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APPLIED THERMODYNAMICS

A TEXT-BOOK COVERING THE SYLLABUSES OF THE B.Sc. (Eng.), Inst.C.E., AND I Mech.E. EXAMINATIONS IN THIS SUBJECT

BY THE LATE

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PREFACE TO THIRD EDITION

WHILE the main structure of the book remains the same, considerable revision has been effected.

Thus the chapter on Air Compression has been modified to include applications to the gas turbine, while the elements of thermodynamics as applied to this latter subject are included in the chapter on Internal Combustion Engines.

The subject matter of Refrigeration has been modernized and amplified, and a new chapter on First Principles of Heat Transmission has been added.

Thanks are due to several authorities for permission to include extracts from their publications. These are acknowledged in footnotes in appropriate places.

I am also indebted to the University of Nottingham for permission to include examples from examinations.

Suggestions and corrections received from time to time are hereby acknowledged, and will be received with gratitude in the future.

J. M. D.

PREFACE TO SECOND EDITION

IN this edition the opportunity has been taken to include a number of improvements suggested by readers of the previous edition. Grateful thanks are due to these gentlemen.

The chapter on Internal Combustion Engines has been rewritten and additional chapters on the use of steam, and on engine testing have been added.

No attempt has been made to include descriptive matter, as, apart from limitations of space, students have ample facilities in this direction offered by the technical press, and by the many admirable publications of engineering manufacturers.

J. M. D.

PREFACE TO FIRST EDITION

THE aim of this book is to aid engineering students and engineers studying heat engines and the scientific principles which condition their performance and thermal efficiency. The scope of the work is that indicated by the syllabus of Applied Thermodynamics for the B.Sc. degree (Engineering) Examination of the University of London, and includes a portion of the work suitable for engineers taking the examinations of the Institutions of Civil and Mechanical Engineers, as well as for the advanced National Diploma and Higher Certificate awarded jointly by the Mechanical Institution and the Board of Education. As a further guide, by the kind permission of the Senate of the University of London, and the University of London Press, Ltd., and of the Councils of the Institutions of Civil and Mechanical Engineers, questions have been selected from papers set at their recent examinations. However, experimental work and accurate measurements in the laboratory are all-important, and the engineering student must still keep in touch with modern practice and the rapid advances of research on the different branches of this subject.

Experience in teaching has encouraged me to endeavour by simple treatment from first principles to make the work useful as a text-book. A sketch, curve or diagram represents relations more clearly and is more easily understood by the average student than a description or general symbols. Accordingly, at the outset, the laws of curves, indicators for tracing the diagrams of the pressure in the engine cylinder, and a few measuring instruments, are briefly dealt with.

Mr. J. M. Dickson, B.Sc., kindly collaborated with me by writing the four chapters on the steam engine and steam turbines; and by drawing the temperature-entropy chart for water and steam.

I have much pleasure in thanking Mr. William Reavell, of Ipswich, who very kindly supplied me with data on air compressors; and Mr. John L. Hodgson, B.Sc., with his assistant, Mr. R. M. Maling, B.Sc., for data on the measurement of compressed air; and Mr. J. Bedford, B.Sc., for the preparation of some of the drawings.

I am also indebted to Professor W. E. Dalby, Professor A. H. Gibson, Mr. Harry R. Ricardo, and other friends and firms who helped me with data, for which acknowledgment is made in the footnotes.

Although care has been taken in correcting proofs and checking numerical examples, errors may remain, and I shall be grateful to the reader for any corrections.

WILLIAM ROBINSON

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CHAPTER I

INTRODUCTORY

A **Heat Engine** is a machine which converts heat into mechanical work. Heat is usually generated by the combustion of a fuel: either coal, oil, or gas. The heat, supplied to the working substance at a high temperature, is reduced in temperature by expansion within the engine, and, in the process, part of the heat is converted into work and part rejected at a lower temperature. As the heat energy in the working substance is gradually changed into mechanical energy, the temperature falls, and the greater the temperature drop of the working substance within the engine, the larger will be the proportion of heat utilized or turned into work.

In the reverse process, work is spent in driving a **Refrigerating Machine** or heat pump, which takes in heat from bodies at a low temperature and discharges heat at a higher temperature, generally by changes of state of the working substance. In this reversed heat engine, the smaller the difference between the high and low temperatures, the less work will be required to extract a given quantity of heat from the cold body. The working substance may be air, or a mixture of liquid and vapour, and may change its state in various ways.

Applied Thermodynamics deals with the relation of heat to mechanical work, and the various physical properties of the working substance. The working substances chiefly used in heat engines are fluids in the gaseous and liquid states: air and other gases or vapours in the *internal combustion engine*; water vapour in the *steam engine* or *turbine*; also ammonia and carbon dioxide, commonly called carbonic acid, mostly in *refrigerating machines.*

Ideal Cycle of Operations. In the action of a heat engine, when a given mass of the working substance receives heat at a high temperature, it undergoes a series of changes of pressure, volume, temperature and physical state, rejects some heat at a lower temperature, and is brought back to its initial state or condition in every respect. It has thus passed through a complete cycle of operations.

Systems of Units. In order to reduce and compare the experimental results from various investigations and the results of modern practice, it is necessary to consider the different systems of units commonly adopted by engineers: (i) British—Foot-pound and Centigrade or Fahrenheit; (ii) Metric—Metre, kilogramme and Centigrade; and (iii) the C.G.S.—Centimetre-gramme-second system, universally adopted for scientific purposes, especially in Chemistry and Experimental Physics.

It is desirable for the engineering student to be familiar, at least, with the first two systems in order to compare the performance of engines given in either of them. Most British engineers still use the Foot-pound-Fahrenheit system, although modern research is expressed in values on the Centigrade scale, which is convenient for comparison with the metric system. Thus quantities of heat are represented by the same figures when given either as lb-degree-Centigrade per lb., or kilo-calorie per kilogramme.

The engineering Institutions of this country are making laudable efforts to promote international agreement on the rating and performance of the various types of heat engines.

UNITS OF MEASUREMENT

Units of Length. One metre = 100 centimetres = 1,000 millimetres = 39·3702 inches; and one-third of the standard yard = 1 foot = 0·3048 metre, or 1 inch = 2·54 centimetres.

Units of Force. British engineers take the gravitational unit of force as the weight of the standard *pound* (1 lb.) avoirdupois of 16 ounces; and Continental engineers the weight of the international standard *kilogramme*, which is equal to 2·20462 lb.; or 1 lb. = 0·4535924 kg. = 453·5924 grammes.

A body of weight W lb. has a mass $m = \frac{W}{g}$, since force = mass × acceleration. The *weight* of a body means the force of attraction of the earth on the mass of the body. The acceleration of gravity, g, is slightly less at the equator than towards the poles, and the weight of 1 lb. is greater in London than in New York by 1 part in 1,000. The intensity of gravity also varies along a given parallel of latitude, so that, for scientific precision, the locality should be given. American and Continental engineers generally take the acceleration of gravity at the sea-level in Lat. 45°, $g = 980·665$ C.G.S. units. In London it is 981·16 C.G.S. or 32·19 ft. per sec. per sec. For nearly all engineering purposes, $g = 32·2$, and the difference is negligible.

The C.G.S. unit of force is 1 *dyne*, or that force which, acting on a gramme mass for a second, gives it a velocity of a centimetre per second, i.e. unit acceleration. This absolute unit of force is independent of gravity, and is the same everywhere.

The unit or measure of *volume* is the British legal* Imperial standard *gallon*, containing 10 lb. weight of distilled water at the temperature of 62° F. and barometer 30 inches. The Imperial gallon contains 8 *pints*, so that, in Great Britain, 1 pint of pure water weighs 1¼ lb. An American gallon of pure water weighs only 8·331 lb., thus 6 American gallons are nearly equal to 5 Imperial

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* See *Weights and Measures Acts* of 1878, 1889, 1892 and 1893; and *Our Weights and Measures*, by H. J. Chaney.

gallons. A standard oil barrel of 42 American gallons contains only 35 British gallons.

Specific Volume. One cu. metre per kg. = 16·0186 cu. ft. per lb.

Pressure is the weight or force per unit of area, and is expressed in pounds weight per square inch and kilogrammes per square centimetre : 1 lb. per sq. in. = 0·070307 kg. per sq. cm. (in London) ; or 1 kg. per sq. cm. = 14·223 lb. per sq. in.

The standard *atmospheric pressure* is that of a column of mercury 760 mm. high at 0° C. in Lat. 45°, nearly 14·689 lb. per sq. in., or 1·0327 kg. per sq. cm. (London), and is practically 30 in. of mercury column at 62° F. in London; and 30 in. of mercury at 62° F. = 14·7 lb. per sq. in., so that 1 in. mercury column is equal to 0·49 lb. per sq. in. Thus the barometer reading in inches multiplied by 0·49 gives the atmospheric pressure in lb. per sq. in. One cm. of mercury column = 0·1933 lb. per sq. in. = 0·01359 kg. per sq. cm. The absolute (C.G.S.) unit of pressure is 1 dyne per sq. cm., and 1 atmosphere is $1{\cdot}0133 \times 10^6$ dynes per sq. cm., which is independent of gravity.

Absolute pressure, reckoned from zero, is obtained by adding the atmospheric pressure to the gauge or indicator pressure.

The *actual* or *absolute pressure* in a condenser is found by subtracting the reading of the vacuum gauge from that of the barometer, both readings being usually expressed either in inches of mercury or pounds per square inch.

The Bourdon pressure gauge is used to measure the pressure of steam in a boiler, also the pressure of air and steam in a condenser.

Pressure gauges and indicator springs are tested or calibrated by comparison with a mercury column subjected to steam pressure at the same temperature as when in use ; or by balancing the pressure against a column of fluid of known vertical height and specific gravity. Also, in the *dead weight tester*, a standard weight acts on a piston of hardened steel, accurately fitting into a steel cylinder, and pressing on oil, the free surface of which is acted on by the pressure to be measured. This tester has been developed by the National Physical Laboratory. The piston and cylinder can be changed, so that a comparatively small weight is used, even at the highest pressures. This instrument will show 1 lb. increase in a pressure of 20,000 lb. per sq. in.

A *water gauge* manometer is employed to measure the slight variation in the pressure of air and gas above or below atmospheric pressure. The difference in the heights of water, oil, or mercury in a glass U-shaped tube is shown in inches, one limb of the tube being open to the atmosphere and the other connected to the gas supply, or to the furnace gas in the base of the chimney to measure the draught. The single tube and curved tube manometers (p. 191) are convenient to use ; other special instruments of great precision in measuring air or gas pressures have been specially designed in various investigations.*

Pure water at 10° C. (50° F.) weighs 62·41 lb. per cu. ft., so that each foot of head of water is equal to a pressure of 62·4 lb. per sq. ft., or 0·4334 lb. per sq. in.; and 1 in. of water column = 0·036 lb. per sq. in. Mercury is 13·596 times heavier than water at standard temperature and pressure.

The engineer's units of work are the foot-pound (ft-lb.), and kilogramme-metre. One ft-lb. is the work done by a force of 1 lb. weight acting through a distance of 1 ft. in its own direction. One ft-lb. = 0·138255 kg-metre (London), or 1 kg-metre = 7·233 ft-lb. The C.G.S. unit of work is an *erg*, being the work done by a dyne acting through a distance of a centimetre. The more convenient *practical* unit of work employed by electrical engineers is 1 joule = 10^7 ergs = 0·7372 ft-lb.; and 1 ft-lb. = 12 × 2·54 cm. × 453·6 grammes × 981 ergs = 1·356 × 10^7 ergs = 1·356 joules.

Power is the *rate of doing work* = $\dfrac{\text{work done}}{\text{time taken}}$; or, work done = power × time.

In order to determine the *power* of an engine, we have to make two measurements, viz. a measurement of the work and of the time during which the work is done, then the power is the quotient of work divided by time. The *unit of power*, introduced by James Watt and commonly adopted by British engineers, is 1 horse-power = 33,000 ft-lb. of work done per minute = 550 ft-lb. per sec., or 1,980,000 ft-lb. per hour.

The *metric* horse-power, or French *force-de-cheval*, is 75 kg-metres per sec. = 542·475 ft-lb. per sec. = 0·9863 British H.P.; or 1 British H.P. = 1·014 metric *force-de-cheval* (approx.).

The C.G.S. unit rate of working is 1 erg per sec., but this is an extremely small unit, and the *practical unit of electrical power is* 1 *joule per second*, and called 1 *watt*, which is equal to the work done per second by a current of 1 ampere in a circuit having 1 volt of electric pressure maintained between its terminals, and is equal to 1 ampere × 1 volt; 1 watt = 0·7372 ft-lb. per sec., or 44·23 ft-lb. per min. 1 British H.P. = $\dfrac{550}{0{\cdot}7372}$ = 746 watts, and 1 metric *force-de-cheval* is 736 watts; 1 kilowatt = 1·3405 H.P.

The *electrical horse-power* expended in a circuit and transformed into heat or mechanical energy is equal to the product of *amperes* of current and *volts* of electric pressure divided by 746. The *Board of Trade Unit* for supply of electrical energy is the work done in a circuit when the power exerted in watts, multiplied by the time in hours, equals 1,000, or 1,000 watt-hours, called 1 *kilowatt-hour*, equal to the supply of energy amounting to 1·3405 horse-power-hour.

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* See, for instance, *Proc. Inst. C. E.*, Vol. CCIV, Part 2 (1918), pp. 126 and 134.

Energy is the capacity for doing work, and the definite quantity of work adopted by British engineers as a *unit* for measuring the performance of a heat engine or power plant is 1 *horse-power-hour* = 33,000 × 60 = 1,980,000 ft-lb.

The comparison is made by the weight or quantity of fuel supplied or needed per horse-power-hour output of the power plant.

CHAPTER II

HEAT, WORK, POWER AND ENERGY

By **temperature** is meant the degree of *hotness* or intensity of heat in a body, and is that property in virtue of which heat tends to flow from a warm body to a colder one.

Measurements of temperature are based on some of its effects on substances—

1. In the ordinary thermometer the property made use of is the expansion of mercury or other liquid enclosed in a fine glass tube of uniform bore, and the temperature is indicated by the rise of the column of mercury. The indications depend on the difference of the expansion of the liquid and that of the glass.

In the mercury thermometer, the glass, after being heated, does not quite return to its original volume at once when cooled, so that the zero point is altered during the slow and gradual shrinkage of the glass.

In order to form a scale, the two definite temperatures selected as *fixed points* are the temperature of melting ice and the temperature of steam as it escapes from pure water boiling under standard atmospheric pressure. On the Centigrade scale the boiling point is marked 100° C. and the freezing point 0° C., while these temperatures on the Fahrenheit scale are 212° and 32° F.

The range of temperature between these fixed points is divided into equal parts called degrees, by comparison with standard instruments; 100° on the Centigrade scale and 212° - 32° or 180° F. Hence to reduce from one scale to the other, if readings of the same temperature are denoted by C and F, we have $\frac{C}{100} = \frac{F - 32}{180}$. Thus $C = \frac{5}{9}(F - 32)$, and $F = 1{\cdot}8\,C + 32$.

The mercury in glass thermometer is portable and convenient for ordinary purposes, where strict accuracy is not required. Although a temperature cannot be determined in this way with very great accuracy, without several corrections, *differences of temperatures* may be measured with more precision.

It is essential for accuracy in indications that mercury or other liquid thermometers should be standardized, that is, calibrated for corrections at all points on their scale by comparison with a standard instrument under the same conditions as when in use.

2. In the gas thermometer the increase of volume of air or gas when heated and kept at constant atmospheric pressure is about 20 times that of mercury for the same change of temperature, and furnishes an accurate scale of temperature as a standard of reference over a very wide range.

The *gas thermometer* is a delicate piece of apparatus requiring careful manipulation, and is not suitable for ordinary work. Perfectly dry air, hydrogen and nitrogen expand nearly the same fraction of their volume at 0° C. for a given rise of temperature, if the pressure is kept constant, and are used to measure temperature in two ways: (*a*) by change of volume while the pressure of the gas is kept constant, and (*b*) by change of pressure while the volume is kept constant. The scale of temperature of the hydrogen thermometer is the standard adopted below 100° C., and that of nitrogen at higher temperatures.

3. The *platinum thermometer** has been made a standard of reference for work of precision by Callendar and Griffiths, and measures temperature by the variation of the electrical resistance of *pure* annealed platinum wire, and is suitable for direct use at very high temperatures,† as well as at extremely low ones. A special advantage secured by this thermometer is the close agreement of the thermodynamical scale with the platinum scale of temperature, as given by the expression $p_t = 100(r - r_0) \div (r_1 - r_0)$, in which p_t denotes platinum temperature Centigrade, and r, r_1, and r_0 are the observed electrical resistances of the thermometer wire at the temperatures p_t, 100°, and 0° C. respectively. The change of resistance of the platinum thermometer wire between 100° C. and 0° C., $r_1 - r_0$, is called the *fundamental interval* of the thermometer, and is taken as 100° C., so that on the platinum temperature scale a rise of 1° C. at any temperature would increase the electrical resistance of the platinum wire by one-hundredth of the difference between its resistance at 100° and 0° C. The reduction to the scale of the gas thermometer is effected by the parabolic *difference formula*

$$t - p_t = 1{\cdot}5t\,(t - 100) \times 10^{-4}$$

4. A *thermo-electric couple* is composed of the junction of two different metals, as a piece of cast iron with wrought iron wire inserted, and leading to a cold junction to complete the electric circuit at the other end. A junction may also be formed by simply twisting together at one end a fine wire of pure platinum and another wire of platinum containing 10 per cent of rhodium or iridium connecting them with a dead-beat D'Arsonval galvanometer or suitable voltmeter, and inserting the other pair of ends in melting ice. On heating this thermo-couple by dipping the junction in melting metals, a small electromotive force is generated, which is plotted against the known fixed temperatures, and a calibration curve obtained giving the relation of electromotive force to temperature. Then the junction may be exposed to hot gases and their temperature measured. The platinum-iridium couple gives

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* *Phil. Trans.*, R.S. 1887, p. 161; 1891 and 1893.

† Heycock and Neville, *Journal of Chem. Soc.*, Feb., 1895. Harker and Chappins, *Phil. Trans.*, 1900. Chree, *Proc. R. S.*, 1900.

about 1½ times the electromotive force of the rhodium alloy, but the iridium volatilizes above 1,000° C., when the platinum-rhodium alloy should be used; while for the highest temperatures a couple of tungsten and tantalum may be used.

The chief use of the thermo-electric couple is for measuring very high and low temperatures or difference of temperatures; and it has proved specially suitable in the determination of the cyclical variation of temperature at any particular *point* or depth in the cylinder walls during the working of the steam engine,* and in the cylinder of the gas or petrol engine.†

5. Lord Kelvin devised a *thermodynamic scale* of temperature which is independent of the physical properties of any substance, and is deduced from the fact that two temperatures on this absolute scale are to one another in the same ratio as the quantities of heat taken in and rejected by a perfect reversible heat engine which works between the higher and lower of these temperatures (p. 122).

Professor H. L. Callendar has, by various investigations, deduced the correction or deviation of the scale reading on the gas thermometer‡ from that of the absolute thermodynamic scale, based on the "cooling effect" observed by Joule and Thomson in their porous plug experiment. A few values of the correction found for hydrogen are given, to show the degree of accuracy—

Temperature, C. . .	– 200°	– 10°	40° to 50°	200°	1,000°
Constant Volume . .	0·0311	0·00021	0·0004	0·00236	0·0438
Constant Pressure . .	0·201	0·00075	0·00132	0·0068	0·1040

At the fixed points, 0° and 100° C., the readings agree, and the greatest difference between the fixed points is at 40° to 50° C. Even at the extremely low temperature – 250° C., the correction of the constant volume hydrogen scale is only 0·1005° C.

Callendar takes the *absolute zero of temperature* 273·1° C. below the freezing point, and for his steam tables the exact values of absolute temperature,

$$T = t^\circ \text{ C.} + 273{\cdot}1 \text{ on the Centigrade scale,}$$

and

$$T = t^\circ \text{ F.} + 459{\cdot}58 \text{ on the Fahrenheit scale.}$$

In ordinary calculations, except for tables, the numbers 273 and 460 may be used, these values being within the limits of accuracy of all practical observations.

Units of Heat.§ Quantities of heat are measured by the heat required to raise the temperature of unit weight of water through

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* Callendar and Nicolson, *Proc. Inst. C. E.*, Vol. CXXXI, p. 1.

† Coker and Scoble, *Proc. Inst. C. E.*, Vol. CXCVI, p. 1.

‡ "Thermodynamical Correction of the Gas Thermometer," by Professor H. L. Callendar, *Proc. Phys. Soc.* (*London*) Vol. XVIII, May, 1903, pp. 282–334.

§ For a full discussion of heat units, see "The Heat Unit" by Dr. Ezer Griffiths, *Proc. Inst. Mech.E.*, 1951, Vol. 164, p. 215.

a given range, namely, between the two physically fixed points, the temperature of melting ice to that at which water boils under the constant pressure of one atmosphere.

The mean pound calorie (C.H.U.) is defined as $\frac{1}{100}$ part of the quantity of heat required to raise the temperature of one pound of water from 0° to 100° C.

The mean British Thermal Unit (B.Th.U.) is defined as $\frac{1}{180}$ part of the quantity of heat required to raise the temperature of one pound of water from 32° F. to 212° F.

Thus $$1 \text{ C.H.U.} = \frac{9}{5} \text{ B.Th.U.}$$

The *kilo-calorie* is one-hundredth part of the quantity of heat required to raise the temperature of 1 kilogramme of water from 0° to 100° C.

Thus 1 kilo-calorie is the heat taken to raise 1 kilogramme of water 1° C.

$$= 2{\cdot}20462 \text{ lb.} \times 1{\cdot}8^\circ \text{ F.} = 3{\cdot}9683 \text{ B.Th.U.}$$

The unit of heat commonly adopted in scientific work is the *gramme-calorie*, which is one-thousandth of the kilo-calorie, and is the quantity of heat required to raise the temperature of 1 gramme of water, under atmospheric pressure, by 1° C. measured on the scale of the standard hydrogen thermometer, either at 15° C. or, preferably, at 20° C.

Although Fahrenheit is still the most familiar scale of temperature, the Centigrade is also used by British engineers because of its international use and convenience, since the same numbers express quantities of heat energy in the British, metric, and C.G.S. systems.

Work can be represented graphically by an area on the pressure-volume diagram.

In the diagram, Fig. 1, at the state point A, the height AA_1 represents, to a suitable scale, the intensity of pressure, p, of a given mass of working substance on the piston which has area of cross-section a. Then $p \times a$ is the total absolute pressure or force in pounds on the piston. The volume at A is v_1, represented by the length OA_1, and the effective volume swept by the piston during its stroke of length, L ft., under the *constant pressure*, p, of the working substance to the state B, is $v_2 - v_1$. The area ABB_1A_1 represents the amount of work done by the fluid in moving the piston, since *work done* = *average force* (lb.) × *distance* (ft.) $= pa \times L$. If p is in lb. per sq. ft., and the piston area a sq. ft., $a \times L$ is the increase of volume of the working substance, $(v_2 - v_1)$ cu. ft., and the work done during this expansion while the pressure remains constant is $p(v_2 - v_1)$ ft-lb. = *average pressure* (*lb. per sq. ft.*) × *change of volume* (*cu. ft.*).

When the *pressure changes* during expansion, let the ordinates of the curve AC, Fig. 1, represent the relation of the pressure to the volume. If p is the average pressure acting on the piston, at any instant, while the piston is moved through a very short distance, l ft., during a small increase of volume $al = \delta v$, as the volume increases from v to $v + \delta v$, the work done by the fluid is $\delta w = pal = p\delta v$, represented by the area of the little shaded strip. In the limit when A_1B_1 is divided into an infinitely great number of equal parts, the change of volume δv for each becomes infinitely small. Then the total work done by the fluid in its expansion, from volume v_1 to v_2, is the sum of all such strips or of the values $p \,.\, dv = \int_{v_1}^{v_2} p \,.\, dv$, represented by the area ACB_1A_1 under the curve, AC, which shows the changes of pressure and volume. The area ACB_1A_1 may also be obtained graphically.

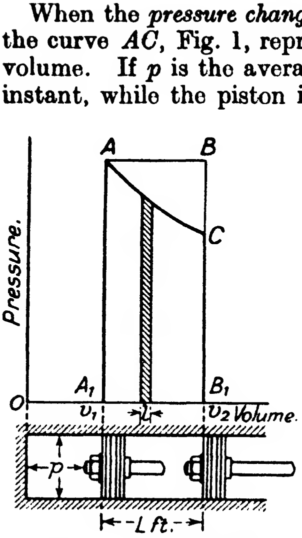

FIG. 1. P-V DIAGRAM

Laws of Curves. Generally, during the expansion and compression of the working substance in an engine cylinder, the pressure and volume follow a law of the form $p \,.\, v^n$ = *a constant*, C, where p is the absolute pressure, and v the total volume including clearance. To find a suitable value of n from the curve, take logarithms, and we have $\log p + n \log v = \log C$, which is the equation of a straight line.

Example 1. During the expansion of a given mass of air, the initial pressure was $p_1 = 80$ lb. per sq. in. when the volume, including clearance, $v_1 = 1$ cu. ft. At two other points measured on the curve, when the volume was 2 and 3 cu. ft., the indicated pressures were 33·36 and 20 lb. per sq. in. respectively.

If the air followed the law $p \,.\, v^n =$ constant, $p_1 \,.\, v_1{}^n = p_2 \,.\, v_2{}^n = p_3 \,.\, v_3{}^n = C$, that is, $80 \times 1^n = 20 \times 3^n = C$; hence $3^n = 4$, and, taking logarithms of each side, we have $n \log 3 = \log 4$, and $n = \dfrac{0{\cdot}60206}{0{\cdot}47712} = 1{\cdot}262$. Obviously, $C = 80$ when $v_1 = 1$, so that the law of expansion is $p \,.\, v^{1{\cdot}262} = 80$.

This rough calculation, from only two points on the curve, may be checked by substituting values of p and v, measured at the other point. Here, $p \,.\, v^{1{\cdot}262} = 33{\cdot}36 \times 2^{1{\cdot}262} = C$, and

$$\begin{aligned}\log C &= \log 33{\cdot}36 + 1{\cdot}262 \log 2 = 1{\cdot}52322 + 1{\cdot}262 \times 0{\cdot}30103 \\ &= 1{\cdot}9031\end{aligned}$$

hence $C = 80$, as before.

It is necessary to take several points on a curve in order to correct errors of observation.

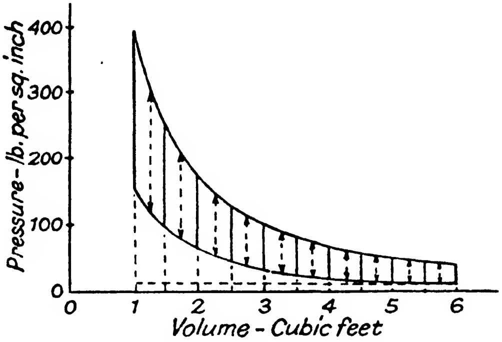

FIG. 2. *P-V* DIAGRAM

Example 2. The pv diagram, Fig. 2, shows the compression of 6 cu. ft. of gas at 14·7 lb. per sq. in. to 1 cu. ft., and after receiving heat at constant volume, which raises the pressure to 400 lb. per sq. in., the gas expands to the original volume.

To find the *law of expansion*, read off and tabulate the pressures and corresponding volumes from the expansion curve, Fig. 3, plotted to an enlarged scale. We have—

v cu. ft.	1	1·2	1·5	2	2·5	3	4	5	6
p lb. per sq. in.	400	318·4	241	168·2	127·3	101·3	70·7	53·5	42·6
log v	0	0·08	0·18	0·30	0·398	0·477	0·60	0·699	0·778
log p	2·6	2·5	2·38	2·23	2·105	2·005	1·85	1·73	1·63

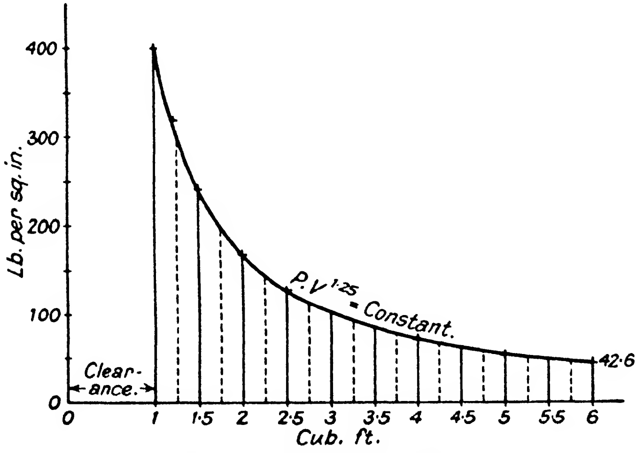

FIG. 3. EXPANSION CURVE

Plotting the values of log p and log v, on squared paper, Fig. 4, the points lie very nearly in a straight line, found by means of a fine thread stretched along them. Take any two points on the line, one near each end, for the corrected values of each pair of logarithms.

log p	2·35	1·6	Difference 0·75
log v	0·2	0·8	0·6

Now $p_1 . v_1^n = p_2 . v_2^n$, and taking logarithms,

$$\log p_1 + n \log v_1 = \log p_2 + n \log v_2$$

$$\therefore n = \frac{\log p_2 - \log p_1}{\log v_1 - \log v_2} = \frac{0{\cdot}75}{0{\cdot}6} = 1{\cdot}25$$

Or, substituting these values in the equation,

$$\log p + n . \log v = \log C$$

gives $1{\cdot}6 + 0{\cdot}8n = \log C$

and $2{\cdot}35 + 0{\cdot}2n = \log C$

Subtracting, we have $0{\cdot}6n = 0{\cdot}75$, and $n = 1{\cdot}25$; also, when $v = 1$, $C = 400$. On this graph the index n gives the *slope* of the line, that is, the ratio of the decrease of log p to the change of log v between any two points on the line.

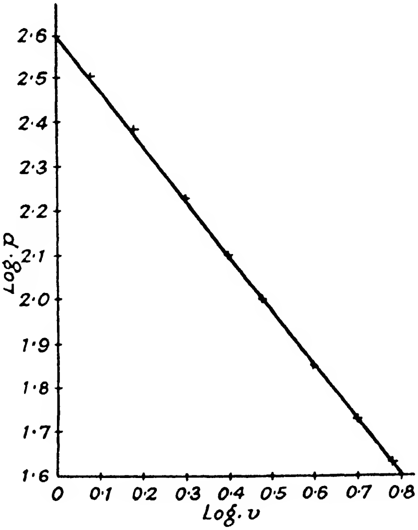

FIG. 4

Hence the *law of expansion* is $p . v^{1{\cdot}25} = 400$.

In the same way the relation between p and v, on the *compression curve*, is determined and expressed by the equation $p \cdot v^{1\cdot 35} = 165$.

These equations give information as to the behaviour of the charge in a gas engine cylinder during compression, explosion, and expansion.

Work done *by a fluid during expansion* according to the law

$$p \cdot v^n = a\ constant = C$$

The work done is measured by the area under the *expansion curve*, Fig. 1, between the given limits v_1 to v_2, and denoted by

$$W = \int_{v_1}^{v_2} p \cdot dv$$

We are given $p = \frac{C}{v^n} = C \cdot v^{-n}$

In this case, by integration,

$$W = C\int_{v_1}^{v_2} v^{-n} \cdot dv \cdot = \frac{C}{1-n}\Big[v^{1-n}\Big]_{v_1}^{v_2} = \frac{C}{1-n}[v_2^{1\cdot n} - v_1^{1\cdot n}],$$

and since $p_1 \cdot v_1^n = p_2 \cdot v_2^n$,

$$W = \frac{1}{1-n}(p_2v_2 - p_1v_1) = \frac{p_1v_1 - p_2v_2}{n-1} \quad . \quad . \quad . \quad . \quad (1)$$

or $$W = \frac{p_1v_1}{n-1}\left\{1 - \left(\frac{v_1}{v_2}\right)^{n-1}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also $\left(\frac{v_1{}^n}{v_2}\right) = \frac{p_2}{p_1}$, so that the expression becomes

$$W = \frac{p_1v_1}{n-1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

expansion

During *compression* the work done *by* the fluid is negative, since work is done *upon it* by the piston, then equation (1) becomes

$$W = \frac{p_2v_2 - p_1v_1}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and equation (3) is changed to

compression

$$W = \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

When the pressure is expressed in pounds per *square foot*, and the volume in *cubic feet*, the work done will be obtained in foot-pounds.

In the case when $n = 1$, the expansion or compression curve is expressed by the equation, pv = constant, and $p_1v_1 = p_2v_2 = k$, say.

Then $$W = \int_{v_1}^{v_2} p \cdot dv = \int_{v_1}^{v_2} k \cdot \frac{dv}{v} = k\int_{v_1}^{v_2}\frac{dv}{v} = k\Big[\log_\varepsilon v\Big]_{v_1}^{v_2}$$

and $$W = k \cdot \log_\varepsilon \frac{v_2}{v_1} = p_1v_1 \cdot \log_\varepsilon r \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

r is called the *ratio of expansion* or compression.

Multiply common logarithms of numbers (to the base 10) by 2·3026 to convert them into " Napierian " or " natural " logarithms, $\log_\varepsilon$, (to the base $\varepsilon = 2{\cdot}71828$) used in equation (6).

Example 3. Given the results in Example 2, from the diagram, Fig. 2, calculate the work done during expansion and compression of the gas ; also the net effective work done by the gas, and the mean effective pressure during the cycle.

During expansion of the gas, $p \, . \, v^{1.25}$ = constant, and when the volume is 1 cu. ft. the pressure is 400 lb. per sq. in., and

$$p_2 \cdot v_2^{1.25} = p_1 \cdot v_1^{1.25}, \quad \therefore p_2 = p_1 \left(\frac{v_1}{v_2}\right)^{1.25} = 400\left(\frac{1}{6}\right)^{\frac{5}{4}}$$

Taking logarithms, $\log p_2 = \log 400 - \frac{5}{4} \log 6$

$$= 2{\cdot}60206 - \frac{5}{4} \times 0{\cdot}77815 = 1{\cdot}6294$$

$\therefore p_2 = 42{\cdot}6$ lb. per sq. in., or $42{\cdot}6 \times 144$ lb. per sq. ft.

Work done during expansion *by* the gas, from equation (1),

$$W = \frac{p_1 v_1 - p_2 v_2}{n - 1}$$

and work done

$$= \frac{144(400 \times 1 - 42{\cdot}6 \times 6)}{1{\cdot}25 - 1} = \frac{144}{0{\cdot}25}(400 - 255{\cdot}6)$$

$= 576 \times 144{\cdot}4 = 83{,}175$ ft-lb. *Answer.*

Equations (2) or (3) may be used to check this answer.

The work done may also be measured directly from the expansion curve, Fig. 3. Divide the area under the curve into ten strips by equidistant ordinates. Read off the mid-heights, and the average value of these gives the mean pressure during expansion 115·52 lb. per sq. in. Work done by the gas is pressure × change in volume

$$= 144 \times 115{\cdot}52\,(6 - 1) = 83{,}175 \text{ ft-lb.}$$

For work done *on* the gas during compression, we have the initial pressure $p_3 = 14{\cdot}7$ lb. per sq. in., and the *ratio of compression* = 6.

The law is $p_4 \times 1^{1.35} = p_3 \times 6^{1.35}$, or $p_4 = 14{\cdot}7 \times 6^{1.35}$.

Taking logarithms, $\log p_4 = \log 14{\cdot}7 + 1{\cdot}35 \log 6$

$$= 1{\cdot}16732 + 1{\cdot}35 \times 0{\cdot}77815 = 2{\cdot}21782$$

and $\therefore p_4 = 165{\cdot}125$ lb. per sq. in.

Substituting these values in equation (4),

$$\text{Work done} = \frac{144(165{\cdot}125 \times 1 - 14{\cdot}7 \times 6)}{1{\cdot}35 - 1} = \frac{144}{0{\cdot}35}(165{\cdot}125 - 88{\cdot}2)$$

$= \frac{144}{0{\cdot}35} \times 76{\cdot}925 = 31{,}650$ ft-lb. *Answer.*

To check this result by equation (5), we have

$$\text{Work done} = \frac{144 \times 14{\cdot}7 \times 6}{1{\cdot}35 - 1}\left\{\left(\frac{165{\cdot}125}{14{\cdot}7}\right)^{\frac{0{\cdot}35}{1{\cdot}35}} - 1\right\}$$

$$= 36{,}288\,(1{\cdot}8722 - 1) = 31{,}650 \text{ ft-lb.}$$

The effective work done *by* the gas during the two operations of compression and expansion is therefore 83,175 – 31,650, or 51,525 ft-lb.

Mean Pressure. Again, since the work done is equal to the mean effective pressure (M.E.P. or p_m), multiplied by the stroke volume, $p_m(v_2 - v_1) = 5p_m = 51{,}525$ ft-lb.; and

$$\therefore p_m = \frac{51{,}525}{5} = 10{,}305 \text{ lb. per sq. ft.} = 71{\cdot}56 \text{ lb. per sq. in.}$$

The *mean effective pressure* (M.E.P.) during compression and expansion, Fig. 2, is represented by the average width between the two curves, measured parallel to the pressure axis of the closed area of diagram. Then the work done is this mean effective pressure (lb. per sq. in.) multiplied by the effective area of the piston in square inches, and by the length of the piston stroke in feet.

The area may be divided into ten strips by equidistant lines at right angles to the atmospheric lines (Fig. 2). Measure the mid-length of each strip as shown by dotted lines, and the average value of these, multiplied by the pressure scale of the diagram, that is, the scale number of the indicator spring used in tracing the diagram, gives the mean effective pressure.

The average width of the diagram is generally measured more accurately and quickly by means of the Amsler *planimeter*, thus—

Set the two points on the adjustable sliding bar to the *exact* length of the diagram, measured on the atmospheric line. Start with the tracer at a point about mid-length of the diagram and having the arms of the planimeter at right angles. Then move the tracing point clockwise around the diagram, and the reading divided by 40 gives the average width of the diagram. The accuracy of the instrument may be tested by using it to measure the known height of a carefully drawn rectangle of about the same area and length.

In order to determine the relation between the *absolute pressure* and volume of the fluid, as in Fig. 2, from an indicator diagram, the barometer reading is noted when the diagram is taken. The height of the mercury column in inches, multiplied by 0·49 (see p. 3), gives the atmospheric pressure in pounds per square inch. The line of zero pressure can then be drawn parallel to the atmospheric line, at a distance below it representing the atmospheric pressure to the appropriate scale of the diagram.

Clearance. The space between the cylinder cover and the piston at the end of the compression or in-stroke, including valve passages in which the working fluid may remain, is called the *clearance volume.* It is found directly in some engines by the measured quantity of water that fills it, taking care to avoid air being locked in the passages. The length, OI, Fig. 2, is set off in the same ratio to the length of the diagram as the clearance volume is to the piston displacement.

Engine Indicators. The function of the engine indicator is to trace and register a pressure-volume diagram, showing the actual changes in pressure and volume of the working substance in the cylinder during a cycle of operations. The *indicator diagram* thus shows the events of the cycle, and by its shape indicates faulty setting of valves, and other irregularities in the action of the engine. From the area of the diagram, the mean effective pressure during the cycle and the rate at which the fluid is doing work upon the piston, called the *indicated horse-power*, may be determined.

The simple *piston and pencil indicator*, invented by James Watt, has been improved and designed to obviate the difficulties due to rapid variation in pressure and increased speed of engines by McNaught, Richards, and other inventors, who introduced the paper drum, light piston and moving parts, short and stiff springs allowing the indicator piston only a short travel and magnifying this motion four- or six-fold by the parallel motion linkage. The other modern types—the *optical indicator* and the *electrical indicator*—are less portable and more especially adapted for scientific research purposes and work in the laboratory.

The Crosby Indicator,* Fig. 5, is a piston and pencil type, designed for steam, gas, or oil engines. The indicator is fixed by the union U on a cock screwed into the end of the engine cylinder, and communicates with the clearance space. The piston P, of ½ or ¼ sq. in. area, on which the fluid pressure acts when the indicator cock is open, is in the form of the central zone of a sphere, having line contact with the cylinder so as to reduce friction. One end of the hollow piston-rod is pivoted at the centre of the piston, and the other end attached to the top of the controlling spring S by ball and socket joints. The helical spring is fixed externally so as not to be exposed to the high temperature of the gases, which would affect the elasticity of the steel. This spring, also shown in Fig. 6, is formed from a piece of steel wire, having a steel bead fixed in the middle, and made into a double spiral, the ends of which are passed through holes drilled helically in four radial wings of the metal head to hold the spring securely. Adjustment of the exact strength is made by screwing the ends of the wire into the head more or less. While the piston is moved rapidly upwards by pressure the spring is in tension. The cylinder of the indicator is held by a screwed flange at the top end only, and is free to expand. The piston needs to be kept clean and carefully lubricated. Its rise or fall is proportional to the change of pressure in the engine cylinder, and its travel is kept very short by the strong spring, but

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* Indicators of a similar type are made by Messrs. Dobbie McInnes, Ltd.

multiplied six times by the light linkage guiding the long, stiff arm carrying the pencil point *R*, which traces a vertical line on the paper drum *D*.

The length of the vertical lines traced by the pencil *R* upon the paper wrapped round the indicator drum, is proportional to the pressure to the scale marked on the spring. With a spring marked 200, a pressure of 200 lb. per sq. in. will give a height of 1 in. on the diagram, but, if the area of the indicator piston is ½ sq. in., then 1 in. vertical height on the diagram will indicate a pressure of 400 lb. per sq. in. in the engine cylinder.

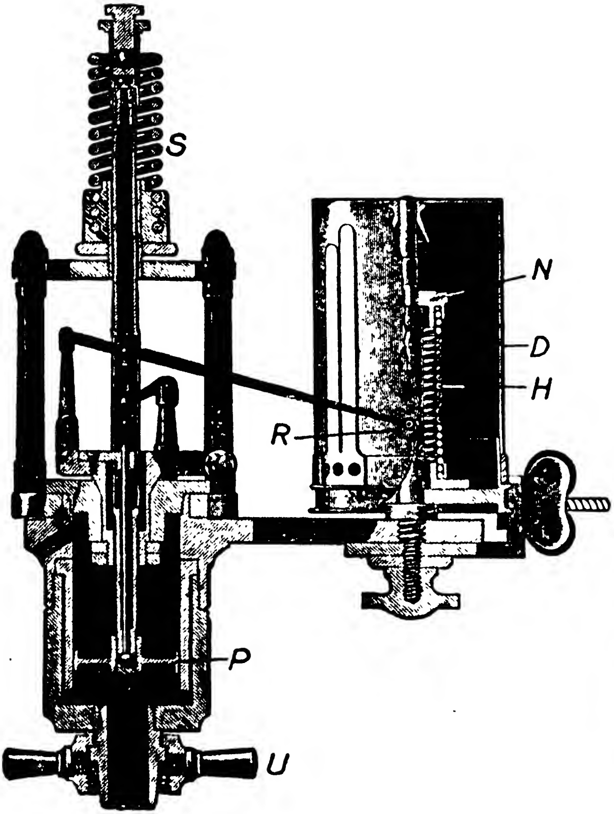

Fig. 5
The Crosby Indicator

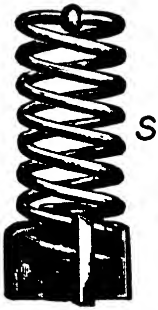

Fig. 6
Crosby Spring

The total actual pressure on the indicator piston should be between 100 and 125 lb., which would be given by a pressure of 500 lb. per sq. in. on a piston area ¼ sq. in. The spring should then be chosen which would give a diagram not more than 1½ in. high.

The spiral spring *H*, inside the paper drum *D*, keeps the cord connected to the *reducing gear* from the engine piston in tension, and can be adjusted by the nut *N* to suit the speed of the engine.

In reducing the reciprocating motion of the engine piston to the indicator drum, the tension in the cord has to overcome the inertia of the drum resisting acceleration and the varying pull of the spring *H*. The special cord used must have the least possible amount of yield or stretch, and steel wire is better, in order to avoid errors of stretching, especially at high speeds, otherwise the horizontal motion of rotation of the paper will not be an exact miniature of the piston's motion.

The whole pencil mechanism can be turned so as to bring the pencil into contact with the paper. The pencil must touch the paper very lightly, since friction causes the pencil to lag behind its true position, so that the area of the indicator diagram is too large.* The fine point of a "Koh-i-noor" pencil on very smooth paper reduces friction and its effects.

Mr. Willans stated that for *steam* engines at 400 r.p.m., the Crosby indicator produced clear diagrams, and the inaccuracy did not exceed 1 per cent. In the case of internal combustion engines, the shock of the explosion may produce errors of at least 2 to 3 per cent, or more at high speeds. If the explosion attains maximum pressure while the crankshaft moves through 15° or $\frac{1}{24}$ of a revolution, and the petrol engine speed is 2,100 r.p.m. or 1 revolution in $\frac{1}{35}$ of a second, then the high maximum pressure is applied in $\frac{1}{840}$ of a second.

Obviously the piston and pencil type of indicator is not suitable for internal combustion engines running at speeds above about 400 r.p.m. when the explosions are rapid and the pressures very high. *Errors in the diagram* are caused by the combined effects of inertia, friction, vibration of the spring, backlash due to slackness in the pin-joints of linkage, and high temperatures in the indicator cylinder.

(1) The effect of *friction* of the piston and pencil, and *inertia* of moving parts, is to increase the mean pressure, because the pencil lags behind its true position, so that the indicated pressure is too low during compression and above the actual pressure during expansion. (2) The position of the drum may not correspond to motion of the engine piston due to *backlash* from slackness and wear in the pin-joints of the pencil mechanism, also to *stretch* of the

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* "Errors of the Indicator," by Osborne Reynolds and H. W. Brightmore, *Proc. Inst.C.E.* (1885), Vol. 83, Part 1; and by J. G. Stewart, *Proc. Inst. Mech.E.*, January, 1913; also "Symposium of Papers on Engine Indicators," *Proc. Inst.Mech.E.*, No. 2 (1923).

indicator cord by sudden pulls. The latter effect is shown in the pencil indicator diagram, Fig. 7, in which the full line is the correct diagram and the broken line the boundary of the actual diagram traced by the indicator pencil. (3) Inertia effects are due to suddenly applied pressure, like blows, which produce *varying* accelerations of the masses of moving parts—the indicator piston, pencil linkage, and the paper drum—tending at first to retard the motion of the tracing pencil, and finally to drive it too high, so that the registered maximum pressure of an explosion is much higher than the actual pressure. Moreover, rapid rate of change of pressure produces vibration of the indicator spring, and sets up waves which vary with the natural or free periodic time of the spring and the frequency of the explosions.

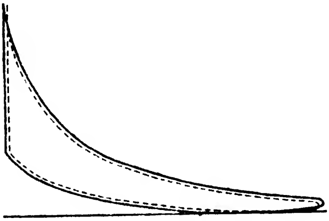

Fig. 7. Indicator Diagram Distorted: Effect of Backlash and Stretch of Cord

Optical Indicators. Professor John Perry, the inventor of the optical indicator, designed a simple type of indicator to get rid of the effects of inertia and friction. The pencil mechanism is replaced by a ray of light, which has neither weight nor inertia, while the indicator diagram can be enlarged by the reflected ray from a very small motion of the mirror. A thin steel disk or circular diaphragm *A*, Fig. 8, is rigidly clamped around the edge, between flanges, *B*, of a shallow cast-iron box *CC*. A small mirror *M*, fixed on the disk, is tilted by the pressure of steam from the engine cylinder, so that a ray of light projected on it is reflected and forms a bright spot on a screen, and the deflections vary as the pressure. The whole box is fixed directly on the indicator cock of the steam engine cylinder, and turned through a small angle proportional to the piston motion by means of reducing gear. This gives a motion of the spot of light at right angles to that due to the pressure. The passage between the indicator and cylinder is made short to avoid lag and undue addition to the clearance volume, which would change the action in the gas engine cylinder. Here the disk is its own spring and practically takes the place of the indicator piston with its spring and pencil mechanism. The spot of light moves so quickly that it gives a continuous bright curve or diagram, which may be recorded on a photographic plate of suitable size or enlarged on a wall or screen, and can be seen by a large audience. *The great advantage of this type of indicator is that the diagrams from any number of consecutive cycles can be observed, any variation noted, and the action in the engine adjusted accordingly, without taking a diagram.* The diagram may also be easily traced on paper against a ground-glass screen.

The pressure scale of the diagram from a flat disk is not the same at high as at low pressures, but the indicator can be readily calibrated by comparison with a Bourdon standard gauge, when both are acted on by steam at increasing and decreasing pressure, a series of readings being taken at steady pressures up and down by throttling the steam. A suitable scale is then drawn for each disk, with parallel lines at equal intervals of pressure.

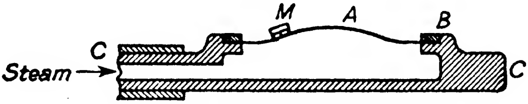

FIG. 8. DIAGRAM OF THE PERRY OPTICAL INDICATOR

The Hospitalier-Charpentier Optical Indicator, or Manograph, uses a flat disk, like that in the Perry indicator. The mirror is pressed by springs on three pivot points, on its back, which form a right-angled triangle. The point at the right angle is fixed, at another point a rod from the centre of the disk presses, and the third point is given a motion proportional to that of the piston by means of a flexible drive from the engine crankshaft. The incident beam of light entering a dark box is reflected by a prism to the pivoted mirror, which reflects the spot of light on a screen.

Professor H. L. Callendar modified this indicator to obviate several of its disadvantages, and obtained perfectly clear diagrams in his researches on a small air-cooled petrol motor* at speeds up to 2,500 r.p.m. He found that when both motions are given to the same mirror, one motion slightly tends to disturb the record of the other, and he suggested a separate mirror for the piston motion.

The Watson Indicator. This disk type of optical indicator was greatly improved by Professor W. Watson. Fig. 9A shows the disk and mirror mechanism diagrammatically. In order to make the pressure scale uniform, he adopted a corrugated disk *A*, made of an alloy steel (silicon steel) gilded upon the under surface exposed to the hot gases and oxidized on its upper side. The corrugations are concentric, and the disk is tightly held by bolts *B* around its edge with a copper washer to keep it air-tight. From the centre of the disk or diaphragm, a small vertical rod *C* passes through guides and acts on one end of a light frame, like the eam of a balance, carrying a concave mirror *M*, while the other end of the frame presses on a cantilever spring *S*. Hardened steel points on each end of the beam rest in hollows on the top of the rod *C*, and on the end of the cantilever *S*. The mirror frame turns about its centre on pivots of steel points, which are pressed upward against the hardened steel under-surface of the beam. The disk *A* is

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* "The Effect of Size upon the Thermal Efficiency of Motors," *Proc. Inst. Auto. Engrs*, 1906–7. See also Hopkinson on "Indicating Gas Engines," *Proc. Inst. Mech. E.*, Oct., 1907, p. 911.

kept cool by water *W* circulating in the jacket below it, while the upper surface is exposed to atmospheric pressure. The pressure of the gases in the engine cylinder forces the disk and rod upwards, and tilts the mirror in the plane of the paper, so that light from an arc lamp is reflected upwards from *M* to a plane mirror, which is turned perpendicular to the paper and proportional to the motion of the piston.

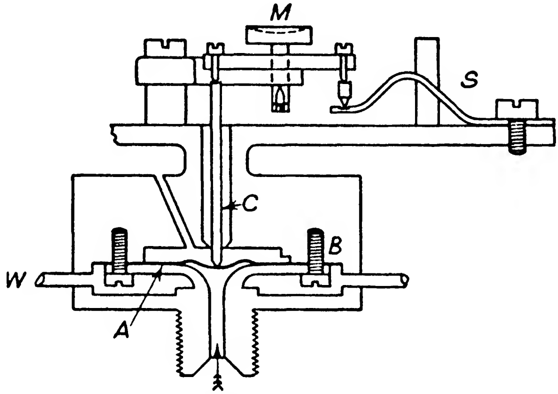

FIG. 9A. DISK AND MIRROR MECHANISM—WATSON OPTICAL INDICATOR

Dr. Watson employed this optical indicator in researches on air-petrol mixtures* in the engine, and in the study of ignition in the petrol motor.

Professor W. E. Dalby has designed this indicator in a commercial form, shown in the diagram Fig. 9B, and devised different types from time to time to meet the various conditions that have arisen, like that illustrated by the diagram, Fig. 9A.

The *Dalby-Watson Indicator* consists essentially of a disk *A* firmly supported between flanges *B*, and arranged for water circulation to keep the disk as cool as possible. The variation of the pressure is measured by the variation in the deflection of the disk. The thickness of the disk must be proportioned so that the deflection corresponding to the maximum pressure is small. The deflection of the disk *A* is transmitted by a rod *C* to a light mirror *D*, suitably mounted and constrained by a spring (not shown) to follow the deflection of the disk so that the angular displacement of this mirror is proportional to the variation of the pressure in the engine cylinder, with practically negligible error.

A plane mirror *E* is mounted so that its axis is at right angles to the axis of the mirror *D*, and is so connected that it tilts proportionally to the displacement of the piston.

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* *Proc. Inst. Auto. E.* III, p. 245, and *Cantor Lectures*, 1908.

A beam of light, originating either in a " Point-o'-Lite " lamp or in an arc, is projected centrally on the mirror D, and reflected by the second mirror E, and finally brought to a focus on the camera screen F.

When the engine is working, the simultaneous tilting of the two mirrors causes the spot of light to describe the pressure-volume diagram on the plate.

By means of an ingenious phase-changing device, designed by Professor Dalby, which is totally enclosed in an oil bath, the phase of the piston displacement may be altered to produce the kind of diagram so useful for examining the dead points in the p.v. diagram. By *setting the phase* at 90° from normal, the period of ignition is at the centre of the diagram, on what is practically a time basis. The print, Fig. 10, from the original plate, taken by Professor Dalby with his optical indicator from a Diesel engine, gives four pressure-volume diagrams placed one over the other, and two superposed cycles differing in phase 90° from the normal cycle. The horizontal line marked " admission " indicates corresponding points on both diagrams. The compression curve in each of the p.v. cycles is coincident on one fine line, and clearly shows the precise point at which the mixture began to burn. The small variations and differences of pressure which followed ignition in these cycles are exactly recorded, and there is very little difference in the expansion curves. The spot of light in the phase-changed diagram moved from left to right, showing clearly for two other cycles the time rate of rise of pressure in the cylinder before and after ignition.

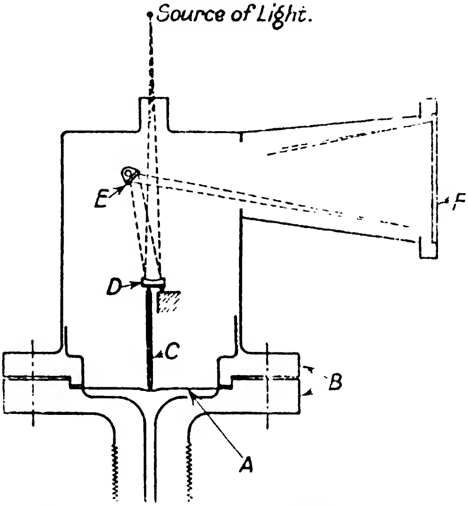

FIG. 9B. DIAGRAM OF THE DALBY-WATSON OPTICAL INDICATOR

The *phase must be adjusted* at the dead centre at the end of the compression stroke, in order that diagrams on a time basis may give useful indications of the action in the engine cylinder. This dead centre may be found either (i) by switching off the spark, when the top end of the compression and the expansion curves is given on the pressure-time diagram ; or (ii) by motoring the engine and taking a pressure-time diagram on the same plate as the power curve. On the motoring curve draw parallels to the base line, cutting the compression and expansion curves. The line joining the points of bisection of these parallels, near the top of the peak, gives the dead centre, provided there is no leakage when the diagrams are taken.

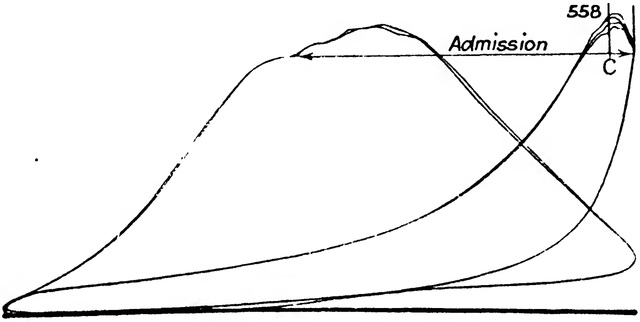

Fig. 10. Diagrams Taken with a Dalby-Watson Optical Indicator

(From a Diesel Engine—R.P.M. 240 ; Air Blast Pressure 715 lb. per sq. in.; Mean Pressure 118 lb. per sq. in.)

Various modifications of mechanical details have been tried to overcome difficulties and weak points found by experience in taking indicator diagrams from some petrol engines having high compression and explosion pressures, and running at very high speed with excessive vibration. For instance, the connection between the cylinder and diaphragm had the effect of slightly increasing the clearance and of changing the phase of the pressures between the cylinder and disk. One of many ways tried by Professor Dalby to get a reliable record of pressure was by partly filling the pressure chamber with a disk turned and fitted exactly to the shape of the diaphragm, leaving as small a clearance as possible.

Another instance of the development of the Watson Optical Indicator is given in the modifications* devised by Dr. Moss and Mr. W. J. Stern, of the Air Ministry Laboratory. The corrugated diaphragms showed fatigue and hysteresis effects when exposed to high explosion pressures, and eventually cracked. A plane, flat disk of silicon steel, containing 1 per cent of silicon and 0·6 to 0·7

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* See *Proc. Inst. Mech. E.*, January, 1925, p. 1.

per cent carbon, is employed, and after hardening at 800° C. by quenching in oil, and tempering to 440° C., is ground down to the thickness 0·05 in., suitable to withstand explosion pressures up to 500 lb. per sq. in. The radius of the disk exposed to the gas pressure is 0·95 in., and Young's modulus $28{\cdot}5 \times 10^6$ lb. per sq. in. Besides cooling the lower side of the disk by circulation of water through a jacket, as in Fig. 9, a wick on the atmospheric side is kept moist by a drip of water, and the temperature, indicated by a rivet of copper-constantan, forming a thermo-couple, in the centre of the disk, is less than 50° C. under full load conditions. The mirror frame M, Fig. 9, is rigid, yet adjustable, and oscillates about two knife-edges kept in grooves of the hardened under-surface of the table, giving greater surface than the steel points originally used. In order to reduce mechanical vibration, the camera is reduced to half size and made of aluminium plate. The indicator diagrams are thereby reduced to half size, and have to be enlarged to find the indicated mean effective pressure.

During a series of tests* of a 40 H.P. single-cylinder Benz engine, this optical indicator and the R.A.E. electrical indicator (see below) were both connected to the engine and diagrams obtained simultaneously. The values of the indicated M.E.P. were identical for pressures up to 80 lb. per sq. in., and for mean pressures up to 115 lb. per sq. in. agreed within less than 1 per cent.

Optical indicators are also made of the *piston type*, with very stiff springs made of steel strips fixed like a beam, and the deflection tilts a mirror. In the *Hopkinson Indicator* the beam rests in grooves at both ends and is held by set screws. The indicator designed by *Professor F. W. Burstall* has a cantilever or beam encastré at one end. The sectional area of the indicator cylinder is $\frac{1}{8}$ sq. in., and the total motion of the piston, controlled by the spring, is about 0·08 in. The spring is not exposed to the hot gases, and both the cylinder and indicator cock are water-cooled. One mirror is attached to the spring and reflects the light to the second mirror, which is driven through a small angle from the engine piston, or other mechanism, and throws the light on the screen of the camera. This indicator is adapted for speeds up to about 1,000 r.p.m. At higher speeds and rapid explosions the inertia effects, friction, and vibration may be reduced but cannot quite be got rid of in the piston indicator.

In this type the indicator spring can be easily calibrated. The instrument has only to be inverted and known weights applied, so that the axis of the load passes through the centre of the spring.

The Electrical Indicator of the "balance" type, for high-speed internal combustion engines, has novel features. This instrument was designed by G. H. Norman, and developed by Harry Wood at the Royal Aircraft Establishment. It consists essentially of a light

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* *The Automobile Engineer*, 1923.

disk *A*, Fig. 11, floating between two seatings about 0·01 in. apart, and guided by a spindle electrically insulated at *D*, and connected to the primary circuit *E* of an induction coil. The current is "earthed" when the disk makes contact with either of the seatings. One side of the disk communicates by the indicator cock *B* with the combustion chamber in the engine, through the cylinder wall *F*, and the other side is exposed to compressed air through *C*. This balance disk-valve acts as a contact-breaker in the primary circuit.

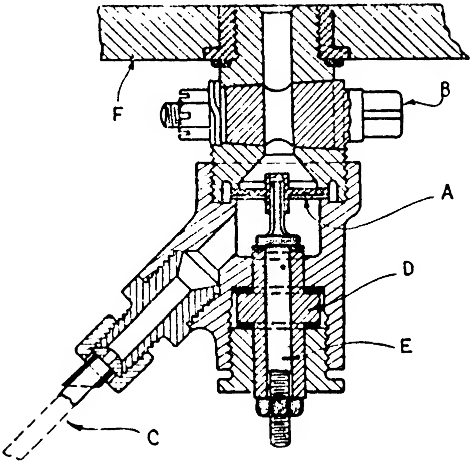

Fig. 11. Balance Disk-valve

At the instant when the varying pressure in the cylinder just exceeds the known air pressure, or in decreasing comes below it, the disk passes from one seating to the other, and, in doing so, breaks the electric circuit, and a spark passes between the pointer and a revolving drum and perforates a sheet of paper on the latter. Thus for any given pressure there are two points of balance in a cycle, one when the pressure is increasing during compression, and the other during the expansion stroke. By gradually increasing the air pressure, a pair of points will be pricked out for each cycle, and these points come nearer together until the peak of the diagram is reached. About 100 cycles in the engine cylinder are required to prick out a single card, Fig. 12, giving the average pressures.

The drum, about 12 in. circumference and 9 in. long, is driven from the engine camshaft, or at half speed of the crankshaft. Compressed air bottles supply air through a distribution box and *C*, Fig. 11, to act on the disk-balance valve, and by another pipe to move the indicator piston, which is controlled by four spiral springs, and operate the multiplying parallel motion. This linkage determines the position of the sparking point, which is moved parallel to the axis of the drum and at a constant distance from the paper.

The linkage makes the motion of the spark point about eight times that of the indicator piston and gives a very large diagram.

The indicator diagram, Fig. 12, is timed or synchronized by turning the engine crankshaft until the piston is on the top centre (T.C.) of its compression stroke, and a line is then scribed on the recording paper drum opposite the spark point.

The autographic record on a time base is readily converted to the ordinary p.v. diagram on a stroke base.

Air bottles are needed to supply air (1) at a pressure higher than that to be measured, and (2) at reduced pressure for recording pressures below atmospheric.

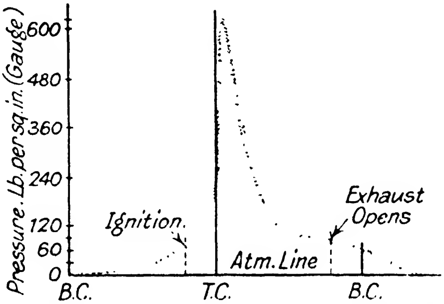

FIG. 12. ELECTRICAL INDICATOR DIAGRAM
Engine speed = 1,400 r.p.m.

The recording mechanism is kept apart from the engine; only the balance disk-valve unit is screwed into the combustion chamber. In the case of a four-cylinder engine, the primary circuit connection of the balance disk-valves on the cylinders were joined through a four-way switch to the induction coil. The compressed air was supplied from a common chamber, and the four cylinders were indicated simultaneously in their firing sequence on one recording drum. The auxiliary apparatus required for this indicator, including battery and induction coil with condenser, is fitted inside a casting carrying recording drum, indicator cylinder, and linkage.

This indicator was fitted in an aeroplane, driven by a 450 H.P. Napier-Lion engine, the recording mechanism being 11 ft. from the cylinder under test. The aeroplane flew to a height of 10,000 ft., and indicator diagrams were taken in flight for the first time by Mr. Harry Wood, on 3rd February, 1923. Fig. 13 is a reproduction of the diagrams taken at 5,000 and 10,000 ft.; Fig. 14 shows the p.v. diagrams obtained by conversion, on a time base, of those recorded. The ratio of the indicated mean effective pressures at different altitudes was the same as the ratio of the air densities.

The Cathode-ray Engine Indicator. This indicator consists of a magnetic pressure pick-up which converts the cylinder pressures into electric potentials, these potentials being magnified by a valve amplifier, the output of which operates a cathode-ray tube, to produce a picture of the changing cylinder pressure.

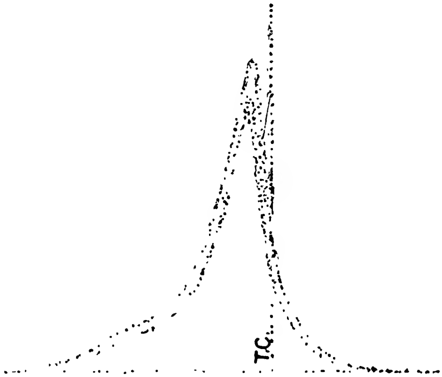

Fig. 13. Diagrams Taken on an Aeroplane in Flight

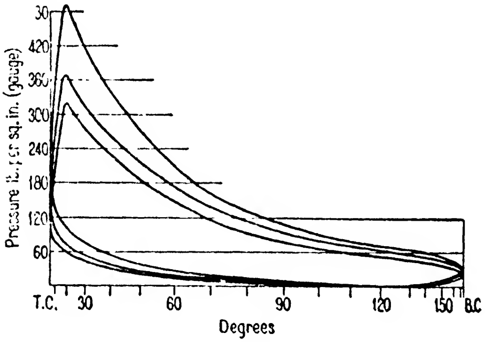

Fig. 14. Diagrams at Different Altitudes

Napier Lion Engine in Aeroplane

Height.	R.P.M.	I.M.E.P.
500 feet	2,150	148·0
5,000 feet	2,140	124·0
10,000 feet	2,100	108·0

The pressure pick-up attached to the engine cylinder contains a diaphragm to which small movements are given by the changing pressure, thus causing small voltage fluctuations in a search coil situated in the pick-up. This varying voltage is magnified in the amplifier, and is applied to the vertical deflecting plates in the cathode-ray tube, producing a vertical movement of the fluorescent spot on the screen, proportional to the rate of change of pressure in the engine cylinder. This vertical movement can be converted to one proportional to the pressure by means of an integrating circuit in the amplifier.

In order to produce a horizontal time base a condenser in the amplifier is gradually charged through a high resistance, the voltage causing a horizontal movement of the fluorescent spot in the tube. This condenser is short circuited once each revolution of the engine, by a contact-breaker attached to and rotating with the engine shaft. Thus the spot gradually moves on the screen from say left to right for the major portion of the engine revolution, and on short circuit flies back almost instantaneously from right to left.

Thus a rate of change of pressure diagram or, by means of the integrating circuit, a pressure-time diagram is obtained.

In order to relate any point on the diagram with the corresponding position of the engine shaft, the rotating portion of the contact breaker is provided with a disk, slotted on the periphery in intervals of one or two degrees. A stationary magnetic pick-up situated at the edge of this disk, produces magnetic variations in a similar manner to the pressure unit. The variations of voltage so produced are applied to the time base circuit, thus giving a degree scale on the cathode-ray tube screen. The necessary motive power is provided by a small electric generator actuated by a battery.

To take Indicator Diagrams with the ordinary pencil indicator, the stiffness of spring is chosen appropriate to the maximum pressure in the engine cylinder (p. 16). The indicator should be tested and the pressure scale calibrated, by comparison with a standard Bourdon gauge or a mercury column when both are acted upon by steam at high temperature, readings being taken at steady pressure which is gradually increased and decreased, giving parallel lines on the paper drum for equal intervals of pressure. When fixed to the indicator cock screwed on the engine cylinder, the passage thereto should be as short and direct as possible, and the indicator piston must be kept well lubricated, clean, and as cool as possible. The pencil or tracing point should touch the paper drum *very lightly* to reduce friction. The diagrams should be taken at regular intervals during an engine trial.

In order to obtain a *mean indicator diagram* by the method of ordinates, carefully measure the average ordinate at each equidistant point along the diagrams taken during the test. Plot to a large scale the pressures so averaged, and the smooth curve drawn through the points will represent the *mean* diagram.

The **mean effective pressure,** p lb. per sq. in., acting in the engine cylinder may also be measured rapidly and directly from the indicator diagrams by the use of a planimeter.

The **indicated horse-power** of an engine is the average rate at which work is done by the gas or vapour on the piston, as calculated from the *mean* indicator diagram. Let a represent the cross-sectional area of the cylinder in square inches, l the length of piston stroke in feet, and n the number of explosion cycles or indicator diagrams traced per minute. Then for every diagram or p.v. cycle pa lb. represents the total mean effective pressure or force driving the piston, and pal ft-lb. the work done by the gas or vapour on the piston in one explosion or for every diagram. Hence the product $plan$ ft-lb. gives the work done on the piston per minute, and this rate of working is the

$$\textit{Indicated horse-power} = \frac{plan}{33{,}000} \quad . \quad . \quad . \quad . \quad (7)$$

When working out the indicated horse-power from a large number of indicator diagrams taken during a trial, the calculation is made easier by finding the value of the *cylinder constant* which is the product of the constant factors $\frac{la}{33{,}000}$, and this, multiplied by $p \times n$, gives the indicated horse-power.

In an internal combustion engine, n represents the number of explosions per minute. Thus the *indicated power* is the area of the indicator diagram multiplied by the number of diagrams or explosions per minute, and by the appropriate constant for reducing to horse-power. In a single-acting steam engine there is one cycle per revolution, so that n is the number of revolutions per minute. In a double-acting steam engine there are two p.v. cycles or diagrams per revolution, so that in this case n is twice the number of revolutions per minute, that is, the number of piston-strokes per minute. Also, on one side of the piston the effective area is a, minus the sectional area of the piston-rod or tail-rod, and the *mean pressures* on each side of the piston will be p_1 and p_2 on effective piston areas a_1 and a_2 respectively, then the

$$\text{Indicated horse-power} = \frac{(p_1a_1 + p_2a_2)ln}{33{,}000} \quad . \quad . \quad (8)$$

where n is the number of revolutions per minute, and l the length of stroke in feet.

Example 4. Explain what is meant by the mean effective pressure of an engine, and show how its value is obtained from an indicator card. Average cards taken during a trial have an area of 4·2 sq. in., and a length of 3·0 in., and the scale of the spring used is 1/60. The engine indicated had a cylinder diameter of 15 in., a stroke of 30 in., and ran at 120 r.p.m. Estimate the average I.H.P. developed during the trial. (*A.M. Inst. C.E.*, 1923.)

The mean effective pressure on the piston is measured by the average width of the indicator diagram multiplied by the scale number of the spring used. In this case, M.E.P. $= \frac{4 \cdot 2}{3} \times 60 = 84$ lb. per sq. in., and the effective piston area $= \frac{\pi}{4} \times 15^2 = 176 \cdot 7$ sq. in.

$\therefore$ the average force on the piston $= 84 \times 176 \cdot 7$ lb.

The work done on the piston per revolution $= 84 \times 176{\cdot}7 \times \frac{30}{12}$ ft-lb.

$$\therefore \text{ the average indicated horse-power} = \frac{84 \times 176{\cdot}7 \times 30 \times 120}{33{,}000 \times 12}$$

$$= 134{\cdot}9 \qquad \textit{Answer.}$$

If the engine is double-acting, giving two diagrams per revolution, the indicated horse-power $= 270$, nearly.

Example 5. A double-acting steam engine has cylinder 18 in. in diameter, piston stroke 30 in., piston-rod 6 sq. in. sectional area, speed 120 r.p.m. By planimeter measurement of the indicator diagram, the mean pressure on one side of the piston is 31, and on the other side 30 lb. per sq. in. Find the indicated horse-power.

The sectional area of cylinder is $\pi \times 9^2 = 254{\cdot}5$ sq. in., and the effective piston area on one side, found by deducting area of piston rod, is 248·5 sq. in. Substitute these values in equation (8), and we have

$$\text{Indicated horse-power} = \frac{(31 \times 254{\cdot}5 + 30 \times 248{\cdot}5) \times 2{\cdot}5 \times 120}{33{,}000}$$

$$= 139{\cdot}4 \qquad \textit{Answer.}$$

Again, if we take the average mean pressure $\frac{1}{2}(30 + 31)$, or 30·5 lb. per sq. in., and the sum of areas on the two sides of piston 503 sq. in., the work done by the steam on the piston per revolution is $30{\cdot}5 \times 503 \times 2{\cdot}5 = 38{,}354$ ft-lb., and the

$$\text{Indicated horse-power} = \frac{38{,}354 \times 120}{33{,}000} = 139{\cdot}47$$

Also, in this case, neglecting the sectional area of the piston-rod which prevents the steam acting on that part of the piston, the average of the mean pressure gives the average force driving the piston $= 30{\cdot}5 \times 254{\cdot}5 = 7{,}762$ lb.; and the work done by the steam per stroke $= pal = 7{,}762 \times 2{\cdot}5$ ft-lb. $= 19{,}405$ ft-lb.

There are 2×120 strokes per minute, and we have

$$\text{Indicated horse-power} = \frac{19{,}405 \times 240}{33{,}000} = 141{\cdot}1$$

Obviously 140 H.P. is a close enough approximation, within the limits of error of the mean pressure obtained by the ordinary piston and pencil indicator, and may be used when the effective piston areas are nearly equal and the mean pressures differ only slightly.

The **brake power** of an engine is the effective power delivered at the crankshaft or brake wheel, and available for external work.

Mechanical efficiency is the *ratio* of the brake to indicated power.

The *difference* between the indicated and brake power is the *mechanical loss*, i.e. that part of the indicated power which is wasted in overcoming the internal resistances of the engine, including:

(1) *piston friction* due to the inertia of the piston and reciprocating parts, and friction of piston-rings; (2) *friction* of the bearings, valves and other parts of the mechanism, caused by resistance to relative motion; (3) *fluid* resistance in the *pumping strokes* of the piston of the internal combustion engine.

In this type of engine the total mechanical losses under good conditions at normal load usually vary from 10 to 20 per cent of the indicated power, or the mechanical efficiency is from 90 to 80 per cent.

Suppose an engine at full load and normal speed develops 100 I.H.P., and of this 10 H.P. is absorbed in internal friction, then the brake horse-power will be 90 and the mechanical efficiency 90 per cent. If the load is reduced to 40 I.H.P. and the mechanical losses remain the same 10 H.P., now the brake horse-power is 30, and mechanical efficiency becomes $\frac{30}{40}$, or 75 per cent. At full load the friction on the bearings may increase slightly, owing to increased pressure, even with good lubrication; on the other hand, the fluid or pumping loss is then less than at no load. The results of careful tests give the total mechanical losses 12 per cent of the indicated power, and of this piston friction absorbs one half, or 6 per cent, bearing friction 3 per cent, and fluid friction 3 per cent. However, in some very high-speed petrol engines, piston friction alone accounts for 50 to 60 per cent of the total mechanical losses.

When this type of engine is running at half load or less than full load, the mean effective pressure in successive cycles is variable. Hence the uncertainty of the correct value of the indicated power at less than full load, which could only be measured directly by an accurate integrating indicator, and the instruments of this kind available are unsuitable for high-speed engines.

Without using the ordinary pencil indicator, the indicated horse-power and *mechanical efficiency* of an internal combustion engine may be estimated indirectly from the brake horse-power, which can be measured accurately.

(1) One method adopted by the Committee of the Institution of Civil Engineers in gas engine trials, depended on the accuracy of measurement of the gas supplied to the engine. Keep the speed constant and assume friction the same at different loads. Measure the brake horse-power and total gas used per hour at full load and half load. Plot the total gas as ordinates against brake horse-power as base line. Thus at full load point, *A*, the vertical *AB* represents total gas used; at half load point, *C*, the vertical *CD* represents the total gas at half load. Produce the line through the points *B* and *D* to meet the vertical through no load point, *E*, at *F*. Draw *DL* parallel to base line *AE*. Then *BL* represents the gas required to change the brake power by the amount *DL*, and, provided the friction remains constant, this quantity of gas per hour would

change the indicated horse-power by the same amount. Hence the ratio BL to DL is the gas per indicated horse-power hour, and

$$\textit{mechanical efficiency} \text{ at full load} = \frac{\text{gas per I.H.P. hour}}{\text{gas per B.H.P. hour}}$$

(2) In high speed multicylinder petrol engines, the method devised by Mr. L. G. E. Morse to find the mechanical efficiency consists in measuring the brake power of the engine when all the cylinders are working at constant speed and full load. Then the ignition of one cylinder is stopped by cutting off the current, and the brake load reduced until the engine speed is brought up to its normal value. The reduction in the brake horse-power is approximately the indicated horse-power of the cylinder which has been cut out. Treat the other cylinders in the same way, and the sum of the results is approximately the indicated horse-power of the engine at constant speed. Mr. Morse points out that the value so obtained is too high, and he deducts the power between the compression and expansion in a cylinder not firing. Also the pumping and fluid losses are usually greater in a cylinder when not firing than when firing.

(3) Another method is to measure the power required to drive the engine at normal speed without external load; then, assuming the mechanical losses constant at all loads, the indicated horse-power at any load is the brake horse-power added to the indicated power at no load. The engine is driven, without load, at normal speed by an electromotor, and the power absorbed is measured; but the working conditions are different, as also the friction and fluid losses, when the engine is firing.

The power wasted in overcoming all the internal resistances of an engine may be expressed as the *equivalent torque* on the crankshaft at a given speed. Thus $\frac{T\omega}{550} = \text{H.P.}$, or $T = \frac{550\ \text{H.P.}}{\omega}$, where T is torque or resultant turning moment in lb-ft., and ω the angular velocity in radians per second. Suppose the difference between the indicated and brake horse-power of an engine is 5 H.P., which is absorbed or wasted in the engine when the speed is 300 r.p.m. or $\frac{2\pi \times 300}{60} = 31{\cdot}42$ radians per second, then

$$\text{the equivalent frictional torque } T = \frac{5 \times 550}{31{\cdot}42} = 87{\cdot}53 \text{ lb-ft.}$$

The total frictional torque T on the crankshaft at a given speed may be made up of the torque T_0 when the engine is running light, and a part, T_1, proportional to the brake horse-power, then

$$T = T_0 + a \,.\, T_1,$$

where a is a constant which can be found, as well as the frictional torque at no load, from two determinations of the indicated and brake horse-power of an engine at different speeds.

The *mechanical efficiency*, $m = \dfrac{\text{Brake horse-power}}{\text{Indicated horse-power}}$, that is, brake horse-power $= m \times$ indicated horse-power.

From equation (7), page 29, we have

$$\text{brake horse-power} = \frac{m \times pa}{33{,}000} \times \frac{s}{12} \times \frac{n}{2}$$

where n = revolutions per minute,

and s = stroke in inches,

d = cylinder diameter in inches

$$\text{or, brake horse-power} = mp \times \frac{\pi}{4} d^2 \times \frac{s}{12} \times \frac{n}{2 \times 33{,}000}$$

$$= \frac{mp \, . \, d^2 \, . \, sn}{1{,}008{,}410}$$

$$\text{and,} \qquad mp = \frac{1{,}008{,}410 \times \text{brake horse-power}}{d^2 sn} \quad . \quad (9)$$

The product mp is called the "brake mean effective pressure" which may be determined from brake tests, and is found to increase with the bore of the cylinder in high-speed petrol engines.

Assume that, in the ordinary petrol engine, mp is 84 lb. per sq. in. at 1,200 r.p.m., then by equation (9), a rough estimate of the brake horse-power $= \dfrac{d^2 s}{10}$.

The brake horse-power is measured by a brake or dynamometer applied to a wheel on the crankshaft. In friction-brake dynamometers all the power of the engine is absorbed or spent in friction producing heat.

A simple and accurate friction brake, invented by Lord Kelvin, consists of a rope, or couple of ropes, carried once round a pulley fitted on the crankshaft, with weights hanging freely on one end to oppose the motion of the engine, and a spring balance gives the vertical pull at the slack end of the rope. The rim of the pulley should be flat and not rounded. The little wooden cross-pieces, with grooves to keep the ropes from slipping off, should be laced to the ropes and not fastened by nails to touch the rim of the wheel, and thereby become heated by friction so as to burn the rope.

When each block of wood is lubricated with tallow and plumbago, the coefficient of friction is steady between 0·14 and 0·25, depending on the load; when the wood is covered with lubricated leather, the coefficient of friction for the different loads is nearly the same as with rope contact from 0·15 to 0·35, and the brake runs smoothly and steadily. The friction may be increased by fixing ferodo on the wood.

For any given load or resistance, W (Fig. 15), the pull S, in the spring balance at the slack end of the rope varies slightly, due to alterations in the coefficient of friction, μ, caused by heating of the wheel and drying of the rope. On this account, frequent readings of the spring balance must be taken to obtain a correct average value.

The work done by the engine in overcoming the frictional resistance of the brake is converted into heat at the surface of the wheel, warming up the rim rapidly, and consequently the brake pulley must be water-cooled to prevent overheating.

A brake pulley is usually made with flanges on the rim, forming a trough outside, which dispenses with the wooden blocks, and inside the rim the trough for cooling water prevents overheating. Water drips continuously into this trough, and centrifugal force, due to speed of rotation, keeps the water against the rim, where it evaporates; or, for cooler running, any surplus water is scooped away by another pipe. The water supply should be turned off before the end of a test to allow time for the water in the rim trough to evaporate, otherwise, as the engine slows down, any water in the trough falls and splashes all around.

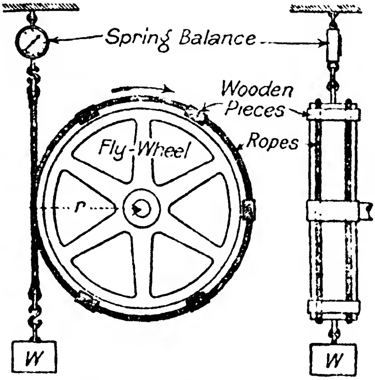

FIG. 15. ROPE BRAKE

If θ is the angle, in radians, subtended at the centre by the arc of contact between the pulley and rope, then $\frac{W}{S} = \varepsilon^{\mu\theta}$, where $\varepsilon = 2{\cdot}71828$, and μ is the coefficient of friction between rope and pulley.

In order to keep S a small fraction of W, the angle θ is made large. Usually, with the rope once round, when $\theta = 2\pi$, the ratio $\frac{S}{W}$ may be constant at $\frac{1}{10}$ to $\frac{1}{6}$ for a dry rope.

The load or resistance is adjusted by the weight, W lb., and the pull, S lb., in the spring balance changes automatically.

The difference of the vertical pulls, $(W - S)$ lb., at the ends of the rope, multiplied by the effective radius, R, measures the total turning moment or mean *torque*, T, on the crankshaft, necessary to overcome the frictional resistance—

$$T = (W - S)R \text{ lb-ft.}$$

R is the effective radius of the brake measured from the axis of rotation to the centre of the rope where the vertical pulls are applied.

At the speed n r.p.m. of the wheel, $2\pi n$ radians is the angle turned through per minute; and the angular velocity $\omega = \frac{2\pi n}{60}$ radians per second. Then the work done per second is $T\omega$, and the

$$\text{brake horse-power} = \frac{T\omega}{550} = \frac{2\pi nR(W - S)}{33{,}000} \quad . \quad . \quad (10)$$

where $2\pi R$ ft. is the circumference measured round the centre of the rope.

Alternative Method. The pull of the freely hanging weight, W lb., may be regarded as the load or resistance opposing the rotation of the brake wheel, while the vertical pull S, at the slack end of the rope, helps the engine, so that the difference of the vertical and opposite pulls on the two ends of the rope, $(W - S)$ lb., represents the net load or frictional resistance of the brake. The work done by the engine in overcoming this frictional resistance is $2\pi R(W - S)$ ft-lb. per revolution, where $2\pi R$ ft. is the effective circumference of the brake, or the distance through which the resistance is overcome during one revolution. When the brake wheel is making n r.p.m., the rate of working or power is $2\pi Rn(W - S)$ ft-lb. per minute, and we have the

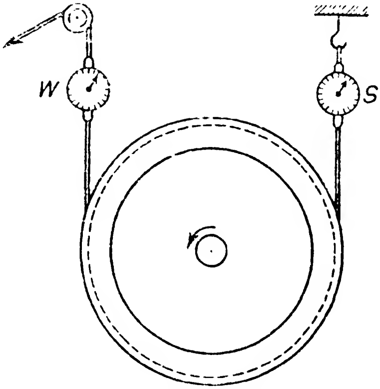

FIG. 16. ROPE BRAKE

$$\text{brake horse-power} = \frac{2\pi Rn(W - S)}{33{,}000}.$$

Thus, *the difference of the pulls, in pounds weight, at the ends of a rope or belt on a pulley, multiplied by the rim speed, or velocity, in feet per minute, and divided by* 33,000, *gives the horse-power either absorbed or transmitted by the rope or belt.*

In the case of high-speed motors up to 40 or 50 H.P., a spring-balance is commonly applied at each end of the rope, which is wound once round the water-cooled brake pulley, Fig. 16.

The load, W lb., opposing the motion of the engine can be readily adjusted by having one end of the rope carried over a pulley, while the pull on the spring-balance, S lb., at the tail end changes automatically, depending on the friction, and must be observed frequently, owing to changes of the frictional resistance between the rope and pulley. Care should be taken to keep a steady water supply to the brake wheel.

Example 6. In a brake test of an engine, the diameter of the water-cooled brake wheel fitted on the crankshaft is 24 in., and that of rope, wrapped round the wheel, is 1 in. The rope is kept at rest by a vertical pull of 315 lb. on a spring balance at one end opposing the rotation of the shaft, and a vertical pull of 85 lb. on the tail end, when the speed is 1,200 r.p.m. Calculate the mean torque on the shaft, and the brake horse-power.

The effective radius of the brake wheel is $\frac{25}{2 \times 12}$ ft., and the torque exerted by the shaft is $(315 - 85)\frac{25}{24} = 239{\cdot}58$ lb-ft.

The angular velocity $\omega = \frac{1{,}200}{60} \times 2\pi = 125{\cdot}66$ radians per second, and therefore the

$$\text{brake horse-power} = \frac{T\omega}{550} = \frac{239{\cdot}58 \times 125{\cdot}66}{550} = 54{\cdot}74$$

Answer.

Otherwise, the net load on the brake is $315 - 85 = 230$ lb., and the work done by the engine in overcoming this resistance through *one* revolution is $230 \times \frac{25\pi}{12}$ ft-lb. Therefore, at 1,200 r.p.m., the

$$\text{brake horse-power} = 230 \times \frac{25 \times 3{\cdot}1416}{12} \times \frac{1{,}200}{33{,}000} = 54{\cdot}74.$$

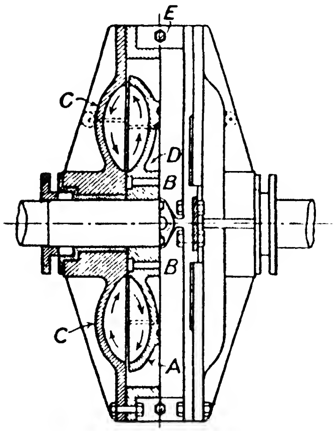

Fig. 17. Froude Water Brake (Half Section)

Water Brake. In 1877 William Froude invented the hydraulic brake to measure the power of large marine engines.* A central wheel *A*, Fig. 17, is keyed to the engine shaft, and consists of two similar parts bolted together at *B*, and is enclosed in the outside casing, *C*, like an annular box which is free to turn on the shaft. The ring of the central wheel, or rotor, is divided into semi-elliptical vanes or cups inclined at 45° to the vertical plane of rotation. Facing these, on both sides, there are similar vanes or baffles around the casing, with clearance from the rotor. The water inlet is at *D* and the outlet at *E*. When the casing is full of water, and the engine turns the central wheel, *A*, the water is under the influence of centrifugal force, and the pairs of

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* *Proc. Inst. Mech. E.*, 1877.

opposite vanes, when they come together, form egg-shaped pockets, in which the centrifugal action produces vortices in a plane at 45° to the axis. The vortices, on one side of the brake, whirl in the opposite direction to those on the other side, and so balance reactions parallel to the axis of the shaft.

The other component of the centrifugal pressure acts on the casing, and the turning moment of the shaft, producing the motion of the water, must balance the moment of resistance of the outer casing, which is measured by the torque applied by weights on a lever rigidly fitted to the casing.

The casing is always full of water, and the resistance is regulated by thin metal sluices between the vanes of the central wheel and casing.

The work done by the engine against hydraulic friction is converted into heat, which warms the water flowing away by the outlet *E*. The work done per minute is equal to the torque multiplied by 2π times the revolutions of the shaft per minute.

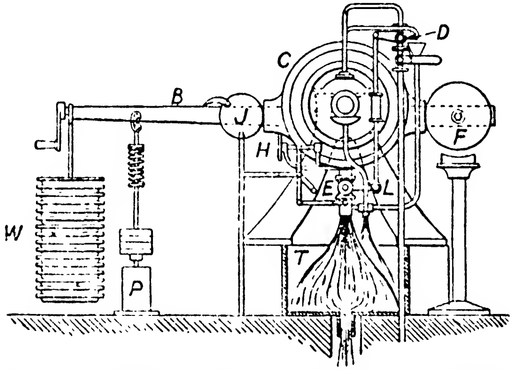

FIG. 18. REYNOLDS-FROUDE HYDRAULIC BRAKE

Reynolds-Froude Hydraulic Brake. Professor Osborne Reynolds designed a modification of the Froude brake,* without the sluices, which has constant atmospheric pressure at the centre of the hollow vortices in the brake wheel, even when the 24 pockets on both faces of the rotor are only partly full of water. The essential feature is that small holes, drilled in the walls of the casing buckets, allow any air brought into the wheel by the water to escape to the atmosphere. Also the resistance of the brake is regulated automatically by the quantity of water in the wheel by the water-cocks *D* and *E*, Fig. 18. A slight clockwise motion of the casing and lever, *B*, about the shaft, partly opens the outlet *E*, and, through the linkage *L*, partly closes the inlet *D*, thus reducing the quantity of water in the brake. By

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* *Proc. Inst. C. E.*, Vol. 99, p. 167; and *Phil. Trans.*, Vol. 190 (1897), pp. 305-422.

adjusting the handle wheel, H, this regulation is made more sensitive. The water supply enters radially, passes through the holes in the vanes of wheels, and by centrifugal action from the central cavity to the vortex pockets, enters the outer chamber between the wheel and casing, which is always full when the wheel is running, and leaves the casing radially through E to the trough T. The weights, W, hang on a knife-edge, and the dash-pot piston, P, moves freely in oil to steady the motion of the lever and damp out oscillations set up by fluctuation in the speed of the engine. The jockey weight, J, on the lever B, is adjusted by a fine screw, while the balance weight, F, compensates the weight of the lever arm B, and the jockey weight, J, when not loaded.

Froude Hydraulic Dynamometer. A typical cross-sectional drawing of a modern Froude Dynamometer as manufactured by Messrs. Heenan & Froude, Ltd., is shown in Fig. 19.

The main shaft is carried by bearings fixed in the casing (not in external supports). The casing in turn is carried by anti-friction trunnions, so that it is free to swivel about the same axis as the main shaft. When on test the engine is directly coupled to the main shaft transmitting the power to a rotor revolving inside the casing, through which water is circulated to provide the hydraulic resistance and simultaneously to carry away the heat developed by destruction of power.

In each face of the rotor are formed pockets of semi-elliptical cross-section divided one from another by means of oblique vanes. The internal faces of the casing are provided with liners which are pocketed in the same way. Thus, the pockets in rotor and liners together form elliptical receptacles round which the water courses at high speed.

When in action the rotor discharges water at high speed from its periphery into the pockets formed in the casing liners, by which it is then returned at diminished speed into the rotor pockets at a point near the shaft.

The resistance offered by the water to motion of the rotor re-acts upon the casing, which tends to turn on its anti-friction roller supports. This tendency is counteracted by means of a lever arm terminating in a weighing device which measures the torque.

From the above description it will be seen that the forces resisting rotation of the dynamometer shaft may be divided into three main classes—

(*a*) The hydraulic resistance created by the rotor.

(*b*) The friction of the shaft bearings, which are usually of the ball type.

(*c*) The friction of the glands.

It will be noticed that every one of these forces re-acts upon the casing, which being free to swivel upon anti-friction trunnions transmits the whole of the forces to the weighing apparatus.

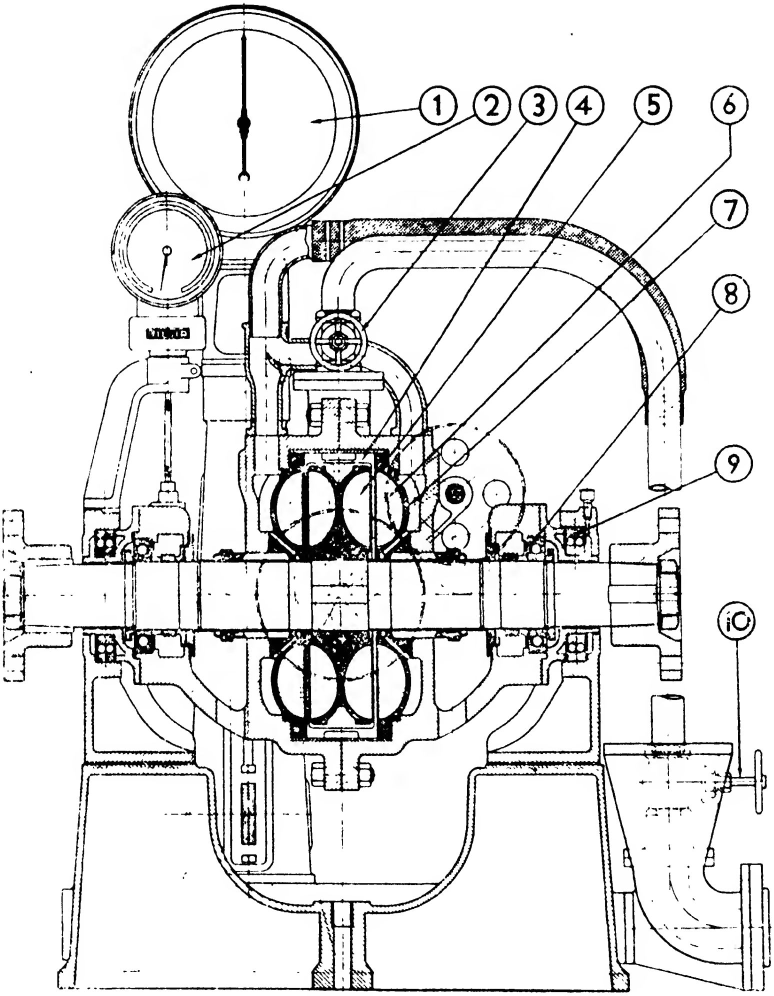

FIG. 19. HEENAN & FROUDE DYNAMOMETER

1. Spring Balance
2. Tachometer
3. Water Outlet Valve
4. Sluice Gates for Load Control
5. Rotor
6. Water Inlet Holes in Vanes
7. Casing Liners
8. Shaft Bearing
9. Casing Trunnion Bearing
10. Water Inlet Valve

By kind permission of Messrs. Heenan & Froude Ltd., Worcester.

Thus, *every force resisting rotation of the engine shaft is caused to re-act upon the weighing apparatus.* This ensures scientific accuracy.

Referring to the diagram, Fig. 19, it will be noticed that between the rotor and the casing liners are interposed thin metal sluice gates, which can be advanced or withdrawn by means of a single-hand-wheel. If these sluice gates be moved towards the main shaft they will cut off communication between the rotor and a number of cups in the casing liners, with the result of diminishing the effective resistance of the dynamometer, and vice versa.

This method of adjusting the load to suit the capacity of the engine can be operated while the dynamometer is running, so that in a short space of time a power curve can be obtained over a wide range of speed.

The Swinging Field Electric Dynamometer. This consists of an electric generator, with the casing and field windings mounted in trunnions. The armature is connected to the engine under test. When running, the casing and field coils tend to rotate in the trunnions, this rotation being prevented by a torque arm loaded with weights or spring balances, from which the engine load can be determined. The output from the generator is dissipated in electric resistances, and the load is controlled by varying these, and the field resistances.

The Heenan-dynamatic Dynamometer. In this machine made by Messrs. Heenan and Froude, the dynamometer rotor has the appearance of a toothed gear wheel of wide face and coarse pitch, the teeth forming magnetic poles. The rotor runs with a slight clearance inside a stator having a smooth cylindrical bore. In the stator are arranged one or more field coils, circumferentially wound, designed for excitation by a small amount of direct current.

Application of this current produces a magnetic flux which links the rotor with the stator. The concentrations of flux thus created give rise to induced eddy currents, which, interacting with the main field concentrations, tend to resist rotation of the rotor. The forces opposing rotation re-act upon the stator assembly which tends to turn upon its trunnion bearings, this tendency being counteracted by means of a lever arm connected to a torque measuring apparatus.

The load imposed on the engine under test is governed by the amount of excitation current passed through the field coil.

The Mechanical Equivalent of Heat. James Prescott Joule, of Manchester, was the first* to prove that when heat is produced by mechanical, electrical, or other means, the quantity of heat generated by the friction of bodies is always proportional to the quantity of energy expended, and that the production of 1 unit of heat always requires a certain number of units of work to be spent. In other words, Joule demonstrated the *constant relation between the mechanical work spent in producing heat and the amount of heat produced.*

Also, in accordance with the **principle of the conservation of energy**

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* *Joule's Scientific Papers,* Vol. I, p. 149.

in the conversion of heat into work, *when work is done by means of heat in the cylinder of any heat engine, a definite quantity of heat disappears for every unit of work done*, and, conversely, *when heat is generated by mechanical energy, the same definite quantity of work is spent which is the equivalent of every unit of heat generated.* The relation between heat and work is called Joule's equivalent (J). Whenever W units of mechanical energy are wholly expended in generating Q units of heat, $W = JQ$. This *law of equivalence* of heat and work is the **first law of thermodynamics.**

Joule showed by experiments, from 1842 to 1849, in which the potential energy of raised weights was spent in turning a paddle, which generated heat by churning water in a vessel, and observing the rise of temperature, that the quantity of heat produced by the friction between the metal paddles and water is always proportional to the mechanical energy expended. The mean result was that 772 ft-lb. of work generated heat enough to raise the temperature of 1 lb. weight of water 1° F. on the mercury thermometer, between 55° and 60° F. Joule concluded that 772 ft-lb. was equivalent to 1 B.Th.U.

Experiments by electrical heating, based on the value of the British Association unit of electrical resistance, gave the result $J = 782{\cdot}5$, which did not agree with the value of J obtained from the friction of fluids. So, in 1878, Joule made another series of experiments. The method adopted was to revolve a paddle in a calorimeter containing water, and suspended from a bearing on the vertical axis of rotation, and to measure the heat produced.

The friction between the water, driven by the paddle, and the walls of the calorimeter produced a turning moment. The work done, W, measured by the product of the lifted weights w, to keep the calorimeter from turning, $2\pi R$ the effective circumference of the groove around the calorimeter, and n the number of revolutions of the paddle, was $W = 2\pi Rnw$. The result, reduced to the sea level at Greenwich, becomes 773·49 ft-lb., equivalent to 1 B.Th.U. for the temperature 60° to 61° F. on the mercury in glass thermometer. Taking weights *in vacuo*, $J = 772{\cdot}55$ ft-lb. When afterwards reduced to the scale of an air thermometer standardized by Professor H. A. Rowland, at Baltimore, and for latitude 45°, the value of J was brought up to about 776 ft-lb. per B.Th.U. at 60° F.

In 1879 Rowland repeated Joule's experiment on a larger scale. The calorimeter was suspended by a torsion wire; the water was churned in a spiral direction by curved paddles with perforations to reduce the jerky resistance, the vertical spindle passing through the bottom of the calorimeter being driven from a steam engine. The work was measured by the friction balance method employed by Joule, by the moment of resistance necessary to prevent the calorimeter from turning owing to the friction of the water while the paddle was rotated.

Rowland discovered that the capacity for heat of water diminishes as the temperature rises from 0° to about 30° C. over the range of temperature 5° to 35° C. He found the B.Th.U. equivalent to 776·8 ft-lb., the kilo-calorie equivalent to 426·2 kilogramme-metres in latitude 45° at 20° C. on the air thermometer, and the gramme-calorie equivalent to 4·18 joules. This absolute value of J also depends on the scale of temperature, whether on the hydrogen, nitrogen, or mercury in glass thermometer.

Professor Reynolds and Mr. W. H. Moorby made a determination of the mechanical equivalent of heat by using the Reynolds-Froude brake driven by a 100 H.P. steam engine at 300 r.p.m. The brake gave the exact amount of work expended in raising the temperature of water between the two physically fixed points, the temperature of melting ice and that of water boiling, under standard atmospheric pressure, by the agitation of the water in the hydraulic brake. The heat so generated was discharged in the water which entered ice-cold and, after circulating through the brake, left at the boiling point; then the water was cooled and allowed to flow into a tank on a weighing machine. Steam was prevented from forming by a pressure head of 11·3 ft. Before every trial the brake was balanced without load, but filled with water in the same conditions as during the trial, when the lever was always kept level. Standardized thermometers indicated the temperature of the water entering and leaving the brake. In heavy trials the torque or moment of resistance was 1,200 lb-ft., and in the light trials 600 lb-ft. Differences were taken for each pair of heavy and light loads to eliminate friction and other losses. The average corrected result of 44 reliable trial runs, each of 62 minutes' duration, was—

Trials	Torque (lb-ft.)	Average Work (ft-lb. per trial)	Heat Given to Water (lb. degree F)
21 heavy .	1200	134,337,403	171,510
23 light .	600	61,355,503	77,710
Differences . .		72,981,900	93,800

$$\text{This gives} \quad \frac{\text{Work done (ft-lb.)}}{\text{Heat generated (lb. deg. F.)}} = 778{\cdot}06 = J. \quad \text{(p. 41).}$$

After various small corrections, the mean value of the MECHANICAL EQUIVALENT OF HEAT, between the temperatures 32° F. and 212° F., is 777·91, taking the weights in air at Manchester. Reduced to sea-level, lat. 45° gives 777·6 ft-lb., equivalent to the *mean British Thermal Unit.* The unit so defined does not depend upon the scale of any particular thermometer.

Joule also measured the *heating effect of the electric current,* and found that the heat generated in an electric conductor is proportional to the work done, W, by the electromotive force or difference of potential at its terminals, e volts, in passing a given quantity of coulombs of electricity through the conductor, $w = eq = eit = i^2rt$; or the work in *joules* done by a steady electric current of i amperes in t seconds, in heating a conductor of r ohms resistance equals i^2rt.

Since the mean gramme calorie is equivalent to 4·1868 joules, it follows that the thermal equivalent of the work done by a steady current of i amperes flowing through a resistance of r ohms, generates in t seconds,

$$\textit{heat in gramme calories} = 0{\cdot}239\, i^2rt, \text{ or } 0{\cdot}239\frac{e^2}{r}\,.\,t \quad . \quad (11)$$

Professor E. H. Griffiths adopted this method of heating water in a calorimeter by passing an electric current* through a thin platinum wire, coated with amber varnish, immersed in the water and having its ends kept at a constant difference of potential by a battery of Clark cells. The rise of temperature was measured by means of a platinum thermometer standardized by the air thermometer, and compared, under the same conditions as when used, with the nitrogen scale of the Bureau International, when the reading was found correct.

The value of J deduced from the electric standards used, subsequently corrected for an error in the electromotive force of the Clark standard cell, was 4·187 joules = the gramme calorie at 20° C.

Griffiths also confirmed Rowland's result that the capacity for heat of water decreases as the temperature rises, over the range 14° to 26° C.

Callendar and Barnes† heated a continuous steady stream of water flowing through a fine tube by passing an electric current through a central wire. The difference of temperature between the inlet and outlet water was given by a single reading of a pair of differential platinum thermometers. The flow-tube and thermometer pockets at each end were sealed in a vacuum jacket to reduce loss of heat. The water jacket, around the vacuum jacket of the calorimeter, was kept at any desired temperature.

The capacity for heat of water, measured by this method, shown in Fig. 20, agrees closely with the values found by Rowland. The result for the mechanical equivalent of the *mean* gramme calorie in absolute (C.G.S.) units between 0° and 100° C. is 4·1868 joules; and the mean pound calorie (0° to 100° C.) is 1,400 ft-lb. The corresponding value for the mechanical equivalent of the mean British Thermal Unit is $1{,}400 \times \frac{5}{9}$; or 777·78 ft-lb. (London), and that of the kilo-calorie = 426·7 kilogramme-metres.

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* *Phil. Trans.*, Vol. 184 (1893); *ibid.* (1895).

† *B. A. Reports*, 1897 and 1899. *Phil. Trans.*, 1902, pp. 58–263; also 1913, pp. 1–32.

By using J, the numerical equivalence of mechanical and heat energy, the heat obtained by the combustion of a fuel may be expressed either in thermal units or foot-pounds; while the work done by a fluid on the piston in an engine cylinder may be calculated in the equivalent heat units. For example, when working at the rate of one I.H.P. for one hour, a heat engine converts

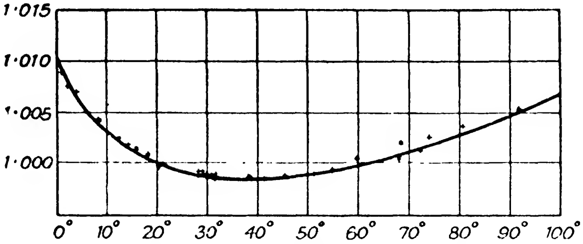

FIG. 20. CAPACITY FOR HEAT OF WATER (CALLENDAR AND BARNES)

$$\frac{33{,}000 \times 60}{778} = 2{,}545 \text{ B.Th.U., or } 1{,}414 \text{ C.H.U.}$$

into work on the piston. Hence, the amount of heat 2,545 B.Th.U. or 1,414 C.H.U., is the *thermal equivalent of one H.P. hour.* The engineer also uses one kilowatt-hour, equivalent to 3,412 B.Th.U. or 1,896 C.H.U.

Example 7. A rope brake on the flywheel of an engine carries a load of 250 lb. at one end, and a load of 10 lb. at the other end. The diameter of the flywheel is 5 ft., and the engine runs at 200 r.p.m. What brake horse-power is the engine developing?

Assuming that 85 per cent of the heat generated by friction between the rope and the flywheel is absorbed by a stream of cooling water, in which the temperature rise is 10° C., calculate the amount of water required per minute.

(*A.M. Inst.Mech.E.*, 1924.)

Given effective circumference of flywheel $= 5\pi$ ft.

Net load on brake $= 250 - 10 = 240$ lb.

$$\therefore \text{ Brake horse-power} = \frac{240 \times 5 \times 22 \times 200}{7 \times 33{,}000} = \frac{160}{7}, \text{ or } 22{\cdot}85 \text{ H.P.}$$

Again, the heat equivalent of 1 H.P. minute is

$$\frac{33{,}000}{1{,}400} = \frac{165}{7} \text{ C.H.U. per minute.}$$

$$\therefore \text{ Heat absorbed by cooling water} = \frac{85}{100} \times \frac{165}{7} \times \frac{160}{7}$$

$$= 458 \text{ C.H.U. per minute.}$$

We have, $10^\circ \times W$ lb. $= 458$, and $W = 45{\cdot}8$ lb. per minute, or 4·58 gallons per minute. *Answer.*

The Thermal Efficiency of a Heat Engine. The thermal efficiency of a heat engine is defined as the ratio $\dfrac{\text{Heat converted into work}}{\text{Heat supplied}}$.

Thus there are two thermal efficiencies—

$$(a)\ \text{Brake thermal efficiency} = \frac{\text{Heat equivalent of B.H.P.}}{\text{Heat supplied}}$$

$$(b)\ \text{Indicated thermal efficiency} = \frac{\text{Heat equivalent of I.H.P.}}{\text{Heat supplied}}$$

The two quantities of heat in these ratios must of course be expressed in the same units, i.e. per hour, per minute, or per lb. of fuel used.

It should be noted that—

$$\text{Mechanical efficiency} = \frac{\text{Brake thermal efficiency}}{\text{Indicated thermal efficiency}}$$

Example 8. A trial of a heavy oil engine gave the following data: duration of test, 6 hours; oil used, 192 lb. of calorific value 18,000 B.Th.U./lb.; brake load, 535 lb., and average pull on spring balance at other end of ropes 45 lb.; at speed 210 r.p.m.; effective diameter of brake wheel, 98 in.; 90 explosions or diagrams per minute of mean effective pressure 96·6 lb./sq. in., diameter of engine cylinder, 14·75 in., and the length of stroke, 24 in.; cooling water circulating through cylinder jacket, 45 lb. per minute, raised from 60° to 110° F.; and water through exhaust calorimeter, 32 lb. per minute, raised 90° F. Calculate: (*a*) the indicated and brake horse-power; (*b*) the mechanical efficiency; (*c*) mean frictional torque on the crankshaft; (*d*) oil consumption per brake horse-power hour; (*e*) thermal efficiency, and brake thermal efficiency; (*f*) draw up a balance sheet showing the heat distribution in the engine.

(*a*) The effective area of piston is $\frac{\pi}{4} \times \left(\frac{59}{4}\right)^2 = 170{\cdot}9$ sq. in., and length of stroke 2 ft.; hence the indicated horse-power

$$= \frac{plan}{33{,}000} = \frac{96{\cdot}6 \times 170{\cdot}9 \times 2 \times 90}{33{,}000} = 90;$$

also net brake load $= 535 - 45$ lb. $= 490$ lb., and circumference of brake wheel is $\pi \times \frac{98}{12}$ ft.

$$\therefore \quad \text{Brake horse-power} = \frac{490 \times 22 \times 98 \times 210}{7 \times 12 \times 33{,}000} = 80.$$

Answer.

(*b*) Mechanical efficiency $= \dfrac{80\ (\text{B.H.P.})}{90\ (\text{I.H.P.})} = 0{\cdot}889$ or 89 per cent.

(*c*) And $90 - 80$, or 10 H.P. are required to overcome all the resistances in the engine at 210 r.p.m.

The corresponding frictional torque on the crankshaft is

$$T = \frac{550 \times \text{H.P.}}{\omega}$$

and
$$\omega = \frac{2\pi \times 210}{60} = 7\pi = 22 \text{ radians per sec.}$$

$$\therefore\ T = \frac{550 \times 10}{22} = 250 \text{ lb-ft.}$$

Answer.

(*d*) The oil used per hour is $\frac{192}{6} = 32$ lb., and per brake horse-power hour $= \frac{32}{80} = 0{\cdot}4$ lb. *Answer.*

(*e*) The oil used per indicated horse-power hour $= \frac{32}{90} = 0{\cdot}356$ lb.

$$\therefore \text{ Thermal efficiency} = \frac{\text{Heat equivalent of 1 H.P. hour}}{\text{Heat in oil supplied per I.H.P. hour}}$$

$$= \frac{2{,}545 \text{ B.Th.U.}}{0{\cdot}3556 \times 18{,}000 \text{ B.Th.U.}} = 0{\cdot}3976, \text{ or } 39{\cdot}76 \text{ per cent.}$$

Similarly the brake thermal efficiency

$$= \frac{2{,}545}{0{\cdot}4 \times 18{,}000} = 0{\cdot}353, \text{ or } 35{\cdot}3 \text{ per cent.}$$

Answer.

(*f*) The *balance sheet* of the total heat supplied to the engine in the oil *per minute* is

$$\frac{32}{60} \times 18{,}000 = 9{,}600 \text{ B.Th.U.} = 100 \text{ per cent.}$$

	Per cent
Heat converted into I.H.P. $= 90 \times 42{\cdot}42$ B.Th.U./min. $= 3{,}818$ B.Th.U.	$= 39{\cdot}8$
Heat converted into useful work on brake $= 80 \times 42{\cdot}42$ B.Th.U./min. $= 3{,}394$ B.Th.U. $= 35{\cdot}3$ per cent.	
Heat taken away in jacket water $= 45(110^\circ - 60^\circ)$ B.Th.U./min. $= 2{,}250$ B.Th.U.	$= 23{\cdot}4$
Heat taken away in exhaust gases $= 32 \times 90^\circ$ B.Th.U./min. $= 2{,}880$ B.Th.U.	$= 30{\cdot}0$
Heat lost by radiation and unaccounted for (by difference)	$= 6{\cdot}8$
	100·0

Mechanical Losses in a Gas Engine. Professor B. Hopkinson* measured the thermal and mechanical efficiency of a Crossley gas engine of cylinder diameter 11·5 in. by 21 in. stroke, intended to give a maximum of 40 b.h.p. when firing a charge every cycle, and running at 180 revolutions, that is, giving an explosion stroke every two revolutions of the crankshaft, or 90 explosions per minute.

The power lost in friction of the mechanism was also estimated.

The engine was run with an average load, which was applied by

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* *Proc. Inst. Mech. E.*, October, 1907.

rope brakes, one all round each flywheel, and as each test only lasted a few minutes, it was unnecessary to have the wheels water-cooled.

While careful observations were taken of the brake load, the dead weight and the spring balance, three photographs of indicator diagrams were taken with the optical indicator, each covering about a dozen explosions. At the same time the number of explosions per minute were noted, and the speed kept constant by the governor at 180 r.p.m.

In the measurement of gas consumption the supply to a small standard gas-holder was cut off, so that the engine took gas only from the holder, and the quantity taken in a definite number of suctions was observed by the number of divisions on the calibration scale in the change of level of the gas-holder. The heating value of the gas was tested during the experiments.

The diagrams taken with the Hopkinson optical indicator were remarkably uniform and made one fine line curve for many consecutive explosions at this load and speed. The *mean effective pressure* (M.E.P.) p_m of the diagram, was measured from the negative on the plate by means of a planimeter.

The effective area of the piston $\frac{\pi}{4} . d^2 = 0{\cdot}7854(11{\cdot}5)^2 = 103{\cdot}9$ sq. in., and the length of stroke l ft. $= \frac{21}{12} = \frac{7}{4}$ ft., so that the work done by the gases on the piston during one explosion cycle is

$$p_m \times 103{\cdot}9 \times \frac{7}{4} = 181{\cdot}825\, p_m \text{ ft-lb.}$$

and at one explosion per minute the horse-power is

$$\frac{181{\cdot}825}{33{,}000} \cdot p_m = 0{\cdot}00551\, p_m$$

When the engine is running at 180 r.p.m., and firing 90 times per minute, the indicated horse-power is

$$0{\cdot}00551 \times 90 p_m = 0{\cdot}4959\, p_m$$

In one test the average number of explosions per minute was 72·3, and the mean effective pressure by the indicator diagrams was $p_m = 100{\cdot}3$ lb. per sq. in., giving the indicated power

$$0{\cdot}00551 \times 72{\cdot}3 \times 100{\cdot}3 = 39{\cdot}95 \text{ h.p.}$$

The useful power on the brake wheels is $\frac{2\pi Rn(W - S)}{33{,}000}$, that is, the effective circumference of each brake wheel in feet multiplied by the number of revolutions per minute of the crankshaft, and by the net load, or difference between the dead weight and pull on the spring balance.

Tabulated results obtained from three experiments with the optical indicator and brake, 180 r.p.m., were as follows—

Water-jacket Exit Temperature	Explosions per Minute	M.E.P. from Diagram lb. per sq. in.	Indicated H.P.	Brake H.P.	Mechanical Loss H.P.	Gas per suction cu. ft.
149° F.(65°C.)	72·3	100·3	39·9	34·0	5·9	0·1196
—	73·8	99·4	40·4	34·6	5·8	0·1182
160°F. (71°C.)	74·1	99·0	40·4	34·9	5·5	0·1164

The mean of the three experiments was 34·5 B.H.P., and 40·2 I.H.P. which gives the mechanical efficiency, $\frac{34\cdot5}{40\cdot2} \times 100 = 85\cdot8$ per cent.

Thermal Efficiency. The average number of explosions was 73·4 per minute, and for each charge 0·118 cu. ft. of coal gas was taken in, having a lower calorific value of 570 B.Th.U. per cu. ft.

The available heat supplied to the engine in the coal gas *per minute* was—

	B.Th.U.	Per cent
$73\cdot4 \times 0\cdot118 \times 570$	= 4,937	= 100
Indicated work $= 40\cdot2 \times \frac{33,000}{778}$	= 1,705·1	= 34·5
Work on the brake $= \frac{34\cdot5 \times 33,000}{778}$	= 1,463·3	= 29·6
Mechanical loss $= 5\cdot7 \times \frac{33,000}{778}$	= 241·8	= 4·9

The mechanical loss $= 40\cdot2 - 34\cdot5 = 5\cdot7$ H.P., which becomes less as the temperature of the water jacket rises. This led to further investigation.

After the tests, the brakes were taken off and the engine run without load, the flow of the jacket water being reduced to prevent the engine from cooling rapidly. Indicator diagrams were at once taken, the explosions noted on the average, 12·65 per min., and the gas consumption 0·1252 cu. ft. per suction stroke. The mean pressure by the indicator diagrams was 105·5 lb. per sq. in., giving the indicated power

$$0\cdot00551 \times 12\cdot65 \times 105\cdot5 = 7\cdot35 \text{ H.P.},$$

or $7\cdot35 - 5\cdot7 = 1\cdot65$ H.P. more than the difference between the indicated and brake power in the full load tests. Had the engine been firing every cycle at full load, instead of about 4 out of every 5 cycles, this difference would have been increased in the ratio $\frac{5}{4}$,

$$\text{or } \frac{90}{73\cdot4} \times 1\cdot65 = 2\cdot02 \text{ H.P.}$$

In the idle cycles, without load, only air is admitted to the engine cylinder, and the back pressure on the piston driving the air out during exhaust is much greater than after an explosion, because in the latter, at release, when the exhaust valve opens, gases under high pressure in the cylinder rush down the exhaust pipe, so that there is little or no back pressure on the piston during exhaust. Besides, there is loss of heat during the compression of air which is not all recovered during expansion, with the result that there is a negative loop on the compression and expansion shown by the light spring diagrams; also the extra power taken to compress the air at light load, when no gas was taken in, increases the negative work to about 3·5 H.P. at 180 r.p.m. That is, the power taken to run the engine *without load* is over 2 H.P. more than the mechanical losses at normal load. This increase of fluid pumping losses nearly balances the decrease in piston friction when running light.

In order to investigate or estimate this power loss in the pumping strokes, i.e. during exhaust and suction of the air, independently of the optical indicator and brake, the gas engine was driven at 180 r.p.m., without load and without allowing it to take gas, by an electromotor, and the power required was measured electrically.

During one test the engine was closed as usual, and in the other the exhaust valve cover was removed, the cylinder being open to the air, so that there was no loss by compression.

Assuming the loss in the driving belt from the motor to the engine, about 0·5 H.P., the same in the two cases, the difference in the power absorbed should be equal to the power absorbed in the pumping strokes, the water jacket being about 180° F. (82° C.) in both cases.

On the 24th and 25th August, the tests gave the following results—

	H.P.	H.P.
Engine closed	7·72	7·1
Engine opened	4·14	3·77
Difference	3·58	3·33

The average difference, 3·45 H.P., as compared with 3·5 H.P., estimated from the indicator diagrams, gives tolerably close agreement.

Next, the temperature of the water jacket was varied, and the engine again motored round with the exhaust valve cover taken off, and the pumping losses measured by the electrical power taken to drive the engine running light at 180 r.p.m.—

	H.P. Absorbed
Engine hot (about 180° F., 82° C.) normal lubrication . .	4·0
Engine cold (70° F., 21° C.) normal lubrication . . .	6·5
Engine cold (70° F.) excess of oil	4·7
Engine cold (70° F.) water injected into cylinder . . .	2·7

Here the frictional losses of the engine vary considerably with the temperature, and are reduced by the injection of water.

A separate determination of frictional loss was also made with the piston and connecting rod removed; the loss would then include the friction of the main bearings, side-shaft, valve gear, and driving belt at 180 r.p.m., and was found to be 1·4 H.P.

Deducting this 1·4 H.P. from the total power absorbed gives the power lost in friction of the piston and crank-pin; the piston friction alone being affected by the changes in the cylinder temperature and lubrication. The normal value of the H.P. absorbed due to piston friction with the jacket water at 82° C. was

$$4 - 1{\cdot}4 = 2{\cdot}6 \text{ H.P.}$$

Other tests, under varying conditions, with a jacket temperature of 82° to 85° C., gave the mechanical loss about 5 H.P., determined by the optical indicator and brake, and fully half of this total mechanical loss is spent in piston friction.

Professor Hopkinson found that, under normal working conditions, at the best jacket temperature 82° C., with the brake 36 H.P., this engine indicated 41 H.P. when running at 180 r.p.m., or piston speed 630 ft. per min., and the mechanical losses were—

	H.P.			
Pumping loss	1·4	=	3·4	per cent I.H.P.
Piston friction	2·5	=	6·1	,, ,,
Other friction, valve lifting, etc.	1·1	=	2·7	,, ,,
Total mechanical loss	5·0	=	12·2	,, ,,

from which the mechanical efficiency was 87·8 per cent.

The motoring test, under nearly actual running conditions, appears to afford one of the best methods of checking the mechanical efficiency and of determining the fluid pumping losses.

With *rise of cylinder temperature* the viscosity of the lubricating oil is reduced, and so also is the piston friction, but the volumetric efficiency is reduced, so that the weight of charge taken into the cylinder is reduced, and with it the indicated horse-power; yet the reduction in piston friction may more than counterbalance the decrease of indicated horse-power, so that the brake horse-power rises slightly.

Conservation of Energy. *Energy*, which means the *capacity for doing work*, may be changed from one kind into another; and, in any transformation of energy, as for instance, from mechanical energy to electrical energy or this into heat energy, a quantity of one form of energy disappears and is converted into another kind, exactly equivalent. The function of a heat engine is to do mechanical work with the least possible expenditure of heat. The ratio of the quantity of heat converted into work in the cylinder to the total heat supplied to an engine, is called the absolute *thermal efficiency* of the engine.

In a complete cycle of operations, by the principle of the CONSERVATION OF ENERGY, when the working fluid in any heat engine returns to the initial state (see pp. 113–4),

Heat taken in = Heat rejected + Work done by the substance

The **internal energy,** E, of a fluid may be defined as the quantity of energy in the fluid in a given state; that is, its value depends only on the state of the fluid and not on how the fluid has reached that state. There is no means of measuring the stock of intrinsic or internal energy which a fluid contains in virtue of its pressure, volume, temperature, and state. Only *changes* of the whole internal energy can be measured when the substance receives heat, does work, or gives out heat. Internal energy is usually expressed in thermal units per unit weight, and, of course, cannot be calculated absolutely, but must be measured from an arbitrary zero. For instance, the internal energy of steam is generally reckoned as zero when the fluid is in the state of liquid water at 0° C. under its own vapour pressure.

If the working substance takes in heat without doing work, its stock of internal energy increases; on the other hand, if the substance does work without receiving heat, it does the work at the expense of its own stock of heat energy, which is reduced by an amount equivalent to the work done (pp. 69–70).

The product pv has a definite value for any state of a fluid, and is therefore a function of the state. It may be defined as the work done in ft-lb. per lb. by forcing unit weight of volume v cu. ft. into a cylinder against a constant pressure p lb. per sq. ft. To convert this into thermal units we must divide by J. Hence, if we use the symbol A for $\frac{1}{J}$, the quantity becomes Apv heat units.

When heat is received by the substance while it is doing external work, we have,

Heat supplied = Increase of internal energy + External work done

$$\text{i.e.} \quad dQ = dE + A\,p\,.\,dv$$

Adiabatic Expansion. An adiabatic expansion is one in which no heat is supplied to or removed from the working substance. From the energy equation just mentioned it will be seen that in an adiabatic expansion, the work done by the working substance is done at the expense of its internal energy.

It will be seen later (p. 114), that the above definition can be amplified by the inclusion of the condition of "reversibility," which condition implies the absence of friction within the working substance. Frictionless adiabatic expansion or compression is therefore better termed "isentropic."

From the fundamental equation

$$dQ = dE + Ap \,.\, dv$$

as dQ by definition is zero, it follows that—

$$dE = - Ap \,.\, dv$$

that is, during a frictionless adiabatic expansion or compression, the work done by or on the fluid, is done at the expense of, or added to the internal energy of the fluid. From this it follows that the area under the pv curve for an adiabatic expansion or compression represents the change in internal energy.

Isothermal Expansion. An isothermal expansion or compression is one in which the temperature of the fluid remains constant. Here again care must be taken to distinguish between frictionless expansion, and one where internal friction is set up in the fluid.

The Total Heat, H, of a fluid is defined as the sum $E + Apv$. It is generally measured in thermal units per pound, and is an important function of the state of a fluid. (See pp. 379 and 566.)

The relation between heat transfer and change in total heat is shown thus—

$$H = E + A\,pv$$

$$\therefore \qquad dH = dE + A\,p \,.\, dv + A\,v \,.\, dp$$

$$= dQ + A\,v \,.\, dp$$

It should be noticed that, by this definition, the total heat of a liquid at 0° C. is not zero, but equal to Ap_0v_0, where p_0 is the vapour pressure in lb. per sq. ft. at 0° C., and v_0 is the specific volume of the liquid at 0° C., and pressure p_0. Water at 0° C. has a vapour pressure of 0·0892 lb. per sq. in., and a specific volume of 0·01602 cu. ft. per lb. Hence, the total heat of water at 0° C. is

$$H_0 = E_0 + Ap_0v_0 = 0 + \frac{144}{1{,}400} \times 0{\cdot}0892 \times 0{\cdot}01602$$

$$= 0{\cdot}000147 \text{ C.H.U.}$$

This quantity is negligible and generally neglected in practice, thus Callendar* assumes that the total heat of water is zero at 0° C., and is 100 at 100° C.

For carbonic acid this quantity is about 1 C.H.U., and for ammonia much smaller at 0° C.

Total Energy of a Fluid. In some applications of thermodynamics, the kinetic energy of a fluid is an important factor. The kinetic energy per lb. weight of a body is $\frac{U^2}{2g}$ ft-lb., where U = velocity in feet per second and g is the acceleration due to gravity in feet per second per second.

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* Some authorities call what is here defined as total heat the total energy and denote it by the letter I instead of H. Another name for total heat is *enthalpy*.

Therefore the total energy per pound of a fluid is

$$E + Apv + A\,\frac{U^2}{2g}\ \text{heat units}$$

Equation of Steady Flow of a Fluid. Imagine an enclosure through which a fluid is passing with a *steady flow*: that is, the mass entering per second, which is equal to the mass leaving per second, remains constant. Let the fluid as it enters have specific volume, velocity, pressure, and internal energy equal to v_1, U_1, p_1, and E_1; also as it leaves the enclosure, v_2, U_2, p_2, and E_2 respectively. Now suppose a turbine or heat appliance within this enclosure that will do an amount of work, W, for every lb. weight of fluid passing through it; also, let an amount of heat, Q, be lost from the enclosure per pound of fluid passing through.

Now the work done *on* each pound of the fluid by that behind it in passing into the enclosure is p_1v_1, while the work done *by* the fluid on that in front of it in passing out is p_2v_2.

By the conservation of energy, the difference between these quantities, together with the change in internal and kinetic energy, between entry and exit must be equal to the work done by the appliance and the heat energy lost from the enclosure. That is,

$$A(p_1v_1 - p_2v_2) + (E_1 - E_2) + A\left(\frac{U_1^2}{2g} - \frac{U_2^2}{2g}\right) = AW + Q$$

or, $$(Ap_1v_1 + E_1) - (Ap_2v_2 + E_2) + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q$$

that is $$H_1 - H_2 + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = A\,W + Q$$

This relation is very useful when dealing with the steam turbine and various appliances in practical thermodynamics.

Throttling. In the case of *steady flow* of a fluid through a throttle valve or reducing valve, the total energy in heat units per pound in the initial state before entering the valve is

$$E_1 + Ap_1v_1 + AU_1^2/2g$$

and, after throttling,

$$E_2 + Ap_2v_2 + AU_2^2/2g$$

By the conservation of energy, the small difference between the total energy of each pound of fluid entering and leaving the valve is the sum of the loss of heat to external bodies and the external work done.

When the valve merely reduces the pressure without external work being done, assuming neither gain nor loss of heat to the valve or external bodies, and the velocity of flow so small that the change in kinetic energy is negligible, as in the porous plug experiments by Joule and Thomson (p. 63), then it follows that the *total heat remains constant.*

Before entering the valve, the work done on one pound of the fluid by the pressure behind it is p_1v_1; when the flow has become steady after throttling, each pound does work, p_2v_2, upon the fluid in front of it. Any difference in the work done must reduce the stock of internal energy, and

$$Ap_2v_2 - Ap_1v_1 = E_1 - E_2$$

that is,
$$E_2 + Ap_2v_2 = E_1 + Ap_1v_1$$

or
$$H_2 = H_1$$

Thus, the *total heat of a fluid does not change in a throttling process* as in a porous plug, expansion valve of a refrigerating machine, or in a reducing valve, during the ideal process.

Transfer of Heat at Constant Pressure. Let a quantity of heat Q be given to or taken away from one pound of a fluid at constant pressure p, the internal energy of the fluid changing from E_1 to E_2, while the volume changes from v_1 to v_2.

Then, by the first law of thermodynamics,

Heat supplied = increase of internal energy + external work done

$$\begin{aligned} Q &= \int_1^2 dE + A\int_1^2 p\,.\,dv, \text{ where } p \text{ is constant} \\ &= E_2 - E_1 + Ap(v_2 - v_1) \\ &= E_2 + Apv_2 - (E_1 + Apv_1) \\ &= H_2 - H_1, \text{ by definition of } H \end{aligned}$$

Hence, when heat is absorbed or rejected by a fluid at constant pressure, the quantity of heat so transferred is equivalent to the change in the total heat of the fluid.

Transfer of Heat at Constant Volume. Let a quantity of heat Q be transferred to or from one pound of fluid at constant volume, the pressure changing from p_1 to p_2 while the internal energy changes from E_1 to E_2. Then in the equation $dQ = dE + A\,p\,.\,dv$ the change in volume dv is zero and $dQ = dE$.

Hence the heat absorbed or rejected by the fluid at constant volume is equivalent to the change in internal energy.

Change of Total Heat during Adiabatic Expansion of a Fluid. (Heat drop.) Let ab, Fig. 21, represent the adiabatic expansion of unit weight of a fluid, from volume and pressure v_1 and p_1 to v_2 and p_2, when no heat enters or leaves the fluid, while it does work at the expense of its own stock of internal energy.

By the energy equation,

$$dQ = dE + A\,p\,.\,dv$$

but in adiabatic expansion $dQ = 0$, and we have

$$dE = -A\;p.\,dv$$

Integrating, $E_1 - E_2 = \int_2^1 dE = A\int_2^1 p\,.\,dv$

which is represented by the area *abfe* in work units, and is the difference between the internal energy of the fluid at *a* and *b*.

Now, by definition,

$$\begin{aligned} H_1 - H_2 &= E_1 + Ap_1v_1 - (E_2 + Ap_2v_2) \\ &= (E_1 - E_2) + A\,p_1v_1 - Ap_2v_2 \\ &= A \times (\text{area } abfe) + A(\text{area } daeo) - A(\text{area } cbfo) \\ &= A \times \text{area } abcd \end{aligned}$$

This area represents the work done in admitting into the cylinder the volume v_1 of the fluid at pressure p_1, expanding adiabatically to pressure p_2 and volume v_2, and then discharging from the cylinder at pressure p_2.

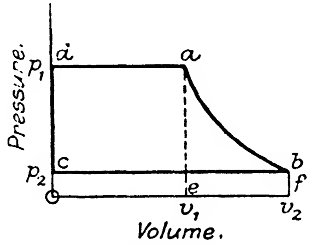

FIG. 21. ADIABATIC EXPANSION

The same result may be obtained from the total heat equation,

$$H = E + A\,pv$$

Differentiating $dH = dE + A\,p\,.\,dv + A\,v\,.\,dp$

but, as above, $dE = -A\,p\,.\,dv$

and, substituting this value of dE, gives

$$\begin{aligned} dH &= -A\,p\,.\,dv + A\,p\,.\,dv + A\,v\,.\,dp \\ &= A\,v\,.\,dp \end{aligned}$$

hence, integrating,

$$H_1 - H_2 = \int_2^1 dH = A\int_2^1 v\,.\,dp$$

This decrease in the total heat energy of the fluid during adiabatic expansion is called the adiabatic "Heat Drop," and is represented by the work area *abcd*. Therefore, the thermal equivalent of the whole work done in the cylinder is measured by the adiabatic Heat Drop.

It must be remembered that absence of friction is assumed.

EXAMPLES II

1. A petrol engine, of cylinder diameter 3·6 in. and 4 in. stroke, gives 600 explosions or cycles per minute. The mean indicator diagram (Fig. 2, p. 11) has an area 1·5 sq. in. and length 3 in.; the scale of the indicator spring is 1/200. Estimate the average indicated horse-power of the engine.

2. In a brake test the diameter of the brake wheel fitted on the crankshaft is 24 in., and that of the rope, around the wheel, 1 in.; the vertical pulls on spring balances at the ends of the brake rope are 24 lb., and 3 lb. when the speed is 1,200 r.p.m. The mean effective pressure on the piston, 80 lb. per sq. in. during each cycle at 600 explosions per minute. The cylinder is 4 in. in diameter and 4 in. stroke. Calculate: (*a*) the indicated and brake horse-power; (*b*) the power spent in engine friction; and (*c*) the mechanical efficiency.

3. A four-cylinder petrol engine at a certain speed develops B_4 B.H.P. When firing is stopped in one cylinder, and the engine made to run under the same conditions as to speed, etc., the brake horse-power is B_3. Neglecting pumping losses and assuming that the indicated horse-power developed in each cylinder is the same, find the approximate indicated horse-power of the engine. Tests on a four-cylinder petrol engine under the conditions indicated above gave the following results: B.H.P. with four cylinders firing, 14·7; B.H.P. with three cylinders firing, 10·0. Find the approximate indicated horse-power of the engine and its mechanical efficiency.
(*Inst. Mech. E.*, 1923.)

4. A test of an oil engine, during *one hour*, gave the following data: Effective circumference of brake wheel and rope, 11·25 ft.; load on brake rope around the wheel, 48 lb., and the pull on the spring balance at the other end of rope, 4 lb.; mean speed, 350 r.p.m.; oil fuel used, 2·84 lb. of calorific value 18,400 B.Th.U. per lb.; mean indicated horse-power, 6·5; jacket cooling water, 252 lb. raised 62·4° F. Calculate: (*a*) the brake horse-power; (*b*) mechanical efficiency; (*c*) the thermal efficiency; and (*d*) give the percentage of heat energy of the fuel—(1) converted into useful mechanical energy at the crankshaft, (2) spent in engine friction, and (3) given to the jacket cooling water.

5. A test of an oil engine gave the following figures. Determine from them the five items of the heat balance, and draw up the balance in tabular form. Duration of trial, 30 min.; oil used, 9·7 lb.; calorific value of oil, 18,000 B.Th.U./lb.; jacket water, 725 lb.; rise of temperature of water, 70° F.; total revolutions, 6,100; brake load, 314 lb. at 2·5 ft. from shaft centre; mean effective pressure, 82 lb./in.^{2}; diameter of cylinder, 12 in., and stroke 18 in. (*Inst. C. E.*, 1923.)

6. A gas engine, working on the constant volume cycle (Fig. 2, p. 11) has clearance 0·25 cu. ft., and the volume swept by piston, 1·25 cu. ft.; compression curve is $p \, . \, v^{1.35}$ constant, and expansion curve $p \, . \, v^{1.3}$ constant. At the beginning of the compression stroke the charge is at 14 lb. per sq. in., absolute pressure. While making 110 explosions per minute of maximum pressure 400 lb. per sq. in., the engine uses 12·5 cu. ft. of gas having calorific value 400 B.Th.U. per cu. ft. Calculate: (*a*) the pressure at end of the compression and expansion strokes; (*b*) the net work done per cycle; (*c*) the mean effective pressure and indicated horse-power; and (*d*) the thermal efficiency of the engine. (*National Diploma of Inst. Mech. E.*, 1922.)

7. The following results were obtained from a test on a tank engine: I.H.P., 187·0; B.H.P., 162·9; fuel used per brake horse-power hour, 0·554 lb.; calorific value of fuel used, 10,350 C.Th.U. per lb.; heat carried away by circulating water, 232,500 C.Th.U. per hour. Determine: (*a*) the thermal efficiency (measured on the I.H.P.); (*b*) the mechanical efficiency; and (*c*) the heat lost in exhaust and radiation per minute.
(*Inst.Mech.E.*, 1924.)

8. The following particulars were obtained from a trial of a four-stroke cycle oil engine : duration of trial, 40 min. ; oil used, 9·77 lb. ; total revolutions, 8,142 ; total combustion cycles, 4,071 ; jacket water, 738 lb. ; rise of temperature of jacket water, 74° F. ; mean effective pressure in cylinder, 96 lb. per sq. in. ; torque due to brake load, 786 lb-ft. ; calorific value of 1 lb. oil fuel, 17,000 B.Th.U.'s ; area of piston, 113 sq. in. ; stroke, 18½ in. Find (*a*) the indicated and brake horse-powers ; (*b*) the oil used per hour per I.H.P. and per B.H.P. ; (*c*) the heat converted into indicated work per minute ; (*d*) the heat rejected by the jacket water per minute ; (*e*) the heat lost by friction, exhaust gases, etc., per minute.

(*U.L.*, *B.Sc.* (*Eng.*).)

9. The following results were obtained from a test on an oil engine : I.H.P., 167 ; B.H.P., 142 ; fuel per B.H.P. hour, 0·457 lb. of calorific value 10,000 C.H.U. per lb. Calculate (*a*) the indicated thermal efficiency, (*b*) the brake thermal efficiency, and (*c*) the mechanical efficiency.

(*Inst.Mech.E.*, 1925.)

10. What is meant by the " pumping horse-power " of an internal combustion engine ? A gas engine, 8 in. piston diameter and 13 in. stroke, when running at 250 r.p.m., gave indicator diagrams showing mean effective pressures in lb. per sq. in. ; firing cycle—positive loop 85, negative loop 5 ; dead cycle—negative loop 9·5. Find (*a*) the frictional horse-power of the engine if, when running without brake load, there are 31 explosions per minute; (*b*) the I.H.P. (from positive loop), and (*c*) the mechanical efficiency.

11. Calculate the diameter and stroke of a gas engine which can develop 25 brake horse-power at 300 r.p.m., assuming a mechanical efficiency of 80 per cent, a mean effective pressure of 85 lb. per sq. in., and a ratio of stroke to diameter of 1·5. (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

CHAPTER III

PROPERTIES OF GAS AND VAPOUR

Properties of a Perfect Gas. A perfect gas may be defined as a fluid which obeys Boyle's Law and the Law of Charles. The so-called permanent gases, such as oxygen, nitrogen, hydrogen, and also air under normal conditions, follow these laws fairly closely, and thus for normal purposes, the various simple equations for a perfect gas may be applied to them.

It will, however, be seen later that this is only true when the conditions of the gas are well removed from what is termed the critical point. Thus if the fluid is below the critical temperature these equations will not hold, as the fluid may be either a liquid, or a wet, dry, or superheated vapour. In such cases other analytical methods must be used as in the case of steam, carbon dioxide, ammonia, etc.

Boyle's Law. Boyle discovered by experiment that if the *temperature* of a given quantity of any gas is kept *constant*, during a change of pressure and volume, the volume, v, varies inversely as pressure, p, of the gas, or $pv = a$ *constant*, which is represented by a rectangular hyperbola called an *isothermal curve*.

Law of Charles. If the *pressure* be kept *constant*, the permanent gases expand nearly the same fraction of their volume at 0° C. for a given increase of temperature. The results of experiments show that hydrogen and other permanent gases, when kept at constant pressure, expand nearly $\frac{1}{273}$ of their volume at 0° C. for every 1° C rise of temperature, or $\frac{1}{492}$ of their volume at 32° F. for every 1° F. rise of temperature. For example, at constant pressure, 273 cu. in. of gas at 0° C. became 273 + 1 cu. in. at 1° C., 273 + 2 cu. in. at 2° C., and 273 + t cu. in. at t° C. Thus at t° C. the volume at constant pressure becomes

$$v_t = v_0\left(1 + \frac{1}{273} \times t\right) = v_0 \cdot \frac{273 + t}{273},$$

and

$$\frac{v_t}{v_0} = \frac{273 + t}{273} = \frac{T}{T_0},$$

where T and T_0 are absolute temperatures.

The equation becomes $\frac{v_0}{T_0} = \frac{v_t}{T} =$ a constant.

Also, at *constant pressure*, when the same quantity of gas is cooled 1° C. below the freezing point to − 1° C., the volume becomes 273 − 1 cu. in., and at − 2° C. the volume is reduced to 273 − 2 cu. in.; and the volume would become nothing at − 273° C. if the law could be held to apply down to the lowest possible temperature.

Before this temperature would be reached any actual gas would become a vapour, change its physical state to a liquid, and then become frozen to a solid. By the equation, if the temperature be reckoned from this absolute zero, the volume of a gas, heated at constant pressure, is proportional to the temperature reckoned from absolute zero. Now if the pressure be changed from p_0 to p, and the gas obeys Boyle's law, it follows that

$$\frac{pv}{p_0v_0} = \frac{T}{T_0} \text{ or } \frac{pv}{T} = \frac{p_0v_0}{T_0} = \text{a constant } (R)$$

for unit mass of gas, and we have

$$pv = w\,.\,RT, \text{ or } \frac{pv}{T} = w\,.\,R \quad . \quad . \quad . \quad . \quad (1)$$

the **characteristic equation** of the relations between pressure, volume, and absolute temperature for w lb. weight of a gas, where

p = absolute pressure in pounds per square foot

v = volume in cubic feet, of w lb.

R = a constant

depending on the particular gas and whether the absolute temperature, T, is expressed on the Centigrade or Fahrenheit scale.

At 0° C. and standard atmospheric pressure of 14·7 lb. per sq. in., or 2,116 lb. per sq. ft., the weight of *dry air* is 0·0807 lb. per cu. ft., hence the volume of 1 lb. weight is $\frac{1}{0{\cdot}0807}$, or 12·391 cu. ft.

$$\text{Hence} \quad R = \frac{pv}{T} = \frac{2{,}116 \times 12{\cdot}391}{273} = 96 \text{ ft-lb.}$$

per lb. weight of air, in the Pound Centigrade system.

Also in the Pound Fahrenheit system,

$$R = \frac{pv}{T} = \frac{2{,}116 \times 12{\cdot}391}{492} = 53{\cdot}2 \text{ ft-lb.}$$

per lb. weight of air (nearly).

In the metric system, Regnault found that 1 cubic metre of dry air at 0° C. and atmospheric pressure of 1·03274 kg. per sq. cm., or 10,327·4 kg. per sq. metre, weighed 1·2928 kg.

Hence the volume of 1 kg. of dry air $= \frac{1}{1{\cdot}2928}$, or 0·7735 cubic metre, and the increase in volume for the rise of temperature 0° C. to 1° C., if the air were a perfect gas, would be $\frac{0{\cdot}7735}{273{\cdot}1}$ cubic metre. The work done by the air during expansion against constant atmospheric pressure while being warmed is $R = \frac{10{,}327{\cdot}4 \times 0{\cdot}7735}{273{\cdot}1} = 29{\cdot}25$ kg-metres per kilogramme.

To check this result, multiply 29·25 by 3·28085, the number of feet in a metre, giving the value of R in ft-lb. per lb. = 95·965.

In the case of hydrogen, the specific volume, at 0° and atmospheric pressure of 2,116 lb. per sq. ft., is 178·8 cu. ft. per lb.

$$\therefore R = \frac{2{,}116 \times 178{\cdot}8}{273{\cdot}1} = 1{,}385 \text{ ft-lb. per lb. (nearly).}$$

The ideal *perfect gas* follows Boyle's law and satisfies the characteristic equation exactly. No actual gas does, but hydrogen comes near to it, since the deviation is very small over a wide range of temperature.

Electrons, Atoms, and Molecules. Any chemical element is built up of very small particles called atoms. Modern science now shows that these atoms consist of a central nucleus positively charged with electricity around which are negatively charged particles termed electrons. The smallest number of atoms that can exist alone is called a molecule. Some atoms such as those of helium can exist alone, and the molecule is termed monatomic. Others such as those of hydrogen form a diatomic molecule, the symbol being H_2, while other molecules are polyatomic. Some substances such as carbon may have a varying number of atoms in the molecule.

The weights of atoms and molecules vary with the particular substances concerned, and it is convenient to consider the relative molecular and atomic weights, taking as a standard of reference that of 16 for the oxygen atom, this molecule being diatomic.

Table I gives in round figures the values for substances commonly used in engineering. The molecular weight of any substance is of course not the actual weight, but a relative weight.

The Mol and the Pound Molecule. Avogadro's Law states that *equal volumes of all gases under the same conditions of temperature and pressure contain the same number of molecules.* Hence a mass of any gas having a weight equal to its molecular weight will

TABLE I

SUBSTANCE	SYMBOL		WEIGHT	
	Atom	Molecule	Atomic	Molecular
Hydrogen	H	H_2	1	2
Oxygen	O	O_2	16	32
Nitrogen	N	N_2	14	28
Carbon	C		12	
Sulphur	S		32	
Carbon Monoxide		CO		28
Carbon Dioxide		CO_2		44
Methane		CH_4		16
Benzene		C_6H_6		78
Water or Steam		H_2O		18
Sulphur Dioxide		SO_2		64

occupy at N.T.P. a fixed volume, no matter what the particular gas is. Actual gases follow this law very closely, and the approximate value of this fixed volume is 359 cu. ft. if the molecular weight is reckoned in pounds. Thus 2 lb. of hydrogen, 32 lb. of oxygen, or 28 lb. of nitrogen occupy at 0° C. and 14·689 lb. per sq. in. a volume of 359 cu. ft.

A weight of gas equivalent to its molecular weight is called the *gramme molecule* or *pound molecule* respectively. The term *mol* is used generally for the gramme molecule, although it is sometimes applied to the pound molecule.

As 1 lb. mol occupies 359 cu. ft.

$$1 \text{ gm. mol occupies } \frac{359}{453{\cdot}6} \text{ cu. ft.}$$

$$= \frac{359 \times 28{,}316}{453{\cdot}6} = 22{,}400 \text{ c.c.}$$

Heat energy of a gas is often expressed in heat units per mol. It should be noticed that—

$$\begin{aligned} &1 \text{ gm. calorie per gm. mol} \\ &\quad = 1 \text{ lb. calorie per lb. mol} \\ &\quad = 1{,}400 \text{ ft-lb. per lb. mol} \\ &\quad = \frac{1{,}400}{359} = 3{\cdot}9 \text{ ft-lb. per cu. ft. at N.T.P.} \end{aligned}$$

$$\begin{aligned} &1 \text{ B.Th.U. per lb. mol} \\ &\quad = 778 \text{ ft-lb. per lb. mol} \\ &\quad = \frac{778}{359} = 2{\cdot}167 \text{ ft-lb. per cu. ft. at N.T.P.} \end{aligned}$$

A cubic foot of gas at N.T.P. is sometimes called a *Standard Cubic Foot* (S.C.F.).

The Universal Gas Constant. The characteristic gas equation states that $pv = wRT$ where v is the volume of the mass w.

If w is the molecular weight in pounds then

$$v = 359 \text{ S.C.F.}$$

$$\therefore \quad 14{\cdot}689 \times 144 \times 359 = wR \times 273$$

whence $wR = 2{,}779$ ft-lb. per lb. mol per ° C.

$$= \frac{2{,}779}{1{,}400} = 1{\cdot}985 \text{ lb. cal. per lb. mol per } ^\circ \text{ C.}$$

In Fahrenheit units

$$\begin{aligned} wR = \frac{14{\cdot}689 \times 144 \times 359}{492} &= 1{,}544 \text{ ft-lb. per lb. mol, per } ^\circ \text{ F.} \\ &= \frac{1{,}544}{778} \\ &= 1{\cdot}985 \text{ B.Th.U. per lb. mol per } ^\circ \text{ F.} \end{aligned}$$

This quantity, which is generally represented by G, is obviously constant for all gases, assuming they obey Avogadro's Law, and is called the *Universal Gas Constant.*

Hence $pv = GT$ where v is the volume of the mol.

To obtain the value of R for any gas we have $R = \frac{G}{w}$. Thus if the molecular weight of air is taken as 28·95, then

$$R \text{ for air} = \frac{2{,}779}{28 \cdot 95} = 96 \text{ ft-lb. per lb. per } ^\circ \text{ C.}$$

$$\text{or} = \frac{1{,}544}{28 \cdot 95} = 53 \cdot 3 \text{ ft-lb. per lb. per } ^\circ \text{ F.}$$

For hydrogen $w = 2$.

$$R \text{ for hydrogen} = \frac{2{,}779}{2} = 1{,}390 \text{ ft-lb. per lb. per } ^\circ \text{ C.}$$

Joule's Law. The **Internal Energy** *of a given quantity of gas* **depends only on its temperature,** *and is practically independent of its pressure and volume.*

Joule first investigated the assumption that practically no *internal work* is done, and no appreciable quantity of heat energy is absorbed by a gas during free expansion into vacuum, without doing external work. To do this, two copper vessels of equal capacity were connected by a short tube with stop-valve. One of them, A, Fig. 22, was filled with dry air compressed to about 21 atmospheres, and the other, B, was exhausted. Both vessels were immersed in a bath containing 16½ lb. of water, which was stirred and its steady temperature taken by a sensitive thermometer reading to $\frac{1}{200}$ of a degree Fahrenheit. The stop-valve at C was opened by means of a key and compressed air allowed to pass from A into the empty vessel B until equilibrium was established. The water was again stirred, and no change of temperature could be detected by the thermometer.

In order to analyse this result, Joule inverted the vessels and immersed them, as well as the connecting pipe and stop-valve, C, each in a separate bath, Fig. 23.

After opening the stop-valve to allow the compressed air in A to expand into the exhausted vessel B, as before, the water was again stirred and a drop of temperature was observed in A, while that around B and C was raised to the same amount, showing that the heat absorbed by part of the air as it rushed out of A was given up to B and C; but, on the whole, there was neither gain nor loss of heat by the air, the heat lost by A being compensated by that gained by B and C. It follows that no internal work is done by a gas during free expansion; or in a perfect gas the molecules are so far apart, compared with their size, that there is no appreciable attraction between them, consequently no internal work has to be done to separate the molecules.

Joule and Thomson (Lord Kelvin) found in later experiments[*] that the 0·13 lb. weight of air in Joule's experiment, under 21 atmospheres expanding to 10·5 atmospheres, without doing work during free or unresisted expansion required only 0·063 heat units to make up for the total internal cooling effect. This quantity of heat taken from 16·5 lb. of water, 28 lb. of copper, and 7 lb. of tinned iron, would produce a drop of temperature of only 0·003° C.

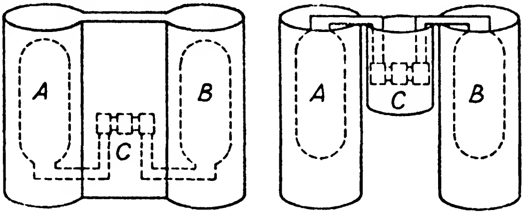

Fig. 22 Fig. 23

Free Expansion of Gas

Little wonder they remarked that no appreciable external thermal effect was observed, and that similar experiments afterwards made by Regnault should have led only to the same negative conclusion.

By *taking the heat from the air itself* the drop of temperature was 2·8° C., that is, 900 times the thermometric effect in the earlier experiments.

In the **porous plug experiments** of Joule and Thomson this extremely sensitive method was devised of detecting the deviations of actual gases from the ideal perfect state, which has led to far-reaching results.

It was proved that when there is molecular attraction or any trace of friction amongst the particles of actual gases, mere expansion, without doing work, produces cooling or reduction of the stock of internal energy equivalent to the internal work done in overcoming the molecular forces or friction.

A steady continuous stream of gas supplied at constant pressure and temperature is forced through a porous plug, or small orifice, from which it issues at a lower pressure, where its temperature is observed. Care is taken to prevent external loss or gain of heat by the plug or orifice, and the walls of the pipe are non-conductors. The issuing jet has kinetic energy of eddies which quickly subside, being converted into heat by fluid friction.

Suppose in the pipe there is a diaphragm with a small orifice. Consider a short length of the stream at cross-sections *A* and *B*

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[*] Joule and Thomson, "Thermal Effects of Fluids in Motion." *Joule's Scientific Papers*, Vol. II, pp. 216–362; or Thomson (Lord Kelvin) Papers, Vol. I, p. 333.

on each side of the orifice but not close to it. At A, approaching the orifice, the internal energy per lb. of fluid is E_1, and p_1, v_1 the pressure and volume, and at B, E_2, p_2 and v_2 respectively. At A the gas forced forward by the pressure p_1, with volume v_1, escapes through the orifice, and pushes the gas in front with pressure p_2 with volume v_2, beyond the region of the "rapids" or eddies which subside at some distance from the orifice. These are greatly reduced by the porous plug device.

In the case of *steady flow* the temperature may be higher or lower at B than that upstream at A, depending on the nature of the fluid. The decrease of energy $E_1 - E_2$ must be equal to the work done by the fluid. Now the work done *by* the gas per pound on the portion in front at B is p_2v_2, while at A, p_1v_1 is the work done *on* the gas by the stream behind it. Therefore,

$$E_1 - E_2 = Ap_2v_2 - Ap_1v_1,$$

that is, $$E_1 + Ap_1v_1 = E_2 + Ap_2v_2,$$

a constant at any two sections of the stream, which is defined as the Total Heat (H) of the gas (see p. 52).

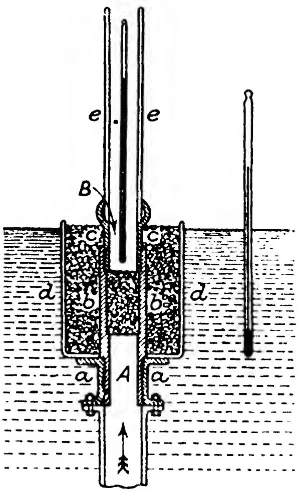

FIG. 24. POROUS PLUG

Suppose the product p_1v_1 has not changed and is equal to p_2v_2, then $E_1 = E_2$, and *if the internal energy* depends only on the temperature, then the temperature will be the same at A and B. Hence, if the temperature be found constant, the gas obeys Boyle's law and Joule's law, $E_1 = E_2$, even though the pressure and volume vary; consequently the internal energy must be a function of the temperature only. This is only strictly true of the ideal perfect gas, but if the temperature changes, then E must vary with p and v, as well as with temperature.

Joule and Thomson had a pump driven by an engine to force dried air or other gas in a tranquil stream through a long spiral of copper pipe immersed in a bath of water, which was constantly stirred and kept at a uniform temperature. A brass cylinder, *aa*, Fig. 24, was secured to the flange on the upper end of the copper pipe, to which a boxwood nozzle, *bb*, 1·5 in. diameter, was screwed, and in this a plug of cotton wool was slightly compressed between two perforated brass plates resting on ledges 2·72 in. apart. A short piece of boxwood, *cc*, was screwed above the top of the upper plate to hold it securely. A tin can, *dd*, screwed on the brass cylinder, *aa*, also filled with cotton wool, prevented heat from the bath passing to the boxwood nozzle.

When the stop-valve was opened to allow the stream of compressed air or gas up through the plug, fluctuations of temperature were observed, due to the sudden cooling effect and changes of temperature at the nozzle, as well as to expansion and compression in the long coil of copper pipe in the bath. After about an hour or two, a steady condition of heat-flow allowed temperatures to be noted on sensitive thermometers, one in the bath and the other the same distance above the plug, observed through the glass tube *ee*; also the pressure gauge was observed, every two minutes, and the mean values recorded.

The experiments showed that for pressures from 1 to 6 atmospheres, hydrogen, nitrogen, oxygen, common dry air, and carbonic acid passed towards, through, and from the porous plug in a tranquil stream, almost without change of temperature, carbonic acid much less so than any of the others; hence the inference that absolute temperature is approximately proportional to the volume of these fluids. The Joule-Thomson *cooling effect* is defined as the ratio of drop of temperature to drop of pressure between the two sides of the plug. For dry air it was at 17° C., nearly 0·0176° C. for each pound per square inch difference of pressure, and decreased at 91·6° C. to 0·014° C., or about a quarter of a degree for each atmosphere drop in pressure.

In the case of CO_2, the cooling effect was 0·0833° C. per lb. per square inch difference of pressure at 12·8° C., while at 91·5° C. it was reduced to 0·0474° C. Callendar finds that a curve for the mean corrected results for carbonic acid gas gives 0·0944° C. per lb. pressure at 0° C., and 0·0421° C. per lb. at 100° C. At higher temperatures these gases approximate more nearly to Boyle's law; and the results showed that *the cooling effect was proportional to the difference of pressure on the two sides of the porous plug, and varied inversely as the square of the absolute temperature for dry air and carbonic acid.*

For air, the drop of temperature (Centigrade) is

$$0{\cdot}275(P_1 - P_2)\left(\frac{273}{T_1}\right)^2$$

where the pressure is in atmospheres. For carbonic acid, the constant in this expression is 1·388.

In the case of hydrogen there was a *heating effect* which agrees with the results of Amagat's experiments that the product *pv* *increases* with the pressure, whereas for carbonic acid and air, *pv* *decreases* nearly uniformly and the curve has a steep slope downwards to a minimum near the critical temperature, where it becomes vertical.

Kelvin also deduced from the cooling effect a method of comparing the absolute thermodynamic scale of temperature with that of the gas thermometer. Taking the work equivalent of the heat required to compensate or neutralize the cooling effect, and Regnault's measurements of the specific heat at constant pressure and the coefficients of expansion between 0° and 100° C., Kelvin concludes that the absolute temperature of melting ice is 273·1° C.,[*] or "for every 100 units of heat converted into work by a perfect thermodynamic engine, 373·1 are taken from the source, and 273·1 rejected to the refrigerator, if the temperature of the source be that at which steam of water has a pressure of 1 atmosphere and the temperature of the refrigerator that at which ice melts."

The cooling effect has been thoroughly investigated and applied by Callendar in the determination of the properties of steam and other vapours at low and moderate pressures common in practice. He pointed out that the cooling effect indicates a corresponding variation of specific heat with pressure by the relations between specific heat, cooling effect, and variation of the total heat of gases.

This Joule-Thomson cooling effect, or small drop in the temperature of compressed air and vapours escaping through a fine orifice, as in a throttling operation, through a valve without gain or loss of heat, is most important in applied thermodynamics.

Professor Linde took advantage of it in a *regenerative process* which accumulates the cooling effect, with an *interchanger*, and air pump for the production of extreme cold below the critical temperature of air − 140° C., and nearly to absolute zero (see p. 588).

Obviously, Joule's law only holds strictly for the ideal perfect gas; the deviation for the permanent gases is so small as to be negligible, except in the case of vapours near their critical temperatures.

Specific Heats of a Perfect Gas. The specific heat of any substance may be defined for elementary purposes, as the quantity of heat required to raise the temperature of one pound of the substance by one degree. The numerical value of specific heat so defined, being in heat units, per degree of temperature, per pound, will be independent of the temperature scale used. We shall, however, see later that specific heat may be measured in other units.

The simple definition given above automatically assumes that the value of specific heat is constant at all temperatures. This assumption is adopted in this chapter for gases, but in practice this is not so, and later on the variation of specific heat with temperature will be considered.

A gas may be considered to have two specific heats, depending on whether it is kept at constant volume, or at constant pressure while it is being heated.

(*a*) *Specific Heat at Constant Volume* (C_v in heat units, K_v in work units). This may be defined as the amount of heat required to raise the temperature of one pound of gas, by one degree, while the volume remains constant, and the pressure increases.

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[*] Article on "Heat," *Ency. Brit.*; also Kelvin's *Mathematical and Physical Papers*, Vol. I, p. 333.

Thus in the equation $dQ = dE + A\,p\,.\,dv$ P. 51. $A = \frac{1}{J}$

$$dQ = C_v dT \text{ and } dv = 0$$

Hence $dQ = C_v dT = dE$

It is thus seen that if heat is transferred to or from a gas at constant volume, then the quantity of heat transferred is equivalent to the change in internal energy of the gas. This statement is in agreement with Joule's Law (p. 62).

(*b*) *Specific Heat at Constant Pressure* (C_p in heat units, K_p in work units). This may be defined as the amount of heat required to raise the temperature of one pound of gas, by one degree, while the pressure remains constant, and the volume increases.

Thus in the equation $dQ = dE + A\,p\,.\,dv$

$$dQ = C_p dT, \text{ while } dE = C_v dT$$

Hence, $C_p dT = C_v dT + A\,p\,.\,dv$

Thus if heat is transferred to a gas at constant pressure while the gas expands, the heat transferred is equivalent to the increase in internal energy of the gas ($C_v dT$) plus the work done by the gas in expanding ($A\,p.\,dv$).

(*c*) *Difference of the Specific Heats.* Let one pound of gas be heated at constant pressure.

Then $p\,v = R\,T$ and $p\,.\,dv = R\,dT$ (as p is constant)

Hence $C_p dT = C_v dT + A\,R\,d\,T$

$$\therefore \quad R = \frac{C_p - C_v}{A} = J\,(C_p - C_v) = K_p - K_v$$

R being in work units.

(*d*) *Ratio of the Specific Heats.* The ratio of the specific heats $\frac{C_p}{C_v}$, denoted by γ is an important factor in the thermodynamics of gases. Its relation to the gas constant R is obtained from the equation $R = J\,(C_p - C_v)$.

Thus, $$\frac{R}{C_v} = J\left(\frac{C_p}{C_v} - 1\right) = J\,(\gamma - 1)$$

or $$C_v = A\,\frac{R}{\gamma - 1} \text{ and } C_p = A\,\frac{\gamma}{\gamma - 1}\,R$$

(*e*) *Units and Values of Specific Heats.* Swann,* by the continuous electric heating method of Callendar, found the value of C_p for air at 0° C. as 0·2413 lb. calorie per lb. per degree C.

As $$R = \frac{96}{1{,}400} = 0{\cdot}0686 \text{ lb. calorie per lb. per degree C.,}$$

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* See *Brit. Assoc. Report,* 1908; *Phil. Trans.,* 1910, p. 199.

then $C_v = 0{\cdot}2413 - 0{\cdot}0686 = 0{\cdot}1727$ lb. calorie, per lb. per degree C.

This gives $\gamma = \frac{C_p}{C_v} = \frac{0{\cdot}2413}{0{\cdot}1727} = 1{\cdot}397$ for air.

In calculations on internal combustion engines, it is often more convenient to express these values in foot pounds per cubic foot at N.T.P. (Standard Cubic Foot). This quantity is sometimes called the *volumetric specific heat*, or more simply the *volumetric heat* of the gas. This term is generally but not always confined to the specific heat at constant volume.

Thus, taking the molecular weight of air as 28·95,

$C_v = 0{\cdot}1727 \times 1{,}400 \times \frac{28{\cdot}95}{359} = 19{\cdot}52$ ft-lb. per S.C.F. per degree C.

Similarly,

$C_p = 27{\cdot}27$ ft-lb. per S.C.F.

The specific heat is also sometimes expressed in pound calories per pound molecule. Thus for air

$C_v = 0{\cdot}1727 \times 28{\cdot}95 = 5{\cdot}00$ lb. cal. per lb. mol. per degree C, or B.Th.U. per lb. mol. per degree F.

and $C_p = 0{\cdot}2413 \times 28{\cdot}95 = 6{\cdot}985$ lb. cal. per lb. mol. per degree C, or B.Th.U. per lb. mol. per degree F.

whence the Universal Gas Constant

$$G = C_p - C_v = 6{\cdot}985 - 5{\cdot}00 = 1{\cdot}985.$$

As a pound mol. of any gas is virtually a unit of volume, these values of specific heat are termed volumetric or molar heats.

The actual values of the specific heats of gases increase with temperature, and this variation is considered later. Table II gives values of the specific heats at 15° C. and atmospheric pressure for a

TABLE II

	A	B	C	D	E	F	H
	C_p C.H.U. per Mol per °C	C_v C.H.U. per Mol per °C	G $= A - B$	γ $= \frac{A}{B}$	C_v ft. lb. per S.C.F. $= B \times 3{\cdot}9$	C_p C.H.U. per lb. $= \frac{A}{\text{Mol Wt}}$	C_v C.H.U. per lb. $= \frac{B}{\text{Mol Wt.}}$
Air	6·943	4·947	1·996	1·403	19·29	0·240	0·171
CO	6·94	4·94	2·00	1·404	19·27	0·248	0·176
N_2	6·925	4·929	1·996	1·405	19·22	0·247	0·176
O_2	7·04	5·04	2·00	1·396	19·66	0·220	0·158
H_2	6·86	4·87	1·99	1·408	18·99	3·43	2·44
CO_2	8·79	6·75	2·04	1·302	26·33	0·20	0·153
CH_4	8·49	6·48	2·01	1·310	25·27	0·531	0·405

number of gases, based on calculations by Messrs. Partington and Shilling.*

From a consideration of the kinetic theory of gases, it is shown that for a perfect diatomic gas (p. 109)

$$C_p = \frac{7}{2}R \text{ and } C_v = \frac{5}{2}R$$

Taking the value of 96 for R

$$C_p = \frac{7}{2} \times 96 = 0{\cdot}24$$

and $$C_v = \frac{5}{2} \times 96 = 0{\cdot}171 \text{ C.H.U. per lb. per } ^\circ \text{C.},$$

or taking the value of 1·985 for G

$$C_p = \frac{7}{2} \times 1{\cdot}985 = 6{\cdot}948$$

and $$C_v = \frac{5}{2} \times 1{\cdot}985 = 4{\cdot}963 \text{ cal. per mol. per } ^\circ \text{C.}$$

These values should be compared with the actual values given in Table II.

It should be noted that specific heats in C.H.U. per ° C., have the same numerical values in B.Th.U. per ° F.

Isothermal Operations of a Perfect Gas. As the internal energy change is measured by $C_v dT$ and dT is zero, it follows that in an isothermal change of a perfect gas the internal energy remains constant and the equation $dQ = C_v dT + A\,p\,.\,dv$ becomes $dQ = A\,p\,.\,dv$.

Thus the work energy produced by or on the gas is provided by an equivalent heat transfer to or from the gas from or to exterior surroundings. Obviously some difference of temperature between the gas and its surroundings would have to be provided, and the operation would have to be performed very slowly. The temperature cannot, however, be kept rigorously constant in practice.

When the *absolute temperature* is kept *constant*, the *isothermal curve*, APB, Fig. 25, gives the relation of pressure and volume of gas during expansion from the initial state p_1, v_1, T_1, to the state p_2, v_2, T_1; the characteristic equation becomes $p_1v_1 = p_2v_2 = wRT_1 = k$ a constant (Boyle's law).

The product of any pair of values of pressure and volume at state points A, P, and B, Fig. 25, on the curve is equal to 50. Hence the curve is a rectangular hyperbola.

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* *The Specific Heat of Gases*, by Partington and Shilling (Benn).

The same quantity of gas at a different constant temperature will have another similar isothermal curve, *CD*, and in this way we can draw any number of isothermals for the same mass of gas, by increasing its stock of internal energy.

The work done, *W*, *by* the gas during expansion, and *on* the gas during compression, is calculated by the equation (6) (p. 13), substituting for k the value wRT; or graphically by measuring the area under the curve to the volume axis, and bounded by the pressure ordinates at the initial and final states, *A* and *B*.

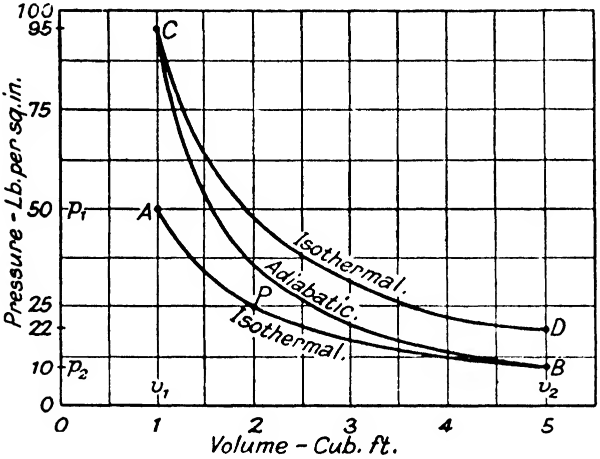

FIG. 25. ISOTHERMALS AND ADIABATICS

$$W = p_1 v_1 \log_\epsilon \frac{v_2}{v_1}, \text{ or } wRT_1 \log_\epsilon r \quad . \quad . \quad . \quad (4)$$

where v_1 = volume of w lb.

and $$Q = E + \frac{W}{J}, \text{ becomes } JQ = O + p_1 v_1 \log_\epsilon r$$

Adiabatic Operations of a Perfect Gas. Expansion or compression is adiabatic when the operation is reversible and the working substance neither gains nor loses heat by conduction and radiation, or by internal chemical action. We must imagine a given quantity of perfect gas to be enclosed in a cylinder, fitted with a piston, both perfect non-conductors of heat, which neither take up any heat themselves nor allow heat to pass through them to or from the gas. An adiabatic operation cannot be quite realized because every substance absorbs heat, more or less, and a cylinder cannot be perfectly heat-tight, or impervious to heat. An operation is nearly adiabatic when *performed quickly* to allow very little time for the transfer of heat between the gas and the surface of the cylinder walls.

In adiabatic expansion *CB*, Fig. 25, work is done by the given weight of gas solely at the expense of its own store of internal energy, hence the temperature and pressure fall below that of the isothermal *CD* for the same change in volume, since during isothermal expansion heat is supplied to keep the temperature constant. During adiabatic *compression*, *BC*, the work done on the gas generates heat, which increases its stock of internal energy since no heat is given out, and the temperature rises accordingly and gives rapid increase of pressure, whereas, during isothermal compression between the same volumes, *B* to *A*, the heat produced must be taken away in order to keep the temperature constant, therefore the isothermal compression curve must be below the adiabatic.

Hence, *the adiabatic curve has a steeper slope than the isothermal, during both expansion and compression.*

Let dQ denote a small quantity of heat taken in by 1 lb. weight of a perfect gas, and dT and dv the resulting small increments of temperature and volume; then the fundamental energy equation, Heat received = Gain of internal energy + External work done, becomes $dQ = C_v \, . \, dT + p \, . \, dv$ if C_v and Q are in work units.

Since in adiabatic expansion or compression no heat is taken in or given out by the gas, $dQ = 0$, the *work done is equivalent to the change of internal energy*, and we have $C_v \, . \, dT + p \, . \, dv = 0$.

Differentiate the characteristic equation of a perfect gas $pv = RT$, and we obtain $p \, . \, dv + v \, . \, dp = R \, . \, dT$, or $dT = (p \, . \, dv + v \, . \, dp)/R$.

Substitute this value of dT in the above equation, and taking $R = C_p - C_v$, we obtain

$$C_p \, . \, p \, . \, dv + C_v \, . \, v \, . \, dp = 0, \text{ and } \frac{C_p}{C_v} = \gamma$$

from which
$$\gamma \, . \, \frac{dv}{v} + \frac{dp}{p} = 0$$

and, by integration, $\gamma \, . \, \log_\varepsilon v + \log_\varepsilon p =$ a constant.

Therefore, $$p \, . \, v^\gamma = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the **equation to the adiabatic curve** of a perfect gas, giving the relation between the absolute pressure and volume during expansion or compression.

The value of the constant can be found for a given state of the working substance from any pair of values of p and v on the adiabatic curve. (See p. 12.)

The work area during an adiabatic change of state can now be found by substituting γ for n in the general equations (1) to (5) (p. 13).

We have $pv = wRT$, and $p_1v_1 - p_2v_2 = wR(T_1 - T_2)$ and, for adiabatic expansion,

$$W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}, \text{ becomes } W = \frac{wR(T_1 - T_2)}{\gamma - 1} \quad . \quad (6)$$

or

$$W = \frac{p_1v_1}{\gamma - 1}\left(1 - \frac{T_2}{T_1}\right) = \frac{p_1v_1}{\gamma - 1}\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right\} \quad . \quad (7)$$

Then $\frac{W}{J}$, represents the reduction of the internal energy of the gas or the amount converted into mechanical energy during expansion from the state p_1, v_1, T_1, to the state p_2, v_2, T_2.

The internal energy of 1 lb. weight of the gas is gradually spent in doing work on the piston as the adiabatic curve is indefinitely prolonged to cut the volume axis at infinity and the pressure falls to zero, when the gas can do no more external work and its stock of energy is used up. If $p_2 = 0$, the equations representing the work area under the adiabatic curve become

$$\text{Work done} = \frac{p_1v_1}{\gamma - 1} = \frac{RT_1}{\gamma - 1} = K_vT_1 \text{ ft-lb.} \quad . \quad . \quad . \quad (8)$$

which may be taken as the mechanical equivalent of the *internal energy* of 1 lb. weight of gas in the initial state at absolute temperature T_1.

Change of Temperature of a Perfect Gas during Adiabatic Expansion and Compression. During an adiabatic operation from the state p_1, v_1, T_1, to that of p_2, v_2, T_2, the relation between pressure and volume is

$$p_1 \cdot v_1^{\gamma} = p_2 \cdot v_2^{\gamma}, \text{ where } \gamma \text{ denotes the ratio } \frac{C_p}{C_v} \quad . \quad . \quad (a)$$

and, for a perfect gas, the characteristic equation is

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

To eliminate p, divide (a) by (b), and we obtain

$$T_1 \cdot v_1^{\gamma - 1} = T_2 \cdot v_2^{\gamma - 1}, \text{ or } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1} \quad . \quad . \quad . \quad (9)$$

where $\frac{v_2}{v_1}$ is the ratio of expansion.

Again, to eliminate v, raise (b) to the power γ, and divide by (a)

which gives $$\frac{p_1^{\gamma - 1}}{T_1^{\gamma}} = \frac{p_2^{\gamma - 1}}{T_2^{\gamma}}, \text{ or } \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma - 1}{\gamma}}$$

and the temperature may be calculated from a given pressure change. Hence, the adiabatic law for a perfect gas may also be written—

$$T \cdot v^{\gamma - 1} = \text{constant, and } \frac{T}{p^{\frac{\gamma-1}{\gamma}}} = \text{constant.} \quad . \quad . \quad (10)$$

In the application of the energy law to a perfect gas—

(*a*) In isothermal expansion $pv = \text{constant}$, when T is constant $dQ = dE + p \cdot dv$; all the heat given to the gas goes to do external work in expansion, and the internal energy remains constant; $dE = 0$, and $dQ = p \cdot dv$.

(*b*) Adiabatic expansion, $dQ = 0$, and $\therefore p \cdot dv = -dE$. In this case, external work is done at the expense of the internal energy of the gas.

(*c*) Heating gas at constant pressure: Internal energy increases and external work is done by the heat $\int_1^2 dQ = \int_1^2 dE + \int_1^2 p \cdot dv$,

$\therefore$ heat given to the gas, $Q = E_2 - E_1 + p(v_2 - v_1)$.

(*d*) Heating at constant volume, $\therefore p \cdot dv = 0$, and $dQ = dE$.

All the heat goes to increase the internal energy of the gas.

✓**Example 1.** The temperature of the mixture of gas and air in a gas engine at the end of the admission stroke is 90° F., and the pressure 15 lb. per sq. in. absolute. The clearance volume is 4·6 cu. ft., and the total volume of clearance plus piston displacement is 12 cu. ft. Assuming adiabatic compression $p.v^{1 \cdot 4} = \text{constant}$, determine the temperature at the end of the compression stroke.

If the pressure after ignition is 240 lb. per sq. in., find the temperature in the cylinder. (*U.L., B.Sc.* (*Eng.*).)

The student should draw a compression curve above the atmospheric line and a vertical to represent increase of pressure at constant volume during explosion after compression and ignition, like the lower curve and constant volume line in Fig. 2 (p. 11).

Here the temperature before compression is

$$T_1 = 90° + 460° = 550° \text{ F. (absolute).}$$

Let T_2 = absolute temperature at end of adiabatic compression.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{1 \cdot 4 - 1} \text{ and } T_2 = 550 \left(\frac{12}{4 \cdot 6}\right)^{0 \cdot 4}$$

Taking logarithms

$$\begin{aligned} \log T_2 &= \log 550 + 0{\cdot}4 (\log 12 - \log 4{\cdot}6) \\ &= 2{\cdot}7404 + 0{\cdot}4(1{\cdot}0792 - 0{\cdot}6628) = 2{\cdot}907 \end{aligned}$$

hence, $T_2 = 807°$ F. (abs.), or $807° - 460° = 347°$ F.

Again, since the *volume* is constant during explosion, by the characteristic equation, the temperature is proportional to pressure.

Now, $p_2 \times 4{\cdot}6^{1{\cdot}4} = 15 \times 12^{1{\cdot}4}$

that is, $p_2 = 15\left(\dfrac{12}{4{\cdot}6}\right)^{1{\cdot}4} = 57{\cdot}42$ lb. per sq. in.

and, at constant volume,

$$\frac{T}{807} = \frac{240}{57{\cdot}42}, \text{ or } T = \frac{807 \times 240}{57{\cdot}42} = 3{,}373^\circ \text{ F. (abs.)}$$

$\therefore$ temperature of the charge at 240 lb. per sq. in. is

$$3{,}373^\circ - 460^\circ = 2{,}913^\circ \text{ F.}$$ *Answer.*

Example 2. If 0·1 lb. of gas occupying 0·5 cu. ft. is expanded in a cylinder at constant pressure of 150 lb. per sq. in. absolute until its volume is 1 cu. ft., and is then expanded adiabatically to 5 cu. ft., find the temperature of the gas (*a*) at the end of the constant pressure stage, (*b*) at the end of the adiabatic expansion; and calculate the heat expended and the work done during each portion of the process. Take K_p = 198 ft-lb., and K_v = 144 ft-lb. (*U.L., B.Sc. (Eng.).*)

Given, $wR = w(K_p - K_v) = \dfrac{1}{10}(198 - 144) = 5{\cdot}4$ ft-lb.

also $pv = wRT$, and initial temperature $T_1 = \dfrac{p_1v_1}{wR}$.

Substituting values,

$$T_1 = \frac{(144 \times 150)0{\cdot}5}{5{\cdot}4} = 2{,}000^\circ \text{ F. (abs.) or } 1{,}540^\circ \text{ F.}$$

(*a*) At constant pressure,

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}, \text{ that is, } \frac{T_2}{T_1} = \frac{v_2}{v_1} = \frac{1}{0{\cdot}5} = 2,$$

therefore, at the end of the constant pressure stage,

$$T_2 = 2T_1 = 4{,}000^\circ \text{ F. (abs.), or } 3{,}540^\circ \text{ F.}$$

Heat energy received by the gas during expansion at constant pressure is

$$w \,.\, K_p(T_2 - T_1) = \frac{1}{10} \times 198(4{,}000^\circ - 2{,}000^\circ) = 39{,}600 \text{ ft-lb.},$$

which is equivalent to 39,600/778 = 50·9 B.Th.U.

The work done at constant pressure

$$= p(v_2 - v_1) = (144 \times 150) \times \tfrac{1}{2}$$
$$= 10{,}800 \text{ ft-lb.}$$ *Answer.*

(*b*) For adiabatic expansion,

$$\gamma = \frac{198}{144} = \frac{11}{8}$$

and $$\frac{T_3}{T_2} = \left(\frac{v_2}{v_3}\right)^{\gamma - 1} = \left(\frac{1}{5}\right)^{\frac{3}{8}}$$

Taking logarithms $\log T_3 = \log 4{,}000 - \frac{3}{8}\log 5 = 3{\cdot}3400$,
and we obtain, $T_3 = 2{,}188^\circ$ F. (abs.), or 1,728° F.

The gas neither receives nor gives out heat during an adiabatic change, therefore the work done *by* the gas is equal to its loss of internal energy due to change of temperature, which is $w.K_v(T_3 - T_2)$, namely,

$$\frac{1}{10} \times 144(4{,}000 - 2{,}188) = 26{,}093 \text{ ft-lb.}$$

which is equivalent to 26,093/778 = 33·54 B.Th.U.

To check this result, for the adiabatic curve, $p_3 \,.\, v_3{}^\gamma = p_2 \,.\, v_2{}^\gamma$,

that is, $$\frac{p_3}{p_2} = \left(\frac{v_2}{v_3}\right)^\gamma = \left(\frac{1}{5}\right)^{\frac{11}{8}},$$

and $$\log p_3 = \log 150 - \frac{11}{8}\log 5 = 1{\cdot}2150.$$

We have the pressure $p_3 = 16{\cdot}41$ lb. per sq. in., and the work done during expansion,

$$\begin{aligned} W &= \frac{p_2 v_2 - p_3 v_3}{\gamma - 1} \\ &= \frac{144(150 \times 1 - 16{\cdot}41 \times 5)}{\frac{3}{8}} \\ &= 384 \times 67{\cdot}95, \text{ or } 26{,}093 \text{ ft-lb.} \end{aligned}$$ *Answer.*

A pressure-volume diagram, drawn to scale, would show these changes clearly.

Example 3. Four cu. ft. of gas at 20° C. and 14·7 lb. per sq. in. is compressed adiabatically to one-fifth of this volume. Determine: (*a*) the weight of the gas; (*b*) the pressure and temperature at the end of the adiabatic compression; (*c*) the work done *on* the gas; (*d*) the heat given out while the gas is cooled to 20° C., at the reduced constant volume; and (*e*) the pressure after cooling. The specific heat at constant pressure is 0·2404, and at constant volume 0·1718.

We have $$\gamma = \frac{0{\cdot}2404}{0{\cdot}1718} = 1{\cdot}4;$$

and the gas constant,

$$R = (C_p - C_v)\,J = (0{\cdot}2404 - 0{\cdot}1718)\,1{,}400 = 96 \text{ ft-lb.}$$

(*a*) Then the weight of the gas is found by the characteristic equation

$$w = \frac{p_1 v_1}{RT_1} = \frac{(144 \times 14{\cdot}7)4}{96 \times 293} = 0{\cdot}301 \text{ lb.}$$

(*b*) By the adiabatic law, $p_2 \,.\, v_2{}^\gamma = p_1 \,.\, v_1{}^\gamma$, or $p_2 = p^1 \left(\frac{v_1}{v_2}\right)^\gamma$, and the compression ratio $\frac{v_1}{v_2} = 5$; substituting the given values

$$\begin{aligned} p_2 = 14{\cdot}7 \times 5^{1{\cdot}4}, \text{ and } \log p_2 &= \log 14{\cdot}7 + 1{\cdot}4 \log 5 \\ &= 2{\cdot}1459, \end{aligned}$$

we obtain the pressure at end of compression,

$$p_2 = 139{\cdot}93 \text{ lb. per sq. in.}$$

Again, the absolute temperature, $T_2 \,.\, v_2{}^{\gamma-1} = T_1 \,.\, v_1{}^{\gamma-1}$

or $$T_2 = T_1 \,.\, \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

which becomes,

$$T_2 = 293 \times 5^{0\cdot4},$$

and $$\log T_2 = \log 293 + 0\cdot4 \log 5 = 2\cdot7465,$$

and $$T_2 = 557\cdot8^\circ \text{ C. (absolute), or } 284\cdot8^\circ \text{ C.}$$

Check this value by the relation

$$\frac{T_2}{T_1} = \left(\frac{139\cdot93}{14\cdot7}\right)^{\frac{1\cdot4-1}{1\cdot4}}$$

(c) Work done *on* the gas during adiabatic compression is

$$\frac{p_2v_2 - p_1v_1}{\gamma - 1} = \frac{144(139\cdot93 \times 0\cdot8 - 14\cdot7 \times 4)}{0\cdot4}$$
$$= 360 \times 53\cdot144 = 19{,}132 \text{ ft-lb.}$$

The equivalent of this work is the heat energy given to the gas

$$= \frac{19{,}132}{1{,}400} = 13\cdot67 \text{ C.H.U.}$$

since, by the energy equation, $Q = E + \frac{W}{J}$.

(d) Now in cooling 0·301 lb. weight of gas at *constant volume*, from 284·8° C. to 20° C., the heat given out by the gas

$$= 0\cdot301 \times 264\cdot8^\circ \times 0\cdot1718, \text{ or } 13\cdot69 \text{ C.H.U.}$$

(e) Also the pressure drops to that on the isothermal curve through the initial state point, and the internal energy of the gas is the same as at the outset, while the pressure is reduced to that on the isothermal, which, by Boyle's law, is 14·7 × 5, or 73·5 lb. per sq. in. *Answer.*

If a *pv* diagram were drawn similar to *BCA*, Fig. 25, but to a suitable scale, the point *B* would represent the initial state of the gas as regards pressure and volume. During compression without gain or loss of heat, the change of pressure and volume would be shown by the *adiabatic curve BC*. Heat is rejected in the constant volume change *CA*, and no work is done. During the isothermal expansion, as the state point moves from *A* to *B* along the isothermal curve, the gas receives heat at the rate required to keep the temperature constant, while the gas does work represented by the area under *AB*. The difference between the work area under *BC* done *on* the gas, and that done *by* the gas under *AB*, is *BCA*. When the state point moves round the closed area in the direction *BCAB*, opposite to the hands of a clock, on the whole cycle, work is converted into heat.

Example 4 One pound of air at 354° F. (178·9° C.) expands adiabatically to three times its original volume, and in the process falls in temperature to 60° F. (15·6° C.). The work done during the expansion is 38,410 ft-lb. Calculate the two specific heats. (*U.L., B.Sc.* (*Eng.*).)

$$T_1 = 178{\cdot}9 + 273 = 451{\cdot}9^\circ \text{ C. (abs.)},$$

and $$T_2 = 15{\cdot}6^\circ + 273 = 288{\cdot}6^\circ \text{ C. (abs.)}.$$

Substitute given values in equation (9) (p. 72), $\frac{451{\cdot}9}{288{\cdot}6} = 3^{\gamma - 1}$

Take logarithms $(\gamma - 1) \log 3 = 0{\cdot}19474$,

and $\gamma \log 3 - \log 3 = 0{\cdot}19474$,

that is, $\gamma \times 0{\cdot}47712 = 0{\cdot}19474 + 0{\cdot}47712$,

from which,

$$\frac{C_p}{C_v} = \gamma = \frac{0{\cdot}67186}{0{\cdot}47712} = 1{\cdot}408, \text{ and } C_p = 1{\cdot}408\, C_v.$$

Again, by equation (6),

$$W = \frac{wR(T_1 - T_2)}{\gamma - 1}, \text{ where } wR = wJ(C_p - C_v)$$

Substituting values,

$$38{,}410 = \frac{1 \times 1{,}400(C_p - C_v)(451{\cdot}9 - 288{\cdot}6)}{\frac{C_p}{C_v} - 1} \text{ ft-lb.},$$

and $\gamma - 1 = \frac{C_p - C_v}{C_v}$, and the equation reduces to

$$38{,}410 = 1{,}400 C_v \times 163{\cdot}3, \text{ and } C_v = \frac{38{,}410}{1{,}400 \times 163{\cdot}3} = 0{\cdot}168.$$

$$\therefore C_p = 1{\cdot}408 \times 0{\cdot}168 = 0{\cdot}23655.$$ *Answer.*

To check calculations, substitute the values obtained, as above, for the work done

$$C_p - C_v = 0{\cdot}23655 - 0{\cdot}168 = 0{\cdot}06855,$$

and $$\gamma = \frac{0{\cdot}23655}{0{\cdot}168} = 1{\cdot}408$$

also $$W = \frac{wR(T_1 - T_2)}{\gamma - 1} = \frac{1 \times 1{,}400 \times 0{\cdot}06855 \times 163{\cdot}3}{1{\cdot}408 - 1}$$
$$= 38{,}410 \text{ ft-lb.}$$

Example 5. The characteristic constant for hydrogen is 1,382 ft-lb. units and its specific heat at constant pressure is 3·41; 3 cu. ft. of hydrogen, measured at 15 lb. per sq. in. and 18° C., are compressed adiabatically to 200 lb. per sq. in. and then expanded isothermally to the original volume of 3 cu. ft. Determine the final pressure of the gas. Calculate the amount of heat which must be added to the gas during isothermal expansion, and also the heat which must be abstracted from the gas after expansion in order to reduce it to its initial state of pressure. (*U.L.*, *B.Sc.* (*Eng.*), 1923).

Let R represent the gas constant, then the difference of the specific heats is

$$C_p - C_v = \frac{R}{J}, \text{ that is, } C_v = 3\cdot41 - \frac{1,382}{1,400} = 2\cdot423$$

Then, $$\gamma = \frac{C_p}{C_v} = \frac{3\cdot41}{2\cdot423} = 1\cdot4074.$$

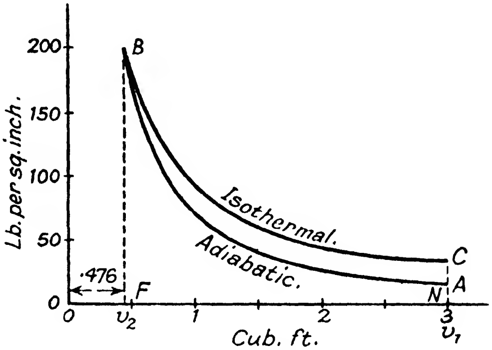

FIG. 26

The adiabatic law is

$$p_1 \cdot v_1^{\gamma} = p_2 \cdot v_2^{\gamma}, \text{ or, } \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{200}{15}\right)^{\frac{1}{1\cdot4074}}$$

Taking logarithms,

$$\log\left(\frac{v_1}{v_2}\right) = \frac{1}{1\cdot4074}(\log 200 - \log 15) = \frac{1\cdot125}{1\cdot4074} = 0\cdot7993$$

Hence the compression ratio $r = \frac{v_1}{v_2} = 6\cdot3$; and the volume at end of the adiabatic compression, at state point, B, on the sketch diagram (Fig. 26),

$$v_2 = \frac{3}{6\cdot3} = 0\cdot4762 \text{ cu. ft.}$$

During isothermal expansion, the state point moves along the curve BC.

Let p_1, p_2 and p_3 be the pressures at A, B and C (Fig. 26).

At B and C, on the isothermal,

$$p_2 \cdot v_2 = p_3 \cdot v_1, \text{ and } p_3 = p_2\left(\frac{v_2}{v_1}\right)$$

Hence, $$p_3 = \frac{200}{6\cdot3} = 31\cdot75 \text{ lb. per sq. in. at } C.$$

By Joule's energy law, the internal energy of a perfect gas depends only on its temperature, and the deviation by volume of hydrogen is practically negligible ; it follows that the internal energy of hydrogen is constant at every point on the expansion curve *BC*, Fig. 26. Therefore, the amount of heat received by the gas, to keep its temperature constant, is equivalent to the work done by the gas during expansion, and represented by the work area *BCNF*. The work done during expansion is

$$\begin{aligned} W &= p_2v_2 \times 2{\cdot}3026 \log_{10}r \\ &= 144 \times 200 \times 0{\cdot}4762 \times 2{\cdot}3026 \log 6{\cdot}3 \\ &= 25{,}240 \text{ ft-lb.} \end{aligned}$$

which is equivalent to

$$\frac{W}{J} = \frac{25{,}240}{1{,}400} = 18{\cdot}03 \text{ C.H.U.} \qquad \textit{Answer.}$$

Hence, also, the heat energy to be extracted from the gas at constant volume, *CA*, to reduce the gas to its initial state must be the equivalent of the work done on the gas during adiabatic compression, *AB*, and represented by the area *ABFN*, equal to

$$\frac{p_2v_2 - p_1v_1}{\gamma - 1} = \frac{144(200 \times 0{\cdot}4762 - 15 \times 3)}{1{\cdot}4074 - 1} = 17{,}760 \text{ ft-lb.}$$

which is equivalent to

$$\frac{17{,}760}{1{,}400} = 12{\cdot}68 \text{ C.H.U.} \qquad \textit{Answer.}$$

The cycle *ABC* is closed, and the gas returned to its initial state.

The *difference* between the isothermal work done *by* the gas, and the work done *on* it, which is 18·03 - 12·68 C.H.U. = 5·35 C.H.U., the heat converted into work, is represented by the area of the indicator diagram *ABC*.

It is to be noted, when the state point moves round a closed curve in the direction of the hands of a clock, the gas, on the whole, receives more heat than it rejects, and the difference is converted into work.

By the principle of the conservation of energy, and Joule's first law of the equivalence of heat and work—

Heat received = Work done by the gas + Heat rejected

becomes, in this case, 18·03 C.H.U. = 5·35 + 12·68 C.H.U.

Further, the temperature in the initial state, *A*, is

$$T_1 = 18^\circ + 273 = 291^\circ \text{ C. (abs.),}$$

and we may calculate the temperature at B, due to the adiabatic compression,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{200}{15}\right)^{0\cdot2895}$$

$$\text{since } \frac{\gamma-1}{\gamma} = \frac{1\cdot4074-1}{1\cdot4074} = 0\cdot2895,$$

from which, $T_2 = 616°$ C. (abs.), that is, 343° C.

The same result is obtained by taking the adiabatic change of volume, thus,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = 6\cdot3^{0\cdot4074}, \text{ and } T_2 = 616°\text{ C. (abs.)}.$$

As a check on the above calculations, estimate the quantity of heat given out by the hydrogen in cooling at constant volume, from 343° C. at the state C, to 18° C., the initial state at A. The temperature drop is 325° C. From the hydrogen gas constant, R, given equal to 1,382 ft-lb. per lb., we may calculate the density or weight of hydrogen in pounds per cubic foot at 0° C. and 14·7 lb. per sq. in., by the characteristic equation $pv = wRT$, which gives 0·00561 lb. per cu. ft. By the same equation, reduce the 3 cu. ft. measured at 15 lb. per sq. in. and 18° C. to its volume at 0° C. and 14·7 lb. per sq. in., which comes to 2·873 cu. ft., and so find its weight 2·873 × 0·00561, or 0·01611 lb. The weight is also obtained directly from the characteristic equation

$$w = \frac{pv}{RT} = \frac{15 \times 144 \times 3}{1{,}382 \times 291} = 0\cdot01611\text{ lb.}$$

The heat given out by this weight of gas in cooling at constant volume is

$$w \times C_v \times \text{drop of temperature} = 0\cdot01611 \times 2\cdot423 \times 325°$$
$$= 12\cdot68\text{ C.H.U.}$$

The pressure at C is also determined from the temperature, since the volume is constant during cooling. The relation is

$$\frac{p_1}{T_1} = \frac{p_3}{T_3}, \text{ that is, } \frac{p_3}{p_1} = \frac{T_3}{T_1}, \text{ and } p_3 = p_1 \times \frac{T_3}{T_1}$$

from which $p_3 = 15 \times \frac{616°}{291°} = 31\cdot75$ lb. per sq. in.

This method is important in the determination of extremely high and quickly varying temperature in the gas engine cylinder, from measurements of pressure and volume on the indicator diagram.

Rate of Heat Reception or Rejection by a gas in the engine cylinder. Assume the specific heats of the gas constant. Given a pv indicator diagram traced to scale, without knowing the temperature.

Let dp be the infinitely small change of pressure while the piston moves through a very short length of stroke and sweeps out the infinitely small change of volume dv. Then $\frac{dp}{dv}$ is the rate of change of pressure per unit change of volume, at the point corresponding to the mean value of dv. If dp gets less, this is negative, as in expansion; if dv is negative, the volume is decreasing, as in compression, and a positive value of $\frac{dQ}{dv}$ is the *rate of rejection of heat* by the gas to the cylinder walls.

Let dQ be the small quantity of heat given to the gas during the small change of pressure and volume, and dT the corresponding change in temperature, then $\frac{dQ}{dv}$ will represent the rate at which the expanding gas receives heat *per unit change of volume.* Also the *rate of heat reception per second*, during the short interval of time dt is $\frac{dQ}{dt} = \frac{dQ}{dv} \cdot \frac{dv}{dt} = \frac{dQ}{dv} \times$ volume swept by the piston per second.

Now a pound weight of perfect gas follows the law

$$pv = RT, \text{ or } T = \frac{pv}{R}$$

By differentiating, we have

$$\frac{dT}{dv} = \frac{1}{R}\left(p + v \cdot \frac{dp}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Again, when heat dQ is given to a pound of gas for an infinitely small change of state, the energy equation is

$$dQ = p \,.\, dv + C_v \,.\, dT$$

and

$$\frac{dQ}{dv} = p + C_v \cdot \frac{dT}{dv}$$

Substituting the value $C_v = \frac{R}{\gamma - 1}$, and the value of $\frac{dT}{dv}$ from (11),

gives

$$\frac{dQ}{dv} = p + \frac{R}{\gamma - 1} \times \frac{1}{R}\left(p + v \cdot \frac{dp}{dv}\right)$$

from which,

$$\frac{dQ}{dv} = \frac{1}{\gamma - 1}\left(\gamma p + v \cdot \frac{dp}{dv}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

the rate of reception of heat by the gas per unit change of volume.

If γ is known for the gas, and the values of p and v given on the indicator diagram, the rate of gain of heat by the gas may be calculated.

Since the expansion or compression curve follows the law,

$$p \, . \, v^n = k, \text{ a constant}$$

by differentiation, we obtain

$$v^n \, . \, \frac{dp}{dv} + np \, . \, v^{n-1} = 0\,;$$

hence $\frac{dp}{dv} = -\frac{np}{v}$, or $v \, . \, \frac{dp}{dv} = -np$,

and this value reduces equation (12) to the form

$$\frac{dQ}{dv} = \frac{1}{\gamma - 1}(\gamma p - np) = \frac{\gamma - n}{\gamma - 1} \, . \, p \qquad . \quad . \quad . \quad (13)$$

or, $dQ = \frac{\gamma - n}{\gamma - 1} \, . \, p \, . \, dv = \frac{\gamma - n}{\gamma - 1} \, . \, dW$

That is, for a very small addition of heat, the increase in heat content of gas is $\frac{\gamma - n}{\gamma - 1}$ × work done on the gas; and for adiabatic expansion or compression $n = \gamma$, and $\frac{dQ}{dv} = 0$, which is obvious.

Alternative. To find the total heat, Q, received or rejected by 1 lb. weight of gas during an operation in which the change of state is from p_1, v_1, T_1, to p_2, v_2, T_2, by expansion or compression, according to the law $p \, . \, v^n =$ a constant.

Let $Q =$ external work done + change of internal energy.

For a very small change,

$$dQ = dW + dE$$

$$= p \, . \, dv + C_v \, . \, dT$$

The total heat change is

$$Q = \int_{v_1}^{v_2} p \, . \, dv + C_v \int_{T_1}^{T_2} dT$$

and becomes

$$Q = \frac{p_1 v_1 - p_2 v_2}{n - 1} + C_v (T_2 - T_1)$$

For a perfect gas,

$$C_v = \frac{R}{\gamma - 1}; \text{ and } T_1 = \frac{p_1 v_1}{R}$$

so that $T_2 - T_1 = \frac{1}{R}(p_2 v_2 - p_1 v_1)$

Substituting, we have

$$Q = \frac{p_1 v_1 - p_2 v_2}{n - 1} + \frac{R}{\gamma - 1} \times \frac{1}{R}(p_2 v_2 - p_1 v_1)$$

$$= (p_1 v_1 - p_2 v_2)\left(\frac{1}{n - 1} - \frac{1}{\gamma - 1}\right)$$

hence $$Q = \frac{p_1v_1 - p_2v_2}{n-1} \times \frac{\gamma - n}{\gamma - 1} = \frac{R\,(T_1 - T_2)}{n-1} \times \frac{\gamma - n}{\gamma - 1}\,. \quad (14)$$

or $$Q = \frac{\gamma - n}{\gamma - 1} \times \text{work done during the change.}$$

For an infinitely small change, we obtain as above (13)

$$dQ = \frac{\gamma - n}{\gamma - 1} \times p\,.\,dv, \text{ or } \frac{dQ}{dv} = \frac{\gamma - n}{\gamma - 1}\,.\,p$$

The latter expression gives the rate of heat reception or rejection per unit change of volume at any point on a *pv* diagram. Thus, for part of the expansion curve *after* the steep slope of the explosion peak of a gas engine indicator diagram, by plotting log p and log v, the value of n is obtained (p. 11).

The rate of heat reception *per second*

$$\frac{dQ}{dt} = \frac{dQ}{dv} \times \frac{dv}{dt}$$

becomes $$\frac{dQ}{dt} = \frac{\gamma - n}{\gamma - 1}\,p \times \text{volume swept by the piston per second.}$$

Example 6. Suppose at part of the expansion curve of an indicator diagram from a gas engine the law is found to be $p\,v^{1\cdot 3}$ = a constant, and the ratio of the specific heats of the mixture is $\gamma = 1{\cdot}37$, then by (13),

$$\frac{dQ}{dv} = \frac{1{\cdot}37 - 1{\cdot}3}{1{\cdot}37 - 1}\,.\,p = \frac{7}{37}p$$

which is positive, and therefore the gas is receiving heat. Now, if the piston sweeps out 72 cu. ft. per min. when the absolute pressure on this expansion curve is 200 lb. per sq. in., what is the rate of heat reception at this instant ?

Here the change of volume per second is $\frac{72}{60}$, = 1·2 cu. ft. per sec. when $p = 200 \times 144$ lb. per sq. ft.

Hence $$\frac{dQ}{dt} = \frac{\gamma - n}{\gamma - 1}\,.\,p\,.\,\frac{dv}{dt} = \frac{1{\cdot}37 - 1{\cdot}3}{1{\cdot}37 - 1}\,(200 \times 144)\,1{\cdot}2 \text{ ft-lb. per second}$$

$$= \frac{7}{37} \times \frac{240 \times 144}{1{,}400}, \text{ or } 4{\cdot}67 \text{ C.H.U. per second}$$

The relative slope of the isothermal and adiabatic lines affords guidance in each case.

If the index, n, for the **expansion curve** is less than γ, the slope of the curve is not so steep as the adiabatic, and the gas is receiving heat, $\frac{dQ}{dv}$ is positive ; but if n is greater than γ, the expansion curve, has a steeper slope than the adiabatic curve, and the gas is losing heat, $\frac{dQ}{dv}$ is negative.

When $n = \gamma$, $\frac{dQ}{dv} = 0$, the curve is adiabatic, the gas neither gains nor loses heat, and it does the work of expansion at the expense of its stock of internal energy. When $n = 1$, the expansion is isothermal, and the heat received is equal to the equivalent of the work done by the gas, while its internal energy remains the same. (See Fig. 25.) Operation from C: during isothermal *compression*, the rate of heat *rejection* will be equivalent to the rate at which work is being done *upon* the gas.

Also when n for the **compression curve** is less than γ, heat is taken from the gas; and if n is greater than γ, the heat *rejection* will be negative, and the gas will be receiving heat during its compression.

For example, if the law of the *compression curve* is $pv^{1\cdot25} =$ a constant, and $\gamma = 1{\cdot}4$ for the gas, we have $\frac{dQ}{dv} = 0{\cdot}375p$. This is the rate at which the gas is giving out heat per unit decrease of volume.

Example 7. An internal combustion engine has the following dimensions: diameter of cylinder, 22 in.; stroke, 30 in.; compression ratio, 13·5. At the end of the suction stroke the pressure is 14 lb. per sq. in. and the temperature is 43° C. Compression follows the law $pv^{1\cdot37} = C$. Determine (*a*) the pressure and temperature at the end of compression; (*b*) the weight of the charge; (*c*) the work done; and (*d*) the heat rejected during compression. Assume the specific heat at constant pressure to be 0·238 and at constant volume to be 0·169. (*U.L., B.Sc.* (*Eng.*), 1921.)

Given $\gamma = \frac{0{\cdot}238}{0{\cdot}169} = 1{\cdot}408$

and $R = (C_p - C_v)\,J = (0{\cdot}238 - 0{\cdot}169)\;1{,}400 = 96{\cdot}6$ ft-lb.

The sectional area of cylinder $= \pi r^2 = 380{\cdot}1$ sq. in., and stroke volume $= \frac{380{\cdot}1}{144} \times \frac{30}{12} = 6{\cdot}6$ cu. ft.; also clearance volume from compression ratio $= \frac{6{\cdot}6}{12{\cdot}5} = 0{\cdot}528$ cu. ft., hence total volume of cylinder is 7·128 cu. ft. = volume of charge.

$$T_1 = 273 + 43^\circ\text{ C.} = 316^\circ\text{ C. (absolute)}$$

(*b*) Weight of the charge

$$w = \frac{p_1 v_1}{RT_1} = \frac{(144 \times 14)\,7{\cdot}128}{96{\cdot}6 \times 316} = 0{\cdot}4708\text{ lb.}$$

(*a*) During compression,

$$p_1 \,.\, v_1^{1\cdot37} = p_2 \,.\, v_2^{1\cdot37},\ \text{or}\ p_2 = p_1\left(\frac{v_1}{v_2}\right)^{1\cdot37} = 14\,(13{\cdot}5)^{1\cdot37}$$

Taking logarithms,

$$\log p_2 = \log 14 + 1{\cdot}37 \log 13{\cdot}5 = 2{\cdot}69468,$$

hence $p_2 = 495{\cdot}1$ lb. per sq. in.

also
$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{1\cdot37-1} = 316 \times 13{\cdot}5^{0\cdot37}$$
$$= 827{\cdot}77^\circ\text{ C. (absolute), or } 554{\cdot}8^\circ\text{ C.}$$

(*c*) Work done during compression

$$= \frac{p_2v_2 - p_1v_1}{n-1} = \frac{144(495{\cdot}1 \times 0{\cdot}528 - 14 \times 7{\cdot}128)}{1{\cdot}37 - 1}$$

$$= 62{,}900 \text{ ft-lb., or } \frac{62{,}900}{1{,}400} = 44{\cdot}93 \text{ C.H.U.}$$

(*d*) Since $\gamma > n$, heat is rejected, and

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{Heat equivalent of work done.}$$

Therefore, heat rejected during compression

$$= \frac{1{\cdot}408 - 1{\cdot}37}{1{\cdot}408 - 1} \times 44{\cdot}93 = 4{\cdot}185 \text{ C.H.U.} \qquad \textit{Answer.}$$

Example 8. Air at a temperature of 59° F. (15° C.) is compressed in a cylinder from 15 lb. pressure (absolute) to 120 lb. pressure (absolute) per square inch. The equation of the compression curve is $pv^{1\cdot25}$ = constant. Find the work done in compressing a pound of air, and the heat that escapes through the cylinder walls. (*U.L.*, *B.Sc.* (*Eng.*).)

Here $T_1 = 15° + 273 = 288°$ C. (absolute)

and $p_1 = 15 \times 144$ lb. per sq. ft.

Now by the characteristic equation for 1 lb. of air, $\frac{pv}{T} = 96$,

and $$v_1 = \frac{96 \times 288}{15 \times 144} = 12{\cdot}8 \text{ cu. ft.}$$

Work done in compression is

$$W = \frac{p_1v_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{120}{15}\right)^{\frac{1\cdot25-1}{1\cdot25}} = 8^{\frac{1}{5}} = 1{\cdot}51572$$

and $$W = \frac{144 \times 15 \times 12{\cdot}8}{1{\cdot}25 - 1}\left\{8^{\frac{1}{5}} - 1\right\}$$

hence $$W = 4 \times 144 \times 15 \times 12{\cdot}8\,(1{\cdot}51572 - 1) = 57{,}034 \text{ ft-lb.}$$

∴ Heat equivalent of work done on the air

$$= \frac{57{,}034}{1{,}400} = 40{\cdot}74 \text{ C.H.U.}$$

By (14) (p. 83), the heat rejected by the air during compression is

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{work done}$$

Substituting values,

$$Q = \frac{1{\cdot}4 - 1{\cdot}25}{1{\cdot}4 - 1} \times 40{\cdot}74 \text{ C.H.U.}$$

$\therefore$ Heat rejected

$$= \frac{0{\cdot}15}{0{\cdot}4} \times 40{\cdot}74 = \frac{3}{8} \times 40{\cdot}74 = 15{\cdot}28 \text{ C.H.U.}$$

or $\quad 15{\cdot}28 \times 1{\cdot}8 = 27{\cdot}5$ B.Th.U. *Answer.*

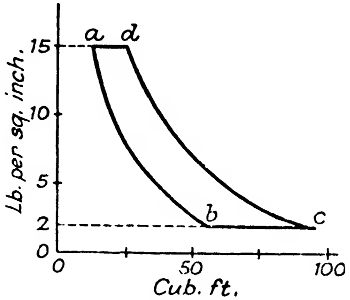

FIG. 27

Otherwise, as a check, the volume after compression may be found.

Since $p_1 \cdot v_1{}^{1\cdot25} = p_2 \cdot v_2{}^{1\cdot25}$, $\dfrac{v_1}{v_2} = \left(\dfrac{p_2}{p_1}\right)^{\frac{4}{5}} = 8^{\frac{4}{5}} = 5{\cdot}278$

and $\quad v_2 = \dfrac{12{\cdot}8}{5{\cdot}278} = 2{\cdot}425$

then, work done,

$$W = \frac{p_2v_2 - p_1v_1}{n-1} = \frac{144(120 \times 2{\cdot}425 - 15 \times 12{\cdot}8)}{1{\cdot}25 - 1}$$

$$= 4 \times 144(291 - 192) = 57{,}024 \text{ ft-lb.}$$

$$= 40{\cdot}73 \text{ C.H.U.}$$

and heat rejected

$$= \tfrac{3}{8} \times 40{\cdot}73 = 15{\cdot}274 \text{ C.H.U., or } 27{\cdot}49 \text{ B.Th.U.}$$

Example 9. Air at 60° F. and atmospheric pressure (15 lb. per sq. in.) is expanded adiabatically till the pressure is 2 lb. per sq. in. The air then receives heat at constant pressure until its temperature is again 60° F., and is afterwards compressed adiabatically to the atmospheric pressure, and exhausted at that pressure. Find the temperature at exhaust and the heat rejected in the exhaust per unit of work done. (*U.L.*, *B.Sc.* (*Eng.*).)

Given $T_a = 60° + 460° = 520°$ F. (absolute),

take $\quad \gamma = 1{\cdot}4$, $\therefore \dfrac{\gamma - 1}{\gamma} = \dfrac{1{\cdot}4 - 1}{1{\cdot}4} = \dfrac{2}{7}$, and $C_p = 0{\cdot}238$

In adiabatic expansion, *ab*, Fig. 27,

$$\frac{T_b}{T_a} = \left(\frac{p_b}{p_a}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore\ T_b = T_a\left(\frac{p_b}{p_a}\right)^{\frac{\gamma-1}{\gamma}} = 520\left(\frac{2}{15}\right)^{\frac{2}{7}} = 292{\cdot}4° \text{ F. (abs.)}$$

Heat received by 1 lb. air at constant pressure, during *bc*,

$$Q_2 = 0\cdot238(520^\circ - 292\cdot4^\circ) = 0\cdot238 \times 227\cdot6 = 54\cdot17 \text{ B.Th.U.}$$

Adiabatic compression, *cd*,

$$T_d = 520^\circ \left(\frac{15}{2}\right)^{\frac{1\cdot4-1}{1\cdot4}} = 924\cdot7^\circ \text{ F. (abs.)}$$
$$-460^\circ$$

$$\therefore \text{ temperature at exhaust } = 464\cdot7^\circ \text{ F.}$$ *Answer.*

Or, since the adiabatics *ab* and *cd* are between the same constant pressures, we have

$$\frac{T_d}{T_c} = \frac{T_a}{T_b}, \quad \therefore T_d = T_c \times \frac{T_a}{T_b} = 520 \times \frac{520}{292\cdot4} = 924\cdot7^\circ\text{F. (abs.)}$$

The *heat rejected* by 1 lb. of air in the exhaust, at constant pressure during *da*, is

$$Q_1 = 0\cdot238(924\cdot7^\circ - 520^\circ) = 96\cdot32 \text{ B.Th.U.}$$

The work done *on* 1 lb. of air during this complete cycle in the counter clockwise direction, is represented by the area *abcd*, and is

$$W = Q_1 - Q_2 = 96\cdot32 - 54\cdot17 = 42\cdot15 \text{ B.Th.U.,}$$

equivalent to $42\cdot15 \times 778 = 32{,}800$ ft-lb. (nearly).

∴ the heat rejected in the exhaust per unit of work done is

$$\frac{Q_1}{W} = \frac{96\cdot32 \text{ B.Th.U.}}{32{,}800 \text{ ft-lb.}} = 0\cdot00293 \text{ B.Th.U. per ft-lb.}$$

Answer.

Otherwise, we may find the volumes of 1 lb. of air at the state points *a*, *b*, *c*, *d*, and calculate the work done in each operation—

The volume of 1 lb. air at 60° F. and 15 lb. per sq. in. pressure is

$$v_a = \frac{RT_a}{p_a} = \frac{53\cdot2 \times 520}{15 \times 144} = 12\cdot81 \text{ cu. ft. (approx.)}$$

In the adiabatic expansion, *ab*,

$$\frac{v_b}{v_a} = \left(\frac{p_a}{p_b}\right)^{\frac{1}{\gamma}} = \left(\frac{15}{2}\right)^{\frac{1}{1\cdot4}}$$

$$\therefore \; v_b = 12\cdot81 \left(\frac{15}{2}\right)^{\frac{5}{7}} = 54\cdot01 \text{ cu. ft.}$$

At constant pressure on *bc*, we have $\dfrac{v_c}{T_c} = \dfrac{v_b}{T_b}$

$$\therefore \; v_c = 520 \times \frac{54\cdot01}{292\cdot4} = 96\cdot06 \text{ cu. ft.}$$

and the volume, after adiabatic compression, *cd*, is

$$v_d = 96\cdot06 \times \left(\frac{15}{2}\right)^{\frac{1}{1\cdot4}} = 96\cdot06 \times \left(\frac{2}{15}\right)^{\frac{5}{7}} = 22\cdot77 \text{ cu. ft.}$$

Hence the work done *by* 1 lb. of air during adiabatic expansion *ab*, represented by the area under *ab* to zero pressure line,

is $$\frac{144}{0{\cdot}4}(15 \times 12{\cdot}81 - 2 \times 54{\cdot}01) = 30{,}280 \text{ ft-lb.}$$

Also, during expansion *bc* at constant pressure, 2 lb. per sq. in., the area under *bc* represents the work done *by* the air

$$= 144 \times 2(96{\cdot}06 - 54{\cdot}01) = 12{,}110 \text{ ft-lb.}$$

Again, the work done *on* the air during adiabatic compression *cd*, is shown by the area under *cd* to zero pressure, and is equal to

$$\frac{p_c v_c}{\gamma - 1}\left\{\left(\frac{p_d}{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = \frac{144 \times 2 \times 96{\cdot}06}{1{\cdot}4 - 1}\left\{\left(\frac{15}{2}\right)^{\frac{2}{7}} - 1\right\}$$

$$= 360 \times 192(1{\cdot}7783 - 1) = 53{,}800 \text{ ft-lb.}$$

and during exhaust *da* at constant pressure 15 lb. per sq. in., the work done *on* the air is represented by the rectangular area under *da*, between ordinates drawn from *d* and *a*, and is equal to pressure × change of volume,

$$144 \times 15(22{\cdot}77 - 12{\cdot}81) = 2{,}160 \times 9{\cdot}96 = 21{,}513 \text{ ft-lb.}$$

∴ net work done *on* the 1 lb. of air, represented by the area *abcd*, is

$$53{,}800 + 21{,}513 - (30{,}280 + 12{,}110) = 32{,}923$$

and equivalent to $\frac{32{,}923}{778} = 42{\cdot}32$ B.Th.U. (nearly).

Hence the heat rejected in the exhaust, per unit of work done, is

$\frac{96{\cdot}32}{32{,}923} = 0{\cdot}002926$ B.Th.U. per ft-lb. *Answer.*

Total Heat of a Perfect Gas. For unit mass of a fluid, by definition, $H = E + A\,p\,.\,v$ (p. 52), where H and E are in heat units.

$$\therefore \quad H_1 - H_2 = E_1 - E_2 + A\,(p_1 v_1 - p_2 v_2)$$

Hence for a perfect gas the change in total heat

$$\begin{aligned} H_1 - H_2 &= C_v\,(T_1 - T_2) + AR\,(T_1 - T_2) \\ &= C_v\,(T_1 - T_2) + (C_p - C_v)\,(T_1 - T_2) \\ &= C_p\,(T_1 - T_2) \end{aligned}$$

(*a*) *For an isothermal change*—

$dT = 0$, i.e. $T_2 = T_1$ and hence H is constant

(*b*) *For an adiabatic change*—

The adiabatic heat drop $H_1 - H_2 = \int_2^1 v\,.\,dp$ (p. 55)

But $p.v^{\gamma} =$ constant K, i.e. $v = \dfrac{K^{\frac{1}{\gamma}}}{p^{\frac{1}{\gamma}}}$

$$\therefore \quad H_1 - H_2 = K^{\frac{1}{\gamma}} \int_{p_2}^{p_1} \frac{dp}{p^{\frac{1}{\gamma}}} = \frac{K^{\frac{1}{\gamma}} \left[p^{\gamma - 1} \right]_{p_2}^{p_1}}{\frac{\gamma - 1}{\gamma}} = \frac{K^{\frac{1}{\gamma}} \left[p_1^{\frac{\gamma-1}{\gamma}} - p_2^{\frac{\gamma-1}{\gamma}} \right]}{\frac{\gamma - 1}{\gamma}}$$

$$\text{But} \qquad K = p_1.v_1^{\gamma} \text{ or } p_2.v_2^{\gamma}$$

$$\therefore \quad H_1 - H_2 = \left[p_1^{\frac{1}{\gamma}} v_1 \times p_1^{\frac{\gamma-1}{\gamma}} - p_2^{\frac{1}{\gamma}} v_2 \times p_2^{\frac{\gamma-1}{\gamma}} \right] \frac{\gamma}{\gamma - 1}$$

$$\therefore \qquad = \frac{\gamma}{\gamma - 1} \left[p_1 v_1 - p_2 v_2 \right]$$

$$= C_p (T_1 - T_2)$$

It should be noted that this adiabatic heat drop is a measure of the work done by unit mass of the gas in entering a cylinder at the higher pressure, expanding adiabatically to the lower pressure, and being rejected from the cylinder at the lower pressure, or the work done on the gas in drawing in at the lower pressure, compressing, and rejecting at the higher pressure.

The work is represented by the area *abcd* in Fig. 21, p. 55.

(*c*) *For a Constant Pressure Change*—

$$H = E + A\,p\,.\,v$$

$$dH = dE + A\,p\,.\,dv$$

$$= dQ$$

$$\text{i.e. } H_1 - H_2 = Q \text{ (True for any fluid)}$$

$$= C_p (T_1 - T_2) \text{ (True for perfect gas)}$$

(*d*) *For a Constant Volume Change*—

$$H = E + A\,p\,.\,v$$

$$dH = dE + A\,v\,.\,dp$$

$$\text{i.e. } H_1 - H_2 = E_1 - E_2 + A\,v\,(p_1 - p_2) \text{ (True for any fluid)}$$

$$= C_v (T_1 - T_2) + A\,v\,(p_1 - p_2) \text{ (True for a perfect gas)}$$

Mixtures of Perfect Gases (Dalton's Law). Dalton formulated the principle that "any gas acts as a vacuum to any other gas mixed with it." This may be more generally stated as follows. In a mixture of perfect gases which do not react chemically with one another, the total absolute pressure exerted by the mixture is the sum of the partial absolute pressures which each gas would exert if it separately occupied the same volume, and was at the same temperature as the mixture. In other words in such a mixture, each constituent gas obeys its own characteristic equation, as if the other constituent gases were absent.

In practice actual gases follow this law as closely as, if not more so than is the case with the other relations of a perfect gas so far considered.

Partial Pressures. Let the volume of w lb. of mixture at absolute pressure p lb. per square foot, and absolute temperature T, be v cubic feet.

Then $p\,v = wRT$

If suffixes 1 and 2 refer to the constituent gases

then $p_1v = w_1R_1T$ and $p_2v = w_2R_2T$

$$\text{and} \quad p = p_1 + p_2 = (w_1R_1 + w_2R_2)\frac{T}{v} = (w_1R_1 + w_2R_2)\frac{p}{wR}$$

i.e. $wR = w_1R_1 + w_2R_2$

$$\text{also} \quad \frac{p_1}{p} = \frac{w_1R_1}{wR} \text{ and } \frac{p_2}{p} = \frac{w_2R_2}{wR} \text{ and } \frac{p_1}{p_2} = \frac{w_1R_1}{w_2R_2}$$

Now the universal gas constant $G = mR$ where m is the molecular weight of the gas.

$$\text{Hence } \frac{p_1}{p} = \frac{w_1R_1}{wR} = \frac{w_1m}{wm_1} \text{ and } \frac{p_2}{p} = \frac{w_2R_2}{wR} = \frac{w_2m}{wm_2}$$

$$\text{also } \frac{p_1}{p_2} = \frac{w_1R_1}{w_2R_2} = \frac{w_1m_2}{w_2m_1}$$

But $\frac{w_1}{m_1}$ and $\frac{w_2}{m_2}$ are the number of pound mols. in each gas respectively.

$$\therefore \frac{p_1}{p_2} = \frac{\text{number of pound mols. of gas 1}}{\text{number of pound mols. of gas 2}}$$

Volumetric Analysis. Let v_1 and v_2 be the volumes of the constituent gases at the mixture pressure p.

$$\text{Then } pv_1 = p_1v \text{ and } pv_2 = p_2v$$

$$\text{i.e.} \quad p_1 = p\frac{v_1}{v} \text{ and } p_2 = p\frac{v_2}{v}$$

Thus the partial pressure of a constituent gas is measured by the product of the mixture pressure, and the fractional volumetric content of that gas.

Combining the two equations given above—

$$\frac{p_1}{p_2} = \frac{v_1}{v_2}$$

which shows that the partial pressures of the constituent gases are in the same ratio as their respective volumetric contents.

Specific Volumes of Mixtures. Let v_s represent specific volume.

Then from Dalton's Law

$$v = wv_s = w_1 v_{s1} = w_2 v_{s2} \text{ and } w = w_1 + w_2$$

thus $$\frac{v}{v_s} = \frac{v}{v_{s1}} + \frac{v}{v_{s2}}$$

i.e. $$\frac{1}{v_s} = \frac{1}{v_{s1}} + \frac{1}{v_{s2}}$$

Molecular Weights of Mixtures. Although a mechanical mixture of gases has strictly speaking no molecular weight, it is sometimes convenient to use its equivalent molecular weight.

Let v_1 and v_2 be the volumes of the constituent gases in a pound mol. of mixture having a volume v at normal temperature and pressure (N.T.P.).

Then as a pound mol. of any gas occupies approximately 359 cubic feet at N.T.P.—

$$\frac{w_1}{v_1} = \frac{m_1}{359} \text{ and } \frac{w_2}{v_2} = \frac{m_2}{359}$$

Hence the weight of the mixture, which is the weight of one pound mol., and therefore the molecular weight of the mixture

is $$m = w_1 + w_2 = \frac{m_1 v_1}{359} + \frac{m_2 v_2}{359}$$

But $$\frac{v_1}{359} = \frac{\text{Percentage volumetric content of gas 1}}{100}$$

$$\frac{v_2}{359} = \frac{\text{Percentage volumetric content of gas 2}}{100}$$

Hence $$m = \frac{(\text{Percentage Volume of gas 1})\, m_1 + (\text{Percentage Volume of gas 2})\, m_2}{100}$$

A formula in terms of weights can also be obtained thus. Let w lb. of mixture contain w_1 and w_2 lb. of constituent gases.

Then $$m = \frac{w}{v} 359 = \frac{w_1 + w_2}{v_1 + v_2} 359$$

$$= \frac{w_1 + w_2}{\frac{w_1}{m_1} + \frac{w_2}{m_2}}$$

Example 10. Air at a pressure of 15 lb. per sq. in. absolute is contained in a closed vessel. If air consists of 23·2 per cent oxygen and 76·8 per cent nitrogen by weight find (*a*) the volumetric analysis of the air, (*b*) the equivalent molecular weight of air, (*c*) the partial pressures of oxygen and nitrogen in the air.

(*a*) From the equation $\frac{w}{v} = \frac{m}{359}$

$$\frac{v_O}{v_N} = \frac{w_O}{w_N} \times \frac{m_N}{m_O} = \frac{23{\cdot}2}{76{\cdot}8} \times \frac{28}{32} = \frac{649{\cdot}6}{2{,}457{\cdot}6}$$

$$\therefore \text{ Volumetric content of oxygen} = \frac{v_O}{v_O + v_N} = \frac{649{\cdot}6}{649{\cdot}6 + 2{,}457{\cdot}6}$$
$$= 21 \text{ per cent}$$

Volumetric content of nitrogen $= 100 - 21 = 79$ per cent

(*b*) From the equation

$$m = \frac{m_1 \text{ (per cent } v \text{ of gas 1)} + m_2 \text{ (per cent } v \text{ of gas 2)}}{100}$$

m for air $= 32 \times 0{\cdot}21 + 28 \times 0{\cdot}79 = 28{\cdot}84$

or from the equation $m = \dfrac{w_1 + w_2}{\dfrac{w_1}{m_1} + \dfrac{w_2}{m_2}}$ taking 1 pound air

$$m = \frac{1}{\dfrac{0{\cdot}232}{32} + \dfrac{0{\cdot}768}{28}} = 28{\cdot}86$$

It should be noted that if other gases in air such as carbon dioxide and argon are taken into account, a more exact value for the equivalent molecular weight of air is 28·95.

(*c*) From the equation $p_1 = p\dfrac{v_1}{v}$

$$p_O = p\frac{v_O}{v} = 15 \times 0{\cdot}21 = 3{\cdot}15 \text{ lb. per sq. in.}$$

$$p_N = p\frac{v_N}{v} = 15 \times 0{\cdot}79 = 11{\cdot}85 \text{ lb. per sq. in.}$$

Total Heat of Mixtures. For one pound of gas $H = E + A\,p\,v_s$

and $$A\,p = \frac{H - E}{v_s} = \frac{wH - wE}{w\,v_s}$$

Hence for a gaseous mixture

$$A\,p = A\,(p_1 + p_2)$$

i.e. $$\frac{wH - wE}{wv_s} = \frac{w_1H_1 - w_1E_1}{w_1v_{s1}} + \frac{w_2H_2 - w_2E_2}{w_2v_{s2}}$$

But each constituent gas occupies the same volume v where

$$v = wv_s = w_1v_{s1} = w_2v_{s2}$$

Hence $$wH - wE = (w_1H_1 - w_1E_1) + (w_2H_2 - w_2E_2)$$

Now from Dalton's Law it follows that the internal energy of a gaseous mixture is equal to the sum of the internal energies of the constituent gases, when each constituent gas alone occupies the volume of the mixture, at the temperature of the mixture.

Hence $$wE = w_1E_1 + w_2E_2$$

and $$wH = w_1H_1 + w_2H_2$$

Thus the total heat of a gaseous mixture is equal to the sum of the total heats of the constituent gases.

Specific Heat of Mixtures

(*a*) Let the specific heat be measured in heat or work units per pound of gas. Then as the internal energy of a mixture is equal to the sum of the internal energies of the constituents,

$$w\,C_v dT = w_1 C_{v1} dT + w_2 C_{v2} dT$$

i.e.

$$C_v = \frac{w_1}{w} C_{v1} + \frac{w_2}{w} C_{v2}$$

(*b*) Let the specific heat be measured in heat or work units per cubic foot of gas. Then—

$$vC_v dT = v_1 C_{v1} dT + v_2 C_{v2} dT$$

i.e.

$$C_v = \frac{v_1}{v} C_{v1} + \frac{v_2}{v} C_{v2}$$

Example 11. The composition of a gas by volume is H_2 45·4, CH_4 20·7, CO 16·1, CO_2 4·3, N_2 13·5 per cent. Find the density of this gas in lb. per cu. ft. at a pressure of 14·7 lb. per sq. in. abs. and a temperature of 0° C. Also calculate the mean volumetric heat, taking the volumetric heat of the diatomic gases as 5, of CO_2 as 7·3, and of CH_4 as 8·7. What is the value of the adiabatic index? $R_{mol} = 1{\cdot}985$.

A comparison of the values of the volumetric heats given in this question with those in Table II, p. 68, indicate that the units are heat units per pound mol.

A perusal of the paragraphs on molecular weights and specific heats of mixtures, make the table given below self explanatory.

1	2	3	4	5
Composition of gas cu. ft.	Molecular Wt. of constituent lb.	Proportion of Molecular Wt.	Specific Heat of constituent C.H.U. per mol per degree C.	Proportion of Specific Heat
v	m	$m.v.$	C_v	vC_v
H_2 0·454	2	0·908	5·0	2·270
CH_4 0·207	16	3·312	8·7	1·800
CO 0·161	28	4·508	5·0	0·805
CO_2 0·043	44	1·892	7·3	0·314
N_2 0·135	28	3·780	5·0	0·675
1·000		14·400		5·864

From the above table (column 3)

(*a*) Molecular weight of gas = 14·4.

Now $p\,v = GT$ where v is the volume of a pound mol. at N.T.P. and G is in work units.

$$\therefore \qquad v = \frac{1{\cdot}985 \times 1400 \times 273}{14{\cdot}7 \times 144} = 358 \text{ cu. ft.}$$

$$\therefore \quad \text{Density of gas} = \frac{\text{molecular weight}}{\text{molecular volume}} = \frac{14{\cdot}4}{358} = 0{\cdot}0402 \text{ lb. per S.C.F.}$$

From the above table (column 5)

(*b*) Specific heat of gas $= 5{\cdot}864$ C.H.U. per mol. per degree C.

$$= \frac{5{\cdot}864 \times 1{,}400}{358} \text{ ft-lb. per S.C.F. per degree C.}$$

i.e. $C_v = 22{\cdot}9$ ft-lb. per S.C.F. per degree C.

$$(c)\ C_p = C_v + G = 5{\cdot}864 + 1{\cdot}985$$
$$= 7{\cdot}849$$

$$\text{Hence } \gamma = \frac{C_p}{C_v} = \frac{7{\cdot}849}{5{\cdot}864} = 1{\cdot}34$$

Properties of a Vapour. So far we have only considered a fluid which under normal conditions remains in a gaseous condition. Such substances are oxygen, nitrogen, etc. We must now consider fluids which under normal working conditions may be either in a liquid or a vapour state, such as H_2O, ammonia, carbon dioxide, etc. The behaviour of such fluids is typically represented by steam and water. With such fluids the simple laws of perfect gases do not even approximately apply.

The various states of such a fluid will now be considered.

(*a*) *Saturated Vapour.* Under normal equilibrium conditions a vapour such as steam if in contact with its liquid (water) is said to be in a saturated condition.

(*b*) *Dry Saturated Vapour.* If a mixture of saturated vapour and liquid is heated at constant pressure, the liquid will gradually change into vapour. Just at the point where all the liquid has been converted into vapour, the stuff is said to be dry saturated. The amount of heat required to convert one pound of liquid into dry saturated vapour at the same temperature and pressure, is called the *Latent Heat.*

For any particular pressure, there is only one temperature at which the liquid will change into vapour, this temperature, called the *saturation temperature*, remaining constant as long as the pressure remains constant, and as long as there is any liquid present. Values of the saturation temperature, and latent heat for various pressures are given in steam tables (see Chap. IX)*.

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* See footnote p. 377.

(*c*) *Superheated Vapour.* As soon as all the liquid has been converted into vapour, the temperature will rise with any further addition of heat at constant pressure, the vapour now being superheated. Thus superheated vapour may be defined as vapour at a temperature higher than the saturation temperature corresponding to its pressure. The behaviour of the vapour will now approximate more and more to that of a perfect gas, the further the temperature is removed from the saturation temperature.

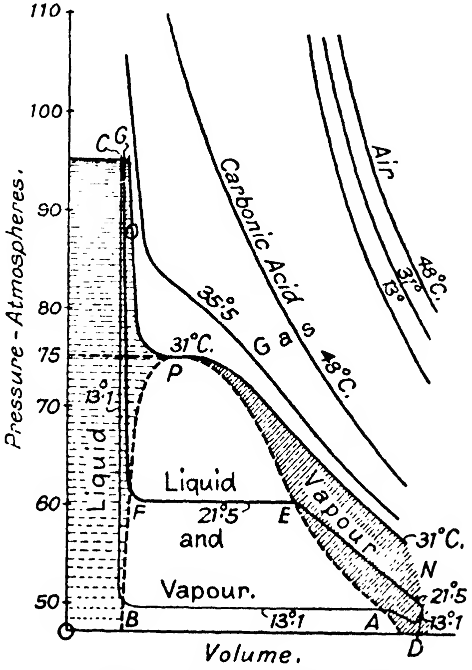

FIG. 28. ISOTHERMALS OF CO_2

(*d*) *Wet Vapour.* If dry saturated vapour has heat abstracted from it while the pressure remains constant, some of the vapour will condense at constant temperature. This mixture is termed wet vapour, and is described by its dryness fraction. The dryness fraction is the fraction of the whole mixture which is vapour.

The various states described above are well illustrated by the isothermal lines in Fig. 28 plotted by Professor Thomas Andrews from the results of his experiments* on the behaviour of carbon dioxide (CO_2), known as carbonic acid. The CO_2 "gas" was compressed over mercury in a capillary glass tube, carefully calibrated. Another similar tube, placed beside it and connected with it,

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* *Phil. Trans.* 1869, *part II.*, p. 575; *ibid.* 1876, *part II.*

contained dry air, which acted as a pressure gauge. Both tubes were kept at any desired temperature in a bath, and the pressure altered by a screw plunger.

Starting with dry CO_2 at a point vertically below *D*, keeping the temperature constant at 13·1° C., the volume is gradually reduced by compression to 49 atmospheres at the point *A*, when the vapour is saturated, and with a slight increase of pressure condensation begins. The pressure remains practically constant as the volume is reduced and the vapour condensed from *A* to *B*. Only a slight increase of pressure is required to convert all the vapour into liquid at *B*. By increasing the pressure up to 95 atmospheres the volume of the liquid is only slightly reduced from *B* to *C*.

At the constant temperature 21·5° C. a similar isothermal, *DEFG*, is obtained. The pressure is gradually increased from *D* to 61 atmospheres at *E*, the saturation pressure of the vapour, when a sharp, clearly marked change is observed to the liquid state, and condensation continues at constant pressure as the volume is reduced from *E* to *F* until all the mixture becomes liquid.

Starting from *N*, at 31·1° C., the compression curve shows that the volume diminishes regularly until a pressure of about 73 atmospheres is reached. The volume is then *reduced very rapidly* to about one-half, as seen by the points of inflection in the isothermal, while the pressure is increased steadily to 75 atmospheres, and there is no evidence of condensation or separation of liquid in the tube, the space being then occupied by a homogeneous fluid. "No apparent liquefaction of CO_2, or separation, into two distinct forms of matter could be effected, even when a pressure of 300 or 400 atmospheres was applied." The isothermal for 35·5° C. is similar, but the inflection by change in volume is less abrupt.

(*e*) *Critical Point.* It is seen from Fig. 28 that if carbon dioxide is below a temperature of 31° C., then conversion from superheated vapour to liquid via a wet vapour stage can be effected by an increase in pressure if the temperature is maintained constant, while if the temperature is above 31° C., then there is no discontinuity as between the liquid and superheated vapour phases. If the fluid is therefore above the temperature 31° C., then increase of pressure alone will not liquify the superheated vapour.

This temperature is termed the critical temperature for CO_2, the corresponding saturation pressure being the critical pressure.

For comparison, the approximate critical values for CO_2, H_2O and air are given in Table III.

The relative positions for the isothermals for air at 13°, 31° and 48° C. are shown in Fig. 28 and indicate why there are more simple laws for air than for CO_2, etc.

(*f*) *Steam Tables and Properties of Refrigerants.* These properties are available in tables and on charts. They are discussed in Chapters IX and XV.

TABLE III

Substance	Critical pressure lb. per sq. in. abs.	Critical Temperature ° F.
CO_2	1,070	87·8
H_2O	3,200	705
Air	575	−221

Van der Waals devised a general equation giving the relation of p, v and T for any fluid through any range of temperature and density from the state of liquid to that of a vapour and perfect gas:

$$p+\left(\frac{a}{v^2}\right)(v-b)=RT, \text{ or } p=\frac{RT}{v-b}-\frac{a}{v^2},$$

where a and b are constants, determined by experiment for any fluid.

Here, a makes allowance for the pressure *in* the gas due to the mutual attraction between the molecules, assumed proportional to the square of the density, but independent of the temperature. This internal molecular attraction assists the external pressure of the enclosure in preventing the fluid from expanding. The effect is to add a term to p of the form a/v^2 where a is assumed constant, although it is probably less the higher the temperature.

The other correction is for the size of the molecules, which reduces the free space and volume of the gas by an amount b per unit volume. This co-volume b represents the least volume the gas can occupy when the pressure is exceedingly high; and by the equation, at absolute zero, when $T = 0$, we have $v = b$.

The equation of Van der Waals, plotted on squared paper, agrees approximately with the general form of the isothermals (Fig. 28) for carbonic acid in the liquid and vapour states obtained by Dr. Andrews, and shows the deviation of vapour from the ideal gas when suitable constants are chosen for each substance, but fails to represent with sufficient accuracy the exact quantitative results of these experiments, or the behaviour of a fluid in all states. There is discrepancy between the actually observed values of the saturation pressures and volumes and those calculated from this equation.

Callendar's Characteristic Equation gives the behaviour of any vapour, saturated or superheated, at low and moderate pressures and temperatures, and may be written in the form

$$v=\frac{RT}{P}-c+b, \text{ or } v=\frac{RT}{P}-(c-b) \quad . \quad . \quad (15)$$

where v = volume in cubic feet per pound; P = pressure in pounds per square foot; T = absolute temperature in degrees Centigrade; c is a function of the temperature only; R and b are constants. This equation does not hold even approximately over an unlimited range, but it is of such a form that *simple thermodynamic* relations are readily deduced from it, between the various properties of steam, and give results which agree very closely with the values observed by experiment. Callendar has applied it to calculate his tables of the properties of dry steam for pressures up to 500 lb. per sq. in., such as are commonly required in modern steam engine practice. This is well below the *critical point* for water, which is about 374° C., and critical pressure 200 atmospheres.*

For 1 lb. of the *ideal* perfect gas $PV = RT$, and the volume $V = \frac{RT}{P}$. The volume of any actual gas, having the same values of P, T, and R, is less than this ideal volume. The diminution of volume from that of the ideal gas is regarded as due to co-aggregation or pairing of the molecules, which reduces the volume. Callendar deduced the "co-aggregation volume," c, from the cooling effect in throttling, and showed that, for a vapour at low and moderate pressure, it may be regarded as a function of the temperature of the form $c = c_1\left(\frac{T_1}{T}\right)^n$, where c_1 is the value of c at 100° C., or 373·1° C. absolute.

All the experiments on a fluid, in the state of gas or vapour, show that the defect $(c - b)$ of volume from the ideal gas volume $\frac{RT}{P}$ is a function of the temperature only at low pressures, and the increment of $\log_\varepsilon p = (c - b) / \frac{RT}{P}$.

The numerical value of n depends on the nature of the fluid, and for steam $n = \frac{10}{3}$. The value of c agrees very closely with the results of experiments on the cooling effect in the porous plug by Joule and Thomson, and is mainly based on the more recent throttling experiments of Jenkin and Pye, Grindley, Peake, as well as those by Nicolson and Callendar. The cooling effect is the ratio of the drop of temperature for every pressure drop of 1 lb. per sq. in. $= \left(\frac{dt}{dp}\right)$ in a throttling process at constant total heat H.

The co-volume, b, is taken as the minimum volume of the fluid, that is, when reduced to the liquid state. For water, the co-volume of 1 lb. at 0° C. = 0·01602 cu. ft.

Taking $n = \frac{10}{3}$ for steam, the constant $R = \frac{PV}{1{,}400T} = 0{\cdot}11012$ mean calorie per lb., or $R = \frac{PV}{T}$ ft-lb. per lb. of vapour,

$$RJ = 0{\cdot}11012 \times 1{,}400 = 154{\cdot}168 \text{ ft-lb.},$$

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* Revised Edition by G. S. Callendar (Edward Arnold & Co.) now gives values up to 3,000 lb. per sq. in.

and $c_1 = 0{\cdot}4213$ cu. ft. per lb. at 100° C.; then the co-aggregation volume

$$c = 0{\cdot}4213\left(\frac{373{\cdot}1}{T}\right)^{\frac{10}{3}} = \frac{157{\cdot}52 \times 10^6}{T^{\frac{10}{3}}}$$

Substituting these values, the Callendar equation for dry saturated and superheated steam is

$$v = \frac{154{\cdot}168T}{P} - \frac{157{\cdot}52 \times 10^6}{T^{\frac{10}{3}}} + 0{\cdot}01602 \quad . \quad (16)$$

Example 12. Calculate the volume of 1 lb. of dry saturated steam at 170° C. The saturation pressure is 115 lb. per sq. in.

We have $T = 170° + 273{\cdot}1° = 443{\cdot}1°$ C. absolute

and $P = 115 \times 144$ lb. per sq. ft.

$$v = \frac{154{\cdot}168 \times 443{\cdot}1}{115 \times 144} - \frac{157{\cdot}52 \times 10^6}{443{\cdot}1^{\frac{10}{3}}} + 0{\cdot}016$$

$$= 4{\cdot}1251 - 0{\cdot}2375 + 0{\cdot}016 = 3{\cdot}9036 \text{ cu. ft.}$$

which agrees closely with the value given in the tables.

Suppose this dry saturated steam is superheated 100° C. at constant pressure 115 lb. per sq. in. The temperature is now

$$T = 270° + 273{\cdot}1° = 543{\cdot}1° \text{ C. absolute}$$

and

$$v = \frac{154{\cdot}168 \times 543{\cdot}1}{115 \times 144} - \frac{157{\cdot}52 \times 10^6}{543{\cdot}1^{\frac{10}{3}}} + 0{\cdot}016$$

$$= 5{\cdot}0561 - 0{\cdot}1205 + 0{\cdot}016 = 4{\cdot}95 \text{ cu. ft.}$$

Again, in Example 12, starting with 1 lb. of dry saturated steam, suppose the temperature is kept constant 170° C., whilst the pressure is reduced from 115 lb. to 20 lb. per sq. in. The term c remains the same, the volume per pound is found by the characteristic equation (16), or from the total heat, H, 673·35 C.H.U. by (17), to be 23·497 cu. ft., and the steam is superheated.

Example 13. Find the volume of 1 lb. of steam at 230° C. and 120 lb. per sq. in.

$T = 503{\cdot}1°$ C. absolute; and the total heat $H = 695{\cdot}95$ C.H.U per lb.

Substituting values in equation (16), gives

$$v = \frac{154{\cdot}168 \times 503{\cdot}1}{120 \times 144} - \frac{157{\cdot}52 \times 10^6}{503{\cdot}1^{\frac{10}{3}}} + 0{\cdot}016$$

$$= 4{\cdot}4885 - 0{\cdot}1555 + 0{\cdot}016 = 4{\cdot}349 \text{ cu. ft.}$$

In this case the steam is superheated, since the saturation temperature for pressure 120 lb. per sq. in. is 171·75° C.

Professor Dalby derived from the characteristic equation and that of total heat (p. 384) a simple expression for the volume in cubic feet per pound of dry saturated or superheated steam—

$$v = \frac{2{\cdot}2436(H - 464)}{\quad} + 0{\cdot}0123 \quad . \quad . \quad . \quad (17)$$

where p = lb. per sq. in., and the corresponding value of H is given in the steam tables.

Thus, for the case of saturated steam at 115 lb. per sq. in. (as in Example 12), from tables the total heat $H = 663{\cdot}44$, and

$$v = \frac{2{\cdot}2436(663{\cdot}44 - 464)}{115} + 0{\cdot}0123$$

$$= 3{\cdot}8910 + 0{\cdot}0123 = 3{\cdot}9033 \text{ cu. ft. per lb.}$$

The constant 0·0123 is negligible at low pressures, and up to 400 lb. per sq. in. is less than 1 per cent.

The volume in Example 13, works out by formula (17), to 4·349 cu. ft.

Supersaturation of Steam. C. T. R. Wilson showed by experiment* that water-vapour, mixed with air free from dust and saturated at 20° C., can be suddenly expanded until its pressure is *eight times the normal saturation pressure* corresponding to the temperature after expansion, and its temperature is much lower than that of saturation at the actual pressure reached, without condensation taking place. With expansion beyond this *limit*, or Wilson curve (p. 464), extremely rapid condensation begins, a dense cloud of thick fog being formed of very fine particles as centres or nuclei of condensation.

Callendar estimates the radius of each nucleus about 5×10^{-8} cm., of the same order of magnitude as the co-aggregated or paired molecules in water-vapour at 20° C.

Professors Callendar and Nicolson first pointed out the loss of heat-drop due to supersaturation in the rapid expansion of steam observed in their experiments.† The adiabatic law of expansion: $P\,(v - b)^{1{\cdot}3}$ = constant, still holds while the steam remains dry and supersaturated. The time taken to reach the throat of the nozzle is only about 0·0001 second, and it is probable that the steam flows through the nozzle in a supersaturated condition, while the condensation at the throat must be very small, seeing it is retarded by the surface tension of the small drops, and it takes place very rapidly after passing the throat.

From data obtained in trials of the performance of marine steam turbines, Callendar defined the *supersaturation limit at 3 per cent equivalent wetness of saturated steam*, from which the total heat, H, is more easily deduced than from the Wilson limit, and gives results in agreement with observations on both high and low pressure steam turbines.

Mixtures of Gases and Vapours. Dalton's Law holds approximately for mixtures of gases and vapours which do not combine chemically. In such a mixture the total pressure of the mixture is

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* *Phil. Trans. R.S.*, 1897, p. 301.

† "On the Law of Condensation of Steam," *Proc. Inst. C. E.*, 1898, Vol. CXXXI, p. 147; also *Proc. Roy. Soc.*, 1900, Vol. LXVII, p. 266; and "Flow of Steam Through a Nozzle," *Inst. Mech. E.*, 1915, p. 53.

equal to the sum of the partial pressures of the constituents, and each constituent acts under its partial pressure as if the other constituents were not present.

(*a*) *Wet Air.* A special case of such a mixture is that of moisture in air, and this will be used to illustrate the problems involved.

If water is present in a vessel containing air, it will slowly evaporate, and become mixed as vapour with the air. In the presence of excess liquid, this evaporation will proceed until a state of equilibrium is reached, when the air is said to be saturated with water vapour, the amount of vapour required to saturate the given quantity of air being dependent on the temperature. In practice complete saturation in this way would take a very long time. Thus on a normal day, the atmosphere is only partially saturated.

As an example if the air is saturated at say 20° C. we see from the steam tables that the partial pressure of the vapour would be 0·34 lb. per square inch absolute (see Steam Table II) and its specific volume 922 cubic feet per pound. Thus the weight of water vapour per cubic foot of vessel volume would be $\frac{1}{922} = 0{\cdot}00108$ pound, and the total pressure in the vessel would be 0·34 plus the partial pressure of the air.

If the water vapour present were less than that necessary to saturate the air present, then the partial pressure exerted by the water vapour would be less than the saturation pressure corresponding to its temperature, that is the water vapour would be superheated.

If such a mixture were cooled, a point would be reached when the air would become saturated, and liquid would commence to form. This temperature is called the *Dew Point.* The partial pressure of the water vapour would be the saturation pressure corresponding to this temperature.

(*b*) *Saturated Air.* It has been stated above that the presence of water as distinct from vapour does not necessarily ensure that the superincumbent air is saturated with water vapour, unless condensation has been effected by cooling. In many cases however it is allowable to assume that if liquid is present, then the air is saturated.

In such a case let—

t = temperature of air-vapour mixture
p_s = saturation vapour pressure corresponding to $t°$
p_a = partial air pressure at $t°$
v_{ss} = specific volume of vapour at $t°$
v_{sa} = specific volume of air at $t°$
p_v = pressure in vessel containing mixture,

Then—

$$p_v = p_s + p_a \text{ and } p_a v_{sa} = R\,(t + 460) \text{ or } R\,(t + 273)$$

Also if—

V = volume of vessel (excluding any liquid)
W_s = weight of vapour
W_a = weight of air
W_m = weight of mixture

then

$$V = W_s v_{ss} = W_a v_{sa}$$

and

$$W_m = W_s + W_a = W_s\left(1 + \frac{v_{ss}}{v_{sa}}\right) - W_a\left(1 + \frac{v_{sa}}{v_{ss}}\right)$$

Example 14. Before the fire is lighted in a boiler, the temperature is 20° C., the pressure in the boiler being atmospheric. What will be the boiler pressure when the temperature is raised to 160° C.? If dry saturated steam is drawn off at this pressure, what weight of air will first come over per pound of steam?

Assume air above water is saturated.

Vapour pressure at 20° C.

$= p_{s1} = 0{\cdot}339$ lb. per sq. in. (from Tables)

Air partial pressure

$= p_{a1} = 14{\cdot}7 - 0{\cdot}339 = 14{\cdot}36$ lb. per sq. in.

Let V = volume of steam and air space.

Then for air

$pV = wRT$ where V and R are constants.

Hence air pressure at 160° C.

$$p_{a2} = p_{a1} \times \frac{160 + 273}{20 + 273} = 21{\cdot}22 \text{ lb. per sq. in.}$$

Steam pressure at 160° C.

$= p_{s2} = 89{\cdot}8$ lb. per sq. in. (from Steam Table II)

∴ Boiler pressure at 160° C.

$= p_{a2} + p_{s2} = 21{\cdot}22 + 89{\cdot}8$
$= 111{\cdot}02$ lb. per sq. in.

Specific volume of air at N.T.P.

$= 12{\cdot}39$ cu. ft. per lb.

∴ Specific volume of air at 160° C. and 21·22 lb. per sq. in.

$$v_{sa} = 12{\cdot}39 \times \frac{14{\cdot}7}{21{\cdot}22} \times \frac{160 + 273}{273} = 13{\cdot}61 \text{ cu. ft. per lb.}$$

Specific volume of steam at 160° C.

$v_{ss} = 4{\cdot}92$ cu. ft. per lb. (from Steam Table II)

$$\therefore \quad \frac{W_a}{W_s} = \frac{v_{ss}}{v_{sa}} = \frac{4{\cdot}92}{13{\cdot}61} = 0{\cdot}36$$

(c) *Humidity and Dew Point.* The dew point is the temperature at which wet air becomes saturated with water vapour. If above this temperature at the same mixture pressure, the vapour in the air is superheated, its partial pressure remaining equal to the saturation vapour pressure corresponding to the dew point. This can be shown to be the case as follows.

Using the same symbols as on pp. 101 and 102, andl etting the suffix *m* refer to the mixture, then as the vapour is superheated we may treat it as a gas.

Hence $$p_s = \frac{R_s T}{v_{ss}} \text{ and } p_m = \frac{R_m T}{v_{sm}}$$

Thus $$\frac{p_s}{p_m} = \frac{R_s}{R_m} \times \frac{v_{sm}}{v_{ss}} = \frac{R_s}{R_m}\frac{W_s}{W_m}$$

that is, the partial pressure of the water vapour remains unchanged as long as the pressure and composition of the mixture is unaltered.

Absolute humidity is measured by the number of pounds of water vapour associated with one pound of dry air.

$$\text{i.e. Absolute humidity} = \frac{\text{Weight of vapour in a given volume}}{\text{Weight of dry air in the same volume}}$$

$$= \frac{W_s}{W_a} = \frac{v_{sa}}{v_{ss}}$$

To find the relative weights of water vapour and air we can use the universal characteristic gas equation for air and the superheated vapour, i.e. $p\, v_{mol} = 2{,}779\ T$ where v_{mol} is the molar volume. (See p. 61).

$$\text{Specific volume} = \frac{\text{Molar volume}}{\text{Molecular weight}} = \frac{2{,}779\ T}{\text{M.W.} \times \text{partial pressure}}$$

Thus for air—

$$\text{Specific volume } v_{sa} = \frac{2{,}779\ T}{28{\cdot}95 \times p_a}$$

for vapour—

$$\text{Specific volume } v_{ss} = \frac{2{,}779\ T}{18 \times p_s}$$

and weight of water vapour per lb. of air

$$= \frac{v_{sa}}{v_{ss}} = \frac{18}{28{\cdot}95} \times \frac{p_s}{p_a}$$

$$\text{Relative humidity} = \frac{\text{Weight of water vapour actually associated with a given volume of dry air}}{\text{Weight of water vapour necessary to saturate the given volume of dry air}}$$

$$= \frac{\text{Specific volume of vapour if air were saturated}}{\text{Actual specific volume of vapour}}$$

With the small pressures involved it is sufficiently accurate to assume that $pv =$ constant, and thus

Relative humidity

$$= \frac{\text{Actual partial pressure of water vapour}}{\text{Saturation pressure corresponding to temperature}}$$

Example 15. A volume of 10,000 ft.^{3} of moist air is at 14·7 lb. per sq. in. abs. and 45° F., and its relative humidity is 0·705. It is required to heat the air at constant pressure to 65° F.

Find (*a*) the heat to be supplied if C_p for air $= 0{\cdot}24$ and C_p for steam $= 0{\cdot}475$; (*b*) the relative humidity after heating.

° F.	p lb./in.^{2}	Heat/lb.		Specific Vol. ft.^{3}/lb.
		h	L	
36	0·1040	4·03	1,073·2	2,837
45	0·1475	13·07	1,068·1	2,037
65	0·3060	33·08	1,056·8	1,022

(*U.L. B.Sc.* (*Eng.*)) 1945.

(*a*) Partial pressure of vapour at 45° F.

$p_s =$ Relative humidity × saturation pressure for 45° F.

$= 0{\cdot}705 \times 0{\cdot}1475 = 0{\cdot}104$ lb. per sq. in.

Partial pressure of air

$p_a = 14{\cdot}7 - 0{\cdot}104 = 14{\cdot}596$ lb. per sq. in.

Weight of air in 10,000 cu. ft.

$$W_a = \frac{p_a v_a}{R_a T_a} = \frac{14{\cdot}596 \times 144 \times 10{,}000}{53{\cdot}3 \times (45 + 460)} = 782 \text{ lb.}$$

Weight of water vapour

$$W_s = W_a \frac{v_{sa}}{v_{ss}} = W_a \frac{18}{28{\cdot}95} \frac{p_s}{p_a} = \frac{782 \times 18 \times 0{\cdot}104}{28{\cdot}95 \times 14{\cdot}596} = 3{\cdot}46 \text{ lb.}$$

Heat supplied to water vapour

$$= W_s C_{ps} (t_2 - t_1) = 3{\cdot}46 \times 0{\cdot}475\,(65 - 45) = 32{\cdot}9 \text{ B.Th.U.}$$

Heat supplied to air

$$= W_a C_{pa} (t_2 - t_1) = 782 \times 0{\cdot}24\,(65 - 45) = 3{,}754 \text{ B.Th.U.}$$

(*b*) In the equation $pv = wRT$ the air and mixture each occupy the same volume at the same temperature, the pressure p_m of the mixture being 14·7 and the partial pressure p_a of the air being 14·596. Hence

$$\frac{p_a}{p_m} = \frac{W_a R_a}{W_m R_m} = \frac{W_a}{W_m} \times \frac{\text{molecular weight of mixture}}{\text{molecular weight of air}}$$

This indicates that the partial pressure of the air, and hence that of the vapour remain constant during heating at constant pressure.

$$\therefore \quad \text{Relative humidity at } 65^\circ \text{ F.} = \frac{\text{Partial pressure of water vapour}}{\text{Saturation pressure for } 65\ ^\circ \text{F.}}$$

$$= \frac{0{\cdot}104}{0{\cdot}306} = 0{\cdot}34$$

Another method of treating this question is as follows—

(*a*) The partial vapour pressure at 45° F. is as before 0·104 lb. per sq. in. The saturation temperature for this pressure is given in the table supplied with the question as 36° F. This is the dew point, and if the mixture were cooled down to this temperature at constant pressure the air would be saturated. The volume of the air would become $10{,}000 \times \frac{36 + 460}{45 + 460} = 9{,}822$ cu. ft.

The specific volume of the vapour would be that corresponding to its partial pressure, that is 2,837 cu. ft. per lb.

Hence the weight of vapour $W_s = \frac{9{,}822}{2{,}837} = 3{\cdot}46$ lb.

(*b*) If the air were heated at constant pressure from 45° to 65° F the volume would become $10{,}000 \times \frac{65 + 460}{45 + 460} = 10{,}400$ cu. ft. The table in the question shows that the specific volume of dry saturated vapour at 65° F. is 1,022 cubic feet. Hence if the air were saturated with vapour at this temperature, there would be $\frac{10{,}400}{1{,}022} = 10{\cdot}18$ lb. vapour. There is, however, only 3·46 lb.

$$\text{Hence relative humidity} = \frac{3{\cdot}46}{10{\cdot}18} = 0{\cdot}34$$

The Kinetic Theory of Gases. In this theory a gas is considered to consist of a large number of molecules moving with high velocity, and behaving as perfectly elastic bodies, there being no dissipation of energy when the molecules collide. Any molecule moves freely with uniform velocity in a straight line, except when it collides with another molecule or with the wall of the containing vessel.

In general when a molecule encounters another molecule or the wall of the vessel, its velocity will alter in direction and amount, the average value of the path traversed between successive encounters is termed the "mean free path," and in the case of a gas this is very long compared to the dimensions of the molecule. The time of movement in the mean free path is large compared to the time of the encounter.

If the gas is compressed, the mean free path is decreased and the encounters become more frequent.

Pressure. The pressure of a gas on the walls of the containing vessel is due to the collisions of the molecules on the walls.

Consider a cubical vessel of side d (Fig. 29) containing N molecules each of mass α. Then at any moment a given molecule will have a velocity U in a given direction. This velocity may be resolved along three mutually perpendicular directions with respect to the walls of the cube, equal to U_x, U_y, U_z where—

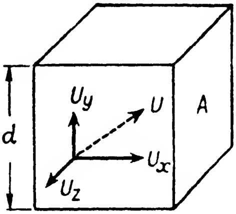

FIG. 29

$$U^2 = U_x^2 + U_y^2 + U_z^2$$

Consider a molecule colliding with face A with a component velocity U_x. The change in velocity produced is $2U_x$, the change in momentum of the molecule is $2\alpha U_x$, and the time between successive encounters with face A is $\frac{2d}{U_x}$ provided the molecule does not encounter another on its way.

Hence Force exerted on face A per collision

$$= \text{Rate of change of momentum}$$

$$= \frac{2\,\alpha\,U_x}{g} \times \frac{U_x}{2d} = \frac{\alpha\,U_x^2}{gd}$$

Let n be the number of molecules which are moving with the same numerical value of U_x.

Then force exerted on face A due to this group of molecules

$$= \frac{n\,\alpha\,U_x^2}{gd}$$

But there are many other groups of molecules with different values of U_x.

Hence total force exerted on face A—

$$F = \Sigma \frac{n\,\alpha\,U_x^2}{gd} = \frac{\alpha}{gd}\Sigma\, nU_x^2 = \frac{\alpha}{gd}\,N\overline{U_x^2}$$

where $\overline{U_x^2}$ is the average value of U_x^2 for all the molecules and N is the total number of molecules in the cubical space. We have already seen that the actual velocity U of any molecule is given by the equation

$$U^2 = U_x^2 + U_y^2 + U_z^2$$

Hence taking average values

$$\overline{U^2} = \overline{U_x^2} + \overline{U_y^2} + \overline{U_z^2}$$

and assuming a state of equilibrium in which the motions take place equally in all directions, then

$$\overline{U^2} = 3\overline{U_x^2}$$

and the pressure on face A is given by the equation

$$P = \frac{F}{d^2} = \frac{\alpha N}{gd^3}\frac{\overline{U^2}}{3} = \frac{1}{3}\frac{w}{V}\frac{\overline{U^2}}{g} \text{ lb. per unit area}$$

where w is the mass and V is the volume of the gas. As $\frac{V}{w}$ = the specific volume V_s of the gas, the above equation may be written.

$$PV_s = \frac{1}{3}\frac{\overline{U^2}}{g} = \frac{2}{3}\frac{\overline{U^2}}{2g}$$

The equation $P = \frac{\alpha N}{gd^3}\frac{\overline{U^2}}{3}$ may be written

$$PV = \frac{1}{3}\alpha N\frac{\overline{U^2}}{g}$$

Temperature. If we combine the equation $PV_s = \frac{1}{3}\frac{\overline{U^2}}{g}$ with the characteristic equation $PV_s = RT$ we obtain the expression

$$\frac{1}{3}\frac{\overline{U^2}}{g} = RT$$

This indicates that for any given gas, $\overline{U^2}$ is proportional to the absolute temperature. As kinetic energy is proportional to $\overline{U^2}$, it is seen that the average kinetic energy of the molecules due to translational velocity is proportional to the absolute temperature. Hence absolute zero temperature is reached when the translational velocities of the molecules have been reduced to zero, and the pressure is also zero.

Boyle's Law. From the equation $PV = \frac{1}{3}w\frac{\overline{U^2}}{g}$, it follows that if $\overline{U^2}$ is constant, that is if T is constant, then PV = constant.

Avogadro's Law. From the equation $PV = \frac{1}{3}\alpha N\frac{\overline{U^2}}{g}$ it is seen that for two gases at the same temperature and pressure

$$\frac{1}{3}\alpha_1 N_1\frac{\overline{U_1^2}}{g} = \frac{1}{3}\alpha_2 N_2\frac{\overline{U_2^2}}{g}$$

Now $\alpha\frac{\overline{U^2}}{2g}$ is the average kinetic energy of the molecule, and Maxwell has shown that for equal temperatures this is a constant for different gases.

Hence $$\frac{\alpha_1\overline{U_1^2}}{g} = \frac{\alpha_2\overline{U_2^2}}{g} \text{ and } N_1 = N_2.$$

Dalton's Law. For a mixture of gases, the kinetic theory would indicate that

$$P = \frac{1}{3}\alpha_1 N_1 \frac{\overline{U^2_1}}{g} + \frac{1}{3}\alpha_2 N_2 \frac{\overline{U_2^2}}{g} + \text{etc.}$$

hence $$P = P_1 + P_2 + \text{etc.}$$

Kinetic Energy of Translation. For unit mass of gas the kinetic energy of translation

$$E_t = \frac{\overline{U^2}}{2g}$$

But $PV_s = \frac{2}{3}\frac{\overline{U^2}}{2g} = RT$ where R is the particular gas constant.

Hence $$E_t = \frac{3}{2}RT$$

The average kinetic energy of translation per pound mol. will therefore be

$$E_t \times \text{molecular weight} = \frac{3}{2}GT$$

where G is the universal gas constant.

If N_m is the number of molecules in a pound mol. then the average kinetic energy of translation per molecule will be

$$\frac{3}{2}\frac{GT}{N_m}$$

Internal Energy and Specific Heat. The kinetic energy of translation is not necessarily the whole internal energy of the gas, for the molecules may have rotation about their own axis, and there may be vibration within the molecule.

If we can assume that there is only energy of translation, then the internal energy $E = E_t = \frac{3}{2}RT$, that is the internal energy of a gas is proportional to its absolute temperature. This agrees with Joule's Law.

If the gas is heated $dE = \frac{3}{2}RdT$

But for a perfect gas $dE = C_v dT$

Hence $$C_v = \frac{3}{2}R \text{ and } C_p = C_v + R = \frac{5}{2}R$$

and $$\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1{\cdot}667$$

It is found by experiment that in the case of monatomic gases, such as argon and helium that the value of γ agrees well with this, thus justifying our assumption of translation only. One can therefore think of any molecule as having three degrees of freedom, as regards translation, with respect to three mutually perpendicular axes. By the theorem of equipartition of energy, each degree of freedom in one group is assumed to have on the average the same amount of kinetic energy as is possessed by a degree of freedom in any other group.

Thus the energy per degree of freedom will be one-third of the total, namely $\frac{1}{2}RT$.

A diatomic molecule may be supposed to consist of two atoms attached rigidly to one another, forming a dumb-bell shaped molecule. As in the case of monatomic molecules it can be assumed that as the result of mutual collisions, no rotational motion about the axis joining the two atoms can be imparted. There can, however, be rotation about each of two axes, mutually perpendicular, and passing through the centre of gravity of the molecule at right angles to its axis. Thus in the diatomic molecule, three degrees of freedom with regard to translation and two with regard to rotation may be considered.

As each degree of freedom accounts for kinetic energy of amount $\frac{1}{2}RT$, the total amount of energy will be $\frac{5}{2}RT$.

Hence $$C_v = \frac{5}{2}R$$

and $$C_p = C_v + R = \frac{7}{2}R$$

and $$\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1{\cdot}4$$

Thus taking the value of $G = 1{\cdot}985$ B.Th.U. per lb. mol. per degree F., then

$$\left.\begin{aligned} C_p &= \frac{7}{2} \times 1{\cdot}985 = 6{\cdot}947 \\ C_v &= \frac{5}{2} \times 1{\cdot}985 = 4{\cdot}962 \end{aligned}\right\} \text{B.Th.U. per lb. mol. per degree F.}$$

These values should be compared with those given in Table II, p. 68, for diatomic gases. The fairly good approximation at comparatively low temperatures would appear to justify the assumption of absence of vibrational energy.

It is found by experiment, however, that with increase of temperature, the values of the specific heats increase and the value of γ decreases. Hence one must assume that as the temperature increases, the energy of vibration cannot be ignored.

In a polyatomic gas, where the structure of the molecule may be considered as polyhedral, we may assume three degrees of freedom as regards rotation in addition to the three translational degrees. Hence ignoring any vibrational energy, we have

$$\left.\begin{aligned} C_v &= 3R = 3 \times 1{\cdot}985 = 5{\cdot}955 \\ C_p &= 4R = 4 \times 1{\cdot}985 = 7{\cdot}940 \end{aligned}\right\} \text{B.Th.U. per lb. mol. per degree F.}$$

$$\gamma = \frac{7{\cdot}940}{5{\cdot}955} = 1{\cdot}333$$

A comparison with the values for CO_2 and CH_4 in Tables II, p. 68 and XVII, p. 307, shows that the practical values of specific heats are considerably larger, and that of γ smaller than the theoretical values. Hence there must be, even at moderate temperatures, vibratory energy in the gas, which is not accounted for in the simple theory of equipartition of energy.

Planck's Quantum Theory modifies the simple theory given above.* Based on this theory, so called quantum values of specific heats have been derived from spectroscopic frequency measurements.†

EXAMPLES III

1. Find the volume of 3 lb. weight of gas at pressure 115 lb. per sq. in. absolute, and temperature 59° F. Take the specific heat at constant pressure as 0·2404 and at constant volume 0·172 per lb. weight.

2. A pound of dry air at 15·5° C. and 14·7 lb. per sq. in. occupies 13·09 cu. ft. and is compressed adiabatically ($pv^{1{\cdot}4}$ = constant) to one-fifth of this volume. Find (*a*) its pressure and temperature at the end of compression, and (*b*) the pressure at the reduced volume when the air is cooled to 15·5° C.

3. One cu. ft. of gas, at 300 lb. per sq. in. absolute, expands to 5 cu. ft. according to the law $pv^{1{\cdot}2}$ = constant. Find the pressure at the end of expansion and the work done by the gas during expansion.

4. Find the temperature at the end of compression when air at 70° F. (21·1° C.) is compressed from 15 to 105 lb. per sq. in. absolute. Assume $n = 1{\cdot}35$.

5. A cubic foot of gas, at pressure 300 lb. per sq. in. absolute, expands until its pressure is 60 lb. per sq. in. absolute, the law of expansion being $pv^{1{\cdot}3}$ = constant; find the work done by the gas during expansion.

6. In an air compressor the air is drawn in at a temperature of 60° F. and pressure 14·7 lb. per sq. in. absolute. The air drawn in per stroke is 12 cu. ft., and the final pressure is 90 lb. per sq. in. absolute. Find the work done during compression (*a*) if the air is compressed isothermally; (*b*) if it is compressed adiabatically. In the second case, find the temperature of the air when the pressure reaches 90 lb. absolute. (*U.L.*, *B.Sc.* (*Eng.*).)

7. If 10·2 cu. ft. of air, at pressure 14 lb. per sq. in. absolute, be compressed isothermally to 1·7 cu. ft., what is (1) the pressure at the end of compression; (2) the work done on the air; and (3) the heat taken from it?

✓8. Twelve cu. ft. of air at 14 lb. per sq. in. absolute and 27° C. is compressed to 1 cu. ft. The compression law is $pv^{1{\cdot}3}$ = constant. Find (*a*) the pressure and temperature at the end of compression; (*b*) the work done in foot-pounds during compression; and (*c*) the amount of heat received or rejected by the air to the cylinder walls when $\gamma = 1{\cdot}4$.

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* See *Thermodynamics for Engineers*, Ewing (C.U.P.).

† See "Fuel Economy in Petrol Engines," David and Leah, *Proc. Inst. Mech.E.*, Vol. 143, 1940.

9. Ten cu. ft. of air at 90 lb. per sq. in. absolute and 65° F. are expanded to 4 times the original volume, the law for expansion being $pv^{1\cdot 25}$ = a constant. Given that the specific heat of air at constant volume is 130·3 ft-lb. per lb., and at constant pressure 183·4 ft-lb. per lb., find (*a*) the temperature of the air at the end of expansion ; (*b*) the work done in foot-pounds ; (*c*) the amount of heat which must have been given by or been rejected to an external source during the expansion. (*U.L.*, *B.Sc.* (*Eng.*).)

10. Air at 60° F. and pressure 15 lb. per sq. in., is compressed to 75 lb. per sq. in. absolute, and the compression curve is $pv^{1.3}$ = constant. Find the work done and heat rejected per pound of the air during compression. (*U.L.*, *B.Sc.* (*Eng.*).)

11. A cylinder contains 0·5 cu. ft. of gas at pressure 15 lb. per sq. in. absolute. What work in foot-pounds is expended in compression to 90 lb. per sq. in. absolute if the compression curve follows the law $pv^{1\cdot 35}$ = constant ? What is the volume after compression ?

12. The pressure is 160 lb. per sq. in. absolute when gas occupies 1 ft. length of cylinder, 12 in. in diameter. What work is done by the gas kept at *constant temperature* in driving the piston through 2 ft. length of stroke, and what amount of heat must be given to the gas ?

13. One pound of air, at atmospheric pressure and 60° F., is compressed adiabatically to 6 atmospheres ; find (*a*) its temperature and the work done by the pump. It is now allowed to cool at this pressure down to 60° F. ; (*b*) what amount of heat is given out, and what further work of compression is done ? If the final state had been brought about by isothermal compression, (*c*) what work would have been done by the pump ? Specific heat of air at constant pressure 0·238, at constant volume 0·169. (*U.L.*, *B.Sc.* (*Eng.*).)

14. State and prove the relation existing between the specific heat of a gas at constant pressure and constant volume, and the characteristic constant for a gas.

If one-tenth of a pound of gas occupies 1·345 cu. ft. at 15° C., and 15 lb. per sq. in., and if after adiabatic compression to 0·5 cu. ft. the pressure is 60·9 lb. per sq. in., what is the value of the specific heat at constant pressure of this gas ? (*U.L.*, *B.Sc.* (*Eng.*), 1925.)

15. An air pump is employed to extract air from a large receiver containing V cu. ft. of air at atmospheric pressure. If the pump draws in air at a uniform rate of v cu. ft. per min., and the temperature inside the receiver remains constant, prove that the time in reducing the pressure in the receiver by one-half is $0{\cdot}693V/v$ minutes. (*U.L.*, *B.Sc.* (*Eng.*), 1933.)

CHAPTER IV

IDEAL REVERSIBLE CYCLES

Carnot's Reflections on the Motive Power of Heat. In 1824, Sadi Carnot published the extraordinary essay, of remarkable insight, entitled *Reflections on the Motive Power of Heat,* containing the fundamental principles from which thermodynamics has been developed.*

Carnot conceived a *closed, reversible cycle,* in which the working substance receives heat *only* at the higher constant temperature of the hot *source,* when expanding and doing external work, is cooled by adiabatic expansion, gives out heat only during isothermal compression at the lower constant temperature of the *condenser* ; is then raised in temperature by adiabatic compression, and brought back again to its initial state of volume, temperature, pressure and physical properties, so that, he states as an *axiom,* the working substance must contain the same quantity of heat as it had originally.

In Carnot's *ideal engine* the piston and cylinder are supposed to be perfect non-conductors, absolutely impervious to heat, but the cylinder is fitted with a perfectly conducting end. The working substance is changed in temperature only by rapid adiabatic expansion and compression, with a perfect non-conducting cover on the end of the cylinder, so that there is no transfer of heat, except while heat is being received from the *source* or hot body at temperature T_1, and rejected to the refrigerator or cold *condenser* at T_2. Both of these bodies must have infinite capacity for heat, in order that each of them may remain at exactly the same constant temperature as the working substance during its isothermal expansion and compression while the transfer of heat is taking place through the perfectly conducting end cover of the cylinder.

In 1834, Clapeyron took up Carnot's work† and made the supposed changes and transformations during the cycle clearer, by an indicator diagram, Fig. 30.

Carnot's Cycle, with a Perfect Gas as the Working Substance. Take 1 lb. weight of a perfect gas as working substance in the cylinder, and let A, Fig. 30, represent the state as regards pressure p_1, and volume v_1, at absolute temperature T_1.

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* Sadi Carnot, *Réflexions sur la puissance motrice du feu et sur les moyens propres à la développer* (Paris, 1824); or, "The Motive Power of Heat" (with a biographical sketch by a brother of Carnot), translated and edited by R. H. Thurston, 1890.

† See "*Memoire sur la puissance motrice du feu,*" in the *Journal de l'école Polytechnique,* Tome XIV (1834), or Translation in Taylor's *Scientific Memoirs,* Part III (1837), p. 347.

1. *Isothermal Expansion*, with the conducting end of the cylinder in contact with the source or hot body at the higher temperature T_1. Allow the gas to expand along the isothermal AB, Fig. 30, from volume v_1 to v_2 at B, receiving heat Q_1 at constant temperature. The external work done *by* the gas in driving the piston is represented by the area $ABba$, and is equal to the heat received $Q_1 = RT_1 \log_\varepsilon r$, $\left(\text{where } r = \dfrac{v_2}{v_1}\right)$ while the temperature and internal energy of the gas remain the same. (See Eq. (4) (p. 70).)

2. *Adiabatic Expansion*, BC, with the non-conducting cover on the cylinder end, the gas expands adiabatically, doing work on the piston at the expense of the internal energy of the gas. Consequently the temperature falls from T_1 to the lower temperature T_2 at C. The pressure and volume are now p_3 and v_3, and the external work done *by* the gas is represented by the area $BCcb$, and is equal to $\dfrac{R(T_1 - T_2)}{\gamma - 1}$, which is also the change in internal energy of the gas. (See p. 72.)

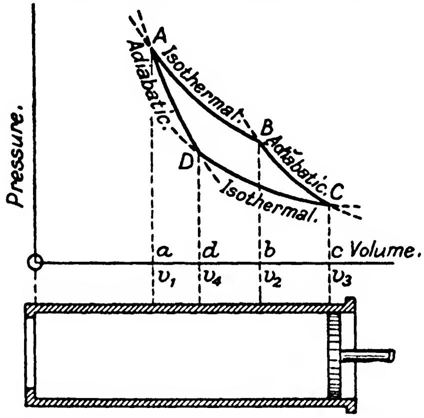

FIG. 30. CARNOT CYCLE WITH A PERFECT GAS

3. *Isothermal Compression*, CD. Bring the conducting end of the cylinder into direct contact with the cold body or condenser at T_2. Force the piston slowly inwards until the state point D of the gas is on the adiabatic curve through A. The work done on the gas is $RT_2 \log_\varepsilon r$, represented by area $CDdc$, and is equal to the heat rejected, $Q_2 = RT_2 \log_\varepsilon r$.

The adiabatic drop in temperature from B to C must be equal to the rise in temperature from D to A; that is, from p. 72

$$\frac{T_1}{T_2} = \left(\frac{v_3}{v_2}\right)^{\gamma - 1} = \left(\frac{v_4}{v_1}\right)^{\gamma - 1}, \text{hence } \frac{v_3}{v_2} = \frac{v_4}{v_1}, \text{ or, } \frac{v_3}{v_4} = \frac{v_2}{v_1} = r,$$

where r is the ratio of isothermal expansion and compression.

4. *Adiabatic Compression, DA.* Again, put the non-conducting cover on the cylinder end, and force the piston rapidly inwards, compressing the gas adiabatically from D to A, until the temperature rises to the original T_1 of the hot body. The pressure and volume are now p_1 and v_1; and the additional work done *on* the gas by the piston is represented by the area *DAad*, equivalent to $\frac{R(T_1 - T_2)}{\gamma - 1}$ which is also the *gain* of internal energy of the gas.

During adiabatic expansion and compression, no heat is received or rejected, and, on the whole, there is no change of internal energy since the external work done *by* the gas, represented by the area under BC, and the work done *upon* it by the area under DA, are each equal to $\frac{R(T_1 - T_2)}{\gamma - 1}$ for the same change of temperature.

During the whole cycle, the heat received by the 1 lb. weight of gas at T_1 from the hot body is $Q_1 = RT_1 \log_\varepsilon r$, and the heat rejected at T_2 to the cold body is $Q_2 = RT_2 \log_\varepsilon r$.

There has been no other transfer, leakage, or waste of heat energy to or from the gas, which is brought to its original state, having the same temperature, internal energy, pressure and volume as at the start. Therefore, the difference

$$Q_1 - Q_2 = R(T_1 - T_2) \log_\varepsilon r$$

is the net amount of heat that has disappeared as heat, and must have been converted into the net amount of external work done by the gas on the piston, and represented by the area $ABCD$ of the diagram.

The *thermal efficiency* of the Carnot ideal cycle is therefore—

$$\frac{\text{Work done } (W)}{\text{Heat received}} = \frac{Q_1 - Q_2}{Q_1} = \frac{R(T_1 - T_2) \log_\varepsilon r}{RT_1 \log_\varepsilon r} = \frac{T_1 - T_2}{T_1} \quad . \quad (1)$$

that is, work done

$$W = Q_1 \left(\frac{T_1 - T_2}{T_1}\right) = \frac{Q_1}{T_1}(T_1 - T_2)$$

gives the largest fraction of the heat received from the hot body that can be converted into work, and this depends only upon the temperature limits at which the gas receives and rejects heat.

The heat is let down through an engine from a source at a high temperature, and some of it is rejected at a lower temperature, the only change of temperature being adiabatic.

Carnot's cycle is **reversible,** that is, when the working substance is forced to perform the same operations in the opposite direction, i.e. counter clockwise, the same *pv* indicator diagram is traced, and, by the same net work expended, the same quantity of heat is taken in at the low temperature and rejected at the higher temperature.

Starting at state A, the gas is cooled by adiabatic expansion from T_1 to T_2 at D; and, by isothermal expansion from D to C, the gas takes heat Q_2 from the cold body at temperature T_2, while doing work represented by $RT_2 \log_\varepsilon r$, or the area $DCcd$. Then adiabatic compression CB raises the temperature of the gas to T_1; and by isothermal compression BA, the gas rejects heat Q_1 at the higher temperature T_1 while *work is done upon* the gas equal to $RT_1 \log_\varepsilon r$, or the area $BAab$, and the gas is restored to its original state at A.

In this reversed cycle, on the whole, there is no change of the internal energy of the working gas; the area of the diagram $ADBC$ indicates the net amount of

$$\textit{Work spent upon the gas} = R(T_1 - T_2) \log_\varepsilon r,$$

whilst heat extracted at T_2, from cold body, is $Q_2 = RT_2 \log_\varepsilon r$, and rejected at higher temperature T_1, to hot body, is $Q_1 = RT_1 \log_\varepsilon r$.

Hence the transfers of heat are exactly reversed, and, during the series of stages, the states of the gas are precisely the same in pressure, volume and temperature as when working direct. In the reversed engine the work spent upon the gas is equal in amount to the work which the gas does on the piston in direct working.

The *performance* of the ideal Carnot heat engine reversed is measured by

$$\frac{\text{Heat extracted from cold body}}{\text{Work expended}} = \frac{Q_2}{Q_1 - Q_2} = \frac{RT_2 \log_\varepsilon r}{R(T_1 - T_2) \log_\varepsilon r}$$

hence $$\frac{Q_2}{\text{Work spent}} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

that is $$Q_2 = \text{Work done} \times \frac{T_2}{T_1 - T_2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This reversed heat engine is also an ideal *refrigerating* machine, and, by the conservation of energy, $Q_1 = Q_2 +$ work done in driving the machine. In other words, the quantity of heat Q_1 rejected to the hot body is equal to the heat Q_2, received from the cold body, added to the work spent upon the gas.

Carnot's Principle. Reversibility is the sole test of perfection of a thermodynamic engine. No engine can perform more work from a given quantity of heat working between the same limits of temperature, i.e. a reversible engine has the highest possible theoretical efficiency.

To prove this: Suppose, if possible, a heat engine, S, to be capable of doing more work from a given quantity of heat than a reversible engine, R, working between the same temperatures of source and condenser.

Couple the two engines together as a compound engine, S working direct and R reversed. R needs part w only of the work $w + w_1$ given out by S, to be able to restore to the source the heat received by S. Thus, during each cycle of the compound engine, the heat rejected to the source by R is equal to that supplied to S, while work w_1 is done. This would go on continuously. Even if we suppose no mechanical friction, heat must have been spent to perform the work w_1, and this heat could only come from the condenser or cold body. The quantity of heat in the source or hot body remains the same, while that in the cold body must be diminished indefinitely, and work would be done continuously by the transfer of heat from the colder body. This result is contrary to all experimental evidence, and to the axiom stated by Clausius: "It is impossible for a self-acting machine, unaided by any external agency, to convey heat from a body at a low temperature to one at a higher temperature." Or by Kelvin: "We cannot transform heat into work merely by cooling a body already below the temperature of the coldest surrounding objects."*

These statements of the **second law of thermodynamics** agree with the facts of experience, and apply to the performance of work continuously by heat engines working in complete cycles.

The *second law* governs the transformation of heat into work, and *gives the greatest fraction of the total heat supplied to any heat engine that can be converted into work done on the piston.*

Therefore no engine can have a higher thermal efficiency than a reversible one. It follows that all heat engines, with perfectly reversible cycles, receiving and rejecting heat at the same higher and lower temperatures, have the same efficiency, whether air or other gas be used in them. The efficiency of a reversible engine is independent of the nature or physical properties of the working substance employed, and depends solely on the temperatures between which the engine works.

The greatest amount of work which can be obtained from Q_1 units of heat received by the working substance at absolute temperature T_1, and rejecting heat at absolute temperature T_2, is

$$JQ_1 \frac{(T_1 - T_2)}{T_1} \text{ ft-lb.}$$

Conditions for Reversibility and Maximum Efficiency. The first condition for reversibility is absence of friction. This is obvious, for whatever the direction of the path taken by the working fluid, friction will mean a conversion of mechanical work into heat. It also means that there must be no "free" expansion by sudden changes of pressure, such as in throttling, where eddying motions are dissipated in heat.

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* Wm. Thomson, "Memoir on the Dynamical Theory of Heat," *Trans. Royal Soc., Edin.*, 1851 and May 1854; and *Phil Trans.* 1854, Vol. 144.

The second condition of reversibility is equality of temperature between the working substance and the source or sink of heat, when taking in or rejecting heat. If there were a difference of temperature, heat could pass down a temperature gradient by itself, but in the reverse cycle, work would be required to "pump" the heat up the temperature gradient. As already mentioned, this is laid down in the statement of the second law of thermodynamics.

There is a further condition for the efficiency of a heat engine cycle to have the maximum value of $\frac{T_1 - T_2}{T_1}$. This condition is that there must be only two temperatures at which heat is transferred, namely one at which heat is taken in, and another at which heat is rejected.

It will be seen later (pp. 131–4) that in the case of the Stirling and Ericsson cycles, the efficiencies are equal to that of the Carnot cycle, while in the Joule cycle, the efficiency is lower than that of the corresponding Carnot cycle. This is because the Joule cycle, while satisfying the first two conditions, does not conform to the third.

Irreversible Processes. By the conservation of energy, in the case of the steady flow of a fluid (p. 53), the output of work done by an engine is $AW = H_1 - H_2 - Q - A\left(\frac{U_2^2 - U_1^2}{2g}\right)$ where $H_1 - H_2$ is the change in total heat, and Q is the heat loss to the surroundings. When the change in kinetic energy and heat loss are negligible, the work done per pound of the working fluid is $W = (H_1 - H_2)J$.

Lord Kelvin pointed out that in all actual transformations of energy there is a universal tendency towards dissipation of energy, and degradation of heat which passes from warm to colder bodies; and as the temperature falls the *heat becomes less available* for doing mechanical work.

Even adiabatic throttling, as through a porous plug, is not reversible. Heat is always lost by friction, whether it be fluid friction within the working substance itself, due to viscosity and eddies, or mechanical friction in passing through small pipes or valves and of steam jets in the turbine. The sudden drop of pressure is irreversible. There is also loss of heat from the hot working substance to the colder surface of the cylinder walls and surroundings, only part of which is recovered.

Similarly in the reversed heat engine or refrigerating machine, where mechanical work is spent in extracting heat from the cold body, heat always tends to leak inwards from the warmer surroundings, and more work has to be done in extracting it from the cold working substance.

Carnot's Cycle, using Vapour as the Working Substance. Consider the ideal perfect heat engine working on the Carnot cycle and using

1 lb. of saturated vapour, namely, vapour in contact with its own liquid. At A, Fig. 31, 1 lb. weight of water or other liquid occupies w cu. ft. at the boiling point T_1 under the corresponding saturation pressure p_1. Heat is applied at this constant pressure until the whole pound of liquid is turned into vapour occupying the *volume* V cu. ft. at B. The heat received is the latent heat, L_1, of the vapour at this pressure. The isothermal expansion follows the horizontal line AB at constant pressure.

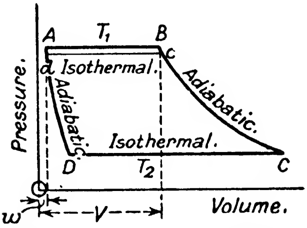

Fig. 31. Carnot Cycle with Vapour as the Working Substance

Then the dry saturated vapour is expanded adiabatically, reducing the pressure until the temperature falls to that of the condenser, T_2, that is, the lowest temperature of the available cooling water in a condenser. The vapour is wet along the adiabatic BC. Isothermal compression, CD, follows at constant lower pressure and temperature T_2. The vapour gradually condenses and its latent heat is rejected to the condenser until a point D is reached, such that adiabatic compression condenses all the vapour and restores the working substance in the liquid state to temperature T_1 at A. The adiabatic curves BC and DA of expansion and compression are usually plotted from tables giving the properties of vapours.

Since the cycle is reversible and heat L_1 is taken in only at the higher temperature T_1, and heat rejected only at T_2, the thermal efficiency is $\frac{T_1 - T_2}{T_1}$, and the work done per pound of the liquid is represented by the area $ABCD$, and equal to $JL_1 \cdot \frac{T_1 - T_2}{T_1}$ ft-lb.

Suppose the difference between the higher and lower temperatures is very small, from T to $T - \delta T$, so that δT is the fall in temperature, and the corresponding difference in pressure δp. The indicator diagram is now reduced to the narrow parallelogram $ABcd$, of height δp, since the adiabatics are nearly straight lines. The length AB of the little diagram is the difference, $V - w$, between the volume of the saturated vapour V and that of the liquid w, of 1 lb. weight.

The work done in this Carnot cycle is equal to the area $ABcd$, and becomes nearly its height × length, or $\delta p(V - w)$. The heat taken in during the cycle is L, the latent heat of evaporation and the work done is $JL \frac{\delta T}{T}$ ft-lb. per lb. of working substance; therefore

$$\delta p(V - w) = JL \cdot \frac{\delta T}{T}$$

In the limit, when δT is infinitely small, the relation becomes

$$\frac{dp}{dT}(V-w)=\frac{JL}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or,

$$V-w=\frac{JL}{T}\cdot\frac{dT}{dp} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

known as **Clapeyron's Equation,** giving the relation between the latent heat of any vapour, the change in cubic feet of 1 lb. of the liquid when vaporized at absolute temperature T, and the rate at which the temperature of saturated vapour varies relatively to the pressure. The latter $\frac{dT}{dp}$ is given by the slope of the tangent to the saturation curve of temperature and pressure at the given absolute temperature, and from steam tables.

The student should plot the temperature-pressure curve of saturated steam or other vapour from tables, and find $\frac{dp}{dT}$ from the curve, also L and w, the results of experiments, from tables; then calculate the specific volume of 1 lb. of dry saturated steam at a few pressures by equation (3).

The accuracy of results for the saturation volumes may be compared with the tabulated values. Or, the temperatures of dry saturated steam may be taken, 2° C. apart, and half the difference of corresponding saturation pressures in pounds per square foot gives the rate at which p is changing per degree. The values of L, and w cu.ft. for the mean temperature, substituted in equation (3), give the saturation volume in cubic feet.

Since $(V-w)$ is positive for steam, it follows from equation (4) that $\frac{dT}{dp}$ is positive, that is, increase of pressure raises the boiling point.

The Clapeyron equation also applies to a change of state from solid to a liquid. If a solid contracts in melting, $\frac{dp}{dT}$ is negative, and the melting point is lowered by increase of pressure.

From the contraction of ice in melting, Professor James Thomson* calculated that the *melting point of ice would be lowered* about 0·0074° C. for every atmosphere increase of pressure, and this was proved experimentally by his brother, Lord Kelvin. Thus, a pound of water at 0° C. changes its volume, in freezing, from 0·016 to 0·0174 cu. ft., and gives out 80 C.H.U., hence

$$\frac{dT}{dp}=\frac{(0{\cdot}0174-0{\cdot}016)273}{1{,}400\times 80}=0{\cdot}00000341$$

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* See Kelvin's *Mathematical and Physical Papers*, Vol. I, p. 156 and p. 165.

and if dp be 1 atmosphere, 15 lb. per sq. in., or 2,160 lb. per sq. ft.,

$$dT = 2{,}160 \times 0{\cdot}00000341 = 0{\cdot}0074^\circ \text{ C. (nearly).}$$

Also, if the atmospheric pressure were reduced to a vacuum, the water vapour at the melting point would have a pressure of 0·09 lb. per sq. in., and ice would melt about 0·0074° C., instead of 0° C. When there is no air pressure, the temperature of melting ice is known as the *triple point*, because water substance may then be in the three states of melting ice, water, and vapour together in equilibrium.

Again, from equation (3),

$$\frac{\frac{\delta p}{\delta T}(V - w)}{JL} = \frac{1}{T}; \text{ or } \frac{\delta p(V - w)}{JL} = \frac{\delta T}{T} \quad . \quad . \quad . \quad (5)$$

taking δT as 1° C., and the work done* per unit of heat received at absolute temperature T, for 1 lb of vapour, with the change of pressure in pounds per square foot corresponding to a drop of temperature 1° C. at that temperature, is simply $\frac{1}{T}$.

Graphic Representation of Energy. The *internal energy* of a pound of gas in the state A, Fig. 32, is represented by the area $ADeZaA$, under the adiabatic curve through A, indefinitely produced to cut the volume axis at infinity, or absolute zero. Imagine the gas, originally in the state A as to pressure and volume, and temperature T, if it were possible, to be expanded adiabatically, doing external work at the expense of its own internal energy, without gain or loss of heat, until its pressure falls to zero. It can do no more external work, and contains no heat. The whole of the internal energy of the 1 lb. of gas is gradually spent in doing work on the piston, and the equations (p. 72) representing the area $ADeZaA$ under the adiabatic, become

$$\text{Work done} = \frac{p_1 v_1}{\gamma - 1} = \frac{RT_1}{\gamma - 1} = K_v T_1 \quad . \quad . \quad . \quad (6)$$

which may be taken as the internal energy in 1 lb. of gas in the state A, at absolute temperature T_1. But we know that all gases would become solid and cease to behave like a perfect gas before reaching this low temperature.

We have no means of measuring the total amount of internal energy stored up in a given quantity of gas or other working substance. We can only measure or estimate the *change of internal*

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* See *Steam Power*, by Professor W. E. Dalby, p. 157. Examples of the physical properties of various working substances (steam, SO_2, NH_4, alcohol, acetone) gave the same result $\frac{1}{T} = \frac{1}{303}$, or 0·0033 ft-lb. per unit of heat supplied at 30° C. (303° absolute) to 1 lb. of the substance for a fall of temperature from 30° to 29° C., illustrating clearly that work done does not depend on the physical properties of any particular working substance.

energy of a given mass of a substance in changing from one state to another.

Now allow the state point to move from A to B along any curve AmB. To find the change of internal energy during this operation, draw through B the adiabatic BCf. The internal energy of the gas in the state B is represented by the area $BCfZbB$; and the change of internal energy is the difference between the areas $BCfZbB$ and $ADeZaA$, which is independent of the shape of the curve AmB. The gas might have changed from A to B along the isothermal. It follows that the internal energy of a substance in a given state must always be the same, and that the change of internal energy in any transformation depends only on the initial and final positions of the state point, and *not* upon the path.

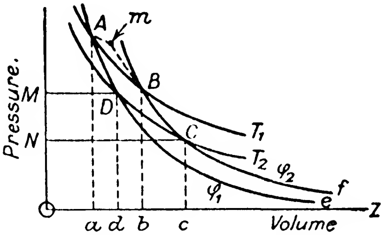

Fig. 32. Energy and Absolute Temperature

But, by Joule's energy equation, the heat absorbed $Q = E$ + external work done. The work done by the gas is represented by the area under the curve, either $AmBbaA$ or $ABbaA$, depending on the path between A and B. In this case the external work done along the isothermal will be less than that along AmB, by the area enclosed between these paths, in a clockwise direction. Therefore, the *heat received* by a given quantity of gas during a change of state from A to B is represented on the pv diagram by the area bounded by the actual curve AB or AmB and the two adiabatic curves through the initial and final points drawn to infinity.

The **Total Heat** of a substance in any state is defined by the equation

$$H = E + \frac{pv}{J}$$

as the sum of the internal energy and the product of the pressure and volume, expressed in heat units, corresponding to the state.

Thus, on the pv diagram, Fig. 32, draw the constant pressure line DM, then the total heat of a gas in the state D, as regards pressure and volume, is shown by the sum of the rectangle $MDdO$ and the area under the adiabatic DeZ, or the total area $MDeZOM$. Along the isothermal DC, for a perfect gas, $p_1v_1 = p_2v_2$, and the heat energy received during expansion is equivalent to the external work done by the gas, $DCcdD$, while its internal energy remains constant.

For practical purposes, it is the difference between the energy in the two states that is considered, and the usual convention is to take an *arbitrary zero* from which the internal energy of a substance is reckoned. When the working substance is a perfect gas, the absolute zero of temperature is taken, and for calculations on internal combustion engines 100° C. or 0° C. When the working substance is a vapour, like steam, the zero temperature chosen corresponds to 0° C., and under pressure equal to its vapour pressure at that temperature. (See pp. 51 and 52.)

Absolute Temperature. In the Carnot reversible engine, as T_2 is lowered, the work area *ABCD* of the diagram, Fig. 32 (p. 121), increases until the temperature is reduced to zero, when all the heat received at the higher constant temperature is converted into work. Since the work done can never exceed the heat received, there must be a zero limit of temperature at which the pressure of the working substance would vanish. This ideal limiting temperature is defined as the absolute zero.

By Carnot's principle, the efficiency of all perfectly reversible heat engines working between the same limits of temperature must be the same and equal to $(T_1 - T_2)/T_1$, and the work done

$$W = Q_1 \cdot \left(\frac{T_1 - T_2}{T_1}\right)$$

This relation, combined with the first law, forms the basis and expression of the absolute scale of temperature devised by Lord Kelvin and now universally adopted.

By the first law, $Q_1 - Q_2 = W$, and the heat rejected at the lower temperature

$$Q_2 = Q_1 - W = Q_1 \left(1 - \frac{T_1 - T_2}{T_1}\right) = Q_1 \left(\frac{T_2}{T_1}\right)$$

that is, $$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \text{ and } Q_2 = Q_1 \times \frac{T_2}{T_1},$$

also $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \text{a constant.} \qquad . \quad . \quad . \quad . \quad . \quad (7)$$

Thus the ratio $\frac{T_1}{T_2}$ of the temperatures of the source and condenser is the same as $\frac{Q_1}{Q_2}$ of the quantities of heat received and rejected at those temperatures in a Carnot cycle, and the *work done for a given drop of temperature is the same at any part of the scale,* which is absolute in the sense of being independent of the properties of any particular substance.

Rankine takes a series of reversible engines, working between equal intervals of temperature, and shows that these would do equal amounts of work when one receives the heat which another rejects.

Between a pair of adiabatic curves *ADe* and *BCf*, draw a series of isothermals for temperatures T_1, T_2, T_3, etc., at equal intervals of temperature $T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = \delta T$. Each little enclosed area is an indicator diagram of one engine working on the Carnot cycle. As above, let Q_1 = heat supplied during isothermal expansion at the highest temperature of the range T_1. The work done

$$= Q_1 \times \frac{T_1 - T_2}{T_1} = \frac{Q_1}{T_1} \cdot \delta T$$

and the heat rejected at T_2

$$Q_2 = \frac{Q_1}{T_1} \times T_2$$

Then this heat Q_2, rejected by the first engine, is supplied to the second engine at T_2. Here work done = heat supplied × efficiency

$$= \left(\frac{Q_1}{T_1} \times T_2\right) \times \left(\frac{T_2 - T_3}{T_2}\right) = \frac{Q_1}{T_1} \times T_2 \times \frac{\delta T}{T_2} = \frac{Q_1}{T_1} \cdot \delta T$$

and heat rejected = heat supplied − work done

$$= Q_1 \times \frac{T_2}{T_1} - Q_1 \times \frac{\delta T}{T_1}$$

$$= Q_1 \left(\frac{T_2}{T_1} - \frac{\delta T}{T_1}\right) = Q_1 \left(\frac{T_2 - T_2 + T_3}{T_1}\right)$$

$$= Q_1 \times \frac{T_3}{T_1}$$

Similarly in the third engine, the efficiency is $\frac{\delta T}{T_3}$, and the work done $= Q_1 \times \frac{\delta T}{T_1}$, with heat rejected $= Q_1 \times \frac{T_4}{T_1}$.

Hence the work done by every engine during a cycle is the same, $\frac{Q_1}{T_1} \cdot \delta T$, when each engine works through the same range of temperature δT on the perfect gas thermometer. Equal intervals of temperature may be defined as those intervals which give equal amounts of work in a series of perfect heat engines. This thermodynamic scale is identical with that of the ideal perfect gas, but does not agree exactly with the scales of actual gases because of the variation in specific heat. (See corrections, p. 8.)

Again, suppose the engines in the series received quantities of heat Q_1, Q_2, Q_3, etc., at absolute temperatures T_1, T_2, T_3, etc., and rejected heat at the lower temperatures. In each little reversible cycle $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$, that is, $\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = 0$, and when the summation is taken all round the large reversible cycle, heat taken in being reckoned as positive and heat rejected as negative, $\Sigma\frac{Q}{T} = 0$.

By taking an indefinitely large number of steps and corresponding reception and rejection at the different temperatures, we come at last to a perfectly gradual or continuous change of temperature. If dQ represents the small quantity of heat taken in or given out by a perfect reversible engine at the absolute temperature T, then the sum of all the quantities $\frac{dQ}{T}$ is zero for a reversible cycle, that is, the integration round the complete cycle

$$\int\frac{dQ}{T} = 0 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Entropy. The quantity $\int\frac{dQ}{T}$ is called *entropy* and is usually represented by ϕ. Thus entropy, ϕ, is that characteristic or function of the state of a substance which does not change in a reversible adiabatic expansion or compression, and which is increased or diminished when heat is taken in or given out in a reversible manner. Since there is neither gain nor loss of heat along adiabatics in a reversible process, these are lines of *constant entropy*, and called *isentropics*. When any substance takes in or gives out a quantity of heat Q at the *absolute* temperature T, its gain or loss of entropy ϕ is measured by the quotient $\frac{Q}{T}$. If a quantity of heat is taken in or rejected at a varying temperature, then the change of entropy

$$\phi_2 - \phi_1 = \int_{T_1}^{T_2}\frac{dQ}{T} \quad . \quad . \quad . \quad . \quad (9)$$

provided the process is reversible.

As the value of $\int\frac{dQ}{T}$ taken round a complete reversible cycle is zero, it follows that the change in entropy between two states will be the same for all reversible paths between those two states. In other words, entropy is a function of the state of a substance in the same sense as pressure, volume and temperature.

Temperature-entropy Diagram. On a temperature-entropy diagram the state of a substance is given by vertical ordinates to represent absolute temperature, and horizontal ordinates to represent entropy. As $dQ = T \, . \, d\phi$ then $Q = \int T \, . \, d\phi$ and hence areas on a $T\phi$ diagram represent quantities of heat taken in or rejected.

An adiabatic, or constant entropy line, will be shown by a vertical straight line AM, Fig. 33, parallel to the axis of temperature; and an isothermal will be a horizontal straight line, AB, parallel to the axis of entropy.

Carnot's Cycle on a T-ϕ Diagram. Suppose a pound weight of working substance, kept at constant temperature $T_1 = 1{,}000°$ C. (absolute), expands and takes in heat from the source in amount $Q_1 = 150$ C.H.U. Its *gain* of entropy is $\frac{Q_1}{T_1} = \frac{150}{1{,}000}$, or 0·15, which the source has lost. This isothermal is represented on the $T\phi$ diagram, Fig. 33, by the horizontal line $AB = 0·15$; the increase of entropy of the working substance is $\phi_2 - \phi_1 = ON - OM$. The area $MABN = T_1(\phi_2 - \phi_1)$, represents the heat Q_1 received by the working substance. During adiabatic expansion BC, the temperature falls to 600° C. (absolute), say, but the entropy does not change. Next, isothermal compression CD at temperature $T_2 = 600°$ C., during which heat is rejected by the working substance, and its *loss* of entropy $\frac{Q_2}{T_2}$ must equal the gain $\frac{Q_1}{T_1}$, since the cycle is reversible, and $DC = AB$, or $\phi_2 - \phi_1 = 0·15$. The quantity of heat rejected is

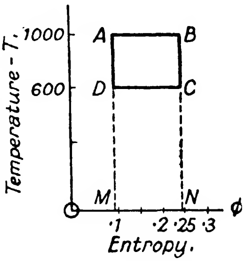

FIG. 33. CARNOT CYCLE

$$Q_2 = \text{area } CDMN = T_2(\phi_2 - \phi_1),$$

that is, $600 \times 0·15$, or 90 C.H.U.

In adiabatic compression DA, the entropy remains constant, while the temperature rises to the initial value T_1, or 1,000° C. (absolute).

The heat turned into work during the cycle is

$$Q_1 - Q_2 = \text{area } ABCD = (T_1 - T_2)(\phi_2 - \phi_1)$$

that is, $150 - 90 = (1{,}000 - 600)\,0·15$, or 60 C.H.U.

and the thermal efficiency

$$= \frac{60}{150} = \frac{1{,}000 - 600}{1{,}000}, \text{ or 40 per cent}$$

Here, heat converted into work

$$= \frac{Q_1}{T_1} \cdot (T_1 - T_2)$$

$$= \text{entropy} \times \text{drop of temperature}$$

Change of Entropy. As we can only determine *differences* of the internal energy of a substance, so in *measuring entropy*, differences are reckoned from some arbitrary standard condition.

Clausius Theorem. When a quantity of heat Q passes from one body, at absolute temperature T_1, to another body at a lower temperature T_2, the warm body *loses* entropy $\frac{Q}{T_1}$, while the colder body *gains* entropy $\frac{Q}{T_2}$. Since T_2 is less than T_1, the *gain of entropy* of the system as a whole is

$$\frac{Q}{T_2} - \frac{Q}{T_1} = Q \cdot \frac{T_1 - T_2}{T_1 T_2}$$

In all natural processes, heat flows from hotter to colder bodies, and it is obvious that *entropy* increases and tends to a maximum as the temperature falls.

We have already seen that, when 1 lb. of a substance takes in or gives out, reversibly, a small quantity of heat energy, dQ, the *absolute temperature* remaining *constant* at T, during the small change, its gain or loss of entropy, $d\phi$, is measured by the ratio $\frac{dQ}{T}$, so that the change in heat energy is $dQ = T \cdot d\phi$.

Also, $dQ = C \cdot dT$, where C is the *constant specific heat* of the substance at that temperature; and for unit weight of a substance changing from T_2 up to T_1, the total change of heat energy

$$\int_{T_2}^{T_1} dQ = C(T_1 - T_2), \text{ since } dQ = C \cdot dT \quad . \quad . \quad (10)$$

If 1 lb. of a substance is changing continuously and reversibly in temperature from T_2 up to T_1, while heat is being taken in, the total gain of entropy is

$$\phi_1 - \phi_2 = \int_{T_2}^{T_1} \frac{dQ}{T} = \int_{T_2}^{T_1} C \cdot \frac{dT}{T} = C \log_\epsilon \frac{T_1}{T_2} \quad . \quad . \quad . \quad (11)$$

At constant volume $C = C_v$, and at constant pressure $C = C_p$, in this expression.

Example 1. Assume that the entropy of 1 lb. of water at 0° C. is taken as zero. Calculate the entropy gained when the water is warmed to 100° C., and converted into dry saturated steam at 100° C.

In case steam tables are available, the entropy of dry saturated steam is given as $\phi_s = \phi_w + \frac{L}{T}$; that is, by adding the entropy of water at 100° C., and that due to the addition of the latent heat, L, at the constant absolute temperature of evaporation.

As a first approximation, suppose the heating is under the constant pressure of 1 atmosphere; and the specific heat of water is the mean thermal unit. Callendar gives the various properties of water vapour from 0° to 100° C. in his steam tables.

1. Gain of entropy of water, $\phi_w = 1 \times 2{\cdot}3026 \log_{10} \frac{373{\cdot}1}{273{\cdot}1} = 0{\cdot}312$.

2. The latent heat is 539·3 lb. calories (Steam Table II) and since

the temperature remains constant until all the water is converted into steam under the pressure of 1 atmosphere, gain of entropy is

$$\frac{L}{T} = \frac{539{\cdot}3}{373{\cdot}1} = 1{\cdot}4454$$

Hence the total entropy gained is $\phi_s = 0{\cdot}3120 + 1{\cdot}4454 = 1{\cdot}7574$

The accurate value by steam tables $\phi_s = 0{\cdot}31186 + 1{\cdot}44546 = 1{\cdot}7573$

The difference is within the limits of error of experiment.

If more heat is given after all the water is converted into dry team, the temperature of the steam would rise above saturation. he steam is then ***superheated***, and the additional gain of entropy ay be calculated by the expression $0{\cdot}48 \log_\varepsilon \frac{T_{su}}{T_{sa}}$, where 0·48 is the cific heat of steam, for which Callendar gives 0·47719 in his tables.

Change of Entropy of a Perfect Gas when changing from state p_1, v_1, T_1 to the state p_2, v_2, T_2. The energy equation of 1 lb. of a perfect gas for a small change in the limit is

$$dQ = C_v \,.\, dT + p \,.\, dv,$$

hence, $\frac{dQ}{T} = C_v \,.\, \frac{dT}{T} + \frac{p}{T} \,.\, dv$, and $pv = RT$, or $\frac{p}{T} = \frac{R}{v}$

Substituting this value gives

$$\frac{dQ}{T} = C_v \,.\, \frac{dT}{T} + R \,.\, \frac{dv}{v},$$ the expression for $d\phi$

Integrating, we have,

$$\int_{T_1}^{T_2} \frac{dQ}{T} = C_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}$$

and change of entropy,

$$\phi_2 - \phi_1 = C_v \,.\, \log_\varepsilon \frac{T_2}{T_1} + R \,.\, \log_\varepsilon \frac{v_2}{v_1} \qquad . \quad . \quad . \quad (12)$$

Again, since $R = C_p - C_v$, substituting for R,

$$\begin{aligned} \phi_2 - \phi_1 &= C_v \,.\, \log_\varepsilon \frac{T_2}{T_1} + (C_p - C_v) \log_\varepsilon \frac{v_2}{v_1} \\ &= C_v \left(\log_\varepsilon \frac{T_2}{T_1} - \log_\varepsilon \frac{v_2}{v_1} \right) + C_p \,.\, \log_\varepsilon \frac{v_2}{v_1} \\ &= C_v \left(\log_\varepsilon \frac{T_2}{T_1} \times \frac{v_1}{v_2} \right) + C_p \,.\, \log_\varepsilon \frac{v_2}{v_1} \end{aligned}$$

But $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$, and $\frac{p_2}{p_1} = \frac{T_2}{T_1} \times \frac{v_1}{v_2}$, hence we obtain

$$\phi_2 - \phi_1 = C_v \, . \log_\varepsilon \frac{p_2}{p_1} + C_p \, . \log_\varepsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad . \quad (13)$$

Similarly, by substituting for $C_v = C_p - R$ in equation (12), we obtain

$$\phi_2 - \phi_1 = C_p \, . \log_\varepsilon \frac{T_2}{T_1} - R \, . \log_\varepsilon \frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad (14)$$

In calculating the change of entropy for 1 lb. of a perfect gas, we may use either (12), (13), or (14), where R, C_p and C_v are expressed in heat units, v_1 and v_2 in cubic feet, p_1 and p_2 in pounds per square foot.

During an *isothermal change* of state, $T_1 = T_2$, and equation (12) becomes

$$\phi_2 - \phi_1 = R \, . \log_\varepsilon \frac{v_2}{v_1}, = (C_p - C_v) \log_\varepsilon \frac{v_2}{v_1} \quad . \quad . \quad . \quad (15)$$

In a *constant volume* change, $v_1 = v_2$, and equations (12) and (13) become

$$\phi_2 - \phi_1 = C_v \, . \log_\varepsilon \frac{T_2}{T_1}, = C_v \, . \log_\varepsilon \frac{p_2}{p_1} \quad . \quad . \quad . \quad . \quad (16)$$

In a *constant pressure* change, $p_1 = p_2$, and equations (13) and (14) become

$$\phi_2 - \phi_1 = C_p \, . \log_\varepsilon . \frac{v_2}{v_1} = C_p \, . \log_\varepsilon \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad (17)$$

Change of entropy of 1 lb. of gas during expansion or compression according to the general law pv^n = a constant.

By equation (13) (p. 82), $dQ = \frac{\gamma - n}{\gamma - 1} . p \, . dv$. Dividing by T, we have

$\frac{dQ}{T} = \frac{\gamma - n}{\gamma - 1} \times \frac{p}{T} \, . dv$, and since $\frac{p}{T} = \frac{R}{v}$ we obtain

$$\frac{dQ}{T} = R \, . \frac{\gamma - n}{\gamma - 1} \times \frac{dv}{v}$$

Integrating, we have

$$\int_{T_1}^{T_2} \frac{dQ}{T} = R \, . \frac{\gamma - n}{\gamma - 1} \, . \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\phi_2 - \phi_1 = R \, . \frac{\gamma - n}{\gamma - 1} \, . \log_\varepsilon \frac{v_2}{v_1}$$

Now $\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$, and $R = C_v(\gamma - 1)$

$$\text{hence } \phi_2 - \phi_1 = C_v(\gamma - 1)\frac{\gamma - n}{\gamma - 1} \times \log_\varepsilon\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$= C_v(\gamma - 1) \times \frac{\gamma - n}{\gamma - 1} \times \frac{1}{n - 1} \times \log_\varepsilon \frac{T_1}{T_2}$$

$$\text{and} \quad \phi_2 - \phi_1 = C_v \cdot \frac{\gamma - n}{n - 1} \cdot \log_\varepsilon \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Hence from measurements of pressure and volume on the indicator diagram, $\phi_2 - \phi_1$ is found, also the corresponding temperature calculated, and plotted on the temperature-entropy, T-ϕ diagram.

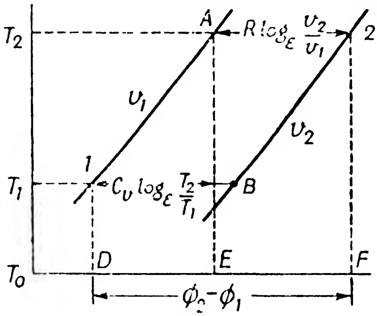

FIG. 34. T-ϕ DIAGRAM

T-ϕ Diagrams for a Perfect Gas. We will now consider constant volume and constant pressure lines on the T-ϕ diagram for a perfect gas with constant specific heat, showing the increase of entropy from a point (1) to a point (2) where T_2 is higher than T_1. Fig. 34 shows the two constant volume lines v_1 and v_2.

The horizontal distance between the two curves, that is the distance $A2$ or $1B$ is given by the constant temperature equation

$$\phi_2 - \phi_A = R \log_\varepsilon \frac{v_2}{v_1}. \text{ See Eq. 15, p. 128.}$$

The change in entropy along either constant volume line due to an increase in temperature from T_1 to T_2 is given by the constant volume equation

$$\phi_A - \phi_1 = \phi_2 - \phi_B = C_v \log_\varepsilon \frac{T_2}{T_1}. \text{ See Eq. 16, p. 128.}$$

The total change of entropy between points (1) and (2) is therefore

$$\phi_2 - \phi_1 = R \log_\varepsilon \frac{v_2}{v_1} + C_v \log_\varepsilon \frac{T_2}{T_1}. \text{ See Eq. 12, p. 127.}$$

It has already been stated that areas under a T-ϕ curve represent heat transferred. Hence in Fig. 34, the area $1AED$ represents

$C_v(T_2 - T_1)$, and the area $A2FE$ represents $T_2R\log_\epsilon\frac{v_2}{v_1}$, that is the heat transferred, which equals the work done in an isothermal change between v_1 and v_2.

In a similar manner constant pressure lines may be obtained as in Fig. 35 using the equations

$$\phi_2 - \phi_B = -R\log_\epsilon\frac{p_2}{p_1}. \text{ See Eq. 14, p. 128.}$$

and

$$\phi_2 - \phi_A = C_p\log_\epsilon\frac{T_2}{T_1}. \text{ See Eq. 17, p. 128.}$$

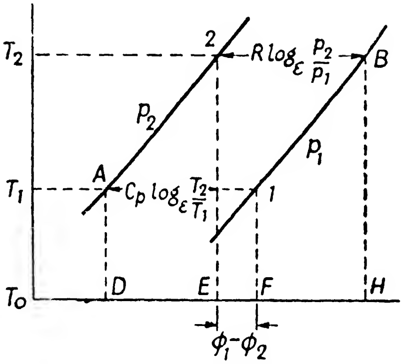

FIG. 35. T-ϕ DIAGRAM

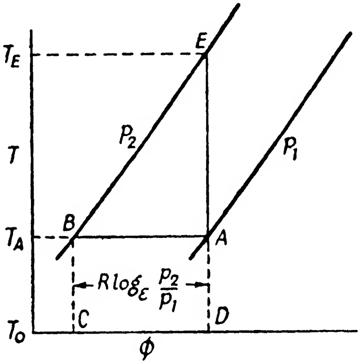

FIG. 36. T-ϕ DIAGRAM FOR ISENTROPIC AND ISOTHERMAL

Comparing the equation $\phi_2 - \phi_1 = C_p\log_\epsilon\frac{T_2}{T_1} - R\log_\epsilon\frac{p_2}{p_1}$ with Fig. 35, it will be seen that the constant pressure line for the higher pressure p_2 is on the left. A comparison of the values of $C_p\log_\epsilon\frac{T_2}{T_1}$ and $C_v\log_\epsilon\frac{T_2}{T_1}$ indicates that as C_p is greater than C_v, then the constant volume lines are steeper than the constant pressure lines.

For an isothermal compression from p_1 to p_2 the process is represented by AB (Fig. 36). The heat rejected and hence the work done is represented by the area $ABCD$ and is equal to $T_AR\log_\epsilon\frac{p_2}{p_1}$. It should be remembered that this work is that done in the compression only, and also that done in drawing in a charge into a cylinder, compressing it and then rejecting it at the higher pressure.

A frictionless adiabatic compression from p_1 to p_2 is represented by *AE* (Fig. 36). The heat transfer is nil, and the total work done in drawing in, compressing, and rejecting the charge is given by the equation

$$W = C_v \, (T_E - T_B). \text{ See pp. 88 and 89.}$$

This work is represented by the area *DEBC*.

It should be noted that areas on the T-ϕ diagram must be measured down to the temperature T_0 at which entropy is taken as zero.

Stirling's Air Engine with Regenerator. In 1827, Dr. Robert Stirling, and his brother James Stirling, patented and made the first hot-air engine to work on a perfect reversible cycle. This appears to be the first application of the *regenerator*, also invented by Stirling, in which heat is stored while the air falls from T_1 to T_2, the higher to the lower temperature, at constant volume before heat is rejected at the lower temperature. This transfer of heat is reversible, and the regenerator gives up the same heat that it took in, so that the air is gradually heated from T_2 to T_1 in its passage back through the regenerator, before receiving heat from the furnace at T_1. To be perfect, the regenerator must have infinite or very large capacity for heat compared with that of the air used, so that there is no sensible rise or fall of temperature when storing and re-storing the heat; and further, the regenerator must be at the same temperature as the air at every part in its passage.

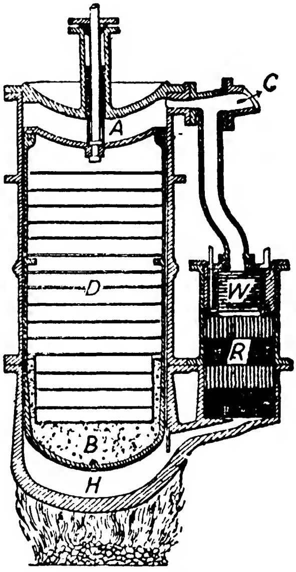

FIG. 37. STIRLING AIR ENGINE REGENERATOR AND DISPLACER

In one design, Fig. 37, a displacer D forces the working air up and down through the regenerator R, which is constructed of sheet iron plates $\frac{1}{40}$ in. thick, and kept $\frac{1}{50}$ in. apart by ridges to offer a large surface. The cooler or refrigerator W consists of copper pipes $\frac{1}{8}$ in. bore, about $\frac{1}{20}$ in. apart, and through these cold water circulates. The displacer D is lined on the lower end with brick dust B.

Compressed air at 150 lb. per sq. in. is delivered into the air vessel A to keep up the average pressure, and supply any waste by leakage, the same air being used over again continuously as working substance. When the plunger D is raised to the top of its stroke, the air passes from the top end A down through the regenerator, taking in heat and rising to the higher temperature T_1 of the furnace.

Then 1 lb. of air receives heat from the furnace at constant temperature T_1, and expands from volume v_1 to v_2, along the isothermal AB on the ideal indicator diagram, Fig. 38. The pressure drives up the piston in the motor cylinder at C, Fig. 37 (cylinder itself not shown), and the heat received is equal to the work done, that is, $Q_1 = RT_1 \log_\varepsilon r$, where the ratio of expansion $r = \frac{v_2}{v_1}$.

During operation BC, the plunger D, in its downstroke, passes the hot air out of H up through the regenerator to the cold end, W, and the top of the air vessel. The air leaves heat stored up in the regenerator, and falls in temperature to T_2 at constant volume, the pressure falling with the temperature. The heat stored in the regenerator is $C_v(T_1 - T_2)$ calories per lb.

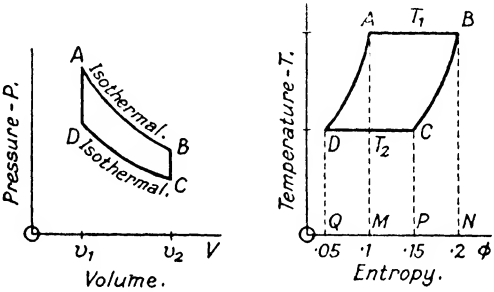

FIG. 38 FIG. 39

STIRLING CYCLE

During stroke CD, the air is compressed from volume v_2 to v_1, at constant temperature T_2, being in contact with the cooler W, Fig. 37. The work done *on* the air is equal to the heat rejected,

$$Q_2 = RT_2 \log_\varepsilon r$$

During DA, the plunger D is raised and the air passed down through the regenerator, at constant volume, picking up the same quantity of heat previously stored there, $C_v(T_1 - T_2)$, and having its temperature raised to T_1 in its initial condition of volume, pressure, and temperature.

During the cycle, the work done = heat supplied − heat rejected,

that is, $Q_1 - Q_2 = RT_1 \log_\varepsilon r - RT_2 \log_\varepsilon r$

and the efficiency is

$$\frac{\text{Heat converted into work}}{\text{Heat supplied}}$$

or,
$$\frac{Q_1 - Q_2}{Q_1} = \frac{RT_1 \log_\varepsilon r - RT_2 \log_\varepsilon r}{RT_1 \log_\varepsilon r} = \frac{T_1 - T_2}{T_1} \quad . \quad (19)$$

Also
$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

If the efficiency of the regenerator is e, the heat received by the air from the regenerator during the operation DA is $e\,C_v(T_1 - T_2)$.

Now total heat supplied is $RT_1 \log_\varepsilon r + (1 - e)\,C_v(T_1 - T_2)$

and efficiency
$$= \frac{R(T_1 - T_2) \log_\varepsilon r}{RT_1 \log_\varepsilon r + (1 - e)\,C_v(T_1 - T_2)} \quad . \quad . \quad (20)$$

In practice, e was about 0·8 to 0·9.

The *ideal temperature entropy diagram* of the Stirling regenerative engine is $ABCD$, Fig. 39. In the isothermal operation, AB, suppose 1 lb. of air takes in $Q_1 = 100$ heat units at $T_1 = 1{,}000°$ C. (absolute), the gain of entropy is $\frac{Q_1}{T_1} = \frac{100}{1{,}000} = 0{\cdot}1$. The heat received is represented by the area $ABNM$.

During BC the air is gradually cooled from T_1 to T_2 in passing through the regenerator where heat is stored at constant volume, measured by the area $NBCP$. If the regenerator action is perfect, it restores the same amount at every point in the passage of the air through it in the opposite direction DA, also at constant volume. Hence the curves BC and DA are precisely alike, and have equal areas $DAMQ$ and $CBNP$ under them. The loss of entropy NP, and gain of entropy QM, is $C_v \log_\varepsilon \frac{T_1}{T_2}$, since the fall and rise of temperature follow the logarithmic curve. (See p. 128.)

During the isothermal compression CD at T_2, the heat rejected is measured by the area $PCDQ$, which is $Q_2 = 600° \times 0{\cdot}1$, or 60 heat units.

The heat converted into work is $Q_1 - Q_2 = 100 - 60$, or 40 heat units.

Hence the thermal efficiency is 40 per cent.

Since $AB = DC$,
$$\frac{Q_1}{Q_2} = \frac{\text{area } MABN}{\text{area } PCDQ} = \frac{T_1}{T_2},$$

and the heat converted into work $= \frac{Q_1}{T_1}(T_1 - T_2)$.

Stirling's cycle is also perfect in the thermodynamic sense, and would be *reversible* if the regenerator had infinitely large capacity for heat.

The device of the regenerator for storing and restoring heat directly, as heat, effects the change from T_1 to T_2, and T_2 to T_1, instead of the adiabatic expansion and compression in the Carnot cycle, and greatly reduces the length of the piston stroke.

Stirling's engine appears to be the *earliest perfect heat engine* constructed, but air is a bad conductor of heat, and there was loss of heat by the *external furnace.*

The Ericsson Regenerative Air Engine. This engine worked on a cycle consisting of isothermal expansion and compression as in the Stirling cycle, but with passage of the air through the regenerator at constant pressure instead of constant volume (see Fig. 123, p. 368). Thus the *pv* diagram consists of two isothermals and two constant pressure lines, while the T-ϕ diagram is similar to that of the Stirling cycle, with constant pressure instead of constant volume lines. With a perfect regenerator the ideal efficiency is $\frac{T_1 - T_2}{T_1}$, that is the maximum possible between the two given temperatures.

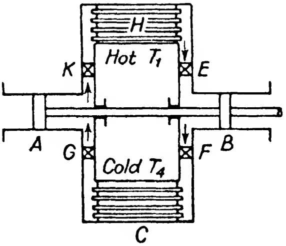

FIG. 40. JOULE'S AIR ENGINE

In **Joule's Air Engine** the air receives heat at the higher constant pressure in a large heated chamber H, after adiabatic compression, and rejects heat into a cold chamber C, at a lower constant pressure. Fig. 40 is a diagram of the essential parts.

Air is kept cool at absolute temperature, T_4, in a large chamber, C, by circulating water. The pump A takes in a little of this air from C, and compresses it adiabatically until its pressure is the same as in the hot chamber H, where it is further heated from T_2 to T_1 at constant pressure in tubes, by burning fuel. A little of the air then passes at constant pressure into the working cylinder B, until the valve E closes, and adiabatic expansion follows to atmospheric pressure. The air, at absolute temperature T_3, which is above the atmospheric temperature T_4, is discharged, through valve F, at the lower constant pressure, back again into the cold chamber. The supply of air from the pump to the hot chamber is so small that it does not sensibly alter the higher constant pressure. Fig. 41 is the pressure-volume diagram of this cycle. The area *FDAE* represents the work done *on* each pound of air by the pump; and area *EBCF* the work done *by* the air. The difference, *ABCD*, is the net amount of work done by each pound of air during one complete cycle. The numbers denote absolute temperatures. At the higher constant pressure *AB*, the ratio

$$\frac{T_2}{T_1} = \frac{\text{Volume } EA \text{ discharge by the pump}}{\text{Volume } EB \text{ admitted to the working cylinder}}$$

and at the lower pressure, *CF* is the volume of air discharged, while *FD* is the volume taken in by the pump. The volume swept by the pump piston is smaller than that of the working cylinder in this proportion.

The highest temperature reached is T_1 in the heated chamber *H*, Fig. 40, and the lowest T_4 at *D*, and between these limits the efficiency of the ideal perfect engine is $\frac{T_1 - T_4}{T_1}$. In the Joule engine the heat is not all taken in at the highest temperature T_1, nor is it all rejected at the lowest temperature, since T_3 is greater than T_4.

The heat taken in per lb. of air, $Q_1 = C_p(T_1 - T_2)$,
and heat rejected per lb. of air, $Q_2 = C_p(T_3 - T_4)$

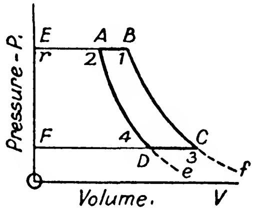

FIG. 41

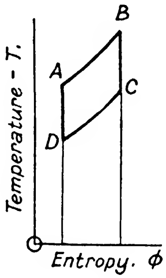

FIG. 42

CYCLE IN JOULE'S AIR ENGINE

In adiabatic expansion and compression between the same terminal pressures, the ratio of expansion and compression is the same; so that

$$\frac{T_1}{T_3} = \left(\frac{p_1}{p_3}\right)^{\frac{\gamma - 1}{\gamma}} = \frac{T_2}{T_4}, \text{ and } \frac{T_1}{T_2} = \frac{T_3}{T_4}, \text{ hence } \frac{T_3 - T_4}{T_4} = \frac{T_1 - T_2}{T_2}$$

Also, $$\frac{T_3 - T_4}{T_1 - T_2} = \frac{T_4}{T_2} = \frac{T_3}{T_1}; \text{ but } \frac{Q_2}{Q_1} = \frac{T_3 - T_4}{T_1 - T_2} = \frac{T_3}{T_1}$$

Therefore the thermal efficiency is

$$\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_3}{T_1} \quad . \quad . \quad . \quad (21)$$

and T_3 is greater than T_4.

Hence the efficiency is less than that of a perfect heat engine working between the same temperature limits.

The *entropy-temperature* diagram for this cycle is given in Fig. 42. The same letters refer to the corresponding operations in Fig. 41. *DA* is the adiabatic compression in the pump at constant entropy; then, during the change *AB*, the air receives heat in the hot chamber, at constant pressure, and the gain of entropy, $\phi_b - \phi_a = C_p \log_\varepsilon \frac{T_1}{T_2}$. *BC* is adiabatic expansion, and *CD* is the other logarithmic curve for the rejection of heat and cooling at constant pressure.

Example 2. Ten cu. ft. of air, at 65° F. and 90 lb. per sq. in. absolute, are expanded to 4 times the original volume, the law of expansion being $pv^{1\cdot 25}$ = constant. Find the change of entropy. Given K_v = 130·2 ft-lb., and $\gamma = 1\cdot 4$.

Here $T_1 = 65 + 460 = 525°$ F. (absolute)

and $\dfrac{T_2}{T_1} = \left(\dfrac{1}{4}\right)^{1\cdot 25 - 1} = 0\cdot 7071$

hence $T_2 = 525 \times 0\cdot 7071 = 371\cdot 2°$ F. (abs.) or $371\cdot 2 - 460 = -89°$ F.

Change of entropy

$$= C_v \cdot \frac{\gamma - n}{n - 1} \cdot \log_\varepsilon \frac{T_1}{T_2}, \text{ by equation (18) p. 129}$$

$$= \frac{130\cdot 2}{778} \times \frac{1\cdot 4 - 1\cdot 25}{1\cdot 25 - 1} \times 2\cdot 3026 \log_{10} \frac{525}{371}$$

$$= \frac{130\cdot 2}{778} \times 0\cdot 15 \times 4 \times 2\cdot 3026 \times 0\cdot 1508 = 0\cdot 0348$$

Answer.

Check by $\phi_2 - \phi_1 = \dfrac{R}{J} \cdot \dfrac{\gamma - n}{\gamma - 1} \cdot \log_\varepsilon \dfrac{v_2}{v_1}$

We have $R = K_v(\gamma - 1) = 130\cdot 2 \times 0\cdot 4 = 52\cdot 08$

and change of entropy

$$= \frac{52\cdot 08}{778} \times \frac{1\cdot 4 - 1\cdot 25}{1\cdot 4 - 1} \times 2\cdot 3026 \log_{10} \left(\frac{4}{1}\right) = 0\cdot 0348$$

Answer.

Example 3. If 1 lb. of air occupying 3 cu. ft. at 180 lb. per sq. in. absolute, and 537° C., expands at constant temperature to 12 cu. ft., what is its pressure after expansion, the work done, heat reception, and change of entropy ?

Here the temperature is kept constant and, by Boyle's law,

$$p_2 v_2 = p_1 v_1, \text{ or } p_2 = p_1 \times \frac{v_1}{v_2} = \frac{180 \times 3}{12} = 45 \text{ lb. per sq. in.}$$

Work done,

$$W = p_1 v_1 \log_\varepsilon r = 144 \times 180 \times 3 \log_\varepsilon \left(\frac{12}{3}\right)$$

but $\log_\varepsilon 4 = 2\cdot 3026 \times 0\cdot 60206 = 1\cdot 3863$,

$\therefore\ W = 144 \times 540 \times 1\cdot 3863 = 107{,}800$ ft-lb.

Since the temperature, and consequently the internal energy, are kept constant, the heat taken in by the air is equivalent to the work done by it, and is

$$\frac{107{,}800}{1{,}400} = 77 \text{ C.H.U., at } 537 + 273 = 810^\circ \text{ C. (abs.)}$$

$\therefore$ gain of entropy

$$= \frac{77}{810} = 0{\cdot}095 \text{ units.}$$

or, by equation (15), gain of entropy

$$= (C_p - C_v) \log_\varepsilon \frac{v_2}{v_1} = 0{\cdot}0685 \log_\varepsilon 4$$

$$= 0{\cdot}0685 \times 1{\cdot}3863 = 0{\cdot}095 \text{ units.} \qquad \textit{Answer.}$$

Example 4. A quantity of air having a volume of 2 cu. ft. at atmospheric conditions of 14·7 lb. per sq. in. and 15° C. is compressed according to the law $pv^{1\cdot15}$ = a constant, until its pressure is 120 lb. per sq. in. Find the change in the internal energy of the air and also the change in entropy. The specific heats for air at constant pressure and constant volume are 0·238 and 0·169. (*U.L., B.Sc.* (*Eng.*), 1925.)

Given $v_1 = 2$ cu. ft.

$p_1 = 14{\cdot}7 \times 144$ lb per sq. ft.

$T_1 = 15^\circ + 273^\circ = 288^\circ$ C. (abs.)

$\frac{R}{J} = 0{\cdot}238 - 0{\cdot}169 = 0{\cdot}069$ C.H.U. per lb.

$\gamma = \frac{C_p}{C_v} = \frac{0{\cdot}238}{0{\cdot}169} = 1{\cdot}408$

Let w = weight of air in lb.

then $w = \frac{p_1 v_1}{RT_1} = \frac{14{\cdot}7 \times 144 \times 2}{0{\cdot}069 \times 1{,}400 \times 288} = \frac{7}{46}$ lb.

Now, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{120}{14{\cdot}7}\right)^{\frac{1\cdot15-1}{1\cdot15}}$

$$\therefore \log T_2 = \log 288 + \frac{3}{23}(\log 120 - \log 14{\cdot}7)$$

$$= 2{\cdot}45939 + \frac{3}{23}(2{\cdot}07918 - 1{\cdot}16732)$$

$$= 2{\cdot}45939 + 0{\cdot}11894 = 2{\cdot}57833$$

$$\therefore T_2 = 378{\cdot}7^\circ \text{ C. (abs.)}$$

and the air gains internal energy $= w \,.\, C_v(T_2 - T_1)$

$$= \frac{7}{46} \times 0{\cdot}169(378{\cdot}7 - 288) = 2{\cdot}33 \text{ C.H.U.}$$

Also, by equation (18), the change of entropy,

$$\phi_2 - \phi_1 = C_v \left(\frac{\gamma - n}{n - 1}\right) \log_\epsilon \frac{T_1}{T_2}$$

$$= 0{\cdot}169 \left(\frac{1{\cdot}408 - 1{\cdot}15}{1{\cdot}15 - 1}\right) \times 2{\cdot}3026 \log_{10}\left(\frac{288}{378{\cdot}7}\right)$$

$$= 0{\cdot}169 \times \frac{0{\cdot}258}{0{\cdot}15} \times 2{\cdot}3026(-0{\cdot}11894) = -0{\cdot}0797$$

The negative sign means that during the change of state the air loses entropy in amount 0·0797 units. *Answer.*

The student should check these results by different methods—

We have $p_1 v_1^{1\cdot15} = p_2 v_2^{1\cdot15}$, or

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{1\cdot15}} = \left(\frac{120}{14{\cdot}7}\right)^{\frac{20}{23}}$$

$$\therefore \log \frac{v_1}{v_2} = \frac{20}{23}(\log 120 - \log 14{\cdot}7) = \frac{20}{23} \times 0{\cdot}91186 = 0{\cdot}79293$$

$$\therefore \quad \frac{v_1}{v_2} = 6{\cdot}208, \text{ and } v_2 = \frac{2}{6{\cdot}208} = 0{\cdot}3222 \text{ cu. ft.}$$

Or $T_1 \,.\, v_1^{n-1} = T_2 \,.\, v_2^{n-1} \quad \therefore \dfrac{v_1}{v_2} = \left(\dfrac{T_2}{T_1}\right)^{\frac{1}{n-1}} = \left(\dfrac{378{\cdot}7}{288}\right)^{\frac{1}{0\cdot15}}$

and $\quad \log \dfrac{v_1}{v_2} = \dfrac{20}{3}(\log 378{\cdot}7 - \log 288) = \dfrac{20}{3} \times 0{\cdot}11894 = 0{\cdot}79293$

$$\log v_2 = \log 2 - 0{\cdot}79293 = 0{\cdot}30103 - 0{\cdot}79293 = \bar{1}{\cdot}5081$$

$$\therefore \quad v_2 = 0{\cdot}3222 \text{ cu. ft.}$$

Also (p. 128), change of entropy,

$$\phi_2 - \phi_1 = \frac{R}{J} \cdot \frac{\gamma - n}{\gamma - 1} \cdot \log_\epsilon \frac{v_2}{v_1}$$

Substituting values,

$$\phi_2 - \phi_1 = 0{\cdot}069 \left(\frac{1{\cdot}408 - 1{\cdot}15}{1{\cdot}408 - 1}\right) 2{\cdot}3026 \log_{10} \frac{v_2}{v_1}$$

$$\therefore \phi_2 - \phi_1 = -0{\cdot}069 \times \frac{0{\cdot}258}{0{\cdot}408} \times 2{\cdot}3026 \times 0{\cdot}79293 = -0{\cdot}0797$$

Alternative Method, useful for students. Work done on the air during compression

$$= \frac{p_1 v_1}{1{\cdot}15 - 1}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{1\cdot15-1}{1\cdot15}} - 1\right\} = \frac{144 \times 14{\cdot}7 \times 2}{0{\cdot}15}\left\{\left(\frac{120}{14{\cdot}7}\right)^{\frac{3}{23}} - 1\right\},$$

$$= \frac{144 \times 14{\cdot}7 \times 2}{0{\cdot}15}(1{\cdot}315 - 1) = 8{,}891 \text{ ft-lb.}$$

Otherwise,

$$\text{Work done} = \frac{p_2v_2 - p_1v_1}{n-1} = \frac{144}{0{\cdot}15}(120 \times 0{\cdot}32218 - 14{\cdot}7 \times 2)$$

$$= 960(38{\cdot}662 - 29{\cdot}4) = 8{,}891 \text{ ft-lb.}$$

$$\text{The heat equivalent of this work} = \frac{8{,}891}{1{,}400} = 6{\cdot}35 \text{ C.H.U.}$$

Since $n < \gamma$, heat is given out by the air or rejected during compression, in amount

$$= \frac{\gamma - n}{\gamma - 1} \times \frac{\text{Work done (ft-lb.)}}{J} = \frac{1{\cdot}408 - 1{\cdot}15}{1{\cdot}408 - 1} \times 6{\cdot}35 \text{ C.H.U.}$$

$$= 4{\cdot}02 \text{ C.H.U.}$$

During compression of the air,

$$\text{Increase of internal energy} = \text{Heat received} - \text{Heat rejected}$$
$$= 6{\cdot}35 - 4{\cdot}02 = 2{\cdot}33 \text{ C.H.U.}$$

Again, the change in entropy may be calculated by either of the equations (12), (13) or (14) (pp. 127 and 128).

Thus, by (12),

$$\phi_2 - \phi_1 = C_v \,.\, \log_\varepsilon \frac{T_2}{T_1} + \frac{R}{J} \,.\, \log_\varepsilon \frac{v_2}{v_1}$$

$$= 0{\cdot}169 \log_\varepsilon \frac{378{\cdot}7}{288} + 0{\cdot}069 \log_\varepsilon \frac{0{\cdot}3222}{2}$$

$$= 0{\cdot}169 \times 2{\cdot}3026 \times 0{\cdot}11894 - 0{\cdot}069 \times 2{\cdot}3026 \times 0{\cdot}79293$$

$\therefore$ change in entropy $= 0{\cdot}04628 - 0{\cdot}12598 = -\,0{\cdot}0797$

Also by (13), when the pressure and volume are known,

$$\phi_2 - \phi_1 = C_v \,.\, \log_\varepsilon \frac{p_2}{p_1} + C_p \,.\, \log_\varepsilon \frac{v_2}{v_1}.$$

Substituting as above

$$= 0{\cdot}169 \times 2{\cdot}3026 \times 0{\cdot}91186 - 0{\cdot}238 \times 2{\cdot}3026 \times 0{\cdot}79293$$

$\therefore$ change in entropy $= 0{\cdot}35481 - 0{\cdot}43454 = -\,0{\cdot}0797$

Or (by 14),

$$\phi_2 - \phi_1 = C_p \,.\, \log_\varepsilon \frac{T_2}{T_1} - \frac{R}{J} \,.\, \log_\varepsilon \frac{p_2}{p_1},$$

$$= 0{\cdot}238 \times 2{\cdot}3026 \times 0{\cdot}11894 - 0{\cdot}069 \times 2{\cdot}3026 \times 0{\cdot}91186$$

$\therefore$ change in entropy $= 0{\cdot}06518 - 0{\cdot}14488 = -\,0{\cdot}0797$

Answer.

Example 5. A Stirling regenerative air engine works between temperatures of 700° F. and 80° F., the ratio of isothermal expansion is 2. Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator;

(*b*) when the efficiency of the regenerator is 0·9. Take $C_p = 0{\cdot}2375$ and $C_v = 0{\cdot}1691$.

Given $T_1 = 700 + 460 = 1{,}160°$ F. (abs.)
and $T_2 = 80 + 460 = 540°$ F. (abs.)
and $T_1 - T_2 = 620°$ F.

Also thermal equivalent of $R = C_p - C_v = 0{\cdot}2375 - 0{\cdot}1691 = 0{\cdot}0684$

$$(a)\ \text{Efficiency} = \frac{T_1 - T_2}{T_1} = \frac{1{,}160 - 540}{1{,}160} = \frac{620}{1{,}160} = 0{\cdot}5345,$$

or 53·45 per cent. *Answer.*

(*b*) By equation (20), (p. 133),

$$\text{Heat supplied, } Q_1 = R\,.\,T_1 \log_\varepsilon r + (1 - e)\,C_v(T_1 - T_2)$$

$$\text{Heat rejected, } Q_2 = R\,.\,T_2 \log_\varepsilon r + (1 - e)\,C_v(T_1 - T_2)$$

$$\text{Efficiency} = \frac{R(T_1 - T_2)\log_\varepsilon r}{R\,.\,T_1 \log_\varepsilon r + (1 - e)\,C_v(T_1 - T_2)}\ (\text{and} \log_\varepsilon 2 = 0{\cdot}69315)$$

$$= \frac{0{\cdot}0684 \times 620 \times 0{\cdot}69315}{0{\cdot}0684 \times 1{,}160 \times 0{\cdot}69315 + 0{\cdot}1 \times 0{\cdot}1691 \times 620}$$

$$= \frac{29{\cdot}4}{55 + 10{\cdot}48} = 0{\cdot}449, \text{ or } 45 \text{ per cent.}$$

Answer.

Example 6. In a double-acting Stirling engine working between the temperatures of 650° F. and 150° F., diameter of cylinder 16 in. by 4 ft. stroke, revolutions per minute 28, ratio of expansion 1·24, piston displacement per pound of air per stroke 1·06 cu. ft., brake horse-power 45·45. Estimate (*a*) the work done per pound of air per stroke; (*b*) thermal efficiency of the engine, assuming the efficiency of regenerator 0·9; (*c*) the mean effective pressure; and (*d*) indicated horse-power and mechanical efficiency.

Take $C_p = 0{\cdot}2375$, $C_v = 0{\cdot}1691$.

Given $T_1 = 650° + 460 = 1{,}110°$ F. (abs.)
and $T_2 = 150° + 460 = 610°$ F. (abs.)
Also $R = 778(0{\cdot}2375 - 0{\cdot}1691) = 778 \times 0{\cdot}0684$
$= 53{\cdot}2$ ft-lb. per lb.

(*a*) Work done per pound of air per stroke

$$= R(T_1 - T_2)\log_\varepsilon r \quad (\text{and } \log_\varepsilon 1{\cdot}24 = 0{\cdot}2151)$$

$$= 53{\cdot}2 \times 500 \times 0{\cdot}2151 = 5{,}722 \text{ ft-lb.}$$

(*b*) Heat supplied

$$= RT_1 \log_\varepsilon r + (1 - e)\,Kv\,(T_1 - T_2)$$

Substituting values,

$$\text{Heat supplied} = 53{\cdot}2 \times 1{,}110 \times 0{\cdot}2151 + 0{\cdot}1 \times 131{\cdot}6 \times 500$$

$$= 12{,}700 + 6{,}580 = 19{,}280 \text{ ft-lb.}$$

Thus the heat energy wasted in the generator, due to imperfections, is 6,580 ft-lb.

Hence the actual thermal efficiency of the engine

$$= \frac{\text{Work done by air on piston}}{\text{Heat supplied to engine}} = \frac{5{,}722}{19{,}280} = 0{\cdot}3 \text{ (nearly).}$$

The ideal efficiency with perfect regenerator having efficiency unity would be

$$\frac{T_1 - T_2}{T_1} = \frac{1{,}110 - 610}{1{,}110} = \frac{500}{1{,}110} = 0{\cdot}45$$

or, neglecting waste in regenerator,

$$= \frac{5{,}722 \text{ ft-lb.}}{12{,}700} = 0{\cdot}45$$

(c) Mean effective pressure

$$= \frac{\text{Work done per pound of air per stroke}}{\text{Volume swept by piston}} = \frac{5{,}722}{1{\cdot}06}$$

$$= 5{,}398 \text{ lb. per sq. ft.} = 37{\cdot}5 \text{ lb. per sq. in.}$$

(d) Sectional area of piston $= \pi(8)^2 = 201$ sq. in.

Work done by air on the piston per minute (two strokes per revolution) $= 37{\cdot}5 \times 201 \times 4 \times 28 \times 2$ ft-lb.

Hence indicated horse-power

$$= \frac{37{\cdot}5 \times 201 \times 4 \times 28 \times 2}{33{,}000} = 51{\cdot}2 \text{ I.H.P.}$$

Mechanical efficiency

$$= \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{45{\cdot}45}{51{\cdot}2} = 0{\cdot}888, \text{ or nearly 89 per cent}$$

and efficiency ratio

$$= \frac{\text{Thermal efficiency}}{\text{Ideal efficiency}} = \frac{0{\cdot}3}{0{\cdot}45} = 0{\cdot}667$$

Example 7. The air engines of the ship *Ericsson* worked on the Ericsson cycle between the temperature limits 122° F. and 414° F. Piston displacement per pound of air, 22 cu. ft.; ratio of expansion, 1·5; revolutions per minute, 9. Diameters of the four cylinders, each 14 ft., stroke 6 ft. Calculate (*a*) work done per pound of air per stroke; (*b*) thermal efficiency of engines; (*c*) heat energy wasted in the regenerator assuming its efficiency $e = 0{\cdot}9$; (*d*) mean effective pressure; (*e*) indicated horse-power. Take $K_p = 184{\cdot}8$ and $K_v = 131{\cdot}6$ ft-lb. per pound of air.

Given $T_1 = 414° + 460° = 874°$ F. (abs.)

$T_2 = 122° + 460° = 582°$ F. (abs.)

and $R = 184{\cdot}8 - 131{\cdot}6 = 53{\cdot}2$ ft-lb.

(*a*) Work done per pound of air per stroke

$$= R(T_1 - T_2) \log_\varepsilon r$$

$$= 53{\cdot}2(874 - 582) \log_\varepsilon 1{\cdot}5$$

$$\therefore \text{Work done} = 53{\cdot}2 \times 292 \times 0{\cdot}4055 = 6{,}300 \text{ ft-lb.}$$

(*b*) Heat supplied

$$= RT_1 \log_\varepsilon r + (1 - e) K_p (T_1 - T_2)$$

$$= 53{\cdot}2 \times 874 \times 0{\cdot}4055 + 0{\cdot}1 \times 184{\cdot}8 \times 292$$

$$= 18{,}860 + 5{,}396 = 24{,}256 \text{ ft-lb.}$$

$\therefore$ Thermal efficiency of engine

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{6{,}300}{24{,}256} = 0{\cdot}26.$$

(*c*) Here, heat wasted in the regenerator = 5,396 ft-lb.
With a perfect regenerator, thermal efficiency

$$= \frac{6{,}300}{18{,}860} = 0{\cdot}334, \text{ or } = \frac{874^\circ - 582^\circ}{874^\circ} = 0{\cdot}334.$$

(*d*) Mean effective pressure

$$= \frac{\text{Work done per pound of air per stroke}}{\text{Volume swept by piston}} = \frac{6{,}300}{22}$$

$$= 286 \text{ lb. per sq. ft.} = 2 \text{ lb. per sq. in. (nearly)}$$

(*e*) Sectional area of each cylinder

$$= \pi r^2 = \pi \times 7^2 = 154 \text{ sq. ft.}$$

Joint area of the four pistons

$$= 154 \times 4 = 616 \text{ sq. ft.}$$

$\therefore$ Work done per minute

$$= 286 \times 616 \times 6 \times 9 \text{ ft-lb.}$$

and therefore indicated horse-power

$$= \frac{286 \times 616 \times 54}{33{,}000} = 288$$

Answer.

EXAMPLES IV

1. Ten cu. ft. of dry air at 18° C. and 90 lb. per sq. in. absolute pressure, are expanded to 4 times the original volume, the law of expansion being $pv^{1\cdot25}$ = constant. Given that the specific heat of air at constant volume is 0·169, and at constant pressure 0·238, find the change in entropy.

2. One lb. weight of dry air occupies 3 cu. ft. at 110 lb. per sq. in. absolute pressure, and 227° C., and is expanded at this constant temperature to 12 cu. ft. Find the heat taken in and the change in entropy.

3. One lb. of dry air occupies 12·39 cu. ft. at 0° C. and 14·7 lb. per sq. in. absolute, and is compressed to 200 lb. per sq. in. absolute, (*a*) isothermally, (*b*) when the compression is $pv^{1\cdot2}$ = constant. What is (1) the work done on

the air in foot-pounds; (2) the amount of heat taken from the air during compression; and (3) the change of entropy in each case? Take the specific heat of air at constant pressure, 0·238, and at constant volume 0·17.

4. The lower limit of temperature of a Stirling engine is 1,000° F. (538° C.). The maximum and minimum pressures are 180 lb. per sq. in. absolute, and 30 lb. per sq. in. absolute, and the expansion ratio is 3. Calculate the ideal efficiency. (*U.L., B.Sc.* (*Eng.*).)

5. A Stirling engine, with perfect regenerator, works between pressures of 135 lb. per sq. in. absolute and 15 lb. per sq. in. absolute, and temperatures 550° F. and 50° F. respectively. Calculate the mean effective pressure on the piston.

6. In a Stirling engine working between the temperatures of 700° F. and 80° F., the ratio of isothermal expansion is 2. Calculate the ideal efficiency when (*a*) the engine is fitted with a perfect regenerator; (*b*) when the efficiency of the regenerator is 0·9. Take $C_p = 0{\cdot}2375$ and $C_v = 0{\cdot}1691$.

CHAPTER V

AIR COMPRESSORS

An air compressor is any machine which takes in air, compresses, and delivers it to a receiver at a higher pressure ; and may be of (1) the ordinary cylinder and reciprocating piston type, or (2) the rotary type, including turbo-compressors.

An air motor is a machine which takes in air at the higher pressure, and in allowing the air to expand to the lower pressure, converts pressure energy of the air into mechanical work.

The Reciprocating Compressor (no internal friction losses). The ideal indicator diagram *ABCD* (Fig. 43) illustrates the action in the piston compressor, assuming water jacket cooling but with no internal losses by friction or eddies.

During the *suction* stroke, *AB*, volume v_1 cu. ft. of free air flows into the cylinder at atmospheric pressure p_1 lb. per sq. ft. absolute, and the work done *on* the piston is p_1v_1 ft-lb.

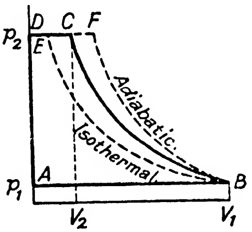

FIG. 43

BC is the *compression* curve, pv^n = a constant, the slope lying between the adiabatic *BF* and isothermal *BE*. The work done *on* the air during compression to absolute pressure p_2, and volume v_2, is $\frac{p_2v_2 - p_1v_1}{n-1}$. (See Eq. 4, p. 13).

When there is a water jacket around the cylinder, *n* may vary from 1·35 to 1·25 for thoroughly efficient cooling ; and in the special case of cooling the air during compression, by spraying water into the cylinder, which is drained away from the discharged air, *n* may be 1·2. Lower values of *n* are probably due to leakage past the piston or valves.

CD is the delivery of the air from the cylinder to a receiver at constant pressure p_2, and the work done during this stroke is p_2v_2.

The total effective work of suction, compression, and discharge in foot-pounds per volume v_1 of free air compressed per cycle is represented by the area *ABCD*, and is

$$W = \frac{p_2v_2 - p_1v_1}{n-1} + p_2v_2 - p_1v_1 = \frac{n}{n-1}(p_2v_2 - p_1v_1) \quad . \quad (1)$$

and since,

$$pv = wRT, \; W = \frac{n}{n-1} w \,.\, R(T_2 - T_1) \quad . \quad . \quad . \quad (2)$$

Also during compression, BC, the temperature is raised from T_1 to T_2 absolute, and

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Substituting this value of T_2 in (2), gives

$$W = \frac{n}{n-1} \cdot RT_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} \text{ per pound of air} \quad . \quad (3)$$

If the compression is adiabatic (no water cooling), the compression curve is BF (Fig. 43) and the total work done W is represented by the area $ABFD$, and is expressed by the equations given above with the value of $n = \gamma = 1{\cdot}4$ approximately for air, the value of v_2 being v_F and the value of T_2 being T_F.

With isothermal compression, the compression curve is BE and the work area is $ABED$ where

$$W = \int vdp = \int pdv = p_1 v_1 \log_\epsilon \frac{p_2}{p_1} = WRT_1 \log_\epsilon \frac{p_2}{p_1} = WRT_1 \log_\epsilon \frac{v_1}{v_E}$$

In assessing work done, one must be careful to differentiate between work done in the compression only ($\int pdv$) and the total work of suction, compression, and delivery ($\int vdp$).

The T-ϕ diagram for water-cooled compression is shown in Fig. 44, assuming no internal friction losses. As during the compression BC (Fig. 43) heat is abstracted from the air by the cooling water, there will be a decrease of entropy, and the compression curve will be bc (Fig. 44).

The heat abstracted Q is given by the equation

$$Q = \frac{\gamma - n}{\gamma - 1} \frac{R(T_2 - T_1)}{n - 1} \text{ per lb. of air. See Eq. 14, p. 83.}$$

The total work done W is given by the equation

$$W = \frac{n}{n-1} R(T_2 - T_1) \text{ per lb. of air}$$

and

$$W - Q = \frac{n}{n-1} R(T_2 - T_1) - \frac{\gamma - n}{\gamma - 1} \frac{R(T_2 - T_1)}{n-1} = \frac{\gamma R}{\gamma - 1}(T_2 - T_1)$$
$$= C_p(T_2 - T_1)$$

In any one of the above equations, Q, W and R must all be expressed in either heat or work units.

In Fig. 44—

Q is represented by the area *bchg*.

$W - Q$ is represented by the area *echk*.

Hence W is represented by the area *bcekg*.

Isothermal compression *BE* (Fig. 43) is shown as *be* in Fig. 44. The heat transferred Q which in this case is equal to the work done is represented by the area *bekg* and is given by the expression

$$Q = T_1 R \log_e \frac{p_2}{p_1}$$

Hence the excess of work in the case of normal water cooling over

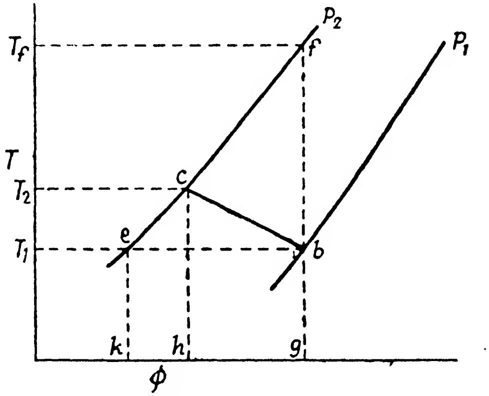

FIG. 44. T-ϕ DIAGRAM FOR RECIPROCATING COMPRESSOR

work in the case of isothermal compression is represented by the area *bce*.

Adiabatic compression *BF* is shown as *BF*, Fig. 43 and *bf*, Fig. 44. The heat transfer is nil and the work done is

$$W = C_p(T_f - T_b) = C_p(T_f - T_e)$$

This work is therefore represented by the area *fekg* and the excess of work in adiabatic compression over the actual work in cooled compression is represented by the area *bcf*.

It should be noted that the equations given above can also be used for a rotary compressor provided internal friction can be neglected.

Free Air Delivery. When stating the volume of air delivered by a compressor it is obviously necessary to state the pressure and temperature. This is generally done by giving the delivery in terms of "free air," namely the delivered volume in terms of standard intake conditions. In the absence of any given conditions these are generally taken as 14·7 lb. per sq. in. and 60° F.

Isothermal Compression Efficiency. In a water-cooled compressor as already shown the final discharge temperature at the point *C* (Fig. 44) will be greater than the isothermal temperature $T_E = T_B$, and the work done (area *ABCD*), Fig. 43, will be greater than the isothermal work (area *ABED*). The isothermal compression efficiency is hence defined as

$$\frac{\text{Isothermal H.P. on free air delivery}}{\text{Indicated H.P.}}$$

The overall isothermal efficiency is measured by the term

$$\frac{\text{Isothermal H.P. on free air delivery}}{\text{B.H.P. delivered to compressor shaft}}$$

Compressed Air Motors. Compressed air is used for a large variety of purposes and, on account of its safety, especially for the transmission of power in mines. After cooling in the compressor receiver, the compressed air flows through supply mains to the motor and, in the cylinder, does work by expansion. The cycle in the motor cylinder is like that of the air compressor reversed.

If the air after expansion is exhausted just above atmospheric pressure the temperature may be below the freezing point, then any moisture in the air would be deposited as snow, which would block the exhaust valves of the motor. In order to prevent snow and ice forming in the motor cylinder the air is warmed by steam and passed through a "preheater," or heating stove, at constant pressure, before admission to the motor. The volume of the air is thereby increased in proportion to the absolute temperature, and part of this heat energy is converted into additional work in the motor cylinder.

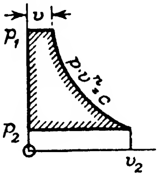

FIG. 45

Let 1 lb. of compressed air from the mains in the state p_1, v_1, and T_1 (absolute) be heated at constant pressure to T, so that

$$\frac{v}{v_1} = \frac{T}{T_1}, \text{ or } v = v_1 \times \frac{T}{T_1}$$

When admitted into the motor cylinder at constant pressure p_1, Fig. 45, the work done is $p_1 v$. Next, during expansion from absolute pressure p_1 to p_2, according to the law pv^n = constant, the work done $= \frac{p_1 v - p_2 v_2}{n - 1}$, and during exhaust at constant pressure p_2 the work done by the piston *on* the air $= -p_2 v_2$.

The total effective work done *by* the air during the cycle is

$$W = p_1 v + \frac{p_1 v - p_2 v_2}{n - 1} - p_2 v_2 = \frac{n}{n - 1}(p_1 v - p_2 v_2)$$

but $pv = RT$, and $W = \frac{n}{n - 1} \cdot R(T - T_2) = \frac{n}{n - 1} \cdot RT\left(1 - \frac{T_2}{T}\right)$

also $$\frac{T_2}{T} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Substituting this value, we have

$$W = \frac{n}{n-1} \cdot RT\left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\} \quad . \quad . \quad . \quad (4)$$

Example 1. An engine is supplied with compressed air at 90 lb. per sq. in. (abs.) and 65° F. The air is expanded according to the law $pv^{1 \cdot 3}$ = constant, down to 15 lb., and then exhausted at that pressure. Determine the pounds of air that will be used per hour per indicated horse-power, and calculate the temperature of the air at the end of expansion. Neglect losses due to clearance, etc.

(*U.L.*, *B.Sc.* (*Eng.*).)

Take 1 lb. of air at

$T_1 = 65° + 460° = 525°$ F. (abs.); and $p_1 = 90 \times 144$ lb. per sq. ft.

$$v_1 = \frac{RT_1}{p_1} = \frac{53 \cdot 2 \times 525}{90 \times 144} = 2 \cdot 1551 \text{ cu. ft. per lb.}$$

During expansion

$$p_1 . v_1^{1 \cdot 3} = p_2 . v_2^{1 \cdot 3} \quad \therefore v_2 = v_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{1 \cdot 3}} = 2 \cdot 1551 \times 6^{\frac{1}{1 \cdot 3}}$$

Taking logarithms,

$$\log v_2 = \log 2 \cdot 1551 + \frac{1}{1 \cdot 3} \log 6 = 0 \cdot 33347 + \frac{0 \cdot 77815}{1 \cdot 3} = 0 \cdot 93204$$

$$\therefore v_2 = 8 \cdot 5515 \text{ cu. ft.}$$

Work done by 1 lb. of air in motor $= \frac{n}{n-1}(p_1 v_1 - p_2 v_2)$

$$= \frac{1 \cdot 3}{0 \cdot 3} \times 144(90 \times 2 \cdot 1551 - 15 \times 8 \cdot 5515)$$

$$= 624(193 \cdot 959 - 128 \cdot 273) = 624 \times 65 \cdot 686 \text{ ft-lb.}$$

∴ Weight of air per indicated horse-power hour

$$= \frac{33{,}000 \times 60}{624 \times 65 \cdot 686} = 48 \cdot 31 \text{ lb.}$$ *Answer.*

$$\text{Now } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{1 \cdot 3 - 1} = \left(\frac{p_1}{p_2}\right)^{\frac{1 \cdot 3 - 1}{1 \cdot 3}} \therefore T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{0 \cdot 3}{1 \cdot 3}} = 525\left(\frac{1}{6}\right)^{\frac{3}{13}}$$

$$\text{and } \log T_2 = \log 525 - \frac{3}{13}\log 6 = 2 \cdot 72016 - \frac{3}{13} \times 0 \cdot 77815 = 2 \cdot 540586$$

$\therefore T_2 = 347 \cdot 2°$ F. (abs.), or $347 \cdot 2° - 460° = -112 \cdot 8°$ F. *Answer.*

Efficiency of a Compressed Air System. As already stated, the slope of the actual compression curve in a compressor is steeper than the isothermal curve, the final temperature is higher than the intake temperature and the actual work is greater than the isothermal work. This would not mean a loss if the air at the higher delivery temperature could be used directly in the air motor without loss of heat in transmission from the compressor to the motor. In practice, however, this is not the case.

It will be useful to consider the effect of this loss on the efficiency of the system as a whole, and for simplicity we will assume that the compression and expansion laws are the same, the pressure ratios in compressor and motor being equal, while the hot air leaving the compressor is cooled down in the transmission line to the original compressor intake temperature.

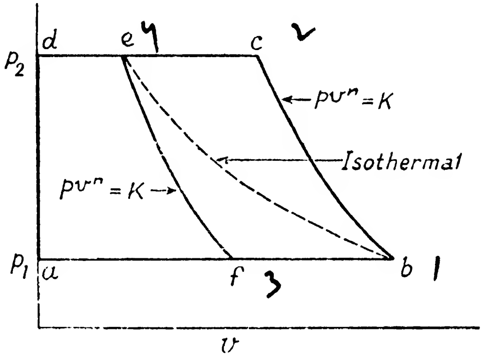

FIG. 46. COMPRESSED AIR SYSTEM

Then in Fig. 46 the work done on the air in the compressor

$$W_c = \frac{n}{n-1}(p_c v_c - p_b v_b) = \text{Area } abcd$$

The work done by the air in the motor is

$$W_m = \frac{n}{n-1}(p_e v_e - p_f v_f) = \text{Area } defa$$

The efficiency of the system

$$\frac{W_m}{W_c} = \frac{p_e v_e^n - p_f v_f^n}{p_c v_c^n - p_b v_b^n} = \frac{p_e v_e^n}{p_b v_b^n} \times \frac{1 - \dfrac{p_f v_f^n}{p_e v_e^n}}{\dfrac{p_c v_c^n}{p_b v_b^n} - 1}$$

But $\qquad p_b v_b = p_e v_e$ and $\dfrac{v_c}{v_b} = \left(\dfrac{p_b}{p_c}\right)^{\frac{1}{n}}$ and $\dfrac{v_f}{v_e} = \left(\dfrac{p_e}{p_f}\right)^{\frac{1}{n}}$

$$\text{Hence efficiency} = \frac{1 - \left(\dfrac{p_e}{p_f}\right)^{\frac{1-n}{n}}}{\left(\dfrac{p_c}{p_b}\right)^{\frac{n-1}{n}} - 1} = \frac{1 - \left(\dfrac{p_1}{p_2}\right)^{\frac{n-1}{n}}}{\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1}$$

Assuming adiabatic compression and expansion ($n = 1{\cdot}4$) and a pressure ratio of 10 this gives an efficiency of 0·518. This indicates that as a means of transmitting power over any but a short distance, the efficiency of the system is low. This is, however, compensated for by its simplicity and safety.

Multi-stage Compressors. In order to reduce the losses during the compression of air to high pressures, above, say, 120 lb. per sq. in. (gauge), the compression is usually carried out by two or three stages in separate cylinders ; and after each stage the air is cooled, at constant pressure, as nearly as possible down to the initial temperature, and thus reduced in volume, by passing through an *intercooler*, with circulation of cold water.

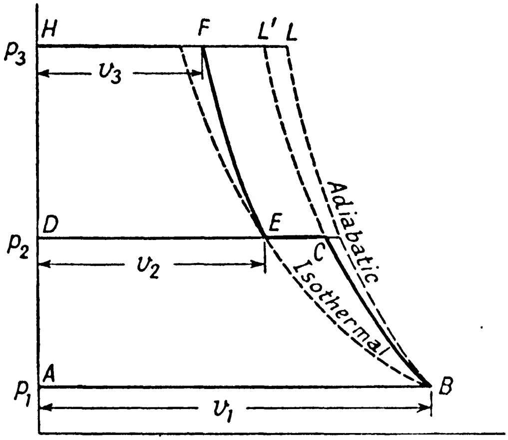

Fig. 47. Two-stage Compression

In modern practice, the copper coils used in small machines are being replaced by straight tube intercoolers, on the counter-current principle, with air passing through the tubes while the cooling water surrounds them and flows in the opposite direction, so that the air leaves the cooler almost at the temperature of the inlet water.

Fig. 47 shows two-stage compression with perfect intercooling. The length *AB* represents the volume, v_1, of air at pressure, p_1, taken into the low pressure cylinder, and *BC* is the compression curve according to some law $pv^n =$ constant.

The air is then cooled at constant pressure p_2 down, as near as possible, to the initial temperature, at *E*, and its volume reduced to v_2. The air is then delivered to the high pressure cylinder where compression follows the curve *EF*, having $pv^n =$ constant, until the pressure is p_3 at *F* and the volume *HF* is v_3.

Fig. 47 shows the saving in work by two-stage compression and intercooling. For single-stage compression the work is represented by the area ABL^1H. For two-stage compression the low pressure work is $ABCD$ and the high pressure work is $DEFH$. Hence the work saved is given by the area ECL^1F.

The work done in foot-pounds per pound of air per cycle, in the two cylinders, is

$$W = \frac{n}{n-1} \cdot RT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1}\ RT_1\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{n}{n-1} \cdot RT_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right] \quad . \quad . \quad (5)$$

The total work expended will be the least possible when

$$y = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}$$

is a minimum. Now, since p_1 and p_3 are constants, differentiate with respect to p_2, and we have

$$\frac{dy}{dp_2} = \frac{n-1}{n} \cdot p_2^{-\frac{1}{n}} \cdot \left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} + \frac{n-1}{n}\, p_3^{1-\frac{1}{n}} \cdot p_2^{\frac{1}{n}-2} = 0$$

for a minimum.

Divide by $p_2^{-\frac{1}{n}}$, and we have

$$\frac{n-1}{n} \cdot p_1^{\frac{1-n}{n}} = \frac{n-1}{n} \cdot p_3^{\frac{n-1}{n}} \cdot p_2^{\frac{2(1-n)}{n}}$$

from which

$$p_2^{\frac{2(1-n)}{n}} = \frac{p_1^{\frac{1-n}{n}}}{p_3^{\frac{n-1}{n}}}; \quad \therefore p_2^2 = p_1 \cdot p_3, \text{ or } \frac{p_2}{p_1} = \frac{p_3}{p_2} \quad . \quad . \quad (6)$$

Hence for the least work to be done, the ratios of compression in the two stages are equal, and $p_2 = \sqrt{p_1 \cdot p_3}$.

At the end of the first-stage compression in the low pressure cylinder the air is cooled in the intercooler from T_2 to T_1, at E on the isothermal and the ratio of the volumes of the two cylinders is

$$\frac{v_1}{v_2} = \frac{p_2}{p_1} = \sqrt{\frac{p_3}{p_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Also, since

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}, \text{ and } \frac{T_3}{T_1} = \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}}$$

we have

$$\therefore \frac{T_2}{T_1} = \frac{T_3}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

showing that the ratio of the initial and final temperatures during compression is the same for each stage when the work of compression is a minimum.

Substituting (6) in (5) and puting $RT_1 = p_1 v_1$, we have the minimum work done in ft-lb. per volume v_1 for two stage compression,

$$W = \frac{2n}{n-1} \cdot p_1 v_1 \left\{ \left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right\} \quad . \quad . \quad . \quad . \quad (9)$$

In *three-stage compression* the volume v_1 of free air at absolute pressure p_1 is compressed to pressure p_2 in the low pressure cylinder, and is then delivered at constant pressure through an intercooler to the intermediate pressure cylinder, its volume shrinking to v_2, and temperature from T_2 to T_1 on the isothermal. The volume of air v_2 at pressure p_2 is now compressed to pressure p_3 in the intermediate cylinder, and is then delivered through another intercooler to the high pressure cylinder, its volume being reduced to v_3, at constant pressure, and temperature T_3 to T_1.

In the high pressure cylinder the air is compressed to pressure p_4, and is then delivered to the receiver at this constant pressure p_4.

The work done in foot-pounds per pound of air per cycle in the three cylinders is

$$W = RT_1 \cdot \frac{n}{n-1} \cdot \left[\left\{ \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\} + \left\{ \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1 \right\} \right.$$

$$\left. + \left\{ \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} - 1 \right\} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

since $RT_1 = p_1 v_1 = p_2 v_2 = p_3 v_3$.

The same conditions hold for least work to be done, as in two-stage compression, and

$$\frac{p_2}{p_1} = \frac{p_3}{p_2} = \frac{p_4}{p_3} = \left(\frac{p_4}{p_1}\right)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Substituting (11) in (10), we have the least work done in foot-pounds per pound of air,

$$W = \frac{3n}{n-1} \cdot RT_1 \left\{ \left(\frac{p_4}{p_1}\right)^{\frac{n-1}{3n}} - 1 \right\} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The ratio of the volumes of the three cylinders will be

$$\frac{v_1}{v_2} = \frac{v_2}{v_3} = \left(\frac{p_4}{p_1}\right)^{\frac{1}{3}}$$

Clearance Volume. So far we have ignored the effect of clearance volume, that is the volume of air left in the clearance space at the end of the delivery stroke. This volume is represented by *ED* at a pressure p_2 in Fig. 48.

It will be seen that on the piston starting to move on its suction stroke, the clearance air must first expand along the curve *DF*, and air will not be drawn in until the pressure of this air has fallen to p_1 at the point *F*. Thus the theoretical volume of air drawn in at a pressure p_1 is represented by *FB* and not by the stroke volume *HB*.

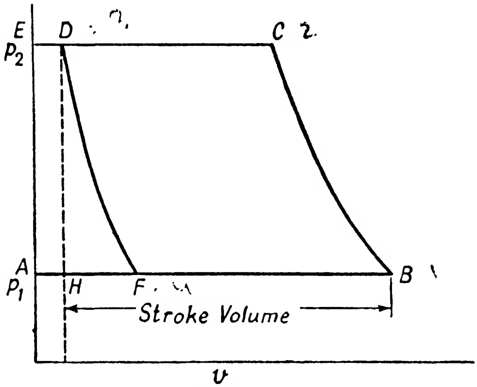

FIG. 48. EFFECT OF CLEARANCE

Let W_1 = weight of air compressed along the curve *BC*
W_2 = weight of air expanded along the curve *DF*

and assume laws of compression and expansion are the same. Then net work done

$$W = \underset{(\text{for } W_1 \text{ lb.})}{\text{Area } ABCE} - \underset{(\text{for } W_2 \text{ lb.})}{\text{Area } EDFA}$$

$$\therefore \quad W = \frac{n}{n-1}\left[(p_C v_C - p_B v_B) - (p_D v_D - p_F v_F)\right]$$

But as $pv = WRT$

then
$$W = \frac{n}{n-1}\left[W_1 R(T_C - T_B) - W_2 R(T_D - T_F)\right]$$

This expression may be simplified if it is assumed that the temperatures during intake and delivery respectively are constant, i.e.

$$T_B = T_F \text{ and } T_D = T_C$$

Then

$$W = \frac{n}{n-1} R(W_1 - W_2)(T_C - T_B)$$

where $W_1 - W_2$ = weight of air delivered per stroke.

Volumetric Efficiency. Volumetric efficiency is measured by the ratio of the volume of air delivered per stroke, corrected to free air, to the stroke volume swept by the piston.

For the ideal conditions assumed in the previous paragraph therefore the theoretical or "indicated" volumetric efficiency is measured by $\frac{FB}{HB}$, Fig. 48.

Practical considerations, however, do not follow these assumptions for various reasons. Thus there are differences of temperature between the clearance air and the fresh air drawn in, while the temperatures during intake and delivery are not necessarily constant. Again automatic spring-loaded valves require force to open them and leakage may take place past valves and pistons. Thus the actual volumetric efficiency which is measured by the ratio

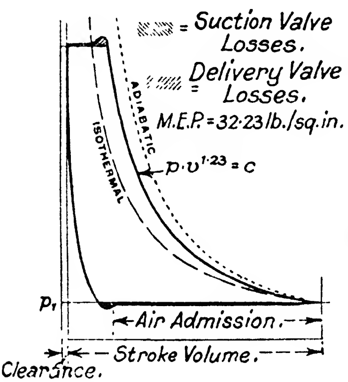

FIG. 49. SINGLE-STAGE COMPRESSION POSITIVELY OPERATED SUCTION VALVES. AUTOMATIC DELIVERY VALVES.

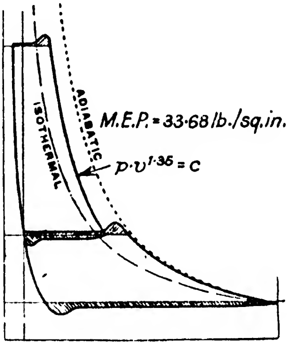

FIG. 50. TWO-STAGE COMPRESSION WITH INTERCOOLING AUTOMATIC SUCTION AND DELIVERY VALVES

$$\left.\begin{array}{r}\text{Power per cu. ft.}\\ \text{free air delivered}\end{array}\right\} = 0{\cdot}202 \text{ B.H.P.}$$

$$\frac{\text{Actual volume of free air delivered per stroke}}{\text{Piston stroke volume}}$$

is different from the indicated volumetric efficiency.

Figs. 49 and 50 show indicator diagrams for a single- and a two-stage machine.

Properly designed mechanical inlet valves ensure the cylinder filling with air at atmospheric pressure, while the spring-loaded automatic valve inevitably causes attenuation due to the fact that the atmospheric pressure has to compress the spring of the suction valve so that the pressure inside the cylinder will be less than atmospheric. Consequently, the equivalent air delivered, measured in terms of free air, will be less, and the volumetric efficiency will be less. It will be noticed, in Fig. 50, that there is loss where the delivery pressure in the first stage is higher than the intercooler pressure; and the suction pressure in the second stage is lower than the intercooler pressure, owing to the resistance of the spring-loaded valves. There are also considerable *inertia losses* due to starting and stopping the air-flow at the suction, and in the air main on delivery of the air.

The volumetric efficiency is about 85 to 90 per cent in well-designed air compressors.

Example 2. What is the horse-power required to drive an air compressor which takes 20 lb. of air per min. at 60° F. and atmospheric pressure, compresses it in one stage to 6 atmospheres, and delivers the air to a receiver at this pressure. Compression curve, $pv^{1\cdot25}$ = constant, and mechanical efficiency of the compressor 80 per cent. Neglect losses due to clearance, cooling, leakage, etc.

Here $T_1 = 60° + 460° = 520°$ F. (abs.), and, as in Fig. 43, we have

$$\text{Work done per pound of air} = \frac{n}{n-1} \cdot RT_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1\cdot25}{1\cdot25 - 1} \times 53\cdot2 \times 520 \left[\left(\frac{6}{1} \right)^{\frac{1\cdot25-1}{1\cdot25}} - 1 \right]$$

$$= 5 \times 53\cdot2 \times 520(1\cdot431 - 1) = 59{,}617 \text{ ft-lb.}$$

$\therefore$ Work for 20 lb. air per min. = 59,617 × 20 ft-lb. per min.

$$\therefore \text{Horse-power required} = \frac{59{,}617 \times 20}{33{,}000} \times \frac{100}{80} = 45\cdot16. \quad \textit{Answer}$$

To check, take the volume of 1 lb. dry air at N. T. P.

$$= 12\cdot39 \text{ cu. ft.} = \frac{1}{0\cdot0807}$$

$\therefore$ volume of 1 lb. dry air at 1 atmosphere and 60° F.

$$= 12\cdot39 \times \frac{520}{492} = 13\cdot09 \text{ cu. ft.}$$

$\therefore$ volume of 20 lb. of free air at 60° F. $= 13\cdot09 \times 20 = 261\cdot8$ cu. ft.

Now $p_1 \cdot v_1^{1\cdot25} = p_2 \cdot v_2^{1\cdot25}$, and $\left(\frac{v_2}{v_1} \right)^{\frac{5}{4}} = \frac{p_1}{p_2} = \frac{1}{6}$

$$\therefore v_2 = v_1 \times \left(\frac{1}{6} \right)^{\frac{4}{5}}, \text{ and } \log v_2 = \log 261\cdot8 - \frac{4}{5} \log 6 = 1\cdot7954$$

$$\therefore v_2 = 62\cdot43 \text{ cu. ft.}$$

$$\therefore\ W = \frac{n}{n-1}(p_2v_2 - p_1v_1) = \frac{1{\cdot}25}{0{\cdot}25} \times 144 \times 14{\cdot}7(6 \times 62{\cdot}43 - 1 \times 261{\cdot}8)$$

$$= 720 \times 1{,}657{\cdot}86 \text{ ft-lb.}$$

and horse-power required $= \dfrac{720 \times 1{,}657{\cdot}86}{33{,}000} \times \dfrac{100}{80} = 45{\cdot}2$ *Answer.*

The amount of heat that escapes through the cylinder walls during the compression may be estimated by Eq. (14), p. 83.

The work done W namely 59,617 ft-lb. per lb. is the total work done in drawing in, compressing, and delivering.

Hence the work done in compression only

$$= \frac{W}{n} = \frac{59{,}617}{1{\cdot}25} = 47{,}700 \text{ ft-lb. per lb.}$$

$$\therefore\ \text{Heat lost} = \frac{\gamma - n}{\gamma - 1} \times \text{work done in compression}$$

$$= \frac{1{\cdot}4 - 1{\cdot}25}{1{\cdot}4 - 1{\cdot}0} \times \frac{47{,}700 \times 20}{1{,}400} = 256 \text{ C.H.U. per min.}$$

Example 3. Air under atmospheric conditions of 15 lb. per sq. in. and 10° C., is drawn into the low pressure cylinder of a two-stage air compressor, and is compressed adiabatically to 100 lb. per sq. in. The air is then cooled at constant pressure to 20° C., and is drawn into the high pressure cylinder, where it is compressed to 600 lb. per sq. in., at which pressure it is discharged into the reservoir. Calculate the horse-power of the compressor which thus deals with 800 cu. ft. of air per hour measured at atmospheric conditions. (*U.L., B.Sc. (Eng.).*)

Let p_1, v_1, T_1 and p_2, v_2, T_2 be the pressure, volume, and temperature, respectively, at start and end of the first stage; also p_2, v_2', T_2' and p_3, v_3, T_3 at the start and end of the second stage.

Neglecting clearance volume, the work done in compressing 1 lb. of air under above conditions, by equation (3), is

$$W = \frac{\gamma}{\gamma-1}\cdot p_1v_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + \frac{\gamma}{\gamma-1}\cdot p_2v_2'\left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= \frac{\gamma}{\gamma-1}\cdot RT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] + \frac{\gamma}{\gamma-1}\cdot RT_2'\left[\left(\frac{p_3}{p_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$= \frac{1{\cdot}4}{0{\cdot}4} \times 96\left\{283\left[\left(\frac{100}{15}\right)^{\frac{0\cdot4}{1\cdot4}} - 1\right] + 293\left[\left(\frac{600}{100}\right)^{\frac{0\cdot4}{1\cdot4}} - 1\right]\right\}$$

$$= 336\left\{283\left(\frac{20^{\frac{2}{7}}}{3} - 1\right) + 293(6^{\frac{2}{7}} - 1)\right\}$$

$$= 336\{283(1{\cdot}7195 - 1) + 293(1{\cdot}6685 - 1)\}$$

$$= 336\{203{\cdot}62 + 195{\cdot}87\} = 336 \times 399{\cdot}49$$

$$= 134{,}230 \text{ ft-lb. per lb. of air}$$

The compressor has to deal with 800 cu. ft. of air per hour, measured under atmospheric conditions, i.e.

$v = 800$ cu. ft. when $p = 15$ lb. per sq. in., and $T = 283°$ C. (abs.)

Now $pv = w \,.\, RT$, where w = weight of air $= \frac{pv}{RT}$

$$\therefore\ w = \frac{144 \times 15 \times 800}{96 \times 283} = \frac{18{,}000}{283} = 63{\cdot}6 \text{ lb.}$$

$\therefore$ work in compressing 63·6 lb. of air per hour

$= 63{\cdot}6 \times 134{,}230$ ft-lb. per hour

$$\therefore \text{ horse-power of compressor} = \frac{63{\cdot}6 \times 134{,}230}{33{,}000 \times 60} = 4{\cdot}31 \text{ H.P.}$$

Answer.

Students may check this result by calculating the changes in volume of air by adiabatic compression, i.e. from 800 to 206·34 cu. ft. in the low pressure cylinder, and from 124·24 to 34·548 cu. ft. in the high pressure cylinder; also determine the amount of heat given per hour by the air to the water in the intercooler.

Example 4. Determine the size of cylinder for a double-acting air compressor of 50 I.H.P., in which the air is drawn in at 15 lb. pressure and 60° F., and compressed, according to the law $pv^{1\cdot2}$ = constant, to 90 lb. pressure. Revolutions per minute, 100; average piston speed, 500 ft. per min. Neglect clearance. (*U.L.*, *B.Sc.* (*Eng.*).)

$$\text{Given } p_1.v_1^{1\cdot2} = p_2.v_2^{1\cdot2},\ \therefore \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{1\cdot2}} = \left(\frac{90^{\frac{5}{6}}}{15}\right) = 6^{\frac{5}{6}} = 4{\cdot}451$$

where p_1 and p_2 are pounds per square inch, and v_1 and v_2 cubic feet.

$$\text{Mean effective pressure } p_m \text{ lb. per sq. in.} = \frac{\text{Work done per cycle}}{\text{Stroke volume}}$$

$$\text{Then } 144 \times p_m = \frac{1{\cdot}2}{0{\cdot}2}\left(\frac{p_2v_2 - p_1v_1}{v_1}\right) \times 144$$

$$\text{and} \qquad p_m = 6\left\{\frac{p_2}{\frac{v_1}{v_2}} - p_1\right\} = 6\left\{\frac{90}{4{\cdot}451} - 15\right\}$$

$$= 6(20{\cdot}2203 - 15) = 31{\cdot}3218 \text{ lb. per sq. in.}$$

Let A be the sectional area of cylinder in square inches; then $(A \times 31{\cdot}3218) \times 500 = 50 \times 33{,}000$ ft-lb. per min.

$$\therefore\ A = \frac{3{,}300}{31{\cdot}3218} = 105{\cdot}36 \text{ sq. in.}$$

$$\text{Diameter of cylinder} = \sqrt{\frac{105{\cdot}36}{0{\cdot}7854}} = 11{\cdot}58 \text{ in.}$$

Answer.

A double-acting compressor at speed 100 r.p.m. gives 200 piston strokes per min., and the average piston speed = 500 ft. per min.

$$\therefore \text{ piston stroke} = \frac{500}{200} = 2{\cdot}5 \text{ ft.} \qquad \textit{Answer.}$$

Example 5. A two-stage air compressor is to deliver air at 800 lb. per sq. in. pressure. The cylinders have the same stroke and the air is cooled to atmospheric temperature, 15° C., in the intercooler. Determine the ratio of the cylinder diameters so that the power required to drive the compressor shall be a minimum. Find the work required to compress and deliver a pound of air. Take atmospheric pressure as 15 lb. per sq. in. Assume adiabatic compression. (*U.L., B.Sc. (Eng.)*, 1924.)

In a two-stage compressor, assuming that air enters the second cylinder at atmospheric temperature, as in Fig. 47, and neglecting friction, leakage, etc., the work done in compressing 1 lb. of air from p_1 to p_2 lb. per sq. ft., by equation (5), p. 151, is

$$W = \frac{n}{n-1} \cdot p_1 v_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2 \right]$$

It has been shown (p. 151), by differentiating and equating to zero, that the above expression for work is a minimum when

$$p_2 = \sqrt{p_1 p_3},$$

from which it follows, by equation (9), p. 152, that

$$W = \frac{2n}{n-1} \cdot p_1 v_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

As air is at the same temperature when admitted to the high pressure cylinder as when admitted to the low pressure cylinder, then $p_1 v_1 = p_2 v_2$, also $p_2 = \sqrt{p_1 p_3} = \sqrt{15 \times 800} = 109{\cdot}544$

$$\therefore \frac{v_1}{v_2} = \frac{p_2}{p_1} = \frac{109{\cdot}544}{15} = 7{\cdot}3,$$

and the cylinders have same stroke.

$$\therefore \frac{d_1}{d_2} = \sqrt{7{\cdot}3} = 2{\cdot}7 = \text{ratio of cylinder diameters.} \qquad \textit{Answer.}$$

The work done to compress and deliver 1 lb. of air, by equation (9), as above,

$$W = \frac{2n}{n-1} \cdot RT_1 \cdot \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1 \right]$$

For air $n = \gamma = 1{\cdot}4$, and $\frac{n-1}{2n} = \frac{1{\cdot}4-1}{2{\cdot}8} = \frac{0{\cdot}4}{2{\cdot}8} = \frac{1}{7}$

$$\therefore W = \frac{2{\cdot}8}{0{\cdot}4} \times 96 \times 288 \left[\left(\frac{800}{15}\right)^{1/7} - 1 \right] \text{ ft-lb.}$$

$$= 193{,}536\,[1{\cdot}764875 - 1] = 148{,}030 \text{ ft-lb.} \qquad \textit{Answer.}$$

Example 6. A single-stage air compressor displaces 1 cu. ft. per working stroke. The clearance is to be taken as 0·15 cu. ft. (this is excessive) for the purposes of this question. During the compression the index n can be taken as 1·3, and at the end of the delivery stroke the air in the clearance space is at a temperature of 100° C. The law of the expansion of the clearance air is $n = 1{\cdot}35$. Find the actual volume of air, at atmospheric pressure and temperature, sucked in per stroke. Assume suction pressure 14·7 lb. per sq. in. and temperature of admitted air 14° C. Delivery pressure at 80 lb. per sq. in.

(*U.L.*, *B.Sc.* (*Eng.*), 1920.)

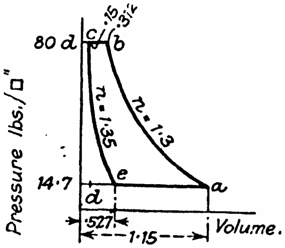

FIG. 51

Suppose the heat gained or lost to the cylinder walls is negligible, and the clearance air expands to 14·7 lb. per sq. in. before atmospheric air is freely admitted.

To find the temperature and volume at end of expansion of clearance air,

$$p_1.v_e^{1.35} = p_2.v_c^{1.35}, \text{ or } \left(\frac{v_e}{v_c}\right)^{1.35} = \frac{p_2}{p_1}$$

$$v_e = v_c\left(\frac{p_2}{p_1}\right)^{\frac{1}{1.35}} = 0{\cdot}15\left(\frac{80}{14{\cdot}7}\right)^{\frac{1}{1.35}} = 0{\cdot}15 \times 3{\cdot}5076 = 0{\cdot}526 \text{ cu. ft.}$$

On Fig. 51, ad = piston displacement or stroke volume = 1 cu. ft.

Weight of clearance air

$$= \frac{p_2 \cdot v_c}{R \cdot T_c} = \frac{80 \times 144 \times 0{\cdot}15}{96 \times 373} = \frac{18}{373} = 0{\cdot}04826 \text{ lb.}$$

Let temperature of clearance air at end of expansion be T_e (abs.)

$$\text{then} \quad \frac{T_c}{T_e} = \left(\frac{p_2}{p_1}\right)^{\frac{1.35-1}{1.35}} = \left(\frac{80}{14{\cdot}7}\right)^{\frac{7}{27}} = 1{\cdot}5515$$

$$\therefore\ T_e = \frac{T_c}{1{\cdot}5515} = \frac{373}{1{\cdot}5515} = 240{\cdot}4^\circ \text{ C. (abs.).}$$

Again,

let T_a = temperature (°C. abs.) of air in cylinder at end of suction stroke,

volume $v_a = 1{\cdot}15$ cu. ft., since piston displacement is *one* cu. ft.

and pressure $p_1 = 14{\cdot}7$ lb. per sq. in.

$$\therefore \text{ weight of air in cylinder} = \frac{p_1 \cdot v_a}{R \cdot T_a} = \frac{14{\cdot}7 \times 144 \times 1{\cdot}15}{96 \times T_a} = \frac{25{\cdot}358}{T_a} \text{ lb.}$$

Neglecting work done during suction stroke, T_a is the resultant temperature obtained by mixing

$$\left(\frac{25{\cdot}358}{T_a} - 0{\cdot}04826\right) \text{ lb. of air at } 14^\circ \text{ C. } (287^\circ \text{ C. abs.})$$

with 0·04826 lb. of air at 240·4° C. (abs.).

$$\therefore T_a = \frac{287\left(\frac{25{\cdot}358}{T_a} - 0{\cdot}04826\right) + 0{\cdot}04826 \times 240{\cdot}4}{\frac{25{\cdot}358}{T_a}}$$

$$25{\cdot}358 = \frac{7{,}277{\cdot}6}{T_a} - 13{\cdot}85 + 11{\cdot}6 = \frac{7{,}277{\cdot}6}{T_a} - 2{\cdot}25$$

$$\therefore T_a = \frac{7{,}277{\cdot}6}{27{\cdot}608} = 263{\cdot}6^\circ \text{ C. (abs.).}$$

Hence weight of air drawn into cylinder

$$= \frac{25{\cdot}358}{263{\cdot}6} - 0{\cdot}0482 = 0{\cdot}0962 - 0{\cdot}0482 = 0{\cdot}048 \text{ lb.}$$

and the volume of 0·048 lb. of air at 14·7 lb. per sq. in. and 14° C.

$$= \frac{0{\cdot}048 \times 96 \times 287}{14{\cdot}7 \times 144} = 0{\cdot}625 \text{ cu. ft.}$$

= volume of free air sucked in per stroke at atmospheric pressure and temperature. *Answer.*

To check this result,

$$p_a \cdot v_a^{1\cdot3} = p_b \cdot v_b^{1\cdot3} \quad \therefore \frac{v_a}{v_b} = \left(\frac{80}{14{\cdot}7}\right)^{\frac{1}{1\cdot3}} = 3{\cdot}6811$$

$$\therefore v_b = \frac{v_a}{3{\cdot}6811} = \frac{1{\cdot}15}{3{\cdot}6811} = 0{\cdot}3124 \text{ cu. ft.}$$

and $$\frac{T_b}{T_a} = \left(\frac{p_b}{p_a}\right)^{\frac{1\cdot3-1}{1\cdot3}} = \left(\frac{80}{14{\cdot}7}\right)^{\frac{3}{13}} = 1{\cdot}48$$

$$\therefore T_b = 263{\cdot}6 \times 1{\cdot}48 = 390^\circ \text{ C. (abs.)}$$

∴ weight of air in cylinder at *b*

$$= \frac{p_b v_b}{RT_b} = \frac{80 \times 144 \times 0{\cdot}3124}{96 \times 390} = 0{\cdot}096 \text{ lb.}$$

But the weight of clearance air = 0·048 lb.

∴ weight of air delivered = 0·096 − 0·048 = 0·048 lb. per cycle, which is the weight of air sucked in per stroke, neglecting friction and leakage past the piston and at valves.

Rotary Compressors. Rotary compressors may be classified into three types—

(i) Positive displacement types as in the Rootes blower.
(ii) Radial flow or centrifugal flow types (Turbo-compressor).
(iii) Axial flow types.

In the positive type the rate of air flow is dependent on the speed and is limited, while the pressure is more or less independent of the speed. The second and third types can be run at high speeds, and can therefore give high rates of flow, although the pressure and rate of flow are interdependent. The pressure can be increased by having more than one stage of compression. Single-stage machines giving a small pressure ratio are termed "blowers" and are used to supply blast air to furnaces, as boosters for gas flow, and for scavenging and supercharging internal combustion engines. They can also be used for exhausting. Multi-stage units are used for supplying air to pneumatic appliances, such as drills and coalcutters, and are also incorporated in gas turbine units.

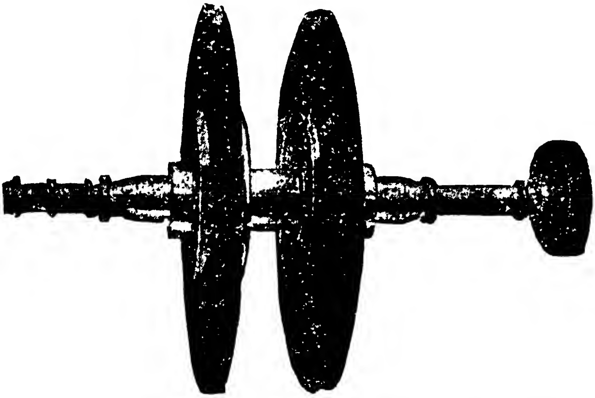

FIG. 52. TWO-STAGE ROTOR OF A TURBO-COMPRESSOR

Fig. 52 shows a two-stage rotor of a turbo-compressor and Figs. 53 and 54 are sections through one stage of a high pressure multi-stage unit. The three essential parts are *A* the rotor with impeller blades, *B* the diffuser guide blades, fitted in the annular space of the stationary casing surrounding the impeller disks, and *C* the fixed guide blades required to guide the air from the periphery of one stage to the eye of the impeller of the next stage. The arrows show the direction of flow of the air into the eye of the impeller through the diffuser, and between the guide blades, so passing into the next stage.

The action of the machine is as follows: the impeller in rotating, produces a centrifugal head on the air within it, and thus creates a flow of air outwards and an increase of pressure from the centre outwards. The fixed diffuser blades are so shaped as to give an increased area of passage to the air in passing outwards, thus reducing the velocity and giving a further increase of pressure. This process is repeated in each stage.

Cooling. In the piston-type compressor, the temperature at the end of compression is usually lower than that due to frictionless adiabatic compression by reason of the heat transfer to the water jackets, but in a rotary compressor, owing to skin friction in the blades and eddying in the air, the maximum temperature at the outlet of the diffuser blades may be above that for adiabatic compression for the same compression ratio. In any case it is not advisable to neglect this internal heating in the case of a rotary compressor, as was done with the reciprocating type.

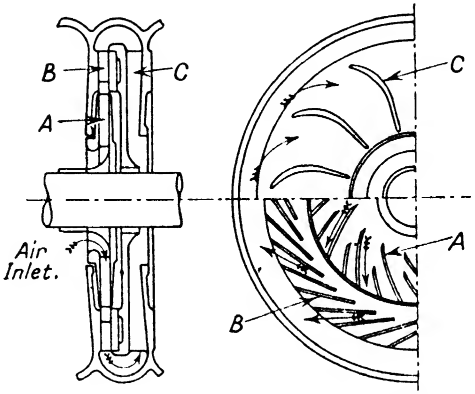

Fig. 53 Fig. 54

Reavell Turbo-Compressor

The casing of a turbo compressor may have a water jacket, and in addition intercoolers may be placed between each group of say three stages. In some cases of blowers and compressors of comparatively moderate pressure ratios, such as those used in gas turbine plants, the machine is uncooled, except for any loss by radiation.

The Uncooled Rotary Compressor (with internal friction). In terms of the original definition of adiabatic compression the uncooled compressor, through which air is passing at a high rate, may be assumed to fulfil the necessary condition of no heat transfer to or from exterior surroundings. Owing to internal friction and eddying of the air, however, a certain amount of mechanical energy is converted into heat energy, which will cause the specific volume of the air at any point during the compression to be greater than what it would have been with frictionless adiabatic compression. To avoid misconception this latter is termed "isentropic" compression, while the actual compression may be termed "polytropic."

Owing to the internal heating, the p-v condition curve may be considered as being *ab* (Figs. 55 and 56), where the isentropic compression is represented by *ab′*.

As heat has been generated in the air there will be an increase of entropy during the compression, and hence the two compression lines with corresponding lettering will be as shown in the two figures.

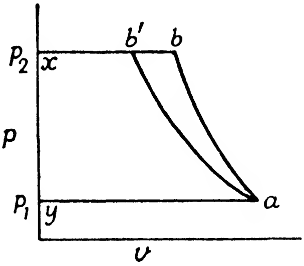

FIG. 55. p-v DIAGRAM

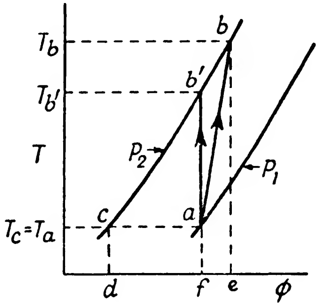

FIG. 56. T-ϕ DIAGRAM

The actual work done on the air may be obtained from the equation of steady flow (where all quantities are in the same units)

$$E_a + p_a v_a + \frac{U_a^2}{2g} + W = E_b + p_b v_b + \frac{U_b^2}{2g} + Q$$

(see p. 53).

As the change in kinetic energy may be considered negligible and Q the heat transfer to or from exterior surroundings is zero, the equation becomes—

$$\begin{aligned} W &= (p_a v_b - p_a v_a) + (E_b - E_a) \\ &= (p_b v_b - p_a v_a) + C_v(T_b - T_a) \\ &= R(T_b - T_a) + C_v(T_b - T_a) \\ &= C_p(T_b - T_a) \text{ heat units per lb. of air} \quad . \quad . \quad (13) \end{aligned}$$

if C_p is in heat units.

As $T_a = T_c$ this work is represented by the area *bcde* (Fig. 56). The work done if the compression were isentropic is given by the equation

$$W_i = C_p(T_{b'} - T_a) \text{ (see p. 131)}$$

As $T_a = T_c$ this work is represented by the area $b'cdf$ (Fig. 56). The extra work due to friction, above the isentropic work is

$$\begin{aligned} W_e &= \text{Actual work} - \text{Isentropic work} \\ &= C_p(T_b - T_a) - C_p(T_{b'} - T_a) = C_p(T_b - T_{b'}) \\ &= \text{Area } bcde - \text{Area } b'cdf = \text{Area } bb'fe. \end{aligned}$$

This work may be considered as in two parts—

(*a*) The heat generated by internal friction (area *abef*)

(*b*) The increase of $\int v \, . \, dp$ (area abb').

It should be noted that the area *abxy* (Fig. 55) does not represent the actual work done on the air, as the area abb' represents the increase of $\int v \, . \, dp$ only, above that for isentropic compression. The area *abxy* can, however, be considered as representing the useful work energy available in the air after compression. This useful or "polytropic" work is given by the equation—

$$\begin{aligned} W_p &= \frac{n}{n-1}(p_b v_b - p_a v_a) = \frac{n}{n-1} R\,(T_b - T_a) \\ &= \frac{n}{n-1} \cdot \frac{\gamma - 1}{\gamma} C_p\,(T_b - T_a) \end{aligned}$$

where the compression *ab* is assumed to follow the law pv^n = constant, *n* being the polytropic index.

$$\begin{aligned} \text{Hence } W_p &= \text{Isentropic work} + \text{Increase of} \int v.dp \\ &= \text{Area } b'cdf + \text{Area } abb' \\ &= \text{Area } abcdfa \end{aligned}$$

There are two efficiencies to consider (*a*) the isentropic efficiency and (*b*) the polytropic efficiency.

(*a*) Isentropic efficiency, ε_i—

$$\begin{aligned} \varepsilon_i &= \frac{\text{Work done if isentropic compression}}{\text{Actual work done on air}} \\ &= \frac{C_p(T_{b'} - T_a)}{C_p(T_b - T_a)} \\ &= \frac{\dfrac{T_{b'}}{T_a} - 1}{\dfrac{T_b}{T_a} - 1} \end{aligned}$$

But $$\frac{T_{b'}}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \text{ and } \frac{T_b}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \qquad \varepsilon_i = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{n-1}{n}} - 1} \text{ where } r \text{ is the pressure ratio } \frac{p_2}{p_1} \quad . \quad (14)$$

(*b*) Polytropic efficiency ε_p—

$$\varepsilon_p = \frac{\text{Work done with polytropic compression}}{\text{Actual work done on air}}$$

$$= \frac{\frac{n}{n-1} \cdot \frac{\gamma-1}{\gamma} C_p (T_b - T_a)}{C_p (T_b - T_a)}$$

$$= \frac{n}{n-1} \cdot \frac{\gamma-1}{\gamma} = \frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{n}} \quad . \quad . \quad . \quad . \quad (15)$$

(*c*) Relation between the efficiencies—

From Eq. 15

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma\, \varepsilon_p}$$

Substituting in Eq. 14

$$\varepsilon_i = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{\gamma-1}{\gamma\varepsilon_p}} - 1}$$

(*d*) Pressure ratio in terms of temperature—

It is required to express the pressure ratio $\frac{p_2}{p_1}$ in terms of the temperatures T_a and T_b, Figs. 55 and 56. For ideal isentropic compression

$$\frac{T_{b'}}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\text{i.e. } \frac{(T_{b'} - T_a) + T_a}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

But the isentropic compression efficiency is given by the expression

$$\varepsilon_i = \frac{(T_{b'} - T_a)}{T_b - T_a}$$

Hence from the last two equations

$$\frac{\varepsilon_i(T_b - T_a) + T_a}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

or

$$r = \frac{p_2}{p_1} = \left(\frac{\varepsilon_i \Delta T + T_a}{T_a}\right)^{\frac{\gamma}{\gamma-1}}$$

where ΔT is the temperature difference due to compression.

The Cooled Rotary Compressor (with internal friction). Let it be assumed that while internal friction may not be neglected, yet the cooling is sufficient to bring the actual compression curve *ab″* (Fig. 57) below isentropic conditions.

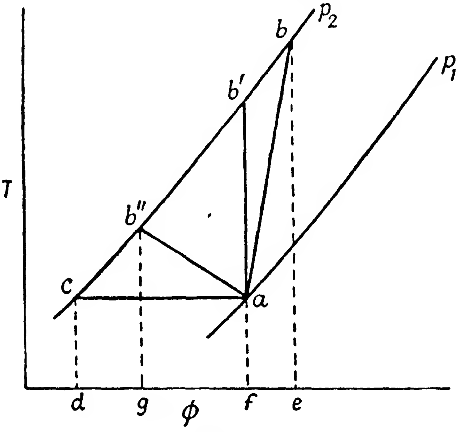

FIG. 57

For this purpose we may combine the two cases already considered, namely cooled compression neglecting internal friction and uncooled compression with internal friction.

The equation of steady flow for this case, neglecting any change in kinetic energy, becomes

$$E_a + p_a v_a + W = E_{b''} + p_{b''} v_{b''} + Q$$

where W is the work done on the air, and Q is the heat abstracted, all quantities being in the same units.

Hence

$$\begin{aligned} W &= Q + (E_{b''} - E_a) + (p_{b''}v_{b''} - p_a v_a) \\ &= Q + C_v(T_{b''} - T_a) + R(T_{b''} - T_a) \\ &= Q + C_p(T_{b''} - T_a) \end{aligned}$$

Now Q is made up of the heat which would be extracted for cooling with no friction (area *ab″gf*), and the extra heat to be extracted due to internal friction (area $abef$). Thus the work W is equivalent to the areas in Fig. 57.

$$ab''gf + abef + cb''gd = ab''cdf + abef$$

The isentropic compression efficiency is represented by the areas

$$\frac{b'cdf}{ab''cdf + abef}$$

The isothermal compression efficiency is represented by the areas

$$\frac{acdf}{ab''cdf + abef}$$

A special case would be that in which the cooling were such as to keep the temperature constant during compression. The condition curve would then be ac (Fig. 57) and the actual work done would be represented by the areas $acdf + abef$. It should be noted that although in this case the compression is at constant temperature, the isothermal efficiency $\dfrac{acdf}{acdf + abef}$ is not unity, as there is extra work equivalent to area $abef$ necessary due to internal friction.

To evaluate the various areas mentioned, we have

$$\text{Area } acdf \equiv RT_a \log\varepsilon \frac{p_2}{p_1}$$

$$b'cdf \equiv C_p(T_{b'} - T_c) = C_p(T_{b'} - T_a)$$

$$b''cdg \equiv C_p(T_{b''} - T_c) = C_p(T_{b''} - T_a)$$

$$abef \equiv \int_{T_a}^{T_b} T \,.\, d\phi$$

$$ab''gf \equiv \int_{T_a}^{T_{b''}} T \,.\, d\phi$$

To find the value of $\int T \,.\, d\phi$ between the temperatures T_1 and T_2 for the law pv^n = constant, we have

$$d\phi = C_p \frac{dT}{T} - R\frac{dp}{p} \text{ (from Eq. 14, p. 128)}$$

But if $\quad pv^n = \text{constant, then } \dfrac{dp}{p} = \dfrac{n}{n-1}\dfrac{dT}{T}$

and $$d\phi = C_p \frac{dT}{T} - R\frac{n}{n-1} \cdot \frac{dT}{T}$$

$$\therefore \quad \int_{T_1}^{T_2} T \,.\, d\phi = C_p \int_{T_1}^{T_2} dT - R\frac{n}{n-1}\int_{T_1}^{T_2} dT$$

$$= \left(C_p - R\frac{n}{n-1}\right)(T_2 - T_1)$$

Summary and Example 7. To summarize the various forms of compression considered let the following figures refer to Fig. 57.

$p_1 = 15$ lb. per sq. in. abs. $\qquad p_2 = 60$ lb. per sq. in. abs.

$T_a = 520°$ F. $\qquad T_{b'} = 866°$ F. $\qquad T_{b''} = 736°$ F. absolute

$\alpha = 1{\cdot}4 \qquad C_p = 0{\cdot}24$ B.Th.U. per lb. per degree F.

$R = 53{\cdot}2$ ft-lb. per lb. per degree F.

As ab' (Fig. 57) represents isentropic compression

$$T_{b'} = T_a\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 520 \times \left(\frac{60}{15}\right)^{\frac{0\cdot4}{1\cdot4}} = 773° \text{ F. abs.}$$

Let the law for uncooled compression with friction (ab, Fig. 57) be $pv^n =$ constant,

then
$$\frac{T_b}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

i.e.
$$\frac{866}{520} = 4^{\frac{n-1}{n}}$$

whence
$$n = 1{\cdot}582$$

Let the law for cooled compression with friction (ab'') be pv^m = constant,

then
$$\frac{T_{b''}}{T_a} = \left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}$$

i.e.
$$\frac{736}{520} = 4^{\frac{m-1}{m}}$$

whence
$$m = 1{\cdot}334$$

We can now evaluate the areas under the curves ab and ab''.

(*a*) The frictional heat represented by the area *abef* is given by the expression

$$\int_{T_a}^{T_b} T\,.\,d\phi = \left(C_p - R\frac{n}{n-1}\right)(T_b - T_a)$$
$$= \left(0{\cdot}24 - \frac{53{\cdot}2}{778} \times \frac{1{\cdot}582}{0{\cdot}582}\right)(866 - 520)$$
$$= 18{\cdot}75 \text{ B.Th.U. per lb.}$$

(*b*) The heat represented by the area *ab″gf* is given by

$$\int_{T_a}^{T_{b''}} T\,.\,d\phi = \left(C_p - R\frac{m}{m-1}\right)(T_{b''} - T_a)$$
$$= \left(0{\cdot}24 - \frac{53{\cdot}2}{778} \times \frac{1{\cdot}334}{0{\cdot}334}\right)(736 - 520)$$
$$= -7{\cdot}17 \text{ B.Th.U. per lb.}$$

This quantity is negative as there is a decrease in entropy. The necessary amount of work in the various cases may now be determined.

(i) Isentropic compression, Path *ab′*, area *b′cdf*

$$\begin{aligned} W &= C_p(T_{b'} - T_a) \\ &= 0{\cdot}24(773 - 520) \\ &= 60{\cdot}72 \text{ B.Th.U. per lb.} \end{aligned}$$

(ii) Isothermal frictionless compression, Path *ac*, area *acdf*

$$\begin{aligned} W &= RT_a \log_\varepsilon \left(\frac{p_2}{p_1}\right) \\ &= \frac{53{\cdot}2}{778} \times 520 \times 2{\cdot}3 \log_{10} 4 \\ &= 49{\cdot}2 \text{ B.Th.U. per lb.} \end{aligned}$$

(iii) Cooled frictionless compression, Path *ab″*, area *ab″cdf*

$$W = \frac{m}{m-1} R\,(T_{b''} - T_a) \quad . \quad . \quad \text{(see p. 145)}$$

$$\begin{aligned} &= \frac{1{\cdot}334}{0{\cdot}334} \times \frac{53{\cdot}2}{778} (736 - 520) \\ &= 59 \text{ B.Th.U. per lb.} \end{aligned}$$

(iv) Uncooled compression with friction, Path *ab*, area *bcde*

$$\begin{aligned} W &= C_p(T_b - T_a) \\ &= 0{\cdot}24(866 - 520) \\ &= 83{\cdot}04 \text{ B.Th.U. per lb.} \end{aligned}$$

The isentropic efficiency for this compression is given by the expression $\dfrac{\text{Work done if compression were isentropic}}{\text{Actual work done}}$

$$= \frac{60{\cdot}72}{83{\cdot}04} = 0{\cdot}731$$

This efficiency is also given by Eq. 14, p. 165,

i.e.
$$\varepsilon_t = \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{r^{\frac{n-1}{n}} - 1} = \frac{4^{\frac{0{\cdot}4}{1{\cdot}4}} - 1}{4^{\frac{0{\cdot}582}{1{\cdot}582}} - 1} = 0{\cdot}733$$

(v) Cooled compression with friction, Path *ab″*, area *ab″gf* + *abef* + *cb″gd*.

$$\begin{aligned} &= \int_{T_a}^{T_{b''}} T \, . \, d\phi + \int_{T_a}^{T_b} T \, . \, d\phi + C_p\,(T_{b''} - T_a) \\ &= 7{\cdot}17 + 18{\cdot}75 + 0{\cdot}24(736 - 520) \\ &= 77{\cdot}76 \text{ B.Th.U. per lb.} \end{aligned}$$

It should be noted that although the numerical value of $\int_{T_a}^{T_{b'}} T\,.\,d\phi$ is negative, the numerical values of the three areas must be added.

The isentropic efficiency of this compression is given by

$$\frac{\text{Work done if compression were isentropic}}{\text{Actual work done}}$$

$$= \frac{60{\cdot}72}{77{\cdot}76} = 0{\cdot}781$$

The Rotary Air Motor (Gas Turbine). In this machine we have the reversed action to the rotary compressor. Air or gas at the higher pressure, after passing through fixed guide blades, expands during the passage through the moving blade of the rotor to a lower pressure, work thus being done by the air on the rotor, the action being similar to that of the steam turbine.

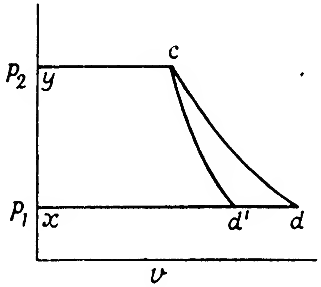

FIG. 58. *p-v* DIAGRAM

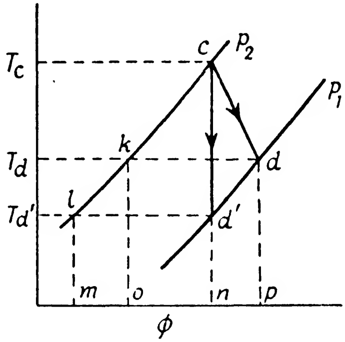

FIG. 59. *T-φ* DIAGRAM

If we assume the same conditions as in the case of the uncooled compressor, then there will be no loss of heat by radiation or cooling, but internal heat will be generated due to friction, eddies, etc. Hence as in the case of the compressor, there will be an increase of specific volume of the air during expansion, and also an increase in entropy.

These conditions are shown in Figs. 58 and 59 where the point *c* represents the state of the air on entry to the turbine, the point *d'* represents the state at exit for isentropic expansion, and the point *d* the actual state of exit.

As work is now done by the air on the rotor, the equation of steady flow becomes

$$E_c + p_c v_c + \frac{U_c^2}{2g} = E_d + p_d v_d + \frac{U_d^2}{2g} + W + Q$$

and making the same assumptions as in the case of the uncooled compressor this reduces to an expression for the useful work

$$\begin{aligned} W &= p_cv_c - p_dv_d + C_v(T_c - T_d) \\ &= R(T_c - T_d) + C_v(T_c - T_d) \\ &= C_p(T_c - T_d) \end{aligned}$$

As $T_d = T_k$ this work is represented by the area *ckon* (Fig. 59). This is the actual work produced by the air on the rotor.

If the expansion were isentropic the work obtained would be

$$W_i = C_p(T_c - T_{d'}) = C_p(T_c - T_l)$$

which is represented by the area *clmn*. This is obviously greater than the actual useful work obtained.

The loss of useful work due to internal friction is therefore

$$\begin{aligned} W_i - W &= C_p(T_c - T_d') - C_p(T_c - T_d) \\ &= C_p(T_d - T_d') \end{aligned}$$

This is represented by the area *dd'np* = area *klmo*.

The frictional heat is represented by the area under the expansion curve *cd* on the T-ϕ diagram, namely *cdpn*.

The increase of $\int v \, . \, dp$ due to friction is represented by the area *cdd'* on both the p-v and the T-ϕ diagrams.

The polytropic work W_p is represented by the area *ycdx* on the p-v diagram (Fig. 58), and if the law of the curve *cd* is taken as pv^n = constant, then

$$W_p = \frac{n}{n-1}(p_cv_c - p_dv_d) = \frac{n}{n-1} R(T_c - T_d)$$

There are two efficiencies to consider, (*a*) isentropic efficiency, and (*b*) polytropic efficiency.

(*a*)
$$\begin{aligned} \varepsilon_i &= \frac{\text{Actual useful work}}{\text{Isentropic work}} = \frac{W}{W_i} \\ &= \frac{C_p(T_c - T_d)}{C_p(T_c - T_{d'})} \\ &= \frac{1 - \dfrac{T_d}{T_c}}{1 - \dfrac{T_{d'}}{T_c}} \end{aligned}$$

But
$$\frac{T_d}{T_c} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} \text{ and } \frac{T_{d'}}{T_c} = \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$\therefore$
$$\varepsilon_i = \frac{1 - \left(\dfrac{1}{r}\right)^{\frac{n-1}{n}}}{1 - \left(\dfrac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}}$$

where r is the pressure ratio $\frac{p_2}{p_1}$ . . . . (16)

(*b*) Polytropic efficiency, ε_p—

$$\varepsilon_p = \frac{\text{Actual useful work}}{\text{Polytropic work}} = \frac{W}{W_p}$$

$$= \frac{C_p(T_c - T_d)}{\frac{n}{n-1} R\,(T_c - T_d)}$$

But $$R = C_p \frac{\gamma - 1}{\gamma}$$

$$\therefore \qquad \varepsilon_p = \frac{\frac{n-1}{n}}{\frac{\gamma-1}{\gamma}} = \frac{1 - \frac{1}{n}}{1 - \frac{1}{\gamma}} \qquad (17)$$

(*c*) Relation between the efficiencies—

From Eq. 17, $$\frac{n-1}{n} = \varepsilon_p \frac{\gamma - 1}{\gamma}$$

Substituting in Eq. 16

$$\varepsilon_i = \frac{1 - \left(\frac{1}{r}\right)^{\varepsilon_p \frac{\gamma-1}{\gamma}}}{1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}}$$

EXAMPLES V

1. A motor is supplied with air at 4 atmospheres pressure and 60° F. The air is expanded adiabatically in the motor cylinder to atmospheric pressure, and then exhausted at this pressure. (*a*) What is the temperature of the exhaust ? (*b*) Calculate the work done per pound of air, and the mean effective pressure in the motor cylinder. (*c*) How many cubic feet per minute of the compressed air are required to give 1 I.H.P., and what is the corresponding volume of "free air" ? Take the specific heat of air at constant pressure and constant volume, 0·24 and 0·1716, respectively.

2. A single-acting, single-stage air compressor is belt-driven from an electromotor at 400 r.p.m. The cylinder diameter is 6 in. and stroke 7 in. The air is compressed from 15 to 105 lb. per sq. in. (abs.), and the law of compression $pv^{1\cdot3}$ = constant. Find the horse-power of the motor, if transmission efficiency is 97 per cent and mechanical efficiency of the compressor 90 per cent. Neglect clearance effects. (*Grad. Inst. C. E.*)

3. Air is drawn into a cylinder and compressed adiabatically to a pressure of 75 lb. above its original pressure (15 lb. per sq. in.), and is then expelled at this pressure into a receiver; its original temperature was 60° F. In the receiver the compressed air cools down to its original temperature, and, in order to maintain a uniform pressure in the receiver, an equal weight of compressed air is constantly drawn off and expanded isothermally in a

working cylinder down to 15 lb. pressure. Calculate: (*a*) the work spent per pound of air in the compressor; (*b*) the work done per pound of air in expanding; (*c*) the temperature of the air as it enters the receiver.
(*U.L.*, *B.Sc.* (*Eng.*).)

4. Estimate the brake horse-power required to drive a single-stage air compressor which takes in 400 cu. ft. per min. at 15 lb. per sq. in. (abs.), compresses to 120 lb. per sq. in., and delivers it into a receiver at the higher pressure. The law of compression curve is $pv^{1 \cdot 26}$ = constant, and the mechanical efficiency of the compressor 88 per cent. Neglect losses due to clearance, cooling, and leakage.

5. An air compressor draws in 260 cu. ft. of air per min. at atmospheric pressure, compresses it adiabatically in one stage to 10 atmospheres, and delivers it to a receiver at this higher pressure. Calculate (*a*) the air horse-power, (*b*) the brake horse-power at the compressor shaft when the mechanical efficiency of the compressor is 86 per cent. Take $y = 1 \cdot 4$, and neglect all losses due to clearance, cooling, etc.

6. A single-stage double-acting air compressor of 40 I.H.P., at 120 r.p.m., takes in air at 15 lb. per sq. in. and delivers it at 120 lb. per sq. in. (abs.); the index of compression curve is $n = 1 \cdot 35$. Find the diameter of cylinder and length of stroke at the average piston speed of 600 ft. per min. Neglect clearance.

7. A two-stage compressor, with perfect intercooler, takes in air at atmospheric pressure and 17° C., and, after adiabatic compression to 9 atmospheres, delivers the air to a receiver at this pressure. Calculate (*a*) the minimum work done, and (*b*) the heat given to the intercooler per pound of air compressed. What would be (*c*) the least work done and heat given to the intercoolers in a three-stage compressor working under the same conditions? Take $C'_p = 0 \cdot 24$ for air.

8. A two-stage air compressor for a Diesel engine is to deal with 200 cu. ft. of air per min. under atmospheric conditions at 250 r.p.m., and to deliver it at 1,000 lb. per sq. in. Assuming complete intercooling, determine the indicated horse-power and the cylinder diameters and stroke. Allow a piston speed of 500 ft. per min., assume $pv^{1 \cdot 3} = C$ during compression, and neglect clearance and wire-drawing losses. Take atmospheric conditions as 14·7 lb. per sq. in., and 15° C. (*U.L.*, *B.Sc.* (*Eng.*).)

9. A single-acting two-stage air compressor compressing air for a Diesel engine has piston diameters 4 in. and 1·5 in., and a stroke in each cylinder of 3 in. Air is drawn into the low pressure cylinder at atmospheric conditions of 14·7 lb. per sq. in. and 15° C., and is cooled in the intercooler to 15° C. From the high pressure cylinder it is delivered into a storage bottle having a pressure of 600 lb. per sq. in. Neglecting clearance volumes, determine the pressure in the intercooler when the compressor is running steadily and the intercooler pressure is steady, and find the horse-power used in compressing air at 350 r.p.m. Take index of compression curves, $n = 1 \cdot 35$.
(*U.L.*, *B.Sc.* (*Eng.*), 1923.)

10. An air compressor working continuously compresses air adiabatically from atmospheric pressure and 288° C. abs. to a pressure at which the absolute temperature is T_2. The compressed air passes through a pipe without loss of pressure and is cooled to an absolute temperature T_3; the air then passes into the cylinder of an air-motor and expands adiabatically down to atmospheric pressure. Prove, neglecting clearance in the two cylinders, that the ratio "work developed by air motor : work put into compressor" is given by T_3/T_2.

If the higher pressure is 10 atmospheres and T_3 is 473° C. abs., calculate the value of this ratio.

$(C_p = 0 \cdot 238,\ C_v = 0 \cdot 169)$.

(*U.L.*, *B.Sc.* (*Eng.*), 1933.)

CHAPTER VI

GAS FLOW AND MEASUREMENT

Gas Flow through a Nozzle. Fig. 60 represents a convergent nozzle through which there is frictionless adiabatic (isentropic) gas flow.

Let p_1 = pressure at inlet in lb. per sq. ft.
p_b = back pressure beyond exit in lb. per sq. ft.
a = cross-sectional area of nozzle in sq. ft.
w = gas density in lb. per cu. ft.
W = gas flow in lb. per sec.
v = specific volume of gas in cu. ft. per lb.

Let (1), (2) and (t) refer to sections shown in Fig. 60.

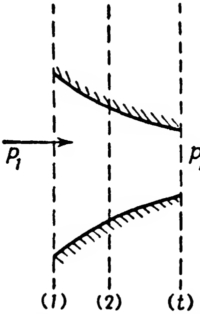

FIG. 60. NOZZLE

If the pressure p_b is only slightly lower than p_1, the flow W will be small, but increases as p_b decreases, until a certain critical value of p_b is reached, beyond which no further decrease of p_b will increase W. The pressure p_t at the throat remains constant as p_b decreases beyond the critical value.

To investigate this phenomenon, consider the two sections (1) and (2).

The equation of steady flow (p. 53), for one pound of gas

$$H_1 + \frac{U_1^2}{2g} = H_2 + \frac{U_2^2}{2g} + W + Q \text{ (all quantities in work units)}$$

becomes

$$\frac{U_2^2 - U_1^2}{2g} = H_1 - H_2 \text{ as } W \text{ and } Q \text{ are zero.} \quad . \quad (1)$$

But from p. 55, the adiabatic heat drop

$$H_1 - H_2 = \int_1^2 v\,.\,dp = \frac{\gamma}{\gamma - 1}(p_1v_1 - p_2v_2)$$

$$= \frac{\gamma}{\gamma - 1} p_1v_1 \left(1 - \frac{p_2v_2}{p_1v_1}\right)$$

$$= \frac{\gamma}{\gamma - 1} p_1v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]$$

Therefore if U_1 is considered negligible compared to U_2

$$U_2 = \sqrt{2g \frac{\gamma}{\gamma - 1} p_1v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \text{ ft. per sec.} \quad (2)$$

But $$W = a_2U_2w_2 \text{ and } \frac{w_2}{w_1} = \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

Hence $$W = a_2U_2w_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = a_2\frac{1}{v_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} U_2$$

$$\therefore \quad \frac{W}{a_2} = \frac{1}{v_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \sqrt{2g \frac{\gamma}{\gamma - 1} p_1v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]} \quad (3)$$

$$= \sqrt{2g \frac{\gamma}{\gamma - 1} \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma + 1}{\gamma}}\right]}$$

lb. per sq. ft. per sec. (4)

The flow per unit area of nozzle $\frac{W}{a_2}$ is obviously a maximum at the throat in plane (t), Fig. 60.

Hence writing (t) instead of (2) in the last equation—

$$\left[\left(\frac{p_t}{p_1}\right)^{\frac{2}{\gamma}} - \left(\frac{p_t}{p_1}\right)^{\frac{\gamma + 1}{\gamma}}\right] \text{ must be a maximum.}$$

Differentiating and equating to zero—

$$\frac{2}{\gamma}\left(\frac{p_t}{p_1}\right)^{\frac{2}{\gamma} - 1} - \frac{\gamma + 1}{\gamma}\left(\frac{p_t}{p_1}\right)^{\frac{\gamma + 1}{\gamma} - 1} = 0$$

whence $$\frac{p_t}{p_1} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} \quad . \quad . \quad . \quad . \quad (5)$$

Substiting this value of $\left(\frac{p_2}{p_1}\right) = \left(\frac{p_t}{p_1}\right)$ in Eq. 2

then $$U_t = \sqrt{\frac{2g\,\gamma}{\gamma + 1} p_1v_1} \text{ ft. per sec.} \quad . \quad . \quad . \quad (6)$$

Substituting the value of $\frac{p_t}{p_1}$ in Eq. 3

$$\frac{W}{a_t} = \frac{1}{v_1}\left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}}\sqrt{\frac{2}{\gamma + 1}\,g\,\gamma p_1 v_1}$$

$$= \sqrt{\left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma + 1}{\gamma - 1}} g\,\gamma\,\frac{p_1}{v_1}} \text{ lb. per sq. ft. per sec.} \qquad (7)$$

It is thus seen that provided the back pressure p_b is less than or equal to the critical pressure p_t as given in Eq. 5, the throat pressure is a definite constant proportion of the initial pressure p_1, irrespective of the throat area. Under these conditions the flow W is dependent only on the initial conditions and the throat area.

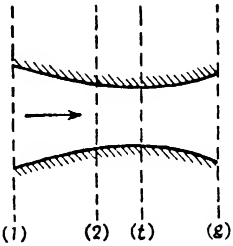

FIG. 61. CONVERGENT-DIVERGENT NOZZLE

If p_b is less than p_t then any addition to the nozzle of a divergent portion as in Fig. 61 will not increase the flow. The divergent portion will, however, increase the final velocity of the gas at exit (U_e), by changing the pressure to some value p_e lower than p_t. The values of U_e and $\frac{W}{A_e}$ in terms of p_1 and p_e can be found by substituting the suffix e for 2 in equations 2 and 4.

On p. 88 it was shown that the adiabatic heat drop

$$H_1 - H_2 = C_p(T_1 - T_2) \text{ where } C_p \text{ and } H \text{ are in work units.}$$

Hence Eq. 1 gives—

$$\frac{U_2^2 - U_1^2}{2g} = C_p(T_1 - T_2)$$

or neglecting U_1 compared with U_2

$$U_2 = \sqrt{2gC_p(T_1 - T_2)}$$

Hence $$\frac{W}{a_2} = U_2 w_2 = w_2\sqrt{2gC_p(T_1 - T_2)} \quad . \quad . \qquad (8)$$

Thus the discharge may be found in terms of temperatures.

Example 1. Air is expanded in a suitably shaped nozzle from a pressure of 100 lb. per sq. in. abs. to 14·7 lb. per sq. in. abs., the initial temperature being 200° F. Given that the critical pressure ratio for air is 0·527 when $\gamma = 1{\cdot}4$ and that the area at the throat is 0·35 sq. in., find the weight of air passing through the nozzle per second.

Assuming that there is no frictional loss before the throat, but that there is a 5 per cent frictional loss in the divergent part of the nozzle, find the

velocity and temperature of the air at exit. The velocity of approach to the nozzle inlet may be neglected. $R = 53{\cdot}3$ ft-lb.° F. units.
(*U.L., B.Sc.* (*Eng.*)), 1947.

Let 1 refer to the initial conditions and t refer to the throat.

Then $C_p = \dfrac{\gamma}{\gamma - 1} R = \dfrac{1{\cdot}4}{0{\cdot}4} \times 53{\cdot}3 = 187$ ft-lb. per lb. per ° F.

and $T_1 = 660°$ F. abs.

$$\therefore \quad T_t = T_1\left(\frac{p_t}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 660 \times (0{\cdot}527)^{\frac{0{\cdot}4}{1{\cdot}4}} = 550° \text{ F. abs.}$$

(*a*) $$U_t = \sqrt{2gC_p(T_1 - T_t)} = \sqrt{64{\cdot}4 \times 187(660 - 550)}$$
$$= 1{,}150 \text{ ft. per sec.}$$

Now $v_t = \dfrac{RT_t}{p_t}$

and $p_t = 0{\cdot}527 \times 100 = 52{\cdot}7$ lb. per sq. in. abs.

$$\therefore \quad v_t = \frac{53{\cdot}3 \times 550}{52{\cdot}7 \times 144} = 3{\cdot}86 \text{ cu. ft. per lb.}$$

and $$W = \frac{a_tU_t}{v_t} = \frac{0{\cdot}35 \times 1{,}150}{144 \times 3{\cdot}86} = 0{\cdot}725 \text{ lb. per sec.}$$

(*b*) Consider the expansion from throat to exit.

Let T_e = actual temperature at exit

T_{e1} = temperature at exit if flow were frictionless

Then $$T_{e1} = T_t\left(\frac{p_e}{p_t}\right)^{\frac{\gamma-1}{\gamma}} = 550\left(\frac{14{\cdot}7}{52{\cdot}7}\right)^{\frac{0{\cdot}4}{1{\cdot}4}} = 382° \text{ F. abs.}$$

and

Frictionless heat drop $= C_p(T_t - T_{e1})$

Actual heat drop $= 0{\cdot}95C_p(T_t - T_{e1})$

$= 0{\cdot}95 \times 187(550 - 382)$

$= 29{,}800$ ft-lb. per lb. per ° F.

$$\therefore \quad U_e = \sqrt{64{\cdot}4 \times 29{,}800 + U_t^2}$$
$$= \sqrt{64{\cdot}4 \times 29{,}800 + 1{,}150^2}$$
$$= 1{,}800 \text{ ft. per second.}$$

(*c*) $$T_t - T_e = 0{\cdot}95(T_t - T_{e1})$$

i.e. $$T_e = T_t - 0{\cdot}95(T_t - T_{e1})$$
$$= 550 - 0{\cdot}95(550 - 382)$$
$$= 390° \text{ F. abs.} = -70° \text{ F.}$$

General Flow Formula for Gases (U_1 not negligible). Let the same symbols and Fig. 61 as in the previous paragraph be used, and let $\frac{p_2}{p_1} = r$ and $\frac{a_2}{a_1} = x$.

Then from Eq. 1 in the previous paragraph

$$U_2^2 - U_1^2 = 2g\frac{\gamma}{\gamma-1}p_1v_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$= 2g\frac{\gamma}{\gamma-1}\frac{p_1}{w_1}\left[1-r^{\frac{\gamma-1}{\gamma}}\right] \quad . \quad . \quad (9)$$

But for continuity of flow

$$W = a_1U_1w_1 = a_2U_2w_2$$

$$\therefore \quad U_1 = \frac{a_2}{a_1}\frac{w_2}{w_1}U_2 = \frac{a_2}{a_1}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}U_2 = xr^{\frac{1}{\gamma}}U_2$$

Substituting in Eq. 9

$$U_2^2\left(1-x^2r^{\frac{2}{\gamma}}\right) = 2g\frac{\gamma}{\gamma-1}\frac{p_1}{w_1}\left(1-r^{\frac{\gamma-1}{\gamma}}\right)$$

i.e.

$$U_2 = \sqrt{\frac{2g\frac{\gamma}{\gamma-1}\frac{p_1}{w_1}\left(1-r^{\frac{\gamma-1}{\gamma}}\right)}{1-x^2r^{\frac{2}{\gamma}}}} \text{ ft. per sec.} \quad (10)$$

But

$$W = a_2w_2U_2 = a_2w_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}U_2 = a_2w_1r^{\frac{1}{\gamma}}U_2$$

$$\therefore \quad W = a_2w_1r^{\frac{1}{\gamma}}\sqrt{\frac{2g\frac{\gamma}{\gamma-1}\frac{p_1}{w_1}\left(1-r^{\frac{\gamma-1}{\gamma}}\right)}{1-x^2r^{\frac{2}{\gamma}}}}$$

$$= a_2\sqrt{\frac{2g\frac{\gamma}{\gamma-1}p_1w_1\left(1-r^{\frac{\gamma-1}{\gamma}}\right)r^{\frac{2}{\gamma}}}{1-x^2r^{\frac{2}{\gamma}}}} \text{ lb. per sec.} \quad . \quad (11)$$

In practice the right-hand side of this equation must be multiplied by a coefficient of discharge C_d which is determined by experiment. The equation can then also be used for flow through a sharp edged orifice where r is the pressure ratio through the orifice, and a_2 is the area of the orifice, and a_1 the area of the pipe.

It is evident from Eq. 11 that for maximum discharge conditions the expression $\dfrac{\left(1 - r^{\frac{\gamma-1}{\gamma}}\right) r^{\frac{2}{\gamma}}}{1 - x^2 r^{\frac{2}{\gamma}}}$ must be a maximum.

Therefore differentiating and equating to zero

$$\frac{2}{r^{\frac{\gamma+1}{\gamma}}} - \frac{\gamma+1}{r^{\frac{2}{\gamma}}} + x^2(\gamma - 1) = 0$$

If it is assumed that a_2 is small compared to a_1, that is that x is negligible, then

$$\frac{2}{r^{\frac{\gamma+1}{\gamma}}} = \frac{\gamma+1}{r^{\frac{2}{\gamma}}} \quad \text{i.e. } r_t = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

This is the same result as that found in the last paragraph.

It has been shown in the previous paragraph that the simple flow formula can be expressed in terms of temperature. This can also be done in the more detailed formula given above.

Thus $$\frac{U_2^2 - U_1^2}{2g} = H_1 - H_2 = C_p(T_1 - T_2)$$

and $$U_2^2\left(1 - x^2 r^{\frac{2}{\gamma}}\right) = 2gC_p(T_1 - T_2)$$

or $$U_2 = \sqrt{\frac{2gC_p(T_1 - T_2)}{1 - x^2 r^{\frac{2}{\gamma}}}} \text{ ft. per sec.}$$

and $$W = w_2 a_2 U_2$$

$$= w_2 a_2 \sqrt{\frac{2gC_p(T_1 - T_2)}{1 - x^2 r^{\frac{2}{\gamma}}}} \text{ lb. per sec.} \qquad (12)$$

Sonic Velocity. Equation 7 may be written

$$W = a_t \sqrt{g\,\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} p_1 w_1} \quad . \quad . \quad (13)$$

To investigate the velocity at the throat, we must put all the pertinent terms in terms applicable to the throat.

Thus $$p_1 w_1 = p_t w_t \times \frac{p_1 w_1}{p_t w_t} = p_t w_t \frac{p_1}{p_t}\left(\frac{p_1}{p_t}\right)^{\frac{1}{\gamma}} = p_t w_t \left(\frac{p_1}{p_t}\right)^{\frac{\gamma+1}{\gamma}}$$

Substituting the throat value of $\frac{p_1}{p_t}$ from Eq. 5

$$p_1 w_1 = p_t w_t \left[\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma+1}{\gamma}} = p_t w_t \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{\gamma-1}}$$

Substituting this value of $p_1 w_1$ in Eq. 13

$$W = a_t\sqrt{g\,\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma+1}{\gamma-1}} p_t w_t}$$

$$= a_t\sqrt{g\,\gamma\,p_t w_t} = a_t w_t\sqrt{g\,\gamma\,\frac{p_t}{w_t}}\,. \qquad . \quad . \quad . \ (14)$$

The expression $\sqrt{g\,\gamma\,\frac{p_t}{w_t}}$ is the velocity of sound (U_s) in the gas where p_t is in lb. per sq. ft. and w_t is in lb. per cu. ft.

Hence $W = a_t w_t U_s$ and the velocity at the throat is the sonic velocity for conditions at the throat, provided the back pressure p_b is not greater than the exit pressure p_e from the nozzle.

Equation 14 may be written

$$W = a_t w_t\sqrt{g\,\gamma\,RT_t}$$

Substituting the values for air as $\gamma = 1{\cdot}4$ and $R = 53{\cdot}3$ or 96

$$W = 49\,a_t w_t\sqrt{T_t} \text{ lb. per sec. for degrees F.}$$

$$\text{or } 65{\cdot}8\,a_t w_t\sqrt{T_t} \text{ lb. per sec. for degrees C.} \quad . \ (15)$$

From the foregoing we can obtain an expression for the velocity at the throat, in terms of the velocity at any other point in the nozzle, as follows.

The Bernoulli theorem for compressible flow states that for all sections along the nozzle

$$\frac{U^2}{2g} + \int\frac{dp}{w} = \text{constant}$$

But for isentropic flow $pv^\gamma = \frac{p}{w^\gamma} = \text{constant } K$, say.

$$\therefore \qquad \int\frac{dp}{w} = K^{\frac{1}{\gamma}}\int\frac{dp}{p^{\frac{1}{\gamma}}} = \frac{K^{\frac{1}{\gamma}}\,p^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} + \text{constant}$$

$$= \frac{\frac{p^{\frac{1}{\gamma}}}{w}\cdot p^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} + \text{constant}$$

$$= \frac{\gamma}{\gamma-1}\,\frac{p}{w} + \text{constant}$$

Hence the Bernoulli equation may be written

$$\frac{U^2}{2g} + \frac{\gamma}{\gamma - 1} \frac{p}{w} = \text{constant}$$

Now the sonic velocity U_s at any point is given by the expression

$$U_s^2 = g\,\gamma \frac{p}{w} \text{ and hence } \frac{p}{w} = \frac{U_s^2}{g\,\gamma}$$

Hence the Bernoulli equation becomes

$$\frac{U^2}{2g} + \frac{U_s^2}{g(\gamma - 1)} = \text{constant}$$

where U_s is the sonic velocity at the place where the actual velocity is U. Thus comparing the conditions at any one point with the conditions at the throat and using the suffix t for the throat, we have

$$\frac{U_t^2}{2g} + \frac{U_{st}^2}{g(\gamma - 1)} = \frac{U^2}{2g} + \frac{U_s^2}{g(\gamma - 1)}$$

It has, however, been shown that the actual velocity at the throat is the sonic velocity for the throat conditions, i.e. $U_t = U_{st}$.

Hence

$$\frac{U_t^2}{2g}\left(1 + \frac{2}{\gamma - 1}\right) = \frac{U_t^2}{2g}\left(\frac{\gamma + 1}{\gamma - 1}\right) = \frac{U^2}{2g} + \frac{U_s^2}{g(\gamma - 1)}$$

But $$U_s^2 = \frac{\gamma\, g p}{w} = \gamma\, gRT$$

where T is the absolute temperature where the velocity is U.

$$\therefore \qquad \frac{U_t^2}{2g}\left(\frac{\gamma + 1}{\gamma - 1}\right) = \frac{U^2}{2g} + \frac{\gamma g R T}{g(\gamma - 1)}$$

and $$U_t^2 = \frac{\gamma - 1}{\gamma + 1} U^2 + \frac{2\gamma g}{\gamma + 1} RT \quad . \quad . \quad . \quad (16)$$

To find the temperature at the throat we have

$$U_t^2 = U_{st}^2 = \gamma\, gRT_t \text{ and } U_s^2 = \gamma\, gRT$$

Hence $$T_t = T\left(\frac{U_t}{U_s}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

To find w_t we have for isentropic flow

$$\frac{w_t}{w} = \frac{v}{v_t} = \left(\frac{T_t}{T}\right)^{\frac{1}{\gamma - 1}} \text{ and } w_t = w\left(\frac{T_t}{T}\right)^{\frac{1}{\gamma - 1}}$$

Example 2. Air flows through a convergent-divergent nozzle. At some point (*a*) at the upstream end of the convergent part the pressure is 2 lb. per sq. in. gauge, and the temperature is 20° C., the sectional area being one square foot.

A manometer containing water connects this point (a) to another point (2) downstream but before the throat, the area at the point (2) being one-half square foot. This manometer shows a difference of pressure head of six feet of water.

The atmospheric pressure is 14 lb. per sq. in. abs., and it can be assumed that $\gamma = 1\cdot4$, and that the density of air is 0·0807 lb. per cu. ft. at N.T.P. The coefficient of discharge of the nozzle is 0·98.

Find (a) The air flow in lb. per minute.

(b) The area of the throat to give this discharge.

(c) The maximum back pressure p_b if the exit area a_e at the divergent end is one-half square foot.

(a) From Eq. 11

$$W = C_d \times 60 \times a_2 \sqrt{\frac{2g\dfrac{\gamma}{\gamma - 1}\, p_a w_a \left(1 - r^{\frac{\gamma-1}{\gamma}}\right) r^{\frac{2}{\gamma}}}{1 - x^2\, r^{\frac{2}{\gamma}}}} \text{ lb. per min.} \qquad (18)$$

where $p_a = (14\cdot7 + 2)144 = 2{,}400$ lb. per sq. ft.

$= 16\cdot7$ lb. per sq. in.

$$p_2 = 16\cdot7 - \frac{6 \times 62\cdot4}{144} = 14\cdot1 \text{ lb. per sq. in.}$$

$\equiv 2{,}030$ lb. per sq. ft.

$$r = \frac{14\cdot1}{16\cdot7} = 0\cdot845$$

$$r^{\frac{2}{\gamma}} = 0\cdot845^{1\cdot43} = 0\cdot786$$

$$r^{\frac{\gamma-1}{\gamma}} = 0\cdot845^{0\cdot286} = 0\cdot953$$

$$x^2 = \left(\frac{0\cdot5}{1}\right)^2 = 0\cdot25$$

$$w_a = 0\cdot0807 \times \frac{16\cdot7}{14\cdot7} \times \frac{273}{293} = 0\cdot0855 \text{ lb. per cu. ft.}$$

then $W = 0\cdot98 \times 60 \times 0\cdot5 \times$

$$\sqrt{64\cdot4 \times \frac{1\cdot4}{0\cdot4} \times \frac{2{,}400 \times 0\cdot0855}{1 - 0\cdot25 \times 0\cdot786}(1 - 0\cdot953)0\cdot786}$$

$= 1{,}355$ lb. per min.

Or alternatively from Eq. 12

$$W = C_d \times 60 \times w_2 a_2 \sqrt{\frac{2gC_p(T_a - T_2)}{1 - x^2 r^{\frac{2}{\gamma}}}} \text{ lb. per min.} \quad . \quad (19)$$

where

$$T_2 = T_a r^{\frac{\gamma-1}{\gamma}} = 293 \times 0\cdot953 = 279\cdot3^\circ \text{ C. abs.}$$

$$w_2 = 0{\cdot}0807 \times \frac{14{\cdot}1}{14{\cdot}7} \times \frac{273}{279} = 0{\cdot}0758 \text{ lb. per cu. ft.}$$

$$T_a - T_2 = 293 - 279{\cdot}3 = 13{\cdot}7^\circ \text{ C.}$$

$$C_p = \frac{\gamma}{\gamma - 1} R = \frac{1{\cdot}4}{0{\cdot}4} \times 96 = 336 \text{ ft-lb. per lb. per }^\circ \text{ C.}$$

then

$$W = 0{\cdot}98 \times 60 \times 0{\cdot}0758 \times 0{\cdot}5 \sqrt{\frac{64{\cdot}4 \times 336 \times 13{\cdot}7}{1 - 0{\cdot}25 \times 0{\cdot}786}}$$

$$= 1{,}355 \text{ lb. per min.}$$

(*b*) Let the suffixes *a* and *t* refer to conditions at the section *a* and the throat respectively.

Then $$U_a = \frac{W}{0{\cdot}98 \times 60} \times \frac{1}{w_a a_a} = \frac{1{,}355}{0{\cdot}98 \times 60} \times \frac{1}{0{\cdot}0855 \times 1}$$

$$= 270 \text{ ft. per sec.}$$

From Eq. 16

$$U_t = \sqrt{\frac{\gamma - 1}{\gamma + 1} U_a^2 + \frac{2\gamma g}{\gamma + 1} RT_a}$$

$$= \sqrt{\frac{0{\cdot}4}{2{\cdot}4} \times 270^2 + \frac{2 \times 1{\cdot}4 \times 32{\cdot}2}{2{\cdot}4} \times 96 \times 293}$$

$$= \sqrt{1{,}069{,}150} = 1{,}034 \text{ ft. per sec.}$$

Now $U_{sa}^2 = \gamma gRT_a = 1{\cdot}4 \times 32{\cdot}2 \times 96 \times 293 = 1{,}269{,}000$

Hence from Eq. 17

$$T_t = T_a \left(\frac{U_t}{U_{sa}}\right)^2 = 293 \times \frac{1{,}069{,}150}{1{,}269{,}000} = 246{\cdot}8^\circ \text{ C. abs.}$$

Also $$w_t = w_a \left(\frac{T_t}{T_a}\right)^{\frac{1}{\gamma - 1}} = 0{\cdot}0855 \left(\frac{246{\cdot}8}{293}\right)^{\frac{1}{0{\cdot}4}}$$

$$= 0{\cdot}0556 \text{ lb. per cu. ft.}$$

Hence the throat area

$$a_t = \frac{W}{C_d \times 60} \times \frac{1}{U_t w_t} = \frac{1{,}335}{0{\cdot}98 \times 60} \times \frac{1}{1{,}034 \times 0{\cdot}0556}$$

$$= 0{\cdot}401 \text{ sq. ft.}$$

(*c*) Using the suffix *e* for the exit conditions in Eq. 18 we obtain

$$\left(\frac{W}{60 \times C_d a_e}\right)^2 \left(\frac{\gamma - 1}{2g\,\gamma\, p_a w_a}\right) = \frac{r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}}}{1 - x^2 r^{\frac{2}{\gamma}}}$$

i.e. $$\left(\frac{1{,}355}{60 \times 0{\cdot}98 \times 0{\cdot}5}\right)^2 \left(\frac{0{\cdot}4}{64{\cdot}4 \times 1{\cdot}4 \times 16{\cdot}7 \times 144 \times 0{\cdot}0855}\right)$$

$$= \frac{r^{1{\cdot}428} - r^{1{\cdot}1714}}{1 - 0{\cdot}25 \times r^{1{\cdot}428}}$$

whence $r = 0{\cdot}245$

and the pressure at the exit to nozzle $= p_e = 0{\cdot}245\, p_a$

$= 0{\cdot}245 \times 16{\cdot}7 = 4{\cdot}09$ lb. per sq. in. abs.

The back pressure outside the exit of nozzle must therefore not exceed this pressure.

To check this result we can use Eq. 19

i.e. $$W = C_d \times 60 \times w_e a_e \sqrt{\frac{2gC'_p(T_a - T_e)}{1 - x^2 r^{\frac{2}{\gamma}}}}$$

where $r = 0{\cdot}245$

$$T_e = T_a r^{\frac{\gamma - 1}{\gamma}} = 293 \times 0{\cdot}245^{0{\cdot}286} = 196^\circ \text{ C. abs.}$$

$$w_e = w_a \left(\frac{T_e}{T_a}\right)^{\frac{1}{\gamma - 1}} = 0{\cdot}0855 \left(\frac{196}{293}\right)^{2{\cdot}5} = 0{\cdot}0313 \text{ lb. per cu. ft.}$$

$$a_e = 0{\cdot}5 \text{ sq. ft.} \qquad x = \frac{0{\cdot}5}{1} = 0{\cdot}5$$

$$C'_p = 336 \text{ ft-lb. per lb. per } ^\circ\text{ C.}$$

and

$$W = 0{\cdot}98 \times 60 \times 0{\cdot}0313 \times 0{\cdot}5 \times \sqrt{\frac{64{\cdot}4 \times 336 \times (293 - 196)}{1 - 0{\cdot}25 \times 0{\cdot}245^{1{\cdot}428}}}$$

$= 1{,}356$ lb. per min.

Practical Measurement of Gas Flow. A more simple formula than those already given may be obtained from a consideration of the Bernoulli Theorem for non-compressible fluid flow between two sections (1) and (2) in a nozzle or between a section (1) in a pipe, and a section (2) at an orifice plate in a pipe.

Thus if $h = \frac{p}{w}$ is the pressure head in feet of the gas being measured, and ignoring changes of datum energy then

$$\frac{U_1^2}{2g} + h_1 = \frac{U_2^2}{2g} + h_2$$

and
$$\frac{U_2^2 - U_1^2}{2g} = h_1 - h_2 = h$$

where h is the difference of pressure head between the two sections measured in feet of the gas concerned.

If it is assumed that the change in pressure of the gas between the two sections has no effect on the density of the gas, that is that $w_2 = w_1$ and hence $U_2a_2 = U_1a_1$,

then
$$U_2^2\left[1 - \frac{a_2^2}{a_1^2}\right] = 2gh$$

and
$$U_2 = \sqrt{\frac{2gh}{1 - x^2}} \text{ ft. per sec.}$$

If U_1 is negligible compared with U_2

then $U_2 = \sqrt{2gh}$ and $Q = a_2U_2 = a_2\sqrt{2gh}$ cu. ft. per sec.

This volume Q may be considered as under the temperature and pressure conditions of either section, for as the density of the gas is considered constant then $\frac{p_1}{T_1} = \frac{p_2}{T_2}$.

Volumes of air are, however, often expressed in terms of cubic feet of "free air," and volumes of a gas in terms of "standard cubic feet." The standard pressure p_s is taken as 14·7 lb. per sq. in. abs. and the standard temperature t_s as 32° F. or 60° F.

Thus
$$Q = a_2\sqrt{2gh} \times \frac{p_1}{p_s} \times \frac{T_s}{T_1}.$$

$$= a_2 \frac{T_s}{p_s}\sqrt{2gh\left(\frac{p_1}{T_1}\right)^2} \text{ standard cu. ft. per sec.}$$

But
$$\frac{p_1}{T_1} = wR$$

$$\therefore \quad Q = a_2 \frac{T_s}{p_s}\sqrt{2gh\, wR\, \frac{p_1}{T_1}} \text{ standard cu. ft. per sec.}$$

It is often convenient to measure the upstream pressure p_1 in inches of mercury h_g above atmospheric pressure, and the difference of pressure head h in inches of water h_w.

Hence as 1 inch mercury = 0·49 lb. per sq. in.

$$p_1 = 0\cdot49 \times 144(h_g + \text{barometer})$$
$$= 70\cdot5(h_g + \text{barometer}) \text{ lb. per sq. ft.}$$

and $$p_s = 70\cdot5 \times 30 \text{ lb. per sq. ft.}$$

Also if w_w = density of water and w = density of gas,

then h_w inches of water $= \dfrac{w_w}{w}\dfrac{h_w}{12}$ ft. of gas

$$= \frac{62\cdot4}{12}\frac{h_w}{w} = 5\cdot2\frac{h_w}{w} \text{ ft. of gas}$$

Hence

$$Q = a_2\frac{T_s}{p_s}\sqrt{2ghwR\frac{p_1}{T_1}}$$

$$= a_2\frac{T_s}{70\cdot5\times30}\sqrt{64\cdot4\times5\cdot2\frac{h_w}{w}\times wR\frac{(h_g+\text{bar.})}{T_1}\times70\cdot5}$$

standard cu. ft. per sec.

Substituting 53·2 ft-lb. per lb. per ° F. for R and multiplying by 60, this expression becomes

$$Q = 954\,a_2\frac{T_s}{30}\sqrt{\frac{h_w(h_g+\text{bar.})}{T_1}} \text{ standard cu. ft. per min.}$$

the temperatures being in degrees F.

It must be remembered that this equation is only even approximately true if the pressure difference between the sections is small, so that the gas density may be assumed constant. In addition the expression should be multiplied by a coefficient of discharge C_d to correct for friction effects. In addition to this, for accurate work, corrections must be made for pipe size, viscosity effects, change of air density, moisture in air, etc.

Much pioneer work in this direction was carried out by Mr. John L. Hodgson.* He found that using one general theoretical formula, it was possible in the case of any one fluid, to determine experimentally the values of the discharge coefficient for various values of the pressure ratio $\frac{p_2}{p_1}$. A typical nozzle and orifice as used by him are shown in Figs. 62 and 63.

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* See "The Commercial Metering of Air, Steam, and Gas," *Proc. Inst. C. E.*, Vol. 204 (1918), pp. 108–193; also "The Orifice as a Basis of Flow Measurement," *Proc. Inst. C. E.*, 1925.

The flow of a fluid through an orifice depends on its viscosity, μ, density, w, and compressibility or adiabatic elasticity. If μ is the coefficient of viscosity in C.G.S. units, the kinematic viscosity is $\frac{\mu}{w}$.

The values of the viscosity of dry air at various temperatures in Table IV are multiplied by 10^6 to reduce the number of figures.

Thus $\mu = 0{\cdot}00001172 = 11{\cdot}7 \times 10^{-6}$.

Suppose that gaseous or liquid flow through an orifice depends only upon the viscosity μ, the density ρ, the average velocity U at the orifice, and the diameter d_2 of the orifice, then the flow conditions are similar for the same values of $Ud_2\rho/\mu$ or Ud_2w/μ. This criterion of flow is known as the " Reynolds' criterion " for viscous flow. Mr. Hodgson adopted a coefficient of discharge Ωx.

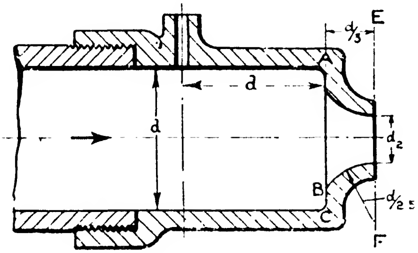

Fig. 62. Shaped Nozzle

If $4G/\pi d^2$ is substituted for $Ud_2\rho$ the flow criterion becomes $G/d_2\mu$, where G is the discharge in grammes per second. Mr. Hodgson* has shown the relation between Ωx and $G/d_2\mu$ by plotting Ωx against $\sqrt{G/d_2\mu}$, for values of the latter from 0 to 300 and 600, also with values of the ratio d_2/d_1 from 0·421 to 0·843, and at pressures of 1 to 4 atmospheres, giving the deviations of the " air curve " and " liquid curve " from the limiting value $\Omega x = 0{\cdot}600$. He has shown by actual measurement that when the value of $\sqrt{G/d_2\mu}$ is greater than 200 C.G.S. units the flow is turbulent, viscosity and compressibility effects are negligibly small, and the

TABLE IV

Values of Viscosity, μ, for Dry Air

Temperature, °F. .	40°	60°	70°	80°	120°	140°	240°
Viscosity, $\mu \times 10^6$ British Engineers' Units	11·72	12·09	12·27	12·46	13·19	13·56	15·36

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* *Proc. Inst. Mech. E.*, No. 4 (1925) p. 885.

discharge through an orifice is proportional to the square root of the difference of pressure across it. The values of $\sqrt{G/d_2\mu}$ usually employed in air, steam, and water metering lie between 200 and 3,000 C.G.S. units, well above the viscous stage, while the ratio p_2/p_1 is between 1·0 and 0·98.

Usually, in the case of air, the kinematic viscosity, μ/w, is so small, and the velocity so high, that corresponding velocities for similar orifices occur at the same value of the expansion ratio p_2/p_1, rather than at the same values of Reynolds' criterion, $Q/d_2\mu$; and, provided that the value of this criterion is greater than 200,000 British Engineers' units, or $\sqrt{G/d_2\mu}$ is greater than 200 C.G.S. units, then the viscosity effects are negligible. Whereas, when orifices or nozzles are used for the measurement of such relatively viscous fluids as water, the corresponding velocities would occur at the identical value of Reynolds' criterion.

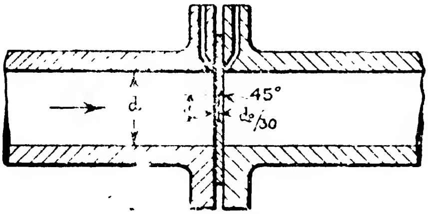

FIG. 63. SQUARE-EDGED CIRCULAR ORIFICE

In the measurement of air flow under ordinary conditions, through an orifice or nozzle of smooth surface, the variations of the discharge coefficient are unaffected by both elasticity and viscosity.

Since this original work was carried out by Mr. Hodgson, considerable standardization has taken place, as exemplified in the *British Standard Code for Flow Measurement.*[*]

Two of the general formulae used are as follows—

$$W = 359{\cdot}1\, CZEd^2\sqrt{h}\sqrt{w} \text{ lb. per hour}$$

$$Q = 7{,}859\, CZEd^2\sqrt{h}\sqrt{\frac{p_a}{T\delta}} \times N \text{ standard cu. ft. per hour}$$

where

C is a coefficient of discharge

Z is a combined correction factor for viscosity, size of pipe and expansion

E is the velocity of approach factor $\dfrac{1}{\sqrt{1-x^2}}$

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[*] "Flow Measurement," B.S. 1042–1943, published by the British Standards Institution.

N is the correction for water vapour in the gas
h is the differential pressure in inches of water at 60° F.
w is the density of the fluid in lb. per cu. ft. at the high pressure tapping
d is the diameter of the orifice or throat of nozzle in inches
p_a is the absolute pressure in lb. per sq. in., measured at the high pressure tapping
T is the absolute temperature in degrees F. at the high pressure tapping
δ is the specific gravity of the dry gas relative to dry air

For details of these corrections the student is referred to the publication concerned, which also includes details of standard orifices, nozzles and venturi tubes.

Correction Factor for Moisture in Gas in the Flow Equation. This is an application of the subject matter of the paragraph on partially wet air on p. 103.

Let p_m = pressure of wet gas (i.e. of mixture)
p_a = partial pressure of dry gas
p_w = partial pressure of vapour
s = density of dry gas relative to air
v_{sm}, v_{sa} and v_{sw} = specific volume of wet gas at p_m, of dry gas at p_a, and of vapour at p_w respectively
v_{mol} = molecular volume
m = molecular weight
W_m, W_a, and W_w = actual weights of wet gas, dry gas, and vapour respectively.

Generally, it may be assumed that the air is not saturated with vapour and hence the vapour is superheated. Hence from p. 103,

$$p\,v_{mol} = 2{,}779T, \text{ hence specific volume } v_s = \frac{v_{mol}}{m} = \frac{2{,}779T}{mp}$$

$$\therefore \quad \frac{v_{sw}}{v_{sa}} = \frac{m_a p_a}{m_w p_w} = \frac{28{\cdot}95s}{18}\,\frac{p_a}{p_w} = \frac{s}{0{\cdot}622}\,\frac{p_a}{p_w}$$

As both the dry gas and the superheated vapour occupy the same volume under their respective partial pressures then

$$\frac{W_a}{W_w} = \frac{v_{sw}}{v_{sa}} = \frac{s}{0{\cdot}622}\,\frac{p_a}{p_w}$$

$$\therefore \quad \frac{\text{Weight of dry gas}}{\text{Weight of wet gas}} = \frac{W_a}{W_w + W_a} = \frac{sp_a}{sp_a + 0{\cdot}622\,p_w}$$

$$= \frac{s(p_m - p_w)}{s(p_m - p_w) + 0{\cdot}622\,p_w} \quad . \quad . \quad (a)$$

To find the ratio of the densities of wet gas to dry gas we have

Actual volume of wet gas $V = W_m v_{sm}$ at pressure p_m . (*b*)

Actual volume of dry gas $= V$ at pressure $p_a = p_m - p_w$

$$\therefore \text{ Actual volume of dry gas at pressure } p_m = \frac{V(p_m - p_w)}{p_m}$$

$$\therefore \quad V\frac{(p_m - p_w)}{p_m} = W_a v_s \text{ where } v_s = \text{sp. vol. of dry gas at } p_m \qquad (c)$$

From Eqs. (*b*) and (*c*), $W_a v_s \times \dfrac{p_m}{p_m - p_w} = W_m v_{sm}$

Hence $\dfrac{\text{Density of wet gas at } p_m}{\text{Density of dry gas at } p_m}$

$$= \frac{v_s}{v_{sm}} = \frac{W_m}{W_a}\frac{p_m - p_w}{p_m}.$$

$$= \frac{s(p_m - p_w) + 0{\cdot}622\,p_w}{s(p_m - p_w)} \times \frac{p_m - p_w}{p_m}. \text{ From Eq. } (a)$$

$$= \frac{s(p_m - p_w) + 0{\cdot}622\,p_w}{s\,p_m} \quad . \quad . \quad . \quad . \quad . \qquad (d)$$

The water vapour affects the flow by

(*a*) the gas measured being only part of the mixture passing;
(*b*) the density of the moist gas being different from that of the dry gas.

The correction for (*a*) is proportional to the weight of gas passing; for (*b*) is proportional to the square root of the density of the mixture.

Thus from equations (*a*) and (*d*) the correction factor is—

$$\frac{s(p_m - p_w)}{s(p_m - p_w) + 0{\cdot}622\,p_w} \times \sqrt{\frac{s(p_m - p_w) + 0{\cdot}622\,p_w}{s\,p_m}}$$

$$= \sqrt{\frac{p_m - p_w}{p_m + \dfrac{0{\cdot}622\,p_m p_w}{s(p_m - p_w)}}}$$

Measurement of Output of Air Compressors. Figs. 64, 65, and 66 show the general arrangements for measuring the air output from an air compressor by means of an orifice or a nozzle. The compressor delivers air into a receiver or receivers of sufficient capacity to damp down the pulsations caused by a reciprocating compressor.

For rotary compressors which give a continuous flow, the necessary capacity can be small, if not dispensed with. The air from the receiver is expanded by the valve down to nearly atmospheric pressure (Figs. 64 and 66). A modification as to the position of the valve is shown in Fig. 65. The air passes along a straight length of pipe containing the orifice or nozzle. In the first arrangement, a perforated plate is situated on the downstream side of the valve to straighten out the flow. A slight pressure difference will be set up between the two sides of the orifice, which difference is measured by some suitable form of manometer. This can be a U-gauge as in Fig. 65, or a curved tube manometer as in Fig. 67. The advantage

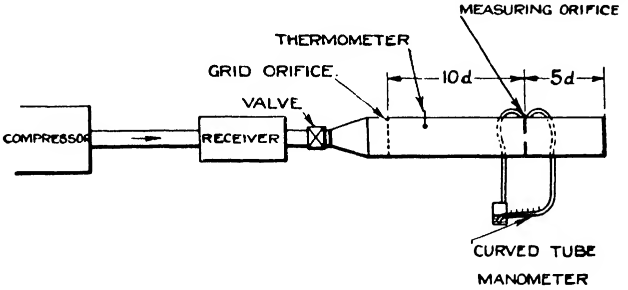

FIG. 64. MEASUREMENT OF AIR-COMPRESSOR DISCHARGE

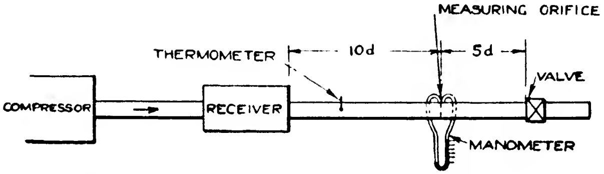

FIG. 65. MEASUREMENT OF COMPRESSED AIR

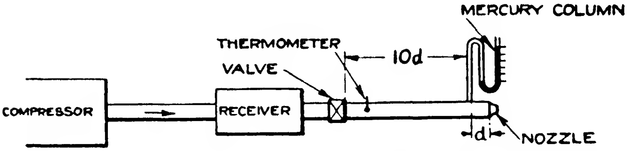

FIG. 66. MEASUREMENT OF COMPRESSED AIR BY NOZZLE

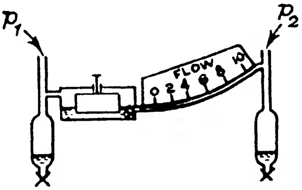

FIG. 67. CURVED TUBE MANOMETER

of the latter is that it gives an equally spaced flow scale down to small fractions of full flow. This pressure difference furnishes the means of determining the volume of air delivered as already dealt with in this chapter. As a result of experiment, certain standard conditions are laid down to ensure uniformity.*

It is necessary to ensure that any moisture in the air does not condense in the pressure piping leading to the manometer. For this reason the pressure pipes should be taken out at the top side of the air main. They should be of sufficient diameter to prevent water hanging in them at bends. There should be no U-bends where water can collect. When using U-tubes for measuring pressure, care should be taken that the bore of the tube is sufficiently large to eliminate surface tension effects at the water surfaces. With a manometer of $\frac{1}{8}$ in. bore, this error may be 0·2 in. of mercury. With a bore of 0·3 in., the error can be 0·2 in. of water. The gauge tubes for water should not be less than $\frac{1}{2}$ in. internal diameter.

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* British Standard Specification, No. 726–1937, "Measurement of Air Flow and the Free Air Delivered by Compressors."

CHAPTER VII

COMBUSTION OF FUELS

THE fundamental law of chemical combination is that all compounds are constant in composition; in other words, the molecule of any substance has always the same atoms similarly combined. Every molecule of water always consists of 1 atom of oxygen combined with 2 atoms of hydrogen, and is therefore represented by the symbol H_2O. These letters tell us the *weight* of the substance. H_2O means 18 parts by weight of the compound water, or steam, containing 2 parts by weight of hydrogen, and 16 parts by weight of oxygen. The *molecular weight* of a compound is the sum of the atomic weights of its constituents.

The following table gives in round numbers, the atomic or combining weights, and the molecular weights of the substances—

Symbol of Substance . .	H_2	O_2	N_2	C
Combining Weight . . .	1	16	14	12
Molecular Weight . . .	2	32	28	—

Thus CO_2 means $12 + 2 \times 16$, that is, 44 parts by weight of carbon dioxide. CO represents 28 parts by weight of carbon monoxide; and CH_4, 16 parts by weight of methane or marsh gas.

All chemical changes take place between fixed definite proportions by weight of substances.

When an electric current is passed through water containing a little sulphuric acid to conduct the current, bubbles of gas are formed at the two platinum plates immersed in the liquid and connected to a battery or other generator of electric current. After making allowance for solubility of the gases and secondary chemical changes, and collecting the gases separately, exactly two volumes of hydrogen are given off at the platinum plate by which the electric current leaves the water, for every volume of oxygen collected at the other plate by which the current enters. The *weights* of the hydrogen and oxygen so set free by electrolysis are always in the fixed proportion of 1 to 8, and their sum is equal to the weight of liquid decomposed. The gases hydrogen and oxygen are entirely different in properties from each other and from water. The fact that distilled water is always composed of these two gases combined in this proportion may be verified by mixing two volumes of hydrogen gas with one volume of oxygen above mercury in a glass tube surrounded by a steam jacket. Passing an electric spark through the mixture causes an explosion; the gases combine, giving out a great amount of heat, whilst steam or water vapour is formed. This steam, on being reduced to the original pressure and temperature, occupies two volumes, and is exactly the same weight as the hydrogen and oxygen combined. Moreover, the heat evolved in the combination is equivalent to the electrical energy spent in the electrolysis of water.

The combination of hydrogen and oxygen to form water is represented by the equation

$$2H_2 + O_2 = 2H_2O + \text{heat evolved}$$

and means that two molecules of hydrogen unite with one molecule of oxygen to form two molecules of water or steam, and heat is generated. Thus 4 parts by weight of hydrogen combine with 32 parts by weight of oxygen and form 36 parts by weight of steam; or, in the same proportion, 1 *lb. of hydrogen unites with* 8 *lb. of oxygen to form* 9 *lb. of steam.*

Again, the molecular volumes of all perfect gases are equal, and Avogadro's law states that equal volumes of different gases, under the same conditions of temperature and pressure, contain the same number of molecules. Therefore the weight of unit volume, that is the density, is proportional to the molecular weight. Taking hydrogen as unit weight, the law holds that the density of every perfectly gaseous compound is half its molecular weight. The density of hydrogen at 0° C. and 14·7 lb. per sq. in. is nearly equal to 0·00559 lb. per cu. ft.

Hence the weight of a gas in pounds per cubic foot, at standard pressure and temperature, is equal to half its molecular weight multiplied by 0·00559. For instance, 1 cu. ft. of methane (CH_4), at 0° C. and 14·7 lb. per sq. in., weighs

$$\frac{12 + 4}{2} \times 0{\cdot}00559 = 0{\cdot}0447 \text{ lb.}$$

The equation agrees with experiment, which also shows that 2 volumes of hydrogen burned with 1 volume of oxygen yield 2 volumes of water vapour at the same temperature and pressure, so that, for complete combustion, 1 *cu ft. of hydrogen requires* $\frac{1}{2}$ *cu. ft. of oxygen to form* 1 *cu. ft. of water vapour.*

Again, when solid carbon burns to carbon dioxide, the equation $C + O_2 = CO_2$ shows that 12 parts by weight of carbon combine with 32 parts by weight of oxygen to form, by complete combustion, 1 molecule or 44 parts by weight of carbon dioxide. Reducing the combining weights in the same proportion, it follows that 1 lb. of carbon requires $2\frac{2}{3}$ lb. of oxygen to form $3\frac{2}{3}$ lb. of carbon dioxide.

Under certain conditions, carbon will combine with oxygen to form carbon monoxide (CO). The combustion equation is then

$$2C + O_2 = 2\,CO$$

Thus 24 parts by weight of carbon combine with 32 parts by weight of oxygen to produce 56 parts by weight of carbon monoxide. It follows that 1 lb. of carbon requires $1\frac{1}{3}$ lb. of oxygen to produce $2\frac{1}{3}$ lb. of carbon monoxide.

The oxygen required for the combustion of fuel is supplied by the air. For the purpose of calculation, atmospheric air, when pure and dry, may be taken as a mixture of

	Per Cent by Weight	Per Cent by Volume
Oxygen	23·1	20·9
Nitrogen	76·9	79·1

There is always water vapour in the air, which helps to start the chemical action, and part of the heat of combustion evaporates the moisture.

Experiment shows that an *inert gas* like nitrogen retards combustion, not only by taking up heat, thereby lowering the temperature of combustion and reducing the velocity of the gaseous molecules, but also by preventing contact between the molecules of the combustible gases with oxygen. Hence an excess of air, usually at least 1·5 times the theoretical quantity required, is drawn through the boiler furnace, because the oxygen in the air supply does not get so intimately into contact with the combustible in the lumps of solid fuel, in order to burn the coal perfectly on the fire-grate, as it does with finely divided coal or liquid fuel when sprayed.

One lb. of oxygen is contained in $\frac{100}{23 \cdot 1} = 4 \cdot 33$ lb. of air, therefore, to find the theoretical weight of air supply for the complete combustion of 1 lb. of a fuel, the necessary weight of oxygen is multiplied by 4·33. The volume of air may then be obtained, since 1 lb. of dry air occupies 13·09 cu. ft. at 15° C. and 14·7 lb. per sq. in.

Table V shows the theoretical minimum weight of oxygen or air required to burn 1 lb. and 1 cu. ft. of each combustible separately.

Example 1. The analysis by weight of petroleum is 86 per cent of carbon and 14 per cent of hydrogen; calculate the quantity of air required for the complete combustion of 1 lb. of this oil fuel.

Given 1 lb. of the oil consists of 0·86 lb. of carbon, and 0·14 lb. of hydrogen, from Table V, the minimum weight of air required is

$$0 \cdot 86 \times 11 \cdot 54 + 0 \cdot 14 \times 34 \cdot 63 = 9 \cdot 92 + 4 \cdot 85 = 14 \cdot 77 \text{ lb.}$$

If the air is at 15° C. and atmospheric pressure, its volume will be

$$14 \cdot 77 \times 13 \cdot 09, \text{ or } 193 \text{ cu. ft.}$$

Quantity of Air Required for the Complete Combustion of Gaseous Fuel. In the case of gaseous fuel the volume of oxygen required for its complete combustion is calculated from the chemical equations, and the cubic feet of oxygen multiplied by $\frac{100}{20\cdot9} = 4\cdot785$ gives the cubic feet of air necessary.

TABLE V

Calorific Values of Substances and Oxygen Required for Complete Combustion

Substance	1 lb. Substance requires:		Products formed with Oxygen		Calorific Values C.H.U. per lb.		Cu. ft of Oxygen required per 1 cu. ft. burned	Calorific Values of Gas			
								C.H.U. per cu ft. burned		Kil-cal. per cu. metre	
	Oxygen lb.	Air lb.	Water lb.	CO_2 lb.	Higher	Lower		Higher	Lower	Higher	Lower
Hydrogen H_2	8	34·63	9		34,500	29,000	$\frac{1}{2}$	191	162	3,055	2,555
Carbon C	$2\frac{2}{3}$	11·54		$3\frac{2}{3}$	8,080	8,015					
Carbon Monoxide CO	$\frac{4}{7}$	2·47		$1\frac{4}{7}$	2,420	2,392	$\frac{1}{2}$	190	188	3,040	3,012
Marsh Gas (Methane) CH_4	4	17·3	$2\frac{1}{4}$	$2\frac{3}{4}$	13,100	11,650	2	591	529	9,470	8,464
Olefiant Gas (Ethylene) C_2H_4	$3\frac{3}{7}$	14·84	$1\frac{2}{7}$	$3\frac{1}{7}$	11,900	11,060	3	930	867	14,890	13,880

Pound-calories (C.H.U.) × 1·8 = B.Th.U.
Kilo-calories per cubic metre × 0·1124 = B.Th.U. per cubic foot.

Example 2. A coal gas has the following percentage analysis by volume: hydrogen, 46; methane, 39·5; ethylene, 2·53; tetrylene, 1·27; carbon monoxide, 7·5; nitrogen, 1·2; and water vapour, 2.

Calculate (*a*) the minimum amount of air required for the complete combustion of the gas; (*b*) the contraction that takes place on combustion when the products are reduced to the same pressure and temperature, without condensation of water vapour.

Taking each combustible constituent in 1 cu. ft. of the gas burnt separately—

(1) *Hydrogen.* $2H_2 + O_2 = 2H_2O$
i.e. 2 cu. ft. H + 1 cu. ft. O form 2 cu. ft. Water vapour
∴ 0·46 ,, ,, + 0·23 ,, ,, ,, 0·46 ,, ,, ,,

(2) *Methane.* $CH_4 + 2O_2 = CO_2 + 2H_2O$
i.e. 1 cu. ft. CH_4 + 2 cu. ft. O form 1 cu. ft. CO_2 + 2 cu. ft. H_2O
∴ 0·395 ,, ,, + 0·79 ,, ,, ,, 0·395 ,, ,, + 0·79 ,, ,,

(3) *Ethylene.* $C_2H_4 + 3O_2 = 2CO_2 + 2H_2O$
i.e. 1 cu. ft. C_2H_4 + 3 cu. ft. O form 2 cu. ft. CO_2 + 2 cu. ft. H_2O
∴ 0·0253 ,, ,, + 0·0759 ,, ,, ,, 0·0506 ,, ,, + 0·0506 ,, ,,

(4) *Tetrylene.* $C_4H_8 + 6O_2 = 4CO_2 + 4H_2O$
i.e. 1 cu. ft. C_4H_8 + 6 cu. ft. O form 4 cu. ft. CO_2 + 4 cu. ft. H_2O
∴ 0·0127 ,, ,, + 0·0762 ,, ,, ,, 0·0508 ,, ,, + 0·0508 ,, ,,

(5) *Carbon Monoxide.* $2CO + O_2 = 2CO_2$
i.e. 2 cu. ft. CO + 1 cu. ft. O form 2 cu. ft. CO_2
∴ 0·075 ,, ,, + 0·0375 ,, ,, ,, 0·075 ,, ,,

Arranging these figures in a table, we get

Constituents in Coal Gas	Volume of Gas in 1 cu. ft. of Coal Gas	Volume of Oxygen required by each Constituent	Volume of Products of Combustion			
			H_2O	CO_2	N_2	
		Cu. ft.	Cu. ft.	Cu. ft.	Cu. ft.	
H_2	0·46	0·23	0·46			
CH_4	0·395	0·79	0·79	0·395		
C_2H_4	0·0253	0·0759	0·0506	0·0506		
C_4H_8	0·0127	0·0762	0·0508	0·0508		
CO	0·075	0·0375		0·075		
N_2	0·012				0·012	
H_2O	0·020		0·0200			
Total	1·000	1·2096	1·3714 +	0·5714 +	0·012	= 1·9548

(*a*) Since 1·2096 cu. ft. of oxygen are required for combustion, the air necessary to supply this volume of oxygen is

$$\frac{100}{20{\cdot}9} \times 1{\cdot}2096 = 5{\cdot}787 \text{ cu. ft. air.}$$

(*b*) This air contains 5·787 − 1·2096 = 4·577 cu. ft. of nitrogen which apparently retards combustion and is added to the burnt products, making a total of 1·955 + 4·577 = 6·532 cu. ft.

A mixture of 1 cu. ft. of coal gas and 5·787 cu. ft. of air becomes after complete combustion 6·532 cu. ft., consisting of water vapour, carbon dioxide, and nitrogen reduced to the same temperature and pressure. Hence the contraction in volume during combustion is

$$6{\cdot}787 - 6{\cdot}532 = 0{\cdot}255 \text{ cu. ft., or}$$

$$\frac{0{\cdot}255}{6{\cdot}787} \times 100 = 3{\cdot}76 \text{ per cent.}$$

In a gas engine, for efficiency in working, there is always an excess of air, in the ratio of about 9 cu. ft. of air to 1 cu. ft. of coal gas, making the charge 10 cu. ft., and this mixture has added to it 2 cu. ft. of residual products from the previous charge in the engine cylinder, hence the total volume would be 12 cu. ft. before combustion, becoming, with the same contraction, 11·745 cu. ft. after combustion, which will now be $\frac{0{\cdot}255}{12} \times 100$, or 2·125 per cent.

Example 3. The analysis by weight of coal is C, 80 per cent; H_2, 5 per cent; S, 0·5 per cent. Calculate the theoretical quantity of air necessary for the complete combustion of 1 lb. of the coal. If 20 lb. of air are supplied per pound of coal, and the combustion is complete, estimate the analysis of the flue gas by weight, and volume.

By Table V, the air required per pound of coal is

$$0{\cdot}8 \times 11{\cdot}54 + 0{\cdot}05 \times 34{\cdot}63 + 0{\cdot}005 \times 4{\cdot}33$$
$$= 9{\cdot}23 + 1{\cdot}73 + 0{\cdot}02 = 10{\cdot}98 \text{ lb. air.}$$

The oxygen needed for the given weights of C, H, and S in 1 lb. of coal might have been calculated, and the sum divided by 0·231 lb., the oxygen per pound of air.

The total weight of flue gas from 20 lb. of air and the weight of combustibles in 1 lb. of the coal is 20·855 lb.

The weights of the burnt products are—

		Lb.		Per cent
$CO_2 = 0{\cdot}8 \times \frac{44}{12}$	=	2·933	=	14·06
$H_2O = 0{\cdot}05 \times 9$	=	0·45	=	2·16
$SO_2 = 0{\cdot}005 \times 2$	=	0·01	=	0·05
$N_2 = 20 \times 0{\cdot}769$	=	15·38	=	73·75
O_2 (by difference)	=	2·082	=	9·98
Total weight	=	20·855		100·00

To find the percentages, divide each constituent by the total weight, thus obtaining the weight of each in 1 lb. of the flue gas; then multiply by 100, thus

$$CO_2 = \frac{2{\cdot}933}{20{\cdot}855} \times 100 \text{ gives } 14{\cdot}06 \text{ per cent.}$$

The composition of the flue gas by volume can easily be obtained, remembering that the molecular weight of any gas occupies 359 cu. ft. at N.T.P. Thus we have

Flue Gas	Constituent by Weight	Constituent by Volume	Percentage
CO_2	2·933	$\frac{2{\cdot}933}{44} \times 359 = 0{\cdot}067 \times 359$	9·48
H_2O	0·450	$\frac{0{\cdot}45}{18} \times 359 = 0{\cdot}025 \times 359$	3·54
SO_2	0·010	$\frac{0{\cdot}01}{32} \times 359 = 0{\cdot}0003 \times 359$	0·04
N_2	15·380	$\frac{15{\cdot}38}{28} \times 359 = 0{\cdot}549 \times 359$	77·74
O_2	2·082	$\frac{2{\cdot}082}{32} \times 359 = 0{\cdot}065 \times 359$	9·20
		Total $= 0{\cdot}7063 \times 359$	100

Determination of Volume of Air Required per cubic foot of gas fuel, given the volumetric analysis of the fuel and the percentage by volume of oxygen in the products of combustion.

Assume the complete combustion of the carbon to carbon dioxide.

Let V_{mp} = volume of dry products with minimum air required per 100 cu. ft. of gas fuel

V_{ea} = volume of excess air per 100 cu. ft. of gas fuel

O = percentage oxygen by volume in dry products (i.e. neglecting the water vapour)

Then the total volume of dry products = $V_{mp} + V_{ea}$.

The oxygen comes from the air and may be assumed as 21 per cent by volume.

$$\therefore \qquad O = \frac{V_{ea} \times 0{\cdot}21}{V_{mp} + V_{ea}} \times 100$$

and
$$V_{ea} = \frac{O}{21 - O} \times V_{mp}$$

Given the volumetric analysis of the fuel, the minimum air required can be determined as in Example 2 and hence the volume of the excess air can be determined.

Solid Fuels. The term **fuel** is applied to any combustible which is sufficiently abundant, and by its combustion with oxygen generates heat energy rapidly, at a cost low enough to be used for industrial purposes. The fuels mostly in common use are wood, peat, coal, petroleum and gas.

Solid fuel is formed by the slow decomposition of wood and decayed vegetable debris buried in the earth for long ages, exposed to great pressure and the internal heat of the earth. In peat bogs this long transformation process from wood to anthracite is seen to begin with decayed marsh plants and mosses as spongy peat near the surface, gradually converted into black dense peat at the bottom of deep bogs.

Lignite, of woody fibrous structure, intermediate between peat and true coal, is found in the tertiary formation of a more recent period than the carboniferous, in which coal occurs. Lignites vary from earthy brown and friable, to " jet " black, contain 50 to 70 per cent carbon, 20 to 28 per cent oxygen, and up to 80 per cent moisture.

Bituminous coals are chiefly derived from *sigillaroid trees*, gigantic ferns, the debris of primeval forests ; whilst the structure of *cannel coal*, with spore-cases of *Lepidodendra*, point to its origin from vegetable mud and club mosses. Bituminous, or " flaming " coal yields 30 to 40 per cent volatile matter, and burns with a long smoky flame Bituminous caking coals soften when heated, and on distillation in coke ovens at high temperatures, like Durham " coking coal," yield strong coke, used in blast furnaces.

Cannel or *parrot coal*, and other *gas coals*, contain 30 to 40 per cent volatile matter, are chiefly used in the manufacture of coal gas, and yield about 12,000 cu. ft. of gas per ton by high temperature carbonization to 1,100° C., and coke, while naphthalene, benzole, toluol, and other products are obtained from the gas tar or oil.

In laboratory tests the products obtained by the *low temperature distillation*, up to 500° or 600° C., of Yorkshire cannel coal and British bituminous coals are crude oil, yielding on fractionation 10 to 15 per cent of their weight of motor spirit, and fuel oil, with 3,000 to 4,000 cu. ft. of rich gas; but Scotch and other cannels yield only from 20 to 40 gallons of crude oil per ton by low temperature carbonization. After distilling off this volatile matter from the coal, there remains about 70 per cent of the coal as a valuable semi-coke, smokeless in its combustion.

Free burning, non-caking bituminous coals are suitable for making Mond gas, but are likely to give trouble in gas producers owing to the tar formed, unless special precautions are taken to convert the tar into fixed gas or to burn it.

The semi-bituminous *steam coals* yield 15 to 20 per cent volatile matter, and burn with short, practically smokeless, flame.

Anthracites contain 85 to 93 per cent carbon, about 8 to 15 per cent volatile matter, are non-caking, and burn with a short non-luminous flame. Anthracite and non-caking coals are suitable for small power gas producers, because the gas is practically free from tar and there is little clinkering trouble.

Anthracite appears to be formed from bituminous coal by heat in the earth's crust.

Table VI gives the average percentage composition and calorific values of typical solid fuels.

TABLE VI*

Air Dried	Wood	Peat	Lignite	Bituminous	Anthracite
Moisture	20	20	18	1–10	1
Volatiles	—	50	47	30–35	8
Fixed C	—	27	28	45–64	88
Ash	0·5	3	7	5–10	3
Dry Ashless					
C	50	60	67	77–87	94
H_2	6·5	6	5·5	5–5·3	3
O_2	43	32	26	4·7–16	2
S_2 and N_2	0·5	2	1·5	2–4	1
Cal. Value B.T.U. per lb. (Higher)	8,000	10,000	13,200	13,400–15,300	15,600

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* See *Fuel* by Brame and King (Edward Arnold) from which figures are extracted by permission of the publishers.

Coal is a very complex material, and its **composition** for industrial purposes is usually given in two ways—

(*a*) By *proximate analysis*, first heating the powdered sample a little over 100° C., when the moisture is driven off and "dry coal" is left. The dry coal is heated in a closed crucible without air being admitted at 500° to 900° C., and the *volatile combustibles* which escape are burned, leaving a combustible residue of fixed carbon or *coke*. When the coke is completely burned in air, only *ash* remains.

(*b*) By *ultimate analysis*, giving the percentages by weight of moisture, carbon, hydrogen, nitrogen and sulphur in a powdered sample.

The *carbon* and *hydrogen* are found by burning a weighed quantity of dry coal in a hard glass combustion tube in a current of dry oxygen, the water and carbon dioxide formed being absorbed by dehydrated calcium chloride and an aqueous solution of caustic potash respectively.

The *nitrogen* can be found as ammonia by heating the coal to redness or burning it in a current of steam. The *sulphur* is determined separately, the *ash* added to the sum of these elementary constituents, and the amount short of 100 is usually returned as "oxygen by difference," and may also cover errors in the analysis.

The combustion of coal is a complicated interaction, and the products of complete combustion are CO_2 from the carbon, H_2O from hydrogen, and SO_2 from the sulphur. Carbon monoxide shows incomplete combustion of the carbon.

In a boiler furnace the volatile hydrocarbons are first driven off the coal, and the oxygen for their combustion is supplied by air admitted over the top of the fuel bed. The solid carbon or coke left is burnt by the air drawn through the fuel bed. It appears from experiments that, in burning bituminous coal, the fuel bed in a steam boiler need not be more than 6 in. thick, with careful stoking. About half the oxygen required for the complete combustion of the coal is used up within the lower 4 in. of the fuel bed, the other half of the air supply, preferably heated, passes at high velocity over the fuel to mingle with and burn the volatile matter. The highest temperature, 1,200° to 1,500° C., depending on the draught, is usually found about 3 to 5 in. above the grate. An increase in the draught increases the rate of combustion and the supply of air, which is usually 1·5 to twice the theoretical quantity. Large excess of cold air tends to cool down the interacting gases, retards combustion, and gives a low percentage of CO_2 in the flue gas.

Conditions for Combustion. *Rapid combustion*, or burning, is chemical combination of combustible substances with oxygen, accompanied by the evolution of heat, and generally of light. When the resulting compound is decomposed or split up into the original combustibles, an amount of heat is absorbed, or disappears as sensible heat, equal to that generated in forming the compound. The actual process of combustion is very complicated, and the chemical equations given above only represent the results.

A combustible gas combines completely with oxygen only when (*a*) the mixture contains the two gases in the proper proportions; and (*b*) the temperature and pressure of the mixture are within fixed limits.

As stated above (p. 193), the electrolytic mixture of hydrogen and oxygen at 18° C. and 14·7 lb. per sq. in. may be ignited by the electric spark, giving an explosion. Sir Humphry Davy showed that if the same mixture be reduced to 0·8 lb. per sq. in. at 18° C., combustion will not take place when the spark is passed; but, keeping the mixture at the same lower pressure, it becomes inflammable when the temperature is raised sufficiently. When the mixture is heated up, slow combustion may take place and the mixture heats itself, and its temperature will rise very rapidly until flame appears.

When this electrolytic, or true *explosive mixture* of hydrogen and oxygen, is fired by adiabatic compression it ignites at 526° C. with a sharp explosion.

During the initial period of inflammation at atmospheric pressure the flame is propagated through the mixture with a uniform velocity of 20 metres per sec (65·6 ft. per sec.). The early experiments of Mallard and Le Chatelier on the velocity of flame propagation in mixtures of hydrogen and air, at constant pressure, showed that the spread or travel of flame is most rapid when there is a slight excess of the inflammable gas in the mixture. Thus the mixture 1 volume of hydrogen to 2·5 volumes of air contains the proportion of oxygen for complete combustion, and the velocity of the flame was 11·5 ft. per sec.; while with excess of hydrogen 1 volume to 1·5 of air, the maximum velocity 14·4 ft. per sec. was attained; but with greater excess, as in a mixture consisting of equal volumes of hydrogen and air, the velocity of flame propagation was reduced to 12·4 ft. per sec. Dr. Neumann, of Dresden, has shown that mixtures of petrol vapour and air in the proportion of 1 to 12 gave the maximum velocity of flame propagation, while the proportion for complete combustion is about 1 to 15.

The effects of different methods of firing upon the rate of combustion in the case of solids were strikingly shown by Sir Frederick Abel with gun-cotton. A train of gun-cotton, laid loosely on a table open to the air, when ignited by a flame, burns rapidly and harmlessly without explosion. When laid in the same way, the gun-cotton, fired by a percussion detonator to produce a high initial pressure or shock, gives a most violent explosion, due to the instantaneous change of the solid into the gaseous state.

Explosion Wave. Berthelot discovered when detonation is set up in the electrolytic mixture of hydrogen and oxygen in a closed tube, fired by an electric spark and a grain of fulminate, the shock and ignition with combustion are transmitted through the whole gaseous mixture by an *explosion wave* of adiabatic compression at a velocity of 2,820 metres per sec. Each layer is compressed before being fired, and then compresses the next layer. It has been proved, by collecting the residual gas and exploding it, that the combustion at the highest temperature in the wave front is not *wholly* complete. The velocity depends on the nature of the mixture. The rate of explosion is retarded by inert gases like nitrogen, according to their volume and density, which, when present in excess, may stop detonation.

Professor H. B. Dixon confirmed these results* for various gaseous mixtures, and showed that in the explosion of a volatile carbon compound with oxygen, the carbon burns first to carbonic oxide, and afterwards, if oxygen is present in excess, the carbonic oxide first formed burns to carbonic acid.

The explosion wave is not easily produced in the gas engine cylinder. The sudden blow or shock by explosion of petroleum vapour, known as "pinking" in the petrol engine, at high temperature has nearly the the same effect, and is called "detonation." Yet the ordinary propagation of inflammation is quite different from the detonation of gun-cotton, dynamite, etc., for which the speed of detonation, observed by Abel, also increases with the density of the charge.

The ignition point of an explosive gaseous mixture is that temperature to which it must be raised locally so that the flame spreads throughout the whole mixture.

Professor H. B. Dixon and H. F. Coward heated the combustible gases and oxygen, or air, separately to the ignition temperature before the streams of gas and oxygen, or air, were allowed to mix, and found the following ignition ranges—

	IGNITION RANGE, °C.	
Gas	*In Oxygen*	*In Air*
Hydrogen	580°–590°	580°–590°
Carbon Monoxide (Moist) . .	637°–658°	644°–658°
Methane	556°–700°	650°–750°
Ethylene	500°–579°	542°–547°
Acetylene	416°–440°	406°–440°

Professor Dixon found that carbon monoxide will not combine with oxygen unless a trace of moisture is present to start the action, although it takes no part in the combination. Professor Baker showed that a mixture of hydrogen and oxygen in the presence of water would not explode without a trace of impurity in the water; and charcoal would not burn unless a trace of water vapour were present.

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* *Phil. Trans. Roy. Soc.*, Vol. 184 (1893); and *Phil. Mag.*, January, 1899.

Each combustible gas or vapour has a well-defined *range of inflammability*, when mixed with air, which varies according to the nature of the hydrocarbons as well as to the temperature and pressure of the mixture. Combustion ceases when a certain excess of either gas is added to the explosive mixture ; thus 9 volumes in excess of oxygen or 8 volumes in excess of hydrogen, added to the electrolytic mixture, render it non-inflammable, and the only portion of hydrogen burned is that lying in the path of the electric spark ; but on raising the temperature of the new mixture to a certain point, it may be ignited by the electric spark. Hence a mixture of hydrogen and oxygen which will not combine at the ordinary atmospheric temperature and pressure may be made to do so by increasing either the pressure or temperature up to certain definite limits, to be determined by experiment.

The addition of excess of an inert gas like nitrogen to the electrolytic mixture of 2 volumes H_2 to 1 of O_2 also retards combustion. The same effect is producĕd by the presence of an excess of the cooled burnt products of a gaseous explosive mixture. In this way the *rate of combustion in the engine cylinder is under control* by : (1) changing the proportion of air to gas in order to determine by experiment the most economical mixture for an engine working at a given compression and speed ; (2) increasing the compression of the charge before ignition, depending upon the nature of the fuel, and enabling weak or dilute mixtures to be burned rapidly and completely ; (3) the time and method of firing the compressed charge in the cylinder ; (4) the shape or design of the combustion chamber ; (5) the temperature of the water jacket and the cooling action of cylinder walls.

Professor W. A. Bone found in his researches* upon the behaviour of hydrocarbons, that their combustion is a complex process, involving slow combustion at temperatures below the ignition point, and that there are clearly defined stages of successive oxidation. Thus, during this slow combustion, the most stable hydrocarbon, methane (CH_4), is oxidized at temperatures between 400° and 500° C., well below its ignition point. In the initial stage *methyl-alcohol* is formed, which breaks up into steam and *formaldehyde* in the second stage. The formaldehyde is subsequently oxidized to *formic acid* (stage 3), which may partly break up into CO and H_2O, to the unstable *carbonic acid* (stage 4), which at once results in CO_2 and H_2O. At temperatures above the ignition point, the *formaldehyde* (stage 2) is so rapidly changed by heat into CO and H_2 that in the final stages of the process these gases are probably oxidized independently.

This view is supported by the presence of formaldehyde found in the condensed steam from the exhaust gases of petrol and oil engines. Also, the "surface effect" in the slow decomposition of

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* *J. Chem. Soc.* (1908), p. 1197 ; and Papers during 1902–6.

methane, according to the reversible reaction $CH_4 \rightleftharpoons C + 2H_2$ between 800° and 1,200° C., explains the formation of that very hard, lustrous, black carbon, "methane carbon," deposited on the vaporizers of some oil engines.

Calculation of the Air Supply from an Analysis of the Flue Gas and an Analysis of the Fuel. The flue gas analysis by volume is conveniently made by means of the Orsat apparatus. A good *average* sample, 100 c.c of the gas, is collected over mercury in a eudiometer, the steam formed by the combustion of the hydrogen in the fuel condenses to water, and the dry flue gas consists of CO_2, CO, O_2, and nitrogen. These are successively absorbed by reagents in separate flasks. The CO_2 is absorbed by an aqueous solution of caustic potash, the oxygen by an alkaline solution of pyrogallic acid or, more slowly, by sticks of phosphorus, and the CO by an acid solution of cuprous chloride, and the remainder is assumed to be nitrogen.

Then, to calculate the *weight* of each constituent per pound of carbon in the flue gas, the proportional weights are found from the molecular weights. A molecule of CO_2 contains 44 parts by weight; i.e. 12 of carbon and 32 of oxygen; CO contains 28 parts by weight; i.e. 12 of carbon and 16 of oxygen.

Example 4. The flue gas analysis by volume in a boiler trial was CO_2, 10·5 per cent; CO, 1 per cent; O_2, 8 per cent; N_2, 80·5 per cent; and the coal analysis, as burned, was C, 82 per cent; H_2, 4·2 per cent; O_2, 4·8 per cent; other matter, 9 per cent. Calculate the following items in the heat balance per pound of coal, the rise in temperature in the flue gases being 300° C.—

(*a*) Heat carried by products of combustion, average specific heat, 0·24.
(*b*) Heat carried by excess air, average specific heat, 0·2375.
(*c*) Heat lost by incomplete combustion.

Calorific value of 1 lb. carbon: when burned to CO_2, 8,080 C.H.U.; burned to CO, 2,413 C.H.U. (*U.L., B.Sc. (Eng.)*.)

The quantity of air drawn through the furnace per pound of fuel may be determined with fair accuracy from the total weight of the burnt products, and the carbon content in these when the ultimate analysis of the fuel is given. In this example the volumetric analysis gives the proportional weights—

		lb.		lb.		lb.
CO_2	10·5 × 44 =	462,	contains 10·5 × 12 =	126 C.,	and 10·5 × 32 =	336 O_2
CO	1·0 × 28 =	28	,, 1 × 12 =	12 C.,	,, 1 × 16 =	16 O_2
O_2	8 × 32 =	256	,,		=	256 O_2
N_2	80·5 × 28 =	2,254				
Total weight	=	3,000	flue gas contains	138 C., 2,254 N_2	and	608 O_2

∴ 1 lb. of carbon is contained in 3,000/138 = 21·74 lb. of dry flue gas.

From the fuel analysis, with every pound of carbon burnt, there is $\frac{0{\cdot}042}{0{\cdot}82} = 0{\cdot}0512$ lb. of H_2, which forms 9 × 0·0512 = 0·461 lb. of steam. Hence, the total weight of flue gas per pound of *carbon* burnt is $21{\cdot}74 + 0{\cdot}461 = 22{\cdot}2$ lb.

The weight of combustibles burnt to form this flue gas is 1 lb. carbon + 0·0512 lb. of H_2, also the oxygen of the coal in the products of combustion per pound of carbon is $\frac{0{\cdot}048}{0{\cdot}82} = 0{\cdot}0585$, making the total 1·11 lb. Hence the air supply, drawn through the furnace, per pound of carbon in the flue gas is $22{\cdot}2 - 1{\cdot}11 = 21{\cdot}09$ lb. And, by the analysis, 1 lb. of coal burnt has 0·82 lb. of carbon; therefore, the weight of air supplied per pound of coal burnt is $21{\cdot}09 \times 0{\cdot}82 = 17{\cdot}294$ lb.

Again, the minimum weight of oxygen needed for complete combustion of the coal is—

$C = 0{\cdot}82$ lb. takes $0{\cdot}82 \times \frac{8}{3} = 2{\cdot}187$ lb. of oxygen
$H_2 = 0{\cdot}042$ „ $0{\cdot}042 \times 8 = 0{\cdot}336$ „ „

∴ Total oxygen required = 2·523 lb.
And the coal itself supplies O_2 = 0·048 „

∴ Oxygen to be supplied by the air = 2·475 lb.

Air contains 23·1 per cent by weight of oxygen, hence the minimum weight of air needed for complete combustion is

$$\frac{2{\cdot}475}{0{\cdot}231} = 10{\cdot}714 \text{ lb.}$$

Therefore the *excess* air drawn through the furnace is

$$17{\cdot}294 - 10{\cdot}714 = 6{\cdot}58 \text{ lb. per lb. of coal burnt.}$$

Now the products of combustion consist of the air supplied, with the weights of combustibles and oxygen in the fuel. The coal furnishes $0{\cdot}82 + 0{\cdot}042 + 0{\cdot}048 = 0{\cdot}91$ lb. Hence the total products of combustion $= 17{\cdot}294 + 0{\cdot}91 = 18{\cdot}2$ lb. per lb. of coal burnt.

$$\left.\begin{array}{l}(a)\ \text{Heat carried away by these gases}\\ \qquad = 18{\cdot}2 \times 0{\cdot}24 \times 300^\circ = 1{,}310\ \text{C.H.U.}\\ (b)\ \text{Heat carried away by excess air}\\ \qquad = 6{\cdot}58 \times 0{\cdot}2375 \times 300^\circ = 469\ \text{C.H.U.}\end{array}\right\}\ \textit{Answer.}$$

(*c*) Incomplete combustion of carbon,

1 lb. carbon burned to CO_2 evolves 8,080 C.H.U.
1 lb. „ „ CO „ 2,413 C.H.U.
∴ 1 lb. carbon in CO burned to CO_2 must give 5,667 C.H.U.

which is lost when the combustion is incomplete. The proportion by weight of carbon in 10·5 parts by volume of CO_2 is

$$10{\cdot}5 \times \frac{12}{44} \times 44 = 126 \text{ parts}$$

and the carbon in 1 part by volume of CO is 12 parts by weight. Thus the total proportional weight of carbon in the furnace gas is $126 + 12 = 138$ parts, and the proportion of carbon burned to CO is $\frac{12}{138}$. The heat lost or not produced in CO per lb. of carbon in dry flue gas $= 5{,}667 \times \frac{12}{138}$ C.H.U., and since 1 lb. of coal contains 0·82 lb. carbon, the heat lost by incomplete combustion per pound of coal burned is

$$0{\cdot}82 \times \frac{12}{138} \times 5{,}667 = 404 \text{ C.H.U.} \qquad \textit{Answer.}$$

The weight of excess air namely 6·58 lb. calculated above is the excess over that required for complete combustion. To be more correct in this example we should find the excess over that required for the given incomplete combustion. This calculation is as follows—

From p. 211 $\dfrac{\text{Wt of C burnt to } CO_2}{\text{Wt of C burnt to CO}}$

$$= \frac{\text{percentage } CO_2 \text{ by volume in products}}{\text{percentage CO by volume in products}} = \frac{10{\cdot}5}{1}$$

Hence weight of C burnt to CO_2 per lb. of fuel

$$= \frac{10{\cdot}5}{11{\cdot}5} \times \text{weight of C per lb. of fuel}$$

$$= \frac{10{\cdot}5}{11{\cdot}5} \times 0{\cdot}82 = 0{\cdot}749 \text{ lb. per lb. of fuel}$$

and weight of C burnt to CO

$$= 0{\cdot}82 - 0{\cdot}749 = 0{\cdot}071 \text{ lb. per lb. of fuel.}$$

Hence

$$\begin{array}{lrl}
\text{C burnt to } CO_2 \text{ requires} & 0{\cdot}749 \times \frac{8}{3} = & 1{\cdot}997 \text{ lb. } O_2 \\
\text{C burnt to CO requires} & 0{\cdot}071 \times \frac{4}{3} = & 0{\cdot}095 \text{ ,,} \\
H_2 \text{ requires} & 0{\cdot}042 \times 8 = & 0{\cdot}336 \text{ ,,} \\
\hline
 & \text{Total} = & 2{\cdot}428 \text{ lb.} \\
 & \text{less } O_2 \text{ in fuel} = & 0{\cdot}048 \\
\hline
 & \text{Total oxygen required} = & 2{\cdot}380 \text{ lb.}
\end{array}$$

Therefore the air required $= 2{\cdot}38 \times \frac{100}{23} = 10{\cdot}32$ lb. per lb. fuel and excess air over that required for the given incomplete combustion

$$= 17{\cdot}294 - 10{\cdot}32 = 6{\cdot}974 \text{ lb. per lb. fuel.}$$

Formulæ for approximate estimate of the weight of air supplied per pound of fuel burned—

1. Given the *volumetric analysis* of dry flue gas, and C per cent by weight of carbon in the fuel. Since air consists of about 77 per cent by weight of nitrogen, N parts by volume of nitrogen are contained in $28 \times \frac{100}{77} \times N$ parts by weight of air.

The proportional weights, as above, are found by multiplying the volumes by the molecular weights in 100 parts of flue gas, and the *ratio* of air to carbon is

$$\frac{N \times \frac{100}{77} \times 28}{\left(CO_2 \times \frac{12}{44} \times 44\right) + \left(CO \times \frac{12}{28} \times 28\right)}$$

and since 1 lb. of the fuel contains $\frac{C}{100}$ lb. of carbon, multiply the ratio of air to carbon by $\frac{C}{100}$, from which the weight of air per pound of fuel burned is nearly

$$\frac{N}{33(CO_2 + CO)} \times C \text{ lb.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In example 4, this gives $\frac{80{\cdot}5}{33 \times 11{\cdot}5} \times 82 = 17{\cdot}39$ lb. of air.

2. When the *percentages* of CO_2, CO, and N in the flue gases are given *by weight*, N parts of nitrogen are contained in $\frac{100}{77} \times N$ parts by weight of air; and in 100 parts by weight of flue gases there are $\left(CO_2 \times \frac{12}{44}\right) + \left(CO \times \frac{12}{28}\right)$ parts of carbon.

Then the ratio of air to carbon is

$$\frac{\frac{100}{77} N}{\left(CO_2 \times \frac{12}{44}\right) + \left(CO \times \frac{12}{28}\right)}$$

and 1 lb. of fuel contains $\frac{C}{100}$ parts of carbon.

Hence, to find the weight of air supplied per pound of fuel burned, multiply the ratio of air to carbon by $\frac{C}{100}$, and the expression becomes

$$\frac{N}{21\ CO_2 + 33\ CO} \times C \text{ lb.} \qquad (2)$$

Usually the value of N is not directly determined from the flue gases, and is only given by difference, including errors, of the analysis ; also small quantities of ammonia formed from the nitrogen of the fuel may be decomposed.

The formulae given above do not allow for any oxygen originally in the fuel.

Example 5. In a trial of a Lancashire boiler with economizer, the following results were obtained—

Volumetric analyses of the flue gases on entering and leaving the economizer

	Entering	*Leaving*
	%	%
CO_2 . . .	8·3	6·2
CO . . .	0·4	0·3
O . . .	11·2	13·7
N . . .	80·1	79·8
Total .	100·0	100·0

Temperature of flue gases on entering and leaving the economizer, 642° F. and 335° F. Temperatures of feed water on entering and leaving the economizer, 134° F. and 234° F. Weight of feed water per hour, 7,370 lb. ; weight of coal stoked per hour, 1,000 lb. Per pound of dry fuel stoked, the carbon burned was 0·735 lb., and the weight of the flue gases, including moisture, entering the economizer was found to be 21·5 lb. The average specific heat of the gas may be taken as 0·25.

Calculate per pound of fuel stoked : (*a*) the air leakage into the economizer ; (*b*) the heat lost by the gases in passing through the economizer ; (*c*) the heat gained by the feed water. (*U.L., B.Sc.* (*Eng.*).)

(*a*) By (1) (p. 208), total air leaving the economizer per pound of fuel

$$= \frac{79{\cdot}8}{33(6{\cdot}2 + 0{\cdot}3)} \times 73{\cdot}5 = 27{\cdot}34 \text{ lb.}$$

Total weight of gases leaving economizer per pound of fuel

$$= 27{\cdot}34 + 1 = 28{\cdot}34 \text{ lb.}$$

Hence, air leakage into the economizer

$$= 28{\cdot}34 - 21{\cdot}5 = 6{\cdot}84 \text{ lb.}$$

(*b*) Heat lost by 21·5 lb. of gases

$$= 21{\cdot}5 \times 0{\cdot}25\,(642° - 335°) = 1{,}650 \text{ B.Th.U.}$$

(*c*) Feed water per pound of fuel

$$= 7{,}370/1{,}000 = 7{\cdot}37 \text{ lb.}$$

∴ heat gained by the feed water

$$= 7{\cdot}37\,(234^\circ - 134^\circ) = 737 \text{ B.Th.U. per lb. of coal}$$

Formula for Air Required from CO_2 by volume in dry products and C and H_2 in fuel (complete combustion) by weight.

Let $C = \frac{\text{percentage}}{100}$ of carbon by weight in fuel

$H = \frac{\text{percentage}}{100}$ of hydrogen by weight in fuel

$CO_2 = \frac{\text{percentage}}{100}$ of carbon dioxide by volume in dry products

Then

Minimum air required per lb. of fuel $= \left(\frac{8}{3}C + 8H\right)\frac{100}{23}$ lb.

and weight of CO_2 formed $= \frac{11}{3}C$ lb.

Let actual weight of air used $= W$ lb. per lb. fuel

Then weight of excess air $= W - \left(\frac{8}{3}C + 8H\right)\frac{100}{23}$ lb.

Weight of oxygen in this $= \left[W - \left(\frac{8}{3}C + 8H\right)\frac{100}{23}\right] \times \frac{23}{100}$ lb.

Weight of nitrogen $= \frac{77}{100}W$ lb.

The dry products are

	By Weight	*By Volume (proportional)*
CO_2	$\frac{11}{3}C$	$\frac{11}{3}C \times \frac{1}{44} = \frac{C}{12}$
N_2	$\frac{77}{100}W$	$\frac{77}{100}W \times \frac{1}{28} = \frac{11}{400}W$
O_2	$\left[W - \left(\frac{8}{3}C + 8H\right)\frac{100}{23}\right]\frac{23}{100}$	$\left[W - \left(\frac{8}{3}C + 8H\right)\frac{100}{23}\right]\frac{23}{100} \times \frac{1}{32}$

Hence

Percentage CO_2 by volume

$$= \frac{\frac{C}{12}}{\frac{C}{12} + \frac{11W}{400} + \left[W - \left(\frac{8}{3}C + 8H\right)\frac{100}{23}\right]\frac{23}{100} \times \frac{1}{32}}$$

from which W can be calculated.

This formula can be simplified to

$$\text{percentage CO}_2 = \frac{80{,}000\,C}{3(111\,W - 800\,H)}$$

Incomplete Combustion.

(*a*) To find the proportions of carbon burnt to CO and CO_2 given the percentages of CO and CO_2 by volume in the products.

$$\frac{\text{Volume of CO}}{\text{Volume of CO}_2} = \frac{\text{percentage CO by volume}}{\text{percentage CO}_2\text{ by volume}}$$

$$\therefore \quad \frac{\text{Weight of CO}}{\text{Weight of CO}_2} = \frac{\text{percentage CO by volume} \times 28}{\text{percentage CO}_2\text{ by volume} \times 44}$$

Hence

$$\frac{\text{Weight of C burnt to CO}}{\text{Weight of C burnt to CO}_2} = \frac{\text{percentage CO by volume} \times 28 \times \frac{12}{28}}{\text{percentage CO}_2\text{ by volume} \times 44 \times \frac{12}{44}}$$

$$= \frac{\text{percentage CO by volume}}{\text{percentage CO}_2\text{ by volume}}$$

(*b*) To find the proportions of carbon burnt to CO and CO_2 given the analysis of fuel and the weight of air actually used.

Let C = weight of carbon per pound of fuel
H = weight of hydrogen per pound of fuel
W = weight of air used per pound of fuel

Then

Weight of oxygen in W lb. of air $= W \times \frac{23}{100}$ lb.

Weight of oxygen required for hydrogen $= 8\,H$ lb.

Oxygen available for carbon $= \frac{23}{100}W - 8\,H$ lb.

Let x = weight of carbon burnt to CO_2
y = weight of carbon burnt to CO

Then

$$\frac{8}{3}x + \frac{4}{3}y = \frac{23}{100}W - 8\,H$$

and

$$x + y = C$$

From these two equations the values of x and y can be calculated. If there is other combustible matter in the fuel, a similar method can be adopted.

Example 6. A fuel has an ultimate analysis of 88 per cent C and 4·4 per cent H, the remainder being incombustible.

A partial volumetric analysis of the dry products of combustion shows 13·2 per cent CO_2 and 3·2 per cent oxygen, and it is suspected some CO is present.

Estimate (*a*) the complete volumetric analysis of the dry exhaust gases; (*b*) the air-fuel ratio by weight based on the fuel as fired.

Air contains 20·8 per cent O_2 by volume and all the H and C have been burned.

(*U.L., B.Sc.,* (*Eng.*).)

If as given in the question 1 cu. ft. of air contains 0·208 cu. ft. O_2 and we assume the rest, i.e. 0·792 cu. ft. is N_2, then

Molecular weight of air $= 0{\cdot}208 \times 32 + 0{\cdot}792 \times 28 = 28{\cdot}83$ (see p. 91).

Hence 1 lb. of air contains $0{\cdot}208 \times \dfrac{32}{28{\cdot}83}$ lb. oxygen $= 0{\cdot}231$ lb.

In 1 lb. fuel let x lb. C be burnt to CO_2.

Then $(0{\cdot}88 - x)$ lb. C is burnt to CO.

Oxygen required to burn 1 lb. fuel will be—

$$\text{for C to } CO_2 = \frac{8}{3}x \text{ lb.}$$

$$\text{C to CO} = \frac{4}{3}(0{\cdot}88 - x) \text{ lb.}$$

$$\text{H to } H_2O = 8 \times 0{\cdot}044 \text{ lb.} = 0{\cdot}352 \text{ lb.}$$

$$\text{Air required} = \frac{100}{23{\cdot}1}\left[\frac{8}{3}x + \frac{4}{3}(0{\cdot}88 - x) + 0{\cdot}352\right]$$

$$= (5{\cdot}77x + 6{\cdot}6) \text{ lb.} \quad . \quad . \quad . \quad (a)$$

$$\text{Nitrogen in this air} = \frac{76{\cdot}9}{100}(5{\cdot}77x + 6{\cdot}6) = (4{\cdot}44x + 5{\cdot}08) \text{ lb.}$$

Let Z = weight of excess air per lb. of fuel containing $0{\cdot}231\,Z$ lb. oxygen and $0{\cdot}769\,Z$ lb. nitrogen.

The products of combustion will contain in proportion by *volume*

$$CO_2 = \frac{11}{3}\,\frac{x}{44} = 0{\cdot}0833x$$

$$CO = \frac{7}{3}(0{\cdot}88 - x)\frac{1}{28} = 0{\cdot}0732 - 0{\cdot}0833x$$

$$O_2 = 0{\cdot}231\,\frac{Z}{32} = 0{\cdot}0072\,Z$$

$$N_2 = \text{Nitrogen in minimum air} + \text{Nitrogen in excess air}$$

$$= (4{\cdot}44x + 5{\cdot}08 + 0{\cdot}769\,Z)\frac{1}{28}$$

$$= 0{\cdot}159x + 0{\cdot}181 + 0{\cdot}0275\,Z$$

$$\text{Total} = 0{\cdot}159x + 0{\cdot}0347\,Z + 0{\cdot}254$$

Now from the given gas analysis by volume

$$\frac{CO_2}{O_2} = \frac{13{\cdot}2}{3{\cdot}2} = \frac{0{\cdot}0833\,x}{0{\cdot}0072\,Z} \text{ from the equations above}$$

$$\therefore \quad Z = 2{\cdot}8\,x$$

Also from the given gas analysis

$$\frac{\text{percentage } CO_2}{100} = 0{\cdot}132 = \frac{0{\cdot}0833\,x}{0{\cdot}159\,x + 0{\cdot}0347\,Z + 0{\cdot}254}$$

From the last two equations

$$x = 0{\cdot}676 \text{ and } Z = 2{\cdot}8 \times 0{\cdot}676 = 1{\cdot}893 \quad . \quad . \quad . \quad (b)$$

Substituting these values in the equations given above, the proportions by volume of the products of combustion are obtained

$$\begin{array}{lllll}
CO_2 & = 0{\cdot}0833 \times 0{\cdot}676 & = 0{\cdot}056 & = & 13{\cdot}1 \text{ per cent} \\
CO & = 0{\cdot}0732 - (0{\cdot}0833 \times 0{\cdot}676) & = 0{\cdot}017 & = & 3{\cdot}98 \text{ per cent} \\
O_2 & = 0{\cdot}0072 \times 1{\cdot}893 & = 0{\cdot}014 & = & 3{\cdot}29 \text{ per cent} \\
N_2 & = (0{\cdot}159 \times 0{\cdot}676) + 0{\cdot}181 + & & & \\
 & \quad (0{\cdot}0275 \times 1{\cdot}893) & = 0{\cdot}340 & = & 79{\cdot}63 \text{ per cent} \\
\hline
 & & \quad 0{\cdot}427 & & 100{\cdot}0
\end{array}$$

From Eq. (*a*) given above

$$\text{Minimum air required} = (5{\cdot}77 \times 0{\cdot}676) + 6{\cdot}6$$
$$= 10{\cdot}5 \text{ lb. per lb. of fuel}$$

From Eq. (*b*)

$$\text{Excess air} = 1{\cdot}893 \text{ lb. per lb. of fuel}$$

$$\therefore \quad \text{Total air used} = 10{\cdot}5 + 1{\cdot}893 = 12{\cdot}393 \text{ lb. per lb. fuel}$$

Natural Draught in a chimney is produced by the difference of weight between the column of hot gas inside a chimney and a column of the same height of cold air outside. The *draught* is measured by the difference in height or head in inches of water in a U-shaped water gauge, one limb of which is connected to a tube inserted at the chimney base and acted on by the pressure of the hot gas, and the other limb is open to the atmospheric pressure in the boiler house outside the ash-pit or below the fire-grate.

A column of water 1 sq. ft. in cross-section and 1 in. high is $\frac{1}{12}$ cu. ft., which, at 17° C., weighs slightly less than $62{\cdot}4 \times \frac{1}{12}$, or 5·2 lb. A draught of 1 in. head of water corresponds to a pressure of 5·2 lb. per sq. ft. If the mean temperature of gas in the chimney is 290° C., and the air outside is 17° C., what height h ft. of chimney 1 sq. ft. in section will produce a draught equal to the pressure of 1 in. of water ?

The difference in weight of a column h cu. ft. of air at 290° and 17° C. is $0{\cdot}0368h$ lb., which must be equal to 5·2 lb., or

$$h = \frac{5{\cdot}2}{0{\cdot}0368} = 141 \text{ ft.}$$

Take the air pressure p lb. per sq. ft. near the boiler below the fire-grate, and p_1 lb. per sq. ft. pressure of the flue gas, inside the chimney at its base.

Since a head or difference of level of 1 in. water column indicates a pressure of 5·2 lb. per sq. ft., the difference of pressure, or the draught, between the air in the ash-pit and at the base of chimney is $p - p_1 = 5{\cdot}2\,h$ lb. per sq. ft., which produces the flow of air through the furnace and up the chimney.

If w lb. of air is drawn through the furnace per pound of fuel burnt, the flue gas formed, together with combustibles in the fuel, will be nearly $w + 1$ lb.; 'and the volume of these products of combustion is practically equal to the volume of air supplied per pound of fuel under the same pressure and temperature. Therefore, the volume of furnace gas from a given air supply may be calculated by the general formula for air, i.e.,

$$pv = 96\,wT$$

where v is the volume in cubic feet of w lb. of air supplied per pound of fuel burnt, and T the absolute temperature in degrees C. Also, when the combustion is at atmospheric pressure, 2,116 lb. per sq. ft., the volume of furnace gas is

$$v = \frac{96wT}{2{,}116} = \frac{wT}{22} \text{ cu. ft. per lb. of fuel burnt} \quad . \quad . \quad (1)$$

and the density of the air or furnace gas is

$$\frac{w}{v} = \frac{22}{T} \text{ lb. per cu. ft. (approximately)}$$

Let A be the sectional area of the flue at the chimney base, where the temperature T is observed, then the total amount of gas entering the chimney in pounds per second is equal to

$$\left(\begin{matrix}\text{Velocity of flow,}\\ \text{ft. per sec.}\end{matrix}\right) \times \left(\begin{matrix}\text{Section } A \text{ sq. ft. at}\\ \text{base of chimney}\end{matrix}\right) \times \left(\begin{matrix}\text{Density of gas,}\\ \text{lb. per cu. ft.}\end{matrix}\right)$$

from which the mean velocity of gas flow across that section is

$$\frac{96\,wTW}{2{,}116A} \text{ ft. per sec.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where W is the pounds weight of fuel burnt per second.

Example 7. A boiler is fired with coal having a percentage composition, carbon 85·1, hydrogen 4·2, oxygen, ash, etc., 10·7. The analysis of dry flue gas shows 10·2 per cent of CO_2. Estimate the weight of air supplied to the furnace per pound of fuel fired. If the measured temperature of the flue gas at the base of the chimney is 410° C. when the coal consumption in the boiler is 1,625 lb. per hour, find the mean speed of the flue gas entering the chimney if the cross-sectional area of the chimney is 18 sq. ft.

(*U.L.*, *B.Sc.* (*Eng.*), 1923.)

Using the formula given on p. 211 if W = weight of air used per lb. fuel

$$\text{percentage CO}_2 = \frac{80{,}000\ \text{C}}{3(111\ W - 800\ \text{H})}$$

i.e.

$$10{\cdot}2 = \frac{80{,}000 \times 0{\cdot}851}{333\ W - 800 \times 3 \times 0{\cdot}042}$$

$$\text{and } W = 20{\cdot}3 \text{ lb.}$$

If the simplified formula is used (neglecting the hydrogen)

$$W = \frac{240\ \text{C}}{\text{per cent CO}_2} = \frac{240 \times 0{\cdot}851}{10{\cdot}2} = 20{\cdot}02 \text{ lb.}$$

The temperature at the base of the chimney = 410 + 273 = 683° C. abs. The gas density at the base of the chimney, from the equation opposite is given by

$$\frac{w}{v} = \frac{22}{T} = \frac{22}{683} = 0{\cdot}0322 \text{ lb. per cu. ft.}$$

The coal fired per second $W_f = \dfrac{1{,}625}{3{,}600} = 0{\cdot}4514$ lb. per sec.

The weight of flue gas per lb. of fuel

W_g = weight of combustible in 1 lb. fuel + weight of air per lb. fuel

$= 0{\cdot}851 + 0{\cdot}042 + 20{\cdot}3 = 21{\cdot}193$ lb. per lb. fuel.

Hence the weight of flue gas per second $= 0{\cdot}4514 \times 21{\cdot}193$

$= 9{\cdot}57$ lb. per sec.

Velocity of flow

$$= \frac{\text{weight of flue gas per second}}{\text{chimney area} \times \text{gas density}}$$

$$= \frac{9{\cdot}57}{18 \times 0{\cdot}0322} = 16{\cdot}5 \text{ ft. per sec.}$$

Volume of flue gases at chimney base conditions (Eq. (1), p. 214)

$$= \frac{wT}{22} = \frac{21{\cdot}193 \times 683}{22} = 658 \text{ cu. ft. per lb. of fuel.}$$

Note—In this example no account has been taken of any oxygen originally in the fuel as this is not known.

Height of Chimney to Produce a Given Draught.

Let

H = height of chimney above the fire-grate

H_1 = height of column of gas of density D equivalent to draught

D = mean density of gases in chimney

D_1 = density of outside air

P_1 = pressure of outside air at the chimney top

then $P_1 + HD$ lb. per sq. ft. is the pressure inside at the base of chimney

$P_1 + HD_1$ lb. per sq. ft. is the pressure outside at entrance to the ash-pit

The difference of these pressures which produces the air flow from the outside into the chimney, through the grate, fuel bed, and chimney is

$$H(D_1 - D) = H_1 D, \quad \therefore H_1 = H\left(\frac{D_1}{D} - 1\right) \quad . \quad . \quad (3)$$

This head H_1 overcomes the frictional resistance F of the fuel bed, etc., and produces the velocity of flow u of the flue gases.

H_1 is measured by the draught h in. of water column in the U-tube inserted at the chimney base, which is equal to $\frac{h \times 62 \cdot 4}{12D}$ ft. of chimney gas $= H_1$, since the density of water is 62·4 lb. per cu. ft. at the atmospheric temperature.

But $$D = \frac{22}{T} \text{ lb. per cu. ft.}$$

and $$D_1 = \frac{22}{T_1} = \left(\begin{matrix}0 \cdot 0764 \text{ lb. per cu. ft. at } 15^\circ \\ \text{C. and } 14 \cdot 7 \text{ lb. per sq. in.}\end{matrix}\right)$$

where T_1 is the absolute temperature of outside air, and T mean temperature of the chimney gas. Substituting these values in equation (3), draught

$$h = 4 \cdot 2H\left(\frac{1}{T_1} - \frac{1}{T}\right) \text{ in. of water column} \quad . \quad . \quad (4)$$

and $$H_1 = H\left(\frac{T}{T_1} - 1\right) \text{ ft. of chimney gas} \quad . \quad . \quad . \quad (5)$$

Now $H_1 = F + \frac{u^2}{2g}$, where $\frac{u^2}{2g}$ is part of the head used to give velocity of flow, u, at the base of the chimney. This can be obtained from the equation

$$u = k\sqrt{H\left(\frac{T}{T_1} - 1\right)} \text{ ft. per sec.} \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

where the constant k is about 2 when $\frac{1}{16}$ of the head is spent in producing the velocity of flow. Professor Dalby* has found from marine engine trials that the value of k for the chimneys of the ships tested varied from 1·6 to 2·7.

Example 8. The composition by weight of the coal used to fire a boiler is: carbon, 0·863; hydrogen, 0·038; oxygen, 0·004; ash, 0·095. Assuming that the ratio of air used to that theoretically required for combustion is 1·6, find the weight of flue gas per pound of coal fired. If the chimney draught is 1 in. of water, and the temperature of flue gas at the base of the chimney is 400° C., what should be the internal cross-sectional area of the chimney if the boiler is to consume 1½ tons of coal per hour. Assume that one-tenth of the draught is available for giving velocity, the rest of the draught being used in overcoming friction of the air passing through the fuel bed.
(*U.L.*, *B.Sc.* (*Eng.*), 1925.)

The oxygen theoretically required for complete combustion of 1 lb. of the coal is $0·863 \times \frac{8}{3} + 0·038 \times 8 = 2·605$ lb., and the coal itself contains 0·004 lb., so that 2·601 lb. is needed in the air supply by $\frac{2·601}{0·231}$, or 11·26 lb. air, the minimum theoretically required. The air used per pound of coal is $1·6 \times 11·26 = 18$ lb.

The total head to overcome frictional resistance and keep up the velocity of flow is $\frac{h \text{ in.} \times 62·4}{12D}$, where D is the mean density of the chimney gas, and the draught h in. of water. Thus $\frac{5·2h}{D} = H\left(\frac{D_1}{D} - 1\right)$, by equation (3), where $D_1 = 0·0764$ lb. per cu. ft. is the density of the outside air at 15° C. and atmospheric pressure. At 400° C., or 673° C. (abs.), the air or gas in the chimney has density

$$D = \frac{22}{T} = 0·0327 \text{ lb. per cu. ft.}$$

and h is given equal to 1 in. of water. Hence

$$5·2 = H(D_1 - D) = H(0·0764 - 0·0327),$$

from which $H = 119$ ft., and one-tenth of this is available for giving velocity. Otherwise, by equation (4), $4·2H\left(\frac{1}{288} - \frac{1}{673}\right) = 1$, hence $H = 12$ ft. The velocity of flow,

$$u^2 = 2g \times 12, \text{ or } u = \sqrt{64·4 \times 12} = 27·8 \text{ ft. per sec.}$$

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* See *Steam Power*, by Professor W. E. Dalby.

Coal is burned at the rate of 3,360 lb. per hour

$$= \frac{3,360}{60 \times 60} = \frac{14}{15} \text{ lb. per second} = W$$

Substituting these values in equation (2) (p. 214) gives the internal cross-sectional area of the chimney

$$\frac{96wTW}{2,116u} = \frac{96 \times 18 \times 673 \times 14}{2,116 \times 27{\cdot}8 \times 15} = 18{\cdot}45 \text{ sq. ft.}$$

Answer.

To check this result: the volume of the flue gas is taken equal to that of the air supplied, and

$$pv = 96wT, \text{ or } v = \frac{96 \times 18 \times 673}{2,116} = 549{\cdot}6 \text{ cu. ft. per lb.}$$

of coal burned; and, at $\frac{14}{15}$ lb. per sec., the rate of gas flow is $549{\cdot}6 \times \frac{14}{15} = 512{\cdot}9$ cu. ft. per sec. This should be equal to the velocity of flow × sectional area of chimney, or

$$27{\cdot}8 \times 18{\cdot}45 = 512{\cdot}9 \text{ cu. ft. per sec.}$$

The natural draught of a chimney draws sufficient air through the fuel to burn coal completely at the rate of 20 lb. of coal per hour per square foot of grate. The rate of firing is increased by **forced draught.** In a locomotive the draught is produced by the *exhaust steam blast* in the smoke-box, which reduces the pressure of the escaping gases below atmospheric and greatly increases the air flow through the grate, tubes, and up the chimney, at an average rate of burning 80 lb. of coal per hour per square foot of grate in an express locomotive.*

In marine boilers the draught is forced by means of fans blowing air into the stoke-hole, which is made air-tight, so that the air can only escape through the ash-pit and fuel bed to the chimney.

In the *Howden system* of forced draught, air driven by a fan passes through heated tubes, before it enters the ash-pit and passes over the grate.

Another way is by **induced draught,** when the fan is placed near the chimney base and draws the air through the furnace, and the hot gases pass through the fan.

The advantages of mechanical draught are: easy control of the draught by regulation of the speed of the fans; ability to meet sudden demands for increased output of steam; greater furnace efficiency, because complete combustion is obtained and the furnace gas may be reduced to a lower temperature than if chimney draught alone is used; natural draught changes with the weather, and forced draught overcomes that difficulty; also a greater rate of burning at the fire-grate, and cheaper coal can be economically used.

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* See "Trials of an Express Locomotive," *Proc. Inst. C. E.*, Vol. CXXV, 291.

The chief disadvantages are the upkeep of the machinery and cost of power in driving the fans; also increased wear and repairs of the boiler owing to the higher rates of combustion.

The brake horse-power of the engine driving a fan may be estimated—

Let v = volume, in cubic feet, of air or gas passing through the fan per minute

h = draught in inches of water column

e = efficiency of the fan

w = pounds of air delivered per pound of fuel burnt

W = weight (pounds) of fuel burned per second

The required brake horse-power $= \dfrac{5{\cdot}2\,hv}{33{,}000 \times e}$ . . . (7)

With *forced draught*, $v_f = 60wWv_0 \times \dfrac{T_f}{273}$

Where v_0 = 12·39 cu. ft., the volume of 1 lb. air at 0° C and 14·7 lb. per sq. in.

and T_f = absolute temperature of air delivered by the fan

With *induced draught* there will be $(w + 1)W$ lb. of flue gas delivered per second, and assuming density to be that of air,

$$v_i = 60(w + 1)Wv_0 \times \frac{T_i}{273}$$

Hence, $\dfrac{\text{Brake horse-power of induced draught fan}}{\text{Brake horse-power of forced draught fan}}$

$$= \frac{w+1}{w} \times \frac{T_i}{T_f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Heat of Combustion (Calorific Value). The calorific value of a fuel is the measure of the amount of heat produced in the complete combustion of unit quality. In the case of a solid or liquid, it is expressed in B.Th.U. or C.H.U. per pound, and for gaseous fuels per standard cubic foot.

For solid or liquid fuels, the value may be determined by means of a bomb calorimeter illustrated in Figs. 68 and 69.

The inner surface of the steel bomb, *B*, is gilt or coated with special enamel to prevent corrosion by the acids formed by combustion of fuel containing sulphur and nitrogen. The cover, *D*, carries a needle valve, *N*, to admit compressed oxygen. A thick platinum wire, *P*, is screwed into the cover, the other wire, *T*, carries the crucible, *C*, at the lower end, which is formed into a ring; the other end of *T* passes through an insulated plug, protected against the high temperature inside the bomb. A fine platinum or nickel wire, with its end twisted round the wires *P* and *T*, making good electrical contact, dips into the fuel, which is ignited when the fine wire is heated to incandescence by an electric current. One gramme of oil fuel, or finely powdered coal, is accurately weighed in the crucible, which is then placed in the ring and with the ignition wire attached; the cover D is then placed on the bomb. The cover is held firmly in position by a large hexagonal nut H, screwed down tightly, a gas-tight joint being made by the spigot S, fitting into a ring of lead in a groove on the top of the bomb. Oxygen is admitted very gradually through a cone and nut attachment at A, by opening the needle valve N, until the pressure is about 25½ atmospheres, when the valve is closed. Wires from an electric circuit are connected to the terminals of P and T, on the cover. The bomb is then placed in a known weight of water in the copper calorimeter, which is insulated near the top to prevent leakage of heat by conduction, and rests on blocks of cork in the air space of an outer double-walled vessel, shown in Fig. 69. This outer vessel and its lid are well lagged on the outside by a thick layer of felt, and forms a water jacket at the temperature of the room. In order to reduce the correction for radiation, the temperature of the bomb and water in the calorimeter should be as many degrees below the jacket temperature at the start as above it when the maximum temperature is reached by burning the fuel.

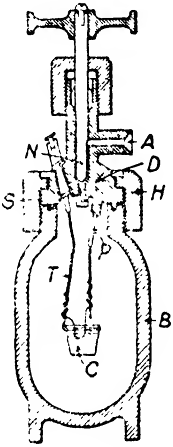

FIG. 68. THE BOMB

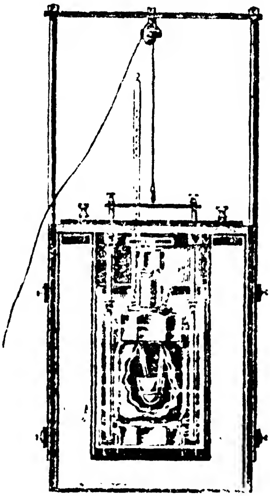

FIG. 69. BOMB CALORIMETER

A standardized Beckmann thermometer, graduated in $\frac{1}{100}$° C., having a range of a little over 5° C., is placed in the water surrounding the bomb. Stirring is then started slowly, and continued throughout the experiment, while readings of the thermometer are noted every minute. After 4 or 5 min., the electric circuit is closed and the fuel fired. The heat evolved is absorbed by the bomb and water in the calorimeter, the temperature rises rapidly, and readings are continually taken until 5 to 10 min. after the maximum temperature is attained, in order to estimate the rate of cooling by radiation and the true rise of temperature. The temperature of the outer water jacket is also noted at the start and finish as a check on the radiation correction.

The water equivalent of the bomb, calorimeter and solid parts therein should be previously found by using a fuel of known calorific value, e.g. naphthalene or benzoic acid. The calorific value of benzoic acid is 6,322 C.H.U. per lb. This water equivalent added to the weight of water in the calorimeter and multiplied by the rise in temperature, measures the total heat generated by combustion of the fuel and by subsequent cooling of the products, then, divided by the weight of fuel taken, gives its *higher calorific value.* The steam formed by burning the hydrogen or hydrocarbons present condenses on the walls of the bomb, and its latent heat is included in the quantity measured.

The **higher or gross calorific value** of a fuel is the amount of heat given out in the complete combustion of unit weight of the fuel in oxygen, when the products of combustion are cooled down to the initial atmospheric temperature (15° C. or 60° F.) at which the oxygen is supplied.

For comparative purposes, the higher calorific value of coal is usually the value found, either by experiment or calculation, on a sample of " dry coal," from which moisture has been driven off by heating the sample to 105° C. until its weight is constant, since the amount of moisture in coal changes with atmospheric conditions and the time of exposure. " Pure dry coal," or " dry ash-free coal," denotes the combustibles left in the coal after the coal is dried and the ash content also subtracted. The calorific value on a " moist coal " basis, i.e. on a sample, as received and used, is the actual value generally required by the engineer.

Moreover, in practice, the furnace gas rejected to the chimney, and the exhaust products of combustion from the cylinder of the internal combustion engine, contain steam not condensed to water, and the exhaust gases are not cooled to the ordinary temperature before leaving the engine.

The **lower or net calorific value** is the heat obtained by the complete combustion of unit weight of the fuel in oxygen when the products are cooled down to 100° C., and the steam is not condensed to water at 100° C.

The latent heat per pound of steam at 100° C. is 539 C.H.U., or 970 B.Th.U., and cooling of the condensed water from 100° C. to 15° C., gives 85 C.H.U. The heat given out by 1 lb. of steam at atmospheric pressure in condensing and cooling to 15° C. is $539 + 85$, or 624 C.H.U. Since 1 lb. of hydrogen when burnt forms 9 lb. of steam, the difference between the higher and lower calorific values of hydrogen is 9×624, or 5,616 C.H.U. Therefore, the lower calorific value of hydrogen is $34{,}500 - 5{,}616$, or 28,884, which may be taken in round numbers as 28,900 or 29,000 C.H.U. per lb., corresponding to $29{,}000 \times 1 \cdot 8$, or 52,000 B.Th.U. per lb.

The mean specific heat of carbon dioxide is 0·21, and 1 lb. of carbon completely burnt gives 11/3 lb. carbon dioxide, which in cooling from 100° to 15° C., gives out $\frac{11}{3} \times 0 \cdot 21 \times 85°$, or 65·45 C.H.U. Thus the lower calorific value of carbon is $8{,}080 - 65$, or 8,015 C.H.U. per lb. This correction for carbon is usually neglected.

Experiment shows that, when carbon is burned in oxygen to carbon dioxide, the amount of heat evolved per unit weight depends on the variety of carbon taken, whether diamond, graphite, wood charcoal, or amorphous carbon, and the higher calorific values vary accordingly, from 7,800 to 8,140 C.H.U. per lb.

The heat given out in converting solid carbon into carbon dioxide is less than that evolved in converting the same weight of carbon in a liquid or gaseous hydrocarbon into the same product, because the heat used up in changing the physical condition of the solid carbon is greater than that required to separate it from the hydrogen.

The calorific value of a simple hydrocarbon, like CH_4 or C_2H_4, is different from the sum of the calorific values of its chemical constituents when burned separately, and depends upon the chemical constitution, or, on the way in which the carbon and hydrogen are combined in the fuel. Thus 1 lb. of marsh gas, CH_4, consists of 0·75 lb. of carbon and 0·25 lb. of hydrogen. The total amount of heat given out by burning these weights of each constituent separately in oxygen, when free, is $0 \cdot 75 \times 8{,}080 + 0 \cdot 25 \times 34{,}500$, equal to 14,685 C.H.U. The actual value found by burning marsh gas in a calorimeter is 13,100 C.H.U. per lb. The difference, 1,585 C.H.U. per lb., must be used up in tearing the constituents in the compound apart in the process of combustion, which is found to be very complicated.

The heat energy spent in doing internal work, tearing the particles of carbon and hydrogen asunder, must necessarily vary with the arrangement or internal combination of the particles in the molecule. Liquid hydrocarbon fuels consisting of light volatile paraffins are chemically unstable compounds, of low ignition point, and their normal rate of burning is very rapid. The aromatic hydrocarbons, benzene and toluene, have greater chemical stability and high ignition temperature.

Again, 1 lb. of acetylene, C_2H_2, consists of 0·923 lb. of carbon and 0·077 lb. of hydrogen. The heat evolved by burning each constituent separately in oxygen is 10,100 C.H.U. per lb. of acetylene. But, in burning 1 lb. of acetylene in a calorimeter, the heat evolved is 12,000 C.H.U., that is, 1,900 C.H.U. more than the constituents give out. Berthelot found that acetylene is an *endothermic* compound, that is to say, it can be formed by direct combination of carbon and hydrogen, with *absorption* of heat at intensely high temperature, as in the electric arc between pencils of retort carbon in an atmosphere of hydrogen. This is an example of the direct *synthesis*, or building up, of a hydrocarbon from its elements.

The *calorific value of a fuel can only be determined with accuracy by direct experiment in the calorimeter.*

It is obvious, from the results of experiment, that a *general formula* given for calculating the calorific value of any kind of fuel from the ultimate analysis can only give approximate results.

Example 9. The volumetric analysis of the flue gas of a locomotive boiler was CO_2, 15·0 ; CO, 2·2 ; O_2, 1·6 ; N_2, 81·2 per cent. The coal had a calorific value of 8,250 C.H.U. per lb. and contained 85 per cent of carbon. The weight of cinders and ashes collected per pound of coal fired was 0·18 lb., and these contained 62 per cent of carbon. Determine the percentage of the calorific value of the coal which was actually produced as heat in the furnace.

If the efficiency of heat transmission through the tubes was 75 per cent, what was the evaporation from and at 100° C. per lb. of coal fired ? One pound of carbon burnt to CO gives 2,420 and to CO_2 8,080 C.H.U.

(*U.L., B.Sc. (Eng.)*, 1924.)

The calorific value of cinders and ashes

$$= \frac{62}{100} \times 8{,}080 \text{ C.H.U. per lb.}$$

∴ the heat lost in cinders and ashes per pound of coal fired

$$= 0{\cdot}18 \times 0{\cdot}62 \times 8{,}080 \text{ C.H.U.} = 901{\cdot}73 \text{ C.H.U.}$$

The proportion by weight of carbon in 15 parts by volume of CO_2 is $15 \times 12 = 180$, and the carbon in 2·2 parts by volume of $CO = 2{\cdot}2 \times 12 = 26{\cdot}4$.

The total weight of carbon is 206·4, and the fraction of this burnt to CO is $\frac{26{\cdot}4}{206{\cdot}4} = \frac{1{\cdot}1}{8{\cdot}6}$. But the heat lost or unproduced by 1 lb. of carbon burnt to CO is $8{,}080 - 2{,}420 = 5{,}660$ C.H.U.

∴ the heat lost per pound of carbon in furnace gas

$$= 5{,}660 \times \frac{1{\cdot}1}{8{\cdot}6} = 724 \text{ C.H.U.}$$

∴ loss per pound of coal fired, by incomplete combustion, is

$$0{\cdot}85(1 - 0{\cdot}18)724 = 504{\cdot}6 \text{ C.H.U.}$$

∴ the heat actually produced in the furnace per pound of coal fired is $8{,}250 - 901{\cdot}73 - 504{\cdot}6 = 6{,}844$ C.H.U., and the percentage of the calorific value actually produced is

$$= \frac{6{,}844}{8{,}250} \times 100 = 83 \text{ per cent.} \qquad \textit{Answer.}$$

Again, the transmission of heat through the tubes is

$$6{,}844 \times 0{\cdot}75 = 5{,}133 \text{ C H.U.,}$$

and, since the latent heat of evaporation is 539 C.H.U. per lb. of water, the equivalent evaporation from and at 100° C. per pound of coal fired is

$$\frac{5{,}133}{539} = 9{\cdot}523 \text{ lb.} \qquad \textit{Answer.}$$

Liquid Fuels. Crude petroleum oil issuing from wells in America, Russia, Iran, Iraq and other countries varies in physical and chemical properties, and is a mixture of many solid, liquid and gaseous hydrocarbons. The individual hydrocarbons may be grouped into different series of compounds, each group having its own particular chemical properties.

Those of most importance as internal combustion engine fuels are—

(*a*) The Paraffins C_nH_{2n+2}	Saturated chain compounds
(*b*) The Olefines C_nH_{2n}	Unsaturated chain compounds
(*c*) The Naphthenes C_nH_{2n}	Saturated ring compounds
(*d*) The Aromatics C_nH_{2n-6}	Unsaturated ring compounds

Examples are—

(*a*) Methane CH_4 (gas), Heptane C_7H_{16} (liquid), Octane C_8H_{18} (liquid).

(*b*) Butylene C_4H_8

(*c*) Cyclohexane C_6H_{12}

(*d*) Benzene C_6H_6.

In general as the value of n increases in the formula, so the substances vary from gases, through liquids of varying density and boiling points, to solids. Methane and other gaseous hydrocarbons are dissolved or held in solution and the lighter constituents are extremely volatile.

Separation is carried out by fractional distillation, and various refining processes. According to the distillation temperature range, the following commercial products are obtained: (i) light oils such as petroleum ether and the petrols, (ii) burning oil such as paraffin, (iii) light and heavy fuel, oils, including so-called Diesel oil, (iv) lubricating oils, (v) paraffin wax and petroleum jelly.

Apart from fractional distillation, the heavier fractions are heated and distilled under pressure at a temperature higher than their normal boiling points, thus decomposing them into lighter fractions. This process called "cracking" is taken advantage of to obtain a larger yield of the lighter oils required in the petrols.

Table VII gives the properties of typical liquid fuels.

TABLE VII*

	Composition per cent by weight			Boiling Range °F.	Specific Gravity	Cal. Value Gross B.Th.U./lb.	Cal. Value Net B.Th.U./lb.
	C	H	S				
Heptane C_7H_{16}	83·9	16·1	—	208·5	0·69	20,808	19,278
Average petrol	85·4	14·6	Small	93–374	0·74	20,232	18,900
Paraffin .	86·3	13·6	0·1	284–536	0·79	20,052	18,756
Diesel oil .	86·3	12·8	0·9	350–572†	0·87	19,800	18,540
Heavy fuel oil	86·1	11·8	2·1	482 upwards	0·95	18,900	17,794

A motor fuel known as "Benzole Mixture," consisting chiefly of benzene, toluene and xylene, is obtained as a by-product from the distillation of coal in gas works. As the aromatics have good anti-knock properties, this mixture has this advantage as a motor fuel. Its freezing point is, however, comparatively high (pure benzole being 41·7° F.), and hence its use mixed with ordinary petrol is advisable. Properties of a typical benzole mixture are—

Composition per cent			Distillation Range °F.	Density S.G.	Calorific Value B.Th.U. per lb.	
C	H	S			Gross	Net
91·65	8·05	0·30	174–338	0·88	18,045	17,280

The alcohols obtained by fermentation of vegetable matter are good motor fuels. The two most common ones are methyl alcohol (CH_3OH) and ethyl alcohol (C_2H_5OH). When used as fuels they are denaturated to make them unfit for drinking. In some cases some small proportion of hydrocarbon fuel is added. Although the calorific value is lower than that of the petrols, the alcohols have a high latent heat. Alcohol has also good anti-knock properties. Table VIII shows approximate figures for ethyl alcohol and a typical methylated spirit mixture. The calorific value of the methylated spirit mixture depends on what additions are made.

Crude shale oil can be obtained from bituminous shales found as natural deposits in many countries. It is distilled from shale oil at high temperatures, and is a dark viscous fluid resembling crude

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* Some figures abstracted from *Technical Data on Fuel* by H. M. Spiers.

† 50 per cent over.

natural petroleum. The fractionation varies in detail and the products include petrol spirit, paraffin oils, and heavier oils which can be used as fuels in oil engines. The resources in Great Britain are, however, very small compared to the demand.

TABLE VIII*

Composition per cent					Distillation Range ° F.	Specific Gravity	Calorific Value B.Th.U. per lb.	
	C	H	O	S			Gross	Net
Ethyl Alcohol .	52	13	35	0	78·3	0·79	12,770	11,530
Methylated Spirit	71	11·5	17·5	Small	70–200	0·82	15,840	14,740

Example 10. The analysis by weight of petrol, of specific gravity 0·72, is C, 84·6 ; H_2, 15·4 per cent. Calculate, (*a*) the higher and lower calorific values of the petrol ; (*b*) the cubic feet of air required for complete combustion of 1 lb. of petrol ; (*c*) the heat energy evolved by the combustion of 1 cu. ft. of the explosive mixture, which contains 2 per cent by volume of petrol vapour with air, in the engine cylinder. Given : the lower calorific value, in B.Th.U. per pound of carbon, and hydrogen in a liquid hydrocarbon is 13,500 and 52,200 respectively. The heat needed to vaporize the petrol is 140 B.Th.U. per pound ; and 1 lb. of air occupies 13 cu. ft. at the atmospheric pressure and temperature ; also the density of petrol vapour is 3·25 times that of air.

(*a*) The net heat given out by 1 lb. of *liquid* petrol in burning to CO_2, the steam formed being cooled only to 100° C. is—

Carbon 0·846 × 13,500 = 11,421 B.Th.U.
Hydrogen 0·154 × 52,200 = 8,039 „

and therefore the lower calorific value = 19,460 B.Th.U. per lb.

The steam formed from the hydrogen is 0·154 × 9 = 1·386 lb. The latent heat of the steam at 212° F. is 970 B.Th.U. per lb., and the condensed water cooled to 60° F. has 212 – 60, or 152 B.Th.U. abstracted ; a total of 1,122 B.Th.U. per lb., that is, for the amount of steam formed, 1·386 × 1,122 = 1,555 B.Th.U. This heat is usually deducted from the higher calorific value to give the lower value ; therefore, the higher calorific value is 19,460 + 1,555 = 21,015 B.Th.U. per lb.

Since petrol vapour at 60° F. is 3·25 times heavier than air, 1 lb. of the vapour occupies $\frac{13}{3\cdot25} = 4$ cu. ft. ; and 1 lb. of liquid petrol, of specific gravity 0·72 at 60° F., occupies 0·0223 cu. ft. The calorific values determined by experiment in the bomb calorimeter are given from the liquid state. If the petrol were all in the

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* Extracted from *Technical Data on Fuel*, by H. M. Spiers.

state of vapour before combustion in the explosion engine cylinder, its lower calorific value would be increased by that part of the heat of combustion which is needed to vaporize the liquid petrol, and which does not appear as heat in the determination by the calorimeter. Hence the latent heat of evaporation should be added to the lower calorific value of the liquid to give the **available heat** of combustion of 1 lb. of petrol vapour: that is, 19,460 + 140 = 19,600 B.Th.U. More correctly, the latent heat of the fuel at *constant volume* only should be added, which is less than the latent heat at constant atmospheric pressure by the equivalent of the work done in the expansion or change in volume from liquid to vapour, namely, $140 - \frac{14{\cdot}7 \times 144}{778}(4 - 0{\cdot}0223) = 130$ B.Th.U. nearly, or about $\frac{14}{15}$ of 140 B.Th.U. at constant pressure. (see p. 245).

(*b*) Again, the oxygen required for the complete combustion of the carbon and hydrogen in 1 lb. of petrol is

$$0{\cdot}846 \times \frac{8}{3} + 0{\cdot}154 \times 8 = 2{\cdot}256 + 1{\cdot}232, \text{ or } 3{\cdot}488 \text{ lb.}$$

Air contains 23·1 per cent by weight of oxygen, and the air required to supply 3·488 lb. of oxygen is $\frac{3{\cdot}488}{0{\cdot}231} = 15{\cdot}1$ lb., i.e. the ratio of air to petrol by weight is 15·1 to 1 lb.; also the volume of this weight of air, at 60° F. and 14·7 lb. per sq. in., is $13 \times 15{\cdot}1 = 196$ cu. ft. This air is 98 per cent of the correct explosive mixture, since the oil vapour occupies 2 per cent, and, therefore, the total volume of explosive mixture of 1 lb. petrol vapour with air is $196 \times \frac{100}{98}$, or 200 cu. ft. at ordinary temperature and pressure. Hence the volume of 1 lb. petrol vapour is 200 - 196, or 4 cu. ft., which agrees with the value found above.

(*c*) The heat energy evolved by burning 1 cu. ft. of this mixture in the engine cylinder is $\frac{19{,}600}{200} = 98$ B.Th.U., or 76,244 ft-lb. If the mixture were at 32° F. or 492° F. (abs.), instead of 60° F., and the same pressure, the heat energy liberated by burning 1 cu. ft. would be $98 \times \frac{520}{492}$, or 103·57 B.Th.U ; and the heat energy per cubic inch of the mixture is equivalent to $\frac{103{\cdot}57 \times 778}{1{,}728} = 46{\cdot}6$ ft-lb.

Further, experiment shows that (at normal temperature and pressure) the specific volume of the products of combustion of petrol vapour and air is 1·05 times that of the mixture before combustion, and the total energy available by combustion becomes 46·6 × 1·05, or 48·9 ft-lb. per cu. in. of the mixture

TABLE IX. OIL FUEL CHARACTERISTICS (*H. R. Ricardo*)

NAME OF FUEL	Specific Gravity at 15° C.	Ratio $\frac{\text{Air}}{\text{Fuel}}$ by Weight for Complete Combustion	Self-ignition Temperature by Adiabatic Compression with Air	Lower Calorific Value, B.Th.U. per lb. of Liquid	Latent Heat of Evaporation, B.Th.U. per lb.	Heat of Combustion ft-lb. per cu. in. of mixture at N.T.P.	Increase of Specific Volume after Combustion	Total Energy Liberated by Combustion ft-lb. per cu. in. of Mixture at N.T.P.
Paraffin Series—								
Hexane, C_6H_{14}	0·670	15·2	366° C.	19,390	156	46·0	1·053	48·33
Heptane, C_7H_{16}	0·688	15·1	330°	19,420	133	46·06	1·056	48·64
Octane, C_8H_{18}	0·704	15·05	—	19,210	128	46·06	1·058	48·73
Decane, $C_{10}H_{22}$	0·757	15·0	—	19,060	108	46·06	1·060	48·82
Aromatic Series—								
Benzene (pure), C_6H_6	0·884	13·2	419°	17,300	172	46·9	1·013	47·51
Toluene (99%), C_7H_8	0·870	13·4	422°	17,520	151	46·9	1·023	47·98
Xylene (91%), C_8H_{10}	0·866	13·6	—	17,800	145	46·85	1·029	48·26
Naphthene Series—								
Cyclohexane (93%)	0·786	14·7	387°	18,800	156	46·08	1·045	48·11
Hexahydrotoluene (80%)	0·770	14·7	378°	18,760	138	46·04	1·049	48·32
Olefine Series—								
Heptylene, C_7H_{14}	0·70	14·7	—	19,170	167	46·8	1·049	49·1
Decylene, $C_{10}H_{20}$	0·76	14·7	—	19,170	—	47·2	—	—
Alcohol Group—								
Ethyl Alcohol, (98%)C_2H_6O	0·798	8·95	514°	11,840	397	44·5	1·065	47·4
Methyl Alcohol (Wood Naphtha), $C_2H_4O_2$	0·829	6·4	457°	9,630	512	42·46	1·061	45·05
Methylated Spirits	0·821	8·0	—	10,200	450	44·0	1·064	46·82

Properties of Hydrocarbons. Sir Harry R. Ricardo carried out an extensive experimental investigation* into the behaviour of various hydrocarbons and their influence as fuels on the performance of internal combustion engines. Table IX gives the heat values of different hydrocarbons, which are constituents of most of the available volatile hydrocarbon fuels.

The heat of combustion of the fuel and air mixture is expressed in the equivalent foot-pounds of heat energy evolved by the combustion of 1 cu. in. of a mixture of fuel and air, measured at standard temperature and pressure, and in the proportion for complete combustion when all the oxygen in the air is combined. In hydrocarbon fuels like "petrol" (containing hexane, heptane, and octane), the specific volume of the mixture after complete combustion is greater than before it. There is thus an increase in the number of molecules in the working fluid mixture during combustion and the total internal energy of the burnt mixture is increased, since, at the same temperature, the pressure will be proportional to the number of molecules. Also the energy will be equal to the product of the heat of combustion of the fuel, when burnt with the necessary quantity of air, and the ratio of the specific volume after combustion to that before it. In the case of benzene and the mixture of the aromatic group (Table IX), known as "benzol," there is only a slight increase in specific volume, about 2 per cent after combustion with the theoretical amount of air, about 5½ per cent with the paraffins, and nearly 6 per cent increase in the alcohol and air mixture; whereas for carbon monoxide and hydrogen the decrease is 15 per cent, and the mixture of coal gas and air also decreases in specific volume (pp. 196 and 197).

Owing to the greater specific gravity of benzene and other aromatic hydrocarbons, their heat values per gallon are greater, and the percentage of the heat converted into work is also greater, than those of the paraffin series, although their calorific values per pound are lower.

The last column gives the remarkable result, confirmed by many experiments, that, provided the fuel is completely evaporated and burned at suitable pressure in the engine cylinder, the energy content and the maximum power obtainable from all the various volatile hydrocarbon fuels, or from any combination of such fuels, is practically the same, within narrow limits, except only the alcohols. In a case where the total internal energy is lower, the latent heat is generally higher, consequently the fuel-air mixture taken into the engine cylinder will be at a lower temperature, allowing a greater weight of charge to be admitted, with greater volumetric efficiency, sufficient to make up for the lower internal energy.

For instance, the energy liberated by the combustion of a cubic inch, at standard temperature and pressure, of benzene–air mixture is a little lower than that of the paraffins which form the greater

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* *The Automobile Engineer*, 1921.

proportion of petrols. But the latent heat of benzene is much greater, i.e. 172, and the mixture called "motor benzole" 164 B.Th.U. per lb. Take the specific heat of the fuel 0·5, and that of air 0·2375. The heat capacity or water equivalent of the air–fuel mixture is

$$13{\cdot}2 \times 0{\cdot}2375 + 1 \times 0{\cdot}5 = 3{\cdot}635 \text{ lb.}$$

If the latent heat of evaporation of benzene were taken from the mixture, its fall in temperature would be $\frac{172}{3{\cdot}635} = 47{\cdot}2^\circ$ F.; whereas with the octane–air mixture the drop in temperature is only 31·4° F.

The light and more volatile fractions of ordinary petrol consist of either hexane, cyclohexane, or benzene, and these render possible the starting of an engine from cold, by "flooding" the carburettor to give an over-rich mixture.

It will be seen by the Tables IX and X that, of these volatile constituents in petrol, the air to fuel ratio is about 14·7 : 1, to form mixtures giving complete combustion. For ethyl alcohol, $\frac{1}{9}$ of the weight of air in the mixture must consist of fuel vapour. The average latent heat of evaporation, in B.Th.U. per pound, of petrols is about 135, motor benzole 164, and ethyl alcohol 397. The latent heat absorbed in vaporizing 1 lb. of alcohol is 2·5 times that taken by 1 lb. of petrol; and 1·6 times the weight of alcohol is required to

TABLE X

OIL FUELS—EXPERIMENTAL RESULTS (*H. R. Ricardo*)

Fuel	Specific Gravity at 15° C.	Approximate Composition Percentage by Weight			Ratio, Air/Fuel by Weight for complete combustion	Lower Calorific Value, including its Latent Heat of evaporation B.Th.U. per lb	Latent Heat of evaporation, B.Th U per lb.
		Paraffins	Aromatics	Naphthenes			
Aromatic Free Petrol	0·718	63	1·7	35	15·05	19,200	133
"A" Petrol . . .	0·782	26	39·0	35	14·3	18,580	142
"B" ,, . . .	0·723	62	14·9	23	14·7	19,020	140
"C" ,, . . .	0·727	61	8·5	30·5	14·8	19,120	135
"D" ,, . . .	0·760	38	14·6	47	14·6	18,890	132
"E" ,, . . .	0·719	68	11·3	20	14·9	19,090	133
"F" ,, . . .	0·704	80	4·3	15·2	15	19,250	134
"H" ,, . . .	0·767	10	4·8	85	14·7	18,920	145
Heavy Aromatics .	0·885	—	71·5	—	13·5	18,030	136
Kerosene . . .	0·813	—	—	—	15·0	19,100	108
Paraffin Series							
Hexane (80% pure) .	0·685	77	2·7	20	15·2	19,390	156
Heptane (97% pure) .	0·691	100	up to 0·2	—	15·1	19,420	133
Alcohol Group		Water					
Ethyl Alcohol, 98·5%	0·798	1·5 to 2	—	—	8·95	11,840	397
Ditto (95 vol %)	0·815	7	—	—	8·4	11,125	—
Methyl Alcohol (Wood Naphtha) . . .	0·829	—	—	—	6·5	10,030	500 app.
Methylated Spirits .	0·821	7	—	—	8·0	10,580	450 app.
Ether (50% with Petrol)	0·727	2·5	—	50% by vol. pet'l aromatic free	13·0	16,830	180 app.

give an inflammable mixture with a given cylinder full of air, so that 4 times the quantity of heat is required to give a combustible vapour mixture with alcohol as compared with petrol.

With petrol " F," a light aircraft spirit of boiling range 60° to 150° C., which has a vapour pressure of 68 mm. mercury at 0° C., an engine is easily started from cold.

The *available energy* of each fuel when burnt in the *state of vapour* in the engine cylinder is taken equal to its lower calorific value in the liquid state as burnt in the bomb calorimeter, plus its latent heat of evaporation, because this amount of heat is added in vaporizing the liquid fuel.

Gaseous Fuel has several well-recognized advantages over solid fuels: (1) Gas can be made in bulk from coal, and economically distributed through pipes; (2) complete combustion is obtained under ideal conditions with a minimum excess of air, and without smoke or dust nuisance; (3) the rate of combustion of gas is completely under control, and can be varied at will; (4) higher heat intensity or temperature is attained with gas than with solid fuel, and the cost of fuel for a regenerative gas-fired steel furnace is much less than that of a coal-fired furnace, owing to the quality and consumption of coal required.

Gas Calorimeter. Instead of making a chemical analysis of liquid or gaseous fuel to estimate its calorific value by calculation from the combustible constituents burnt separately, a far simpler, better, and more practical way is to determine the calorific value of the gas directly during the engine trial by burning it with air in a **gas calorimeter** similar to that devised by Junker or Professor C. V. Boys.

The Junker calorimeter, Figs. 70 and 71, consists of a combustion chamber, *C*, surrounded by a water jacket, fitted with a large number of thin copper tubes shown in sectional plan, Fig. 71. A current of cold water enters the water jacket at the bottom by an annular space, with perforated ring to distribute and steady the flow. The water surrounds the tubes and *flows upwards* at a steady rate under a " head," kept constant by the overflow at the funnels in the supply and discharge vessels. There are also baffle plates at the top of the water jacket. The rate of flow is regulated by means of a cock with a pointer and graduated dial at the inlet to the calorimeter, in order to adjust the rise of temperature in the calorimeter.

The products of combustion of the gas or oil vapour, from the burner in the combustion chamber, enter the copper tubes at the top, as shown by the arrows, *pass downward,* and escape by a butterfly valve. The heat evolved by the combustion passes through the large surface of thin copper to the cooling water, and the burnt products are reduced practically to atmospheric temperature by the coldest water around the lower end of the tubes. The steam, formed by burning the hydrogen in the hydrocarbons, is condensed inside the tubes, and after their surface is all wetted, the water drips down and is collected in a small measuring glass. The latent heat of this steam, given to the circulating water and to be subtracted from the total heat, is determined from the amount of condensed water formed per cubic foot of gas burned. This water is also a measure of the hydrogen content of the gas or oil fuel. The water jacket is surrounded by an annular air space, closed at the top, which, with the polished nickel-plated outer surface, greatly reduces loss by radiation.

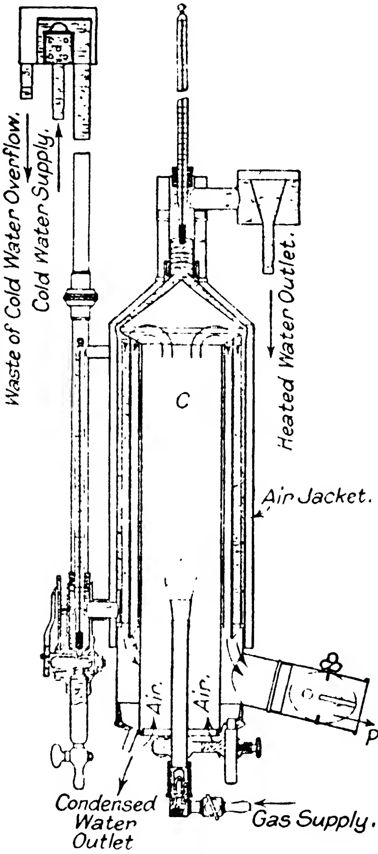

Fig. 70. Junker Gas and Oil Calorimeter

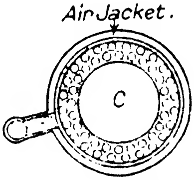

Fig. 71. Cross-section

The gas to be burned is passed through a small meter, reading to one hundredth of a cubic foot, and fitted with a gas governor or regulator to ensure constant pressure, which is measured by a

U-shaped water gauge manometer tube. The barometer reading must also be taken to obtain the absolute pressure of the gas supply. The temperature of the gas leaving the meter is observed so that the cubic feet of gas burned may be reduced to standard temperature and pressure. The size of the flame and its position in the combustion chamber can be adjusted. The flame can be seen during a test by a mirror placed below the combustion chamber. When the flow of water is adjusted, the temperature of the outlet water is constant, and the drip of the condensed water steady, measurements are to be made of the cubic feet of gas burned, and the cooling water passed through the calorimeter in the same time is collected and weighed; the outlet and inlet temperatures are observed every minute to obtain the mean rise in temperature, and the condensed water collected and weighed. The latter may be collected from the gas burned for a longer period. The temperature and pressure of the gas supply is noted; also the reading of the barometer. In this way the higher and lower calorific values are determined without the trouble of collecting samples and without any chemical analysis.

The Boys calorimeter works on the same principles as the Junker, but the water passes through a spiral coiled tube, the hot gases passing up and down round these tubes before passing to the atmosphere.

Example 11. The gas used in a gas engine trial was tested in a Junker calorimeter and the following results were obtained: gas burned in calorimeter, 2·13 cu. ft.; pressure of gas supplied, 2·1 in. of water; temperature of gas, 11·7° C.; weight of water heated by gas, 50·3 lb.; mean temperature of water at inlet, 8·6° C., and at outlet, 22·4° C.; steam condensed during test, 0·116 lb.; and temperature of steam condensed during test, 25° C.

Determine the higher and lower calorific values of the gas per cubic foot at 15·6° C., and barometric pressure 30 in. of mercury. (Specific gravity of mercury = 13·6.) (*U.L., B.Sc.* (*Eng.*).)

Barometer, 30 in of mercury = 14·7 lb. per sq. in.; 1 in. of mercury is $\frac{14{\cdot}7}{30} = 0{\cdot}49$ lb. per sq. in.; and 1 in. of water column $= \frac{0{\cdot}49}{13{\cdot}6} = 0{\cdot}036$ lb. per sq. in.; so that 2·1 in. of water $= 0{\cdot}036 \times 2{\cdot}1$, or 0·0756 lb. per sq. in. Therefore, absolute pressure of gas supply is 14·7756 lb. per sq. in.

The temperature of gas 11·7° C. + 273° = 284·7° C. (abs.), and the volume 2·13 cu. ft. is to be reduced to that at 15·6° + 273, or 288·6° C. (abs.).

Let p_1, v_1, T_1 and p, v, T be the observed and standard pressure, volume, and temperature respectively, then

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1} \text{ or } v = v_1 \times \frac{p_1 T}{p T_1}$$

$$= \frac{2{\cdot}13 \times 14{\cdot}7756 \times 288{\cdot}6}{14{\cdot}7 \times 284{\cdot}7} = 2{\cdot}17 \text{ cu. ft.}$$

Heat given to raise 50·3 lb. water from 8·6° C. to 22·4° C. is $50{\cdot}3(22{\cdot}4 - 8{\cdot}6) = 694{\cdot}14$ C.H.U. This includes the latent heat of steam, 539 C.H.U. per lb. at atmospheric pressure, and cooling of condensed water from 100° C. to 25° C. = 75 C.H.U. per lb., or a total 614 C.H.U. per lb. Hence the condensed steam gives out $0{\cdot}116 \times 614 = 71{\cdot}22$ C.H.U. The heat available in the engine cylinder from the gas burned = $694{\cdot}14 - 71{\cdot}22$, or 622·92 C H.U., when the steam in the burnt products is cooled only to 100° C.

Therefore, higher calorific value of the gas is

$$\frac{694{\cdot}14}{2{\cdot}17} = 320 \text{ C.H.U. per cu. ft.}$$

and the lower calorific value of the gas is

$$\frac{622{\cdot}92}{2{\cdot}17} = 287 \text{ C.H.U. per cu. ft.} \qquad \textit{Answer.}$$

These results, multiplied by 1·8, give the calorific values in B.Th.U. per cubic foot.

In order to determine the *calorific value of liquid fuels,* when burned *with air at constant pressure,* a *Primus* lamp, fitted with a vaporizing coil and special burner, may be used. The oil to be tested is poured into the cistern of the coil lamp and, when the coil is heated by burning a little methylated spirit under the coil to start the lamp, air is pumped into the cistern. When the flame is steady and indicating complete combustion, the lamp is clamped to one arm of a special balance with counterpoise weight. The coil burner hangs freely in the combustion chamber of the Junker calorimeter, which is raised on longer legs than when a Bunsen burner is used for gaseous fuels.

The weights in the scale pan are adjusted for balance when the heated water from the calorimeter has attained a constant temperature. At the instant when the long pointer of the balance is passing zero on the dial, remove a weight from the scale pan and collect the water flowing through the calorimeter at a steady rate; observe its mean rise of temperature until the pointer is again passing the zero, balance having been restored by the weight of oil burned. The heat given to the water is thus measured for a known weight of liquid fuel burned, the usual readings having been taken as when gas is burned.

Again, after adjustment of the flame and balance, another weight may be taken off the scale pan, and further readings noted to verify the previous determination. The steam formed by burning the liquid fuel is condensed in the water jacket and collected in order to obtain the lower calorific value of the fuel.

Heavy oil fuels, that are viscous and difficult to evaporate, are mixed with petrol or other light oil of known calorific value, and the calorific value of the mixture obtained, care being taken to avoid partial volatilization of the petrol or light oil in the mixture before combustion.

Calculation of Calorific Value. In order to *calculate* the *calorific value* of a gaseous fuel, average samples of the gas are taken, and complete chemical analysis made to find its composition. By reckoning the sum of the calorific values of all the constituent combustible gases when burned separately as free gas, an estimate is made of the amount of heat evolved by the complete combustion of a cubic foot of the gaseous mixture, and in the subsequent cooling of the burnt products. The complete analysis requires skilful chemical manipulation in the separation and determination of the quantity of the various *heavy hydrocarbons* present. The chemical analysis gives the volume or weight of each gas present in the compound or mixture, but affords no clue as to the chemical constitution, or the way in which the elements are held together, or how the constituent gases interact during combustion at the high temperatures in the cylinder of the internal combustion engine.

An example or two will illustrate the method of calculation—

Example 12. A coal gas gives the following volumetric analysis : H_2, 0·48 ; CH_4, 0·31 ; CO, 0·110 ; N_2, 0·020 ; CO_2, 0·035 ; C_2H_4, 0·025 ; O_2, 0·020. Calculate the higher calorific value of this gas, employing the usual carbon and hydrogen figures, viz. 1 lb. C, burning to CO, evolves 2,420 C.H.U. ; burning to CO_2, 8,080 C.H.U. ; 1 lb. H_2, burning to H_2O, evolves 34,000 C.H.U. State the assumptions made in such a calculation. Compare the result with the value obtained when the following calorific values for the different constituents are used. Calorific values, C.H.U. per cubic foot, H_2, 191 ; CH_4, 575 ; CO, 188 ; C_2H_4, 930.

(*U.L.*, *B.Sc.* (*Eng.*), 1921.)

Given the calorific values of C and H_2 per pound in order to estimate the calorific values of the hydrocarbons present, it is assumed that the calorific values of CH_4 and C_2H_4 are equal to the sum of the calorific values of the carbon and hydrogen present, as *free* elements, when burned separately, without knowing either the amount of heat energy used up in the decomposition of the gases or the interaction that may take place between the different gases in the process of combustion.

Taking the density of hydrogen at normal temperature and pressure equal to 0·00559 lb. per cu. ft., the weight of 1 cu. ft. of any gas is equal to half its molecular weight multiplied by 0·00559 lb.

Hence the density of $CH_4 = \frac{12 + 4}{2} \times 0{\cdot}00559$, or 0·0447 lb. per cu. ft.; and of $C_2H_4 = \frac{2 \times 12 + 4}{2} \times 0{\cdot}00559 = 0{\cdot}07826$ lb. per cu. ft.; also $CO = \frac{12 + 16}{2} \times 0{\cdot}00559 = 0{\cdot}07826$ lb. per cu. ft.

Now 1 lb. of CH_4 consists of C $= \frac{12}{16}$ or $\frac{3}{4}$ lb., and hydrogen $\frac{1}{4}$ lb., so that these constituents, burning separately, evolve

$$\tfrac{3}{4} \times 8{,}080 + \tfrac{1}{4} \times 34{,}000 \text{ C.H.U.} = 6{,}060 + 8{,}500,$$

or 14,560 C.H.U. Therefore the calorific value of 1 cu. ft. of CH_4, which weighs 0·0447 lb., is 14,560 × 0·0447 = 650 C.H.U.

Similarly, 1 lb. of C_2H_4 contains carbon, $\frac{24}{28}$ or $\frac{6}{7}$ lb., and hydrogen $\frac{1}{7}$ lb., which, in burning, evolve

$$\tfrac{6}{7} \times 8{,}080 + \tfrac{1}{7} \times 34{,}000 = 6{,}926 + 4{,}857,$$

or 11,783 C.H U., and 1 cu. ft. of C_2H_4 has calorific value

$$0{\cdot}07826 \times 11{,}783 = 922 \text{ C.H.U.}$$

Again, the weight of C in 1 cu. ft. of CO $= 0{\cdot}07826 \times \frac{12}{28} = 0{\cdot}03354$ lb.

∴ calorific value of CO

$$= 0{\cdot}03354(8{,}080 - 2{,}420) = 190 \text{ C.H.U. per cu. ft.}$$

The calorific value of hydrogen

$$= 0{\cdot}00559 \times 34{,}000 = 190 \text{ C.H.U. per cu. ft.}$$

Hence the calorific value of this coal gas

$$\begin{aligned} &= 0{\cdot}48 \times 190 + 0{\cdot}31 \times 650 + 0{\cdot}11 \times 190 + 0{\cdot}025 \times 922 \\ &= 91{\cdot}2 + 201{\cdot}5 + 20{\cdot}9 + 23{\cdot}0 \\ &= 336{\cdot}6 \text{ C.H.U. per cu. ft., at N.T.P.} \end{aligned}$$

Using the calorific values given for the combustible constituents, the calorific value of the coal gas

$$\begin{aligned} &= 0{\cdot}48 \times 191 + 0{\cdot}31 \times 575 + 0{\cdot}11 \times 188 + 0{\cdot}025 \times 930 \\ &= 91{\cdot}68 + 178{\cdot}25 + 20{\cdot}68 + 23{\cdot}25 \\ &= 313{\cdot}86, \text{ or nearly 314 C.H.U. per cu. ft.} \\ &= 314 \times 1{\cdot}8, \text{ or 565 B.Th.U. per cu. ft.} \end{aligned}$$

Answer.

Example 13. A producer gas has the following percentage analysis by volume: hydrogen, 16; carbon monoxide, 20; carbon dioxide, 6; nitrogen, 58. Determine, (*a*) its higher and lower calorific value per cubic foot at standard temperature and pressure; (*b*) the minimum amount of air for complete combustion: (*c*) the volumetric analysis of the products, if combustion is complete. Calorific value of 1 lb. of carbon, burning to CO_2, is 14,500; burning to CO, 4,400; of hydrogen, 62,000 B.Th.U. Composition of air by volume: oxygen, 21; nitrogen, 79 per cent. Volume occupied by 2 lb. of hydrogen is 357 cu. ft. at standard temperature and pressure.

(*U.L., B.Sc. (Eng.).*)

(*a*) Given the weight of 1 cu. ft. of hydrogen $= \frac{2}{357} = 0{\cdot}0056$ lb.

Density of CO is $\left(\frac{12 + 16}{2}\right) \times 0{\cdot}0056 = 0{\cdot}0784$ lb. per cu. ft.

Weight of carbon in 1 cu. ft. CO is $0{\cdot}0784 \times \frac{12}{28} = 0{\cdot}0336$ lb.

The calorific value of 1 lb. carbon burning from CO to CO_2

$$= 14{,}500 - 4{,}400 = 10{,}100 \text{ B.Th.U.}$$

and the calorific value of 1 cu. ft. of CO

$$= 0{\cdot}0336 \times 10{,}100 = 339{\cdot}36 \text{ B.Th.U.}$$

Again, the higher calorific value of H_2 is

$$0{\cdot}0056 \times 62{,}000 = 347{\cdot}2 \text{ B.Th.U. per cu. ft.}$$

Now 1 lb. H_2 forms 9 lb. steam, which gives out 970 B.Th.U. per lb. in condensing at 212° F. (100° C.), and, in cooling as water from 212° to 60° F., 152 B.Th.U., that is, 1,122 B.Th.U. per lb. Hence 9 lb. of steam gives out 1,122 × 9 = 10,098 B.Th.U.

∴ the lower calorific value of hydrogen

$$= 62{,}000 - 10{,}098 = 51{,}902 \text{ B.Th.U. per lb.}$$

and the lower calorific value of hydrogen

$$= 51{,}902 \times 0{\cdot}0056 = 290{\cdot}65 \text{ B.Th.U. per cu. ft.}$$

Hence the lower calorific value of the producer gas, by burning its content of H and CO,

$$0{\cdot}16 \times 290{\cdot}65 + 0{\cdot}2 \times 339{\cdot}36 = 46{\cdot}5 + 67{\cdot}87$$
$$= 114{\cdot}37 \text{ B.Th.U per cu. ft.}$$

and the higher calorific value is

$$0{\cdot}16 \times 347{\cdot}2 + 0{\cdot}2 \times 339{\cdot}36 = 123{\cdot}4 \text{ B.Th.U. per cu. ft.}$$

Answer.

(*b*) $2H_2 + O_2 = 2H_2O$ and heat produced

2 volumes H takes 1 volume O to form 2 volumes of steam

∴ 0·16 cu. ft. H takes 0·08 cu. ft. O to form 0·16 cu. ft. of steam

Also $2CO + O_2 = 2CO_2$ and heat evolved

2 cu. ft. H takes 1 cu. ft. O to form 2 cu. ft. CO_2

∴ 0·2 cu. ft. H takes 0·1 cu. ft. O to form 0·2 cu. ft. CO_2

Hence the oxygen required for complete combustion of 1 cu. ft. producer gas is

$$0{\cdot}08 + 0{\cdot}1 = 0{\cdot}18 \text{ cu. ft.}$$

and the minimum amount of air necessary to supply this oxygen is

$$0{\cdot}18 \times \frac{100}{21} = \frac{6}{7} \text{ cu. ft.}$$

Answer.

(*c*) The products from the combustion of 1 cu. ft. of producer gas are—

	cu. ft.		%
Steam (H_2O)	= 0·16	. . $\frac{0{\cdot}16}{1{\cdot}677} \times 100 =$	9·53
CO_2 = 0·2 cu. ft. 0·06 cu. ft. in gas	= 0·26	. . $\frac{0{\cdot}26}{1{\cdot}677} \times 100 =$	15·51
N_2 in air supply $= \frac{6}{7} \times \frac{79}{100} = 0{\cdot}677$ N_2 in gas itself $= 0{\cdot}58$	= 1·257	. . $\frac{1{\cdot}257}{1{\cdot}677} \times 100 =$	74·96
∴ Total Products	= 1·677		100·00

Answer.

Natural gas consists mainly of methane or marsh gas, formed by the decomposition of organic matter, and may be disengaged in any stagnant pool by stirring the mud. The escaping gas contains nearly equal parts by volume of methane and nitrogen, with 3 per cent carbon dioxide. Coal seams contain occluded gases, and methane in a " blower " is known to the coal miner as *fire damp*.

The occluded gas in some freshly-won steam coals contains: CH_4, 60 to 80 per cent; other hydrocarbons, 6 to 8 per cent; and CO_2, 1 to 12 per cent by volume; also CO.

Natural gas dissolves in petroleum, and escapes therefrom, carrying with it part of the oil vapour, which condenses when the gas is compressed.

The paraffin constituents, chiefly methane and ethane, in the Pittsburg natural gas contain by weight, carbon 75·4 per cent, and hydrogen 24·6 per cent. An average analysis is: methane, 87; ethane, 6·5; ethylene and carbon monoxide, each 0·2 per cent; helium, 0·1 per cent; the balance being made up of H_2, O_2, and N_2. The specific gravity is about 0·61, and the calorific value 493 C.H.U. per cu. ft. Many tests in the Junker calorimeter of natural gas containing CH_4 up to 95 per cent, give, in round numbers, 555 C.H.U., or 1,000 B.Th.U. per cu. ft. at N.T.P.

Coal gas is obtained by the distillation or carbonization of bituminous gas-coal in closed retorts, heated externally at high temperature to distil the hydrocarbons from the coal out of contact with the air, and yield crude coal gas which, when purified, is distributed in towns for lighting, heating, and power purposes. The quality and quantity of the coal gas per ton of coal vary greatly with the *composition* of the coal carbonized, as well as with the *temperature* of the retorts, and the *time* the distillation is allowed to continue.

Low temperature carbonization at 450° to 600° C. yields 2,000 to 4,000 cu. ft. of gas per ton of coal; and high temperature, 900° to 1,100° C. yields 10,000 to 13,000 cu. ft. of gas per ton of coal.

The gas formed at low temperature is rich in hydrocarbons, and the semi-coke contains up to 15 per cent volatile matter. The gas made at 950° C. consists of the combustibles H_2, 40 to 50; CH_4, 25 to 35; other hydrocarbons, 2 to 5; and CO, 5 to 10 per cent, and has calorific value 500 to 600 B.Th.U. per cu. ft.

The scarcity of cannel and rich gas coal for admixture with ordinary bituminous coal in the retorts, led to the use of hydrocarbons from oils to enrich the simple "straight" coal gas. The latter is also mixed with carburetted *water gas*, produced by the action of steam on incandescent coke and enriched by the products from the vaporization of cheap hydrocarbon oils. This mixed gas contains a larger percentage of carbon monoxide.

Table XI gives approximate properties of various gases.

TABLE XI

Composition of Gaseous Fuels

	Composition (per cent by volume)							Calorific Value B.Th.U. per S.C.F.	
	CH_4	C_6H_6	H_2	CO	CO_2	N_2	O_2	Higher	Lower
Town Gas (Range)	15–30	1·5–3·5	48–52	6–20	1·5–6·5	5–10	0–1	—	—
(Typical)	25·9	2·2	49·6	11·3	2·4	7·9	0·7	511	459
Coke Oven Gas	25·0	2·0	54·5	6·0	2·0	10·0	0·5	494	443
Water Gas	0·6	—	49·2	40·7	4·5	4·9	0·1	310	286
Producer Gas	4·9	—	20·5	21·0	8·3	45·3	—	190	178
Blast Furnace Gas	—	—	1·0	30·0	11·0	58·0	—	95	—
Natural Gas	86·7	1·8	5·4	—	—	6·1	—	915	832

A **Gas Producer** converts solid fuel into combustible gas. When 1 lb. of carbon is burnt in air or oxygen to CO, the heat liberated is 2,420 C.H.U., and, by completely burning this gas to CO_2, additional heat, 5,660 C.H.U., is liberated, making the total heat of combustion of 1 lb. solid carbon 8,080 C.H.U. Thus the heat available in the CO gas, and evolved by burning it, is 70 per cent of the total heat obtained by completely burning the solid carbon. The remaining 30 per cent of the total heat of the carbon is evolved and goes to raise the temperature in the fuel-bed.

Rhead and Wheeler showed that when dry air or oxygen is passed over purified wood-charcoal or incandescent carbon, the primary product is a complex of carbon and oxygen, which rapidly decomposes into a mixture of both CO and CO_2 in proportions which depend upon the temperature. Karl Wendt found by drilling holes through the wall of a gas producer, working with an air blast upon a fuel-bed of coke 7 ft. 6 in. in depth, that the gas at the fire-grate consisted of CO_2, 15; CO, 9·7; and N_2, 75·3 per cent, and at 10 in. above the grate the temperature was 1,400° C., and the gas at that level consisted of CO, 34·1; CO_2, 0·2; and N_2, 65·7 per cent.

There are two ways by which solid carbon may be converted into CO gas.

With excess of carbon above 1,200° C., or insufficient oxygen, by the reaction—

$$2C + O_2 = 2CO \text{ and heat liberated} \quad . \quad . \quad . \quad . \quad (1)$$

that is, 24 lb. of C and 32 lb. of O_2 form 56 lb. of CO, and the heat evolved is 24 × 2,420 C.H.U.

Thus, 1 volume of oxygen combines with carbon to form 2 volumes of CO gas. If all the oxygen in the air is used up in forming CO, the nitrogen in the air with 1 volume of oxygen is $\frac{79 \cdot 1}{20 \cdot 9}$, or 3·785 volumes, and the producer gas will consist of 2 volumes of CO mixed with 3·785 volumes of nitrogen. The composition of the gas is—

N_2	= 3·785	= 65·4	per cent by volume
CO	= 2·0	= 34·6	,, ,,
	5·785	100·0	,, ,,

The composition by weight will be the same as by volume, since CO and N_2 are of the same density.

For the complete combustion of carbon with free oxygen in the lower layers of the fuel-bed, the reaction is

$$C + O_2 = CO_2 \text{ and heat liberated} \quad . \quad . \quad . \quad . \quad (2)$$

that is, 12 lb. of carbon combined with 32 lb. of O_2 give 44 lb. CO_2 and liberate 12 × 8,080 C.H.U.

Again, this CO_2 may be reduced by carbon above 1,200° C.,

$$CO_2 + C = 2CO \text{ and heat absorbed 39,000 C.H.U.} \quad . \quad . \quad (3)$$

This double process gives the final result, from 24 lb. of carbon,

$$C + O_2 + C = 2CO + (97{,}000 - 39{,}000) \text{ or } 58{,}000 \text{ C.H.U.},$$

as in equation (1).

All the oxygen in the air blast is quickly used up near the fire-bars and the mixture of CO_2 and CO formed in contact with the carbon, follows the reversible reaction, depending on the temperature,

$$C + CO_2 \rightleftharpoons 2CO$$

From 850° C. upwards, CO_2 is reduced to CO, and very little CO_2 remains above 1,200° C.; whereas, from 250° to 500° C., the reaction is reversed and the products are CO_2 and C to the left.

By the conservation of energy, the amount of energy required to decompose a compound substance is equal to the energy evolved in its formation.

The 30 per cent (2,420 C.H.U. per lb.) of the total heat of the carbon evolved in its partial combustion to form CO, goes to keep the fire alight and maintain the high temperature necessary. About 8 per cent is lost by radiation and conduction, and a large proportion will raise the sensible heat of the gas. But the high temperature in the producer may become excessive, fuse the ash and slag, and form clinker, which may cause serious trouble by blocking the air passages. In order to avoid too high a temperature and to increase the efficiency of the producer by reducing the 30 per cent loss, part of the sensible heat may be used to generate steam to be added to the air blast. By the admixture of steam with the air passed into the producer, heat is absorbed in the decomposition of the steam, the oxygen of the steam combines with the carbon, and free hydrogen is added to the gas, while the temperature of the producer is under control by the proportion of steam in the blast.

The gas formed in a producer burning carbon in a mixed blast of air and steam will consist of a greater proportion of the combustible gases H_2 and CO to the total volume of the producer gas than when the blast is air alone, and therefore the calorific value of gas will be higher.

Water gas is produced by the interaction of superheated steam with incandescent carbon, and consists of a mixture, in nearly equal parts by volume, of the combustible gases hydrogen and carbon monoxide. Experiments by Dr. Bunte give the proportion of steam decomposed and the composition by volume of the gases formed at various temperatures—

Temperature Degree C.	Percentage of Steam Decomposed	Percentage Composition by Volume of Gas Formed		
		Hydrogen (H_2)	Carbon Monoxide (CO)	Carbon Dioxide (CO_2)
674°	8·8	65·2	4·9	29·8
758°	25·3	65·2	7·8	27·0
838°	41·0	61·9	15·1	22·9
954°	70·2	53·3	39·3	6·8
1,010°	94·0	48·8	49·7	1·5
1,060°	95·0	50·7	48·0	1·3
125°	99·4	50·9	48·5	0·6

The reaction between steam and carbon commences about 500° C. and below 600° C. only 8 per cent of steam is decomposed, and the products are principally carbon dioxide and hydrogen, i.e. the reaction is

$$C + 2H_2O = CO_2 + 2H_2 + 12 \times 8{,}080 - 4 \times 29{,}000 \text{ C.H.U.} \quad (4)$$
$$= CO_2 + 2H_2 - 19{,}040 \text{ C.H.U. absorbed}$$

12 lb. C decompose 36 lb. steam, absorbing heat.

As the temperature is raised, the CO_2 is reduced in the presence of carbon to CO, until above 1,000° C., the ideal reaction of carbon and steam in the manufacture of *water gas* gives CO and H_2 in equal volumes, thus—

$$C + H_2O = CO + H_2 + 12 \times 2{,}420 - 2 \times 29{,}000 \text{ C.H.U.} \quad . \quad (5)$$
$$= CO + H_2 - 28{,}960 \text{ C.H.U.}$$

12 lb. C decompose 18 lb. steam, absorbing nearly 29,000 C.H.U. from the heat of the incandescent fuel in the producer.

The mixture of gases formed by the reactions (4) and (5) also interact, thus—

above 1,000° C, $CO_2 + H_2 = CO + H_2O - 10{,}000$ C.H.U. absorbed,
at 500° C, $CO + H_2O = CO_2 + H_2 + 10{,}000$ C.H.U. liberated,

and the reaction is reversible—

$$CO + H_2O \rightleftharpoons CO_2 + H_2 \pm 10{,}000 \text{ C.H.U.}$$

In the practical working of a gas producer, these gases undergo a chemical change, depending on the temperature, which results in a mixture of all four gases. Generally, increase of temperature favours the formation of CO, giving a richer gas and higher efficiency, while at low temperatures a higher percentage of CO_2 is obtained.

At any temperature above 500° C. the ratio $\dfrac{CO \times H_2O}{CO_2 \times H_2} = K$, the "equilibrium constant," increases with the temperature.

Oscar Hahn found the values of K to change from about 1 to 2·49 at various temperatures between 800° C. and 1,400° C.

The manufacture of *water gas* from coke is *intermittent*; by alternate air blast or blow during 1 min. to raise the temperature of the coke to incandescence, followed by the superheated "steam blow" about 4 min., when the ideal gas by reaction (5) is collected until the temperature falls too low. Then the "air blow" is repeated, and the "steam blow" passed alternately up and down through the incandescent fuel.

Theoretical Weight of Steam Required per Pound of Carbon in a producer with mixed air and steam blast. The richest gas and maximum efficiency will be obtained by the ideal reactions (1) p. 240 and (5) above. Assume no heat loss by radiation, and that all the sensible heat of the gases leaving the producer are used in raising steam utilized in the reactions, so that all the heat energy of the fuel is available. Then the amount of heat absorbed by the interaction of steam and carbon will be equal to the heat evolved by the formation of CO from carbon with the oxygen in the blast.

By reaction (1) in the direct combination of 24 lb. of carbon with oxygen to form CO, the amount of heat liberated is 24 × 2,420, or 58,080 C.H.U.

By (5), above 1,000° C., every 12 lb. of carbon decompose 18 lb. of steam, and the heat absorbed is 28,960 C.H.U. Since H_2O is supplied to the producer as water, not as steam, and taking the temperature of water supply 17° C., the amount of heat required to raise 1 lb. of this feed water to 100° C., and generate 1 lb. of steam at atmospheric pressure, is $(100 - 17) + 539 = 622$ C.H.U.

Therefore 18 lb. of steam require 18×622, or 11,200 C.H.U.

Hence the reaction of steam with 12 lb. of carbon absorbs $28{,}960 + 11{,}200$, or 40,160 C.H.U.; and the weight of water needed to take up the heat evolved in reaction (1) is $18 \times \frac{58{,}080}{40{,}160} = 26$ lb.

The carbon required for this weight of H_2O will be, by reaction (1) 24 lb., and by reaction (5) $12 \times \frac{26}{18} = 17{\cdot}34$, or the total carbon, 41·34 lb.

∴ weight of water to form the steam decomposed per lb. of carbon is $\frac{26}{41{\cdot}34} = 0{\cdot}63$ lb.

In case it is necessary to work at the lower temperature, for the recovery of the nitrogen in the fuel as ammonia in the Mond type of gas producer, by reaction (4), 12 lb. of carbon, in decomposing 36 lb. steam, absorb 19,040 C.H.U. + heat of formation of steam from water at 17° C., that is 36×622, or 22,390 C.H.U., or total heat 41,430 C.H.U.

∴ Weight of water required is $36 \times \frac{58{,}080}{41{,}430} = 50{\cdot}47$ lb. and the total weight of carbon is

$$24 + 12 \times \frac{50{\cdot}47}{36} = 24 + 16{\cdot}82 \text{ or } 40{\cdot}82 \text{ lb.}$$

∴ Weight of steam decomposed per lb. of carbon is

$$\frac{50{\cdot}47}{40{\cdot}82} = 1{\cdot}236 \text{ lb.}$$

Composition of the Ideal Producer Gas. By reaction (1) p. 240, 24 lb. of carbon yield 56 lb. of CO gas. The weight of 1 cu. ft. of CO gas is $14 \times 0{\cdot}00559$, or 0·07826 lb.; hence 24 lb. of carbon give

$$\frac{56}{0{\cdot}07826} = 716 \text{ cu. ft. of CO at N.T.P.}$$

From reaction (5), 17·34 lb. carbon give

$$\frac{28 \times 17{\cdot}34}{12 \times 0{\cdot}07826} = 517 \text{ cu. ft. of CO at N.T.P.}$$

making the total yield of CO 1,233 cu. ft.

Also, from the steam and 17·34 lb. C, the yield of hydrogen is

$$\frac{2 \times 17{\cdot}34}{12 \times 0{\cdot}00559} = 517 \text{ cu ft. at N.T.P.}$$

Again, by (1), the total oxygen supplied is 32 lb.

$$\text{or,} \quad \frac{32}{16 \times 0{\cdot}00559} = 357{\cdot}8 \text{ cu. ft. at N.T.P.}$$

and, since air contains 79·1 per cent by volume of nitrogen, the total nitrogen in the air with 357·8 cu. ft. of oxygen is

$$357{\cdot}8 \times \frac{79{\cdot}1}{20{\cdot}9} = 1{,}354 \text{ cu. ft. at N.T.P.}$$

The gas formed will be made up of

CO = 1,233 cu. ft.	= 39·73 per cent	Combustible gases 56·4 per cent	
H_2 = 517 ,,	= 16·65 ,,		
N_2 = 1,354 ,,	= 43·62 ,,		
Total 3,104 ,,	100·00		

having higher calorific value = 107·3 C.H.U. per cu. ft.

and its lower calorific value = 101·7 C.H.U. per cu. ft.

The proportion of steam to the air supply controls the temperature of these reactions, as well as the yield of gas and its composition.

Example 14. A gas engine uses producer gas which has a volumetric analysis: CO, 11; H_2, 29; CH_4, 1·8; CO_2, 16·1; N_2, 42·1. Calculate the volume of air required to completely burn a cubic foot of this gas.

If the air supplied to the gas engine be 50 per cent in excess of this quantity, what is the calorific value of a cubic foot of cylinder mixture? Calorific values of CO, H_2, and CH_4 are 190, 162, and 535 C.H.U. per cubic foot respectively. What is the gas consumption per horse-power hour if the engine efficiency is 23·6 per cent?

(*U.L., B.Sc.* (*Eng.*), 1922.)

The oxygen required for complete combustion of 1 cu. ft. of producer gas is—

$$\text{For } CO = \frac{0{\cdot}11}{2} = 0{\cdot}055 \text{ cu. ft. (p. 196)}$$

$$\text{,, } H_2 = \frac{0{\cdot}29}{2} = 0{\cdot}145 \text{ ,,}$$

$$\text{,, } CH_4 = 2 \times 0{\cdot}018 = \underline{0{\cdot}036} \text{ ,,}$$

$$\therefore \text{ total oxygen} = 0{\cdot}236 \text{ cu. ft.,}$$

and air contains 20·9 per cent by volume of oxygen,

$$\therefore \text{ air required} = \frac{0{\cdot}236 \times 100}{20{\cdot}9} = 1{\cdot}1292 \text{ cu. ft.} \qquad \textit{Answer.}$$

$$\text{and 50 per cent excess air} = \underline{0{\cdot}5646} \text{ ,, ,,}$$

$$\text{making a total of } 1{\cdot}6938 \text{ cu. ft.}$$

The mixture of 1 cu. ft. of producer gas with excess air occupies 2·6938 cu. ft. in the engine cylinder, and the combustible content evolves in burning,

$$0{\cdot}11 \times 190 + 0{\cdot}29 \times 162 + 0{\cdot}018 \times 535 \text{ C.H.U.}$$

$$= 20{\cdot}9 + 46{\cdot}98 + 9{\cdot}63 = 77{\cdot}51 \text{ C.H.U.}$$

Hence the calorific value of a cubic foot of cylinder mixture is

$$\frac{77{\cdot}51}{2{\cdot}6938} = 28{\cdot}77 \text{ C.H.U.} \qquad \textit{Answer.}$$

Again, 1 H.P. hour = 33,000 × 60 ft-lb., and its thermal equivalent

$$= \frac{33{,}000 \times 60}{1{,}400} = \frac{9{,}900}{7} \text{ C.H.U.}$$

If the thermal efficiency of engine is

$$\frac{23{\cdot}6}{100} = \frac{\text{Heat converted into work}}{\text{Heat supplied}}$$

$$= \frac{9{,}900}{7} \times \frac{1}{77{\cdot}51 \times \text{cu. ft. gas}}$$

∴ gas consumption per horse-power hour is

$$\frac{9{,}900 \times 100}{7 \times 77{\cdot}51 \times 23{\cdot}6} = 77{\cdot}32 \text{ cu. ft.} \qquad \textit{Answer.}$$

Calorific Values and Latent Heats at Constant Volume and Constant Pressure. When the calorific value of a fuel is tested in a calorimeter it is the internal energy which is absorbed by the water passing through, or in the calorimeter. The fundamental energy equation states that Q = change of internal energy $+ \frac{1}{J}\int p \, . \, dv$. Therefore a fuel which on combustion produces an increase in volume will show a smaller calorific value when burnt at constant pressure than when burnt at constant volume, although the heat liberated in both cases is the same.

Hence when burning at constant volume

Heat liberated = change in internal energy

and when burning at constant pressure

$$\text{Heat liberated} = \text{change in internal energy} + \frac{p \times \text{increase in volume}}{J}$$

Hence for the same amount of heat liberated

$$\text{Calorific value at constant volume} = \text{Calorific value at constant pressure} + \frac{p\int dv}{J}$$

Let the increase in volume be x mol. per mol. burnt.

Then the increase in volume will be xv, where v = molar volume

But $pv = 1{\cdot}985\ T$ (see p. 61), and therefore

$$\text{Calorific value at constant volume} = \text{Calorific value at constant pressure} + 1{\cdot}985 \times x \times T$$

In the same manner there are two latent heats, the heat necessary to vaporize a liquid being less at constant volume than at constant pressure.

Thus L at C.V. per mol. = L at. C.P. per mol. − 1·985 × x × T

Example 15. What is the calorific value of hydrogen burnt at constant volume, if its calorific value at constant pressure is 34,500 C.H.U. per lb.? Assume cooling to 0° C.

If it is assumed that air contains 21 per cent oxygen and 79 per cent nitrogen, the combustion equation is

$$2\ H_2 + O_2 + \frac{79}{21}\ N_2 \rightarrow 2\ H_2O + \frac{79}{21}\ N_2$$

As a mol. of all gases occupy the same volume, 2 mols. of H_2 + 1 mol. of $O_2 + \frac{79}{21}$ mols. of nitrogen become 2 mols. of $H_2O + \frac{79}{21}$ mols. of N_2 of which the two mols. of H_2O are condensed.

Hence the change in volume per mol. of hydrogen is $\frac{3}{2}$ mols., and the work of expansion $= -\ 1{\cdot}985 \times \frac{3}{2} \times 273$ C.H.U. per mol. H_2

$= -\ 1{\cdot}985 \times \frac{3}{4} \times 273$ C.H.U. per lb. H_2

$= -\ 406$ C.H.U. per lb. H_2.

Hence the calorific value of hydrogen at constant volume is

34,500 − 406 = 34,094 C.H.U. per lb.

Example 16. The combustion at constant volume of 1 lb. mol. of benzene (C_6H_6) with the necessary O_2 liberates 1,403,000 B.Th.U. if both C_6H_6 and the resulting H_2O are in liquid form at 60° F.

(*a*) Given that $R = 1{\cdot}985$ and that the latent heat of benzene at 60° F., and constant pressure is 14,600 B.Th.U. per lb. mol., find the heat released at constant volume if the C_6H_6 were originally in the form of vapour at 60° F., and the H_2O again in the liquid form.

(*b*) Given that the latent heat of H_2O at 60° F., and constant pressure is 1,060 B.T.U. per lb., how much heat would be released at constant volume if the C_6H_6 were originally in the form of vapour at 60° F. and the H_2O also in the vapour form at 60° F.?

(*U.L., B.Sc.* (*Eng.*), 1945.)

The calorific value of benzene is given at constant volume and the required quantities of heat liberated are also at constant volume, hence the only corrections to be made are to the latent heats.

(*a*) Here the work correction due to expansion is that equivalent to the increase in volume of benzene when changing from liquid to vapour at 60° F. Neglecting the volume of the liquid, the expansion is therefore 1 mol. per mol. of benzene.

Hence $L_{c.v.} = L_{c.p.} - 1{\cdot}985 \times x \times T$ (where $x = 1$)

$= 14{,}600 - 1{\cdot}985 \times 520$

$= 13{,}568$ B.Th.U. per lb. mol.

$\therefore$ Heat released $= 1{,}403{,}000 + 13{,}568$

$= 1{,}416{,}568$ B.Th.U. per lb. mol. of benzene

(*b*) Here the benzene starts and ends as vapour, therefore the work correction is only that due to the vaporizing of the H_2O, the expansion again being 1 mol. per mol. Hence for the H_2O, as the latent heat $L_{c.p.}$ is given per lb.

$$\begin{aligned} L_{c.v.} &= L_{c.p.} - 1{\cdot}985\,T \\ &= 18 \times 1{,}060 - 1{\cdot}985 \times 520 \\ &= 18{,}048 \text{ B.Th.U. per mol. } H_2O. \end{aligned}$$

The combustion equation is $2C_6H_6 + 15O_2 \rightarrow 6H_2O + 12\,CO_2$

i.e. with one lb. mol. of benzene there is associated 3 lb. mols. H_2O

$\therefore$ $L_{c.v.} = 3 \times 18{,}048$ B.Th.U. per lb. mol. of benzene

and heat released $= 1{,}416{,}158 - (3 \times 18{,}048)$

$= 1{,}362{,}424$ B.Th.U. per lb. mol. of benzene.

Molar Heats and the Combustion Equation. In Example 11, p. 93, it was shown how the specific heats of a gaseous mixture may be obtained from the volumetric analysis of the gas, and the individual specific heats of the constituent gases. If the combustion equation of a combustible mixture is known, the specific heat of the mixture may be obtained in a somewhat different manner. Thus for methane the equation is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

As a lb. mol. of any gas occupies the same volume under the same pressure and temperature conditions, we may write

1 mol. CH_4 + 2 mols. O_2 produce 1 mol. CO_2 + 2 mols. H_2O

If the nitrogen in the air producing the necessary oxygen is included, and assuming air contains 21 per cent O_2 and 79 per cent N_2 by volume the equation becomes

$$CH_4 + 2O_2 + 2 \times \frac{79}{21} N_2 = CO_2 + 2H_2O + 2 \times \frac{79}{21} N_2$$

The specific heat of the mixture may be obtained by taking the proportions of the specific heats of the constituents in the ratios of the number of mols. of each constituent. Similarly the equivalent molecular weight and density of the mixture may be obtained.

Example 17. Find the value of the adiabatic index γ and also that of R in $PV = WRT$ where W is the mass in lb., for a 10 per cent weak mixture of hexane vapour, C_6H_{14}, and air, given that air contains 21 per cent of O_2 by volume.

(*U.L., B.Sc.* (*Eng.*) 1945.)

	Molar	
	C_v	C_p
C_6H_{14}	40·3	42·285
Diatomics	5·18	7·165

The equation for complete combustion is

$$2C_6H_{14} + 19O_2 + 19 \times \frac{79}{21} N_2 \rightarrow \text{Products}$$

For a 10 per cent weak mixture this becomes

$$1{\cdot}8\ C_6H_{14} + 19O_2 + 71{\cdot}5\ N_2 \rightarrow \text{Products}$$

Hence allotting to each constituent its proportion of its individual specific heat, and dividing by the total number of lb. mols.—

$$C_v = \frac{(1{\cdot}8 \times 40{\cdot}3) + (19 \times 5{\cdot}18) + (71{\cdot}5 \times 5{\cdot}18)}{1{\cdot}8 + 19 + 71{\cdot}5}$$

$$= 5{\cdot}865 \text{ B.Th.U. per mol. per } ^\circ \text{ F.}$$

$$C_p = \frac{(1{\cdot}8 \times 42{\cdot}285) + (19 \times 7{\cdot}165) + (71{\cdot}5 \times 7{\cdot}165)}{1{\cdot}8 + 19 + 71{\cdot}5}$$

$$= 7{\cdot}85 \text{ B.Th.U. per mol. per } ^\circ \text{ F.}$$

$$\therefore \quad \gamma = \frac{7{\cdot}85}{5{\cdot}865} = 1{\cdot}34$$

The equivalent molecular weight of the mixture is

$$m = \frac{(1{\cdot}8 \times 86) + (19 \times 32) + (71{\cdot}5 \times 28)}{1{\cdot}8 + 19 + 71{\cdot}5} = 29{\cdot}95$$

The universal gas constant G is given by

$$G = C_p - C_v = 7{\cdot}85 - 5{\cdot}865 = 1{\cdot}985 \text{ B.Th.U. per lb. mol. per } ^\circ \text{ F.}$$

$$\therefore \quad R = \frac{G}{m} \times J = \frac{1{\cdot}985 \times 778}{29{\cdot}95} = 51{\cdot}5 \text{ ft-lb. per lb. per } ^\circ \text{ F.}$$

Partial Pressures in Combustible Mixtures. This is best explained by means of an example.

Example 18. A closed vessel of 10 cu. ft. capacity contains a mixture consisting of one-tenth of a pound of hydrogen and sufficient air for its combustion.

(*a*) When the temperature in the vessel is 20° C., determine the pressure.

(*b*) At what temperature would the pressure in the vessel reach 200 lb. per sq. in. abs.?

Take the density of hydrogen as 0·0056 lb. per cu. ft. at N.T.P. and the percentage of oxygen in air as 23 per cent by weight.

(*U.L., B.Sc.* (*Eng.*) 1937.)

(*a*) We first require the percentage of oxygen in air by volume and this is given by $\dfrac{\frac{23}{32}}{\frac{23}{32}+\frac{77}{28}} = 20{\cdot}75$ per cent O_2 and 79·25 per cent N_2.

The expression for the combustion is—

$$2H_2 + O_2 + \frac{79{\cdot}25}{20{\cdot}75} N_2 \rightarrow \text{Products}$$

i.e. 1 cu. ft. H_2 + 0·5 cu. ft. O_2 + 1·91 cu. ft. $N_2 \rightarrow$ products

$$\text{The total volume of the mixture} = 1 + 0{\cdot}5 + 1{\cdot}91$$
$$= 3{\cdot}41 \text{ cu. ft.}$$

$$\therefore \quad \text{Proportion of hydrogen in mixture} = \frac{1}{3{\cdot}41} = 0{\cdot}293$$

$$\text{The volume of } 0{\cdot}1 \text{ lb. of } H_2 \text{ at N.T.P.} = \frac{0{\cdot}1}{0{\cdot}0056} = 17{\cdot}85 \text{ cu. ft.}$$

$$\therefore \quad \frac{14{\cdot}7 \times 17{\cdot}85}{273}$$

$$= \frac{\text{Partial pressure of } H_2 \times \text{Total volume of mixture}}{\text{Temperature}}$$

$$= \frac{p_h \times 10}{293}$$

whence partial pressure of hydrogen $p_h = 28{\cdot}2$ lb. per sq. in.

But p_h = Total mixture pressure × proportion by volume of H_2 (see p. 90).

$$\therefore \quad \text{Mixture pressure} = \frac{28{\cdot}2}{0{\cdot}293} = 96{\cdot}3 \text{ lb. per sq. in. } \textit{Answer.}$$

(*b*) As the volume remains constant, $\dfrac{p_2}{T_2} = \dfrac{p_1}{T_1}$

$$\therefore \quad T_2 = T_1 \frac{p_2}{p_1} = 293 \times \frac{200}{96{\cdot}3} = 608° \text{ C. abs. } \textit{Answer.}$$

Example 19. In a petrol engine the combustible mixture is compressed adiabatically through a compression ratio of 7·5 to 1. The mean value of γ is 1·32. The combustion, which is under constant volume conditions, releases 31,800 B.Th.U. per mol. of mixture and produces a molecular increase of 7½ per cent. The pressure and temperature at the start of compression are 14 lb. per sq. in. abs. and 180° F. respectively.

Estimate the pressure and the temperature at the end of combustion, assuming there is no heat lost to the jackets.

The mean volumetric heats per mol. of the products of combustion reckoned from 180° F. are given below—

C_v	6·89	7·0	7·1	7·15
t° F.	4,000	4,500	5,000	5,500

(*U.L., B.Sc.* (*Eng.*) 1948.)

Let the points 1, 2, and 3, Fig. 72, refer to the start of the compression, end of compression, and end of combustion respectively.

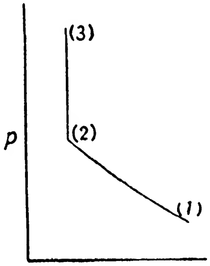

Fig. 72

Then $p_1 = 14$ lb. per sq. in., $T_1 = 640$° F. abs. and volume compression ratio $r = 7·5$.

For the adiabatic compression from 1 to 2

$$p_2 = p_1 r^{\gamma} \quad = 14·7 \times 7·5^{1·32} = 200 \text{ lb. per sq. in.}$$

$$T_2 = T_1 r^{\gamma - 1} = 640 \times 7·5^{0·32} = 1{,}220\ ^{\circ}\text{ F. abs.}$$

Work done during adiabatic compression

$$\int p \,.\, dv = \frac{p_2 v_2 - p_1 v_1}{J(\gamma - 1)} = \frac{wR}{J}\left(\frac{T_2 - T_1}{\gamma - 1}\right) \text{ B.Th.U. per } w \text{ lb.}$$

$$= \frac{G(T_2 - T_1)}{\gamma - 1} \text{ B.Th.U. per lb. mol. with } G \text{ in heat units.}$$

$$= \frac{1·985(1{,}220 - 640)}{0·32}$$

$$= 3{,}598 \text{ B.Th.U. per lb. mol.}$$

During the adiabatic compression from 1 to 2

Heat taken in by gas = increase in internal energy
+ work done by gas

i.e.
$$Q = E_2 - E_1 + \int p \, . \, dv$$

But $Q = 0$ and $E_1 = 0$ being reckoned from 640° F. abs. while the work is done on the gas.

Hence $0 = E_2 - 3{,}598$, i.e. $E_2 = 3{,}598$ B.Th.U. per lb. mol.

During the constant volume combustion from 2 to 3

$$Q = 31{,}800 \text{ B.Th.U. per mol., and no work is done.}$$

hence $E_3 = 3{,}598 + 31{,}800 = 35{,}398$ B.Th.U. per lb. mol.

But due to combustion there is a molecular increase of 7·5 per cent.

Hence, reckoned from 640° F. abs. = 180° F.,

$$E_3 = 1{\cdot}075\, C_v(t_3 - 180)$$

$$\therefore \quad 35{,}398 = 1{\cdot}075\, C_v(t_3 - 180)$$

or
$$\frac{32{,}928}{C_v} = t_3 - 180$$

Plotting each side of this equation using values of C_v and t from the table in the question, against values of t, we obtain the value of t_3, as 4,840° F.

To find p_3, we can use the equation

$$pv = wRT = \frac{w}{m} mRT = nGT$$

where w = mass of gas,

m = molecular weight of gas,

n = number of lb. mols.

Hence $p_3v_3 = n_3GT_3$ and $p_2v_2 = n_2GT_2$

$$\therefore \quad p_3 = p_2 \frac{n_3}{n_2} \frac{T_3}{T_2} = 200 \times 1{\cdot}075 \times \frac{4{,}840 + 460}{1{,}220}$$

$$= 935 \text{ lb. per sq. in. abs.}$$

EXAMPLES VII

1. In a boiler trial the dry coal contained 84 per cent of carbon and 3 per cent of free hydrogen. The flue gas analysis gave 11·5 per cent CO_2, 8·4 per cent O_2, and 80·1 per cent of nitrogen. Calculate per pound of dry fuel, the weight of necessary air, and the weight of excess air.

(*U.L., B.Sc. (Eng.)*.)

2. The volumetric analysis of a producer gas is: H_2, 14 per cent; CH_4, 2 per cent; CO, 22 per cent; CO_2, 5 per cent; O_2, 2 per cent; N_2, 55 per cent. Find the air required for the perfect combustion of 1 cu. ft. of the gas. If 40 per cent excess air is supplied, find the volume analysis of the dry products. Air contains O_2, 20·9 per cent; N_2, 79·1 per cent by volume. (*U.L., B.Sc. (Eng.).*)

3. The volumetric analysis of a producer gas supplied to a gas engine is: H_2, 20·2; CH_4, 2·8; CO, 22·2; CO_2, 7·7; N_2, 47·1 per cent. Calculate the volume of air necessary for complete combustion of 1 cu. ft. of the gas. If 50 per cent excess air is admitted, find the actual percentage contraction in volume after combustion in the engine cylinder, and the volumetric analysis of the dry products of combustion.

4. The ultimate analysis by weight of a petrol is: C, 0·835; H_2, 0·147; O_2, 0·018. Calculate the ratio of air to petrol consumption by weight, if the volumetric analysis of the dry exhaust gas is: CO_2, 12·1; CO, 1·7; O_2, 0·8; N_2, 85·4 per cent. (*U.L., B.Sc. (Eng.)*, 1925.)

5. Analysis of coal in a boiler trial was: C, 88; H, 3·6; O_2, 4·8 per cent.; and other matters 3·6 per cent. Flue gas by volume: CO_2, 10·9; CO, 1·0; O_2, 7·1; N_2, 81 per cent. Find (*a*) the proportion of C burned to CO; and (*b*) air required per pound of fuel for combustion as it actually occurred; also (*c*) the weight of flue gas per pound of fuel burned. (*U.L., B.Sc. (Eng.).*)

6. In a boiler trial the percentage analysis of dry coal as burned was: C, 83; H_2, 4; O_2, 8; ash, etc., 5; and the volumetric analysis of the flue gas was: CO_2, 10; CO, 1·7; O_2, 8·1; N_2, 80 per cent. The rise in temperature of the flue gases was 290° C. Calculate the following items: (*a*) the proportion of C burned to CO, and the heat lost through incomplete combustion, expressing the latter as a percentage of the heat in the fuel; (*b*) the heat carried away in the flue gas per pound of coal burnt, average specific heat 0·24; (*c*) heat carried away in excess air, average specific heat, 0·2375. Calorific value of 1 lb. carbon, when burnt to CO_2, 8,080 C.H.U.; when burned to CO, 2,420 C.H.U. (*U.L., B.Sc. (Eng.).*)

7. During a boiler trial, the average analysis of the flue gases gave the following results: CO_2, 10·7; CO, 1·1; O_2, 6·85; N_2, 81·35 per cent. The chemical analysis of the coal as fired gave: C, 87·6; H_2, 3·4; O_2, 4·5; ash, etc., 4·5 per cent. Determine the number of pounds of air supplied to the boiler furnace per pound of coal; also calculate the weight of flue gas per pound of coal. (*U.L., B.Sc. (Eng.)*)

8. An oil fuel of specific gravity 0·875 contained carbon 86·9 per cent, and hydrogen 13·1 per cent. From a bomb calorimeter test, the following data were obtained: weight of oil taken, 0·9034 gramme; total weight of water, including water equivalent of bomb and calorimeter, etc., 2,800 grammes. Observed rise of temperature by combustion in the bomb calorimeter, 3·47° C., and loss by radiation, 0·025° C. Calculate the higher and lower calorific values of the oil fuel, (*a*) in C.H.U. per pound; also (*b*) in B.Th.U. per pound, and (*c*) per gallon.

9. During an engine trial, the gas, as used, was tested in a calorimeter and gave the following results: gas burned, 0·75 cu. ft., at pressure 14·8 lb. per sq. in. and 18° C, which raised the temperature of 24·5 lb. of water from 15° C. to 23·5° C.; and the steam formed was condensed to 0·04 lb. of water at 18° C. Calculate the higher and lower calorific values per cubic foot of the gas at N.T.P. Take the latent heat of steam as 539 C.H.U. per lb. at atmospheric pressure.

10. In a gas producer 1 ton of coal, having calorific value 14,000 B.Th.U. per lb., yields 208,000 cu. ft. of gas of calorific value 125 B.Th.U. per cubic foot. A gas engine takes 67·8 cu. ft. of this gas per brake horse-power hour.

Calculate: (*a*) the thermal efficiency of the gas producer; and (*b*) the brake thermal efficiency of the gas engine; (*c*) the pounds weight of coal consumed per brake horse-power hour; and (*d*) the overall thermal efficiency of the gas plant.

11. In a gas producer fed with bituminous coal of lower calorific value 13,400 B.Th.U. per lb., the yield of gas per ton of coal is 134,000 cu. ft. at N.T.P. of lower calorific value 172 B.Th.U. per cubic foot. The percentage analysis by volume of the gas is: H_2, 18·3; CH_4, 3·4; CO, 25·4; CO_2, 6·9; N_2, 46; and a gas engine takes 49·9 cu. ft. of this gas per brake horse-power hour. Calculate: (*a*) the cubic feet of air necessary for the complete combustion of 1 cu. ft. of the gas; (*b*) the thermal efficiency of the producer from the lower calorific value of the gas; (*c*) the weight in pounds of coal used per brake horse-power hour; (*d*) the brake thermal efficiency of the gas engine; and (*e*) the overall efficiency of the power plant.

12. Find the maximum efficiency of a suction gas producer, the composition of the gas produced, and its calorific value per cubic foot, assuming that the fuel is carbon, and that only dry air is passed through the fuel. Given that 1 lb. of H_2 occupies 178·8 cu. ft., that the calorific value of CO is 342·4 B.Th.U. (190·2 C.H.U.) per cubic foot; and that the calorific value of 1 lb. C is 14,544 B.Th.U. (8,080 C.H.U.).

What is the effect of admitting steam in addition to the air (*a*) on the working; (*b*) on the efficiency of the producer?

(*U.L.*, *B.Sc.* (*Eng.*).)

13. In a test of a suction gas producer and gas engine under ordinary working conditions, the following results were obtained: duration of trial, 6 hours; gas engine cylinder, 17 in. diameter by 24 in. stroke; average speed of engine, 209·5 r.p.m.; explosions per minute, 102; mean effective pressure from indicator cards, 83·4 lb. per sq. in. Brake horse-power, 102·1. Total coal consumed, 444·7 lb., of higher calorific value 14,580 B.Th.U. per lb. Work out: (*a*) fuel per brake horse-power hour; (*b*) indicated horse-power; (*c*) mechanical efficiency of engine; (*d*) thermal efficiency of gas producer and engine combined; (*e*) percentage heat of fuel converted into work on the brake.

CHAPTER VIII

INTERNAL COMBUSTION ENGINES AND GAS TURBINES

In the internal combustion engine the fuel is either gas or petrol, or other oil. The characteristic of such engines is that the combustion takes place inside the cylinder, and therefore the maximum temperature available is considerably higher than is the case with steam turbines and steam engines. Thus it is estimated that, in an internal combustion engine, a maximum temperature of about 2,500° C. is attainable. At the same time the minimum practical exhaust temperature which is possible under ordinary conditions is about 500° C. With these temperatures it will be seen that the Carnot efficiency $\frac{T_1 - T_2}{T_1} = \frac{2{,}773 - 773}{2{,}773} = 72$ per cent. The corresponding limits for a modern steam turbine may be taken as about 450° C. and 40° C. giving a Carnot efficiency of $\frac{723 - 313}{723} = 56{\cdot}7$ per cent. It must be remembered, however, that the practical efficiencies are considerably less than these figures, the highest values for a modern internal combustion engine being about 38 per cent, and for a modern steam turbine, including the boiler, about 28 per cent.

Another noteworthy point is that, whereas in the steam engine and turbine precautions are taken to prevent as far as possible any loss of heat from the cylinder, in the case of the internal combustion engine the cylinder must be cooled either by air or water, owing to the limiting temperature to which we may subject the materials at present at our disposal. This consideration is therefore at present one of the factors which prevent us from taking full advantage of the high temperatures available in this form of motive power.

From a theoretical standpoint any form of fuel which will give an explosive mixture in the cylinder should be satisfactory. In order to obtain reasonable economy, however, certain conditions must be satisfied. In the first place, owing to the practical difficulty of efficient scavenging of the cylinder after combustion, it is at present necessary that the fuel should have no ash content. For this reason the use of pulverized coal in an internal combustion cylinder is still little more than a dream. Secondly, it is essential that the fuel should be in sufficiently intimate contact with the necessary air to give efficient combustion in the very short time available. In the case of the gas engine this is a simple matter, while with a volatile fuel like petrol there is no great difficulty since the fuel vaporizes as it enters the warm cylinder. With heavier oils, however, the fuel enters the cylinder as a finely atomized liquid, and it is round the problems involved in obtaining atomization and subsequent combustion that the main development in the heavy oil engine has taken place.

Ideal Cycles. A consideration of the Carnot cycle shows that a heat engine converts heat into work by the working substance taking in heat at a high temperature or temperatures and rejecting a lesser quantity of heat at a lower temperature or temperatures, the work obtained being equal to the difference between these two quantities of heat, provided there is no exchange of heat with exterior surroundings other than the two quantities just mentioned. In practice there is always an exchange of heat between the working substance and the cylinder walls, and this complicates calculations. We will therefore first consider a number of ideal cycles in which the heat is taken in and rejected under several different conditions, and in which expansion and compression of the working substance takes place without exchange of heat; that is, they are adiabatic. Such cycles are—

1. *The Constant Temperature Cycle* in which heat is taken in and rejected at constant temperature.
2. *The Constant Volume Cycle* in which heat is taken in and rejected at constant volume.
3. *The Constant Pressure Cycle* in which heat is taken in and rejected at constant pressure.
4. *The Modified Constant Pressure or Diesel Cycle* in which heat is taken in at constant pressure and rejected at constant volume.
5. *The Dual Combustion or Composite Cycle* in which heat is first taken in at constant volume, then at constant pressure and is rejected at constant volume.
6. *The Atkinson Cycle* in which heat is taken in at constant volume, and rejected at constant pressure, expansion continuing to atmospheric pressure.

It should be noted that No. 4 is sometimes called the Diesel cycle for want of a better term. The word Diesel is at present loosely applied to most types of heavy oil engines, and we shall see in due course that this is not completely justifiable. We shall first consider these cycles on the assumption that the specific heat of the working substance remains constant at all temperatures.

1. The Constant Temperature Cycle. This is of course the Carnot cycle which is described on pp. 112–7 (Fig. 30). The efficiency was here shown to be $\frac{T_1 - T_2}{T_1}$. This efficiency may be expressed in a different form, for in Fig. 30

$$\frac{T_1 - T_2}{T_1} = \frac{T_A - T_D}{T_A} = 1 - \frac{T_D}{T_A} = 1 - \frac{p_D v_D}{p_A v_A}$$

But as the points A and D are on the same adiabatic, $\dfrac{p_D}{p_A} = \left(\dfrac{v_A}{v_D}\right)^\gamma$ where γ is the ratio of the specific heats.

$$\text{Thus efficiency} = 1 - \left(\frac{v_A}{v_D}\right)^\gamma \frac{v_D}{v_A} = 1 - \left(\frac{v_A}{v_D}\right)^{\gamma-1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

where r is the Adiabatic Compression Ratio.

It is shown on p. 113 that r also is the Adiabatic Expansion Ratio.

2. THE CONSTANT VOLUME CYCLE. In this cycle (Fig. 73) heat is taken in at constant volume from points 2 to 3. Adiabatic expansion then takes place from 3 to 4. Heat is rejected at constant volume from 4 to 1, after which the working substance is compressed adiabatically from 1 to 2.

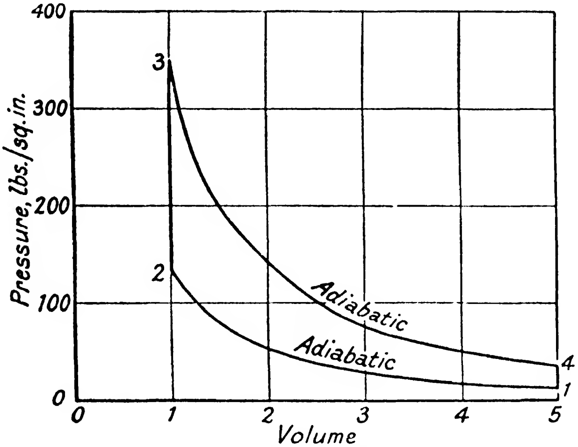

FIG. 73. CONSTANT VOLUME CYCLE

We will first consider the relation between the pressures, volumes, and temperatures of the four points on the pv diagram.

Now $\quad p_4v_4^\gamma = p_3v_3^\gamma$ or $\dfrac{p_4}{p_3} = \left(\dfrac{v_3}{v_4}\right)^\gamma$

also $\quad p_1v_1^\gamma = p_2v_2^\gamma$ or $\dfrac{p_1}{p_2} = \left(\dfrac{v_2}{v_1}\right)^\gamma$

thus $\quad \dfrac{p_4}{p_3} = \dfrac{p_1}{p_2} = \left(\dfrac{1}{r}\right)^\gamma$

where r is the adiabatic compression or expansion ratio.

As $\quad pv = wRT$

then $\quad \dfrac{T_1}{T_2} = \dfrac{p_1v_1}{p_2v_2} = \left(\dfrac{v_2}{v_1}\right)^\gamma \dfrac{v_1}{v_2} = \left(\dfrac{1}{r}\right)^{\gamma-1}$

Similarly $$\frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

hence $$\frac{T_1}{T_2} = \frac{T_4}{T_3} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

To calculate the efficiency of the cycle, we have

$$\text{Work done} = \text{Heat taken in} - \text{heat rejected}$$
$$= C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

$$\text{Efficiency} = \frac{\text{Work done}}{\text{Heat taken in}} = \frac{C_v(T_3 - T_2) - C_v(T_4 - T_1)}{C_v(T_3 - T_2)}$$
$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

But since $\frac{T_4}{T_1} = \frac{T_3}{T_2}$ or $\frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2}$

hence $$\frac{T_4 - T_1}{T_3 - T_2} = \frac{T_1}{T_2} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

and efficiency $$= 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

In Fig. 73 the following values are assumed—

$p_1 = 14·7$ lb. per sq. in.

$T_1 = 100 + 273 = 373°$ C. (abs.).

Compression ratio = 5

Maximum pressure after combustion $p_3 = 350$ lb. per sq. in.

C_v for air = 0·173

Ratio of specific heats = 1·4

Then $$p_2 = p_1 \cdot \left(\frac{v_1}{v_2}\right)^{\gamma} = 14·7 \times 5^{1·4} = 139·8 \text{ lb. per sq. in.}$$

and $$p_4 = p_3 \cdot \left(\frac{v_3}{v_4}\right)^{\gamma} = 350 \times \left(\frac{1}{5}\right)^{1.4} = 36·8 \text{ lb. per sq. in.}$$

also $$T_2 = T_1 \cdot \frac{p_2}{p_1} \cdot \frac{v_2}{v_1} = 373 \times \frac{139·8}{14·7} \times \frac{1}{5} = 709° \text{ C. (abs.)}$$

$$T_3 = T_1 \cdot \frac{p_3}{p_1} \cdot \frac{v_3}{v_1} = 373 \times \frac{350}{14·7} \times \frac{1}{5} = 1{,}775° \text{ C . (abs.)}$$

$$T_4 = T_1 \cdot \frac{p_4}{p_1} = 373 \times \frac{36·8}{14·7} = 933° \text{ C. (abs.)}$$

$$\begin{aligned}
\text{Heat taken in} &= C_v(T_3 - T_2) = 0{\cdot}173(1{,}775 - 709) \\
&= 184{\cdot}4 \text{ C.H.U. per lb.} \\
\text{Heat rejected} &= C_v(T_4 - T_1) = 0{\cdot}173(933 - 373) \\
&= 96{\cdot}9 \text{ C.H.U. per lb.} \\
\text{Work done} &= 184{\cdot}4 - 96{\cdot}9 = 87{\cdot}5 \text{ C.H.U. per lb.} \\
\text{Efficiency} &= \frac{87{\cdot}5}{184{\cdot}4} = 0{\cdot}475
\end{aligned}$$

$$\text{Also efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1} = 1 - \left(\frac{1}{5}\right)^{0{\cdot}4} = 0{\cdot}476.$$

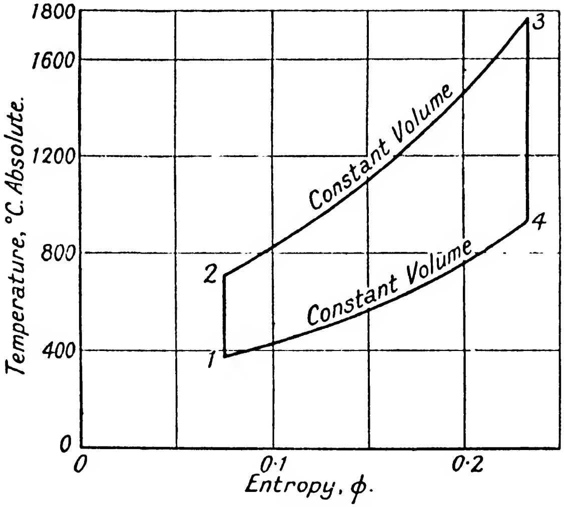

FIG. 74. T-φ DIAGRAM FOR CONSTANT VOLUME CYCLE

THE TEMPERATURE ENTROPY ($T\phi$) DIAGRAM FOR THE CONSTANT VOLUME CYCLE. It is convenient to reckon the values of entropy of the working substance from a zero value at N.T.P. The entropy of 1 lb. of air at the point 1, Fig. 73, can now be calculated from equation 11, p. 126.

$$\text{Hence } \phi_1 = C_p \log_\varepsilon\left(\frac{T_1}{273}\right) = 0{\cdot}241 \times 2{\cdot}3 \times \log_{10}\left(\frac{373}{273}\right) = 0{\cdot}075$$

This fixes the point 1 on the $T\phi$ diagram, Fig. 74.

The curve 1, 2 on the pv diagram, Fig. 73, is an adiabatic. Hence the curve 1, 2 on the $T\phi$ diagram is a vertical straight line, the position of the point 2 being given by the temperature $T_2 = 709°$ C. abs.

We now have to transfer the constant volume line 2, 3 from the pv diagram to the $T\phi$ diagram. Taking the temperature T for any point between the points 2 and 3, we can calculate the entropy from equation 16, p. 128.

$$\text{Thus}\quad \phi_3 - \phi_2 = C_v \log_\varepsilon\left(\frac{T_3}{T_2}\right) = 0{\cdot}173 \times 2{\cdot}3 \log_{10}\frac{1{,}775}{709} = 0{\cdot}159$$

$$\phi_3 = 0{\cdot}159 + 0{\cdot}075 = 0{\cdot}234.$$

Intermediate values can be obtained by taking different values of T_3 between 709° C. (abs.) and 1,775° C. (abs.).

The line 3, 4 is again a constant entropy line, and the constant volume line 4, 1 can be calculated in the same way as for the line 2, 3.

THE AIR STANDARD CYCLE. The Constant Volume cycle is the basis of the Air Standard Cycle recommended by a committee of the Institution of Civil Engineers* as a basis of comparison for internal combustion engines. The following assumptions are made in formulating this standard.

(*a*) The working substance to be considered as air.

(*b*) Expansion and compression to be considered adiabatic.

(*c*) The combustion to be considered as taking place without molecular contraction.

(*d*) The specific heat of the working substance to be considered constant at all temperatures.

(*e*) No dissociation to take place.

For air we may take the value of γ as $\frac{0{\cdot}2413}{0{\cdot}1727} = 1{\cdot}396$ at 0° C. (p. 68). Hence the efficiency of the air standard cycle may be taken as $e = 1 - \left(\frac{1}{r}\right)^{0{\cdot}396}$. A curve showing the values of e for various compression ratios is given in Figs. 80 and 101.

We will now consider how far these assumptions are applicable in practice.

(*a*) Calculations by Tizard† on petrol-air mixtures show values of γ decreasing from 1·35 for a mixture containing 80 per cent of the necessary fuel for complete combustion, to 1·32 when 50 per cent excess fuel is used. For gas engine mixtures it may be taken as varying from 1·30 to 1·38.

(*b*) No real expansion or compression is adiabatic owing to the transfer of heat between the working substance and the cylinder walls. The value of n in the law pv^n = constant in an actual expansion varies considerably owing to various causes too complicated to investigate here. Work by Tizard† on petrol-air mixtures suggests values of n for the compression curve varying from 1·32 with 80 per cent of the necessary fuel for complete combustion to 1·29 for 50 per cent excess fuel.

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* *Proc. Inst. C. E.*, Vol. 152 (1905). See also Report on Heat Engine Trials presented by Inst. C. E., 1927.

† *Proc. N.E. Coast Inst. of E.*, 1921.

(*c*) This may best be explained by reference to Example 2, p. 196. Here we see that on combustion of the hydrogen, three volumes of the original gas mixture become two volumes of products. It will also be seen that there is a contraction on the combustion of carbon monoxide, while an expansion takes place on the combustion of tetrylene. The calculation of the total percentage contraction is also shown. It will be obvious that owing to this contraction, the value of $\frac{pv}{T}$ does not remain constant round the cycle, and hence the simple expression of efficiency does not hold in practice.

A correction can be made for this discrepancy for, if p_1, v_1, and T_1 are values before combustion, and p_2, v_2, and T_2 the values after combustion, and x the volume ratio before and after combustion,

then
$$\frac{xp_1v_1}{T_1} = \frac{p_2v_2}{T_2}$$

(*d*) The specific heat of any gas varies with the temperature. With the value of specific heat increasing with temperature, the temperature to which the working substance will be raised will be lower than would be the case with constant specific heat. This matter will be dealt with more fully later, but it will be useful to consider here the effect of a given change in the specific heat.

Since $C_p - C_v = R$, and $\gamma = \frac{C_p}{C_v}$, $\gamma - 1 = \frac{R}{C_v}$

and the efficiency $e = 1 - \left(\frac{1}{r}\right)^{\frac{R}{C_v}}$

that is, $1 - e = \left(\frac{1}{r}\right)^{\frac{R}{C_v}}$, or $\frac{1}{1-e} = r^{\frac{R}{C_v}}$

$\log (1-e)$

$: - \frac{R}{C_v} \log_e r$

Differentiating with respect to C_v, we have

$$-\frac{1}{1-e} \cdot \frac{de}{dC_v} = \frac{R}{C_v^2} \cdot \log_\varepsilon r$$

$$\therefore \quad \frac{de}{dC_v} = -\frac{R(1-e)}{C_v^2} \cdot \log_\varepsilon r$$

Hence
$$de = -\frac{dC_v}{C_v^2}\{R(1-e)\log_\varepsilon r\}$$

that is,
$$\frac{de}{e} = \frac{-dC_v}{C_v}\left\{\frac{R}{C_v} \cdot \left(\frac{1-e}{e}\right) . \log_\varepsilon r\right\}$$
$$= -\frac{dC_v}{C_v}\left\{(\gamma - 1)\left(\frac{1-e}{e}\right)\log_\varepsilon r\right\}.$$

Suppose $\gamma = 1{\cdot}4$, and compression ratio $r = 5$, what is the fractional change of efficiency for 1 per cent increase in C_v? We have,

$$e = 1 - \left(\frac{1}{5}\right)^{0\cdot4} = 0\cdot475,$$

Then,

$$\frac{de}{e} = -\frac{1}{100}\left\{0\cdot4 \times \frac{1-0\cdot475}{0\cdot475} \times 2\cdot303 \log_{10} 5\right\}$$

$$= -\frac{1}{100}\left\{\frac{0\cdot4 \times 0\cdot525}{0\cdot475} \times 2\cdot303 \times 0\cdot699\right\}$$

$$= -\frac{1}{100}(0\cdot711) = -0\cdot711 \text{ per cent}$$

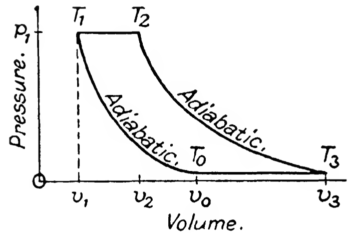

FIG. 75. IDEAL CONSTANT PRESSURE AIR CYCLE

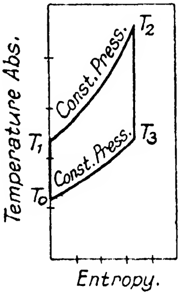

FIG. 76 T-ϕ DIAGRAM

i.e. the efficiency would decrease 0·711 per cent when C_v is increased 1 per cent.

(*e*) At the high temperatures obtained in an engine cylinder, the products of combustion can partially dissociate into CO and O_2 from the CO_2, and H_2 and O_2 from the H_2O. This dissociation is accompanied by an absorption of heat. It is true that recombination takes place during the expansion of the working substance, but as this return of heat is at a lower temperature than the maximum, there will be a loss on the whole due to this factor. As the amount of dissociation is reduced by an increase in pressure, the loss due to this cause is lessened by an increased compression ratio.

3. THE CONSTANT PRESSURE CYCLE. This cycle is shown in Fig. 75.

The ideal operations are: (*a*) adiabatic compression from p_0, v_0, T_0 to p_1, v_1, T_1; (*b*) heat is received at constant pressure, the temperature rising to T_2; (*c*) adiabatic expansion from pressure $p_2 = p_1$ to p_3, the temperature falling to T_3; (*d*) heat is rejected at pressure $p_3 = p_0$, the temperature falling to T_0.

The specific heat of the air is assumed to be constant throughout the cycle.

The heat taken in is $C_p(T_2 - T_1)$, the heat rejected is $C_p(T_3 - T_0)$, and the ideal efficiency

$$\frac{C_p(T_2 - T_1) - C_p(T_3 - T_0)}{C_p(T_2 - T_1)} = 1 - \frac{T_3 - T_0}{T_2 - T_1}$$

The ratio of adiabatic expansion is equal to the ratio of adiabatic compression, $r = \frac{v_0}{v_1}$

since $$\frac{T_0}{T_1} = \frac{T_3}{T_2} = \left(\frac{v_1}{v_0}\right)^{\gamma - 1} = \left(\frac{p_0}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

from which $$\frac{T_3 - T_0}{T_2 - T_1} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

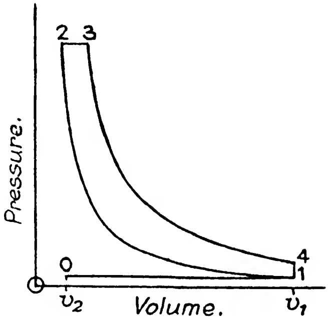

FIG. 77. IDEAL CYCLE FOR DIESEL ENGINE

Hence the air standard efficiency of this cycle becomes $1 - \left(\frac{1}{r}\right)^{\gamma - 1}$

The temperature-entropy diagram, Fig. 76, for this cycle is constructed in a similar manner to that for the constant volume cycle, with the exception that the curves indicating heat transfer are now constant pressure lines, and in their calculation C_p must be taken instead of C_v.

This cycle is of no practical utility owing to the very large value of the volume v_3 necessary for complete expansion.

It will be seen that for the three cycles so far considered the efficiencies increase with the compression ratio and are the same for equal adiabatic compression ratios. The ranges of temperature, however, would be different in the three cases for any given efficiency, the Carnot cycle having the least temperature range for any given efficiency.

4. **THE MODIFIED CONSTANT PRESSURE OR DIESEL CYCLE.** In this cycle the practical difficulty of the large volume necessary for complete expansion in the constant pressure cycle is obviated by stopping the expansion at a point 4 (Fig. 77) so that subsequent

rejection of heat at constant volume from points 4 to 1 takes place. The working fluid is then compressed adiabatically from points 1 to 2 heat is taken in at constant pressure from points 2 to 3, after which adiabatic expansion proceeds from points 3 to 4.

On these assumptions, the heat received is $C_p(T_3 - T_2)$ and the heat rejected is $C_v(T_4 - T_1)$. The efficiency of this ideal air engine is

$$\frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\gamma} \cdot \frac{T_4 - T_1}{T_3 - T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

To express this efficiency in terms of the volume ratios—

Let ratio of compression $r = \frac{v_1}{v_2}$, and $\rho = \frac{v_3}{v_2} = \frac{\text{Volume at cut-off}}{\text{Volume of clearance}}$

then, during the adiabatic operation, $T_2 = T_1 \,.\, r^{\gamma - 1}$; at constant pressure $\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho$, and $T_3 = \rho \,.\, T_2 = \rho \,.\, T_1 r^{\gamma - 1}$;

also $\quad T_4 = \rho^{\gamma} \,.\, T_1$

Substituting these values, $\dfrac{T_4 - T_1}{T_3 - T_2} = \dfrac{T_1(\rho^{\gamma} - 1)}{T_1 r^{\gamma - 1}(\rho - 1)}$

and equation (1) becomes

$$\text{Ideal efficiency} = 1 - \frac{1}{r^{\gamma - 1}}\left\{\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which is independent of the temperature.

The factor $\dfrac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}$ depends upon the cut-off, and is greater than unity; hence the ideal efficiency of this constant pressure cycle, for a given compression ratio, is less than $1 - \left(\frac{1}{r}\right)^{\gamma - 1}$, but increases as $\rho = \frac{v_3}{v_2}$ is reduced.

Example 1. An air engine works on the following cycle; air is taken in at atmospheric pressure (15 lb. per sq. in.) and temperature 60° F., and is compressed adiabatically, the pressure at the end of the stroke being 500 lb. per sq. in. Heat is taken in at constant pressure, and expansion afterwards takes place adiabatically, the ratio of expansion being 5. The air is exhausted at the end of the stroke, the heat assumed to be rejected at constant volume. Find the efficiency. Take the specific heats of air $C_p = 0{\cdot}238$, and $C_v = 0{\cdot}17$. (*U.L.*, *B.Sc.* (*Eng.*).)

Here $\gamma = \dfrac{0{\cdot}238}{0{\cdot}17} = 1{\cdot}4 = \dfrac{7}{5}$, and $\dfrac{\gamma - 1}{\gamma} = \dfrac{0{\cdot}4}{1{\cdot}4} = \dfrac{2}{7}$

$T_1 = 60° + 460° = 520°$ **F (abs.)**

In Fig. 77, the ratio of compression,

$$r = \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{5}{7}} = \left(\frac{500}{15}\right)^{\frac{5}{7}} = 12{\cdot}24$$

Also, the ratio of expansion $\frac{v_4}{v_3} = 5$, and $v_1 = v_4$

$$\therefore \text{ cut-off } \rho = \frac{\text{ratio of compression}}{\text{ratio of expansion}} = \frac{v_3}{v_2} = \frac{12{\cdot}24}{5} = 2{\cdot}448$$

In adiabatic compression $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{2}{7}} = \left(\frac{500}{15}\right)^{\frac{2}{7}}$

Taking logarithms, $\log T_2 = \log 520 + \frac{2}{7}(\log 100 - \log 3)$

$$= 2{\cdot}71600 + \frac{2}{7}(2 - 0{\cdot}47712) = 3{\cdot}15111$$

and $T_2 = 1{,}416°$ F. (abs.)

At constant pressure $\frac{T_3}{T_2} = \frac{v_3}{v_2} = 2{\cdot}448$

and $T_3 = 1{,}416 \times 2{\cdot}448 = 3{,}466°$ F. (abs.)

also $\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = \left(\frac{1}{5}\right)^{0{\cdot}4}$, and $T_4 = 3{,}466\left(\frac{1}{5}\right)^{0{\cdot}4}$

$= 1{,}821°$ F. (abs.)

To check this result,

$$T_4 = T_1 \times \rho^{\gamma} = 520 \times 2{\cdot}448^{1{\cdot}4} = 1{,}821° \text{ F. (abs.)}$$

Now, heat taken in per cycle is

$$C_p(T_3 - T_2) = 0{\cdot}238(3{,}466° - 1{,}416°) = 487{\cdot}9 \text{ B.Th.U.}$$

and heat rejected per cycle is

$$C_v(T_4 - T_1) = 0{\cdot}17(1{,}821° - 520°) = 221{\cdot}1 \quad ,,$$

Hence, heat converted into work during cycle $= 266{\cdot}8$ B.Th.U.

$$\therefore \text{ ideal thermal efficiency} = \frac{266{\cdot}8}{487{\cdot}9} = 0{\cdot}547, \text{ or } 54{\cdot}7 \text{ per cent.}$$

Alternative method, by expression (2) (p. 263); the ideal air efficiency is

$$1 - \left(\frac{1}{r}\right)^{\gamma - 1} \times \left\{\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right\}, \text{ where ratio of compression } r = 12{\cdot}24.$$

Here $\left(\frac{1}{r}\right)^{\gamma - 1} = \left(\frac{1}{12{\cdot}24}\right)^{0{\cdot}4} = 0{\cdot}3672$; and $\rho = \frac{v_3}{v_2} = 2{\cdot}448$

thus $\rho^{\gamma} = 2{\cdot}448^{1{\cdot}4} = 3{\cdot}5$, and $\gamma(\rho - 1) = 1{\cdot}4 \times 1{\cdot}448 = 2{\cdot}0272$

Substituting these values in the above expression, the efficiency is

$$1 - 0{\cdot}3672\left\{\frac{3{\cdot}5 - 1}{2{\cdot}0272}\right\} = 1 - 0{\cdot}3672 \times 1{\cdot}2334 = 0{\cdot}547$$

or 54·7 per cent.

5. **The Dual Combustion or Composite Cycle.** In this cycle the working fluid is compressed adiabatically from points 1 to 2, Fig. 78. Heat is taken in first at constant volume from 2 to 3, and then at constant pressure from 3 to 4, then there is adiabatic expansion from points 4 to 5, and rejection of heat at constant volume from points 5 to 1.

Then Heat taken in $= C_v(T_3 - T_2) + C_p(T_4 - T_3)$

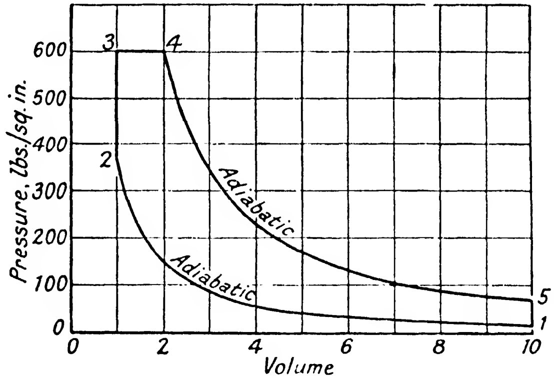

Fig. 78. *P-V* Diagram for Composite Cycle

$$\text{Heat rejected} = C_v(T_5 - T_1)$$

$$\text{Efficiency} = \frac{C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{T_5 - T_1}{T_3 - T_2 + \gamma(T_4 - T_3)}$$

$$= 1 - \frac{T_1\left(\frac{T_5}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1 + \gamma\left[\frac{T_4}{T_2} - \frac{T_3}{T_2}\right]\right)}$$

Let σ = Pressure Ratio due to Constant Volume Combustion $= \frac{p_3}{p_2}$

ρ = Volume Ratio due to Constant Pressure Combustion $= \frac{v_4}{v_3}$

$r =$ Adiabatic Compression Ratio $= \dfrac{v_1}{v_2}$

Then as $p_5 v_5^{\gamma} = p_4 v_4^{\gamma}$ and $p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$ and $v_5 = v_1$

$$\frac{T_5}{T_1} = \frac{p_5}{p_1} = \frac{p_4 v_4^{\gamma}}{p_2 v_2^{\gamma}} = \frac{p_3}{p_2}\left(\frac{v_4}{v_3}\right)^{\gamma} = \sigma\rho^{\gamma}$$

also

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1}$$

$$\frac{T_4}{T_2} = \frac{p_4 v_4}{p_2 v_2} = \frac{p_3 v_4}{p_2 v_3} = \sigma\rho$$

and

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \sigma$$

$$\text{Efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\sigma\rho^{\gamma} - 1}{\sigma - 1 + \gamma(\sigma\rho - \sigma)}\right]$$

It should be noticed that if $\rho = 1$ in this expression, as holds for the constant volume cycle, then

$$\text{Efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}\left(\frac{\sigma - 1}{\sigma - 1}\right) = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$$

whereas if $\sigma = 1$, as for the Diesel cycle,

$$\text{Efficiency} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}\left\{\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right\}$$

In Fig. 78 the following values are assumed—

$p_1 = 14{\cdot}7$ lb. per sq. in.

$T_1 = 373°$ C. (abs.)

Compression Ratio $= 10$

Maximum Pressure $p_3 = 600$ lb. per sq. in. $\qquad \rho = \dfrac{v_4}{v_3} = 2$

$C_p = 0{\cdot}241 \qquad C_v = 0{\cdot}173 \qquad \gamma = 1{\cdot}395 = 1{\cdot}4$. approx.

Then $p_2 = p_1 \cdot \left(\dfrac{v_1}{v_2}\right)^{1\cdot4} = 14{\cdot}7 \times 10^{1\cdot4} = 369$ lb. per sq. in.

$$\sigma = \frac{p_3}{p_2} = \frac{600}{369} = 1{\cdot}628$$

$$p_5 = p_4 \cdot \left(\frac{v_4}{v_5}\right)^{1\cdot4} = 600 \times \left(\frac{2}{10}\right)^{1\cdot4} = 63 \text{ lb. per sq. in.}$$

Also $T_2 = T_1 \cdot \dfrac{p_2 v_2}{p_1 v_1} = 373 \times \dfrac{369 \times 1}{14{\cdot}7 \times 10} = 936°$ C. (abs.)

$$T_3 = T_2 \cdot \frac{p_3}{p_2} = 936 \times \frac{600}{369} = 1{,}523^\circ \text{ C. (abs.)}$$

$$T_4 = T_3 \cdot \frac{v_4}{v_3} = 1{,}523 \times 2 = 3{,}046^\circ \text{ C. (abs.)}$$

$$T_5 = T_1 \cdot \frac{p_5}{p_1} = 373 \times \frac{63}{14{\cdot}7} = 1{,}600^\circ \text{ C. (abs.)}$$

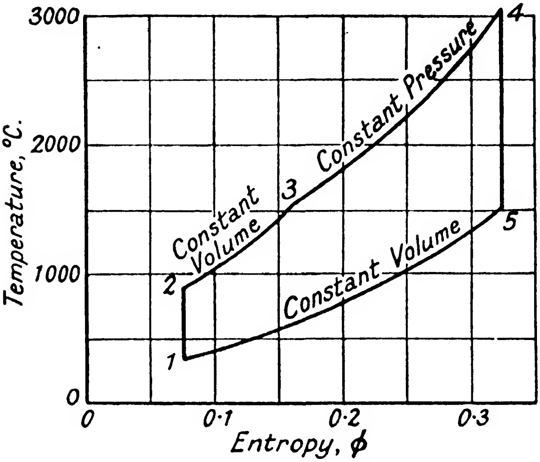

FIG. 79. T-ϕ DIAGRAM FOR COMPOSITE CYCLE

$$\begin{aligned}\text{Heat taken in} &= C_v(T_3 - T_2) + C_p(T_4 - T_3)\\ &= 0{\cdot}173(1{,}523 - 936) + 0{\cdot}241(3{,}046 - 1{,}523)\\ &= 468{\cdot}5 \text{ C.H.U. per lb.}\end{aligned}$$

$$\begin{aligned}\text{Heat rejected} &= C_v(T_5 - T_1)\\ &= 0{\cdot}173(1{,}600 - 373) = 212{\cdot}3 \text{ C.H.U. per lb.}\end{aligned}$$

$$\text{Efficiency} = \frac{468{\cdot}5 - 212{\cdot}3}{468{\cdot}5} = 0{\cdot}547$$

$$\begin{aligned}\text{Also Efficiency} &= 1 - \left(\frac{1}{10}\right)^{0{\cdot}395} \times \frac{1{\cdot}628 \times 2^{1{\cdot}395} - 1}{0{\cdot}628 + 1{\cdot}395(1{\cdot}628 \times 2 - 1{\cdot}628)}\\ &= 0{\cdot}547\end{aligned}$$

The temperature-entropy diagram, Fig. 79, is calculated in a similar manner to that for the constant volume cycle, p. 256. As the curve 2, 3 is a constant volume, and the curve 3, 4 is a constant pressure line, the curve 2, 3, 4 is not a continuous curve on the $T\phi$ diagram.

The Effect of Compression Ratio on the Composite Cycle. In this cycle the heat taken in is divided into two portions, and for a given quantity of heat per cycle there is an arbitrary ratio of the amount taken in at constant pressure to that taken in at constant volume. The proportion of the heat taken in at constant volume depends on the maximum allowable value of the pressure p_3.

Let us assume that the maximum pressure is to have a constant value, and that a constant amount of heat is to be taken in by the working fluid per cycle. Then, if the compression ratio is increased, the compression pressure p_2 is raised, and hence the amount of heat taken in at constant volume is decreased, and that taken in at constant pressure increased.

To illustrate this let (Fig. 78)—

$$p_3 = 1{,}000 \text{ lb. per sq. in.}$$

$$p_1 = 14{\cdot}7 \text{ lb. per sq. in.}$$

$$T_1 = 373^\circ \text{ C. (abs.)}$$

$$C_v = 0{\cdot}17 \qquad C_p = 0{\cdot}24 \text{ and } \gamma = 1{\cdot}4 \text{ (approx.)}$$

If $$r = \frac{v_1}{v_2} = 10$$

then $$p_2 = p_1\left(\frac{v_1}{v_2}\right)^{1\cdot 4} = 369 \text{ lb. per sq. in.}$$

$$T_2 = T_1\left(\frac{p_2v_2}{p_1v_1}\right) = 936^\circ \text{ C. (abs.)}$$

$$T_3 = T_2\left(\frac{p_3}{p_2}\right) = 2{,}537^\circ \text{ C. (abs.)}$$

Heat taken in at constant volume $= C_v(T_3 - T_2) = 272$ C.H.U. per lb.

Let this be the total amount of heat taken in.

Then efficiency $= 1 - \left(\frac{1}{r}\right)^{\gamma - 1} = 0{\cdot}6$

If $$r = \frac{v_1}{v_2} = 12$$

then $$p_2 = 14{\cdot}7 \times 12^{1\cdot 4} = 476 \text{ lb. per sq. in.}$$

$$T_2 = 373 \times \frac{476}{14{\cdot}7} \times \frac{1}{12} = 1{,}006^\circ \text{ C. (abs.)}$$

$$\sigma = \frac{p_3}{p_2} = \frac{1{,}000}{476} = 2{\cdot}1$$

$$T_3 = \sigma T_2 = 2{\cdot}1 \times 1{,}006 = 2{,}113^\circ \text{ C. (abs.)}$$

Heat taken in at constant volume

$$= 0{\cdot}17(2{,}113 - 1{,}006) = 188 \text{ C.H.U. per lb.}$$

Heat taken in at constant pressure

$$= 272 - 188 = 84 \text{ C.H.U. per lb.}$$

But $C_p(T_4 - T_3) = 0{\cdot}24(T_4 - 2{,}113) = 84 \quad \therefore T_4 = 2{,}463^\circ$ C. (abs.)

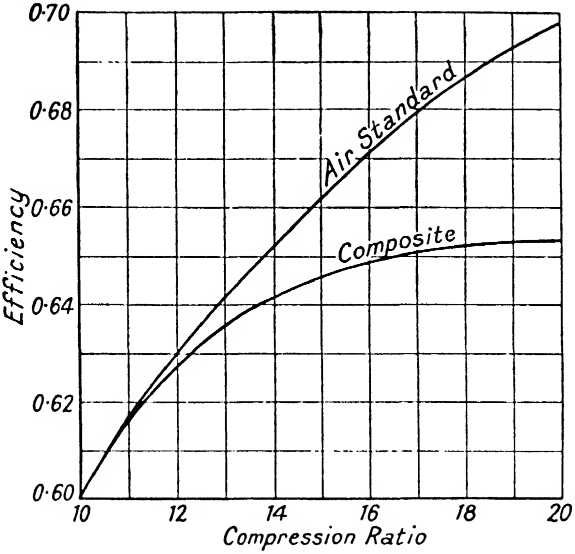

FIG. 80. COMPARISON OF COMPOSITE AND AIR STANDARD CYCLES

$$\rho = \frac{v_4}{v_3} = \frac{T_4}{T_3} = \frac{2{,}463}{2{,}113} = 1{\cdot}116$$

Efficiency of composite cycle

$$= 1 - \left(\frac{1}{12}\right)^{0\cdot4} \times \frac{2{\cdot}1 \times 1{\cdot}116^{1\cdot4} - 1}{2{\cdot}1 - 1 + 1{\cdot}4 \times 2{\cdot}1(1{\cdot}116 - 1)}$$

$$= 0{\cdot}628$$

If all the heat had been taken in at constant volume, that is, if the value of p_3 had not been limited to 1,000 lb. per sq. in., the Air Standard Efficiency would be $1 - \left(\frac{1}{12}\right)^{0\cdot4} = 0{\cdot}63$.

In Table XII the various values calculated as above are shown for various compression ratios, the efficiencies for the two cycles being plotted against compression ratio in Fig. 80.

TABLE XII

r	p_2	T_2	$\sigma = \frac{p_3}{p_2}$	T_3	Heat taken in at const. vol.	Heat taken in at const. press.	T_4	$\rho = \frac{v_4}{v_3}$	Efficiencies	
									Composite Cycle	Air Standard
10	369	936	2·710	2,537	272	0		1·000		0·600
12	476	1,006	2·100	2,113	188	84	2,463	1·116	0·628	0·630
14	591	1,071	1·692	1,812	126	146	2,421	1·336	0·641	0·652
16	713	1,131	1·402	1,586	77·4	195	2,398	1·512	0·649	0·671
20	974	1,236	1·026	1,269	5·6	267	2,380	1·875	0·653	0·698

6. The Atkinson Cycle. In this ideal cycle, the heat is taken in at constant volume from point 2 to 3 (Fig. 81). Adiabatic expansion right down to atmospheric pressure at point 4 then takes place, followed by constant pressure rejection of heat from 4 to a point 1, such that subsequent adiabatic compression brings the working fluid back to point 2.

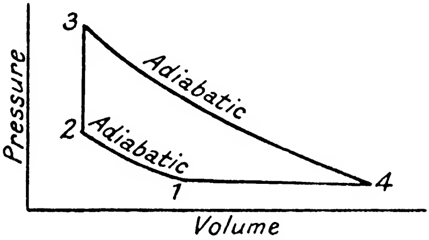

Fig. 81. Atkinson Cycle

$$\text{The efficiency} = \frac{C_v(T_3 - T_2) - C_p(T_4 - T_1)}{C_v(T_3 - T_2)} = 1 - \gamma\frac{T_4 - T_1}{T_3 - T_2}$$

$$= 1 - \gamma\left(\frac{p_4v_4 - p_1v_1}{p_3v_3 - p_2v_2}\right) = 1 - \gamma\left(\frac{p_1v_1}{p_2v_2}\right)\frac{\left(\frac{p_4v_4}{p_1v_1}\right) - 1}{\left(\frac{p_3v_3}{p_2v_2}\right) - 1}$$

But $$\frac{p_1v_1}{p_2v_2} = \left(\frac{1}{r}\right)^{\gamma - 1}, \quad p_4 = p_1, \quad v_3 = v_2$$

and $$\frac{v_4}{v_1} = \left(\frac{p_3}{p_2}\right)^{\frac{1}{\gamma}}$$

$$\text{Efficiency} = 1 - \gamma\left(\frac{1}{r}\right)^{\gamma-1}\frac{\left(\dfrac{v_4}{v_1}\right) - 1}{\left(\dfrac{p_3}{p_2}\right) - 1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1}\gamma\,\frac{(\rho^{\frac{1}{\gamma}} - 1)}{(\rho - 1)}$$

where r is the adiabatic compression ratio, and $\rho = \dfrac{p_3}{p_2}$.

This cycle (but allowing for the variation in specific heat) was

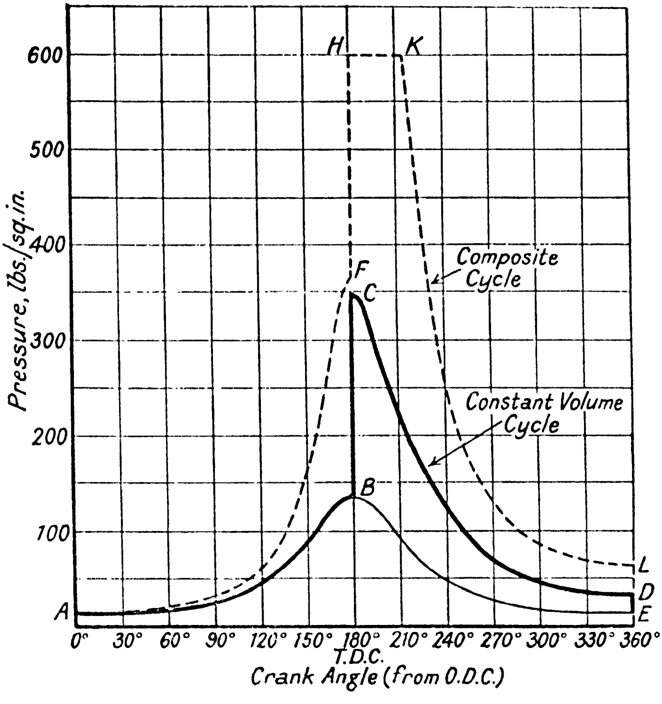

FIG. 82. PRESSURE/CRANK-ANGLE DIAGRAMS

recommended by the Heat Engines Trials Committee* as an ideal standard of comparison for all types of internal combustion engines.

Pressure/Crank-angle Diagrams. In the practical investigation of the condition of the working substance in an internal combustion engine, the most important part of the cycle is from about the point of ignition to the beginning of expansion, that is, about 30° or 40° either side of the top dead centre. About this point a considerable

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* *Report on Heat Engine Trials to the Inst. Civil E*, 1927. Also "Standards of Thermal Efficiency for Internal Combustion Engines," Sir Dugald Clerk, *Proc. Civil E.*, Vol. 225, 1928.

movement of the crank is accompanied by a relatively small movement of the piston, and hence the ordinary indicator diagram which shows the pressure of the working substance plotted to a piston displacement base, does not give a sufficiently accurate picture of what is happening in the cylinder during this important period. This difficulty can be obviated to a certain extent by changing the phase of the stroke base as indicated on p. 23. The "Farnboro" Indicator (p. 24) gives us an even better and more convenient method, in that a diagram showing pressure plotted to a crank-angle base is obtained.

Fig. 82 shows two ideal diagrams plotted on these lines. To obtain these, if the ratio of the length of the connecting rod to that of the crank is known, it is simply necessary to obtain the piston positions corresponding to various crank angles graphically, and then to plot values of pressure against these crank angles.

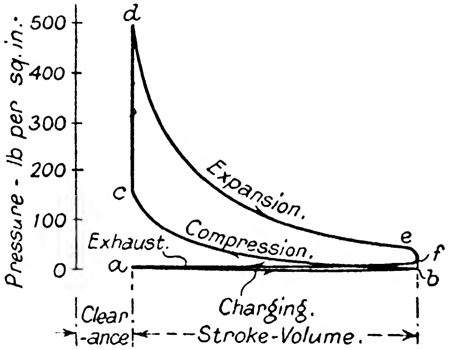

Fig. 83. Gas Engine Indicator Diagram

The curve *ABCDE* shows the constant volume cycle corresponding to Fig. 73. If no combustion took place, the condition of the working substance would be given by the expansion curve *BE*. The diagram *AFHKLE* shows the composite cycle corresponding to Fig. 78.

Practical Cycles

1. The Gas Engine. The gas engine may be said to work approximately on the constant volume cycle. The four-stroke constant volume cycle, sometimes called the "Otto" cycle, invented by Beau de Rochas in 1862, was adopted by Dr. A. N. Otto and made a practical success in 1876, being developed in this country by Messrs. Crossley Bros., Manchester, and later by many others. In a single-acting engine the cycle consists of four operations in the working cylinder on one side of the piston, during four consecutive strokes, giving one explosion or working stroke for every two

revolutions of the crank-shaft. The action is shown by the indicator diagram, Fig. 83, from an engine of 40 B.H.P.

Suction. During the first forward or *out*stroke, *ab*, of the piston a mixture of gas with excess of air is drawn into the cylinder, so that the cylinder is filled with the explosive mixture at atmospheric pressure.

Compression. During the return or *in*stroke, *bc*, the mixture is compressed into the clearance space at the end of the cylinder.

Explosion and Expansion. At *c* the compressed charge is ignited just before the crank passes the dead centre, and the pressure due to the heat energy developed by the combustion at *constant volume* rises so rapidly that the maximum pressure is reached before the piston has moved appreciably on its second outstroke. This is the *working stroke*, *cde*, and the piston is driven forward by the expanding gas.

Exhaust (*efa*). The exhaust valve is opened at *e* to give " release," and during the second back-stroke the burnt products are discharged from the cylinder, except what remains in the clearance space. These burnt products mix with the next incoming charge of gas and air, and the cycle is repeated, except when the *governor* cuts off the supply of gas ; then air is admitted, compressed, and expanded, without explosion, and discharged.

In some modern gas engines, *scavenging air* under slight pressure is admitted to the cylinder towards the end of the exhaust stroke, to sweep out the burnt products from the clearance space through the exhaust pipe, to cool the exhaust valve and leave the clearance space full of pure air.

Scavenging is specially desirable when working with gas of low heating value, such as blast furnace gas, containing a small proportion of combustibles which require to be well mixed with air to ensure complete combustion, retarded by the inert gases present. Usually there is no special arrangement for scavenging.

Beau de Rochas first gave the **chief conditions** necessary to obtain the best results and greatest economy from the elastic force of gas in the engine cylinder—

(*a*) Largest ratio of cylinder volume to boundary surface, that is, reduce to a minimum all cooling surfaces in contact with hot gases, or make the combustion space as nearly spherical as possible.

(*b*) Piston speed as high as possible.

(*c*) Long range of expansion, within practical working limits, in order to have the heat converted into work rapidly.

(*d*) Highest possible compression of charge before ignition, which is only limited in practice by pre-ignition or spontaneous combustion.

The outstanding disadvantage of this cycle is, that it gives only one impulse for every two revolutions of the crankshaft, the motor piston acting as a pump during suction and exhaust strokes, although this type is convenient for small powers and tends to keep the cylinder comparatively cool with good mechanical and volumetric efficiencies.

The light spring indicator diagrams, Fig. 84, taken from the same engine as Fig. 83, shows the bottom loop for the pumping strokes during exhaust and suction. The full line is the negative part of the diagram, Fig. 83, in which gas and air are taken; and the mean pressure of this full load suction loop is 2·9 lb. per sq. in., corresponding to 1·4 H.P. at 180 r.p.m. The dotted lines form the diagram of the pumping strokes when the governor stops the gas supply and only air is admitted to the cylinder. In this case the mean pressure of the loop is 5 lb. per sq. in. taking 2·5 H.P., that is more power than is required for the exhaust and suction strokes after an explosion.

As during the compression stroke there is an explosive mixture in the cylinder, the risk of pre-ignition in practice limits the compression ratio. This limit, which depends on the nature of the fuel, is about 7 to 1 for coal gas, but it may be increased with advantage for gases of low-heating value, such as producer or blast furnace gas.

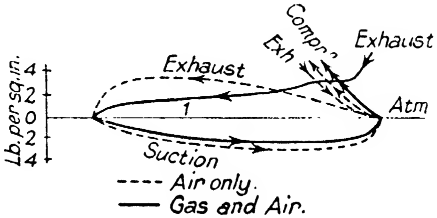

FIG. 84. LIGHT SPRING DIAGRAMS OF PUMPING STROKES

When the four-stroke cycle was introduced, attempts to overcome its defects led to the two-stroke cycle giving an explosion every revolution of the crankshaft. Such an engine was described by James Robson in Patent No. 2334 of 1877. Sir Dugald Clerk designed an engine of the two-stroke type in 1878, improving on this in 1886.

In the two-stroke cycle for a gas engine, combustion causes the working substance to expand, pushing the piston forward. Towards the end of this working stroke exhaust ports in the cylinder walls are opened by the piston, thus relieving the residual pressure of the products of combustion. This is followed by the opening of an inlet valve or inlet ports through which the explosive mixture enters under slight pressure, driving the residual exhaust gases before it out of the exhaust ports. This process continues to the end of the working stroke and for a small portion of the return stroke. The exhaust and inlet ports are then closed, and the working fluid compressed for the remainder of the return stroke, toward the end of which ignition takes place.

This cycle has two practical disadvantages: separate cylinder is necessary to put the entering explosive mixture under the necessary slight pressure, and it is difficult to avoid direct loss of explosive mixture through the exhaust ports before the beginning of compression.

The two-stroke cycle appears to be best adapted for gas engines of large output, such as the Koerting type.

2. **The Petrol Engine.** The pioneer petrol engine may be said to be that of Gottlieb Daimler described in Patent No. 4315 of 1885.

The modern petrol engine, working on the four-stroke cycle, draws in an explosive mixture of petrol and air during the suction stroke through a carburettor, the function of which is to provide a mixture of constant richness at all engine speeds. This mixture is compressed on the return stroke, being ignited somewhat before the end of this stroke by means of an electric spark. Combustion takes place, followed by expansion of the working substance during the working stroke, after which the products of combustion are pushed out of the cylinder by the returning piston. The useful compression ratio when an explosive mixture is compressed is limited, for ordinary fuels, to between 5 and 6 to 1, although this limit may be increased by the use of special dopes such as tetra-ethyl lead.

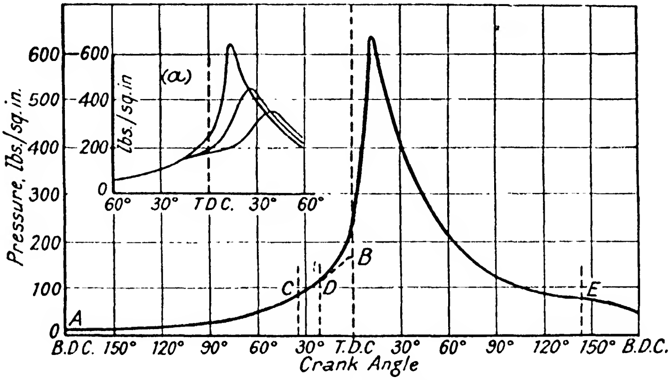

FIG. 85. PRESSURE/CRANK-ANGLE DIAGRAM FOR PETROL ENGINE

Typical indicator diagrams are shown in Fig. 14, p. 27. At first sight it appears from these that the practical cycle approximates to the constant volume cycle. Fig. 85, which shows a typical pressure/crank-angle diagram, indicates how far this assumption is justified. The compression curve, assuming no combustion, is shown at *ACDB*. If ignition takes place at *C*, shown about 35° before top dead centre, no immediate pressure rise is noticeable, but the actual curve follows the compression curve for a period *CD*. At the point *D* the pressure begins to rise rapidly, and reaches a maximum value when the crank has moved about 12° past the top dead centre. After the maximum pressure is reached, expansion proceeds until the exhaust valve opens at E somewhat before the end of the working stroke.

The period CD, during which there is no appreciable pressure rise above the normal compression pressure, indicates that there is a delay between the ignition and the start of active combustion. Investigations by Tizard and Pye* showed this period to be very consistent with various liquid hydrocarbon fuels, it being dependent on the excess of the compression temperature over the necessary ignition temperature.

Ricardo regards the combustion process in petrol engines as taking place in two distinct phases†—

(*a*) The delay period, which is more or less independent of engine speed.

(*b*) The spread of the flame throughout the combustible mixture.

The first phase depends on—

(i) The chemical nature of the fuel.

(ii) The mixture strength.

(iii) The temperature and pressure of the combustible mixture.

The second phase depends on—

(i) The shape of the combustion chamber.

(ii) The amount and nature of the turbulence, and hence on the engine speed.

The delay period is one reason why the point of ignition must occur, for maximum efficiency, before the piston reaches top dead centre, another reason being the necessity to compensate for the fact that the pressure rise after the start of active combustion takes a definite time to reach a maximum. As the time of the delay period for any given compression ratio is approximately constant, it follows that, as the engine speed increases, the ignition advance in terms of crank angle must be increased.

Fig. 85 (*a*) shows the effect of a variation in the point of ignition. As the ignition is retarded, the point of maximum pressure comes later in the working stroke. This means that, as work is being done by the working substance during the rise of pressure due to the combustion, the rate of the pressure rise will be reduced, and the maximum value of the pressure will be lessened. In practice it is found that the best point for the maximum pressure is about 10° or 12° after top dead centre. It must be remembered that the area under the pressure curve to the left of the T.D.C. in Fig. 85 corresponds to negative work in the cycle, and hence too early ignition may make this negative work excessive. Another effect is to make the pressure rise too rapidly, which will give rough running.

Petrol engines can be designed to work on the two-stroke cycle, but its use in practice is chiefly confined to engines of small size.

3. THE DIESEL ENGINE. In 1892, Herr Rudolph Diesel patented

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* *Phil. Mag.*, May, 1926.

† *Proc. Inst. Auto. E.*, March, 1930.

(No. 7421) a *Rational Heat Motor* to work on the ideal Carnot cycle described in his book of 1893. The proposed "rational" motor was to use all kinds of fuel—solid, liquid, and gas—and to operate as follows: (*a*) isothermal compression of excess air, the heat produced being carried off by water spray; (*b*) adiabatic compression of the air to the highest temperature of the cycle far greater than the ignition temperature of the fuel, to 250 atmospheres in a pulverized coal engine, modified to "only" 90 atmospheres (1,300 lb. per sq. in.); (*c*) gradual and regulated injection of the fuel, such as pulverized coal, into the hot air to produce gradual *isothermal combustion*, so that the temperature will not exceed the compression temperature during expansion at *constant pressure*; and (*d*) after the fuel cut-off *adiabatic expansion* of the surplus air and products of combustion would cool the gases to the lower limit of temperature, without artificial cooling of the cylinder walls, but, on the contrary, they should be "lagged to protect against loss by radiation of heat. By special arrangement the exhaust temperature can be made lower than the atmospheric temperature, and can be used for refrigerating purposes." "The combustion, in opposition to all hitherto known processes of combustion, does not produce any increase of temperature, or at least only an unessential one, the highest temperature in the cylinder is produced by compression of air."

This first Diesel engine patent was based on a practically impossible thermodynamic cycle, and the difficulties were found too great without a water jacket. In 1895, air injection of paraffin oil was adopted, at a pressure higher than that of adiabatic compression, which was reduced to 35 or 40 atmospheres, the isothermal compression by water spray cooling was abandoned, and external cooling by water jacket introduced.

A reliable Diesel oil engine was built in 1897, after four years of experimental work in the Maschinenfabrik Augsburg-Nurnberg (M.A.N.).

In this engine the cylinder is thoroughly water jacketed. An excess of air is drawn into the motor cylinder and compressed to 500 or 600 lb. per sq. in. Another supply of air is raised by an air compressor to about 1,000 lb. per sq. in., and stored in a reservoir for starting the engine, and for the blast or injection air which pulverizes the oil fuel by forcing it, through finely perforated disks fitted around a fluted needle valve, into the compressed air in the motor cylinder. The air blast, at a much higher pressure than that in the cylinder, causes turbulence in the combustion chamber. The expansion or drop in pressure cools the air blast 50° or 60° C., owing to the lower pressure of the compressed air. The finely divided oil mist ignites at once in contact with the hot air, and the rate of admission is so regulated that, about full load, the burning takes place at a slightly rising pressure, and when the load is decreased the burning pressure gradually decreases, although the temperature rises, during the first part of the working stroke. After cut-off the air and products of combustion continue to expand, driving the piston forward until released at constant volume, when the exhaust gases are pushed out during the return stroke in the four-stroke cycle.

In the two-stroke cycle, at the beginning of the outward stroke the fuel valve is open as in the four-stroke cycle. As the piston nears the end of its outward stroke, a set of exhaust ports in one side of the cylinder liner is uncovered and the exhaust gases commence to pass through this. On the opposite side of the cylinder liner is a set of ports, through which enters the scavenging air which expels the exhaust gases and fills the cylinder with a charge of air. As the piston moves inwards on the next stroke the two sets of ports are covered and the air is compressed to the end of the stroke. The next working stroke then commences.

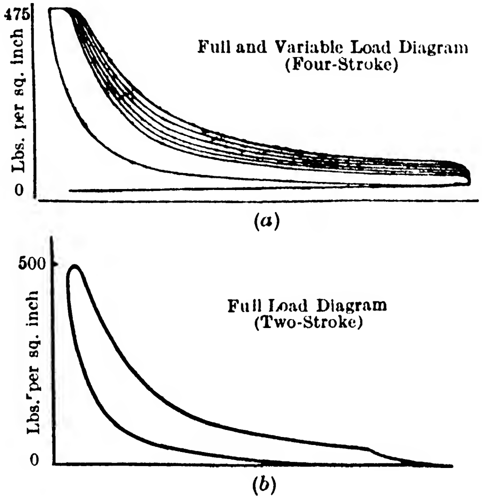

FIG. 86. INDICATOR DIAGRAMS FROM AIR INJECTION CYLINDERS (*Mirrlees, Bickerton & Day Ltd.*)

Typical indicator diagrams from air injection cylinders are shown in Fig. 86 (*a*) and (*b*). These are for engines running at from 200 to 300 r.p.m.

It should be noted that the compressed air added in the blast with the fuel increases the pressure and weight of the charge in the cylinder, and therefore gives a higher expansion curve, thus increasing the area of the indicator diagram. The work done in the compressor cylinder should therefore be deducted from that in the power cylinder in calculating the indicated horse-power. If this is not done, the indicated thermal efficiency will be too high and the mechanical efficiency too low.

This engine has now been almost entirely superseded by the airless injection type.

4. THE COMPRESSION IGNITION AIRLESS INJECTION OIL ENGINE. Prior to 1890 the compressed combustible mixture in the cylinder of gas and oil engines was ignited by the electric spark, or various devices of flame, heated platinum coil, and tubes kept hot by external burners.

Such engines were the Brayton Oil Engine, introduced into England by Messrs. Simon of Nottingham in 1878, and the Priestman Oil Engine of 1888, which was the first reliable engine with compression to work successfully on paraffin or ordinary burning oils. In 1890 a new type of oil engine, of simple construction, was introduced, to work with heavy residual oils. Mr. H. Akroyd Stuart was the originator of the method of compressing air alone in the motor cylinder previous to the injection of the fuel oil by means of a pump and spraying nozzle, and of **automatic ignition by compression** of the air in the combustion space at the end of engine cylinder, the essential pioneer work being carried out in his works at Bletchley, Buckinghamshire. He began experimental work on oil engines in 1886, and in 1889 he tried to eliminate the possibility of pre-ignition of the mixture of air and oil vapour during compression by taking a lean mixture with pure air alone next the piston. During these attempts he conceived the idea of filling the engine cylinder with air alone on the suction stroke, and experiment proved that the oil spray could be automatically ignited by injection into the hot compressed air in the vaporizer or combustion chamber when the piston was near the end of the compression stroke, and before commencing its second outward stroke. The engine had no cylinder clearance, except the vaporizer, and all the excess air was compressed into this combustion chamber before the oil spray was injected. The combustion chamber was not jacketed; the walls were corrugated and provided with internal webs, giving a large heating surface. At starting it was heated externally by an oil lamp until the temperature attained was sufficient to ignite a few charges by compression. Then the combustion chamber was maintained at a high enough temperature by the heat retained from the explosions, together with the heat of the compressed air, to ensure

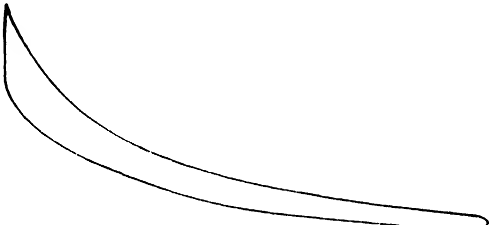

FIG. 87. INDICATOR DIAGRAM WITH SMALL CHARGE FROM AKROYD EXPERIMENTAL ENGINE (1905 TYPE)

regular automatic ignition. In fact, the spring fitted on the air inlet valve at the end of the combustion chamber got overheated, and its position was changed to the side of the cylinder near the exhaust valve. By this method the Akroyd oil engine worked on the four-stroke cycle: air alone was drawn into the cylinder, compressed into the vaporizer, followed by oil spray injection into the compressed air near the end of the compression stroke, giving perfectly regular automatic ignition and explosion, and then exhaust as stated in the Patent No. 7146 of 8th May, 1890. This engine would work without pre-ignition *only* when the oil spray was injected near the end of the compression at firing time.

In 1891, Mr. W. Robinson tested the Akroyd type of compressed air automatic ignition oil engine,* at Bletchley, and working as described in the patent specification No. 7146. The engine used intermediate shale oil of specific gravity 0·854 and flash point 225° F.

The quantity of oil injected was regulated to suit the load on the engine, and the speed was controlled by a sensitive centrifugal governor which opened a by-pass valve and allowed the oil delivered by the pump to flow back to the oil tank, instead of passing through the sprayer into the combustion chamber. The cylinder was water-jacketed, the working parts few, of neat design, and remarkable simplicity. The engine, rated at 6 H.P., worked smoothly and steadily at 216 r.p.m., developing 7·6 B.H.P. The consumption was considerably less than a pint of the heavy oil fuel per brake horse-power hour.

This early type of Akroyd oil engine worked on the same principle as the Diesel engine of 1897, inasmuch as excess air was compressed in the cylinder before oil injection, and differed from it in that the compression was not so high and the combustion chamber retained a greater proportion of heat from successive explosions; but the simple, prompt, and direct airless injection of oil by a pump through a spraying nozzle, at the proper moment, produced explosion at approximately constant volume, and did not necessitate the use of a two- or three-stage air compressor with intercoolers, air receivers, and accessories as in the Diesel oil engine.

Akroyd Stuart worked out another method of keeping the oil vapour and air apart, and gradually mixing them by compressing a portion of the air from the cylinder into the oil vapour in the vaporizer until an explosive mixture was formed, giving automatic ignition as described in the patent specification No. 15994 of 8th October, 1890. A novel and important feature of this device was a **contracted passage** or **bottle-neck** between the combustion chamber and the motor cylinder, which is still a characteristic of many modern oil engines. The air was kept separate from the oil vapour until the compression stroke.

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* See *Journal of the Society of Arts*, April, 1891, on "The Uses of Petroleum in Prime Motors."

During his experiments with the bottle-neck vaporizer, he found that the oil spray or jet injection could be timed so as to occur *not only* at firing time, but during any part of the compression stroke, or even of the suction stroke, to allow more time for vaporization, without any sign of pre-ignition. An adjustable cam was used to *time the injection* of the charge of oil, and deliver it quickly into the vaporizer at any desired point of the suction stroke or during the compression of the air to ensure the best conditions of working.

The vaporizer, once heated, was maintained at a sufficient temperature by the successive explosions and by the hot residual products, to vaporize the oil spray and ignite the compressed mixture.

Only part of the air drawn into the cylinder was compressed through the narrow neck into the oil vapour in the vaporizer to form an explosive mixture, when automatic ignition and explosion followed in the next out-stroke, and the flame was projected with great turbulence into the surplus air in the motor cylinder.

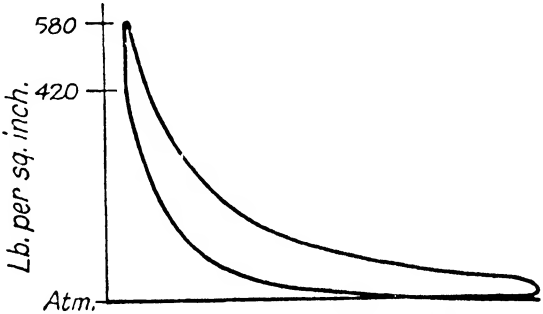

FIG. 88. INDICATOR DIAGRAM FROM RUSTON HEAVY-OIL ENGINE
(*Ruston & Hornsby*)

A clearance space, of a quarter of the cylinder diameter, was left between the piston and the back end of the cylinder, which was filled with relatively pure compressed air to prevent contact of the unburnt or partially burnt products with the piston and cylinder walls; also to supply "an excess of oxygen to complete the combustion originated in the vaporizer, and thereby burn up any carbon which might otherwise form deposit."

In practice, low working pressure, combined with simple and substantial design, were important factors for ensuring reliability and durability in self-contained prime movers of comparatively small output for agricultural purposes, and Messrs. Hornsby, the licensees, were content to develop the "hot bulb" type of Akroyd engine (Patent No. 15994).

In recent years, however, the heavy oil engine has been developed on the original idea of Akroyd Stuart. In this modern type of oil engine "airless injection" of the fuel into the highly compressed air in the engine cylinder is carried out by means of a positively driven pump plunger driving the oil at high liquid pressure through a nozzle or atomizer. This type of oil engine, which has been developed to a considerable extent in this country, can be designed for either the two-stroke or four-stroke cycle. Unfortunately this engine is often described as a Diesel engine. Strictly speaking, this name should be limited to the engine using compressed air injection of the fuel. The two types can therefore best be differentiated by the terms *airless injection* and *air injection* engines.

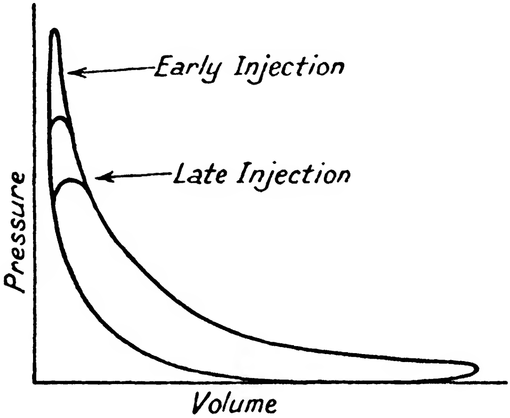

FIG. 89. INJECTION TIMING

A typical indicator diagram for an airless injection oil engine is given in Fig. 88.

It will be noticed that this diagram has a much sharper top than is the case with the air injection type. As a matter of fact the type of diagram obtained depends, other conditions being equal, on the timing of the point of injection. The engine may be said to work on the dual combustion cycle, the proportion of the heat taken in at constant pressure to that taken in at constant volume depending on the timing of the injection. This is illustrated in Fig. 89, which shows that as the injection point is advanced so the proportion of the heat taken in at constant volume is increased.

The tendency in modern high-speed compression ignition engines is to advance the injection well before the end of the compression stroke, and in this case the major portion of the heat is taken in at approximately constant volume. It is found that by this means the fuel economy is improved due to the greater range of expansion. It must be remembered that the compression ratio also affects the proportioning of the heat taken in as explained on p. 268.

The Combustion Process in Compression Ignition Engines. In the air injection system, the use of compressed air at a pressure considerably higher than the compression pressure in the engine cylinder gives very efficient atomization of the fuel, and a large amount of turbulence, resulting in excellent mixing of the fuel with the necessary air. This fact, combined with the possibility of a small advance of the injection point, gives a more or less flat top to the indicator diagram with a relatively low maximum pressure. In other words, the rise of pressure due to combustion is small, the fuel burning as it enters and the pressure remaining more or less constant.

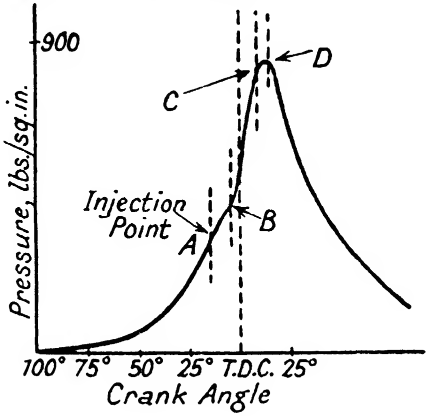

FIG. 90. COMBUSTION IN C.I. ENGINES

In the airless injection system Ricardo,* as a result of his investigations, regards the process as taking place in three distinct phases as indicated in Fig. 90.

These phases are—

(i) A delay period *AB* much the same as in the case of the petrol engine.

(ii) A period *BC* covering the mechanical spread of the flame from the initial nucleus to the main body of the cylinder contents.

(iii) A period *CD* during which the remainder of the fuel burns as it enters the cylinder from the atomizer.

The first phase, the time of which appears to be constant, depends on the nature of the fuel, the compression pressure and temperature, and the degree of pulverization of the finest particles. It is the finest particles that matter during this phase, for there is merely a formation of nuclei in this period, from which the flame is propagated during the second period.

As the second period depends on turbulence, and this in its turn is automatically increased by a rise in engine speed, this phase

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* *Proc. Inst. Auto. E.*, March, 1930. Also *The High-speed Internal Combustion Engine*, by Ricardo (Blackie & Son Ltd.).

remains constant with crank angle rather than with time. Any device to increase the turbulence will speed up this phase, but as only a part of the fuel is in the cylinder, the maximum pressure will not be attained during this period. The actual pressure reached at the end of this phase will depend on the extent of the delay period, on the rate of the fuel injection, and on the compression pressure and temperature.

By the beginning of the third phase, the flame has spread throughout the combustion space, and hence the rate of combustion is so accelerated that the fuel burns as it enters, thus producing a further pressure rise, the rate of which depends on the rate of fuel delivery. The third period is therefore under direct mechanical control.

It should be noted that as the delay period is constant in time, the crank angle turned through during this period increases as the engine speed rises. The amount of fuel injected during the delay period will therefore become greater with increased engine speed, hence the amount of fuel available for the second phase is increased. This means that with any given conditions the maximum pressure attained will increase with increased engine speed.

Fuel Injection Methods in Compression Ignition Engines. The methods of fuel injection may be divided into three general classes—

1. The Pressure Injection or Penetration Method.
2. The Pre-combustion Chamber Method.
3. Organized Air Turbulence Method.

We will now consider these methods in detail.

1. *The Pressure Injection or Penetration Method.* The fuel is injected into the combustion space through a nozzle with sufficient pressure to ensure complete penetration with high velocity of the particles of fuel into the hot mass of compressed air in the cylinder. This method gives easy starting from cold and, owing to the simple form of the clearance space and the absence of excessive turbulence, the heat losses are minimized. It has, however, the practical objection of high injection pressure which may be dissipated in atomizing the fuel rather than producing the high velocity necessary for efficient penetration. It also appears that the velocity that can be imparted to the liquid spray is limited, and thus a limit is imposed on the maximum engine speed.

This method is chiefly used for medium and slow speed engines.

2. *The Pre-Combustion Chamber Method.* To the cylinder head is attached a small combustion chamber, separated from the main cylinder by a narrow neck or a number of small holes. The fuel is injected into the pre-combustion chamber where the first two phases of the combustion take place. The high pressure produced during the second phase projects the burning mass at a high velocity through the neck or holes into the main clearance space in the cylinder. In this method the liquid pressure need not be unduly

high, and the nozzle orifices need not be so small as in the previous method. The speed of the injected fuel, therefore, does not limit the engine speed. Owing to the separate combustion chamber, however, the heat losses are increased, and as part of this loss occurs during the compression stroke, the difficulty of starting from cold is increased.

This method is not now used to any extent in this country.

3. *The Organized Air Turbulence Method.* The air taken into the cylinder is given a swirling motion. This can be done by arranging the air ports tangentially or inclined to the cylinder walls. This swirl in the air persists during the compression stroke, and on injection the air sweeps past the entering fuel particles, thus removing the products of combustion and replacing them with fresh air. This method is favoured by Ricardo, who points out* that the velocity of the air can be enormously greater than that of the liquid fuel, and is at all times proportional to the engine speed; further it can be maintained more nearly constant throughout the combustion process.

This method is used for high speed engines.

Volumetric Efficiency. The volumetric efficiency of an internal combustion engine may be defined as the ratio of the actual volume of combustible mixture or charge drawn into the engine per stroke at N.T.P., to the volume swept by the piston per stroke. The volume of the combustible mixture drawn in is made up of two parts, the volume of the air and the volume of the gas or the fuel vapour. In the case of a gas engine the volume of the gas used can be readily measured. The volume of the air supplied can also be determined practically, and hence a direct measurement of the volumetric efficiency may be obtained. In the case of a petrol or oil engine, however, the amount of liquid fuel is measured by weight, and the volume of the vapour produced is not so readily determined. The volumetric efficiency is therefore often given in terms of the air consumption only.

In the practical method of air consumption, the fact that the air flow is pulsating, necessitates special precautions and corrections.

The most convenient methods are—

(*a*) The air-box orifice method.†
(*b*) The direct measurement method.‡
(*c*) The electric flow-meter method.§

In the air-box orifice method the engine draws its air from a large box, into which the air flows through a standard orifice. The flow

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* *Proc. Inst. Auto. E.*, March, 1930.

† "Air Supply to Internal Combustion Engines," by Watson and Schofield, *Proc. Inst. Mech. E.*, May, 1912. Also "An Investigation of the Air-box Method of Measuring the Air Consumption of Internal Combustion Engines," by L. J. Kastner, *Proc. Inst. Mech. E.*, 1947, Vol. 157.

‡ Paper by Ricardo, *Automobile Engineer*, September, 1922.

§ "Air consumption in I.C. Engines," by Moss, *Proc. Inst. Mech. E.*, April, 1924.

into the box is determined from the difference of pressure across the orifice. In the direct method air is drawn from a form of gasometer. In the electrical method the air flows over a standard electrical resistance, through which an electric current is passing, the rise in temperature of the air being measured. If W lb. of air per sec. are raised in temperature by an amount dT by an electric current flowing through a resistance R ohms with a potential difference of V volts, then $WC_p dT = \frac{V^2}{JR}$.

Example 2. Describe a method of obtaining the air consumption of an internal combustion engine.

A single-cylinder gas engine with an explosion in every cycle, used 7·8 cu. ft. of gas per min. during a test, the pressure and temperature of the gas at the meter being 3 in. of water and 17° C.; the calorific value at N.T.P. was 281 C.H.U. per cu. ft. The air consumption was 6·3 lb. per min., the barometer reading 29·3 in. The bore of the engine was 10 in. and the stroke 19 in.; the speed was 240 r.p.m.

Find its volumetric efficiency relative to air at N.T.P., (*a*) taking air and gas into account, (*b*) taking only air into account. What is the heat value of 1 cu. ft. of the mixture at N.T.P.? (*U.L., B.Sc. (Eng.)*, 1934.)

3 in. water

$$= \frac{30 \times 1}{34 \times 4} = 0{\cdot}23 \text{ in. mercury}$$

Pressure of gas

$$= 29{\cdot}3 + 0{\cdot}23 = 29{\cdot}53 \text{ in. mercury}$$

Gas used per minute

$$= \frac{7{\cdot}8 \times 29{\cdot}53 \times 273}{30 \times 290} = 7{\cdot}23 \text{ cu. ft. at N.T.P.}$$

Gas used per stroke

$$= \frac{7{\cdot}23}{120} = 0{\cdot}06 \text{ cu. ft. at N.T.P.}$$

Taking the volume of air as 12·39 cu. ft. per lb. at N.T.P.

Air used per stroke

$$= \frac{6{\cdot}3 \times 12{\cdot}39}{120} = 0{\cdot}65 \text{ cu. ft. at N.T.P.}$$

Mixture used per stroke

$$= 0{\cdot}65 + 0{\cdot}06 = 0{\cdot}71 \text{ cu. ft. at N.T.P.}$$

Stroke Volume

$$= \frac{3{\cdot}14 \times 10^2 \times 19}{4 \times 1{,}728} = 0{\cdot}865 \text{ cu. ft.}$$

Volumetric Efficiency (mixture)

$$= \frac{0{\cdot}71}{0{\cdot}865} = 0{\cdot}821 = E_m$$

Volumetric Efficiency (air)

$$= \frac{0{\cdot}65}{0{\cdot}865} = 0{\cdot}751 = E_a$$

Gas used per cu. ft. of mixture

$$= \frac{E_m - E_a}{E_m} = \frac{0{\cdot}821 - 0{\cdot}751}{0{\cdot}821} = 0{\cdot}085 \text{ cu. ft. at N.T.P.}$$

Heat value of mixture

$$= 0{\cdot}085 \times 281 = 23{\cdot}9 \text{ C.H.U. per cu. ft. at N.T.P.}$$

In practice the volumetric efficiency depends on many factors, including—

(*a*) The temperature of the charge as it enters the cylinder.

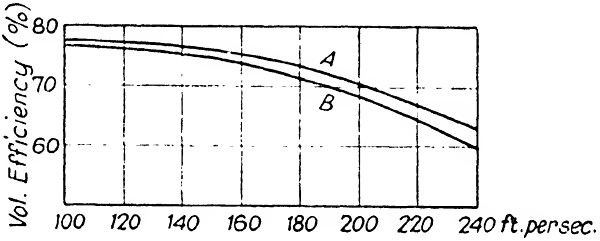

FIG. 91. RELATION BETWEEN VOLUMETRIC EFFICIENCY AND VELOCITY OF FLOW THROUGH VALVES (*Ricardo*)

(*b*) The resistance of the inlet valves and pipes, and the back pressure in the exhaust.

(*c*) The timing of the valves.

(*d*) The mixture strength and latent heat of the fuel.

(*e*) The compression ratio.

Obviously the external conditions of temperature and pressure, and any artificial heating of the induction system, as well as the cylinder and water jacket temperature, will all have their effect on the density of the charge and hence on the volumetric efficiency.

The effect of valve resistance is shown in Fig. 91 where the curves give the average volumetric efficiency at standard pressure and temperature, obtained by Sir Harry R. Ricardo from a large number of direct tests, and from many indicator diagrams. With valves in the cylinder head, the volumetric efficiency *A* is higher than *B* when they are in the side pockets. Experience shows that, with a gas velocity of 100 ft. per sec. through the inlet valves in the head, the average volumetric efficiency is 77 per cent, and the highest recorded is 79 per cent.

In practice it is difficult at high speeds to keep the inlet velocity down to 100 ft. per sec., as this would require too large valves, with the added bad effect on an effective turbulence. The higher the velocity of the gas through the inlet valve the greater will be the turbulence, and the rate of propagation of the flame after ignition, but the lower the volumetric efficiency. Experience tends to show that for effective turbulence and rapid combustion, the velocity of the fresh charge at entry must be at least 130 to 150 ft. per sec.

The effect of mixture strength is due to the cooling caused by the evaporation of a liquid fuel, and to the effect of the flame temperature on the surface temperature of the cylinder walls. With both petrol and gas fuels, there is a decrease in the volumetric efficiency as the mixture strength increases from weak to the correct proportions, the chief reason being that the increasing flame temperature increases the temperature of the cylinder walls. For mixtures richer than the correct mixture, the variation is very slight, in the case of gas, but with petrol the volumetric efficiency rises, due to the increased cooling effect of the evaporation, and to the increase of the specific heat of the charge keeping down the flame temperature. This increase of efficiency is very marked in the case of alcohol, because of its greater latent heat.

The effect of increased compression ratio is to lower the volumetric efficiency.

Suction Temperature. In the analysis of any practical cycle, a knowledge of the temperature of the cylinder contents at the commencement of the compression stroke is important. An exact determination of this temperature is not simple, as the ingoing charge of air or combustible mixture is raised in temperature by the hot metal surfaces with which it comes into contact and also by mixing with the hot residual gases left in the clearance space. In the case of liquid fuels, there will also be a lowering of temperature due to the latent heat of the fuel.

An approximate value of the suction temperature can be found if the temperature of the residual gases and the volumetric efficiency are known. For let v_s = Stroke Volume

v_c = Clearance Volume

v = Volume of charge drawn in at N.T.P.

$$E = \text{Volumetric Efficiency} = \frac{v}{v_s}$$

Then if we first consider the heat given to the charge by the cylinder walls, the volume v on entry is raised to some temperature T and expands to the volume v_s, and $\frac{v_s}{v} = \frac{T}{273}$, that is, $T = \frac{273}{E}$.

The residual gases of volume v_c and temperature T_c, in giving up heat to the incoming charge, are cooled to some final temperature T_s, and contract to a volume $v_1 = v_c \frac{T_s}{T_c}$.

The fresh charge on taking in this heat, will rise in temperature from T to T_s, and expand to a volume $v_2 = v_s \frac{T_s}{T}$.

But the volume in the cylinder

$$= v_1 + v_2 = v_s + v_c = v_c\frac{T_s}{T_c} + v_s\frac{T_s}{T}$$

$$\therefore T_s = \frac{v_s + v_c}{\dfrac{v_c}{T_c} + \dfrac{v_s \times E}{273}}$$

The value of T_c may be taken as the temperature of the exhaust gases close to the exhaust valve.

Let $T_c = 800°\text{ C.}$, $r = \dfrac{v_c + v_s}{v_c} = 5$, i.e. $v_c = \dfrac{v_s}{4}$, and $E = 0{\cdot}82$.

$$\text{Then } T_s = \frac{1 + 0{\cdot}25}{\dfrac{0{\cdot}25}{1{,}073} + \dfrac{0{\cdot}82}{273}} = 387°\text{ C. (abs.)} = 114°\text{ C.}$$

If higher values of the compression ratio are put in this equation, the suction temperature will become less.

Another expression for the suction temperature given by Morgan is

$$T_s = \frac{T_0 T r}{T_0\dfrac{P_0}{P_1} + T\left(r - \dfrac{P_0}{P_1}\right)}$$

where T_0 = the absolute temperature of the charge before mixing with the residual gases

P_0 = atmospheric pressure

P_1 = cylinder pressure at close of inlet valve

T = absolute temperature of the exhaust gases

Taking the above figures,

$$T_0 = \frac{273}{E} = \frac{273}{0{\cdot}82} = 333°\text{ C. (abs.)}$$

The value of P_1 can be obtained from the light spring indicator diagram, and will in general be slightly lower than the atmospheric pressure. For our purpose we will take it as equal to the atmospheric pressure. Then $T_s = \dfrac{333 \times 1{,}073 \times 5}{333 + 1{,}073(5-1)} = 386°\text{ C. (abs.)}$.

The difficulty in the early attempts at the *direct measurement* of temperature within the engine cylinder, by means of a platinum wire fine enough and of so small thermal capacity as to follow the rapid variation of temperature without appreciable lag, was that the high explosion temperature at once melted the fine platinum wire.

Professors Callendar and Dalby have overcome this difficulty by the ingenious device* of fitting the most delicate platinum thermometer in a jack-in-the-box valve, which exposes the fine wire

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* "On the Measurement of Temperatures in the Cylinder of a Gas Engine," *Proc. Roy. Soc.*, Series A, Vol. 80 (1907), p. 57.

to the mixture in the cylinder during compression, withdraws and screens the wire from the action of the gas momentarily at the highest temperature of explosion, and exposes it again during each suction and compression stroke.

The ordinary admission valve *AB*, Fig. 92, has an axial hole drilled through the stem and head, in which a tube moves to and fro, carrying the insulated leads from *B* to the platinum thermometer loop at *P*. This tube forms the spindle of the thermometer valve *T*, which fits into the little conical seating on *A*, and two thin ribs join the head to the spindle so as to allow the heat in the cylinder to act on the fine wire loop *P*, when the valve *T* is pushed out as shown in Fig. 92. When this valve is momentarily closed it shields the thermometer wire from the highest explosion temperatures. The thermometer valve is pushed out and closed at the correct times by a separate cam and mechanism.

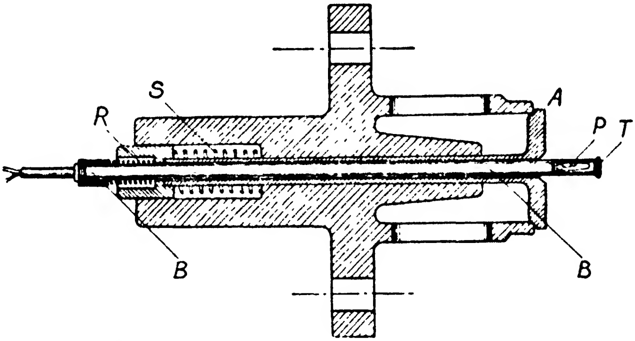


FIG. 92. ADMISSION AND THERMOMETER VALVES
(*Callendar & Dalby*)

The electrical resistance or temperature of the platinum thermometer is measured by the *bridge* and *contact-maker* shown in the diagram, Fig. 93. *P* is the pair of leads to the fine platinum loop, and *C* equal and similar leads to compensate the resistance of the other pair alongside of them in the tube, while the short piece of fine platinum wire across their ends compensates the end effect or cooling action by conduction from the thermometer wire to the thick leads. Thus the difference of resistance will be that of the hottest part of the thermometer wire exposed to the gas within the cylinder. The bridge wire *BW* is carefully calibrated.

The *contact-maker* consists of a pair of springs fitted with platinum contacts and carried on the arm *A* of a graduated and adjustable disk *O*, which is secured by clamping screw *L* for any desired point *I* of the cycle. A pair of stepped cams *D* rotate with the lay shaft, and allow the platinum point of one spring to make electric contact with the other for an interval of about 10° of the crank angle, depending upon the distance between the steps.

Readings are taken at several points in the compression stroke when the index *I* is set for the springs to make contact, and the electric balance is adjusted on the bridge wire by the indication of the galvanometer *G*.

Obviously, with rapidly varying temperatures, there will be time lag between the true temperature of the gas at any moment and that of the finest wire. On this account, Professors Callendar and Dalby made measurements on the compression curve where the rate of variation of temperature is a minimum, rather than on the expansion curve after explosion when the change of temperature may be 1,000° C. in 1/10 sec., that is, at the rate of 10,000° C. per sec. The platinum thermometer wire used was 1/1,000 inch in diameter.

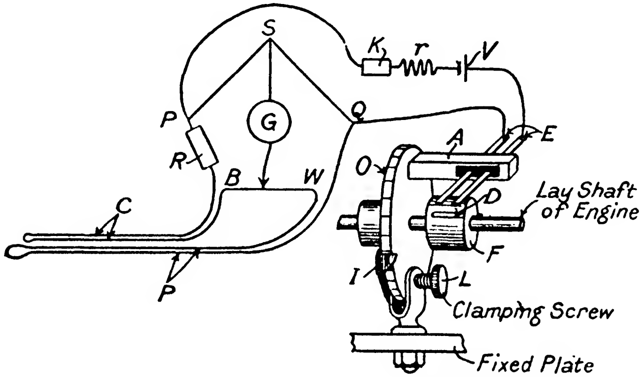


FIG. 93. CONTACT-MAKER AND THERMOMETER BRIDGE
(*Callendar & Dalby*)

At engine speed 130 r.p.m., the lag of the thermometer, for points on the compression curve, was not more than 10° of crank angle, with a temperature variation of 200° C. in half a revolution, which corresponds to a lag of about $\frac{10^\circ}{360^\circ} \times \frac{130}{60} = 0{\cdot}06$ sec. To determine the exact correction for this lag effect, the engine was motored round with the platinum thermometer exposed and readings noted when only air was compressed and expanded, while indicator diagrams were taken, from which the temperatures were calculated. Comparison of these temperatures gave the lag effect correction.

By this method it was found that the suction or charge temperature varied from 95° C. at light load to 125° C. at full load, the atmospheric temperature being 20° C. and water jacket 27° C.

Example 3. In an attempt to estimate the temperature at the end of the suction stroke of a petrol engine having a compression ratio of 5, the following assumptions are made—

(i) The clearance is filled with residuals at 15·4 lb. per sq. in. abs., and 1,600° F., and that these first expand adiabatically to 12·8 lb. per sq. in. abs.

behind an imaginary insulating diaphragm separating them from the incoming charge.

(ii) For the remainder of the induction stroke, fresh mixture is drawn in at 12·8 lb. per sq. in. abs. and 140° F.

(iii) The diaphragm is now withdrawn and the gases allowed to mix while the piston is at the out end of the stroke.

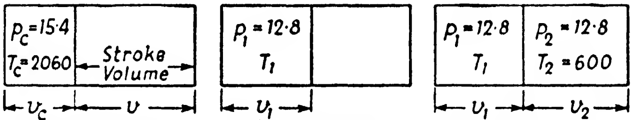
Find (*a*) the resulting temperature of the charge,

(*b*) the volumetric efficiency based on N.T.P. conditions.

	Residuals	Incoming Gas
C_p	0·250	0·24
C_v	0·179	0·17

(*U.L., B.Sc. (Eng.)* 1944.)

The three stages are illustrated in Fig. 94.



Original Residuals

After Adiabatic Expansion

Fresh Mixture In

FIG. 94

(*a*) *Adiabatic Expansion of Residuals*

$$\gamma = \frac{0{\cdot}250}{0{\cdot}179} = 1{\cdot}4$$

$$\therefore \qquad T_1 = T_c \left(\frac{12{\cdot}8}{15{\cdot}4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 2{,}060 \left(\frac{12{\cdot}8}{15{\cdot}4}\right)^{0{\cdot}286} = 1{,}954^\circ \text{ F. abs.}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_c v_c}{T_c}$$

$$\therefore \qquad v_1 = v_c \frac{p_c}{p_1} \frac{T_1}{T_c}$$

$$= \frac{15{\cdot}4}{12{\cdot}8} \cdot \frac{1{,}954}{2{,}060} v_c = 1{\cdot}14 v_c$$

As the compression ratio is 5, then $\frac{v + v_c}{v_c} = 5$ and $v = 4v_c$.

As $\quad v_1 + v_2 = v + v_c$

$$\therefore \quad v_2 = v + v_c - v_1$$
$$= 4v_c + v_c - 1\cdot14\, v_c = 3\cdot86\, v_c$$

(*b*) *After Withdrawal of the Diaphragm*

As the system is an isolated one, no heat being transferred from or to the exterior surroundings, and no external work being done—

Loss of internal energy of residuals = gain of internal energy of charge

therefore the values of R and the weights of residuals and charge are required.

For residuals, $R_r = (C_p - C_v)778 = (0\cdot25 - 0\cdot179)778$
$= 55\cdot24$ ft-lb. per lb. per ° F.

For charge, $R_c = (0\cdot24 - 0\cdot17)778$
$= 54\cdot46$ ft-lb. per lb. per ° F.

For residuals, $w_r = \dfrac{p_r v_r}{R_r T_r} = \dfrac{p_c v_c}{R_r T_c}$

$$= \frac{15\cdot4 \times 144}{55\cdot24 \times 2{,}060}\, v_c = 0\cdot0195 v_c \text{ lb.}$$

For charge, $w_c = \dfrac{p_2 v_2}{R_c T_2} = \dfrac{12\cdot8 \times 144}{54\cdot46 \times 600}\, v_2$

$$= \frac{12\cdot8 \times 144}{54\cdot46 \times 600} \times 3\cdot86 v_c = 0\cdot218 v_c \text{ lb.}$$

If T_s = final temperature of mixture

$w_r \times$ sp. ht. of residuals $(T_1 - T_3) = w_c \times$ sp. ht. of charge $\times (T_s - T_2)$

i.e. $0\cdot0195 \times 0\cdot179(1{,}954 - T_s) = 0\cdot218 \times 0\cdot17(T_s - 600)$

i.e. Final temperature of mixture $T_s = 716°$ F. abs.

Volumetric Efficiency

The volume of charge drawn in = v_2 at 12·8 lb. per sq. in. and 600° F. At N.T.P. this volume becomes

$$v_2 \times \frac{12\cdot8}{14\cdot7} \times \frac{492}{600} = 3\cdot86 v_c \times \frac{12\cdot8}{14\cdot7} \times \frac{492}{600} = 2\cdot75 v_c$$

The stroke volume $= 4v_c$

$$\therefore \quad \text{Volumetric efficiency} = \frac{2\cdot75}{4} = 0\cdot69$$

Determination of Thermal Efficiency from Air Consumption. In measuring the thermal efficiency of an engine it is customary to measure the fuel consumption, and, knowing its calorific value, so to determine the quantity of heat supplied. With this method, if some fuel is unburnt, or if some lubricating oil is burnt as fuel, the value of the efficiency so obtained is somewhat indefinite in its meaning. Ricardo has pointed out that this is especially the case in a multi-cylinder engine where a waste of fuel is possible due to irregular distribution to the cylinders, a fault which does not arise from the thermodynamic characteristics of the engine.

If, in burning 1 lb. of fuel with its requisite quantity of air, instead of regarding the heat generated H as identified with the 1 lb. of fuel, we think of this heat as identified with the x lb. of air necessary for complete combustion, we may speak of the calorific value of the air instead of that of the fuel. This quantity will be $\frac{H}{x}$. Thus, by measuring the air consumption instead of the fuel consumption, we obtain an efficiency to which it has been suggested the term *combustion chamber efficiency* be given.

Ricardo has pointed out that the calorific value of air so defined is remarkably constant for liquid fuels. This is shown in Table XIII where it will be seen that the energy liberated per pound of air is approximately 1,300 B.Th.U. for all fuels, while for the petrols, the variation is only from 1,275 to 1,295. Ethyl alcohol gives the greatest amount of heat energy per pound of air.

TABLE XIII

AIR CONSUMPTION (*H. R. Ricardo*)

FUEL	Effective Lower Calorific Value, B.Th.U. per lb.	Air to Fuel Ratio at Correct Mixture Strength	Heat Liberated by 1 lb. of Air, B.Th.U.
Petrol or Gasolene—			
Sample *A* . . .	19,200	15·05	1,275
,, *B* . . .	19,020	14·7	1,295
,, *C* . . .	19,120	14·8	1,293
,, *D* . . .	18,900	14·6	1,295
,, *F* . . .	19,250	15·0	1,285
,, *G* . . .	18,920	14·7	1,288
Kerosene . . .	19,100	15·0	1,275
Hexane	19,390	15·2	1,275
Heptane	19,420	15·1	1,285
Benzene	17,460	13·2	1,320
Toluene	17,660	13·4	1,315
Cyclohexane . .	18,940	14·7	1,290
Ether	16,830	13·0	1,295
Ethyl Alcohol, 99%	11,950	8·95	1,335

Tables XIV and XV show the results of some experiments by Ricardo in which he measured the air consumption in order to determine the combustion efficiency, and from the observed maximum mean effective pressure developed, so ascertained the thermal efficiency. It will be noticed that no matter what the excess of fuel present, there is little variation in the air consumption, while the maximum efficiency is for a mixture with about 20 per cent excess of fuel.

TABLE XIV

Petrol, Sample *D*, Compression Ratio, 5 : 1 (*H. R. Ricardo*)

Mixture Strength	Total Air lb. per hr.	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency Per Cent
Correct . . .	196·0	132·0	32·0	6·13	32·1
Excess fuel 5%	196·5	135·0	32·7	6·0	32·8
,, ,, 10%	197·0	136·5	33·1	5·95	33·1
,, ,, 15%	197·5	137·5	33·3	5·93	33·2
,, ,, 20%	198·1	138·0	33·4	5·94	33·15
,, ,, 25%	198·5	138·0	33·4	5·96	33·05
,, ,, 30%	199·5	137·5	33·3	5·98	32·85
,, ,, 35%	200·0	136·5	33·1	6·04	32·7

TABLE XV

Ethyl Alcohol, 99%, Compression Ratio, 5 : 1 (*H. R. Ricardo*)

Mixture Strength	Total Air lb. per hr.	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency Per Cent
Correct . . .	199·0	141·0	34·2	5·82	32·9
Excess fuel, 5%	199·5	143·0	34·6	5·77	33·3
,, ,, 10%	200·0	144·5	35·0	5·72	33·65
,, ,, 15%	201·0	145·5	35·25	5·70	33·7
,, ,, 20%	202·0	146·5	35·5	5·69	33·75
,, ,, 25%	203·0	147·0	35·6	5·70	33·7
,, ,, 30%	204·0	147·3	35·7	5·72	33·65
,, ,, 35%	205·0	147·6	35·8	5·73	33·6

A reference to Table XX shows that the indicated thermal efficiency based on fuel is less than that based on air. This is due to the layer which escapes complete combustion by adhering to the cold cylinder walls.

Example 4. In reckoning the thermal efficiency of an internal combustion engine from the air consumption, what is meant by the "calorific value" of a pound of air?

Calculate the thermal efficiency of a petrol-engine which developed 32 I.H.P., using 196 lb. per hour of air. The liquid fuel may be assumed to be heptane

(C_7H_{16}), and the fuel-air ratio may be assumed to have been "correct." The effective heating value of 1 lb. of heptane (i.e. the lower calorific value plus latent heat of the vapour) is 10,775 C.H.U. (*U.L.*, *B.Sc.* (*Eng.*), 1933.)

The combustion equation for heptane is—

$$C_7H_{16} + 11O_2 = 7CO_2 + 8H_2O$$

$$\left.\begin{array}{l}\text{Relative}\\ \text{weights}\end{array}\right\}\quad 100 + 352 = 308 + 144$$

Hence weight of air required per lb. of fuel for complete combustion is $\dfrac{352 \times 100}{100 \times 23} = 15{\cdot}3$ lb.

Hence higher calorific value of air

$$= \frac{10{,}775}{15{\cdot}3} = 704 \text{ C.H.U. per lb.}$$

$$\text{Energy supplied} = 704 \times 196 = 138{,}000 \text{ C.H.U. per hour}$$

$$\text{Energy given out} = \frac{32 \times 33{,}000 \times 60}{1{,}400}$$

$$= 45{,}250 \text{ C.H.U. per hour}$$

$$\text{Thermal efficiency} = \frac{45{,}250}{138{,}000} = 0{\cdot}328$$

In any internal combustion engine, the H_2O formed by the combustion is still vapour in the exhaust. Hence it is often the practice to take the lower calorific value of the fuel in calculating the thermal efficiency. In this case—

Weight of H_2O formed per lb. of fuel = 1·44 lb.

Hence lower calorific value of air

$$= \frac{10{,}775 - 1{\cdot}44 \times 624}{15{\cdot}3} \quad \text{(see p. 221)}$$

$$= 645 \text{ C.H.U. per lb.}$$

Thermal efficiency

$$= \frac{45{,}250}{196 \times 645} = 0{\cdot}358.$$

Variation of Specific Heat with Temperature. We shall see later that the specific heat of a gas increases with the temperature. The practical determination of this variation is extremely difficult, but as experimental technique becomes more perfect so we are gradually obtaining more reliable data.

For any but the most exact work it is generally assumed that the value of specific heat is a linear function of the temperature, that is, that $C_p = a + kT$ and $C_v = b + kT$ where a, b and k are constants.

Thus the characteristic constant $R = C_p - C_v = a - b$.

Instead of considering the actual value of the specific heat of an internal combustion engine mixture, it is often more convenient to use the internal energy content of the gas. As $dE = C_v dT$ the gain in internal energy of a gas when the temperature rises from absolute temperature T_1 to T_2 may be calculated from the equation

$$E_2 - E_1 = \int_{T_1}^{T_2} C_v dT = \int_{T_1}^{T_2} (b + kT)dT = \left[bT + \frac{kT^2}{2}\right]_{T_1}^{T_2}$$

If we consider the internal energy as zero at a temperature of 100° C., then the internal energy of a gas at a temperature T absolute is equal to $b(T - 373) + \frac{k}{2}(T^2 - 373^2)$.

At very high temperatures the variation of specific heat is sometimes expressed in the following form—

$$C_v = a + bT + cT^2 \text{ and } C_p = a_1 + bT + cT^2$$

making $\quad R = a_1 - a$

It will now be useful to consider the effect of these variations on the various equations that we have so far obtained on the assumption of constant specific heat.

Adiabatic Expansion with Variable Specific Heats.

1. Assuming $C_p = a + kT$ and $C_v = b + kT$

then for an adiabatic expansion $C_v v . dp + C_p p . dv = 0$ (p. 71)

$$(b + kT)v.dp + (a + kT)p.dv = 0$$

Dividing by pv and substituting the value of $T = \frac{pv}{R}$

$$\left(b + \frac{kpv}{R}\right)\frac{dp}{p} + \left(a + \frac{kpv}{R}\right)\frac{dv}{v} = 0$$

$$b\frac{dp}{p} + a\frac{dv}{v} + \frac{k}{R}(v . dp + p . dv) = 0$$

$$b\frac{dp}{p} + a\frac{dv}{v} + \frac{k}{R}d(pv) = 0$$

Integrating $\quad b \log_\varepsilon p + a \log_\varepsilon v + \frac{k}{R}(pv) = \text{Constant}$

Taking antilogs

$$p^b v^a \varepsilon^{k\frac{pv}{R}} = \text{Constant}$$

or

$$p v^{\frac{a}{b}} \varepsilon^{\frac{k}{b}T} = \text{Constant}$$

This expression may be put in another useful form by substituting for $p = \frac{RT}{v}$ then

$$\frac{RT}{v} \cdot v^{\frac{a}{b}}\, \varepsilon^{\frac{k}{b}T} = \text{Constant}$$

$$\therefore\ T v^{\left(\frac{a}{b} - 1\right)}\, \varepsilon^{\frac{k}{b}T} = \text{Constant} \quad ✓$$

This is useful in finding the temperature rise due to adiabatic expansion.

2. Assuming $C_v = a + bT + cT^2$ and $C_p = a_1 + bT + cT^2$, then $$R = a_1 - a$$

In adiabatic expansion $dE = C_v \,.\, dT = -p \,.\, dv$

$$C_v \cdot \frac{dT}{T} + \frac{p}{T} . dv = 0, \text{ that is, } C_v \,.\, \frac{dT}{T} + R \,.\, \frac{dv}{v} = 0$$

Substituting for C_v and R,

$$a\frac{dT}{T} + b \,.\, dT + cT \,.\, dT + (a_1 - a)\frac{dv}{v} = 0.$$

Integrating,

$$a \log_\varepsilon T + bT + \frac{c}{2}T^2 + (a_1 - a) \log_\varepsilon v = \text{Constant}$$

$$\therefore\ T^a v^{(a_1 - a)}\, \varepsilon^{\left(bT + \frac{c}{2}T^2\right)} = \text{Constant}$$

Rate of Heat Reception with Variable Specific Heats.

Here $\quad dQ = C_v . dT + p \ dv$ (p. 81)

$$\therefore \frac{dQ}{dv} = C_v \cdot \frac{dT}{dv} + p, \text{ but } \frac{dT}{dv} = \frac{1}{R}\left(p + v \,.\, \frac{dp}{dv}\right) \text{ by differentiating } T = \frac{pv}{R},$$

$$\therefore\ \frac{dQ}{dv} = \frac{C_v}{R}\left(p + v \cdot \frac{dp}{dv}\right) + p \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, take $C_p = a + kT$, and $C_v = b + kT$, $R = C_p - C_v = a - b$, and when $T = 0°$ C. (abs.), $\frac{C_p}{C_v} = \gamma_0 = \frac{a}{b}$

Substituting these values of C_v and R in (1)

$$\frac{dQ}{dv} = \frac{b + kT}{a - b}\left(p + v \cdot \frac{dp}{dv}\right) + p$$

$$= \frac{1}{a - b}\left\{pa - pb + pb + pkT + bv \,.\, \frac{dp}{dv} + kTv \,.\, \frac{dp}{dv}\right\} \quad (2)$$

$$= \frac{1}{\frac{a}{b} - 1}\left\{p\frac{a}{b} + v \cdot \frac{dp}{dv} + \frac{pkT}{b} + \frac{kTv}{b} \cdot \frac{dp}{dv}\right\}$$

$$\therefore \frac{dQ}{dv} = \frac{1}{\gamma_0 - 1}\left(p \cdot \gamma_0 + v \cdot \frac{dp}{dv}\right) + \frac{kT}{a-b}\left(p + v \cdot \frac{dp}{dv}\right) \quad . \quad (3)$$

For constant specific heats, $k = 0$, and this equation becomes

$$\frac{dQ}{dv} = \frac{1}{\gamma - 1}\left(p \cdot \gamma + v \cdot \frac{dp}{dv}\right), \text{ as in (12) (p. 81).}$$

Rate of heat reception when the expansion or compression follows the law $p \cdot v^n = c$, a constant. Then $v \cdot \frac{dp}{dv} = -np$ (p. 82), and substituting in (3),

$$\frac{dQ}{dv} = \frac{1}{\gamma_0 - 1}(p \cdot \gamma_0 - np) + \frac{kT}{a-b}(p - np) = \frac{\gamma_0 - n}{\gamma_0 - 1} \cdot p + \frac{kT}{a-b}(1-n)p$$

$$= \left\{\frac{\gamma_0 - n}{\gamma_0 - 1} - kT \cdot \frac{n-1}{a-b}\right\} p \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For constant specific heats, $k = 0$, and this equation becomes

$$\frac{dQ}{dv} = \frac{\gamma - n}{\gamma - 1} \cdot p, \text{ the same as (13) (p. 82).}$$

Change of Entropy with Variable Specific Heats. The equation for entropy given on p. 127 is

$$d\phi = \frac{dQ}{T} = C_v \cdot \frac{dT}{T} + R \cdot \frac{dv}{v}$$

Substituting the values of C_v and R from p. 297,

$$d\phi = \frac{b + kT}{T} \cdot dT + (a - b)\frac{dv}{v}$$

$$= \frac{b}{T} \cdot dT + k \cdot dT + (a - b)\frac{dv}{v}$$

Integrating

$$\phi_2 - \phi_1 = b \cdot \log_\epsilon \frac{T_2}{T_1} + k(T_2 - T_1) + (a - b)\log_\epsilon \frac{v_2}{v_1}$$

To obtain the expression in terms of T and p we have $\frac{T_2}{T_1} = \frac{p_2 v_2}{p_1 v_1}$

so that $\log_\epsilon \frac{T_2}{T_1} = \log_\epsilon \frac{p_2}{p_1} + \log_\epsilon \frac{v_2}{v_1}$

Hence substituting

$$\phi_2 - \phi_1 = b \log_\epsilon \frac{T_2}{T_1} + k(T_2 - T_1) + (a - b)\log_\epsilon \frac{T_2}{T_1} - (a - b)\log_\epsilon \frac{p_2}{p_1}$$

$$= a \log_\epsilon \frac{T_2}{T_1} + k(T_2 - T_1) - (a - b)\log_\epsilon \frac{p_2}{p_1}$$

Also, substituting in this equation for $\log_\varepsilon \frac{T_2}{T_1}$ we have

$$\phi_2 - \phi_1 = a \log_\varepsilon \frac{p_2}{p_1} + a \log_\varepsilon \frac{v_2}{v_1} + k(T_2 - T_1) - (a - b) \log_\varepsilon \frac{p_2}{p_1}$$

$$= a \log_\varepsilon \frac{v_2}{v_1} + k(T_2 - T_1) + b \log_\varepsilon \frac{p_2}{p_1}$$

If specific heat is constant then $a = C_p$, $b = C_v$ and $k = 0$. By putting these values in the above equations, the equations for constant specific heat can be obtained.

Example 5. An engine working on the constant volume cycle has a compression ratio of 10 to 1. The temperature rise during compression is from 120° F. to 1,050° F. and at the beginning and end of expansion the temperatures are 2,640° F. and 780° F., both the compression and expansion being adiabatic and occupying the whole stroke.

Assuming that the specific heat at constant volume has the form $b + sT$ where b and s are constants and T is the absolute temperature, and that the value of R is 53·3, find (i) the values of b and s, and (ii) the theoretical efficiency.

(*U.L.*, *B.Sc.* (*Eng.*) 1947.)

(i) Let Fig. 95 represent the p-v diagram where

$$\frac{v_1}{v_2} = \frac{v_4}{v_3} = 10$$

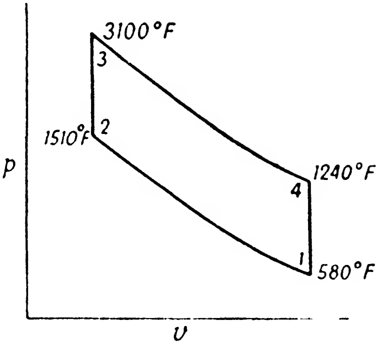


FIG. 95

Then

$$\frac{p_2 v_2}{T_2} = \frac{p_1 v_1}{T_1} \text{ and } \frac{p_2}{p_1} = \frac{v_1}{v_2} \cdot \frac{T_2}{T_1} = 10 \times \frac{1{,}510}{580} = 26{\cdot}03$$

$$\text{and } \frac{p_3}{p_4} = \frac{v_4}{v_3} \frac{T_3}{T_4} = 10 \times \frac{3{,}100}{1{,}240} = 25{\cdot}0$$

The adiabatic equation for variable specific heats (p. 297) may be written $p^b v^a \varepsilon^{sT} = \text{constant}$.

Hence for the compression 1 to 2, Fig. 95

$$b \log_e p_1 + a \log_e v_1 + ST_1 = b \log_e p_2 + a \log_e v_2 + ST_2$$

or
$$-2{\cdot}3b \log_{10} \frac{p_2}{p_1} + 2{\cdot}3a \log_{10} \frac{v_1}{v_2} = S(T_2 - T_1)$$

$$-2{\cdot}3 \times (\log_{10} 26{\cdot}03)b + 2{\cdot}3(\log_{10} 10)a = S(1{,}510 - 580)$$

and
$$-3{\cdot}2557b + 2{\cdot}3a = 930S \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly for the expansion from 3 to 4

$$-3{\cdot}2152b + 2{\cdot}3a = 1{,}860S \quad . \quad . \quad . \quad (2)$$

From Eqs. 1 and 2 eliminating a

$$b = 22{,}960S \quad . \quad . \quad . \quad . \quad (3)$$

But
$$C'_v = b + ST \text{ and } C'_p = a + ST$$

Hence
$$R = C'_p - C'_v = a - b \text{ and } a = R + b = \frac{53{\cdot}3}{778} + b$$
$$= 0{\cdot}0685 + b$$

and from Eq. 3

$$a = 0{\cdot}0685 + 22{,}960S \quad . \quad . \quad . \quad (4)$$

From Eqs. 2 and 4

$$-3{\cdot}2152b + 2{\cdot}3(0{\cdot}0685 + 22{,}960S) = 1{,}860S$$

Substituting the value of b from Eq. 3

$$S = 0{\cdot}00000689$$

and from Eq. 3

$$b = 22{,}960 \times 0{\cdot}00000689 = 0{\cdot}158$$

$\therefore$ $C'_v = 0{\cdot}158 + 0{\cdot}00000689T$ B.Th.U. per lb. per ° F.

(ii) During the constant volume combustion from 2 to 3 Fig. 95
Heat supplied = increase in internal energy

$$= \int_{T_2}^{T_3} C'_v dT = \int_{T_2}^{T_3} (b + ST)\, dT$$
$$= \left[bT + \frac{ST^2}{2}\right]_{T_2}^{T_3}$$
$$= b(T_3 - T_2) + \frac{S}{2}(T_3^2 - T_2^2)$$

$\therefore$ Heat supplied $= 0{\cdot}158(3{,}100 - 1{,}510)$
$$+ \frac{0{\cdot}00000689}{2}(3{,}100^2 - 1{,}510^2)$$
$$= 276{\cdot}5 \text{ B.Th.U. per lb.}$$

Similarly for points 4 and 1

$$\begin{aligned}\text{Heat rejected} &= b(T_4 - T_1) + \frac{S}{2}(T_4^2 - T_1^2)\\ &= 0{\cdot}158(1{,}240 - 580)\\ &\quad + \frac{0{\cdot}00000689}{2}(1{,}240^2 - 580^2)\\ &= 108{\cdot}4 \text{ B.Th.U. per lb.}\\ \therefore \quad \text{Work done} &= \text{Heat supplied} - \text{Heat rejected}\\ &= 276{\cdot}5 - 108{\cdot}4\\ &= 168{\cdot}1 \text{ B.Th.U. per lb.}\\ \therefore \quad \text{Efficiency} &= \frac{168{\cdot}1}{276{\cdot}5} = 0{\cdot}608\end{aligned}$$

Practical Determinations of the Increase of Specific Heat. If a combustible mixture is ignited in a closed vessel, the temperature and pressure will obviously rise. The rise of pressure recorded in practice is always considerably lower than that calculated from the equations—

$$\text{Heat taken in} = C_v(T_2 - T_1) \text{ and } pv = RT$$

The possible reasons for this discrepancy may be summarized as follows—

1. *Cooling Due to Loss of Heat.* Heat may be lost to the walls of the vessel before the greatest pressure is reached. This loss will depend on the position of the point of ignition, and the amount of turbulence created by the combustion, which in its turn affects the rate of propagation of the flame through the mixture. This loss should also depend on the size and shape of the vessel.

2. *Dissociation.* A part of the normal products of combustion may be dissociated into CO, H_2, and O_2 until after the highest temperature has been reached and some heat lost to the walls, recombination only taking place when the temperature has thus been lowered below the dissociation temperature. If this were the only cause of the pressure loss, one would expect this loss to be increased as the richness of the combustible mixture was increased, and hence as the maximum temperature was raised.

3. *After-burning.* The completion of combustion may not be instantaneous, and thus will not be complete by the time the maximum pressure is reached. This presupposes that the chemical reaction of combustion, once started, takes a definite time to become complete. There is some doubt whether this is the case.

4. *Increase of Specific Heat with Temperature.* If the value of the specific heat of the products of combustion increases with temperature, then the rise of temperature actually produced by a given quantity of heat will obviously be less than would be the case with constant specific heat.

While all these facts play their part in lowering the maximum pressure obtained on combustion, by far the most important factor is the increase of specific heat. In practical determinations, the great difficulty at the high temperatures required with modern work is to obtain an accurate estimate of the heat lost to the walls of the vessel before the maximum pressure is reached. This is especially the case with a slow-burning mixture.

There are two general methods of approaching the problem practically. One is to obtain experimental results for each constituent gas, and from these results calculate values for any required mixture. The objection to this method is that the properties of the individual gases may not be a reliable criterion of the behaviour of the mixture in an engine cylinder. The second method is to determine the values of the apparent specific heat of the actual mixtures under conditions as near as possible to the working conditions in the engine cylinder.

It is impossible to deal here with all the practical work already carried out in this connection, and only a brief outline can be given. The practical methods may conveniently be divided into four classes—

(i) Sir Dugald Clerk's experiments.
(ii) Constant Pressure experiments.
(iii) Constant Volume experiments.
(iv) Spectroscopic measurements.

(i) Sir Dugald Clerk* used a method of alternately compressing and expanding the products of combustion in a gas engine cylinder.

By means of a special arrangement, the exhaust and inlet valves were kept closed at the end of an explosion stroke, and the burnt products were heated by compression on the return stroke and cooled by the next expansion, the speed was kept up by the energy of the flywheel, while a continuous indicator diagram was taken for a series of successive compressions and expansions.

Fig. 96 shows the ordinary compression and explosion curves, with one compression, *ABC*, and expansion, *CD*, of the exhaust gases in the cylinder. The change of internal energy in any portion *BC* of a compression stroke is equal to the work done, less the heat lost to the cylinder walls ; in an expansion stroke *CD* it is the work done plus the heat lost. The loss of heat comes in as a correction, of some uncertainty, on the work done and was estimated by a comparison of the compression line *BC* and the following expansion *CD*. The total heat loss in the first partial compression and expansion line in the diagram *BCD*, Fig. 96, is estimated from the fall of temperature and from the net work done, represented by the area *BCD* in the double operation, and amounts roughly to half the work done in expansion. This loss has to be divided between

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* *Proc. Roy. Soc.*, Sec. A, Vol. 77, p. 500. *Proc. Inst. C.E.*, Vol. 169 (1907).

compression and expansion. The calculation is based on the assumption that the total heat loss from the hot gases, during any given portion of a stroke is the same in expansion and compression if the mean temperature is the same.

(ii) In constant pressure experiments, the gas under constant pressure flows steadily through a heater and then through a calorimeter, where it is cooled. The temperature before entering and just after leaving the calorimeter, and the quantity of heat evolved, are measured. This quantity, less the work done in contraction, is the change in the internal energy of the gas corresponding to the fall of temperature. Such experiments were carried out by Regnault, Wiedermann, Wilkowski, Holborn, and Henning,* and others. Swann determined the specific heats of air and CO_2 by the continuous electric heating method of Callendar.†

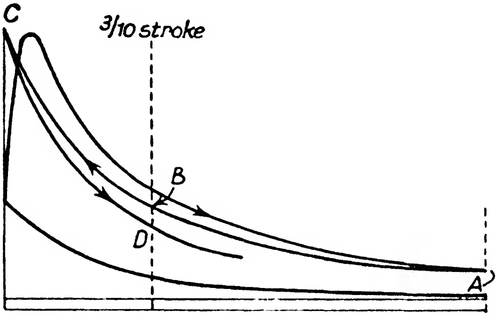

FIG. 96. INDICATOR DIAGRAM OF EXPLOSION, FOLLOWED BY COMPRESSION AND EXPANSION OF THE BURNT PRODUCTS WITHIN THE CYLINDER (*Clerk*)

In experiments of this type, the highest temperature practically attainable is limited.

(iii) In constant volume determinations the gaseous mixture is heated by internal combustion in a closed vessel, a pressure record being obtained. In this method the apparent specific heat including the effect of any dissociation can be determined, but the difficulty of determining the heat lost to the walls of the vessel still remains, especially at the high temperatures required for modern work. Workers on these lines include Joly,‡ Mallard and Le Chatelier,§ Langen,|| Hopkinson,¶ Pier and Bjerrum,** David and Leah.††

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* *Ann. der Phys.*, Vol. 23 (1907).

† *British Assoc. Report* (1908) and *Phil. Trans.* (1910).

‡ *Phil. Trans.* A, Vol. 182 (1891), p. 73, and A (1894), p. 943.

§ *Comptes Rendus*, Vols. 91, 93 (1880–81).

|| *Z.V.D.I.*, No. 18 (1903) and Vol. IX (1916).

¶ *Proc. Roy. Soc.*, Vol. 77, A (1906).

** *Phys. Zeit.*, p. 973 (1913). *Zeit. für Elektrochem.* (1909–1912). *Zeit. für Phys. Chem.* (1912).

†† *Phil. Mag.*, 1934, Seventh Series, Vol. 9, p. 307.

(iv) The modern values of specific heats are derived from quantum statistics, and verified experimentally by spectroscopic analysis.

Numerous authorities have attempted to express the values of the specific heats of the gases concerned in the working fluid of an internal combustion engine in the form of an equation. These results are still somewhat inconsistent among themselves, and only typical results can be quoted here. The student is, however, referred to the various authorities* for more detailed information.

The results given by Langen are as follows—

Air $C_v = 4{\cdot}8 + 0{\cdot}0012t$ cal. per mol.

CO_2 $C_v = 6{\cdot}7 + 0{\cdot}0052t$ cal. per mol.

H_2O $C_v = 5{\cdot}9 + 0{\cdot}0043t$ cal. per mol.

where t is ordinary temperature in ° C.

Partington and Shilling† give the following equations—

O_2, N_2, CO, Air

$$C_v = 4{\cdot}97 + 0{\cdot}00017t + 0{\cdot}00000031t^2 \text{ cal. per mol.} \quad (1)$$

CO_2 $$C_v = 6{\cdot}7 + 0{\cdot}0045t - 0{\cdot}00000102t^2 \text{ cal. per mol.} \quad (2)$$

H_2O (from 0 to 1,700° C.)

$$C_v = 6{\cdot}75 - 0{\cdot}00119t + 0{\cdot}00000234t^2 \text{ cal. per mol.} \quad (3)$$

(from 1,700 to 2,500° C.)

$$C_v = -\,12{\cdot}652 + 0{\cdot}02214t - 0{\cdot}00000467t \text{ cal. per mol.} \quad (4)$$

An equation expressing the specific heat of any given gaseous mixture can be calculated from these or similar equations if the composition is known. Thus the mixture used by Clerk had a weight of 0·07833 lb. per cu. ft., and was of the following percentage composition by volume, N_2 75, O_2 8, CO_2 5, H_2O 12. Remembering that the specific heats given above in calories per mol are for a volume of 359 cu. ft. for any gas, we multiply each equation by the appropriate proportion by volume, thus obtaining the following equations—

For $0{\cdot}75\ N_2 + 0{\cdot}08\ O_2$,

$$C_v = 4{\cdot}125 + 0{\cdot}000140t + 0{\cdot}000000257t^2 \text{ from equation (1)}$$

For $0{\cdot}05\ CO_2$

$$C_v = 0{\cdot}335 + 0{\cdot}000225t - 0{\cdot}000000051t^2 \text{ from equation (2)}$$

For $0{\cdot}12\ H_2O$

$$C_v = 0{\cdot}810 - 0{\cdot}000143t + 0{\cdot}000000281t^2 \text{ from equation (3)}$$

Adding for the mixture

$$C_v = 5{\cdot}27 + 0{\cdot}000222t + 0{\cdot}000000487t^2 \text{ cal. per mol.} \quad (4a)$$

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* *The Specific Heat of Gases*, by Partington and Shilling (Benn). "Specific Heats of Gases at High Temperatures," by D. R. Pye in *Dict. of App. Physics*, Vol. 1. "The Development of Variable Specific Heat Charts," Walker and Rogers, *Proc. Inst. Mech. Eng.*, 1948, Vol. 159, p. 301.

† *The Specific Heat of Gases*, by Partington and Shilling (Benn).

Sir Alfred Ewing[*] by another method obtained for this mixture,

$$C_v = 5{\cdot}2 + 0{\cdot}00086t + 0{\cdot}0000006t^2 \text{ cal. per mol.}$$

To obtain a value of the specific heat in ft-lb. per cu. ft., we can multiply equation (4*a*) by 3·9.

Thus $C_v = 20{\cdot}55 + 0{\cdot}000866t + 0{\cdot}0000019t^2$ ft-lb. per cu. ft. (5)

Taking a density for this mixture as given above of 0·07833 lb. per cu. ft. and dividing equation (5) by this figure, and by 1,400, we obtain

$$C_v = 0{\cdot}187 + 0{\cdot}0000079t + 0{\cdot}000000018t^2 \text{ lb. cal. per lb.}$$

Instead of using a value of the actual specific heat at any temperature, it is often more convenient to know values of the mean specific heat between two given temperatures.

Values of the mean volumetric specific heats in ft-lb. per S.C.F. between 100° C. and the given temperatures, published in the Empire Motor Fuels Committee Report† are as follows—

TABLE XVI ‡

100° C. up to	500°	1000°	1500°	2000°	2500°	3000°
Diatomic Gases .	20·2	20·6	21·5	22·4	23·4	24·6
Water Vapour .	24·4	27·1	29·8	32·8	37·9	43·6
Carbon Dioxide .	32·2	37·25	39·25	40·9	42·4	42·7

To convert these figures to calories per mol, they should be divided by 3·9 (see p. 61).

It should be noticed that the rise of mean specific heat with temperature is definitely greater in the case of H_2O than in the case of CO_2.

It should be noted that the equations in this section are expressed with ordinary temperature. A simple calculation will convert them to absolute temperature. Thus equation (1), p. 305 can be expressed as—

For diatomic gases

$$\begin{aligned} C_v &= 4{\cdot}97 + 0{\cdot}00017(T - 273) + 0{\cdot}00000031(T - 273)^2 \\ &= 4{\cdot}9467 + 0{\cdot}0000007T + 0{\cdot}00000031T^2 \quad . \quad . \quad (6) \end{aligned}$$

If we assume a value of $\gamma = 1{\cdot}4$ at 0° C. (abs.), we obtain

$$C_p = 6{\cdot}9254 + 0{\cdot}0000007T + 0{\cdot}00000031T^2. \quad . \quad . \quad (7)$$

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[*] *Thermodynamics for Engineers.*

† *Proc. Inst. Auto. E.*, 1923.

‡ From *The Internal Combustion Engine* by D. R. Pye (Oxford University Press).

In modern work it is considered preferable to use tables of the properties of gases, rather than equations showing the relations between the various properties and temperature.[*]

The Ratio of the Specific Heats. It has been shown on p. 109 that from a consideration of the kinetic theory of gases, for a perfect diatomic gas

$$C_p = \frac{7}{2}R \text{ and } C_v = \frac{5}{2}R, \text{ whence } \gamma = \frac{7}{5} = 1{\cdot}4$$

It has also been shown that for gases with molecules of more than two atoms, the theoretical value of the ratio of the specific heats is 1·33. These theoretical values should be compared with the practical values given in Table XVII.

In actual gases the value of γ decreases with a rise of temperature. This will be evident from an examination of the equations (6) and (7). Thus from these equations we see that for a temperature of 1,000° C. (abs.)

$$C_p = 7{\cdot}236 \text{ and } C_v = 5{\cdot}257, \text{ whence } \gamma = 1{\cdot}376$$

The practical determination of γ has been carried out in various ways. Table XVII gives a few values quoted from the work of Messrs. Partington and Shilling.[†]

The classic experiment for the determination of γ was made by Clément and Desormes. They used a large glass reservoir furnished with a tap and a manometer. The reservoir was partially exhausted and its contents were allowed to assume room temperature, when the pressure was measured. The tap was then opened for a moment to establish equilibrium with the external atmospheric pressure and was then closed. When the temperature had again steadied down to the room temperature, the pressure was again taken.

Let the original pressure and volume be p_1 and v_1. When the tap was opened, the pressure rose to atmospheric p_a and the volume of the original contents was reduced to v_a by the entering air. If this change is assumed to be adiabatic, $p_1v_1^{\gamma} = p_av_a^{\gamma}$.

TABLE XVII

GAS	Temp. t° C.		
	0	1000	2000
Diatomic Gases .	1·402	1·365	1·303
CO_2 . . .	1·306	1·195	1·171
H_2O . . .	—	1·252	1·153
H_2 . . .	1·410	1·358	1·318

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[*] *Gas Tables*, by Keenan and Kaye (John Wiley and Sons). *Tables of Properties of Gases* by Geyer and Bruges (Longmans Green and Co.).

[†] *The Specific Heat of Gases*, by Partington and Shilling (Benn).

When the temperature thus produced falls again to the original room temperature, the final volume and pressure are given by $p_2 v_a = p_1 v_1$.

Hence
$$\gamma = \frac{\log p_1 - \log p_a}{\log p_1 - \log p_2}$$

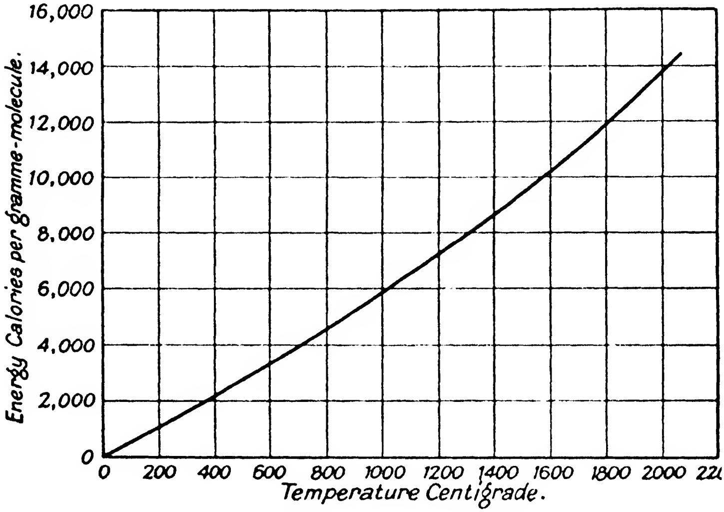

FIG. 97. INTERNAL ENERGY OF BURNT PRODUCTS IN A GAS ENGINE

Example 6. Air is forced into a vessel fitted with a thermometer and a pressure gauge. After the air has assumed the atmospheric temperature, the pressure is 150 lb. per sq. in. The vessel is then opened for a very short time, in which the pressure in the vessel falls to that of the atmosphere, 15 lb. per sq. in. When the temperature in the vessel is again restored to that of the atmosphere, the pressure is observed to be 29 lb. per sq. in.

Find from this data, the ratio of the specific heats of air.

(*U.L.*, *B.Sc.* (*Eng.*), 1934.)

$$\gamma = \frac{\log 150 - \log 15}{\log 150 - \log 29}$$
$$= 1{\cdot}401$$

The Internal Energy Curve. It has been shown on p. 297 that if we know the value of the specific heat of a gas for any temperature, we can calculate the internal energy from the expression

$$E_2 - E_1 = \int_{T_1}^{T_2} C_v \, . \, dT$$

The curve, Fig. 97, shows the internal energy E of the working fluid in the gas engine taken from that given by the British Association Committee on Gaseous Explosions after discussion of the best available results of experiments by Langen, Mallard, and Le Chatelier at the highest temperatures; Clerk, Holborn, and Henning over the lower range. It should be noted that the internal energy is taken as zero at $t = 0°$ C.

We will now calculate some values for the internal energy from equation (4*a*), p. 305, which was calculated for this mixture. We shall, however, consider internal energy as zero at $t = 100°$ C.

Thus $C_v = 5{\cdot}27 + 0{\cdot}000222t + 0{\cdot}000000487t^2$ cal. per mol. per ° C.

$$E = 5{\cdot}27(t - 100) + \frac{0{\cdot}000222}{2}(t^2 - 100^2) + \frac{0{\cdot}000000487}{3}(t^3 - 100^3) \text{ cal. per mol. per } ° \text{ C.} \quad (8)$$

Substituting for t in this equation we obtain

t	1,000° C.	2,000° C.	3,000°C.
E	5,015 19,558	11,752 45,833	20,655 lb. cal. per mol. 80,554 ft-lb. per cu. ft.

These values should be compared with those shown in Fig. 98.

The internal energy can also be calculated from the values of mean specific heat given in Table XVI. The first step is to find values of the mean specific heat of the mixture. This is done by multiplying the specific heat of each constituent gas by its proportion by volume, and adding the proportionate values so obtained. This is done in the table given below, the working being shown for 500° C.

The value of the internal energy can now be obtained by multiplying these values of specific heats by the rise in temperature from 100° C. as in the table on p. 310.

Gas Composition by Volume	Proportional Mean Specific Heat (ft-lb. per cu. ft.) from 100° C. to					
	500°	1,000°	1,500°	2,000°	2,500°	3,000°
N_2 0·75 O_2 0·08 } 0·83	0·83 × 20·2 = 16·77	17·10	17·85	18·59	19·42	20·42
H_2O 0·12	0·12 × 24·4 = 2·93	3·25	3·58	3·94	4·55	5·23
CO_2 0·05	0·05 × 32·2 = 1·61	1·86	1·96	2·05	2·12	2·14
Mixture (by addition)	21·31	22·21	23·39	24·58	26·09	27·79

Temperature t° C.	Internal Energy (ft-lb. per cu. ft.)
500	21·31 × 400 = 8,524
1,000	22·21 × 900 = 19,989
1,500	23·39 × 1,400 = 32,746
2,000	24·58 × 1,900 = 46,702
2,500	26·09 × 2,400 = 62,616
3,000	27·79 × 2,900 = 80,591

These values are shown plotted on the curve *AB*, Fig. 98.

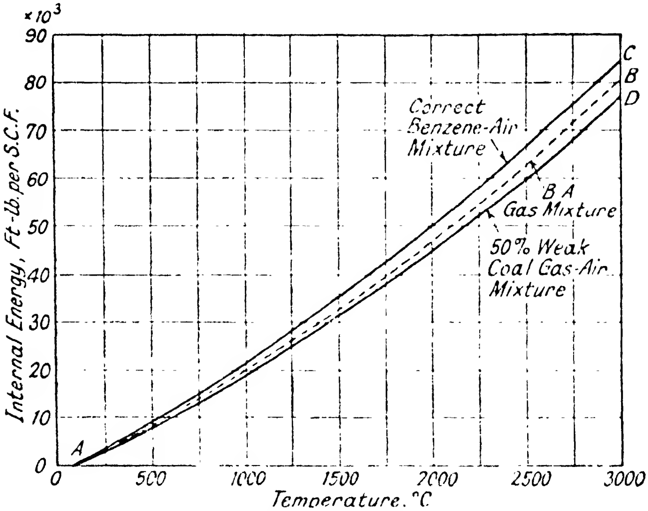

FIG. 98. INTERNAL ENERGY CURVES

In a similar manner the internal energy curve for any gaseous mixture may be calculated. Thus for a benzene-air mixture, the combustion equation is—

$$2C_6H_6 + 15O_2 \rightarrow 12CO_2 + 6H_2O$$

Proportions by volume are—

$$2 + 15 \rightarrow 12 + 6$$

With 15 volumes of O_2 we have $\frac{79}{21} \times 15 = 56{\cdot}4$ volumes of N_2.

Hence the complete equation is

$$2C_6H_6 + 15O_2 + 56{\cdot}4N_2 \rightarrow 12CO_2 + 6H_2O + 56{\cdot}4N_2$$

Proportions by volume are

$$2 + 15 + 56{\cdot}4 \rightarrow \underbrace{12 + 6 + 56{\cdot}4}_{74{\cdot}4}$$

From this the composition by volume of the products of combustion can be obtained.

The curves *AC* and *AD* (Fig. 98) show values of internal energy for a correct benzene-air mixture, and for a 50 per cent weak coal gas-air mixture, given by Pye.* It will be noticed that although the two fuels are quite different, the energy curve for the products of combustion do not differ greatly, owing to the fact that the major portion of the final mixture is nitrogen.

Working on these values, Pye gives as a close approximation for the internal energy at an absolute temperature T for a range of temperature from 100° C. to maxima between 2,000° and 3,000° C. for benzene

$$E_T = 549\left[\frac{T-373}{100}\right] + 7{\cdot}03\left[\frac{T-373}{100}\right]^2 \text{ lb. cal. per mol.}$$

The Determination of Cycle Temperatures. If the temperature of one point on an indicator diagram is known, then the temperatures of all other points can be determined if we assume that the relation $\frac{pv}{T}$ = Constant, holds all round the cycle. Thus, in Fig. 73, if we know the temperature T_1, then $T_2 = T_1\frac{p_2v_2}{p_1v_1}$. If molecular contraction during combustion takes place, then $T_3 = \frac{T_2p_3v_3}{xp_2v_2}$ (equation (1), p. 260). This method, apart from the assumption that the working substance behaves as a perfect gas, depends on the accuracy with which measurements of volume and pressure can be taken from the indicator diagram.

The problem can be approached from a somewhat different point of view if the calorific value of the fuel and particulars of the specific heat of the working substance are known. In any practical case the variation of specific heat and the effect of dissociation must be allowed for, but it will be instructive first of all to consider a simple case where the specific heat is assumed constant and where the effect of dissociation is neglected. Let us also assume that there is no heat loss to the cylinder walls.

Let Specific heat = 0·173

Compression ratio = 5

Volumetric efficiency = 1

Calorific value of fuel (Petrol) = 10,500 C.H.U. per lb., requiring 14·6 lb. of air per lb. of fuel

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* *The Internal Combustion Engine*, by D. R. Pye (Oxford University Press).

Temperature of exhaust products left in clearance space, at 14·7 lb. per sq. in. = 1,200° C. = 1,473° C. (abs.)

Temperature of fresh charge = 20° C. = 293° C. (abs.).

$$\gamma = 1{\cdot}4$$

We must first find the temperature of the cylinder contents, made up of the ingoing charge and the products of combustion left in the clearance space at the end of the exhaust stroke. For a compression ratio of 5, the clearance volume is 0·2, and the ingoing charge is 0·8 of the total cylinder contents. Hence, at the end of the suction stroke, the volume of the total cylinder contents, if reduced to N.T.P., would equal $\left(\frac{273}{1{,}473} \times 0{\cdot}2\right) + \left(\frac{273}{293} \times 0{\cdot}8\right)$

$$= 0{\cdot}037 + 0{\cdot}745$$
$$= 0{\cdot}782$$

That is, if we call the total cylinder contents unity, they would be reduced to a volume of 0·782 if their temperature were 0° C. instead of the actual suction temperature.

Hence the suction temperature $= 273 \times \frac{1}{0{\cdot}782} = 349°$ C. (abs.)

Thus p_1 (Fig. 73) = 14·7 lb. per sq. in.

$T_1 = 349°$ C. (abs.)

These cylinder contents are compressed adiabatically from p_1 and T_1 to p_2 and T_2, where

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = 349 \times 5^{0{\cdot}4} = 664° \text{ C. (abs.)}$$

and $$p_2 = p_1 \left(\frac{v_1}{v_2}\right)^{\gamma} = 14{\cdot}7 \times 5^{1{\cdot}4} = 140 \text{ lb. per sq. in.}$$

The temperature rise due to this compression = 664 - 349 = 315° C., and the corresponding increase in internal energy, that is the work done during the compression,

$$= C_v(T_2 - T_1) \times 1{,}400$$
$$= 0{\cdot}173 \times 315 \times 1{,}400$$
$$= 76{,}290 \text{ ft-lb. per lb. of cylinder content}$$

Now the calorific value of the ingoing charge is for 1 part of petrol to 14·6 of air by weight. Therefore the calorific value of the ingoing charge $= \frac{10{,}500}{15{\cdot}6} = 673$ C.H.U. per lb.

But of the cylinder contents equivalent to a volume of 0·782, the volume of the ingoing charge is equivalent to 0·745. Hence the heat liberated per pound of working substance in the cylinder is equal to

$$\frac{0{\cdot}745}{0{\cdot}782} \times 673 = 641 \text{ C.H.U.} = 897{,}400 \text{ ft-lb.}$$

Hence the temperature rise due to combustion

$$T_3 - T_2 = \frac{641}{C_v} = \frac{641}{0{\cdot}173} = 3{,}705^\circ \text{ C.}$$

Therefore $T_3 = 3{,}705 + 664 = 4{,}369^\circ$ C. (abs.)

and $p_3 = p_2\frac{T_3}{T_2} = 140 \times \frac{4{,}369}{664} = 921$ lb. per sq. in.

The working substance now expands adiabatically to p_4 and T_4

where $$T_4 = T_3\left(\frac{v_4}{v_3}\right)^{\gamma-1} = 4369\left(\frac{1}{5}\right)^{0{\cdot}4} = 2{,}295^\circ \text{ C. (abs.)}$$

and $$p_4 = p_3\left(\frac{v_4}{v_3}\right)^{\gamma} = 921\left(\frac{1}{5}\right)^{1{\cdot}4} = 96{\cdot}8 \text{ lb. per sq. in.}$$

The work done during this expansion

$$= C_v(T_3 - T_4)1{,}400$$
$$= 0{\cdot}173(4{,}369 - 2{,}295)1{,}400$$
$$= 502{,}323 \text{ ft-lb. per lb. of cylinder content}$$

The net work done in the cycle $= 502{,}323 - 76{,}290$
$= 426{,}033$ ft-lb.

$$\text{The efficiency therefore} = \frac{426{,}033}{897{,}400} = 0{\cdot}475$$

This will, of course, be the same value as that for the air standard cycle efficiency found for a compression ratio of 5 on p. 258; for we have taken a value for γ of 1·4, and we have assumed the same constant value of the specific heat for both compression strokes.

We will now consider the problem, taking into account the variation of specific heat of the working substance. For example, let the fuel be gas of calorific value 300 C.H.U. per standard cubic foot, with an air-fuel ratio of 4 to 1. Then the calorific value of the mixture will be $\frac{300}{5} = 60$ C.H.U. per S.C.F.

$= 60 \times 359 = 21{,}540$ C.H.U. per lb. mol.

Taking (Fig. 73, p. 256) $t_1 = 100^\circ$ C., $p_1 = 14{\cdot}7$ lb. per sq. in., and $r = 5$; also in order to approximate to actual conditions, assuming the index of the compression between p_1 and p_2 to be 1·35 (p. 259), we have

$$p_2 = 14{\cdot}7 \times 5^{1{\cdot}35} = 129 \text{ lb. per sq. in.}$$
$$T_2 = 373 \times 5^{0{\cdot}35} = 655^\circ \text{ C. (abs.)}$$

The work done on the mixture during this compression is

$$\frac{p_2v_2 - p_1v_1}{\gamma - 1} = \frac{(129 \times 1 - 14{\cdot}7 \times 5) \times 144}{1{\cdot}35 - 1} = 22{,}834 \text{ ft-lb.}$$

This is for a volume of 5 cu. ft. at atmospheric pressure and at 100° C., that is for $5 \times \frac{273}{373} = 3{\cdot}66$ cu. ft. at N.T.P. Hence the

$$\text{work done in the compression} = \frac{22{,}834}{3{\cdot}66} \text{ ft-lb. per cu. ft.}$$
$$= \frac{22{,}834}{3{\cdot}66 \times 3{\cdot}9}$$
$$= 1{,}600 \text{ C.H.U. per mol.}$$

It is convenient to consider internal energy as zero at atmospheric pressure and at 100° C. Then the internal energy E_2 at point 2, Fig. 73, will be 1,600 C.H.U. per mol, while at point 3, the internal energy $E_3 = 1{,}600 + 21{,}540 = 23{,}140$ C.H.U. per mol.

But from equation (8), p. 309, we have,

$$E_3 = 5{\cdot}27(t_3 - 100) + 0{\cdot}000111(t_3^2 - 100^2) + 0{\cdot}000000162(t_3^3 - 100^3)$$

Substituting the above value of E_3, we obtain $t_3 = 3{,}250°$ C., whence $T_3 = 3{,}523°$ C. (abs.). It should be noted that if an internal energy curve such as Fig. 98 is available, the temperature can be obtained direct.

During the expansion from points 3 to 4, the values of the index γ and the specific heat C_v will vary. We can, however, obtain a satisfactory value for T_4 by assuming a constant mean value of γ, which must be consistent with a constant mean value of C_v. Let us assume that $\gamma = 1{\cdot}21$.

Then $T_4 = \frac{T_3}{r^{\gamma - 1}} = \frac{3{,}523}{5^{0{\cdot}21}} = 2{,}513°$ C. (abs.) and $t_4 = 2{,}240°$ C.

$$\text{Then } E_4 = 5{\cdot}27(2{,}240 - 100) + 0{\cdot}000111(2{,}240^2 - 100^2)$$
$$+ 0{\cdot}000000162(2{,}240^3 - 100^3)$$
$$= 13{,}659 \text{ C.H.U. per mol.}$$

The mean specific heat during the expansion will be obtained by dividing the change in internal energy by the corresponding fall in temperature. Hence

$$\text{Mean } C_v = \frac{23{,}140 - 13{,}659}{3{,}523 - 2{,}513} = 9{\cdot}387 \text{ C.H.U. per mol.}$$

This value of C_v must, however, be consistent with the equation

$$\gamma = 1 + \frac{R}{C_v} = 1 + \frac{1{\cdot}985}{9{\cdot}387} = 1{\cdot}211$$

If this value had not been consistent with the assumed value 1·21, then another value of γ must be assumed, and the calculation repeated until the assumed and calculated values were in agreement.

The work done by the working substance during the expansion is given by $E_3 - E_4 = 23{,}140 - 13{,}659 = 9{,}481$ C.H.U. per mol.

The net work obtained from the cycle, that is the difference between the work done by and the work done on the working substance, equals 9,481 - 1,600 = 7,881 C.H.U. per mol.

Hence the efficiency $= \dfrac{7{,}881}{21{,}540} = 0{\cdot}366.$

The above problem has been considered on the assumption that there is no dissociation. In practice, the carbon dioxide normally formed by the combustion of carbon can, at high temperatures, be split up into carbon monoxide and oxygen. In the same way water vapour can be dissociated into hydrogen and oxygen. In both cases the amount of dissociation taking place increases with the temperature. In the products of combustion of a fuel, therefore, there is an equilibrium mixture of CO_2, H_2O, CO, H_2, and O_2, the proportions of the various constituents being dependent on the temperature. As dissociation is accompanied by an absorption of heat, its effect is to lower the maximum temperature below the theoretical value calculated on the assumption that no hydrogen or carbon monoxide is present in the products of combustion.

TABLE XVIII

CO_2

Temp. ° C.	Pressure (Atmospheres)		
	0·1	10	100
1,500	0·104	0·0224	0·01
3,000	77·1	32·2	16·9

H_2O

Temp. ° C.	Pressure (Atmospheres)		
	0·1	10	100
1,500	0·043	0·009	0·004
3,000	28·4	7·04	3·33

Table XVIII* on page 315, shows a few values of the degree of dissociation of CO_2 and H_2O for different temperatures and pressures.

It will be noticed that the degree of dissociation is far greater in the case of CO_2 than with H_2O, and hence from the point of view of dissociation a fuel containing a large proportion of carbon is at a disadvantage compared with one containing a large proportion of hydrogen.

Calculations on the effect of dissociation on the maximum temperature obtained by the combustion of various fuels have been made by Messrs. Tizard and Pye† in the following manner—

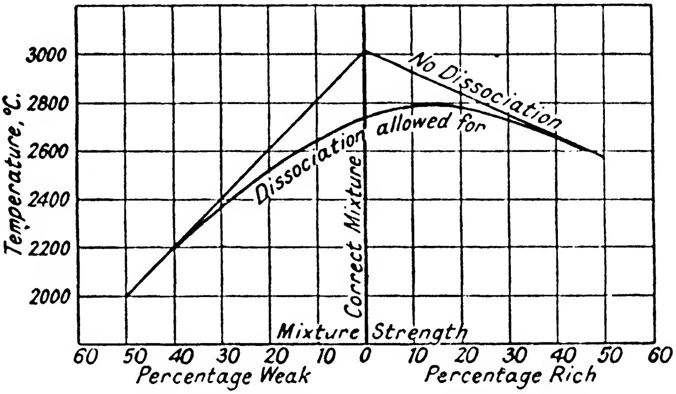

FIG. 99. CALCULATED MAXIMUM TEMPERATURES FOR BENZENE AIR MIXTURES

The combustion of benzene with the correct amount of air, is represented by the equation

$$2C_6H_6 + 15O_2 + 56{\cdot}4N_2 = 12CO_2 + 6H_2O + 56{\cdot}4N_2$$

while for a mixture of any strength,

$$2(1 + x)C_6H_6 + 15O_2 + 56{\cdot}4N_2$$
$$= aCO_2 + bCO + cH_2O + dH_2 + eO_2 + 56{\cdot}4N_2$$

where $x = +0{\cdot}1$ for a 10 per cent rich mixture, and $-0{\cdot}1$ for a 10 per cent weak mixture, etc. The authors show how, knowing the dissociation constants, the values of the constants a to e can be calculated, and hence how the maximum temperatures obtained on combustion can be calculated. Fig. 99 shows their results for a benzene-air mixture.‡

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* Bjerrium: *Zeitschrift für Phys. Chemie*, 1912.

† *Report of Empire Motor Fuels Committee*, Inst. Auto. E., 1923, and *The Internal Combustion Engine*, D. R. Pye.

‡ From *The Internal Combustion Engine*, by D. R. Pye (Oxford University Press).

With no dissociation, the maximum temperature attained will be for the correct mixture, but allowing for dissociation the maximum temperature is for a mixture 15 to 20 per cent rich. It will be seen later that this is the mixture strength which in practice gives maximum power. As would be expected, for very weak or very rich mixtures, where the temperatures are comparatively low, there is practically no effect.

An internal energy diagram* (Fig. 100) for the working fluid of an internal combustion engine has been developed by Ricardo from the work of Tizard and Pye. From this diagram the temperatures at the beginning and end of the expansion stroke can be read off with a fair degree of accuracy.

Although this curve applies strictly only to the chemically correct benzene-air mixture for complete combustion at a compression ratio of 5 : 1, it may be taken as applicable within negligible error to any other hydrocarbon fuel at the compression ratios commonly used in the ordinary petrol engine working on the constant-volume cycle. Owing to the different specific heat of the products of combustion of alcohol or ether and air mixtures, the curve does not apply to these, nor to very rich or weak mixtures, though, with these, the error is very small within the working range of a homogeneous mixture.

The curve gives the heat energy in equivalent foot-pounds per standard cubic inch plotted against the temperature on the horizontal scale. Zero energy is taken at 100° C., this being the average temperature of the charge at the beginning of compression. The difference between the upper part of the two curves, on a vertical line, shows the chemical energy stored in the products of dissociation.

A graphic construction, by Mr. Alcock, is given by which the range of temperature throughout the expansion stroke may be estimated from the heat of combustion of the mixture and thermal efficiency of the engine. The ideal thermal efficiency of the petrol constant-volume cycle is taken as $1 - (1/r)^{0 \cdot 258}$ See p. 321.

An example is worked out to explain the use of the diagram—

Given the compression ratio $R = 5$; energy content of charge $= 46{\cdot}2$ ft-lb. per standard cu. in.; assuming heat loss to the cylinder walls during combustion, 6 per cent; and heat loss during expansion, 6 per cent. The heat added to the mixture during compression is allowed for various ratios by the scale PP_1, near the bottom of the diagram.

The point $r = 5$, representing 3·6 ft-lb. heat of compression, is joined to that value above the energy content of the mixture on the vertical O_1Y_1 from the 100° C. starting-point, i.e. $46{\cdot}2 + 3{\cdot}6$, or 49·8 ft-lb. From this gross energy content per cubic inch must be deducted: (*a*) the loss due to the weakening of the mixture by dilution with the residual exhaust products, assumed to be at

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* *The Automobile Engineer*, November, 1922. Reproduced by the kind permission of Sir Harry R. Ricardo.

1,000° C. ; and (*b*) the loss to the walls of the combustion chamber during combustion.

At the top of the diagram on the horizontal scale *C* is marked the

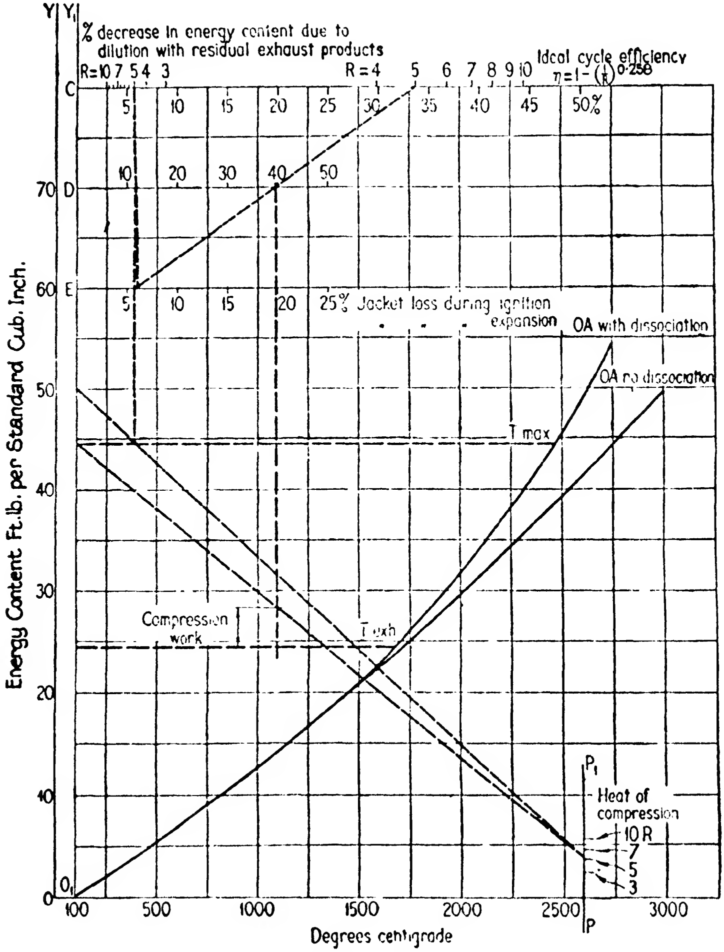

FIG. 100. INTERNAL ENERGY OF A PETROL OR BENZENE-AIR MIXTURE

decrease in energy due to dilution, and scale *E* shows the loss due to cooling during combustion. A line is drawn between the two points on these scales representing 6 per cent, and the point of intersection of this line with the scale *D* gives the total loss due to these two causes, 11·5 per cent.

Drop a vertical from this point of intersection to the line joining the total energy content 49·8 ft-lb., and the heat of compression on the scale PP_1. Then a horizontal line is drawn to the energy scale O_1Y_1 on the left, and to the energy curve OA. The point on the energy scale shows the net energy available for expansion, 44·5 ft-lb. per standard cu. in. of charge, and from the curve OA can be read off the actual maximum temperature, 2,475° C.

The drop in temperature during expansion depends on (1) the work done on the piston, and (2) the heat loss to the walls. The net power output as a percentage of the heat content of the mixture given on scale C, does not cover losses to the walls during expansion, which is laid off on scale E. A line drawn between the points on scales E and C gives their sum on scale D, as before. A vertical from this on D is drawn to meet a line from the net energy point on O_1Y_1 to the suitable compression point on PP_1. As the gross work done during expansion is the sum of the net work and that during compression, the latter (3·6 ft-lb. in example) must be laid off below the point of intersection to find, by the horizontal, the energy content at the end of expansion, 24·5 ft-lb.; and the corresponding final temperature, 1,675° C., is read off the curve OA.

In an actual test of the variable compression engine, having compression ratio 5 : 1, the observed indicated thermal efficiency is 31 per cent; 5 per cent of loss during expansion is due to the change of specific volume of the mixture during combustion, so that the heat drop is $46{\cdot}2 \times 0{\cdot}31 \times \frac{100}{105} = 13{\cdot}6$ ft-lb. per cu. in. Adding to this the 3·6 ft-lb. of compression work restored during expansion, and 6 per cent of 46·2, or 2·8 ft-lb. of wall loss, makes the total heat drop during expansion $13{\cdot}6 + 3{\cdot}6 + 2{\cdot}8 = 20$ ft-lb. per cu. in., leaving a final energy content, $44{\cdot}5 - 20 = 24{\cdot}5$ ft-lb. per cu. in., which agrees with the result found graphically, under the same conditions.

Professor W. T. David and A. S. Leah have published charts* from which may be found attainable indicated thermal efficiencies, fuel consumptions and mean effective pressures for a petrol engine of specified limits of dimensions and working conditions. Characteristic curves are suggested which appear to be capable of yielding useful information in regard to the functioning of engines.

Ideal Efficiency for a Liquid Hydrocarbon Fuel (Tizard and Pye). It has already been explained (p. 259) how a simple ideal efficiency may be adopted as a basis of comparison for any internal combustion engine, but it is obvious that the assumptions made render the ideal performance so obtained quite unattainable in practice, and this limits its value from a practical standpoint. The provision of a more

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* "Fuel Economy in Petrol Engines," David and Leah, *Proc. Inst. Mech. Eng.*, 1940, Vol. 143, p. 289.

approachable standard is therefore very desirable, namely, one in which, while still assuming no exchange of heat between the working substance and the cylinder walls, yet takes account of the practical imperfections of the working substance.

Messrs. Tizard and Pye* have investigated the ideal thermal efficiency of the petrol engine, assuming no loss of heat to the cylinder walls, but allowing for losses due to increase of specific heat at high temperatures, and for dissociation with recombination during explosion and expansion. It has been shown (p. 311) how the cycle temperatures and efficiencies for any fuel can be calculated assuming no dissociation, and it was indicated how the further effect of dissociation on the explosion temperature can be allowed for. Working on these lines, Messrs. Tizard and Pye have calculated the ideal efficiencies for benzene-air mixtures of different fuel-air ratios and varying compression ratios, and their results are shown in Fig. 101.†

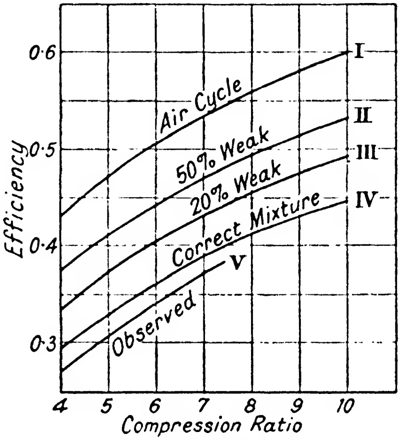

FIG. 101. THERMAL EFFICIENCY
(*Tizard and Pye*)

Curve I represents the air standard cycle calculated from the equation $E = 1 - \left(\frac{1}{r}\right)^{0 \cdot 396}$; while curves II, III, and IV have been calculated for mixtures 50 per cent weak, 20 per cent weak, and correct air-fuel ratio.

The authors show that the curve for the correct mixture may be

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* *Report of the Empire Motor Fuels Committee.* "The Internal Combustion Engine," D. R. Pye.

† From *The Internal Combustion Engine,* by D. R. Pye (Oxford University Press).

closely represented by the equation $E = 1 - \left(\frac{1}{r}\right)^{0\cdot258}$, while for the 20 per cent weak mixture $E = 1 - \left(\frac{1}{r}\right)^{0\cdot296}$.

It should be noted that these equations are empirical, that is, the average value of γ is not necessarily 1·296 for a 20 per cent weak mixture, but that the formula given is a convenient one, and is sufficiently accurate for practical purposes of comparison.

Curve V shows the highest thermal efficiencies actually observed with a fuel-air ratio about 15 per cent weak. If we take a 20 per cent mixture as the practical standard of comparison, then the vertical distances between the curves III and V will represent the loss of efficiency due to the loss of heat to the cylinder walls, and due to incomplete combustion.

Ricardo has expressed the highest thermal efficiency attainable as*

$$E = 1 - \left(\frac{1}{r}\right)^{0\cdot25}$$

This formula represents the limiting thermal efficiency obtainable under the best possible conditions in a cylinder of comparatively large capacity, with engine speed not less than 1,500 r.p.m., assuming (1) perfect carburation and distribution; (2) that the compression and expansion ratios are equal; and (3) that the mixture is homogeneous and of the most economical strength.

Table XIX below summarizes the various thermal efficiencies discussed.

In the curves shown in Fig. 102, the Ideal is plotted from the results of Messrs. Tizard and Pye for a 20 per cent weak mixture,

TABLE XIX

IDEAL THERMAL EFFICIENCY

Compression Ratio, r	Air Cycle $\gamma = 1\cdot4$	Tizard & Pye Correct Mixture $n = 1\cdot258$	Tizard & Pye 20% weak $n = 1\cdot295$	Ricardo Limit $n = 1\cdot25$	Results Observed for Benzene 15% weak, by Ricardo
4·0	0·4256	0·300	0·336	0·296	0·277
4·5	0·4521	0·322	0·359	0·314	0·297
5·0	0·4747	0·340	0·378	0·332	0·316
5·5	0·4944	0·356	0·396	0·348	0·332
6·0	0·5116	0·370	0·411	0·361	0·346
6·5	0·5270	0·383	0·424	0·375	0·360
7·0	0·5398	0·395	0·437	0·386	0·372
7·5	0·5534	0·406	0·449	0·396	0·383
8·0	0·5647	0·416	0·460	0·406	—

* *Society of Automotive Engineers*, U.S.A.; also *The Automobile Engineer*, September, 1922.

this being about the most economical mixture in practice. The Possible Limit curve is plotted from the Ricardo formula with $n = 1{\cdot}25$.

To calculate the above curves some particular fuel such as a benzene-air mixture must be taken, and hence, strictly speaking, these figures will only hold for that particular fuel. It has been shown in practice, however, that the efficiency for all hydrocarbon fuels is the same within narrow limits, provided the supply temperature is the same and that the vaporized fuel is subjected to the same compression ratio. This is what is to be expected when it is remembered that, whereas a fuel containing a large proportion of carbon is at a disadvantage compared to one containing a large proportion of hydrogen with respect to dissociation (p. 316), it has the advantage with respect to specific heat. The two factors seem to balance out, and as long as the fuel consists of carbon and hydrogen only, its nature has no appreciable effect on the cycle efficiency.

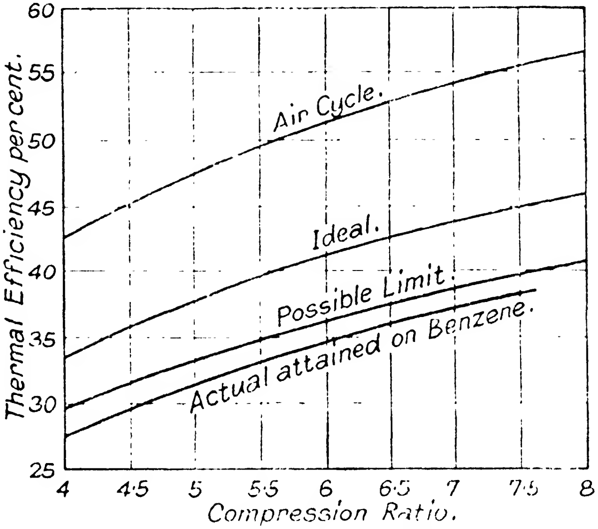

FIG. 102. IDEAL AND INDICATED THERMAL EFFICIENCIES

The Effect of Mixture Strength (Fuel-air Ratio)

1. THE GAS ENGINE. When discussing the process of combustion in the petrol engine (p. 276), it was pointed out that on ignition of the explosive mixture not only did the combustion require a definite time to complete itself, but also there was a delay period between the ignition and the actual start of the pressure rise due to the combustion. This also holds for the working substance in a gas

engine cylinder. A definite time is required to complete the combustion, and this time will vary between considerable limits dependent on the fuel-air ratio. Hence, if the maximum pressure is to be reached before the piston has moved through any appreciable portion of the working stroke, the ignition must be advanced as the mixture strength is reduced. It has already been stated that, in general, the best compromise for maximum power with maximum efficiency is obtained if the maximum pressure is reached when the crank is about 12° past top dead centre (p. 276), and this was the standard condition adopted by A. F. Burstall in his experiments on coal gas mixtures.*

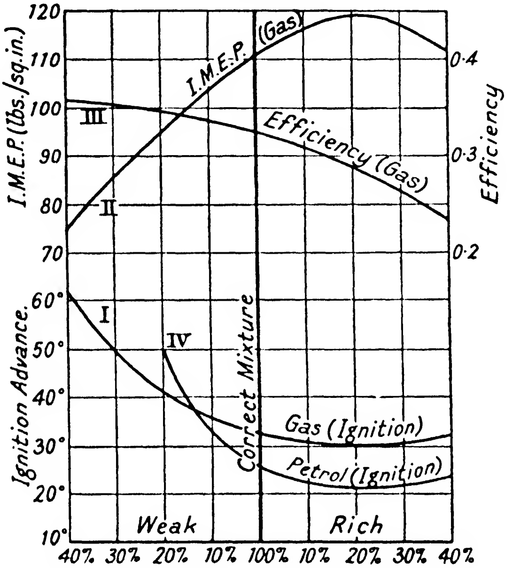

FIG. 103. EFFECT OF MIXTURE STRENGTH

The curve I, Fig. 103, shows the relation between the mixture strength and the point of ignition required to give the above standard condition for coal gas mixtures. It will be noticed that the minimum advance of about 30° is necessary for a mixture strength about 20 per cent rich, and that by a suitable adjustment of the ignition combustion will take place between wide limits of mixture strength. It must be remembered that with the large ignition advance necessary with weak mixtures, combustion and hence the pressure rise will be taking place to an appreciable extent before the top dead centre, that is, during the latter part of the compression stroke, thus increasing the negative work on the piston.

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* *Proc. Inst. Auto. E.*, March, 1925.

The effect of the fuel-air ratio on the indicated mean effective pressure is shown by the curve II, Fig. 103. It will be seen that maximum power, which at constant speed is proportional to M.E.P., is obtained with a mixture about 20 per cent rich, the power falling off both with rich and with weak mixtures. It is interesting to note that the mixture strength for maximum power closely agreed with that for maximum explosion temperature as calculated when allowing for dissociation (Fig. 99). This seems to indicate that the rise in power as the mixture strength increases from the correct mixture to one 20 per cent rich can be explained by dissociation, for, of the other variable factors governing power, namely, volumetric efficiency and the change of specific volume due to combustion, the first in the case of a gas engine is very small, and the second will not account for the variation.

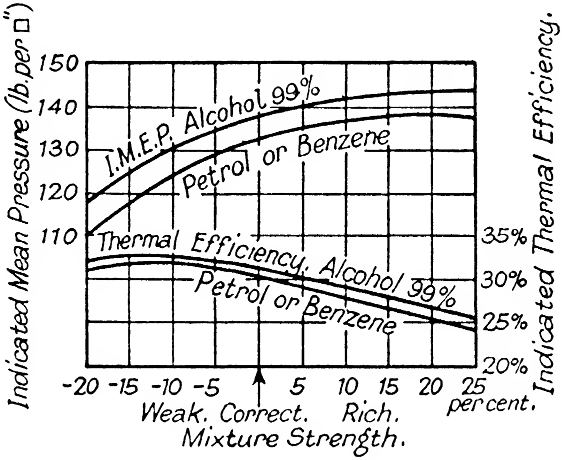

FIG. 104. OBSERVED VARIATION IN MEAN PRESSURE AND THERMAL EFFICIENCY (*Ricardo*)

Heat input to carburettor = 65 B.Th.U. per min.
Speed = 1,500 r.p.m. Compression ratio = 5 : 1.

The effect of the fuel-air ratio on thermal efficiency is shown on the curve III, Fig. 103. There is an increase of efficiency as the mixture becomes weaker. This is due in the first place to the fact that with the lower temperatures associated with the weaker mixtures the heat loss to the cylinder walls is reduced, and secondly to the lowering of the specific heat of the working mixture as the temperature is reduced.

2. THE PETROL ENGINE. The relation between mixture strength and the ignition advance necessary to produce maximum pressure about 10° after top dead centre is shown in curve IV, Fig. 103. With rich mixtures the effect is very much the same as with gas,

the minimum advance being for a mixture about 20 per cent rich. With weak mixtures, however, the necessary ignition advance increases rapidly as the proportion of fuel is reduced, and no matter how far ignition is advanced the combustion becomes so slow that it is incomplete throughout the expansion stroke.

The effect of mixture strength on the I.M.E.P. and on efficiency is shown in Fig. 104.

For rich mixtures the power remains comparatively constant as the richness is increased. Thus a petrol engine will run satisfactorily with a too rich mixture, although the fuel economy will of course be adversely affected. A reference to the power curve for petrol indicates that maximum power is obtained as in the case of gas, with a mixture about 20 per cent rich. On the weak side, however, petrol shows a difference from gas. If the mixture is more than about 20 per cent weak, the combustion becomes so slow that it continues throughout the working stroke, thus producing backfiring of the incoming charge and consequently irregular working.

Sir Harry Ricardo showed by experiment that in practice the highest thermal efficiency is obtained with about 15 per cent excess air in homogeneous mixtures of a volatile liquid fuel. This variation of efficiency is shown in Fig. 104 for both petrol and alcohol. It must be remembered that this inability to work efficiently on weak mixtures is a characteristic of homogeneous mixtures of petrol and air.

By means of stratification it is possible to use much weaker mixtures. In this method the mixture strength is reduced by using a small charge of combustible mixture and admitting separately a large charge of air, and preventing them mixing before ignition. After ignition turbulence is needed in the combustible part of the charge to ensure rapid combustion. By this means Ricardo has shown practically that the efficiency can be increased as the mixture is reduced down to very weak mixtures, and he has actually obtained a maximum efficiency of about 37 per cent at one-third full load corresponding to a fuel consumption of just under 0·36 pint of benzol per I.H.P. hour. Not only is the efficiency on reduced loads far higher than could be obtained by other means, but the heat lost is so low that a water-cooled engine can be run continuously on low loads without cooling water.

A very useful method of showing the effect of mixture strength is illustrated in Fig. 105, where fuel consumption in pounds per horsepower hour is plotted against mean effective pressure. The curves *A* and *B* are typical consumption "loops" for a petrol engine, the curve *A* being for full throttle, and the curve *B* for the throttle partly closed. To obtain such a curve, continuous readings of the power are taken while the mixture strength is reduced by means of an adjustable jet on the carburettor, the speed being kept constant. Starting with a very rich mixture, we know from p. 324 that the power will increase as the mixture strength is reduced, to a maximum at the point X, Fig. 105, where the mixture is about 20 per cent rich. The fuel consumption continues at first to drop rapidly with a small fall in power, and then the power decreases quickly with a slight further reduction in fuel consumption. The correct fuel-air ratio will be represented by a point Y where the fuel consumption will be some 20 per cent less than for the point X. The point Z where the fuel consumption is a minimum represents maximum efficiency referred to the fuel. After this the mixture strength can still be reduced by a small amount, after which the firing will become irregular as already explained. The length and flatness of the horizontal portion of the curve illustrates an important feature of the engine, for this is obviously a measure of the power range over which the efficiency can be maintained reasonably high. The limit of this portion to the left is of course determined by the mixture becoming so weak that efficient combustion is no longer possible.

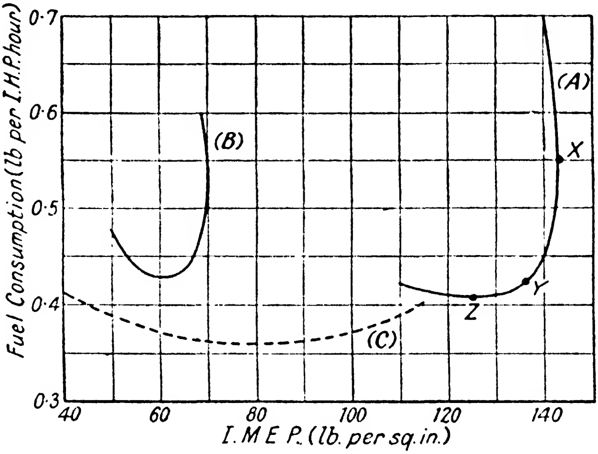

FIG. 105. CONSUMPTION LOOPS

3. THE COMPRESSION IGNITION ENGINE. In this engine the fuel is injected into the hot compressed air in the engine cylinder. It burns as it enters, combining with as much oxygen as it requires, the excess air being simply heated by the combustion. Theoretically, therefore, as long as there is sufficient air present to supply the necessary oxygen, efficient combustion should be possible. In practice at present, however, it is found that compression ignition engines can only utilize about 80 per cent of the available air in the engine cylinder. With a larger proportion of fuel than this, part of the oil remains unburnt, causing a smoky exhaust with the presence of carbon monoxide, while the power is not appreciably increased.

A typical consumption curve is shown in the curve C*, Fig. 105,

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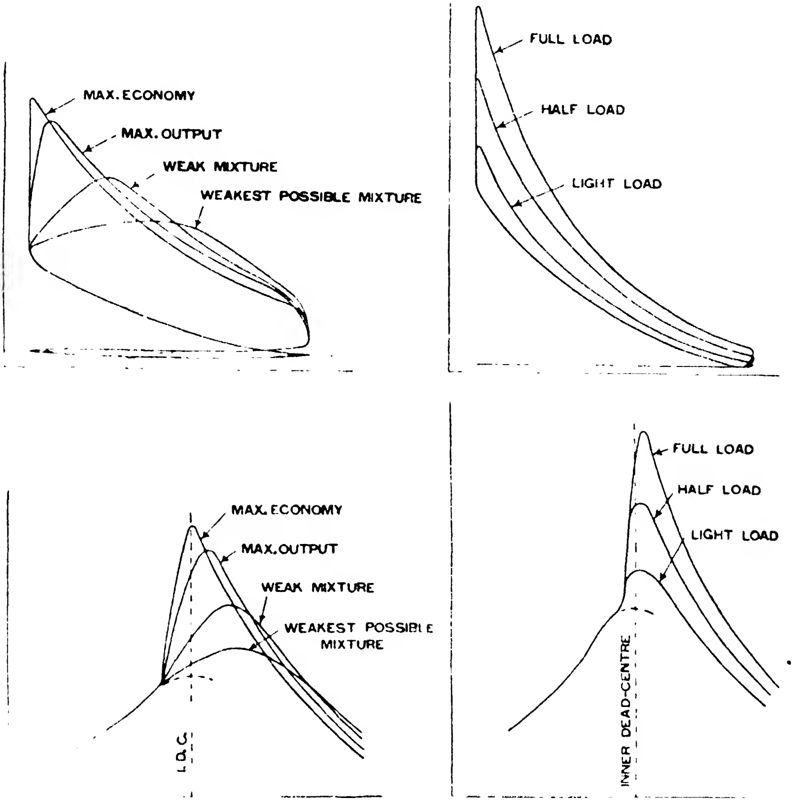
* "High-speed Heavy-oil Engines," by S. J. Davies, Ph.D., M.Sc., *Proc. Inst. Mech. E.*, June, 1932.

where it will be noticed that the range of power is considerably greater than is the case with the petrol engine. The efficiency also remains much more constant as the fuel-air ratio is altered, while it actually increases for a considerable reduction in the loading. This



Petrol Engine C.I. Engine

Fig. 106. Comparison of Indicator Diagrams for Petrol and Compression-ignition Engines

(*Institution of Mechanical Engineers*)

is due to the fact that while efficient combustion is maintained for weak mixtures, the proportion of heat lost to the cylinder walls is lessened due to the lower temperature, and the specific heat of the working substance is also reduced. It should be remembered that in order to obtain the best conditions the injection timing should be advanced to compensate for the weaker mixture, and consequent slower burning.

A very interesting comparison of petrol and compression ignition engines appears in the paper by Mr. S. J. Davies, already referred to,* from which Fig. 106 is reproduced. It will be noticed that in the case of the petrol engine, although of very different shapes, the indicator diagrams are not very different in area owing to the limited range of efficient combustion. In the case of the compression ignition engine, owing to the greater range of efficient combustion, the differences of the indicator areas are considerable.

Example 7. In a trial on a petrol engine, the speed and throttle were kept constant while the jet was varied by means of an adjustable needle, so that the B.H.P. could be determined for varying mixtures.

The following figures were obtained on test, the air consumption being sensibly constant at 1,090 lb. per hour.

	1	2	3	4	5	6	7	8
A. Fuel, lb. per hour	56	57	59	63·6	71·2	82·5	87·7	90·8
B. B.H.P. . . .	95	100	105	110	112	112	110	105

The composition of the fuel by weight was C 83 per cent, H_2 14·5 per cent O_2 2·5 per cent, with a calorific value of 19,000 B.Th.U. per lb.

Draw the following curves—

(*a*) Fuel consumption in lb. per H.P. hour against B.H.P.
(*b*) Thermal efficiency against B.H.P.
(*c*) Thermal efficiency against mixture strength.
(*d*) B.H.P. against mixture strength.

(*a*) Fuel in lb. per H.P. hour

$$= \frac{\text{Line } A}{\text{Line } B} = \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0{\cdot}59 & 0{\cdot}57 & 0{\cdot}562 & 0{\cdot}578 & 0{\cdot}637 & 0{\cdot}736 & 0{\cdot}796 & 0{\cdot}864 \end{array}$$

These figures are plotted in curve *W*, Fig. 107.

(*b*) Thermal efficiency

$$= \frac{\text{B.H.P.} \times 33{,}000 \times 60}{\text{Fuel per hour} \times \text{C.V.} \times 778} = \frac{\text{B.H.P.} \times 33{,}000 \times 60}{\text{Fuel} \times 19{,}000 \times 778}$$

$$= 0{\cdot}1335\,\frac{\text{Line } B}{\text{Line } A}$$

$$= \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 22{\cdot}6 & 23{\cdot}4 & 23{\cdot}8 & 23{\cdot}1 & 21{\cdot}0 & 18{\cdot}15 & 16{\cdot}71 & 15{\cdot}41 \end{array}$$

These figures are plotted in curve *X*, Fig. 107.

(*c*) and (*d*)

Oxygen required per lb. fuel

$$= 0{\cdot}83 \times \frac{8}{3} + 0{\cdot}145 \times 8 - 0{\cdot}025 = 3{\cdot}348 \text{ lb.}$$

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* From Mr. S. J. Davies' Paper in *Proc. Inst. Mech. E.*, June, 1932.

∴ Correct amount of air required

$$= 3{\cdot}348 \times \frac{100}{23} = 14{\cdot}56 \text{ lb. per lb. fuel}$$

∴ Correct mixture is $\dfrac{1}{14{\cdot}56}$

$$= 0{\cdot}0688 \text{ lb. fuel per lb. air.}$$

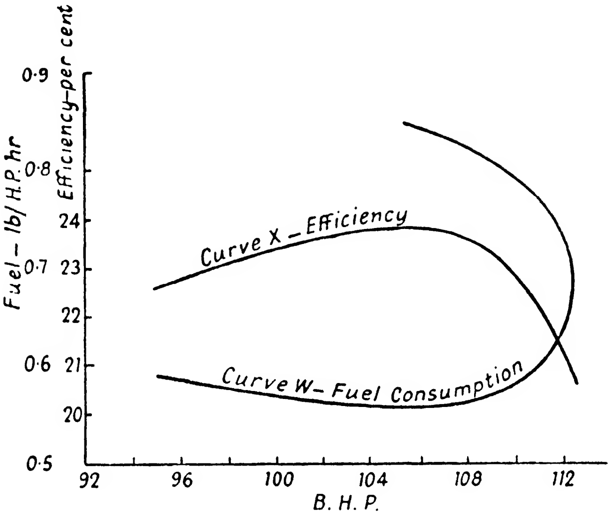

FIG. 107. FUEL CONSUMPTION AND EFFICIENCY CURVES

Let x = actual weight of fuel per lb. air

$$= \frac{\text{Weight of fuel per hour}}{\text{Weight of air per hour}}$$

Then

$$x = \frac{\text{Line } A}{1{,}090}$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ = 0{\cdot}0513 & 0{\cdot}0523 & 0{\cdot}0542 & 0{\cdot}0584 & 0{\cdot}0653 & 0{\cdot}0756 & 0{\cdot}0805 & 0{\cdot}0833 \end{array}$$

Percentage richness or weakness

$$= \frac{x - 0{\cdot}0688}{0{\cdot}0688} \times 100$$

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ = -\ 25{\cdot}4 & -\ 23{\cdot}9 & -\ 21{\cdot}2 & -\ 15{\cdot}1 & -\ 5{\cdot}08 & +\ 9{\cdot}9 & +\ 17 & +\ 21{\cdot}1 \end{array}$$

Thermal efficiency and B.H.P. are plotted against the above values in curves *Y* and *Z*, Fig. 108.

Governing. Any system of governing an engine should aim at maintaining a high efficiency at all loads. From the previous paragraphs it will be seen that from a theoretical point of view the necessary requirements are inclined to become somewhat complicated, involving possibly stratification, and a simultaneous alteration of mixture strength and the point of ignition or injection. In practice the problem becomes one of compromise between theoretical requirements and practical simplicity.

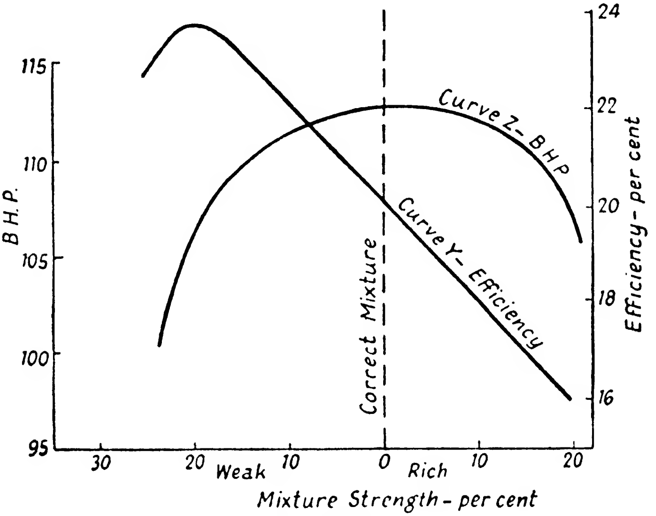

FIG. 108. EFFECT OF MIXTURE STRENGTH

There are three general methods, namely—

Hit and miss governing.

Quantitative governing.

Qualitative governing.

In the *hit and miss* method, which is used on gas engines, the air valve is opened for each suction stroke, but the gas valve, under the control of the governor, is left closed for one or more suction strokes if the engine speed rises above the normal. Thus on light loads there will be idle cycles in which air only is drawn in. One characteristic of this method therefore is that during an idle cycle the cylinder is scavenged by cool clean air which displaces the hot products of combustion. The result is that the efficiency at light loads is somewhat greater than at normal full load. The necessary gear is also light and simple, allowing for sensitive governing, while the adjustments for mixture strength do not require alteration. The drawback is that the torque on the crankshaft is very irregular on light loads, necessitating a heavy flywheel, and hence this method is limited to comparatively small engines.

Quantitative governing is used in many gas engines and is the common method for petrol engines. In this method, which has the merit of practical simplicity, the mixture strength is maintained constant but the quantity is varied by means of a throttle valve or by a variation in the lift of the main inlet valve. This reduces the weight of the charge drawn in and hence the mean effective pressure. By reducing the weight of the incoming charge the weight of the hot exhaust gases left in the cylinder becomes greater in proportion to the whole cylinder content at the end of the suction stroke. This means that in the first place the temperature of the working substance at the beginning of the compression stroke, and hence the temperatures all round the cycle, are raised, and secondly the combustible mixture is diluted, thus tending to delay completion of combustion. The effect of these two conditions is to lower the efficiency on light loads. An adverse effect on the efficiency is also produced by the fact that the mean effective pressure is reduced by a decrease in the weight of the working substance, and not, as is desirable, by a lowering of the cycle temperatures. The effect of delayed combustion due to the dilution of the working substance can be compensated for by advancing the point of ignition as the throttle is closed. This is illustrated by tests carried out on a Ricardo single-cylinder research engine, which showed that as the load is reduced by throttling and with the ignition set to give the best results on full load, the fuel consumption per indicated horsepower hour increases slightly as the load is reduced, but that, if at each throttle opening, the ignition timing is advanced as the load is reduced, then the fuel consumption per indicated horse-power hour remains nearly constant throughout the range from 30 per cent to 100 per cent full load torque.

" In all these tests the same procedure was adopted, viz. at each throttle opening the whole range of mixture strength was explored. The circulating water was maintained throughout at the same temperature ; the heat input to the carburettor was proportional to the load, i.e. at full load the heat input was at the rate of 65 B.Th.U. per min. ; at half load, 32·5 B.Th.U. per min., etc. ; also, at each throttle position the mechanical losses were measured at intervals by motoring, which was done by simultaneously switching off the ignition and changing over the armature circuit of the dynamometer, the combined operation taking less than 1 sec. and being effected without any appreciable change in speed."

Curves A, A_1, Fig. 109, show the results obtained with fixed ignition ; and curves B, B_1 with ignition adjusted for each change in load. The gain in thermal efficiency by advancing the spark when running on light loads is considerable.

In practice, however, the correct adjustment of the ignition for each throttle position is a difficult matter, especially if the engine speed is variable, and hence this method of governing, while in common use owing to its extreme simplicity, may leave something to be desired as regards efficiency.

The effect of throttling is also shown in the consumption loops in Fig. 105 where the curve *A* is for full load and the curve *B* is for the throttle partially closed. It will be seen that not only is the fuel consumption increased somewhat at the reduced load, but the power range is also reduced.

In *qualitative governing* the amount of air taken in per stroke is constant, but the amount of fuel is varied with the load. In considering the curves for gas (Fig. 103), it must be remembered that in obtaining these the point of ignition was advanced as the mixture became weaker, and this in practice is an added complication as far as "carburettor" engines are concerned. With fixed ignition the range over which governing by this method can be effective is very limited if the mixture in the cylinder is homogeneous. With weak mixtures, and the consequent slowing up of the combustion, firing back can occur due to the ignition of the fresh charge by the still burning exhaust gases. The possible range of effective governing can be increased considerably by stratification. The practical attainment of efficient stratification by simple means, is, however, difficult, and the more extensive use of this economical method of governing awaits practical development in this direction.

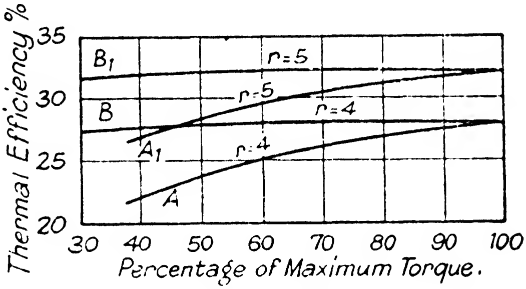

FIG. 109. POWER AND THERMAL EFFICIENCY CURVES FOR A RICARDO THROTTLE IN A PETROL ENGINE

The compression ignition engine, owing to the fact that it draws in air only during the suction stroke, is very adaptable to qualitative governing. Both the metering of the fuel and the injection timing can be controlled by the injection system. This consists of a fuel jerk pump actuated by a cam, the pump delivering the oil to an atomizer or injector. The jerk pump which consists of a plunger oscillating in a barrel, draws in oil on its suction stroke and delivers it on the delivery stroke to the injector via valves or ports. The metering of the fuel can be carried out (*a*) by varying the stroke of the plunger, or (*b*) by by-passing part of the oil back to the storage tank.

The modern injection system will meter the fuel with accuracy, and not only maintains uniformity as between individual cycles, but will keep the maximum cylinder pressures closely within the required range, for varying speeds and loads.

Compression Ratio. By increasing the compression ratio, the weight of exhaust gases left in the clearance space is reduced. This has two mutually opposed effects, for in the first place the reduction of the proportion of these gases mixing with the incoming charge lessens the dilution, and hence increases the rate of combustion, while in the second place, the rise of temperature produced by the hot residual exhaust gases is reduced. Another effect of the presence of the residual gases is to increase the specific heat of the combustible mixture due to the presence of CO_2 and H_2O. This effect is lessened by an increase in compression ratio. On the whole, the effect is to increase the rate of combustion at the higher ratios. This is shown in Fig. 110, the curves of which were obtained from a Ricardo variable compression engine running at 1,500 r.p.m., with a constant ignition advance of 30°, the fuel being petrol.

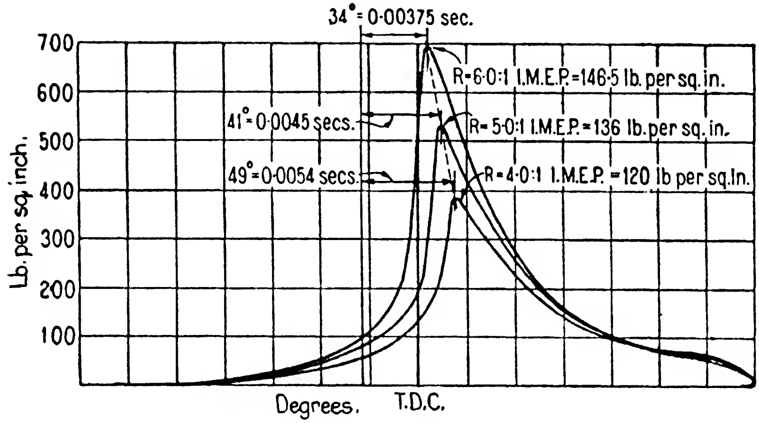

FIG. 110. EFFECT OF COMPRESSION RATIO

This shows how the time required after ignition to reach the maximum pressure is reduced as the compression ratio is increased.

Theoretically, the variation in the temperature and mass of the exhaust products should not affect the volumetric efficiency, for the heat interchange between the residual gases and the incoming charge should merely be accompanied by a contraction of the residual, and a corresponding expansion of the incoming gases (see p. 288). In practice, however, the volumetric efficiency is found to fall off with an increase of compression ratio. Mr. J. F. Alcock has suggested that with a low compression the volume and temperature of the residual gases are greater, with the result that there is a greater loss of heat to the cylinder walls. It is doubtful whether this is a complete explanation, for the question is also affected by valve timing.

As regards thermal efficiency, all theoretical equations show that this should increase with increased compression ratio, and this is verified in practice. Table XX gives some figures obtained by Ricardo.

TABLE XX

VARIED COMPRESSION RATIO. BENZOL ABOUT 20 PER CENT RICH
(*H. R. Ricardo*)

Compression Ratio	Total Air lb. per Hour	Indicated Mean Pressure, lb. per sq. in.	Indicated Horse-power	Lb. of Air per I.H.P. Hour	Indicated Thermal Efficiency by Air Measurement. Per Cent	Indicated Thermal Efficiency by Fuel, 15% Weak. Per Cent
4	200·5	125·0	30·3	6·62	29·2	27·7
5	194·0	136·5	33·1	5·32	33·2	32·0
6	188·0	145·0	35·2	5·34	36·2	35·0
7	164·0	152·0	36·8	5·0	38·5	37·3

It will be seen that not only is the efficiency raised by an increase in compression ratio, but that the volumetric efficiency is reduced. In the last column is shown the maximum thermal efficiency as found from the fuel consumption with the mixture adjusted to give highest economy, i.e. about 15 per cent weak.

The curves in Fig. 101 also show how thermal efficiency increases with the compression ratio.

When dealing with the theoretical cycles, it will be remembered that the expansion and compression ratios were equal in the case of the constant temperature, volume, and pressure cycles, while they were nearly equal in the case of the Diesel and composite cycles. Now if the expansion ratio can be increased without a corresponding increase in the compression ratio, or in the heat input, the efficiency will be increased without the attendant practical difficulties of a high compression. For example, the Atkinson cycle shows efficiencies greater than the air standard efficiencies for corresponding compression ratios. The practical difficulty in attempting to carry out such a cycle is, of course, one of mechanical complication.

In a carburettor engine, that is, one in which the cylinder contains an explosive mixture during the compression stroke, the maximum compression ratio possible in practice is limited by the possibility of pre-ignition and detonation. This tendency depends on the nature of the fuel, and will be discussed later. For an ordinary petrol the compression limit is about 5 or 6 to 1. In compression ignition engines there is a lower limit of the compression ratio of about 10 to 1, below which the temperature of the compressed air in the cylinder will not be high enough to give efficient combustion of the injected fuel. As regards the higher limit, we have seen (p. 268) that raising the compression ratio also raises the maximum pressure due to the combustion, and as this is limited by practical considerations, the injection must be so timed that part of the combustion must take place at more or less constant pressure. This reduces the expansion ratio, and hence there is a practical limit of the useful compression ratio from this point of view. Apart from these aspects of the case, the increase of the thermal efficiency due to an increase of compression is comparatively small at the higher ratios. At present the limit in practice is about 16 to 1.

Detonation. When an explosive mixture is ignited by means of a spark under normal conditions, the flame at first spreads by the ordinary process of flame propagation throughout the bulk of the mixture, the speed of propagation being chiefly dependent on the amount of turbulence, and to a certain extent on the pressure and temperature before ignition. Ignition should of course not start before the spark has passed, but this may sometimes occur owing to hot spots of carbon or to local overheating of the sparking plug points, and it tends to become accentuated by an increase in the compression ratio. This is what is meant by "pre-ignition." Another phenomenon is known as "detonation." This is due to the rate of flame propagation increasing to such an extent as to set up a wave of high pressure, travelling at very high speed, and causing a blow to be delivered against the cylinder walls, producing what is commonly called "knocking" or "pinking." This in its turn further compresses the portion of the charge first ignited, and raises its temperature and that of the ignition points to so high a degree as eventually to cause persistent pre-ignition, which may stop the engine.

Investigations on the conditions governing detonation by Ricardo, Tizard, and others have led to the following conclusions—

1. Detonation depends primarily upon the *time rate of burning* of that portion of the charge first ignited.

2. That the rate of burning increases very rapidly with slight increase of *flame temperature*, and that whether it will prove sufficiently rapid to produce detonation or not depends upon the ratio between the rate of evolution of heat by the burning portion of the mixture and the *rate of heat loss* to the cylinder walls. An engine with water-cooled exhaust valves, or with sleeve-valves and combustion chamber without side pockets, and hemispherical

cylinder head with ample water cooling, is comparatively exempt from detonation, and will stand a high compression ratio.

3. Detonation also depends upon the *distance* the flame has to travel from the ignition point before it has passed through the whole mixture. Thus the more nearly hemispherical in shape the *combustion chamber*, the better to avoid detonation; and the *smaller the cylinder* the less the tendency to detonation of the charge.

4. For any given mixture strength the maximum flame temperature which causes detonation depends upon the *compression temperature* and the proportion of *residual inert exhaust products* present in the cylinder, which, like CO_2. exert a great influence in diluting the charge, *retarding the rate of burning*, and so lowering the temperature of the flame that a higher compression can be used.

5. The tendency to detonate varies with the type of fuel.

The fact that the maximum compression pressure possible without producing detonation depends on the type of liquid fuel, has led to the use of the term *highest useful compression ratio* by Ricardo. This quantity may be defined as the maximum compression ratio at which the fuel can be used in an engine working on full throttle, without detonation being produced. While the absolute value of this quantity depends on the type of engine and the working conditions, it affords a very useful comparison of different fuels in any one engine, and Table XXI gives particulars of different types of fuel. In this table the value given for benzene is not for detonation but for pre-ignition.

Owing to the comparatively non-detonating property of benzene,

TABLE XXI

Fuel	Specific Gravity at 15° C.	Detonation Point in Engine			Self-ignition Temperature by Adiabatic Compression with Air Mixture °C.
		Compression Ratio	Compression Pressure, lb. per sq. in. (Gauge)	Compression Temperature °C.	
Petrol *A*	0·782	6·1	148·5	430	367
Petrol *F*	0·704	5·05	111·5	400	—
Petrol *G*	0·750	4·55	96	381	—
Kerosene	0·813	4·2	86	369	—
Aromatic Series					
Benzene, 98%	0·884	6·9	179	450	419
Toluene, 99%	0·870	> 7	> 183	> 452	422
Alcohol Group					
Ethyl Alcohol, 98%	0·798	> 7·5	> 204	> 424	514
Methyl Alcohol (Purified Wood Naphtha)	0·829	5·2	116·5	342	457
Methylated Spirits	0·821	6·5	163·5	382	—
Ether	0·735	2·95	47·5	305	256

it is often added to petrol to improve the H.U.C.R. Toluene is a still more effective "anti-detonator" whilst the alcohols are best of all. A small proportion of alcohol to petrol stops detonation, but the alcohol must not contain any water. A very efficient anti-detonator has been discovered in tetra-ethyl lead ($Pb(C_2H_5)_4$), which is generally used in conjunction with ethylene-dibromide ($C_2H_4Br_2$), this constituent being added to prevent the deposition of lead in the engine. By the addition of a few cc. to a gallon of petrol, the H.U.C.R. can be raised by about 15 per cent. There are a considerable number of other dopes, such as the metal carbonyls, aniline, phenol, etc., but none of those known at present are as effective as tetra-ethyl lead.

With gaseous fuels the compression ratio is limited, not so much by the possibility of detonation as by pre-ignition. The approximate limits of compression ratio are; town gas 5½ to 1, coke oven gas 5 to 1, producer gas 6 or 7 to 1, blast furnace gas 8 to 1.

Octane and Performance Numbers. Pure iso-octane which possesses good anti-knock properties is rated as having 100 octane number. This is mixed with heptane which has poor anti-knock properties, and is rated as zero octane number. The mixture is adjusted to have the same H.U.C.R. as the petrol under test. The percentage of iso-octane in the mixture is taken as the octane number.

For petrol with anti-knock properties better than 100 octane, the performance number is used. This is measured by the relative power that the engine will develop with equal anti-knock tendencies compared to a fuel rated at 100 octane. Thus 130 performance number means that the engine will develop 1·3 times the power it would develop when using a 100 octane fuel.

Engine Fuels. The composition and general properties of liquid fuels have been discussed on pp. 224–31, and it was there pointed out that petrol varies considerably in composition. One important property of any fuel is, as we have seen, its tendency to detonate, and from this point of view the aromatic compounds are the best, and the paraffins are the worst. The olefines, obtained by cracking, are good, but as they are unsaturated compounds, they tend to polymerize on storage, forming more complicated compounds, producing gummy deposits in the engine. Another good fuel from the same standpoint is alcohol, but as we shall see there is practical difficulty in its production. We will now consider the advantages or otherwise of the various liquid fuels.

Table XXII gives the results of numerous experiments oft-repeated with a very high degree of accuracy in measurement, all the conditions being checked before and after each trial run. It was found from experiments that the characteristic of the torque curve over the range of speed 800 to 1,800 r.p.m. was the same for every fuel tested with the correct mixture strength and ignition setting for each speed. In each series of tests the engine was run on wide-open throttle at 1,500 r.p.m., with the best ignition setting, mixture strength, and compression ratio for the development of the maximum power obtainable on each sample of fuel; the temperature of, the circulating water and the heat input to the carburettor being kept constant.

TABLE XXII

Results of Tests (*H. R. Ricardo*)

Fuel	Specific Gravity at 15° C.	Compression Ratio 5:1 Maximum I.M.E.P. lb. per sq. in.	Highest Useful Compression		Maximum I.M.E.P at highest Useful Compression lb./sq. in.	Minimum Consumption lb. per I.H.P. hour	Thermal Efficiency at highest Useful Compression Per Cent
			Ratio, $\frac{\text{Total Vol.}}{\text{Clearance}} = r$	Pressure, lb. per sq. in. (Gauge)			
A Petrol . . .	0·782	131·2	6·0	148·5	140·1	0·393	34·9
B ,, . . .	0·723	131·5	5·7	133·5	137·5	0·393	34·1
C ,, . . .	0·727	131·0	5·25	118·0	133·9	0·410	32·5
D ,, . . .	0·760	131·2	5·35	121·5	134·9	0·407	33·1
F ,, . . .	0·704	131·8	5·05	111·5	132·7	0·412	32·1
H ,, . . .	0·767	131·0	5·9	140·5	139·5	0·389	84·6
Heavy Aromatics .	0·885	130·7	6·5	163·5	142·5	0·447	31·5
Paraffin Series							
Hexane (80% pure)	0·685	132·3	5·1	113·5	133·1	0·405	32·4
Heptane (97% pure)	0·691	131·2	3·75	72·0	119	0·491	26·7
Aromatic Series							
Benzene (pure). .	0·884	131·6	6·9	179·0	146·5	0·392	37·2
Toluene (99% pure)	0·870	131·5	>7	>183	147	0·385	37·5
Naphthene Series							
Cyclohexane (93% pure) . . .	0·786	131·3	5·9	140·5	139·2	0·385	34·9
Hexahydrotoluene (80%) . . .	0·780	131·0	5·8	136·5	137·9	0·394	34·3
Olefines							
Cracked Spirit (53% unsat.) . . .	0·757	131·6	5·55	128·0	136	0·405	33·9
Alcohol Group							
Ethyl Alcohol (98%) . . .	0·798	137·8	>7·5	>204	156·5	0·532	40·4
Methyl Alcohol (Wood Naphtha)	0·829	144·8	5·2	116·5	146·6	0·725	35·1

A constant supply of heat was added to the air at the carburettor by electrical resistance coils, giving 65 B.Th.U. per min. when running at 1,500 r.p.m.

We will first consider the effect of the latent heat of evaporation. Starting with a mixture of fuel and air at atmospheric temperature, heat is added by (*a*) contact with the inlet valve and hot walls of the cylinder, (*b*) admixture with the residual exhaust gases in the combustion chamber, and by (*c*) external heating of the carburettor or induction system. Heat is absorbed by (*d*) the latent heat of evaporation of the fuel; (*a*) and (*b*) may be regarded as constant.

The weight of charge taken into the cylinder will be inversely proportional to its absolute temperature when the inlet valve closes. Experiment proves that, with the exception of alcohol, all fuels boiling below 200° C. are completely evaporated at the end of the suction stroke, except a very small negligible proportion entering the cylinder in coarse drops, which are neither evaporated nor burned completely. The temperature, and therefore the weight of the charge taken into the cylinder, depends upon the quantity of the fuel and its latent heat; also upon the heat added to it before entry. With a suitable fixed amount of pre-heating, the volumetric efficiency, and therefore the *power output, will vary directly with the latent heat of the fuel,* since it is immaterial, as regards power output, whether the pre-heating is expended in raising the temperature of the mixture or in vaporizing the fuel at low temperature, provided it is completely evaporated before the inlet valve closes. The fall in temperature of the mixture due to evaporation of the liquid is shown on p. 230. Variations due to the differences in the latent heat of evaporation were most marked in the alcohol group.

All the tests proved that the power obtainable was proportional to the internal energy of the fuel. It will be seen from Table XXII that the observed power, given in terms of the indicated mean effective pressure, was practically the same for all fuels boiling below 200° C., except alcohol, *when used at the same safe suitable compression ratio* 5 : 1, with the ignition fully advanced. The three columns on the right give the indicated mean effective pressure, the fuel consumption, and thermal efficiency when full advantage was taken of increasing the compression ratio up to the limit set by detonation or by pre-ignition for each particular fuel, under the same constant conditions of pre-heating.

In the case of alcohol, the very high latent heat of evaporation, and the larger proportion of fuel to air in the mixture, caused the evaporation to continue throughout the compression stroke, which was nearly isothermal. The low temperature in the cylinder increased the weight of charge, or volumetric efficiency, and the small rise of temperature at the end of compression, combined with the low calorific value of alcohol, conspired to keep the cylinder temperature low throughout the cycle. Hence the direct heat losses by conduction and increase of specific heat were reduced. The power obtainable also increased as the mixture strength was enriched and continued to increase until the over-rich mixture was about 20 per cent more than that required for complete combustion; apparently because more fuel was then evaporated and the temperature of the charge lowered, the gain in weight of charge or volumetric efficiency more than counter-balanced the increased loss due to change in the specific heat of the products of combustion.

Although ethyl alcohol gives greater maximum power, and can be used with a much higher compression ratio and thermodynamic efficiency than either petrol or benzol, yet the calorific value per unit weight or per unit volume of the liquid is so much lower than that of petrol or benzol, that the rate of fuel consumption of alcohol. per horse-power hour is greater.

The advantages of alcohol as a motor fuel are enumerated in the 1907 Report of the Fuels Committee of the Motor Union, U.S.A.: (*a*) high compression pressure over 200 lb. per sq. in. before ignition; (*b*) the wide range of explosive mixtures of alcohol vapour with air, from 4 to 13·6 per cent by volume; also the small range of boiling point at which industrial alcohol distils from 80° to 110° C., whereas that of petrol extends at least 50° to 150° C. or higher, and benzol 80° to 120° C.; (*c*) high thermal efficiency and safety in use, the flash point being 60° C. as compared with that of petrol − 10° C, and a fire from alcohol can be extinguished by water, which only spreads the flames of petrol.

It is obvious from the results in the above tables that ethyl alcohol is a more suitable fuel for power purposes, at high compression, than methyl alcohol or wood spirit, because of the low calorific value and high latent heat of evaporation of the latter.

An admixture of industrial alcohol with some highly volatile spirit appears necessary not only for easy starting from cold, but also to increase the heating value of the mixture, in order to obtain, with high compression, a lower rate of fuel consumption per horse-power hour. Industrial or power alcohol is never pure, but nearly always contains water and, in order to render it undrinkable, a small proportion of acetone, pyridine, or other unpalatable substance not easily separated, is added as a denaturant. The aromatic hydrocarbons, benzol or benzene and toluene, are readily soluble in alcohol, even at low temperatures, have comparatively high heating values, and can be used to best advantage at high compression with good excess of air, giving complete combustion and tending to prevent acetic acid being formed. Thus, a mixture of industrial alcohol, with 20 or 30 per cent of benzol, has a high ignition temperature, and can be used in an engine with a compression ratio of about 8 : 1. The high percentage of carbon in benzol compensates for the low carbon content of alcohol.

Ethyl alcohol is obtained by the fermentation and distillation of any starchy or sugary substance like sugar-cane or edible grains. The supply of alcohol is inexhaustible while the sun shines, but it is not likely to prove a commercial success to a large extent until it can be produced cheaply on a large scale from some form of vegetation without destroying foodstuffs.

Extensive research and experimental work was carried out by the Empire Motor Fuels Committee on various alcohol mixtures, and the results reported in the *Proceedings of the Institution of Automobile Engineers.*

It will be noticed from Table XXII that the fuel economy with benzene is greater than with most of the petrols. Also, as the H.U.C.R. is greater, the thermal efficiency is in advance of that of petrol. The use of benzole as a fuel is, however, limited by its price, and by the fact that it freezes at 5° C. It is therefore used mixed with petrol, and not in its undiluted form.

In the last column of Table XXII are given the thermal efficiencies, which vary with the fuel. These have, however, been obtained at the best compression ratio possible with the fuel. For equal compression ratios it has been found by experiment that all volatile liquid fuels give practically the same thermal efficiency, with the possible exception of alcohol. This fuel gives an efficiency of about 2 per cent greater than that for petrol at a compression ratio of 5 to 1. This, as we have already seen, is due to its high latent heat of evaporation.

Desirable Properties of Petrol. These may be enumerated as follows—

(*a*) Not too high a proportion of volatile constituents, involving evaporation losses. For easy starting, a certain minimum is required. Too high a percentage produces vapour locks.

(*b*) Not too large a proportion of high-boiling hydrocarbons, otherwise condensation takes place in the engine cylinder, and lubricants are diluted.

(*c*) Freedom from water and corrosive matter, and objectionable odour.

(*d*) Freedom from residue on evaporation.

(*e*) Freedom from detonation.

(*f*) High calorific value and latent heat.

(*g*) Freedom from "gum" due to polymerization of constituents in cracked spirit.

The readiness of a fuel to start an engine from cold is determined by the vapour pressure and the latent heat of evaporation, as well as the ratio of air to fuel required to give complete combustion. These values for the more volatile fractions of petrol in comparison with alcohol and ether, from experiments by Ricardo, are as shown in Table XXIII.

TABLE XXIII

	Specific Gravity at 15° C.	Boiling Point, °C.	Vapour Pressure at 0° C.; mm. Mercury	Latent Heat, B.Th.U. per lb.	Ratio Air/Fuel by Weight
Hexane . . .	0·670	69	45·0	156	15·2
Cyclohexane .	·780	81	27·5	155	14·7
Benzene . .	·884	80	26·0	172	13·2
Ethyl Alcohol .	·794	78	12·7	397	8·95
Ether . . .	·719	35	185·0	158	11·14

The addition to alcohol of a small proportion of ether, with its high vapour pressure, renders starting easy in a suitably designed engine and carburettor.

Carburation. The ordinary type of carburettor consists of a jet tube to which the liquid fuel is supplied at a constant level from a float chamber, the jet tube being surrounded by a choke tube which is shaped to act as a venturi nozzle. When at rest, the level of the liquid in the float chamber is slightly below the top of the jet tube, in order to prevent the fuel being spilt by vibration. The suction of the engine draws air through the choke tube, the venturi effect of which is to lower the pressure at the top of the jet tube thus causing the air to carry the petrol or other volatile fuel with it into the engine cylinder. The velocity of the air past the jet tube pulverizes the petrol, which is then evaporated by the heat from the induction system and the cylinder walls. This simple method of pulverizing the fuel is not ideal, and some carburettors contain a diffuser tube in which air is drawn through the liquid in order to form an emulsion.

The first and most important function of a carburettor is to supply a mixture to the cylinder of constant fuel-air ratio at all engine speeds and all loads, and we will first consider this problem.

For steady motion of a fluid, the Bernoulli equation of constant energy is

$$\frac{v^2}{2g} + h + \int \frac{dp}{w} = \text{constant}$$

Let v_a = the air velocity past the jet in the choke tube.

p_a = the pressure at the top of the jet tube.

p = the pressure at the petrol surface in the float chamber.

w_a = the mass per cubic foot of air.

w_p = the mass per cubic foot of petrol.

v_p = the velocity of the petrol leaving the jet.

h = the distance between top of jet tube and float chamber level.

Then applying the energy equation to the flow of air from rest to the velocity v_a

$$\frac{v_a^2}{2g} + \frac{p_a}{w_a} = 0 + \frac{p}{w_a}$$

i.e.

$$p - p_a = \frac{w_a v_a^2}{2g}$$

Equating the energy of the liquid at the top of the jet tube to that at the float chamber level

$$\frac{v_p^2}{2g} + h + \frac{p_a}{w_p} = 0 + 0 + \frac{p}{w_p}$$

i.e. $$w_p\left(\frac{v_p^2}{2g}+h\right)=p-p_a=\frac{w_a v_a^2}{2g}$$

$$\therefore\ v_p^2=\frac{w_a}{w_p}v_a^2-2gh \qquad . \quad . \quad . \quad . \quad . \qquad (1)$$

If $$v_p=0 \text{ then } v_a=\sqrt{2gh\frac{w_p}{w_a}} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

This is the minimum or critical air velocity at which fuel will just begin to flow.

Example 8. Find the critical air velocity, and the effective throat area of the choke tube for a carburettor with a jet area of 2 mm. used on a petrol engine consuming 12 lb. of petrol per hour. The density of air at atmospheric pressure and 60° F. is 0·0765 lb. per cu. ft., the specific gravity of petrol at 60° F. is 0·72, the fuel-air ratio by weight is 1 to 15, and the top of the jet tube is ¼ in. above the level in the float chamber. What is the drop in pressure in the choke tube?

Density of petrol, $w_p = 62{\cdot}4 \times 0{\cdot}72 = 44{\cdot}9$ lb. per cu. ft.

Density of air $w_a = \frac{1}{13{\cdot}07} = 0{\cdot}0765$ lb. per cu. ft.

$$\text{Critical air velocity} = \sqrt{2gh\frac{w_p}{w_a}} = \sqrt{64{\cdot}4 \times \frac{1}{48} \times \frac{44{\cdot}9}{0{\cdot}0765}}$$

$$= 28 \text{ ft. per sec.}$$

$$\text{Area of jet} = 0{\cdot}0000215 \text{ sq. ft.}$$

$$\text{Weight of petrol used} = \frac{12}{3{,}600} = 0{\cdot}00333 \text{ lb. per sec.}$$

$$= \text{Area of jet} \times v_p \times w_p$$

Hence $$v_p = \frac{0{\cdot}00333}{0{\cdot}0000215 \times 44{\cdot}9} = 3{\cdot}45 \text{ ft. per sec.}$$

Now $$v_a^2 = (v_p^2 + 2gh)\frac{w_p}{w_a} = \left(3{\cdot}45^2 + \frac{64{\cdot}4}{48}\right)\frac{44{\cdot}9}{0{\cdot}0765}$$

$$v_a = 88{\cdot}1 \text{ ft. per sec.}$$

Weight of air used = 15 × Weight of petrol = 15 × 0·00333
= 0·05 lb. per sec.

Hence $0{\cdot}05 = \text{Area of choke tube} \times v_a \times w_a$

Assuming the air in the choke tube is at atmospheric pressure and 60° F.,

$$\text{Effective area of choke tube} = \frac{0{\cdot}05}{88{\cdot}1 \times 0{\cdot}0765}$$

$$= 0{\cdot}00742 \text{ sq. ft.} = 1{\cdot}07 \text{ sq. in.}$$

The pressure drop in the choke tube

$$= p - p_a = \frac{w_a v_a^2}{2g}$$

$$= \frac{0{\cdot}0765 \times 88{\cdot}1^2}{64{\cdot}4}$$

$$= 9{\cdot}22 \text{ lb. per sq. ft.}$$

$$= 0{\cdot}064 \text{ lb. per sq. in}$$

It will be seen from equation (1), p. 343, that this may be written

$$\frac{v_p}{v_a} = \sqrt{\frac{w_a}{w_p} - \frac{2gh}{v_a^2}}$$

Now as the engine speed increases, the value of v_a on the right-hand side of this equation is increased, and hence the value of $\frac{v_p}{v_a}$, which is proportional to the fuel-air ratio, is raised, that is, the mixture becomes richer. For this reason modern carburettors contain a compensating jet, which if working alone would give a weaker mixture as the engine speed increased; the two jets working together therefore give a mixture of approximately constant mixture strength. In addition to this function, an ideal carburettor should also provide an over-rich mixture when starting, when warming up, and when idling on light load. It should also give momentarily an over-rich mixture when the throttle is suddenly opened to give quick acceleration. The reason for this last requirement is that when the throttle is nearly closed the pressure in the induction system is low and the petrol supplied is easily evaporated. When the throttle is suddenly opened, the pressure in the induction system is suddenly increased, and unless the induction pipes are very hot the incoming petrol cannot all be evaporated; some of it is therefore deposited on the walls of the induction system, thus causing the mixture in the system to be too weak to burn efficiently. The result of this is that the engine will backfire and splutter for a short time, unless an over-rich mixture is supplied at the moment the throttle is suddenly opened.

Another point for consideration is that the rapid pulsation of flow due to the varying suction of the piston at very high speeds will give greater momentum to the petrol than to the air which, being lighter, responds more readily to the pulsations, and a surging action may take place, tending to precipitate the finely divided petrol held in suspension only by the velocity of the air. These inertia effects may be damped out by increasing the friction of the petrol orifices, and this can be done by making the orifices small and increasing their number. Another advantage of this device is to expose a greater surface of petrol spray to the air, and so to produce a more intimate homogeneous mixture.

The evaporation of the petrol is of course assisted by pre-heating the air and by heating the induction system, which can be done by the hot water from the cooling system or by the exhaust gases. It must be remembered that this will reduce the volumetric efficiency of the engine, and therefore it should be avoided as far as possible. In the case of alcohol some form of pre-heating is essential owing to its high latent heat.

Atmospheric conditions of temperature and pressure will affect carburation. The viscosity of air increases with its temperature, while that of petrol diminishes, the rate of flow of a liquid being proportional to the reciprocal of its viscosity. The relation between fluidity and temperature for various liquids is shown in Fig. 111.*

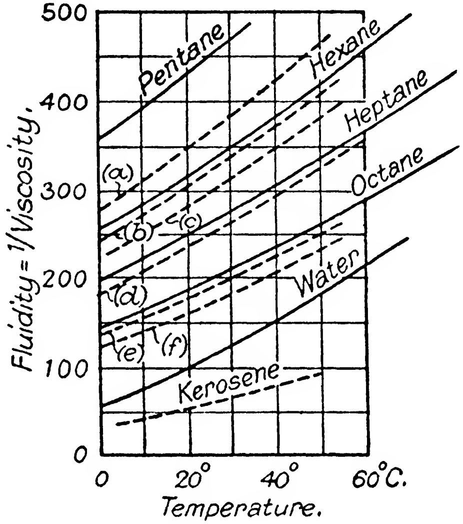

FIG. 111. RELATION BETWEEN FLUIDITY AND TEMPERATURE OF PETROL

The pressure effect due to a change in altitude is very important in the case of aircraft engines, and this is compensated for by some means of altitude control. One method uses a sealed float chamber, the space above the petrol being connected to two points, one between the choke tube and the engine, the other between the choke tube and the throttle. For normal running the first connection is closed, and the pressure in the float chamber is that of the atmosphere. As the altitude increases and the atmospheric pressure becomes less, the mixture would become richer. The connection

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* *Bureau of Standards Technologic*, U.S.A., Paper No. 125.

between the float chamber and the first point mentioned above can now be opened, thus reducing the pressure above the petrol surface in the float chamber, and hence reducing the proportion of petrol supplied. Another method is to have a hand-controlled valve between the float chamber and the jet.

Engine Testing and the Heat Balance. The object of a complete test on an engine is not only to determine what the engine is capable of doing, but also to discover if possible whether there are any means of improving its overall efficiency. For this reason in addition to fuel consumption, practical measurements of as many quantities as possible are required in order that it may be seen how the original heat supplied by the fuel has been distributed into useful work and mechanical and thermal losses. In this way we can obtain what may be termed a "heat balance," in which the different quantities of energy are expressed as a percentage of the heat supplied.

The main headings under which this information may be grouped are—

1. Brake horse-power delivered by the engine.
2. Energy wasted due to mechanical losses, i.e. engine friction, and pumping losses.
3. Heat energy carried away by the cooling water in a water-cooled engine or by the air-flow in an air-cooled engine.
4. Heat carried away in the exhaust gases.
5. Heat lost by radiation other than that removed by the normal cooling arrangements.

1. Measurement of B.H.P. (see pp. 33–40). This quantity can easily be determined by measuring the torque delivered by the engine shaft by means of some form of dynamometer. Of these the mechanical forms are the simplest, but they are somewhat limited in their application to large powers and speeds. The hydraulic dynamometer, which is made in various sizes, is extremely convenient for any power and speed, but its characteristics place both a maximum and a minimum limit of power absorbable by any one size of brake. The electrical dynamometer approaches the ideal but is somewhat costly.

2. Measurement of Mechanical Losses. The mechanical losses include (*a*) piston friction; (*b*) bearing friction and any power used by auxiliaries; (*c*) pumping losses. A direct measurement of the total losses may be made by means of an electro-dynamometer which will determine the torque necessary to turn the engine when it is not firing. This must of course be done while the engine is hot and as far as possible under normal running conditions. Even with these precautions the conditions are not exactly the same, for while the friction losses will on the whole be reduced when motoring, the pumping losses will be increased. This increase is due to the fact that when running under load the pressure in the cylinder on the opening of the exhaust valve is higher than when motoring, and this accelerates the expulsion of the gases. The "negative loop" (p. 274) will therefore be less when the engine is working under load. According to Ricardo the reduced friction loss balances the average increase of the pumping losses.

It would appear that the pumping loss could be measured from the indicator diagram taken with a light spring, but as the accuracy of any indicator is open to doubt, this method is more useful for purposes of comparison than for absolute measurement.

In the absence of such a method, the difference between the I.H.P. and the B.H.P. is required. It has already been pointed out that a really exact direct measurement of indicated mean effective pressure is almost impracticable. Apart from the inherent errors of the indicator, the necessity of accurate phasing makes the problem an extremely difficult one.

When taking the difference between the two horse-powers, it should be noted whether the I.H.P. includes the negative pumping loop. Also it must be remembered that the piston friction appears as heat which passes to the cylinder walls, and hence is really included in the heat carried away by the cooling system.

An approximation to the I.H.P. may be obtained from measurements of the B.H.P. and the fuel consumption with different loads at constant speed, on the following assumptions—

(*a*) That the friction loss is constant at all loads at any one constant speed.

(*b*) That the fuel consumption is a linear function of the I.H.P. and the B.H.P. at any one constant speed.

This method is illustrated in the following example.

Example 9. In an oil engine running at 550 r.p.m., the following data were obtained—

B.H.P.	Fuel Consumption lb. per hour
10	6·8
20	9·8
30	12·75
40	16·0
50	19·5
60	23·4
70	28·0

Assuming a straight line relationship between the brake and indicated horse-powers, plot to a B.H.P. base, curves showing the thermal performance and the mechanical efficiency of the engine.

The variation of thermal performance with B.H.P. is shown in curve *A* (Fig. 112), in which is plotted the fuel consumption in pounds per horse-power hour. If the calorific value of the fuel were known, the actual thermal efficiency could of course be plotted.

Curve B shows the fuel consumption plotted in pounds per hour against B.H.P. It will be noticed that with the exception of the two last points, the relationship approximates to a straight line.

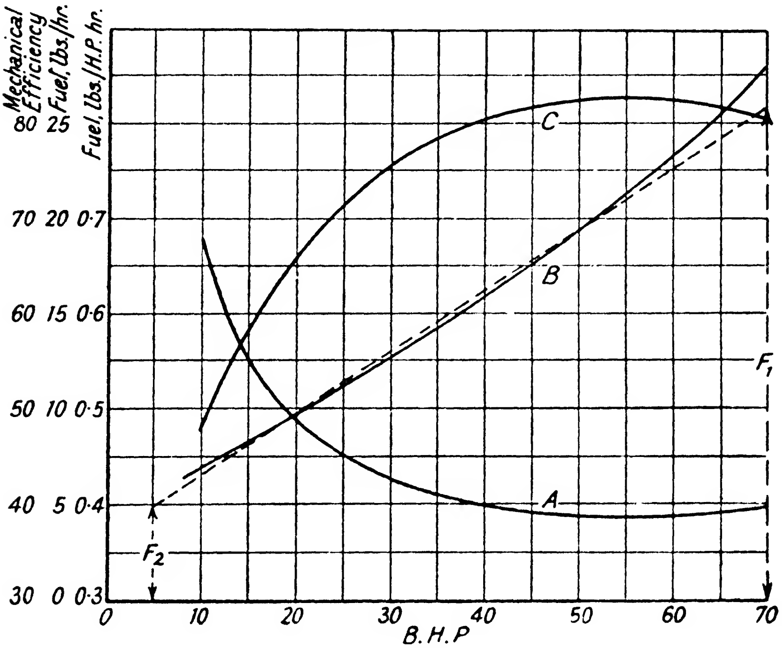

FIG. 112. OIL ENGINE PERFORMANCE CURVES

Hence if F = fuel consumption in lb. per hour, we may assume that B.H.P. = $mF + n$ where m and n are constants.

$$\text{Hence} \quad m = \frac{B.H.P._1 - B.H.P._2}{F_1 - F_2} = \frac{65}{21} = 3{\cdot}09$$

As there is a straight line relationship between B.H.P. and I.H.P., we can assume that I.H.P. = $MF + N$.

We must now make one more assumption, namely, that as the engine is running at constant speed, the mechanical losses may be taken as constant at all loads.

$$\text{Hence} \quad I.H.P._1 - B.H.P._1 = (MF_1 + N) - (mF_1 + n)$$
$$I.H.P._2 - B.H.P._2 = (MF_2 + N) - (mF_2 + n)$$

whence $M = m = 3{\cdot}09$ and $I.H.P. = 3{\cdot}09F + N$.

The value of N should obviously be zero, and hence we obtain the following figures—

B.H.P.	I.H.P.	Mechanical Efficiency Per Cent
10	21	47·6
20	30·3	65·9
30	39·5	76·0
40	49·6	80·7
50	60·4	82·7
60	72·5	82·7
70	86·5	80·9

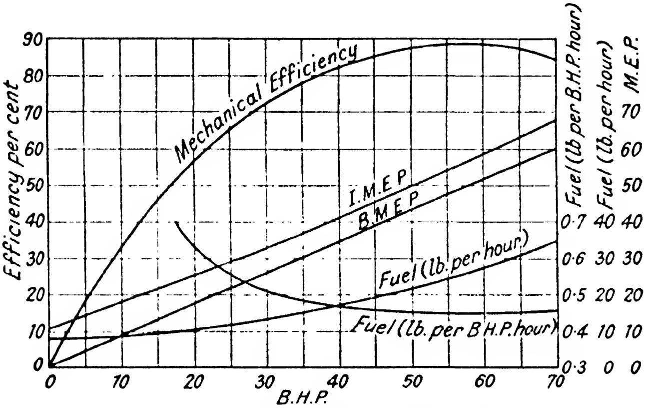

FIG. 113. OIL ENGINE TEST RESULTS

The mechanical efficiency is plotted in curve C (Fig. 112). It must be remembered that the assumed straight line laws are not strictly true in practice, and hence the above results are only approximate to this extent.

As a comparison with practical figures, Fig. 113 shows the test results of an airless injection oil engine.

This engine which runs at a speed of 230 r.p.m. has a cylinder diameter of 14¾ in. and a stroke of 25 in. The compression pressure is 390 lb. per sq. in. and the maximum pressure is 525 lb. per sq. in.

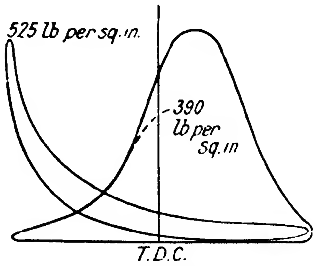

FIG. 114. INDICATOR DIAGRAMS FOR C.I. OIL ENGINE

Fig. 114 shows the type of indicator diagrams obtained with this engine.

It will be noticed in Fig. 113 that one curve shows the brake mean effective pressure or B.M.E.P. This quantity bears the same relation to B.H.P. as the indicated mean effective pressure or I.M.E.P. bears to I.H.P., that is

$$\text{B.M.E.P.} = \frac{\text{B.H.P.} \times 33{,}000}{\text{L.A.N.}}$$

In Fig. 115 are plotted some typical results of a test on a petrol engine. These indicate how the horse-power, the mechanical efficiency, and the losses vary with the speed.

3. Measurement of Heat Carried Away in the Cooling System. In the case of a water-cooled engine it is necessary to provide means of measuring the quantity of water passing through the cooling system of the engine, and the temperatures of the water at inlet and outlet of the engine.

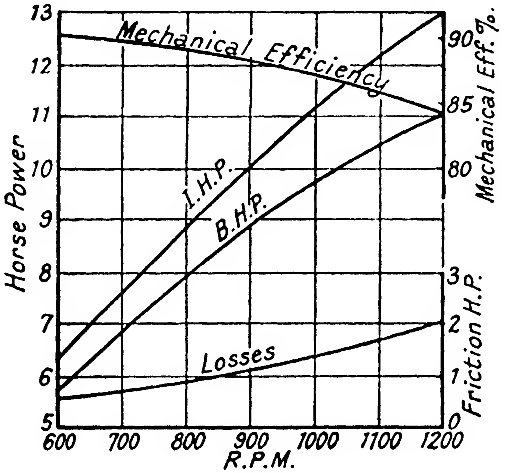

FIG. 115. TEST RESULTS FOR PETROL ENGINE

Then very nearly,

Heat carried away per min. = Weight of water per min. × Rise of Temperature

There is no reliable method of measuring directly the heat carried away by the air-flow over an air-cooled engine, and this quantity must therefore be included in the radiation losses which are estimated by difference between the quantity of heat supplied and the total of the heat quantities measured directly.

4. Measurement of Heat Carried Away by the Exhaust Gases. It is instructive to separate this quantity into (*a*) the heat carried away by the products of combustion made up of the fuel and the minimum quantity of air required for complete combustion, and (*b*) the heat carried away by the excess air.

The minimum quantity of air required may be calculated from the analysis of the fuel (see pp. 195–8). The total quantity of air used by the engine may be measured directly by one of the methods indicated on p. 285. This is not always convenient owing to the lead in of the air to the engine not being adaptable for the attachment of the necessary apparatus. In this case the total air may be calculated from an analysis of the exhaust products (see pp. 205–8 and Example 11, p. 352).

The mean specific heat of the products of combustion will also be required. This can be calculated from a knowledge of the constituent products, by allowing the appropriate proportion of the specific heat of each constituent (see Example 11, p. 352). Then in each case, Heat carried away

$$= \text{Weight of products} \times \text{Specific heat} \times \left(\begin{matrix}\text{Temp. of} \\ \text{Products}\end{matrix} - \begin{matrix}\text{Temp. of} \\ \text{Inlet air}\end{matrix}\right)$$

A direct measurement of the total heat carried away by the exhaust products may be made by means of an exhaust gas calorimeter. This consists of a vessel containing a number of tubes through which water is passing. The products pass round these tubes and are thereby cooled. By measuring the fall in temperature of the products, and the rise in temperature and the quantity of the water, the necessary determination may be made (see Example 10). It is very difficult to design an efficient calorimeter which will not baffle the exhaust, and hence this method is limited in its application.

5. Heat Lost by Radiation. The heat lost by radiation cannot be measured directly, and hence it must be estimated by difference between the total heat supplied and the total heat measured. In the case of an efficient water-cooled engine it is comparatively small. As already mentioned, in the case of an air-cooled engine it will be a much larger quantity as it then includes the total heat rejected from the cylinder walls.

Example 10. In a test on an oil engine the heat in the exhaust gas was measured by means of a calorimeter which consisted of a number of water-cooled tubes; in this the gas in passing through the tubes had its temperature lowered while heating the water. When the engine was developing 11·3 B.H.P., the water absorbed heat at the rate of 107 C.H.U. per min., and the observed gas temperatures were: at exit from the cylinder, 309° C.; at inlet to the calorimeter, 257° C.; and at exit from the calorimeter, 145° C.; and the temperature of the laboratory was 18·6° C. Find the heat in the exhaust gas in C.H.U. per min. above the room temperature, on the assumption that the specific heat of the gas is constant.

The fuel consumption during the test was 5·14 lb. per hr. of oil of calorific value 10,140 C.H.U. per lb., and the cylinder jacket-water was 8·2 lb. per min. with a rise of temperature of 34° C. Make out a heat balance for the test. (*U.L.*, *B.Sc.* (*Eng.*), 1933.)

Let M = Mass of exhaust gas in lb. per min.

C_p = Specific heat of exhaust gas.

Then $$107 = M \times C_p(257 - 145)$$

and $$M \times C_p = \frac{107}{112}$$

Hence Heat in exhaust gas above room temperature

$$= M \times C_p(309 - 18{\cdot}6)$$

$$= \frac{107}{112} \times 290{\cdot}4 = 277{\cdot}4 \text{ C.H.U. per min.}$$

The heat balance is therefore as follows—

	C.H.U. per min.	Percentage of heat supplied
Heat supplied by combustion of fuel $= \frac{5{\cdot}14 \times 10{,}140}{60} = 868{\cdot}7$		
Heat equivalent of B.H.P. $= \frac{11{\cdot}3 \times 33{,}000}{1{,}400} =$	266·4	30·7
Heat rejected in exhaust gases =	277·4	31·9
Heat rejected in cooling water = 8·2 × 34 =	278·8	32·1
Heat equivalent of engine friction, lost by radiation, and unaccounted for	46·1	5·3
	868·7	100·0

Example 11. The following particulars were obtained during a test on a gas engine. Draw up a complete heat account for the engine.

Indicated horse-power	100
Mechanical efficiency	76%
Gas used per hour	1,435 cu. ft. at 15° C., and 6 in. water pressure
Lower calorific value	255 C.H.U. per S.C.F.
Analysis of gas (volume)	CH_4 28·1%, H_2 49·6%, CO 11·3%, CO_2 2·4%, N_2 8·6%
Analysis of exhaust gas (volume)	CO_2 5·9%, N_2 84·3%, O_2 9·8%
Temperature of air	24° C.
Barometer	751 mm.
Dew point	20° C.
Temperature of exhaust	500° C.
Cooling water per hour	2,550 lb.
Cooling water inlet temp.	15° C.
outlet temp.	55° C.

$$\text{Gas used per hour} = 1{,}435 \times \frac{273}{273 + 15} \times \frac{(34 \times 12) + 6}{(34 \times 12)}$$

$$= 1{,}380 \text{ S.C.F.}$$

We will calculate all heat quantities as per standard cubic foot of gas supplied, and reckon them from a temperature of 0° C., and so allow for the fact that the temperatures of the gas and air supplies are not the same. In this connection it must be remembered that the air supply will carry a certain amount of water vapour with it and this represents a certain amount of heat supply to the engine. This quantity can be calculated from a knowledge of the dew point.

Minimum Air required (see p. 196)

1 cu. ft. gas contains (cu. ft.)		O_2 required (cu. ft.)	Products of Combustion (cu. ft.)		
			CO_2	N_2	H_2O
CH_4	0·281	0·562	0·281	—	0·562
CO	0·113	0·0565	0·113	—	—
H_2	0·496	0·248	—	—	0·496
CO_2	0·024	—	0·024	—	—
N_2	0·086	—	—	0·086 3·260 (from air)	—
Total	1·000	0·8665	0·418	3·346	1·058

Minimum air required per cu. ft. gas

$$= 0{\cdot}8665 \times \frac{100}{21} = 4{\cdot}126 \text{ cu. ft.}$$

Nitrogen in this air $= 4{\cdot}126 - 0{\cdot}866 = 3{\cdot}26$ cu. ft.

Dry products $= 0{\cdot}418\ CO_2 + 3{\cdot}346 N_2 = 3{\cdot}764$ cu. ft.

Excess Air used

Let x = the number of cu. ft. of excess air used per cu. ft. gas.

Then the total volume of dry exhaust products

$$= \left(\begin{array}{c}\text{Volume of dry products}\\ \text{from minimum air}\end{array}\right) + \text{Volume of excess air}$$

$$= 3{\cdot}764 + x$$

Hence percentage CO_2 in total volume of dry products

$$= \frac{0{\cdot}418}{3{\cdot}764 + x} = 0{\cdot}059 \text{ (from analysis)}$$

Hence excess air = 3·322 cu. ft. per cu. ft. of gas

$$\text{Total air} = 4{\cdot}126 + 3{\cdot}322$$
$$= 7{\cdot}448 \text{ cu. ft. per cu. ft. of gas}$$

Mean Specific Heat of Gas

A 1 cu. ft. of Gas Contains (cu. ft.)		B Specific Heat C_v from Table II (ft-lb. per S.C.F.)	C Proportion of Specific Heat contributed by Constituent Gas = A × B (ft-lb. per S.C.F.)
CH_4	0·281	25·27	7·101
CO	0·113	19·27	2·177
H_2	0·496	18·99	9·419
CO_2	0·024	26·33	0·632
N_2	0·086	19·22	1·653
			Total 20·982

Hence mean specific heat of gas

= 20·982 ft-lb. per S.C.F.

= 0·015 lb. cal. per S.C.F.

Mean Specific Heat of Exhaust Products

We will consider the products of combustion obtained with the minimum quantity of air necessary for complete combustion.

	A Composition of Products per cu. ft. of Gas (cu. ft.)	B Composition of Products per cu. ft. of Products $= \frac{A}{4 \cdot 822}$ (cu. ft.)	C Specific Heat of Constituent Gas, from Table XVI, col. 2, (ft-lb. per S.C.F.)	D Proportion of C_p contributed by Constituent = B × C (ft-lb. per S.C.F.)
CO_2	0·418	0·087	37·25	3·240
N_2	3·346	0·694	20·6	14·296
H_2O	1·058	0·219	27·1	5·935
Total	4·822	1·000		23·471

Hence the mean specific heat of the exhaust products with the minimum quantity of air, between temperatures of 100° C. and 1,000° C. may be taken as 23·47 ft-lb. per S.C.F. = 0·0168 lb. cal. per S.C.F.

The mean specific heat of the excess air between the same temperatures = 20·6 ft-lb. per S.C.F. = 0·0147 lb. cal. per S.C.F.

Weight of Water Vapour taken into the Engine by the Air Supply

$$\text{Atmospheric pressure} = 14 \cdot 7 \times \frac{751}{760} = 14 \cdot 5 \text{ lb. per sq. in.}$$

Atmospheric temperature = 24° C.

Dew point = 20° C.

Pressure of water vapour at 20° C. (from steam tables)

= 0·34 lb. per sq. in.

Hence pressure of dry air (see p. 505)

= 14·5 – 0·34 = 14·16 lb. per sq. in.

Hence 1 cu. ft. of air at 14·16 lb. per sq. in. and 24° C.

$$= 1 \times \frac{14 \cdot 16}{14 \cdot 7} \times \frac{273}{297} = 0 \cdot 885 \text{ S.C.F.}$$

But 1 lb. of water vapour at 0·34 lb. per sq. in. occupies 922 cu. ft. (from steam tables).

Hence weight of water vapour associated with 1 S.C.F. of air

$$= \frac{1}{922} \times \frac{1}{0{\cdot}885} \text{ lb.}$$

Hence weight of water vapour taken in by the air supply (7·448 lb.) per cu. ft. of gas

$$= \frac{7{\cdot}448}{922 \times 0{\cdot}885} = 0{\cdot}0091 \text{ lb.}$$

The heat carried in 1 lb. of water vapour is the total heat at 0·34 lb. per sq. in. (from steam tables) = 603·7 lb. cal.

Heat Balance

		Lb. cal. per S.C.F. of gas	
Heat supplied by combustion		255	
Heat carried in by gas = 1 × 0·015 × 15	=	0·23	
Heat carried in by air = 7·448 × 0·0147 × 24	=	2·63	
Heat carried in by moisture = 0·0091 × 603·7	=	5·49	
Heat carried in by cooling water = $\frac{2{,}550}{1{,}380} \times 15$	=	27·72	
Heat rejected in cooling water = $\frac{2{,}550}{1{,}380} \times 55$	=		101·63
Heat rejected in exhaust products (minimum air) = 4·822 × 0·0168 × 500	=		40·51
Heat rejected in excess air = 3·322 × 0·0147 × 500	=		24·42
Heat equivalent of B.H.P. $= \frac{0{\cdot}76 \times 100 \times 1{,}414}{1{,}380}$	=		77·87
Heat equivalent of engine friction, etc. $= \frac{(100 - 76) \times 1{,}414}{1{,}380}$	=		24·59
Heat lost by radiation and unaccounted for			22·05
		291·07	291·07

A convenient arrangement of these figures is as follows—

		Lb. cal. per S.C.F. of gas	Percentage of heat supplied
Heat supplied by combustion of fuel		255	
Net heat rejected in exhaust gases = (40·51 + 24·42) - (0·23 + 2·63 + 5·49)	=	56·58	22·2
Net heat rejected in cooling water = 101·63 - 27·72	=	73·91	29·0
Heat equivalent of B.H.P.	=	77·87	30·5
Heat equivalent of engine friction	=	24·59	9·6
Heat lost by radiation and unaccounted for	=	22·05	8·7
		255·00	100·0

As an example of the heat distribution in a high-speed petrol engine the following results of observations, taken with a high degree of accuracy, in tests of the Ricardo variable compression engine, show the distribution of heat in that engine under different conditions of working on the same sample of petrol (*A*). In the first test at compression ratio 3·8 : 1, with constant fuel to air ratio, but varying speed, the results obtained were—

Revolutions per minute	975	1,500	1,700
Piston Speed, feet per minute	1,300	2,000	2,266
Heat converted to indicated horse-power, per cent	25·9	26·1	26·1
Heat lost to cooling water, per cent	30·4	28·0	27·0
Heat lost in exhaust, radiation, etc., per cent.	43·7	45·9	46·9
	100·0	100·0	100·0

Similar tests with ethyl alcohol (95 per cent) give a thermal efficiency up to 38·3 per cent, with a compression ratio 7 : 1.

The second test with petrol (*A*) at compression ratio 5·45: 1, with constant fuel to air ratio, and at constant speed 1,500 r.p.m. and piston speed 2,000 ft. per min., but with power varied by throttling, gave—

Percentage of maximum indicated horse-power	100%	80%	60%	40%
Heat converted to indicated horse-power, per cent	33·5	34·0	34·1	33·5
Heat lost to cooling water, per cent	26·5	28·2	31·8	35·5
Heat lost in exhaust, radiation, etc., per cent.	40·0	37·8	34·1	31·0
	100·0	100·0	100·0	100·0

In both cases the circulating water was at constant temperature 60° C. ± 2° C. The heat input to the carburettor was adjusted to bear a constant proportion to the weight of petrol used. The fuel-air ratio was such as to give nearly 10 per cent of air in excess of that required for complete combustion of the petrol, the air consumption being measured and adjusted in each case. No readings were recorded until all temperature conditions had been steady for a considerable time after each change of working condition.

The heat lost in the exhaust, radiation, etc., was obtained by difference.

The indicated thermal efficiency may be taken as accurate to within about half of 1 per cent, and the heat to the cooling water to within 1 per cent. In both tests the heat produced by piston friction and that lost by radiation balanced at nearly 1,500 r.p.m., the cooling water temperature at which the readings were taken being that which the cylinder attains when motored continuously at 1,500 r.p.m., i.e. 45° C. above atmospheric temperature.

The results of a very valuable series of oil engine trials may be found in the various reports of the Marine Oil Engine Trials Committee. As an example may be quoted that carried out on a Scott-Still Marine Oil Engine, which consists of a combined internal combustion and steam engine, in which heat from the exhaust gases and that usually lost from the combustion of the fuel through the walls of the cylinder liner to the jacket cooling water, is recovered and utilized in the generation of steam, which does work in the steam engine cylinders, and the exhaust steam from these drives a low pressure steam turbine.

This particular installation is only one example of the application of the Still principle of waste heat recovery, and is chosen on account of the very complete and independent tests published.*

The oil engine works on the two-stroke cycle, and automatic ignition of the charge is ensured at the comparatively low compression pressure of 360 lb. per sq. in., owing to the high temperature of the jacket and cylinder walls.

The oil fuel is supplied by airless injection through a spring-loaded valve from a pump operated by a cam, which fixes the timing of the fuel spray injection into the cylinder at a pressure of about 3,500 lb. per sq. in. produced by the pump.

The fuel pump mechanism opens a spill-valve at a variable position of the pump stroke, which is controlled by the governor, thus regulating the period of injection and the quantity of fuel injected. When the spill valve is opened, the delivery of oil fuel from the pump to the cylinder is cut off, and the oil flows back to the supply tank.

The cam rollers are moved sideways by a ram under oil pressure, to engage with either the " ahead " or " astern " cams for reversal.

The exhaust gases from the cylinder pass direct through the small steam generator, or "primary regenerator," fixed on the exhaust pipe of each cylinder, wherein part of the heat carried away in the gases is given to the water as it flows up a group of vertical tubes on its way to the cylinder jacket. After leaving the primary regenerator, the gases from the cylinders enter a common exhaust manifold, and pass along to the water drum of the main regenerator. The water drum contains straight tubes, through which the gases flow and give up more heat. The last quantity of heat is taken from them in passing through a rectangular counter-current feed water heater, which consists of a nest of water tubes. The exhaust gases thus reduced in temperature to about 300° F., finally escape up the funnel to the atmosphere.

The water in the cylinder jacket is at practically constant temperature, corresponding to the steam saturation pressure, about 120 lb. per sq. in. (gauge) in the boiler and regenerators. The heat from the explosion, transmitted through the liner and head of the cylinder

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* *Proc. Inst. Mech. E.*, 1925.

to the water jacket, is absorbed as latent heat of evaporation of the steam generated in the jacket.

The natural circulation of water, from the water drum of the boiler through the primary regenerators and cylinder jackets, and the mixture of water and steam led up a rising main to the steam drum, where the steam is separated, is by thermo-siphon, by the difference of density of the mixture of steam and water.

The steam, separated in the steam drum, passes to the cylinder jacket and through valves, to the underside of the piston in the high pressure cylinder. In a four-cylinder engine, the aft cylinder of the set is high pressure and exhausts into a receiver which supplies the other three cylinders.

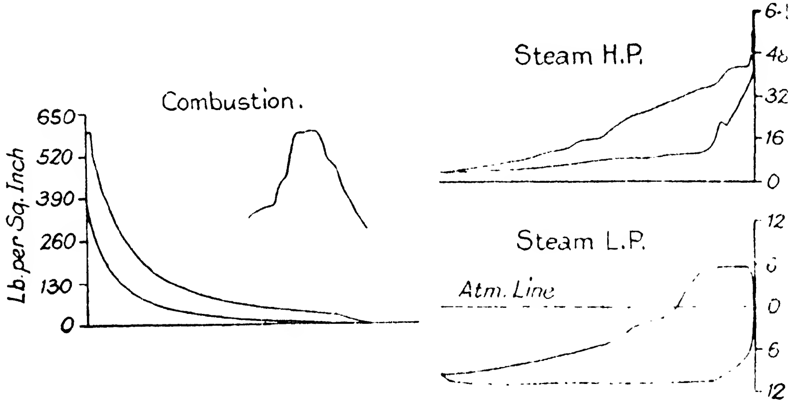

FIG. 116. INDICATOR DIAGRAMS —SCOTT-STILL MARINE OIL ENGINE

The main regenerator is a modified Yarrow type boiler, fitted with oil burners, in the combustion chamber under the water tubes connecting the steam and water drums, to augment the steam supply when more power is required, as well as in starting and manœuvring. To start the engine, the steam raised in this oil-fired boiler passes direct to all the single-acting steam cylinders, which exhaust through the turbine to condenser.

The rating of each main engine is 1,250 B.H.P. at 120 r.p.m., the power being developed in four cylinders, working on the two-stroke single-acting cycle. The dimensions of the port engine are : stroke, 36 in. ; bore of each combustion, and steam cylinder, 22 in. ; piston-rods, 6·25 in. ; and ratio of compression, 8·56. The fuel was Anglo-American Diesel oil, of specific gravity 0·864 at 59° F. ; flash point, 176° F. ; burning point, 198° F. ; and higher calorific value, 19,500 B.Th.U. per lb. Ultimate analysis : Carbon, 86·09 ; H_2, 12·95 ; S, 0·4 ; O_2, 0·52 ; and N_2, 0·04 per cent.

In the tests at full rated power, of 4 hours' duration, at average speed 121·9 r.p.m., the indicator diagrams, Fig. 116, were taken.

The combustion is partly at constant volume and partly at constant pressure, on the dual cycle.

Results obtained from the *combustion cylinders*—

Pressure at end of compression	355 lb. per sq. in.
Pressure at maximum explosion	582 ,, ,,
Pressure at end of expansion	39·3 ,, ,,
Mean indicated pressure, average of all cylinders	76·6 ,, ,,
Indicated horse-power (combustion only)	1,290
Thermal equivalent of indicated horse-power for 1 min.	54,700 B.Th.U.

On the *steam side*—

Temperature at exit from water-drum	344° F.
Pressure in steam drum	115·8 lb. per sq. in.
Pressure at admission, high pressure cylinder	47·5 ,, ,,
Pressure at admission, low pressure cylinders	4·8 ,, ,,
Pressure at exhaust (below atmosphere)	− 10·6 ,, ,,
Average, mean indicated pressure of all cylinders	6·9 ,, ,,
Indicated horse-power (steam only)	116
Thermal equivalent of indicated horse-power for 1 min.	4,920 B.Th.U.
Total indicated horse-power, combustion and steam	1,406

Work done by exhaust steam from one engine on turbo-blower (intended for both sets of engines)—

Heat drop in turbine per pound of steam	88 B.Th.U.
Effective brake horse-power by Heenan & Froude absorption dynamometer	1,271
Brake mean effective pressure	75·4 lb. per sq. in.

$$\text{Mechanical efficiency} = \frac{\text{Brake horse-power}}{\text{Total indicated horse-power}} = \frac{1{,}271}{1{,}406}$$
$$= 90{\cdot}4 \text{ per cent}$$

Oil fuel consumption per brake horse-power hour = 0·352 lb.
Oil fuel consumption per indicated horse-power = 0·322 lb.

a remarkably good result.

In order to enable comparisons to be made with the performance of other engines, and to subdivide the thermal equivalents of the work done in the Thermal Balance Sheets, the combustion side of the engine is debited by the Committee with all the frictional resistances, and so may be said to give at the brake a quantity called the "combustion brake horse-power." Then the effect of the assistance of the steam is to add all its power (indicated horse-power steam) to the combustion brake horse-power to make up the observed effective brake horse-power.

Thus, Friction H.P. = 1,406 − 1,271 = 135 H.P.
and Combustion B.H.P. = 1,290 − 135 = 1,155 H.P.
Effective B.H.P. = 1,155 + Steam I.H.P. 116 = 1,271

Thermal equivalent of effective B.H.P. for 1 min. is 53,910 B.Th.U.

Thermal efficiency on total I.H.P. (not including the L.P. turbine) = 40·5 per cent

Thermal efficiency on effective B.H.P. = 37·1 per cent.

$$\text{Efficiency ratio} = \frac{\text{Thermal efficiency on total I.H.P.}}{\text{Efficiency of ideal engine (50 per cent)}} = 0{\cdot}81$$

Thermal Balance Sheet, from Test No. 10 of the Marine Oil Engine Trials Committee (Second Report)—

Fuel supplied—0·352 lb. per brake horse-power hour		100 per cent
Distribution of Heat—	Per cent	
Effective brake horse-power, 1,271 at 121·9 r.p.m. (Combustion B.H.P. 1,155 + steam I.H.P. 116)	37·12	
Power for scavenger blower from L.P. turbine .	2·37	
Friction	3·93	
Total indicated horse-power . . .		43·42 per cent

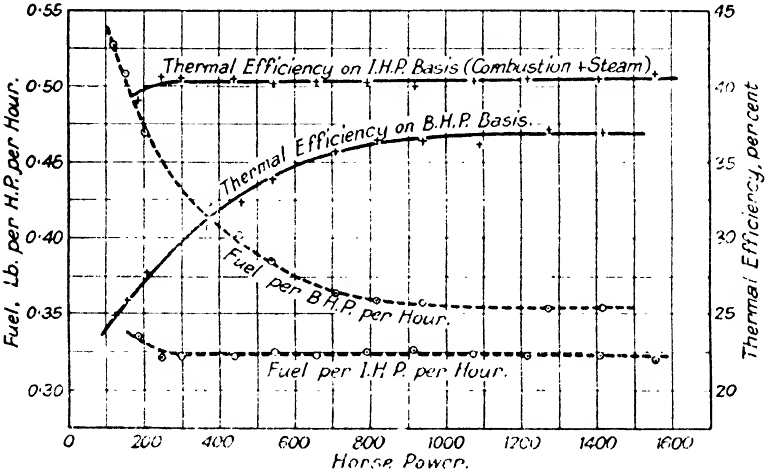

FIG. 117. FUEL CONSUMPTION AND THERMAL EFFICIENCY CURVES (SCOTT-STILL REGENERATIVE OIL ENGINE)

Heat Losses—		
Finally carried away in exhaust gases . .	23·07	
Rejected in condenser cooling water . .	25·82	
Radiation	7·69	56·58 per cent
Heat in fuel.	100·00	

The indicated power is made up as follows—

Combustion indicated horse-power	37·67 per cent
Regenerated steam indicated horse-power . .	3·38 „
Regenerated steam in L.P. turbine (heat drop) . .	2·37 „
Total indicated horse-power	43·42 per cent

The power developed by the low pressure steam turbine is obtained from the fuel burned in the combustion cylinder, and is here added to the indicated power of the engine, as part of that from the steam generated by the waste heat. This makes the total indicated horse-power 1,457, and the rate of fuel consumption 0·31 lb. per indicated horse-power hour, increasing the thermal efficiency to 42·1 per cent, instead of 40·5 per cent (see Fig. 117).

The mechanical efficiency becomes 87·2 per cent.

The Gas Turbine. The essential difference between the reciprocating internal combustion engine and the gas turbine, apart from the difference in reciprocating and rotary motion, lies in the fact that while in the former, combustion, expansion and compression take place in one and the same cylinder, in the latter each of these operations is produced in a separate component.

Thus the main operations consist of (*a*) the compression of cool air in a rotary compressor, (*b*) the heating of this air by the combustion of fuel in a combustion chamber, and (*c*) the expansion of this hot high pressure gas in a turbine. To these main components may be added heat exchangers, heaters and coolers, so that in dealing with the thermodynamics of a gas turbine system we have to deal with the efficiencies of the various components, and their effect on the overall efficiency of the system. Limitations of space confine our treatment of the subject to the basic principles underlying some of the systems in use.

The Open Circuit, Constant Pressure System. The system is illustrated in Figs. 118, 119, 120 in which polytropic compression and expansion with no cooling, but with internal friction, is assumed as in the paragraphs on compressors (p. 162) and the air motor (p. 170).

For simplicity it is also assumed that the compressor and turbine work between the same pressures.

Air from the atmosphere is drawn into the compressor at a temperature T_a and a pressure p_1 and compressed to a pressure p_2 and a temperature T_b. As already explained (p. 162) the actual pressure-volume curve *ab* will be steeper than the ideal isentropic compression curve *ab'*, while the corresponding T-ϕ curves with corresponding lettering are shown in Fig. 120. This air now passes through a heat exchanger, and takes up heat from the turbine exhaust which would otherwise be wasted, thus raising the temperature to some value T_x. The air now enters the combustion chamber, where it receives further heat from the combustion of the fuel (assumed at constant pressure p_2) the products of combustion rising in temperature to T_c in which condition they enter the turbine. The products expand in the turbine along the actual expansion curve *cd*, the ideal isentropic expansion curve being *cd'*. The products leave the turbine at T_d and pass through the heat exchanger to exhaust.

Owing to the heat produced from the combustion of the fuel, the work delivered, by the expansion in the turbine, to the turbine shaft is greater than that absorbed by the compressor, the balance being available for the exterior load.

If the weight of fuel is neglected, and the properties of air and the combustion products are assumed the same, then from pp. 171 and 163 we have—

Turbine work $= C_p(T_c - T_d)$ and compressor work $= C_p(T_b - T_a)$ in heat units per pound of air, if C_p is in heat units.

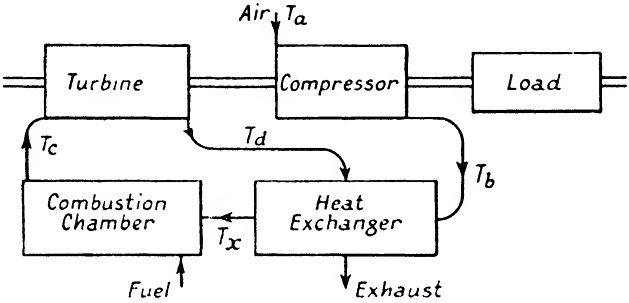

FIG. 118 SYSTEM

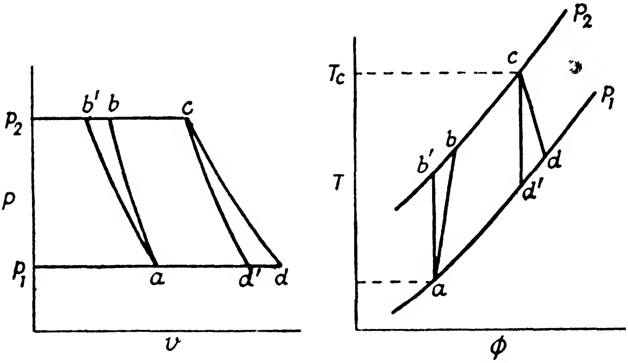

FIG. 119. p-v DIAGRAM FIG. 120. T-ϕ DIAGRAM

Net work available $= C_p[(T_c - T_d) - (T_b - T_a)]$ heat units per lb.

Heat supplied from fuel $= C_p(T_c - T_x)$ heat units per lb.

The heat available from the turbine exhaust is $C_p(T_d - T_b)$ as obviously the lowest possible temperature of the exhaust is T_b.

Let it be assumed that the fraction y of this heat is put back into the air leaving the compressor.

Then heat saved in the heat exchanger $= yC_p(T_d - T_b)$

where $yC_p(T_d - T_b) = C_p(T_x - T_b)$ and hence $T_x = T_b + y(T_d - T_b)$

Hence

$$\text{Heat supplied from fuel} = C_p(T_c - T_x)$$
$$= C_p[(T_c - T_b) - y(T_d - T_b)] \text{ per lb.}$$

$$\text{and efficiency of system} = \frac{\text{Net work available}}{\text{Heat supplied from fuel}}$$
$$= \frac{(T_c - T_d) - (T_b - T_a)}{(T_c - T_b) - y(T_d - T_b)} \quad . \quad (1)$$

In the absence of a heat exchanger the efficiency of the system would be

$$\frac{(T_c - T_d) - (T_b - T_a)}{(T_c - T_b)} \quad . \quad . \quad . \quad (2)$$

If in addition the compression and expansion were isentropic then

$$\text{Efficiency of system} = \frac{(T_c - T_{d'}) - (T_{b'} - T_a)}{(T_c - T_{b'})}$$
$$= \frac{(T_c - T_{b'}) - (T_{d'} - T_a)}{(T_c - T_{b1})} \quad . \quad . \quad . \quad (3)$$

This it will be seen is the efficiency of the Joule Air Cycle.

In the equations given above, the temperatures which would probably be fixed are T_a the compressor inlet air temperature and T_c the maximum allowable gas temperature at the turbine inlet. If the compressor and turbine efficiencies and the pressure ratio r are known, the other temperatures can be determined as follows.

$$\text{Isentropic turbine efficiency} = \varepsilon_t = \frac{T_c - T_d}{T_c - T_{d'}}$$

i.e.
$$T_c - T_d = \varepsilon_t(T_c - T_{d'}) = \varepsilon_t T_c\left(1 - \frac{T_{d'}}{T_c}\right)$$
$$= \varepsilon_t T_c\left[1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\therefore \quad T_d = T_c\left[1 - \varepsilon_t\left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)\right] \quad . \quad . \quad (4)$$

$$\text{Isentropic compressor efficiency} = \varepsilon_c = \frac{T_{b'} - T_a}{T_b - T_a}$$

i.e.
$$T_b - T_a = \frac{1}{\varepsilon_c}(T_b - T_a) = \frac{T_a}{\varepsilon_c}\left(\frac{T_{b'}}{T_a} - 1\right) = \frac{T_a}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$\therefore \quad T_b = T_a\left[1 + \frac{1}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right] \quad . \quad . \quad . \quad . \quad (5)$$

Specific Heats and Weight of Fuel. In the simple treatment given above, no account has been taken of the weight of fuel injected, and of the difference of the specific heats of air and the products after the fuel combustion.

Let w_f = weight of fuel per lb. of air used

C_m = specific heat of products and C_a = specific heat of air

Then the expressions in the last paragraph become—

Turbine work $= (1 + w_f)C_m(T_c - T_d)$

Compressor work $= C_a(T_b - T_a)$

Heat supplied (approx.) $= C_m(1 + w_f)(T_c - T_b) - C_a y(T_d - T_b)$ with heat exchanger

$= C_m(1 + w_f)(T_c - T_b)$ without heat exchanger

all expressions being in heat units per pound of air used.

Example 12. An open circuit constant pressure gas turbine plant works on a pressure ratio of 5. The inlet air temperature is 60° F. and the maximum allowable temperature in the turbine is, 1,000° F. The turbine isentropic efficiency is 87 per cent and that of the compressor is 85 per cent.

What is the overall efficiency of the plant (*a*) if no heat exchange is used, (*b*) with a heat exchange returning 60 per cent of the available heat to the system. Assume $\gamma = 1{\cdot}4$.

$$r^{\frac{\gamma-1}{\gamma}} = 5^{0\cdot286} = 1{\cdot}584 \text{ and } \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} = 0{\cdot}631$$

$$\therefore \quad T_d = T_c\left[1 - \varepsilon_t\left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)\right] = 1{,}460[1 - 0{\cdot}87(1 - 0{\cdot}631)]$$

$$= 991° \text{ F. abs.}$$

$$T_b = T_a\left[1 + \frac{1}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right] = 520\left[1 + \frac{1}{0{\cdot}85}(1{\cdot}584 - 1)\right]$$

$$= 877° \text{ F. abs.}$$

$$(a) \text{ Efficiency} = \frac{(T_c - T_d) - (T_b - T_a)}{T_c - T_b}$$

$$= \frac{(1{,}460 - 991) - (877 - 520)}{1{,}460 - 877}$$

$$= 0{\cdot}193$$

$$(b) \text{ Efficiency} = \frac{(T_c - T_d) - (T_b - T_a)}{(T_c - T_b) - y\,(T_d - T_b)}$$

$$= \frac{(1{,}460 - 991) - (877 - 520)}{(1{,}460 - 877) - 0{\cdot}6(991 - 877)}$$

$$= 0{\cdot}218$$

Effect of Compressor and Turbine Efficiences on Plant Efficiencies. An examination of the equations given in the last paragraph will show that if the compressor and turbine efficiencies are below certain values, there will be no net work available from the system. Thus in Eq. 1 (p. 363) for the net work to be zero $T_c - T_d = T_b - T_a$. Substituting the values of T_d and T_b from Eqs. 4 and 5 we have

$$T_c\,\varepsilon_t\left[1-\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right] = \frac{T_a}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}}-1\right)$$

or

$$\varepsilon_t\varepsilon_c = \frac{T_a}{T_c}\cdot\frac{r^{\frac{\gamma-1}{\gamma}}-1}{1-\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= \frac{520}{1{,}460}\cdot\frac{1{\cdot}584-1}{1-0{\cdot}631} = 0{\cdot}565$$

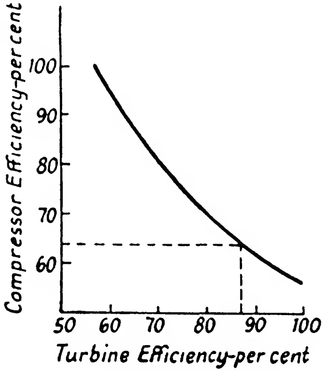

FIG. 121

Fig. 121 shows values of ε_t and ε_c plotted from this equation and indicates that in order that there shall be a positive amount of net work for the system, values of ε_t and ε_c must be taken from the right-hand side of the curve. Thus, in the previous example, if the turbine efficiency is 87 per cent, the minimum necessary efficiency for the compressor is 64 per cent.

Effect of Compression Ratio on Work Done and Efficiency. This is best shown by means of an example, so referring to Fig. 120

let $T_a = 520°$ F. abs. and $T_c = 1{,}460°$ F. abs.

$\varepsilon_t = 0{\cdot}87$ and $\varepsilon_c = 0{\cdot}85$

Specific heat $C_m = C_a = 0{\cdot}25$ and $\gamma = 1{\cdot}4$

For simplicity ignore the weight of fuel compared to that of air. Then for a compression ratio of 2—

$$r^{\frac{\gamma-1}{\gamma}} = 2^{0\cdot 286} = 1\cdot 219 \text{ and } \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} = 0\cdot 82$$

$$T_d = T_c\left[1 - \varepsilon_t\left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)\right] = 1{,}460\,[1 - 0\cdot 87\,(1 - 0\cdot 82)]$$

$$= 1{,}230° \text{ F. abs.}$$

Turbine work $= 0\cdot 25(T_c - T_d) = 0\cdot 25(1{,}460 - 1{,}230)$

$= 57\cdot 5$ B.Th.U. per lb. air

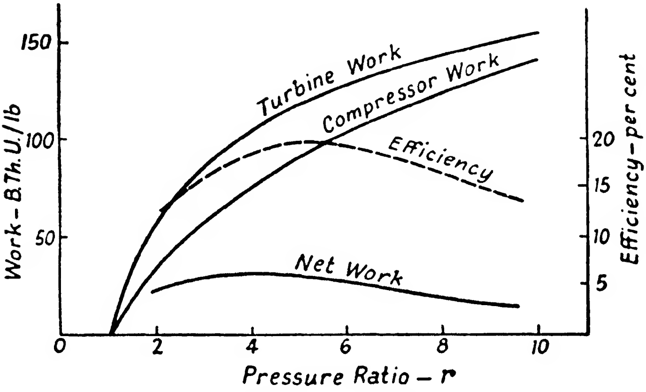

FIG. 122. EFFECT OF PRESSURE RATIO

$$T_b = T_a\left[1 + \frac{1}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right] = 520\left[1 + \frac{1}{0\cdot 85}\,(1\cdot 219 - 1)\right]$$

$$= 653° \text{ F. abs.}$$

Compressor work $= 0\cdot 25(T_b - T_a) = 0\cdot 25(653 - 520)$

$= 33\cdot 3$ B.Th.U. per lb. air

Net work $= 57\cdot 5 - 33\cdot 3 = 24\cdot 2$ B.Th.U. per lb. air

Heat supplied by combustion assuming no heat exchange

$= 0\cdot 25(T_c - T_b)$

$= 0\cdot 25(1{,}460 - 653)$

$= 201\cdot 8$ B.Th.U. per lb. air

Efficiency of system $= \dfrac{24\cdot 2}{201\cdot 8} = 0\cdot 12$

If this is repeated for various values of the pressure ratio r as below, the curves in Fig. 122 will be obtained,.

r	2	4	6	8	10
$r^{0{\cdot}286}$	1·219	1·486	1·699	1·811	1·932
$\left(\frac{1}{r}\right)^{0{\cdot}286}$	0·820	0·673	0·600	0·552	0·517
$1-\left(\frac{1}{r}\right)^{0{\cdot}286}$	0·180	0·327	0·400	0·448	0·482
$\varepsilon_t\left[1-\left(\frac{1}{r}\right)^{0{\cdot}286}\right]$	0·157	0·284	0·348	0·390	0·419
$1-\varepsilon_t\left[1-\left(\frac{1}{r}\right)^{0{\cdot}286}\right]$	0·843	0·716	0·652	0·610	0·581
$T_d = T_c\left[1-\varepsilon_t\left(1-\left(\frac{1}{r}\right)^{0{\cdot}286}\right)\right]$	1,230	1,045	951	893	850
$W_e = 0{\cdot}25\,(T_c - T_d)$	57·5	103·7	127·2	141·7	152·2
$r^{0{\cdot}286} - 1$	0·219	0·486	0·669	0·811	0·932
$1 + \frac{r^{0{\cdot}286} - 1}{\varepsilon_c}$	1·257	1·572	1·786	1·955	1·092
$T_b = T_a\left[1 + \frac{r^{0{\cdot}286} - 1}{\varepsilon_c}\right]$	652	818	928	1,016	1,080
$W_c = 0{\cdot}25\,(T_b - T_a)$	33	74·5	102	124	140
Net work $= W = W_t - W_c$	24·5	29·2	25·2	17·7	12·2
Heat supplied $= 0{\cdot}25\,(T_c - T_b)$	202	160	133	111	95
Efficiency $= \frac{W_t - W_c}{\text{Heat in}}$	0·121	0·184	0·189	0·159	0·128

Multi-stage Compression and Expansion. It has been shown (p. 134) that the efficiency of the Ericsson Cycle is equal to that of the Carnot Cycle between the same extreme temperatures. This is owing to the fact that the heat transfers are made during the isothermal compression and expansion only (Paths *ab* and *cd*, Fig. 123), the heat transferred from the regenerator to the air along the constant pressure line *bc* (Fig. 123) being returned to the regenerator along the path *da*.

Fig. 124 shows how multi-stage compression with intercooling can approximate to isothermal compression if the number of stages is sufficiently numerous. Thus the first-stage isentropic compression would be along the path aB, the intercooling at constant pressure to the original temperature being represented by $Ba_{,}$, so that $Ta_{,} = T_a$. The next stage compression would be $a_{,}B_{,}$ and so on. It is obvious therefore that as the number of stages increases, the

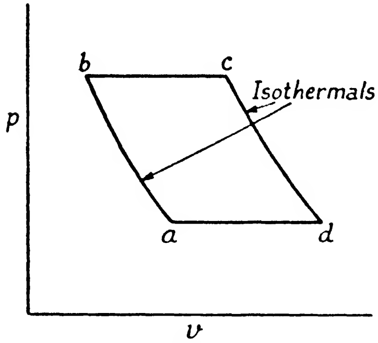

FIG. 123. ERICSSON CYCLE

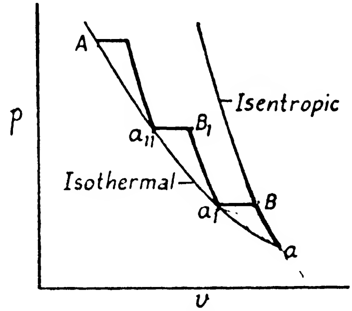

FIG. 124. MULTI-STAGE COMPRESSION

serrated line aBa_1B_1, etc., approximates more and more to the smooth isothermal curve aA. The same argument applies to an expansion curve with the substitution of reheating for intercooling.

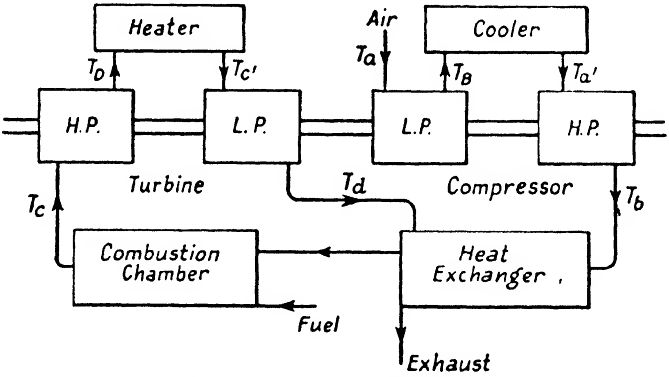

FIG. 125 MULTI-STAGE SYSTEM

The arrangement as applied to the open circuit system is shown in Fig. 125, while Fig. 126 shows the pv diagram. In this figure, the thick lines indicate the actual path of the working substance, the thin lines are isentropics and the broken lines are isothermals.

The process through which the working substance passes is as follows. Air at a pressure p_1 and temperature T_a enters the low pressure compressor and is compressed to a pressure p' and a temperature T'_B, and passing through the intercooler is cooled to its original temperature T_a along the path Ba'. In passing through the high pressure compressor, its pressure is raised along the path $a'b$ to a pressure p_2 the temperature rising to T_b. After passage at constant pressure p_2 through the heat exchanger and combustion chamber, the temperature is raised to T_c. In the high pressure turbine the gas expands along the path cD to the pressure p' and the temperature T'_D. In the passing through the reheater at constant pressure, the temperature is raised along the path Dc' to its original temperature T_c after which expansion in the low pressure turbine along $c'd$ reduces the pressure to p_1 and the temperature to T'_d. The gas then passes through the heat exchanger to exhaust.

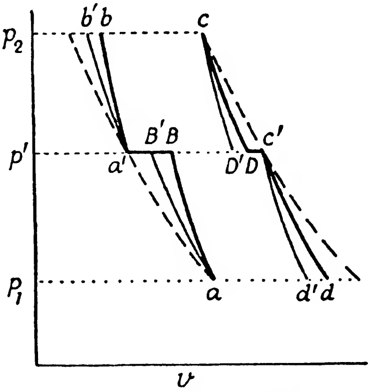

FIG. 126. p-v DIAGRAM

In order to obtain an expression for the efficiency of the system, let the pressure ratio be the same for each stage of compression or expansion so that $r = \frac{p_2}{p'} = \frac{p'}{p_1}$

Hence from Fig. 126—

As a and a' are on the same isothermal aa' then $T_{a'} = T_a$

As c and c' are on the same isothermal cc' then $T_{c'} = T_c$

Assuming the laws for the two compressions aB and $a'b$ are the same, then $T_b = T_B$

Similarly for the two expansions cD and $c'd$ $T_d = T_D$

For the two isentropic compressions aB' and $a'b'$ $T_{b'} = T_{B'}$

For the two isentropic expansions cD' and $c'd'$ $T_{d'} = T_{D'}$

A comparison with the equations on pp. 262 and 263 will indicate the following, all expressions being in terms of heat units per pound of air.

Work done in each turbine stage

$$= C_p(T_c - T_D)$$

Work done in each compressor stage

$$= C_p(T_B - T_a)$$

Net work $= C_p[(T_c - T_D) - (T_B - T_a)]$

Heat supplied by fuel

$$= C_p[(T_c - T_b) - y(T_d - T_b)]$$

$$= C_p[(T_c - T_B) - y(T_D - T_B)]$$

Heat supplied by reheater

$$= C_p(T_c - T_D)$$

Efficiency of system

$$= \frac{2C_p[(T_c - T_D) - (T_B - T_a)]}{C_p[(T_c - T_B) - y(T_D - T_B) + (T_c - T_D)]}$$

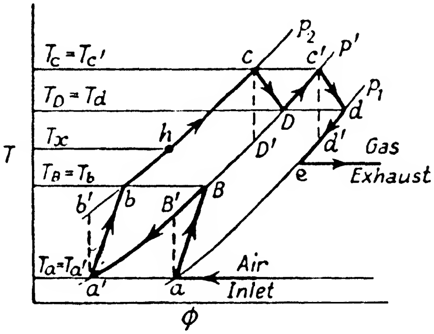

FIG. 127. T-ϕ DIAGRAM MULTI-STAGE SYSTEM

The temperatures T_B and T_D may be expressed in terms of T_a and T_c and the turbine and compressor efficiencies as follows.

$$\text{Since } \varepsilon_c = \frac{T_{B'} - T_a}{T_B - T_a} \text{ then } T_B = T_a\left[1 + \frac{1}{\varepsilon_c}\left(\frac{T_{B'}}{T_a} - 1\right)\right]$$

$$= T_a\left[1 + \frac{1}{\varepsilon_c}\left(r^{\frac{\gamma - 1}{\gamma}} - 1\right)\right]$$

$$\text{Since } \varepsilon_t = \frac{T_c - T_D}{T_c - T_{D'}} \text{ then } T_D = T_c\left[1 - \varepsilon_t\left(1 - \frac{T_{D'}}{T_c}\right)\right]$$

$$= T_c\left[1 - \varepsilon_t\left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma - 1}{\gamma}}\right)\right]$$

The temperature-entropy diagram for this cycle is shown in Fig. 127 with corresponding lettering for comparison with the pressure-volume diagram Fig. 126. It should be noticed that the path *bh* on the T-ϕ diagram represents the heat absorbed by the air on passing through the heat exchanger, and the path *hc* represents the heat produced by the combustion of the fuel. The path *de* represents the heat saved from the exhaust from the turbine.

Example 13. In a two-stage compression and expansion gas turbine system with exhaust heat recovery the pressures concerned are 1, $\sqrt{5}$, and 5 atmospheres respectively. The inlet air temperature is 60° F. and the maximum permissible gas temperature is 1,000° F. The isentropic efficiencies of turbine and compressor are 87 per cent and 85 per cent respectively, while 60 per cent of the available heat in the turbine exhaust is recovered in the heat exchanger. Determine the efficiency of the system. Assume $\gamma = 1{\cdot}4$.

For each stage $r = \sqrt{5}$

$$\therefore \qquad r^{\frac{\gamma-1}{\gamma}} = 5^{\frac{0{\cdot}286}{2}} = 1{\cdot}259$$

and

$$\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}} = 0{\cdot}794$$

$$\therefore \qquad T_B = T_a\left[1 + \frac{1}{\varepsilon_c}\left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right]$$

$$= 520\left[1 + \frac{1}{0{\cdot}85}(1{\cdot}259 - 1)\right] = 678{\cdot}6^\circ \text{ F. abs.}$$

$$T_D = T_c\left[1 - \varepsilon_t\left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)\right]$$

$$= 1{,}460[1 - 0{\cdot}87(1 - 0{\cdot}794)] = 1{,}199^\circ \text{ F. abs.}$$

Efficiency of system

$$= \frac{2\,[(T_c - T_D) - (T_B - T_a)]}{(T_c - T_B) - y(T_D - T_B) + (T_c - T_D)}$$

$$= \frac{2[(1{,}460 - 1{,}199) - (679 - 520)]}{(1{,}460 - 679) - 0{\cdot}6(1{,}199 - 679) + (1{,}460 - 1{,}199)}$$

$$= 0{\cdot}279$$

A comparison of this result with those in Ex. 12, p. 364, shows the effect on the plant efficiency, of the heat exchange, and of compound compression, and expansion with intercooling and reheating, if the pressure ratio and the extreme temperatures are the same.

With no heat exchanger, efficiency is 19·3 per cent
With heat exchanger, efficiency is 21·8 per cent
With heat exchanger with compound
compression, etc., efficiency is 27·9

The Closed Cycle (Escher Wyss). In the closed cycle (Fig. 128) compressed air leaves the compressor *C*, and passes via the heat exchanger *HE* through tubes in the air heater *H* which are heated from some external source. The hot high pressure air then passes through the turbine *T* and then through the heat exchanger. As the air is still at a high temperature when leaving the heat exchanger, it is cooled in a pre-cooler *PC* before passing to the compressor intake.

There are several advantages of this closed circuit. The air is not contaminated by the combustion, and hence fouling of the heat exchange tubes and the turbine blades is obviated. The intake pressure to the compressor can be above atmospheric thus raising all the pressures in the circuit. This enables smaller machines to be used, which again means lower centrifugal stresses on the compressor and turbine blades. All forms of fuel can be used.

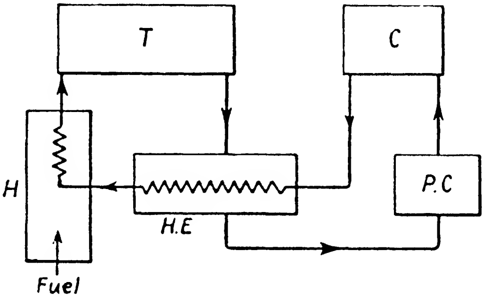

FIG. 128. CLOSED CYCLE

A disadvantage is that large heating surfaces are required in the air heater and pre-cooler which enhance the initial cost, so counterbalancing the reduced cost in other directions.

EXAMPLES VIII

1. Prove that the ideal efficiency of an internal combustion engine working on the Otto cycle is $1 - \left(\frac{1}{r}\right)^{\gamma - 1}$, where r is the ratio of compression. Calculate this efficiency in the case of an engine having a stroke 16 in., a piston diameter 12 in., and a clearance volume of 485 cu. in.; and find the gas consumption per indicated horse-power hour if the gas has a calorific value of 260 C.H.U. per cu. ft., and the efficiency ratio of the engine is 56 per cent.

(*U.L., B.Sc. (Eng.)*, 1923.)

2. A gas engine, working on the Otto cycle, has clearance 0·25 cu. ft. and the volume swept by piston 1·25 cu. ft. From the indicator cards the compression curve is $p \, . \, v^{1\cdot35}$ = constant, and the expansion $p \, . \, v^{1\cdot3}$ = constant. When the compression stroke begins, the charge is at 14 lb. per sq. in. absolute and 100° C. The maximum pressure of explosion at constant volume is 400 lb. per sq. in. (abs.), and at 110 explosions per minute the engine takes in 10 cu. ft. per min. of gas, having lower calorific value 275 C.H.U. per cu. ft. as used.

Calculate: (*a*) the net indicated work per cycle; (*b*) the indicated horse-power; (*c*) the thermal efficiency; and (*d*) the efficiency relative to that of the ideal air cycle for the engine ($\gamma = 1\cdot4$).

3. An air engine works on an ideal cycle in which heat is received at constant pressure and rejected at constant volume. The pressure at the end of the suction stroke is 14 lb. per sq. in. (abs.); the ratio of compression is 15·3,

and the ratio of expansion is 7·5. If the expansion and compression curves are given by $p \,.\, v^{1\cdot 4}$ = constant, find the mean pressure for the cycle.

An oil engine working on this cycle, and with the above expansion and compression ratios, and initial pressure, gives a mean pressure of 114 lb. per sq. in. Account for the difference. (*U.L.*, *B.Sc.* (*Eng.*).)

✓ 4. Calculate the ideal efficiency of a Diesel engine with a compression ratio 14 and fuel cut-off at 1/20th stroke. If the efficiency ratio for the engine is 0·72 at full load find the oil consumption per horse-power hour when using oil of calorific value 10,000 C.H.U. per lb.

What change in efficiency ratio would you expect as the load on the engine is reduced? (*U.L.*, *B.Sc.* (*Eng.*), 1927.)

5. The cycle of an engine consists of three stages: an isothermal compression, an increase of pressure at constant volume, and an adiabatic expansion. If r is the ratio of expansion and compression, show that the efficiency is—

$$1 - \frac{R \log_\epsilon r}{J \,.\, C_v(r^{\gamma - 1} - 1)}$$

In such a cycle air is supplied at 15 lb. per sq. in. and 27° C., and compressed to 75 lb. per sq. in. $C_p = 0{\cdot}238$; $C_v = 0{\cdot}17$.

Find, per lb. of air, the heat supplied, in C.H.U., and the work done, in ft-lb. per cycle. (*U.L.*, *B.Sc.* (*Eng.*), 1934.)

✓ 6. Calculate the stroke and diameter of a gas engine which is to develop 15 H.P. at 300 r.p.m. Assume a compression ratio of 4·5, adiabatic compression, a pressure at the end of explosion of 350 lb. per sq. in., and an expansion index of 1·3. The stroke is to be 1·5 times the diameter. (*U.L.*, *B.Sc.* (*Eng.*), 1923.)

✓ 7. A gas engine has to develop 20 B.H.P. with a mechanical efficiency of 80 per cent, when there are 80 explosions per min. Determine the necessary diameter of the cylinder if the stroke is 2 ft. The equations to the expansion and compression curves are of the form $pv^{1\cdot 25}$ = constant; the clearance is one-fourth of the volume swept by the piston, and the maximum pressure after ignition is three times the compression pressure. (*U.L.*, *B.Sc.* (*Eng.*).)

8. The swept volume of a mixed cycle oil engine is 805 cu. in. and the clearance volume 70 cu. in. An indicator diagram shows a mean effective pressure of 91 lb. per sq. in. and that the pressure at the end of a compression is 385 lb. per sq. in., the maximum pressure is 570 lb. per sq. in. and cut-off of fuel occurs at 4 per cent of the stroke. Assuming that compression begins at the beginning of the stroke and that the pressure and temperature of the cylinder charge is then 14 lb. per sq. in. and 90° C., find the temperature (*a*) at the end of compression, (*b*) after heat addition at constant volume, (*c*) at cut-off after heat addition at constant pressure. Neglect molecular contraction.

Assuming constant specific heats at constant volume and constant pressure of 0·18 and 0·25 respectively, estimate the heat supplied per cycle to the point of cut-off and find the fraction of this heat which appears as indicated work.

Why would the actual thermal efficiency of the engine be much lower than is given by this estimate? (*U.L.*, *B.Sc.* (*Eng.*), 1931.)

9. A single-cylinder oil engine working on the four-stroke cycle is to develop 40 B.H.P. when running at 250 r.p.m. The fuel supplied has a calorific value of 18,500 B.Th.U. per lb., and contains 85 per cent of carbon and 15 per cent of hydrogen by weight. The following efficiencies may be assumed: thermal efficiency on B.H.P., 30 per cent; mechanical efficiency, 85 per cent; volumetric efficiency, 80 per cent. The air temperature is 70° F., and the pressure

14·2 lb. per sq. in.; and the weight of excess air supplied is to be 80 per cent of that theoretically required for perfect combustion. Calculate the cylinder diameter, assuming a stroke-bore ratio of 2 : 1. (*I.Mech.E.*, 1932.)

10. Derive a general expression for the change of entropy taking place in a mass of gas when it expands.

In a gas engine the compression ratio was 6 : 1. The temperature and pressure at the beginning of compression were 80° C. and 14 lb. per sq. in. The index of compression was 1·32, and combustion took place approximately at constant volume, the maximum pressure attained being 360 lb. per sq. in. The specific heats of the mixture at constant pressure and at constant volume can be taken as 0·25 and 0·18 respectively.

Find the change of entropy per lb. (*a*) during compression, (*b*) during combustion. (*U.L.*, *B.Sc.* (*Eng.*), 1932.)

11. A six-cylinder petrol engine, 100 mm. diameter by 100 mm. stroke, running at 1,500 revolutions per minute uses a mixture of air to petrol by weight of 13·5 to 1. Assuming that the air drawn into the cylinder per stroke and measured at atmospheric pressure and 80° C. is equal to $\frac{7}{8}$ths of the swept cylinder volume, and that the thermal efficiency of the engine is 22 per cent, find the power developed at ground level where the barometer reads 30 in. of mercury. The calorific value of the petrol is 9,000 C.H.U. per lb.

What would be the power developed at an altitude of 5,000 ft.? A drop of 1 in. in barometric reading may be assumed for each 900 ft. rise in altitude. (*U.L.*, *B.Sc.* (*Eng.*), 1934.)

12. Show by means of curves the general manner in which the power and efficiency of a petrol engine vary with the mixture strength, and indicate the reasons for these variations.

A four-cylinder petrol engine has cylinders 2·48 in. diameter by 4·72 in. stroke and runs at 1,000 r.p.m. The petrol consumption is 7 lb. per hour, and the volumetric efficiency of the engine may be taken as 80 per cent. Find the percentage by which the mixture is stronger or weaker than the theoretical mixture, assuming that the petrol has the following composition (by weight): H 15 per cent, C 85 per cent. Air contains 23 per cent of oxygen by weight, and the volume of 1 lb. of air in the condition under which the volumetric efficiency is given is 13 cu. ft. (*I.Mech.E.*, 1928.)

13. Describe the difficulties that are encountered in obtaining indicator diagrams from high-speed internal combustion engines.

A four-cylinder petrol engine is coupled to a brake having an effective radius of 1·90 ft. When running at 1,600 r.p.m. the brake load was 27·1 lb. When the sparking plug of each cylinder was short-circuited in turn, and the load adjusted to bring the speed to the same value, the brake loads were 18·2, 18·9, 17·7 and 19·6 lb. respectively. Estimate the indicated horse-power of the engine and the mechanical efficiency, and comment upon the method. (*I.Mech.E.*, 1932.)

14. A trial of an internal combustion engine, during *one hour*, gave the following results: indicated horse-power, 24·7; brake horse-power, 20·5; total oil fuel used, 1·35 gallon of specific gravity 0·750, and lower calorific value 18,700 B.Th.U. per lb.; total cooling jacket water, 523 lb., of temperature at inlet 60° F. and at exit 150° F. The exhaust gases leaving the cylinder were passed through an exhaust calorimeter, raising 923 lb. of water from 60° F. to 140° F.

Calculate (*a*) the mechanical and (*b*) thermal efficiencies of the engine, and draw up a heat balance sheet for the engine, showing the distribution of heat in B.Th.U. per minute.

15. A gas engine, working on the four-stroke constant volume cycle, gave the following results when loaded by a friction brake during a test of 1 hour duration: cylinder diameter, 9·5 in.; stroke, 19 in.; clearance volume, 272 cu. in.; effective circumference of brake wheel, 12·8 ft.; net load on

brake, 277 lb., at average speed 226·7 r.p.m.; average explosions per minute, 77; and mean effective pressure of indicator cards, 106 lb. per sq. in. Gas used, 455·5 cu. ft. at 15° C., and pressure 771 mm. mercury. Lower calorific value of gas, 329 C.H.U. per cu. ft. at N.T.P. Cooling jacket water, 1,380 lb., raised 34·2° C.

Calculate, (*a*) the indicated horse-power; (*b*) brake horse-power; (*c*) the mechanical efficiency and indicated thermal efficiency; (*d*) the ideal air cycle efficiency ($\gamma = 1{\cdot}4$) and efficiency ratio; and (*e*) give the heat balance for this engine per minute.

16. The exhaust gas from a gas engine passes through a waste-heat boiler installed for the purpose of providing a supply of hot water. The water initially at 50° F. passes first through the engine jackets, and then through the waste-heat boiler. The engine develops 400 B.H.P., and consumes 18 cu. ft. of gas per B.H.P. per hour, the gross calorific value being 500 B.T.U. per cu. ft., both measured under the same conditions of temperature and pressure. It may be assumed that 25 per cent of the heat supplied is transferred to the water in the jackets, that the loss by radiation is 5 per cent, and that 60 per cent of the heat in the exhaust gas is recovered by the waste-heat boiler. If the quantity of water supplied is 1,200 gallons per minute, find the temperature at which the water will leave the waste-heat boiler.

(*I.Mech.E.*, 1932.)

17. In a jet propulsion unit, air is compressed by means of an uncooled rotary compressor, the pressure at delivery being 3½ times that at entrance, and the temperature rise during compression is 1·15 times that for frictionless adiabatic compression. The air is then led to a combustion chamber where the fuel is burned under constant pressure conditions. The products of combustion at 900° F. pass through a turbine which drives the compressor. The exhaust gases from the turbine are expanded in a nozzle down to atmospheric pressure. The atmospheric pressure is 14·7 lb. per sq. in. abs., and the temperature is 50° F.

Assuming that the values of R and γ after combustion remain the same as for air, estimate (*a*) the power required to drive the compressor per lb. of air per second, (*b*) the air-fuel ratio if the calorific value of the fuel is 18,750 B.Th.U. per lb., and (*c*) the thrust developed per lb. of air per second. The velocity of approach may be neglected and the gases are expanded isentropically in both the turbine and the nozzle.

(*U.L., B.Sc. (Eng.)*, 1947.)

CHAPTER IX

STEAM

Properties of Steam. In steam we have to deal with a fluid which may be either in the liquid or gaseous condition under normal conditions of temperature and pressure, and we can no longer even approximately apply the simple laws of perfect gases. Under normal equilibrium conditions, a vapour, such as steam, if in contact with its liquid, is said to be in a saturated condition. If the mixture is now heated at constant pressure, the liquid will gradually change into vapour, until just at the point where all the liquid has turned into vapour, the stuff is called dry saturated vapour. Any further addition of heat to the dry saturated steam will superheat it, and we have what is called a superheated vapour. For any particular pressure, there is only one temperature at which the liquid will change into vapour, this temperature (called the saturation temperature) remaining constant, provided the pressure is kept constant, so long as there is any liquid left. As soon as all the liquid has been converted into vapour, the temperature will rise with any further addition of heat at constant pressure. The behaviour of the superheated vapour will now approximate more and more to that of a perfect gas the higher the temperature is removed from the saturation temperature corresponding to the pressure to which the vapour is subjected.

We have to consider the production of steam from water, but it may be mentioned that the general principles which will be applied to steam can be applied to other fluids with which we have to deal in both the liquid and vapour conditions, as in refrigeration.

In steam we are by no means dealing with a perfect gas, and therefore the simple characteristic equation

$$PV = RT$$

does not apply.

Various characteristic equations have been formulated to express the properties of a fluid in any state, none of which are applicable over an unlimited range of temperature and pressure. The subject is beyond the scope of this work, and it must suffice to mention the characteristic equation given above (p. 97) as formulated by Callendar, which may be written

$$V = \frac{RT}{P} + b - c$$

where R and b are constants and c is a function of the temperature; and V = volume in cubic feet per pound; P = pressure in pounds per square foot; T = absolute temperature in degrees C.

The properties of steam which have been calculated by Callendar are embodied in his Steam Tables, excerpts from which are included at the end of this volume.* If values from these tables are compared with those of other authorities, such as the Marks and Davis, or the Peabody Tables, discrepancies will be found. It may be pointed out, however, that any such tables must be calculated from empirical formulae based on experimental results, and it is found that the Callendar Tables are not only in good agreement with modern research, but also give values which are mutually consistent among themselves.

Relation of Pressure and Temperature in Saturated Steam. The temperature at which a liquid is converted into vapour by the addition of heat depends on the pressure to which the fluid is subjected, and the liquid boils when heated to the saturation temperature corresponding to the external pressure to which it is subjected. Regnault carried out classical investigations on the relation between temperature and pressure, when water was being converted into steam. The values determined, up to a temperature of 220° C., agree within narrow limits with those of later experimenters. Modern applications, however, require values at higher temperatures, and a considerable amount of work has been carried out since Regnault's time, culminating in that of Callendar, who has determined the corresponding values of saturation temperature and pressure from his characteristic equation for steam. These values are given in the first two columns of the Steam Tables. Thus, in Table I, (p. 634) at a pressure of 14·689 lb. per sq. in. absolute, water boils at a temperature of 100° C., while at 100 lb. per sq. in. absolute, water boils at 164·28° C.

Various attempts have been made from time to time to formulate an empirical formula expressing the relation between temperature and pressure for saturated steam. An empirical formula being based on experimental results, its justification is that it gives values consistent with those results. A formula given by Callendar, consistent with his characteristic equation, is

$$\log p = 21{\cdot}07449 - \frac{2903{\cdot}39}{T} - 4{\cdot}71734 \log T + 0{\cdot}4057(c - b)\frac{p}{T}$$

where p = saturation pressure in pounds per square inch, T = absolute saturation temperature C., and b and c have the values given above (p. 98). This formula is somewhat cumbersome and, for elementary purposes, intermediate values not given in the tables may be obtained by direct interpolation.

The Specific Volume of Steam. Direct experimental determination of the volume per unit mass of dry saturated steam is difficult,

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* *Abridged Callendar Steam Tables* in Centigrade and Fahrenheit units are published by Edward Arnold and Co. It should be noted that answers to numerical examples will vary with the particular table used.

although a certain number of measurements have been carried out, the best known being those of Knoblauch, Linde, and Klebe. Values of V_{sa} for the Steam Tables are more easily found by calculation from experimental values of other properties, the direct experimental results serving as a check on these values with which they are in very close agreement.

The specific volume of dry saturated steam may be found from Clapeyron's equation (p. 119),

$$V_{sa} = V_w + \frac{JL}{T} \cdot \frac{dT}{dP}$$

where $\frac{dT}{dP}$ is the rate of change of saturation temperature with pressure at the absolute temperature T; and V_w is the specific volume of water at temperature T, and L is the latent heat.

Various empirical formulæ expressing the relation between the specific volume of dry steam and the saturation pressure have been put forward, among which are Rankine's formula

$$PV_{sa}^{\frac{17}{16}} = \text{constant}$$

and Mollier's formula, deduced from Callendar's equation,

$$pV_{sa}^{\frac{16}{15}} = 490$$

where p is in pounds per square inch, and V_{sa} is in cubic feet per pound.

The specific volume of steam, either dry, saturated, or superheated, may be calculated from the Callendar characteristic equation

$$V = 154{\cdot}168\frac{T}{P} - 0{\cdot}4213\left(\frac{373{\cdot}1}{T}\right)^{\frac{10}{3}} + 0{\cdot}01602 \text{ (see p. 99)}$$

where T is in Centigrade absolute and P is in pounds per square foot.

Latent Heat. In the formation of steam from 1 lb. of water at 0° C. under constant atmospheric pressure of 14·689 lb. per sq. in., as heat is given to the water the temperature rises gradually to 100° C. According to the usual definition of the pound calorie, the heat necessary to raise the temperature to 100° C. would be 100 calories. When the temperature has reached 100° C. it remains stationary, any further addition of heat converting the water, at 100° C., into steam at 100° C. The amount of heat necessary to just change the 1 lb. of water at 100° C. into steam at 100° C. is called the "latent heat" at 100° C.

If steam is to be formed at any other pressure P, the water must first be heated to the corresponding saturation temperature T. When the water is at this temperature, then any further addition of heat will change the water into steam at the same temperature. The values of the latent heat at various pressures are given in Table I of the Steam Tables.

Formation of Steam at Constant Pressure. Let us imagine that we have 1 lb. of water at 0° C. at the bottom of an upright cylinder of cross-sectional area 1 sq. ft. (Fig. 129). On this water rests a frictionless, weightless piston, the constant pressure on the top side of the piston being that due to normal atmospheric pressure, namely, 14·689 lb. per sq. in. By definition (p. 52) the total heat of this water is

$$E_0 + APV_0 = 0 + \frac{144 \times 14{\cdot}689 \times 0{\cdot}01602}{1{,}400}$$

$$= 0{\cdot}0242 \text{ calories (approximately)}$$

E_0 is really not quite zero, as the pressure is not the vapour pressure at 0° C., but the increased pressure has no appreciable effect on E_0.

Let the water now be given a quantity of heat Q calories, which changes the temperature of the water from 0° C. to 100° C., the pressure above the piston remaining the same throughout, and the water expanding from V_0 to V_{100}. The total heat of the water will now be $h_{100} = E_{100} + APV_{100}$.

As the heat has been taken in at constant pressure, then from page 54,

$$Q = h_{100} - h_0$$
$$= E_{100} + APV_{100} - (E_0 + APV_0)$$
$$= E_{100} - E_0 + AP(V_{100} - V_0)$$

Now as the area of the piston is 1 sq. ft., it will rise a distance $V_{100} - V_0$, due to the expansion of the water, and therefore $AP(V_{100} - V_0)$ is the external work done by the water expanding when taking in the quantity of heat Q. This external work is equal to $\frac{144 \times 14{\cdot}689\,(0{\cdot}01671 - 0{\cdot}01602)}{1{,}400} = 0{\cdot}00105$ calories, which is negligible compared with Q. Thus practically the whole of Q goes to increase the internal energy of the water. It will be noticed that the equation $Q = (E_{100} - E_0) + AP(V_{100} - V_0)$ directly follows from the energy law $dQ = dE + dW$, for $E_{100} - E_0$ is the increase in internal energy and $AP(V_{100} - V_0)$ is the external work done.

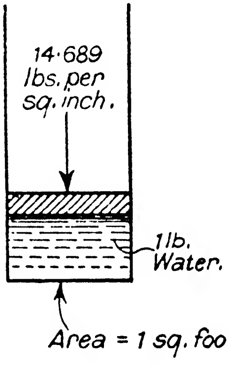

FIG. 129

At the point where the water is at a temperature of just 100° C., we have the total heat of the water $h_{100} = Q + h_0 = Q + APV_0$, if E_0 is taken as zero. Any further addition of heat will now gradually change the water at 100° C. into steam at 100° C. In order to just change all the water at 100° C. into dry saturated steam at 100° C. we must give a quantity of heat, L, to the water, L being the latent heat at 100° C. = 539·3 calories.

The total heat of the dry saturated steam $H_{100} = h_{100} + L$, as the heat has been taken in at constant pressure, and hence the change in total heat is equivalent to the heat taken in. During the formation of the steam from the water, the piston will rise a distance

$(V_{sa} - V_{100})$ where V_{sa} is the specific volume of dry saturated steam at 100° C., namely, 26·788 cu. ft. per lb.

Thus the part of the latent heat L which is used up in doing the external work due to the expansion of the water at 100° C. into steam at 100° C. equals $AP(V_{sa} - V_{100})$

$$= \frac{144 \times 14{\cdot}689\,(26{\cdot}788 - 0{\cdot}01671)}{1{,}400} = 40{\cdot}45 \text{ calories}$$

It will be noticed that the volume of the water is practically negligible compared to the volume of the steam, and hence it can be neglected in elementary calculations. The increase in internal energy of the stuff as it changes from water at 100° C. to dry steam at 100° C. is, therefore, 539·3 − 40·45 = 498·85 calories.

If dry saturated steam is to be formed at any other constant pressure from water at 0° C., then the above calculations still hold if the values for V and L, corresponding to the pressure under which the steam is being formed, are taken.

Thus, if V_0 = Specific volume of water at 0° C,

V_w = Specific volume of water at saturation temperature corresponding to the pressure P at which steam is to be formed,

V_{sa} = Specific volume of dry saturated steam at pressure P,

E_0 = Internal energy of water at 0° C,

E_w = Internal energy of water at saturation temperature,

E_{sa} = Internal energy of dry saturated steam,

L = Latent heat of steam at pressure P,

h = Total heat of water at saturation temperature,

H_{sa} = Total heat of dry saturated steam,

we have, from the fact that heat taken in at constant pressure is equivalent to the change in total heat, and also from the basic energy law,

$$\left.\begin{array}{l} \left.\begin{array}{ll} E_0 + APV_0 & = h_0 \\ (E_w - E_0) + AP(V_w - V_0) & = Q \end{array}\right\} = h \\ (E_{sa} - E_w) + AP(V_{sa} - V_w) = L \end{array}\right\} = H_{sa}$$

Also from the definition of total heat,

$$H_{sa} = \begin{cases} AP\,V_{sa} = AP(V_{sa} - V_w) + AP(V_w - V_0) + APV_0 \\ E_{sa} = (E_{sa} - E_w) + (E_w - E_0) + E_0 \end{cases}$$

We see that the total heat of dry saturated steam $H_{sa} = h + L$, where h is the total heat of the liquid at saturation temperature and pressure, and L is the latent heat of steam at the saturation pressure. It should be noticed that by our definition the total heat of dry steam is not the latent heat plus the heat given to the water while its temperature rises from 0° C. to the saturation temperature. It is made up of these two quantities together with the term APV_0, although this term for normal pressures is small compared with the values of Q and L.

Specific Heat of Water and the Mean Thermal Unit.* The variation of specific heat of water with temperature, as determined by Callendar and Barnes, is shown above in Fig. 20 (p. 44). Hence the heat taken in by water while being raised from temperature t_1 to temperature t_2 is not exactly equal to $(t_1 - t_2)$. The minimum specific heat is now taken as 0·99666 and it has this value at a temperature of 37·5° C. Owing to this variation of specific heat, it has been found convenient to formulate a mean thermal unit. The mean pound calorie may be defined as the one-hundredth part of the amount of heat necessary to raise the temperature of 1 lb. of water from 0° to 100° C. under a constant pressure of 1 standard atmosphere.

In view of the fact that the heat is assumed to be taken in *at constant pressure*, this unit may also be defined as the one-hundredth part of the change of the total heat of 1 lb. of water when heated from 0° to 100° C., under constant pressure of 1 standard atmosphere. By this definition, if the total heat of water at 100° C. under a pressure of 1 standard atmosphere (14·689 lb. per sq. in., London) is taken as 100 lb. calories, then water would have zero total heat at 0° C. and under a pressure of 14·689 lb. per sq. in. But from the last paragraph we see that, by the definition, the total heat of this water will be equal to

$$APV_0 = \frac{144 \times 14{\cdot}689 \times 0{\cdot}01602}{1{,}400} = 0{\cdot}0242 \text{ calorie}$$

Callendar therefore defines the mean pound calorie as the one-hundredth part of the change in total heat of water between 0° C. and 100° C., when the water is heated *under the varying pressure of its own vapour only*.

By the first definition we have

$$100 = E_{100} + AP_{100}V_{100} - (E_0 + AP_{100}V_0)$$

By Callendar's definition,

$$100 = E_{100} + AP_{100}V_{100} - (E_0 + AP_0V_0)$$

Callendar's definition therefore exceeds the usual definition by the term

$$A(P_{100} - P_0)V_0 = \frac{144(14{\cdot}689 - 0{\cdot}0892)0{\cdot}01602}{1{,}400}$$

$$= 0{\cdot}024 \text{ lb. calorie}$$

This difference between the usual definition and the Callendar definition of the mean unit of heat, namely, 0·024 lb. calorie in 100 lb. calories, is beyond the limit of experimental accuracy in the

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* See "The Heat Unit" by Dr. Ezer Griffiths, *Proc. Inst. Mech. Eng.*, 1951, Vol. 164, p. 215.

determination of the heat unit, and hence, at the expense of this error, we have the convenience that the total heat of water at 0° and 100° C., is zero and 100 lb. calories respectively.

Formation of Superheated Steam at Constant Pressure. If dry saturated steam takes in heat at constant pressure the temperature will rise and the steam will become superheated. If the specific heat C_p of steam were constant for all temperatures and pressures, then the heat taken in would be $C_p(t_{su} - t_{sa})$, where t_{su} is the temperature to which the steam is superheated and t_{sa} is the saturation temperature corresponding to the pressure at which the steam is being heated. The specific heat of superheated steam, however, falls with rise of temperature and rises with rise of pressure, as shown in Fig. 130.

An approximate value of the heat taken in may be found by putting for C_p in the above expression the mean value between the temperatures of saturation and superheat for the particular pressure in question. The required values of C_p may be found from a chart or from tables. Thus, if dry saturated steam at an absolute pressure of 100 lb. per sq. in. (saturated temperature 164·28° C.) be superheated to 300° C., the mean specific heat will be $\frac{0{\cdot}562 + 0{\cdot}504}{2} = 0{\cdot}533$. The heat taken in while superheating 1 lb. of dry saturated steam by this amount will therefore be approximately $0{\cdot}533(300 - 164{\cdot}28) = 72{\cdot}34$ lb. calories.

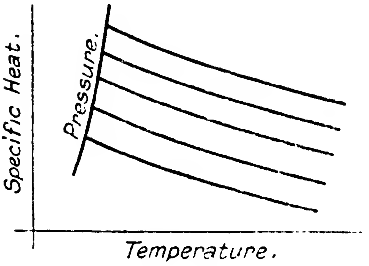

FIG. 130

As the heat taken in by a fluid at constant pressure is equal to the change in total heat of a fluid, an approximate expression for the change of total heat of a fluid while being heated from the dry saturated state is $C_p(t_{su} - t_{sa})$. Hence the total heat of superheated steam is approximately $H_{su} = h + L + C_p(t_{su} - t_{sa})$. More exact values of H_{su} are given in the steam tables (Table III), and hence the amount of heat Q_2 taken in during the superheating from the dry state may be calculated from the equation $Q_2 = H_{su} - H_{sa}$. For example, the amount of superheat in steam at 100 lb. per sq. in. and 300° C. is $t_{su} - t_{sa} = 300 - 164{\cdot}28 = 135{\cdot}72°$ C. Hence, from Table III, the total heat H_{su} is, by interpolation, 733·15 calories, while the total heat of dry saturated steam at the same pressure is 661·83. Hence the heat taken in during the superheating is $733{\cdot}15 - 661{\cdot}83 = 71{\cdot}32$ lb. calories per lb.

The heat Q_2 is used in raising the internal energy of the steam, and also in doing external work as the steam expands. The external work done will be $AP(V_{su} - V_{sa})$ where V_{su} is the specific volume of the superheated steam. The volume of 1 lb. of superheated steam at 100 lb. per sq. in. and 300° C. is 6·053 cu. ft. (Table C). The external work done in the above example is therefore $\frac{144 \times 100(6{\cdot}053 - 4{\cdot}451)}{1{,}400} = 16{\cdot}47$ lb. calories per lb. The change in internal energy will therefore be $71{\cdot}32 - 16{\cdot}47 = 54{\cdot}85$ lb. calories per lb.

The mean specific heat of superheated steam can be obtained from Table III. Thus, suppose the specific heat of steam at 100 lb. per sq. in. superheated 20° C. above the saturation temperature is required. The total heat of dry saturated steam at 100 lb. per sq. in. $= H_{sa} = 661{\cdot}83$ lb. calories; while the total heat of the steam if at 100 lb. per sq. in. and superheated 40° C. $= H_{su} = 683{\cdot}70$ lb. calories. Hence, as heat taken in at constant pressure is measured by the change in total heat, the mean specific heat

$$= \frac{H_{su} - H_{sa}}{40} = \frac{683{\cdot}70 - 661{\cdot}83}{40} = 0{\cdot}547$$

This is, therefore, the approximate value of the specific heat of superheated steam at 100 lb. per sq. in. pressure, and at a temperature of $164{\cdot}28 + 20 = 184{\cdot}28$° C.

Empirical Formulæ for Total Heat. The formula used by Callendar to represent the total heat of water is

$$h = st + L\frac{V_w}{V_{sa} - V_w} - 0{\cdot}003$$

The constant 0·003 is the value of the term $L\frac{V_w}{V_{sa} - V_w}$ at 0° C., namely, $\frac{594{\cdot}27 \times 0{\cdot}01602}{3{,}275{\cdot}9 - 0{\cdot}01602}$, hence the formula gives zero value for h at 0° C. For particulars as to derivation of this formula, reference should be made to Professor H. L. Callendar's work.*

An oft-quoted formula for dry saturated steam is that of Regnault

$$H_{sa} = 606{\cdot}5 + 0{\cdot}305t \text{ calories}$$

where t is the temperature in degrees Centigrade.

This equation, however, while giving a fair value between 100° C. and 200° C., is inconsistent below about 100° C., and is more or less useless for exact calculations.

The formula† of Professor E. H. Griffiths for saturated steam at t° C.

$$H_{sa} = 596{\cdot}73 + 0{\cdot}4t^\circ$$

agrees more closely with the results of experiment, taking the thermal unit at 15° C.; and with Joly's determinations gives consistent results.

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* *Properties of Steam and Thermodynamic Theory of Turbines*, by H. L. Callendar.

† *Phil. Trans.* Vol. CLXXXVI, 1895, p. 319; and *Proc. Phy. Soc.* 26, 1913.

A formula given by Callendar for steam either dry or superheated is

$$H = 0{\cdot}4772T - \left(\frac{13}{3}c - 0{\cdot}016\right)\frac{144}{1{,}400}p + 464 \text{ calories}$$

$$= 0{\cdot}4772T - \frac{(c - 0{\cdot}0037)}{2{\cdot}2436}p + 464\,^*$$

where p is in pounds per square inch.

From this equation a value for H in terms of pressure and volume can be derived,

$$H_{sa} = p\frac{(V_{sa} - 0{\cdot}0123)}{2{\cdot}2436} + 464 \text{ calories.}\,^*$$

By inverting this formula a convenient expression for the specific volume of dry steam is obtained,

$$V_{sa} = \frac{2{\cdot}2436(H_{sa} - 464)}{p} + 0{\cdot}0123 \text{ cu. ft.}\,^*$$

(See Examples, pp. 99–100).

Entropy of Water. The increase of entropy of a fluid when taking in a quantity of heat dQ in a reversible manner at a constant absolute temperature T, is measured by $d\phi = \frac{dQ}{T}$. If the heat is taken in at a temperature varying from T_1 to T_2, then the change of entropy is

$$\phi_2 - \phi_1 = \int_{T_1}^{T_2}\frac{dQ}{T}$$

When heat is taken in at constant pressure

$$\phi_2 - \phi_1 = \int_{T_1}^{T_2}\frac{dQ}{T} = \int_{T_1}^{T_2}\frac{C_p dT}{T}$$

where C_p is the specific heat of the fluid at constant pressure. If C_p is assumed constant for all temperatures, this quantity reduces to

$$C_p\int_{T_1}^{T_2}\frac{dT}{T} = C_p \log_\varepsilon\frac{T_2}{T_1}$$

Entropy is generally taken as zero for a fluid in the liquid state at 0° C. Hence, taking the specific heat of water as constant and equal to unity, the entropy of 1 lb. of water ϕ_w at absolute temperature T is approximately $\log_\varepsilon (T/273)$. The change in entropy while heating 1 lb. of water from T_1 to T_2 will be $\log_\varepsilon T_2/T_1$.

Values of entropy of water measured from 0° C. are given in Table I of the Steam Tables.

Entropy of Steam. When 1 lb. of water at the saturation temperature is changed into 1 lb. of dry saturated steam, the latent

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* These equations have been slightly modified in the Callendar Steam Tables (Edward Arnold & Co.), revised by G. S. Callendar.

heat L is taken in at the constant saturation temperature T_{sa}. Hence the change of entropy in evaporating 1 lb. of water at T_{sa} to 1 lb. of dry saturated steam is $\frac{L}{T_{sa}}$. This quantity is sometimes called the "entropy of evaporation." The entropy of dry saturated steam is therefore $\phi_{sa} = \phi_w + \frac{L}{T_{sa}} = \log_\varepsilon \frac{T_{sa}}{273} + \frac{L}{T_{sa}}$ approx.

Suppose the 1 lb. of dry saturated steam is now superheated at constant pressure to a temperature T'_{su}. The increase of entropy during superheating will be

$$\phi_{su} - \phi_{sa} = \int_{T_{sa}}^{T_{su}} \frac{C_p\,dT}{T}$$

As has already been pointed out, the specific heat for superheated steam is by no means constant, but a fairly approximate value for the increase of entropy may be obtained by taking for the value of C_p the mean value between the temperature of saturation and superheat. With this assumption

$$\phi_{su} - \phi_{sa} = C_p \log_\varepsilon \frac{T_{su}}{T_{sa}}$$

and the entropy of superheated steam at a temperature T_{su} may be written

$$\phi_{su} = \log_\varepsilon \frac{T_{sa}}{273} + \frac{L}{T_{sa}} + C_p \log_\varepsilon \frac{T_{su}}{T_{sa}} \text{ approx.}$$

Values of entropy of dry saturated steam are given in Table I, and values of entropy of superheated steam are given in Table IV of the Steam Tables.

Wet Steam. When steam vapour is in contact with water it is said to be wet. The dryness fraction of steam is measured by the proportion of the stuff which consists of steam vapour. Thus, 1 lb. of steam of 0·9 dryness fraction would contain 0·9 lb. steam vapour and 0·1 lb. water, all, of course, at the same temperature. From this it follows that to produce 1 lb. of steam, of dryness fraction say 0·9, from 1 lb. water at the temperature of saturation corresponding to the external pressure, the heat required would be 0·9 L, when L is the latent heat of steam at the pressure considered. Hence the total heat of wet steam is

$$H_x = h + xL$$

where x is the dryness fraction and h is the total heat of the water at the saturation temperature.

As $\quad H_{sa} = h + L$ and $H_x = h + xL$

then $\quad x = \frac{H_x - h}{L} = \frac{H_x - h}{H_{sa} - h}$

an expression for dryness fraction in terms of total heat.

The specific volume of wet steam must obviously be

$$xV_{sa} + (1-x)V_w$$

As the specific volume V_w of the water is small compared with the specific volume of steam, it is for elementary purposes generally sufficiently accurate in the case of nearly dry steam to take the specific volume of wet steam as equal to xV_{sa}.

In the formation of 1 lb. of wet steam of dryness fraction x, the heat taken in is xL, and hence the entropy of 1 lb. of wet steam will be $\phi_x = \phi_w + \frac{xL}{T_{sa}}$.

As $\phi_{sa} = \phi_w + \frac{L}{T_{sa}}$, $\phi_x = \phi_w + \frac{xL}{T_{sa}} = \phi_{sa} - (1-x)\frac{L}{T_{sa}}$.

From these equations, remembering that $H_x = h + xL$, we get

$$\phi_x - \phi_w = \frac{H_x - h}{T_{sa}}$$

and

$$\phi_{sa} - \phi_x = \frac{H_{sa} - H_x}{T_{sa}}$$

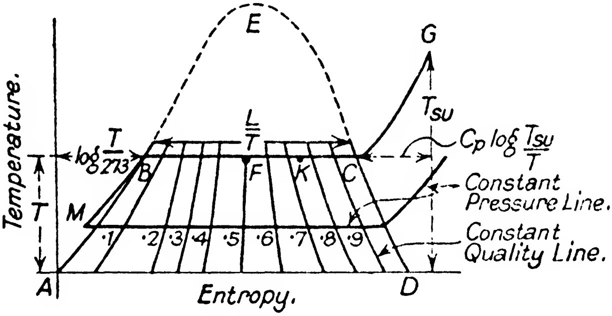

FIG. 131. SIMPLE TEMPERATURE-ENTROPY DIAGRAM FOR STEAM

The Temperature-Entropy Diagram for Steam. In this diagram values of entropy for 1 lb. of fluid are plotted horizontally and values of temperature vertically. If the values of entropy of 1 lb. of water at various temperatures are plotted against the corresponding temperatures, the line AB (water line), Fig. 131, is obtained. In the same way, plotting corresponding values of entropy and temperature for dry saturated steam will produce the line CD (saturation line). These two lines merge into one another at E, the temperature of which is the critical temperature for steam (365° C.). Charts for ordinary practical purposes, however, are not carried up as far as this point. In the absence of tables, the approximate value of ϕ at any point B on the water line at an absolute temperature T can be calculated from the expression $\log_e \frac{T}{273}$.

If water at T° is heated at constant pressure it gradually changes into steam at the same temperature, and attains a dry saturated condition when it has received a quantity of heat L, its entropy increasing by the amount $\frac{L}{T}$. Hence the horizontal line BC of length $\frac{L}{T}$ represents the conversion of water into steam at constant pressure, the point B representing 1 lb. of water and C 1 lb. of dry steam, any intermediate point representing wet steam. Thus at F, half-way between B and C, the difference in entropy between F and B is $\frac{1}{2}\frac{L}{T}$, the heat given to the water in passing from B to C is $\frac{1}{2}L$, and the dryness fraction is 0·5. Thus, by dividing the line BC into, say, 10 equal parts, the various points so obtained will represent steam of dryness fractions varying by one-tenth. A little thought will show that the dryness fraction at any point K will be measured by the ratio $\frac{BK}{BC}$.

If the dry saturated steam at the point C is heated at constant pressure it becomes superheated to some temperature T_{su}. The actual value of the entropy of superheated steam at the temperature T_{su} and pressure P, can be obtained from the steam tables and hence various points on the constant pressure line CG may be found. An approximate value of the change in entropy as the fluid passes along the constant pressure line from C to G can be calculated from the expression

$$\phi_g - \phi_c = \phi_{su} - \phi_{sa} = C_p \log_e \frac{T_{su}}{T_{sa}}$$

By drawing constant pressure lines such as BCG for various pressures, then dividing each horizontal portion between the boundary curves into 10 equal portions, and joining correspondingly numbered points, we can obtain constant quality lines as shown in Fig. 131.

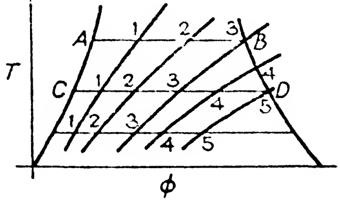

FIG. 132

Constant volume lines may be drawn on the chart in the following manner—

At a pressure of 152 lb. per sq. in., the volume of 1 lb. of dry saturated steam is practically 3 cu. ft. If AB, Fig. 132, is therefore the 152 lb. per sq. in. pressure line, the specific volume of the fluid at the point B is 3 cu. ft. Neglecting the volume of the water compared with that of the steam, the volume of the fluid at the point A may be taken as zero. Now, in passing from A to B, the volume of the fluid increases as steam is formed, in proportion to the heat taken in, and hence in proportion to the increase of entropy. Hence, if *AB* is divided into three equal parts, points corresponding to volumes of 1 and 2 cu. ft. are obtained. At 89 lb. per sq. in. the specific volume of dry steam is practically 5 cu. ft. Hence, if *CD* is the 89 lb per sq. in. constant pressure line, and it is divided into 5 equal parts, points representing volumes from 1 to 5 are obtained. By proceeding in the same way for other pressures and joining correspondingly numbered points, the various constant volume lines may be obtained. It must be remembered that if exact conditions are required, the volume of the water must be taken into account, although this correction is very small under normal conditions.

On a temperature-entropy chart (Fig. 254), it will be seen that for any point between the boundary lines the fluid is wet steam, and its dryness fraction, pressure, temperature, specific volume, and entropy may be read directly, Points to the right of the saturation line represent superheated steam, and pressure, temperature and entropy may be read off the chart directly.

Points to the left of the water line represent water. If any constant pressure line such as *GCB* is produced into the water area it will come down some such curve at *BM*, Fig. 131. Owing to the incompressibility of water, however, these constant pressure curves in the water region are practically indistinguishable from the water curve *AB*, so that, for all practical purposes, in heating water from one temperature to another at constant pressure we can assume we pass along the curve *AB*.

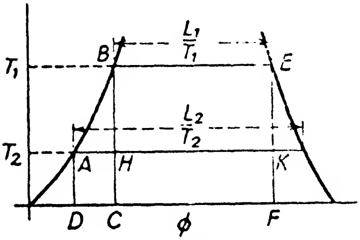

FIG. 133

Let us now consider what the various areas on the chart represent. It will be remembered that areas under a temperature-entropy curve represent the heat taken in or given out as the state of the substance passes along the curve (p. 124).

Thus, in Fig. 133, the area *ABCD* represents the heat necessary to change 1 lb. of water from the state *A* to the state *B*. If *A* corresponds to a temperature T_2 and *B* to a temperature T_1, then the area *ABCD* practically represents the heat necessary to raise the temperature of 1 lb. of water from T_2 to T_1. If the heating is carried out at constant pressure this is equal to $h_1 - h_2$ where h_1 and h_2 are the values of the total heat of water at T_1 and T_2 respectively, the pressure being the same in each case. Hence area $ABCD = h_1 - h_2$. Neglecting the variation in the specific heat of water, this may be taken as approximately equal to $T_1 - T_2$.

The area $BEFC$ will represent the heat necessary to convert 1 lb. of water at T_1 to 1 lb. of dry saturated steam at the same temperature. This quantity is, of course, L_1. This also follows from the fact that

$$BEFC = BE \times EF = \frac{L_1}{T_1} \times T_1 = L_1$$

It may also be noted that AH is the increase of entropy of water when it is heated from T_2 to T_1. Hence $AH = \log_\varepsilon \frac{T_1}{T_2}$ approximately, and therefore $AK = AH + HK = \log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}$.

The Total Heat-Entropy Diagram for Steam (Mollier Chart). Fig. 134 shows a H-ϕ chart devised by Mollier, in which total heat is plotted vertically and entropy horizontally. The water and saturation curves can be plotted directly from the tables, the two portions really forming one curve if plotted far enough. Within the wet region, between the two boundary curves, any point A, corresponding to a saturation pressure p and dryness fraction x, can be determined from the expressions

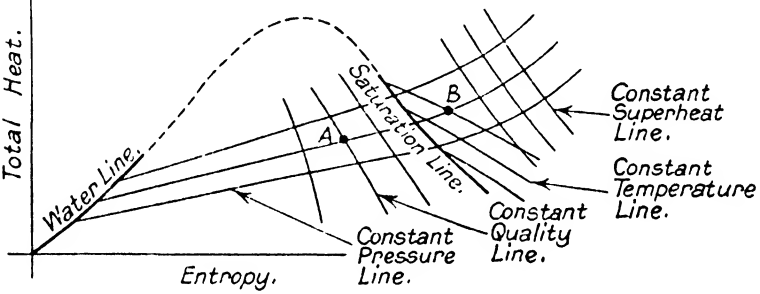

FIG. 134. ELEMENTARY HEAT-ENTROPY DIAGRAM FOR STEAM

$$H_x = h + xL$$

$$\phi_x = \phi_w + \frac{xL}{T}$$

By joining up points of corresponding dryness fraction, the constant quality lines are obtained, while a constant pressure line passes through all points representing the same pressure.

The slope of a constant pressure line is obviously measured by $\frac{dH}{d\phi}$. Now it has been shown (p. 54) that when heat is taken in by a fluid at constant pressure, the heat taken in is equivalent to the change of total heat of the fluid. Hence $\frac{dH}{d\phi} = \frac{dQ}{d\phi} = T$ by the definition of entropy. Therefore the slope of a constant pressure line at any point measures the temperature at that point. In the wet region, as water changes to steam at constant pressure, the temperature remains constant until the fluid becomes dry steam. Constant pressure lines in the wet region are therefore straight, and their slope measures the saturation temperature corresponding to the pressure.

To the right of the saturation line is the superheated region. Any such point as B at a temperature T_{su} and pressure P can be plotted from the tables of total heat and entropy for superheated steam. In the absence of such tables, approximate values may be obtained from the equations

$$H_{su} = H_{sa} + C_p(T_{su} - T_{sa})$$

$$\phi_{su} = \phi_{sa} + C_p \log_e \frac{T_{su}}{T_{sa}}$$

By joining up points of corresponding temperatures and corresponding pressures respectively, constant temperature and constant pressure lines are obtained.

It should be noticed that, as an adiabatic expansion of a fluid is one in which the entropy remains constant, adiabatics on this chart, as well as on the temperature entropy chart, are vertical straight lines.

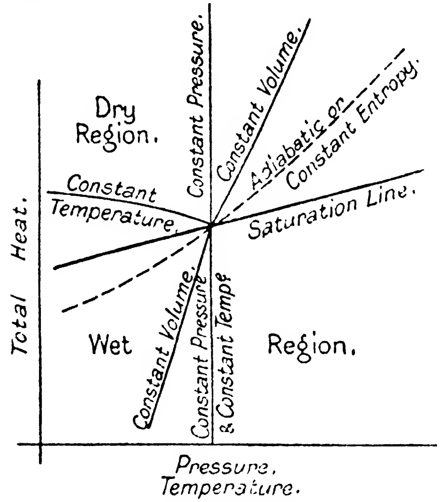

FIG. 135. ELEMENTARY CALLENDAR STEAM CHART

Only the region round the saturation curve is included in an H-ϕ chart, as this is all that is necessary for practical purposes, the very wet region not being often required.*

The Callendar Steam Chart. In this diagram* pressure is plotted horizontally, not on a linear scale but to a logarithmic scale, and total heat is plotted vertically to the usual linear scale. Alongside the pressure scale is the corresponding saturation temperature scale, the temperatures on this scale being the saturation temperatures at the pressures coinciding on the pressure scale. By the use of the logarithmic scale, equal horizontal intervals on the diagram represent equal ratios of pressures, a great convenience in

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* The "Callendar Steam Diagram," and the "H-ϕ chart," published by Messrs. Edward Arnold & Co., London, can be purchased separately.

practice. The diagram also gives a more open scale for pressure than is the case in the Mollier diagram.

For the sake of clearness, one line of each type included in this diagram is shown in Fig. 135. The saturation line with the wet region below and the superheated region above is nearly straight, and slightly inclined to the horizontal, the constant quality lines running below and more or less parallel to it. Constant pressure lines are obviously vertical straight lines through the diagram. As water changes into steam at constant temperature if the pressure is kept constant, the constant temperature lines in the wet region will also be vertical straight lines. They become slightly curved in the superheated region and run nearly horizontal, any such constant temperature line in the superheated region, of course, starting on the saturation line at a point vertically above where its particular temperature is indicated on the horizontal temperature scale. The constant volume lines are slightly curved and change their direction at the saturation line. The adiabatics or lines of constant entropy are curved and run down from right to left.

Interpolation can be easily carried out by the use of scales. Logarithmic scales are required for interpolating between the pressure lines and also between the volume lines. For interpolating between the adiabatic lines, a curved paper template may be used. It will be found that these curves are not all of the same curvature, but a template cut to the mean curvature of the various adiabatics on the chart will be found sufficiently accurate for most purposes.

Example 1. Find the internal energy of a cubic foot of steam at 150 lb. per sq. in. absolute pressure and of dryness 0·7. Calculate the change in internal energy if this steam is expanded until its pressure is 1 lb. per sq. in. absolute, (1) if the expansion is hyperbolic; (2) if the expansion is at constant entropy. (*U.L., B.Sc.* (*Eng.*).)

Total heat of 1 lb. of steam at 150 lb. per sq. in. and 0·7 dryness fraction by Table I,

$$H_x = h + xL = 183{\cdot}59 + 0{\cdot}7 \times 482{\cdot}9 = 521{\cdot}62 \text{ lb. calories}$$

This value can be taken direct from the H-ϕ chart by finding the point where the 150 lb. per sq. in. line intersects the 0·7 dryness line and passing horizontally across to the total heat scale. From Table I the volume of 1 lb. of dry steam at 150 lb. per sq. in. $= 3{\cdot}041$ cu. ft. $= V_{sa}$.

The saturation temperature at 150 lb. per sq. in. $= 181{\cdot}31°$, and at this temperature the volume of 1 lb. water $= 0{\cdot}018$ approx. $= V_w$.
$\therefore$ volume of 1 lb. of steam at 150 lb. per sq. in. of 0·7 dryness $= xV_{sa} + (1-x)V_w = 0{\cdot}7 \times 3{\cdot}041 + 0{\cdot}3 \times 0{\cdot}018 = 2{\cdot}1341$ cu. ft.

$\therefore$ 1 cu. ft. of this steam weighs $\dfrac{1}{2{\cdot}13}$ lb.

$\therefore$ total heat $H_1 = \frac{H_2}{2{\cdot}13} = \frac{521{\cdot}62}{2{\cdot}13} = 244{\cdot}9$ lb. calories per. cu. ft.

$\therefore$ internal energy $E_1 = H_1 - AP_1V_1 = 244{\cdot}9 - \frac{144}{1{,}400} \times 150 \times 1$

$= 244{\cdot}9 - 15{\cdot}43 = 229{\cdot}47$ lb. calories per cu. ft.

1. Before hyperbolic (PV = constant) expansion $P_1V_1 = 150 \times 1$.
$\therefore$ after hyperbolic expansion to 1 lb. per sq. in.,

$$\text{Volume } V_2 = \frac{150 \times 1}{1} = 150 \text{ cu. ft.}$$

This volume is part steam and part water.

Volume of 1 lb. dry steam at 1 lb. per sq. in. = 333·1 cu. ft. = V_{s1}
Volume of water at saturated temperature corresponding to 1 lb. per sq. in. = 0·0161 cu. ft.

$$\therefore 150 = xV_{s1} + (1 - x)V_w = \frac{x \times 333{\cdot}1 + (1 - x)0{\cdot}0161}{2{\cdot}13}$$

$$\therefore x = 0{\cdot}96$$

$\therefore$ total heat after hyperbolic expansion

$$H_2 = \frac{h + xL}{2{\cdot}13} \text{ (from tables at 1 lb. per sq. in.)}$$

$$= \frac{38{\cdot}63 + 0{\cdot}96 \times 573{\cdot}83}{2{\cdot}13} = 276{\cdot}76 \text{ lb. calories per cu. ft.}$$

This value can be obtained more quickly from the Callendar chart as follows. The volume after expansion is 150 cu. ft.

$\therefore$ the volume of 1 lb. is 150 × 2·13 = 320 cu. ft. approximately.

The constant volume line corresponding to 320 cu. ft. is found on the chart by interpolation. From the point of intersection of the line thus found with the 1 lb. per sq. in. pressure line pass horizontally to the total heat scale on the left. The value of total heat thus obtained must be divided by 2·13.

The internal energy $E_2 = H_2 - AP_2V_2$

$$= 276{\cdot}76 - \frac{144}{1{,}400} \times 1 \times 150$$

$$= 276{\cdot}76 - 15{\cdot}43$$

$$= 261{\cdot}33 \text{ lb. calories per cu. ft.}$$

$\therefore$ change in internal energy due to hyperbolic expansion

$$= E_2 - E_1 = 261{\cdot}33 - 229{\cdot}47 = 31{\cdot}86 \text{ lb. calories increase}$$

2. The value of the total heat after expansion at constant entropy can be calculated or obtained direct from the H-ϕ chart thus: from the point of intersection of the 150 lb. per sq. in. line with the 0·7 dryness line, drop a vertical line to cut the 1 lb. per sq. in. line. The value of the total heat of this point read off on the H scale is

thus $H_3 = 388{\cdot}5$ lb. calories per lb. The dryness fraction is read off the chart as 0·61. Hence the volume V_3 is $0{\cdot}61 \times 333{\cdot}1$ (neglecting the volume of the water) = 203·2 cu. ft. per lb.

$$\therefore \text{ internal energy } E_3 = H_3 - AP_3V_3$$

$$= 388{\cdot}5 - \frac{144 \times 1}{1{,}400} \times 203{\cdot}2$$

$$= 388{\cdot}5 - 20{\cdot}9 = 367{\cdot}6 \text{ lb. calories per lb.}$$

∴ internal energy per original cubic foot

$$E_4 = \frac{367{\cdot}6}{2{\cdot}13} = 172{\cdot}58 \text{ lb. calories}$$

∴ change in internal energy due to expansion at constant entropy

$$= E_1 - E_4 = 229{\cdot}47 - 172{\cdot}58 = 56{\cdot}89 \text{ lb. calories, decrease}$$

Example 2. Calculate the internal energy of 1 cu. ft. of steam at 128 lb. per sq. in., absolute pressure, and having dryness 0·95. If this steam expand hyperbolically until it is just dry and saturated, find the final pressure of the steam. How much heat must be added to the steam during expansion? (*U.L., B.Sc. (Eng.)*, 1925.)

Volume of 1 lb. of dry steam at 128 lb. per sq. in.

$$= 3{\cdot}531 \text{ cu. ft. (from tables)}$$

∴ volume of 1 lb. of this wet steam = $3{\cdot}531 \times 0{\cdot}95 = 3{\cdot}354$ cu. ft.

$$\therefore \text{ weight per cubic foot} = \frac{1}{3{\cdot}354} \text{ lb.}$$

Total heat of 1 lb. of this steam

$$H_x = h + xL = 176{\cdot}37 + 0{\cdot}95 \times 488{\cdot}28$$

(by interpolation from tables)

$$= 640{\cdot}24 \text{ lb. calories per lb.}$$

$$\therefore \text{ total heat per cubic foot, } H_1 = \frac{640{\cdot}24}{3{\cdot}354} = 191 \text{ lb. calories}$$

$$\therefore \text{ internal energy, } \quad E_1 = H_1 - AP_1V_1$$

$$= 191 - \frac{144}{1{,}400} \times 128 \times 1$$

$$= 177{\cdot}8 \text{ lb. calories per cubic foot}$$

To find pressure P_2 after hyperbolic expansion, we have

$$P_1V_1 = 128 \times 1 \times 3{\cdot}354 = 429, \text{ for 1 lb. steam}$$

From the tables we have, for 1 lb. dry saturated steam—

P	*V*	*PV*
100	4·451	445·1
70	6·218	435·3
50	8·520	426
40	10·5	420
20	20·08	401·6

By plotting the value of PV against the corresponding value of P we find that $PV = 429$ for dry saturated steam when the pressure $P_2 = 56$ lb. per sq. in.

It is interesting to compare this with the value obtained from the equation connecting pressure and volume for dry saturated steam, $PV^{\frac{16}{15}} = 490$. We also have $PV = 429$.

From these two equations, $P = \dfrac{429^{16}}{490^{15}}$

whence $P_2 = 58{\cdot}4$ lb. per sq. in.

At 56 lb. per sq. in. the total heat H_2 of this dry steam $= 654{\cdot}95$ lb. calories per lb.

The specific volume $V_2 = 7{\cdot}663$ cu. ft. per lb.

The heat added

$$Q = \int_{E_1}^{E_2} dE + \int_{V_1}^{V_2} Pdv = E_2 - E_1 + AP_1V_1 \log \frac{V_2}{V_1}$$

$$= (H_2 - AP_2V_2) - (H_1 - AP_1V_1) + AP_1V_1 \log_\varepsilon \frac{V_2}{V_1}$$

$$= H_2 - H_1 + AP_1V_1 \log_\varepsilon \frac{V_2}{V_1}, \text{ since } P_1V_1 = P_2V_2$$

$$= \frac{654{\cdot}95 - 640{\cdot}24 + \frac{144}{1{,}400} \times 429 \times 2{\cdot}3 \times \log_{10} \frac{7{\cdot}663}{3{\cdot}354}}{3{\cdot}354}$$

$$= 15{\cdot}3 \text{ lb calories}$$

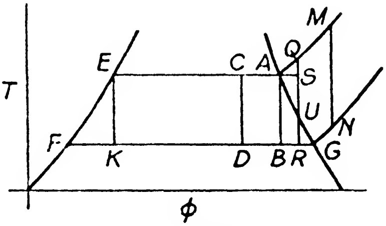


FIG. 136. ADIABATIC EXPANSION OF STEAM

The Adiabatic Expansion of Steam. The normal adiabatic expansion of steam is best shown by means of the temperature entropy chart. Let A, Fig. 136, represent the state of 1 lb. of dry saturated steam at pressure P_1 and saturation temperature T_1. Let this steam expand adiabatically to B at some lower pressure P_2 and temperature T_2. Since in adiabatic expansion the entropy remains constant, AB is a vertical straight line, the point B being in the wet region, thus showing that on adiabatic expansion, dry steam becomes wet, the dryness fraction being measured by the ratio $\dfrac{FB}{FG}$.

In order to obtain a general expression for the dryness fraction after expansion, let wet steam, represented by the point C, expand adiabatically to D. Then, if 1 refers to the higher temperature T_1 and 2 to the lower temperature T_2, we have

Entropy at C = entropy at D

$$\phi_{w1} + \frac{x_1 L_1}{T_1} = \phi_{w2} + \frac{x_2 L_2}{T_2}$$

Therefore $$x_2 = \left(\phi_{w1} - \phi_{w2} + \frac{x_1 L_1}{T_1}\right)\frac{T_2}{L_2}$$

If the steam is originally dry, then $x_1 = 1$.

In the absence of tables, remembering that the entropy of water is approximately given by $\log_\varepsilon \frac{T}{273}$

$$x_2 = \left(\log_\varepsilon \frac{T_1}{273} - \log_\varepsilon \frac{T_2}{273} + \frac{x_1 L_1}{T_1}\right)\frac{T_2}{L_2} \text{ approximately}$$

$$= \left(\log_\varepsilon \frac{T_1}{T_2} + \frac{x_1 L_1}{T_1}\right)\frac{T_2}{L_2}$$

It is instructive to obtain this expression direct from Fig. 136. The length FK represents the change of entropy as water is heated from T_2 at a pressure P_2 to T_1 at a pressure P_1, that is $FK = \phi_{w1} - \phi_{w2}$.

Now $$x_2 = \frac{FD}{FG} = \frac{FK + KD}{FG} = \frac{FK + EC}{FG} = \frac{\phi_{w1} - \phi_{w2} + \frac{x_1 L_1}{T_1}}{\frac{L_2}{T_2}}$$

In order to investigate the adiabatic expansion of superheated steam, let AM and GN be the constant pressure lines for pressures P_1 and P_2 respectively in the superheated region. Then, after expansion from some superheat temperature T_{su}, the steam may be either still superheated, as in the expansion MN, just dry, or wet, as in the expansion QR. In order to calculate the dryness fraction of the point R, we have

Entropy at Q = entropy at R

$$\phi_{su} = \phi_{w2} + \frac{x_2 L_2}{T_2}$$

$$x_2 = (\phi_{su} - \phi_{w2})\frac{T_2}{L_2}$$

If we use the approximations given in a previous paragraph, we have

$$\phi_{su} = \log_\varepsilon \frac{T_1}{273} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1}, \text{ and } \phi_{w2} = \log_\varepsilon \frac{T_2}{273}$$

whence $\log_\varepsilon \frac{T_1}{273} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1} = \log_\varepsilon \frac{T_2}{273} + \frac{x_2 L_2}{T_2}$

$$x_2 = \left\{\log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1}\right\}\frac{T_2}{L_2}$$

As in the previous case, this expression may be derived direct from the diagram for

$$x_2 = \frac{FR}{FG} = \frac{FK + KB + BR}{FG} = \frac{FK + EA + AS}{FG},$$

$$= \frac{\log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1} + C_p \log_\varepsilon \frac{T_{su}}{T_1}}{\frac{L_2}{T_2}}$$

If a pressure-volume curve for the adiabatic expansion of wet or dry saturated steam from some pressure P_1 is required, it may be obtained by calculating from the foregoing equations the dryness fraction x due to expansion from the pressure P_1 down to some lower pressure. The volume at this lower pressure may then be calculated from the equation

$$V_x = xV_{sa} + (1-x)V_w$$

This may be done for expansion from the pressure P_1 down to various pressures, and hence the curve may be plotted.

An approximation to this curve may be obtained by assuming that adiabatic expansion of steam follows the law PV^m = a constant, where m can be calculated from Zeuner's equation, $m = 1{\cdot}035 + 0{\cdot}1x$, where x is the dryness fraction at the beginning of the expansion. This method will obviously be only an approximation, for m depends on the value of the dryness fraction at the start of the expansion, and therefore curves obtained in this way starting at various dryness fractions will not coincide.

A curve correct at the initial and final pressures will be obtained if we make $P_1V_1^m = P_2V_2^m$, whence

$$\log P_1 + m \log V_1 = \log P_2 + m \log V_2$$

$$m = \frac{\log P_1 - \log P_2}{\log V_2 - \log V_1}$$

In the case of the adiabatic expansion of steam which is initialy superheated and finally wet, we have two stages, QU and UR, Fig. 136. In the first stage, down to the point U where it is just dry, the fluid behaves more or less as a gas, and Callendar shows that the expansion closely follows the law

$$P(V-b)^{1{\cdot}3} = \text{a constant}$$

where $b = 0{\cdot}016$ cu. ft. per lb. and for ordinary work is negligible, except for high pressures. The second stage, *UR*, of the expansion, proceeds as described above for wet steam.

Experiments at McGill, in 1895, on the law of condensation of steam, led to the conclusion that, for the adiabatic expansion of dry steam, the law connecting pressure and temperature was given by $\frac{P}{T^{\frac{13}{3}}} = \text{constant}$.

Now Callendar's characteristic equation for steam can be written

$(V - b) = \frac{RT}{P} - c$, where R is a constant, and c varies as $\frac{1}{T^{\frac{10}{3}}}$

Combining these two equations, we obtain for the adiabatic expansion of dry steam

$$\frac{P}{T^{\frac{13}{3}}} = \text{constant}\ ; \quad P(V - b)^{1\cdot3} = \text{constant}$$

$$T^{\frac{10}{3}}(V - b) = \text{constant}\ ; \quad \frac{P(V - b)}{T} = \text{constant}$$

It may therefore be assumed that superheated steam expanding adiabatically closely follows these equations as long as it remains dry.

Example 3. Calculate as accurately as the steam tables permit the final dryness fraction of steam which has expanded adiabatically from an initial state of 100 lb. per sq. in. absolute and 200° C. to a pressure of 15 lb. per sq. in. absolute. The specific heat of steam at 100 lb. per sq. in. is 0·55.

If this expansion be represented by the equation $PV^n = \text{constant}$, what is the value of n, satisfying initial and final conditions?

(*U.L.*, *B.Sc.* (*Eng.*), 1925.)

(*a*) Saturation temperature at 100 lb. per sq. in.

= 164·28° C. = 437·38° C. (abs.)

∴ steam is superheated 200 − 164·28 = 35·72° C.

Entropy per pound of this steam (reckoned from water at 0° C.)

$$= \phi_1 = \log_\varepsilon \frac{T_{sa}}{273\cdot1} + \frac{L}{T_{sa}} + C_p \log_\varepsilon \frac{T_{su}}{T_{sa}}$$

$$= 2\cdot3 \log_{10} \frac{437\cdot4}{273\cdot1} + \frac{496\cdot1}{437\cdot4} + 0\cdot55 \times 2\cdot3 \log_{10} \frac{473\cdot1}{437\cdot4}$$

$$= 1\cdot6477$$

This entropy can be obtained direct from Table IV thus—

ϕ for steam at 100 lb. per sq. in. and 30° C. of superheat = 1·6447

ϕ „ „ „ „ 40° C. „ = 1·6560

Difference for 10° C. „ = 0·0113

$\therefore$ ϕ_1 for steam at 100 lb. per sq. in. and 35·72° of superheat

$$= 1\cdot6447 + 0\cdot00113 \times 5\cdot72 = 1\cdot6512$$

Entropy after expansion $= \phi_2 = \log_e \frac{T_2}{273\cdot1} + x\frac{L_2}{T_2}$

where T_2 = saturation temperature at 15 lb. per sq. in. = 373·7° C. (abs.)

L_2 = latent heat at T_2 = 538·95

$$\therefore \qquad \phi_2 = 2\cdot3 \log_{10}\frac{373\cdot7}{273\cdot1} + x\frac{538\cdot9}{373\cdot7}$$

But $\phi_2 = \phi_1 = 1\cdot65$; hence $x = 0\cdot93$

This dryness fraction can be taken direct from the H-ϕ chart, thus: from the point of intersection of the 100 lb. per sq. in. pressure line with the 200° C. temperature line in the superheated region, draw a vertical line to cut the 15 lb. per sq. in. line. The dryness fraction of the point thus obtained is then read off as 0·926.

(*b*) To find the value of n we first require the specific volume V_1 of superheated steam at 100 lb. per sq. in. and 200° C. This can be obtained from tables, $V_1 = 4\cdot8901$ cu. ft.

In the absence of tables it can be calculated from the Callendar equation, $V_1 = \frac{2\cdot2436\,(H - 464)}{p} + 0\cdot0123$

$$= \frac{2\cdot2436\,(681\cdot4 - 464)}{100} + 0\cdot0123 = 4\cdot89 \text{ cu. ft.}$$

The specific volume of dry steam at 15 lb. per sq. in. = 26·27 cu. ft.

$\therefore$ specific volume of wet steam after the adiabatic expansion

$$= V_2 = 0\cdot93 \times 26\cdot27 = 24\cdot4 \text{ cu. ft.}$$

$$\therefore \qquad n = \frac{\log P_1 - \log P_2}{\log V_2 - \log V_1} = \frac{\log 100 - \log 15}{\log 24\cdot4 - \log 4\cdot89}$$

$$\therefore \qquad n = 1\cdot183$$

Supersaturated Steam. In the last paragraph we have dealt with adiabatic expansion where the fluid is always in a state of equilibrium at any point during the expansion. As the expansion proceeds the steam condenses and there is always a stable mixture of steam and water. We have now to deal with an expansion which is adiabatic in the sense that no heat passes into or out of the mixture, but during which there is no separation of water as the expansion proceeds.

As we have already seen, if a dry saturated vapour is expanded under adiabatic conditions, the vapour will normally condense more and more as the expansion proceeds. It can be shown, however, that if a dry saturated vapour be suddenly expanded, it may continue for a time to remain in the state of a homogeneous vapour, although the conditions are such that it should normally be partly liquid. Under these conditions, the vapour is said to be supersaturated. The matter will perhaps be made clearer by means of the T-ϕ chart (Fig. 137).

Consider dry saturated steam expanding adiabatically from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2). Normal adiabatic expansion is represented by AB (Fig. 137), where any point on AB lower than A lies in the wet region, the dryness fraction at any point G of the expansion being measured by $\frac{FG}{FK}$.

Now, if the expansion proceeds under supersaturated conditions, we may look on the saturation line AD, that is, the boundary line between the wet and superheated regions, as not being in existence. If, therefore, DE is the continuation of the constant pressure line MD, supersaturated expansion will proceed from A to C, where DC is the continuation of the constant pressure line ED, into what is normally the wet region. Thus, while AB represents normal equilibrium adiabatic expansion from pressure P_1 to P_2, AC represents supersaturated expansion from pressure P_1 to P_2. It will be noticed that the temperature at C, after supersaturated expansion, is lower than the saturation temperature T_2, corresponding to the pressure P_2, that is, the supersaturated vapour is said to be supercooled or under cooled.

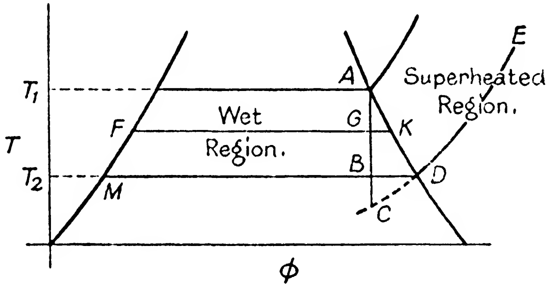


FIG. 137. SUPERSATURATION OF STEAM

In this supersaturated condition, the density at any pressure is higher than the density of saturated vapour at that particular pressure. As the expansion proceeds, therefore, not only does the actual temperature become considerably lower than the corresponding saturation temperature, but the density increases above the density of saturated vapour. As the supersaturated condition is metastable, there will be a limit of expansion below which the supersaturated condition can no longer hold, and condensation takes place.

During supersaturated adiabatic expansion of steam, as the steam remains dry, the same equations hold as in the case of the adiabatic expansion of superheated steam, namely,

$$P(V-b)^{1\cdot3} = \text{constant}\,; \quad \frac{P}{T^{\frac{13}{3}}} = \text{constant}\,; \quad T^{\frac{10}{3}}(V-b) = \text{constant}.$$

From these equations the actual temperature T of the supersaturated steam can be calculated, and hence the degree of undercooling can be determined; that is, the difference between this temperature and the saturation temperature corresponding to the pressure of the steam.

It will be seen in a later chapter that supersaturated expansion can take place through a suitably shaped nozzle.

The Throttling of Steam. If steam is allowed to pass through a small orifice so that the pressure drops as it passes through, it is said to be throttled. In an ideal throttle there should be no change in velocity on the two sides of the orifice; also no external work will be done by the steam, and no heat should be lost to external bodies.

Under these circumstances the equation of steady flow (p. 53)

$$H_1 - H_2 + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q$$

gives us $H_1 - H_2 + 0 = 0 + 0$ that is, $H_1 = H_2$.

Hence in the ideal throttling of steam the total heat remains constant.

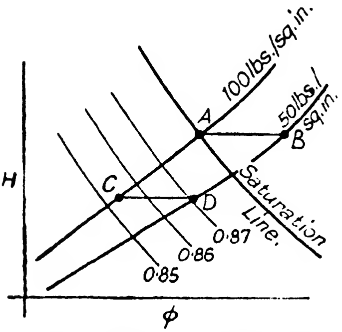


FIG. 138. EFFECT OF THROTTLING STEAM

Let us consider what happens when dry saturated steam at, say, 100 lb. per sq. in. is throttled to 50 lb. per sq. in. The total heat of dry steam at 100 lb. per sq. in. = 661·83, while the total heat of dry steam at 50 lb. per sq. in. is only 653·6. But during throttling the total heat of the fluid remains constant. Hence the fluid must become superheated. Similarly, throttling wet steam tends to dry it. This may best be shown on the total heat-entropy chart.

In this chart, as total heat is plotted vertically, throttling will be represented by a horizontal straight line. Hence, in Fig. 138, dry saturated steam at 100 lb. per sq. in. will be represented by A where the 100 lb. per sq. in. constant pressure line crosses the saturation line. In being throttled to 50 lb. per sq. in., the state of the steam will pass along a horizontal straight line to the point B where this line cuts the 50 lb. per sq. in. constant pressure line. This point will obviously be in the superheated region, and its exact state can be read off from the chart.

Similarly, steam of dryness fraction equal to 0·854 at a pressure of 100 lb. per sq. in. will be represented by the point *C*. In throttling, the state point of the steam will pass along the horizontal straight line until it cuts the 50 lb. per sq. in. pressure line at *D*. The dryness fraction of the point *D* is read off the chart as equal to 0·87. Hence throttling wet steam dries it.

In order to calculate the condition of steam after throttling, we may consider two cases. Let suffixes 1 and 2 refer to the state of the steam before and after throttling.

Case I. Wet steam of dryness fraction x_1 still wet after throttling. Total heat before throttling = total heat after throttling.

$$h_1 + x_1L_1 = h_2 + x_2L_2$$

$$\therefore\ x_2 = \frac{h_1 + x_1L_1 - h_2}{L_2}$$

Case II. Wet steam of dryness fraction x_1 superheated to T_{su} after throttling.

$$h_1 + x_1L_1 = h_2 + L_2 + C_p(T_{su} - T_{sa}) \text{ (assuming } C_p \text{ is constant)}$$

$$T_{su} = \frac{h_1 + x_1L_1 - h_2 - L_2}{C_p} + T_{sa}$$

The Throttling Calorimeter. The use of this instrument to find the dryness fraction of steam depends on the constancy of total heat during a throttling process.

Steam from the main steam pipe passes by way of a valve *F* (Fig. 139) through the orifice *A* into the calorimeter *B*, and so away at *C*. The pressure of the steam after throttling at *A* can be determined from the manometer at *D*. The temperature of the steam after throttling is given by the thermometer *E*. If the steam is nearly dry in the main steam pipe it will become superheated after passing through the orifice.

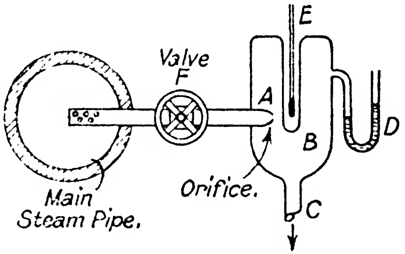


FIG. 139. THROTTLING CALORIMETER

Let p_1 = pressure in main steam pipe

h_1 = total heat of water at p_1

x_1 = dryness fraction in main steam pipe

L_1 = latent heat at p_1

p_2 = pressure in calorimeter *B*, obtained from manometer

H_2 = total heat of dry steam at p_2

T_{su} = temperature from thermometer *E*

T_{sa} = saturation temperature at pressure p_2

Then $h_1 + x_1L_1 = H_2 + C_p(T_{su} - T_{sa})$

hence $$x_1 = \frac{H_2 + C_p(T_{su} - T_{sa}) - h_1}{L_1}$$

Care must be taken that the valve F is fully opened to prevent the steam from being throttled in passing through it.

This instrument will only be of use if the steam, after expansion, is superheated. It is useless if the steam in the main steam pipe is so wet that it remains wet after expansion. The limiting case will be when the steam is just dry after expansion, that is, when the temperature on the thermometer E is just the saturation temperature corresponding to the pressure p_2. In this case $h_1 + x_1L_1 = H_2$. Hence one may know when the instrument is not superheating by the fact that the thermometer E is not giving a temperature greater than the saturation temperature corresponding to the pressure in the calorimeter B.

Provided that a true sample of the steam is obtained, this method is an accurate one for determining the dryness fraction of steam within the limits of its functioning. The main difficulty is that of obtaining a true sample. A common method is as shown in Fig. 139, where a pipe closed at the end and perforated with holes facing the direction of the steam, lies across the main steam pipe. In the case of a horizontal steam pipe, however, an error may be caused by a large proportion of the water in the steam being carried along the bottom of the pipe.

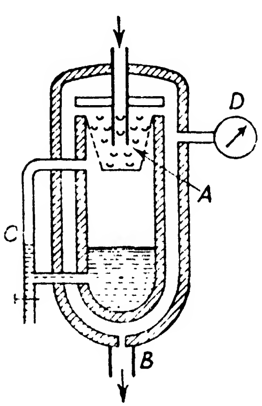


FIG. 140. SEPARATING CALORIMETER

The Separating Calorimeter. The dryness fraction of steam too wet for use in a throttling calorimeter can be determined by means of a separating calorimeter. This consists of a double-walled chamber, the inner chamber containing a perforated metal cup A (Fig. 140). The inner chamber is connected to the outer through an opening at the top of the perforated cup. The outer chamber has an outlet at B. The steam to be tested enters the inner chamber at the top and, in discharging against the bottom of the perforated cup, is forced to make a sharp turn, thus separating the water from it. The steam then passing through the sides of the cup passes to the outer chamber, and so to the outlet B. The separated water collects at the bottom of the inner chamber, and its amount may be determined by the graduated gauge glass C. The amount of steam passing through the outer chamber can be determined by condensing it in a weighed quantity of cold water, as it passes from the outlet B. For quick measurements, which may be somewhat unreliable, the flow may be determined by a pressure gauge D carrying a second scale, which shows directly the weight of steam flowing in a certain time. This scale can be calibrated by weighing the steam actually passing in this time, for several rates of flow. The whole calorimeter must be well lagged. Then, assuming that the steam passing into the outer chamber is dry, if

w = weight of water removed from steam in a certain time

then W = weight of dry steam passing in same time

$$\text{Dryness fraction} = \frac{W}{W + w}$$

If the steam passing into the outer chamber is not quite dry, the dryness fraction can be determined by the throttling calorimeter, and allowance made accordingly.

Example 4. Describe carefully any method of determining the dryness fraction of the steam supplied to an engine. Discuss fully the precautions which have to be adopted and state what degree of accuracy you expect to obtain.

Steam passes through a sampling pipe in a steam pipe to a separator and from the separator it passes through a wire drawing calorimeter. The following observations were made—

(1) Temperature of the wet steam in the main, 168° C.

(2) Temperature of the steam after wire drawing to atmospheric pressure 108° C.

(3) Weight of water caught in the separator per minute, 0·035 lb.

(4) Weight of water passing through the wire drawing orifice per minute, 0·81 lb.

Determine the original dryness fraction of the steam.

(*U.L., B.Sc.* (*Eng.*).)

Saturation pressure corresponding to 168° C. = 110 lb. per sq. in. absolute (very nearly).

Let x = dryness fraction as the steam leaves the separator and before it is wire drawn.

Before being wire drawn—

$$\text{Total heat at 110 lb. per sq. in.} = h_{110} + xL_{110} = 169{\cdot}75 + x \times 493{\cdot}18$$

After being wire drawn—

$$\text{Total heat at 14·7 lb. per sq. in.} = H_{14\cdot 7} + C_p(t_{su} - t_{sa}) = 639{\cdot}3 + 0{\cdot}5\,(108 - 100)$$

$$\therefore 169{\cdot}75 + 493{\cdot}18x = 639{\cdot}3 + 0{\cdot}5 \times 8$$

$$\therefore x = 0{\cdot}96$$

Hence 0·81 lb. of steam of dryness fraction 0·96 leaves the separating calorimeter per minute.

Hence amount of dry steam leaving the separator per minute = 0·96 × 0·81 = 0·778 lb.

Total stuff passing per minute $= 0{\cdot}035 + 0{\cdot}81 = 0{\cdot}845$

$$\therefore \text{ original dryness fraction} = \frac{0{\cdot}778}{0{\cdot}845} = 0{\cdot}92$$

It should be noted that the dryness fraction before wire drawing can be determined direct from the H-ϕ chart thus : from the point of intersection in the superheated region of the 14·7 lb. per sq. in. pressure line with the 108° C. temperature line, draw a horizontal line to cut the 110 lb. per sq. in. pressure line. The dryness fraction of the point so obtained can then be read.

Gain of Entropy During Throttling of Steam. From Fig. 138 it is obvious that the entropy of steam increases due to throttling. To calculate this increase of entropy we can make use of the temperature-entropy chart, and we will consider two cases.

Case I. Steam of dryness fraction x_1 still wet after throttling.

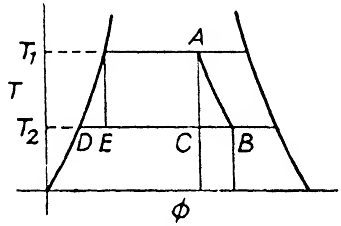


FIG. 141

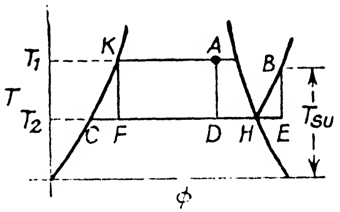


FIG. 142

Let the steam represented by the state point A, Fig. 141, be throttled from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2). Since entropy increases the final state, B will be to the right of A on the chart. Let the final dryness fraction be x_2.

$$\text{Then entropy at } B = \phi_{w2} + \frac{x_2L_2}{T_2} = \log_e \frac{T_2}{273} + \frac{x_2L_2}{T_2} \text{ (approx)}$$

$$\text{entropy at } A = \phi_{w1} + \frac{x_1L_1}{T_1} = \log_e \frac{T_1}{273} + \frac{x_1L_1}{T_1}$$

Hence increase of entropy due to throttling

$$= \phi_{w2} - \phi_{w1} + \frac{x_2L_2}{T_2} - \frac{x_1L_1}{T_1}$$

$$= \frac{x_2L_2}{T_2} - \frac{x_1L_1}{T_1} - \log_e \frac{T_1}{T_2} \text{ (approx.)}$$

This result may be obtained direct from the diagram

$$\text{Increase in entropy} = CB = DB - DE - EC$$

$$= \frac{x_2L_2}{T_2} - \log_e \frac{T_1}{T_2} - \frac{x_1L_1}{T_1}$$

In order to find x_2 in terms of the initial conditions, we have, since total heat remains constant during throttling,

$$h_2 + x_2L_2 = h_1 + x_1L_1$$

$$\therefore x_2 = \frac{h_1 - h_2 + x_1L_1}{L_2}$$

$$\therefore \text{increase of entropy} = \phi_{w2} - \phi_{w1} + \frac{h_1 - h_2 + x_1L_1}{T_2} - \frac{x_1L_1}{T_1}$$

$$= \frac{T_1 - T_2 + x_1L_1}{T_2} - \frac{x_1L_1}{T_1} - \log_\varepsilon \frac{T_1}{T_2} \text{ (approx.)}$$

Case II. Steam of dryness fraction x_1 superheated after throttling.

Let the steam represented by the state point A, Fig. 142, of dryness fraction x_1 be throttled from a pressure P_1 (saturation temperature T_1) to a pressure P_2 (saturation temperature T_2).

As we are assuming that the steam becomes superheated, the final state B will be on the continuation of the constant pressure line for P_2 in the superheated region. Let the temperature at this point be T_{su}. The increase in entropy required will therefore be represented by the distance DE.

Increase in entropy $= CE - CD = (CH + HE) - (CF + FD)$

But CH represents the change in entropy when 1 lb. of water at T_2 is changed into 1 lb. of dry steam at $T_2 = \frac{L_2}{T_2}$; and HE represents the change in entropy when 1 lb. of dry steam at saturation temperature T_2 is superheated at constant pressure to a temperature $T_{su} = C_p \log_\varepsilon \frac{T_{su}}{T_2}$, approximately; and CF = change in entropy when 1 lb. of water is heated from T_2 to $T_1 = \log_\varepsilon \frac{T_1}{T_2}$, approximately; and $FD = KA$ = change in entropy when 1 lb. of water at T_1 is changed into dry saturated steam of dryness fraction x_1 at the same temperature $= \frac{x_1L_1}{T_1}$.

$\therefore$ Change in entropy during throttling

$$= \frac{L_2}{T_2} + C_p \log_\varepsilon \frac{T_{su}}{T_2} - \log_\varepsilon \frac{T_1}{T_2} - \frac{x_1L_1}{T_1}$$

In order to find the value of T_{su} in terms of the conditions before throttling, we have, as total heat remains constant during throttling,

$$h_1 + x_1L_1 = H_2 + C_p(T_{su} - T_2)$$

from which T_{su} can be calculated and the value inserted in the previous equation.

EXAMPLES IX

1. Determine from the steam tables the amount of heat in 1 lb. of steam—

(*a*) At a pressure of 152 lb. per sq. in. abs. and 0·9 dry.

(*b*) At a pressure of 215 lb. per sq. in. abs. with 250° F. (139° C.) of superheat.

If in both cases the steam was generated from water at 90° F. (32° C.), estimate the respective gains of internal energy. (*I.Mech.E.*, 1925.)

2. Estimate the amount of heat necessary to convert 10 lb. of water at 212° F. into steam at a pressure of 190 lb. per sq. in. abs. with 280° F. of superheat. Determine the proportion of this heat used in doing external work. (*I.Mech.E.*, 1923.)

3. Calculate the total energy of 1 lb. of steam at 126 lb. per sq. in. abs. pressure and dryness 0·985, and find also its internal energy. If steam in this state be expanded in a cylinder to 3 lb. per sq. in. abs., the expansion being hyperbolic, find the final temperature of the steam. Use Callendar's equation

$$V - 0{\cdot}016 = 1{\cdot}0706\frac{T}{p} - 0{\cdot}4212\left(\frac{373{\cdot}1}{T}\right)^{\frac{10}{3}}$$

where p = pressure in pounds per square inch
V = volume in cubic feet
T = absolute temperature of the steam.

(*U.L.*, *B.Sc.* (*Eng.*). 1923.)

✓4. Steam at a pressure of 15 lb. per sq. in. abs., but slightly superheated, reaches a surface condenser at 232° F. (111° C.) and leaves as water at 202° F. (94·4° C.). The circulating water enters at 55° F. (12·8° C.), and the quantity has to be regulated so that its maximum outlet temperature does not exceed 160° F. (71° C.). Calculate the amount of circulating water that will be required per pound of steam. (*I.Mech.E.*, 1925.)

5. Steam originally dry is allowed to fall in pressure from 150 to 60 lb. per sq. in. abs. under the following conditions—

(*a*) In a closed vessel which loses heat by radiation and conduction.

(*b*) By passing through a throttle valve.

Estimate the final condition of the steam (superheat or dryness fraction) in both cases. (*I.Mech.E.*, 1924.)

6. Steam at a pressure of 190 lb. per sq. in. abs., with 200° F. (111° C.) of superheat, has its pressure lowered to 80 lb. per sq. in. abs. by passing through a reducing valve. Estimate the temperature of the low pressure steam. If 10 lb. of steam per second are passing through the pipes, calculate their diameters so that the steam speed should not exceed 120 ft. per sec. (*I.Mech.E.*, 1925.)

7. Steam at a pressure of 200 lb. per sq. in. abs., with 200° F of superheat, is throttled to 120 lb. per sq. in. abs. and then expanded adiabatically to 50 lb. per sq. in. abs. Determine the temperature of the steam (*a*) after throttling, (*b*) after expansion. (*I.Mech.E.*, 1923.)

8. Calculate the internal energy of one pound of steam at 8·0 lb. per sq. in. pressure and dryness 0·94. If this steam is compressed to 150 lb. per sq. in. according to the law $PV^{1{\cdot}18}$ = a constant, find the work done during the compression and the change in internal energy.

(*U.L.*, *B.Sc.* (*Eng.*), 1935.)

9. A cubic foot of steam of dryness fraction 0·9 and at a pressure of 200 lb. per sq. in. abs. expands to 50 lb. per sq. in. abs. Find the final state of the steam if the expansion is (*a*) hyperbolic; (*b*) adiabatic. Find the value of n in the equation PV^n = constant for initial and final conditions in the case (*b*).

10. Define the term "total heat" of a fluid and prove that this quantity does not change during a throttling process.

Steam, after throttling, has a pressure of 15 lb. per sq. in. abs. and a temperature of 150° C. If the pressure of the steam before throttling was 200 lb. per sq. in. abs., find its dryness fraction.

What is the minimum dryness fraction which may be determined by means of a throttling calorimeter if the steam is throttled from 200 lb. per sq. in. abs. to 15 lb. per sq. in. abs.? (*U.L.*, *B.Sc.* (*Eng.*), 1926.)

11. A pipe line between a boiler and engine is equivalent to a straight length of 200 ft. of 4 in. external diameter piping covered with lagging. The engine takes 4,860 lb. of steam per hour. The steam pressure is 120 lb. per sq. in. abs. and at the boiler end of the pipe the steam is by test ¾ per cent wet. A throttling calorimeter at the engine steam chest expands a steam sample to 15 lb. per sq. in. abs., at which pressure the temperature reading is 134° C. The drainage from the pipe line is 30 lb. per hour. Estimate the heat loss per square foot of metal surface, per hour, per degree Centigrade difference between steam and air temperature (22° C.) for the pipe.

(*U.L.*, *B.Sc.* (*Eng.*), 1921.)

12. In an industrial plant steam is generated at 150 lb. per sq. in., dry saturated, and supplied direct to the power units. Wet steam at 100 lb. per sq. in. is required for process work, and for this purpose, some of the boiler steam is cooled in a closed vessel, of 300 cu. ft. volume by spraying water at 1,000 lb. per sq. in. and 20° C. into the vessel.

Assuming no external losses, find the weight of water to be injected, and the final dryness fraction of the steam in the vessel.

(*U.L.*, *B.Sc.* (*Eng.*), 1933.)

CHAPTER X

STEAM CYCLES AND THE STEAM ENGINE

The Carnot Cycle with Steam as Working Substance. It will be seen above (pp. 112 and 118) that the Carnot cycle consists of an isothermal and adiabatic expansion and an isothermal and an adiabatic compression. Let us consider this cycle, using water and steam as the working substance.

Let us have 1 lb. of water at an absolute temperature T_1 in the cylinder, the state being represented on the pressure-volume diagram by the point a, and on the temperature-entropy diagram by the point A (Fig. 143).

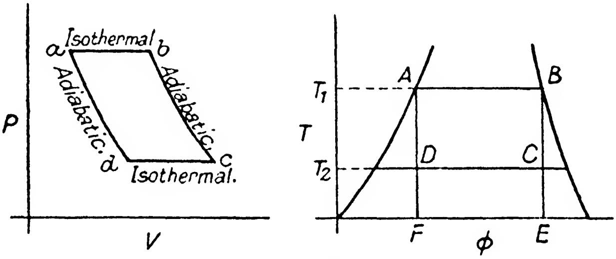


FIG. 143

Stage 1. Path ab on p-v diagram, and AB on T-ϕ diagram.

The water at T_1 is gradually changed into dry saturated steam at the same temperature, the dry state being represented by b and B. The isothermal expansion of the substance takes place at constant pressure (the saturation pressure corresponding to T_1).

The heat taken in is L_1 and the change of entropy $AB = \frac{L_1}{T_1}$.

Stage 2. Path bc on p-v diagram, BC on T-ϕ diagram.

The dry steam expands adiabatically, the pressure and temperature falling and the volume increasing. No heat is transferred to or from the working substance, and there is no change in entropy. BC is therefore a vertical straight line and the state at the end of expansion, as represented by C, is that of wet steam. The temperature of the steam falls to some lower temperature T_2.

Stage 3. Path cd on p-v diagram, CD on T-ϕ diagram.

The wet steam is compressed at constant temperature T_2, i.e. isothermally. The pressure remains constant at the saturation pressure corresponding to T_2, while the steam gradually changes to water, heat being rejected from the working substance. Let this compression be completed at some point (d and D), such that the subsequent adiabatic compression completes the cycle.

Stage 4. Path *da* on p-v diagram and DA on T-ϕ diagram.

The wet mixture at D is compressed adiabatically to A, pressure and volume returning to the original state (a and A). No heat is transferred to or from the working substance, and there is no change in entropy. The temperature rises, and the final state is once again that of water at T_1.

It will be noticed that the whole cycle is reversible and the heat is taken in at one temperature and rejected at a lower temperature and hence the efficiency is a maximum and should be equal to $\frac{T_1 - T_2}{T_1}$.

The heat taken in during stage 1 is represented by the area

$$ABEF = AB \times BE = \frac{L_1}{T_1} \times T_1 = L_1$$

The heat rejected in stage 3 is represented by the area

$$DCEF = DC \times CE = \frac{L_1}{T_1} \times T_2$$

The work done in the cycle = heat taken in − heat rejected

$$= \text{Area } ABCD = AB \times BC = \frac{L_1}{T_1}(T_1 - T_2)$$

$$\therefore \text{Efficiency of cycle} = \frac{\text{Work done}}{\text{Heat taken in}}$$

$$= \frac{\frac{L_1}{T_1} \cdot (T_1 - T_2)}{L_1} = \frac{T_1 - T_2}{T_1}$$

Clapeyron's Equation. The equation which has already been referred to (pp. 118 and 119) gives a means of calculating the specific volume of dry saturated steam, a quantity very difficult to measure experimentally.

Consider a Carnot engine working between two temperatures differing by a small amount δT, the corresponding difference of saturation pressure being δP. Let this cycle be represented by *abcd*, Fig. 144. The efficiency of this cycle is $\frac{\delta T}{T}$ and the work done per pound of working substance $= \frac{L\delta T}{T}$. This work is represented by the area of the diagram *abcd*. If the temperature difference is very small this area approximates to a rectangle of height δP and width $(V_{sa} - V_w)$ where V_{sa} is the specific volume of dry saturated steam and V_w is the specific volume of water.

Hence $(V_{sa} - V_w)\delta P = JL\frac{\delta T}{T}$, approximately

$$\text{i.e. } V_{sa} - V_w = \frac{JL}{T}\frac{\delta T}{\delta P}$$

In the limit, as δT and δP approach zero, this equation becomes exactly true and hence

$$V_{sa} - V_w = \frac{JL}{T}\frac{dT}{dP}, \text{ or } V_{sa} = V_w + \frac{JL}{T}\frac{dT}{dP}$$

From this equation the volume of 1 lb. of dry steam may be calculated for any pressure. The values of L and T can be obtained from the steam tables, the value of $\frac{dT}{dP}$, which is the rate of change of saturation temperature with pressure, being found by calculation from the steam tables or by finding the slope of the saturation temperature-pressure curve for the temperature T.

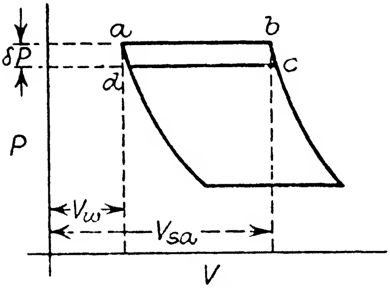


FIG. 144

Example 1. Establish the equation $V = \frac{JL}{T}\frac{dT}{dP}$, where V is the volume of 1 lb. of dry steam, L latent heat, J is 1,400 ft-lb. per C.H.U., T the absolute temperature, and P the pressure in pounds per square foot. From the tables supplied work out the volume of 1 lb. of steam using the above formula, taking the case of steam at 5 lb. per sq. in. pressure, and give any reasons you consider valid to account for the difference between your answer and that given in the steam tables supplied. (*U.L.*, *B.Sc.* (*Eng.*), 1920.)

It will be noticed that the small quantity $V_w = 0{\cdot}01602$ cu. ft. has been neglected in this question.

From the tables, when $P = 4$ lb. per sq. in. $T = 340{\cdot}33°$ C.

$P = 6$ lb. per sq. in. $T = 349{\cdot}82°$ C.

$$\therefore \frac{\delta T}{\delta P} = \frac{349{\cdot}82 - 340{\cdot}33}{2 \times 144} = \frac{4{\cdot}745}{144}$$

This is not the true value of $\frac{dT}{dP}$, for if values of T and P are plotted, it will be seen that the slope of the curve, that is $\frac{dT}{dP}$, varies considerably, especially at the low pressures.

Taking this value, and finding L and T for 5 lb. per sq. in. from the tables, we have

$$V = \frac{1{,}400 \times 555{\cdot}38}{345{\cdot}48} \times \frac{4{\cdot}745}{144}$$

$$= 74{\cdot}16 \text{ cu. ft.}$$

The chief reason for the difference between this value and that in the tables is due to the inaccuracy of the determination of the value of $\frac{dT}{dP}$. If a smaller difference of pressure δP is taken, the value will be nearer the true one, which is exactly given by the tangent to the pressure-temperature curve at the point where $P = 5$ lb. per sq. in.

With some steam tables there may also be another reason for the difference. In the older steam tables the various properties of steam have been calculated from various empirical formulæ obtained from experimental results. These formulæ are not necessarily mutually consistent. In the Callendar steam tables the various formulæ have all been derived to conform to the Callendar characteristic equation for steam, and hence the values for the various properties of steam in these tables are mutually consistent.

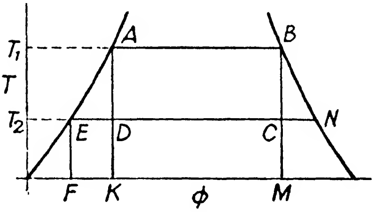


FIG. 145

The Rankine Cycle. An obvious practical difficulty in the Carnot cycle is the stopping of the isothermal compression of the working substance at the correct point D (Fig. 143).

A simple modification is to continue the compression CD to E (Fig. 145) until all the steam has been condensed to water and then to convert this water at the low pressure and temperature T_2 to water at the higher pressure and temperature T_1. The cycle would then be $ABCE$, Fig. 145, where CE represents the condensation of the wet steam at C to water at E, and EA represents the conversion of the water at low pressure and temperature T_2 to high pressure and temperature T_1.

The ideal cycle thus modified will be as follows—

Stage 1. Path AB. Water at temperature T_1 and saturation pressure P_1 is converted into dry saturated steam at constant temperature and pressure. The heat taken in by working substance is represented by the area $ABMK$ and is equal to L_1.

Stage 2. Path BC. The dry saturated steam expands adiabatically from pressure P_1 to P_2.

The dryness fraction of the working substance after expansion $= \frac{EC}{EN}$

Stage 3. Path CE. The steam is now condensed by the removal of heat at constant pressure P_2 and constant saturation temperature T_2, until the working substance is wholly water.

The heat given out by the working substance is represented by the area $ECMF$.

Stage 4. Path EA. The water at temperature T_2 and saturation pressure P_2 is raised by the addition of heat to temperature T_1 and saturation pressure P_1, the pressure at any stage being the saturation pressure corresponding to the temperature at that stage. The heat taken in by the working substance is represented by the area $EAKF$.

The work done in the complete cycle is therefore equal to heat taken in - heat rejected, and is represented by the area $EAKF + ABMK - ECMF =$ area $ABCE$.

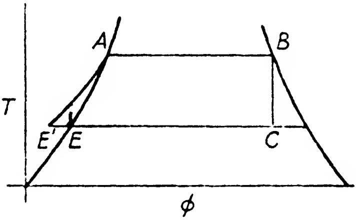


FIG. 146

The efficiency of the cycle $=$

$$\frac{\text{Work done}}{\text{Heat taken in}} = \frac{\text{Area } ABCE}{\text{Area } EABMF}$$

In the cycle just considered the working substance is assumed to remain in the cylinder throughout the whole cycle. We will now consider what modification is necessary to conform more nearly to practice, where heating is carried out in a boiler and the condensation of the steam is carried out in a condenser apart from the working cylinder. Under these circumstances, the fourth stage just considered can be assumed to be carried out in two operations.

The condensed water at E, Fig. 145, at P_2 and T_2 will first be pumped by means of a feed pump into the boiler, where the pressure is P_1. The water then takes in heat in the boiler at constant pressure P_1 until its temperature rises to T_1. These two stages are represented by EE' and $E'A$, Fig. 146, where AE' is the continuation in the water region of the P_1 constant pressure line AB. Owing to the small compressibility of water, however, the points E and E' are practically coincident, and hence, to all intents and purposes, the cycle is still represented by $ABCE$.

We will now consider this cycle, assuming it is carried out not all in the working cylinder, but consecutively in the boiler, working cylinder, condenser, and feed pump. The cycle is shown at *abce* in the *p-v* diagram and $ABCE$ in the T-ϕ diagram, Fig. 147. The line *ea* in the *p-v* diagram really slopes to the left as it goes from *e* to *a*, owing to the compression of the water reducing the volume.

This is, however, so slight that *ea* may be taken as a vertical straight line. The cycle is as follows—

Stage 1. Path *ab* and *AB*. Water is converted in the boiler into steam at constant pressure P_1 and constant temperature T_1, and enters the working cylinder as it is produced. Work done by steam as it enters cylinder, *fbmo*, during formation from water. Heat taken in by working substance, *ABUW*.

Stage 2. Path *bc* and *BC*. Boiler is shut off from the cylinder and the steam expands adiabatically in cylinder. Work done by steam, *bcnm*. Heat taken in, *nil*. Pressure falls to p_2 and volume increases.

Stage 3. Path *ce* and *CE*. Steam is rejected from cylinder and is ejected to the condenser, where it is condensed at constant pressure p_2 and constant temperature T_2 to water. Work done on steam, *gcno*. Heat given out by working substance, *ECUS*.

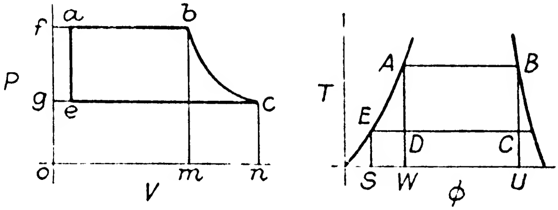


FIG. 147

Stage 4. Water at pressure p_2 and temperature T_2 is pumped into boiler, where the pressure is p_1. It takes in heat in the boiler and its temperature rises to T_1. Work done by feed-pump, *faeg*. Heat taken in by water *EAWS*.

The net work done in the cycle is represented by

$$fbmo + bcnm - gcno - faeg = abce$$

on the p-v diagram. On the T-ϕ diagram this work, which is equal to the difference between the heat taken in and the heat rejected by the working substance, is represented by

$$ABUW + EAWS - ECUS = ABCE$$

In order to obtain expressions for the work done in the cycle, it will be remembered that on p. 55 it was proved that the area *fbcg*, Fig. 147, which represents $\int VdP$ over the adiabatic expansion *bc* is measured by the heat drop during this expansion. The area *fbcg* represents the value of $H_B - H_C$ in work units, where H_B and H_C are the values of total heat of the working substance before and after expansion. Now the area *faeg* represents the work done by the feed-pump, namely, $V_w(P_1 - P_2)$, where V_w is the volume of the water at *e*.

Hence the work done during the cycle

$$= H_B - H_C - \frac{V_w(P_1 - P_2)}{J} \text{ heat units}$$

The heat taken in is the heat necessary to change the water at P_1 and T_2 into dry saturated steam at P_1 and T_1. As this heat is given at constant pressure, its value will be $H_B - h_{E'}$, Fig. 146, where $h_{E'}$ is the total heat of water at P_1 and T_2. Now $h_{E'}$ will be greater than h_E, the total heat of water at P_2 and T_2, by the thermal equivalent of the amount of work necessary to bring the water at P_2 and T_2 to P_1 and T_2, that is, by $\frac{V_w(P_1 - P_2)}{J}$.

Hence the heat taken in $= H_B - \left(h_E + \frac{V_w(P_1 - P_2)}{J}\right)$

The efficiency of the cycle is therefore

$$\frac{H_B - H_C - \dfrac{V_w(P_1 - P_2)}{J}}{H_B - h_E - \dfrac{V_w(P_1 - P_2)}{J}}$$

The term $\frac{V_w(P_1 - P_2)}{J}$ is so small compared to the other quantities that it can usually be ignored, and hence the efficiency which is represented on the T-ϕ diagram by $\frac{\text{area } ABCE}{\text{area } EABUS}$ may be taken as equal to $\frac{H_B - H_C}{H_B - h_E}$. In this expression H_B is the total heat of dry saturated steam at P_1, and h_E is the total heat of water at P_2, both values being taken from the steam tables. In order to calculate the value of H_C, the dryness fraction x_c at C must first be calculated from the expression $x_c = \left(\phi_A - \phi_E + \frac{L_1}{T_1}\right)\frac{T_2}{L_2}$ where ϕ_A and ϕ_E are the values of the entropy of water at A and E respectively.

There is no need for this calculation, however, if the H-ϕ chart is used, for H_B can be read off direct for the point where the P_1 constant pressure line cuts the saturation line. If a vertical straight line is now drawn from this point to cut the P_2 constant pressure line, the second point so obtained will give the value of H_C direct from the chart.

This cycle, called the Rankine cycle, is of great importance, as it has been adopted as the basis of comparison for any real steam engine. Unlike the Carnot cycle, it allows the use of a condenser apart from the working cylinder. Although the Rankine cycle is reversible, the efficiency is less than that of the Carnot cycle.

This can be seen roughly by comparing areas in the T-ϕ diagrams of Figs. 143 and 145. It will be seen that, although a larger amount of work per pound of steam is obtained, yet the extra amount of heat taken in is in a greater proportion. The reason for this lower efficiency is that all the heat is not taken in at the highest temperature, as in the Carnot cycle, the water taking in heat between the two temperatures T_1 and T_2.

It is interesting to calculate the Rankine efficiency for roughly normal conditions. Let the boiler pressure be 300 lb. per sq. in. absolute and the condenser pressure be 2 lb. per sq. in. absolute.

From the H-ϕ chart we find the total heat of dry saturated steam at 300 lb. per sq. in. to be 674 C.H.U. per lb. Drawing a vertical straight line from this point on the chart until it cuts the 2 lb. per sq. in. pressure line, we find the total heat is 491 C.H.U. per lb. The work done in the Rankine cycle between these two pressures is therefore $674 - 491 = 183$ C.H.U. per lb.

The total heat of water at the saturation temperature corresponding to a pressure of 2 lb. per sq. in. (52·27° C.) = 52·16 C.H.U. per lb. This heat in the water is theoretically available for use again in the boiler.

$$\text{Hence the Rankine efficiency} = \frac{674 - 491}{674 - 52{\cdot}16} = 29{\cdot}4 \text{ per cent}$$

With an ideal steam plant therefore only 29·4 per cent of the available energy would be converted into work under these conditions. In practice, as will be seen later, even this efficiency could not be reached for the given pressures. The ratio of the actual work done in a steam engine compared to the work done in the corresponding Rankine cycle is termed the efficiency ratio.

$$\text{Thus efficiency ratio of a steam engine} = \frac{\text{Heat converted into work}}{\text{Adiabatic heat drop}}$$

The ideal steam consumption for any two pressures is easily obtained from the adiabatic heat drop.

$$\text{Work done per pound of steam in Rankine cycle}$$
$$= \text{Adiabatic heat drop}$$

$$\text{and 1 h.p. hour} = \frac{33{,}000 \times 60}{J} \text{ heat units}$$

$$\therefore \text{ Steam per horse-power hour} = \frac{33{,}000 \times 60}{J\,(\text{adiabatic heat drop})} \text{ lb.}$$

Suppose an actual engine working with dry steam between the pressures of 300 lb. per sq. in. and 2 lb. had an efficiency ratio of 62 per cent. We have just seen that the Rankine efficiency is 29·4 per cent for these pressures. The actual thermal efficiency of the engine would therefore be $29{\cdot}4 \times 0{\cdot}62 = 18{\cdot}23$ per cent.

The ideal steam consumption would be

$$\frac{33{,}000 \times 60}{1{,}400 \times 183} = 7{\cdot}73 \text{ lb. per H.P. hour}$$

The actual steam consumption would be

$$\frac{7{\cdot}73}{0{\cdot}62} = 12{\cdot}47 \text{ lb. per H.P. hour}$$

Effect of Condensing on the Rankine Cycle. This will be explained best by an actual example. Suppose instead of condensing down to 2 lb. per sq. in. pressure as in the last paragraph, the exhaust is at atmospheric pressure, namely, 14·7 lb. per sq. in. From the H-ϕ chart we see that the total heat after expanding adiabatically from 300 lb. per sq. in. to 14·7 lb. per sq. in. is 550 C.H.U. per lb.

The work done in the Rankine cycle in this case is therefore 674 − 550 = 124 C.H.U.

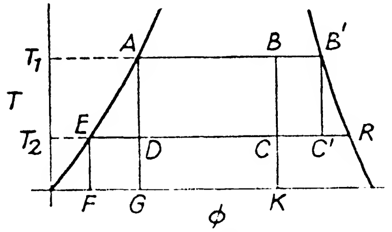


FIG. 148

The total heat of water at 14·7 lb. per sq. in. = 100 C.H.U. per lb.

$$\text{Hence the Rankine efficiency} = \frac{124}{674 - 100} = 21{\cdot}6 \text{ per cent}$$

$$\text{The ideal steam consumption} = \frac{33{,}000 \times 60}{1{,}400 \times 124} = 11{\cdot}4 \text{ lb. per H.P. hour}$$

As there is no condenser, the total heat of the water, namely, 100 C.H.U. per lb., would not be directly available in practice for using again in the boiler. In modern plants, however, the heat in the exhaust steam of a non-condensing engine is utilized for feed-water heating, or to do work in a low pressure turbine.

Efficiency of the Rankine Cycle in Terms of Temperature.

Case I. Steam saturated on admission to cylinder.

Let the steam on admission to the cylinder be of dryness fraction x_1. Its state will then be represented by the point B, Fig. 148, and $\frac{AB}{AB'} = x_1$. The cycle will be $ABCE$. Let the temperature of the steam on admission be T_1 and on condensation be T_2.

Then AB = Gain in entropy when water at T_1 is converted into

$$\text{steam of dryness fraction } x_1 = \frac{x_1 L_1}{T_1}$$

EC = Gain in entropy when water at T_2 is converted into

$$\text{steam of dryness fraction } x_2 = \frac{x_2 L_2}{T_2}$$

ED = Gain of entropy when water at T_2 is converted into water at T_1

$$= \log_\varepsilon \frac{T_1}{T_2}, \text{ approximately}$$

Area $EAGF$ = Heat taken in when water is heated from T_2 to T_1

$$= T_1 - T_2 \text{ approximately.}$$

Then work done in cycle = Area $ABCE = FEABK - ECKF$

$$= (EAGF + ABKG) - (EDGF + DCKG)$$

$$= (T_1 - T_2) + (AB \times BK) - (DG \times ED) - (AB \times DG)$$

$$= T_1 - T_2 + \frac{x_1 L_1}{T_1} \times T_1 - T_2 \log_\varepsilon \frac{T_1}{T_2} - \frac{x_1 L_1}{T_1} T_2$$

$$= (T_1 - T_2)\left(1 + \frac{x_1 L_1}{T_1}\right) - T_2 \log_\varepsilon \frac{T_1}{T_2}$$

The heat taken in

$$= \text{Area } EABKF = EAGF + ABKG$$

$$= (T_1 - T_2) + \frac{x_1 L_1}{T_1} \times T_1 = T_1 - T_2 + x_1 L_1$$

$$\text{Efficiency of cycle} = \frac{\text{Work done}}{\text{Heat taken in}}$$

$$= \frac{(T_1 - T_2)\left(1 + \frac{x_1 L_1}{T_1}\right) - T_2 \log_\varepsilon \frac{T_1}{T_2}}{T_1 - T_2 + x_1 L_1}$$

The dryness fraction after expansion

$$= x_2 = \frac{EC}{ER} = \frac{ED + DC}{ER} = \frac{\log_\varepsilon \frac{T_1}{T_2} + \frac{x_1 L_1}{T_1}}{\frac{L_2}{T_2}}$$

If the steam is dry saturated on admission to the cylinder then the cycle is $AB'C'E$ and $x_1 = 1$ in the above expressions.

Case II. Steam superheated on admission to cylinder.

Let the steam be superheated to a temperature T_{su} at a constant pressure corresponding to the saturation temperature T_1. Then the state of the steam before adiabatic expansion will be represented by B, Fig. 149, on the continuation in the superheat region of the constant pressure line corresponding to T_1. The cycle will then be $ASBCE$.

SW = Increase in entropy when dry saturated steam at T_1 is superheated to T_{su} at constant pressure

$= C_p \log_\varepsilon \dfrac{T_{su}}{T_1}$ (approx.) where C_p = Mean specific heat of steam between T_1 and T_{su}

Area $SBKX$ = Heat taken in when dry saturated steam at T_1 is superheated at constant pressure to T_{su} $= C_p(T_{su} - T_1)$ (approx.)

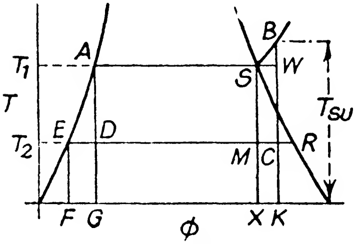


FIG. 149

The work in cycle = Area $ASBCE = EASBKF - ECKF$

$$= (EAGF + ASXG + SBKX) - (EDGF + DMXG + MCKX)$$

$$= (T_1 - T_2) + AS \times SX + C_p(T_{su} - T_1) - DG \times ED - DM \times MX - CK \times MC$$

$$= T_1 - T_2 + \frac{L_1}{T_1} \times T_1 + C_p(T_{su} - T_1) - T_2 \log_\varepsilon \frac{T_1}{T_2} - \frac{L_1}{T_1} T_2 - T_2 C_p \log_\varepsilon \frac{T_{su}}{T_1}$$

$$= (T_1 - T_2)\left(1 + \frac{L_1}{T_1}\right) + C_p(T_{su} - T_1) - T_2\left(\log_\varepsilon \frac{T_1}{T_2} + C_p \log_\varepsilon \frac{T_{su}}{T_1}\right)$$

The heat taken in = Area $EASBKF = EAGF + ASXG + SBKX$

$$= T_1 - T_2 + L_1 + C_p(T_{su} - T_1)$$

∴ Efficiency of cycle

$$= \frac{(T_1 - T_2)\left(1 + \dfrac{L_1}{T_1}\right) + C_p(T_{su} - T_1) - T_2\left(\log_\varepsilon \dfrac{T_1}{T_2} + C_p \log_\varepsilon \dfrac{T_{su}}{T_1}\right)}{T_1 - T_2 + L_1 + C_p(T_{su} - T_1)}$$

If a comparison is made of the efficiency of a Rankine cycle using in one case dry saturated steam and in the other superheated steam between the same pressures, it will be found that there is little gain in thermodynamic efficiency due to the superheat.

The chief value of superheating is due to practical considerations resulting from the fact that as the steam in transmission loses heat, condensation does not take place, and heat losses are reduced.

Example 2. Sketch the pressure-volume and temperature-entropy diagrams of the Rankine cycle for a steam engine. Determine the Rankine efficiency of a steam engine working between pressure limits of 100 and 5 lb. per sq. in., and supplied with dry saturated steam. By how much per cent is this efficiency increased by superheating the steam 100° C. ? What effect would superheating produce on the efficiency ratio of an actual steam engine, and what are the reasons for this effect ? (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

1. Total heat of dry steam at 100 lb. per sq. in. absolute $= H_B$ $= 661\cdot83$ lb. calories (from tables or H-ϕ chart).

Dryness fraction after adiabatic expansion to 5 lb. per sq. in. abs.

$$= \left(\begin{array}{c}\text{Entropy of}\\ \text{water at}\\ \text{100 lb.}\\ \text{per sq. in.}\end{array} - \begin{array}{c}\text{Entropy of}\\ \text{water at}\\ \text{5 lb.}\\ \text{per sq. in.}\end{array} + \frac{\begin{array}{c}\text{Latent heat at}\\ \text{100 lb. per sq. in.}\end{array}}{\begin{array}{c}\text{Saturation tem-}\\ \text{perature at}\\ \text{100 lb. per sq. in.}\end{array}} \right) \times \frac{\begin{array}{c}\text{Saturation tempera-}\\ \text{ture at 5 lb. per sq. in.}\end{array}}{\begin{array}{c}\text{Latent heat at}\\ \text{5 lb. per sq. in.}\end{array}}$$

$$= \left(\phi_{w1} - \phi_{w2} + \frac{L_1}{T_1}\right)\frac{T_2}{L_2}$$

From the tables $\phi_{w1} - \phi_{w2} = 0\cdot4739 - 0\cdot2346$
$= 0\cdot2393$

In the absence of tables an approximate value can be calculated from $2\cdot3026 \times \log_{10}\frac{T_1}{T_2} = 2\cdot3026 \times \log_{10}\frac{437\cdot38}{345\cdot48} = 0\cdot236$

Taking the first value—

$$\text{Dryness fraction} = \left(0\cdot2393 + \frac{496\cdot12}{437\cdot38}\right) \times \frac{345\cdot48}{555\cdot38} = 0\cdot855$$

The dryness fraction can be found directly from the H-ϕ chart. Taking a vertical line down from the intersection of the 100 lb. per sq. in. line with the saturation line, we see that the 5 lb. per sq. in. line is cut at a dryness fraction of 0·855.

Total heat of steam after adiabatic expansion to 5 lb. per sq. in.

$$= H_C = h + xL$$
$$= 72\cdot26 + 0\cdot855 \times 555\cdot38 = 547\cdot11 \text{ lb. calories}$$

The value of H_C can also be obtained from the H-ϕ diagram, the point on the 5 lb. per sq. in. line of dryness fraction 0·855 having a total heat of 547·5 lb. calories.

The total heat of water at 5 lb. per sq. in. $= h = 72\cdot26$ (from tables).

$$\therefore \text{Rankine efficiency} = \frac{\text{Adiabatic heat drop}}{\text{Heat put in}} = \frac{H_B - H_C}{H_B - h}$$

$$= \frac{661\cdot83 - 547\cdot11}{661\cdot83 - 72\cdot26} \times 100 = 19\cdot4 \text{ per cent}$$

2. The saturation temperature at 100 lb. per sq. in. = 164·28° C.

∴ Temperature of steam at 100 lb. per sq. in. superheated 100° C.
= 264·28° C.

From the point on the H-ϕ chart, where the 100 lb. per sq. in. line is cut by the constant temperature line corresponding to 264·28° C., we find that total heat before expansion = 715 lb. calories.

Passing vertically down from this point until we cut the 5 lb. per sq. in. line, the point so obtained gives total heat after expansion = 584 lb. calories.

Hence, Rankine efficiency with superheated steam

$$= \frac{715 - 584}{715 - 72} \times 100 = 20{\cdot}4 \text{ per cent}$$

∴ Gain in efficiency due to superheating = 20·4 − 19·4 = 1 per cent.

Rankine Cycle in Terms of Pressure and Volume. The work done in the Rankine cycle may be expressed approximately in terms of pressure and volume. The result, which will be in foot-lb. per lb. of steam, will be approximate, for the assumption is made that, in the adiabatic expansion law of steam, namely, pv^n = constant, the value of n is constant throughout the expansion.

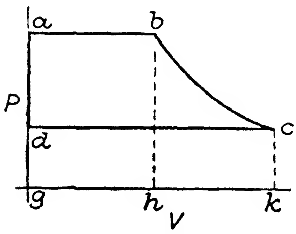


FIG. 150. RANKINE CYCLE

Let *abcd*, Fig. 150, represent the Rankine cycle for 1 lb. of steam (the feed-pump work being neglected).

The work done = area *abcd* = *abhg* + *bckh* − *dckg*

$$= p_b v_b + \frac{p_b v_b - p_c v_c}{n-1} - p_c v_c$$

$$= \frac{n}{n-1}(p_b v_b - p_c v_c)$$

$$= \frac{n}{n-1} p_b v_b \left(1 - \frac{p_c v_c}{p_b v_b}\right)$$

But $\quad p_b v_b{}^n = p_c v_c{}^n \quad \therefore \frac{v_c}{v_b} = \left(\frac{p_b}{p_c}\right)^{\frac{1}{n}}$

$$\therefore \quad \text{Work done} = \frac{n}{n-1} p_b v_b \left\{1 - \frac{p_c}{p_b} \times \left(\frac{p_b}{p_c}\right)^{\frac{1}{n}}\right\}$$

$$= \frac{n}{n-1} p_b v_b \left\{1 - \left(\frac{p_b}{p_c}\right)^{\frac{1-n}{n}}\right\}$$

Hence if 1 lb. of steam of specific volume v_1 before the adiabatic expansion be caused to pass through a Rankine cycle between the pressures p_1 and p_2, the work done is given approximately by the expression

$$\frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_1}{p_2}\right)^{\frac{1-n}{n}}\right\} = \frac{n}{n-1}p_1v_1\left\{1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}$$

Another method of obtaining this expression is as follows—

The area *abcd* is measured by $\int_{P_c}^{P_b} V\,dP$

$$= \int\left(\frac{K}{P}\right)^{\frac{1}{n}}dP \text{ where } PV^n = K$$

$$= K^{\frac{1}{n}}\int_{P_c}^{P_b} P^{-\frac{1}{n}}\,dP = K^{\frac{1}{n}} \times \left(\frac{P^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right)_{P_c}^{P_b}$$

$$= K^{\frac{1}{n}} \times \frac{n}{n-1}\left(P_b^{\frac{n-1}{n}} - P_c^{\frac{n-1}{n}}\right)$$

But $K^{\frac{1}{n}} = P_b^{\frac{1}{n}}V_b \;\therefore\; abcd = \frac{n}{n-1}P_b^{\frac{1}{n}}V_b\left\{P_b^{\frac{n-1}{n}} - P_c^{\frac{n-1}{n}}\right\}$

$$= \frac{n}{n-1}P_bV_b\left\{1-\left(\frac{P_c}{P_b}\right)^{\frac{n-1}{n}}\right\}$$

Incomplete Expansion. In the Rankine cycle the steam is expanded right down to the pressure at which condensation takes place, as at *C*, Fig. 151. If the steam is released from the cylinder at a higher pressure than that of the condenser, as at *f*, there will be a sudden drop of pressure *fg* at constant volume, the amount of work *fcg* being lost. The heat taken in being the same as for complete expansion, there is a corresponding loss in efficiency of the cycle. In order to show the effect of incomplete expansion on the T-ϕ diagram, we note that the drop of pressure *fg* on release takes place at constant volume, that is, instantaneously while the piston in the ideal engine is at the end of its expansion stroke. In the ideal Rankine cycle, where the whole operation takes place within the cylinder, the same effect as the above will be produced if a certain amount of the steam is condensed by removing heat while the piston is stationary at the end of the expansion stroke. The state of the steam in the cylinder would pass along the constant

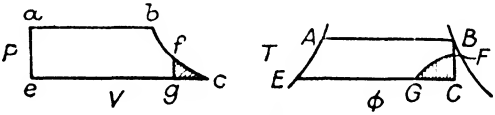


Fig. 151

volume line *FG* in the T-ϕ diagram. The cycle would therefore be represented by *ABFGE*, the loss of available work due to incomplete expansion being represented by the area *FCG*, this loss of work being equivalent to the extra amount of heat given up to the condenser.

Example 3. A boiler supplies steam at 400 lb. per sq. in. abs. and 600° F. The steam passes through a pipe line (1) Fig. A to a reducing valve, where the pressure is dropped to 200 lb. sq. in. abs., the temperature now being 420° F. The steam now passes through the pipe line (2) to a turbine. In order to test the quality of the steam entering the turbine, a small quantity is passed through a throttling calorimeter as shown in Fig. A. The pressure and temperature after throttling are 15 lb. per sq. in. abs. and 295° F. respectively. The steam passing through the turbine expands with an efficiency ratio of 75 per cent compared to the Rankine efficiency, to a pressure of 4 lb per sq. in. abs. The steam consumption of the turbine is 100 lb. per minute.

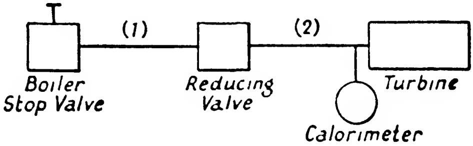


FIG. A

Indicate on a free-hand sketch of the Total Heat —Entropy chart the paths taken by the steam, and from the chart find the following—

(*a*) State of the steam entering the turbine.
(*b*) Heat losses in pipes (1) and (2) (ignore any other losses).
(*c*) Horse power of the turbine.
(*d*) State of steam leaving the turbine.

(*U. Nottm.*)

Fig. 152 extracted from Callendar's H-ϕ chart illustrates the process.

The points *A*, *C* and *E* are fixed from the data in the question, by the intersections of the appropriate pressure and temperature lines in the superheated region.

Drawing *CB* horizontally to the left (constant total heat) gives the point *B*, before throttling from 400 to 200 lb. per sq. in. abs. Hence the heat lost in pipe 1 at constant pressure is measured by $H_A - H_B$ B.Th.U. per lb. of steam.

Similarly drawing *ED* horizontally to the left from the point *E* gives the point *D*, and the heat lost in pipe 2 is given by $H_C - H_D$ B.Th.U. per lb.

A vertical line drawn downwards from *D* to *K* to cut the 4 lb. per sq. in. pressure line in the point *K*, gives the adiabatic heat drop from 200 to 4 lb. per sq. in. as $H_D - H_K$ B.Th.U. per lb. The actual heat drop in the turbine is measured by $DF = 0{\cdot}75\,DK$, and the final state of the steam leaving the turbine is represented by the point *L*.

All the necessary values may now be read off the chart.

$T_A = 600°$ F. $\quad H_A = 1{,}305$ B.Th.U. per lb.
$T_C = 420°$ F. $\quad H_B = H_C = 1{,}224$ B.Th.U. per lb.
$T_E = 295°$ F. $\quad H_D = H_E = 1{,}192$ B.Th.U. per lb.
$x_D = 0{\cdot}99 \quad H_F = H_L = 992$ B.Th.U. per lb.
$x_L = 0{\cdot}865 \quad H_K = \quad = 926$ B.Th.U. per lb.

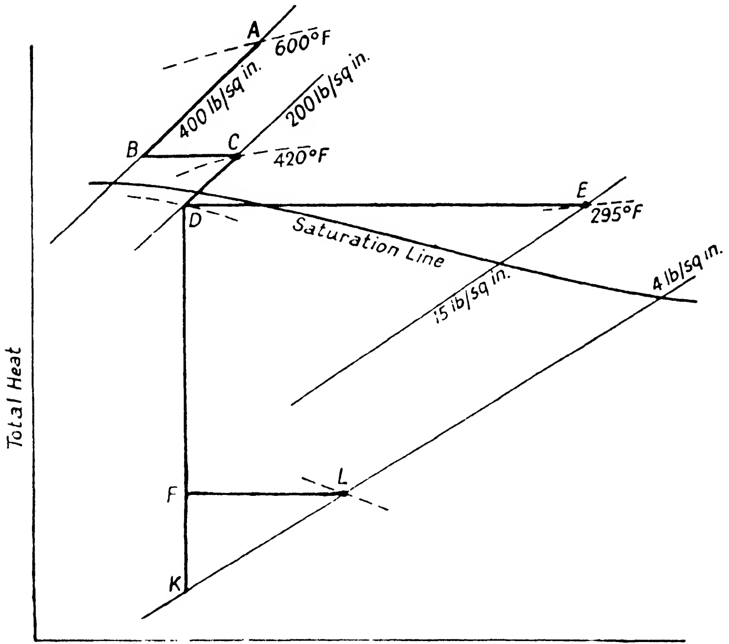


FIG. 152. H-ϕ CHART FOR EXAMPLE 3

(*a*) Dryness fraction $= x_D = 0{\cdot}99$

(*b*) Heat loss in pipe 1 $= H_A - H_B = 1{,}305 - 1{,}224$
$= 81$ B.Th.U. per lb.
$= 8{,}100$ B.Th.U. per min.

Heat loss in pipe 2 $= H_C - H_D = 1{,}224 - 1{,}192$
$= 32$ B.Th.U. per lb.
$= 3{,}200$ B.Th.U. per min.

(c) Actual heat drop $= H_D - H_F = 1{,}192 - 992$
$= 200$ B.Th.U. per lb.

$$\therefore \qquad \text{H.P.} = \frac{100 \times 200 \times 778}{33{,}000} = 472$$

(*d*) Final dryness fraction $= x_L = 0{\cdot}865$

The Theoretical Indicator Diagram. In practice, the expansion in a steam engine is not complete. The volume of the cylinder necessary would be excessive, as the specific volume of steam is large at low pressures compared with that at higher pressures. In practical cases, therefore, the pressure at release is greater than the pressure at the exhaust pipe and approximates to the cycle *abfge*, Fig. 151.

Now in practice there are, among others, two important modifications to be considered. In the first place, when the piston is at either end of the stroke there is the clearance volume, that is, the volume of working substance contained between the piston and the cylinder end, and in the ports leading from the cylinder to the valve chest. In the second place, the working substance is not pushed out to exhaust for the whole of the return stroke, but a certain amount is compressed into the clearance space. The theoretical indicator diagram so modified is shown in Fig. 153.

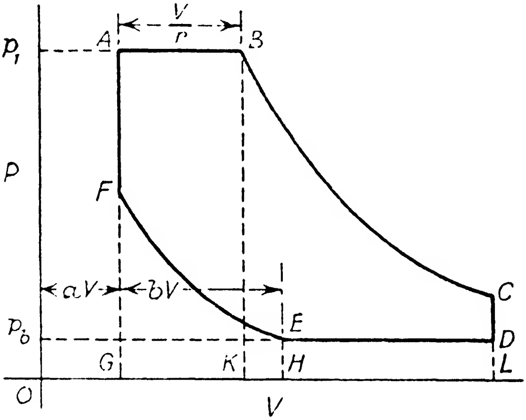


FIG. 153. THEORETICAL INDICATOR DIAGRAM

In the diagram, OG represents the clearance volume, AB shows the admission of steam to the cylinder, BC the expansion which is assumed hyperbolic for this purpose, CD is the pressure drop at release, while DE is the exhaust and EF the compression portions of the return stroke. After compression the pressure is represented by GF. Admission of fresh steam at F then raises the pressure to the initial pressure GA.

Mean Effective Pressure from the Theoretical Diagram. Let the stroke volume $GL = V$; clearance volume $OG = aV$; compression volume $GH = bV$; admission volume, $AB = \frac{V}{r}$; initial pressure, $AG = p_1$; back pressure $DL = p_b$.

Then, as the area $ABCDEF$ represents the work done during the cycle, the mean effective pressure

$$p_m = \frac{\text{Area } ABCDEF}{V}, \text{ (the area being measured in } p \text{ and } v \text{ units)}$$

$$= \frac{ABKG + BCLK - EDLH - FEHG}{V}$$

$$= \frac{(BK \times AB) + \left(BK \times OK \log_\varepsilon \frac{OL}{OK}\right)}{V} - \frac{(DL \times HL) + EH \times OH \log_\varepsilon \frac{OH}{OG}}{V}$$

$$= \frac{p_1 \frac{V}{r} + p_1\left(aV + \frac{V}{r}\right) \log_\varepsilon \frac{V + aV}{aV + \frac{V}{r}}}{V} - \frac{p_b(V - bV) + p_b(aV + bV) \log_\varepsilon \frac{aV + bV}{aV}}{V}$$

$$= p_1 \left(\frac{1}{r} + \left(a + \frac{1}{r}\right) \log_\varepsilon \frac{1 + a}{a + \frac{1}{r}}\right) - p_b \left(1 - b + (a + b) \log_\varepsilon \frac{a + b}{a}\right)$$

If there is no compression, then $b = 0$

$$\text{and } p_m = p_1 \left(\frac{1}{r} + \left(a + \frac{1}{r}\right) \log_\varepsilon \frac{1 + a}{a + \frac{1}{r}}\right) - p_b$$

If there is no compression and no clearance, then $a = 0$ and $b = 0$

$$\text{and } p_m = \frac{p_1}{r}(1 + \log_\varepsilon r) - p_b.$$

Steam Consumption from the Theoretical Diagram (the Willans Line). At the point of cut-off B the volume of steam in the cylinder is $\left(aV + \frac{V}{r}\right)$. If W_1 is the weight of a cubic foot of steam at pressure p_1, then the weight of steam in the cylinder at cut-off is $\left(a + \frac{1}{r}\right) V W_1$. At the point of compression E, the volume of steam in a cylinder is

$(aV + bV)$. If W_b is the weight of a cubic foot of steam at the pressure p_b, then the weight of steam in the cylinder at the beginning of compression is $(a + b)VW_b$.

The weight of steam used per stroke is therefore

$$\left(a + \frac{1}{r}\right) VW_1 - (a + b)VW_b$$

Assuming for a double-acting engine that the diagrams for each end of the cylinder are the same, then, if the engine is running at N r.p.m., the steam consumption in pounds per hour is given by

$$W = 2N \times 60 \times V\left\{\left(a + \frac{1}{r}\right)W_1 - (a + b)W_b\right\}$$

If the assumption is made that steam is dry at the points B and E, the values of W_1 and W_b can be obtained from the steam tables.

Now a trial calculation will show that, for ordinary ranges of pressure, the density of steam is a linear function of the pressure, that is, $W = C_1 + C_2p$, where C_1 and C_2 are constants.

Hence the steam consumption W may be written

$$W = 2N \times 60 \times V\left\{\left(a + \frac{1}{r}\right)(C_1 + C_2p_1) - (a + b)(C_1 + C_2p_b)\right\}$$

If the engine is governed by throttling, that is, by varying p_1 while the speed N and the cut-off are kept constant, then

$$W = C_3p_1 + C_4, \text{ where } C_3 \text{ and } C_4 \text{ are constants.}$$

With the same assumption, the mean effective pressure may be written $p_m = C_5p_1 - C_6$, where C_5 and C_6 are constants.

But I.H.P. $= \dfrac{p_mLA2N}{33{,}000}$ $\quad\therefore$ I.H.P. $= C_7p_m = C_8p_1 - C_9$

$$\therefore p_1 = \frac{\text{I.H.P.} + C_9}{C_8}$$

Hence $\quad W = C_3p_1 + C_4 = \dfrac{C_3(\text{I.H.P.} + C_9)}{C_8} + C_4$

i.e. $\quad W = (A \times \text{I.H.P.}) + B$, where A and B are constants.

That is, the steam consumption is a linear function of the indicated horse-power.

The Effect of Governing on the Theoretical Indicator Diagram. A reciprocating steam engine may be governed by means of a throttle valve, or by alteration of the cut-off. It is instructive to consider the effect of these two methods, by use of the equations given in the last two paragraphs. The matter will best be made clear by means of a numerical example. Assume the following conditions for a double-acting engine—

Piston speed $= 2LN = 500$ ft. per min.; area of piston $A = 80$ sq. in.; stroke $L = 15$ in.

$$\therefore \text{Speed } N = \frac{500 \times 12}{2 \times 15} = 200 \text{ r.p.m.}$$

$$\text{Stroke volume } V = LA = \frac{15}{12} \times \frac{80}{144} = 0{\cdot}695 \text{ cu. ft.}$$

Initial pressure, $p_1 = 100$ lb. per sq. in. abs.; back pressure $p_b = 15$ lb. per sq. in. abs.; ratio of expansion $r = 3$; clearance volume $= 0{\cdot}05 \times$ stroke volume, where $a = 0{\cdot}05$. Zero compression assumed.

Mean effective pressure

$$p_m = 100\left\{0{\cdot}333 + (0{\cdot}05 + 0{\cdot}333)\,2{\cdot}3 \log_{10} \frac{1{\cdot}05}{0{\cdot}333 + 0{\cdot}05}\right\} - 15$$

$$= 56{\cdot}9 \text{ lb. per sq. in. absolute}$$

$$\text{I.H.P.} = \frac{p_m LA\, 2N}{33{,}000} = \frac{56{\cdot}9 \times 80 \times 500}{33{,}000} = 68{\cdot}9$$

$$\text{Steam consumption} = W = 2N \times 60V\left\{\left(a + \frac{1}{r}\right)w_1 - aw_b\right\}$$

$$= 2 \times 200 \times 60 \times 0{\cdot}695\left(\frac{0{\cdot}05 + 0{\cdot}333}{4{\cdot}451} - \frac{0{\cdot}05}{26{\cdot}27}\right)$$

$$= 1{,}400 \text{ lb. per hour.}$$

Case I. Throttle Governing. Values of p_m, I.H.P., and W may be calculated for various values of the initial pressure p_1, all other conditions being kept constant. In this way the following figures will be obtained—

p_1	p_m	I.H.P.	W (lb. per hr.)
100	56·9	68·9	1,400
80	42·5	51·4	1,135
70	35·3	42·7	1,000
60	28·1	34	852
40	13·8	16·7	584

Case II. Cut-off Governing. Values of p_m, I.H.P. and W may be calculated for various values of the expansion ratio r with the constant value $p_1 = 100$ and all other conditions constant.

In this way the following figures are obtained—

$\frac{1}{r}$	p_m	I.H.P.	W (lb. per hr.)
0·333	56·9	68·9	1,400
0·25	47·6	57·5	1,100
0·20	40·8	49·4	903
0·15	33·1	40	716
0·1	24·2	29·8	517

Corresponding values of W and I.H.P. can be plotted as in Fig. 154, which clearly indicates the theoretical advantage of cut-off governing over throttle governing.

The Actual Indicator Diagram. We will now consider how the actual indicator diagram varies from the ideal. If the boiler pressure is represented by the height of the line AB, Fig. 515, the pressure in the cylinder at the beginning of the stroke will be somewhat less, as at F, because a certain difference of pressure is necessary to produce a flow of steam from the boiler to the engine cylinder. As the stroke proceeds there will generally be a slight fall of pressure, as shown at FG, owing to wire-drawing through the steam ports. The point of cut-off will not be sharp but rounded off as at G. This wire-drawing at cut-off will be more marked in the case of a slide valve, where cut-off is not instantaneous, than in the case of a Corliss or a drop valve provided with a trip mechanism, where cut-off is performed much more quickly. Release must occur at H before the end of the stroke, owing to wire-drawing through the exhaust ports, thus causing a rounding off of the toe of the diagram. During the exhaust stroke the pressure in the cylinder will be somewhat higher than that of the condenser in the case of a condensing engine, and somewhat higher than atmospheric pressure in the case of a non-condensing engine. The exhaust valve closes at some point K, before the end of the exhaust stroke. The reason for this is twofold. In the first case the steam left in the cylinder forms a cushion, which gradually brings the piston to rest

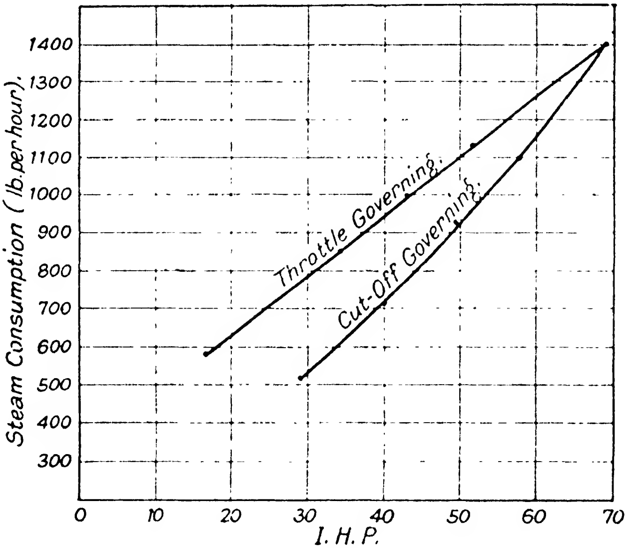


FIG. 154. COMPARISON OF THROTTLE AND CUT-OFF GOVERNING

and thus obviates the shock on the bearings which would otherwise be produced. In the second case, the resulting increase of pressure reduces the amount of live steam necessary to bring the pressure at the beginning of the working stroke up to the admission pressure. Owing to wire-drawing effects, the live steam is admitted just before the end of the exhaust stroke at L, the pressure produced by cushioning up to this point being thus raised to the admission pressure by the time the piston has arrived at the end of the exhaust stroke.

The effect of wire-drawing is obviously to decrease the area of the diagram, that is, the work done by the engine is decreased. This, however, is to some extent compensated for by the fact that wire-drawing or throttling dries the steam slightly.

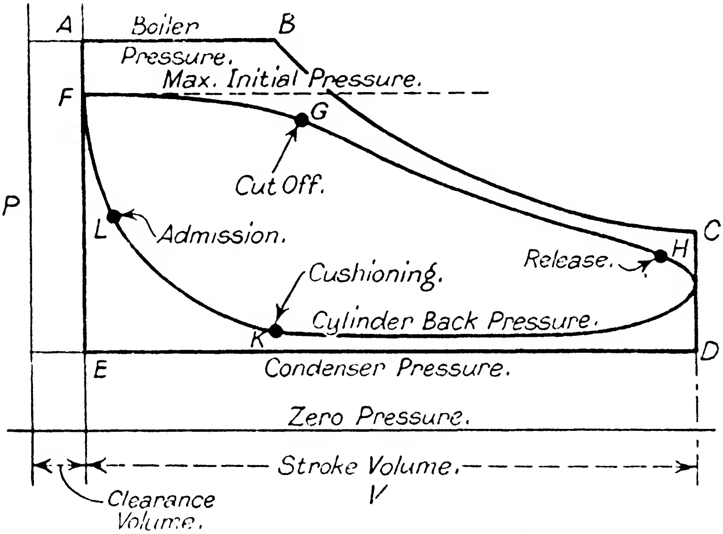


FIG. 155. COMPARISON BETWEEN THEORETICAL AND ACTUAL INDICATOR DIAGRAMS

The effect of clearance would, at first sight, appear to be simply to increase the steam consumption, as before the steam at admission can do work it must raise the pressure from that at the point L to that at the point F but, from p. 425, we can see that clearance increases the mean effective pressure and thus increases the work done. The net effect of clearance, however, is to decrease the efficiency. The increase in steam consumption due to clearance can be reduced by making the point of cushioning earlier, and thus increasing the pressure obtained at L when the live steam is admitted to the cylinder. Earlier cushioning, however, obviously decreases the area of the diagram, that is, the work done is lessened.

We will now discuss the admission and expansion curve *FGH*.

During the exhaust stroke the cylinder and piston have been in contact with steam at a low pressure and a corresponding lower saturation temperature than the temperature of the live steam. If the steam on admission is dry saturated it must give up heat to the walls of the clearance space, and hence a certain amount of initial condensation takes place. This initial condensation will, of course, be greater the larger the amount of clearance surface exposed to the incoming steam. The condensation of the steam continues for a short time after cut-off, due to the fact that energy is being taken out of the steam as work is done. As the expansion proceeds the pressure drops and hence the saturation temperature falls, eventually below the temperature of the cylinder walls, thus causing re-evaporation to take place during the latter portion of the working stroke. This re-evaporation is increased at release, due to the sudden pressure drop and the consequent drop in saturation temperature at the end of the stroke. This re-evaporation may be enough to make the steam left in the cylinder during the exhaust stroke practically dry. If the steam were admitted to the cylinder in a superheated condition this effect of condensation and re-evaporation would be modified, for the cooling effect of the clearance surface would be to lower the temperature of the steam and not to condense it until the saturation temperature was reached.

The Diagram Factor k. From the previous paragraph it is obvious that the ideal mean effective pressure obtained from the expression

$$p_m = p_1 \left(\frac{1}{r} + \left(a + \frac{1}{r} \right) \log_e \frac{1 + a}{\frac{1}{r} + a} \right) - p_b \left\{ 1 - b + (a + b) \log_e \frac{a + b}{a} \right\}$$

will in general be greater than the actual mean effective pressure obtained in an engine. In order to obtain the probable actual mean effective pressure p_a, we must multiply the ideal value p_m by a factor k. Thus $p_a = kp_m$.

The value of the diagram factor to be used in any particular case depends, as we have seen in the last paragraph, on a number of considerations. The particular formula used for calculating p_m will affect the value of k. Again, the value will depend on the condition of the steam, the initial pressure, the back pressure, the speed of the engine, the type of engine, the type of valves, whether the cylinder is jacketed or not, etc.

An average value for k for a simple slide valve condensing engine is about 0·8; for a compound condensing engine 0·7; for a compound condensing engine using superheated steam 0·65.

Example 4. A double-acting steam engine has a single cylinder of diameter 33½ in. by 3·25 ft. stroke, and develops 600 I.H.P. at 100 r.p.m. Assuming a diagram factor of 0·82, find the ratio of expansion if the initial steam pressure is 155 lb. per sq. in. absolute and the back pressure 2 lb. per sq. in. absolute.

In this case a rough estimate may be made by assuming no clearance and no compression.

The actual M.E.P., or p_a, is given by

$$\text{Indicated horse-power} = \frac{p_a LA2N}{33{,}000}$$

where N = the revolutions per minute for a double-acting engine.

$$\therefore p_a = \frac{600 \times 33{,}000}{3{\cdot}25 \times 200 \times 861} = 35{\cdot}3$$

Also, $$p_a = k\left\{\frac{p_1}{r}(1 + \log_\varepsilon r) - p_b\right\}$$

where k is the diagram factor.

$$\therefore \frac{35{\cdot}3}{0{\cdot}82} = 43 = \frac{155}{r}(1 + \log_\varepsilon r) - 2$$

$$\therefore 155 + 155 \log_\varepsilon r = 45r$$

$$31 + 31 \times 2{\cdot}3 \log_{10} r = 9r$$

and $$\log_{10} r = \frac{9r - 31}{71{\cdot}3} = x$$

Take different values of r and plot values of $\log r$ and x, and so find $r = 12$, which makes $\log r = x$. Therefore ratio of expansion $r = 12$.

Example 5. A non-condensing steam engine with a single cylinder has to develop 75 B.H.P. Determine from the data given below the diameter of the cylinder and the stroke of the piston.

(1) Boiler pressure, 80 lb. by gauge.
(2) Cut-off steam occurs at 28 per cent of piston stroke.
(3) Clearance volume, 6·5 per cent of volume swept by piston.
(4) Piston speed, 750 ft. per min.
(5) Diagram factor, 0·88.

You are to assume hyperbolic expansion and that the stroke of the piston is 1·5 times the diameter. (*U.L., B.Sc.* (*Eng.*), 1917.)

Let atmospheric pressure be 15 lb. per sq. in. absolute.

Then boiler pressure $p_1 = 80 + 15 = 95$ lb. per sq. in. absolute

$$\text{Mean effective pressure} = p_m = p_1\left\{\frac{1}{r} + \left(a + \frac{1}{r}\right)\log_\varepsilon \frac{1 + a}{\frac{1}{r} + a}\right\} - p_b$$

$$= 95\left\{0{\cdot}28 + (0{\cdot}065 + 0{\cdot}28)2{\cdot}3 \log_{10} \frac{1{\cdot}065}{{\cdot}28 + 0{\cdot}065}\right\} - 15$$

$$= 48{\cdot}5 \text{ lb. per sq. in.}$$

$\therefore$ Actual M.E.P. $= p_a = 0{\cdot}88 \times 48{\cdot}5 = 42{\cdot}7$ lb. per sq. in.

Piston speed $= 2LN = 750$ ft. per min. (for a double-acting engine).

If we assume a mechanical efficiency of 90 per cent, then

$$\text{I.H.P.} = \frac{\text{B.H.P.}}{\text{M.E.}} = \frac{75}{0{\cdot}9} = 83, \text{ say}$$

$$\text{Now I.H.P.} = \frac{p_a LA2N}{33{,}000}\ ;\ \therefore\ A = \frac{33{,}000 \times 83}{42{\cdot}7 \times 750} = 85{\cdot}5 \text{ sq. in.}$$

$$\therefore \text{ Diameter } = \sqrt{\frac{4 \times 85{\cdot}5}{3{\cdot}14}} = 10{\cdot}4 \text{ in.}$$

$$\therefore \text{ Stroke } = 1{\cdot}5 \times 10{\cdot}4 = 15{\cdot}6 \text{ in.}$$

Application of the Saturation Curve to the Indicator Diagram (the Missing Quantity). The weight of working substance (steam and water) in the cylinder at any point between cut-off and release is made up of the clearance steam and the stuff which is admitted to the cylinder between admission and cut-off, that is—

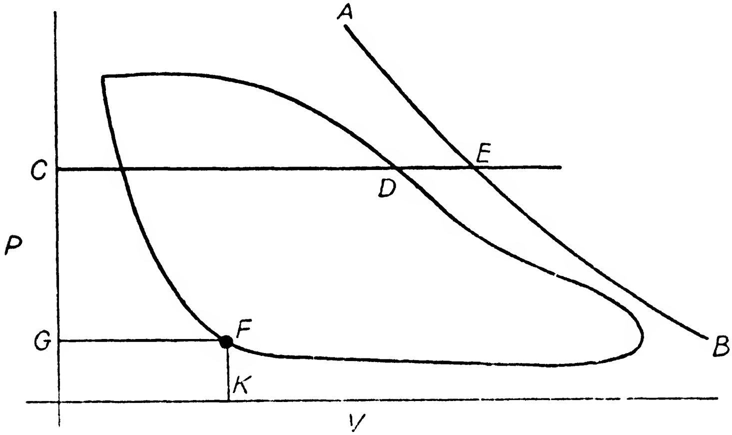


FIG. 156. APPLICATION OF SATURATION CURVE TO INDICATOR DIAGRAM

Weight of stuff = weight of clearance steam + cylinder feed,

where the cylinder feed is the weight of stuff passing through the cylinder per stroke.

If a pressure volume curve for this weight of dry saturated steam were plotted from the steam tables on to the indicator diagram, some such curve as *AB*, Fig. 156, would be obtained, the reason being that the stuff in the cylinder would not in general be dry saturated.

The volume of the water is negligible compared with that of the steam, and hence *CD* represents the volume of steam in the cylinder at the point *D*. Were all the stuff in the cylinder at this point dry saturated steam, its volume would be equal to *CE*. Hence the dryness fraction of the working substance in the cylinder at the point D is measured by $\frac{CD}{CE}$. It will thus be evident that the indicator diagram, while accounting for that part of the working substance which is steam, gives no indication of how much water is present, unless the saturation curve is drawn on it. The volume of steam represented by DE is called the missing quantity. To apply the saturation curve we must know the weight of the clearance steam and the weight of the cylinder feed.

The weight of the clearance steam can be found from the indicator diagram as follows: it has been explained that just after the beginning of cushioning the steam left in the cylinder is practically dry. Let F, Fig. 156, be a point just after cushioning. Then GF is the volume of the clearance steam at a pressure represented by FK. Hence if W = weight of 1 cu. ft. of dry steam at the pressure represented by FK, then

$$\text{Weight of clearance steam} = (\text{Volume represented by } GF) \times W$$

Steam tables may not always give values of W, but they generally give values of V, the volume of 1 lb. of dry saturated steam at various pressures. Obviously then, weight of clearance steam $= \frac{\text{Volume } (GF)}{V}$.

If the cylinder feed is now known, the saturation curve AB for a weight of dry saturated steam equal to weight of (clearance steam + cylinder feed) can be drawn on the indicator diagram.

If all the steam that passed into the steam chest of the engine also passed through the cylinder, and none leaked past the valve or the piston, then the cylinder feed could be measured by weighing the condensate in the case of a condensing engine. Experimental evidence, however, goes to show that there is a certain amount of leakage past the valve and the piston, and hence, in this case, if the saturation line AB be drawn from the weight of condensate, then the missing quantity DE will not only be due to condensation but also to leakage. In this case, $\frac{CD}{CE}$ will no longer accurately represent the dryness fraction in the cylinder.

Example 6. The following results were obtained by measurements taken on an indicator card from a double-acting steam engine.

(1) Immediately after cut-off—

Volume = 3·37 cu. ft.; pressure = 178 lb. per sq. in. absolute.

(2) Immediately after compression has begun—

Volume = 1·03 cu. ft.; pressure = 57 lb. per sq. in. absolute.

The speed of the engine was 105 r.p.m. and the measured steam supply per minute was 405·3 lb. Find the missing quantity in pounds per minute.

(*U.L.*, *B.Sc.* (*Eng.*), 1917.)

Volume of 1 lb. of dry steam at 57 lb. per sq. in.

= 7·539 cu. ft. (from tables)

$$\therefore \text{ weight of cushion steam} = \frac{1\cdot03}{7\cdot539}$$
$$= 0\cdot137 \text{ lb. (assuming steam is dry)}\ .$$

Weight of steam passing through cylinder per stroke

$$= \frac{405\cdot3}{2 \times 105} = 1\cdot93 \text{ lb.}$$

$\therefore$ weight of stuff in cylinder just after cut-off

$$= 1\cdot93 + 0\cdot137 = 2\cdot067 \text{ lb.}$$

Volume of 1 lb. of dry steam at 178 lb. per sq. in.

$$= 2\cdot59 \text{ cu. ft. (from tables)}$$

$$\therefore \text{ Indicated weight of steam just after cut-off} = \frac{3\cdot37}{2\cdot59} = 1\cdot301 \text{ lb.}$$

$$\therefore \text{ missing quantity} = 2\cdot067 - 1\cdot301 = 0\cdot766 \text{ lb. per stroke}$$
$$= 0\cdot766 \times 2 \times 105 = 161 \text{ lb. per min.}$$

It should be noted that this is the missing quantity at the beginning of the expansion. Owing to re-evaporation during expansion the missing quantity at the end of expansion would generally become less. This quantity could be calculated if the indicated pressure and volume just before release were known.

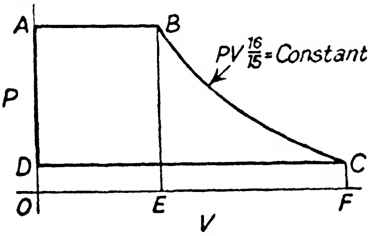


FIG. 157

The Steam Jacket. We have seen in a previous paragraph that the steam in a steam engine cylinder is wet during expansion. This condensation can be reduced to some extent by steam jacketing the cylinder. It must be remembered, however, that although by this means the actual expansion curve on the indicator diagram can be brought nearer to the saturation curve, steam must be used to pass through the jacket.

Let us first consider the theoretical case where dry saturated steam is supplied to the engine and this steam is kept dry throughout the cycle by means of a steam jacket. The law of the expansion curve can be taken as $PV^{\frac{16}{15}} = \text{constant}$.

Let $ABCD$, Fig. 157, be the cycle, there being no clearance. Then, work done during the cycle = Area $ABCD$

$$= ABEO + EBCF - DCFO = P_B V_B + \frac{P_B V_B - P_C V_C}{\frac{16}{15} - 1} - P_C V_C$$

$$= 16(P_B V_B - P_C V_C) \text{ work units} = \frac{16}{J}(P_B V_B - P_C V_C) \text{ heat units}$$

The heat taken from the boiler = Total heat at $B = H_B$
(measured from 0° C.)

The heat taken in from jacket = Q

The total heat rejected = Total heat at $C = H_C$

∴ net heat taken in = $H_B + Q - H_C$

$$\therefore\ H_B + Q - H_C = \frac{16(P_B V_B - P_C V_C)}{J}$$

∴ heat taken in from jacket during cycle is

$$Q = \frac{16(P_B V_B - P_C V_C)}{J} + H_C - H_B$$

Let Q' = heat taken in from jacket during expansion. Then work done during expansion $= \dfrac{P_B V_B - P_C V_C}{\frac{16}{15} - 1} = 15(P_B V_B - P_C V_C)$

Heat in steam at $B = H_B$; and heat in steam at $C = H_C$

$$\therefore Q' + H_B = \frac{15(P_B V_B - P_C V_C)}{J} + H_C$$

$$\therefore \qquad Q' = \frac{15(P_B V_B - P_C V_C)}{J} + H_C - H_B$$

In the above equations the values must be calculated for the total amount of steam in the cylinder. In this case, as there is no clearance, this will be the weight of cylinder feed.

In practice, even if steam were dry on entering the cylinder, it would be wet at cut-off owing to initial condensation. The clearance steam can be found from the indicator diagram, and if the cylinder feed is known, then the total weight of stuff in the cylinder can be obtained, and hence the dryness fraction just after cut-off and just before release can be calculated. The values of pressure and volume just after cut-off and just before release being found from the indicator diagram, the value of n in the expansion law PV^n = constant can be calculated. Values of total heat of the stuff at cut-off and release are calculated from the equation $H = h + xL$, and hence the amount of heat supplied by the jacket during expansion can be found.

Example 7. The pressure indicated at cut-off in the cylinder of a jacketed steam engine is 75 lb. per sq. in. absolute, the volume is 0·45 cu. ft. and the dryness fraction of the steam is 0·73. If the values of pressure and volume at release are 34·2 lb. per sq. in. and 1·05 cu. ft., find the heat which passes through the cylinder walls during expansion. Assume PV^n = constant.

Volume of 1 lb. of dry steam at 75 lb. per sq. in. absolute is 5·83 cu. ft.

∴ indicated weight of steam at cut-off is

$$\frac{\text{Indicated volume at cut-off}}{5{\cdot}83} = \frac{0{\cdot}45}{5{\cdot}83} = 0{\cdot}077 \text{ lb.}$$

∴ weight of stuff in cylinder during expansion is

$$\frac{\text{Indicated weight of steam}}{\text{Dryness fraction}} = \frac{0{\cdot}077}{0{\cdot}73} = 0{\cdot}105 \text{ lb.}$$

Volume of 1 lb. of dry steam at 34·2 lb. per sq. in. absolute is 12·16 cu. ft.

∴ volume of stuff in cylinder at release if all were dry is

$$12{\cdot}16 \times 0{\cdot}105 = 1{\cdot}277 \text{ lb.}$$

Dryness fraction at release is

$$\frac{\text{Indicated volume of steam}}{\text{Volume of stuff if it were all dry}} = \frac{1{\cdot}05}{1{\cdot}277} = 0{\cdot}823$$

From the steam tables the following values are obtained—

p (lb. per sq. in. absolute) . .	h (C.H.U.)	L (C.H.U.)	x (dryness fraction)
75 ,, ,, ,, . .	154·07	504·38	0·73
34·2 ,, ,, ,, . .	125·82	523·26	0·823

∴ Heat in steam just after cut-off

$$= 0{\cdot}105(154{\cdot}07 + 0{\cdot}73 \times 504{\cdot}38) = 54{\cdot}84 \text{ lb. calories}$$

Heat in steam just before release

$$= 0{\cdot}105(125{\cdot}82 + 0{\cdot}823 \times 523{\cdot}26) = 58{\cdot}43 \text{ lb. calories}$$

Now the value of n in the law for expansion PV^n = constant is found from

$$n = \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} = \frac{\log 75 - \log 34{\cdot}2}{\log 1{\cdot}05 - \log 0{\cdot}45} = 0{\cdot}927$$

∴ Work done during expansion

$$= \frac{144(75 \times 0{\cdot}45 - 1{\cdot}05 \times 34{\cdot}2)}{1{,}400(0{\cdot}927 - 1)} = 3{\cdot}04 \text{ lb. calories}$$

Heat in steam just after cut-off + Heat supplied from steam jacket = Heat in steam just before release + Heat equivalent of work done.

∴ Heat supplied from steam jacket is

$$58{\cdot}43 + 3{\cdot}04 - 54{\cdot}84 = 6{\cdot}6 \text{ lb. calories}$$

The Heat Balance in a Steam Engine. The equation of steady flow, namely,

$$H_1 - H_2 + A\left(\frac{U_1^2 - U_2^2}{2g}\right) = AW + Q$$

holds for any heat appliance, and hence can be applied to the steam engine.

The velocities U_1 and U_2 in the admission and exhaust pipes of the steam engine are both comparatively small enough to be neglected, and hence we have for the steam engine the relation $H_1 - H_2 = AW + Q$, that is, the difference between the total heat H_1 in the admission pipe and the total heat H_2 in the exhaust pipe is equivalent to the work done AW in the engine, together with the heat lost Q to the surroundings. It must be noted that Q is the whole amount of heat lost between the two points where H_1 and H_2 are measured. It can be reduced in practice by lagging the steam cylinder. If the loss of heat Q could be reduced to zero then $H_1 - H_2 = AW$, that is, the work done in the engine is equal to the actual total heat drop in the steam in passing from the admission pipe to the exhaust pipe.

This actual heat drop in a practical case is not the same as the adiabatic heat drop for the admission and exhaust pressures. For this to be the case, two conditions must be fulfilled. In the first place no heat must pass from or to the surroundings, and secondly all processes through which the steam passes must be reversible. Now in a practical case, although we may consider the heat lost Q as negligible, the process through which the steam passes in the cylinder is by no means reversible. As we have seen, owing to initial condensation and re-evaporation, there is a constant interchange of heat between the steam and the cylinder walls. This takes place while the steam is at a different temperature to that of the walls, and hence, although there may not be any loss of heat to the surroundings, yet the action is an irreversible one. In the next place, throttling or wire-drawing taking place in the steam and exhaust ports involves internal friction, which also produces an irreversible action. Both these effects, therefore, reduce the actual heat drop to a value less than the corresponding adiabatic heat drop.

If, therefore, we assume that no heat is lost to the surroundings in a practical case, the efficiency ratio of the engine, which is measured by $\dfrac{\text{Work done}}{\text{Adiabatic heat drop}}$ may be expressed by $\dfrac{\text{Actual heat drop}}{\text{Adiabatic heat drop}}$.

If there is a quantity of heat Q lost to the surroundings, then work done $AW = H_1 - H_2 - Q$,

$$\text{and the efficiency ratio} = \frac{\text{Actual heat drop} - Q}{\text{Adiabatic heat drop}}$$

Application of the Indicator Diagram to the Temperature-Entropy Chart. If a temperature-entropy chart is provided with constant volume and constant pressure lines it is a comparatively simple matter to transfer the indicator diagram of a steam engine to the T-ϕ chart. It must be remembered that, while the indicator diagram represents the actual weight of the steam in the cylinder between cut-off and release, the T-ϕ chart is drawn for 1 lb. of stuff.

The expansion line can be transferred direct from one chart to another if the saturation line is drawn on the indicator diagram. Thus, in Fig. 158, the dryness fraction for any point b on the P-V diagram between cut-off and release is measured by $\frac{ab}{ac}$. Hence, if $a'\,c'$ and ac are at corresponding pressures, and $\frac{a'b'}{a'c'}$ is made equal to $\frac{ab}{ac}$, the point b' on the T-ϕ chart will correspond to the point b on the P-V chart. In this way the whole expansion line may be drawn. For any other point the weight of stuff in the cylinder during expansion (including clearance) must be calculated as shown in a previous paragraph. If this weight be W, then, volume to be plotted on T-ϕ chart $= \frac{\text{Volume from } P\text{-}V \text{ chart}}{W}$. Having in this way corrected the volumes from the P-V chart to volumes corresponding to 1 lb. of working substance, each pair of values of pressure and volume can be plotted on the T-ϕ chart, by the aid of the constant pressure and constant volume lines.

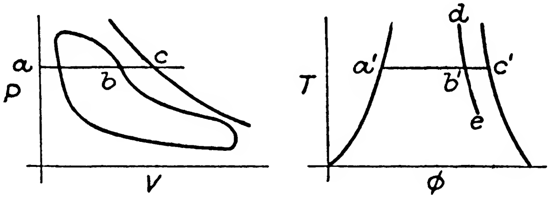


FIG. 158

Boulvin's Method of Drawing the T-ϕ Diagram. In this method four scales for entropy, temperature, pressure, and specific volume are drawn from a common centre C, Fig. 159.

The following curves are now drawn in the four sections so obtained—

Section I. The curves connecting temperature and entropy for water (A) and for dry saturated steam (B).

Section II. The curve connecting temperature and pressure for dry saturated steam (C).

Section III. The curve connecting pressure and specific volume for dry saturated steam (*D*).

The weight of stuff in the cylinder during expansion must now be calculated as shown in a previous paragraph. Knowing this weight *W*, the volumes on the indicator diagram can now be reduced to volumes corresponding to 1 lb. of stuff, and the indicator diagram so reduced can be drawn in Section III of the chart.

The indicator diagram so obtained for 1 lb. of stuff can be transferred to Section I as follows. In Section I draw any constant pressure line *ab* and produce it across to cut the pressure temperature curve *C* in *c*. From this point *c* drop a vertical line to cut the pressure volume curve *D* in *d*. Draw a horizontal line through *d* and a vertical line down through *b* to cut one another in *e*. Project *a* vertically down to cut the entropy scale at *a'*.

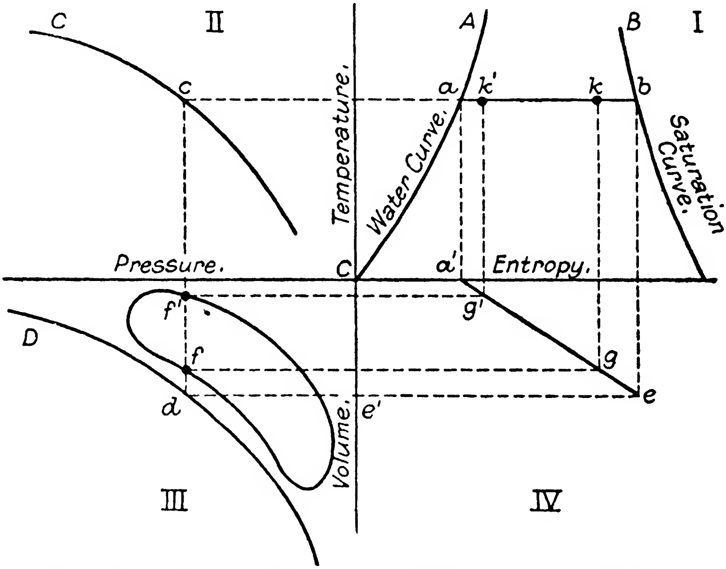


FIG. 159. BOULVIN'S METHOD OF DRAWING THE T-ϕ DIAGRAM

A little thought will show that the point *e* on the volume scale measures the volume of 1 lb. of dry steam of a temperature and entropy given for the point *b*. The volume of 1 lb. of stuff corresponding to *a* can be taken as zero, as it is there all water. Hence, if we join *ea'* we obtain a line *ea'* which gives the volume of 1 lb. of stuff at a pressure and temperature corresponding to the point *b* for all conditions from dry steam to water. The two points on the indicator diagram corresponding to this pressure are *f* and *f'*. Hence, if we project these two points *f* and *f'* across to *g* and *g'*, and then up to *k* and *k'*, we obtain the points *k* and *k'* on the T-ϕ

chart, corresponding to the points f and f' on the P-V chart. By proceeding thus for various pressures, the whole T-ϕ diagram may be obtained.

Compound Engines. The tendency in modern practice is to use higher and higher boiler pressures. This makes it necessary to have a large ratio of expansion which, if carried out in a single cylinder, would be uneconomically and practically disadvantageous. In the first place the large ratio of expansion would necessitate a very early cut-off and a very large and heavy cylinder, because not only would it be necessary for the cylinder to accommodate the large volume of steam at the low exhaust pressure, but the walls would have to be thick enough to withstand the high initial pressure. The initial stress on the moving parts at the beginning of the stroke would also be excessive, and heavy moving parts would be necessary. Again, with a single cylinder engine the turning moment on the crankshaft is very variable, and a heavy flywheel would be required to even out the variation of torque during the cycle. Another disadvantage would be the amount of initial condensation, which, of course, increases with the temperature variation of the cylinder walls, which increases with the difference of pressure of incoming and exhaust steam.

By dividing the pressure drop between two or more cylinders, the cut-off in any one of the cylinders is reduced, the low pressure cylinders need not have such thick walls, while, as the range of stress on any one piston is reduced, the moving parts may be of smaller dimensions. By arranging the cranks at 90° to one another in the case of two cylinders, or at 120° in the case of three cylinders, the torque on the crankshaft will be more uniform during the cycle. This, however, is not the case if the cranks are in line or at 180° to one another.

The reduction of initial condensation is due in the first case to the smaller pressure difference in any one cylinder, and secondly to the fact that, as evaporation occurs during the exhaust stroke, the steam leaving one cylinder is practically dry again as it enters the succeeding cylinder.

Compound engines have two, three, or four cylinders. Two-cylinder compound engines may have the two cranks at 0°, 180°, or 90° to one another. In the first case there need be only one crank, the two pistons being in tandem on the same piston rod. In this case, and in the case where the cranks are 180° apart, the two pistons are at the end of their respective strokes at the same time, and hence the steam can be exhausted from the high pressure cylinder directly into the low pressure cylinder, the two cylinders being connected throughout the stroke. In this case there will be continuous expansion throughout the stroke in the low pressure cylinder and no independent cut-off. When an independent cut-off in the low pressure cylinder is required, or when the cranks are 90°

apart, a receiver must be provided between the two cylinders. This receiver takes the steam from the high pressure cylinder and holds it until the low pressure cylinder is ready to draw from it. The pressure in the receiver will obviously vary throughout the stroke, according to the supply of steam from the high pressure cylinder and the demand of the low pressure cylinder. In modern engines the receiver volume is generally amply provided for by the connecting pipe and low pressure steam chest between the cylinders. Receiver space is also necessary for triple expansion engines where the three cranks are 120° apart.

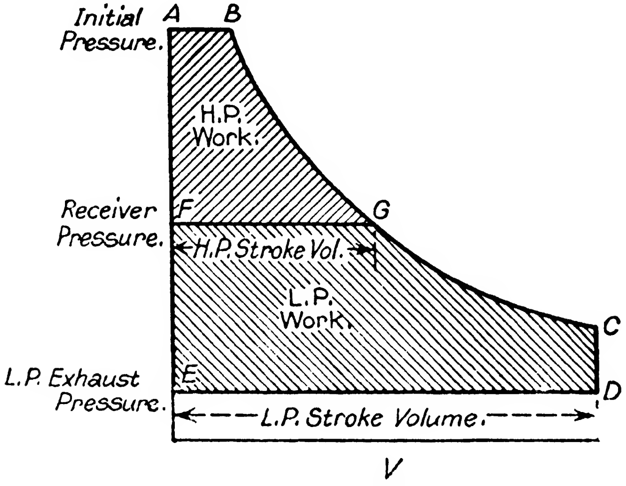


FIG. 160. THEORETICAL INDICATOR DIAGRAM FOR COMPOUND ENGINE

Two-cylinder Compound Expansion. In Fig. 160, let $ABCDE$ represent the hypothetical pressure-volume diagram for the complete expansion in a two cylinder compound engine, neglecting clearance and compression, and let the pressure at the point F be the mean receiver pressure. It has already been pointed out that the receiver pressure will vary throughout the stroke. If the volume of the receiver space is large compared with that of the cylinders we may assume that the receiver pressure remains constant throughout the stroke. In this case $ABGF$ will represent the P-V diagram for the high pressure cylinder and $FGCDE$ will represent the P-V diagram for the low pressure cylinder. The stroke volume of the low pressure cylinder is independent of the value of the mean receiver pressure, that is, it is not affected by the position of the line FG which divides the diagram up between the two cylinders. In other words, the stroke volume of the low pressure cylinder is such as if the total number of expansions $\frac{ED}{AB}$ takes place in the low pressure cylinder alone, and it is not affected by the ratio of the cylinder stroke volumes $\frac{ED}{FG}$. It will be noticed that in this case, where the volume of steam in the low pressure cylinder at cut-off, namely FG, is the same as the stroke volume in the high pressure cylinder, there is complete expansion (BG) in the high pressure cylinder. In practice, however, for reasons which will appear later, there is generally a drop of pressure at release in the high pressure cylinder, as shown at HK, Fig. 161.

For the purposes of calculation it is generally assumed that the expansions BH and GC in the two cylinders lie on the same curve,

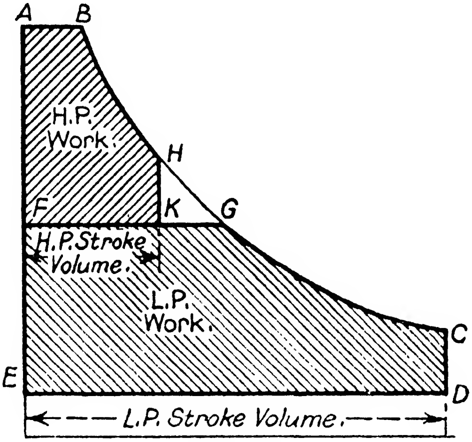


FIG. 161

and the law assumed for this curve is that for hyperbolic expansion, namely, $PV =$ constant.

Ratio of the Cylinder Volumes (R). The total number of expansions in the engine is represented by the ratio $\frac{ED}{AB}$, Fig. 161. The number of expansions in the high pressure cylinder is represented by $\frac{FK}{AB}$, or the cut-off in the high pressure cylinder is represented by $\frac{AB}{FK}$.

$$\text{Hence the cylinder volume ratio} = R = \frac{ED}{FK} = \frac{ED}{AB} \times \frac{AB}{FK}$$

$$= \text{Total number of expansions} \times \text{Cut-off in high pressure}$$

$$\text{or, Total number of expansions, } r = \frac{R}{\text{Cut-off in high pressure}}$$

Distribution of the Pressure Drop Between the Two Cylinders. Let p_1 = initial pressure in high pressure cylinder ; p_r = receiver pressure ; p_b = exhaust pressure.

In determining the receiver pressure, two conditions are desirable. In the first place, the initial load on the piston exerted by the steam at the commencement of the stroke should be approximately the same for the two cylinders. That is—

$$AF \times \text{Area of high pressure} = FE \times \text{Area of low pressure (Fig. 161)},$$

$$\text{i.e} \qquad (p_1 - p_r) = (p_r - p_b) \frac{\text{Area of low pressure}}{\text{Area of high pressure}}$$

If the stroke is the same for the two cylinders, the ratio of the areas will be the same as the ratio of the cylinder volumes R.

Hence, in order that the initial loads on the two pistons should be equal $(p_1 - p_r) = (p_r - p_b) R$.

The second condition is that the work done in the engine should be equally divided between the two cylinders.

Now work in a cylinder per stroke

$$= \text{Mean effective pressure} \times \text{Stroke volume}$$

Hence, for equal distribution of work, and assuming the simplest formula for the mean effective pressure, we have

$$\frac{p_1}{r_1}(1 + \log_\epsilon r_1) - p_r = \left\{\frac{p_r}{r_2}(1 + \log_\epsilon r_2) - p_b\right\} R$$

If clearance and compression are known, the more complicated formulæ for the mean effective pressure may be used.

The two conditions just discussed cannot generally be satisfied when the expansion curve is continuous, as in Fig. 160, and hence a compromise is made by having a pressure drop at release in the high pressure cylinder as at HK, Fig. 161.

Example 8. Estimate the cylinder dimensions of a compound steam engine to develop 500 I.H.P. at 120 r.p.m. Initial pressure 120 lb. per sq. in. absolute, back pressure 4 lb. per sq. in. absolute, allowable piston speed 500 ft. per min., cylinder ratio 3·5, diagram factor 0·85, cut-off in high pressure cylinder at 0·4 stroke. If the point of cut-off in the low pressure cylinder is at 0·53 stroke, determine the approximate low pressure receiver pressure and compare the initial loads on the two pistons. Assume no clearance.

$$\text{Total number of expansions} = \frac{\text{Cylinder volume ratio } (R)}{\text{Cut-off in high pressure cylinder}}$$

$$= \frac{3{\cdot}5}{0{\cdot}4} = 8{\cdot}75$$

$$\text{Mean effective pressure for whole engine} = \frac{p_1}{r}(1 + \log_\epsilon r) - p_b$$

$$= \frac{120}{8{\cdot}75}(1 + 2{\cdot}3 \log_{10} 8{\cdot}75) - 4 = 39{\cdot}5 \text{ lb. per sq. in.}$$

$\therefore$ actual mean effective pressure $= K \times 39{\cdot}5 = 0{\cdot}85 \times 39{\cdot}5$

$= 33{\cdot}6$ lb. per sq. in.

$$\text{I H.P.} = \frac{PLA2N}{33{,}000}, \text{ where } 2LN = 500 \text{ ft. per min.}$$

$$\therefore \quad A = \frac{33{,}000 \times \text{I.H.P.}}{P \times 2LN} = \frac{33{,}000 \times 500}{33{\cdot}6 \times 500} = 982 \text{ sq. in.}$$

$$\therefore \text{ Diameter of low pressure cylinder} = \sqrt{\frac{982 \times 4}{3{\cdot}14}} = 35{\cdot}4 \text{ in.}$$

$$\therefore \text{ Diameter of high pressure cylinder} = \frac{35{\cdot}4}{\sqrt{R}} = \frac{35{\cdot}4}{\sqrt{3{\cdot}5}} = 18{\cdot}93 \text{ in.}$$

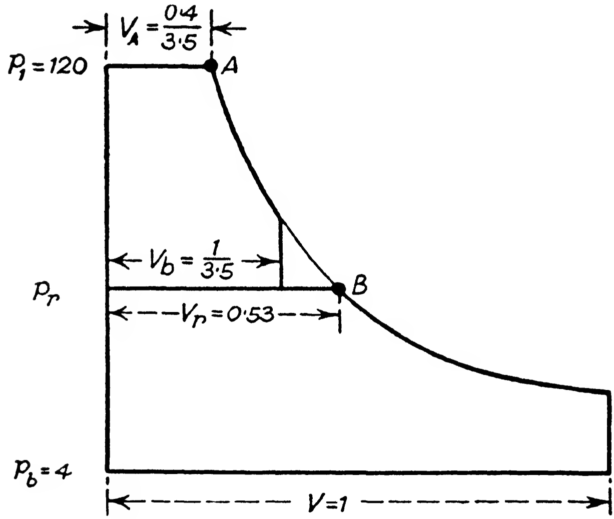


FIG. 162

Hence, the diameter of high pressure cylinder is 19 in.; diameter of low pressure cylinder, 35½ in.; and the stroke, 25 in.

Fig. 162 shows the hypothetical indicator diagram for the engine taking the stroke volume of the low pressure cylinder as unity.

To find the value of p_r we have, for the point A

$$p_a v_a = 120 \times \frac{0{\cdot}4}{3{\cdot}5}$$

$$\therefore \text{ pressure at } B = p_r = \frac{120}{0{\cdot}53} \times \frac{0{\cdot}4}{3{\cdot}5} = 25{\cdot}9 \text{ lb. per sq. in.}$$

$$\therefore \frac{\text{Load on high pressure piston}}{\text{Load on low pressure piston}} = \frac{(p_1 - p_r)}{(p_r - p_b)\,R} = \frac{(120 \; - 25{\cdot}9)}{(25{\cdot}9 - 4)3{\cdot}5}$$

$$= \frac{1{\cdot}22}{1}$$

Governing of Compound Engines. We will consider three cases, namely, governing by throttle, by variation of cut-off in the high pressure cylinder, and by variation of cut-off in the low pressure cylinder. For simplicity, consider an engine with a cylinder volume ratio of 5 : 2, and with cut-off in the high pressure cylinder at full load of 1/2. Let the back pressure in the low pressure cylinder be zero

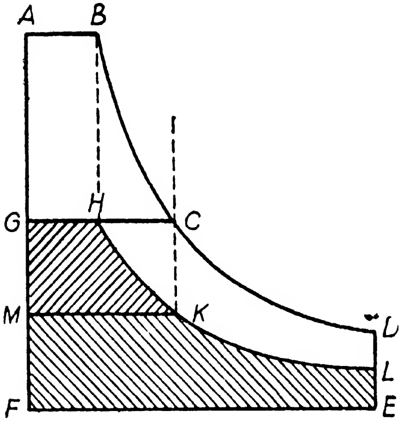


FIG. 163A. THROTTLE GOVERNING

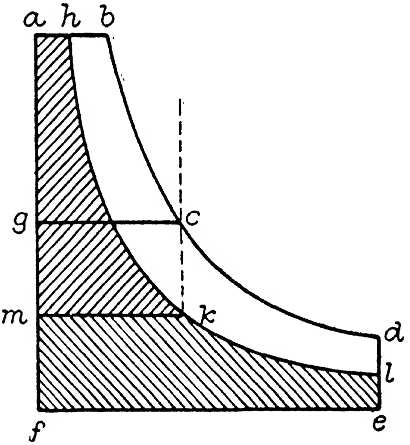


FIG. 163B. H.P. CUT-OFF GOVERNING

Figs. 163 (*A*, *B* and *C*) illustrates the three cases, the full load diagrams being *ABDEF*, *abdef*, *a'b'd'e'f'* respectively, the dividing line between the high pressure and low pressure diagrams being *GC*, *gc*, and *g'c'* respectively.

Case I. Throttle Governing. Suppose the full load pressure *FA* reduced to half by throttling, that is, to *FG*, the cut-off in each cylinder remaining the same. The light load diagram will become *GHLEF*. The high pressure work will be reduced from *ABCG* to *GHKM* and the low pressure work will be reduced from *GCDEF* to *MKLEF*.

Case II. High Pressure Cut-off Governing. Suppose for the light load the cut-off in high pressure is reduced from 1/2 to 1/4, the initial pressure and the cut-off in low pressure remaining constant. The light load diagram will become *ahlef*, the high pressure work becoming *ahkm*, which is not very different from *abcg*, while the low pressure work is reduced from *gcdef* to *mklef*. Governing by varying the cut-off in the high pressure cylinder will therefore reduce the proportion of the work done in the low pressure cylinder on light loads. With condensing engines at very light loads this may cause the average pressure in the low pressure cylinder to fall below that necessary to overcome the back pressure and the frictional resistance, thus reducing the efficiency of the engine.

A comparison of the diagrams Figs. 163A and 163B shows that the high pressure cut-off governing is more economical than throttle governing. The light load diagrams for the low pressure cylinder, namely, *MKLEF* and *mklef*, are the same in each case, the release pressure *LE* or *le* also being the same. Hence the same volume of steam at the same release pressure is exhausted from the engine in each case, whereas work done with throttle governing is represented by *GHLEF*, while the work done with high pressure cut-off governing is represented by the larger area *ahlef*.

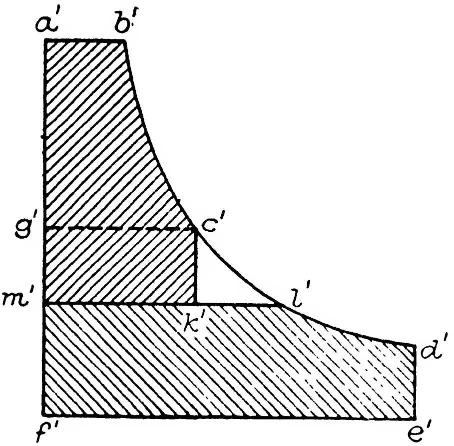


FIG. 163C. L.P. CUT-OFF GOVERNING

Case III. Low Pressure Cut-off Governing. Suppose the high pressure cut-off is kept constant at 1/2, while the low pressure cut-off is changed from 2/5 to 3/5. By this change of cut-off the high pressure work is increased from *a'b'c'g'* to *a'b'c'k'm'*, while the low pressure work is decreased from *g'c'd'e'f'* to *m'l'd'e'f'*, Fig. 163C. Thus, by making the cut-off in the low pressure cylinder earlier, the total work done by the engine is only slightly affected, but the proportion of the total work done in the low pressure cylinder is reduced, while the work done in the high pressure cylinder is increased.

Example 9. Obtain the ratio of cylinder areas for a compound steam engine receiving steam at 100 lb. per sq. in. absolute and exhausting at 3 lb. per sq. in. absolute. Cut-off in high pressure at 0·4 stroke and low pressure at 0·66 of stroke. Take the clearance in each case as 1/10 of the volume swept through by piston. Assume the cylinder feed to be 0·6 lb. per stroke and the weight shut in the high pressure and low pressure cylinders to be 0·05 and 0·03 respectively. Draw the probable combined diagram, putting in the intermediate pressure line so that the work shall be approximately equally divided between the cylinders. Estimate the horse-power of the combined engine and the steam used per horse-power hour. Assume a dryness fraction of 0·75 in the high pressure cylinder for initial condensation.

(*U.L., B.Sc. (Eng.)*, 1920.)

The diagram, Fig. 164, is obtained in the following manner—
Weight of stuff in high pressure cylinder at cut-off (B)

$$= \text{cylinder feed} + \text{clearance} = 0{\cdot}6 + 0{\cdot}05 = 0{\cdot}65 \text{ lb.}$$

The dryness fraction at $B = 0{\cdot}75$.

Volume of 1 lb. of dry steam at 100 lb. per sq. in. $= 4{\cdot}451$ cu. ft.

$\therefore$ indicated volume at $B = 4{\cdot}451 \times 0{\cdot}75 \times 0{\cdot}65 = 2{\cdot}17$ cu. ft.

Now the volume $AB = 0{\cdot}4 \times$ stroke volume of high pressure.

$\therefore$ Stroke volume of high pressure $= \dfrac{AB}{0{\cdot}4}$

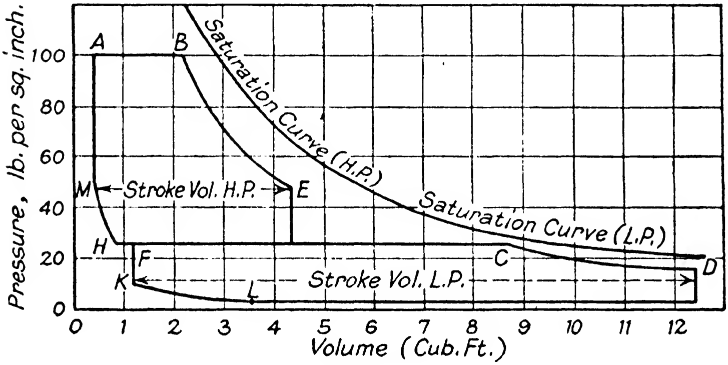


FIG. 164

Also, AB + Clearance of high pressure = Volume at $B = 2{\cdot}17$

i.e. $$AB + \frac{\text{Stroke volume of high pressure}}{10} = 2{\cdot}17$$

$$\therefore \quad AB + \frac{AB}{0{\cdot}4 \times 10} = 2{\cdot}17 \qquad \therefore \text{ volume } AB = 1{\cdot}736 \text{ cu. ft.}$$

$$\therefore \text{ Stroke volume of high pressure} = \frac{1{\cdot}736}{0{\cdot}4} = 4{\cdot}34 \text{ cu. ft.}$$

Clearance volume of high pressure is 0·434 cu. ft., and indicated volume at $E = 4{\cdot}34 + 0{\cdot}434 = 4{\cdot}774$ cu. ft.

For the work to be equally distributed between the cylinders we have

$$p_1\left(\frac{1}{r_1} + \left(a + \frac{1}{r_1}\right)\log_\varepsilon \frac{1+a}{\dfrac{1}{r_1}+a}\right) - p_r$$

$$= \left\{p_r\left(\frac{1}{r_2} + \left(a + \frac{1}{r_2}\right)\log_\varepsilon \frac{1+a}{\dfrac{1}{r_2}+a}\right) - p_b\right\} R$$

where, from the question, $p_1 = 100$; $p_b = 3$; $\frac{1}{r_1} = 0{\cdot}4$; $\frac{1}{r_2} = 0{\cdot}66$; $a = 0{\cdot}1$; $R =$ cylinder volume ratio.

$$\therefore\ 100\left\{0{\cdot}4 + (0{\cdot}1 + 0{\cdot}4)2{\cdot}3 \log_{10}\frac{1{\cdot}1}{0{\cdot}5}\right\} - p_r$$

$$= \left\{p_r\left(0{\cdot}66 + (0{\cdot}1 + 0{\cdot}66)2{\cdot}3\log_{10}\frac{1{\cdot}1}{0{\cdot}76}\right) - 3\right\} R$$

i.e. $$79{\cdot}4 - p_r = (0{\cdot}941\, p_r - 3)R \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

To find R. Let V be the stroke volume of low pressure ; then volume $FC = 0{\cdot}66V$.

$\therefore$ indicated volume at $C = V_C =$ volume FC + clearance volume of low pressure $= 0{\cdot}66\ V + 0{\cdot}1 V = 0{\cdot}76\ V$. $\therefore\ V = \frac{V_C}{0{\cdot}76}$

Now $P_B V_B = 100 \times 2{\cdot}17$, and $P_C V_C = P_B V_B$. $\therefore\ V_C = \frac{217}{P_C} = \frac{217}{p_r}$

$\therefore$ stroke volume of low pressure $= V = \frac{V_C}{0{\cdot}76} = \frac{217}{0{\cdot}76\, p_r}$

$$\therefore \text{ cylinder volume ratio } R = \frac{\text{Volume of low pressure}}{\text{Volume of high pressure}}$$

$$= \frac{217}{0{\cdot}76} \times \frac{1}{4{\cdot}34} = \frac{65{\cdot}8}{p_r}$$

Substituting this value of R in equation (1), we have

$$79{\cdot}4 - p_r = (0{\cdot}941\, p_r - 3)\frac{65{\cdot}8}{p_r}$$

whence $p_r = 25{\cdot}3$ lb. per sq. in. ; $\therefore\ R = \frac{65{\cdot}8}{25{\cdot}3} = 2{\cdot}6$

Stroke volume of low pressure $= 2{\cdot}6 \times$ stroke volume of high pressure
$= 2{\cdot}6 \times 4{\cdot}34 = 11{\cdot}284$ cu. ft.

$\therefore$ Volume of point $D = 11{\cdot}284 + 1{\cdot}128 = 12{\cdot}41$ cu. ft.

Volume of point $C = \frac{217}{p_r} = \frac{217}{25{\cdot}3} = 8{\cdot}577$ cu. ft.

Pressure at $D = \frac{217}{12{\cdot}41} = 17{\cdot}5$ lb. per sq. in.

The volumes at H and L can be obtained if we assume that the steam at these points is dry.

Then, volume at $H =$ weight of clearance steam in high pressure multiplied by specific volume of steam at 25 lb. per sq. in.
$= 0{\cdot}05 \times 16{\cdot}29 = 0{\cdot}815$ cu. ft.

Volume at L = weight of clearance steam in low pressure multiplied by specific volume of steam at 3 lb. per sq. in.

$$= 0{\cdot}03 \times 118{\cdot}6 = 3{\cdot}56 \text{ cu. ft.}$$

Assuming PV = constant for the compression curves, the pressures at M and K can be found.

Horse-power. Assume speed at N r.p.m.

In the high pressure cylinder, mean effective pressure = 79·4 − 25·3 from Equation (1) = 54·1 lb. per sq. in.

$$\therefore \text{Horse-power} = \frac{PLA2N}{33{,}000} \ (A \text{ being in square inches})$$

$$= \frac{54{\cdot}1 \times 4{\cdot}34 \times 144}{33{,}000} \times 2N = 2{\cdot}05N$$

In low pressure cylinder mean effective pressure

$$= 0{\cdot}941 \times 25{\cdot}3 - 3 = 20{\cdot}8 \text{ lb. per sq. in.}$$

$$\therefore \text{Horse-power per stroke} = \frac{20{\cdot}8 \times 11{\cdot}28 \times 144}{33{,}000} \times 2N = 2{\cdot}05N$$

$\therefore$ Total horse-power = $4{\cdot}1N$, and

$$\text{steam consumption} = \frac{\text{Cylinder feed} \times 2N \times 60}{\text{Horse power}}$$

$$= \frac{0{\cdot}6 \times 2N \times 60}{4{\cdot}1N} = 18 \text{ lb. per H.P. hour}$$

EXAMPLES X

1. An engine takes in dry saturated steam at 80 lb. per sq in. absolute, and exhausts at 15 lb. per sq. in. absolute. Determine the work done per lb. of steam and the efficiency when working on (*a*) the Carnot cycle and (*b*) the Rankine cycle. (*I.Mech.E.*, 1925.)

2. Compare the Rankine and Carnot cycles for a vapour like steam, illustrating your answers with sketches of the pressure-volume, and temperature-entropy diagrams. Obtain an expression for the work available per lb. of steam by an engine working on the Rankine cycle. A steam engine takes in steam at 100 lb. per sq. in. absolute, and exhausts it at 5 lb. per sq. in. absolute. Assume that its efficiency ratio using the Rankine cycle standard is 62 per cent, and estimate its steam consumption per I.H.P. hour. (*I.Mech.E.*, 1923.)

3. Explain why the increase in efficiency obtained by using superheated steam in reciprocating engines is more than the gain which would be expected from consideration of the increase in efficiency of the corresponding Rankine cycle.

A steam engine, working over a constant pressure range of 90 to 14·7 lb. per sq. in. absolute, has a constant ratio of expansion, and is supplied with steam at various degrees of superheat. The steam consumption was observed to be as follows—

Temp. of steam supply ° C.	dry	202°	245°	290°
Steam per I.H.P. hour, lb.	42·0	37·4	32·3	27·1

Plot curves showing the variation with degree of superheat of the actual efficiency and the Rankine efficiency. (*U.L.*, *B.Sc.* (*Eng.*), 1923.)

4. Explain why the Rankine cycle rather than the Carnot cycle is employed as the standard with which to compare the performance of actual steam engines.

In an efficient steam engine plant the boiler supplies the engine with dry saturated steam at a pressure of 180 lb. per sq. in. absolute. If the condenser pressure is 3 lb. per sq. in. absolute, calculate the Rankine efficiency of the engine and, selecting your own values for the boiler efficiency, the mechanical efficiency, and the efficiency ratio, estimate the probable overall efficiency of the plant from coal to brake. (*U.L.*, *B.Sc.* (*Eng.*), 1926.)

5. Steam reaches a turbine at a pressure of 180 lb. per sq. in. absolute, and 2 per cent wet, but before entering the nozzle chest it is throttled to a pressure of 120 lb. per sq. in. absolute. In the turbine it expands to 20 lb. per sq. in. absolute, with an efficiency ratio of 55 per cent. The steam flow amounts to 5·2 lb. per sec. Determine (*a*) the condition of the steam at exhaust, and (*b*) the power developed by the turbine. (*U.L.*, *B.Sc.* (*Eng.*), 1921.)

6. State what is meant by the "Willans Line." A steam engine governed by throttling, uses 1,180 lb. of steam per hour when developing 55 I.H.P., and 4,800 lb. per hour when developing 280 I.H.P. Find the approximate thermal efficiency of the engine when developing 200 I.H.P., assuming the steam supplied to be dry and saturated at 180 lb. per sq. in. absolute, and the back pressure to be 2 lb. per sq. in. absolute. (*I.Mech.E.*, 1926.)

7. Explain what is meant by the "Willans Line." The test results from a turbine governed by throttling can be represented by a line whose equation is

$$G = 0{\cdot}0021\,N + 1{\cdot}15,$$

where G is lb. of steam per second, and N is horse-power. The maximum power is 5,000 H.P., and the conditions of steam supply at this load are 200 lb. per sq. in. absolute, and 100° F. superheat. Throttling is at constant heat and the condenser pressure is 1·0 lb. per sq. in. absolute. Assuming that the steam pressure varies directly as the steam flow, calculate the efficiency ratio of the turbine at one-third power. (*I.Mech.E.*, 1924.)

8. Calculate the cylinder diameter and stroke of a double-acting steam engine that will develop 120 I.H.P. under the following conditions: initial pressure 80 lb. per sq. in absolute; back pressure, 16 lb. per sq. in. absolute; revolutions per minute, 100; average piston speed, 500 ft. per min.; cut-off at 0·4 stroke. Take the diagram factor as 0·85. (*I.Mech.E.*, 1925.)

9. A single cylinder double-acting steam engine has a piston diameter of 11 in., a stroke of 18 in., and the clearance volume is 8·5 per cent of the swept volume. The steam supply is at 70 lb. per sq. in. gauge pressure, and the exhaust at atmospheric pressure (30 in. of mercury). Cut-off is at 40 per cent of the stroke and compression at 80 per cent. Assuming that the actual indicator card has an area 90 per cent of that of the theoretical card having hyperbolic expansion and compression, find the indicated horse-power of the engine when running at 185 revolutions per min. (*U.L.*, *B.Sc.* (*Eng.*), 1925.)

10. Show how the "missing quantity" in a steam engine cylinder may be determined for any point of the stroke between cut-off and release.

Determine the "missing quantity" at cut-off from the following particulars: cylinder diameter, 12 in.; stroke, 30 in.; clearance 8 per cent of volume swept by piston, consumption at 104 R.P.M.; 2,490 lb. of steam per hour.

Event	Fraction of stroke	Pressure
Cut-off . . .	0·26	112 lb. per sq. in. abs.
Compression . .	0·82	24 lb. per sq. in. abs.

(*I.Mech.E.*, 1924.)

11. The effective volume of the cylinder of a steam engine is 0·92 cubic ft., and the clearance volume is 8·5 per cent of this volume. The weight of steam in the cylinder at half stroke is 0·063 lb., and the pressure is 42 lb. per sq. in. absolute. The steam expands hyperbolically to 0·95 stroke. Calculate the final dryness of the steam, and determine the number of heat units that pass through the walls into the cylinder during expansion.

(*U.L.*, *B.Sc.* (*Eng.*), 1922.)

12. Calculate the cylinder diameter and stroke of a compound engine to develop 400 I.H.P., under the following conditions. Admission pressure, 165 lb. per sq. in. absolute; back pressure, 2 lb. per sq. in. absolute; revolutions per minute, 120; average piston speed, 720 ft. per min.; number of expansions, 15; diagram factor, 0·85; cut-off in H.P. at 0·27 stroke. How would you fix the point of cut-off in the L.P. cylinder?

(*I.Mech.E.*, 1925.)

13. A compound steam engine is to develop 125 horse-power at 110 revolutions per minute. The steam supply is at 105 lb. per sq. in. absolute, and the condenser pressure is 3 lb. per sq. in. absolute. Assuming hyperbolic expansion, an expansion ratio of 15, a diagram factor of 0·7, and neglecting clearance volume and receiver losses, determine the diameters of the cylinders so that they may develop equal powers. The stroke of each piston may be taken equal to the low pressure cylinder diameter.

(*U.L.*, *B.Sc.* (*Eng.*), 1922.)

14. The two cylinders of a compound steam engine have the same stroke, and the ratio of piston diameters is 1·8. Assuming hyperbolic expansion, and neglecting clearance, find the point of cut-off in the low pressure cylinder, which will theoretically give maximum efficiency if the cut-off in the high pressure cylinder is at half stroke.

A series of trials on this engine made at approximately constant speed, and in which the point of low pressure cut-off was varied gave result as follows—

Steam supply dry and saturated at 92 lb. per sq. in. absolute.

Condenser pressure 2·75 lb. per sq. in. absolute.

L.P. cut-off . .	0·225	0·254	0·339	0·350	0·508	0·67
B.H.P. . . .	49·0	57·0	56·5	56·1	54·4	47·6
Steam per hour, lb. .	1,060	1,214	1,180	1,165	1,190	1,090

Plot on a low pressure cut-off base a curve of thermal efficiency (B.H.P. basis). (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

CHAPTER XI

STEAM NOZZLES

Velocity after Expansion Through a Nozzle. If steam is allowed to expand through a suitably shaped nozzle, part of the heat energy in the steam may be converted into kinetic energy. In order to investigate the process let us consider the equation of steady flow, the heat appliance in our imaginary enclosure being a nozzle. The ideal nozzle, being an appliance to convert heat energy of the steam into kinetic energy, should neither gain heat from nor lose it to the surroundings, and no external work should be done by the expansion. Therefore, in the equation of steady flow (p. 53),

$$H_1 - H_2 + A\frac{U_1^2 - U_2^2}{2g} = AW + Q$$

both W and Q are zero.

Hence $$\frac{U_2^2 - U_1^2}{2g} = J(H_1 - H_2)$$

This equation shows that the gain in kinetic energy is equal to the change in total heat of the fluid, J being introduced, as both sides of the equation must be expressed in the same units.

If, for the sake of simplicity, we assume that the initial velocity U_1 is negligible compared to the final velocity U_2, then

$$U_2 = \sqrt{2gJ(H_1 - H_2)}.$$

If, in addition to the assumption that there is no heat transfer from or to the enclosure and surroundings, we also assume absence of friction, then at first sight it would appear that $H_1 - H_2$ is the adiabatic heat drop for the difference in pressure considered. It will be shown, later on, that the ordinary type of adiabatic expansion so far considered does not, in general, happen in the practical case of expansion through a nozzle ; but it will be convenient to assume, in the first instance, that this type of expansion does take place.

Hence the velocity of steam leaving a nozzle is

$$U = \sqrt{2gJ\,(\text{Adiabatic heat drop in the nozzle})}$$

If we, therefore, wish to find the velocity obtained by expansion through a nozzle from a pressure P_1 to a pressure P_2, then the adiabatic heat drop between these two pressures must be either calculated by first finding the dryness fraction after expansion, and hence the value of the total heat after expansion, or it may be read off the H-ϕ chart direct.

Example 1. Dry saturated steam, at a pressure of 200 lb. per sq. in. absolute, expands adiabatically through a suitably shaped nozzle to a final pressure of 2 lb. per sq. in. absolute. What is the final velocity of the steam ?

Initial total heat $H_1 = 669{\cdot}69$ C.H.U., at 200 lb. per sq. in. (from Table I). Dryness fraction after expansion

$$x = \left(\log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}\right)\frac{T_2}{L_2}$$

$$= \left(2{\cdot}3 \times \log_{10}\frac{467{\cdot}46}{325{\cdot}37} + \frac{472{\cdot}2}{467{\cdot}46}\right)\frac{325{\cdot}37}{566{\cdot}51} = 0{\cdot}788$$

$\therefore$ Total heat after expansion

$= H_2 = h + xL$, at 2 lb. per sq. in. (from Table I)

$= 52{\cdot}16 + (0{\cdot}788 \times 566{\cdot}51) = 498{\cdot}6$ C.H.U.

$\therefore$ Adiabatic heat drop $H_1 - H_2 = 669{\cdot}69 - 498{\cdot}6 = 171$ C.H.U.

This heat drop can be measured directly on the H-ϕ chart. The initial state point is given by the intersection of the 200 lb. per sq. in. pressure line with the saturation line. A vertical line through this point will cut the 2 lb. per sq. in. pressure line at the final state point. The vertical distance between these two points, measured on the total heat scale, gives the heat drop.

Then velocity $U = \sqrt{2gJ \text{ (heat drop)}}$

$= \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times 171} = 3{,}926$ ft. per sec.

The velocity may also be expressed in terms of pressure and volume. It has already been shown that if a fluid expands according to the law $PV^n =$ constant, the value of the heat drop $(H_1 - H_2)$ for an expansion from pressure P_1 to pressure P_2 is given by

$$\int_{P_2}^{P_1} VdP = \frac{n}{n-1}(P_1V_1 - P_2V_2). \quad \text{(See Eq. 1, p. 144)}$$

Hence $$U_2 = \sqrt{2g\frac{n}{n-1}(P_1V_1 - P_2V_2)}$$

$$= \sqrt{2g \cdot \frac{n}{n-1} \cdot P_1V_1\left(1 - \frac{P_2V_2}{P_1V_1}\right)}$$

But $\dfrac{V_2}{V_1} = \left(\dfrac{P_1}{P_2}\right)^{\frac{1}{n}}$, and substituting for $\dfrac{V_2}{V_1}$

$$U_2 = \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

This formula should give the same value for U_2 as that calculated from $U_2 = \sqrt{2gJ \text{ (adiabatic heat drop)}}$

The two values will not be quite the same for, as we have already seen, the value to be taken for n when steam expands adiabatically is not definite. If the initial state of the steam is taken as dry saturated then the value $n = 1{\cdot}135$ may be taken.

Area of Cross-section of Nozzle. Let the area of cross-section of the nozzle at the point where the pressure is P_2 be A_2 sq. ft. If the weight of steam passing per second be M lb., then the total volume of steam passing the point considered is MV_2 cu. ft. per sec., where V_2 is the specific volume of the steam at that point in cubic feet per pound The total volume of the steam passing per second is obviously also given by A_2U_2 cu. ft.

$$\therefore A_2U_2 = MV_2, \text{ or } \frac{M}{A_2} = \frac{U_2}{V_2}$$

But as $P_2V_2{}^n = P_1V_1{}^n$, $\dfrac{1}{V_2} = \dfrac{1}{V_1}\left(\dfrac{P_2}{P_1}\right)^{\frac{1}{n}}$

Hence
$$\frac{M}{A_2} = \frac{1}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} U_2$$

$$= \frac{1}{V_1}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

or
$$\frac{M}{A_2} = \sqrt{2g\frac{n}{n-1}\frac{P_1}{V_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right\}}$$

From this equation, if M is known, the area of the nozzle at any point where the pressure is P_2 may be calculated.

Another method for finding the nozzle area is as follows. We have, as before,

$$A_2U_2 = MV_2$$

and
$$U_2 = \sqrt{2gJ(H_1 - H_2)}$$

If the dryness fraction x after expansion to the pressure P_2 is found, then

$$V_2 = xV_{s2}$$

where V_{s2} is the specific volume of dry saturated steam at the pressure P_2.

Hence,
$$\frac{M}{A_2} = \frac{U_2}{V_2} = \frac{\sqrt{2gJ(H_1 - H_2)}}{xV_{s2}}$$

Example 2. Find the shape of a nozzle of circular cross-section to expand dry saturated steam from a pressure of 200 lb. per sq. in. absolute to a back pressure of 2lb. per sq. in. absolute. Assume frictionless adiabatic expansion, with the pressure falling uniformly along the nozzle. Discharge = 10 lb. per sec.

The figures are shown in the table on p. 456. The values of the dryness fraction were obtained from the H-ϕ chart. The values of heat drop from an initial pressure of 200 lb. per sq. in. down to the various back pressures were also obtained from the H-ϕ chart.

The values of V_s for dry saturated steam were obtained from the tables.

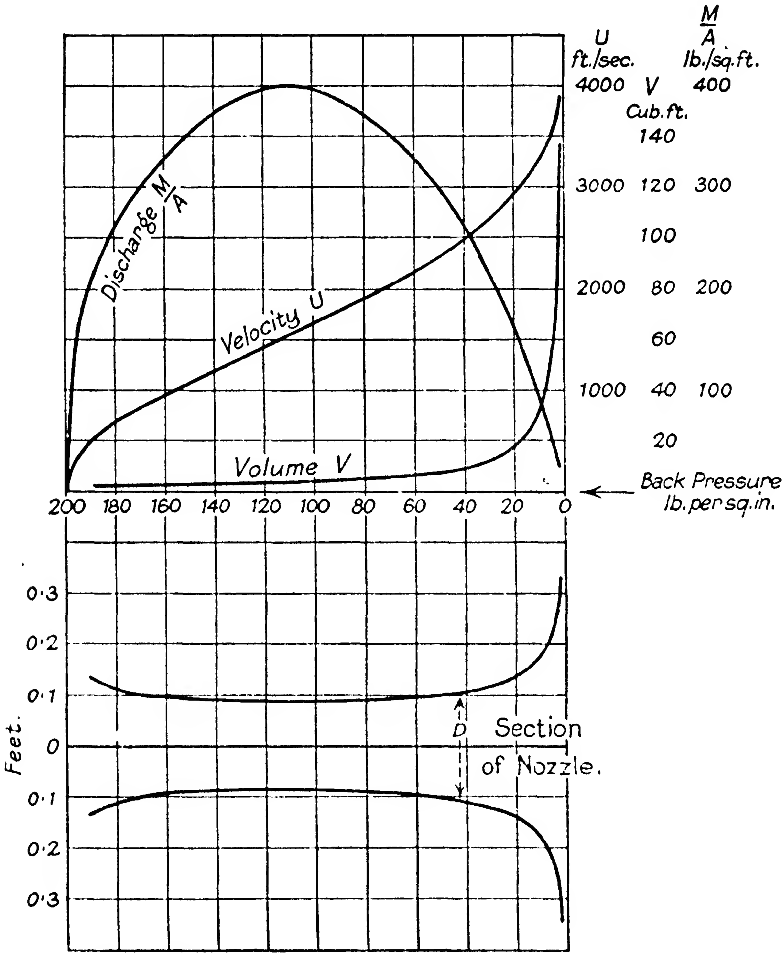


Fig. 165

In Fig. 165 the values of specific volume, velocity and discharge per unit area are shown plotted with the axis of the nozzle as the horizontal co-ordinate. The section of the nozzle obtained by plotting the various values of D along the axis of the nozzle is also given.

NOZZLE FOR FRICTIONLESS ADIABATIC FLOW OF STEAM

Pressure P lb./sq. in.	Dryness fraction x	Heat Drop $H_1 - H_2$ C.H.U.	Velocity U ft./sec.	Vs cu.ft./lb.	Specific Volume $V = xVs$ cu. ft./lb.	Discharge per unit area $\frac{M}{A} = \frac{U}{V}$ lb./sq. ft.	Area A sq. ft.	Diameter D ft.
200	1	—	—	—	—	—	—	—
190	0·995	2·9	511	2·435	2·423	211	0·0475	0·246
180	0·992	5·0	671	2·562	2·54	264	0·0378	0·219
150	0·98	12·5	1,061	3·041	2·98	356	0·0281	0·189
100	0·954	31·5	1,685	4·451	4·25	397	0·0252	0·179
50	0·915	60·5	2,335	8·520	7·80	300	0·0334	0·206
40	0·904	69	2,494	10·50	9·49	263	0·0381	0·220
30	0·89	80·5	2,695	13·74	12·23	220	0·0455	0·241
20	0·873	95	2,926	20·08	17·53	167	0·0600	0·277
10	0·847	119	3,275	38·39	32·52	101	0·0990	0·355
6	0·828	135·5	3,494	61·91	51·25	68	0·1470	0·433
2	0·794	168	3,891	173·50	137·76	28·5	0·3520	0·670

From these curves it will be seen that at the higher pressures the specific volume V increases at first slowly as the pressure drops, while the velocity U increases at a greater rate. As the expansion proceeds the increase of the specific volume V becomes greater than the increase of velocity U. Hence, as we proceed from the high pressure end to the low pressure end of the nozzle, the area A, which is proportional to $\frac{V}{U}$, decreases to a minimum and then increases. In other words, the nozzle is of a convergent-divergent section. The point of the nozzle where the area is a minimum is called the throat. From the curve it will be seen that the pressure at the throat is about 115 lb. per sq. in. The ratio of the pressure at the throat to the initial pressure is $\frac{115}{200} = 0{\cdot}57$, approximately. The conditions at the throat will now be considered more fully.

Conditions at the Throat. At the throat, where the area is a minimum, the mass of steam passing per second per unit of area is a maximum.

Hence, $\frac{M}{A} = \sqrt{2g\,\frac{n}{n-1}\,\frac{P_1}{V_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right\}}$ must be a maximum.

For any given initial conditions the variable quantity in this equation is $\left(\frac{P_2}{P_1}\right)$. Differentiating with respect to $\left(\frac{P_2}{P_1}\right)$ and equating to zero, we have

$$\frac{2}{n}\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}-1} - \frac{n+1}{n}\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}-1} = 0$$

This particular value of P_2 is the pressure at the throat. If we call this pressure P_t, we have

$$\frac{P_t}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

From this equation it follows that the throat pressure is dependent solely on the initial pressure and the value of n.

If this value of $\left(\frac{P_t}{P_1}\right)$ is substituted in the general equation for velocity, namely,

$$U_2 = \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

we have for the value of the velocity at the throat

$$U_t = \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1-\left[\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]^{\frac{n-1}{n}}\right\}}$$

$$= \sqrt{2g\frac{n}{n+1}P_1V_1}$$

The specific volume of the steam at the throat

$$V_t = V_1\left(\frac{P_1}{P_t}\right)^{\frac{1}{n}}$$

$$\therefore \quad \frac{1}{V_t} = \frac{1}{V_1}\left(\frac{P_t}{P_1}\right)^{\frac{1}{n}} = \frac{1}{V_1}\left[\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]^{\frac{1}{n}} = \frac{1}{V_1}\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}$$

The mass of steam passing per second per unit area at the throat

$$\frac{M}{A_t} = \frac{U_t}{V_t} = \frac{1}{V_1}\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}}\sqrt{2g\frac{n}{n+1}P_1V_1}$$

$$= \sqrt{gn\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\frac{P_1}{V_1}}.$$

From this equation it follows that, provided the final pressure is not greater than $P_t = P_1\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$, the mass of steam passing per second through a convergent-divergent nozzle is dependent solely on the initial conditions of the steam and the throat area, and is independent of the final pressure at the end of the nozzle. This pressure is called the *critical pressure.* Thus, the addition of the divergent part of the nozzle after the throat does not affect the discharge of the nozzle, but it does affect the final velocity of the steam. This final velocity will be determined from the equation

$$U_2 = \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}}$$

where P_2 is the pressure at the end of the nozzle, the value of P_2 being determined by the final area of the nozzle, provided the back pressure outside the end of the nozzle is not greater than this value of P_t so determined. If the final back pressure is lower than the critical pressure the nozzle will be convergent-divergent. If the back pressure is equal to or higher than the critical pressure, then the nozzle should be convergent only and the mass flow through such a nozzle will be determined by the final mouth area of the nozzle.

The Value of n. The Zeuner equation for n in the adiabatic expansion of steam is $n = 1{\cdot}035 + 0{\cdot}1x$, and gives a value of 1·135 for n for originally dry saturated steam. If this value of n is substituted in the equation

$$P_t = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \times P_1$$

we obtain for the critical pressure, $P_t = 0{\cdot}577 P_1$

If the same value of n is substituted in the equation

$$\frac{M}{A_t} = \sqrt{gn\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\frac{P_1}{V_1}}$$

we obtain $M = 3{\cdot}604\, A_t \sqrt{\frac{P_1}{V_1}}$; A_t being in square feet, P_1 in pounds per square foot, and M in pounds per second.

If p is in pounds per square inch and A_t in square inches, this reduces to $M = 0{\cdot}3003\, A_t \sqrt{\frac{p_1}{V_1}}$.

Now, for dry saturated steam

$$p_1 V_1^{\frac{16}{15}} = 490, \text{ where } p \text{ is in pounds per square inch.}$$

Substituting this value of V_1 in the above equation,

$$M = 0{\cdot}0165\, A_t\, p_1^{\frac{31}{32}}$$

an equation connecting the discharge M in pounds per second and the throat area in square inches for a convergent-divergent nozzle with steam initially dry and at a pressure p_1 lb. per sq. in.

As will be seen later, the values of M determined from these equations are not confirmed by practical measurement, showing that ordinary adiabatic expansion does not take place.

We must now consider the case where superheated steam is expanded.

The value of n for the adiabatic expansion of superheated steam is given by Callendar as 1·3. If this value is substituted in the equation

$$P_t = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \times P_1$$

we obtain for the critical pressure $P_t = 0{\cdot}5457\, P_1$.

If the same value of n is substituted in the equation,

$$\frac{M}{A_t} = \sqrt{gn\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\frac{P_1}{V_1}}$$

we obtain $M = 3{\cdot}786\, A_t \sqrt{\frac{P_1}{V_1}}$; A_t being in square feet; P_1 in pounds per square foot; M in pounds per second.

The Effect of Friction in a Nozzle. If there were no friction during expansion through a nozzle, then, assuming the expansion were adiabatic, the expansion line would be AB and ab, Fig. 166. The heat energy transformed into kinetic energy would be represented by the area $ABDE$. This area is $\int V\,dP = H_A - H_B$; where H_A and H_B are the total heats at A and B respectively.

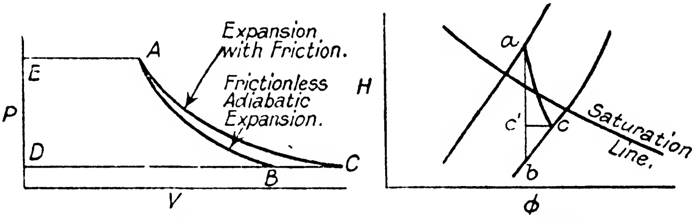


FIG. 166. THE EFFECT OF FRICTION IN A NOZZLE

In a practical case, however, there is friction between the fluid and the sides of the nozzle, as well as dissipation of energy due to eddies being formed in the fluid itself. At any point in the expansion, some of the energy which, in the absence of friction would be in the form of kinetic energy, has been dissipated in overcoming friction and reappears in the form of heat. At this point, therefore, there is less kinetic energy and more heat energy than would be the case in frictionless flow. The specific volume of the fluid will be greater than in the case of frictionless flow, and the actual expansion curve would be some curve AC on the P-V chart, Fig. 166. Owing to the loss by nozzle friction the expansion is not reversible, and hence there will be a progressive increase in entropy; the actual expansion on the H-ϕ chart follows some such curve as ac, the final state point c being on the constant pressure line through b. From the H-ϕ chart we see that, owing to friction, the actual heat drop is reduced from ab to ac', and hence the final kinetic energy generated will be less than in the case of frictionless adiabatic flow. The H-ϕ chart also shows that the effect of friction is to re-heat the fluid, the final state of the steam being drier in the case of wet steam, and at a higher temperature in the case of superheated steam, than would be the case in the absence of friction.

Considering the P-V diagram, Fig. 166, the amount of heat energy converted into kinetic energy with frictionless flow is represented by the area $ABDE = \int_{P_B}^{P_A} V\,dP$.

With frictionally resisted flow the heat energy converted into kinetic energy is not represented by the area $ACDE$. To illustrate this, consider the cycle ABC on the P-V diagram, Fig. 166. Starting at the state point A, and allowing the steam to expand with frictional resistance along AC, there will be a certain amount of kinetic energy generated due to the expansion. Let this be K. In order to pass from the state point C to the state point B, heat must be extracted from the fluid at constant pressure. This amount will be $H_C - H_B$. On compressing the steam back to its original condition at A along BA, the energy required will be the same as the kinetic energy obtained by expanding from A to B without friction, namely, $H_A - H_B$. Since the fluid has passed round a closed cycle and returned to its original condition,

Energy taken out = energy put in

i.e. $$K + H_C - H_B = H_A - H_B$$

i.e. $$K = (H_A - H_B) - (H_C - H_B)$$

Now, $H_A - H_B$ is represented by the area $ABDE = \int_{P_B}^{P_A} V\,dP$

The kinetic energy produced with friction, therefore, is less than that produced without friction by the amount $(H_C - H_B)$. This amount is represented on the H-ϕ chart by $c'b$. It also follows that $K = H_A - H_C$ is the actual heat drop, represented on the H-ϕ chart by ac'.

The nozzle efficiency ε_n may be defined as the ratio of the heat equivalent of the kinetic generated in the expansion to the isentropic heat drop. Thus in Fig. 166

$$\varepsilon_n = \frac{K}{H_a - H_b} = \frac{H_a - H_c}{H_a - H_b}$$

If the initial velocity of the steam entering the nozzle can be neglected compared with the final velocity U leaving the nozzle then

$$\varepsilon_n = \frac{U^2}{2gJ(H_a - H_b)}$$

Example 3. Find the throat and exit diameters of a convergent-divergent nozzle which will discharge 120 lb. of steam per hour from a pressure of 120 lb. per sq. in. absolute into a chamber having a pressure of 30 lb. per sq. in. absolute. Assume the steam is originally dry saturated and that the flow is abiabatic (*a*) without friction, (*b*) with the friction loss in the divergent part of the nozzle equal to one-tenth of the total heat drop. What will be the final velocity of the steam in each case ?

(*a*) *Frictionless Adiabatic Flow.* To find the throat area we may use the formula

$$\frac{M}{A_t} = 0{\cdot}3003\sqrt{\frac{p_1}{V_1}}$$

where $M = \frac{120}{60 \times 60} = \frac{1}{30}$ lb. per sec., and $V_1 = 3{\cdot}751$ cu. ft. per lb. (from Table I).

$$\therefore \text{ Area of throat } A_t = \frac{1}{30 \times 0{\cdot}3003} \times \sqrt{\frac{3{\cdot}751}{120}} = 0{\cdot}0196 \text{ sq. in.}$$

$$\therefore \text{ Diameter of throat } D_t = \sqrt{\frac{4 \times 0{\cdot}0196}{3{\cdot}14}} = 0{\cdot}158 \text{ in.}$$

As the steam is originally dry saturated, the throat area may also be calculated from the equation

$$M = 0{\cdot}0165\, A_t p_1^{\frac{31}{32}}$$

$$\therefore \quad A_t = \frac{120}{60 \times 60 \times 0{\cdot}0165} \times \frac{1}{120^{\frac{31}{32}}} = 0{\cdot}01955 \text{ sq. in.}$$

To find the *exit area* we may use the equations

$$\frac{M}{A_2} = \sqrt{2g\frac{n}{n-1}\frac{P_1}{V_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right\}}$$

$$\frac{M}{A_t} = 3{\cdot}604\sqrt{\frac{P_1}{V_1}}$$

whence

$$\frac{A_2}{A_t} = \frac{3{\cdot}604\sqrt{\frac{P_1}{V_1}}}{\sqrt{2g\frac{n}{n-1}\frac{P_1}{V_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right\}}}$$

$$= \frac{3{\cdot}604}{\sqrt{\frac{2 \times 32{\cdot}2 \times 1{\cdot}135}{0{\cdot}135}\left\{\left(\frac{30}{120}\right)^{\frac{2}{1{\cdot}135}} - \left(\frac{30}{120}\right)^{\frac{2{\cdot}135}{1{\cdot}135}}\right\}}} = 1{\cdot}34$$

$$\therefore \text{ Area of outlet } A_2 = 1{\cdot}34 \times 0{\cdot}0196 = 0{\cdot}0263 \text{ sq. in.}$$

whence diameter of outlet $= 0{\cdot}183$ in.

From the H-ϕ chart the final state of the steam is given by the point *b*, Fig. 167, showing a dryness fraction of 0·92.

Heat drop for expansion *ab* = 60·1 C.H.U. (from H-ϕ chart).

$$\therefore \text{ Velocity at exit } U_2 = \sqrt{2gJ\,(\text{heat drop})}$$

$$= \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times 60{\cdot}1} = 2{,}328 \text{ ft. per sec.}$$

It should be noted that the exit area could be calculated from this value of U_2, using the equation

$$U_2 A_2 = M \times V_s \times \text{dryness fraction } x$$

where V_s is the specific volume of dry saturated steam at 30 lb. per sq. in.

$$\therefore \quad A_2 = \frac{M \times V_s}{U_2} \times x = \frac{120}{60 \times 60} \times \frac{0{\cdot}92 \times 13{\cdot}74}{2{,}328} \times 144$$

$$= 0{\cdot}0261 \text{ sq. in.}$$

(*b*) *Frictionally Resisted Flow.* As the friction is only in the divergent part of the nozzle, the throat area is the same as in the

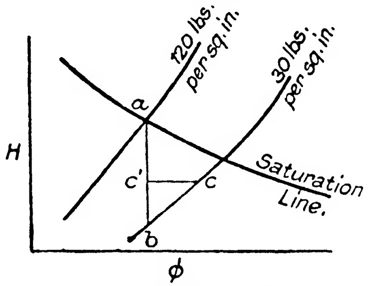


FIG. 167. FRICTIONLESS ADIABATIC FLOW THROUGH NOZZLES

first case. To find the exit area we cannot use the formula containing n as we do not know its value. The actual heat drop with friction

$$= 0{\cdot}9 \times 60{\cdot}1 = 54{\cdot}09 \text{ C.H.U.}$$

∴ the final velocity

$$U_2 = \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times 54{\cdot}09} = 2{,}208 \text{ ft. per sec.}$$

The dryness fraction at exit may be obtained from the H-ϕ chart as follows. Find the point c' (Fig. 167) such that $c'b = \frac{1}{10} ab$. Draw a horizontal line through c' to cut the 30 lb. per sq. in. pressure line in c. This point will show the final state of the steam to be in this case 0·932.

$$\therefore \text{ Exit area} = \frac{M \times V_s}{U_2} = \frac{120}{60 \times 60} \times \frac{0{\cdot}932 \times 13{\cdot}74}{2{,}208} \times 144$$

$$= 0{\cdot}0278 \text{ sq. in., whence diameter of exit} = 0{\cdot}188 \text{ in.}$$

Supersaturated Expansion in Nozzles. As already pointed out, if the value of n for dry saturated steam is taken as 1·135 the value of the discharge calculated from the equation

$$M = 3{\cdot}604 \, A_t \sqrt{\frac{P_1}{V_1}}$$

is less than the discharge actually measured by experiment, although the calculated discharge from this theoretical equation, which assumes the absence of friction, should obviously be greater than the discharge obtained in practice where friction is present. This fact seems to show that the usual type of adiabatic expansion, where there is at any point during the expansion an equilibrium mixture of vapour and liquid, does not take place. It is now generally assumed that supersaturated expansion takes place. This type of expansion has already been touched on in a previous paragraph (page 398), but we must now go further into the matter. We have already seen how this type of expansion can be shown on the T-ϕ chart, and we will now indicate it on the H-ϕ chart.

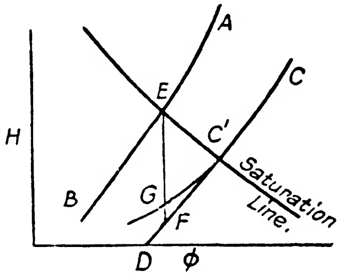


FIG. 168

Let AB and CD, Fig. 168, be two constant pressure lines at P_1 and P_2 lb. per sq. in. respectively. It will be remembered that these lines are curved in the superheated region and straight in the wet region. The ordinary type of adiabatic expansion will be represented for a pressure drop from P_1 to P_2 by the vertical line EF. Supersaturated expansion will be represented by EG, where G is the point on the continuation of the curved portion CC' of the constant pressure line CD into what is normally the wet region. In this case of supersaturated expansion we must imagine a state of affairs in which the saturation line, which normally divides the superheated region from the wet region, has no existence. Under these circumstances, at the point G, after supersaturated expansion, the fluid will be a homogeneous gas, whereas, had the ordinary adiabatic expansion taken place, the state of the fluid given by the point F would have been that of a wet mixture. It should be remembered that the temperature in the state indicated by the point G will be lower than the saturation temperature corresponding to the pressure P_2.

Supersaturated expansion does not, however, proceed indefinitely. As we have already seen, the density of supersaturated steam at any point in the expansion is greater than it would be at the same point had the expansion proceeded under ordinary adiabatic conditions. The limit of supersaturation is attained when the density of the supersaturated vapour is about eight times that of the ordinary saturated vapour at the corresponding pressure. As soon as this limit is reached, the vapour will begin to condense and will eventually become an equilibrium mixture of vapour and liquid. The time taken for this condensation is a debatable point and is too complicated for discussion here, but for the present purpose we may assume that, when the supersaturation limit is reached, condensation to the proper equilibrium mixtures takes place instantaneously.

The limit of supersaturated expansion was first shown by the experiments of C. T. R. Wilson in 1897, and the subsequent work of H. M. Martin has enabled a curve, termed by him the *Wilson line*, to be applied to the H-ϕ chart. If expansion is carried below this line it can be here assumed for practical purposes that instantaneous condensation takes place when the Wilson line is reached. Such an expansion is shown in Fig. 169.

Suppose expansion takes place from a pressure P_1 to a pressure P_3. The first stage of the expansion AB is supersaturated, the point B being the intersection of the curved extension of that constant pressure line P_2 which cuts the Wilson line vertically below A. At this point condensation takes place. If this is instantaneous it will take place at constant pressure, and as there is no heat interchange, the total heat will remain constant, and the condensation is represented by the horizontal line BC. When equilibrium is attained by condensation there will be a quantity of heat available due to the condensation. This heat will cause the steam to be drier at C than at E, the point which would have been reached had ordinary adiabatic expansion taken place down to the pressure P_2. The remainder of the expansion is indicated by CD, this being ordinary adiabatic expansion. The actual state of affairs is more complicated than this, but with the present state of knowledge, for practical purposes, the steam may be considered to attain the equilibrium condition as soon as the Wilson line is reached.

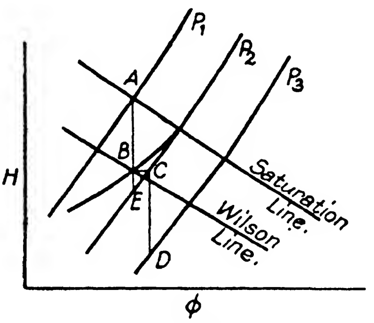


FIG. 169

As we have already seen, (p. 400) during supersaturated expansion the same equations held as in the case of adiabatic expansion of superheated steam, namely,

$$P(V-b)^{1\cdot 3} = \text{constant}\ ; \quad \frac{P}{T^{\frac{13}{3}}} = \text{constant}\ ; \quad T^{\frac{10}{3}}(V-b) = \text{constant}$$

It will be remembered that the temperature of supersaturated steam at any particular pressure during the expansion is lower than the saturation temperature corresponding to this pressure. The degree of undercooling at a pressure P_2 due to supersaturation can therefore be obtained by first finding the temperature T_2 from the equation

$$\frac{P_1}{T_1^{\frac{13}{3}}} = \frac{P_2}{T_2^{\frac{13}{3}}}$$

and then subtracting this temperature from the saturation temperature corresponding to the pressure P_2.

In order to calculate the total heat of supersaturated steam we may use the Callendar equation (p. 384)

$$H = \frac{p(V - 0{\cdot}0123)}{2{\cdot}2436} + 464 \text{ calories}$$

p being in pounds per square inch, and V in cubic feet.

The critical pressure and discharge of a nozzle with supersaturated steam will be the same as in the case of superheated steam, namely,

$$P_t = 0{\cdot}5457\ P_1, \text{ and } M = 3{\cdot}786\ A_t \sqrt{\frac{P_1}{V_1}}$$

The value of the discharge calculated from this equation will be 4 or 5 per cent greater than the discharge actually obtained by experiment.

Heat Drop in Supersaturated Expansion. The heat drop between two pressures P_1 and P_2 can be calculated from the equation for total heat just given, the value of V for the lower pressure being calculated from the equation

$$P_1(V_1 - b)^{1{\cdot}3} = P_2(V_2 - b)^{1{\cdot}3}$$

The value of $b = 0{\cdot}01602$ cu. ft. can be neglected for elementary purposes.

From previous work we know that the heat drop

$$H_1 - H_2 = \int_{P_2}^{P_1} \frac{V\,dP}{J} = \frac{n}{n-1}\,\frac{(P_1V_1 - P_2V_2)}{J}$$

where $n = 1{\cdot}3$.

This gives another method of calculation.

A third equation in terms of pressures can be obtained as follows

$$P(V - b)^{1{\cdot}3} = P_1(V_1 - b)^{1{\cdot}3}$$

$$\therefore \quad V = \left(\frac{P_1}{P}\right)^{\frac{10}{13}}(V_1 - b) + b$$

$\therefore$ Heat drop,

$$H_1 - H_2 = \int_{P_2}^{P_1} \frac{V\,dP}{J} = \frac{1}{J}\int_{P_2}^{P_1}\left\{\left(\frac{P_1}{P}\right)^{\frac{10}{13}}(V_1 - b) + b\right\}dP$$

$$= \frac{P_1^{\frac{10}{13}}(V_1 - b)}{J}\int_{P_2}^{P_1}\frac{dP}{P^{\frac{10}{13}}} + \frac{b}{J}\int_{P_2}^{P_1} dP$$

$$= \frac{13}{3}\,\frac{P_1^{\frac{10}{13}}(V_1 - b)}{J}\left[P_1^{\frac{3}{13}} - P_2^{\frac{3}{13}}\right] + \frac{b}{J}(P_1 - P_2)$$

Example 4. Explain what is meant by the supersaturated expansion of steam and give some idea of the limits within which this condition is possible. Steam is expanded from a pressure of 60 lb. per sq. in. absolute and 170° C.

to a pressure of 20 lb. per sq. in. absolute. If the expansion is supersaturated and occurs with a friction loss of 5 per cent, determine the actual and isentropic drops and degree of undercooling.

For the supersaturated state you may use the approximate Callendar equations—

$$V = \frac{2{\cdot}2436\,(H - 464)}{P}\,;\ \frac{P}{T^{\frac{13}{3}}} = \text{constant}\,;\ \text{and } PV^{1{\cdot}3} = \text{constant}$$

Take the specific heat as 0·52. Volume of 1 lb. of steam at 60 lb. per sq. in. and 170° C. is 7·685 cu. ft. (*U.L., B.Sc. (Eng,.)* 1921.)

Supersaturated Flow. $PV^{1{\cdot}3} = K$

$\therefore$ specific volume after expansion $V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1{\cdot}3}} V_1$

$$= \left(\frac{60}{20}\right)^{\frac{1}{1{\cdot}3}} \times 7{\cdot}685 = 17{\cdot}89 \text{ cu. ft.}$$

But $\quad V = \dfrac{2{\cdot}2436\,(H - 464)}{P}$

$\therefore$ total heat before expansion,

$$H_1 = \frac{VP}{2{\cdot}2436} + 464 = \frac{7{\cdot}685 \times 60}{2{\cdot}2436} + 464 = 669{\cdot}5 \text{ C.H.U.}$$

$\therefore$ total heat after expansion

$$H_2 = \frac{17{\cdot}89 \times 20}{2{\cdot}2436} + 464 = 623{\cdot}5 \text{ C.H.U.}$$

$\therefore$ heat drop (without friction) $= 669{\cdot}5 - 623{\cdot}5 = 46$ C.H.U.

$\therefore$ actual heat drop $= 0{\cdot}95 \times 46 = 43{\cdot}7$ C.H.U.

The heat drop (without friction) can also be calculated from the equation

$$H_1 - H_2 = \frac{n}{n-1}\left(\frac{P_1V_1 - P_2V_2}{J}\right)$$

$$= \frac{1{\cdot}3}{0{\cdot}3}\left(\frac{60 \times 7{\cdot}685 - 20 \times 17{\cdot}89}{1{,}400}\right) \times 144 = 46 \text{ C.H.U.}$$

Isentropic Flow. Total heat before expansion, $H_1 = 669{\cdot}5$ C.H.U.

Dryness fraction after expansion

$$x = \left(\log_e \frac{T_1}{T_2} + \frac{L_1}{T_1} + C_p \log_e \frac{T_{su}}{T_1}\right)\frac{T_2}{L_2}$$

$$= \left(2{\cdot}3 \log_{10} \frac{417{\cdot}89}{381{\cdot}97} + \frac{510{\cdot}22}{417{\cdot}89} + 0{\cdot}52 \times 2{\cdot}3 \log_{10} \frac{443}{417{\cdot}89}\right)\frac{381{\cdot}97}{533{\cdot}87}$$

$$= 0{\cdot}959$$

$\therefore$ Total heat after expansion

$$H_2 = h_2 + xL_2 = 108{\cdot}95 + 0{\cdot}959 \times 533{\cdot}87 = 621{\cdot}14 \text{ C.H.U.}$$

∴ isentropic heat drop (with friction)

$$= 0{\cdot}95(H_1 - H_2) = 0{\cdot}95\,(669{\cdot}5 - 621{\cdot}1) = 46 \text{ C.H.U.}$$

Degree of Undercooling. $\dfrac{P}{T^{\frac{13}{3}}} = \text{constant}.$

∴ Temperature after supersaturated expansion

$$T_2 = \left(\frac{P_1}{P_2}\right)^{\frac{3}{13}} T_{su} = \left(\frac{20}{60}\right)^{\frac{3}{13}} \times 443 = 343{\cdot}8^\circ \text{ C. (abs.)}$$

But saturation temp. at 20 lb. per sq. in. (abs.) = 381·97° C. (abs.).

∴ undercooling = 381·97 − 343·8 = 38·2° C.

Example 5. A nozzle is supplied with steam at 100 lb. per sq. in. absolute pressure and 275° C. Find the temperature and velocity of the steam at the throat of the nozzle.

If the diverging portion of the nozzle is 2 in. long and the throat diameter ¼ in., determine the angle of the cone so that the steam may leave the nozzle at 15 lb. per sq. in. absolute. Assume a friction loss of 15 per cent of the heat drop used in this diverging part. (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

Saturation temperature for 100 lb. per sq. in. = 164·28° C.

∴ amount of superheat = 275 − 164·28 = 110·72° C.

First find the state of the steam after ordinary adiabatic expansion to 15 lb. per sq. in. from the *H*-ϕ chart. A vertical line drawn from the intersection of the 275° C. temperature line with the 100 lb. per sq. in. pressure line (or of the 110·72 superheat line with the 100 lb. per sq. in. pressure line) will cut the 15 lb. per sq. in. pressure line at a point where the dryness fraction is 0·982. An *H*-ϕ chart on which the Wilson line is drawn will show that this point is above this line. Hence the whole expansion may be taken as supersaturated.

The critical pressure, $P_t = 0{\cdot}5457\,P_1 = 0{\cdot}5457 \times 100 = 54{\cdot}57$

Hence, temperature at throat,

$$T_t = \left(\frac{P_t}{P_1}\right)^{\frac{3}{13}} T_1 = \left(\frac{54{\cdot}57}{100}\right)^{\frac{3}{13}} \times (275 + 273) = 476{\cdot}5^\circ \text{ C. (abs.)}$$

∴ temperature at throat, $T_t = 476{\cdot}5 - 273 = 203{\cdot}5^\circ$ C.

The temperature of saturation at 54·57 lb. per sq. in. = 141·4° C.

∴ degree of superheat at throat = 203·5 − 141·4 = 62·1° C.

(This temperature can be read direct from the *H*-ϕ chart, from the point of intersection of the vertical expansion line with the 54·57 lb. per sq. in. pressure line.)

To obtain the velocity at the throat, the heat drop from 100 lb. per sq. in. to 54·57 lb. per sq. in. is required.

Total heat of superheated steam at 100 lb. per sq. in. and 275° C. (i.e. 110·72° of superheat),

H_1 = 720·42 C.H.U. (by interpolation from tables)

Total heat of superheated steam at 54·57 lb. per sq. in. and 203·5° C. (62·1° of superheat),

$$H_2 = 686{\cdot}92 \text{ C.H.U.}$$

$\therefore$ Heat drop $H_1 - H_2 = 720{\cdot}42 - 686{\cdot}92 = 33{\cdot}50$ C.H.U.

(This value can be read direct from the H-ϕ chart.)

$\therefore$ Velocity at throat $= \sqrt{2gJ \text{ (heat drop)}}$

$$= \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times 33{\cdot}5} = 1{,}738 \text{ ft. per sec.}$$

Now $V = \dfrac{2{\cdot}2436\,(H - 464)}{p} + 0{\cdot}0123$

$\therefore$ specific volume at throat

$$= \frac{2{\cdot}2436\,(686{\cdot}92 - 464)}{54{\cdot}57} + 0{\cdot}0123 = 9{\cdot}18 \text{ cu. ft.}$$

$\therefore$ Using suffixes t and e for throat and exit respectively (neglecting friction),

$$P_t V_t^{1\cdot3} = P_e V_e^{1\cdot3}, \text{ and } V_e = \left(\frac{P_t}{P_e}\right)^{\frac{1}{1\cdot3}} V_t$$

$$= \left(\frac{54{\cdot}57}{15}\right)^{\frac{1}{1\cdot3}} \times 9{\cdot}18 = 24{\cdot}77 \text{ cu. ft.}$$

$\therefore$ Total heat at exit, H_e (neglecting friction)

$$= \frac{p_e(V_e - 0{\cdot}0123)}{2{\cdot}2436} + 464$$

$$= \frac{15(24{\cdot}77 - 0{\cdot}0123)}{2{\cdot}2436} + 464 = 629{\cdot}52 \text{ C.H.U.}$$

$\therefore$ Heat drop between throat and exit (no friction)

$$= 686{\cdot}92 - 629{\cdot}52 = 57{\cdot}4 \text{ C.H.U.}$$

$\therefore$ actual heat drop in diverging portion

$$= 0{\cdot}85 \times 57{\cdot}4 = 48{\cdot}8 \text{ C.H.U.}$$

But heat drop in converging portion $= 33{\cdot}5$ C.H.U.

$\therefore$ total heat drop along nozzle $= 33{\cdot}5 + 48{\cdot}8 = 82{\cdot}3$ C.H.U.

$\therefore$ final velocity $= \sqrt{2gJ \text{ heat drop}}$

$$= \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times 82{\cdot}3} = 2{,}724 \text{ ft. per sec.}$$

$$\text{Now } MV = UA\,;\quad \therefore \frac{A_e}{A_t} = \frac{V_e}{U_e} \times \frac{U_t}{V_t}$$

$$\text{or } \frac{\text{Diameter at exit}}{\text{Diameter at throat}} = \sqrt{\frac{A_e}{A_t}} = \sqrt{\frac{V_e}{U_e} \times \frac{U_t}{V_t}}$$

$$\therefore \text{ Diameter at exit} = \frac{1}{4}\sqrt{\frac{24{\cdot}77}{2{,}724} \times \frac{1{,}738}{9{\cdot}18}} = 0{\cdot}328 \text{ in.}$$

Hence if ϕ = angle of cone, $\tan \phi = \frac{0·328 - 0·25}{2 \times 2}$

$= 0·0195$

and $\therefore \phi = 1° 6'$

The Steam Injector. The principles of the steam nozzle may be applied to the steam injector, although in what follows it must be understood that the given formulae are only approximate. The action of the injector is illustrated diagramatically in Fig. 170.

Steam from the boiler is supplied to the convergent nozzle A, and this steam issuing at a high velocity into the combining tube is condensed by the cold water flowing from the feed tank C. This tank may be either above or below the level of the injector. The mixture issuing from the nozzle B, then flows through the delivery nozzle D in which its kinetic energy is reduced and converted into pressure energy, until on leaving at E this pressure energy is sufficient to overcome the boiler pressure and to lift the water through a height H_2. Round the gap between the nozzles B and D is a chamber provided with an outlet through which any excess water may overflow during the starting of the injector.

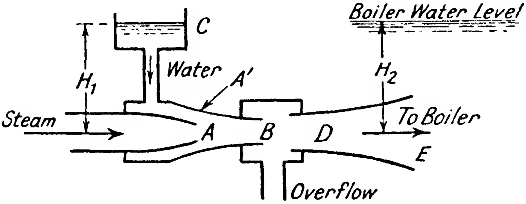


FIG. 170. PRINCIPLE OF STEAM INJECTOR

Let W_s = Weight of steam supplied to injector in lb. per sec.
W_w = Weight of water entering at A' in lb. per sec.
U_a = Velocity of steam leaving nozzle at A in ft. per sec.
U_w = Velocity of water entering at A' in ft. per sec.
U_b = Velocity of mixture leaving nozzle at B in ft. per sec.

Then

$$\left.\begin{matrix}\text{Momentum of}\\ \text{steam entering}\\ \text{combining nozzle}\end{matrix}\right\} + \left\{\begin{matrix}\text{Momentum of}\\ \text{water entering}\\ \text{combining nozzle}\end{matrix}\right\} = \left\{\begin{matrix}\text{Momentum of}\\ \text{mixture leaving}\\ \text{combining nozzle}\end{matrix}\right.$$

$$\frac{W_sU_a}{g} + \frac{W_wU_w}{g} = \frac{(W_s + W_w)U_b}{g}$$

or
$$W_sU_a + W_wU_w = (W_s + W_w)U_b$$

If the water level in C is below the injector, then

$$W_sU_a - W_wU_w = (W_s + W_w)U_b$$

Hence $$\frac{W_w}{W_s} = \frac{U_a - U_b}{U_b \pm U_w}$$

according to whether the water level is below or above the injector level.

This formula gives the amount of water injected per pound of steam, if the velocities are known.

To find the velocities, let

P_1 = initial pressure of steam supply in lb. per sq. ft.

P_2 = steam pressure at the throat A in lb. per sq. ft.

V_1 = specific volume of initial steam supply in cu. ft. per lb.

Then assuming adiabatic expansion, the velocity at any place in the nozzle where the pressure is P_2 is given by the equation

$$U_2 = \sqrt{2g\frac{n}{n-1}P_1V_1\left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}} \text{ (p. 453)}$$

or $$U_2 = \sqrt{2gJ \text{ (adiabatic heat drop)}}$$

If we assume that the pressure in the throat of the nozzle A is the critical pressure, then the steam velocity at the exit of A is

$$U_a = \sqrt{2g\frac{n}{n+1}P_1V_1} \text{ (p. 457)}$$

Hence if the steam supply is dry saturated, we may take $n = 1{\cdot}135$

and $$U_a = 5{\cdot}85\sqrt{P_1V_1}$$

If the steam supply is superheated, then $n = 1{\cdot}3$

and $$U_a = 6{\cdot}03\sqrt{P_1V_1}$$

This velocity can also be calculated from the adiabatic heat drop read off the total heat-entropy chart.

It must be understood that these equations will not give a strictly accurate result as the expansion in the steam nozzle will obviously be affected by the water surrounding it.

The velocity of the water entering the annular space at A' will be given by the equation $U_w = \sqrt{2gH_1}$.

To find the velocity U_b, let

P = boiler pressure in lb. per sq. ft. = p lb. per sq. in.

P_b = pressure at B in lb. per sq. ft.

w = density of water at B in lb. per cu. ft.

Then the total energy per pound of water at $B = \frac{P_b}{w} + \frac{U^2_b}{2g}$.

This energy must be enough to lift the water through a height H_2 feet and inject it into the boiler. The final pressure on leaving at E must therefore be somewhat greater than this height plus the boiler pressure.

Let h = the necessary excess head in feet

$P_b = 15 \times 144$ lb. per sq. ft.

$w = 62{\cdot}4$ lb. per cu. ft.

Then $\dfrac{15 \times 144}{62{\cdot}4} + \dfrac{U^2_b}{2g} = \dfrac{P}{w} + H_2 + h$

and $U_b = \sqrt{2g\left[\dfrac{144}{62{\cdot}4}(p - 15) + H_2 + h\right]}$

To find the nozzle areas, let

A_a = steam nozzle area at A in sq. ft.

A_b = combining nozzle area at B in sq. ft.

A_a' = area of water annulus at A' in sq. ft.

V_t = specific volume of steam after expansion in nozzle A

Then the value of V_t can be found from the formula

$$\frac{1}{V_t} = \frac{1}{V_1}\left(\frac{2}{n+1}\right)^{\frac{1}{n-1}} \quad \text{(p. 457)}$$

Hence $\dfrac{W_s}{A_a} = \dfrac{U_a}{V_t}$ and $A_a = \dfrac{W_s V_t}{U_a}$

also $W_s + W_w = A_b U_b \times 62{\cdot}4$ and $A_b = \dfrac{W_s + W_w}{62{\cdot}4 U_b}$

and $W_w = A_a' U_w \times 62{\cdot}4$ and $A_a' = \dfrac{W_w}{62{\cdot}4 U_w}$

The heat balance may be determined as follows—

Let H_s = total heat per pound of steam entering the injector

h_w = sensible heat per pound of water supplied to injector

h_m = sensible heat per pound of water leaving at B

Then

$$\left\{\begin{matrix}\text{Heat supplied} \\ \text{in steam}\end{matrix}\right\} + \left\{\begin{matrix}\text{Heat supplied} \\ \text{in water}\end{matrix}\right\} \pm \left\{\begin{matrix}\text{Kinetic energy of} \\ \text{water at } A'\end{matrix}\right\}$$

$$= \left\{\begin{matrix}\text{Heat in mixture} \\ \text{at B}\end{matrix}\right\} + \left\{\begin{matrix}\text{Kinetic energy of} \\ \text{mixture at B}\end{matrix}\right\}$$

that is

$$W_s H_s + W_w h_w \pm \frac{W_w U^2_w}{2gJ} = (W_s + W_w)h_m + \frac{(W_s + W_w)U_b^2}{2gJ}$$

according to whether the water supply level is above or below the injector level.

From this equation the temperature of the mixture may be found.

EXAMPLES XI

1. Steam flows through a correctly designed nozzle with a pressure drop from 200 lb. to 2 lb. per in. absolute. Assuming adiabatic frictionless flow find the dryness fraction and the velocity of the steam as it leaves the nozzle when the steam entering the nozzle is (*a*) dry saturated, (*b*) superheated 50° C. above the saturation temperature.

2. Find the velocity of the steam, in Question (1), assuming the flow is supersaturated throughout the expansion.

3. Find the correct throat and exit diameters of a convergent-divergent nozzle which will discharge 120 lb. of steam per hour from a pressure of 120 lb. per sq. in., superheated to 220° C., into a chamber having a pressure of 30 lb. per sq. in. absolute. The friction loss in the divergent part of the nozzle may be taken as one-tenth of the total heat drop.
(*U.L.*, *B.Sc.* (*Eng.*), 1923.)

4. A convergent-divergent nozzle is to be designed to discharge 0·15 lb. of steam per sec. into a vessel in which the pressure is 20 lb. per sq. in. absolute, when the nozzle is supplied with steam at 100 lb. per sq. in. absolute, superheated to 200° C. Find the throat and exit diameters of the nozzle on the assumption that the friction loss in the diverging part is 10 per cent of the total heat drop. (*U.L.*, *B.Sc.* (*Eng.*), 1925.)

5. A nozzle discharges steam dry saturated at 160 lb. per sq. in. absolute pressure into a reservoir where the pressure is 1 lb. per sq. in. absolute. The diameter of the nozzle at the throat is ⅜ in. If the flow is frictionless adiabatic, what weight of steam will pass through the nozzle per minute ? If 10 per cent of the energy goes in friction, what weight of steam is delivered per minute ? (*U.L.*, *B.Sc.* (*Eng.*).)

6. Dry and saturated steam at 25 lb. per sq. in. absolute pressure is allowed to discharge through a long convergent nozzle into the atmosphere (15 lb. per sq. in.). Assuming frictionless adiabatic flow, find the dryness of the steam at discharge and calculate the weight of steam which should be discharged per second if the exit diameter of the nozzle is 0·5 in. If the weight of steam actually discharged be 94 per cent of this calculated quantity, estimate the percentage of the heat drop which is wasted in friction.
(*U.L.*, *B.Sc.* (*Eng.*), 1923.)

7. A divergent nozzle for a steam turbine has to deliver 800 lb. of steam per hour under a supply condition of 160 lb. per sq. in. absolute, with 200° F. (111° C.) of superheat, and a stage pressure of 1·5 lb. per sq. in. absolute. Assuming a coefficient of discharge of 0·97 and a frictional loss of 12 per cent, determine the throat and outlet areas. (*I.Mech.E.*, 1926.)

8. The throat and exit diameters of a nozzle are 0·300 and 0·375 inches respectively. If dry and saturated steam at 100 lb. per sq. in. absolute is allowed to expand through the nozzle into a chamber in which the pressure is 15 lb. per sq. in. absolute, find the weight of steam passing per second. If 10 per cent of the heat drop is wasted in friction, what is the probable pressure of the steam at its exit from the nozzle ? (*U.L.*, *B.Sc.* (*Eng.*), 1922.)

9. Steam at 150 lb. per sq. in. pressure and dryness 0·95 expands adiabatically through a nozzle to a pressure of 12 lb. per sq. in. Assuming that PV^n = constant is the law for the expansion, find the value of n which satisfies the initial and final state points, and use this value to calculate the pressure and velocity of the steam at the throat of the nozzle.
(*U.L., B.Sc.* (*Eng.*), 1934.)

CHAPTER XII

STEAM TURBINES

The Velocity Diagram. We have seen in the last chapter that when steam expands through a suitably shaped passage or nozzle, some of the heat energy in the steam may be converted into kinetic energy, the steam thus attaining a high velocity. If this steam is now allowed to pass through blades situated in a wheel so arranged that it may revolve freely about its axis (Fig. 171), motion will be imparted to the wheel and mechanical work can be thus obtained.

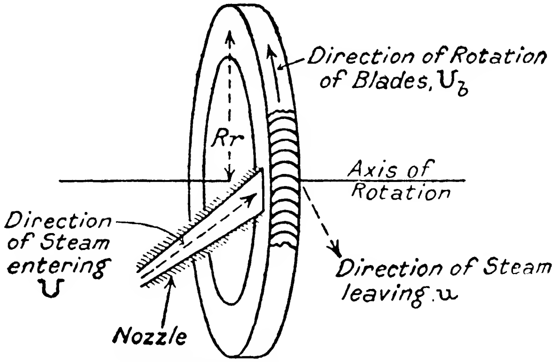


FIG. 171. ELEMENTARY SINGLE-STAGE TURBINE

As the blades are situated radially round the wheel, the linear velocity of the outside edges will be greater than that of the inside edges. Let the linear velocity of the blades at the mean radius R_r, Fig. 171, be U_b. We can now draw velocity triangles for the steam on entering and leaving the blading.

In Fig. 172, let CD be a moving blade.

At Entry. Let $AB = U$ = Velocity of steam leaving nozzle and entering moving blade (ft. per sec.)

$CB = U_b$ = Mean velocity of moving blade (ft. per sec.)

$\therefore AC = R$ = Velocity of steam relative to moving blade at entry (ft. per sec.)

In order that the steam should enter the moving blade without shock, there must be no component velocity of the steam relative to the blade at right angles to the profile of the blade at entry. In other words, the entry angle of the moving blade must be the same as the angle β made by the direction of the steam AC at entry relative to the direction of motion CB of the moving blade.

At Exit. $FD = U_b$ = Mean velocity of moving blade (ft. per sec.)
$DE = r$ = Velocity of steam relative to moving blade at exit (ft. per sec.)
$\therefore FE = u$ = Actual velocity of steam at exit (ft. per sec.)

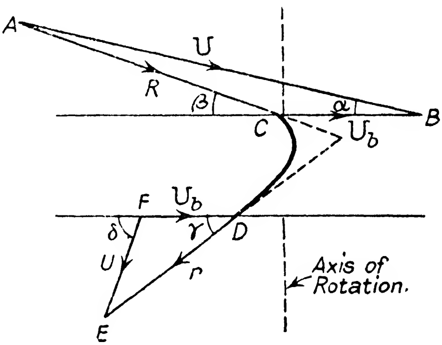


FIG. 172

It is obvious that the direction of the relative velocity at exit DE will be at an angle γ to the direction of motion of the moving blade, the angle γ being the exit angle of the blade.

These two velocity triangles can be combined for convenience in one diagram (Fig. 173).

From this diagram it will be seen that the change in velocity produced in the steam by its passage through the moving blade is ED. If this change in velocity is resolved parallel and at right angles to the direction of motion of the moving blade, then CD is the component change in velocity produced in the steam parallel to the direction of motion of the moving blade. Let this velocity be U_c.

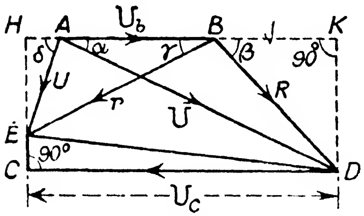


FIG. 173

Work Done in Blading. According to Newton's second law of motion, force is measured by the rate of change of momentum.

Hence, if W lb. of steam pass through the blading per second, the force exerted in the direction of motion by the steam on the moving blades $= \frac{W}{g} \times$ change in velocity of the steam in the direction of motion (in feet per second)

$$= \frac{W}{g} \times U_c \text{ lb.} \quad \text{(Fig. 173)}$$

$\therefore$ work done per second

$$= \text{Force} \times \text{Distance moved per second}$$

$$= \frac{W}{g} \times U_c \times U_b \text{ ft-lb. per second}$$

$$\therefore \text{ horse-power of blading} = \frac{W}{g} \times \frac{U_c U_b}{550}$$

It should be noted that if Fig. 173 is drawn to scale, the values U_c and U_b can be read from the diagram.

Another expression for the work done can be found as follows—

From Fig. 173 we have

$$AD^2 = DB^2 + AB^2 + 2DB \times AB \cos \beta$$

$$\therefore \quad U^2 = R^2 + U_b^2 + 2RU_b \cos \beta$$

$$\text{also } AE^2 = EB^2 + AB^2 - 2EB \times AB \cos \gamma$$

$$\therefore \quad u^2 = r^2 + U_b^2 - 2rU_b \cos \gamma$$

Subtracting these two equations,

$$2U_b(R \cos \beta + r \cos \gamma) = U^2 - u^2 - R^2 + r^2$$

But from Fig. 173 we see that

$$U_c = HA + AK = HB + BK$$

$$= u \cos \delta + U \cos \alpha = r \cos \gamma + R \cos \beta$$

But $2U_b(R \cos \beta + r \cos \gamma) = U^2 - u^2 - R^2 + r^2$

$\therefore$ $2U_b U_c = U^2 - u^2 - R^2 + r^2$

$\therefore$ Work done per second

$$= \frac{W}{g} U_c U_b = W\left(\frac{U^2 - u^2 - R^2 + r^2}{2g}\right)$$

This result can be obtained direct from a consideration of kinetic energy. The kinetic energy of the steam on entry to the moving blades is $\frac{U^2}{2g}$ per lb. If r is greater than R, as in the reaction turbine, then this increase of velocity must have taken place by an expansion of steam in the moving blades, similar to the expansion in the nozzle. Hence the increase in kinetic energy due to expansion in the moving blades is $\left(\frac{r^2}{2g} - \frac{R^2}{2g}\right)$ per lb. The total kinetic energy supplied to the moving blades is therefore $\frac{U^2 + r^2 - R^2}{2g}$ per lb. of steam. The final kinetic energy of the steam on leaving the moving blades is $\frac{u^2}{2g}$ per lb. Hence the energy imparted to the moving blades is $\frac{U^2 + r^2 - R^2 - u^2}{2g}$ per lb.

Hence, work done per second $= \dfrac{W(U^2 - u^2 - R^2 + r^2)}{2g}$ ft-lb.

$\therefore$ Horse-power $= \dfrac{W}{2g}\left(\dfrac{U^2 - u^2 - R^2 + r^2}{550}\right)$

It should be noted that if r is less than R, as in the case of an impulse turbine, then $\left(\dfrac{r^2}{2g} - \dfrac{R^2}{2g}\right)$ is negative and is the loss of energy due to friction as the steam passes through the moving blades.

The Impulse Turbine. In an impulse turbine expansion of the steam occurs practically only in the nozzles or fixed blades. The expansion converts heat energy into kinetic energy, as already explained, and the steam leaving the nozzles at a high velocity enters the moving blades, giving up some of its kinetic energy to them, and hence producing motion of the rotor. The state of affairs in a single stage simple impulse turbine is shown diagrammatically in Fig. 174, which should be compared with Fig. 171.

It is obvious that several nozzles, all supplied with steam from the same source, may be situated round the moving blade ring. The curves in Fig. 174 indicate the variations of pressure and velocity throughout the stage.

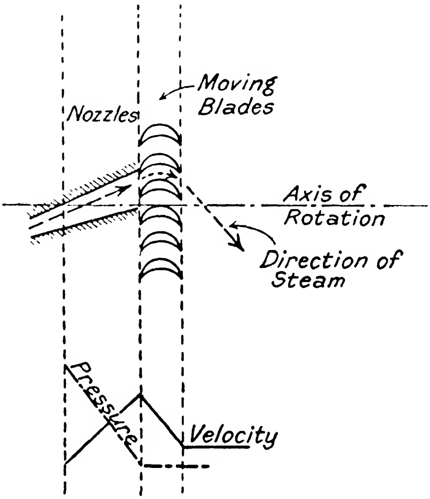


FIG. 174. DIAGRAM OF SINGLE-STAGE IMPULSE TURBINE

The pressure will drop during the passage of the steam through the nozzles, and will then remain practically constant in the moving blades. The velocity will increase, due to the pressure drop in the nozzles, and will then decrease as kinetic energy is given to the moving blades.

Efficiency of Impulse Turbine Blading. As the pressure remains constant through the moving blades, any difference between R and r (Fig. 173) will be due to friction during the passage of the steam through the moving blades. Hence the value of r will be less than that of R.

Let $r = KR$. Then work done per second

$$= \frac{W(U^2 - u^2 - R^2 + r^2)}{2g} = \frac{W(U^2 - u^2)}{2g} - \frac{WR^2}{2g}(1 - K^2)$$

$$\text{Kinetic energy supplied per second} = \frac{WU^2}{2g}$$

$$\therefore \text{ Efficiency} = \frac{\text{Work done per second}}{\text{Kinetic energy supplied per second}}$$

$$= \frac{\dfrac{W(U^2 - u^2 - R^2 + r^2)}{2g}}{\dfrac{WU^2}{2g}} = \frac{U^2 - u^2 - R^2 + r^2}{U^2}$$

$$= \frac{U^2 - u^2}{U^2} - \frac{R^2}{U^2}(1 - K^2)$$

If we assume that there are no losses in the moving blades, then $R = r$ and

$$\text{Efficiency} = \frac{U^2 - u^2}{U^2}$$

The efficiency can also be expressed in other terms, as follows. It has already been shown that

$$\text{Work done per second} = \frac{W}{g} \times U_c \times U_b. \quad \text{(Fig. 173)}$$

$$\therefore \text{ Efficiency} = \frac{2\,U_c \times U_b}{U^2}$$

Maximum Efficiency of Impulse Turbine Blading (U and α given).

$$\text{Work done per second} = \frac{W}{g} \times U_c \times U_b$$

But $U_c = BK + BH$ (Fig. 173) $= R\cos\beta + r\cos\gamma$

$$= R\cos\beta + KR\cos\gamma = R\cos\beta\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

and $R\cos\beta = AK - AB = U\cos\alpha - U_b$

$$\therefore \quad U_c = (U\cos\alpha - U_b)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$\therefore \text{ Work done per second} = \frac{W}{g} \times U_c \times U_b$$

$$= \frac{W}{g}(U_bU\cos\alpha - U_b^2)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$\therefore \text{ Efficiency} = \frac{\dfrac{W}{g}(U_bU\cos\alpha - U_b^2)\left(1 + K\dfrac{\cos\gamma}{\cos\beta}\right)}{\dfrac{WU^2}{2g}}$$

$$= 2\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^2}{U^2}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

If we assume that the exit blade angle γ is a constant proportion of the inlet blade angle β, then $\dfrac{\cos\gamma}{\cos\beta} = \text{constant}$.

$$\therefore \text{ Efficiency} = 2\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^{\,2}}{U^2}\right) C, \text{ where } C \text{ is constant.}$$

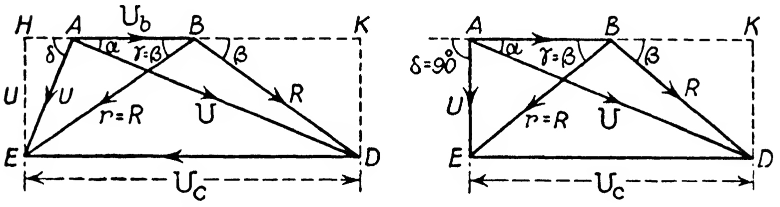
$\therefore$ Differentiating and equating to zero we have, for maximum efficiency

$$\cos\alpha = \frac{2U_b}{U} \text{ or blade velocity } U_b = U\frac{\cos\alpha}{2}$$

Substituting this value of U_b in the general equation for efficiency just obtained

$$\text{Maximum efficiency} = 2\left(\frac{\cos^2\alpha}{2} - \frac{\cos^2\alpha}{4}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$= \frac{\cos^2\alpha}{2}\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$



(a) (b)

FIG. 175

Maximum Efficiency of Impulse Turbine Blading (U and α given, and assuming symmetrical blading with no losses). If there are no losses, $R = r$, $\therefore K = 1$, and, with symmetrical blading, $\gamma = \beta$.

$$\therefore \text{ Efficiency} = 2\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^{\,2}}{U^2}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$= 4\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^{\,2}}{U^2}\right)$$

$$\therefore \text{ Maximum efficiency} = 4\left(\frac{\cos^2\alpha}{2} - \frac{\cos^2\alpha}{4}\right) = \cos^2\alpha$$

These results can also be obtained by modifying Fig. 173. The diagram so modified is shown in Fig. 175 (*a*).

The triangle *DBE* becomes isosceles and the actual change of velocity *DE* (Fig. 173) becomes parallel to the direction of motion of the blades.

$$\therefore r \cos \gamma = R \cos \beta$$

$$\therefore U_c = 2R \cos \beta = 2(AK - AB) = 2(U \cos \alpha - U_b)$$

$$\therefore \text{Work done per second} = \frac{W}{g} U_c U_b = \frac{W}{g} 2(U \cos \alpha - U_b) U_b$$

$$= 2\frac{W}{g}(U_b U \cos \alpha - U_b{}^2)$$

$$\therefore \text{Efficiency} = \frac{2\dfrac{W}{g}(U_b U \cos \alpha - U_b{}^2)}{\dfrac{WU^2}{2g}} = 4\left(\frac{U_b}{U}\cos \alpha - \frac{U_b{}^2}{U^2}\right)$$

Differentiating and equating to zero,

$$\cos \alpha = 2\frac{U_b}{U} \text{ for maximum efficiency}$$

This will be the case when the diagram is as Fig. 175 (*b*), where $\delta = 90°$; for here $\cos \alpha = \frac{AK}{AD} = \frac{2AB}{AD} = \frac{2U_b}{U}$.

Hence maximum efficiency

$$= \frac{\dfrac{W}{g} U_c U_b}{\dfrac{W}{2g} U^2} = 2 \times \frac{DE \times AB}{AD^2} \quad \text{(Fig. 175 (}b\text{).)}$$

$$= \frac{2 \times AK \times \dfrac{AK}{2}}{AD^2} = \cos^2 \alpha$$

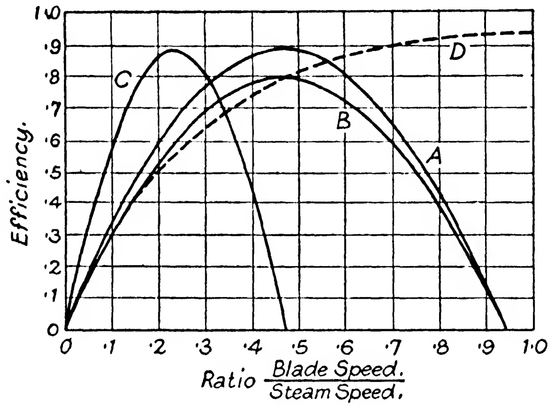
The fact that for maximum efficiency the angle δ must be 90° also follows from the expression for efficiency, namely, $\frac{U^2 - u^2}{U^2}$. For this to have a maximum value for any given value of U, the value of u must be a minimum. Fig. 175 shows that this will be the case where $\delta = 90°$, that is, when the direction of the steam on leaving the moving blades is at right angles to the direction of motion of the blades, the steam then having no velocity in the direction of motion of the blades.

Effect of Blade Speed on Efficiency of Impulse Turbine Blading. It is instructive to plot curves showing the variation of blade efficiency with the value of the ratio $\frac{\text{Blade speed}}{\text{Steam speed}} = \frac{U_b}{U}$. For simplicity we will assume symmetrical blades, i.e. $\gamma = \beta$. If then, in the general equation,

$$\text{Efficiency} = 2\left(\frac{U_b}{U}\cos\alpha - \frac{U_b^2}{U^2}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

$$= 2\frac{U_b}{U}\left(\cos\alpha - \frac{U_b}{U}\right)\left(1 + K\frac{\cos\gamma}{\cos\beta}\right)$$

we substitute constant values for U and K, we can calculate the values of the efficiency for various values of the ratio $\frac{U_b}{U}$. This has been done in Fig. 176, curves A and B for a value of $\alpha = 20°$. Curve A is the efficiency curve for no losses, i.e. $R = r$, making $K = 1$. Curve B is the efficiency curve for a value of $K = 0{\cdot}8$.



Efficiency.

$\frac{U_b}{U}$	K=1	K=0·8
0	0	0
0·1	0·336	0·302
0·2	0·592	0·533
0·3	0·768	0·691
0·4	0·864	0·778
0·5	0·879	0·791
0·6	0·815	0·734
0·7	0·671	0·604
0·8	0·447	0·402
0·9	0·143	0·129

FIG. 176. EFFICIENCY CURVES

We see that the efficiency is zero when

$$\frac{U_b}{U} = 0, \text{ i.e. when } U_b = 0$$

and when $$\frac{U_b}{U} = \cos 20° = 0{\cdot}9397$$

The maximum efficiency is equal to $\cos^2 20° = 0{\cdot}883$ in the case where there are no losses. When $K = 0{\cdot}8$ the maximum efficiency is equal to $\frac{\cos^2 20°}{2}(1 + 0{\cdot}8) = 0{\cdot}795$. In both cases the maximum efficiency is obtained when

$$\frac{U_b}{U} = \frac{\cos 20°}{2} = 0{\cdot}4698$$

Example 1. A stage in an impulse turbine consists of converging nozzles and one ring of moving blades. The nozzle angles are 22° and the moving blades have both blade tip angles of 35°. If the velocity of the steam at the exit from the nozzle is 1,500 ft. per sec., find the blade speed so that the

steam shall pass on to the blade without shock, and find the stage efficiency, neglecting frictional losses, if the blades run at this speed.

If the relative velocity of steam to blade is reduced by 15 per cent in passing through the blade ring, find the actual efficiency and the end thrust on the shaft when the blade ring develops 50 H.P.

(*U.L.*, *B.Sc.* (*Eng.*), 1924.)

1. Let the mass of steam passing per second $= W = 1$ lb. Then Fig. 177 shows the velocity diagram, assuming no losses. Draw the angle $CAD = 22°$ and let the length AD represent 1,500 ft. per sec. to some convenient scale. As there are no losses and the blade angles are equal, the direction of the actual change of velocity of the steam will be parallel to the direction of motion of the blades. Therefore draw DF parallel to AC and make the angles DBC and ABE each equal to 35°, and draw $BE = BD$. Then DE represents U_c, the change in velocity produced in the steam, and AB represents U_b, the velocity of the blades. The values of the various vectors are shown on the diagram.

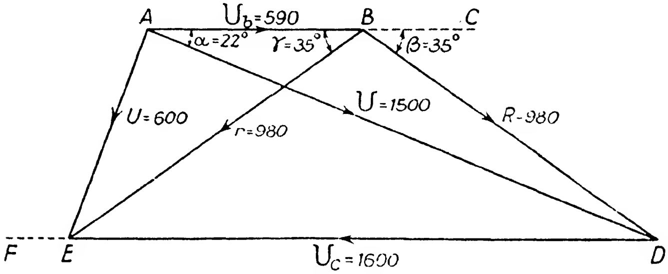


FIG. 177

Blade speed for no shock and no losses $= U_b = 590$ ft. per sec.

$$\text{Efficiency} = \frac{2\,U_cU_b}{U^2} = \frac{2 \times 1{,}600 \times 590}{(1{,}500)^2} = 83{\cdot}9 \text{ per cent}$$

$$\text{or} \quad \text{Efficiency} = \frac{U^2 - u^2}{U^2} = \frac{1{,}500^2 - 600^2}{1{,}500^2} = 84 \text{ per cent}$$

2. With 15 per cent loss in passing through moving blades the triangle ABD (Fig. 177) remains the same as in the first part of the question, but the triangle ABE is modified as $r = 0{\cdot}85\ R = 0{\cdot}85 \times 980 = 834$ ft. per sec.

Hence with the angle $\gamma = 35°$ and $r = 834$ ft. per sec., the modified triangle ABE may be drawn to scale and the diagram becomes similar to Fig. 173 with $r < R$.

The required velocities can now be read off the diagram and the problem solved.

These velocities can, however, be calculated without drawing the diagram to scale, the new velocities required being U_c and EC (Fig. 173).

$$\text{Thus } BK = R\cos\beta = 980\cos 35^\circ = 803 \text{ ft. per sec.}$$

$$BH = r\cos\gamma = 834\cos 35^\circ = 683 \text{ ft. per sec.}$$

$$\therefore \quad U_c = BK + BH = 803 + 683 = 1{,}486 \text{ ft. per sec.}$$

$$\text{Also } EC = HC - HE = R\sin\beta - r\sin\gamma$$

$$= 980\sin 35^\circ - 834\sin 35^\circ$$

$$= 84 \text{ ft. per sec.}$$

$$\text{Hence efficiency} = \frac{2U_cU_b}{U^2} = \frac{2 \times 1{,}486 \times 590}{1{,}500^2} = 0{\cdot}778$$

and $\text{End Thrust} = \frac{W}{g} EC$ where W = Steam flow in lb. per sec.

To find W—

$$\text{H.P.} = \frac{W}{g}\,\frac{U_cU_b}{550}$$

i.e. $$50 = \frac{W}{32{\cdot}2} \times \frac{1{,}486 \times 590}{550}$$

whence $$W = 1{\cdot}01 \text{ lb. per sec.}$$

and $$\text{End Thrust} = \frac{1{\cdot}01}{32{\cdot}2} \times 84 = 2{\cdot}63 \text{ lb.}$$

Velocity Compounding. We have seen in the last chapter that when steam expands in a suitably shaped nozzle, a large velocity can be generated in the steam. Thus, if dry steam expands through a nozzle from 200 lb. per sq. in. (abs.) to 2 lb. per sq. in. (abs.), the final velocity of the steam is about 3,900 ft. per sec. We have just seen that, if this steam is passed through a single row of blades situated round the periphery of a wheel, the linear velocity of the blades must be somewhere about half that of the steam if the efficiency is to be a maximum. The speed of rotation of a rotor carrying a single row of blades would therefore be very high, and reduction gearing would be required to bring the speed down for most practical purposes. One method of reducing the speed of the turbine rotor is to provide the wheel with two rows of blades, between which there is a row of fixed blades attached to the stationary turbine casing. This row of fixed blades simply changes the direction of the steam leaving the first row of moving blades back to a direction suitable for entering the second row of moving blades. Fig. 178 shows diagrammatically a section of a velocity compounded wheel. Curves are also given showing the variation of pressure and velocity of the steam as it passes through the blading.

It will be noticed that the pressure drop takes place in the nozzles, there being no further pressure drop in either the fixed or moving blades. The high velocity produced in the steam is reduced on passing through each row of moving blades. As there is no pressure drop in the intermediate fixed row of blades, there is no change in the value of the velocity but only in its direction. In practice, friction occurs and modifies matters to some extent.

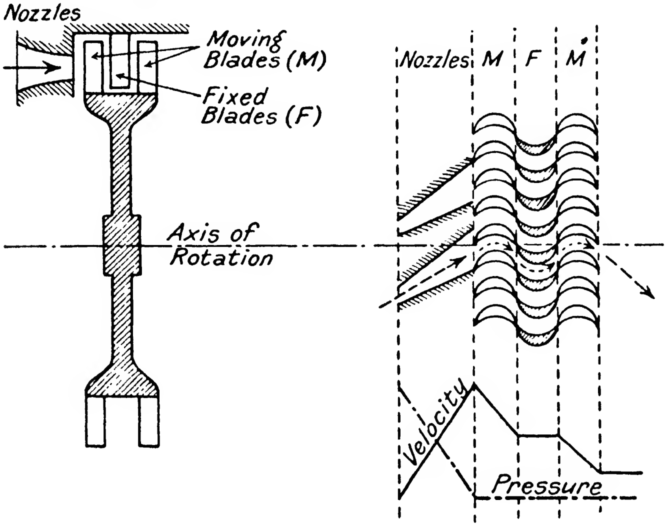


FIG. 178. DIAGRAMMATIC SECTION OF A VELOCITY COMPOUNDED TURBINE WHEEL

Efficiency of a Velocity Compounded Stage. The complete velocity diagram will consist of two diagrams, one for each set of moving blades. Let us assume for simplicity that the blading is symmetrical and that there is no friction. Then Fig. 179 shows the velocity diagram.

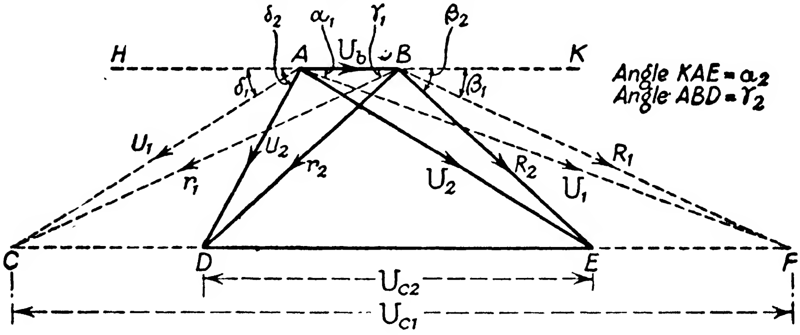


FIG. 179. VELOCITY DIAGRAM

Let AF represent U_1 the velocity of the steam leaving the nozzles and entering the first row of moving blades. Then $ABFC$ is the velocity diagram for the first row of moving blades, the actual velocity of the steam on leaving this first row being represented by $AC = u_1$. The intermediate fixed blades must have an entry angle equal to the angle HAC if the steam is to enter without shock and an equal exit angle if the blade is symmetrical. Hence, if the angle KAE is made equal to the angle HAC and AE is made equal to AC, then AE will represent the velocity of the steam leaving the intermediate fixed blades, and this will be the velocity U_2 of the steam entering the second row of moving blades. It is obvious that, with the simple conditions assumed here, the point E will be on the line CF. The blade velocity U_b being the same for both rows of moving blades, the velocity diagram $ABED$ for the second row of moving blades may now be drawn, making $r_2 = R_2$ and angle HBD = angle KBE. The point D will then also be on the line CF. The change in velocity produced in the steam by its passage through the first row of moving blades U_{c1} is represented by CF, while that produced by the passage through the second row of moving blades U_{c2} is represented by DE.

In Fig. 179 we have $U_2 = u_1$, $\alpha_2 = \delta_1$, $r_2 = R_2$, $\gamma_2 = \beta_2$.

$$\begin{aligned}
\text{Then}\quad CE &= 2u_1 \cos \delta_1 \\
DE &= 2R_2 \cos \beta_2 = 2(U_2 \cos \alpha_2 - U_b) = 2(u_1 \cos \delta_1 - U_b) \\
\therefore CD &= CE - DE = 2u_1 \cos \delta_1 - 2(u_1 \cos \delta_1 - U_b) = 2U_b \\
\text{Also}\quad CF &= 2R_1 \cos \beta_1 = 2r_1 \cos \gamma_1 = 2(u_1 \cos \delta_1 + U_b) \\
\therefore EF &= CF - CE = 2(u_1 \cos \delta_1 + U_b) - 2u_1 \cos \delta_1 = 2U_b \\
\therefore DE &= CF - (CD + EF) \\
\therefore U_{c2} &= U_{c1} - 4U_b
\end{aligned}$$

$\therefore$ Total change of velocity $U_c = U_{c1} + U_{c2} = 2U_{c1} - 4U_b$

$$\begin{aligned}
\text{But}\quad U_{c1} &= 2R_1 \cos \beta_1 = 2(U_1 \cos \alpha_1 - U_b) \\
\therefore U_c &= 4(U_1 \cos \alpha_1 - U_b) - 4U_b \\
&= 4(U_1 \cos \alpha_1 - 2U_b)
\end{aligned}$$

$\therefore$ If W = mass of steam passing per second (in pounds)

$$\begin{aligned}
\text{Work done per second} &= \frac{W}{g} U_c U_b \\
&= \frac{W}{g} 4(U_1 U_b \cos \alpha_1 - 2U_b^2) \text{ (ft-lb.)}
\end{aligned}$$

$$\text{Efficiency} = \frac{\frac{W}{g} 4(U_1 U_b \cos \alpha_1 - 2U_b^2)}{\frac{WU_1^2}{2g}}$$

$$= 8\left(\frac{U_b}{U_1} \cos \alpha_1 - 2\left[\frac{U_b}{U_1}\right]^2\right)$$

This will be a maximum when $\cos\alpha_1 = 4\dfrac{U_b}{U_1}$

$\therefore$ For maximum efficiency $\dfrac{U_b}{U_1} = \dfrac{\cos\alpha_1}{4}$

and Maximum Efficiency $= \cos^2\alpha$

In a similar manner it can be shown that, with n rows of moving blades, maximum efficiency will be obtained when $\dfrac{U_b}{U_1} = \dfrac{\cos\alpha_1}{2n}$.

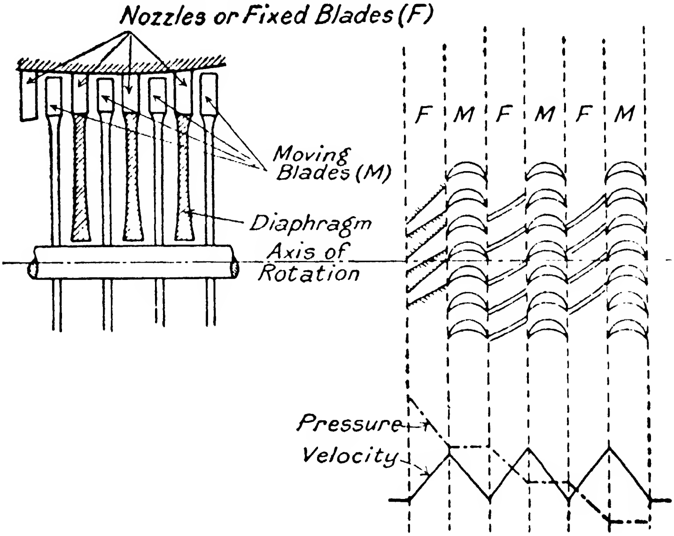


FIG. 180. PRESSURE COMPOUNDING

Fig. 176, curve C, shows the variation of efficiency with the value of the ratio $\dfrac{\text{Blade speed}}{\text{Steam speed}}$ for a value of $\alpha_1 = 20°$.

It will be seen that the maximum efficiency equals $\cos^2 20° = 0{\cdot}883$ when the value of $\dfrac{\text{Blade speed}}{\text{Steam speed}} = \dfrac{\cos 20°}{4} = 0{\cdot}2349$ and the efficiency is zero when the value of $\dfrac{\text{Blade speed}}{\text{Steam speed}} = \dfrac{\cos 20°}{2} = 0{\cdot}4698$.

Pressure Compounding. Another method of obviating the difficulty of the high velocity produced by a large drop in pressure is that of pressure compounding. With this method the whole pressure drop does not take place in one set of nozzles, but it is split up into a number of stages each consisting of a set of nozzles or fixed blades with a ring of moving blades, the various rings of moving blades all being fixed to the same rotor shaft. In this way the pressure drop in any one set of nozzles or fixed blades may be small and hence the velocity produced in the steam may be kept to any desirable value. The general arrangement is shown diagrammatically in Fig. 180.

The pressure drops take place only in the nozzles or fixed blades; in fact a pressure compounded turbine is simply a number of simple turbines in series. A stage consists of one set of nozzles or fixed blades, together with a ring of moving blades. There are thus four stages shown in Fig. 180. By suitably adjusting the number of stages it can be arranged that the rotor runs at any required speed for any given pressure drop in the steam through the whole turbine. In the first one or two stages, where the pressure of the steam is high and its specific volume small, the nozzles or fixed blades do not always extend all round the periphery of the wheel. In subsequent stages, as the steam expands the nozzles cover the whole circumference of the wheel. In modern turbines working with steam at a high initial pressure, the first stage is sometimes velocity compounded. In this way a comparatively large pressure drop can take place in the first stage nozzles, thus reducing the pressure inside the main turbine casing.

Example 2. Steam is supplied to an impulse steam turbine at a pressure of 180 lb. per sq. in. absolute, and superheated to 250° C. The pressure in the wheel chamber is 80 lb. per sq. in. absolute, and in the chamber there are two rings of moving blades separated by fixed blades. The tips of the moving blades are inclined 30° to the plane of motion. Assuming a 10 per cent friction loss in the nozzle and also a reduction of 8 per cent in the velocity of the steam relative to the blade due to frictional resistances in passing through a blade ring, determine the speed of the blades so that the final velocity of discharge shall be axial. Also state what should be the inclination of the nozzles to the plane of motion of the blades.

(*U.L., B.Sc. (Eng.)*, 1922.)

Since the pressure drop in the nozzles is from 180 lb. per sq. in. to 80 lb. per sq. in., the velocity of the steam on leaving the nozzles is given by

$$U_1 = \sqrt{2gJ\,(\text{Adiabatic heat drop from 180 to 80 lb. per sq. in.}) \times 0{\cdot}9}$$

$$= \sqrt{2 \times 32{\cdot}2 \times 1{,}400 \times (703 - 667) \times 0{\cdot}9}$$

$$= 1{,}709 \text{ ft. per sec.}$$

This is, therefore, the value of U_1, the velocity of the steam on entering the first ring of moving blades.

The velocity diagram (Fig. 181) can now be drawn, starting from the final velocity u_2 of the steam on leaving the second ring of moving blades.

Draw a triangle ABC, Fig. 181, with angle $BAC = 90°$ and angle $ABC = 30°$. Make angle $EBD = 30°$ and make $BD = \frac{CB}{0{\cdot}92}$.

Join AD. Then, as CB is 8 per cent less than BD, the figure $ABCD$ is the velocity diagram for the second ring of moving blades, drawn to some scale yet to be determined. Produce BC and let $AF = \frac{AD}{0{\cdot}92}$ cut BC produced in F. Produce BD to H, making $BH = \frac{FB}{0{\cdot}92}$. Join AH. Then the figure $ABHF$ will be the velocity diagram for the first ring of moving blades. AB will represent the velocity of the moving blades. AF will represent the velocity of the steam on leaving the first ring of moving blades. This velocity will be changed in direction and reduced by 8 per cent in magnitude by the passage through the fixed blades. Hence the velocity on leaving the fixed blades will be given by AD, which, therefore, also represents the velocity of the steam U_2 on entering the second ring of moving blades.

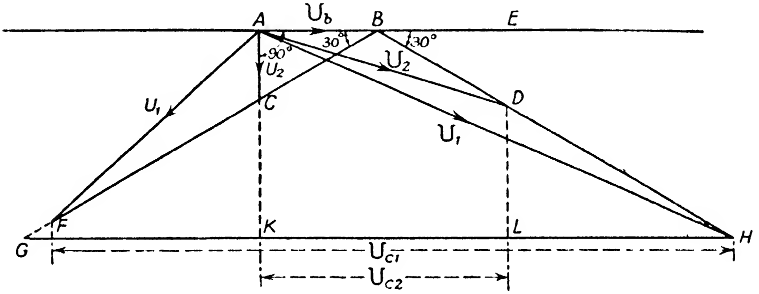


FIG. 181. VELOCITY DIAGRAM

The velocity U_1 of the steam on leaving the nozzles and entering the first row of moving blades is represented by AH. This must be the value calculated at the beginning of the example, namely, 1,709 ft. per sec. This value, compared to the length of AH, fixes the velocity scale of the diagram.

From the diagram, Speed of blades = 392 ft. per sec.

Nozzle angle $= BAH = 23{\cdot}5°$

$$\text{Also, efficiency} = \frac{2(U_{c1} + U_{c2})U_b}{U_1^2} = \frac{2(2{,}260 + 820) \times 392}{1{,}709 \times 1{,}709}$$

$$= 82{\cdot}7 \text{ per cent.}$$

Losses and Efficiencies in an Impulse Stage. These can be explained with the help of the H-ϕ chart (Fig. 182) where AB represents the isentropic heat drop (I.H.D.) in an impulse stage consisting of a set of nozzles or fixed blades, and a set of moving blades, the expansion taking place in the nozzles or fixed blades.

Let U, U_c and U_b have the meanings already assigned to them and consider one pound of steam passing per second. Also let it be assumed that the velocity of the steam entering the nozzles is negligible compared with the velocity leaving.

It has already been explained on p. 459 that if the isentropic heat drop is represented by AB (Fig. 182), and the friction loss in the nozzles by $C'B$, then the actual heat drop in the nozzles and hence in the stage is given by AC'.

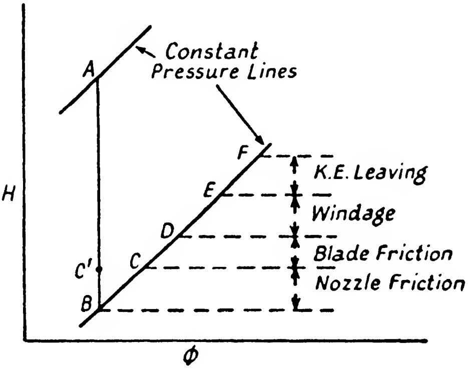


FIG. 182. TURBINE LOSSES

Hence

Isentropic heat drop (I.H.D.) $= H_A - H_B$

Friction loss in nozzles $= H_C - H_B$

Actual heat drop $= H_A - H_C = \dfrac{U^2}{2gJ}$

Nozzle efficiency, $\varepsilon_n = \dfrac{H_A - H_C}{H_A - H_B} = \dfrac{U^2}{2gJ\,(\text{I.H.D.})}$

The blading efficiency is a measure of the mechanical conversion of the kinetic energy of the steam into kinetic energy of the moving blades, allowing for the friction loss as the steam passes through the blades. This friction loss can be allowed for in the velocity diagram, by making $r = KR$ where K is the friction coefficient (p. 477).

Thus if

Friction loss in the moving blades $= H_D - H_C$ (Fig. 182)

then

Work done on moving blades $= H_A - H_D = \dfrac{U_c U_b}{gJ}$

and

Blading efficiency $\varepsilon_b = \dfrac{H_A - H_D}{H_A - H_C} = \dfrac{2U_c U_b}{U^2}$

The gross stage efficiency may be defined as the ratio of the work done on the moving blades as defined above, to the isentropic heat drop.

Hence gross stage efficiency

$$= \frac{H_A - H_D}{H_A - H_B} = \frac{H_A - H_D}{H_A - H_C} \times \frac{H_A - H_C}{H_A - H_B} = \varepsilon_b \times \varepsilon_n$$

or
$$= \frac{U_c U_b}{2gJ(\text{I.H.D.})} = \frac{2U_c U_b}{U^2} \times \frac{U^2}{2gJ(\text{I.H.D.})} = \varepsilon_b \times \varepsilon_n$$

There is also a disk and blade windage loss to be taken into account.

Thus if windage loss $= H_E - H_D$

then Net work supplied to turbine shaft $= H_A - H_E$

and Net stage efficiency $= \dfrac{\text{Work to shaft}}{(\text{I.H.D.})} = \dfrac{H_A - H_E}{H_A - H_B}$

Terminal Velocity and Carry-over. The steam leaving a stage will have a certain velocity, the equivalent kinetic energy of which is wasted unless the steam passes on to a further stage. In any case, for the final stage of a multi-stage turbine, there will be this leaving loss.

If Fig. 182 apply to such a final stage, then from the equation of steady flow

$$H_A + \frac{U_A^2}{2gJ} = H_E + \frac{U_E^2}{2gJ} + \text{work on turbine shaft}$$

i.e. Work done in heat units $= (H_A - H_E) + \dfrac{U_A^2 - U_E^2}{2gJ}$

If U_A is negligible

Work done on shaft $= (H_A - H_E) - \dfrac{U_E^2}{2gJ}$

and S.H.P. $= \dfrac{778}{550}\left[(H_A - H_E) - \dfrac{U_E^2}{2gJ}\right]$ per lb. steam if total heat is in B.Th.U. per lb.

Referring to Fig. 182, this loss $= \dfrac{U_E^2}{2gJ} = H_F - H_E$

When this kinetic energy is passed over to the next stage it is termed "carry-over."

The Reaction Turbine. For reasons which will be mentioned later this turbine should more strictly be termed impulse reaction. In this type the steam expands in both the fixed and moving blades, the pressure dropping continuously right through the turbine. The velocity diagram for one reaction stage, that is, for one set of fixed and one set of moving blades, is similar to Fig. 173, with the modification that, as expansion takes place in the moving blades, r will be greater than R.

$$\text{The work done per second} = \frac{W}{g} \times U_c \times U_b$$

$$\text{or} = \frac{W(U^2 - u^2 + r^2 - R^2)}{2g}$$

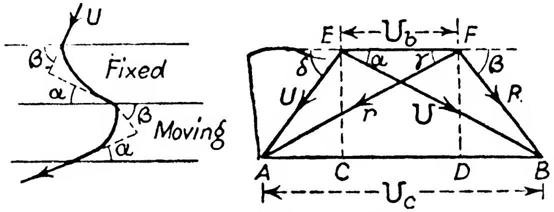


FIG. 183. VELOCITY DIAGRAM

The energy supplied to the stage per second

$$= W\left(\frac{U^2}{2g} + \begin{array}{c}\text{Kinetic energy generated, due to} \\ \text{expansion in the moving blades}\end{array}\right)$$

$$= W\left(\frac{U^2}{2g} + \frac{r^2 - R^2}{2g}\right)$$

$$\therefore \text{ Efficiency} = \frac{U^2 - u^2 + r^2 - R^2}{U^2 + r^2 - R^2}$$

The efficiency can obviously also be expressed as equal to

$$\frac{\frac{W}{g} \times U_c \times U_b}{\frac{W(U^2 + r^2 - R^2)}{2g}} = \frac{2 \times U_c \times U_b}{U^2 + r^2 - R^2}$$

Efficiency of Reaction Blading with Fixed and Moving Blading Identical. We will assume there are no losses due to friction. If the fixed and moving blades of a stage are identical, the velocity diagram reduces to Fig. 183.

In this case, $\gamma = \alpha$, $\delta = \beta$, $U = r$, $u = R$

The fixed blade will receive steam from the preceding moving blade at a velocity u The velocity of the steam after leaving the fixed blade is U. Hence—

$$\text{Kinetic energy of supply to fixed blade} = \frac{Wu^2}{2g}$$

Kinetic energy generated in fixed blade $= \dfrac{W(U^2 - u^2)}{2g}$

Kinetic energy generated in moving blade $= \dfrac{W(r^2 - R^2)}{2g}$

$= \dfrac{W(U^2 - u^2)}{2g}$

$\therefore$ Total energy supplied $= 2\dfrac{W(U^2 - u^2)}{2g} + \dfrac{Wu^2}{2g} = \dfrac{W}{2g}(2U^2 - u^2)$

But $\quad FB^2 = EB^2 + EF^2 - 2EB \times EF \cos FEB \quad$ (Fig. 183)

$\therefore \quad R^2 = U^2 + U_b{}^2 - 2UU_b \cos \alpha.\quad$ But $u = R$

$\therefore$ Total energy supplied

$$= \frac{W}{2g}(2U^2 - u^2)$$

$$= \frac{W}{2g}(2U^2 - [U^2 + U_b{}^2 - 2UU_b \cos \alpha])$$

$$= \frac{W}{2g}(U^2 - U_b{}^2 + 2UU_b \cos \alpha)$$

Work done per second $= \dfrac{W}{g} U_c U_b$

But $\quad U_c = AB = CB + AD - CD \quad$ (Fig. 183)

$$= U \cos \alpha + r \cos \gamma - U_b = 2U \cos \alpha - U_b$$

$\therefore$ Efficiency

$$= \frac{\dfrac{W}{g}(2U \cos \alpha - U_b)U_b}{\dfrac{W}{2g}(U^2 - U_b{}^2 + 2UU_b \cos \alpha)}$$

$$= \frac{2\left(2\dfrac{U_b}{U}\cos \alpha - \dfrac{U_b{}^2}{U^2}\right)}{1 - \dfrac{U_b{}^2}{U^2} + 2\dfrac{U_b}{U}\cos \alpha}$$

$$= \frac{2(2\lambda \cos \alpha - \lambda^2)}{1 - \lambda^2 + 2\lambda \cos \alpha}$$

where $\lambda = \dfrac{\text{Blade speed}}{\text{Steam speed leaving fixed blading}}$

To obtain the maximum efficiency, this expression must be differentiated and equated to zero. This gives, for maximum efficiency,

$$\lambda = \frac{U_b}{U} = \cos \alpha$$

whence maximum efficiency $= \dfrac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$

A curve D, shown in Fig. 176, indicates the variation of efficiency with the ratio $\frac{\text{Blade speed}}{\text{Steam speed}}$.

Degree of Reaction. It has been pointed out that the heat drop in the case of a reaction turbine can be considered as in two parts. In the impulse turbine the whole heat drop occurs in the nozzles or fixed blades, the final velocity of the steam leaving the nozzles being that corresponding to the heat drop. In the reaction turbine part of the heat drop occurs in the fixed blades as in the impulse turbine, thus increasing the steam velocity to U on leaving the fixed blades, the equivalent kinetic energy producing an impulse on the moving blades. There is then a further heat drop in the moving blades, increasing the relative velocity of the steam from R to r (Fig. 183) the reaction of which also produces motion of the moving blades. For this reason the term "impulse-reaction" more correctly describes this type of turbine.

The degree of reaction is defined as the ratio of the isentropic heat drop in the moving blades to the sum of the isentropic heat drops in the fixed and moving blades. In the simple case discussed on pp. 490–2, as $\frac{U^2 - u^2}{2g}$ equals $\frac{r^2 - R^2}{2g}$, the total heat drop across the stage is divided into two equal parts and the degree of reaction is 50 per cent, or half-degree.

Efficiency of Reaction Blading with Half-degree Reaction and Losses. Let Fig. 183 represent the velocity diagram, and let K, the velocity coefficient, be the same for both fixed and moving blades. Consider a steam flow of 1 lb. per second.

The kinetic energy of the steam leaving the previous stage is $\frac{u^2}{2g}$, but owing to losses, the fraction of this which is usefully used in the stage under consideration is $\frac{K^2u^2}{2g}$.

Hence, kinetic energy supplied to the fixed blading (carry over)

$$= \frac{K^2u^2}{2g}$$

$$= \frac{K^2R^2}{2g}$$

The velocity of the steam leaving the fixed blades is U.

Hence, kinetic energy supplied by expansion in fixed blades

$$= \frac{U^2 - K^2R^2}{2g}$$

In the moving blades if the stage were impulse only, the relative velocity in the direction FA (Fig. 183) would be KR, but owing to expansion in the moving blades, the final relative velocity r is greater than R.

Hence kinetic energy supplied in moving blades

$$=\frac{r^2-K^2R^2}{2g}=\frac{U^2-K^2R^2}{2g}$$

and total energy supplied

$$=\frac{K^2R^2+U^2-K^2R^2+U^2-K^2R^2}{2g}=\frac{2U^2-K^2R^2}{2g}$$

From Fig. 183, $R^2=U^2+U_b{}^2-2UU_b\cos\alpha$

Hence total energy supplied

$$=[2U^2-K^2(U^2+U_b{}^2-2UU_b\cos\alpha)]\times\frac{1}{2g}$$

$$=\frac{U^2}{2g}[2-K^2(1+\lambda^2-2\lambda\cos\alpha)]$$

where $\lambda=\frac{U_b}{U}$

The work done on the moving blading

$$=\frac{U_cU_b}{g}=\frac{(U\cos\alpha+r\cos\gamma-U_b)U_b}{g}$$

$$=\frac{(2U\cos\alpha-U_b)U_b}{g}=\frac{U^2}{g}(2\,\lambda\cos\alpha-\lambda^2)$$

Hence blading efficiency

$$=\frac{\frac{U^2}{g}(2\,\lambda\cos\alpha-\lambda^2)}{\frac{U^2}{2g}[2-K^2(1+\lambda^2-2\,\lambda\cos\alpha)]}=\frac{2\,(2\,\lambda\cos\alpha-\lambda^2)}{2-K^2(1+\lambda^2-2\,\lambda\cos\alpha)}$$

If there were no losses, then $K=1$

and $\qquad$ Blading efficiency $=\dfrac{2(2\,\lambda\cos\alpha-\lambda^2)}{1-\lambda^2+2\,\lambda\cos\alpha}$

which agrees with the equation on page 492.

If the carry-over is neglected

$$\text{Blading efficiency}=\frac{2\,\lambda\cos\alpha-\lambda^2}{1-K^2(1+\lambda^2-2\,\lambda\cos\alpha)}$$

and this is 100 per cent if $\qquad K=1$

Example 3. In an impulse-reaction turbine with half-degree reaction the outlet angles for fixed and moving blades are 20°. The mean diameter of the

blade ring is 6 ft. 6 in., and the speed of rotation is 700 r.p.m. The ratio $\frac{\text{blade speed}}{\text{steam speed}} = 0{\cdot}452$.

The velocity loss factor is 0·85, and the nozzle efficiency in both fixed and moving blades is 0·9. The steam consumption is 10 lb. per sec.

Neglecting carry-over and windage loss find—

(*a*) The horse-power developed per stage.

(*b*) The blade and stage efficiencies.

The mean blade speed

$$U_b = \frac{\pi DN}{60} = \frac{3{\cdot}14 \times 5{\cdot}5 \times 700}{60} = 202 \text{ ft. per sec.}$$

∴ Steam velocity

$$U = \frac{U_b}{\lambda} = \frac{202}{0{\cdot}452} = 447 \text{ ft. per sec.}$$

The velocity diagram can now be drawn and will be as in Fig. 186 from which any necessary velocity can be read,

$$(a) \text{ Work done in a stage} = \frac{U_c U_b}{gJ} = \frac{637 \times 202}{32{\cdot}2 \times 778}$$

$$= 5{\cdot}13 \text{ B.Th.U. per lb. steam}$$

$$\text{Hence} \quad \text{H.P.} = \frac{10 \times 5{\cdot}13 \times 778}{550} = 72{\cdot}6$$

(*b*) As there is half-degree reaction the kinetic energy supplied in fixed and moving blades is the same.

$$\text{Hence K.E. supplied to the pair} = \frac{2(U^2 - K^2R^2)}{2gJ}$$

$$= \frac{447^2 - 0{\cdot}85^2 \times 265^2}{32{\cdot}2 \times 778}$$

$$= 5{\cdot}94 \text{ B.Th.U. per lb. steam.}$$

$$\text{Hence blade efficiency} = \frac{\text{Work done}}{\text{Energy supplied}} = \frac{5{\cdot}13}{5{\cdot}94} = 0{\cdot}864$$

The given nozzle efficiency of 0·9 is the efficiency of conversion of pressure energy into kinetic energy.

Hence isentropic heat drop across stage

$$= \frac{\text{K.E. supplied}}{\text{nozzle efficiency}}$$

$$= 6{\cdot}6 \text{ B.Th.U. per lb. steam}$$

$$\text{and stage efficiency} = \frac{\text{Work done}}{\text{Isentropic heat drop}}$$

$$= \frac{5{\cdot}13}{6{\cdot}6} = 0{\cdot}778$$

Height of Blading. (*a*) *Impulse Blading.* Let Fig. 184 (*a*) be the section of two consecutive blades.

Let V = Volume of steam at point considered (cu. ft. per lb.)
h = Height of blade (in.) (Fig. 184 *b*)
r = Velocity of steam on leaving moving blade (ft. per sec.)
γ = Exit angle of moving blade
t = Thickness of edge of blade (in.) (Fig. 184 *a*)
n = Number of blades covered by nozzle
W = Mass of steam passing per second (lb.)
p = pitch (in.) (Fig. 184 *a*)
D = Mean diameter of blade ring (in.) (Fig. 184 *b*)
N = Number of revolutions per minute of blade ring.

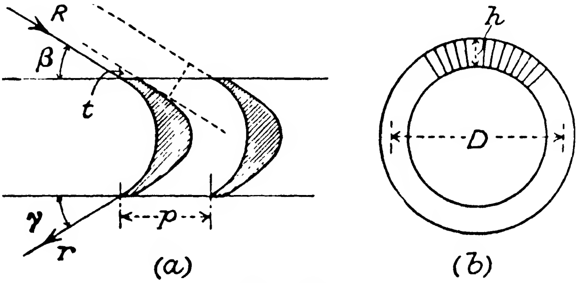


FIG. 184

If we consider the outlet end of the blades, we have the relation—

Leaving velocity of steam relative to blade × Area available to steam at right angles to this velocity = Mass of steam passing per second × Volume per pound of steam.

i.e. $r \times nh(p \sin \gamma - t) = WV \times 144$

(*b*) *Reaction Blading.* In a reaction turbine the admission of steam is all round the rotor. The blades, as usually constructed, are also sharp, so that t may be neglected.

Hence, area available to steam per blade $= hp \sin \gamma$

$\therefore$ Total area $= nhp \sin \gamma$

But $np = \pi D$, $\therefore$ Total area $= \pi Dh \sin \gamma$

$\therefore r \times \pi Dh \sin \gamma = WV \times 144$

If the fixed and moving blades are similar, as in Fig. 183, then

$$r = U, \text{ and if } \frac{\text{Blade velocity}}{\text{Steam velocity}} = \frac{U_b}{U} = \lambda,$$

then $$r = U = \frac{U_b}{\lambda} = \frac{\pi D N}{12 \times 60\,\lambda}$$

$$\therefore \frac{\pi^2 D^2 N h \sin \gamma}{720 \times 144\,\lambda} = WV$$

Re-heating and the Condition Curve. Modern impulse turbines for any but small powers are multi-stage machines. The pressure drop in the steam is divided up into a comparatively small number of stages, as already described under pressure compounding. In some cases the first stage is velocity compounded. The reaction turbine consists of a large number of stages, the pressure drop being continuous right through the blading.

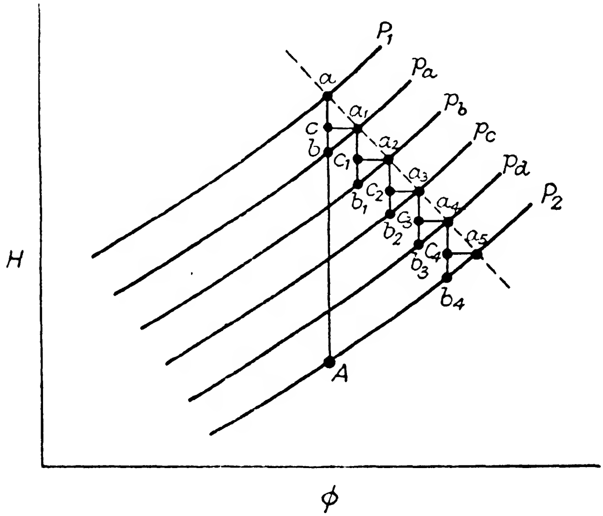


FIG. 185. RE-HEATING AND THE CONDITION CURVE

We must now consider how the condition of the steam changes as it passes through the blading. Suppose the supply pressure to a turbine be P_1 and the discharge pressure P_2. For clearness, let us consider, say, five stages, the intermediate pressures being p_a, p_b, p_c, p_d. These pressure lines are indicated on the H-ϕ chart, Fig. 185, which, it will be remembered, is drawn for 1 lb. of steam.

In the ideal machine the work done on the turbine rotor would be equal to the adiabatic heat drop aA. We have already seen, however, that, owing to friction losses, etc., the full adiabatic heat drop is not converted into mechanical work on the rotor. Thus, after expansion ab in the first stage, the state of the steam on leaving that stage will not be represented by the point b, but by some point a_1, the actual heat drop, and hence the work done on the rotor being represented by ac and not by ab, which represents the adiabatic heat drop for the stage. The total heat on leaving the first stage, given by the point c, is greater than would be the case were the expansion ideal. The steam is thus said to have been re-heated. Considering the complete expansion through the five stages, the state of the steam may be considered to change as represented by the path, a, a_1, a_2, a_3, a_4, a_5. The sum of the separate adiabatic heat drops, namely, Σab (called the cumulative heat drop) is, obviously, greater than the adiabatic heat drop aA. The ratio $\frac{\Sigma ab}{aA}$ is termed the re-heat factor. Let this be represented by R. It will be noticed that, owing to this re-heating, there is an increase of entropy and an additional amount of heat is lost to the turbine exhaust.

Efficiency of a Compound Turbine. The stage efficiency ε_s will be measured by the ratio of the work done on the rotor in that stage to the stage adiabatic heat drop, i.e. $\varepsilon_s = ac/ab$.

This efficiency must not be confused with the blading efficiency, which is a measure of the conversion of kinetic energy of the steam into kinetic energy of the blades.

The internal efficiency of the turbine is measured by the ratio of the total work done on the rotor (measured from the diagram) to the total adiabatic heat drop, i.e. $\varepsilon_i = \frac{\Sigma ac}{aA} = \frac{\Sigma \varepsilon_s \times ab}{aA}$.

If we assume that the stage efficiency ε_s is the same for all stages, then $\varepsilon_i = \varepsilon_s \frac{\Sigma ab}{aA} = \varepsilon_s R$.

The internal efficiency accounts for all the internal losses due to friction, etc. It obviously does not take account of outside losses due to friction at the bearings, etc. These are all accounted for by the "efficiency ratio," which is measured by the ratio of total work produced at the driving end of the shaft per pound of steam to the adiabatic heat drop across the turbine. The absolute thermal efficiency of the turbine is the ratio of the actual output of the turbine to the heat supplied to the turbine.

Example 4. In a reaction steam turbine the fixed and moving blades have inclinations at the receiving and discharging tips of 35° and 20° respectively. If steam passes through the blades without shock, what would be the work developed in a ring of moving blades per pound of steam if the mean diameter of the blade ring circle is 5 ft. 6 in., and the speed of rotation 700 r.p.m. ?

Assuming an efficiency of 85 per cent for the one pair of fixed and moving blades, find the heat drop which takes place in the pair, and find the state of the steam when arriving at the fixed blades if the steam has a pressure of 20 lb. per sq. in. and dryness 0·92 when leaving the moving blades.

(U.L., B.Sc. (Eng.), 1922.)

Blade speed at the mean periphery

$$= \frac{\pi \times 5{\cdot}5 \times 700}{60} = 202 \text{ ft. per sec.}$$

Fig. 186 shows the velocity diagram, where AB represents this blade speed. Make the angle $EBC = 35°$ and the angle $EAC = 20°$. The diagram can now be completed. The values of the various velocities read off the diagram are given in the figure.

Work done per pound of steam

$$= \frac{U_b U_c}{g} = \frac{202 \times 637}{32{\cdot}2} = 4{,}000 \text{ ft-lb. (approx.)}$$

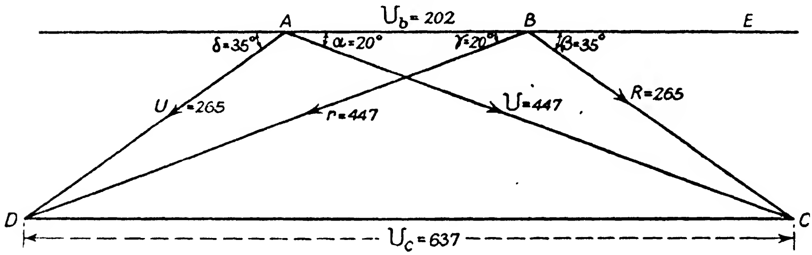


FIG. 186. VELOCITY DIAGRAM

The work done per pound of steam is also given by

$$\frac{U^2 - u^2}{2g} + \frac{r^2 - R^2}{2g} = 2\left(\frac{447^2 - 265^2}{2 \times 32{\cdot}2}\right) = 4{,}024 \text{ ft-lb.}$$

The discrepancy between the two results is, of course, due to slight inaccuracy in the drawing of the velocity diagram.

$$\text{Stage efficiency} = \frac{\text{Work done on rotor}}{\text{Adiabatic heat drop}}$$

$$\therefore \text{ adiabatic heat drop } (H_1 - H_2) = \frac{4{,}000}{0{\cdot}85 \times 1{,}400} = 3{\cdot}36 \text{ C.H.U.}$$

Total heat after expansion

$$H_2 = h_2 + x_2 L_2 \text{ at 20 lb. per sq. in. (from tables)}$$
$$= 108{\cdot}95 + 0{\cdot}92 \times 533{\cdot}87 = 600{\cdot}11 \text{ C.H.U.}$$

$$\therefore \text{ total heat before expansion } H_1 = 600{\cdot}11 + 3{\cdot}36 = 603{\cdot}47 \text{ C.H.U.}$$

The state point of the steam before expansion could now be obtained from the H-ϕ chart. The heat drop is, however, so small that a large scale diagram would be required for accuracy. The state can be calculated as follows—

Entropy during expansion

$$= 2{\cdot}3 \log_{10} \frac{T_2}{273{\cdot}1} + \frac{x_2 L_2}{T_2} \text{ at 20 lb. per sq. in. (from tables)}$$

$$= 2{\cdot}3 \log_{10} \frac{381{\cdot}97}{273{\cdot}1} + \frac{0{\cdot}92 \times 533{\cdot}87}{381{\cdot}97} = 1{\cdot}62$$

$\therefore$ Entropy during expansion $= 1{\cdot}62 = 2{\cdot}3 \log_{10} \dfrac{T_1}{273{\cdot}1} + \dfrac{x_1 L_1}{T_1}$ (1)

Total heat before expansion

$$= 603{\cdot}47 = h_1 + x_1 L_1 = (T_1 - 273{\cdot}1) + x_1 L_1 \quad . \quad . \quad (2)$$

$\therefore$ From (2), $x_1 L_1 = 603{\cdot}47 + 273{\cdot}1 - T_1 = 876{\cdot}57 - T_1$

Substituting in (1), $1{\cdot}62 = 2{\cdot}3 \log_{10} \dfrac{T_1}{273{\cdot}1} + \dfrac{876{\cdot}57 - T_1}{T_1}$

Solving this equation, $T_1 = 384{\cdot}6°$ C. (abs.)

Pressure at this saturation temperature = 22 lb. per sq. in. approx.

$\therefore H_1 = h_1 + x_1 L_1$

$603{\cdot}47 = 111{\cdot}83 + x_1 \times 532{\cdot}09$ at 22 lb. per sq. in. (from tables)

$\therefore \quad x = 0{\cdot}924$

State of steam at entrance to fixed blades : Pressure = 22 lb. per sq. in. (abs.) ; dryness fraction = 0·924.

Example 5. A reaction steam turbine is supplied with steam at 150 lb. per sq. in. absolute pressure, superheated to 250° C., and exhausts at 1·5 lb. per sq. in. absolute. Assuming that the frictional loss in the blades is 25 per cent of the adiabatic heat drop, and taking a re-heat factor of 1·05, find the steam used per horse-power hour developed.

If the turbine develops 15,000 H.P. at 350 r.p.m., calculate the blade height and drum diameter at the point in the turbine where the pressure is 15 lb. per sq. in. absolute and the dryness factor 0·985. The discharge tip angle of the blades is 20°, and the blade velocity is 0·7 of the relative velocity of the discharging steam, and the blade height is one-twelfth of the drum diameter. (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

Stage efficiency = 0·75

$\therefore$ internal efficiency of turbine = Re-heat factor × Stage efficiency

$$= 1{\cdot}05 \times 0{\cdot}75 = 0{\cdot}7875$$

Adiabatic heat drop from 150 lb. per sq. in. and 250° C. to 1·5 lb. per sq. in. = 179 C.H.U. (from H-ϕ chart).

$\therefore$ Neglecting outside losses, Work done on rotor per pound of steam

$$= 0{\cdot}7875 \times 179 \times 1{,}400 = 197{,}350 \text{ ft-lb.}$$

$\therefore$ Weight of steam required per horse-power hour

$$= \frac{33{,}000 \times 60}{197{,}350} = 10 \text{ lb. (approx.)}$$

Steam used per second

$$= \frac{\text{H.P.} \times 10}{60 \times 60} = \frac{15{,}000 \times 10}{60 \times 60} = 41{\cdot}67 \text{ lb. per sec.}$$

Specific volume of steam at 15 lb. per square inch

$= V = xV_s$ where $V_s =$ Specific volume of dry steam at 5 lb. per sq. in.

$= 0{\cdot}985 \times 26{\cdot}27 = 25{\cdot}88$ cu. ft. per lb.

Blade height

$$h = \frac{WV \times 720 \times 144 \times \lambda}{\pi^2 D^2 N \sin\gamma} \quad \left(\begin{array}{c}\text{where } D \text{ is mean diameter} \\ \text{of blade ring}\end{array}\right)$$

$$= \frac{WV \times 720 \times 144 \times \lambda}{\pi^2 h^2 \times 144 \times N \sin\gamma} \quad \left(\begin{array}{c}\text{neglecting difference between} \\ D \text{ and actual drum diameter}\end{array}\right)$$

$$\therefore \quad h = \sqrt[3]{\frac{WV \times 720 \times \lambda}{\pi^2 \times N \sin\gamma}}$$

$$= \sqrt[3]{\frac{41{\cdot}67 \times 25{\cdot}88 \times 720 \times 0{\cdot}7}{3{\cdot}14 \times 3{\cdot}14 \times 350 \times 0{\cdot}342}} = 7\tfrac{3}{4} \text{ in.}$$

$\therefore$ Mean drum diameter = 7 ft. 9 in.

Example 6. A reaction turbine runs at 300 r.p.m., and its steam consumption is 34,200 lb. per hour. The pressure of the steam at a certain pair is 27 lb. per sq. in., its dryness is 0·93, and the horse-power developed by the pair is 4·5. The discharging blade tip angle is 20° for both fixed and moving blades and the axial velocity of flow is 0·72 of the blade velocity.

Find the drum diameter and the blade height. Take the tip leakage steam as 0·8 per cent, but neglect blade thickness. (*U.L.*, *B.Sc.* (*Eng.*), 1934.)

Weight of steam passing through blades

$$= W = \frac{0{\cdot}92 \times 34{,}200}{3{,}600} = 8{\cdot}74 \text{ lb. per second}$$

Specific volume of steam (from tables)

$= V = 0{\cdot}93 \times 15{\cdot}18 = 14{\cdot}12$ cu. ft. per lb.

Velocity of blades

$$= U_b = \frac{\pi DN}{60} = \frac{3{\cdot}14 \times D \times 300}{60} = 15{\cdot}7\,D \text{ ft. per sec.}$$

where D = mean diameter of blade ring in feet.

But horse-power $= \dfrac{W}{g}\,\dfrac{U_c U_b}{550}$

Hence $$4{\cdot}5 = \frac{8{\cdot}74 \times U_c \times 15{\cdot}7D}{32{\cdot}2 \times 550}$$

and $$U_c = \frac{582}{D} \text{ ft. per sec.}$$

Now in Fig. 183 $EC = 0{\cdot}72U_b$

Hence $U_b + AC = EC \times \cot\gamma$

$\therefore \; AC = U_b(0{\cdot}72 \cot 20° - 1) = 4{\cdot}49U_b$

But $$U_c = U_b + 2AC = 9{\cdot}98U_b = 9{\cdot}98 \times 15{\cdot}7D$$
$$= 156{\cdot}7D$$

Hence $$\frac{582}{D} = 156{\cdot}7D$$

and $$D = 1{\cdot}93 \text{ ft.}$$

Now $$\text{Axial velocity} \times \pi Dh = WV$$

Hence $$0{\cdot}72U_b \times 3{\cdot}14 \times 1{\cdot}93 \times h = 8{\cdot}74 \times 14{\cdot}12$$

Hence $$h = \frac{8{\cdot}74 \times 14{\cdot}12}{0{\cdot}72 \times 15{\cdot}7 \times 1{\cdot}93 \times 3{\cdot}14 \times 1{\cdot}93}$$
$$= 0{\cdot}93 \text{ ft.} = 11{\cdot}6 \text{ in.}$$

EXAMPLES XII

1. The nozzles of a turbine stage deliver 9 lb. of steam per second at an outlet angle of 18° and a speed of 1,400 ft. per second. If the blading outlet angle is 22°, and the blading velocity coefficient is 0·76, determine the blade power developed and the inlet blade angle. Take the peripheral speed of the wheel as 550 ft. per second. (*I.Mech.E.*, 1924.)

2. The steam supplied to a single row impulse wheel turbine expands in the nozzles over such a range that the adiabatic heat drop is 38 B.Th.U. per lb. The nozzle efficiency is 93 per cent and the nozzle angle is 15°. If the blading speed is 575 ft. per second, the outlet blade angle 18°, and the velocity coefficient for the blading 0·82, determine a suitable inlet angle for the blade and the speed and direction of the steam after discharge from the blading. (*I.Mech.E.*, 1923.)

3. At a stage of an impulse turbine the wheel diameter is 39 in. and the r.p.m. 3,000. The ratio, blade speed/steam speed is 0·34, the nozzle efficiency 0·93, disk and vane friction 3 per cent of stage heat drop; blade entrance and exit angles 29° and 25° respectively, and velocity coefficient for blading 0·78. Draw the velocity diagram and calculate (*a*) the work done on blading per lb. of steam; (*b*) the stage efficiency. (*I.Mech.E.*, 1927.)

4. Steam issues from a nozzle on to the blade ring of an impulse steam turbine with a velocity of 1,500 ft. per second. The receiving and discharging tips of the moving blades are inclined at 30° to the plane of motion and the nozzle is inclined at 20° to that plane. The mean diameter of the blade ring circle is 3 ft. The discharge from the nozzle is 0·23 lb. per second. Assuming a friction loss of 15 per cent in the velocity of the steam relative to the blade whilst passing through the blade ring, find: (1) the instantaneous torque on the shaft when the turbine is beginning to move; (2) the speed of the turbine so that the steam shall discharge axially, and the horse-power developed at this speed. (*U.L., B.Sc.* (*Eng.*), 1925.)

5. In a De Laval turbine in which the blades of the rotor have equal inlet and outlet angles, steam is supplied dry and saturated at 150 lb. per sq. in. (abs.) and the exhaust pressure is 2 lb. per sq. in. (abs.). The nozzle makes an angle of 20° with the direction of rotation of the blades and the peripheral speed of the blades is 1,160 ft. per second. Assuming that the losses due to friction and supersaturation effects in the nozzle amount to 15 per cent, and that the relative velocity of the steam at exit from blades is 80 per cent of that at inlet, calculate the velocity of discharge of the steam from the nozzles, the inlet angle of the blades for the condition that there should be no shock at entry, and the work done per lb. of steam. (*U.L., B.Sc.* (*Eng.*), 1926.)

6. In an impulse steam turbine the steam issues from the nozzle with a speed of 1,520 ft. per second. The velocity is compounded by passing the steam through a ring of moving blades, then through a ring of fixed blades, and finally through a second ring of moving blades. The tips of the moving blades at entry and exit are inclined 30° to the direction of motion of the blades, and the steam is to pass on and off the blades without shock.

Assuming that the speed of the steam relative to a blade is reduced by friction by 10 per cent whilst passing over the blade, find the speed of the moving blades so that the final discharge shall be axial, and calculate the horse-power developed in the turbine under these conditions per lb. of steam supplied per second.

What is the magnitude of the angle between the nozzle and the plane of the moving blades? (*U.L., B.Sc.* (*Eng.*), 1920.)

7. In a stage of an impulse turbine in which the velocity is compounded in two rings of moving blades separated by fixed blades the moving blades have tip angles of 30°, and the blade speed and the nozzle and fixed blade angles are designed on the assumption that the velocity of discharge from the nozzle is 1,800 ft. per second, and the relative velocity of steam to blade is reduced by 10 per cent in passing through each of the three blade rings, and that the final discharge shall be axial. Determine the blade velocity and find the efficiency of the stage.

What would be the approximate reduction in efficiency caused by reducing the blade speed by 10 per cent? (*U.L., B.Sc.* (*Eng.*), 1924.)

8. The first stage of an impulse steam turbine is supplied with steam at 200 lb. per sq. in. superheated to 250° C. The velocity is compounded, there being two rings of moving blades separated by fixed blades. The pressure in the wheel chamber is 100 lb. per sq. in. Assuming an 8 per cent loss in the nozzle, find the velocity of the steam as it leaves the nozzle.

Given that the angle of the tips of the moving blades is 30°, that the blade velocity is one-fifth of that of the steam discharging from the nozzle, that the velocity of steam relative to a blade is reduced by 12 per cent during its passage through a blade ring, and that the nozzle and fixed blade angles are designed so that the steam shall pass through without shock, find the work developed per lb. of steam and the efficiency of the stage. (*U.L., B.Sc.* (*Eng.*), 1923.)

9. In a reaction steam turbine the blade tips are inclined 35° and 20° to the direction of motion. The guide blades are of the same shape as the moving blades but reversed in direction. At a certain place in the turbine the drum diameter is 3 ft. 6 in. and the blades are 4 in. high, and at this place the steam has a pressure of 25 lb. per sq. in. and dryness 0·935. If the speed of the turbine is 250 r.p.m. and the steam passes through the blades without shock, find the power developed in the ring of moving blades.
(*U.L., B.Sc.* (*Eng.*), 1923.)

10. Explain carefully what is meant by "re-heat factor" in turbine work, and indicate how this factor involves two efficiency ratios.

An impulse turbine installation is to be arranged in three casings, H.P., I.P., and L.P., and to work between pressures of 210 lb. per sq. in., with 80° C. superheat and 1·1 lb. per sq. in. Allowing a re-heat factor of 1·075, and a loss of available heat (in receiver pipes, etc.) of 5 C.H.U. per lb., determine the heat to be allocated to each unit if the H.P. and I.P. are each to develop quarter power and the L.P. half power. Allow for hydraulic or stage efficiencies of H.P. 0·79, I.P. 0·76, L.P. 0·72. (*U.L., B.Sc.* (*Eng.*), 1921.)

11. Explain what is meant by the "re-heat factor" in steam turbine design.

An impulse steam turbine consists of eight stages and the efficiency of each stage is 70 per cent. The steam supply is at 215 lb. per sq. in. and 250° C., and the pressure in successive stages are in geometrical progression, the condenser pressure being 1·5 lb. per sq. in. Find the re-heat factor for the turbine and the steam consumption per horse-power-hour.

What is the thermal efficiency of the turbine ?

(*U.L., B.Sc.* (*Eng.*), 1925.)

12. The low-pressure turbine of a naval installation receives 30·7 lb. of steam per second from the high-pressure unit at 27 lb. per sq. in. and 0·915 dry. This steam expands to 12·5 lb. per sq. in. in the first two stages of the L.P. turbine with an efficiency ratio of 0·69. Into this steam at 12·5 lb. per sq. in. pressure there also passes 8·2 lb. per second of exhaust steam from the auxiliaries at 0·94 dry. The total steam quantity then expands through the succeeding stages to an exhaust pressure of 1·3 lb. per sq. in. If the horse-power developed in the turbine is 6,120, determine (*a*) the efficiency ratio of these later stages ; (*b*) the condition of the steam at entrance to the third stage ; (*c*) the condition of the steam at exhaust from the last stage. Assume a leaving velocity at exhaust of 600 ft. per second.

(*U.L., B.Sc.* (*Eng.*), 1921.)

13. The condition of the steam at the beginning of an expansion in a reaction turbine is 50 lb. per sq. in., dryness 0·97 and the blade velocity is 125 ft. per sec. If the efficiency is 75 per cent and the ratio $\frac{\text{axial steam velocity}}{\text{blade velocity}}$ varies in the expansion from 0·58 to 0·78, find the pressure at the end of the expansion and the useful heat drop per pound of steam.

If the exit blade angle is 20° for both fixed and moving blades, find the work done per pound of steam for a pair halfway along the expansion and estimate the number of pairs in the expansion. (*U.L., B.Sc.* (*Eng.*), 1935.)

CHAPTER XIII

CONDENSING, FEED WATER HEATING, RE-HEATING, ETC.

Measurement of Vacuum. A vacuum gauge generally measures the vacuum in inches of mercury. Thus a mercury gauge reading of 28 in. with the barometer standing at 30 in., would mean that the absolute pressure being measured was that due to 2 in. of mercury, namely $\frac{14{\cdot}689}{30} \times 2 = 0{\cdot}49 \times 2 = 0{\cdot}98$ lb. per sq. in.

Hence if H_b = Barometric height in inches of mercury
H_g = Gauge reading in inches of mercury
p = Absolute pressure in lb. per sq. in.
then $p = 0{\cdot}49(H_b - H_g)$

The barometric height is a variable quantity, and hence it is more convenient for purposes of comparison, to refer vacuum readings to a standard barometer of 30 in. Hence to correct any vacuum to standard we have the relation

$$\text{Standard vacuum} = (30 - H_b) + H_g$$

Mixtures of Air and Water Vapour. Dalton showed that if a vessel contained a mixture of any gas with water vapour, the pressure of the vapour was the same for the same temperature, whether there was any gas present or not, and that the pressure of such a mixture was equal to the sum of the partial pressures which each constituent would exert if it were present in the same space by itself.

Thus if t = temperature of an air-vapour mixture
p_s = vapour pressure corresponding to $t°$
p_a = partial air pressure corresponding to $t°$
v_s = specific volume of vapour at $t°$
v_a = specific volume of air at $t°$
p_v = pressure in vessel containing the mixture
then $p_v = p_s + p_a$ and $p_a v_a = R(t + 273)$ or $R(t + 460)$
Also if V = volume of vessel containing the mixture
W_s = weight of vapour
W_a = weight of air
W_m = weight of mixture
then $V = W_s v_s = W_a v_a$
and $W_m = W_s + W_a = W_s\left(1 + \frac{v_s}{v_a}\right) = W_a\left(1 + \frac{v_a}{v_s}\right)$

Example 1. Before the fire is lighted in a boiler the temperature is 20° C., the pressure in the boiler being atmospheric. What will be the boiler pressure when the temperature has been raised to 160° C.? If dry saturated steam is drawn off at this pressure, what weight of air will first come over per pound of steam? Assume air is saturated.

Vapour pressure at 20° C. $= p_{s1} = 0{\cdot}339$ lb. per sq. in. (Table B)

Air pressure $= p_{a1} = 14{\cdot}7 - 0{\cdot}339 = 14{\cdot}36$ lb. per sq. in.

Let $V =$ volume of steam and air space

then for the air $pV = wRT$ where V, w, and R are constant for the two temperatures.

Hence air pressure at 160° C.

$$= p_{a2} = p_{a1} \times \frac{160 + 273}{20 + 273} = 14{\cdot}36 \times \frac{433}{293}$$

$$= 21{\cdot}22 \text{ lb. per sq. in.}$$

But steam pressure at 160° C.

$$= p_{s2} = 89{\cdot}64 \text{ lb. per sq. in. (Table } B\text{)}$$

Hence boiler pressure

$$= p_{s2} + p_{a2} = 89{\cdot}64 + 21{\cdot}22 = 110{\cdot}86 \text{ lb. per sq. in.}$$

Now specific volume of air at 160° C. and 21·22 lb. per sq. in.

$$= v_a = 12{\cdot}39 \times \frac{14{\cdot}7}{21{\cdot}22} \times \frac{(160 + 273)}{273}$$

$$= 13{\cdot}61 \text{ cu. ft. per lb.}$$

and specific volume of steam at 160° C.

$$= v_s = 4{\cdot}93 \text{ cu. ft. per lb. (Table } B\text{)}$$

Hence weight of air

$$= W_a = W_s \times \frac{v_s}{v_a} = W_s \times \frac{4{\cdot}93}{13{\cdot}61} = W_s \times 0{\cdot}36$$

that is weight of air per pound of steam $= 0{\cdot}36$ lb.

Classification of Condensers. Condensers may be classified broadly into two groups—

1. Jet condensers in which the cooling water mixes with the steam which it condenses.
2. Surface condensers in which the mixture of the condensing water and the steam is prevented by means of metal tubes.

The first type may be subdivided into—

(i) Parallel flow condensers in which both steam and water enter at the top, the mixture being removed from the bottom.

(ii) Contra flow condensers in which the steam entering the bottom and flowing upwards, meets the down-coming cooling water.

(iii) Ejector condensers in which the steam and water mix in passing through a series of metal cones.

The second type may be subdivided into—

(i) The surface condenser in which the steam passes over a series of tubes through which the cooling water is flowing.

(ii) The evaporative condenser in which the steam passes through a series of tubes down the outsides of which the cooling water is falling while a stream of air passes up outside the tubes.

It is obvious that while in the jet condenser more intimate heat exchange between the cooling water and the steam is ensured, there is the practical disadvantage due to the mixing of the cooling water and the condensate, which disadvantage is obviated in the surface condenser.

The pressure in a condenser can be lowered by removing the condensate as it is formed by means of an extraction pump. If only this one pump is used, any air carried over with the steam will tend to accumulate in the condenser, thus limiting the vacuum obtainable. For this reason in modern steam turbine plants the air is extracted separately from the condenser by means of an air pump or an air ejector.

THE SURFACE CONDENSER. One form of the surface condenser is shown in Fig. 187. The steam enters at the top, and, passing downwards over the tubes through which the cooling water is flowing, is condensed, the condensate being extracted at the bottom.

The cooling water enters at one end of the tubes situated in the bottom half of the condenser, and after flowing to the other end, returns through the tubes situated in the top half of the condenser. The temperature of condensation therefore decreases as the steam passes downwards, and hence the partial pressure of the steam decreases from top to bottom of the condenser. The air exit is shielded from the downstream of condensate by means of a baffle, and thus it is extracted with only a comparatively small amount of water vapour. As the air passes downwards, it is progressively cooled and becomes denser, and hence it is extracted from the lowest convenient point. In a condenser of this type, therefore, the partial pressure of the steam decreases, the partial pressure of the air increases, while the total vacuum pressure falls as the mixture passes from top to bottom of the condenser. The result of these effects is that the condensate temperature falls below the vacuum temperature at the top.

The regenerative condenser (Fig. 188) is designed on the principle that some of the exhaust steam can pass unimpeded to the bottom of the condenser, and in passing upwards can meet the cooler condensate falling through the nest of cooling tubes. In this way the condensate is re-heated to a temperature more nearly that of the steam. The air is drawn from the centre of the condenser, but in order to effect the maximum cooling, it is passed through a nest of cooler tubes before final extraction.

From the above remarks it will be realized that the exact value of the condensation temperature is somewhat ambiguous, but a simple expression for the amount of cooling water may be found as follows—

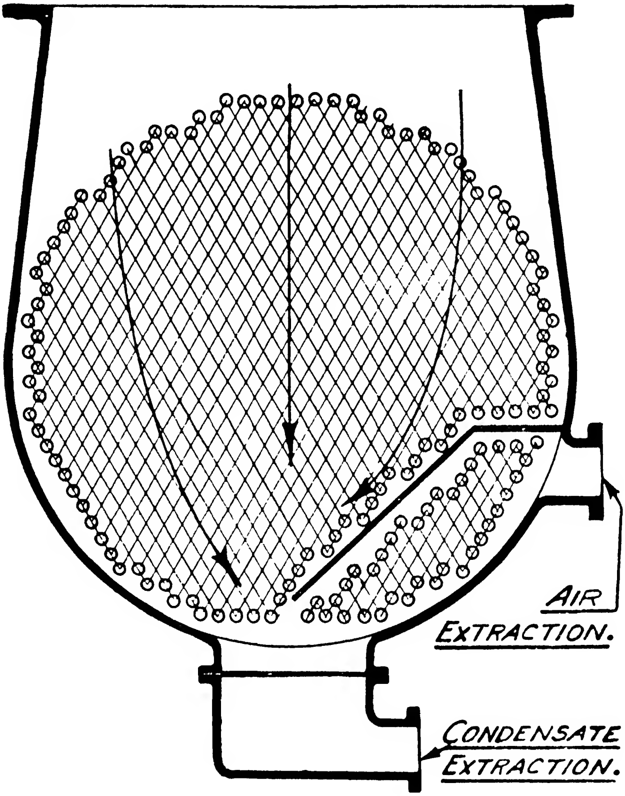


FIG. 187. DIAGRAMMATIC VIEW SHOWING THE STEAM PATH IN THE ORDINARY DOWNFLOW TYPE OF CONDENSER
(Metropolitan-Vickers)

Let W_s = weight of steam in pounds per minute
x = dryness fraction of steam entering condenser
h = sensible heat of water at temperature of condensation
L = latent heat of steam at temperature of condensation
h_c = sensible heat of condensate
t_1 and t_2 = inlet and outlet temperatures of cooling water

then, Weight of cooling water $= \dfrac{W_s(h + xL - h_c)}{t_2 - t_1}$ lb. per min.

This quantity can be obtained by another method if the steam consumption and initial conditions of the steam supply to the engine or turbine is known.

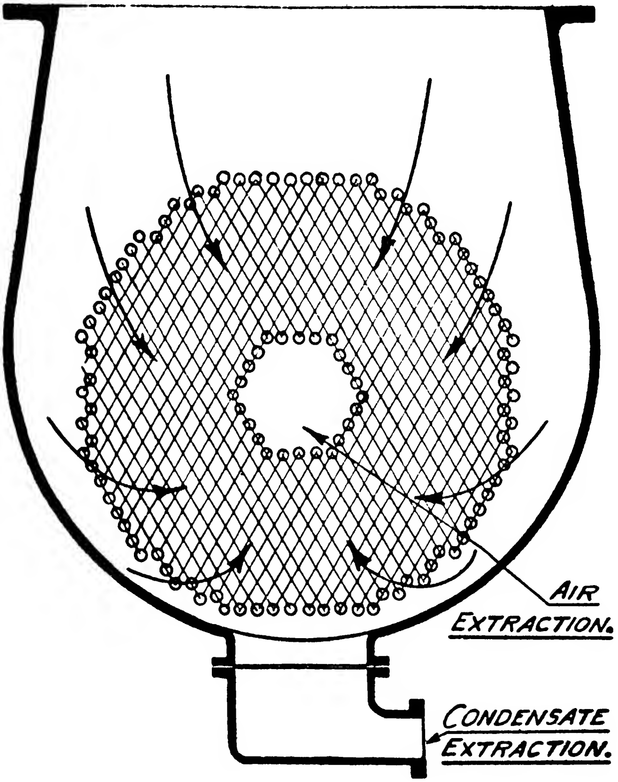


FIG. 188. DIAGRAMMATIC VIEW SHOWING THE STEAM PATH IN A METRO-VICK CENTRAL FLOW SURFACE CONDENSER

(*Metropolitan-Vickers*)

Let W_s = weight of steam condensed in pounds per minute

w_s = steam consumption of turbine in pounds per H.P.-hour

H_s = total heat of steam entering turbine in lb. cal. per lb.

h_c = sensible heat of condensate in lb. cal. per lb.

then heat rejected in condenser per pound of steam condensed

$$= H_s - \frac{1{,}414}{w_s} - h_c$$

and weight of cooling water

$$= \frac{W_s\left(H_s - \frac{1{,}414}{w_s} - h_c\right)}{t_2 - t_1} \text{ lb. per min.}$$

The expressions given above give the amount of water required to change the entering steam into liquid condensate. The air with a small amount of water vapour being drawn from a separate exit is generally cooled as it is removed by passing over a separate nest of cooling water tubes. The amount of this additional heat which must be removed by the cooling water can be determined approximately as follows—

Let the symbols have the meanings given to them on page 505 and let the suffix 1 apply to the entrance of the integral cooler while the suffix 2 applies to the exit.

Then $p_{a1} = p_{v1} - p_{s1}$ and $p_{a2} = p_{v2} - p_{s2}$

Weight of vapour at entrance of integral cooler

$$= W_{s1} = W_{a1} \times \frac{v_{a1}}{v_{s1}}$$

Weight of vapour at exit of integral cooler

$$= W_{s2} = W_{a1} \times \frac{v_{a2}}{v_{s2}}$$

since the weight of air does not alter.

Hence weight of vapour condensed

$$= W_s' = W_{a1}\left(\frac{v_{a1}}{v_{s1}} - \frac{v_{a2}}{v_{s2}}\right)$$

Then if we take the latent heat of the vapour as L (assumed constant)

Heat removed to cool vapour $= W_s' \times L$

Heat removed to cool air $= W_{a1} \times C_p(t_1 - t_2)$

where t_1 and t_2 are the temperatures of the air at inlet and outlet of the integral cooler.

Hence the total amount of heat removed by the integral cooler

$$= W_{a1}\left[L\left(\frac{v_{a1}}{v_{s1}} - \frac{v_{a2}}{v_{s2}}\right) + C_p(t_1 - t_2)\right]$$

The weight of air W_{a1} can be determined as shown in the following example.

Example 2. State the law of partial pressures and show how it applies to the condenser of a steam plant.

The following observations were made on a condenser plant in which the temperature of condensation was measured directly by thermometers. The recorded condenser vacuum was 28·1 in. of mercury and the barometer read 30·2 in. Temperature of condensation 33° C. Temperature of hot well 27·6° C. Weight of condensate per hour 3,935 lb. Weight of cooling water per hour 126,700 lb. Inlet temperature 8·51° C., outlet temperature 26·24° C.

Find the state of the steam entering the condenser and the weight of air present per cubic foot of condenser volume. (*U.L., B.Sc. (Eng.)*, 1931.)

The condenser pressure p_v

$$= (30{\cdot}2 - 28{\cdot}1) \times 0{\cdot}49 = 1{\cdot}03 \text{ lb. per sq. in. (abs.)}$$

The vapour pressure corresponding to 33° C. $= p_s$

$$= 0{\cdot}75 \text{ lb. per sq. in. (abs.) (Table } B)$$

The latent heat corresponding to this pressure $= L$

$$= 577{\cdot}3 \text{ lb. cal. per lb.}$$

Hence $$W_w(t_2 - t_1) = W_s(h + xL - h_c)$$

$$126{,}700(26{\cdot}24 - 8{\cdot}51) = 3{,}935(33 + 577{\cdot}3x - 27{\cdot}6)$$

whence $$x = 0{\cdot}98$$

The partial air pressure p_a

$$= p_v - p_s = 1{\cdot}03 - 0{\cdot}75 = 0{\cdot}28 \text{ lb. per sq. in. (abs.)}$$

Let V = the volume occupied by vapour and air

Then $p_aV = wRT$

Hence, Weight of air per cubic foot of condenser volume

$$= \frac{w}{V} = \frac{0{\cdot}28 \times 144}{96 \times (273 + 33)} = 0{\cdot}0014 \text{ lb. per cu. ft.}$$

THE JET CONDENSER. The principles of the parallel flow jet condenser are illustrated in Fig. 189.

The cooling water entering at *A* is sprayed through the nozzles *B*. The steam enters at *C*, and, in mixing with the water spray, is condensed, the mixture of condensate and cooling water falling to the bottom where it is extracted by the pump *E*. The air passing through a cooling water spray *F* is extracted by an air ejector *H*. If for any reason the water level in the condenser rises above a certain height, the float *L* operates a valve *M*, which allows air to enter at the top of the condenser through the pipe *N*, thus breaking the vacuum.

The amount of cooling water required may be obtained by a similar method to that used for the surface condenser. Thus, if we use the same symbols as on page 508, the final temperature of the cooling water t_2 is the same as the final temperature of the condensate t_c.

Hence

$$\text{Weight of cooling water} = \frac{W_s(h + xL - h_c)}{t_c - t_1} \text{ lb. per min.}$$

or Weight of cooling water $= \dfrac{W_s\left(H_s - \dfrac{1{,}414}{w_s} - h_c\right)}{t_c - t_1}$ lb. per min.

In addition to this water, an additional quantity is required for air and vapour cooling. This may be calculated in a similar manner to that described for the surface condenser.

In a parallel flow jet condenser there will generally be a difference between the temperatures of the incoming steam and the outgoing condensate. Hence in the equations given above h is greater than h_c. In the contra flow type the cooling water enters at the top and falls through a series of perforated trays, meeting the steam which enters near the bottom, the condensate being extracted from the bottom and the air being extracted from the top of the condenser. In this type, therefore, it is possible to make the temperature difference between the incoming steam and the condensate very nearly zero, and hence h_c approximates to h. This reduces the quantity of cooling water required, and leaves more heat in the condensate. At the same time, the air in passing upward is

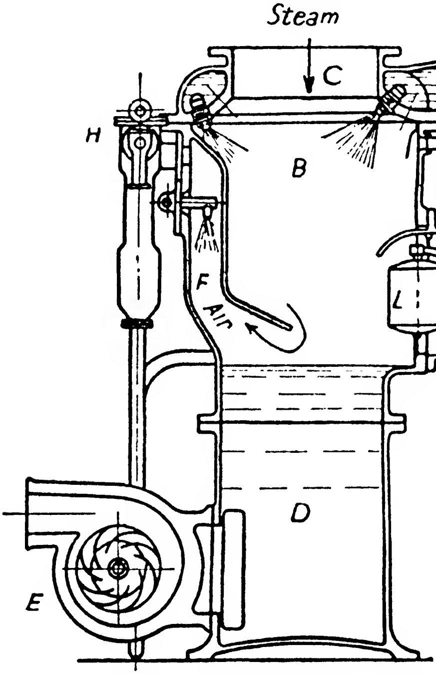


FIG. 189. HICK-BREGUET LOW LEVEL JET CONDENSER

progressively cooled by the down-flowing water and hence increases in density as it rises.

Jet condensers may be either of the low-level or of the high-level type. The low-level condenser is placed on the same level or immediately below the engine or turbine, and hence the condensate must be extracted by means of a pump, while the cooling water will enter under the suction produced by the vacuum. The high-level or barometric condenser is placed at a height greater than that of the water barometer and is provided with a long tail pipe. No extraction pump is therefore needed as the condensate will flow out under gravity and thus produce the necessary vacuum in the condenser. A water pump will, however, be required to lift the cooling water up to the condenser level.

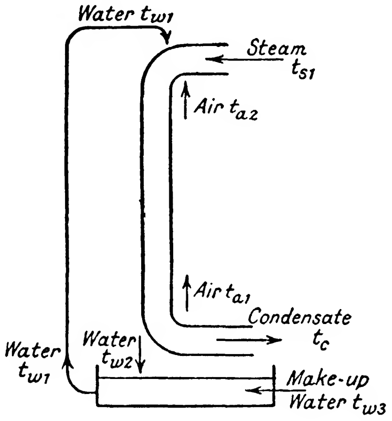


FIG. 190. PRINCIPLE OF EVAPORATIVE CONDENSER

THE EVAPORATIVE CONDENSER. The principle of the evaporative condenser is shown in Fig. 190. The steam to be condensed enters at the top of a series of tubes round the outsides of which a film of cold water is falling. At the same time, a current of air circulates over the water film.

Some of the cooling water is evaporated and, in passing off with the heated air, carries away a quantity of heat. The remainder of the cooling water at an increased temperature is collected and used again, its temperature being restored to the original value by the addition of the requisite quantity of cold make-up water.

Let p_b = the barometric pressure

p_{s1} and p_{s2} = the partial atmospheric vapour pressures at inlet and outlet of the air stream

p_{a1} and p_{a2} = the partial atmospheric air pressures at inlet and outlet of the air stream

v = specific volume
w = specific weight
t = temperatures as shown in Fig. 190

Also let us assume for simplicity that the ingoing air is saturated with moisture.

Then $p_{a1} = p_b - p_{s1}$ and $v_{a1} = \frac{RT}{P} = \frac{96 \times T_{a1}}{144 \times p_{a1}}$

As the ingoing air is assumed to be saturated, the partial vapour pressure p_{s1} is that corresponding to the temperature t_{a1} and can therefore be found from the steam tables. The corresponding value of the specific volume of the vapour v_{s1} can also be found from the steam tables, or from some formula such as Callendar's characteristic equation (p. 378).

Then $\frac{\text{Weight of vapour}}{\text{Weight of air}} = \frac{v_{a1}}{v_{s1}}$

and the weight of vapour entering with 1 lb. of air $= \frac{v_{a1}}{v_{s1}}$.

The total heat of this vapour H_1 can be found from the steam tables, and hence

Heat entering with 1 lb. of dry air $= C_p \times t_{a1}$ (from 0° C.)

Heat entering with accompanying vapour $= \frac{v_{a1}}{v_{s1}} \times H_1$

Hence

Total heat entering with 1 lb. of dry air

$$= C_p \times t_{a1} + \left(\frac{v_{a1}}{v_{s1}}\right) H_1$$

In the same way, if all values are calculated for the temperature t_{a2}, then

Weight of vapour leaving with 1 lb. of dry air $= \frac{v_{a2}}{v_{s2}}$

and Total heat leaving with 1 lb. of dry air

$$= C_p \times t_{a2} + \left(\frac{v_{a2}}{v_{s2}}\right) H_2$$

Hence

Total heat extracted from the steam by the saturated air stream $= C_p(t_{a2} - t_{a1}) + \left(\frac{v_{a2}}{v_{s2}}\right) H_2 - \left(\frac{v_{a1}}{v_{s1}}\right) H_1$ lb. cal. per lb. dry air

and Amount of water evaporated

$$= \frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}} \text{ lb. per lb. dry air.}$$

Let W_e = Total amount of water evaporated in pounds per hour

W_a = Total amount of dry air passing through condenser in pounds per hour

W_w = Total amount of circulating water entering at t_{w1} in pounds per hour

W_s = Total amount of steam condensed in pounds per hour

Then Weight of circulating water leaving at t_{w2}

$$= W_w - W_e \text{ lb. per hour}$$

and Heat carried away by circulating water

$$= (W_w - W_e)(t_{w2} - t_{w1})$$

Hence

Heat from steam = Heat to air stream + Heat to circulating water

$$W_s(t_{s1} + xL - t_c) = W_a\left[C_p(t_{a2} - t_{a1}) + \left(\frac{v_{a2}}{v_{s2}}\right) H_2 - \left(\frac{v_{a1}}{v_{s1}}\right) H_1\right] + (W_w - W_e)(t_{w2} - t_{w1})$$

If this process is to be continuous, an amount of make-up water equal to W_e at some temperature t_{w3} must be supplied so that

Heat to raise temperature of make-up from t_{w3} to t_{w1} = Heat to lower temperature of circulating water from t_{w2} to t_{w1}

that is $W_e(t_{w1} - t_{w3}) = (W_w - W_e)(t_{w2} - t_{w1})$

$$\text{Hence}\quad W_w - W_e = W_e \frac{t_{w1} - t_{w3}}{t_{w2} - t_{w3}}$$

$$= W_a\left(\frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}}\right)\left(\frac{t_{w1} - t_{w3}}{t_{w2} - t_{w1}}\right)$$

Therefore

$$W_s(t_{s1} + xL - t_c) = W_a\left[C_p(t_{a2} - t_{a1}) + \left(\frac{v_{a2}}{v_{s2}}\right) H_2 - \left(\frac{v_{a1}}{v_{s1}}\right) H_1 + \left(\frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}}\right)(t_{w1} - t_{w3})\right]$$

and

$$W_a = \frac{W_s(t_{s1} + xL - t_c)}{C_p(t_{a2} - t_{a1}) + \left(\frac{v_{a2}}{v_{s2}}\right) H_2 - \left(\frac{v_{a1}}{v_{s1}}\right) H_1 + \left(\frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}}\right)(t_{w1} - t_{w3})}$$

also

$$W_w = W_e\left(\frac{t_{w1} - t_{w3}}{t_{w2} - t_{w1}} + 1\right) = W_a\left(\frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}}\right)\left(\frac{t_{w1} - t_{w3}}{t_{w2} - t_{w1}} + 1\right)$$

Example 3. A low-vacuum surface condenser uses the film evaporation method (evaporative condenser). The heat from the steam is transmitted through the tube walls and is taken up by a water film moving over the tube surface. The water partly evaporates into an air stream. The temperature of the water film remains constant at 50° C. The air enters in a saturated condition at 12° C. and leaves, saturated at 30° C. The heat to be extracted from the steam amounts to 500 C.H.U. per lb.

It is estimated that an evaporation rate of 0·0007 lb. per sec. per sq. ft. of surface should be allowed. Determine the air flow and the surface necessary per 1,000 lb. of steam per hour.

The saturation pressures of steam at 12° C. and 30° C. are, respectively, 0·42 in. Hg. and 1·26 in. Hg. For dry air take $pv = 0{\cdot}666\,T$; and for steam $pv = 1{\cdot}071\,T$, where p is in lb. per sq. in., v in cu. ft. per lb. and T is absolute temperature. Specific heat air, 0·24; barometer, 30 in. Hg.

(*U.L.*, *B.Sc.* (*Eng.*), 1932.)

$$t_{w1} = t_{w2} = 50^\circ \text{ C.}$$

$$t_{a1} = 12^\circ \text{ C.} = 285^\circ \text{ C. (abs.)}$$

$$t_{a2} = 30^\circ \text{ C.} = 303^\circ \text{ C. (abs.)}$$

$$p_{s1} = 0{\cdot}49 \times 0{\cdot}42 = 0{\cdot}206 \text{ lb. per sq. in.}$$

$$p_{s2} = 0{\cdot}49 \times 1{\cdot}26 = 0{\cdot}617 \text{ lb. per sq. in.}$$

$$H_1 = 600 \text{ C.H.U. per lb.}$$

$$H_2 = 609 \text{ C.H.U. per lb. (Table A)}$$

$$p_{a1} = 14{\cdot}689 - 0{\cdot}206 = 14{\cdot}483 \text{ lb. per sq. in.}$$

$$p_{a2} = 14{\cdot}689 - 0{\cdot}617 = 14{\cdot}072 \text{ lb. per sq. in.}$$

$$v_{s1} = \frac{1{\cdot}071 \times 285}{0{\cdot}206} = 1{,}481 \text{ cu. ft. per lb.}$$

$$v_{s2} = \frac{1{\cdot}071 \times 303}{0{\cdot}617} = 526 \text{ cu. ft. per lb. (from formula)}$$

$$v_{a1} = \frac{0{\cdot}666 \times 285}{14{\cdot}483} = 13{\cdot}1 \text{ cu. ft. per lb.}$$

$$v_{a2} = \frac{0{\cdot}666 \times 303}{14{\cdot}072} = 14{\cdot}3 \text{ cu. ft. per lb. (from formula)}$$

$$\frac{v_{a1}}{v_{s1}} = \frac{13{\cdot}1}{1{,}481} = 0{\cdot}0088 \text{ cu. ft. per lb.}$$

$$\frac{v_{a2}}{v_{s1}} = \frac{14{\cdot}3}{526} = 0{\cdot}0273 \text{ cu. ft. per lb.}$$

Then Heat from steam = Heat to air stream and cooling water.

$$W_s \times 500 = W_a[0{\cdot}24(30 - 12) + (0{\cdot}0272 \times 609 - 0{\cdot}0088 \times 600)]$$

Hence the air flow

$$W_a = W_s\frac{500}{15{\cdot}6} = \frac{1{,}000 \times 500}{15{\cdot}6} = 32{,}051 \text{ lb. per hour.}$$

and water evaporated

$$W_e = W_a\left(\frac{v_{a2}}{v_{s2}} - \frac{v_{a1}}{v_{s1}}\right) = 32{,}051(0{\cdot}0272 - 0{\cdot}0088)$$

$$= 589{\cdot}74 \text{ lb. per hour}$$

$$\text{Hence heating surface} = \frac{589{\cdot}74}{0{\cdot}0007 \times 3{,}600} = 234 \text{ sq. ft.}$$

Regenerative Feed Heating. When considering the Rankine cycle it was seen that its efficiency was lower than that of the corresponding Carnot cycle working between the same two extreme temperatures. In the case of the Stirling cycle (p. 131), however, an efficiency equal to that of the Carnot cycle can be obtained by means of a regenerative process. This regenerative effect can be approximated to in practice, in the case of steam, by abstracting heat from the steam at one or more stages of the expansion in the turbine and giving this heat to the feed water during its passage from the condenser to the boiler.

We will first consider the ideal case in which the number of stages is infinite; that is, there is a continuous abstraction of heat from the steam during expansion, this heat progressively raising the temperature of the feed water from the condensate temperature to that of the boiler steam.

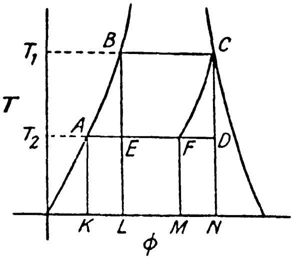


FIG. 191. IDEAL REGENERATIVE CYCLE

In Fig. 191 let *ABCD* be the temperature-entropy diagram of the Rankine cycle working between the temperatures T_1 and T_2. If heat is continuously abstracted from the steam during expansion, this expansion will no longer be down the adiabatic line *CD*, but along some line *CF*, the steam becoming progressively wetter than would be the case with adiabatic expansion. As this heat is to be just enough to heat the feed water from T_2 to T_1, the area *FCNM* must equal the area *ABLK* and the curve *FC* will be parallel to the curve *AB*.

Hence $FD = AE$ and Area $FCNM$ = Area $ABLK$

Therefore

$$\text{Efficiency} = \frac{\text{Net heat supplied} - \text{Heat rejected}}{\text{Net heat supplied}}$$

$$= \frac{\text{Area } BCNL - \text{Area } AFMK}{\text{Area } BCNL}$$

$$= \frac{\text{Area } BCNL - \text{Area } EDNL}{\text{Area } BCNL}$$

$$= \frac{L_1 \times T_1 - L_1 \times T_2}{L_1 \times T_1}$$

$$= \frac{T_1 - T_2}{T_1}$$

This is the Carnot efficiency.

This ideal regenerative cycle is impracticable, and in practice steam is extracted from the turbine at one or more stages of the expansion. The extracted steam passes through heat exchangers and gives up its heat to the feed water, the condensed extracted steam being then returned to the condenser or to some part of the feed-water circuit.

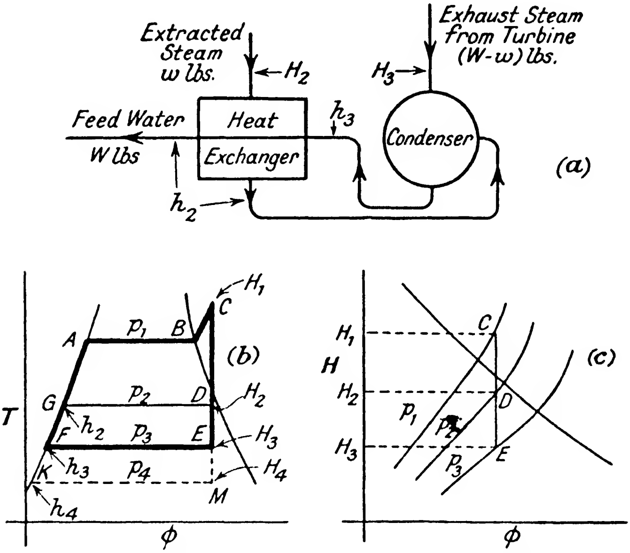


FIG. 192. FEED WATER RE-HEATING BY EXTRACTED STEAM

As a practical example let us consider a case where there is one steam extraction, the condensed extracted steam being returned to the condenser (Fig. 192 (*a*)). We will assume that the heated feed water and the condensed extracted steam both leave the heat exchanger at the same temperature.

The temperature-entropy diagram is *ABCEF* (Fig. 192 (*b*)), the expansion in the turbine being represented by *CE*, while some steam is extracted at *D* and in passing through the heat exchanger condenses from *D* to *G*, thus heating the feed water from *F* to *G*.

Let W = Weight of steam entering turbine at pressure p_1

w = Weight of steam extracted from turbine at pressure p_2

then

$W - w$ = Weight of stuff leaving turbine exhaust at pressure p_3

If the total heats per pound at the various points of the circuit *ABCEF* are as shown in Fig. 192 (*a*), then as the boiler receives the feed water with sensible heat h_2

Heat supplied to turbine $= W(H_1 - h_2)$

Heat rejected to condenser by turbine

$$= (W - w)\,(H_3 - h_3)$$

Heat rejected to condenser by heat exchanger condensate

$$= w(h_2 - h_3)$$

Hence Heat per pound of steam converted into work

$$= \frac{W(H_1 - h_2) - (W - w)\,(H_3 - h_3) - w(h_2 - h_3)}{W}$$

$$= (H_1 - h_2) - \left(1 - \frac{w}{W}\right)(H_3 - h_3) - \frac{w}{W}(h_2 - h_3)$$

Therefore

$$\text{Efficiency} = \frac{\text{Heat per pound of steam converted into work}}{\text{Heat supplied per pound of steam}}$$

$$= \frac{(H_1 - h_2) - \left(1 - \frac{w}{W}\right)(H_3 - h_3) - \frac{w}{W}(h_2 - h_3)}{H_1 - h_2}$$

The value of the quantity $\frac{w}{W}$ can be found as follows—

Heat given to feed water = Heat given up by extracted steam

That is $W(h_2 - h_3) = w(H_2 - h_2)$

Hence $$\frac{w}{W} = \frac{h_2 - h_3}{H_2 - h_2}$$

The case just considered deals with only one feed reheat. In practice this is generally carried out at more than one pressure, the condensates from the heaters being disposed of in various ways. Let us consider the case of two heaters in cascade, that is when the condensate from the first heater passes through the second heater and then to the condenser. Then referring to the temperature-entropy diagram (Fig. 192 (*b*)), steam is extracted from the turbine at *D* and passes through the first heater. In doing so it condenses and heats the feed water from *F* to *G*, and leaves the heater with sensible heat h_2. This condensate passes through the second heater where it gives up heat equal to $h_2 - h_3$ to the feed water after which it passes to the condenser. The second heater receives bled steam from the turbine at *E*, this steam giving up heat to the feed water and leaving the heater with sensible heat equal to h_3. In the second heater the feed water is heated from *K* to *F*.

Let W = Weight of steam entering turbine at pressure p_1

w_2 = Weight of steam extracted from turbine at pressure p_2

w_3 = Weight of steam extracted from turbine at pressure p_3

then $W - (w_2 + w_3)$ = Weight of stuff leaving turbine exhaust at pressure p_4

Then

Heat supplied to turbine

$$= W(H_1 - h_2)$$

Heat rejected to condenser by turbine

$$= [W - (w_2 + w_3)](H_4 - h_4)$$

Heat rejected to condenser by heater condensates

$$= (w_2 + w_3)(h_3 - h_4)$$

Hence Heat per pound of steam converted into work

$$= \frac{W(H_1 - h_2) - (W - w_2 - w_3)(H_4 - h_4) - (w_2 + w_3)(h_3 - h_4)}{W}$$

$$= H_1 - h_2 - \left(1 - \frac{w_2}{W} - \frac{w_3}{W}\right)(H_4 - h_4) - \left(\frac{w_2}{W} + \frac{w_3}{W}\right)(h_3 - h_4)$$

In the first heater

Heat given to feed water = Heat given up by extracted steam

Hence $\quad W(h_2 - h_3) = w_2(H_2 - h_2)$

and
$$\frac{w_2}{W} = \frac{h_2 - h_3}{H_2 - h_2}$$

In the second heater

$$W(h_3 - h_4) = w_3(H_3 - h_3) + w_2(h_2 - h_3)$$

and
$$\frac{w_3}{W} = \frac{h_3 - h_4}{H_3 - h_3} - \frac{w_2(h_2 - h_3)}{W(H_3 - h_3)}$$

In the cases just considered, the feed-water heaters are assumed to be constructed in a manner similar to that of a surface condenser, that is the extracted steam and the feed water to be heated do not come into equal contact, but are separated by the tube walls. In practice the temperature of the condensed extracted steam leaving the heater would be higher than the temperature of the feed water entering the heater, and hence it has been assumed that the sensible heat of the condensed extracted steam is h_2 and not h_3. (See Fig. 192.) Sensible heat equal to $h_2 - h_3$ is therefore rejected to the condenser. Let us now assume the case of a single extraction where the condensed extracted steam is not rejected to the condenser, but is added to the feed water leaving the condenser. This could be done by having a contact feed heater in which the extracted steam and the feed water came into actual contact in the heater, the mixture being pumped into the boiler. In these circumstances the extracted steam rejects no heat to the condenser.

Then

Heat converted into work $= W(H_1 - h_2) - (W - w)(H_3 - h_3)$

and $$\text{Efficiency} = \frac{(H_1 - h_2) - \left(1 - \frac{w}{W}\right)(H_3 - h_3)}{H_1 - h_2}$$

also Heat given up by extracted steam = Heat given to feed water

$$w(H_2 - h_2) = (W - w)(h_2 - h_3)$$

whence $$\frac{w}{W} = \frac{h_2 - h_3}{H_2 - h_3}$$

This case is illustrated in Example 4.

The equations just given also apply to the case of the surface type of feed-water heater when the condensed extracted steam is pumped into the feed-water line instead of being rejected to the condenser.

Example 4. In some modern steam-power plants the feed water is heated by live steam extracted from the turbines at some suitable point in the expansion range. The work obtained from each pound of boiler steam is thus reduced, but as the heat supplied to each pound of steam in the boiler is also reduced a gain in thermal efficiency generally results from the process. Show that this is the case in the following example—

Steam enters h.p. turbine at 500 lb. per sq. in. with 200° F. (111° C.) superheat; steam leaves h.p. and enters l.p. turbine at 60 lb. per sq. in. abs., dry; steam leaves l.p. turbine at 0·5 lb. per sq. in. abs., 0·84 dry. Steam is extracted between the two turbines and used in a contact feed heater, mixture leaves heater and enters boiler at 292·6° F. (144·8° C.) (*I.Mech.E.*, 1929.)

Total heat of steam at 500 lb. per sq. in. with 111° C. superheat

$= H_1 = 744{\cdot}2$ C.H.U. per lb. (Table D)

Total heat of steam at 60 lb. per sq. in. dry

$= H_2 = 655{\cdot}5$ C.H.U. per lb. (Table A)

Sensible heat of water corresponding to this (Sat. Temp. 144·9° C.)

$= h_2 = 145{\cdot}7$ C.H.U. per lb. (Table A)

Total heat of stuff at 0·5 lb. per sq. in. and 0·84 dry

$= H_3 = 26 + 0{\cdot}84 \times 581 = 514$ C.H.U. per lb. (Table A)

Sensible heat corresponding to this

$= h_3 = 26$ C.H.U. per lb.

Then $$\frac{w}{W} = \frac{h_2 - h_3}{H_2 - h_3} = \frac{145{\cdot}7 - 26}{655{\cdot}5 - 26} = 0{\cdot}19$$

$$\text{and efficiency} = \frac{(H_1 - h_2) - \left(1 - \frac{w}{W}\right)(H_3 - h_3)}{H_1 - h_2}$$

$$= \frac{(744{\cdot}2 - 145{\cdot}7) - (1 - 0{\cdot}19)(514 - 26)}{744{\cdot}2 - 145{\cdot}7}$$

$$= 0{\cdot}339$$

If there is no feed-water heating

$$\text{Rankine efficiency} = \frac{H_1 - H_3}{H_1 - h_3}$$

$$= \frac{744{\cdot}2 - 514}{744{\cdot}2 - 26}$$

$$= 0{\cdot}321$$

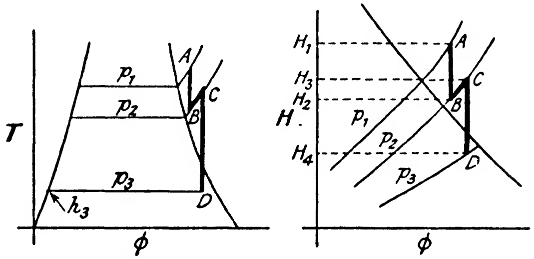


FIG. 193. STEAM RE-HEATING

Steam Re-heating. In the ordinary Rankine cycle the steam supplied to the high-pressure end of the turbine has received all its heat from the boiler. If the expansion in the turbine were adiabatic, the steam would progressively lose its superheat (if any), and then become gradually wetter. It must be remembered, however, that this simple adiabatic expansion is modified in practice by the effect of friction, which causes an internal re-heating effect as explained on page 497. In modern practice the steam also receives heat at one or more stages during the expansion from an external source. This can be done by withdrawing the steam from the turbine at some stage in the expansion, passing it through heating coils in the boiler where it is resuperheated, at the extraction pressure, and returning it to the turbine at this pressure, where it continues its expansion. Further extractions and re-heats can then be carried out at lower pressures. The process for one re-heat is shown in Fig. 193, where true adiabatic expansion is assumed.

The state of the steam entering the turbine is represented by the point A and adiabatic expansion from the pressure p_1 to the pressure p_2 takes place from A to B. The steam is then extracted from the turbine and reheated at the pressure p_2, the consequent change in state of the steam being represented by *BC*. The steam now re-enters the turbine in a condition represented by the point *C*, and expands adiabatically along *CD* from the pressure p_2 to the pressure p_3, final extraction to the condenser occurring at the point *D*.

The work obtained in the turbine per pound of steam is measured by the sum of the adiabatic heat drop from the initial pressure p_1 to the extraction pressure p_2, and the adiabatic heat drop from the pressure p_2 to the final exhaust pressure p_3. The heat supplied is that put into the working substance by the boiler at the pressure p_1, and that added during the re-heat.

Hence

$$\text{Work done per pound of steam} = (H_1 - H_2) + (H_3 - H_4)$$

$$\text{Heat supplied per pound of steam} = (H_1 - h_3) + (H_3 - H_2)$$

$$\text{Cycle efficiency} = \frac{(H_1 - H_2) + (H_3 - H_4)}{(H_1 - h_3) + (H_3 - H_2)}$$

Example 5. Describe the construction of a heat-entropy chart for steam. Select sufficient values from the steam tables to construct so much of such a chart as will enable you to trace out on it the following changes in steam condition. Steam at 250 lb. per sq. in. abs. with 100° F. (55° C.) superheat expanded in an H.P. turbine to 35 lb. per sq. in. abs. with an efficiency ratio of 0·7, then reheated at 35 lb. per sq. in. to the original temperature and expanded in an L.P. turbine to 1·2 lb. per sq. in. abs. with an efficiency ratio of 0·78. Determine the steam condition at the final pressure (1·2 lb. per sq. in.) and estimate the power that would be developed in the two turbines from a steam flow of 10 lb. per sec. (*I.Mech.E.*, 1927.)

The required details from the total heat-entropy chart are shown in Fig. 194, where the path taken by the steam is represented by the curve *ACDF*.

The steam is supplied to the h.p. turbine in a condition represented by the point *A*, where

Total heat at 250 lb. per sq. in. and 55° C. of superheat

$$= H_A = 706 \text{ C.H.U. per lb.}$$

After adiabatic expansion to 35 lb. per sq. in., total heat

$$= H_B = 614 \text{ C.H.U. per lb.}$$

Hence Adiabatic heat drop

$$= H_A - H_B = 706 - 614 = 92 \text{ C.H.U. per lb.}$$

Hence Actual heat drop

$$= H_A - H_C = 0{\cdot}7 \times 92 = 64{\cdot}4 \text{ C.H.U. per lb.}$$

and Total heat after actual expansion to 35 lb. per sq. in.

$$= H_C = 706 - 64{\cdot}4 = 641{\cdot}6 \text{ C.H.U. per lb.}$$

Saturation temperature at 250 lb. per sq. in. = 205° C. (Table 1)

Hence Temperature at *A* = 205 + 55 = 260° C.

Saturation temperature at 35 lb. per sq. in. = 126° C.

Hence Superheat at 35 lb. per sq. in. and 260° C.

$$= 260 - 126 = 134° \text{ C.}$$

Either this superheat or the actual temperature fixes the point D representing the condition of the steam after reheating.

Total heat at $D = H_D = 717$ C.H.U. per lb.

Adiabatic heat drop to 1·2 lb. per sq. in.

$$= H_D - H_E = 717 - 576 = 141 \text{ C.H.U. per lb.}$$

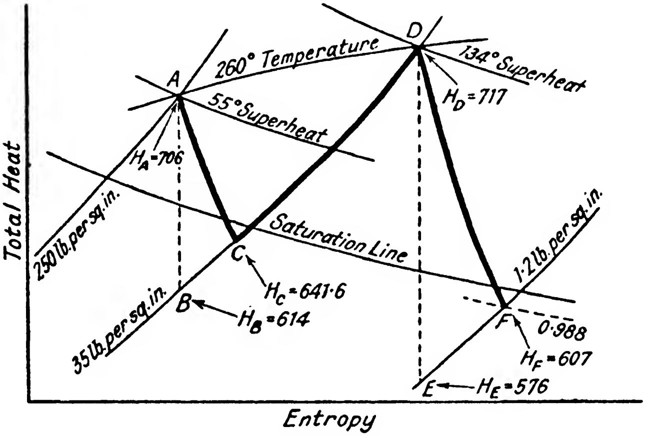


Fig. 194

Hence actual heat drop

$$= H_D - H_F = 0{\cdot}78 \times 141 = 110 \text{ C.H.U. per lb.}$$

and final total heat

$$= H_F = 717 - 110 = 607 \text{ C.H.U. per lb.}$$

also final dryness fraction = 0·988

Heat equivalent of work done in the two turbines

$$= (H_A - H_C) + (H_D - H_F) = 64{\cdot}4 + 110 = 174{\cdot}4 \text{ C.H.U. per lb.}$$

$$\text{and} \quad \text{Total power} = \frac{10 \times 174{\cdot}4 \times 1{,}400}{550}$$

$$= 4{,}439 \text{ H.P.}$$

Example 6. In a power plant the boiler supplies 10 lb. of steam per second to a high-pressure turbine at 300 lb. per sq. in. abs., with 50° F. superheat, and it is there expanded to 40 lb. per sq. in. abs. 0·98 dry. The exhaust from the turbine is mixed with boiler steam throttled to 40 lb. per sq. in. abs. supplied at the rate of 1 lb. per sec., and the mixture is expanded in a low-pressure turbine to 1 lb. per sq. in. abs., 0·95 dry. Determine the total power developed in the turbines and their overall thermal efficiency.

(*I.Mech.E.*, 1933.)

This example can be solved by means of the H-ϕ chart from which the various values of total heat can be read directly.

Thus

Total heat of boiler steam at 300 lb. per sq. in. and 50° F. superheat = 692 C.H.U. per lb.

Total heat after expansion to 40 lb. per sq. in., 0·98 dry = 642 C.H.U. per lb.

This steam is now mixed with throttled boiler steam at the same pressure.

Let H = Total heat of final mixture

Then $10(H - 642) = 1(692 - H)$

and $$H = \frac{7{,}112}{11} = 647 \text{ C.H.U. per lb.}$$

Total heat after further expansion to 1 lb. per sq. in., 0·95 dry = 585 C.H.U. per lb.

Hence Work in H.P. turbine

$$= 10(692 - 642) = 500 \text{ C.H.U. per sec.}$$

Work in L.P. turbine

$$= 11(647 - 585) = 682 \text{ C.H.U. per sec.}$$

and Total power

$$= \frac{(500 + 682)1{,}400}{550} = 3{,}009 \text{ H.P.}$$

Sensible heat at 1 lb. per sq. in.

$$= 39 \text{ C.H.U. per lb.}$$

Hence $$\text{Efficiency} = \frac{500 + 682}{11(692 - 39)} = 0{\cdot}164$$

Production of Steam Above the Critical Point. It is possible to produce superheated steam from water without passing through the wet vapour stage. In order to do this the water must first be brought up to a pressure just above the critical pressure (3,200 lb. per sq. in.), and then heated above the critical temperature (374° C.). The process is illustrated in Fig. 195, given by Schüle.*

If water is heated under a rising pressure so that the pressure at any moment is the saturation value corresponding to the temperature at that moment, it would pass along the liquid curve *OFC*. If, however, the water is first raised in pressure to some value p_s less than the critical, and then heated, it will pass along the curve *DF*. This curve will meet the liquid curve at the point *F*, where the

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* *Technical Thermodynamics*, by Schüle, translated by Geyer (Pitman).

temperature is the saturation value corresponding to the pressure p_s. Further heating will cause the water to evaporate in the usual way along the horizontal line through F. On the other hand, if the water is first brought to the critical pressure p_c and then heated, its state throughout the heating is indicated by the curve D_cC extended into the superheated region. In this case the curve just touches the boundary curve to the wet vapour region, but superheated steam is produced without passing through the wet vapour state. If the cold water is first brought to a pressure above the critical value, the substance passes along the curve $D_1A_1A_2$ and the water changes into superheated steam, without there being any definite point where the liquid state ceases and the gaseous state begins.

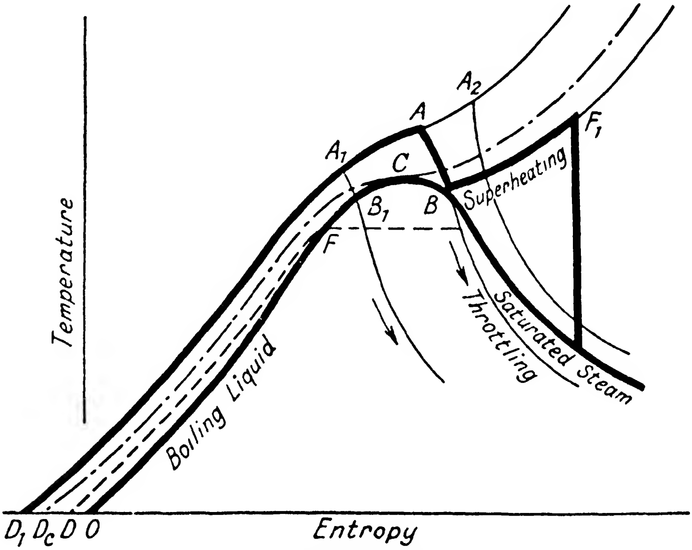

FIG. 195. PRODUCTION OF STEAM ABOVE THE CRITICAL POINT

In this way superheated steam can be produced without the production of ebullition due to the wet vapour stage. The steam, however, is at too high a pressure for practical use, but this can easily be reduced by throttling, during which the total heat remains constant. The choice of the point at which throttling commences is, however, important. If it is started at A_1, then the state curve will meet the liquid curve at B_1 and liquefaction will take place. If throttling starts at A, the state curve meets the saturation curve at B and this steam could be expanded from B in a turbine. If one considers adiabatic expansion from B, however, it will be seen that the steam would soon become very wet. It could be superheated at B along the constant pressure line BF_1, and then it could be expanded satisfactorily from the point F_1. If throttling started at the point A_2 the steam would never become wet during the throttling.

This process has been used in the Benson Steam Generator.

The Binary Vapour Cycle. The Carnot cycle efficiency indicates that the higher the initial temperature of the working substance, the greater will be the efficiency of the cycle. The critical temperature of steam is 374° C. at which the pressure is 3,200 lb. per sq. in. It would seem at first sight that, by superheating, high steam temperatures could be obtained without the necessity of such high pressures. It is found, however, that the thermal efficiency depends on the saturation temperature corresponding to the supply pressure rather than on the superheat temperature.

The critical temperature of mercury is above 1,000° C. and its saturation temperatures are considerably higher than those for steam at the same pressures, as shown in Table XXIV.

TABLE XXIV

Vapour Pressure lb. per sq. in.	Temp. ° C.	h	L	H	ϕ Liquid	ϕ Vapour
0·2	184	6·8	71·37	78·17	0·0169	0·1748
0·8	225	7·44	70·94	78·38	0·0200	0·1631
1·0	235	7·79	70·87	78·66	0·0205	0·1612
10·0	336	11·1	69·91	81·01	0·0265	0·1425
50·0	434	14·4	69·22	83·62	0·0316	0·1305
70·0	453	15·1	69·07	84·17	0·0327	0·1281
100·0	487	16·3	68·88	85·18	0·0340	0·1257
150·0	520	17·4	68·67	86·07	0·0356	0·1229

From these figures it will be seen that mercury at moderate pressures can be used at temperatures to attain which with steam would mean a very considerable amount of superheat.

The Binary Vapour Cycle is illustrated in Figs. 196 and 197. Gaseous mercury supplied from a mercury boiler at 70 lb. per sq. in. (saturation temperature 453° C.) is allowed to expand in a turbine to a pressure of 0·8 lb. per sq. in., and so do work. For simplicity this expansion will be assumed to be adiabatic and it is represented in Figs. 196 and 197 by the line CD. The final dryness fraction of the mercury vapour at D is given by $\frac{AD}{AM} = 0{\cdot}75$. This wet mercury vapour now enters a mercury condenser supplied with water at a pressure of 350 lb. per sq. in. (saturation temperature 222° C.) and represented by the point E. The mercury condenses and the water evaporates. The condensation of the mercury is represented by the line *DA* and the evaporation of the water by the line *EF*. The liquid mercury at *A* is pumped into the mercury boiler where it is heated to *B* and evaporated to *C*. The steam represented by the point *F* expands in a steam turbine to a pressure of 0·5 lb. per sq. in. Again assuming adiabatic expansion, this is shown by the line *FK*. The steam is then condensed in the usual way, and returned to the water heater where it is heated to a temperature of 222° C. at a pressure of 350 lb. per sq. in. In this example it is assumed that the heat given up by the condensed mercury is just sufficient to produce dry steam, although in practice the steam would probably be superheated.

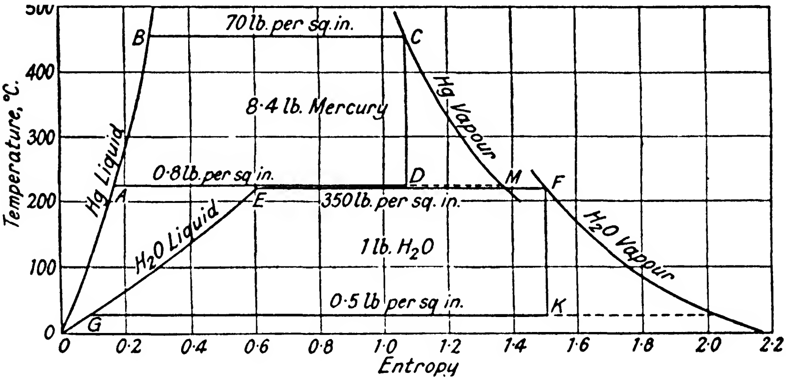

FIG. 196. TEMPERATURE-ENTROPY CHART FOR BINARY VAPOUR CYCLE

Let L_s = Latent heat of steam at 350 lb. per sq. in.
= 447·4 C.H.U. per lb.
L_m = Latent heat of mercury at 0·8 lb. per sq. in.
= 71 C.H.U. per lb.

Then

Heat rejected by mercury = Heat received by water

Weight of mercury × Dryness × L_m = Weight of water × L_s

and
$$\frac{\text{Weight of mercury}}{\text{Weight of water}} = \frac{447 \cdot 4}{0 \cdot 75 \times 71} = 8 \cdot 4.$$

The values taken for Figs. 196 and 197 are therefore for 8·4 lb. of mercury and for 1 lb. of water.

From Table XXIV, p. 527,

Total heat of 8·4 lb. mercury at 70 lb. per sq. in. (Point *C*)
$= 8 \cdot 4 \times 84 \cdot 17 = 707$ C.H.U.

Total heat after adiabatic expansion to 0·8 lb. per sq. in. (Point *D*)
= 8·4(7·44 + 0·75 × 70·94) = 509 C.H.U.

Hence Work done by 8·4 lb. mercury
= 707 - 509 = 198 C.H.U.

Total heat of 1 lb. steam at 350 lb. per sq. in. (Point *F*)
= 675 C.H.U.

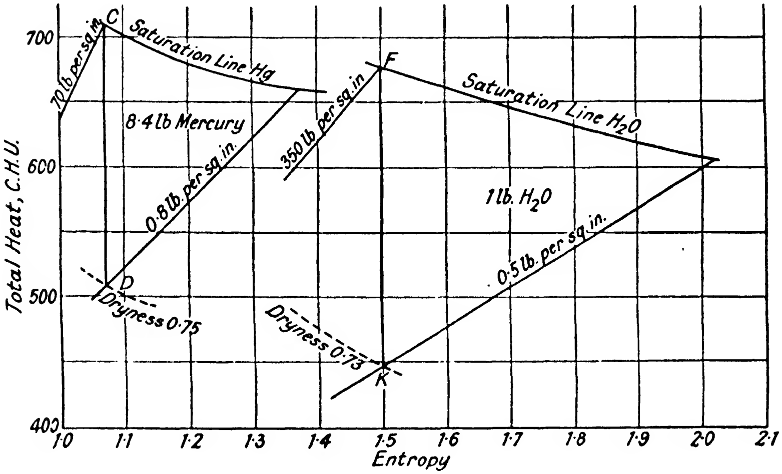

FIG. 197. TOTAL HEAT-ENTROPY CHART FOR BINARY VAPOUR CYCLE

Total heat after adiabatic expansion to 0·5 lb. per sq. in. (Point *K*)
= 449 C.H.U.

Hence Work done by 1 lb. steam = 675 - 449 = 226 C.H.U.
and Total work done = 198 + 226 = 424 C.H.U.
Heat supplied to mercury = 8·4($H_0 - h_A$) (Fig. 196)
= 8·4(84·17 - 7·44) = 644 C.H.U.
Heat supplied to steam = $h_B - h_G$
= 228 - 26 = 202 C.H.U.
Hence Total heat supplied = 644 + 202 = 846 C.H.U.
and Efficiency of cycle $= \frac{424}{846} = 0{\cdot}50$

In a practical case there would be a certain amount of reheating due to friction during expansion, and hence the points *D* and *K* would be to the right of the positions shown in Figs. 196 and 197.

EXAMPLES XIII

1. How does the presence of air affect the working of a surface condenser? The vacuum at the bottom of a surface condenser is 26·8 in. of mercury (barometer 29·8 in.), the temperature at the air pump suction is 100° F. (37·7° C.). If the rate of air leakage into the condenser is 1 lb. per 1,000 lb. of steam, estimate the weight of air and vapour removed by the air pump per minute when the engine consumption is 30,000 lb. of steam per hour. Take R for air as 53·3 in equation $pv = RT$. (*I.Mech.E.*, 1930.)

2. A steam engine is supplied with dry steam at 300 lb. per sq. in. abs., exhausts against a back pressure of 2 lb. per sq. in. abs. and develops 750 I.H.P. with a consumption rate of 15 lb. of steam per I.H.P. hour. Determine the indicated thermal efficiency and also the condition of the steam entering the condenser, neglecting all external losses. Estimate the amount of cooling water required at the surface condenser if the permissible rise of temperature is 36° F. (20° C.) and the condensate leaves at 118° F. (47·8° C.). (*I.Mech.E.*, 1931.)

3. The temperature in a surface condenser near to the air-pump suction is 115·7° F. (46·49° C.) when the vacuum is 26·12 in. with a 30-in. barometer. The condenser receives 40,000 lb. of steam per hour and the air leakage is 0·3 lb. per 1,000 lb. of steam. Estimate the necessary swept volume per minute of the dry air-pump, assuming that its volumetric efficiency is 0·85. (*I.Mech.E.*, 1932.)

4. A barometric jet condenser deals with 10,000 lb. of steam, 0·96 dry, per hour, and maintains a vacuum of 26·5 in. (barometer 30 in.). The cooling water enters at 55° F. and the mixed condensate and cooling water leave at 108° F. Sketch a typical condenser showing the connections to the air-pump and hotwell. Estimate (*a*) the cooling water required per hour, and (*b*) the minimum height of the tail pipe above the level of the hotwell. (*I.Mech.E.*, 1933.)

5. A steam turbine is supplied with 6 lb. of steam per second at a pressure of 200 lb. per sq. in. abs. with 160° F. (89° C.) superheat. Exhaust takes place at 1·5 lb. per sq. in. abs. At a stage of pressure of 20 lb. per sq. in. abs. steam is tapped off to heat the feed in a surface heater to the corresponding saturation temperature. Assuming the efficiency for any range of expansion as 0·75, determine the power developed and the thermal efficiency of the plant. (*I.Mech.E.*, 1935.)

6. In a marine plant the boilers supply steam at 250 lb. per sq. in. abs. with 120° F. (67° C.) superheat to the main engines and auxiliaries. The main engines develop 2,000 I.H.P. at a consumption rate of 10·2 lb. of steam per I.H.P. hour, and exhaust at 2 lb. per sq. in. abs. The auxiliaries develop 130 I.H.P. at a consumption rate of 30 lb. per I.H.P. hour and exhaust at 15 lb. per sq. in. abs. The exhaust steam from the auxiliaries is used to heat the feed water, the condensate from the heater being drained to the hotwell. The temperature of the condensate from the engines entering the hotwell is 100° F. Estimate (*a*) the total heat (above 32° F.) per pound of steam leaving the auxiliaries, (*b*) the total heat per pound of feed leaving the exhaust heater and entering the boiler, and (*c*) the net indicated thermal efficiency of the plant if the boiler efficiency is 0·73. (*I.Mech.E.*, 1935.)

7. A turbine is supplied with steam at 250 lb. per sq. in. abs., superheated to 300° C. Adiabatic expansion then takes place to 100 lb. per sq. in. abs. after which the steam is removed and reheated to the original temperature. It is then expanded adiabatically in the turbine to 15 lb. per sq. in. abs., at which point a fraction is extracted to heat the feed water, the remainder expanding in the turbine to a final pressure of $\frac{1}{2}$ lb. per sq. in. abs. The condensate from the feed-water heater is pumped back into the feed-water line. What is the efficiency of the cycle compared with the corresponding Rankine efficiency?

8. A surface condenser has separate air and steam extraction, the air being cooled by an integral cooler before passing to the air extraction pump. The condensation temperature at entry to condenser is 90° F., the steam entering dry and the temperatures at entry and exit of the cooler are 88° F. and 79·6° F. respectively. Steam is condensed at the rate of 200 lb. per minute, while the air leakage is 0·15 lb. per minute. It may be assumed that the vacuum is constant throughout the condenser.

Find the necessary pump cylinder capacity if single acting and running at a speed of 100 r.p.m., and the heat to be carried away in the cooling water of the air cooler.

Assume R for air = 53·2 ft-lb. per lb. per degree F. and C_p for air = 0·24.
(*U. Nottm., B.Sc., Pass* (*Eng.*), 1950.)

9. In an evaporative condenser the steam enters the condensing tubes dry saturated at 100 lb. per square inch absolute and leaves as water without undercooling. The cooling water film may be considered as at a constant temperature. The air enters at 54° F. and leaves at 86° F. and may be considered as saturated at exit and entry.

If 1,000 pounds of steam is to be condensed per hour, find the necessary air flow, and the amount of cooling water evaporated per hour.

Assume a barometer of 30 inches, and the specific heat of air as 0·24 B.Th.U. per lb., per degree F. Take R = 53·3 foot pounds per pound per degree F. The necessary properties of steam are—

Saturation Temperature ° F.	Saturation Pressure lb. per sq. in. abs.	Total Heat of dry vapour B.Th.U. per lb.	Specific Volume Cu. ft. per lb.
54	0·21	1,081	1,500
86	0·62	1,099	527

(*U. Nottm., B.Sc.* (*Eng.*), 1951.)

CHAPTER XIV

STEAM ENGINE AND BOILER TRIALS

LIMITATIONS of space allow only a brief survey of the vast subject of heat engine trials in this chapter, and the student is referred for more detailed information to the report of the Heat Engines Trials Committee,* and to standard works on engine testing.†

Heat Account for a Steam Engine. In order to draw up a heat account for a steam engine the following measurements are necessary—

(i) The amount of steam supplied.
(ii) The pressure of the steam supply at the engine stop valve.
(iii) The dryness fraction (if wet), or the temperature (if superheated) of the steam supply.
(iv) The indicated horse-power of the engine.
(v) The brake horse-power of the engine.
(vi) The amount of steam jacket drainage (if any).
(vii) The temperature or pressure of the exhaust steam (if non-condensing).

If the engine is used with a condenser, it is usual to include the condensing system in the heat balance, and the following additional particulars are required—

(viii) The amount of cooling water used.
(ix) The initial and final temperatures of the cooling water.
(x) The temperature of the condensate.

If the engine is fitted with a surface condenser, the amount of steam supplied can be determined by measuring the condensate either by direct weighing, by the use of calibrated tanks or by means of a V notch. Care must be taken that there are no leaks between the engine and the point where the condensate is collected. In the case of a non-condensing engine, the feed to the boiler may be determined, but this quantity will include—

(*a*) The steam leakage between the boiler and the engine.
(*b*) The steam condensed in the pipe line between the boiler and the engine.
(*c*) The steam used for driving the boiler feed pump and any other auxiliaries.

The use of steam flow meters is not very satisfactory with reciprocating engines owing to the dependence of such meters for accuracy on constant steam conditions.

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* Report on tabulating results of Heat Engines Trials (*Inst. C.E.*, 1927).
† *The Testing of Motive Power Engines*, by Royds.

If the steam supply is superheated, it is a simple matter to measure the pressure and temperature of the steam at the engine stop valve. If the steam is slightly wet, a throttling calorimeter can be used to determine the dryness fraction. The great difficulty, however, is to obtain a representative sample of the steam. A plain sampling pipe simply screwed flush into the side of the main steam pipe will give too high a reading of the dryness fraction, owing to the entry of the sampling pipe being at right angles to the steam flow, and the water therefore not turning into the pipe so readily as the vapour. A sampling pipe provided with holes and screwed into the main steam pipe so that it is situated across the steam flow is more satisfactory, although even here there is some uncertainty as to the accuracy of the steam sample.

The methods of measuring I.H.P. and B.H.P. are dealt with in Chapter II. In fixing the indicator, care should be taken that the connecting pipe between the engine and the indicator is as short and straight as possible, and is of adequate internal diameter. For this reason, when testing a double-acting engine, it is more satisfactory to have a separate indicator for each end of the cylinder.

The type of brake used must be determined by circumstances, and the apparatus available. If the engine drives an electric generator, the power output can easily be determined from the electrical load, provided an efficiency curve for the generator is available.

The condenser cooling water may be measured by any convenient means such as calibrated tanks, or V notch. If the circulating pump is of the centrifugal type, the use of a venturi meter is very convenient.

A trial should last for several hours, the plant being run under test load for some time before the commencement of taking data, so that everything may settle down. Conditions should be maintained as constant as possible during the test. Observations should be taken frequently, say every ten minutes, and the readings of any one quantity should be averaged up for the period of the trial.

It must be emphasized that when meters are used, it is essential that they should be calibrated if there is the slightest doubt of their accuracy.

The method of setting out the heat account is illustrated in the following example.

Example 1. The following observations were made during a trial on a jacketed double-acting compound engine supplied with dry saturated steam. Cylinder dias. H.P. 9 in., L.P. 16 in., stroke 23 in., M.E.P. 35·1 and 19·9 lb. per sq. in. in H.P. and L.P. cylinders respectively. Speed 92·4 r.p.m. Brake torque 3,440 lb-ft. Steam pressures: admission and receiver 77·7 and 23·3 lb. per sq. in. by gauge, condenser vacuum 24 in. of mercury. Barometer, 30·3 in. Steam measured as discharge from air pump 18·10 lb. per minute, from jackets 1·89 lb. per minute, from receiver drain 1·07 lb. per minute. Cooling water 360 lb. per minute rising 29° C. Temperature of condensate 53° C.

Make out a heat balance sheet giving heat quantities per minute and calculate the thermal and relative efficiencies of the engine.

(*U.L.*, *B.Sc.* (*Eng.*), 1928.)

H.P. area = 63·6 sq. in. L.P. area = 201 sq. in.

$$\text{I.H.P.} = \frac{[(35{\cdot}1 \times 63{\cdot}6) + (19{\cdot}9 \times 201)] \times 23 \times 2 \times 92{\cdot}4}{33{,}000 \times 12} = 66{\cdot}9$$

$$\text{B.H.P.} = \frac{3{,}440 \times 2 \times 22 \times 92{\cdot}4}{33{,}000 \times 7} = 60{\cdot}5$$

$$\text{I.H.P.} - \text{B.H.P.} = 66{\cdot}9 - 60{\cdot}5 = 6{\cdot}4$$

Total heat in steam at 92·4 lb. per sq. in. abs.
= 661 C.H.U. per lb. (Table 1).

Sensible heat in steam at 92·4 lb. per sq. in. abs.
= 162·4 C.H.U. per lb.

Sensible heat at 38 lb. per sq. in. abs.
= 129·3 C.H.U. per lb.

Steam supplied = 18·10 + 1·89 + 1·07 = 21·06 lb. per minute

HEAT BALANCE

		C.H.U. per minute	Per Cent
Gross heat supplied = 21·06 × 661	=	13,920	
Heat equivalent of B.H.P. = $\frac{60{\cdot}5 \times 33{,}000}{1{,}400}$	=	1,426	10·2
Heat equivalent of I.H.P. - B.H.P. = $\frac{6{\cdot}4 \times 33{,}000}{1{,}400}$	=	151	1·1
Heat in jacket drainage = 1·89 × 162·4	=	307	2·2
Heat in receiver drainage = 1·07 × 129·3	=	138	1·0
Heat removed by cooling water = 360 × 29	=	10,440	75·0
Heat in condensate = 18·1 × 53	=	959	6·9
Heat unaccounted for	=	499	3·6

If it is assumed that the drainage mixes with the condensate and can be used in the boiler again, then

Net heat supplied

$$= 13{,}920 - 959 - 307 - 138 = 12{,}516 \text{ C.H.U. per min.}$$

Hence brake thermal efficiency

$$= \frac{1{,}426}{12{,}516} = 0{\cdot}114$$

Indicated thermal efficiency

$$= \frac{(1{,}426 + 151)}{12{,}516} = 0{\cdot}126$$

Pressure in condenser

$$= (30{\cdot}3 - 24)0{\cdot}49 = 3{\cdot}1 \text{ lb. per sq. in. abs.}$$

From H-ϕ chart

Adiabatic heat drop from 92·4 lb. per sq. in. to 3·1 lb. per sq. in.

$$= 661 - 534 = 127 \text{ C.H.U. per lb.}$$

$$\text{Rankine efficiency} = \frac{127}{661 - 162} = 0{\cdot}255$$

$$\text{Hence} \quad \text{Brake efficiency ratio} = \frac{0{\cdot}114}{0{\cdot}255} = 0{\cdot}45$$

$$\text{Indicated efficiency ratio} = \frac{0{\cdot}126}{0{\cdot}255} = 0{\cdot}49$$

Heat Account for a Turbine. In order to draw up a heat account for a turbine the following data are required—

(i) The amount of steam supplied.
(ii) The pressure of the steam supply at the turbine stop valve.
(iii) The quality of the steam supply.
(iv) The brake horse-power of the turbine.
(v) The power used in bearing friction and windage.
(vi) The temperature and pressure of the steam at the turbine exhaust.
(vii) The amount of condenser cooling water used.
(viii) The initial and final temperatures of the cooling water.
(ix) The temperature of the condensate extracted from the condenser.
(x) The amount of steam used for driving any auxiliaries.

The general methods already enumerated for steam engines also apply in the case of turbines, although, of course, the use of the indicator is not applicable. If the turbine is of the multi-stage impulse type, and is supplied with superheated steam, then readings of pressure and temperature at each stage may be taken, if thermometer pockets and points for the attachment of pressure gauges are available. As long as the steam remains superheated, the condition curve (see p. 497) may be plotted on the *H*-ϕ chart from these readings. When the steam becomes wet, this can still be done if the dryness fraction at each stage pressure is known, but this is not such a simple matter, owing to the difficulty in obtaining an accurate sample of the steam.

When testing a turbine fitted with a jet condenser, it would appear that the steam consumption could easily be determined by making separate determinations of the total condensate and of the cooling water supplied, and taking the difference as a measure of the steam consumption of the turbine. It must be remembered, however, that the amount of cooling water is considerably greater than the amount of condensed steam, and hence a small percentage error in the measurement of the total condensate means a large percentage error in the figure so obtained for the amount of condensed steam. In these cases a direct determination of the steam consumption may be made by means of a steam flow meter, their use being quite justifiable under the constant steam conditions possible with a turbine.

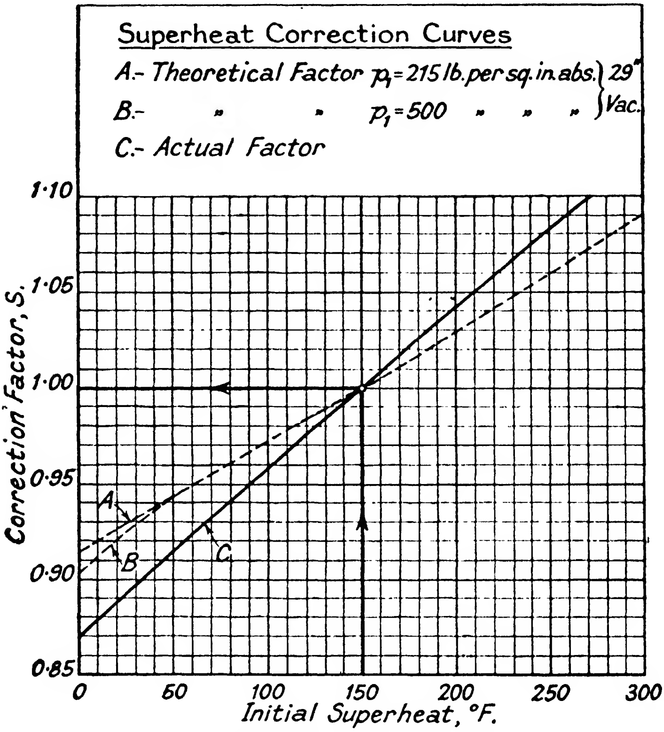

FIG. 198. SUPERHEAT CORRECTION FACTORS

The determination of the B.H.P. must be guided by circumstances and the apparatus available. In the case of a marine turbine, the power output can be measured by means of a torsion dynamometer applied to the propeller shaft.

The power absorbed by bearing friction, windage, etc., may be estimated by motoring the turbine when unloaded, and measuring the torque required by means of an electrical dynamometer.

ACCEPTANCE TESTS. It is usual when dealing with turbine plants of any considerable size, for the makers to give a guarantee within certain limits for the steam consumption at various loads. This guarantee is based on specified steam conditions. When an acceptance test is carried out, either on the maker's test bed or after final erection, it may be impossible or inconvenient to reproduce these conditions. In these circumstances it is usual to employ correction factors for any variation in the steam supply pressure and temperature, and in the exhaust vacuum. The theoretical change in steam consumption due to a variation in any one of these quantities, the others being constant, can easily be calculated from the H-ϕ chart. It is found, however, that the actual change in consumption is somewhat different from the theoretical values. Typical correction curves are given in Figs. 198, 199, and 200* where the theoretical and actual values are given.†

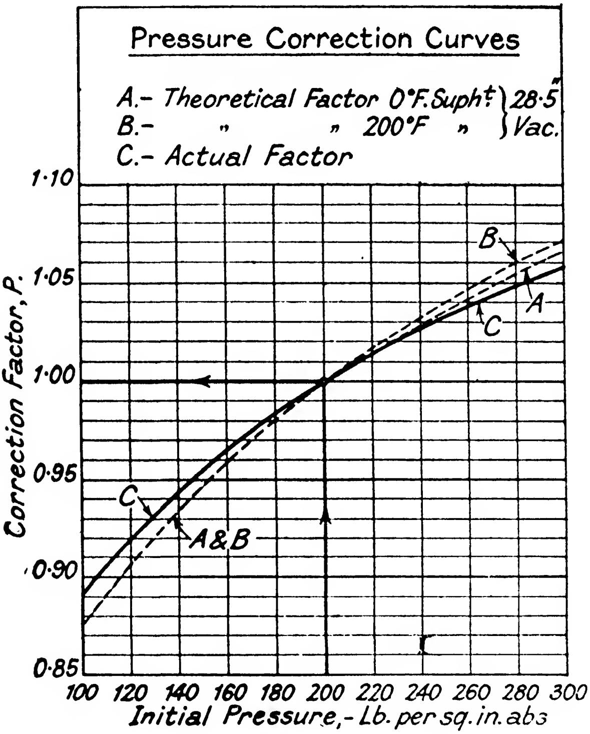

FIG. 199. PRESSURE CORRECTION FACTORS

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* The figures are taken from *Steam Turbine Operation*, by Kearton (Pitman).

† "Recent Developments in Steam Turbine Practice," by K. Baumann, *Proc.I.E.E.*, 1912.

Example 2. The following particulars refer to a trial on a small two-stage impulse turbine supplied with superheated steam—

Pressure and temperature of steam in the nozzle box, 154 lb. per sq. in. and 210° C. respectively; pressure and temperature of the steam in the first expansion, 14·2 lb. per sq. in. and 108° C.; pressure in the condenser, 1·13 lb. per sq. in.; revolutions per minute, 2,900; brake torque, 252 lb-ft.; friction torque, 17·5 lb-feet; steam per minute, 46·53 lb.; cooling water per minute, 1,665 lb. raised 15·3° C.; temperature of hotwell, 32·5° C.

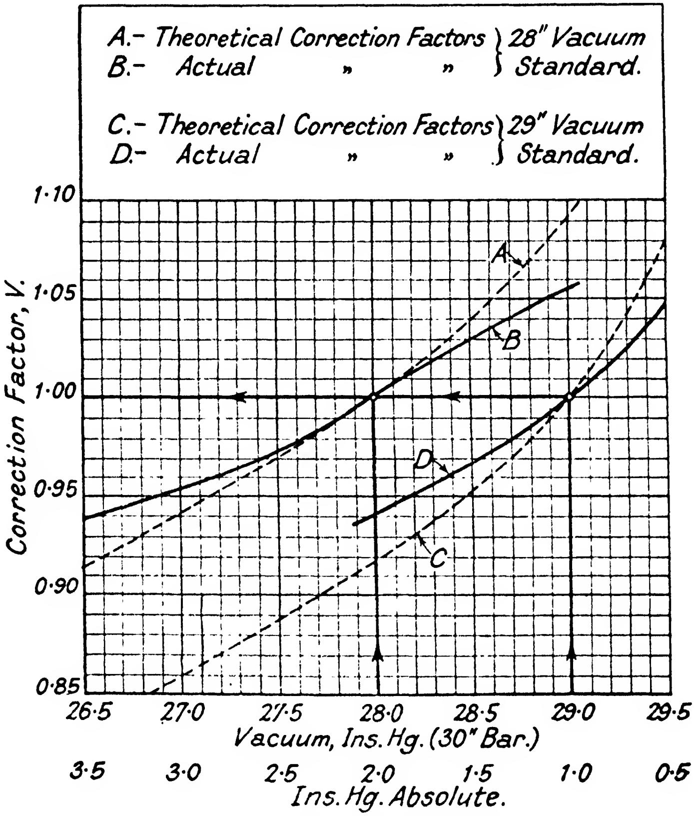


FIG. 200. VACUUM CORRECTION FACTORS

Make a heat balance sheet for the turbine expressing the values in lb-cals. per minute; and find the thermal efficiency.

What is the percentage of reheat in the first stage?

(*U.L.*, *B.Sc.* (*Eng.*), 1934.)

Total heat at 154 lb. per sq. in., and 210° C.
= 682 C.H.U. per lb. (Table D)

Total heat at 14·2 lb. per sq. in. and 108° C.
= 644 C.H.U. per lb. (estimated from H-ϕ chart).

Total heat after adiabatic expansion from 154 lb. per sq. in. and 210 ° C. to 14·2 lb. per sq. in.

$$= 584 \text{ C.H.U. per lb. (from } H\text{-}\phi \text{ chart).}$$

Hence Adiabatic heat drop $= 682 - 584 = 98$ C.H.U. per lb.

and Actual heat drop $= 682 - 644 = 38$ C.H.U. per lb.

Amount of reheat $= 98 - 38 = 60$ C.H.U. per lb.

which is $\frac{60 \times 100}{98} = 61{\cdot}2$ per cent of the adiabatic heat drop

Heat Balance

		C.H.U. per minute	Per Cent
Gross heat supplied $= 46{\cdot}53 \times 682$	=	31,733	
Heat equivalent of B.H.P. $= \frac{252 \times 2 \times 22 \times 2{,}900}{7 \times 1{,}400}$	=	3,281	10·3
Heat equivalent of friction loss $= \frac{17{\cdot}5 \times 2 \times 22 \times 2{,}900}{7 \times 1{,}400}$	=	228	0·7
Heat removed by cooling water $= 1{,}665 \times 15{\cdot}3$	=	25,475	80·3
Heat in condensate $= 46{\cdot}53 \times 32{\cdot}5$	=	1,512	4·8
Heat unaccounted for	=	1,237	3·9

Thermal efficiency of whole plant

$$= \frac{3{,}281}{31{,}733 - 1{,}512} = 0{\cdot}108$$

The sensible heat corresponding to the turbine exhaust pressure of 1·13 lb. per sq. in.

$$= 40{\cdot}4 \text{ C.H.U. per lb. (Table 1)}$$

Hence, Sensible heat corresponding to turbine exhaust pressure

$$= 46{\cdot}53 \times 40{\cdot}4 = 1{,}880 \text{ C.H.U. per minute}$$

Hence, Thermal efficiency of turbine only

$$= \frac{3{,}281}{31{,}733 - 1{,}880} = 0{\cdot}11$$

Heat Account for a Boiler. In order to draw up a heat account for a boiler plant, the following measurements are required—

(i) The amount of fuel burnt.
(ii) The complete analysis and the calorific value of the fuel.
(iii) The analysis of the flue gases at the exit from the boiler.
(iv) The amount of steam produced.
(v) The pressure of the steam at the boiler stop valve.
(vi) The quality of the steam at the boiler stop valve.

(vii) The amount of ash left after combustion.
(viii) The amount of combustible left in this ash.
(ix) The humidity of the air supply.
(x) The temperature of the boiler room (or air supply).
(xi) The temperature of the feed water.
(xii) The temperature of the flue gases at the boiler exit.
(xiii) The amount of steam used for driving the feed pump, and any other auxiliaries.

If the boiler is provided with an economizer, the following additional data should be obtained—

(xiv) The inlet and outlet water temperatures on the economizer.
(xv) The inlet and outlet flue temperatures on the economizer.

The boiler should be run under load for some considerable time before the commencement of the trial, which should last for at least five hours, and steaming conditions should be kept as constant as possible during the trial. When the trial actually starts, the water level in the boiler gauge glasses and in any feed-water storage tanks, such as the hotwell, should be noted and these levels should be the same at the end as at the commencement of the trial.

The methods adopted for determining the various measurements must depend to a great extent on circumstances and the apparatus available, but the general methods given below should be followed as closely as possible.

Amount of Fuel Burnt. If the boiler is hand-fired with coal, the fire should be cleaned about fifteen minutes before the commencement of the trial, when the thickness of the fuel bed should be noted. At the end of the trial the thickness and state of the fuel bed should be as near as possible the same as at the start. During the time of the trial, all fuel should be taken from a weighed quantity. It is desirable that the weight of the fuel fired up to the end of say each half-hour be noted. If the load on the boiler is steady, this will serve as a check on the weighing. If the boiler is fitted with a mechanical stoker, then the hopper supplying the stoker must be full at the commencement and at the end of the trial, all coal fed to the hopper during this period being weighed. If the stoker is fitted with a coal meter, this should not be relied upon without calibration. With an oil-fired boiler, the level of the oil in the supply tank can be noted by means of a hook gauge, and this level should be restored at the end of the trial by the addition of a weighed quantity of oil.

Analysis and Calorific Value of the Fuel. When weighing the coal, a handful should be taken haphazard before each weighing and put on one side. The total amount so collected at the end of the trial should be broken up, well mixed, and quartered several times, so that eventually a small representative sample is obtained. When sampling, care should be taken that extraneous dirt and moisture is not added from the boiler house floor, and the sample should be kept in a closed container until required. The method for determining the calorific value is described on page 220. For strict accuracy, the difference between the gross and net calorific value should be allowed for. The calorific value obtained from the bomb calorimeter is the gross value, including the latent heat and the sensible heat of the H_2O formed from the combustion of the hydrogen, and of any natural moisture left in the fuel even after drying, from 100° C. down to the final temperature of the calorimeter. This H_2O when formed by combustion in the boiler, however, passes off as a gas. Although this correction may be considered rather a refinement, it can of course be allowed for.

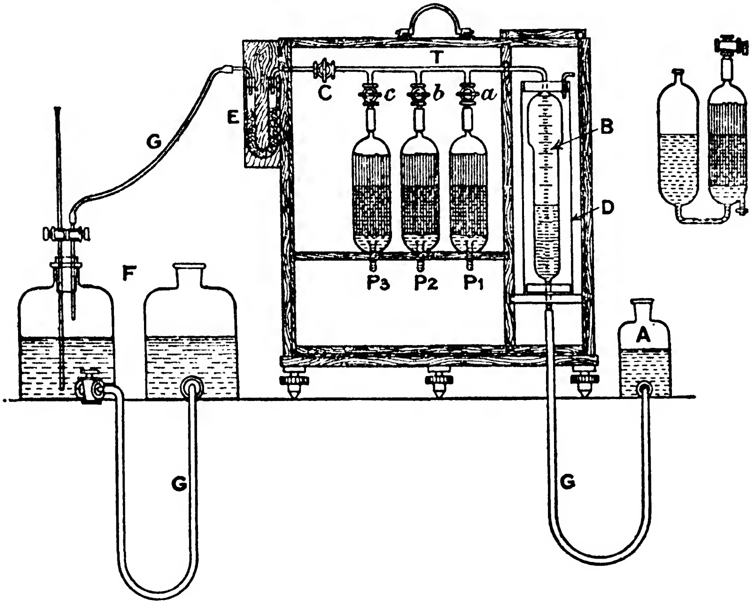


Fig. 201. Orsat Apparatus

Analysis of the Flue Gases. In order to obtain an average sample of the flue gases, the sampling pipe should consist of a pipe blanked off on the inside end and provided with a number of holes along the length which is situated in the boiler flue across the path of the gases. In order to obtain a live sample of the gases, some device such as a small water-operated ejector should be used to produce a small continuous outward flow of the gases, the sample being taken from a point between the ejector and the boiler flue.

Analysis of the gases may conveniently be made by means of the Orsat apparatus which is illustrated in Fig. 201.

A eudiometer *B*, graduated to 100 c.c. and surrounded by a water jacket to ensure uniform temperature is used to measure the gas. The pipettes P_1, P_2, and P_3 contain solutions for the absorption of CO_2, O_2, and CO respectively. An aspirator *A*, containing water, is used to manipulate the gas. *C* is a three-way cock whereby the eudiometer *B* can be connected to the sampling pipe or to the atmosphere, the third position of the cock closing the end of the pipe *T* altogether.

In order to draw in a sample, the cock *C* is turned to connect the eudiometer *B* to the atmosphere. The aspirator *A* is raised until the eudiometer is full of water. The cock *C* is now turned to connect the eudiometer with the flue gas and the aspirator is lowered to draw in the gas. The eudiometer is now connected to the atmosphere and the gas sample is expelled. This procedure is repeated a few times to ensure a fair sample of the gas. The aspirator is now lowered until the water levels in *A* and *B* are the same and the cock *C* is finally closed. The cock above the pipette P_1 is now opened and the gas is driven over into P_1 by raising the aspirator, the CO_2 being now absorbed. The gas is passed backwards and forwards several times, the reagent in P_1 is drawn up to its original position and the cock above P_1 is then closed. The water levels in *A* and *B* are brought level and the contraction is read off on the eudiometer *B*. This procedure is repeated for O_2 and then for CO.

The absorbing solutions may be made as follows—

A (absorbing CO_2). 1 part of caustic potash to 2 parts of water by weight.

B (absorbing O_2). 5 grammes of pyrogallic acid in 15 c.c. of water, mixed with 120 grammes of caustic potash in 80 c.c. of water. This solution must be made up just before use.

C (absorbing CO). A solution of cuprous chloride in hydrochloric acid, made by dissolving copper oxide (CuO) in twenty times its weight of strong hydrochloric acid, and allowing to stand in a corked bottle in contact with copper wire until colourless.

Another method for the determination of CO is to substitute for the absorption pipette P_3 a similar vessel provided with a platinum wire heating coil which can be heated from outside by means of an electric current. After absorption of the CO_2 and the O_2, some of the gas in the eudiometer is expelled leaving say 50 c.c. in the apparatus. 50 c.c. of air is then drawn in and the mixture is driven over into the new pipette P_3. The platinum coil is heated to burn the CO to CO_2, the gas is then drawn back into the eudiometer, and the amount of CO can be determined from the consequent contraction.

$$(C + O_2 \rightarrow CO_2)$$

Analyses should be taken as frequently as possible during the trial.

AMOUNT OF STEAM PRODUCED. The weight of steam produced can best be determined by measuring the feed water. This can be done by passing the water alternately through two calibrated tanks, each tank capacity being about fifteen minutes' supply. The water after measurement passes to a well from which the feed pump draws. Care must be taken that the level in the well is the same at the commencement and at the end of the trial, and that there is no loss of water between the point where the measurement is made and the boiler check valve. A V-notch if properly calibrated is also a very convenient piece of apparatus for this purpose. The use of mechanical meters is not desirable unless their accuracy is definitely ascertained. If such a meter is used it should be calibrated both before and after the trial. Automatic water machines if of a reliable make are also very convenient.

PRESSURE AND QUALITY OF THE STEAM. In the absence of recorders, readings of pressure and temperature (if superheated) should be taken every ten minutes. If these values vary, they should be averaged for the period of the trial.

If there is no superheater, the dryness fraction should be determined on the engine side of the boiler stop valve by means of a throttling calorimeter, although as already mentioned there is some doubt as to the possibility of obtaining an accurate sample.

ASH LEFT AFTER COMBUSTION. At the start of the trial the ash pit should be clean and at the end the pit should be cleared and the ashes weighed. With some forms of mechanical stoker, the ashes tend to accumulate at the back of the stoker. In such a case these should be removed and added to the quantity being weighed. A fair sample for analysis should be obtained by mixing and quartering as in the case of the coal.

HUMIDITY OF THE AIR SUPPLY. The amount of moisture carried in by the air supply may be determined by means of a wet and dry bulb thermometer. From these readings the dew point, that is the temperature at which the air would be saturated can be obtained.

TEMPERATURES. Temperatures should be taken every ten minutes, and averaged for the period of the trial. When taking flue gas temperatures, these may vary over the flue cross-section and this leads to some uncertainty as to the true value.

STEAM USED TO DRIVE FEED PUMP, ETC. If possible the feed pump should be driven by steam from a boiler separate from that under test. Failing this, some means must be devised to ascertain the steam consumption of the pump. As the amount of steam so consumed is a small amount of the total steam produced a small error in the estimation will not have much effect on the heat balance.

THE HEAT BALANCE. The method of drawing up the heat balance is illustrated in Example 3. The proportions of carbon left unburnt, burnt to CO_2 and burnt to CO must first be determined. If the proportions of CO_2 and CO by volume in the flue gases are known then if CO_2 = Percentage CO_2 in flue gas by volume

CO = Percentage CO in flue gas by volume

then $$\frac{CO_2 \times 44}{CO \times 28} = \frac{\text{Weight of } CO_2}{\text{Weight of CO}}$$

Hence

$$\frac{\text{Weight of carbon burnt to } CO_2}{\text{Weight of carbon burnt to CO}} = \frac{CO_2 \times 44 \times \frac{12}{44}}{CO \times 28 \times \frac{12}{28}} = \frac{CO_2}{CO}$$

The amount of heat lost due to the presence of CO can now be determined from the difference of the calorific value of carbon when burnt to CO_2 and when burnt to CO.

The weight of air actually used can be calculated from the formula 1, p. 208. It is essential that the analysis of the flue gases should be made accurately as the large quantity is the nitrogen, and this being determined by difference, it contains all the errors made in the determination of the other quantities.

The minimum weight of air required to produce the given combustion can be determined as shown in the table in Example 3. If the analysis of the fuel were given as per pound of fuel as fired, this should be corrected to per pound of dry fuel.

The calculation of the products of combustion is also shown in the table. The specific heat of the products of combustion could be calculated by assigning to each constituent its specific heat in its correct proportion as shown in Example 3.

In order to find the amount of moisture carried in by the air supply, the dew point is required (see p. 103). As an example let this be 10° C., and let the room temperature be 23° C. and flue temperature be 300° C. Then

Vapour pressure at 10° C.

= 0·178 lb. per sq. in. from steam table B.

Hence partial air pressure

= 14·7 − 0·178 = 14·522 lb. per sq. in.

and Volume of 1 lb. air at 23° C.

$$= \frac{RT}{P} = \frac{96(23 + 273)}{14{\cdot}522 \times 144} = 13{\cdot}8 \text{ cu. ft.}$$

Volume of 1 lb. water vapour at 10° C.

= 1,704 cu. ft. from steam table B.

Hence weight of moisture in 1 lb. air $= \frac{13{\cdot}8}{1{,}704} = 0{\cdot}008$ lb.

and heat carried away by moisture being heated from 23° C. to 300° C. $= 0{\cdot}008 \times 0{\cdot}5 \times (300 - 23) = 1{\cdot}108$ C.H.U. per lb. of air. where 0·5 is taken as the specific heat of superheated water vapour.

Example 3. The fuel used in a boiler trial gave the following percentage weight analysis, dry: $C = 84{\cdot}84$; $H_2 = 5{\cdot}22$; $O_2 = 0{\cdot}59$; $N_2 = 1{\cdot}29$; $S = 1{\cdot}39$; Ash = 6·67, and, when fired, contained 3·43 per cent of moisture. The steam produced amounted to 10·4 lb. per lb. of dry fuel. The steam pressure and temperature were 235 lb. per sq. in. and 320° C.; the feed temperature was 86° C.

The air for combustion entered at 28° C. The percentage volumetric analysis of the dry flue gases gave $CO_2 = 14{\cdot}65$; $CO = 0{\cdot}27$; $O_2 = 4{\cdot}36$; $N_2 = 80{\cdot}72$; and the ash contained 6 per cent of combustibles (assumed carbon). The outlet gas temperature was 300° C.

The calorific value of the dry fuel was 8,350 C.H.U.

Draw up a complete heat balance, allowing 68 C.H.U. per lb. dry fuel for combustibles in ash and moisture in air. Take that 1 lb. C burning to CO_2 liberates 8,080 C.H.U., and to CO, 2,420 C.H.U.; specific heat of dry products of combustion 0·265, and of excess air 0·24; O_2 in air, 23 per cent by weight. (*U.L.*, *B.Sc.* (*Eng.*), 1932.)

Weight of unburnt carbon.

Let x = Weight of unburnt carbon per lb. of dry fuel left in ash.

Then $$\frac{x}{0{\cdot}0667 + x} = \frac{6}{100} \text{ and } x = 0{\cdot}0043 \text{ lb.}$$

Hence Weight of C burnt $= 0{\cdot}8484 - 0{\cdot}0043$
$= 0{\cdot}8441$ lb. per lb. of dry fuel

Proportion of carbon burnt to CO_2 and CO.

$$\frac{\text{C burnt to } CO_2}{\text{C burnt to CO}} = \frac{14{\cdot}65}{0{\cdot}27}$$

$$\text{Hence C burnt to } CO_2 = \frac{14{\cdot}65}{14{\cdot}92} \times 0{\cdot}8441$$
$$= 0{\cdot}8288 \text{ lb. per lb. of dry fuel}$$

$$\text{C burnt to CO} = \frac{0{\cdot}27}{14{\cdot}92} \times 0{\cdot}8441$$
$$= 0{\cdot}0153 \text{ lb. per lb. of dry fuel}$$

Heat loss due to presence of CO.

For every pound of C burnt to CO instead of CO_2
heat loss $= 8{,}080 - 2{,}420 = 5{,}660$ C.H.U.

Moisture in fuel as fired.

0·0343 lb. H_2O is associated with $(1 - 0{\cdot}0343)$
$= 0{\cdot}9657$ lb. per lb. of dry fuel

Hence

$$\text{Moisture in fuel} = \frac{0{\cdot}0343}{0{\cdot}9657} = 0{\cdot}0355 \text{ lb. per lb. of dry fuel}$$

Total heat of this at atmospheric pressure and 300° C.
= 735 C.H.U. per lb.

Total heat of steam raised.

Total heat at 235 lb. per sq. in., and 320° C. = 736 C.H.U. per lb.

Weight of air used.

$$\text{Weight of air used} = \frac{80{\cdot}72}{33(14{\cdot}65 + 0{\cdot}27)} \times 84{\cdot}41$$
$$= 13{\cdot}84 \text{ lb. per lb. of dry fuel}$$

Oxygen Required, and Products of Combustion

1 lb. Dry Fuel lb.		O_2 Required lb.	Products of Combustion (lb.)			
			CO_2 and CO	N_2	H_2O	SO_2
C unburnt	0·0043					
C to CO_2	0·8288	0·8288 × $\frac{8}{3}$ = 2·2101	0·8288 × $\frac{11}{3}$ = 3·0389			
C to CO	0·0153	0·0153 × $\frac{4}{3}$ = 0·0204	0·0153 × $\frac{7}{3}$ = 0·0357			
H_2	0·0522	0·0522 × 8 = 0·4176			0·0522 × 9 = 0·4698	
S	0·0139	0·0139 × 1 = 0·0139				0·0139 × 2 = 0·0278
		2·6620				
O_2	0·0059	less 0·0059				
		2·6561				
N_2	0·0129			0·0129 + N_2 From air		
Ash	0·0667					
	1·0000					

Minimum weight of air required to produce given combustion.

Oxygen required = 2·6561 lb. per lb. of dry fuel

$$\text{Minimum air required} = \frac{2{\cdot}6561 \times 100}{23}$$
$$= 11{\cdot}5483 \text{ lb. per lb. of dry fuel}$$

$$\text{Nitrogen in this air} = 11{\cdot}5483 - 2{\cdot}6561$$
$$= 8{\cdot}8922 \text{ lb. per lb. of dry fuel}$$

Weight of excess air.

Excess air = 13·84 – 11·55 = 2·29 lb. per lb. of dry fuel

Weight of dry products with minimum air.

CO_2 = 3·0389
CO = 0·0357
N_2 from fuel = 0·0129
N_2 from air = 8·8922

Total 11·9797 lb. per lb. of dry fuel

Weight of condensable products with minimum air.

SO_2 = 0·0278
H_2O = 0·4698

Total 0·4976 lb. per lb. of dry fuel (Assume specific heat = 0·48)

Check on total weight of products.

C to CO_2 =	0·8288		
C to CO	0·0153		
H_2	0·0522		
S	0·0139		
O_2	0·0059		
N_2	0·0129	Dry products =	11·9797
Air	11·5483	Wet products	0·4976
Total	12·4773	Total	12·4773

Heat Balance

		C.H.U. per lb. of dry fuel	Per Cent
Heat supplied	=	8,350	
Heat used to form steam = 10·4(736 − 86)	=	6,760	81·00
Heat carried away by dry products = 11·9797(300 − 28)0·265	=	863	10·30
Heat carried away by condensable products = 0·4976(300 − 28)0·48	=	65	0·78
Heat carried away by excess air = 2·29(300 − 28)0·24	=	149	1·78
Heat lost due to presence of CO = 0·0153 × 5,660	=	87	1·04
Heat lost due to unburnt carbon in ash = 0·0043 × 8,080	=	35	0·42
Heat carried away by moisture in air = 68 − 35	=	33	0·38
Heat used in evaporating and superheating moisture in fuel = 0·0355(735 − 28)	=	25	0·30
Heat lost by radiation and unaccounted for	=	333	4·00

CHAPTER XV

REFRIGERATION

Refrigeration is the process of taking heat from a body, or from the contents of a cold insulated chamber, and keeping that body at a low temperature by continuously extracting heat as fast as it leaks in from the surroundings. Mechanical refrigeration requires the expenditure of energy, commonly as work spent in driving the machine which acts like a heat pump, on a reversed heat engine cycle, and transfers the heat to a body at a higher temperature.

The *coefficient of performance* of a refrigerating machine is measured by the ratio

$$\frac{\text{Heat extracted}}{\text{Work spent}}$$

both quantities being reckoned in the same units of heat or work.

It has been shown (p. 115) that the ideal heat engine works on a reversible cycle, and may take in heat Q_2 from a cold body at the low temperature T_2 (abs.), and reject heat Q_1 at the higher temperature T_1 (abs.). Let W be the thermal equivalent of the work required to drive this refrigerating machine. Since the machine works on a reversible cycle, by the conservation of energy,

$$Q_1 = Q_2 + W, \text{ or } W = Q_1 - Q_2$$

Also (see p. 122)

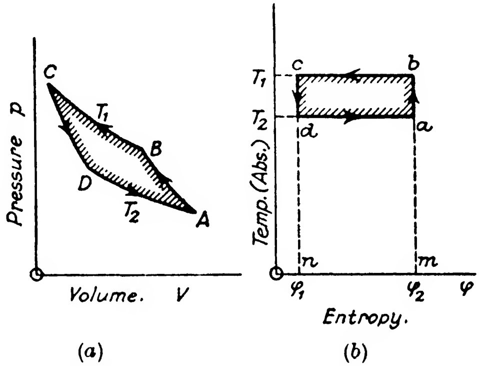
$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}, \text{ and } \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Hence the coefficient of performance of this ideal reversible refrigerating machine is

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}, \text{ and } Q_2 = W \times \frac{T_2}{T_1 - T_2}$$

By this relation it is clear that, for a given quantity Q_2 of heat to be taken from a cold body, the expenditure of work, W, is proportional to the difference of temperature through which the heat is raised, or the machine works. In other words, the smaller the range of temperature the less work will be required to cool a body by extracting a given amount of heat; or, *for a given amount of work W spent, the heat Q_2 extracted will be greater the smaller the range of temperature, $T_1 - T_2$.*

Consider a machine working on the ideal Carnot cycle reversed. Starting at the point D or d (Fig. 202), the cycle is in a counter-clockwise direction *abcd*. The engine takes in a quantity of heat Q_2, represented by the area *damn*, during isothermal expansion *da*, at the lower temperature T_2; that is, the gain of entropy from the cold body is $Q_2/T_2 = \phi_2 - \phi_1$, represented by the line *da* or *nm*. During adiabatic compression *ab* there is no transfer of heat, and the temperature of the working fluid is raised from T_2 to T_1. Then, during isothermal compression *bc* at the higher temperature T_1, a larger quantity of heat, Q_1, represented by the area *bcnm*, is rejected to the hot body, so that the entropy given to the warmer body is $\frac{Q_1}{T_1} = \phi_2 - \phi_1$, represented by the line *bc*.



(*a*) (*b*)

FIG. 202. CARNOT CYCLE REVERSED

During adiabatic expansion *cd*, the fluid does work and drops in temperature from T_1 to T_2.

The work required to drive this refrigerating machine is represented by the area *ABCD* or *abcd*, equivalent to $Q_1 - Q_2$, which is converted into heat in the machine.

In fact, the reversed perfect engine acts like a heat pump taking heat Q_2 from a cold body at T_2, and delivering more heat Q_1 at a higher temperature T_1, whilst the entropy Q_2/T_2 taken from the cold body is equal in amount to the entropy Q_1/T_1 given to the warmer body.

The *coefficient of performance* is

$$\frac{Q_2}{Q_1 - Q_2} = \frac{\text{Area } damn}{\text{Area } bcnm - \text{area } damn}$$

or

$$\frac{Q_2}{W} = \frac{T_2(\phi_2 - \phi_1)}{T_1(\phi_2 - \phi_1) - T_2(\phi_2 - \phi_1)} = \frac{T_2}{T_1 - T_2}$$

No refrigerating machine, working between the same limits of temperature, can have a higher coefficient of performance than the reversible one. If a refrigerating machine *B* could be conceived to have a greater coefficient of performance than the reversible machine A, then the ratio of Q_2/W would be greater in B than in A. Let the reversible machine A, acting as a heat engine, be coupled to and drive the more than perfect refrigerating machine B that would extract more heat from the cold body than A gives it, while B driven by A would also reject more heat to the hot body than A takes from it. Thus heat would be continuously taken from the cold body to the hotter body by means of this self-acting machine, which would be contrary to the second law of thermodynamics; or the axiom that it is impossible for a self-acting machine, unaided by any external agency, to convey heat from a cold body to one at a higher temperature.

Hence all ideal reversible refrigerating machines, working between the same limits of temperature, have the same coefficient of performance.

No refrigerating machine used in practice works on this reversed Carnot cycle.

Highest Efficiency for Refrigeration. The ideal greatest ratio of heat Q_2, extracted from a cold body at T_2, to the heat Q supplied at the high temperature T, for a given temperature T_1 of the condensing water to which the heat is rejected, is obtained by the combination of a perfect reversible heat engine driving a perfect reversible heat pump.

The perfect heat engine takes in heat Q at T, and rejects heat at T_1, the temperature of the condenser, hence $W = \frac{Q}{T}(T - T_1)$.

The perfect heat pump extracts heat Q_2 from the cold body at the low temperature T_2, also delivers heat at T_1, and, since it is reversible, $W = \frac{Q_2}{T_2}(T_1 - T_2)$, where W is the heat equivalent of the work given by the heat engine, and also that used in driving the heat pump.

$$\text{Hence } \frac{Q}{T}(T - T_1) = \frac{Q_2}{T_2}(T_1 - T_2)$$

$$\text{from which} \qquad \frac{Q_2}{Q} = \frac{T_2(T - T_1)}{T(T_1 - T_2)}$$

No other combination could extract more heat than Q_2 at the temperature T_2, by means of the same amount of high-temperature heat Q. If any other machine could extract more heat than Q_2, by the expenditure of the same amount of heat Q at the high temperature, then with the above combination working reversed, and the other refrigerating machine working direct, the result would be a transfer of heat from the cold body at T_2 to a body at a higher temperature T_1 without the expenditure of energy, by a self-acting agency, which is impossible by the second law of thermodynamics.

Cold Air Machine. Joule's air engine reversed was developed as a refrigerating machine by Bell, Coleman, and Lightfoot, and was one of the earliest used for mechanical refrigeration on board ship and cold storage.

The diagram of the essential parts of the plant, Fig. 203, and the ideal indicator diagram, Fig. 204, may serve to explain the action of this air machine.* Air from the cold chamber *R* at 18° F. or 32° F. is compressed in a cylinder *A*, with water-jacket, to about 65 lb. per sq. in. (abs.), and 270° F. or 290° F. The air is passed at this constant pressure through tubes in a cooler *C*, and heat is extracted by circulating water. This cold air at high pressure is further cooled by expansion in cylinder *B*, and delivered at about - 80° F. and initial pressure to the cold chamber. During expansion the air does work which helps to drive the machine.

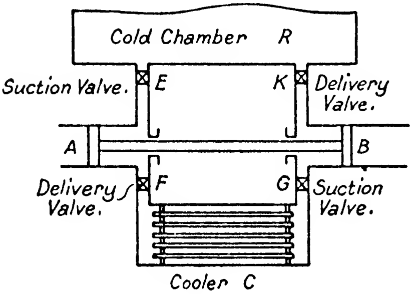


FIG. 203. COLD AIR MACHINE (Diagram of Essential Parts)

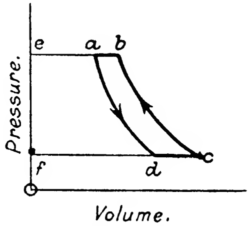


FIG. 204. CYCLE OF COLD AIR MACHINE

The ideal cycle is that of Joule's air engine reversed (p. 134), as in Fig. 204. In the compression cylinder, *A*, Fig. 203, during the suction stroke *fc*, air, from the cold chamber *R*, is drawn into the cylinder *A*, and compressed adiabatically *cb*. When the valve *F* opens, the pump delivers the air into the cooler *C*, completing its stroke from *b* to *e*, and the temperature falls, whilst the air gives up heat at constant pressure to the circulating water. Then an equal quantity of air, from *C*, is taken into the expansion cylinder *B* at temperature T_a and expands adiabatically, doing work on the piston shown by area under *ea* and *ad*. During the return stroke *df*, in *B*, the chilled air is discharged into the cold chamber at the initial pressure.

During the cycle, the work done *on* the air in the compressor cylinder *A* is represented by the area *fcbe*, and the work done *by* the air in the expansion cylinder *B* is given by the area *eadf*, hence the net work in driving the machine is given by the area *abcd*.

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* For the construction and performance of these machines, see Lightfoot, *Proc. Inst. Mech. Eng.* (1881), p. 105, and 1886, p. 201; also Coleman, *Min. Proc. Inst. C.E.*, Vol. LXVIII (1882), p. 146.

The net amount of heat extracted from the cold chamber per pound of air is

$$Q_2 = C_p(T_c - T_d)$$

and the heat rejected to the cooling water per pound of air is

$$Q_1 = C_p(T_b - T_a)$$

where T_a, T_b, T_c, and T_d denote the absolute temperature of the working air at the points *a*, *b*, *c*, *d* of the diagram, Fig. 204.

The net amount of work spent in driving the machine is $Q_1 - Q_2$; and the coefficient of performance is $\frac{Q_2}{Q_1 - Q_2}$.

Also, since the ratio of expansion in *B* is the same as the ratio of compression in *A*, and both are adiabatic,

$$\frac{T_a}{T_d} = \frac{T_b}{T_c}, = \frac{T_b - T_a}{T_c - T_d}$$

$$\therefore \text{ coefficient of performance} = \frac{Q_2}{Q_1 - Q_2}$$

$$= \frac{C_p(T_c - T_d)}{C_p(T_b - T_a) - C_p(T_c - T_d)} = \frac{T_c - T_d}{T_b - T_a - (T_c - T_d)}$$

$$= \frac{1}{\frac{T_b}{T_c} - 1} = \frac{T_c}{T_b - T_c} = \frac{T_d}{T_a - T_d}$$

This coefficient of performance is less than that of a reversed Carnot cycle between the limits T_a and T_c, namely, $\frac{T_c}{T_a - T_c}$, since T_b is greater than T_a. All the heat is not taken in at T_c and rejected at T_a; besides, the transfer of heat from the compressed air at T_b to the circulating water at T_a is not reversible.

In practice, the compression of the air in *A* is not adiabatic, because the water-jacket on the cylinder reduces the temperature, so that the slope of the compression curve is not so steep as the adiabatic, and the work spent in compression is reduced.

The actual coefficient of performance obtained with this machine varies from 0·5 to 0·7, and is very much lower than that of a vapour compression machine owing to (1) the necessity of working with a wide range of temperature, since air is a poor conductor and absorber of heat; (2) air has a small capacity for heat and all machines using air as the working substance must be bulky, and consequently the loss of heat by friction is large. Not only is power lost in friction, but the working air takes up most of the heat so generated, and has less capacity left for taking up heat in useful refrigeration.

Example 1. In a Bell-Coleman refrigerating plant air is drawn into the cylinder of the compressor at atmospheric pressure of 15 lb. per sq. in. and temperature − 5° C., and it is compressed adiabatically to 75 lb. per sq. in., at which pressure it is cooled to 15° C. It is then expanded in an expansion cylinder to atmospheric pressure and discharged into the refrigerating chamber. If the law for expansion is $pv^{1 \cdot 2}$ = a constant, find the work done on the air per pound, and the coefficient of performance of the refrigerating plant. Specific heat of air at constant pressure is 0·238. (*U.L.*, *B.Sc.* (*Eng.*), 1924.)

On the indicator diagram, Fig. 204 (p. 551), we have

$$T_c = -5^\circ + 273^\circ = 268^\circ \text{ C. (abs.)}$$

$$\frac{T_b}{T_c} = \left(\frac{75}{15}\right)^{\frac{1 \cdot 4 - 1}{1 \cdot 4}} = 5^{\frac{2}{7}} \quad \therefore T_b = 268 \times 5^{\frac{2}{7}} = 424 \cdot 45^\circ \text{C. (abs.)}$$

and $$T_a = 15^\circ + 273^\circ = 288^\circ \text{ C. (abs.)}$$

Also $$\frac{T_a}{T_d} = \left(\frac{75}{15}\right)^{\frac{1 \cdot 2 - 1}{1 \cdot 2}} = 5^{\frac{1}{6}} = 1 \cdot 3077$$

$$\therefore \quad T_d = \frac{T_a}{1 \cdot 3077} = \frac{288}{1 \cdot 3077} = 220 \cdot 24^\circ \text{ C. (abs.)}$$

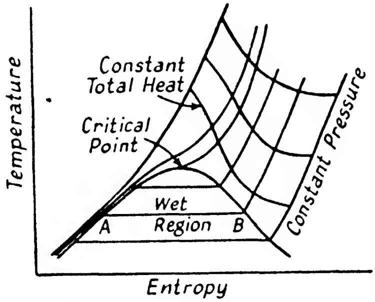

FIG. 205. T-ϕ CHART FOR AIR

Now the heat rejected per pound of air at the higher pressure

$$Q_1 = C_p(T_b - T_a) = 0 \cdot 238(424 \cdot 45 - 288) = 32 \cdot 475 \text{ C.H.U.}$$

and the heat extracted per pound of air from the cold chamber

$$Q_2 = 0 \cdot 238(T_c - T_d) = 0 \cdot 238(268 - 220 \cdot 24) = 11 \cdot 367 \text{ C.H.U.}$$

Heat taken in during expansion *ad*

$$Q_3 = \frac{1 \cdot 4 - 1 \cdot 2}{1 \cdot 4 - 1} \times \frac{0 \cdot 068}{0 \cdot 2} \times (288 - 220) = 11 \cdot 52 \text{ C.H.U.}$$

$$\therefore \text{ Work done} = 32 \cdot 47 - (11 \cdot 37 + 11 \cdot 52) = 9 \cdot 58 \text{ C.H.U.}$$

$$\therefore \text{ Coefficient of performance} = \frac{11 \cdot 37}{9 \cdot 58} = 1 \cdot 18$$

Temperature-Entropy Chart for the Joule Cycle. It is instructive to follow the reversed Joule Cycle on the T-ϕ chart for air. This chart in broad outline is shown in Fig. 205*. Comparing this with

---

* A complete chart is given in *Refrigeration Theory and Applications* by H. G. Venemann (Nickerson and Collins, Chicago).

the corresponding chart for steam (Fig. 254), three main differences will be noted—(*a*) Owing to the low critical temperature for air (— 221° F. at 40 atmospheres), when working under normal conditions, we should be using the right-hand top corner of the chart. (*b*) The evaporation line AB in the wet region is not horizontal, and coinciding with the constant temperature line. This is due to the fact that during evaporation of liquid air, nitrogen boils at a higher temperature than oxygen. (*c*) The chart for air includes total heat lines, which are not generally shown on that for steam.

Example 2. Ten pounds of air at 80° F. and 5 atmospheres pressure are expanded per minute in an expansion cylinder to 1 atmosphere. At this pressure it passes into a refrigerator where it absorbs heat and rises in temperature to — 20° F. at constant pressure. It then passes to a compressor, where the pressure is raised to 5 atmospheres, and hence to a cooler where the temperature is lowered to 80° F. at constant pressure.

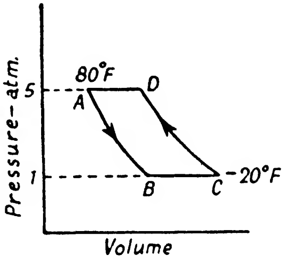

FIG. 206. JOULE CYCLE
p-v DIAGRAM

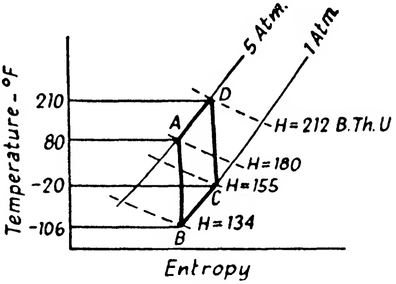

FIG. 207. JOULE CYCLE
T-ϕ DIAGRAM

Find (*a*) the horse-power required, (*b*) the refrigerating effect, (*c*) the heat rejected to the cooler and (*d*) the coefficient of performance.

This problem can be solved in terms of total heat. Referring to p. 54 it was shown that the transfer of heat at constant pressure is measured by the change in total heat. Hence referring to Fig. 206

Heat rejected from D to $A = H_D - H_A$ per lb.

Heat taken in from B to $C = H_C - H_B$ per lb.

On p. 55 it was seen that the work done in a frictionless adiabatic expansion or compression is measured by the adiabatic heat drop. Hence if the expansion and compression in this problem are assumed to be isentropic, then referring to Fig. 206

Work done on air by compressor $= H_D - H_C$ B.Th.U. per lb.

Work done by air in expansion cylinder

$= H_A - H_B$ B.Th.U. per lb.

∴ Net work done on air $= (H_D - H_C) - (H_A - H_B)$ B.Th.U. per lb.

and refrigerating effect $= (H_C - H_B)$ B.Th.U. per lb.

$$\text{Hence coefficient of performance} = \frac{H_C - H_B}{(H_D - H_C) - (H_A - H_B)}$$

Fig. 207 gives the appropriate part of the T-ϕ chart for air with figures extracted from the chart mentioned in the footnote on p. 553. Starting at point A, particulars of which are given in the question, a constant entropy line AB is drawn to cut the constant pressure line for one atmosphere at B. This gives H_B as 134 B.Th.U. per lb. The taking in of heat along BC in Fig. 206 is shown in Fig. 207 by passing up the constant pressure line for one atmosphere until the horizontal temperature line for — 20° F. is reached. This gives $H_C = 155$ B.Th.U. per lb. Similarly the isentropic compression line CD is drawn upwards from C to cut the constant pressure line for five atmospheres at D, giving $H_D = 212$ B.Th.U. per lb.

Hence

(i) Work in expansion $= M(H_A - H_B) = 10(180 - 134)$

$= 460$ B.Th.U. per minute

Work in compression $= M(H_D - H_C) = 10(212 - 155)$

$= 570$ B.Th.U. per minute

Net work $= 570 - 460$

$= 110$ B.Th.U. per min.

$$= \frac{110 \times 778}{33{,}000} = 2{\cdot}59 \text{ H.P.}$$

(ii) Refrigerating effect $= M(H_C - H_B) = 10(155 - 134)$

$= 210$ B.Th.U. per minute

(iii) Heat rejected to cooler $= M(H_D - H_A) = 10(212 - 180)$

$= 320$ B.Th.U. per minute

$$\text{(iv) Coefficient of performance} = \frac{\text{Ref. effect}}{\text{Net work}} = \frac{210}{110} = 1{\cdot}91$$

As a check we can calculate the coefficient of performance in terms of temperatures as shown on p. 552. Thus reading off the temperatures from Fig. 207.

$$\text{Coefficient of performance} = \frac{T_C}{T_D - T_C} = \frac{-20 + 460}{210 + 20} = 1{\cdot}91$$

The Vapour Compression Cycle. In Fig. 145 p. 411, *ABCE* in a clockwise direction represents the Rankine cycle as applied to a heat engine. If we proceed round the cycle in an anti-clockwise direction we have a refrigerating cycle as follows—

CB, adiabatic compression of slightly wet vapour to a higher pressure thus drying it.

BA, condensation of dry vapour to liquid at constant pressure and temperature.

AE, expansion of liquid from high pressure and temperature to low pressure and temperature.

EC, evaporation of liquid at constant pressure and temperature by absorption of heat from some external medium.

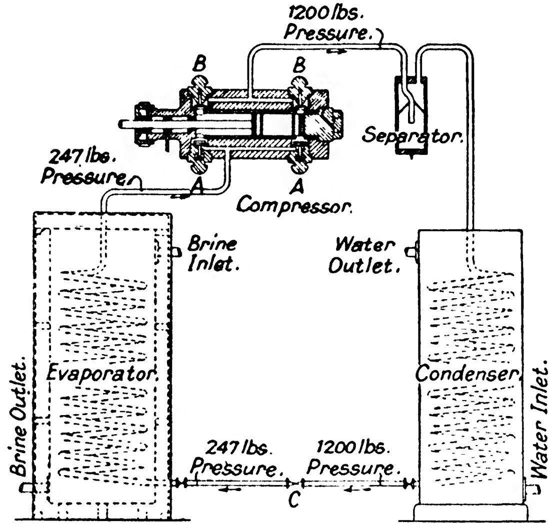

FIG. 208. CARBONIC ACID REFRIGERATING MACHINE (*Haslam*)

By using a suitable working substance (termed *refrigerant*) the temperature at the lower pressure can be a refrigerating temperature for the external medium from which heat is extracted.

The refrigerating effect = heat extracted between E and C

$$= H_C - H_E$$

The work done = Area $ABCE = H_B - H_C$

The coefficient of performance $= \dfrac{H_C - H_E}{H_B - H_C}$

This cycle is not used practically owing to the complication introduced by the necessity of expanding the liquid along the line *AE*.

In the practical vapour compression cycle, instead of using an expansion cylinder to lower the pressure from A to E (Fig. 145) this pressure drop is effected by passing the refrigerant through a throttle valve. It should be noted that this throttling is a non-reversible process.

The machine thus consists of four essential parts, shown in the diagram (Fig. 208).

1. A *compressor*, into which vapour is drawn through the suction valve A, from the evaporator at a low temperature T_2, compressed to the higher pressure and temperature T_1, and discharged through the delivery valve B, as shown by the indicator diagram, Fig. 209.

A *separator* collects oil, etc., in the vapour from the compressor.

2. A *condenser*, or cooler, consisting of coils of pipe in which the compressed vapour is cooled and condensed at T_1, and gives up its heat to circulating water.

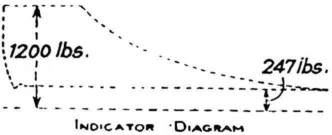

FIG. 209

3. An adjustable throttle valve C, called a *regulator* or *expansion valve*, through which the liquid expands from T_1 to T_2 at the lower pressure.

4. An *evaporator*, or *refrigerator*, consisting of pipes in which the liquid evaporates at the lower temperature and takes up heat from cold brine, which is circulated round the pipes and conveys heat to the working fluid from a cold chamber or from water to be frozen.

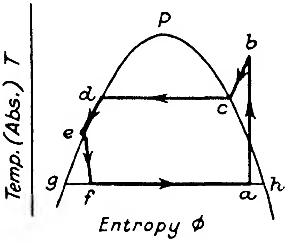

FIG. 210. CO_2 COMPRESSION CYCLE

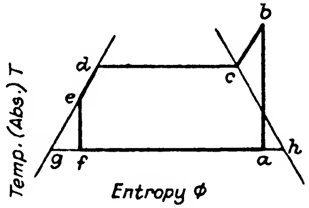

FIG. 211. AMMONIA COMPRESSION CYCLE

The cycle is shown on a T-ϕ diagram in Figs. 210 and 211. In Fig. 210 P is the critical point of carbon dioxide, and Pch the dry saturation line. Suppose adiabatic compression ab begins at a, where the dryness fraction is ga/gh. If the compression is completed at b the vapour is now superheated, the compression first drying and then superheating the vapour, to a temperature somewhat higher than the saturation temperature corresponding to the pressure.

It will be seen that the starting point a of the compression determines the final state of vapour at the end of compression given by the point b. Thus referring to Fig. 212, if the start were at *a'* the refrigerant would be finally in the state of dry saturated vapour given by the point *c*. This is termed "wet compression." If the start is at *a"* it is called "dry compression," the normal start of compression being between these two states.

After compression follows cooling at the constant high pressure. This takes place in three stages as shown in Figs. 210 and 211 namely—

bc, the superheated vapour at *b* is cooled to the temperature of dry saturated vapour at *c*, when condensation begins.

cd, the vapour is completely condensed at constant temperature

de, undercooling takes place, the liquid being cooled below the saturation temperature as far as possible, depending on the temperature of the cooling water.

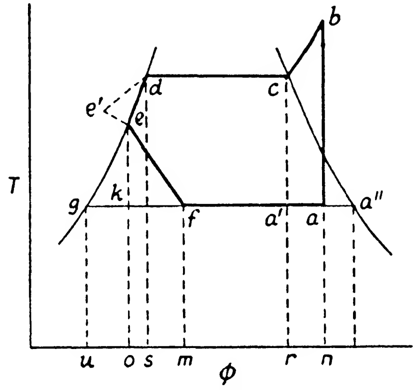

FIG. 212

It will be noticed in Fig. 211 that *de* the undercooling of the ammonia liquid is practically along the liquid boundary line, whereas with CO_2, the constant pressure cooling curve is quite distinct from the liquid line *dg*, due to the greater compressibility of CO_2.

The line *ef* shows unresisted expansion through the throttle valve, that is throttling at constant total heat (p. 53) to the lower constant pressure and temperature of the evaporator. The fraction converted into vapour by passing through the expansion valve is *gf*/*gh*.

During the phase *fa* we have evaporation of the liquid in the refrigerator coils, when heat is extracted from the brine or other substance to be cooled, this taking place at constant pressure and temperature. This heat which is absorbed by the refrigerant is called the refrigerating effect.

Heat Areas on the T-ϕ Chart. It is instructive to consider the various heat quantities involved in terms of areas on the *T*-ϕ

diagram. This is illustrated in Fig. 212 where the cycle *abcdefa* will be considered.

In this cycle it will be assumed that the refrigerant is sufficiently incompressible, to allow the constant pressure under cooling curve *de* to be taken as coincident with the liquid line *dg*. As an area under a T-ϕ curve represents heat, and remembering that heat transferred at constant pressure is measured by the change in total heat, we see that the area under a constant pressure T-ϕ curve between two points is a measure of the change in total heat between those two points. Thus in Fig. 212 as the total heat before and after throttling is the same then $H_e = H_f$ and therefore

$$\text{Area } geou = \text{Area } gfmu$$

$$\text{i.e.} \qquad \text{Area } gek = \text{Area } kfmo$$

$$\text{The refrigerating effect} = \text{Area } fanm$$

The heat rejected to cooling water

$$= \text{Areas}$$

$$\underset{\left(\begin{array}{c}\text{From superheated}\\ \text{vapour}\end{array}\right)}{cbnr} + \underset{\left(\begin{array}{c}\text{From condensation}\\ \text{of vapour}\end{array}\right)}{dcrs} + \underset{\left(\begin{array}{c}\text{From undercooling}\\ \text{of liquid}\end{array}\right)}{edso}$$

$$= \text{Area } bnoedcb$$

The work done

$$\begin{aligned}
&= \text{Heat rejected to cooling water} - \text{Refrigerating effect}\\
&= \text{Area } bnoedcb \qquad - \text{Area } fanm\\
&= \qquad bnoedcb \qquad - kano + kfmo\\
&= \qquad bnoedcb \qquad - kano + gek\\
&= \text{Area } abcdga
\end{aligned}$$

Thus the work done in the compression is represented by the area of the true Rankine cycle, and not the area of the cycle modified by the use of a throttle valve.

The loss of refrigerating effect due to the substitution of the throttle valve, for an expansion cylinder is represented by the area *gfmu* = area *geou*.

The Vapour Compression Cycle in terms of Latent and Specific Heats and Temperatures. Assume constant specific heats of liquid and vapour and consider the performance of the vapour compression refrigerating machine with expansion valve, in the three cases in practice represented by the temperature-entropy diagram, Fig. 213, when the vapour, at the end of adiabatic compression is (1) just dry and saturated at the state point *b*; (2) at *h* the vapour is still wet;

and (3) the dry saturated vapour after complete evaporation at g is superheated by compression gv to the temperature T_3.

1. Assume neither gain nor loss of heat in passing through the expansion valve. The heat in 1 lb. of the working fluid will be the same before and after throttling. The heat before expansion is represented by the area *cepn*, and that after expansion by the area *efrp*. Hence these areas are assumed to be equal, and the effective refrigeration per pound during evaporation *fa* is the heat extracted, as represented by the area *famr*. Take s the specific heat of the liquid.

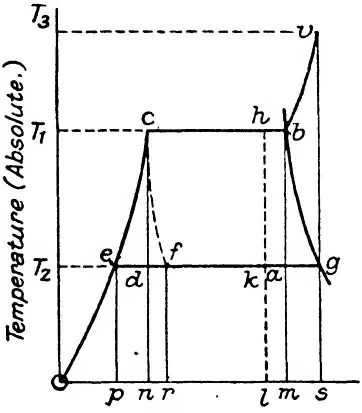

FIG. 213. VAPOUR COMPRESSION CYCLE

Now the area *cepn* = area *efrp* $= s(T_1 - T_2)$, and

$$ef = s \,.\left(\frac{T_1 - T_2}{T_2}\right)$$

Also

$$ea = ed + da = s \,.\log_e \frac{T_1}{T_2} + \frac{L_1}{T_1}$$

$$\therefore fa = ea - ef = s \,.\log_e \frac{T_1}{T_2} + \frac{L_1}{T_1} - s \,.\left(\frac{T_1 - T_2}{T_2}\right)$$

and heat extracted

$$= T_2 \times fa = s \,.\, T_2 \,.\log_e \frac{T_1}{T_2} + T_2 \times \frac{L_1}{T_1} - s(T_1 - T_2) \quad (1)$$

Work done = area *abcea* = area *pecbm* – area *eamp*

= area *pecn* + area *cbmn* – area *ednp* – area *damn*

$$= s(T_1 - T_2) + T_1 \times \frac{L_1}{T_1} - T_2 \,.\, s \,.\log_e \frac{T_1}{T_2} - T_2 \times \frac{L_1}{T_1}$$

$$= (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) - T_2 \,.\, s \,.\log_e \frac{T_1}{T_2} \quad . \quad . \quad (2)$$

$$\text{Coefficient of performance} = \frac{\text{Heat extracted}}{\text{Work done}}$$

$$= \frac{T_2\left(s \,.\log \frac{T_1}{T_2} + \frac{L_1}{T_1}\right) - s(T_1 - T_2)}{(T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) - T_2 \,.\, s \,.\log_e \frac{T_1}{T_2}} \quad . \quad . \quad . \quad (3)$$

2. When the vapour is wet at the end of compression. Let the compression begin at the state point k when the dryness fraction is ek/eg; and end at h with dryness $ch/cb = x_1$, and the latent heat $= x_1L_1$ per lb. of stuff.

We have, heat extracted

$$= \text{area } fklr$$

$$= s \cdot T_2 \cdot \log_\varepsilon \frac{T_1}{T_2} + T_2 \cdot \frac{x_1L_1}{T_1} - s(T_1 - T_2) \quad . \quad . \quad (4)$$

Also, work done

$$= echke = (T_1 - T_2)\left(s + \frac{x_1L_1}{T_1}\right) - T_2 \cdot s \cdot \log_\varepsilon \frac{T_1}{T_2} \quad . \quad (5)$$

$$\therefore \text{ coefficient of performance} = \frac{\text{Expression (4)}}{\text{Expression (5)}}$$

3. When the compressor draws in dry saturated vapour from the refrigerator. The " dry " compression begins at the state point g on the saturation line, Fig. 213, and ends at v, temperature T_3, on the superheat curve of constant pressure drawn from b at saturation temperature T_1.

Heat extracted = area $fgsr$; here $eg = \frac{L_2}{T_2}$, and $ef = s\,\frac{(T_1 - T_2)}{T_2}$.

Now $\quad fg = eg - ef = \frac{L_2}{T_2} - s\,\frac{(T_1 - T_2)}{T_2}$

and heat extracted

$$= fg \times T_2 = L_2 - s(T_1 - T_2) \quad . \quad . \quad . \quad . \quad (6)$$

Work done = area $ecbvge$ = area ecd + area $dcbvg$

$$= s(T_1 - T_2) - T_2 \cdot s \cdot \log_\varepsilon \frac{T_1}{T_2} + \frac{L_1}{T_1}(T_1 - T_2)$$

$$+ C_p(T_3 - T_1) - T_2 \cdot C_p \cdot \log_\varepsilon \frac{T_3}{T_1}$$

$$= (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) + C_p(T_3 - T_1)$$

$$- T_2\left(s \cdot \log_\varepsilon \frac{T_1}{T_2} + C_p \cdot \log_\varepsilon \frac{T_3}{T_1}\right) \quad . \quad (7)$$

$$\text{Coefficient of performance} = \frac{\text{Expression (6)}}{\text{Expression (7)}}$$

To calculate T_3, in superheating from b to v, at constant pressure, the gain of entropy,

$$ag = C_p \cdot \log_\varepsilon \frac{T_3}{T_1}$$

But $$ag = eg - ea = \frac{L_2}{T_2} - s \, . \log_\varepsilon \frac{T_1}{T_2} - \frac{L_1}{T_1}$$

Equate these values of ag, we have

$$C_p \, . \log_\varepsilon \frac{T_3}{T_1} = \frac{L_2}{T_2} - s \, . \log_\varepsilon \frac{T_1}{T_2} - \frac{L_1}{T_1} \quad . \quad . \quad . \quad . \qquad (8)$$

from which T_3 may be calculated.

Example 3. In an ammonia refrigerating machine the temperature in the refrigerator is 15° F., and after compression 90° F. In the cooler the vapour is condensed at 90° F., and then passes through an expansion valve. Calculate the coefficient of performance when the vapour at the end of adiabatic compression is (*a*) just dry and saturated; (*b*) 85 per cent dry; (*c*) just dry and saturated when compression begins. Take the specific heat of liquid ammonia as 1·1, and of the vapour $C_p = 0{\cdot}508$, and the latent heat of evaporation as $566 - 0{\cdot}8t°$ F.

Given $L_{90°} = 566 - 0{\cdot}8 \times 90 = 494$ B.Th.U. per lb.

$T_1 = 90° + 460° = 550°$ F. (abs.), and $T_2 = 475°$ F. (abs.)

(*a*) By equation (1) (p. 560), heat extracted

$$= s \, . T_2 \, . \log_\varepsilon \frac{T_1}{T_2} + T_2 \times \frac{L_1}{T_1} - s(T_1 - T_2)$$

$$= 1{\cdot}1 \times 475 \times 2{\cdot}303 \log_{10} \frac{550}{475} + \frac{475 \times 494}{550} - 1{\cdot}1(90 - 15)$$

$$= 76{\cdot}65 + 426{\cdot}6 - 82{\cdot}5 = 420{\cdot}75 \text{ B.Th.U. per lb.}$$

By equation (2), work done $= (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) - T_2 \, . \, s \log_\varepsilon \frac{T_1}{T_2}$

$$= 75(1{\cdot}1 + 0{\cdot}898) - 1{\cdot}1 \times 475 \times 2{\cdot}303 \log_{10} \frac{550}{475}$$

$$= 75 \times 1{\cdot}998 - 76{\cdot}65 = 73{\cdot}2 \text{ B.Th.U. per lb.}$$

$\therefore$ coefficient of performance $= \frac{420{\cdot}75}{73{\cdot}2} = 5{\cdot}74$ *Answer.*

(*b*) By equation (4), heat extracted

$$= s \, . T_2 \, . \log_\varepsilon \frac{T_1}{T_2} + T_2 \, . \frac{x_1 L_1}{T_1} - s(T_1 - T_2)$$

$$= 76{\cdot}65 + 0{\cdot}85 \times 426{\cdot}6 - 82{\cdot}5 = 356{\cdot}76 \text{ B.Th.U. per lb.}$$

And by equation (5),

Work done $= (T_1 - T_2)\left(s + \frac{x_1 L_1}{T_1}\right) - T_2 \, . \, s \, . \log_\varepsilon \frac{T_1}{T_2}$

$= 75(1{\cdot}1 + 0{\cdot}85 \times 0{\cdot}89818) - 76{\cdot}65 = 63{\cdot}1$ B.Th.U. per lb.

$$\therefore \text{ coefficient of performance} = \frac{356\cdot76}{63\cdot1} = 5\cdot65 \qquad \textit{Answer.}$$

(c) Again, when the dry saturated vapour at g is compressed adiabatically to the state point v, and superheated to T_3, to find this temperature, we have $L_{15^\circ} = 566 - 0\cdot8 \times 15 = 554$ B.Th.U per lb.

and, by equation (8), $C_p \,.\, \log_\varepsilon \dfrac{T_3}{T_1} = \dfrac{L_2}{T_2} - s \,.\, \log_\varepsilon \dfrac{T_1}{T_2} - \dfrac{L_1}{T_1}$

that is, $0\cdot508 \times 2\cdot303 \log_{10} \dfrac{T_3}{550} = \dfrac{554}{475} - 1\cdot1 \times 2\cdot303 \log_{10} \dfrac{550}{475} - \dfrac{494}{550}.$

$$1\cdot17 \log_{10} \frac{T_3}{550} = 1\cdot1663 - 0\cdot1613 - 0\cdot8982 = 0\cdot1069$$

and $\qquad \log_{10} T_3 - \log_{10} 550 = \dfrac{0\cdot1069}{1\cdot17} = 0\cdot09137$

from which $\qquad T_3 = 678\cdot8^\circ$ F (abs.)

Then, by equation (6), heat extracted

$$= L_2 - s(T_1 - T_2)$$
$$= 554 - 1\cdot1(90 - 15) = 471\cdot5 \text{ B.Th.U. per lb}$$

and, by equation (7), work done

$$= (T_1 - T_2)\left(s + \frac{L_1}{T_1}\right) + C_p(T_3 - T_1) - T_2\left(s \,.\, \log_\varepsilon \frac{T_1}{T_2} + C_p \,.\, \log_\varepsilon \frac{T_3}{T_1}\right)$$

$$= 75\left(1\cdot1 + \frac{494}{550}\right) + 0\cdot508 \times 128\cdot5 - 475\ (\ .\ \ .\ \ .\ \ )$$

$$\left(1\cdot1 \times 2\cdot303 \log_{10} \frac{550}{475} + 0\cdot508 \times 2\cdot303 \log_{10} \frac{678\cdot8}{550}\right)$$

$$= 75 \times 1\cdot99812 + 0\cdot508 \times 128\cdot5 - 475(0\cdot1613 + 0\cdot1069)$$

$$= 149\cdot859 + 65\cdot278 - 127\cdot39 = 87\cdot74 \text{ B.Th.U. per lb.}$$

$$\therefore \text{ coefficient of performance} = \frac{471\cdot5}{87\cdot74} = 5\cdot37 \qquad \textit{Answer.}$$

Example 4. The following are approximate expressions for the entropy of ammonia liquid and dry saturated vapour: liquid, $0\cdot00184\,(t - 32)$; vapour, $1\cdot158 - 0\cdot00192\,(t - 32)$, t being the temperature on the Fahrenheit scale. Obtain corresponding expressions of the form $a + bt_c$, t_c being the temperature on the Centigrade scale. Draw the θ-ϕ chart between temperatures 14° F. and 77° F. (− 10° C. and 25° C.). Find the coefficient of performance of a refrigerator working on a reversed Rankine cycle between these limits, the vapour being 5 per cent wet at the end of compression. If the actual performance is 0·6 of the amount in the above ideal case, calculate the pounds of ice produced per horse-power from water at the freezing point. Latent heat of ice, 144 B.Th.U. (80 C.H.U.).

(*U.L.*, *B.Sc.* (*Eng.*).)

$$\text{At } 77^\circ \text{ F. or } 25^\circ \text{ C.}\left\{\begin{array}{l}\text{liquid } \phi = 0{\cdot}00184(77-32) = 0{\cdot}0828\\ \text{vapour } \phi = 1{\cdot}158 - 0{\cdot}00192 \times 45 = 1{\cdot}0716\end{array}\right.$$

$$\text{At } 14^\circ \text{ F. or } -10^\circ\text{C.}\left\{\begin{array}{l}\text{liquid } \phi = 0{\cdot}00184(14-32) = -0{\cdot}03312\\ \text{vapour } \phi = 1{\cdot}158 - 0{\cdot}00192 \times (-18)\\ \qquad\quad = 1{\cdot}158 + 0{\cdot}03456 = 1{\cdot}19256\end{array}\right.$$

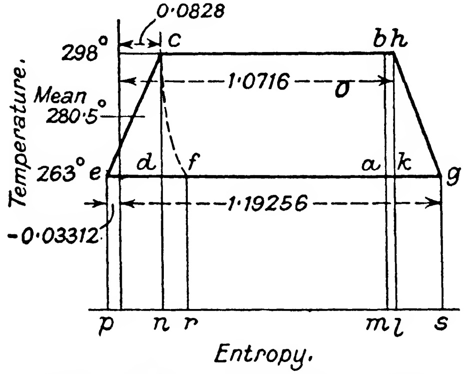

FIG. 214. AMMONIA COMPRESSION CYCLE

On C. scale, *liquid*

$$\left.\begin{array}{r}-0{\cdot}03312 = -10b\\ +0{\cdot}08280 = 25b\\ \hline 0{\cdot}04968 = 15b\end{array}\right\}\quad \therefore b = 0{\cdot}003312 \text{ and } \phi = 0{\cdot}003312t_c$$

Answer.

vapour

$$\left.\begin{array}{r}1{\cdot}07160 = a + 25b\\ 1{\cdot}19256 = a - 10b\\ \hline -0{\cdot}12096 = 35b\end{array}\right\}\quad \therefore b = -0{\cdot}003456$$

$$a = 1{\cdot}19256 - 0{\cdot}03456 = 1{\cdot}158$$

$$\therefore \phi = 1{\cdot}158 - 0{\cdot}003456t_c$$

Answer.

Plotting the above values on θ-ϕ chart, as shown in Fig. 214,

Dryness fraction $bc/ch = 95$ per cent

$$bc = 0{\cdot}95(1{\cdot}0716 - 0{\cdot}0828) = 0{\cdot}95 \times 0{\cdot}9888 = 0{\cdot}93936$$

$$ed = 0{\cdot}0828 + 0{\cdot}03312 = 0{\cdot}11592$$

As Temperature range = 35° C, and area $ecd = dfrn$,

$$df = \frac{17{\cdot}5 \times 0{\cdot}11592}{263} = 0{\cdot}00771$$

$$\therefore \quad fa = 0{\cdot}93936 - 0{\cdot}00771 = 0{\cdot}93165$$

Refrigeration per pound $= 0{\cdot}93165 \times 263 = 245{\cdot}024$ C.H.U.

$$\text{Work done} = 17{\cdot}5 \times 0{\cdot}11592 + 0{\cdot}93936 \times 35$$

$$= 2{\cdot}0286 + 32{\cdot}8776 = 34{\cdot}906 \text{ C.H.U. per pound}$$

$$\therefore \text{coefficient of performance} = \frac{245{\cdot}024}{34{\cdot}906} = 7{\cdot}02$$

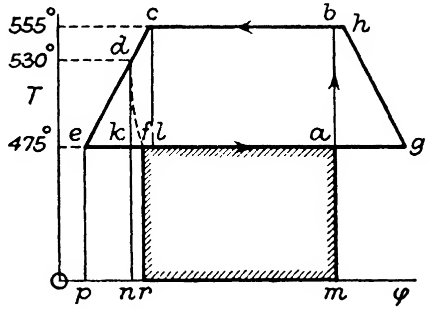

FIG. 215. AMMONIA COMPRESSION CYCLE

Actual coefficient $= 7{\cdot}02 \times 0{\cdot}6 = 4{\cdot}212$

Hence, ice produced per horse-power hour

$$= \frac{33{,}000 \times 60}{1{,}400} \times \frac{4{\cdot}212}{80} = 74{\cdot}46 \text{ lb.} \qquad \textit{Answer.}$$

Example 5. In an ammonia refrigerating machine, the temperature of evaporation of the liquid is 15° F. The vapour is compressed and eventually delivered to the cooler at a temperature of 95° F., the dryness at the end of compression being 0·95. In the cooler the vapour is condensed at 95° F., and then cooled to 70° F.; after which the liquid is passed through a throttling valve into the evaporating coils. Calculate for this machine the ratio of the heat abstracted to the work done if the specific heat of the liquid is 1·1, and the latent heat of the vapour is given by $566 - 0{\cdot}8t$, where t is the temperature of the vapour in degrees Fahrenheit. (*U.L., B.Sc. (Eng.)*.)

At 95° F., $L = 566 - 0{\cdot}8 \times 95 = 566 - 76 = 490$ B.Th.U.

Fig. 215 shows the cycle, *abcdfa*, on the temperature entropy diagram or chart.

$bc = 0{\cdot}95 \times \dfrac{490}{555} = 0{\cdot}84$, since the dryness fraction is $\dfrac{bc}{ch} = 0{\cdot}95$.

The total heat is constant during throttling, from d to f.

$\therefore$ area $pedn$ = area $pefr = ef \times 475 = 1{\cdot}1(70 - 15) = 60{\cdot}5$ B.Th.U.

$$\therefore \; ef = \frac{60{\cdot}5}{475} = 0{\cdot}1274$$

Also $el = 1{\cdot}1 \times 2{\cdot}303 \times \log_{10}\frac{555}{475} = 0{\cdot}1712$

$\therefore fa = ea - ef = bc + el - ef = 0{\cdot}84 + 0{\cdot}1712 - 0{\cdot}1274 = 0{\cdot}8838$

$\therefore$ Heat extracted per pound of NH_3, represented by the area *famr*, down to absolute zero

$$= 0{\cdot}884 \times 475 = 419{\cdot}8 \text{ B.Th.U. per lb.}$$

Work done, represented by the area *abcde* = area *cel* + area *abcl*

$$= (1{\cdot}1 \times 80 - 0{\cdot}1712 \times 475) + 0{\cdot}84(95 - 15)$$

$$= 88 - 81{\cdot}32 + 67{\cdot}2 = 73{\cdot}88 \text{ B.Th.U. per lb.}$$

$$\therefore \text{coefficient of performance} = \frac{\text{Heat abstracted}}{\text{Work done}} = \frac{419{\cdot}8}{73{\cdot}88}$$

$$= 5{\cdot}68$$ *Answer.*

Refrigerating Effect and Work Done in Terms of Total Heat. During a reversible operation at *constant pressure*, like evaporation or condensation, the heat taken in or given out by a fluid is the change of its total heat (see p. 54). In the evaporating coils, the amount of heat taken in from the cold body by 1 lb. of refrigerant is $H_a - H_f$ (Fig. 210); also the heat rejected to the condenser and cooler is $H_b - H_e$. During throttling in passing the expansion valve, the total heat is unchanged, and $H_f = H_e$. Therefore the heat rejected is $H_b - H_f$. The work done in driving the compressor, represented by the area of the p-v indicator diagram (Fig. 21, p. 55) or its thermal equivalent, $A\int_a^b v\,.\,dp = H_b - H_a$. Hence the quantities of heat dealt with during the whole refrigerating cycle are

$$\text{Work expended} = \text{Heat rejected} - \text{Heat taken up}$$

$$H_b - H_a = (H_b - H_f) - (H_a - H_f)$$

$$\text{The coefficient of performance} = \frac{H_a - H_f}{H_b - H_a}$$

Example 6 Solve Example 4 by calculation of Total Heats.

$ed = 0{\cdot}11592 = s \log_{\varepsilon}\frac{298}{263}$ where s = specific heat of liquid.

Hence $s = \frac{0{\cdot}11592}{2{\cdot}3 \times 0{\cdot}0542} = 0{\cdot}93$

Take total heat at $e = H_e$ as zero.

Then total heat at $c = H_c = s(T_c - T_e) = 0{\cdot}93(298 - 263)$

$$= 32{\cdot}55 \text{ C.H.U. per lb.}$$

Let x = Dryness fraction

L = Latent heat

Then for point b

$$cb = 0{\cdot}95(1{\cdot}0716 - 0{\cdot}0828) = \frac{x_b L_b}{298}$$

and $x_b L_b = 280$ C.H.U. per lb.

Hence total heat at b

$$= H_b = H_c + x_b L_b = 32{\cdot}55 + 280$$
$$= 312{\cdot}55 \text{ C.H.U. per lb.}$$

Also for point a

$$ea = \frac{x_a L_a}{263} = 0{\cdot}9888 \times 0{\cdot}95 - 0{\cdot}11592$$

and $x_a L_a = 277{\cdot}54$ C.H.U. per lb.

Hence total heat at $a = H_a = 277{\cdot}54$ C.H.U. per lb.

Then refrigerating effect

$$= H_a - H_f = H_a - H_c = 277{\cdot}54 - 32{\cdot}55$$
$$= 245 \text{ C.H.U. per lb.}$$

Work done $= H_b - H_a = 312{\cdot}55 - 277{\cdot}54 = 35$ C.H.U. per lb.

and Coefficient of performance $= \dfrac{245}{35} = 7$

Standardization. The coefficient of Performance as given in the last paragraph was chosen as the performance of the standard refrigerating machine working on the ideal cycle by a Refrigeration Research Committee under the chairmanship of Sir Alfred Ewing.* A sub-committee appointed to report on the accuracy of the charts issued by the former committee issued a report in 1940† which contains a considerable amount of more up-to-date information. The general recommendations of the former committee may be summarized as follows—

The standard unit of refrigeration proposed for rating refrigerating machines is 1 *Calorie per second*, the Calorie being the amount of heat required to change the temperature of 1 kilogramme of water by 1° C.; 1 Calorie is $2{\cdot}2046 \times \frac{9}{5}$, or 3·9683 B.Th.U.; and 1 Calorie per second is equivalent to $3{\cdot}9683 \times 60 \times 60 \times 24$, or about 342,860 B.Th.U. per day of 24 hours.

The refrigeration may be stated for particular temperature conditions, or for *standard conditions* defined as follows: the temperature limits to be steady; the temperature of the cooling water

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* *Proc. Inst. Mech. Eng.*, Oct., 1914.

† *Proc. Inst. Mech. Eng.*, 1940, Vol. 143, p. 261.

to range from 15° C. (59° F.) at inlet to 20° C. (68° F.) at outlet, and the temperature of the brine to range from 0° C. (32° F.) to - 5° C. (23° F.). The refrigeration produced under standard conditions is called the *rated capacity* of the machine. *A machine of unit rated capacity produces a refrigeration of* 1 *Calorie per second in steady working under standard conditions.* In the case of cooling by direct expansion, without the use of brine, the lower limit of temperature of the vapour in the evaporator is taken as - 10° C. (14° F.), and the temperature of the cooling water the same as those for machines using brine.

"In the *ideal standard cycle* for comparison with actual performance, the most efficient cycle employing adiabatic compression is that in which the pressures of suction and condensation, measured just after and just before the substance passes the expansion valve, are the same as those that occur in the actual process of working, and in which the substance is cooled before passing the expansion valve to a temperature equal to that at which, in the actual process, the cooling water is supplied."

The *efficiency ratio* is the fraction—

$$\frac{\text{Coefficient of actual performance}}{\text{Coefficient of ideal performance}}$$

The unit of refrigeration as proposed above has not been generally adopted by the industry, and performances are often reckoned in "tons of refrigeration per day," meaning a rate of heat abstraction equal to that required to freeze so many tons of water per day. As the ton may be either English or American it would appear to be better to give the duty in terms of B.Th.U. per hour under given conditions of brine and cooling water temperatures.

Total Heat-Entropy Charts. For the calculation of the ideal performance, the Mollier charts of entropy and total heat for CO_2, NH_3, and SO_2, the three substances in practical use, have been drawn on a large scale, and tables giving the pressure, temperature and volume of saturated vapour prepared by Professor C. F. Jenkin for the Committee.

Mollier, in his total heat-entropy chart, uses oblique co-ordinates in order to open up or spread out the diagram and exhibit the cycle more clearly. When plotting on ordinary squared paper, the co-ordinates are found by drawing lines parallel to the direction of the axes through each state point to cut the scales. The rectangular co-ordinates of the point B, Fig. 216, are $y = 4$, $x = 3{\cdot}5$, and of C, 4, 4·5. By drawing the axis of x, inclined at a small angle α in Fig. 217, the vertical scale is not changed, but the horizontal scale is enlarged 4 times, the line BC is made 4 times its original length, and the triangle ABC is enlarged for the same size of paper, since the values of x can be given as shown on the constant ϕ lines to the right.

Fig. 218 shows, on a small scale with a few lines, a total heat-entropy chart for carbonic acid with oblique co-ordinates. Adiabatics (ϕ constant) are drawn parallel to the oblique axis, and lines representing constant total heat are horizontal. During evaporation or condensation of the CO_2 at *constant pressure* and temperature, the change of total heat, $dH = dQ = T \,.\, d\phi$, and $\frac{dH}{d\phi} = T$, so that the slope of a constant pressure line measures the temperature.

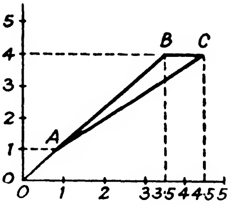

FIG. 216. RECTANGULAR CO-ORDINATES

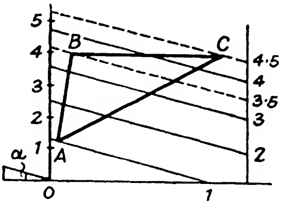

FIG. 217. OBLIQUE CO-ORDINATES

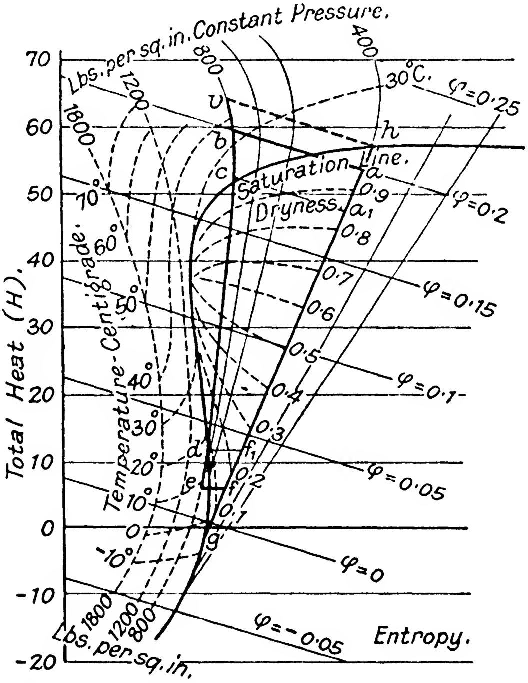

FIG. 218. H-ϕ CHART FOR CARBON DIOXIDE

Within the saturation line or boundary curve, representing the mixture of liquid and saturated vapour, the temperature of saturation along any constant pressure line is constant, and therefore any constant pressure line in the wet region is straight, but, upon crossing the dry saturation line into the region of superheat, the temperature rises and the line curves. Each of the fan-shaped lines is thus a line of constant pressure and temperature, and represents evaporation in the refrigerating coils. The effect of drawing the oblique axis is to open out the fan. Again, since the heat taken in by the liquid during evaporation is proportional to the dryness, equal distances along any constant temperature line *gh* correspond to equal increments of total heat, or equal changes of dryness. Any evaporation line like *gh* may be divided into 10 equal parts, representing dryness fractions.

Vapour Compression Cycle on the H-ϕ Chart. In Fig. 218, which shows the changes of total heat in relation to entropy for carbonic acid, the same letters *a*, *b*, *c*, *d*, *e*, *f* represent the state points of the refrigerant throughout the cycle as are given in the T-ϕ diagram, Fig. 210 (p. 557). The point *a* represents the state of the fluid mixture in the evaporating coils before it enters the compressor.

During evaporation along *fa* the constant pressure is 400 lb. per sq. in., and the temperature about $-8{\cdot}6°$ C. At *a* the wetness of the vapour is ah/gh, and the dryness fraction ag/gh is 0·95, when the mixture is drawn into the compressor. The line *ab* parallel to the oblique lines of constant entropy represents adiabatic compression, and is drawn to intersect at *b* the curve giving the higher limit of pressure, namely, that in the condenser, 800 lb. per sq. in. The temperature of the superheated vapour is then 35° C.

If evaporation were complete and only dry saturated vapour drawn into the compressor, then "dry" adiabatic compression in the region of superheat would be shown by the broken line *hv*, drawn from *h* on the dry saturation line; and the temperature at *v*, 45° C.

On the other hand, if the dryness fraction of the mixture were only 0·83, at a_1 when taken into the compressor, such that the line a_1c would not pass the dry saturation line but meet it just at the condenser pressure, then there would be no superheating, and the compression is called entirely "wet" compression, and the temperature of the dry saturated vapour 18·6° C.

The difference between the readings on the total heat scale of the ends *b* and *a* of the compression line, namely $H_b - H_a = 6$, gives the amount of work in heat units necessary to compress 1 lb. of the refrigerant. For *hv*, the work done is equivalent to 6·7 C.H.U.; and for a_1c it is 5·2 C.H.U.

The superheated vapour at *b* is cooled in the condenser at constant pressure, 800 lb. per sq. in., to dry saturated vapour at *c*; followed by condensation *cd* until all is liquid at *d*, and temperature 18·6° C., and the liquid is further cooled to 10° C. at the state point *e*, found by the intersection of the constant pressure line from *d* with the line of constant temperature appropriate to the cooling water. Now the liquid at 10° C. passes through the expansion valve into the evaporating coils. From *e* draw a line *ef* parallel to the horizontal lines of total heat to meet, at *f*, the line representing the temperature − 8·6° C. and pressure 400 lb. per sq. in. in the evaporator. The line *ef* of constant total heat represents throttling through the expansion valve, and determines the state point *f*, giving the conditions when evaporation starts in the coils of the refrigerator. The dryness fraction *gf*/*gh* indicates the fraction of liquid which has evaporated in the throttling process to reduce the mixture to the temperature and pressure in the evaporating coils.

The line *fa* shows evaporation at the lower constant temperature and pressure during the effective refrigeration. The difference between the readings on the vertical total heat scale, corresponding to the state points *a* and *f*, the ends of the evaporation line, gives the amount of heat which 1 lb. of the carbonic acid in the evaporator takes up from the brine or cold body.

In this case the refrigerating effect, $H_a - H_f = 48$ C.H.U. per lb., and the coefficient of performance is 48/6, or 8. When the evaporation is carried on to completely dry vapour at *h*, $H_h - H_f = 51$ C.H.U. per lb., and the coefficient of performance is 51/6·7, or 7·6. In the third case, that of "wet" compression, $H_{a1} - H_{f1} = 40{\cdot}8$ C.H.U. per lb., giving a coefficient of performance 40·8/5·2, or 7·85.//

Sir Alfred Ewing points out* a simple method of determining the dryness before compression which gives the maximum coefficient of performance in the ideal theoretical refrigerating machine.

"The refrigerating effect of any degree of dryness *a*, Fig. 218, before compression is proportional (on some scale) to the length *fa*. The work done is proportional (on another scale) to the length *ab*. Hence the position of *b* which will give the highest coefficient of performance is that which gives the smallest ratio of *ab* to *fa*. This is found by drawing a tangent from *f* on the evaporation line to the line of constant pressure for the condenser, meeting that line in *b*. Then the compression line *ab* is the one which gives the maximum coefficient of performance in the ideal cycle."

In order to determine the standard cycle a point *e* is taken, which gives the condition of the liquid carbonic acid at 10° C. in this example before it enters the evaporator. Draw the line *ef* parallel to the horizontal lines of total heat to meet the line representing the required temperature and pressure in the evaporator.

Then the tangent from *f* to the condenser pressure curve determines the point *b* in the Fig. 218, hence the state point *a* of dryness

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[*] See Appendix II of the *Report of the Refrigeration Committee of the Institution of Mechanical Engineers*, 1914, p. 681.

at which compression should begin. It is advisable to use the large scale charts, drawn up by Professor C. Frewen Jenkin, from which the figures given in this example were found by measurement.

Again, suppose the condensed liquid at 18·6° C., in the state d, instead of being cooled to 10° C., as in the previous example, is allowed to pass directly through the expansion valve. The throttling is represented by the constant total heat line df_1, and effective refrigeration starts at f_1, when the dryness fraction is already gf_1/gh. Now, for maximum performance, draw a tangent from f_1 to the condenser pressure curve bc, and the constant ϕ line from the point of contact meets the evaporation line gh, where the dryness is 0·975. The maximum coefficient of performance is found to be 7·25, instead of 8. Obviously, the performance of the machine is improved by cooling the liquid refrigerant after condensation and before it passes the regulating or expansion valve to the lowest temperature to take up heat from the cold body.

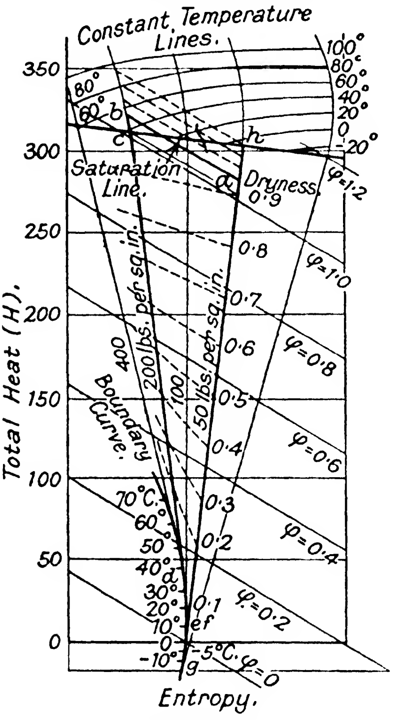

FIG. 219. H-ϕ CHART FOR AMMONIA

Another example is given in Fig. 219, to illustrate the use of the H-ϕ diagram for ammonia. The temperature of evaporation taken is − 5° C. at the corresponding constant pressure 50 lb. per sq. in., and the ammonia is condensed at 35·5° C., the pressure being 200 lb. per sq. in., and the liquid cooled to 15° C.

Here the cooling curve de of the liquid ammonia practically coincides with the boundary curve, even on the large scale chart, while the points e and f coincide on the diagram, Fig. 219.

When there is uncertainty as to the exact point of contact of the tangent fb to the curve, the point a for the greatest coefficient of performance may be found by taking several values of the dryness before compression, so that the vapour is dry or slightly superheated at the end of adiabatic compression.

Starting with the vapour of dryness 0·9 when drawn into the compressor, which gives dry saturated vapour when the pressure is 200 lb. per sq. in., it is only necessary to take the two readings of H_a, H_b on the chart for each compression line, and of H_f on the total heat scale. The following results were obtained—

Dryness before Compression	Refrigerating Effect	Work in the Compressor	Coefficient of Performance
	$H_a - H_f$	$H_b - H_a$	
1·0	286·6	45·5	6·30
0·975	277·6	43·7	6·35
0·95	270·2	41·7	6·48
0·925	258·6	39·0	6·63
0·9	255·6	38·8	6·58

Thus, according to the ideal standard cycle for ammonia, on the assumption of *adiabatic* compression, the greatest coefficient of performance is found by having the vapour slightly superheated at the end of compression.

From the H-ϕ charts it is easy to determine the amount of heat which 1 lb. of ammonia or carbonic acid can extract from the cold body when working under given conditions as to temperature and pressure of evaporation and condensation. It is also necessary to know the volume of 1 lb. of the working fluid in the state of vapour from tables. Then, making due allowances for loss of heat, mechanical and volumetric efficiencies, the power and size of the compressor may be estimated.

In the **actual machine,** instead of the ideal adiabatic compression there is interchange of heat between the vapour and the walls of the compressor cylinder, and this transfer of heat is less with dry compression than with wet, because the liquid is a better heat conductor and has greater specific heat than the dry vapour. In addition to the indicated work in the compressor, there is heat, given to the vapour, equivalent to work spent in friction of the piston and glands. Every part of the plant at a temperature below that of the atmosphere must gain heat by leakage into it from the surroundings.

With saturated vapour in the compressor at the end of the stroke, the loss due to **clearance** may be greater than with superheated vapour. It is all-important to have the clearance between the piston and cylinder cover, reduced to the smallest practicable. Any vapour remaining in the clearance at the end of the discharge stroke is at the higher pressure, and the piston must travel some distance in the suction stroke before this vapour expands below the lower pressure to allow the fresh charge to be admitted.

Superheat at the end of compression is also desirable, to avoid liquid being pumped idly round the whole cycle, and to ensure that the clearance space does not contain liquid which would evaporate during the following suction stroke, and seriously diminish the circulation of the vapour and the volumetric efficiency of the compressor. Thus by careful adjustment for slightly wet compression, a greater quantity of refrigerant would be circulated and greater refrigerating effect apparently produced than with all dry compression for the same volume swept by the piston. Generally, in practice, the liquid is completely evaporated in order to avoid the absorption of heat by evaporation in the compressor cylinder, which is found to counterbalance the slight theoretical gain by partially wet compression.

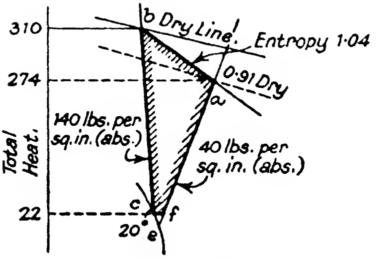

FIG. 220. H-ϕ CHART

When the temperature of water entering the condenser is near the critical point of carbonic acid, superheating is necessary. With cooling water at a lower temperature, and under ordinary working conditions, the record of tests shows the best results when the CO_2 vapour is discharged from the compressor at a temperature about 50° to 70° C.

Example 7. An ammonia vapour compression refrigeration plant is to effect a refrigeration of 20 lb. calories per second, and the working limits are as shown in the heat-entropy chart, Fig. 220. The net heat from the cold chamber is only 85 per cent of the possible amount shown on the chart, and the mechanical efficiency of the compressor is 65 per cent. Determine the horse-power required to drive the compressor and the amount of fluid circulation required for the stated performance.

If the specific volume of saturated ammonia vapour at 40 lb. per sq. in. is 7 cu. ft. per lb., determine a suitable size of compressor at 80 r.p.m., single-acting, with a piston speed not exceeding 150 ft. per min.

(*U.L., B.Sc. (Eng.)*, 1921.)

By the chart, at the state points a, b, c, e, f of the cycle, we have

$$\text{Heat extracted} = H_a - H_f = 274 - 22$$

$$= 252 \text{ calories per lb. of } NH_3 \text{ evaporated}$$

$$\text{Work done} = H_b - H_a = 310 - 274 = 36 \text{ calories per lb.}$$

$$\text{Net refrigeration per lb. } NH_3 = 252 \times 0{\cdot}85$$

$$= 214{\cdot}2 \text{ C.H.U. per cycle or rev.}$$

$$\text{Work} = \frac{\text{Refrigerating effect}}{\text{Coefficient of performance}} = \frac{20 \times 36}{214{\cdot}2} \text{ C.H.U. per sec.}$$

$$\therefore \text{ H.P.} = \frac{20 \times 36 \times 1{,}400 \times 100}{214{\cdot}2 \times 550 \times 65} = 13{\cdot}16$$

Let V = specific volume of gas leaving evaporator
s = specific volume of liquid
x = dryness fraction of gas leaving evaporator

Then effective volume per lb. of NH_3

$$= Vx + s(1 - x)^* = 7 \times 0{\cdot}91 = 6{\cdot}37 \text{ cu. ft.}$$

∴ refrigeration per cubic foot of compressor cylinder volume

$$= \frac{0{\cdot}85(H_a - H_f)}{Vx} = \frac{214{\cdot}2}{6{\cdot}37} = 33{\cdot}62 \text{ calories}$$

The refrigeration required is 20 calories per sec., which would be given by

$$\frac{20}{33{\cdot}62} \times 60 \text{ cu. ft. of } NH_3 \text{ per min.} = 35{\cdot}7 \text{ cu. ft. per min.}$$

Now, work done per lb. NH_3 per cycle = 36 C.H.U. = 36 × 1,400 ft-lb.

and the work done per cubic foot $= \dfrac{36 \times 1{,}400}{6{\cdot}37}$ ft-lb.

∴ Horse-power required

$$= \frac{36 \times 1{,}400 \times 35{\cdot}7}{6{\cdot}37 \times 33{,}000} \times \frac{100}{65} = 13{\cdot}16 \text{ H.P.}$$

Again, at 80 rev. or 160 piston strokes per min., and piston speed 150 ft. per min., the length of one stroke is $\frac{15}{16}$ ft. But the *volume swept* by the piston during one suction and one compression stroke must be 2 × 35·7, or 71·4 cu. ft. per min.

Let A = sectional area in square feet of compressor cylinder,

then $A \times \frac{15}{16} \times 160 = 71{\cdot}4$, and $A = 0{\cdot}476$ sq. ft.

∴ Diameter of cylinder in inches $= \sqrt{\dfrac{0{\cdot}476 \times 144}{0{\cdot}7854}} = 9{\cdot}342$ in.,

and piston stroke $= \frac{15}{16} \times 12 = 11\frac{1}{4}$ in.

Pressure-Total Heat Charts. The most convenient chart for studying the behaviour of a refrigerant is that in which the two co-ordinates are pressure and total heat. A typical chart is shown in Fig. 221 in broad outline.

The saturated liquid line and the saturated vapour line merge into one another at the critical point. A saturated liquid is one

---

* The term $s(1 - x) = 0{\cdot}02563(1 - 0{\cdot}91) = 0{\cdot}0023$ for NH_3 is negligible for practical purposes.

which has a temperature equal to the saturation temperature corresponding to its pressure. The space to the left of the saturated liquid line will therefore be a supercooled liquid region, the space between the liquid and vapour lines being a wet vapour region and that to the right of the vapour line being the superheated vapour region. A typical constant temperature line is shown by the line *CABD*. In the wet region the portion *AB* of the constant temperature line is also a constant pressure line, the vapour at *A* condensing to liquid at *B*. The general trend of the various lines is shown in the figure although their exact slopes depend on the co-ordinate scales.* Thus pressure is generally plotted to a log scale.

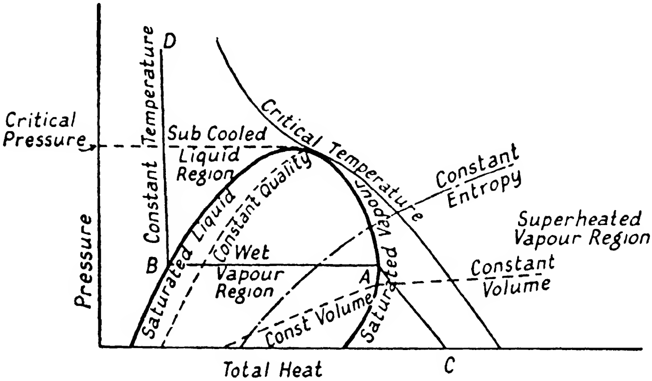

FIG. 221. PRESSURE-TOTAL HEAT CHART

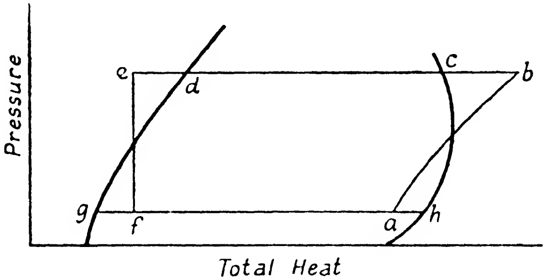

FIG. 222. VAPOUR COMPRESSION CYCLE

The vapour compression cycle on a p-H diagram is illustrated in Fig. 222 the letters corresponding with those in Figs. 210 and 211.

The various processes are—*ab*, adiabatic (isentropic) compression; *bc*, extraction of superheat; *cd*, condensation of vapour; *de*, subcooling of liquid; *ef*, throttling (constant total heat); *fa*, evaporation (refrigeration).

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* A set of charts for various refrigerants is issued in *Refrigeration Theory and Applications* by H. G. Venemann, published by Nickerson and Collins (Chicago).

Pre-cooling. The replacement of the expansion cylinder by an expansion valve simplifies the machine, but reduces the refrigeration effects in the evaporator. Not only is there less liquid left to evaporate usefully and absorb heat from the cold body, but the work done in expansion is lost. On the other hand very little work could be recovered by adiabatic expansion of the liquid, and this would be reduced by mechanical loss in the expansion cylinder, apart from the necessity of adjustment of the expansion for varying temperature conditions.

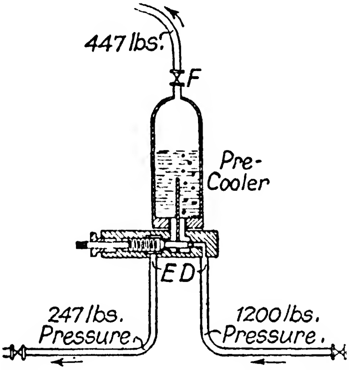

FIG. 223. PRE-COOLER FOR LIQUID CO_2 (*Haslam*)

The loss is negligibly small in the ammonia machine, but is greater in the CO_2 machine.

In order to reduce the loss, a pre-cooler (Fig. 223) can be used between the condenser and evaporator. The liquid from the condenser is passed through valve *D* into the pre-cooler at a lower pressure. Hence a portion of the liquid evaporates, cooling the rest of the liquid which collects at the bottom of the receiver, and passes on through valve *E* to the evaporator. The vapour collects at the top of the pre-cooler and passes through pipe *F* direct to the compressor cylinder, where the piston uncovers ports at the end of the suction stroke and admits the high pressure vapour.

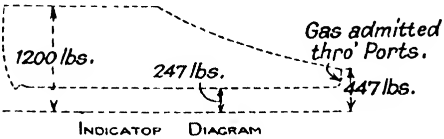

FIG. 224. INDICATOR DIAGRAM WITH PRE-COOLER

This *super-charge* greatly increases the pressure of the charge drawn from the evaporator and already in the cylinder, before the compression stroke begins. The result is increased refrigerating effect and efficiency of the machine. The *p-v* diagram, Fig. 224, compared with that of Fig. 209, indicates the increase of work required to drive the compressor. The results of careful tests show a marked increase of refrigerating effect per unit of work done. There is a larger proportion of liquid in the fluid entering the evaporating coils from the pre-cooler, and the coil surface is more efficient in absorbing heat from the brine.

Example 8. A vapour compression refrigerating plant uses ammonia as the working fluid, and the pressure and temperature of delivery from the compressor are 160 lb. per sq. in. and 100° F. After condensation, the liquid at 60° F. is passed through a throttle valve to the evaporator in which the pressure is 35 lb. per sq. in. Using the following information abstracted from tables using − 40° F. as datum, find the coefficient of performance. Sketch the Total Heat-Pressure diagram for the cycle.

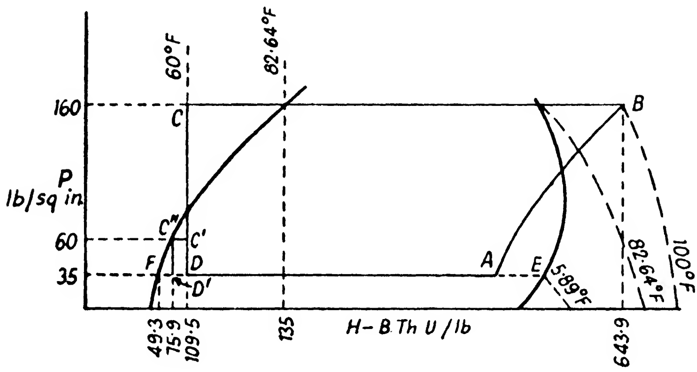

FIG. 225

(lb. per sq. in.)	(° F.)	(B.T.U.)	entropy	$\frac{L}{T}$
p sat.	t sat.	h (liquid)	(liquid)	
160	82·64	135·0	0·2804	0·9148
35	5·89	49·3	0·1113	1·2123

At 160 lb. per sq. in. and 100° F., the total heat is 643·9 B.Th.U. and the total entropy is 1·2186; at the same pressure the liquid heat at 60° F. is 109·5 B.Th.U.

If the throttling process had been carried out by first throttling to 60 lb. per sq. in. and then throttling the liquid only to the evaporator, the vapour being passed back to the compressor, find the increase in the refrigeration effect per lb. of the fluid passing through the evaporator. Assume the vapour leaves the evaporator in the same condition as previously, and at 60 lb. per sq. in., the liquid heat at the saturation temperature of 30·2° F. is 75·9 B.Th.U. All pressures are absolute.

(*U.L.B.Sc.* (*Eng.*), 1945)

Fig. 225 illustrates the p-H diagram where

$$T_A = 5{\cdot}89 + 460 = 465{\cdot}9° \text{ F. abs.}$$

$$\phi_A = \phi_B = 1{\cdot}2186$$

$$\phi_F = 0{\cdot}1113$$

Let x = dryness fraction at A.

(*a*) For 35 lb. per sq. in.,

$$\frac{xL}{T_A} = \phi_A - \phi_F$$

Then $xL = (\phi_A - \phi_F)T_A = (1{\cdot}2186 - 0{\cdot}1113)465{\cdot}9$
$= 516$ B.Th.U. per lb.

$H_A = h_F + xL = 49{\cdot}3 + 516 = 565{\cdot}3$ B.Th.U. per lb.

Hence work done

$= H_B - H_A = 643{\cdot}9 - 565{\cdot}3 = 78{\cdot}6$ B.Th.U. per lb.

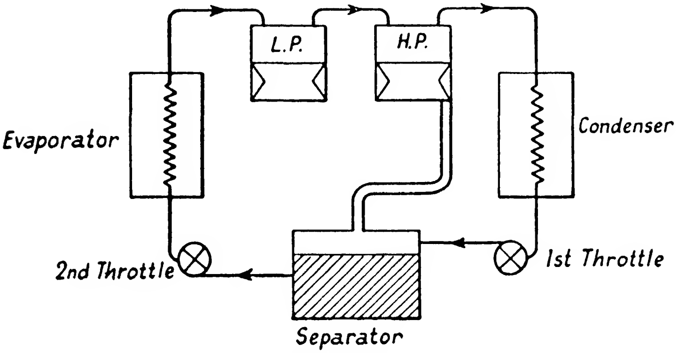

FIG. 226. COMPOUND COMPRESSION

Refrigerating effect $= H_A - H_D = 565{\cdot}3 - 109{\cdot}5$
$= 455{\cdot}8$ B.Th.U. per lb.

Coefficient of Performance

$$= \frac{455{\cdot}8}{78{\cdot}6} = 5{\cdot}8$$

(*b*) If removal of vapour takes place at 60 lb. per sq. in., the initial expansion is CC'. On removal of the vapour, the state of the remaining fluid is given by the point C''. The subsequent expansion is therefore $C''D'$. The increase in refrigerating effect is given by DD' where $H_{D'} = H_{C''} = 75{\cdot}9$ B.Th.U. per lb. (given in question).

Hence increase in refrigerating effect per pound of fluid passing through the evaporator $= H_D - H_{D'} = 109{\cdot}5 - 75{\cdot}9 = 33{\cdot}6$ B.Th.U. It should be noted that if the extracted vapour is passed back to the compressor, the mass of fluid passing through the compressor is greater than that passing through the evaporator.

Compound Compression. Compound compression is used in conjunction with pre-cooling. This is illustrated in Figs. 226, 227 and 228.

There are two throttle valves with an intermediate separator and two stages of compression, arranged as shown in Fig. 226, the arrows showing the direction taken by the refrigerant.

Figs. 227 and 228 show the compound cycle $ABCDD''EFKM$ compared to the single stage cycle $AD'D''EN$. For simplicity no undercooling of the refrigerant in the condenser is included.

In the compound cycle, the refrigerant leaving the condenser in the state represented by the point E (Figs. 227, 228) is throttled through the first valve along EF, being partially vaporized to a dryness fraction x_F. The wet vapour passes into the separator where the mass of liquid is $(1 - x_F)$ lb. for every pound of refrigerant leaving the condenser. This liquid represented by the point K passes through the second valve (along KM), is evaporated along MA and then compressed in the L.P. cylinder along AB whence it passes into the H.P. suction together with the x_F pound of dry vapour from the separator. This x_F lb. of dry vapour cools the $(1 - x_F)$ lb. of superheated vapour from the L.P. compressor, but this cooling is not necessarily enough to remove all the superheat from B to C. Hence some external cooling is generally necessary for this purpose. Assuming the refrigerant is cooled to dry saturated vapour at C, it is then compressed along CD, and condensed along DE to liquid, and possibly under-cooled. (Not included in diagrams.)

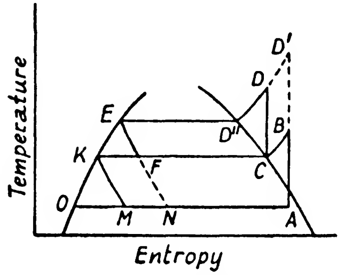

FIG. 227. COMPOUND COMPRESSION T-ϕ DIAGRAM

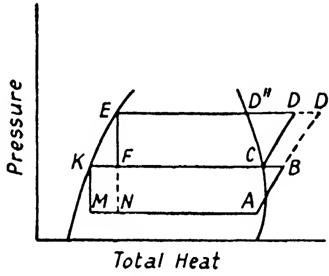

FIG. 228. COMPOUND COMPRESSION p-H DIAGRAM

Assume 1 lb. of fluid passes through the H.P. compressor. Then $(1 - x_F)$ lb. of liquid passes through the second throttle, and hence to the evaporator.

$$\therefore \quad \text{Refrigerating effect} = (1 - x_F)\,(H_A - H_M)$$
$$= (1 - x_F)\,(x_A - x_M)L$$

where L is the latent heat of the refrigerant at the evaporator pressure.

$$\text{The H.P. work} = H_D - H_C$$
$$\text{L.P. work} = (1 - x_F)\,(H_B - H_A)$$

If the loss in intercooling mentioned above is neglected then

$$\text{Coefficient of performance} = \frac{(1 - x_F)\,(x_A - x_M)L}{(H_D - H_C) + (1 - x_F)\,(H_B - H_A)}$$

For single-stage working without pre-cooling

$$\text{Work} = H_{D'} - H_A$$

$$\text{Refrigerating effect} = H_A - H_N$$

$$\text{C.O.P.} = \frac{H_A - H_N}{H_{D'} - H_A}$$

The amounts of work done may be shown in terms of areas on the T-ϕ diagram (Fig. 227). Thus for 1 lb. of fluid passing through the H.P. cylinder in two stage compression,

$$\text{H.P. work} = \text{Area } CDD''EKC$$

$$\text{L.P. work} = \text{Area } ABCKOA$$

For single stage working without pre-cooling

$$\text{Work} = \text{Area } AD'D''EOA$$

Hence the extra work required for single stage compression is measured by the area $BD'DC$.

Ammonia Absorption Machine. Ammonia vapour (NH_3) is very soluble in cold water, and heat is given out when the NH_3 is absorbed by water. The ammonia vapour may be driven out of the solution by heating it, and a larger quantity of heat is needed to evaporate the liquid when in solution with water than to evaporate the same weight of pure liquid ammonia, at the same pressure and temperature.

The difference is the heat energy required to overcome the affinity of the substances and to separate the NH_3 from the water solution. It takes about 450 to 470 C.H.U. to separate 1 lb. of NH_3 from the solution in water and change its state from that of liquid to vapour, whereas the latent heat of evaporation from pure liquid ammonia is only 300 to 320 C.H.U. per lb. (See Table XXVII, p. 594.) When the NH_3 vapour, driven out of its water solution by steam heating in the generator, Fig. 229, is cooled by water circulation in a *condenser*, it is readily liquefied by its own pressure above 150 lb. per sq. in. (abs.) at the ordinary temperature. In the process of refrigeration, the liquid NH_3, at this pressure, is allowed to pass through a *regulating* or *expansion valve* into the evaporating coils in the *refrigerator*, where it extracts its latent heat of evaporation from the brine or other body to be cooled.

The vapour pressure is reduced, say to about 30 lb. per sq. in., and evaporation encouraged by *absorption* of the vapour in the solution in the *absorber* at 25 lb. per sq. in., which acts like the suction stroke in the vapour compression machine. The water solution, rich in NH_3, is pumped from the *absorber* back through the *interchanger*, where it is heated, to the *analyser*, where it drips down over trays to the generator.

While the NH_3 is driven out of the solution in the generator, *heated by steam coils*, the liquid becomes denser and falls to the bottom. This weak solution from the bottom of the generator at the higher pressure and temperature is allowed to pass the *check valve* and through coils in the interchanger or economizer, where it gives up heat to the rich solution pumped from the absorber, and, thus *cooled*, passes again to the absorber, which requires to be cooled because of the amount of heat given out by the NH_3 in dissolving.

The wet ammonia vapour leaving the generator passes up through the *analyser*, or separator, containing a series of trays which separate more water from the NH_3. The vapour is further cooled and dried in a *rectifier*, not shown in the diagram, Fig. 229, on the top of the condenser, and the water condensed is returned to the upper part of the analyser.

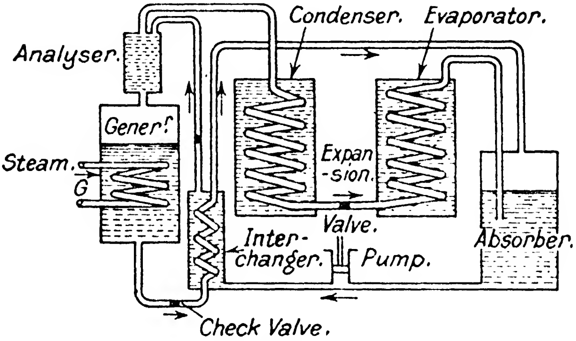

FIG. 229. AMMONIA ABSORPTION REFRIGERATING MACHINE

In this absorption process of refrigeration the efficiency is measured by the ratio Q_2/Q, where Q_2 is the refrigeration effect, that is, the amount of heat extracted from the brine in the evaporator, and Q is the heat supplied to the refrigerating machine at high temperature in the steam, not only in the generator coils, but also that required to drive the pump. It is obvious this efficiency must be less than unity, on account of the additional heat needed to drive the ammonia from the solution in water, above the latent heat of evaporation which gives the effective refrigeration. In practice, taking into account all the losses, the refrigeration effect is about a quarter of the heat supplied in the steam. For instance tests show that an ammonia absorption machine is capable of making 15 tons of ice per ton of coal. The calorific value of ordinary coal is about 12,000 B.Th.U. per lb., and the heat taken from 1 lb. of water in cooling and freezing, is about 200 B.Th.U. of effective refrigeration. Then the heat ratio, $\frac{Q_2}{Q} = \frac{15 \times 200}{12{,}000}$, or 25 per cent.

In the *absorption refrigerating machine* invented by Mr. W. W. Seay, anhydrous NH_3 is absorbed by anhydrous salt like sulphocyanide of ammonium, for which it has great affinity. The proportion in the mixture is 1 lb. of NH_3 to 3 lb. of the dry salt, which unite to form a liquid solution, and the NH_3 vapour can be driven off by heating, leaving the salt dry again. There is no water in this working substance, but in the absorber cold water circulates through a coil of pipe to take up the heat given out during absorption of the ammonia vapour by the salt. This heat is much less than the latent heat of the vapour, for part of it is used to liquefy the salt. In the same way, in the generator, steam or hot water through a coil of pipe supplies the heat required to drive off the NH_3 vapour. The amount of heat required is also less than the latent heat of the vapour, for part of the heat required is given out in the solidification of the salt. The NH_3 vapour from the generator is cooled by circulating water and liquefied in the condenser, then it passes through an expansion valve to the evaporator, where it extracts heat from the brine in the usual way.

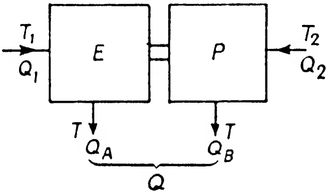

FIG. 230. IDEAL MACHINE

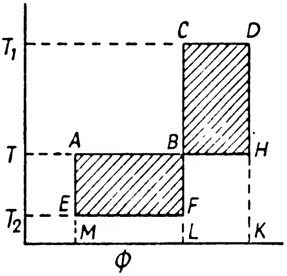

FIG. 231. IDEAL CYCLE

For small domestic installations using the ammonia absorption process, the pump may be omitted by the introduction of hydrogen into the low pressure side. The ammonia acts normally under its partial pressure. The total pressure is arranged to be practically uniform throughout the system. Thus the weak solution passing from the boiler to the absorber moves under gravity, while the flow of strong solution in the opposite direction is assisted by a vertical pipe between boiler and absorber, heated at its lower end by a small heating coil or gas jet.

The Ideal Absorption Process. The ideal process may be considered as a combination of an ideal heat engine E driving an ideal heat pump P (Fig. 230). A quantity of heat Q_1 is taken in by the engine at a temperature T_1 absolute. This in the absorption process is the heat required to drive off the ammonia in the generator. Another quantity of heat Q_2 at a low temperature T_2 is taken in by the heat pump. This is equivalent to the refrigerating effect in the absorption process. A third quantity of heat Q at some intermediate temperature T is rejected, being equivalent to the heat rejected in the condenser, and the cooling of the absorber.

The ideal cycles are shown in Fig. 231. The work done by the heat engine must be equal to the work absorbed by the pump.

i.e. $Q_1\frac{(T_1 - T)}{T_1} = Q_2\frac{(T - T_2)}{T_2}$ and area $CDHB$ = area $ABFE$

The ratio of the refrigerating effect to the heat taken in at the higher temperature T_1 is therefore $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \times \frac{T_1 - T}{T - T_2}$.

The heat areas are given on the T-ϕ chart (Fig. 231) as under—

Heat absorbed by engine $= Q_1$ at T_1 = Area $CDKL$
Heat rejected by engine $= Q_A$ at T = Area $BHKL$
Work done by engine $= Q_1 - Q_A$ = Area $CDHB$
Heat absorbed by pump $= Q_2$ at T_2 = Area $EFLM$
Heat rejected by pump $= Q_B$ at T = Area $ABLM$
Work absorbed by pump $= Q_B - Q_2$ = Area $ABFE$

As the work done by the engine is equal to the work absorbed by the pump,

$$Q_1 - Q_A = Q_B - Q_2$$

that is

$$Q_A + Q_B = Q = Q_1 + Q_2$$

From the equation $Q_1\frac{(T_1 - T)}{T_1} = Q_2\frac{(T - T_2)}{T_2}$ it follows that

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{Q_1 + Q_2}{T} = \frac{Q}{T}$$

This means that the entropy as a whole does not change; that is we may regard the process as a transfer of entropy from two sources at T_1 and T_2 to a common sink at an intermediate temperature T.

Refrigeration by Sublimation

(*a*) *The Triple Point.* One may represent the relation between temperature and pressure of saturated aqueous vapour by a curve (the steam line) such as AB, Fig. 232. A similar curve AC shows the relation of the same quantities, for which a mixture of ice and water will remain in equilibrium. This is termed the ice line. A third curve AD showing the same relation for the equilibrium of ice and aqueous vapour is termed the hoar-frost line. These three lines meet in a point A called the "triple point," which may be defined as the point indicating the temperature and pressure at which water, ice and aqueous vapour can simultaneously exist in contact. The same phenomenon of course occurs for other substances which can exist in the liquid, solid, and vapour conditions.

The pressure and temperature at the triple point for water are approximately 0·089 lb. per sq. in. abs. and 32·013° F. respectively.

From Fig. 232 it will be seen from the slope of the ice line *AC* that if the pressure is raised, the melting point of ice is lowered. This also follows from Claperyon's equation

$$v_w = v_i + \frac{JL}{T}\frac{dT}{dp}$$

for as v_w is less than v_i then $\frac{dT}{dp}$ is negative.

(*b*) *Charts for* CO_2 *including the Triple Point.* The T-ϕ chart for CO_2 in broad outline (not to scale) is shown in Fig. 233*. The portion above *BD* is similar to that for water and steam as in Fig. 131. *C* is the critical point, *DC* the saturated vapour line, and *BC* is the saturated liquid line, that is liquid at a temperature equal to the saturation temperature corresponding to its pressure. In passing along *DB*, dry saturated vapour at *D* is condensing into liquid at *B* by the abstraction of heat at constant pressure and temperature.

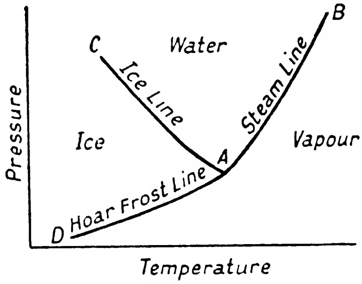

FIG. 232. THE TRIPLE POINT

Any further removal of heat at constant temperature will change the liquid at *B* into solid at *A*. The line connecting temperature and entropy for the saturated solid state is shown at *AO*. A solid is said to be saturated when at a certain temperature, it is ready to sublime upon the addition of heat. The area *OADKO* represents the sublimation zone, wherein solid and vapour may exist together. Thus while passing horizontally from the curve *BC* to the curve *CD* represents evaporation from liquid to vapour, so passing horizontally from the curve *AO* to the curve *DK* represents sublimation, that is the change from the solid to the vapour state without passing through the liquid phase. The triple point conditions for CO_2 are 75·1 lb. per sq. in. abs., and – 69·9° F. Thus at atmospheric pressure solid CO_2 will sublimate and is therefore called "dry ice."

To illustrate the use of the T-ϕ chart, certain values of entropy have been included on Fig. 233 in order to calculate the latent heat of fusion Q_F of CO_2, that is the heat to melt 1 lb. of solid CO_2 into liquid at the same temperature. This heat will be the heat absorbed in passing from *A* to *B* at a constant temperature of – 69·9° F.

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* A T-ϕ chart for CO_2 is given in *Thermodynamics for Engineers* by the late Sir Alfred Ewing (C.U.P).

Thus $Q_F = (\phi_B - \phi_A)\,T = (-0{\cdot}1115 + 0{\cdot}3275)\,(460 - 69{\cdot}9)$
$= 84{\cdot}3$ B.Th.U. per lb.

The latent heat of evaporation L is the heat absorbed in passing from B to D. Thus

Thus $L = (\phi_D - \phi_B)\,T = (0{\cdot}2724 + 0{\cdot}1115)\,(460 - 69{\cdot}9)$
$= 149{\cdot}7$ B.Th.U. per lb.

In the above it has been assumed that the CO_2 has passed from A to D by first melting from A to B and then evaporating from B to D. Taking a horizontal line very slightly below AD the substance may be considered to pass from solid at A to vapour at D through the sublimation zone. The heat of sublimation Q_S will therefore be given by

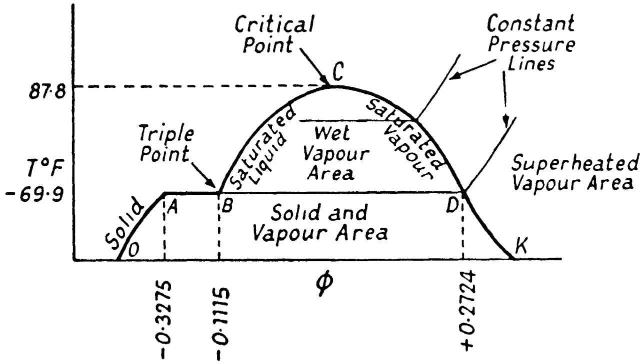

FIG. 233. T-ϕ CHART FOR CO_2

$Q_S = (\phi_D - \phi_A)\,T = (0{\cdot}2724 + 0{\cdot}3275)\,(460 - 69{\cdot}9)$
$= 234$ B.Th.U. per lb.

The pressure-total heat chart for CO_2 is illustrated in Fig. 234* The portion above *ab* is similar to the chart illustrated in Fig. 221. The lettering in this chart corresponds with that in Fig. 233. Thus the area *bcd* is the evaporation area, and the area *adko* is the sublimation area. Four constant temperature lines are shown for the following temperatures.

87·8° F. Corresponding to the critical temperature
32° F.
– 69·9°F. Corresponding to the triple point
– 109·3° F. Corresponding to atmospheric pressure of 14·7 lb. per sq. in.

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* A complete chart is included in those mentioned in footnote to page 576.

The values of total heat for several points are also shown for calculating the following examples.

(i) As a check on the value of the latent heat of fusion of CO_2 under triple point conditions of temperature and pressure.

$$Q_F = H_b - H_a = -13{\cdot}7 + 97{\cdot}9 = 84{\cdot}2 \text{ B.Th.U. per lb.}$$

(ii) To calculate the heat absorbed in the sublimation of dry ice, let it be assumed that we have 1 lb. at a pressure of 14·7 lb. per sq. in. abs. in the solid conditions represented by the point *e* (Fig. 234). The temperature of the solid CO_2 will be – 109·3° F. If it now absorbs heat, the process of sublimation follows the line *ef*, the heat absorbed in this sublimation from solid vapour being given by the increase in total heat.

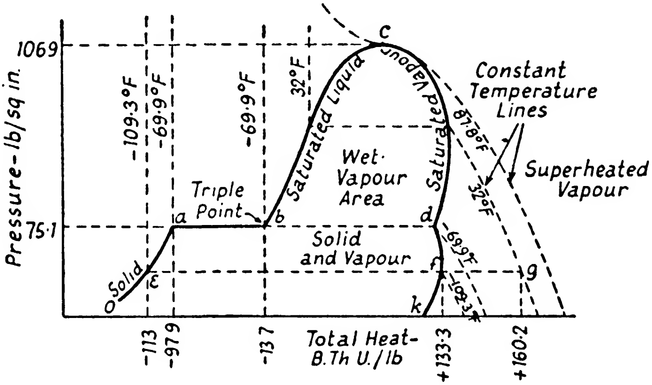

Fig. 234. *p-H* Chart for CO_2

Thus $\quad H_f - H_e = 133{\cdot}3 + 113 = 246{\cdot}3$ B.Th.U. per lb.

If this refrigerant were cooling any substance at 32° F. then the extra heat absorbed by the CO_2 vapour in raising temperature from – 109·3° F. to 32° F. would be $H_g - H_f = 160{\cdot}2 - 133{\cdot}3 = 26{\cdot}9$ B.Th.U. per lb. Thus the total heat absorbed by the CO_2 would be $246{\cdot}3 + 26{\cdot}9 = 273{\cdot}2$ B.Th.U. per lb. The latent heat of fusion of water ice is 143·3 B.Th.U. per lb. This illustrates the advantage of the use of CO_2 over water ice, apart from the fact that there is no liquid phase.

Cascade Method for very Low Temperatures. In this method a series of compression refrigeration circuits is used in which the refrigerant in the first circuit, after throttling, on subsequent evaporation acts as the condensing medium for the next circuit, and so on. The working substances in the circuits are so chosen, that the evaporation temperature of any one is lower than the critical temperature of the next in the series.

Regenerative Cooling. The Linde regenerative process of refrigeration, applied to attain the lowest temperatures for the liquefaction of air and gases, is based on the Joule-Thomson cooling effect.

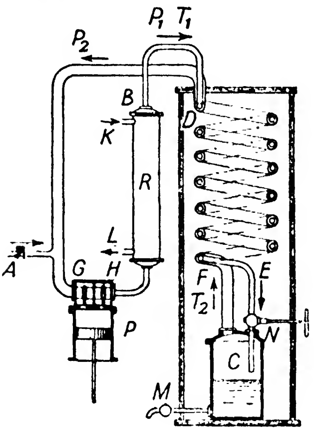

FIG. 235. LINDE REGENERATIVE MACHINE

In the diagram, Fig. 235, the compression pump, P, draws in air, or other gas, by the suction valve G, and delivers it under high pressure, p_1, of 100 to 200 atmospheres, by the valve H to the cooler R, through which cold water circulates by the inlet K and the outlet L. The compressed air at high pressure, p_1, and temperature T_1, then passes through the *counter current interchanger* by the inner spiral pipe DE, to the throttle valve or orifice N, into the vessel C. During throttling the drop of temperature to T' occurs with the pressure drop from p_1 to p_2.

Then the air returns through the outer pipe FD, taking up heat from the compressed air flowing in the opposite direction through the inner coil DE, and is raised very nearly to the initial temperature T_1 at D. The same air is again drawn into the compression pump P, and goes through the cycle of compression, cooling throttling, and taking up heat during its return.

The apparently slight cooling effect or drop of temperature by throttling, which in the case of air is only ¼° C. for every atmosphere fall in pressure, when repeated continuously under the difference of pressure, $p_1 - p_2$, enables the air after throttling to take up heat in the interchanger from the other portion of compressed air on its way through DE to the throttle valve. By repeating the cycle with continuous abstraction of the same quantity of heat from the interchanger coils, and air in them, any gas is gradually reduced below its critical temperature and to T_2, the boiling point at the lower pressure p_2, which produces liquefaction. Thus the steady state of thermal equilibrium is established at the lowest temperature in the vessel C.

The ideal conditions are: no heat leakage from outside to the interchanger, and complete transfer of heat in the interchanger at every point along the inner coil at the high pressure, p_1, to the outer coil. Then there will be a gradual fall of temperature from T_1 at D, to E, where the drop of temperature by throttling eventually reduces it to T_2 at F in the low pressure coil FD, in which the returning gas at pressure p_2 receives heat from the high pressure coil and gas, until at D its temperature is raised to T_1.

There are two losses: (i) That due to leakage of heat from the outside by conduction and radiation, which thorough insulation may reduce, but never quite prevent. (ii) Imperfect interchange of heat, which is overcome by the counter current in very long pipes of copper or other good conductor of heat, one inside the other, formed into a spiral coil or worm, giving close thermal connection.

The Joule-Thomson cooling effect, $T_1 - T'$, for a given pressure drop may be measured by the quantity of heat, Q per lb. of gas, necessary to raise the gas after throttling to the temperature T_1, at which it enters the interchanger. That is, $Q = C_p(T_1 - T')$, where C_p is the mean specific heat of the gas between these temperatures and at the lower pressure p_2.

Now, the total heat, H, is a function of the pressure, and for a given temperature the total heat of a gas is greater at the lower pressure. Each pound of gas entering the interchanger at p_1, T_1, has internal energy E_1, and the work done upon it is p_1v_1. If each pound of returning gas leaves the interchanger at the same point D, under pressure p_2, and at the same temperature T_1, having taken up a quantity of heat, Q, in the interchanger, it has internal energy E_2, and does work against external pressure equal to p_2v_2. Therefore, the difference in the total heat is the amount of heat taken from the apparatus per pound of gas that flows through in one cycle—

$$H_2 - H_1 = E_2 + Ap_2v_2 - (E_1 + Ap_1v_1)$$

This amount of heat Q is that necessary to raise the gas on leaving to the temperature T_1, hence $Q = H_2 - H_1$.

The value of Q depends only on the pressures p_1 and p_2, and the temperature T_1, and may be determined experimentally by observing the Joule-Thomson drop of temperature, $T_1 - T'$, during throttling from p_1 to p_2, without any interchange of heat.

When the gas begins to liquefy, the part that is liquid collects in the vessel C, and is drawn off by the stop-cock M. More air is pumped in through the stop valve A by an auxiliary low pressure pump, to replace the fraction that is liquefied.

The effective refrigeration is greatly increased with a lower initial temperature, T_1, hence the compressed air or gas is pre-cooled by separate refrigeration before being supplied through the stop valve A. This pre-cooling is essential for hydrogen, in which throttling at the ordinary temperature has a heating effect that becomes a cooling effect at a low initial temperature.

Linde also showed that, while the cooling effect in throttling is increased by a larger pressure drop or difference of pressures $p_1 - p_2$, the work done in compression depends on the ratio of the pressures $r = \frac{p_1}{p_2}$, which should be kept small to obtain greater practical efficiency. In the isothermal compression of a perfect gas the work spent is $RT_1 \log_\varepsilon r$, and in adiabatic compression from constant initial temperature, $\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$ is the variable factor. Hence, by working at a comparatively high back pressure p_2, the work spent is greatly reduced. If p_1 is 100 atmospheres and p_2 is one atmosphere, the cooling effect is proportional to 99, and the work of compression is nearly proportional to log 100. Whereas, when p_2 is 50 atmospheres, the cooling effect is proportional to 50 and the work of the compression pump to log 2. For adiabatic compression the work is still more reduced. Thus, working with p_2 at 50 atmospheres gives higher thermodynamic and practical efficiency. When p_1 is 200 atmospheres, p_2 may be 100 for efficient working.

As liquefaction proceeds, the apparatus remains at a steady temperature condition in thermal equilibrium, not getting any colder, neither gaining nor losing heat on the whole. At this stage the larger fraction of gas not liquefied continues to go through the same cycle, and the gas leaves the interchanger at the same temperature T_1 as the gas that enters at D.

If x is the fraction not liquefied, and $1 - x$ the small fraction drawn off as liquid at T_2; then the total heat H_1 of the gas entering the interchanger is equal to xH_2, that of the gas returning at D_1, together with $(1 - x)H_c$ of the liquid leaving at C.

$$H_1 = xH_2 + (1 - x)H_c$$

Suppose the liquid leaving at C were evaporated at its boiling point T'_2, and the vapour heated at the same pressure p_2, from T_2 to T_1, to bring the whole of the fluid leaving the interchanger to the temperature T_1 at D; the heat required would be

$$(1 - x)\{L + C_p(T_1 - T_2)\} = Q$$

which gives the fraction $(1 - x)$ that is liquefied.

In Norway the Linde plant liquefies air on a large scale for the purpose of supplying nitrogen, which is passed over hot calcium carbide in the manufacture of a nitrogeneous fertilizer.

Rectification of Liquid Air. In the Linde process of rectification of liquefied air, the more volatile nitrogen evaporates more readily than oxygen, and the liquid becomes richer in oxygen. Under atmospheric pressure the boiling point of nitrogen is about − 194°C. or 79° C. (abs.), and that of oxygen − 182° C., or 91° C. (abs.).

In one process the upper part of a tall rectifying column is filled with glass balls, through which liquid air trickles down; and at the bottom there are condenser pipes, where nearly pure liquid oxygen collects. Highly compressed air, cooled by passing through a counter current interchanger, enters at the bottom, becomes liquefied in the condenser pipes, and its latent heat serves to evaporate some of the liquid oxygen. A stream of oxygen gas passes up the rectifying column and meets the liquid air trickling down. The oxygen gas condenses and returns with the down-stream, while the latent heat given out by the condensation evaporates the nitrogen, which passes off at the top of the column mixed with about 7 per cent oxygen; while nearly pure oxygen collects at the bottom. The gas from the top of the column is passed through a counter current interchanger to take heat from the incoming compressed air.

In a modification of this process, a fractional method of liquefaction is adopted to separate a liquid rich in nitrogen into practically pure nitrogen, which passes off as gas at the top of the rectifying column, and pure oxygen at the bottom.

Properties of Refrigerants. The chief desirable properties of an ideal refrigerant are as follows—

1. High critical point compared with the condenser temperature and pressure. If the condenser temperature is too near the critical temperature the coefficient of performance will be low.
2. Low freezing point. The freezing point of the refrigerant should be below that of any temperature produced in the process.
3. Suitable condensing and evaporating pressures. The condensing pressure should be as low as possible to keep the necessary strength and therefore the weight of the machine as low as practicable. The evaporating pressure should be above atmosphere to avoid leakage of air into the system. The compression ratio should be as small as possible (see p. 548).
4. High latent heat of evaporation. This reduces the weight of refrigerant required.
5. Low specific volume. This tends to make machines smaller.
6. High coefficient of performance. Practical coefficients are given in Table XXV. The Carnot coefficient for the temperatures 5° and 86° F. is 5·74.
7. Inertness, stability, and non-corrosiveness.
8. High thermal conductivity. This increases the efficiencies of the condensers and evaporators.
9. Non-toxic and non-explosive properties.
10. Leaks easily detectable.
11. Low cost.

A list of refrigerants with useful data is given in Table XXV* and the following general remarks may be added—

Ethane, Ethylene and Propane with their low boiling points are suitable for obtaining low temperatures by the Cascade method.

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* Extracted from the Thomas Hawksley lecture "Recent Developments in Refrigeration," by Lord Dudley Gordon, *Proc. Inst. of Mech. Eng.*, 1943, Vol. 149, p. 49.

TABLE XXV

Properties of Refrigerants

	For Low Temperature			For Large Cold Stores			For Domestic and Air Conditioning Plants				Steam Jet and Centrifugal Plants	Centrifugal Plants
	Ethane C_2H_6	Ethylene C_2H_4	Propane C_3H_8	Ammonia NH_3	Carbon dioxide CO_2	Sulphur dioxide SO_2	Methyl chloride CH_3Cl	Freon 12 CCl_2F_2	Methylene chloride CH_2Cl_2	Freon 11 CCl_3F	Water H_2O	Dichloroethylene $C_2H_2Cl_2$
Critical Temperature (° F.) . . .	89·8	49·1	204·1	271·4	87·8	315·1	289·6	232·7	421	388·4	706	470
Triple Point (° F.) .	− 298·5	− 272·92	− 309·8	− 107·86	− 69·9	− 98·9	− 144	− 247	− 142	− 168		
Boiling Point at Atmospheric Pressure (° F.) . . .	− 127·5	− 155·02	− 48	− 28	− 109·3 Solid	13·99	− 10·76	− 21·7	103·7	74·66	212	122
Vapour Pressure at 5° F. . . . . (lb. per sq. in. abs.)	236·5	416·4	42·1	34·27	331·9	11·71	21·15	26·51	1·17	2·93	0·016	0·8
Vapour Pressure at 86° F. . . . . (lb. per sq. in. abs.)	682·7		155·3	169·2	1,043	66·85	94·7	107·9	10·6	18·28	0·62	6·9
Volume at 5° F. . (cu. ft. per lb.)	0·629	0·273	2·48	8·15	0·266	6·5	4·471	1·485	50	12·195	9.880	63
Volume at 86° F. . (cu. ft. per lb.)	0·122		0·717	1·772	0·0479	1·265	1·081	0·389	6·667	2·242		
Latent Heat at 5° F. (B.Th.U. per lb.)				565	115·3	170·7	178·5	69·5			1,088	136
Coefficient of Performance . . .				4·76	2·56	4·73	4·85	4·61		4·82	4·1	

SO_2 has low vapour pressures, and fairly large latent heats. It is highly toxic, and is corrosive in contact with moisture.

NH_3 has fairly low vapour pressures, and high latent heat. It is toxic and attacks non-ferrous metals. It is in large supply.

CO_2 has high vapour pressures, but low volume. Hence machines are smaller. It has a low critical temperature. It is non-toxic but will suffocate in high concentration.

CH_3Cl while being thermodynamically suitable can be inflammable, and causes illness in concentration.

The Freons have low vapour pressures and F12 has a small specific volume. They have small latent heats. They are odourless and colourless and non-toxic. They are comparatively costly.

$C_2H_2Cl_2$ has a large specific volume for a given heat-carrying capacity and its use is therefore chiefly confined to plants with centrifugal compressors.

H_2O can be used for heat pump purposes at temperatures around 212° F. Below this temperature the specific volumes are so big as to make the machine bulky. It is therefore confined to centrifugal plants, or to the use of steam jets.

Typical values of the properties of carbon dioxide, ammonia and methyl chloride are given in Tables XXVI, XXVII and XXVIII.*

Reversed Heat Engine as a Warming Machine.† LORD KELVIN, in 1852, made the suggestion that the reversed heat engine cycle

TABLE XXVI

PROPERTIES OF CARBONIC ACID (CO_2)

Temp. t° C.	Pressure in lb. per sq. in. p	Volume of liquid in cu. ft. per lb.	Volume of satu'd vapour in cu.ft. per lb.	Heat of liquid in lb. calories C.H.U. per lb. h	Latent Heat of satu'd vapour C.H.U. per lb. L	Total Heat C.H.U. per lb. H	Entropy (ϕ)		Specific Heat at Constant Pressure	
							Liquid	Vapour	Vapour	Liquid
-15°	331	0·0159	0·258	-8·7	63·5	54·8	-0·032	0·214	—	—
-10°	383	0·0163	0·220	-5·9	60·6	54·7	-0·021	0·209	0·33	0·58
-5°	441	0·0168	0·188	-3·0	57·5	54·5	-0·011	0·204	—	—
0°	505	0·0173	0·161	0	54·1	54·1	0	0·198	0·40	0·63
5°	576	0·0180	0·137	3·1	50·4	53·5	0·011	0·192	—	—
10°	653	0·0187	0·117	6·5	46·2	52·7	0·022	0·185	0·54	0·74
15°	737	0·0196	0·099	10·0	41·4	51·4	0·034	0·178	—	—
20°	829	0·0208	0·083	13·9	35·7	49·6	0·047	0·169	0·92	0·99
30°	1,037	0·0264	0·048	24·5	15·4	39·9	0·081	0·132	6·83	—
Critical Point 31·5°	1,071	0·0344		31·4	0	31·4	0·103	0·103	Inf.	—

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* Table XXVI is abridged from results by Professor H. L. Callendar. Most of the data in Table XXVII are from Dr. Mollier. Table XXVIII is extracted from *Refrigeration Theory and Applications* by H. G. Venemann (Nickerson and Collins). This book includes extensive details of properties of refrigerants. See also "Report of Sub-committee appointed to report on the accuracy of the Refrigeration Research Committee's Charts," *Proc. Inst. Mech. Eng.*, 1940, Vol. 143, p. 261.

† *Collected Papers*, Vol. I, p. 515; or *Proc. of the Phil. Soc. of Glasgow*, Vol. III, p. 269.

TABLE XXVII

PROPERTIES OF AMMONIA (NH_3)

Temp. t° C.	Pressure in lb. per sq. in. p	Volume of saturated vapour in cu. ft. per lb.	Heat of liquid in lb. calories C.H.U per lb. h	Latent Heat of saturated vapour in C.H.U. per lb. L	Total Heat, lb. calories C.H.U. per lb. H	Entropy (ϕ)	
						Liquid	Vapour
10°	41·5	6·8	−8	322	314	−0·033	1·193
5°	51	5·6	−4·5	319·5	315	−0·017	1·175
0°	61	4·7	0	316	316	0	1·158
5°	74·5	4·0	4·5	312·5	317	0·017	1·141
10°	89	3·3	9·2	308·6	317·8	0·033	1·123
15°	106	2·7	14	304	318	0·050	1·107
20°	125	2·3	18·6	300	318·6	0·066	1·089
30°	170	1·7	28·2	290	318·2	0·098	1·055

TABLE XXVIII

PROPERTIES OF METHYL CHLORIDE (CH_3Cl)

Temp. ° F.	Pressure in lb. per sq. in. abs.	Volume in cu. ft. per lb.		Total Heat B.Th.U. per lb.		Entropy from − 40° F.	
		Liquid	Vapour	Liquid	Vapour	Liquid	Vapour
− 80	1·953	0·01493	41·08	− 13·888	184·75	− 0·0351	0·4882
− 40	6·878	0·01553	12·72	0	190·66	0	0·4544
− 20	11·71	0·01583	7·761	7·146	193·49	0·0166	0·4405
0	18·9	0·01613	4·969	14·39	196·23	0·0327	0·4284
4	20·68	0·01619	4·568	15·85	196·78	0·0359	0·4262
5	21·15	0·01622	4·471	16·21	196·92	0·0367	0·4257
20	29·16	0·01647	3·312	21·73	198·84	0·0484	0·4177
40	43·25	0·01684	2·286	29·17	201·17	0·0636	0·4079
60	62·0	0·01724	1·624	36·71	203·33	0·0784	0·3991
68	71·01	0·0174	1·427	39·76	204·15	0·0842	0·3958
70	73·41	0·01744	1·382	40·52	204·34	0·0856	0·3950
90	100·6	0·01786	1·018	48·21	206·13	0·0908	0·3872
110	134·5	0·01833	0·7672	56·0	207·7	0·1138	0·3801
140	199·6	0·01915	0·5189	67·87	209·58	0·1341	0·3705

might be used to keep a room warm. A machine of the Bell-Coleman type might take in air from the atmosphere at a low temperature, expand it to a lower temperature and pressure, and allow the temperature to rise again by conduction from the external air, after which it would be compressed to atmospheric pressure, and its temperature thereby raised above that of its surroundings. The heated air might then be discharged into the room to be warmed. In this way a reversed heat engine might take heat Q_2 from the atmosphere at a low temperature T_2, and by means of expenditure of work W for expansion and compression, deliver to the room heat Q_1 at a slightly higher temperature T_1. If the machine were reversible,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}, \text{ and } \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T}$$

$$\therefore \quad Q_1 = W \frac{T_1}{T_1 - T_2}$$

The Heat Pump. The idea of using a heat pump for space heating did not lead to any practical application in this country until T. G. N. Haldane reviewed it in 1929.* In more recent years a long term experiment has been carried out on a large scale at Norwich† using the vapour compression process, for heating a large block of buildings. The essential parts of the plant are illustrated in Fig. 236.

The vaporized refrigerant leaving the compressor C at a high pressure and temperature, in passing through the condenser B gives out heat to the water circulating through the radiators in the building to be heated. In leaving the condenser, the condensed refrigerant is expanded to a low temperature and pressure through the expansion valve D. The liquid thus formed, in passing through the evaporator A absorbs heat from the circulating river water, and is thus vaporized at low temperature and pressure, in which state it is drawn into the compressor. The choice of refrigerant was largely governed by mechanical considerations and availability at the time, sulphur dioxide being used.

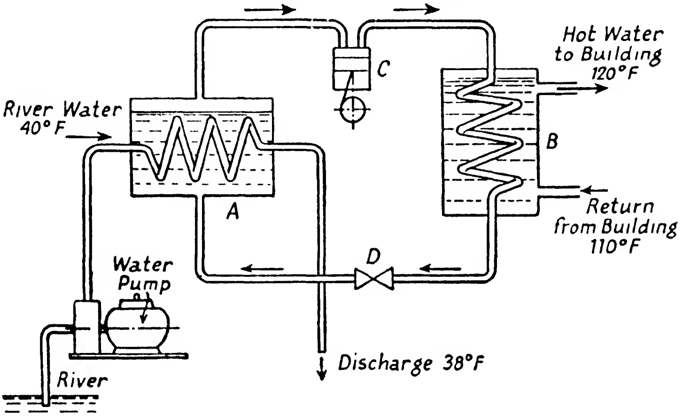

FIG. 236. ESSENTIAL ELEMENTS OF A HEAT PUMP INSTALLATION

In order to assess the value of the heat pump as a means of providing heat at a higher temperature than that available from existing sources, we may use the expression

$$\frac{\text{Heat delivered at the higher temperature}}{\text{Work done in compression}}$$

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* "The Heat Pump" by T. G. N. Haldane, *Journal I.E.E.*, 1930, Vol. 65, p. 666.

† "The Norwich Heat Pump" by J. A. Sumner, *Proc. Inst. Mech. Eng.*, 1948, Vol. 158, p. 22.

In the ideal case if Q_2 = heat absorbed by the refrigerant in the evaporator and Q_1 = heat rejected by the refrigerant in the condenser, then—

$$\frac{\text{Heat delivered}}{\text{Work done}} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

where T_1 and T_2 are the saturation temperatures corresponding to the pressures in the condenser and evaporator respectively.

This is the reciprocal of the ideal efficiency $\frac{T_1 - T_2}{T_1}$ of a heat engine working between the same two temperatures. Mr. Sumner has therefore suggested that the term as applied to a heat pump should be called the "reciprocal thermal efficiency." Professor S. J. Davies suggests the term "performance energy ratio."

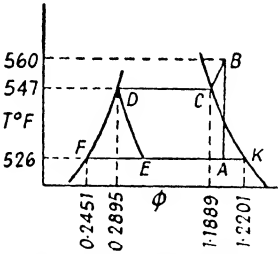

FIG. 237. T-ϕ CHART

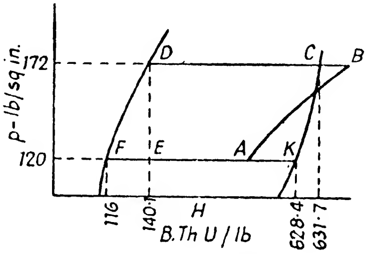

FIG. 238. p-H CHART

Example 9. A heat pump operates between the pressures of 120 and 172 lb. per sq. in. abs. The refrigerant has the properties given below reckoned from − 40° F. It enters the compressor as slightly wet vapour at the lower pressure, and is compressed to the higher pressure and a temperature of 100° F., the compression being assumed as isentropic. The superheated vapour is then cooled and condensed at the higher pressure to liquid without undercooling. This liquid is then expanded through a throttle valve to the lower pressure, at which pressure it is evaporated to its original slightly wet condition.

Sketch the cycle on a temperature-entropy diagram and on a pressure-total heat diagram.

How much heat is delivered per H.P. minute?

Assume that the specific heat of the superheated ammonia vapour is 0·7 B.Th.U. per lb. per degree F.

Pressure lb. per sq. in. abs.	Temp. ° F.	Liquid Heat B.Th.U. per lb.	Latent Heat B.Th.U. per lb.	Entropy	
				Liquid	Vapour
172	87	140·1	491·6	0·2895	1·1889
120	66	116	512·4	0·2451	1·2201

The cycle is shown as *ABCDEA* in Figs. 237 and 238.

It should be noted that the statement that the properties of the refrigerant are reckoned from — 40° F. means that the liquid heat and entropy are taken as zero at — 40° F.

$$\phi_B - \phi_C = C_p \log_e \frac{T_B}{T_C} = 0{\cdot}7 \times 2{\cdot}3 \times \log_{10}\frac{560}{547} = 0{\cdot}0164$$

$$\phi_A = \phi_B = \phi_C + 0{\cdot}0164 = 1{\cdot}1889 + 0{\cdot}0164 = 1{\cdot}2053$$

Dryness fraction at A

$$= x_A = \frac{\phi_A - \phi_F}{\phi_K - \phi_F} = \frac{1{\cdot}2053 - 0{\cdot}2451}{1{\cdot}2201 - 0{\cdot}2451} = 0{\cdot}985$$

$$H_B - H_C = C_p(T_B - T_C) = 0{\cdot}7(560 - 547) = 9{\cdot}1 \text{ B.Th.U. per lb.}$$

$$H_B = H_C + 9{\cdot}1 = 631{\cdot}7 + 9{\cdot}1 = 640{\cdot}8 \text{ B.Th.U. per lb.}$$

Heat delivered

$$= H_B - H_D = 640{\cdot}8 - 140{\cdot}1 = 500{\cdot}7 \text{ B.Th.U. per lb.}$$

$$H_A = h_f + x_A L = 116 + 0{\cdot}985 \times 512{\cdot}4$$

$$= 620{\cdot}7 \text{ B.Th.U. per lb.}$$

Work done

$$= H_B - H_A = 640{\cdot}8 - 620{\cdot}7 = 20{\cdot}1 \text{ B.Th.U. per lb.}$$

1 H.P. minute

$$= \frac{33{,}000}{778} = 42{\cdot}4 \text{ B.Th.U.}$$

∴ Heat delivered per H.P. minute

$$= 500{\cdot}7 \times \frac{42{\cdot}4}{20{\cdot}1} = 1{,}056 \text{ B.Th.U.}$$

Note. The value of the ratio $\frac{\text{Heat delivered}}{\text{Work done}}$ for this cycle is

$$\frac{500{\cdot}7}{20.1} = 24{\cdot}9.$$

The value of the same expression for the reversed Carnot cycle between the same pressure limits is $\frac{T_C}{T_C - T_A} = \frac{547}{547 - 526} = 26.$

The Air Cycle Heat Pump. The cycle as a closed circuit may be considered as the reverse of the Joule air cycle, as illustrated in Figs. 206 and 207. The necessary plant (Fig. 239) consists of a compressor cylinder C, an expansion cylinder E and a motor M, together with two heat exchangers X and Y. The function of the motor is to supply the difference between the work required to drive the compressor and the work returned by the expansion cylinder.

Referring to Figs. 206 and 207 where the temperatures are shown with corresponding lettering, the air leaving the compressor at an absolute temperature T_D passes through the heat exchanger X, giving up heat to the room. The air therefore enters the expansion cylinder at some temperature T_A less than T_D. In the expansion cylinder the air does work and is lowered in temperature to T_B. It now passes through the heat exchanger Y and is warmed to T_C by the surrounding atmosphere, at which temperature it enters the compressor.

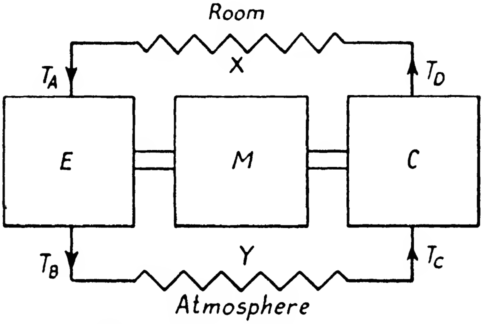

FIG. 239. CLOSED CIRCUIT AIR CYCLE

The heating load rejected to the room

$$= C_p(T_D - T_A) \text{ per lb. of air} = Q_1$$

Heat taken in from the atmosphere

$$= C_p(T_C - T_B) \text{ per lb. of air} = Q_2$$

Work required from the motor

$$= Q_1 - Q_2 = C_p\{(T_D - T_A) - (T_C - T_B)\}$$

The criterion of performance of the machine as a heat pump, namely $\dfrac{\text{Heating Load}}{\text{Work done by motor}}$ has been mentioned in the last paragraph. Mr. T. F. Thomas who discusses the air cycle heat pump in a paper to the Institution of Mechanical Engineers* calls this ratio the Performance Ratio. Thus

Performance Ratio as a heat pump

$$= \frac{Q_1}{W} = \frac{W + Q_2}{W} = 1 + \frac{Q_2}{W}$$

$$= 1 + \text{Coefficient of Performance as a refrigerator}$$

---

* "The Air Cycle Heat Pump" by T. F. Thomas, *Proc. Inst. Mech. Eng.*, 1948, Vol. 158, p. 30.

From the equations given above the performance ratio can be expressed in terms of temperatures.

Thus $$\frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_D - T_A}{(T_D - T_A) - (T_C - T_B)}$$

In a similar manner to that on page 552, this can be shown to be equal to $\frac{T_A}{T_A - T_B}$

In the open circuit constant pressure air cycle (Fig. 240), the heat delivered is not transferred direct to the room to be heated, but is transmitted via a heat exchanger to a current of air produced by a fan from the atmosphere.

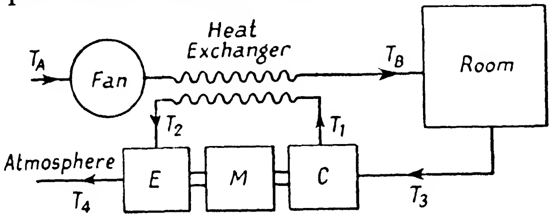

FIG. 240. OPEN CIRCUIT CYCLE

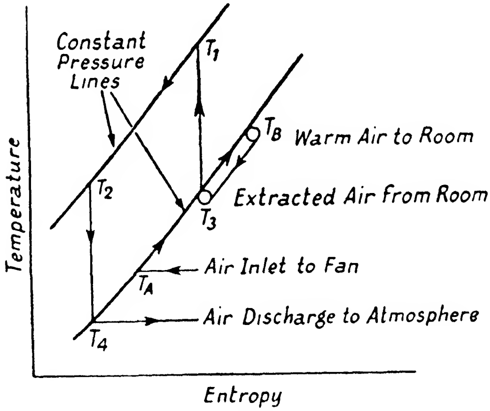

FIG. 241. OPEN CIRCUIT T-ϕ CHART

The air thus transmitted is raised in temperature from T_A to T_B.

The T-ϕ diagram assuming isentropic compression and expansion, and heat exchanges at constant pressure is shown in Fig. 241. In practice the compression and expansion lines are not vertical but are at a slope indicating increase of entropy due to internal friction, as the compression or expansion proceeds. This has already been discussed on pages 163 and 170. The heat transfer lines are also not exactly along the constant pressure lines owing to the pressure drop during heat transfer.

Referring to the ideal diagram (Fig. 241) it will be seen that

Heating load $= C_p(T_B - T_3)$ per lb.

Work by motor = Compressor work — expansion work

$$= C_p\{(T_1 - T_3) - (T_2 - T_4)\}$$

$$\text{Performance Ratio} = \frac{\text{Heating load}}{\text{Motor work}} = \frac{T_B - T_3}{(T_1 - T_3) - (T_2 - T_4)}$$

Mr. Thomas points out that this circuit, in addition to heating, provides also a ventilating effect. To assess this he proposes a ratio

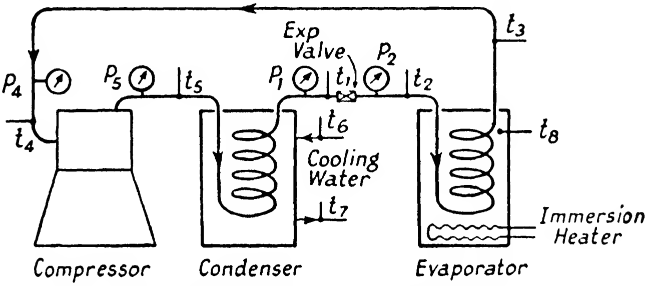

FIG. 242. ARRANGEMENT FOR REFRIGERATOR TEST

$$\frac{\text{Rate of heating fresh air}}{\text{Heat equivalent of work}} = \text{Fresh air heating ratio}$$

This value of this ratio from Fig. 241 is $= \dfrac{T_B - T_A}{(T_1 - T_3) - (T_2 - T_4)}$

Test on a Vapour Compression Refrigerating Plant. The general arrangement of the plant using methyl chloride (CH_3Cl) is shown in Fig. 242 where p indicates a pressure gauge and t a thermometer.

The compressor was driven by an electric motor, the input to the motor being measured by a watt meter. The evaporator took the form of a brine bath, which was continuously stirred by an electric motor. Situated in the bottom of the evaporator were three electric immersion heaters connected with a selector switch, and a variable resistance. A watt meter measured the electric input to the immersion heaters. The evaporator was carefully lagged, as well as the appropriate connecting pipes. The cooling water supplied to the condenser was measured by weighing. The compressor was air-cooled and, as it was not indicated, a value of 70 per cent was assumed for its mechanical efficiency.

In a test the compressor was started, and as the brine became cooler, the power to the immersion heaters was adjusted to keep the temperature t_8 of the brine constant at some required value.

When all the temperatures and pressures had settled down to constant values, readings were taken as shown below.

Room temperature 60·5° F. Barometer reading 15 lb. per sq. in. abs.

Refrigerant Temperatures and Pressures

	Point	Temp. (° F.)	Pressure (lb. per sq. in.) gauge	Pressure (lb. per sq. in.) Absolute
Out of condenser	1	59	56	71
Into evaporator	2	4	7	22
Out of evaporator	3	33	—	—
Into compressor	4	48	6	21
Out of compressor	5	170·5	57	72

Cooling water—Inlet temperature $t_6 = 50°$ F.
Outlet temperature $t_7 = 56{\cdot}8°$ F.
Flow measurement $w_w = 12$ lb. per min.
Compressor motor input = 575 watts
Immersion heater input = 1,200 watts
Brine bath temperature (constant) = $t_8 = 30°$ F.
Mechanical efficiency of compressor (assumed) = 70 per cent
Compressor motor efficiency = 70 per cent

It will be noticed that there is a drop of pressure of 1 lb. per sq. in. between the inlet and outlet of the condenser, and a similar drop between the inlet to the evaporator and the inlet to the compressor. These pressure drops have been ignored in this test, and pressures of 72 and 21 lb. per sq. in. abs. have been taken as the condensing and evaporation pressures respectively.

By comparing the temperatures of points 3 and 4 with the saturation temperature for a pressure of 21 lb. per sq. in., it will be seen that the conditions for these points are in the superheated region. The same applies to point 5 for the pressure of 72 lb. per sq. in. The details given in Table XXVIII are therefore not sufficient, as this table does not include the superheated region. These details are best obtained from a *p-H* chart such as that illustrated in Fig. 243.* By means of tracing paper, the complete cycle may be plotted from such a chart as shown in Fig. 244 and as described below.

(*a*) *Condensation and Evaporation Lines.* These can be drawn in for the two pressures of 72 and 21 lb. per sq. in. The pertinent parts of the saturated vapour and liquid lines can also be drawn.

---

* Extracted from *Refrigeration Theory and Applications* by Venemann (Nickerson and Collins).

(*b*) *Ideal Cycle with Isentropic Compression.* Point 1 ($t_1 = 59°$ F.) can be located by the intersection of the 59° F. temperature line with the 72 lb. per sq. in. pressure line. The position of this point indicates that the refrigerant is cooled in the condenser, below the saturation temperature corresponding to 72 lb. per sq. in. A vertical line drawn downwards from point 1 represents throttling at constant total heat through the expansion valve, and hence point 2 on the lower pressure line is located.

The points 3 and 4 can be located from the temperatures 33° and 48° at 21 lb. per sq. in. given in the experimental data. The positions of these points indicate that the refrigerant is superheated on leaving the evaporator (point 3), while as it is still at a temperature below that of the room, it becomes slightly more superheated in its passage to the compressor inlet (point 4).

If the compression were truly isentropic, the constant entropy line through point 4 would represent the compression and the refrigerant would be in the state at point 5′ on leaving the compressor. The cycle would then be 1245′1.

(*c*) *Actual Cycle.* The actual conditions at exit from the compressor are represented by point 5 which can be plotted from the experimental data $t_5 = 170{\cdot}5°$ F. and $p_5 = 72$ lb. per sq. in. abs. The conditions at entry to the condenser are taken at the same values, as the pipe line between is very short.

From the various points now located the values of total heat may be read off the *p-H* chart (Fig. 243) and these have been transferred to Fig. 244.

(*d*) *Work done with Isentropic Compression.* This is measured by $H_{5'} - H_4 = 232{\cdot}3 - 205 = 27{\cdot}3$ B.Th.U. per lb.

(*e*) *Weight of Refrigerant Used.* Let w_w = weight of cooling water passing through the condenser per min. Heat carried away by cooling water $= w_w(t_7 - t_6) = 12(56{\cdot}8 - 50) = 81{\cdot}6$ B.Th.U. per min.

From the *p-H* chart—

Heat rejected by refrigerant in the condenser $= H_5 - H_1$ $= 227 - 35{\cdot}5 = 191{\cdot}5$ B.Th.U. per lb.

Assuming a perfect condenser—

$$\text{Weight of refrigerant used} = \frac{81{\cdot}6}{191{\cdot}5} = 0{\cdot}426 \text{ lb. per min.}$$

(*f*) *Actual Work of Compression.* Measured input to motor = 575 watts

Indicated work in compressor

$$= \text{Compressor mechanical efficiency} \times \text{motor efficiency} \times 575$$

$$= 0{\cdot}7 \times 0{\cdot}7 \times 575 \text{ watts}$$

$$= \frac{0{\cdot}49 \times 575 \times 44{\cdot}23}{778} \text{ B.Th.U. per min.}$$

$$= \frac{0{\cdot}49 \times 575 \times 44{\cdot}23}{778 \times 0{\cdot}426} = 37{\cdot}5 \text{ B.Th.U. per lb. refrigerant.}$$

(*g*) *Heat Loss from Compressor.* By taking a horizontal distance equivalent to 37·5 B.Th.U. from the point 4 (Fig. 244), we obtain the point A of total heat value

$$H_A = 205 + 37{\cdot}5 = 242{\cdot}5 \text{ B.Th.U. per lb.}$$

This point A would represent the state of the refrigerant if there were no heat loss from the compressor during compression.

The actual loss from the compressor and the line up to point 5 is given by

$$H_A - H_5 = 242{\cdot}5 - 227 = 15{\cdot}5 \text{ B.Th.U. per lb.}$$

(*h*) *Heat Absorbed in Line from Evaporator to Compressor.* As the temperatures of points 3 and 4 are below the room temperature some heat will be absorbed by the refrigerant in the line between these points from the atmosphere, and heat absorbed = 205 − 202·5 = 2·5 B.Th.U. per lb.

(*i*) *Heat Removed in Condenser.* This is measured by $H_5 - H_1 = 227 - 35{\cdot}5 = 191{\cdot}5$ B.Th.U. per lb.

(*j*) *Actual Refrigerating Effect.* This is equal to the heat supplied to the brine by the immersion heaters which is 1,200 watts for a flow of 0·426 lb. per minute. Actual refrigerating effect

$$= \frac{1{,}200 \times 44{\cdot}23}{778} \text{ B.Th.U. per min.}$$

$$= \frac{1{,}200 \times 44{\cdot}23}{778 \times 0{\cdot}426} = 160 \text{ B.Th.U. per lb.}$$

(*k*) *Actual Coefficient of Performance.* This is given by

$$\frac{\text{Actual refrigerating effect}}{\text{Actual work done}} = \frac{160}{37{\cdot}5} = 4{\cdot}26$$

(*l*) *Refrigerating Effect with Isentropic Compression.* This from the *p-H* chart $= H_3 - H_2 = 202{\cdot}5 - 35{\cdot}5 = 167$ B.Th.U. per lb.

(*m*) *Coefficient of Performance with Isentropic Compression.* This from the *p-H* chart $= \dfrac{H_3 - H_2}{H_{5'} - H_4} = \dfrac{167}{27{\cdot}3} = 6{\cdot}12$

(*n*) *Isentropic Efficiency of Compression.* This quantity may be defined as the ratio—

$$\frac{\text{Work done in isentropic compression}}{\text{Actual work done}} = \frac{27{\cdot}3}{37{\cdot}5} = 0{\cdot}728$$

(*o*) *Evaporator Efficiency.* This may be defined as

$$\frac{\text{Actual refrigerating effect}}{\text{Ref. effect from } p\text{-}H \text{ chart}} = \frac{160}{167} = 0{\cdot}958$$

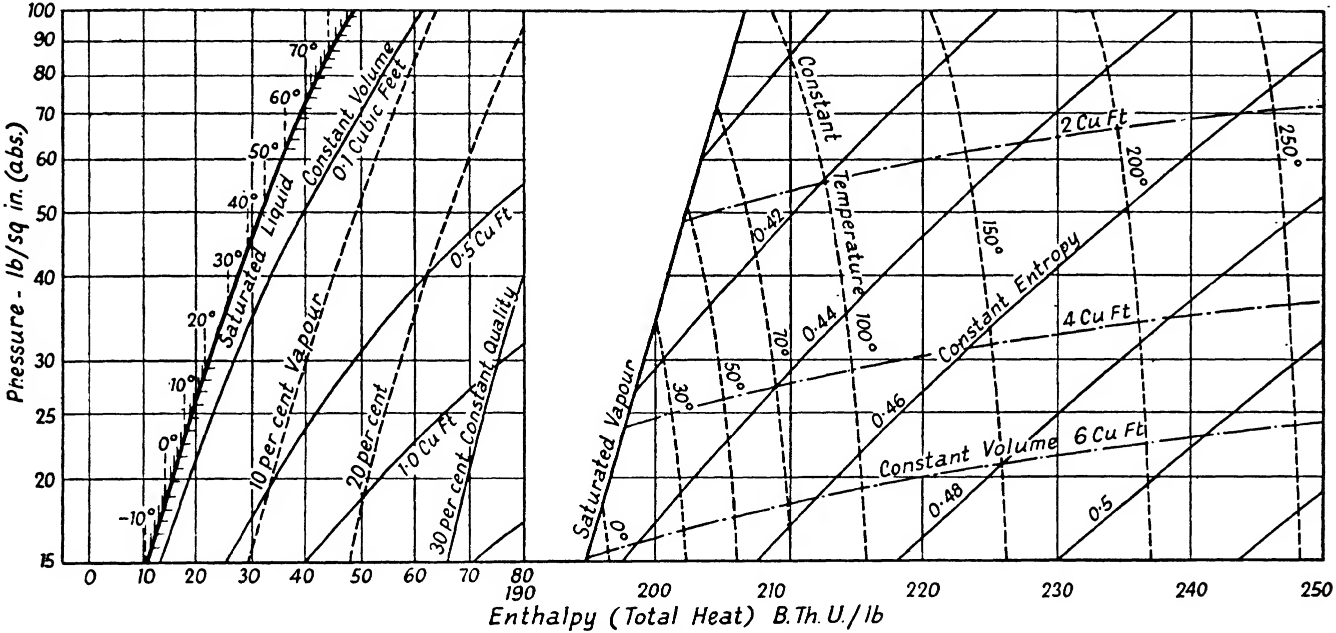

FIG. 243. *p-H* CHART FOR METHYL CHLORIDE

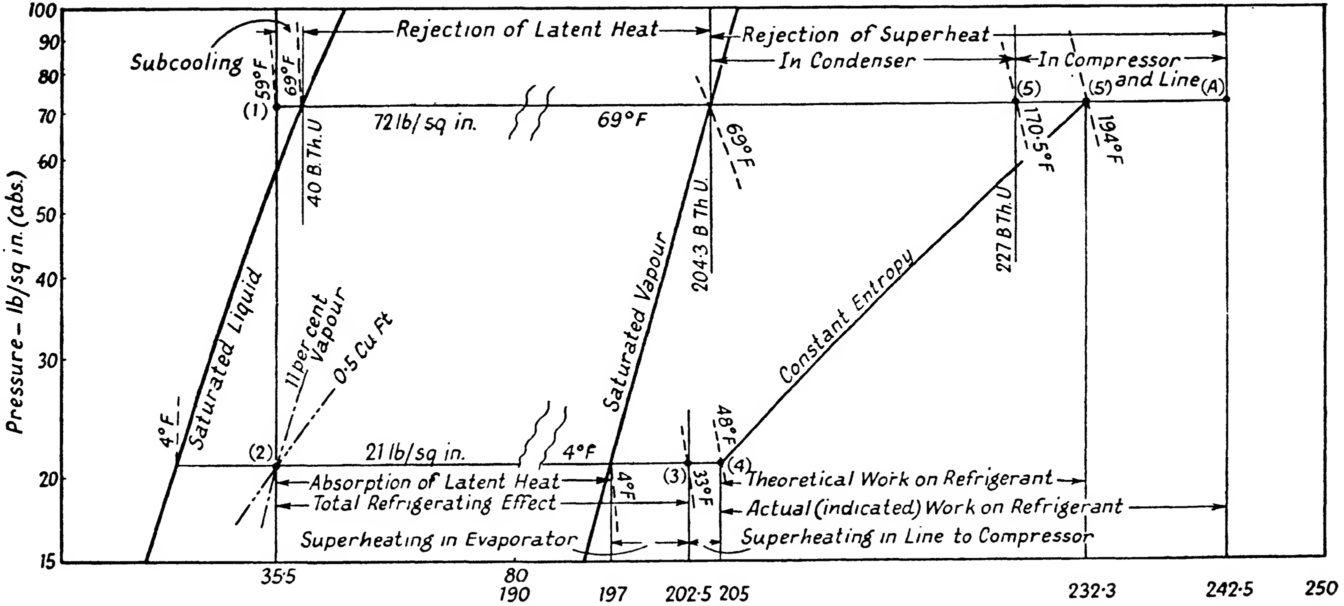

Fig. 244. Methyl Chloride Refrigeration Cycle

(*p*) *Heat Balance.*(*B.Th.U. per lb. of refrigerant*)

	In	Out
Heat absorbed in evaporator .	160	
Heat equivalent of compressor work	37·5	
Heat absorbed in line to compressor	2·5	
Heat rejected to condenser . .		191·5
Heat lost from compressor . .		15·5
Heat unaccounted for . . .	7·0	
	207·0	207·0

EXAMPLES XV

1. An ammonia compression refrigerating machine has to do an amount of refrigeration equal to the production of 25 tons of ice per 24 hours from and at 32° F. If the temperature limits in the compressor are 75° F. and – 5° F., calculate the horse-power of the compressor (*a*) on the assumption that the cycle is a perfect one, and (*b*) if the actual performance is 60 per cent of the ideal. The latent heat of ice, 142 B.Th.U. per lb. (*U.L.*, *B.Sc.* (*Eng.*).)

2. Find the horse-power of a perfect reversed heat engine that will make 900 lb. of ice per hour at 27° F. from water at 60° F. What amount of heat must be extracted per pound of ice ? Given the specific heat of ice, 0·5 ; and the latent heat 142 B.Th.U. per lb.

3. In a Bell-Coleman refrigerating machine the air is drawn in from the cold chamber at 23° F. and atmospheric pressure of 15 lb. per sq. in. (abs.), and, after adiabatic compression to 60 lb. per sq. in. (gauge), the air, at this constant pressure, is cooled to 60° F. Then the air is expanded adiabatically to atmospheric pressure and discharged to the cold chamber. What is (*a*) the net amount of work expended per pound of air ; (*b*) the heat extracted from the cold chamber ; and (*c*) the coefficient of performance ? $C_p = 0{\cdot}238$ for air.

4. In an open cycle air type of refrigerating machine circulating 1,500 lb. of air per hour, air is drawn from a cold chamber at a temperature of 10° C. (50° F.), and compressed adiabatically to 67 lb. (abs.). It is afterwards cooled at this pressure to 25° C., the temperature of the condenser, and then expanded adiabatically to atmospheric pressure and returned to the cold chamber. Find the number of units of heat (*a*) extracted per hour from the cold chamber, and (*b*) the heat rejected. If the indicated horse-power of the compressor is 25·0, find (*c*) the coefficient of performance of the machine. The specific heat of air may be taken as 0·241, and the law of expansion and compression as $pv^{1\cdot4}$ = constant. (*U.L.*, *B.Sc.* (*Eng.*).)

5. The temperature range in an ammonia plant is from 20° C. to – 10° C. Find the coefficient of performance (*a*) for the corresponding reversed Carnot cycle, and also (*b*) for the ammonia plant if, after compression, the ammonia gas is 0·95 dry. (*c*) What improvement in the coefficient of performance would be obtained if the plant were so managed that the temperature of the gas after compression were 35° C., the pressure range remaining as before ?

The values of the entropy for the liquid and dry vapour at 20° C. are 0·066 and 1·089 ; and at – 10° C. they are – 0·033 and 1·193 respectively ; and the specific heat of the vapour is 0·508. (*U.L.*, *B.Sc.* (*Eng.*), 1925.)

6. A heat pump works between the pressures of 73·5 and 100·5 lb. per sq. in. absolute. The working substance enters the compressor as slightly wet vapour, and is compressed adiabatically to the higher pressure and a temperature of 100° F. The superheated vapour is then cooled and condensed at the higher pressure to liquid without undercooling. The liquid is then expanded to the lower pressure, at which it is evaporated to its slightly wet condition.

Sketch the cycle on a free-hand temperature-entropy diagram and on a pressure-total heat diagram.

What is the amount of heat delivered per H.P. minute

(*a*) if the expansion is through a throttle valve,

(*b*) if the expansion is done in an expansion cylinder?

Assume the specific heat of the vapour is 0·25 B.Th.U. per lb. per degree F., and use the table of properties below.

Pressure lb. per sq. in. abs.	Sat. Temp. ° F.	Total Heat Liquid B.Th.U. per lb.	Total Heat Vapour B.Th.U. per lb.	Entropy Liquid	Entropy Vapour
73·5	70	40·52	204·34	0·0856	0·3950
100·5	90	48·21	206·13	0·0998	0·3872

(*U. Nottm., B.Sc.* (*Eng.*).)

CHAPTER XVI

HEAT TRANSFER

HEAT can be transferred in three ways: (*a*) by radiation, (*b*) by conduction, (*c*) by convection. The most obvious example of radiation is that from the sun. This radiant energy consists of electromagnetic waves which differ in wave-length, varying from the short ultra-violet waves, through light waves, to the long heat waves and wireless waves.

Radiation. In the elementary molecular theory, all matter is assumed to be made up of molecules which are in a state of vibration, depending on the temperature. These vibrations will set up radiant heat motions from a hot body, which become apparent when they encounter a cooler body whose molecules, if their period of vibration is suitable, will have their vibrations increased, the cooler body thus absorbing heat and rising in temperature. Any heat energy not absorbed will be reflected, or transmitted through the material of the body.

Different substances vary in their radiating and absorbing properties, and as a standard the term "black body" is adopted. This is defined as opaque and non-reflective and therefore absorbs all incident radiation. No perfectly black body exists, but an example of a nearly black body is lamp black. It must be remembered that a substance can absorb nearly all the incident thermal radiation without absorbing all light rays, and therefore may not be black to the eye.

Absorptivity, A, is defined as the fraction of the radiant energy incident on the surface of a body which is absorbed by that body. Hence the absorptivity of a black body is unity.

Emissive power, E, is defined as the total radiant energy emitted per unit time, per unit area, of radiating surface.

Emissivity, Σ, is defined as the ratio of the total emissive power of a body to the total emissive power of a black body at the same temperature.

The heat emitted per unit time, per unit area by a black body is given by the expression $Q = \sigma T^4$(abs.) where σ is the Stefan-Boltzmann constant and is equal to $5{\cdot}71 \times 10^{-5}$ ergs per sq. cm., per sec., per $(^\circ\ \mathrm{C.})^4$.

To convert to British units, as 1 erg. equals $0{\cdot}7372 \times 10^{-7}$ ft-lb., the conversion factor is

$$\frac{0{\cdot}7372 \times 10^{-7}}{778} \times \frac{30{\cdot}48^2 \times 3{,}600}{\left(\frac{9}{5}\right)^4} = 3{\cdot}021 \times 10^{-5}$$

Hence $\sigma = 5{\cdot}71 \times 10^{-5} \times 3{\cdot}021 \times 10^{-5}$

$= 17{\cdot}25 \times 10^{-10}$ B.Th.U. per sq. ft. per hour, per $(^\circ\ \mathrm{F.})^4$

and $Q = 17{\cdot}25 \times 10^{-10}T^4$ B.Th.U. per sq. ft., per hour.

Thus for a body of emissivity Σ, at a temperature T_1 abs. with surroundings at a temperature T_2 abs., the heat emitted

$$Q = \Sigma \times 17{\cdot}25 \times 10^{-10}(T_1^4 - T_2^4) \text{ B.Th.U. per sq. ft. per hour.}$$

Kirchhoff has shown that for two bodies the ratio of the emissive power to the absorptivity is the same. Thus if the suffixes 1 and 0 refer to any given body and the black body respectively then

$$\frac{E_1}{A_1} = \frac{E_0}{A_0} \text{ where } A_0 \text{ for a black body is unity}$$

Hence $A_1 = \dfrac{E_1}{E_0} = \Sigma_1$ by definition, and thus it follows that the emissivity and the absorbity of a body have the same value.

The Law of Conduction. The thermal conductivity of a material may be defined as the quantity of heat passing per unit time between the opposite faces of a unit cube of the material, when unit temperature is maintained between the surfaces.

The quantity of heat Q passing per unit time is therefore proportional to

(i) The conductivity K of the material.
(ii) The area A of the material surface normal to the heat flow.
(iii) The reciprocal of the thickness x between the two surfaces.
(iv) The temperature difference $T_1 - T_2$ between the surfaces.

Thus $Q = \dfrac{KA(T_1 - T_2)}{x}$ or for an elemental thickness dx

$Q = -KA \cdot \dfrac{dT}{dx}$ the minus sign signifying that T decreases as x increases.

As $K = \dfrac{Qx}{A(T_1 - T_2)}$ the conductivity can be expressed in British units as B.Th.U. per hour, per sq. ft. area, per ft. thickness, per ° F. or B.Th.U. per hour, per foot, per ° F.

The value of K depends on the temperature, and Table XXIX gives some approximate values, in the units given above, for ordinary temperatures.

Radial Flow of Heat through a Thick Hollow Cylinder

Let l = length of cylinder

R_1 and R_2 = internal and external radii of cylinder

T_1 and T_2 = temperatures of internal and external faces of cylinder, T_1 being the higher
K = conductivity of material
Q = heat flow per unit time

TABLE XXIX*

MATERIAL	CONDUCTIVITY
Wrought Iron .	35
Steel (0·5–1·5 C.) .	20–30
Copper . .	223
Brass . . .	60
Cork . . .	0·025
Asbestos . .	0·09–0·13
Water . . .	0·3 –0·4
Air . . .	0·015

Consider the passage of heat through an elemental shell of thickness *dr* (Fig. 245).

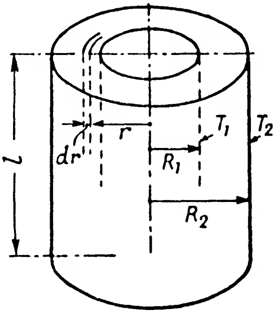

FIG. 245. THICK CYLINDER

Then $$Q = -K2\pi rl\frac{dT}{dr} \quad\text{or}\quad Q\frac{dr}{r} = -K2\pi l dT$$

Hence $$Q\int_{R_1}^{R_2}\frac{dr}{r} = -K2\pi l\int_{T_1}^{T_2}dT \text{ or } Q\log_e\frac{R_2}{R_1} = K2\pi l(T_1 - T_2)$$

$$\therefore \qquad Q = \frac{K2\pi l(T_1 - T_2)}{\log_e\dfrac{R_2}{R_1}}$$

---

* For more detailed values see *Introduction to the Transfer of Heat and Mass*, E. R. G. Eckert (McGraw Hill).

If the radial flow is expressed in terms of the same flow through an equivalent plate of thickness $R_2 - R_1$, for which the area is A_e then

$$Q = \frac{2\pi Kl(T_1 - T_2)}{\log_e \frac{R_2}{R_1}} = \frac{KA_e(T_1 - T_2)}{R_2 - R_1}$$

i.e. $$A_e = \frac{R_2 - R_1}{\log_e \frac{R_2}{R_1}} 2\pi l$$

The expression $\dfrac{R_2 - R_1}{\log_e \frac{R_2}{R_1}}$ is called the *Logarithmic mean radius.*

Radial Heat Flow through a Thick Spherical Shell

Let R_1 and R_2 be the internal and external radii of the shell.
T_1 and T_2 be the temperature of the inside and outside surfaces, the temperature T_1 being the greater.

Consider the heat flow outwards through an elemental thin shell of radius r and thickness dr.

Then $$Q = -K4\pi r^2\frac{dT}{dr} \text{ or } Q\frac{dr}{r^2} = -K4\pi dT$$

Hence

$$Q\int_{R_1}^{R_2}\frac{dr}{r^2} = -4\pi K\int_{T_1}^{T_2}dT \text{ or } Q\left[-\frac{1}{r}\right]_{R_1}^{R_2} = 4\pi K\left[T\right]_{T_1}^{T_2}$$

$$\therefore\quad Q\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 4\pi K(T_1 - T_2)$$

and $$Q = \frac{4\pi KR_1R_2(T_1 - T_2)}{R_2 - R_1} \text{ per unit time}$$

Lagging of a Hot Pipe. In order to determine the heat lost from the surface of the lagging of say a steam pipe to the surrounding air, it is necessary to know some property of the lagging material which measures heat lost from its surface. This may be defined as the heat lost by radiation, conduction and convection from unit area of the lagging surface, per unit time, per unit difference of temperature between the suface and the surrounding air. It will be represented by the letter e. This coefficient is sometimes wrongly called "emissivity," which should only be used in connection with radiation. In the case under consideration a large proportion of the heat lost is due to convection, that is by the bodily carrying away of heat by the moving particles of air.

In Fig. 245 let

R_1 = radius of the outside of the steam pipe at a temperature T_1

R_2 = radius of the outside of the lagging at a temperature T_2

K = conductivity of the lagging

e = as defined above

T_a = air temperature

Q_1 = heat lost per unit time, per unit area of pipe surface

Q = heat lost through lagging per unit time.

Then $$Q_1 = \frac{Q}{2\pi R_1 l} = \frac{K 2\pi l(T_1 - T_2)}{\log_e \frac{R_2}{R_1}(2\pi R_1 l)} = \frac{K(T_1 - T_2)}{R_1 \log_e \frac{R_2}{R_1}}$$

and $$T_2 = T_1 - \frac{Q_1}{K} R_1 \log_e \frac{R_2}{R_1}$$

By the definition of e—

$$Q_1 = \frac{Q}{2\pi R_1 l} = \frac{e 2\pi R_2 l(T_2 - T_a)}{2\pi R_1 l} = e\frac{R_2}{R_1}(T_2 - T_a)$$

or $$T_2 = T_a + \frac{Q_1}{e}\frac{R_1}{R_2}$$

Hence $$T_1 - \frac{Q_1}{K} R_1 \log_e \frac{R_2}{R_1} = T_a + \frac{Q_1}{e}\frac{R_1}{R_2}$$

and $$Q_1 = \frac{T_1 - T_a}{R_1\left(\frac{1}{eR_2} + \frac{1}{K}\log_e \frac{R_2}{R_1}\right)}$$

Example 1. A pipe carrying wet steam at 200 lb. per sq. in. abs. has an external radius of 3 in. It is lagged to a radius of 6 in. with asbestos of conductivity 0·04 B.Th.U. per ft.^{2} of surface, per ft. thickness, per hour per ° F. The surrounding air is at 70° F. and the loss from the lagging surface is 1·8 B.Th.U. per ft.^{2} of surface, per hour, per ° F. difference of temperature between lagging surface and air.

Find the loss in B.Th.U. per hour, per 100 ft. length of pipe. If you use a formula, prove it.

(*U.L., B.Sc.* (*Eng.*).)

Heat lost per sq. ft. of pipe surface per hour

$$Q_1 = \frac{T_1 - T_a}{R_1\left(\frac{1}{eR_2} + \frac{1}{K}\log_e \frac{R_2}{R_1}\right)}$$

From the steam tables for 200 lb. per sq. in. abs. $T_1 = 382°$ F.

$$\therefore \quad Q_1 = \frac{382 - 70}{0{\cdot}25\left(\dfrac{1}{1{\cdot}8 \times 0{\cdot}5} + \dfrac{2{\cdot}3 \log_{10} 2}{0{\cdot}04}\right)}$$

$$= 67{\cdot}7 \text{ B.Th.U. per sq. ft., per hour}$$

Heat lost from pipe $= 67{\cdot}7 \times 2 \times 3{\cdot}14 \times 0{\cdot}25 \times 100$

$= 10{,}630$ B.Th.U. per hour.

Series of Conductors

(*a*) *Equal Areas.* Consider a series of conductors as shown in Fig. 246 having equal areas normal to the heat flow, but having different thickness x and different conductivities K. If the surfaces a and b were in perfect thermal contact, the thermal gradients would be as shown in the figure. In practice, however, there would be a temperature drop at the surfaces of contact. We will therefore assume perfect contact, and then the heat flow through each of the conductors will be the same.

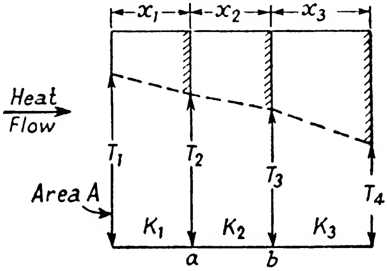

FIG. 246. SERIES OF CONDUCTORS

$$\therefore \quad Q = \frac{K_1A(T_1 - T_2)}{x_1} = \frac{K_2A(T_2 - T_3)}{x_2}$$

$$= \frac{K_3A(T_3 - T_4)}{x_3} \text{ per unit time}$$

Hence $T_1 - T_2 = \dfrac{Qx_1}{K_1A}$ and $T_2 - T_3 = \dfrac{Qx_2}{K_2A}$ and $T_3 - T_4 = \dfrac{Qx_3}{K_3A}$

$$\therefore \quad T_1 - T_4 = \frac{Q}{A}\left(\frac{x_1}{K_1} + \frac{x_2}{K_2} + \frac{x_3}{K_3}\right)$$

or

$$Q = \frac{T_1 - T_4}{\dfrac{x_1}{K_1A} + \dfrac{x_2}{K_2A} + \dfrac{x_3}{K_3A}} = \frac{T_1 - T_4}{r_1 + r_2 + r_3}$$

where $r = \dfrac{x}{KA}$ may be termed the thermal resistance.

(*b*) *Varying Areas.* Let Fig. 247 represent two concentric thick cylinders A and B of equal length l and of thermal conductivities K_1 and K_2 respectively, the surface temperatures being as shown in the figure, the temperature T_1 being the highest.

Then for an elemental cylinder of radius r and thickness dr the rate of heat flow $Q = -K2\pi rl\dfrac{dT}{dr}$

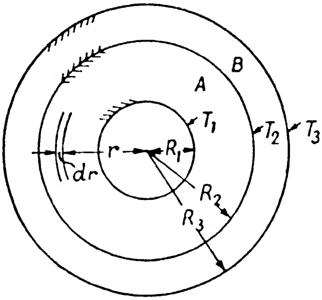

FIG. 247. CONDUCTORS WITH VARYING AREAS

Hence for cylinder A,

$$Q = \frac{K_1 2\pi l(T_1 - T_2)}{\log_e \dfrac{R_2}{R_1}} \text{ i.e. } T_1 - T_2 = \frac{Q \log_e \dfrac{R_2}{R_1}}{K_1 2\pi l}$$

and for cylinder B,

$$Q = \frac{K_2 2\pi l(T_2 - T_3)}{\log_e \dfrac{R_3}{R_2}} \text{ i.e. } T_2 - T_3 = \frac{Q \log_e \dfrac{R_3}{R_2}}{K_2 2\pi l}$$

Hence $$T_1 - T_3 = \frac{Q}{2\pi l}\left(\frac{1}{K_1}\log_e \frac{R_2}{R_1} + \frac{1}{K_2}\log_e \frac{R_3}{R_2}\right)$$

or $$Q = \frac{2\pi l(T_1 - T_3)}{\dfrac{1}{K_1}\log_e \dfrac{R_2}{R_1} + \dfrac{1}{K_2}\log_e \dfrac{R_3}{R_2}}$$

If, in Fig. 247, A and B are considered as being two thick hollow spheres then

$$Q = \frac{4\pi(T_1 - T_3)}{\dfrac{R_2 - R_1}{K_1 R_1 R_2} + \dfrac{R_3 - R_2}{K_2 R_2 R_3}} \text{ for outward flow}$$

or
$$Q = \frac{4\pi(T_3 - T_1)}{\dfrac{R_2 - R_1}{K_1R_1R_2} + \dfrac{R_3 - R_2}{K_2R_2R_3}} \text{ for inward flow}$$

Example 2. A spherical vessel of 20 in. radius contains a liquefied gas at − 297° F. It has two jackets of lagging each 4 in. thick. The inner layer has a conductivity of 0·35 and the outer of 0·45 in B.Th.U./ft.^{2}/hour/° F. difference/inch thickness.

Find the rate of heat leakage into the container if the outmost surface is at 60° F.

Establish any formula used.

(*U.L., B.Sc.* (*Eng.*), 1948)

Referring to Fig. 247 and the equation $Q = \dfrac{4\pi(T_3 - T_1)}{\dfrac{R_2 - R_1}{K_1R_1R_2} + \dfrac{R_3 - R_2}{K_2R_2R_3}}$

$T_3 = 60°$, $T_1 = -297°$, and $T_3 - T_1 = 357°$ F.

$R_1 = \frac{20}{12} = 1{\cdot}67$ ft., $R_2 = \frac{24}{12} = 2$ ft., and $R_3 = \frac{28}{12} = 2{\cdot}33$ ft.

$R_2 - R_1 = 0{\cdot}33$ ft., and $R_3 - R_2 = 0{\cdot}33$ ft.

$$K_1 \text{ per ft. thickness} = \frac{0{\cdot}35}{12} = 0{\cdot}0292$$

$$K_2 \text{ per ft. thickness} = \frac{0{\cdot}45}{12} = 0{\cdot}0375$$

$$Q = \frac{4 \times 3{\cdot}14 \times 357}{\dfrac{0{\cdot}33}{0{\cdot}0292 \times 1{\cdot}67 \times 2} + \dfrac{0{\cdot}33}{0{\cdot}0375 \times 2 \times 2{\cdot}33}}$$
$$= 851 \text{ B.Th.U. per hour}$$

Note that if the radii are given to three places of decimals $Q = 841$.

Steady Film Heat Transfer. Let Fig. 248 represent a wall of thickness x_w separating a hot fluid from a cold one. In practice it is found that the temperature gradients in the fluids are confined to comparatively thin films x_1 and x_2 in contact with the surfaces of the wall. Outside these films the temperature is constant in the bulk of the fluid. The temperature gradients therefore will be approximately as shown in Fig. 248.

For the heat flow per unit time in a fluid film we may write

$$Q = \frac{K}{x} A(T_a - T_b)$$

If $h = \frac{K}{x}$, this may be termed the heat transfer coefficient, and its units will be B.Th.U. per hour, per sq. ft., per ° F.

Then for the two films

$$Q = h_1A(T_1 - T_2) \text{ where } h_1 \text{ refers to the hot fluid}$$

$$Q = h_2A(T_3 - T_4) \text{ where } h_2 \text{ refers to the cold fluid}$$

and for the wall,

$$Q = \frac{K}{x_w} A(T_2 - T_3) \text{ where } K \text{ refers to the wall}$$

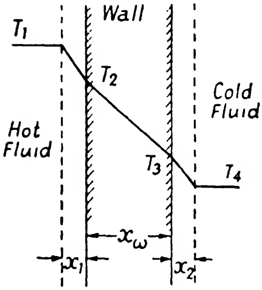

Fig. 248. Film Transfer

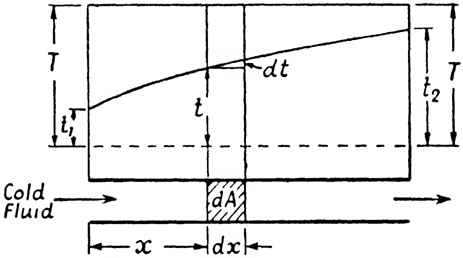

Fig. 249. Condenser Tube

Hence by adding the three equations

$$T_1 - T_4 = Q\left(\frac{1}{h_1A} + \frac{x_w}{KA} + \frac{1}{h_2A}\right)$$

$$= \frac{Q}{AF}$$

where

$$\frac{1}{F} = \frac{1}{h_1} + \frac{x_w}{K} + \frac{1}{h_2}$$

Heat Flow through a Condenser Tube. Let Fig. 249 illustrate a tube through which a cold fluid is passing, while round it is a hot fluid condensing at a constant temperature T, the temperature of the cold fluid rising from t_1 to t_2 in flowing through the tube. At a distance x from the entry the temperature of the cold fluid is t rising by dt in a small length of tube dx. It will be assumed that the flow of heat through the tube is proportional to the temperature difference on the two sides of the tube, and that no heat passes lengthways along the tube.

Let q = the average rate of heat transfer per unit area of tube surface per degree temperature difference

A = surface area of tube

w_c = weight of cold liquid passing per unit time

S_c = specific heat of cold liquid

Then for the elemental area dA of tube surface

Heat passing through small length of tube = Heat gained by cold fluid

i.e.
$$q(T - t)dA = w_c S_c dt$$

and for the whole tube surface

$$\frac{qA}{w_c S_c} = \int_{t_1}^{t_2} \frac{dt}{T - t} = \log_\varepsilon \frac{T - t_1}{T - t_2}$$

If t_m is the mean temperature difference between the fluids for the whole tube surface, then the rate of heat passing from the hot to the cold fluid is

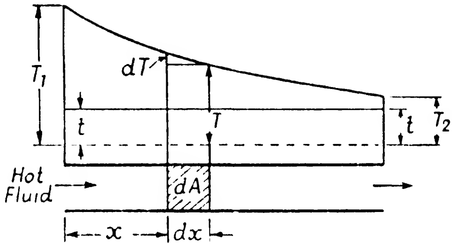

FIG. 250. EVAPORATOR TUBE

$$Q = qt_m A = w_c S_c (t_2 - t_1)$$

$$\therefore \quad t_m = \frac{w_c S_c}{qA}(t_2 - t_1) = \frac{t_2 - t_1}{\log_\varepsilon \dfrac{T - t_1}{T - t_2}} = \frac{(T - t_1) - (T - t_2)}{\log_\varepsilon \dfrac{T - t_1}{T - t_2}}$$

Thus the mean temperature difference, sometimes called the logarithmetic mean, is given by the expression,

$$\frac{\text{Initial temperature difference} - \text{Final temperature difference}}{\log_\varepsilon \dfrac{\text{Initial temperature difference}}{\text{Final temperature difference}}}$$

This will not have the same value as the arithmetic mean temperature difference $T - \dfrac{t_1 + t_2}{2}$.

Heat Flow through an Evaporator Tube. Here (Fig. 250) we have a hot fluid entering the tube at a temperature T_1, and leaving at a lower temperature T_2, while the tube is surrounded with a colder fluid which is evaporating at a constant temperature t.

Let w_h and S_h be the weight of hot fluid passing per unit time, and the specific heat of the hot fluid respectively, and let q and Q have the same meanings as in the previous paragraph.

For the elemental area dA of tube surface

$$qdA(T - t) = w_hS_hdT$$

i.e.
$$\frac{qA}{w_hS_h} = \int_{T_2}^{T_1}\frac{dT}{T - t} = \log_e\frac{T_1 - t}{T_2 - t}$$

For the whole tube surface

$$Q = qAt_m = w_hS_h(T_1 - T_2)$$

$$\therefore \quad t_m = \frac{w_hS_h}{qA}(T_1 - T_2) = \frac{T_1 - T_2}{\log_e\dfrac{T_1 - t}{T_2 - t}} = \frac{(T_1 - t) - (T_2 - t)}{\log_e\dfrac{T_1 - t}{T_2 - t}}$$

This is the same type of formula as in the previous paragraph.

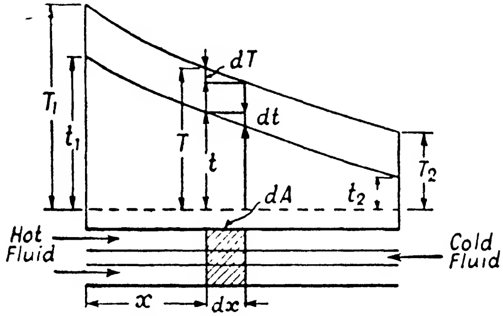

FIG. 251. COUNTER CURRENT FLOW

Counter Current Flow. Fig. 251 illustrates two concentric tubes with hot fluid entering the outer annulus at a temperature T_1 and leaving at T_2, while the cold fluid enters the inner tube at the other end at a temperature t_2 and leaves at t_1.

At a distance x from the left-hand end let the hot fluid be at a temperature T and let the cold fluid be at t. Then for a positive value of the elemental tube area dA, the value of t decreases.

Hence for the heat flow through the elemental tube area dA,

$$q(T - t)dA = - w_cS_cdt$$

or
$$qA = - w_cS_c\int_{t_1}^{t_2}\frac{dt}{T - t} \qquad . \quad . \quad (1)$$

For the heat exchange through the length x of the tube,

$$w_hS_h(T_1 - T) = w_cS_c(t_1 - t)$$

or $T_1 - T = \dfrac{w_cS_c}{w_hS_h}(t_1 - t) = a(t_1 - t)$ where $a = \dfrac{w_cS_c}{w_hS_h}$ = constant

Hence

$$T - t = T_1 - at_1 + (a - 1)t \tag{2}$$

and from the equations 1 and 2

$$qA = -w_cS_c\int_{t_1}^{t_2}\frac{dt}{T_1 - at_1 + (a-1)t}$$

$$= \frac{w_cS_c}{a-1}\log_e\frac{T_1 - at_1 + (a-1)t_1}{T_1 - at_1 + (a-1)t_2}$$

$$= \frac{w_cS_c}{a-1}\log_e\frac{T_1 - t_1}{T_1 - at_1 + (a-1)t_2} \tag{3}$$

But for the whole length of the tube,

$$w_hS_h(T_1 - T_2) = w_cS_c(t_1 - t_2)$$

$$\therefore \quad a = \frac{w_cS_c}{w_hS_h} = \frac{T_1 - T_2}{t_1 - t_2} \text{ and } a - 1 = \frac{(T_1 - T_2) - (t_1 - t_2)}{t_1 - t_2}$$

Substituting these values of a and $a - 1$ in Eq. 3,

$$T_1 - at_1 + (a-1)t_2 = T_2 - t_2$$

and

$$qA = \frac{w_cS_c}{a-1}\log_e\frac{T_1 - t_1}{T_2 - t_2} \tag{4}$$

For the whole heat flow through the tube,

$$Q = qt_mA = w_cS_c(t_1 - t_2)$$

or

$$t_m = \frac{w_cS_c}{qA}(t_1 - t_2)$$

and substituting from Eq. 4

$$t_m = \frac{w_cS_c(t_1 - t_2)}{\dfrac{w_cS_c}{a-1}\log_e\dfrac{T_1 - t_1}{T_2 - t_2}}$$

$$= (a-1)\frac{t_1 - t_2}{\log_e\dfrac{T_1 - t_1}{T_2 - t_2}}$$

$$= \frac{(T_1 - T_2) - (t_1 - t_2)}{(t_1 - t_2)} \times \frac{(t_1 - t_2)}{\log_e\dfrac{T_1 - t_1}{T_2 - t_2}}$$

$$= \frac{(T_1 - t_1) - (T_2 - t_2)}{\log_e\dfrac{T_1 - t_1}{T_2 - t_2}}$$

This is the same type of formula as in the previous paragraphs.

If the cold fluid is at a constant temperature t the formula becomes that for an evaporator tube,

i.e.
$$t_m = \frac{(T_1 - t) - (T_2 - t)}{\log_\varepsilon \dfrac{(T_1 - t)}{(T_2 - t)}}$$

If the hot fluid is at a constant temperature T the formula becomes that for a condenser tube,

i.e.
$$t_m = \frac{(T - t_1) - (T - t_2)}{\log_\varepsilon \dfrac{T - t_1}{T - t_2}}$$

Parallel Current Flow. For parallel current flow, the temperature of the cold fluid is rising from t_1 to t_2, while that of the hot fluid is falling from T_1 to T_2. In a similar manner to that in the previous paragraph it can be shown that

$$t_m = \frac{(T_1 - t_1) - (T_2 - t_2)}{\log_\varepsilon \dfrac{T_1 - t_1}{T_2 - t_2}}$$

Example 3. A condenser has a vacuum of 29 in. and deals with 50,000 lb. of steam per hour, the steam entering the condenser 0·95 dry. The cooling water enters at 55° F. and leaves at 75° F., the condensate leaving at the saturation temperature corresponding to the vacuum.

Assuming the coefficient of heat transmission through the tubes is 600 B.Th.U. per sq. ft., per hour, per ° F. temperature difference, find the total tube surface required.

Reference to the steam tables at 29 in. and 79·6° F. gives

Total heat of steam entering condenser

$$= h + xL = 47{\cdot}6 + 0{\cdot}95 \times 1{,}048{\cdot}5$$
$$= 1{,}043{\cdot}7 \text{ B.Th.U. per lb.}$$

Total heat left in condensate

$$= 79{\cdot}6 - 32 = 47{\cdot}6 \text{ B.Th.U. per lb.}$$

$\therefore$ Heat carried away by cooling water

$$= 50{,}000\ (1{,}043{\cdot}7 - 47{\cdot}6)$$
$$= 50{,}000 \times 996 \text{ B.Th.U. per hour.}$$

If T = temperature of condensing steam

t_1 = temperature of entering cooling water

t_2 = temperature of leaving cooling water

Then the mean temperature difference on the two sides of a tube

$$t_m = \frac{(T - t_1) - (T - t_2)}{\log_e \dfrac{T - t_1}{T - t_2}} = \frac{(79{\cdot}6 - 55) - (79{\cdot}6 - 75)}{2{\cdot}3 \log_{10} \dfrac{79{\cdot}6 - 55}{79{\cdot}6 - 75}}$$

$$= 11{\cdot}94^\circ \text{ F.}$$

But $Q = qt_mA$

$$\therefore \quad A = \frac{Q}{qt_m} = \frac{50{,}000 \times 996}{600 \times 11{\cdot}94} = 6{,}950 \text{ sq. ft.}$$

Note. If the condensate left at a somewhat lower temperature than the saturation temperature corresponding to the vacuum at steam entry, then the problem could be treated as one of counter current flow and $t_m = \dfrac{(T_1 - t_1) - (T_2 - t_2)}{\log_e \dfrac{T_1 - t_1}{T_2 - t_2}}$

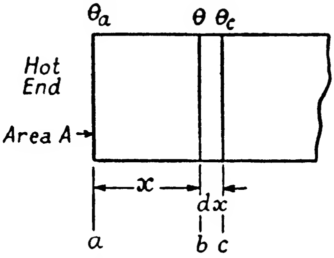

FIG. 252. HEAT FLOW ALONG A BAR

where T_1 = temperature of entering steam

T_2 = temperature of condensate leaving

General Equation for Conduction. Consider a uniform bar as in Fig. 252 heated at one end.

Let ϕ = the temperature difference between the bar and its surroundings, the latter temperature being constant

A = the sectional area of the bar

P = the perimeter of the bar

q = the heat flow along the bar

K = the conductivity of the bar

The change in the value of ϕ between sections b and $c = \dfrac{d\phi}{dx} \,.\, dx$

$$\therefore \qquad \phi_c = \phi + \frac{d\phi}{dx} \,.\, dx$$

The heat flow at section $b = q_b = -KA\dfrac{d\phi}{dx}$

The heat flow at section $c = q_c = -KA\dfrac{d}{dx}\left(\phi + \dfrac{d\phi}{dx}\,.\,dx\right)$

$$= -KA\left(\frac{d\phi}{dx} + \frac{d^2\phi}{dx^2}\,.\,dx\right)$$

The difference of heat flow between the two sections b and c must be lost from the small sectional surface Pdx.

Thus if e = heat lost from this surface per unit time, per unit area, per unit difference of temperature

then $-ePdx\,\phi = q_b - q_c = -KA\left(\dfrac{d\phi}{dx} + \dfrac{d^2\phi}{dx^2}\,dx\right) + KA\dfrac{d\phi}{dx}$

i.e. $ePdx\,\phi = KA\dfrac{d^2\phi}{dx^2}\,.\,dx$

or $\dfrac{\frac{d^2\phi}{dx^2}}{\phi} = \dfrac{eP}{KA} = \mu^2$ say, where μ is constant

The solution of the equation

$$\frac{d^2\phi}{dx^2} = \mu^2\phi \text{ is } \phi = a\varepsilon^{\mu x} + b\varepsilon^{-\mu x}.$$

But when $x = 0$ then $\phi = \phi_a$

and when $x = \infty$ then $\phi = 0$

Hence $a = 0$ and $b = \phi_a$

Thus the equation for steady heat flow along the bar is

$$\phi = \phi_a\varepsilon^{-\mu x} = \phi_a\varepsilon^{-\sqrt{\frac{eP}{KA}}\,.\,x}$$

Dimensions of Thermal Quantities. For the present purpose let us assume the following symbols—

Q = Quantity of heat in heat units

q = Quantity of heat per unit time in heat units

W = Quantity of work, or heat in work units

J = Mechanical equivalent of heat

h = Heat transfer coefficient, or heat transferred per unit time, per unit area, per unit temperature difference

K = Thermal conductivity from $K = \frac{Q}{at}\frac{dx}{d\theta}$

θ = Temperature or temperature difference

x = Length or distance a = Area

d = Distance or diameter

t = Time v = Velocity

S = Specific heat L = Latent heat

m = Mass ρ = Density

μ = Coefficient of viscosity

Although temperature almost certainly depends on the fundamental dimensions of Mass, Length and Time (*M.L.T.*), the nature of this dependence is not clear. One method of avoiding this difficulty is to treat temperature as a fourth fundamental dimension, represented hereafter by ϕ.

If this is done then some decision must be made as to the treatment of specific heat, which can be defined in two ways—

(*a*) as the ratio of the heat necessary to raise a mass m of a body through a temperature difference θ, to the heat necessary to raise the same mass of water through the same temperature difference,

(*b*) as the quantity of heat necessary to raise unit mass of the body through unit temperature difference.

In the first case the specific heat has no dimensions, as it is a mere number, while in the second case, we shall see that it has dimensions in terms of the four fundamental dimensions assumed.

There are thus three cases to consider—

(i) Four fundamental dimensions ($M.L.T.\phi$) with specific heat dimensionless.

(ii) Four fundamental dimensions with J dimensionless.

(iii) Three fundamental dimensions with S and J dimensionless.

(i) *Four fundamental dimensions. Specific heat dimensionless.* As temperature is adopted as a fourth fundamental dimension, this as already stated will be represented by ϕ.

A quantity of heat Q given to a mass m is measured by the product mass × specific heat × change in temperature. As specific heat is considered dimensionless, then the dimensions of Q will be $M\phi$.

The mechanical equivalent of heat J is defined by the ratio $\frac{\text{work units}}{\text{heat units}}$. Hence the dimensions of J are

$$\frac{MLT^{-2} \times L}{M\phi} = L^2T^{-2}\phi^{-1}$$

In a similar way the dimensions of other thermal quantities may be found as given below in Table XXX.

TABLE XXX

Quantity	Definition	Dimensions
Temperature . . .	θ	ϕ
Specific heat . . .	S	Dimensionless
Heat	$Q = mS\theta$	$M\phi$
Mechanical equivalent .	$J = \frac{W}{Q}$	$\frac{ML^2T^{-2}}{M\phi} = L^2T^{-2}\phi^{-1}$
Thermal capacity . .	mS	M
Latent heat . . .	$L = \frac{Q}{m} = \frac{mS\theta}{m}$	ϕ
Thermal conductivity .	$K = \frac{Q\,dx}{ta\,d\theta}$	$\frac{M\phi}{TL^2}\frac{L}{\phi} = ML^{-1}T^{-1}$
Coefficient of heat transfer	$h = \frac{Q}{ta\theta}$	$\frac{M\phi}{TL^2\phi} = ML^{-2}T^{-1}$
Diffusivity . . .	$c = \frac{K}{\rho S}$	$\frac{ML^{-1}T^{-1}}{ML^{-3}} = L^2T^{-1}$
Coefficient of absorption .	$\frac{Q}{ta\theta}$	$\frac{M\phi}{TL^2\phi} = ML^{-2}T^{-1}$

In the above table it will be seen that as J is a dimensioned quantity, the dimensions of any heat quantity will have different values according to whether it is expressed in heat or work units. In the two following systems, where J is considered dimensionless, this does not arise.

(ii) *Four fundamental dimensions. Mechanical equivalent dimensionless.* In this system, as J is dimensionless, then a quantity of heat Q has the dimensions of work, namely ML^2T^{-2}. Specific heat now becomes a dimensioned quantity. The dimensions of other thermal quantities are given below in Table XXXI.

TABLE XXXI

Quantity	Definition	Dimensions
Temperature . . .	θ	ϕ
Mechanical equivalent .	J	Dimensionless
Heat	$Q = \frac{W}{J}$	ML^2T^{-2}
Specific heat . . .	$S = \frac{Q}{m\theta}$	$\frac{ML^2T^{-2}}{M\phi} = L^2T^{-2}\phi^{-1}$
Thermal capacity . .	mS	$ML^2T^{-2}\phi^{-1}$
Latent heat . . .	$\frac{Q}{m}$	$\frac{ML^2T^{-2}}{M} = L^2T^{-2}$
Thermal conductivity .	$K = \frac{Q\,dx}{ta\,d\theta}$	$\frac{ML^2T^{-2}}{TL^2}\,\frac{L}{\phi} = MLT^{-3}\phi^{-1}$
Coefficient of heat transfer	$h = \frac{Q}{ta\theta}$	$\frac{ML^2T^{-2}}{TL^2\phi} = MT^{-3}\phi^{-1}$
Diffusivity . . .	$c = \frac{K}{\rho S}$	$\frac{MLT^{-3}\phi^{-1}}{ML^{-3}L^2T^{-2}\phi^{-1}} = L^2T^{-1}$
Coefficient of absorption .	$\frac{Q}{ta\theta}$	$\frac{ML^2T^{-2}}{TL^2\phi} = MT^{-3}\phi^{-1}$

(iii) *Three fundamental dimensions* (*M.L.T.*). *S and J dimensionless.* The dimensions of the various quantities in this system are given in Table XXXII. It will be seen that temperature now becomes a dimensioned quantity, with the dimensions of the square of a velocity.

TABLE XXXII

Quantity	Definition	Dimensions
Specific heat . . .	S	Dimensionless
Mechanical equivalent .	J	Dimensionless
Heat	$Q = \frac{W}{J}$	ML^2T^{-2}
Temperature . . .	$\theta = \frac{Q}{mS}$	$\frac{ML^2T^{-2}}{M} = L^2T^{-2}$
Thermal capacity . .	mS	M
Latent heat . . .	$L = \frac{Q}{m}$	$\frac{ML^2T^{-2}}{M} = L^2T^{-2}$
Thermal conductivity .	$K = \frac{Q\,dx}{ta\,d\theta}$	$\frac{ML^2T^{-2}}{TL^2}\ \frac{L}{L^2T^{-2}}$ $= ML^{-1}T^{-1}$
Coefficient of heat transfer	$h = \frac{Q}{ta\theta}$	$\frac{ML^2T^{-2}}{TL^2L^2T^{-2}} = ML^{-2}T^{-1}$
Diffusivity . . .	$c = \frac{K}{\rho S}$	$\frac{ML^{-1}T^{-1}}{ML^{-3}} = L^2T^{-1}$
Coefficient of absorption .	$\frac{Q}{ta\theta}$	$\frac{ML^2T^{-2}}{TL^2L^2T^{-2}} = ML^{-2}T^{-1}$

Dimensional Homogeneity applied to Heat Transfer. Let the variables concerned be v, d, ρ, S, μ, K, h, where the symbols have the meanings given on pp. 622 *et seq.*

Assume four fundamental dimensions (M, L, T, ϕ), and let J be dimensionless. There will then be $7 - 4 = 3$ dimensionless coefficients obtainable. Let these be C_1, C_2, and C_3.

(*a*) If $C_1 = v^\alpha d^\beta \rho^\gamma \mu^\delta S^\varepsilon$

then using the dimensions of thermal quantities given on p. 625

$$M^\circ L^\circ T^\circ \phi^\circ = L^\alpha T^{-\alpha} L^\beta M^\gamma L^{-3\gamma} M^\delta L^{-\delta} T^{-\delta} L^{2\varepsilon} T^{-2\varepsilon} \phi^{-\varepsilon}$$

and equating appropriate indices—

$$\begin{array}{lll}
\text{For } M & 0 = \gamma + \delta & \text{whence } \gamma = -\delta \\
\phi & 0 = -\varepsilon & \varepsilon = 0 \\
T & 0 = -\alpha - \delta - 2\varepsilon & \alpha = -\delta - 2\varepsilon = -\delta \\
L & 0 = \alpha + \beta - 3\gamma - \delta + 2\varepsilon & \beta = -\alpha + 3\gamma + \delta \\
 & & \quad - 2\varepsilon \\
 & & \quad = \delta - 3\delta + \delta = -\delta
\end{array}$$

and substituting in the original equation

$$C_1 = v^{-\delta} d^{-\delta} \rho^{-\delta} \mu^\delta = f\left(\frac{vd\rho}{\mu}\right)$$

(*b*) If $C_2 = v^\alpha d^\beta \mu^\gamma S^\delta K^\varepsilon$

$$M^\circ L^\circ T^\circ \phi^\circ = L^\alpha T^{-\alpha} L^\beta M^\gamma L^{-\gamma} T^{-\gamma} L^{2\delta} T^{-2\delta} \phi^{-\delta} M^\varepsilon L^\varepsilon T^{-3\varepsilon} \phi^{-\varepsilon}$$

$$\begin{array}{lll}
\text{For } M & 0 = \gamma + \varepsilon & \text{whence } \gamma = -\varepsilon \\
\phi & 0 = -\delta - \varepsilon & \delta = -\varepsilon \\
T & 0 = -\alpha - \gamma - 2\delta - 3\varepsilon & \alpha = -\gamma - 2\delta - 3\varepsilon \\
 & & \quad = \varepsilon + 2\varepsilon - 3\varepsilon = 0 \\
L & 0 = \alpha + \beta - \gamma + 2\delta + \varepsilon & \beta = -\alpha + \gamma - 2\delta - \varepsilon \\
 & & \quad = 0 - \varepsilon + 2\varepsilon - \varepsilon \\
 & & \quad = 0
\end{array}$$

$$C_2 = \mu^{-\varepsilon} S^{-\varepsilon} K^\varepsilon = f\left(\frac{\mu S}{K}\right)$$

(*c*) If $C_3 = v^\alpha d^\beta \rho^\gamma K^\delta h^\varepsilon$

$$M^\circ L^\circ T^\circ \phi^\circ = L^\alpha T^{-\alpha} L^\beta M^\gamma L^{-3\gamma} M^\delta L^\delta T^{-3\delta} \phi^{-\delta} M^\varepsilon T^{-3\varepsilon} \phi^{-\varepsilon}$$

$$\begin{array}{lll}
\text{For } \phi & 0 = -\delta - \varepsilon & \text{whence } \delta = -\varepsilon \\
M & 0 = \gamma + \delta + \varepsilon & \gamma = -\delta - \varepsilon = \varepsilon - \varepsilon \\
 & & \quad = 0 \\
T & 0 - \alpha - 3\delta - 3\varepsilon & \alpha = -3\delta - 3\varepsilon \\
 & & \quad = 3\varepsilon - 3\varepsilon = 0
\end{array}$$

$$L \quad 0 = \alpha + \beta - 3\gamma + \delta \qquad \beta = -\alpha + 3\gamma - \delta = 0 + 0 + \varepsilon = \varepsilon$$

$$C_3 = d^{\varepsilon} K^{-\varepsilon} h^{\varepsilon} = f\left(\frac{hd}{K}\right)$$

and hence $\dfrac{hd}{K} = f\left(\dfrac{vdp}{\mu} \cdot \dfrac{\mu S}{K}\right)$

Reynolds' Analogy. Reynolds' originated the idea of two distinct forms of fluid flow, namely laminar or viscous flow, and turbulent flow.

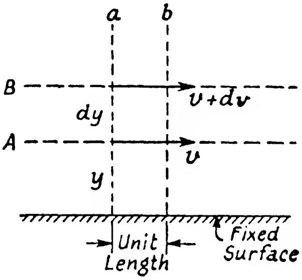

FIG. 253

As regards laminar flow, consider a fixed solid surface (Fig. 253) past which fluid is flowing, the velocities of the fluid in two parallel planes A and B, at distances y and $y + dy$ from the surface, being u and $u + du$. Let a and b be two parallel planes normal to the surface and unit length apart. Consider also a thickness of fluid normal to the paper equal to unit length.

Then by the definition of the coefficient of viscosity μ, the shear stress (i.e. fluid resistance per unit area of surface) is given by $R_1 = \mu \dfrac{du}{dy}$.

As regards turbulent flow, consider a small mass α having a velocity u in plane A moving into plane B where the flow velocity is $u + du$. To take its place we can imagine an equal small mass α in plane B being transferred to plane A. The virtual turbulent shear stress equals the increase or decrease of momentum of the mass α per unit time. If therefore a mass m passes per unit area of surface, per unit time, then the shear stress for turbulent flow $R_2 = mdu$.

Hence the total shear stress $R = R_1 + R_2 = \mu \dfrac{du}{dy} + m \cdot du$

In passing it is worth while to check dimensions of each side of this equation thus—

$$MLT^{-2}L^{-2} = ML^{-1}T^{-1}\frac{LT^{-1}}{L} + \frac{M}{L^2}T^{-1}LT^{-1}$$

i.e. $$ML^{-1}T^{-2} = ML^{-1}T^{-2} + ML^{-1}T^{-2}$$

As regards heat flow Reynolds considered that the heat carried off per unit time by any fluid from a surface was due to—

(a) the natural heat diffusion through the fluid when at rest.

(b) the eddies caused by turbulent motion, which mixes up the fluid and continually brings fresh particles into contact with the surface (i.e. convection).

As regards (*a*) the heat flow may be considered as normal to the fixed surface from which heat is being transferred, and the ordinary conduction equation gives the heat transferred per unit time, per unit area of surface as $Q_1 = -K\dfrac{d\theta}{dy}$, the minus sign indicating that as y increases, the temperature θ decreases.

As regards (*b*) we can imagine a small mass α in the plane A (Fig. 253), at a temperature θ being transferred to the plane B where the temperature is $\theta + d\theta$, and carrying a quantity of heat $\alpha S\theta$, while a similar particle which takes its place, carries a quantity of heat $\alpha S(\theta + d\theta)$ from plane B to plane A. The net quantity of heat transferred is therefore $-\alpha S d\theta$, the minus sign indicating that as y increases θ decreases. Thus if m has the same significance as previously, the heat flow due to turbulence per unit time per unit area of surface is given by the equation $Q_2 = -mSd\theta$.

Hence the total heat flow is $Q = Q_1 + Q_2 = -K\dfrac{d\theta}{dy} - mSd\theta$.

Checking dimensions, using $MLT\phi$ and making J dimensionless,

$$\frac{ML^2T^{-2}}{L^2T} = MLT^{-3}\phi^{-1}\frac{\phi}{L} + \frac{M}{L^2T}L^2T^{-2}\phi^{-1}\phi$$

$$MT^{-3} = MT^{-3} + MT^{-3}$$

Considering the two equations for R and Q it would appear that there is a simple analogy between fluid flow and heat flow, a temperature gradient being analogous to a velocity gradient. There are, however, limitations to its validity. Modern work has shown that in turbulent flow there exists a laminar sub-layer in the immediate neighbourhood of the solid surface. The heat from the solid surface must flow through this sub-layer before it can be carried away by turbulent mixing. The simple analogy has therefore been considerably modified.

EXAMPLES XVI

1. A sphere of 12 in. radius is at a temperature of 400° F. It is evenly covered with lagging which is 3 in. thick. The conductivity of the lagging is 0·5 B.Th.U. per sq. ft. per hour, per degree F. temperature difference, per inch thickness. The outside surface of the lagging is at a temperature of 100° F. Find the rate of heat leakage, proving any formula you use.

(*U. Nottm., B.Sc. (Eng.) Pass*, 1951.)

2. A thin pipe of 3 in. external diameter is covered with two layers of lagging, each layer being 2 in. thick. The conductivity of the inner layer of lagging is 0·48, and that of the outer layer is 0·6 B.Th.U. per sq. ft., per hour, per degree F. per in. thickness. The pipe contains a fluid at 200° F., the temperature of the outside surface of the outer layer of lagging being 60° F. What is the heat leakage per hour, per foot run of pipe?

(*U. Nottm., B.Sc. (Eng.)*, 1951.)

3. A cold room has one of the walls which measures 15 ft. by 7 ft. 6 in. constructed of brick 4½ in. thick, insulated externally by cork slabbing 3 in. thick. The cork is protected externally by wood 1 in. thick.

Estimate the heat leakage through the wall per 24 hours, if the interior temperature is 28° F., and the exterior 65° F.

The thermal conductivities of brick, cork and wood are 6·5, 0·3 and 1·2 respectively, these values being measured in B.Th.U. per hour for an area of 1 sq. ft., thickness 1 in., and a temperature gradient of 1° F.

What will be the temperatures of the interfaces?

(*U.L.*, *B.Sc.* (*Eng.*), 1944.)

4. Describe the difference between a parallel-flow and a counter-flow cooler, showing graphically how the temperatures vary with the length of the cooler in each case.

Explain the term "mean effective temperature difference" and show that in the case of a tubular oil cooler where the rise in temperature of the water may be neglected, the value of the mean effective temperature difference is given by $(\theta_1 - \theta_2)/\log_e \frac{\theta_1}{\theta_2}$, where θ_1 and θ_2 are the initial and final temperature differences respectively.

In such an oil cooler, the oil enters the tubes at a temperature of 160° F. and leaves at 75° F., the water temperature being sensibly constant at 55° F. Find the weight of oil which can be cooled per sq. ft. of active area per hour, if the coefficient of heat transmission is 75 B.Th.U. sq. ft. hr.° F. units and the specific heat of the oil is 0·55.

(*U.L.*, *B.Sc.* (*Eng.*), 1947.)

APPENDIX

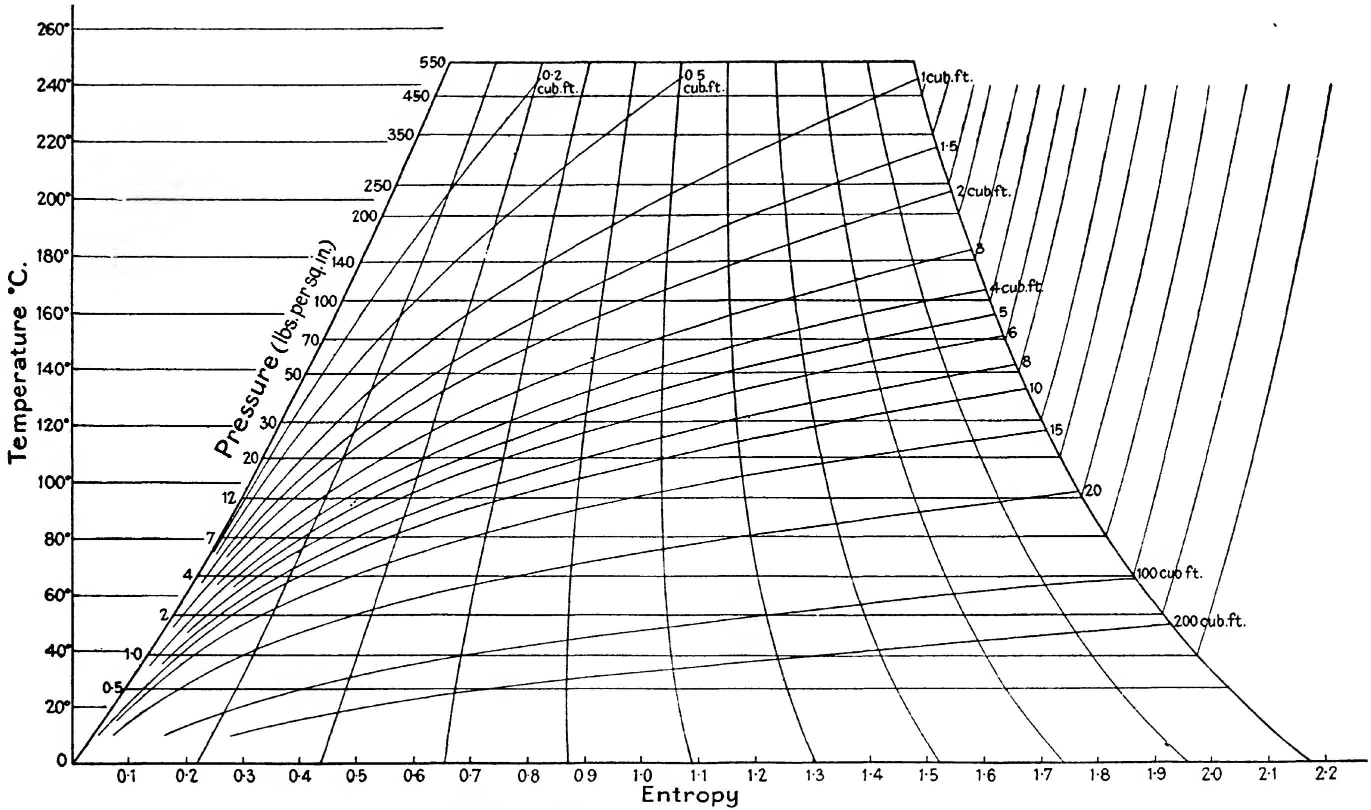

Fig. 254. Temperature-entropy Diagram for Water and Steam

TABLES OF THE PROPERTIES OF STEAM

The values in Tables I and II are taken from *Steam Power* (by Professor W. E. Dalby, F.R.S.), by the kind permission of the author and the publishers, Messrs. Edward Arnold & Co.

The values in Tables III and IV are taken from the *Abridged Callendar Tables Centigrade Units* by the kind permission of the author and the publishers, Messrs. Edward Arnold & Co.

More modern figures are given in *Abridged Callendar Steam Tables* revised by G. S. Callendar and published by Messrs. Edward Arnold & Co.

TABLE I

PROPERTIES OF DRY SATURATED STEAM, CENTIGRADE UNITS

Pressure lb. per sq. in. (Abs.) p	Temperature t° C.	Volume cu. ft. per lb. Steam v_s	Total Heat of Water C.H.U. per lb. h or I_w	Latent Heat of Steam C.H.U. per lb. L	Total Heat of Dry Steam C.H.U. per lb. I_s	Internal Energy C.H.U. per lb. E	ENTROPY PER LB.		
							Water ϕ_w	Increase Due to L L/T	Steam ϕ_s
0·0892	0·0	3276	0·0	594·27	594·27	564·21	0·0	2·1760	2·1760
0·1	1·59	2940	1·59	593·44	595·03	564·79	0·0057	2·1605	2·1662
0·2	11·69	1524·4	11·67	588·14	599·81	568·45	0·0417	2·0651	2·1068
0·3	17·99	1037·7	17·94	584·83	602·77	570·74	0·0635	2·0092	2·0727
0·4	22·66	790·7	22·60	582·37	604·97	572·44	0·0794	1·9688	2·0482
0·5	26·41	640·50	26·34	580·39	606·73	573·80	0·0922	1·9377	2·0299
1	38·74	333·12	38·63	573·83	612·46	578·20	0·1323	1·8401	1·9724
2	52·27	173·54	52·16	566·51	618·67	582·98	0·1747	1·7412	1·9159
3	60·83	118·56	60·70	561·83	622·53	585·95	0·2007	1·6826	1·8833
4	67·23	90·54	67·10	558·28	625·38	588·14	0·2197	1·6403	1·8600
5	72·38	73·44	72·26	555·38	627·64	589·88	0·2346	1·6076	1·8422
6	76·72	61·91	76·61	552·92	629·53	591·33	0·2472	1·5805	1·8277
7	80·49	53·59	80·39	550·76	631·15	592·58	0·2579	1·5577	1·8156
8	83·84	47·30	83·75	548·82	632·57	593·67	0·2673	1·5376	1·8049
9	86·84	42·36	86·76	547·08	633·84	594·64	0·2757	1·5199	1·7956
10	89·58	38·39	89·51	545·50	635·01	595·55	0·2833	1·5041	1·7874
12	94·44	32·37	94·40	542·61	637·01	597·08	0·2967	1·4764	1·7731
14	98·66	28·02	98·66	540·12	638·78	598·46	0·3081	1·4530	1·7611
14·689	100·00	26·79	100·00	539·30	639·30	598·85	0·31186	1·44546	1·7573
16	102·41	24·73	102·43	537·82	640·25	599·58	0·3184	1·4322	1·7506
18	105·79	22·16	105·84	535·75	641·59	600·61	0·3274	1·4140	1·7414
20	108·87	20·08	108·95	533·87	642·82	601·56	0·3356	1·3977	1·7333
22	111·71	18·37	111·83	532·09	643·92	602·41	0·3430	1·3828	1·7258
24	114·34	16·93	114·50	530·43	644·93	603·18	0·3499	1·3690	1·7189
26	116·80	15·71	116·98	528·87	645·85	603·89	0·3563	1·3563	1·7126
28	119·11	14·66	119·32	527·42	646·74	604·58	0·3622	1·3447	1·7069
30	121·28	13·74	121·51	526·01	647·52	605·18	0·3679	1·3337	1·7016
32	123·35	12·94	123·63	524·67	648·30	605·78	0·3732	1·3234	1·6966
34	125·31	12·22	125·63	523·40	649·02	606·34	0·3782	1·3137	1·6919
36	127·17	11·59	127·52	522·17	649·69	606·85	0·3830	1·3044	1·6874
38	128·96	11·02	129·34	521·00	650·34	607·35	0·3874	1·2957	1·6831
40	130·67	10·500	131·08	519·87	650·95	607·82	0·3918	1·2874	1·6792
50	138·30	8·520	138·89	514·71	653·60	609·87	0·4109	1·2511	1·6620
60	144·79	7·184	145·54	510·22	655·76	611·53	0·4269	1·2210	1·6479
70	150·46	6·218	151·37	506·23	657·60	612·96	0·4407	1·1952	1·6359
80	155·52	5·487	156·61	502·59	659·20	614·20	0·4530	1·1726	1·6256
90	160·09	4·913	161·35	499·24	660·59	615·27	0·4620	1·1525	1·6165
100	164·28	4·451	165·71	496·12	661·83	616·24	0·4739	1·1343	1·6082
110	168·15	4·070	169·75	493·18	662·93	617·09	0·4831	1·1176	1·6007
120	171·75	3·751	173·52	490·40	663·92	617·85	0·4915	1·1023	1·5938
130	175·13	3·479	177·07	487·76	664·83	618·55	0·4994	1·0882	1·5876
140	178·31	3·245	180·42	485·27	665·69	619·23	0·5068	1·0750	1·5818
150	181·31	3·041	183·59	482·90	666·49	619·85	0·5138	1·0627	1·5765
160	184·16	2·862	186·61	480·61	667·22	620·42	0·5204	1·0511	1·5715
170	186·88	2·703	189·50	478·40	667·90	620·95	0·5266	1·0400	1·5666
180	189·48	2·562	192·27	476·26	668·53	621·44	0·5326	1·0294	1·5620

TABLE I

PROPERTIES OF DRY SATURATED STEAM, CENTIGRADE UNITS (*contd.*)

Pressure lb. per sq. in (Abs.) p	Temperature t° C.	Volume cu. ft. per lb. Steam v_s	Total Heat of Water C.H.U. per lb. h or I_w	Latent Heat of Steam C.H.U. per lb. L	Total Heat of Dry Steam C.H.U. per lb I_s	Internal Energy C.H.U. per lb E	ENTROPY PER LB.		
							Water ϕ_w	Increase Due to L L/T	Steam ϕ_s
190	191·97	2·435	194·94	474·19	669·13	621·91	0·5383	1·0194	1·5577
200	194·35	2·320	197·49	472·20	669·69	622·35	0·5437	1·0101	1·5538
210	196·66	2·216	199·94	470·26	670·20	622·75	0·5490	1·0012	1·5502
220	198·87	2·120	202·32	468·38	670·70	623·14	0·5540	0·9925	1·5465
230	201·02	2·034	204·64	466·55	671·19	623·52	0·5588	0·9841	1·5429
240	203·09	1·954	206·88	464·76	671·64	623·88	0·5635	0·9760	1·5395
250	205·10	1·880	209·07	463·01	672·08	624·23	0·5680	0·9682	1·5362
260	207·04	1·811	211·17	461·31	672·48	624·54	0·5724	0·9608	1·5332
270	208·93	1·748	213·23	459·65	672·88	624·86	0·5767	0·9536	1·5303
280	210·77	1·689	215·25	458·02	673·27	625·17	0·5808	0·9466	1·5274
300	214·32	1·583	219·12	454·85	673·97	625·72	0·5887	0·9332	1·5219
350	222·45	1·368	228·08	447·44	675·52	626·95	0·6067	0·9029	1·5096
400	229·75	1·206	236·21	440·63	676·84	628·02	0·6228	0·8763	1·4991

TABLE II

PROPERTIES OF DRY SATURATED STEAM

Tem-perature t° C.	Pressure lb. per sq. in. p	VOLUME CU. FT. PER LB.		Total Heat of Water lb.-calories per lb. h	Latent Heat of Steam lb.-calories per lb. L	Total Heat of Steam lb.-calories per lb. H_{sa}	Internal Energy lb.-calories per lb. E_{sa}	ENTROPY PER LB.		
		Dry Steam V_{sa}	Water v_w					Water ϕ_w	Increase Due to Latent Heat L/T	Steam ϕ_{sa}
0	0·0892	3275·9	0·01602	0·0	594·27	594·27	564·21	0·0	2·17602	2·17602
10	0·1788	1693·8	0·01603	9·98	589·03	599·01	567·85	0·03585	2·08064	2·11650
20	0·3400	922·2	0·01605	19·94	583·78	603·72	571·48	0·07046	1·99174	2·06220
30	0·6162	525·8	0·01609	29·91	578·49	608·40	575·07	0·10393	1·90855	2·01248
40	1·0703	312·4	0·01614	39·89	573·15	613·04	578·64	0·13631	1·83057	1·96688
50	1·7888	192·7	0·01621	49·88	567·75	617·63	582·17	0·16770	1·75720	1·92490
60	2·8873	122·9	0·01629	59·87	562·29	622·16	583·66	0·19815	1·68805	1·88620
70	4·5156	80·8	0·01638	69·88	556·72	626·60	589·07	0·22774	1·62265	1·85039
80	6·8627	54·6	0·01648	79·90	551·05	630·95	592·41	0·25652	1·56061	1·81713
90	10·161	37·82	0·01659	89·94	545·25	635·19	595·67	0·28454	1·50165	1·78619
100	14·689	26·79	0·01671	100·00	539·30	639·30	598·83	0·31186	1·44546	1·75732
110	20·777	19·37	0·01684	110·09	533·17	643·26	601·86	0·33853	1·39174	1·73027
120	28·808	14·271	0·01698	120·22	526·85	647·07	604·78	0·36460	1·34026	1·70485
130	39·213	10·696	0·01713	130·40	520·32	650·72	607·58	0·39011	1·29081	1·68092
140	52·482	8·143	0·01729	140·62	513·57	654·19	610·23	0·41511	1·24321	1·65832
150	69·150	6·289	0·01746	150·91	506·56	657·47	612·73	0·43963	1·19726	1·63689
160	89·80	4·923	0·01765	161·26	499·29	660·55	615·08	0·46373	1·15284	1·61657
170	115·06	3·902	0·01785	171·69	491·75	663·44	617·27	0·48743	1·10981	1·59724
180	145·59	3·128	0·01807	182·21	483·93	666·14	619·30	0·51078	1·06806	1·57884
190	182·08	2·534	0·01831	192·83	475·82	668·65	621·19	0·53381	1·02747	1·56128
200	225·24	2·074	0·01856	203·55	467·41	670·96	622·91	0·55654	0·98798	1·54452
210	275·78	1·713	0·01885	214·40	458·69	673·09	624·48	0·57904	0·94947	1·52851
220	334·38	1·428	0·01914	247·74	449·69	675·06	625·93	0·60128	0·91198	1·51326
240	478·74	1·018	0·01980	225·37	430·21	678·55	628·43	0·64517	0·83962	1·48479

TABLE III

Total Heat of Superheated Steam

(For values of total heat of superheated steam at high temperatures and pressures see Table D, p. 644)

Pressure Absolute lb. per sq. in.	Sat.	Superheat in Degrees (Centigrade)									
	0°	+ 10°	+ 20°	+ 30°	+ 40°	+ 50°	+ 60°	+ 70°	+ 80°	+ 90°	+ 100°
15	639·54	644·54	649·53	654·49	659·43	664·36	669·25	674·15	679·04	683·92	688·78
20	642·83	647·89	652·91	657·91	662·90	667·85	672·79	677·72	682·63	687·52	692·42
30	647·55	652·71	657·81	662·89	667·94	672·95	677·95	682·93	687·88	692·82	697·75
40	650·96	656·19	661·37	666·50	671·61	676·69	681·73	686·76	691·76	696·73	701·69
50	653·60	658·90	664·15	669·35	674·50	679·63	684·71	689·78	694·81	699·84	704·83
60	655·78	661·14	666·44	671·69	676·89	682·07	687·20	692·29	697·37	702·42	707·45
70	657·61	663·04	668·39	673·69	678·94	684·14	689·33	694·46	699·56	704·64	709·70
80	659·20	664·67	670·07	675·43	680·73	685·97	691·18	696·35	701·49	706·59	711·68
90	660·59	666·11	671·57	676·96	682·30	687·58	692·83	698·03	703·20	708·33	713·44
100	661·83	667·39	672·88	678·33	683·70	689·02	694·30	699·53	704·73	709·90	715·03
120	663·93	669·60	675·19	680·70	686·14	691·54	696·88	702·17	707·42	712·64	717·81
140	665·70	671·46	677·13	682·71	688·23	693·68	699·08	704·42	709·73	714·98	720·20
160	667·21	673·03	678·77	684·43	690·01	695·52	700·98	706·37	711·72	717·02	722·28
180	668·53	674·43	680·24	685·96	691·60	697·17	702·66	708·12	713·51	718·85	724·16
200	669·68	675·64	681·52	687·30	693·00	698·61	704·17	709·66	715·10	720·48	725·82
250	672·07	678·20	684·23	690·15	695·97	701·73	707·38	712·98	718·51	723·99	729·41
300	673·96	680·24	686·40	692·45	698·39	704·25	710·01	715·69	721·32	726·88	732·38
350	675·52	681·94	688·22	694·38	700·44	706·38	712·24	718·01	723·72	729·36	734·93
400	676·84	683·38	689·77	696·05	702·19	708·22	714·16	720·02	725·81	731·51	737·17
450	677·97	684·62	691·12	697·49	703·72	709·84	715·88	721·81	727·65	733·44	739·14
500	678·97	685·72	692·32	698·77	705·09	711·30	717·40	723·41	729·32	735·17	740·93

TABLE IV

Entropy of Superheated Steam

Pressure lb. per sq. in.		Degrees of Superheat (Centigrade)									
Abs.	Gauge	+ 10°	+ 20°	+ 30°	+ 40°	+ 50°	+ 60°	+ 70°	+ 80°	+ 90°	+ 100°
15	0·31	1·7689	1·7817	1·7941	1·8062	1·8180	1·8295	1·8406	1·8515	1·8622	1·8725
20	5·31	1·7463	1·7590	1·7713	1·7832	1·7948	1·8061	1·8171	1·8279	1·8384	1·8486
30	15·31	1·7146	1·7271	1·7392	1·7509	1·7624	1·7734	1·7843	1·7949	1·8052	1·8153
40	25·31	1·6922	1·7045	1·7165	1·7282	1·7395	1·7505	1·7612	1·7716	1·7818	1·7918
50	35·31	1·6748	1·6870	1·6990	1·7105	1·7218	1·7327	1·7433	1·7537	1·7638	1·7737
60	45·31	1·6605	1·6728	1·6846	1·6961	1·7073	1·7182	1·7288	1·7390	1·7491	1·7589
70	55·31	1·6486	1·6608	1·6726	1·6841	1·6952	1·7060	1·7165	1·7268	1·7368	1·7465
80	65·31	1·6381	1·6503	1·6621	1·6736	1·6846	1·6954	1·7059	1·7161	1·7261	1·7358
90	75·31	1·6290	1·6412	1·6530	1·6644	1·6754	1·6862	1·6966	1·7068	1·7167	1·7264
100	85·31	1·6207	1·6329	1·6447	1·6560	1·6670	1·6778	1·6882	1·6984	1·7083	1·7179
120	105·31	1·6065	1·6187	1·6304	1·6418	1·6528	1·6635	1·6739	1·6840	1·6938	1·7034
140	125·31	1·5944	1·6066	1·6183	1·6297	1·6407	1·6513	1·6617	1·6717	1·6815	1·6911
160	145·31	1·5840	1·5962	1·6079	1·6194	1·6303	1·6409	1·6512	1·6613	1·6711	1·6806
180	165·31	1·5748	1·5870	1·5987	1·6101	1·6210	1·6317	1·6420	1·6520	1·6618	1·6713
200	185·31	1·5665	1·5787	1·5904	1·6018	1·6127	1·6234	1·6337	1·6437	1·6535	1·6630
250	235·31	1·5490	1·5612	1·5730	1·5843	1·5953	1·6059	1·6162	1·6262	1·6359	1·6454
300	285·31	1·5346	1·5468	1·5587	1·5700	1·5810	1·5917	1·6020	1·6120	1·6217	1·6312
350	335·31	1·5224	1·5348	1·5466	1·5580	1·5690	1·5797	1·5900	1·6000	1·6097	1·6191
400	385·31	1·5120	1·5243	1·5362	1·5476	1·5586	1·5693	1·5796	1·5896	1·5993	1·6087
450	435·31	1·5026	1·5150	1·5269	1·5384	1·5494	1·5601	1·5704	1·5805	1·5902	1·5996
500	485·31	1·4943	1·5068	1·5187	1·5302	1·5413	1·5519	1·5623	1·5723	1·5820	1·5915

EXTRACTS FROM THE *MOLLIER STEAM TABLES**

The values in Tables A, B, C, D, and E are taken from *The Mollier Steam Tables and Diagrams* (Third Edition) by the kind permission of the publishers, Sir Isaac Pitman & Sons, Ltd. Much more detailed figures and information will be found in the *Complete Tables*, published in med. 8vo, with two large charts, at 7s. 6d. net.

The first set of Mollier steam tables was founded upon the characteristic equation of Professor H. L. Callendar.

Owing to the tendency, in modern steam practice, to use higher pressures and high superheat, there is need for information on the properties of steam above 500 lb. per square inch.

After consideration of the available data, Dr. Mollier has devised a new characteristic equation by the addition of one term to the original equation and, by extrapolation, he has extended the range of these steam tables to the critical point.

The equation for superheated steam applicable up to 2,000 lb. per square inch is

$$\frac{pV}{T} = 1{\cdot}0731 - 0{\cdot}3204 \frac{p}{(T/100)^{13/3}} - 15{\cdot}044 \frac{p^3}{(T/100)^{15}}$$

Taking coefficients, $k_1 = \frac{32{\cdot}04}{(T/100)^{10/3}}$, and $k_2 = \frac{15{\cdot}044 \times 10^6}{(T/100)^{14}}$

$$V = 1{\cdot}0731 \frac{T}{p} - k_1 - k_2 \left(\frac{p}{100}\right)^2$$

where p = pressure in lb. per sq. in. (abs.)
V = volume of steam in cu. ft. per lb.
$T = t^\circ$ C. $+ 273$ = absolute temperature (Centigrade)

The coefficients for the various equations, from zero to 500° C., are given in the *Complete Tables*.

Dr. H. Moss has amplified and adapted the tables to the British Centigrade system of units.

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* These tables are now out of print.

TABLE A

PROPERTIES OF SATURATED STEAM

Pressure lb./sq. in. (absolute) p	Temperature t° C.	Volume of Steam cu. ft/lb. V_{sa}	TOTAL HEAT		Latent Heat of Steam lb. cal./lb. $L = H - h$	Internal Energy of Steam lb. cal./lb. E	External Work of Vaporization lb. cal./lb.	ENTROPY PER LB.		
			Water lb. cal./lb. h	Steam lb. cal./lb. H_{sa}				Water ϕ_w	Increase Due to L/T	Steam ϕ_{sa}
0·2	11·7	1526	11·7	600·4	588·7	569·0	31·38	0·0418	2·0686	2·1104
0·3	17·9	1040	17·9	603·3	585·4	571·2	32·07	0·0636	2·0122	2·0758
0·4	22·6	791·9	22·6	605·5	582·9	572·9	32·57	0·0793	1·9721	2·0514
0·6	29·5	540·1	29·5	608·6	579·1	575·3	33·32	0·1026	1·9148	2·0174
0·8	34·6	411·9	34·6	611·0	576·4	577·1	33·87	0·1193	1·8741	1·9934
1·0	38·7	334·0	38·7	612·9	574·2	578·5	34·31	0·1326	1·8423	1·9749
2·0	52·3	174·0	52·3	619·0	566·7	583·3	35·74	0·1753	1·7426	1·9179
3·0	60·9	118·8	60·9	622·8	561·9	586·2	36·64	0·2012	1·6837	1·8849
4·0	67·2	90·71	67·2	625·6	558·4	588·3	37·30	0·2200	1·6418	1·8618
5·0	72·4	73·6	72·4	627·8	555·4	590·0	37·82	0·2351	1·6086	1·8437
6	76·7	62·06	76·7	629·7	553·0	591·5	38·26	0·2476	1·5815	1·8291
8	83·9	47·41	83·9	632·7	548·8	593·7	38·96	0·2679	1·5384	1·8063
10	89·6	38·48	89·6	635·1	545·5	595·6	39·52	0·2837	1·5048	1·7885
12	94·4	32·44	94·4	637·1	542·7	597·1	39·98	0·2969	1·4772	1·7741
14	98·6	28·08	98·6	638·8	540·2	598·4	40·38	0·3083	1·4536	1·7619
16	102·4	24·79	102·4	640·4	538·0	599·7	40·74	0·3185	1·4329	1·7514
18	105·8	22·20	105·9	641·7	535·8	600·7	41·05	0·3277	1·4145	1·7422
20	108·9	20·12	109·0	642·8	533·8	601·5	41·33	0·3359	1·3981	1·7340
22	111·7	18·40	111·9	643·9	532·0	602·2	41·58	0·3433	1·3832	1·7265
24	114·3	16·96	114·6	644·9	530·3	603·0	41·81	0·3502	1·3695	1·7197
30	121·3	13·76	121·7	647·5	525·8	605·0	42·40	0·3683	1·3338	1·7021
35	126·3	11·92	126·7	649·3	522·6	606·4	42·81	0·3810	1·3090	1·6900
40	130·7	10·52	131·1	650·8	519·7	607·5	43·17	0·3922	1·2874	1·6796
50	138·4	8·531	139·0	653·4	514·4	609·5	43·75	0·4115	1·2505	1·6620
60	144·9	7·190	145·7	655·5	509·8	611·2	44·23	0·4277	1·2199	1·6476
80	155·6	5·484	156·8	658·6	501·9	613·5	44·96	0·4536	1·1711	1·6247
100	164·4	4·443	166·0	660·9	494·9	615·2	45·48	0·4747	1·1321	1·6068
120	171·8	3·738	173·7	662·8	489·1	616·7	45·88	0·4922	1·0997	1·5919

140	178·3	3·228	180·6	664·2	483·6	617·7	46·20	0·5074	1·0718	1·5792
160	184·2	2·842	186·7	665·4	478·7	618·7	46·44	0·5209	1·0471	1·5680
180	189·5	2·539	192·3	666·3	474·0	619·4	46·64	0·5331	1·0248	1·5579
200	194·3	2·295	197·5	667·0	469·5	619·9	46·79	0·5441	1·0048	1·5489
220	198·8	2·093	202·2	667·6	465·4	620·3	46·90	0·5539	0·9866	1·5405
240	203·0	1·924	206·7	668·1	461·4	620·7	46·99	0·5633	0·9695	1·5328
260	206·9	1·779	210·9	668·4	457·5	620·9	47·05	0·5721	0·9535	1·5256
300	214·1	1·547	218·7	668·8	450·1	621·1	47·11	0·5881	0·9242	1·5123
340	220·6	1·367	225·8	669·0	443·2	621·2	47·10	0·6022	0·8982	1·5004
380	226·5	1·224	232·2	668·9	436·7	621·1	47·04	0·6151	0·8745	1·4896
420	231·9	1·107	238·1	668·6	430·5	620·9	46·94	0·6268	0·8528	1·4796
460	237·0	1·010	243·8	668·2	424·4	620·5	46·81	0·6376	0·8326	1·4702
500	241·7	0·9275	249·0	667·8	418·8	620·1	46·64	0·6477	0·8138	1·4615
600	252·4	0·7679	261·0	665·9	404·9	618·6	46·12	0·6703	0·7710	1·4413
700	261·7	0·6528	271·6	663·7	392·1	616·7	45·49	0·6900	0·7334	1·4234
800	270·1	0·5656	281·3	661·0	379·7	614·5	44·77	0·7075	0·6994	1·4069
900	277·8	0·4968	290·1	658·2	368·1	612·2	43·97	0·7233	0·6684	1·3917
1,000	284·8	0·4410	298·3	655·2	356·9	609·9	43·12	0·7377	0·6398	1·3775
1,200	297·3	0·3574	313·3	648·4	335·1	604·4	41·27	0·7634	0·5878	1·3512
1,400	308·3	0·2966	327·2	641·3	314·1	598·7	39·30	0·7868	0·5403	1·3271
1,600	318·2	0·2505	340·4	633·9	293·5	592·8	37·21	0·8088	0·4960	1·3048
1,800	327·2	0·2142	353·2	626·1	272·9	586·4	35·04	0·8295	0·4543	1·2838
2,000	335·4	0·1850	365·9	618·3	252·4	580·2	32·77	0·8491	0·4150	1·2641
2,400	350·1	0·1406	391·0	601·0	210·0	566·4	27·7	0·8877	0·3370	1·2247
2,800	362·9	0·1024	421·6	576·1	154·5	546·7	20·3	0·9344	0·2432	1·1776
3,000	368·6	0·0814	444·0	556·2	112·2	530·8	14·6	0·9684	0·1750	1·1434
3,200	374·0	0·0497	501·1	501·1	0	484·8	0	1·0558	0	1·0558

$T = t + 273$ p (absolute) = p (gauge) + 14·69 = 0·49(30 − Hg), where Hg = inches of mercury vacuum.

To convert to British Thermal Units per pound, multiply by 1·8. To convert to foot-pounds per pound, multiply by 1,400.

TABLE B

PROPERTIES OF SATURATED STEAM

Temp t° C.	Pressure lb./sq. in. (abs.) p	VOLUME CU. FT./LB.		TOTAL HEAT		Latent Heat of Steam lb. cal./lb. $L = H - h$	Internal Energy of Steam lb. cal./lb. E	ENTROPY PER LB.		
		Water V_w	Steam V_{sa}	Water lb. cal./lb. h	Steam lb. cal./lb. H_{sa}			Water ϕ_w	Increase Due to L/T	Steam ϕ_{sa}
0	0·088	0·016	3308	0	595·0	595·0	564·9	0	2·1800	2·1800
10	0·178	0·016	1704	10·0	599·6	589·6	568·4	0·0361	2·0839	2·1200
20	0·339	0·016	926·3	20·0	604·3	584·3	572·0	0·0708	1·9944	2·0652
30	0·616	0·016	527·5	30·0	608·9	578 9	575·5	0·1043	1·9108	2·0151
40	1·070	0·016	313·2	40·0	613·5	573·5	579·1	0·1367	1·8324	1·9691
50	1·789	0·016	193·1	50·0	618·0	568·0	582·5	0·1682	1·7586	1·9268
60	2·889	0·016	123·1	60·0	622·5	562·5	585·9	0·1986	1·6894	1·8880
70	4·519	0·017	80·92	70·0	626·8	556·8	589·2	0·2283	1·6236	1·8519
80	6·87	0·017	54·69	80·0	631·1	551·2	592·6	0·2570	1·5614	1·8184
90	10·17	0·017	37·88	90·0	635·3	545·3	595·7	0·2848	1·5024	1·7873
100	14·69	0·017	26·83	100·0	639·4	539·4	598·9	0·3121	1·4461	1·7582
120	28·80	0·018	14·28	120·3	647·0	526·7	604·7	0·3649	1·3404	1·7053
140	52·41	0·018	8·163	140·7	653·9	513·2	610·0	0·4155	1·2427	1·6583
160	89·64	0·018	4·925	161·4	659·9	498·5	614·5	0·4642	1·1514	1·6156
180	145·5	0·018	3·114	182·3	664·6	482·3	618·0	0·5112	1·0648	1·5760
200	225·4	0·019	2·043	203·5	667·7	464·2	620·4	0·5567	0·9816	1·5383
220	336·5	0·019	1·381	225·1	669·0	443·9	621·2	0·6010	0·9005	1·5015
240	485·4	0·20	0·956	247·1	668·0	420·8	620·2	0·6442	0·8204	1·4646
260	680·6	0·0205	0·6727	269·6	664·2	394·5	617·1	0·6864	0·7403	1·4267
280	930·5	0·0215	0·4786	292·7	657·3	364·6	611·5	0·7278	0·6595	1·3873
300	1,246	0·0227	0·3414	316·6	646·8	330·2	603·3	0·7690	0·5763	1·3454
320	1,639	0·0243	0·2429	343·0	632·5	289·5	591·6	0·8128	0·4881	1·3009
340	2,119	0·0266	0·1704	373·3	613·5	240·2	576·4	0·8608	0·3918	1·2526
360	2,707	0·0306	0·1115	413·0	583·4	170·4	552·4	0·9214	0·2692	1·1906
365	2,873	0·0327	0·0958	428·5	570·1	141·6	542·0	0·9450	0·2220	1·1670
374	3,200	0·0497	0·0497	501·1	501·1	0	484·8	1·0558	0	1·0558

TABLE C

VOLUME OF SUPERHEATED STEAM

Pressure lb./sq. in. (abs.) p	Saturation Temperature t° C.	Volume, V, cu.ft./lb., of Superheated Steam at Temperature—								
		t_s = 100° C.	150° C.	200° C.	250° C.	300° C.	350° C.	400° C.	450° C.	500° C.
2	52·3	199·7	226·7	253·6	280·5	307·4	334·2	361·0	387·9	414·7
4	67·2	99·67	113·2	126·7	140·2	153·6	167·1	180·5	194·0	207·4
6	76·7	66·33	75·40	84·42	93·42	102·4	111·4	120·3	129·3	138·2
8	83·9	49·63	56·46	63·26	70·02	76·76	83·49	90·22	96·93	103·6
10	89·6	39·63	45·14	50·58	56·00	61·39	66·77	72·16	77·55	82·93
12	94·4	32·97	37·56	42·13	46·65	51·16	55·65	60·13	64·61	69·09
16	102·4	—	28·11	31·54	34·95	38·33	41·71	45·08	48·44	51·80
20	108·9	—	22·44	25·20	27·92	30·64	33·35	36·05	38·75	41·44
30	121·3	—	14·87	16·75	18·59	20·41	22·22	24·03	25·83	27·62
40	130·7	—	11·08	12·51	13·90	15·28	16·64	18·00	19·35	20·70
50	138·4	—	8·81	9·97	11·09	12·20	13·30	14·39	15·47	16·55
60	144·9	—	—	8·278	9·227	10·16	11·07	11·98	12·89	13·79
80	155·6	—	—	6·159	6·885	7·590	8·284	8·970	9·653	10·33
90	160·1	—	—	5·456	6·108	6·737	7·356	7·969	8·576	9·182
100	164·4	—	—	4·890	5·483	6·053	6·614	7·167	7·714	8·259
120	171·8	—	—	4·041	4·546	5·028	5·499	5·963	6·422	6·878
140	178·3	—	—	3·434	3·877	4·296	4·703	5·103	5·498	5·890
160	184·2	—	—	2·978	3·375	3·747	4·106	4·458	4·804	5·148
180	189·5	—	—	2·622	2·986	3·321	3·643	3·957	4·266	4·573
200	194·3	—	—	—	2·672	2·978	3·270	3·554	3·835	4·114
300	214·1	—	—	—	1·730	1·951	2·156	2·351	2·542	2·731
400	229·2	—	—	—	1·252	1·436	1·597	1·749	1·896	2·039
500	241·7	—	—	—	0·9606	1·125	1·262	1·388	1·507	1·623
1,000	284·8	—	—	—	—	0·4831	0·5852	0·6625	0·7306	0·794
2,000	335·4	—	—	—	—	—	0·2170	0·2900	0·3384	0·3776
3,200	374·0	—	—	—	—	—	—	0·1306	0·1841	0·2185

$t_s - t$ = degrees of superheat.

TABLE D

TOTAL HEAT OF SUPERHEATED STEAM

Pressure lb./sq. in. (abs.) p	Saturation Temperature t_s° C.	Saturation Total Heat lb. cal./lb. H_{sa}	Total Heat, H, in lb cal./lb , of Superheated Steam at Temp.							
			t = 150° C.	200° C.	250° C.	300 C°.	350° C.	400° C.	450° C.	500° C.
15	100·6	639·5	663·7	687·8	711·6	735·4	759·0	782·6	806·2	829·8
20	108·9	642·8	663·1	687·4	711·4	735·2	758·9	782·5	806·1	829·7
30	121·3	647·5	661·9	686·6	710·8	734·7	758·6	782·3	805·9	829·5
40	130·7	650·8	660·7	685·8	710·2	734·3	758·2	782·0	805·7	829·3
50	138·4	653·4	659·5	685·0	709·6	733·9	757·9	781·8	805·5	829·1
60	144·9	655·5	658·3	684·1	709·0	733·5	757·6	781·5	805·3	829·0
70	150·5	657·2	657·0	683·3	708·5	733·0	757·2	781·3	805·1	828·9
80	155·6	658·6	—	682·4	707·9	732·6	756·9	781·0	804·9	828·8
90	160·1	659·9	—	681·6	707·3	732·2	756·6	780·8	804·7	828·6
100	164·4	660·9	—	680·7	706·7	731·8	756·3	780·5	804·5	828·4
120	171·8	662·8	—	678·8	705·5	730·9	755·6	780·0	804·1	828·1
140	178·3	664·2	—	677·0	704·2	730·0	755·0	779·5	803·7	827·8
160	184·2	665·4	—	675·0	703·0	729·1	754·3	779·0	803·4	827·5
180	189·5	666·3	—	672·8	701·7	728·3	753·7	778·5	803·0	827·2
200	194·3	667·0	—	—	700·4	727·4	753·0	778·0	802·6	826·9
250	205·0	668·2	—	—	697·1	725·1	751·3	776·7	801·6	826·1
300	214·1	668·8	—	—	693·4	722·7	749·6	775·5	800·6	825·3
350	222·1	669·0	—	—	689·4	720·3	748·0	774·2	799·6	824·5
400	229·2	668·8	—	—	685·0	717·8	746·3	772·9	798·6	823·7
450	235·8	668·3	—	—	680·3	715·2	744·5	771·6	797·6	822·9
500	241·7	667·8	—	—	675·3	712·5	742·7	770·3	796·6	822·1
600	252·4	665·9	—	—	—	706·2	738·9	767·7	794·6	820·6
800	270·1	661·0	—	—	—	692·2	730·7	762·1	790·5	817·3
1,000	284·8	655·2	—	—	—	674·7	721·6	756·2	786·2	814·0
2,000	335·4	618·3	—	—	—	—	648·5	717·6	761·6	796·5
3,200	374·0	501·1	—	—	—	—	—	638·9	720·3	770·7

To convert to foot-pounds per pound, multiply by 1400
To convert to British Thermal Units per pound, multiply by 1·8.
$t_s - t$ = degrees of superheat.

TABLE E

ENTROPY OF SUPERHEATED STEAM

Pressure lb./sq. in. (abs) p	Saturation Temp. t_s° C.	Entropy of Superheated Steam at Temperature—							
		$t = 150$°C	200° C.	250° C.	300° C.	350° C.	400° C.	450° C.	500° C.
15	100·6	1·8169	1·8712	1·9191	1·9624	2·0019	2·0384	2·0722	2·1037
20	108·9	1·7839	1·8386	1·8868	1·9302	1·9699	2·0064	2·0402	2·0718
30	121·3	1·7370	1·7926	1·8412	1·8850	1·9248	1·9614	1·9953	2·0269
40	130·7	1·7030	1·7595	1·8087	1·8527	1·8927	1·9294	1·9634	1·9950
50	138·4	1·6762	1·7335	1·7832	1·8275	1·8677	1·9046	1·9386	1·9703
60	144·9	1·6539	1·7121	1·7622	1·8068	1·8472	1·8842	1·9183	1·9500
70	150·5	—	1·6937	1·7444	1·7892	1·8298	1·8668	1·9011	1·9329
80	155·6	—	1·6775	1·7287	1·7739	1·8146	1·8518	1·8861	1·9180
90	160·1	—	1·6631	1·7148	1·7604	1·8012	1·8385	1·8729	1·9048
100	164·4	—	1·6502	1·7024	1·7482	1·7892	1·8266	1·8610	1·8930
120	171·8	—	1·6270	1·6805	1·7269	1·7683	1·8060	1·8405	1·8727
140	178·3	—	1·6068	1·6617	1·7087	1·7505	1·7884	1·8231	1·8554
160	184·2	—	1·5886	1·6451	1·6928	1·7350	1·7731	1·8079	1·8403
180	189·5	—	1·5720	1·6302	1·6786	1·7212	1·7595	1·7945	1·8270
200	194·3	—	—	1·6166	1·6658	1·7088	1·7473	1·7825	1·8150
250	205·0	—	—	1·5868	1·6381	1·6821	1·7213	1·7568	1·7896
300	214·1	—	—	1·5611	1·6146	1·6599	1·7997	1·7357	1·7687
350	222·1	—	—	1·5379	1·5943	1·6408	1·6812	1·7177	1·7509
400	229·2	—	—	1·5162	1·5761	1·6239	1·6650	1·7019	1·7354
450	235·8	—	—	1·4957	1·5594	1·6086	1·6505	1·6878	1·7216
500	241·7	—	—	1·4761	1·5440	1·5947	1·6375	1·6751	1·7092
600	252·4	—	—	—	1·5151	1·5698	1·6140	1·6528	1·6875
800	270·1	—	—	—	1·4634	1·5274	1·5758	1·6166	1·6526
1,000	284·8	—	—	—	1·4122	1·4908	1·5444	1·5874	1·6247
1,200	297·3	—	—	—	—	1·4565	1·5165	1·5625	1·6012
1,600	318·2	—	—	—	—	1·3882	1·4671	1·5201	1·5626
2,000	335·4	—	—	—	—	1·3137	1·4204	1·4836	1·5303
2,400	350·1	—	—	—	—	—	1·3730	1·4497	1·5019
2,800	362·9	—	—	—	—	—	1·3224	1·4169	1·4759
3,200	374·0	—	—	—	—	—	1·2666	1·3837	1·4513

Entropy is numerically the same in all temperature systems.

ANSWERS TO EXAMPLES

EXAMPLES II

1. I.H.P., 6·17.
2. (*a*) I.H.P., 6·09 ; B.H.P., 5 ; (*b*) 1·095 H.P. ; (*c*) 0·82, or 82 per cent.
3. I.H.P., 18·8 ; 78·2 per cent.
4. (*a*) 5·25 ; (*b*) 80·77 per cent ; (*c*) 31·66 per cent ; (*d*) (1) 25·57 per cent ; (2) 6·09 per cent ; (3) 30·09 per cent.
5. I.H.P., 42·86 ; B.H.P., 30·39 ; mechanical loss, 12·47 H.P. ; mechanical efficiency, 70·9 per cent ; oil per I.H.P. hour, 0·453 lb. ; per B.H.P. hour, 0·6384 lb.

Balance Sheet. Heat Flow per Hour

	B.Th.U. per hour	Per Cent
Heat supplied in oil fuel	349,200	100
Indicated work	109,079	31·24
B.Th.U. Per cent		
Including { Useful work on brake 77,343 = 22·15		
Including { Mechanical loss . 31,736 = 9·09		
Heat to jacket cooling water	101,500	29·07
Heat in exhaust gases and radiation (by difference) .	138,621	39·69
Total	349,200	100·00

6. (*a*) 157·3, and 38·94 lb. per sq. in. ; (*b*) 12,423 ft-lb. ; (*c*) 69 lb. per sq. in. and 41·4 I.H.P. ; (*d*) 35 per cent.
7. (*a*) 0·2831 or 28·31 per cent ; (*b*) 87·1 per cent ; (*c*) 7,285 C.H.U. or 46·8 per cent.
8. (*a*) I.H.P., 50·88 ; B.H.P., 30·46 ; (*b*) oil 0·288 lb. and 0·48 1lb. ; (*c*) 2,158 B.Th.U. ; (*d*) 1,365 B.Th.U. ; (*e*) by difference, 629 B.Th.U.
9. (*a*) 36·4 per cent ; (*b*) 30·95 per cent ; (c) 85·03 per cent.
10. (*a*) 2·62 ; (*b*) 17·53 ; (c) 0·85 or 85 per cent.
11. Diameter $9\frac{3}{8}$ in. × stroke $14\frac{1}{16}$ in.

EXAMPLES III

1. 5 cu. ft.
2. (*a*) 139·9 lb. per sq. in., 276·2° C. ; (*b*) 73·5 lb. per sq. in.
3. 43·49 lb. per sq. in. ; 59,445 ft-lb.
4. 417·8° F. (214·2° C.)
5. 50,825 ft-lb.
6. (*a*) 46,026 ft-lb. ; (*b*) 43,062 ft-lb., and 412·6° F.
7. (1) 84 lb. per sq. in. ; (2) 36,845 ft-lb. ; and (3) 26·3 C.H.U.
8. (*a*) 354 lb. per sq. in. ; 359·24° C. ; (*b*) 89,300 ft-lb. ; (*c*) rejected, 15·94 C.H.U. or 28·69 B.Th.U.
9. (*a*) − 88·8° F. ; (*b*) 151,866 ft-lb. ; (*c*) heat received by air, 42 C.H.U., or 75·6 B.Th.U.
10. 42,560 ft-lb. and 15·2 C.H.U.

11. 1,824 ft-lb.; 0·1326 cu. ft.
12. 19,880 ft-lb., and 14·2 C.H.U.
13. (*a*) 414° F., 46,548 ft-lb.; (*b*) 84·252 B.Th.U., 19,000 ft-lb. (*e*) 50,000 ft-lb. (Here, 46,548 + 19,000 = 65,548 ft-lb. = 84·252 × 778)
14. $C_p = 0{\cdot}24526$; $C_v = 0{\cdot}1732$; and $\gamma = 1{\cdot}416$.

EXAMPLES IV

1. 0·037.
2. 47·05 C.H.U., and gain 0·094.
3. (*a*) 68,466 ft-lb.; 48·9 C.H.U.; entropy loss 0·179; (*b*) 71,480 ft-lb.; 25·53 C.H.U.; entropy loss 0·074.
4. 0·5, or 50 per cent.
5. 28·54 lb. per sq. in.
6. (*a*) 0·5345, or 53·45 per cent; (*b*) 0·4489, or 44·89 per cent.

EXAMPLES V

1. (*a*) − 110° F.; (*b*) 31,670 ft-lb.; 39·78 lb. per sq. in.; (*c*) 3·405 cu. ft. 13·62 cu. ft.
2. 8·44 B.H.P.
3. (*a*) 64,860 ft-lb. (approx.); (*b*) 49,660; (*c*) 407·6° F.
4. 76·7 B.H.P.
5. (*a*) Air H.P., 54·6; B.H.P., 63·5.
6. Diameter, 8·232 in.; stroke, 30 in.
7. (*a*) 71,850 ft-lb. and 25·66 C.H.U.; (*b*) 68,060 ft-lb. and 32·4 C.H.U.
8. Air I.H.P., 69·74; stroke, 12 in.; diameter, low pressure cylinder, 12·11 in., high pressure cylinder, 4·22 in.
9. 93·91 lb. per sq. in.; air I.H.P., 2·33.
10. 0·84.

EXAMPLES VII

1. 10·736 lb., and excess air 7·032 lb.
2. Air, 0·957 cu. ft.; dry products, CO_2, 14·65; O_2, 4·04; N_2, 81·31 per cent.
3. Air, 1·2823 cu. ft.; 7·25 per cent; dry products: CO_2, 13·3; O_2, 5·5; N_2, 81·2 per cent.
4. 15·43 : 1.
5. (*a*) 8·4 per cent; (*b*) 17·865 lb.; (*c*) 18·83 lb.
6. (*a*) 14·53 per cent, 8·82 per cent; (*b*) 1,254 C.H.U.; (*c*) 444 C.H.U.
7. Air, 17·9 lb.; flue gas, 18·856 lb. per pound of coal.
8. (*a*) 10,833 and 10,097 C.H.U. per lb.; (*b*) 19,500 and 18,175 B.Th.U. per lb.; (*c*) 170,625, and 159,030 B.Th.U. per gallon.
9. 294 and 258·9 C.H.U. per cu. ft. at N.T.P.
10. (*a*) 82·9, and (*b*) 30 per cent; (*c*) 0·73 lb.; (*d*) 24·9 per cent.
11. (*a*) 1·37 cu. ft.; (*b*) 77 per cent; (*c*) 0·832 lb.; (*d*) 29·65 per cent; (*e*) 22·84 per cent.
12. 70 per cent; CO, 34·6; N_2, 65·4 per cent; 119 B.Th.U. per cubic foot.
13. (*a*) 0·726 lb.; (*b*) I.H.P., 117; (*c*) 87·26 per cent; (*d*) 27·55 per cent; (*e*) 24 per cent.

EXAMPLES VIII

1. 0·463; 20·98 cu. ft.
2. (*a*) 12,423 ft-lb.; (*b*) 41·4; (*c*) 35·5; (*d*) 69·5.

3. 98·5 lb. per sq. in.
4. 0·611; 0·322 lb.
5. 46·1 C.H.U.; 18,620 ft-lb.
6. 7·4 in. diam.; 11·1 in. stroke.
7. 9·7 in.
8. 5·89 C.H.U.; 74 per cent.
9. 10·8 in.
10. (*a*) − 0·0222; (*b*) 0·1586.
11. 42·5; 34·8.
12. 8·3 per cent rich (calculated on petrol).
13. 19·7; 79·8 per cent.
14. (*a*) 83 per cent; (*b*) 33·2 per cent.

(*c*) HEAT BALANCE FOR THE ENGINE PER MINUTE

			B.Th.U. per Min.	Per Cent
Heat supplied in fuel			3,155·6	100
Indicated work	B.Th.U.	Per cent	1,047·7	33·2
Including: Work on brake	. 869·6	= 27·56		
Including: Mechanical loss	. 178·2	= 5·64		
Heat to jacket cooling water			784·5	24·9
Heat in exhaust gases			1,230·7	39·0
Radiation loss, etc. (by difference)			92·7	2·9
Total			3,155·6	100·0

15. (*a*) I.H.P., 27·76; (*b*) B.H.P., 24·36; (*c*) 87·75 per cent, thermal efficiency, 27·24 per cent; (*d*) ideal air cycle 0·51, or 51 per cent; efficiency ratio, 53·4 per cent.

(*e*) HEAT BALANCE SHEET FOR THE ENGINE

			C.H.U. per Min.	Per Cent
Heat supplied by gas			2,401·7	100
Indicated work	C.H.U.	Per cent	654·3	27·24
Including: Useful work on brake	574·2	= 23·91		
Including: Mechanical loss .	80·1	= 3·33		
	654·3	= 27·24		
Heat lost to cooling jacket water			786·6	32·75
Heat loss in exhaust and radiation (by difference) .			960·8	40·01
Total			2,401·7	100·00

16. 52·5° F.
17. (*a*) 86 H.P., (*b*) 130, (*c*) 83 lb. for $\gamma = 1{\cdot}4$ and $C_p = 0{\cdot}24$.

EXAMPLES IX

1. (*a*) 618·4 C.H.U., (*b*) 747·4 C.H.U., (*c*) 544·5 C.H.U., (*d*) 668·2 C.H.U.
2. 6,539 C.H.U., 670 C.H.U.
3. 657·1 C.H.U., 611·4 C.H.U., 143° C.
4. 9·456 lb.
5. (*a*) dryness 0·423, (*b*) 20·1° C. superheat.
6. 293·5° C., 7 in. and 10¾ in.
7. 333·3° C., 222·4° C.
8. 563 C.H.U. per lb.; 114 C.H.U.; 91 C.H.U.
9. 0·98; 0·83; 1·138.
10. 0·988; 0·936.
11. 1·094 C.H.U.
12. 113 lb.; 0·32.

EXAMPLES X

1. (*a*) 90,200 ft-lb., 12·8 per cent; (*b*) 95,450 ft-lb., 12·2 per cent.
2. 19·9 lb.
3. Rankine efficiency, per cent: 13·3, 13·45, 13·77, 14·31.
   Actual efficiency, per cent: 6·0, 6·49, 7·23, 8·3.
4. Rankine efficiency, 25·5 per cent.
   Boiler ,, 80 ,,
   Mechanical ,, 85 ,,
   Efficiency ratio, 65 ,,
   Overall efficiency, 11·27 ,,
5. Dryness fraction, 0·95, 542 H.P.
6. 13 per cent.
7. 0·866.
8. Diameter 16·75 in., stroke 30 in.
9. 72·1.
10. 0·063 lb.
11. Dryness fraction, 0·883; 1·84 C.H.U.
12. Diameter of H.P., 13·25 in., L.P., 26·75 in.; stroke, 36 in.
13. Diameter of H.P., 15·25 in., L.P., 26·25 in.
14. Curve.

EXAMPLES XI

1. (*a*) Dryness 0·79; 3,915 ft. per sec.; (*b*) 0·828, 4,000 ft. per sec.
2. 3,493 ft. per sec., 3,730 ft. per sec.
3. 0·159 in., 0·187 in.
4. 0·366 in., 0·456 in.
5. 14·88 lb., 13·56 lb.
6. Dryness 0·972, 0·0734 lb. per sec., 10·7 per cent.
7. 0·378 in., 0·468 in.
8. 0·0835 lb. per sec.
9. 1·133; 86·6 lb. per sq. in.; 1,462 ft. per sec.

EXAMPLES XII

1. 394 H.P., 29°.
2. 26°, 205 ft. per sec. at 101°.
3. 18·45 C.H.U., 65·6 per cent.
4. 26·7 lb-ft., 4,255 rev. per min., 12·2 h.p.
5. 3,480 ft. per sec., 30°, 136,714 ft-lb.
6. 340 ft. per sec., 52 H.P., 23·5°.
7. 400 ft. per sec., 79 per cent.

8. **1,710 ft. per sec., 60·5 H.P., 73 per cent.**
9. **4·3 H.P.**
10. **Heat drops: 49·6, 51·4, 109·1 C.H.U.**
    Work done: 39·2, 39·2, 78·4 C.H.U.
11. **1·04, 10·2 lb. per H.P.-hour, 19·8 per cent.**
12. **64 per cent, 0·9, 0·87.**
13. **36 lb. per sq. in.; 11 C.H.U.; 0·93 C.H.U.; 12.**

EXAMPLES XIII

1. Air 0·5 lb. per min.; Vapour 0·57 lb. per min.
2. 0·15; 0·93; 4,980 lb. per min.
3. 286 cu. ft. per min.
4. (*a*) 187,400 lb.; (*b*) 30 ft.
5. 2,110 H.P.; 0·226
6. (*a*) 664 C.H.U.; (*b*) 128·2 C.H.U.; (*c*) 0·2.
7. 0·42; Rankine efficiency, 0·34.
8. 1·51 cu. ft.; 1,010 B.Th.U. per min.
9. 31,500; 585 lb. per hour.

EXAMPLES XV

1. (*a*) 22·9 I.H.P.; (*b*) 38·2 I.H.P.
2. 172·5 B.Th.U. per lb. of ice; 4·13 I.H.P.
3. (*a*) 16,727 ft-lb.; (*b*) 36·81 B.Th.U.; (*c*) 1·712.
4. (*a*) 32,060 C.H.U.; (*b*) 49·164 C.H.U. per hour; (*c*) 0·907.
5. (*a*) 8·77; (*b*) 8·29; (*c*) 8·28.
6. (*a*) 1,150 B.Th.U. per min.; (*b*) 1,180 B.Th.U. per min.

EXAMPLES XVI

1. 785 B.Th.U. per hour.
2. 29·2 B.Th.U. per hour.
3. 8,670 B.Th.U.; 62·2° and 30·2° F.
4. 82·4 lb. per hour.

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