kips. Allowable rivet stresses are controlled by A.I.S.C. Specification (1946). Connection angles are $\frac{3}{8}$-in. thick. Find the number of $\frac{3}{4}$-in. rivets required in

(*a*) the web of the 14-in. beam,

(*b*) the flange of the 8-in. column.

Make a sketch of the connection.

11. Problem statement similar to Problem 8 above, except that $\frac{3}{4}$-in. unfinished bolts are to be used in place of rivets. A.I.S.C. Specification (1946) controls. (Answer given in Appendix E.)

12. Problem statement similar to Problem 10 above, except that $\frac{3}{4}$-in. unfinished bolts are to be used in place of rivets. A.I.S.C. Specification (1946) controls.

CHAPTER V

PLATE GIRDERS

50. Introduction. — Although the heavier Wide Flange beams now available are adequate for most of the longer spans encountered in typical building construction, plate girders are frequently used for long spans, especially where unusually heavy loads are involved. Figure 64 shows the cross sections of riveted plate girders made up of web plate, flange angles, and cover plates. Other types of riveted girders are described in Art. 66. Welded plate girders are discussed in Chapter IX.

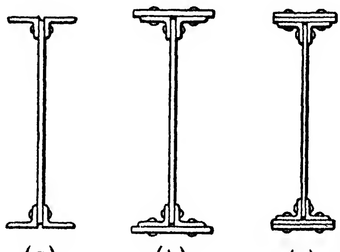


FIG. 64. Riveted Plate Girders.

51. Methods of Design. — Two common methods used in the design of plate girders for determining their resistance to bending are the chord method, frequently known as the flange area method, and the moment of inertia method. In the first of these the bending stresses are assumed to be uniformly distributed over the flange areas,[1] whereas in the second method the resisting moment

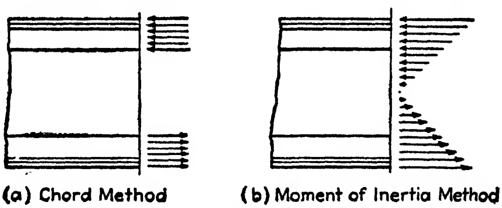


FIG. 65. Plate Girder Analysis.

is determined by the moment of inertia of the entire built-up section in the same manner as for I-beams. Figure 65 shows the assumed stress distribution for each method. The moment of

[1] Some specifications permit ⅛ of the web area to be counted as effective flange area.

inertia method is the more accurate of the two for the relatively shallow girders used in building construction, and many current specifications require that they be designed by this method.

It is sometimes convenient to make a trial design by the flange area method and then check it by investigating its moment of inertia.[2] However, the preliminary design may frequently be selected or interpolated from tables of "Properties of Sections of Plate and Angle Girders." Table VIII in Appendix D is a specimen of such tables. More complete tables will be found in the A.I.S.C. Manual and in other structural handbooks.

In the A.I.S.C. Specification (1928), plate girders were required to be designed by the moment of inertia of the net section, that is, the moment of inertia of the cross section after the moment of inertia of the rivet holes had been deducted. The 1936 and 1946 editions of the A.I.S.C. Specification were revised so as to permit the design of plate girders by the gross moment of inertia of the cross section, with certain reservations for special cases. Although the reception of this revision has been generally favorable, the demonstration of plate girder design under net area rules has been retained in this chapter so that the student may have such a treatment available.

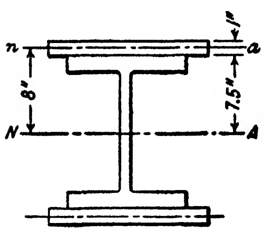


Fig. 66.

52. Moment of Inertia. — In Art. 18, the idea of moment of inertia was developed in connection with the derivation of the beam formula. Methods for computing moments of inertia of plane areas are given in standard textbooks on mechanics, and moments of inertia of rolled shapes will be found in the steel handbooks.

The simple built-up section shown in Fig. 66 is composed of a 15″ I 50 lb. with two 7-in. by 1-in. plates welded to its flanges. The moment of inertia of the built-up section about its neutral axis is equal to the sum of the moments of inertia of all the members about the same axis. The first step in determining the moment of inertia of the built-up section is to find the moment of inertia of the 15″ I 50 lb., since the neutral axis of the beam coincides with the

[2] This system is illustrated in the A.I.S.C. Manual, and in the eighteenth edition of the Kidder-Parker "Architects' and Builders' Handbook," Chapter XIX.

neutral axis of the built-up section. From Table II this is found to be 481. The moment of inertia of one of the plates 1 in. thick and 7 in. wide, about its own gravity axis parallel to the longer side (*na* in Fig. 66), is found from the formula for rectangular sections, $I = bt^3/12$. Therefore, $I = (7 \times 1^3)/12 = 0.583$. It is now necessary to find the moment of inertia of the plate about the neutral axis of the built-up section. This is accomplished by means of the transfer of axis equation,[3] which states that

THE MOMENT OF INERTIA OF A MEMBER ABOUT ANY AXIS PARALLEL TO AN AXIS THROUGH ITS OWN CENTER OF GRAVITY IS EQUAL TO THE MOMENT OF INERTIA OF THE MEMBER ABOUT ITS OWN GRAVITY AXIS, PLUS THE AREA OF THE MEMBER TIMES THE SQUARE OF THE DISTANCE BETWEEN THE TWO AXES.

Expressed mathematically,

$$I = I_0 + Az^2$$

In the above equation,

I = the moment of inertia of the member about the required axis;
I_0 = the moment of inertia of the member about its own gravity axis parallel to the required axis;
A = the area of the member;
z = the distance between the two parallel axes.

Applying this equation to the two plates of the built-up section shown in Fig. 66,

$$\begin{aligned} I &= 2[I_0 + Az^2] \\ &= 2[0.583 + (7 \times 8 \times 8)] \\ &= 2[0.583 + 448] = 2 \times 448.583 = 897.166 \text{ or } 897 \end{aligned}$$

The moment of inertia of the entire section about the neutral axis, NA is $481 + 897 = 1378$.

It is evident from this discussion that the moment of inertia (I_0) of a thin plate about its own gravity axis is negligible when that axis is parallel to the longer side of the plate. When the members considered are angles or other shapes, however, the value of I_0 is appreciable and should not be omitted.

[3] For a derivation of this formula see Chapter XII of Young and Baxter's "Mechanics of Materials," The Macmillan Company, or any standard textbook on mechanics of materials.

If the moment of inertia of a section composed of several members had to be determined in this way, the computations would be quite tedious, but when tables similar to VI and VII of Appendix D are available the work is greatly simplified. For example, to obtain the moment of inertia of the pair of plates about the axis NA in Fig. 66, it is necessary to know the area of one plate (7 sq. in.) and the distance between their centers of gravity (16 in.). Entering Table VI with $d = 16.0$ in., the moment of inertia of a "pair of unit areas" is found to be 128. Multiply this by the area of *one* of the plates, $I_{NA} = 7 \times 128 = 896$. The moment of inertia of the entire section about axis NA (neglecting I_0 of the plates) $= 481 + 896 = 1377$.

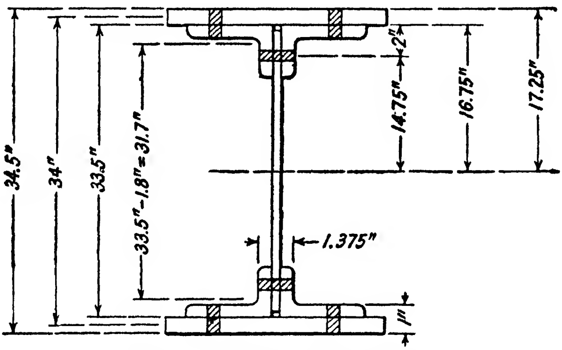


FIG. 67. Plate Girder Section.

53. Investigation of Plate Girders. — The first step in the investigation of the bending strength of a plate girder is to make a sketch of the cross section such as that shown in Fig. 67, giving the dimensions that are required in the solution. It should be noted that the backs of the flange angles are usually set $\frac{1}{4}$ in. beyond the web. This is done to facilitate fabrication as the edges of the web plate are seldom perfectly straight.

In order to find the load a girder will carry it is necessary to know its resisting moment. This is found from the flexure formula, $M = \frac{fI}{c}$. The value of f is given in the specifications, c is half the total depth, and I must be computed. The moment of inertia used is that of the net section, that is, the I of the cross section after the I of the rivet holes has been deducted. It is assumed that the holes in the tension flange weaken the girder but that those in the compression flange do not since the rivets are assumed to fill the holes completely. However, for convenience the upper half of the girder is usually made identical with the lower half. This method of design supplies a somewhat larger area than necessary in the compression flange, thus reducing the extreme fibre stress. The reduction is frequently helpful in meeting requirements for provision against buckling of the top flange as discussed in Art. 32 under lateral support. The diameter of a rivet hole is taken $\frac{1}{8}$ in. larger than the nominal diameter of the rivet. The holes are actually punched $\frac{1}{16}$ in. larger than the rivet diameter as explained in Art. 40. The additional $\frac{1}{16}$ in. is to compensate for the metal surrounding the hole which is damaged in punching.

EXAMPLE

Find the total uniformly distributed load as governed by bending, that the girder shown in Fig. 67 will carry on a 32-ft. span. The girder is composed of a $33 \times \frac{3}{8}$-in. web plate, four angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in., and two $12 \times \frac{1}{2}$-in. cover plates. There is one line of $\frac{3}{4}$-in. rivets connecting the flange angles to the web and two lines connecting the cover plates to the angles. The maximum allowable fibre stress is 20,000 lb. per sq. in.

Solution:

(1) Make a sketch similar to Fig. 67.

(2) Compute the I of the gross section using Tables V, VI, and VII.

The I of the web plate may be found directly from Table V. Entering the table with $d = 33$ in., the value for a $\frac{3}{8}$-in. plate is found to be 1123.

The I of the cover plates is found from Table VI, as demonstrated in the preceding article. Entering the table with $d = 34$ in. (see Fig. 67), the I of a " pair of unit areas " is found to be 578. Multiplying this by the area of *one* plate (6 sq. in.) gives a value of $6 \times 578 = 3468$.

The I of the four angles (two sets) about their own gravity axes is found from Table VII to be 16. The distance between the gravity axes of the two sets of angles is found by deducting the distance $2x = 1.8$ in. (found from Table VII) from the total distance back to back of angles. From Fig. 67 this is seen to be 31.7 in. The I of a " pair of unit areas," 31.7 in. apart, is found from Table VI to be 502. If this is multiplied by the area of *one* set of angles ($2 \times 4 = 8$ sq. in. — Table III), the value is $8 \times 502 = 4016$. Hence, the total I of the four angles $= 4016 + 16 = 4032$.

The total gross $I = 1123 + 3468 + 4032 = 8623$.

(3) The moment of inertia of the holes is computed by using the equation $I = I_0 + Az^2$, except that the moment of inertia of the holes

about their own gravity axis is so small that it may be neglected and the formula becomes $I = Az^2$ where the area, A, is the diameter of the hole times the length of the hole. In this problem, the diameter of the holes is $\frac{3}{4} + \frac{1}{8} = \frac{7}{8}$ in. or 0.875 in. and the length equals the total thickness of metal through which they extend. The distances from the neutral axis to the centers of gravity of the holes are given in Fig. 67. The 2-in. distance from the back of the angles to the web holes was found from Table IX, Gages for Angles. The center of gravity of the holes through the cover plate and angles lies at the back of the angles. This is not usually the case but occurs here because the plates and angles are of the same thickness.

$$\begin{aligned} I \text{ (web holes)} &= 2[0.875 \times 1.375 \times \overline{14.75^2}] &= 524 \\ I \text{ (c.pl. holes)} &= 4[0.875 \times 1.0 \times \overline{16.75^2}] &= \underline{982} \\ & \qquad\qquad \text{Total } I \text{ of holes} &= 1506 \end{aligned}$$

(4) The net I of the girder is $8623 - 1506 = 7117$

(5) The resisting moment is

$$M = \frac{fI}{c} = \frac{20{,}000 \times 7117}{17.25} = 8{,}240{,}000 \text{ in.-lb.} \quad \text{or} \quad 686{,}700 \text{ ft.-lb.}$$

(6) The total allowable uniformly distributed load, including the girder weight, is found by solving the equation $M = WL/8$ for W, or

$$W = \frac{8M}{L} = \frac{8 \times 686{,}700}{32} = 172{,}000 \text{ lb.}$$

54. Design of Plate Girders. — Plate girders, like beams, must be designed to resist bending, shear, flange buckling, and web crippling. In I-beam design the investigation of shearing stress is necessary only for short, heavily loaded spans or for spans on which large concentrated loads occur near the ends, but the design of a plate girder depends as much upon shear considerations as upon those of bending. In I-beams the relation of web to flange area is fixed by a few standard proportions, one of which must be used, while in plate girders this relation is subject to extensive variation.

The economical depth of a plate girder is dependent upon so many conditions that no simple rule for computing it can be given. In general, the depth increases with an increase in shear, bending moment, or span length. The depth may be decreased by increasing the thickness of the web. A shallow girder is preferable from the standpoint of lateral stiffness but will have a greater deflection than a deeper one.

The most economical range of the depth of a girder is from

$\frac{1}{12}$ to $\frac{1}{7}$ of the span, although depth as shallow as $\frac{1}{20}$ of the span may be used under certain conditions. In building construction it is generally desirable to make the girders as shallow as practicable.

The procedure for the design of a plate girder carrying a uniformly distributed load includes the following steps, each of which is discussed separately in the design of the girder shown in Fig. 68.

(1) Make a sketch of the girder showing the loads.

(2) Compute the end reactions and construct the shear and bending moment diagrams. It is of course necessary to assume a trial girder section so that the effect of the weight of the girder will be included in the total bending moment. The accuracy of this assumed weight will depend upon the designer's judgment and experience.[4]

(3) Design the web plate.

(4) Compute the required moment of inertia by solving the flexure formula for I which gives $I = \frac{Mc}{f}$, and select the component parts of the girder so that the section will have a gross moment of inertia somewhat larger than that found by the formula.

(5) Compute the moment of inertia of the holes and subtract this value from that supplied in Step (4). If the net moment of inertia is equal to or slightly larger than that required and the weight of the combined sections is in reasonable agreement with that assumed in Step (2), accept the design; otherwise, another trial section must be assumed and the work repeated.

(6) If cover plates are used determine their required length.

(7) Design the web stiffeners.

(8) Design the end connection.

(9) Compute the spacing of the rivets connecting the flange angles to the web.

(10) Compute the spacing of the rivets connecting the cover plates to the flange angles.

(11) Make a design drawing of the girder.

Because of the indeterminate nature of stress distribution in a plate girder, most methods of design involve some simplifying

[4] Tables giving the section moduli of various built-up shapes are found in some handbooks. When such tables are available they furnish an excellent guide for selecting the trial section. See Table VIII in Appendix D.

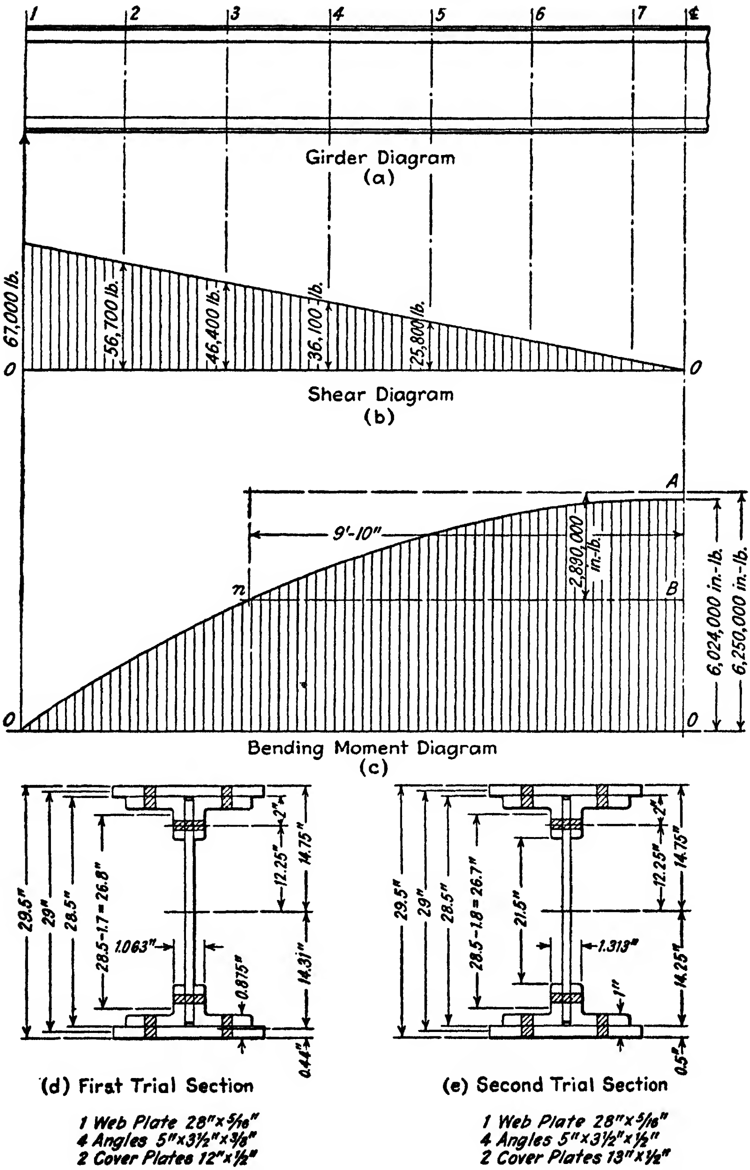


Fig. 68. Plate Girder Design Data.

assumptions and empirical rules which experience has shown to be satisfactory. It is beyond the scope of this book to discuss the different specifications controlling plate girder design or the theory upon which many of the assumptions are based. Steps similar to those outlined in the foregoing procedure are necessary to any method, although the details of the solutions may differ. The following example is given to illustrate the design of plate girders:

Design a plate girder to carry a uniformly distributed load of 130,000 lb. on a 30-ft. span. Head room under the girder restricts the over-all depth to 30 in. The allowable extreme fibre stress equals 18,000 lb. per sq. in.; the allowable average shearing stress on the gross area of the web equals 10,000 lb. per sq. in.; the allowable rivet stresses are 13,500 lb. per sq. in. in shear and 27,000 lb. per sq. in. in bearing. The girder is to be connected to the 1-in. flange of a steel column by means of web angles; $\frac{3}{4}$-in. rivets are to be used throughout.

The solution of this example is given in the following articles. The numbers in parentheses after the article captions refer to steps of the procedure. Figure 68*a* is a diagram of the girder.

55. Bending Moment, Reactions, and Shear. — (Step 2.) The bending moment due to the *superimposed* load is $M = WL/8 = 130{,}000 \times 30/8 = 488{,}000$ ft.-lb. The required section modulus is $S = M/f = 12 \times 488{,}000/18{,}000 = 325$. From Table VIII in Appendix D, under section moduli for net sections, it is found that an S_x of 348 is provided by a girder weighing 133.8 lb. per foot, or an S_x of 342 by one weighing 126.6 lb. Estimating the probable weight as 130 lb. per ft., the total weight will be $130 \times 30 = 3{,}900$ lb., and the *total load* becomes $130{,}000 + 3{,}900$ or approximately 134,000 lb.

The end reactions are $134{,}000/2 = 67{,}000$ lb.

The maximum design bending moment is

$$M = \frac{WL}{8} = \frac{134{,}000 \times 30}{8} = 502{,}000 \text{ ft.-lb.} \quad \text{or} \quad 6{,}024{,}000 \text{ in.-lb.}$$

With these values, the shear and bending moment diagrams may be constructed as shown in Fig. 68*b* and *c*.

56. Web Plate Design. — (Step 3.) Assuming that the web takes all of the shear, the required area of the web plate is found by dividing the end reaction by the allowable average unit shearing stress. The formula used is the same as that given in Art. 25.

$$v = \frac{V}{A_w} \quad \text{or} \quad A_w = \frac{V}{v} = \frac{67{,}000}{10{,}000} = 6.7 \text{ sq. in.}$$

Assuming a web plate depth of 28 in. for the trial design, the required thickness is $\frac{6.7}{28} = 0.239$ in. It is considered good practice in building work to use a minimum of $\frac{5}{16}$ in. for the web thickness. Inasmuch as the required thickness comes within this minimum, a $\frac{5}{16}$-in. web is adopted.

57. Required Moment of Inertia. — (Step 4.) The required moment of inertia is found from the beam formula $I = \frac{Mc}{f}$. However, it will be necessary to select a trial girder section in order to obtain a value for c. In making the trial selection, it will be observed that one of the girders used to establish a weight allowance in Art. 55 (the one weighing 133.8 lb. per ft. found in Table VIII) is composed of a $27\frac{1}{2} \times \frac{5}{16}$-in. web plate; 4 angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in.; and 2 cover plates $12 \times \frac{1}{2}$ in. *With this as a guide* and with the 28-in. web plate assumed in Art. 56, the First Trial Section (Fig. 68*d*) with $\frac{3}{8}$-in. angles is selected. The required moment of inertia is then

$$I = \frac{Mc}{f} = \frac{6{,}024{,}000 \times 14.75}{18{,}000} = 4940$$

The *gross* moment of inertia of this section is found from Tables V, VI, and VII, by the method demonstrated in Step (2) of the example on page 93, and is

$$\begin{aligned} I \text{ of web plate } 28 \times \tfrac{5}{16} \text{ in.} &= 572 \\ I \text{ of 4 angles } 5 \times 3\tfrac{1}{2} \times \tfrac{3}{8} \text{ in.} &= 2203 \\ I \text{ of 2 cover plates } 12 \times \tfrac{1}{2} \text{ in.} &= \underline{2526} \\ \text{Gross } I &= 5301 \end{aligned}$$

58. Net Moment of Inertia. — (Step 5.) The net moment of inertia is found by the same method as illustrated in Step (3) of the example following Art. 53. Using $\frac{3}{4}$-in. rivets the diameter of the hole is $\frac{7}{8}$ in. or 0.875 in. From Fig. 68*d*,

$$\begin{aligned} I \text{ (web holes)} &= 2[0.875 \times 1.063 \times \overline{12.25^2}] = 279 \\ I \text{ (c.pl. holes)} &= 4[0.875 \times 0.875 \times \overline{14.31^2}] = \underline{629} \\ \text{Total } I \text{ of holes} &= 908 \\ \text{Net } I \text{ supplied} = 5301 - 908 &= 4393 \\ \text{Net } I \text{ required (as found in Art. 57)} &= 4940 \end{aligned}$$

The net I supplied is *insufficient* by an amount equal to 4940 - 4393 = 547. Consequently, the trial section must be increased.

The Second Trial Section (Fig. 68*e*) is obtained by increasing the thickness of the angles to $\frac{1}{2}$ in. and the width of cover plates to 13 in.

$$
\begin{aligned}
I \text{ of web plate } 28 \times \tfrac{5}{16} \text{ in.} &= 572 \\
I \text{ of 4 angles } 5 \times 3\tfrac{1}{2} \times \tfrac{1}{2} \text{ in.} &= 2864 \\
I \text{ of 2 cover plates } 13 \times \tfrac{1}{2} \text{ in.} &= \underline{2737} \\
\text{Gross } I &= 6173
\end{aligned}
$$

$$
\begin{aligned}
I \text{ (web holes)} &= 2[0.875 \times 1.313 \times \overline{12.25}^2] &= 345 \\
I \text{ (c.pl. holes)} &= 4[0.875 \times 1 \times \overline{14.25}^2] &= \underline{710} \\
&\qquad\qquad \text{Total } I \text{ of holes} &= 1055
\end{aligned}
$$

$$
\begin{aligned}
\text{Net } I \text{ supplied} = 6173 - 1055 &= 5118 \\
\text{Net } I \text{ required (as found in Art. 57)} &= \underline{4940} \\
\text{Excess} &= 178
\end{aligned}
$$

The weight[5] of this section, exclusive of rivet heads, stiffeners, etc., is 3860 lb. as against 3900 lb. assumed in Art. 55. These auxiliary pieces will probably make the total weight somewhat in excess of 3900 lb., but this allowance is sufficiently accurate for purposes of design. Therefore the Second Trial Section is adopted.

59. Length of Cover Plates. — (Step 6.) The built-up section adopted in the preceding article was the one required at the point of maximum bending moment. It is evident from the bending moment diagram (Fig. 68*c*) that at some point between the center and ends of the span the resisting moment of the four angles and web plate will be sufficient to take care of the bending moment. The point at which the cover plates are no longer necessary may be found as follows.

First compute the resisting moment of the adopted section by the formula $M = \dfrac{fI}{c}$, in which I is the *net* moment of inertia supplied. Plot this value in Fig. 68*c* to the same scale as the bending moment diagram and draw a horizontal line through A.

$$M = \frac{fI}{c} = \frac{18{,}000 \times 5118}{14.75} = 6{,}250{,}000 \text{ in.-lb.}$$

[5] The weight of angles will be found in tables of properties of sections, and the weight of the web and cover plates is computed from tables giving the weight of flat-rolled steel.

The resisting moment of the cover plates is now computed from the above formula by substituting the net moment of inertia of the plates for I. The net I of the plates is found as follows (see Fig. 69).

$$
\begin{aligned}
I \text{ (gross) of the two plates (from Art. 58)} &= 2737 \\
I \text{ (4 holes)} = 4 \times (0.875 \times 0.5 \times 14.5^2) &= \underline{368} \\
\text{Net } I \text{ (cover plates)} &= 2369
\end{aligned}
$$

$$M = \frac{fI}{c} = \frac{18{,}000 \times 2369}{14.75} = 2{,}890{,}000 \text{ in.-lb.}$$

This value is laid off to scale (line AB of Fig. 68c) along the center line of the span and a horizontal line drawn through B. Cover plates are unnecessary beyond the point where this line intersects the bending moment curve. The distance Bn is scaled from the diagram and is 9 ft. 10 in.

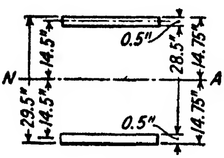


Fig. 69.

The accuracy of this method for determining the length of cover plates depends upon the accuracy with which the bending moment curve is plotted. If the intersection does not occur near one of the plotted points, an additional point may be computed. This method is sufficiently exact for most practical purposes inasmuch as the plates are usually extended 1 or 1½ ft. beyond the theoretical cut-off point.

60. Web Stiffeners. — (Step 7.) To prevent buckling of the web plate it is usually necessary to employ stiffener angles at intervals along the span length. There are two classes of stiffeners: intermediate stiffeners, and those used under concentrated loads. The design of the latter is considered in Art. 65. The most common type of intermediate stiffener consists of two angles riveted to opposite sides of the web.

Specification requirements for intermediate stiffeners vary. One typical specification requires that stiffeners be used when the height of the web between flange angles is more than 60 times its thickness and calls for spacing not to exceed the full depth of the web with a maximum of 5 ft. In this problem the web thickness is $\frac{5}{16}$ in. This value multiplied by 60 gives 18.75 in. The unsupported height of the web plate is 21½ in. (see Fig. 68e), which is more than 60 times the thickness; hence, intermediate stiffeners are required.

Angles with unequal legs are used for stiffeners. The leg in contact with the web is made wide enough to accommodate a single row of rivets ($2\frac{1}{2}$ or 3 in.). The width of the outstanding leg is determined arbitrarily. Some specifications require that this leg be at least 2 in. greater than $\frac{1}{30}$ of the nominal depth of the girder, and that the thickness of the stiffener angles be at least equal to the thickness of the web plate. Angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ in. fulfill these conditions in the present problem. The spacing of the rivets connecting the stiffener angles to the web plate is frequently made twice that required in the end connection with a maximum of 6 in. For this problem web stiffeners composed of 2 angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ in. with the longer legs outstanding should be spaced not more than 28 in. on centers. The required spacing will seldom go into the span length an even number of times. In such cases the end panels (that is, the distance from the ends of the girder to the first stiffeners) are made less than the other panels, or the spacing is decreased and all panels made equal (see Fig. 71). Filler plates must be used under the stiffener angles as illustrated by Fig. 70*a* or the angles crimped as shown in Fig. 70*b*, a method no longer commonly employed.

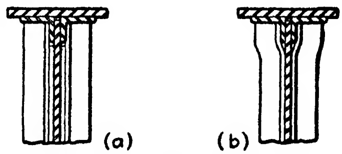


FIG. 70. Web Stiffeners.

61. End Connections. — (Step 8.) When a girder is connected to a column or another girder by means of web angles, the design is similar to that of an ordinary I-beam web connection. If the girder rests on top of a column or on a masonary wall, the design of the end stiffeners is substantially the same as that of stiffeners under concentrated loads (see Art. 65). In this problem the girder is to be connected by $\frac{3}{4}$-in. rivets to the 1-in. flange of a column. Assuming $\frac{3}{8}$-in. angles and using the specified stresses, the design is as follows:

(1) Rivets required in web or girder.

R.V. (Double Shear) = 11,928 lb.
R.V. ($\frac{5}{16}$-in. bearing in web) = 6,328 lb.

The number of rivets required is equal to the end reaction

The resisting moment of the cover plates is now computed from the above formula by substituting the net moment of inertia of the plates for I. The net I of the plates is found as follows (see Fig. 69).

$$\begin{aligned} I \text{ (gross) of the two plates (from Art. 58)} &= 2737 \\ I \text{ (4 holes)} = 4 \times (0.875 \times 0.5 \times 14.5^2) &= \underline{368} \\ \text{Net } I \text{ (cover plates)} &= 2369 \end{aligned}$$

$$M = \frac{fI}{c} = \frac{18{,}000 \times 2369}{14.75} = 2{,}890{,}000 \text{ in.-lb.}$$

This value is laid off to scale (line AB of Fig. 68c) along the center line of the span and a horizontal line drawn through B. Cover plates are unnecessary beyond the point where this line intersects the bending moment curve. The distance Bn is scaled from the diagram and is 9 ft. 10 in.

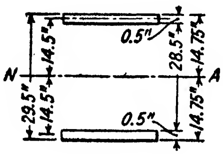


Fig. 69.

The accuracy of this method for determining the length of cover plates depends upon the accuracy with which the bending moment curve is plotted. If the intersection does not occur near one of the plotted points, an additional point may be computed. This method is sufficiently exact for most practical purposes inasmuch as the plates are usually extended 1 or $1\frac{1}{2}$ ft. beyond the theoretical cut-off point.

60. Web Stiffeners. — (Step 7.) To prevent buckling of the web plate it is usually necessary to employ stiffener angles at intervals along the span length. There are two classes of stiffeners: intermediate stiffeners, and those used under concentrated loads. The design of the latter is considered in Art. 65. The most common type of intermediate stiffener consists of two angles riveted to opposite sides of the web.

Specification requirements for intermediate stiffeners vary. One typical specification requires that stiffeners be used when the height of the web between flange angles is more than 60 times its thickness and calls for spacing not to exceed the full depth of the web with a maximum of 5 ft. In this problem the web thickness is $\frac{5}{16}$ in. This value multiplied by 60 gives 18.75 in. The unsupported height of the web plate is $21\frac{1}{2}$ in. (see Fig. 68e), which is more than 60 times the thickness; hence, intermediate stiffeners are required.

Angles with unequal legs are used for stiffeners. The leg in contact with the web is made wide enough to accommodate a single row of rivets ($2\frac{1}{2}$ or 3 in.). The width of the outstanding leg is determined arbitrarily. Some specifications require that this leg be at least 2 in. greater than $\frac{1}{30}$ of the nominal depth of the girder, and that the thickness of the stiffener angles be at least equal to the thickness of the web plate. Angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ in. fulfill these conditions in the present problem. The spacing of the rivets connecting the stiffener angles to the web plate is frequently made twice that required in the end connection with a maximum of 6 in. For this problem web stiffeners composed of 2 angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ in. with the longer legs outstanding should be spaced not more than 28 in. on centers. The required spacing will seldom go into the span length an even number of times. In such cases the end panels (that is, the distance from the ends of the girder to the first stiffeners) are made less than the other panels, or the spacing is decreased and all panels made equal (see Fig. 71). Filler plates must be used under the stiffener angles as illustrated by Fig. 70*a* or the angles crimped as shown in Fig. 70*b*, a method no longer commonly employed.

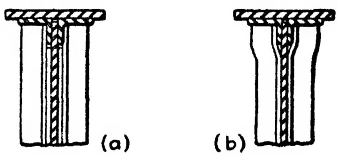


FIG. 70. Web Stiffeners.

61. End Connections. — (Step 8.) When a girder is connected to a column or another girder by means of web angles, the design is similar to that of an ordinary I-beam web connection. If the girder rests on top of a column or on a masonary wall, the design of the end stiffeners is substantially the same as that of stiffeners under concentrated loads (see Art. 65). In this problem the girder is to be connected by $\frac{3}{4}$-in. rivets to the 1-in. flange of a column. Assuming $\frac{3}{8}$-in. angles and using the specified stresses, the design is as follows:

(1) Rivets required in web or girder.

R.V. (Double Shear)	= 11,928 lb.
R.V. ($\frac{5}{16}$-in. bearing in web)	= 6,328 lb.

The number of rivets required is equal to the end reaction divided by 6328 or $\frac{67{,}000}{6328} = 10.6$ or 11 rivets. Inasmuch as 11 rivets in a single row would be spaced too close together, two rows are used.

(2) Rivets required in outstanding legs.

R.V. (Single Shear) $= 5964$ lb.
R.V. ($\frac{3}{8}$-in. bearing in outstanding leg) $= 7594$ lb.

The number of rivets required is $\frac{67{,}000}{5964} = 11.2$ or 12 rivets. Use 6 rivets in each leg.

(3) Two angles $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in. with the shorter legs outstanding will be satisfactory. In girders with large end shears, the filler plate should be made wide enough to allow for an extra row of rivets beyond the edges of the angles. Such a filler plate is known as a "tight filler" (see Fig. 71). Detailing of the connection is left to the fabricating shop.

62. Spacing of Flange Rivets. — (Step 9.) In order to make the flanges and web of a plate girder act as a unit they must be riveted together. If the flange angles were not connected to the web, the members would slide past each other as explained in Art. 23 under the discussion of horizontal shear. It is therefore necessary to provide enough rivets to transmit the horizontal stresses from the web to the flanges. The increase in the horizontal flange stress per unit length is directly proportional to the rate of increase in the bending moment and hence is greatest at the ends of the span where the value of the moment is changing most rapidly. Assuming that the flanges resist all the bending stress, it can be shown from the general shear equation (Art. 25) that the increase in flange stress per linear inch (called the horizontal increment of flange stress) at any section of a girder is expressed by the equation

$$K_h = \frac{V}{d}$$

in which K_h = horizontal increment of flange stress;
V = vertical shear at the section;
d = distance between center lines of rivets in top and bottom flanges.

This horizontal increment of flange stress must be resisted by the rivets connecting the flange angles to the web. If R.V. is the value of one rivet as determined by shear or bearing in the web, the maximum horizontal rivet spacing (c. to c. of rivets) is

$$p = \frac{\text{R.V.}}{K_h}$$

The term p is commonly known as the rivet pitch.

It is evident from the above equations that there is a change in pitch wherever the vertical shear changes. Theoretically, the spacing between rivets in girders carrying uniformly distributed loads should vary from one rivet to the next. In practice, however, the pitch is computed from the maximum shear in each panel and made constant throughout the panel length.

When the load carried by a girder rests on the top flange, as in the case of a girder supporting a masonry wall, the flange rivets must transmit this load to the web. Hence, there will be a vertical stress on the rivets in addition to the horizontal stress. The resultant increment K_r is found from the expression

$$K_r = \sqrt{(K_h)^2 + u^2}$$

where u is the vertical load per linear inch of span. Under such conditions the formula for rivet pitch becomes

$$p = \frac{\text{R.V.}}{K_r}$$

The rivet pitch equations developed above are based on the assumption that the web of the girder takes no bending moment. When designing by the moment of inertia method, however, the horizontal increment of flange stress to be transmitted from the flange to the web is reduced by the ratio of the gross moment of inertia of the flanges, I_f, to that of the entire girder section, I_g. The horizontal increment to be taken by the rivets then becomes

$$K_h = \frac{V}{d} \times \frac{I_f}{I_g}$$

All vertical stress due to loads applied to the flange must, however, be transmitted to the web by the rivets, and the equation for K_r does not change.

In the problem under consideration the load is assumed to be applied through the top flange. Theoretically the rivet pitch in the bottom flange could be computed for horizontal flange stress only, but the rivet spacing is usually made the same in both flanges in order to simplify the fabrication. The approximate location of the web stiffeners is shown in Fig. 68*a*. Inasmuch as the girder is symmetrical about the center line, only one half need be considered. The bearing value of a $\frac{3}{4}$-in. rivet in the $\frac{5}{16}$-in. web plate is 6328 lb. The load per linear inch of span is $\frac{134{,}000}{30 \times 12} =$ 372 lb. From Art. 58, the gross moment of inertia, I_g, of the adopted girder section is 6173 and the gross moment of inertia of the flanges, I_f, equals I_g minus the moment of inertia of the web, or $I_f = 6173 - 572 = 5601$. Referring to Fig. 68*a* and *b*,

Shear at (1) = 67,000 lb.

$$K_h = \frac{V}{d} \times \frac{I_f}{I_g} = \frac{67{,}000}{24.5} \times \frac{5601}{6173} = 2480$$

$$K_r = \sqrt{(K_h)^2 + u^2} = \sqrt{\overline{2480}^2 + \overline{372}^2} = 2510 \text{ lb.}$$

$$p = \frac{\text{R.V.}}{K_r} = \frac{6328}{2510} = 2.52 \quad \text{or} \quad 2\tfrac{1}{2} \text{ in.}$$

The pitch is made $2\frac{1}{2}$ in. from (1) to (2). See Fig. 71.

Shear at (2) = 56,700 lb.

$$K_h = \frac{V}{d} \times \frac{I_f}{I_g} = \frac{56{,}700}{24.5} \times \frac{5601}{6173} = 2110$$

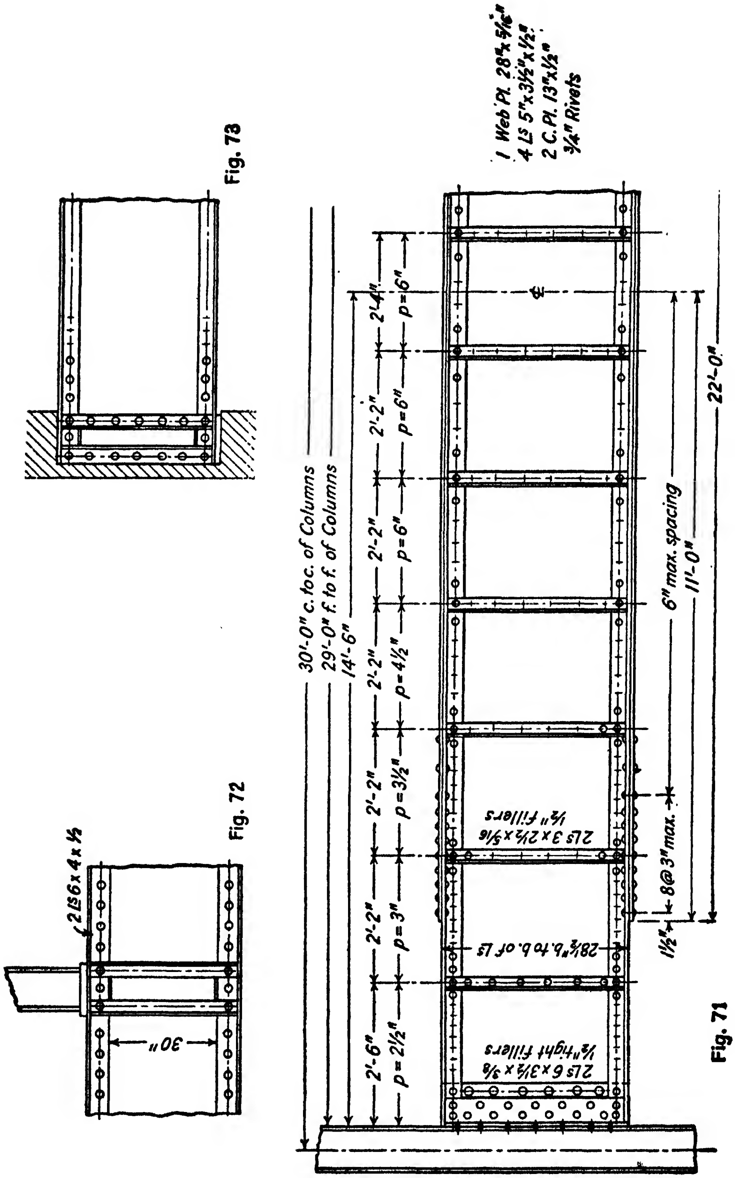
$$K_r = \sqrt{(K_h)^2 + u^2} = \sqrt{\overline{2110}^2 + \overline{372}^2} = 2140 \text{ lb.}$$

$$p = \frac{\text{R.V.}}{K_r} = \frac{6328}{2140} = 2.96 \quad \text{or} \quad 3 \text{ in.}$$

The pitch is made 3 in. from (2) to (3). See Fig. 71.

The rivet pitch in the remaining panels is found in a similar manner. Most specifications allow a maximum pitch of 6 in. and limit the minimum spacing between centers of rivet holes to 3 times the diameter of the rivet, although slightly more than this is desirable. For example, the preferred minimum spacing for $\frac{3}{4}$-in. rivets is $2\frac{1}{2}$ in. If the required pitch in the end panel had been smaller than the allowable minimum the girder would have had to be redesigned using a thicker web plate, since two rows of rivets cannot be placed in the $3\frac{1}{2}$-in. legs of the flange angles.

63. Rivets in Cover Plates. — (Step 10.) One of the simplest methods of determining the number of rivets required to connect



FIGS. 71, 72, 73. Design Drawing and Details.

the cover plates and flange angles is to provide sufficient rivets at the ends of the plate to transmit the allowable stress on the net section of the plate to the angles. In the present problem the computations are as follows:

(1) The net area of 1 plate $13 \times \frac{1}{2}$ in. when $\frac{3}{4}$-in. rivets are used (see Fig. 68e) is

$$(13 \times \tfrac{1}{2}) - 2(\tfrac{7}{8} \times \tfrac{1}{2}) = 6.5 - 0.875 = 5.63 \text{ sq. in.}$$

(2) The allowable stress in the plate is $5.63 \times 18{,}000 = 101{,}000$ lb.

(3) The rivet values for a $\frac{3}{4}$-in. rivet are

R.V. (Single Shear) = 5,964 lb.
R.V. ($\frac{1}{2}$ in. bearing on plate or angles) = 10,125 lb.

(4) No. of rivets at each end $= \dfrac{101{,}000}{5964} = 17.$

Use two lines of 9 rivets at each end of the plate.

The usual rules for spacing the cover-plate rivets state that the spacing at the ends of the plates shall not exceed 4 diameters of the rivet and that the maximum distance between rivets in the cover plates, in a direction parallel to the length of the girder shall not exceed 6 in. Assuming that the cover plate extends 1 ft. beyond the theoretical cut-off point as determined in Art. 59, a 3-in. spacing would be used for the first nine rivets followed by a 6-in. spacing to the center of the span. Opposite halves of the girder are, of course, identical (see Fig. 71).

64. The Design Drawing. — (Step 11.) Before a plate girder is built, detail drawings must be made showing such information as the exact location of rivets, clearances near stiffeners to allow for driving the rivets, etc. These details are prepared by the fabricating shop. A design drawing similar to that shown in Fig. 71, indicating the general make-up of the girder, is usually supplied by the designer and furnishes the basis for the working drawings.

65. Plate Girders with Concentrated Loads. — The design of plate girders supporting concentrated loads follows closely the procedure for uniform loads. The principal differences are in the spacing of the flange rivets and the design of stiffeners under the concentrated loads. Inasmuch as the bending moment diagram for girders carrying concentrated loads varies nearly as a straight line between the points of application of the loads,[6] the rivet pitch is more nearly constant between these points, and it is usually unnecessary to change the pitch at each intermediate stiffener.

Stiffeners under concentrated loads are used to transmit the load to the web as directly as possible. If the loads rested on the flanges alone, the number of flange rivets would have to be increased as explained in Art. 62, but, as it is not usually possible to supply enough rivets in the angles to care for the concentrated loads, stiffeners have to be used. Such stiffeners should have filler plates as shown in Fig. 70*a* and the outstanding legs should fit tightly against the outstanding legs of the top flange angles.

One method of design is to consider the stiffeners as short compression members, using an allowable unit stress of 12,000 lb. per sq. in. on the gross area of the angles. The stiffeners are connected to the web plate with sufficient rivets to transmit the load. This method is illustrated by the following example.

EXAMPLE

Figure 72 represents an H-column resting on a girder. The load on the column is 120,000 lb. and the girder web is $\frac{5}{16}$ in. thick. Design a stiffener of 4 angles using a unit stress of 12,000 lb. per sq. in. in compression. The rivet stresses are 13,500 lb. per sq. in. in shear and 27,000 lb. per sq. in. in bearing; $\frac{3}{4}$-in. rivets are to be used.

Solution:

(1) The area required in the 4 stiffener angles is

$$A = \frac{120{,}000}{12{,}000} = 10 \text{ sq. in.}$$

Angles should be selected so that the outstanding legs will not project beyond the outstanding legs of the flange angles. Four angles $5 \times 3 \times \frac{3}{8}$ in. supply an area of 11.44 sq. in. and will be used.

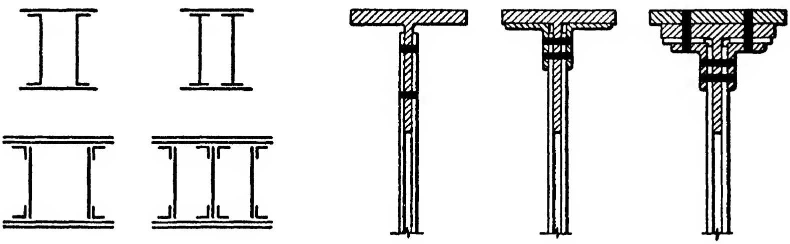
(2) The rivet value is limited by bearing in the $\frac{5}{16}$-in. web. Therefore, the total number of rivets required is $\frac{120{,}000}{6328} = 19$, or 10 in each pair of stiffener angles. The 30-in. distance between flange angles provides ample space for this number of rivets. It should be noted that the flange rivets through the stiffeners are not counted in the required number

[6] The bending moment diagram for the concentrated loads alone is, of course, a series of straight inclined lines, but the moment due to the girder weight (uniformly distributed) causes the line to curve slightly.

because their value is already being developed by the horizontal flange stresses.

When a girder rests on the top of a column or on a masonary wall (Fig. 73) the design of the end stiffeners is similar to that for stiffeners under concentrated loads, the end reactions being considered as concentrated loads. In this case the stiffener angles must fit tightly against the bottom flange.

66. Other Types of Riveted Girders. — The principles of design discussed in the preceding articles may be applied to other forms of riveted girders such as I-beams reinforced with plates, and box girders, which are essentially plate girders having more than one



(*a*) Typical Box Girders. (*b*) T-type Girder Flanges.

FIG. 74.

web, shown in Fig. 74*a*. The use of T-sections for plate girder flanges is a relatively recent development.[7] The Tees are manufactured by splitting Wide Flange sections or other beams. When used in girder design the stem of the Tee is riveted between two web plates as shown in Fig. 74*b*. In the lighter class of girders, where shear considerations require a thin web, this system of construction may not prove so efficient as the conventional type; however, for heavier girders the double web construction may be found distinctly advantageous. Furthermore, the T-flange permits a reduction in the amount of riveting required, and the double web plates are particularly useful where it is necessary to transmit large horizontal shearing stresses from the flange to the web. An example illustrating the design of girders with T-type flanges is given in Appendix C.

[7] See " A Comparative Analysis of Plate Girders " by Walter H. Weiskopf and John W. Pickworth in the November, 1934, issue of *Civil Engineering*. See also Appendix C of this book. The use of T-sections for girder flanges is covered by patents.

PROBLEMS

1. Table VIII of Appendix D gives the properties of selected plate girder sections. Carry out the necessary computations to verify the tabulated values of moment of inertia and section modulus for the following sections:

(*a*) Web = $39\frac{1}{2} \times \frac{3}{8}$ in. Angles = $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in.
(*b*) Web = $27\frac{1}{2} \times \frac{5}{16}$ in. Angles = $4 \times 3 \times \frac{1}{2}$ in.
(*c*) Add $12 \times \frac{3}{8}$-in. cover plates to (*a*) above.
(*d*) Add $10 \times \frac{1}{2}$-in. cover plates to (*b*) above.

2. A plate girder of the type shown in Fig. 64*b* is composed of a $41\frac{1}{2} \times \frac{3}{8}$-in. web plate, 4 angles $6 \times 4 \times \frac{5}{8}$ in., and 2 cover plates $14 \times \frac{5}{8}$ in. Rivet diameter = $\frac{3}{4}$ in.

(*a*) Determine the net moment of inertia.

(*b*) Find the allowable uniformly distributed load, in addition to its own weight, that the girder will support on a span of 70 ft. if the allowable extreme fibre stress is 20,000 lb. per sq. in. Full lateral support is provided.

3. A plate girder of the type shown in Fig. 64*b* consists of a $37\frac{1}{2} \times \frac{3}{8}$-in web plate, 4 angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$ in., and 2 cover plates $12 \times \frac{1}{2}$ in. Rivet diameter = $\frac{3}{4}$ in.

(*a*) Calculate the net moment of inertia.

(*b*) Determine the allowable uniformly distributed load, in addition to its own weight, that the girder will support on a span of 60 ft. if the allowable extreme fibre stress is 20 kips per sq. in. Full lateral support is provided.

4. (*a*) Using the data developed in Problem 2 above, find the required length of the cover plates and design the cover plate riveting. Allowable rivet stresses are controlled by the A.I.S.C. Specification (1946).

(*b*) Assuming that web stiffeners are spaced so as to divide the girder into panels approximately 3 ft. long, determine the spacing of the flange rivets in the end panel.

CHAPTER VI

COLUMNS AND STRUTS

67. Introduction. — A column or strut is a compression member, the length of which is several times greater than its least lateral dimension. The term column usually applies to relatively heavy vertical members, while relatively light or inclined members carrying compressive stresses, such as braces and the compression members of roof trusses, are called struts.

The ultimate load that can be carried by a short block, having a square cross section such as the one shown in Fig. 75*a*, may be determined experimentally by increasing the applied load until failure by crushing occurs. The unit compressive stress in the block at the time of failure is found by dividing the ultimate load by the area of the block or, $f = \frac{P}{A}$ where f is expressed in lb. per sq. in.; P in lb.; and A in sq. in.

If a piece of the same material, having the same cross section as the block, but with a height several times greater than the width, is subjected to a similar test it will fail before the applied load reaches the value that caused failure of the short block. In this case the failure is due to bending. Figure 75*b* represents the long piece supporting a light load. As the load is increased the column begins to bend (Fig. 75*c*). If the load which first causes the bending is not increased the column will continue to support it and the deflection, z, will remain constant. The column is, however, in a state of unstable equilibrium and any slight increase in the load or application of a horizontal force due to accidental jarring may destroy this condition. In such a case the deflection, z, becomes greater, thereby increasing the

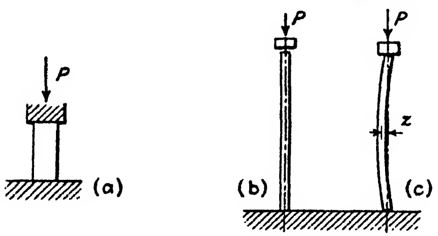


Fig. 75

bending stresses due to the moment Pz which in turn causes z to increase still more. The bending stresses continue to increase in this manner until the column fails.

In general, the tendency of a column to bend varies with the ratio of the length to the least lateral dimension, such as the diameter of a circle or the side of a square or rectangle. For tall, slender columns this ratio is large, hence the failure of such columns is due almost wholly to bending. For short columns or blocks this ratio is small and failure is due to crushing. Between these extremes are the so-called intermediate columns that fail due to a combination of bending and crushing.

68. Bending in Columns. — Bending occurs in a column due to an uneven distribution of stress over its cross section. If a column were perfectly straight and built of a perfectly homogeneous material, the stress would be uniformly distributed and there would be no bending if the loads were applied exactly along the axis. These conditions, however, are ideal and cannot be attained in practice. If the distribution of stress over the cross section is not uniform, a slight eccentricity is introduced which will cause a bending moment and hence bending stresses in addition to the stress due to axial compression.

Most columns used in building construction come under the intermediate class where failure, when it does occur, is due to a combination of crushing and bending. The value of the actual maximum unit stress in such cases is difficult to determine but it is evident that the *average* unit stress $\frac{P}{A}$ will be *less* than the crushing strength of the material by an amount dependent upon the tendency of the column to bend. For columns of this class there is no completely rational method of analysis. The "reduction" formulas in practical use are based to a large extent upon the results of experimental tests.

69. Column Shapes. — From the foregoing discussion it is evident that the strength of a column (because of its tendency to bend) depends upon the shape of the cross section as well as the area. The property of a section dependent upon its shape is the moment of inertia. All columns tend to fail in a direction perpendicular to the axis about which the moment of inertia is least. Hence the ideal cross section is one having the same moment of inertia about any axis through its center of gravity.

As material near the center of gravity of a section contributes little to the moment of inertia, the most efficient column is one having as small an amount of material as possible placed near the axis. A hollow circular section (a pipe) closely approaches the ideal section. Pipe columns are used to a limited extent in some types of building work but they are rarely seen in multi-story steel frame structures. One objection to their use is the difficulty of rigidly connecting beams to them. A rolled H-column is shown at Fig. 76*a* and the same section reinforced with flange plates at *b*. Columns built up of angles, plates, and channels are shown at *c*, *d*, and *e*, and angle sections used as struts at *f*, *g*, and *h*.

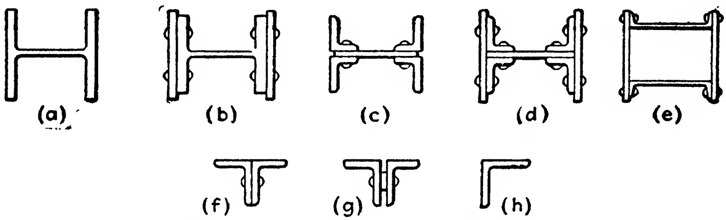


FIG. 76. Typical Column Sections.

70. Radius of Gyration and Slenderness Ratio. — In the design of beams (Art. 19) the section modulus was found to be an index of the strength of a member in bending. From its definition, $S = \frac{I}{c}$, it is evident that the value of the section modulus depends upon the size and shape of the section. In column design, there is a term analogous to this, called the radius of gyration, which is also dependent upon the size and shape of the section and is a measure of its effectiveness in resisting bending. The radius of gyration is expressed in inches by the formula $r = \sqrt{\frac{I}{A}}$ in which I is the moment of inertia of the section about the given axis and A is the area.[1]

In Art. 67 it was stated that the tendency of a column to bend varies in general with the ratio of its unstayed length to its least lateral dimension. For structural shapes such as those shown in Fig. 76, the least lateral dimension is not an accurate criterion.

[1] For a more extended treatment of radius of gyration, see any standard textbook on mechanics.

Consequently, the radius of gyration, which relates more precisely to the stiffness of column sections in general, is used in column formulas. The ratio of the length to the *least* radius of gyration, $\frac{L}{r}$, is called the *slenderness ratio.* L and r are both expressed in inches.

71. Column Formulas. — It was stated in Art. 68 that the average unit stress in a column at the time of failure is less than the crushing strength of the material by an amount dependent upon the tendency of the column to bend. It follows that the *allowable* average stress for use in design must likewise be influenced by this factor and will therefore depend upon the slenderness ratio as well as upon the compressive strength of the material. One of the earlier formulas which was widely used for determining the allowable average stress in building columns is

$$f = 16{,}000 - 70\frac{L}{r}$$

with the maximum value of f limited to 14,000 lb. per sq. in., and the maximum value of the slenderness ratio limited to 120 for main members and 150 for secondary members such as wind bracing. When the basic unit stress for structural-steel design was increased from 16,000 to 18,000 lb. per sq. in., this change was reflected in some column formulas. The formula recommended by the U. S. Department of Commerce in 1926 gave

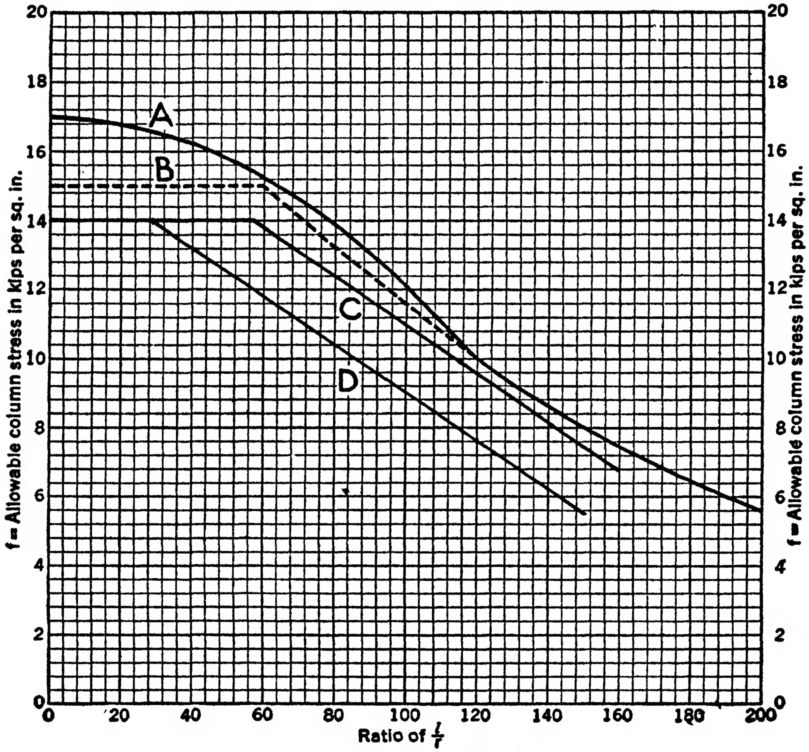
$$f = 18{,}000 - 70\frac{L}{r}$$

with a maximum of 14,000 lb. per sq. in., and with the slenderness ratio limited to 160 for both main and secondary members. Formulas of this type are known as straight-line formulas because the curve obtained by plotting values of f for different values of the slenderness ratio is a straight line.

The 1928 specification of the American Institute of Steel Construction required a formula of the Rankine type using a basic stress of 18,000 lb. per sq. in. This formula gives

$$f = \frac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}} \quad \text{or} \quad \frac{18{,}000}{1 + \dfrac{1}{18{,}000}\left(\dfrac{L}{r}\right)^2}$$

with a maximum of 15,000 lb. per sq. in., and with the slenderness ratio limited to 120 for main members and 200 for secondary members. The maximum of 15,000 lb. per sq. in. occurs when the slenderness ratio is 60. Hence for values below 60 an average stress of 15,000 lb. per sq. in. is used. By referring to Fig. 77, it will be observed that the allowable stresses given by this formula



A = A.I.S.C. (1946) $C = 18{,}000 - 70L/r$
B = A.I.S.C. (1928) $D = 16{,}000 - 70L/r$

FIG. 77. Column Formulas.

are not greatly different from those given by the 1926 Department of Commerce straight-line formula. They are, however, considerably in excess of those given by the first straight-line formula mentioned above. This difference is due primarily to the different basic stresses used rather than to the type of formula.

When the American Society for Testing Materials raised the requirement for tensile strength and yield point for structural steel for buildings (see Art. 4), many design specifications naturally took the higher strength steel into account. The 1936 and 1946 revisions of the A.I.S.C. Specifications use the parabolic formula

$$f = 17{,}000 - 0.485 \frac{L^2}{r^2} \quad \text{or} \quad 17{,}000 - 0.485 \left(\frac{L}{r}\right)^2$$

instead of the former one of the Rankine type, for slenderness ratios up to 120. This is the range through which the increased strength of the material is effective. For slenderness ratios above 120 (for secondary members), where stiffness rather than strength is the controlling factor, the original A.I.S.C. formula was retained. However, Sec. 16(*b*) of the 1946 revision permits the use of main members with L/r values between 120 and 200 under certain conditions. (Consult the A.I.S.C. Manual, 5th edition.)

72. Investigation of Columns. — To determine the safe axial load that a column will carry according to any particular specification, it is usually convenient first to compute the slenderness ratio. This value is then substituted in the appropriate column formula, and the allowable average stress determined. The safe load on the column will be equal to the allowable average stress times the area of the column section.

EXAMPLE

Determine the total safe load on a 12-in. 72-lb. Wide Flange section used as a column on an unstayed height of 14 ft. The A.I.S.C. Specification (1946) controls.

Solution:

(1) From a table of properties or elements of sections given in a steel handbook, the area and radii of gyration of this section are found to be

$$A = 21.16 \text{ sq. in.} \qquad r_{xx} = 5.31 \qquad r_{yy} = 3.04$$

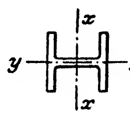


FIG. 78.

(2) The slenderness ratio is $\dfrac{L}{r} = \dfrac{14 \times 12}{3.04} = 55.3$

(3) Since the above value is less than 120, the allowable average stress according to the A.I.S.C. Specification (1946) is

$$f = 17{,}000 - 0.485 \left(\frac{L}{r}\right)^2 = 17{,}000 - 0.485 \times (55.3)^2 = 15{,}520 \text{ lb. per sq. in.}$$

(4) The allowable load on the column is equal to the allowable average stress times the area, or

$$P = fA = 15{,}520 \times 21.16 = 328{,}000 \text{ lb. or } 328 \text{ kips}$$

In order to expedite design, tables are usually computed giving values of the allowable stress for different values of the slenderness ratio, for the particular column formula in use. Such a table for the A.I.S.C. (1946) formulas will be found in the fifth edition of the A.I.S.C. Manual. A table for the 1928 A.I.S.C. formula is given in the second edition of the Manual and in other handbooks. Similar tables may be made up for any formula, or graphs similar to Fig. 77 may be constructed at a suitable scale and the allowable stresses determined therefrom, without having to solve the formula each time.

73. Built-up Sections. — In the example of Art. 72 the column used was a rolled Wide Flange or H-section such as that shown in Fig. 76*a*. Sections of this type comprise the vast majority of columns encountered in steel building construction. It is sometimes necessary, however, to reinforce ordinary rolled sections with plates as shown in Fig. 76*b*, and conditions may arise making desirable a section built up entirely of plates and angles. The strength of such columns is investigated by the same procedure as that outlined in Art. 72. Tables giving properties of built-up sections of many types are now published in the steel handbooks, so that the area and radii of gyration may be looked up as easily as for the rolled shapes. When such tables are not available, the moment of inertia of the built-up section must be determined in the same manner as for reinforced beams and plate girders except that the gross rather than the net section is used. (See Arts. 52 and 53.) It is usually necessary to find the moment of inertia about *both* principal axes of the section and then compute the least radius of gyration by substituting the least I in the equation $r = \sqrt{I/A}$. (See Art. 70.) With the least radius of gyration and the area determined, the methods of Art. 72 apply.

PROBLEMS

1. Find the allowable load on an 8-inch., 40-lb. Wide Flange section used as a column with an unstayed height of 18 ft., if the allowable stress is controlled by formula C, Fig. 77. (Answer given in Appendix E.)

2. Find the allowable load on a 12-in., 65-lb. Wide Flange section used as a column with an unstayed height of 22 ft., if the allowable stress is controlled by formula B, Fig. 77.

3. Determine the allowable unit compressive stress on the columns of Problems 1 and 2 above in accordance with the A.I.S.C. Specification

(1946). Obtain the values by solving the appropriate column formula, and check these results by using the corresponding graph in Fig. 77.

4. Find the allowable load on a 5-in., 10-lb. American Standard *I*-beam used as a column on an unstayed height of 6 ft., if the allowable stress is controlled by formula *C*, Fig. 77.

5. Find the allowable unit compressive stress and allowable load on a 6-in., 25-lb. Wide Flange section used as a column on an unstayed height of 12 ft., if the allowable stress is controlled by the A.I.S.C. Specification (1946).

6. A built-up plate and angle column similar to Fig. 76*d* is used on an unstayed height of 20 ft. It is composed of a web plate $12 \times \frac{1}{2}$ in., 4 angles $6 \times 4 \times \frac{9}{16}$ in., and two cover plates $13 \times \frac{5}{16}$ in. Each pair of angles is set $\frac{1}{4}$ in. beyond the edge of the web plate. If the A.I.S.C. Specification (1946) controls, find the allowable unit compressive stress and the allowable load.

74. Column Design. — The design of columns is an indirect process. The length of the column and the load it is to support are given; the designer must select a rolled shape or built-up section that will carry the load without having the average stress exceed that given in the specifications. There are two unknowns, A and r, and the value of one cannot be computed without knowing the other. Hence, the design must be made by trial and error. A trial section is assumed and then investigated by the method described in Art. 72. If the allowable load that this trial section is found to carry is less than the required load or if it is enough larger so that the section would be uneconomical, another trial section is assumed, using the first as a guide. This process is repeated until a satisfactory section is found. The steel handbooks contain tables of safe loads for different column sections of different lengths. These tables are a great aid in design but they should not be used until the underlying principles are understood. It should be borne in mind that such tables can be used directly only when they have been computed by the formula laid down in the specifications controlling the design. However, a set of safe load tables may frequently be employed as a guide for selecting the trial section even though they are not made up from the specifications called for by the design. Different handbooks have special arrangements of the safe load tables, but a short study of any of them will reveal their method of use.

75. Column Design Procedure. — The steps necessary in the design of a column, when safe load tables are not used, may be summarized in the following procedure.

(1) Assume a trial section. (The load and unstayed height are known.)

(2) Find the area and least radius of gyration of the trial section from a table of elements of sections in a steel handbook.

(3) Compute the slenderness ratio, L/r.

(4) Compute the allowable average stress by the formula given in the specifications.

(5) Compute the allowable load on the column by multiplying the stress found in Step (4) by the area of the section.

(6) Compare the allowable load as found in Step (5) with the required load. If the result is unsatisfactory, repeat the work using a new trial section.

76. Design and Investigation of Struts. — The lighter compression members of roof trusses and other types of struts are designed in the same manner as columns. Figure 76*f*, *g*, and *h* illustrates the more common types of cross sections. Figure 76*f* shows a strut composed of two angles riveted together with their legs in contact. In the type shown by *g*, the angles are separated at intervals by ring fillers and "stitch rivets," spaced not to exceed two feet on centers. A single angle strut is shown at *h*.

EXAMPLE

A strut 9 ft. long is composed of two $5 \times 3\frac{1}{2} \times \frac{1}{2}$-in. angles. The 5-in. legs of the angles are spaced $\frac{3}{8}$ in. back to back as shown in Fig. 79. Find the allowable load on the strut if the average stress is controlled by the A.I.S.C. Specification (1946).

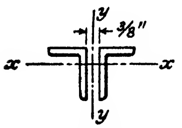


FIG. 79.

Solution:

(1) From a table in the steel handbooks giving the "properties of members composed of two angles,"

$A = 8.0$ sq. in. $\quad r_{xx} = 1.58 \quad r_{yy} = 1.49$

(2) The slenderness ratio is $\dfrac{L}{r} = \dfrac{9 \times 12}{1.49} = 72.5$

(3) The allowable average stress is

$$f = 17{,}000 - 0.485\left(\frac{L}{r}\right)^2 = 17{,}000 - 0.485 \times (72.5)^2 = 14{,}500 \text{ lb. per sq. in.}$$

(4) The allowable load on the strut is equal to the average stress times

the area or

$$P = fA = 14{,}500 \times 8.0 = 116{,}000 \text{ lb. or } 116 \text{ kips}$$

In the design of single angle struts it should be noted that the least radius of gyration of a single angle section is about a diagonal axis (see axis z–z, Table III).

The design of struts composed of two angles is greatly facilitated by safe load tables such as Table X of Appendix D, based on the A.I.S.C. Specification (1928), and those in the fifth edition of the A.I.S.C. Manual, based on the 1946 revised specification. Two-angle compression members will be discussed more fully under the design of roof trusses.

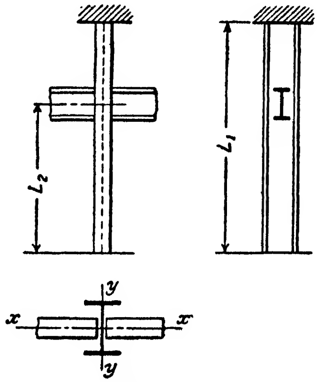


FIG. 80.

77. Unstayed Height. — In multistory, steel frame buildings, the unstayed height of a column is usually taken as the distance from floor to floor. It occasionally happens, however, that a column is braced against deflection in one direction at smaller intervals than in the other, as shown in Fig. 80. It is desirable in such cases to place the column so that the axis having the smaller radius of gyration will be braced by the closer supports. The slenderness ratio used in the design of the column in the figure is $\frac{L_1}{r_{xx}}$ or $\frac{L_2}{r_{yy}}$, whichever is greater.

78. Loads on Columns. — The load carried by a column in any story of a building is made up of (1) the live and dead load on the floor immediately over the column being designed; (2) the load transmitted by the column above; (3) the weight of the column itself. The portion of the column load contributed by the floor that it supports is found by adding together the reactions of the beams and girders framing into it, or by multiplying the area of the supported bay by the live and dead load per square foot of floor. The design load for a column is taken as the total load at the base.

Under certain conditions, when the character of the occupancy is such that all floors in the building will not be subjected simultaneously to the full live load, most building codes permit 5 per cent reduction of the live load on each floor below the top floor, for buildings more than five stories high, provided that the total reduction is not more than 50 per cent of the live load. It should be clearly understood that this reduction is applied only in computing the live load carried by the columns. The beams and girders of each floor are, of course, designed to take the full load.

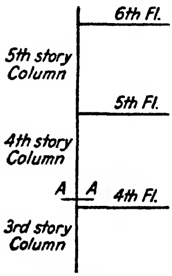


FIG. 81.

Where this reduction is permitted, the live load on the floor below the top floor may be assumed as 95 per cent of that for which the floor is designed. On the next lower floor 90 per cent is used, and correspondingly decreasing percentages for each succeeding lower floor until a 50 per cent reduction is reached. This percentage is used for each of the remaining floors. The columns supporting the top floor and roof are designed to take the full live load.

Columns are designated by the story through which they run. For example, a column between the fourth and fifth floors of a building, supporting the fifth floor, is known as a fourth story column. The design load on a fourth story column is the load at its base just above the fourth floor, as shown by the line A–A in Fig. 81.

EXAMPLE

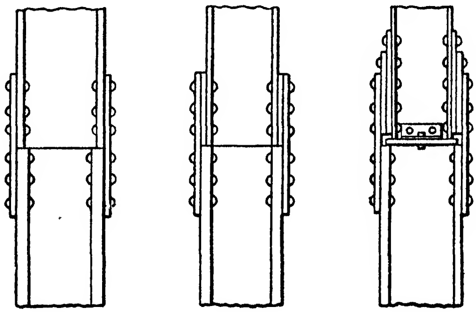
A steel frame building is laid out in bays 20 ft. square. The live load is 70 lb. per sq. ft., and the dead load, including an allowance for the weight of beams, girders, and columns, is 60 lb. per sq. ft. The building comprises eight floors and roof. The live load on the roof is 40 lb. per sq. ft., and the dead load 50 lb. per sq. ft. Determine the design load on a typical fifth story column using the allowable 5 per cent reduction in live load.

Solution:

Live and Dead Load Contributed by Each Floor in Pounds			**Design Load in Pounds**	
Roof	$LL = 400 \times 40 =$	16,000		
	$DL = 400 \times 50 =$	20,000		
		36,000........	36,000	8th Story Column
8th Fl.	$LL = 400 \times 70 =$	28,000		
	$DL = 400 \times 60 =$	24,000		
		52,000........	88,000	7th Story Column

Live and Dead Load Contributed by Each Floor in Pounds			**Design Load in Pounds**	
7th Fl.	$LL = 28{,}000 \times 0.95$	$= 26{,}600$		
	$DL = 400 \times 60$	$= \underline{24{,}000}$		
		50,600.	138,600	6th Story Column
6th Fl.	$LL = 28\,000 \times 0.90$	$= 25{,}200$		
	$DL = 400 \times 60$	$= \underline{24{,}000}$		
		49,200. .	187,800	5th Story Column

79. Column Splices. — Columns are usually spliced about 1 ft. 6 in. above the floor level, the detail of the splice depending upon the relative sizes of the members to be connected. In building construction it is usual practice to mill the ends of the columns since most of the stress is transmitted by direct bearing. When



(a) (b) (c)

FIG. 82. Typical Riveted Column Splices.

the entire cross-sectional area of the upper column has full bearing on the one below it, the splice plates are used merely to hold the columns in position or to transmit bending stresses (see Fig. 82*a* and *b*). Where full bearing cannot be secured, special splices similar to that shown in Fig. 82*c* must be designed. Welded column splices are discussed in Chapter IX.

PROBLEMS

1. Design a 16-ft. Wide Flange column to support a total load of 290,000 lb. The allowable average stress is that given by the A.I.S.C. Specification (1946). (Answer given in Appendix E.)

2. Design a 14-ft. column to support a total load of 250,000 lb. The allowable average stress is that given by the A.I.S.C. Specification (1928). Use a Wide Flange section.

3. A 12-ft. column is to support a total load of 280 kips. Using the

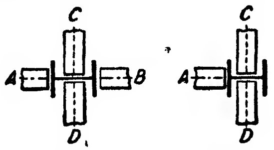
A.I S.C. (1946) formula, find the lightest-weight 8-in, 10-in, and 12-in. rolled column sections that might be used.

4. A strut 6 ft. long is composed of two $5 \times 3 \times \frac{3}{8}$-in. angles placed with the long legs $\frac{3}{8}$ in. back to back. Find the total safe load using the A.I.S.C. Specification (1946). What is th safe load if the long legs are placed in contact?

5. A 10-in. 24.5-lb. Standard I-beam is used as a column 12 ft. long. Five feet from the top it is braced by intermediate beams in a direction perpendicular to its web. Using the A.I.S.C. (1946) formula, find the total safe load on the column. Make a sketch showing the arrangement.

6. A steel frame building has ten floors and roof. A typical interior bay is 24 ft square. The live load on a typical floor is 80 lb. per sq. ft., and the dead load, including an allowance for beams and girders, is 65 lb. per sq. ft. The columns are to be designed with the usual 5 per cent reduction in live load. Find the design load for the sixth story column. (Neglect the weight of the column.) Roof live and dead load is 110 lb. per sq. ft.

80. Columns with Eccentric Loads. — So far in the discussion of columns all the loads have been concentric, that is, applied along the axis of the column. This condition exists when the load is applied uniformly over the top of the column or when



(a) (b)

FIG. 83.

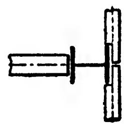


FIG. 84.

beams having equal reactions frame into the column opposite each other as shown in Fig. 83*a*. Beams *A* and *B* in the figure have equal reactions, and so do *C* and *D*. If beam *B* were omitted as shown in Fig. 83*b* (or if the reaction of *B* were considerably less than that of *A*), it is evident that the loads on the column would no longer be symmetrical and that the column flange on the left would be subjected to a greater unit stress than the one on the right. This condition occurs frequently in the wall columns of buildings where a floor beam is supported on the interior face without a corresponding load opposite it. Figure 84 illustrates a method of framing which may be used to balance the loads when the total reaction of the two spandrel beams is nearly the same as that of the floor beam.

In practice, when a column receives a floor beam on one side only (Fig. 83*b*), the resulting eccentricity is usually neglected if the beam frames directly to the column flange. If the reaction of such a beam is exceptionally heavy, however, or if beams are supported on brackets (Fig. 61), the stresses induced by the eccentric loading must be considered.

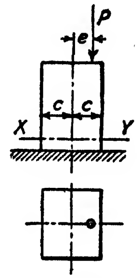


FIG. 85.

Figure 85 represents a rectangular block with the load P eccentrically applied. The distance e is called the eccentricity, and c is the distance from the axis of the block to the extreme fibres.The stress on any cross section of the block such as $X-Y$ may be considered the sum of the average stress $\frac{P}{A}$, which is the load divided by the area of the block, and a stress caused by the moment Pe. To the right of the axis of the block, that is, on the same side as P, this moment causes a compressive stress on the section and to the left of the axis a tensile stress, in accordance with the ordinary theory of bending. The unit stress at Y is equal to the average stress $\frac{P}{A}$ plus the extreme fibre stress $\frac{Mc}{I}$ caused by the moment Pe. Substituting Pe for M, the intensity of stress at Y is expressed by the formula

$$f_Y = \frac{P}{A} + \frac{Pec}{I}$$

and the intensity of stress at X is

$$f_X = \frac{P}{A} - \frac{Pec}{I}$$

The above expressions are applicable to sections symmetrical about two axes such as rectangles, I-sections, and H-sections. They may be stated in the general form

$$f = \frac{P}{A} \pm \frac{Pec}{I}$$

in which f is the unit stress at either edge of the section depending on whether the plus or minus sign is used and I is the moment of inertia in the direction of the eccentricity.

In the design of eccentrically loaded columns the maximum compression is usually the stress wanted. It is seldom that the stress due to the moment Pe causes enough tension on the far edge of the column to counteract the compression caused by the direct stress $\frac{P}{A}$. Where this does occur it is of importance only when the column is to be spliced.

In building work, columns usually carry a direct axial load in addition to any eccentric ones. Where such a condition exists a more convenient form of the expression is

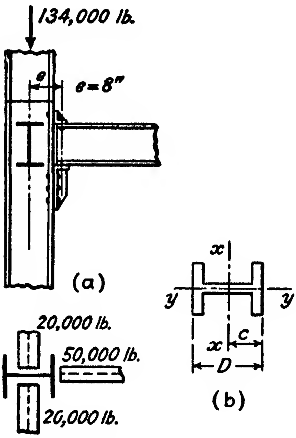


FIG. 86.

$$f = \frac{P}{A} + \frac{P'ec}{I}$$

in which P is the total vertical load *including the eccentric load,* and P' is the eccentric load alone. The investigation of a column carrying an eccentric load is illustrated by the following example.

EXAMPLE

A 10-in., 72-lb. Wide Flange column 13 ft. long supports the loads shown in Fig. 86. The center of bearing of the 50,000-lb. load is assumed to be 8 in. from the axis of the column. Neglecting the weight of the column, determine the maximum unit compressive stress.

Solution:

(1) The *total* vertical load $P = 134{,}000 + 20{,}000 + 20{,}000 + 50{,}000 = 224{,}000$ lb. The eccentric load $P' = 50{,}000$ lb.

(2) The following properties of the 10-in., 72-lb. Wide Flange section are found from a steel handbook:

$I_{xx} = 420.7$	$r_{xx} = 4.46$	$D = 10.5$	$A = 21.18$ sq. in.
$I_{yy} = 141.8$	$r_{yy} = 2.59$	$c = 5.25$	

(3) The maximum value of the unit compressive stress is

$$f = \frac{P}{A} + \frac{P'ec}{I} = \frac{224{,}000}{21.18} + \frac{50{,}000 \times 8 \times 5.2}{420.7}$$

$$= 10{,}600 + 4940 = 15{,}540 \text{ lb. per sq. in.}$$

To ascertain whether or not the column is overstressed, it will be necessary to determine the *allowable* unit stress on the column under these conditions of loading. The determination of this allowable stress is discussed in the following article.

81. Allowable Combined Stresses. — Some specifications have required that the maximum compressive stress in a column subjected to combined axial and bending stresses be limited to the stress that would be permitted if only axial stress existed, that is, to the stress given directly by the column formula controlling the design. Applying this requirement to the column investigated in the preceding article and assuming that the A.I.S.C. (1928) column formula controls, the allowable stress is determined as follows:

(1) The slenderness ratio is

$$\frac{L}{r} = \frac{13 \times 12}{2.59} = \frac{156}{2.59} = 60.2$$

(2) The allowable unit stress is

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\left(\frac{L}{r}\right)^2} = \frac{18{,}000}{1 + \frac{(60.2)^2}{18{,}000}} = 14{,}980 \text{ lb. per sq. in.}$$

(3) Comparison of this value with the actual stress developed, as computed in Step (3) of the example in Art. 80, indicates that the column is overstressed.

It will be noted in the foregoing operations that the value of r used to determine the allowable stress is the *least* radius of gyration for the section, that is r_{yy}, whereas the bending due to eccentricity takes place about axis x–x. This is an approximate method that has been widely used.

SECOND METHOD

A more precise method, which takes into account the allowable bending stress as well as the allowable axial stress permitted by the controlling column formula, is described below. Let

F_a = allowable axial unit stress;
F_b = allowable bending unit stress;
f_a = actual axial unit stress developed;
f_b = actual bending unit stress developed.

Additional notation conforms to that used in Art. 80. It should also be noted from the definition of radius of gyration (Art. 70) that moment of inertia may be expressed as $I = Ar^2$. Then, when *investigating* an eccentrically loaded column,

$$f_a = \frac{P}{A} \quad \text{and} \quad f_b = \frac{P'ec}{I} = \frac{P'ec}{Ar^2}$$

In *design*, if a column is subjected to *axial* loading only, the required cross-sectional area (A_a) will be

$$A_a = \frac{P}{F_a}$$

If a column is subjected to *bending* action only, the required cross-sectional area (A_b) will be

$$A_b = \frac{P'ec}{F_b r^2}$$

If a column is subjected to *both* axial loading and bending action, the total required area becomes

$$(A_a + A_b) = A = \frac{P}{F_a} + \frac{P'ec}{F_b r^2}$$

Dividing both sides of the equation by A,

$$1 = \frac{P}{F_a A} + \frac{P'ec}{F_b A r^2}$$

This may be written

$$\frac{P/A}{F_a} + \frac{P'ec/Ar^2}{F_b} = 1$$

or, substituting the symbols for actual unit stresses,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

This expression indicates that the column will not be overstressed if the section is so proportioned that the *actual* axial and bending unit stresses developed by the loading, when substituted in the above expression, result in a value equal to or less than unity.

Using this method and the A.I.S.C. (1928) formula, the column investigated in Art. 80 will be tested to determine whether the section is overstressed. Step (3) of the example is repeated below, and subsequent operations are given step numbers in the same sequence.

(3) The maximum value of the unit compressive stress is

$$f = \frac{P}{A} + \frac{P'ec}{I} = 10{,}600 + 4940 = 15{,}540 \text{ lb. per sq. in.}$$

(4) Therefore, the actual unit stresses being developed are

$$f_a = 10{,}600 \text{ lb. per sq. in.} \qquad f_b = 4940 \text{ lb per sq. in.}$$

(5) The allowable axial stress is found as before from the A.I.S.C. (1928) formula, using the least r of the section.

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}\left(\frac{L}{r}\right)^2} = \frac{18{,}000}{1 + \frac{(60.2)^2}{18{,}000}} = 14{,}980 \text{ lb. per sq. in.}$$

or, using the notation developed above, $F_a = 14{,}980$ lb. per sq. in.

(6) The allowable bending stress according to this same specification is 18,000 lb. per sq. in., provided the ratio of unbraced length to flange width (L/b) does not exceed 15 (see Art. 32). The actual flange width of this 10-in., 72-lb. section is 10.17 in. (found from a steel handbook), making $L/b = (13 \times 12)/10.17 = 15.35$. Therefore, it is necessary to reduce the allowable stress below 18,000 lb. per sq. in. in accordance with the first formula given in Art. 32. Solving this formula,[2]

$$f = \frac{20{,}000}{1 + \frac{L^2}{2000b^2}} = \frac{20{,}000}{1 + \frac{(15.35)^2}{2000}} = 17{,}900 \text{ lb. per sq. in.}$$

or, using the notation developed above, $F_b = 17{,}900$ lb. per sq. in.

(7) Testing the stress ratios,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{10{,}600}{14{,}980} + \frac{4940}{17{,}900} = .708 + .276 = .984$$

Since this value is less than 1, the column is not overstressed.

It should be noted that this conclusion is not in agreement with that reached in Step (3) of the first method for determining allowable combined stresses given at the beginning of this article. It is to be expected that this second (more precise) method will frequently permit the employment of lighter column sections.

82. Equivalent Concentric Load Method. — A common method for determining the unit bending stress in columns supporting eccentric loads is to consider the effect of eccentricity in terms of an equivalent concentric load. That is, the eccentric load is replaced by a concentric load of sufficient magnitude to give the same maximum stress as that produced by the eccentric one. If W_q is the equivalent concentric load and A the cross-sectional area of the column, the unit stress produced by W_q may be set

[2] Tables or graphs for flange-stress reduction formulas and the regular column formulas should be used for obtaining allowable stresses in all practical work. Such tables are contained in the steel handbooks. Care must be exercised, however, to see that the table or graph is based on the specifications controlling the design.

equal to the unit bending stress produced by the eccentric load, or

$$\frac{W_q}{A} = \frac{P'ec}{I} \quad \text{and} \quad W_q = \frac{P'ecA}{I}$$

Dividing both numerator and denominator of the right-hand member by c and noting that I/c is the section modulus,

$$W_q = \frac{P'eA}{I/c} = \frac{P'eA}{S}$$

In the three foregoing equations I is the moment of inertia in the direction of the eccentricity, that is, I for the axis about which the load is eccentric. S is the corresponding section modulus.

The ratio A/S is called the *bending factor* and will be found tabulated in the steel handbooks. For convenience, bending factors are usually listed in connection with column safe load tables rather than with the other "properties of sections." Their values, however, are in no way dependent on the column formula used in computing the tables. If A/S is denoted by B, the expression for equivalent concentric load becomes

$$W_q = P'eB$$

where B is the bending factor in the direction of the eccentricity. Applying this method to Step (3) of the example in Art. 80,

$$W_q = P'eB = 50{,}000 \times 8 \times 0.264 = 105{,}600 \text{ lb.}$$

$$f = \frac{P + W_q}{A} = \frac{224{,}000 + 105{,}600}{21.17} = 15{,}560 \text{ lb. per sq. in.}$$

This agrees with the result previously obtained (within the limits of slide rule accuracy).

The equivalent concentric load method greatly facilitates design when safe load tables are employed. It should be noted that the effect of an eccentric load is assumed to disappear at each story height. This method of designing eccentrically loaded columns extending through several stories is discussed in Art. 159.

83. Steel Slab Column Bases. — Rolled steel slabs are used to distribute the load at the base of a column over the concrete footing. The bottom of the column and the surface of the plate under the column are planed so that the load is transmitted to the plate by direct bearing. The underside of the plate is not planed but rests on cement grout from 1 to 2 in. thick laid on top of the concrete. Angles are used to fasten the column to the plate by means of anchor bolts (Fig. 87).

The required area of the slab is found by dividing the column load by the allowable unit bearing pressure on the masonry. It is assumed that the bearing pressure on the bottom of the slab is of uniform intensity and that the load from the column is uniformly distributed over a rectangular area on the top of the slab. The dimensions of this "equivalent rectangle" are taken as 0.95 of the column depth and 0.8 of the column flange width. The portion of the slab projecting beyond the sides of the equivalent rectangle acts as an inverted cantilever with the maximum bend-

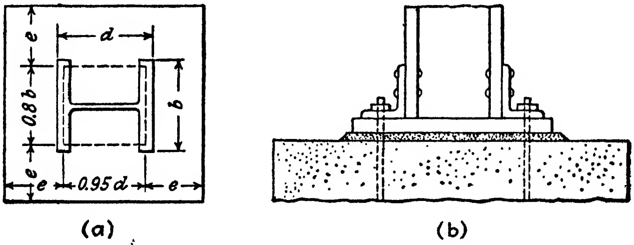


FIG. 87. Steel Slab Column Base.

ing moment occurring at that side for which the overhand, e, is greater (see Fig. 87a). This is not an exact method of analysis but the results obtained by its use have proved satisfactory in practice. The following notation is used in the derivation of the formulas.

P = total load on the column in pounds;
L = length of slab in inches;
K = width of slab in inches;
d = depth of the column section in inches;
b = width of column flange in inches;
A = area of the slab in square inches;
u = unit bearing pressure on the underside of the slab (i.e., the allowable pressure on the masonry);
t = thickness of the slab in inches;
f = allowable extreme fibre stress in bending.

The required area of the slab is $A = P/u$. The dimensions L and K are chosen so as to give this area. If a square footing is used, L and K should be equal.

Considering a strip of slab 1 in. wide, the bending moment on the strip at the edge of the equivalent rectangle is

$$M = u \times e \times \frac{e}{2} = \frac{ue^2}{2}$$

The section modulus is $$S = \frac{M}{f} = \frac{ue^2}{2f}$$

But the section modulus is also equal to I/c and the moment of inertia of a rectangle 1 in. wide and t inches thick is

$$I = \frac{1 \times t^3}{12} = \frac{t^3}{12}$$

The value of c is $t/2$; hence,

$$S = \frac{I}{c} = \frac{t^3}{12} \div \frac{t}{2} = \frac{2t^3}{12t} = \frac{t^2}{6}$$

Equating I/c to M/f, $$\frac{t^2}{6} = \frac{ue^2}{2f}$$

$$t^2 = \frac{6ue^2}{2f} = \frac{3ue^2}{f}$$

Therefore, the required thickness is

$$t = \sqrt{\frac{3ue^2}{f}}$$

84. Design of Steel Slab Column Bases. — The operations involved in the design of steel slab column bases are illustrated by the following example.

Design a rolled steel slab base for a 14-in., 87-lb. Wide Flange column that supports a load of 383,000 lb. The allowable unit bearing pressure on the concrete footing is 500 lb. per sq. in. and the allowable extreme fibre stress in the slab is 20,000 lb. per sq. in.

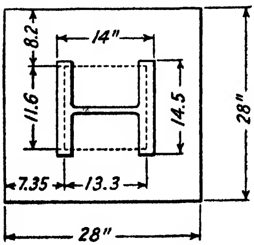


FIG. 88.

Solution:

(1) The required area is $A = \frac{P}{u} = \frac{383{,}000}{500} = 766$ sq. in.

(2) This area requires a slab 27.7 in. square. Working to the nearest inch, a 28 × 28 in. slab is selected.

(3) The values of d and b for the column section are obtained from a handbook, and a sketch is made as shown in Fig. 88. The larger value of e is found to be 8.2 in.

(4) $t = \sqrt{\dfrac{3ue^2}{f}} = \sqrt{\dfrac{3 \times 500 \times 8.2 \times 8.2}{20{,}000}} = \sqrt{5.05} = 2.24$ in.

(5) If the thickness is taken to the next greater $\frac{1}{4}$ in., the finished dimensions of the required slab are $28 \times 28 \times 2\frac{1}{4}$ in.

85. Grillage Foundations. — Steel grillages are sometimes used to distribute column loads over the foundation bed although, at present, reinforced concrete footings are more commonly employed for this purpose except in cases where extremely heavy loads are supported on rock. Grillage foundations are made up of one or more layers of steel beams arranged as shown in Fig. 89, and the entire grillage is encased in concrete. Pipe separators are placed near the ends of the beams and under points where concentrated loads occur.

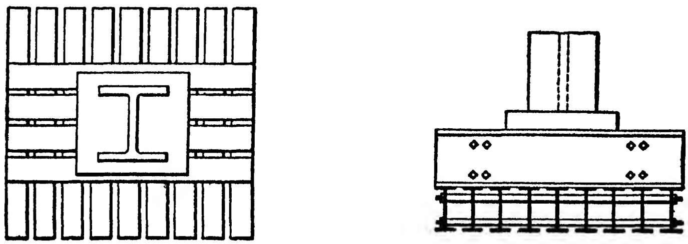


FIG. 89. Grillage Foundation.

Sufficient space is left between the flanges of the beams in any one tier to permit proper tamping of the concrete. The unit bearing pressure on the bottom of the grillage is limited to 500 or 600 lb. per sq. in. inasmuch as a thin concrete mat is usually placed under the grillage even where the supporting material is bedrock. The design of steel grillages and the analysis of stresses in grillage beams will be found in other books on structural design.

PROBLEMS

1. A 12-in. 72-lb. Wide Flange column section is used on an unstayed height of 13 ft. The column is to support an axial load of 250,000 lb. and an additional load of 40,000 lb. carried on a bracket. The distance from the *flange face* to the center of bearing of the eccentric load is 12 in. Using the A.I.S.C. Specification (1928):

(*a*) Determine whether the column is overstressed.

(*b*) If it is overstressed, select another 12-in. section that will carry the load safely.

2. Using the data given in Problem 1, determine whether the column is overstressed according to the A.I.S.C. Specification (1946).

3. An 8-in. 40-lb. Wide Flange section is used on an unstayed height of 12 ft. The only load to be supported is the 60,000-lb. reaction of a beam framing to one flange at the top of the column. The connection is so arranged that the center of bearing of the beam is 5 in. from the *axis* of the column. Determine whether the column is overstressed ccording to the A.I.S.C. Specification (1946).

4. An 8 × 8 WF 40 column with an unstayed length of 11 ft. 4 in. supports an axial load of 104 kips and an *eccentric* load of 9 kips applied 12 in. from the column axis. The column is turned so that bending is about the strong axis. Using A.I.S.C. Specification (1946) for combined stress, check whether the column is safe as loaded.

5. An 8 × 10 WF 41 column carries a load of 122 kips. A base plate 16 in. × 16 in. is to be used. Determine the thickness of the base plate required using A.I.S.C. Specification (1946).

CHAPTER VII

ROOF TRUSSES

86. Introduction. — A complete roof structure consists of the roof deck, the system of rafters and purlins which support the deck and the roof trusses which carry the purlins and span the distance between supporting walls or columns. The area of the roof between two trusses is called a bay.

Purlins are the horizontal beams which span the distance between trusses. In some types of roofs the deck is supported directly on the purlins while in others rafters or subpurlins, running parallel to the trusses, carry the deck and these in turn are supported by the purlins.

Roof trusses are generally made up of an inclined top chord, a horizontal lower chord and a web system. The lower chord may also be inclined or "cambered" in order to increase the headroom under the truss. The joint at the top of the truss supporting the ridge is called the peak and the joint at either end is the heel. The joints at which the web members intersect the chords are called the panel points and the distance between these joints is a panel length. The plates by means of which the members are connected to each other are called gusset plates.

The rise or height of a truss is the vertical distance from the peak to the horizontal lower chord. In trusses with cambered lower chords the rise is measured from the peak to a horizontal line passing through both supports.

The pitch is the ratio of the rise to the span length. For example, a truss having a rise of 10 ft. and a span of 40 ft. has a pitch of $\frac{10}{40}$ or $\frac{1}{4}$. This should not be confused with the slope which is the ratio of the rise to one-half of the span. In the case mentioned above the slope is $\frac{10}{20}$ or $\frac{1}{2}$. These terms are often used synonymously especially by men in the field. A common way of expressing the slope of a roof is to give the amount of rise in a foot of run. For example, a slope of $\frac{1}{2}$ is the same as a rise of 6 inches in 12 inches or a slope of "6 in 12." The slope, or pitch of a roof, is usually determined by architectural considerations but in the absence of any governing factor a rise equal to one-quarter of the span will generally prove economical.

87. Types of Roof Trusses. — There are a great many forms of steel trusses used in building construction. One of the most

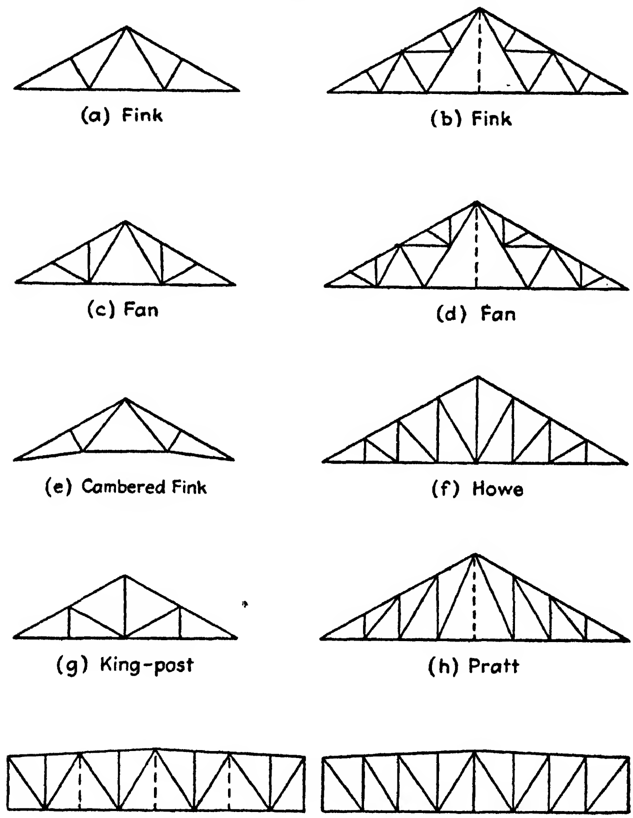


FIG. 90. Types of Roof Trusses.

common types for spans of ordinary length is the Fink truss shown in Fig. 90 (*a* and *b*). To lay out the Fink truss shown in (*a*), a perpendicular to the top chord is erected at its center and extended until it meets the lower chord. The remaining two web members are drawn in as shown. The dotted line at the center indicates the position of a sag rod often added for the sake of stiffness but which theoretically carries no stress. The number of panels in half the truss may be increased to 4 by sub-dividing each panel into 2 panels as shown in Fig. 90*b*. This process may be repeated again, resulting in 8 panels. One of the disadvantages of this type of truss is that the number of panels can be increased only by doubling the previous number. A modification of the Fink truss that permits greater flexibility in the number of panels is the fan truss shown in Fig. 90 (*c* and *d*). In this type the web members do not intersect the top chord at right angles. Figure 90 also illustrates several other types of roof trusses frequently encountered.

Where trusses support purlins at the panel points only, a desirable length for the panels of the top chord is about 8 ft. This limitation corresponds to a maximum span of about 30 ft. for the truss shown at (*a*) in Fig. 90; 40 ft. for that shown at (*c*); 55 ft. for (*b*); and 75 ft. for (*d*). These maximum spans are, of course, only approximate and may be varied to meet special conditions.

88. Spacing of Trusses. — The most economical spacing of trusses insofar as the roof structure as a whole is concerned depends upon so many factors that a simple rule for finding it cannot be given. It is usually desirable, however, to have bays of approximately equal lengths over any one portion or wing of a building so that as many trusses as possible may be identical. For spans up to about 50 ft. a spacing of 15 to 18 ft. is satisfactory for loads in the neighborhood of 30 lb. per sq. ft. In a great many cases the distance between trusses is determined by the location of windows, piers, etc., without regard to the economical spacing.

89. Loads on Roof Trusses. — The dead load carried by a roof truss consists of the weight of the roof deck and covering, the rafters and purlins, and the weight of the truss itself.[1] The live load is made up of the probable wind and snow loads.

All of the loads mentioned above (except that due to its own weight) are brought to the truss by the purlins. There is usually a purlin connection at every panel point of the top chord, and

[1] Trusses frequently support suspended ceilings and sometimes suspended floors. Ceilings may be hung from the purlins or from a light framework supported on the lower chord. In the former case the weight of the ceiling reaches the truss through the purlins, but in the latter the loads are considered as applied at the lower-chord panel points.

frequently one or more purlins are supported between panel points. In the first case, the top chord is an ordinary compression member, while in the second there is bending combined with the direct stress. For the present the discussion will be confined to trusses receiving loads at the panel points only.

The area of roof surface supported by one purlin is equal to the panel length (distance between purlins) multiplied by the length of a bay (distance between trusses). Tables giving the weights of various building materials used for the roofing and deck will be found in most handbooks.

The weight of roof trusses for spans less than about 75 ft. is usually a small part of the total load to be carried. All of the load except the truss weight can be determined directly from the specifications and the layout of the roof. The weight of the truss for ordinary spans and bay lengths can be approximated by using a value of 8 to 12 per cent of the load to be carried. This load is distributed among the panel points of the top chord. After the truss has been designed the actual weight should be computed and compared with the assumed weight. If the two are not in reasonable agreement, a new weight should be assumed, using the weight of the first truss as a guide, and the design repeated.

90. The Snow Load. — The snow load to be carried by a roof depends upon the geographical location of the structure and the roof slope. On this continent, the greater snow falls occur in

ALLOWANCE FOR SNOW LOADS IN POUNDS PER SQUARE FOOT OF ROOF SURFACE

Location	Pitch	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
	Slope	12 in 12	8 in 12	6 in 12	4.8 in 12	4 in 12
		* †	* †	* †		
Southern States and Pacific Slope		0–0	0–5	0–5	5	5
Central States		0–5	7–10	15–20	22	30
Rocky Mountain States		0–10	10–15	20–25	27	35
New England States		0–10	10–15	20–25	35	40
North West States		0–12	12–18	25–30	37	45

* For slate, tile or metal roofs. † For shingle roofs.

northern latitudes and high altitudes. Snow tends to slide off of steep roofs but may accumulate on relatively flat ones. Freshly fallen snow may weigh as much as 10 lb. per cu. ft. and when it becomes packed or wet the weight increases. The amount of snow retained on a roof also depends upon the type of roof covering. For example, snow slides off of a metal or slate roof more readily than from a wood shingle surface. The accompanying table gives snow loads for various locations and roof slopes.

91. The Wind Load. — When determining the wind pressure on the inclined surface of a roof, it is customary to consider only the pressure acting in a direction normal (perpendicular) to the surface. The friction of the air on comparatively smooth surfaces is so small that the component of the wind pressure parallel to the roof may be neglected. The direction of the wind is assumed horizontal but it should be noted that the normal pressure cannot be obtained by resolving the horizontal pressure into components normal and parallel to the roof surface.

The normal pressure is calculated from formulas which have been derived from experiments. One of the most common formulas is that based upon the investigations of a French army officer by the name of Duchemin. The Duchemin formula is

$$P_n = P \times \frac{2 \sin A}{1 + \sin^2 A}$$

in which P is the unit pressure in pounds per square foot on a vertical surface; P_n is the normal pressure in pounds per sq. ft.; and A is the angle that the roof surface makes with the horizontal (see Fig. 91).

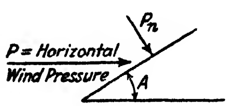


FIG. 91

Building codes of different cities show considerable variation in the value to be used for the horizontal pressure P. The following table gives values of P_n for various slopes when $P = 20$ and $P = 30$. The value of P_n for any other specified value of P can be computed from the Duchemin formula.

The amount of wind load brought to each panel point of the truss is found by multiplying the normal pressure by the product of the panel length times the distance between trusses. In order to find the maximum stresses in the members due to wind pressure, it is frequently necessary to assume the wind acting first in

NORMAL WIND PRESSURES IN POUNDS PER SQUARE FOOT OF ROOF SURFACE

Pitch	Slope	Normal Pressure P_n	
		$P = 20$	$P = 30$
$\frac{1}{6}$	4 in 12	11.5	17.2
$\frac{1}{5}$	4.8 in 12	13.1	19.8
$\frac{1}{4}$	6 in 12	14.9	22.4
	30°	16.0	24.0
$\frac{1}{3}$	8 in 12	17.0	25.5
$\frac{1}{2}$	12 in 12 (45°)	18.9	28.3
	60° and over	20.0	30.0

one direction on one side of the roof and then in the reverse direction on the opposite side. (See footnote 4 of this chapter.)

One method widely used in the design of ordinary roof trusses is to replace the normal wind load by an equivalent vertical load acting over the entire roof surface. This method is explained in the following article.

92. Combinations of Loading. — It is generally recognized as unreasonable to expect that the full snow load and the wind load will occur simultaneously on a roof. If the wind were blowing hard enough to produce the maximum pressure for which the roof structure is designed, much of the snow would be blown off. The following combinations of loads have been used in practice and probably approximate the actual conditions.

(1) Dead load and maximum snow load.
(2) Dead load, maximum wind load and $\frac{1}{2}$ the snow load.
(3) Dead load, maximum snow load and $\frac{1}{3}$ or $\frac{1}{2}$ the wind load.

For ordinary roof trusses of the types shown in Fig. 90, supported on masonry walls, the maximum stresses in the members due to the wind and snow loads are practically the same as for a uniform vertical load acting over the entire roof surface. The method of equivalent vertical loading necessitates only one analysis of the truss (or the drawing of one stress diagram) to find the stresses in the members, and will answer for any probable conditions of dead, wind and snow loads when the proper equivalent vertical live load is chosen. Most building codes specify a minimum vertical live load which must be used for buildings within their jurisdiction, but in the absence of such codes the following table gives suitable minimum values for ordinary conditions. In this book, the equivalent vertical live load is considered uniformly distributed over the actual area of the roof surface, although some specifications permit this load to be based on the horizontal projection of the area.

EQUIVALENT VERTICAL LIVE LOADS FOR COMBINED WIND AND SNOW LOADS

(Pounds per Square Foot of Roof Surface)

Location	Pitch		$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$
	Slope	60°	45°	8 in 12	6 in 12	4.8 in 12	4 in 12
Southern States and Pacific Slope........		30	30	25	25	22	20
Central States.........		30	30	25	25	22	30
Rocky Mountain States		30	30	25	25	27	35
New England States...		30	30	25	25	35	40
North West States.....		30	30	25	30	37	45

The American Standard Building Requirements for Minimum Design Loads in Buildings and Other Structures, A58.1–1945, requires that ordinary roofs, either pitched or flat, be designed for a load of not less than 20 lb. per sq. ft. of horizontal projection, including snow load but in addition to wind load. This figure is a minimum and must be increased in many localities. A U. S. Weather Bureau map in the Appendix to A58.1–1945 indicates roughly that such an increase is in order north of latitude 40 degrees. At the northeastern and north central boundaries of the United States and in parts of Washington, Oregon, and Idaho, a value of 40 lb. per sq. ft. appears reasonable. The local building code or, in its absence, local practice should always be consulted when establishing the equivalent vertical live load allowance to be used in roof design.

The *total* uniform vertical load is the sum of the dead load (weight of roof deck and purlins) and the equivalent vertical load due to combined wind and snow loads.[2]

93. Panel Loads. — In an ordinary roof truss, where the top chord panels are of equal length and there is only one slope between the heel and the peak, all the vertical panel loads due to dead and snow loads are equal except at the heel of the truss, where there is a half panel load. If the method of combined wind and snow loads acting vertically is used, then the same relation holds for the wind load. To determine the value of the panel load, the area of one panel is multiplied by the sum of the dead and the combined wind and snow loads expressed in pounds per square foot. To this must be added the weight of the purlins and the truss. The weight of the truss is computed as explained in Art. 89 and distributed among the panel points. When the panel loads have been determined the stresses in the members may be found by drawing a stress diagram or by any other method of analysis.

94. Determination of Stresses in Members. — Four common methods of determining the stresses in the members of a roof truss are the algebraic method of sections, the algebraic method of successive joints, the method of stress coefficients, and the graphic method of successive joints. Descriptions of these methods and discussions of the theory upon which they are based will be found in textbooks on statics, or stresses in structures.[3] Only the application of the graphic method of successive joints (stress diagram) will be considered here. For this method of analysis the procedure is as follows:

(1) Draw a diagram of the truss to scale showing the loads and positions of the reactions (Fig. 92*a*). The lines of this diagram theoretically represent the center of gravity lines of the truss members.

[2] An interesting tabulation, comparing the stresses in a roof truss for different combinations of loading with those produced by the equivalent vertical load method, is contained in the article on determination of stresses in members, in Section 3 of Hool and Johnson's "Handbook of Building Construction," McGraw-Hill Book Co., New York.

[3] See especially "The Theory and Practice of Modern Framed Structures," Part 1, Johnson, Bryan, and Turneaure, John Wiley & Sons, New York; "Design of Simple Steel Structures," C. T. Morris, McGraw-Hill Book Co., New York; or Chapter 25 of the Kidder-Parker "Architects' and Builders' Handbook," John Wiley & Sons, New York.

(2) Compute the reactions.

(3) Starting at the left end, letter the spaces between forces and members in a clockwise direction as shown in Fig. 92*a*. Each force and each member is then designated by two letters. This

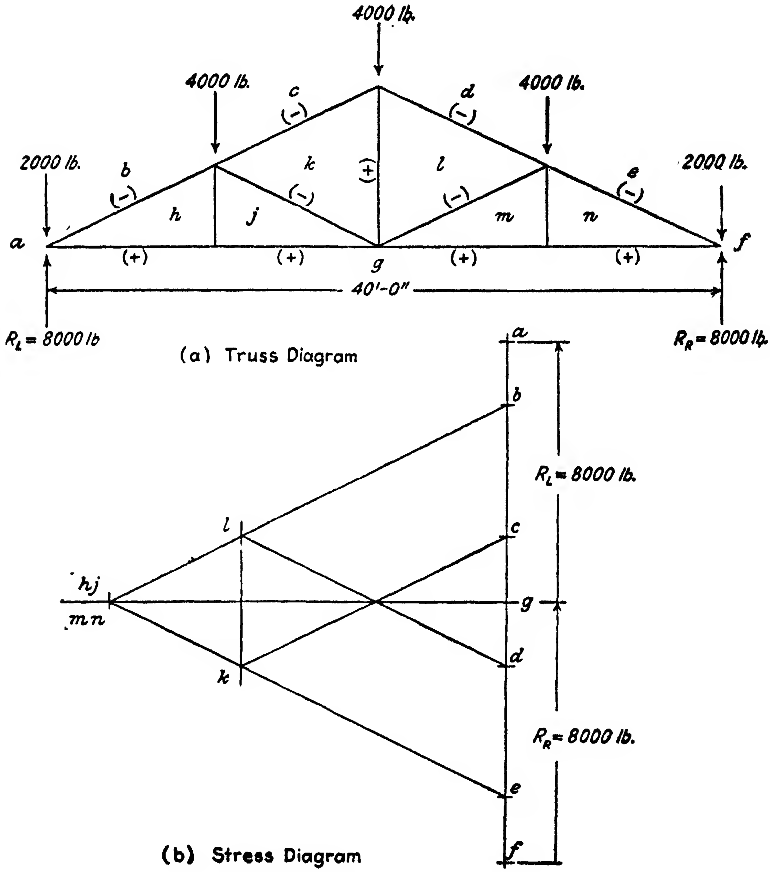


FIG. 92.

system is known as Bow's notation. It is sometimes convenient to modify this method so that the corresponding members in opposite halves of the truss have similar designations.

(4) Draw to some convenient scale the force polygon for the external forces, plotting them in the order obtained by proceeding around the truss diagram in a clockwise direction. For a

system of vertical loads the polygon is a straight line as shown in Fig. 92*b*. In this case ab = 2000 lb., bc = 4000 lb., cd = 4000 lb., de = 4000 lb., ef = 2000 lb. The reactions fg and ga each equal 8000 lb., thus closing the polygon.

(5) Select a joint such as the left heel of the truss where there are only two unknown forces. In the truss shown in Fig. 92*a* the two unknown forces are the stresses in members bh and gh; their direction, however, is known. The force polygon for this joint is then drawn as follows:

Through b (stress diagram) draw a line parallel to bh (truss diagram). Through g (stress diagram) draw a line parallel to gh (truss diagram). The intersection of these two lines locates h, thereby completing the force polygon for this joint. The stress in the member bh is obtained by scaling the line bh from the stress diagram.

(6) Select another joint where there are only two unknown forces and proceed as above. For the truss in Fig. 92*a* the next joint to choose is that formed by the intersection of member jh and the lower chord.

Through h (stress diagram) draw a line parallel to hj (truss diagram). Through g (stress diagram) draw a line parallel to jg (truss diagram). The point j will coincide with h since gh and jg have the same line of action; that is, the stress in jh is zero.

The stresses in the remaining members are found in a similar manner and recorded on the truss diagram.

(7) After the magnitude of the stress in each member is found, it is necessary to determine whether the member is in tension or compression. A minus sign ($-$) usually denotes compression and a plus sign ($+$) tension. To determine the sign of the stress in a member (say bh) read the letters clockwise about one of its joints in the truss diagram. In Fig. 92*a*, using the heel joint, we read from b to h. On the stress diagram, reading from b to h, the direction is downward and to the left or toward the joint under consideration. This indicates that the member is in compression.

Using the heel joint again to find the nature of the stress in member hg, we read clockwise h to g on the truss diagram. On the stress diagram h to g reads toward the right or away from the joint. This indicates that the member is in tension. The sign of the stress in each of the remaining members is found similarly and recorded on the truss diagram.

Where the equivalent vertical load method is not permitted, a common procedure is to determine the stresses in the members due to dead, snow, and wind loads separately by drawing three stress diagrams.[4] The resulting stresses are then combined as explained in Art. 92. The maximum value obtained from the three load combinations is the stress to be considered in design.

95. Procedure for the Design of a Roof System. — The operations necessary for the design of the purlins and trusses of a roof system, when the equivalent vertical load method is employed, are outlined in the following procedure.

(1) Select the type of roof truss best suited to the roof covering and deck that are to be used. Compute the weight of the covering and deck in pounds per square foot of roof surface. Add to this the proper allowance for the combined wind and snow load, thus obtaining the total load per square foot. Make a line diagram of the truss to scale.

(2) Design the purlins.

(3) Determine the panel loads and the reactions.

(4) Draw the stress diagram and prepare a design table.

(5) Design the compression members.

(6) Design the tension members.

(7) Design the joints.

(8) Design the end bearing and anchorage.

(9) Design the bracing.

(10) Make a design drawing.

The design of a roof system is illustrated by the example stated below, each step of the solution being discussed in a separate article. The numbers in parentheses after the article titles refer to corresponding steps of the procedure. The equivalent vertical live load method has been selected because of its wide adoption for trusses of this general classification and because its simplicity permits emphasis on matters of *design* rather than on stress analysis. For more detailed studies of stress analysis the reader

[4] If one end of the truss is considered hinged and the other supported on rollers (free to move laterally), two wind stress diagrams are required in order that the maximum stress in a member due to the wind coming from either side may be determined. Under such conditions the reaction at the hinged end will have a horizontal component.

is referred to the textbooks cited in footnotes 2 and 3 of this chapter.

The distance between the center lines of the wall piers in a brick building is 50 ft. Design the roof system to meet the following conditions.

A Fink truss having the top chord divided into 8 panels is to be used. The rise is 12 ft. 6 in. The trusses are to be spaced 16 ft. on centers. The roofing is to be tile and its weight, including bedding material, is 15 lb. per sq. ft. Decking will consist of precast light-weight concrete slabs on Tees welded to the purlins (see Fig. 111) weighing 20 lb. per sq. ft. The structure is located in southern New England, but the local building code requires a minimum vertical live load of 30 lb. per sq. ft.

The allowable unit stresses in tension and compression are those given by the A.I.S.C. Specification (1928). The minimum section shall consist of two angles $2\frac{1}{2} \times 2 \times \frac{5}{16}$ in. Gusset plates are to be $\frac{3}{8}$ or $\frac{1}{2}$ in. thick, and $\frac{3}{4}$-in. rivets will be used throughout. The allowable rivet stresses are 13,500 lb. per sq. in. in shear and 27,000 lb. per sq. in. in bearing.

96. Design Load. — (Step 1.) Determine the total load per square foot of roof surface.

Tile roofing	= 15	lb. per sq. ft. of roof surface
Roof deck	= 20	" " " "
Combined wind and snow	= 30	" " " "
Total	= 65	lb. per sq. ft. of roof surface

A diagram of the truss giving the controlling dimensions and the joint numbers is drawn as shown in Fig. 95*a*. (The loads and notations are added later.)

97. The Design of Purlins. — (Step 2.) The design of purlins may be treated under two headings: purlins free to bend in any direction and purlins fixed laterally. It is evident that the channel purlin shown in Fig. 93, when subjected to a vertical load, does not tend to bend about either axis x–x or y–y but about some other axis. The purlin is subjected to unsymmetrical bending and comes under the first of the two cases mentioned above.

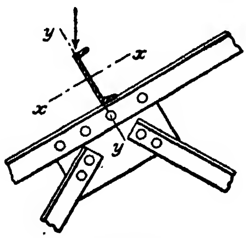


FIG. 93.

When the roof deck is of such a nature as to supply lateral support in a direction parallel to the roof surface, the vertical load on the purlin may be resolved into components normal and parallel to the roof slope. The component parallel to the slope may then be assumed as carried by the deck and the purlin is designed for the normal component only. This method may also be followed, when the roof deck is not sufficiently rigid to furnish lateral support, by employing tie rods at the center or third-points of the purlin span. These rods (usually $\frac{5}{8}$ or $\frac{3}{4}$ in. in diameter) run from the purlin nearest the heel on one side of the truss up over the ridge and down the other side. Since the stress in such a rod is greatest at the ridge, it is frequently necessary to increase the size of the ridge purlin in order to provide for the concentrated load due to the vertical component of the tie-rod stresses.[5]

Tie rods may also be used with rigid roof decks to provide lateral support during erection. They are seldom necessary on roofs having a slope of less than 3 in 12. In the problem under consideration the welded Tees furnish adequate lateral support.

Each purlin carries a rectangular area of roof equal to the distance between trusses, 16 ft., multiplied by the panel length of about 7 ft. (scaled from Fig. 95*a*). The load brought to the purlins by this type of roof deck may be considered uniformly distributed and is equal to $W = 7 \times 16 \times 65 = 7280$ lb. For roofs with slopes not greater than 6 in 12, purlins are usually designed for the full load, but the following paragraph illustrates the design when only the normal component is considered.

The component perpendicular to the roof is found graphically as shown in Fig. 94. The vertical load (7280 lb.) is drawn to any convenient scale and is represented by V in the figure. A line is drawn through the upper end of V parallel to the roof surface and one through the lower end normal to the roof surface. The values of P and N are then scaled from the diagram. P is found to equal 3200 lb., and N, 6600 lb. (approximately). The component P will be taken care of by the roof deck and tie rods so the purlin may be designed as a simple beam carrying a total uniform load of 6600 lb. on a span of 16 ft. By means of a table of safe loads or the method given in Arts. 21 and 22, a 9-in. channel weighing 13.4 lb. per ft. is found to be satisfactory. The weight of the purlin is $16 \times 13.4 = 214$ lb. (A 10-in. 15-lb. light beam

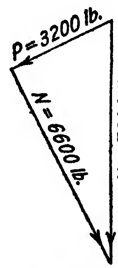


FIG. 94.

[5] For a more complete discussion of unsymmetrical bending and the use of tie rods, see pp. 191–197 of Hool and Johnson's "Handbook of Building Construction," McGraw-Hill Book Co., New York.

could also be selected. Such beams have wider flanges than channels and are frequently used as purlins.)

98. Panel Loads and Reactions. — (Step 3.) The panel loads are computed as explained in Art. 93. The amount of the panel load due to the wind, snow, and weight of the roof deck and covering is the same as that supported by one purlin. In Art. 97 this was found to be 7280 lb.[6] The weight of one purlin is 214 lb., making the total load brought to the panel point 7494 lb. Assuming that each heel of the truss takes a half panel load, there are 8 panel loads in all (see Fig. 95). The total superimposed load on the truss is $8 \times 7494 = 60{,}000$ lb. If the trial weight of the truss is assumed as 8 per cent of the total load to be carried, the weight will be $0.08 \times 60{,}000 = 4800$ lb. This weight divided by 8 gives 600 lb., and the revised panel load becomes $7494 + 600$ or approximately 8000 lb.

In this problem the two reactions of the truss are equal and have a value of $4 \times 8000 = 32{,}000$ lb. The reactions and panel loads are usually recorded on the truss diagram in kips. (1 kip = 1000 lb.) (See Fig. 95*a*.)

99. Stress Diagram and Design Table. — (Step 4.) With the loads and reactions recorded on the truss diagram (Fig. 95*a*) the stress diagram may be drawn. Inasmuch as the truss and loads are symmetrical it is necessary to draw the diagram for half the truss only. The notation consisting of the letters A to N may be considered as repeated on the right half of the truss by the addition of primes ($'$). The stress diagram is constructed as explained in Art. 94 until the joint U_2 is reached. Here there are three unknowns, the stresses in members DL, LK, and JK.

The unknowns at this joint may be reduced temporarily to two by replacing members ML and LK by the member MX, shown dotted in Fig. 95*a*. On the stress diagram (Fig. 95*b*) a line is drawn through D parallel to DX and one through J parallel to JX, thus locating X. Through X a line is drawn parallel to MX and through E one parallel to EM. The intersection gives point M. The line EM gives the true stress in member EM.[7]

[6] It should be noted that the entire load due to wind, snow, and weight of the roof deck is used when computing panel loads, whereas in the design of purlins only the vertical component of this load was considered.

[7] It is evident that the stress in member EM is not affected by any change in the form of the truss to the left of joint U_3 so long as the positions of the loads remain the same.

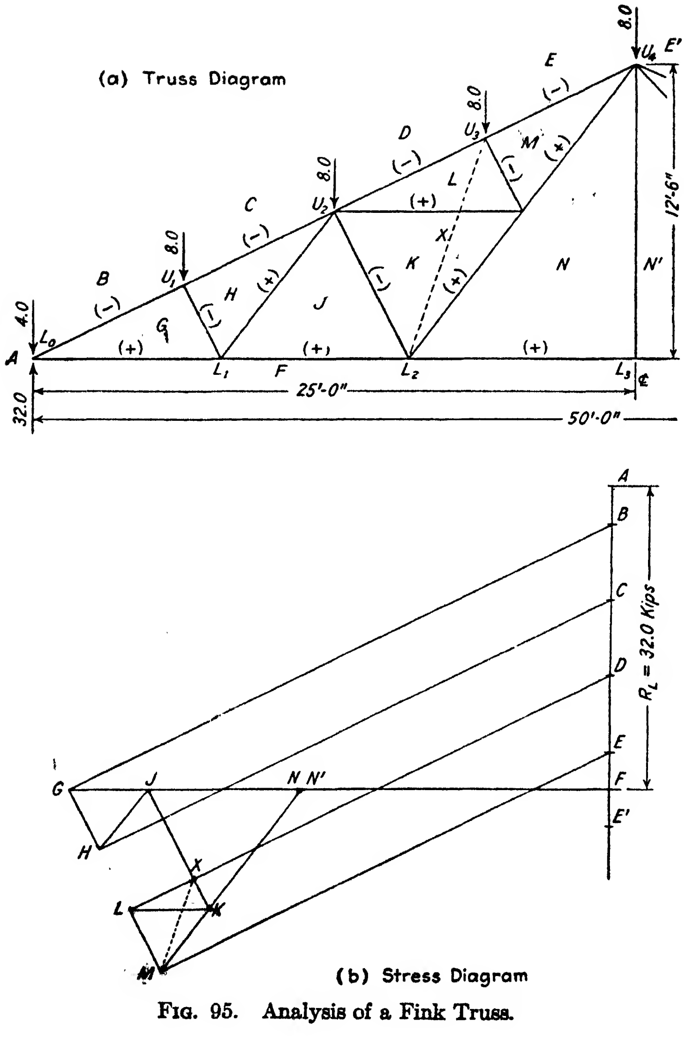


Fig. 95. Analysis of a Fink Truss.

Member *MX* is now removed and *ML* and *LK* are replaced. Knowing the position of point *M* in the stress diagram the stresses in the remaining members are found in the usual manner.

When the panel loads on a Fink truss are equal and symmetrically placed, as in the case under consideration, a simpler method not requiring the auxiliary member may be used for constructing the stress diagram. In Fig. 95*b* it is observed that the points *G*, *H*, *L*, and *M* lie on a straight line perpendicular to *BG*. When the point *G* has been determined, points *H*, *L*, and *M* are located by the intersection of this perpendicular with lines *CH*, *DL*, and *EM*. When the panel loads are not equal this method cannot be used as the points *G*, *H*, *L*, and *M* will no longer lie on the same straight line.

The fact that lines *LK*, *JK*, and *MN* intersect in a common point will serve as a check on the accuracy of the construction of the stress diagram. Additional checks will become evident from a study of Fig. 95 and the stresses tabulated below.

DESIGN TABLE

Member	Stress	Length, ft.-in.	Required section	Adopted section	Weight
BG	−62.8	7–0	2 ⌊s 4 × 3½ × $\frac{5}{16}$	2 ⌊s 4 × 3½ × $\frac{5}{16}$	108
CH	−59.5	7–0	"	"	108
DL	−56.8	7–0	"	"	108
EM	−52.0	7–0	"	"	108
FG	+56.2	7–10	2 ⌊s 3 × 2½ × $\frac{3}{8}$	2 ⌊s 3 × 2½ × $\frac{3}{8}$	103
FJ	+48.0	7–10	2 ⌊s 3 × 2½ × $\frac{5}{16}$	"	103
FN	+32.0	9–4	2 ⌊s 2½ × 2 × $\frac{5}{16}$	2 ⌊s 2½ × 2 × $\frac{5}{16}$	84
GH	− 7.2	3–6	"	"	32
HJ	+ 8.2	7–10	"	"	70
JK	−14.4	7–0	"	"	63
KL	+ 8.2	7–10	"	"	70
LM	− 7.2	3–6	"	"	32
MN	+24.2	7–10	"	"	70
KN	+16.0	7–10	"	"	70
NN′	0.0	12–6	1 ⌊ 2½ × 2 × $\frac{5}{16}$	1 ⌊ 2½ × 2 × $\frac{5}{16}$	56
			Total weight	= 2 × 1185 = 2370 lb.	

The signs of the stresses in the members are determined as explained in Art. 94, and the values are scaled from the stress diagram. It is then desirable to prepare a table in which design information may be recorded: that shown above is only one of many types that might have been chosen. Up to this point in the solution of the present problem enough information has been obtained to fill out the first three columns. As the members are designed, the required section is entered in the fourth column. When all the members have been designed slight variations in sections are eliminated to simplify the truss. In trusses of this type it is usual practice to use one section for the entire top chord, the waste in material being offset by saving in fabrication.

100. Design of Compression Members. — (Step 5.) The top chord of a roof truss similar to the one under consideration is usually composed of two angles. Top chord sections composed of two channels are discussed in Art. 109. The design of the upper chord or any other compression member is an indirect process unless safe load tables are used. A section is assumed and then investigated, following the same procedure as that given for columns in Art. 75. For purposes of illustration, member *BG* of the truss will be designed.

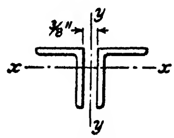


FIG. 96.

The stress in this member, as found in Art. 99, is 62,800 lb. compression. Following the procedure of Art. 75,

(1) Assume two angles $4 \times 3\frac{1}{2} \times \frac{5}{16}$ in. with the longer legs vertical and spaced $\frac{3}{8}$ in. back to back ($\frac{3}{8}$ in. equals thickness of gusset plate).

(2) The following properties of this section are taken from a steel handbook (Fig. 96).

$$A = 4.5 \text{ sq. in.} \qquad r_{xx} = 1.26 \qquad r_{yy} = 1.55$$

(3) The slenderness ratio is $\dfrac{L}{r} = \dfrac{7 \times 12}{1.26} = 66.6$

(4) The allowable average stress according to the A.I.S.C. Specification (1928) is

$$f = \frac{18{,}000}{1 + \dfrac{1}{18{,}000}\left(\dfrac{L}{r}\right)^2} = \frac{18{,}000}{1 + \dfrac{(66.6)^2}{18{,}000}} = 14{,}440 \text{ lb. per sq. in.}$$

(5) The allowable load on the member is

$$P = fA = 14{,}440 \times 4.5 = 65{,}000 \text{ lb.}$$

(6) The stress in the member as given in the design table is 62,800 lb. Since these values are in close agreement, the trial section is adopted.

When a safe load table for double angle struts is available, the required section may be selected directly provided the table is based on the specifications controlling the design. Table X of Appendix D gives safe loads for double angle struts based on the A.I.S.C. Specification (1928). A similar table based on the 1946 revision is contained in the fifth edition of the A.I.S.C. Manual.

The section selected for *BG* will be accepted for the entire top chord. Hence it is not necessary to design members *CH*, *DL*, and *EM*. The remaining compression members of the truss are designed in the same manner and the required section recorded in the fourth column of the design table.

It should be borne in mind that the minimum section to be used consists of two angles, $2\frac{1}{2} \times 2 \times \frac{5}{16}$ in.

In general the most efficient two angle strut will be composed of angles having unequal legs with the longer legs parallel to each other and separated by the thickness of the gusset plate. When angles are arranged in this manner the radii of gyration about the two axes are more nearly equal.

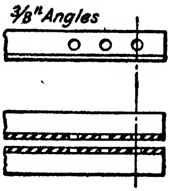


FIG. 97.

101. Design of Tension Members. — (Step 6.) The strength of a member in tension depends on the area of its least cross section. In riveted tension members built up of two angles, the least area is obtained when the cross section is taken through the rivet holes (Fig. 97). This is called the net area. The procedure in design is as follows:

(1) Find the required area by dividing the total stress to be carried by the allowable unit tensile stress. This may be expressed by the formula $A = \frac{P}{f}$, where A is in square inches, P in pounds, and f in pounds per square inch.

(2) Select two angles that have a combined gross area somewhat larger than that found in Step (1).

(3) Determine the net area by deducting the area of the rivet holes from the gross area. The net area must be equal to or slightly larger than the required area found in Step (1). If the two are not in reasonable agreement another trial must be made.

It should be noted that the area deducted for one rivet hole is not the area of the hole but the area of a rectangle, the dimensions of which are the diameter of the hole and the thickness of the metal. The actual hole diameter is made $\frac{1}{16}$ in. larger than the nominal diameter of the rivet, but is considered $\frac{1}{8}$ in. larger for design purposes in order to compensate for the metal surrounding the hole, which is damaged in punching.

The lower chord member *FG* of the roof truss under consideration will be used to illustrate the design of tension members. The total stress in this member was found to be 56,200 lb.

(1) The required area is

$$A = \frac{P}{f} = \frac{56{,}200}{18{,}000} = 3.12 \text{ sq. in.}$$

(2) As a trial section, assume two angles $3 \times 2\frac{1}{2} \times \frac{3}{8}$ in. The gross area of these two angles is 3.84 sq. in.

(3) The specifications call for $\frac{3}{4}$-in. rivets, making the diameter of the hole $\frac{7}{8}$ in. Referring to Fig. 97, the net area is

$$\begin{aligned} \text{N.A.} &= 3.84 - 2(\tfrac{7}{8} \times \tfrac{3}{8}) \\ &= 3.84 - 0.656 = 3.18 \text{ sq. in.} \end{aligned}$$

This is in satisfactory agreement with the required area, and so the trial section is adopted.

When tables of safe loads and net areas of angles with different diameter rivet holes taken out are available, the work is greatly facilitated. Tables of this nature usually state that the angles must be connected to the gusset plates by both legs in order to develop the full strength. However, this consideration is not usually serious in light trusses and may be provided for where necessary by selecting a slightly larger section than that called for. In large, heavy trusses, additional clip angles are used and the members are connected by both legs as shown in Fig. 98.

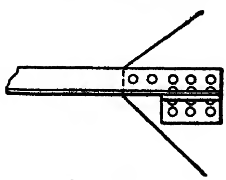


FIG. 98.

The remaining tension members of the truss are designed in a similar manner and the results recorded in the design table. One angle $2\frac{1}{2} \times 2 \times \frac{5}{16}$ in. is used for NN' as this member carries no stress but simply stiffens the lower chord. When all the members have been designed, the fifth column of the table is filled out and the weight of the truss computed.

The weight of the truss in this case is found to be about 2400 lb., and the assumed weight was 4800 lb. It must be borne in mind that the weight of the connection angles, bracing, etc., is not included in the first figure so that the total weight of the completed truss will be somewhat more than 2400 lb. Even though there were still a discrepancy of 2000 lb. after all factors were considered this would represent an error of only $\frac{2000}{60,000}$ or 3.3 per cent of the total load.

102. Design of Joints. — (Step 7.) The lines of action of all truss members entering a joint should meet in a common point. If this cannot be arranged additional rivets must be provided to take care of the resulting eccentricity. Theoretically, the rivets should be placed on the center of gravity lines but ordinarily it is not possible to do so where angle members are used, owing to the difficulty of driving rivets close to the out-standing legs; hence, they are usually placed on the standard gage lines (see Table IX).

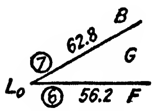


FIG. 99. Joint L_0.

Two general cases must be considered when designing the joints in a truss: first, that of members extending simply from one joint to another; and, second, that of members continuous over the joints.

The first case is illustrated by the design of joint L_0. The stresses in the top chord member *BG* and in the lower chord member *FG* are shown in Fig. 99. The number of rivets necessary in *BG* is found as follows:

BG is composed of two $\frac{5}{16}$-in. angles. A $\frac{1}{2}$-in. gusset plate will be used at this joint. The strength of one rivet will be governed by double shear or bearing in the $\frac{1}{2}$-in. plate. Referring to Table IV, the values of a $\frac{3}{4}$-in. rivet for the allowable stresses specified (see problem data in Art. 95) are

R.V. (Double Shear) = 11,928
R.V. ($\frac{1}{2}$-in. Bearing) = 10,125

$$\text{No. of rivets required} = \frac{62,800}{10,125} = 6.2 \text{ or } 7 \text{ rivets.}$$

Bearing in the $\frac{1}{2}$-in. gusset plate also governs the strength of one rivet in member *FG*.

$$\text{No. of rivets required} = \frac{56,200}{10,125} = 5.5 \text{ or } 6 \text{ rivets.}$$

A detail of the connection is shown in Fig. 102.

The second case is illustrated by the design of joint U_1, a $\frac{3}{8}$-in. gusset being used. The stresses in the members meeting at this joint are shown in Fig. 100. The number of rivets required to connect GH to the gusset plate is found as in the first case. Although only one rivet is required, a minimum of two is used. The same number will, of course, be necessary to connect the gusset plate to the top chord. In Art. 99 it was stated that the top chord is one continuous member from heel to peak.

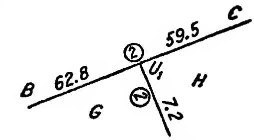


FIG. 100. Joint U_1.

Therefore, the rivets at U_1 do not transmit to the gusset the chord stress in the panel on either side of the joint but only the *difference* between these stresses. The number of rivets required (using a $\frac{3}{8}$-in. gusset) is

$$\frac{(62{,}800 - 59{,}500)}{7594} = \frac{3300}{7594} \text{ or less than 1 rivet}$$

Therefore, the two rivets used to transmit the stress in GH from the gusset to the chord angles will be adequate for this consideration also. See Fig. 102 for detail. It sometimes happens that the maximum allowable distance between rivets in a gusset plate (usually 6 in.) will determine the number of rivets. See joint U_2, Fig. 102.

The remaining connnections are designed in a similar manner, and the number of rivets is indicated on the design drawing (Fig. 102). The design of gusset plates may usually be left to the fabricating shop. The number of rivets in a joint such as L_2 may be reduced by using a thicker gusset or splice plates on the horizontal legs of the angles.

103. End Bearing and Anchorage. — (Step 8.) The number of rivets required in the upper and lower chords at the joint L_0 was determined in the preceding article. The number of rivets in the shoe angle (see Fig. 102) is made large enough to transmit the end reaction; in this case $\frac{32{,}000}{10{,}125}$ or 4 rivets. When roof trusses rest on masonry walls a sole plate is usually riveted to the underside of the shoe angles and a bearing plate rests on the wall. These plates are made somewhat wider than the shoe angles in order to provide holes for the anchor bolts, and a minimum thickness of $\frac{1}{2}$ in. is commonly used for each plate. They are designed in the same manner as bearing plates for beams (Art. 37), each plate being considered to resist one-half the bending moment developed at a vertical section through the edge of the fillet on a shoe angle. The area of the bearing plate must be sufficient to distribute the end reaction on the masonry.

In this problem, if the allowable bearing pressure on the masonry is 250 lb. per sq. in., the required area of the plate is $\frac{32,000}{250} = 128$ sq. in. A 12 × 12-in. plate is selected. The actual bearing pressure on the underside of the plate is $\frac{32,000}{144} = 220$ lb. per sq. in. The critical moment section through the angle fillet edge is 1 in. from the center line of the gusset (allowing $\frac{1}{4}$ in. for the fillet radius). The bending moment for a strip 1 in. wide is $220 \times 5 \times \frac{5}{2} = 2750$ in.-lb. Since each plate is considered to resist half of this moment, the required section modulus for one plate is

$$S = \frac{M}{f} = \frac{1375}{18,000} = 0.076$$

The required thickness of each plate is $t = \sqrt{6 \times 0.076}$ or 0.675 in. (see Art. 37). Hence, two 12 × 12-in. plates, each $\frac{3}{4}$ in. thick, are satisfactory for this case.

Two $\frac{3}{4}$-in. round anchor bolts about 18 in. long are generally provided to fasten the truss to the wall. The slotted holes in the sole plate provide for movement due to expansion.

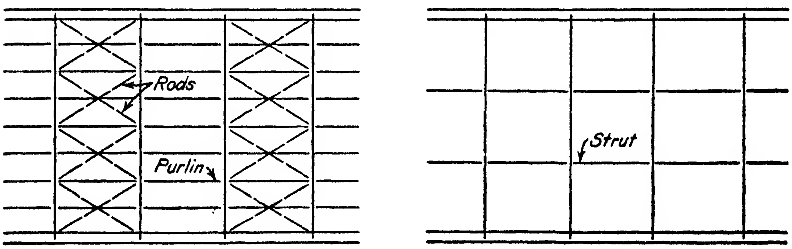
Figure 102 also shows another type of end bearing in which the lower chord angles act as shoe angles. It will be noted from the figure that the lower chord has been dropped about 8 in. and that the lines of action of the upper and lower chords and the end reaction do not meet in a common point, thus causing a moment in the connection. This may be taken care of in light trusses by making the gusset plate larger and providing additional rivets. There must also be sufficient rivets in the lower chord to transmit the end reaction. It is better to avoid this type of end detail in heavy trusses, but where it must be used the joint should be especially designed to resist the stresses due to eccentricity. The procedure in such cases is similar to that discussed in Art. 48.[8]

[8] For a detailed discussion of this type of connection, see p. 1468 of the Kidder-Parker "Architects' and Builders' Handbook," John Wiley & Sons, New York.

104. Design of Bracing. — (Step 9.) Roof trusses resting on masonry walls usually have diagonal bracing, consisting of $\frac{3}{4}$-in. round rods, in the plane of the top chord in alternate bays (Fig. 101*a*). In addition, the lower chord frequently is braced with one or two lines of continuous struts, depending on the span (Fig. 101*b*). These struts may consist of angles, channels, or light beams and are designed on a basis of a maximum L/r of 200 (see Art. 71). In this problem the trusses are 16 ft. apart, thereby requiring struts with a least radius of gyration of

$$r = \frac{L}{200} = \frac{16 \times 12}{200} = 0.96$$

From a table of properties of double angle struts, two angles $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in. with their longer legs vertical and $\frac{3}{8}$ in. back to



(*a*) Top Chord Bracing (*b*) Lower Chord Bracing

FIG. 101.

back would be satisfactory. Because of the indefinite nature of the forces which the bracing is called upon to resist, an accurate stress analysis cannot be made. The designer must rely upon his judgment and experience in selecting a satisfactory type.

105. The Design Drawing. — (Step 10.) The size of members, number of rivets in each joint, thickness of gusset plates, and all other details determined by the calculations are recorded on the design drawing shown in Fig. 102. The shop drawings of the fabricating company are made from the design drawing and give complete details such as exact rivet spacing, size of gusset plates, length of members, etc.

106. Trusses with Loads between the Panel Points. — It frequently happens that the limiting span of the roof covering is such that purlins must be placed between the panel points of the top chord. The lower chord is also often called upon to support

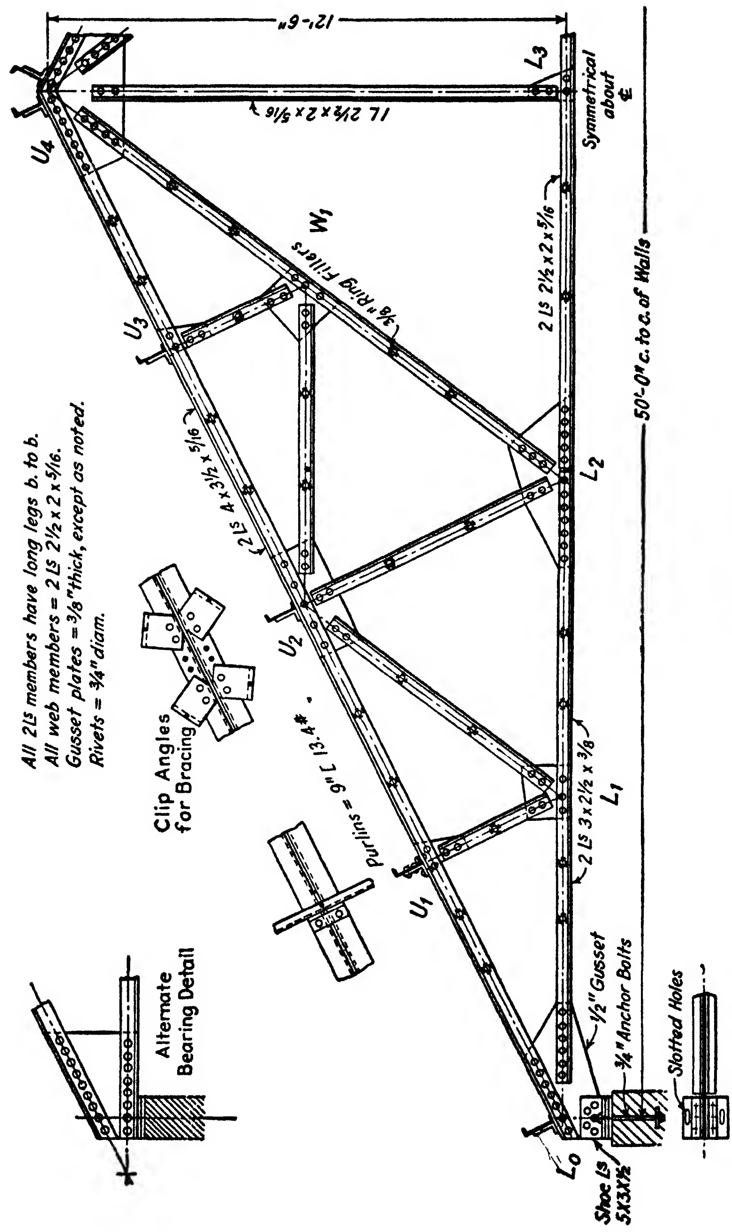


FIG. 102. Design Drawing.

loads such as suspended ceilings between the lower panel points. Where such a condition exists the chord must act as a beam as well as a direct compression or tension member; that is, it is subjected to bending and direct stress.

The stress diagram for the axial stress in the members is drawn in the same manner as explained in Art. 94 except that the panel loads must include the load brought to the truss by the intermediate purlins. For example, let the actual top chord loading on a truss be that shown in Fig. 103. The panel load at U_1 for the stress diagram is the sum of 1000 lb. due to the purlins fram-

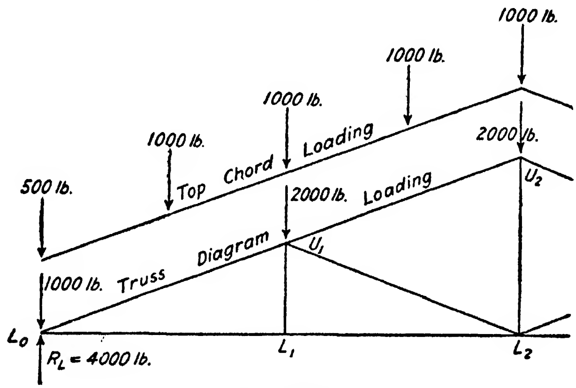


FIG. 103.

ing at the joint, 500 lb. from the intermediate purlin to the left, and 500 lb. from the intermediate purlin to the right, or a total of 2000 lb. When the panel loads have been determined in this manner, the stress diagram is drawn as previously explained.

In the design of the members subjected to bending and direct stress, a section is assumed and the unit stress investigated by the formula

$$f = \frac{P}{A} + \frac{Mc}{I}$$

in which $\frac{P}{A}$ represents the unit stress due to axial tension or compression and $\frac{Mc}{I}$ (the beam formula) the unit stress due to beam action. The design of such members is demonstrated in Arts. 107–109.

107. Bending Moments for Continuous Members. — It is beyond the scope of this book to give a detailed discussion of bending in members continuous over supports, but the ideas underlying continuity and bending under restraint are illustrated in Fig. 104. Figure 104*a* represents a single beam resting on three supports and carrying equal loads at the centers of the two spans. Now imagine the beam to be cut over the middle support as shown in Fig. 104*b*, thereby making two simple beams. Each of these simple beams will deflect as shown. When the beams are made continuous over the support the deflection curve has a shape similar to that indicated by the dotted line in the first figure.

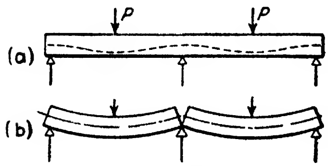


FIG. 104.

It is evident that there is no bending moment developed over the center support in Fig. 104*b*, while there must be a moment over the support in Fig. 104*a*. A study of the figure shows that there is tension in the bottom of the beam at mid-span in both cases but that in addition to this there is tension developed in the top of the beam of Fig. 104*a* over the center support. In other words, the beam shown at *a* has a positive bending moment at the center of the span and a negative moment over the support (see Figs. 16 and 17 and the definition in Art. 12). It can be shown that the positive bending moment at mid-span for the beam at *a* is less than the mid-span moment for the condition shown at *b*.

The value of the bending moments in continuous beams cannot be found by the usual equations of equilibrium, and hence the problem is statically indeterminate, requiring additional equations which involve the elasticity of the material. Various formulas have been developed that give the values of the bending moments for different conditions of loading and restraint, but the designer must exercise his judgment in using them, bearing in mind that the actual conditions under which the structure is built may not duplicate the theoretical ones. In the design of the top chord of the truss shown in Fig. 103 the members would be made continuous over the joint U_1. The load due to the intermediate purlins in the member $L_0 - U_1$ then creates a positive bending moment under the point of application of the load and a negative moment at joint U_1. For simple roof trusses of ordinary span and panel lengths, sufficiently accurate results may be obtained by taking the values of both the positive moment at mid-span and the negative moment over the support as $\frac{8}{10}$ of the moment for a simple beam, similarly loaded. The error is on the side of safety.[9]

108. Combined Tension and Bending. — The design of members to resist combined bending and tensile stresses is illustrated by the following example.

Design the lower chord of a roof truss carrying a direct tensile stress of 50,000 lb. and supporting a uniform load of 600 lb. per lin. ft. (including the weight of the member). The distance between panel points is 9 ft. 6 in. The member is to be built of two angles placed $\frac{3}{8}$ in. back to back

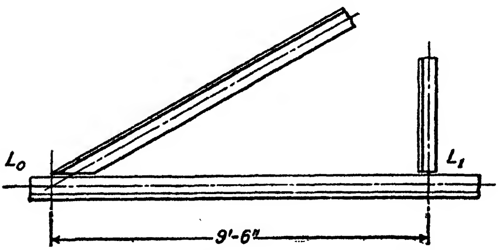


FIG. 105. Combined Tension and Bending.

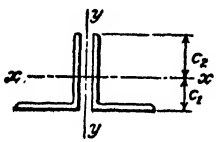


FIG. 106.

and made continuous over the first interior panel point. The allowable unit stress in tension is 18.000 lb. per sq. in. The arrangement is indicated diagrammatically in Fig. 105.

Solution:

(1) Assume as a trial section two angles $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in. placed with the longer legs vertical (Fig. 106). The following properties are found from a steel handbook.

$$A = 6.84 \text{ sq. in.} \qquad c_1 = 2.04 \text{ in.}$$
$$I_{xx} = 2 \times 12.86 = 25.72 \qquad c_2 = 3.96 \text{ in.}$$

(2) The direct stress P is 50,000 lb.; hence, the unit stress due to axial tension is $f = \dfrac{P}{A} = \dfrac{50{,}000}{6.84} = 7320$ lb. per sq. in.

[9] For an excellent treatment of the analysis of continuous beams by the area-moment method and the theorem of three moments, see "Mechanics of Materials," by Laurson and Cox, John Wiley & Sons, New York.

(3) The value of the bending moment (positive at mid-span and negative at joint L_1) is

$$M = 0.8 \times \frac{wL^2}{8} = \frac{0.8 \times 600 \times 9.5 \times 9.5 \times 12}{8} = 65{,}000 \text{ in.-lb.}$$

(4) The maximum unit tensile stress at mid-span occurs at the lower edge of the angles, because at this point the stress due to direct tension is augmented by the maximum tensile stress due to bending. The equation becomes

$$f = \frac{P}{A} + \frac{Mc}{I} = 7320 + \frac{65{,}000 \times 2.04}{25.72} = 12{,}470 \text{ lb. per sq. in.}$$

Note that the value of c_1 was substituted for c in the equation, c_1 being the distance from the neutral axis to the extreme fibre in tension.

(5) Over the joint L_1 the tension due to the negative bending moment is greatest at the upper edge of the angles; hence, the total maximum unit tensile stress at this point is

$$f = \frac{P}{A} + \frac{Mc}{I} = 7320 + \frac{65{,}000 \times 3.96}{25.72} = 17{,}320 \text{ lb. per sq. in.}$$

(6) The unit stress both at mid-span and at the support comes within the allowable limit; hence, the assumed section is adopted. If the maximum stress had not been in reasonable agreement with the allowable (18,000 lb. per sq. in), another section would have to be assumed, with the first as a guide, and the work repeated.

It will be noted that the maximum stress at mid-span is considerably less than at joint L_1. This condition is due to the fact that the member is not symmetrical about the neutral axis (axis x–x). A symmetrical section composed of two channels placed back to back makes a more efficient section for tension members subject to bending and direct stress. In such a section the unit stresses at mid-span and at the support are the same since $c_1 = c_2$.

109. Combined Compression and Bending. — The design of top chord members to resist combined bending and direct compression is illustrated by the following example.

Design a top chord member composed of two angles with the long legs riveted $\frac{3}{8}$ in. back to back. The direct compression as a member of the truss is 55,000 lb. The arrangement and dimensions are shown in Fig. 107. The load P due to an intermediate purlin is 3000 lb. The allowable unit stress in compression is governed by the A.I.S.C. Specification

(1928). In this problem the intermediate purlin will not be considered as furnishing lateral support to the top chord member.

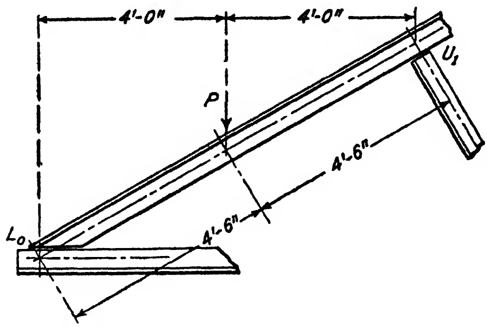


FIG. 107. Combined Compression and Bending.

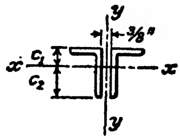


FIG. 108.

Solution:

(1) Assume two angles $6 \times 4 \times \frac{7}{16}$ placed with the long legs $\frac{3}{8}$ in. back to back (Fig. 108). The following properties are found from a steel handbook.

$$\text{Area} = 8.36 \text{ sq. in.} \qquad c_1 = 1.96$$

$$\text{Weight} = 28.6 \text{ lb. per ft.} \qquad c_2 = 4.04$$

$$I_{xx} = 2 \times 15.46 = 30.92 \qquad r_{xx} = 1.92$$

$$\frac{L}{r} = \frac{9 \times 12}{1.63} = 66.2 \qquad r_{yy} = 1.63$$

(2) The maximum allowable unit compressive stress for this section is

$$f = \frac{18{,}000}{1 + \dfrac{1}{18{,}000}\left(\dfrac{L}{r}\right)^2} = \frac{18{,}000}{1 + \dfrac{(66.2)^2}{18{,}000}} = 14{,}480 \text{ lb. per sq. in.}$$

It is not necessary to solve this formula if a graph such as that in Fig. 77 (curve B) or a table giving allowable working stresses for various ratios of $\frac{L}{r}$ is available.

(3) The direct stress due to axial compression is 55,000 lb.; hence

$$f = \frac{P}{A} = \frac{55{,}000}{8.36} = 6570 \text{ lb. per sq. in.}$$

(4) Compute the maximum bending moment considering the member as a simple beam. Note that the span of the member is the hori-

zontal distance between supports and not the distance along the slope.

$$M \text{ (purlin load)} = \frac{PL}{4} = \frac{3000 \times 8}{4} = 6000 \text{ ft.-lb.}$$

$$M \text{ (beam weight)} = \frac{WL}{8} = \frac{(28.6 \times 9) \times 8}{8} = 258 \text{ ft.-lb.}$$

$$\text{Total simple beam moment} = \overline{6258} \text{ ft.-lb.}$$

Continuous beam moment = 0.8 × 6258 = 5010 ft.-lb. or 60,200 in.-lb.

(5) The maximum unit compressive stress at mid-span occurs at the upper edge of the angles because at this point the direct compression is augmented by the maximum compression due to bending.

$$f = \frac{P}{A} + \frac{Mc}{I} = 6570 + \frac{60{,}200 \times 1.96}{30.92} = 10{,}390 \text{ lb. per sq. in.}$$

(6) The maximum unit compressive stress at joint U_1 occurs at the lower edge of the angles.

$$f = \frac{P}{A} + \frac{Mc}{I} = 6570 + \frac{60{,}200 \times 4.04}{30.92} = 14{,}440 \text{ lb. per sq. in.}$$

(7) The maximum unit stress as computed in Steps (5) and (6) comes within that allowed by Step (2); hence, the assumed section is adopted.

Here, again, the maximum stress at mid-span differs considerably from that at the support, because the member is not symmetrical about the neutral axis (axis x–x). However, if a section composed of two channels were used in this case, the radius of gyration about axis y–y would be too small. This difficulty is overcome by framing the channel purlins so that their top flanges are flush with the top flange of the channels used for the upper chord as shown in Fig. 109. This arrangement gives lateral support to the top chord member between panel points and reduces the length of the top chord to the distance between purlins for purposes of figuring the $\frac{L}{r}$ about axis y–y.

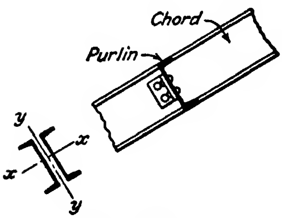
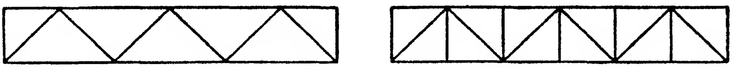


FIG. 109.

110. Parallel Chord Trusses. — Trusses with parallel chords are frequently used for flat roof construction and also for supporting floors over long spans when plate girders are undesirable. One of the chief advantages of parallel chord trusses over plate girders in the latter case is the comparatively clear working space provided for pipes and ventilating ducts by the open web system. The depth of trusses of this type is usually determined by architectural considerations, but, in the absence of other governing factors, a depth of $\frac{1}{8}$ to $\frac{1}{12}$ the span will be found satisfactory. Figure 110*a* illustrates a Warren truss with parallel chords. The distance between joints on the upper or lower chords may be reduced by the addition of vertical web members as shown at *b* of the same figure.

Roof trusses of this same general type are often built with the top chord slightly inclined, as shown at Fig. 90 *i* and *j*. Where



(a) (b)

FIG. 110. Warren Truss with Parallel Chords.

some slope is required in the roof deck to insure proper drainage, this arrangement permits the purlins to be connected directly to the truss without blocking. Here, again, a depth at the center of $\frac{1}{8}$ to $\frac{1}{12}$ the span length will usually prove economical. The depth at the wall may be varied to suit local conditions.

111. Roof Deck Construction. — Although a 4-in. cinder concrete slab spanning the distance between purlins forms an entirely adequate roof deck from the structural viewpoint, such a system is comparatively heavy and is difficult to place on steeply pitched roofs. This type of construction has been largely supplanted by precast slabs of gypsum or light-weight concrete supported on T-bar subpurlins spaced about 30 in. on centers and welded or clipped to the main purlins (Fig. 111). A 2-in coating of " nailing concrete " may be applied over the slabs to provide nailing for the roofing material although this is unnecessary with some types of material. Some precast units will safely span as much as 7 ft. and are supported directly on the purlins. End joints should occur at the purlins, and the slabs should be securely clipped to the supporting members. Complete details concerning the slab thickness, purlin spacing, and weight of construction will be found in the various manufacturers' catalogues.

Ribbed steel plates covered with an insulating material and a membrane roofing are frequently used on flat roofs. The plates are either clipped or welded to the purlins.

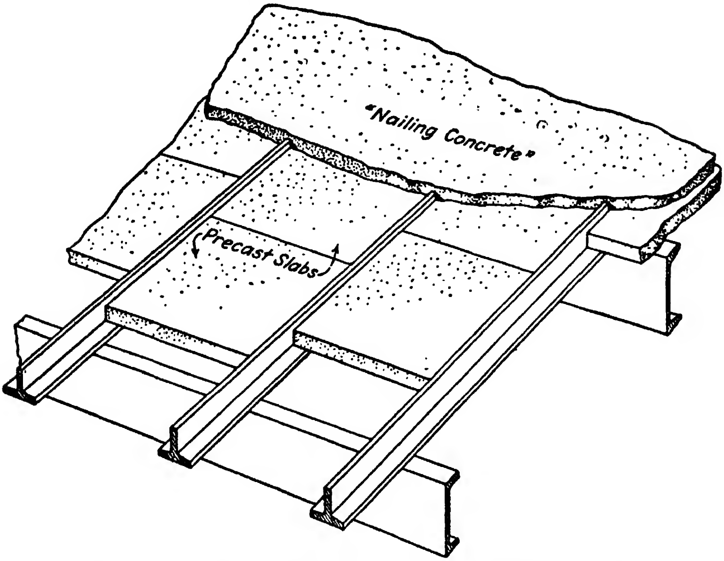


FIG. 111. Precast Slabs Supported on Tee Subpurlins.

112. Roof Truss with Knee-Braces. — The discussion in the preceding articles is confined to roof trusses resting on masonry walls. Such walls, as ordinarily constructed, are sufficiently rigid to resist lateral forces due to wind. In certain types of industrial

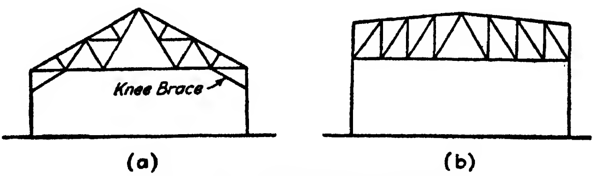


FIG. 112. Mill-Building Bents.

structures such as mill buildings, however, the trusses are supported on steel columns, the exterior wall surface consisting largely of windows or a thin curtain wall.

Under such conditions, the truss and its supporting columns, commonly known as a bent, must be designed to resist lateral forces. Rigidity is usually obtained by means of a knee-brace, as shown in Fig. 112*a*, or by making the supporting columns act as end posts when trusses of the type shown in Fig. 112*b* are employed. The bent then becomes a rigid frame and the stresses are statically indeterminate.

Inasmuch as industrial structures are not treated in this volume, the reader is referred to more detailed books on structural design for the analysis of bents of this type.[10]

113. Long Span Roof Construction. — When the span which a roof must cover is appreciably greater than 120 ft., some form of arch or rigid frame construction is usually necessary. In the United States, structural-steel arches are probably more common than any other type of arched roof system, although in Europe

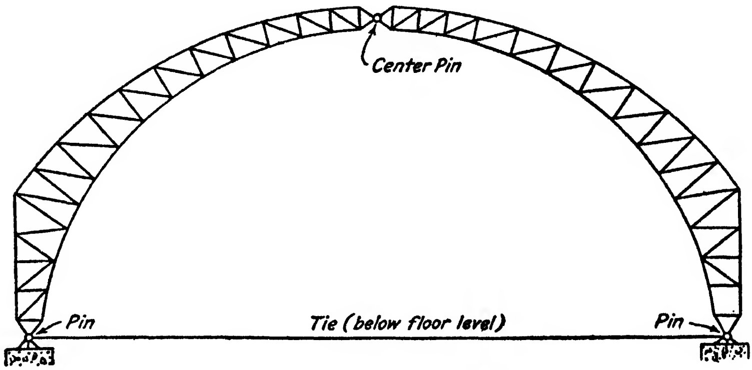


FIG. 113. Three-Hinged Arch.

reinforced concrete arches of long span are frequently employed for this purpose. The three-hinged steel arch truss (Fig. 113) is widely used for the roofs of exhibition halls, armories, and gymnasiums, the general shape of the truss being determined by the architectural features of the structure. The chord and web members may be built up of channels, or plates and angles similar to those in ordinary roof trusses, although I-beams and H-sections are frequently more satisfactory.

Inasmuch as a great deal of shop work is required in the fabrication of trusses of this type and as the truss itself will usually be quite heavy, the distance between trusses should be relatively

[10] See especially Morris and Carpenter's "Structural Frameworks" (Chapter X), John Wiley & Sons, New York; Hool and Johnson's "Handbook of Building Construction," McGraw-Hill Book Co., New York; Kidder-Parker's "Architects' and Builders' Handbook," John Wiley & Sons, New York.

large. For spans of 100 ft. and over a spacing of 22 ft. to about 40 ft. is generally employed, the greater spacing being more economical for the longer spans. Such a spacing necessitates the use of framed trusses for purlins; however, the purlin trusses may also be made a part of the bracing system between arches.

During recent years steel rib arches and rigid frames fabricated from Wide Flange sections have been widely adopted for long-span construction. Bents of this type are essentially two-hinged arches with the outward thrust at the base of the columns taken by tie rods under the floor. A detailed discussion of the design of rigid frames and arches is outside the scope of this book, since the stresses in all but the three-hinged type are statically indeterminate.[11]

PROBLEMS

1. Find the stresses in the members of the King-post truss shown in Fig. 90*g* if the span = 40 ft., rise = 6 ft., and top chord panel loads = 6000 lb. (*Note:* The general geometrical relationships of the stress diagram will be similar to Fig. 92*b*. The slopes will be different, however, because of the flatter pitch of this truss.)

2. Find the stresses in the members of the Fan truss shown in Fig. 90*c* if the span = 45 ft., rise = 8 ft., and top chord panel loads = 5000 lb.

3. Find the stresses in the members of the Pratt truss shown in Fig. 90*h* if the span = 64 ft., rise = 10 ft., and top chord panel loads = 6000 lb.

4. Design an 8-panel Warren roof truss of the type shown in Fig. 90*i* using verticals at each panel point. The truss is to rest on masonry piers with the span from center to center of bearing equal to 70 ft. The depth of the truss at mid-span is 10 ft. with the depth at each end 8 ft. The top chord panel loads (including an allowance for the truss weight) are 8000 lb., and the lower chord panel loads due to a suspended ceiling are 2500 lb. All other features of the design are controlled by the A.I.S.C. Specification (1946).

5. Solve the example of Art. 108 using two channels for the lower chord member subjected to direct tension and bending, and basing the design on the A.I.S.C. Specification (1946).

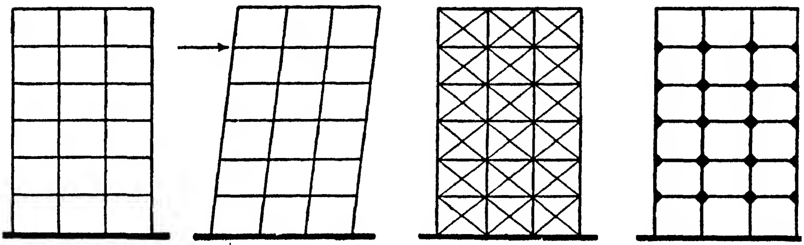
6. Solve the example given in Art. 109 using two channels for the top chord member subjected to direct compression and bending, and basing the design on the A.I.S.C. Specification (1946). Assume that the intermediate purlins frame into the chord as shown in Fig. 109.

[11] For a well-illustrated, comparative description of steel long-span roof construction see Chapter 6, Art. 2, of Crane's "Architectural Construction," John Wiley & Sons, New York. For a general discussion of methods of analysis of rigid frames, see Chapter 6 of Morris and Carpenter's "Structural Frameworks," John Wiley & Sons, New York.

CHAPTER VIII

WIND STRESSES IN TALL BUILDINGS

114. Introduction. — Tall buildings of skeleton construction must be designed to resist lateral forces due to wind pressure. If the joints in the building frame shown in Fig. 114*a* are considered pin-connected, it is evident that the application of horizontal forces will cause the frame to collapse as illustrated at



(a) (b) (c) (d)

FIG. 114. Wind Bracing.

(*b*) of the same figure. The frame may be made stable by the use of diagonal bracing (Fig. 114*c*) or by making the joints rigid (Fig. 114*d*).

Diagonal bracing is very efficient from a structural viewpoint but cannot be used extensively because of its interference with windows and doors. The second method, involving the use of rigid connections, is more adaptable to the architectural requirements of a building and is the one commonly employed at present. Bracing of this type induces bending stresses in the columns and beams which must be provided for when these members are designed.

115. Wind Pressure. — Although most municipal building codes specify a minimum wind pressure which must be used in the design of tall buildings, there is a wide variation in their requirements. The minimum pressure to be provided for varies from 15 to 30 lb. per sq. ft. over the entire exposed surface and in some specifications from 10 to 35 lb. over certain parts of the structure. Some codes state that the wind pressure is to be resisted entirely by the steel framework, others permit a certain portion to be allocated to walls and partitions, while still others are silent on this subject, thus leaving the matter to the judgment of the designer.

When the height of a steel skeleton building is not more than about twice the least lateral dimension of its base, special provision for wind bracing is unnecessary under normal conditions. In buildings having such proportions, the stiffening effect of masonry walls and partitions, together with the rigidity of standard riveted beam connections, affords ample resistance to wind forces.

There is also considerable variation in the requirements affecting wind load distribution. Some specifications require the wind pressure to be considered as acting uniformly on an exposed surface from the ground to the top; others neglect the pressure on the first 100 ft. of height, while some require that different pressures be used at different heights. The last-mentioned distribution recognizes the fact that the wind velocity and hence the pressure increase with the altitude. Virtually all assumptions as to wind pressure and distribution consider the wind to blow with a uniform velocity in a direction perpendicular to the face of the building. This, of course, is not strictly correct. The pressure constantly changes with the variation in velocity due to gusts, and the direction of the wind is greatly influenced by the location and height of adjacent buildings.[1]

116. Routing the Stresses. — The stresses in a building induced by wind pressure must ultimately be resisted by the foundations. The wind load, acting against the exterior walls, is carried to the floor construction, which is usually sufficiently rigid to transfer the load to the parts of the steel frame selected to transmit it to the foundations. The braced frames designed to resist wind stresses are known as *wind bents.* It is not necessary that all bents of a building be designed to resist wind stresses, although such an arrangement is economical where it can be used.

In any but the simplest type of buildings, the choice and de-

[1] For a report on an exhaustive study of wind bracing in steel buildings, see the 1940 *Transactions of the American Society of Civil Engineers.*

sign of bents to carry the wind stresses requires wide knowledge and experience. The discussion in a text of this nature must obviously be confined to simple, typical conditions which illustrate the fundamental principles involved.

117. Methods of Analysis. — When the wind pressure which a steel frame is to withstand has been determined, there are several methods that may be used to analyze the stresses. In general, all methods fall into one of two classes, the so-called "exact" methods and the "approximate" methods. The stresses in a modern building frame with rigid connections are, of course, highly indeterminate. Any system of analysis such as the slope-deflection method, least work, or the method of moment distribution, that gives theoretical stresses in the frame by considering the elastic distortions of the members, is called an exact method. Solutions by any of the exact methods except that of moment distribution,[2] are so laborious that they are seldom employed in the analysis of building frames. The disadvantages of the exact methods have led to the development of the approximate methods. Such methods introduce simplifying assumptions which, when used with the equations of statics, make possible the calculation of the shear, direct stress, and bending moment in each member of the frame. Two approximate methods which have been widely used, the portal and cantilever, are discussed in subsequent articles.

It should be clearly understood that, insofar as the actual stresses existing in a wind bent are concerned, the "exact" methods may give no more accurate results than the "approximate" methods, since all methods contain certain inaccuracies. For example, it is known that walls and partitions have a pronounced effect from the fact that the deflections of buildings are far less than would be the case if the steel frame acted alone. Furthermore, the rigid floor construction commonly employed in tall buildings tends to make the deflections of all bents equal and hence may cause loads on any one bent to be quite different from those it would carry on the ordinary assumption that the bent

[2] The method of moment distribution, frequently referred to as the Cross method, is recommended in the "Second Progress Report of Sub-Committee No. 31, Committee on Steel of the Structural Division" of the American Society of Civil Engineers. This report is published under the title "Wind Bracing in Steel Buildings," in the February, 1932, Proceedings of that Society.

loads are proportional to the adjacent wall areas. Although heavy connections are provided to make beams and girders carry the wind load end moments, these members are usually considered as simply supported under vertical dead and live loads. The restraint furnished by such connections causes large end moments from the vertical loads alone and hence the connections must be overstressed when maximum live and wind loads occur simultaneously.

118. The Portal Method. — The portal method has probably been used more widely than any other for the analysis of wind stresses in building frames. The simplicity of the calculations, and the checks which are afforded as the work progresses, are largely responsible for its popularity. It is the simplest of the approximate methods.

The assumptions on which the portal method is based are given below:

(1) A bent of a frame acts as a series of independent portals.

(2) The point of contraflexure of each column (i.e., the point of zero bending moment) is at mid-height of the story.

(3) The point of contraflexure of each beam is at its mid-length.

(4) The horizontal shear on any plane is divided equally between the number of aisles. An exterior column takes one-half the shear of an interior column.

(5) The floors remain plane and level.

(6) The change in length of the beams and columns may be neglected.

(7) The wind load is resisted entirely by the steel frame.

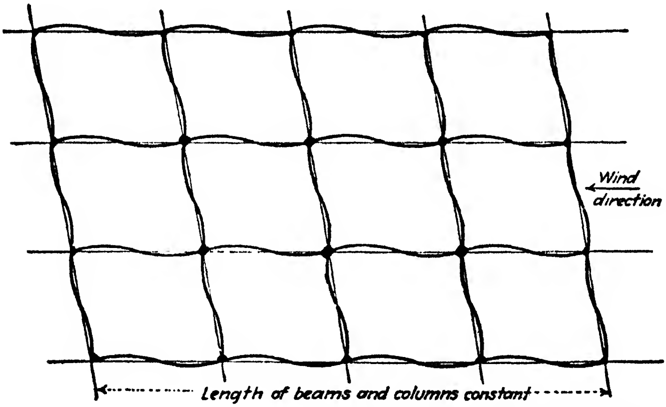
The distortion of a wind bent under the portal theory assumptions is shown in Fig. 115.[3] When applying this method to the analysis of a bent, the first step is to compute the column shears. Next, the direct compressive stresses in the beams are found by deducting from the panel load that portion of the panel load taken as shear by intervening columns.

The bending moment at the top and bottom of each column is equal to the column shear multiplied by half the story height.

[3] Taken from an article entitled "Wind Design for Tall Buildings, A Review of Current Theories," by David C. Coyle, published in the *Engineering News-Record* for June 4, 1931.

The bending moment in an interior column of any story is twice that in an exterior column since the shears are twice as large. The bending moments at the ends of the floor beams are equal to the sum of the moments in the exterior columns immediately above and below the floor under consideration. From this it is evident that the end moments in the beams of the same floor are equal and are independent of the aisle width, thereby permitting the duplication of beam connections.

The shears in the floor beams may now be computed from the relation that the shear multiplied by half the beam span equals the end moment in the beam. Since the moments in the beams



Courtesy of the Engineering News-Record.

Fig. 115. Distortion of Wind Bent under Portal Theory Assumptions.

of any one floor are equal, the shears are also equal when the aisles are of the same width.

The direct or vertical axial stresses in the columns due to overturning wind moment are readily computed from the beam shears. When the aisles of a bent are of equal width, the shears in the beams of any one floor are equal; hence the direct stress is taken entirely by the exterior columns and there is no direct stress in the interior columns. When the aisles vary in width, the interior columns also carry direct stress.

The application of the portal method is illustrated by the solution of the example stated in Art. 122. The relationships discussed above will become evident from a study of Fig. 118.

119. The Cantilever Method. — The cantilever method is based upon the following assumptions:

(1) A bent of a frame acts as a cantilever.

(2) The point of contraflexure of each column is at mid-height of the story.

(3) The point of contraflexure of each beam is at its mid-length.

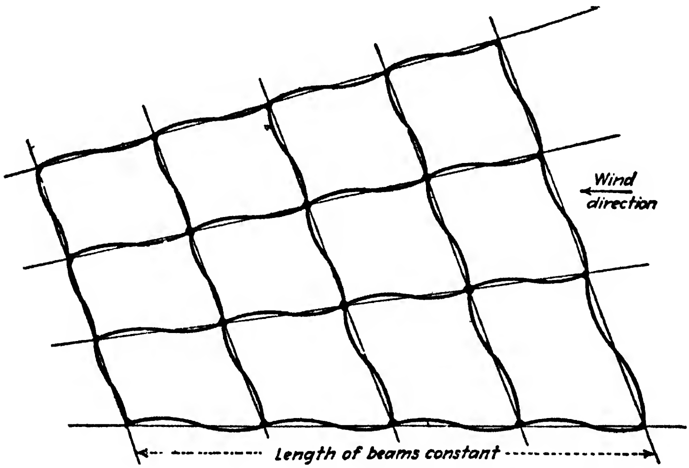
(4) The direct stress in a column is proportional to its distance from the neutral axis of the bent. It is usually further assumed that all columns in a story are of the same cross-sectional area.

(5) The floors remain plane but not level.

(6) The columns change in length but the beam lengths remain constant. (The change in column lengths is ordinarily neglected.)

(7) The wind load is resisted entirely by the steel frame.

The distortion of a wind bent under the cantilever assumptions is shown in Fig. 116.[4] Assumptions (2) and (3) as stated above



Courtesy of the Engineering News-Record.

FIG. 116. Distortion of Wind Bent under Cantilever Theory Assumptions.

locate the points of contraflexure. By assumption (1) there is tension in the windward columns and compression in the leeward columns. From assumption (4), for a bent of three equal spans,

[4] See footnote 3 of this chapter.

the stresses in the interior columns are numerically equal and are one-third the stresses in the exterior columns. By taking a section through the points of contraflexure of the columns in any story and considering the wind forces on the portion of the building above that section, it is possible to write a moment equation and solve for the values of the direct stresses in the columns of that story.

When the direct column stresses have been found, the beam shears may be calculated at once by applying $\Sigma V = 0$ at each joint. Knowing the values of the beam shears, the beam moments are determined directly, being equal to the shear in each beam times half the span length.

The equation $\Sigma M = 0$ must be satisfied at each joint, hence the sum of the girder moments must equal the sum of the column moments. Using this relation, the moments in the top-story columns at the roof may be obtained from the roof beam moments. By assumption (2) the points of contraflexure in the columns are at mid-height. Therefore the value of the moment at the base (top floor level) of each top-story column is equal to the moment at the roof. Knowing the moments in the top floor girders and the moment at the base of the top-story column, the moment in the column of the next story below may be computed by solving $\Sigma M = 0$. This process is repeated for the columns in each story below.

The column shears are obtained directly from the column moments, using the relation that the column shear equals the column moment divided by half the story height. The fact that the sum of the column shears in any one story equals the sum of the horizontal external forces above that story will serve as a check on the computations at this point.

When applying this method to bents having aisles of unequal width, the neutral axis must first be located. The direct stresses in the columns are then found and the work proceeds as outlined above.[5]

120. Limitations of Portal and Cantilever Theories. — As stated in Art. 116, the design of wind bracing in any but the simplest types of buildings requires much knowledge and expe-

[5] Applications of the cantilever method will be found in "Wind Stresses in Buildings," by Robins Fleming and "Structural Theory," by Sutherland and Bowman, John Wiley & Sons, Inc.

rience. In structures where the column arrangement is unsymmetrical, or the continuity of columns is broken due to set-backs or the presence of large rooms requiring unobstructed floor areas, the foregoing methods cannot be applied directly. Such cases require special investigation and no set of rules for their solution can be given.

The portal and cantilever methods as well as most of the so-called exact methods assume that the changes in length of the columns may be neglected. This assumption probably introduces no serious error in cases where the height of a building is less than four or five times the least width, but is less applicable to the analysis of tall slender towers where the column deformations under wind load are an appreciable factor. For structures of the latter type either the Spurr method[6] or the method of moment distribution as modified in the report cited in the footnote on page 169 provides a more rational analysis, although many of the tall buildings now in existence have been designed by the portal method or one of its modifications.

121. Working Stresses. — Since wind loads are intermittent in character and seldom maintain their maximum intensity for any length of time, most building codes permit the use of higher unit stresses in members carrying combined wind, live, and dead load as well as for those carrying wind load alone. The A.I.S.C. Specification (1928) states:

> For combined stresses due to wind and other loads, the permissible working stress may be increased $33\frac{1}{3}\%$, provided the section thus found is not less than that required by the dead and live loads alone.
>
> For members carrying wind stresses only, the permissible working stresses may be increased $33\frac{1}{3}\%$.

Since the basic unit tensile stress in these specifications is 18,000 lb. per sq. in., the increase of $33\frac{1}{3}$ per cent permits a working stress of 24,000 lb. per sq. in. in tension and on the extreme fibre in bending. The 1946 revision of the A.I.S.C. Specification, using a basic stress of 20,000 lb. per sq. in., permits the same pro-

[6] The Spurr theory is presented in "Wind Bracing," by Henry V. Spurr, the McGraw-Hill Book Co., New York, 1930. A concise statement of this method is also given in the paper by David C. Coyle referred to in the footnote on page 170.

portional increase, thereby bringing the working stress to approximately 26,600 lb. per sq. in.

122. Wind Bent Design — Portal Method. — The analysis and design of a simple wind bent by the portal method will be illustrated by an example. Figure 117 represents the framing plan of a typical floor in a building. The columns are spaced 20 ft. on centers in one direction and 14 ft. in the other with the beams and girders arranged as indicated. There are six stories, each

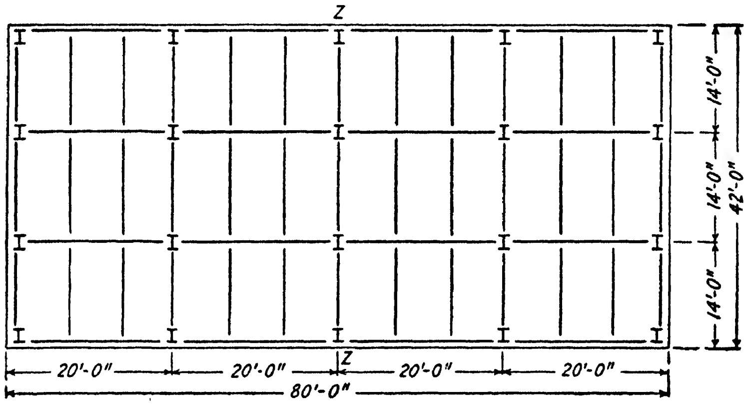


FIG. 117. Typical Framing Plan.

having a height of 12 ft. from floor to floor. In order to simplify the computations, the building is assumed to have no basement and details such as grade elevation, varying story heights and increased live load on the first floor have been omitted.

The floor and roof slabs are built of stone concrete 4 in. thick, and the exterior walls consist of a 4-in. brick facing backed with 8-in. terra cotta blocks. The dead load on the floors and roof, including an allowance for fill, finish, partitions, steel work, etc, is taken as 100 lb. per sq. ft. and the wall load as 85 lb. per sq. ft. of gross area. The live load on the roof is 40 lb. per sq. ft. and on the floors 60 lb. per sq. ft. The unit stresses to be used in design are those of the A.I.S.C. (1928) Specification, unless otherwise noted. The wind pressure to be provided for is 30 lb. per sq. ft., acting over the entire wall surface.

Although the height of this building does not exceed twice the least width, the principles involved in the wind stress analysis can be illustrated equally well as for a higher building. The loads to be resisted have purposely been taken rather large so that their individual effects will be made evident in the design. In this problem the only case considered will be that with the wind blowing against the longer side of the building. A wind bent then consists of three 14-ft. aisles and six 12-ft. stories. Bent *Z–Z* shown in Fig. 117 is selected as typical.

DETERMINATION OF STRESSES

A line diagram of the bent such as that shown in Fig. 118 is constructed. The amount of wind load applied at each floor (called the panel load) is found by multiplying the wind pressure by the story height times the sum of half the distances to adjacent bents or

$$\text{Panel load} = 30 \times 12 \times (10 + 10) = 7200 \text{ lb.}$$

The panel loads have been recorded in Fig. 118 with the wind assumed blowing from the left. The roof panel load was taken somewhat larger than half a typical floor panel load in order to take the effect of a parapet into account.

The remainder of this analysis follows the procedure outlined in Art. 118. The various operations have been given article numbers to facilitate cross-reference.

123. Column Shears. — Since the shear in any story is equally divided between the number of aisles, an exterior column taking half as much shear as an interior column, (assumption 4, Art. 118), the procedure is as follows:

Shear in Sixth Story Columns.

$$\text{Total shear in 6th story} = 5400 \text{ lb.}$$

$$\text{Shear in each aisle} = \frac{5400}{3} = 1800 \text{ lb.}$$

$$\text{Shear in exterior columns} = \frac{1800}{2} = 900 \text{ lb.}$$

$$\text{Shear in interior columns} = 1800 \text{ lb.}$$

The shears for the sixth story columns are recorded in Fig. 118 just to the right of the columns, at the assumed point of contraflexure (mid-height of story).

Shear in Fifth Story Columns.

$$\text{Total shear in 5th story} = 5400 + 7200 = 12{,}600 \text{ lb.}$$

$$\text{Shear in each aisle} = \frac{12{,}600}{3} = 4{,}200 \text{ lb.}$$

$$\text{Shear in exterior columns} = \frac{4200}{2} = 2{,}100 \text{ lb.}$$

$$\text{Shear in interior columns} = 4{,}200 \text{ lb.}$$

The shears for the fifth story columns are recorded in Fig. 118. The shears in the columns of the remaining stories have been similarly computed and recorded in Fig. 118.

124. Beam Direct Stress. — The direct compressive stress in the floor beams which distribute the panel load as shear to columns on the right of the beam is found by deducting from the panel load that portion of the panel load taken as shear by columns on the left of the beam.

Direct stress in roof beam of aisle AB

$$\begin{aligned} \text{Stress} &= \text{Panel load} - \text{shear in column } A \\ &= 5400 - 900 = 4500 \text{ lb.} \end{aligned}$$

Direct stress in roof beam of aisle BC

$$\begin{aligned} \text{Stress} &= \text{Panel load} - \text{shear in columns } A \text{ and } B \\ &= 5400 - (900 + 1800) = 2700 \text{ lb.} \end{aligned}$$

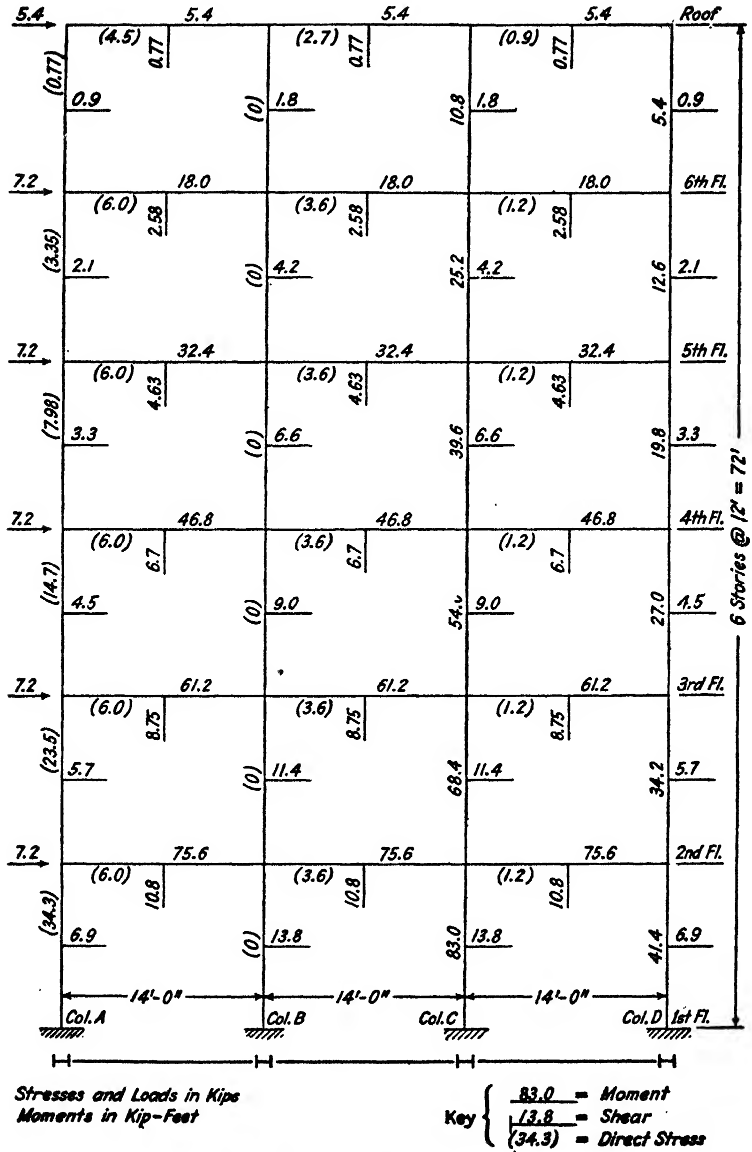
Direct stress in roof beam of aisle CD

$$\text{Stress} = 5400 - (900 + 1800 + 1800) = 900 \text{ lb.}$$

These stresses are recorded in parentheses directly below the beams (Fig. 118).

The direct compressive stresses in the remaining floor beams are found in a similar manner. It should be noted that in computing this direct stress the panel load at each floor is used and not the total shear in the story. Hence all floors having the same panel load have the same direct stress in corresponding floor beams of the bent.

125. Column Bending Moments. — The bending moment at the top and bottom of each column is equal to the column shear multiplied by one-half the story height. The bending moment in an interior column is twice that in an exterior column since the shears are twice as large.



Stress Sheet for Portal Method

FIG. 118

Bending moment in sixth story of column D

$$M = 900 \times 6 = 5400 \text{ ft.-lb.}$$

Bending moment in sixth story of column C

$$M = 1800 \times 6 = 10{,}800 \text{ ft.-lb.}$$

These moments are written parallel to the columns in Fig. 118. The moments in columns A and B are equal to those in D and C respectively. It is not necessary to record them on the figure.

The bending moments for the remaining columns of the bent have been computed in a similar manner and recorded on the right half of Fig. 118.

126. Beam Bending Moments. — The bending moments at the ends of the floor beams are equal to the sum of the moments in the exterior columns immediately above and below the floor under consideration. Moments in beams of the same floor are equal and are independent of the width of the aisle.

Bending moment in roof beams

$$M = 5400 + 0 = 5400 \text{ ft.-lb.}$$

Bending moment in sixth floor beams

$$M = 5400 + 12{,}600 = 18{,}000 \text{ ft.-lb.}$$

The bending moments in the remaining floor beams are computed in the same manner and recorded directly above the beams in Fig. 118.

127. Beam Shears. — The shears in the beams are computed from the relation that the shear multiplied by half the span length equals the end moment of the beam. Since the moments in the beams of any one floor are equal, the shears are also equal when the aisles are of the same width.

Shear in roof beams

$$V = \frac{5400}{7} = 770 \text{ lb.}$$

Shear in sixth floor beams

$$V = \frac{18{,}000}{7} = 2580 \text{ lb.}$$

These computations have been carried out for all the beams of the bent and recorded in Fig. 118 opposite the assumed point of contraflexure (mid-span).

128. Column Direct Stresses. — The direct stresses in the columns are computed directly from the beam shears. Since the shears in the beams of any one floor are equal when the aisles are of equal width, there is no direct stress in the interior columns of this bent. With the wind blowing from the left as shown, the direct stress in column A is tension and that in column D compression. The numerical values, however, are equal.

Direct stress in sixth story of columns A and D

$$P = 770 \text{ lb.}$$

Direct stress in fifth story of columns A and D

$$P = 770 + 2580 = 3350 \text{ lb.}$$

The direct stresses in the remaining stories have been similarly computed and recorded in parentheses just to the left of columns A and B in Fig. 118.

DESIGN OF MEMBERS

129. Beam Design. — The procedure used in the design of beams which resist combined wind and gravity stresses is given below.

(1) Considering the beam as simply supported, compute the maximum bending moment of the gravity loads.

(2) Compute the maximum bending moment due to the combined effect of wind and gravity loads. This moment occurs at

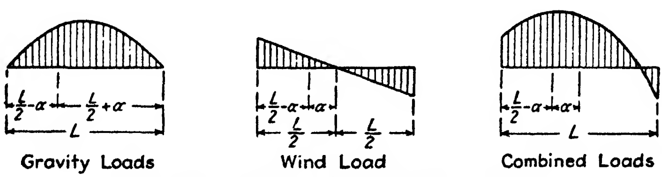


FIG. 119. Bending Moment Diagrams.

some distance a to one side of the center. See Fig. 119. When the gravity load is uniformly distributed,

$$a = \frac{2M_w}{W}$$

in which M_w = moment due to wind load;

W = total uniformly distributed gravity load.

The bending moment at a due to the gravity load is found from the expression

$$M = \frac{W}{2L}\left(\frac{L}{2} - a\right)\left(\frac{L}{2} + a\right)$$

in which L = length of the beam.

The bending moment at a due to the wind load is

$$M = \frac{aM_w}{\frac{L}{2}}$$

The total bending moment at a is the sum of these two moments, or

$$M = \frac{W}{2L}\left(\frac{L}{2} - a\right)\left(\frac{L}{2} + a\right) + \left(\frac{aM_w}{\frac{L}{2}}\right)$$

These relations may be easily derived from bending moment diagrams shown in Fig. 119. The expression for a is obtained by equating the first derivative of the equation immediately above, to zero.

(3) Select the beam section as governed by (A) the bending moment at mid-span due to gravity loads alone and using the normal unit stresses or (B) the combined moment at a using the increased unit stress for members carrying wind loads.

In beams carrying wind loads only or having large wind moments in proportion to the gravity load moments, the maximum moment will occur at the ends.

Theoretically, in case A, the direct stress in the beam should also be included. However, since the unit stresses for members carrying wind and gravity loads are increased $33\frac{1}{3}$ per cent, this factor is usually negligible.

130. Design of Roof Beams. — The gravity load carried by one beam of the bent is equal to the live and dead load per square foot multiplied by the area supported. From Art. 122, the total live and dead load including an allowance for the beam weight is 40 + 100 or 140 lb. per sq. ft. From Fig. 117, the area supported by one beam is $14 \times \frac{40}{6}$ or 93.3 sq. ft. Hence the total gravity load is

$$W = 140 \times 93.3 = 13{,}100 \text{ lb.}$$

The bending moment at the center is

$$M = \frac{WL}{8} = \frac{13{,}100 \times 14 \times 12}{8} = 275{,}000 \text{ in.-lb.}$$

The section modulus required at the center is

$$S = \frac{M}{f} = \frac{275{,}000}{18{,}000} = 15.3 \qquad \text{(A)}$$

The point of maximum combined moment is at distance a from the center

$$a = \frac{2M_w}{W} = \frac{2 \times 5400}{13{,}100} = 0.8 \text{ ft. (approximately)}$$

The value of M_w used in this equation was taken from Fig. 118.[7]

The total bending moment at a is

$$\begin{aligned} M &= \frac{W}{2L}\left(\frac{L}{2} - a\right)\left(\frac{L}{2} + a\right) + \left(\frac{aM_w}{\frac{L}{2}}\right) \\ &= \frac{13{,}100}{2 \times 14} \times 6.2 \times 7.8 + \frac{0.8 \times 5400}{7} \\ &= 22{,}600 + 617 \\ &= 23{,}200 \text{ ft.-lb. or } 278{,}000 \text{ in.-lb.} \end{aligned}$$

The section modulus required at a is

$$S = \frac{M}{f} = \frac{278{,}000}{18{,}000 \times 1.33} = \frac{278{,}000}{24{,}000} = 11.6 \qquad \text{(B)}$$

Since the section modulus for case (A) found by using the mid-span moment due to gravity loads and the normal unit stress is greater than that found in case (B) where the maximum combined moment and the increased unit stress was used, the former

[7] In this example the moment at the end of the beam is taken as equal to that at the column center line. The resulting error is on the side of safety and where the columns are small is negligible. However, in the lower stories of a high building where the columns become quite large, the moment at the column face is often used.

value of 15.3 governs. An 8-in. 20-lb. Wide Flange section having a section modulus of 17 is selected.

131. Design of Floor Beams. — The design of floor beams is carried out in a similar manner. In general, the beam sizes in the top few floors will be governed by the mid-span moment due to gravity loads, and the normal unit stress. A few floors lower, the maximum combined moment and the increased allowable unit stress will govern, while in the lower floors of the building where the wind moments are large in proportion to the gravity load moments the distance *a* will be more than half the span length and hence fall outside the beam. This means that the beam section will be governed by the wind moment alone and the increased allowable stress.

If the floor framing in this building (Fig. 117) had been arranged so that the girders instead of the beams were members of the three-aisle bent, it is probable that no increase in their sizes, at least in the upper floors, would have been necessary in order to carry the wind stresses. Although it is customary to place the girders parallel to the shorter side of the building, they were placed parallel to the longer side in this problem in order to better illustrate the effect of wind load. When the girders are members of the wind bent, the point of maximum moment cannot be found by the equation given in Art. 129, since the loads are not uniformly distributed. The maximum moment due to gravity and wind loads is determined graphically by plotting the wind moment and gravity load moment diagrams and measuring the greatest ordinate.

132. Column Design. — The procedure employed in the design of columns to resist the direct stress due to gravity loads and the bending moment due to wind load is given below.

(1) Select a column section strong enough to carry the axial gravity loads. The required section may be taken from a safe load table or designed as explained in Art. 74 and 75. In cases where the wind load stresses are relatively large, the trial section should be taken somewhat greater than that required by the gravity loads alone.

(2) Investigate the stress due to combined gravity and wind loads in the selected section by the formula

$$f = \frac{P + P'}{A} + \frac{Mc}{I}$$

in which

P = axial gravity load;
P' = direct stress in column due to wind load;
A = area of assumed column section;
M = bending moment due to wind load;
c = half the depth of the section in the direction of bending;
I = moment of inertia of section in the direction of bending.

(3) The value of f found in Step (2) must not exceed the allowable stress given by the specified column formula, plus the increase permitted for members carrying combined gravity and wind loads. If the unit stress found in Step (2) does exceed this value, a heavier section must be selected and the investigation repeated.

The columns of the bent used in this example have their webs parallel to the plane of the bent (see Fig. 117 and 118). It is assumed that the spandrel beams frame to the outside flanges of the wall columns as shown in Fig. 84 thereby practically balancing the floor beam reaction on the inside flange. When the wind is blowing from the left, as indicated in Fig. 118, the direct stresses due to wind in columns A and D are equal but opposite in sign, column A being in tension and D in compression. Columns B and C carry no direct stress. Inasmuch as the direct stress due to gravity loads is much larger than that due to wind load, the direct tensile stresses are usually negligible. Since the wind may blow from either direction, columns A and D are designed for the same direct compressive stress.[8]

133. Design of Columns *A* and *D*. — The application of the foregoing procedure is illustrated by the design of columns A and D in the third and fourth stories. Inasmuch as the columns run continuously through two stories, the sections are designed for the lower story only, of the two-story length. Referring to Fig. 117 and the loads given in Art. 122, the gravity loads on column A or D in the third story are

[8] The direct compressive stresses in the columns are frequently so small, in comparison with the gravity stresses, that they may be neglected. They have been included in this demonstration, however, for the sake of completeness.

From roof $20 \times 7 \times 140$	=	19,600 lb.
From 6th, 5th and 4th floors		
$3 \times (20 \times 7 \times 160)$	=	67,200 lb.
From parapet (4 ft. high) $20 \times 4 \times 85$	=	6,800 lb.
From 6th, 5th and 4th story walls		
$3 \times (20 \times 12 \times 85)$	=	61,200 lb.
		154,800 lb.

(1) A 12-in. 45-lb. Wide Flange section is selected from a safe load table based on the unit stresses specified in Art. 122. This section will sustain a direct axial load of 182,000 lb. on an unstayed height of 12 ft. (see Art. 132, Step 1).

(2) The gravity load as found above is 154,800 lb. Referring to Fig. 118, the direct stress due to wind is 14,700 lb. and the wind moment in the column is 27,000 ft.-lb.[9] The area of the section selected = 13.24 sq. in.; the moment of inertia in the direction of bending (I_{xx}) = 350.8; and c (half the depth of the column in the direction of bending) = 6.03 in. Substituting these values in the equation given in Step (2) of the procedure, the combined unit stress is

$$f = \frac{P + P'}{A} + \frac{Mc}{I} = \frac{154{,}800 + 14{,}700}{13.24} + \frac{27{,}000 \times 12 \times 6.03}{350.8}$$

$$= 12{,}800 + 5600 = 18{,}400 \text{ lb. per sq. in.}$$

(3) The slenderness ratio for this column is $\frac{L}{r} = \frac{12 \times 12}{1.94}$ = 74.2. This value, substituted in the A.I.S.C. (1928) column formula, gives a stress of 13,820 lb. per sq. in. The allowable increased unit stress for combined gravity and wind loads is

$$f = 13{,}820 \times 1.33 = 18{,}400 \text{ lb. per sq. in.}$$

Since this value agrees with that found in Step (2) the 12-in. 45-lb. column is adopted.

134. Stability. — When the wind blows against the side of a building there is a tendency for the entire structure to rotate

[9] This is the value of the column moment at the center line of the girder, found by multiplying the shear (4.5 lb.) by half the story height (6 ft.). In practice, the moment is usually taken somewhat smaller, allowance being made for the reduced arm due to the depth of the beam.

about a horizontal axis through the bases of the columns on the leeward side. This tendency is resisted by the weight of the building. The New York City Building Code effective January 1, 1938, requires that

"The overturning moment due to wind pressure shall not exceed 70 per centum of the moment of stability of the structure as measured by the dead loads in the columns, unless the structure is securely anchored to the foundations. Anchors shall be of sufficient strength to carry safely the excess overturning moment without exceeding the working stresses prescribed."

The total wind force on one bent of the building used in this example is equal to the unit wind pressure (30 lb. per sq. ft.) multiplied by the distance between bents (20 ft.) times the total height of the building. If the height of the building, including the parapet, is taken as 76 ft. (see Fig. 120) the total wind force is

$$P = 30 \times 20 \times 76 = 45{,}600 \text{ lb.}$$

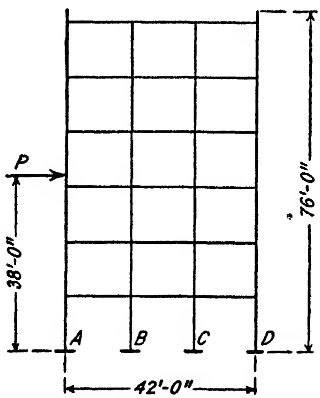


Fig. 120

The resultant of this force acts at mid-height, making the over-turning wind moment equal to

$$M_{\text{O.T.}} = 45{,}600 \times 38 = 1{,}730{,}000 \text{ ft.-lb.}$$

The moment of stability of the bent, as measured by the dead load in the windward wall column only, is equal to the total dead load on the column at its base multiplied by the width of the building. The total dead load on column A is equal to the dead load brought to the column at the roof and each floor. Inasmuch as there is no basement in this building, the first floor and first story walls are supported directly on the ground, thereby contributing no load to the column. Referring to Art. 122 and Fig. 117 for unit loads and panel dimensions, the dead load is determined as follows:

$$\begin{aligned} &\text{D.L. (floors and roof)} = 6(100 \times 20 \times 7) &= \ \ 84{,}000 \text{ lb.} \\ &\text{D.L. (exterior wall)} = 85 \times 20 \times (76 - 12) &= \underline{109{,}000} \text{ lb.} \\ &\qquad \text{Total} &= 193{,}000 \text{ lb.} \end{aligned}$$

The width of the bent between center lines of wall columns is 42 ft. Hence the moment of stability is

$$M_S = 193{,}000 \times 42 = 8{,}110{,}000 \text{ ft.-lb.}$$

Since the overturning wind moment is well within the limit stated above (70 per cent of the moment of stability), special anchorages are unnecessary.

135. Wind Bracing Connections. — The rigidity of a building frame depends largely upon the stiffness of its connections. Great care should be exercised in design of details so that they will be strong enough and stiff enough to transmit the moments for which the girders are designed. The effect of poorly designed details was brought out clearly in the wreck of the Meyer-Kiser Building at Miami, Florida, during the hurricane of September 17 and 18, 1926.

"Based on the number and size of the rivets in the wind braces and assuming the elastic limit of the steel at 36,000 lb. per sq. in., the steelwork alone, without any help from the walls, should have resisted a wind pressure of about 15 lb. per sq. ft. at the fifth floor. However, the clip angles were so thin that in bending they did not develop more than 25 per cent of the value of the rivets. These clip angles in many cases bent or broke, thus destroying all the necessary strength; and even if they had not broken they would have been so limber as to have given the building but little stiffness."[10]

The design of wind bracing connections is not treated in this text. The reader is referred to a discussion of the subject by Walter H. Weiskopf in the *Engineering News-Record*, Vol. 99, p. 396. The article contains a number of drawings and tables giving the moment capacity for several types of details.[11] An interesting discussion is also contained in a paper entitled "Wind-Bracing Connection Efficiency," by U. T. Berg, published in the January, 1932, Proceedings of the American Society of Civil Engineers. Figure 121 illustrates two common types of wind bracing connections. A bracket connection is shown at (*a*) and

[10] From the Final Report of the Committee from the Structural Division of the American Society of Civil Engineers, appointed to study the effects of the Florida hurricane. Printed in the August, 1928, Proceedings of that Society, p. 1757.

[11] This article is also reprinted in "Wind Stresses in Buildings," by Robins Fleming, Chapter VIII.

one composed of split I-beams at (*b*). The latter type is frequently preferable from the architectural viewpoint since it is easily concealed in the fireproofing of beams and columns.

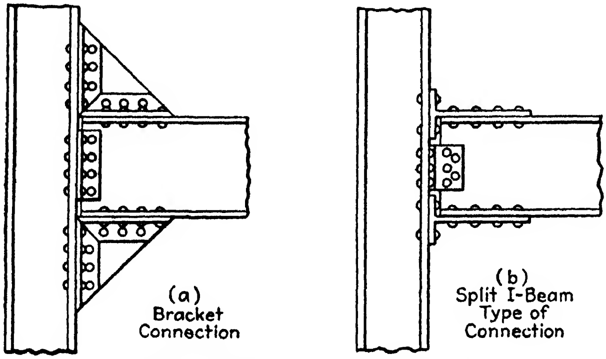


FIG. 121. Wind Bracing Connections.

136. Earthquake Resistance. Earthquake shocks produce movements (or, more precisely, accelerations) in the ground near the surface of the earth that may be resolved into vertical and horizontal components. In general, it is the effect of the horizontal component only that is taken into account when providing against *earthquake loads*. Although earthquake loads are similar to wind loads, in that both act laterally, it does not follow that a building designed to resist wind loads will also resist earthquake shocks. This will be readily apparent when it is considered that the total wind shear in any story of a building is proportional to the surface area exposed to wind action above the story in question, whereas the lateral earthquake load is proportional to the weight of the building and its contents above the same horizontal plane.

The American Standard Building Requirements for Minimum Design Loads in Buildings and Other Structures, A58.1, approved by the American Standards Association, June 19, 1945, recommends that it be made general practice in this country to design buildings to withstand a static lateral earthquake force from any horizontal direction equal to at least 5 per cent of the dead load. This moderate coefficient, together with the usual increase in working stress permitted for members subjected to both vertical and earthquake loads, will permit the attainment of satisfactory stability of construction with only slight rise in cost. For localities that have experienced earthquakes of major or near-major intensity more severe provisions are recommended, the minimum lateral load being determined by the expression

$$F = CW$$

in which F = horizontal lateral force in pounds; W = total dead load in pounds at and above the plane under consideration (except that for buildings used for storage 50 per cent of the live load shall be added); and C = a numerical coefficient varying from 0.10 to 1.00 for different elements of the building structure. Section 7 of the above-mentioned A.S.A. Standard A58.1 contains a table of values for C.

Since earthquake shocks result in accelerations and vibrations in the earth's surface, and in buildings and other structures erected thereon, the problem of earthquake-resistant design is really one of dynamical rather than statical theory. Research indicates, however, that the complex stresses set up in structures during destructive earthquake motions can be expressed in terms of equivalent static forces as recommended above.

Considerable general information on earthquakes and earthquake-resistant construction is contained in the Appendix of A.S.A. Standard A58.1. For an excellently illustrated, comprehensive survey of the earthquake problem in the United States from the engineering and insurance viewpoints, the reader is referred to "Earthquake Damage and Earthquake Insurance," by J. R. Freeman (McGraw-Hill Book Co., New York).

PROBLEMS

1. Compute the wind stresses in a bent similar to that shown in Fig. 118 except that the aisles are each 18 ft. wide and the stories 11 ft. high. The wind panel load at each floor is 9600 lb., and that at the roof 6500 lb. Record the moments, shears, and direct stresses on a diagram of the bent.
2. Design the fourth floor beams of the bent analyzed in the text.
3. Design the second floor beams of the bent analyzed in the text.
4. Design columns B and C in the third story of the bent analyzed in the text.
5. Design columns A and D in the first story of the bent analyzed in the text.

CHAPTER IX

WELDED CONSTRUCTION

137. Introduction. — The use of fusion welding to join component elements of steel structures, either to supplement or replace riveting, has materially increased since about 1930. Plate girders and roof trusses designed and fabricated as welded units are more economical than similar riveted members and may well be employed even for structures on which all other connecting will be done with rivets. Welding should be considered for field connecting of structural frames near or joined to existing buildings in order to eliminate the noise of the riveting operations, even though the individual frame members are shop fabricated by riveting. More freedom in architectural design is possible when welding is employed because special forms with smooth, pleasing surfaces can easily be constructed.

The electric-arc process is most generally used in structural welding operations in the field of building construction, and, unless otherwise qualified, the term "welding" means fusion joining by this process. An arc established across the gap between the material or work to be joined and a steel rod or electrode heats the work and melts the electrode so that additional metal is deposited on and completely fused with the work to form a connection. One terminal of a motor-generator is attached to the work, which may be an individual truss or column being fabricated in the shop or an entire building frame being connected in the field. The other terminal is fastened through a flexible cable to the electrode, which, in ordinary manual welding, is gripped in a special holder by the operator. With the generator running, the electrode is brought in contact with the work and then withdrawn to provide the proper arc gap. As the electrode metal melts and is deposited on the locally fused metal of the work, the operator must continually move his hand closer to the work to maintain a constant gap.

The power supply at the arc may be either alternating or direct current. With direct current the work is normally the positive pole and the electrode is the negative pole of the arc circuit, although for some types of welds reversed polarity may be desirable. Welding electrodes used on structural material are normally $\frac{5}{32}$, $\frac{3}{16}$, or $\frac{1}{4}$ in. in diameter although the larger size is satisfactory only for heavy welds. The electrodes are about 18 in. long and have a heavy coating material except on a short length at one end which clamps into the holder. The coating serves to stabilize the arc and, by producing a protective gaseous envelope around the molten metal, improves the properties of the finished weld. These coated rods are known as shielded arc electrodes and are the only type allowed on welded work meeting the requirements of the A.I.S.C. Specification (1946). Electrodes should be supplied in containers that indicate the manufacturer's recommendations regarding voltage and amperage for all uses and positions for which they are suitable.

It is of utmost importance that structural welding be done by qualified operators who have had experience on building construction. If there is any question of a welder's ability to perform satisfactory work, either before or during the progress of a job, he should be required to weld up test specimens which can be checked for yield-point and ultimate-breaking-strength values in a testing machine and which will indicate general weld qualities on inspection after being broken. Welding operations should be performed under favorable conditions. Complete supervision by able inspectors is definitely recommended to insure constant good performance and compliance with the design drawings as regards size and length of welds.

The design for welding as discussed in this book is applicable to members and joints of ordinary building frames in which the loads to be carried are substantially static in nature. Special members, such as crane girders in industrial buildings and framing for elevator machinery or vehicular ramps, are subjected to dynamic loading. Adequate provision for impact loading in building members can usually be made by increasing the assumed total load to be carried in accordance with the A.I.S.C. Specification (1946). However, a thorough understanding of the behavior of materials subjected to fatigue conditions is essential in the design of structures subject to dynamic loadings which cause many repetitions of maximum design stress.

138. Types of Welds. — Complete information on accepted weld types, terms used to define weld details and techniques, and symbols employed to explain weld data has been prepared by the American Welding Society. The table of welding symbols and the sketches of welded joints and of recommended standard details illustrated in the latest edition of the A.I.S.C. Manual are suggested for supplementary study.

Welds used to join structural elements are of three general types, defined as butt welds, fillet welds, and slot welds. A plug weld is a special form of slot weld with the weld material deposited in a circular hole rather than in a slot. Each of these general types is illustrated in Fig. 122.

Butt welds are classified according to the method of grooving, or preparing of the base metal form before deposition of weld metal. Nine standard forms of butt (or groove) welds are recognized in practice, and two of these are shown in Fig. 122*b* and *d*. Complete-penetration butt welds have an effective thickness or throat dimension equal to the thickness of the thinner member joined. Except for the square-edge type made in relatively thin material, complete-penetration joints generally must be welded from both sides with the root of the initial layer thoroughly chipped out on the reverse side before welding is started from that side. For several types of butt joints, complete-penetration requirements can be met by welding from one side with a backing strip of the same material as the base metal and with an increased root opening. Incomplete-penetration butt welds, usually of the single-V or single bevel type welded one side only, have an effective throat dimension equal to $\frac{3}{4}$ of the thickness of the thinner member joined. In all cases the minimum requirements for groove or bevel angle, root opening, root face, and reinforcement must be fulfilled and the quality of the completed weld must indicate satisfactory workmanship.

A fillet weld has a cross section which is approximately triangular, and it is used to join two surfaces normally at right angles to each other. This type of weld is employed more frequently in structural connections than either of the other types and is always assumed to be an equal leg, or 45-degree, weld unless clearly noted otherwise. The size of a 45-degree fillet weld is the leg length of the largest isosceles triangle that can be inscribed in the weld cross section, and the throat dimension equals the size multiplied by sine 45°. When an unequal leg fillet weld is necessary for a special design condition, the throat

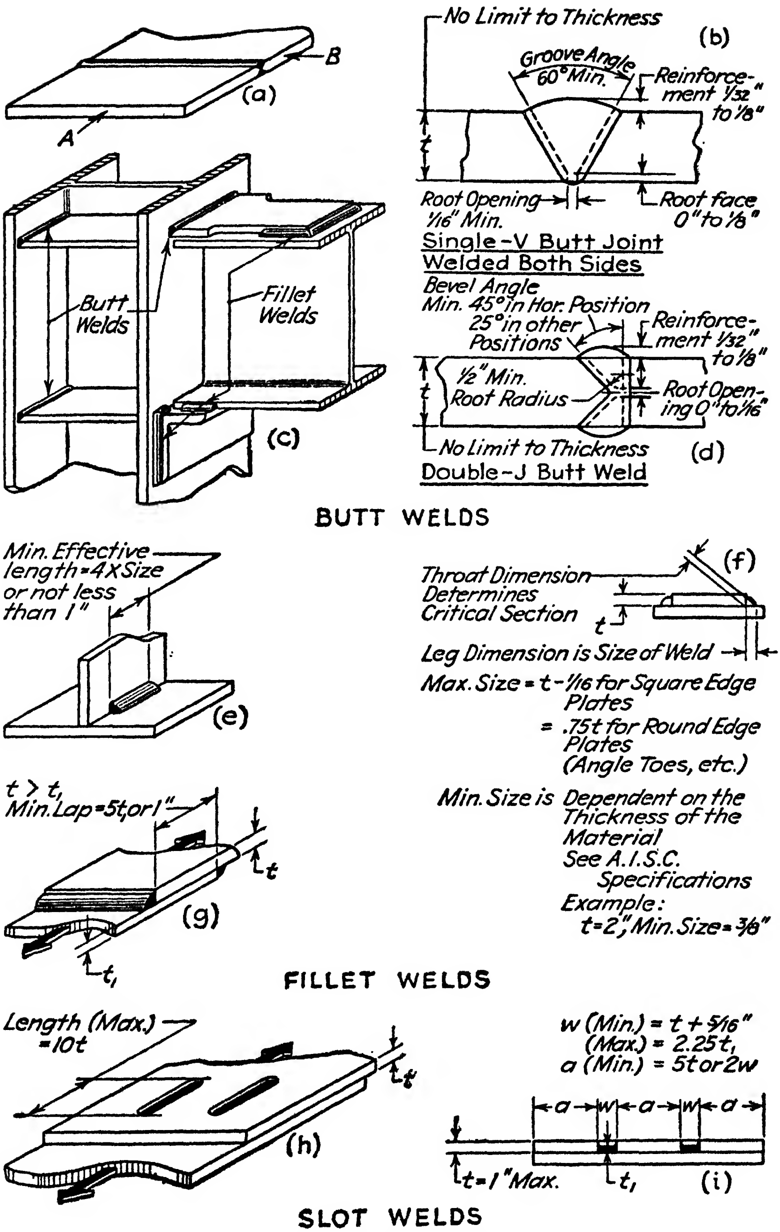


Fig. 122. Types of Welds.

dimension can be computed as the least dimension from the root apex to the hypotenuse of the right triangle determined by the leg dimensions used. Small fillet welds are most economical, since the strength is determined by the throat dimension, which is proportional to the leg size, while the amount of weld metal to be deposited varies approximately as the square of the leg size. A $\frac{5}{16}$-in. fillet weld is the maximum size that may normally be made in one pass (one progression of the electrode along the axis of the weld). Larger welds are made in several passes with the surface of the weld metal thoroughly chipped and wire brushed to remove all slag before each successive pass.

The effective area of a slot weld is the nominal area of the slot in the plane of the contact, or faying, surfaces of the parts joined. Specification requirements controlling the slot dimensions are listed in Fig. 122*h* and *i*. Fillet welds made in slots are often used and are generally preferable to slot welds for joining heavy members.

The positions of welding recognized in practice are classified as flat, horizontal, vertical, and overhead. Referring to Fig. 122*a*, with the plates held horizontal as shown, the upper weld would be made in a flat position and the lower weld would be made in an overhead position. If the plates were held vertical on edge *A* as a base, both welds would be made in a horizontal position. Again, if the plates were held vertical on edge *B* as a base, both welds would be made in a vertical position. In Fig. 122*c* the fillet welds connecting the top plate to the beam flange and those connecting the lower beam flange to the outstanding leg of the seat angle would be made in a horizontal position. The beam seat angle would normally be connected to the column flange in the shop, in which case the side welds, which are shown as vertical, could be made in a horizontal position. Overhead welds cannot always be avoided, but details should be worked out to keep their number to the minimum in the field.

139. Design of Simple Welded Connections. — The allowable unit working stress values for butt welds, expressed in pounds per square inch of effective throat section, are given in the A.I.S.C. Specification (1946) as follows: tension = 20,000; compression = 20,000; shear = 13,000. Since these values correspond to the working stresses permitted for the base material, it is possible to fully develop the strength of members at splice points by a complete-penetration type of butt weld.

The connection of a double-channel hanger member to the lower flange of a 36-in. Wide Flange beam is illustrated in Fig. 123. A plate width of 8 in. is selected to provide a 1-in. extension on each side for field welding the channels. This plate is joined to the girder with a double bevel butt weld. The full 8-in. length of the weld cannot be counted as effective, unless the ends are cut down to solid metal and side welds applied to furnish weld reinforcement similar to that provided at the faces; or

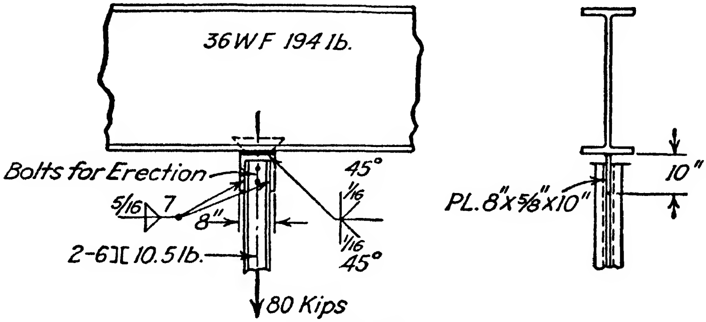


FIG. 123.

unless short extension bars are used to eliminate reduction of weld due to crater effects by continuing the full weld section beyond the ends. Normally it is more economical to assume an effective weld length somewhat shorter than the total length in order to avoid special welding. The effective throat section required is $\frac{80{,}000}{20{,}000} = 4$ sq. in. Using an effective length of 7 in., the throat thickness required is $\frac{4}{7} = 0.57$ in.; use a $\frac{5}{8}$ in. plate and a complete-penetration weld ($\frac{5}{8}$-in. throat dimension). The symbol on the pointer line in Fig. 123 shows the type of groove, the root opening, and the bevel angle. Where only one member at a joint is grooved the arrow points to that member; therefore, the arrow is drawn to the plate rather than to the beam.

In Fig. 126 complete-penetration butt welds join several elements into a continuous plate for each flange of the girder. Detail *A* illustrates the manner in which the larger plate is beveled and tapered by flame cutting to avoid serious stress concentrations due to abrupt change in cross section.

The resistance of a fillet weld is determined on the basis of the allowable shear on the section through the throat of the weld, regardless of the direction of the applied loading relative to the line of the weld. A unit shear value of 13,600 lb. per sq. in. on the throat section is given by the A.I.S.C. Specification (1946). The throat, or critical, resisting section is equal to the throat dimension multiplied by the weld length. For a 45-degree fillet weld the allowable total stress (pounds) per inch of weld length = $0.707 \times$ weld size $\times 1 \times 13{,}600 = 9615 \times$ weld size. The value generally used is 9600 times weld size, or 600 lb. for each $\frac{1}{16}$ in. of weld size.

Vertical fillet welds along each side of the double channel member, Fig. 123, will be used to field connect the member to the $\frac{5}{8}$-in. plate. No portion of the load can be considered to be transferred by the bolts that are necessary in erection of the members to hold them in position. Each weld must carry $\frac{80{,}000}{4} = 20{,}000$ lb. Where only longitudinal fillet welds are used in an end connection, the A.I.S.C. Specification (1946) requires that the length of each weld be at least equal to the perpendicular distance between the welds. For a $\frac{5}{16}$-in. weld, the value per inch $= 5 \times 600 = 3000$ lb. Each weld must have a length of $\frac{20{,}000}{3000} = 6.67$; use 7 in. The symbol indicates the size and length of the required welds, both in front of and behind the plate as seen in elevation, at each of two points and also shows that these welds are to be made in the field.

When the connected member is not symmetrical, welds should be proportioned to avoid eccentricity if possible. In Fig. 124 a double angle strut with a total stress of 63 kips is welded to a $\frac{1}{2}$-in. gusset plate. The action line of the load coincides with the gravity axis of the angles, which is 1.7 in. from the back of the 3-in. outstanding legs. By taking moments about a point on the action line of R_h, the amount of weld resistance along the toe can be determined.

$$R_t \times 5 - 63{,}000 \times 1.7 = 0$$

$$R_t = \frac{63{,}000 \times 1.7}{5} = 21{,}400 \text{ lb.}$$

$$R_h = 63{,}000 - 21{,}400 = 41{,}600 \text{ lb.}$$

Each angle must be connected for at least 10,700 lb. along the toe and 20,800 lb. along the heel. The toe of the angle is a rolled edge, and the weld size is limited to $0.75t = 0.75 \times 0.375 = 0.281$ in. For a $\frac{1}{4}$-in. weld, the length required $= \dfrac{10{,}700}{4 \times 600} = 4.46$ in.; select 5 in. (min.). A larger weld can be used along the back of the angle. For a $\frac{3}{8}$-in. weld, the length required $= \dfrac{20{,}800}{6 \times 600} = 5.8$; use 6 in.

Welds cannot always be arranged to eliminate eccentricity by having the line of the load coincide with the gravity line of

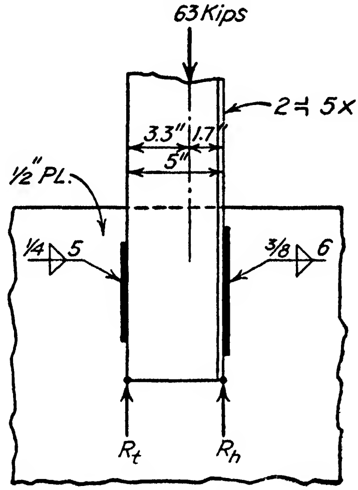
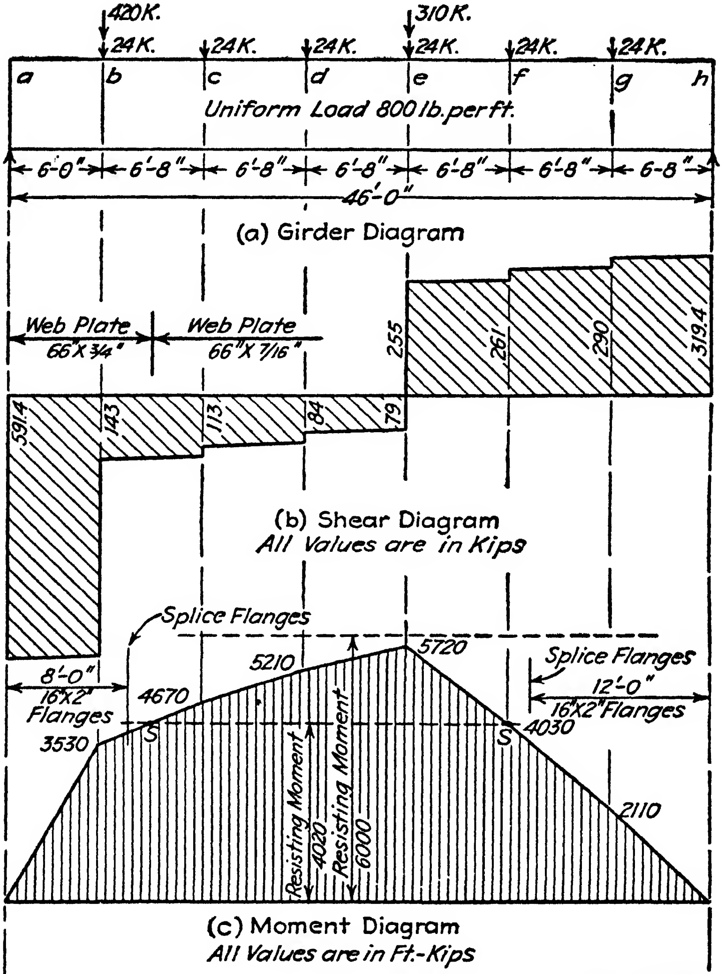
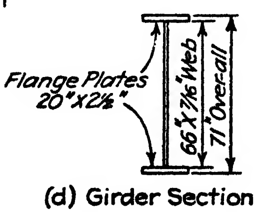


FIG. 124.

the weld or weld group. Figure 60 illustrates an eccentric riveted connection designed to support a beam framing 9 in. off the column center line. In this figure the plate might readily be fillet welded to each edge of the column flange by using an increased plate width. The welds must be designed so that the combined critical unit stress due to the direct load and the moment does not exceed the allowable unit weld stress value. The basic general formula, $f = \dfrac{P}{A} \pm \dfrac{Pec}{I}$, is applicable in all problems involving combined stress. Several books on welding[1] have com-

[1] Among these are "Manual of Design for Arc Welded Steel Structures," La Motte Grover, Air Reduction Sales Co., New York, and "Arc-Welded Steel Frame Structures," Gilbert D. Fish, McGraw-Hill Book Co., New York.





Panel	$\frac{64000000}{(h/t)^2}$	V	$v=\frac{V}{A_w}$	$d=\frac{11000t}{\sqrt{v}}$
a-b	8260	591,400	11,950	75.5
b-c	2810	143,000	4,950	68.5
c-d	2810	113,000	3,915	76.9
d-e	2810	84,000	2,905	89.2
e-f	2810	261,000	9,040	50.6
f-g	2810	290,000	10,020	48.0
g-h	2810	319,400	11,040	45.8

(e)

FIG. 125. Plate Girder Design Data.

plete charts and tables that are useful in designing or checking welded connections subjected to eccentric loadings.

140. Welded Plate Girder Design. — The design procedure outlined in Art. 54 for riveted girders is, in general, applicable when welding is used to join the various elements. The design of the welded girder in Figs. 125 and 126 illustrates this procedure and the size determination of component elements as required by the A.I.S.C. Specification (1946).

Load concentrations as shown in Fig. 125*a* assume a floor framing system such as that in Fig. 48*a* with columns spaced 20 ft. on centers in each direction and unit loads corresponding to the weight of normal fireproof floor construction plus live load allowance as required for office-building type of occupancy. The large concentrations are loads from multi-story columns which are carried by the girder to increase the clear space in the story below. An allowance for the weight of girder and fireproofing is indicated as a uniform load. The span is chosen arbitrarily to illustrate a condition of unsymmetrical loading.

The high shear value in panel *a–b* due to the heavy concentrated load at *b*, and the high bending moment value at *e* due to the heavy concentrated load near the center of the span, indicate the need for a girder with a relatively large depth to span ratio. An approximate over-all depth of 6 ft. is used, which is between $\frac{1}{7}$ and $\frac{1}{8}$ of the span (see Art. 54). A 66-in. web plate is selected and the thickness determined for the shear value in panel *g–h*. To provide for the greater shear resistance necessary in panel *a–b* a heavier plate is selected for this end. This plate is spliced to the normal web plate by a complete-penetration butt weld for the full girder web depth at or near the center of panel *b–c*. The splice should be located well to the right of point *b* to avoid excessive stresses due to the 444-kip load which must be transferred to the web.

Web plate design for right end:

V in panel *g–h* = 319,400 lb. Allowable unit shear = 13,000 lb. per sq. in.

Web area required $= \dfrac{319{,}400}{13{,}000} = 24.57$ sq. in. Min. $t = \dfrac{24.57}{66} = 0.372$ in.

Ratio $\dfrac{h}{t}$ must be less than 170. Min. $t = \dfrac{66}{170} = 0.388$ in.

Use $\frac{7}{16}$-in. web plate. Actual $\frac{h}{t} = \frac{66}{0.4375} = 151$, satisfactory.

Web plate design for left end:

V in panel a–b = 591,400 lb. Required $t = \frac{591{,}400}{13{,}000 \times 66} = 0.69$ in.

Use $\frac{3}{4}$-in. web plate from left end to 3 ft. 4 in. beyond point b.

The next step in design is to determine the flange areas required to provide a resisting moment of the gross cross section as dictated by the bending moment diagram in Fig. 125c. Each flange will consist of a single plate at any section although the size determined for the section of maximum moment need not continue for the entire length. Smaller plates are used as the moment decreases with the plates laid end to end and butt welded to form a continuous flange, as shown in Fig. 126, Detail A. The corresponding reduction of flange areas for a riveted girder is discussed in Art. 59. In the following computations handbook tables are used to facilitate the determining of the moment of inertia of the various trial sections.

Flange area design:

Section modulus required at point $e = \frac{5{,}720 \times 12{,}000}{20{,}000} = 3432$ in.^{3}

Trial section with flange plates 18 × $2\frac{1}{2}$ in.

$$
\begin{aligned}
I\text{ (web)} &= \tfrac{1}{12} \times \tfrac{7}{16} \times 66^3 = \dots\dots\dots\dots & 10{,}482 \\
I\text{ (flange)} &= I_0 + Az^2\ (I_0 \text{ negligible, Art. 52}) \\
&= 2 \times 18 \times 2.5 \times 34.25^2 = 45 \times 2346 = & \underline{105{,}570} \\
I\text{ (gross)} &= \dots\dots\dots\dots\dots\dots\dots\dots & 116{,}052 \text{ in.}^4
\end{aligned}
$$

$$\frac{I}{c} = \frac{116{,}052}{35.5} = 3270 \text{ in.}^3, \text{ which is not satisfactory.}$$

Trial section with flange plates 20 × $2\frac{1}{2}$ in.

I (gross) = 10,482 + 117,300 = 127,782 in.^{4}

$$\frac{I}{c} = \frac{127{,}782}{35.5} = 3600 \text{ in.}^3, \text{ satisfactory.}$$

The points at which the flange plates can be reduced in size to 16 × 2 in. are determined as explained in Art. 59. The moment of inertia of the reduced section = 10,482 + 73,984 = 84,466 in.^{4}; $c = 35$ in.; $\frac{I}{c} = 2413$ in.^{3}; resisting moment = 4020 ft.-kips. Splice locations are fixed to provide at least 1 ft. beyond the theoretical points marked s (Fig. 125c). The girder web at the

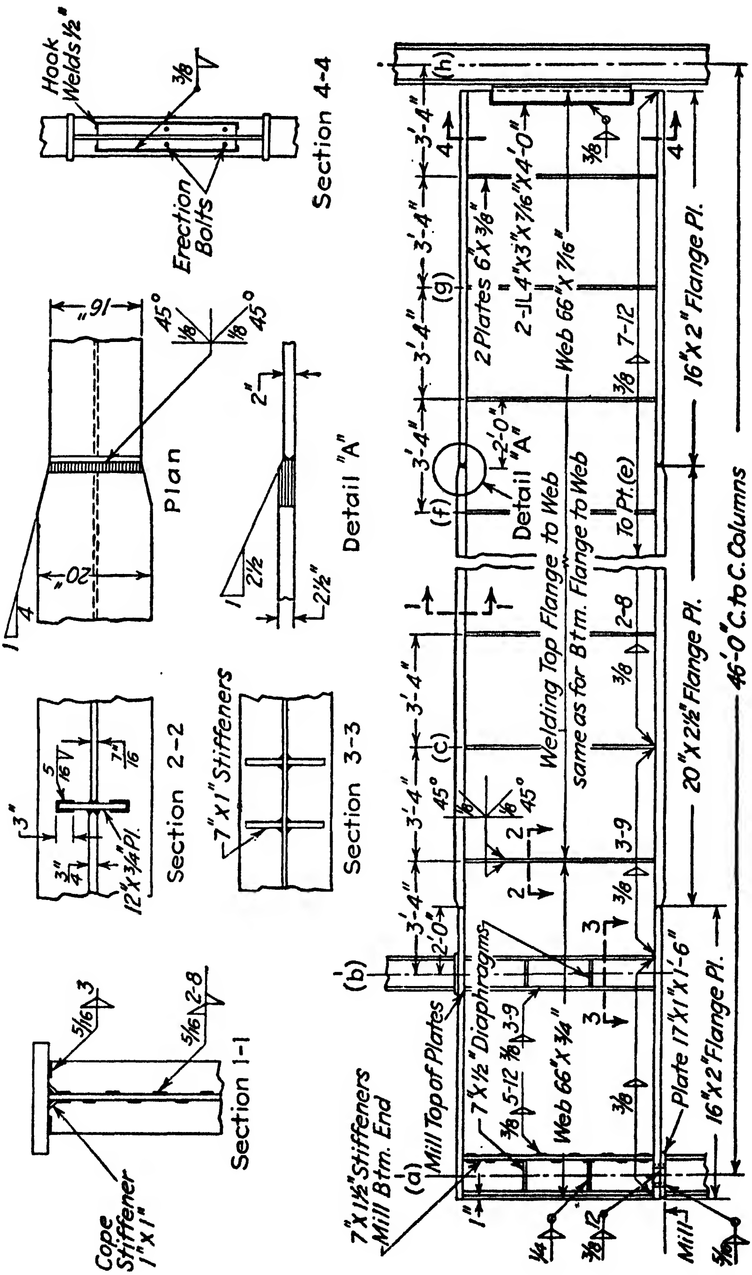


FIG. 126. Welded Plate Girder Details.

flange splice in panel b–c is $\frac{3}{4}$ in. thick, and the resisting moment for this section is almost 9 per cent greater than the value computed for a section with a $\frac{7}{16}$-in. web. However, to avoid having the web and flange splices occur in the same section, the location as determined is accepted.

The welds connecting the flanges and the web must be sufficient to develop the horizontal shear stresses as explained in Art. 62. The maximum increment of flange stress per inch of length is computed for each panel, and the welding, as determined for this maximum value, is used for the entire length of panel. Design of welding is shown for four panels only, although complete flange welding is indicated in Fig. 126.

Welding, flange to web, panel a–b:

$V = 591{,}400$ lb. $Q = 2 \times 16 \times 34 = 1088$ in.^{3} $I = 91{,}953$ in.^{4}

$$v_h = \frac{VQ}{I} = \frac{591{,}400 \times 1088}{91{,}953} = 7000 \text{ lb. per in. of length.}$$

Use $\frac{3}{8}$-in. fillet weld continuous each side. Resisting value $= 2 \times 3600 = 7200$ lb. per in.

Welding, flange to web, panel b–c:

In this panel three conditions of girder cross section exist and the maximum value of flange stress increment must be obtained by computing $\frac{VQ}{I}$ for each cross section. Since the shear variation in the panel is less than 4 per cent all computations are based on the maximum value of 143,000 lb.

For 2 ft. at left end of panel, $v_h = \dfrac{143{,}000 \times 1088}{91{,}953} = 1695$ lb. per in.

For remainder of left half of panel, $v_h = \dfrac{143{,}000 \times 1712}{135{,}269} = 1810$ lb. per in.

For right half of panel, $v_h = \dfrac{143{,}000 \times 1712}{127{,}782} = 1920$ lb. per in. (max.).

In accordance with A.I.S.C. Specification recommendations covering relation of fillet weld size and thickness of connected material, a $\frac{3}{8}$-in. weld should not be used for material over 2 in. thick. It is reasonable to assume that, for the welding conditions existing in shop fabrication, slight deviations may be justified in the interest of uniformity.

Intermittent $\frac{3}{8}$-in. fillet welds, 3 in. long and spaced 9 in. on centers, are used on each side. The average resisting value per inch of length $= \dfrac{2 \times 3 \times 3600}{9} = 2400$ lb.

Welding, flange to web, panels c–d and d–e:

Max. $V = 113{,}000$ lb. $v_h = \dfrac{113{,}000 \times 1712}{127{,}782} = 1515$ lb. per in.

Use $\frac{3}{8} \times 2$-in. welds, 8 in. on centers, each side minimum.

Resisting value = 1800 lb. per in. average.

Web bearing stiffeners are required at points b and e to transfer the heavy concentrations into the girder web. Where the supporting column is not continuous, similar stiffeners are required to effect load transfer from the web to the column, as shown at the left end of the girder in Fig. 126. These stiffeners are extended as close to the edge of the flanges as practicable and are milled at the bearing end to insure close contact against the loaded flanges.

The design procedure followed to determine the size of these members is not in accordance with the A.I.S.C. Specification (1946), which permits a portion of the girder web to be considered as acting with each pair of stiffeners to form a cross-shaped column(+). The method used is more conservative and provides that all load be taken by the stiffeners acting as individual columns. Since the radius of gyration of a narrow rectangle with respect to the long gravity axis is small, intermediate support is provided for the stiffener columns by welding horizontal diaphragm plates to these members and to the web. At least two diaphragms located at the third-points of the stiffener length are needed. The column length is taken as $\frac{2}{3}$ of the distance between support points in computing the ratio $\dfrac{L}{r}$, since a reasonably complete condition of end fixity is obtained. In the following design of the stiffeners at point b, the load from the floor members framing to the girder has been combined with the load of the supported column.

Design of bearing stiffeners:

Total load = 444 kips. Use 2 pairs of stiffeners arranged to center under flanges of supported column. Load per stiffener = 111 kips.

Required contact bearing area $= \dfrac{111}{30} = 3.7$ sq. in. Try 7×1 in. plate. (Allow for 1-in. triangular cope on all stiffeners as shown in Section 1–1, Fig. 126.) Bearing area furnished $= 6 \times 1 = 6.0$ sq. in., satisfactory.

Min. $r = 0.288$ in. With 2 diaphragms, $\frac{L}{r} = \frac{2}{3} \times \frac{22}{0.288} = 51.$

f (allowable) = 15.74 kips per sq. in.

Area required as column $= \frac{111}{15.74} = 7.05$ sq. in. Area furnished = 7.0 sq. in., accept as satisfactory.

Welding, stiffener to web:

Use $\frac{5}{16}$-in. fillet. Weld length required $= \frac{111}{3} = 37$ in.

Use intermittent welds, 3 in. long, spaced 9 in. on centers.
Total length furnished $= \frac{1}{3} \times 66 \times 2 = 44$ in.

At intermediate points additional stiffeners are required to prevent web buckling, as explained in Art. 60. Since the ratio $\frac{h}{t}$ is 88 for the left end where the web is $\frac{3}{4}$-in. thick and 151 for the remainder of the girder length, both of which values are greater than 70 as given in the A.I.S.C. Specification, stiffeners will be needed wherever the unit shear exceeds the quantity $\frac{64{,}000{,}000}{\left(\frac{h}{t}\right)^2}$. The spacing of these stiffeners shall not exceed 84 in. or a distance of $d = \frac{11{,}000\,t}{\sqrt{v}}$. In Fig. 125*e* the necessary computations are presented in tabular form. A pair of stiffeners is located at each concentrated load (points *c*, *d*, *f*, and *g*) to provide for connecting the floor framing beams. Additional stiffeners, located at the panel center line, are provided in each panel except *a–b* and *d–e* to fulfill the spacing requirements.

The stiffeners serve to square up the girder and to hold the flanges, to which they are welded, during fabrication and handling. For this reason a pair of plates is used, and a 6-in. plate width, or projection normal to the web face, is selected. To maintain a maximum width to thickness ratio of 16, the size necessary is $6 \times \frac{3}{8}$ in. To meet A.I.S.C. Specification requirements, the moment of inertia of the stiffener section, figured with a common axis at the web center line for stiffeners in pairs or with the axis at the interface between the stiffener and the web for single stiffeners, must be not less than $16 \times 10^{-8} \times h^4$. With $h = 66$ in., the minimum allowable moment of inertia $= 0.00000016 \times 66^4 = 3.04$ in.^{4} A single stiffener plate $3\frac{1}{2} \times \frac{1}{2}$ in., would meet this requirement, the section having a moment of inertia about an axis in the face of the web equal to $(\frac{1}{12} \times 3.5^3 \times \frac{1}{4}) + (\frac{1}{4} \times 3.5 \times 1.75^2) = 3.57$ in.^{4} For the pair of stiffeners selected, the moment of inertia of the section about an axis on the center line of the web $= 2\,(\frac{1}{12} \times 6^3 \times \frac{3}{8} + \frac{3}{8} \times 6 \times 3.22^2) = 60.1$ in.^{4}

Connection of the intermediate stiffeners to the girder web is made by using $\frac{5}{16}$-in. welds, 2 in. long, spaced 8 in. on centers on each side of the plates. With intermittent welding, the length of each segment should be at least 4 times the weld size. The spacing of the welds is based on the maintenance of a clear distance between the segments of not more than 16 times the thickness of the thinnest member joined. All stiffeners are connected to the top and bottom flanges with 3 in. of $\frac{5}{16}$-in. weld on each side. Since these welds are not required to provide for any stress transfer, the recommended weld-size-to-material-thickness relationship is disregarded. The magnitude of the reactions from the floor framing beams is not sufficient to require any additional stiffener welding at the concentration points.

At the center of panel *b–c* a $12 \times \frac{3}{4}$-in. plate serves as the stiffener and as a butt plate for the web splice, as shown in Fig. 126, Section 2–2. This detail is considered advantageous in that it provides additional rigidity and lateral support for the web at the splice. Since the web stress must be transferred through the butt plate it is necessary to specify that this plate be of steel, like that manufactured for boiler or firebox use, to eliminate any possibility of lamination, sometimes found in ordinary steel plates. If the stiffener detail at this location is made similar to that at the typical intermediate points, it will be necessary to move the web plate butt splice slightly to either right or left of the panel center line. The butt weld directly joining the two sections of web plate must provide a gradual transition of thickness within the weld to avoid stress concentrations due to abrupt changes in cross section.

At the right end of the girder a welded double angle web connection transfers the reaction to the flange of the supporting column. This type of framing connection is discussed in Art. 142. The heavy reaction applied at the column flange must be considered an eccentric load. The resulting moment must be resisted either by the column, as explained in Art. 80, or by a rigidly connected beam framed to the opposite flange.

141. Welded Roof Trusses. — The general steps to be followed in designing a roof truss as a component part of a complete roof system are the same for a truss fabricated by welding as they are for a riveted truss. The factors that determine the shape or type of truss, spacing of trusses, and the magnitude of design loadings, as discussed in Chapter VII, apply for either form. It is only in the actual selection of the truss members and design of the truss joints that the method of fastening the members together must be considered.

The chord and web members of riveted roof trusses are usually composed of double angle members to provide for stress transfer at the joints through gusset plates placed between the angles and riveted to them. This general arrangement is illustrated in Fig. 102. A comparable, though uneconomical, welded truss could be fabricated merely by substituting adequate fillet welds along the edges of each angle at the gussets in place of the rivets.

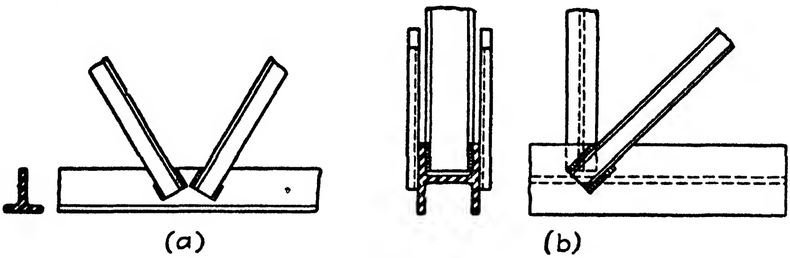


Fig. 127.

For the loading and span conditions normally encountered in roof construction, properly designed welded trusses will show substantial savings in weight of truss material over corresponding riveted designs. Tension members are selected on the basis of gross cross-sectional area as no reduction need be made for rivet holes. Gusset plates can be reduced in number and size or, often, totally omitted. One method is to use Tee-sections for the chord members and weld the web members directly to them, as shown in Fig. 127*a*. A possible joint arrangement for heavy trusses is illustrated in Fig. 127*b*. The chord consists of an H-section placed with the web horizontal. The vertical web members are I-beams or Wide Flange sections welded to the inside of the chord member flanges. Channels welded to the flange faces are used for the inclined web members.

The controlling design details for a 100-ft.-span truss for an industrial building are illustrated in Fig. 128. Plan requirements determined the 25-ft. truss spacing and the location of the tramrail support points on the lower chord. The line diagram of a half truss indicates the member arrangement used to provide for a purlin spacing of 5 ft. Membrane roofing and 1-in. insulation board are supported by ribbed metal deck plates welded to 10-in. channel purlins.

Stresses shown on the line diagram were computed analytically and are maximum values obtained due to either full loading on the entire span or one-half live load plus full tramrail loading on one half of the truss. The compression stress of 12 kips in the center diagonal is obtained under the latter condition of loading.

The following design of the members and welding details is in accordance with the requirements of the A.I.S.C. Specification (1946).

Upper chord:

Max. stress 201 kips compression.

For projecting elements under compression the effective width to thickness ratio should not exceed 16.

Try ST 9 WF 42.5. Area = 12.49 sq. in. Min. $r = 2.00$ in.

Stem projection = depth $- k = 9.16 - 1.5 = 7.66$ in.

Stem thickness = 0.526 in. Ratio $= \dfrac{7.66}{0.526} = 14.6$, satisfactory.

$\dfrac{L}{r} = \dfrac{60}{2.0} = 30.0$ f (allowable) = 16.56 kips per sq. in.

Area required $= \dfrac{201}{16.56} = 12.13$ sq. in. Trial section satisfactory.

Lower chord:

Max. stress 197 kips tension. Area required $= \dfrac{197}{20} = 9.85$ sq. in.

Max. projection of stem beyond edge of fillet will be limited to 24 times stem thickness. Projection greater than 18 times thickness will not be counted to resist stress.

Try ST 10 WF 36.5. Area = 10.73 sq. in.

Stem projection $= 10.62 - 1.31 = 9.31$ in.

Stem thickness = 0.455 in. Ratio = 20.4, satisfactory.

Consider only $18 \times 0.455 = 8.2$ in. of stem for effective area.

Area reduction $= (9.31 - 8.2) \times (0.455) = 0.51$ sq. in.

Effective area furnished $= 10.73 - 0.51 = 10.22$ sq. in. Trial section satisfactory.

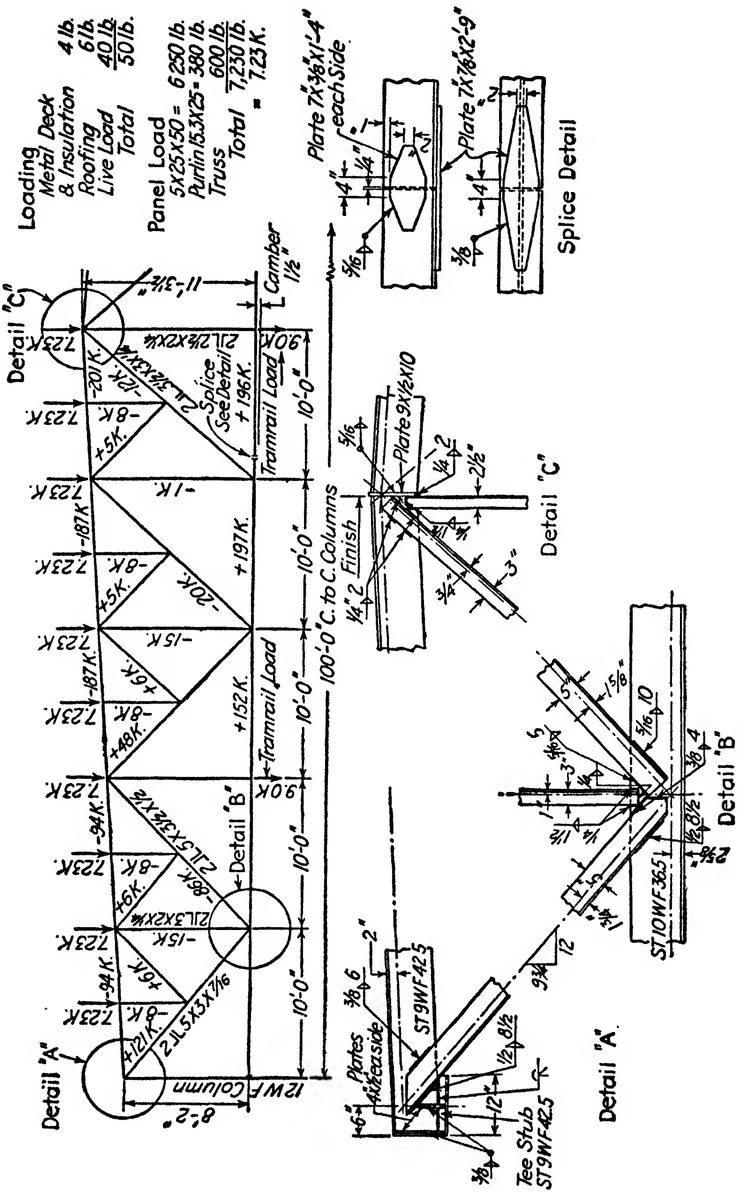


Fig. 128. Welded Roof Truss Details.

1st diagonal:

Max. stress 121 kips tension. Area required $= \dfrac{121}{20} = 6.05$ sq. in.

Use 2 angles $5 \times 3 \times \frac{7}{16}$. Area furnished $= 6.62$ sq. in.

1st main vertical:

Max. stress 15 kips compression. Length $= 105.5$ in.

Max. $\dfrac{L}{r} = 120$. Min. $r = \dfrac{105.5}{120} = 0.88$ in.

Try 2 angles $3 \times 2 \times \frac{1}{4}$ with 3-in. legs separated $\frac{7}{16}$ in.

$r = 0.91$ in. $\dfrac{L}{r} = 116$. f (allowable) $= 10.47$ kips per sq. in.

Area required $= \dfrac{15}{10.47} = 1.43$ sq. in.

Trial section furnishes 2.38 sq. in. Selected for min. r requirements.

2nd diagonal:

Max. stress 86 kips compression. Length $= 165$ in.

Min. $r = \dfrac{165}{120} = 1.375$ in.

Try 2 angles $5 \times 3\frac{1}{2} \times \frac{1}{2}$ with 5-in. legs separated $\frac{7}{16}$ in.

$r = 1.52$ in. $\dfrac{L}{r} = \dfrac{165}{1.52} = 108.5$. f (allowable) $= 11.29$ kips per sq. in.

Area required $\dfrac{86}{11.29} = 7.62$ sq. in.

Trial section furnishes 8.0 sq. in., satisfactory.

5th diagonal:

Max. stress 12 kips compression. Length $= 181$ in.

Min. $r = \dfrac{181}{120} = 1.51$ in.

Use 2 angles $3\frac{1}{2} \times 3 \times \frac{1}{4}$ with 3 in. legs separated $\frac{7}{16}$ in.

$r = 1.67$ in. For the other principal gravity axis of the double angle member, $L = 90.5$ in., $r = 0.91$ in., Min. r required $= \dfrac{90.5}{120} = 0.76$ in.

Welding for 1st diagonal:

At angle heel, weld for $\dfrac{121}{2} \times \dfrac{3.25}{5} = 39.4$ kips.

Use $\frac{1}{2}$-in. fillet, length $= \dfrac{39.4}{4.8} = 8.21$, say 8.5 in.

At angle toe, weld for $\dfrac{121}{2} - 39.4 = 21.1$ kips.

Use $\frac{3}{8}$-in. fillet, length $= \dfrac{21.1}{3.6} = 5.86$, say 6 in.

At the lower chord joint, (Fig. 128, Detail B), the necessary length of weld cannot be provided along the angle toe. Eight inches of weld length is used, one half along the toe and one half across the end of the angle. Although this detail furnishes an excess of welding for strength requirements, the center of gravity of the total weld resistance is maintained on the gravity axis of the connected angles. Note that the $\frac{3}{8}$-in. fillet size along the angle toe exceeds $\frac{3}{4}$ of the angle thickness, and, in accordance with specification requirements, care must be exercised to attain an actual weld throat dimension equal to 0.707 times the size specified.

Welding for vertical member (Detail B):

Stress in member = 15 kips or 7.5 kips per angle. The arrangement of members as indicated permits a weld length of only $1\frac{1}{2}$ in. along the edges of the vertical member angles. With a $\frac{3}{8}$-in. weld along the heel and a $\frac{3}{16}$-in. weld along the toe, the resistance values of $1.5 \times 3600 = 5400$ lb. and $1.5 \times 1800 = 2700$ lb., respectively, would satisfy strength requirements. In order to comply with the A.I.S.C. Specification, these welds should each have a length equal to the member width or 3 in. when used alone. The detail used provides a weld across the end of each angle to supplement the side welds. Although all welds are designated as $\frac{1}{4}$ in., the toe and end fillets are considered to have an effective size of $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ in. On this basis the total weld value per angle is $4.5 \times 1800 + 1.5 \times 2400$, or 11,700 lb., with a resultant action line 1.62 in. from the angle toe. This does not coincide with the gravity axis of the members, but the eccentricity, which is less than would prevail if rivets were used on the standard gage line for a 3-in. angle leg, is resisted by the excess welding furnished.

Welding for 2nd diagonal (Detail B):

At angle heel, weld for $\frac{86}{2} \times \frac{3.375}{5} = 29$ kips.

Use $\frac{5}{16}$-in. fillet, length $= \frac{29}{3} = 9.67$, say 10 in.

At angle toe, weld for $\frac{86}{2} - 29 = 14$ kips.

Use $\frac{5}{16}$-in. fillet, length $= \frac{14}{3} = 4.67$, say 5 in.

Lower chord splice detail:

The stress at the splice point is 196 kips, or nearly equal to the maximum chord stress. The splice is designed to develop the full section of the chord member to provide connection for whatever reserve strength is available due to excess area in the member. Chord splices made up with fillet welded lap splice plates, as used in this design, are satisfactory only when the truss stresses are due essentially to static loading. For dynamic loading conditions full penetration butt welds should be used for these splices. The lower chord stress due to the three tramrail loading concentrations indicated in Fig. 128 is 35.8 kips, or only about 18 per cent of the total stress. An additional consideration in interpreting the relative dynamic effect is that these loads, which include an allowance for impact, would be applied simultaneously at all three locations at very infrequent intervals.

Area of flange plate required = area of Tee flange = $8.295 \times 0.74 = 6.13$ sq. in. For plate 7 in. wide, thickness required $= \dfrac{6.13}{7} = 0.876$ in. Use $7 \times \frac{7}{8}$-in. plate. Weld for $6.13 \times 20 = 122.6$ kips to develop full plate strength. Use $\frac{3}{8}$-in. weld, length required $= \dfrac{122.6}{3.6} = 34$ in. With a plate 2 ft. 9 in. long, tapered to a width of 2 in. at each end as shown, the actual weld length furnished on each side of the center line of the splice is approximately 35 in. The circle on the welding symbol line indicates that a fillet weld of the size shown shall be applied continuously along all edges of the plate.

Area of plate each side of stem $= \dfrac{10.73 - 6.13}{2} = 2.3$ sq. in. For plate 7 in. wide, thickness required $= \dfrac{2.3}{7} = 0.33$ in. Use $7 \times \frac{3}{8}$ in. plates.

Max. weld size $= \frac{3}{8} - \frac{1}{16} = \frac{5}{16}$ in. Length required $= \dfrac{7 \times .375 \times 20}{3} = 17.5$ in.

The detail shown provides approximately 18.5 in. of weld length on each side of center line or a total weld length of 37 in. per plate.

Top chord detail at center of truss:

Transfer of the horizontal component of the chord stresses at the peak joint is obtained by direct bearing of the chord members against a $\frac{1}{2}$-in. butt plate as shown in detail *C*. The ends of the members are milled, or finished, to provide true bearing against the plate. The $\frac{5}{16}$-in. continuous weld on each side holds the elements together as a unit and is not determined on the basis of any stress requirements. Welding indicated for diagonal and vertical members at this joint is somewhat arbitrary and, in each case, more than required for stress transfer to provide satisfactory detail.

The design loading and stress data, including welded fabrication details, for a wall-bearing truss on a short span is shown in Fig.

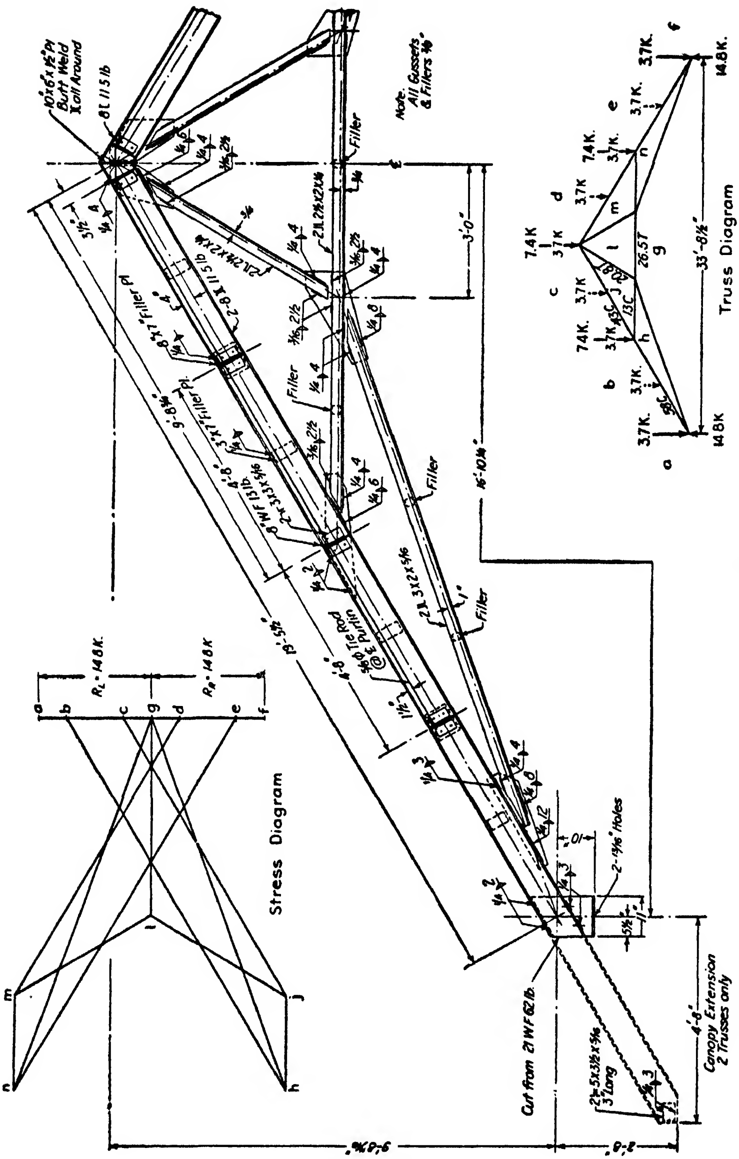

Stress Diagram
Truss Diagram
Note: All Gussets & Fillers 3/8"
Canopy Extension 2 Trusses only
Cut from 21 WF 62 lb.
10"x6"x1/2" Pl Butt Weld Roll Around
33'-8 1/2"
Filler

FIG. 129.

129. Architectural requirements determined the basic truss outline and a truss spacing of 16 ft. Precast concrete plank, supported by and clipped to the 8-in. beam or channel purlins, are covered with cement asbestos shingles.

The purlin reaction values shown on the truss diagram include an allowance for weight of truss. Loads applied to the truss chord at mid-panel lengths are proportioned to adjacent panel joints for purposes of stress analysis.

The procedure outlined in Art. 109 is followed in the design of the upper chord. With the purlins framed directly to the chord the unsupported length is reduced to 4 ft. 8 in. for the y–y axis (see Fig. 109).

Try two 8-in. 11.5-lb. channels.

Direct stress $= f_a = \dfrac{58}{6.72} = 8.63$ kips per sq. in.

Moment $= 0.8 \times 3.7 \times \dfrac{16.85}{19.46} \times \dfrac{9.73}{4} = 6.2$ ft.-kips.

Bending stress $= f_b = \dfrac{6.2 \times 12}{16.2} = 4.60$ kips per sq. in.

Max. stress $= f_a + f_b = 8.63 + 4.6 = 13.23$ kips per sq. in.

$r_{y-y} = 0.99$ in. $\quad \dfrac{L}{r} = \dfrac{56}{0.99} = 56.6.$

f (allowable) $= 15.45$ kips per sq. in. (A.I.S.C. Specification, 1946).

Trial section is satisfactory.

Note: Compare this design with the illustrative problem in Art. 109 in which total stresses are nearly the same.

The actual stress in the lower chord member h–g, with 2 angles $3 \times 2 \times \frac{5}{16}$, is 18.2 kips per sq. in. All other members are 2 angles $2\frac{1}{2} \times 2 \times \frac{1}{4}$. Weld sizes and lengths are based on strength requirements or, in some cases, on minimum specification requirement that length equal width of member. Where the latter condition controls, heel welds are increased to maintain center of weld resistance on the gravity axis of members.

142. Welded Framing Connections. — The design of the members in a steel frame building and the design of the connections joining the members must be carried out in accordance with the same basic assumptions. Members assumed to be simply supported or pin-connected to facilitate design cannot, logically, be rigidly fastened together in the actual structure. Again, if all frame joints are considered rigid or continuous in the design analysis they must be constructed to provide a reasonably complete degree of continuity.

The A.I.S.C. Specification (1946) clearly defines three basic types of frame design in terms of definite assumptions regarding rigidity of joint details. Each type is permissible under certain conditions, and each will specifically determine the sizes of members and the types and strength of connections required.

Type 1, or *rigid-frame,* design assumes that joint details are sufficiently rigid to prevent appreciable change in the angle between any two connected members at their juncture throughout the entire range of the design loading.

Type 2, or *conventional,* design assumes that joint details at the ends of beams and girders are capable of transferring the end reactions to the supporting members, either girders or columns, without preventing end rotation of the connected members. This type of design is also called unrestrained, free-ended, or simple framing.

Type 3, or *semi-rigid,* design assumes that end connections of beams and girders are detailed to provide an end moment resistance of known value less than that required for complete frame continuity.

In the past comparatively few steel buildings in the United States have been designed and built as fully continuous, or Type 1, structures. A building frame with rigid connections is statically indeterminate. Bending moment and shear stress intensities in any one member of the frame cannot be determined by considering that member as a separate unit and using only the fundamental equations of static equilibrium according to the methods discussed in Chapter II. The analysis of any member must consider the restraining effects, based on the relative elastic deformations, of all adjacent members in the same bent. Since elastic deformations depend upon cross section, a preliminary selection of member sizes in the bent must be made before the correct stresses can be determined. An extended discussion of the analysis of rigid frames is beyond the scope of this book. Several approximate methods of analysis often used to determine wind bracing requirements in buildings otherwise designed as Type 2 structures are discussed in Chapter VIII. These methods are not applicable, however, to the complete analysis of a rigid structure. The method of moment distribution,[2] which has been widely used in the design analysis of continuous reinforced concrete frames, is a practical method for readily determining stresses in a rigid steel frame with an acceptable degree of accuracy.

It is a reasonable assumption that Type 1 design will become more common as the application of welding to shop fabrication and field erection of structural steel is increased. There has been considerable discussion in technical literature regarding the overall economy of fully continuous frames for steel tier buildings. The additional design time required and the increased field welding costs incurred tend to offset the savings in material weight that can be effected by rigid-frame construction. However, for buildings over ten or fifteen stories high, the ability of the structural frame to resist lateral wind forces is an important design factor. The trend in modern buildings to replace heavy masonry walls and partitions with large glass areas and prefabricated panels necessitates a careful review of the empirical design methods now used. Steel frames, designed for full continuity and joined by welding, will result in stiffer buildings with a corresponding decrease in exterior and interior maintenance costs.

Type 1 construction is unconditionally permitted under the A.I.S.C. Specification. It is necessary that the frame be considered statically indeterminate, that a careful stress analysis be made, and that the normal unit working stress values allowed by the specification are not exceeded in the design of members and connections. Types of welded connections for continuous structures are illustrated in the references noted in Art. 139.

The design methods and procedures discussed throughout this book are applicable to the analysis and design of Type 2 steel frames. However, the design of welded connections to provide end reaction transfer for simply supported beams and girders must be given careful consideration. The angle of end rotation of a beam can be determined as readily as the maximum deflection. For example, the tangent of this angle for a 12-in. Wide Flange 27-lb. beam, simply supported at each end on a span of 20 ft. and uniformly loaded to produce a maximum fibre stress of 20,000 lb. per sq in., will equal 0.009. This value multiplied by the 12-in.

[2] See "Analysis of Continuous Frames by Distributing Fixed End Moments," by Prof. Hardy Cross, *Trans. Am. Soc. C. E.*, Vol. 96, 1932.

beam depth, or more than $\frac{1}{10}$ in., is the distance that the end of the top flange moves from a vertical line drawn through the end of the bottom flange. Connections must, therefore, be designed to be flexible and to allow this natural end rotation to take place.

The use of Type 2 construction is permitted under the A.I.S.C. Specification although certain stipulations are made regarding the design of wind bracing connections. In general beam-to-column connections must be flexible. At joints required to resist wind moments, computed according to a recognized empirical method as discussed in Chapter VIII, the connections must be designed to fulfill either of the following conditions:

(*a*) The safe capacity of the connections must be adequate to resist the moments induced by the gravity loading and the wind loading, at the increased unit stresses permitted therefor, or

(*b*) The connections, if welded and if designed to resist the computed wind moments, are so detailed that larger moments, induced by the gravity loadings, will be relieved by deformation of the connection material without overstress in the welds.

Standard double angle connections provide a reasonable degree of flexibility and constitute the most common connection type employed on riveted or bolted conventional steel structures. Both stiffened and unstiffened seat connections, often used for members framing to a column web, are satisfactory for riveted work if designed in accordance with the methods illustrated in the A.I.S.C. Manual

Corresponding types of flexible welded connections are shown in Fig. 130. In the design of each of these types it is necessary to proportion the welds for combined stress due to the eccentric distance from the center of the weld to the action line of the applied load. Although this eccentricity is relatively small in each case, the resultant weld stress may be very much greater than the value obtained by dividing the reaction by the total weld length. Since stress due to bending is always a function of the square of the depth of the resisting section, the combined effect of bending and direct stress is more critical for short weld lengths.

Formulas generally used in the design of welded connections, together with tables and charts showing safe design loads for many standard types, are included in the references mentioned in Art. 139. The design value of a double angle connection similar to Fig. 130*a*, using angles 8 in. long with $3\frac{1}{2}$-in. outstanding legs and

$\frac{1}{4}$-in. fillet welds placed as shown in the illustration, is equal to 20.6 kips. As a comparison, to show the effect of eccentricity,

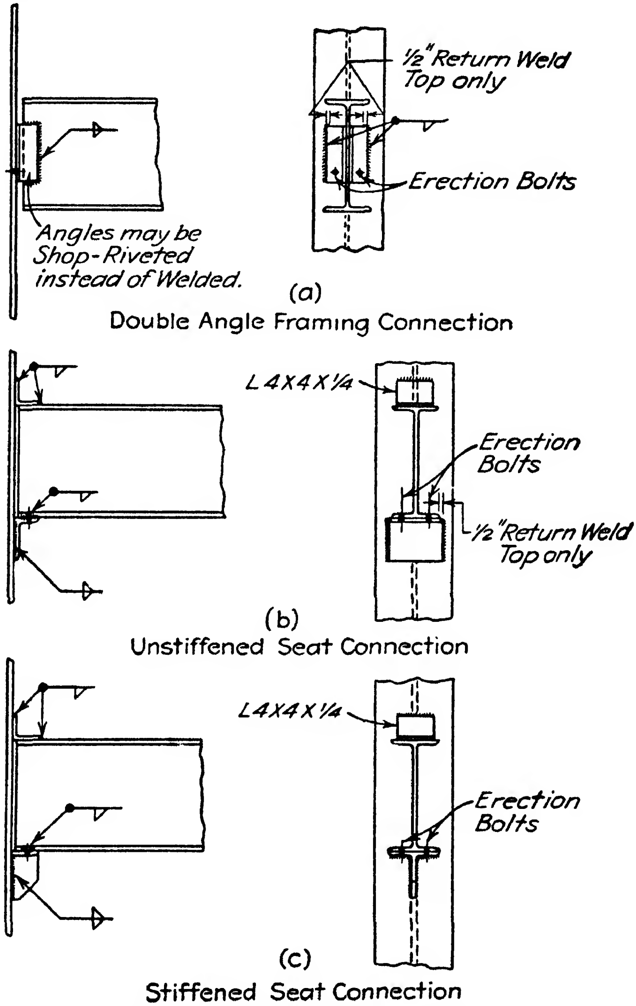


FIG. 130. Welded Framing Connections.

the field welding would have a design resistance to direct loading in the plane of the welds equal to $2 \times 8 \times 2.4$, or 38.4 kips.

The returns shown at the top of the vertical welds in Fig. 130*a* and *b* are not considered in the design length but are used to eliminate crater effect at the point of critical weld stress. For the double angle type the return length should be no greater than shown, as the flexibility of the connection would be correspondingly decreased. For the unstiffened seat connection the return is limited to provide proper seating for the beam. The top clip angle shown for each type of seat connection serves only to provide lateral support for the beam. For this reason, heavy angles with short legs should not be selected and welding should be placed only along the toe edges as shown. These angles are shipped to the job as loose pieces and welded to both members in the field. Field welds connecting the lower flange of the beam to the seat are not required for stress transfer. A $\frac{1}{4}$-in. fillet, 2 or 3 in. long on each side, is satisfactory.

Type 3 construction is permitted by the A.I.S.C. Specification only upon evidence that the connections are capable of developing definite resisting moments without overstress of the welds. Designs of beam-to-column connections with a statement of the experimentally determined bending resistance thereof will probably be published by the American Institute of Steel Construction. In the absence of such designs at the present writing, further discussion of this type of construction is not justified.

143. Column Details. — It is accepted practice to design and fabricate column sections in steel tier buildings in two-story lengths and to locate splices at a definite distance, usually 1 ft. 6 in. above the finished floor level. The ends of all sections are milled or planed at right angles to the column axis to provide for transfer of stress by bearing.

A rolled-steel plate is used to distribute the column load at the base over sufficient area to maintain the unit bearing within the allowable value for the footing concrete. These plates are designed according to the procedure outlined in Arts. 83 and 84. On large buildings the base plates are shipped loose, set to proper level, and grouted in place before erection of the columns. Anchor bolts, imbedded in the concrete footing, project above the base plate to engage lug angles, shop welded to the column flanges or web, and secure the column as illustrated in Fig. 87*b*. If the base plate is welded to the column at the fabricating shop care must be exercised to provide a uniform bearing under the plate in its final position. The base plates can be set on wedge-shaped steel or hardwood shims and grouted after the entire first tier of columns is plumbed and leveled. Another method is to prepare the top of the concrete accurately to proper level and insert a thin lead plate between the concrete and the base plate to compensate for minor irregularities.

In some instances a thin steel plate equal in size to the base plate is grouted in place at the correct elevation and the base plate set directly thereon. This method is not recommended for large plate sizes because of the difficulty of attaining true bearing over the entire area.

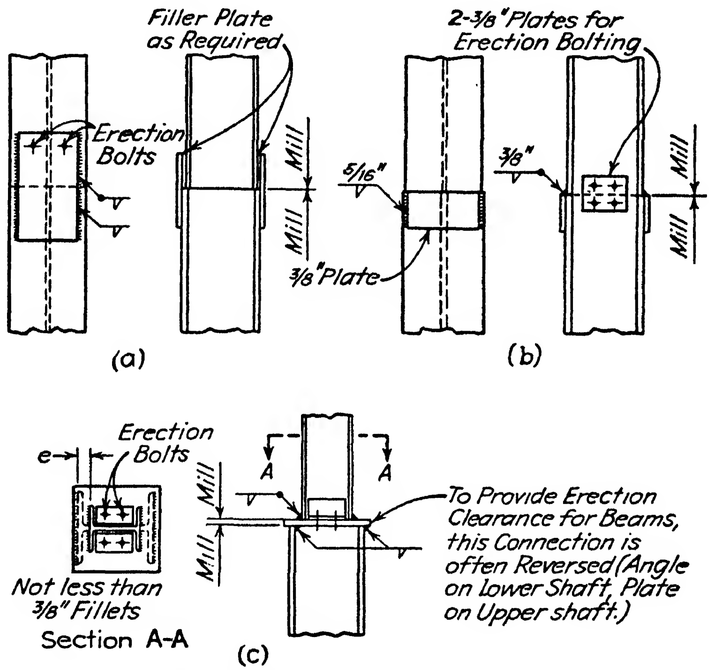


FIG. 131. Typical Welded Column Splice Details.

Several typical welded column splice details are shown in Fig. 131. When both members are Wide Flange sections of the same nominal depth the clear dimension inside of the flanges is constant and full bearing is provided for the upper member. When detail (*a*) is used fillers must be welded to the flanges of the upper section to compensate for any appreciable variation in over-all depth. The plate welded to the lower member in detail (*b*) is provided to receive the horizontal leg of the fillet weld when the depth variation between upper and lower members is less than twice the weld size, which is the usual condition. Detail (*b*) may be employed when the steel is shop riveted and field welded, in which case the plate is riveted to the lower column section in the shop. The splice in detail (*c*) is frequently used when the upper and lower sections are not the same nominal depth. The thickness of the butt plate is computed to provide for a bending moment equal to the load in the upper column flange multiplied by the offset distance *e* between the centers of the upper and lower flanges. All these standard details employ nominal amounts of welding and are satisfactory for normal designs. Where the analysis indicates high bending moments in the columns at the splice sections, the welding must be sufficient to provide for any resultant tension stresses.

PROBLEMS

1. Determine the safe design load per inch of length, based on an allowable unit shear value of 11,300 lb. per sq. in. on the throat section, for each of the following sizes of fillet welds: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{3}{4}$ in.

2. Compare the values obtained in Problem 1 with the safe design loads for corresponding fillet weld sizes based on using the unit shear value permitted under the A.I.S.C. Specification (1946).

Note: Use A.I.S.C. Specification (1946) for the following problems.

3. Design the third and fourth main diagonal members and the third main vertical member for the truss in Fig. 128. Use double angles for each member.

4. Design the welding for each of the members in Problem 3, and detail the lower chord joint at which they meet.

5. For the vertical member in Problem 3, determine the maximum allowable spacing between points at which the two angles should be tied together. Show in detail an acceptable method of effecting the tie for welded trusses.

6. Determine the maximum horizontal increment of flange stress in panels *f–g* and *g–h* of the girder shown in Fig. 126. Compare these results with the safe resistance per unit of length furnished by the actual web-to-flange welding as shown.

CHAPTER X

BUILDING DESIGN PROJECT

144. Introduction. — The first step in the design of the structural frame for any building is the determination of the loads that the structure must support. The live load to be provided for depends on the conditions of occupancy, and the dead load, on the type of floor and wall construction. As stated in Art. 3, the live load is usually expressed in pounds per square foot of floor area, the value being taken large enough to cover the effect of ordinary concentrations. The dead load on any member is computed from the weight of the permanent construction supported by that member, the quantities being estimated from the architectural and structural plans as they are developed.

145. Effect of Beam Weight. — Since the weights of beams and girders, together with their fireproofing, are not known until after the members are designed, some system for estimating such weights must be adopted. It is customary practice among experienced designers to include the weight of beams and fireproofing in the design load by assuming an equivalent load per square foot, acting over the floor area. The value of the equivalent load depends upon the type of floor construction and the general arrangement of the framing. It is determined for any particular case by dividing the total weight of structural steel and fireproofing in a typical floor of a building of similar construction by the floor area. Considerable judgment must be exercised with this method, if accurate results are to be obtained, since special conditions of framing often require beams of greater weight than the allowance provided by the equivalent load.

In general, the procedure of Art. 21 has been followed in the design of the floor framing for the building considered in this chapter, except that safe load tables have also been used for members carrying uniform loads. A table has been prepared in Art. 149 to facilitate the determination of the beam weight and fireproofing for Step (7) of the procedure.

146. Design Problem. — The remainder of this chapter is devoted to the design of typical members of the steel framework for a small business building. The architectural drawings are shown on Plates I to V and the structural drawings on Plates VI to IX. Although additional drawings would be required in order to make a complete design, the ones shown in Plates I to IX are sufficient for the purposes of this example.

General Arrangement. The building is of the type that might be used as a clothing store or a small department store. The shorter side faces a main thoroughfare and the longer side an important side street. The rear wall forms one side of a narrow alley and the remaining wall is against an adjacent building. The walls of the first story that face the two streets are devoted to show-window space.

Elevator and Stairs. The elevator at the rear is for both passengers and freight. The rear stairway is enclosed with fireproof walls but the front one is open, except the flight running from the first floor to the basement. The second and third floor plans are similar to the first except that there is no show-window space. The front stairway terminates at the third floor while the rear one continues to the pent house. It is assumed that the basement is used for storage purposes, toilets, etc., and that no heating plant is required since steam is supplied to the building from a main in the street.

Walls. The exact distance from the center lines of exterior columns to the outside face of the wall depends upon the spandrel detail but is taken as 12 in. for design purposes. The exterior walls on the two street facades consist of a 4-in. ashlar limestone veneer backed up with 8 in. of common brick. The inner surface is furred with 2-in. split tile and plastered. The limestone veneer returns on the alley wall a distance of 3 ft., the remainder being faced with a light colored brick. The fourth wall consists of 12 in. of common brick plus the furring and plaster. The basement walls are of reinforced concrete 16 in. thick. The pent house walls are of brick 12 in. thick, plastered on the inside.

Partitions. Interior partitions around the elevator well, stairway, etc., are built of terra cotta block 6 in. thick. The inside of the elevator shaft is not plastered but all other interior wall surfaces have a $\frac{3}{4}$-in. plaster coat.

Floors and Roof. The floor construction consists of a 4-in. stone concrete slab supported on steel beams spaced not more than 8 ft. apart. There is a 3-in. cinder fill and 1-in. cement finish on top of the floor slabs and the undersides are plastered. The roof slab is also 4 in. thick covered with 4-ply roofing felt and gravel. An allowance for 3 in. of cinder fill over the entire roof has been included in order that a slight pitch may be given to the finished surface. The underside of the roof slab is not plastered but an allowance has been made for a suspended ceiling in the top story.

Fireproofing. All beams and girders are surrounded with 2 in. of concrete fireproofing. The interior columns are fireproofed with terra cotta and the wall columns with brick.

Loads. The live load on the first floor is 100 lb. per sq. ft.; on the second and third floors, 50 lb. per sq. ft.; and on the roof, 40 lb. per sq. ft. In addition, an allowance of 20 lb. per sq. ft. for movable partitions must be included in the second and third floor loads. All stairs are designed for a live load of 100 lb. per sq. ft. of horizontal projection. The weights of building materials used in computing the dead loads are taken from the table in Art. 3. The structural design is governed by the specifications of the American Institute of Steel Construction (1946 revision) unless otherwise noted.

147. Floor Loads. — The design load in pounds per square foot, supported by the framing of any floor, is equal to the sum of the live and dead loads on that floor. For example, the live load on the first floor is 100 lb. per sq. ft. The weight of stone concrete as given in the table of Art. 3 is 144 lb. per cu. ft. Since the slab in this building is 4 in. thick, its weight per square foot of floor area is $\frac{4}{12} \times 144 = 48$ lb. The cinder fill weighs $\frac{3}{12} \times 60 = 15$ lb. per sq. ft. The weight of the 1-in. cement finish and the plaster are given in the table as 12 and 5 lb. per sq. ft., respectively. Hence the design load for the first floor framing is $100 + 48 + 15 + 12 + 5 = 180$ lb. per sq. ft. The design loads in pounds per square foot for the remaining floors and the roof have been computed in a similar manner and recorded below.

First Floor

Live Load	100
4-in. Concrete Slab	48
3-in. Cinder Fill	15
1-in. Cement Finish	12
Plaster	5
Total Load Per Sq. Ft.	180 lb.

Second and Third Floors

Live Load	50
Partitions	20
Slab, Fill, Finish, Plaster	80
Total Load Per Sq. Ft.	150 lb.

Roof

Live Load	40
4-in. Concrete Slab	48
3-in. Cinder Fill	15
Roofing (4-ply felt and gravel)	6
Suspended Ceiling	10
Total Load Per Sq. Ft.	119 lb.

Stairs

It is assumed that steel stairs with concrete or terrazzo treads are used throughout the building. The weight of such construction varies with the particular type employed but an allowance of 50 lb. per sq. ft. of horizontal projection is ample for ordinary cases. Using this value, the design load is:

Live Load	100
Dead Load	50
Total Load Per Sq. Ft.	150 lb.

148. Wall Loads. — The weights of walls and partitions per square foot of surface may also be computed and tabulated for reference. For example, the weight of one square foot of the exterior stone-faced wall is determined as follows:

Limestone weighs 160 lb. per cu. ft. Since the stone facing is 4 in. thick, the weight of a piece 1 ft. square is $\frac{4}{12} \times 160 = 54$ lb. (approximately). The 8-in. common brick backing weighs $\frac{8}{12} \times 120 = 80$ lb. per sq. ft. The table of Art. 3 gives 8 lb. per sq. ft.

for the weight of 2-in. terra cotta split-furring, and 5 lb. per sq. ft. for plaster. Hence the weight of the exterior stone-faced wall is $54 + 80 + 8 + 5 = 147$ lb. per sq. ft. of surface. Similar weights for other typical wall and partition sections have been computed and recorded below.

Exterior Wall — Stone Faced

4-in. Limestone	54
8-in. Common Brick	80
2-in Terra Cotta Furring	8
Plaster	5
Total Weight per Sq. Ft. of Surface	147 lb.

Exterior Wall — Face Brick

4-in. Face Brick	47
8-in. Common Brick	80
2-in. Terra Cotta Furring	8
Plaster	5
Total Weight per Sq. Ft. of Surface	140 lb.

Exterior Wall — Common Brick

12-in. Common Brick	120
2-in. Terra Cotta Furring	8
Plaster	5
Total Weight per Sq. Ft. of Surface	133 lb.

Partitions — 6-in Terra Cotta

6-in Terra Cotta Block	22	22
Plaster (two sides)	10	
Plaster (one side)		5
Total Weight per Sq. Ft. of Surface	32 lb.	27 lb.

149. Weight of Fireproofing. — Inasmuch as the equivalent load method discussed in Art. 145 is not used in this chapter, it will be found convenient to prepare a table of fireproofing weights for beams of different depth. Figure 132*a* shows a section taken through the floor beams supporting the concrete slab, and Fig. 132*b* is a detail of the beam section drawn to a larger scale. The area of the floor panel supported by any beam is equal to the span length multiplied by the sum of half the distances to adjacent beams. Since panel dimensions are usually measured from center to center of supporting members, the product obtained by multiplying the design load in pounds per square foot by the panel area gives the total load brought to the beam. However, this load does not include the weight of the beam itself nor that portion of the fireproofing which projects below the under surface of the slab.

The fireproofing specifications for this building, as stated in Art. 146, require the beams to be surrounded by 2 in. of concrete. Referring to Fig. 132*b*, the cross-sectional area of the fireproofing in square inches is equal to $a \times (b + 4)$, where b is the flange

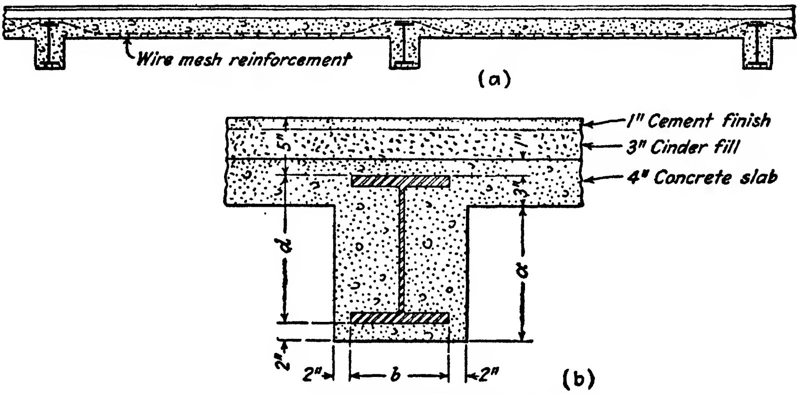


FIG. 132. Typical Beam Fireproofing.

width of the beam and a is the projection below the under surface of the slab. With the beam set 1 in. below the top of the structural slab, it is evident from the figure that

$$a = d + 2 - 3 = (d - 1)$$

making the expression for cross-sectional area of fireproofing equal

$$(d - 1) \times (b + 4) \text{ sq. in.}$$

Dividing this expression by 144 gives the area in square feet, and multiplying by 1 ft. gives the *volume* of fireproofing per linear foot of beam. The *weight* of fireproofing per linear foot is then obtained by multiplying this value by the weight of 1 cu. ft. of concrete (144 lb.). Introducing a symbol for fireproofing weight,

$$W_{FP} = \frac{(d - 1) \times (b + 4)}{144} \times 1 \times 144$$

or

$$W_{FP} = (d - 1) \times (b + 4) \text{ lb. per lin. ft.}$$

It should be noted that the numbers in the above expressions depend upon the thickness of fireproofing, the thickness of the structural slab, and the distance from the tops of the steel beams to the top of the slab. Consequently, where conditions differ materially from those shown in Fig. 132, another formula should be set up.

Using the above relation, the weight of fireproofing for a 12-in. 27-lb. Wide Flange section is computed by substituting its depth, 12 in., and flange width, 6.5 in. (found from a steel handbook) in the formula. Hence

$$W_{FP} = (d - 1) \times (b + 4) = 11 \times 10.5 = 115 \text{ lb. per lin. ft.}$$

The weights of fireproofing for several other beams have been computed in a similar manner and recorded in the following table. Minor variations in depth and flange width for beams not listed may be neglected. The area of the steel section has not been deducted from the fireproofing area as refinements of this nature are not warranted.

Weight of Concrete Fireproofing per Linear Foot for Certain Selected I-Beams and Wide Flange Sections

Section	*Flange Width*	*Lb. per Lin. Ft.*
6 I 12.5	$3\frac{3}{8}$	37
7 I 15.3	$3\frac{5}{8}$	46
8 I 18.4	4	56
8 WF 17	$5\frac{1}{4}$	65
8 WF 31	8	84
10 WF 25	$5\frac{3}{4}$	88
10 WF 33	8	105
12 WF 27	$6\frac{1}{2}$	115
12 WF 40	8	132
14 WF 30	$6\frac{3}{4}$	140
14 WF 43	8	152
16 WF 36	7	165
18 WF 50	$7\frac{1}{2}$	196
21 WF 62	$8\frac{1}{4}$	245
24 WF 76	9	300
27 WF 94	10	360

150. Choice of Sections. — In general, Wide Flange sections and the Miscellaneous Light Beams will be found more satisfac-

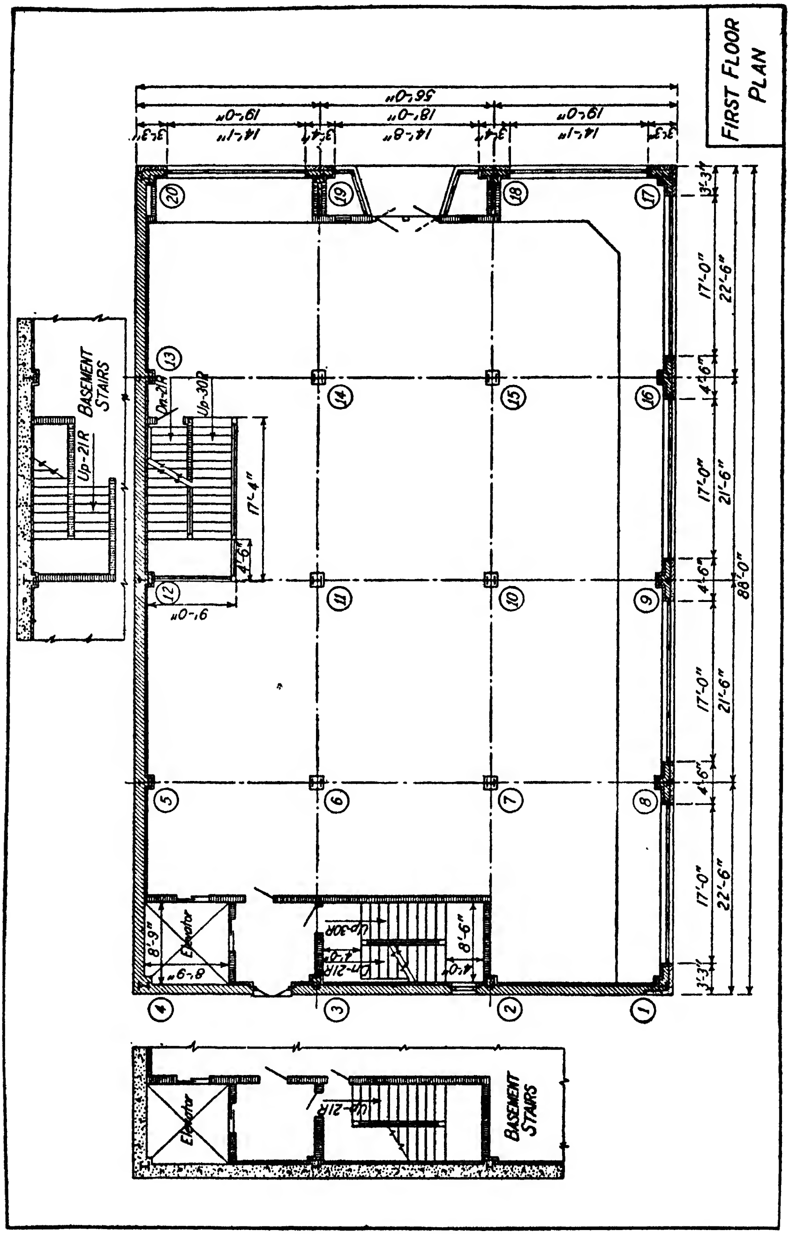


PLATE I. First Floor Plan.

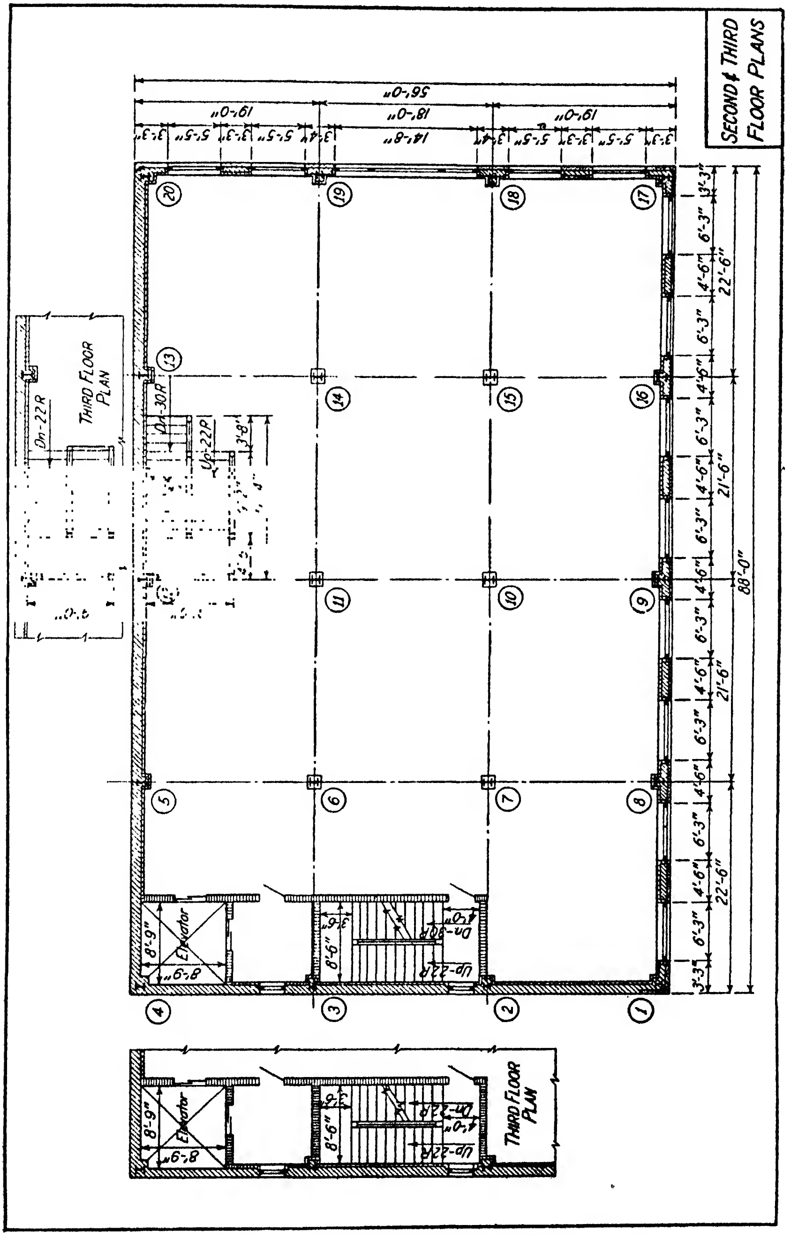


Plate II. Second and Third Floor Plans.

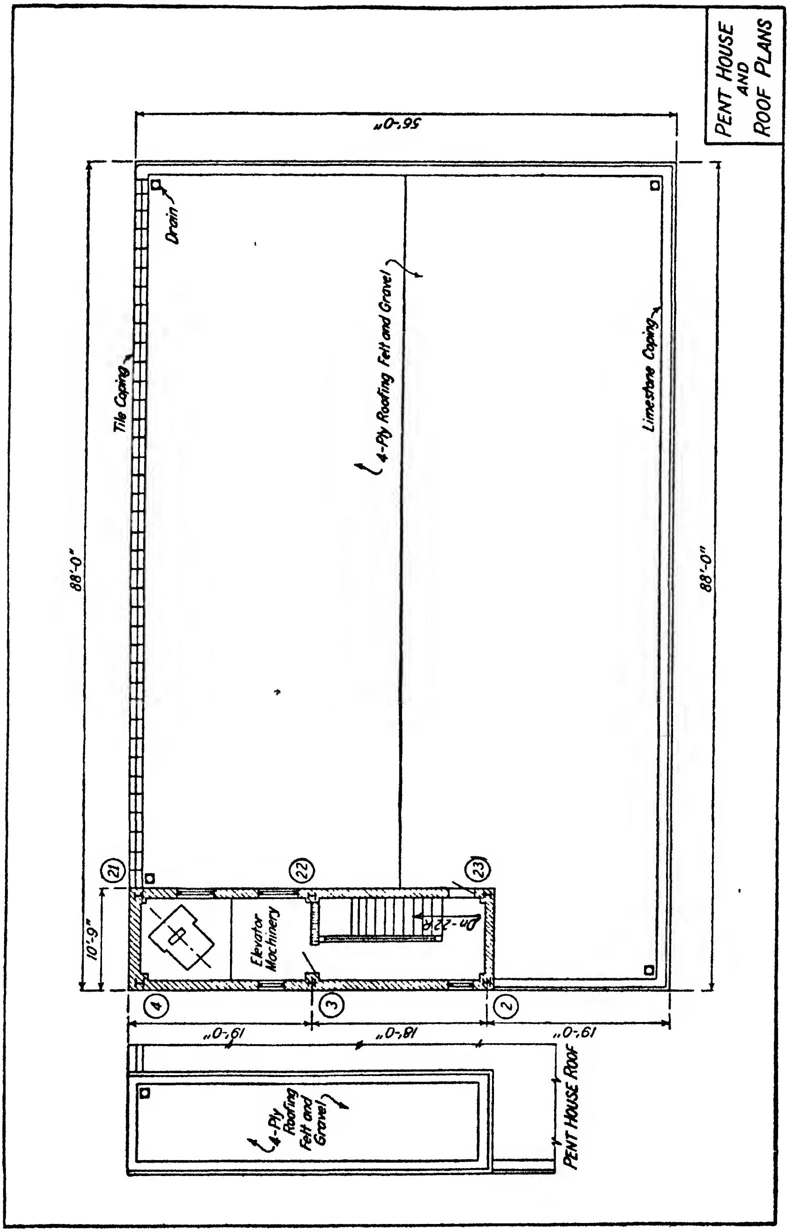


PLATE III. Pent House and Roof Plans.

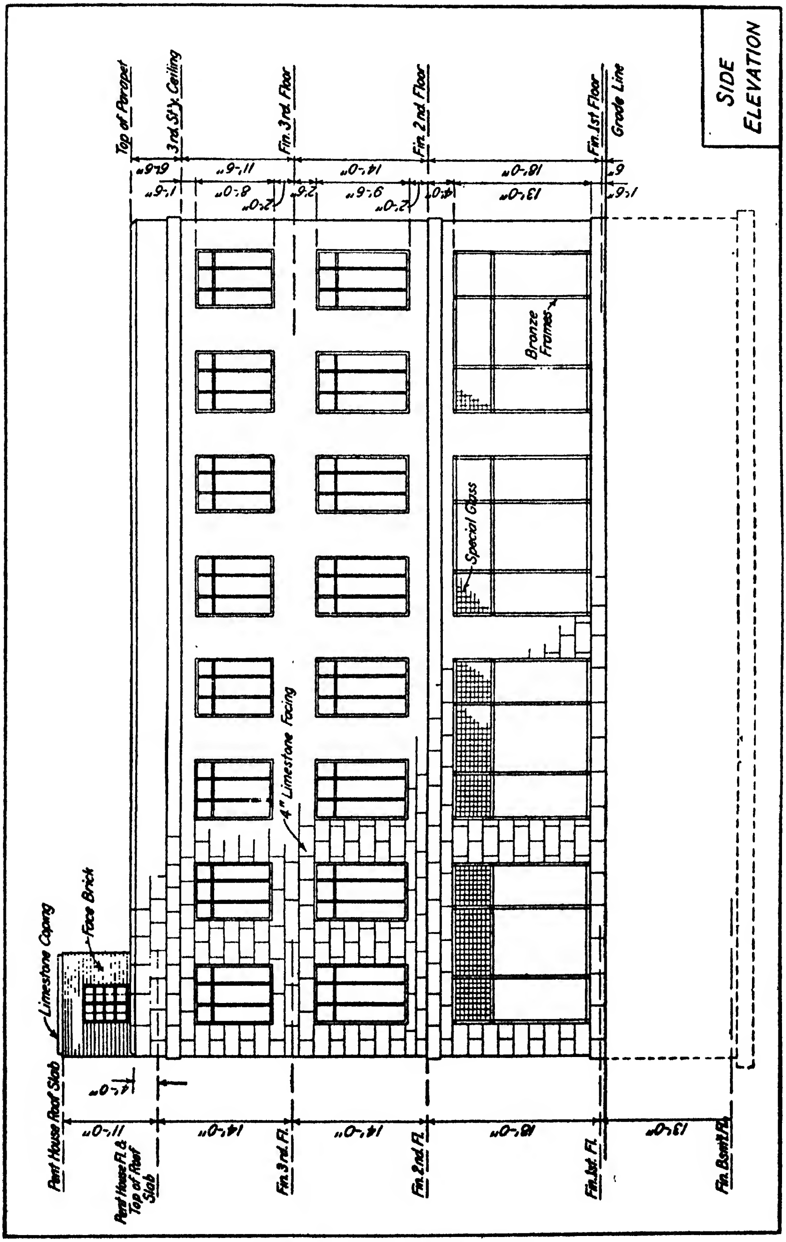


Plate IV. Side Elevation.

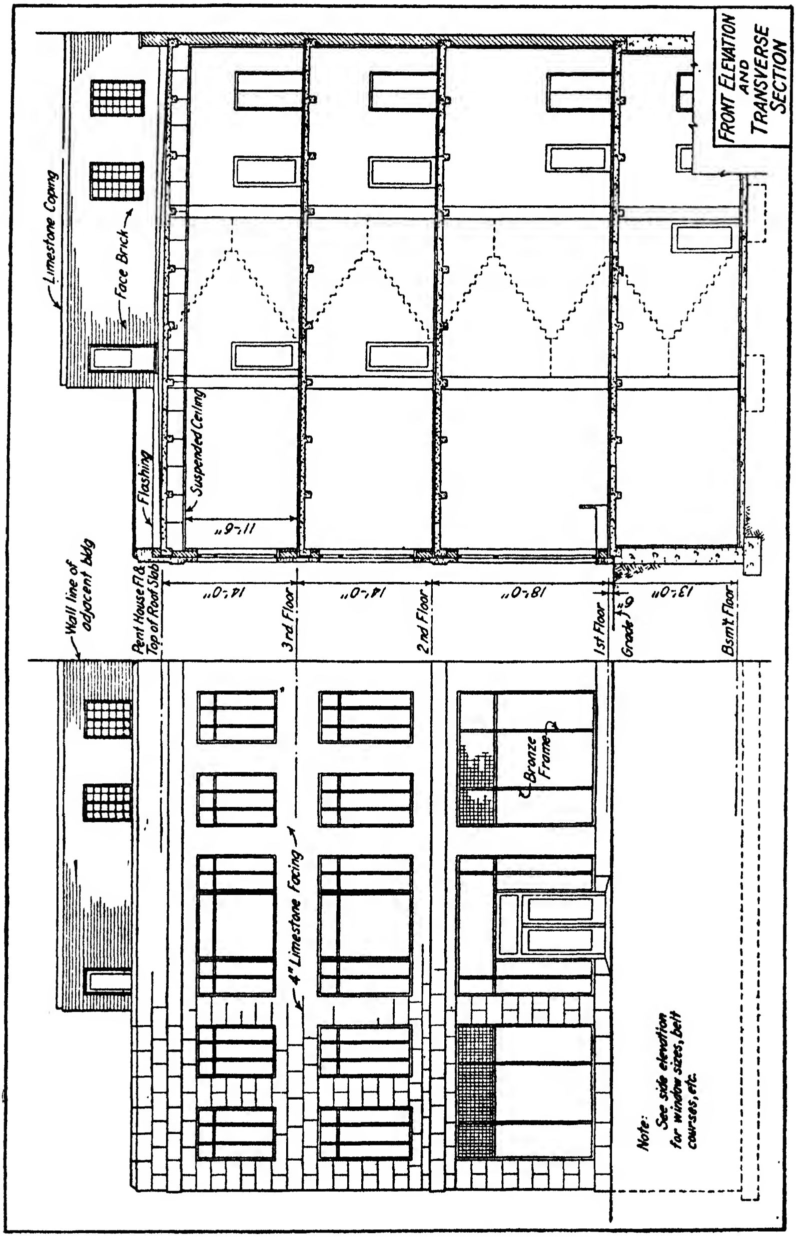


Plate V. Front Elevation and Section.

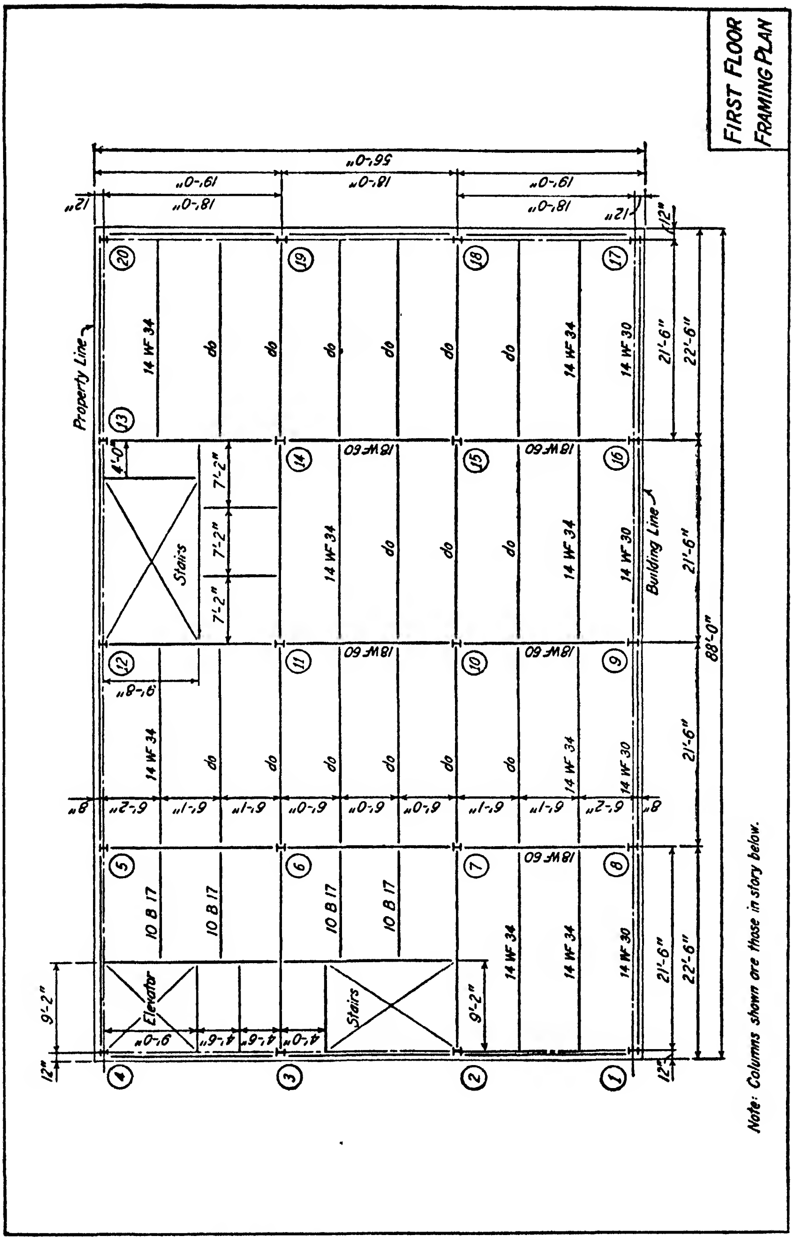


PLATE VI. First Floor Framing Plan.

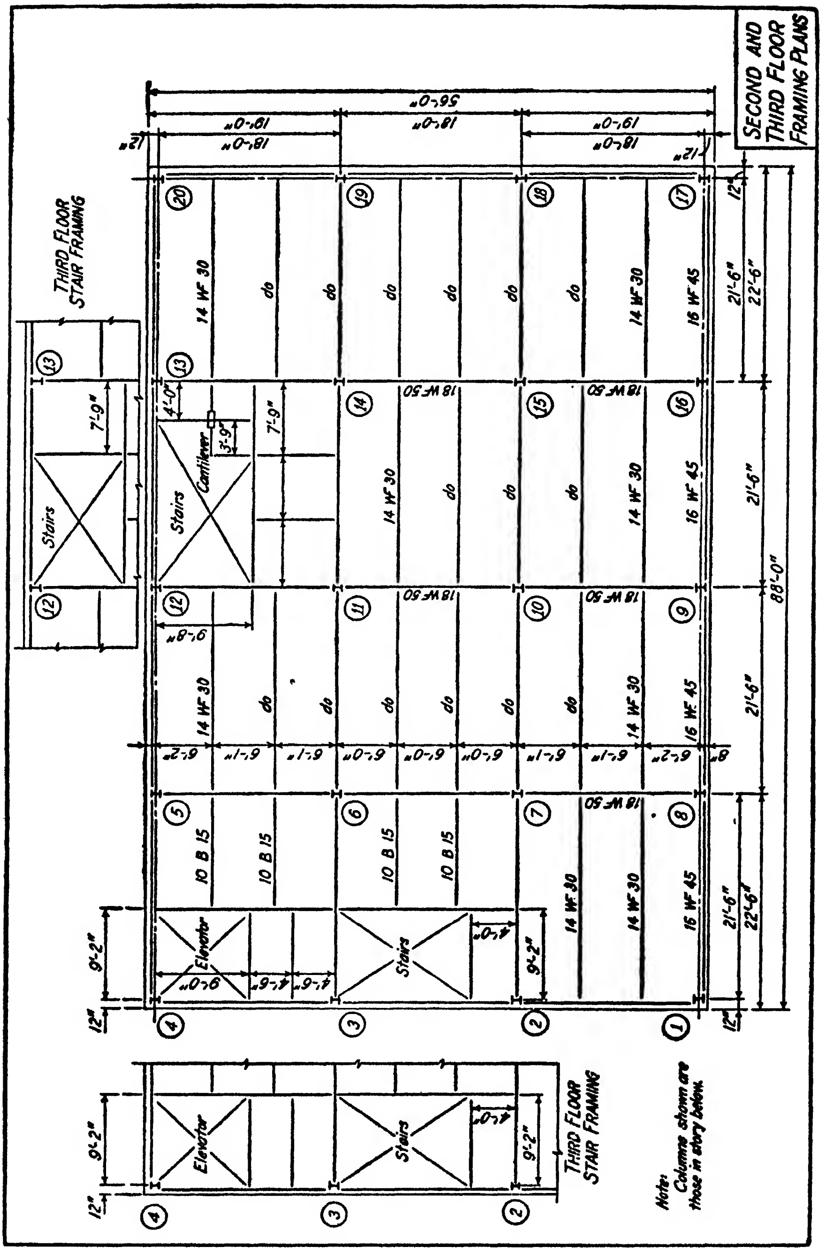


PLATE VII. Second and Third Floor Framing Plans.

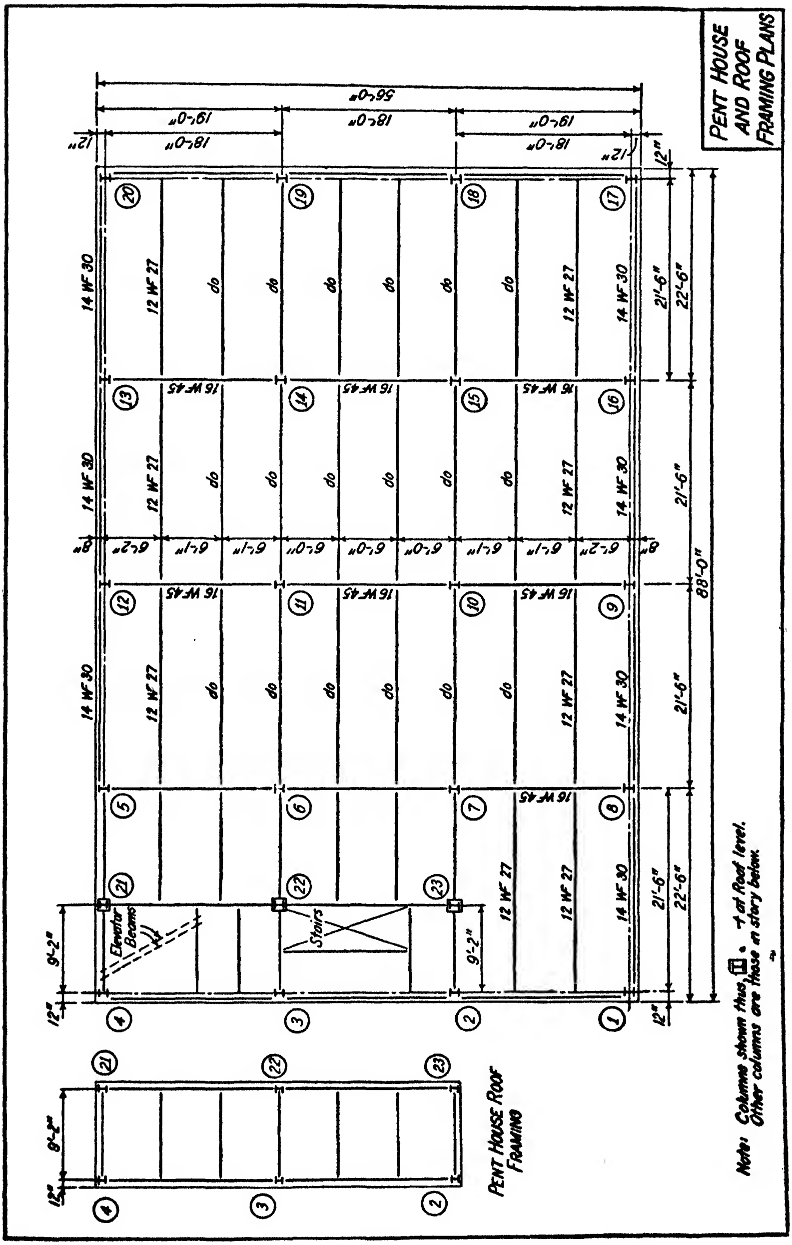
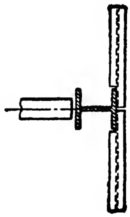


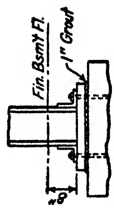
PLATE VIII. Pent House and Roof Framing Plans.

COLUMN SCHEDULE AND DETAILS

Story \ Col. No.	Story height	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Pent House Roof																								
Roof & P.H. Floor	11'-0"																					6 x 6B 15½	6 x 6B 15½	6 x 6B 15½
Third Floor	14'-0"								47 8 WF 40	47 8 WF 40	62 8 WF 35					62 8 WF 35	47 8 WF 40							
Second Floor	14'-0"								119	119	137					137	119							
First Floor	18'-0"								191 10 WF 60	191 10 WF 60	213 10 WF 72					213 10 WF 72	191 10 WF 60							
Bsm't. Floor	13'-0"								252	252	302					302	252							
Rolled Steel Slab Bases									24 x 22 x 2½	24 x 22 x 2½	27 x 24 x 2½					27 x 24 x 2½	24 x 22 x 2½							



SPANDREL FRAMING



DETAIL OF COLUMN BASE

PLATE IX. Column Schedule.

tory for building construction than American Standard Beams and Channels. The manufacturers of Wide Flange shapes have standardized their respective sections so that corresponding sections from the different mills have practically the same properties. The system of designating structural shapes on drawings, recommended in the A.I.S.C. Manual, is used throughout this chapter.

FIRST FLOOR FRAMING

151. The Structural Plan. — The framing plan for the first floor is shown on Plate VI. The most suitable arrangement of beams and columns is determined from a study of the architectural plan (Plate I). In practice, the exact dimensions are not recorded on the framing plans until most of the members have been designed, since clearances around elevator wells, stairways, etc., often necessitate slight changes in the beam locations. In this problem, several dimensions have been recorded on the structural plans in order to facilitate the computations, but it should be borne in mind that small changes may be made later without redesigning the members.

Beams and girders are not usually numbered on the framing plans since numbers are subsequently assigned to members on the erection plans prepared by the fabricating company. Separate *key* plans with beam numbers are sometimes used by the designer. In this problem, the beam under design is designated by its position in relation to a column. For example, the beam extending between columns 6 and 11 is denoted thus —(6)→. The beam directly below this one (when viewed from the lower edge of the drawing) is —(6)—. In all cases the line with the arrow indicates the member under consideration.

Unless otherwise noted, the tops of all floor beams and girders are set 5 in. below the elevation of the finished floor as shown in Fig. 132, the beams being coped as shown in Fig. 56*c* where they frame into the girders. The roof beams and girders frame with their top flanges flush, 1 in. below the top surface of the concrete structural slab.

Design computations for typical members only are given in the text which follows. The correct sizes of certain other beams and girders have been recorded on the plans; many others have been left blank to be used as individual problems.

152. Typical Members. — Typical members are those which occur most frequently in the design. It is evident from a study of Plate VI that the conditions governing the design of beam 6 apply to several other members. Hence this beam is designed first. Although the panel on one side is 6 ft. 1 in. and on the other 6 ft. 0 in., this slight difference may be neglected.

Beam 6

Span = 21.5 ft.
Floor area supported = 6 × 21.5 = 129 sq. ft.
Load (see Art. 147) = 180 × 129 = 23,200 lb.

From a safe load table based on the 20,000-lb. unit stress permitted by the A.I.S.C. Specification (1946) (Art. 146), a 14 WF 30 is found to carry 26 kips on a span of 21.5 ft. The weight of the beam and fireproofing (see table on page 227) is 30 + 140 = 170 lb. per lin. ft., making the total *weight* 170 × 21.5 = 3660 lb. The total *load* to be carried is, therefore, 23,200 + 3660 = 26,860 lb. or 26.9 kips. Since this value exceeds the allowable load (26 kips) given in the safe load table, the 14 WF 30 is too light and a 14 WF 34 (allowable load 30 kips on 21.5-ft. span) is selected. This section is recorded on the framing plan above all beams subjected to similar loading conditions. Each end reaction is approximately 13.5 kips.

Beam 19

The span of this member will be a few inches longer than 21.5 ft., owing to offsetting the spandrel beam from the column center line. However, as this distance will be something less than 6 in. in this case, it may be neglected and a 14 WF 34 selected as above. The end reactions may be assumed as 13,500 lb.

Girder 11

Span = 18 ft. (between center lines of columns).

Loads. The girder supports two typical beams at each third-point of the span. Since the reactions of the typical beams each equal 13,500 lb., the load at each third-point is 27,000 lb. The load diagram is shown in Fig. 133.

From Table I, the maximum bending moment occurs at the center and is

$$M = \frac{PL}{3} = \frac{27{,}000 \times 18}{3} = 162{,}000 \text{ ft.-lb. or } 1{,}950{,}000 \text{ in.-lb.}$$

The required section modulus is

$$S = \frac{M}{f} = \frac{1{,}950{,}000}{20{,}000} = 97.5$$

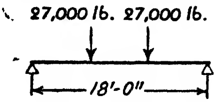


FIG. 133.

Try an 18 WF 60. This beam has a section modulus of 107.8. The total weight of the beam and fireproofing is $18 \times (60 + 196) = 4610$ lb. The bending moment due to the beam weight and fireproofing is

$$M = \frac{WL}{8} = \frac{4610 \times 18 \times 12}{8} = 125{,}000 \text{ in.-lb.}$$

The total moment is $125{,}000 + 1{,}950{,}000 = 2{,}075{,}000$ in.-lb.

The total required section modulus is

$$S = \frac{M}{f} = \frac{2{,}075{,}000}{20{,}000} = 104$$

Since the 18 WF 60 has an S greater than 104, it is adopted. End reactions $= 27{,}000 + \frac{4610}{2}$ or approximately 30 kips.

The above design may readily be checked by using safe load tables as explained in Art. 33. The Equivalent Tabular Load Factor for equal concentrated loads at the third-points of the span is 2.67 (see table on page 57). Therefore, the equivalent uniform load is $2.67 \times 27 = 72.1$ kips. The weight of the beam and fireproofing was found to be 4610 lb. or 5.0 kips, making the *total* equivalent uniform load $72.1 + 5.0 = 77.1$ kips. By scanning a set of safe load tables based on a 20 kip per sq. in. stress, it will be found that an 18 WF 60 will support 80 kips on an 18-ft. span.

153. Spandrel Beams. — There are two general types of spandrel beams as illustrated by —8→ and 1— on Plate VI. The former supports the wall and a uniform floor load whereas in the latter the floor load is applied at the third-points. This second type is sometimes called a spandrel girder. Spandrel beams are designed like any other beam, once the load diagram is drawn. For purposes of illustration, beam —⑧→ will be designed.

The portion of wall between the top of the large show windows and the second floor level (see Plate IV, Side Elevation) would probably be supported by a lintel hung from the second floor spandrel beams, or else carried on a separate lintel framing directly to the columns. Hence it may be assumed that none of the wall above the window heads is supported by the first floor spandrel beams. With this in mind, the load diagram for beam —⑧→ is constructed as follows (see Fig. 134):

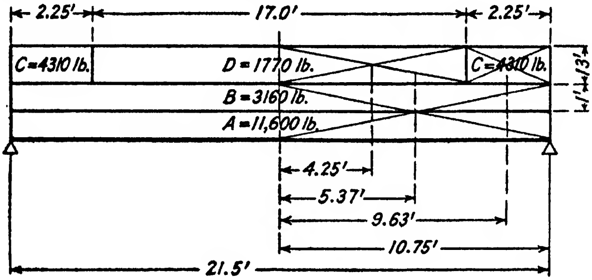


FIG. 134. Load Diagram — Spandrel Beam —⑧→.

The span length, area of window, distance between beams, etc., are found from Plates I, IV, and VI, and the unit loads from Arts. 147 and 148. The inch-dimensions are reduced to decimals of a foot for convenience. The area *A* represents the floor load; *B*, the weight of wall below the sill; *C*, the weight of the wall areas adjacent to the window; and *D*, the weight of the window sash, frame, and glass (see table in Art. 3).

$$
\begin{aligned}
A &= 180 \times (21.5 \times 3) &&= 11{,}600 \\
B &= 147 \times (21.5 \times 1) &&= 3{,}160 \\
C &= 147 \times (2.25 \times 13) && \begin{cases} = 4{,}310 \\ = 4{,}310 \end{cases} \\
D &= 8 \times (17 \times 13) &&= \underline{1{,}770} \\
& \qquad\qquad \text{Total load} &&= 25{,}150 \text{ lb.}
\end{aligned}
$$

Since the beam is symmetrically loaded, each reaction due to applied load is approximately equal to 12,600 lb. and the maximum bending moment occurs at mid-span. From Fig. 134, the value of the maximum moment is

$$M = (12{,}600 \times 10.75) - \left(\frac{11{,}600 + 3160}{2}\right) \times 5.37 - (4310 \times 9.63)$$
$$- \left(\frac{1770}{2} \times 4.25\right) = 135{,}000 - 39{,}600 - 41{,}500 - 3760$$
$$= 50{,}140 \text{ ft.-lb.}$$

This moment requires a section modulus of

$$S = \frac{M}{f} = \frac{50{,}140 \times 12}{20{,}000} = 30.1$$

Try a 14 WF 30. The section modulus of this beam is 41.8. The weight of the beam and fireproofing is 30 + 140 = 170 lb per ft. The bending moment at the center due to this load is

$$M = \frac{wL^2}{8} = \frac{170 \times 21.5 \times 21.5}{8} = 9820 \text{ ft.-lb.}$$

The total bending moment is 9820 + 50,140 = 59,960 ft.-lb. The total section modulus required is

$$S = \frac{M}{f} = \frac{59{,}960 \times 12}{20{,}000} = 36.0$$

Since this value is in reasonable agreement with the section modulus supplied, the 14 WF 30 is adopted. The end reactions each equal $12{,}600 + 170 \times \frac{21.5}{2} = 14{,}400$ lb. or 14.4 kips.

The first floor members previously designed are recorded below for reference when designing columns.

Member	Adopted Section	Reaction
6	14 WF 34	13.5 kips
19	14 WF 34	13.5 kips
11	18 WF 60	30.0 kips
8	14 WF 30	14.4 kips

154. Stair Wells. — The design of beams around stair wells presents no special difficulty, but the work is likely to be tedious because of the irregular spacings and loads. Steel stairs are either carried on stringers which transmit the entire load to the beams at the ends of the well, or by hangers and struts supported by the side beams. The system to be used in any particular case is usually undetermined at the time the structural plans are being prepared. Hence it is necessary to design the stair well framing so that the load may be supported by either the side or end beams.

When there are two landings in any flight, as is the case with the rear stairway of this building between the first and second floors, (see Transverse Section, Plate V) care must be taken to provide support for both landings.

SECOND AND THIRD FLOOR FRAMING

155. Typical Beams and Girders. — The second and third floor framing plans are identical except for the beams around the front stair well (see Plates II and VII). The stair framing will not be designed, but attention is called to the cantilever at the second

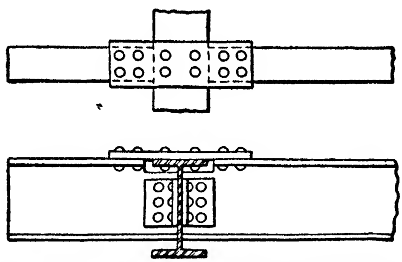


FIG. 135. Cantilever Detail. (Plate VII)

floor level. One possible arrangement for such a detail is shown in Fig. 135. The size of the plate and the number of rivets required are determined by computing the total tensile stress to be transmitted across the joint by the plate. The rivets through the flange of the supporting beam stiffen the connection.

The design computations for typical members and spandrel beams will not be repeated but the required sizes and the end reactions of beams and girders corresponding to those designed for the first floor are recorded below. In general, the second and third floor framing, except spandrels, is lighter than the first since the design load is 150 lb. per sq. ft. in place of 180 lb.

Member	Adopted Section	Reaction
6	14 WF 30	11.5 kips
19	14 WF 30	11.5 kips
11	18 WF 50	25.0 kips
8	16 WF 45	22.4 kips[1]

ROOF AND PENT HOUSE FRAMING

156. Roof Framing. — The typical roof members are designed in the same manner as the floor members except that the design load is 119 lb. per sq. ft. (see Art. 147). The spandrel beams support a parapet 4 ft. high, the weight of which may be considered the same as the walls in the third story. A section through the roof construction is shown on Plate V. The framing plan is shown on Plate VIII. Beams and girders corresponding to those designed for each floor are recorded below.

Member	Adopted Section	Reaction
6	12 WF 27	9.3 kips
19	12 WF 27	9.3 kips
11	16 WF 45	20.4 kips
8	14 WF 30	12.0 kips

[1] Although the wall area between the heads of third story windows and the roof framing is greater than that between the second story window heads and the third floor framing, the spandrel beams will be about the same size in both cases, since the second floor spandrels also carry the wall area between the second floor framing and the heads of the show windows in the first story. (See Art. 153 and Plates IV and V.)

157. Pent House Framing. — The principal special feature involved in the design of the pent house floor framing is the load due to the elevator beam reactions (see Plate VIII). The values to be used for the reactions are furnished by the elevator company. As stated in Art. 3, these loads should be increased 100 per cent when designing the members which support the elevator beams directly, such as ④→ and ④— shown on Plate VIII. These loads may be reduced somewhat on beams ④— and ④. It should be noted that beams ④→, ③→, and ②→ support columns 21, 22, and 23 respectively which carry the pent house roof. The brick walls of the pent house (see Plates III and V) must also be considered in the design of certain members.

Once the elevator beam reactions are determined, the pent house floor framing presents no special problems, but the design of members is quite laborious because of the variety of loads involved.

DESIGN OF COLUMNS

158. Interior Columns. — Column 10 is selected to illustrate the design of typical interior columns. The total load brought to the column at any floor is equal to the reactions of all beams framing into it.[2] The load brought to column 10 (third story) by the roof framing (see Plate VIII and the reactions tabulated in Art. 156) is $20.4 + 20.4 + 9.3 + 9.3 = 59.4$ kips. Since the beam reactions balance one another, the load is concentric. The loads brought to the column at each floor have been similarly computed and recorded on the second line of the accompanying table.

In addition to these loads, the weight of the column section and fireproofing must be considered. For structures of this type, where the column weight is small in relation to the load supported, an average weight per foot of length may be computed and used throughout the building. For example, the total load due to beam reactions, transmitted to the first story section[3] of

[2] See Art. 78 and Fig. 81. Since this building is less than five stories high, no live load reduction is permitted.

[3] This particular section is selected because it represents the average condition. In an eight or ten story building the fourth or fifth story column would be used.

DESIGN TABLE FOR COLUMN 10

		3rd Story Ht. = 14 ft.	2nd Story Ht. = 14 ft.	1st Story Ht. = 18 ft.	Basement Story Ht = 13 ft.
1	Load from column above	....	61.6	136.8	212.7
2	Beam reactions........	59.4	73.0	73.0	87.0
3	Column weight allowance	2.2	2.2	2.9	2.1
4	Design load............	61.6	136.8	212.7	301.8
5	Adopted section........	8 WF 35	8 WF 35	10 WF 72	10 WF 72

column 10, is $59.4 + 73.0 + 73.0 = 205.4$ kips. The floor-to-floor height of this column is 18 ft. From a safe load table, a 10-in., 54-lb. Wide Flange column is found to be satisfactory (allowable load = 215 kips). The weight of 4 in. of terra cotta fireproofing around the entire column is approximately 100 lb. per ft. of height,[4] making the total weight 160 lb. per ft. The value obtained in this manner is usually too large in the upper stories and too small in the lower but is satisfactory for design purposes.

The weight allowance for third story columns is equal to the story height, 14 ft., multiplied by 160 or 2240 lb. (2.2 kips). Similar weights are determined for the other stories and recorded on the third line of the design table for column 10.

The design load for the third story section of column 10 is equal to the sum of the beam reactions plus the column weight. This value is recorded on the fourth line of the table. The design load for the third story section is equal to the " load from column above " for the second story section and is recorded on the first line. The design loads for the other stories are determined in a similar manner and entered in the table.

With the loads and story heights known, the columns are selected directly from a table of safe loads. Since each column

[4] The outside dimensions of the fireproofing are approximately 18 in. by 18 in. Hence the total surface (four sides) of a section 1 ft. high is 4×1.5 or 6 sq. ft. Four-inch terra cotta block weighs 16 lb. per sq. ft. of surface, hence $16 \times 6 = 96$ lb. per ft. of height. The 100-lb. allowance will also cover the filling between flanges, since in this method of computation, a portion of the terra cotta has been counted twice.

will extend through two stories, it is necessary to select sections for the lower story only, of the two story length. WF sections for the second and basement stories have been chosen from a steel handbook and recorded in the design table.

159. Wall Columns. — Column 9 is selected to illustrate the design of wall columns. Figure 136 shows the loads in kips brought to the column at each floor level. It is assumed that the spandrel beams frame against the outside face of the column as shown in Fig. 84 and on Plate IX. At the roof and first floor levels, the

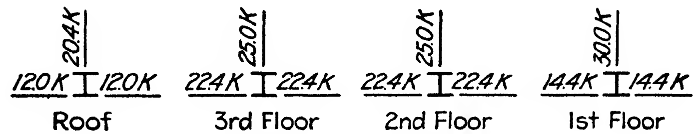


FIG. 136. Loads on Column No. 9.

sum of the spandrel beam reactions practically balances those from the girders, hence the eccentricity due to unequal loading may be neglected. At the second and third floors, however, the eccentricity will be considered.

Inasmuch as the effect of an eccentric load is not cumulative (see Art. 82), the accompanying table for the design of column 9 is prepared and the first, second, third, and sixth lines are filled out in the same manner as for column 10 (Art. 158).[5] Since the basement story column is not subjected to eccentric loading, the section may be selected directly from a safe load table. A 10 WF 60 is satisfactory (allowable load = 269 kips).

Since the basement column section will be used in the first story also, it will be tested for the eccentricity induced by the second floor loads (Fig. 136). The unbalanced load ($P' = 19.8$ kips) may be considered applied at the face of the column. The actual depth of this section is 10.25 in., making $e = 5.12$ in., and the bending factor is 0.263. Referring to Art. 82, the equivalent concentric load is

$$W_q = P'eB = 19.8 \times 5.12 \times 0.263 = 26.6 \text{ kips.}$$

[5] The average weight of column and fireproofing may be taken the same as that for the interior columns. Although the wall columns are fireproofed with brick, at least half of the load has been considered when designing the spandrel beams. This compensates for the difference in weight between brick and terra cotta.

This value is recorded in line 4 of the table and the design load for the first story column determined as 217.9 kips. The allowable axial load for a 10 WF 60 on a story height of 18 ft. is 240 kips (from a safe load table). Hence the section is satisfactory.

It will be sufficiently accurate to assume the same equivalent concentric load for the second story column as that used for the first, since the unbalanced load is similar. The design load then becomes 145.2 kips. From a safe load table, a 10 WF 39 will sustain an axial load of 155 kips on this story height (14 ft.) and an

DESIGN TABLE FOR COLUMN 9

		3rd Story Ht. = 14 ft.	2nd Story Ht. = 14 ft.	1st Story Ht. = 18 ft.	Basement Story Ht. = 13 ft.
1	Load from column above	....	46.6	118.6	191.3
2	Beam reactions........	44.4	69.8	69.8	58.8
3	Column weight allowance	2.2	2.2	2.9	2.1
4	Eccentric effect (W_q) ...	....	26.6	26.6	.....
5	Design load...........	46.6	145.2	217.9	252.2
6	Design load minus eccentric effect...........	46.6	118.6	191.3	252.2
7	Adopted section........	8 WF 40	8 WF 40	10 WF 60	10 WF 60

8 WF 40 will carry 161 kips. Either of these sections is therefore satisfactory. Since there is so little difference in weight, the 8 WF 40 is selected because it will project into the room less than the 10-in. section.

Where the "eccentric effect" forms a larger proportion of the design load than in the case under consideration, it may be found worth while to select a trial section with an allowable load somewhat less than the design load, and then investigate it by the second method for determining *allowable combined stresses*, discussed in Art. 81.

160. Column Schedule. — All information pertaining to the columns of a building is tabulated in a *column schedule*, similar to the one shown on Plate IX. The data recorded are loads, sections adopted, lengths, position of splices, size of base plates, etc.

The steel slab bases may be selected from a manufacturer's table of standard sizes, or designed according to the method described in Arts. 83 and 84. In this problem, it is assumed that all columns are supported on reinforced concrete footings.

APPENDICES

APPENDIX A

UNITED STATES DEPARTMENT OF COMMERCE

MINIMUM DESIGN LOADS IN BUILDINGS AND OTHER STRUCTURES

FROM AMERICAN STANDARD BUILDING CODE REQUIREMENTS A58.1—1945
NATIONAL BUREAU OF STANDARDS, SPONSOR.

UNIFORMLY DISTRIBUTED FLOOR LOADS

The live loads assumed for purposes of design shall be the greatest loads that probably will be produced by the intended occupancies or uses, provided that the live loads to be considered as uniformly distributed shall be not less than the values given in the following table.

Occupancy or Use	Live Load Lb. per Sq. Ft.	Occupancy or Use	Live Load Lb. per Sq. Ft.
Apartment houses:		Hotels:	
Private apartments	40	Guest rooms	40
Public stairways	100	Corridors serving public rooms	100
Assembly halls:		Public rooms	100
Fixed seats	60		
Movable seats	100	Loft buildings	125
Corridors, upper floors	100		
Corridors:		Manufacturing, light	125
First floor	100	Office buildings:	
Other floors, same as occupancy served except as indicated		Offices	80
		Lobbies	100
Courtrooms	80	Schools:	
Dance halls	100	Classrooms	40
		Corridors	100
Dining rooms, public	100		
Dwellings	40	Stores	125
Hospitals and Asylums:		Theatres:	
Operating rooms	60	Aisles, corridors, and lobbies	100
Private rooms	40	Orchestra floor	60
Wards	40	Balconies	60
Public space	80	Stage floor	150

PROVISION FOR PARTITIONS

In office buildings or other buildings where partitions might be subject to erection or rearrangement, provision for partition weight shall be made, whether or not partitions are shown on the plans, unless the specified live load exceeds 80 pounds per square foot.

CONCENTRATED LOADS

In the design of floors, consideration shall be given to the effects of known or probable concentrations of load to which they may be subjected. Floors shall be designed to carry the specified distributed loads, or the following minimum concentrations, whichever may produce the greater stresses. The indicated concentrations shall be assumed to occupy areas 2½ feet square and to be so placed as to produce maximum stresses in the affected members.

Floor Space	Load
Office floors, including corridors	2,000 lb.
Garages	Maximum wheel load
Trucking space within building	Maximum wheel load

PARTIAL LOADING

When the construction is such that the structural elements thereof act together in the nature of an elastic frame due to their continuity and the rigidity of the connections, and the live load exceeds 150 pounds per square foot or twice the dead load, the effect of partial live load such as will produce maximum stress in any member shall be provided for in the design.

IMPACT LOADS

The live loads tabulated above may be assumed to include a sufficient allowance to cover the effects of ordinary impact. For special occupancies and loads involving unusual impacts, such as those resulting from moving machinery, elevators, crane-vehicles, ways, etc., provision shall be made by a suitable increase in the assumed live load.

REDUCTION OF LIVE LOAD

(*a*) No reduction shall be applied to the roof live load.

(*b*) For live loads of 100 pounds or less per square foot, the design live load on any member supporting 150 square feet or more may be reduced at the rate of 0.08 percent per square foot of area supported by the member, except that no reduction shall be made for areas to be occupied as places of public assembly. The reduction shall exceed neither R as determined by the following formula, nor 60 percent:

$$R = 100 \times \frac{D + L}{4.33L}$$

in which R = reduction in percent
D = dead load per square foot of area supported by the member
L = design live load per square foot of area supported by the member

For live loads exceeding 100 pounds per square foot, no reduction shall be made except that the design live loads on columns may be reduced 20 percent.

ROOF LOADS (INCLUDING SNOW LOADS)

(*a*) Ordinary roofs, either flat or pitched, shall be designed for a load of not less than 20 pounds per square foot of horizontal projection in addition to the dead load, and in addition to either the wind or the earthquake load, whichever produces the greater stresses.

(NOTE: The figure of 20 pounds per square foot is a minimum snow load and should be increased in many localities. A U. S. Weather Bureau map in the Appendix to A58.1–1945 indicates roughly that such an increase is in order north of the 40th parallel of latitude; attaining 40 pounds per square foot at the northeastern and north central boundaries of the United States and in parts of Washington, Oregon and Idaho.)

(*b*) Roofs to be used for promenades shall be designed for a minimum load of 60 pounds per square foot in addition to the dead load. Roofs to be used for other special purposes shall be designed for appropriate loads as directed or approved by the building official.

OTHER LIVE LOADS

(*a*) Stair treads shall be designed to support a uniformly distributed load of 100 pounds per square foot, or concentrated loads of 300 pounds spaced 3 feet center to center, each occupying an area 1 foot wide by the depth of the tread, whichever will produce the greater stresses.

(*b*) Sidewalks shall be designed to support either a uniformly distributed load of 250 pounds per square foot, or a concentrated load of 8,000 pounds on an area 2½ feet square placed in any position, whichever will produce the greater stresses.

(*c*) Driveways shall be designed to support a uniformly distributed load of 100 pounds per square foot for vehicles weighing less than 3 tons with load, 150 pounds per square foot for vehicles weighing 3 to 10 tons with load, 200 pounds per square foot for vehicles weighing over 10 tons with load, or a concentrated load equal to the maximum expected wheel load on an area 2½ feet square placed in any position, whichever will produce the greater stresses.

(*d*) Accessible ceilings, scuttles, and ribs of skylights shall be designed to support a concentrated load of 200 pounds occupying an area 2½ feet square and so placed as to produce maximum stresses in the affected members.

(*e*) Stairway and balcony railings, both exterior and interior, shall be designed to resist a horizontal thrust of 50 pounds per linear foot applied at the top of the railing.

APPENDIX B

STRESS AND DEFORMATION

The relation between the elastic limit, yield point and ultimate strength of a material is clearly illustrated by means of a *stress-strain diagram.* Such a diagram is obtained from the data of a static tensile test, by plotting simultaneous values of the unit stress (total load on the specimen divided by original area of cross section) and the unit elongation (increase in gage length divided by original gage length).

The upper curve of Fig. 137 represents the stress-strain diagram for a test of structural grade steel. The lower curve is a portion of the upper one drawn to larger scale. The portion of the curve from the origin O to A is a straight

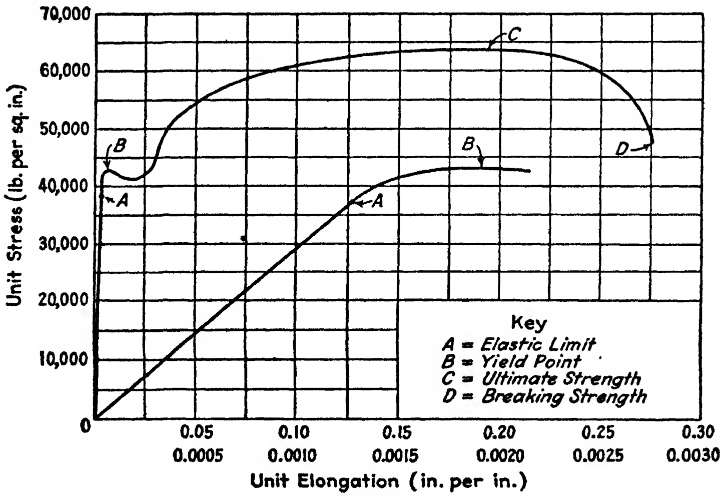


FIG. 137. Stress-Strain Diagram.

line, since within this range the material behaves according to Hooke's Law, "Stress is proportional to deformation." In the vicinity of point A, the curve deviates from a straight line, indicating that the proportional relationship no longer holds. Beyond point A, if the load had been released during the test, the specimen would not have recovered its original length, but would have taken a permanent set. The unit stress corresponding to point A on the curve is the *elastic limit* of the material.

At point B, a short distance beyond A, the curve becomes horizontal. This indicates that the specimen elongated without any increase in the load. The stress at B is called the *yield point*. Shortly beyond the yield point, the unit stress increased until the *ultimate strength* was reached at C. Failure actually begins at the ultimate strength as evidenced by "necking" of the specimen, although rupture does not occur until the *breaking stress* is reached at D.

It is evident from Fig. 137 that an exact determination of the elastic limit (point A) would be extremely difficult whereas the yield point may be ascertained quite accurately, since the balance beam of the testing machine drops suddenly when this stress is reached. For this reason it is customary to base allowable working stresses on the yield point rather than the elastic limit.

In addition to the stresses mentioned above, the stress-strain diagram also indicates the *modulus of elasticity* of the material tested. Modulus of elasticity is defined as the ratio of unit stress to unit deformation or, expressed mathematically,

$$E = \frac{f}{\delta}$$

If any point on the straight line portion (O to A) of the stress-strain diagram is selected, the value of E is found by dividing the ordinate (unit stress) by the abscissa (unit strain). For example, in the diagram shown, an elongation of 0.001 in. corresponds to a unit stress of about 29,000 lb. per sq. in. Hence the modulus of elasticity is

$$E = \frac{f}{\delta} = \frac{29{,}000}{0.001} = 29{,}000{,}000 \text{ lb. per sq. in.}$$

More complete discussions of this subject will be found in textbooks on mechanics of materials.

APPENDIX C

T-FLANGE GIRDERS

Some of the advantages inherent in the T-flange girder were enumerated in Art. 66. It appears, from data based on the many girders of this type built to date, that the T-flange permits a reduction in depth over the conventional type of about 25 per cent, or a reduction in cost of about 10 per cent where depth is not restricted. Although the use of T-sections for girder flanges is covered by patents, the royalties charged have been nominal. Further information in this connection, as well as design data, may be obtained from Weiskopf and Pickworth, Consulting Engineers, New York City, originators of the T-flange girder. The following example illustrates the features involved in the design of girders of this type.

EXAMPLE

Design a plate girder with T-flanges to carry a concentrated load of 580,000 lb. located 8 ft. from one end of a 32-ft. span. The floor, plus an assumed weight for the girder and its encasement, creates a uniform load of 3000 lb. per ft. The concentrated load is imposed by a 14-in. Wide Flange column section. Headroom under the girder limits the over-all depth to 4 ft. The allowable extreme fibre stress is 18,000 lb. per sq. in.; the allowable shearing stress on the gross area of the web is 12,000 lb. per sq. in.; the allowable rivet stresses are those given by the A.I.S.C. (1928) specification.

Solution: Unless otherwise indicated, all loads in this solution are expressed in kips (1 kip = 1000 lb.); stresses in kips per square inch; and moments in foot-kips or inch-kips.

Loads, Shear, and Moment

(1) Figure 138*a* is the load diagram. The reactions have been determined in the usual manner and recorded on the load diagram. The shear diagram has been plotted in Fig. 138*b*. The maximum bending moment occurs under the concentrated load and is

$$M = 483 \times 8 - (3 \times 8) \times 4 = 3768 \text{ ft.-kips.}$$

The bending moment diagram is indicated by the broken line in Fig. 138*c*. The required section modulus is

$$S = \frac{M}{f} = \frac{3768 \times 12}{18} = 2512 \text{ in.}^3$$

Trial Section

(2) It is next necessary to select a trial section. This selection will naturally be influenced by the designer's experience and judgment. The approximate *area* required in the flanges may be found by the flange-area method (see

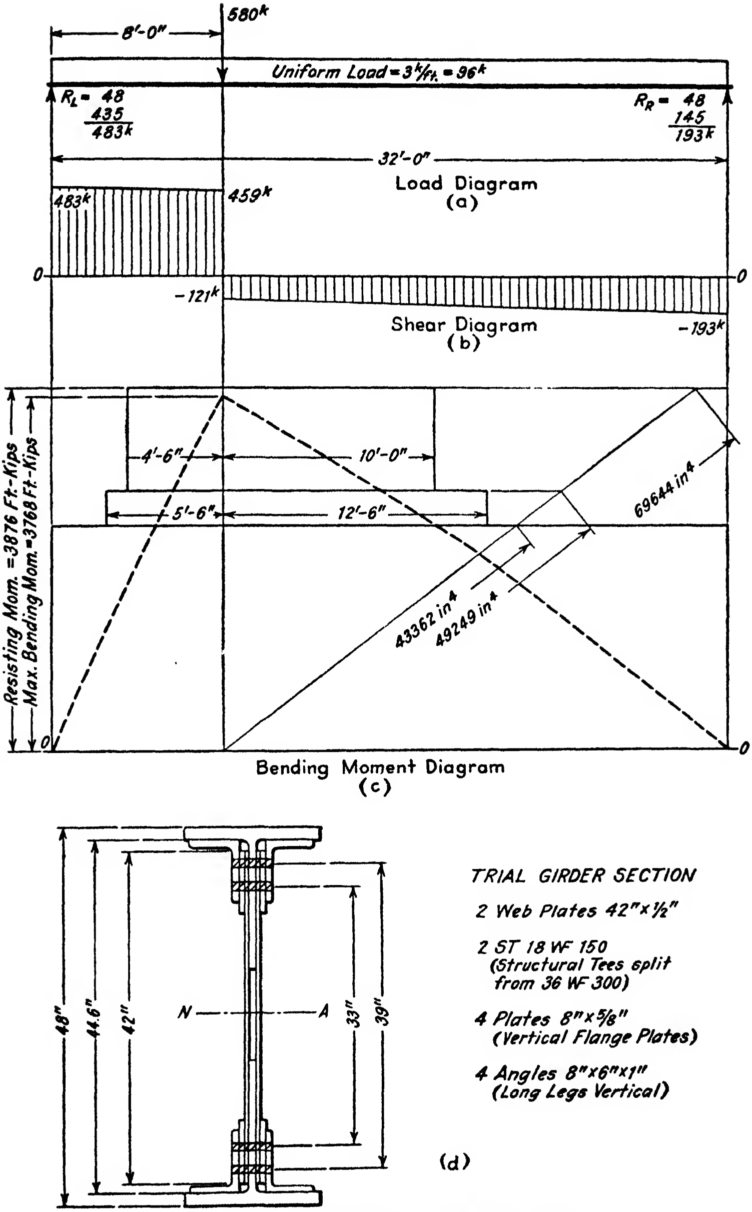


FIG. 138. T-Flange Girder Design Data.

footnote 2 in Art. 51) or by consulting tables similar to Table VIII in Appendix D, although such tables are not generally available for girders of large capacity. When the approximate area of material necessary in the flanges has been determined, the trial section is made up, using double web plates and a structural Tee, reinforced with angles and plates if necessary. The trial section for the girder under discussion is shown in Fig. 138*d*.

Moment of Inertia

(3) The net moment of inertia of the trial girder is now determined. This computation has been set down in tabular form accompanied by explanatory notes. The use of Tables V, VI, and VII in Appendix D was demonstrated in the example following Art. 53, Chapter V. The notation *o.* to *o.* = distance out to out; d = distance between centers of gravity of areas; and A = gross area of the section in *one* flange.

Section	*o.* to *o.* in.	$2y$ in.	d in.	Coef.	A in.^{2}	Ad^2 in.^{4}	I_0	I_{NA}	Totals
2 Webs 42 × $\frac{1}{2}$							6174 Ⓐ	6174	
2 ST 18 WF 150	48.0	8.26Ⓑ	39 74	788 Ⓓ	44.09	34743	2445 Ⓔ	37188	43362
4 Pls. 8 × $\frac{5}{8}$	42.0	8.00	34.00	578 Ⓓ	10.00	5780	107 Ⓕ	5887	49249
4 Ls 8 × 6 × 1	44.6	5.30Ⓒ	39 30	772 Ⓓ	26 00	20072	323 Ⓖ	20395	69644
1 Holes [Area = ($\frac{15}{16}$ + 1 + $1\frac{1}{4}$ + 2) × $1\frac{1}{8}$]			39.00	761 Ⓓ	5.84	4444	.....		
			33.00	545 Ⓓ	5.84	3183	.....	7627	−7627
									62017

Ⓐ Table V, Appendix D, or similar table in the A.I.S.C. Manual.

Ⓑ See data on Fig. 139, taken from a table of Properties of Structural Tees cut from WF Sections, A.I.S.C. Manual, *third edition* (1937).

Ⓒ Table VII, Appendix D, or similar table in the A.I.S.C. Manual.

Ⓓ Table VI, Appendix D, or similar table in the A.I.S.C. Manual.

Ⓔ See note Ⓑ above, I = 1222.7 and $2I$ = 2445.4 or 2445 (approx.).

Ⓕ I of 4 plates $8 \times \frac{5}{8} = 4 \times \frac{bd^3}{12} = 4 \times \frac{.625 \times 8^3}{12} = 107.$

Ⓖ Table VII, Appendix D, or similar Table in the A.I.S.C. Manual.

Section Modulus and Shear Check

(4) The section modulus of the trial section may now be computed from the net moment of inertia and half the over-all depth in inches.

$$S = \frac{I}{c} = \frac{62017}{24} = 2584 \text{ in.}^3$$

This value is in reasonable agreement with the required section modulus found in Step (1). Before accepting the trial section, however, the web plates should be checked for shear. The two plates have a combined area of $2 \times (42 \times \frac{1}{2})$ = 42 sq. in. The permissible end shear is, then, 42 multiplied by 12 kips per sq. in. or 504 kips. This value is in excess of the maximum shear developed, 483 kips (see Fig. 138*b*). Hence the trial section is adopted.

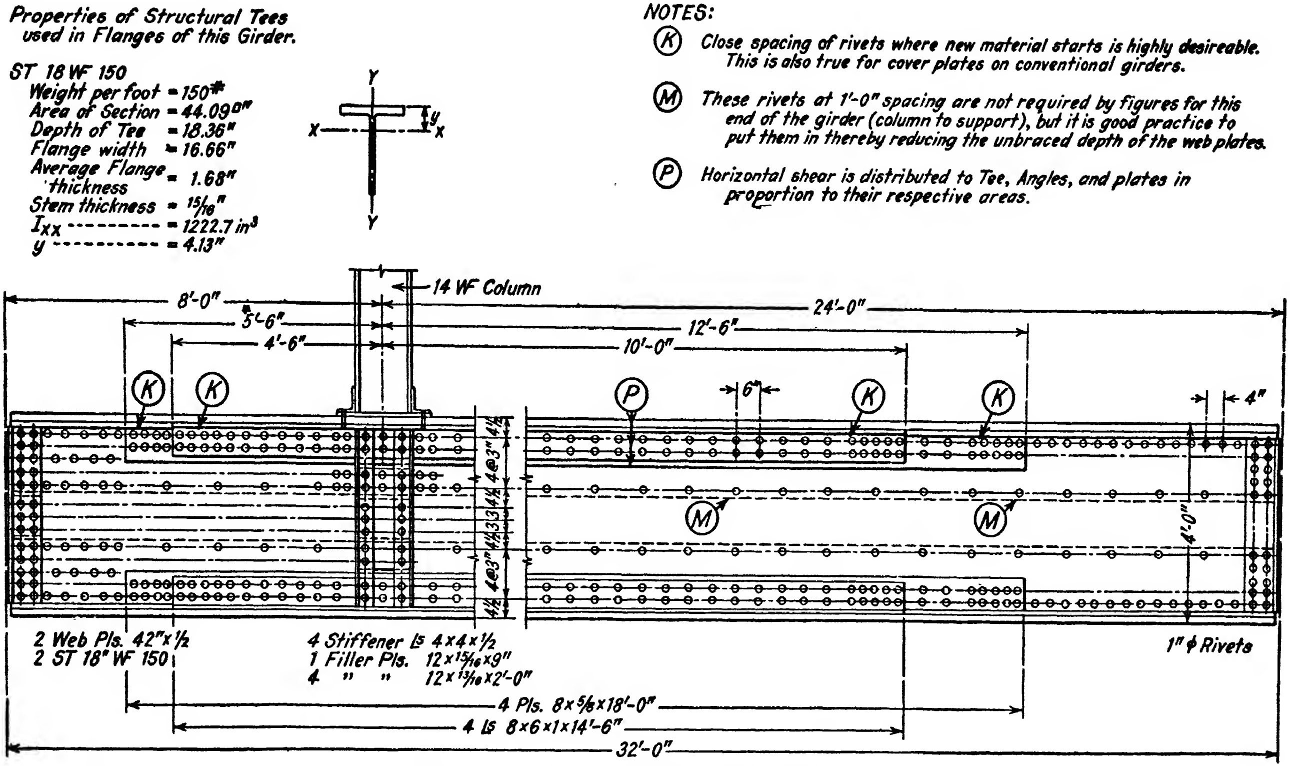


FIG. 139. T-Flange Girder Design Drawing.

Cut-offs for Flange Reinforcement

(5) The girder section adopted in the preceding step was determined from the maximum bending moment which occurs under the concentrated load. It is evident from the moment diagram (broken line in Fig. 138c) that at some points between the concentrated load and the ends of the span the angles and side plates will not be required. The theoretical points at which these members may be cut off are found by plotting their *resisting* moments and noting the points of intersections with the *bending* moment curve. The total resisting moment of the girder is

$$M_R = fS = 18 \times 2584 = 46{,}500 \text{ in.-kips or } 3876 \text{ ft.-kips.}$$

This value has been plotted in Fig. 138c. The resisting moments of the webs and Tees, the four angles, and the four plates will be proportional to their moments of inertia. These quantities are taken from the foregoing tabulation and plotted on the moment diagram. Actual cut-offs are made about 2 ft. 6 in. beyond the theoretical intersections.

Flange Riveting

(6) The rivet pitch equation for unloaded flange developed in Art. 62, Chapter V is $p = \frac{\text{R.V.}}{K_h}$. This may be written $p = \frac{\text{R.V.} \times d}{V}$, where R.V. is the rivet value, V the total vertical shear at the section considered, and d the effective depth of the girder. In conventional plate girders the effective depth (distance between centers of gravity of flanges) is so nearly equal to the distance between rivet lines that it is customary to use the latter value as a convenient and safe procedure. However, in T-flange girders where the stem portions of the flanges are long, thereby permitting many of the rivets to be placed on lines much nearer the neutral surface, it is important to recognize the more correct value of d in order to avoid extravagance. Therefore, if 1-in. rivets were to be used in a single line, the pitch at the left end would be

$$p = \frac{\text{R.V.} \times d}{V} = \frac{21.2 \times 39.74}{483} = 1.75 \text{ in.}$$

This effective pitch is maintained by placing the rivets in three rows. (It should be noted that R.V. in the equation is the double shear value for 1-in. rivets according to the 1928 A.I.S.C. Specification.) The required pitch at the right end or at any other point is found in a similar manner. The layout of the riveting, together with additional design notes, is indicated on the design drawing, Fig. 139.

Stiffeners

(7) The only stiffener angles required for this girder are those under the supported column. Most of the load from the column is transferred to the girder webs directly through the stem of the Tee and the 8 × 6 × 1-in. angles. The riveting indicated under the column in Fig. 139 will be found more than sufficient to transfer the load to the webs. Hence the stiffeners are a matter of judgment. The detail is indicated on the design drawing.

APPENDIX D

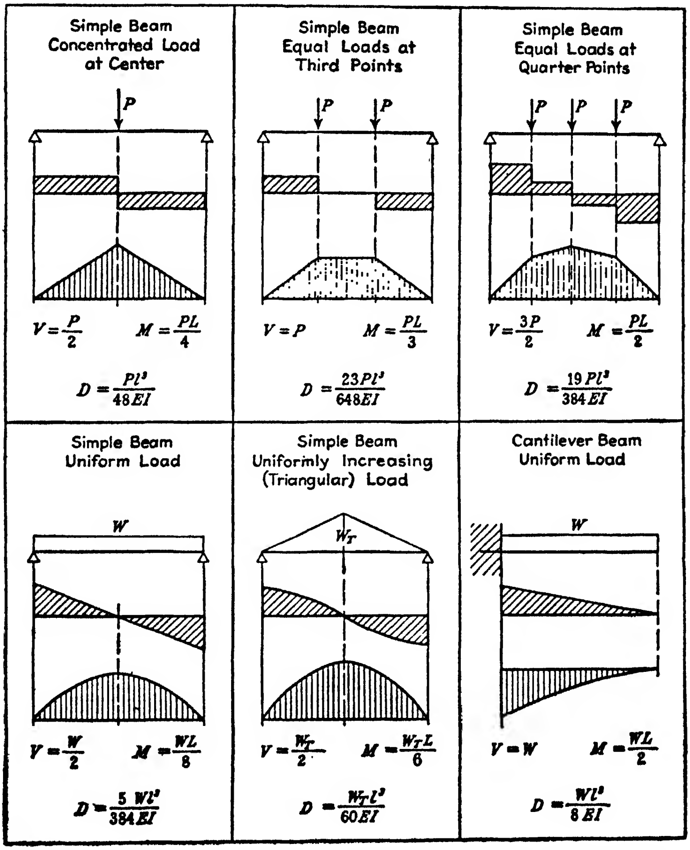
TABLES

Note. In general, the tables which follow are abstracted from those in the steel handbooks. All information required for the solution of the illustrative examples is included, but more extensive tables are necessary to solve many of the exercise problems.

TABLE I

Diagrams and Formulas for Static Loads on Beams

V = Max. Shear (lb.) M = Max. Moment (ft.-lb.) D = Max. Deflection (in.)*
P & W = Loads in lb. L = Length in ft. l = Length in in.

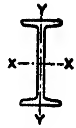


* Deflection at center of simple beams and free end of cantilevers.

TABLE II*

American Standard Beams

Properties for Designing



Nominal Size	Weight per Foot	Area	Depth	Flange Width	Flange Thickness	Web Thickness	Axis X-X I	Axis X-X S	Axis X-X r	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r
In.	Lb.	In.^{2}	In.	In.	In.	In.	In.^{4}	In.^{3}	In.	In.^{4}	In.^{3}	In.
24 × 7⅛	120.0	35.13	24.00	8.048	1.102	.798	3010.8	250.9	9.26	84.9	21.1	1.56
	105.9	30.98	24.00	7.875	1.102	.625	2811.5	234.3	9.53	78.9	20.0	1.60
24 × 7	100.0	29.25	24.00	7.247	.871	.747	2371.8	197.6	9.05	48.4	13.4	1.29
	90.0	26.30	24.00	7.124	.871	.624	2230.1	185.8	9.21	45.5	12.8	1.32
	79.9	23.33	24.00	7.000	.871	.500	2087.2	173.9	9.46	42.9	12.2	1.36
20 × 7	95.0	27.74	20.00	7.200	.916	.800	1599.7	160.0	7.59	50.5	14.0	1.35
	85.0	24.80	20.00	7.053	.916	.653	1501.7	150.2	7.78	47.0	13.3	1.38
20 × 6¼	75.0	21.90	20.00	6.391	.789	.641	1263.5	126.3	7.60	30.1	9.4	1.17
	65.4	19.08	20.00	6.250	.789	.500	1169.5	116.9	7.83	27.9	8.9	1.21
18 × 6	70.0	20.46	18.00	6.251	.691	.711	917.5	101.9	6.70	24.5	7.8	1.09
	54.7	15.94	18.00	6.000	.691	.460	795.5	88.4	7.07	21.2	7.1	1.15
15 × 5½	50.0	14.59	15.00	5.640	.622	.550	481.1	64.2	5.74	16.0	5.7	1.05
	42.9	12.49	15.00	5.500	.622	.410	441.8	58.9	5.95	14.6	5.3	1.08
12 × 5¼	50.0	14.57	12.00	5.477	.659	.687	301.6	50.3	4.55	16.0	5.8	1.05
	40.8	11.84	12.00	5.250	.659	.460	268.9	44.8	4.77	13.8	5.3	1.08
12 × 5	35.0	10.20	12.00	5.078	.544	.428	227.0	37.8	4.72	10.0	3.9	.99
	31.8	9.26	12.00	5.000	.544	.350	215.8	36.0	4.83	9.5	3.8	1.01
10 × 4⅝	35.0	10.22	10.00	4.944	.491	.594	145.8	29.2	3.78	8.5	3.4	.91
	25.4	7.38	10.00	4.660	.491	.310	122.1	24.4	4.07	6.9	3.0	.97
8 × 4	23.0	6.71	8.00	4.171	.425	.441	64.2	16.0	3.09	4.4	2.1	.81
	18.4	5.34	8.00	4.000	.425	.270	56.9	14.2	3.26	3.8	1.9	.84
7 × 3⅝	20.0	5.83	7.00	3.860	.392	.450	41.9	12.0	2.68	3.1	1.6	.74
	15.3	4.43	7.00	3.660	.392	.250	36.2	10.4	2.86	2.7	1.5	.78
6 × 3⅜	17.25	5.02	6.00	3.565	.359	.465	26.0	8.7	2.28	2.3	1.3	.68
	12.5	3.61	6.00	3.330	.359	.230	21.8	7.3	2.46	1.8	1.1	.72
5 × 3	14.75	4.29	5.00	3.284	.326	.494	15.0	6.0	1.87	1.7	1.0	.63
	10.0	2.87	5.00	3.000	.326	.210	12.1	4.8	2.05	1.2	.82	.65
4 × 2⅝	9.5	2.76	4.00	2.796	.293	.326	6.7	3.3	1.56	.91	.65	.58
	7.7	2.21	4.00	2.660	.293	.190	6.0	3.0	1.64	.77	.58	.59
3 × 2⅜	7.5	2.17	3.00	2.509	.260	.349	2.9	1.9	1.15	.59	.47	.52
	5.7	1.64	3.00	2.330	.260	.170	2.5	1.7	1.23	.46	.40	.53

* Taken from the fifth edition of the A.I.S.C. Manual.

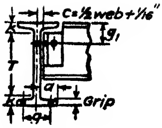


TABLE II — (*Continued*)

American Standard Beams

Dimensions for Detailing

Depth of Section	Weight per Foot	Flange: Width	Flange: Mean Thickness	Web: Thickness	Web: Half Thickness	Distance: a	Distance: T	Distance: k	Distance: g_1	Distance: c	Grip	Max. Flange Rivet	Usual Gage g
In.	Lb.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.	In.
24	120.0	8	1 1/8	13/16	7/16	3 5/8	20 1/8	1 15/16	3 1/2	1/2	1 1/8	1	4
	105.9	7 7/8	1 1/8	5/8	5/16	3 5/8	20 1/8	1 15/16	3 1/2	3/8	1 1/8	1	4
24	100.0	7 1/4	7/8	3/4	3/8	3 1/4	20 3/4	1 5/8	3	7/16	7/8	1	4
	90.0	7 1/8	7/8	5/8	5/16	3 1/4	20 3/4	1 5/8	3	3/8	7/8	1	4
	79.9	7	7/8	1/2	1/4	3 1/4	20 3/4	1 5/8	3	5/16	7/8	1	4
20	95.0	7 1/4	15/16	13/16	7/16	3 1/4	16 1/2	1 3/4	3 1/4	1/2	15/16	1	4
	85.0	7	15/16	11/16	3/8	3 1/4	16 1/2	1 3/4	3 1/4	3/8	7/8	1	4
20	75.0	6 3/8	13/16	5/8	5/16	2 7/8	16 7/8	1 9/16	3	3/8	13/16	7/8	3 1/2
	65.4	6 1/4	13/16	1/2	1/4	2 7/8	16 7/8	1 9/16	3	5/16	3/4	7/8	3 1/2
18	70.0	6 1/4	11/16	3/4	3/8	2 3/4	15 1/4	1 3/8	2 3/4	7/16	11/16	7/8	3 1/2
	54.7	6	11/16	1/2	1/4	2 3/4	15 1/4	1 3/8	2 3/4	5/16	11/16	7/8	3 1/2
15	50.0	5 5/8	5/8	9/16	5/16	2 1/2	12 1/2	1 1/4	2 3/4	3/8	9/16	3/4	3 1/2
	42.9	5 1/2	5/8	7/16	1/4	2 1/2	12 1/2	1 1/4	2 3/4	5/16	9/16	3/4	3 1/2
12	50.0	5 1/2	11/16	11/16	3/8	2 3/8	9 3/8	1 5/16	2 3/4	7/16	11/16	3/4	3
	40.8	5 1/4	11/16	1/2	1/4	2 3/8	9 3/8	1 5/16	2 3/4	5/16	5/8	3/4	3
12	35.0	5 1/8	9/16	7/16	1/4	2 3/8	9 3/4	1 1/8	2 1/2	5/16	1/2	3/4	3
	31.8	5	9/16	3/8	3/16	2 3/8	9 3/4	1 1/8	2 1/2	1/4	1/2	3/4	3
10	35.0	5	1/2	5/8	5/16	2 1/8	8	1	2 1/2	3/8	1/2	3/4	2 3/4
	25.4	4 5/8	1/2	5/16	3/16	2 1/8	8	1	2 1/2	1/4	1/2	3/4	2 3/4
8	23.0	4 1/8	7/16	7/16	1/4	1 7/8	6 1/4	7/8	2 1/4	5/16	7/16	3/4	2 1/4
	18.4	4	7/16	5/16	1/8	1 7/8	6 1/4	7/8	2 1/4	3/16	7/16	3/4	2 1/4
7	20.0	3 7/8	3/8	7/16	1/4	1 3/4	5 3/8	13/16	2	5/16	3/8	5/8	2 1/4
	15.3	3 5/8	3/8	1/4	1/8	1 3/4	5 3/8	13/16	2	3/16	3/8	5/8	2 1/4
6	17.25	3 5/8	3/8	1/2	1/4	1 1/2	4 1/2	3/4	2	5/16	3/8	5/8	2
	12.5	3 3/8	3/8	1/4	1/8	1 1/2	4 1/2	3/4	2	3/16	5/16		
5	14.75	3 1/4	5/16	1/2	1/4	1 3/8	3 5/8	11/16	2	5/16	5/16	1/2	1 3/4
	10.0	3	5/16	1/4	1/8	1 3/8	3 5/8	11/16	2	3/16	5/16	1/2	1 3/4
4	9.5	2 3/4	5/16	5/16	3/16	1 1/4	2 3/4	5/8	2	1/4	5/16	1/2	1 1/2
	7.7	2 5/8	5/16	3/16	1/8	1 1/4	2 3/4	5/8	2	3/16	5/16		
3	7.5	2 1/2	1/4	3/8	3/16	1 1/8	1 7/8	9/16		1/4	1/4	3/8	1 1/2
	5.7	2 3/8	1/4	3/16	1/8	1 1/8	1 7/8	9/16		3/16	1/4	3/8	1 1/2

Gage g_1 is based on $k + 1\frac{1}{2}''$, to nearest $\frac{1}{4}$ in.

Gage g is permissible near ends of beam; elsewhere Specification may require reduction in rivet size.

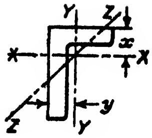


TABLE III
Standard Angles
Technical Functions

Size in In.	Thickness	Wt. per Ft.	Area Sq. In.	Axes								
				X–X				Y–Y				Z–Z
				I	S	r	x	I	S	r	y	r
2½×2	1/8	1.86	0.55	0.35	0.20	0.80	0.74	0.20	0.13	0.61	0.49	0.43
	3/16	2.75	0.81	0.51	0.29	0.79	0.76	0.29	0.20	0.60	0.51	0.43
	1/4	3.62	1.06	0.65	0.38	0.78	0.79	0.37	0.25	0.59	0.54	0.42
	5/16	4.50	1.31	0.79	0.47	0.78	0.81	0.45	0.31	0.58	0.56	0.42
2½×2½	1/8	2.08	0.61	0.38	0.20	0.79	0.67	......	......	......	......	0.50
	3/16	3.07	0.90	0.55	0.30	0.78	0.69	......	......	......	......	0.49
	1/4	4.1	1.19	0.70	0.39	0.77	0.72	......	......	......	......	0.49
	5/16	5.0	1.47	0.85	0.48	0.76	0.74	......	......	......	......	0.49
	3/8	5.9	1.73	0.98	0.57	0.75	0.76	......	......	......	......	0.48
3×2½	1/4	4.5	1.31	1.17	0.56	0.95	0.91	0.74	0.40	0.75	0.66	0.53
	5/16	5.6	1.62	1.42	0.69	0.94	0.93	0.90	0.49	0.74	0.68	0.53
	3/8	6.6	1.92	1.66	0.81	0.93	0.96	1.04	0.58	0.74	0.71	0.52
	7/16	7.6	2.22	1.88	0.93	0.92	0.98	1.18	0.66	0.73	0.73	0.52
3×3	1/4	4.9	1.44	1.24	0.58	0.93	0.84	......	......	......	......	0.59
	5/16	6.1	1.78	1.51	0.71	0.92	0.87	......	......	......	......	0.59
	3/8	7.2	2.11	1.76	0.83	0.91	0.89	......	......	......	......	0.58
	7/16	8.3	2.43	1.99	0.95	0.91	0.91	......	......	......	......	0.58
	1/2	9.4	2.75	2.22	1.07	0.90	0.93	......	......	......	......	0.58
3½×2½	1/4	4.9	1.44	1.80	0.75	1.12	1.11	0.78	0.41	0.74	0.61	0.54
	5/16	6.1	1.78	2.19	0.93	1.11	1.14	0.94	0.50	0.73	0.64	0.54
	3/8	7.2	2.11	2.56	1.09	1.10	1.16	1.09	0.59	0.72	0.66	0.54
	7/16	8.3	2.43	2.91	1.26	1.09	1.18	1.23	0.68	0.71	0.68	0.54
	1/2	9.4	2.75	3.24	1.41	1.09	1.20	1.36	0.76	0.70	0.70	0.53
3½×3½	5/16	7.2	2.09	2.45	0.98	1.08	0.99	......	......	......	......	0.69
	3/8	8.5	2.48	2.87	1.15	1.07	1.01	......	......	......	......	0.68
	7/16	9.8	2.87	3.26	1.32	1.07	1.04	......	......	......	......	0.68
	1/2	11.1	3.25	3.64	1.49	1.06	1.06	......	......	......	......	0.68
	9/16	12.4	3.62	3.99	1.65	1.05	1.08	......	......	......	......	0.68
	5/8	13.6	3.98	4.33	1.81	1.04	1.10	......	......	......	......	0.68
4×3	5/16	7.2	2.09	3.38	1.23	1.27	1.26	1.65	0.73	0.89	0.76	0.65
	3/8	8.5	2.48	3.96	1.46	1.26	1.28	1.92	0.87	0.88	0.78	0.64
	7/16	9.8	2.87	4.52	1.68	1.25	1.30	2.18	0.99	0.87	0.80	0.64
	1/2	11.1	3.25	5.05	1.89	1.25	1.33	2.42	1.12	0.86	0.83	0.64
	9/16	12.4	3.62	5.55	2.09	1.24	1.35	2.66	1.23	0.86	0.85	0.64
	5/8	13.6	3.98	6.03	2.30	1.23	1.37	2.87	1.35	0.85	0.87	0.64
4×3½	5/16	7.7	2.25	3.56	1.26	1.26	1.18	2.55	0.99	1.07	0.93	0.73
	3/8	9.1	2.67	4.18	1.49	1.25	1.21	2.99	1.17	1.06	0.96	0.73
	7/16	10.6	3.09	4.76	1.72	1.24	1.23	3.40	1.35	1.05	0.98	0.72
	1/2	11.9	3.50	5.32	1.94	1.23	1.25	3.79	1.52	1.04	1.00	0.72
	9/16	13.3	3.90	5.86	2.15	1.23	1.27	4.17	1.68	1.03	1.02	0.72
	5/8	14.7	4.30	6.37	2.35	1.22	1.29	4.49	1.83	1.02	1.04	0.72
	11/16	16.0	4.68	6.86	2.56	1.21	1.32	4.86	2.00	1.02	1.07	0.72
	3/4	17.3	5.06	7.32	2.75	1.20	1.34	5.18	2.15	1.01	1.09	0.72

Smaller angles are rolled but not listed here

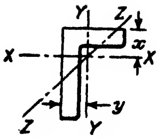


TABLE III — (*Continued*)

Standard Angles

Technical Functions

Size in In.	Thick-ness	Wt. per Ft.	Area Sq. In.	Axes X-X I	X-X S	X-X r	X-X x	Y-Y I	Y-Y S	Y-Y r	Y-Y y	Z-Z r
4×4	5/16	8.2	2.40	3.71	1.29	1.24	1.12	.....	.....	.....	.....	0.79
	3/8	9.8	2.86	4.36	1.52	1.23	1.14	.....	.....	....	.....	0.79
	7/16	11.3	3.31	4.97	1.75	1.23	1.16	.....	.....	.....	.....	0.78
	1/2	12.8	3.75	5.56	1.97	1.22	1.18	...	.....	.....	.....	0.78
	9/16	14.3	4.18	6.12	2.19	1.21	1.21	.....	.....	.....	.....	0.78
	5/8	15.7	4.61	6.66	2.40	1.20	1.23	.....	.....	.....	.....	0.77
	11/16	17.1	5.03	7.17	2.61	1.19	1.25	.....	.....	.....	.....	0.77
	3/4	18.5	5.44	7.66	2.81	1.19	1.27	.....	.....	.....	.....	0.77
5×3	5/16	8.2	2.40	6.26	1.89	1.61	1.68	1.75	0.75	0.85	0.68	0.66
	3/8	9.8	2.86	7.37	2.24	1.61	1.70	2.04	0.89	0.84	0.70	0.65
	7/16	11.3	3.31	8.43	2.58	1.60	1.73	2.32	1.02	0.84	0.73	0.65
	1/2	12.8	3.75	9.45	2.91	1.59	1.75	2.58	1.15	0.83	0.75	0.65
	9/16	14.3	4.18	10.43	3.23	1.58	1.77	2.83	1.27	0.82	0.77	0.65
	5/8	15.7	4.61	11.37	3.55	1.57	1.80	3.06	1.39	0.82	0.80	0.64
	11/16	17.1	5.03	12.28	3.86	1.56	1.82	3.29	1.51	0.81	0.82	0.64
	3/4	18.5	5.44	13.15	4.16	1.55	1.84	3.51	1.62	0.80	0.84	0.64
5×3½	5/16	8.7	2.56	6.60	1.94	1.61	1.59	2.72	1.02	1.03	0.84	0.77
	3/8	10.4	3.05	7.78	2.29	1.60	1.61	3.18	1.21	1.02	0.86	0.76
	7/16	12.0	3.53	8.90	2.64	1.59	1.63	3.63	1.39	1.01	0.88	0.76
	1/2	13.6	4.00	9.99	2.99	1.58	1.66	4.05	1.56	1.01	0.91	0.75
	9/16	15.2	4.47	11.03	3.32	1.57	1.68	4.45	1.73	1.00	0.93	0.75
	5/8	16.8	4.92	12.03	3.65	1.56	1.70	4.83	1.90	0.99	0.95	0.75
	11/16	18.3	5.37	12.99	3.97	1.56	1.72	5.20	2.06	0.98	0.97	0.75
	3/4	19.8	5.81	13.92	4.28	1.55	1.75	5.55	2.22	0.98	1.00	0.75
6×3½	3/8	11.7	3.42	12.86	3.24	1.94	2.04	3.34	1.23	0.99	0.79	0.77
	7/16	13.5	3.97	14.76	3.75	1.93	2.06	3.81	1.41	0.98	0.81	0.76
	1/2	15.3	4.50	16.59	4.24	1.92	2.08	4.25	1.59	0.97	0.83	0.76
	9/16	17.1	5.03	18.37	4.72	1.91	2.11	4.67	1.77	0.96	0.86	0.75
	5/8	18.9	5.55	20.08	5.19	1.90	2.13	5.08	1.94	0.96	0.88	0.75
	11/16	20.6	6.06	21.74	5.65	1.89	2.15	5.47	2.11	0.95	0.90	0.75
	3/4	22.4	6.56	23.34	6.10	1.89	2.18	5.84	2.27	0.94	0.93	0.75
	13/16	24.0	7.06	24.89	6.55	1.88	2.20	6.20	2.43	0.94	0.95	0.75
	7/8	25.7	7.55	26.39	6.98	1.87	2.22	6.55	2.59	0.93	0.97	0.75
6×4	3/8	12.3	3.61	13.47	3.32	1.93	1.94	4.90	1.60	1.17	0.94	0.88
	7/16	14.3	4.18	15.46	3.83	1.92	1.96	5.60	1.85	1.16	0.96	0.87
	1/2	16.2	4.75	17.40	4.33	1.91	1.99	6.27	2.08	1.15	0.99	0.87
	9/16	18.1	5.31	19.26	4.83	1.90	2.01	6.91	2.31	1.14	1.01	0.87
	5/8	20.0	5.86	21.07	5.31	1.90	2.03	7.52	2.54	1.13	1.03	0.86
	11/16	21.8	6.40	22.82	5.78	1.89	2.06	8.11	2.76	1.13	1.06	0.86
	3/4	23.6	6.94	24.59	6.25	1.88	2.08	8.68	2.97	1.12	1.08	0.86
	13/16	25.4	7.47	26.15	6.70	1.87	2.10	9.23	3.18	1.11	1.10	0.86
	7/8	27.2	7.98	27.73	7.15	1.86	2.12	9.75	3.39	1.11	1.12	0.86

Larger angles are rolled but not listed here.

TABLE IV

Power-Driven Rivets

Shearing and Bearing Values, in Pounds

$\frac{3}{4}$-In. Rivets, Area = 0.4418 Sq. In.

Shear	Unit Stress, Lb./Sq. In.		13500	13500*		15000†	
	Single Shear per Rivet		5964	5964		6627	
	Double Shear per Rivet		11928	11928		13254	
Bearing	Bearing Stress, Lb./Sq. In.		27000	24000	30000	32000	40000
	Thickness $\frac{3}{16}$	.1875	3797	3375	4219	4500	5626
		.2188	4435	3938	4923	5251	6564
	$\frac{1}{4}$	.250	5063	4500	5625	6000	7500
		.2813	5690	5063	6329	....	8439
	$\frac{5}{16}$	.3125	6328	5625	7031	....	9376
		.3438	6966	....	7736	....	10314
	$\frac{3}{8}$	.375	7594	....	8438	....	11250
		.4063	8222	....	9142	....	12189
	$\frac{7}{16}$	.4375	8859	....	9844	....	13126
	$\frac{1}{2}$	.500	10125	....	11250	....	.....
	$\frac{9}{16}$	.5625	11391	....	.....	....	.....

$\frac{7}{8}$-In. Rivets, Area = 0.6013 Sq. In.

Shear	Unit Stress, Lb./Sq. In.		13500	13500*		15000†	
	Single Shear per Rivet		8118	8118		9020	
	Double Shear per Rivet		16236	16236		18040	
Bearing	Bearing Stress, Lb./Sq. In.		27000	24000	30000	32000	40000
	Thickness $\frac{1}{4}$	.250	5906	5250	6563	7000	8750
	$\frac{5}{16}$	.3125	7383	6563	8203	8750	10938
	$\frac{3}{8}$	.375	8859	7875	9844	....	13126
	$\frac{7}{16}$	.4375	10336	....	11484	....	15312
	$\frac{1}{2}$	.500	11813	....	13125	....	17500
	$\frac{9}{16}$	.5625	13289	....	14766	....	.....
	$\frac{5}{8}$	.625	14766	....	.....	....	.....

* A.I.S.C. Specification (1928). Use 24,000-lb. bearing stress when rivets are in single shear; 30,000 when in double shear.

† A.I.S.C. Specification (1946). Use 32,000-lb bearing stress when rivets are in single shear; 40,000 when in double shear.

More complete tables will be found in the A.I.S.C. Manual and other steel handbooks.

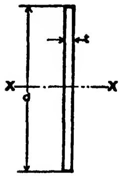


TABLE V*

Moment of Inertia of One Plate About Axis X–X

To obtain the moment of inertia for any thickness of plate not listed below, multiply the value for a plate one inch thick by the desired thickness.

Depth d, Inches	Thickness t, Inches							
	3/8	7/16	1/2	9/16	5/8	3/4	7/8	1
10	31.3	36.5	41.7	46.9	52.1	62.5	72 9	83.3
11	41.6	48.5	55.5	62.4	69.3	83.2	97 1	110.9
12	54.0	63.0	72.0	81.0	90.0	108.0	126.0	144.0
13	68.7	80.1	91.5	103.0	114.4	137.3	160.2	183.1
14	85.8	100.0	114.3	128.6	142.9	171.5	200.1	228.7
15	105.5	123.0	140.6	158.2	175.8	210.9	246.1	281.3
16	128.0	149.3	170.7	192.0	213.3	256.0	298.7	341.3
17	153.5	179.1	204 7	230.3	255 9	307.1	358.2	409.4
18	182.3	212.6	243.0	273.4	303 8	364.5	425.3	486.0
19	214.3	250.1	285.8	321.5	357.2	428.7	500.1	571.6
20	250.0	291.7	333.3	375.0	416 7	500.0	583.3	666.7
21	289.4	337.6	385.9	434 1	482 3	578.8	675.3	771.8
22	332.8	388.2	443 7	499.1	554 6	665 5	776 4	887.3
23	380.2	443 6	507 0	570 3	633 7	760 4	887.2	1013.9
24	432.0	504.0	576.0	648 0	720.0	864.0	1008.0	1152.0
25	488.3	569.7	651.0	732.4	813.8	976 6	1139.3	1302.1
26	549.3	640.8	732.3	823 9	915.4	1098 5	1281.6	1464.7
27	615.1	717.6	820.1	922 6	1025.2	1230.2	1435.2	1640.3
28	686.0	800.3	914.7	1029 0	1143.3	1372 0	1600 7	1829.3
29	762.2	889.2	1016.2	1143 2	1270.3	1524.3	1778.4	2032.4
30	843.8	984.4	1125 0	1265 6	1406 3	1687.5	1968.8	2250 0
31	931.0	1086.1	1241 3	1396.5	1551.6	1861.9	2172 3	2482.6
32	1024.0	1194 7	1365 3	1536 0	1706.7	2048 0	2389.3	2730.7
33	1123 0	1310 2	1497.4	1684.5	1871.7	2246.1	2620.4	2994 8
34	1228.3	1433.0	1637.7	1842.4	2047.1	2456.5	2865.9	3275.3
35	1339.8	1563.2	1786 5	2009.8	2233.1	2679 7	3126.3	3572 9
36	1458.0	1701.0	1944 0	2187.0	2430.0	2916.0	3402.0	3888.0
37	1582.9	1846.7	2110 5	2374 4	2638 2	3165.8	3693.4	4221.1
38	1714.8	2000.5	2286 3	2572.1	2857.9	3429 5	4001 1	4572.7
39	1853.7	2162.7	2471 6	2780.6	3089.5	3707.4	4325.3	4943.3
40	2000.0	2333 3	2666.7	3000 0	3333.3	4000.0	4666 7	5333.3
41	2153.8	2512.7	2871.7	3230 7	3589.6	4307.6	5025 5	5743.4
42	2315.3	2701.1	3087 0	3472 9	3858.8	4630.5	5402.3	6174 0
43	2484.6	2898.7	3312 8	3726.9	4141.0	4969.2	5797.4	6625.6
44	2662.0	3105.7	3549.3	3993.0	4436.7	5324.0	6211.3	7098.7
45	2847.7	3322.3	3796 9	4271.5	4746.1	5695 3	6644.5	7593.8
46	3041.8	3548.7	4055.7	4562.6	5069.6	6083.5	7097 4	8111.3
47	3244.5	3785.2	4326.0	4866.7	5407.4	6488.9	7570.4	8651 9
48	3456.0	4032 0	4608 0	5184.0	5760.0	6912.0	8064.0	9216.0
49	3676.5	4289.3	4902.0	5514.8	6127.6	7353.1	8578.6	9804.1
50	3906.3	4557.3	5208.3	5859.4	6510 4	7812.5	9114 6	10417
51	4145.3	4836.2	5527.1	6218.0	6908 9	8290.7	9672.5	11054
52	4394.0	5126.3	5858.7	6591.0	7323.3	8788.0	10253	11717
53	4652.4	5427.8	6203.2	6978.6	7754 0	9304.8	10856	12406
54	4920.8	5740.9	6561.0	7381.1	8201.3	9841.5	11482	13122

* Taken from a more complete table in the A.I.S.C. Manual, "Steel Construction," third edition.

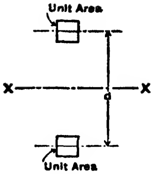


TABLE VI*

Moment of Inertia of a Pair of Unit Areas About Axis *X–X*

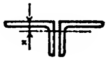
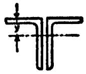
d	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
10	50	51	52	53	54	55	56	57	58	59
11	61	62	63	64	65	66	67	68	70	71
12	72	73	74	76	77	78	79	81	82	83
13	85	86	87	88	90	91	92	94	95	97
14	98	99	101	102	104	105	107	108	110	111
15	113	114	116	117	119	120	122	123	125	126
16	128	130	131	133	134	136	138	139	141	143
17	145	146	148	150	151	153	155	157	158	160
18	162	164	166	167	169	171	173	175	177	179
19	181	182	184	186	188	190	192	194	196	198
20	200	202	204	206	208	210	212	214	216	218
21	221	223	225	227	229	231	233	235	238	240
22	242	244	246	249	251	253	255	258	260	262
23	265	267	269	271	274	276	278	281	283	286
24	288	290	293	295	298	300	303	305	308	310
25	313	315	318	320	323	325	328	330	333	335
26	338	341	343	346	348	351	354	356	359	362
27	365	367	370	373	375	378	381	384	386	389
28	392	395	398	400	403	406	409	412	415	418
29	421	423	426	429	432	435	438	441	444	447
30	450	453	456	459	462	465	468	471	474	477
31	481	484	487	490	493	496	499	502	506	509
32	512	515	518	522	525	528	531	535	538	541
33	545	548	551	554	558	561	564	568	571	575
34	578	581	585	588	592	595	598	602	606	609
35	613	616	620	623	627	630	634	637	641	644
36	648	652	655	659	662	666	670	673	677	681
37	685	688	692	696	699	703	707	711	714	718
38	722	726	730	733	737	741	745	749	753	757
39	761	764	768	772	776	780	784	788	792	796
40	800	804	808	812	816	820	824	828	832	836
41	841	845	849	853	857	861	865	869	874	878
42	882	886	890	895	899	903	907	912	916	920
43	925	929	933	937	942	946	950	955	959	964
44	968	972	977	981	986	990	995	999	1004	1008
45	1013	1017	1022	1026	1031	1035	1040	1044	1049	1053
46	1058	1063	1067	1072	1076	1081	1086	1090	1095	1100
47	1105	1109	1114	1119	1123	1128	1133	1138	1142	1147
48	1152	1157	1162	1166	1171	1176	1181	1186	1191	1196
49	1201	1205	1210	1215	1220	1225	1230	1235	1240	1245

* Taken from a more complete table copyrighted by Weiskopf & Pickworth, Consulting Engineers, New York City, and published in the A.I.S.C. Manual, "Steel Construction," third edition.

TABLE VII*

Properties of Four Angles

For computing moment of inertia of flange angles

Size	Thick-ness					Size	Thick-ness				
		I-4 ⌊s	$2y$	I-4 ⌊s	$2x$			I-4 ⌊s	$2y$	I-4 ⌊s	$2x$
In.	In.	In.^{4}	In.	In.^{4}	In.	In.	In.	In.^{4}	In.	In.^{4}	In.
8 × 8	1⅛	392	4.8			6 × 4	1	123	4.3	43	2.3
	1	356	4.7				⅞	111	4.2	39	2.2
	⅞	318	4.6				¾	98	4.2	35	2.2
	¾	279	4.6				⅝	84	4.1	30	2.1
	⅝	238	4.5				½	70	4.0	25	2.0
	½	195	4.4				⅜	54	3.9	20	1.9
8 × 6	1⅛	356	5.4	170	3.4	5 × 5	1	78	3.2		
	1	323	5.3	155	3.3		⅞	71	3.1		
	⅞	289	5.2	140	3.2		¾	63	3.0		
	¾	254	5.1	123	3.1		⅝	54	3.0		
	⅝	216	5.0	105	3.0		½	45	2.9		
	½	177	4.9	87	2.9		⅜	35	2.8		
8 × 4	1	278	6.1	46	2 1	5 × 3½	¾	56	3.5	22	2.0
	⅞	249	6.0	42	2.0		⅝	48	3.4	19	1.9
	¾	219	5.9	38	1.9		½	40	3.3	16	1.8
	⅝	187	5.8	32	1.8		⅜	31	3.2	13	1.7
	½	154	5.7	27	1.7						
						4 × 4	¾	31	2.6		
7 × 4	1	191	5.2	45	2.2		⅝	27	2.5		
	⅞	172	5.1	41	2.1		½	22	2.4		
	¾	151	5.0	36	2.0		⅜	17	2.3		
	⅝	130	4.9	31	1.9						
	½	107	4.8	26	1.8	4 × 3½	⅝	25	2.6	18	2.1
	⅜	82	4.7	20	1.7		½	21	2.5	15	2.0
							⅜	17	2.4	12	1.9
6 × 6	1	142	3.7								
	⅞	128	3.6			4 × 3	½	20	2.7	10	1.7
	¾	113	3.6				⅜	16	2.6	8	1.6
	⅝	97	3.5				¼	11	2.5	5	1.5
	½	80	3.4								
	⅜	62	3.3								

* Taken from a more complete table in the A.I.S.C. Manual, "Steel Construction," third edition.

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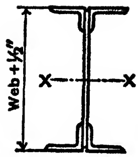
TABLE VIII*
Plate and Angle Girders
PROPERTIES OF SECTIONS

Short Legs Connected to Web Plate

⅞" Rivets



MATERIALS			Weight per Foot including Rivets	GROSS SECTION			NET SECTION			Maximum Allowable Shear	Rivet Factor
One Web Plate	Four Angles	Two Cover Plates		I_x	S_x	Max. Allowable Moment	I_x	S_x	Max. Allowable Moment		
			Lb.	In.⁴	In.³	Ft. Kips	In.⁴	In.³	Ft. Kips	Kips	In. Kips
39½" × ⅜" Web Plate	5×3½×⅜		94 9	6400	320	533	5762	288	480	122	617
	5×3½×7/16		101.4	7100	355	592	6391	319	531	122	592
	5×3½×½		107.8	7773	389	648	6993	349	581	122	573
	5×3½×9/16		114.2	8447	422	703	7596	380	632	122	558
	5×3½×⅝		120.6	9087	454	757	8166	408	680	122	546
	5×3½×11/16		126 6	9726	486	810	8734	437	728	122	536
	5×3½×¾		132.6	10334	517	862	9271	464	773	122	527
	5×3½×½	12×¼	134.2	10203	504	840	9166	453	755	122	546
	5×3½×½	12×⅜	144.4	11441	562	937	10224	502	836	122	539
	5×3½×½	12×½	154.6	12694	619	1032	11294	551	918	122	535
	6×4 ×⅜		101.7	7191	360	600	6588	329	548	130	588
	6×4 ×7/16		110.6	8010	400	667	7340	367	611	130	567
	6×4 ×½		118.2	8818	441	735	8081	404	673	130	550
	6×4 ×9/16		125.8	9613	481	802	8809	440	733	130	537
	6×4 ×⅝		133.4	10391	520	867	9520	476	793	130	526
	6×4 ×11/16		140.6	11141	558	930	10203	510	850	130	517
	6×4 ×¾		147.8	11899	595	992	10894	545	908	130	510
	6×4 ×13/16		155.0	12636	632	1053	11564	578	963	130	504
	6×4 ×⅞		162.2	13343	667	1112	12204	610	1016	130	499
	6×4 ×⅝	14×½	187.0	16132	787	1312	14567	711	1185	130	533
	6×4 ×⅝	14×⅝	198.9	17611	854	1423	15861	770	1283	130	531
	6×6 ×⅜		114.0	7766	388	647	7146	357	595	170	546
	6×6 ×7/16		123.2	8804	440	733	8115	406	677	170	534
	6×6 ×½		132.8	9725	486	810	8967	448	767	170	520
	6×6 ×9/16		142.0	10618	531	885	9791	490	816	170	508
	6×6 ×⅝		151.2	11516	576	960	10620	531	885	170	499
	6×6 ×11/16		160.4	12395	619	1031	11430	572	953	170	491
	6×6 ×¾		169.2	13246	662	1103	12212	611	1018	170	485
	6×6 ×13/16		178.4	14090	704	1173	12988	649	1082	170	479
	6×6 ×⅞		186.8	14917	746	1243	13746	687	1145	170	475
	6×6 ×⅝	14×½	204.8	17256	841	1402	15694	785	1308	170	484
	6×6 ×⅝	14×⅝	216.7	18735	908	1513	16979	849	1415	170	482

To obtain rivet pitch in any panel divide Rivet Factor by Shear in that panel. CAUTION: Not applicable for rivets carrying both horizontal and vertical shearing stresses.

Maximum Allowable Bending Moments are permissible only when compression flange is fully supported laterally.

Weight of rivets is based on spacing of 4 inches.

* Taken from a more complete table in the A.I.S.C. Manual, "Steel Construction," third edition. Values for Moment, Shear, and Rivet Factor are for A.I.S.C. (1936) Specification only.

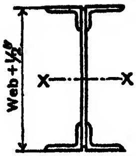


TABLE VIII* — (*Continued*)

Plate and Angle Girders

PROPERTIES OF SECTIONS

Short Legs Connected to Web Plate

¾″ Rivets

28

MATERIALS			Weight per Foot including Rivets	GROSS SECTION			NET SECTION			Maximum Allowable Shear	Rivet Factor
One Web Plate	Four Angles	Two Cover Plates		I_x	S_x	Max. Allowable Moment	I_x	S_x	Max. Allowable Moment		
			Lb.	In.⁴	In.³	Ft. Kips	In.⁴	In.³	Ft. Kips	Kips	In. Kips
27½″ × 5/16″ Web Plate	4×3 ×¼		55.4	1736	124	207	1523	109	182	111	363
	4×3 ×5/16		61.0	2014	144	240	1768	126	210	111	341
	4×3 ×⅜		66 2	2283	163	272	2003	143	238	111	327
	4×3 ×7/16		71 4	2551	182	303	2238	160	267	111	316
	4×3 ×½		76 6	2806	201	335	2460	176	293	111	307
	4×3 ×9/16		81.8	3056	218	363	2678	191	318	111	301
	4×3 ×⅝		86.6	3298	236	393	2887	206	346	111	296
	4×3 ×½	10×¼	99 6	3804	267	445	3299	232	387	111	295
	4×3 ×½	10×⅜	108.1	4314	300	500	3719	259	432	111	292
	4×3 ×½	10×½	116.6	4836	334	557	4150	286	477	111	289
	5×3½×5/16		67.0	2326	166	277	2090	149	248	112	324
	5×3½×⅜		73.8	2661	190	317	2393	171	285	112	311
	5×3½×7/16		80 2	2987	214	357	2688	192	320	112	302
	5×3½×½		86.6	3300	236	393	2969	212	353	112	296
	5×3½×9/16		93.0	3613	258	430	3251	232	387	112	290
	5×3½×⅝		99.4	3912	279	465	3518	251	418	112	286
	5×3½×11/16		105 4	4209	300	500	3784	270	450	112	282
	5×3½×¾		111.4	4492	321	535	4035	288	480	112	279
	5×3½×½	12×¼	113 0	4497	316	527	3992	280	467	112	287
	5×3½×½	12×⅜	123 6	5110	356	593	4515	314	523	112	284
	5×3½×½	12×½	133.8	5736	396	660	5050	348	580	112	283
	6×4 ×⅜		81 4	3024	216	360	2778	198	330	112	301
	6×4 ×7/16		89.4	3407	243	405	3131	224	373	112	293
	6×4 ×½		97.0	3783	270	450	3479	248	413	112	287
	6×4 ×9/16		104.6	4153	297	495	3819	272	453	112	282
	6×4 ×⅝		112.2	4515	322	537	4153	296	493	112	279
	6×4 ×11/16		119.4	4861	347	578	4471	319	532	112	275
	6×4 ×¾		126.6	5210	372	620	4790	342	570	112	273
	6×4 ×13/16		133.8	5551	396	660	5102	364	607	112	270
	6×4 ×⅞		141.0	5876	420	700	5399	385	641	112	268
	6×4 ×⅝	14×½	165.8	7358	507	845	6593	454	756	112	286
	6×4 ×⅝	14×⅝	177.7	8099	554	923	7241	496	827	112	284

To obtain rivet pitch in any panel divide Rivet Factor by Shear in that panel. CAUTION: Not applicable for rivets carrying both horizontal and vertical shearing stresses.

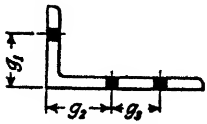
Maximum Allowable Bending Moments are permissible only when compression flange is fully supported laterally.

Weight of rivets is based on spacing of 4 inches.

* Taken from a more complete table in the A.I.S.C. Manual, " Steel Construction," third edition. Values for Moment, Shear, and Rivet Factor are for A.I.S.C. (1936) Specification only.

TABLE IX

Gages for Angles



Leg.	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	5	6	7	8
g_1	1	$1\frac{1}{8}$	$1\frac{3}{8}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
g_2	..	..	..	..	..	..	2	$2\frac{1}{2}$	$2\frac{1}{2}$	3
g_3	..	..	..	..	..	..	$1\frac{3}{4}$	$2\frac{1}{4}$	3	3
Max. Riv.	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	1	$1\frac{1}{8}$

TABLE X*

Double Angle Struts

ALLOWABLE CONCENTRIC LOADS IN KIPS

Long Legs Back to Back

1/2″ BACK TO BACK

Size	Thickness	Two Angles		Radius of Gyration		AXIS X-X: Unsupported Length in Feet										AXIS Y-Y: Unsupported Length in Feet											
		Area	Weight	X-X	Y-Y	9	10	12	14	16	18	20	22	24	26	6	7	8	9	10	12	14	16	18	20	22	24
6×3½	3/8	6.84	23.4	1.94	1.43	103	102	94	87	80	73	67	61	55	50	103	103	99	94	89	79	70	62	54	48	43	
	7/16	7.94	27.0	1.93	1.44	119	118	109	101	92	84	77	70	64	58	119	119	115	109	103	92	81	72	64	56	50	
	1/2	9.00	30.6	1.92	1.46	135	133	123	114	104	95	87	79	72	66	135	135	131	124	118	105	93	83	73	65	58	
	9/16	10.06	34.2	1.91	1.47	151	148	138	127	116	106	96	88	80	73	151	151	146	139	132	118	105	93	82	73	65	
	5/8	11.10	37.8	1.90	1.48	167	164	152	139	127	116	106	96	88	80	167	167	162	154	146	131	116	103	92	81	72	
	11/16	12.12	41.2	1.89	1.49	182	178	165	152	139	126	115	105	95	87	182	182	177	169	160	144	128	113	101	89	80	
	3/4	13.12	44.8	1.89	1.51	197	193	179	164	150	137	125	113	103	94	197	197	193	184	175	157	140	124	111	98	88	
	13/16	14.12	48.0	1.88	1.52	212	207	192	176	161	147	133	121	110	101	212	212	208	199	189	170	151	135	120	107	95	
	7/8	15.10	51.4	1.87	1.53	227	221	205	188	172	156	142	129	117	107	227	227	223	213	203	182	163	145	129	115	102	
6×4	3/8	7.22	24.6	1.93	1.67	108	107	99	91	84	77	70	64	58	53	108	108	108	106	101	92	83	75	67	61	54	49
	7/16	8.36	28.6	1.92	1.68	125	124	115	106	97	88	81	73	67	61	125	125	125	122	117	107	97	87	78	71	64	57
	1/2	9.50	32.4	1.91	1.69	143	140	130	120	110	100	91	83	76	69	143	143	143	139	134	122	110	100	90	81	73	65
	9/16	10.62	36.2	1.90	1.70	159	156	145	133	122	111	101	92	84	77	159	159	159	156	150	137	124	112	101	91	82	74
	5/8	11.72	40.0	1.90	1.71	176	173	160	147	135	123	112	102	93	85	176	176	176	173	166	151	137	124	112	101	91	82
	11/16	12.80	43.6	1.89	1.73	192	188	174	160	146	134	121	111	101	92	192	192	192	189	182	166	151	137	124	111	101	91
	3/4	13.88	47.2	1.88	1.74	208	204	188	173	158	144	131	119	108	99	208	208	208	206	198	181	165	149	135	122	110	99
	13/16	14.94	50.8	1.87	1.75	224	219	202	186	170	155	141	128	116	106	224	224	224	222	213	195	178	161	146	132	119	107
	7/8	15.96	54.4	1.86	1.76	239	233	215	198	181	164	149	136	123	112	239	239	239	238	228	209	191	173	156	141	128	116
7×3½	3/8	7.60	26.0	2.27	1.37	114	114	112	105	98	91	84	78	72	67	114	113	108	102	96	85	75	65	58	51	45	
	7/16	8.80	30.0	2.26	1.38	132	132	129	121	113	105	97	90	83	77	132	131	125	118	112	99	87	76	67	59	52	
	1/2	10.00	34.0	2.25	1.39	150	150	147	137	128	119	110	102	94	87	150	150	142	135	127	113	99	87	77	68	60	
	9/16	11.18	38.2	2.25	1.41	168	168	164	154	143	133	123	114	105	97	168	168	160	152	144	128	113	99	87	77	68	
	5/8	12.34	42.0	2.24	1.42	185	185	181	169	158	146	136	125	116	107	185	185	177	168	159	141	125	110	97	86	76	
	11/16	13.50	46.0	2.23	1.43	203	203	197	185	172	160	148	137	126	116	203	203	194	185	175	155	138	121	107	95	84	
	3/4	14.62	49.8	2.22	1.45	219	219	213	200	186	173	160	147	136	125	219	219	212	201	191	170	151	133	118	104	93	
	13/16	15.74	53.6	2.21	1.46	236	236	229	215	200	185	171	158	146	134	236	236	228	217	206	184	163	144	128	113	101	
	7/8	16.84	57.4	2.20	1.47	253	253	245	229	213	197	183	168	155	143	253	253	245	233	221	198	176	156	138	122	109	
	15/16	17.94	61.0	2.19	1.49	269	269	260	243	226	210	194	179	165	152	269	269	262	250	237	213	189	168	149	132	118	
	1″	19.00	64.6	2.19	1.50	285	285	276	258	240	222	205	189	174	161	285	285	279	266	252	226	202	179	159	141	126	
8×6	7/16	11.86	40.4	2.57	2.48	178	178	178	173	163	153	144	135	126	117	178	178	178	178	178	178	170	160	150	140	131	122
	1/2	13.50	46.0	2.56	2.48	203	203	203	196	185	174	163	153	143	133	203	203	203	203	203	203	194	182	171	160	149	139
	9/16	15.12	51.4	2.55	2.50	227	227	227	219	208	195	182	171	159	148	227	227	227	227	227	227	218	205	192	180	168	157
	5/8	16.72	57.0	2.54	2.51	251	251	251	242	228	215	201	188	176	164	251	251	251	251	251	251	241	227	213	200	186	174
	11/16	18.30	62.4	2.54	2.52	275	275	275	265	250	235	220	206	192	179	275	275	275	275	275	275	264	249	234	219	205	191
	3/4	19.88	67.6	2.53	2.52	298	298	298	287	271	255	239	223	208	194	298	298	298	298	298	298	287	271	254	238	222	207
	13/16	21.44	73.0	2.52	2.54	322	322	322	309	292	274	257	240	224	208	322	322	322	322	322	322	310	293	275	258	241	225
	7/8	22.96	78.2	2.51	2.55	344	344	344	331	312	293	274	256	239	223	344	344	344	344	344	344	333	316	295	277	259	242
	15/16	24.50	83.4	2.50	2.56	368	368	368	353	332	311	292	272	254	236	368	368	368	368	368	368	356	336	316	296	277	259
	1″	26.00	88.4	2.49	2.57	390	390	390	374	352	330	309	288	269	250	390	390	390	390	390	390	378	357	336	315	295	276

* Abstracted from the first edition of the A.I.S.C. Manual, "Steel Construction." Loads by A.I.S.C. (1928) Specification. Values to right of heavy broken lines are for secondary members only.

TABLE X* — (*Continued*)

Double Angle Struts

ALLOWABLE CONCENTRIC LOADS IN KIPS

3/8" BACK TO BACK

Long Legs Back to Back

Size	Thickness	Two Angles: Area	Two Angles: Weight	Radius of Gyration: X-X	Radius of Gyration: Y-Y	AXIS X-X, Unsupported Length in Feet: 4	5	6	7	8	9	10	12	14	16	18	AXIS Y-Y, Unsupported Length in Feet: 4	5	6	7	8	9	10	12	14	16	18
2½×2	3/16	1.62	5.5	.79	.92	24	22	20	18	16	14	13	10	...	...	...	24	24	22	20	18	17	15	12	10		
	1/4	2.12	7.2	.78	.93	32	29	26	23	21	19	17	13	...	...	...	32	31	29	26	24	22	20	16	14		
	5/16	2.62	9.0	.78	.95	39	36	32	29	26	23	20	16	...	...	...	39	39	36	33	30	27	25	21	17		
3×2½	1/4	2.62	9.0	.95	1.13	39	39	36	33	30	28	25	20	17	...	...	39	39	38	36	34	31	29	25	21	18	16
	5/16	3.24	11.2	.94	1.14	49	48	44	40	37	34	31	25	21	...	...	49	49	48	45	42	39	36	31	26	23	19
	3/8	3.84	13.2	.93	1.16	58	56	52	48	43	40	36	30	25	...	...	58	58	57	53	50	47	43	37	32	27	24
	7/16	4.42	15 2	.92	1.17	66	64	59	54	50	45	41	34	28	...	...	66	66	66	62	58	54	50	43	37	32	27
3½×2½	1/4	2.88	9.8	1.12	1.09	43	43	42	39	37	34	32	27	23	20	17	43	43	42	39	36	34	31	26	22	19	16
	5/16	3.56	12.2	1.11	1.10	53	53	52	49	45	42	39	33	28	24	21	53	53	52	48	45	42	39	33	28	24	20
	3/8	4.22	14 4	1.10	1.11	63	63	61	57	53	50	46	39	33	28	24	63	63	62	58	54	50	46	39	33	29	24
	7/16	4.86	16.6	1.09	1.12	73	73	70	66	61	57	52	44	38	32	26	73	73	71	67	62	58	53	46	39	33	29
	1/2	5.50	18.8	1.09	1.13	83	83	80	75	69	64	59	50	43	36	31	83	83	81	76	71	66	61	52	44	38	33
4×3	5/16	4.18	14.4	1.27	1.30	63	63	63	61	57	54	50	44	38	33	29	63	63	63	61	58	54	51	45	39	34	30
	3/8	4.96	17.0	1.26	1.31	74	74	74	72	68	63	59	52	45	39	34	74	74	74	73	69	65	61	53	47	41	36
	7/16	5.74	19.6	1 25	1.32	86	86	86	83	78	73	68	60	52	45	39	86	86	86	84	80	75	71	62	54	47	42
	1/2	6.50	22.2	1.25	1.33	98	98	98	94	88	83	77	67	58	51	44	98	98	98	96	91	86	81	71	62	54	47
	9/16	7.24	24.8	1.24	1.35	109	109	109	104	98	92	86	75	65	56	49	109	109	109	107	102	96	91	80	70	61	54
	5/8	7.96	27.2	1.23	1.36	119	119	119	114	107	100	94	81	70	61	53	119	119	119	118	112	106	100	88	78	68	60
4×3½	5/16	4.50	15.4	1.26	1.55	68	68	68	65	61	58	54	47	41	35	31	68	68	68	68	67	64	61	55	49	44	39
	3/8	5.34	18.2	1.25	1.56	80	80	80	77	72	68	64	55	48	42	36	80	80	80	80	79	76	72	65	58	52	47
	7/16	6.18	21.2	1.24	1.57	93	93	93	89	83	78	73	64	55	48	41	93	93	93	93	92	88	84	76	68	61	54
	1/2	7.00	23.8	1.23	1.58	105	105	105	100	94	88	82	72	62	54	46	105	105	105	105	105	100	95	86	77	69	62
	9/16	7.80	26.6	1 23	1.59	117	117	117	112	105	98	92	80	69	60	52	117	117	117	117	117	112	107	96	87	78	69
	5/8	8.60	29.4	1 22	1.60	129	129	129	123	115	108	101	87	75	65	56	129	129	129	129	129	123	118	107	96	86	77
	11/16	9.36	32 0	1.21	1.62	140	140	140	133	125	117	109	94	81	70	61	140	140	140	140	140	135	129	117	105	95	85
	3/4	10.12	34.6	1.20	1 63	152	152	152	143	134	126	117	101	87	75	65	152	152	152	152	152	146	140	127	115	103	92
5×3	5/16	4.80	16.4	1.61	1.22	72	72	72	72	72	69	66	60	54	48	43	72	72	72	68	64	60	56	49	42	36	32
	3/8	5.72	19.6	1.61	1 23	86	86	86	86	86	82	79	71	64	57	51	86	86	86	82	77	72	67	59	51	44	38
	7/16	6.62	22.6	1.60	1 24	99	99	99	99	99	95	91	82	74	66	59	99	99	99	95	89	84	78	68	59	51	44
	1/2	7.50	25.6	1.59	1.25	113	113	113	113	112	107	103	93	83	75	67	113	113	113	108	102	95	89	78	67	58	51
	9/16	8.36	28.6	1 58	1.26	125	125	125	125	125	119	114	103	92	83	74	125	125	125	121	114	107	100	87	76	66	57
	5/8	9.22	31.4	1.57	1.28	138	138	138	138	137	131	125	113	101	91	81	138	138	138	134	126	119	112	98	85	74	64
	11/16	10.06	34.2	1.56	1.29	151	151	151	151	150	143	136	123	110	98	88	151	151	151	147	138	130	122	107	93	81	71
	3/4	10.88	37.0	1.55	1 31	163	163	163	163	161	154	147	132	118	106	94	163	163	163	159	151	142	134	117	102	89	78
5×3½	5/16	5.12	17.4	1.61	1.45	77	77	77	77	77	74	70	64	57	51	46	77	77	77	77	74	71	67	60	53	47	41
	3/8	6.10	20.8	1.60	1.46	92	92	92	92	92	88	84	76	68	61	55	92	92	92	92	89	84	80	71	63	56	50
	7/16	7.06	24.0	1.59	1.47	106	106	106	106	106	101	97	87	78	70	63	106	106	106	106	103	98	93	83	74	65	58
	1/2	8.00	27.2	1.58	1.49	120	120	120	120	120	114	109	99	88	79	71	120	120	120	120	117	112	106	95	84	75	66
	9/16	8.94	30.4	1.57	1.50	134	134	134	134	133	127	121	110	98	88	78	134	134	134	134	131	125	119	107	95	84	75
	5/8	9.84	33.6	1.56	1.51	148	148	148	148	146	140	133	120	108	96	86	148	148	148	148	145	138	131	118	105	93	83
	11/16	10 74	36.6	1.56	1.52	161	161	161	161	160	153	146	131	118	105	94	161	161	161	161	158	151	144	129	115	103	91
	3/4	11.62	39.6	1.55	1.54	174	174	174	174	172	165	157	141	127	113	101	174	174	174	174	172	164	156	141	126	112	100

* Abstracted from the first edition of the A.I.S.C. Manual, "Steel Construction." Loads by A.I.S.C. (1928) Specification. Values to right of heavy broken lines are for secondary members only.

TABLE X* — (*Concluded*)

Double Angle Struts

ALLOWABLE CONCENTRIC LOADS IN KIPS

Long Legs Back to Back

3/8″ BACK TO BACK

Size	Thickness	Two Angles Area	Two Angles Weight	Radius of Gyration X–X	Radius of Gyration Y–Y	AXIS X–X Unsupported Length in Feet 9	10	12	14	16	18	20	22	24	26	AXIS Y–Y Unsupported Length in Feet 6	7	8	9	10	12	14	16	18	20	22	24
6×3½	3/8	6.84	23.4	1.94	1.39	103	102	94	87	80	73	67	61	55	50	103	102	97	92	87	77	68	60	53	46	41	
	7/16	7.94	27.0	1.93	1.40	119	118	109	101	92	84	77	70	64	58	119	119	113	107	102	90	79	70	62	54	48	
	1/2	9.00	30.6	1.92	1.41	135	133	123	114	104	95	87	79	72	66	135	135	129	122	116	103	91	80	70	62	55	
	9/16	10.06	34.2	1.91	1.42	151	148	138	127	116	106	96	88	80	73	151	151	144	137	130	115	102	90	79	70	62	
	5/8	11.10	37.8	1.90	1.43	167	164	152	139	127	116	106	96	88	80	167	167	160	152	144	128	113	100	88	78	69	
	11/16	12.12	41.2	1.89	1.45	182	178	165	152	139	126	115	105	95	87	182	182	175	167	158	141	125	110	98	87	77	
	3/4	13.12	44.8	1.89	1.46	197	193	179	164	150	137	125	113	103	94	197	197	190	181	172	153	136	120	107	94	84	
	13/16	14.12	48.0	1.88	1.48	212	207	192	176	161	14[illegible]	133	121	110	101	212	212	206	196	186	167	148	131	116	103	92	
	7/8	15.10	51.4	1.87	1.49	227	221	205	188	172	156	142	129	117	107	227	227	221	211	200	179	159	141	125	111	99	
6×4	3/8	7.22	24.6	1.93	1.62	108	107	99	91	84	77	70	64	58	53	108	108	108	104	100	90	81	73	65	59	53	47
	7/16	8.36	28.6	1.92	1.63	125	124	115	106	97	88	81	73	67	61	125	125	125	121	116	105	95	85	76	68	61	55
	1/2	9.50	32.4	1.91	1.65	143	140	130	120	110	100	91	83	76	69	143	143	143	138	132	120	109	98	88	79	71	64
	9/16	10.62	36.2	1.90	1.66	159	156	145	133	122	111	101	92	84	77	159	159	159	155	148	135	122	110	98	88	80	72
	5/8	11.72	40.0	1.90	1.67	176	173	160	147	135	123	112	102	93	85	176	176	176	171	164	149	135	122	109	98	88	80
	11/16	12.80	43.6	1.89	1.68	192	188	174	160	146	134	121	111	101	92	192	192	192	187	180	164	148	134	120	108	97	87
	3/4	13.88	47.2	1.88	1.69	208	204	188	173	158	144	131	119	108	99	208	208	208	204	195	178	161	146	131	118	106	96
	13/16	14.94	50.8	1.87	1.70	224	219	202	186	170	155	141	128	116	106	224	224	224	220	211	192	174	158	142	128	115	104
	7/8	15.96	54.4	1.86	1.71	239	233	215	198	181	164	149	136	123	112	239	239	239	235	226	206	187	169	152	137	124	112
7×3½	3/8	7.60	26.0	2.27	1.33	114	114	112	105	98	91	84	78	72	67	114	112	106	100	94	83	72	63	55	49	43	
	7/16	8.80	30.0	2.26	1.34	132	132	129	121	113	105	97	90	83	77	132	130	123	116	110	96	85	74	65	57	50	
	1/2	10.00	34.0	2.25	1.35	150	150	147	137	128	119	110	102	94	87	150	148	141	133	125	110	97	85	74	65	58	
	9/16	11.18	38.2	2.25	1.36	168	168	164	154	143	133	123	114	105	97	168	166	158	149	140	124	109	96	84	74	65	
	5/8	12.34	42.0	2.24	1.37	185	185	181	169	158	146	136	125	116	107	185	184	175	165	156	138	121	106	93	82	72	
	11/16	13.50	46.0	2.23	1.39	203	203	197	185	172	160	149	137	126	116	203	202	192	182	172	152	134	118	104	92	81	
	3/4	14.62	49.8	2.22	1.40	219	219	213	200	186	173	160	147	136	125	219	219	209	198	187	166	146	129	113	100	88	
	13/16	15.74	53.6	2.21	1.41	236	236	229	215	200	185	171	158	146	134	236	236	225	214	202	179	158	140	123	109	96	
	7/8	16.84	57.4	2.20	1.42	253	253	245	229	213	197	183	168	155	143	253	253	242	229	217	193	170	150	133	117	104	
	15/16	17.94	61.0	2.19	1.44	269	269	260	243	226	210	194	179	165	152	269	269	259	246	233	208	184	162	144	127	113	
	1″	19.00	64.6	2.19	1.45	285	285	276	258	240	222	205	189	174	161	285	285	275	261	248	221	196	173	153	136	121	
8×6	7/16	11.86	40.4	2.57	2.43	178	178	178	173	163	153	144	135	126	117	178	178	178	178	178	178	169	158	148	138	129	120
	1/2	13.50	46.0	2.56	2.44	203	203	203	196	185	174	163	153	143	133	203	203	203	203	203	203	192	181	169	158	147	137
	9/16	15.12	51.4	2.55	2.46	227	227	227	219	208	195	182	171	159	148	227	227	227	227	227	227	216	203	191	178	166	155
	5/8	16.72	57.0	2.54	2.46	251	251	251	242	228	215	201	188	176	164	251	251	251	251	251	251	239	225	211	197	184	171
	11/16	18.30	62.4	2.54	2.47	275	275	275	265	250	235	220	206	192	179	275	275	275	275	275	275	262	247	231	216	202	188
	3/4	19.88	67.6	2.53	2.48	298	298	298	287	271	255	239	223	208	194	298	298	298	298	298	298	285	268	252	235	220	205
	13/16	21.44	73.0	2.52	2.49	322	322	322	309	292	274	257	240	224	208	322	322	322	322	322	322	308	290	272	255	238	222
	7/8	22.96	78.2	2.51	2.51	344	344	344	331	312	293	274	256	239	223	344	344	344	344	344	344	331	312	293	274	256	239
	15/16	24.50	83.4	2.50	2.51	368	368	368	353	332	311	292	272	254	236	368	368	368	368	368	368	353	333	312	293	273	255
	1″	26.00	88.4	2.49	2.52	390	390	390	374	352	330	309	288	269	250	390	390	390	390	390	390	375	354	332	311	291	271

* Abstracted from the first edition of the A.I.S.C. Manual, "Steel Construction." Loads by A.I.S.C. (1928) Specification. Values to the right of heavy broken lines are for secondary members only.

APPENDIX E

ANSWERS TO SELECTED EXERCISE PROBLEMS

The answers given below are the results of slide-rule computations carried to three significant figures, except in certain cases where extension of the numerical answer seemed desirable as an aid in interpreting the result. (See footnote on page 10.)

Pages 23, 24. Chapter II

1. R_L = 10,000 lb. R_R = 11,000 lb.

2. R_L = 15,550 lb. R_R = 11,650 lb. For all practical purposes these may be considered as 15,600 lb. and 11,700 lb. respectively, although their sum (27,300) will not then be *exactly* equal numerically to the sum of the loads (27,200 lb.). Minor discrepancies of this sort in the third significant figure will, of necessity, occur frequently when computations are carried out on the 10-in. slide rule. (See footnote on page 10.)

3. R_L = 7,700 lb. R_R = 18,100 lb.

7. R_L = 10,000 lb. R_R = 11,000 lb. Shear: at left end = 10,000 lb., 6 ft. from left = 10,000 lb. and −2000 lb., 14 ft. from left = −2000 lb. and −11,000 lb., at right end = −11,000 lb. Bending moment: at left end = zero, 6 ft. from left = 60,000 ft.-lb., 14 ft. from left = 44,000 ft.-lb., at right end = zero.

8. R_L = 15,550 lb. R_R = 11,650 lb. Shear: at left end = 15,550 lb., 5 ft. from left = 15,550 lb. and −4550 lb., 8 ft. from left = −4550 lb., at right end = −11,650 lb. Bending moment: at left end = zero, 5 ft. from left = 77,750 ft.-lb., 8 ft. from left = 64,400 ft.-lb., 12 ft. from left = 39,400 ft.-lb., at right end = zero.

9. R_L = 7700 lb. R_R = 18,100 lb. Shear: at left end = 7700 lb., 5 ft. from left = 7700 lb. and −8300 lb., 15 ft. from left = −8300 lb. and 9800 lb., at right end = 5000 lb. Bending moment: at left end = zero, 5 ft. from left = 38,500 ft.-lb., at right reaction = −44,400 ft.-lb., 3 ft. from right end = −18,600 ft.-lb., at right end = zero. (B.M. = zero at 9.64 ft. from left end.)

Page 35. Chapter III

1. W = 28,300 lb.
2. V = zero at 11.5 ft. from right end. f = 18,000 lb./sq. in.
3. Required section modulus = 83.6 in.^{3} 18 I 54.7, 18 WF 50.

Pages 51, 52. Chapter III

1. V = 42,200 lb. 3. v = 10,700 lb./sq. in. N = 5.3 in.

6. Unif. ld. D = 0.322 in. Conc. ld. D = 0.293 in. Tot. D = 0.615 in.

8. (*a*) D = 0.828 in. (*b*) 12 WF 27. (*c*) 10 WF 39, 8 WF 58.

Page 59. Chapter III

1. (*a*) 8 WF 20. (*b*) 10 WF 21.

Pages 70, 71. Chapter III

1. Two 10 WF 45. 2. (*a*) 11.4 kips. (*b*) 6.6 kips.
9. $t = 0.79$ in.; therefore, use $6 \times 7 \times \frac{13}{16}$-in. plate or next greater standard thickness available.

Pages 87, 88. Chapter IV

2. Shear = 12,500 lb. per sq. in. Bearing = 34,300 lb. per sq. in.
6. $R = 48{,}800$ lb. 11. (*a*) 6 bolts. (*b*) 6 bolts.

Page 109. Chapter V

1. This problem is self-checking.

Page 116. Chapter VI

1. $P = 124$ kips.

Page 121. Chapter VI

1. 10×10 WF 72 or 12×12 WF 72.

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