ELECTRICITY SUPPLY METERS

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by

A. E. B. PERRIGO

M.Sc , A.C.G.I., A.M.I.E.E., A.M.I.Mech.E.

Head of the Research Department, H. Frost and Company Ltd., Walsall

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PREFACE

The following chapters have primarily been written in an attempt to aid the Meter Engineer who, equipped with a sound fundamental knowledge of electrical phenomena, trigonometry, elementary calculus, and vector summation, wishes to become more familiar with the fundamentals of electricity meters and all instruments employed, directly or indirectly, in their calibration.

They have also been written, however, with a view to being of service to the Engineer or Experimentalist who employs electrical instruments in his investigations, and desires information as a guide in the choice of the most suitable apparatus for his particular purpose.

Finally, I should like to record my gratitude to the manufacturers who have kindly furnished me with information, blocks, and photographs of their products; to the I.E.E., for permission to reproduce diagrams appearing in the Journal; to the C.E.B., for their helpful reading of the sections upon Summation Metering; and, last but not least, to Mr. W. Reynolds, for his most valuable and enthusiastic help in the preparation of the diagrams.

A. E. B. PERRIGO

CONTENTS

CHAPTER I

Units and Definitions associated with the Measurement of Electrical Quantities

1.1. Development of Electrical Units. Before commencing the measurement of any electrical quantity, one must first be familiar with the definitions of the various units involved. It is therefore proposed to devote this chapter to the derivation and definition of the units used in the study of electrical phenomena. A primary consideration in the development of electrical units is the observed effects of electric currents. These will therefore receive our attention first.

1.2. Effects of an Electric Current. The most important effects observed when an electric current is flowing are.

(*a*) *Heating.* A wire carrying current is heated. This property is made use of in hot-wire instruments. It is not, however, suitable for tandardising purposes because of the difficulty of measuring quantities f heat accurately.

(*b*) *Magnetic.* A wire carrying current is surrounded by a magnetic eld. This property is of very great importance and, as will be seen ter, most of the instruments described in this book owe their action o this particular phenomenon of electric current.

(*c*) *Chemical.* The passage of an electric current through an electrolyte is accompanied by a deposition of the substances composing the hemical solution, and the amounts deposited are proportional to the urrent flowing and the length of time during which the action takes lace. As the measurement of both time and weight can be performed ith great accuracy, the chemical effect is very suitable for determining nd defining a standard of current. In practice, the chemical employed silver nitrate dissolved in water, and the unit derived is termed the nternational ampère.

1.3. The International Ampère is that unvarying electric current which, when passed through a solution of silver nitrate in water, eposits silver at the rate of 0·00111800 gramme per second.

The other unit of current is based upon the magnetic effect and is ermed the Absolute electro-magnetic unit.

1.4. The Absolute Electro-magnetic Unit of Current is that urrent which, when flowing in an arc of 1 cm. length and 1 cm. radius, xerts a force of 1 dyne upon a unit magnetic pole placed at the centre round which the arc is described. This current is equivalent to 10 nternational ampères.

1.5. Coulomb. The idea of a current, or rate of flow, suggests uantity of electricity. Quantity of electricity is therefore defined as the amount passed when a certain rate of flow is maintained for a given period. The unit is the Coulomb, which is the quantity of electricity passing a certain point per second when the current is 1 international ampère.

The following units are also of importance in the measurement of electrical quantities:

1.6. Ohm. The international ohm (i.e. the standard fixed by international agreement) is defined as "the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, the column being 14·4521 grammes in mass, of constant cross-sectional area, and 106·300 cm. in length.

1.7. Volt. The international volt is that steady e.m.f. which, applied to the ends of a conductor whose resistance is 1 international ohm, causes a current of 1 international ampère to flow.

The international units are the legal units and, in all practical work, one adheres to these. Hence, throughout the remainder of this book, all reference will be to the international units.

1.8. Ampère-hour. The ampère-hour is the quantity of electricity passing a point in an electric circuit, when a steady current of 1 ampère flows for one hour.

1.9. Watt. The practical unit of power is the watt. It is the power dissipated when a steady current of 1 ampère flows through a resistance of 1 ohm. As the watt is a small unit for the measurement of power, a larger unit—the kilowatt—is also used. As its name implies, a kilowatt is 1,000 watts.

1.10. Watt-hour. If power is supplied to a circuit at the rate of 1 watt for a period of one hour, the total energy supplied is 1 watt-hour. This unit is too small for supply work and therefore the kilowatt-hour (kWh) is employed. The latter unit corresponds to 1,000 watts developed over a period of one hour, and is also known as the Board of Trade Unit (B.T.U.).

1.11. Electro-motive Force. The practical standards of electro-motive force are the Weston cadmium cell and the Clark cell. The former has the value

$$E_t = E_{20} - 0{\cdot}0000406\,(t-20) - 0{\cdot}00000095\,(t-20)^2 + 0{\cdot}00000001\,(t-20)^3 \text{ volts}$$

for any temperature between the limits of 0°C. and 40°C. For the Clark cell

$$E_t = E_{15} - 0{\cdot}00119\,(t-15) - 0{\cdot}000001\,(t-15)^2 \text{ volts}$$

for any temperature between the limits of 10°C. and 25°C.

In the above expressions, E_t is the e.m.f. at temperature t, t is the temperature in degrees Centigrade, E_{20} is the e.m.f. of the Weston cell at 20°C., i.e. 1·0183 volts, and E_{15} is the e.m.f. of the Clark cell at 15°C. (1·4326 volts).

1.12. Construction of Standard Cells. It is not proposed to describe fully the construction of the Weston and Clark standard cells in this book, but the interested reader is recommended to consult Chapter XXVIII of *A Text Book of Practical Physics*, by Watson, for full details. As the standard cell plays such an important part, however, in all work involving the use of the d.c. and a.c. potentiometers, we will consider certain relevant details.

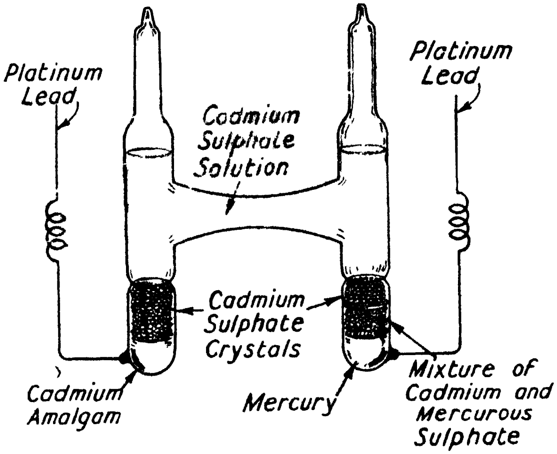


FIG. 1.—Weston Cadmium Cell.

The Weston cell is the more generally used, its advantages over the Clark cell being:

(*a*) A much lower temperature coefficient (less than one-twentieth that of the Clark cell).

(*b*) It has a much longer life than the Clark cell; the latter having a tendency to crack at the junction of the negative terminal wire.

(*c*) In the Clark cell, a layer of gas is formed which tends to interrupt the circuit. No such layer occurs in the Weston cell.

(*d*) Temperature variations cause only small hysteresis effects in the Weston cell, but large effects in the Clark cell.

Both types of cell are formed of mercury, zinc, and cadmium of a high degree of purity. This quality is essential for a cell required to give, as far as possible, a permanent unvarying e.m.f.

An illustration of a Weston cadmium cell is given in Fig. 1. The positive element consists of mercury, and the negative element of cadmium amalgam—a solution of 1 part of cadmium in 7 parts of mercury. A saturated solution of cadmium sulphate forms the electrolyte; the cadmium sulphate crystals being added to ensure saturation of the solution. The depolariser consists of mercurous sulphate. This must be exceedingly pure, as any impurities in this chemical have a much greater effect upon the permanence of the cell e.m.f.

than is the case for impurities existing in any of the other chemicals employed.

1.13. Precautions in the Use of a Standard Cell. Great care must be taken to ensure that no appreciable current is given by a standard cell, as the e.m.f. is only constant when the cell is on open circuit. When a heavy current is taken, the voltage falls, and it is a matter of time before recovery is complete. During this interval large errors may occur in any measurements undertaken. In consequence, standard cells are only employed for null methods of measurement (such as with the potentiometer) and, in order to protect the cell, a high resistance should be connected in series with it and this should only be taken out of circuit when one is nearing the point of balance.

The cells should be stored in a dry place, having a fairly uniform temperature between about 12°C. and 22°C.

1.14. Ohm's Law. If any given conductor is kept at constant temperature, and the potential difference at its ends is compared with the current flowing through the conductor, there is found to be a constant relationship between the two. This relationship is called Ohm's Law, and may be stated symbolically as

$$I=\frac{E}{R}$$

where I is the current in ampères, E is the potential difference in volts between the terminals of the conductor, and R is the resistance in ohms of the conductor.

1.15. Alternating Current. As the above units and definitions have been derived from steady values of supply (direct current) we will consider the effect of employing alternating current. Given a supply e.m.f. with a pure sine wave, the wave-form of the current will, in general, be similar and may be represented by the continuous line curve of Fig. 2. If this current were passed through a d.c. permanent magnet moving coil instrument, the inertia of the moving system would be too great for it to follow the rapid alternations of the current, and it would register the average value, which is zero. This value is obvious

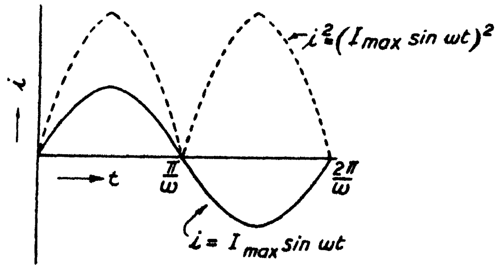


FIG. 2.

from the shape of the curve $i=I_{max}\sin\omega t$ of Fig. 2. However, we know that the power expended in producing heat, by a current of i ampères flowing through a resistance of R ohms, is i^2R watts, and therefore the heating effect is independent of the direction in which the current is flowing. Hence, we may use a hot-wire instrument for the purpose of comparing the relative heating effects of direct and alternating currents, and therefrom deduce the relationship which exists between the two. Assuming the maximum value of the alternating current to be I ampères, it will be found that the hot-wire instrument registers the same deflection for this current as for a direct current of 0·707 (i.e. $1/\sqrt{2}$) I ampères. From this experiment, we learn that the effective value, or virtual value, of the alternating current is $1/\sqrt{2}$ of the maximum value. This effective value is usually termed the root mean square (r.m.s.) value of the current since it is the square root of the average value of the square of the current over a complete cycle.

1.16. R.M.S. Value of Alternating Current, by Mathematical Means. The results of the last paragraph may be quickly and accurately obtained by utilising the calculus. Let the current at any time t be given by

$$i=I_{max}\sin\omega t$$

then the energy dissipated in time $dt=i^2R.dt$.

The total energy dissipated in one complete cycle

$$=\int_0^{\frac{2\pi}{\omega}} i^2R.dt$$

where $\omega = 2\pi$ times the frequency in cycles per second.

But $i = I_{max}\sin\omega t$.

Therefore, the total energy dissipated per cycle

$$=\int_0^{\frac{2\pi}{\omega}} I^2_{max}R\sin^2\omega t.dt$$

$$=I^2_{max}R\int_0^{\frac{2\pi}{\omega}} \tfrac{1}{2}(1-\cos 2\omega t).dt$$

$$=\frac{I^2_{max}R}{2}\left(t-\frac{1}{2\omega}\sin 2\omega t\right)_0^{\frac{2\pi}{\omega}}$$

$$=\frac{I^2_{max}R\pi}{\omega}$$

but this energy would be dissipated in the same time by a direct current I, if

$$\left(\frac{2\pi}{\omega}\right)\ I^2R=\frac{I^2_{max}R\pi}{\omega}$$

$$\text{that is, if } I^2=\frac{I^2_{max}}{2}$$

$$\text{or } I=\frac{I_{max}}{\sqrt{2}}$$

$$=0{\cdot}707\ I_{max}$$

Hence, the r.m.s. value of an alternating current is 0·707 times its maximum value.

1.17. Alternating Voltages are likewise measured by their maximum or their r.m.s. values, and similar reasoning to that of section 1.16 will show that $E_{r.m.s.}=0{\cdot}707\ E_{max}$.

The rated value of a supply voltage is always the r.m.s. value, and an a.c. voltmeter indicates r.m.s. values only. Thus, if a voltmeter indicates 200 volts, the voltage is rising and falling between zero and a maximum value of $\pm 200\sqrt{2}=(282{\cdot}8)$ volts.

For power calculations, and for determining the size of cable required to carry a certain current, one is concerned with r.m.s. values of voltage and current. On the other hand, for the determination of the thickness of insulation necessary for a conductor carrying alternating current, one is concerned with the maximum value of the voltage.

1.18. Power in Direct Current Circuits. In any d.c. circuit, the expenditure of power is easily determined in any one of the three following ways, depending upon which two of the three quantities—voltage, current, and resistance—are known.

(*a*) If the current and applied voltage are known, then the power expenditure $=EI$ watts, where E is measured in volts and I in ampères.

(*b*) If the resistance of the circuit is known, and also the current flowing in the circuit, then the power expenditure

$$=I^2R \text{ watts}$$

where R is the resistance in ohms.

(*c*) If the resistance of the circuit and the applied voltage are known, then the power expenditure

$$=EI=\frac{E.E}{R}=\frac{E^2}{R} \text{ watts.}$$

1.19. Power in Alternating Current Circuits. In alternating current circuits, the above simple relationships do not hold. The values of the applied voltage and the current flowing in the circuit are continuously changing. They are, in general, out of phase with one another and the power supplied to the circuit is a continuously variable quantity. In fact, as we shall see later, in all circuits containing reactance, the circuit will be returning power to the supply for certain portions of each cycle. Formulae, therefore, have to be derived which give the mean power expenditure over a complete cycle.

1.20. A.C. Circuits containing Resistance only. We will first consider the case of a.c. circuits containing resistance only (Fig. 3).

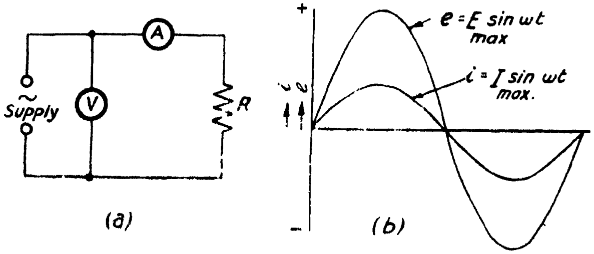


FIG. 3.—Circuit containing resistance only.

The current will be in phase with the applied voltage and its value will be given by

$$I = \frac{E}{R}$$

where E and I are the r.m.s. values of the voltage and current respectively.

The power dissipated in the circuit $= I^2R$ watts. Therefore, since $I = E/R$

$$I^2R = EI$$

and the power expended is the product of the voltage and current.

1.21. A.C. Circuits containing Resistance and Reactance. We will now consider the general case of a.c. circuits where there is reactance as well as resistance. A simple circuit of this nature is illustrated in Fig. 4, consisting of a resistance of R ohms and an inductance of L henrys. The ammeter A will indicate the r.m.s. value of the current in the circuit, whilst the voltmeter V will indicate the value of the applied voltage.

The voltage V must be the vector sum of the voltage across the inductance L and that across the resistance R. Furthermore, the voltage drop E_1 across R will be in phase with the current. The voltage drop E_2 across the inductance L will be $I\omega L$ volts, and since this portion of the load is purely inductive, it will lead the current by 90°. The conditions may therefore be represented by the vector diagram of Fig. 4 (*b*).

We see from this diagram that the current is lagging the applied voltage V by angle ϕ, and that

$$\phi = \tan^{-1}\frac{\omega L}{R}$$

The applied voltage is performing two duties:

(*a*) It is supplying a component $E_1=IR$ to force the current through the resistance of the circuit. This voltage is in phase with the current and is known as the "in-phase" component of the total voltage.

(*b*) It is supplying a component $E_2=\omega LI$ to overcome the counter e.m.f. of the inductance. This component is 90° ahead of the current and is known as the wattless or quadrature component of the total voltage.

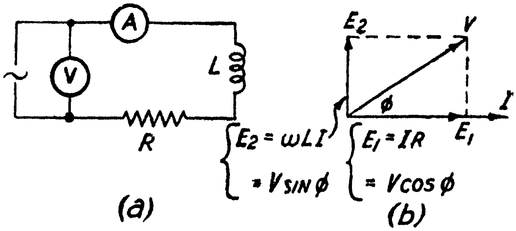


FIG. 4.—Circuit containing resistance and reactance.

One may consider inductance as the electro-magnetic equivalent of the mechanical phenomenon of inertia. The energy required to create the magnetic field around the inductance during one-quarter cycle is completely restored to the supply during the succeeding quarter-cycle (this is assuming the ideal inductance, which possesses no resistance and has no iron-losses). Hence, the mean expenditure of energy over the complete cycle is zero.

The only power dissipated in the circuit is due to the current flowing through the resistance R. Therefore, the power expended

$$=I^2R \text{ watts}$$

$$\text{but } E_1=IR \text{ volts}$$

$$=V\cos\phi$$

$$\text{and the power expended } =I^2R=VI\cos\phi \text{ watts}$$

where V is the supply voltage.

When a condenser is connected in the circuit, the component of the voltage required to overcome the counter e.m.f. of the condenser lags the current by 90° and is wattless, since the energy associated with this voltage and the current is alternately taken from and returned to the supply. As there are no losses, the mean power supply to the condenser is zero over a complete cycle.

1.22. Power Factor. For the general a.c. circuit, with reactance as well as resistance, an a.c. voltmeter across the supply would read V, the r.m.s. value of the applied voltage, and the ammeter connected in series with the load would indicate I, the r.m.s. value of the current.

The product of these readings, VI, is termed the "apparent watts", since this is the power which apparently is being expended in the circuit. We have already seen, however, that the actual power is $VI \cos \phi$, which is known as the "true watts".

The ratio of mean power supplied ($VI \cos \phi$) to the product of the r.m.s. values of the voltage and current is

$$\frac{VI \cos \phi}{VI}$$

$$= \cos \phi$$

This ratio of true watts : apparent watts is termed the power factor of the circuit and its value, as can be seen from the above expression, is $\cos \phi$; where ϕ is the angle of lag or lead of the current relative to the supply voltage.

CHAPTER II

Absolute and Secondary Measuring Instruments—Potentiometers

Although the Meter Engineer, for whom this book is primarily written, is mainly concerned with the measurement of electrical energy, quantity, etc., he is largely dependent for his accuracy upon instruments other than integrating meters. Therefore, before we consider integrating meters, we will concern ourselves with the principles and construction of instruments designed for the measurement of current, voltage, and power. These instruments may be divided into two classes, namely, absolute instruments and secondary instruments.

2.1. Absolute Instruments give the value of the electrical quantity being measured in terms of the deflection, or setting required, due to the passage of the current through the instrument, and the constants of the instrument. No comparison with any other instrument is necessary, since it does not require initial calibration.

Two examples of this class of instrument are the Rayleigh Current Balance and the Tangent Galvanometer, both of which will be described presently.

2.2. Secondary Instruments depend upon calibration against either an absolute instrument or another secondary instrument which has already been calibrated. Until such a calibration has been made, the deflection of the secondary instrument bears no known relationship to the electrical quantity being measured.

The secondary instrument is in most general use, as the absolute instrument is almost entirely confined to the laboratory. However, we will consider briefly the two absolute instruments already mentioned.

2.3. The Tangent Galvanometer. A simplified illustration of this instrument is given in Fig. 5. It consists of a coil in the centre of which is suspended a very small magnetic needle. The plane of the coil is adjusted to lie exactly in the magnetic meridian and in the vertical position. The axis of the needle is horizontal.

Let the radius of the coil be r cm., the number of turns be N, the horizontal component of the earth's magnetic field be H gauss, and the length of the needle d cm. Then, when a current of I international ampères flows through the coil, the magnet will deflect to such a position that the respective torques, due to the earth's field and the current in the coil, are equal and opposite. Under these conditions

$$H \sin \theta = \frac{2\pi IN}{10r} \cos \theta$$

$$\text{Hence, } \tan \theta = \frac{2\pi NI}{10rH}$$

and I, the current (in international ampères) will be given by

$$I = \frac{10rH}{2\pi N} \tan \theta$$

The accuracy of the measurement depends directly upon the accuracy in determination of the value of H, and the correct alignment of the coil. Thus, in general, a separate and very accurate determination of the value of the earth's horizontal magnetic field is essential as a prelude to the above test. For further details, the interested reader is recommended to read the article by Kohlbrausch (*Phil. Mag.*, Vol. XXXIX).

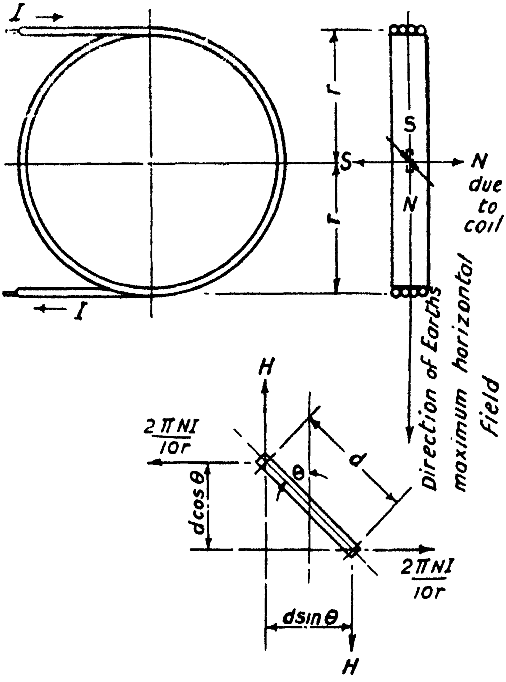


FIG. 5.—Tangent Galvanometer.

2.4. Rayleigh Current Balance. This apparatus, due to Lord Rayleigh, consists of two fixed coils (similar in construction to a Helmholtz coil) with a moving coil suspended between them; all three coils having a common vertical axis. The moving coil is somewhat smaller than the fixed coils, and the remote end of its suspension is fixed to the arm of a balance. A simplified illustration of the arrangement of the coils is given in Fig. 6.

When such a system of coils is carrying a common current, there will be a force of attraction or repulsion between the moving coil and each of the fixed coils, which will be proportional to the square of the current. As the movable coil is suspended from the arm of a balance, it is an easy matter to "weigh" the force and, from a knowledge of the constants of the instrument, determine the value of the current. In consequence, the instrument is sometimes termed a current "weigher".

An absolute instrument of this type was employed by Lord Rayleigh and Mrs. Sidgwick, in 1884, in their determination of the electrochemical equivalent of silver. Since that time, the instrument has been developed to a high degree of perfection due to the work of Ayrton, Mather, and Smith at the National Physical Laboratory, England, and Dorsey, Miller, and Rosa at the Bureau of Standards, Washington, U.S.A.

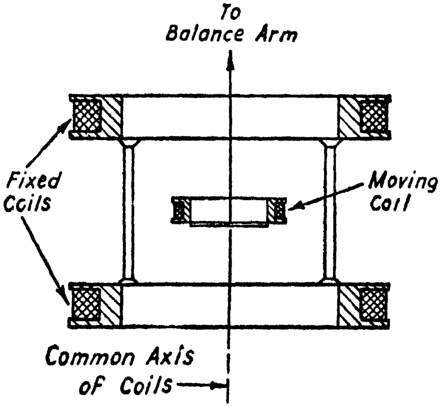


FIG. 6.—Rayleigh Current Balance.
(Diagrammatic Arrangement)

The Rayleigh current balance is not intended to be used as a normal current-measuring instrument, but as an absolute instrument for current measurement in special investigations. It has given great service in the determination of standards, and is considered to be the most accurate device which has been developed for this purpose.

The advantages of the instrument are:

(*a*) The constant of the instrument depends principally upon the ratio of the effective radii of the coils.

(*b*) The measurements are independent of the local field and its variations.

(*c*) There are no torsion constants to be determined.

It has been found that when the distance between the fixed coils and the movable coil is equal to one-half the radius of the fixed coils, slight inaccuracies in the location of the movable coil produce very small errors in the calculated constant of the instrument.

When the balance is used in the manner described above,

$$I^2K=\frac{mg}{2}$$

where m is the change in weights, corrected for the buoyancy of air, which is necessary to restore balance to equilibrium upon reversal of the current in the fixed coils, and g is the acceleration due to gravity. The factor K depends principally upon the ratio of the radii of the fixed coils to that of the movable coil and is equal to dm/dx, i.e. the rate of change of mutual inductance with respect to axial displacement of the movable coil.

2.5. Kelvin Current Balance. Lord Kelvin designed a secondary current balance, a diagrammatic arrangement of which appears in Fig. 7. It consists of four fixed and two movable coils; the two movable coils being carried on a beam free to rotate in a vertical plane in a similar manner to the beam of an ordinary chemical balance. The beam is suspended by means of a collection of hard-drawn copper wires, this collection forming a flexible conducting ligament; the number of wires used depending upon the current capacity of the balance. In manufacture, precautions are taken so that the tension upon each element of the total ligament is the same. The six coils (Fig. 7) are connected in series and the current flows in such a direction in each coil that the movable coil B is attracted by fixed coil 2 and repelled by fixed coil 4. Likewise, movable coil A is attracted by fixed coil 3

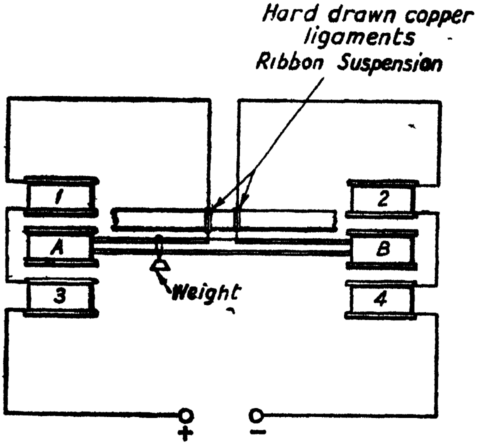


FIG. 7.—Kelvin Current Balance. (Diagrammatic Arrangement)

and repelled by fixed coil 1. The effect of these forces is to create an anti-clockwise moment upon the beam carrying the movable coils. This is balanced by the displacement of small weights which are carried on a small carriage running along a graduated bar attached to the beam. In order that the weights shall always be placed in the same position on the carriage, the latter is fitted with small conical locating pins which fit into similar shaped recesses in the weights.

The operation of measuring the current is performed in the following manner. A suitable known weight, for the approximate magnitude of the current, is placed on the carriage and a counterpoise, equal in weight to the loaded carriage, is placed in the V-shaped trough attached to the right-hand arm of the beam. The carriage is then moved to zero at the left-hand end of the graduated scale, and the clamp is freed. The carriage is thereupon moved along the graduated bar until a balance is obtained for the particular current passing through the coils.

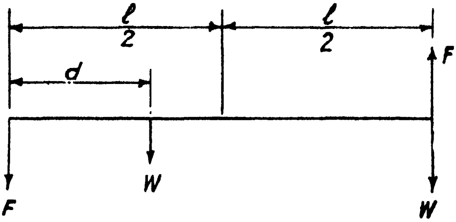


FIG. 8.—Kelvin Current Balance—Force Diagram.

Suppose Fig. 8 represents the conditions when balance is obtained. Then, if the length of the beam is l cm. and, at balance, the distance of the carriage from zero is d cm., the total moment in a clockwise direction due to the weights (say, each of w grammes) will be

$$\left(\frac{wl}{2}-w\left(\frac{l}{2}-d\right)\right)$$

$$=wd \text{ gramme-cm.}$$

At balance, this turning moment must be equal and opposite to that due to the current, and if we consider the force due to the current to be applied at the zero points of the bar, the turning moment of the latter is

$$F\frac{l}{2}+F\frac{l}{2}$$

$$=Fl$$

which is in an anti-clockwise direction. Furthermore, the force due to the current is proportional to the square of the current.

$$\text{Therefore, } K_1I^2=wd$$

and, since w and K_1 are constants,

$$I=\sqrt{\frac{wd}{K_1}}$$

$$=K_2\sqrt{d} \text{ ampères.}$$

The Kelvin balance may also be used for the measurement of alternating current. Since all coils are in series, the electro-magnetic fields of the six coils will reverse simultaneously. The turning moment is therefore always in the same direction.

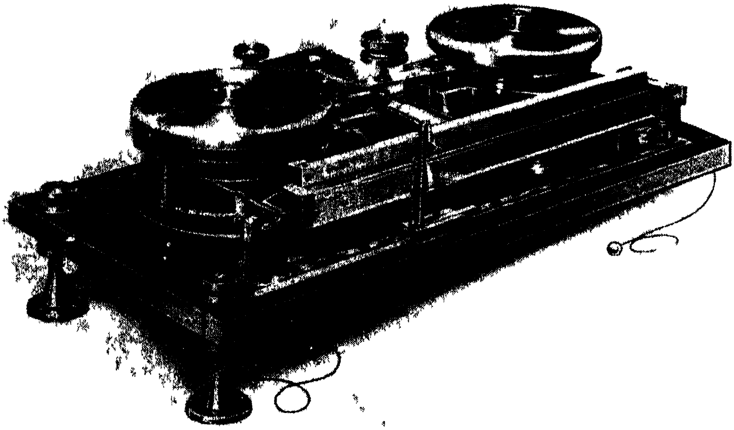


FIG 9.—Kelvin Current Balance

An instrument of the above type, manufactured by Kelvin, Bottomley and Baird Ltd., is illustrated in Fig. 9. These instruments are manufactured with ten different ranges, the maximum range obtainable being 2,500 ampères.

2.6. Galvanometers. An instrument which is employed for the detection and also measurement of small currents is the galvanometer. It is used extensively in potentiometer work and has been developed for use in a.c., as well as d.c., circuits. The most generally used galvanometer for d.c. measurements is that of the D'Arsonval type, whilst for the a.c. measurements it is modified in order to employ the principle of electrical and mechanical resonance.

2.7. The D'Arsonval Galvanometer consists essentially of a coil of many turns of very fine wire suspended between the poles of a permanent magnet; a cylindrical iron core often being placed inside the coil in order to keep the magnetic flux in the air gap as large and uniform as possible. A simple illustration of such an instrument appears in Fig. 10.

The air gap between the sides of the coil and the pole faces is made as small as possible, consistent with the permitting of suitable clearance, and the pole faces are shaped in such a manner as to ensure a radial field. The suspension consists of a single strand of very fine phosphor-bronze wire, which also serves as an electrical connection to the coil.

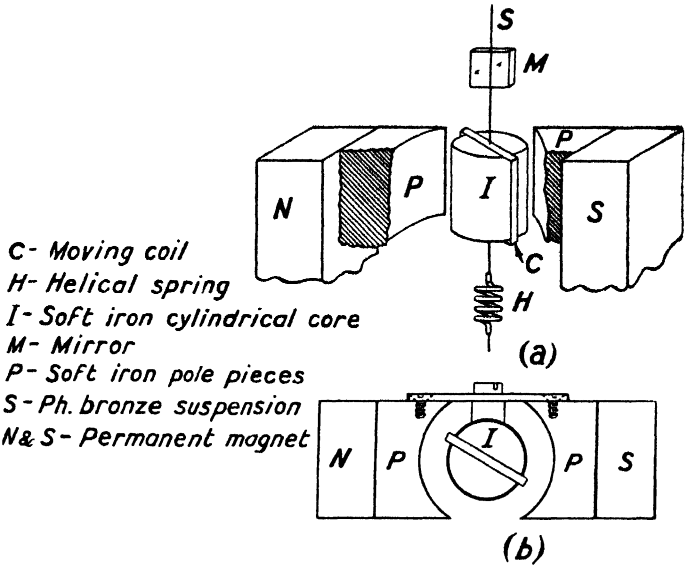


FIG. 10.—Arrangement of D'Arsonval Galvanometer.

To the bottom of the coil is fitted a loosely coiled spiral of wire, acting as the other lead to the coil. The extremities of both the suspension and the spiral wire are connected one to each of the terminals of the instrument. A small mirror is attached to the suspension and, when the instrument is in use, a beam of light from an external lamp is focused upon it. This beam is reflected by the mirror to an external scale, and a torsion head is provided upon the galvanometer to permit of zero adjustment. In the case of short period galvanometers (five seconds or less) the loosely coiled helix of wire, attached to the bottom of the moving coil, is often replaced by a phosphor-bronze strip similar to that of the suspension. This is effected in order to keep the moving system taut, and it improves the zero-keeping qualities of the instrument, especially if the latter is not seated upon a steady foundation.

The action of the galvanometer is very simple. When a current is flowing through the coil, the interaction of this current with the magnetic field due to the permanent magnet produces a torque. The coil then rotates until it finally reaches a position of equilibrium such that this torque is equal and opposite to that due to the torsion of the suspension; the latter endeavouring to bring the coil back to its zero position.

2.8. Damping is produced by means of eddy currents induced in the coil, and also in the metal former upon which the coil is wound, as they rotate in the magnetic field of the permanent magnet. The length of time taken for the coil to reach its position of equilibrium depends upon the extent of damping and upon the period of a complete swing when the instrument is on open circuit

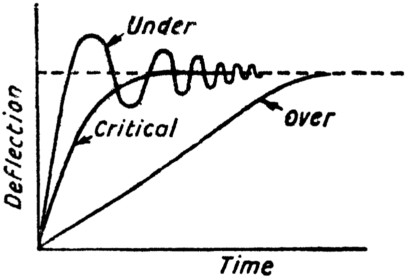


FIG. 11.—Response Curves—Damping.

If an instrument is over-damped, it will be sluggish in action and gradually creep to its final position. This is due to the creation of comparatively large eddy currents which oppose the motion of the coil. If under-damped, the coil will swing past its position of equilibrium on account of its inertia, and will then oscillate around this position with gradually decreasing amplitudes before finally coming to rest. In this case, the eddy currents induced in the coil are insufficient to overcome the inertia of the coil.

In most work it is advantageous to have the instrument critically damped, that is, swinging to the position of equilibrium, but just not oscillating at this point.

Fig. 11 illustrates the above three degrees of damping. It can be seen from these curves that it is advisable to aim for critical damping since the galvanometer will accurately respond to changes of current much more quickly in this condition than with under-damping or over-damping. As a consequence, one determines the external resistance necessary to give critical damping and endeavours to keep the galvanometer circuit at this value during service. Most manufacturers specify the external resistance necessary for critical damping. It can, however, easily be determined in a simple manner by giving the galvanometer a large initial deflection and observing whether or not the coil oscillates when it reaches its position of equilibrium. The external resistance is varied, and the test repeated, until it is found that the coil just does not oscillate as it comes to rest. If the resistance is increased, the damping will decrease, and vice versa.

2.9. Periodicity. The periodicity of the galvanometer is the time taken for it to make one complete oscillation when the coil is on open circuit. It can be ascertained by timing a number of complete oscillations whilst the coil is swinging freely. For meter-testing purposes, 1·0–2·0 seconds will be found to be a suitable period.

2.10. Sensitivity. The sensitivity of a galvanometer is usually defined as the scale deflection produced by one micro-ampère, when the scale is placed 1 metre from the galvanometer mirror. It is usually expressed as mm./micro-ampère. Similarly, the voltage sensitivity is defined as the mm. deflection per micro-volt under the same conditions.

The great advantage of the galvanometer is its high sensitivity, thereby permitting of a very large deflection for a very small current.

2.11. Vibration Galvanometer. It was suggested by Max Wien, in 1891, that a very sensitive detector of alternating currents could be developed if the principle of resonance was followed in the design of a detector. In other words, if one requires the maximum sensitivity, the moving member must be tuned mechanically, so that its natural period is the same as that of the electro-magnetic forces which cause its deflection. Thus, the moving member must have a very short natural period of vibration, and the damping must be small in order that the resonance curves shall be sharp.

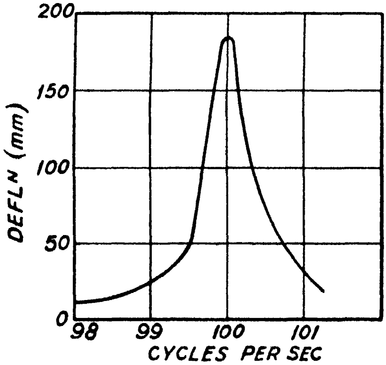


FIG. 12.—Response Curves Vibration Galvanometer. Cambridge Inst. Co.

Such a galvanometer has been developed and is termed a vibration galvanometer. It is of the D'Arsonval type, having a moving coil suspended between the poles of a very strong magnet. The moving system carries a small mirror, upon which a beam of light is cast.

When an alternating current, the frequency of which is the same as the natural frequency of the galvanometer, is flowing through its coil, the coil and mirror vibrate and a band of light is reflected upon the scale. Thus, to effect a balance in the circuit, adjustments are made until the length of the band becomes a minimum.

A great advantage of the instrument is that it is practically insensitive to all frequencies except that band which is very near its natural frequency. This can be seen from the response curve of Fig. 12, which demonstrates the sensitivity of the moving coil vibration galvanometer developed by Campbell and manufactured by the Cambridge Instrument Co. Since it is insensitive to harmonics, it is exceedingly useful in many measurement operations.

The instrument is tuned to the required frequency by varying the tension, and also the length of the suspension. A spiral spring is provided below the moving system for the former adjustment; whilst the latter is performed by alteration of the position of the bridge situated above the coil. In practice, a suitably small e.m.f. at the required frequency is applied to the galvanometer terminals, and the above adjustments are effected until the band of light seen on the screen is of maximum length. The adjustment is very critical and must be made with care.

It should be mentioned that, for 50 cycle per second measurements, the Cambridge Instrument Co. have developed their Simple Vibration Galvanometer, which does not require initial tuning.

2.12. The Potentiometer. A potentiometer is an instrument designed for the comparison of potential differences and utilises the null method of measurement. It is only used for current measurements inasmuch as the p.d. developed across a standard resistance, carrying a current of unknown value is compared with the e.m.f. of a standard cell; generally of the Weston cadmium type already discussed (section 1.12). Thus, since the value of the standard resistance is known, and also the voltage drop across its terminals, it is only necessary to apply Ohm's Law (section 1.14) in order to determine the value of the current.

2.13. The D.C. Potentiometer. Fig. 13 illustrates the principle of the potentiometer and gives the circuit in its most elementary form. The battery B_s sends a current through the slide-wire AC, of uniform cross-section, the value of the current being regulated by adjustment of the variable resistance R. The battery for test (or the voltage to be measured) is connected in series with a galvanometer and key S to the terminal A of the slide-wire, and also to a sliding contact B along the slide-wire. The battery B_t is connected so that its e.m.f. at AB is in opposition to that due to the battery B_s. This is clearly shown in the diagram.

Let us consider the resistance per unit length of the slide-wire as r ohms, and the current flowing through the slide-wire as I ampères.

Then the p.d. per unit length of the slide-wire will be rI volts, and that between the points A and B will be rId_t volts, where d_t is the distance between the points A and B. When the switch S is closed, there will be two p.d.s in opposition in the galvanometer circuit, i.e. the e.m.f. due to the battery under test and that due to the voltage drop between the points A and B on the slide-wire; this latter p.d. being due to the passage of the slide-wire current through the resistance of AB.

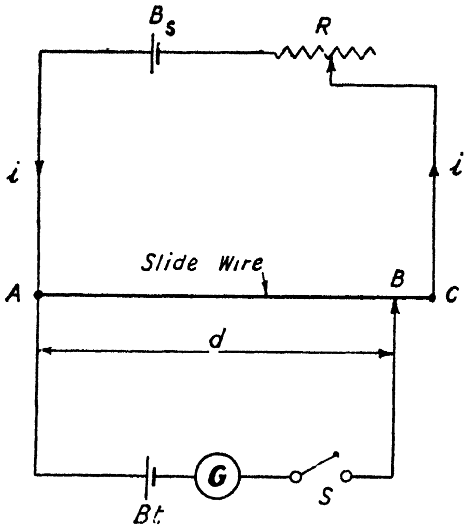


FIG. 13.—Simple d.c. Potentiometer.

As the position of the sliding contact B is altered, so d_t will change in value and, when rId_t is equal in value to the e.m.f. of B_t, no current will flow in the galvanometer circuit, and the galvanometer will have zero deflection. It therefore follows that, at balance,

$$E_t = Ird_t \text{ volts.}$$

If the battery under test is replaced by a second battery B_v, the e.m.f. of which is known, and the sliding contact is again adjusted until zero deflection of the galvanometer is obtained for the same current in the slide-wire circuit, it follows that

$$E_v = Ird_v \text{ volts,}$$

where d_v is the new distance between A and B.

$$\text{Thus, } \frac{E_t}{E_v} = \frac{d_t}{d_v}$$

$$\text{or } E_t = E_v \frac{d_t}{d_v}$$

from which expression the value of E_t can be determined.

In this elementary form of potentiometer, a scale is fitted along the slide-wire and the distance between the points A and B can be read off directly in cm.; the ratio $d_t : d_v$ being that of the two e.m.f.s subjected to comparison.

It is essential that the supply battery B_s shall be of ample capacity, so that the current in the slide-wire circuit will remain constant throughout the test. During the initial adjustment of the sliding contact, a resistance of high value should be placed in series with the galvanometer in order to limit the current to a value which will not damage the instrument. As the zero point is approached, the resistance should be taken out of the galvanometer circuit, thereby providing full sensitivity for the final adjustment. The resistance also fulfils the important function of protecting the standard cell which should not, under any circumstances, be permitted to give appreciable current.

2.14. Crompton Potentiometer. The simple slide-wire form of potentiometer was first modified by Crompton, in the manner illustrated in Fig. 14. The graduated slide-wire AC is connected in series with fourteen or more coils; the resistance of each coil being exactly equal to that of the slide-wire (10 ohms). When the potentiometer is standardised, there is 0·1 volt drop across each of the coils and also across the slide-wire. The two moving contacts C_2 and C_1 make contact with the slide-wire and the resistance coil studs respectively. The e.m.f. of the supply battery is 2 volts. The variable resistances R_1 and R_2 are for adjustment of the current; the former giving course adjustment whilst the latter gives a continuous fine adjustment by means of a slide-wire.

The galvanometer G is connected in series with the switch S and a multiple circuit switch, the latter being provided to facilitate the selection of either the standard cell, or the e.m.f. to be measured, for connection into the galvanometer circuit. It is very important that the e.m.f. to be measured is connected with care. Otherwise, there is a possibility of damage to the apparatus due to the connecting of wrong polarities.

The method of standardising the instrument is as follows:

A standard cell, usually of the Weston cadmium type, is connected to the terminals SC. For reasons already mentioned, the galvanometer and cell are connected in series with a high value resistance. The setting of contacts C_1 and C_2 depends upon the voltage of the standard cell employed. In the case of the Weston cadmium cell, contact C_1 is set at stud 1·0 and C_2 at 0·0183 on the slide-wire, since the voltage of the cell is 1·0183 volts. The current in the main battery circuit is then adjusted, by means of R_1 and R_2, until the galvanometer has zero deflection at full sensitivity, i.e. with the series protective resistance out of circuit.

The resistances R_1 and R_2 are thereafter kept at this value and the selector switch is rotated to the contacts to which the voltage for measurement is connected. The galvanometer is again initially rendered insensitive and C_1 and C_2 are readjusted until, finally, there is no deflection of the galvanometer at maximum sensitivity. The summation of the readings C_1 and C_2 will now give the value of the voltage being measured, since the potentiometer current has already been standardised with the standard cell.

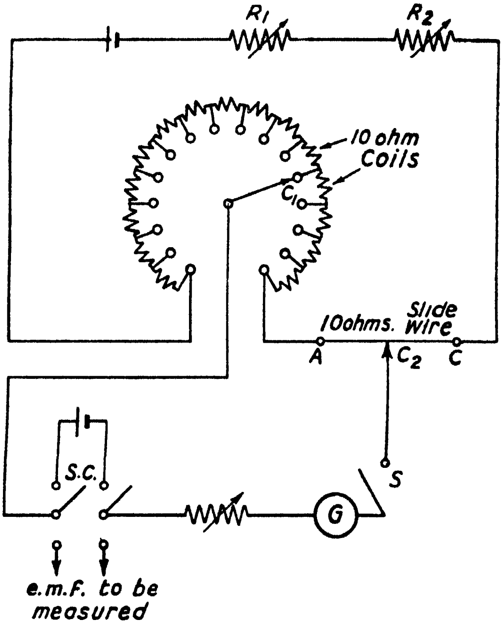


FIG. 14.—Circuit of Crompton Potentiometer.

In the case of current measurement, the reading of C_1 and C_2 will give the value of the p.d. developed across the standard resistance, due to the passage of the current being measured. It is therefore a simple matter, since the value of the standard resistance is known, to calculate the value of the current. This will be

$$\frac{C_1+C_2}{R}$$

where R is the value of the standard resistance, and C_1 and C_2 are the potential values which correspond to the positions of C_1 and C_2. It is advisable to choose a value of R so that the p.d. developed will result in a convenient reading for C_1 and C_2.

2.15. Tinsley Vernier Potentiometer. Fig. 15 illustrates another form of d.c. potentiometer. This instrument, manufactured by H.

Tinsley & Co. Ltd., possesses two ranges; the normal range being from 1·90100 volts down to 0·00001 volt and the lower range from 0·190100 volt down to 1 micro-volt. There are three measuring dials. The left-hand dial covers 1·8 volts in steps of 0·1 volt, the middle dial covers 0·1 volt in steps of 0·001 volt, and the right-hand dial gives 0·001 volt in 100 equal steps. No slide-wire is provided in this instrument since the smallest steps are sufficiently fine to be considered continuously variable for most purposes.

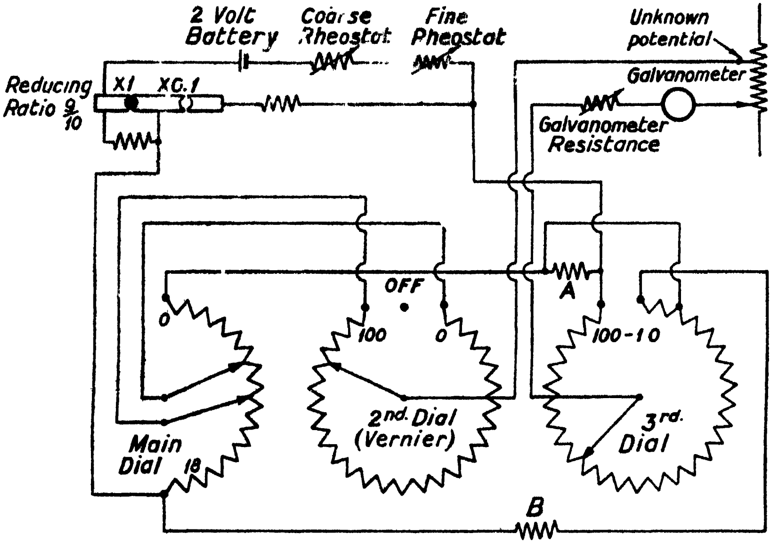


FIG. 15.—Circuit Arrangement of Tinsley Vernier Potentiometer.

The resistances of the middle dial shunt two coils of the first dial; the moving arm of the latter carrying two contacts spaced two studs apart. It thus employs the principle of the Varley slide, the purpose of which is as follows. The first dial possesses nineteen coils, each of 5 ohms giving a p.d. of 0·1 volt with a current of 20 milliampères. The coil of the second dial, as previously stated, shunts two coils of the first dial, and has a total resistance equal to that of the two shunted coils, i.e. 10 ohms. This coil is equally divided into 100 sections. Therefore, the effective resistance of the parallel circuit is equal to that of one coil of the first dial, and the p.d. across each section of the middle dial coil is 1/100th that across each coil of the first dial. In consequence, it is equivalent to dividing each of the first dial coils into 100 sections.

The Varley slide method necessitates the employment of exceptionally good contacts on both brushes but, providing this is so, it is a very convenient method of extending the scale of a potentiometer. Small zero errors, however, are very liable to exist in such an arrangement.

The simplified circuit diagram of Fig. 16 illustrates how the third dial operates. One step of this resistance is in series with resistance *B* across the main dial, whilst the remaining 100 steps are in parallel with resistance *A*. The values of *A* and *B* are such as to give the necessary p.d. across each division of the third dial when the potentiometer has been standardised. On the normal range this value, as previously stated, is 0·00001 volt. The advantage of this arrangement is that a small negative setting of 0·00001 volt is provided and also, since the 0 stud of the first dial would not be at exactly the same potential as the 0 stud of the second dial, it provides a ready means of compensation for this difference; making it possible to attain a perfect zero potential for zero setting of the potentiometer.

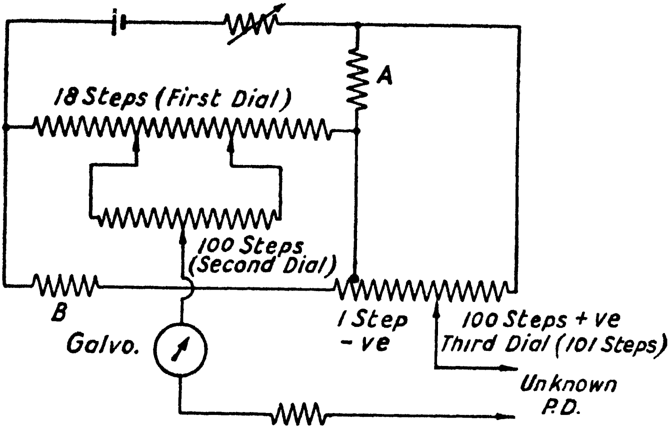


FIG. 16.—Simplified Arrangement—Tinsley Vernier Potentiometer.

The total resistance of this instrument is approximately 100 ohms, and it is provided with a four-position selector switch for the selection of either the standard cell or any one of three p.d.s to be measured.

The potentiometer has an independent standardising circuit, and it is only necessary to set the selector switch to standard cell, set the standard cell dial to the appropriate value at the ambient temperature (20 steps of 0·00005 volt each) and adjust the current until balance is obtained. The lower range of the potentiometer is obtained by moving the plug in the contact recesses, marked "reducing ratio", from the position "X by 1" to "X by 0·1". This operation shunts the working portion of the potentiometer with a resistance of one-ninth its value; thus making the resistance of the working portion virtually one-tenth its previous value. In order to keep the total resistance constant in the current circuit, and therefore the total current at a constant value, a resistance of nine-tenths that of the normal working resistance of the potentiometer is connected in series with the parallel circuit. Consequently, as the current is of the same value as it was for the "X by 1" setting of the potentiometer, and the resistance of the working portion of the potentiometer is one-tenth its previous value, the total volt drop across the coils will be one-tenth its normal value.

2.16. Cambridge Meter Testing Potentiometer. The Cambridge Instrument Co. has developed a potentiometer specially for meter-testing purposes. It is robust and capable of withstanding rough usage without its accuracy being endangered. On the normal range, steps of 0·1 volt are given upon an engraved switch, and lower values of voltage are covered by a large disc, graduated in millivolts, which carries a rolling contact over a circular wire. Each millivolt graduation is subdivided into five divisions. The potentiometer current is supplied by a 2-volt accumulator and, at any time during a test, it is possible to check or readjust the standardisation of the potentiometer without disturbing the main dial settings. The normal range of the instrument is 0 to 1·8 volts and a rotary switch is provided to give one-tenth of this range. Compensation for temperature (between 10°C. and 30°C.) of the standard cell can be effected by means of a calibrated slide-wire provided for this purpose.

2.17. The A.C. Potentiometer. This instrument operates by means of the same principle as the d.c. potentiometer. The only difference is that, as the former must take into account the relative phase as well as magnitude of the alternating voltage, it must have an arrangement whereby the p.d. in the slide-wire circuit can be altered in phase relative to that of the p.d. being measured. It is therefore necessary to modify the d.c. potentiometer if it is to be suitable for alternating current measurements. In consequence, the operation of the a.c. potentiometer must of necessity be more complicated.

It is essential that the frequency and wave-form of the current in the slide-wire circuit of the potentiometer be exactly the same as that of the voltage to be measured. Hence, it is advisable to take the supply for the potentiometer from the same source of supply as the voltage to be measured.

There are two general classes of the a.c. potentiometer, and they differ in the method by which the unknown voltage is measured. They are:

(*a*) Those which measure the magnitude and phase angle of the unknown voltage, indicating them in polar form.

(*b*) Those which measure the rectangular co-ordinates of the voltage (i.e. the "in-phase" and the 90° out-of-phase components), thus indicating them in cartesian form.

2.18. Drysdale-Tinsley A.C. Potentiometer. This instrument belongs to class (*a*) and consists of an ordinary potentiometer (the coils, however, being non-inductively wound), a Drysdale phase-shifting transformer, and a precision type electro-dynamometer ammeter. The D'Arsonval galvanometer is replaced by a vibration galvanometer (section 2.11).

2.19. Drysdale Phase-shifting Transformer. This phase-shifting transformer consists of a ring-shaped stator wound with either a two-phase or three-phase winding. A wound rotor is located in the stator; only a small air gap being provided. When current flows in the stator windings, a rotating field is produced, thereby inducing an e.m.f. in the rotor winding. Thus, the stator winding is the primary and the rotor winding the secondary of the transformer. In addition, the rotor can be adjusted at will through any required angle, the phase displacement of the secondary e.m.f. being equal to the angle through which the rotor has been moved from its zero position. A scale and pointer are provided on top of the instrument to indicate the angle through which the rotor has travelled from its zero position; the scale being graduated in degrees and cosines of the angles.

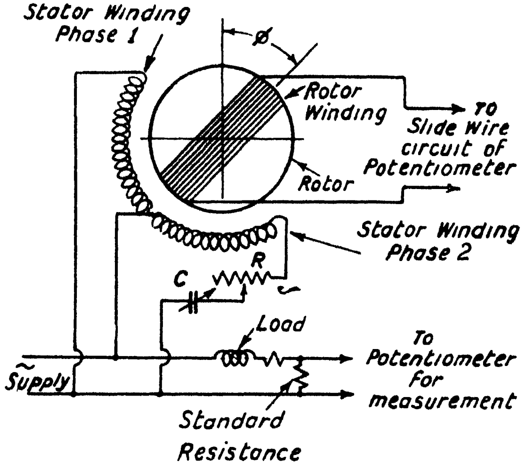


FIG. 17.—Circuit Arrangement—Drysdale Phase-shifter, Single-phase.

The phase-shifter may be employed from a single-phase supply by means of a phase-splitting device, consisting of a resistance and variable condenser, arranged as shown in Fig. 17. It is then only necessary to have two separate pairs of windings on the stator, set 90 geometrical degrees apart, instead of the three windings required for the three-phase instrument. One pair of windings is fed directly from the supply, at the phase of the supply voltage, whilst the other pair is connected in series with the resistance and variable condenser, the p.d. at the terminals of the winding thereby being displaced by 90° from that of the supply voltage. Correct phase-splitting is attained by adjusting the values of the resistance *R* and the condenser *C* until the magnitude of the e.m.f. induced in the rotor winding is constant, irrespective of the angular position of the rotor from zero. When a three-phase supply is available, it is usual to employ three-phase windings for both stator and rotor, thereby dispensing with the necessity of a phase-splitting device. The connections of such an instrument are illustrated diagrammatically in Fig. 18.

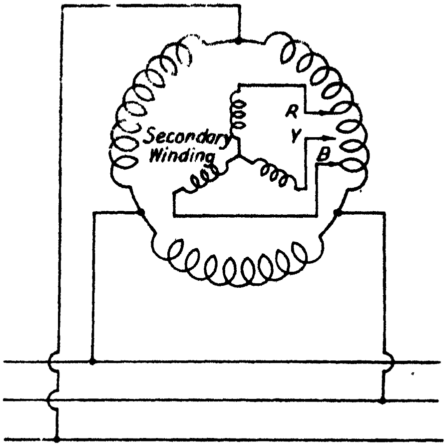


Fig. 18.—Drysdale Phase-shifter—
Three-phase.

Now, consider the two coils in the diagram of Fig. 17. Let the stator winding of phase 1 be energised by a current of value $I \sin \omega t$, at any time t, and the stator winding of phase 2 by a similar current, but 90° out of phase with the former, i.e. of value $I \cos \omega t$. Then the rectangular components of the resultant field at the centre are

$$x = H \sin \omega t$$

$$y = H \cos \omega t$$

The resultant field $R = \sqrt{x^2 + y^2}$

$$H\sqrt{\sin^2 \omega t + \cos^2 \omega t}$$

$$H$$

which is constant, and equal to the maximum field at the centre due to each individual coil.

At any instant, the tangent of the inclination of this resultant to the vertical axis is x/y.

$$\text{Therefore, } \tan\gamma = \frac{x}{y} = \frac{\sin\omega t}{\cos\omega t} = \tan\omega t$$

$$\text{Hence, } \gamma = \omega t.$$

Therefore, the resultant field at the centre is of constant magnitude and rotates with constant angular velocity. Also, since the flux threading the rotor winding is proportional to cos $(\omega t-\theta)$ and the induced e.m.f. to sin $(\omega t-\theta)$ the time-phase displacement is equal to the actual displacement of the rotor winding from its zero position. In other words, providing the currents in the two stator windings are exactly 90° apart, and of the same amplitude, the phase angle of the e.m.f. induced in the rotor winding will correspond to the angular displacement of the rotor from its zero position, whilst the magnitude of the e.m.f. will remain constant for every position of the rotor.

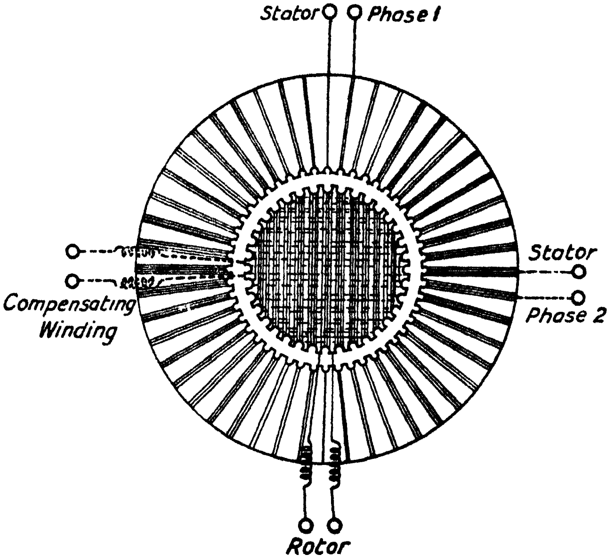


FIG. 19.—Drysdale Phase-shifter—Schematic Arrangement.

The actual instrument, for use with a single-phase supply, consists of a laminated stalloy stator with 48 slots; the windings of the two phases being as shown diagrammatically in Fig. 19. The number of turns in each slot is graded according to a sine law (i.e. in a slot $\theta°$ from the zero position the number of turns will be $N \sin\theta$, where N is the maximum allowable turns per slot). The object of this is to produce a practically uniform field in the portion surrounded by the stator.

Since the windings are displaced by 90°, both geometrically and electrically, the stator will be influenced by two uniform fields in time and space quadrature, in the manner already described. The rotor is of such dimensions as to give only a small air gap between its periphery and the stator, and it has 36 slots which are almost totally enclosed. Two windings, also graduated according to a sine law, are symmetrically distributed over the rotor, but at right angles to one another, as indicated in the winding diagram. Only one of these windings is connected to the potentiometer circuit; the other being connected to a coil of equal resistance and reactance to that of the potentiometer circuit. The purpose of this second winding is to compensate, in every position, for rotor reaction. Good constancy of voltage and accuracy of phase angle can, however, be attained without this compensating circuit by careful adjustment of the phase-splitter already described. To ensure a smooth variation, it is usually necessary to incline the slots to the vertical.

The rotor is mounted in parallel bearings concentric with the stator, and its motion is controlled by means of a handle and worm gear. The dial, which is on top of the instrument, has one-half graduated in degrees and the other half with a corresponding cosine scale. Four pointers, at right angles, are provided with the scale to facilitate rapid conversion from polar to cartesian form, without reference to trigonometrical tables. An index pointer is also provided to indicate the magnetic axis of the rotor.

The electro-dynamometer ammeter is necessary for the accurate measurement of the potentiometer current, and is chosen for this purpose on account of its indicating accurately for both alternating and direct current. It is of the torsion-head type described in section 4.12.

2.20. Circuit of Drysdale-Tinsley A.C. Potentiometer. A simplified diagram of connections of the Drysdale-Tinsley potentiometer is given in Fig. 20. It can be seen from this diagram that the potentiometer incorporates the Varley device (section 2.15). A vibration galvanometer replaces the D'Arsonval galvanometer when the potentiometer is connected to the a.c. supply. R_1 and R_2 are the shunt and series resistances respectively, and when these are brought into circuit, the potentiometer has one-tenth its normal range. This arrangement has already been described (section 2.15) in connection with the Tinsley Vernier d.c. potentiometer. The stator terminals of the phase-shifter are connected to the supply, via a phase-splitting device if only a single-phase supply is available, and the leads so indicated on the diagram go from the rotor terminals of the phase-shifter to the working portion of the potentiometer circuit. Usually, the potentiometer is connected to the supply via an isolating transformer.

The potentiometer is standardised with a d.c. supply and a standard cell (see section 2.15), the galvanometer switch S_2 being located so as to bring the D'Arsonval galvanometer into circuit in place of the vibration galvanometer. The standardising current, indicated by the electro-dynamometer ammeter, is carefully noted.

Switch S_2 is now changed over to the alternating current supply and, by means of switch S_3, the D'Arsonval galvanometer is replaced by the vibration galvanometer. The stator windings of the phase-shifter are now adjusted to exact quadrature and the resistance R_3 varied until the electro-dynamometer ammeter indicates the same alternating current value as that noted during the d.c. standardisation of the potentiometer. Balance against the p.d. being measured is obtained by successive adjustment of the potentiometer knobs and the phase-shifter until, at maximum sensitivity, the vibration galvanometer gives minimum deflection. The phase angle of the voltage being measured (relative to the supply voltage) will then be indicated by a pointer upon the calibrated scale of the phase-shifter; whilst the magnitude may be read directly from the setting of the potentiometer dials.

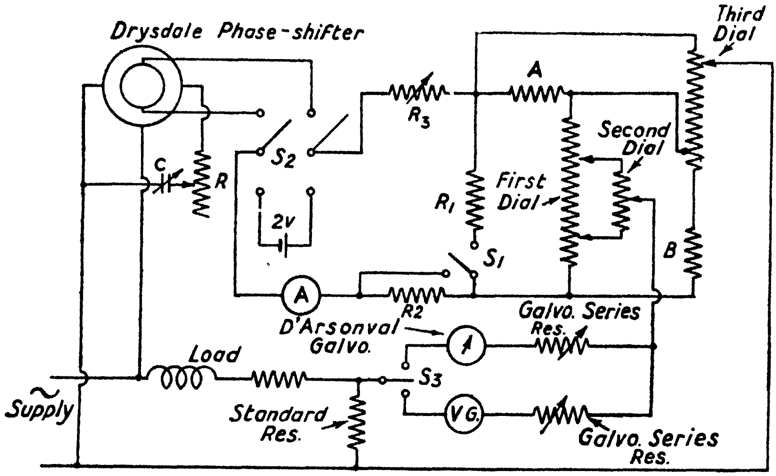


FIG. 20.—Circuit Diagram—Drysdale-Tinsley a.c. Potentiometer.

Where the voltage to be measured is that due to a current passing through a standard resistance, the magnitude of the current is—as in the d.c. case—the potentiometer reading divided by the value of the resistance, and the reading on the scale of the phase-shifter gives the phase angle of the current relative to the supply voltage. Since the cosine of the phase angle gives the power factor of the load, the power dissipated in the circuit is equal to the product of the supply voltage, current, and cos ϕ, where ϕ is the indication on the scale of the phase-shifter. It is assumed, of course, that the phase-shifter is energised by the supply voltage responsible for the current being measured.

In order to maintain the sensitivity of the potentiometer, it is essential that the frequency of the supply be kept constant. This is because the vibration galvanometer is a tuned instrument with a highly selective response curve, i.e. it only responds freely to currents of one particular frequency. If the vibration galvanometer is tuned to the fundamental frequency of the circuit, it will only indicate a balance when the fundamentals, and not the r.m.s. values, of the p.d.s are of the same magnitude. If the wave-form is very bad, the galvanometer may vibrate at some other than its natural frequency, thus making it impossible to obtain an exact balance In consequence, good wave-forms are necessary for the successful operation of the a.c. potentiometer.

2.21. Campbell-Larsen A.C. Potentiometer. This belongs to the second class of a.c. potentiometers, in which the in-phase and quadrature components of the voltage are measured. It was developed by Campbell, and is a modification of the Larsen potentiometer. The latter potentiometer had the disadvantage of not giving a direct reading of the quadrature component of the voltage being measured. The reason for this is that the e.m.f. induced in the secondary of a mutual inductometer (which Larsen employed for balance of the quadrature component of the voltage) is proportional to ωMI, where ω is 2π times the frequency, M is the mutual inductance, and I is the primary current. As I was of constant value in Larsen's potentiometer, the induced voltage, for any given value of M, was directly proportional to the frequency.

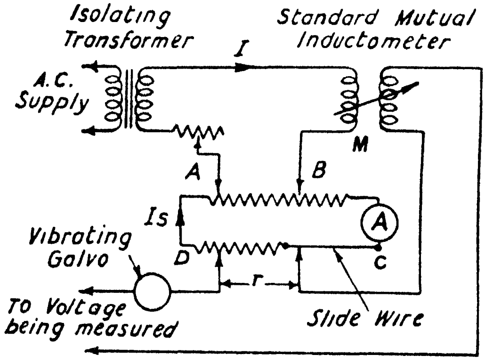


FIG. 21.—Circuit Diagram—Campbell-Larsen Potentiometer.

Campbell improved the instrument by an ingenious arrangement which makes I inversely proportional to the frequency. Thus, the mutual inductometer can be directly calibrated in volts, corresponding to the e.m.f. induced in the secondary winding of the mutual inductometer. Campbell incorporated a variable loop shunt, graduated in cycles per second, in the resistance portion of the potentiomet A simplified circuit of the modified potentiometer is illustrated Fig. 21.

When a current I passes through the primary winding of the mutu inductometer, part of the current passes through the working portic of the potentiometer. Let this be denoted by I_s. If the resistanc between the movable contacts AB is P, and that round the pat $BCDA$ is Q, it follows that

$$I_s = \frac{I\,P}{P+Q}$$

$$\text{therefore, } I = \frac{(P+Q)}{P}.I_s$$

Now, $P+Q$ is constant, and if we make I_s constant, the value of I is inversely proportional to P. Thus, if we make the value of P directly proportional to the frequency, I will be inversely proportional to the frequency and the induced voltage will be proportional to M, the mutual inductance.

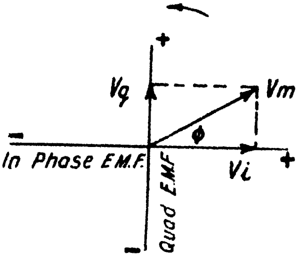


Fig. 22.—Vector Diagram—Cartesian type a.c. Potentiometer.

$$\text{Hence, } E_s = KM$$

where E_s is the induced secondary voltage of the mutual inductometer and K is a constant.

In the actual instrument, P is adjusted by means of a dial, calibrated directly in terms of the frequency.

The instrument is initially standardised with direct current in the same manner as described for the Drysdale-Tinsley a.c. potentiometer. The dial of P is then set to correspond to the frequency of the voltage to be measured, and the current in the working portion of the potentiometer is adjusted until the ammeter A indicates the value found to be correct during the d.c. standardisation of the potentiometer.

The voltage to be measured is now switched into the vibration galvanometer circuit, the galvanometer initially being rendered very insensitive. The movable contacts on the slide-wire circuit and the mutual inductometer are then adjusted by successive approximation until balance is obtained, i.e. until at maximum sensitivity the deflection of the vibration galvanometer is a minimum.

If V_i is the voltage of the in-phase component, and V_q is that of the quadrature component (Fig. 22), then

$$V_m = \sqrt{V_i^2 + V_q^2}$$

$$\text{and } \phi = \tan^{-1} \frac{V_q}{V_i}$$

where V_m is the magnitude of the voltage being measured and ϕ is the phase angle of this voltage relative to the potentiometer supply voltage.

The same remarks apply to the measurement of current with this potentiometer as applied to the Drysdale-Tinsley a.c. potentiometer.

2.22. The Gall-Tinsley A.C. Potentiometer. This instrument, which was developed by D. C. Gall and is manufactured by H. Tinsley and Co. Ltd., measures the in-phase and quadrature components of the voltage. This is effected by means of two potentiometers, one of which is termed the "in-phase" potentiometer, and the other the "quadrature" potentiometer.

A simplified diagram of the potentiometer circuit is given in Fig. 23. The transformers T_1 and T_2 are for the purpose of isolating the potentiometers from the line potential and also for stepping down the voltage to 6 volts. The resistance R_3 and condenser C are for phase-splitting, and quadrature is obtained by their variation. Alternatively, one may feed the transformers from a two-phase alternator.

VG is, as the letters imply, a vibration galvanometer, and *A* is a reflecting electro-dynamometer ammeter. The vibration galvanometer is tuned to resonance at the supply frequency. S_1 is a switch for the galvanometer circuit. S_3 and S_4 are switches for reversal, if necessary, of the e.m.f. under measurement to the working portions of the respective potentiometers. They may be operated individually. R_1 and R_2 are variable resistances for adjustment of the magnitude of the currents in the in-phase and quadrature potentiometers respectively. S_2 is a selector switch for connection of the voltage to be measured to the potentiometer circuits, or for comparison of the two potentiometer currents during quadrature adjustment. For this latter purpose, the mutual inductance *M* is brought into circuit.

The operation of the potentiometer is as follows:

The in-phase potentiometer is first standardised with direct current against a standard cell, in the manner already described, and the value of current required for this purpose is carefully noted. This is indicated by the electro-dynamometer ammeter, which is of the torsion-head type described in section 4.12. During standardisation, the torsion-head is rotated until there is zero deflection for the correct in-phase potentiometer current. Throughout all subsequent operations the torsion-head is not moved.

The potentiometer is now connected to the a.c. supply and resistance R_1 is adjusted until the electro-dynamometer ammeter again registers

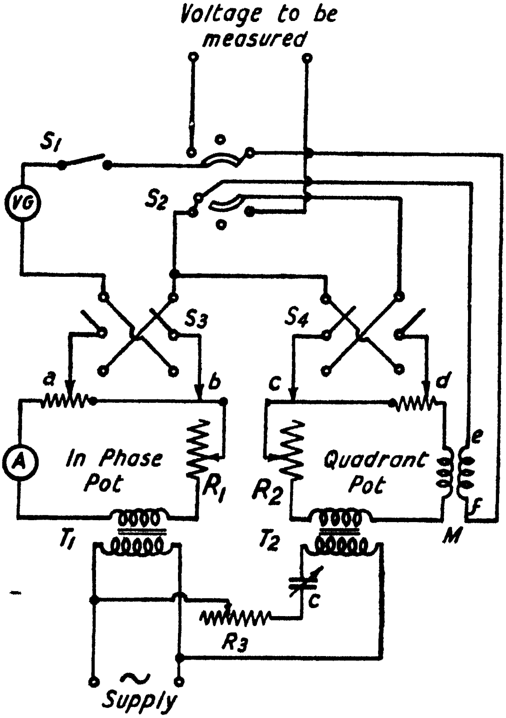


FIG. 23.—Gall-Tinsley a.c. Potentiometer.

zero deflection, i.e. until the torque due to the current is equal to that due to the torsion-head. The selector switch S_2 is now rotated into the test position illustrated in Fig. 23, thus bringing the mutual inductance M in series with the vibration galvanometer circuit; the entire circuit receiving a p.d. from the points a and b of the in-phase potentiometer. Now, if a current of r.m.s. value I ampères passes through the working portion of the quadrature potentiometer, it follows that the induced voltage in the secondary of the mutual inductance will be ωMI, where M is the value of the mutual inductance in henrys. Therefore, if the standard value of the current in each of the potentio meter circuits is 50 milliampères (as is the case in the actual instrument the induced voltage in the secondary of the mutual inductance will be

$$E_s = \omega MI \text{ volts.}$$

At 50 cycles per second, this will be

$$2\pi.50.50.10^{-3}M$$
$$5\pi M \text{ volts.}$$

Thus, if we adjust the contacts *a* and *b* so that there is a p.d. of 1 volt between them, the inducted voltage in the secondary winding of the mutual inductance will be exactly equal and opposite when the following conditions are observed

(*a*) The induced voltage in the secondary of the mutual inductance is exactly 180° out of phase with the p.d. between the points *a* and *b* of the in-phase potentiometer.

As the e.m.f. in the secondary winding of the mutual inductance lags the primary current by 90° (assuming no losses in the mutual inductance), this will only occur when the current in the quadrature potentiometer is exactly 90° out of phase with that of the in-phase potentiometer. That is, when the two potentiometer currents are in exact quadrature.

(*b*) The magnitude of the induced e.m.f. in the secondary winding of the mutual inductance must be equal to the p.d. across the points *a* and *b* of the in-phase potentiometer. Therefore, for the e.m.f. induced in the secondary winding of the mutual inductance to be, say, 1 volt when a current of 50 milliampères is passing through the primary winding and the quadrature potentiometer,

$$\omega M 50.10^{-3} = 1.$$

At 50 cycles per second, $5\pi M = 1$

$$\text{and } M = 0{\cdot}0636 \text{ henry.}$$

In the actual instrument, *M* is constant at 0·0318 henry and the value of E_s, the induced voltage in the secondary winding of the mutual inductance, is

$$\begin{aligned} E_s &= 2\pi f MI \\ &= 2\pi f 0{\cdot}0318 \times 50.10^{-3} \text{ volts} \\ &= f/100 \text{ volts,} \end{aligned}$$

where *f* is the frequency in cycles per second.

Hence, for adjustment of the quadrature potentiometer, the p.d. between the points *a* and *b* of the in-phase potentiometer is set at *f*/100 volts. At 50 cycles per second this will be 0·5 volt.

To standardise the quadrature potentiometer, after the points *a* and have been set for the correct p.d., successive adjustments of the condenser *C* and the resistance R_3 are made until, at full sensitivity, the deflection of the vibration galvanometer is a minimum. If the frequency is not determinable with sufficient accuracy, it will be necessary to install a second electro-dynamometer ammeter in the quadrature potentiometer circuit.

When the above adjustments have been made, the voltage to be measured is connected in circuit by means of the selector switch (which simultaneously opens the circuit of the mutual inductance secondary winding). This connects the tappings from the potentiometer circuits, the voltage to be measured, and the vibration galvanometer, in series. Balance is obtained by successive adjustment of the contacts *ab* and *cd*, and reversal of either or both S_3 and S_4, if this is found to be necessary. The settings of the contacts *ab* and *cd*, together with the positions of their respective reversing switches, give the magnitude and direction of the in-phase and quadrature components of the voltage.

If the setting of the in-phase potentiometer is found to be V_i, and that of the quadrature potentiometer V_q, as represented in the vector diagram of Fig. 22, the magnitude V_m, of the voltage being measured, is given by

$$V_m=\sqrt{V_i^2+V_q^2}$$

and the phase angle of this voltage, with respect to the potentiometer supply voltage, is given by

$$\phi=\tan^{-1}\frac{V_q}{V_i}$$

If these are both positive, as indicated in Fig. 22, the voltage under measurement will lead the potentiometer supply voltage by the angle ϕ.

2.23. Calibration of Ammeters with Potentiometer. In the case of the calibration of an ammeter, the instrument is connected in series with a standard resistance of such a value that when the calibration current is flowing a p.d. of the order of 1 volt is developed across its terminals. The potentiometer, after standardisation, is set at the p.d. which would be developed across the standard resistance for the exact value of calibration current. The p.d. across the standard resistance is then applied to the potentiometer circuit in the normal manner and the calibration current is varied until balance is obtained. The deflection of the ammeter is noted and the percentage error will be given by

$$\frac{I_A-I}{I}\times 100$$

where I is the calibration current and I_A is the indication of the ammeter.

Alternatively, the calibration current may be set at an approximate value and the p.d. developed across the standard resistance measured by means of the potentiometer. When the potentiometer is balanced the current in the standard resistance and ammeter circuit will be given by

$$I=\frac{E_p}{R}$$

where E_p is the potentiometer setting and R is the value of the standard resistance.

When calibrating an ammeter throughout its range, it is advisable to change the value of the standard resistance at suitable stages during the test, in order to retain a convenient setting of the potentiometer.

2.24. Calibration of Voltmeters with Potentiometer. It has already been seen that the potentiometer is only suitable for the direct measurement of voltages between the limits of 0 and 2 volts. For greater values of voltage, a voltage ratio box must be employed in conjunction with the potentiometer. The principle of the ratio box can be seen from the simple diagram of Fig. 24, illustrating the internal connections of such a piece of apparatus. We will assume that the voltage of the supply is V_s. Then the current flowing through the circuit *abcd* will be given by

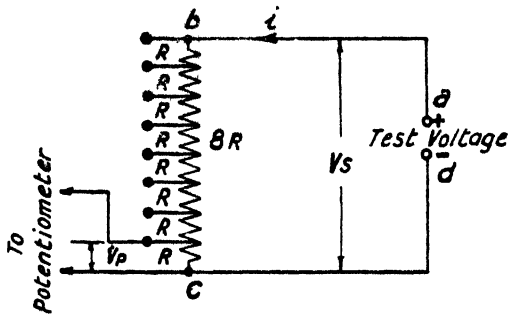


FIG 24.—Voltage Ratio Box.

$$i_s = \frac{V_s}{8R}$$

where $8R$ is the total resistance across the test voltage.

Now, if this resistance consists of eight portions, each of value R, the voltage drop across each portion R will be

$$\begin{aligned} V_p &= i_s R \\ &= \frac{V_s R}{8R} \\ &= \frac{V_s}{8} \end{aligned}$$

From this it follows that any value of voltage between zero and the full supply voltage may be tapped off a voltage ratio box, or potential divider, providing a tapping is made such that the ratio of the resistance tapped off, to the total resistance across the supply, bears the same ratio relationship as does the required voltage to the supply voltage.

During the calibration of a voltmeter, a volt box, or potential divider, is connected across its terminals and the ratio adjusted until there is a p.d. of the order of 1 volt applied to the potentiometer, for the particular calibration voltage supplied to the voltmeter terminals.

If, after balance of the potentiometer is obtained, E_p is the potentiometer setting, V_r the voltmeter reading, and r_a the ratio of the potential divider, the error of the voltmeter will be

$$V_r - r_a E_p$$

$$= \left(\frac{V_r - r_a E_p}{r_a E_p} . 100 \right) \text{ per cent.}$$

By choosing suitable ranges of the voltage ratio box, and various settings of the potentiometer, it is possible to calibrate the voltmeter throughout its range for any required number of readings.

2.25. Calibration of Wattmeters with Potentiometer. A simplified diagram of connections of a circuit for the testing of a.c. wattmeters and watt-hour meters, with a potentiometer, is given in Fig. 25. The potential coil of the meter under test is supplied through a step-up transformer from the rotor winding of a phase-shifting transformer. In parallel with the potential coil of the meter is connected a voltage ratio box and also a voltmeter V; the former being for the purpose of tapping off a given ratio of the potential coil voltage to the potentiometer. As already stated, the ratio should be such that about 1 volt is applied to the potentiometer. The voltmeter is connected to give an indication of the voltage across the potential coil of the instrument under test. In the current coil circuit of the instrument under test are connected an ammeter, a standard resistance, and a variable resistance for adjustment of the current. The voltage applied to the potentiometer, for measurement of the current in the current coil of the instrument, is taken from the terminals of the standard resistance; the value of the latter being chosen to give a suitable p.d. to the potentiometer circuit.

Variation of power factor of the phantom load upon the meter is obtained by rotation of the rotor of the phase-shifting transformer, by means of which the potential coil voltage may be made to lead or lag the current in the current coil circuit, by any required angle. Once the pointer on the phase-shifter dial has been correctly set at unity or zero power factor of the phantom load, the reading on the dial will give the correct phase angle between the current in the current coil of the instrument and the voltage applied to its pressure coil terminals.

In order to ensure accuracy of measurement at zero power factor, the primary winding of a mutal inductance is connected in series with the current coil circuit of the wattmeter and, since the e.m.f. induced in the secondary winding of the mutual inductance will lag the current by 90°, zero power factor lagging will be obtained when the induced voltage is exactly 180° out of phase with the applied voltage to the pressure coil.

In the case of the wattmeter, if r_a is the ratio of the potential divider, R the value of the standard resistance in the current coil circuit, E_p the reading (in volts) of the potentiometer when connected to the portion of the pressure coil voltage, E_c the reading of the potentiometer when connected across the standard resistance, ϕ the phase angle of the current in the current coil circuit relative to the pressure coil voltage,

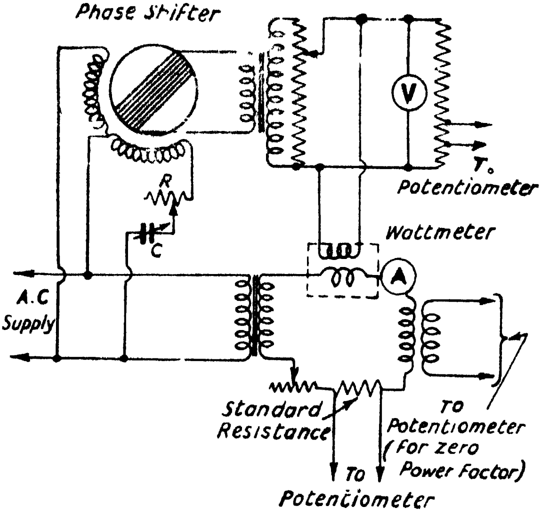


FIG. 25.—Calibration of Wattmeters—Potentiometer Method.

and W is the reading of the wattmeter under these conditions, then the wattmeter error is

$$W - (r_aE_p)\left(\frac{E_c}{R}\right)\cos\phi$$

$$= \frac{\left(W - (r_aE_p)\left(\frac{E_c}{R}\right)\cos\phi\right) \times 100}{r_aE_p\left(\frac{E_c}{R}\right)\cos\phi} \text{ per cent.}$$

In the above expression, r_aE_p is the value of the applied voltage to the pressure coil, E_c/R is the current in the current coil of the wattmeter, and cos ϕ is the power factor of the load.

For the watt-hour meter, using the same nomenclature, but where

E is the total energy registered in kWh, and T is the time in hours during which the load is dissipated, the error is

$$\frac{\left(E - \frac{T}{1000} r_a E_v \left(\frac{E_c}{R}\right) \cos \phi\right) 100}{\frac{T}{1000} r_a E_v \left(\frac{E_c}{R}\right) \cos \phi} \text{ per cent.}$$

2.26. Voltage Standardiser. H. Tinsley & Co., Ltd., have developed an instrument for the standardisation of d.c. voltages to within very narrow limits, such as is required in the calibration of sub-standard wattmeters, etc. By this means the applied voltage can be accurately and conveniently checked at the same time as potentiometer measurement of the current is being effected. A circuit diagram of the instrument is given in Fig. 26 (*a*). It is similar to the potential divider previously described, the only difference being that, when definite predetermined voltages are applied to the "live terminals", the voltage drop across the "low potential terminals" is equal to the e.m.f. of a standard Weston cadmium cell, providing the main dial selector switch is set at the correct range. The p.d. tapped off from the resistance across the live terminals is connected in series-opposition to the standard cell. A galvanometer is also connected in this circuit, and the main voltage is adjusted to its correct value when, at maximum sensitivity, the deflection of the galvanometer is zero. It is thus similar in principle to a potentiometer and, as a consequence, has the advantages of such an instrument.

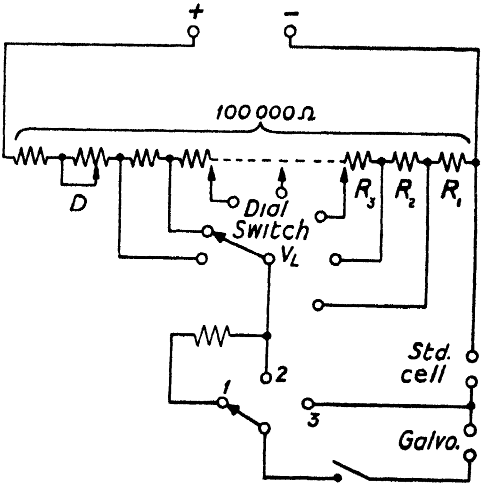


FIG. 26 (*a*).—Voltage Standardiser—Tinsley Inst. Co.

Referring to Fig. 26 (*a*), if the selector switch V_L is in the extreme clockwise position, such that it only brings in the p.d. across the resistance R_1, then at balance

$$\frac{100{,}000}{R_1}=\frac{V_1}{E_c}$$

where E_c is the voltage of the standard cell and V_1 is the voltage across the main terminals (+ and −).

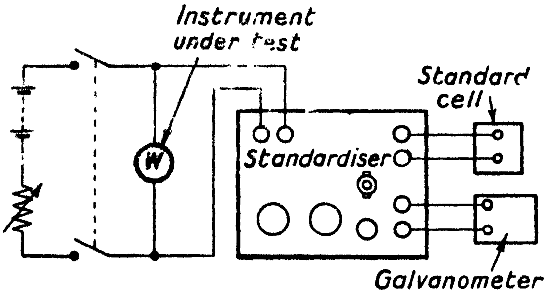


FIG. 26 (*b*).—Voltage Standardisation—Tinsley Inst. Co.

Similarly, when the selector switch is rotated one stud anti-clockwise

$$\frac{100{,}000}{R_1+R_2}=\frac{V_2}{E_c}$$

The values of R_1, R_1+R_2, $R_1+R_2+R_3$, etc., are so chosen in relation to 100,000 ohms as to require a predetermined value of voltage across the main terminals (+ and −) for each setting of the selector switch, when the galvanometer circuit is balanced.

When the selector switch is at its *r*th position, commencing from the most clockwise setting, the voltage will be given by

$$V_R(R_1+R_2+R_3+\ +\ +R_r)=100{,}000\,E_c$$

The limits of accuracy within which the adjustment of the voltage can be carried out are governed by the same considerations as apply to measurements with precision potentiometers and the makers claim the accuracy of the instrument as of the order of 1 in 10,000 (i.e. ±0·01 per cent). Correction for variation of the standard cell voltage between the limits of 1·0187 and 1·0177 international volts is provided for on a continuously variable slide-wire dial (*D* in Fig. 26) which is calibrated in standard cell voltage values. Fig. 26 (*b*) illustrates the external connections of such an instrument.

The instrument is capable of standardising up to twelve specified voltages between the limits of 100 and 1,000 volts. It therefore follows that, as the resistance across the line terminals is 100,000 ohms, the current taken from the line is always between the values of 10 milliampères and 1 milliampère.

It should be mentioned that of late the voltage standardiser described

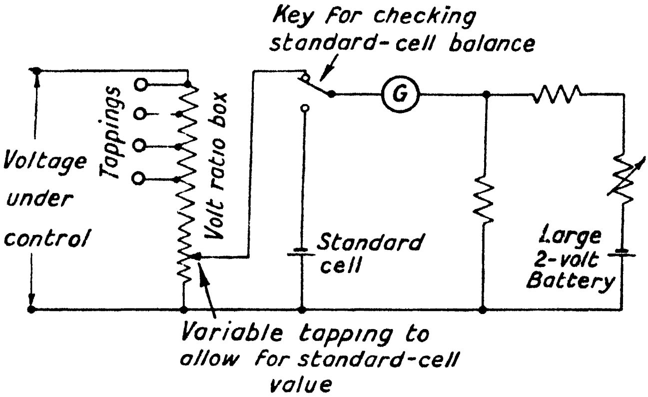


FIG. 27 —Voltage Standardiser—With Auxiliary Potential Circuit Tinsley Inst Co

above has been somewhat altered. It has been found that if the standard cell is left in circuit, there is a considerable drain on it, so it is used to standardise an auxiliary potential circuit. A circuit diagram of this version is given in Fig. 27.

CHAPTER III

Indicating Instruments

3.1. Electro-magnetic Instruments. Most indicating instruments owe their action to the electro-magnetic phenomenon which occurs when a conductor passing current is placed in a magnetic field. Under these conditions a force acts upon the conductor in such a direction as can be determined by application of the well-known left-hand rule. Thus, if a coil is placed between the poles of a magnet, those conductors which carry current in a direction perpendicular to that of the magnetic field will experience a force which will create a torque upon the coil and tend to make it rotate.

3.2. Forces upon the Moving System. In order that the instrument will indicate satisfactorily, it is required that the moving system shall be acted upon by:

(*a*) A deflecting force
(*b*) A controlling force
(*c*) A damping force.

The deflecting force causes the moving system to be displaced angularly from its zero position and, unless some controlling force were exerted upon the moving system, the magnitude of the deflection would in general bear no simple relationship to the deflecting force. It is this controlling force which ensures that the magnitude of the deflection is always the same for any given current passing through the coil. In most indicating instruments, this force is usually obtained by means of a hair-spring or by the action of gravity upon a deliberately unbalanced moving system.

In order to bring the moving system quickly to rest, at its correct angle of deflection for the current passing through the coils, a "damping" or "resistance to velocity" force is necessary. Otherwise, the moving system would have such a high velocity when it reached its position of equilibrium that it would overshoot this position and then oscillate around it with successively decreasing amplitudes, before finally coming to rest. This would prove most inconvenient to the observer and, on account of the time which would elapse before the pointer took up its final position, only slow variations of the quantity being measured could be observed on the scale.

Damping may be provided by means of air friction, fluid friction, or eddy currents. Its effect (see section 2.8) can be seen by inspection of the deflection-time curves given in Fig. 11 for different values of damping, i.e. over-damping, under-damping, and critical damping.

It can be seen from these curves that when the system is critically damped it reaches its final position much earlier than when under-damped or over-damped, thereby making it possible to follow more rapid variations of the quantity being measured than can be followed in either of the other cases. One can therefore describe critical damping as being that damping, or resistance to motion, which permits the moving system most quickly to reach its final position, without oscillating around this point. When the damping is critical, the instrument is said to be "dead-beat". In practice, however, the damping is made a little less than its critical value, since experience has proved that this leads to slightly better results. As the damping is only a resistance to motion, it should have no effect upon the final deflection of the moving system.

3.3. Controlling Torque. When an electric current produces a torque upon the moving system of an instrument, deflection follows and, as a result, a torque is set up by the controlling medium; the magnitude of which will depend upon the angular displacement of the moving system from its zero position. Furthermore, this controlling torque will act in opposition to the deflecting torque and will increase in magnitude as the moving system departs farther from its zero position. In consequence, the moving system takes up a position such that the controlling torque is equal and opposite to the deflecting torque. There will thus be a different position of equilibrium for every value of current flowing through the coils, from zero to the maximum range of the instrument.

3.4. Spring Control. In most indicating instruments, the controlling torque is obtained by means of a hair-spring, the best type of hair-spring being manufactured from phosphor-bronze. To obtain the best results, it is finally annealed. Experiments have been carried out with many other metals and alloys, but all have proved inferior to phosphor-bronze, mainly on account of their higher temperature coefficient or greater tendency to fatigue.

The spring, which is co-axial with the moving system, should contain a large number of turns, thereby keeping the maximum deflection per turn at a very small value. In these circumstances, the controlling torque is directly proportional to the angular deflection of the moving system. The dimensions of the spring should be such that the maximum stress on the spring (i.e. that at full-scale deflection) is well within the elastic limit of the material from which the spring is made. The spring should also be non-magnetic, not subject to fatigue and, when used as a lead to the moving coil, its resistance and temperature coefficient should be small. Often, in order to compensate for the effects of temperature variation, two springs are coiled in opposite directions—generally one on each side of the moving system—and when the moving system is deflected one spring is extended and the other compressed.

This gives twice the controlling torque of one spring and at the same time eliminates errors due to variation of the spring temperature.

3.5. Gravity Control. In gravity controlled instruments, a small weight is fitted to the moving system in such a position as to create a restoring torque when the latter is deflected from its zero position. Fig. 28 provides an illustration of such a method. From the triangle of forces of Fig. 28, it can be seen that the control torque is $Wl \sin \theta$, where θ is the angle of deflection from the zero position, l is the distance of the centre of gravity of the controlling arm from the pivot, and W is the control weight. The controlling torque of this type, therefore, suffers from the disadvantage of being proportional to the sine of the angle of deflection, instead of being directly proportional to the angle of deflection—as is the case with the spring control. This gives rise to a close scale at the lower end of the instrument. Another disadvantage of the gravity control system is that any instrument fitted with this method of control must, for successful operation, be used in a vertical position.

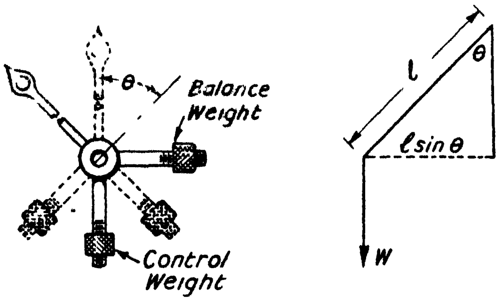


FIG. 28.—Gravity Control—Indicating Instruments.

3.6. Eddy Current Damping. Of the three methods of damping in general use, i.e. air-friction, fluid-friction, and eddy current damping, the last mentioned is the best when it can conveniently be employed. Unfortunately, it can only be provided when the introduction of a small permanent magnet, for the creation of eddy currents, will not distort the magnetic field responsible for the deflecting torque upon the moving system. If such distortion did occur, inaccuracies would be introduced due to the interaction of the two magnetic fields. Where a disc or permanent magnet is included in the operating system, it is very convenient to employ eddy current damping. Its theory will be found in section 8.19.

3.7. Air-friction Damping. This consists of the movement of a light metal vane or piston in a restricted space, thereby utilising the resistive properties of air to motion. One such method is by the use of a light aluminium piston which is fixed to the moving system and free to travel in a curved cylindrical box which envelops the piston; the clearance between the piston and the cylinder walls being of the order of a few thousandths of an inch. Another method is the employment of two thin aluminium vanes, mounted on the spindle of the moving system and totally enclosed in a double V-shaped box. Both these methods are illustrated in Fig. 29.

When the piston is moving rapidly towards the closed end of the cylinder, the air between the piston and this end will be compressed and the pressure thus created will oppose the motion of the piston and the moving system to which it is attached. Conversely, when the piston travels away from the closed end the enclosed air expands and its pressure decreases. As the pressure at the open end remains more or less constant, it will be greater, and once again the difference in air pressure between the two sides of the cylinder will oppose the motion of the piston and thereby the motion of the whole moving system. The same remarks apply in the case of the method employing vanes, except that the pressure will always increase on the forward side of each vane (forward with respect to the direction of travel) and decrease on the opposite side of each vane. Thus, the pressure in these circumstances will also oppose motion. Care must be taken to ensure that the damping system is never deformed in any way, as the introduction of solid friction (i.e. friction due to the rubbing of the vanes or piston against the walls of the chamber) would introduce very serious deflectional errors.

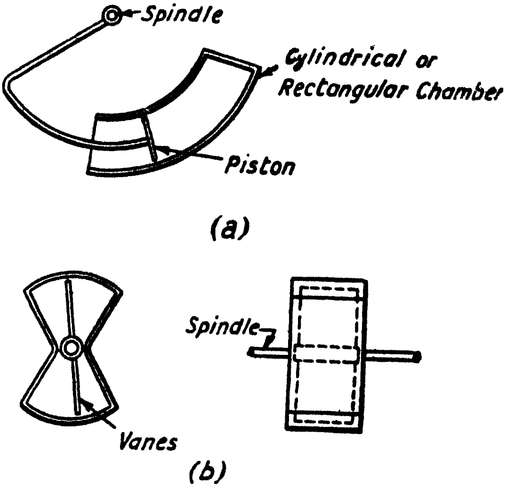


FIG. 29.—Damping—Air-friction.

3.8. Fluid-friction Damping. In this method a light vane, fitted to the spindle of the moving system, is completely submerged in a pot of damping oil. Owing to the viscosity of the oil, the motion of the vane will meet with resistance opposing its direction, and this resistance will increase with increased speed of the vane. Sometimes a horizontal disc is employed and this rotates in its surface plane. Oil used for damping should have no corrosive action upon the metals with which the instrument is constructed, the viscosity should change but little with temperature, evaporation should be negligible, and it should be a good insulator.

Fluid friction has only one real advantage, and that is when the whole of the moving system has to be immersed in oil for the purpose of insulation. Its disadvantages are.

(*a*) The instrument can only be used vertically.

(*b*) Even when used vertically, there is a tendency for the instrument to become dirty due to the creeping of the oil.

The moving system of an instrument is usually supported by pivots or a thread suspension. In the former case, the bearings are of case-hardened steel and are conical in shape, fitted into conical holes or recesses in synthetic or natural sapphires. When the deflecting force is very small compared with the weight of the moving system, thread suspension is used for the purpose of avoiding bearing friction. The thread suspension is generally made of phosphor-bronze.

The pointer should be as light as possible in order to keep down the inertia of the moving system, thereby keeping the necessary damping down to a minimum. A light pointer also has the advantage of minimising the load upon the bearings and thereby the friction due to the load.

Ammeters and voltmeters both owe their action to exactly the same principle. A voltmeter actually measures current, but as this current is directly proportional to the voltage applied to its terminals, it follows that the instruments can easily be calibrated in terms of voltage. Thus, the only real difference between an ammeter and voltmeter is the magnitude of current necessary to produce the deflecting torque.

3.9. A.C. and D.C. Instruments. Instruments may be divided into three categories, in connection with their suitability for the measurement of d.c. only, a.c. only, or both d.c. and a.c.

If the deflecting torque of an ammeter or voltmeter is proportional to the current passing through its circuit, then, when the instrument is connected in an alternating current circuit, it follows that the torque on the moving system will also be alternating. As the inertia of the moving system is too great for it to follow these oscillations, the system will take up a mean position and will tend to oscillate around this point. In the case of a sinusoidal current, the average value will be zero, hence no deflection will be indicated by the pointer of the instrument. If, however, the torque is proportional to the square of the current, it will always be unidirectional and, providing the alternations are of a sufficiently high frequency compared with the natural frequency of the instrument, the moving system will take up a position depending upon the average value of the torque, i.e. the average value of the square of the current. The instrument can therefore be calibrated for r.m.s. values of the current or voltage.

The criterion as to whether an instrument is suitable for use with both a.c. and d.c., or only d.c., is that if the torque is proportional to the square of the current it may be used for both a.c. and d.c., but if the torque is directly proportional to the current, it is only suitable for direct current. An exception to this rule is the induction type of instrument. This can only be employed upon alternating current, for reasons to be given later in the chapter (section 3.19). It should, however, be mentioned that the torque upon this type of instrument is proportional to the square of the current, and therefore it partly fulfils the above-mentioned conditions.

3.10. Rectifier Bridge Circuit. An instrument, the torque of which is proportional to the current, can be made suitable for the measurement of alternating current by employing it in a rectifier bridge circuit, such as that illustrated in Fig. 30. The rectifiers are connected to pass current in the directions shown. Thus, during one half-cycle the current will pass through two opposite arms of the bridge network and also through the meter. During the following half-cycle, the current will pass through the other two arms, but through the meter in exactly the same direction as was the case in the first half-cycle. Full-wave rectification consequently takes place, and the moving system takes up a deflection depending upon the average value of the current over half a cycle. Since the ratio of r.m.s. to average value

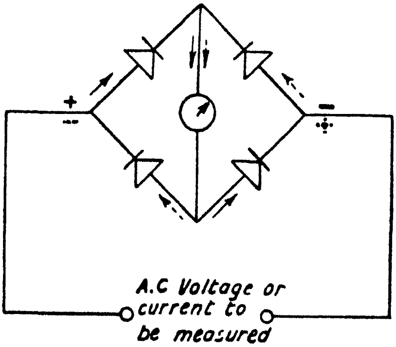


FIG. 30.—Rectifier Bridge Circuit for Permanent Magnet Moving-coil Instrument.

of a sine wave is constant (1·11 : 1) the instrument can be calibrated in r.m.s. values.

The rectifiers are generally of the copper oxide type, and are used fairly extensively with permanent magnet moving-coil instruments for the measurement of small alternating currents and voltages. The rectifiers may have a current range up to 10 milliampères and are therefore only suitable for light current work where the resistance is high. One great disadvantage of the rectifier is that its resistance is a function of the current passing through it; the resistance increasing rapidly as the current approaches zero

3.11. Ammeters and Voltmeters. The types of ammeter and voltmeter in common use are:

(*a*) Moving-coil which may be subdivided into two classes:
  (1) Permanent magnet type.
  (2) Electro-dynamometer type.
(*b*) Moving-iron, which may also be divided into two classes:
  (1) Attraction type.
  (2) Repulsion type.
(*c*) Induction.
(*d*) Electrostatic (depending entirely upon voltage for operation, and not upon current).
(*e*) Hot-wire.

Of these instruments, the moving-coil permanent-magnet type can only be used for d.c. measurements, and the induction type can only be used for a.c. measurements. The others are universal in their application.

3.12. Moving-coil Instruments. As already stated, there are two types of moving-coil instrument. These are the permanent magnet type, which is only suitable for d.c. measurements, and the electro-dynamometer type, which can be used for both direct and alternating current.

The principle of the permanent magnet type of moving-coil instrument is exactly the same as that of the D'Arsonval galvanometer. A light rectangular coil, placed over a soft iron cylinder, is so pivoted that it is free to rotate in a small air gap between the soft iron cylinder and the poles of the permanent magnet. It can be seen from Fig. 31 that when the current passes through the coil in the direction indicated the conductors will experience a force which will create a moment upon the coil, and the moving system will be deflected in a clockwise direction.

If r is the mean radius of rotation of the coil, in cm., N the number of turns, I the current in ampères, l the active length of each coil under the influence of the magnetic field, H the field strength of the air gap in c.g.s. units, then the force P acting upon the coil will be given by

$$P=\frac{NHlI}{10}\text{ dynes.}$$

The torque $T=2Pr$

$$=\frac{2rNHIl}{10\times 981}\text{ grm. cm.}$$

$$=\text{Area of coil}\times\frac{\text{Ampère-turns}}{10\times 981}\times\text{field strength of air gap.}$$

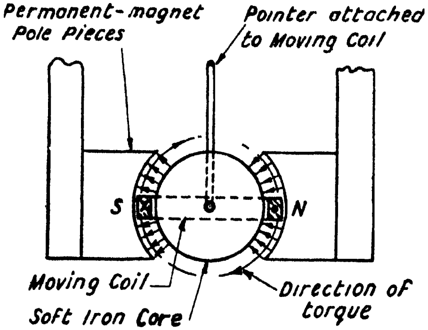


FIG 31 —Permanent Magnet Moving-coil Instrument.

Moving-coil instruments, in general, are spring controlled. Two phosphor-bronze springs are placed in opposition for this purpose, and they also act as leads to the moving coil. They are usually placed one on each side of the moving system and, after the instrument has left the manufacturers, it is only possible to alter the front spring—by means of the zero adjuster fitted to the front of the case.

3.13. Unipivot Instrument—Cambridge Instrument Co. Ltd. One further type of permanent magnet moving-coil instrument worthy of mention is the Unipivot Instrument, manufactured by the Cambridge Instrument Co. Ltd. An illustration of the working portion of such an instrument appears in Fig. 32. The moving coil is circular and is carried by one pivot which rests in a recess at the top of the spherical soft-iron core. As a laboratory instrument it is of inestimable value, since it is accurate to a high degree.

The main advantages of permanent magnet moving-coil instruments are:

(*a*) High torque-weight ratio.
(*b*) Low power consumption.

(*c*) Suitability for use with shunts and resistances, i.e. as multi-range instruments.

(*d*) They are free from hysteresis errors and affected but slightly by stray magnetic fields.

(*e*) Possess uniform scale and, since a large angular deflection is possible, the scale can also be long.

(*f*) Damping is ideal, due to the eddy currents induced in the metal former.

A disadvantage of the instrument is that the accuracy depends upon the permanent magnet retaining its magnetism and, if the magnet is not carefully "aged" (section 8.22), considerable errors may be introduced. However, with the recent progress made in this direction by the use of chromium and cobalt steels, etc., and electrical ageing, the errors due to the above have been greatly minimised. Another disadvantage of the instrument is that when used in conjunction with a badly designed shunt it is subject to errors due to thermo-electric e.m.f.s.

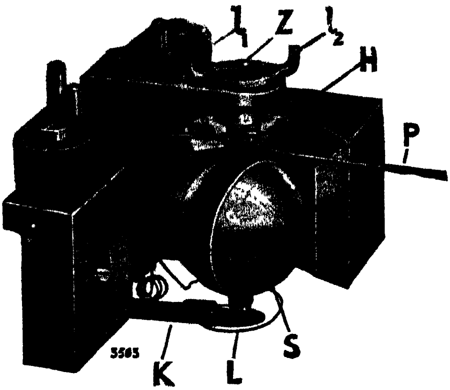


FIG. 32.—Unipivot Instrument—Cambridge Inst. Co.

3.14. Electro-dynamometer Instrument. In this type of instrument the permanent magnet is replaced by either one or two fixed coils. These coils carry the whole, or a portion of, the current to be measured and are connected in series or parallel with the moving coil. In order to avoid hysteresis and other errors, when the instrument is used in a.c. circuits, the coils are air-cored. Fig. 33 shows the connections of an electro-dynamometer ammeter and an electro-dynamometer voltmeter. The general arrangement of the coils is also illustrated.

As the torque is proportional to the product of the currents in the fixed and moving coils, i.e. the square of the load current, it follows that the torque is always unidirectional, irrespective of the direction of the current. Thus, the instrument is suitable for both alternating and direct current measurements.

The controlling force consists of two springs in opposition, which are also used as leads for the current to the moving coil. Damping may be pneumatic or by means of eddy currents (sections 3.6 and 3.7).

The instrument is usually connected so that the swamping resistance and moving coil are in parallel with the fixed coils and shunt. By such an arrangement it is possible to make the time constants of the parallel paths approximately equal. This ensures that the current distribution between the two paths is independent of frequency, which is essential for alternating current measurements.

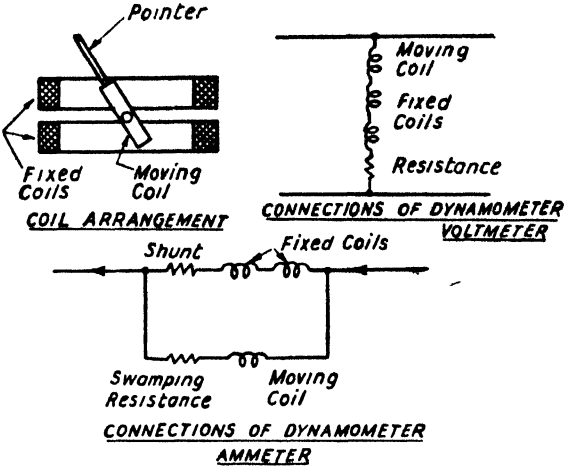


FIG 33 —Connections—Electro-dynamometer Instruments.

3.15. Determination of Torque for Electro-dynamometer Instrument. The fundamental relationship underlying the behaviour of electro-dynamometer instruments is that the torque is proportional to the product of the currents and $dM/d\theta$, where M is the coefficient of mutual inductance and θ is the deflection from zero.

$$\text{Thus,}\quad T_D = \frac{10^9}{981 \times 100} . I_1 I_2 . \frac{dM}{d\theta} \text{ grm. cm.}$$

where I_1 and I_2, the respective currents in the fixed and moving coils, are in ampères, θ is in radians, and M is the coefficient of mutual inductance in henrys. Thus, providing we can calculate the rate of change of the mutual inductance, we can deduce the torque.

We will assume that the fixed coil is a long helix, and that for our purpose the moving coil is inside the helical coil. Then, when a current of I_1 ampères flows through the helical coil of N_1 turns, length l cm., and the field in the centre is uniform, the value of the latter is given by

$$H=\frac{4\pi N_1 I_1}{10l}$$

The number of linkages in the moving coil, of area A sq. cm. and N_2 turns, at an angle θ to the axis of the fixed coil, will be

$$\begin{aligned}\Phi N_2 &= HAN_2 \sin\theta \\ &= \frac{4\pi N_1 N_2 A I_1 \sin\theta}{10l}\end{aligned}$$

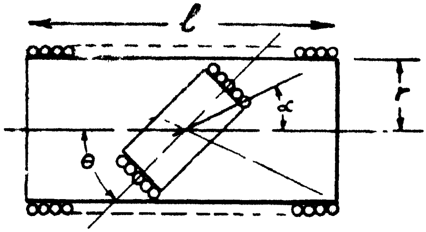


FIG 34.

and the coefficient of mutual inductance will be given by

$$\begin{aligned}M &= \frac{\Phi N_2}{I_1} \\ &= \frac{4\pi N_1 N_2 A \sin\theta}{10l} \text{ c.g.s. units} \\ &= \frac{4\pi N_1 N_2 A \sin\theta}{10^9 l} \text{ henrys.}\end{aligned}$$

$$\text{Thus,} \quad \frac{dM}{d\theta}=\frac{4\pi N_1 N_2 A \cos\theta}{10^9 l}$$

In practice, the fixed coil is never long in comparison with its diameter, and this has the effect of modifying the field inside it. The field is consequently smaller and non-uniform, but the value of H at any point can be determined, providing one knows the solid angle subtended by the surface of the winding at the point in question. This may be calculated from the expression

$$H=\frac{I_1 N_1 \phi}{l}$$

where ϕ is the solid angle. Suppose that Fig. 34 represents the coil system of the instrument, then at the centre of the helix $\phi=4\pi\cos\alpha$,

α being the semi-angle of the cone which the end of the helix makes at the centre.

$$\text{Hence,}\quad \frac{dM}{d\theta}=\frac{4\pi N_1N_2A}{10^9 l}\cos\alpha\cos\theta$$

$$=\frac{4\pi N_1N_2A}{10^9 l}\left(\frac{l/2}{\sqrt{(l/2)^2+r^2}}\right)\cos\theta\cdot$$

$$=\frac{4\pi AN_1N_2\cdot}{10^9\sqrt{l^2+4r^2}}\cos\theta$$

$$\text{and the torque}=\frac{I_1I_2\,4\pi AN_1N_2}{10^9\sqrt{l^2+4r^2}}\cos\theta$$

It can be seen from the above expression that, for a given value of current in the electro-dynamometer circuit, the torque will be maximum when $\cos\theta=1$, i.e. $\theta=0$. Advantage of this is taken in the torsion-head type of electro-dynamometer instrument. Torsion is applied by means of the suspension, in order that the moving system will have zero deflection when the current to be measured is passing through the circuit, thereby ensuring that the current will create a maximum torque upon the moving system. Further details of such an instrument are given in section 4.12.

For direct current measurements, the electro-dynamometer deflectional instrument has the following disadvantages compared with the permanent magnet type.

(*a*) It requires a much larger number of ampère-turns, giving rise to a high moment of inertia, small torque/weight ratio and high power loss, due to the absence of iron in the circuit. Thus, the possibility of friction errors is greater than with the permanent magnet type.

(*b*) The scale is not uniform, on account of the deflecting torque varying approximately proportional to the square of the current. This causes the lower part of the scale to be cramped.

(*c*) It is much more costly than the permanent magnet type.

The instrument, however, is very useful for precision measurements in alternating current circuits. When used for voltage measurements, its inductive term will be negligible at normal supply frequencies, i.e. up to 100 cycles per second, but when used for measurements at higher frequencies the inductive term will increase in importance. Furthermore, its value will depend upon the angular position of the moving coil. Errors will also be introduced due to the eddy currents induced in the metal formers, and the capacity effect in the series resistance and between the coils.

3.16. Moving-iron Instruments. There are two types of this instrument. One is dependent upon the attraction which occurs between two sections of iron of opposite polarity, whilst the other depends upon the repulsion of like polarities. In the former type, a small piece of soft iron is drawn into a coil energised by the current, or a definite proportion, to be measured. As the iron is deflected towards the coil, so the controlling force tends to return it to its zero position. The moving system will therefore take up a position of equilibrium, such that the controlling and deflecting torques are equal and opposite in direction. The pointer, which is attached to the moving system, will then indicate a position upon the scale corresponding to the current flowing in the circuit.

In the repulsion type of instrument there are two thin rods of iron in the coil; one fixed and the other movable. When the coil is energised by a current, the two iron rods are similarly magnetised and the movable rod is repelled from the fixed one the repelling force being approximately proportional to the square of the current flowing through the coil.

As the deflecting force of both the attraction type and repulsion type moving-iron instruments is independent of the direction of the current, it follows that they are suitable for both alternating and direct current measurements.

3.17. Theory of Attraction type Moving-iron Instrument. Let us assume that when a current of I ampères flows through the coil a magnetic field of strength H is produced. We will also assume that the field is uniform and parallel to the axis of the coil.

If the soft iron arm (Fig. 35) initially makes an angle of α with the direction perpendicular to the axis of the coil, and β is the deflection from this position when current I ampères flows through the coil, then the longitudinal induced magnetism in the soft iron disc will be proportional to H sin $(\alpha+\beta)$ and the force pulling the disc into the coil will be proportional to H^2 sin $(\alpha+\beta)$. Assuming constant permeability, this force F is proportional to I^2 sin $(\alpha+\beta)$, since H is proportional to I. If the equivalent point of application of this force is D cm. from the pivot, the deflecting torque

$$\begin{aligned} T_d &= FD \cos(\alpha+\beta) \\ &\propto H^2 \sin(\alpha+\beta)\cos(\alpha+\beta) \\ &\propto I^2 \sin 2(\alpha+\beta) \end{aligned}$$

since D is a constant and H is proportional to I.

$$\text{Hence, } T_d = K_1 I^2 \sin 2(\alpha+\beta)$$

where K_1 is a constant.

Now, if spring control is employed, the torque due to this control will be proportional to the angle β, say $K_2\beta$, where K_2 is a constant. Therefore, a steady deflection will be obtained when

$$K_2\beta = K_1 I^2 \sin 2(\alpha+\beta)$$

$$\begin{aligned} \text{and } I &= \sqrt{\frac{K_2\beta}{K_1 \sin 2(\alpha+\beta)}} \\ &= K_3\sqrt{\frac{\beta}{\sin 2(\alpha+\beta)}} \end{aligned}$$

If, on the other hand, gravity control is employed, the deflecting torque will be $K_4 \sin \beta$.

$$\text{Therefore, } K_4 \sin \beta = K_1 I^2 \sin 2(\alpha + \beta)$$

$$\text{Hence, } I = K_5 \sqrt{\frac{\sin \beta}{\sin 2(\alpha + \beta)}}$$

K_2, K_3, K_4, and K_5 being constants.

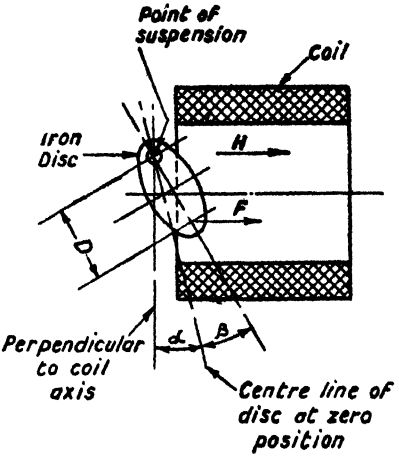


FIG. 35.—Attraction type Moving-iron Instrument.

From the above expressions, it can be seen that the deflection is maximum when the angle α is zero, i.e. when the longitudinal axis of the soft iron arm, at its zero position, is perpendicular to the axis of the coil.

3.18. Theory of Repulsion type Moving-iron Instrument. If we assume that the two iron rods are of equal dimensions, and that the distance between them is small compared with their lengths, we can neglect the forces of attraction which exist between the opposite poles and merely consider the forces of repulsion between the adjacent poles.

When a current of I ampères flows through the coil there will be a magnetic field set up in the coil of, say, H c.g.s. units. Suppose, under these conditions, that the magnetism induced in the fixed and free rod is such that their pole strengths are m_1 and m_2 respectively, and that their relative positions are as illustrated in Fig. 36, say, d cm. apart. Then the force of repulsion existing between the rods will be

$$\frac{2m_1m_2}{d^2} \text{ dynes}$$

$$\text{But } d=2r\sin\frac{\alpha+\beta}{2}$$

where r is the distance of the rods from the axis of rotation of the movable rod.

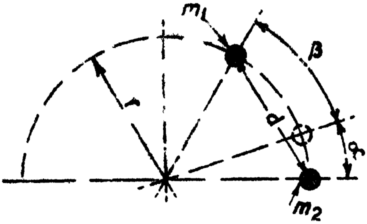


FIG. 36 —Repulsion type Moving-iron Instrument.

The deflecting torque T_d is given by

$$T_d=r\cos\left(\frac{\alpha+\beta}{2}\right)\frac{2\,m_1m_2}{d^2}$$

$$=r\cos\left(\frac{\alpha+\beta}{2}\right)\frac{2\,m_1m_2}{4\,r^2\sin^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$=\frac{m_1m_2}{2r\tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

But m_1 and m_2 are both proportional to the current, hence

$$T_d=\frac{KI^2}{\tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

where K is a constant.

For a spring-controlled instrument, when a steady deflection has been attained, we have

$$K_1\beta=\frac{KI^2}{\tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

$$\text{whence } I=K_2\sqrt{\beta\tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

In the case of the gravity-controlled instrument, the controlling torque will be equal to $K_3 \sin \beta$

$$\text{Therefore, } K_3 \sin \beta = \frac{KI^2}{\tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

$$\text{and } I = K_4 \sqrt{\sin \beta \tan\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)}$$

In the above expressions, K, K_1, K_2, K_3, and K_4 are constants.

The disadvantages of moving-iron instruments are:

(*a*) High hysteresis errors, in both a.c. and d.c. measurements. It will be found that the instruments give a higher reading for decreasing values of current than for increasing values.

(*b*) They are very susceptible to stray magnetic fields. This, however, can be considerably minimised by magnetically screening the working portion of the instrument.

(*c*) When employed as an a.c. voltmeter, errors are likely to occur with change of frequency, due to the change in impedance of the coil.

(*d*) Errors occur, with change of frequency, due to the change in magnitude of the eddy currents set up near the working portion of the instrument, thereby affecting the magnetic field in the coil.

3.19. Induction Meters. This type of instrument depends for its action upon the torque produced by the interaction of an alternating flux with the eddy currents induced in a metal disc by another magnetic flux; the magnitude of both fluxes being dependent upon the value of the current or voltage being measured.

Let us consider a magnetic flux $\phi = \Phi \sin \omega t$ producing a force by interaction with an eddy current of the same frequency and lagging the flux by an angle θ. We will assume the eddy current to have the same wave-form as the magnetic flux. Its instantaneous value may therefore be represented by $i = I \sin (\omega t - \theta)$. The instantaneous torque will therefore be proportional to the product of the instantaneous values of the flux and the eddy current.

$$\text{Therefore, } T_{inst.} \propto \phi i$$

The mean torque will be proportional to

$$\frac{1}{t}\int_0^t \phi i dt$$

$$\text{Therefore, } T_M \propto \frac{\omega}{\pi}\int_0^{\frac{\pi}{\omega}} \Phi I \sin \omega t \sin (\omega t - \theta) dt$$

$$\propto \frac{\omega}{\pi}\Phi I \int_{0}^{\frac{\pi}{\omega}} \tfrac{1}{2}\left[\cos\theta - \cos(2\omega t - \theta)\right]dt$$

$$\propto \frac{\omega}{2\pi}\Phi I \left[t\cos\theta - \frac{\sin(2\omega t - \theta)}{2\omega}\right]_{0}^{\frac{\pi}{\omega}}$$

$$\propto \frac{\omega}{2\pi}\Phi I \left(\frac{\pi}{\omega}\cos\theta\right)$$

$$\propto \frac{\Phi I}{2}\cos\theta$$

But if Φ is proportional to I.

$$T_M \propto (I_{r.m.s})^2 \cos\theta.$$

As θ approaches $\frac{\pi}{2}$ radians, or 90°, the torque will approach zero. It therefore follows that in all induction instruments some means must be provided for producing eddy currents which are much less than (or much more than) 90° out of phase with the flux. Thus, the trigonometrical term of the expression ($\cos\theta$) should approach as near as possible to unity.

Two general methods have been devised to fulfil the above condition. One method is to split the winding of the electro-magnet into two sections, one being highly inductive and the other non-inductive. This is adopted in the Ferraris type of instrument.

The other method is to split the poles of the electro-magnet into two sections; lagging the flux in the one section by means of a short-circuited turn in the form of a copper band. In this manner, two effective fluxes are created with a suitable phase angle between them. Such an instrument is of the shaded-pole type.

3.20. Ferraris Motor. This operates by means of the same principle as the induction motor. Two pairs of coils are wound upon a laminated magnetic system, as illustrated in Fig. 37. The windings are all supplied from the same source, but there is a phase displacement of approximately 90° between the currents in the coils *AA* and *BB*. This is attained by placing a high-value non-inductive resistance in series with the coils *AA* and an inductance in series with the coils *BB*. Thus, the current in coils *BB* lags almost 90° behind the current in coils *AA*. The magnetic flux of *BB* will therefore lag that of *AA* by the same angle. In this manner a rotating field is produced and, providing the drum was free to rotate, it would do so at a slightly lower speed than that of the magnetic field, the difference in the speeds (i.e. the slip) being necessary for the creation of a torque to overcome friction, etc. Furthermore, its direction would be the same as that of the rotating field. In the case of an ammeter or voltmeter employing the above principle, a spring control prevents continuous rotation of the drum, and the latter will come to rest when the controlling torque is equal and opposite to the deflecting torque. A cylindrical laminated iron core is placed inside the drum and its diameter is within about thirty-thousandths of an inch of the internal diameter of the drum. The whole moving system is carried by a spindle, the ends of which seat into jewelled cups. The damping is effected by means of eddy currents induced in an aluminium disc attached to the spindle and rotating between the gap of a permanent magnet.

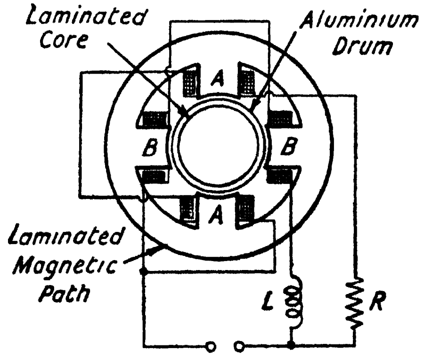


FIG. 37.—Diagrammatic Arrangement—Ferraris Motor

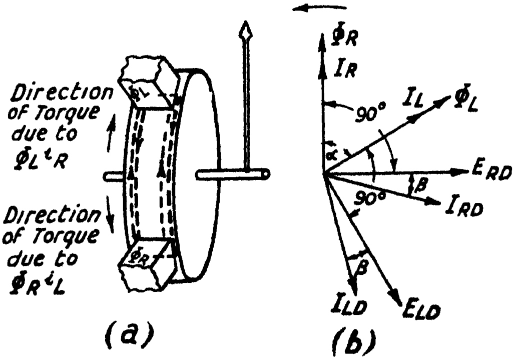


FIG. 38.—Torque and Vector Diagrams—Ferraris Motor.

Fig. 38 (*b*) gives a vector diagram in which I_R and I_L represent the currents in the non-inductive and inductive circuits respectively, Φ_R

and Φ_L being the fluxes due to these currents. The phase angle between the currents will be almost 90° and is represented by α. The e.m.f.s induced in the drum due to the fluxes Φ_R and Φ_L will be 90° behind their respective fluxes and are represented by E_{RD} and E_{LD}. These e.m.f.s will set up eddy currents in the drum which, owing to the inductance of their paths, will lag their respective e.m.f.s by angle β. They are represented by I_{RD} and I_{LD}. Fig. 38 (*a*) illustrates diagrammatically the relative directions of the fluxes and induced currents, and it can be seen that the total torque is the sum of two separate torques. One is proportional to the product of ϕ_I and i_{RD}, and the other, in the opposite direction, is proportional to $\phi_L i_{RD}$. Thus, the total torque equals $K(\phi_L i_{RD}-\phi_R i_L)$; K being a constant.

It Φ_L, Φ_R, I_{LD}, and I_{RD} are the r.m.s. values of the respective magnetic fluxes and induced currents, the value of the mean torque, T_M, will be given by

$$T_M=K[\Phi_L I_{RD}\cos(90+\beta-\alpha)-\Phi_R I_{LD}\cos(90+\beta+\alpha)]$$

where $(90+\beta-\alpha)$ is the phase angle between Φ_L and I_{RD}, and $(90+\beta+\alpha)$ is the phase angle between Φ_R and I_{LD}.

$$\text{Thus, } T_M \propto [\Phi_L I_{RD}\sin(\alpha-\beta)+\Phi_R I_{LD}\sin(\alpha+\beta)]$$

But $I_{LD}=\dfrac{E_{LD}}{z}$ and $I_{RD}=\dfrac{E_{RD}}{z}$; z being the impedance of the eddy current paths. Also, I_{LD} is proportional to the frequency and the magnetic flux Φ_L, similarly I_{RD} is proportional to $f\Phi_R$.

$$\text{Therefore, } T_M \propto \left[\frac{\Phi_R\Phi_L f}{z}\sin(\alpha-\beta)+\frac{\Phi_R\Phi_L f}{z}\sin(\alpha+\beta)\right]$$

$$\propto \Phi_R\Phi_L f[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$$

$$\propto \Phi_R\Phi_L f\cos\beta\sin\alpha.$$

But ΦL and ΦT are proportional to the current being measured, therefore

$$T_M \propto I^2 f\cos\beta\sin\alpha.$$

From this expression, it can be seen that the torque is proportional to $\cos\beta\sin\alpha$ for any given current and frequency. Furthermore, the inductance of the eddy current path is very small, and so $\cos\beta$ approximates to unity. It therefore follows that, for any given current and frequency, $\sin\alpha$ must be as large as possible for maximum torque, i.e. the currents in the inductive and non-inductive circuits should approach as near as possible to 90° difference in phase.

3.21. Shaded-pole Type. The construction of this type of instrument is illustrated in Fig. 39. A thin aluminium disc, situated between the poles of an electro-magnet, is mounted on a spindle supported by jewelled bearings. Attached to the spindle is a pointer. The poles of the electro-magnet are each split into two equal sections, as indicated in the figure; a copper band being fitted to one section of each pole. When the coil is energised, the copper bands act as short-circuited secondary turns of a transformer. Heavy currents consequently flow in the bands. These currents in turn set up a flux which opposes the main flux. This has the effect of lagging the flux in the shaded-pole by an amount depending upon the constants of the copper band. In general, the dimensions of the bands are such that the shaded-pole flux lags the unshaded-pole flux by about 45°.

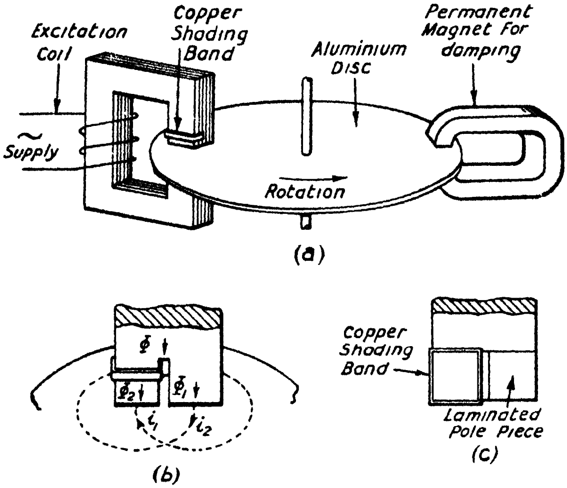


FIG. 39.—Shaded-pole Induction Motor

Fig. 40 depicts vectorially the relative phases of the fluxes, and also the induced e.m.f.s and eddy currents in the disc. The flux in the unshaded pole is represented by Φ_1, that in the shaded pole by Φ_2, and the main flux, which is the vector sum of these fluxes, is represented by Φ. The e.m.f. induced in the disc due to Φ_1 lags this flux by 90° and is represented by e_1. Similarly, e_2, which is the induced

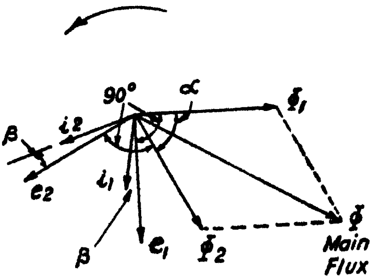


FIG. 40.—Vector Diagram—Shaded-pole Induction Motor.

e.m.f. due to Φ_2, lags the latter by 90°. Owing to the inductance of the eddy current paths in the disc, the eddy currents will lag their respective e.m.f.s by angle β, and they are represented by i_1 and i_2 respectively. The displacement of the shaded-pole flux relative to that of the unshaded pole is represented by the angle α, which is usually of the order of 40°.

From Fig. 39, which illustrates the operation of the instrument, it can be seen that there are two components of the effective torque, acting in opposite directions. One is due to Φ_1 acting upon i_2, and the other is due to Φ_2 acting upon i_1. Therefore, the mean torque T_M is given by

$$T_M = K[\Phi_2 i_1 \cos(90 + \beta - \alpha) - \Phi_1 i_2 \cos(90 + \alpha + \beta)]$$

K being a constant.

$$\text{Therefore } T_M \propto [\Phi_1 i_2 \sin(\beta + \alpha) + \Phi_2 i_1 \sin(\alpha - \beta)]$$

But $e_1 \propto \Phi_1 f$, and $i_1 \propto e_1$, therefore $i_1 \propto \Phi_1 f$. Similarly, $i_2 \propto \Phi_2 f$.

$$\begin{aligned}\text{Therefore } T_M &\propto [\Phi_1\Phi_2 f \sin(\alpha+\beta) + \Phi_1\Phi_2 f \sin(\alpha-\beta)] \\ &\propto \Phi_1\Phi_2 f[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \\ &\propto \Phi_1\Phi_2 f \cos\beta \sin\alpha.\end{aligned}$$

Also, Φ_1 and Φ_2 may be considered to be directly proportional to the current I in the coil. Hence,

$$T \propto I^2 f \cos\beta \sin\alpha.$$

The torque is therefore proportional to the square of the current at any given frequency, and proportional to the frequency at any given current.

It can be seen from the above expression that, in order to make the torque as large as possible, it is required that $\sin\alpha$ shall be as large as possible. In other words, the angle α—which is the phase angle between the flux in the shaded pole and that of the unshaded pole —should be as large as possible.

In the case of either type of induction meter, when employed as an ammeter, compensation for change of frequency and temperature is provided in the form of a non-inductive shunt. Thus, when the frequency increases, the increase in torque is counterbalanced by the increase in impedance of the meter, thereby resulting in a lower current being taken by the meter (since the impedance of the shunt remains constant at all normal frequencies).

When used as a voltmeter, the induction meter tends to be self-compensating, since the increase in frequency also increases the impedance of the instrument, thereby causing less current to be taken.

The main advantages of the induction instrument are:

(*a*) A large-scale deflection can be obtained, often more than 270°.

(*b*) The damping is very simple and efficient.

the work done upon the moving system will be $T_\theta d\theta$, resulting in an increase of energy stored. This increase may be represented by dW and, since the work done on the moving system is equal to the increase in energy,

$$T_\theta d\theta = dW$$

$$T_\theta = \frac{dW}{d\theta}$$

$$= \frac{d}{d\theta}\left[\tfrac{1}{2}C_1(V_3-V_1)^2+\tfrac{1}{2}C_2(V_3-V_2)^2\right]$$

$$= \frac{(V_3-V_1)^2}{2}.\frac{dC_1}{d\theta}+\frac{(V_3-V_2)^2}{2}.\frac{dC_2}{d\theta}$$

Assuming that the plane of the needle is equidistant from that of the top and bottom quadrants, say, d cm. from each, and θ and r are respectively the angle and radius of the needle sector, then since the plates are in air,

$$C_1 = \frac{2\left[\frac{1}{2}r^2\left(\frac{\phi}{2}+\theta\right)\right]}{4\pi d}$$

$$= \frac{r^2}{4\pi d}\left[\frac{\phi}{2}+\theta\right]$$

where ϕ and θ are in radians.

Therefore, $$\frac{dC_1}{d\theta} = \frac{d}{d\theta}\cdot\frac{r^2}{4\pi d}\left[\frac{\phi}{2}+\theta\right]$$

$$= \frac{r^2}{4\pi d}$$

Similarly, $$C_2 = \frac{r^2}{4\pi d}\left[\frac{\phi}{2}-\theta\right]$$

and $$\frac{dC_2}{d\theta} = -\frac{r^2}{4\pi d}$$

But $$T_\theta = \frac{(V_3-V_1)^2}{2}\cdot\frac{dC_1}{d\theta}+\frac{(V_3-V_2)^2}{2}\cdot\frac{dC_2}{d\theta}$$

$$= \frac{r^2}{8\pi d}\left[(V_3-V_1)^2-(V_3-V_2)^2\right]$$

It can be seen from the above expression that the value of T_θ is dependent upon the value of V_3, since V_1 and V_2 are of definite value depending upon the voltage to be measured. Thus, when the electrometer is connected heterostatically, the torque can be increased by increasing the potential of the needle.

From the diagram it is apparent that, when the instrument is connected idiostatically, $V_3 = V_1$. Therefore, the above expression for the torque simplifies to

$$T_\theta = -\frac{r^2}{8\pi d}(V_2 - V_1)^2$$

Also, since $(V_2 - V_1)$ is the voltage to be measured, and considering the four quadrants instead of two,

$$T = -\frac{r^2}{4\pi d} \times (\text{Voltage to be measured})^2$$

and the needle will rotate in the opposite direction to that indicated in Fig. 42.

The value of T_θ is in dyne-cm., providing the voltage is in electrostatic c.g.s. units. Thus, the value of the torque is given by

$$T = \frac{r^2 V^2}{1\cdot 11 \times 10^9 d} \text{ . grm. cm.}$$

3.23. Attracted Disc Electrometer. This instrument, which is primarily for the measurement of voltages between the values of 20 kV and 200 kV, relies for its action upon the fact that when two parallel plates are, say, d cm. apart, with a potential difference of V volts between them, the force of attraction existing between the plates is given by

$$F = \frac{AK\phi^2}{8\pi} \text{ dynes}$$

where F is the force in dynes, A is the area of the plates in square cm., K is the dielectric constant of the medium, and ϕ is the uniform field intensity between the plates, i.e. the voltage gradient between the plates.

$$\text{Thus, } \phi = \frac{V}{d}$$

$$\text{and } F = \frac{AKV^2}{8\pi d^2} \text{ dynes}$$

Therefore, the potential difference between the plates is given by

$$V = d\sqrt{\frac{8\pi F}{AK}} \text{ electrostatic c.g.s. units.}$$

If the plates are in air, K is unity and, since 300 volts = 1 electrostatic c.g.s. unit,

$$V = 1504\, d\sqrt{\frac{F}{A}} \text{ volts}$$

$$\text{and } F = A(V/1504d)^2 \text{ dynes.}$$

An example of the attracted disc voltmeter is the Abraham-Villard instrument, manufactured by Everett Edgcumbe & Co. Ltd. It consists of two curved metal discs, somewhat similar in shape to an ordinary lighting switch cover. The right-hand disc is connected to the higher potential. The centre portion of the left-hand disc is cut away to house a small movable disc which is geared to the pointer of the instrument and acts against a spring. The small movable disc is at the lower potential. The two large discs give protection to the working parts of the instrument against external electrostatic fields, and the range of the instrument is varied by setting the right-hand disc at different distances from the other.

The attracted disc type of instrument is an absolute instrument and is based on Lord Kelvin's Volt Balance.

The advantages of electrostatic instruments are:

(*a*) They are equally accurate for a.c. and d.c. measurements.

(*b*) The wave-form of the supply and variation of frequency do not in any way affect the accuracy of the instrument.

(*c*) The power loss is very small.

As one might assume from (*c*), the instruments have the disadvantage of very small operating forces, and this particularly applies to the measurement of voltages below 500 volts. However, for measurements from 500 volts to practically 1,000,000 volts they are extremely useful.

3.24. Hot-wire Instruments. This type of instrument depends upon the thermal effect of an electric current. A proportion, or the whole, of the current to be measured is passed through a wire and, since the heating effect is directly proportional to the square of the current, a deflection approximately proportional to the square of the current is produced through the linear expansion of the wire.

When a current of I ampères passes through a wire of, say, resistance R ohms, the power expended will be given by

$$P = I^2R \text{ watts}$$

$$= I^2R10^7 \text{ ergs per second,}$$

and the heat developed will be

$$H = \frac{I^2 R\, 10^7}{J} \text{ calories}$$

$$= 0{\cdot}24\, I^2R \text{ calories per second}$$

where J is the mechanical equivalent of heat ($4{\cdot}2 \times 10^7$ ergs per calorie).

In order that the conductor will attain a steady temperature very soon after the current is flowing, it is always made of very small cross-sectional area and the whole of the heat is lost by conduction, convection, and radiation.

If s is the area of the conductor surface in sq. cm., θ is the rise in temperature above the surrounding air, then the loss of heat, h_c, due to cooling will be given by

$$h_c = Us\theta \text{ calories per second,}$$

where U is the coefficient of emissivity, or loss of heat per sq. cm. per second per °C. above the surrounding air temperature.

Assuming that U is constant for the range of temperature of the wire, then, as soon as the temperature becomes constant,

$$h_c = 0\cdot24 i^2 R = Us\theta$$

$$\text{and } \theta = \frac{0\cdot24\, R.\, I^2}{Us}$$

$$= \frac{0\cdot24 V^2}{Us\, R}$$

or, providing it is small, the rise in temperature will be proportional to the square of the current in the wire.

We are also assuming, in the above statement, that:

(*a*) The resistance remains constant

(*b*) The coefficient of emissivity is also constant.

The change in length, dl, will therefore be given by

$$dl = l\theta\alpha$$

$$= \frac{0\cdot24 R I^2}{Us} l\alpha \text{ cm.}$$

where l is the original length, and α is the coefficient of expansion of the conductor.

With a rapid alternation of the current, the expansion will be proportional to the average value of the square of the current and, therefore, if the instrument is initially calibrated with d.c. it will indicate the r.m.s. value of the current from an a.c. supply.

In commercial instruments, owing to the fine wire being too short for its expansion to be used directly for the deflection of the pointer, a geometrical magnifying device is employed. This is illustrated in Fig. 43 (*a*).

Suppose that the length of the fine wire is x cm., and its expansion, when the current is passing and a state of equilibrium has been reached, is dx. Then, if it be pulled at its centre, so that the slack is taken up and there is no appreciable sag between the points AC and BC,

$$y^2 = \left(\frac{x+dx}{2}\right)^2 - \left(\frac{x}{2}\right)^2$$

$$= \frac{2x.dx + (dx)^2}{4}$$

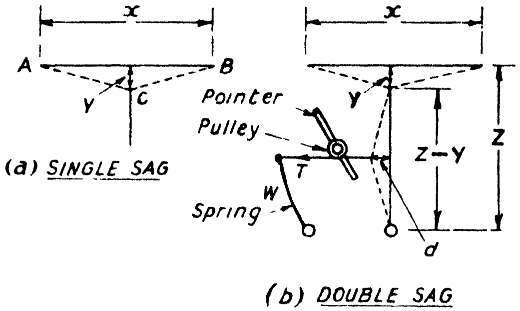
where y is the distance of the centre from its original position.

Neglecting $(dx)^2$, since it will be very small,

$$y = \sqrt{\frac{x}{2}dx}$$

Thus, y will be proportional to the current, since $\sqrt{\frac{x}{2}}$ is a constant and dx is proportional to the square of the current.

In order further to increase the magnification, a double-sag type has been devised; the further magnification being obtained by means of a second device similar to the above. This is illustrated in Fig. 43 (*b*).



(a) SINGLE SAG

(b) DOUBLE SAG

FIG. 43.—Hot-wire Instrument—Magnifying Device.

Suppose the length of the second wire is z, and this is also pulled at the centre to take up the slack due to the expansion of the hot wire. Then d, the sag of the wire z, is derived from

$$d^2 = \left(\frac{z}{2}\right)^2 - \left(\frac{z-y}{2}\right)^2$$

$$= \frac{2zy - y^2}{4}$$

Now, y^2 may be neglected in comparison with $2zy$

Therefore $d = \sqrt{\frac{zy}{2}}$

But we have already found that $y = \sqrt{\frac{x.dx}{2}}$

Therefore $d = \sqrt{\frac{z}{2}\sqrt{\frac{x\,dx}{2}}}$

$$= \left(\frac{z}{2\sqrt{2}}\right)^{\frac{1}{2}} \left(x.dx\right)^{\frac{1}{4}}$$

$$= K\,(dx)^{\frac{1}{4}}$$

From which it can be seen that the final deflection is proportional to $\sqrt[4]{dx}$ or, in other words, to the square root of the current.

In most commercial instruments the hot wire is of platinum-iridium; the advantage of this alloy being that no deterioration occurs due to oxidation at high temperatures. The diameter of the wire is usually of the order of 0·1 mm. The wire z, of Fig. 43 (*b*), is generally of phosphor-bronze, and attached to its centre is a fine silk thread which passes round a small pulley and thence to a spring which keeps the whole system taut. Attached to the pulley is a light pointer and a thin aluminium damping disc, the latter rotating between the gaps of a permanent magnet. Good damping is particularly necessary in the hot-wire instrument in order to damp out movements due to vibration or sudden changes of current.

As the hot wire expands, so d increases (Fig. 43 (*b*)), and the slack is taken up by the action of the spring W, this causing angular deflection of the pulley and pointer. It can be seen that the pulley and pointer are a magnifying device in themselves.

When this type of instrument is used as an ammeter, it requires a shunt for all values of current above 5 ampères. When employed as a voltmeter, a high value non-inductive resistance is connected in series with the hot wire.

This type of instrument has the advantage of being unaffected by stray magnetic fields. Furthermore, it can be used for both a.c. and d.c. measurements; the calibration being the same for both. Its greatest advantage, however, is that its deflection depends upon the heating effect of the current. Therefore it will measure the r.m.s. value of an alternating current irrespective of the wave-form or frequency.

The main disadvantages of the instrument are:

(*a*) The scale is cramped.

(*b*) It requires frequent zero adjustment, this being due to the stretching of the wires and also to the difference of temperature between the working wire and the base of the instrument.

(*c*) It is very sluggish, due to the length of time taken for the wire to heat up.

(*d*) It cannot withstand overloads as the wire fuses when the current is greatly in excess of the maximum working value.

(*e*) The power consumption is high.

(*f*) The instrument is very fragile.

CHAPTER IV

Wattmeters

4.1. Measurement of Power. The measurement of power in a d.c. circuit is a simple matter, since the product of the current and voltage gives the power directly. Thus, the measurement of power may be made with an ammeter and voltmeter only.

In an a.c. circuit, the product of the instantaneous voltage and the instantaneous current still gives the instantaneous power, but, owing to the frequency of alternation of these values, the voltmeter and ammeter merely take up r.m.s. values of voltage and current respectively which, in themselves, give no indication as to the instantaneous power, or mean power, dissipated in a circuit. Their product only gives the *VA* of the circuit, or, in other words, the apparent watts being dissipated.

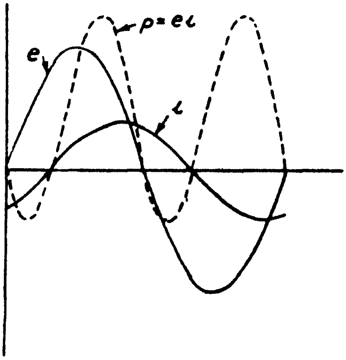


Fig. 44.—Voltage, Current and Power Curves.

Now, the instantaneous power p in an a.c. circuit is given by $p=ei$ where e and i are the instantaneous values of the voltage and current respectively. These conditions are represented by the curves of Fig. 44; all three values being plotted to different ordinate scales.

If the wave-form of the supply is sinusoidal, the value of e at any time t will be E_{max} sin ωt, and that of the current i will be I_{max} sin $(\omega t-\theta)$, where θ is the angle of lag of the current with respect to the voltage.

Therefore, the instantaneous power p, at any time t, is given by

$$\begin{aligned} p &= ei \\ &= E_{max} \sin \omega t.\ I_{max} \sin (\omega t-\theta) \\ &= E_{max}\ I_{max} \sin \omega t.\ \sin (\omega t-\theta) \end{aligned}$$

$$\text{The mean power } P=\frac{\omega}{2\pi}\int_{0}^{\frac{2\pi}{\omega}} E_{max}I_{max}\sin\omega t.\sin(\omega t-\theta)dt.$$

$$=\frac{\omega E_{max}I_{max}}{2\pi}\int_{0}^{\frac{2\pi}{\omega}}\tfrac{1}{2}[\cos\theta-\cos(2\omega t-\theta)].dt$$

$$=\frac{\omega E_{max}I_{max}}{4\pi}\left[t.\cos\theta-\frac{1}{2\pi}\sin(\omega t-\theta)\right]_{0}^{\frac{2\pi}{\omega}}$$

$$=\tfrac{1}{2}E_{max}I_{max}\cos\theta$$

$$=EI\cos\theta$$

where E and I are the r.m.s. values of the voltage and current respectively (since $E=E_{max}/\sqrt{2}$ and $I=I_{max}/\sqrt{2}$). As can be seen, the voltmeter and ammeter only take into account the first two quantities in the expression for power. Thus, an instrument is essential which also takes into account the expression $\cos\theta$; in other words, the power factor. Such an instrument is the wattmeter. Its instantaneous torque is proportional to the product of the instantaneous values of the current and the voltage and, since it cannot follow the rapid alternations of the torque, it takes up a deflection proportional to the mean torque, i.e. to the mean power.

4.2. Single-phase A.C. Circuits. The two obvious methods for the connection of the wattmeter are as illustrated in Fig. 45. Connection (*a*) has the disadvantage that the current coil carries the small current taken by the pressure coil, as well as the load current. In connection (*b*), the pressure coil is connected on the side of the current coil remote from the load. Thus, the applied voltage to the pressure coil is greater than that to the load by an amount equal to the voltage drop in the current coil; the applied voltage to the pressure coil being the vector sum of the voltage drop in the current coil and applied voltage to the load. Thus, in connection (*a*), the wattmeter measures the watts lost in the pressure coil and, in connection (*b*), the watts lost in the current coil, in addition to the power dissipated in the load.

When the load to be measured is large, it is preferable to connect the wattmeter in method (*a*), as the watts lost in the pressure coil will be small compared with the total load. If the load is small, connection (*b*) is better, since the volt-drop in the current coil will be small, thereby only introducing a small error.

Some wattmeters are constructed as indicated diagrammatically in Fig. 46. A compensating coil is connected in series with the pressure coil and so arranged that it sets up a magnetic field in opposition to that of the current coil, and neutralises the component due to the pressure coil current in the current coil. This is adjusted correctly when the deflection is zero; the pressure coil being energised and no other current passing through the current coil of the instrument.

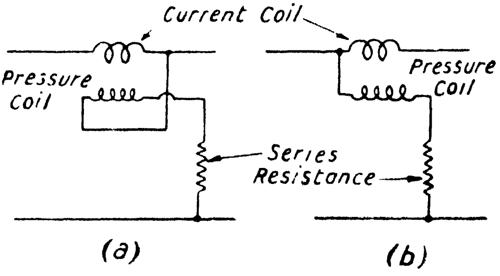


FIG. 45.—Alternative Wattmeter Connections.

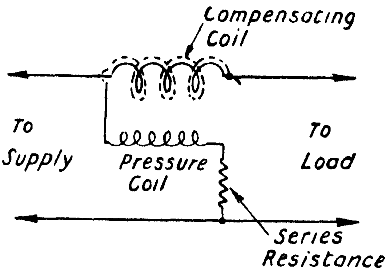


FIG. 46.—Compensating Winding for Potential Coil.

4.3. The Measurement of Power in a Polyphase Circuit. At first sight, the measurement of power in a polyphase circuit might appear somewhat complicated. We will therefore derive a rule which will be applicable to all balanced and unbalanced multi-wire systems.

Let us first consider a system such as is illustrated in Fig. 47 (*a*), consisting of two wires from the supply to the load. No matter what the variation of the current, the instantaneous value in wire 1 will be equal and opposite to that in wire 2 and, as we have already seen, the instantaneous power expended in the load will be the product of the instantaneous values of the load current and the potential drop across the load due to the passage of the current through the load.

The mean power will be the average value of this product, and this is the value which the wattmeter will indicate, when connected as shown in Fig. 45, since the inertia of the moving system is too great to follow the rapid alternations in power which occur at normal alternating current supply frequencies.

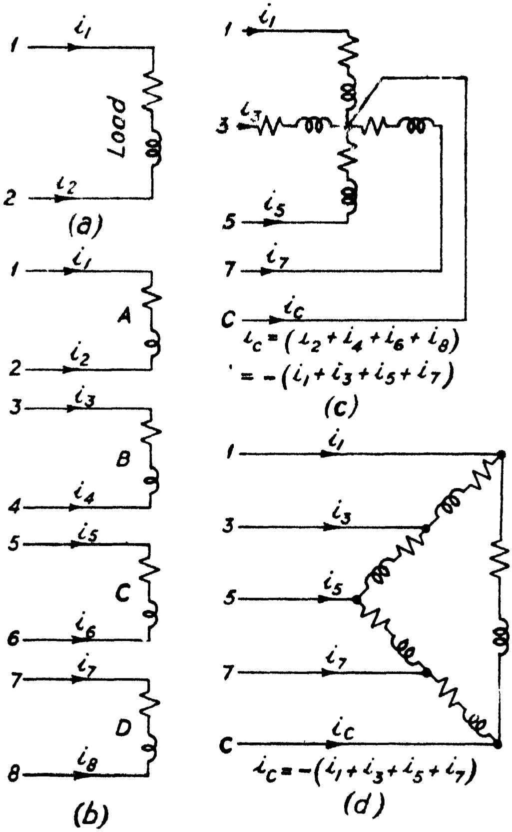


FIG. 47.—Multi-wire Supply Circuits.

Now consider a number of such systems, entirely independent of one another, as illustrated in Fig. 47 (*b*). The current in wire 1 will be equal and opposite to that in wire 2 and the current in wire 3 will be equal and opposite to that in wire 4, etc. Also, the total power expended in the four circuits will be the sum of the powers expended in the individual circuits.

If we now merge the lines 2, 4, 6, and 8 into one common line *C*, as illustrated in Fig. 47 (*c*), the currents which previously returned through the separate lines will now return through the line C, and the total instantaneous value of the current flowing in this line will be the sum of the return currents which were previously flowing in the individual lines 2, 4, 6, and 8. Thus, the instantaneous current in the line C will be given by

$$i_c = i_2 + i_4 + i_6 + i_8$$

$$\text{but } i_2 = -i_1,\ i_4 = -i_3, \text{ etc.}$$

$$\text{Therefore, } i_c = -i_1 - i_3 - i_5 - i_7$$

$$\text{or } i_c + i_1 + i_3 + i_5 + i_7 = 0$$

which agrees with Kirchoff's Law, which states that the algebraic sum of the currents which meet at any point is zero.

The total instantaneous power expended in the receiving network will remain unaltered and will be

$$i_1(V_1 - V_c) + i_3(V_3 - V_c) + i_5(V_5 - V_c) + i_7(V_7 - V_c)$$

since $V_c = V_2$, V_4, etc.,

and the summation of the indications of four separate wattmeters, connected so that their respective current coils are one in each of four lines, and their potential coils are connected each to the remaining line and the line carrying the corresponding current coil, will give the total average power dissipated in the system. Furthermore, if four wattmeter elements are constructed to impart their torque upon one moving system, the indication will be proportional to the average total expended in the system.

4.4. Polyphase Systems—General Case. By similar reasoning, we can generalise and state that if a system consists of n wires, $(n-1)$ wattmeters (or a polyphase wattmeter consisting of $(n-1)$ elements) are sufficient, under any load conditions, to measure the total power expended in the network. We shall see later, however, that when the line currents and voltages are balanced it is possible to dispense with one or more wattmeters.

In three-phase four-wire circuits it is usual to connect the wattmeters with the neutral wire as the common potential point. From the above, we see that this is not necessary, since any line would do for the common potential point. Such a practice, however, has the advantage that all wattmeter potential circuits are energised by more or less the same value of voltage.

It should be stated that the above rule has been derived for a star-connected load, but should a mesh-connected load, such as is illustrated in Fig. 47 (*d*), be employed, the same argument would still hold, since on the supply side of the system it is exactly similar to that of a group of two-wire systems with a common return. The passing of the current through the load effects a drop in potential in exactly the same manner as with the star-connected system of Fig. 47 (*c*). The only difference is that the current is distributed through two parallel circuits in the mesh-connected load, whilst in the case of the star-connected load there is only one circuit. The total instantaneous power will still be the summation of the products of the instantaneous values of the current in each line and the potential of the line relative to that of the common return. The polyphase wattmeter with $(n-1)$ elements will accordingly indicate the average value of the total instantaneous power whilst, as we shall see later, the polyphase watt-hour meter, with $(n-1)$ elements similarly connected, will integrate the total consumption of energy dissipated in the circuit in any given period.

4.5. Three-phase Three-wire Systems—Two Wattmeter Method of Measuring Power. The most common method of measuring power in a three-phase three-wire circuit is by the two wattmeter method; the connections for measurement of a star-connected load being as shown in Fig. 48. As we have already seen, in section 4.4, where there are only three lines, and two wattmeters are connected in the manner indicated, the sum of their readings will give the total load. Taking the case of a balanced load, such as is represented in the vector diagram of Fig. 49, the sum of the wattmeter readings will be

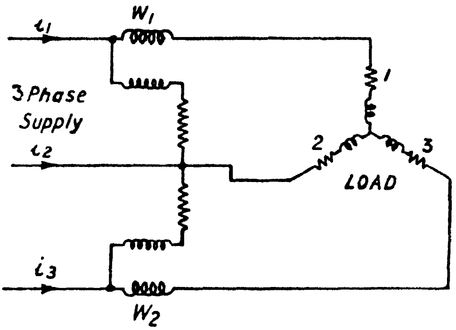


FIG. 48—Two-wattmeter Method of Measuring 3-wire Load.

$$E_{12}I_1 \cos(30+\theta)+E_{32}I_3 \cos(30-\theta)$$
$$=\sqrt{3}E_pI_1 \cos(30+\theta)+\sqrt{3}E_pI_3 \cos(30-\theta)$$
$$\text{since } E_{12}=E_{32}=\sqrt{3}E_p, \text{ and } I_1=I_3=I$$
$$\text{Therefore, } W_1+W_2=\sqrt{3}E_pI\,[\cos(30+\theta)+\cos(30-\theta)]$$
$$=\sqrt{3}E_pI\,(2\cos 30\cos\theta)$$
$$=3E_pI\cos\theta$$

which is the total power dissipated in the load.

Fig. 49 reveals that, when θ is 30°, W_2 reads its maximum value, i.e. $E_{32}I_3$ for any given current. Also, when θ is 60° (at 0·5 power factor) W_1 will read $E_{12}I_1 \cos(60+30)$, which is zero. For any value of power factor below 0·5 lagging, W_1 will read a negative quantity. In such a case the connections to the pressure coil of W_1 must be reversed in order that the wattmeter will indicate in the correct direction. The mean power will then be the reading of W_2 minus that of W_1.

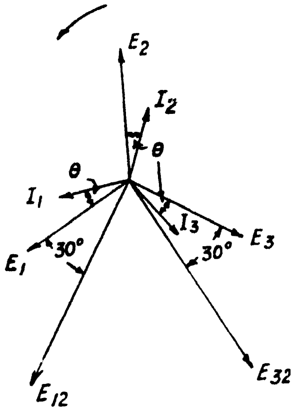


FIG. 49.—Vector Diagram—Two-wattmeter Method of Measuring 3-wire Load.

4.6. Three-phase Three-wire Systems. Three Wattmeter Method. Fig. 50 illustrates the method of connecting three wattmeters for the measurement of power in a three-phase three-wire circuit; the load being star-connected.

The instantaneous power in the load

$$=e_R i_R + e_Y i_Y + e_B i_B$$

where e_R is the potential between the red line and the star point of the load, e_Y is that between the yellow line and the star point of the load, and e_B is the potential of the blue line relative to the star point. Now, suppose there exists a potential difference of V between the star point and the common connection of the wattmeters. Then we have

$$e_{W1} + V = e_R$$

$$e_{W2} + V = e_Y$$

$$e_{W3} + V = e_B$$

Therefore, the instantaneous power will be

$$(e_{W1}+V)i_R + (e_{W2}+V)i_Y + (e_{W3}+V)i_B$$

$$= e_{W1}i_R + e_{W2}i_Y + e_{W3}i_B + V(i_R + i_Y + i_B)$$

but, in any balanced or unbalanced three-wire system,

$$i_R + i_Y + i_B = 0$$

and, in consequence, the total instantaneous power will be

$$e_{W1} i_R + e_{W2} i_Y + e_{W3} i_B$$

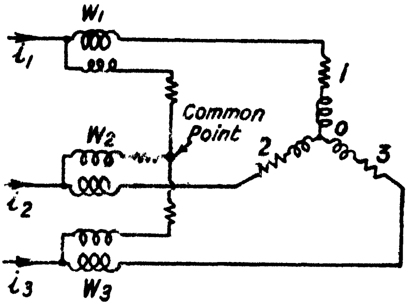


FIG. 50.—Connections for Measuring 3-wire Load with 3 wattmeters.

Hence, the summation of the readings of the three wattmeters, when connected as shown in Fig. 50, will give the mean value of the total power expended in the circuit.

4.7. Three-phase Four-wire Systems. Three Wattmeter Method of Measuring Power. The connections of the three wattmeters, or three-element wattmeter, are as shown in Fig. 51. The total instantaneous torque will be proportional to

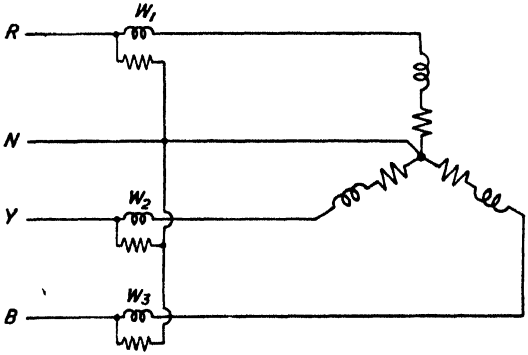


FIG. 51.—3-Phase 4-Wire System—3-Wattmeter Method of Measuring Power.

$$e_1 i_1 + e_2 i_2 + e_3 i_3$$

which is proportional to the instantaneous power dissipated in the circuit and, as we have already seen, the summation of the wattmeter readings will be correct for both balanced and unbalanced load conditions.

4.8. Balanced Loads. One Wattmeter Method of Measuring Power. Fig. 52 gives the connections for the measurement of power in a balanced three-phase circuit when only one wattmeter is available. The current coil of the wattmeter is connected in one line, whilst the potential is connected for alternate readings between that line and each of the other two lines. This is attained by means of the pressure coil circuit change-over switch shown in the diagram.

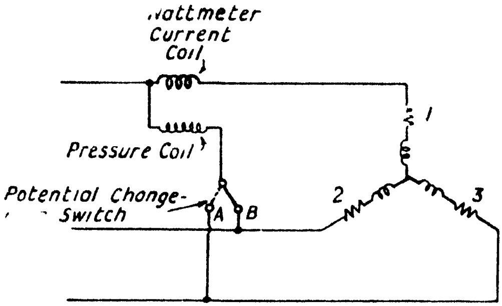


FIG. 52.—3-Wire Balanced Load—One Wattmeter Method of Measuring Power.

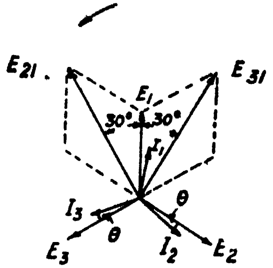


FIG. 53.—Vector Diagram—One Wattmeter Method of Measuring Balanced 3-wire Load.

If the phase voltages are E_1, E_2, and E_3 respectively, and the line currents are I_1, I_2, and I_3, the currents lagging their respective voltages by θ degrees, then the reading of the wattmeter when the pressure coil is connected between the lines 1 and 2 will be

$$W_{12}=E_{21}I_1 \cos (30+\theta)$$

which can be verified from the vector diagram of Fig. 53.

Similarly, the reading when connected with the pressure coil between lines 1 and 3 will be given by

$$W_{13}=E_{31}I_1 \cos (30-\theta)$$

Therefore, $W_{12}+W_{13}=E_{21}I_1 \cos (30+\theta)+E_{31}I_1 \cos (30-\theta)$

but $E_{21}=E_{31}=\sqrt{3}E$, where E is the phase voltage,

and $I_1=I$, where I is the line current.

Therefore, $W_{12}+W_{13}=\sqrt{3}EI[\cos (30-\theta)+ \cos (30+\theta)]$

$$=3EI \cos \theta$$

which is the total power applied to the load.

Thus, a single wattmeter connected in the above manner may be used for the measurement of power in a balanced three-phase circuit.

4.9. Wattmeters. There are three main classes of wattmeter, and they are as follows:

(*a*) Electro-dynamometer.
  (i) Pivoted coil deflectional type.
  (ii) Torsion-head type.
(*b*) Induction.
(*c*) Electrostatic.

We will now consider each class in turn.

4.10. Electro-dynamometer Wattmeters. We have already seen (section 4.1) that the instantaneous power is the product of the instantaneous current and the instantaneous voltage. Therefore, if two coils, one carrying the circuit current and the other carrying a current proportional to the applied voltage—and in phase with the applied voltage—are placed in such a position that they react upon one another, then an instantaneous torque will be developed which will be proportional to the instantaneous power dissipated in the load. Also, providing the instrument has a long natural period compared with the periodicity of the alternating current, the moving system will take up a position dependent upon the mean power expended in the circuit.

In order that the current in the pressure coil shall be in phase with the applied voltage, the pressure coil must be as non-inductive as possible. This is usually attained by placing a non-inductive resistance in series with the actual pressure coil of the instrument, thus making the reactance of the pressure coil negligible compared with its resistance. A further advantage of this non-inductive resistance in series with the pressure coil is that it minimises the current change in the pressure coil, with change of frequency.

Suppose r_p = the resistance of the pressure coil
L_p = the inductance of the pressure coil
E = voltage applied to the pressure coil circuit
r_s = resistance in series with the pressure coil.

Then the current in the pressure coil, I_p, is given by

$$I_p=\frac{E}{\sqrt{(r_p+r_s)^2+\omega^2L_p{}^2}}$$

and the phase angle by which the current is lagging the voltage is given by

$$\theta=\tan^{-1}\frac{\omega L_p}{(r_p+r_s)}$$

From these expressions, it can be seen that an increase in the value of the non-inductive resistance reduces the current in the pressure coil and also reduces the value of $\tan^{-1}\omega L_p/(r_p+r_s)$, i.e. the angle of lag of the current relative to the applied voltage. Furthermore, the greater the value of r_p+r_s, the less effect, with change of frequency, has the term $\omega^2L_p^2$ upon the value of the current.

Increase of frequency increases θ and also very slightly reduces the current in the pressure circuit.

Suppose we are measuring an inductive load, the current lagging the voltage by angle ϕ. The wattmeter deflection will be proportional to $I\,I_p \cos(\phi-\theta)$, where I is the current in the current coil, I_p is the current in the pressure coil, and θ is the angle of lag of the current I_p relative to the applied voltage. Since I_p is proportional to E, the supply voltage, the torque is proportional to

$$\frac{IE}{Z_p}\cdot\cos(\phi-\theta)$$

where Z_p is the impedance of the pressure coil circuit.

$$\text{But } Z_p=\frac{r_p+r_s}{\cos\theta}$$

Therefore, the torque is proportional to

$$\frac{IE\cos\theta\cos(\phi-\theta)}{r_p+r_s}$$

The wattmeter will only read correctly at all frequencies providing the torque is proportional to $IE\cos\phi/(r_p+r_s)$. Therefore, the ratio of the true reading to the actual reading of the instrument

$$=\frac{\dfrac{IE}{(r_p+r_s)}\cdot\cos\phi}{\dfrac{IE}{r_p+r_s}\cdot\cos(\phi-\theta)\cos\theta}$$

$$=\frac{\cos\phi}{\cos(\phi-\theta)\cos\theta}$$

which is the factor by which the actual reading must be multiplied in order to allow for the inductance of the pressure coil circuit.

Errors are likely to be introduced on account of the induction of eddy currents in the solid metal parts, in the neighbourhood of the coils, thereby distorting the magnetic field set up by the current coil of the instrument. The phase of the eddy e.m.f.s will be 90° behind the flux creating them and the eddy currents will be almost in phase with their e.m.f.s. These currents will create a magnetic field which combines with the magnetic field due to the current coil. The resultant magnetic field will be slightly less in magnitude than the magnetic field due to the current coil alone, and it will lag the latter by a small angle.

In order to ensure that the error is not serious, all solid metal parts should be excluded from the vicinity of the coils and, when the wattmeter is designed for passing heavy currents through the current coil, the conductors should be stranded. This will decrease the eddy currents by increasing the resistance of their path.

The pressure coil circuit may have self-capacity as well as inductance. This tends to lead the current in the pressure coils relative to the applied voltage. In this case there will be one frequency at which the capacitive reactance is equal to the inductive reactance, and there will be no phase angle between the pressure circuit current and the applied voltage.

Such constitutes the principles of the electro-dynamometer wattmeter and illustrates its main errors. The actual instrument is similar in principle and design to the electro-dynamometer ammeter and voltmeter described in section 3.14. The current is carried in the fixed coils of the instrument, whilst the moving coils are connected across the applied voltage, in series with a high non-inductive resistance.

4.11. The Deflectional type of Electro-dynamometer Wattmeter. In this type of instrument the moving coil is carried on a pivoted spindle and its movement is spring-controlled; the moving pressure coil being almost entirely surrounded by the fixed current coils. The damping is usually by air-friction, a vane in an enclosed sector-shaped compartment generally being employed. For currents up to 100 ampères, the instrument is usually directly connected. Beyond this range, either shunts or current transformers have to be employed, depending upon whether the measurements are for direct or alternating current.

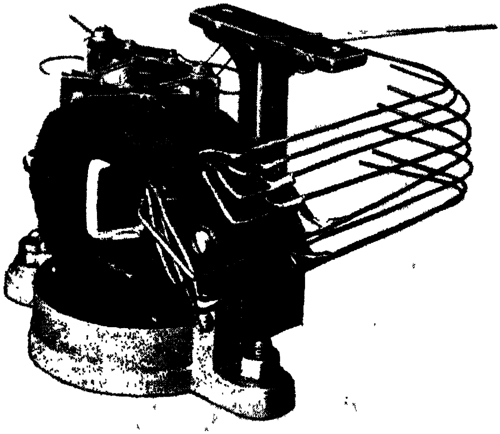


Fig. 54 (*a*).—Deflecting Electro-dynamometer Wattmeter (Salford Inst Co.)

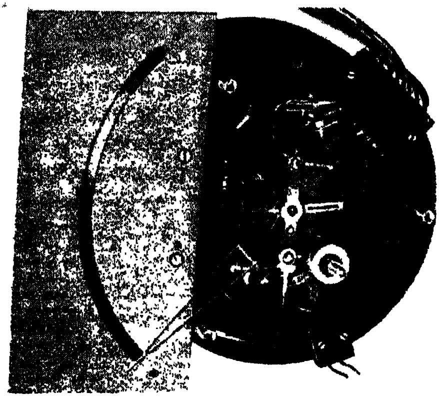


FIG. 54 (*b*).

In order to minimise the effects of eddy currents, the coil conductors are usually stranded or laminated. This particularly applies to coils designed for heavy current working. The use of metal in the working parts is kept to a minimum, consistent with freedom from warping or other movement of the working coils.

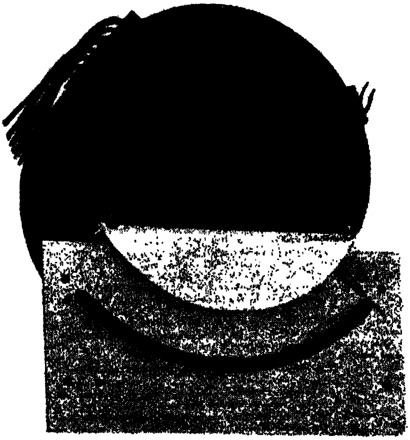


FIG 54 (*c*).—Deflecting Electro-dynamometer Wattmeter (Salford Inst. Co.)

Fig. 54 illustrates a substandard precision electro-dynamometer wattmeter manufactured by the Salford Instrument Co. Ltd. No iron or other magnetic substance is employed in the construction of either the fixed or moving coils of the instrument. The pointer is girder-shaped, and it is claimed that this special shape of pointer prevents vibration due to resonance, which in some instruments tends to occur between 10 and 100 cycles per second. A pointer of this design also has the advantage of great mechanical strength. The damping of the instrument is aperiodic (see section 2.8).

Fig. 54 (*a*) illustrates the element of the instrument, whilst Figs. 54 (*b*) and (*c*) show the laminated shield which protects the instrument from stray magnetic fields and is sufficiently remote from the element to have no appreciable effect upon it.

The instrument also has a low volt-ampère loading of the current coils which permits of its being employed with standard instrument transformers without fear of burdening the transformers by overloading.

The advantages of the pivoted coil electro-dynamometer wattmeter over the induction wattmeter are:

(*a*) Greater accuracy
(*b*) Less weight of moving system
(*c*) Less power consumption
(*d*) The instrument can be used for either a.c. or d.c. measurements.

4.12. The Torsion-head types of Electro-dynamometer Wattmeter. This class of instrument differs from the deflectional type inasmuch as the torque of the moving system is measured whilst the moving system is in its zero position, the significance of which will be appreciated in the succeeding paragraphs. The pressure coil is suspended from an adjustable torsion-head, which serves the dual purpose of a suspension and also leading in the current to the moving coil. Thus, any deflectional torque may be balanced by a controlling torque such that when the system is in a state of equilibrium, due to the deflecting and controlling torques being equal and opposite, it still retains its zero position.

The pressure coil is entirely embraced by the fixed, or current, coils, and in most cases is wound in such a manner that the system is astatic. This entirely eliminates any error due to external magnetic fields. A scale is set upon the torsion-head and, for any required load, the torsion-head is turned through an angle which will create the same controlling torque, when the moving coil is at its zero position, as the deflectional torque due to the predetermined load.

$$\text{Thus, } \alpha = KP$$

where K is a constant, P is the load, and α is the angle through which the torsion-head must be turned. It should be mentioned that the controlling torque is proportional to α, hence the validity of the above expression.

The load is then applied and varied until the deflection of the moving system is zero, as indicated by the position of the pointer on the scale.

Alternatively, if a load of unknown value is to be measured, the torsion-head is turned until the moving system returns to its zero position, and the load can be determined from the expression

$$P=K_1\alpha$$

where K_1 is a constant for the instrument.

One well-known instrument of this type is that due to Dr. C. V. Drysdale and manufactured by H. Tinsley & Co. Ltd. Figs. 55 and 56 show the arrangement of the coils for single-phase and polyphase wattmeters respectively. The moving coil is stitched to a flat strip of mica, and is carried by the latter. It is wound in two equal sections, the current flowing in the one direction in one section and in the opposite direction in the other. This makes the system astatic, and ensures that the instrument will not be influenced by stray magnetic fields. The suspension consists of a silk fibre, attached to which is a spiral spring for transmission of the required torsion to the moving system.

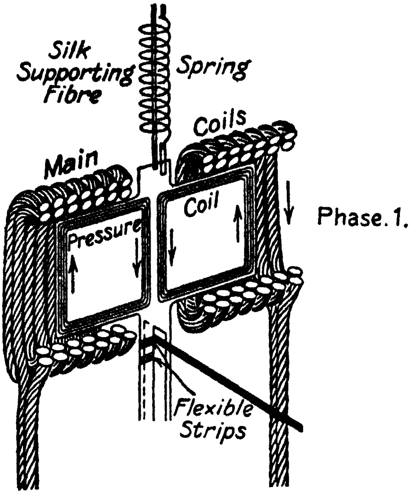


FIG. 55.—Drysdale Single-phase Torsion-head Electro-dynamometer Wattmeter (H. Tinsley & Co. Ltd.)

The fixed coil is also in two sections, and these are astatically wound. The conductors consist of ten strands, each of which is individually insulated. The strands are brought out to a commutator, by virtue of which it is possible to connect all the strands in series, parallel, or a series-parallel combination; thus permitting of several ranges. Two fine phosphor-bronze ligaments are employed to lead the current into the moving coil. The spring, which is made of German silver, merely acts as a torsion control. By very careful adjustment of the length of this spring, which comprises several turns, the constant of the instrument is made an exact figure. A knife-edge pointer is attached to the moving coil and a short scale is provided so that the zero can be accurately determined. A large scale, equally divided in divisions and subdivisions, is provided for the adjustment of the torsion-head. When the deflection is zero for any given load, the value of the load is directly proportional to the scale reading of the torsion-head. Damping is provided by means of the mica vane which carries the moving coil.

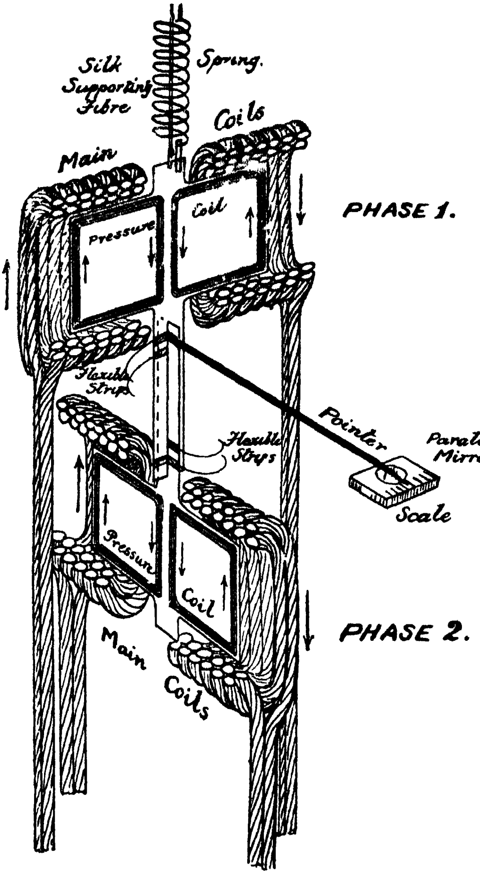


FIG. 56 - Drysdale Polyphase Torsion-head Electrodynamometer Wattmeter (H. Tinsley & Co. Ltd)

The above instrument can be constructed to cover a current range up to 500 ampères. Beyond this value, the cross-section of the conductors would be so great that the current would be non-uniformly distributed. When employed in a.c. circuits, eddy currents would also be introduced with conductors of large cross-section.

The torsion-head wattmeter is largely used for standardising purposes.

4.13. Expression for Torque—Electro-dynamometer Instruments. The fundamental difference between the deflectional type of electro-dynamometer instrument and the torsion-head type is that the relative positions of the fixed and moving coils of the former change with load whilst, in the latter type of instrument, the relative position of the coils—when finally adjusted—is the same for all loads. Thus, it is only in the latter type of instrument that the mutual inductance between the coils remains constant. The following argument will enable the reader to appreciate the significance of this difference in principle between the two instruments.

Suppose the instantaneous current in the fixed coil is i and, at any time t, $i=I_{max}\sin(\omega t-\theta)$, where θ is the angle of lag of the current relative to the applied voltage. Also, let the instantaneous value of the applied voltage (e) be given by $e=E_{max}\sin\omega t$. Then, if the inductance of the pressure coil is negligible compared with its resistance, the pressure coil current i_p will be given by $i_p=(E_{max}\sin\omega t)/R$, since it is in phase with the voltage; R being the resistance of the pressure coil circuit.

Now, the flux threading the pressure coil, when the latter is placed with its plane parallel to that of each current coil, is given by

$$\phi=K_l N_c i$$

where K_l is a constant and N_c the number of turns in the current coils.

The maximum value of the mutual inductance between the current and pressure coils is given by

$$M_{max}=K_l N_c N_p$$

where N_p is the number of turns in the pressure coil.

The energy potential of the coil, when its plane is parallel to that of the fixed coil, is

$$V=i_p N_p \phi$$

and when the coil is turned through an angle β from this position

$$\begin{aligned} V&=i_p N_p \phi\cos\beta \\ &=i_p N_p K_1 N_c i\cos\beta \\ &=M_{max} i_p i\cos\beta \end{aligned}$$

When the pressure coil moves through a small angle $d\beta$ from an angular deflection of β, the work done, which is equal to the torque times the angle moved through, is $Td\beta$, and the change in potential energy is dV.

$$\text{Therefore, } Td\beta = dV$$

$$\text{and } T = \frac{dV}{d\beta}$$

$$= \frac{d\, M_{max} i_p i \cos\beta}{d\beta}$$

$$= -M_{max} i_p i \sin\beta \text{ dyne-cm}$$

providing M_{max}, i_p, and i are in electro-magnetic c.g.s. units.

$$\text{But } i_p = \frac{E_{max}}{R} \sin\omega t$$

$$\text{and } i = I_{max} \sin(\omega t - \theta)$$

Therefore, the mean torque

$$T_m = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \frac{M_{max} E_{max} \sin\omega t . I_{max} \sin(\omega t - \theta) \sin\beta . dt}{R}$$

$$= \frac{\omega}{2\pi} . \frac{M_{max} E_{max} I_{max} \sin\beta}{R} \int_0^{\frac{2\pi}{\omega}} \sin(\omega t - \theta) \sin\omega t . dt$$

$$= \frac{\omega}{2\pi} \cdot \frac{M_{max} E_{max} I_{max} \sin\beta}{R} \int_0^{\frac{2\pi}{\omega}} \tfrac{1}{2} (\cos\theta - \cos(2\omega t - \theta)) dt$$

$$= \frac{\omega}{4\pi} . \frac{M_{max} E_{max} I_{max} \sin\beta}{R} \left[t . \cos\theta - \frac{1}{2\omega} (\sin(2\omega t - \theta)) \right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{\omega}{4\pi} . \frac{M_{max} E_{max} I_{max} \sin\beta}{R} \left[\frac{2\pi}{\omega} \cos\theta \right]$$

$$= \frac{M_{max} E_{max} I_{max} \sin\beta \cos\theta}{2R}$$

$$\text{Therefore, } T_m = \frac{\text{Load in Circuit}}{R} . \; M_{max} \sin\beta$$

Thus, in the deflectional type of electro-dynamometer instrument, the torque decreases as the moving system is reaching its position of equilibrium. In the case of the torsion-head type of wattmeter, the maximum possible torque is utilised for any load, i.e. the axis of the moving coil is perpendicular to that of the fixed coil.

$$\text{Thus (Load in Circuit) } \frac{M_{max} \sin 90^\circ}{R} = K^1\alpha$$

$$\text{Load in Circuit} = \frac{K^1\alpha R}{M_{max}}$$

$$= K^{11}\alpha$$

where K^1 and K^{11} are constants and α is the angle of twist of the torsion-head. This gives a uniform scale and the maximum possible torque for any load, which is a decided advantage of the torsion-head electro-dynamometer instrument over the deflectional type.

4.14. Induction Wattmeter. The induction wattmeter depends for its operation upon the same principle as the induction ammeter and voltmeter, and has the same disadvantage of only being suitable for measurements in alternating current circuits.

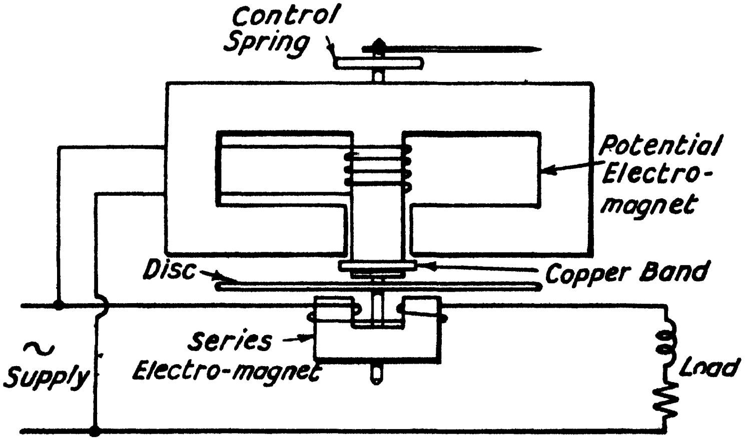


FIG. 57.—Diagrammatic Arrangement—Induction Wattmeter.

A diagrammatic arrangement of such an instrument is given in Fig. 57. It consists of two laminated electro-magnets, one of which carries a coil containing a few turns of comparatively heavy copper wire, whilst the other is embraced by a coil consisting of many turns of fine wire. The coil consisting of the heavier wire carries the load current, or a definite proportion of it; the other coil being connected across the supply and carrying a current proportional to the applied voltage. An aluminium disc is mounted between the poles of the two electro-magnets and a deflecting torque is created by the interaction of the magnetic flux of each electro-magnet with the eddy currents induced in the disc by the flux of the other electro-magnet.

The pressure coil is made as inductive as possible and a copper ring is fitted to the centre limb of the laminated core. This is adjusted so that the effective magnetic flux will lag behind the applied voltage by exactly 90° (see section 9.5).

A simplified vector diagram of the induction wattmeter is given in

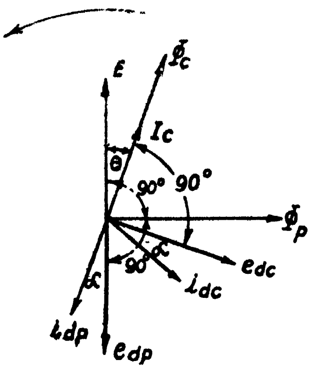


Fig. 58.—Vector Diagram —Induction Wattmeter

Fig. 58. Consider the applied voltage to the load as E, and the load current I_c as lagging the voltage by an angle θ. These are represented by vectors E and I_c respectively. Then Φ_p will be the flux due to E and (after correct adjustment of the copper band) this will be exactly 90° behind E. This flux will induce an e.m.f. e_{dp} in the disc, lagging Φ_p by 90°. The eddy currents due to this e.m.f. are represented by i_{dp} and will lag e_{dp} by angle α, due to the inductance of the eddy current paths in the disc.

The flux Φ_c, due to the current coil, will be in phase with the load current, and will lag behind the applied voltage by angle θ; assuming $\cos\theta$ to be the power factor of the load. This flux will induce an e.m.f. e_{dc} in the disc, lagging Φ_c by 90°. The eddy currents due to this e.m.f. are represented by i_{dc} and they will lag e_{dc} by angle α (approximately) due to the reasons mentioned above.

Now, the instantaneous torque acting upon the disc will be

$$\phi_p i_{dc} + \phi_c i_{dp}$$

where the above quantities are the instantaneous values (see section 3.20).

Suppose the applied voltage $e=E_{max}\sin\omega t$, then the current

$$i_c=I_{max}\sin(\omega t-\theta)$$

$$\text{and } \phi_c=K_1I_{max}\sin(\omega t-\theta)$$

$$\text{similarly, } \left[\phi_p=K_2\frac{E_{max}\cos\omega t}{\omega}\right]$$

$$e_{dc}=-K_3\frac{d\phi_c}{dt}$$

$$=-K_4I_{max}\omega\cos(\omega t-\theta)$$

$$i_{dc}=-K_5I_{max}\omega\cos(\omega t-\theta-\alpha)$$

$$e_{dp}=-K_6\frac{d\phi_p}{dt}$$

$$=-K_7E_{max}\sin\omega t$$

Therefore $i_{dp}=-K_8E_{max}\sin(\omega t-\alpha)$

where $K_1, \ldots K_8$ are constants.

The mean torque T_m upon the disc

$$T_m\infty K^1\Phi_pI_{dc}\cos(\theta+\alpha)+K^{11}\Phi_cI_{dp}\cos(180+\alpha-\theta)$$

where Φ_p, Φ_c, I_{dp}, and I_{dc} are r.m.s. values.

$$\infty K^1\left[K_2\frac{E_{max}}{\omega}K_5I_{max}\omega\cos(\theta+\alpha)\right]+K^{11}\left[K_1I_{max}K_8E_{max}\cos(\theta-\alpha)\right]$$

Therefore, since α is small

$$T_m=E_{max}I_{max}\cos\theta\,(K^1K_2K_5+K^{11}K_1K_8)$$

Therefore $T_m\propto E_{max}I_{max}\cos\theta$

$$\propto EI\cos\theta$$

Thus, the mean torque is proportional to the power in the circuit.

One would assume from the above expression that the deflection of the instrument was independent of frequency, but this is not so. In order to simplify the calculations, the inductance of each part of the circuit has been considered independent of frequency, which is not strictly true.

Instruments of the above type are usually spring-controlled and, as the deflecting torque is directly proportional to the power being measured, the scale is uniform. Furthermore, the instrument can be designed with a scale covering 300° deflection.

The instruments can be self-contained up to 100 ampères loading, but for greater currents transformers are necessary. The induction wattmeter, however, is only satisfactory when the supply voltage and frequency are reasonably constant.

The advantages of such an instrument are:

(*a*) Long uniform scale.
(*b*) High working torque.
(*c*) Very useful for switchboard purposes.

The disadvantages are:

(*a*) High power consumption.

(*b*) The instrument is only capable of first grade accuracy at a stated frequency and under certain conditions of temperature.

(*d*) It can only operate on alternating currents.

4.15. The Lipman Induction Wattmeter. Fig. 59 represents diagrammatically the electrical and magnetic circuits of the induction wattmeter developed by C. L. Lipman and manufactured by Nalder Bros. & Thompson Ltd. V_+ and V are the terminals of the potential winding, whilst M and L are those of the current winding. Power factor compensation is obtained by means of the coils F. These form a closed circuit in themselves, and the resistance R is so adjusted that the fluxes F_p and F_c are electrically displaced by 90°.

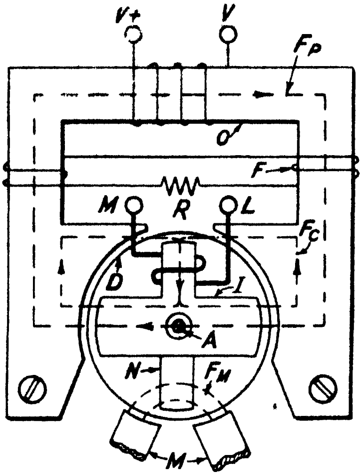


FIG 59.—Diagrammatic Arrangement—Lipman Induction Wattmeter

The useful flux F_p, due to the potential coil, circulates along the path indicated by the chain line and acts upon the drum D in a horizontal direction. The flux F_c, due to the current coil, acts upon the drum in a vertical direction. Thus the two fluxes are 90 geometrical degrees apart and, since they are also electrically displaced by 90 time-phase degrees, a torque, which is proportional to the power in the circuit, is exerted upon the drum D. Damping is performed by the electro-magnet M.

4.16. Three-voltmeter Method of Measuring Power. At this point it would be advantageous to consider the method by which three voltmeters can be employed for the measurement of power. The method of connection is illustrated in the circuit diagram of Fig. 60 (*a*). V_1, V_2, and V_3 are three voltmeters, and R is a non-inductive resistance which is connected in series with the load. From the vector diagram of Fig. 60 (*b*), which represents the conditions when current is flowing in the circuit, we have

$$V_1^2 = V_2^2 + V_3^3 + 2V_2V_3 \cos \theta$$

If we neglect the currents taken by the voltmeters V_2 and V_3, since they will be small compared with the load current, the current in the non-inductive resistance will be the same as the load current, and also

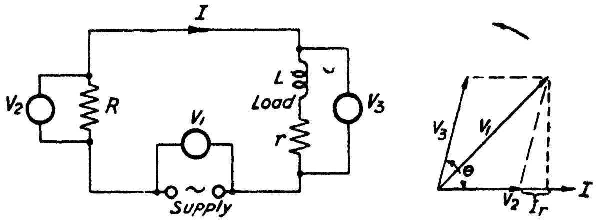


FIG. 60.—Three Voltmeter Method of Measuring Power.

in phase with it. Thus, the reading of V_2 will be IR volts, where I is the load current.

$$\text{Therefore, } 2V_2V_3 \cos \theta = 2IRV_3 \cos \theta$$

$$\text{and } V_1^2 = V_2^2 + V_3^2 + 2IRV_3 \cos \theta$$

But $IV_3 \cos \theta$ is the power dissipated in the load, therefore

$$V_1^2 = V_2^2 + V_3^2 + 2RP$$

$$\text{and } P = \frac{V_1^2 - V_2^2 - V_3^2}{2R}$$

where P is the power expended in the load.

We see from the above that it is possible to measure power by means of three voltmeters. It only holds, however, when the resistance R is non-inductive.

4.17. Electrostatic Wattmeter. This type of wattmeter is essentially a delicate reflecting instrument, for laboratory rather than general use. Nevertheless, since it has been in constant use at the National Physical Laboratory for the standardisation of wattmeters, watt-hour meters, etc., it is well worth brief description in this chapter. The development of the electrostatic ammeter, voltmeter, and wattmeter, as accurate standard instruments, is due to Drs. Patterson and Rayner, at the National Physical Laboratory.

The electrostatic wattmeter consists of a quadrant electrometer used in conjunction with a non-inductive resistance, and the simplified connections are as shown in Fig. 61.

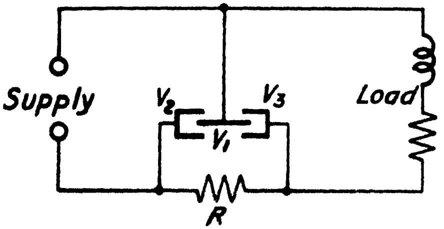


FIG. 61.—Simplified Connections—Electrostatic Wattmeter.

4.18. Expression for Torque—Electrostatic Wattmeter. We will consider the instantaneous load current as i ampères and V_1, V_2, and V_3 as the instantaneous potentials of the needle and two opposite quadrants, as illustrated in the figure. Then the instantaneous torque

$$T=K[(V_1-V_2)^2-(V_1-V_3)^2]$$

K being a constant.

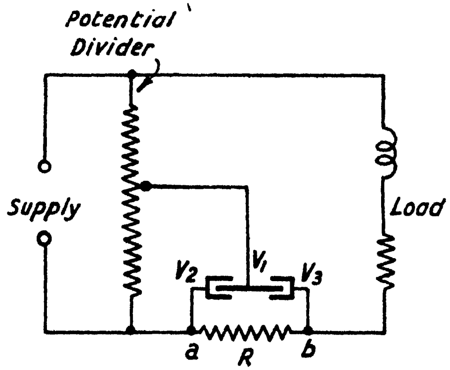


FIG. 62.—Electrostatic Wattmeter—Alternative Connections.

From the diagram of Fig. 61 we see that (V_1-V_3) is the instantaneous value of the voltage applied to the load, which we will call e, and

$$V_1-V_2=(V_1-V_3)+(V_3-V_2)=e+iR$$

Hence, the instantaneous torque

$$\begin{aligned}T&=K[(e+iR)^2-e^2)]\\&=K(2e+iR)(iR)\\&=K(2eiR+i^2R^2)\\&=K2R(ei+\frac{i^2R}{2})\end{aligned}$$

Thus, the instantaneous torque is proportional to the instantaneous power supplied to the load, plus half the power dissipated in the non-inductive resistance R.

There are several methods by which this disadvantage may be overcome, but we will only concern ourselves with two of them. One is illustrated in Fig. 62. In this arrangement a potential divider, consisting of a high value non-inductive resistance, is placed across the supply terminals, and the electrometer needle is connected to its mid-point. The instantaneous potential across the divider will therefore be

$$e+(V_3-V_2)$$

where e is the p.d. across the load.

The potential of the electrometer needle relative to the point a will be

$$\frac{e+(V_3-V_2)}{2}$$

and relative to the point b

$$\frac{e+(V_3-V_2)}{2}-(V_3-V_2)$$

$$\frac{e-(V_3-V_2)}{2}$$

Therefore, the instantaneous torque

$$\begin{aligned}T&=K\left[\left(\frac{e+(V_3-V_2)}{2}\right)^2-\left(\frac{e-(V_3-V_2)}{2}\right)^2\right]\\&=\tfrac{1}{4}K[2e(2V_3-2V_2)]\\&=Ke(V_3-V_2)\\&=KeiR\end{aligned}$$

and the torque is proportional to the true power dissipated in the load.

4.19. Miles Walker's Method. Occasionally, it is not possible to use the mid-point of the potential divider. For example, when the voltage would be too high to apply to the instrument. To overcome such difficulties, the arrangement given in Fig. 63, which is due to Miles Walker, may be employed. In this method a further non-inductive resistance R_1 is connected in series with the load, but on the opposite side of the load to the non-inductive resistance R, and the needle of the electrometer is connected to a convenient point on the potential divider.

Suppose $$\frac{\text{Voltage between } a \text{ and } d}{\text{Voltage between } a \text{ and } c}=x$$

Then the instantaneous torque

$$T=K[(V_1-V_2)^2-(V_1-V_3)^2]$$

but $V_1-V_2=\dfrac{e+iR_1+iR}{x}$

and $V_1-V_3=V_1-V_2-iR$

$$=\frac{e+iR_1+iR}{x}-iR$$

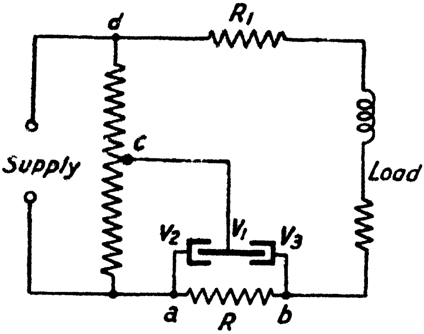


FIG. 63.—Electrostatic Wattmeter
(Miles Walker.)

Therefore, $T=K\left[\left(\dfrac{e+iR_1+iR}{x}\right)^2-\left(\dfrac{e+iR_1+iR}{x}-iR\right)^2\right]$

$$=K\left[2\left(\frac{e+iR_1+iR}{x}-\frac{iR}{2}\right)\left(iR\right)\right]$$

$$=K\left[\frac{2}{x}(eiR+i^2R_1R+i^2R^2)-(i^2R^2)\right]$$

and, by making the value of $R_1=R(x-2)/2$, we have

$$T=K\left[\frac{2}{x}\left(eiR+i^2R^2\frac{x-2}{2}+i^2R^2\right)-i^2R^2\right]$$

$$=K\frac{2}{x}\left(eiR+i^2R^2\frac{x-2}{2}+i^2R^2-\frac{x}{2}i^2R^2\right)$$

$$=K\left(\frac{2}{x}ieR\right)$$

$$T\propto ei$$

In other words, the instantaneous torque is proportional to the true instantaneous power in the load.

In the above argument, the current taken by the electrometer has been neglected, which is sufficiently accurate for general purposes, particularly at low frequencies.

4.20. Calibration of Wattmeters or Watt-hour Meters with Electrostatic Wattmeter. Fig. 64 gives the diagram of connections for the circuit employed by the National Physical Laboratory for wattmeter and watt-hour meter-testing. One great advantage of this wattmeter is that it may be used for circuits with potentials varying from 100 volts to many kilovolts, and currents from less than a milliampère to thousands of ampères. A lantern projects a beam of light upon a mirror attached to the moving system, and the latter reflects the beam on to a scale at a distance of three metres. The scale is in the shape of a horseshoe and is approximately five metres long. The voltage applied to the needle is generally 100 volts, and that between the quadrants in the neighbourhood of 2 volts. In order to obtain the high sensitivity required of the instrument, the quadrants are only 2 mm. apart, and the difference of potential is obtained from the ends of the non-inductive resistance *R*, through which the current passes.

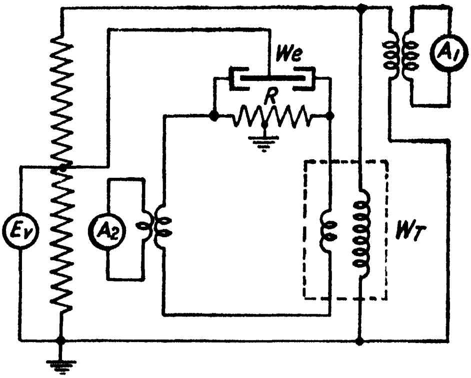


FIG. 64.—Calibration of Wattmeters and Watt-hour Meters with Electrostatic Wattmeter.

When the pressure coil voltage is above 100 volts, the needle of the voltmeter and that of the electrostatic wattmeter are connected to the potential divider, in order to give some convenient ratio of the total voltage in the pressure circuit of the instrument under test.

In order to maintain the instruments at earth potential, one end of the potential divider and the centre point of the non-inductive resistance *R* are connected to earth.

A phantom load is applied to the wattmeter under test, the voltage coil being supplied from one alternator and the current from another, as illustrated in the diagram. The two alternators are mechanically coupled and driven by the same motor.

In Fig. 64, E_v is the electrostatic voltmeter for determination of the applied voltage to the pressure coil of the instrument under test, and also to the electrostatic wattmeter, W_e. W_T is the wattmeter or watt-hour meter under test, whilst the pressure and current circuit alternators are represented by A_1 and A_2 respectively. Adjustment of the phase angle of the current, relative to the voltage applied to the pressure circuit, is obtained by rotation of the stator of one of the alternators, relative to the other. Provision for this is in the form of a worm gear, which permits of the leading or lagging of the stator of the one alternator relative to that of the other. Alternatively, one could employ a phase-shifting transformer, such as the Drysdale-Tinsley type described in section 2.19. It has the advantage of being simpler and less costly than the alternators.

The electrostatic wattmeter is very convenient for polyphase measurements, since it is only necessary to have two similar non-inductive resistances, one in each of two lines, and to change over the connections to the needle and quadrants by means of a selector switch. The total power will then be the summation of the two readings.

4.21. Polyphase Wattmeters. This class of instrument has been developed in order that only one wattmeter need be employed for the measurement of power in a polyphase circuit. It consists of two separate wattmeter elements mounted in the same case, the moving coils being mechanically coupled. Thus, the total deflecting torque is the sum of the torques due to the separate wattmeter elements, and the pointer of the instrument indicates directly the total power in the circuit.

In such an instrument, it is essential that there shall be no interaction between the fixed coils of the one element and the moving coils of the other (see section 11.13) and vice versa. This may be reduced to a negligible amount by means of an astatic coil system, such as is employed in the torsion-head type of electro-dynamometer polyphase wattmeter manufactured by H. Tinsley & Co. Ltd. (section 4.12). In some types of instrument a laminated iron screen is placed between the two elements, thereby providing a magnetic screening.

CHAPTER V

Frequency Meters—Power Factor Meters—Insulation Testers—Recorders

5.1. Measurement of Power Factor, Frequency, Insulation, etc. In this chapter, we will consider the action of instruments employed to determine the frequency and the power factor of alternating current circuits, an instrument for the measurement of the insulation of a circuit and, finally, one which gives a continuous record of the variation of such electrical quantities as voltage, current, power, etc., over a period of time.

5.2. Frequency Meters. There are three main classes of frequency meter and they respectively depend for their operation on:

(*a*) Electrical resonance
(*b*) Mechanical resonance
(*c*) Variation of impedance with frequency.

5.3. Electrical Resonance Type. This type of instrument is illustrated diagrammatically in Fig. 65. It consists of a magnetising coil, wound on a horn-shaped laminated iron core, and energised from the supply voltage. A moving coil is pivoted over the core and the terminals of the coil are connected directly across a condenser. In the vector diagrams I is the current in the magnetising coil, Φ is the flux in the laminated core due to this current, e_c is the e.m.f. induced in the pivoted coil, and i_c is the current in the coil due to e_c.

Diagram (*a*) represents the conditions when the frequency is such that the moving coil circuit is capacitive, the current leading the induced e.m.f. e_c by angle α. The torque acting upon the coil will therefore be proportional to $Ii_c \cos(90-\alpha)$.

Diagram (*b*) illustrates the conditions when the moving coil circuit is inductive. In this case, the torque will be proportional to $Ii_c \cos(90+\beta)$, and since the values of $\cos(90+\beta)$ and $\cos(90-\alpha)$ are opposite in sign, it follows that the torque will be opposite in direction to that when the moving coil circuit is capacitive.

Diagram (*c*) represents the case when the reactance due to the inductance of the moving coil is equal and opposite to the reactance due to the condenser, i.e. when $\omega L = 1/\omega C$, where L is the inductance of the coil in henrys, and C is the capacitance of the condenser in farads.

The current i_c will now be in phase with the induced e.m.f. and the torque will be $Ii_c \cos 90$, which is zero. Therefore, under such conditions, the coil will be in a state of equilibrium.

In the actual instrument, the value of ωL changes with the position of the moving coil on the laminated iron core. In consequence, the coil alters its position on the core until $\omega L = 1/\omega C$ at the particular frequency of the supply. The condenser is chosen to be of such a value that when the frequency is at its normal value, say 50 cycles, the pointer is at the central position upon the scale.

When the laminated iron core is designed so that the inductance changes slowly with change of position of the pivoted coil, great sensitivity can be obtained.

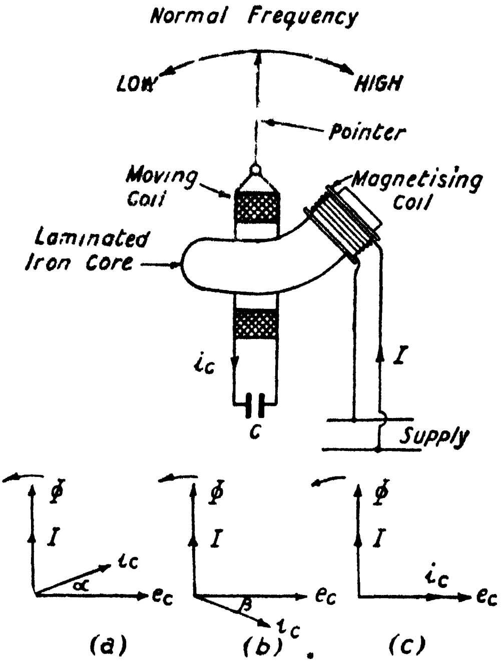


FIG 65 —Frequency Meter-Electrical Resonance Type.

5.4. The Vibrating Reed Type. This instrument depends for its operation upon the phenomenon of mechanical resonance. It possesses a number of thin steel reeds arranged so that they come under the influence of an electro-magnet which is energised by a coil connected across the supply voltage. The reeds are graded according to their natural frequency, in predetermined and equal steps, and the flag of each reed is set in front of a scale which indicates the supply frequency at which each individual reed will resonate. Fig. 66 shows the construction of such an instrument.

The reeds are of slightly different dimensions and are loaded at the remote end from the pivot to give the required natural frequency. This is determined from the expression

$$f = \frac{1}{2\pi}\,\frac{b}{l^2}\sqrt{\frac{E}{\delta(1+4.1K)}}$$

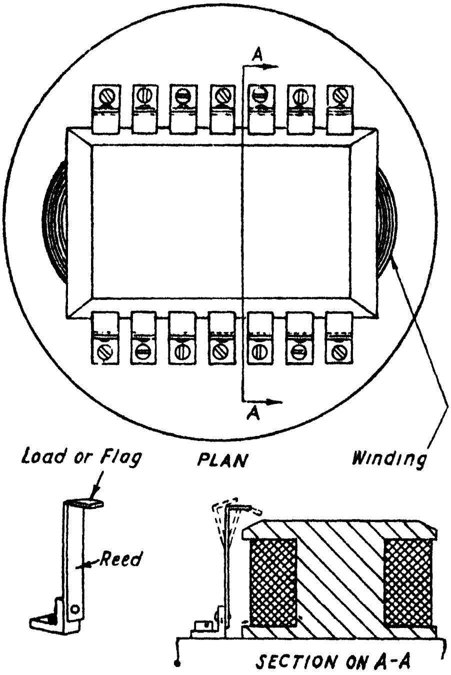


Fig. 66.—Frequency Meter—Vibrating Reed Type

where l is the length of the reed in cm.
b ,, breadth of the reed in cm.
E ,, modulus of elasticity of the material, constituting the reed, in dynes/sq. cm.
δ ,, density of the reed in grm./cu. cm.
K ,, ratio of the mass of the load at the end of the reed to the mass of the reed.

In the case of unpolarised reeds, the electro-magnet attracts the reeds every half-cycle, since they are of a magnetic material. The reeds then all tend to vibrate and the one with a natural frequency twice that of the supply frequency will vibrate most, due to the effect of mechanical resonance.

If the reeds are polarised (i.e. magnetised) they will only be attracted once per cycle, in which case the reed with the same natural frequency as that of the supply voltage will vibrate most. Thus, for either type of instrument, the frequency being measured will have the value indicated on the scale by the flag which is vibrating with the greatest amplitude. The instrument is entirely independent of the wave-form and magnitude of the supply voltage, providing the latter is sufficient to cause reasonable vibrations of the reeds, and will indicate the fundamental frequency of the supply voltage.

5.5. Variable Impedance type of Frequency Meter. This type of instrument depends for its operation upon the variation in the distribution of current between two parallel circuits with change of frequency, when one circuit is resistive and the other is inductive. Such an instrument is manufactured by Nalder Bros. & Thompson Ltd., a diagrammatic arrangement of which is given in Fig. 67. The instrument consists essentially of two parts, one of which is fixed and the other moving. The fixed system includes an external U-shaped electro-magnet *E*, which carries the primary and secondary windings, *P* and *S* respectively. *P* is energised directly, or through a suitable current transformer. On two diametrically opposite arms of the core *C* are mounted two auxiliary arms *l* and *n*. The thin aluminium drum *D* comprises the moving system, and this is free to rotate between the poles of the two electro-magnets *E* and *C*. The rotor is pivoted between

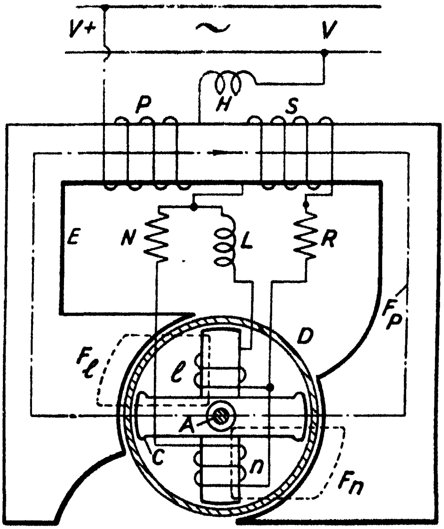


FIG. 67.—Frequency Meter—Variable Impedance Type. (Lipman.)

jewelled centres and carries the pointer and balance weight arm; the weight of the whole movement only being 7 grammes.

The operation of the instrument is as follows:

The primary circuit *P*, which, as previously stated, is energised directly or through a suitable potential transformer, produces the main flux F_p acting upon the rotor in a horizontal direction. An e.m.f. is produced in coil *S*, wound around the main electro-magnet, and this energises the parallel coils *l* and *n*, producing fluxes F_l and F_n respectively, which act upon the drum in a vertical direction. The coils are so wound that in combination with the main flux they produce torques in opposite directions which, at the normal frequency, balance one another when the moving system pointer is at the centre of the scale.

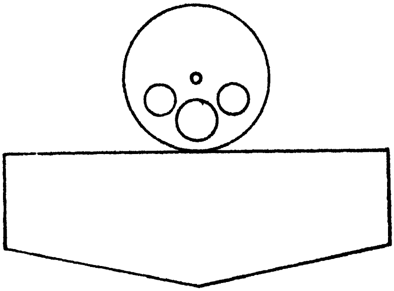


FIG. 68.—Lipman Frequency Meter—Arrangement of Rotor.

An inductance *L* is connected in series with *l*, and a non-inductive resistance *N* is inserted in series with *n*. Thus, an increase of frequency will cause the current in the inductive circuit *Ll* to decrease. At the same time, the current in the circuit *Nn* increases, owing to the special construction of the transformer. This change in current values causes the drum to rotate to the new position of equilibrium, i.e. until the torques are again equal and opposite.

No control spring is employed in this instrument; the rotor being of the special construction illustrated in Fig. 68. A device is also fitted whereby the pointer returns to the off-scale position when the supply is cut off, and the inductance *H* is provided to create a high impedance to the higher harmonics.

5.6. Power Factor Meters. Power factor meters indicate directly the power factor of a load and consist essentially of a current circuit and a potential circuit. Usually, the potential circuit is split into two parallel paths, one of which is inductive and the other non-inductive. The current coil carries a definite proportion of the load current and the deflection of the instrument depends upon the phase angle between this current and the currents in the two parallel paths of the potential circuit, i.e. upon the power factor of the load.

There are two types of power factor meter in general use, these are:

(*a*) The moving-iron type

(*b*) The electro-dynamometer type.

The latter is the older type, but nowadays the moving-iron type is the more generally used.

5.7. Moving-Iron type Power Factor Meter. One such instrument has been developed by C. L. Lipman and is manufactured by Nalder Bros. & Thompson Ltd. A simple diagrammatic arrangement of a three-phase balanced load instrument is given in Fig. 69, in order to illustrate the principle of the meter. In general, the instrument consists of two coils in parallel planes, one each side of the moving system. In the single-phase instrument they are energised by the load current, whilst in the polyphase instrument they are energised by one of the line currents. The moving system, which has the advantage of carrying no moving coils, consists of an appropriate number of magnetic units, each insulated from its neighbour by a non-magnetic material; the combination forming a spindle. Each magnetic system has a thin iron vane mounted at each extremity, and they are set on the spindle 180° apart from one another. All the vanes are mounted in parallel planes, and their axes of symmetry are fixed to correspond with the phase displacements of the respective pressure coil currents which energise each individual system. Thus, in the case of the three-phase balanced load instrument of Fig. 69, there is a displacement of 120° between the axes of symmetry of the vanes in the three magnetic systems. The actual vanes are 120° sectors of circles.

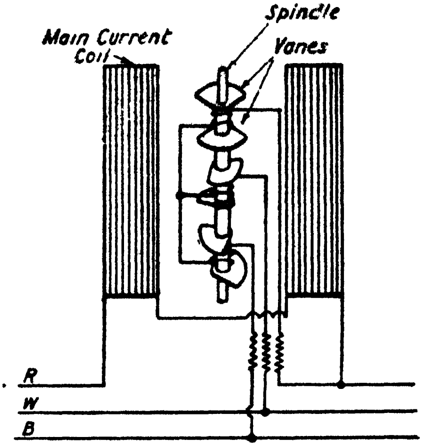


FIG. 69.—Power Factor Meter—Moving Iron Type (Nalder Bros. & Thompson Ltd.)

The magnetic fluxes due to the current coils oscillate to and fro in parallel planes with the common axis of the coils. The fluxes due to the pressure coils oscillate up and down co-axially, magnetising the vanes of the component magnetic portions of the moving system. Thus, a turning moment is developed on the system, due to the interaction of the alternating fields of the current coils and the vanes, the latter being magnetised periodically and in correct sequence by the pressure coils. The moving system will therefore take up a position of neutral equilibrium, depending upon the phase angle of the current in the main coils relative to the supply voltages. Consequently this position is independent of the magnitude of the load.

No controlling force is necessary in this type of instrument, and the damping is obtained by air-friction; four light mica vanes being attached to the spindle for this purpose.

The above type of instrument is made for single-phase and polyphase unbalanced load circuits. The great advantage of the instrument is that by virtue of the pressure coils being at different levels no resultant rotating magnetic field is produced. In consequence, no "drag-torque" is exerted through the induction of eddy currents in the moving system.

Other advantages of this type of instrument are:

(*a*) No moving coils are employed. Hence, such controlling forces as those due to ligaments, for leading in the current to moving coils, are entirely eliminated.

(*b*) A 360° scale is possible, on account of the perfect freedom of rotation of the spindle and its pointer.

(*c*) It indicates accurately on low loads by virtue of the lightness of the moving system.

5.8. The Electro-dynamometer Power Factor Meter. This instrument is really a development of the Tuma phase meter and consists of two fixed coils (Fig. 70) carrying the load current, or a definite proportion of it, and two further coils which are pivoted within the magnetic field of the two fixed coils. The two latter coils, which are rigidly fixed 90° apart, are connected across the load. The one coil has a non-inductive resistance R in series with it, whilst the other is connected in series with an inductance L. The number of turns and the area of the two pivoted coils are identical. Furthermore, the value of R and L are such that the same value of current, but at 90° phase difference, flows in each coil at the normal frequency. At other frequencies, however, the currents will be different in the inductive and non-inductive circuits for similar reasons to those given in section 5.5.

Actually, the current in the inductive coil will not be quite 90° behind that in the resistance circuit, since the former circuit will include a slight resistive component and the latter a slight inductive component. However, for our purpose, we will consider that they are exactly 90° apart. We will also assume that the fixed coils are large compared with the two moving coils, thereby giving a sensibly uniform magnetic field in the neighbourhood of the moving coils.

When the coils are energised, and the angle of the load current relative to the applied voltage is θ, the turning moment at any instant on the coil in the non-inductive circuit will be

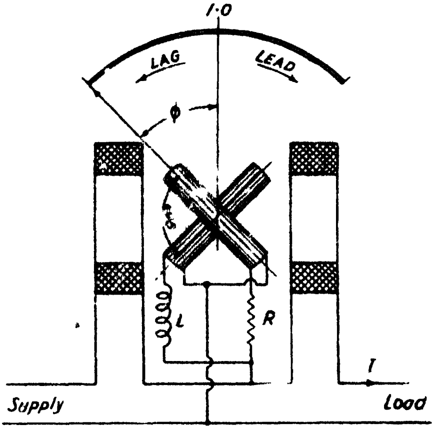


FIG. 70.—Power Factor Meter—Single-phase Electro-dynamometer.

$$T_N = K_1\left[I \sin(\omega t - \theta)\frac{E}{R}\sin\omega t\right]\sin\phi$$

where I is the main current and E is the applied voltage to the pressure circuit terminals.

The turning moment on the coil in the inductive circuit will be

$$T_L = K_2\left[I \sin(\omega t - \theta)\frac{E}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)\right]\sin\left(\phi + \frac{\pi}{2}\right)$$

These turning moments tend to turn the moving system in opposite directions: hence, the system will be in equilibrium when the torques are equal, i.e. when

$$\frac{K_1 IE.}{R}\sin\phi.\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\sin(\omega t - \theta)\sin\omega t.dt$$

$$= \frac{K_2 IE}{\omega L}.\cos\phi.\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\sin(\omega t - \theta)\sin\left(\left(\omega t - \frac{\pi}{2}\right)\right)dt.$$

where K_1 and K_2 are winding constants of the coils

$$\text{Therefore } \frac{K_1}{R} \cdot \sin \phi \int_0^{\frac{2\pi}{\omega}} \cos \theta - \cos (2\omega t - \theta) dt$$

$$= \frac{K_2}{\omega L} \cos \phi \int_0^{\frac{2\pi}{\omega}} \cos\left(\frac{\pi}{2} - \theta\right) - \cos \left(2\omega t - \frac{\pi}{2} - \theta\right) dt$$

$$\frac{K_1}{R} \cdot \sin \phi \left[t \cos \theta - \frac{\sin (2\omega t - \theta)}{2\omega} \right]_0^{\frac{2\pi}{\omega}}$$

$$= \frac{K_2}{\omega L} \cos \phi \left[t \sin \theta - \frac{\sin\left(2\omega t - \frac{\pi}{2} - \theta\right)}{2\omega} \right]_0^{\frac{2\pi}{\omega}}$$

$$\frac{K_1}{R} \sin \phi \left[\frac{2\pi}{\omega} \cos \theta \right] = \frac{K_2}{\omega L} \cos \phi \left[\frac{2\pi}{\omega} \sin \theta \right]$$

$$\text{Therefore } \frac{K_1}{R} \tan \phi = \frac{K_2}{\omega L} \tan \theta$$

Thus, by suitably choosing the values so that $K_1/R = K_2/\omega L$

$$\phi = \theta$$

and the instrument reads directly the phase angle of the current relative to the supply voltage. The power factor will be the cosine of this angle.

When the above type of instrument is made for single-phase circuits, it is essential that it be used at the normal frequency for which it was constructed, since any departure from this frequency will alter both the phase and magnitude of the current in the inductive circuit coil, thereby rendering the readings inaccurate. For this reason single-phase power factor meters of this type are not in common use.

5.9. Polyphase Electro-dynamometer Power Factor Meter. The polyphase balanced load type of power factor meter has the two main current coils connected in one of the lines, and the moving coils are connected across two different phases of the supply circuit. In this case, there is no necessity to employ a phase-splitting device, since the voltages applied to the moving coils will be 120° apart. The coils are therefore fixed at 120° apart relative to one another, instead of 90°—as in the single-phase case. Such an instrument is illustrated in Fig. 71.

The torque equation can be derived in a similar manner to that of the single-phase instrument. The polyphase instrument has the advantage of being independent of wave-form and frequency, since the two moving coils have exactly the same characteristics and their currents are affected similarly by change of frequency. The readings of the polyphase instrument, however, are only correct when the load is balanced.

No controlling torque is necessary for the electro-dynamometer power factor meter, since the moving system finds its position of equilibrium and any departure from this position gives rise to an out-of-balance torque which causes it to take up its correct setting.

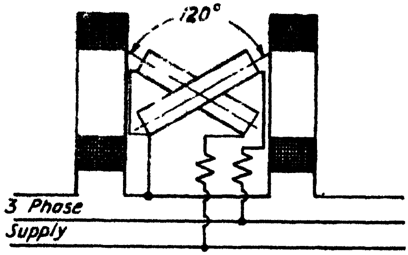


FIG. 71.—Power Factor Meter—Three-phase Electro-dynamometer.

5.10. Measurement of Insulation Resistance. As the insulation resistance of an electrical circuit must always be sufficient to avoid the possibility of a leakage to earth, or between any two parts of the circuit which are at a different potential, it is important to possess some convenient instrument, preferably portable, for high resistance measurement. It is also desirable that the instrument be direct reading and simple to operate, in order that it may be used by persons not necessarily accustomed to the handling of delicate instruments. Furthermore, it must be capable of supplying a sufficiently high testing voltage for the location of high resistance leakage paths. Such an instrument has been developed by Evershed & Vignoles Ltd.

5.11. Megger Insulation Testing Set—Evershed & Vignoles Ltd. A diagrammatic illustration of this instrument appears in Fig. 72. The instrument consists of a hand-driven generator and a direct reading ohmmeter. The generator, which produces the testing voltage, is of the two-pole permanent magnet type; the armature shaft rotating between roller bearings. The armature is turned by means of the handle illustrated in the diagram, and a free-wheel device is provided to ensure that the armature will only rotate in a clockwise direction. The free-wheel also has the advantage of allowing the armature to come to rest slowly when the handle is stopped abruptly. The ohmmeter, which measures the value of the insulation resistance, is of the moving-coil type.

There are two coils connected in parallel across the generator. One is in series with the fixed control resistance and the other is in series with the resistance under test; the former being termed the control coil and the latter the deflecting coil. The coils are mounted at a fixed angle relative to one another and are free to rotate in the field of the

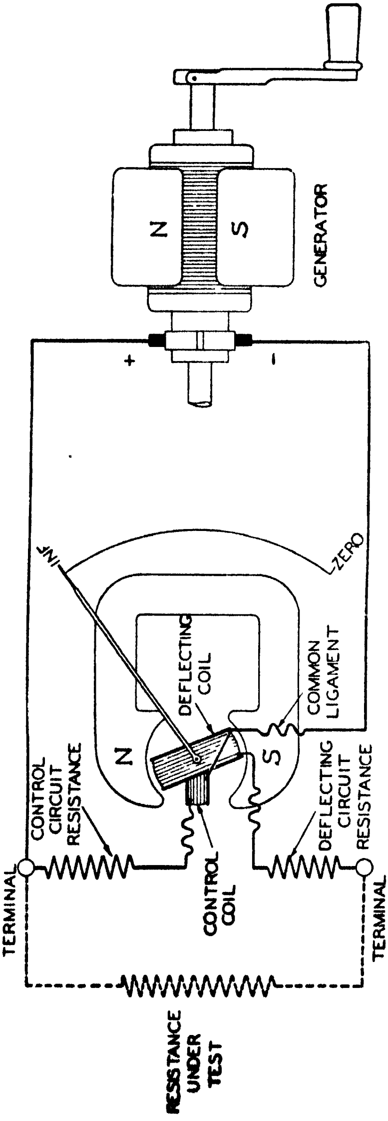


Fig. 72.—Diagram showing principle of operation of Megger Testing Set (Evershed & Vignoles Ltd.)

permanent magnet *NS*. They are so arranged that when a current is flowing through the control coil, i.e. when the line and earth terminals are not connected to any circuit or resistance under test, the moving system will come to rest at the "infinity" position of the scale, while

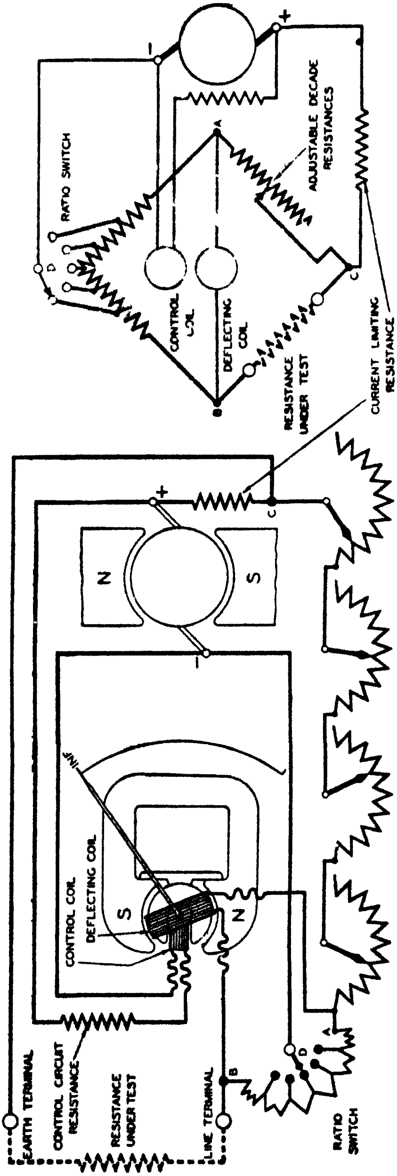


FIG. 73 —Schematic Diagram, Bridge setting, "Bridge-Meg" Testing Set (Evershed & Vignoles Ltd)

a current flowing at the same time through the deflecting coil will cause the pointer to move down the scale towards "zero". The instrument therefore measures the ratio of the torques exerted by the two coils, and this will only depend upon the value of the resistance under test, since variation in the pressure generated—due to variation of handle speed—affects both coils in the same proportion. Hence, there will be a different position of the pointer for every value of resistance between zero and infinity, the latter being the position of the moving system when the ohmmeter terminals are on open circuit. The dials of such instruments are individually calibrated.

Evershed & Vignoles Ltd. manufacture several models of instrument in their "megger" series. Some include the constant pressure clutch. This is a centrifugal clutch in the driving gear of the generator, and was introduced to eliminate errors due to capacity in the circuits under test. Providing the handle is turned in excess of the slipping speed of the centrifugal clutch, the generator will run at a constant speed, and once the capacity has been charged the reading of the instrument will remain constant. Below this speed, the pointer may wander owing to variation of current in the deflecting coil, due to capacity.

One very useful instrument in the "megger" series is the "Bridge-Meg" testing set, which combines the functions of an insulation tester with those of a Wheatstone bridge. By the operation of a single switch the instrument can be used for either insulation or circuit resistance measurements. When used for the former purpose, the instrument is exactly similar to the "Megger" described above. When in use as a bridge, the generator forms the source of supply and the ohmmeter acts as a galvanometer.

Fig. 73 shows the connections and schematic diagram of the instrument when set to operate as a Wheatstone bridge. The two diagrams are correspondingly lettered for the points *A*, *B*, *C*, and *D*. The two ratio arms and the adjustable decade resistances form three arms of the bridge, the resistance to be measured forming the remaining arm. The deflecting coil of the ohmmeter serves as the out-of-balance detector, or galvanometer, and the controlling force is provided by the control coil, which is independently supplied from the generator terminals.

When the bridge system has been balanced, by correct adjustment of the ratio arms and decade resistances, no current will flow through the deflecting coil. In consequence, the control coil brings the moving system into the position at which the pointer comes to rest at the "infinity" mark on the scale .

5.12. Recording Instruments This type of instrument is one which automatically records its indications upon a uniformly moving chart of paper. In this manner a continuous and permanent record of the electrical quantity, for which the instrument is adapted to measure, is obtained. Such an instrument, therefore, gives a record which is extremely useful in providing accurate data which by observation of an indicating instrument would be almost impossible to obtain under conditions where the electrical quantity fluctuated rapidly. For instance, the records obtained with a recording wattmeter show a consumer's power consumption throughout the day and are extremely useful in the determination of a suitable tariff.

The moving system of such an instrument is similar in construction to its indicating counterpart. In place of the pointer, a light arm is attached to the moving system. This arm carries a small pen at its extremity, which rests lightly upon a chart travelling at a predetermined speed, from one drum to another in a direction perpendicular to the deflection of the pen. Thus, the length of the chart is a time-base, and the inked pen gives a continuous trace of the variations of deflection of the instrument. Therefore, as the deflection is proportional to some electrical quantity, it gives a continuous record of the variation of this quantity with respect to time.

It follows that increased inertia will be introduced into the moving system due to the greater weight and increased moment of inertia. There will also be an increase of friction due to the contact of the pen with the chart. The design of a normal indicating instrument, therefore, must be modified if the instrument is to be used for recording purposes.

First of all, the deflecting torque of the instrument must be increased, and consequently the controlling force, in proportion to the increased friction torque. Also, as the inertia of the system has to be increased, greater damping is necessary. Mechanical considerations, such as the size of the bearings necessary for the greater load, must also be taken into account.

The chart-driving mechanism must be capable of creating uniform speed of travel of the chart under the pen. In general, this consists of a good quality spring-driven, or synchronous motor, clock, suitably geared to a revolving metal drum having projecting pins which engage in perforations at the edges of the record chart. Thus, the speed of the chart is kept the same as that of the drum, and the chart remains in alignment-

The damping may be magnetic, or in the form of an oil dashpot; the former having the advantage of being free from the "messiness" of oil and giving a true response free from creep.

CHAPTER VI

Instrument Shunts, Voltage Multipliers, and Transformers

We will consider, in this chapter, the methods employed for increasing the range of an instrument, and the calibration of apparatus employed for this purpose.

6.1. Shunts. A low resistance shunt is frequently placed in parallel with a d.c. instrument in order to increase the current range of the latter. For instance, in the case of a permanent magnet moving-coil instrument, it is always used with a shunt when the current to be measured is in excess of a few milliampères.

We will only deal briefly with shunts for this type of instrument. Since the current flowing through the moving coil is very small, any thermal e.m.f.s set up at the junctions of the shunt will seriously affect the accuracy of the instrument, and also its zero when the current is switched off and the shunt is still warm. In view of this, it is advisable to choose an alloy with a low temperature coefficient and a low thermo-e.m.f. to copper.

The effect of these thermo-e.m.f.s will be accentuated on account of the Peltier effect, or unequal heating of the two junctions, due to the fact that the current is in the same direction as the thermo-e.m.f. at one junction and in the opposite direction at the other junction, thereby causing the junctions to be at different temperatures.

The simplest method to overcome the above difficulties is to use manganin, which combines an extremely low temperature coefficient with a low thermo-e.m.f. (about 1·4 microvolts per 1° C.) with copper. To overcome the readiness of manganin to oxidise, it is usually annealed.

The disadvantage of the copper-nickel alloys, for use in shunts, is their high thermo-e.m.f.s to copper (usually in the region of 40 microvolts per 1° C.).

In order that a shunt will be satisfactory, its essential requirements are:

(*a*) Low temperature coefficient.
(*b*) Sufficient cooling surface for the radiation of heat.
(*c*) Simple and permanent construction.
(*d*) Low or compensated thermo-e.m.f. and Peltier effect.
(*e*) The current and potential terminals must be well designed.

We have already discussed the advantages of shunts when employed in conjunction with induction ammeters, on account of their giving some measure of compensation for change of frequency (section 3.21).

Let R_M be the resistance of the ammeter, I_M the current for maximum deflection, and I the current which is required to pass through the main circuit when I_M is passing through the shunted meter; the ratio of I to I_M being n. Then the current passing through the shunt will be

$$I-I_M$$

Therefore, as the potential drop across the meter and its leads must be the same as that across the shunt,

$$(I-I_M)\,R_S=I_M R_M$$

where R_S is the resistance of the shunt.

$$\text{Hence, } R_S=\left(\frac{I_M}{I-I_M}\right)R_M$$

$$\left(\frac{1}{n-1}\right)R_M$$

If the meter reading is multiplied by the factor n, we shall have the value of the total current in the circuit.

When the shunt is to be used with an ammeter for alternating current measurements, it is essential that the impedances of the shunt and meter bear the same ratio at all frequencies for the current distribution to remain constant. In other words, their inductances must bear the same ratio to one another as do their resistances. Thus, if L_M is the inductance of the meter, and L_S that of the shunt,

$$R_S+j\omega L_S=\frac{1}{n-1}\left(R_M+j\omega L_M\right)$$

which will only occur when $L_S=\dfrac{L_M}{n-1}$

and the current ratio $\dfrac{I_S}{I_M}=\dfrac{L_M}{L_S}=\dfrac{R_M}{R_S}$

from which $n=\dfrac{R_M+R_S}{R_S}=1+\dfrac{R_M}{R_S}=1+\dfrac{L_M}{L_S}$

6.2. Shunts for Integrating Meters. It is also the common practice to shunt d.c. integrating meters of the mercury motor type, when the rated maximum is greater than 10 ampères. Provided the shunt is well designed and there are no troubles due to bad contacts at the shunt potential terminals, such a practice has the following advantages:

(*a*) Smaller and lighter armatures may be used, thereby making it possible to reduce the width of the magnet gap. As a consequence, the magnetic field can be strengthened, resulting in a reduction in speed of the disc and less wear on the pivots and bearings.

(*b*) The temperature coefficient of the meter can be reduced.

(*c*) The shunt tends to protect the meter against momentary overload.

Meters fitted with external shunts also have the following advantages:

(*a*) The cost of installation is reduced, since the heavy cables need not be brought to the meter. This is a very great advantage when the meter cannot be fixed in the immediate vicinity of the circuit.

(*b*) The meter may be removed at any time for examination or overhaul, without in any way interrupting the main circuit supply.

(*c*) Only one current size of meter need be made by the manufacturer, the counting train alone being modified to suit the shunt. The advantages of this are obvious, since it decreases the cost and simplifies production.

It is essential that the shunts have a negligible temperature coefficient, such as that of manganin, German silver, or constantan, in order that their resistance will not vary with change of ambient temperature, or due to self-heating. Furthermore, since at high currents a considerable expenditure of power takes place in the shunt, it must so be constructed that the cooling surface is sufficiently large to avoid risk of injury to the shunt. For this reason, shunts are frequently constructed in the form of laminations, parallel rods, or parallel tubes, so spaced that there is plenty of ventilation for cooling. Massive copper blocks are provided at their extremities for the current to enter or leave the resistance plates. Suitable attachments are made for the cables and these are so designed that there is, as near as possible, an even distribution of current over the plates.

Suppose the resistance of the meter be R_M, that of its leads R_L, and the shunt resistance R_S. If we wish the shunt to increase the maximum range of the instrument to n times its self-contained value,

$$\text{then } I_S = nI_M - I_M$$
$$= (n-1)I_M$$

where I_M is the maximum rated current flowing through the meter, and I_S is the current flowing through the shunt.

Now, since the volt drop at the shunt potential terminals must be the same as for the instrument plus its leads,

$$I_M(R_M + R_L) = I_S R_S$$
$$= (n-1)I_M R_S$$

$$\text{Therefore } R_M + R_L = (n-1)R_S$$

and the resistance of the shunt

$$R_S = \frac{R_M + R_L}{n-1}$$

Alternatively, if a current I is flowing through the circuit, the meter current

$$I_M = I\frac{R_S}{R_M + R_L + R_S}\,.$$

From the above expression one very important point is obvious, and should be mentioned at this stage. When a meter and shunt have been calibrated with certain leads, these leads should under no circumstances be shortened, lengthened, or changed, without subsequent re-calibration of the instrument, since any alteration in the value of R_L will affect the current distribution between the meter and the shunt. Several cases of erroneous registration have come to the notice of the writer where it has been found that the length of the meter leads has been altered, on account of the meter being placed nearer or farther away from the shunt than was originally intended.

6.3. Testing of Shunts. There are several methods of testing shunts. One is by means of a Kelvin double bridge, another is with a d.c. potentiometer. In the latter case, a standard shunt and the shunt under test are placed in series, and the millivolt drop of each is measured in turn, the current being kept constant for both measurements. Thus, the resistance of the shunt under test

$$R_T - R_S \frac{V_T}{V_S}$$

where R_T is the resistance of the shunt under test
R_S ,, resistance of the standard shunt
V_T ,, millivolt drop of the shunt under test
V_S ,, millivolt drop of the standard shunt.

The shunt may also be tested in the following manner, where one has heavy current facilities.

If I_M is the maximum meter current and I is the maximum rated current of the shunt and meter, then the current $I - I_M$ is passed through the shunt and the millivolt drop between the potential terminals of the shunt is measured. Manufacturers often stamp this on the copper end of the shunt. This may be, say, 100 millivolts. The full load meter current is then passed through the meter and its leads and the millivolt drop is measured. The resistance of the leads is then adjusted until the total millivolt drop is 100 millivolts. Thus, when the meter is connected in parallel with its shunt, the correct current distribution will be assured. A great advantage of this method is that, when the meter is returned for examination or overhaul, it is only necessary to know the millivolt drop stamped upon the shunt. It is then unnecessary to disconnect the shunt from the circuit and the meter can be calibrated as a self-contained unit; the resistance of the leads or meter finally being adjusted so that the full load meter current flowing through the meter and leads gives the required millivolt drop. Extra provision for such an adjustment is usually fitted inside the meter case.

The leads to the shunt must have a sufficient contact area to avoid overheating, and the contact surfaces must be perfectly clean when the joints are made, since a bad contact is virtually an increase in the resistance of the instrument leads.

6.4. Voltage Multipliers. The range of a voltmeter can also be increased by connecting a non-inductive resistance in series with the instrument.

If the current taken is I ampères, when the latter is giving full-scale deflection, and the resistance of the voltmeter is R_M, it follows that the volt drop across the voltmeter terminals is IR_M.

Now, suppose a resistance R_S be connected in series with the meter. When full-scale deflection is again registered the applied voltage E will be given by

$$E=I(R_M+R_S)$$

Therefore, for the applied voltage to be n times its initial value, in order to cause a full-scale deflection of the instrument (i.e. pass I ampères)

$$\begin{aligned} nIR_M &= E \\ &= I(R_M+R_S) \\ \text{and } nR_M &= (R_M+R_S) \\ \text{Therefore } R_S &= (n-1)R_M \end{aligned}$$

It follows that the essential requirement of such a multiplying resistance is that it will remain constant in value. The temperature coefficient must therefore be very small and sufficient provision for cooling must be made, since it will dissipate an appreciable amount of power ($[n-1]$ times that of the meter).

When used for alternating current measurements, the total impedance must remain as nearly constant as possible for all frequencies. Voltage multipliers, therefore, should be as non-inductive as possible and, in view of this, they are often non-inductively wound upon flat strips of mica.

Suppose R_M is the voltmeter resistance, L_M is the voltmeter inductance, and R_S is the resistance of the multiplier, then, for full-scale deflection of the instrument, when it is directly connected,

$$E=I\sqrt{R^2_M+\omega^2L^2_M}$$

When the multiplier is included

$$E_S=I\sqrt{(R_M+R_S)^2+\omega^2L^2_M}$$

Therefore, the multiplying ratio

$$\frac{E_S}{E}=\frac{\sqrt{(R_M+R_S)^2+\omega^2L^2_M}}{\sqrt{R^2_M+\omega^2L^2_M}}$$

which illustrates that, if the inductance of the instrument be neglected in the calculations, an appreciable error might ensue.

6.5. Multi-range Test Sets. Several manufacturers have marketed instruments with internal shunts and multipliers so arranged that measurements over wide ranges of current, voltage, and resistance can be made in both a.c. and d.c. circuits by simple manipulation of various selection switches on the control panel. The instrument is usually of the permanent-magnet moving-coil type and the a.c. measurements are obtained by means of a transformer and rectifier. Such an instrument is extremely useful to the meter engineer who may wish to make measurements on site. This normally would require several instruments or, in other words, a miniature test-room within his reach. They are therefore made very compact and portable.

6.6. Century Set—Elliott Bros. (London) Ltd. One instrument of the multi-range type is the "Century" set manufactured by Elliott Bros. (London) Ltd. The instrument can be supplied with substandard or first-grade accuracy and is for d.c. measurements only. The voltage ranges have provision for any measurements between 0·5 millivolt and 750 volts; a special form of selector switch being fitted to facilitate the changing of the range. The series resistances are housed in the instrument case. For the current ranges, two small shunts are carried in the lid of the box, and two larger shunts, together with the flexible leads, are contained in the wooden tray which is fitted to the bottom of the case.

Certain other models of testing sets, also manufactured by the above company, are suitable for a.c. and d.c. measurements.

6.7. "A.C. Test" Instrument—Crompton Parkinson Ltd. A multi-range instrument of many and varied applications is the "A.C. Test", manufactured by Crompton Parkinson Ltd. It is a moving-iron instrument of the iron-cored type, and differs from the conventional types previously described (section 3.16-18) inasmuch as no winding is provided. The conductor which carries the current to be measured is embraced by an almost closed laminated iron circuit, which is magnetised by the field due to the current in the conductor. An air gap is provided in the iron circuit, and this contains a moving-iron system which depends for its position upon the amount of magnetic flux in the iron circuit. In other words, its position depends upon the magnitude of the current passing through the conductor.

The core (Fig. 74) consists of a number of iron stampings of a similar shape to those of a ring-type transformer. It is modified, as already stated, by means of the circular air gap *A* and also by the two butt joints *BB*, the former being to create the poles of an electro-magnet and the latter to permit of the entry of a conductor, or conductors, in the iron ring without breaking the electrical circuit.

The moving portion consists of two curved irons spaced diametrically opposite one another. Their radius of curvature is slightly less than that of the air gap; the air gap and movement being co-axial. The movement is spring-controlled and the damping is pneumatic.

As we have already seen (section 3.16-18), any magnetic flux produced in the air gap of such a system would tend to draw the moving iron into the field of maximum intensity. The reluctance of the butt joints has little effect upon the accuracy of such an instrument since it will be very small in comparison with that of the cylindrical gap *A*.

The instrument, since it is of the moving iron type and has no transformer, will also indicate in d.c. circuits. Its accuracy, however, will be impaired on account of the hysteresis which is associated with a magnetic circuit containing so much iron. It is therefore advisable to make any d.c. tests by closing the magnetic circuit over a live conductor or switching on the full energising current in one step: the readings should not be made when increasing the current slowly or in small increments.

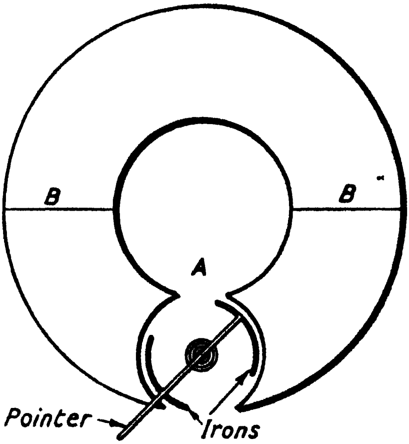


FIG. 74.—Iron Circuit of "A.C. Test" Instrument
(Crompton Parkinson Ltd.)

The instrument is manufactured in various ranges between 0/100 and 0/300 ampères. A full-scale reading can, of course, be got for a smaller current than that of the maximum range by threading several turns of the conductor through the iron arch of the instrument. Thus, 20 ampères can be made to give a full-scale reading on a 0/100-ampère instrument by threading the conductor five times through the orifice of the iron circuit.

In order to extend the application of the instrument, by making it into a voltmeter, an attachment is provided for clamping around the iron core. In the case of the 0/100-ampère instrument, this comprises a sufficient number of turns to permit of full-scale reading when

75 milliampères is flowing through its windings and, with the swamping resistance, it requires 100 volts across its terminals for this purpose. A resistance box is also provided, for connecting in series with the volt attachment, as a multiplier for extending the voltage range (section 6·4). Probably the 0/120-ampère instrument has the most useful selection of ranges with 0/120, 0/480—bringing 240 to the best part of the scale—and 0/600 volts.

Such an instrument has many and novel applications apart from the measurement of voltage and current. One such application, due to R. G. Isaacs, is the determination of power and power factor. The conductor carrying the current in the circuit is passed one or more times through the orifice of the laminated iron core and the reading (R_1) of the instrument is noted. The potential is then also applied to the volt attachment and resistance box and the new reading (R_2) is also noted. By means of a reversing switch, the current in the volt attachment is reversed and the further reading (R_3) is taken. Finally, the conductor carrying the circuit current is removed from the arch of the instrument and the reading (R_4) due to the potential winding alone, is noted.

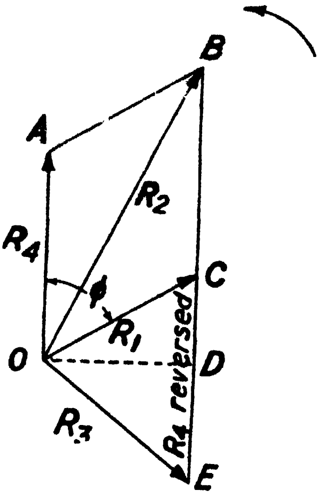


FIG. 75.—Vector Diagram—Determination of Power and Power Factor, "A.C. Test" Inst. (Crompton Parkinson Ltd.)

It should be mentioned that for the power factor test the indications are relative and not quantitative. Thus, it is advantageous to bring the reading of R_1 near the middle of the scale. This may be attained either by increasing the turns or introducing, by means of spacers, a small gap in the magnetic circuit, thereby increasing the reluctance and decreasing the indication. The 600-volt range should be used for R_2 and R_3, otherwise the pointer might go off scale.

Now, R_1 is the reading due to the ampère-turns of the current winding, R_2 is the vector sum of the ampère-turns in the current and potential windings, R_3 is that due to the vector difference of the ampère-turns in the current and potential windings, and R_4 is due to the ampère-turns in the potential windings only. From the vector diagram of Fig. 75, representing these conditions, we see that cos ϕ will be the power factor of the load, since ϕ is the angle between the voltage and the current.

$$\text{Thus, } \cos\phi=\frac{R_1{}^2+R_4{}^2-R_3{}^2}{2R_1R_4}$$

The above expression may be modified in order to eliminate R_4, by giving the latter in terms of R_1, R_2, and R_3.

Let $BD=x$, $DE=y$, $OD=z$. Then $BC=CE=\frac{1}{2}(x+y)$

and $CD=\frac{1}{2}(x+y)-y=\frac{1}{2}(x-y)$

In terms of x, y, and z,

$$R_1{}^2=\frac{(x-y)^2}{4}+z^2,\ R_2{}^2=x^2+z^2,\ R_3{}^2=y^2+z^2,\ R_4{}^2=\frac{(x+y)^2}{4}$$

Therefore,

$$\begin{aligned}R_1{}^2+R_4{}^2-R_3{}^2&=\frac{(x-y)^2+4z^2+(x+y)^2-4y^2-4z^2}{4}\\&=\frac{2x^2-2y^2}{4}\\&=\frac{x^2-y^2}{2}\\&=\frac{(x^2+z^2)-(y^2+z^2)}{2}\\&=\frac{R_2{}^2-R_3{}^2}{2}\end{aligned}$$

$$\begin{aligned}\text{and } R_4&=\sqrt{\frac{(x+y)^2}{4}}\\&=\sqrt{\frac{(2x^2+2z^2)+(2y^2+2z^2)}{4}\left(-z^2+\frac{(x-y)^2}{4}\right)}\\&=\sqrt{\frac{R_2{}^2+R_3{}^2}{2}-R_1{}^2}\end{aligned}$$

$$\text{Therefore, } \cos\phi = \frac{R_1^2+R_4^2-R_3^2}{2R_1R_4}$$

$$= \frac{R_2^2-R_3^2}{4R_1R_4}$$

$$\text{or} \quad \frac{R_2^2-R_3^2}{4R_1\sqrt{\dfrac{R_2^2+R_3^2}{2}-R_1^2}}$$

Since the actual current and potential can be determined from the readings of R_1 and R_4, it is a simple matter to calculate the power for either a single-phase or polyphase system.

Other uses for this type of instrument include the checking of the ratio of a current transformer by means of differential windings connected to the primary and secondary windings of the transformer under test (see the article by R. G. Isaacs, p. 272, *Mining Electrical Engineer*, February 1931). It is also suitable for the location of cable faults.

Crompton Parkinson have also produced another instrument, similar in principle to the above, termed a "Tong Test". It has been designed with a view to its being employed in awkward places where it is not possible to use the "A.C. Test" instrument, i.e. for fuses in feeder pillars, etc. The iron circuit, as the name "Tong Test" suggests, is in the form of a pair of tongs and these are actuated by a lever which opens the butt joint sufficiently to permit of the passing of the conductor into position. The butt joint is then closed and the measurement made.

One interesting point about the above instrument is that the moving system is detachable from the iron circuit by means of the operation of a pair of clips, and permits of the replacement of the moving system by one of a more suitable range for the current to be measured. Apart from the convenience of the "Tong Test" for the measurement of current in conductors situated in awkward positions, the "A.C. Test" instrument is considered the more generally useful.

6.8. Instrument Transformers. As the range of self-contained a.c. instruments is limited, it is generally increased by the use of either current or potential transformers, or in some cases by both.

Instrument transformers have many advantages, the most important being:

(*a*) Ammeters and voltmeters for use with transformers may be standardised at 5 ampères and 110 volts respectively.

(*b*) The single-range instrument may be used to cover a large current or voltage range (in the case of a wattmeter or watt-hour meter, it may cover both a large current and voltage range) with a multi-range transformer or several single-range transformers.

(*c*) In the case of instruments measuring high-tension voltages or loads, the secondary winding of the potential transformer is isolated from the high potential and the instrument need only be insulated for 110 volts a.c. This ensures greater safety for the operators, providing the secondary windings of all current and potential transformers in H.T. circuits are earthed. Such a procedure is most important.

(*d*) The current flowing in the bus-bar, or any other conductor, can be measured by means of a current transformer of the split-core type, without interrupting the main circuit current.

6.9. Split-core type Transformer—Price & Belsham Ltd. A transformer of the split-core type, manufactured by Price and Belsham Ltd., is illustrated in Fig. 76. The laminated iron core is in two parts, one of which carries the secondary winding. To assemble the transformer round the conductor, it is only necessary to give a half-turn to the core lever, which releases the top of the core, push back the flat spring clip, place the transformer in position, and reassemble.

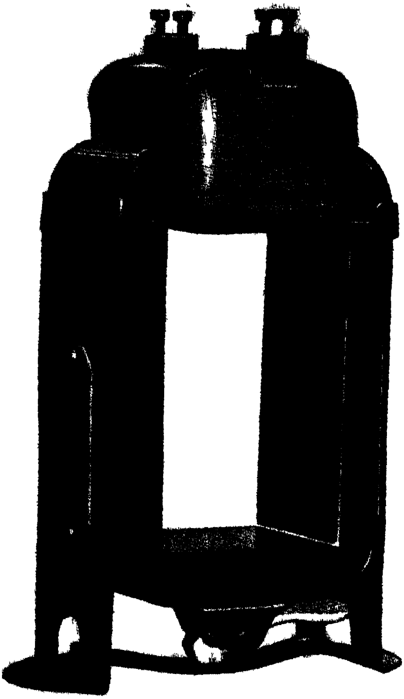


FIG. 76.—Split-core type Current Transformer (Price & Belsham Ltd.)

6.10. Tong type Current Transformer. This type of transformer has been designed for use where it is not possible to open the circuit of which one wishes to measure the load. As the name implies, it is in the shape of a pair of tongs, and the iron circuit opens and slips over the conductor, the secondary winding being connected to an indicating instrument whilst the conductor carrying the load acts as the primary. One such instrument, due to G. Shotter, has been specially adapted for measuring the current passing through fuses in feeder pillars, where space is limited and it is not permissible to open the circuit. The iron circuit consists of a hinged iron section at one end of which is a butt joint; the other end having an air gap.

In order to make a test, the handles are pressed inwards, thus overcoming the pressure of the spring and permitting the butt joint to open. The iron section is then passed over the fuse and the pressure on the handles is discontinued. The spring thereupon takes control and closes the butt joint. The subsequent readings indicated on the instrument are almost independent of the position of the fuse within the iron section.

The indicating instrument, which is of the moving-coil permanent-magnet type, is connected in a bridge circuit employing Westinghouse metal-oxide rectifiers (section 3.10).

There are two ranges to the instrument, one of 100 ampères maximum and the other 400 ampères maximum. As the readings can be relied upon down to one-fifth of full scale, it follows that accurate measurements can be made for any currents within the limits of 20 ampères and 400 ampères. Change of range is provided for by terminals upon the indicating meter; the transformer being of the single-ratio type. The latter is only suitable for operating at 50 cycles, but is insulated for use in any circuits where the working voltage does not exceed 750 volts.

6.11. Transformers. A transformer consists essentially of two insulated coils surrounding a common magnetic circuit, and is supplied with four terminals—two for the extremities of each winding. Such a circuit is represented diagrammatically in Fig. 77.

Alternating current is supplied to one of the windings, say *PP*, and this winding is termed the primary. An alternating flux is therefore produced in the magnetic circuit and this flux links with the other winding *SS*, which is termed the secondary. The alternations of the flux induce an alternating e.m.f. in the secondary winding, of the same frequency as that of the alternating current supplied to *PP*.

If we neglect the small effect due to the resistance of the primary winding, then the whole of the applied voltage is used in overcoming the self-induced e.m.f. of the primary winding created by the flux in the magnetic circuit. This self-induced e.m.f. will vary as the maximum flux, the number of turns, and the frequency (since the average rate of change is directly proportional to the frequency). In exactly the same way, the induced e.m.f. in the secondary winding will vary as the maximum flux, the number of turns of the secondary, and the frequency.

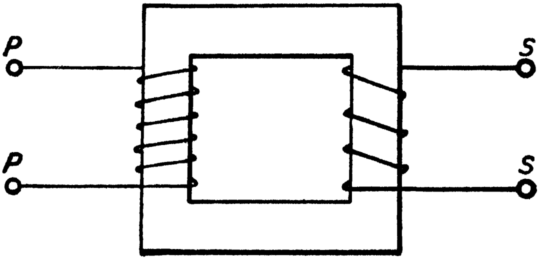


FIG. 77.—Diagrammatic Arrangement—Transformer.

Thus, if Φ_M is the maximum flux

f ,, frequency of the supply, in cycles per second
N_1 ,, number of primary turns
N_2 ,, number of secondary turns

then the back e.m.f. induced in the primary will be

$$E_p = K\Phi_M f N_1$$

K being constant.

The e.m.f. induced in the secondary winding will be

$$E_S = K\Phi_M f N_2$$

$$\text{The ratio } \frac{\text{secondary e.m.f.}}{\text{induced primary e.m.f.}} = \frac{K\Phi_M f N_2}{K\Phi_M f N_1} = \frac{N_2}{N_1}$$

$$\text{or } = \frac{\text{number of secondary turns}}{\text{number of primary turns}}$$

and if we neglect the resistance of the primary winding and also that of the secondary winding, since they are very small, we have

$$\frac{\text{Secondary terminal voltage}}{\text{Primary terminal voltage}} = \frac{\text{No. of secondary turns}}{\text{No. of primary turns}}$$

In one cycle the flux changes twice from zero to maximum and back again to zero, and therefore the change is $4\Phi_M$. This takes place in $1/f$ seconds. Therefore, the average change of flux is

$$\frac{4\Phi_M}{1/f} \text{ lines per second}$$

and the rate of change in linkages in the primary winding is

$$\frac{4\Phi_M N_1}{1/f} \text{ per second.}$$

Therefore, the average e.m.f. induced in the primary winding is

$$4\Phi_M f N_1 10^{-8} \text{ volts.}$$

The r.m.s. value of the e.m.f., E_p, induced in the primary will be

$$E_p = 4(1{\cdot}11)\Phi_M f N_1 10^{-8} \text{ volts,}$$

$$\text{since } \frac{\text{r.m.s. value}}{\text{average value}} = 1{\cdot}11 \text{ for a sinusoidal wave.}$$

Therefore, $E_p = 4{\cdot}44\Phi_M f N_1 10^{-8}$ volts.

Similarly, it can be proved that the induced e.m.f. E_S, in the secondary winding, is given by

$$E_S = 4{\cdot}44\Phi_M f N_2 10^{-8} \text{ volts.}$$

Providing the transformer were 100 per cent efficient, the output at the secondary terminals would be the same as the input at the primary terminals, i.e.

$$E_p I_p \cos\theta_1 = E_s I_s \cos\theta_2$$

and, assuming the two power factors were the same,

$$E_p I_p = E_s I_s$$

$$\text{Therefore, } \frac{E_s}{E_p} = \frac{I_p}{I_s} = \frac{\text{No. of secondary turns}}{\text{No. of primary turns}}$$

$$\text{and } \frac{\text{Secondary current}}{\text{Primary current}} = \frac{\text{No. of primary turns}}{\text{No. of secondary turns}}$$

We have made several assumptions in obtaining the above relationships. These are:

(*a*) That the primary and secondary windings are free of resistance

(*b*) The same value of flux links the secondary winding as that which links the primary winding

(*c*) The transformer is 100 per cent efficient.

Actually, (*c*) is involved in (*a*) and (*b*).

In practice, the efficiency of well-designed transformers is very high, and the above relationships hold, within a few per cent, for commercial transformers. The voltage relationship is most nearly true when the secondary is on open-circuit. The current relationship, on the other hand, becomes nearer the above as the load is increased.

6.12. Theory of Current Transformers. A vector diagram for a current transformer is given in Fig. 78. In order to make the diagram clear, the magnetising component I_M and the iron-loss component I_i of the excitation current I_e are magnified and the diagram has been constructed for a 1.1 ratio.

$$\text{Let } n = \frac{\text{No. of secondary turns}}{\text{No. of primary turns}} = \frac{T_s}{T_p}$$

where T_s is the number of secondary turns and T_p is the number of primary turns.

Also, let E = applied voltage
I_p = primary current
I_s = secondary current
Φ = working flux of the transformer
r_p = resistance of the primary winding
x_p = reactance of the primary winding
r_s = resistance of the secondary winding
x_s = reactance of the secondary winding.

For the construction of the vector diagram, suppose that E_p is that

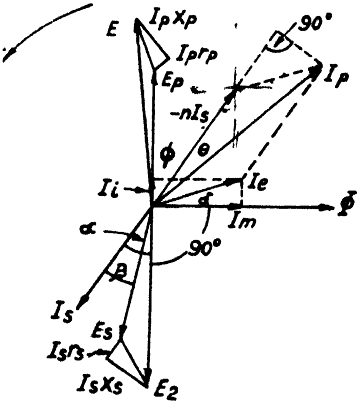


FIG. 78—Vector Diagram—Current Transformer.

component of the applied voltage which is available for creating the magnetic flux. Then the flux will lag this voltage by 90° and this is represented by the vector Φ. Now, there will be two components of the excitation current: I_i, which is necessary to overcome the iron losses and is in phase with E_p, and I_M, which is necessary for magnetising purposes, and is in phase with the magnetic flux, i.e. 90° behind E_p. The total excitation current will be the vector sum of these two components and it is represented by I_e which leads the flux by angle δ. The e.m.f. induced in the secondary winding will be 90° behind the magnetic flux; this is represented by E_2. The voltage available at the terminals (E_s) will be the vector difference between E_2 and the voltage drop across the secondary impedance; the latter being composed of the voltage drops across the resistance of the secondary winding ($I_s r_s$) and the reactance ($I_s x_s$). These voltage drops will be 90° out of phase with one another; the resistance drop $I_s r_s$ being in phase with the secondary current. The latter lags E_s by angle β where β is the angle of the current relative to the secondary terminal voltage.

The current in the primary winding will be I_e plus a current equal and opposite to the secondary current, and this is represented by I_p, the vector sum of I_e and $-nI_s$. The angle θ between $-nI_s$ and I_p is termed the phase angle of the transformer, and α is the phase angle of the total burden (including the impedance of the secondary winding). The applied voltage E will be E_p in addition to that required to overcome the impedance of the primary winding, i.e $I_pr_p+I_px_p$.

6.13. Transformer Ratio. The current transformation ratio of the transformer is the ratio of the primary current to the secondary current. We will consider the most usual case in practice, that is, when the secondary current is lagging its voltage. Then, from the diagram, we can determine the ratio sufficiently accurate for our purpose in the following manner.

From the vector diagram

$$\begin{aligned} I_p^2 &= (I_c+nI_s\cos\phi)^2+(I_M+nI_s\sin\phi)^2 \\ &=(I_e\sin\delta+nI_s\cos\phi)^2+(I_e\cos\delta+nI_s\sin\phi)^2 \\ &=I_e^2+n^2I_s^2+2nI_eI_s\sin\delta\cos\phi+2nI_eI_s\cos\delta\sin\phi \\ &=I_e^2+n^2I_s^2+2nI_eI_s(\sin\delta\cos\phi+\cos\delta\sin\phi) \end{aligned}$$

and $I_p=\sqrt{I_e^2+n^2I_s^2+2nI_eI_s\sin(\delta+\phi)}$

Therefore, the transformation ratio

$$T=\frac{I_p}{I_s}=\frac{\sqrt{I_e^2+n^2I_s^2+2nI_eI_s\sin(\delta+\phi)}}{I_s}$$

which, since I_e is small compared with nI_s, approximates to

$$\sqrt{n^2+\frac{2nI_sI_e\sin(\delta+\phi)}{I_s^2}}$$

which further approximates to

$$n+\frac{I_e}{I_s}\sin(\delta+\phi)$$

6.14. Phase Angle Error. In practice, due to the excitation current, the transformer usually introduces a further change of phase in the secondary current, in addition to the normal 180° phase change which can be allowed for by changing over the leads to the transformer secondary terminals. This angle is termed the phase angle of the transformer. Thus, if, after the secondary current has been changed through 180°, the secondary current leads the primary current by some angle, the error is considered positive. If the secondary current lags behind the primary current, it is considered negative. In general, this angle is very small, but under certain conditions when the transformer is used in conjunction with a wattmeter or watt-hour meter, serious errors might arise. We will investigate these conditions a little later in the chapter.

From the vector diagram, it can be seen that

$$\tan\theta = \frac{I_e \sin[90-(\delta+\phi)]}{nI_s+I_e\cos[90-(\delta+\phi)]}$$

$$= \frac{I_e\cos(\delta+\phi)}{nI_s+I_e\sin(\delta+\phi)}$$

but, when θ is small, $\tan\theta\to\theta$

$$\text{hence } \theta = \frac{I_e(\cos\delta\cos\phi-\sin\delta\sin\phi)}{nI_s+I_e\sin[\delta+\phi]}$$

and, since $I_e \sin[\delta+\phi]$ is small compared with nI_s,

$$\theta = \frac{I_M\cos\phi-I_i\sin\phi}{nI_s} \text{ radians}$$

$$\text{or } \frac{I_M\cos\phi-I_i\sin\phi}{nI_s}\left(\frac{180}{\pi}\right) \text{ degrees}$$

The change of phase angle introduces no error in current measurements, the only error that matters being that due to the transformation ratio. From our vector diagram, we can see that the transformation ratio depends upon the magnitude of the exciting current, and the current and power factor of the secondary circuit. Thus, the variation of ratio under different conditions of load may lead to appreciable errors.

In the measurement of power, we also have to take into consideration the phase angle of the transformer, i.e. the angle by which the reversed secondary current leads or lags the primary current. In this case, if ϕ were the angle of lag of the load current relative to the supply voltage, and θ were the phase angle of the transformer, the wattmeter would indicate

$$EI\cos(\phi-\theta)$$

instead of $EI\cos\phi$.

This would probably be negligible at power factors approaching unity, but as ϕ became greater, so the error would become more appreciable.

Similar errors would also occur in a watt-hour meter, the current coil of which was energised from the secondary of the current transformer.

6.15. Nickel-iron Alloys for Magnetic Circuit. In order to keep the phase angle as small as possible, the core must have a low reluctance and small iron loss. It is frequently made of "Stalloy", which is a silicon steel alloy. In the case, however, when it is essential that the transformer ratio error and phase angle be very small, "Mumetal" is generally used. Mumetal is a nickel-iron alloy containing a small percentage of copper. There are also other nickel-iron alloys useful for this purpose, such as "Permalloy"; their names, like that of Mumetal, actually being the trade-names given to them by the various manufacturers. They have the advantage of high permeability, small retentivity, and low iron loss.

In order to reduce the current ratio error, it is usual to reduce the secondary turns by about 1 per cent. As long as this alteration in the turns ratio is small, it has little effect upon the phase angle error.

If a very fine adjustment of ratio is necessary, it can be obtained by making one of the secondary turns encircle only a portion of the laminations, instead of them all. Such a procedure demands great care in ensuring that the conductor is sufficiently well insulated and free from mechanical strain.

6.16. Current Transformer Secondary Winding—Open Circuit. It should be remembered that, in a current transformer, the number of primary ampère-turns is not appreciably reduced when the secondary is on open circuit, since the current flowing in the primary winding remains practically the same, i.e. the primary ampère-turns are a fixed value. Therefore, if a current transformer has its secondary circuit opened, while current is flowing through the primary, there will be no demagnetisation flux due to the secondary current. This will result in a very high flux density in the core and, in consequence, a high induced voltage in the secondary winding; the latter severely straining the insulation and possibly endangering the life of the engineer. It is therefore essential that the secondary winding be given its normal burden, or be short-circuited when the primary current is flowing. The short-circuiting of the secondary will not cause damage, since the impedance of the ammeter or wattmeter coils, which are in general connected in with the secondary winding, is exceedingly low.

After the secondary winding of a current transformer has been opened, with the primary carrying its normal current, it is possible for the current ratio and phase angle of the transformer to be altered. One should therefore demagnetise the transformer, before further use, in the following manner:

A high value variable resistance, capable of carrying the rated secondary current, is placed across the secondary terminals of the transformer and the rated full load current of the transformer is allowed to flow through the primary winding. The resistance is initially adjusted to its maximum value and gradually decreased to zero. In this manner the alternating magnetic flux commences with high amplitude and gradually decreases until it reaches its normal value.

6.17. Potential Transformers—Theory. The theory of the potential transformer is similar to that of the power transformer, the main difference being that, in the case of the former, the secondary current is of the same order in magnitude as the magnetising current of the transformer, whilst in the power transformer the secondary current depends upon the load and therefore varies greatly.

A vector diagram of a potential transformer of 1 : 1 ratio is given in Fig. 79.

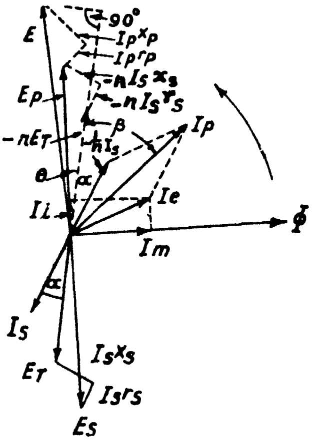


FIG. 79.—Vector Diagram—Potential Transformer.

Let E be the primary voltage
E_p be that portion of the primary voltage which is available for transformation
Φ be the working flux of the transformer
E_s be the voltage induced in the secondary winding
E_T be the secondary terminal voltage
I_i be the component of the no-load current, due to iron losses
I_M be the component of the no-load current, required for magnetisation purposes
I_e be the no-load current
r_p be the resistance of the primary winding
x_p be the reactance of the primary winding
r_s be the resistance of the secondary winding
x_s be the reactance of the secondary winding
I_p be the primary current
I_s be the secondary current
θ be the phase angle of the reversed secondary terminal voltage with respect to the primary applied voltage.

The voltage E_p, which is that available for transformation, will be the vector difference between E and the voltage drop due to the primary current flowing through the impedance of the primary winding. This voltage E_p will cause a magnetic flux of Φ which lags E_p by 90°. This flux will induce an e.m.f. E_s in the secondary winding which will lag the flux by 90°. The secondary terminal voltage E_T will be the vector difference between the induced voltage E_s and the voltage drop in the secondary winding due to the passage of I_s, the secondary current, through the secondary winding impedance. The value of the voltage drop in the secondary winding will be $I_s\sqrt{r_s^2+x_s^2}$. I_sr_s will be in phase with the secondary current and I_sx_s will be 90° ahead of it. The primary current is represented by I_p, and is the vector sum of I_e and $-\frac{1}{n}I_s$, where n is the ratio of primary turns to secondary turns.

The phase angle of the potential transformer is that between the applied voltage E and the reversed secondary terminal voltage E_T, and is shown as θ.

The voltage ratio E/E_T may be deduced in the following manner from the vector diagram:

If we assume that θ is small, as is the case for a potential transformer, then the projection of E on $-nE_T$ continued will be approximately of the same magnitude as E. The magnitude of E will then be given by

$$E=n(E_T+I_sr_s\cos\alpha+I_sx_s\sin\alpha)+I_pr_p\cos\beta+I_px_p\sin\beta$$

$$=nE_T+nI_s(r_s\cos\alpha+x_s\sin\alpha)+I_p(r_p\cos\beta+x_p\sin\beta)$$

$$\text{but } I_p\cos\beta=I_i\cos\theta^1+\frac{I_s}{n}\cos\alpha$$

where θ^1 is smaller than θ.

As θ is very small, the angle θ^1 will approximate to unity, therefore

$$I_p\cos\beta=I_i+\frac{I_s}{n}\cos\alpha$$

Similar reasoning gives $I_p\sin\beta=I_M+\frac{I_s}{n}.\sin\alpha$

$$\text{Therefore } E=nE_T+nI_s(r_s\cos\alpha+x_s\sin\alpha)$$

$$+r_p\left(I_i+\frac{I_s}{n}\cos\alpha\right)+x_p\left(I_M+\frac{I_s}{n}\sin\alpha\right)$$

$$=nE_T+I_s\left[\left(nr_s+\frac{r_p}{n}\right)\cos\alpha+\left(nx_s+\frac{x_p}{n}\right)\sin\alpha\right]+r_pI_i+x_pI_M$$

Now, the reflected secondary resistance of the primary $=r_p/n^2$ and the equivalent resistance R_{es} of the transformer referred to the secondary is given by

$$R_{es}=r_s+\frac{r_p}{n^2}$$

Similarly, the equivalent reactance X_{es} of the transformer, referred to the secondary, is given by

$$X_{es}=x_s+\frac{x_p}{n^2}$$

Therefore $E=nE_T+nI_s(R_{es}\cos\alpha+X_{es}\sin\alpha)+r_pI_t+x_pI_M$

$$\text{and}\quad \frac{E}{E_T}=n+\frac{nI_s(R_{es}\cos\alpha+X_{es}\sin\alpha)+r_pI_t+x_pI_M}{E_T}$$

It therefore follows that the difference between the voltage ratio and the turns ratio, i.e. the ratio error

$$=\frac{nI_s(R_{es}\cos\alpha+X_{es}\sin\alpha)+r_pI_t+x_pI_M}{E_T}$$

The phase angle may be determined from the diagram in the following manner:

$$\sin\theta=\frac{I_p(x_p\cos\beta-r_p\sin\beta)+nI_s(x_s\cos\alpha-r_s\sin\alpha)}{E}$$

but, since θ is small, $\sin\theta$ approximates to θ.

Therefore

$$\theta=\frac{x_p\left(I_t+\frac{I_s}{n}\cos\alpha\right)-r_p\left(I_M+\frac{I_s}{n}\sin\alpha\right)+nI_s(x_s\cos\alpha-r_s\sin\alpha)}{E}$$

$$=\frac{I_s\left(\frac{x_p}{n}+nx_s\right)\cos\alpha-I_s\left(\frac{r_p}{n}+nr_s\right)\sin\alpha+x_pI_t-r_pI_M}{E}$$

$$=\frac{nI_sX_{es}\cos\alpha-nI_sR_{es}\sin\alpha+x_pI_t-r_pI_M}{E}\text{ radians}$$

$$=\left[\frac{nI_s}{E}(X_{es}\cos\alpha-R_{es}\sin\alpha)+\frac{x_pI_t-r_pI_M}{E}\right]\frac{180}{\pi}\text{ degrees}$$

6.18. Transformer Tests—Methods. The tests for transformers may be divided into two classes; these are:

(*a*) Absolute methods.
(*b*) Comparison methods.

There are several variations of each method, but we will briefly consider one example of each relative to the testing of current and potential transformers respectively.

6.19. Testing of Current Transformers. The American Bureau of Standards has developed a method termed the "Mutual Inductance Method" for the testing of current transformers. A circuit suitable for this test is given in Fig. 80.

R and R_1 are non-inductive resistances of low value (R may be a universal shunt), the former being suitable for carrying the primary current, and the latter including a slide-wire for fine adjustment. The ratio of R_1 to R must also be approximately equal to the current ratio of the transformer under test since, when the in-phase component of the out-of-balance voltage of the circuit is balanced,

$$I_pR \doteqdot I_sR_1$$

$$\text{i.e. } \frac{R_1}{R} \doteqdot \frac{I_p}{I_s}$$

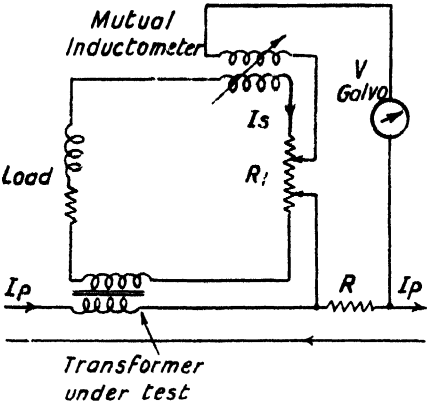


FIG. 80.—Current Transformer Testing—Mutual Inductance Method.

The reactive component is balanced by adjustment of the mutual inductance, which injects into the circuit an e.m.f. 90° out of phase with the voltage drop of I_sR_1.

The conditions of balance are illustrated in Fig. 81, which gives a

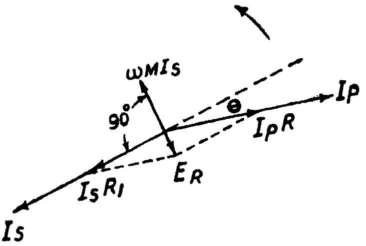


FIG. 81.—Vector Diagram—Mutual Inductance Method of Test for C.T.'s.

vector diagram of the voltages and currents in the circuit. If I_p represents the primary current, and I_s that of the secondary, then I_pR and I_sR_1 are the voltage drops across R and R_1 respectively, and each will be in phase with its current. E_R, the vector sum of I_pR and I_sR_1, will be the resultant voltage in the circuit and upon the correct adjustment of R_1 will be 90° ahead of I_s. Now, this can only be balanced by a voltage equal and opposite to E_R, i.e. a voltage lagging I_s by 90°. This voltage is supplied by the mutual inductometer, since its secondary e.m.f., which is injected into the vibration galvanometer circuit, is always 90° behind I_s and of magnitude dependent upon the value of M (ωMI_s, where M is the mutual inductance and ω is 2π times the frequency in cycles per second). Therefore M is adjusted until $\omega MI_s = -E_R$. When this condition occurs, the voltage in the vibration galvanometer circuit will be zero and no current will pass through the galvanometer.

From the vector diagram, it can be seen that the phase angle may be derived from

$$\tan\theta = \frac{E_R}{I_sR_1} = \frac{\omega MI_s}{I_sR_1} = \frac{\omega M}{R_1}$$

$$\text{The transformation ratio} = \frac{I_p}{I_s}$$

$$\text{and, as } \cos\theta = \frac{I_sR_1}{I_pR}$$

$$\frac{I_p}{I_s} = \frac{R_1}{R\cos\theta}$$

$$\text{which approximates to } \frac{R_1}{R}$$

since θ is small.

As the phase angle is $\omega M/R_1$, it follows that the supply frequency must be accurately known. Also, the impedance of the primary of the mutual inductometer must be included with the load when deciding the burden to which the errors of the transformer apply.

6.20. Watt-hour Meter Method. In this method, which is of the comparison class, the transformer is compared with a standard transformer through the medium of two substandard watt-hour meters. The equipment required is as follows:

(*a*) A standard transformer of the same nominal ratio as the transformer under test.

(*b*) Two substandard 5-ampère meters, the relative errors of which are known at unity and 0·5 power factor, for certain loads.

The apparatus is connected as shown in Fig. 82, the standard and test transformer primary windings being connected in series, their secondary windings being connected one to each of the current coils of the substandard watt-hour meters.

A predetermined value of current is passed through the primary windings of the two transformers, thus energising the current coils of the meters; the potential coils being energised by a separate supply of the same wave-form and frequency. In general, this supply will come from the rotor winding of a phase-shifting transformer. In this manner a fictitious or phantom load is thrown upon the meters.

The phase-shifting transformer is initially adjusted so that the potential supply to the meters is in phase with the current flowing in the primary windings of the transformers. The meters are then run against one another (for, say, 25 to 100 revolutions of the disc) and the error between the registration of the two meters is noted. The phase-shifting transformer is then adjusted until the current in the transformer primaries lags the applied voltage to the potential coils of the meters by 60°. Another twenty-five or more revolutions of the two discs are made and the difference in the registration of the two discs is again noted.

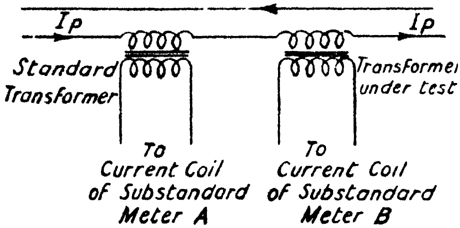


FIG. 82.—Current Transformer Testing—Watt-hour Meter Method.

This procedure is followed for any required current loadings of the primary windings of the transformers, in order that the ratio and phase angle errors may be determined for these conditions.

6.21. Determination of Ratio Error—Watt-hour Meter Method. Let us suppose that, when the full rated current is passing through the primaries of the two transformers, in phase with the applied voltage to the meter potential coils, that the meter M_T carrying the secondary current from the test transformer registers x per cent greater than the meter M_S which carries the secondary current of the standard transformer. Suppose, also, that when the meter current coils are connected in series without their transformers, and 5 ampères is flowing through their current coils in phase with the applied voltage to the potential coils, that M_T reads y per cent greater than M_S. Then, since the meters are operating upon a fictitious load at unity power factor, the phase angles of the transformers will have little effect upon the registration of the meters and the errors introduced will be due to the difference in ratio errors of the transformers. This will be

$$(x-y) \text{ per cent}$$

and if the ratio error of the standard transformer is z per cent, the ratio error of the test transformer will be

$$(z+x-y) \text{ per cent}$$

to a first approximation.

The current ratio of the test transformer will be

$$N\left(1+\frac{(z+x-y)}{100}\right):5$$

where $N:5$ is the nominal ratio of the transformer.

6.22. Determination of Phase Angle Error—Watt-hour Meter Method. With the transformers and meters still connected as above and the primary windings of the transformers still passing the same value of current, the phase-shifter is adjusted until the primary current lags the applied voltage to the meter potential coils by 60°, thus giving a phantom load of 0·5 power factor, full load. Suppose, under these circumstances, that M_T registers a per cent greater than M_S. Then if, when the current coils of the two meters are fed directly with 5 ampères at 60° lag relative to the applied voltage to the potential coils, M_T registers b per cent greater than M_S, the relative error introduced by the transformers will be

$$(a-b) \text{ per cent}$$

If R_T is the ratio error of the test transformer and R_S is that of the standard transformer, the relative percentage error introduced, due to the difference in ratio errors of the two transformers, will be

$$R_T-R_S$$

to a first order approximation.

The percentage error due to the difference in phase angle will be

$$[(a-b)-(R_T-R_S)] \text{ per cent}$$

If the phase angle error of the standard transformer is ϕ_S, the error recorded by M_S, due to this, at 0·5 power factor, will be

$$\left[\frac{\cos(60-\phi_S)-\cos 60}{\cos 60}\times 100\right] \text{ per cent}$$

Similarly, if the phase angle error of the test transformer is ϕ_T, the error of M_T, due to the phase angle of its transformer, will be

$$\left[\frac{\cos(60-\phi_T)-\cos 60}{\cos 60}\times 100\right] \text{ per cent}$$

But we have already found that the difference in phase angle errors of the two transformers causes M_T to register $[(a-b)-(R_T-R_S)]$ per cent greater than M_S.

Therefore, if $[(a-b)-(R_T-R_S)]=Q$

$$Q=100\left[\frac{\cos(60-\phi_T)-\cos 60-\cos(60-\phi_S)+\cos 60}{\cos 60}\right] \text{ per cent}$$

$$=2\,[\cos(60-\phi_T)-\cos(60-\phi_S)]\;100 \text{ per cent}$$

$$=200\ [\cos 60 \,.\, \cos \phi_T+\sin 60 \,.\, \sin \phi_T-\cos 60 \cos \phi_S-\sin 60 \sin \phi_S] \text{ per cent}$$

$$=200\ [\sin 60 \sin \phi_T-\sin 60 \sin \phi_S]$$

since ϕ_T and ϕ_S are small angles.

Therefore $Q=100\sqrt{3}\ [\sin \phi_T-\sin \phi_S]$.

Since ϕ_S, the phase angle of the substandard transformer, and Q are known, ϕ_T can easily be determined.

Example. If the phase angle error of the standard transformer is $-0° \ 30'$ and, at 0·5 power factor, it is found that the difference in phase angle errors between the standard and test transformers causes M_T to register 1·2 per cent less than M_S, due allowance having been made for transformer ratio errors and meter errors,

$$\text{then } -1{\cdot}2=100\sqrt{3}\ (\sin \phi_T+\sin 0° \ 30')$$

$$-1{\cdot}2=100\sqrt{3}\ (\sin \phi_T+0{\cdot}00873)$$

$$\text{Therefore } -\sin \phi_T=\frac{1{\cdot}2}{100\sqrt{3}}+0{\cdot}00873$$

$$=0{\cdot}00674+0{\cdot}00873$$

$$=0{\cdot}01547$$

$$\text{Therefore } \phi_T=-\sin^{-1} 0{\cdot}01547$$

$$=-0° \ 54'$$

It can be seen from the cosine tables that the cosine of any angle around 60° differs from the cosine of 60° by approximately 0·015 per degree, i.e. 3 per cent of cos 60° per degree. A simple approximation may therefore be made, in the case of small variations, by dividing the percentage relative error (introduced by the difference in phase angle of the two transformers) by three and considering the value as degrees. Hence, if this error be Q per cent, the difference in phase angle between the two transformers, i.e. $\phi_T-\phi_S$, will be given by

$$(\phi_T-\phi_S)=\frac{Q}{3} \text{ degrees.}$$

Thus, in the above example, as M_T registered 1·2 per cent less than M_S,

$$\phi_T-\phi_S=-\frac{1{\cdot}2}{3} \text{ degrees}$$

$$=-0° \ 24'$$

$$\text{Therefore } -\phi_T=\phi_S+0° \ 24'$$

$$=0° \ 30'+0° \ 24'$$

$$\text{and } \phi_T=-0° \ 54'$$

which agrees with the previous result.

$$\text{which is } \frac{AD}{OA}$$

Since OA is constant, the ratio error of the transformer will be proportional to the current in the ratio error winding.

The difference in phase angle between the two transformers will be given by

$$\theta = \angle DOB$$

By virtue of this angle being very small, sin θ will approach θ, therefore

$$\theta = \frac{BD}{OD}$$

and, since AD is very small,

$$\theta = \frac{BD}{OA}$$

Furthermore, OA is constant. Therefore the phase angle between the two transformers will be proportional to BD.

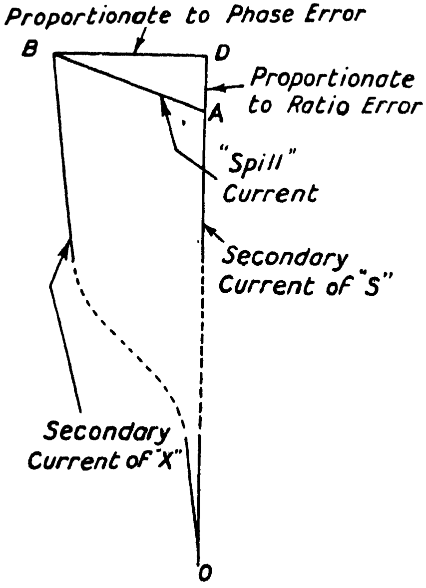


FIG. 84.—Vector Diagram—Petch-Elliott Current Transformer Testing Set.

We thus have the ratio and phase angle errors of the test transformer relative to those of the standard transformer and, as the errors of the latter will be known, the actual errors of the test transformer can easily be determined.

It should be mentioned that, as the toroidal transformer is not magnetised at balance, the "spill" winding will be non-inductive under these conditions and the burden will be low, only comprising the resistance of the winding.

The maximum range of the apparatus is ±2·5 per cent for ratio error and ±100 minutes for phase angle error (this is at the 5 : 1 setting) and on the direct setting 4 mm. of the right-hand dial represents a ratio error of 0·01 per cent, whilst 9 mm. of the left-hand dial represents a phase angle error of 1 minute. Reversing keys are provided in order that both positive and negative errors may be determined.

When the transformer is under conditions of balance, the loading on the X transformer, due to the "spill" winding, is less than 0·1 V.A. and may be neglected in comparison with the rated burden of the transformer. Provision is therefore made, in the X transformer circuit, for the addition of an actual burden, the value being that for which the errors are required.

The test set is built up to impose a non-inductive burden of 1·5 V.A. on the standard transformer and extra non-inductive burden can be added to make up the actual burden at which the errors of the standard transformer are known. This is attained by removing the test set load links and connecting to the "Std. Load" terminals a non-inductive burden to make up the required value. Similarly, the non-inductive load of the required value for the X transformer is connected to the "X load" terminals. An instrument is also provided on the panel to indicate the load current of the transformers.

The apparatus is very simple to use and has the advantage that no phase-shifting transformer is necessary. Furthermore, since there are no precision readings to be taken, the test may be made by a semi-skilled operator.

6.24. The Testing of Potential Transformers. In the absolute methods of testing potential transformers, the terminal voltage of the transformer secondary winding is compared with a suitable fraction of the applied voltage to the primary winding, which is obtained from a non-reactive potential divider. Usually, the potential divider is connected across the primary winding of the transformer, and the reversed terminal voltage of the secondary winding is balanced against a portion of the primary voltage; provision being made for the difference in phase between the two voltages. A suitable circuit is illustrated in Fig. 85. T is the potential transformer under test, and the terminal voltage of the secondary winding is balanced against a portion of the primary voltage in the manner indicated; the burden of the transformer being that to be used in service. The potential divider consists of a high non-inductive resistance R_2+R_3 in series with an inductance L, and across a part R_1 of the potential divider is connected the condenser C.

Now, the circuit between the points *a* and *b* will have an impedance of

$$j\omega L + \frac{1}{\frac{1}{R_1} + j\omega C}$$

$$= j\omega L + \frac{R_1}{1 + j\omega C R_1}$$

$$= j\omega L + \frac{R_1(1 - j\omega C R_1)}{1 + \omega^2 C^2 R_1^2}$$

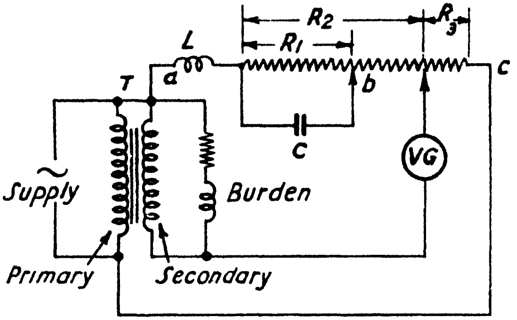


FIG. 85 —Potential Transformer Testing—Absolute Method

Therefore, the effective series inductance L_e, of this part of the circuit, is given by

$$L_e = L - \frac{CR_1^2}{1 + \omega^2 C^2 R_1^2}$$

and, since C is very small, $\omega^2 C^2 R_1^2$ may be neglected with respect to unity.

$$\text{Hence, } L_e = L - CR_1^2$$

Since L and C are constant, it follows that the effective series inductance L_e will depend upon the value of R_1. Thus, by adjusting the value of R_1, the phase of the voltage tapped off from the primary voltage and applied to the galvanometer circuit may be matched with that of the secondary terminal voltage of the transformer, under any required burden. Balance will occur when the vibration galvanometer has minimum deflection, and this is obtained by successive adjustment of R_1 and R_2.

Under conditions of balance, we see from the vector diagram of Fig. 86 that

$$\frac{E_p}{E_s} = \frac{R_2 + R_3}{R_2} \cdot \frac{\cos(\theta_2 + \delta)}{\cos \theta_2}$$

$$=\frac{R_2+R_3}{R_2}\cdot\frac{\cos\theta_2\cos\delta-\sin\theta_2\sin\delta}{\cos\theta_2}$$

$$=\frac{R_2+R_3}{R_2}(\cos\delta-\tan\theta_2\sin\delta)$$

where E_p is the applied voltage to the primary winding, E_s is the secondary terminal voltage and δ is the phase angle of the transformer.

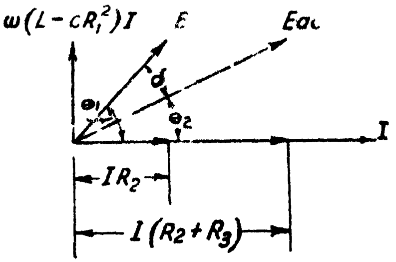


FIG. 86.—Vector Diagram—Absolute Method of Testing Potential Transformers.

Also, since δ will be small, cos δ approximates to unity and $\omega(L-CR_1^2)I$ will also be relatively small, whence θ_2 will be small. Therefore sin δ and tan θ_2 approach zero. Thus

$$\frac{E_p}{E_s}=\frac{R_2+R_3}{R_2}$$

The phase angle is determined in the following manner:

During the above approximation to the effective series inductance L_e in the circuit, we found that

$$z=j\omega L+\frac{R_1(1-j\omega R_1)}{1+\omega^2C^2R_1^2}$$

where z is the impedance between the points a and b in Fig. 85.

We have already seen that $1+\omega^2C^2R_1^2$ approximates to unity,

$$\text{therefore } z=j\omega L+R_1-j\omega CR_1^2$$

Assuming that the vector diagram of Fig. 86 indicates the conditions at balance, then δ_1, the phase angle of the voltage of the secondary relative to the applied voltage, will be $\theta_1-\theta_2$.

$$\text{But } \theta_1=\tan^{-1}\frac{\omega(L-CR_1^2)}{R_2}$$

$$\text{and } \theta_2=\tan^{-1}\frac{\omega(L-CR_1^2)}{R_2+R_3}$$

Therefore, $\delta_1 = \theta_1 - \theta_2$

$$= \tan^{-1}\left[\omega(L - CR_1{}^2)\left(\frac{1}{R_2} - \frac{1}{R_2+R_3}\right)\right] \text{ approx.}$$

6.25. Comparison Method. One of the methods which depend upon comparison of the transformer under test with a standard transformer is illustrated by the diagram of Fig. 87. T_S and T_T are the standard and test transformers respectively. Their primary windings are connected in parallel and their secondaries in series-opposition to the potential coil of the wattmeter W_1. The desired burden of the test transformer is connected across its secondary terminals, whilst the potential coil of wattmeter W_2 is connected across the secondary winding of the standard transformer. P is a phase-shifting transformer.

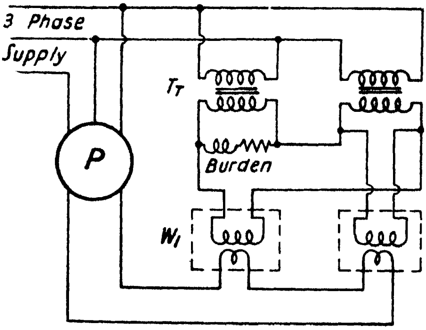


FIG. 87—Potential Transformer Testing—Comparison Method.

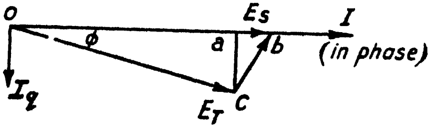


FIG. 88.—Vector Diagram—Comparison Method of Testing Potential Transformers.

The potential coil of W_1 receives a potential equal to the vector difference of the secondary terminal voltage E_S, of the standard transformer and the secondary terminal voltage E_T of the test transformer.

It can be seen from the vector diagram of Fig. 88 that if the current coil is energised by a current in phase with E_S, the reading of W_1 will be proportional to the difference in transformation ratios of the two transformers. Furthermore, if the difference in phase of E_S and E_T is small, *Oa* will be approximately equal to *OC* or E_T. Therefore, if

R_S is the ratio of the standard transformer and K is the constant of W_1, representing the volts per scale division, the ratio R_T of the test transformer will be given by

$$\left[R_T = \frac{R_S(E_S - K\delta_1)}{E_S}\right]$$

where δ_1 is the number of divisions deflection of W_1.

The phase-shifter is now adjusted until the current in the wattmeter current coils is in quadrature with F_S. This current is represented by I_q in Fig. 88. It can be seen from the figure that the reading of W_1 will be proportional to the voltage ac, which is a component of the voltage cb. In other words, it is proportional to the tangent of the angle ϕ, where ϕ is the difference in phase angles between the two transformers.

$$\text{Therefore } \phi = \tan^{-1}\frac{K\delta_2}{E_S}$$

where δ_2 is the number of divisions deflection of the wattmeter W_1 and K is the same constant of volts/division already mentioned.

Then, if θ_S is the phase angle of the standard transformer, the phase angle θ_T of the test transformer will be given by

$$\theta_T = \theta_S + \phi$$

$$= \theta_S + \tan^{-1}\frac{K\delta_2}{E_S}$$

The wattmeter W_2 is only in circuit for the purpose of adjusting the phase of the current in the wattmeter current coils for the "in-phase" and quadrature tests. For the "in-phase" tests, the phase shifter is adjusted until W_2 gives a maximum reading. Quadrature is obtained when the wattmeter W_2 reads zero. It therefore follows that, unless W_2 has only a small phase error, there will be serious errors introduced due to the incorrect reading of W_2 at zero power factor. Allowance, however, can be made for such a case.

The wattmeter W_1 is preferably a high-grade instrument, the potential coil of which has a very low voltage range in order to ensure a large deflection.

6.26. Ratio (or Primary) Correction Factor. In the reports of the National Physical Laboratory the ratio error of a transformer is given as

$$\frac{\text{True Ratio}}{\text{Nominal Ratio}}$$

This value is termed the Ratio (or Primary) Correction Factor, which is abbreviated to R.C.F.

CHAPTER VII

Integrating Meters—Mathematical Treatment

7.1. Measurement of Energy. Let us consider the curve of Fig. 89 as representative of the power taken by a consumer over a period of time, say T hours. Now, since energy is the dissipation of power over a period of time, the total energy supplied to the consumer over the period of T hours will be proportional to the area enclosed by the curve.

If we take a small element of time dt, during which the power is constant at a value of P, then the energy supplied during this period will be $P.dt$, and the total energy supplied during the T hours will be the summation of all such elements from $t=0$ to $t=T$ hours. Therefore, if P is in kilowatts, the total energy supplied will be

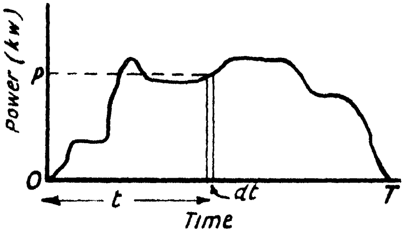


Fig. 89.—Power-Time Curve.

$$\int_0^T P.dt \text{ kWh}$$

From the above it is apparent that for an instrument to measure energy it must integrate the energy over the elements which constitute the period of measurement. Thus, if we have an instrument, the disc of which rotates with an angular velocity proportional to the mean power being supplied at any given instant, then the number of revolutions R_H made in T hours will be given by

$$R_H = \int_0^T \phi dt$$

where ϕ is the angular velocity of the disc, at instant t, in revolutions per hour.

If, at any instant, the velocity is proportional to the power expended, the power

$$P=K\phi$$

K being a constant.

The total energy supplied

$$=\int_0^T P.dt \text{ kWh}$$

$$=\int_0^T K\phi dt \text{ kWh}$$

$$=K\int_0^T \phi dt \text{ kWh.}$$

Thus, a counting train will read directly in kWh, providing that it is geared so that 1/K revolutions of the disc register one unit on the counting train dial.

7.2. D.C. Systems. In the case of a two-wire d.c. system, the power is the product of the load current and the supply voltage. Therefore, providing the voltage remains constant and at its rated value, the power is proportional to the current. That is $p=Ki$, where p and i are the instantaneous values of the power and current respectively, and K is a constant. Hence, if the angular velocity of the spindle in revolutions per hour is ϕ, and this is at all times proportional to the current, the quantity of electricity supplied in time T hours will be

$$Q=\int_0^T idt \text{ A.H.}$$

and if $i=K_2\phi$

$$Q=K_2\int_0^T \phi dt \text{ A.H.}$$

and the total energy supplied in time T

$$=K_2 \cdot \frac{V}{1000}\int_0^T \phi \, dt \text{ kWh}$$

where K_2 is a constant and V is the value of the declared voltage.

Hence, if the revolution counter is so geared that $1000/K_2V$ revolutions of the disc register one unit on the counting train dial, the meter will register directly in kWh. If, on the other hand, it is geared so that $1/K_2$ revolutions of the disc register one unit, the meter will register directly in ampère-hours.

7.3. Electrolytic Method of Determining Energy Supplied. We saw in section 1.2 (*c*) that when a direct current passes through an electrolyte the rate of deposition at the cathode is directly proportional to the current flowing in the circuit.

Let the rate of deposition at time t be r units of weight or volume, per unit of time, then the current

$$i = Kr$$

The ampère-hours supplied

$$= \int_0^T i\,dt$$

where T is measured in hours.

$$\text{But} \int_0^T i\,dt = K \int_0^T r.dt$$

$$\text{Therefore, Quantity} = K\int_0^T r.dt \text{ A.H.}$$

Since $K \int_0^T r.dt$ is proportional to the weight or volume of the deposition, the ampère-hours are also directly proportional to the weight or volume of the deposition. Thus, by suitable methods of measuring this deposition, it is possible to determine the quantity of electricity supplied in any period of time.

Furthermore, if the voltage V of the supply is constant, the total energy supplied will be

$$K \cdot \frac{V}{1000} \int_0^T r.dt \text{ kWh}$$

and, since $KV/1000$ is constant, the energy supplied is directly proportional to the weight or volume of deposition.

It should be mentioned that the accuracy of the above method of measuring the energy supplied depends upon the supply voltage remaining at its rated value. For example, by such methods of measurement, if the supply voltage were x per cent below the declared value, the meter would register x per cent in excess of the actual energy supplied.

7.4. Measurement of Energy by means of Differential Pendulums. Suppose two pendulums, with the same normal frequency of beat, are geared to the sun-wheels of a differential. Then, since they are swinging at the same frequency, the planet arm will not rotate. Now, suppose forces are applied to the pendulums such that the period of the one is increased and that of the other reduced, directly in proportion to the power supplied to the circuit. The planet wheel, which revolves with an angular velocity directly proportional to the difference in periodicity of the two pendulums, will consequently rotate with an angular velocity directly proportional to the power supplied. Hence, by suitable gearing of the revolution counter it could be arranged for the counting train to register directly in kWh.

We shall discuss in the following chapters of this book the different classes of meters which depend for their action upon the above methods of integration of various electrical quantities, for the determination of an equitable cost to the consumer for the supply of electrical energy.

CHAPTER VIII

D.C. Ampère-hour and Watt-hour Meters

8.1. Electrolytic Meters. The simplest form of ampère-hour meter is undoubtedly of the electrolytic type. This type of meter depends for its action upon the chemical effect which is associated with the passage of a current through an electrolytic solution. Such a meter is cheap to construct, is simple in design, and is accurate over a wide range. Unfortunately, it is very fragile on account of the amount of glass used in its construction.

We will consider briefly two of the main types of electrolytic meter.

8.2. Bastian Meter. A simplified illustration of this meter appears in Fig. 90. It consists of a glass bulb, the upper portion of which has a uniform bore. The bulb is filled with water containing a small percentage of sulphuric acid, the acid being introduced, since water is a non-conductor, for the purpose of decreasing the resistance of the electrolyte and assisting the action of decomposition. The electrolyte also occupies most of the space in the uniform glass bore and a thin film of paraffin is run on the surface of the electrolyte in order to

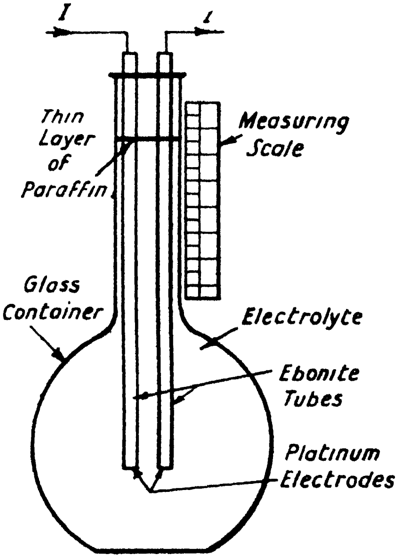


FIG. 90.—Bastian Electrolytic Meter.

prevent the normal evaporation which would otherwise occur, due to contact with the atmosphere.

Two leads are threaded down the ebonite rods and at their lower extremities are connected two platinum electrodes; the other ends of the leads being connected in the supply circuit in series with the load. A uniform scale is placed alongside the glass tube for determination of the height of the electrolyte.

When a current passes through the electrodes the water is decomposed into its constituents, oxygen and hydrogen; the gases escaping into the atmosphere. Now, the rate of decomposition is proportional to the rate of current flowing, and for a given quantity of electricity passing through the electrolyte there will be a definite decomposition of the water which will be directly proportional to the quantity of electricity supplied to the consumer. Therefore, the glass tube may be graduated directly in ampère-hours. Also, if the supply voltage remains constant at its declared value, the tube may be calibrated directly in kWh, since the energy supplied to the consumer will be directly proportional to the ampère-hours supplied.

This type of meter is now obsolete on account of its many disadvantages. There is a chemical back e.m.f. of polarisation of 1·5 volts and also a voltage drop due to the passage of the current through the resistance of the electrolyte. This voltage drop, naturally, is proportional to the load. It therefore follows that there is a large dissipation of energy in the meter and, as a consequence, it is only suitable for small currents.

Another disadvantage is that the meter requires periodic refilling (the decomposition of water being at the rate of 0·346 cu. cm. per ampère-hour), and once this is overlooked no record of the consumer's consumption of electricity remains. The inside of the tube is also liable to get very dirty after a long period of service, thereby making it very difficult—and sometimes impossible—to read the meter.

In order to overcome the disadvantages due to the high internal resistance and volt drop of the meter, a later type employed an alkaline electrolyte of caustic soda, the platinum electrodes being replaced by nickel ones. A definite improvement resulted, but the meter still retained the disadvantages of (*a*) requiring periodic refilling, (*b*) a tendency towards a dirty tube after long service, and (*c*) fragility, due to the amount of glass employed in its construction.

For such a meter to be properly shunted, the back e.m.f. of polarisation must be very small. For example, if e is the back e.m.f. of the meter, R_m the resistance of the meter, and R_S the resistance of the shunt, we shall have conditions which can be represented by the simple circuit diagram of Fig. 91. In this diagram, I is the supply current and I_M the meter current.

Now, the potential drop across the meter will be the same as that across the potential terminals of the shunt. Therefore,

$$(I-I_M)R_S=I_MR_M+e$$

and $IR_S=I_M(R_M+R_S)+e$

$$\frac{I_M}{I}=\frac{R_S}{R_M+R_S}-\frac{e}{I\,(R_M+R_S)}$$

$$=\frac{R_S}{R_M+R_S}\left(1-\frac{e}{IR_S}\right)$$

$$=\frac{1}{N}\left(1-\frac{K}{I}\right)$$

where N is the nominal multiplying power of the shunt and K is a constant of value e/R_S; assuming the back e.m.f. of polarisation to be constant.

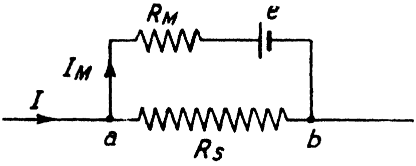


FIG. 91.—Back e.m.f. of Polarisation—Electrolytic Meter.

The above expression shows that the meter current is not a constant proportion of the load current, but decreases as the load current decreases.

8.3. Reason Electrolytic Meter. This meter has probably been the most successful of the electrolytic meters, and a schematic diagram is given in Fig. 92. The electrolyte is a saturated solution of mercury and potassium iodides. The anode consists of an annular ring of mercury contained in a shallow trough at the top of the tube, and the cathode is a cone of specially prepared carbon or a ring of iridium. During electrolytic action, mercury is transferred from the anode to the cathode and, after being deposited upon the latter, it falls down the funnel-shaped mouth into the glass measuring tube. Mercury so used from the anode is replaced from the reservoir of mercury, the latter keeping the anode mercury at a constant level. In order to prevent any risk of the mercury being thrown by vibration into the registering tubes, a glass fence is fitted between the anode mercury and the cathode and, in addition, the complete tube is spring-suspended and mounted in spring buffers. The whole of the tube, with the exception of the space occupied by the mercury, is filled with the electrolyte.

In order to compensate for the negative temperature coefficient of the meter, a resistance is connected in series with the tube on the tube side of the shunt, the value of the resistance being such that the overall resistance of it and the tube remains sensibly constant over a wide range of temperature.

Alongside the glass measuring tube is a scale calibrated in ampère-hours or kWh. When calibrated in the latter units, due regard must be paid to the declared voltage of the circuit in which the meter is to be installed.

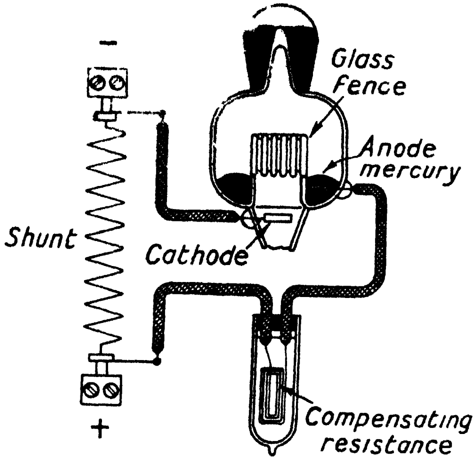


FIG 92.—Reason Electrolytic Meter—Schematic Diagram.

For the larger instruments, the measuring tube is made in the form of a siphon, and when full it automatically empties itself into the lower portion of the outer measuring tube, which is graduated in hundreds of units.

The current is conveyed to the electrolyte by means of a platinum wire which dips into the mercury anode, and it is led out again by another platinum wire which is embedded in the carbon cathode, or welded to the iridium cathode. As stated previously, the passage of the current deposits mercury upon the cathode and this loss is made good by the mercury anode; the electrolyte remaining a saturated solution.

Since the bore of the measuring tube is not always uniform, but the deposition of the mercury is directly proportional to the quantity of electricity passed through the meter, it is necessary for the manufacturers to calibrate individually each instrument of the higher capacity types having siphon tubes; the bore of the single-stem types, however, is uniform and a uniform scale is provided with the meter.

The Reason meter has a very low back e.m.f. of polarisation (about 10^{-4} volts) and, as a consequence, it is almost always shunted. This enables the rating of the meter to be increased, and the ratio of load current/tube current is always very high.

The tube is hermetically sealed and the meter is easily reset merely by tilting the tube. Owing to the increased rating of the meter, due to the provision of a shunt, this operation need rarely be performed. The larger sizes, for example, can be scaled to register up to 500,000 units for one setting.

This meter has the advantage of no moving parts to wear out, and there are no frictional losses to cause non-registration errors at low loads. It must, however, be handled with care on account of the amount of glass employed in its construction.

8.4. Mercury Motor Ampère-hour Meters. The mercury motor ampère-hour meter consists of a copper disc carrying current between the poles of a permanent magnet. The disc and the pole-pieces of the magnet are totally enclosed in a bath of mercury, the mercury being the medium of introduction, and exit, of the current to the disc. The width of the magnet gap is made about twice the thickness of the disc, and since the resistance of mercury is about fifty-nine times that of copper, most of the current flows through the copper disc.

Suppose I_M is the current flowing through the meter
Φ ,, flux density within the gap of the permanent magnet, in lines per sq. cm.
l ,, radial width, in cm., of the magnet gap
N ,, angular velocity of the disc, in r.p. sec
T ,, driving torque
r ,, mean radius of the disc, relative to the meter current
r_1 ,, mean radius of the disc cutting the flux of the permanent magnet
$K, K_1, K_2, K_3 \ldots$ are constants.

Then the torque $T = \dfrac{\Phi I_M l r}{10}$ dyne-cm.

$= K\Phi I_M$ dyne-cm.

This current will cause the disc to rotate, but since the disc thereby cuts the magnetic lines of force due to the permanent magnet, an e.m.f. will be induced in it. The value of this e.m.f. Eb, will be

$$Eb = \Phi 2\pi N r_1 l 10^{-8} \text{ volts}$$

Therefore, $Eb = K_1 \Phi N$ volts

Now, the eddy currents i_d set up in the disc will be proportional to Eb and

$$i_d = K_2 \Phi N$$

This current will react with the magnetic flux and create a force tending to oppose motion. The value of this force F will be proportional to $i_d\Phi$

$$\text{and } F = K_3 i_d \Phi$$
$$= K_4 \Phi^2 N \text{ dynes}$$

The braking torque T_b, required to overcome this force, will be Fr_1,

$$\text{therefore } T_b = K_4 \Phi^2 N r_1$$
$$= K_5 \Phi^2 N \text{ dyne-cm.}$$

As the disc is rotating in a medium of mercury, there will be a fluid-friction resistance between the mercury and the armature. This is approximately proportional to the square of the angular velocity of the disc, and the force will be in such a direction as to oppose motion.

The torque T_m necessary to overcome the fluid friction of the mercury will therefore be given by

$$T_m = K_6 N^2 \text{ dyne-cm.}$$

Lastly, there will be a torque T_f necessary to overcome the solid friction of the counter, pivots, and mercury. This will be fairly constant at all loads. It should be mentioned, however, that when the disc is stationary, the torque necessary to overcome solid friction will increase slightly, since the static friction between solid bodies is greater than the dynamic friction.

When the disc is running at a steady speed, the above-mentioned torques must balance. Hence

$$T = T_b + T_m + T_f$$
$$\text{or } K\Phi I_M = K_5 \Phi^2 N + K_6 N^2 + K_7$$

From the above expression, it follows that an increase of Φ will decrease N. In other words, if we require to decrease the speed of the meter, for any given current, it is necessary to increase the flux density within the permanent magnet gap. Conversely, if it is necessary to increase the speed of the meter, for any given current, one must decrease the flux density within the gap.

Some manufacturers employ one magnet as the driving and braking magnet of the meter, others employ an additional magnet for the sole purpose of providing a braking torque. In the latter case, the current passing through the disc does not come under the influence of the second magnet. The expression for the torque equation then becomes

$$K\Phi I_M = K_5(\Phi^2 + K^1 \Phi_2{}^2)N + K_6 N^2 + K_7$$

where Φ_2 is the flux density due to the additional magnet.

In the above expression it is assumed that there is no reaction between the eddy currents due to one magnet and the flux due to the other, and it can be seen that the variation in strength of the second—or brake—magnet does not affect the driving torque of the meter.

There will be two forces at work to prevent the meter from registering accurately at all loads. These forces will be due to the fluid friction and the solid friction of the meter. Since the solid friction is reasonably constant at all loads, it will have the greatest effect at low loads. The fluid friction, on the other hand, due to its being proportional to the square of the angular velocity of the disc, will become more apparent with increase of load. We can therefore expect that a meter which is not compensated for these characteristics will register low at light loads and also at high loads. This is found to be so in practice.

8.5. Compensation for Fluid Friction. In order to bring a mercury motor meter within the limits of accuracy prescribed by the Electricity Commissioners, it usually has to have compensation for the effect of fluid friction. This is arranged in the form of an iron-cored solenoid fitted within close proximity of the poles of the permanent magnet. The coil of the solenoid carries the meter current, and its polarity is so arranged that it decreases the magnetic flux acting upon the disc. An air gap is left between the solenoid and the permanent magnet in order that the decrease in the magnetic flux will be approximately proportional to the meter current.

In the case of the double-magnet type of meter, where both the magnets act as braking agents, but only one as the driving magnet, the compensating coil is so arranged that it increases the strength of the driving magnet, but weakens the other magnet by an amount of the same order. The result is that the total braking flux remains constant, but the driving flux is increased by an amount sufficient to compensate for the effect of fluid friction.

Considering the case of the meter with a single magnet, the flux density in the gap will now be $(\Phi-K_8I_M)$ and

$$K(\Phi-K_8I_M)I_M=K_5(\Phi-K_8I_M)^2N+K_6N^2+K_7$$
$$=K_5(\Phi^2-2K_8\Phi I_M+K_8^2I^2_M)N+K_6N^2+K_7$$

If we assume that $K_8^2I_M^2$ is sufficiently small to be neglected, we have

$$K(\Phi-K_8I_M)I_M=K_5\Phi^2N-2K_5K_8\Phi I_MN+K_6N^2+K_7$$

and, since I_M is approximately proportional to N,

$$K(\Phi-K_8I_M)I_M=K_5\Phi^2N-K_9N^2+K_6N^2+K_7$$
$$\text{or } K\Phi I_M=K_5\Phi^2N+K_7+(K_{10}N^2-K_9N^2+K_6N^2)$$
$$=K_5\Phi^2N+K_7+(K_{10}-K_9+K_6)N^2$$

Thus, by adjusting K_8, the constant of the solenoid, so that

$$KK_8I_M{}^2-2K_5K_8I_M\Phi N+K_6N^2=0$$

$$\text{i.e. } K_{10}-K_9+K_6=0$$

we have the relationship

$$K\Phi I_M=K_5\Phi^2N+K_7$$

In other words, except for the effect of solid friction, which cannot be compensated for in this type of meter, the speed can be made proportional to the meter current. With certain types of meter the speed is kept very low (about 20 r.p.m. at full load), and by so doing the mercury fluid friction is not excessive even at the higher loads. In fact, with such a low speed the mercury fluid friction often has an advantageous effect upon the meter.

Since, neglecting friction, the angular velocity is proportional to the current I_M,

$$I_M=K^1N$$

$$\text{and quantity} = \int_{T_1}^{T_2} I_M dt$$

$$= K^1\int_{T_1}^{T_2} N.dt \text{ ampère-hours}$$

Therefore, providing the gearing of the counting train is such that $1/K^1$ revolutions of the disc cause a registration of one unit on the counting dial, the meter will read directly in ampère-hours.

Assuming the voltage of the supply in which the meter is installed remains constant at V volts, the energy supplied to the consumer between T_1 and T_2 hours will be

$$K^1 . \frac{V}{1000}\int_{T_1}^{T_2} N.dt \text{ kWh}$$

and, providing the counting mechanism is so geared that $1000/K^1V$ revolutions of the disc spindle register one unit on the counting train dial, the meter will read directly in kWh.

8.6. Variations due to Change of Temperature. We will first consider the effects of change of temperature upon an unshunted meter.

The strength of a permanent magnet weakens with increase of temperature to the extent of 0·03 per cent per degree C. We have already seen that the driving torque is proportional to the magnetic flux, whilst the braking torque is proportional to the square of the flux. Therefore, neglecting the effects of friction,

$$\Phi(1+\alpha t)I_M=K\Phi^2(1+\alpha t)^2N$$

$$\text{and } N=K^1\left(\frac{I_M}{1+\alpha t}\right)$$

where t is the increase in temperature and α is $-0{\cdot}0003$, the temperature coefficient of the magnet.

$$\text{Thus } N = KI_M(1+3.10^{-4}t)$$

since the value of αt is small.

The meter will therefore tend to increase its overall speed by 0·03 per cent per degree C. rise in temperature, due to the change in strength of the magnet.

As the temperature coefficient of copper is greater than that of mercury (0·4 per cent per degree C. rise for copper and 0·07 per cent per degree C. rise for mercury), increase of temperature will result in a slightly different current distribution between the copper disc and the mercury; more current now passing through the mercury and less through the copper. The actual effect of this is very small and, at the most, is about 0·01 per cent per degree C.

Another effect of increase of temperature, due to the increase in resistance of the copper disc, is the consequent reduction in the eddy currents. This is about 0·4 per cent per degree C. Therefore, the braking effect will decrease proportionally and, neglecting friction, the meter will tend to increase in speed by 0·4 per cent per degree C. rise in temperature.

As the density of mercury decreases with increase of temperature, so the solid friction of the mercury is reduced and the meter speeds up slightly throughout the curve. This is particularly noticeable on the lower loads, and the lower the load the greater the percentage increase. Furthermore, the effect of fluid friction will be slightly less with increase of temperature, and this will cause a small increase in registration at the higher loads.

8.7. Shunted Meters. It is the practice of most manufacturers to shunt meters with a full load current rating of 10 ampères or more. In the case of heavy current meters, from about 100 ampères upwards, the shunt is generally situated external to the meter. The advantages of shunting meters have already been discussed in section 6.2.

Since the shunt will be made of a low temperature-coefficient alloy, its resistance change due to temperature will be negligible. On the other hand, the resistance of the meter circuit will increase, as this is mainly composed of copper; the value of this increase being approximately the same as that of copper (0·4 per cent per degree C. rise in temperature). The ratio of meter current to load current will therefore decrease by practically the same coefficient. The result is that the shunt improves the overall temperature coefficient of the meter, reducing it to about one-quarter of its original amount.

8.8. Self-heating Errors. Both unshunted and shunted meters have certain self-heating errors. These errors can be determined by running a meter, which has been off load for several hours, on full load for a period of about an hour. If the meter is initially timed throughout its curve, from low load to full load, and then re-timed after the hour's run, it will be found that there is a considerable difference between the results of the two tests. This difference in registration is termed the self-heating effect.

The whole curve will be about 1 per cent faster, due to the increased resistance of the disc after it has carried the full load meter current for this length of time. At low loads it will generally be found that the disc speed has increased by more than 1 per cent, due to the reduction of friction of the mercury, with increase in temperature. Therefore, in order to have standard conditions to which the recorded errors of a meter apply, it is usual to give the meters an hour or more on full load before commencing tests.

8.9. Ferranti Mercury Motor Ampère-hour Meter, Type FH. A meter of this type and manufacture is illustrated in Fig. 93. The

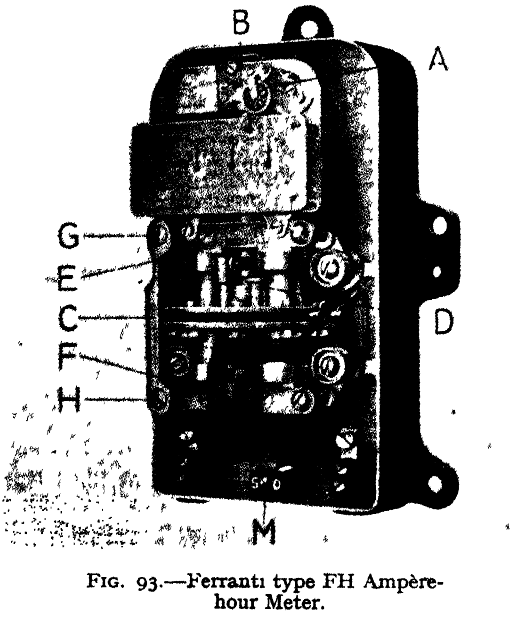
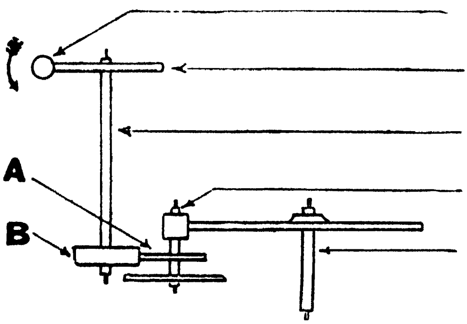


Fig. 93.—Ferranti type FH Ampère-hour Meter.

ıagnetic system consists of two similar permanent magnets. Their djacent poles, which are of like polarity, are connected by means of on bars; the lower bar being embraced by a coil carrying the meter ırrent. The meter is so designed that the current only passes through ne magnet gap. The magneto-motive force created in the coil by the ıeter current acts upon the bar in such a direction that it increases the flux in the driving and braking magnet, but decreases the flux in the magnet used only to produce a braking force. In consequence, compensation is provided for the effect of the mercury fluid friction by increasing the driving flux with increase of load. The braking flux also increases with load, but not being in as great a proportion as the driving flux, the overall effect is to speed up the meter. In order to prevent demagnetisation of the permanent magnets, in the event of a short circuit occurring on the load side of the meter, a coil is wound around the lower pole of the driving magnet. This coil is connected in series with the meter and compensating coil and carries the meter current in such a direction as to sustain the strength of the permanent magnet.



Meter rotor spindle single worm

Worm wheel 50 teeth.

Worm wheel spindle, carrying pinion "B".

Ratio wheel "A" spindle, also carrying upper test circle

Spindle of lower test circle (on dial plate).

FIG. 94 —Gear Train—Ferranti type FH Meter

The magnetic shunt *C* provides a small adjustment of the overall speed of the meter and its position can be varied by means of the screws *G* and *H*. In order to do this, it is first necessary to unlock screws *E* and *F*. To increase the speed of the meter, screws *G* and *H* have to be turned equally in a clockwise direction, thereby increasing the shunting effect of *C* upon the brake magnet. After completion of the adjustment, screws *E* and *F* must be re-locked. A special clamping device is fitted to the meter, and this fulfils the dual purpose of clamping the disc (thereby taking the pressure away from the jewelled seating) and sealing the mercury bath when the meter is in transit. This device is controlled by screw *M* and is not in operation when its pointer is in the "O" position. Extra mercury can be run into the mercury bath by means of screw-hole *D*, provided for filling the mercury bath after assembly, should any mercury leak during transit.

The revolution counter is driven by means of a single or double worm attached to the disc spindle. This arrangement is illustrated in Fig. 94. The ratio wheel *A* can easily be replaced, as it is fitted with an adjustable counter-shaft arrangement to permit of alteration of the gear ratio during initial calibration. For this purpose the manufacturer supplies tables indicating the ratio wheels required for each meter constant; due allowance being made for the declared voltage of the system in which the meter is to be installed.

8.10. The Meter Constant —K— is given by the expression

$$K=\frac{\text{ampères}\times\text{seconds}}{\text{revolutions}}$$

In other words, it is the ampère-seconds of electricity required per revolution of the disc. Thus, if a meter takes 120 seconds for the disc to make forty revolutions, with a current loading of 5 ampères, the meter constant

$$K=\frac{5\times 120}{40}$$

$$=15{\cdot}0$$

We have already defined the meter constant as the ampère-seconds per revolution. Hence, the reciprocal of this will be the revolutions per ampère-second, i.e. $1/K$.

The revolutions per ampère-hour will be $3{,}600/K$ and, where V is the supply voltage, the revolutions per kWh will be

$$\frac{3600\times 1000}{K\times V}$$

$$=3{\cdot}6\frac{10^6}{KV}$$

In the previous example, where K is $15{\cdot}0$, if the supply voltage is 200 volts, the revolutions per kWh will be given by

$$\text{r.p.u.}=\frac{3{\cdot}6\times 10^6}{15\times 200}$$

$$1200$$

and for the meter to read directly in kWh it must be geared so that 1,200 revolutions of the disc spindle register one unit on the counting rain dial.

The meter element is insulated from the case, but it must be remembered that the mercury bath, magnets, and counting train, etc., are alive when the meter is connected to the mains.

8.11. Chamberlain & Hookham Mercury Motor Ampère-hour Meter. A sectional view of this meter is given in Fig. 95. The magnetic system consists of a permanent magnet, having the wrought-ron pole-pieces B^1B^1 attached to it by means of the locating screws hown dotted in the diagram. These pole-pieces terminate in the ircular poles BB, the faces of which are amalgamated to provide good ontact with the mercury; thereby preventing the possibility of air bubbles lodging between the pole faces and the disc. These circular poles *BB* are screwed one in each of the two circular brass castings *EE*, which form the top and bottom of the mercury bath and project into the mercury. Both the circular poles *BB* and the brass castings *EE* are suitably insulated.

The bottom pivot of the spindle rests upon a cup jewel housed in the screw H, whilst the top consists of a pin-type bearing. Attached to the spindle is the copper disc *D* which rotates between the circular pole pieces; the clearance on each side being of the order of half the width of the disc. The disc is enamelled all over, with the exception of a thin annulus around the spindle and the rim; these portions being amalgamated. This has for its object the concentration of current in the disc, and also tends to reduce the mercury friction. The jacket of the mercury chamber consists of the metal ring *C*, which is lined on the inner side with cork or leather; this acting as a washer and insulator. Four insulated locating pins are provided in the top casting *E* to ensure correct alignment of the metal ring *C*. *G* is a brass weight sufficient to overcome the buoyancy of the copper disc in the mercury.

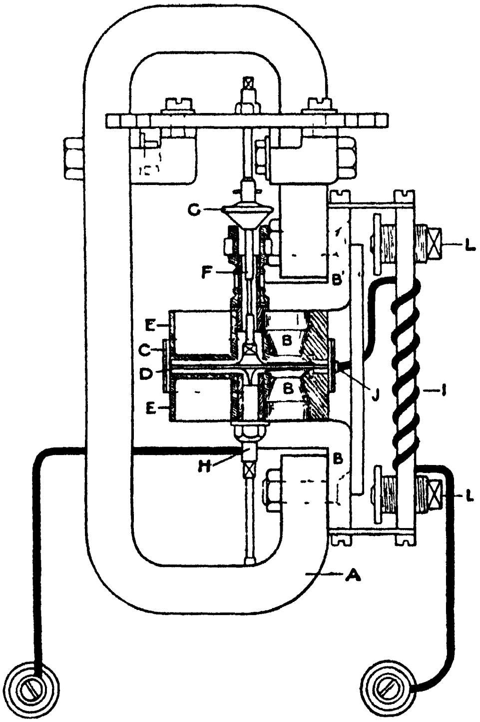


FIG. 95.—Schematic Diagram (Chamberlain and Hookham Ampère-hour Meter.)

Compensation for the effect of fluid friction is provided in the form of the coil *I*, which embraces an iron core, the latter being mounted on the pole-pieces B^1B^1 by means of brass plates. This coil carries the meter current and is wound in such a direction that it weakens the flux within the gap between the circular pole-pieces. Adjustment is provided in the form of the iron screws *LL*, the positions of which determine the influence of the coil upon the magnet. We have already seen that a weakening of the magnetic flux within the magnet gap will increase the overall speed of the meter and that when a coil carrying the meter current is employed for this purpose the effect will increase with load.

Fig. 96 gives a horizontal section showing the position of the pole, and disc. The current is introduced into the bath at the centre of the bottom casting *F* and leaves at the point *J*; thereby traversing the magnet gap. From *J* it flows to the negative terminal of the meters via the compensating coil.

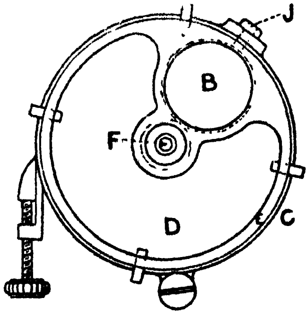


FIG. 96.—Horizontal Section of Mercury Bath (Chamberlain and Hookham Ampère-hour Meter).

The bracket carrying the counting train and top bearing is fixed to the permanent magnet. Attached to the spindle is a small pinion and this meshes into a wheel of the counting train. This wheel is termed the "change-wheel" and the number of its teeth, which are between 60 and 120, determine the gearing constant of the meter. This change-wheel is easily detachable from its spindle, being located by a coned seating and nut, and the initial calibration of the meter is performed by varying the size of this wheel. The manufacturer provides tables giving the meter constant, corresponding to each number of teeth, from 60 to 120; due allowance being made for the declared voltage at which the meter is to operate.

When the initial calibration has been made, the straightening of the error curve may be carried out by means of the iron screws *LL*. Thus, if the meter exhibits an undesirable negative error at full load, the screws are brought nearer the magnet.

The top circular casting *E* is provided with a valve for sealing the mercury bath during transit. It is controlled by means of an eccentric disc at the bottom of the meter case, operating a lever in a similar manner to that described in section 8.9.

8.12. Metropolitan-Vickers Mercury Motor Ampère-hour Meter, Type DM. This meter differs from the two previously described, inasmuch as all of the same size run at a constant speed for any given load. Figs. 97 (*a*) and (*b*) respectively give the diagrammatic circuit arrangement of up to 25 ampères and 40 to 600 ampères rating.

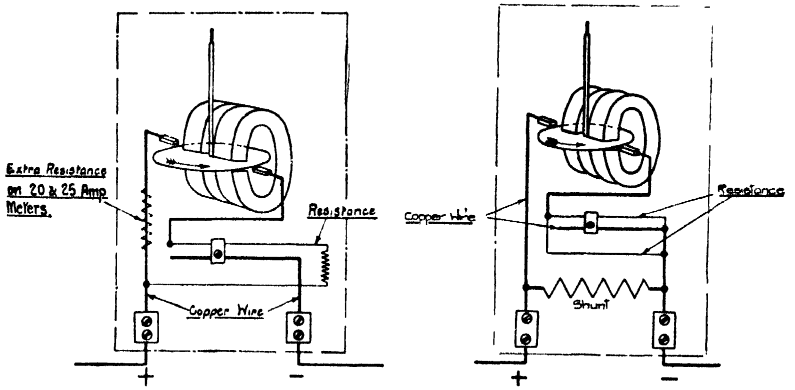


FIG. 97 (*a*). FIG. 97 (*b*).
Diagrammatic Circuit Arrangement (Metropolitan-Vickers Type DM Meter).

Adjustment of the speed of the meter is performed by alteration of the position of the movable clamp; movement to the left increasing the speed and movement to the right decreasing the speed of the meter.

In the case illustrated by Fig. 97 (*a*), movement to the left will decrease the resistance of the meter circuit between the clamp and the positive terminal, simultaneously increasing the value of the resistance which is virtually shunting the meter. This results in a greater percentage of the total load current passing through the meter, with a consequent increase in speed. The meters of 40 ampères or greater rating have a shunt of constant value, and movement of the clamp towards the left merely decreases the meter circuit resistance, resulting in a greater proportion of the total load current passing through the meter, thereby causing increase of overall speed.

The adjustment in both cases provides for a total change of speed of about 20 per cent, which should be of sufficient range for the meter to be recalibrated even after long periods of service, without changing the registration gear.

The meter consists of two small circular magnets moulded in a bakelite block which fits exactly into the main moulded bakelite container. A brass base-plate is screwed to the magnet block and bakelite container, and attached to the base-plate are two supporting pillars. To these are screwed the bridge piece containing the top bearing for the disc spindle, and the supports for the counting train. Mercury is poured into the filling hole until it reaches the underside of the base-plate.

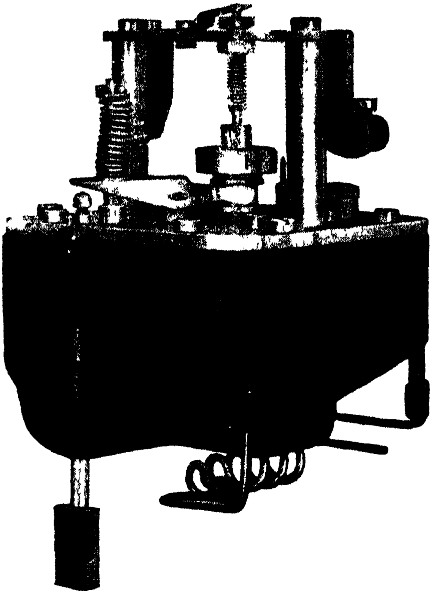


FIG. 98.—Working Parts Assembly (Metropolitan-Vickers Type DM Meter).

The disc runs centrally in the magnet gap, and attached to its spindle is a worm which drives the registration gear, the latter being fitted to the supports previously mentioned. A counter-weight is also attached to the spindle for the purpose of overcoming the buoyancy of the copper disc in mercury, and this is so shaped that it acts as the seating for the mercury bath sealing mechanism. This latter is of the conventional type described in sections 8.9 and 8.11.

An illustration of the working parts assembly is provided in Fig. 98. The current, in crossing the bath, passes between the poles of both permanent magnets and therefore both magnets act as driving and braking agents. A high torque is consequently attained together with a low speed (18·75 r.p.m. for 2·5 ampère meters, 25 r.p.m. for 5 ampère meters, and 37·5 r.p.m. for all meters of greater rating). In consequence, it has been found unnecessary to provide compensation for the effect of mercury fluid friction.

The temperature coefficient of this meter is exceptionally small, and this is attained by mounting the two cobalt magnets in opposite senses and providing a magnetic shunt, the permeability of which varies with temperature. Increase of temperature slightly diminishes the driving torque and simultaneously increases the braking torque; making thereby the registration more or less independent of temperature variations.

Ferranti Ltd. also manufacture a constant overall speed mercury motor ampère-hour meter. This meter, the type *FK*, is more compact than the meter described in section 8.9. Its constructional features are generally similar to those of the type *FH* meter except that a single permanent magnet is used to provide both the driving and braking torques. Overall speed adjustment is performed by means of a sliding bridge piece located under the mercury bath.

8.13. The Mercury Motor Watt-hour Meter. The mercury motor watt-hour meter differs from the ampère-hour meter inasmuch as the driving magnetic field of the former is supplied by an electromagnet energised by a winding carrying a current directly proportional to the supply voltage, instead of by a permanent magnet. Braking is provided by means of a second disc which rotates between the poles of a permanent magnet.

In general, the disc submerged in mercury, which carries the meter current, is slotted radially to prevent the circulation of eddy currents in it. Thus, the braking effect due to this disc is reduced to a minimum.

We have found, in the case of the ampère-hour meter, that the driving torque

$$T = K\Phi I_m$$

and the torque necessary to overcome the braking effect is

$$T_b = K_5\Phi^2 N$$

Now suppose, in the case of the watt-hour meter, that the effective flux due to the electro-magnet is Φ_v when the applied voltage to its winding is V, and that the flux due to the permanent magnet is Φ_p.

Then, where K_1, K_2, etc., are constants, the driving torque

$$T = K_1\Phi_v I_m$$

and the braking torque

$$T_b = K_2\Phi_p{}^2 N + K_3\Phi_v{}^2 N$$

If Φ_v is made small compared with Φ_p (K_2 and K_3 being more or less of the same order, depending upon the resistance offered to eddy currents by the respective discs) it can be neglected, and we have

$$T_b = K_2\Phi_p{}^2 N$$

Therefore neglecting the effects of fluid and solid friction

$$T = T_b$$

$$\text{and } K_1\Phi_v I_m = K_2\Phi_p{}^2 N$$

But Φ_v is proportional to the applied voltage, i.e. $\Phi_v = K_4V$. Hence

$$K_1K_4VI_m = K_2\Phi_p{}^2N$$

The power P expended in the load is equal to VI_m and Φ_p is constant, therefore

$$P = K_5N$$

Thus, neglecting friction, the disc rotates at a speed directly proportional to the power expended in the load and, as we have already found in section 7.1, if suitable gearing is provided to the registration train, the meter will read directly in kWh irrespective of (reasonable) fluctuations in the supply voltage.

8.14. Compensation for Fluid Friction. In the mercury motor d.c. watt-hour meter, compensation for fluid friction is provided in the form of a coil, carrying the meter current I_m, wound around the electro-magnet in such a direction that it strengthens the field of the electro-magnet. The driving torque will therefore be

$$T = I_mK_1(\Phi_v + K_7I_m)$$

and the braking torque

$$T_b = K_2\Phi_p{}^2N + K_3(\Phi_v + K_7I_m)^2N$$

but $K_3(\Phi_v + KI_m)^2$ will be small compared with $K_2\Phi_p{}^2$ and may therefore be neglected.

The total torque equation will now be

$$I_mK_1(\Phi_v + K_7I_m) = K_2\Phi_p{}^2N + K_8N^2 + K_9$$

where K_8N^2 and K_9 are the torques required to overcome respectively the fluid and solid friction.

Providing the supply voltage does not substantially depart from its rated value, I_m will be proportional to N

$$\text{or, } K_1K_7I_m = K_{10}N^2$$

Therefore, if the value of K_7 is so arranged that $K_1K_7I_m{}^2 = K_8N^2$, the torque equation will become

$$K_1I_m\Phi_v = K_2\Phi_p{}^2N + K_9$$

and the meter will be compensated for the effect of fluid friction.

At all normal loads, the torque K_9 necessary to overcome the solid friction will be small compared with the driving and braking torques, and the speed of the meter will be approximately proportional to the power expended in the load.

8.15. Compensation for Solid Friction. Compensation for the solid friction torque may be provided in the form of a thermo-couple which is heated by means of a coil connected in series with the pressure coil across the supply voltage; the thermo-couple being connected to the current terminals of the mercury bath in such a manner that its current assists the meter current. Since the current passing through the pressure coil circuit will be reasonably constant, the e.m.f. of the thermo-couple will also remain constant and, by suitable resistance adjustment, it can be made to compensate for solid friction. Care must be taken to ensure that the torque created by the thermo-couple current will not be sufficient to permit of the meter registering on no-load.

8.16. Temperature Variations. The variations due to temperature change will be the same as those for the ampère-hour meter with the addition of a further variation due to the winding of the pressure coil, since the latter is wound with copper wire.

If the pressure coil is connected directly across the supply voltage, its current will decrease by 0·4 per cent per degree C. rise in temperature. If, on the other hand, it is connected in series with a swamping resistance of negligible temperature coefficient, the current will decrease to a lesser degree.

Suppose R_p is the resistance of the pressure coil

and R_s is the resistance of the swamping resistance,

then the decrease in current per degree C. rise in temperature will be

$$\frac{R_p}{R_p+R_s} \times 0{\cdot}4 \text{ per cent}$$

This decrease in current has the effect of slowing the meter down, and as the other errors due to increase of temperature tend to increase the overall speed of the meter it is possible to make the overall temperature coefficient negligible. Suppose the meter speeds up to x per cent per degree C. rise in temperature, due to the variations mentioned in the section dealing with ampère-hour meters, then if

$$x = \frac{R_p}{R_p+R_s} \times 0{\cdot}4 \text{ per cent}$$

there will be no change of overall speed with change of temperature.

8.17. Self-heating will also introduce errors in the watt-hour meter, particularly at low loads, due to variation with change of temperature of the solid friction of the mercury. It is therefore advisable to run the meter for a considerable time on full load before commencing calibration. The self-heating effect is pronounced by the length of time required to heat up the great mass of metal which forms the electro-magnet core.

8.18. Hysteresis Error. Another error may arise, due to hysteresis when the meter is connected in a variable voltage supply such as that of a traction circuit. This will tend to make the meter read high with decreasing voltage and low with increasing voltage.

8.19. Theory of the Eddy Current Brake. We have already noted that a braking force can be attained by the rotation of a disc between the poles of a permanent magnet. The great advantage of such a device is that the braking force, for any given position of the magnet, is directly proportional to the velocity of the disc

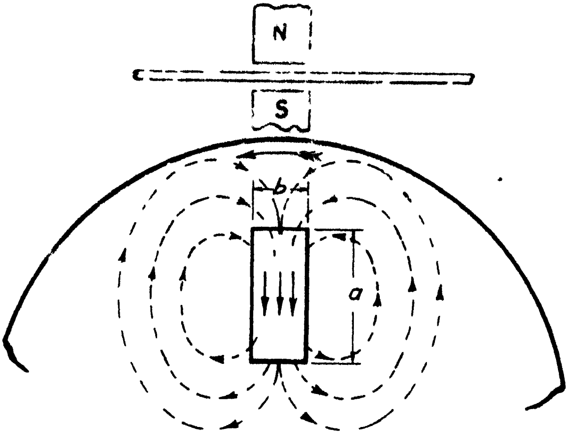


FIG. 99.—Eddy Current Brake.

Consider a disc rotating between the poles of a permanent magnet of flux density B, and with pole faces of dimensions a by b cm., as illustrated in Fig. 99; the position of the poles being as shown. Then the e.m.f. induced in the portion of the disc passing through the magnetic field of the permanent magnet will be $aB\omega r10^{-8}$ volts, where ω is the angular velocity of the disc in radians per second and r is the distance between the axis of the disc and the centre of the magnet pole faces.

This will produce a current in the disc, in the direction indicated by the arrow, and its width will be b cm., i.e. the width of the pole faces. If t is the thickness of the disc, and ρ its specific resistance, the resistance of the portion of the disc between the pole faces, to the passage of the current, will be

$$\frac{a\rho}{bt} \text{ ohms}$$

Since the current must have a return path, it follows that the resistance of the total path must be greater than this. We will call this ratio, of total resistance of the path to resistance within the magnet gap, K. For a disc of infinite area, the value of K would be unity.

Providing the dimensions of the pole faces are small compared with the diameter of the disc, and the poles are not near to the edge of the disc, the current distribution will be similar to that illustrated in Fig. 99. If the disc is very large compared with the pole face area, the resistance of the return path will be negligible and we can assume that $a\rho/bt$ ohms is the total resistance to the current, i.e. that $K=1$. In general, K will have a greater value than unity, and the resistance of the total current circuit will be

$$R=\frac{Ka\rho}{bt}\text{ ohms}$$

The current will be

$$I=\frac{E}{R}=\frac{aB\omega r10^{-8}bt}{Ka\rho}$$

$$=\frac{bB\omega rt10^{-8}}{K\rho}\text{ ampères}$$

The braking force will be given by

$$F=\frac{aBI}{10}$$

$$=\frac{abB^2\omega rt10^{-9}}{K\rho}\text{dynes}$$

but $abB=\Phi$, the total flux of the magnet, therefore

$$F=\frac{\Phi^2r\omega t10^{-9}}{abK\rho}\text{dynes}$$

The torque $T=Fr$

$$=\frac{\Phi^2\omega r^2t10^{-9}}{abK\rho}\text{ dyne-cm.}$$

and the damping constant

$$=\frac{\text{torque}}{\omega}$$

$$=\frac{\Phi^2r^2t10^{-9}}{abK\rho}\text{dyne-cm./radian/sec.}$$

The value of K will increase as the magnet reaches the edge of the disc, since the return path for the current will be restricted.

We see from the above that there will be an optimum position for the magnet poles, in order to obtain the maximum damping constant. As the magnet is moved from the centre of the disc, so the damping constant will increase, until the magnet reaches a position where K begins to increase. As the magnet is brought still nearer the edge of the disc, the damping constant will increase at a lower rate until it reaches a maximum at a position a little from the edge. It will then drop rapidly as the centre of the magnet pole reaches the edge of the disc.

The shape and size of the magnet poles are of considerable importance. Since the resistance of the eddy current path increases rapidly as the magnet approaches the edge of the disc, it follows that the smaller the magnet and the higher the flux density, the greater will be the efficiency of damping. It also follows that conically shaped pole pieces on the pole faces of a circular magnet would give a very high efficiency of damping. On the other hand, small pole faces tend to reduce the permanency of a magnet. Therefore, although the large pole faces of the damping magnets for electricity supply meters are not efficient as regards the ratio torque/ω, they have the outstanding advantage of being much more permanent than the conical magnet. For this purpose, the gaps are made as small as possible, and the pole faces are sufficiently large to avoid a high demagnetising factor.

8.20. Demagnetising Coefficient. Du Bois and Lehmann have shown that the demagnetising effect of almost-closed magnets is approximately proportional to the width of the gap. The value of the demagnetising coefficient K is given by

$$K = \frac{4\pi}{\lambda}\left(\frac{l_a/A_a}{l_m/A_m}\right)$$

where λ is the leakage factor of the magnet (i.e. ratio of the flux at the middle cross-section of the iron circuit to the useful flux in the gap) l_a is the length of the air gap, A_a is the area of each pole face, l_m is the length of the magnet, and A_m is the cross-sectional area of the magnet. Thus, to have a small value of K, the ratios l_a/A_a and A_m/l_m must be small, and the larger the value of λ the better.

8.21. Safety Factor of a Magnet. Heinrich and Bercovitz have termed the ratio $(A_a/l_a) / (A_m/l_m)$ the safety factor of a magnet. The value for supply meter brake magnets is in the region of 100, and may be even better for cobalt steel magnets.

8.22. Ageing of Magnets. When a magnet is first magnetised, usually to saturation, there will be a temporary component in its magnetism which it will lose in time. The magnet is therefore initially unstable and, if wanted for such a purpose as that of producing a damping flux for an electricity supply meter, it is of paramount importance that it be rendered stable before calibration of the instrument. This is attained by demagnetising it by about 15 per cent of its initial strength. It should then be left to age for about three months and, if its strength remains constant over this period, it may then be regarded as reasonably permanent.

It is possible artificially to age the magnet to a more or less permanent condition. This may be done by

(*a*) Heat treatment
(*b*) Mechanical shock
(*c*) Partial demagnetisation in an alternating field.

Artificial ageing is often effected in addition to the above-mentioned natural ageing. Of the three methods, (*c*) is the most convenient and, needless to say, the one in most general use. Some manufacturers completely assemble their meters first, and then finally age the magnets by placing the whole meter in a rotating magnetic field, increasing the field strength until the desired stability has been obtained. It should, however, be mentioned that only time can show beyond doubt that a magnet is stable.

Much improvement has been effected in the manufacture of brake magnets within the last few years, especially since the advent of the cobalt-steel magnet with its high coercivity of about 500 c.g.s. units. This has led to a great reduction in the ratio $\Phi./\Phi_p$ and a decrease in overall speed of meters, with a consequent improvement in the load curve of the meter (section 9.7).

8.23. Magnetic Definitions. We will briefly define certain quantities related to magnets. Although these can be found in text-books, a repetition may not be entirely wasted.

The pole strength of a magnet is equal to the force in dynes between it and a unit pole situated 1 cm. away.

The moment of a magnet is equal to the product of its pole strength and the distance between its poles.

The magnetic flux (Φ) is the number of lines of force between the poles of the magnet. In the case of a brake magnet, the number of lines which cut the disc.

The flux density (B) is the amount of flux per unit area between the pole faces. It is generally expressed in lines per sq. cm.

The intensity of magnetisation (I) is the magnetic moment per cu. cm. of magnet.

The permeability (μ) is the ratio of the flux density (B) induced, to the magnetising field (H).

The magnetic susceptibility is the ratio $I : H$.

Remanence is the magnetisation which exists in the magnet when, under cyclic conditions, the magnetising field is reduced to zero from some maximum value of H. This is usually expressed in terms of B or H.

Coercivity is the reversed magnetising field required to reduce the intensity of magnetisation to zero.

8.24. Metropolitan-Vickers Mercury Motor D.C. Watt-hour Meter, Type DE. This type of meter is illustrated schematically in Fig. 100. It is comprised of a spindle carrying two discs, the lower of which is slotted radially and is enveloped by a mercury chamber in which are situated the poles of an electro-magnet. The upper disc rotates between the poles of a permanent magnet and acts as a damping agent. A pinion is cut on the common disc spindle and this gears into a counting train.

The electro-magnet consists of two vertical limbs suspended from poles in the mercury chamber. Each of these limbs carries a coil, and they are connected in series with the swamping and thermo-couple heater resistances across the supply voltage. The gap between the lower ends of the electro-magnet limbs is almost closed by means of an adjustable yoke.

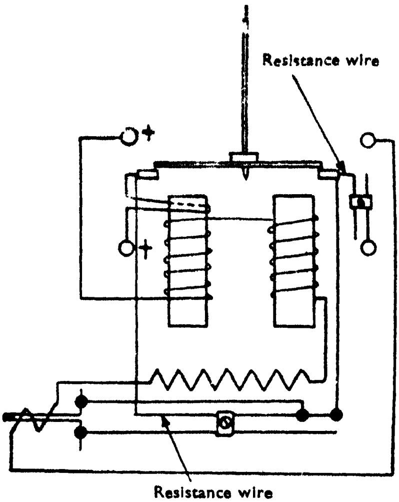


FIG. 100.—Diagrammatic Arrangement of D.C. Watt-hour Meter (Metropolitan-Vickers).

The common disc spindle is supported by a hardened steel pivot seated upon a sapphire jewel, and has a journal upper bearing. An arrangement is also provided for the sealing of the mercury bath to ensure safe transit. This is operated by means of a screw which is accessible without opening the case of the meter.

A compounding turn is placed around one electro-magnet pole. This is shunted by means of a fixed low resistance; the turn carrying a definite proportion of the meter current. It is arranged to assist the electro-magnet flux and is of such a value that it compensates for the effect of fluid friction.

A thermo-couple is provided. This is heated by means of the pressure circuit current, and it is connected across the mercury bath terminals, in the manner described in section 8.15, in order to compensate for solid friction. Movement to the left increases the compensating torque and movement to the right decreases it. This adjustment is in the form of a clamp which short-circuits a section of the resistance in series with the thermo-couple.

The meter has a six-pointer register, the dial of the smallest denomination being of an extremely large diameter and having 100 divisions.

A modification of the above meter is made suitable for the wider range of voltage experienced in a traction circuit. In this meter a biasing permanent magnet is introduced into the electro-magnetic field for the purpose of compensating for the errors which would arise from hysteresis with the variable voltage of a traction circuit.

Adjustment of the overall speed of the meter is performed by means of the permanent magnet acting upon the aluminium upper disc. This operates upon the edge of the disc and decreases the overall speed of the latter as more of the magnet is brought to act upon the disc.

On all standard meters the full load speed is 20 r.p.m. and the revolutions of the disc, per kWh, are given by

$$\text{r.p.u.} = \frac{(\text{r.p.m of the disc at full load}) \times 60}{\text{Full load rating of meter in kW}}$$

8.25. Chamberlain & Hookham Mercury Motor D.C. Watt-hour Meter. A section through the working portion of this meter is given in Fig. 101. The copper disc *A*, which is slotted radially to minimise the eddy currents created by the electro-magnetic flux, is carried by the main spindle and rotates in the mercury bath. The magnetic system consists of a U-shaped laminated electro-magnet *K*, which is located so that its poles just clear the underside of the driving disc, and its magnetic circuit is completed by the iron ring *Q* placed immediately above the centre of the disc. Thus, the magnetic flux cuts the driving disc twice. The coils *GG*, which are carried one on each of the limbs of the electro-magnet, are connected in series across the supply voltage. Hence, the current passing through the coils, and the flux thereby created in the electro-magnet, will be directly proportional to the applied voltage. Between the upper and lower portions of the mercury bath is fitted a ring *W* of insulating material, and this carries the two terminals *XX*, by means of which the current enters and leaves the bath. A second disc, *O*, of aluminium is also carried upon the main spindle, and this runs between the poles of a permanent magnet, for the purpose of creating a brake torque for overall speed control.

The lower end of the spindle has a hard steel pivot which rests upon a jewelled bearing housed in the screw *F*. This screw can be adjusted for the purpose of setting the driving disc in its correct position in the mercury chamber. The upper end of the spindle consists of a pin-type bearing which fits into the journal *M*.

The mercury chamber is constructed on the unspillable inkwell principle, but to avoid losses due to severe shaking during transit, the valve *J* is provided. This is lifted into contact with the leather washer on the underside of the disc, by means of a flat forked spring which is introduced before transit between the top of the mercury bath and the underside of the valve. Before commencing tests, this spring must be removed.

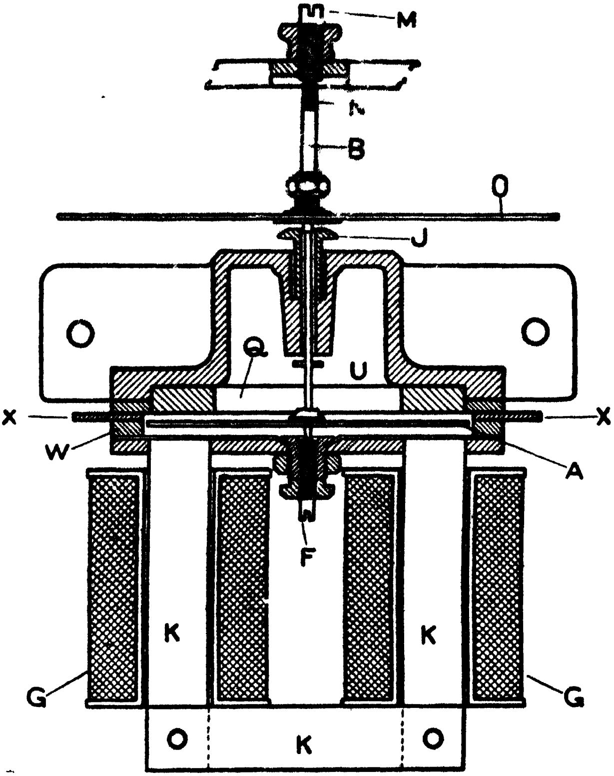


FIG. 101.—Schematic arrangement of D C Watt-hour Meter (Chamberlain & Hookham Ltd)

The motion of the spindle is transmitted to the registration gear by means of the pinion *N*, and compensation for fluid friction is obtained by compounding the electro-magnet with several turns carrying the meter current.

All meters of the same size are geared to run at the same full load speed (of the order of 20 r.p.m.), and an illustration of a switchboard pattern meter is given in Fig. 102. The clamps, shown one on each side of the electro-magnet, are provided for adjustment of the meter resistance when the meter is used in conjunction with a shunt. It is usually of such a value that, when the full load meter current is flowing, the millivolt drop across the meter current circuit is 100 mv. For this adjustment, the full load meter current is passed through the meter circuit and the clamps are altered in position until the voltage drop across the meter and leads is the same as that across the shunt when the rated full load current, minus that passing through the meter, is flowing through the shunt. In other words, if I_M is the full load meter current, and I is the actual full load of the meter and shunt, the drop across the meter, when carrying I_M ampères, must be the same as that across the shunt when the latter is carrying $I-I_M$ ampères. In practice, this adjustment must be made whilst the disc is rotating.

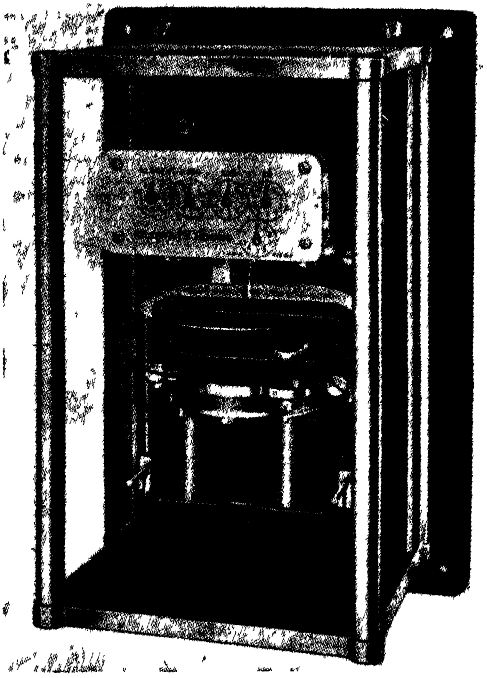


FIG. 102.—Switchboard pattern D.C. Watt-hour Meter showing Adjusting Clamps (Chamberlain & Hookham Ltd.)

8.26. Sangamo Mercury Motor D.C. Watt-hour Meter, Type D5. This d.c. watt-hour meter is similar in general arrangement to the two types already described. The effect of mercury fluid friction is compensated for by compounding the electro-magnet in the manner described in section 8.25. Solid friction is compensated for in the manner described in section 8.15. It differs, however, from the other two watt-hour meters inasmuch as a float is fitted to the spindle immediately above the driving disc in the mercury chamber. In this manner, a force of about 3 grammes is obtained to press the moving system against the top bearing, which contains a sprung jewel. A pin type bearing is provided at the lower end of the spindle.

8.27. Commutator type D.C. Watt-hour Meter. As this type of meter is now obsolete in this country, we will only consider its general action. A diagrammatic arrangement is given in Fig. 103. In principle, the meter consists of an armature *A* wound on a non-conducting former, the windings of which are connected to a commutator. The armature carries a current I_p proportional to the supply voltage E, since it is connected across the mains, and the two large coils CC carry the load current. The action is therefore similar to that of a separately excited d.c. shunt motor, and the torque on the armature will be proportional to the product of the armature current and the magnetic flux due to the large coils. Hence

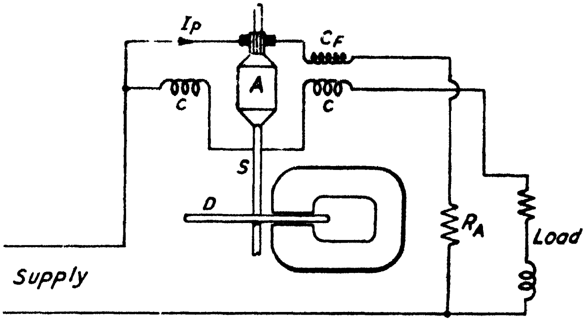


Fig. 103.—D.C. Commutator Meter.

$$T \propto I_p \Phi_c$$

where I_p is the armature current and Φ_c is the flux in the current coils.

$$\text{But } I_p \propto E \text{ and } \Phi_c \propto I$$

where E is the supply voltage and I is the load current.

$$\text{Therefore, } T \propto EI$$

which is the power expended in the circuit.

Braking is effected by means of the copper disc *D*, attached to the main armature spindle *S*. The disc rotates between the poles of a permanent magnet; the latter being magnetically screened from the armature field. Thus, the torque T_b necessary to overcome the braking force will be given by

$$T_b = KNr$$

where K is a constant and r is the effective radius of the braking disc.

Calibration is performed by variation of the armature current. Provision for this is made in the form of a resistance R_A in series with the armature circuit, the value of which can be varied. Compensation for friction is usually in the form of a movable coil C_F, carrying the pressure circuit current. This is placed in such a position relative to the armature that a torque is created upon the armature, due to the interaction of the flux in the movable coil with the current in the armature, of a value just sufficient to overcome the friction. No registration must occur on no-load, due to this compensating device, and sometimes an iron wire is placed on the spindle in such a position that it will come under the influence of the permanent magnet and prevent the armature from creeping forward a whole revolution.

The moving system of this type of meter is essentially very heavy and, as a consequence, rapid wear of the jewels is likely to take place, particularly if any vibration of the system occurs.

With increase of temperature, the following variations tend to occur in the commutator meter:

(*a*) The armature speeds up 0·4 per cent per degree C., due to increase in the resistance of the brake disc.

(*b*) An increase of speed of 0·06 per cent, due to the decrease in strength of the brake magnet, for every 1° C. rise in temperature.

(*c*) The speed tends to decrease by 0·4 $R_c/(R_c+R_s)$ per cent per degree C., due to the increase in pressure circuit resistance; R_c being the resistance of the copper section and R_s being the value of the series swamping resistance of negligible temperature coefficient. This value is usually very small, owing to the employment of large calibrating resistances.

The use of a shunt improves the temperature coefficient, since rise of temperature will decrease the meter current and compensate to a certain extent for the errors (*a*) and (*b*). When the ratio meter current/load current is small, the decrease in speed will be 0·4 per cent per degree C. rise in temperature, but as this ratio increases, so the compensating effect will decrease. Furthermore, in order to obtain a large driving torque, the meter current is usually fairly large.

In most commutator type meters, the self-heating errors are also very large, due to the considerable watt-loss in the pressure and current coils. This is accentuated by the fact that there is little possibility of the heat getting away, as the meter is fitted with an airtight case.

8.28. Pendulum Type Watt-hour Meter. This type of meter consists of two pendulums which have the same normal beat frequency.

Attached to each pendulum is a coil, which is co-axial with the pendulum rods, the coils being energised by the supply voltage. Beneath each pendulum, and co-axial with the pendulum rod when the latter is in its vertical position, is a fixed coil carrying the load current. The two systems are mechanically symmetrical; the fixed coils being in series with the load and the pendulum coils being in series across the supply voltage. The coil connections are so arranged that the force exerted by the action of the one fixed coil assists the gravitational force upon its own pendulum, and the action of the other fixed coil opposes the gravitational force upon the other pendulum. For a given current in the fixed coils, the magnitudes of the forces exerted on the pendulums are equal.

When the current is flowing through the fixed coils, the one pendulum will increase its speed of oscillation and the speed of the other will decrease by the same amount; the difference in speed being proportional to the product of the voltage and the current. Therefore the difference in speed of oscillation is proportional to the power expended in the circuit, and this is integrated through the medium of a differential gear suitably geared to a counting mechanism.

8.29. Theory of Simple Pendulum. Let us first consider the action of a simple pendulum (Fig. 104 (*a*)). Suppose *m* is the mass of

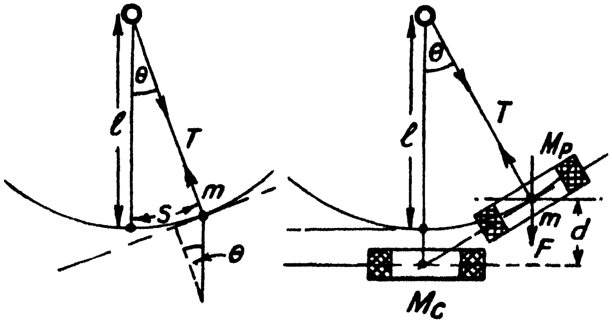


FIG. 104 (*a*) FIG. 104 (*b*)
Simple and Electrical Pendulums

the pendulum, *l* is its length, *θ* is the angle which, at any time *t*, the pendulum makes with the vertical, and *T* is the tension in the rod of the pendulum; the rod being sufficiently light compared with the weight of the pendulum that its mass may be neglected.

The free end of the pendulum will move along the arc of a circle whose radius is *l*, and the length of the arc measured from the lowest point will be

$$s = l\theta$$

$$\text{The velocity } v=\frac{ds}{dt}=l\frac{d\theta}{dt}$$

θ and $\frac{d\theta}{dt}$ being considered positive when moving outwards.

There will only be two forces acting on the end of the pendulum, these will be its weight mg acting vertically downwards, and the tension T in the rod.

Resolving outwards along the tangent of the path, there will be no component due to T, therefore

$$ml\frac{d^2\theta}{dt^2}=-mg\sin\theta$$

$$\text{Therefore } l\frac{d^2\theta}{dt^2}=-g\sin\theta$$

Since θ will always be small, $\sin\theta=\theta$

$$\text{and } l\frac{d^2\theta}{dt^2}=-g\theta$$

$$\text{or } \frac{d^2\theta}{dt^2}=-\frac{g}{l}\theta$$

This is in the form of $d^2\theta/dt^2=-\omega^2$, where $\omega^2=g/l$. It therefore represents simple harmonic motion, and its frequency of oscillation

$$f_o=\frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

8.30. Differential Pendulum—Theory. Let us now consider the action of the pendulum when the mass at its free end consists of a coil of mass m, carrying a current proportional to the supply voltage, acted upon by the magnetic flux of a fixed coil carrying the load current. Then Fig. 104 (*b*) will represent the conditions when the pendulum is displaced by an angle θ from the vertical.

Providing the air gap is large, or there is an absence of magnetic material, the magnetic moments of the coils will be proportional to the respective currents passing through them. Furthermore, if the angle of swing is very small, the coils may be considered to be in their end-on position at all positions of swing, and the force exerted upon the pendulum will be

$$F=\frac{M_pM_c}{d^2}$$

where M_p is the magnetic moment of the pressure coil, M_c is the magnetic moment of the current coil, and d is the distance between the two coils when the pendulum is in the vertical position.

$$\text{But } M_p=KV, \text{ and } M_c=K_1I$$

where V is the supply voltage and I is the load current.

$$\text{Therefore, } F=\frac{KK_1VI}{d^2} \text{ dynes}$$

$$K_2VI$$

and, since VI is the power P expended in the circuit,

$$F=K_2P \text{ dynes.}$$

This force F will act at all positions of the swing, since the distance between the two coils will remain sensibly constant under all conditions.

Resolving outwards, along the tangent to the path, we have

$$ml\frac{d^2\theta}{dt^2}=-mg\sin\theta\pm K_2P\sin\theta$$

$$\text{or } \frac{d^2\theta}{dt^2}=-\left[(g/l)\mp K_3P\right]\sin\theta$$

which approximates to

$$\frac{d^2\theta}{dt^2}=-\left[(g/l)\mp K_3P\right]\theta$$

and the frequency of oscillation

$$f=\frac{1}{2\pi}\sqrt{\frac{g}{l}\pm K_3P}$$

In the case of the coils being wound so that the force due to the magnetic moments assists the force of gravity, the frequency of oscillation, f_1, will be given by

$$f_1=\frac{1}{2\pi}\sqrt{\frac{g}{l}+K_3P}$$

When they are wound so that the force opposes that due to gravity, the frequency f_2 will be given by

$$f_2=\frac{1}{2\pi}\sqrt{\frac{g}{l}-K_3P}$$

Therefore, the difference in frequency of oscillation, f_d, of the two pendulums will be given by

$$\begin{aligned} fd&=f_1-f_2\\ &=\frac{1}{2\pi}\left[\left(\frac{g}{l}+K_3P\right)^{\frac{1}{2}}-\left(\frac{g}{l}-K_3P\right)^{\frac{1}{2}}\right]\end{aligned}$$

and, expanding by the binomial theorem, we have

$$f_d=\frac{1}{2\pi}\left\{\left[\left(\frac{g}{l}\right)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{g}{l}\right)^{-\frac{1}{2}}K_3P-\frac{1}{8}\left(\frac{g}{l}\right)^{-\frac{3}{2}}\left(K_3P\right)^2+\frac{1}{16}\left(\frac{g}{l}\right)^{-\frac{5}{2}}\left(K_3P\right)^3-+-\right]\right.$$
$$\left.-\left[\left(\frac{g}{l}\right)^{\frac{1}{2}}-\frac{1}{2}\left(\frac{g}{l}\right)^{-\frac{1}{2}}K_3P-\frac{1}{8}\left(\frac{g}{l}\right)^{-\frac{3}{2}}\left(K_3P\right)^2-\frac{1}{16}\left(\frac{g}{l}\right)^{-\frac{5}{2}}\left(K_3P\right)^3---\right]\right\}$$

$$-\frac{1}{2\pi}\left\{\left(\frac{g}{l}\right)^{-\frac{1}{2}}K_3P+\frac{1}{8}\left(\frac{g}{l}\right)^{-\frac{5}{2}}K_3P^3++\right\}$$

$$=\frac{1}{2\pi}\left\{\left(\frac{g}{l}\right)^{\frac{1}{2}}\left[\left(\frac{g}{l}\right)^{-1}K_3P+\frac{1}{8}\left(\frac{g}{l}\right)^{-3}\left(K_3P\right)^3++\right]\right\}$$

$$f_d-f_o\left[\left(\frac{g}{l}\right)^{-1}K_3P+\frac{1}{8}\left(\frac{g}{l}\right)^{-3}\left(K_3P\right)^3++\right]$$

where f_o is the frequency of the pendulums when under no other influence than that of gravity.

We can see from the above expression that if the difference in frequency is to be directly proportional to the load, the terms which have higher powers than the first power of P must be negligible in comparison with the first power term. Thus, $\frac{1}{8}\left(\frac{g}{l}\right)^{-2}\left(K_3P\right)^2$ must approach zero.

Suppose the second term causes the frequency difference to depart from linearity by only 0·125 per cent at full load. Then

$$\frac{1}{8}\left(\frac{g}{l}\right)^{-2}\left(K_3P\right)^2-0{\cdot}125 \text{ per cent}$$

$$\left(\frac{g}{l}\right)^{2}\left(K_3P\right)^2=\frac{10}{1000}$$

$$\text{Therefore}\quad\left(\frac{g}{l}\right)^{-1}\left(K_3P\right)=\frac{1}{10}$$

and the difference in frequency, f_d, at full load will be of the order of 10 per cent of the normal frequency, f_o, of each pendulum.

The higher order terms of the power P will obviously be negligible, and it follows that this will give a more or less linear law relating power expenditure in the circuit to the difference in frequency between the two pendulums. Providing this difference in frequency is employed to drive a suitably geared registration train, through the medium of a differential gear, the meter will read directly in kWh.

8.31. Differential Gear—Theory. A simple differential gear is illustrated in Fig. 105. The sun-wheels S_1 and S_2 ride freely on the shaft B, and the planet-wheel P rides freely on the arm A which is rigidly fixed to shaft B. The planet-wheel meshes with both sun-wheels.

With the arm A fixed, we will give one revolution to the sun-wheel S_1. The revolutions of each wheel will then be as follows:

$$P=\frac{S_1}{P}$$

$$S_2=-\frac{S_1}{P}\times\frac{P}{S_2}$$

$$= -\frac{S_1}{S_2}$$

where the nomenclature S_1, S_2, and P represent the number of teeth, as well as the identity of the wheels.

The shaft B will obviously not rotate since it is rigidly fixed to the arm A.

From the above we can construct a table giving the number of revolutions of each component for certain conditions.

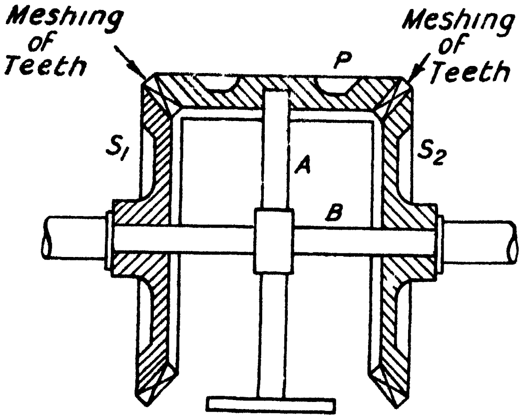


FIG 105—Simple Differential Gear

Operation	Condition	Revolutions made by				
		A	S_1	S_2	P	B
1	A fixed, give S_1 1 rev.	O	1	$-\frac{S_1}{S_2}$	$\frac{S_1}{P}$	O
2	Multiply by X	O	X	$-\frac{XS_1}{S_2}$	$\frac{XS_1}{P}$	O
3	Add Y revs. to all	Y	$X+Y$	$Y-\frac{XS_1}{S_2}$	$Y+\frac{XS_1}{P}$	Y

In the case of the pendulum meter, the sun-wheels S_1 and S_2 are geared to the escapement shafts in such a manner as to rotate in opposite directions, and providing we know the number of teeth on every wheel and the motion of two of the wheels, we can find the motion of the whole system.

Suppose that the sun-wheels are each driven at 90 r.p.m. by their respective escapement shafts when there is no load upon the meter (i.e. when the pendulums are both beating at the same frequency). Then, from the above table

$$X+Y=90$$

$$\text{and } -\frac{S_1}{S_2}X+Y=-90$$

We must have no rotation of the shaft B when no load is passing through the meter, therefore

$$Y=O$$

$$\text{from which } X=90$$

$$\text{Also, since } -\frac{S_1}{S_2}X=-90$$

$$S_1 = S_2$$

In other words, there must be the same number of teeth upon each sun-wheel.

Now, suppose a load is switched on, and this speeds up S_1 by R r.p.m. and slows down S_2 by the same amount. Then

$$X+Y=+(90+R)$$

$$\text{and } -\frac{S_1}{S_2}X+Y=-(90-R)$$

$$\text{i.e. } -X+Y=-(90-R)$$

which gives the value of Y as

$$\frac{(90+R)-(90-R)}{2}$$

$$\text{Therefore, } Y=R$$

Thus the speed of the shaft B is directly proportional to the difference in speed between S_1 and S_2 and therefore directly proportional to the load.

8.32. Aron Clock type Watt-hour Meter. This is the most important of the pendulum type meters and, whilst the fundamental principle of this meter is the same as that described above, certain modifications have been made in order to overcome the difficulties which arise in practice in making the meter suitable for long periods of service, without requiring attention for such purposes as:

(*a*) Winding of the mainspring

(*b*) Synchronising of the pendulums at the same natural frequency.

In this meter, a differential gear is also employed between the mainspring and the pendulums, in order that both pendulums may be driven by the one mainspring, even when the load causes the pendulums to beat at different frequencies.

The pendulums (Fig. 106) each consist of a long brass rod with a coil on the lower end. The coil is co-axial with the rod, and the current is supplied by means of flexible leads. Adjustable weights are attached to the upper end of the pendulum for the purpose of giving it the required natural frequency. The pendulum is pivoted to the bearing plates of the counting mechanism by means of a steel spindle, and on the boss is fixed a pallet arm which meshes with the escapement wheel, the shaft of which is geared to the revolution counter differential and the driving differential.

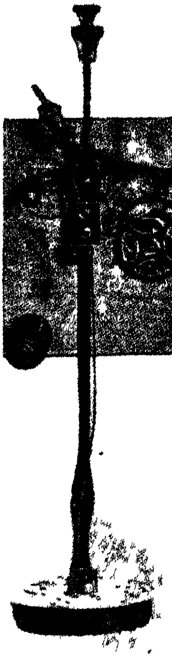


FIG. 106 — Pendulum with Pallet and Escapement Wheel (Aron Clock Meter)

The driving differential is required for the purpose of employing only one mainspring to drive both pendulums, since they will swing at different frequencies when the meter is on load. The mainspring drives the central shaft of this differential, and the energy is transmitted to the pendulums by means of the two sun-wheels which ride freely on the arm which is rigidly attached to the central shaft. In this manner the sun-wheels can both be driven by the main shaft, even though they are rotating at different speeds, and the planet-wheel will rotate around its axis at a speed proportional to the difference in frequency of the two pendulums.

Figs. 107 and 108 give diagrammatic arrangements of the winding

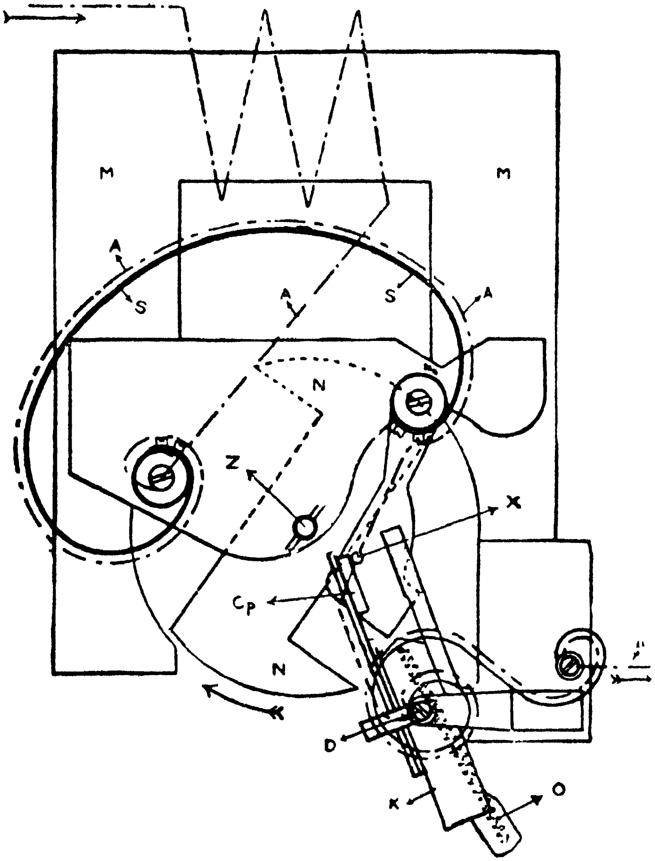


FIG. 107.—Diagram of Winding Gear in Normal Position (Aron Clock Meter).

gear in the normal position and with the spring fully wound respectively. One end of the mainspring is fixed to the electro-magnet *M*, whilst the other end is fixed to the rotor *N* at a point near the latter's periphery. When the spring *S* reaches its "run-down" position, the pin *X*, which is rigidly attached to the rotor, makes contact with the silver plate C_p, thereby completing the circuit of the electro-magnet coil. This creates a flux in the electro-magnet, which causes the rotor *N* to revolve into the position illustrated in Fig. 108, where it comes under the influence of the maximum flux.

The spring *O* is provided for the purpose of pulling the switch *K* sharply on or off, according to whether it is on the right- or left-hand side of the pivot *D*, thereby accelerating the make and break action.

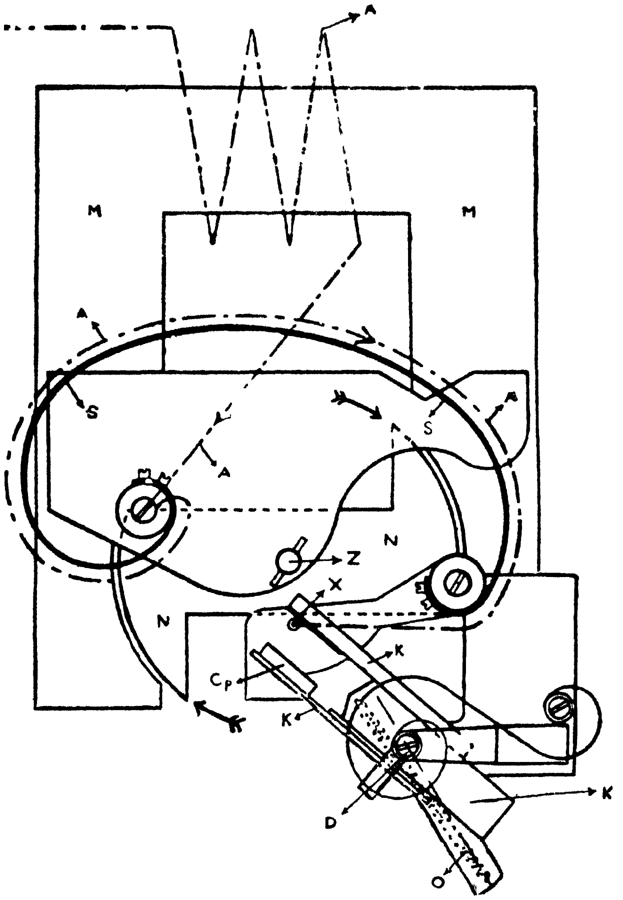


FIG. 108.—Diagram of Winding Gear; Spring fully wound (Aron Clock Meter)

In order that the mainspring may be wound without interfering with the drive to the pendulums, the power of the spring is transmitted to the driving spindle *Z* (Fig. 108) by means of a ratchet and pawls, and the spring cannot unwind without its potential energy being supplied to the pendulums.

The winding gear operates every thirty seconds, and during the short winding interval a drive is maintained upon the driving differential by means of a spiral spring which connects the driving spindle to the shaft of the planet-wheel arm. The spiral spring retains sufficient energy to supply the pendulums during this period.

The advantage of an intermittent driving gear, such as the above, is that it avoids the continuous waste of power which would be present with a small d.c. motor.

Since it is difficult to obtain exactly equal natural periods of swing for the two pendulums (as a commercial proposition) it follows that there will in general be a slight registration, either forward or backward, when the meter is on no-load, and also an error in registration when the meter is on load, due to the same cause. A device is therefore employed for neutralising this effect, and it consists of the reversing gear illustrated in Fig. 109. Every ten minutes the current in the pendulum coils is reversed by means of a commutator, the spindle of which is turned through 180° by the mainspring. This makes each pendulum alternately the faster and then the slower pendulum, due to the load. In turning, the commutator spindle operates the reversing gear bracket, thereby engaging an alternate gear train to take up the drive from the differential gear to the registration train. The one gear train has an extra wheel than the other, and in this manner gives the required reversal to the registration mechanism.

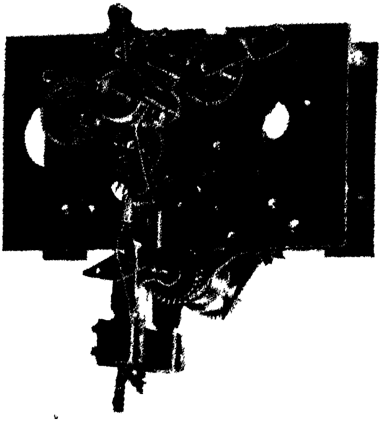


FIG. 109.—Reversing Mechanism (Aron Clock Meter).

Since the reversal causes the faster pendulum to become the slower, and vice versa, the differential gear planet arm will rotate in the opposite direction, but the reversal due to the reversing mechanism between the differential and the counting train will automatically correct for this and permit the meter to register in the correct direction for any load.

Thus, for a reversal period of ten minutes the difference between the natural frequencies of the pendulums will add to the effect due to the load, but in the following ten minutes it will subtract from it. Therefore the meter will register slightly high for one interval of ten minutes and then the same amount low for the next ten minutes. It follows that, at the end of every even ten-minute period, the effect due to want of synchronisation between the two pendulums will be neutralised, and the error due to this cause will be a maximum at the end of every odd ten minutes. In the case of meters incorporating a maximum demand indicator, a period other than ten minutes is sometimes chosen.

When the load is such that the sun-wheels of the revolution counter differential gear are rotating at almost the same velocity, the force thereby created on the planet-wheel arm may be insufficient to overcome the friction and inertia of the registration mechanism. This will cause the planet-wheel arm to remain stationary and the sun-wheels will consequently run in synchronism. This also occurs in the larger differential and, in order to avoid these effects acting at the same time, the sun-wheels of the former differential have 89 and 91 teeth respectively, instead of 90 each.

Suppose S_1 has 89 teeth and S_2 91 teeth (Fig. 105), then if the pendulums are swinging at the same frequency on no-load, S_1 will rotate 91 revolutions while S_2 rotates 89 revolutions.

From our table (section 8.31)

$$X+Y=91$$

$$-\frac{89}{91}X+Y=-89$$

$$\left(1+\frac{89}{91}\right)X=180$$

$$X=91$$

$$\text{and } Y=0$$

The meter will therefore not register on no-load.

If a load is applied, such that the speed of S_1 increases by R revs. and that of S_2 decreases by the same amount, in the time previously taken for S_1 to rotate 91 revolutions

$$X+Y=91+R$$

$$\text{and } -\frac{89}{91}X+Y=-(89-R)$$

$$\text{Therefore } \left(1+\frac{89}{91}\right)X=180$$

$$\text{and } X=91$$

$$\text{therefore } Y=91+R-X$$

$$=R$$

Therefore, the speed of the shaft B is directly proportional to the difference in speed between S_1 and S_2, and consequently directly proportional to the load.

8.33. Temperature Variations—Pendulum Meter. The resistance of the copper coils in the pressure circuit will increase with rise in temperature, and this will result in less current being taken by the pressure circuit. The meter will therefore tend to register low by

$$0{\cdot}4\frac{R_c}{R_c+R_b}\text{ per cent per degree C.}$$

where R_c is the resistance of the copper portion of the pressure coil circuit, R_b is the resistance of the zero temperature coefficient ballast resistance, and 0·4 per cent is the temperature coefficient of copper per degree C. rise in temperature. In the average meter the value of R_b is large compared with R_c, and the error due to temperature variation of the unshunted meter is of the order of $-0{\cdot}025$ per cent per degree C. rise in temperature.

In the case of the shunted meter, since the shunt is made of an alloy having a negligible temperature coefficient, there will be a decrease in current in the meter circuit due to the increase in resistance of the current coils with change of temperature. The meter will therefore tend to register low by

$$0{\cdot}4\frac{R_M}{R_M+R_S}\text{ per cent per degree C. rise in temperature}$$

where R_M is the resistance of the copper portion of the meter circuit, and R_S is the value of the swamping resistance. This is only so when the resistance of the shunt is small compared with R_M+R_S.

From the above it can be seen that the greater the value of R_S compared with that of R_M the less effect will temperature variation have upon the meter. In general, the error due to temperature variation of a shunted meter is of the order of 0·2 per cent per degree C. rise in temperature.

The error due to change of natural frequency of the pendulums, with change of temperature, is negligible, since the coefficient of linear expansion of brass is only of the order of 10^{-5} per degree C. Care must be taken, however, to ensure that the coils are absolutely secure, or errors may occur due to the alteration of their position with change of temperature.

On overload, the meter will tend to over-register, since the higher powers of P in the expression

$$f_a = f_o\left[\left(\frac{g}{l}\right)^{-1} K_3 P + \frac{1}{8}\left(\frac{g}{l}\right)^{-3}\left(K_3 P\right)^3 + +\right]$$

will become more important.

It is of paramount importance that the pendulum meter be set level, and for this purpose a plumb-bob is attached to the meter.

The pendulum type of meter has the following advantages:

(*a*) It is capable of registering the smallest possible loads.

(*b*) It possesses no permanent magnets which might change their strength with age.

(*c*) There are no starting troubles, as the meter is in continual oscillation.

(*d*) It possesses a very straight curve.

(*e*) It is astatic, and therefore independent of external magnetic fields.

8.34. Three-wire D.C. Systems. Energy may be supplied by means of a three-wire system, the circuit of which is as illustrated in Fig. 110. Two similar generators, or parallel sets of generators, are connected in series. The cables carrying the load current are taken away from them, as illustrated in the diagram. The power supplied to the load

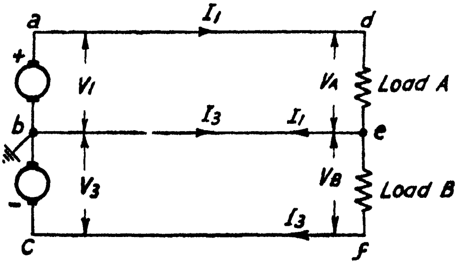


FIG. 110.—Three-wire D.C. System.

$$= V_A I_1 + V_B I_3$$

We have seen in section 4.3 that the current in line 2 will be $I_3 - I_1$ and, when the load is balanced (i.e. the currents of loads A and B are equal), the current in line 2 will be zero.

If r is the resistance of each cable, the voltage of point d above that of the earth point b will be

$$V_1 - I_1 r$$

and that of f will be

$$-V_3 + I_3 r$$

The voltage of e relative to the earth point b

$$=(I_1-I_3)r$$

When the load is balanced, $I_1=I_3$, and the current in line 2 is zero. Hence, the potential of e will be the same as that of b.

The potential at d will be

$$V_1-I_1r=V_A$$
$$=-(V_3-I_3r)=-V_B$$
$$\text{The load }=V_AI_1+V_BI_3$$
$$=2V_AI_1$$

From this it follows that an ordinary two-wire d.c. meter will register

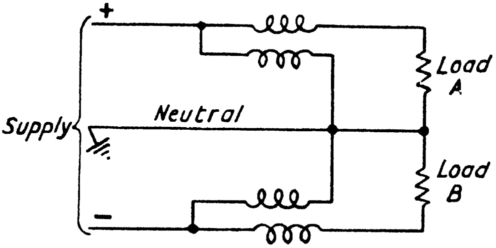


FIG. 111.—Measurement of Three-wire D.C. Load.

the correct supply of energy, in a balanced three-wire system, if its current circuit is connected in one of the two outer lines and the potential circuit between an outer and the middle line, providing the counting train registration is multiplied by two, or the gearing suitably modified.

If the system became unbalanced (i.e. when I_1 is not equal to I_3) such a meter would give a false registration. Therefore, since an unbalanced load occurs very frequently in practice, it is advisable to meter the load by two separate meters, or two separate elements driving one common disc spindle geared to the registration train; the connections being as illustrated in Fig. 111.

The advantages of a three-wire system are:

(*a*) When the load is truly balanced the losses in line 2 (called the neutral line) are zero. Under these circumstances the voltage drop between the load terminals and the generator terminals will only be one-half of what it would be if the load were connected to a two-wire system. Even when the system is not balanced, the cable losses are considerably reduced.

(*b*) Since line 2 only carries the difference between the currents of loads *A* and *B*, its cross-sectional area may be made considerably smaller than that of lines 1 and 3, thereby resulting in a great saving in copper.

(*c*) Voltages of the order of V_1 and $2V_1$ are available (the latter by connecting the load across lines 1 and 3).

(*d*) When the point B is earthed, the greatest potential difference from that of earth is only of the order of V_1, although twice this supply voltage is available, if required. This effects a saving in insulation costs.

8.35. Three-wire D.C. Watt-hour Meters. When a consumer is supplied with energy by means of a three-wire system, it is most convenient to employ one meter for measurement purposes. This may consist of two elements, one of which associates the current in the one outer wire with the pressure between that wire and the neutral; the other element associating the current in the other outer wire with the pressure between that outer and the neutral. We have already seen that such a method of metering will register the true energy supplied by the system. There are also other methods of metering the load some of which, although employed in practice, are not quite so accurate.

The power taken by the load

$$\begin{aligned} &= V_A I_1 + V_B I_3 \\ &= I_1(V_A + V_B) + V_B(I_3 - I_1) \\ &= I_1(V_A + V_B) + I_2 V_B \end{aligned}$$

where I_2 is the current in the middle wire, i.e. $I_3 - I_1$, the out-of-balance current.

Thus, if a two-element meter were connected so that the current of line 1 interacted with the magnetic flux set up by the current in a pressure coil connected across the two outer lines, and the current in the middle line interacted with the flux due to a pressure coil across the middle line and line 3, the meter would, if suitably geared and calibrated, register correctly for a balanced or unbalanced load.

8.36. Mercury Motor Type Three-wire Meter. In general, this type of three-wire meter consists of two elements mounted upon one common spindle, the elements so being connected that the driving torque transmitted to the spindle is proportional to

$$\frac{I_1}{2}(V_A + V_B) + \frac{I_3}{2}(V_A + V_B)$$

This is attained by connecting the current circuits of the meter one in each of the outer lines, and the common pressure coil across the outer lines. Since the baths of the meter are at different potentials, the common spindle possesses an insulating coupling between the two discs. The braking system is also common to both elements and consists of the usual aluminium disc rotating between the poles of a permanent magnet.

Suppose the three-wire system is in an unbalanced condition, then the current in the middle line

$$=I_2=I_3-I_1$$

The total power will be $I_1V_A+I_3V_B$

$$=I_1(V_A+V_B)+I_2V_B$$

The driving torque of the meter, however, will be proportional to

$$\frac{I_1}{2}(V_A+V_B)V+\frac{I_3}{2}(V_A+V_B)$$

$$=I_1(V_A+V_B)+\frac{I_2}{2}(V_A+V_B)$$

and the error in registration will be

$$\frac{\frac{I_2}{2}(V_A+V_B)-I_2V_B}{I_1V_A+I_3V_B}\times 100 \text{ per cent}$$

$$=\frac{\frac{I_3-I_1}{2}(V_A+V_B)-(I_3-I_1)V_B}{I_1V_A+I_3V_B}$$

$$=\frac{I_3(V_A-V_B)-I_1(V_A-V_B)}{2(I_1V_A+I_3V_B)}\times 100 \text{ per cent}$$

$$=\frac{+I_2Vu}{2(I_1V_A+I_3V_B)}\times 100 \text{ per cent}$$

where $Vu=V_A-V_B$

The error due to the unbalanced load is not usually so serious as it would at first appear. For example, if the voltage V_A is 220 v., V_B is 180 v., I_1 is 11 ampères, and I_3 is 9 ampères, the percentage error will be

$$\frac{-(11-9)\ (220-180)\ 100}{2(11\times 220+9\times 180)} \text{ per cent}$$

$$=-\frac{80\times 100}{8080}$$

$$=-1{\cdot}0 \text{ per cent}$$

In the case of the two elements not being properly balanced, there will be a further error introduced from this source.

8.37. Pendulum Type Three-wire Meter. This type of meter is easily adapted to register the total energy supplied in a three-wire circuit, since it normally consists of two elements. The connections for the meter, when used in a three-wire circuit, are as illustrated in Fig. 112.

No. 1 pendulum will beat with a frequency

$$f_1=\frac{1}{2\pi}\left[\left(\frac{g}{l}\right)^{\frac{1}{2}}+\frac{1}{2}\left(\frac{g}{l}\right)^{-\frac{1}{2}}K_3P_1-\frac{1}{8}\left(\frac{g}{l}\right)^{-\frac{3}{2}}(K_3P_1)^2+-\ldots\right]$$

$$=f_0\left[1+\frac{1}{2}\left(\frac{g}{l}\right)^{-1}K_3P_1-\frac{1}{8}\left(\frac{g}{l}\right)^{-2}(K_3P_1)^2+-\ldots\right]$$

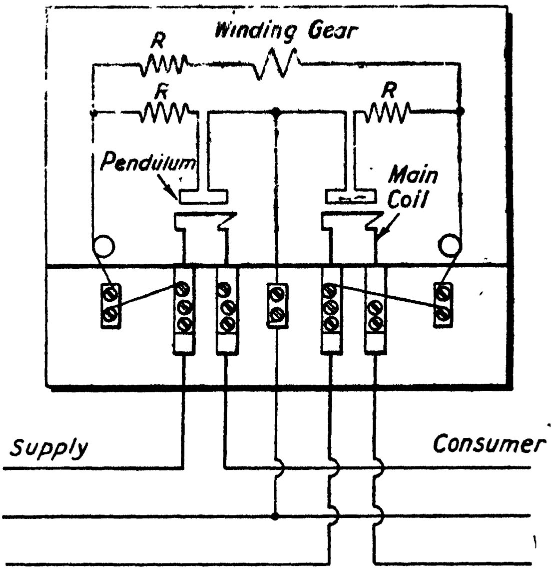


FIG. 112.—Schematic Arrangement—Aron Three-wire D.C. Clock Meter.

and the other will beat with a frequency

$$f_2=f_0\left[1-\frac{1}{2}\left(\frac{g}{l}\right)^{-1}K_3P_2-\frac{1}{8}\left(\frac{g}{l}\right)^{-2}(K_3P_2)^2--\ldots\right]$$

Therefore difference in frequency

$$f_d=f_1-f_2=f_0\left[\frac{1}{2}\left(\frac{g}{l}\right)^{-1}K_3(P_1+P_2)-\frac{1}{8}\left(\frac{g}{l}\right)^{-2}K_3^2(P_1^2-P_2^2)\right]$$

When the reversing gear is brought into operation the difference in frequency

$$f_d=f_2-f_1=f_0\left[\frac{1}{2}\left(\frac{g}{l}\right)^{-1}K_3(P_2+P_1)-\frac{1}{8}\left(\frac{g}{l}\right)^{-2}K_3^2(P_2^2-P_1^2)+\right]$$

Therefore, the mean angular velocity of the differential shaft, over an even number of reversal intervals, will be

$$v = K^1 f_o \left[\frac{1}{2} \left(\frac{(g)}{l} \right)^{-1} K_3 (P_2 + P_1) \right]$$
$$= K^{11} (P_2 + P_1)$$

Thus, providing the differential shaft is suitably geared to the registration mechanism, the meter will correctly register the energy supplied by the circuit.

8.38. The Testing of D.C. Ampère-hour Mercury Motor Meters. Before actually commencing tests, one should, without dismantling the meter, make a thorough inspection of its mechanical condition. All nuts and screws should be tried to ensure that they are tight, and one should make certain that the first wheel of the revolution counting mechanism meshes properly with the worm or pinion of the spindle. The insulation resistance of the meter should then be tested by means of a "Megger" (section 5.11), or by subjecting the meter to a "flash" test. This should be made between the meter circuits and the case, and also between any circuits which should be electrically isolated.

When the "Megger" is used for this test, it should register at least 5 megohms resistance between the meter and case, or between the individual circuits.

The circuit for the "flash" tester may be as shown in Fig. 113. The apparatus consists of a voltage step-up transformer which gives a secondary voltage of 1,000 or 2,000 volts when the primary winding is connected across the mains. In series with the primary winding is connected a circuit breaker, usually set to trip at some value of current between 1 and 2 ampères.

The indicating lamp is connected across the primary winding of the transformer to give a warning when the latter is energised. The leads on the secondary side are well insulated with flexible "cab-tyre" and at their remote ends from the transformer are fitted copper or brass testing probes which are provided with thick rubber or bakelite handles with large hand-guards (Fig. 113 (*b*)). The low-tension leads terminate with a suitable plug-in arrangement for connection to the mains.

For convenience, the whole of the apparatus is usually mounted on a trolley and, with the exception of the control switch and circuit breaker, it is contained in a wooden box, a window being provided for the indicator lamp.

When the insulation under test is faulty, current flows in the secondary winding of the transformer. As we have seen in section 6.11 dealing with transformers, this creates demagnetising ampère-turns which endeavour to decrease the flux in the core, thereby causing more current to flow from the mains to keep the flux density at its initial value. When the primary current reaches the value for which the circuit breaker is set to trip, the latter operates and warns the tester that the insulation is faulty. On account of the quick action of the circuit breaker, further damage to the insulation is prevented.

Sometimes a resistance is placed in parallel with the circuit breaker and is made of such a value that when the circuit breaker trips the secondary terminal voltage is equal to that of the mains supply. If this resistance is in the form of a lamp, it will indicate when the circuit breaker has tripped.

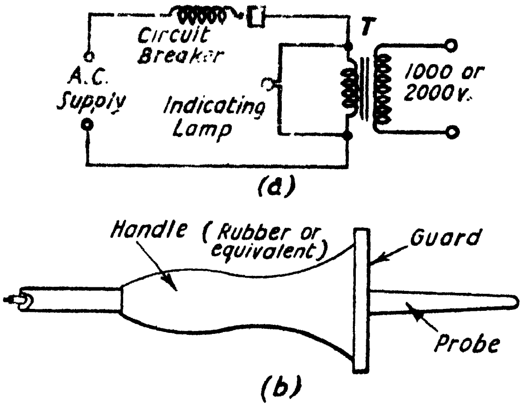


FIG. 113.—"Flash" Insulation Tester.

After satisfactorily passing the insulation test, the meters are ready for connecting up. When this is being done, care should be taken to ensure that all connections are tight. This is essential if the load is to remain perfectly steady.

The sealing device is now unclamped and the meters are run for a period on full load, in order to free the mercury baths of air bubbles which have been introduced during transit, and also to take up any back-lash which may be in the counting train. They are then placed upon starting current. This is 1/100th full load, but some supply undertakings insist upon the meters registering upon 1/200th full load. They are then observed for a convenient period. Any meter failing to register is given attention until it runs satisfactorily upon this load. In view of the importance of this test, and the fact that care taken at this stage will be well repayed by the subsequent performance of the meter, we will consider very carefully the various conditions which may be responsible for non-registration of the meter upon starting current. They are:

(*a*) Air bubbles in the mercury bath.

(*b*) Dirty mercury, or loss of mercury during transit, or a leaky bath.

(*c*) Friction in the counting mechanism.

(*d*) Bad gearing between the pinion or worm of the spindle and the first wheel of the counting train gear.

(*e*) Dirty or faulty top bearing.

(*f*) Faulty top pin. This may be bent, or possess a rough surface due to rust or wear.

(*g*) Incorrect adjustment of the top bearing relative to the top pin of the spindle.

(*h*) Rough bottom pivot.

(*i*) Cracked or rough bottom jewel.

(*j*) Foreign magnetic substance in disc (usually iron).

(*k*) Disc fouling one of the permanent magnet pole-pieces.

(*l*) Floating of the disc, due to insufficient weight of the sinker attached to the disc spindle.

(*m*) Buckled disc or bent spindle.

(*n*) Faulty insulation between components in mercury bath.

We will consider those of the above faults which are not entirely self-explanatory.

The air bubbles may sometimes be removed by tilting the meter slightly and vigorously oscillating the disc spindle between the thumb and first finger. If this is unsuccessful, the bath must be emptied of mercury, cleaned, and refilled. Occasionally, it may be found that the fault lies in the disc, or the pole-pieces of the permanent magnet, not being properly amalgamated. When this is the case the meter should be returned into the workshop in order that it may be dismantled and the offending component cleaned with nitric acid before re-amalgamation.

Friction in the revolution counter may be due to a bent tooth, faulty spindle or bearing, dirt in one or more of the teeth, "sticky" bearings, or a rough worm. In the case of a bent tooth, it is a serious fault on account of the fact that the meter may be run for several hours—or even days—without the tooth coming into gear. It is therefore advisable rigorously to inspect every counting mechanism individually. "Sticky" bearings are often caused by too liberal an application of lubricating oil, which not only absorbs the dirt but often, due to evaporation, changes its viscosity.

Experience alone will teach one when the gearing between the spindle and counting mechanism is correct, merely by the "feel" of the mesh. Either very loose or very tight gearing will give trouble.

Bad adjustment of the top bearing may cause it to rub against the shoulder or pinion of the spindle, thereby creating considerable friction. On the other hand, the top of the pin might be pressing against the spring jewel inserted in the top bearing. Obviously, the remedy in both these cases is to raise the top bearing to a suitable position. In other cases, the top bearing might be too high, and if the upper end of the pin is slightly burred and in contact with the walls of the bearing, unnecessary friction will result. Lowering of the top bearing will frequently remedy this.

Iron in the disc can be ascertained by disconnecting the meter and turning the spindle backwards slightly from the stopping position. If iron is in the disc, the latter will move forward with increasing velocity until it locks itself in the stopping position. If it is then placed the same amount forward it will repeat the previous performance, but in the opposite direction, again locking itself in the stopping position. Since air bubbles in the mercury bath often produce the same effect, it is advisable to make the test with the bath empty of mercury

With practice, it is quite an easy matter to determine whether the disc is in its correct position in the mercury bath; all that is required being a sensitive touch. One first lowers the bottom jewel very gradually until one feels that the underside of the disc is just making contact with the bottom pole-piece. The bottom jewel is then raised with equal care until the disc just touches the top pole-piece; slight rotation of the disc will advise one when this occurs. The bottom jewel is then set at the position which permits of equal clearance of the disc between the top and bottom pole-pieces.

If the sinker is of insufficient weight to create a downward force on the bottom pivot of the spindle, the pivot is liable to foul the jewel seating. This applies to certain types of meter in which the driving force is unbalanced. The obvious solution is to increase the weight of the sinker.

When the insulation of the mercury bath is faulty, a certain amount of current which should be going through the copper disc is by-passed. This decreases the driving torque and, at starting current, it may reduce it to an amount insufficient to overcome friction. In this case it is necessary to replace the bath.

It is strongly recommended to subject meters to an all-night dial test on starting current. This should be done at the conclusion of all other tests and adjustments, and will well repay for the extra trouble taken, both by minimising the number of rejects from service and increasing the revenue capacity of the meters at very low loads. Providing the meters are still running the next morning, and register within 25 per cent of the correct reading, they may be deemed satisfactory for service.

The meters are now ready for calibration, but before commencing they should be run at full load for about an hour in order to get constant conditions for calibration purposes (see Self-heating Errors, section 8.8). At the end of the hour the current is accurately set at its full-load value by means of an ammeter or potentiometer (section 2.13). Each meter is now timed for a suitable number of revolutions of the disc for a period of 60 to 100 seconds. The value of the expression

$$K = \frac{\text{ampères} \times \text{seconds}}{\text{revolutions}}$$

at this load is now calculated for each meter (see Meter Constant, section 8.10) and subsequently recorded. This procedure is repeated with the correct current for half or quarter full load, and then again for one-twentieth full load (one-tenth full load in the case of 2·5 and 5 ampère meters).

Example. Consider the case of a 10-ampère meter which takes 68·0 seconds for 100 revolutions at full load, 67·2 seconds for 25 revolutions at quarter full load, and 69·5 seconds for 5 revolutions at one-twentieth full load.

At full load the value of the expression (ampères × seconds)/(revolutions) will be

$$\frac{10 \times 68}{100} = 6{\cdot}8$$

$$\text{At } \tfrac{1}{4} \text{ load } = \frac{2{\cdot}5 \times 67{\cdot}2}{25} = 6{\cdot}72$$

$$\text{At } 1/20\text{th load } = \left[\frac{\tfrac{1}{2} \times 69{\cdot}5}{5} = 6{\cdot}95\right]$$

It should be mentioned that due allowance should be made for the difference of room temperature from 15°C. Thus, if t is the room temperature in °C. and α is the temperature coefficient of the meter (section 8.16)

$$K_L = K^1[1 + \alpha(t - 15)]$$

where K^1 is the value of the expression (ampères × seconds) per revolution, at any given load, and K_L is the corrected value of this expression for a temperature of 15°C.

One next consults the tables, provided by the manufacturer of the meter, giving each size of change-wheel and its corresponding meter constant, due allowance being made for the declared voltage of the installation in which the meter is to operate. A meter constant K is then chosen of such a value that where K_L is the value of the expression (ampères × seconds)/(revolutions)

$$\frac{K - K_L}{K_L}$$

falls within the limits of ± 2 per cent for loads greater than one-twentieth load, and ± 2·5 per cent for one-twentieth load.

The change-wheel or gear compound corresponding to the constant K of the tables is then fitted to the registration mechanism.

In the example already given, suppose the tables indicate that the most suitable constant is 6·82, then the wheel corresponding to this constant is inserted in the registration mechanism and the errors (providing the meter was timed at 15°C.) will be

$$\text{at } \tfrac{1}{1} \text{ load } = \frac{6{\cdot}82-6{\cdot}80}{6{\cdot}80} = +0{\cdot}29 \text{ per cent}$$

$$\text{at } \tfrac{1}{4} \text{ load } = \frac{6{\cdot}82-6{\cdot}72}{6{\cdot}72} = +1{\cdot}5 \text{ per cent}$$

$$\text{at } \tfrac{1}{20} \text{ load } = \frac{6{\cdot}82-6{\cdot}95}{6{\cdot}95} = -1{\cdot}9 \text{ per cent}$$

If the meter is outside the limits at one load it is often possible to straighten the error curve by adjustment of the fluid friction compensating device. Alternatively, one may adjust the whole curve of the meter by the magnetic shunt of the brake magnet, when such an adjustment is provided.

In the case of meters which have previously been tested (this will be so with most meters in the test-room of a supply undertaking) the previous constant of the meter will be clearly marked upon a label fixed to the counting dial. Therefore, the percentage error of the meter at each load can be calculated immediately from

$$\frac{K-K_L}{K_L}$$

and if the error at each load falls within the prescribed limits, there is no need to replace the change-wheel. If the error falls outside these limits at one or more loads, one can either fit a wheel giving a more suitable constant to the meter or make use of the adjustments provided to improve the curve.

At the conclusion of the tests, one must see that every meter carries a label bearing the constant K corresponding to the change-wheel inserted in the registration mechanism.

We have already seen (section 8.10) that this constant K

$$= \frac{\text{amperes} \times \text{seconds}}{\text{revolutions}}$$

$$\text{Therefore, ampères} \times \text{seconds} = K \text{ revolutions}$$

$$\text{ampère-hours} = \frac{K \times \text{revolutions}}{3600}$$

$$\text{kW hours} = \frac{K \times V \times \text{revolutions}}{3600 \times 1000}$$

$$\text{Revolutions per unit (r.p.u.)} = \frac{3600 \times 1000}{V \times K}$$

Therefore, providing the registration mechanism is geared so that $3{\cdot}6 \times 10^6/(V \times K)$ revolutions of the disc register one unit on the counting dial, the meter will register correctly in kWh.

In the above example, the r.p.u., when the declared voltage is 200 volts, will be

$$\frac{3600 \times 1000}{200 \times 6{\cdot}82}$$
$$=2640$$

In other words, the change-wheel is such that 2,640 revolutions of the disc register one unit on the counting dial.

8.39. Dial Test. All meters that have been refitted with new change-wheels must be re-timed at one-twentieth load (or one-tenth load in the case of 2·5 and 5 ampère meters) and, providing they are within the limits, the batch will be ready for dial test. This may be carried out by connecting a substandard meter in series with the meters under test, and comparing the registration of the latter with that of the substandard meter, after running them on full load for such a period that the pointer of the dial of smallest denomination has completed at least ten revolutions. Alternatively, the current may be set at its correct value by means of an ammeter or potentiometer and held at this value until the meters have registered this amount. The readings of the meters under test are then compared with that of the substandard meter.

In either test the pointers are initially set at zero and are read at the end of the test, each meter being corrected individually, according to the make and type, for the error due to difference in temperature from 15°C. The final error should not differ greatly from that found when timing the meter, since the dial test serves as a check on the gearing of the entire revolution counting mechanism.

8.40. Mercury Motor Meter with Constant Full Load Speed. The initial tests are similar in every way to those described for the change-wheel type of meter, and the same faults are liable to cause the meter not to register on starting current. The counter gearing, however, is fixed, i.e. the revolutions per kWh are the same for every meter of the same size. Therefore

$$\text{r.p.m.} = \frac{\text{r.p.u.} \times I \times V}{60 \times 1000}$$

where I is the full load current and V is the declared voltage of the circuit in which the meter is to be installed.

The meters are timed on full load and adjusted by means of the calibrating device, usually in the form of a movable clamp, until each meter has a full load error within such narrow limits as the tester knows from experience are most likely to bring it within the prescribed limits at all points of its curve. The meters are then tested at one intermediate load, either quarter or half full load, and again at low load (one-twentieth for all meters above 5 ampères rating and one-tenth full load for 2·5 and 5 ampère meters) and all errors are recorded.

If T is the nominal time which the meter should take for N revolutions, and T^1 is the actual time taken, due allowance being made for temperature variation, the percentage error

$$=\frac{(T-T^1)100}{T^1}$$

If the error at one or more loads is outside the prescribed limits (or the narrower limits which the tester sets for himself) it may be possible, by readjustment of the calibration device, to bring the meters within the limits at all loads. If this is not possible, it is advisable to return the meter to the workshop for inspection and overhaul.

Dial test is carried out in the same manner as is described in section 8.39, and this should be followed by an all-night test on starting current.

8.41. Electrolytic Meters. The testing of this type of meter is a very simple, although long, procedure. They are tested against a substandard electrolytic meter, the load on the meters under test being three-quarters their full load value. The duration of the test should be at least 100 hours; the error being recorded after about 50 hours and at the end of the test (also at siphoning, for the siphon type of meter).

All meters found to be within the prescribed limits are ready for service. Any others should have their scales regraduated and be subjected to a further test during which periodic observation of the registration should be made in order to ensure that the entire scale calibration is correct.

8.42. Mercury Motor D.C. Watt-hour Meters. The meters are first connected in circuit and the pressure coils energised to heat up the considerable mass of metal forming the electro-magnet. In order to avoid an expenditure of power equal to the maximum rating of the meter, separate supply circuits are connected to the pressure and current coils. Thus, the only power expended is the small amount dissipated in the meters under test. Such a load applied to a meter is termed a fictitious, or phantom, load (section 9.17).

The meters are then run at full load (or full load meter current in the case of shunted meters tested without their shunts) for sufficient time to eliminate the back-lash which may be in the counting mechanism. Starting current is then applied and any meters which fail to register satisfactorily are examined for such faults as are enumerated in section 8.38.

When the meters have passed the above test they are run at full load for about an hour and the load is then accurately set by means of a wattmeter, an ammeter and voltmeter, or a potentiometer and voltage stabiliser (section 2.25 and 26) and the meters are timed for a convenient number of revolutions. The respective errors are then calculated and the meters are adjusted in turn, by means of the brake magnets, until the errors fall within the narrow limits set by the tester, from his experience of the characteristics of each make and type of meter.

We have already seen that the speed of the disc is directly proportional to the power, i.e.

$$kW=\frac{K\times\text{revolutions}}{\text{seconds}}$$

$$\text{Therefore}\quad\frac{kW\times\text{seconds}}{\text{revolutions}}=K$$

where *K* is a constant, termed the *kW* seconds per revolution constant.

$$\text{Now,}\quad kWh=\frac{\text{revolutions of disc}}{\text{revs. per unit}}$$

$$\text{and}\quad kW\text{-seconds}=\frac{\text{revs. of disc}\times 3600}{\text{revs. per unit}}$$

$$\text{From which}\quad\frac{kW\times\text{seconds}}{\text{revolutions}}=\frac{3600}{\text{r.p.u.}}=K$$

and the time taken for *N* revolutions

$$=\frac{N\times K\times 1000}{\text{amps.}\times\text{volts}}$$

$$\text{or}\quad\frac{N\times 3600\times 1000}{\text{r.p.u.}\times\text{amps.}\times\text{volts}}$$

Suppose the actual time taken for *N* revolutions of the disc $=T^1$ seconds (after correction for effects of temperature) and the correct time for these revolutions is *T* seconds, then the percentage error

$$=\frac{(T-T^1)\,100}{T^1}$$

Each meter is now timed on one medium load, say quarter load, and then at one-twentieth load. The calculated errors are recorded and any meter, not fitted with a solid-friction compensating device, which registers low on light load must be speeded up throughout the curve if this will bring it within the prescribed limits throughout the curve. Otherwise, it must be returned to the workshop for overhaul.

In the case of meters fitted with friction compensating devices, adjustment is made at low load to bring the meter within the prescribed limits; due attention being paid to the characteristics of the particular type and make of meter under test. The current is then switched off, and a voltage of 10 per cent in excess of the declared voltage is applied to the pressure coils. Any meter which creeps under these conditions, for a revolution or more, must be adjusted until the disc stops in one position. It must then be re-timed on one-twentieth load, and if it is outside the limits at this load the tester must re-time it throughout the curve and ascertain whether it is possible to bring it within the required limits at all loads by overall adjustment of the curve. If this is not possible, the meter must be returned to the workshop for overhaul.

It should be mentioned that any meter which has been adjusted at low load should be re-timed at the other two points of the curve.

When the meters have passed the above test they are subjected to a dial test. For this purpose, the pointers are set at zero and the meters are run on an accurately set full load for a period sufficiently long to give at least ten complete revolutions of the pointer of smallest denomination. The errors are then corrected for differences of temperature from 15°C. and are duly recorded.

The correct dial reading will be

$$\frac{I \times V \times t}{60 \times 1000} kWh$$

where I is the current in ampères passed through the meters
V ,, applied voltage to the pressure coils
t ,, time of the dial test in minutes.

The meters should now be left for an all-night run at starting current after which, providing the registration is satisfactory, they would be ready for service.

It has already been stated that meters tested without their shunts are, for the purpose of the tests, considered as meters of a rating equal to the full load meter current. On dial test, however, the meter should read

$$\frac{N \times I \times V \times t}{60 \times 1000} kWh$$

where N is the multiplying power of the shunt
I ,, current, in ampères, passing through the meter circuit
V ,, voltage applied to the pressure coils.

8.43. Pendulum Meters. As the Aron Clock Meter is the only one of the pendulum type in general use in this country, we will concern ourselves only with the testing of this type of meter. It is first tested for creep with 10 per cent in excess of the declared voltage applied to the potential coils and no load on the current coils. If, during any ten minutes period between the operation of the reversing gear, there is a false registration of more than two divisions of the dial of smallest denomination, due to lack of synchronisation of the two pendulums, the balance weights are adjusted. This operation is continued until the false registration is within these limits.

All calibration tests upon this type of meter are made by means of dial tests, since it is obviously impracticable to time two pendulums simultaneously.

The initial test is at full load. The pointers are set to zero and the meters are run for a sufficient length of time for an initial adjustment to be made, bearing in mind that the period of the test must be a multiple of twenty minutes, in order that the difference in natural frequency between the two pendulums may be neutralised by the reversing gear. The adjustment is performed by alteration in value of the resistance in series with the pressure coil across the applied voltage. Thus, if the meter is registering low, the resistance is decreased, thereby increasing the current in the pressure coil circuit.

The expression for the dial reading will be the same as that given in section 8.42, and where R is the correct reading and R_1 the actual reading of the meter (after correction for temperature) the percentage error will be

$$\frac{(R_1-R)\ 100}{R}$$

When the meters have been adjusted to within the limits which the tester considers suitable for full load, they are given a long period dial test (a multiple of twenty minutes) until the pointer of the dial of smallest denomination has made at least ten revolutions. The meters should next be checked at half and one-twentieth full load, in a similar manner, and it is advisable to make an accurate observation of the dial reading every twenty minutes to ensure that equal advances of the pointers are made each period.

If the meter is within the limits at each load it is ready for service. It is usual, however, to give the meters an all-night test on voltage alone, to make sure that the false registration is not greater than the two divisions previously mentioned.

8.44. Pendulum Meters—Indirectly Connected. In the case of indirectly connected meters, it is advisable to test them in a cupboard at the average temperature of the place of installation, as this avoids the necessity of making allowances for the temperature coefficient of the meter. The full load meter current is then passed through the meter and its leads, and the resistance between the lead terminals is adjusted until the voltage drop across these points is the same as that across the potential terminals of the shunt, when $(N-1)$ times the meter current is passing through the shunt; N being the multiplying factor of the shunt. Usually, the voltage drop of the shunt at full load is expressed in millivolts and is clearly stamped at the end. Apart from this, the testing of the meter is exactly the same as for the directly connected meter, except that the dial will read N times the value of the applied load.

8.45. Three-wire D.C. Meters. The test for a three-wire d.c.

meter is similar to that for a two-wire meter, except that the elements must first be balanced. This is attained by running each element separately at full load—as a two-wire meter—with the pressure coil (or coils) energised. Each is timed in turn and then one or other of the elements is adjusted until they both run at the same speed for the same load. In the case of a meter employing two pressure coils, the balance is effected by adjustment of the resistance in series with one of the pressure coils. When there is a common pressure coil, the adjustment will depend upon the construction of the meter.

When balance is satisfactory, they are connected as ordinary two-wire meters, the current coils being connected in series. This gives conditions similar to those of a balanced three-wire system, in which case there is no current in the middle wire. From here onwards the procedure is exactly the same as for a two-wire meter, but it must be remembered that the equivalent load on the meter is

$$2VI$$

where V is the voltage applied to the pressure circuit and I is the current in ampères passing through the current circuit.

On dial test the meters will register

$$\frac{2VIt}{60 \times 1000}\ kWh$$

the nomenclature being the same as that given in section 8.42.

CHAPTER IX

A.C. Single-phase Electricity Supply Meters

9.1. Meters Suitable for the Measurement of Energy to A.C. Circuits. For measurement of energy in a.c. circuits, there are three types of meter which could be used. These are:

(*a*) Commutator
(*b*) Pendulum
(*c*) Induction.

These are all of the watt-hour type. There is, however, an ampère-hour type manufactured by the Reason Manufacturing Co. Ltd. This employs a transformer and rectifier unit in conjunction with their standard electrolytic meter.

The induction meter is by far the most used, as it has the advantages of being inexpensive, simple in operation, and possessing a high torque/weight ratio. Furthermore, it requires little maintenance, has a long life, is not greatly affected by temperature variations, and registers correctly down to very low power factors.

9.2. The A.C. Commutator Meter. Since this instrument suffers from even greater disadvantages than the d.c. type, such as rapid jewel wear due to vibration, and inaccuracy at low power factors, we will not discuss the meter here. Like its d.c. counterpart, it has not received the approval of the Electricity Commissioners.

9.3. The A.C. Pendulum Meter. The main modification to the d.c. pendulum meter, in order to make it suitable for use in a.c. circuits, is in the winding gear; the coil being altered to suit the particular voltage and frequency of the supply in which it is to be installed.

As the pressure coil circuit will possess a certain amount of reactance, it follows that there will be a phase difference between the current in the pressure coil and the applied voltage. Unless compensation is provided for this, there will be an error of registration (section 4.10) of

$$\frac{\cos(\theta-\beta)-\cos\theta}{\cos\theta}$$

where θ is the angle of lag of the load current and β is the angle of lag of the current in the pressure coil, relative to the applied voltage.

In practice, compensation is provided by making the flux in the current coil lag the current by the same angle as that by which the pressure coil current lags the applied voltage. This is attained by means of the brass cheeks on the current coil formers, which are slotted on one side; the circuit being completed by means of a resistance wire which joins the two sides of the slot. The angle by which the flux in the current coil is made to lag the load current is determined by the value of this resistance, the length of which can easily be adjusted. The conditions, when the required phase angle is obtained, are illustrated in the vector diagram of Fig. 114.

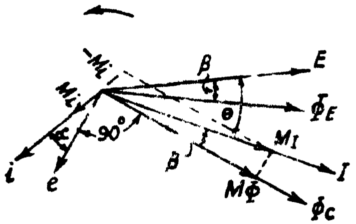


Fig. 114 —Vector Diagram—Reactance Compensation of Pendulum Meter

Let I be the current in the current coils
M_I ,, m.m.f. due to I
Φ_c ,, magnetic flux in the current coil
E ,, applied voltage
Φ_E ,, magnetic flux due to E
$M\Phi$,, m.m.f. required to produce the magnetic flux Φ_c
e ,, induced e.m.f. in the brass cheeks due to Φ_c
i ,, eddy currents due to e
M_i ,, m.m.f. due to i
α ,, angle of lag of i relative to e, due to the reactance of the eddy current path
β ,, angle of lag of Φ_E relative to the applied voltage E
θ ,, angle of lag of the load current relative to the applied voltage

The flux Φ_c due to the current I will create an e.m.f. e in the brass cheeks, which will lag Φ_c by 90°. Eddy currents will therefore be created and they will lag e by angle α, this angle depending upon the ratio reactance/resistance of the eddy current path. The m.m.f. M_i, due to the eddy currents i, will be in phase with i and will, according to Lenz's Law, oppose the force setting up the flux. Hence, the m.m.f. M_I, due to the current I, will be the vector sum of $-M_i$ and $M\Phi$ and will be in phase with the load current.

From the vector diagram it can be seen that by suitable adjustment of the resistance of the eddy current path (i.e. adjustment of the resistance value across the slots in the brass cheeks) it is possible to lag the flux Φ_c in the current coil by the same amount as the current in the pressure coil lags the applied voltage. When the adjustment has been made, the phase angle between the fluxes Φ_c and Φ_E will be the same as that between the current I and the applied voltage E. Under these conditions the meter can be calibrated to register correctly at all power factors which will occur in practice.

9.4. The Induction Watt-hour Meter. The induction watt-hour meter is similar in principle to the induction wattmeter (section 4.14). We have already seen that, in an arrangement such as is illustrated in Fig. 57 (section 4.14) where an aluminium disc is mounted between the poles of two electro-magnets, one of which is fitted with a coil carrying the load current—or definite proportion of it—and the other with a coil carrying a current proportional to the applied voltage and lagging the latter by 90°, the mean torque is proportional to $EI \cos \theta$; E being the applied voltage, I the load current, and θ the phase angle between these two quantities.

In the induction watt-hour meter the spring control is replaced by a permanent magnet which operates as a magnetic brake upon the aluminium disc. This has already been described in connection with d.c. watt-hour meters (section 8.19). The magnet is usually placed diametrically opposite the electro-magnet system, in order that it might not be damaged should an accidental short-circuit current pass through the current coil of the meter.

9.5. Lag Adjustment. In order that the applied voltage and the effective pressure coil flux shall be in exact time quadrature (i.e. 90° apart) a copper loop is mounted around the potential electro-magnet pole tip. This may be in the form of a plain copper band, or a coil whose ends are connected together by means of an adjustable resistance. The operation of this loop can be seen from the vector diagram of Fig. 115, in which

Φ is the main pressure coil flux
M ,, driving m.m.f.
Φ_l ,, flux passing through the loop (the effective flux)
M_l ,, m.m.f. required to overcome the reluctance of the magnetic path
e ,, induced e.m.f. in the loop
i ,, eddy currents due to e
M_i ,, m.m.f. due to i
α ,, angle of lag of i relative to e, on account of the reactance of the eddy current path
β ,, angle by which Φ_l lags Φ.

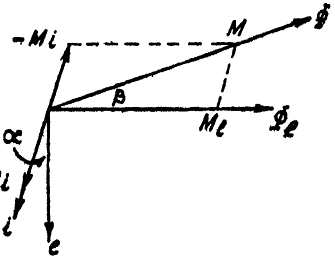


FIG. 115.—Vector Diagram—Lag Adjustment of Induction Meter.

The flux Φ_l, which passes through the loop, induces an e.m.f. in the latter, and this lags Φ_l by 90°. This e.m.f. gives rise to a current i in the loop, which lags e by angle α; the current creating an m.m.f. M_t which is in opposition to the force creating the flux. Therefore, the driving m.m.f. M must be equal to the vector sum of $-M_t$ and M_l, and will be ahead of the latter by angle β. It can be seen from the diagram that alteration of the magnitude of i and/or angle α will alter the value of β. Thus, by alteration of the resistance of the loop circuit, it is possible to adjust the value of the angle β until Φ_l lags the applied voltage by 90°.

Sometimes a copper band is placed around the potential electromagnet limb, instead of a loop of adjustable resistance. In this case, adjustment of quadrature is performed by movement of the band along the axis of the limb. As it is moved farther up the limb it will come under the influence of a greater amount of flux. This will increase the value of e and i, although α will remain at the same value. The m.m.f. M_t will also increase, thereby increasing the angle of lag of Φ_l relative to Φ and to E, the applied voltage.

The aluminium disc also acts in the same manner as the quadrature loop, but its effect is much less pronounced and may therefore be neglected.

We will now consider the conditions in the meter when the quadrature loop has been adjusted so that Φ_l lags the applied voltage by $(90-\Delta)°$;

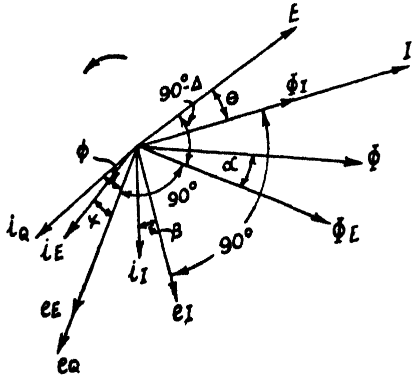


FIG. 116.—Vector Diagram—Induction Watt-hour Meter.

Δ being a small angle. These conditions are illustrated in the vector diagram of Fig. 116.

Let E represent the voltage applied to the potential circuit
I ,, ,, current in the series coil (load current)
θ ,, ,, phase angle between E and I
Φ ,, ,, main pressure coil flux
Φ_I ,, ,, flux due to I

Let Φ_E represent the effective pressure coil flux due to E
e_I ,, ,, e.m.f. induced in the disc by Φ_I
i_I ,, ,, eddy currents in the disc, due to e_I
e_E ,, ,, e.m.f. induced in the disc by Φ_E
i_E ,, ,, eddy currents in the disc, due to e_E
e_q ,, ,, e.m.f. induced in the quadrature loop by Φ_E
i_q ,, ,, eddy currents in the quadrature loop, due to e_q
α ,, ,, angle of lag of Φ_E relative to Φ
β ,, ,, angle of lag of i_I relative to e_I
ψ ,, ,, angle of lag of i_E relative to e_E
ϕ ,, ,, angle of lag of i_q relative to e_q

The flux Φ_I will, as far as we are concerned, be in phase with I and will induce an e.m.f. e_I in the disc. This will give rise to the eddy currents i_I which will lag e_I by angle β, due to the reactance of their path in the disc. The flux Φ will not be in exact quadrature with E. on account of the pressure coil circuit not being wholly inductive. The effective pressure coil flux acting upon the disc is represented by Φ_E, and this lags Φ by angle α, due to the action of the quadrature loop, This flux Φ_E will induce an e.m.f. e_q in the quadrature loop and e_E in the disc; both these e.m.f.s lagging Φ_E by 90°. The eddy currents i_q in the quadrature loop will lag e_q by angle ϕ, whilst those in the disc, due to e_E, will lag the latter by angle ψ; the lag in both cases being due to the reactance of their respective eddy current paths. The angles β and ψ are not necessarily equal, since the eddy current paths in the disc—due to Φ_I and Φ_E—may not be symmetrical. In most meters it will be found that the mean path of the eddy currents due to the effective potential electro-magnet flux covers a larger area than that of the eddy currents due to the series electro-magnet flux. Since the angle of lag is equal to

$$\tan^{-1}\frac{\text{reactance}}{\text{resistance}} = \tan^{-1}\frac{\omega L}{R}$$

where L is the inductance of the path and R is the resistance, it follows that this angle will be directly proportional to the frequency for small angles. Generally, it is about 10° at 50 cycles per second.

9.6. Determination of Torque. As in the case of the induction wattmeter (section 4.14) the driving torque T will consist of two components; one will be due to the interaction of Φ_E with i_I and the other due to the interaction of Φ_I with i_E, and the value of the torque

$$T = K_1 \Phi_E i_I \cos x - K_2 \Phi_I i_E \cos. y$$

where x is the angle between Φ_E and i_I and y is the angle between Φ_I and i_E. K_1 and K_2 are constants which may, or may not, be equal.

From our vector diagram of Fig. 116, we see that the value of T will be given by

$$T=K_1\Phi_E i_I \cos(\theta+\beta+\Delta)-K_2\Phi_I i_E \cos(180+\psi-\Delta-\theta)$$
$$=K_1\Phi_E i_I \cos(\theta+\beta+\Delta)+K_2\Phi_I i_E \cos(\theta-\psi+\Delta)$$

Assuming that the flux density in each electro-magnet is such that they are working upon the straight part of the B—H curve, Φ_E and i_E will be proportional to the applied voltage E, whilst Φ_I and i_I will be proportional to the load current I, at any given frequency. Hence,

$$T=K_3EI\cos(\theta+\beta+\Delta)+K_4EI\cos(\theta-\psi+\Delta)$$
$$=K_3EI\,[\cos(\theta+\beta+\Delta)+\cos(\theta-\psi+\Delta)]\text{ approx.}$$

By careful design the angles β and ψ can be made small and, although not equal, are usually of the order of 10° at 50 cycles. The angle Δ can be eliminated by careful adjustment of the quadrature loop. Therefore, for all power factors likely to occur in any commercial installation, the torque can be made to approximate to $EI\cos\theta$, and providing the angular speed of the disc is directly proportional to the torque the meter can be geared to register directly in kWh.

Figs. 117 (*a*) and (*b*) illustrate the directions of the fluxes and eddy currents in the working portions of the meter, at time t (Fig. 117 (*c*)), as indicated upon the graph of E, I, Φ_E, and Φ_I, to a base of time. Assuming that the power factor of the load is $\cos\theta$, I will lag behind E by θ°. Providing the quadrature loop has been correctly adjusted, Φ_E, the effective flux due to the potential electro-magnet, will lag E by 90°, i.e. by $1/4f$ seconds, where f is the frequency of the supply. We can see from the graph that, at t seconds, Φ_I will be negative and increasing in magnitude. From Lenz's Law, we know that the eddy currents induced in the disc will be in such a direction as to create an m.m.f. which will oppose the change in Φ_I. The eddy currents will therefore be in the direction indicated in Fig. 117 (*a*) and, from application of Fleming's left-hand rule, we know that the direction of the torque, due to the interaction of Φ_E with i_I, will tend to rotate the disc in an anti-clockwise direction viewed from the top of the meter. We also see from Fig. 117 (*c*) that, at time t, the flux Φ_E will be positive, but decreasing in value. Thus, the m.m.f. due to the eddy currents i_E will oppose this change (thereby tending to increase the value of Φ_E) and i_E will be in the direction indicated in Fig. 117 (*b*). Once more applying the left-hand rule, we see that the torque due to the interaction of Φ_I with i_E will also tend to rotate the disc in an anti-clockwise direction, viewed from the top of the meter.

Fig. 44 (section 4.1) gives the values of e, i, and their product plotted (with different scales) to a base of time. At any instant t, the value of the applied voltage is given by $e=E_{max}\sin\omega t$ and that of the current is $i=I_{max}\sin(\omega t-\theta)$; the power factor of the load being $\cos\theta$. From the curve for p, we see that the power will be alternately positive and negative. This is because the load circuit for a portion of each cycle will be returning power to the supply (except at unity power factor).

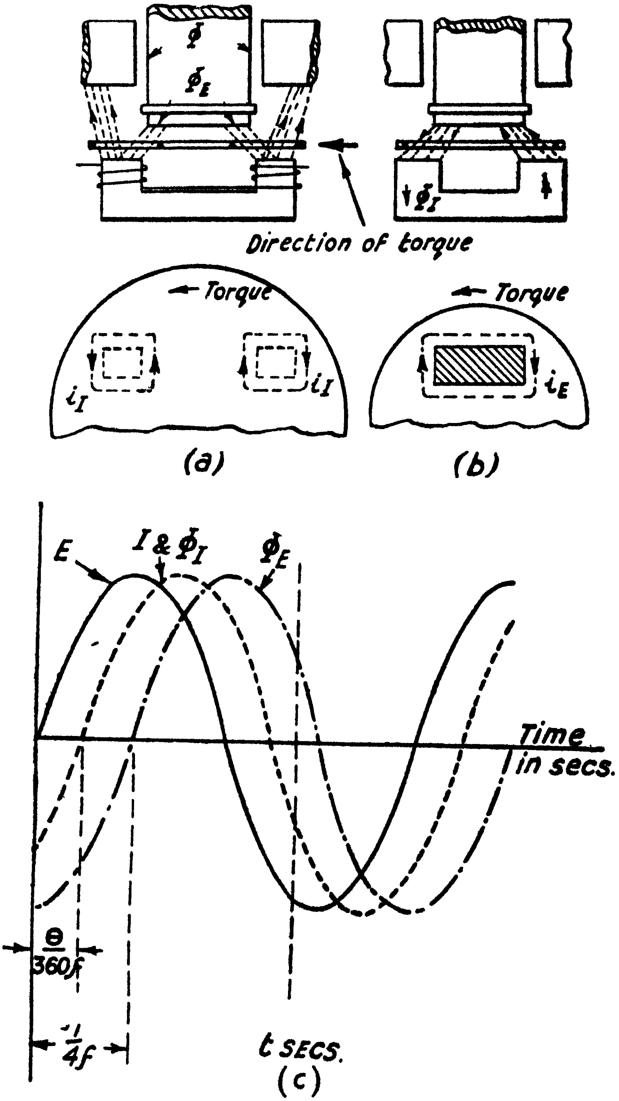


FIG. 117.—Determination of Torque—Induction Watt-hour Meter.

We will now consider the torques acting upon the disc. In Fig. 118 (*a*) the instantaneous values of E, Φ_E, Φ_I, i_E, and i_I are plotted to a base of time; the current lagging the applied voltage E by angle θ. For simplicity, we will assume that all wave-forms are identical, no harmonics existing. The instantaneous torques will be respectively proportional to the products of $\Phi_E i_I$ and $\Phi_I i_E$, where, for the sake of this argument, we will use the preceding nomenclature for the instantaneous values of the sinusoidal quantities. These products are given

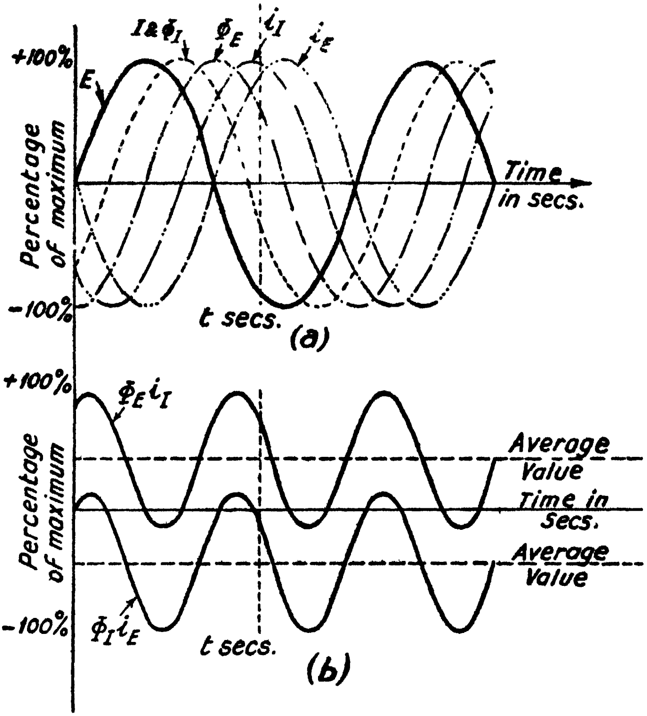


FIG. 118.—Determination of Torque—Induction Watt-hour Meter.

in Fig. 118 (*b*). From the diagram it can be seen that the torques will also be sinusoidal, having a frequency equal to twice that of the supply and an average value greater than zero, around which the wave-form is symmetrical. This is not surprising, since

$$
\begin{aligned}
T &= (K_1 \sin \omega t)\,(K_2 \sin (\omega t - x)) \\
&= K_1K_2(\sin \omega t.\ \sin (\omega t - x)) \\
&= \tfrac{1}{2}K_1K_2(\cos x - \cos (2\omega t - x))
\end{aligned}
$$

and cos x will have a constant value for any particular power factor. Therefore the mean value of the above product will be $\tfrac{1}{2}K_1K_2 \cos x$.

We now have to ascertain in which direction the component torques of the watt-hour meter will operate. Taking time t, in Fig. 118 (*a*), the conditions will be as shown in Fig. 117 (*a*) and (*b*). We find that at this instant t, the two torques will each tend to rotate the disc in the same direction. Thus, the torque proportional to $\Phi_E i_I$ is positive when the curve is above the base-line and that proportional to $\Phi_I i_E$ is positive when the curve is below the base-line. The total instantaneous torque will therefore be the difference in magnitude (taking into account the sign) between the torques as represented in Fig. 118 (*b*) at any given instant.

Inspection of Fig. 118 (*b*) tells us that, neglecting the impedance of the eddy current paths in the disc (which will be small) and assuming that the meter has been adjusted so that Φ_E is in exact quadrature with E, there will be a phase displacement between the two torques of 180 electrical degrees of the torque frequency. Also, each component of the driving torque will give a negative torque for

$$\frac{\theta}{360f} \text{ seconds}$$

for every cycle of the torque frequency. At low power factors it follows that, if the average values of the two components are not equal, the resultant driving torque may be negative at certain portions of the cycle. The disc, however, would not respond to this change of direction of the driving torque, owing to the inertia of the moving system, but would take up a speed proportional to the mean value of the torque, i.e. $EI \cos \theta$.

If the average values of the two components of the total torque are equal, that is

$$\Phi_E i_I = \Phi_I i_E$$

where Φ_E, Φ_I, i_E, and i_I are r.m.s. values, and the conditions are such that there is a phase displacement of 180° between the respective torques (i.e. when I and Φ_I are in phase, E and Φ_E are in quadrature, and the reactance of the eddy current paths in the disc is negligible) the total torque for any given load will be approximately constant at all instants.

When the disc has reached a steady angular velocity the torque will be used up to overcome the braking forces due to

(*a*) the permanent magnet flux
(*b*) the effective potential electro-magnet flux
(*c*) the series electro-magnet flux.

It will also have to overcome the friction of the revolution counter and bearings of the disc spindle, which is practically constant at all disc speeds. The effect of windage will be negligible at all normal speeds of the disc. We have already found (section 8.19) that the braking force F, due to the disc cutting a magnetic flux Φ, is directly proportional to Φ^2N, where N is the speed of the disc. Thus, where Φ_p represents the flux due to the permanent magnet and f is the torque necessary to overcome friction, we have

$$T=N(K_4\Phi_p{}^2+K_5\Phi_E{}^2+K_6\Phi_I{}^2)+f$$

9.7. Compensation for Friction. In order to compensate for the frictional force of the disc spindle bearings and the revolution counter, a small torque is created upon the disc by means of the potential electro-magnet flux. We will assume that this torque has a value T_F and is constant for any given value of applied voltage. Since it will assist the driving torque, the above equation will be modified to

$$T+T_F=N(K_4\Phi_P{}^2+K_5\Phi_E{}^2+K_6\Phi_I{}^2)+f$$

$$K_1\Phi_E i_I \cos(\theta+\beta+\Delta)+K_2\Phi_I i_E \cos(\Delta+\theta-\psi)+T_F$$
$$=N(K_4\Phi_P{}^2+K_5\Phi_E{}^2+K_6\Phi_I{}^2)+f$$

Assuming that the value of T_F is equal to that of f, i.e. it is just sufficient to overcome the frictional forces in the meter, we have

$$K^1EI(\cos[\theta+\beta+\Delta)+K^{11}\cos[\theta+\Delta-\psi)]=N(K_4\Phi_p{}^2+K_5\Phi_E{}^2+K_6\Phi_I{}^2)$$

which, for reasons already given, approximates to

$$K^{111}EI\cos\theta=N(K_4\Phi_P{}^2+K_5\Phi_E{}^2+K_6\Phi_I{}^2)$$

whence $$N=\frac{EI\cos\theta}{K_7\Phi_P{}^2+K_8E^2+K_9I^2}$$

since $\Phi_E \propto E$ and $\Phi_I \propto I$

This expression for the speed of the meter shows that N is not directly proportional to the power, since the denominator possesses two terms which respectively vary as the square of the applied voltage and the square of the load current. In general, the voltage will change but little, whereas the current will vary between zero and full load (or a greater value). In consequence, we get the characteristic droop of the error curve as full load is approached and exceeded. The amount of the droop will depend upon the ratio of Φ_I/Φ_P, and in modern meters this ratio is kept as small as possible by employment of very strong permanent magnets. In fact, the modern meter has an almost straight load error curve from one-quarter to twice full load.

The friction in a meter tends to cause it to register low at light loads, unless some form of compensation is provided. The latter is in the form of a torque due to the potential flux and, providing the applied voltage is constant, it is of the same value at all loads. There are many methods of effecting this torque, but we will only consider certain elemental methods in the subsequent paragraphs.

9.8. Friction Compensating Loops. Fig. 119 (*b*) illustrates the method employed in the Metropolitan-Vickers' type *NA* meter, for providing a torque to overcome the effect of friction. The system comprises two loops which are placed one in each of the potential electro-magnet air gaps. Each loop acts in a similar manner to the quadrature loop (section 9.5). The amount of flux enclosed by each loop can be varied by adjustment of the position of the loop. Thus, when the loops are moved anti-clockwise to the position shown dotted, loop *A* will enclose more flux and loop *B* will enclose less than was previously the case. This will have the effect of lagging the flux across the air gap *A* relative to that across *B*. Now Φ, the main flux, is the vector sum of the fluxes passing across the potential electro-magnet air gap and the corresponding portion of the effective flux Φ_E cutting the disc and, since it is more or less constant in phase and magnitude, alteration of the phase of the flux in the right- or left-hand potential electro-magnet air gap will cause an opposite phase change in the corresponding portion of the effective flux Φ_E. In the case illustrated in the diagram, the adjustment will cause the right-hand portion of the effective flux to lag the left-hand portion. We have already seen, in the case of the shaded-pole motor (section 3.21), that such a phase displacement between two fluxes operating upon the same disc in such a position will exert a torque upon the disc tending to rotate it from the unshaded to the shaded flux. Thus, the adjustment of the loops in an anti-clockwise direction (around their points of fixture) will increase the torque in a clockwise direction viewed from above.

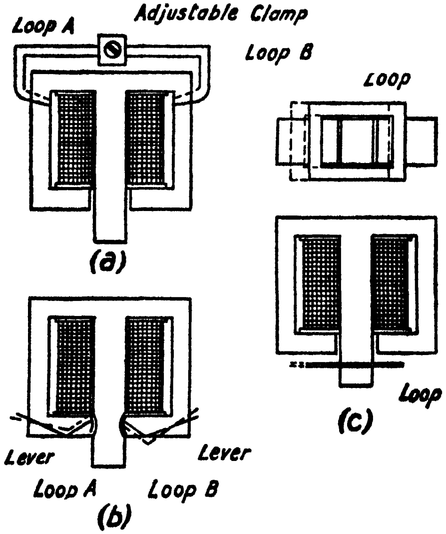


FIG. 119.—Friction Compensation Loops—Elementary Methods. Induction Watt-hour Meter.

Fig. 119 (*c*) illustrates the method employed in the Chamberlain and

Hookham type B meter. In this case a loop similar to the quadrature loop embraces the centre limb of the potential electro-magnet. It is only capable of adjustment, however, in a plane perpendicular to the axis of the potential electro-magnet centre limb. It therefore produces a small shaded-pole effect upon the effective potential electro-magnet flux Φ_E, by encircling more flux on one side than the other. Thus, movement to the left of the central position will cause the left-hand portion of the flux Φ_E to be shaded relative to the right-hand portion and a torque is thereby created upon the disc tending to rotate it in an anti-clockwise direction viewed from above (i.e. more positive). It should be mentioned that this loop will also act as a quadrature loop, but since it is only free to move in a plane perpendicular to the axis of the potential limb, alteration of its position will have no effect upon quadrature.

9.9. Adjustment of Overall Speed of Disc. This may be attained by alteration of the position of the magnet relative to the centre of the disc, and it is usually performed by sliding the magnet diametrically across the disc. Where two magnets are employed for the purpose of braking, they are usually fitted so that their unlike poles are adjacent; an adjustable shunt being provided. Thus, movement of the shunt towards the magnets will cause a greater leakage flux, thereby decreasing the flux in the magnet gaps and speeding up the meter.

The system whereby two magnets are employed has the advantages (*a*) that the possibility of instability of both magnets is more remote than is the case for only one magnet, thereby reducing the chances of error, and (*b*) the system is less susceptible to the influence of an external magnetic field.

An interesting form of fine adjustment employed in the Chamberlain and Hookham type H meter is that of a keeper passing across the magnet gap. This is of the micrometer type and, providing the screw thread keeps contact with the edge of the magnet gap, is very effective and adds to the permanency of the magnet.

We will now consider errors introduced due to variation of operating conditions.

9.10. Variation of Voltage. From the expression deduced in section 9.7, for the speed of the disc, we see that an increase of the applied voltage E (thereby increasing the value of Φ_E) will decrease the speed of the meter for any given load. This, as we have already observed, is due to the braking force exerted by Φ_E, and the amount by which it will decrease the speed of the meter will depend upon the ratio of $\Phi_E : \Phi_p$. The smaller this ratio is made, the smaller will be the error introduced in the meter with change of voltage.

Since the value of the frictional compensating torque is directly proportional to the square of Φ_E, it will increase with increase in value of E. This will have the effect of increasing the speed of the meter throughout the curve, but since this torque will be constant for any given voltage, the percentage increase in speed will be greater the lower the load. The total effect will, in general, be to quicken the meter up on low loads and to decrease the speed at the higher loads.

We have seen that increase of voltage increases the friction compensating torque, and it is therefore necessary to ensure that this torque will not increase, with any reasonable change of voltage, to such a value that the disc creeps continuously when the potential coil alone is energised. On account of this possibility, the Electricity Commissioners lay down that all meters must, at no load, be tested with 10 per cent in excess of the marked voltage applied to the potential coil of the meter.

9.11. Anti-creep Devices. Sometimes the compensation required of the friction compensating device, in order to give the meter a good calibrating curve, is so great that it would cause the disc to creep at no-load and give a false registration, unless some device were employed to prevent the disc creeping more than one revolution.

One of the best methods of preventing creep is to have one or two holes, or slots, in the disc. At certain positions of the disc these holes disturb the eddy current paths in the disc to such an extent that a backward torque is developed upon the disc.

Another method is to fix a piece of iron to the aluminium disc, and when the iron is under the direct influence of the permanent magnet the force of attraction overcomes the friction compensating torque. Sometimes the iron is in the form of a wire wrapped around the disc spindle, its end going out radially from the spindle. At certain positions of the disc this comes under the influence of the permanent magnet or a projecting lamination from the potential electro-magnet, and the length of the wire jutting out from the spindle is such that the force of attraction between the wire and the magnet—or laminations—is just sufficient to overcome the friction compensating torque.

Such a device must not be sufficiently effective to prevent the meter from registering on starting load, but must be able to overcome the friction compensating torque when the applied voltage to the pressure coil is 10 per cent in excess of the declared value.

9.12. Compensation at High Loads. Besides being able to improve the overload performance of a meter by using a low current flux, it is possible to improve it still further by the use of a bridge-piece in close proximity to the limbs of the series electro-magnet. Within certain limits of current, this bridge-piece shunts a proportion of the series coil flux, thereby reducing the driving torque. When the current exceeds a certain value, the bridge becomes saturated and thus a greater proportion of the series coil flux is effective in creating a driving torque. Such a device, unless well designed, is liable to create curious "kinks" in the error curve of the meter.

9.13. Variations Due to Change of Temperature. The errors introduced with change of temperature are as follows:

(*a*) The brake magnet decreases in strength by about 0·03 per cent per degree C. rise in temperature. This will decrease the braking force, thereby tending to increase the speed of the meter.

(*b*) The resistance of the pressure coil will increase with increase of temperature to the extent of 0·4 per cent per degree C., since the coil is constructed of copper. The value of the resistance is small, however, compared with the reactance of the coil and the impedance change is therefore negligible. Now, the angle between the applied voltage and the effective pressure coil flux will be

$$\phi=\left(\tan^{-1}\frac{\omega L}{R}\right)+\alpha$$

where L is the inductance of the coil, R is the resistance, and α is the angle of lag of Φ_E relative to Φ, due to the action of the quadrature loop.

If we neglect the variation in the value of α, due to temperature change, the angle between the applied voltage and the effective pressure coil flux, for a change of temperature t, will be given by

$$\phi_t=\left(\tan^{-1}\frac{\omega L}{R_t}\right)+\alpha$$

Therefore, the change of angle

$$\phi_t-\phi=\tan^{-1}\frac{\omega L}{R_t}-\tan^{-1}\frac{\omega L}{R}$$

We know that, with an increase of temperature, R_t is greater than R. In consequence $(\phi_t-\phi)$ will be negative and will decrease the angle between E and Φ_E. The meter will therefore register as if the load current had been lagged by an angle $(\phi_t-\phi)$. Thus, at lagging power factors, the disc will tend to decrease its overall speed with increase of temperature. At leading power factors the tendency will be opposite.

(*c*) The resistance of the disc and quadrature loop will increase with rise in temperature. Therefore the induced eddy currents due to Φ_E will be smaller, and this will cause the effective flux Φ_E to lag behind the main pressure coil flux Φ by a smaller angle and make the meter register as if the load current had been lagged by this change in angle. This will introduce errors which will tend to increase the overall speed of the meter on leading power factors and have the reverse effect on lagging power factors. The reduction in value of the eddy currents in the quadrature loop, due to its increase in resistance with rise in temperature, will increase the value of Φ_E, thereby slightly increasing the speed of the meter.

(*d*) The resistance of the friction compensating loops will also increase with rise of temperature, thereby diminishing the magnitude of the friction compensating torque. This effect, of course, will be most pronounced at low loads.

To sum up the above, we may say that the overall effect of increase of temperature is to increase the overall speed of the meter at unity and leading power factors and also for lagging power factors where the effect due to (*a*) is greater than that due to (*b*) and (*c*). The power factor at which the increase of speed due to (*a*) is equal to the decrease due to (*b*) and (*c*) is generally in the neighbourhood of 0·8 lagging. At all lagging power factors lower than this value, the meter will decrease in overall speed with increase in temperature.

Provision is made in the Sangamo single-phase meter, type HMT, for compensation for each of the errors introduced with variation of temperature by (*a*), (*b*), and (*c*) described above. That for (*a*) consists of magnetic supports, formed of nickel-steel, attached to the grid of the assembly and so constructed that they form a leakage path for the main permanent magnet flux. Since the reluctance of the nickel-steel increases with increase of temperature, it follows that the leakage flux will decrease with increase of temperature, thereby tending to keep the value of the braking flux constant. In the actual meter, by careful design of the supports, the braking force has been kept sensibly constant over a wide range of temperature.

Compensation for the errors introduced due to (*b*) and (*c*) is provided in the form of an inductance in series with the pressure lag coil. The core of this inductance consists of an alloy, the permeability of which changes in such a manner, with change of temperature, that the value of the inductance decreases with increase of temperature. Thus, the current in the pressure lag coil increases, thereby causing Φ_E to lag behind E by a greater angle and this tends to counterbalance the errors introduced by (*b*) and (*c*).

In the Landis and Gyr single-phase meter, type CF6, compensation for variation of temperature is provided in the form of a bi-metallic, tongue-shaped, counter pole fitted underneath the centre pole of the potential electro-magnet, but on the opposite side of the disc. This operates in conjunction with the pressure coil flux. With increase of temperature the tongue moves away from the disc, thereby reducing the value of the effective potential electro-magnet flux and slightly reducing the speed of the disc.

9.14. Variation of Frequency. At first sight, since the standardisation of the British "Grid" system frequency at 50 cycles per second, it would appear that the errors introduced with change of frequency are not important. Such reasoning would be correct if the wave-form of the supply voltage only possessed the fundamental frequency. Unfortunately, this is rarely the case and, where the wave-form of the local supply voltage is distorted, errors are likely to be introduced in the registration of the meter. These errors will depend upon the accuracy of the meter at the harmonic frequencies and the power factors of the harmonics, as well as upon the percentage harmonic content.

We will first of all consider to what extent the fluxes and eddy currents, and their respective displacements, are dependent upon frequency. When the frequency of the supply is increased, the reactance of the pressure coil circuit will increase approximately in direct proportion to the increase in frequency. Therefore, since the resistance of the pressure coil circuit is negligible compared with its reactance, we have

$$Z \propto f$$

$$\text{and} \quad \Phi_E \propto \frac{E}{Z} \propto \frac{E}{f}$$

where Z is the impedance of the pressure coil circuit and f is the frequency of the supply.

$$\begin{aligned} \text{Also} \quad e_E &\propto \Phi_E f \\ &\propto \frac{Ef}{f} \\ &\propto E \end{aligned}$$

$$\text{and} \quad i_L = \frac{e_E}{R} \cos \psi \propto \text{E} \cos \psi$$

The flux in the series electro-magnet will be practically proportional to the load current and therefore independent of frequency.

$$\text{Therefore} \quad \Phi_I \propto I$$

$$\text{and} \quad e_I \propto f\Phi_I$$

$$i_I = \frac{e_I}{R} \cos \beta \propto fI \cos \beta$$

From the above we see that the product of $\Phi_E i_I$ is given by

$$\begin{aligned} \Phi_E i_I &\propto \frac{E}{f} fI \cos \beta \\ &\propto EI \cos \beta \end{aligned}$$

$$\text{and } \Phi_I i_E \propto IE \cos \psi$$

The reactance of the eddy current paths in the disc will be directly proportional to the change in frequency. Thus, if L be the inductance of the eddy current path and R be the resistance, the angle of lag of the eddy currents relative to the induced e.m.f. in the disc, will be given by

$$\lambda = \tan^{-1} \frac{\omega L}{R} = \tan^{-1} 2\pi f T$$

where T is the time constant of the eddy current path (L/R). Thus

$$\tan \lambda = 2\pi f T$$
$$\propto f$$

For small angles, tan λ is approximately equal to λ, and under such conditions the angle is proportional to the frequency.

We see from the above that the smaller the time-constant of the eddy current path in the disc (i.e. the higher the resistance) the less effect will change of frequency have upon the angle λ.

The reactance of the quadrature loop and friction compensating loops will also increase with increase of frequency, and this will respectively affect the angle of Φ_E relative to E, and the magnitude of the friction compensating torque.

Summarising, the main effects of increasing the supply frequency are:

(*a*) Decrease of Φ_E inversely proportional to the increase of frequency and causing Φ_E to lag E by a slightly larger angle.

(*b*) Increase of i_I directly proportional to the increase in frequency multiplied by cos β^1/cos β, where β^1 is the angle of i_I relative to e_I at the new frequency and β is the angle at the initial frequency.

(*c*) Decrease of i_E inversely proportional to the ratio (cos ψ)/(cos ψ^1) where ψ is the angle of lag of i_E relative to e_E at the initial frequency and ψ^1 is the angle at the new frequency.

(*d*) Increase in the reactance of the quadrature loop, thereby slightly modifying the value of Φ_E and causing the angle between Φ_E and E to decrease slightly.

With reference to the vector diagram of Fig. 116, we derived the following expression for torque (section 9.6):

$$T = K_1 \Phi_E i_1 \cos(\theta + \beta + \Delta) + K_2 \Phi_I i_E \cos(\Delta + \theta - \psi)$$

$$\text{whence } T = K_3 EI \cos\beta \cos(\theta + \beta + \Delta) + K_4 EI \cos + \cos(\Delta + \theta - \psi)$$

and ascertained that for a given frequency $\Phi_E i_I$ and $\Phi_I i_E$ are proportional to EI. The increase in the reactance of the eddy current paths of the disc, however, will affect the driving torque since the angles β and ψ will be increased with increase of frequency. The increase in reactance of the quadrature loop will tend to decrease the angle $(90 - \Delta)$, i.e. increase Δ, the angle by which Φ_E leads quadrature with E. The increase in reactance of the pressure coil current will, on the other hand, tend to decrease Δ; thus the effects are more or less mutually compensating.

At unity power factor, where $\theta = 0$, the torque T^1 for the new frequency will be given by

$$T^1 = K_3 EI \cos\beta^1 . \cos(\beta^1 + \Delta^1) + K_4 EI \cos\psi^1 . \cos(\Delta^1 - \psi^1)$$

and, assuming that the meter had been correctly adjusted so that Φ_E

was in quadrature at the initial frequency, and that the respective alterations in the time-phase of Φ_E are mutually compensating, we have

$$T^1 = K_3EI \cos{}^2\beta^1 + K_4EI \cos{}^2\psi^1$$

Now, the constants K_4 and K_3 will be more or less equal, therefore

$$T^1 = K_5EI\ (\cos{}^2\beta^1 + \cos{}^2\psi^1)$$

$$\text{and } \frac{T^1}{T} = \frac{\cos{}^2\beta^1 + \cos{}^2\psi^1}{\cos{}^2\beta + \cos{}^2\psi}$$

where T, β, and ψ are the respective values at the initial frequency.

Inspection of the above expression shows that at unity power factor the driving torque will decrease with increase of frequency. From knowledge of the time-constants of the eddy current paths in the disc one could easily calculate the values of β, β^1, ψ, ψ^1 and therefrom the ratio of $T^1 : T$.

Since Φ_E is inversely proportional to the frequency, it follows that the braking force due to the potential electro-magnet flux Φ_E will be inversely proportional to the square of the frequency. This will tend to increase the overall speed of the meter with increase of frequency, and the percentage increase will depend upon the ratio $\Phi_E : \Phi_P$.

Suppose, at the initial frequency f, the ratio $\Phi_E : \Phi_P$ is n and that the braking effect of the current flux at all loads is sufficiently small to be neglected, then the torque necessary to overcome the braking effect due to the fluxes will be given by

$$T_B \propto (\Phi_P{}^2 + K^1\Phi_I{}^2 + K\Phi_E{}^2)N$$

$$\propto (1 + Kn^2)N, \text{ since } K^1\Phi_I{}^2 \text{ may be neglected.}$$

and, at frequency f^1

$$T_B{}^1 \propto \left(1 + K\left[\frac{fn}{f^1}\right]^2\right)N^1$$

Thus, the braking force due to Φ_E will decrease with increase of frequency and will tend to increase the overall speed of the meter by the ratio

$$\frac{1 + Kn^2}{1 + K\left[\frac{fn}{f^1}\right]^2}$$

We can see from this expression that when the frequency is reached such that $K\left(\frac{fn}{f^1}\right)^2$ is negligible compared with unity, further increase in frequency will have no appreciable effect upon the braking torque and, in consequence, no tendency further to increase the overall speed of the meter. This has been verified by experiment and is described in an excellent article upon this subject by Moore and Slater (*Jour. I.E.E.*, Vol. 68, p. 1023).

There are thus two opposite effects with increase of frequency, one due to the decrease in driving torque—which tends to decrease the overall speed of the meter with increase of frequency—and the other due to the decrease in breaking torque which tends to increase the overall speed of the meter with increase of frequency. We have already seen that

$$T^1 \propto (1+K\left[\frac{f}{f^1}n\right]^2 N^1$$

$$\text{and } T \propto (1+Kn^2)N$$

$$\text{therefore} \quad \frac{N^1}{N} = \frac{T^1(1+Kn^2)}{T\left(1+K\left[\frac{f}{f^1}n\right]^2\right)}$$

which, at unity power factor, when the meter has been initially adjusted so that Φ_E is in exact quadrature with E, gives us

$$\frac{N^1}{N} = \frac{(\cos{}^2\beta^1+\cos{}^2\psi^1)\,(1+Kn^2)}{(\cos{}^2\beta+\cos{}^2\psi)\left(1+K\left[\frac{f}{f^1}n\right]^2\right)}$$

As one might imagine, there is an optimum frequency at which the overall speed of the meter is a maximum. This value will depend upon the design of the particular meter under consideration and will vary considerably with different makes of meters. Generally, it is between 30 and 100 cycles per second.

The error due to change in braking force with change in frequency will be more or less independent of the power factor of the load, whereas the change in driving torque will be dependent upon the power factor. The latter follows from the expression

$$\frac{T^1}{T} = \frac{\cos\beta^1\cos(\beta^1+\theta+\Delta^1)+\cos\psi^1\cos(\theta+\Delta^1-\psi^1)}{\cos\beta\cos(\beta+\theta+\Delta)+\cos\psi\cos(\theta-\psi+\Delta)}$$

where $\cos\theta$ is the power factor of the load.

If we consider the time constants of the eddy current paths of i_E and i_I as of the same value, this expression simplifies to

$$\frac{T^1}{T} = \frac{\cos\beta^1\,[\cos(\beta^1+\theta+\Delta^1)+\cos(\theta-\beta^1+\Delta^1)]}{\cos\beta\,[\cos(\beta+\theta+\Delta)+\cos(\theta-\beta+\Delta)]}$$

$$= \frac{2\cos{}^2\beta^1\cos(\Delta^1+\theta)}{2\cos{}^2\beta\cos(\Delta+\theta)}$$

and, if the constants of the meter can be so arranged that Δ and Δ^1 are equal, i.e. that the increase in the angle of lag of Φ_E due to the increased reactance of the pressure coil is equal to the decrease in the angle of lag of Φ_E, due to the increased reactance of the quadrature loop, we have

$$\frac{T^1}{T}=\frac{\cos^2\beta^1}{\cos^2\beta}$$

which will be independent of the power factor of the load.

9.15. Compensation for Errors due to Change of Frequency. Compensation for variation of frequency is difficult to apply in practice. The effects of variation of frequency can, however, be greatly minimised by suitable design such that the ratio $\Phi_E : \Phi_r$ is small and by employing a disc as thin, or of as low a conductivity, as possible, consistent with high torque. It is also an advantage to make Φ_E as near as possible in quadrature with E before artificial compensation is provided. This will decrease the variation of the ratio $T^1 : T$ at different power factors, due to the change in value of Δ with change in frequency.

The accuracy of a meter upon distorted wave-forms obviously depends upon its accuracy at the frequency and power factor of each harmonic. We can take it as a general rule that the meter will register slightly low for distorted wave-forms.

9.16. Special Features of Commercial Meters. Since the majority of induction watt-hour meters exhibit a marked similarity in principle, if not in actual construction and design, little purpose would be served in describing each type of meter to be found in service. We will therefore confine ourselves to those features, to be found in various meters, which have not been covered in a general manner.

In the Aron single-phase meter, type *el*, the low load adjustment consists of a non-magnetic bar carrying two small steel shoes and a central iron-piece. This bar slides across the poles of the potential electro-magnet and is operated by means of a cam which is slotted to take a screw-driver. The iron-piece creates a leakage field which, owing to its hysteresis, lags the working flux Φ_E of the meter. When the system is unsymmetrical relative to the potential electro-magnet, a phase difference between the two sides of the pressure coil flux will result and, as we have already seen, a torque will be created upon the disc.

Fig. 120 gives a schematic diagram of the magnetic circuits of the Siemens Bros. single-phase meter, type 21. The bulk of the potential flux Y crosses the gaps G_1, whilst the effective flux X crosses the gap to the limbs embraced by the series coils, thereby cutting the disc. The flux X then divides, one small portion returning via the gap and re-cutting the disc, whilst the remainder Z leaks back to the potential coil core via the iron circuit specially provided for the purpose. The main series coil flux B takes the normal path across the gap G_2 to complete its circuit, cutting the disc in so doing, whilst the portions A cut the disc once and return via the leakage paths as illustrated. The low load adjustment (Fig. 121) is similar in principle to that illustrated in Fig. 119 (*c*). It consists of a small shading vane placed immediately below the centre potential limb and is capable of movement in a plane perpendicular to the axis of the limb. This produces the shaded-pole effect (section 3.21) by means of which the required friction compensating torque can be applied to the disc. The adjustment is actuated by a micrometer worm gear, as illustrated. Quadrature adjustment is effected in the conventional manner, by means of a copper band capable of movement along the axis of the centre potential limb. This, too, is provided with a micrometer worm gear for fine and accurate adjustment.

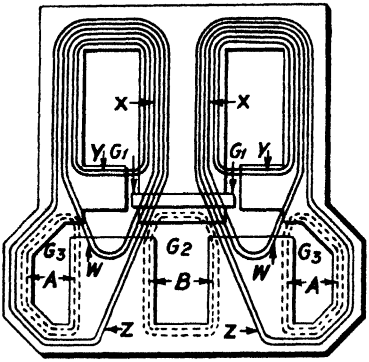


FIG. 120.—Magnetic Flux Circuits—
Siemen's type 21 a.c. Single-phase Meter.

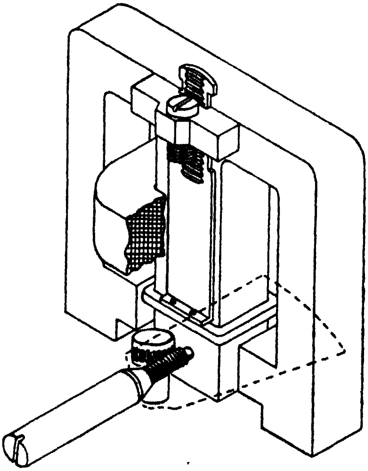


FIG. 121.—Low Load Adjustment.
Siemen's type 21 Single-phase Meter.

An interesting feature of the Landis & Gyr meter (Fig. 122) is the quadrature device. This is in the form of two thin copper vanes (1) on the same shaft and rigidly fixed relative to one another. Quadrature adjustment is effected by variation of the position of the vanes in the potential electro-magnet gaps. A return path for the pressure flux is provided by means of leakage from each of the series limbs to a limb of the potential electro-magnet, as shown. The path *A* (Fig. 123) will have a higher reluctance than path *B*, due to its having a greater air gap, and also because the thick aluminium disc will have a greater lagging effect than the thin copper plate *P*. Thus, the effective flux

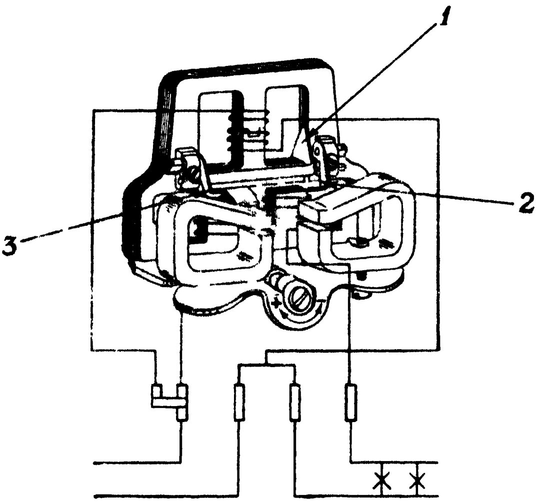


FIG. 122.—Landis & Gyr Single-phase Meter.

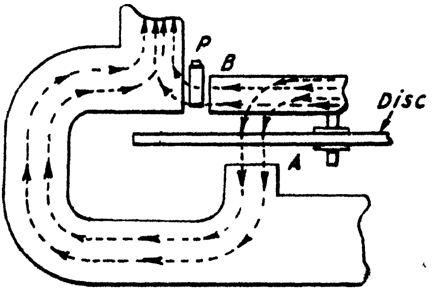


FIG. 123.—Working Flux Circuit—Landis and Gyr Single-phase Meter.

will be smaller than the flux passing across *B*. Adjustment of the copper vane will alter the phase of the latter flux and, since the phase and magnitude of the total flux will remain practically constant, it will also alter the phase of the effective flux, i.e. the flux across the gap *A*.

The copper vanes which, as previously stated, are symmetrically situated one in each of the air gaps of the potential electro-magnet, can be adjusted without any phase displacement of the one-half of the effective flux relative to the other half. Thus, no undesired torque will be exerted upon the disc. We can see from the above that entrance of the vanes into the air gaps will decrease the overall speed of the meter on inductive loads and that their withdrawal will have the opposite effect.

The friction compensating device is in the form of two iron vanes (2 and 3) which are individually operated. These vanes create leakage fields which lag the flux Φ_E on account of their hysteresis. Thus, the vanes can be so adjusted that such a phase displacement is attained, between the two portions of the effective flux Φ_E, as to give the required torque to compensate for friction. Movement of the left-hand vane towards the disc lags the left-hand portion of the effective flux, thereby tending to create a positive torque. Movement of the right-hand vane towards the disc has the reverse effect; the latter vane being employed for fine adjustment and the left-hand vane for the initial course adjustment.

In the Sangamo meter, type HMT, the friction compensating device is in the form of a non-magnetic vane situated between the disc and the series electro-magnet. This is only capable of movement in a plane perpendicular to the axis of the centre limb of the potential electro-magnet, and when it is moved away from the central position it shades the one side of the effective potential electro-magnet flux more than the other side, thereby producing a slight shaded-pole effect (section 3.21) tending to rotate the disc from the less shaded to the more shaded flux. The adjustment is of the micrometer type and the system can be positively locked after correct setting.

The power factor adjustment consists of a loop, to the ends of which is connected a resistance which can be adjusted in value by means of a slider.

The handle is fixed to the base and has the advantage, when the meter is being carried, of relieving the jewel of the weight of the disc.

In the Metropolitan-Vickers single-phase meter, type NES, coarse adjustment of the overall speed of the meter is effected by movement of the entire permanent magnet system towards, or away from, the disc, according to whether the overall speed is to be increased or decreased, whilst a fine micrometer adjustment is obtained by movement of the magnetic shunt.

An interesting feature of the Chamberlain & Hookham meter, type JX, is the unusual form which the brake magnet takes and the method of adjustment of the braking flux. The braking system consists of a U-shaped magnet, in alignment with which is located a keeper; the latter being on the opposite side of the disc. Adjustment is performed by movement of the braking system towards or away from the centre of the disc, according to whether it is required to increase or decrease the speed of the disc. A further adjustment is possible by alteration of the length of the air gap between the keeper and the magnet.

The type J meter, made by the same company, is similar in general construction to the type JX, except that a more conventional form of magnet is employed. The other adjustments of both types of meter are similar in principle to the elemental types already discussed.

9.17. Fictitious Loads. The testing of a.c. single-phase watt-hour meters is somewhat more involved than that of d.c. meters, since the former type of meter also has to allow for the phase difference which is likely to exist between the applied voltage and the load current (sections 1.21 and 4.1). Obviously, a test at only unity power factor would not be sufficient and therefore some means must be provided for adjustment of the phase angle between the applied voltage and the load current.

The most apparent method of testing a.c. watt-hour meters is to connect them, together with a substandard watt-hour meter or wattmeter, across the supply in exactly the same manner as that in which they would be connected in service. This would require special non-inductive and inductive resistances which would be ungainly and expensive, on account of their high dissipation on all except very light loads. The expenditure of power would be at least equal to the rated full load of the instrument, and this would involve heavy electricity costs. It is therefore most advantageous to test the meters with separate voltage and current supplies (i.e. a fictitious load). Thus, the only energy required would be that to supply the losses of the meter coils and the testing equipment at the various loads. In order that the meters may be tested in this manner, an isolating link is fitted in the pressure coil circuit. This is removed for testing purposes, but must be remade when the meter is connected for service. In addition to the above, a device is necessary for varying the phase difference between the applied voltage and the current to the current coils. This may be in the form of a phase-shifter (section 2.19) of the Drysdale type, the stator being energised by the supply voltage and the rotor being connected to the potential coils of the meters. In this manner, the applied voltage to the potential circuits is made to lead or lag the current in the series coils by any required angle.

In the past the device most frequently employed for phase adjustment was two mechanically coupled alternators driven by a motor.

One alternator supplied the voltage to the potential circuits of the meters, whilst the other supplied the current for the series coils through the medium of a step-down transformer. The stator of the one alternator was constructed in such a manner that it could be rotated, by means of a worm gear, relative to the stator of the other alternator. Thus, it was possible to alter the phase of the e.m.f. generated in the one alternator relative to the e.m.f. generated in the other, in order to attain any desired angle between the current in the series coils of the meters and the applied voltage to their potential circuits. Such a system was more costly and involved than that of the phase-shifting transformer and also had the added disadvantage that the wave-forms of the e.m.f.s generated in the respective alternators were required to be identical.

9.18. Testing of A.C. Single-phase Meters. The method adopted for the testing of a.c. single-phase meters will largely depend upon the size of the Supply Undertaking and the facilities possessed by its testing department.

The Electricity Commissioners have laid down certain requirements as to the limits of error permitted for a.c. house-service meters. The errors must not exceed $3\frac{1}{2}$ per cent plus or $3\frac{1}{2}$ per cent minus at any load at which the meter may be operating in service. No compensating allowances are required for varying conditions of temperature, etc.

The loads at which the meters must be tested are (*a*) at 5 per cent of the marked current, (*b*) at one intermediate load, (*c*) at 100 per cent or 125 per cent of the marked current. The above tests are to be made at unity power factor and, in addition, the meters must be tested at 0·5 lagging power factor with marked current and marked voltage. The meters must also be tested to ensure that they do not creep a whole revolution with 10 per cent in excess of the marked voltage applied to their pressure coil terminals.

The meter engineer is permitted to adopt any one of three approved methods for testing the meters. We will consider each method in turn.

9.19. Method A. Long-period Dial Tests using Substandard Rotating Meters. This is a test in which the advance of the pointers of the meters under test is compared with that of a rotating substandard meter connected in series with them. The meters are tested at each of the prescribed loads, the duration of each test being of sufficient length of time to allow the last pointer of each meter under test to make at least ten revolutions.

The load on the substandard rotating meter must be not less than 25 per cent, nor more than 125 per cent, of its full load for any test. Furthermore, if only one rotating substandard per bench is employed, it must be re-tested against a substandard wattmeter at least once per month. On the other hand, if two rotating substandard meters in series are employed, they must be tested against the substandard watt-meter whenever the corrected meter readings show a mutual difference of 0·25 per cent at any load, or after an interval of three months, whichever is the earlier. The performance of the rotating substandard must be within the limits prescribed by the Electricity Commissioners and its construction must also comply with the specification laid down by the Commissioners.

9.20. Method B. Comparison of Disc Revolutions with those of a Rotating Substandard Meter at all Prescribed Loads, and One Long-period Dial Test. In this test, the disc revolutions of the meters under test are compared with those of a substandard rotating meter, the load on the substandard being not less than 25 per cent nor more than 125 per cent of its full load. The number of complete revolutions of each meter under test, for any given load, is ascertained by multiplying the percentage of the marked current at that load by 2/5. Thus, if the load were 50 per cent of the marked current, the meter would be run for 50 × 2/5 = 20 revolutions of the disc. The duration of the test, however, shall not correspond to less than five complete revolutions of the disc and need not be greater than twenty-five complete revolutions of the disc of the meter under test. In addition, the meter must be subjected to one long-period dial test, as laid down in Method A, at one of the prescribed loads.

9.21. Method C. Testing by Substandard Indicating Instrument and Stop-watch, and One Long-period Dial Test. In this method the disc of the meter under test is timed for three revolutions or 100 seconds, whichever is the longer period; whilst each of the specified loads in turn is held steady at a value giving the required deflection of the indicating substandard wattmeter. In addition, one long-period dial test, as for Method A, must be run at one of the specified loads.

For each of the above methods of test, the voltage circuit of the meters under test and that of the substandard meters must be energised for at least one hour prior to commencing the test.

9.22. Choice of Method to be Adopted. Economical considerations usually determine which one of the three methods is to be adopted by the meter engineer, for his particular test-room. Each method has its advantages and disadvantages. In consequence, we will consider each in turn.

The advantages of Method A are:

(*a*) The meter is on test for a longer period than is the case for either of the other methods. This particularly applies to low load, and it thereby gives the tester a greater opportunity of discovering any intermittent mechanical trouble which may occur in the meter. For example, if a tooth on one of the counting train wheels of higher denomination were bent, the possibility of it making itself felt on short period runs would be very remote.

(*b*) Every test is made upon the dial. This is particularly important, since it is the advance of the pointers which determines the amount payable by the consumer.

(*c*) Errors due to the starting and stopping of stop-watches, or rotating substandard meters, are entirely eliminated.

(*d*) There is no necessity to re-set meters after commencement of the first test. Thus, all tests may be run consecutively. This makes it possible for each tester to control several benches at once without producing undue strain.

(*e*) The load need not be held absolutely steady for the tests.

(*f*) For a large undertaking, which is well equipped with bench space and can usually fill each bench with meters of the same size, it is a very economical method, causing the ratio Meter Output : Testing Staff to be a maximum.

The disadvantages are:

(*a*) There must, in general, be sufficient meters of one size passing through the test-room to fill a bench.

(*b*) The initial capital cost is heavy, since the meters spend a longer time on the bench, thereby necessitating many test benches and the carrying of a large stock of meters.

(*c*) Meters which have been overhauled in the workshop must initially be calibrated by a modified form of either *B* or *C*. In general, it will be by disc revolutions of the meters under test against those of a rotating substandard, or against a meter specially calibrated in the test-room for the purpose. Experience has taught one that there is an optimum number of meters which a tester can efficiently (with respect to time) calibrate in one batch. It therefore would appear that in many cases Method A would not be so convenient for overhauled meters as Method B.

The advantages of Method B are:

(*a*) The meters occupy bench space for the shortest period of any method. Therefore less stock need be held and the test bench accommodation required is reduced to a minimum.

(*b*) The load need not be held steady once it has been set at the required value.

The disadvantages are:

(*a*) Only one dial test is run. This would in all probability be made at a high load and, in consequence, little opportunity would be afforded for mechanical friction to reveal itself during the few disc revolutions run at 5 per cent full load.

(*b*) It is very inconvenient when several different makes of meters, having different full load disc speeds, are tested in the same batch.

(*c*) The meter is not on test for a sufficient length of time to attain a steady temperature at each load.

The advantages of Method C are:

(*a*) The meters are run against an indicating substandard wattmeter which is less likely to introduce errors due to self-heating, wave-form distortion, or voltage variation than even the best of rotating substandard meters.

(*b*) It is very convenient where many different makes of meters, with different full load disc speeds, are likely to constitute a batch for test.

The disadvantages are:

(*a*) The accuracy of the test depends greatly upon the human element and enacts a greater strain upon the testing staff. The load must be held perfectly steady, and this is an extremely tedious occupation for the load-holder, especially over a long period of time.

(*b*) Stop-watch errors are introduced.

(*c*) This method is longer than Methods A and B, and much greater concentration is required of the testing staff. Furthermore, two people are necessary for each batch of meters and the responsibility for accuracy is consequently divided.

We see from the above that the method of test adopted by the meter engineer will, in general, depend upon the size of the undertaking and the test-room facilities available. Method A obviously subjects the meter to the most rigorous test and is very suitable for a large undertaking with many meters of the same size simultaneously passing through the test-room. For medium-sized undertakings, the choice will probably lie between Methods B and C and will depend greatly upon the equipment at the disposal of the tester and also upon his natural bias in favour of one or the other of these two methods. Should there be many makes of meters with different full load disc speeds he will quite possibly choose Method C. For very small undertakings, the probability is that Method B alone will be chosen, as the only substandard apparatus required is in the form of rotating meters.

The above remarks only apply to the final testing of meters, and the meter engineer is at liberty to perform his initial tests in any manner he thinks fit. In the case of meters which have come directly from the workshop after overhaul, they will in all probability be recalibrated by disc revolutions against a rotating substandard or against a meter specially calibrated in the test-room for this purpose. Even in the case of new meters direct from the manufacturer it would be advisable initially to calibrate the meters in this manner, performing any adjustments necessary before the final method of testing is commenced.

Before calibration, each meter should be tested to ensure that the disc does not creep a whole revolution when 10 per cent in excess of the marked voltage is applied to the pressure circuit terminals. Any meter found to creep above this amount should be adjusted by means of the friction compensating loop. Meters which have been overhauled should also, where the facilities are available, be tested for quadrature. Full load, at zero power factor, is applied to the meters and the quadrature loop of each meter is adjusted until the disc stands still or rotates backwards or forwards slightly according to the particular type and make of meter under test.

It is next advisable to ensure that the meters run satisfactorily on starting current. Each spot is set at the same position relative to the magnet gap and the discs are left to run a convenient number of revolutions at the starting load. In the meantime a careful watch is kept upon each meter, and any which stop or run intermittently are inspected in order to ascertain the trouble. The discs may stop running due to any of the following causes:

9.23. Common Faults in A.C. Single-phase Meters. (*a*) Foreign matter in the permanent magnet gap, or between the disc and one of the electro-magnet limbs.

(*b*) Iron in disc.

(*c*) Faulty top bearing or top pin. The former may be dirty, or the latter may possess a rough surface due to rust or wear.

(*d*) Rough or worn bottom pivot.

(*e*) Cracked or rough bottom jewel.

(*f*) Friction in counting mechanism. This may be due to a bent tooth, a faulty spindle or bearing, dirt in one or more of the teeth, sticky bearings, or a rough worm.

(*g*) Incorrect adjustment of the top bearing, or bottom jewel.

(*h*) Buckled disc or bent spindle.

(*i*) Incorrect adjustment of friction compensating loop.

(*j*) Discontinuity in pressure coil or series coil circuit.

9.24. Initial Calibration of Meters. When each meter has satisfactorily passed the starting load test, it may initially be calibrated by:

(*a*) Comparing the disc revolutions with those of a substandard meter at each of the prescribed loads.

(*b*) Using an indicating wattmeter and a stop-watch.

In the case of test (*a*), an ordinary meter of the same size and type as the meters under test is frequently calibrated for the purpose of replacing the substandard meter during the initial tests. This is particularly convenient when the nominal overall disc speed of the meters is different from that of the substandard meter. The calibrated meter and the meters under test are set with their discs in the same position, and the load is applied for the requisite number of revolutions. If R_S is the number of disc revolutions of the calibrated meter and R_T is the number of the meter under test, for any given load, the nominal error of the latter is

$$\left[\frac{(R_T - R_S)\,100}{R_S}\right] \text{ per cent}$$

If the error of the calibrated meter at this load is $+x$ per cent, the actual meter error is

$$\left[\frac{(R_T-R_S)\ 100}{R_S}+x\right] \text{per cent}$$

This error is exclusive of the limitations in stability of the calibrated meter due to self-heating errors, etc.

All meters should be calibrated with their covers on, but in the initial tests it is convenient to make due allowance for the average effect of a meter cover and to test the meters with their covers off.

Rotating substandard meters are usually provided with a dial setting device which permits of the pointers being returned to zero for the commencement of each run. Thus, the tester need only count the number of disc revolutions of the meters under test and open the current and potential switches when the requisite number of disc revolutions of the meters under test have been made. The number of disc revolutions of the rotating substandard meter will not of necessity be a whole number, but since the disc is graduated in one-hundredths of a revolution, this is immaterial.

Suppose N_S is the number of revolutions per unit of the rotating substandard meter, N_T is the revolutions per unit of the meters under test, R_S the revolutions made by the rotating substandard meter (not necessarily a whole number), and R_T is the number of revolutions made by the meter under test. The error of the latter meter relative to that of the substandard meter will be

$$\left[\frac{\left(R_T.\frac{N_S}{N_T}-R_S\right)\ 100}{R_S}\right] \text{per cent}$$

This error must be corrected for the error of the substandard meter, and if this be $+y$ per cent at the particular load applied to the meters, the actual error of the meter will be

$$\frac{\left(R_T.\frac{N_S}{N_T}-R_S\right)\ 100}{R_S}+y \quad \text{per cent}$$

When a wattmeter is used for the purpose of testing the meters, the load is adjusted to give the correct reading upon the wattmeter by means of the rheostats, and this is kept at the same value throughout the period of this test. The meters are each timed for a convenient number of revolutions (the period may be less than 100 seconds for the initial tests). The timing constant of each meter is then calculated and the necessary adjustments made.

Now we have already found, in the case of the d.c. watt-hour meter (section 8.42), that the timing constant of a watt-hour meter is

$$\frac{\text{kW}\times\text{seconds}}{\text{revolutions}}$$

i.e. the kW-seconds per revolution.

We also found that the nominal constant of the meter is 3,600 divided by the revolutions per unit of the meter. This is so, since

$$\frac{\text{kW}\times\text{seconds}}{\text{revolutions}}=\frac{\text{kW}}{\text{revolutions per second}}$$

$$=\frac{3600\times\text{kW}}{\text{revolutions per hour}}$$

$$=\frac{3600}{\text{revolutions per kWh}}$$

$$\text{or }\frac{3600}{\text{r.p.u.}}$$

Suppose K^1 is the actual value of the expression (kW × seconds) per revolution and

$$K=\frac{3600}{\text{r.p.u.}}$$

then the nominal meter error will be

$$\left[\frac{K-K^1)\ 100}{K^1}\right]\text{ per cent}$$

If the wattmeter has an error of $+y$ per cent, i.e. an error which causes it to read y per cent high at the particular load of the test, then the value K^1 must be modified to

$$\frac{K^1(100)}{(100+y)}$$

and the true error of the meter will be

$$\left[\frac{\left[K-K^1\left(\frac{100}{100+y}\right)\right]100}{K^1\left(\frac{100}{100+y}\right)}\right]\text{ per cent}$$

$$=\left[\frac{(100+y)K-(100K^1)}{100K^1}\times 100\right]\text{ per cent}$$

which, to a first approximation, is

$$\left[\left(\frac{K-K^1}{K^1}\times 100\right)+y\right]\text{ per cent}$$

The meters should first be tested at 100 or 125 per cent of the marked current, unity power factor, and adjustment made by means of the brake magnet until a suitable error is obtained (bearing in mind the characteristic load/error curve of the particular type and make of meter under test. When the final adjustment of the brake magnet has been effected, it should be securely clamped.

Full load at 0·5 power factor lagging is then applied to the meters, and any that are not within the limits which the tester sets himself (generally, limits narrower than those prescribed by the Electricity Commissioners) must be corrected by adjustment of the quadrature loop. It may be found that a meter is outside the permissible limits of error even though the loop was adjusted correctly for quadrature at zero power factor. This initial quadrature adjustment, however, is only made for convenience, since it often avoids the necessity of changing the quadrature loop at this stage of the tests, which would entail readjustment of the meter at full load unity power factor, as well as adjustment of the friction compensating loop.

After the above tests have been carried out satisfactorily, the meters are tested at one intermediate load (say half or quarter marked current, unity power factor) and then at 5 per cent of the full load current, unity power factor. Any meters which are not within the prescribed limits at the latter load are corrected by adjustment of the friction compensating loops. Also, if a meter is within the limits on this load, but has a rather large positive error, it is advisable to reduce this error by means of the friction compensating loop, thereby decreasing the possibility of creep at no-load due to too great a friction compensating torque.

At the conclusion of the above tests, it is advisable to re-test all meters at the high-load, which have had any adjustment made other than that to the brake magnet. The whole batch should then be tested to ensure that none creeps a whole revolution at no-load; 10 per cent in excess of the marked voltage being applied to the pressure coil circuits.

In the above initial tests it is advisable to keep the limits of error well within the limits prescribed by the Electricity Commissioners. This will materially assist both the Electricity Commissioners' examiner and the undertaking's testing staff.

The writer recommends that all meters be given an all-night test at starting load. This will give mechanical trouble a better opportunity to reveal itself, since during the night the test-room is likely to be free from the vibration usually associated with a room in which people are working.

9.25. Measurement of Torque. Fig. 124 (*a*) illustrates a simple method of determining the torque exerted upon a meter disc by any given load. One end of a cotton thread *OB* is fixed, and suspended from the other end is a weight of *m* grammes. At a point *l* cm. below the fixed end of the thread is attached a second thread. The other end of the second thread is passed through the hole *D* which is drilled diametrically through a brass sleeving and the disc spindle. The brass sleeving is a tight fit on the spindle; its object being to increase the effective radius of the spindle.

The points *C* and *D* are arranged to be in the same horizontal plane and the required load is applied to the meter. This creates a tension in the thread, between *C* and *D*, which causes the suspension thread between *O* and *C* to make an angle θ with the vertical. The junction *C* is thereby displaced horizontally by *y* cm.; this value being made of convenient magnitude by alteration of the value of *m*.

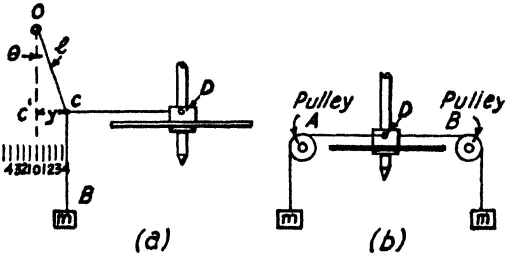


FIG. 124.—Arrangement for Measuring Torque.

When the system is in equilibrium, the tension *T* in the thread, between *C* and *D*, will be

$$\frac{my}{\sqrt{l^2-y^2}}$$

Thus, the load torque

$$=\frac{mry.981}{\sqrt{l^2-y^2}} \text{ dyne-cm.}$$

where *m* is in grammes and *r*, the outer radius of the sleeving, is in cms.

9.26. Measurement of Retarding Torque due to the Brake Magnet. A very convenient apparatus for this purpose is illustrated in Fig. 124 (*b*). The same brass sleeve is employed as for the measurement of torque, described in section 9.25, and a cotton thread passes through the hole drilled diametrically through the sleeve and spindle. Tension is applied to the thread by means of the two weights *m* attached one to each end of the thread, as shown in the diagram. The pulleys are so positioned that the thread between *A* and *D*, and between *D* and *B*, is sensibly in the same horizontal plane. Thus, the weights create a couple of *2mr* grm.-cm. upon the disc; *r* being the radius of the sleeve in cm. and *m* being measured in grammes.

The rotor is turned in a clockwise direction until the weights *mm* are near the pulleys. The system is then released, and as soon as the disc has acquired a constant speed the time taken to complete a convenient number of revolutions is determined by means of a stop-watch.

Thus, if it takes t seconds for N revolutions of the disc, the speed of the disc is

$$\frac{60N}{t} \text{ r.p.m.}$$

and, at this speed, the braking torque is

$$2\times 981mr \text{ dyne-cm.}$$

Thus, at x revolutions per minute the retarding torque T_X, due to the brake magnet, is given by

$$T_X=\frac{2\times 981mrtx}{60N} \text{ dyne-cm.}$$

since the retarding torque is strictly proportional to the speed.

For the above test, the pressure and series coils of the meter are not energised. If it is required to measure the retarding torque due to the braking effect of the pressure coil flux, the brake magnet is taken off the meter, the pressure coil only is energised, and a suitable value of m is employed to give a convenient constant speed of rotation to the disc, due to the couple $2mr$ grm.-cm. The test for the retarding torque due to the braking effect of the series electro-magnet flux is similar, but in this case the series coil alone is energised. Since the magnitude of the latter torque depends upon the load current, it is advisable to obtain the retarding torque for various values of current and construct a curve.

9.27. Stroboscopic Testing. This method of testing is, in general, only employed by meter manufacturers, since its main advantage lies in the fact that it lends itself to the mass-production testing of meters. It can be highly accurate and is sufficiently simple in application for the employment of comparatively unskilled operators under the supervision of a skilled person, to calibrate the meters.

The stroboscope itself is a device which permits of a rotating object to be viewed intermittently and thus produces the optical effect of slowing down or stopping motion. If, for example, a disc is graduated in 100 divisions at its periphery and rotates at a speed of 20 r.p.m., the graduations will appear to be standing still if viewed under a light which flashes 2,000 times per minute. Conversely, if a disc be rotating it is only necessary to determine the number of flashes per minute—at equal intervals—required to produce the optical effect of the graduations standing still, to ascertain the speed of the disc. If, however, the speed of flash is not exactly in synchronism with the graduations, the latter appear to be rotating very slowly backwards or forwards, according to whether the speed of the disc is respectively low or high.

The modern stroboscope consists of a lamp filled with one of the rare gases, such as neon, the speed of the flash being controlled by a thermionic oscillator, or a motor-driven contactor. The flashing rate is usually controlled by means of a knob which, for convenience, is calibrated in r.p.m., corresponding to the number of flashes per minute of the lamp.

Such an instrument, manufactured by General Radio Co., U.S.A., and sold in this country by Claude Lyons Ltd., is the Strobotac type 631-B. It is a small portable stroboscope calibrated to read speed directly in r.p.m. (which, in general, must be divided by 100 for the purpose of determining the speed of a meter disc). A Stroboton neon lamp is employed as the light source and it is mounted in a parabolic reflector. The flashing speed of the neon lamp is controlled by the frequency of a vacuum tube relaxation oscillator which can be adjusted by means of a direct reading dial to any value between 600 and 14,400 flashes per minute.

A feature of the Strobotron is that it is designed to give an extremely short flash (between 5 and 10 micro-seconds) thereby ensuring sharp images.

The accuracy of the scale is guaranteed to be within 1 per cent when standardised in terms of a frequency controlled supply, such as that of the British Grid system.

Several manufacturers have their own system of stroboscopic testing, but unfortunately it is not possible to describe them within these pages. In all cases, however, the principle is essentially the same.

Owing to the difficulties arising from the lack of persistence of vision at speeds below 600 images per minute, the stroboscope is not suitable for the testing of meters at disc speeds less than six revolutions per minute (where the disc is graduated in 100 divisions). One-twentieth load must therefore be tested by one or other of the methods already described.

Even if it were possible to go down to one-twentieth load speed for stroboscopic testing, difficulties would arise due to the effect of the varying friction compensating torque, at various positions of the disc, making itself apparent.

Procedure in the testing of meters by stroboscopic means may be as follows.

The load is first set at the required value (say, at full load, unity power factor) for the adjustment of the overall speed of the disc and, if the corresponding disc speed to this load is 20 r.p.m., the stroboscope dial is set to give a reading of $20n$ r.p.m., where n is the number of graduations upon the disc. The lamp is then focused upon the disc and the permanent magnet braking system is adjusted until the graduations upon the disc appear to remain stationary. If the graduations appear to be moving forward, the overall speed of the disc has to be reduced, and if they appear to be moving backwards the reverse adjustment has to be applied. This procedure is repeated for the setting of full load, at 0·5 power factor lagging, only this time the stroboscope will be set at $10n$ revolutions per minute and adjustment of the disc speed is made by means of the quadrature loop.

In general, the number of graduations marked upon the disc is 100, and this will be the value of n in the above paragraph.

For the intermediate load, say one-half full load at unity power factor, the correct setting of the load is made and the stroboscope dial is adjusted until the disc graduations appear to be standing still. If the number of graduations upon the disc is 100, and the nominal disc speed at this load is ten revolutions per minute the error of the meter will be

$$\frac{(R-1000)\,100}{1000} \text{ per cent}$$

where R is the stroboscope reading when the graduations appear to be standing still.

One-twentieth load and dial test must be run by one or other of the methods previously described.

The range of the Strobotac is far greater than that required for meter testing, as a stroboscope of 600 to 5,000 r.p.m. would satisfactorily fulfil the requirements demanded for the testing of meters. It is, however, illustrated as being a standard model of instrument which could be employed for the stroboscopic testing of meters.

CHAPTER X

Prepayment Mechanisms

10.1. Prepayment Meters. The prepayment meter is popular with many household consumers of electricity on account of the fact that it gives them the opportunity of payment for the energy at the time of supply, thereby avoiding the unpleasant arrival of a quarterly bill. It is particularly convenient in the case of the weekly wage-earner, since it fits in well with the scheme of a weekly budget to which, in many households, one must rigorously adhere. A one-time disadvantage of the prepayment meter was its acceptance of coins of only one denomination, owing to inconvenience when consumers did not have the foresight to keep in hand a coin of the particular value required to operate the mechanism. Since the advent of the multi-coin and dual-coin prepayment mechanisms, however, this objection has largely been overruled. Many prepayment mechanisms of modern design accept coins of a penny, sixpence, and a shilling, and we shall see presently how this is accomplished. Another objection to the prepayment mechanism was that, if a coin of different denomination from that which the mechanism was designed to accept was inserted in the slot, the meter was out of commission until a mechanic arrived from the undertaking to retrieve the offending coin. This has been overcome (e.g. in the Sangamo type B mechanism, and the Chamberlain and Hookham type JPmva mechanism, etc.) by the provision of a push-button attachment which permits the release of rejected coins into the coin box, or a compartment specially reserved for the purpose, and leaves the mechanism free for the acceptance of coins of the correct denomination.

Manufacturers have exhibited marked ingenuity in the development of suitable prepayment devices. A high standard of reliability is called for and, furthermore, the overall dimensions must be kept within the limits enforced by modern requirements. For the mechanism to be successful, it must satisfy the following conditions:

(*a*) Simplicity of operation by the consumer.

(*b*) The accuracy of the meter must not be impaired, due to the attachment of the prepayment device.

(*c*) The mechanism must be fool-proof and fraud-proof.

(*d*) Consecutive insertion of several coins, up to the "coins unused" capacity of the meter, must be possible.

(*e*) Total coins inserted, and coins or units unused, must clearly be registered upon dials provided for the purpose.

(*f*) For every coin of the same value inserted, the meter must deliver the same amount of energy.

(*g*) The switch operation must be certain and positive, and must not unduly deteriorate with service.

(*h*) The changing of the price per unit should be a simple operation, and should not under any circumstances involve the dismantling of the meter.

10.2. General Principle of the Prepayment Mechanism. We will first of all consider, in a general manner, the mechanism of a prepayment meter and then devote a few pages to the special forms of mechanism introduced by various manufacturers.

A prepayment meter consists of a normal quarterly meter to which is attached a coin mechanism, the latter containing a switch which is closed by means of coins inserted via a slot in the meter cover, and which is opened when the advancement of the revolution counter corresponds to the number of coins inserted at the price per unit charged.

The insertion of a coin of the correct value permits of a mechanical coupling between the external operating handle and a coin-rest shaft to which is geared the sun-wheel of a differential gear (section 8.31). Rotation of the operating handle now turns the shaft, thereby advancing the sun-wheel through an angle proportional to the value of the inserted coin, simultaneously driving the planet-wheel arm through a proportionate angle. When the coin rest shaft has rotated through a certain angle, the coin is released and drops into the coin box. The other sun-wheel of the differential gear is driven in the opposite direction by the revolution counter of the meter. Thus, the planet-wheel arm is driven in one direction by the inserted coin and in the opposite direction by the revolution counter, at a rate proportional to the supply of energy to the consumer. When the planet-wheel arm is in its zero position, i.e. when the consumer has exhausted his credit, the switch-tripping mechanism operates. The first coin inserted after this, in addition to advancing the coin sun-wheel of the differential gear, and the planet-wheel arm, causes the switch to close again. Further coins merely advance the coin sun-wheel and the planet-wheel arm in the normal manner.

The gearing between the revolution counter and the energy sun-wheel determines the price per unit to be paid by the consumer. Thus, if the gear ratio between the revolution counter and the energy sun-wheel is $x:y$, the changing of this ratio to $x:2y$ is equivalent to doubling the price charged per unit. Usually, the manufacturer inserts a small gear compound between the revolution counter and the energy sun-wheel; this being designed to facilitate replacement. These gear compounds each correspond to a certain price per unit and are easily interchangeable.

Since the advance of the coin sun-wheel of the differential gear is proportional to the number of coins inserted, a wheel is meshed with it and geared to a further train of wheels, the spindles of which are fitted with pointers which indicate directly, upon a series of dials, the total coins inserted. This provides the engineering and accountancy staffs of the undertaking with very useful information.

We have already seen that the deflection of the planet-wheel arm (and therefore the main differential shaft, to which the arm is rigidly attached) from the position at which the switch trips, is proportional to the coins unused, or to the amount of energy to the credit of the consumer. A pointer may therefore be attached to this shaft, and if provided with a suitable dial it will indicate the coins unused. If the units unused are to be indicated, suitable gearing must be provided between the main differential shaft and the units unused pointer. This gearing must be capable of simple modification with change of price per unit.

Indication of coins unused has the advantage that the average consumer on a prepayment tariff probably appreciates information in monetary, rather than electrical, terms.

Where prepayment mechanisms are designed to take coins of more than one denomination, mechanical coupling will occur for each of the different diameters of the acceptable coins. The gearing between the coin-rest shaft and the coin wheel is selected according to the diameter of the coin inserted, so that the coin sun-wheel may be advanced by an angle proportional to the value of the coin.

We will now briefly consider prepayment devices adopted by various meter manufacturers.

10.3. The Aron Type eP Single-phase Prepayment Meter.

A schematic arrangement of the prepayment mechanism, which is linked to the normal quarterly meter, is illustrated in Fig. 125. When a coin is dropped through the slot provided in the meter cover, it falls through a chute into slot 1 of the coin-rest, where it lodges vertically. As the operating handle is turned, it engages with the cross-pin 3 in the spindle, which is rigidly attached to the semi-cylindrical sleeve 2. In the absence of a coin this sleeve, which co-axially encloses the coin-rest, is free to rotate around the latter. After the introduction of a coin into the coin slot, turning of the operating handle brings the edge 2*A* of the sleeve 2 into contact with the coin and uses it as a lever for the turning of the coin-rest. When half a revolution of the coin-rest has been described, the coin is released into the coin box and the handle, when released, automatically returns to the position for the next coin insertion. The turning of the coin-rest, and wheel 4 which is fixed to the end of it, rotates wheel 5 and the double-throw cam; the latter being double-heart-shaped on account of only half a revolution being necessary for the operation of each coin. The rotation of the cam is against a pressure due to the roller 7, which is located at the end of a lever controlled by a fairly strong spring. This pressure persists until one of the two tips of the cam passes the roller and thereafter forces the completion of half a revolution of wheel 5. This also results in the completion of half a revolution of wheel 4 and the coin-rest 1, causing gravitational ejection of the coin from the now lower face of the slot, and leaving the opposite face ready to receive the next coin.

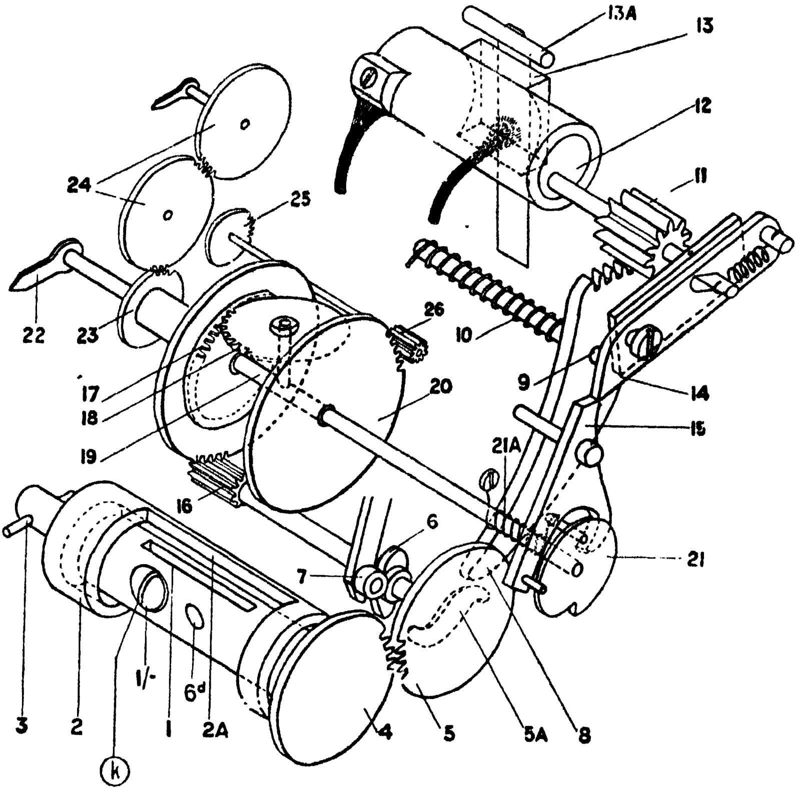


Fig. 125.—Schematic Diagram—Aron type eP Prepayment Mechanism.

When the switch has previously opened, rotation of wheel 5, due to the operation of the following coin, rotates cam 5*A* which is fixed to wheel 5, and one or other end of the cam abruptly deflects the tail of switch quadrant lever 8. This rotates around pivot 9 against the tension of spring 10 and turns pinion 11, thereby turning switch drum 12 into contact with the copper shoe 13, simultaneously lifting switch arm click 14 from a vertical to a horizontal position, where it is trapped by lever 15, and retained in position to keep the switch closed.

The differential gear is similar in principle to that already described, it and geared to a further train of wheels, the spindles of which are fitted with pointers which indicate directly, upon a series of dials, the total coins inserted. This provides the engineering and accountancy staffs of the undertaking with very useful information.

We have already seen that the deflection of the planet-wheel arm (and therefore the main differential shaft, to which the arm is rigidly attached) from the position at which the switch trips, is proportional to the coins unused, or to the amount of energy to the credit of the consumer. A pointer may therefore be attached to this shaft, and if provided with a suitable dial it will indicate the coins unused. If the units unused are to be indicated, suitable gearing must be provided between the main differential shaft and the units unused pointer. This gearing must be capable of simple modification with change of price per unit.

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10.3. The Aron Type eP Single-phase Prepayment Meter.

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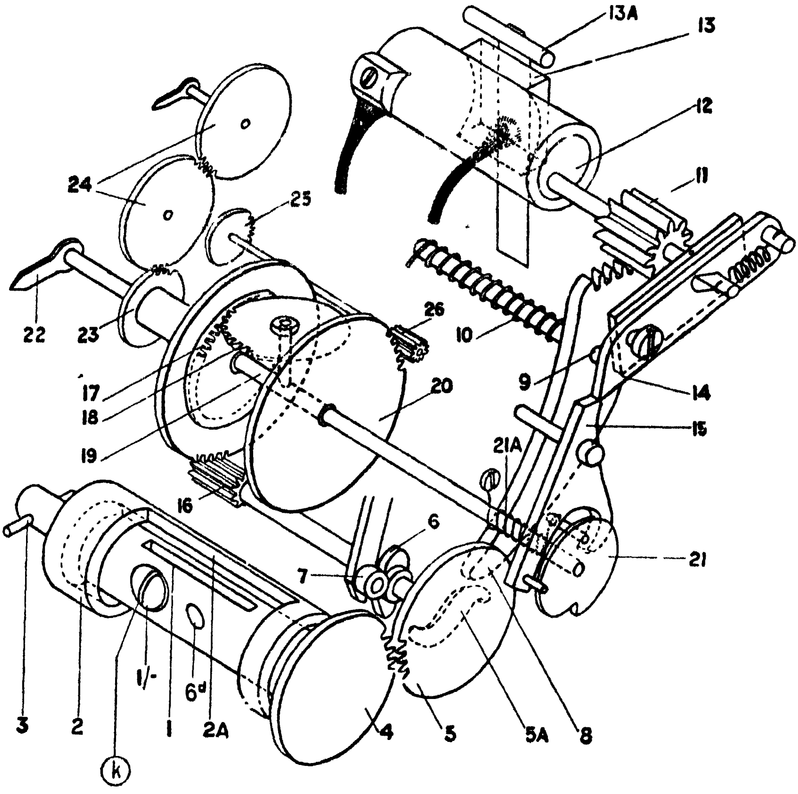

Fig. 125.—Schematic Diagram—Aron type eP Prepayment Mechanism.

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The differential gear is similar in principle to that already described, the coin sun-wheel being driven by pinion 16, and the energy sun-wheel by wheel 25 from the revolution counter of the meter, via the price-change gear compound. We have already seen that the spindle 19 will vary in position according to the coins to the credit of the consumer. Attached to this spindle is cam 21, and when the coins credit is exhausted the pin on the inner face of cam 21 deflects lever 15 and releases the tip of click 14. This abruptly opens the switch contacts through the action of spring 10 operating through the rack in the end of lever 8, and through pinion 11.

10.4. Chamberlain & Hookham Prepayment Switch. An interesting feature of the Chamberlain & Hookham prepayment mechanism is the mercury switch illustrated in Fig. 126. This consists of a tube of insulating material, plugged at each end and pivoted in the centre. Two copper contacts are fitted in the manner shown, and the tube is partly filled with mercury so that, when the axis of the tube is horizontal, the mercury connects the contacts, thereby completing the series circuit of the meter. When the tube is inclined, the mercury rests in the lower portion of the tube, thereby breaking the series circuit.

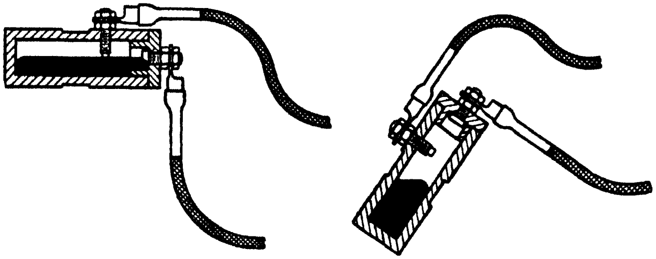


FIG. 126.—Mercury Switch (Chamberlain & Hookham Ltd.)

10.5. Metropolitan-Vickers Type DM Prepayment Meter. In the Metropolitan-Vickers type DM prepayment meter, the drive from the revolution counter to the energy sun-wheel of the differential gear is assisted by a helical spring which is wound up by the turning of the coin-operating handle when a coin is passed through the mechanism. This has the dual advantage of relieving the meter of the prepayment mechanism friction and taking up the back-lash in the gear train, thereby reducing the individual error in the electricity delivered for each first coin. A single-pole switch of the quick-make quick-break contactor pattern is employed, and this has a self-aligning rolling butt contact, and asbestos arc shields. When fitted to a d.c. meter it is provided with a magnetic blow-out.

10.6. Ferranti Switch. The Ferranti switch is of the open type and of unusual, yet simple, construction. It has a very narrow break and relies for its action upon rapid de-ionisation which is induced as the current wave passes through zero. The actual break occurs in 2.10^2 micro-seconds, and once the gap has been de-ionised the current cannot re-establish itself.

The object in making the gap small is twofold. It leaves only a small volume of gas to be de-ionised and also keeps the pressure drop between the contacts (which is proportional to the length of the gap) very small, thereby keeping the energy available for ionisation at a low value. The switch is actually constructed so that there are two gaps in series (Fig. 127 (*a*)), thus enabling the energy to be dealt with in two separate places. The electrodes are formed of comparatively large masses of high conductivity copper, which rapidly cool the electrode "hot-spot" and, by virtue of their possessing a large surface area, quickly absorb the heat of the ionised gases. The absorption is accentuated by the lateral spreading of the arc in the partial vacuum created by the receding contacts. When the switch closes, a wiping action takes place between the contact surfaces, thereby removing any particles of foreign matter from between the surfaces, which would otherwise increase the contact resistance. The largest duration for the arc to be maintained is 1/2*f* seconds, where *f* is the frequency of the supply. Thus, at 50 cycles per second, the maximum arcing period is 1/100th of a second.

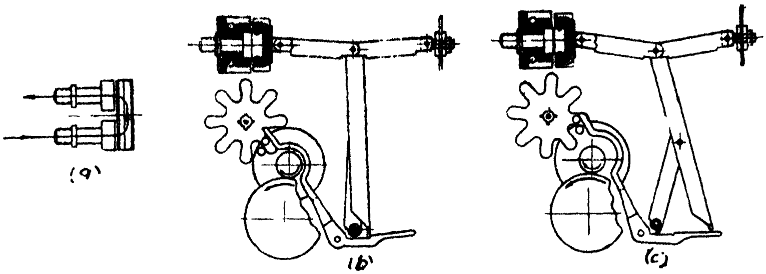


FIG. 127.—Prepayment Mechanism Switch (Ferranti).

Fig. 127 illustrates diagramatically the general principles of the mechanical operation of the switch. The advantage of the double-toggle device is that it reduces the mechanical loading of the tripping system, at the same time closing the switch contacts with a pressure of 60 lb. per square inch. The switch operates satisfactorily up to 50 ampères 230 volts.

10.7. Two-part Tariff Prepayment Devices. One method of charging a consumer for the supply of electricity is by means of a two-part tariff (section 12.7), whereby the consumer pays a fixed charge per year to cover his share of the capital and standing charges of the undertaking, and a certain price per unit for the energy supplied. Generally, the price charged per unit is considerably less than when the consumer is on a flat rate, i.e. when his only form of payment is at a certain price per unit, irrespective of how much or how little he takes. Such a tariff is particularly beneficial to the consumer who makes full use of his installation, since the price per unit is relatively small, and providing he uses sufficient electrical energy, the overall cost per unit is affected but little by the standing charge. Such a system also has its advantages for the undertaking, since it guarantees a certain annual income towards the meeting of the interest and sinking fund charges and, to a certain extent, covers the general maintenance, wages costs, and managerial expenses inevitably incurred by such an establishment. Furthermore, it is apparent that the consumer who demands very little energy during the course of the year is only a liability to the undertaking, as his bill would possibly not even cover his fair share of the cost involved in guaranteeing him a supply of electricity, apart from the fuel costs due to the generation of the electrical energy supplied to him.

In order to meet the requirements of such a tariff, and still permit consumers to remain upon a prepayment basis, manufacturers have developed fixed-charge collectors which have been designed to operate in conjunction with the normal prepayment meters. These collectors operate by subtracting a prearranged sum, over a given period, from the amount credited to the consumer by the prepayment mechanism.

10.8. Two-part Tariff Meter—Shunt-torque Type. One of the earliest forms of two-part tariff prepayment a.c. meter consisted of a normal quarterly meter given a very powerful shunt torque which rotated the disc at a predetermined speed; the meter having a normal prepayment attachment. Since this rotation was superimposed upon the whole curve of the meter, it was equivalent to reducing the consumer's coin credit at a constant rate, whilst charging a certain price per unit for energy supplied. The speed of rotation of the disc was dependent upon the predetermined fixed charge, and was directly proportional to the latter. Obvious disadvantages of such a meter were that each had to be tested and adjusted to suit the individual tariff requirements of the consumer in whose premises the meter was to be installed, and that the alteration of the fixed charges of a meter was a somewhat involved operation. Furthermore, alteration of the price per unit of the prepayment mechanism, unless accompanied by an adjustment of the no-load speed of the meter to an inverse ratio, altered the fixed charge of the meter. Another disadvantage of the meter was that, as the shunt torque is proportional to the square of the applied voltage, the meter was rendered inaccurate with departure from the declared value.

10.9 Synchronous Motor Type Fixed Charge Collector. Generally, in the modern a.c. two-part tariff meter, the fixed-charge side of the mechanism consists of a small, self-starting, synchronous motor which is connected across the mains on the input side of the meter, thereby ensuring that it will run continuously. The usual speed of such a motor is 200 r.p.m. (i.e. it is fitted with fifteen pairs of poles for use at 50 cycles), and it will remain in synchronism for any applied voltage within the wide limits of ±25 per cent of the declared value. This motor is utilised, through a gear train, to drive one side of an auxiliary differential, whilst the other sun-wheel of this differential gear is driven by the revolution counter. Both of the drives are positive and, as a result, the planet-wheel arm and the spindle to which it is rigidly attached advance at a rate proportional to the summation of the advancement of the two sun-wheels. The spindle is then geared to the energy sun-wheel of the main differential gear of the prepayment mechanism, the latter being identical with that already described in section 10.2, etc.

It will be appreciated that the standing-charge tariff will be more flexible than the price per unit (the former often being based upon a certain percentage per year of the rateable value or, in some cases, upon the floor area of the house in which the meter is to be installed). Therefore, some convenient means of altering the rate of exhaustion of the consumer's coin credit, to suit individual conditions, is necessary. This must be capable of simple adjustment and, preferably, avoid the necessity of fitting special gear compounds between the synchronous motor and the fixed-charge sun-wheel of the auxiliary differential. Since no variable speed transmission, covering a very large range and including very small change steps, has been devised, several ingenious arrangements have been developed to overcome this difficulty. Broadly, they all work upon the same principle whereby only an intermittent portion of the synchronous motor drive is transmitted to the coin credit reducing mechanism; the remainder of the travel of the synchronous motor being idle. The cycles of useful and idle time are predetermined, the total overall periods being equal and of comparatively short duration, and the ratio (useful time) : (useful+idle time) is constant for any given setting of the fixed charge. Usually, an arm carrying a pawl is rotated at a constant speed by the synchronous motor, through a train of gears. During each revolution of the arm the pawl engages with a wheel geared to the fixed-charge sun-wheel of the auxiliary differential gear and advances the wheel through a certain angle, after which the pawl is disengaged. This advancement can be varied and, as the rate of exhaustion of the consumer's coin credit is directly proportional to the angular advancement of the above-mentioned wheel, per revolution of the constant speed operating arm, it forms a very simple method of variation of the fixed charge applied to the consumer.

10.10. Hotel-service Prepayment Mechanisms. In view of the demand by hotel authorities for a prepayment mechanism, the coin credit of which can be mechanically exhausted when a resident permanently vacates his room, manufacturers have modified their normal prepayment mechanism to meet this requirement. A push-button is attached to the cover of the meter, and upon being pressed the differential gear is so released that the planet-wheel arm and main differential spindle return to the zero position, thereby exhausting the consumer's credit and tripping the switch. When the room is again occupied, the new resident must insert a suitable coin in the prepayment mechanism before he can receive a supply of electricity.

11.11. Prepayment Collectors for Billiard Tables, etc. A novel application of the prepayment meter is for the collection of charges for the hire of billiard tables in certain places of amusement. The table lighting system is connected to the output side of the prepayment meter and the price per unit is governed by the rate of charge for the billiard table and by the load upon the meter. Thus, if the charge is at the rate of two shillings per hour for the table, and the table lighting system consumes 500 watts, the price per unit can be determined from the expression

$$\text{Price per hour} = (\text{Price per unit})\ (\text{Units consumed per hour})$$

$$\text{Price per unit} = \frac{\text{Price per hour}}{\text{Units consumed per hour}}$$

$$= \frac{2/-}{0{\cdot}5}$$

$$= 4/-$$

When the billiard table has been in use for a period corresponding to the money inserted in the meter, the switch trips and it is necessary to insert a further coin in order to have sufficient light for play to be resumed.

10.12. The Testing of Prepayment Mechanisms. Since the testing of the meter element is exactly the same as for the ordinary quarterly meter of the same make and type, we will only consider the method of testing the mechanism.

Each tester is likely to evolve his own particular method, and this will largely depend upon the type and make of meter passing through his hands, and the faults to which it is most prone. A general form of test, however, may be made in the following manner.

The mechanism is run down to zero credit by connecting a load to the meter. Alternatively, the gearing between the pinion, or worm, on the disc spindle and the first wheel of the revolution counting mechanism may be disengaged. The wheel is then turned by hand, in the same direction as it would be driven by the disc spindle, until the coin credit is almost zero. The gearing is now remade and the switch is allowed to trip by means of an electrical load applied to the meter. The revolution counter and prepayment mechanism pointers are now set to zero (or, where necessary, their registrations are recorded). A coin is then inserted and during the cycle of operations careful observation is made to ascertain whether the switch makes contact in the correct manner. The coin credit is again reduced by hand until it is almost zero; the final tripping being made by a load upon the meter. The registration of the revolution counting dial should now be equal to

$$\frac{\text{The value of coin inserted (pence)}}{\text{Price per unit (pence)}}\text{kWh}$$

within the limits allowed and the actual reading should be recorded. A further coin is inserted and the above procedure repeated, except that the pointers are left untouched. The revolution counter pointers should now read twice the previous value.

This test is repeated until the tester has assured himself that the switch is operating satisfactorily and that the "first-coin" error is within the prescribed limits (usually ±5 per cent). It should be mentioned that one of the tripping tests should be made with one-twentieth full load upon the meter, and another with reasonable vibration at full load.

Coins are then inserted to the "coins-unused" capacity of the mechanism, and inspection must be made to ensure that the device prohibiting the insertion of further coins now comes into operation. The mechanism may now be run down electrically, or by hand, but the final tripping must be effected by a load upon the meter. When it has finally tripped, the advance of the revolution counter pointers should bear the following relationship to the total coins inserted:

$$\frac{\text{Value of total coins inserted (pence)}}{\text{Reading of meter in kWh}}=\text{Price per unit (pence)}$$

Throughout the above process it is essential to ensure that the operation of each coin is correctly recorded upon the "Total coins inserted" and "Coins unused" or "Units unused" dials of the prepayment mechanism, and the overall accuracy in the delivery of energy by the meter should be much higher than that per individual coin. In fact, the advance of the revolution counter dial should be well within ±1 per cent of the value corresponding to the total coins inserted.

If the mechanism fails to pass the above tests, the cause of the trouble must be located and removed. The remedy will obviously depend upon the type and make of mechanism, and it is therefore advisable to purchase meters with as simple, but reliable, mechanisms as possible.

The most frequent source of trouble with the prepayment mechanism is the variation in quantity of energy delivered for a single coin. This is because most prepayment devices are designed for the insertion of ten or more coins to the credit of the consumer. This usually represents an advance of 180° of the main differential spindle, and since the tripping mechanism does not operate at an absolutely definite point of the cam, a very small variation in the tripping point represents an appreciable portion of the cam motion for one coin. The design of a mechanism with a small "coins-unused" capacity would greatly minimise this trouble. Furthermore, in the case of a meter designed to operate with shillings, it is most unlikely that the consumer would have a coin credit approaching ten shillings; this giving a strong case for a meter with a smaller "coins-unused" capacity.

In practice, the above characteristic of the prepayment mechanism is not so serious as it would at first appear, since what a consumer loses upon one coin he gains upon another and, taken over the average of a great number of coins, the energy delivered will be correct.

In the case of a hotel-service prepayment meter, the "coins unused" capacity is generally four coins, thereby greatly minimising the first-coin error and giving each consumer a better chance of receiving his correct amount of energy.

The above method of testing prepayment devices would appear very lengthy where a batch of, say, fifty meters are to be tested. The tripping tests, however, can be arranged so that the switches open at different times. When a meter switch trips the advance of the revolution counter should be recorded and a further coin inserted, thereby completing the series circuit of the whole batch of meters.

10.13. The Testing of Two-part Tariff Meters. The testing of the prepayment mechanism can be performed in the manner described in section 10.12, it only being necessary to open the synchronous motor circuit of the fixed-charge collector. The latter may be tested by noting the time taken for it to trip the switch after the insertion of a convenient number of coins. For the purpose of measuring the time accurately, an electric clock of the type suitable for operation from time-controlled mains may be connected across the load terminals of the meter (with the testing link between the pressure and series coils connected). The energy taken by the clock will be too small to cause any but the smallest sizes of meter to register and will therefore have no effect upon the accuracy of the test. The clock can be arranged to commence as soon as the meter switch is closed, and will stop as soon as the switch opens.

In view of the fact that the synchronous motor only intermittently drives the fixed-charge sun-wheel of the auxiliary differential gear, the above tests must be run for a multiple of the period taken for one complete cycle of useful and idle drive of the synchronous motor.

In general, a combined test of the prepayment mechanism and the fixed-charge collector is more convenient (as well as approaching more nearly the actual conditions of service). In this case, one or more coins are inserted and a convenient load is placed upon the meter. The fixed-charge collector is allowed to operate for a multiple of the period between successive drives of the synchronous motor. At the completion of the period the fixed-charge collector is rendered inoperative by disconnecting the synchronous motor, and the meter is allowed to continue running until the switch trips. If E units of electricity are registered upon the revolution counter dial, the price per unit being P pence, and the coins inserted are equivalent to C pence, then the reduction in credit by the fixed-charge collector is

$$(C-EP) \text{ pence.}$$

If H is the number of hours of operation of the fixed-charge collector, the fixed-charge S, in pence per week, is given by

$$S=(C-EP)\frac{168}{H}$$

which should agree with the setting of the fixed-charge indicated upon the tariff dial.

In order to facilitate a more speedy test than the above, certain manufacturers have produced small gear units for attachment to the fixed-charge collector. The function of such a unit is to replace a section of the gearing between the synchronous motor and the drive of the wheel before the intermittent driving device. These can sometimes be operated manually.

CHAPTER XI

Measurement of Energy in Polyphase Circuits

11.1. Number of Watt-hour Meters Required for Energy Measurement in Polyphase Circuits. In section 4.4, we derived a rule concerning the number of wattmeters, or elements in a polyphase wattmeter, which would be sufficient for the measurement of power in a polyphase circuit. This rule is that if there are n wires in the supply network, $(n-1)$ wattmeter elements will be sufficient for the measurement of power expended in the circuit, irrespective of whether or not the system is balanced. Furthermore, it is entirely independent of the law of variation of the current in the respective lines.

We have since found that if the wattmeters, or polyphase wattmeter, be replaced by watt-hour meters, or a polyphase watt-hour meter connected in a similar manner, the total consumption of energy will be integrated over any given period. Thus, we may state that if there are n wires in the supply network of a polyphase system, $(n-1)$ single-phase watt-hour meters (or one polyphase watt-hour meter comprising $(n-1)$ elements) are capable of measuring the total energy dissipated in a circuit.

In general, a single meter is used for such purposes, and it usually consists of two or more driving elements, mounted in such a manner as to have one common spindle and one common braking system. A separate disc is usually provided for each element, but in certain meters the individual elements operate upon one common disc.

We will now consider the methods employed in practice for the measurement of energy in the various polyphase a.c. systems.

11.2. Two-phase Three-wire System. Since this system is not so common as the others about to be described, we will first consider the arrangement of such a network. The circuit diagram is given in Fig. 128. The system consists of two phases, commoned at one end of each, the p.d.s being 90° out of phase, as shown in the vector diagram of Fig. 128 (*b*). Thus, if the voltage across each phase is V, that across the outer wires of the system will be $1{\cdot}41V$, leading the phase voltages by 45° and 135° respectively. Similarly, if it is a balanced system, the current in the common wire will be 1·41 times the current in either of the separate phases.

One method of measuring the energy in such a system is by means of two single-phase meters, connected as shown in Fig. 128 (*a*). The total energy will be

$$\frac{1}{1000}\int_{t_1}^{t_2}(e_1i_1+e_2i_2)dt.\ \text{kWh}$$

where t_1 and t_2 are measured in hours.

But meter 1 will register $\frac{1}{1000}\int_{t_1}^{t_2} e_1 i_1 dt.$ kWh

and meter 2 will register $\frac{1}{1000}\int_{t_1}^{t_2} e_2 i_2 dt.$ kWh

Hence, the summation of their readings will give the total energy supplied between t_1 and t_2 hours.

This method is not suitable for the purpose of measuring total maximum demand, and where this is required it is essential to use a two-element meter. The connections of the elements are similar to those for the single-phase meters, and the driving torque will therefore be proportional to $E_1I_1 \cos \theta_1 + E_2I_2 \cos \theta_2$ and, since the speed will be proportional to the torque, the meter will register the total energy supplied. Actually, the common spindle has the effect of summating the torques of both elements, thereby making it possible to integrate the total energy upon one revolution counter dial.

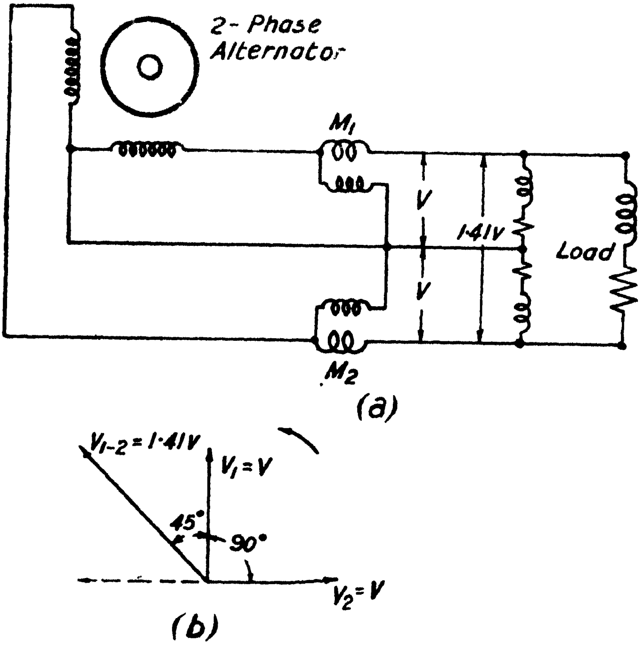


FIG. 128.—Two-phase Three-wire System.

When the loads are equal, each of the single-phase meters will read half the total load. Thus, one meter only need be employed, providing its revolution counter is geared to register twice the previous amount. It should be mentioned that such a method has not met with the approval of the Electricity Commissioners as a means of measuring the supply of electricity.

11.3. Three-phase Three-wire System. Three-element Meter. We have already seen (section 4.6) that three single-phase wattmeters may be employed for the measurement of power in a three-phase three-wire circuit. Similar reasoning shows that three single-phase watt-hour meters, connected in a similar manner, will measure the total energy supplied in a circuit. A polyphase watt-hour meter, comprising three elements, and connected in the same manner, would also be capable of measuring the total energy in a three-phase three-wire circuit.

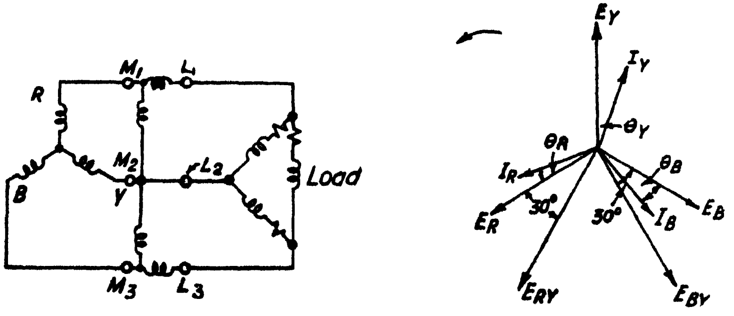
11.4. Three-phase Three-wire System. Two-element Meter. Referring to section 4.3, we found that the total instantaneous power in a three-phase three-wire circuit is

$$i_1(v_1-v_2)+i_3(v_3-v_2)$$

Therefore, the total energy is

$$\frac{1}{1000}\int_{t_1}^{t_2} i_1(v_1-v_2)+i_3(v_3-v_2)dt. \text{ kWh}$$

If the rotational speed of a disc is made proportional to the mean



(a) (b)

FIG. 129.—Connections and Vector Diagram for Three-phase Three-wire Two-element Meter.

value of $i_1(v_1-v_2)+i_3(v_3-v_2)$, then providing it is suitably geared to a revolution counter, the latter will register the total energy directly in kWh. In practice, this is attained by employing one common disc and two separate watt-hour meter elements, the elements being connected in the manner illustrated in Fig. 129 (*a*); whilst the vector diagram of Fig. 129 (*b*) represents the conditions when the load is balanced.

Where it is required to measure the total maximum demand, a polyphase meter of the above type is essential. In other cases, however, two single-phase meters could be utilised for the purpose.

Under the above conditions the "red" element will register

$$\frac{1}{1000}\int_{t_1}^{t_2} E_{RY} I_R \cos(30+\theta_R)dt. \text{ kWh}$$

and the "blue" element

$$\frac{1}{1000}\int_{t_1}^{t_2} E_{BY} I_B \cos(30-\theta_B)dt. \text{ kWh}$$

and the summation of their readings will give the total energy delivered between t_1 and t_2 hours.

When the load is balanced, $E_{RY}=E_{BY}=E_{RB}$, $I_R=I_Y=I_B$, $\theta_R=\theta_Y=\theta_B$ and the total power

$$=E_{RY}I_R\cos(30+\theta)+E_{BY}I_B\cos(30-\theta)$$

$$=EI[\cos(30+\theta)+\cos(30-\theta)]$$

where $E=E_{RY}$, etc., $I=I_R$, etc., $\theta=\theta_R$, etc.

Total power $=EI\,[\cos 30\cos\theta-\sin 30\sin\theta+\cos 30\cos\theta+\sin 30\sin\theta]$

$$=EI\,[2\cos 30\cos\theta]$$

$$=\sqrt{3}\,EI\cos\theta$$

and the total energy

$$=\frac{\sqrt{3}}{1000}\int_{t_1}^{t_2} EI\cos\theta dt. \text{ kWh}$$

Under these balanced conditions the energy may be measured by a single-element meter with two transformers, connected as shown in Fig. 130 (*a*) or, alternatively, by a meter with two series windings upon the same electro-magnet limb, connected as illustrated in Fig. 130 (*b*).

In both cases the conditions will be as represented by the vector diagram of Fig. 130 (*c*), and the resultant energising current of the series electro-magnet (I_{R-B}) will lag E_{R-B} by angle θ. Therefore, the meter will register

$$\frac{1}{1000}\int_{t_1}^{t_2} E_{R-B} I_{R-B}\cos\theta dt. \text{ kWh}$$

but $E_{R-B}=E$ and $I_{R-B}=\sqrt{3}\,I_R=\sqrt{3}\,I$

$$\text{therefore registration } = \frac{\sqrt{3}}{1000}\int_{t_1}^{t_2} EI \cos \theta dt. \text{ kWh}$$

which will be correct as long as the system remains balanced.

For the directly connected meter, the same number of ampère-turns must be provided upon each of the series windings. Similarly, in the case of the meter employed in conjunction with current transformers, the latter must have the same ratios.

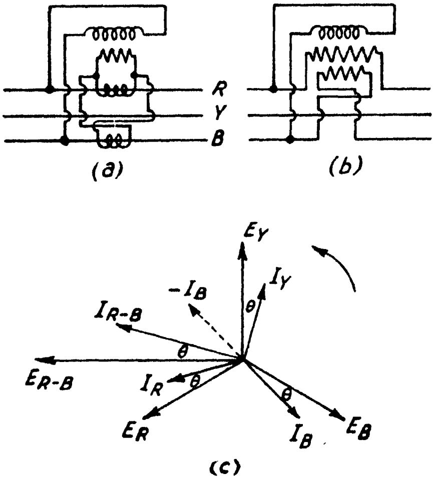


FIG. 130.—Connections and Vector Diagram for Balanced Load Single-element Three-phase Three-wire Meter.

The above type of meter has not received the approval of the Electricity Commissioners. It is, however, used frequently for costing purposes in which a second party is not involved.

11.5. Three-phase Four-wire System. In the four-wire system, the neutral is available, and therefore three single-phase meters may be connected so as to measure the energy in each phase. The connections

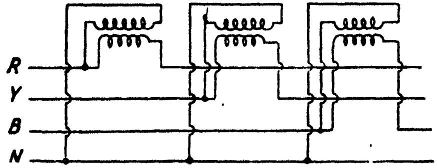


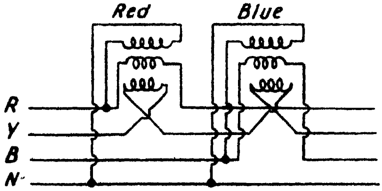
FIG. 131.—Connections for Three-element Meter—Three-phase Four-wire System.

are as shown in Fig. 131, and the total energy is the summation of their respective registrations.

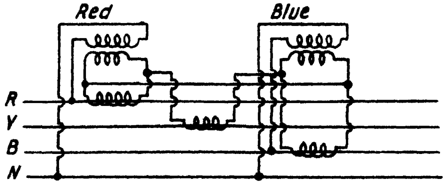
Alternatively, a single meter comprising three separate single-phase elements may be employed and so mounted as to drive one common moving system, the speed of which (neglecting friction and the varying current fluxes) is proportional to the mean power. The meter therefore registers $\frac{1}{1000}\int_{t_1}^{t_2}(E_R I_R \cos\theta_R + E_Y I_Y \cos\theta_Y + E_B I_B \cos\theta_B)dt$. kWh which is the total energy delivered to the circuit.

Such a meter has the advantages over three single-phase meters of being more compact, registering the total energy on one dial, and being suitable for the measurement of average total maximum demand.

11.6. Two-element Meter for Three-phase Four-wire Systems. A two-element meter is frequently employed for the measurement of energy in a three-phase four-wire circuit. The connections of



(a). Directly connected Meter



(b). Current Transformer connected Meter

FIG. 132 (a) and (b).—Circuit Diagrams for Two-element Meters—Three-phase Four-wire System.

such a meter are illustrated in Figs. 132 (a) and (b) for directly connected and transformer connected meters respectively. As can be seen from the diagram, the "red" element has its potential circuit energised by means of the red to neutral voltage, and the series electro-magnet is energised by the red current and the reversed yellow current. The "blue" element has its potential circuit energised by the blue to neutral voltage, whilst its series electro-magnet is energised by the blue current and the reversed yellow current.

In the directly connected meter there are two windings on the series electro-magnet of each element, and these are suitably insulated to withstand the difference in potential between the respective lines. The transformer connected meters have only one series winding upon each series electro-magnet; this being possible by virtue of the isolation of the secondary windings of the transformers from the line potentials.

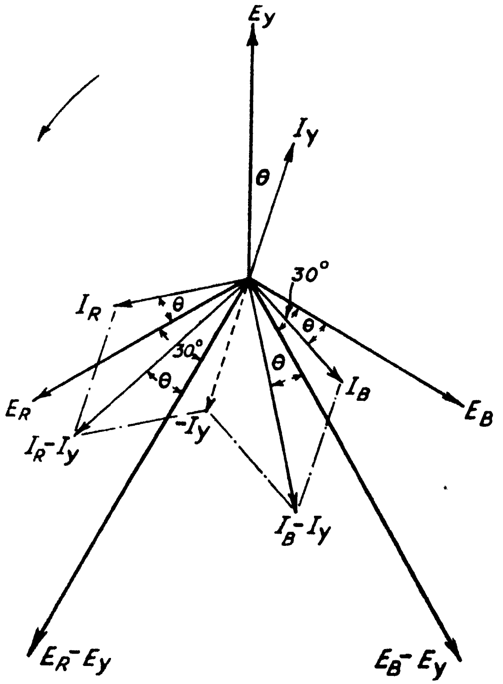


Fig. 132 (*c*).—Vector Diagram for Two-element Meters—Three-phase Four-wire System.

Considering the case of a balanced load, where the power factor is cos θ, the conditions will be as represented in the vector diagram of Fig. 132 (*c*). The driving torque of the "red" element will be proportional to

$$E_R I_{R-Y} \cos (30-\theta)$$

$$\text{or } \sqrt{3} E_P I \cos (30-\theta)$$

where $E_P = E_R$, etc., and $I = I_R$, etc.

The driving torque due to the "blue" element will be proportional to

$$E_B I_{B-Y} \cos (30+\theta)$$

$$\text{or } \sqrt{3} E_P I \cos (30+\theta)$$

Therefore, the total driving torque on the moving system will be proportional to

$$\sqrt{3}E_PI\,[\cos(30-\theta)+\cos(30+\theta)]$$
$$=K\sqrt{3}E_PI\,[2\cos 30\cos\theta]$$
$$=3KE_PI\cos\theta$$
$$\text{or }\sqrt{3}KEI\cos\theta,\text{ since }E=\sqrt{3}E_P$$

and the total registration will be

$$\frac{\sqrt{3}}{1000}\int_{t_1}^{t_2}EI\cos\theta dt.\ \text{kWh}$$

which is the total energy supplied.

Should the conditions depart from balance, the registration will be

$$\frac{1}{1000}\int_{t_1}^{t_2}E_R[I_R\cos\theta_R+I_Y\cos(60-\theta_Y)]+E_B[I_B\cos\theta_B+I_Y\cos(60+\theta_Y)]dt.\ \text{kWh}$$

whereas, it should be

$$\frac{1}{1000}\int_{t_1}^{t_2}(E_RI_R\cos\theta_R+E_YI_Y\cos\theta_Y+E_BI_B\cos\theta_B)dt.\ \text{kWh}$$

Therefore, the error in registration will be

$$\frac{1}{1000}\left[\int_{t_1}^{t_2}(E_RI_Y\cos(60-\theta_Y)+E_BI_Y\cos(60+\theta_Y)dt-\int_{t_1}^{t_2}E_YI_Y\cos\theta_Y dt\right]\text{kWh}$$

If $E_R=E_Y=E_B=E_P$, the error

$$=\frac{1}{1000}\left[\int_{t_1}^{t_2}E_PI_Y[\cos(60-\theta_Y)+\cos(60+\theta_Y)]dt-\int_{t_1}^{t_2}E_PI_Y\cos\theta_Y dt\right]\text{kWh}$$

$$=\frac{E_PI_Y}{1000}\left[\int_{t_1}^{t_2}2\cos 60\cos\theta_Y-\int_{t_1}^{t_2}\cos\theta_Y dt\right]\text{kWh}$$

$$=\frac{E_PI_Y}{1000}\left[\int_{t_1}^{t_2}\cos\theta_Y dt-\int_{t_1}^{t_2}\cos\theta_Y dt\right]$$

$$=0$$

The meter will therefore register accurately as long as the voltages remain balanced. To determine the error when the voltages are unbalanced, it is necessary to evaluate the expression

$$\text{Per cent error} = \frac{\int_{t_1}^{t_2} E_R I_Y \cos(60+\theta_Y) + E_B I_Y \cos(60+\theta_Y)dt - \int_{t_1}^{t_2} E_Y I_Y \cos\theta_Y dt}{\int_{t_1}^{t_2} (E_R I_R \cos\theta_R + E_Y I_Y \cos\theta_Y + E_B I_B \cos\theta_B)dt}$$

We see from the above that, from the point of view of accuracy, a three-element meter is superior to the two-element meter in a four-wire circuit. On account of its limitations, the latter meter has not received the approval of the Electricity Commissioners.

Two single-phase meters, in conjunction with three transformers connected in the same manner as for the two-element meter, or two single-element meters with their series windings suitably modified, may be employed for the measurement of energy in a three-phase four-wire circuit. They have the same limitations, however, as the two-element meter and, in addition, are not suitable for use when it is required to measure the average total maximum demand.

11.7. Constructional Details of Polyphase Meters. Each driving element of a polyphase meter is similar in construction to that of the single-phase meter of the same make. It is modified, however, in order that the driving torque can be adjusted. This is essential, since the various elements of the meter have one common braking system, and each element must exert the same driving torque upon the moving system, for any given load, to effect accurate registration.

A device is therefore provided for the purpose of increasing or decreasing the driving torque of an element relative to that of the other elements. It may operate upon the series or potential fluxes of the meter. In the Chamberlain & Hookham two-element polyphase meters, types BT and BT4, the lower element is inverted and the series laminations of the two elements are mounted on the same base plate. In this manner they are fixed relative to one another. By means of two eccentric studs, operating in slots cut in the base plate, the series laminations can be moved up or down. When moved upwards, the gap between the series and potential laminations of the upper element is narrowed, thereby increasing the torque of that element. At the same time, the gap of the lower element is widened and this reduces the driving torque of the lower element. Thus, any adjustment of the balance device affects both elements and necessitates the complete recalibration of the meter.

11.8. Type JT4 Polyphase Meter—Chamberlain & Hookham Ltd. In the three-element meter, type JT4, manufactured by Chamberlain & Hookham Ltd., only top and bottom elements are fitted with balance adjusters; the middle being the reference element. The adjustment is similar to that already described for the types BT and BT4 meters, except that the series laminations are not mechanically coupled and can therefore be individually adjusted. It should be mentioned that both the eccentrics operating the series laminations should be adjusted by the same amount, otherwise the series laminations will become tilted, thereby altering the friction compensating torque of the potential flux.

11.9. Type NE Meter—Metropolitan-Vickers Ltd. Balance of elements in the Metropolitan-Vickers, type NE, polyphase meter is attained by adjustment of the position of an iron bar which is located behind and across the poles of the potential electro-magnet. As the bar is brought nearer the poles, so the driving torque of the element is decreased. The position of the bar is adjusted by means of screws accessible from the front of the meter.

11.10. Sangamo Polyphase Meter. The Sangamo polyphase meter is so designed that the return plates (which, in the single-phase meters, are fixed in positions relative to the series laminations) are adjustable by means of levers. It is therefore possible to alter the value of the series flux operating upon the disc and in this manner vary the driving torque of one element relative to that of the other elements of the meter. Another feature of this meter is the iron shields cast in the base and positioned between elements; their purpose being to prevent interaction between elements.

11.11. Types FLx and FLy Meters—Ferranti Ltd. An interesting feature of the Ferranti polyphase meters, types FLx and FLy, is that the driving elements operate upon one common disc, thereby allowing of a more compact meter. This is made possible by virtue of the disc having a glass centre-portion and an aluminium outer rim. It is claimed that this composite form of disc reduces the interaction between the elements to a minimum.

11.12. Aron Pendulum Meter. This, being essentially a two-element meter, lends itself to the measurement of power in a three-phase circuit. The connections for the three-phase three wire and three-phase four-wire meters are given in Figs. 133 (*a*), (*b*), and (*c*); the last mentioned giving the connections for a three-phase four-wire meter which is transformer connected. Balance of the elements is performed by adjustment of the ballast resistances, in exactly the same manner as for the single-phase meter (except that the elements are adjusted separately). This type of meter has the advantage of being sensibly

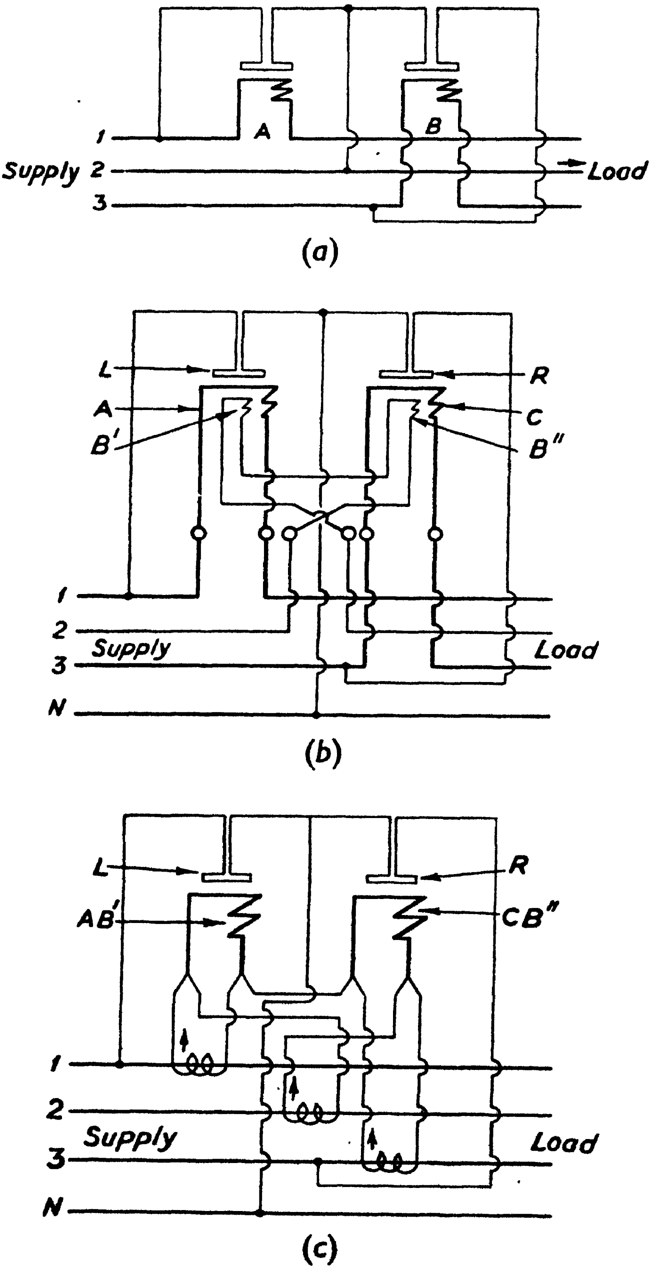


FIG. 133.—Diagrams of Connections for Aron Three-phase Clock Meters.

free from interaction, and it follows that, providing the elements are balanced, its accuracy will be independent of phase rotation.

11.13. Interaction between Elements. One of the greatest disadvantages of the induction type polyphase meter is the interaction between the elements, which creates a torque upon the moving system. Experiment has shown that this is mainly due to the stray potential flux of one element interacting with the eddy currents created by the working fluxes of the other elements.* Thus, in a three-phase, three-wire, two-element meter the torque due to these stray fluxes will be proportional to

$$K_1(E_{RY}I_B \cos[90-\theta_B]) + K_2(E_{BY}I_R \cos[90+\theta_R]) + K_3(E_{BY}E_{RY} \cos 60) + K_4(E_{BY}E_{RY} \cos 60)$$

$$= K_1E_{RY}I_B \sin\theta_B - K_2E_{BY}I_R \sin\theta_R + K_5E_{RY}E_{BY}$$

where E_{RY} is the read to yellow voltage
E_{BY} ,, blue to yellow voltage
I_R ,, current in "red" current coil
I_B ,, current in the "blue" current coil
θ_R ,, phase difference between the red current and the red to neutral voltage
θ_B ,, phase difference between the blue current and the blue to neutral voltage.

Inspection of the above expression tells one that there are two varying components and one constant component of the resultant driving torque, created by the interaction between the elements. The constant component, $K_5E_{RY}E_{BY}$, will have similar characteristics to the friction compensating torque, especially when $E_{RY}=E_{BY}$. We may therefore consider it as modifying the friction compensating torque, which can be compensated for by suitable adjustment of the friction compensating device upon the meter.

The values of K_1 and K_2 may be determined in the following manner:

A pre-determined load, say full load at 0·5 power factor, is applied to the "blue" element, and the potential coil of the "red" element is energised by the blue to yellow pressure. The meter is then timed for a convenient number of revolutions of the disc, in order to determine the percentage error. The potential coil of the "red" element is now reversed relative to the blue to yellow pressure and the meter is re-timed, in order to determine the percentage error under these conditions. Suppose t^1 seconds be the time for n revolutions of the disc when the interaction is positive and t^{11} is the time in seconds for the same number of revolutions when the interaction is negative.

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* Howarth, *Electrical Power Engineer*, Vol. 9, 1927.

Then, at 0·5 power factor,

$$t^{1} \propto \frac{1}{EI(1+K_1 \sin 30)}$$

where $E=E_{RY}=E_{BY}$, etc., and $I=I_R=I_B$, etc.,

$$\text{and } t^{11} \propto \frac{1}{EI(1-K_1 \sin 30)}$$

$$\text{Therefore } \frac{t^1}{t^{11}}=\frac{2-K_1}{2+K_1}$$

$$2t^1+K_1t^1=2t^{11}-K_1t^{11}$$

$$K_1=\frac{2(t^{11}-t^1)}{t^{11}+t^1}$$

The value of K_2 can be determined in a similar manner.

In the case of a balanced load, the power factor of which is cos θ, the total torque due to interaction will be proportional to

$$K_1EI \sin \theta-K_2EI \sin \theta+K_5E^2$$

where $E=E_{RY}$, etc., $I=I_R$, etc., $\theta=\theta_R$, etc.

Assuming that the friction compensating devices have been adjusted to counteract the effect of K_5E^2, the torque due to the interaction is

$$(K_1-K_2)EI \sin \theta$$

and, since the driving torque for a balanced load is proportional to $\sqrt{3}EI \cos \theta$, the effect of interaction will be

$$\frac{(K_1-K_2)EI \sin \theta}{\sqrt{3}EI \cos \theta}$$

$$=\left[\frac{(K_1-K_2) \tan \theta \times 100}{\sqrt{3}}\right] \text{ per cent}$$

For balanced conditions, the above expression becomes zero when $K_1=K_2$. For unbalanced loads, the error due to interaction will depend upon the extent of unbalance, the values of K_1 and K_2, and the power factor.

The above expressions are also applicable to the three-phase four-wire two-element meter, and expressions for the error due to interaction in a three-element meter may be derived in a similar, if a little more laborious, manner.

The effects of interaction may be minimised by magnetically screening each element and, in the case of the two-element meter, it is quite a common practice to invert the lower element as this also tends to reduce the interaction in the meter.

11.14. Phase Rotation Indicators. One can see from the above that it is essential that the test voltage and that of the circuit in which the meter is finally to be installed are of the same phase rotation. Furthermore, when the meter is installed it is extremely important to ensure that the supply has the correct phase rotation. For this purpose, small phase rotation indicators have been developed to facilitate quick checking of the rotation. The terminals of the instrument are connected to the live wires of the supply and the meter responds in a manner dependent upon the phase rotation.

If, when the instrument is connected in accordance with the nominal phase rotation of the supply, the indicator shows the wrong phase rotation, then two of the line wires must be reversed. After this operation the rotation will be correct.

One such phase rotation indicator consists of three small electro-magnets which operate upon a pivoted aluminium disc. The electro-magnets are star-connected across the live wires of the supply and are symmetrically spaced 120° apart relative to the aluminium disc. We thus have a small induction motor, the electro-magnets taking the place of the stator and the aluminium disc acting as the rotor. The latter will therefore be under the influence of a rotating magnetic field, due to the electro-magnets, and its direction will depend upon the phase sequence of the line wires. We have already seen (section 3.20) that the disc will rotate in the same direction as the magnetic field. This direction can be determined by observation of the arrow painted upon the disc and one can therefrom decide upon the actual phase sequence of the supply.

11.15. Metering of Three-phase Systems—Most Suitable Methods. We will now briefly consider the most suitable methods of metering three-phase systems, bearing in mind the interaction which occurs in a polyphase meter and also that the polyphase meter has additional friction errors to those of a single-phase meter, due to its heavier moving system.

The most accurate method of metering a three-phase four-wire supply is by means of three separate single-phase meters. The initial cost is slightly higher than for a polyphase meter, but when one also takes into account the higher testing cost of the polyphase meter, incurred by the supply undertaking, there is little difference in overall cost. Furthermore, the use of single-phase meters results in a smaller number of types and sizes of meters to be stocked, thereby involving less capital in meter stock.

The main disadvantage of the polyphase meter is that the rating required is governed by the maximum load on any one of the phases and, if the phase loading is greatly out of balance, it is quite possible that one of the elements will only be driven by a relatively small load. For example, in the case of a three-phase four-wire supply, metered by a 20-ampère three-element meter, suppose that two of the phases supply power loads up to the full load capacity of each element and only a lighting load is connected across the other phase. It is apparent that, when only a lamp of, say, 60 watts is switched in circuit, the lighting load will be approximately 1/80th of the full load of the element and it will only supply 1/240th of the full load driving torque of the meter. It would be very optimistic to expect a meter to register accurately under such conditions. The alternative of three single-phase meters, two of 20-ampère rating and one of 5-ampère rating, would be infinitely more suitable, since the 60 watts lighting load would then create a driving torque of approximately 1/20th full load upon the 5-ampère meter, at which load the meter should register with reasonable accuracy.

When it is required to measure the total maximum average demand the employment of a polyphase meter is essential, in order that the total torque may be summated upon one spindle and the total energy delivered registered upon one dial.

In the case of a three-phase three-wire supply, where the neutral is not available, the two-element meter has certain advantages over the two separate single-phase meters. For example, when the power factor is below 0·5 lagging (which is liable to occur in certain industrial plant, such as induction motors) the one single-phase meter will register in the reverse direction and, since the friction compensating torque is adjusted for forward registration, the meter is liable to register very inaccurately under such conditions. Furthermore, reverse registration is likely to lead to errors on the part of the meter reading staff, and certain minor confusion in the accounts department of the supply undertaking. On the other hand, it might be said in favour of the two single-phase meters that, where they are employed to measure a balanced load, they give a valuable guide as to the power factor of the system. We have already seen that the watt-hour meter M_1 will register

$$\frac{1}{1000}\int_{t_1}^{t_2} EI\cos(30+\theta)dt.\ \text{kWh}$$

and M_2 will register

$$\frac{1}{1000}\int_{t_1}^{t_2} EI\cos(30-\theta)dt.\ \text{kWh}$$

Now, the difference in speeds of the two discs, at any instant, will be given by

$$\begin{aligned}(N_2-N_1) &= K[EI\cos(30-\theta)-EI\cos(30+\theta)]\\ &= KEI\,[\cos(30-\theta)-\cos(30+\theta)]\\ &= 2KEI\sin 30\sin\theta\end{aligned}$$

$$=KEI\sin\theta$$

but $N_2+N_1=K\sqrt{3}EI\cos\theta$

therefore $\dfrac{N_2-N_1}{N_2+N_1}=\dfrac{EI\sin\theta}{\sqrt{3}EI\cos\theta}$

and $\tan\theta=\dfrac{\sqrt{3}(N_2-N_1)}{(N_2+N_1)}$

$$\theta=\tan^{-1}\left(\frac{\sqrt{3}(N_2-N_1)}{(N_2+N_1)}\right)$$

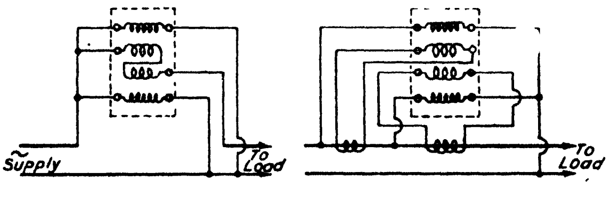
From the above it can be seen that the average power factor of the system is approximately given by

$$\theta=\tan^{-1}\left(\frac{\sqrt{3}(M_2-M_1)}{(M_2+M_1)}\right)$$

where M_1 is the advance of meter M_1 and M_2 is the advance of M_2, during the period over which the average was calculated.

11.16. The Testing of Polyphase Meters. Prior to the Electricity Commissioners specifying that all polyphase meters were to be tested under three-phase conditions, it was the practice of some supply authorities to test polyphase meters with single-phase loading. In consequence, we will consider both methods of test, commencing with the single-phase method.

Three-phase three-wire meters are first connected as shown in Fig. 134, according to the type of meter, and allowed to run on full load for a suitable period for reasons already mentioned in section 9.13. The "red" element potential coils are then energised with 10 per cent in excess of the marked voltage, whilst the "blue" element potential coils, and all series coils, are left on open circuit. The friction compensating device of each "red" element is now adjusted until the disc behaves in such a manner as the tester has found from experience gives the required friction compensating torque (the correct behaviour of the disc is usually backwards slightly in one position and forward in all



(a) Directly connected (b) Transformer connected

FIG. 134.—Connections for Single-phase Testing of Three-phase Three-wire Meters (Phantom Load Arrangement Omitted).

other positions). This procedure is then repeated for the "blue" element potential coils, after which both potential coils are energised and the tester must ascertain that the effect of the friction torque of the combined elements of each meter is satisfactory.

The elements of each meter are now adjusted so that equal loads applied to each element result in equal driving torques upon the meter. For this purpose, the meters are connected as shown in Fig. 134, except that the current in the lower element is reversed, thereby reversing the driving torque of that element. Full load current, in phase with the voltage applied to the potential coils, is passed through the series coils and the balance adjusters are operated until the disc is virtually stationary, i.e. until the driving torques of the two elements are balanced. Each element is now individually timed on full load, slight adjustment of the balance adjuster being made, if necessary, until both elements rotate at the same speed at full load. For this test, the series coil of the element not being timed is left on open circuit, but the potential coils of both elements are energised.

Full load is now applied to both elements and the brake magnet is adjusted until the error is in keeping with the characteristic curve of the particular make and type of meter under test. This error is recorded and each of the elements is individually re-timed and its error recorded; any slight adjustment of the elements being made by means of the balance adjusters.

The individual elements are now tested at 0·5 power factor lagging, still leaving the potential coil of each element energised, and the quadrature loops are adjusted until each element has a suitable error on this load. This inductive load is now applied to both elements and the meter is re-timed in order to determine the error under these conditions. Should a slight alteration prove necessary, it is advisable to perform equal adjustment upon the quadrature loops of both elements.

One intermediate load, say one-half or one-quarter full load, unity power factor, is applied simultaneously to both elements, and the meter error determined. Providing the full load meter error has been adjusted to within reasonable limits, the meter will in all probability be within the limits at this intermediate load.

One-twentieth full load, unity power factor, is now applied to both elements and the combined error is determined. Should the error fall outside the limits which the tester considers suitable, equal adjustment of the friction compensating loops of both elements is made until the meter is within these limits. Any meter which has received adjustment of the friction compensating loops must be re-timed at one of the higher non-inductive loads and also tested for "creep" with 10 per cent in excess of the marked voltage applied to the potential coils.

11.17. Allowance for Interaction. In order to eliminate the effects of interaction from the results of the above tests, the pressure and current coils of the one element are reversed (the driving torque of that element still remaining in the same direction) and the meter is re-timed throughout the curve. The error at each load is determined for the new conditions, and from the two sets of results the mean error of each load is calculated. In the case of the loading of individual elements, either the leads of the series and potential coil of the element under test, or the potential coil of the other element, must be reversed.

(This mean error, when it refers to a combined loading upon the two elements, is taken as the error of the meter on a balanced three-phase load. This is based upon the assumption that K_1 is equal to K_2 (section 11.14). If, however, K_1 does not equal K_2, the interaction will not be zero under balanced load conditions and the mean error mentioned above will have no meaning relative to three-phase conditions.)

11.18. Dial Test. When the meter has satisfactorily passed the above tests, it is placed on dial test. Suppose E is the voltage, I the current in ampères, and T the period of the dial test in hours. Then the dial reading, when both the elements are energised by a unity power factor single-phase load, will be

$$\frac{2EIT}{1000}\ \text{kWh}$$

$$\text{instead of}\ \frac{\sqrt{3}EIT}{1000}\ \text{kWh}$$

which would have been registered if a three-phase load at unity power factor had been applied to the meter.

Similarly, the ratio of the disc speed when the meter has a single-phase load to that of a three-phase load, will be

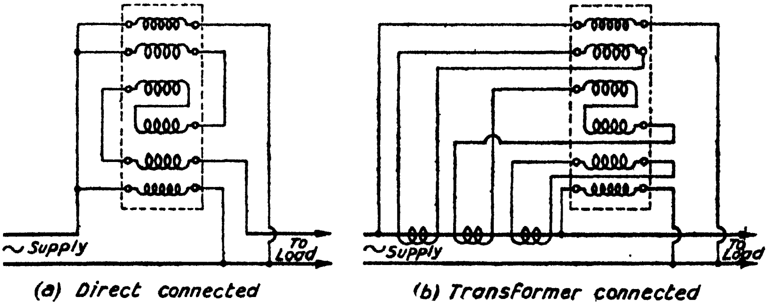
$$\frac{2EI}{\sqrt{3}EI\cos 30}$$

$$\frac{2}{\sqrt{3}}$$

Owing to the interaction of the stray flux of each potential electro-magnet with the induced eddy currents in the disc, due to the potential electro-magnet flux of the other element, thereby creating a constant torque at all current loadings, it is advisable to check the meter for creep at no-load with a three-phase supply at 10 per cent in excess of the marked voltage. Such a test is essential, and the phase rotation should be the same as that of the supply in which the meter is to be installed for service.

11.19. Three-phase Four-wire Two-element Meters are tested in exactly the same manner as described above, except that the "yellow" series winding of each element is only energised for the combined loading tests. The connections are then as illustrated in Fig. 135, according to whether the meter is directly or transformer connected. It will be noted that the reversed yellow current acts with the red to neutral and blue to neutral voltages respectively. It therefore follows that, in order to attain a positive torque from the interaction of the current in the "yellow" series coils with the applied voltages to the potential coils of the "red" and "blue" elements, it is necessary, on single-phase loading, to make a reversal as indicated.

The effects of interaction can be eliminated in exactly the same manner as for the three-wire meter (section 11.17). It should be noted, however, that the ratio of disc speed on the single-phase test to disc speed on the balanced three-phase test will be



(a) Direct connected (b) Transformer connected

FIG. 135.—Connections for Single-phase Testing of Three-phase Four-wire Two-Element Meters (Phantom Load Arrangement Omitted).

$$4EI \cos \theta : 3EI \cos \theta$$

$$4 : 3$$

where E is the applied voltage, I the load current in ampères, and $\cos \theta$ the power factor of the load. Likewise, the advance of the pointers under similar conditions will bear the same ratio. The increase in reading on single-phase loading is due to the two "yellow" series coils associating with the applied voltages to the potential coils, such that the torque exerted is $2EI \cos \theta$ instead of $EI [\cos (60+\theta) + \cos (60-\theta)]$

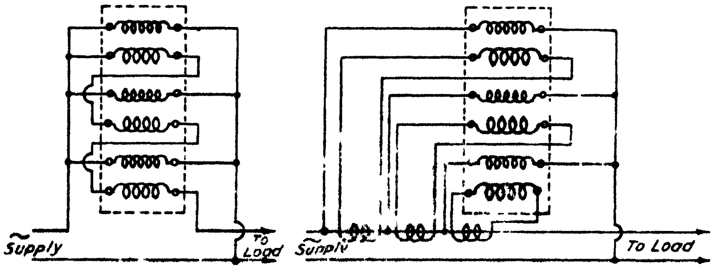
$$\text{i.e. } 2EI \cos \theta \cos 60$$

$$\text{or } EI \cos \theta$$

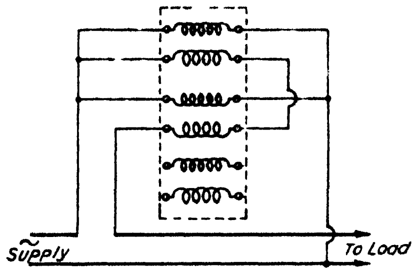
which would be the case for three-phase loading.

11.20. Three-phase Four-wire Three-element Meter. The testing of three-element meters on single-phase loading is not such a simple matter as that of the two-element meter, since the estimation of the interaction becomes more involved.

Balancing of the elements (see Fig. 136 (*c*)) is effected by opposing the driving torque of each element in turn against that of the reference element (which usually has no balance adjuster) in the same manner as for the two-element meter; the balance adjuster being altered in position until equalisation of the driving torques is obtained. Each meter is then connected as illustrated in Fig. 136 (*a*) or (*b*) and the elements are successively timed and adjusted on full load. During the test every potential coil of the meter remains energised. The first element to be tested is the reference element, and this is brought within the limits by means of the brake magnet adjustment. The other elements are then tested in turn and they are brought within the predetermined limits by means of the balance adjusters.



(a) Directly Connected (b) Transformer Connected



(c) Balancing (Top against middle element)

FIG. 136.—Connections for the Single-phase Testing of Three-phase Four-wire Three-element Meters (Phantom Load Arrangement Omitted).

The meter is then timed with full load applied to the three elements and the error is determined. Should any alteration be necessary, it is effected by means of the brake magnets.

The testing of inductive load and the other prescribed loads is made in a similar manner to that described for the two-element meters.

The single-phase method of test is not recommended, and it is imperative that the no-load test (i.e. potential only) be made from a three-phase four-wire supply. This is obvious, since the component of the interaction giving constant torque at all loads is

$$K_1E_RE_Y\cos 60+K_2E_BE_Y\cos 60+K_3E_RE_B\cos 60$$

which, under balanced conditions, where $E = E_R$, etc., will be

$$K_4 E^2$$

(section 11.13)

11.21. Three-phase Testing of Polyphase Meters. It has already been mentioned that the Electricity Commissioners require three-phase meters to be tested in three-phase circuits. Actually, the test is very little more involved than the single-phase test previously described and, in general, it has only been lack of suitable equipment which has prevented supply undertakings from testing their three-phase meters under three-phase conditions.

Three-phase four-wire meters, as well as three-wire meters, can be tested in a three-wire circuit; the connections for each type of meter being given in Fig. 137. As can be seen, in the case of the former type of meter, the common terminal of the three potential coils is connected to the neutral of the potential supply.

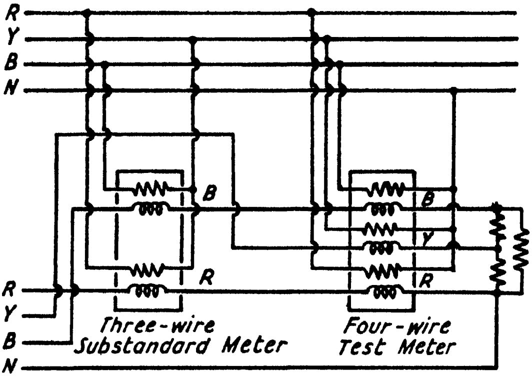


FIG. 137.—Three-phase Testing—Meter Connections.

When tested in a three-wire circuit, the virtual power of the fictitious load can be measured by means of two single-phase wattmeters or a two-element wattmeter. An alternative method of carrying out the tests is by comparison with a substandard three-phase three-wire two-element meter, whence either Method A or Method B (section 9.19) may be employed. Each element is tested individually against the substandard at each of the specified loads and the errors are then determined when the balanced loads are applied to the meters.

If the meters are tested against a two-element substandard wattmeter, or two single-phase wattmeters, the total load must be kept constant during the course of the test, by adjustment of the rheostats connected in the lines for variation of the line currents. A polyphase wattmeter is the more convenient instrument for the measurement of the total load, since the latter is indicated upon one dial. Thus, only one observer is required to hold the load at its correct value, thus ensuring greater accuracy.

The test for starting current, and the initial adjustment of the friction compensating torque to ensure that the disc will not creep on no-load, is performed in exactly the same manner as for the single-phase test, except, of course, that each winding is energised from its correct phase.

The meter is next tested for balance of elements. In the case of the three-wire meter, both potential coils are energised by their correct voltages and full load current is passed through each series coil in turn, whilst a number of disc revolutions are timed. The error of each element is adjusted within suitable limits under these conditions, by manipulation of the balance adjusters. This balance test is made at an equivalent single-phase power factor of unity; in other words, the current in the series winding of the element under test is adjusted to be in phase with the applied voltage to the potential coil of the same element. Thus, in the case of the "red" element, this will occur when the red current leads its phase voltage by 30°, i.e. it is in phase with the red to yellow voltage. The loading upon the element will be the reading of the polyphase wattmeter or the corresponding single-phase wattmeter.

Full load is now applied to each element in turn, at an equivalent single-phase power factor of 0·5 lagging (i.e. the current in the series coil of the element under test being adjusted to lag the applied voltage to the potential coil of that element by 60°); the potential coils being energised by their correct voltages. The respective quadrature loops are adjusted until each element is within the limits of error decided upon by the tester, due regard being paid to the make and type of meter under test.

The meters are now tested, under balanced conditions, at each of the following loads: (*a*) 125 or 100 per cent of the marked current, (*b*) one-quarter or one-half of marked current, and (*c*) one-twentieth of marked current; each of these tests being made at unity power factor. In addition, a balanced load is applied to the meter, of full load 0·5 power factor lagging, and the error at this load is determined.

Providing the tests for the balance of the elements has been carried out carefully, and the tester has successfully estimated the allowable limits of error for each load, the meter should require no adjustment at any of the balanced loads except, perhaps, one-twentieth full load. If any adjustment of the friction compensating devices has to be performed to bring the meter within the limits at low load, the meter must be re-tested at one of the higher non-inductive loads.

Dial test may be run at full load, under balanced conditions at unity power factor, and the advance of the pointers should be

$$\frac{\sqrt{3}EIT}{1000}\text{ kWh}$$

where E is the line voltage, I is the load current in ampères, and T is the length of the dial test in hours.

11.22. Three-phase, Four-wire, Two-element Meters—Three-phase Test. Four-wire two-element meters are tested in a similar manner to three-wire meters except that, when the elements are tested for balance, the "yellow" series coils may be disconnected.

Each element is tested for balance with the current in the series winding in phase with the voltage applied to the potential coil of the element and then with the current in the series winding lagging the applied voltage to the associated potential coil by 60°. Adjustments are similar to those of the three-wire meter and, during the balance tests, both potential coils are energised by their correct voltages. Tests at balanced load are also similar to those for the three-wire meter and therefore need not be discussed further.

11.23. Three-element Meters—Three-phase Tests. The testing of three-phase four-wire three-element meters is similar to the testing of three single-phase meters connected one in each phase of the system. For balancing of the elements, the potential coils are energised by their correct voltages and full load current at unity power factor is passed through the series coil of each element in turn, and the errors determined. It is advisable to commence with the reference element and this is brought within the required limits by means of the brake magnet. The other elements are then tested in turn, the errors being made similar to those of the reference element by alteration of the balance adjusters.

With balanced conditions and full load unity power factor applied to the meter, the error is again determined. Should this error be outside the limits allowed by the tester, from his knowledge of the characteristics of the make and type of meter under test, the necessary alteration is performed by adjustment of the brake magnet, and the recorded error for each individual element is modified by an amount equal to this alteration.

Full load at 0·5 power factor lagging is now applied to each element in turn and any desired alteration of the error is performed by adjustment of the quadrature loop.

Balanced loads at (*a*) full load 0·5 power factor lagging, (*b*) one-half or one-quarter full load at unity power factor, and (*c*) one-twentieth full load at unity power factor are applied in turn to the meter and the error at each load is determined. Any adjustment required under these conditions is similar to that already described for three-wire meters.

The writer prefers the testing of three-phase meters initially in single-phase circuits, and then under three-phase conditions. This is particularly convenient for the testing of a meter which has been completely overhauled, or a new meter never previously calibrated.

Experience soon teaches one the characteristics of any particular type and make of meter, and the most likely errors for single-phase loading which will bring the meter within the limits in a three-phase test.

11.24. Errors due to Potential Flux not being in Quadrature with Applied Voltage. If the effective flux of the potential electromagnet is not in quadrature with the applied voltage to the potential coil, certain errors in registration will result. We will therefore consider the error introduced in a three-phase three-wire meter when the flux of the "red" potential coil lags its applied voltage by $(90-\Delta_R)^\circ$ and the flux of the "blue" potential coil lags its applied voltage by $(90-\Delta_B)^\circ$. The meter torque will be proportional to

$$E_{RY}I_R\cos(30+\theta_R+\Delta_R)+E_{BY}I_B\cos(30-\theta_B-\Delta_B)$$

which, at balanced load, will be

$$EI\,[\cos(30+\theta+\Delta_R)+\cos(30-\theta-\Delta_B)]$$

$$=EI\,[\cos(30+\theta)\cos\Delta_R-\sin(30+\theta)\sin\Delta_R+\cos(30-\theta)\cos\Delta_B$$
$$+\sin(30-\theta)\sin\Delta_B]$$

When Δ_R and Δ_B are small, we can make the approximations $\cos\Delta_R=\cos\Delta_B=1$, $\sin\Delta_R=\pi\Delta_R/180$, $\sin\Delta_B=\pi\Delta_B/180$, and the driving torque is proportional to

$$EI\left[\cos(30+\theta)+\cos(30-\theta)+\sin(30-\theta)\frac{\pi\Delta_B}{180}-\sin(30+\theta)\frac{\pi\Delta_R}{180}\right]$$

$$=\sqrt{3}EI\cos\theta+\frac{\pi}{180}EI\,[\Delta_B\sin(30-\theta)-\Delta_R\sin(30+\theta)]$$

but $\sqrt{3}EI\cos\theta$ corresponds to correct torque

$$\text{Therefore error}=\frac{\frac{\pi}{180}.EI[\Delta_B\sin(30-\theta)-\Delta_R\sin(30+\theta)]}{\sqrt{3}EI\cos\theta}$$

$$=\frac{\pi}{\sqrt{3}.180}\left[\frac{\Delta_B(\sin30\cos\theta-\cos30\sin\theta)-\Delta_R(\sin30\cos\theta+\cos30\sin\theta)}{\cos\theta}\right]$$

$$=\frac{\pi}{\sqrt{3}.180}\,[\Delta_B(\sin30-\cos30\tan\theta)-\Delta_R(\sin30+\cos30\tan\theta)]$$

$$=\frac{\pi}{\sqrt{3}.180}\,\frac{(\Delta_B-\Delta_R)}{2}-\left[\frac{(\Delta_B+\Delta_R)}{2}\sqrt{3}\,.\tan\theta\right]$$

$$=\frac{\pi}{180}\left[\frac{(\Delta_B-\Delta_R)}{2.\sqrt{3}}-\frac{(\Delta_B+\Delta_R)}{2}\tan\theta\right]$$

This expression shows that the error is formed of two components, one of which is proportional to $\tan\theta$ and the other which is entirely independent of the power factor of the balanced load. Furthermore, when $\Delta_R=-\Delta_B$, the meter will be free from any resultant phase error since $\left(\frac{\Delta_B+\Delta_R}{2}\right)\tan\theta$ is equal to zero.

11.25. The Testing of Meters for Use with Transformers. It has been specified by the Electricity Commissioners that a meter must be tested with the instrument transformers with which it is to be used in service. An exception is made concerning the testing of meters for high voltage supplies. In consequence, these may be tested without their transformers. Otherwise, a three-phase high voltage testing equipment would be required. A high voltage meter will therefore generally be tested without its potential transformers; 110 volts being applied directly to the potential circuits of the meter.

When calibrating a meter under these conditions, due allowance must be made for the transformer errors. We will therefore consider the errors for which allowance must be made when a three-phase three-wire meter is tested without its current and potential transformers.

Let E be the nominal terminal voltage of the potential transformers and I be the nominal current in the secondary windings of the current transformers for any load at which the test is made. Then, if this be applied to the meter when it is tested under balanced load conditions, without its transformers, the driving torque of the meter will be proportional to

$$EI\,[\cos(30+\theta)+\cos(30-\theta)]$$

(section 11.4)

Now let us assume that

α_R is the phase angle error of the "red" current transformer
α_B ,, phase angle error of the "blue" current transformer
β_R ,, phase angle error of the "red" potential transformer
β_B ,, phase angle error of the "blue" potential transformer
C_R ,, ratio error of the "red" current transformer
C_B ,, ratio error of the "blue" current transformer
P_R ,, ratio error of the "red" potential transformer
P_B ,, ratio error of the "blue" potential transformer

where each error applies to the particular load at which the meter is subjected to test and, in the case of phase angle errors, is considered positive when the reversed secondary leads the primary voltage or current.

The driving torque, under balanced load conditions, will be proportional to

$$EI\,[(1+C_R)(1+P_R)\cos(30+\theta-\alpha_R+\beta_R)+(1+C_B)(1+P_B)\cos(30-\theta+\alpha_B-\beta_B)]$$

C_R, P_R, C_B, and P_B will be small, therefore C_RP_R, etc., may be neglected and the torque will be proportional to

$$EI\,[(1+C_R+P_R)\cos(30+\theta-\alpha_R+\beta_R)+(1+C_B+P_B)\cos(30-\theta+\alpha_B-\beta_B)]$$

$$\begin{aligned}\text{or } EI\ &[(1+C_R+P_R)(\cos[30+\theta]\cos[\alpha_R-\beta_R]+\sin[30+\theta]\sin[\alpha_R-\beta_R])\\ &+(1+C_B+P_B)(\cos[30-\theta]\cos[\alpha_B-\beta_B]-\sin[30-\theta]\sin[\alpha_B-\beta_B])]\end{aligned}$$

$$\begin{aligned}=EI\Big[&(1+C_R+P_R)\Big(\cos[30+\theta]+\sin[30+\theta]\frac{(\alpha_R-\beta_R)\pi}{180}\Big)\\ &+(1+C_B+P_B)\Big(\cos[30-\theta]-\sin[30-\theta]\frac{(\alpha_B-\beta_B)\pi}{180}\Big)\Big]\end{aligned}$$

since α_R, etc., are small, $\sin\alpha_B=\frac{\alpha_B\pi}{180}$, $\sin\alpha_R=\frac{\alpha_R\pi}{180}$ etc., and $\cos\alpha_R=\cos\alpha_B=1$, and the percentage change in driving torque, due to use of transformers

$$\begin{aligned}=\frac{100}{\sqrt{3}\cos\theta}\Big\{&(C_R+P_R)(\cos[30+\theta])+(C_B+P_B)(\cos[30-\theta])\\ &+\frac{\pi}{180}(C_R+P_R+1)(\sin[30+\theta)])(\alpha_R-\beta_R)-(C_B+P_B+1)(\alpha_B-\beta_B)\\ &\sin[30-\theta]\Big\}\text{ per cent}\end{aligned}$$

$$\begin{aligned}=\frac{100}{\sqrt{3}\cos\theta}\Big\{&(C_R+P_R)(\cos 30\cos\theta-\sin 30\sin\theta)+(C_B+P_B)\\ &(\cos 30\cos\theta+\sin 30\sin\theta)\\ &+\frac{\pi}{180}\Big((C_R+P_R+1)(\alpha_R-\beta_R)(\sin 30\cos\theta+\cos 30\sin\theta)\\ &-(C_B+P_B+1)(\alpha_B-\beta_B)(\sin 30\cos\theta-\cos 30\sin\theta)\Big)\Big\}\text{ per cent}\end{aligned}$$

$$\begin{aligned}=\Big\{&50(C_R+P_R+C_B+P_B)+\frac{50}{\sqrt{3}}(C_B+P_B-C_R-P_R)\tan\theta\\ &+\frac{\pi.100}{360\sqrt{3}}\Big[(C_R+P_R+1)(\alpha_R-\beta_R)-(C_B+P_B+1)(\alpha_B-\beta_B)\Big]\\ &+\frac{\pi.100}{360}\Big[(C_R+P_R+1)(\alpha_R-\beta_R)+(C_B+P_B+1)(\alpha_B-\beta_B)\Big]\tan\theta\Big\}\text{ per cent}\end{aligned}$$

At unity power factor, this becomes

$$\begin{aligned}50(C_R+P_R+C_B+P_B)+\frac{\pi}{3\cdot 6\sqrt{3}}\Big[&(C_R+P_R+1)(\alpha_R-\beta_R)-(C_B+P_B+1)\\ &(\alpha_B-\beta_B)\Big]\end{aligned}$$

and the meter must be so adjusted that its apparent error, when tested without transformers, is equal and opposite to the above.

At 0·5 power factor lagging, the meter must be adjusted so that its apparent error is

$$\left\{100(C_B+P_B)+\frac{\pi}{3{\cdot}6\sqrt{3}}\left[(C_R+P_R+1)(\alpha_R-\beta_R)-(C_B+P_B+1)(\alpha_B-\beta_B)\right]+\frac{\sqrt{3}.\pi}{3{\cdot}6}\left[C_R+P_R+1)(\alpha_R-\beta_R)+(C_B+P_B+1)(\alpha_B-\beta_B)\right]\right\}$$

The necessary correction for a four-wire meter, tested without its transformers, may be determined in a similar manner to the above expression for a three-wire meter. In the case of the three-element meter, the derived expression will prove more simple since, at unity power factor, the series winding current of each element is in phase with the applied voltage to the potential coil of the same element.

We have seen in section 6.17 that the load on a potential transformer is constant and, providing the potential coil of the meter element does not overburden it, the phase angle error will be small. Furthermore, the ratio error will be constant at all meter loads and it is a very easy matter to allow for it.

CHAPTER XII

The Cost of Electricity Supply

12.1. Standing Charges and Running Costs. It will be advantageous, before we proceed further in our study of electricity supply meters, to consider briefly the costs associated with the supply of electricity and the various systems of charging which have been designed with a view towards the ideal of a consumer paying an amount equal to the total cost of supplying him, whilst keeping the tariff as simple as possible for the sake of expediency.

The cost involved in the supply of electrical energy may be divided into two components. These are respectively termed the standing charges and the running costs.

The standing charges represent all expenditure involved in maintaining the generating and distributing plant in readiness to supply any power, up to the rated capacity of the system, required by the consumers. It includes the capital costs, such as interest and sinking fund charges, the major portion of the salary and wages expenses, and a slight proportion of the fuel bill.

The running costs represent the expenditure involved, in addition to the above, when the generating plant is operating and supplying the consumers with energy. This component comprises the bulk of the fuel costs, and a small fraction of the salary and wages bill involved in repairs, etc.

12.2. Load Factor. It has been found in practice that the standing charges are sensibly proportional to the kW rating of the generating plant, whilst the running costs are roughly proportional to the supply of energy from the plant. We may therefore say that, when a generating plant delivers n.kWh of energy in a given period, the total expenditure involved will be

$$\pounds\left(x+\frac{ny}{240}\right)$$

where $\pounds x$ represents the standing charges of the system during this period, and y pence represents the running costs per unit of electricity.

Thus, the total cost per unit

$$=\left(\frac{240x}{n}+y\right)\text{ pence}$$

and it follows that the greater the value of n, for any given plant, the less the cost per unit of electricity. Now, the value of n can only increase until it corresponds to the output of the plant when working continuously on full load. Suppose this maximum amount of energy which could be supplied to the consumers be represented by N, then the ratio

$$\frac{n}{N}=L$$

where L is the load factor of the consumers' use of the available plant. We may therefore define the load factor as the ratio of energy supplied per day by the system to the total amount which the plant is capable of supplying.

Let us consider the hypothetical case where every consumer receives energy up to the full capacity of his connected apparatus for four hours per day, and that the time of each consumer's demand coincides with that of every other consumer. Then, if T is the total kW loading, the kWh of energy delivered per day is $4T$ and, with the standing charges at £x per day, the cost per unit

$$=£\left(\frac{x}{4T}+\frac{y}{240}\right)$$

$$=\left(\frac{60x}{T}+y\right)\text{ pence}$$

12.3. Advantage of Staggered Demands. If the above consumers now arranged their demands so that one-sixth T kW of power were supplied for the whole period of the 24-hour day, the same amount of energy would be delivered and, for the same generating plant, the cost would be

$$\left(\frac{60x}{T}+y\right)\text{ pence per unit}$$

which is the same as under the previous conditions. This is because the load factor still remains

$$\frac{4T}{24T}=\frac{1}{6}$$

There is, however, one big difference between the two conditions. Whereas, in the case of the uneven demand, the plant is working at its full capacity for a period of four hours each day, in the second case the generating plant is never operating at above one-sixth its maximum capacity. Thus, six times such a load could be put on the plant and, since the load factor would now be unity, the overall cost per unit of electricity would be a minimum at

$$£\left(\frac{x}{24T}+\frac{y}{240}\right)$$

$$=\left(\frac{10x}{T}+y\right)\text{ pence per unit.}$$

It can be seen from the above that it is advantageous for the consumers to stagger their demands so that the combined load is as near as possible the same throughout the twenty-four hours.

12.4. Preferential Tariffs. It is the ideal of every supply authority to attain a load factor as near unity as possible, and with this end in view special preferential tariffs have been created. They are designed to encourage consumers to use electricity at light load periods, rather than at the peak load period of the generating plant.

In practice, there is a great variation in the use made of the supply by the numerous individual consumers. Their load factors will not be the same, and what is more important, their hours of peak load will not generally occur at the same time each day. Thus, the sum of the consumers' maximum demands will be greater than the maximum imposed upon the generating plant This will have the advantage of decreasing the rated capacity required of the generating plant, thereby improving the load factor of the system.

12.5. Effect of Power Factor. When the supply is alternating, there is an additional factor which affects the cost of the electricity, and this is the power factor of the consumer's load. The reason for this is that the capacity of electrical plant depends upon the rating in kVA and not kW. Furthermore, for a given load, the current will be inversely proportional to the power factor and, since the heat-losses are proportional to the square of the current, it follows that the running costs to supply energy to the consumer are substantially increased with decrease of power factor.

12.6. Unbalanced Polyphase Loads. In the case of three-phase supplies, there is a further factor which influences the cost of the supply of electricity, and this is the extent of unbalance of the consumers' loads. For example, in a system the line voltage of which is E, the power, at unity power factor, which can be transmitted per ampère of the line is

$$\sqrt{3}E \text{ watts}$$

for a balanced load.

Under the worst conditions of unbalance, when the power is only transmitted by two of the lines, the capacity per ampère of the line is

$$E \text{ watts.}$$

From this we see that the rating of the three-phase plant required to supply the consumer with a kW of power will increase with departure of the load from balanced conditions. Fortunately, unbalance of individual consumers' loads tend mutually to compensate one another, and the overall effect is not so serious as it would at first appear.

From our brief consideration of the factors influencing the cost of supplying electrical energy to the consumer, we can appreciate the difficulties which arise when attempts are made to assess an equitable cost for the supply of electricity to each individual consumer. In consequence, only those of the above electrical quantities which are capable of being satisfactorily defined, and accurately measured, are taken into account in the determination of the amount which a consumer is required to pay for his supply of electrical energy.

12.7. Two-part Tariffs. The main electrical quantity involved in the determination of the amount payable by the consumer is energy. Fortunately, this is a very simple quantity to measure, as we have seen in previous chapters of this book. The consumer may therefore pay a certain price per unit of energy delivered to him, all other considerations concerning the cost of supply being ignored. On the other hand, if he prefers a two-part tariff, he pays a lower price per unit of electricity supplied to him, plus a second component based upon one of the following factors:

(*a*) The rateable value of the premises
(*b*) The floor area of the premises
(*c*) The kW rating of the electrical equipment installed.

The above component is usually a fixed charge. Alternatively, he may be charged upon some electrical quantity such as the maximum demand in kW, since this is easily determinable, in addition to a certain price per unit of energy.

12.8. A Tariff to Promote Improvement of Power Factor. Should the undertaking wish to encourage industrial consumers of a.c. supply to keep the power factor as high as possible, the charges will partly be made dependent upon the power factor of the load and will be so designed that it is to the financial benefit of the consumer to improve his power factor to the most economical value for the particular tariff. Such a tariff would consist of one component proportional to the energy consumption, plus a second component dependent upon the kVA maximum demand or the kVArh (wattless component) of the load. Alternatively, this second component may be estimated from the average power factor of the load.

We have already considered the measurement of energy. Therefore in the following chapters we will consider methods adopted for the measurement of the other electrical quantities involved in the assessment of the amount which a consumer should pay for his supply of electrical energy.

CHAPTER XIII

The Measurement of Reactive kVAh (kVArh) and kVAh, etc.

13.1. Displacement of Current or Potential Coil Flux for Measurement of KVArh. In the preceding chapter we noted that certain tariffs involved the measurement of reactive kVAh (kVArh) as well as kWh. Such a measurement may be required in order to provide sufficient data for the determination of the average power factor of the consumer's load. Alternatively, it may be used as the direct basis in charging the consumer.

We have already observed that when an energy meter is connected in a single-phase circuit, it will register

$$\int_{t_1}^{t_2} ei \cos \theta . dt$$

where e and i are the instantaneous values of the voltage and current.

Now, suppose that either the potential flux or the current flux of the energy meter is displaced by 90°, relative to its previous phase for energy measurement, then the speed of the disc will be proportional to

$$EI \sin \theta$$

and the meter will register

$$\pm \int_{t_1}^{t_2} ei \sin \theta dt \text{ kWh},$$

the sign depending upon whether the power factor is lagging or leading. Such a quantity is termed the "wattless component" or reactive VAh.

13.2. Methods of attaining 90° Phase-change of Potential or Current Flux—Single-phase Meters. There are several methods of altering the phase relationship between the potential and current fluxes in order that the speed of the disc will be proportional to $EI \sin \theta$. These include:

(*a*) The use of two potential windings, one being in series with a non-inductive resistance across the mains and the other being the normal winding of an energy meter connected across the mains. The windings are connected so that the directions of the currents are in opposite sense. The conditions of the applied voltage and fluxes may be represented by the vector diagram of Fig. 138 (*a*), where E is the applied voltage, Φ_L the flux due to the coil connected directly across the mains and Φ_{RL} the flux due to the reversed coil connected in series with the resistance across the mains. By adjustment of the value of the series resistance, the magnitude and phase of Φ_{RL} can be made such that the resultant potential flux lags the applied voltage by 180°. In other words, the potential flux has been displaced 90° relative to its position for energy measurement.

(*b*) The current coil of the meter may be supplied with current from the secondary of a mutual inductance. Thus, providing the secondary circuit is non-inductive, this will effect a 90-degrees lag in the current flux. This is illustrated in the vector diagram of Fig. 138 (*b*), where E represents the supply voltage, I the load current, Φ_E the pressure coil flux, and Φ_1 represents the current coil flux due to the reversed secondary current of the mutual inductance.

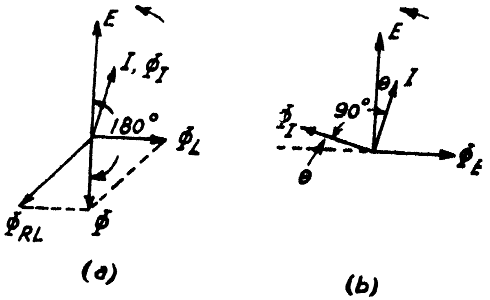


FIG. 138.—Vector Diagrams of Methods of obtaining 90° Phase Change of Working Fluxes.

13.3. Electrolytic Meter. In practice, one rarely wishes to measure the reactive kVAh of a single-phase circuit. However, the kVAh, the quantity in which we are ultimately interested, can be measured by a mercury type electrolytic meter (section 8.3) in conjunction with a current transformer and full-wave copper-oxide rectifier (section 3.10). Such a meter will register correctly, providing:

(*a*) The wave-form of the current supplied to the consumer is the same as that by which the calibration was made (since the rate of deposition will be proportional to the r.m.s. value of the current).

(*b*) The supply voltage remains constant at its declared value.

When the electrolytic meter is operating upon alternating current its performance will not be so good as when on direct current, since errors will be introduced due to the transformer and the rectifier.

13.4. Three-phase Meters. In a three-phase circuit, the kVArh may be measured by means of elements constructed in the manner previously described for the single-phase induction meters (section 13.2), so that the voltage and current fluxes are exactly in phase when a non-inductive load is applied to the element. The meter is constructed and connected in a similar manner to the corresponding polyphase watt-hour meter and, as we can see in continuation of our argument of section 4.4, for a circuit of n wires, $(n-1)$ meter elements are sufficient for the registration of the kVArh, under all conditions of balanced or unbalanced loads.

Such a meter would involve special construction and would be susceptible to considerable error with change of frequency, on account of the non-inductive resistance inserted in the voltage circuit. It is therefore usual to take advantage of the various voltage combinations of a three-phase circuit, in order to effect the 90° phase change of the pressure or current coil fluxes. There are a number of methods, and we will discuss several of these in the succeeding paragraphs.

13.5. Auto-transformer Method. Two auto-transformers are connected in open delta across the supply, the common connection being made to the yellow line and the other line connections being made to the 100 per cent taps, as shown diagrammatically in Fig.

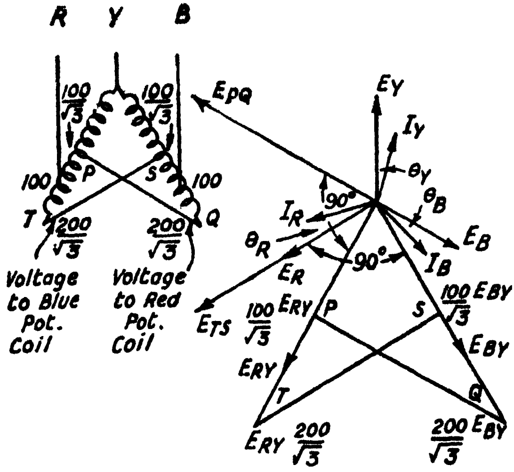
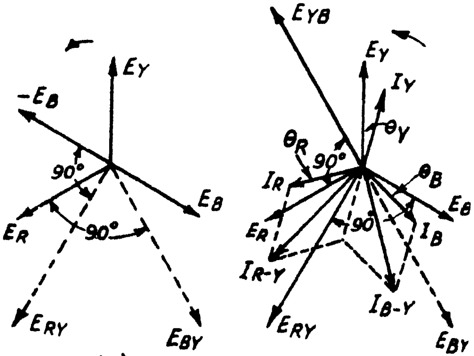


FIG. 139.—Circuit Arrangement and Vector Diagram of Auto-transformer Method for kVArh Measurement.

139 (*a*). Each potential circuit of the meter is then connected between a $200/\sqrt{3}$ tapping of one transformer and a $100/\sqrt{3}$ tapping of the other transformer, so that the potential coil fluxes remain constant in magnitude, but are lagged 90° relative to the phases for energy measurement. From our knowledge of the properties of equilateral triangles, it is quite easy to see from Fig. 139 (*b*) how the phase quadrature occurs and the magnitude of the voltage remains the same. This method is employed in the Westinghouse kVArh meter.

13.6. Three-phase Three-wire Meter—Neutral Available. Another method, if the neutral is available, is illustrated vectorially in Fig. 140 (*a*). The "red" element is energised by the neutral to blue voltage and the "blue" element by the red to neutral voltage. These



(a) Three-Phase Three-Wire

(b) Three-Phase Four-Wire

FIG. 140.—Vector Diagrams Depicting Methods Employed in kVArh Measurement.

voltages lag those employed for energy measurement by 90°, but are only $1/\sqrt{3}$ the magnitude of the normal energy meter voltages. Allowance can, of course, be made for this in the gearing and calibration of the meter.

13.7. Three-phase, Four-wire, Two-element Meter. For this type of kVArh meter, a method of connection is illustrated in the vector diagram of Fig. 140 (*b*). In this case, the meter currents are supplied from delta connected current transformers. The meter currents are respectively I_{R-Y}, the resultant of the red line current and the reversed yellow current, and I_{B-Y}, the resultant of the blue line current and the reversed yellow line current. I_{R-Y} is associated with the yellow to blue voltage and I_{B-Y} with the red to yellow voltage. It will be seen from the vector diagram that the voltages applied to the potential coils will lag the respective energy meter voltages by 90°. The voltages, however, will be $\sqrt{3}$ times those applied to the watt-hour meter, but providing due allowance is made for this in the calibration or gearing, the meter will correctly measure kVArh in a three-phase four-wire circuit, if the phase voltages are symmetrical.

13.8. Metropolitan-Vickers kVArh Meter. An interesting method, employed in the Metropolitan-Vickers kVArh meter, consists of the association of the red current with the yellow to blue voltage and the blue current with the red to blue voltage. This has the effect of lagging the applied voltages to the potential coils by 120° relative to the phases of the respective applied voltages for energy measurement.

To obtain the effect of 90° lag it is necessary to give the voltage flux a virtual lead of 30°; this is done by connecting series resistance in the voltage coil circuit and adding short-circuited windings on the series coil poles. The conditions are illustrated in the vector diagram of Fig. 141.

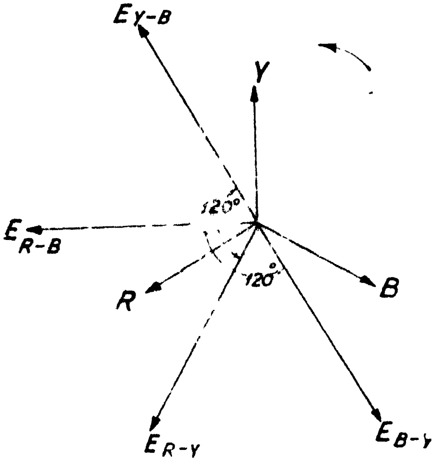


FIG. 141.—Vector Diagram of M.V. Method for kVArh Measurement.

13.9. Cheetham's Method. Another method of virtually lagging the voltage flux by 60° relative to its voltage, instead of by 90°, is due to Cheetham (see *Jour. I.E.E.*, Vol. 73, p. 233, September 1933). It is attained by placing short-circuited turns around the series laminations. These are of such constants that the current flux is made to lag the load current by approximately 30°; a small value of resistance being connected in series with the pressure coil for fine adjustment of this angle. The advantage of such a method is that the errors due to frequency variation are sensibly diminished.

The only circuit condition requiring to be satisfied, in the above methods of kVArh measurement, is that the voltages are symmetrical in phase and magnitude. We will now briefly consider other, though less important, methods which require that the currents also shall be balanced.

13.10. Reversal of "Red" Element—Balanced Load. If the connections of one of the coils in the "red" element are reversed, thereby reversing the torque of that element, the resultant torque T exerted upon the moving system will be proportional to

$$-I_R E_{R-Y} \cos (30+\theta_R)+I_B E_{B-Y} \cos (30-\theta_B)$$

In the case of a balanced load where $I=I_R=I_B$, etc., and $E=E_{R-Y}=E_{B-Y}$, etc., the torque

$$T\propto EI\,[\cos(30-\theta)-\cos(30+\theta)]$$
$$\propto EI[2\sin 30\sin\theta]$$
$$\propto EI\sin\theta$$

and the total registration will be

$$\int_{t_1}^{t_2} EI\sin\theta,$$

which is $1/\sqrt{3}$ times the lagging kVArh. This, of course, will only be correct under balanced load conditions.

13.11. Single-element Meter. A single-element meter may be used for the measurement of kVArh in a balanced three-phase circuit, providing its series coil carries the current in one line, and the voltage between the other two lines is applied to its pressure circuit. The applied voltage to the pressure circuit will be in quadrature with the phase voltage corresponding to the series current, and the meter will

$$\text{register}\quad \frac{1}{1000}\int_{t_1}^{t_2} EI\cos(90-\theta).dt$$

$$\frac{1}{1000}\int_{t_1}^{t_2} EI\sin\theta.\ dt\ \text{kVArh}$$

which is $1/\sqrt{3}$ of the kVArh in a balanced load.

13.12. Crossed-phase Method. Another method of kVArh measurement, termed the "crossed phase" method, consists of a three-phase two-element meter; each element of which is connected in a similar manner to the single-element meter mentioned above. The meter thus registers $2/\sqrt{3}$ times the kVArh in a balanced three-phase load. It need hardly be stated that such a meter has no advantage over the single-element meter and is certainly more costly.

13.13. The Measurement of kVAh. We have already seen that, in an a.c. single-phase circuit with sinusoidal wave-forms of both voltage and current, the energy component

$$=EI\cos\theta$$

and the reactive volt-ampères

$$=EI\sin\theta$$

Therefore, since $EI=\sqrt{(EI\cos\theta)^2+(EI\sin\theta)^2}$

$$\text{the volt-ampères} =\sqrt{(\text{watts})^2+(\text{reactive volt-ampères})^2}$$

The relationships of the above components are therefore similar to those of a right-angled triangle, in the manner illustrated in Fig. 142 (*a*).

When time is taken into account, a further variable is involved, namely, the power factor. This can be seen by reference to the graph of Fig. 142 (*b*), which is obtained by noting the respective advances, at equal intervals of time, of a watt-hour meter and a reactive volt-ampère hour meter. From this graph we see that the expression

$$\text{kilovolt-ampère-hours} = \sqrt{(\text{kWh})^2 + (\text{kVArh})^2}$$

does not hold for the general case, but is only true when the power factor has remained constant throughout the period of investigation. The continuous line represents the conditions for varying power factors, whilst the dotted line represents the condition of unvarying power factor.

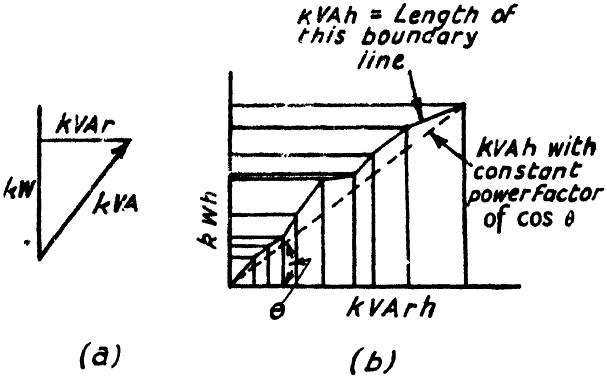


FIG. 142.—kW, kVArh, etc. Relationships.

13.14. Definitions of Three-phase kVAh. Since a straight line is the shortest distance between two points, it follows that $\sqrt{(\text{kWh})^2+(\text{kVArh})^2}$ will always be less than the kVAh (Fig. 142 (*b*)), except in the particular case when the power factor has remained constant. Under these circumstances the above expression will hold.

Three-phase kVA is generally defined as $\sqrt{(\text{kW})^2+(\text{kVAr})^2}$. This conventional definition has the advantage that both the energy and reactive components can be easily and accurately determined, and is based upon the assumption that the total reactive VA is the algebraic sum of the separate single-phase reactive VA. Such a definition, of course, is incorrect in principle since the total equivalent VA is not a vector sum.

13.15. Root Mean Square Definition—KVA. In practice, there is also another method of defining three-phase VA. This is as follows:

$$\text{VA} = \sqrt{3}\left[\sqrt{(I_R E_R)^2 + (I_Y E_Y)^2 + (I_B E_B)^2}\right]$$

where I_R, I_Y, and I_B are the phase currents and E_R, E_Y, and E_B are the phase voltages.

Suppose $I_R E_R = x$, $I_Y E_Y = x+y$, $I_B E_B = x-z$

Then the three-phase VA will be considered as

$$\sqrt{3x^2+3(x^2+2xy+y^2)+3(x^2-2xz+z^2)}$$
$$=\sqrt{9x^2+6x(y-z)+3(y^2+z^2)}$$

The value given by the first-mentioned definition, when the three single-phase power factors are all equal, is

$$(3x+y-z)$$
$$=\sqrt{9x^2+6x(y-z)+y^2+z^2-2yz}$$
$$=\sqrt{9x^2+6x(y-z)+(y-z)^2}$$

Since y and z are both positive, the root mean square definition is the greater (except when the load is balanced, i.e. when y and z are both zero) by an amount depending upon the extent of unbalance. An advantage of the root mean square definition, if without scientific justification from the point of view of correctness of definition, is that the measurement favours the supply undertaking when the load is unbalanced. This is to a certain extent justifiable since, as we have already seen, an unbalanced load is not to the advantage of the supply undertaking (section 12.6), and therefore such an extra charge due to out-of-balance would tend to react fairly for all concerned.

Most kVAh meters register a value corresponding to the definition of sections 13.14, whilst, as we shall see later, there is a kVA meter of the M.D. type which gives an indication corresponding to the root mean square definition.

13.16. Westinghouse kVAh Meter. We have already seen that the expression

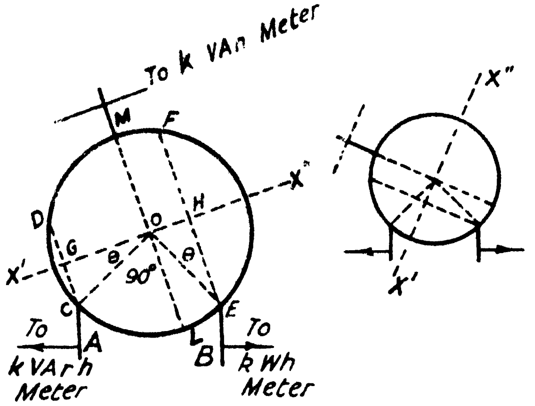
$$kVAh=\sqrt{(kWh)^2+(kVArh)^2}$$

is only correct for a load of unvarying power factor and that, by definition

$$kVA=\sqrt{(kW)^2+(kVAr)^2)}$$

is correct under all conditions. It therefore follows that, providing the spindle of a meter rotates proportionately to $\sqrt{(kW)^2+(kVAr)^2}$ and is suitably geared to a revolution counter, it will (by definition) register correctly in kVAh, irrespective of the normal fluctuations in the value of the power factor.

The Westinghouse Co. have achieved this condition in a very ingenious manner. Their instrument consists of a kWh and a kVArh meter element which drive separate, but identical, friction wheels in contact with an aluminium sphere. The size of the sphere is such that the points of contact subtend a right angle at the centre of the sphere, as illustrated in Fig. 143. A third friction wheel is attached to the framework of the sphere and it is in contact with the surface of the latter. This wheel is geared to a kVAh counter. The framework of the sphere is free to turn about an axis through the centre of the sphere and at right angles to the plane of the paper, taking up a position such that the rotation of the spindle of the kWh meter and that of the kVArh meter produce the same angular velocity of the sphere about



(a) Lagging Power Factor (b) Leading Power Factor

FIG. 143.—Diagrammatic Arrangement of Westinghouse kVAh Meter.

the axis X^1X^{11}. This occurs automatically, due to the frictional forces created by the sphere resting on the rotating wheels A and B.

After a change of power factor, the sphere and its framework swing round until the lengths of the circular friction paths, CD and EF, are proportional to the velocities of their respective friction wheels, thereby producing the same angular velocity around the axis X^1X^{11}. We already know that the ratio of

$$\frac{\text{peripheral speed of kW friction wheel}}{\text{peripheral speed of kVAr friction wheel}} = \frac{\cos\theta}{\sin\theta}$$

$$\text{Hence } \frac{EF}{CD} \text{ must equal } \frac{\cos\theta}{\sin\theta}$$

This will occur when angles GOC and HEO are equal to θ. The peripheral speed of the kVA friction wheel will be proportional to the velocity of the friction ring ML, but

$$ML = \frac{FE}{\cos\theta} \text{ or } \frac{DC}{\sin\theta}$$

which is proportional to $\dfrac{kW}{\cos\theta}$

$$\text{or } \frac{kVAr}{\sin\theta}$$

$$\text{i.e. } kVA.$$

Thus, since the kVA friction wheel rotates at a speed proportional to the kVA in the circuit, the meter will register the defined total kVAh. The meter is therefore fundamentally exact in principle, although such a friction gear has an obvious disadvantage, especially at low loads.

Fig. 143 (*b*) represents conditions when the meter is operating on a leading power factor. In this case the moving system of the kVArh meter will reverse its direction of rotation and the point of contact of the kVAr friction wheel must move to the opposite side of the axis in order to allow for this. The reversal in direction of rotation of the kVAr moving system is no disadvantage since only two counters are fitted to the instrument, one for the registration of kWh and the other for kVAh.

It is obvious that the position of the axis X^1X^{11} relative to that of the kW and kVAr friction wheels is a measure of the angle θ and, in consequence, a pointer is attached to the framework of the sphere. This pointer moves over a calibrated (cosine) scale, thereby enabling one to read the power factor directly. The instrument registers correctly on lagging and leading (above 0·5) power factors and also has provision for the recording of average kW and kVA demand; the recording pointers being operated by means of shaftings from the two counters. In addition, the average power factor can very easily be determined from the ratio of the advance of the kWh counter to that of the kVAh counter, over the period.

13.17. Landis & Gyr Trivector Meter. We will now consider methods of kVAh measurement which are only approximate in fundamental principle, but which give sufficiently accurate results in practice.

Suppose, in a three-phase circuit, that a register advances at a rate proportional to

$$a\,(\text{kW})+b\,(\text{kVArh})$$

where a and b are proper fractions. Then, if the power factor of the system is $\cos\theta$, the advance will be proportional to

$$a\cos\theta+b\sin\theta$$

If the values of a and b are such that the following equations are satisfied

$$a^2+b^2=1$$

$$\tan^{-1}\frac{b}{a}=\alpha$$

$$\text{then } a=\cos\alpha \text{ and } b=\sin\alpha$$

The register will therefore advance at a rate proportional to

$$kVA\ (\cos\alpha\cos\theta+\sin\alpha\sin\theta)$$
$$=kVA\ [\cos(\theta-\alpha)]$$

and, providing the values of *a* and *b* are such that θ and α are of the same order, the register will advance at a rate approximately proportional to the total kVA of the circuit.

This principle has been developed in the Landis & Gyr Trivector meter. The instrument consists of a kWh and a kVArh meter mounted in the same case. Both meters drive a summator situated between them. The summator, in turn, drives a kVAh counter and a kVA maximum demand indicator at a rate proportional to

$$a\ (\text{kW})+b\ (\text{kVAr})$$

This summator consists of a very ingenious system of differential gears (section 8.31) by which means the fractions *a* and *b* can have any one of five different sets of values, which correspond to values for α of 0, $22\frac{1}{2}°$, $45°$, $67\frac{1}{2}°$, or $90°$. The combinations of *a* and *b*, which give the greatest rate of registration, drive the kVAh counter and the kVA maximum demand indicator; interference between the combinations being avoided by driving the first wheel of the counting mechanism by means of ratchet wheels.

It will be clear that, since there are $22\frac{1}{2}°$ steps between the values of α, corresponding to the various combinations, the greatest value of $(\theta-\alpha)$, at a maximum rate of registration, will be $11\frac{1}{4}°$. Therefore, the greatest limits of error introduced by the approximation will be

$$-[1-\cos 11\tfrac{1}{4}°]\ 100 \text{ per cent}$$
$$=-[1-0{\cdot}98]\ 100$$
$$=-2{\cdot}0 \text{ per cent.}$$

By making a 1 per cent positive error for maximum rate of registration for any kVA load (i.e. where $\theta=0°$, or $22\frac{1}{2}°$, etc.) the above maximum error due to the summation can be brought to within ± 1 per cent.

We have previously noted that a kVAh meter will reverse its direction of rotation when the power factor is changed from lagging to leading. It therefore follows that, when there is a possibility of the meter operating on a leading power factor, a relay must be employed to reverse the kVArh meter connections when the load changes from inductive to capacitive, in order to obtain continual positive registration of the meter.

The average power factor can be determined from the ratio of the kWh advance to that of the kVAh register in the same manner as described for the Westinghouse kVAh meter.

13.18. Metropolitan-Vickers "Volt-ampère-hour" Meter. It will be appreciated that, if the voltages applied to the potential coils of a normal watt-hour meter are shifted in phase by an amount equal to the phase displacement of the load current relative to its voltage, the meter will register true kVAh. This principle has been developed in the Metropolitan-Vickers "Volt-ampère-hour" meter. The instrument consists of a normal watt-hour meter, in conjunction with a phase-shifting relay. A diagrammatic arrangement of the apparatus is illustrated in Fig. 144. The relay consists essentially of a power factor relay *P*, a phase-shifting device *S*, and a Ferraris motor *F* for driving the phase-shifting device. Three suitably tapped resistances, delta-connected between the lines of the three-phase supply, form the phase-shifting device; the tapping points being connected to a circular selector switch in the relay. The voltages applied to the potential coils of the meter are selected by two pairs of brushes rigidly spaced 60° apart.

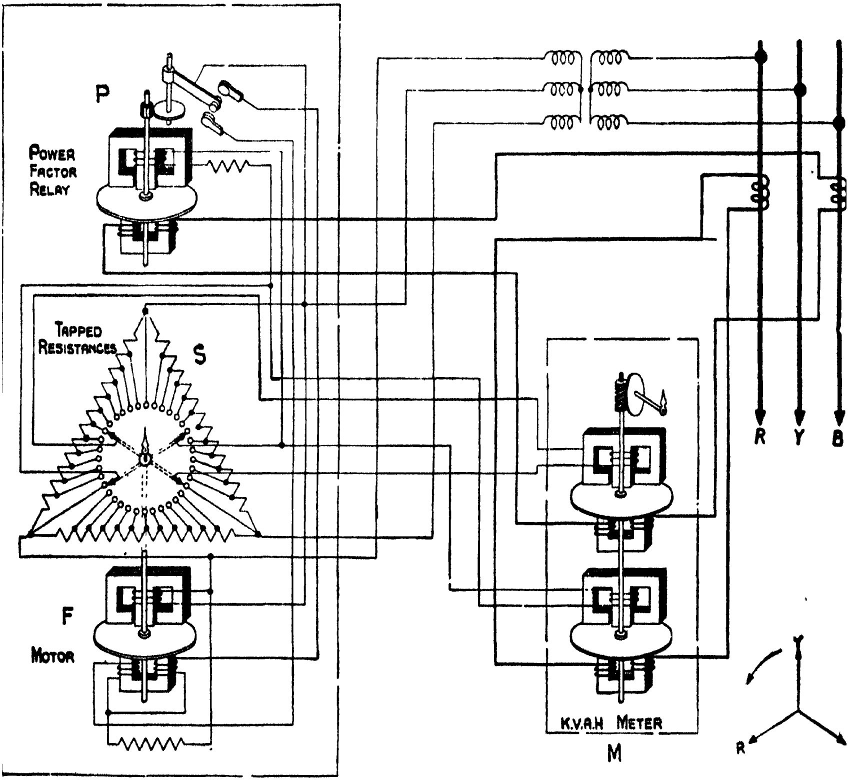


FIG. 144.—Diagram of M.V. Volt-ampère-hour Meter.

The tappings are arranged so that voltages of equal magnitude, but displaced in phase by 10° from those of the adjacent positions of the selector switch, are combined throughout the whole of the 360°.

Suppose, in Fig. 145, *AB*, *BC*, and *CA* represent the three voltages of the three-phase system, and *BD* represents one voltage selected from the delta, i.e. the voltage between one line and the mid-point of the resistance across the other two lines. Then, if a voltage is to be equal in magnitude to *BD*, but displaced in phase by ϕ,

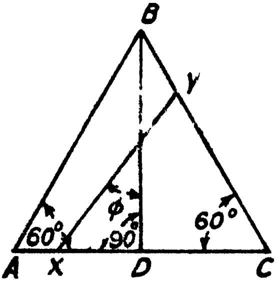


FIG. 145.—Vector Diagram —M.V. Volt-ampère-hour Meter.

$$\frac{XY}{\sin 60} = \frac{CY}{\sin(90-\phi)} = \frac{XC}{\sin(30+\phi)}$$

$$\text{but } XY = BD \quad \text{and} \quad \frac{BD}{\sin 60} = AC$$

$$\text{Therefore } CY = AC\sin(90-\phi)$$

$$\text{and } XC = AC\sin(30+\phi)$$

Thus, the tappings must be arranged according to the above sine law, and in the actual instrument ϕ is made to correspond to multiples of 10°.

The selection of the correct taps for the feeding of the meter potential coils is made by the Ferraris motor, which is capable of rotation in either direction, under the control of the power factor relay. The rotation of the motor is obtained by the energisation of one or other of the two poles of the lower electro-magnet.

When there is no phase displacement between the voltage and current energising the power factor relay (i.e. in the case of the three-wire meter, when I_R lags the voltage applied to the "red" element potential coil by 30°, and I_B leads the voltage applied to the "blue" element potential coil by 30° (the phase rotation being standard) the movable contact floats. When a phase displacement occurs, the disc rotates until the moving contact engages with one or other of the two fixed contacts. This completes the electrical circuit of one of the coils of the lower electro-magnet. Rotation of the Ferraris motor results and the two pairs of brushes are driven around the voltage selector contacts through a system of gearing. This continues until the position of the contacts is such that the voltage impressed upon the power factor relay is, as near as possible, in phase with the supply current. The relay will now lose its torque, the moving contact will have the required phase relative to the supply current, and the meter will register kVAh.

It will be apparent that, since the taps correspond to phase changes of 10°, the rate of registration will always be at least

$$\text{kVAh} (\cos 5^\circ)$$
$$= 0{\cdot}9962 \text{ kVAh}$$

and the maximum error due to defective phase compensation is 0·4 per cent.

As the voltage supply to the meter must of necessity be continuous, certain errors are inevitable due to the short-circuiting of the contacts, thereby decreasing the effective value of the delta resistance. This error is minimised by changing the continuous rotation of the Ferraris motor into a series of intermittent drives, through the medium of a Geneva escapement; thereby reducing the short-circuiting period to a very small value.

The instrument has the advantage that the meter element may be tested as a normal kWh meter, the allowance for the voltage reduction ($\sqrt{3}/2$) being made in the meter voltage coil and the registration mechanism. The phase-shifting relay may be subjected to a separate routine test.

13.19. kVAh Meters for Limited Power Factor Ranges. We have already seen that the torque of an induction meter, for any given kVA loading, is proportional to the sine of the angle between the current and voltage fluxes. Furthermore, the rate of change of the sine function around 90° is very small. For example, through a range of 20°, from 80° to 100°, it only changes approximately 1·5 per cent from the maximum value.

If the relationship between the voltage and current fluxes of a standard watt-hour meter is modified from the normal value by an angle ϕ, the meter will register at a rate proportional to

$$EI \cos (\theta - \phi)$$

instead of $EI \cos \theta$

Also, if $\theta - \phi$ is equal to zero, or approximately equal to zero, the meter will register at a rate corresponding to EI. In other words, it will register the kVAh.

If in practice it is found that the power factor of the three-phase circuit does not vary by any great amount, it is possible to employ such a phase compensated meter for the measurement of kVAh. Providing the value of ϕ, the angle of compensation, is made approximately equal to the mean value of θ, then $EI \cos(\theta-\phi)$ will not differ from EI by more than $1\frac{1}{2}$ per cent (or $\pm 0{\cdot}75$ per cent if the meter is made $+0{\cdot}75$ per cent at the power factor which corresponds to $\theta=\phi$) through 20° range of phase angle When wider limits of error are permissible, the power factor range may be increased. If, for instance, the limits of ± 2 per cent error in kVAh registration are allowable (excluding, of course, normal meter errors) $\theta-\phi$ may have any value between $\pm 16°$. In other words, the meter will be within the permissible limits of error throughout a range of 32° phase angle.

The necessary compensation will depend upon the mean phase displacement of the load current relative to the supply voltage, and may be effected by one or more of the following methods:

(*a*) Selection of voltages with suitable phase displacement
(*b*) Adjustment or modification of normal quadrature loop
(*c*) Short-circuited turns on series coil laminations
(*d*) Resistance inserted in series with potential coils.

Suppose that it is required to measure the kVAh in a three-phase three-wire system, when the power factor of the load is known to remain within the limits of 0·7 and 0·9 lagging. Then the load currents will lag their respective voltages by an angle within the limits of

$$\cos^{-1} 0{\cdot}7 \quad \text{and} \quad \cos^{-1} 0{\cdot}9$$

i.e. 45° and 25°.

Now, the mean angle of lag will be 35°, and if the voltage flux is made to lag its normal phase for energy measurement by this angle, the meter will register at the rate of $EI \cos(\theta°-35)°$ and, since the maximum value of $(\theta-35)°$ is 10°, then $EI \cos(\theta-35)°$ will not differ from EI, or the kVA, by more than 1·5 per cent. By suitable adjustment of the error curve, the meter may be brought within the limits of $\pm 0{\cdot}75$ per cent (excluding normal meter errors).

The most convenient method of lagging the voltage flux by 35° from its normal phase for energy measurement would be by means of a thick copper band around the centre limb of the potential electro-magnet. The action of such a loop would be similar to that of the normal quadrature loop (section 9.5).

If the meter were required for a power factor range of 0·5 to 0·9 lagging, the load current would lag the supply voltage by an angle varying between the limits of

$$\cos^{-1} 0{\cdot}5 \quad \text{and} \quad \cos^{-1} 0{\cdot}9$$

i.e. 60° and 25°.

The mean angle of lag would be $42\frac{1}{2}°$ and, in the case of a three-phase three-wire meter, this could be obtained by associating the red to yellow voltage with the blue current and the red to blue voltage with the red current (thereby lagging the respective voltage fluxes by 60° from their normal phases for energy measurement) and securing the 17½° under-compensation by means of one of the two following methods:

(*a*) Resistance in series with the potential coil.

(*b*) Short-circuited windings around the series electro-magnet, to cause the current flux to lag its normal phase by 17½°.

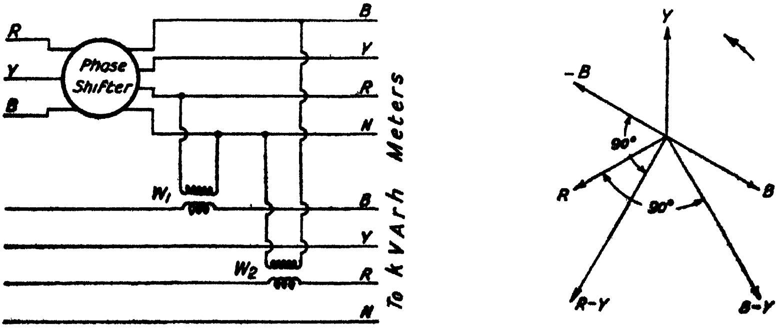
Such a meter, correctly adjusted, would register within the errors of

$$\pm\tfrac{1}{2}\,(1-\cos 17\tfrac{1}{2}^\circ)\text{ per cent}$$

$$\pm 2{\cdot}4\text{ per cent}$$

Meters for limited power factor ranges should only be employed when the power factor limits are definitely known, as the errors increase rapidly outside the specified limits.

13.20. The Testing of kVArh Meters. The meter under test is first connected in a three-phase circuit and 10 per cent in excess of the marked voltage is successively applied to each potential coil, in order that the friction compensating torque can be adjusted to a suitable amount, in exactly the same manner as for the energy meter. Care must be taken to ensure that the meter does not register more than one revolution with any or all of the potential coils energised.



(*a*) (*b*)

FIG. 146.—Watt-meter Connections for kVArh Meter Testing.

To facilitate the testing of the meters, two single-phase wattmeters (or one two-element wattmeter) are connected in such a manner that the red current is associated with the neutral to blue voltage and the blue current with the red to neutral voltage, as illustrated in Fig. 146 (*a*). These conditions are shown in the vector diagram of Fig. 146 (*b*), from which it can be seen that the voltages are thereby lagged 90° from their normal phases for energy measurement. It therefore follows that the speed due to each element will correspond to $\sqrt{3}$ times the apparent power indicated by the relative wattmeter.

13.21. Artificial Neutral for Wattmeters. Where the neutral point is not readily available, it can be obtained by using two wattmeters with identical potential coil resistances and star-connecting them with a non-inductive resistance of similar value. Obviously, there will be some slight error introduced due to difference in time-constants of the three circuits, but in practice this will be found to be negligible since the inductance of the wattmeter potential coils is very small.

The potential circuits of the wattmeters and the meter under test should be supplied from the rotor of a phase-shifting transformer (section 2.19), thereby making it possible to obtain any virtual power factor of the phantom load. The adjustment of the potential flux, until it is displaced exactly 90° from its correct phase for energy measurement, is attained by disconnecting one series element and applying full marked current to the other element; the potential coils of both elements being energised. The phase-shifter is now adjusted until the wattmeter connected in the loaded element circuit reads zero and the quadrature adjustment is utilised until there is no rotation of the disc. This adjustment provides for exact displacement, by 90°, of the potential flux relative to its phase for energy measurement. The other element is then connected in circuit, the adjusted one being disconnected, and the procedure is repeated.

13.22. Balancing of Torques. The balancing of torques is effected by applying full load, under balanced conditions, to the meters and adjusting the phase-shifter until both wattmeters indicate the same value. The connections to the series coil of the one element are now reversed and the balance adjusters are operated until there is no resultant rotation of the disc. This is rather a coarse balance, and it is therefore advisable to time each element separately with the load applied to only the one element. Both potential coils are energised by their correct (kVArh) voltages and the load is applied at zero power factor.

The connections are now returned to normal and, with the phase-shifter still in the position for equal load of the elements (i.e. full load at zero power factor), the meter is timed for a suitable number of revolutions. The meter is now producing its maximum torque, the conditions being similar to those of a watt-hour meter on full load at unity power factor, and the speed of the disc is adjusted to its correct value by means of the brake magnets. After this initial adjustment, the meter is timed at such loads as one and a quarter, half, and one-twentieth full load, at zero power factor, and one and a quarter full load at 0·866 power factor. Providing the elements have been balanced and the quadrature bands have been set to give the correct phase displacement of the pressure coil fluxes, there should be little likelihood of any necessity for further adjustment, except at low load. This, of course, should be effected by means of the friction compensating loops, after which the meter should be timed at one of the higher zero power factor loads. After each balanced load, the meter must be tested for its errors at extreme out-of-balance, i.e. for each element separately. It is carried out in exactly the same manner as described for full load at zero power factor. In the case of 0·866 power factor, the current must be in phase with its phase voltage for the "red" element and lagging its phase voltage by 60° for the "blue" element. The reactive load on the element will again correspond to the indication of the respective wattmeter.

The meter is now subjected to a dial test. A balanced full load at zero power factor is applied to the meter for a sufficiently long period to advance the pointer, of the dial of smallest denomination, by ten revolutions. Let T be the time of the dial test in hours, E the applied voltage, and I the load current in ampères, the dial test reading will be

$$\frac{\sqrt{3}EIT \sin 90^\circ}{1000}$$

$$= \frac{\sqrt{3}EIT}{1000} \text{ kVArh}$$

It is usual to permit a greater error, in the case of a kVArh meter, than is allowed for a kWh meter. Any well-designed commercial meter, however, should be capable of an accuracy within the limits of ±3·0 per cent. In the case of meters specially designed for the British Grid, an error within ±1·0 per cent has been made possible.

The meters should be given a critical running on starting current which, for a kVArh meter, will be one-hundredth load at zero power factor.

It should be mentioned that a kVArh meter will reverse its direction of rotation when the power factor changes from lagging to leading. Where there is a possibility of this happening, it is advisable to fit a ratchet and pawl and thereby avoid reverse running. The ratchet is usually fitted on the rotor spindle, whilst the pawl is mounted on the frame in such a direction that it does not interfere with forward running, but definitely prevents registration in the reverse direction.

13.23. The Testing of kVAh Meters. When the kVAh meter is of the summator type, the kWh and kVArh meters are tested separately. The whole instrument is then connected in circuit with a three-phase balanced full load at 0·5 lagging power factor and the advance of the kVAh counter is noted for a given period of time. It will be remembered that the kVAh meter of this type can only be checked by dial tests. Should the meter require adjustment, it is advisable to return to the kWh and kVArh elements to determine by how much each element is to be adjusted. When the meter has been adjusted satisfactorily, it is given further dial tests at one-quarter and one-tenth full load 0·5 power factor lagging. The above tests are then repeated at 0·866 power factor lagging.

13.24. Automatic Phase-compensation Type. When the meter is of the automatic phase-compensation type, it is first tested as a normal kWh meter without its phase-shifting unit. It is then tested as a complete instrument at 0·866 and 0·5 power factor lagging, with various loads applied to the meter. The loads may be full load, one-quarter and one-tenth full load. An advantage of such a kVAh meter is that it may be tested for accuracy by timing disc revolutions or by means of dial tests.

13.25. Meters for Limited Power Factor Ranges. In the case of kVAh meters for limited power factor ranges, they are first connected in a three-phase circuit in their normal working manner and adjustment of the friction compensating devices is made to give the required compensating torque when 10 per cent in excess of the marked voltage is applied to the potential coils of the meter. It is then advisable to connect the meters in a single-phase circuit, as illustrated in Fig. 134, section 11.16. One series element of each meter is disconnected and full load current is applied to the other element. The phase-shifter is then adjusted until the current lags the voltage by $(90+\psi)^\circ$, where ψ is the angle of over-compensation of the potential flux, or the angle by which the current coil flux lags that for energy measurement. It should be noted that in the latter case ψ is negative. The element must now be adjusted until there is no rotation of the disc under these conditions, and the manner in which this is effected will depend upon which of methods (*b*), (*c*), and (*d*)—section 13.19—is employed to obtain the necessary phase displacement. When this has been accomplished the procedure is repeated for the other element.

While still employing a single-phase circuit, the connections of the one element are altered so that the torques due to the elements will oppose one another. Full load at a lagging power factor of cos ψ is then applied (each element thereby producing maximum torque) and the balance adjusters are altered in position until there is no resultant rotation of the disc. When the phase displacement of angle ψ is effected by lagging the current flux, a leading power factor of cos ψ must be applied.

The meters are then connected in a three-phase circuit in their normal working manner and a balanced load is applied. The wattmeters are connected in the circuit to measure power, in the normal manner. The phase-shifter is then adjusted until the power factor is cos ϕ lagging, with full load applied to the meters, and the latter are timed for a convenient number of disc revolutions. The kVA is then calculated from the ammeter and voltmeter readings and its value will be $\sqrt{3}EI/1{,}000$, where E is the line voltage and I the line current in ampères. The speed of the disc is now adjusted to its correct value by means of the brake magnets, after which the full load error at both the maximum and minimum power factors is determined. These power factors will be $\cos(\phi-\alpha)$ and $\cos(\phi+\alpha)$ where 2α is the phase angle range of the meter as a kVAh instrument. The individual elements are also tested in a similar manner to that described for watt-hour meters, the single-phase loading power factor being $\cos\phi$, the current in the series coil lagging its associated potential coil voltage by ϕ°.

The above tests are then repeated at one-quarter and one-twentieth full load, at each of the power factors $\cos(\phi+\alpha)$, $\cos\phi$, and $\cos(\phi-\alpha)$ and, for the low load tests, adjustment is performed by means of the friction compensating devices. Should the meter be outside the allowable limits at any load, a suitable adjustment should be made and the entire test repeated.

If the meter exhibits equal errors at power factors $\cos(\phi+\alpha)$ and $\cos(\phi-\alpha)$, the adjustment must be performed by means of the brake magnets. On the other hand, if the errors at power factor $\cos(\phi+\alpha)$ are greatly different from those at $\cos(\phi-\alpha)$, the adjustment must be made in the phase displacement of the voltage flux; depending upon which method of compensation is employed.

When the errors are all within the permitted limits, depending upon the discretion of the tester (who will naturally bear in mind the range of phase angle which the meter has to cover) the meters are subjected to a dial test.

In conclusion, the meters should be tested for registration at starting current (the power factor being within the limits of $\cos(\phi+\alpha)$ and $\cos(\phi-\alpha)$ lagging), after which they are again subjected to a test at no-load, with 10 per cent in excess of the declared voltage applied to the potential coils.

CHAPTER XIV

Maximum Demand Indicators

14.1. Measurement of Maximum Demand. The measurement of maximum demand is next in importance to that of energy measurement. Up to the present time, the former quantity has not acquired an exact nor universal definition. For practical purposes, maximum demand represents the maximum average power demand over a predetermined interval. It is a sustained rather than an instantaneous demand, such as would occur through an accidental short-circuit in the consumer's installation.

14.2. Merz Method. There are two fundamental methods of measuring the maximum power demand of a consumer. These are:

(*a*) Merz method
(*b*) Thermal method.

In method (*a*), a separate counter driven by the spindle of a kWh meter registers the average kW supplied during a definite time interval (which may be of any pre-determined value, and is generally between fifteen and sixty minutes). Provision is made for the drive to return to zero for the commencement of each period; the pointer being friction-held in the maximum position attained by the drive during its previous excursions. Thus, the pointer is only driven when the drive exceeds the previous maximum position.

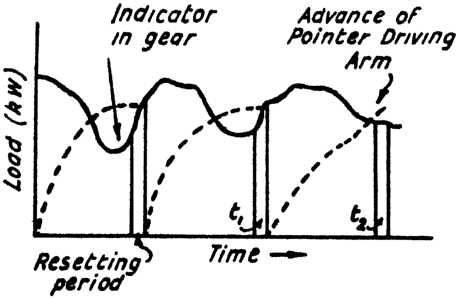


FIG. 147.—Merz Method—Maximum Demand Indicator.

The time intervals are provided by means of a clock mechanism, which controls an arrangement whereby, at equal intervals, the maximum demand indicator is disconnected from the disc spindle in order that the drive to the pointer may be reset to zero. Such a method is illustrated in Fig. 147.

The drive to the pointer will, during the interval between t_1 and t_2 hours, advance by an amount proportional to

$$\int_{t_1}^{t_2} P.dt \text{ kWh}$$

where P is the instantaneous power.

The average power supplied during this period will be

$$\frac{1}{t_2-t_1}\int_{t_1}^{t_2} P.dt \text{ kW}$$

$$= \frac{\text{kWh supplied during interval } t_1 \text{ to } t_2}{t_2-t_1}$$

It therefore follows that the pointer will indicate the average power demand during an interval, providing suitable gearing is employed and the time interval of the indicator remains constant. Such a register attached to a meter is capable of indicating the kVA demand in a similar manner. It will require very little consideration to appreciate that, although the above type of indicator will give a true maximum average demand for definite clock periods, the indication for short peaks will depend upon the averaging period and upon the clock time at which it occurs. Furthermore, the shorter the averaging period, the greater the tendency for the indication to favour the supply authority.

14.3. Thermal Method. In method (*b*), an instrument is employed which depends for its operation upon the heating effect of the load current. Such an instrument is designed so that for a steady current the temperature will only reach its maximum value after a predetermined period, which is generally from fifteen to thirty minutes. It is therefore essentially a maximum current demand indicator, but it can be scaled to indicate kW in a d.c. system, kVA in an a.c. system, or kW in an a.c. system where the power factor is known.

It is difficult to define the type of maximum demand recorded by the thermal instrument, since the heating of an element by a constant current causes an exponential approach to the conditions of equilibrium. From this it follows that a high peak of small duration (compared with the nominal time-lag of the instrument) will give a greater indication than a peak of half the magnitude for twice the duration, although the kWh supply is identical in the two cases. The thermal instrument, however, has one advantage over the Merz type of indicator, this being that the indication is entirely independent of the clock time at which the peak occurs. On the other hand, a disadvantage of most thermal indicators is their square law characteristics.

14.4. Operation of the Merz-type Indicator. We will now consider practical examples of the above types of indicators. In the

Merz type, the drive to the pointer is usually maintained through a wheel which is held in mesh by gravity. At each resetting interval a time-switch completes an electrical circuit whereby the wheel is electro-magnetically disengaged for a period of sufficient duration to enable it, and subsequent gearing to the pointer, to return to its zero position. In some patterns of indicator the above procedure is reversed, the wheel being electro-magnetically held in mesh during the driving period and disengaged by the action of gravity when the time-switch opens the circuit for the brief resetting interval. Of the two types, one would favour the electro-magnetically held-off pattern, since the vibration attending the operation of the electro-magnet is thereby only transmitted for a few seconds of every averaging period, instead of continuously.

14.5. Metropolitan-Vickers M.D. Indicator. A diagrammatic

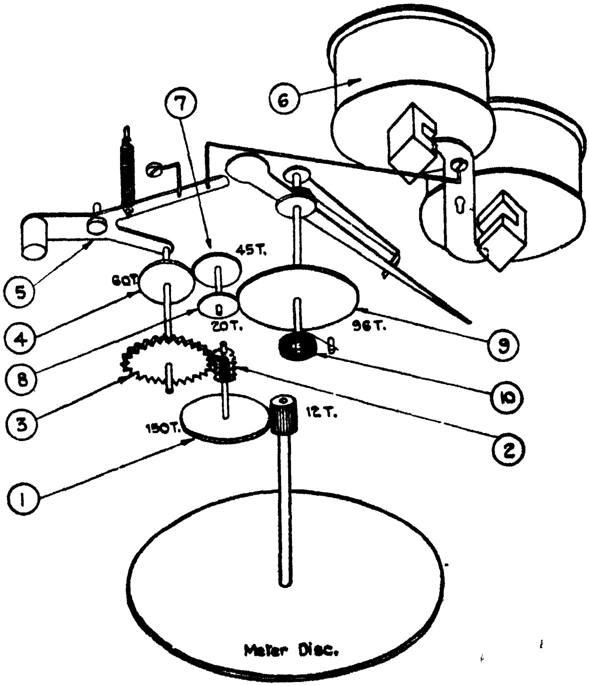


FIG. 148.—Perspective Drawing of Maximum Demand Indicator (Metropolitan-Vickers).

arrangement of the Metropolitan-Vickers maximum demand indicator is illustrated in Fig. 148. It is of the electro-magnetically held-off pattern. The advance of the meter is transmitted to wheel (1), via the pinion on the rotor spindle, thence to worm (2) which is attached to the shaft of wheel (1). Wheel (3) engages with worm (2) and upon a common shaft with the former is wheel (4), which engages with wheel (7). The advance of the latter wheel is transmitted via wheels (8) and (9) to the short driving pointer against the light control spring (10). The common shaft of wheels (3) and (4) is mounted in rocking frame (5) and at the completion of each operating period the electro-magnet (6) is energised for a few seconds, thereby causing the armature of the latter to rotate. This results in the rocking frame moving wheel (4) out of mesh with wheel (7). Spring (10) now takes control and returns the driving pointer and wheels (7), (8), and (9) to their zero position.

There are two pointers to the indicator. The shorter one is rigidly attached to the shaft of wheel (9) and acts as a driving arm to the longer pointer. The latter is friction-held and thus remains in position when the driving pointer returns to zero. At the completion of the resetting period, which is between five and twelve seconds, the electro-magnet circuit is opened, the rocking frame returns to its normal position, wheel (4) re-engages wheel (7) and the driving pointer is again advanced by rotation of the meter spindle. The maximum demand indicator pointer will advance only when the average demand during that operating period exceeds the maximum average demand of each of the previous averaging periods since the last resetting of the m.d.i. pointer.

The ratio of the worm (2) to wheel (3) is selected to suit the averaging period of the demand mechanism, and the m.d.i. pointer may be returned to zero by means of a milled plunger which is mounted in the window covering the meter registration dial. The operation of resetting the pointer can be effected without removing the meter cover; the plunger being sealed to prevent unauthorised resetting.

14.6. Time-switches. There are several types of time-switches for controlling the operating period of a maximum demand indicator, both of the open-circuiting and short-circuiting type, and these include:

(*a*) Hand-wound escapement type
(*b*) Electrically wound escapement type without storage spring
(*c*) Electrically wound escape type with storage spring
(*d*) Synchronous motor type.

The advantage of types (*b*) and (*d*) is that they will stop if the supply is interrupted, thereby avoiding the risk of obtaining a false reading on the demand indicator. Furthermore, the synchronous motor type is essential when it is required that a number of demand indicators in an a.c. system shall operate in synchronism. The obvious disadvantage of (*a*) is that it requires periodic visits for the purpose of re-winding; whilst (*d*) can only be employed when the supply is a.c.

Generally, the time-switch is provided with a dial which advances a complete revolution every hour. Fitted to the dial are a number of pins, depending upon the length of the averaging period. These are symmetrically spaced near the edge of the dial and number one for a one-hour interval, two for thirty minutes, three for twenty minutes, etc. The tripping period is controlled by means of two arms of different length, free to move around the same point. As the dial rotates, both arms are raised together by one of the pins, but they drop at different times due to their different lengths. Adjustment is such that an interval of five to twelve seconds elapses between the tripping of the two arms.

Maximum demand indicator mechanisms frequently include a built-in time-switch of the synchronous motor type. The operating period may be controlled by means of an hour dial provided with pins, etc., or, as is sometimes the case, the synchronous motor may be geared directly to give the required operating period.

14.7. Thermal Maximum Demand Indicators. The two most common types of thermal indicators are:

(*a*) The bi-metallic strip type

(*b*) Differential thermometer type.

A maximum demand indicator of the bi-metallic strip type, manufactured by Price & Belsham Ltd., is illustrated in Fig. 149. Two coils of bi-metallic strip comprise the movement, the lower one being the actuating coil. The upper one is fitted for the purpose of providing compensation for change in ambient temperature. These coils are supported by a stainless steel rod attached to a zero adjuster, and the inner end of the actuating coil is rigidly fixed to the bottom end of the

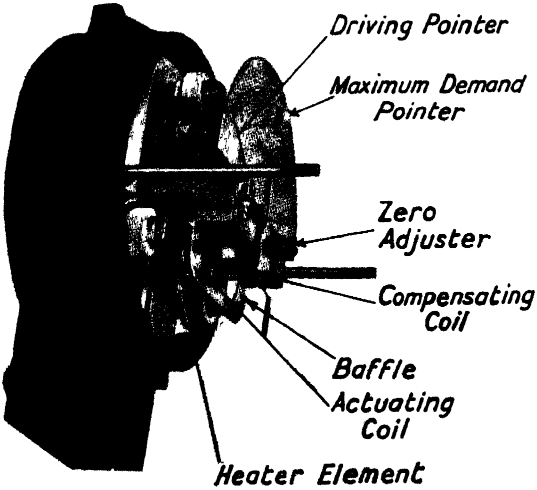


FIG. 149.—Thermal type Maximum Demand Indicator (Price & Belsham Ltd.)

rod. The inner end of the upper, or compensating, coil carries bearings which are free to rotate around it. A baffle is also provided between the coils, and this is bridged by a yoke attached to the outer ends of the two coils; the needle being fixed to the bearings of the top coil.

The heater block is comprised of two discs of insulating material with a heater element, stamped from resistance material, riveted between them. This element carries the load current, or definite proportion of it, and is fixed near the actuating coil in such a position that its heat only acts upon this coil. The needle is therefore advanced from its zero position at a rate depending upon the current passing through the element, and it carries forward a pointer which remains at the position of maximum excursion of the needle. To prevent the needle moving to a false reading, either high or low, due to intentional or accidental vibration of the instrument, a chain friction device is provided. This consists of a fine chain wound around a drum on the idle needle, both ends of the chain being attached to a light spring. Such an arrangement ensures equal friction in both directions on the maximum demand needle.

The two coils are accurately matched and, as a result, any change in ambient temperature rotates both coils through the same angle. The position of the needle is consequently unaffected.

The time-lag is obtained by means of the refractory material, acting as a heat reservoir, which surrounds the heating element.

14.8. Wright M.D. Indicator. The Wright maximum demand indicator, manufactured by the Reason Manufacturing Co. Ltd., works on the differential thermometer principle. The instrument, which is illustrated schematically in Fig. 150, consists of a U-shaped glass tube containing a bulb at the extremity of each limb. At the junction of

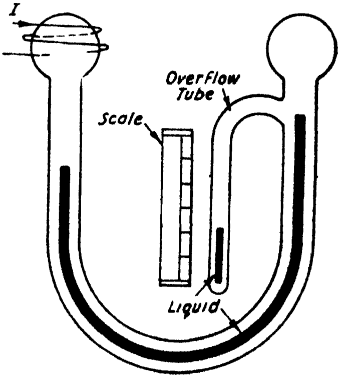


FIG. 150.—Diagrammatic Arrangement of Wright Maximum Demand Indicator.

the right-hand bulb and limb is fitted an overflow tube, suitably graduated in the units of the demand being measured; whilst a coil which carries a definite proportion of the load current embraces the left-hand bulb.

The U-tube is initially filled with a suitable liquid, until it is level with the neck of the overflow tube, and the whole system is hermetically sealed. The proportions of the instrument are such that there is approximately the same volume of air at each end of the column of liquid. Hence, changes in ambient temperature affect both bulbs equally and the position of the column of liquid is unaffected, but the heat produced by the current passing through the coil expands the air in the left-hand bulb, thereby forcing the liquid towards the overflow tube. That which reaches the junction of the right-hand limb and the graduated overflow tube overflows into the latter. The quantity of liquid thus deposited will equal in volume the expansion of air in the left-hand bulb. This expansion will be proportional to the increase in temperature of the air in the left-hand bulb and, in consequence, will be proportional to the square of the load current.

The time lag of such an instrument is usually thirty minutes, but it can be increased by the placing of iron or glass in a pocket in the heating bulb.

14.9. The Testing of Maximum Demand Indicators. The testing of a maximum demand indicator of the thermal type is a very simple procedure. A known current, say full load, is passed through the instrument for a period sufficiently long to permit of the reading attaining its maximum value. The maximum demand indication must correspond to the actual load, and the time taken for the indicator to attain its maximum reading is compared with the nominal time-lag of the instrument. If there is too great an error, either in reading or time-lag, adjustments must be made, depending upon the pattern of indicator. It is advisable to repeat this procedure at about one-quarter full load.

The testing of a Merz type maximum demand indicator may be proceeded with whilst the meter is on dial test. Where the m.d. indicator has an external time-switch, it is advisable where possible to use the actual time-switch to be employed in service. When no time-switch is available for the test, the tripping period will have to be manually controlled by means of a switch.

The m.d.i. pointer is initially set at zero, and the instrument is allowed to run for several averaging periods. At the completion of each averaging period it is advisable to bring the pointer back a little from its final reading, in order that the advance of the indicator—as well as the tripping mechanism—may be checked each time. When the timing device possesses an hour-dial and pins, one should run the apparatus for sufficient length of time to check the averaging periods right around the dial. When the averaging period is controlled by hand, the length of the tripping period should only be five seconds, as this represents the shortest and most stringent tripping interval which is likely to occur in practice.

In order to check the back-lash of the indicator, one should run the m.d.i. test at one-quarter as well as full load. The error of the m.d.i. should be within ±2 per cent of that of the meter at full load, and ±3 per cent at one-quarter full load. Theoretically, where the demand charges are of the same order as the energy charges, the same accuracy is demanded of the m.d.i. as of the meter.

When testing the meter, the m.d.i. pointer should be well up the scale as, with the exception of the initial averaging period and small intervals at the end of various subsequent averaging periods, the meter disc will not be working against the friction spring of the m.d.i. pointer. The error, for purposes of record, will therefore be that obtained when the m.d.i. pointer load is off the meter, but it is advisable to ensure that when the meter is driving the pointer the retarding effect is not too pronounced.

14.10. Synchronous Motor Time-switches. It should be mentioned that, where synchronous motor time-switches are involved, the meters should be tested upon a supply from the mains or, alternatively, due allowance must be made for the difference in frequency between that of the test supply set and that of the mains. For example, if the former's frequency were 49 cycles per second and the mains were 50 cycles per second, the time-switch would operate in a period 2 per cent greater than nominal, and this would increase the registration of the indicator by a similar amount.

14.11. Back-lash—Merz System. The manufacturer frequently allows in his gearing for back-lash and the length of the tripping interval. The importance of the latter can be seen when we consider the case of a nominal averaging period of twenty minutes and a tripping interval of twelve seconds. If no allowance were made for the tripping period, since the actual averaging period would be nineteen minutes forty-eight seconds, the indicator would tend to read 1 per cent low.

14.12. Shaded-pole Type kVA Meter with Maximum Demand Indicator. This type of three-phase kVA meter receives its driving torque from three shaded-pole electro-magnets operating upon a common aluminium disc. The windings of the electro-magnets carry the three line currents of the three-phase circuit in which the instrument is installed. We have already seen (section 3.21) that a driving torque produced by a shaded-pole electro-magnet is proportional to the product of the square of the current and sine of the angle by which the shaded flux lags the unshaded flux. Furthermore, providing the frequency remains constant, the phase angle between the fluxes will remain constant, and the torque will be proportional to the square of the current.

The braking torque is provided by the rotation of the disc in the field of a permanent magnet, as well as in the fields of the shaded-pole electro-magnets. The driving torque, providing the constants of the three electro-magnetic circuits are identical, will be proportional to the sum of the square of the line currents, i.e. proportional to I_v^2 where I_v is the root mean square value of the line currents. The braking flux will be proportional to $N(\Phi_p^2+k_1\Phi_v^2)$ where Φ_p is the permanent magnet flux and N is the velocity of the disc in r.p.m. Hence, the speed of the disc will be given by

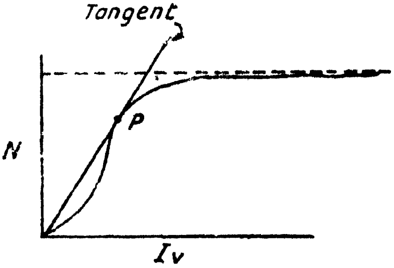


FIG. 151.—Speed-Load Curve—
Shaded-pole type kVA Meter.

$$N=\frac{k_2I_v^2}{\Phi_p^2+k_1I_v^2}$$

The above expression reveals that when $k_1I_v^2$ is small compared with Φ_p, N approximates to $k_2I_v^2/\Phi_p^2$, the curve of which is a parabola. When $k_1I_v^2$ is large compared with Φ_p^2, the expression approximates to $N=k_2/k_1$, i.e. the speed will remain constant for any increase in current.

A general graphical representation of the above expression is given in Fig. 151, and it can be seen from the shape of this curve that there is one point whose tangent passes through the origin, this point being represented by P on the curve. Around P, the curve approximates to that of the expression $N=kI_v$, which is represented by the tangent at P.

It is this portion of the curve, which approximates to the straight line of proportionality of N and I_v, which is employed as the working range of the instrument, but owing to the very approximate proportionality of speed to I_v, the instrument is only utilised as a maximum average kVA demand indicator. When the value of I_v does not depart greatly from that represented by point P, it gives a sensibly accurate indication of the three-phase kVA when defined as $\sqrt{3}$ times the square root of the sum of the squares of the single-phase kVA. As we have already seen, this value is somewhat greater than that of $\sqrt{(kW)^2+(kVArh)^2}$ except in the case of a balanced load; the difference increasing the greater the departure of the load from balanced conditions.

The indicator is scaled to read three-phase kVA at the declared voltage of the system, and compensation for variations in the actual voltage is provided in the form of shading pieces capable of movement in the air gaps of the shaded-pole electro-magnets; the movement of each being controlled by an electro-magnet energised by the corresponding phase voltage.

14.13. The Testing of Time-switches. It is advisable to maintain a separate department of the test-room and repair shop for the testing and overhauling of time-switches. They should be tested and adjusted to operate with the correct averaging period for several successive trippings, and the tripping period must be adjusted to be within five and twelve seconds. This work should be undertaken by a qualified clock repairer.

CHAPTER XV

Summation Metering

15.1. Summation of Electrical Energy, etc., from Several Feeders—Methods. When the total kWh, kVAh, or kVArh, transmitted by a number of feeders is required to be registered upon one meter, it is essential that some arrangement be employed for the summation. The methods employed vary according to the circuit conditions; each of those we are about to consider having its own sphere of usefulness.

There are three principle methods of summation metering, namely:

(*a*) Electrical (transformer, multi-wound series coil meters, multi-element meters)

(*b*) Mechanical impulse

(*c*) Electro-mechanical impulse.

15.2. Summation by Transformer. We have already seen (section 6.8) that one of the greatest advantages of the transformer is the complete isolation of the secondary winding from the primary winding. Thus, if the primary windings of two equal ratio transformers are connected in the corresponding lines of two separate single-phase feeders, and the secondary windings to the same series coil of a meter, the current passing through the series coil will be proportional to the vector sum of the currents in the two feeders. Furthermore, if the supply voltage of the feeders is the same in magnitude and phase, and the potential coil of the meter is connected across one of the feeders, the speed of the disc will be proportional to the total energy supplied by both feeders. Thus, the meter can be designed to register the total energy supplied. The total maximum demand can also be determined by incorporating a suitable indicator.

This method may also be satisfactorily employed for two three-phase feeders, such as would most likely be the case in practice. The current transformer secondaries of the corresponding lines of each feeder are connected to the same series coil of the meter. Thus, in the three-wire case the two secondaries of the transformers to the red lines would be connected to the series coil of the "red" element, etc. The potential circuits would be connected in the normal manner to one or other of the feeders. A diagram of connections for electrical summation with paralleled current transformers is illustrated in Fig. 152; a relay being included to ensure that the meter potential coils are always connected to a live feeder.

Such a scheme could be extended to any number of feeders, except for the errors introduced when the transformers are unequally loaded, particularly when one or more of the transformers is idle. The conditions for the extreme case in a two-feeder summation are given in

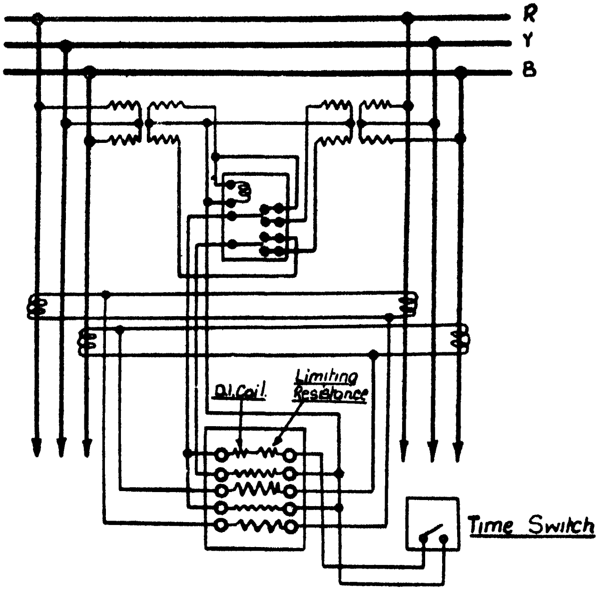


Fig. 152.—Diagram of Connections for Electrical Summation Paralleled Current Transformers.

Fig. 153. Transformer *A* is under full load whilst *B* is idle. The secondary current of *A* is represented by I_S, and this is the vector sum of I_M passing through the meter coil and I_I which flows through the

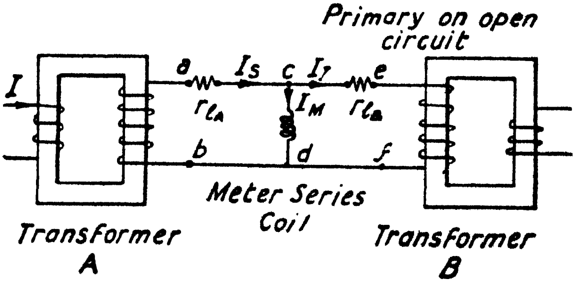


Fig. 153.—Electrical Summation with one Transformer Idle.

parallel secondary winding of the idle transformer. The virtual effect of this is to change the ratio and phase angle errors of the loaded transformer. In general, the former will be changed but slightly, whilst the latter error will be increased appreciably compared with its normal value. In the case of similar transformers, the current in the secondary of the idle transformer will be almost $1/n$ times its primary magnetising current; where n is the current ratio of the transformer.

The value of the current passing through the secondary winding of the idle transformer will be kept at a minimum by connecting the transformer secondaries in parallel at the meter terminals (as illustrated in Fig. 153) instead of at the terminals of the secondary windings, since the p.d. at *ef* is less than that at *ab*. Furthermore, the lower the ratio magnetising current/load current, the smaller will be the errors under the above conditions of unbalance. One can therefore appreciate that mumetal-cored transformers are infinitely superior to the soft-iron type, since the magnetising current is kept very small. With this type of transformer, the errors introduced by the extreme out-of-balance conditions are not considerable, especially if the transformer is not too heavily burdened and is liberally designed. Generally, it is advisable to employ the precision type of transformer if reasonable accuracy is desired.

15.3. Separate Windings on Series Laminations. The method of section 15.2 can be improved if each transformer secondary winding is connected to a separate winding on the series laminations of the corresponding element of the meter. There will then be no change of burden with alteration of load, but mechanical considerations determine the number of coils which can be interleaved on one set of series laminations.

15.4. Transformer with Multi-primary Windings. Special summating transformers have been constructed to overcome the disadvantage of the method of summation of section 15.3. These consist of several primary windings and only one secondary winding. The primary windings are each connected to a corresponding line of the separate feeders, or to the secondary of a transformer connected in the line, and the secondary winding of the summating transformer is connected to the series coil of the corresponding meter element. Since the back ampère-turns of the secondary winding will be approximately equal to the vector sum of the ampère-turns of the primary windings, it follows that the transformer will prove an effective medium for summation of the current, and it has the advantage of being suitable for employment for several feeders.

It should be mentioned that when the primary windings of the summation transformer are connected to secondary windings of transformers in the lines of feeders, care must be taken to ensure that the overall ratio between the current in each line and the current flowing in the corresponding series coil of the meter, due to that line current, is the same in all cases.

In all the above methods provision must be made so that, if the supply fails to the feeder from which the potential circuits of the meter are energised, there is an automatic and instantaneous change-over of the potential circuit connections to a live feeder. This may be effected by an electro-magnetic relay.

The electrical method of summation is simple, reliable, and, except in the more complicated cases, the cheapest form of summation. It relies, of course, upon the corresponding voltages of the different feeders being equal in phase and magnitude. For further information upon this subject, the interested reader is recommended to read the article entitled "Current Transformer Summations," by Hill and Shotter (*Jour. I.E.E.*, Vol. 69, p. 1251, October 1931).

15.5. Summation by Four-element Meter. Another method for determining the total energy delivered by two three-phase feeders is by means of a four-element meter. It really consists of a pair of two-element meters mounted in such a manner as to have one common moving system; two of the elements being connected in one feeder circuit and the other two in the second feeder circuit. The total torque, and therefore the speed of the disc, is arranged to be proportional to the total power supplied by the feeders. The counting mechanism therefore registers the total energy supplied by the two feeders during any given time.

The maximum average total demand on the two feeders can be determined by incorporating in the meter a maximum demand indicator of the Merz pattern (section 14.2).

An advantage of summation by four-element meters, over transformer summation, is that the frequency, phase, and magnitude of the supply voltages from the two feeders need not be identical. Unfortunately, the method labours under the disadvantage of increasing the weight of the moving system, thereby creating extra wear on the pivot and jewel, and also of introducing interaction between the two pairs of elements (unless, of course, the feeders are operating at different frequencies). On account of the former disadvantage, this method of summation is not extended to more than two feeders.

15.6. Summation by Impulses. In multiple circuits, the total energy cannot always be satisfactorily summated by the above methods on account of the limitations already mentioned. A method is accordingly employed whereby each meter is fitted with a device for the communication of an electrical impulse from the meter to a summation register; each impulse representing a definite number of units. The impulse is transmitted by pilot wires between the meter and the summation register, and the impulse current may be either a.c., d.c., or rectified a.c. If the pilot wires are long, it is advisable to employ a smooth d.c. supply, as trouble may otherwise be experienced due to capacitance.

Upon receipt of an impulse, the summation register is advanced by an amount corresponding to a definite number of units integrated by one of the circuit meters.

15.7. Receiving Elements. In the British Grid metering scheme, which is probably the most accurate and largest of its kind in the world, the summator comprises a number of receiving elements, each of which acts as a receiver of impulses from its associated kWh or kVArh meter. A register is fitted to each element, and the elements are mechanically or electrically coupled to the maximum demand indicator (which is of the Merz type) and its recording gear.

There are three forms of contactor employed for impulsing in the grid metering scheme; these are:

(*a*) Commutator.
(*b*) Gravity-operated device.
(*c*) Cam-operated contactor.

15.8. Commutator Contactors. The commutator, which is incorporated with the corridor switching arrangement (Fig. 154) comprising three pilot wires, is fitted to the spindle of the meter. Its function is to close the solenoid circuit, which is then opened by a switch controlled by the solenoid. Contact is therefore only made by the commutator, and not broken, and as a result it has not introduced the trouble experienced in the past with commutator meters.

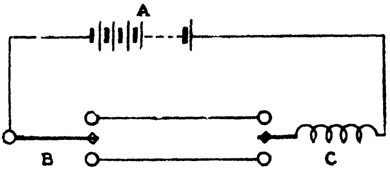


FIG 154 —Corridor Switching Arrangement.

15.9. Gravity-operated Contactor. In the gravity-operated contact arrangement, the rotation of the disc spindle raises a weight which, after the meter has registered a definite amount of energy, falls back into its original position. In so doing, the weight establishes contact for a fraction of a second and energises the corresponding solenoid in the summator.

15.10. Cam-operated Contactor. In arrangement (*c*), a switch is operated by means of a cam fitted to the disc spindle, the object of the cam being to establish contact once per revolution of the disc. In this type the cam also breaks contact, but sparking is prevented by resistances connected across the contacts.

15.11. Prevention of False Registration—Ratchet and Pawl. It is clear that in each of the above arrangements the summation register is incapable of differentiating between backward and forward registration of the circuit meters, since the impulse would be the same in both cases. In order to avoid advance of the summation element, with backward registration of the disc, circuit meters are fitted with ratchet devices. The ratchet is designed to operate upon the main disc spindle, so that there will be no possibility of the disc making several revolutions before the ratchet becomes effective. If this reverse rotation were possible, oscillation of the meter disc due to alternate forward and reverse power would cause a large registration upon the summation meter.

15.12. Graded Impulsing. In some of the Grid metering installations, the number of units per impulse of each circuit meter is constant, being determined by the rated load of the largest circuit. In other cases, the number of units per impulse is graded in proportion to the rated full load of the individial circuit. The disadvantage of the former system is that the rate of impulsing will vary with the rated load of each individual circuit. This will lead to unequal wear of the contacts and, in the case of the smaller circuits, impulses will tend to be included in the wrong period, thereby increasing the errors in the determination of the maximum demand. Graded impulsing, on the other hand, tends to uniform accuracy, wear, and reliability of the entire equipment.

15.13. Impulse Receiving Elements. There are three general types of impulse receiving elements in use. These are:

(*a*) Double-coil rotary armature
(*b*) Double-coil oscillating armature
(*c*) Single-coil attracted armature.

15.14. Double-coil Rotary Armature Impulse Receiving Element. In type (*a*), the rotary armature is connected through a ratchet device to the receiving element driving spindle (Fig. 155). One end of this spindle is geared to a differential shaft, and the other end to the counting train mechanism. The receiving elements are differentially connected together in pairs, in order that their advances might be

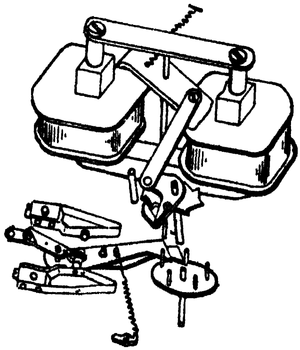


FIG. 155 —Impulse Receiving Element—Double Coil Rotating Armature.

summated and, in turn, these pairs are coupled in pairs as illustrated diagrammatically in Fig. 156. Theoretically, this method may be extended indefinitely, but in practice it is limited to about fourteen elements, on account of the weight and size of the assembled unit. The Grid Authorities have satisfactorily employed up to twelve-element summators in this manner.

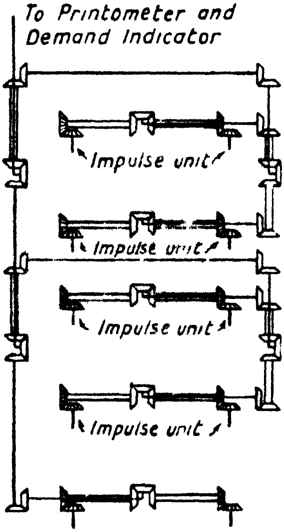


FIG. 156.—10-Element Summator.

15.15. Oscillating Armature Impulse Receiving Element. A diagrammatic arrangement of the oscillating armature of type (*b*) is given in Fig. 157. Impulses are received by the armature and are

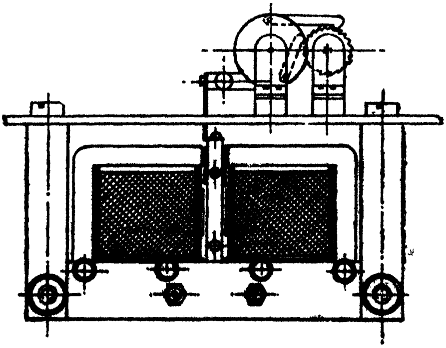


FIG. 157.—Impulse Receiving Element—Oscillating Armature.

transmitted via the two ratchets, one rotating the ratchet wheel in a positive direction for movement of the armature in one direction, and the other continuing the rotation in the same direction when the armature is reversed by the next impulse.

The summation is carried out in a similar manner to that already described.

15.16. Single-coil Attracted Armature Impulse Receiving Element. The principle of type (*c*) is illustrated in Fig. 158. Operation of the armature releases an arm connected to a ratchet wheel, the arm being reset against a spring by a cam-shaft continuously rotated by means of a shaded-pole motor. During the resetting period, the impulse is transmitted to the summator.

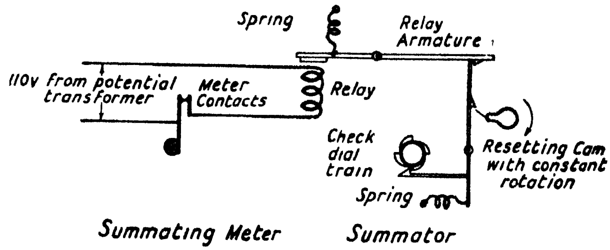


FIG. 158.—Impulse Receiving Element. Single-coil Attracted Armature.

In this type of summator, the ratchet wheels on the receiving elements are all directly connected to the same shaft. It is therefore essential that no two impulses occur simultaneously. Consequently, only four solenoids work on one shaft, these being 90° apart. This number may be increased to eight by arranging another band of four to be operated by the same cams, on the opposite side of the cam-shaft.

15.17. Electrical Impulse System. Another interesting type of summator is one which entirely impulses electrically from meter to demand indicator, and this is also to be found in service in the grid metering scheme. It possesses a double-coil relay with a vertical oscillating armature, and its diagram of electrical connections is given in Fig. 159. When an impulse is received, the armature is deflected, thereby releasing the armature of the restoring relay. When this drops, an electrical circuit connected to a rotary contact-maker is completed; the contact-maker controlling the transmission of all impulses between the integrating meters and the summator. A series of contacts are provided in the rotary contact-maker and these are each connected to restoring relays. Contact of the switch creates an impulse which is transmitted through three notching coils, which respectively operate a totalisor, a repeater counter, and a demand indicator. The action of the rotary switch is similar to that of the cam-shaft described in section 15.16. In the latter case, however, the impulses are mechanically controlled whereas, in this type, they are electrically controlled by the rotary switch.

Care must be taken to ensure that the speed of the rotary switch and rate of transmission of impulses is such that only one impulse arrives for every contact on the rotary switch. Should it not be arranged in this manner, the arrival of two impulses between two contacts on the same transmission circuit would only be recorded as one impulse, thereby introducing an error in summation.

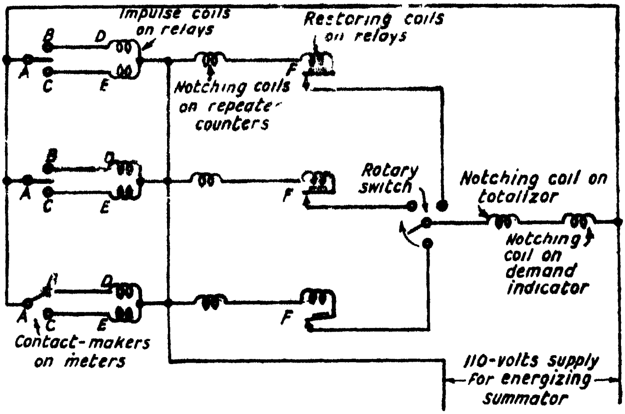


FIG. 159.—Diagrammatic Arrangement of Electrical Impulse System.

15.18. Maximum Demand Indicators. The maximum demand indicators used in conjunction with the above systems are of the Merz type, and differ from the commercial type of indicator by virtue of the provision of a long-scale arrangement, together with a large dial for accurate reading.

One type of m.d.i. is fitted with two pointers which are geared in the same manner as the hands of a clock, the arrangement being such that ten revolutions of the one pointer occur during one revolution of the other.

15.19. Printometers and Maxigraphs. In certain of the Grid summators, the demand recorders are of the printometer type, whilst the remainder are fitted with maxigraphs.

An interesting feature of the maxigraph is that no ink is required for the purpose of making a record, as this is effected by means of a silver marking-wheel which rolls over specially prepared paper during the brief resetting interval. At other times the wheel is clear of the paper.

15.20. Monthly Records. Each monthly accounting period, the following records are required from the British Grid metering equipment, these are:

(*a*) kWh of energy

(*b*) kW of maximum demand during any consecutive thirty minutes (either thirty minutes from the commencement or thirty minutes from the middle of any hour of the twenty-four hours of any day).

(*c*) Power factor at the time of maximum demand (wherever involved in the Board's agreements).

The determination of (*c*) is made by reference to kVArh meters and their maximum demand indicators, and this practically involves a duplication of the kW summating equipment. The diagram of connections of the kWh and kVArh meters, in one circuit, to their respective summators is given in Fig. 160. The maximum average demand is obtained upon the indicator, and the time of this demand from the kW record chart. The corresponding kVAr demand at this time is then determined from the kVAr chart, and from these two demands the power factor can be calculated.

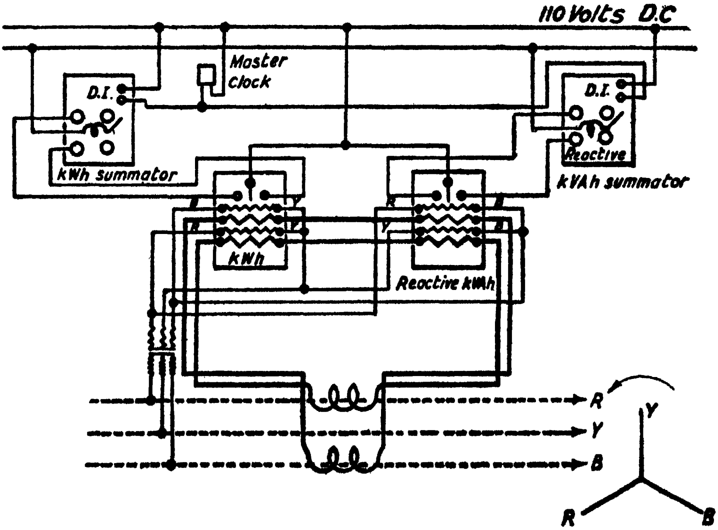


FIG. 160.—Circuit Arrangement—Grid Metering Scheme.

For further particulars of the metering arrangements of the grid system, the interested reader is recommended to read the article entitled "Grid Metering," by Henderson (*Jour. I.E.E.*, pp. 185-99, Vol. 75, August 1934).

A summator, manufactured by Metropolitan-Vickers Ltd., is illustrated in Fig. 161. This is arranged for the summating of six circuits

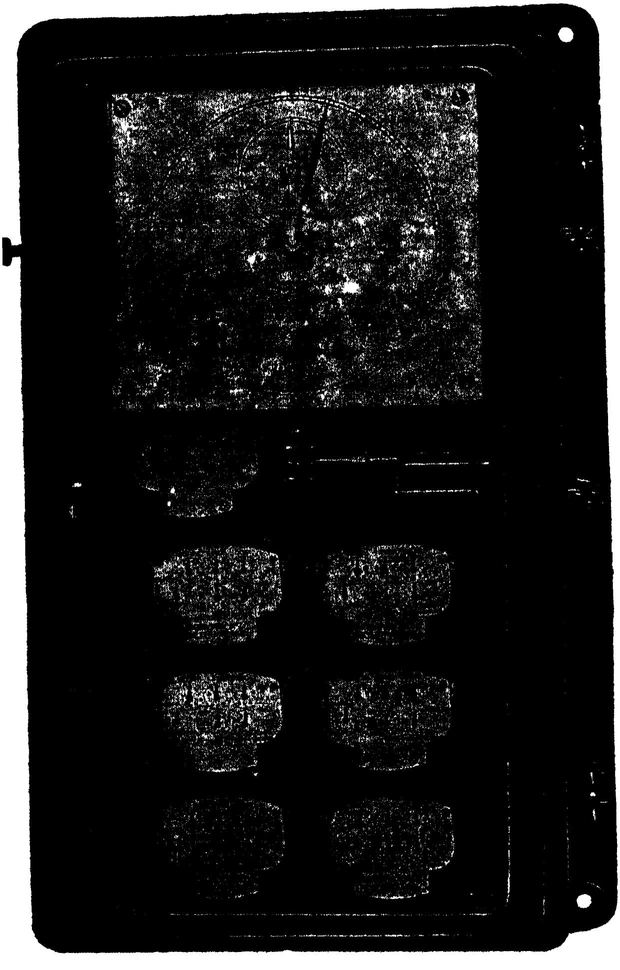


FIG. 161.—Summator for summating Six Circuits, Recording simultaneous Maximum Demand and providing a Printed Record of the Average Demand during Successive Half-hour Periods (Metropolitan-Vickers.)

and records the simultaneous maximum demand, as well as providing a printed record of the average demand during successive half-hour periods. The summation of the total energy is registered upon the top left-hand dial, while the remaining six dials register the energy supplied by the respective individual feeders.

CHAPTER XVI

Special Meters

16.1. Two-rate Meters. This type of meter is employed in connection with the double-tariff method of charging for electrical energy, which differs from the maximum demand system and the two-part tariff system, inasmuch as there are two different rates of charge per unit of energy, depending upon the period of the day at which the energy is supplied (section 12.4). The meter is of the normal quarterly type, but has two separate registering trains. An arrangement is embodied in the meter by means of which they are alternately brought into gear with the meter spindle, at pre-determined times of the day; the arrangement being controlled by a time-switch. The latter alternately opens and closes a solenoid circuit. When the solenoid is energised, an armature is operated and this moves a rocking device, thereby displacing a driving wheel from one registration train to the other. Generally, the time-switch is adjusted to operate in such a manner that the registration of energy occurs upon the "high rate" dial during the hours of peak demand upon the station, and upon the "low rate" register for the remainder of the day. The consumption of energy measured upon the "low rate" register is charged for at a preferential rate.

16.2. Ferranti Type FLDI Two-rate Meter. One such two-rate meter is illustrated in Fig. 162. It will be noted that this meter incorporates the time-switch and change-over mechanism. This consists of a synchronous motor and gearing, forming a twenty-four-hour clock, which drives the operating arms, each of which rotates once every twenty-four hours. The position of the operating arms on the twenty-four-hour dial determines the periods of operation of the relative rates.

An alternative and, in some ways, better method of metering for such a tariff is the employment of two normal quarterly meters in conjunction with a time-switch. The meters are connected in series with the load and for pre-determined periods of the day the time-switch connects the potential circuit of the one meter to the main supply voltage and opens the potential circuit of the other meter. During the remainder of the day, when the other tariff is in operation, the latter potential circuit is energised whilst the other potential circuit is opened by the time-switch.

16.3. Battery Meters. These are usually of the normal ampère-hour mercury motor type and are employed in battery circuits for the purpose of giving information as to the state of charge, etc., of the battery. When one such meter is connected in the battery circuit, it will run in either direction, according to whether the battery is being charged or discharged. It is therefore essential, for accurate operation of the meter, that no fluid-friction compensation is provided. Otherwise, in the one direction, the compensation would be negative and detract from the accuracy of the meter.

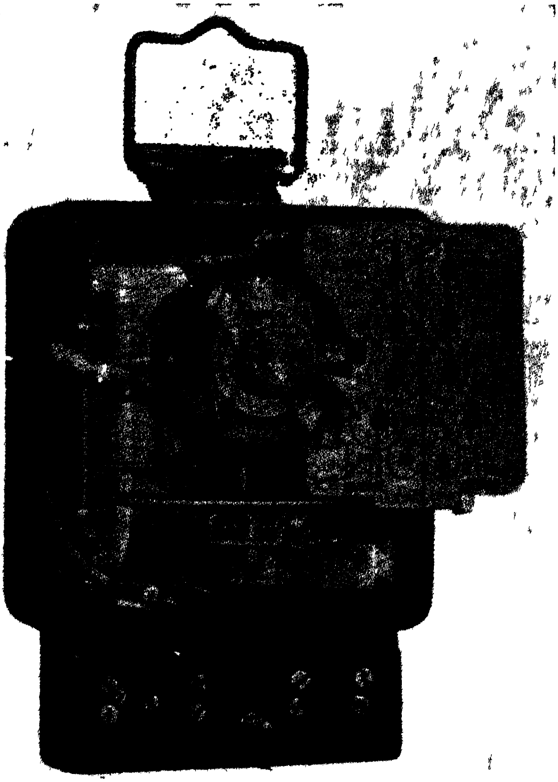


FIG 162 —Ferranti type FLDI Two-rate Meter

Two such meters are frequently employed in series in the battery circuit, each being fitted with a ratchet and pawl in order only to permit of forward registration of the disc. In such circumstances, the connections of the respective meters are such that the one measures the quantity of electricity supplied during the charging of the battery, whilst the other measures the quantity of electricity supplied by the battery during discharge. It will be appreciated that, for forward registration of both discs, the one meter must have its connections reversed from those normally employed for the ampère-hour meter.

16.4. Load-rate Prepayment Meter. An interesting type of meter, developed by Fawsett and Parnall, is the load-rate prepayment meter manufactured by the Metropolitan-Vickers Co. Ltd. The purpose of this meter is to encourage small consumers to install, and use, more apparatus suitable for connection to their normal lighting circuits. The meter is capable of operation at two different prices per unit of energy delivered, one tariff being in operation when the consumer's demand is below a certain value (say, 0·5 kW) and the other for any demand exceeding this value. Usually, the price per unit at the higher demand is about one-quarter that of the other tariff.

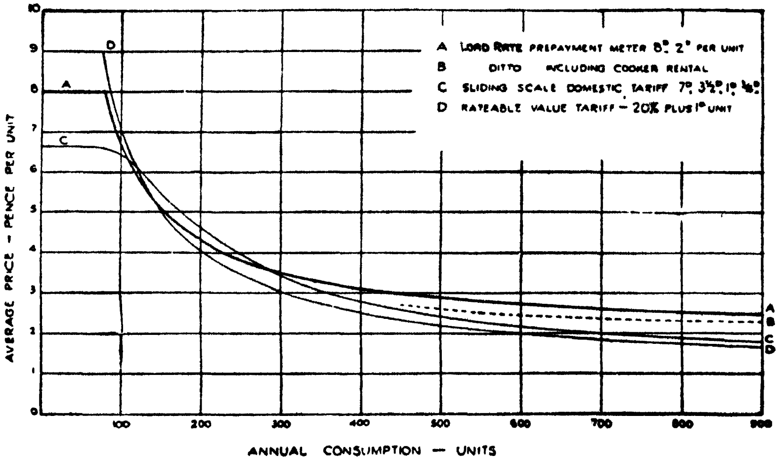


FIG. 163.—Load-Rate and Two-part Tariff Meters. Comparison Curves.

The prepayment mechanism is operated by the consumer in the normal manner, and the instrument is similar to the standard prepayment meter manufactured by the Company, except that a special automatic gear-changing device is incorporated for the selection of the appropriate price per unit. This gear-changing device consists of an electro-magnet controlling a lever which introduces or withdraws a set of gears in the train, between the energy registering mechanism and the main differential of the prepayment device. A pointer is provided upon the instrument to indicate the tariff at which the meter is operating.

The demand at which the tariff changes is determined with reference to the maximum lighting load of the consumer, and is so arranged that the revenue obtained at the threshold demand for operation of the price-reducing gear shall not be lower than that received for the maximum lighting load alone. It is claimed that the revenue curve of such a meter closely follows the cost curve and Fig. 163, which gives comparative curves of average prices received from load-rate meters and ordinary two-part tariff meters, illustrates how closely the results obtained with a suitable load-rate meter compare with those from rateable value tariff.

16.5. Light and Power Prepayment Meters. This type of mete is similar to the ordinary prepayment meter, except that two interlace series windings are provided in the meter element. The one winding i connected in the consumer's lighting circuit, whilst the other, whic consists of less turns, is connected in the power circuit.

If n is the ratio of the lighting circuit winding turns to the powe circuit turns, it follows that the speed of the disc per kW of lightin demand will be n times the speed per kW of power demand. Thus, b designing the meter to register correctly for the lighting load, th registration for the power load will only be $1/n$th of its correct value An obvious disadvantage of such a meter is that its registration give no real indication as to the units consumed, and for this reason it ha not been approved by the Electricity Commissioners.

16.6. Arno Meter. It was first suggested by Professor Arno tha the charge for electricity should be made on a "complex load" tim basis, defining "complex load" as $\frac{2}{3}EI\cos\theta+\frac{1}{3}EI$. Such a tariff woul obviously provide an order of compensation to the supply authorit for the extra cost involved in the supply of electrical energy when th power factor departed from unity (section 12.5). The only modificatio involved in making the normal meter suitable for measuring the abov quantity is the adjustment of the quadrature loop so that the pressur flux lags the applied voltage by an angle slightly greater than 90°, sa $(90+\delta)$. On a lagging power factor of $\cos\theta$, the modified meter wi register proportional to

$$EI\cos(\theta-\delta)$$

which will be greater than that for energy measurement by the factor o

$$\frac{\cos(\theta-\delta)}{\cos\theta}$$

$$\frac{\cos\theta\cos\delta+\sin\theta\sin\delta}{\cos\theta}$$

$$\cos\delta+\tan\theta\sin\delta$$

which, where δ is small, will approximate to

$$1+\tan\theta\sin\delta$$

Now, for $1+\tan\theta\sin\delta$ to be equal to $(\frac{2}{3}EI\cos\theta+\frac{1}{3}EI)/EI\cos\theta$

$$\sin\delta=\frac{\left(\frac{2}{3}+\frac{1}{3\cos\theta}\right)-1}{\tan\theta}$$

$$=\frac{\frac{1}{3}\left(\frac{1}{\cos\theta}-1\right)}{\tan\theta}$$

$$=\frac{1}{3}\left(\frac{1}{\sin\theta}-\cot\theta\right)$$

At 0·866 P.F., $\theta=30°$, and

$$\sin\delta=\frac{1}{3}\,(2-1·73)$$
$$=0·09$$
$$\therefore\ \delta=5·1°$$

Over a considerable range, by choosing a suitable value for the angle δ, the meter will register a quantity dependent upon the value of $\frac{2}{3}EI\cos\theta+\frac{1}{3}EI$. Although the speed of the disc will not always be proportional to this quantity, it will always give a greater registration than that for energy measurement, and this value will increase the

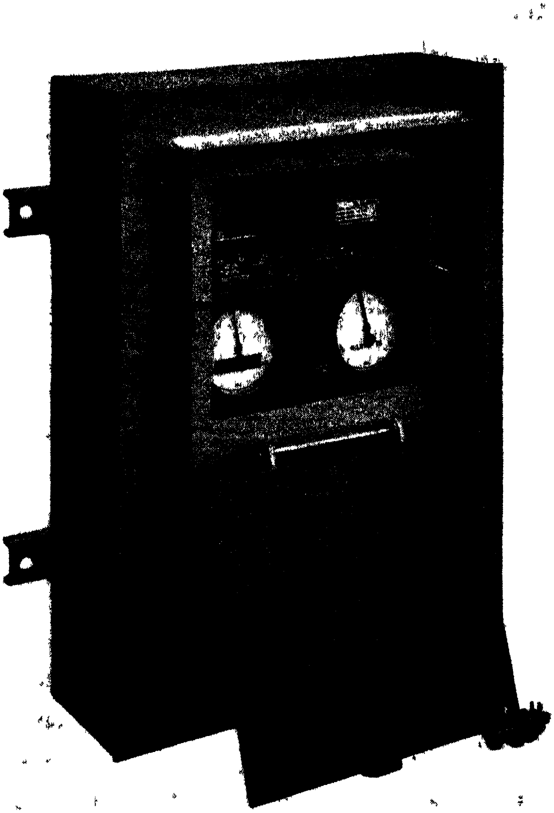


FIG. 164.—Outdoor Unit (Chamberlain & Hookham Ltd.)

lower the power factor; the increase being proportional to tan θ, since sin δ is constant. Where the supply is likely to have a power factor between the limits of unity and 0·8 lagging, it has been suggested that δ should be given a value of 5° and, for power factors varying between 0·9 and 0·5 lagging, δ should be given a value of 14°.

Such a system of metering has much to commend it, since it has the advantage of being simple and inexpensive.

16.7. Outdoor Units. It is sometimes the case that a meter is required for outdoor operation. Weatherproof meter-boxes have accordingly been manufactured, and one, as supplied by Chamberlain and Hookham Ltd., is illustrated in Fig. 164. It consists of a welded steel chamber with a hinged front panel, provision being made for locking and sealing the box and its contents. The entry of moisture is prevented by means of a cork gasket fitted inside the front panel, together with a rain shed. The illustration is of such a box housing a combined kWh and kVAh meter with their corresponding maximum demand indicators.

16.8. Showroom Demonstration Meters. These are of the normal type of quarterly meter, except that the usual kWh counting train and dial are replaced by gearing with a large single pointer and circular dial; the latter usually being graduated in units or pence. The meters are utilised in the showrooms of supply undertakings, when cookery demonstrations are in progress, to give the consumer a simple illustration of the cost involved when an electric cooker is in operation.

CHAPTER XVII

Testing Equipment—Test-room Lay-out

17.1. Approved Apparatus. The test-room equipment of Supply Authorities has, in general, improved tremendously within the past few years, and this is in no small measure due to the Electricity Commissioners. The latter have laid down that the apparatus provided in any testing station, in which supply meters are tested before being submitted for certification, must include the following:

(*a*) *Standard Apparatus.* Direct current potentiometer, standard cells, standard resistances for current measurement, voltage dividers.

(*b*) *Substandard Apparatus.* Indicating wattmeters, ammeters, and voltmeters. Rotating integrating meters. Electrolytic meters (where electrolytic meters have to be tested).

(*c*) Stop-watches or any other suitable timing devices.

(*d*) Apparatus for the control of current, voltage and, in the case of alternating current meters, phase.

This apparatus must in all respects conform to the specifications laid down by the Electricity Commissioners in their supplementary publications to the Electricity Supply (Meters) Act, 1936. The reader is therefore recommended to consult the entire publications, which can be purchased from H.M. Stationery Office at the cost of a few pence. We will only consider the main requirements laid down therein.

17.2. D.C. Potentiometer. This must have a range of at least 0 to 1·5 volts and a range-checking device for a 10 : 1 ratio must be provided. The minimum reading on the normal range must not be greater than 0·0002 volt. In the vernier type of instrument (section 2.15) 0·0002 volt must correspond to at least one step on the last dial on the normal range, whilst in the slide-wire type it must represent at least 1 mm. length of the slide-wire. The resistance of the potentiometer circuit must not be greater than 200 ohms, nor less than 100 ohms per volt. The error due to departure from nominal resistance of any coil of the potentiometer circuit must not exceed 0·0002 volt on the normal range. When the instrument is used on the lower range, this error must not be altered by an amount greater than 0·2 per cent. If the potentiometer includes an independent system for balancing against the standard cell, this section must be capable of adjustment to compensate for the change of e.m.f. of the cell with change in ambient temperature. This provision must be made for a range of e.m.f. of at least 1·0177 to 1·0187 volts, and if it is not continuous at least six positions must be included. Finally, any exposed insulating material must be of such a type as is unaffected by light. Ordinary black ebonite is unsuitable.

For the best operation of the instrument, the accumulator should be of such a rating that it is well able to supply the potentiometer current at a constant value. It is also important that the accumulator be kept in a good condition.

17.3. Voltage Standardisers. Direct current potentiometers which do not conform to the above requirements may be employed in the capacity of voltage standardisers (section 2.26). Under these conditions, the range of the instrument must be at least 0 to 1·5 volts, and not less than 0·75 volt must be applied to the potentiometer for voltage standardisation purposes. If of the slide-wire type, one division of the slide-wire must correspond to not more than 0·001 volt, and shall have a length of not less than 2 mm. In other types of potentiometer one division of the lowest reading dial must not be greater than 0·0005 volt.

If any exposed insulating material is of a type which is adversely affected by light, a plate must be fitted to the potentiometer stipulating that the instrument is to be protected from light when not in actual use.

17.4. Standard Cells. These are to be of the Weston or mercury cadmium type (section 1.11). At least two must be provided for the purpose of mutual comparison.

17.5. Voltage Divider. Voltage dividers must be of the constant current type and the value of the resistance must be 50 to 100 ohms per volt. Furthermore, the error in voltage division must not exceed 0·02 per cent.

17.6. Standard Resistances for Current Measurement. The resistances are to be designed for a voltage drop of 1·0 or 1·5 volts when the rated current flows through them. Thus, the power wasted is much greater than in ammeter and supply meter shunts, and the dissipation of heat in the larger standard resistances is made as large as possible by enclosing them in oil-baths. The resistance elements must be constructed of manganin, or a material with similar properties, hard soldered to the end blocks or copper pieces. The latter may be soft soldered to the terminal blocks.

The thermal e.m.f.s (section 6.1) of a resistance standard must not exceed 0·01 per cent of the voltage drop at rated current, when the shunt has been passing this current for half an hour. The value of this thermal e.m.f. can be estimated by observing the deflection of the galvanometer immediately after the rated current has been interrupted (the potentiometer being set to zero by short-circuiting the leads from its tappings) and comparing it with the deflection produced when the potentiometer is set at its minimum reading.

For current ratings up to 100 ampères, the resistance error must not exceed 0·03 per cent, and for current ratings over 100 ampères the maximum allowable error is 0·05 per cent for any current up to the rated value.

17.7. Voltage Standardisers. These may be of either the constant current or constant resistance type. In the former type the resistance must not be less than 50 ohms, nor more than 100 ohms, per volt. In the constant resistance type of instrument, the current taken from the line, on any range, shall not be less than 1 milliampère nor more than 20 milliampères. The error in the ratio of voltage division shall not exceed 0·02 per cent on any range and, if no adjustment is provided to give compensation for variation in temperature of the standard cell, the e.m.f. of the standard cell will be assumed to have a value of 1·01825 volts. The same remarks which applied to the d.c. potentiometer, concerning exposed insulating material, also apply to the voltage standardiser.

17.8. Standard Clocks. Ships' chronometers, or pendulum clocks, may be employed for the checking of stop-watches. The former shall be provided with a seconds hand, the tip of which shall move in steps of not less than 0·5 mm. Unless the pendulum of the pendulum clock is visible, it must also comply with the above requirement. The chronometer or clock must be adjusted so that its error does not exceed thirty seconds in twenty-four hours at any temperature between 10°C. and 30°C.

17.9. Substandard Stop-watches. These must have dials readable to 0·05 seconds, although dials having divisions of 0·1 seconds will be acceptable providing the length of a division is not less than 1 mm. and a suitably thin pointer is employed. The error developed by watches shall not exceed 0·15 per cent of the interval measured, at any temperature between 10°C. and 30°C.; an extra combined error of 0·05 seconds being allowed for the starting and stopping of the watch.

Suitable timing devices, other than stop-watches or chronograph watches, may be submitted to the Electricity Commissioners for approval as time substandards, but mains-driven synchronous stop-clocks or stop-watches must not be employed as time standards or substandards.

17.10. Substandard Wattmeters. The Electricity Commissioners have specified electro-dynamometer instruments which, unless of the torsion-head (section 4.12), shall have scales not less than 5 inches long, with knife-edged pointers and anti-parallax mirrors.

When such an instrument is to be used for alternating current measurements, the corrections at the declared frequency and unity power factor must be identical with those on direct current, within the accuracy of reading the instrument; the indication on direct current being taken as the mean of the two values obtained with opposite polarities.

We have already considered the factors which determine the accuracy of d.c./a.c. transfer of an electro-dynamometer wattmeter, and a method of testing it on direct current (section 2.25) by means of a d.c. potentiometer.

For other requirements of the Electricity Commissioners, the reader is advised to consult the publications already mentioned.

17.11. Substandard Rotating Meters. The specifications concerning substandard integrating meters are that they shall comply in all respects with the requirements of the British Specification No. 37, 1930, for substandard meters, as far as they are applicable, except that meters intended for long-period dial tests may be fitted with modified registering mechanisms. In such a mechanism the three dials of highest denomination may be omitted. For other particulars concerning the specification of substandard integrating meters, the reader is advised to consult the publications of the Electricity Commissioners.

17.12. Sources of Energy. In addition to the standard and substandard instruments already mentioned, the equipment of a testing station must include suitable and sufficient sources of energy. There must also be means to facilitate the regulation of the voltage and current as well as, in the case of alternating current, means of altering the phase of the load current relative to the supply voltage, or vice versa. As we have already seen (section 9.17), it is usual to apply a phantom load to the meters, i.e. separate sources of voltage and current to the potential and series coil circuits.

17.13. Voltage Supply. The voltage requirements are very modest, all that is needed being supplies (a.c., d.c., or both) at the rated voltages of the meters to be tested, with provision for varying the voltages ±10 per cent of the declared value. The current requirements are more exacting, and sources of current must be provided with means of continuously varying the magnitude from a value equivalent to the starting current of the smallest capacity meter to the full load current of the meters of highest rating.

17.14. Testing of Meters with Potential Transformers. In the case of meters employing potential transformers, it is common practice to test them as 110-volt instruments, and to make allowance for the errors of the transformers (section 6.17). These errors may be obtained from an actual test upon the transformer, or estimated from a knowledge of the characteristics of other transformers of the same type.

17.15. Power Factor. Provision must be made for the adjustment of the artificial power factor of the phantom load. This may be by means of a phase-shifting transformer (section 2.19) or any other device whereby the phase of the testing voltage can be varied independently of that of the current, or vice versa.

17.16. Current Supply for D.C. Meter Testing. In testing stations handling d.c. meters, the capacity of the battery employed for the supply of current to the test circuits will depend upon the maximum current rating of the d.c. meters likely to pass through the station. It must be sufficiently in excess of this to enable the normal d.c. routine testing of the establishment to continue while the large capacity meters are under test. Alternatively, two separate sources of supply might be provided.

In testing stations only concerned with a.c. meters, the battery requirements will be more modest; the capacity need only be sufficient for the d.c. testing of transfer instruments.

The voltage of batteries for direct current supply should be sufficiently high to permit of the major portion being absorbed in loading resistances of negligible temperature coefficient, such as manganin. This will minimise the variation in the value of the testing current, due to changes in temperature, when the copper portion heats up on load. In general, the battery voltage should be between 12 and 20 volts, and preferably of the order of the higher figure. Suitable charging plant must also be provided and great care should be taken in the charging and maintenance of the batteries. This duty should only be given to a capable person and, since it will thereby ensure a steady and reliable source of testing current, it will more than repay for the trouble taken.

17.17. Voltage Supply for D.C. Meter Testing. A battery must be provided capable of supplying at least 10 per cent in excess of the maximum declared voltage of any d.c. energy meter to be tested at the station. The capacity of the battery need only be small, since in general such meters are only tested in small quantities. When an a.c. supply is available, the battery could be kept in a charged condition by means of a trickle charger.

It is essential that the batteries and framework of an H.T. battery system be kept in an exceedingly clean condition. Otherwise, a leakage of charge is likely to occur.

17.18. Current and Voltage Regulation for D.C. Meter Testing. The regulation of the current for d.c. meter testing should be continuously variable over the whole range of the battery. Since the majority of d.c. ampère-hour meters are of no greater capacity than 20 ampères, it is advantageous to make certain benches suitable for the testing of meters of these sizes only. The circuit diagram of a convenient current control panel for such a bench is illustrated in Fig. 165. For the testing of larger meters, up to 200 ampères capacity, a suitable control panel is illustrated in Fig. 166. In both cases the control is such that a continuously variable output from zero to the maximum current rating is provided. An advantage of fitting sockets for test bench plugs is that parallel battery supplies can be given to any test bench. This is particularly advantageous when it is required to test heavy current meters. The function of the ammeter is merely

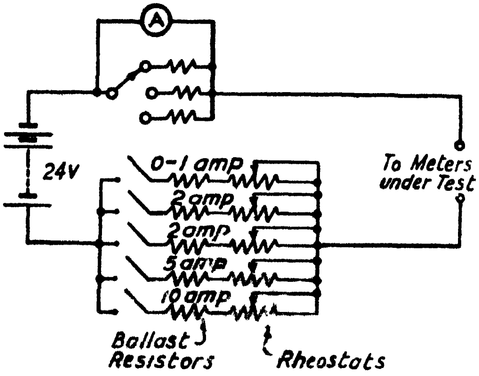


FIG. 165 -- D.C. Control Panel for Light Current Meters

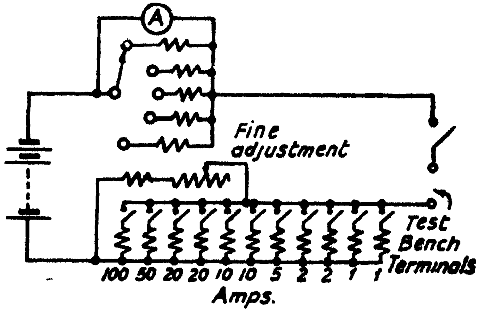


FIG 166.—D C Control Panel for Heavier Current Meters.

to give an approximate indication of the load, and a system of shunts is therefore unnecessary if the instrument is scaled with sufficient divisions.

17.19. Voltage Control Panel. A circuit diagram for a control panel for the voltage supply to the potential circuits of energy meters is given in Fig. 167. This provides continuously variable adjustments around each of the d.c. declared voltages of the undertaking, with allowance for a ±10 per cent departure from these declared values.

17.20. Supplies for the Testing of A.C. Meters. The most satisfactory source of supply for the testing of a.c. meters is a motor generator driven from a battery. Unfortunately, such a scheme is very costly on account of the necessity of a battery capable of driving the generator for long periods, at a pressure of at least 100 volts. A very fine speed control must also be provided in order that frequencies above and below the declared frequency can be obtained, and also to permit of accurate adjustment of the frequency to the required value.

The motor generator may include a synchronous motor to replace the batteries. The advantage of the synchronous motor is that it runs at constant speed irrespective of load. Unfortunately, this speed is not adjustable, being governed directly by the frequency of the supply, and so the frequency of the alternating current from the generator will be directly proportional to the frequency of the supply.

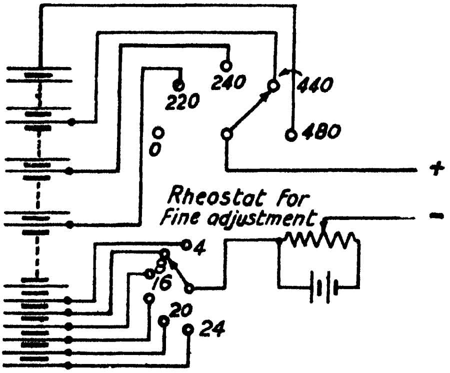


Fig. 167.—Voltage Supply Arrangements for D.C. Control Panel.

The a.c. generator, which must be capable of giving an almost sinusoidal wave throughout its entire load range (see B.S.S. No. 225) usually possesses a three-phase winding.

17.21. Motor Generator Set—Synchronous Motor Type. It is now possible to obtain a motor generator set, of the synchronous motor type, incorporating automatic voltage regulation of such an efficiency that the delivered a.c. pressure is balanced and constant to ±0·05 per cent with supply pressure variations of ±5 per cent. This regulation is obtained by means of thermionic valve amplifiers incorporated in a voltage control circuit, and they are capable of a speed of regulation of 0·3 seconds. Such a motor generator consists of a synchronous motor, a synchronous generator, and a generator for field excitation of the synchronous machines.

17.22. Double Generator Set. We have already considered the method of employing a double generator set (section 9.17) for the supply of a.c. phantom loads. The main advantage of such a method was that the phase of the applied voltage could be altered relative to that of the current, or vice versa. This method has given place to that of the phase-shifting transformer (section 2.19) on account of the ease of operation of the latter instrument and its lower cost. The double generator itself was very expensive, apart from the high cost of the gear for remote control of the relative position of the stators.

17.23. Step-down Transformers. The supply of current, from the generator to the meters under test, is best obtained through the medium of step-down transformers; thereby making it only necessary for the generator to be of sufficient current rating to supply the primary currents of these transformers.

There are two fundamental methods of regulating the current to the meters under test. One is by connecting the primary windings of the transformers directly across the generator supply, and providing suitable current limiting resistances in the secondary circuits in series with the meter current coils. Such a method is illustrated in Fig. 168 (*a*). The alternative, illustrated in Fig. 168 (*b*), is to connect the instrument current coils directly across the secondary terminals of the step-down transformer, and to provide suitable current limiting resistances in series with the primary winding of the transformer across the supply terminals of the generator.

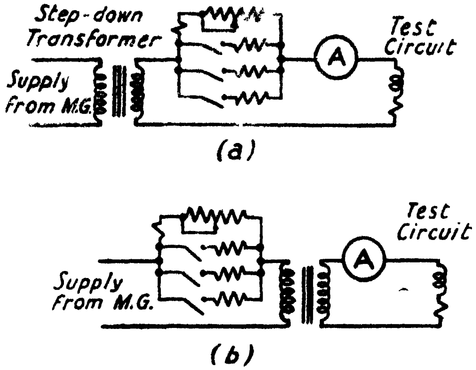


FIG. 168.—Methods for Regulation of Step-down Transformer.

In both cases the current in the test circuit is controlled by variation of the resistance, the value of which should be continuously variable between the limits.

In the latter arrangement the transformer is virtually operating as a current step-down transformer. It will be delivering its current through a low impedance load. Under these conditions the primary current will have a sinusoidal wave-form providing, of course, that the applied voltage is also sinusoidal. The wave-form of the secondary current, however, will not be quite so good as that of the primary side. This is due to the fact that the latter comprises two components, i.e. the excitation current which is distorted, and the current which is transformed. This distortion of the excitation current will cause the secondary current to be distorted, the extent of which will depend upon the magnitude of the excitation current relative to that which is transformed. Fortunately, the excitation current will be comparatively small and, as a result, the wave-form of the secondary current will only be slightly affected and will closely resemble that of the primary side.

If the above conditions are satisfied, by employing a suitable transformer, this method of current regulation is quite effective. No transformer should be used for this purpose, however, if its rated voltage is appreciably less than the supply voltage since, when the resistance in the primary circuit is small, the p.d. at the primary winding terminals is liable to be greater than the rated value. This would create magnetic saturation of the core and a relatively large excitation component of the primary current, tending to distort the wave-form of the secondary voltage.

In the case where adjustment of the meter current is made by alteration of the resistance in the secondary circuit of the transformer, the arrangement provides constant voltage step-down transformation. Under these conditions, provided the supply voltage is sinusoidal, the secondary voltage and current will have a sinusoidal wave-form over the whole current range.

17.24. Heavy-current Control Panel. An advantage of providing the constant current controlling resistance on the primary side of the transformer is that the current rating of the resistance is kept small. This is particularly advantageous for heavy-current test circuits. This can be seen by reference to Fig. 169, which illustrates a suitable circuit for providing variable current control up to 100 ampères. The fixed resistances give nominal primary current values of 2, 2, 1, and 0·5

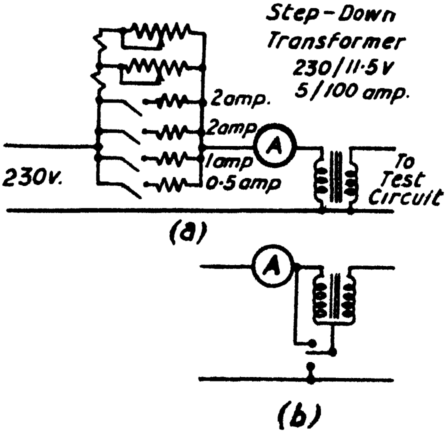


FIG. 169.—A.C. Control Panel Arrangement for Heavy Current Single-phase Meters.

ampères respectively, and these are virtually increased to twenty times this value by the transformer. Two continuously variable resistances are also provided to permit of fine adjustment to any intermediate value not catered for by the fixed resistances. For the testing of 5-ampère meters, suitable switching arrangements can be provided so that the transformer is isolated and the current taken directly from the supply. Alternatively, the primary winding of the transformer could be short-circuited, and the testing current passed through the secondary of the transformer, as illustrated in Fig. 169 (*b*). There would then be little impedance offered by the secondary winding.

17.25. Three-phase Current Supply. In sections 11.16 and 11.21 it was stated that the Electricity Commissioners stipulate that three-phase meters should normally be tested in a circuit for which they have been designed. As a result it is standard practice to employ three-phase balanced currents and voltages for the purpose of calibrating such meters. In consequence, the control circuit for the test load must be designed to provide currents of any required value within the rating of the equipment. It should also be capable of delivering currents which are balanced in magnitude and phase.

Three such circuits as that illustrated in Fig. 169 would supply the necessary current regulation for the testing of such meters, providing the lines to the controlling resistances were taken to the line terminals of the supply, and the returns commoned. For a four-wire circuit, the commoned returns would be connected to the neutral of the supply.

17.26. Control Panel for Small Capacity Three-phase Meters. For the testing of small capacity meters, such a control circuit as that illustrated in Fig. 170 would be satisfactory. The diagram and circuit require little explanation. The resistances are adjusted until the currents in the three lines are equal and of the required value. The neutral switch would be open for the testing of three-wire meters and closed for four-wire meters. When a two-element wattmeter (or two

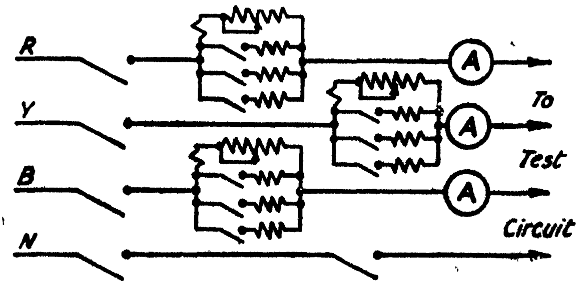


Fig. 170.—A.C. Control Panel Arrangement for Light Current Three-phase Meters.

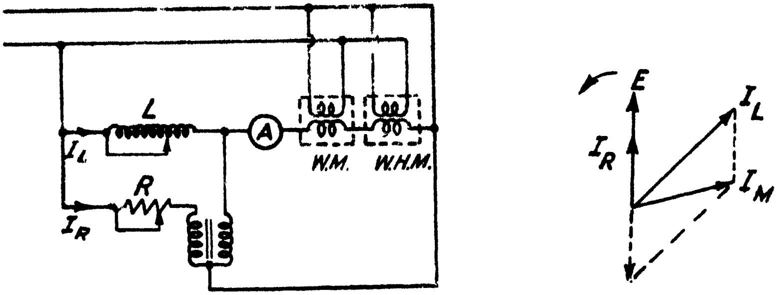
single-phase wattmeters) is employed for the testing of the meters, it is essential to ensure that there is no current flowing in the neutral wire. This condition can be attained by disconnecting the neutral wire from the supply terminal after the load has been balanced. The above remarks also apply when four-wire meters are tested against a three-wire substandard integrating meter.

17.27. Voltage Supply. The supply of voltages for testing purposes will, in general, be a more simple proposition than is the case for the supply of current, since comparatively few values of the former are required. Usually, only 110, 230, and 400 volts will be necessary, together with means for varying the voltages by ±10 per cent from these values. A variable ratio transformer is suitable for this purpose, when supplied from a constant voltage source. In the case of a three-phase supply, a double-wound transformer, delta-star connected, is suitable since it ensures that the phase voltages employed for the testing of two-element four-wire meters correspond in wave-form to the line voltages of the supply, and are also free from residual voltages. As a phase-shifting transformer is generally employed in the potential circuit, the phase displacement due to this method of connection can easily be rectified.

17.28. Testing of Large Quantities of Meters—Single-phase. When it is required to test large quantities of single-phase meters of the same rating, it is convenient to employ special test benches which are simple to operate and yet provide all the essential facilities of a more complicated test bench. Furthermore, since the Electricity Commissioners permit a tolerance of ±10 per cent in power factor for the nominal value of 0·5 lagging (i.e. a phase tolerance of 3°), no elaborate methods are necessary for the attainment of this power factor. Therefore a simple switching arrangement may be provided to give the required power factor, by variation of the connections of the circuit.

17.29. Cross-phasing. Where a three-phase supply is available, the power factor is usually obtained by what is termed "cross-phasing". In this method the series coils of the meters are fed with the current of one line, and the potential coils are connected across any phase or line voltages of the supply which give the required phase angle of the current relative to the potential coil voltage. For example, if 0·5 power factor is required, it is only necessary to supply the voltage with a 60° lead in phase, relative to the current in the meter series coils. Thus, with standard phase rotation, if the meter current coils are in a non-inductive circuit carrying the red line current, the potential coil would require to be energised by the reversed yellow phase voltage, in order to get the necessary phase displacement.

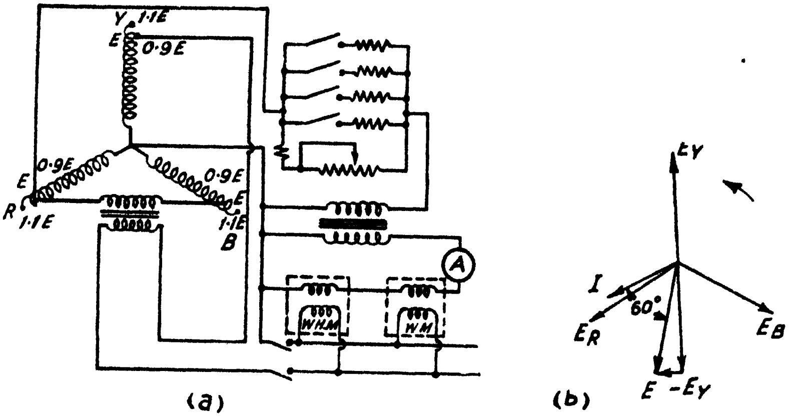
17.30. Artificial Power Factor—Single-phase Supply. Metropolitan-Vickers Ltd. have developed a method whereby an artificial power factor of 0·5 lagging can be obtained from a single-phase supply. In fact, it permits of the variation of the current through the whole range of 90°; the circuit being as given in Fig. 171. The instrument current coils carry the current I_L of the adjustable reactance, and the reversed current I_R of the non-inductive resistance R; the reversal of I_R being obtained by means of a transformer of 1 : 1 ratio. The current I_M, passing through the meters, is therefore the vector sum of $-I_R$ and I_L, and, by suitable adjustment of the value of these components, the phase of I_M can be made to lag I_R by any angle between 0° and 90°. This can be seen from the vector diagram of Fig. 171 (*b*). The obvious disadvantage of such a scheme is that variation of either current component affects both the magnitude and phase of I_M. It is, however, useful when the supply is single-phase and other methods of power factor control are not available.



(*a*) (*b*)

FIG. 171.—Artificial Power Factor from Single-phase Supply (Metropolitan-Vickers.)

17.31. Compensation for Secondary Current Lag—Cross-phasing. When a step-down transformer is employed to supply the current to meters of large rating, and the current controlling resistance is connected in series with the primary winding of the transformer, the equivalent impedance of the transformer and its load may be sufficient to cause the secondary current to lag the supply pressure by an angle of more than 3°. The phase displacement between the secondary current and its associated voltage, for an artificial power factor of 0·5, will then exceed 60° by an amount greater than the tolerance permitted by the Electricity Commissioners. Compensation can be provided for this by suitably lagging the voltage applied to the potential coil, in the manner illustrated in Fig. 172. The secondary voltage of a step-down transformer, the primary of which is across the red and blue lines, is combined with the reversed yellow to neutral voltage applied to the potential coils. This compensating voltage will be in quadrature with the yellow to neutral voltage, and as it will only require to be small it will not sensibly affect the magnitude of the resultant. This can be seen from the vector diagram of Fig. 172 (*b*). Suitable switching arrangements are made for the change-over from the normal to "cross-phase" connections, and provision is also made for the ±10 per cent variation of the applied voltage by means of tappings on the star connected winding of the supply transformer, or generator.



(a) (b)

FIG. 172.—Artificial 0·5 Power Factor—Compensations for Secondary Current Lag

The slight lag of the secondary current relative to its phase voltage will have little effect upon the errors at nominal unity power factor.

17.32. Power Factor Control. We have already considered the application of a phase-shifting transformer (section 2.19). In general, such an instrument will be employed for the control of the power factor of three-phase phantom loads. It is also convenient for the control of the power factor of single-phase loads, providing a phase-splitting device (section 2.19) is employed.

The phase-shifter will be connected in the circuit supplying the potential coils of the meters; a diagram of connections of the control circuit being as given in Fig. 173. The phase-shifting transformer supplies a nominal phase voltage of 63 volts, at the star-connected secondary, to the three star-connected auto-transformers. From the latter can be obtained 63/110 or 230/400 volts, for supply to the potential coils of the meters. When the four-pole, double-throw, switch is to the left, the former voltages are available and, in the opposite position, 230/400 volts are applied to the meters. The variation of the voltage by ±10 per cent is obtained by means of the voltage regulator included in the circuit.

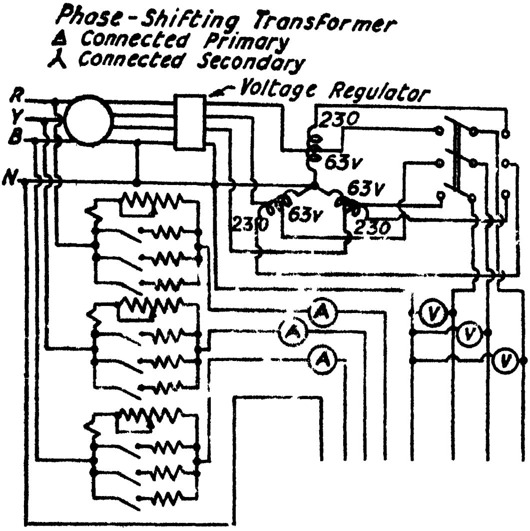


FIG. 173.—Circuit Diagram of Control Panel for A C Three-phase Meters.

17.33. Determination of the Wave-form of the Supply Voltage. When the testing engineer has designed and built his own equipment for load control, he will be particularly anxious to determine the voltage and current output wave-forms for various conditions of loading. Obviously, the most suitable instrument for this purpose is the cathode ray oscillograph, with a camera attachment. For photographic purposes, the vertical trace is made by the p.d. whose characteristics are about to be investigated, and the camera film provides the horizontal time-base. Even the most inexpensive model of modern commercial oscilloscope is quite satisfactory for this purpose, and can be relied upon to give satisfactory service at normal supply frequencies. The standard type of camera for such work takes a 35-mm. film from which, of course, enlargements can be taken.

The larger and more prosperous undertakings would, no doubt, prefer the double beam oscilloscope, which is a little more expensive but has the advantage of also providing a trace of the wave-form of the supply voltage, for the purpose of reference.

Unfortunately, it is outside the scope of this book to discuss the cathode ray oscillograph, but interested readers are advised to read one of the booklets published by the leading manufacturers of such apparatus, or *The Cathode Ray Tube and its Applications* by Parr.

The wave-form of an alternating current can be determined by passing the current through the primary of a mutual inductance (air-cored transformer) and applying the secondary e.m.f. to the Y-plates of the cathode ray tube.

The Electricity Commissioners specify that the control apparatus of a.c. test loads must not vary the peak factor of the voltage wave-form by more than 3 per cent.

17.34. The Lay-out of a Meter Test-room. The cost of meter-testing and maintenance is to a great extent dependent upon the general lay-out and facilities of the test-room. The benches should be well spaced and, for routine testing, of adequate proportions to permit of the testing of large batches of meters at one time. The test control panels should be simple to operate, and the arrangements for lighting should be adequate. The testing department should be divided into two sections (where size and numbers permit of it), one of which is for laboratory work and general investigation, and the other for normal routine testing. Under no circumstances should any repair work be undertaken in the test-room, as a separate workshop should be kept for this purpose.

A very important requirement in a meter-testing department is cleanliness. There should be adequate apparatus and labour for stringent vacuum-cleaning of the department at least once per day, and under no circumstances should the presence of dust be tolerated. There should be no nooks suitable for the congregation of dirt and any apparatus entering the test-room, either from the workshop or the manufacturer, should be carefully cleaned before being brought into the test-room.

Where both a.c. and d.c. meters are tested at the same station it is advisable to have separate sections of the room for each, in order to keep the aluminium discs, etc., of the a.c. meters free from mercury.

Excellent accommodation should be made for the storage of indicating and integrating substandard instruments, when not in use, and a card-index system should be employed so that a record might be kept of the entire history of each substandard, including a copy of each calibration curve taken throughout its life.

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