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COMMERCIAL ARITHMETIC AND ACCOUNTS

PART I

COMMERCIAL ARITHMETIC AND ACCOUNTS

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PART I

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PREFACE

The present treatise on Commercial Arithmetic and Accounts has been written to meet the needs of that great and ever-increasing army of students which is receiving a thorough commercial training in our modern schools and colleges before entering upon a business career.

It is becoming increasingly evident that purely arbitrary methods have to be abandoned, and that students must enter commercial life well equipped with a knowledge of the principles on which their calculations will depend, and endowed with the power of applying those principles. The changes and modifications in methods of calculation are so frequent and subtle that only students who have had a sound mathematical education are able to cope with the exigencies that continually arise in modern mercantile transactions. The day of the man who can do no more than "look up the tables" is past.

While endeavouring first and foremost to train the student for his future commercial work, we have yet kept in view throughout the requirements of various examinations, *e.g.* those of the Union of Institutes of Lancashire and Cheshire, the Society of Arts, etc. In each section of the book the fundamental principles are carefully explained, and copious "drill" examples are set to ensure that the student has thoroughly mastered them; following immediately on these "drill" examples are "applicational" examples, drawn for the most part from actual commercial transactions, and not evolved *ex cathedra*; the methods given being, as far as possible, in accordance with the recommendations of the Mathematical Association. No mention is made, *e.g.*, of "recurring decimals," as they are of academic interest and not of commercial importance. On the other hand, "rough checks," "rough estimates," and the degree of accuracy to which a decimal should be taken receive careful attention throughout the work, and are illustrated by a large number of concrete examples.[1]

In order to make the book commercial in spirit an attempt has been made to present, in perspective, *the commerce of the British Empire*, and on almost every page appear facts relating to the Colonies or Dependencies which have been linked up with the Mother Country, with one another, and with foreign countries by means of trade relationships. Among the latter, considerable prominence has been given to Russia, France, and Brazil.

In many cases it has seemed desirable to translate into English parts of records published in foreign languages, so as to place the most recent data at the disposal of the student, and, in addition, rather more than a thousand authorities in the various departments of commerce have given information which has proved invaluable to us.

We may quote the following as instances of the pedagogic method and spirit of the book:—

After the Metric System has been taught (Section XI.), its important bearing upon commerce is never again lost sight of. Mensuration is dealt with in Sections XII. to XIV., and we hope that some pleasure may be derived from learning it through its applications to commerce. The diagrams, many of which are original, the facsimile documents, and the coloured plates may prove useful in making difficult points clear, while the method of presentation in Section XIX. will, we hope, contribute to the simplification of the important subject of Double Entry in Accounts.

The Coloured Industrial Map will probably be found valuable

[1] Some "drill" examples and a few problems have been taken from examination papers set by the following authorities, to whom we tender our acknowledgments: The Lancashire and Cheshire Union of Institutes, the Society of Arts, the Institute of Chartered Accountants, the Chartered Institute of Secretaries, the London Chamber of Commerce, the Chartered Accountants of Scotland, and the Institute of Bankers.

if it is kept constantly open for reference as the examples are being worked.

The pleasure of paying a tribute to those who have so generously and willingly helped us is, indeed, very great.

We owe to Dr. W. P. Milne our sincerest thanks, not only for placing his wide experience at our disposal, but also for the *constructive* criticism which he has brought to bear upon the book.

Our acknowledgments are also due to the official representatives of Foreign Governments (attached specially to Commerce) accredited to this country, to the High Commissioners of the Colonies, and to some Government Departments at home, who have either lent (or given) us books which would otherwise have been inaccessible.

Mr. Gordon Groom, B.Sc., has rendered us invaluable assistance in working the examples, and we are also indebted to Mr. G. F. New, B.Sc., for help in this connection.

Our thanks are also due to the Controller of His Majesty's Stationery Office for permission to reproduce the War Loan Voucher and the Inland Money Order Form; to the Great Northern Railway Company for authorising the reproduction of their Goods Consignment Note; to the United Kingdom Temperance and General Provident Institution for similar permission in regard to the Insurance Policy; and to the Board of Trade, whose publications we have consulted.

A. R. P.
J. S.

NOTE TO THE SECOND EDITION

The reprinting of this book affords us an opportunity of correcting a few errors which had crept in, and of thanking those readers who have brought them to our notice.

We may say that the book will be found more useful if students will compare the commercial data given with the figures for subsequent years.

In this way they will see where British Trade is increasing or diminishing, and their minds will be stimulated to inquire into the reasons for these fluctuations. The comparison will *indicate*, but not demonstrate conclusively, the directions in which our Trade may be developed; and this, together with the conclusions to be drawn, should enhance the value of the questions.

Part III., which is in the course of active preparation, will carry the application of Arithmetic to Commerce to an advanced stage. It will cover the third to the fifth year's course of the Lancashire and Cheshire Union (and similar examinations) at least, and it will be copiously illustrated with plates in colour.

A. R. P.
J. S.

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PART I

"Clear and round dealing is the honour of man's nature."

"Of Truth."

FRANCIS BACON.

COMMERCIAL ARITHMETIC AND ACCOUNTS

PART I

SECTION I

ADDITION AND SUBTRACTION

1. We are so familiar with numbers that we scarcely think, much less wonder, what their origin may be.

Many centuries before the Christian era there was in use among certain tribes in India a system of vertical and horizontal strokes by means of which the natives were able to count. In process of time another system, called "Cave numerals," developed from the former, and passed into Europe, where it was modified and largely used.

2. In the twelfth century we borrowed our system of numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 0,

called "Hindu-Arabic numerals," from the Arabic scholars of Spain, who were, of course, actually using the modified numerals of the East.

The figures which we now employ, called for brevity "Arabic Numerals," appear to have been formed from the figure[1]  in the manner shown below:

3. The symbols I., II., III., IV., V., X., L. (50), C. (Latin, *centum*, 100), and M. (Latin, *mille*, 1000) are Roman numerals.

The numbers 1, 3, 5, 7, . . . are odd numbers; 2, 4, 6, 8, . . . are even numbers.

All even numbers are divisible by 2, but odd numbers are not.

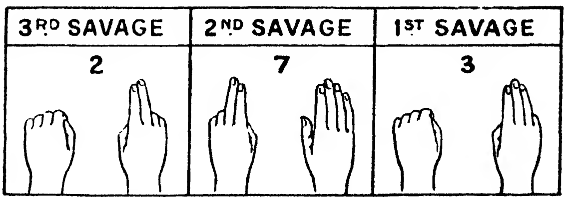
[1] See Sir Thomas Browne, *Garden of Cyrus*.

4. Our ancestors counted with the help of shells, or by making notches in pieces of wood. The latter practice was not restricted to savages, for as late as the seventeenth century, when our kings wished to borrow money, they did so by means of "Exchequer tallies," each of which consisted of a piece of stick about 5 feet long and an inch or so square. Notches cut on one side of the tally showed the amount borrowed by the king, those cut on two of the other sides the amount which he paid back. The stick was split, one half being held by the person lending the money, while the other half, or counterfoil, was retained by the Exchequer. Here we have not only the fundamental idea of accounts, but also the origin of the modern cheque system of our banks.[1] The student should observe, also, that this system, which has long fallen into desuetude, rendered forgery practically impossible, while our modern system of cheques (§ 166, Plate VI.) is not so unassailable.

5. Savages counted in the following way. A number of men stood in a row; for every object to be counted, the first man raised one finger. When he had raised ten the second man raised one, which indicated "one ten"; the first man then repeated the process, and finally the second man raised 2, 3, 4, . . . 10 fingers, indicating ultimately 10 tens or 1 hundred. The third man then put up one finger to mark one hundred.

The "first savage," then, gives us our units figure, "the second" gives us our tens figure, "the third" our hundreds figure.

Representing the number 273 in this way, we have, diagrammatically:



6. The numbers used in commercial calculations can be treated in any of the following ways:

1. Added to one another (§§ 8 and 9; 60 and 61; 83 and 84, etc.).
2. Subtracted from one another (§§ 65 and 66; 83 and 84, etc.).
3. Multiplied together (§§ 67–70; 85–88; 91–93, etc.).
4. Divided one by the other (§§ 71–73; 89 and 90; 94, etc.).

[1] See Francis W. Hirst, *The Stock Exchange.*

7. Numbers may be :

1. Whole numbers—2, 4, 567.
2. Simple fractions—$\frac{1}{2}$ (one-half), $\frac{5}{6}$ (five-sixths).
3. Mixed numbers—$2\frac{1}{4}$ (two and one-quarter), $8\frac{9}{10}$ (eight and nine-tenths).
4. Decimals—4·876 (four, decimal, eight, seven, six).

They can be represented by letters—a, b, x, which can denote a number of pounds (money or weight), of gallons, miles, degrees of temperature, or hours, and the velocity of a shot or the speed of a motor-car, etc.

A. Addition

8. The following rules may be helpful in learning to add rapidly and correctly, the importance of which cannot be over-estimated :

1. If possible, detect combinations of numbers which make ten, and then add the tens. The sign + (plus) means "add." Thus, $(8+2)+(6+4)+(9+1)$ is really three tens. So also, $5+3+2$, $3+4+3$, are easily added.

2. To simplify the addition of $7+8+6+5+9$, take the order as $7+8+5+6+9$, *i.e.* $15+5+6+9$, or $20+6+9$, which equals $26+9$.

3. To add 9 : add 10 and deduct 1.

4. To add 8 : add 10 and deduct 2.

5. In setting down any sum it is necessary to put the figures in their proper positions—tens figures in the tens column, thousands in the thousands column, and so on.

If the numbers are arranged in a line, as in "cross tots," it is then necessary to be very careful to add all the units figures, then all the tens figures, and so on.

9. Accountants frequently prefer to check their totals by beginning to add, first with the right-hand column and then with the left, finally checking one total against the other. Care should be taken to put the totals of the various columns in their proper position. Thus—

(5)	(4)	(3)	(2)	(1)
		5	8	6
	1	7	9	4
6	8	3	2	5
			5	8
		3	7	9
7	1	1	4	2

FROM RIGHT.

			3	2	= sum of (1)
		3	1		= ,, (2)
	1	8			= ,, (3)
	9				= ,, (4)
6					= ,, (5)
7	1	1	4	2	= Total.

FROM LEFT.

6					= sum of (5)
	9				= ,, (4)
	1	8			= ,, (3)
		3	1		= ,, (2)
			3	2	= ,, (1)
7	1	1	4	2	= Total.

The method is: FROM RIGHT, total column (1), put down **32** ; total column (2), and put down **31**, and since the **1** and **3** are tens figures they are put in the same column ; so also **18**, the total of column (3), is placed with the 8 under the 3. In adding from the left, column (5), put down the 6 first; then **9**, the sum of column (4); then 18, that of column (3), with the **1** under the **9**, for they are both thousands figures. Then continue as before. All this work can be done on scrap paper.

B. Subtraction

10. Subtraction consists in finding out what must be added to one number to give another. The sign – (minus) means subtract.

If 2 inches be cut off a piece of magnesium ribbon 6 inches long, 4 inches remain. That is, 4 inches must be added to 2 inches to give 6 inches.

There are several ways of stating a subtraction sum:

1. Subtract 250 from 564.
2. What must be added to 250 to give 564?
3. What is the excess of 564 over 250?
4. Find the difference between 564 and 250.
5. 564 – 250, *i.e.* 564 minus 250.

11. EXAMPLE 1.—(i) **Subtract 4 from 9.**

Employing the **Austrian method,** we ask what must be added to 4 to give 9?

The answer is **5**.

(ii) **What is the difference between 79 and 64?**

$$\text{Since } 64 + \mathbf{15} = 79,$$
$$\therefore \text{ the difference is } 15.$$

(iii) **175 – 94.**

$$\text{Since } 94 + \mathbf{81} = 175,$$
$$\therefore \mathbf{81} \text{ is the difference required.}$$

12. EXAMPLE 2.—**Subtract 68724 from 95643.**

We employ the Austrian method again, and find what must be added to 68724 to give 95643.

	Number to be put up.	Number to be carried.
95643	$4 + \mathbf{9} = 13$	
68724	$2 + 1 + \mathbf{1} = 04$	
26919	$7 + 0 + \mathbf{9} = 16$	
	$8 + 1 + \mathbf{6} = 15$	
	$6 + 1 + \mathbf{2} = 9$	

It is clear that a sum of this kind can be proved by adding the answer,—the difference,—to the smaller number.

EXAMPLE 3.—**What is the excess of 88976354 over 74987538?**

		Number to be put up.	Number to be carried.
88976354	8	+ **6** =	**1**4
74987538	3 + **1**	+ **1** =	**0**5
13988816	5 + **0**	+ **8** =	**1**3
	7 + **1**	+ **8** =	**1**6
	8 + **1**	+ **8** =	**1**7
	9 + **1**	+ **9** =	**1**9
	4 + **1**	+ **3** =	**0**8
	7 + **0**	+ **1** =	8

EXAMPLES. I.

ADDITION AND SUBTRACTION.

1. Write down all the even numbers between—(*a*) 8 and 24, (*b*) 88 and 102, (*c*) 396 and 424, (*d*) 1562 and 1598, (*e*) 85984 and 86010.

2. Write down all the odd numbers between the numbers given in Question 1.

3. Write down six consecutive numbers beginning with—(*a*) 597, (*b*) 10568, (*c*) 15999, (*d*) 999, (*e*) 10000, (*f*) 101010.

4. Write down the four consecutive odd numbers after those given in Question 3, *a*, *b*, and *c*, and then the four consecutive even numbers.

5. Write down, and find the sum of, the even numbers between 28 and 40.

6. What is the sum of the numbers of the first fifteen houses in a street if they are numbered consecutively?

7. The even numbered houses are on one side of a street, the odd numbered the opposite side. The highest number is 28.

Find the sum of the numbers on each side and the difference between the sums.

8. The number of candidates who sat at an examination in ten successive years was—3702, 4106, 4777, 6111, 6919, 7636, 8750, 8894, 8797, 9020. Find the total number of examinees.

9. By how many did the number of examinees in the last year exceed the number in each of the three previous years (see Question 8)?

10. Add the following numbers without setting them down in columns:

(1) 73600, 927, 19, 84302, 354, 482, 3690, 824.
(2) 5432, 983, 598, 1109, 768, 543, 890, 374.
(3) 2194, 2002, 31, 458, 71, 989, 5408, 927, 4814.
(4) 4827532, 771, 84, 2508, 4647, 321, 80843, 907.
(5) 2735, 548, 97, 1752, 329, 406, 58, 5213.
(6) 1076534, 52805705, 827436, 50938.
(7) 5294, 807, 1352, 77, 913, 4125, 256, 413.
(8) 517236, 38417209, 7246513, 803428.
(9) 5724, 831, 20795, 23, 9516, 857.
(10) 843176, 790214, 5340679, 90103506.

11. Find the sum of the following numbers:

(1) 217, 1508, 35, 407, 2094, 77, 134, 908.
(2) 215417, 8417296, 1874382, 707405.
(3) 2167, 593, 72631, 74, 904, 3471.
(4) 91305067, 530577, 8037564, 21004379.
(5) 6347, 21568, 13954, 2713, 179426.
(6) 83674, 217586, 3842, 98751, 63718.
(7) 5036, 284, 27136, 47, 409, 1473.
(8) 72364192, 809726, 4545736, 350008

12. Add the following numbers first in rows then in columns. Find the total of the "row totals" and also of the "column totals," and show that they agree.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	TOTALS OF ROWS.
(1)	11244139	8346231	36318	20289	138409	118053	90756	82333	
(2)	2969369	27946	170	195	4654	1510	6194	5809	
(3)	250712	2374235	1460	259	52766	5246	2918	2109	
(4)	1037415	157192	3669	100	500	51897	1485	1208	
(5)	1772	992	851	3226	3240	1680	706	2213	
(6)	105212	36507	250	75	289	8960	2119	1007	
(7)	442786	273953	248	231	1536	16175	513	23608	
(8)	39888	82160	1144	1313	9991	50464	26496	1303	
(9)	5751637	4352644	3471	3431	7545	6813	2609	169	
(10)	1050	881	616	350	49411	30004	2904	173129	
(11)	42535	30026	307	277	3680	2537	139494	196	
(12)	215100	170889	2975	6479	7139	2805	832	1011	

13. Add in rows:

(1) 46459, 93356, 67874, 324452, 39391, 66665, 163754, 7000, 195.

(2) 1500, 6019, 3000, 536144, 9917, 121353.

(3) 320818, 1267699, 73, 20651, 848586, 10560.

(4) 484582, 914276, 403129, 1398115, 35825.

(5) 500, 2013, 3000, 380139, 9917, 108334.

14. Total the following in columns:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
82085	100501	243967	87564	214006	7404638	8701117	8859992	7471899
...	6793	16800	6420	4943	24520	51900	566540	341250
33105	2206	126224	32400	53600	104360	405270	1088510	614826
1069	...	22491	23975	33979	...	683	329980	156390
91902	1208	349000	418899	586766	1238135	1103780	3439054	2489630
157	2716	283864	83396	57306	235685	315050	995342	285153
...	...	...	10500	9500	20	47345	21450	48100
413	1007	93416	118730	205160	46185	97340	1190	...
...	401	...	...	...	...	...	1950	...
...	...	54680	316028	335411	30150	6090	407160	229170
...	26608	...	...	...	150	...	37000	7410
...	1860	...	...	...	...	...	46800	30680
...	169	...	2866	4655	1750	...	728	280
...	...	...	...	...	4500	10500	40544	42872
...	...	8680	200	...	...	...	161922	89087
3996	...	47040	11880	5985	...	...	500629	340538
3	...	25760	3200	2500	...	...	245144	164749
...	...	...	600	...	...	...	83430	42186
...	...	...	...	...	...	...	8238	3604
...	...	9744	28	100	...	...	246746	122068
238725	192449	35840	296111	317295	...	122950	5252049	4203063
707	303	33601	4918	3086	...	...	507287	420486
...	...	50400	196927	407926	...	...	84000	23852
...	...	...	2755	2900	975325	1013297	...	...
...	210	...	306	...	...	...	133514	102964
...	...	...	...	...	8927	6340	...	...
510	1127	716	...	...	...	1100	...	...
22	...	...	...	...	...	...	...	...
23	...	...	...	...	...	...	...	...
...	...	...	...	100	...	...	240	260
2120	...	...	...	...	3000	2000	...	...

15. Find the sum of the numbers given in the following:

(1)	(2)	(3)	(4)	(5)	(6)
10516642	9193867	74514161	74233629	83924508	84521400
566540	397930	226061	179522	226061	205088
1095010	721166	40688	81778	40688	90178
329980	173290	598500	461279	602551	533565
3439054	3004820	386977	388550	386977	440329
998592	379273	27022	12879	27022	15899
21450	48100	14549	12444	14549	15027
1190	...	27342	21692	37342	26958
1950	...	10536703	10981864	10536703	12378245
459160	268086	28773	36916	30073	39216
37000	7410	5900	11466	5900	12116
46800	30680	70583	79918	70583	83488
728	280	305361	121371	306161	160183
57770	55998	545818	508696	636262	594027
175772	103531	1203356	990988	1320962	1170781
585479	517448	4585252	4362762	5399036	5913627
321488	221959	7076021	6466450	7642881	3125672
92404	84262	661956	592894	761034	637644
8238	3604	33098	22205	34498	29055
322670	147154	3892188	3353948	4339857	3888752
6276057	5958493	6303234	5293717	7979818	6101876
580531	465966	3740183	4073775	4470703	4679885
84000	36352	98490	89927	106490	113511
...	...	405794	347431	465821	402244
133514	124394	1613025	1509655	1614025	1795501
...	...	19416	22909	24016	32397
...	...	1044877	819045	1154630	1173138
...	...	289728	294296	352387	363091
...	...	4366333	5352853	4392511	6423994
240	260	43099	36974	47657	41612
...	...	32435	36372	36435	37728
...	...	57939	50314	67624	63564
...	...	55053	67333	59863	88908

16. The area of the various South American States is given below. Arrange them in order of magnitude, placing the largest first, and find the total area in square miles.

Name of State.	Area in Thousands of Square Miles.	Name of State.	Area in Thousands of Square Miles.
Argentine . .	1136	Ecuador . .	116
Bolivia . .	708	French Guiana .	34
Brazil . .	3219	Paraguay . .	171
British Guiana .	90	Peru . .	696
Chile . .	293	Uruguay . .	72
Colombia . .	438	Venezuela .	394
Dutch Guiana .	46		

17. Taking the areas given in the last question, find by how many thousand square miles the area of the largest of the States is greater than that of each of the next three in size.

18. The position of the Bank of England on 5th August and 2nd September 1914 is given below. Find—

(1) The total value in the Bank on the two dates named, and the difference between the two totals.

(2) The increase or decrease in the amount of each item on 2nd September.

(3) The total net increase or decrease.

(4) On which date was the Bank in the better position?[1]

	August 5.	Sept. 2.	Increase.	Decrease.
	£	£	£	£
Gold . . .	27622069	47772712		
Note Circul., Active.	36105420	32287760		
Reserve . . .	9966649	30934952		
Other Securities .	65351656	121820692		
Other Deposits . .	56749610	133818826		
Government Securities	11041152	28023971		
Total .				

19. The gross receipts for British railways from 1st January to 1st August 1913 and 1914 are given below. Find—(1) The total receipts for all the railways in the two periods and the excess of one over the other. (2) The increase or decrease for each railway in the same period. Which did best and which worst so far as these figures show?

	Gross Receipts.	Same Period, 1913.	Increase.	Decrease.
	£	£	£	£
Great Central . .	3449600	3482700		
Great Eastern . .	3382200	3346200		
Great Northern . .	3907300	3843700		
Great Western . .	8936000	8768000		
Lancashire and York .	3700350	3779072		
North-Western . .	9367000	9390000		
South-Western . .	3056400	3031400		
Brighton . . .	2019453	1955533		
Midland . . .	8167000	8246000		
North-Eastern . .	6489000	6559490		
South-Eastern . .	2994348	2896681		
Caledonian . .	3024000	2991400		
Glasgow and South-Western . .	1174200	1145000		
North British . .	3019600	2968400		
Total .				

[1] The student should, wherever possible, tabulate results.

20. The imports and exports of British goods to and from Great Britain from the year 1900 to 1914 inclusive are given below. In what year were—(*a*) the imports and (*b*) the exports greatest and smallest respectively? Is there any year in which the exports were greater than the imports? If not, can you suggest a reason?

Year.	Imports.	Exports of British Goods.
	£	£
1900 . . .	41200000	24600000
1901 . . .	38200000	22000000
1902 . . .	41300000	23800000
1903 . . .	45500000	23300000
1904 . . .	43100000	25900000
1905 . . .	45700000	29300000
1906 . . .	45100000	30500000
1907 . . .	45300000	35100000
1908 . . .	48000000	31600000
1909 . . .	49500000	32800000
1910 . . .	51600000	36900000
1911 . . .	53700000	36800000
1912 . . .	57200000	43200000
1913 . . .	61300000	42400000
1914 . . .	45100000	26700000
Total .		

Find the total value of the imports and exports for the time given, and the excess of the one over the other.

21. From the following data write down the twelve estates which produced the greatest weight of rubber (in pounds), and find the total weight produced by them and then by all the estates given.

Estate.	Production in Pounds, 1914.
Kepong	22000
Shelford . . .	12000
Mergui Crown . .	46327
Bukit Mertajam . .	30428
Bakap	23904
Ceylon (Para) . .	60240
Jong-Landor . . .	28302
Kuala Kubu . . .	7200
Hidden Str ams . .	13213
Nagolle	37516
Neboda	33290
Merlimau . . .	89516
Hayoep	30542
United Sua Betong .	11688
Linggi	126500
Kombok . . .	25510
Kerala	11150

22. From the data given below find—

(1) The area of the British Empire in square miles;

(2) The population.

Arrange the various parts—

(1) In order of their size;

(2) „ their population.

Does it appear to you that the largest area has the largest population, the second largest area the second largest population, and so on? If not, can you offer a reason for its not being so?

Parts of the British Empire.		Extent in Thousands of Square Miles.	Population.
Europe	United Kingdom of Great Britain and Ireland	121	45216665
European Stations	Gibraltar, Malta, and Cyprus	4	450000
Asia	Empire of India and its Dependencies	1900	314955240
	Ceylon and Maldives	26	4100000
	Eastern Asia	2	1000000
America	Dominion of Canada, Newfoundland	3750	7081869
	West India Islands, South America, Falkland Islands	130	1502000
Australasia	Commonwealth of Australia	3100	4449983
	Dominion of New Zealand	105	1050000
Africa	South Africa	1238	6300000
	West Africa	600	33000000
	East and Central Africa	750	8000000

23. By how many square miles is the area of the British Empire in America greater or less than that in (1) Europe, (2) Asia, (3) Africa? (See Question 22.)

How many more or less people are there in our Asiatic possessions than in (1) our European, (2) our African, and (3) our Australasian possessions?

24. The number of British residents in the United States, France, Belgium, and Switzerland is given below. Find how many more are resident in the States than in each of the other

three countries, and how many fewer there are in Switzerland than in Belgium.

Country.	Number of Resident British Subjects.
United States . . .	2791403
France	34892
Belgium	5096
Switzerland	3898

25. The population of England and Wales for 1811, 1861, and 1911 is given. Find the increase between 1811 and 1861, and between 1861 and 1911.

Which of the two increases is the greater and by how much?

1811 . . .	10164256
1861 . . .	20066224
1911 . . .	36075269

What is the increase in the century 1811–1911—that is, roughly, from the time of Nelson to our own time?

26. Taking the populations just given as being 10, 20, and 40 (for 1912), draw a line 2 inches long to represent the population in 1811, namely, 10 millions. Then draw two other lines to represent 20 and 40 millions.

27. The Government borrowed the following sums of money from the year 1801 to 1816 inclusive:

Year.	Amount of Loan in Pounds.	Year.	Amount of Loan in Pounds.
1801 . .	27305271	1809 . .	12298375
1802 . .	14638254	1810 . .	7792444
1803 . .	8752761	1811 . .	19143953
1804 . .	14570763	1812 . .	24780697
1805 . .	16649801	1813 . .	39649282
1806 . .	13035344	1814 . .	34563603
1807 . .	10432934	1815 . .	20241807
1808 . .	12095044	1816 . .	514059

In which year was the greatest loan raised (*i.e.* the greatest amount borrowed)?

What was the total amount borrowed in the last five years of the period referred to?

The population of England and Wales in 1813 was roughly 10 millions. What do you notice as regards the amount of money borrowed and the population in that year?

In what *three* successive years was the greatest loan raised?

28.

BRITISH WEST AFRICA.

Province.	Gross Revenue, 1913.
	£
Nigeria (North)	749310
,, (South)	2668198
Gold Coast	1301566
Sierra Leone	618383
Gambia	124995
Total gross revenue for British West Africa, 1913	

From the table given above, calculate the total gross revenue of British West Africa for the year 1913.

29. The total public revenue of the West India Islands in British occupation was £2935666, and the total public expenditure £2921981 for the year 1913. By what amount did the revenue exceed the expenditure?

30. The population of the British Empire was approximately 343963000 in 1901 and 371897000 in 1911. What was the increase in population in the decade?

Why are the numbers not given more exactly than to the nearest thousand?

31. There were 41594 marriages in the Australian Commonwealth in 1913 and 42147 in 1912. How many fewer marriages were there in 1913 than in 1912?

32. The gross amount of the public revenue of New Zealand was £12313610 in 1913 and £11817581 in 1912. What was the increase in revenue in 1913 over that for 1912?

33. The total shipping entered and cleared from the ports of Canada in 1913–14 was valued at £29568486, and that from Australia at £10601948. By what amount is the latter value below the former?

34. £11483663 was the value of the shipping entered at Victoria (Hong-Kong) in 1913–14, of which £11478244 was in steam vessels. Calculate the value of the sailing vessels entered (these figures do not include Chinese junks).

35. The value of the total exports from the West India

Islands is given in the following table, except for Jamaica, which has to be calculated from the figures supplied:

Total Exports, West India Islands.	
Islands.	**Value of Exports.**
	£
Bahamas	263954
Turk's and Caicos Islands .	27808
Jamaica	
Cayman Islands . . .	10000
St. Lucia	133421
St. Vincent	115201
Barbadoes	760699
Grenada	367149
Leeward Islands . . .	563963
Trinidad and Tobago . .	5205673
Total, West India Islands .	9878075

What is the value of the exports from Trinidad and Tobago to the nearest £1000?

36. From the following table calculate the value of the exports from Canada to Newfoundland in the year 1914, and write the total correct to the nearest £1000000.

Canadian Exports to British Possessions.	
Possession.	**Value of Exports.**
East Indies—	£
British India . . .	85815
Straits Settlements . .	47597
Other East Indies . . .	8170
Hong-Kong	386913
Commonwealth of Australia .	967308
Dominion of New Zealand .	397930
West Africa	8019
South Africa	788222
East Africa	11743
Newfoundland	
British West Indies . . .	922918
Bermuda	83272
British Guiana . . .	134174
Other British Possessions . .	57140
Total .	4879762

37. Complete the following table (taken from *Whitaker's Almanack*):

TABLE A.

THE REVENUE AS CONTRIBUTED (THOUSANDS OF POUNDS).

Heading.	The Revenue as Contributed, 1912–1913.				
	England and Wales.	Scotland.	Ireland.	Other Sources.	Total.
Customs	26909	3442		...	
Excise	29491	5311	3258	...	
Estate, etc., Duties . .		3170	960	308	
Stamps	9000	672	373	53	
Land Tax	655	32	nil	...	
House Duty	1826	130	nil	...	
Income Tax	38509	4261	1463	467	
Land Value Duties . .	389	43	5	...	
Total Tax Revenues .	127722		9255		

38. Fill in, and draw some conclusions from, the following table:

MAURITIUS: PRINCIPAL EXPORTS.

Article.	Value in £'s.		
	1912.	1913	Inc. (+) Dec. (–)
Aloe fibre		56905	
Coal	nil	2	
Grain, rice	6824	5456	
Hardware and cutlery . .	4509	1593	
Molasses	11759	4087	
Cocoa-nut oil	2094	4232	
Spirits, rum	1959	1808	
Sugar, raw	2350479		
Vanilla	1420	2355	
Specie, silver	19461	84967	
Total .	2443970	2210126	

NOTE.

The student is warned against drawing far-reaching conclusions from figures such as have been given above. The following observations, *in which we assume a knowledge of the imports*, will perhaps be helpful:

Coal export valued at £2; no export of iron at all; ∴ any produced is used in the island.

Now population is 375000; ∵ iron and coal will probably be required; and we find £178330 of coal and £30754 of iron were imported in 1913.

Again, the value of exported silver rose enormously in 1912-1913, but the imported value also rose from £14000 to £22000 in the same time.

Clearly, too, sugar is the chief export, and we find NONE imported; while £5456 was the value of the rice exported and £568000 the value imported. On looking into this still further, we find that the population consists of a very large proportion (⅜) of Indian coolies (for the military garrison is not included in the population given), and this fact accounts for the importation of rice, and also, in part, for the fact that only £5000 of salted beef was imported. Cotton worth £100000 was imported, and the same value of machinery, as against £63000 for the latter in 1912, while large amounts of ammonium sulphate, nitrates of soda and potash, stationery, and timber were imported. The railway (Government) receipts have increased by £8000; the postal telegraph service has been largely augmented and is being worked profitably; the amount standing to the credit of depositors in Government Savings Banks has increased by £3000, in the last year to £20000, and the number of depositors by 500.

These facts suggest:

(1) A desire for European clothing and a consequent civilising influence at work.

(2) A manufacturing spirit (machinery) and a modernising spirit.

(3) The cultivation of the soil and an attempt to enhance its productive value (chemicals).

(4) Greater trade (railway revenue).

We have chosen this island intentionally so that the student may apprehend at the outset that this little outpost, this speck amid the waves of the Indian Ocean, is a part, and an important part, of a stupendous whole, and that it has a place to fill in the commerce of the world.

39. The following is the gold production, in ounces, for the chief British possessions and colonies for 1912 and 1913. Calculate the increase or decrease in production for each possession, and the total weight produced in Australia in each of the two years.

Gold Production (Ounces).

Possession.	1912.	1913.	Inc. (+) Dec. (−)
India	590555	595761	
New South Wales	165295	149657	
Victoria	480131	434933	
South Australia	6592	6545	
Northern Territory	5337	3119	
Western Australia	1282659	1314044	
Tasmania	37973	33400	
Queensland	347946	265735	
Total for Commonwealth			
Transvaal	9107512	8798336	
Canada	611885	784525	
Southern Rhodesia	642807	689954	
New Zealand	316671	343595	
Gold Coast	352118	388126	

C. Approximation

ADDITION AND SUBTRACTION[1]

13. It is stated that the population of British North America is 389**2399**, and so the returns presume that the population is known accurately to *one* individual. It is clear that, even if such a degree of accuracy were possible, it would have very little commercial value. This leads us to the fact that the population should be approximated to (say) the nearest thousand, which is sufficiently close for our purpose.

Hence we say that the population is 3892000, observing that **2399** is nearer to 2000 than to 3000, and consequently 389**2000** is the population **correct to 1000.**

14. Again, let us consider the value of the goods exported last year from Canada to South Africa, namely, £788222, where the value is regarded as correct to £1, while it would be sufficiently accurate if given to £1000.

The value is then £788000 correct to £1000, but £7**9**0000 correct to £10000, for 788222 is nearer to 790000 than it is to 780000.

Had the figures been £78**4**222, the value correct to £10000 would have been £78**0**000.

If £785000 had been given, it is necessary to note that the figure is exactly half-way between 780 thousand and 790 thousand, and we reckon it to the thousand **ABOVE**, namely, £790000 correct to £10000.

Again, (i) 749 correct to 10 is 750, while (ii) 749 correct to 100 is 700, and it would be wrong in writing (ii) to say 749 correct to 10 is 750, and then 750 correct to 100 is 800.

Hence, in correcting any number we must correct the 100's or 1000's figure **in the original number**, and not attempt to correct the latter (ii) from the result of the former (i).

15. It is very important to recognise that no figure can be corrected unless the one to the right of it is known.

If, for example, the population of Oxford be given as 436854, we presume that the units figure is correct and can then write it (i) as 436850 correct to 10, (ii) as 436900 to 100; (iii) 437000 to 1000; (iv) 440000 to 10000; and (v) 400000 to 100000.

(ii) is correct to four significant figures;
(iii) „ „ three „ „ ; while
(v) „ „ one „ figure.

[1] Cf. also §§ 91 to 94.

If 0's are **not** followed by a figure they are **NOT** significant figures; *e.g.*, 536000 is correct to three, not to six significant figures; 530600 to four, not six; and 530006 to six significant figures.

16. We might illustrate the application of these remarks as follows:

Example 1.—**Express the area of British India, namely, 1802112 square miles, correct to two significant figures.**

Clearly the third figure is an 0.
∴ the number required is 1800000,

and the error made is roughly 2000 square miles in 1800000, or 1 square mile in 900, which is a very small error.

Example 2.—**Write down the population of Hull correct to 1000, 10000, 100000, six significant figures, and to one million, if it be given in Whitaker as 1301618.**

Correct to 1000 . . .	it is 1302000
" " 10000 . . .	" 1300000
" " 100000 . .	" 1300000
" " six significant figures	" 1301620
" " one million . .	" 1000000

17. The *practical value* of approximations in addition and substraction is that a good deal of work is often saved by their help.

Example 3.—**The rateable value of five towns is as follows: £1565323, £956348, £756838, £856743, £1356530. Calculate their total rateable value to three significant figures.**

If we write down each value correct, not to three but to *four* figures, and thus allow for the carrying figure to the third significant figure, we shall get the correct result.

Correct to four figures, leaving out three 0's.	In full.
£156|5	£1565323
95|6	956348
75|7	756838
85|7	856743
135|7	1356530
£549|2	£5491782
Or £5490000 correct to three significant figures.	Or £5490000 correct to three significant figures as before, but with much more work.

18. Subtraction can be performed in a similar way:

Example 4.—**The capital of two Banks is respectively £2536846 and £3630300. By how much is the capital of the former less than that of the latter to £1000?**

Writing the amounts correct to the hundreds figure and subtracting we have:

£3630	300
2536	800
£1093	500

Or £1094000 to £1000. (Note the necessity of retaining the hundreds figure here to correct the thousands.)

EXAMPLES. II.

1. Write down the following correct to 10:

363, 574, 865, 530, 1056, 3125.

2. What is the value of the following correct to 1000?

5200, 3564, 8215, 4100, 3425, 2553, 3829.

3. Write down the numbers given:

(*a*) correct to 10000;
(*b*) „ „ 100000;
(*c*) „ „ 1000000:

6587623, 12784831, 295736500.

4. What are the following correct to two significant figures?

426, 735, 864, 931, 862, 784, 1065, 2794.

5. Write down the following correct to three significant figures:

8629, 7513, 5034, 6001, 7820, 15604, 1552, 30001.

6. What are the following

(*a*) correct to five significant figures?
(*b*) „ „ six „ „
(*c*) „ „ three „ „

8695432, 86274315, 80000001.

7. The number of Britons in Chile is estimated at 3639, in China at 3287, in Egypt at 2766, and in Portugal at 2278. Find the total number of Britons in these four countries to the nearest hundred.

8. The capital of the Bank of England is said to be £14553000, of the Bank of Montreal £3287671, of the Capital

and Counties Bank £8750000, of Lloyd's Bank £26304200 Calculate the total capital of these Banks to two significant figures.

9. From the data given in Question 35, p. 13, write down the sum of the value of the exports from the last four places named to four significant figures.

10. From Question 37, p. 15, find the total value of the revenue contributed by Scotland to the nearest million £.

11. Use the data given in Question 38, p. 15, to find the value of raw sugar exported from Mauritius in 1913 to four significant figures.

12. The imports into the United Kingdom from Argentina were valued at £40807685 last year, and from Belgium at £23615740. By how much did the former exceed the latter to the nearest £1000?

13. Find to the nearest hundred thousand tons the amount of the coasting trade of West Africa from the following data:

State.	Coasting Trade in Tons.
Southern Nigeria . . .	1738459
Gold Coast	2986553
Sierra Leone	2931085
Gambia	625132

14. By how much did the trade of the Gold Coast exceed that of Gambia to the nearest 1000 tons? (See Question 13.)

15. India exported 79373 and 56624 thousand lb. of cotton to the United Kingdom in two successive years. Find the excess of the former over the latter to the nearest 1000 lb.

16. The net traffic receipts for the railways of the United Kingdom in the past four years have been: £45136464, £47355889, £48581746, £47329074. Calculate the total returns in the years given to four significant figures.

17. Find from the last question to the nearest £100 the amount by which the highest figure exceeds the lowest.

SECTION II

MULTIPLICATION AND DIVISION

A. Multiplication

19. Multiplication affords a means of adding very rapidly any set of numbers composed of the same digits. Thus 86, 86, 86, 86, 86, 86, 86, 86, 86 is a set of numbers, 9 in all, composed of the same digits, 8 and 6. The sum can be found—

1. By adding them up;
2. „ multiplying 86 by 9.

The sign $\times$ means multiply,

$\therefore$ 86 multiplied by 9 is written 86×9.

20. If a merchant purchases 48 boxes of Tasmanian apples, each of which contains 146 apples, the total number can be found quickly by multiplying the two numbers 146 and 48 together, *i.e.* by finding the PRODUCT of 146 and 48.

21. Having written down these numbers, it is possible to multiply first, either by the 8 or by the 4; the simple rule to observe is the great and fundamental one of "place value."

If you are multiplying by a *units* figure place the first figure of the product in the *units* place, if by the *tens* figure, in the *tens* place, and so on.

22. Before proceeding with the actual multiplication it is always best to make a rough check or approximation.

Thus 146 times 48 is rather less than
146 „ 50, or 7300;
$\therefore$ our rough, or approximate, value would be 7300

The symbol $\fallingdotseq$ will be used to denote "approximately equal to," and we shall ask the student to employ it **whenever possible.**

23. Since in 146×48 the 4 is a tens figure, and therefore the more important, we shall commence multiplying by the 4 rather than by the 8. It is convenient to arrange the work as follows:

$$\begin{array}{r} 146 \\ 48 \\ \hline 584 \\ 1168 \\ \hline 7008 \\ \hline \end{array}$$

(The dark type figures are put up and the small ones are carried.)

$6 \times 4 = \mathbf{24}$ (The 4 is a tens figure, since the multiplying figure is a tens figure.)

$4 \times 4 + 2 = \mathbf{18}$

$1 \times 4 + 1 = \mathbf{5}$

and $6 \times 8 = \mathbf{48}$ (A units figure.)

$4 \times 8 + 4 = \mathbf{36}$

$1 \times 8 + 3 = \mathbf{11}$

On adding the two lines 584 and 1168 together, the result, 7008, is obtained.

Example 1.—**An egg merchant has 854 customers, to each of whom he sells 61 dozen eggs. What is the total number of eggs sold ?**

Each of the 854 customers purchases 61 dozen, or 61×12, *i.e.* 732 eggs. So the total number sold is 854×732. Multiplying as before and omitting the explanation we have—

$$\begin{aligned} & 854 \times 732 \\ \stackrel{\frown}{=}\ & 854 \times 700 \\ =\ & 597800\,; \\ \text{say}\ & 600000. \end{aligned}$$

$$\begin{array}{r} 854 \\ 732 \\ \hline 5978 \\ 2562 \\ 1708 \\ \hline 625128 \\ \hline \end{array}$$

Rapid Methods of Multiplying.[1]

Example 2.—**In a case such as 9567×842—**

It will be well to note that the 4 of the multiplier is twice the *2*, and the **8** twice the 4. The sum then reduces itself to this:

Multiply 9567 by $2 = 19134$ . . . . . . (1)

,, 19134 ,, $2 = 38268$ . . . . . . (2)

and ,, 38268 ,, $2 = 76536$ . . . . . . (3)

Then (1) is twice 9567 ;
(2) ,, four times 9567 ;
and (3) ,, eight ,, 9567.

Proceeding with the sum in the ordinary way, and remembering that the first figure of (2) is a *tens* figure, since the multiplying figure is the 4 and the first of (3), a hundreds figure, we have—

$$9567 \times 842 \stackrel{\frown}{=} 9567 \times 800 = 7653600.$$

and

$$\begin{array}{rl} 9567 & \\ 842 & \\ \hline 19134 & = \text{Twice } 9567. \\ 38268 & = \text{,, (ten) times } 19134 \text{ or } 40 \text{ times } 9567. \\ 76536 & = \text{,, ,, } 38268 \text{ ,, } 800 \text{ ,, ,,} \\ \hline 8055414 & \\ \hline \end{array}$$

[1] See also § 46.

Example 3.—**The advantage of the method of Example 2 is emphasised by the following:**

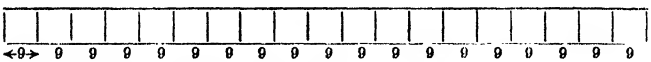
$$8568964 \times 3672\mathit{3}6$$

Here we multiply first by the *3* (tens), then take twice the product for the 6 (units), six times that result for the 36 (to the extreme left), and then twice the last result for the 72.

$$8568964 \times 367236$$
$$\simeq 8600000 \times 400000$$
$= 344$ and ten noughts, *i.e.* 13 figures in the answer.

8568964	
367236	
25706892	= 8568964 × 3 (tens) (line 1).
51413784	= Twice (line 1) or 8568964 × 6 (line 2).
308482704	= Six times (,, 2) ,, ,, × 36 (ten thousand).
616965408	= Twice (,, 3) ,, ,, × 72 (hundred).
3146832063504	

Example 4.—**A two-pound bag of Demerara sugar is 9 inches long. Can a grocer put 19 of them end to end on a shelf 170 inches long?**



We have 9 inches 19 times;
19 times 9 is 171;
The 19 bags occupy 171 inches;
∴ they cannot be put on a shelf 170 inches long.

24. The following examples may illustrate **the use** of multiplication:

Example 5.—**A box which with packing material weighs 56 lb., contains 126 nineteen-pound packages of tea. The whole is weighed and said to be 2449 lb. Is this true? If not, by how much is it wrong?**

1 package contains 19 lb. of tea;
126 packages contain 19 × 126, or 2394 lb.;
The Tare (*i.e.* the weight of the box and packing material) is 56 lb. (a quarter of a hundredweight);
∴ the total weight is 2394 + 56, or 2450 lb.;
And it was said to be 2449 lb.

EXAMPLE 6.—**In course of transit the tea referred to in Example 5 was injured by water, and the *consignee* (the one *to whom* it was sent) returned all but seven packages in a box which weighed 36 lb. What total weight was sent back? How much less was this than the weight sent?**

126 packages were sent: 7 were kept;
∴ 119 „ „ returned;
Each weighed 19 lb.;
So $119 \times 19 =$ the weight of tea returned;
$= 2261$ lb.;
The box weights 36 lb.;
∴ total weight returned $= 2261 + 36$ lb.;
$= 2297$ lb.;
Now 2450 lb. were sent;
And 2297 „ „ returned;
∴ $2450 - 2297$, or 153 lb., is the difference between the weight (wt.) sent and the weight returned.

If the merchant were sending the goods carriage paid he would pay for 1 lb. less than he should do, supposing the charges were made very exactly. In a general way, a carrier or a railway company would call 2449 lb. "2450," and charge accordingly.

25. The accuracy of a multiplication sum may be tested by the famous old method known as **casting out nines** after having multiplied in the ordinary way. Thus:

$$8567 \times 525 = 4497675.$$

The method of casting out nines is as follows:

1. Sum of digits in multiplicand $= 26$;
and $26 \div 9$ gives remainder **8**
2. „ „ multiplier $= 12$;
and $12 \div 9$ gives remainder **3**
3. Product of remainders $= 3 \times 8 = 24$;
and $24 \div 9$ gives remainder **6**
4. Sum of digits in answer $= 42$;
and $42 \div 9$ gives remainder **6**

If the remainders from 3 and 4 are NOT equal the product is wrong; if they are equal it is probably (but not certainly) right.

B. Division

I. FACTORS

26. The number 12 is equal to

(1) 12×1, (2) 4×3, (3) $2 \times 2 \times 3$, or (4) 6×2.

So $40 = 5 \times 8 = 5 \times 4 \times 2 = 5 \times 2 \times 2 \times 2$.

$7008 = 146 \times 48,$[1] but $146 = 2 \times 73$;

$\therefore\ 7008 = 2 \times 73 \times 12 \times 4$ . . . . (1)

$= 2 \times 73 \times 3 \times 4 \times 4$ . . . . . . (2)

$= 2 \times 73 \times 3 \times 2 \times 2 \times 2 \times 2$ . . . . . (3)

or $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 73$ . . . . . (4)

These facts are expressed by saying that the factors of 12 are 12 and 1; 4 and 3; 2, 2, and 3; or 6 and 2, while 7008 has one of the sets of numbers (1), (2), (3), (4) for its factors.

27. Those numbers which, when multiplied together, give another number are called the **factors** of that number.

12 and 1; 4 and 3; 2, 2, and 3; or 6 and 2 are **factors** of 12.

28. A number which cannot be broken up into factors is called a **prime number**,[2] *e.g.*, 3, 17, 29, 31.

29. If the *factors* of a number *cannot* themselves be broken up into other factors they are called **prime factors**.

Thus, $12 = 4 \times 3$;

and 3 is a prime factor of 12;

but 4 is NOT, for it can be "resolved" into 2×2.

Again, $7008 = 146 \times 48$;

Neither 146 nor 48 is a prime factor, for $146 = 2 \times 73$, and $48 = 12 \times 4$.

Of these factors 2 and 73 are **prime**, 12 and 4 are NOT.

30. It is usual to write the factors either in ascending order (that is, beginning with the smallest) or in descending order (beginning with the largest).

$7008 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 73$ (ascending order);

or $= 73 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2$ (descending order).

31. It is very convenient to *agree* upon a means of representing a number of two's (say) multiplied together.

2×2 is represented by 2^2 (two squared).

$2 \times 2 \times 2$ " " 2^3 (two cubed).

$2 \times 2 \times 2 \times 2 \times 2$ " " 2^5 (two the **power** five, or two to the fifth).

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ is represented by 10^7 (ten to the **power** seven or ten to the seventh).

The small number (^{2}, ^{3}, ^{5}, or ^{7}) placed above is called an

[1] § 23.

[2] It should be observed that *every* number is equal to itself multiplied by unity (one), *e.g.* $3 = 3 \times 1$, but 3 and 1 are not spoken of as the factors of 3.

index. Applying this notation we write: $7008 = 2^5 \times 3 \times 73$, which is a very neat way of putting down the result.

$$\begin{aligned} 144 &= 12 \times 12 = 4 \times 3 \times 4 \times 3; \\ &= 2 \times 2 \times 3 \times 2 \times 2 \times 3; \\ &= 2 \times 2 \times 2 \times 2 \times 3 \times 3; \\ &= 2^4 \times 3^2. \end{aligned}$$

$$\begin{aligned} 1000 &= 10 \times 100 = 10 \times 10 \times 10 = 10^3; \\ \text{or } 1000 &= 2 \times 5 \times 2 \times 5 \times 2 \times 5; \\ &= 2 \times 2 \times 2 \times 5 \times 5 \times 5; \\ &= 2^3 \times 5^3. \end{aligned}$$

32. The following tests of exact divisibility should be remembered:

A number is divisible—

1. By 2 if the last digit to the right is even.
2. " 4 " two digits are divisible by 4
3. " 8 " three " " " 4
4. " 3 " sum of the digits is " " 3
5. " 9 " " " " " 9
6. " 11 " sum of the even digits equals the sum of the odd ones, or if the difference between the sum of the odd digits and the sum of the even ones is divisible by 11
7. " 5 " last digit is . . . . 5 or 0
8. " 6 if rules 1 and 4 both apply.
9. " 10 if the units digit is an 0

II. SQUARE ROOT

33. The student has learnt that

$$3^2 = 3 \times 3 = 9.$$
$$12^3 = 12 \times 12 \times 12 = 1728.$$
$$15^2 = 15 \times 15 = 225.$$

So the answer to the question, "What is the square of 56"? is $56 \times 56 = 3136$, while 28 cubed $= 28 \times 28 \times 28 = 21952$.

34. If we consider such numbers as 9 and 25, we see that their factors are respectively 3×3 or 3^2, and 5×5 or 5^2. We may say then that the **Square Root** of 9 is 3, for $3^2 = 9$, and that the square root of 25 is 5, for $5^2 = 25$.

The **Square Root** of a number is that number which, when multiplied **by itself**, gives the original number.

35. We shall find the square root of a few numbers by resolving them into factors here, and refer the student to Section XIII. B., where he will learn how to find the square root of any number, and also why it is necessary to know the rule at all.

EXAMPLE 1.—**Find the square root of 625.**

Factorising, we have

$$625 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 25^2;$$

$$\therefore \text{ the square root of } 625 = 25.$$

This is always written

$$\sqrt{625} = 25.$$

(Check this and subsequent results by squaring, thus: $25 \times 25 = 625$.)

EXAMPLE 2.—**What is the square root of 1936?**

$$\begin{aligned} 1936 &= 11 \times 176 = 11 \times 11 \times 16; \\ &= 11 \times 11 \times 4 \times 4 = 11^2 \times 4^2; \\ \therefore \sqrt{1936} &= 11 \times 4 = 44. \end{aligned}$$

EXAMPLE 3.—**What is the square root of 9604?**

$$\begin{aligned} 9604 &= 7 \times 1372 = 7 \times 7 \times 196; \\ &= 7 \times 7 \times 4 \times 49 = 7 \times 7 \times 4 \times 7 \times 7; \\ &= 7^2 \times 2^2 \times 7^2; \\ \therefore \sqrt{9604} &= 7 \times 2 \times 7 = 98. \end{aligned}$$

36. EXAMPLE 4.—**Is 13824 a perfect cube?**

That is to say, is there a number which when multiplied by itself *3 times* gives 13824?

$$\begin{aligned} 13824 &= 4 \times 3 \times 2 \times 4 \times 3 \times 2 \times 4 \times 3 \times 2; \\ &= 4^3 \times 3^3 \times 2^3; \\ \therefore \sqrt[3]{13824} &= 4 \times 3 \times 2 = 24. \end{aligned}$$

Prove that this is so by cubing 24.

EXAMPLE 5.—**What number cubed will give 32768?**

$$\begin{aligned} 32768 &= 4 \times 4 \times 4 \times 8 \times 8 \times 8; \\ &= 4^3 \times 8^3; \\ &= 32^3; \\ \therefore \sqrt[3]{32768} &= 32. \end{aligned}$$

III. DIVISION

37. EXAMPLE 1.—**Draw a line 6 inches long, divide it into three equal parts, and find the length of each part.**

Example 2.—**An office desk is 4 feet long, how many stools each 1 foot wide can be set along it?**

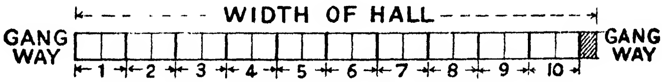
Example 3.—**A builder has a piece of land 200 feet long and wishes to build 10 houses on it. What is the frontage of each house?**

In these three cases we perform a division sum. The builder, in Example 3, must find how many times 10 feet is contained in 200. He then knows that each house can have a frontage of 20 feet.

Example 4.—**A hall is 21 feet wide excluding gangways; sofa-stalls, each 2 feet wide, have to be put in. How many will there be in each row, and what space will there be to spare?**

Draw two lines 21 inches long (see figure below) on a sheet of paper, then 1 inch on the paper will represent 1 foot in the hall.

Mark off the line into 2-inch lengths. Every 2 inches represents one seat.



There are in all ten seats and a space left over, which is 1 foot wide. So that 21 feet contains 2 feet ten times, and leaves 1 foot over, or

$$21 \div 2 = 10 \text{ and } 1 \text{ over.}$$

Example 5.—**If 100 sacks of flour are divided among 3 people each one will have 33 sacks and 1 will be left over.**

38. The actual operation of division can be performed in two ways:

(i)	(ii)
By Short Division	**By Long Division.**
(*a*) **By one number.**	
(*b*) **By factors.**	

(i) **Short Division.**

39. (*a*) By One Number

Example 6.—**Divide 80 by 6.**

$$6\,\underline{|80}$$
$$\underline{13} + 2 \text{ (over)}$$

Method.

6 into 8 = **1** and 2 over.

Put up the 1 and bring down the 0 by the side of the 2.

6 into 20 = **3** and 2 over.

Put up the **3** and indicate the 2 over, as shown.

40. In the last sum

80 is called the Dividend (denote it by V),
6 ,, the Divisor (,, D),
13 ,, the Quotient (Q), and
2 ,, the Remainder (R).

If now Q be multipled by D and R added, we have—

$$\begin{aligned}(Q \times D) + R &= (13 \times 6) + 2 \\ &= 78 + 2 \\ &= 80;\end{aligned}$$

and 80 is the Dividend (V).

Hence we have a most valuable rule both for proving a division sum and for other purposes too, namely:

$$\begin{aligned}\text{Dividend} &= \text{Quotient} \times \text{Divisor} + \text{Remainder}; \\ \text{or } \mathbf{V} &= (\mathbf{Q} \times \mathbf{D}) + \mathbf{R}.\end{aligned}$$

After the section on long division has been mastered, the following will be obvious:

$$\begin{array}{l} D)\ V\ (Q \\ \quad \underline{Q \times D} \\ \quad\ \ R \end{array}$$

and so $V = (Q \times D) + R$, just as before.

EXAMPLE 7.—**A merchant has 878 lb. of rolled oats. He orders them to be put up in 7-lb. bags, and his salesman tells him that 1 lb. remains. Is that so?**

Every bag contains 7 lb.

$\therefore$ $878 \div 7$ is the number of 7-lb. bags which can be filled.

Now $878 \div 7 \doteqdot 900 \div 7$, which is rather more than 100.

$$\begin{array}{l} 7\underline{|878} \\ \quad \underline{\underline{125}} + 3 \text{ lb. over.} \end{array}$$

125 bags can be filled, and 3 lb. will remain over.

The salesman may therefore have given slightly more than 7 lb. in each case, for although he should have had 3 lb. left he has only 1 lb.

What is far more likely is, that he would have **more** left than he ought, for the oats having been weighed in the bags would not give 7 lb. of **oats** in each, but 7 lb. would be the weight of oats **and** bag.

In this sum, too,
$$\begin{aligned}& Q \times D + R; \\ &= 125 \times 7 + 3; \\ &= 875 + 3; \\ &= 878, \text{ which is equal to } V.\end{aligned}$$

EXAMPLE 8.—**A philanthropist gives 87645 gallons of soup to be distributed equally among the poor in 12 of the London Boroughs. How many gallons will there be per borough?**

$87645 \div 12 \simeq 90000 \div 12$, which is rather more than 7000.

$$\begin{array}{r|l} 12 & 87645 \\ \hline & 07303 + 9 \text{ gallons over.} \end{array}$$

Each will have 7303 gallons and there will be 9 gallons to spare.

The METHOD is

12 into 8 = **0** and 8 over; bring down the 7;
12 ,, 87 = **7** + 3 over; bring down the 6;
12 ,, 36 = **3** + 0 over; bring down the 4;
12 ,, 04 = **0** + 4 ,, ; ,, ,, ,, 5;
12 ,, 45 = **3** + 9 over.

Here also

$$\begin{aligned} & Q \times D + R; \\ &= 7303 \times 12 + 9; \\ &= 87636 + 9; \\ &= 87645. \end{aligned}$$

(i) Short Division

41. (*b*) BY FACTORS

EXAMPLE 9.—**Forty-eight chests have to be filled with equal quantities of tea, of which there are 7455 lb. What weight will each chest contain?**

It is necessary to divide 7455 by 48.
The factors of 48 are 6×8 (or 12×4).

Taking 6×8; if 7455 be divided by 6 and then the quotient obtained, divided by 8, it is just the same as if 7455 were divided by 48.

$7455 \div 48 \simeq 7500 \div 50 = 150.$

$$48\begin{cases} 6 \,|\, 7455 \\ 8 \,|\, \overline{1242} + 3 \text{ (over)} \\ \quad\;\; \overline{155} + 2 \text{ (,,)} \end{cases}$$

There are two remainders, one from each division.
The rule for finding the true remainder is:
Multiply the **second remainder** by the **first divisor** and add the **first remainder** to the product.

The second remainder is 2; the first divisor, 6;

$2 \times 6 = 12$. The first remainder is 3;

$12 + 3 = 15$, which is the **true** remainder.[1]

42. EXAMPLE 10.—**Had the numbers been (i) 7453 ÷ 36 or (ii) 9876 ÷ 72 the results would have been:**

(i) $7453 \div 36 \doteqdot 7500 \div 40 = 190$ roughly.

$$36\left\{\begin{array}{r|l} 6 & 7453 \\ \hline 6 & 1242 + 1 \\ \hline & \underline{207} + 0 \end{array}\right. \left.\begin{array}{l} \\ \\ \end{array}\right\} 1 \text{ (over)}$$

2nd R. × 1st D. + 1st R.

$0 \times 6 + 1$

$= 1$

for $0 \times 6 = 0$, not 6.

(ii) $9876 \div 72 \doteqdot 9800 \div 70 = 140$.

$$72\left\{\begin{array}{r|l} 12 & 9876 \\ \hline 6 & 823 + 0 \\ \hline & \underline{137} + 1 \end{array}\right. \left.\begin{array}{l} \\ \\ \end{array}\right\} 12 \text{ (over)}$$

2nd R. × 1st D. + 1st R.

$1 \times 12 + 0$

$= 12 + 0 = 12$

Note carefully the remainders in these two cases.

EXAMPLE 11.—**If it had been necessary to divide 7455 by 144 using three factors, namely 6, 8, and 3, the procedure would have been:**

$7455 \div 144 \doteqdot 7500 \div 150 = 50$.

$$144\left\{\begin{array}{l} 48\left\{\begin{array}{r} 6 \\ 8 \end{array}\right. \\ \\ \end{array}\right. \begin{array}{r|l} 6 & 7455 \\ \hline 8 & 1242 + 3 \\ \hline 3 & 155 + 2 \\ \hline & \underline{51} + 2 \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} 15, \textit{ i.e. } (2 \times 6) + 3,$$

The **true** remainder will then be $(48 \times 2) + 15$;

$= 96 \times 15 = 111$.

43. (ii) Long Division

EXAMPLE 12.—**A piece of land is 87625 acres in area. If it be divided among 684 purchasers in equal lots, how much will each one have, and what area will remain over?**

It might be possible to do this sum by the factor method, but we employ long division.

$87625 \div 684 \doteqdot 88000 \div 700 = 126$ nearly.

[1] The METHOD of finding the remainder is explained as follows:

Regard the 7455 lb. of tea as being divided into packets containing 6 lb. each. There will be 1242 packets and 3 lb. over.

Then let the 1242 packets be placed in 8 parcels; there will be 155 such parcels and 2 packets over.

The total left over will be—

2 packets and 3 lb.;

2 packets contain 6 lb. each;

∴ the weight left over is 12 lb. + 3 lb. = 15 lb.

Method.

	128	
684	87625	684 into 876 goes **1**; place this 1 over the 6, as shown, in order that it may have its proper "place value."[1] Put down 684 under 876, and subtract.
	684	Bring down the 2.
	1922	684 into 1922 goes **2**.
	1368	Multiply 684 by 2 and subtract from 1922. Bring down the 5.
	5545	684 into 5545 goes **8**.
	5472	$684 \times 8 = 5472$.
	73	Subtract 5472 from 5345, and 73 remains over.

Each purchaser will have 128 acres and 73 acres will be left over.[2]

44. With a little practice it will be possible to omit the figures resulting from the multiplication of the divisor by the various figures in the quotient, and to put down simply the results of the subtraction. This is called the Italian Method.

Method.

The figures in the dark type are to be put up.

	128
684	87625
	1922
	5545
	73

$4 \times 1 = 4$; $4 + \mathbf{2} = 6$

$8 \times 1 = 8$; $8 + \mathbf{9} = 17$

$6 \times 1 = 6$; $6 + 1 + \mathbf{1} = 8$

Bring down 2*.

$4 \times 2 = 8$; $8 + \mathbf{4} = 12$

$8 \times 2 = 16$; $16 + 1 + \mathbf{5} = 22$

$6 \times 2 = 12$; $12 + 2 + \mathbf{5} = 19$

Bring down 5°.

$4 \times 8 = 32$; $32 + \mathbf{3} = 35$

$8 \times 8 = 64$; $64 + 3 + \mathbf{7} = 74$

$6 \times 8 = 48$; $48 + 7 + \mathbf{0} = 55$.

[1] Really this is 684 into 876 *hundred*, which goes 1 hundred times, and therefore the 1 is put in the *hundreds* place.

[2] We might write down this result in a slightly different way, thus:

$87625 \div 684 = 128\frac{73}{684}$, which means that 684 goes into 87625, $128\frac{73}{684}$ times, or roughly, $128\frac{1}{9}$ times.

Hence, if we take 684, $128\frac{1}{9}$ times, we shall have 87625 (nearly), or 128 times $684 + \frac{1}{9}$ times $684 = 87625$ (nearly).

45. It is often necessary to give the result of a division sum to the nearest whole number. This is done by observing whether the remainder is (1) less than half the divisor, or (2) exactly one-half of it or greater than one-half.

Thus: Divisor, 584; quotient, 600; remainder, 100; 100 is less than half 584;

$\therefore$ The quotient remains 600 to the nearest whole number.

If the remainder were 292, that is, exactly half 584, or 293, 386, 583,—that is, more than half 584,—we should write the quotient 601 to the nearest whole number.

46. The following methods of rapid multiplication and division will be found useful:[1]

1. To multiply

by	10,			add one 0.
„	100,	*i.e.*	10^2,	„ two 0's.
„	1000,	„	10^3,	„ three 0's, and so on.
„	5,	„	$10 \div 2$,	„ one 0 and divide by 2.
„	25,	„	$100 \div 4$,	„ two 0's „ „ „ 4.
„	125,	„	$1000 \div 8$,	„ three 0's „ „ „ 8.
„	99,	„	$100 - 1$,	„ two 0's and subtract the multiplicand.

e.g. 5898×99
$= 5898 \times (100 - 1) = (5898 \times 100) - (5898 \times 1) = 589800 - 5898$
$= 583902.$

To multiply by 97, *i.e.* $100 - 3$, add two 0's and subtract three times the multiplicand.

By 39 or 49 or 59, etc., *i.e.* by $40 - 1$, $50 - 1$, or $60 - 1$, etc., multiply by 40, 50, or 60 and subtract the multiplicand.

In a similar way, $999 = 1000 - 1$, $998 = 1000 - 2$, so that to multiply by any such number we simply add three 0's and subtract once or twice the multiplicand as the case may be.

2. To divide

by	5,	multiply by 2	and divide by 10.
„	25,	„ „ 4	„ „ „ 100.
„	125,	„ „ 8	„ „ „ 1000.

(See Sections VI.–VIII.)

[1] See also § 23.

EXAMPLES. III.

MULTIPLICATION AND DIVISION

A number of mental questions should be asked upon § 46 (*e.g.* 28 times 100; 39 times 25; 750 ÷ 125, and so on), before proceeding with these examples.

1. Multiply—

Begin multiplying by the tens figure—

(1) 86 by 74 and by 36.
(2) 126 „ 49 „ „ 98.
(3) 150 „ 80 „ „ 128.

Begin multiplying by the figure with the highest place value—

(4) 1964 by 24 and by 168.
(5) 2865 „ 58 „ „ 522.
(6) 5683 „ 124 „ „ 992.
(7) 73954 „ 738 „ „ 2952.
(8) 62741 „ 8631 „ „ 43155.
(9) 758642 „ 10356.
(10) 988699 by 100, by 125, by 99, by 89, and by 9998.

2. Multiply parts 6 and 7 in Question 1, beginning with the tens figure, and so prove the results already obtained.

3. Multiply parts 8 and 9 in Question 1, beginning with the hundreds figure.

4. Multiply—

(1) 86572 by 204, by 205, and by 208.
(2) 68583 „ 819, „ 918, „ „ 981.
(3) 58634 „ 426 and by 412.
(4) 65312 „ 525 „ „ 2514.
(5) 132564 „ 32649.
(6) 562301 „ 129636.

5. Find the product of—

(1) 987 and 2021. (2) 86942 and 5051.
(3) 76843 „ 6008. (4) 793 „ 886473.
(5) 998678 and 659763.

(Multiply by the smaller number in all cases except such as 987×2021, where it is clearly easier to multiply by the larger 2021.)

6. Evaluate (find the value of)—

(1) 86954×248. (2) 86954×496. (3) 86954×1488.

What relationship is there between the results? How could those of the last two have been obtained from the first without multiplying by 496 and by 1488?

7. Determine by inspection which of the following numbers is divisible by 2, 3, 4, 5, 8, 9, 10, or 11.

(1) 86.	(2) 45.	(3) 98.	(4) 123.
(5) 8658.	(6) 58654.	(7) 5269.	(8) 582.
(9) 19668.	(10) 720.	(11) 1566.	(12) 968759.
(13) 1962.	(14) 18544.	(15) 165.	(16) 185.
	(17) 96584.	(18) 756828.	

8. Write down in ascending order the prime factors of the following numbers:

(1) 21, 36, 48. (2) 86, 74, 98.
(3) 126, 150, 175. (4) 525, 396, 999.
(5) 470, 364, 532.

9. Find the prime factors of the following numbers:

1001, 493, 1323, 1404, 2662, 3042, 35802, 667.

10. What is the square of 86, 152, and 99?
11. Find the cube of 36, 79, 94, 138.
12. What is the cube of 39, 67, 49, 121?
13. Find the square root of—

324, 361, 441, 1024, 1369, 1681, 2209, 2704, 4096, 5329, 5625, 9216, 8836, 7396, 9801.

14. Find the cube root of the following numbers:

64, 512, 1331, 3375, 12167, 39304, 97336, 405224, 421875, 357911, 493039, 729000, 830584, 531441.

15. Divide (employing factors)—

(1)	523	by	36	and	by	72.
(2)	7853	,,	49	,,	,,	98.
(3)	9685	,,	80	,,	,,	128.
(4)	38764	,,	144	,,	,,	432.
(5)	85963	,,	363	,,	,,	392.

16. Work parts (1)–(5) of Question 15 by long division, and confirm the results already obtained.

17. Apply the relationship $V = (Q \times D) + R$ to prove the results of Question 15.

18. Divide by long division—

(1) 47136 by 24 and 329952 by 168.
(2) 166170 by 58 and 1495530 by 522.
(3) 704692 by 124 and 5637536 by 992.
(4) 54578052 by 738.
(5) 218312208 by 2952.
(6) 7856496552 by 10356.

19. Divide, using the method of long division—

(1) 17660688 by 204. (2) 62959194 by 918.
(3) 24157208 by 412. (4) 34288800 by 525.
(5) 1994727 by 2021. (6) 439144042 by 5051.
(7) 461672744 by 6008. (8) 164194368 by 2514.
(9) 702973089 by 793. (10) 4328082036 by 32649.
(11) 129387552 by 1488. (12) 9885012602 by 9998.

20. Prove parts (10)–(12) of the previous question, using $V = (Q \times D) + R$.

21. A publisher arranges for an author to write a book containing 50000 words. Each page will contain 35 lines and each line 10 words. There will be in addition 22 pages of illustrations. How many pages will there be in the book?

22. A building contains 8 floors (including the basement). Find, from the data given, the total number of radiator tubes necessary to heat the whole of the rooms.

Floor.	Number of Workshops or Rooms.	Number of Tubes for Each.	Total Number of Tubes for Each Floor.
Basement . .	5 workshops	35	
Ground . .	3 rooms	22	
First . . .	5 ,,	18	
Second . .	6 ,,	16	
Third . .	6 ,,	16	
Fourth . .	7 ,,	15	
Fifth . . .	8 ,,	13	
Sixth . . .	15 ,,	7	
	Total number of tubes required .		

23. If each tube of Question 22 has a surface area of 2 square feet, find the total area of heating surface in square yards if 9 square feet = 1 square yard.[1]

24. A river is 2754 yards wide. What number of spans will be required in a bridge which crosses it, if each span is 48 yards long?

25. A railway company wishes to construct a siding to contain 990 trucks, each of which is 18 feet long, and there is room for 15 rows of trucks side by side. What length must the siding be?

If the trucks are 7 feet wide and a space of 3 feet is left between the rows, what is the distance from the outside of the first truck in the first row to the outside of the first one in the farthest row?

26. A bank has 1865 London and provincial branches. Each of these costs on the average £88 a year in rent. What is the total rent paid by the company?

[1] Section XIII.

27. The average distance between the stations on a tube railway is 352 yards. How far does a man travel by tube every day to and from his business if he passes through thirteen stations and alights at the fourteenth from his own terminus?

28. A carrier is instructed to cart 8926 rolls of paper from the docks to the printing-office of a newspaper company. Each roll weighs 1120 lb. What is the total weight carried in pounds and in tons, if 1120 lb. is half a ton?[1]

29. The cart of the last question makes 1124 journeys, each of which costs 9s. (for cart and horses), while the men's time costs 6s. a journey. What is the cost (in £'s) of removing the paper?

30. In a library there are 756 shelves holding 50 books, 876 holding 93, and 1576 holding 124 books each. What is the total number of books in the library? Tabulate this sum.

31. Linoleum is sold in lengths which are 6 feet wide. A corridor is 12 feet wide and 365 feet long. What length of linoleum will be required to cover the floor?

32. Thirty-six pens are used every day in an office. How many are used each week, and also during a year of 365 days (allowing for 52 Sundays)?

33. How many dozen (12), score (20), and gross (144) pens are used in the circumstances of Question 32?

34. The average number of adult male members of fifteen principal Friendly Societies in the United Kingdom is 221654. Find the total number of members.

35. If the value of the total funds of these Friendly Societies is £31963876, what does this amount represent for each society?

36. A wholesale draper has 96 rolls of calico, each of which is 16 inches deep. He arranges them in twelve piles. How many rolls will there be in each pile? Compare the height of each pile with that of a man 6 feet high.

37. Each person in the United Kingdom consumes 5975 ounces of *imported* wheat per year of 365 days. How much is this per day (to the nearest ounce)?

38. A torpedo boat destroyer can steam at 32 knots—that is, 32 nautical miles per hour.[2] How many complete hours will be necessary for it to travel

(1)	From Dover to Calais . .	22	nautical miles?
(2)	„ Folkestone to Flushing .	95	„
(3)	„ Southampton to Havre .	106	„
(4)	„ Queenboro' to Flushing .	114	„
(5)	„ Southampton to Cherbourg	83	„

[1] Section XI., § 40, and Section XV. [2] Section XI., § 130.

39. At a flying ground visitors' taxis are drawn up end to end with 1 foot between them. How far will it be from the back of the last one to the front of the first, if there are 178 cars, each of which is 14 feet long?

40. A house decorator is instructed to put French paper which is 18 inches wide (and cut to the proper length) on to a wall 75 feet long. How many pieces must he use if 12 inches = 1 foot? (See also Question 42.)

41. If the value of each piece of paper in the previous question is 11d., what will it cost to buy the paper for the room? Taking the price for hanging the paper as 8d. a piece, find the total cost of papering the room.

42. Take a sheet of drawing paper,[1] an inch scale (ruler), and a pencil. Let half an inch represent 3 feet, and draw a line to illustrate the wall referred to in Question 40. Mark off distances to represent 18 inches along it, and show that the number of divisions thus obtained graphically corresponds with the number of pieces of paper calculated.

43. The total area of India is 1802657 square miles, and the most recent estimate of the population places it at 315156396. Calculate the number of people per square mile to the nearest whole number.

44. The area of British North America is 3892399 square miles, and the population 8832574. Calculate the number of people to each square mile to the nearest whole number.

45. By how many people per square mile is the density of population in India greater or less than in Canada? Comment on the result.

46. The Customs revenue[2] of the Commonwealth of Australia for the year 1913–14 was £13055925, and the population was 4872059. Calculate to the nearest £1 the Customs revenue per head of the population.

47. The total imports of the thirty-nine self-governing Dominions, Colonies, Possessions, and Protectorates of the Empire were valued at £562692000 in 1913–14. What was the average value of the imports of each, to the nearest £1000?

48. 91958 cwt. of lard were imported into the Straits Settlements during the last fiscal[3] year, and the value was £204317. Calculate to the nearest £1 the value of the lard per cwt.

[1] Foolscap paper or a double sheet of exercise-book paper will do.

[2] See § 243.

[3] "Fiscal" means "pertaining to the revenue of the State," and the fiscal year often runs from March to March rather than from December to December.

49. If there are 240 pence in £1 and 112 lb. in 1 cwt., calculate the value of the lard in pence per lb. (Question 48.)

50. 1374 tons of coal and coke were exported from British Honduras in the last fiscal year, and the value of the exportation was £1696. Find the average value in shillings of every ton exported to the nearest shilling.

51. In 1913–14 Jamaica imported 10769 cwt. of butter and its compounds, valued at £44462. Find the price of 1 cwt. in shillings to the nearest shilling.

52. Under the same circumstances as the last question, Cyprus imported 2990 cwt. at a cost of £11324. Was the price per cwt. greater or less than for Jamaica, and by how many shillings?

53. What was the value of gold in shillings per ounce if 79847 ounces were exported from British Guiana in 1913–14 and they were worth £289638? (Answer to the nearest shilling.)

54. In 1914 British East Africa imported goods to the value of £1609 from Australia, and to the value of £907410 from the United Kingdom. How many times is the latter greater than the former, to the nearest whole number?

55. Last year there were the names of 495 depositors on the books of the Government Savings Bank in the Falkland Islands, and they had £70402 to their credit. How much had each depositor on the average (to the nearest £1)?

56. In the year 1913 there were 458600 depositors in Government Savings Banks in the Dominion of New Zealand, and they had £17131400 to their credit. For the year given were the average savings per inhabitant of the Falkland Islands greater or less than for New Zealand? (Question 55.)

57. The Government of British India spent £2026576 on its Post Office, which dealt with correspondence to the extent of 1014788745 letters, etc. (including parcels but excluding telegrams), in one year. Calculate the number dealt with for every £1 expenditure.

58. The Government in Australia owns 17775 miles of railway, and private companies own 1912 miles, while in Canada the Government owns 1742 and private companies 28919 miles. Find how many times the length of Government lines exceeds that of private lines in Australia, and the length of private lines exceeds that of the Government lines in Canada.

59. There were 763114 acres under wheat in Ontario last year, and 18014668 bushels of wheat were produced. Calculate the number of bushels per acre to the nearest bushel.

60. In the latest returns for Australia we find that there are 5664 post offices, which deal with 453885000 letters and post cards annually. Calculate the average number of letters handled at each post office.

61. The corresponding figures (see the last question) for Japan are 7166 and 1652942800. Calculate the average number of letters "stamped" at each post office in Japan.

62. What information do the results of the last two questions give you in conjunction with the fact that the population of Australia is 5 millions and of Japan 53 millions (1913)?

C. Averages

MEDIAN: MODE

47. If you take up the seed catalogue of almost any large seed-supplying firm you will find that one column is headed "Average height of plants," *e.g.* a particular kind of geranium may be (say) a foot or 18 inches in height. The information given in the column referred to is such that a purchaser may buy plants suited to the various parts of his garden or to the shelves of his conservatory. Some plants will be less than the average height, some more. How then do they get the average?

48. Example 1.—**What is the average height of 10 geraniums whose heights are found to be 15, 18, 24, 12, 14, 17, 22, 10, 16, and 12 inches?**

To find the average required, we add all the heights together and divide by the number of plants. This gives $160 \div 10 = 16$.

The average height is 16 inches.

Example 2.—**Seven boxes of fish weighed respectively 120, 115, 136, 125, 98, 130, and 95 lb. What was the average weight?**

Sum of weights	= 819 lb.
Number of boxes . . .	= 7
Average weight per box . .	$= 819 \div 7$
	= 117 lb.

Example 3.—**A builder ordered 8 rolls of sheet lead, the average weight of which was to be 18 cwt. He weighed the first 3 and found their average weight to be 12 cwt. What was the average weight of the remaining 5, supposing that he received what he ordered?**

Average weight of first 3 . .	=	12 cwt.
Total „ „ $3 = 12 \times 3$	=	36 „
Average weight of all 8 . .	=	18 „
Total „ „ $8 = 18 \times 8$	=	144 „
∴ „ „ of last 5 . .	=	$144 - 36$ cwt.
	=	108 cwt.
∴ Average weight of last $5 = 108 \div 5$	=	$21\frac{3}{5}$ cwt.

EXAMPLE 4.—**The Statistical Abstracts give the population of the Union of South Africa as 5175824 in 1901 and 5973394 in 1911. Find, correct to the nearest thousand, the average annual increase in population.**

Increase in 10 years . . $= 5973000 - 5176000$
$= 797000.$
Average increase per annum . $= 79700$,
or 80000 to the nearest 1000.

EXAMPLE 5.—**A newsagent sells 750 periodicals on 8 days in a month, 375 on 12 days, 180 on 5 days, and 177 on 6 days. What were his average daily sales?**

We find the total sales and divide by 31, the number of days.

750 on 8 days . .	=	6000	altogether.
375 „ 12 „ . .	=	4500	„
180 „ 5 „ . .	=	900	„
177 „ 6 „ . .	=	1062	„
Total	=	12462	

Average $= 12462 \div 31$
$= 402$ periodicals a day.

EXAMPLE 6.—**The average number of inhabitants in each of three towns is 460500, and in each of four others 640925. What is the average for all seven towns?**

Total for first 3 towns $= 460500 \times 3$
$= 1381500$;..................(1)
„ other 4 „ $= 640925 \times 4$
$= 2563700$;..................(2)
„ all 7 „ $= 3945200$ (*i.e.* $(1) + (2)$);

$$\text{Average per town} = \frac{3945200}{7} = 563600 \text{ inhabitants.}$$

49. In some statistical work it is better to take a number called the median rather than the average.

The median is that number which has as many of the given numbers above as below it.

If, for example, a builder bought a number of scaffold poles of length

40, 60, 50, 60, 50, **120**, 70, 90 feet,

it is clear that the 120-feet pole is far and away longer than the others, The average of these numbers is $67\frac{1}{2}$. Now the median is found by arranging them in order thus:

40, 50, 50, 60, ↑ 60, 70, 90, 120,

and noting that there are four numbers left and right of the arrow, and the median lies midway between the two 60's.

∴ The median is 60.

The average is, then, $67\frac{1}{2}$, and the median is 60.

The latter is the much fairer number to take in paying a contract price for the whole, for if the average be taken the 120-feet pole exerts a greater influence on the result than is due to it.

50. Sometimes a number called the **MODE**[1]—**THAT IS, THE NUMBER WHICH OCCURS MOST FREQUENTLY** — is taken, but it appears, perhaps, to a greater extent in sociological statistical work than in purely commercial problems. If a man shoots at a target the **mode**—that is, the score he makes most often — can be used to express his power as a marksman better than the average or the median. If, now, a merchant wished to measure the power of his employees as salesmen it would again be more relevant to take the mode, and if he wished to work really scientifically he would take perhaps 6 salesmen and record their sales in pounds, or their takings in shillings, over a long period involving, say, 200 or 300 observations. It might appear then that a few of the sales were:

48, 5, 87, 26, 9, 66, 7, 9, 60, 12, 66, 2, 77, 17, 33, 122, 16, 73, 10, 41, 4, 72, 66, 0, 53, 22, 137, 61, 42, 17, 28,

where 66 is the mode.

By doing the same for the other 5 men he might get 10, 80, 40, 55, 70, for the modes, and then 80 for the third man would suggest that he was the best salesman. The average of the above numbers is $41\frac{1}{2}$ and the median 33, facts which emphasise the value of the mode.

Note that, generally,

$$\text{mode} - \text{average or mean} = 3\,(\text{average} - \text{median})$$

and here

$$\text{mode} - \text{mean} = 66 - 41{\cdot}5 = 24{\cdot}5\,;$$
$$\text{mean} - \text{median} = 41{\cdot}5 - 33 = 8{\cdot}5\,;$$

showing that in this case the rule is true.

EXAMPLES. IV.

1. If the average price of silver was 23d., 24d., 22d., 23d., 25d., 26d., 25d. per ounce in seven successive years, find the average price during the period given.

2. Find, to ten thousand tons, the average weight of salt consumed in India during the years 1903–1904 to 1912–1913 from the following table:

Year.	Weight of Salt consumed to the Nearest Thousand Tons.	Year.	Weight of Salt consumed to the Nearest Thousand Tons.
1903–1904 .	1403	1908–1909 .	1602
1904–1905 .	1460	1909–1910 .	1610
1905–1906 .	1440	1910–1911 .	1512
1906–1907 .	1511	1911–1912 .	1780
1907–1908 .	1570	1912–1913 .	1788
		Annual Average	

[1] Cf. Lee, *Primer of Statistics* (Elderton).

3. The ordinary expenditure on account of the Russian Budgets 1908–1914 is as follows:

Year.	Expenditure in Thousands of Roubles.
1908 . . .	2387751
1909 . . .	2451424
1910 . . .	2473157
1911 . . .	2535996
1912 . . .	2721763
1913 . . .	3012264
1914 . . .	3302675

Find the average expenditure in roubles during the seven years given. (1 rouble is worth 2s. 1d.)

4.

AVERAGES FOR THE COMMONWEALTH OF AUSTRALIA.

Year.	Imports (Tons).	Exports (Tons).
1903 . . .	91530	2365
1904 . . .	38035	2944
1905 . . .	24933	11158
1906 . . .	41976	9254
1907 . . .	6167	18261
1908 . . .	19552	14741
1909 . . .	99698	8051
1910 . . .	34008	6584
1911 . . .	33325	7331
1912 . . .	98480	2256
Average for Period .		

Find the averages for which spaces are provided.

5. The number of chests of opium sold in Bengal (India) for export is given below for the last ten years. Calculate the average number of chests sold per annum.

Year.	Number of Chests of Opium sold for Export.	Year.	Number of Chests of Opium sold for Export.
1903–1904 .	48000	1908–1909 .	45900
1904–1905 .	48000	1909–1910 .	42300
1905–1906 .	49200	1910–1911 .	37590
1906–1907 .	52800	1911–1912 .	26860
1907–1908 .	48900	1912–1913 .	17890

6. Find the average value of the window glass imported into China, from Belgium and Hong-Kong (chiefly), in the years 1911–13. Value in Haikuan taels,[1] 632540, 644755, 1141644.

Comment upon the probable meaning of these figures to the people of China.

7. A company has 3 shops in various parts of Leeds, and the daily takings were respectively—

(1) £25, £35, £56, £35, £28, £78.
(2) £38, £41, £27, £18, £54, £38.
(3) £35, £52, £25, £15, £38, £35.

In which shop were the average daily takings greatest and least respectively?

8. Find the median in the case of the figures given in the last question, parts (1), (2), and (3).

9. Calculate the average number of readers in a reference library per day during a month from the following:

On 8 days there were 156 readers.
„ 10 „ „ 200 „
„ 3 „ „ 48 „
„ 9 „ „ 182 „

10. From the table given below calculate the difference between the average municipal income and expenditure in India for the last five years.

Year.	Income.	Expenditure.
	£	£
1908–1909 . .	6703522	6553804
1909–1910 . .	7766121	7544936
1910–1911 . .	8383887	8295355
1911–1912 . .	8591035	8358278
1912–1913 . .	11727041	11345635

11. The value of timber (square)[2] exported from Canada in the years 1908–1914 (both inclusive) was, in thousands of dollars:

1520, 1179, 935, 1043, 1268, 1363, 536.

Find both the average value per annum and also the median. Which appears to you the more useful number to take?

12. From the data given below calculate the difference between the average temperature[3] on 1st July and 2nd July.

[1] A Haikuan tael is not a coin, but a weight of silver the value of which varies.

[2] § 139. [3] § 224.

The following tables appeared in the London *Daily Chronicle* of 1st and 2nd July 1914:

1st July.		2nd July.	
London	86 degrees	London	90 degrees.
Harrogate	79 ,,	Southend	85 ,,
Scarborough	80 ,,	Bath	86 ,,
Skegness	83 ,,	Weston-super-Mare	83 ,,
Leamington	84 ,,	Hastings	82 ,,
Rhyl	77 ,,	Dover	82 ,,
Colwyn Bay	78 ,,	Hythe	82 ,,
Llandudno	76 ,,	Margate	80 ,,
Southend	84 ,,	Cromer	81 ,,
Margate	80 ,,	Bournemouth	80 ,,
Ramsgate	78 ,,	Weymouth	82 ,,
Folkestone	76 ,,	Brighton	78 ,,
Hythe	80 ,,	Ilfracombe	78 ,,
Weston-super-Mare	81 ,,	Ramsgate	78 ,,
Torquay	74 ,,	Torquay	76 ,,

13. If the average temperature for 14 towns be 80°, and for 15 towns 81°, calculate, to the nearest degree, the temperature at the fifteenth town.

14. A tailor wishes to contract for the uniforms of a company of soldiers, and to do so he measures the heights of 15 men to the nearest inch, and finds them as follows:

60, 63, 58, 65, 62, 64, 59, 61, 75, 56, 61, 57, 58, 60, 56.

Should he take the average height, the median, or the mode as the number upon which to base his calculations?

15. If the tailor of the last question reckons that it costs 2s. for every inch in the height of a soldier to supply his uniform, what difference would it make to him if he took the average height instead of the median or the mode on a contract for 1000 uniforms?

16. The times of my trains down from town of an evening are, from station to station (in minutes):

Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
17	14	15	16	28	26

If the times up in the morning are the same, calculate the average length of time I spend in the train every day.

17. In buying some iron rods a merchant measures 8 and finds their average length to be 15 feet, and that of the next 7 to be 12 feet, and that of the next 6 to be 16 feet, while the average of 24 rods is 14 feet. Calculate the average length of the last 3 of them.

18. The net expenditure on the army in India during the last six years has been (in thousands of pounds):

1907–1908	£19248
1908–1909	19603
1909–1910	19112
1910–1911	19265
1911–1912	19559
1912–1913	19565

By how much does the greatest annual expenditure exceed the average?

19. Find the average yearly increase for the last four years from the figures given in Question 18.

SECTION III

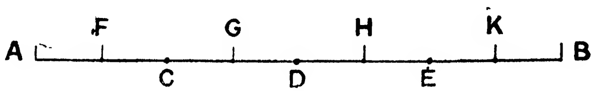
FRACTIONS

INTRODUCTORY

51. A clerk earns £100 a year and spends £79. He therefore spends **a fraction** of his income, namely, £79 out of £100, which we write $\frac{79}{100}$, and read 79 hundredths. He saves £21, or $\frac{21}{100}$, 21 hundredths, of his income, and since what he spends added to what he saves gives his whole income, we have $\frac{79}{100}+\frac{21}{100}=\frac{100}{100}$, and so 100 hundredths equals his whole income.

A piece of gas pipe is 15 inches long, and we want a piece 13 inches long; we therefore cut off 2 inches, which is a fraction of the whole, and we write it as $\frac{2}{15}$. If we cut off $\frac{2}{15}$ of a pipe 15 inches long, $\frac{13}{15}$ remain, for $\frac{15}{15}$ is the length of the whole pipe.

52.



AD is a **fraction** of the line **AB**, namely, one-half, or $\frac{1}{2}$; **AC** is $\frac{1}{4}$ (one-fourth, or one quarter).

Now, **AD** = 2 quarters, or $\frac{2}{4}$,
but **AD** = one-half, or $\frac{1}{2}$,
$\therefore \quad \frac{1}{2}=\frac{2}{4}$.

Again, **AF** = one-eighth of **AB**, or $\frac{1}{8}$ **AB**,
and **AD** = four-eighths „ or $\frac{4}{8}$ **AB**;
but, **AD** = $\frac{1}{2}$ **AB**,
$\therefore \quad \frac{4}{8}=\frac{1}{2}$,
$\therefore \quad \frac{4}{8}=\frac{2}{4}=\frac{1}{2}$.

From this (and all such results) we conclude that **if the top (NUMERATOR) and bottom (DENOMINATOR) of any fraction be multiplied or divided by the same number, the value of the fraction is unchanged.**

53. If you look carefully at the fractions $\frac{6}{12}$ and $\frac{7}{12}$ you will notice that both numerator and denominator of the former can be divided by 2, while the latter cannot be so treated, and that $\frac{6}{12}$ becomes $\frac{1}{2}$ on dividing by 6, but that $\frac{7}{12}$ cannot be reduced.

When both numerator and denominator of a fraction cannot be divided by the same number, the fraction is in its lowest terms.

All fractions should be left in their lowest terms.

54. We know that $\frac{1}{2}$d. + $\frac{1}{4}$d. = $\frac{3}{4}$d.

We have really said two farthings + one farthing = three farthings, or $\frac{2}{4}$d. + $\frac{1}{4}$d. = $\frac{3}{4}$d.

In the same way—

$$\tfrac{3}{4}\text{d.} + \tfrac{1}{2}\text{d.},$$
$$= \tfrac{3}{4}\text{d.} + \tfrac{2}{4}\text{d.} = \tfrac{5}{4}\text{d.},$$
$$= 1\tfrac{1}{4}\text{d.};$$

and

$$\tfrac{3}{4}\text{d.} - \tfrac{1}{2}\text{d.},$$
$$= \tfrac{3}{4}\text{d.} - \tfrac{2}{4}\text{d.} = \tfrac{1}{4}\text{d.}$$

In every case **we have put the same denominator to each fraction** (unconsciously), **and then simply added or subtracted the numerators.**

It is, of course, important to notice that we cannot change the denominator and leave the numerator what it was, but we must multiply or divide BOTH NUMERATOR AND DENOMINATOR by the SAME number.

55. Again, let us add together $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{3}$.

We write

$$\tfrac{1}{2} + \tfrac{3}{4} + \tfrac{1}{3},$$
$$= \tfrac{6}{12} + \tfrac{9}{12} + \tfrac{4}{12}$$

(we choose 12 because no **smaller** number will do),

$$= \tfrac{19}{12} = 1\tfrac{7}{12}.$$

56. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$ are called **Simple Fractions,** for the numerator IS LESS THAN the denominator; $\frac{19}{12}$, $\frac{5}{4}$, etc. are **Improper Fractions,** and the numerator IS GREATER THAN the denominator; while $1\frac{1}{4}$, $1\frac{7}{12}$, $100\frac{1}{18}$ are called **Mixed Numbers, for they contain a whole number (1 or 100) and a fraction.**

57. We now work a few examples:

Example 1.—**Express $\frac{5}{8}$ in 24ths; *i.e.* make the denominator 24 instead of 8.**

$$\frac{5}{8}=\frac{5\times3}{8\times3}=\frac{15}{24},$$
$$\therefore\quad \frac{5}{8}=\frac{15}{24}.$$

Example 2.—**What fraction of a barrel of wine containing 88 gallons has been drawn off to fill an 11-gallon cask?**

11 gallons in 88 have been drawn;

$\therefore$ $\frac{11}{88}$ or $\frac{11\div11}{88\div11}=\frac{1}{8}$ has been drawn off.

Example 3.—**Reduce $\frac{54}{1458}$ to its lowest terms.**

We merely divide numerator and denominator by the same number until no further division is possible, applying the tests given in § 32.

$$\frac{54}{1458}=\frac{27}{729}=\frac{9}{243}=\frac{3}{81}=\frac{1}{27}.$$

Example 4.—**Express $\frac{5}{6}$, $\frac{8}{9}$, $\frac{1}{2}$ with the same denominator.**

We see at once that 18 is the *lowest* number that 6, 9, and 2 will **all** divide into exactly, so we write—

$$\frac{15}{18},\ \frac{16}{18},\ \frac{9}{18}.$$

We might use also 36, but not 27, for 6 and 2 will not divide into it, nor is there any number between 18 and 36 into which all three will divide. If we had used 36, the figures would have been $\frac{30}{36}$, $\frac{32}{36}$, $\frac{18}{36}$, which reduce to $\frac{15}{18}$, $\frac{16}{18}$, $\frac{9}{18}$, as before, and **therefore we should have given ourselves more work by using the higher denominator, 36.**

58. Example 5.—**What is the *Ratio* of 10 oz. to 1 lb.? (1 lb. = 16 oz.)**

We write the ratio as $\frac{10 \text{ oz.}}{16 \text{ oz.}}, =\frac{5}{8}.$

Note.—Quantities of the same kind can alone be expressed as a ratio; *e.g.* it is impossible to have the ratio of a mile to a gallon, but we can have the ratio of 176 yards to a mile, for a mile contains 1760 yards and the ratio is $\frac{176}{1760}=\frac{1}{10}$, and 176 yards $=\frac{1}{10}$th of a mile.

To find the ratio of one quantity to another reduce them to the same unit, yards, feet, gallons, etc., and then write one as a fraction of the other.

59. Another way of expressing a ratio is

5 : 8, which we read

"five *is to* eight";

and since $\frac{5}{8}=\frac{10}{16}$,

we say 5 is to 8 as 10 is to 16,

or 5 : 8 :: 10 : 16.

It is worth noticing that the product of the two outside numbers equals that of the two inside:

$$5 \times 16 = 80$$

and $$8 \times 10 = 80.$$

This is always true.

We could do this another way, thus:

$$\frac{5}{8} = \frac{10}{16}$$

where the dotted lines make a multiplication sign, and if we write

5 × 16 and 8 × 10 we get

$$5 \times 16 = 8 \times 10;$$

for $$80 = 80.$$

This is called Cross Multiplication, and it is often very useful in applying algebra to commercial problems.

EXAMPLES. V.[1]

1. Draw a line 6 inches long and, by dividing it into 24 parts, show that

$$\frac{12}{24}=\frac{6}{12}=\frac{3}{6},$$

and that $$\frac{6}{8}=\frac{3}{4}=\frac{18}{24}=\frac{9}{12}.$$

2. A merchant owns $\frac{2}{3}$rds of a business. How many 9ths has he, and how many 36ths?

3. In travelling up an incline the coupling breaks between two coaches of a train, so that eight coaches go on with the engine and two run back. How many 10ths of the train go forwards and how many backwards?

4. Draw a line 8 inches long, divide it into 32 parts, and prove, geometrically, the following relationships:

$$\frac{4}{32}=\frac{1}{8},\ \frac{1}{8}=\frac{2}{16}; \qquad \frac{1}{4}=\frac{2}{8}=\frac{8}{32}; \qquad \frac{16}{32}=\frac{1}{2}=\frac{4}{8}.$$

5. With the help of a line 10 inches long, prove

(*a*) that $\frac{88}{100}=\frac{44}{50}=\frac{22}{25}$;

(*b*) that $\frac{75}{100}=\frac{15}{20}=\frac{3}{4}$;

and (*c*) that $\frac{64}{80}=\frac{32}{40}=\frac{16}{20}=\frac{8}{10}=\frac{4}{5}$.

[1] **Many of these questions can be answered mentally.**

6. Draw three straight lines, one 7 inches long, the next 14 inches, the third 6 inches, and show, by suitably dividing them, that the following statements are true:

$$(1)\ \tfrac{2}{14}=\tfrac{1}{7}=\tfrac{4}{28}, \qquad (2)\ \tfrac{3}{21}=\tfrac{6}{42}=\tfrac{1}{7},$$
$$\text{and } (3)\ \tfrac{7}{8}=\tfrac{3\frac{1}{2}}{4}=\tfrac{14}{16}.$$

7. Express all the numbers in Examples VII., Question 2, Nos. 6–8, as improper fractions where possible.[1]

8. How many 12ths are there in $6\frac{1}{6}$?

9. How many 8ths are there in $5\frac{5}{16}$?

10. Find the number of 15ths in $7\frac{7}{90}$.

11. Express $3\frac{1}{8}$ as a number of 16ths.

12. Add together $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$, expressing the result in fourths.

13. What is the number of 65ths in $6\frac{2}{13}$?

14. How many packets of tea each containing $\frac{1}{8}$ lb. can be made up from 2 lb.?

15. Express 20 lb. as a fraction of 56 lb.

16. What fraction is 5 grains of 2 carats, if 6 carats equal 24 grains?

17. What fraction of £100 is £4?

18. Is 13 the same fraction of 78 as 13 + 2 is of 78 + 2?

19. What fraction is 55 tons of 1331 tons?

20. There are 366 days in a leap year. What fraction of this is the number of days in a week?

21. Reduce the following fractions to their lowest terms:

$$(1)\ \tfrac{4}{8},\ \tfrac{14}{16},\ \tfrac{21}{42},\ \tfrac{86}{124},\ \tfrac{50}{125},\ \tfrac{65}{525}.$$
$$(2)\ \tfrac{99}{1089},\ \tfrac{323}{785},\ \tfrac{194}{3866},\ \tfrac{121}{1331},\ \tfrac{3000}{4000}.$$
$$(3)\ \tfrac{22}{297},\ \tfrac{819}{3509},\ \tfrac{896}{964},\ \tfrac{26}{676},\ \tfrac{729}{3136}.$$
$$(4)\ \tfrac{81}{6561},\ \tfrac{225}{1225},\ \tfrac{676}{1521},\ \tfrac{841}{3364},\ \tfrac{169}{2704}.$$
$$(5)\ \tfrac{216}{1728},\ \tfrac{729}{5832},\ \tfrac{361}{1444},\ \tfrac{529}{2116},\ \tfrac{961}{8649}.$$

22. In 1855 the traffic receipts per mile for railways of the United Kingdom were £2600, and in 1912 £5500. What fraction is the former of the latter?

23. The traffic receipts per mile in 1888 were $\frac{7}{11}$ of those in 1912. What were the receipts in 1888?[2] (Question 22.)

[1] The student should write down a hundred or more mixed numbers and express them to improper fractions, and then, a day or so after, convert the latter back again to mixed numbers and so check his work. Thus (i) $2\frac{1}{2}=\frac{5}{2}$, and then, without looking at (i), bring $\frac{5}{2}$ back to a mixed number and see that the result is right by comparing with (i).

[2] The figure for 1912 includes receipts from all sources, those for previous years exclude rent, tolls, navigation, steamboats. (*Whitaker's Almanack.*)

24. The revenue received from estate duties is £20937000 from England and Wales, £3170000 from Scotland, and £960000 from Ireland. What fraction of the whole revenue from these duties does each country contribute?

25. Income-Tax in England and Wales contributes £38509000, and Scotland £4261000 to the revenue. To what simple fraction is the ratio of these two sums nearly equal?

26. Ireland contributes £1453000 in Income-Tax. If it were taken as $\frac{1}{3}$ of that contributed by Scotland, by what amount would the value so obtained be greater or less than the true amount received from Ireland? (See Question 25.)

27. A man is 6 feet in height, his son is 4 feet 5 inches. Taking 12 inches as equal to 1 foot, express the son's height in inches as a fraction of the father's height.

28. Measure in inches the length of the two edges of this page. Express those lengths to the nearest inch, and write the breadth as a fraction of the length.

29. A coal vase holds 37 lb. of coal. What fraction is this of 1 cwt. (112 lb.)?

30. How many times could the vase referred to in the previous question be filled from 2 cwt. of coal?

31. A merchant has motor-vans of two sizes—one holds 10 cwt., the other 35 cwt. Express the weight carried by the former, when fully loaded, as a fraction of that carried by the latter.

32. A soldier has 100 rounds of ammunition. He fires 25 rounds. What fraction of the whole has he fired, and what fraction is left?

33. When he has fired 15 rounds more than the 25, what will the fractions be in 10ths and in 5ths?

34. If he had fired $\frac{1}{5}$ of his whole stock and 7 rounds more, how many cartridges would he still have had?

35. How many rounds must he fire so as to have (1) $\frac{1}{2}$, (2) $\frac{1}{5}$, (3) $\frac{1}{10}$ of his original 100 rounds left?

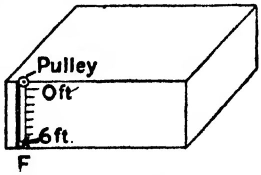
36. Half of a sack of flour is given to one man and the remainder divided among six other men. How many 6ths does each receive? If one of the latter gives his share to the first man, how many 12ths has he now?

37. If in a certain Slate Club the men share out in proportion to the amount each has paid in, and one man takes $\frac{1}{20}$, two other men $\frac{1}{40}$ each, and the remaining nine men divide what is left equally, what fraction of the whole does each man get?

38. If in the previous question there had been 16 men, and the first man had had $\frac{1}{25}$, the next $\frac{1}{50}$, the next three $\frac{3}{50}$ each, what fraction would each of the other men have had?

39. If a sack of flour contain 280 lb., what weight would be contained in each of the divisions, $\frac{5}{28}$, $\frac{1}{7}$, and $\frac{9}{28}$ of a sack?

40. The figure represents a tank such as one often sees at provincial railway stations for supplying water to locomotives. F is an indicator (attached to a float) which shows the height of water in the tank. What distance down will F be—



(1) When the tank is $\frac{1}{2}$ full? (2) When the tank is $\frac{1}{6}$ full?
(3) " " $\frac{1}{3}$ " (4) " " $\frac{2}{3}$ "
(5) " " $\frac{5}{6}$ " (6) " " $\frac{7}{12}$ "

41. Take a 12″ scale and show that—

(1) $3\frac{1}{8}'' = 3\frac{2}{16}'' = 3\frac{4}{32}''$. (2) $4\frac{1}{5}'' = 4\frac{2}{10}''$.
(3) $7\frac{1}{2}'' = 7\frac{2}{4}'' = 7\frac{16}{32}''$.

42. How many $\frac{1}{2}$ oz. of tea can a grocer sell out of 2 lb.?

43. A rasher of bacon weighs $1\frac{1}{2}$ oz., how many can be cut from $2\frac{1}{4}$ lb.?

44. What would be the weight of 8 rashers of bacon as in Question 43?

45. If 2 lb. of tea are divided into 8 parcels—

(1) How many ounces will each parcel contain?
(2) What fraction of the whole will each contain?
(3) What fraction of half the weight will 2 parcels contain?

EXAMPLES. VI.[1]

1. If $\frac{3}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$ a lb. of tea are put into a parcel, what is its weight, and what is its value at 1s. 4d. a lb.?

2. If $\frac{1}{2}$ lb. of sugar costs 2d., what will $\frac{7}{8}$ lb. cost?

3. If a parcel contain $1\frac{1}{2}$ lb. of raisins, $\frac{3}{4}$ lb. of rice, $\frac{7}{16}$ lb. of tea, what will be its weight?

4. One man pays $\frac{1}{3}$ of the cost of an aeroplane, another pays $\frac{1}{8}$, another $\frac{1}{4}$, another $\frac{1}{12}$, another $\frac{1}{24}$. For what fraction do they pay altogether?

5. A house furnisher marks a bedroom suite £60. The wardrobe is priced at $\frac{1}{2}$ of this amount, the dressing-table at $\frac{2}{5}$ of the remainder. Find the price of the wardrobe and of the dressing-table, and also the price of the rest of the suite. What articles would usually be included in "the rest of the suite"?

[1] Many of these questions can be answered mentally.

6. The value of the glass exported by Belgium to the United Kingdom in 1912 was valued at £1440000, the value of gloves (leather) $\frac{1}{6}$ of the former, that of bar iron $\frac{13}{36}$, that of line yarn $\frac{13}{18}$. By how many £'s does the value of the sum of the last three exports exceed £1440000?

7. What was the value of the two greatest exports named in the last question?

8. In the year 1912[1] the amount of the marketed iron-ore production of the United States was 57 million (long) tons[2]; the production in 1909 was $\frac{17}{19}$ of this amount. Find the total iron-ore production in 1909 and 1912.

9. If the total production in 1910 and 1911 of iron ore in the United States was $\frac{49}{54}$ of the production in 1909 and 1912 taken together, find the weight produced in 1910 and 1911. (See last question.)

10. What is the ratio of 15 to 20, of 18 to 23, and of 13 to 91?

11. What number bears to 5 the same ratio that 12 does to 6?

12. What number bears to 12 the same ratio that 25 does to 50?

13. What is the difference between $\frac{1}{3}$ and $\frac{1}{2}$ of £600?

14. A grocer finds that the amount of gas consumed from January to March is $\frac{3}{7}$, from March to June $\frac{3}{14}$, from June to September $\frac{8}{35}$ of the total annual consumption. What fraction is consumed from September to Christmas? (Express fractions as 70ths.)

15. In which quarter is the grocer's gas bill (see Question 14) greatest? Arrange the quarters in descending order of consumption.

16. The value of the exports from Brazil to the United Kingdom in 1912 was £9000000, of which $\frac{34}{45}$ was rubber, $\frac{17}{180}$ raw cotton, $\frac{1}{18}$ raw coffee. The remaining exports consisted of raw cocoa, seeds (cotton), nuts, sugar (unrefined), and wax, in order of importance. What fraction of the whole did these represent?

17. From the data of the previous question, find to £10000 the value of the remaining exports referred to.

18. A field is 150 yards long and 100 yards wide. What is the ratio of its length to its breadth?

19. The total income of the twelve great City Companies is £625000.[3] The income of the Mercers' Company is the largest, namely, £111000, while that of the Grocers' Company is $\frac{19}{257}$ of the

[1] *Statistical Abstract of United States.*
[2] A long ton = 2240 lb. ; a short ton = 2000 lb.
[3] See *Whitaker's Almanack*, p. 548.

remainder; the Merchant Taylors' $\frac{2}{25}$, and the Clothworkers' $\frac{12}{125}$ of the total income. What fraction of the income remaining, after deducting that of the Mercers' Company, belongs to each of the last two companies, and what is the income of the Grocers' Company?

20.

INDIAN EMPIRE.

RECEIPTS BY CIVIL DEPARTMENTS.

Revised Estimate, 1912–1913.

Total Receipts, £1300.

Law and Justice	$\frac{6}{13}$	of the	total.
Police	$\frac{34}{325}$	,,	,,
Ports and Pilotage	$\frac{79}{650}$	,,	,,
Education	$\frac{11}{650}$	,,	,,

What fraction of the total remains for medical and minor departments?

21. Taking the data of the previous question, find what are the receipts under each head given, correct to £10.

SECTION IV

FRACTIONS—(*continued*)

A. Addition and Subtraction of Fractions

60. We summarise the results of the last section for the students' convenience:

1. If numerator and denominator of a fraction be multiplied or divided by the same number, the value of the fraction is unchanged.

2. If numerator and denominator cannot **both** be divided by the **same** number, the fraction is in its lowest terms.

3. To add or subtract fractions, reduce them to the same denominator and then add or subtract the numerators as the case may be.

4. A ratio can exist between quantities **of the same kind only**, and can be expressed as a fraction.

Example 1.—$3\frac{1}{2} + 4\frac{1}{4} + 5\frac{1}{8} + 3\frac{1}{5}$.

1. Add the whole numbers (15).
2. Express all fractions with the same denominator (40).
3. Add the fractions.

Sum—

$$= 15\ (\tfrac{20}{40} + \tfrac{10}{40} + \tfrac{5}{40} + \tfrac{8}{40})$$
$$= 15\tfrac{43}{40} = 15 + 1\tfrac{3}{40} \text{ (cf. footnote 1, page 50)}$$
$$= 16\tfrac{3}{40}.$$

EXAMPLES. VII.

ADDITION OF FRACTIONS

1. Add together—

(1) $\frac{1}{2}+\frac{3}{4}+\frac{1}{4}+\frac{5}{8}$.
(2) $\frac{3}{8}+\frac{1}{4}+\frac{7}{8}+\frac{15}{16}+\frac{1}{8}$.
(3) $\frac{7}{16}+\frac{5}{8}+\frac{1}{4}+\frac{3}{8}$.
(4) $\frac{17}{64}+\frac{15}{32}+2\frac{5}{8}+7\frac{1}{4}$.
(5) $2\frac{7}{30}+\frac{3}{5}+6\frac{1}{10}+8\frac{11}{60}$.

2. Find the sum of—

(1) $\frac{1}{2}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}+\frac{11}{12}$.
(2) $2\frac{3}{8}+\frac{11}{12}+\frac{15}{16}+3\frac{5}{24}$.
(3) $3\frac{5}{6}+8\frac{7}{12}+\frac{31}{48}+2\frac{5}{18}$.
(4) $5\frac{3}{10}+20\frac{18}{25}+56\frac{2}{5}+\frac{1}{2}$.
(5) $7\frac{2}{3}+8\frac{5}{123}+2\frac{21}{41}$.
(6) $3\frac{5}{7}+1\frac{9}{112}+2\frac{5}{28}+4\frac{3}{14}$.
(7) $25\frac{1}{3}+8\frac{5}{9}+\frac{1}{81}+2\frac{1}{6}$.
(8) $22\frac{3}{22}+6\frac{8}{121}+27\frac{1}{2}$.
(9) $\frac{1}{26}+5\frac{9}{13}+15\frac{4}{65}+9\frac{1}{10}$.
(10) $14\frac{5}{34}+20\frac{5}{136}+5\frac{3}{4}+18\frac{7}{17}$.

3. Find the weight of each of the following parcels in pounds and a fraction of a pound, the contents being as stated:

(1) Raisins, $5\frac{1}{2}$ lb.; sultanas, $7\frac{3}{4}$ lb.; tea, $3\frac{1}{2}$ lb.; sugar, $5\frac{1}{4}$ lb.; cheese, $4\frac{7}{8}$ lb.; bacon, $2\frac{1}{8}$ lb.

(2) Rice, $5\frac{3}{8}$ lb.; tapioca, $7\frac{1}{4}$ lb.; saffron, $\frac{1}{4}$ oz. (express in pounds)[1]; lard, 2 lb. 7 oz. (express the ounces as a fraction of a pound); currants, $7\frac{3}{4}$ lb.; pepper, 11 oz.

(3) Beef, 7 lb. 10 oz.; suet, $3\frac{1}{4}$ lb.; ox tails, 6 lb. 5 oz.; dripping, $3\frac{3}{4}$ lb.; pork, 6 lb. 1 oz.

4. Find the cost of delivering each parcel by motor-van at a cost of $1\frac{1}{2}$d. per lb. delivered (charge *any* fractions of a pound as 1 lb.).

5. A draper employs six clerks, who receive annually $\frac{1}{8}$, $\frac{5}{16}$, $\frac{5}{12}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{12}$ of £384. Find the income of each clerk, and also their total income.

6. A train runs from Waterloo to Plymouth on five successive days in the times given:

Day . .	Monday.	Tuesday.	Wednesday.	Thursday.	Friday.
Time in hours.	$4\frac{4}{11}$	$4\frac{1}{3}$	$4\frac{13}{66}$	$4\frac{13}{33}$	$4\frac{37}{110}$

On which days did it travel most quickly and most slowly respectively?

[1] There are 16 oz. in 1 lb.; $\frac{1}{4}$ oz. $=\frac{1}{8}$ of 2 oz. $=\frac{1}{12}$ of 3 oz., and so $\frac{1}{64}$ of 16 oz. or of 1 lb.

7. The rations of a soldier per diem are:

$1\frac{1}{4}$ lb. fresh meat.	4 oz. jam.
$1\frac{1}{4}$ „ bread.	3 „ sugar.
4 oz. bacon.	2 „ dried vegetables.
3 „ cheese.	$\frac{5}{8}$ „ tea, coffee, or cocoa.

What is the total weight of food consumed by each soldier per diem to the nearest pound?

8. If the average price per pound for the rations referred to in Question 7 is 6d., what does it cost the nation to feed a soldier for a week (neglecting the cost of the tea)?

9. How many soldiers will 5 lb. of tea provide for per diem?

10. The weight of tin ore dug in Cornwall and offered for sale in September 1914 was, for twelve different mines, as follows:

Levant Mine	No. 1	$15\frac{1}{4}$ tons.
„	„ 1A	$15\frac{1}{16}$ „
Carn Brea and Tincroft	„ 1	$12\frac{1}{8}$ „
„ „	„ 1A	$12\frac{1}{4}$ „
E. Pool and Agar	„ 1	$12\frac{1}{16}$ „
„ „	„ 1A	$12\frac{3}{16}$ „
Tresavean		$13\frac{7}{8}$ „
Bassett Mines No. 1		$10\frac{1}{4}$ „
„ „ 1A		$10\frac{3}{64}$ „
St. Ives Consolidated		$10\frac{7}{8}$ „
„ „	No. 1A	$10\frac{1}{8}$ „
West Kitty		$12\frac{1}{8}$ „

Find the total weight of ore sold in tons and the fraction of a ton.

11. Taking the weight to the nearest ton, find the money received by the vendors at £72 a ton (Question 10).

12. The value of the export of animals, food, drink, and narcotics from Jamaica, from 1903 to 1912, in millions of dollars and fractions of a million dollars, is appended. Find the total value of the exports for the ten years in millions of dollars and the fraction of a million.

Year	1903.	1904.	1905.	1906.	1907.	1908.	1909.	1910.	1911.	1912.
Value of exports of animals, etc., in millions of dollars	$9\frac{9}{10}$	$6\frac{1}{5}$	$5\frac{3}{5}$	$7\frac{1}{2}$	$8\frac{1}{40}$	$9\frac{11}{20}$	$9\frac{1}{10}$	$10\frac{1}{5}$	$11\frac{9}{10}$	$10\frac{17}{20}$

13. According to Chinese returns the total tonnage of ships

entered and cleared, and the total tonnage of British ships entered and cleared, is given below:

Year ended 31st December.	Total Tonnage of Vessels Entered and Cleared.	Tonnage of British Vessels Entered and Cleared.
	Millions of Tons.	
1901 . .	$48\frac{2}{5}$	$26\frac{3}{20}$
1902 . .	$53\frac{99}{100}$	$26\frac{9}{16}$
1903 . .	$57\frac{2}{9}$	$28\frac{1}{10}$
1904 . .	$63\frac{7}{8}$	$32\frac{9}{10}$
1905 . .	$72\frac{3}{4}$	35
1906 . .	$75\frac{4}{9}$	$33\frac{1}{2}$
1907 . .	$80\frac{1}{16}$	$33\frac{31}{100}$
1908 . .	$83\frac{8}{9}$	$34\frac{2}{5}$
1909 . .	$86\frac{7}{10}$	34
1910 . .	$88\frac{7}{9}$	$34\frac{1}{4}$
1911 . .	$85\frac{7}{8}$	$34\frac{1}{5}$
1912 . .	$86\frac{1}{8}$	$38\frac{1}{16}$

Find— (1) The total tonnage entered and cleared.

(2) " " " " in British vessels.

(3) Write the totals to the nearest million tons, and express the second total as a fraction of the first, and state what this result represents.

14. Find from the following data the total weight of hay exported from Canada in the five years given:

Fiscal Year.	Weight of Hay Exported in Thousands of Tons.
1910 . .	191
1911 . .	$326\frac{1}{10}$
1912 . .	$784\frac{4}{5}$
1913 . .	$394\frac{1}{5}$
1914 . .	$191\frac{1}{2}$

15. Neglecting the fraction in the total weight—

(1) Find the value of the export in the five years at 8 dollars per ton.

(2) Taking a dollar as being of value 4s. (approximately), find what the total export is worth in £'s.

16. The United States imported from Canada goods as follows :

Article.	Weight (Thousands of Tons). Eleven Months ending February		
	1912.	1913.	1914.
Coal (bituminous) . .	$837\frac{2}{5}$	$1207\frac{28}{55}$	$929\frac{3}{5}$
Iron ore	$49\frac{4}{5}$	108	$180\frac{1}{3}$
Pig iron	...	$4\frac{1}{3}$	$3\frac{1}{2}$
Wood pulp . . .	$232\frac{73}{100}$	$227\frac{23}{48}$	$232\frac{7}{60}$

What is the total weight of these imports in each of the three periods?

17. Fish (fresh, or preserved by the cold process) was imported into the Australian States in 1913 as follows :

State.	Weight Imported in Hundreds of Thousands of Pounds.	Gross Value of Duty Received in Thousands of £'s.
New South Wales . .	$12\frac{1}{2}$	$5\frac{1}{4}$
Victoria . . .	$12\frac{1}{3}$	$5\frac{7}{60}$
Queensland . . .	$1\frac{17}{24}$	$\frac{4}{5}$
South Australia . .	$5\frac{1}{6}$	$2\frac{1}{5}$
Western Australia . .	$6\frac{23}{48}$	$2\frac{3}{4}$

Find—(1) The total weight of fish imported in hundreds of thousands of pounds.

(2) The total duty paid in thousands of £'s.

18. The Queensland Government expended money as under during the year 1912–13. Find the total money spent in thousands of £'s.

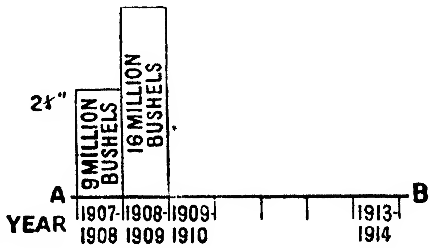
Expenditure on	Thousands of £'s.
Buildings	$52\frac{1}{3}$
Water supply	$2\frac{53}{100}$
Loans to local bodies . . .	$303\frac{4}{5}$
Advances to Central Sugar Mills .	$3\frac{1}{4}$
Wire Netting (Rabbit Boards Act) .	$17\frac{9}{25}$
Railways	$2066\frac{81}{100}$

19. The yield of wheat from New South Wales is given below. Find the total number of bushels produced in the seven years given:

Season.	Produce in Millions of Bushels.
1907–1908 . .	$9\frac{3}{20}$
1908–1909 . .	$15\frac{1}{2}$
1909–1910 . .	$28\frac{11}{20}$
1910–1911 . .	$27\frac{9}{10}$
1911–1912 . .	$25\frac{1}{30}$
1912–1913 .	$32\frac{12}{25}$
1913–1914 . .	$38\frac{1}{90}$

(Express fractions as 900ths.)

20. Take $\frac{1}{4}$ inch to represent a million bushels. Approximate each year's produce to the nearest million bushels. Represent each year's produce as suggested in figure shown. The lengths along AB are to be 1 inch. The heights of the rectangles are, for 9 million bushels $2\frac{1}{4}$ inches, for 16 million 4 inches (*i.e.* $\frac{1}{4}$ inch for 1 million bushels). Complete the figure for the other years, and *find from* IT the difference between the greatest and least production. (See Question 19.)



EXAMPLES. VIII.[1]

SUBTRACTION OF FRACTIONS

1. What is the difference between $\frac{5}{6}$ and $\frac{1}{4}$?

2. By how much does $\frac{3}{4}$ exceed $\frac{2}{5}$?

3. What remains when $2\frac{3}{4}$d. is spent out of $10\frac{1}{2}$d.?

4. If in the course of a drawing lesson a student uses $\frac{3}{30}$th of his pencil, how many thirtieths remain?

5. If a man eats 3 oz. of bread each day for his lunch, what fraction of a 2-lb. loaf does he eat in three days?

[1] Work as many of these examples as possible mentally.

6. What is the excess of $8\frac{5}{16}$ over $4\frac{1}{4}$?

7. By how much is $13\frac{5}{18}$ less than $17\frac{1}{9}$?

8. Express the two fractions in the last sum as improper fractions (1) with denominators 18, (2) with denominators 54.

9. If $\frac{3}{8}$, $\frac{1}{16}$, and $\frac{3}{32}$ of a hundredweight of coal are used successively, what fraction remains after each amount has been used?

10. From the sum of £$5\frac{1}{8}$, £$3\frac{7}{16}$, and £$15\frac{9}{10}$ take the difference between £$7\frac{5}{8}$ and £$3\frac{7}{40}$.

11. A grocer sells $\frac{2}{3}$ of his stock of tea, how many twelfths has he left?

12. If the grocer referred to in Question 11 now added to his stock as much as he had left, how many twenty-fourths would he then have had?

13. If $\frac{28}{75}$ of a tree are broken off in a gale of wind, what fraction remains standing?

14. A tank contains a quantity of oil. $\frac{5}{16}$, $\frac{7}{64}$, and $\frac{1}{8}$ are drawn off successively, and then $\frac{1}{4}$ is added. How many 64ths of the original quantity are now in the tank?

15. The London, Chatham, and Dover Railway Company paid dividends[1] as follows:

Year	1907.	1908.	1909.	1910.	1911.	1912.
Dividend in £'s per cent. .	$3\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{3}{4}$	$4\frac{1}{2}$	$4\frac{1}{2}$

Find by how much the dividend declared in 1912 is greater than that in each of the other years.

16. In Question 3, p. 55, find which of the three parcels is the heaviest, and by how much it is heavier than each of the others.

17. If from a ream of foolscap $\frac{1}{15}$, $\frac{3}{5}$, and $\frac{4}{45}$ are taken successively, what fraction will remain?

18. A motor-van is laden with 10 cwt. of goods in a number of parcels. They are delivered, and the weight carried is reduced as follows: first by $\frac{2}{65}$, then successively by $\frac{3}{13}$, $\frac{1}{5}$, $\frac{4}{26}$, $\frac{1}{26}$, $\frac{2}{13}$, $\frac{7}{130}$ of the total load. What fraction of the total weight has then been delivered, and what remains?

19. Express the results of the previous question (1) in 65ths, (2) in 390ths, and (3) in 13ths.

20. The estimated revenue of Belgium for 1912 was 28155,

[1] Section XXII., § 354.

and the expenditure 28137 thousand £'s sterling. Express the expenditure as a fraction of the revenue.

21. The population and area of Egypt (proper) and of France are :

	Egypt.	France.
Population . .	11290000	39600000
Area	363000 square miles.	207000 square miles.

Find—(1) The number of people per square mile in each country.

(2) Express the population per square mile in Egypt as a fraction of that in France, and state what information the result gives you.

22. In the previous question, how many people would there be to a square mile in France for every one to a square mile in Egypt?

23. The exports from the countries given below to the United Kingdom in 1912 were of the value shown :

Country.	Exports to the United Kingdom in Millions of £'s Sterling.
Germany . .	$70\frac{9}{125}$
France . . .	$45\frac{1}{2}$
Norway . . .	$6\frac{9}{10}$
Italy . . .	$8\frac{1}{5}$
Japan . . .	$3\frac{23}{25}$

Find by how much the exports from Germany exceeded those of each of the other countries.

24. Taking the data of the previous question—

(1) Write the value of the exports to the nearest million pounds for each country.

(2) Take $\frac{1}{10}$ of an inch to represent £1000000, and draw a series of horizontal lines, one below the other, to represent by their lengths the value of the exports.

25. Find from the diagram drawn in the last question the difference between the exports from Japan and those from Norway in the year 1912.

B. (1) Addition of Fractions: More Difficult

L.C.M.

61. In the questions already answered it has been fairly easy to express the fractions with a common denominator, but with larger denominators this is not always possible.

The student will have seen that it is best to use the **Least** denominator, for it involves less work than a greater one.

We find therefore the *Least Common Denominator*,—the least number into which all the denominators will divide,—which is often called the Least Common Measure and denoted, for brevity, by the letters L.C.M.

It is easy to see that the use of the term measure is justifiable, for if we had 3 pints, 5 pints, and 6 pints, the smallest **measure** which would contain each of them an exact number of times—that is, the **Least *Common* Measure**—would be one capable of holding 30 pints.

62. To add the fractions

$$\tfrac{1}{15},\ \tfrac{2}{45},\ \tfrac{7}{360},\ \tfrac{19}{120},$$

we find the L.C.M. as follows:

1. Factorise each denominator—

$$15 = 3 \times 5\,;$$
$$45 = 3^2 \times 5\,;$$
$$360 = 2^3 \times 3^2 \times 5\,;$$
$$120 = 2^3 \times 3 \times 5.$$

2. Take the product of the **highest powers** of every prime factor, namely, $2^3 \times 3^2 \times 5$, as the L.C.M.

That this must give the result is obvious, for if we take (*e.g.*) the HIGHEST power of 2 which occurs, all the lower powers *must* divide into it.

3. Multiply up and express the L.C.M. as a whole number—

$$2^3 \times 3^2 \times 5 = 360.$$

63. We set out the sum:

$$\tfrac{1}{15} + \tfrac{2}{45} + \tfrac{7}{360} + \tfrac{19}{120}$$

with its denominator 360, thus:

$$\tfrac{1\times 24}{15\times 24} + \tfrac{2\times 8}{45\times 8} + \tfrac{7\times 1}{360\times 1} + \tfrac{19\times 3}{120\times 3} \quad . \quad . \quad . \quad (a)$$

$$= \tfrac{24}{360} + \tfrac{16}{360} + \tfrac{7}{360} + \tfrac{57}{360} \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

$$= \tfrac{24+16+7+57}{360}$$

$$= \tfrac{104}{360} = \tfrac{52}{180} = \tfrac{13}{45}.$$

With very little practice lines (a) and (β) can be omitted.

64. If there are whole numbers they are added first, and placed before the fraction, thus:

$$3\tfrac{1}{15}+8\tfrac{2}{45}+12\tfrac{7}{360}+15\tfrac{19}{120}$$
$$=38\tfrac{24+16+7+57}{360}=38\tfrac{104}{360}$$
$$=38\tfrac{13}{45}.$$

EXAMPLES. IX.

1 Add together—

(1) $7\tfrac{1}{6}+8\tfrac{5}{16}+21\tfrac{7}{15}+\tfrac{3}{5}$.

(2) $21\tfrac{4}{5}+8\tfrac{5}{6}+9\tfrac{24}{35}+8\tfrac{14}{15}$.

(3) $7\tfrac{1}{3}+25\tfrac{7}{8}+6\tfrac{2}{9}+13\tfrac{4}{15}$.

(4) $9\tfrac{4}{7}+13\tfrac{5}{6}+12\tfrac{7}{10}+3\tfrac{5}{8}$.

(5) $6\tfrac{7}{12}+14\tfrac{13}{14}+\tfrac{5}{7}+12\tfrac{13}{15}$.

(6) $12\tfrac{13}{22}+18\tfrac{7}{9}+15\tfrac{5}{11}+21\tfrac{11}{24}$.

(7) $15\tfrac{7}{15}+21\tfrac{17}{22}+8\tfrac{7}{12}+5\tfrac{1}{3}$.

(8) $18\tfrac{10}{21}+56\tfrac{7}{15}+8\tfrac{13}{35}+3\tfrac{3}{7}$.

(9) $24\tfrac{17}{24}+45\tfrac{19}{56}+86\tfrac{7}{32}+15\tfrac{1}{8}$.

(10) $36\tfrac{19}{25}+53\tfrac{7}{40}+128\tfrac{3}{65}+5\tfrac{9}{26}$.

2. Find the sum of—

(1) $25\tfrac{12}{19}+13\tfrac{8}{171}+25\tfrac{3}{209}$.

(2) $53\tfrac{9}{29}+6\tfrac{4}{21}+5\tfrac{6}{7}+8\tfrac{43}{261}$.

(3) $28\tfrac{3}{87}+15\tfrac{1}{185}+1\tfrac{2}{3}$.

(4) $21\tfrac{14}{27}+161\tfrac{29}{33}+58\tfrac{10}{121}+3\tfrac{5}{11}$.

(5) $3\tfrac{4}{5}+12\tfrac{9}{10}+15\tfrac{13}{15}+5\tfrac{4}{135}$.

(6) $7\tfrac{5}{18}+9\tfrac{8}{17}+3\tfrac{1}{9}+25\tfrac{8}{153}$.

(7) $33\tfrac{15}{22}+79\tfrac{17}{44}+16\tfrac{3}{4}+5+32\tfrac{18}{121}$.

(8) $23\tfrac{5}{18}+38\tfrac{15}{34}+4\tfrac{7}{60}+28\tfrac{7}{153}$.

3. On opening business a merchant spends his capital[1] as follows:

Purchase of lease	. . .	$\tfrac{1}{12}$
" fittings	. . .	$\tfrac{1}{18}$
" stock	. . .	$\tfrac{5}{84}$
" motor-vans	. .	$\tfrac{1}{63}$

What fraction of his capital is absorbed in this expenditure?

[1] Section XVII. B. (1), p. 366.

4. In the first three days in the week a grocer takes $\frac{16}{123}$, $\frac{2}{41}$, and $\frac{1}{20}$ of his total weekly takings. On the last three days he takes $\frac{1}{8}$, $\frac{8}{27}$, $\frac{11}{48}$, the amount not included being on credit.

Find—(1) The fraction of the whole which is received on the first three days.
(2) The fraction received on the last three.
(3) The day when he took most.
(4) The day when he took least.

5. The area of the five largest London boroughs is given below. Find their total area to the nearest thousand acres.

Borough.	Area in Thousands of Acres.
Wandsworth . .	$9\frac{5}{48}$
Woolwich . .	$8\frac{5}{14}$
Lewisham . .	$7\frac{1}{20}$
Camberwell . .	$4\frac{3}{16}$
Lambeth . .	$4\frac{1}{10}$

6. There are seven coaches of the same size on the last "Workman" train from a suburban station to Baker Street Station. On a foggy morning it was observed that the first coach had $\frac{1}{2}$ of its ordinary complement too many; the second, $\frac{1}{3}$; the third, $\frac{1}{9}$; the fourth, $\frac{3}{8}$; the fifth, $\frac{7}{18}$; the sixth, $\frac{1}{4}$; the seventh, $\frac{7}{36}$. By how many times the total normal complement is there an excess of passengers?

7. A firm of contractors for hospital requisites is instructed to provide goods in accordance with the details given below:

Equipment.	Fraction of Total Expenditure Allowed.
Wards—Beds	$\frac{2}{15}$
Bedding	$\frac{1}{72}$
Tiled-top tables . . .	$\frac{1}{171}$
Chairs and lockers . . .	$\frac{1}{90}$
Toys for Children's Ward .	$\frac{1}{342}$

What fraction of the total expenditure is thus employed?

8. Find the total revenue (for the ten years given) of British Honduras from the following table:

Years ending 31st December.	Revenue in Thousands of Dollars.
1903 . . .	$259\frac{3}{5}$
1904 . . .	$301\frac{3}{14}$
1905 . . .	$309\frac{79}{84}$
1906 . . .	$332\frac{11}{98}$
1907 . . .	$391\frac{2}{3}$
1908 . . .	$395\frac{3}{18}$
1909 . . .	$360\frac{9}{10}$
1910 . . .	$395\frac{23}{28}$
1911 . . .	$459\frac{3}{10}$
1912 . . .	$498\frac{3}{10}$
Total .	

9. Taking the value of the result in the last question to the nearest thousand, and reckoning 4s. to a dollar, find the value of the revenue—(1) In shillings, and (2) in £'s sterling.

10. The value of the exports and imports for the same period were 22 and 23 million dollars, respectively. (See Question 8.) Express these in £'s sterling at 4s. to the dollar, and the value of the imports as a fraction of the exports.

11. The values of exports from South Africa to the United Kingdom, the United States, and to Canada, for 1913, are given in thousands of dollars:

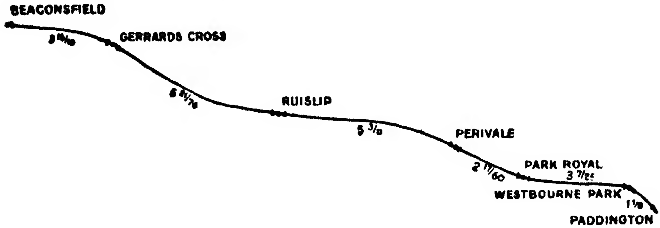
Article.	To United Kingdom.	To United States.	To Canada.
Aloes	$23\frac{1}{21}$	$4\frac{1}{10}$	—
Diamonds	$58332\frac{13}{40}$	—	—
Feathers	$11640\frac{13}{100}$	$2166\frac{13}{90}$	$78\frac{83}{100}$
Fish	$250\frac{1}{5}$	$35\frac{59}{100}$	—
Gold (raw)	$182935\frac{1}{10}$	—	—

(1) Find the total exports to each of the three countries.
(2) Find the total value of each export.

12. Offer some explanation of the fact that the United Kingdom is, in the commodities mentioned in Question 11, a so much larger customer of South Africa than the United States or Canada.

5

13. Find the distance to the nearest mile from Paddington to Beaconsfield (one of the centres of the Penn country), given the following plan with the distances in miles:



B. (2) Subtraction of Fractions

65. Subtraction of fractions is performed just as addition, except that the numerators are subtracted, where necessary, and not added.

Thus—

Example 1.—$3\frac{5}{8} - 2\frac{3}{32}$.

Expressing the mixed numbers as improper fractions, we have:

$$3\tfrac{5}{8} - 2\tfrac{3}{32} = \tfrac{29}{8} - \tfrac{67}{32} = \tfrac{116-67}{32} = \tfrac{49}{32} = 1\tfrac{17}{32}.$$

66. The student will usually find it better to express mixed numbers as improper fractions at first, but after a little experience he may proceed as follows:

Example 2.—**Find the difference between $5\frac{3}{8}$ and $4\frac{5}{8}$.**

$$5\tfrac{3}{8} - 4\tfrac{5}{8}, \text{ or } (5+\tfrac{3}{8}) - (4+\tfrac{5}{8})$$
$$= 1 + \tfrac{3}{8} - \tfrac{5}{8}, \text{ or } 1 + \tfrac{3-5}{8}.$$

Now the 5 cannot be subtracted from the 3, and the student will not think he may take the 3 from the 5! We therefore call the one whole number $\frac{8}{8}$, and $\frac{8}{8} + \frac{3}{8} - \frac{5}{8} = \frac{11}{8} - \frac{5}{8} = \frac{6}{8} = \frac{3}{4}$.

Example 3.—**Find the difference between $15\frac{9}{16}$ and $6\frac{3}{4}$.**

Again— $15\frac{9}{16} - 6\frac{3}{4}$

$$= 9\tfrac{9-12}{16}, \text{ or } 8 + \tfrac{16}{16} + \tfrac{9}{16} - \tfrac{12}{16} = 8 + \tfrac{25}{16} - \tfrac{12}{16}$$
$$= 8\tfrac{25-12}{16}$$
$$= 8\tfrac{13}{16}.$$

EXAMPLES. X.

1. Find the difference between the following mixed numbers:

(1) $7\frac{1}{6}$ and $8\frac{5}{48}$. (2) $21\frac{7}{15}$ and $3\frac{169}{180}$. (3) $21\frac{4}{5}$ and $28\frac{1}{6}$.
(4) $34\frac{4}{7}$ and $30\frac{1}{84}$. (5) $15\frac{7}{15}$ and $21\frac{5}{22}$. (6) $8\frac{7}{12}$ and $6\frac{7}{22}$.
(7) $18\frac{1}{21}$ and $16\frac{4}{15}$. (8) $86\frac{83}{96}$ and $15\frac{7}{8}$. (9) $30\frac{19}{25}$ and $53\frac{19}{40}$.
(10) $138\frac{3}{65}$ and $5\frac{9}{13}$. (11) $53\frac{9}{28}$ and $6\frac{19}{21}$. (12) $5\frac{6}{7}$ and $8\frac{43}{77}$.
(13) $7\frac{5}{18}$ and $9\frac{8}{17}$. (14) $3\frac{8}{9}$ and $25\frac{8}{153}$. (15) $33\frac{15}{22}$ and $79\frac{5}{48}$.
(16) $16\frac{3}{4}$ and $32\frac{18}{121}$.

2. What is the difference between $8\frac{1}{792}$ and $6\frac{12}{121}$?

3. Add together $20\frac{15}{16}$, $150\frac{9}{154}$, and $125\frac{3}{770}$, and subtract $58\frac{5}{7}$ from the sum.

4. A Canadian dollar is worth about $4\frac{4}{37}$ shillings in English money, and a Newfoundland dollar $4\frac{1}{6}$ shillings. Find the difference in value in shillings English money between the two coins.

5. A gasometer has $\frac{29}{70}$ of its normal volume of gas on Friday evening. On Saturday $\frac{3}{8}$ are added. On Sunday $\frac{19}{28}$ are consumed. What fraction remains in it on Monday?

6. What is the sum of the following fractions of a shilling: $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{15}{19}$, $\frac{28}{57}$?

7. Arrange the fractions in the last question in descending order of magnitude.

8. If a franc is worth $9\frac{50}{99}$ pence and a mark $11\frac{73}{98}$ pence, find the difference in value between the French and German coins.

9. What difference is there between a franc and a shilling and between a mark and a shilling?

10.

Year	1908.	1909.	1910.	1911.
The average price of a 4-lb. loaf in England and Wales . . .	$5\frac{3}{5}$d.	$6\frac{1}{25}$d.	$5\frac{83}{105}$d.	$5\frac{57}{100}$d.

From the above table find the increase or decrease from each year given to the one next following it.

11. Show, with the help of rectangles, $\frac{1}{2}$ inch wide, how the price of bread in Lancashire and Cheshire varied in the years

given in the previous question, the price per 4-lb. loaf being $5\frac{4}{5}$, $6\frac{1}{5}$, $5\frac{9}{10}$, $5\frac{3}{5}$ pence. (Take a rectangle 1 inch long to represent 1 penny.)

12. Find by how much the amount received by the British Government for Probate Duty in 1907 in the United Kingdom exceeds that received in 1913, given that the duty was $79\frac{90}{343}$ thousand £'s in 1907 and $36\frac{30}{49}$ in 1913.

13. The total Excise receipts from the United Kingdom in 1912 were $38\frac{1}{4}$ million £'s sterling, and, in 1913, $38\frac{8}{83}$. What is the excess of the former over the latter ?

14. The average wage of a railway employee in 1902 was $24\frac{4}{63}$ shillings a week, and in 1912, $27\frac{1}{81}$. By how many shillings a week has the weekly wage increased in the decade ?

15.

City.	Population.	Rateable Value.
Leeds . . .	446 thousand	2212 thousand £'s.
Belfast . . .	392 ,,	1558 ,, ,,

From the details given above find—

(1) The rateable value of each city per thousand of the population (in thousands of £'s).

(2) The difference between the rateable value per thousand of the population of the two cities.

(3) For which of the two the rateable values is the higher per thousand of the population.

EXAMPLES. XI.

1. The foreign trade of China (for 1912) is given as follows :

Great Britain . . .	$\frac{1}{8}$	Japan	$\frac{1}{6}$
Hong-Kong . . .	$\frac{3}{10}$	United States . . .	
India	$\frac{1}{10}$	Europe (including Russia)	$\frac{11}{50}$
Other British Possessions	$\frac{1}{50}$	Other countries . .	$\frac{1}{50}$

What fraction of Chinese trade is with the United States ?

2. The latitude of Edinburgh is $55\frac{29}{30}°$ N. ; that of London, $51\frac{1}{2}°$ N. ; that of Constantinople, $40\frac{9}{10}°$ N. Find the difference between the latitude of London and that of Edinburgh, and of Constantinople.

3. The latitude of Valencia, on the east coast of Spain, is $39\frac{3}{5}°$ N. How many degrees is the city south of Edinburgh ?

4. In Question 4, p. 64, the daily takings of a grocer were

given. Find by what fraction of his total weekly takings his greatest daily receipts exceeded his smallest.

5. Imports to the United Kingdom (merchandise only) were for the years 1700, 1800, and 1900 (in millions of £'s) $4\frac{3}{5}$, $30\frac{1}{2}$, $523\frac{76}{101}$. What is the excess of the imports in 1900 over each of the others given, and over the sum of the others?

6. By how much did the increase of imports in the century 1800–1900 exceed the increase from 1700–1800? What does this imply?

7. The number of students in training colleges in 1912 is as follows:

Country.	Number of Women Students.	Total Number.
England and Wales . .	3804	5908
Scotland	2063	2570
Ireland	660	1160

What fraction of the total number of students in each country were women in the year given?

8. In the last question arrange the fractions in order of magnitude. What conclusions can be drawn from the result as regards the personnel of the colleges in 1912?

9. The revenue of British Honduras was $395\frac{3}{16}$ thousand dollars in 1908 and $395\frac{11}{13}$ in 1910. Did the revenue increase or decrease in the two years?

10. In the year 1912 we exported to Egypt $9\frac{3}{5}$ million £'s worth of merchandise and imported $25\frac{69}{80}$ million £'s worth. Find the excess of the value of the imports over that of the exports.

11. In 1912, $10\frac{4}{5}$ million £'s worth of shipping entered the port of London and $8\frac{72}{95}$ million £'s worth cleared. How much greater value was entered than cleared?

12. In the same year, 1912, $7\frac{25}{98}$ million £'s worth was entered at Liverpool. What amount was entered at London and at Liverpool together? (See Question 11.)

13.

Year	1840.	1850.	1860.	1870.	1880.	1890.	1900.	1910.
Average price of British wheat in the United Kingdom in shillings per quarter . .	$66\frac{1}{3}$	$40\frac{1}{4}$*	$53\frac{1}{4}$*	$46\frac{11}{12}$	$44\frac{1}{3}$	$31\frac{11}{12}$	$26\frac{11}{12}$	$31\frac{2}{3}$

* There was a duty of 1s. per quarter in these years.

Draw rectangles $\frac{1}{2}$ inch wide and of a length which will represent the prices given to the nearest shilling, taking $\frac{1}{10}$ inch to represent 1s. Find from your diagram the difference between the prices in 1840 and 1870, and 1840 and 1900. Compare the results obtained from the diagram with those obtained by subtracting the figures given for the particular years.

14. In the previous question find for which ten years the variation in price was greatest. Tabulate the work thus:

Year.	Average Price of British Wheat in the United Kingdom in Shillings per Quarter.	Increase or Decrease for the Ten Years.
1840	$66\frac{1}{3}$	...
1850	$40\frac{1}{4}$	$-20\frac{1}{12}$
1860	$53\frac{1}{4}$	+
Etc.	Etc.	Etc.

SECTION V

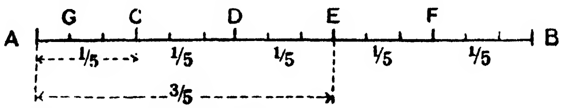
MULTIPLICATION AND DIVISION OF FRACTIONS

A. Multiplication

67. 5×3 means 5 taken 3 times.

$\frac{1}{5} \times 3$ „ $\frac{1}{5}$ „ 3 „

If you take AC, which is $\frac{1}{5}$ of the line AB, three times (*i.e.* $\frac{1}{5} \times 3$),



you have AE, which is $\frac{3}{5}$ of AB.

In the same way, $\frac{4}{5} \times 3 = \frac{12}{5} = 2\frac{2}{5}$.

The student should draw a line containing 12 fifths of AB, and show that it really contains twice AB and $\frac{2}{5}$ of AB, *i.e.* $2\frac{2}{5}$ AB.

Now $\frac{4}{5} \times 3$, which is really $\frac{4}{5} \times \frac{3}{1} = \frac{4 \times 3}{5 \times 1} = 2\frac{2}{5}$, that is to say, **the product of two fractions is given by the product of the numerators over the product of the denominators;**

$$\therefore \frac{4}{5} \times \frac{1}{3} = \frac{4 \times 1}{5 \times 3} = \frac{4}{15},$$

and, in like manner, $\frac{5}{8} \times 9 = \frac{5}{8} \times \frac{9}{1} = \frac{5 \times 9}{8 \times 1} = \frac{45}{8} = 5\frac{5}{8}$,

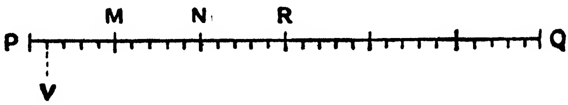
and $\frac{5}{8} \times \frac{1}{9} = \frac{5}{72}$.

68. From the Fig. of § 67 we have—

$$AG = \tfrac{1}{3} \text{ of } \tfrac{1}{5} \text{ of } AB; \text{ and } AG \text{ also equals } \tfrac{1}{15}\, AB;$$

$$\therefore\ \tfrac{1}{3} \text{ of } \tfrac{1}{5} = \tfrac{1}{15}, \text{ but } \tfrac{1}{3} \times \tfrac{1}{5} = \tfrac{1}{15};$$

and so $\tfrac{1}{3}$ of $\tfrac{1}{5}$ is just the same as $\tfrac{1}{3} \times \tfrac{1}{5}$.



Also, $PM = \tfrac{1}{6}$ of PQ,

$PV = \tfrac{1}{5}$ of $PM = \tfrac{1}{5}$ of $\tfrac{1}{6}$ of PQ;

but PV clearly equals $\tfrac{1}{30}\, AB$;

$\therefore\ \tfrac{1}{5}$ of $\tfrac{1}{6} = \tfrac{1}{30}$.

Hence generally $\tfrac{1}{5}$ of $\tfrac{1}{6} = \tfrac{1}{5} \times \tfrac{1}{6}$, and **"of" means "multiply."**

69. On p. 27 we showed how to square and how to cube whole numbers, and we now draw attention to the fact that fractions can be similarly treated.

Thus:

$$(\tfrac{2}{3})^2 = \tfrac{2}{3} \times \tfrac{2}{3} = \tfrac{4}{9},$$

and

$$(\tfrac{5}{8})^3 = \tfrac{5}{8} \times \tfrac{5}{8} \times \tfrac{5}{8} = \tfrac{125}{512}.$$

Also,

$$\sqrt{\tfrac{4}{16}} = \frac{\sqrt{4}}{\sqrt{16}} = \tfrac{2}{4} = \tfrac{1}{2}, \text{ or } \sqrt{\tfrac{4}{16}} = \sqrt{\tfrac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \tfrac{1}{2},$$

and

$$\sqrt[3]{\tfrac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \tfrac{3}{5}.$$

70. The student should study the following examples:

EXAMPLE 1.—**What is the value of $\tfrac{5}{8}$ of $\tfrac{28}{35}$?**

NOTE.—**When fractions are associated with one another by a multiplication sign or by "of,"** ***any*** **numerator and** ***any*** **denominator can be divided by the same number. (This** ***cannot*** **be done when a + or a − intervenes.)**

$\therefore$ 5 and 35 can be divided by 5, and 8 and 28 by 4, thus:

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \text{ of } \frac{\overset{7}{\cancel{28}}}{\underset{7}{\cancel{35}}}$$

Then the 7's in numerator and denominator cancel—that is, divide by the same number—and the result is $\tfrac{1}{2}$.

EXAMPLE 2.—**Simplify** $\frac{5}{8} \times \frac{12}{35} \times \frac{24}{96}$.

1. Cancel (1) 5 into 35, (2) 8 into 24, and then (3) 12 into 96, for they divide exactly.

2. Perform any more cancelling possible.

$$\frac{\cancel{5}}{\cancel{8}} \times \frac{\cancel{12}}{\underset{7}{\cancel{35}}} \times \frac{\overset{3}{\cancel{24}}}{\underset{8}{\cancel{96}}}$$

3. Multiply all the numerators together and then all the denominators, and express the result as a fraction in its lowest terms.

Product of numerators $= 1 \times 1 \times 3 = 3$;

„ „ denominators $= 1 \times 7 \times 8 = 56$;

$\therefore$ the result is $\frac{3}{56}$.

Note carefully that the numerator is $1 \times 1 \times 3$, not $0 \times 0 \times 3$, for 5 into 5 goes 1, and into 35 goes 7, but we just say 5 into 35 goes 7. We ought really to put a 1 above the 5 and the 12, but we save work by leaving it out and merely remembering that it should be there.

EXAMPLE 3.—**Simplify** $3\frac{1}{8} \times 5\frac{1}{5} \times 2\frac{1}{2}$.

Express as improper fractions and **we have**—

$$\frac{25}{8} \times \frac{\overset{13}{\cancel{26}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}} = \frac{25 \times 13}{8} = \frac{325}{8} = 40\frac{5}{8}.$$

EXAMPLE 4.—$(\frac{5}{8}$ of $\frac{10}{12}) + (\frac{3}{5}$ of $\frac{5}{6})$.

In questions containing a $\times$ sign and a $+$ or a $-$ sign it is universally **agreed to** work the parts connected by "of" or the $\times$ sign ***first***, and then to add or subtract.

UNDER NO CIRCUMSTANCES COULD WE CANCEL ACROSS A + OR A − SIGN.

Thus, in the above example, 8 and 10 or 3 and 6 can be cancelled, but 3 and 12 can **not**, for that would mean crossing over the + sign.

Hence—

$$\left(\frac{5}{\underset{4}{\cancel{8}}} \text{ of } \frac{\overset{5}{\cancel{10}}}{12}\right) + \left(\frac{\cancel{3}}{\cancel{5}} \text{ of } \frac{\cancel{5}}{\underset{2}{\cancel{6}}}\right) = \frac{25}{48} + \frac{1}{2} = \frac{25 + 24}{48} = \frac{49}{48} = 1\frac{1}{48}.$$

EXAMPLE 5.—**Had we taken**

$$\left(\tfrac{5}{8} \text{ of } \tfrac{10}{12}\right) - \left(\tfrac{3}{5} \text{ of } \tfrac{5}{6}\right),$$

the result would have been

$$\tfrac{25}{48} - \tfrac{1}{2}$$
$$= \tfrac{25-24}{48} = \tfrac{1}{48}.$$

EXAMPLES. XII.

1. What is the value of each of the following?

(1) $\frac{3}{5} \times \frac{15}{21}$. (2) $\frac{4}{10} \times \frac{25}{32}$. (3) $\frac{5}{12} \times \frac{36}{25} \times \frac{50}{82}$.

(4) $\frac{8}{72} \times \frac{91}{64} \times \frac{54}{84}$. (5) $\frac{25}{30}$ of $\frac{1}{2}$ (6) $\frac{8}{26}$ of $\frac{9}{36}$.

(7) $\frac{23}{25}$ of $\frac{500}{207}$. (8) $\frac{5}{8} \times \frac{96}{100} \times \frac{75}{108}$. (9) $\frac{25}{85} \times \frac{153}{230} \times \frac{35}{36}$.

(10) $\frac{8}{46} \times \frac{25}{174} \times \frac{92}{350} \times \frac{87}{23}$. (11) $\frac{10}{35} \times \frac{55}{960} \times \frac{777}{222}$.

(12) $2\frac{1}{2} \times 3\frac{3}{4} \times 4\frac{2}{15}$. (13) $3\frac{5}{8} \times \frac{21}{45} \times 4\frac{28}{56} \times 2\frac{1}{29}$.

(14) $2\frac{1}{8} \times 3\frac{24}{48} \times 2\frac{1}{4} \times 5\frac{3}{153}$. (15) $8\frac{5}{12} \times \frac{3}{909} \times 25\frac{8}{60}$ of $\frac{6}{1508}$.

(16) $\frac{5}{8}$ of $\frac{24}{30} + \frac{6}{7}$ of $\frac{28}{84}$. (17) $3\frac{1}{2}$ of $\frac{3}{49} + \frac{5}{12}$ of $3\frac{3}{7}$.

(18) $2\frac{1}{2} \times \frac{3}{4} - \frac{7}{8}$ of $\frac{4}{5}$. (19) $5\frac{3}{8} + \frac{3}{119}$ of $8\frac{1}{2} + \frac{5}{8}$ of $\frac{15}{25}$

(20) $3\frac{1}{6} + 5\frac{1}{8} \times 7\frac{1}{41} + \frac{1}{6} \times 3\frac{1}{4}$. (21) $(\frac{3}{5})^2$, $(\frac{8}{10})^2$, $(\frac{25}{75})^2$.

(22) $(\frac{14}{15})^2 - (\frac{5}{6})^2$. (23) $(\frac{1}{4})^3 \times (\frac{2}{5})^2$.

(24) $(\frac{2}{3})^3 + (\frac{3}{5})^2 + 2\frac{1}{4} \times \frac{1}{5}$. (25) $(\frac{3}{5} - \frac{1}{10})^2 \times (\frac{1}{5} + \frac{3}{10})^2$.

2. An estate is worth £785, what are $\frac{1}{5}$ and $\frac{4}{5}$ worth?

3. A plantation contains 9882 trees, and a timber merchant purchases $\frac{2}{27}$ of it. How many trees does he get?

4. If the wholesale value of the trees in the previous question is £50139, what should the timber merchant pay for his purchase?

5. In the purchase of a picture for 60000 guineas the nation contributed $\frac{1}{9}$, and then three private individuals contributed $\frac{1}{8}$ of the remainder of the purchase money each. What amount remained to be raised by subscription?

6. A farmer owns 8950 acres; $\frac{17}{25}$ of this is arable land. What is the area of the remainder?

7. If 179 acres of the farm mentioned in Question 6 are valued at £2400, what is the value of the whole farm?

8. The goodwill of a confectionery business is valued at £3600, and three men agree to purchase it, the first taking $\frac{1}{8}$, the second $\frac{5}{9}$ of the remainder, and the third what then remains. Find what each pays.

9. What is a tenth of $\frac{3}{7}$ of a puncheon of brandy of 105 gallons?

10. A clerk earns $\frac{7}{12}$ of £186 in six months. What does he earn in a year at the same rate?

11. A truck and $\frac{3}{8}$ of its normal load weigh $17\frac{3}{8}$ tons; the truck weighs $8\frac{3}{4}$ tons. What is its full load?

12. The engines of a warship can develop 36000 horse-power (written H.P.) on forced draught, but the captain usually runs her on $\frac{95}{108}$ of this power. What is the normal running horse-power?

13. A powerful locomotive consumes $4\frac{1}{2}$ tons of coal on a 220-mile run. Reckoning 20 cwt. to the ton, find the coal consumption per mile as a fraction of a cwt.

14. An Axminster stair carpet costs 4s. 11d. a yard, and a Wilton pile of the same width 6s. 11d. Find how many times dearer the latter is than the former.

15. Brass, stamped, polished, reeded stair-nosing $2\frac{1}{4}$ inches over all costs 1s. $1\frac{1}{2}$d. a foot; brass, polished, chequered stair-nosing of the same size costs 1s. 6d. a foot. Express the cost of 3 feet of the former as a fraction of the cost of 5 feet of the latter.

16. A fumed oak bedstead 4 feet 6 inches long costs £$3\frac{3}{8}$, and a mahogany one £$5\frac{1}{8}$. How much dearer is the latter than the former?

17. An Axminster stair carpet is $20\frac{1}{2}$ yards long, a Wilton $30\frac{1}{4}$; it costs 2s. 3d. a yard to clean the former and 1s. 6d. a yard to clean the latter. Which costs the more to clean, and by how much? (Answer in pence.)

18. Express $\frac{5}{8}$ of 1 cwt. as a fraction of 10 cwt.

19. If a purchaser spends $\frac{5}{6}$ of £5 on a gramophone and $\frac{3}{8}$ of £1 on records, what fraction is the latter amount of the former?

20. Take a piece of squared paper with 10 squares to 1 inch, draw on it a rectangle 1 inch wide and 10 inches long, and prove that $\frac{3}{5}$ of $\frac{1}{4} = \frac{3}{20}$.

21. In the same manner as in Question 20, only with a rectangle 8 inches long, show that $\frac{5}{8}$ of $\frac{3}{4} = \frac{15}{32}$.

22. Draw a rectangle 7 inches by 2 inches, and by dividing it up and cutting out the appropriate parts show that $\frac{3}{7}$ of $\frac{2}{3} = \frac{6}{21}$, and that $\frac{6}{21} = \frac{2}{7}$.

23. If $\frac{2}{7}$ of 40 guineas is spent on a plate-glass mirror, $\frac{5}{11}$ of £99 on a sideboard, $\frac{1}{20}$ of £15 on ornaments for a dining-room, what fraction of £100 remains?

24. What fraction is that from which if $\frac{2}{3}$ of $\dfrac{3 - 1\frac{1}{4}}{2\frac{2}{5}}$ be taken and the remainder divided by $\dfrac{5\frac{1}{8}}{16\frac{2}{5}}$ the result is $\frac{1}{3}$?

B. Division of Fractions

71.

AB is divided into 8 parts,

and AE $= \frac{1}{8}$ of AB ;

$\therefore$ AB $\div 8 = \frac{1}{8}$ of AB.

If AB were 3 inches long,

then, $3 \div 8 = \frac{1}{8}$ of $3 = \frac{3}{8}$ inch.

If AB were $\frac{3}{4}$ inch long,

then, $\frac{3}{4} \div 8 = \frac{1}{8}$ of $\frac{3}{4} = \frac{3}{32}$ inch.

In precisely the same way, since $8 = \frac{16}{2}$,

we have $\frac{3}{4} \div \frac{16}{2} = \frac{2}{16}$ of $\frac{3}{4} = \frac{1}{8}$ of $\frac{3}{4} = \frac{3}{32}$.

72. Now $\frac{1}{8}$ is called the reciprocal of 8, and $\frac{2}{16}$ the reciprocal of $\frac{16}{2}$.

Hence from these, and all similar illustrations, we conclude that to divide by any number we *multiply* by its reciprocal.

Hence, $\frac{5}{8} \div 12 = \frac{5}{8} \times \frac{1}{12} = \frac{5}{96}$;

also, $\frac{5}{8} \div \frac{1}{12} = \frac{5}{8} \times \frac{12}{1} = 7\frac{1}{2}$.

for $\frac{12}{1}$ or 12 is the reciprocal of $\frac{1}{12}$;
just as $\frac{1}{12}$ is the reciprocal of 12,
or $\frac{1}{12}$ is 12 **inverted**, and 12 is $\frac{1}{12}$ **inverted.**

After the divisor has been **inverted**, we proceed just as in multi plication, thus :

EXAMPLE 1.—**Simplify** $\frac{5}{16} \div 1\frac{1}{48}$.

$$\frac{5}{16} \div 1\frac{1}{48}$$

$$= \frac{5}{16} \div \frac{49}{48} = \frac{5}{\cancel{16}} \times \frac{\cancel{48}^{\,3}}{49}$$

$$= \frac{15}{49}.$$

Note.—The student should note that the 48 is cancelled as one number although, in printing, we have to use two strokes, one for the 4 and the other for the 8. This applies throughout the book.

EXAMPLE 2.—**Evaluate** $\left(\frac{5}{8} \text{ of } \frac{16}{25}\right) \div 5$

$$\left(\frac{5}{8} \text{ of } \frac{16}{25}\right) \div 5$$

$$= \frac{\cancel{5}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{25}}} \times \frac{1}{5} = \frac{2}{25}.$$

EXAMPLE 3.—**Simplify** $\left(\frac{3}{5} \text{ of } \frac{15}{16}\right) + \left(\frac{4}{5} \div \frac{8}{35}\right)$.

$$\left(\frac{3}{5} \text{ of } \frac{15}{16}\right) + \left(\frac{4}{5} \div \frac{8}{35}\right)$$

$$= \left(\frac{3}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{16}\right) + \left(\frac{\cancel{4}}{\cancel{5}} \times \frac{\overset{7}{\cancel{35}}}{\underset{2}{\cancel{8}}}\right)$$

$$= \frac{9}{16} + \frac{7}{2}$$

$$= \frac{9+56}{16} = \frac{65}{16} = 4\tfrac{1}{16}.$$

EXAMPLE 4.—**Simplify** $\dfrac{8\frac{1}{2}}{5\frac{1}{4}}$ (*i.e.* $8\frac{1}{2} \div 5\frac{1}{4}$).

$$\frac{8\frac{1}{2}}{5\frac{1}{4}} = \frac{\frac{17}{2}}{\frac{21}{4}} = \frac{17}{\cancel{2}} \times \frac{\overset{2}{\cancel{4}}}{21} = \frac{34}{21} = 1\tfrac{13}{21}.$$

73. The student should study the following examples very carefully:

EXAMPLE 1.—$\left(\frac{5}{8} \times \frac{16}{25}\right) + \left(\frac{1}{15} \div \frac{2}{5}\right)$.

The brackets mean that the quantities they contain are to be worked out first, thus—

$$\left(\frac{\cancel{5}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{25}}}\right) + \left(\frac{1}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{2}\right)$$

$$= \frac{2}{5} + \frac{1}{6} = \frac{17}{30}.$$

EXAMPLE 2.—$\frac{5}{8} \times \left(\frac{16}{25} + \frac{1}{15}\right) \div \frac{2}{5}$

$$= \frac{5}{8} \times \left(\frac{48+5}{75}\right) \div \frac{2}{5} \qquad \text{(Note this line carefully.)}$$

$$= \frac{\cancel{5}}{8} \times \frac{53}{\underset{\underset{3}{\cancel{15}}}{\cancel{75}}} \times \frac{\cancel{5}}{2} = \frac{53}{48} = 1\frac{5}{48}.$$

EXAMPLE 3.—$\left\{\left(\frac{5}{8} \times \frac{16}{25}\right) + \frac{1}{15}\right\} \div \frac{2}{5}$

$$= \left\{\left(\frac{\cancel{5}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{25}}}\right) + \frac{1}{15}\right\} \div \frac{2}{5} = \left(\frac{2}{5} + \frac{1}{15}\right) \div \frac{2}{5}$$

$$= \frac{7}{15} \div \frac{2}{5} = \frac{7}{\underset{3}{\cancel{15}}} \times \frac{\cancel{5}}{2} = \frac{7}{6} = 1\frac{1}{6}.$$

EXAMPLES. XIII.

1. Find the value of each of the following:

(1) $\frac{1}{2} \div \frac{1}{3}$. (2) $\frac{1}{5} \div \frac{5}{8}$. (3) $\frac{7}{12} \div \frac{56}{144}$.

(4) $\frac{5}{8} \div 3\frac{5}{16}$. (5) $\frac{7}{15} \div \frac{56}{75}$. (6) $\frac{15}{16} \div \frac{5}{32}$.

(7) $\frac{4}{5} \div \frac{18}{25}$. (8) $\frac{8}{15} \div \frac{96}{225}$. (9) $\frac{75}{99} \div \frac{3}{5}$.

(10) (*a*) Write down the reciprocals of 10, 56, 2, $\frac{1}{2}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{56}{99}$, $12\frac{7}{8}$, $196\frac{15}{16}$.

(*b*) Invert $\frac{16}{7}$, $\frac{186}{9}$, $\frac{36}{5}$, $\frac{\frac{1}{2}}{\frac{1}{4}}$, $\frac{3\frac{1}{5}}{4\frac{1}{6}}$.

(11) $(\frac{1}{2} \text{ of } \frac{4}{5}) \div \frac{4}{9}$. (12) $\frac{3}{4} \div (\frac{5}{8} \text{ of } \frac{15}{25})$. (13) $\frac{3}{5} \div (\frac{5}{8} \times \frac{15}{25})$.

(14) $\frac{3}{4}$ of $(\frac{5}{8} \div \frac{15}{25})$. (15) $(2\frac{7}{8} \times 5\frac{1}{23}) \div \frac{29}{30}$.

(16) $(8\frac{1}{8} \div 3\frac{1}{4}) \times (\frac{5}{8} \div \frac{125}{200})$. (17) $3\frac{3}{4} + 5\frac{1}{8} \times \frac{3}{41} - 2\frac{1}{8} \div \frac{85}{90}$.

(18) $(2\frac{1}{4})^2 \div (13\frac{1}{2})^2$. (19) $(\frac{1}{2})^2 \div (\frac{2}{5})^3$.

(20) $(\frac{5}{8})^3 \div \frac{125}{576}$. (21) $(1\frac{3}{4})^3 \div \left(\frac{7}{10\frac{2}{3}}\right)$.

(22) $\frac{\frac{5}{8}}{\frac{7}{16}}$. (23) $\frac{\frac{10}{12}}{\frac{18}{24}}$. (24) $\frac{3\frac{5}{8}}{\frac{29}{31}}$. (25) $\frac{(\frac{2}{3})^2}{(\frac{1}{2})^2}$.

(26) $\frac{(\frac{1}{2})^2}{(\frac{3}{4})^2} \times 2\frac{1}{4}$. (27) $\frac{3\frac{1}{7}}{6\frac{1}{2}}$ of $2\frac{2}{5}$. (28) $28\frac{5}{16} \div \frac{151}{152}$.

(29) Which is greater, $\frac{5}{6}$ of $\frac{16}{85}$ or $7\frac{1}{2} \div \frac{5}{8}$?

(30) Express—(i) 81 as a fraction of 1620;
(ii) 2 „ „ $2\frac{1}{2}$;
(iii) $3\frac{1}{2}$ „ „ 4.

2. Express $3\frac{7}{16}$ as a fraction of $\frac{25}{28}$.

3. What fraction is $3\frac{3}{4}$ of $\frac{90}{92}$?

4. To the sum of $\frac{1}{3}$ and $\frac{1}{5}$, add the product of $2\frac{1}{4}$ and $5\frac{1}{9}$.

5. To the sum of $5\frac{1}{4}$ and $7\frac{1}{10}$, add the product of $\frac{3}{4}$ and $\frac{16}{21}$.

6. To the difference between $\frac{7}{8}$ and $\frac{3}{4}$, add the quotient of $5\frac{1}{8}$ by $\frac{123}{168}$.

7. Find the sum, the difference, the product, and the quotient of $20\frac{5}{64}$ and $18\frac{2}{257}$.

8. Find the product of the reciprocals (i) of 8 and 5, (ii) of $\frac{1}{2}$ and $\frac{5}{8}$, (iii) of $2\frac{1}{4}$ and $3\frac{3}{7}$.

9. To the sum of the reciprocals of $3\frac{2}{5}$ and $5\frac{1}{10}$, add the product of $3\frac{5}{8}$ and $\frac{6}{145}$.

10. To the difference between the reciprocals of $3\frac{2}{5}$ and $5\frac{1}{10}$, add the quotient of $7\frac{3}{8} \div 5\frac{15}{56}$.

(Work Questions 11 to 14 on squared paper.)

11. With the help of a rectangle 8 inches long and 2 broad, prove geometrically that (i) $\frac{5}{8} \div 3 = \frac{5}{24}$, (ii) $\frac{7}{32} \div 14 = \frac{1}{64}$.

12. Draw a square whose side is 6 inches long, and show by dividing it up and then by cutting out the required part that $\frac{10}{15} \div 4 = \frac{1}{6}$.

13. Taking a rectangle whose sides are 7 and 2 inches, show that $\frac{3}{7} \div 9 = \frac{1}{21}$.

14. By means of a rectangle 5 inches long and 1 inch wide, show that the reciprocal of $3\frac{1}{3} \div 3$ is $\frac{1}{10}$.

15. What fraction is £$5\frac{1}{8}$ of £$7\frac{1}{5}$?

16. Express 10s. 6d. as a fraction of £1. (Write 10s. 6d. = $10\frac{1}{2}$s., and £1 = 20s.)

17. What fraction of £5 is 12s. 6d.?

C. Miscellaneous Examples

EXAMPLES. XIV.

1. Reduce each of the following to its simplest form:

(1) $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{3}}$. (2) $\dfrac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{3}}$. (3) $\dfrac{\frac{1}{2}\div\frac{1}{3}}{\frac{1}{3}}$. (4) $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}$.

(5) $\dfrac{\frac{3}{4}\times 1\frac{5}{8}}{\frac{13}{24}\times\frac{4}{16}}$. (6) $\dfrac{\frac{5}{8}\times 2\frac{1}{4}\times\frac{1}{3}}{1\frac{2}{16}\times\frac{5}{9}\times\frac{3}{25}}$. (7) $\dfrac{\frac{7}{8}\times\frac{56}{28}}{2\frac{1}{5}\times\frac{5}{22}}$.

(8) $\dfrac{3\frac{1}{4} \times 4\frac{3}{13} \times \frac{2}{55}}{(8\frac{1}{2} - 7\frac{1}{4}) \div 1\frac{1}{4}}$.

(9) $(\frac{7}{10} \times \frac{15}{16}) + (\frac{1}{49} \div \frac{25}{98})$.

(10) $(\frac{3}{4} \times \frac{7}{8}) - (\frac{5}{16} \div \frac{50}{88})$.

(11) $(\frac{1}{8} \div \frac{3}{4}) \times \frac{12}{45}$.

(12) $(\frac{1}{8} \div \frac{3}{4}) \div \frac{12}{45}$.

(13) $\frac{1}{8} \div (\frac{3}{4} \times \frac{12}{45})$.

(14) $3\frac{5}{12} \div (\frac{82}{36} \times \frac{24}{72})$.

(15) $(7\frac{1}{20} \div 6\frac{4}{15}) \times 3\frac{1}{13}$.

(16) $(8\frac{5}{12} - 3\frac{7}{16} + 2\frac{8}{64} - 3\frac{1}{32}) \times 10\frac{1}{4}$.

(17) $\dfrac{\frac{5}{8} + \frac{7}{16}}{3\frac{7}{8} - 2\frac{1}{4}} \times \dfrac{1\frac{4}{9}}{\frac{2}{3} \div \frac{8}{5}}$.

(18) $\dfrac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{1}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{1}{3}} \div \dfrac{2\frac{1}{2}}{3\frac{1}{3}}$.

(19) $\dfrac{1\frac{1}{8} - \frac{5}{6}}{1\frac{1}{6} + 1\frac{11}{24}} \div \dfrac{1\frac{7}{10} + 2\frac{1}{10}}{2\frac{1}{3} + 1\frac{5}{12}}$.

(20) $\dfrac{\frac{2}{3} + \frac{5}{12} - \frac{5}{6}}{3\frac{11}{18} - 3\frac{2}{9}} \times \dfrac{3\frac{1}{2} - 1\frac{3}{4}}{1\frac{5}{6} - \frac{2}{3}}$.

2. There are twenty-five business houses each of which subscribes £12 12s. to a relief fund. What is the total amount subscribed? (Express the 12s. as a fraction of £1.)

3. A grocer has 6 quarts of vinegar in a barrel. He sells 1 pint to each of three customers, and then $\frac{1}{3}$ of what remains to another customer. How many quarts are left? (1 quart = 2 pints.)

4. If vinegar is sold at $1\frac{1}{2}$d. a pint, what amount did the grocer in Question 3 receive from his four customers, and what was the value of the vinegar which remained?

5. Taking the same grocer, if he bought the vinegar at $1\frac{1}{5}$d. per pint, what profit would he make by selling 6 quarts at $1\frac{1}{2}$d. a pint?

6. Four of us travel to town together every day, and the cost of each journey works out at $3\frac{3}{4}$d. What amount does the railway company receive from us per week if we make two journeys on each working day? Taking the daily cost from this question, and supposing that we travel on 299 days in the year, find the annual cost of each season ticket to the nearest shilling.

7. A spectacle maker charges £3 15s. for a pair of pince-nez; $\frac{2}{5}$ of this amount is the cost of the gold frames, of the remainder $\frac{9}{10}$ is the cost of the lenses and of the labour involved. What amount remains as profit?

8. Express the profit in the previous question as a fraction of the amount paid by the purchaser.

9. A draper buys a lot of ladies' hose at 15s. a dozen, and sells each pair at a profit of $\frac{2}{5}$ of what he gave for it. What is the selling price per pair?

10. What profit does the draper in the previous question make in selling two dozen pairs?

11. The value of the coal exported from the United Kingdom in 1911 was $\frac{64}{71}$ of that exported in 1912, which was £42580023. Find the value of the coal exported in 1911 to the nearest £100.

12. Calico is bought at $2\frac{1}{14}$d. a yard wholesale, and the draper sells it at $2\frac{3}{4}$d. a yard. What profit does he make on 1000 yards?

13. Suppose the draper of the previous question sold the calico in 12-yard lots at 2s. 5d., what would be his profit per dozen yards?

14.

Year.	Merchandise Only.		
	Imports.	Exports.	Total.
1810 . . .	$\frac{11}{12}$ of exports	£43568760	
1912 . . .	$\frac{37}{30}$ "	£598961100	

The trade of the United Kingdom is given above for the years 1810 and 1912. Find the value of the imports to the nearest £10 sterling, and also the total value of the trade.

15. Fill in the following table from the data given and obtained in the last question:

Year.	Merchandise Only.		
	Imports.	Exports.	Excess of Imports over Exports.*
1810 . . .			
1912 . . .			

* Employ a – sign to denote the fact that the exports are greater than the imports if they are.

Attempt to explain the meaning of the results in this question, given that the population of the United Kingdom was, roughly, 12 millions in 1810, and 41 millions in 1912.

16. A bookseller allows his customer $\frac{1}{4}$ off the published price of a book. The wholesale house allows him $\frac{1}{3}$th off the published price and gives him thirteen books for a dozen. What is the bookseller's profit on selling thirteen books published at 2s. each?

17. Find the prices of a No. 2, No. 3, and No. 4 lantern, to

the nearest penny, and fill in the following table, given that the price of No. 2 is found by adding $\frac{2}{5}$ of the price of No. 1 to 2s. 6d. (the cost of No. 1); that of No. 3 by taking $\frac{5}{2}$ of the cost of No. 1 and adding it to 2s. 6d.; and that of No. 4 by taking $\frac{10}{8}$ of the cost of No. 1 and adding it to 2s. 6d.

No. of Magic Lantern.	Size of Slides.	Price.
1 . . .	$1\frac{1}{4}$	2s. 6d.
2 . . .	$1\frac{3}{4}$	
3 . . .	$2\frac{3}{4}$	
4 . . .	$3\frac{1}{8}$	

18. Half a dozen coffee cups and saucers in a case cost 126s. The case costs $\frac{3}{14}$ of the whole. Find the price of each cup and saucer.

19. A quire of notepaper costs $2\frac{2}{3}$d. wholesale. If it contain twenty-four sheets, find the cost of one sheet as a fraction of a penny.

20. A billposter takes up one-hundredth of the amount of paste in his bucket each time he posts a bill. How many bills can he post if he uses three-quarters of his paste?

21. If his pail contains 7 pints of paste, what fraction of a pint does he use for each bill, and how many can he post with 1 gallon (8 pints) of paste? (See Question 20.)

22. How many times is the population of the United States greater than that of (1) the United Kingdom, (2) France, and (3) Italy, if they are, respectively, 92, 41, $39\frac{1}{2}$, $34\frac{1}{2}$ millions?

23. The latitude of Belgrade is $44\frac{13}{20}^\circ$ N., and of Cape Town $34\frac{1}{30}^\circ$ S. Taking 1° of latitude as $69\frac{3}{25}$ miles, find how many miles Belgrade is north of Cape Town. (Why must you assume both places to be of the same longitude?)

24. Find to the nearest dollar the number of Newfoundland dollars in £100 sterling. (1 Newfoundland dollar $= 4\frac{1}{6}$s.)

25. The length of the touch-line on a "Soccer" ground is $125\frac{2}{3}$ yards. What is its width in yards if it is $\frac{17}{30}$ of the length?

26. How many bottles, each containing $\frac{7}{8}$ pints, can be filled from a hogshead of claret which contains 46 gallons?

27. A man and his wife living in London are considering the wisdom of emigrating to Brisbane. Their food consumption per week *each*, together with the prices of various necessary things in

London and Brisbane, are given. In which city could they live more cheaply per week?

Article.	Consumption per Person per Week (unless otherwise stated). Weights in Pounds.	Cost per Pound (unless otherwise stated)	
		In London.	In Brisbane.
Fresh meat	$3\frac{1}{16}$	11d.	8d.
Bacon . .	$1\frac{11}{16}$	$10\frac{1}{2}$d.	$9\frac{1}{4}$d.
Butter . .	$\frac{15}{32}$	1s. 3d.	1s. 0d.
Eggs . .	6	1s. 8d. doz.	1s. 1d. doz.
Bread . .	$7\frac{19}{64}$	$1\frac{1}{2}$d.	$1\frac{3}{4}$d.
Flour . .	$3\frac{1}{24}$	$1\frac{3}{4}$d.	2d.
Tea . . .	$\frac{5}{16}$	1s. 4d.	1s. 2d.
Sugar . .	$1\frac{1}{3}$	3d.	3d.
Potatoes .	$5\frac{1}{8}$	1d.	$\frac{3}{4}$d.
Jam . . .	$1\frac{7}{8}$	$2\frac{1}{2}$d.	$4\frac{1}{2}$d.
Milk . .	5 quarts	4d.	5d.
Coal . .	$\frac{1}{16}$ ton	22s. ton	16s. ton
Gas . . .	100 cubic feet	2s. 9d. per thousand	5s. per thousand
Rent . .	per week for family	15s.	17s. 6d.

Tabulate your answer.

SECTION VI

DECIMALS

INTRODUCTORY

I. Converting Decimals into Fractions

74. We have already explained that the value of any figure in a number depends not only on that figure but also upon the *position* that it holds.

In the number 5864, 4 is the units figure, while the 6, one place to the left of the 4, is the tens figure; it is therefore perfectly logical to put a 2 to the **right** of the 4 and call it the **tenths** figure, thus:

$$\underset{\text{tens}}{6}\;\underset{\text{units}}{4}\;\underset{\text{tenths}}{\mathbf{2}} \quad (5\,8\,6\,4\,\mathbf{2})$$

75. In order to make it clear that the 2 is a **tenths** and not a units figure, we simply put a dot between the 4 and the 2, thus, 5864·2. On the Continent a comma is frequently used: 5864,2; and 5864·2 means $5864\frac{2}{10}$, which equals $5864\frac{1}{5}$. The dot is called a decimal (Latin, *decem*, ten), for it enables us to mark off tenths. In a similar way the second figure to the right is the hundredths figure, and so on.

76. Again, if we take $\frac{1}{10}$ we can write it as $\cdot 1$, or $0\cdot 1$, the 0 being inserted to avoid the possibility of reading $\cdot 1$ as 1.

The truth of the following is now, perhaps, obvious:

$$3\cdot 4 = 3\tfrac{4}{10} = 3\tfrac{2}{5};\quad 8\cdot 04 = 8\tfrac{04}{100} = 8\tfrac{4}{100} = 8\tfrac{1}{25};$$
$$12\cdot 906 = 12\tfrac{906}{1000} = 12\tfrac{453}{500};\quad 0\cdot 01 = \tfrac{01}{100} = \tfrac{1}{100};$$
$$0\cdot 0005 = \tfrac{0005}{10000} = \tfrac{5}{10000} = \tfrac{1}{2000}.$$

3·4 said to have *one* decimal place, 8·04 two, 12·906 three, and so on.

77. Further illustrations will now be given.

EXAMPLE 1.—**Find the value of 0·75 of £1.**

$$0\cdot 75 = \tfrac{75}{100},$$

$$\text{and } \tfrac{75}{100} \text{ of } £1 = \frac{\overset{3}{\cancel{75}}}{\underset{4}{\cancel{100}}} \text{ of } 20\text{s.} = 15\text{s.}$$

EXAMPLE 2.—**What is the value of 0·84 cwt.?**

$$1 \text{ cwt.} = 112 \text{ lb.},$$

$$\therefore\ 0\cdot 84 \text{ cwt.} = \frac{\overset{21}{\cancel{84}}}{\underset{25}{\cancel{100}}} \text{ of } 112 \text{ lb.}$$

$$= \tfrac{2352}{25} = 94\tfrac{2}{25} \text{ lb.}$$

EXAMPLE 3.—**Express 0·5 of £1 as a fraction of 0·85 of 15s.**

$$0\cdot 5 \text{ of } £1 = \tfrac{5}{10} \text{ of } £1 = 10\text{s.};$$

$$0\cdot 85 \text{ of } 15\text{s.} = \frac{\overset{17}{\cancel{85}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}} \text{ of } \overset{3}{\cancel{15}}\text{s.} = \tfrac{51}{4} = 12\tfrac{3}{4}\text{s.};$$

$$\therefore \text{ fraction required} = \frac{10}{12\frac{3}{4}} = \frac{10\times 4}{51} = \tfrac{40}{51}.$$

EXAMPLES. XV.[1]

1. Convert the following decimals into fractions or mixed numbers, reducing to their lowest terms where possible:

(1) 0·5, 0·86, 0·94, 0·186, 0·574.
(2) 0·896, 0·974, 0·899, 0·758, 0·878.
(3) 0·05, 0·086, 0·094, 0·0186, 0·0574.
(4) 0·009, 0·007, 0·008, 0·006, 0·0072.
(5) 0·0004, 0·0005, 0·0050, 0·0008, 0·0003.
(6) 0·900, 0·700, 0·940, 0·1860, 0·5740.
(7) 0·909, 0·708, 0·805, 0·906, 0·8002.
(8) 0·7003, 0·8005, 0·9060, 0·6001, 0·9008.
(9) 6·8, 9·64, 12·863, 15·732, 10·006.
(10) 7·880, 8·9600, 15·7202, 18·9601.
(11) 0·01, 0·001, 0·00801, 0·9008.
(12) 0·0001, 0·00001, 3·0003, 33·0033.
(13) 9·854, 73·652, 86·015, 290·014.
(14) 285·736, 864·001, 900·004, 8500·0101.
(15) 9700·860, 7501·850, 9000·0003.

2. Write down each of the following decimals as a fraction and the pounds as shillings, expressing the result in shillings and a fraction of a shilling:

(1) 0·5 of £1, 0·25 of £1, 0·6 of £2, 0·75 of £1.
(2) 0·34 of 10s., 0·5 of 10s., 0·25 of 10s., 0·6 of 10s.
(3) 0·55 of 5s., 0·505 of 5s., 0·62 of 5s., 0·94 of 5s.
(4) 0·3 of 8s., 0·4 of 8s., 0·6 of 8s., 0·91 of 8s.
(5) 0·76 of 15s., 0·92 of 16s., 0·725 of 12s.

3. How many cwt. are there in 0·85 ton? and how many shillings in 0·56 of £1?

4. How many pounds are there in 0·75 cwt., and in 0·86 ton?

5. If a bale of East Indian cotton contains 390 lb. roughly, what weight will 0·65 of the bale contain?

6. Express 0·6 of £2 10s. as a fraction of 0·8 of £8 12s.

7. A chest of Turkish gum contains 4 cwt., how many pounds will 0·95 of a chest contain?

8. A chest of gum-arabic (East Indian) contains 6 cwt., express 0·95 of a chest of Turkish gum as a fraction of 0·75 of a chest of East Indian gum.

[1] Many of these examples can be worked mentally.

9. In the financial review of a continental Bank, the following details were given recently :

Cheques.	July 1914.	November 1914.
Paris cheque . .	25·18	25·13
Italy ,, . .	25·3	25·50
Madrid ,, . .	26·15	26·10

The figures mean that £1 English money was worth 25·18 francs in Paris and 25·3 lira in Italy and 26·15 peseta in Spain in July 1914, but see Part III.

Find by how many hundredths of a franc, lira, or peseta the value of the cheque has altered between July and November 1914 in each of the three cases, and express the change in centimes, etc., if 100 centimes = 1 franc.

10. The time at several cities corresponding with twelve noon at Greenwich is given below. Express the times in hours and minutes correct to the nearest minute, and refer to Section XVI. (p. 325) for a fuller explanation.

GREENWICH MEAN TIME—NOON.

City.	Corresponding Time.	Time—Hours and Minutes.
Adelaide . . .	9·23 p.m.	
Brussels . . .	0·28 ,,	
Chicago	6·16 a.m.	
Edinburgh . . .	11·78 ,,	
Vancouver . . .	3·63 ,,	

11. The monthly report of the Department of Trade and Commerce of Canada for March 1914 gives the following figures for St. Vincent, showing the percentage of each class to the total trade·

YEAR ENDING 31ST MARCH 1912.

Class.	Articles.	Imports.	Exports.
1 . .	Animals, food, and drink . .	44·40	61·22
2 . .	Raw material	10·72	37·49
3 . .	Manufactured articles . .	43·10	1·29
4 . .	Coin and bullion . . .	1·75	—

Express the percentage of coin and bullion imported as a fraction of the animals, food, and drink imported. Treat classes 2 and 3 in the same way.

12. Offer some comments on the commercial status of St. Vincent as revealed by the above figures.

13. The following is a statement of the rainfall for the months given as observed in Westminster and South Kensington. Find (i) what fraction of 1 foot fell in the three months taken together, and (ii) by what fraction of a foot the rainfall in the three months was below the normal (see § 84).

RAINFALL (1913).

Month.	Amount.	Difference from Normal.
June	0·46 inch	1·75 inch below.
August . . .	1·20 ,,	1·19 ,, ,,
October . . .	2·95 inches	0·22 ,, above.

14. It is stated that in the Glenelg and Wannon River Districts of Australia the average rainfall was—

1910.	1911.	1912.
32·96 inches	27·51 inches	24·73 inches.

By what fraction of 1 foot did the rainfall in 1910 exceed that in 1911 and in 1912?

15. Express the decimal as a fraction in each of the following:

RAILWAYS OF THE UNITED KINGDOM.

Quinquennial Period.	Proportion of Total Net Earnings to Capital per Cent.*
1896–1900 . . .	3·64
1901–1905 . . .	3·38
1906–1910 . . .	3·45

* The student is informed that the figures in this column show a decrease from 4·59 in 1871–75 to 3·38 in 1901–5, while 3·45 in 1906–10 marks an increase which suggests a rise in the proportion of net earnings to capital (see Section XVII. B (1)).

16. Express the consumption per head of population for the Commonwealth of Australia in pounds and ounces correct to the nearest ounce from the data given. (Thus 130·12 lb. = 130 lb. + 0·12 lb., and 0·12 lb. = $\frac{12}{100}$ of 16 oz. = $1\frac{23}{25}$ oz. = 2 oz., to the nearest ounce.)

AUSTRALIA.

Year.	Consumption per Head of Population.
1910	130·12 lb.
1911	108·04 ,,
1912	109·22 ,,

II. Converting Fractions into Decimals

(a) Denominator a Power of 10

78. Just as we have converted decimals into fractions, so we can convert fractions into decimals. Thus—

$$0{\cdot}1=\tfrac{1}{10}\ \therefore\ \tfrac{1}{10}=0{\cdot}1\,;$$

$$\text{so } \tfrac{2}{10}=0{\cdot}2,\ \tfrac{9}{10}=0{\cdot}9,\ \tfrac{9}{100}=0{\cdot}09,\ \tfrac{9}{1000}=0{\cdot}009\,;$$

for on reconverting, $0{\cdot}009=\frac{9}{1000}$.

Hence, generally, if the denominator of a fraction is 10 or a power of 10, there are as many figures after the decimal point in the corresponding decimal as there are 0's in the denominator.

EXAMPLES. XVI.[1]

1. To what decimal is each of the following fractions equivalent?

(1) $\frac{2}{10}$, $\frac{9}{10}$, $\frac{3}{100}$, $\frac{25}{100}$, $\frac{79}{100}$, $\frac{86}{100}$, $\frac{92}{100}$, $\frac{99}{100}$.

(2) $\frac{60}{100}$, $\frac{70}{100}$, $\frac{80}{100}$, $\frac{90}{100}$.

(3) $\frac{3}{1000}$, $\frac{50}{1000}$, $\frac{70}{1000}$, $\frac{40}{1000}$, $\frac{300}{1000}$, $\frac{200}{1000}$.

(4) $\frac{439}{1000}$, $\frac{530}{1000}$, $\frac{503}{1000}$, $\frac{790}{1000}$, $\frac{709}{1000}$.

(5) $\frac{56}{10000}$, $\frac{780}{10000}$, $\frac{1}{10000}$, $\frac{10}{10000}$, $\frac{25}{10000}$, $\frac{8899}{10000}$.

(6) $2\frac{7}{10}$, $8\frac{9}{10}$, $12\frac{8}{10}$, $15\frac{9}{100}$, $17\frac{90}{100}$.

(7) $28\frac{909}{1000}$, $56\frac{900}{1000}$, $73\frac{8000}{10000}$, $98\frac{8008}{10000}$.

(8) $156\frac{1}{1000}$, $574\frac{1}{10000}$, $742\frac{1}{100000}$.

(9) $963\frac{1}{1000000}$, $856\frac{340}{1000000}$, $9880\frac{13500}{1000000}$.

2. Write down $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$ of 2 feet, 5 feet, 8 yards, 80 inches, 200 tons, as decimals.

3. What decimal of £1 results from dividing each of the

[1] Many of these questions can be answered mentally.

following by 1000: £0·6, £0·84, £0·906, £0·0085, £0·1004, £0·01014?

4. What decimal of 8·7 tons of coal would each of 100 people receive if it were divided equally among them?

5. How many tons of coal would have to be divided among 1000 people if each received just as much as in the previous question?

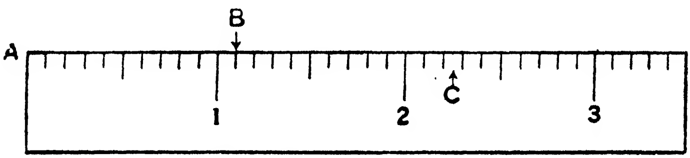
6. If 1·56 lb. of gum-arabic dissolved in 100 oz. of water make a sufficiently strong gum for ordinary purposes, how much would have to be dissolved (i) in 10 oz. of water, (ii) in 1 oz. of water, to give a solution of similar strength?

7. The tare of twenty trucks of equal weight is 234 tons. What is the tare of each in tons and the decimal of 1 ton?

8. In the process of silver-plating commercially, a solution is employed, consisting of 24 oz. of silver nitrate (a white solid substance) dissolved in 300 oz. of distilled water to which is added drop by drop a solution of 50 oz. of potassium cyanide in 500 of water, till no more precipitate is formed. Find out how much silver nitrate there is in 1 oz. of water, and how much cyanide there is in 50 oz. of water.

9. Take a scale marked in inches and tenths of an inch, and measure the length and width of this page in inches and the decimal of an inch.

Note.—If the length comes between two lines which mark the tenths, the student should estimate, "hit off," or judge the length of the extra piece. Thus AB = 1·1 inch, but AC = 2·2 + a little piece more, which is rather less than 0·5 of one division, so we judge AC to be 2·24 inches.



10. Measure the length and width of a sheet of foolscap in the manner of the previous question.

11. A chest of Congou tea contains 80 lb. What decimal of a pound will there be in each one of 200 equal-sized packets filled from it?

12. If a harvest hand receives £8 2s. for 25 days' work in Australia, how many shillings does he receive per day?

13. If the wholesale price of Australian mutton is 112 shillings for 600 lb., what is the price per pound in the Commonwealth?

II. (*b*) DENOMINATOR CAPABLE OF BEING EXPRESSED AS A POWER OF 10

79. If the denominator of a fraction be not 10, or a power of 10, it is **often** possible to convert it into such a number by suitably multiplying numerator and denominator. For example:

$$\tfrac{2}{5}=\tfrac{4}{10}=0{\cdot}4,\text{ and }\tfrac{3}{50}=\tfrac{6}{100}=0{\cdot}06.$$

So, also,
$$\frac{8}{25}=\frac{8\times 4}{25\times 4}=\frac{32}{100}=0{\cdot}32\,;$$

and
$$5\frac{6}{250}=5\frac{24}{1000}=5{\cdot}024.$$

Now, in the case of $\frac{2}{5}$ let us divide the 2 by the 5, observing that with a whole number, such as 2, the decimal point is omitted, for convenience, though 2 really means 2·00000 . . . Hence, we have $\dfrac{5\,|\,2{\cdot}0000}{\;\;0{\cdot}4000}$, which shows that $\frac{2}{5}$ can be expressed as a decimal by reducing it to $\frac{4}{10}$ or by dividing 5 into 2 and putting down the point in the proper place. In just the same way $\frac{8}{25}=8\div 25$, which we find thus: $\dfrac{25\,|\,8{\cdot}0000}{\;\;0{\cdot}3200}$; $\therefore\ \frac{8}{25}=0{\cdot}32$ as before.

80. Some fractions can be expressed with their denominators powers of 10 only, while others cannot; thus, $\frac{7}{19}$ cannot be so expressed, nor can $\frac{2}{225}$, for neither 19 nor 225 will divide exactly into any power of 10.

Now,
$$\frac{3}{250}=\frac{3}{2\times 5^3}=\frac{3\times 2^2}{2\times 5^3\times 2^2}=\frac{12}{1000}=0{\cdot}012.$$

If, then, a fraction in its lowest term have a denominator whose factors are powers of 2 and 5 *alone*, then it can be reduced to a decimal by the method described above—such a decimal is called a **Terminating Decimal**; but if the denominator contains any factor other than 2 or 5 when the fraction is in its lowest terms, then, as we shall see (II. (*c*)), the decimal does *not* terminate, and must be taken to the number of places required by the question.

EXAMPLES. XVII.

1. Decimalise the following without going through the process of division:

(1) $\frac{1}{25}$, $\frac{3}{25}$, $2\frac{6}{25}$, $21\frac{18}{25}$, $156\frac{19}{25}$.

(2) $\frac{8}{50}$, $21\frac{49}{50}$, $156\frac{37}{50}$, $\frac{1}{50}$, $20\frac{20}{50}$.

(3) $8\frac{37}{250}$, $\frac{74}{500}$, $28\frac{38}{250}$, $86\frac{394}{500}$.

(4) $26\frac{98}{2500}$, $58\frac{3}{20}$, $78\frac{19}{20}$, $152\frac{76}{2500}$.

(5) $89\frac{1984}{5000}$, $93\frac{79}{250}$, $88\frac{5}{25000}$, $1\frac{1}{50000}$.

2. In the year 1913 the price per pound of Canadian exported cheese was such that 25 lb. sold for 13s. (This was the highest price reached for twenty-four years.) Find the price per pound in pence and the decimal of a penny.

3. From a hogshead of Moselle which contains 30 gallons one-sixth has been sold, and the value of the remainder is £27. What is the sale price in £'s per gallon and in shillings per pint?

4. Twenty-five feet of oak picture moulding cost 6s., what is the price per foot in pence?

5. Fifty sheets of "elephant" brown paper (*i.e.* paper 34 × 24 inches) cost 4s., find the price per sheet in pence.

6. A ship-load of coal consists of 424 tons, which is sold to 25 customers in equal quantities, how much does each receive?

7. If 75 yards of chintz cost £4 10s., what is the cost in shillings per yard?

8. Twelve score and ten of mackerel cost 15·75 shillings, what is the cost of each as a decimal of a shilling?

9. Pilchards, caught off Cornwall, are packed in oil and exported to Italy in small tin boxes. It is found that 1250 boxes contain, on the average, 9752 fish. How many will there be in each box?

10. In the year 1913 the price of 75 bushels of Canadian wheat exported was 72 dollars. Express the price per bushel as a decimal of a dollar.

11. A merchant imports a quantity of glacé fruits into England and pays £22 18s. duty on 250 cwt., what is the import duty on this commodity in shillings per cwt.?

II. (*c*) Denominator any Number other than 10 or a Power of 10

81. If the denominator of a fraction is any number other than 10 or a power of 10, and is also one that cannot be reduced to that form, it is not a terminating decimal, so that to reduce it to a decimal we divide, put down the decimal point when we come to it, and take it to the required number of places (see § 79).

82. Example 1.—**$\frac{2}{3}$ is converted to a decimal to three places by writing down 2·0000 and dividing by 3.**

Thus—

$$\begin{array}{r|l} 3 & 2{\cdot}0000 \\ \hline & 0{\cdot}6666 \end{array}$$

$\therefore$ **$\frac{2}{3}$ = 0·6666, or 0·667 to three places.**

Example 2.—**Convert $\frac{15}{16}$ into a decimal to two places.**

$$\begin{array}{r|l} 16 & 15{\cdot}000 \\ \hline & 0{\cdot}937 \end{array}$$

$\therefore\ \frac{15}{16}=0{\cdot}94$, correct to two places.

In this question the student should observe that just as in pp. 17 to 20 (Approximation) we had to find one figure beyond that which was necessary in the result so as to correct the last figure required, so here too **we must find the third decimal place and then correct the second.**

Example 3.—**Evaluate $3\frac{5}{18}$ correct to three places.**

We find the value of $\frac{5}{18}$ just as before, and prefix the **3** thus:

$$\begin{array}{r|l} 18 & 5{\cdot}0000 \\ \hline & 0{\cdot}2777 \end{array}$$

$\therefore\ 3\frac{5}{18}=3{\cdot}278$, correct to three places.

EXAMPLES. XVIII.

1. Decimalise each of the following: parts 1 to 8 correct to three places; parts 9 and 10 correct to four places.

(1) $\frac{2}{3}, \frac{3}{5}, \frac{7}{8}, \frac{9}{11}, \frac{10}{12}, \frac{15}{16}, \frac{11}{18}$.

(2) $\frac{21}{25}, \frac{18}{28}, \frac{23}{36}, \frac{48}{124}, \frac{39}{62}, \frac{88}{123}$.

(3) $\frac{79}{732}, \frac{86}{89}, \frac{720}{898}, \frac{600}{754}$. (4) $\frac{1}{586}, \frac{2}{8964}, \frac{21}{7324}$.

(5) $\frac{886}{981}, \frac{798}{894}$. (6) $\frac{758}{963}, \frac{981}{1001}$.

(7) $3\frac{24}{89}, 7\frac{86}{900}, 9\frac{73}{2010}$. (8) $128\frac{56}{94}, 156\frac{92}{158}, 379\frac{158}{159}$.

(9) $536\frac{795}{3868}, 5964\frac{1962}{7345}$. (10) $22\frac{7}{9581}, 3600\frac{2}{5684}$.

2. Decimalise the following fractions correct to two places, and arrange them in descending order of magnitude: $\frac{3}{4}, \frac{5}{6}, \frac{6}{7}, \frac{7}{9}, \frac{7}{10}$.

3. Decimalise $\frac{10}{13}$ and $\frac{253}{329}$, and determine which is the larger.

4. In the year 1912, 13674000 tons of shipping entered the port of New York, and 13549000 tons were cleared (to the nearest thousand tons). Express the latter as the decimal of the former correct to three places of decimals.

5. The data for London on the lines of the last question, as stated in *Whitaker's Almanack*, are "entered 10801000 tons, cleared 8748000 tons." Express the latter as the decimal of the former to three places of decimals, and compare with Question 4.

6. In 1912 the total wheat production of the world was 452 million quarters (of 480 lb. each). Of this total Russia produced 90 million quarters; the United States, 87; India, 45; France, 40; Canada, 27. Express the production of each of these countries as a decimal of the whole world's production.

7. If in the year 1912 Japan produced 21 million pounds of silk, and China $14\frac{1}{2}$, what decimal is the production of the latter of that of the former?

8. The total silk production of the world in 1912 was 57 million pounds. What decimal of that amount did Japan produce?

9. The total value of the general trade of the United Kingdom in 1911 was 1237 million £'s sterling, and that of the German Empire 942. Express the trade of the German Empire as a decimal of that of the United Kingdom. (Note that this question does not relate to the BRITISH EMPIRE.)

10. Taking Questions 4 and 5, express the total shipping entered at London as a decimal of that entered at New York in the year 1912, to three places.

11. An auctioneer has to pay a licence of £10 in the United Kingdom, while a wholesale beer-dealer has to pay £10 10s. What decimal is the former of the latter?

12. A nobleman's estate was sworn at £1220905, of which £654989 was net. What decimal of the sworn value was the net value?

13. The value of a Newfoundland dollar is 4s. 2d., and of a gold boliviano (Bolivia), 4s. Express the value of the latter in terms of the former.

14. The population of Mauritius is 375 thousand, and the value of its imports (1912) £2300000. What is the value of the imports per thousand of the population, to two decimal places?

15. In 1906 there was accommodation for 7059262 children in the public elementary schools of England and Wales. In 1912 there was accommodation for 6862876. How many times is the former greater than the latter? (Three significant figures.)

16. The 1913–14 Budget estimated that £9800000 would be received in stamp duties during the year, while the receipts in 1912–13 were £10059000. How many times did the actual receipts in 1912–13 exceed the estimate for 1913–14? (Three places of decimals.)

17. The area of Cyprus is 3600 square miles, and the population is estimated at 276 thousand. What is the density of the population (that is to say, what is the number of people per square mile), to four significant figures?

18. Taking the following details, find which country is the more densely populated:

Country.	Area in Square Miles.	Population in Millions.	Density of Population.
The Indian Empire .	1900000	315	
United Kingdom . .	121090	46	

19. In 1911, 5020510 tons of iron were extracted from ore dug in the United Kingdom, and the value of this was £16146340. Find to two decimal places the average price per ton.

SECTION VII

DECIMALS—(*continued*)

Addition and Subtraction

83. There is no difference whatever in principle between simple addition and subtraction and the corresponding rules in decimals. We simply place figures with the same place value under one another and add up. Since all the units figures come under one another all the decimal points **must** do likewise, and so all the tenths figures.

84. EXAMPLE 1.—**Add 0·86, 5·46, 8·3705, 100·00014.**

Arranging in columns we have—

$$\begin{array}{r} 0{\cdot}86 \\ 5{\cdot}46 \\ 8{\cdot}3705 \\ 100{\cdot}00014 \\ \hline 114{\cdot}69064 \\ \hline \end{array}$$

EXAMPLE 2.—**Subtract 18·573 from 560·94032.**

Arranging as before—

$$\begin{array}{r} 560{\cdot}94032 \\ 18{\cdot}573 \\ \hline 542{\cdot}36732 \\ \hline \end{array}$$

EXAMPLES. XIX.

1. Add the following numbers, arranging them in columns:

(1) 0·86, 2·54, 7·38, 8·97, 9·324.

(2) 5·793, 8·906, 93·873, 56·02, 7·0031.

(3) 86·954, 91·9801, 76·8864, 986·90001.
(4) 986·1, 1968·01, 59864·001, 28·00001.

2. Fill in the totals in the spaces provided—

	(1)	(2)	(3)	(4)	(5)	Totals.
(1) . .	5·037	107·73	1·6084	59·387	6·5203	
(2) . .	217·04	9·2563	56·607	4·728	1·3748	
(3) . .	27·059	209·3146	8·1078	16·983	—	
(4) . .	3·0726	82·8254	136·27	6·304	—	
(5) . .	28·5284	0·725	7·036	154·82	—	
(6) . .	12·407	8·1275	6·098	135·46	84·076	
(7) . .	2·1384	65·6	720·7	6·435	79·125	
(8) . .	2·15	0·0052	45·2164	3·1285	25·1341	
(9) . .	1·85	76·473	952·57	368·9	16·54	

3. Add horizontally and vertically—

	(1)	(2)	(3)	(4)	Totals.
(1) . .	3·008	5·886	38·9462	—	
(2) . .	576·89043	25·52401	2·701	7·86	
(3) . .	27·5724	721·30154	504·008	80·704	
(4) . .	986·0001	364·72150	28·8621	3·005	
(5) . .	500·0038	51·30024	8·0004	270·6	
(6) . .	751·8642	178·00512	1014·3061	58·091	
(7) . .	25·7913	356·00034	2·853	—	

4. In parts (1) to (9) of Question 2 find the difference between the numbers in columns (1) and (2) and between those in columns (3) and (4).

5. Find the difference between the numbers in columns (1) and (2) of Question (3), parts (1) to (7).

6. Find the sum of $\frac{2}{10}$, $\frac{9}{10}$, $\frac{3}{100}$, $\frac{25}{100}$, $\frac{79}{100}$, giving the answer as a decimal to two places.

7. Decimalise, and find the difference between, $6\frac{8}{46}$ and $4\frac{5}{36}$, to three decimal places.

8. Find the sum and difference of 896·00943 and 987·000000125.

9. Find the difference between the sum and the difference of the numbers in Question 8.

Decimalise, where necessary, and evaluate (Questions 10–12)—

10. $3\frac{5}{6} + 56·843 + 73·90132 - 78·8906$.

11. $18·89764 - 21·96543 + 4\frac{5}{8}$.

12. $74\frac{3}{8} + 91·86943 - 72·70000091$.

In the following questions (13–15)—

(1) Add up the + terms, (2) add up the − terms, and then find the difference between them. (The first term in each sum is +):

13. $6{\cdot}8964 + 8{\cdot}96401 - 3{\cdot}86 + 2{\cdot}73 - 6{\cdot}9301$.
14. $18{\cdot}705 + 9{\cdot}86 - 8{\cdot}864 - 7{\cdot}632 + 5{\cdot}681$.
15. $25693{\cdot}8694 - 9621{\cdot}0081 + 2{\cdot}86 - 9{\cdot}732$.

16. By how much is $8{\cdot}96 - 7{\cdot}84$ less than $7{\cdot}62$?
17. How much is $7{\cdot}62$ greater than $8{\cdot}96 - 7{\cdot}84$?
18. From the sum of 8·869 and 19·7304 take the sum of 7·68 and 9·93.
19. From the sum of 8·0869 and 28·9374 take away the difference.
20. What is the difference between the differences of 18·964 and 15·864; and of 121·8691 and 121·869?
21. What is the sum of the sums of 3·86, 21·964, 128·86324; and of 0·001, 0·00999, 0·090909?

Decimalise the fractions in Questions 22–24, and proceed as in Questions 13 to 15—[1]

22. $3\frac{1}{2} + 5\frac{1}{4} - 6\frac{5}{8} + 2\frac{4}{9} - 1\frac{1}{2}$.
23. $2\frac{1}{12} + 3\frac{5}{16} - 5\frac{1}{8} + 12\frac{8}{10}$.
24. $3\frac{1}{8} - 5\frac{2}{3} - 15\frac{3}{5} + 7\frac{3}{15}$.

25. Decimalise, and evaluate $2\frac{3}{8} + 5\frac{1}{4} - 2\frac{1}{2} + 3\frac{7}{16} - 6\frac{7}{100}$.
26. The quantity of barley imported into the United Kingdom in the years 1912–14 is as follows, for the countries named:

Country.	Weight of Barley Imported into the United Kingdom in Millions of Hundredweights.		
	1912.	1913.	1914.
Roumania	3·551	1·139	1·622
Russia	6·024	3·831	6·295
Turkey	4·503	2·987	2·115
United States	2·266	2·862	2·944
Other countries	8·848	9·541	7·396

Find the total weight of barley imported in each of the three years given to 10000 cwt.

[1] If, in any sum, the − terms added together are greater than the + terms, the numbers are subtracted in the ordinary way and a − sign placed before the answer, thus: $2{\cdot}5 - 6{\cdot}8 + 3{\cdot}5 = +6 - 6{\cdot}8$; where the − number is the greater. Subtracting, we have 0·8, and prefixing the − sign, the result is $-0{\cdot}8$.

27. Taking the data of the previous question, find the amount by which the importation in 1914 was greater or less than that in 1913.

28. Find from the following table the total rateable value of the five largest towns in Ireland, expressing the result in £'s to the nearest £100:

Town.	Rateable Value in Thousands of £'s.
Belfast	1557·809
Dublin	1002·401
Cork	187·111
Londonderry	114·000
Limerick	75·072

29. From the information given in the previous question, find by how many thousand £'s the rateable value of Belfast exceeds that of Dublin.

30. Find the total rateable value of the five cities or boroughs in England given in the following table, correct to £100.

City or Borough.	Rateable Value in Thousands of £'s.
City of Westminster . .	6619·069
City of London . . .	5716·954
Liverpool	4798·516
Manchester . . .	4702·283
Birmingham . . .	4463·404

31. By how much does the rateable value of the city of Westminster exceed that of Belfast?

32. The weight of wheat imported into the United Kingdom in 1912 was 109572·539 thousand cwt. Of this weight 18783·7 came from Argentina and 19973·994 from the United States. By how many hundred cwt. did the total importation exceed that of *each* of the two countries named?

33. By how many **tons** was the weight of wheat imported from Argentina less than that imported from the States?

34. The receipts from Excise for the year 1913 (United Kingdom) were, in thousands of £'s, from—

Beer.	Spirits.	Railway Duty.	Patent Medicines.	Licences.	
				Liquor Licences.	Other Licences.
13200·343	18432·492	283·929	328·319	4595·203	1059·502

Find the total Excise receipts for the year 1913 to the nearest £1000. (See § 14.)

35. The average price of a 4-lb. loaf in various parts of England and Wales is given below for the year 1911. Find the average price throughout the country.[1]

Division.	Price of a 4-lb. Loaf in Pence.
The North and Yorkshire .	6·29
Lancashire and Cheshire . .	5·64
North Midlands . . .	5·18
West Midlands . . .	5·45
South Midlands . . .	5·21
Eastern Counties . . .	5·58
London	5·48
South-eastern Counties . .	5·92
South-western Counties . .	5·58
Wales and Monmouth . .	5·58

36. The average price of a 4-lb. loaf in Scotland for the year 1911 was 6·04 pence. By how much did the average price of bread in Scotland exceed that in England and Wales?

37. Employing the data of Question 35, arrange the districts in order, placing the one in which the price of bread was highest first, and find by how much the highest price exceeded the lowest in the year.

38. Find to £100000 the total amount estimated to be raised in India by taxation in 1912–13, from the figures given:

Year 1912–1913.	
Taxation.	Value in Millions of £'s.
Salt	3·078
Stamps	5·024
Excise	8·104
Provincial rates . .	0·556
Customs	6·857
Income Tax . . .	1·717
Registration . . .	0·480

39. By how much did the estimated receipts from Excise exceed those from Customs in India in 1912–13?

40. Taking the revenue from Excise in 1912–13 in the United Kingdom to be 38063·687 thousand £'s, and that from

[1] For division of decimals see page 100.

Customs as 33517·495 thousand £'s, find the difference between the Customs and Excise receipts for the United Kingdom for the year given to the nearest £100.

41. By how much did the Excise receipts of the United Kingdom for 1912–13 exceed those of India?

SECTION VIII

DECIMALS—(*continued*)

A. Multiplication and Division

I. MULTIPLICATION

85. It is in multiplying or dividing by 10 or a power of 10 that decimals are very convenient, *e.g.*—

(1) $8{\cdot}56 \times 10 = 85{\cdot}6.$
(2) $8{\cdot}56 \times 100 = 856.$

In (1) we multiplied by 10 and the decimal point in 8·56 was simply moved one place to the **right** to give 85·6. In (2) we multiplied by 100 (10^2) and the decimal was two places to the right.

So that any number can be *multiplied* by 10 or 100 or 1000, etc., by moving the decimal point one place, two places, three places, etc., to the *right*.

86. In the same way we have $25{\cdot}032 \div 10$
$= 2{\cdot}5032$ by short division.

$25{\cdot}032 \div 100 = 0{\cdot}25032,$

and $56 \div 10 = 5{\cdot}6,$

or $56 \div 100 = 0{\cdot}56.$

We *divide* a number by 10, 100, etc., by moving the decimal point one, two, etc., places to the *left*.

87. We now show how to multiply or divide a decimal by any whole number. The **rough check** should **always** be made.

EXAMPLE 1.—**Find the product of 5·864 by 5.**

$$5{\cdot}864 \times 5 \doteqdot 6 \times 5 = 30.$$

$$\begin{array}{r} 5{\cdot}864 \\ 5 \\ \hline 29{\cdot}320 \\ \hline \end{array}$$

There are three decimal places in the multiplicand, none in the multiplier, total 3, and there are three in the product. The $\doteqdot$ confirms this by giving two digits before the point.

EXAMPLE 2.—What is the value of 5·864 ÷ 5, to four places.

$$5{\cdot}864 \div 5 \doteq 6 \div 5 = 1\tfrac{1}{5} = 1{\cdot}2.$$

$$5\,|\,\underline{5{\cdot}864000}\ \ldots$$
$$\underline{\underline{1{\cdot}1728}}$$

where we put down the point, when, in the process of division, we come to it.

EXAMPLE 3.—Evaluate $\frac{15}{16}$ of 8·56.

$$\frac{15}{16} \text{ of } 8{\cdot}56 \doteq \frac{15}{16} \text{ of } 8 = 7{\cdot}5.$$

$$\frac{15}{\cancel{16}_{\,2}} \times \overset{1{\cdot}07}{\cancel{8}{\cdot}\cancel{5}\cancel{6}}$$

$$= 15 \times \frac{1{\cdot}070}{2} = 15 \times 0{\cdot}535$$

$$= 8{\cdot}025.$$

The $\doteq$ confirms the position of the decimal.

88. If both the multiplier and multiplicand contain a decimal, we proceed as follows:

EXAMPLE 4.—Evaluate 3·4 × 5·6.

$$3{\cdot}4 \times 5{\cdot}6 \doteq 3 \times 6 = 18.$$

(1) Multiplying, and ignoring the point, we have

$$\begin{array}{r} 34 \\ 56 \\ \hline 170 \\ 204 \\ \hline 1904 \end{array}$$

and the point can be inserted from $\doteq$, which tells us that there are two figures before the point, thus giving us 19·04.

(2) Again, $\quad 3{\cdot}4 \times 5{\cdot}6$

$$= 3\tfrac{4}{10} \times 5\tfrac{6}{10} = \frac{34}{10} \times \frac{56}{10}$$

$$= \frac{34 \times 56}{100} = \frac{1904}{100}$$

$$= 19{\cdot}04.$$

There is *one* decimal place in the multiplier and *one* in the multiplicand, that is *two* altogether, and there are also *two* in the product.

This is true in all cases.

EXAMPLE 5.—**What is the value of $6{\cdot}04 \times 21{\cdot}012$?**

$$6{\cdot}04 \times 21{\cdot}012 \doteqdot 6 \times 21 = 126,$$

that is, three figures before the decimal point.

$$604 \times 21012 = 12691248\,;$$

the $\doteqdot$ places the decimal, 126·91248.

Here, too, 6·04 has two places,
21·012 „ three „
total, five „

$\therefore$ there will be five places in the answer. The $\doteqdot$ confirms the fact that—

There are as many decimal places in the product as there are in the multiplier and multiplicand taken together.

II. DIVISION

89. In division it is best (1) to make a rough check; (2) to divide, ignoring the decimal points; (3) to put in the point from $\doteqdot$. Thus—

EXAMPLE 6.—**Evaluate $58{\cdot}34 \div 72$.**

$$58{\cdot}34 \div 72 \doteqdot 58 \div 72 = 0{\cdot}8.$$

Ignoring the point and dividing 5834·000 by 72 we find the quotient to be 81, and $\doteqdot$ enables us to write 0·81 as the value required.

EXAMPLE 7.—**Divide 5·84 by 2·5, to two places.**

$5{\cdot}84 \div 2{\cdot}5 \doteqdot 6 \div 3 = 2$ (one figure before the point).

Ignoring the points, we have $584 \div 25$.
Now $584 = 584{\cdot}000$,
$\therefore$ the procedure is

$$\begin{array}{r|l} 5 & 584000 \\ \hline 5 & 116800 \\ \hline & \underline{23360} \end{array}$$

but $\doteqdot$ tells us that there is ONE figure before the point,
$\therefore$ the result is 2·336,
or 2·34 to two places

Example 8.—**Evaluate 15·093 ÷ 0·00034, to five significant figures.**

$$15\cdot093 \div 0\cdot00034 \bumpeq 15 \div 0\cdot0003 = \frac{15}{0\cdot0003}$$
$$= \frac{150000}{3}$$
$$= 50000.$$

This becomes 15093 ÷ 34.

$= 443911$

or 44391 to five significant figures by comparing with $\bumpeq$.

90. Some problems may involve combinations of the four rules or of some of them. Thus—

Example 9.—**Simplify to three places** $\dfrac{13\cdot8 - 5\cdot7}{8\cdot1 \times 5\cdot2}$.

$$\frac{13\cdot8 - 5\cdot7}{8\cdot1 \times 5\cdot2} \bumpeq \frac{14-6}{8 \times 5} = \frac{8}{40} = 0\cdot2.$$

$$\frac{13\cdot8 - 5\cdot7}{8\cdot1 \times 5\cdot2} = \frac{8\cdot1}{8\cdot1 \times 5\cdot2} = \frac{1}{5\cdot2} = \frac{10}{52}$$
$$= 0\cdot1923$$
$$= 0\cdot192 \text{ to three places.}$$

Example 10.—**Evaluate to two places** $\dfrac{8\cdot6 \div (5\cdot3 - 4\cdot2)}{7\cdot5 + 9\cdot7}$

$$\frac{8\cdot6 \div (5\cdot3 - 4\cdot2)}{7\cdot5 + 9\cdot7} \bumpeq \frac{9 \div 1}{17} = 0\cdot53.$$

$$\text{Expression} = \frac{8\cdot6 \div 1\cdot1}{17\cdot2}$$
$$= \frac{\cancel{8\cdot6}}{\underset{2}{\cancel{17\cdot2}} \times 1\cdot1} = \frac{1}{2.2} = 0\cdot4545$$
$$= 0\cdot45 \text{ to two places.}$$

EXAMPLES. XX.

1. Write down the results of the following:

(1) 8·732, 9·856, and 8·904, multiplied by 10.
(2) 9·563, 8·972, and 12·834 „ „ 100.

(3) 15·009, 29·001, and 18·0001, multiplied by 10000.
(4) 18·1, 29·01, 0·00803 „ „ 1000.
(5) 67·856, 85·973, 980·00104 „ „ 10000.

2. Write down the value of—

(1) 80·43, 76·92, 58·34, divided by 10.
(2) 28·99, 738·84, 836·58 „ „ 10.
(3) 21·56, 88·94, 936·73 „ „ 100.
(4) 8·7304, 9986·03, 8813·5 „ „ 10000.
(5) 0·0004, 0·00101, 0·00005 „ „ 1000.

3. How many francs[1] are there in 50 francs 75 centimes, 120 francs 86 centimes, 1058 centimes, 5896 centimes, 86 francs 29 centimes?

4. How many centimes are there in each of the above (Question 3)?

5. How many drachmæ[1] are there in 85 drachmæ 75 lepta, 986 drachmæ 38 lepta, 3586 lepta, 98215 lepta, 863 drachmæ 25 lepta?

6. How many lepta are there in each of the above (Question 5)?

7. How many dollars[1] are there in 85 dollars 76 cents, 56 dollars 38 cents, 963 dollars 29 cents, 8685 cents, 7213 cents, 19846 dollars 3 cents?

8. Convert each price in Question 7 into cents.

9. How many francs must a lady in Paris pay for the following: A pair of shoes, 15 francs 75 centimes; an ostrich feather boa, 306 francs 36 centimes; a dozen handkerchiefs, 15 francs 25 centimes; three pairs of kid gloves, 10 francs 12 centimes? What change would she have out of 20 gold twenty-franc pieces?

10. Multiply—

(1) 5·863 by 2 and by 4.
(2) 3·856 by 7 and by 9.
(3) 15·782 by 3 and by 5.
(4) 18·04 by 5 and by 2.
(5) 115·003 by 8 and by 7.
(6) 134·0102 by 3 and by 4.
(7) 0·813 by 2 and by 5.
(8) 0·943 by 8 and by 11.
(9) 0·0013 by 5 and by 8.
(10) 3·0401 by 9 and by 12.

11. Divide in each of the parts 1 to 10 of Question 10 above.

12. Find the value of the following:

(1) $\frac{3}{5}$ of 5·86, $\frac{4}{7}$ of 15·92, to two places.
(2) $\frac{7}{8}$ of 19·843, $\frac{8}{9}$ of 12.96, to three places.
(3) $\frac{3}{11}$ of 15·01, $\frac{4}{9}$ of 18·046, to three places.
(4) $\frac{3}{16}$ of 0·004, $\frac{5}{8}$ of 0·931, to four places.

[1] See § 97.

13. Find the product of—

(1) 0·38 and 0·8, 0·86 and 0·12, 1·94 and 0·25.
(2) 6·84 and 3·72, 90·85 and 5·86, 78·51 and 8·04.
(3) 151·001 and 4·6, 170·001 and 8·01, 151·0101 and 1·01.
(4) 172·1101 and 3·01, 189·0113 and 5·002, 89·00003 and 0·0004.
(5) 1654·804 and 2·4002, 8974·00140 and 0·02.

14. Find the product[1] of—

(1) 151·001 and 9·2, correct to three places.
(2) 340·002 and 16·02, correct to six significant figures.
(3) 12·04 and 172·1101, correct to seven significant figures.
(4) 18·90113 and 100·02, correct to nine significant figures.
(5) 454·25 and 58·6, correct to six significant figures.
(6) 89000·03 and 0·0964, correct to $\frac{1}{10}$th.
(7) 165·4804 and 496·04, correct to $\frac{1}{1000}$th.
(8) 8·0042 and 8·69451, correct to seven places.
(9) 56·84, 28·07, and 0·0041, correct to $\frac{1}{100}$th.
(10) 158·001, 21103·4, and 15·301, correct to $\frac{1}{10}$th.

15. Divide the first number in each part 1–5 of Question 13 by the second (0·38 ÷ 0·8; 0·86 ÷ 0·12, and so on) to four decimal places, where possible.

16. Divide the first number in each of the parts 1–8 of Question 14 by the second to four decimal places, where possible.

17. Evaluate the following (apply ≏):

(1) $2{\cdot}8 \times 3{\cdot}6 \div 2{\cdot}4$. (2) $(7{\cdot}6 \div 9{\cdot}5) \times 2{\cdot}5$.
(3)[2] $(5{\cdot}6 + 2{\cdot}3)7{\cdot}6$. (4) $(5{\cdot}6 - 2{\cdot}3)7{\cdot}6$.
(5) $8{\cdot}3(2{\cdot}7 + 9{\cdot}6)$. (6) $8{\cdot}3(9{\cdot}6 - 2{\cdot}7)$.
(7) $7{\cdot}2(5{\cdot}8 + 6{\cdot}2 - 8{\cdot}8)$.
(8) $2{\cdot}1(7{\cdot}3 + 2{\cdot}8 - 3{\cdot}1 + 4{\cdot}2) - 6{\cdot}5 + 2{\cdot}3$.
(9)[3] $17{\cdot}6 - 3{\cdot}5 \times 7{\cdot}6 + 18{\cdot}5$. (10) $27{\cdot}8 + 5{\cdot}8 \div 2{\cdot}4 - 48{\cdot}3$.
(11) $\dfrac{2{\cdot}15}{4{\cdot}3}$. (Cancel: be careful to keep the decimal point in its proper position.)

(12) $\dfrac{5{\cdot}5}{12{\cdot}1}$ and $\dfrac{0{\cdot}48}{14{\cdot}4}$. (13) $\dfrac{2}{24{\cdot}3} + \dfrac{0{\cdot}0054}{89{\cdot}1} - \dfrac{0{\cdot}0004}{0{\cdot}081}$.

(14) $\dfrac{2{\cdot}5 - 1{\cdot}6}{8{\cdot}3 + 4{\cdot}5}$. (15) $\dfrac{5{\cdot}6 + 7{\cdot}2}{6{\cdot}7 - 3{\cdot}5}$.

[1] It is intended that the multiplication shall be performed in full, and the answer corrected as required.

[2] When a number is written **immediately** before or after a bracket, we agree that it means that the numbers inside the bracket are to be simplified, and **then** multiplied by the number outside, thus: $(5{\cdot}6 + 2{\cdot}3)7{\cdot}6 = 7{\cdot}9 \times 7{\cdot}6$.

[3] Note that 3·5 and 7·6 are bound by the × sign.

(16) $\dfrac{3{\cdot}5+2{\cdot}4-3{\cdot}1}{3{\cdot}6\times4{\cdot}5}$.

(17) $\dfrac{5{\cdot}6\times7{\cdot}3}{8{\cdot}6+3{\cdot}2-1}$.

(18) $\dfrac{2{\cdot}4(5{\cdot}6-3{\cdot}1)}{6{\cdot}6\div(3{\cdot}2-2{\cdot}1)}$.

(19) $\dfrac{9{\cdot}7\div(3{\cdot}5-2{\cdot}1)}{5{\cdot}8\times3{\cdot}2\div2}$.

(20) $2{\cdot}5+\dfrac{3{\cdot}2}{4{\cdot}1}$.

(21) $3{\cdot}6-\dfrac{5{\cdot}2}{3{\cdot}4}$.

(22) $\dfrac{4{\cdot}8}{5{\cdot}3}-1{\cdot}6$.

(23) $\dfrac{7{\cdot}6}{3{\cdot}2}+7{\cdot}5$.

(24) $\dfrac{6{\cdot}3}{7{\cdot}5}\times\dfrac{3{\cdot}9}{2{\cdot}1}$.

(25) $\dfrac{7{\cdot}9}{8{\cdot}6}\div\dfrac{2{\cdot}37}{0{\cdot}43}$.

(26) $\dfrac{48{\cdot}358+24{\cdot}1446}{9{\cdot}82-1{\cdot}97}$. (Square the result of this sum.)

18. Write down the following statements, and place the decimal point in each result *by inspection*—

$$253{\cdot}6\times2{\cdot}04=517344.$$
$$7{\cdot}54\times0{\cdot}35=2639.$$
$$15{\cdot}4\div10{\cdot}8=1426.$$

19. Write down the squares of 0·1, 0·8, and the square root[1] of 0·16.

20. Simplify $\dfrac{3{\cdot}125\times0{\cdot}085}{0{\cdot}01875}$.

21. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, 0·09375, and 2·46?

22. Decimalise each of the following:

(i) $\dfrac{1\frac{1}{8}-\frac{5}{6}}{1\frac{1}{6}+1\frac{11}{24}}$, (ii) $\dfrac{1\frac{7}{10}+2\frac{1}{10}}{2\frac{1}{3}+1\frac{5}{12}}$,

and divide the first by the second.

23. Find the value of—

(*a*) $3{\cdot}84\times0{\cdot}106$.
(*b*) $0{\cdot}1648\times0{\cdot}0112$.
(*c*) $0{\cdot}46376\div0{\cdot}0162$.

24. What is the ratio[2] of 3·06 to 0·306?

25. Evaluate $(3{\cdot}06+0{\cdot}306+306{\cdot}06)\times0{\cdot}12$.

26. Simplify $\dfrac{0{\cdot}0703\times0{\cdot}07}{0{\cdot}019}$.

[1] See § 35. [2] See § 58.

27. Find the value of $\frac{0.232 \times 0.006 \times 1.6}{0.012 \times 0.029 \times 0.08}$.

28. Decimalise, and find the product of—

$$\text{(i) } \frac{\frac{2}{3}+\frac{5}{12}-\frac{5}{6}}{3\frac{11}{18}-3\frac{2}{9}}, \quad \text{and (ii) } \frac{3\frac{1}{2}-1\frac{3}{4}}{1\frac{5}{6}-\frac{2}{3}}.$$

29. Which of the two vulgar fractions $\frac{10}{17}$ or $\frac{17}{29}$ is nearer in value to the decimal fraction 0·587221?

EXAMPLES. XXI.

Miscellaneous Questions

1. The price of Canadian imported wheat in 1911 was, according to the statistical return of the Canadian Government, 0·994 dollars per bushel. Convert this price into shillings and the decimal of a shilling at 4s. to the dollar, and determine whether it is higher or lower than the wholesale price of wheat in Australia at 3·52s. per bushel for the same year.

2. 11·5 feet of mahogany picture-frame moulding cost 4s. 9d., find the price per foot in pence.

3. In Question 6, Examples XVII., if the coal is sold at £1 5s. per ton, what is the total value of the cargo?

4. It requires 60·85 yards of chintz to cover a suite of furniture with "loose covers," what will be the cost of material at 1s. 6d. per yard? (Result in £'s and decimal of a £ to three places.)

5. If pieces of oak 1 inch square and 5 feet long work out at 9·84s. per dozen, what is the cost of 22 five-foot lengths?

6. In Question 4, if the cost of making the "loose covers" at home be at the rate of 9·65 pence per yard, find in £'s the cost of making up the whole 60 yards.

7. The rateable value[1] of a house is £45 and the rates per annum are 8·66 shillings in the £. What amount is paid by the tenant each year as rates?

8. A man occupies business premises for which he pays a rent of £375 per annum. He removes to more commodious ones, where his rental is 538 guineas. How many times is his rental increased? (To three significant figures.)

9. A clerk, employed by a firm which deals in precious metals, being told that gold and iridium are respectively 19·32 and 22·38 times heavier than water, finds that the latter is 1·16 times

[1] See § XVII. B. (2), p. 370.

heavier than the former. Is this correct? If not, by what decimal is it too great or too small?

10. Platinum is 21·45 times heavier than water, how many times is it heavier than gold?

11. A bag of fifty sovereigns[1] is paid into the Mint by a banker and the coins are found to weigh 6153·6845 grains. The least current weight of a sovereign must be 122·500 grains. Is the average weight of each coin above or below the least current weight, and by how many grains?

12. The value of a franc is 9·513 pence, how many francs are there in 18s. 9d.?

13. A year's crop of rubber on a certain plantation was 1185200 lb. The average cost of production was 1s. 7·95d. per pound and the average selling price was 3s. 6·67d. per pound. What was the total profit?

14. A wine merchant has a tun of wine (252 gallons). He sells $\frac{1}{8}$ to one customer, $\frac{1}{9}$ of the remainder to each of two others, and $\frac{1}{27}$ of what then remains to a fourth customer. How many gallons and the decimal of a gallon has he left?

15. The amount of corn required to sow a field of 12·5 acres was 31·25 bushels, and the yield was 453·125 bushels. Find the yield per acre. Find also how many times over the farmer gets his seed back.

16. A nurseryman has an indiarubber plant for which he can obtain now 2·75s. If he keeps it, it grows 0·6 foot per annum and its value increases by 1s. for every 0·15 foot it grows. It costs him 0·75s. per annum to keep. How much more will he get by keeping it for five years and then selling it than if he sold it now?

17. On 31st December 1912 the net receipts from the railways in the United Kingdom were, in millions sterling, 47·329 and the working expenses 81·224. Find the working expenses for every million net receipts (to one decimal place).

18. Cheap nail-brushes, the backs of which are made of soft wood costing 1·45s. per 144 feet, are 6·75 inches long. Find the cost of the wood in each brush, as a decimal of a penny.

19. On certain classes of life insurance,[2] offices pay their agents £2 5s. on every £100 paid by their clients in premiums. The amount received by one company was 3·875 hundred thousand £'s. What amount of money was paid to the agents who secured this business?

[1] A sovereign, when coined, must not be more than 0·2 grains above or below the standard weight of 123·27447 grains. The 0·2 grain is called the "Remedy of Weight." See Part III.

[2] See Section XVII. B. (5).

20.

Liner.	Gross Tonnage (Thousands of Tons).	Length—Hundred Feet.	Line.
Olympic . .	46·359	8·52	White Star.
Mauretania .	31·938	7·62	Cunard.

Find from the above data the gross tonnage for every 100 feet in the length of each of the ships.

21. In the year 1700 the total value of the imports and exports of the United Kingdom was 10·7 million £'s, and the value per head of the population was £1·125. Find the population of the United Kingdom in 1700.

22. In 1912 the figures corresponding with those of the last question were 1343·6 million £'s, and £26·98. Find the population of the United Kingdom in 1912.

23. From the results of the two previous questions find how many times the population of the United Kingdom in 1912 exceeded that in 1700.

24. The area of Malta (acquired in 1814 by treaty cession) is 120 square miles, and its population is 211000. Compare the density of its population with that of Ceylon, the area of which is 25·5 thousand square miles and the population 4·1 millions (1911).

25. The regular forces in India in 1912–13 were 75884, and in 1913–14, 75897. Express the increase in the year as a decimal of the number of troops in 1912–13.

26. The area of the British Empire in Asia and in Africa, together with the respective population, is, according to *Whitaker's Almanack*:

British Empire in	Area in Millions of Square Miles.	Population in Millions.
Asia	2·188	323·158
Africa	3·618	49·458

How many times is the population of the Empire in Asia denser than that in Africa? Comment upon this condition of affairs.

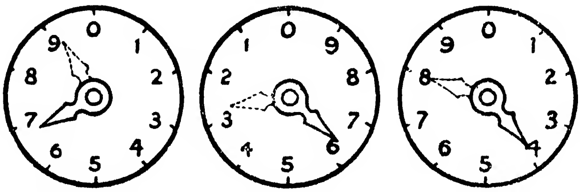
27. A labourer's wife allows half an ounce of tea a day for herself and family. The duty is increased 3d. per lb. How much more does she contribute to the revenue per year than she did before. (Decimalise the numbers and give the answer in shillings and the decimal of a shilling.)

28. If the labourer drinks $3\frac{1}{2}$ pints of beer a day, how much more does he pay in a year when the duty is raised by $6\frac{1}{2}$d. a gallon?

29. From the results of the two previous questions determine by how much the two increased duties, namely, that on tea and that on beer, increase the weekly expenditure of the people referred to.

30. If the wages of a man are 32s. 6d. a week, his rent 0·34 of his wages, firing 0·28, food 0·295, what amount of money has he left for other things?

31. The consumption of gas is read off on dials fixed to the meter. The readings on 1st January are shown by the thick



arrows, those on 1st April by the dotted ones. Find how many cubic feet of gas were consumed in the three months given, and calculate the cost in £'s and the decimal of a £ at 2s. 3d. per thousand cubic feet.

B. Contracted Methods[1]

I. MULTIPLICATION

91. Very often long decimals enter into the working of a problem while it is necessary to give the answer correct to, say, two figures only. Contracted methods enable us to work so as to get the two places and no more.

92. EXAMPLE 1.—**Find the product of 6·3216 and 20·24241, correct to three places.**

It is obvious that we can proceed to multiply in full and correct the third place thus:

[1] This section may be left till the second reading.

$6{\cdot}3216 \times 20{\cdot}24241 \simeq 6 \times 20 = 120$, which shows that there will be three digits in the answer before the decimal point. We therefore work the sum ignoring the decimal, and then put it in the product from the $\simeq$.

We can now write down the result correct to three places **if we know the fourth correctly**, *i.e.* we keep one more figure than is required, and the product is $127{\cdot}964$.

2024241	
63216	
1214544	6
60727	23
04048	482
00202	4241
00121	45446
1279644	19056

93. All the figures to the right of the vertical line are unnecessary, and in employing a contracted method they are simply left out. We now explain the procedure.

1. Make a rough check.

2. Multiply right through by the first figure, ignoring the decimal points, draw the vertical line by comparing with the $\simeq$, and then work to one more figure than is required in the result.

The student should now set down the sum and work step by step as follows:—

Method—

(1) Line (1) enables us to draw the vertical line in the proper place by comparing with $\simeq$. Now the 6 (units), in line (1), is unnecessary, and it results from multiplying the units figure, 1, by 6. ∴ we put a cross over the units figure, or a line through it, as it is not required, **and we correct the 4** (tens) in line (1) to a **5**.

×××××		
2024241		
63216		
1214545	$\cancel{6}$. . .	(1)
60727	2. . .	(2)
04048	4. . .	(3)
00202	4. . .	(4)
00121	2. . .	(5)
		(6)

(2) The next multiplying figure is **3**. 1×3 would be **two** places to the right of the line. 4 (tens) $\times$ 3, one place to the right. Perform this mentally, 12, and carry 1 to the 2 (hundreds) $\times$ 3, then $(2 \times 3) + 1 = 7$. The 1 (units) and 4 (tens) are now "crossed" and the line proceeds as usual.

(3) Now omit the "crossed" figures and say 2 (hundreds) $\times$ 2 $= 4$, one place to the right of the line, cross the 2 hundreds; then 4 (thousand) $\times 2 = 8$; there is nothing to carry, and, when corrected for the following (mental) 4, remains 8. Then finish the line.

(4) 4 thousand $\times 1 = 4$, one place to right of the line; cross the 4. 2 (ten thousand) $\times 1 = 2$, put it down, and on correcting it remains 2.

Continue the multiplication, finish the sum, and then place the decimal point from $\bumpeq$.

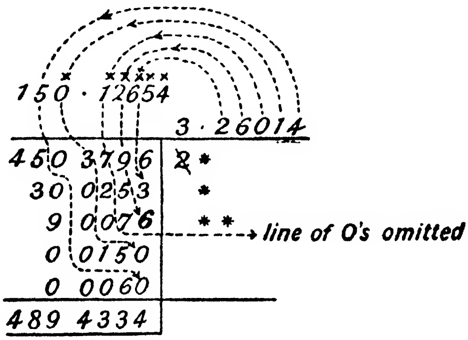
The *principle* of this work is this: **cross out one figure in the multiplicand, as you come to it, for every multiplier.**

We work one other sum, but omit the explanation.

EXAMPLE 2.—**Evaluate $3{\cdot}26014 \times 150{\cdot}12654$, to six significant figures.**

$$3{\cdot}26014 \times 150{\cdot}12654 \bumpeq 150 \times 3 = 450.$$

There are then three figures before the point, so that we shall want three places of decimals, but we work to four for safety.



The product is $489{\cdot}433$, to six significant figures.

* Observe that the carrying figures have been allowed for.
** Corrected from 5 to 6 because of the next figure being over 5.

The line in full is $9{\cdot}007592$ . . .

EXAMPLES. XXII. (a)[1]

Use contracted methods in the following questions, 1 to 25.

Find the value of each of the following:

1. $5{\cdot}803 \times 3{\cdot}705$, to three places of decimals.
2. $3{\cdot}0154 \times 2{\cdot}805$, to four significant figures.
3. $12{\cdot}7134 \times 3{\cdot}523$, to five significant figures.
4. $111{\cdot}5331 \times 5{\cdot}824$, to three places of decimals.
5. $101{\cdot}6215 \times 10{\cdot}336$, to six significant figures.
6. $32{\cdot}8104 \times 0{\cdot}3421$, to four places of decimals.
7. $156{\cdot}72201 \times 0{\cdot}00156$, to four places of decimals.

[1] Many of the drill examples in XXII. (*a* and *b*) have been taken from commercial examination papers.

8 $28{\cdot}38625 \times 3{\cdot}00105$, to five significant figures.

9. $510{\cdot}6321 \times 0{\cdot}12003$, to five places of decimals.

10. $78{\cdot}8424 \times 0{\cdot}0431$, to four significant figures.

11. $1051{\cdot}863 \times 5{\cdot}30043$, to seven significant figures.

12. $2{\cdot}8038 \times 0{\cdot}00137$, to three significant figures.

13. $25{\cdot}6824 \times 3{\cdot}512$, to four places of decimals.

14. $22{\cdot}7541 \times 5{\cdot}7384$, to three places of decimals.

15. A sovereign should weigh 123·27445 grains. Express its weight in grams, correct to four places, if 1 grain = 0·0647989 grams.

16. If £1 = 25·2215 francs, calculate the value of £150 6s. 8d. in francs, to three places of decimals.

17. A small consignment of grain is invoiced at 8 Hectolitres 50 litres. Express this in bushels to three decimal places, if 1 Hl. = 2·75121 bushels.

18. The population of Bradford was 289·618 thousand last year, and its birth-rate 19·275 per thousand. What was the number of births in the city?

19. Taking the death-rate of Bradford to be 14·48 per thousand, calculate the net increase in population, using the data of Question 18.

20. Find the weight of a consignment of dried fruit weighing 1 ton 5 cwt. 2 qrs. in Kilograms, correct to $\frac{1}{10}$ Kgm. (1 cwt. = 0·5080238 quintals.)

21. If platinum foil be sold at the rate of £6 10s. 9d. per ounce, calculate to the nearest penny the value of a piece 1·56 in. square, given that 1 sq. ft. weighs 30 oz.

22. If the width of the Suez Canal, which was opened in 1869, be 121 ft. 5 in., and 1 yard = 0·91438 metre, calculate to two places of decimals the width of the canal in metres.

23. 1 cubic inch equals 16·38618 cub. cm. Calculate to $\frac{1}{100}$ of a cubic centimetre the volume of a cube of glass, such as an optician might use, the edge of which is 2·64 in.

24. Find the cost of covering a sphere, which is 8·58 cm. in radius, with pure gold leaf at the rate of 90 sq. cm. for a franc, correct to two decimal places. ($\pi = 3{\cdot}14159$.)

25. Gold and iridium are respectively 19·32 and 22·38 times heavier than water. Calculate to four places of decimals what weight of iridium is equal in volume to 3 dwt. 8·563 grs. of gold.

II. CONTRACTED DIVISION

94. In contracted multiplication we gradually diminished the multiplicand so as to avoid unnecessary work, and in contracted division we diminish the **divisor** in the same way and arrange that **the *divisor* shall contain 1, or at the most 2, more figures than are required in the answer.** This is effected, sometimes, by adding 0's.

EXAMPLE 1.—**Find the value of 3·58621 ÷ 0·573426, to four significant figures.**

$$3{\cdot}58621 \div 0{\cdot}573426 \doteq 3{\cdot}5 \div 0{\cdot}6 = 5{\cdot}8.$$

We want four significant figures and so we keep $4+2=6$ figures in the divisor.

We ignore the decimal in dividing and insert it from ≏ when the sum is finished.

```
   ×××
573426|3586210|625400
      |3440556|
       -------
       145654   Cross the 6 in the divisor, and begin to
       114685      multiply by 2 at the 2 (tens), allowing
                   for carrying figure from the 6.
       ------
        30969   Cross the 2 tens in the divisor and begin
        28671      with the 4.
        -----
         2298   Cross the 4 hundreds and begin with the
         2294*     3 thousands.
         ----
            4
```

* Note that we are here multiplying 573 by 4, which gives us 2292, but (1) we must allow for the previous carrying figure, namely, 1, and (2) we must correct the first figure we put down by means of the following one. Really, then, $5734 \times 4 = 22936$, which is 2294 to four significant figures. The student should note these points carefully.

We now refer to ≏ for the decimal point, and find that the quotient required is 6·254. It is of the greatest possible importance in answering questions of this kind to give the result as it is asked for. If three decimal places are required, or four significant figures, be careful **not** to give the result to four places or five significant figures, **and always correct the last figure.**

EXAMPLES. XXII. (b)

Use contracted methods.

Find the value of the following:

1. $44{\cdot}789 \div 12{\cdot}7134$, to four significant figures.
2. $21{\cdot}500 \div 3{\cdot}705$, to three places of decimals.
3. $649{\cdot}569 \div 5{\cdot}824$, „ „ „
4. $0{\cdot}2445 \div 0{\cdot}00156$, to one place of decimals.
5. $8{\cdot}4582 \div 2{\cdot}805$, to five significant figures.
6. $11{\cdot}2244 \div 0{\cdot}3421$, to three places of decimals.
7. $85{\cdot}189 \div 3{\cdot}00105$, „ „ „
8. $1050{\cdot}359 \div 10{\cdot}336$, „ „ „
9. $61{\cdot}29117 \div 510{\cdot}6231$, to five „ „
10. $3{\cdot}398 \div 78{\cdot}8424$, to four „ „
11. $5575{\cdot}326 \div 1051{\cdot}863$, to five „ „
12. $0{\cdot}00384 \div 2{\cdot}8038$, to three significant figures.
13. $90{\cdot}1966 \div 3{\cdot}512$, to five significant figures.
14. $130{\cdot}572 \div 22{\cdot}7541$, to four places of decimals.
15. Multiply $845{\cdot}6$ by $23{\cdot}1783$, correct to one decimal place.
16. Divide $0{\cdot}0001$ by $3141{\cdot}592654$, correct to ten places of decimals.
17. Multiply $567{\cdot}23$ by $9{\cdot}845$, correct to one decimal place.
18. Divide $7{\cdot}325$ by $489{\cdot}6$, correct to four decimal places.
19. Divide $2{\cdot}516$ by $8{\cdot}479$, correct to three decimal places.
20. Multiply $537{\cdot}84$ by $9{\cdot}25$, correct to the units place.
21. Find the value of the following to two places of decimals: (*a*) $3{\cdot}1416 \times 5{\cdot}61$; (*b*) $25{\cdot}08 \times 0{\cdot}7854$.
22. Find the value of $3{\cdot}14159 \times 62{\cdot}4817$, to three decimal places.
23. Multiply $56{\cdot}78125$ by $3{\cdot}0125$, correct to one decimal place.
24. Divide $9{\cdot}613425$ by $768{\cdot}26123$, correct to four decimal places.
25. Evaluate $3{\cdot}5618 \div 81{\cdot}79$, to three decimal places.

Find correct to five significant places the value of (26–29)—

26. $64{\cdot}357 \times 395{\cdot}68$.
27. $4{\cdot}8763 \div 395{\cdot}68$.
28. $469{\cdot}34 \times 63{\cdot}874$.
29. $39{\cdot}735 \div 597{\cdot}03$.
30. Multiply $856{\cdot}234$ by $17{\cdot}93$, to one decimal place.
31. Divide 1 by $3{\cdot}14159265$, to five decimal places.
32. Evaluate $32{\cdot}6784 \times 0{\cdot}08769$, to three decimal places.
33. Multiply $3{\cdot}58067$ by $289{\cdot}3785$, to three decimal places.
34. Divide $1{\cdot}765489$ by $26{\cdot}493876$, to three significant figures.

35. Compute by contracted methods to four significant figures the value of $(0{\cdot}4215 \times 8{\cdot}318) \div 9{\cdot}223$.

36. Evaluate to three decimals $(346{\cdot}25 \times 3{\cdot}2164) \div 73{\cdot}296$.

37. In the year 1912, 1,294,337,046 passengers were carried on the railways of the United Kingdom (exclusive of season ticket holders), and the working expenses were £81,224,343. Calculate to three places of decimals the average expense per passenger.

38. The price of bread has increased by 15·3 % above the level of prices in 1915, and milk has increased 9·4 %. Calculate to three places of decimals the number of times the former increase is greater than the latter.

39. If 31·25 bushels of corn are required to sow 12·5 acres, calculate to three decimal places the quantity required per acre.

40. If 1 franc = 9·513d., calculate to three decimal places the value of 18s. 6½d. in francs.

41. An estate produced 500·7653 tons of rubber valued at £90000. Calculate to four significant figures the average value in pence per pound.

42. Determine the weight in Kilograms, correct to two places, of a quantity of cloth consigned from Bradford to Paris, if the railway company returns the weight as 15 cwt. 1 qr. 18 lb., and you know that 1 Hectogram = 3·527394 oz. Av.

43. The area of an estate in France is given as 5·8046 sq. Kms. Express the area in English measure for a firm of estate agents, given that 1 acre = 0·40467 Hectares.

44. If £1 be left at 5 % compound interest for 50 years, it amounts to £11·4674. Find what amount of money must be deposited to amount to £3000 16s. 8d. in the same time.

45. Mount Everest is 29002 ft. high, and is situated 480 miles, as the crow flies, from the Bay of Bengal. Calculate the average rise in the level of the land in going from the sea to the summit of the mountain, in feet per hundred yards.

46. The gross tonnage of the *Olympic* is 46·359 thousand tons, and her length 8·52 hundred feet. Calculate to three places the tonnage per hundred feet.

47. At an iron foundry there is 8 cwt. 2 qrs. 12 lb. of iron run off from one of the blast furnaces. If 1 cubic foot of iron weighs 490·632 lb., calculate to three places of decimals the number of cubic feet of metal there are.

48. If 1 Km. = 0·621382 miles, express 2 mi. 3 fur. 6 chs. in Kilometres, to four decimal places.

49. A hemisphere, 10 in. in diameter, is plated with 23·5624 grains of gold. Calculate the weight of gold per sq. inch, to four places of decimals. ($\pi = 3{\cdot}14159$.)

50. In the last question calculate the average thickness of the gold, in millimetres, to two significant figures, if the specific gravity of that metal be 19·32. (1 inch = 2·53995 cm.; 1 gram = 15·43235 grains.)

SECTION IX

ADDITION, SUBTRACTION, AND REDUCTION OF MONEY

A. Addition and Subtraction

95. We have already asked the student to work problems involving francs and other **foreign** monies, because the procedure involved is based upon decimals and is much simpler than our own.

96. In the Union of South Africa,[1] New Zealand, and some small British possessions, the coins in circulation are just the same as in the United Kingdom. In Australia there are special designs for many of our *silver* coins, and the gold in use there, coined in the mints of Perth, Melbourne, or Sydney, has very small "mint marks," P., M., or S., just above the date.

In India and the East Indies the rupee is the chief coin, and its value is fixed at 1s. 4d., although, being a silver coin, its value varies with the price of silver.

At Hong-Kong and in the Malay States a silver dollar of fixed value, 2s. 4d., is used, while in Canada and Newfoundland the dollar is valued at 4s. $1\frac{1}{3}$d. and 4s. 2d. respectively.

In speaking of a dollar it is necessary to know the country in which it was coined before we can tell its value.

In some countries paper money is very largely used instead of coin,[2] and its value varies very considerably, particularly in the case of South and Central American States, which are not infrequently the scenes of revolution. The gold milreis of Brazil is worth 2s. 3d., but the paper milreis, which is in common use, is worth only 1s. $4\frac{1}{4}$d.

[1] A silver fourpenny-piece is also in use in the Union.

[2] In England this is not so; hence, when in 1914 Treasury notes were issued, people in some parts of England refused to accept them for many weeks.

97. The following tabular statement may be useful:[1]

MONIES OF THE PRINCIPAL NATIONS.

Country.	Standard Coin.	Value in English Currency.	Approximate Value in English Currency.
I. BRITISH POSSESSIONS.			
		£ s. d.	£ s. d.
CANADA	Dollar	0 4 1⅓	0 4 1
INDIA, etc.	Rupee = 16 Annas	0 1 4	0 1 4
NEWFOUNDLAND	Dollar	0 4 2	0 4 2
STRAITS SETTLEMENTS, etc.	Dollar	0 2 4	0 2 4
EGYPT	Pound	1 0 3¾	1 0 4
II. EUROPE.			
BELGIUM	Franc of 100 Centimes		
Bulgaria	Lev of 100 Stotinki		
FRANCE AND COLONIES	Franc of 100 Centimes		
GREECE	Drachma of 100 Lepta		
ITALY	Lira of 100 Centesimi	0 0 9·513	0 0 9½
Roumania	Leu of 100 Bani		
Servia	Dinar of 100 Paras		
SPAIN	Peseta		
SWITZERLAND *	As for France		
Austria-Hungary	Krone of 100 Heller	0 0 10	0 0 10
GERMANY	Mark of 100 Pfennige	0 0 11·7483	0 0 11¾
Denmark	Krone of 100 Ore		
Norway	,, ,,	0 1 1½	0 1 1½
Sweden	,, ,,		
Netherlands	Florin or Gulden of 100 Cents	0 1 7·824	0 1 8
Portugal	Escudo, Gold	0 4 5¼	0 4 5¼
	,, Paper	0 3 4	0 3 4
RUSSIA	Rouble of 100 Kopecks	0 2 1⅓	0 2 1
III. ASIA.			
China	Yuan of 100 Cents	0 2 0	0 2 0
JAPAN	Yen of 100 Sen	0 2 0·582	0 2 0½

* The above nine States are in the Latin Union, and their coins, being of the same value, circulate freely among them all.

[1] *Whitaker's Almanack*, pp. 460–462.

MONIES OF THE PRINCIPAL NATIONS—(*continued*).

IV. AMERICA.			
Country.	Standard Coin.	Value in English Currency.	Approximate Value in English Currency.
		£ s. d.	£ s. d.
UNITED STATES .	**Dollar of 100 Cents** .	0 4 1·32	0 4 1
SOUTH AMERICA .	Various		

98. The value of £1 sterling in foreign money is as follows:

£1 = 25·22 Francs.
,, = 20·43 Marks.
,, = 24·02 Kronen.
,, = 4·8 United States Dollars.
,, = 4·8 Canadian Dollars.

99. The details printed in dark type in the last two paragraphs should be remembered *by continual usage*, and we shall assume them unless otherwise stated. With their help **the student can** at least prepare himself to **quote prices for continental customers in their currency and not in his own.**

For an exhaustive treatment of this question, see Part III.

100. It is not possible to deal with the English money system as readily as with that of many countries, for although 2s. is $\frac{1}{10}$th of £1, and a farthing is very nearly $\frac{1}{1000}$th of £1, it has not yet been reduced to a decimal system. We have

4 farthings (fourthings) = 1 penny (1d.)
12 pence[1] . . = 1 shilling (1s.)
20 shillings . . = 1 pound.

£1 is frequently called "£1 sterling"[2]—that is to say, it is "of standard value."

101. On adding a number of farthings together we divide by 4 to bring to pence, thus: 86 farthings = $\frac{86}{4}$ = $21\frac{1}{2}$d.—that is, 21 pence halfpenny. In the same way 21 pence = $\frac{21}{12}$ shillings

$= 1\frac{9}{12}$,,
= 1s. 9d.

for 9d. is $\frac{9}{12}$th of one shilling.

[1] The symbols £ s. d. are derived from the Latin words—*libræ*, Roman "pounds"; *solidi*, which were Roman coins, really SOLID pieces of money, and hence our word "soldier"; *denarii* = "containing ten," for the silver denarius of the Romans contained, at first, ten asses.

[2] "Sterling" is derived from the word "Easterling," a name which was applied to traders who came from certain parts of the Continent to England, and whose money was of peculiar purity.

If, on adding, we have, say, 356 shillings, we reduce to pounds by dividing by 20, thus: $356 \div 20 = £17\frac{16}{20} = £17$ 16s.

EXAMPLE 1.—**Add—**

£	s.	d.
56	15	$8\frac{1}{4}$
21	6	$5\frac{1}{2}$
103	0	$4\frac{3}{4}$
56	10	$6\frac{1}{2}$
28	12	$3\frac{1}{4}$
£266	5	$4\frac{1}{4}$

The totals are: 9 farthings $= 2\frac{1}{4}$d.

26d. + 2d. = 28d. = 2s. **4**d.

43s. + 2s. = 45s. = £2 **5**s.

£264 + £2 = **£266**.

The figures in dark type give the total required.

EXAMPLE 2.—**What is the difference between £56 15s. $8\frac{1}{2}$d. and £126 17s. $9\frac{3}{4}$d. ?**

£	s.	d.
126	17	$9\frac{3}{4}$
57	15	$8\frac{1}{2}$
£70	2	$1\frac{1}{4}$

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$d.
8d + **1**d. = 9d.

15s. + **2**s. = 17s.
£56 + **£70** = £126.

EXAMPLE 3.—**By how much does £785 12s. $7\frac{1}{4}$d. exceed £358 15s. $10\frac{1}{2}$d. ?**

We write down the amounts as before.

£	s.	d.
785	12	$7\frac{1}{4}$
358	15	$10\frac{1}{2}$
£426	16	$8\frac{3}{4}$

$\frac{1}{2} + \frac{3}{4}$ $= 1\frac{1}{4}$.

10d. + 1d. + **8** = 1s. 7d., *i.e.* 19d.

15s. + 1s. + **16**s. = 32s. = £1 12s.

and £358 + £1 + **£426** = £785.

It is clear that we cannot subtract $\frac{1}{2}$d. from $\frac{1}{4}$d., and therefore we ask what must be added to $\frac{1}{2}$d. to give $1\frac{1}{4}$d.? and so get $\frac{3}{4}$d.; and then proceed just as in simple subtraction and say 10d. + **1**d. = 11d.; what must be added to 11d. to give, not 7d., but **1**s. 7d. or 19d.? and so get **8**d.; and so on.

EXAMPLES. XXIII.

	£	s.	d.		£	s.	d.		£	s.	d.
1.	5632	4	2	2.	492	4	11	3.	526	3	5
	4187	16	$6\frac{1}{2}$		4016	3	9		89	4	11
	437	15	$7\frac{1}{2}$		51200	1	3		729	15	6
	596	13	$1\frac{3}{4}$		6271	18	3		3416	8	8
	5910	7	8		9416	9	8		17	17	7
	4728	6	11		1067	13	8		893	12	6
	59	12	$10\frac{3}{4}$		25643	9	9		56	3	9
	19403	13	8		27768	12	4		255	13	4
	27391	4	$6\frac{1}{2}$		5712	14	4		5317	14	10
	40572	16	4		975	17	7		5	3	7

	£	s.	d.		£	s.	d.		£	s.	d.
4.	423	3	0	5.	1327	8	10	6.	37	4	$8\frac{1}{4}$
	75	6	10		20315	7	9		375	13	5
	59	19	8		5237	0	4		118	3	$10\frac{1}{2}$
	17	2	7		82	15	3		2729	17	4
	2526	12	$10\frac{3}{4}$		707	10	10		94317	8	$11\frac{1}{4}$
	118	4	6		55	13	8		82	14	6
	93	17	5		173	16	7		593	11	$7\frac{3}{4}$
	906	14	$8\frac{1}{2}$		84	9	11		52113	7	10
	3	16	7		6	19	5		324	12	8
	47	9	4		2245	5	6		39	6	$7\frac{1}{2}$

	£	s.	d.		£	s.	d.		£	s.	d.
7.	13874	15	10	8.	28374	14	10	9.	21772	14	4
	649	7	8		12681	7	$7\frac{1}{2}$		8259	6	11
	2735	5	0		13955	9	6		545	12	1
	10867	11	11		267	6	$4\frac{1}{2}$		1728	0	5
	2935	19	5		385	13	9		15419	8	7
	4672	4	6		462	18	3		3742	11	6
	19863	6	4		8793	7	$4\frac{1}{2}$		85	5	2
	274	16	9		1956	19	7		137	15	8
	8553	7	8		728	6	$5\frac{1}{2}$		1027	13	9
	21758	3	5		375	5	8		416	8	4

	£.	s.	d.		£.	s.	d.		£.	s.	d.
10.	13245	13	4½	11.	517	13	4	12.	37	11	2½
	8027	8	10		89	17	11		176	15	5
	592	11	3¾		2054	8	5		2089	14	7¼
	1728	0	8½		709	13	7		592	11	6
	24317	13	7		5427	4	10		13259	4	10¾
	4056	18	4¾		23486	16	3		729	8	8½
	809	9	9		59	12	8		2058	19	1½
	2056	13	5		15476	8	9		72	12	5
	92	19	7½		427	17	2		746	7	7¾
	186	4	4		95	19	7		59	17	8½

	£	s.	d.		£	s.	d.		£	s.	d.
13.	4526	13	5	14.	23	3	4½	15.	348	13	6
	89	4	11		175	6	10		95	8	11
	729	15	6		59	19	8¼		3516	12	7
	23416	8	8		317	2	7		808	7	10
	17	17	7		2526	12	10¾		27247	16	5
	893	12	6		118	4	6		8	13	2
	56	3	9		93	17	5¼		971	7	7
	255	13	4		906	14	8½		1606	16	8
	5317	14	10		3	16	7		59	19	4
	5	3	7		47	9	4¼		217	5	9

	£	s.	d.		£	s.	d.		£	s.	d.
16.	18	3	4¼	17.	7249	11	7	18.	942	3	7½
	729	7	6½		506	3	10		2056	12	4¼
	2314	14	10		21452	16	5		98	7	10
	846	13	5¾		78	7	6		31483	13	8¾
	99	8	10½		942	12	8		576	6	9½
	1417	17	3		8329	8	9		4117	12	5
	6	16	11¼		41525	15	4		823	10	7
	47	10	5½		348	9	11		15462	17	11½
	204	9	4		7254	11	3		6509	18	3
	72	12	3¼		245	6	10		172	4	2¾

	£	s.	d.
19.	193	14	9
	84	14	7
	119	18	4
	74	9	7
	89	7	10
	101	19	2
	76	6	11
	49	17	5
	166	8	7
	195	17	0
	54	7	10
	74	13	5
	296	12	9
	37	14	11
	85	10	4

	£	s.	d.
20.	2639	18	7
	798	16	4
	9109	8	6
	6198	19	2
	527	6	5
	1693	18	9
	7174	15	7
	678	8	8
	9193	17	10
	657	7	8
	793	10	11
	5179	4	3
	98	15	7
	84	16	8
	8163	10	6

	£	s.	d.
21.	9035	2	10
	4123	17	6
	211	6	8
	94	3	4
	4321	18	9
	6409	12	8
	492	13	7
	56	4	10
	109	19	10
	9603	2	5
	362	3	4
	5976	2	5
	1397	7	1
	563	4	1
	4287	9	11
	46	3	6
	200	14	8

	£	s.	d.
22.	7245	6	2
	8139	7	4
	15065	7	3
	279	13	8
	8904	12	9
	4561	10	8
	485	2	2
	163	9	11
	752	15	3
	287	6	8
	2307	4	2
	1528	3	6
	691	13	9
	6127	9	10
	4201	6	8
	232	1	11
	360	5	5

23. Add the following in lines, horizontally :

(1) £13 14s. 6d. ; £98 12s. 7d. ; £213 4s. 11d. ; £77 17s. 7d. ; £8 15s. 9d.

(2) £77 11s. 5d.; £148 16s. 10d.; £7 18s. 2d.; £34 18s. 6d.

(3) £9 13s. 11d.; £58 11s. 7d.; £215 5s. 9d.; £82 6s. 7d.; £47 19s. 2d.

(4) £25 16s. 8d.; £187 14s. 5d.; £19 15s. 9d.; £84 8s. 5d.; £19 12s. 11d.

(5) £7 16s. 3d.; £74 5s. 9d.; £235 11s. 4d.; £82 5s. 10d.; £32 17s. 7d.

(6) £81 13s. 7d.; £295 16s. 10d.; £12 13s. 5d.; £84 11s. 8d.

(7) £59 15s. 7d.; £343 18s. 6d.; £517 3s. 9d.; £48 18s. 5d.

(8) £8; £45 13s. 8d.; £76 7s. 6d.; £21 5s. 8d.

(9) £11 8s. 2d.; £23; £20 5s. 6d.; £38 16s. 5d.

(10) £12 8s. 1d.; £21 5s.; £22 7s. 3d.; £11 15s. 4d.

(11) £28 14s. 11d.; £20 6s. 3d.; £22 1s. 6d.; £40 15s. 6d.

(12) £75 6s. 9d.; £19 10s.; 11s.; £1 1s.; £91 11s. 8d.; £2 15s. 1d.

(13) £12 6s. 3d.; £5 7s. 9d.; £5 4s. 3d.; £16 7s. 9d.; £4 13s. 5d.

(14) £10 5s.; £22 2s. 6d.; £4; £2 17s. 6d.; £38 14s. 8d.

24. Total the following, and prove by cross additions (b/f and c/f mean "brought" and "carried" forward respectively):

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	Totals.		
			b/f			b/f			b/f					
0	15	9½	1	19	7½	3	8	6	16	9	7¾			
3	17	6¼	0	16	5¾	4	9	7	4	0	8½			
0	18	3½	0	13	7½	4	3	11¼	0	3	6¼			
9	3	7¾	0	3	9¼	0	5	7½	8	9	7½			
c/f £			c/f £			c/f £			Total £					

25. In each of the parts 1–14 of Question 23 write down the largest amount and then the largest but one, and find the difference between them.

26. In each of the parts 1–14 of Question 23 write down the smallest amount and the smallest but one, and find the difference between them.

27. By how much does 19s. 10¾d. exceed (1) 18s. 11½d. and (2) 1s. 0½d.?

28. An arm chair costs 55s. 11d. and a small chair 39s. 6d. How much cheaper is the latter than the former?

29. A suite of bedroom furniture cost 25 guineas. The ward-

robe costs £12 19s. 6d., the dressing table, £4 15s. 9d., the washstand, £5 10s. 11d. Find the cost of the three pieces named, and also that of the remainder of the suite.

(*For further problems, see Examples XXV.*)

B. Reduction of Money

102. It is possible to express 2500 farthings in pounds, shillings, and pence, or £56 18s. $5\frac{1}{4}$d. in farthings.

The process of doing either of these things is called **Reduction.**

Example 1.—**Reduce 2500 farthings to £ s. d.**

There will be fewer pennies than farthings;

∴ we divide by 4 to bring farthings to pence, then by 12 to reduce to shillings, and then by 20 to reduce to pounds.

$$\begin{array}{r|l} 4 & 2500 \text{ farthings} \\ \hline 12 & 625 \text{ pence} \\ \hline 20 & 52 \text{ shillings} + 1 \text{ penny (over)} \\ \hline & £2 + 12 \text{ shillings (over)} \end{array}$$

∴ 2500 farthings = £2 12s. 1d.

Example 2.—**Reduce £12 18s. $8\frac{1}{2}$d. to farthings.**

Here we reduce pounds and shillings to shillings, then shillings and pence to pence, and finally pence and farthings to farthings.

There will be **more** farthings than there are pounds;

∴ we multiply.

	£	s.	d.	
	12	18	$8\frac{1}{2}$	
	20			
Shillings,	258	*i.e.* £12 × 20		= 240 shillings,
	12	and 240 shillings + 18 shillings		= 258 shillings.
Pence,	3104	*i.e.* 258 shillings × 12		= 3096 pence,
	4			
		and 3096 pence + 8 pence		= 3104 pence.
Farthings,	12418	*i.e.* 3104 pence × 4		= 12416 farthings.
		and 12416 farthings + 2 farthings		= 12418 farthings.

103. The following examples are of a somewhat harder nature:

EXAMPLE 3.—**Reduce 8654 threepences to pounds.**

There will be fewer pounds; ∴ we divide.
Reduce to shillings, then shillings to pounds.
Since four 3d. = 1s., we proceed thus:

$$\begin{array}{r|l} 4 & 8654 \text{ threepences} \\ \hline 20 & 2163 \text{ shillings} + 2 \text{ threepences, or } 6\text{d.} \\ \hline & £108 + 3 \text{ shillings} \end{array}$$

∴ 8654 threepences = £108 3s. 6d.

EXAMPLE 4.—**Reduce 0·5 of 48 farthings to the decimal of 0·75 of £1 10s.**

0·5 of 48 farthings = 0·5 × 48 = 24 farthings;
and 0·75 of £1 10s. = 0·75 × £1·5 = £1·125.

Now we cannot express 24 **farthings** as the decimal of £1·125, so we reduce the pounds to farthings, and write the decimal required as

$$\frac{24}{1{\cdot}125 \times 20 \times 12 \times 4}$$

now, 20 × 12 × 4 = 960, and there are **960 farthings in £1**;

∴ we have the result $\dfrac{24}{1{\cdot}125 \times 960}$

$$= \frac{1}{1{\cdot}125 \times 40} = \frac{1}{45}$$

$= 0{\cdot}0222$, correct to the fourth place.

EXAMPLES. XXIV.

1. Reduce 5834 and 19658 farthings to pounds, shillings, and pence.
2. How many pounds, shillings, and pence are there in 7586 and 90000 halfpennies?
3. How many farthings are there in 100 francs?
4. What is the difference between $\frac{5}{8}$ths of 9856 farthings and $\frac{3}{5}$ths of 25 francs?
5. Express 0·55 of 500 farthings as a fraction of 0·75 of £10.
6. Express 15864 sixpences in pounds, shillings, and pence.
7. How many shillings are there in 580 halfpennies?
8. How many pounds, shillings, and pence are there in 7964 pence?

9. Express 1795 farthings in pounds, shillings, and pence.
10. Reduce 78564 half-crowns to pounds, shillings, and pence.
11. How many pounds, shillings, and pence are there in 17856 sixpences?
12. Express 97642 threepences in pounds, shillings, and pence.
13. What fraction is 18s. 6d. of £1?
14. Express $6\frac{1}{2}$d. as the fraction of 1s.
15. What is 15s. as the decimal of £1?
16. Express 14s. 6d. as the decimal of 15s.
17. Reduce 17s. 6d. to the fraction of 5 guineas.
18. Which is the greater, $\frac{3}{8}$ths of 17s. 4d., or $\frac{6}{9}$ths of 18s. 9d.?
19. How many half-crowns would be required to pay a bill of £1 5s.?
20. A merchant draws, on the average, 356 cheques a week; how much does this cost him in stamps when each cheque has a penny stamp upon it?
21. The tenant of a small house has paid his rent quarterly, for sixteen years, by means of money orders, the commission on which is 4d.; what amount of money has he paid for the orders in that time?
22. A clerk mistakes $\frac{1}{5}$ of £70 for $\frac{1}{4}$ of £70; by how much is he wrong?
23. Reduce each of the amounts given in Question 23 (pp. 121–2), part (2), to pence.
24. Reduce each of the amounts given in Question 23 (pp. 121–2), part (6), to farthings.
25. Reduce each of the amounts given in Question 23 (pp. 121–2), part (8), to halfpennies.
26. How many shillings are there in 1000 pence?
27. How many pence are there in 876 farthings?
28. Express 90000 pence in pounds, shillings, and pence.
29. If a mark is valued at 11d., how many will there be in £6 12s.?
30. Taking the value of a franc as $9\frac{1}{2}$d., find how many pounds, shillings, and pence there will be in 894 francs.
31. A Spanish silver peseta is of the same value as a franc; how many shillings and pence are there in a 10-peseta piece (gold)?
32. How many pence are there in £25 15s. 8d.?
33. Reduce 3275 guineas to half-crowns.
34. How many men can be paid with £15 6s. if each has earned 8s. 6d.?
35. A merchant employs a number of men who earn, on the average, 32s. 6d. a week each. If his wages bill is £40 12s. 6d., how many men has he?

36. How many cheques for 15s. 8d. can be drawn upon a banking account amounting to £78 6s. 8d.?

37. The total amount subscribed in sums of 10s. 6d. each to a relief fund was £44 12s. 6d. How many subscribers were there?

38. How many brass inkstands, valued at 2s. $6\frac{1}{2}$d. each, can be bought for £5 15s. 3d., and what change will there be?

39. If small bottles of red ink cost $\frac{7}{8}$d. to make, how many can be made for an expenditure of 50 guineas?

40. How many tons of coal, at 15s. 7d. per ton, can be bought for 95 guineas?

EXAMPLES. XXV.

Miscellaneous Questions

1. A Wilton carpet is valued at £10 19s. 6d. but is sold at a sale for £7 15s. 9d.; what difference does this make to the purchaser?

2. Express the sale price of the Wilton carpet mentioned in Question 1 as a decimal of the ordinary price.

3. By how much does a guinea and a half exceed (1) 5s. $9\frac{1}{4}$d., (2) three-fourths of a guinea.

4. Determine the excess of $2\frac{1}{2}$ guineas over (1) the sum of 8s. $9\frac{1}{4}$d. and 17s. $11\frac{1}{2}$d., (2) the difference of 8s. $9\frac{1}{4}$d. and 17s. $11\frac{1}{2}$d.

5. Coal is sold retail at 1s. 6d. per cwt., or at 27s. a ton. How much cheaper is it to buy a ton at the latter rate than at the former?

6. Find the cost of providing the following articles for one's dining-room: 1 carpet, £4 4s.; 1 suite of furniture, £12 19s. 6d.; 1 sideboard, £10 15s. 6d.; 1 overmantel, £22s. 6d.; sundry ornaments, £3 5s. 8d.; marble clock, £5 18s. 6d.; mahogany table, £6 11s. 9d.; pictures, £3 12s. 8d.; fender and fire-irons, £2 12s. 7d.; curtains, £1 15s. $11\frac{3}{4}$d. How much change would there be out of £65?

7. Out of what sum of money would the change in Question 6 have been (1) $6\frac{3}{4}$d., (2) 11s. $11\frac{1}{4}$d., (3) £11 11s. $11\frac{1}{4}$d.?

8. If the table costing £6 11s. 9d. had been left out, and a cheaper one at £4 15s. 6d. had been purchased, what would the total cost have been then, and how much would have been left out of a cheque for 100 guineas?

9. If the price of the articles in Question 6 had been quoted correct to a pound, what would the cost have been, and how much more or less would the purchaser have paid than he actually did pay?

10. For what amount must a grocer draw a cheque[1] to pay for the following? (Tabulate this question.) 1914, May 1. Lump sugar, £58 16s. 8d.; tea, £173 19s. 6d.; black tea, £738 15s. 9d.; coffee (berry), £98 18s. 8d.; coffee (ground), £149 16s. 5d.; cocoa, £89 19s. 6d.; dried fruit, £976 15s. 8d.

11. If the merchant in Question 10 had £1500 18s. to his credit at his bank, by how much would he have had to increase his account there to meet his liabilities?

12. Two salesmen agreed to compare their sales during a particular week and to share equally the difference between their profits, which were as follows: A. £7 15s. $8\frac{1}{2}$d.; £9 17s. $1\frac{1}{2}$d.; £22 15s. $6\frac{1}{2}$d.; £18 12s. 10d.; £25 14s. $8\frac{1}{2}$d.; £30 6s. $4\frac{1}{2}$d. B. £6 10s. $4\frac{1}{2}$d; £8 15s. 9d.; £21 14s. $6\frac{1}{2}$d.; £19 13s. $8\frac{1}{2}$d.; £26 1s. $6\frac{1}{2}$d.; £41 6s. $4\frac{1}{2}$d. Without setting these amounts down, find (1) A's profits, (2) B's profits, (3) the difference between these amounts, and (4) A's total profit after the division of the difference found in (3).

13. In the previous question find (1) the difference between the greatest and least days profits for both A and B; (2) the difference between A's greatest and B's greatest profits, and also between A's least and B's least.

14. A draper owes £20 16s. 8d.; £19 7s. 10d.; £17 8s. 8d.; and £6 15s. 9d. to the same wholesale firm, and he wishes to pay these four amounts by drawing two cheques for amounts as nearly as possible equal. What will be the value of each cheque?

15. Employing the figures of the previous question, find by how much the draper's total debt exceeds the highest item in it.

16. Write down the four amounts in Question 14, correct to £1 and also correct to 1s.

17. A man stands outside Euston Station selling newspapers; the details of his receipts are:

	Morning Papers.		Evening Papers.									
			Noon Edition.		3 o'clock Edition.		5 o'clock Edition.		7.30 o'clock Edition.		Final Edition.	
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
Monday	10	$6\frac{1}{2}$	8	8	7	7	5	8	9	6	7	3
Tuesday	8	$5\frac{1}{2}$	7	6	8	9	9	2	8	6	5	8
Wednesday	9	$8\frac{1}{2}$	9	3	10	7	11	8	10	1	8	6
Thursday	10	$7\frac{1}{2}$	8	2	15	3	16	4	17	11	10	10
Friday	8	8	7	6	9	4	7	11	8	10	5	8
Saturday	15	6	5	4	6	9	5	6	5	8	6	3

[1] § 166 and Plate VI,

Find (1) The total receipts per day; (2) the total receipts for each edition (morning papers to final edition) each day; (3) his weekly receipts.

18. The financial position of a company is given below, find the amount of the Reserve Account omitted from the last line of the Creditor side, so that the latter may balance with the Debtor side.

TRIAL BALANCE.[1]

	Dr. £	s.	d.	Cr. £	s.	d.
Ordinary Share Capital Account	—			30000	0	0
Preference Share Capital Account	—			25000	0	0
Stock (January 1, 1911)	16200	4	3			
Cash in hand	106	2	0			
Cash at Bank	3196	13	0			
Purchases and sales	25123	7	0	63200	4	8
Returns (inwards and outwards)	901	7	9	308	17	6
Manufacturing expenses	5314	2	0			
Manufacturing wages	13210	0	0			
Salaries	1525	0	0			
Travellers' salaries, commission, and expenses	3210	0	0			
Rates and taxes	210	0	0			
Insurance	70	0	0			
General expenses	1420	0	0			
Discounts	1283	2	0	578	3	0
Bad debts	280	0	0			
Interest and Bank charges	87	0	0			
Land and buildings	10000	0	0			
Machinery and plant	17000	0	0			
Debtors and creditors	28397	4	2	9843	2	0
Patents	3000	0	0			
Bad debts reserve (January 1, 1911)	—			400	0	0
Profit and Loss Account (balance, December 1, 1910)	—			810	0	0
Preference Dividend paid	706	5	0			
Interim Ordinary Dividend paid	900	0	0			
Reserve Account	—					
	£			£		

19. Exercise books are retailed at 2d. each and bought at 1s. 6½d. a dozen. What does the retailer make on selling half a dozen?

20. Incandescent mantles are sold at 2¼d. each or at 2s. 1½d. a dozen. What saving is there on buying a dozen rather than in buying twelve singly?

21. If one mantle in every dozen is broken accidentally while none is broken if bought singly, which is the cheaper way to buy, and by how much?

[1] Section XIX., §§ 283 and 306.

22. 100–130 volt, 50 candle-power Osram[1] lamps cost 2s. 6d. each. How many can be bought for 5 guineas?

23. How much money will be left when the greatest number of lamps that can be bought have been bought for 3 guineas?

24. What is the difference between £7 15s. 8d. and the cost of 47 100–130 volt, 16 C.P. tantalum[2] lamps at 2s. each?

25. What is the difference between the cost of a duplex reading lamp in polished brass at 45s. 9d. and one in oxidised brass at 48s. 6d.? Express the difference as the decimal of £1.

26. What fraction of £55 1s. is £45 17s. 6d.?

27. The details given below relate to the sizes and prices of the tubes and tyres for motor-cars as supplied by a company. (1) Find the cost of (i) One tube of each size; (ii) one heavy plain, of one heavy grooved, and of one steel-studded tyre of each size. (2) If motor-car dealers purchase the articles in sets of four (one article of each kind), find the cost of one set of each of the sizes given. By how much does the sum of (1) (i) and (ii) differ from the cost in (2)?

Size.	Inner Tubes.			Heavy Plain.			Heavy Grooved.			Steel-Studded Non-Skid.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
650 × 65 . .	0	9	0	1	9	0	1	15	8	2	8	4
750 × 85 . .	0	15	6	1	16	2	2	4	2	3	3	8
870 × 90 . .	0	17	7	2	18	11	3	12	3	4	5	6
915 × 105 . .	1	5	5	4	8	2	5	8	1	6	0	3
935 × 135 . .	1	14	10	5	19	3	7	6	0	7	17	9

28. In the previous question find by how much the cost of a studded tyre exceeds that of a heavy grooved in the last two sizes.

29. In an interesting pamphlet issued under the authority of the Minister for External Affairs, Melbourne, the following details are given:

[1] Osram filaments are made from the metal tungsten.

[2] Tantalum is a metal used to make the wire for electric lamps. The advantage of metal filament lamps is that they consume much less current than the old carbon filament lamps, though they involve more initial outlay. C.P. means candle-power.

COST OF STARTING A DAIRY FARM OF 160 ACRES.

	£	s.	d.
Rent—First year's deposit and survey fee.	3	16	0
Fencing (2 miles at £20 per mile, posts 12 feet apart, and three barbs) . . .			
12 cows at £6 per cow			
2 horses at £15 each			
Plough	6	0	0
Harness	5	0	0
Swingle bars and chains	1	5	0
House—24 × 12 feet, slabbed and floored at £1 per foot	24	0	0
Milking shed	5	0	0
Yard	10	0	0
30-gallon separator	12	10	0
Cart (second-hand)	5	0	0

Fill in the spaces and find the cost involved, in Australia, in starting a 160-acre dairy farm.

30. In the previous question, if the emigrant erects his own fencing and it costs him £5 10s. a mile, what would his expenses be then?

31. If he builds his own house it costs him £4 18s. Suppose he does this, and also fences his own land, what are his expenses?

32. What does he save by building his own house and fencing his land?

33. The import duty on coffee in the United Kingdom, whether kiln-dried, roasted, or ground, is 2d. per lb. How many shillings and the decimal of a shilling is this duty per cwt. of 112 lb.? What change would there be out of 24s. 6d. after paying the duty on 1 cwt.?

34. Express the result of the last question as the decimal of £1.

35. In the year 1911 every person in the United Kingdom consumed, on the average, 18·09 lb. of fresh and refrigerated beef, according to *Whitaker's Almanack*, while in 1912 the consumption was 19·49 lb. per head. Taking the price per lb. in 1911 to be 9d. per lb. and in 1912 to be 10d., find the difference between

the amount spent per head in the two years given, in shillings and pence and the decimal of a penny.

36. Find the cost of an officer's equipment from the following, after having tabulated the account properly: Tunic, worsted or serge, £2 7s. 6d.; breeches, Bedford cords, £1 17s. 6d.; breeches, serge, £1 2s. 6d.; slacks, serge, 15s.; British warm coats, £2 10s.; overcoats, £3; caps, 15s.; putties, 6s. 6d.; leggings, 15s.; marching boots, 14s.; sleeping caps, 3s.; khaki shirts, all wool, with collar, 8s. 6d.; "Sam Browne" belts, £1 17s. 6d.; sleeping valise, £2 10s.; sleeping bags, £1; army kitbags, £3 10s.; folding beds, £1; mattress, "cork," £1 1s.

37. Find the cost of an oil stove and utensils suitable for a house-boat from the following: Stove, 38s. 4d.; kettle, 1s. 5d.; stewpan, 2s. 4½d.; steamer, 1s. 2d.; fry-pan, 1s. 1½d.; toast hanger, 1s. 3d.; cottons (1 dozen), 1s. 4d.

38. A company supplies clean towels to clubs and offices at the rate of 3s. 3d. for 50 per week, 5s. for 75 per week, 6s. 3d. for 100 per week, 1s. 9d. for 25 per week. If it supplies four clubs with 100 per week, eight offices with 75 per week, four with 50, and twelve with 25, find the weekly income from these sources.

39. A cleaning company received the following orders on one particular day:

CLEAN—

8 ladies' evening gowns .	@ 6s.	each.
6 ladies' dresses . . .	@ 5s. 6d.	,,
6 ladies' skirts . . .	@ 3s.	,,
10 silk blouses . . .	@ 2s.	,,
4 long coats . . .	@ 3s.	,,
8 feather boas . . .	@ 4s. 6d.	,,

What amount of money should have been received for cleaning the articles named?

40. If, in the previous question, £1 15s. 3d. proved a bad debt, what would have been the amount actually received?

41. Determine the cost of providing household linen for a

house of seven rooms as offered by a large London firm. The details are:

	£	s.	d.
2 pairs cotton sheets for one 4 feet 6 inch bed, 8s. 11d.	0	17	10
2 pairs cotton pillow-slips, 1s. 1½d.	0	2	3
1 pair top blankets. .	0	14	11
1 under blanket . .	0	4	6
1 white toilet quilt. .	0	8	11
2 toilet covers, 1s. 0½d. .	0	2	1
2 pairs linen sheets for one 4 feet 6 inch bed, 17s. 9d.	1	15	6
2 pairs linen pillow-slips, 2s. 7d.	0	5	2
1 pair top blankets. .	0	17	9
1 under blanket . .	0	4	6
1 white quilt . . .	0	8	11
2 toilet covers, 10¾d. .	0	1	9½
2 pairs cotton sheets, for one 3 feet servant's bed, 4s. 6d.	0	9	0
1 pair cotton pillow-slips	0	0	10

	£	s.	d.
1 pair top blankets. .	0	6	11
1 under blanket . .	0	2	0
1 coloured quilt . .	0	2	6½
2 toilet covers, 8¾d. .	0	1	5½
2 tablecloths, 2 yards square, 3s. 11d. . .	0	7	10
1 tablecloth, 2 by 2½ yards	0	5	11
½ dozen table napkins, 8s. 6d.	0	4	3
2 kitchen tablecloths, 1½ yards square, 1s. 6½d.	0	3	4
1 dozen huckaback towels	0	8	6
3 bath towels, 1s. 0½d. .	0	3	1½
½ dozen servants' towels, 6s. 11d.	0	3	5½
2 roller towels, 1s. 0½d. .	0	2	1
½ dozen tea cloths, 3s. 11d.	0	1	11½
½ ,, glass cloths, 4s. 11d.	0	2	5½
½ ,, kitchen cloths, 5s. 6d.	0	2	9
½ ,, dusters, 2s. 6d. .	0	1	3
£			

42. Two merchants, one in England and one abroad, send respectively 9 cwt. and 15 cwt. of biscuits to Melbourne. The Government of the colony charges 1d. per lb. duty on the former (being a preferential tariff for British goods) and 1½d. per lb. on the latter. What is the difference between the duties paid on the weights sent.

43. How much more per ton of biscuits imported into Australia has to be paid on foreign than on British biscuits?

44. The price-list of a large London company gives the following:

HAVANA CIGARS.

	Crop.	Per 100.			Per 5.		
		£	s.	d.	£	s.	d.
Coquetas	1908	1	9	0	0	1	8
Divinos de la Crême . .	1905	4	5	0	0	4	9
Excepcionales Superfinos .	1908	4	18	0	0	5	4
Aguilas Rusas . . .	1909	9	0	0	0	10	0

How much cheaper is it to buy 100 of each sort by the hundred than to buy the same number of each sort by fives?

45. The current year's subscriptions to a society are as follows:

£47; £63 10s.; £3 10s.; £30 7s. 6d.; £26 2s. 6d.; £9 10s.; £21 7s. 6d.; £2 5s.; £2 17s. 6d.; £10 15s.; £9 2s. 6d.; £15; £4 10s.; £9 2s. 6d.; 15s.; £2 15s.

Arrange the amounts given above in descending order, and find the total.

46. The statement given below is taken from the accounts of an important insurance company. What is the value of the assets on 31st December?

ASSETS, 31st DECEMBER.

		£	*s.*	*d.*
Mortgages on property within the United Kingdom		1088279	3	10
Loans on Parochial and other Public Rates		332627	7	6
,, Life Interests		11960	0	0
,, Reversions		33927	19	6
,, Stocks and shares		40000	0	0
,, Institution's policies within their surrender values		1033182	10	0
Investments—				
Deposit with the High Court (£28933 India 2½ per Cent. Stock)		20000	0	0
British Government Securities		18047	2	2
Municipal and County Securities, United Kingdom		374350	0	2
Indian and Colonial Government Securities		213849	5	4
,, Municipal Securities		34884	13	5
Foreign Government Securities		191744	10	6
,, Provincial Securities		19650	0	0
,, Municipal Securities		121052	18	4
Railway and other Debentures and Debenture Stocks, Home and Foreign		1204472	4	9
Railway and other Preference and Guaranteed Stocks, Home and Foreign		696732	12	10
Rent charges		43147	13	0
Freehold ground rents		1992622	19	3
Leasehold ,,		430169	7	6
House property		606797	9	7
Reversions		177795	3	11
Indian Railway Stocks and Annuities		252851	17	10
Bank of England Stock		101541	18	5
New River Company's Securities		402	14	8
Agents' Balances		22933	7	7
Outstanding Interest, Dividends, and Rents		21130	19	6
Interest accrued but not payable		68928	9	0
Bills receivable		44[illegible]	6	3
Cash—				
On Deposit	£65000 0 0			
In hand and on Current Account	28375 11 0			
		93375	11	0
Other Assets—				
Loans on Credit Policies		18336	8	1
Postage stamps		11	1	3
	£			

Note.—The student should regard this, and other similar questions, as a

means of familiarising himself with commercial terms, and should endeavour to understand their meaning as he reads on through the book.

47. What is the cost of providing for a secretary's office, given the following details:

	£	s.	d.
Floor in figured oak, $1 \times 4\frac{1}{2}$ inches, ready for polishing	5	8	6
Labour	2	3	8
Cost of papering walls and fixing moulding	1	15	6
Satin walnut desk, finished in mahogany .	9	9	0
Four-tier filing cabinet, finished in mahogany	5	9	6
Chair, in mahogany	1	19	6
Enclosed washstand, in mahogany . .	2	3	6
Bookcase, in mahogany	18	18	0
3 extra chairs, in mahogany	1	13	6

48. The expenditure of a society was—

	£	s.	d.
Postages and petty cash	19	16	11
Clerical assistance	20	9	7
Printing and stationery	44	9	11
Travelling expenses—			
Council meetings	58	2	5
Delegate meetings	19	5	10
Deputations	4	1	10
Report of conference	5	0	0
Use of rooms, etc.	0	18	0
Inauguration expenses of country branch .	4	18	0
Cheques and sundries	0	9	1
Donations	14	11	0
Printing journal	56	0	0
Postages of journal	14	14	2
Legal expenses	1	1	0
	£		

Find the total expenditure.

49. The import duty on chloroform[1] is, for the United Kingdom, 4s. 4d. per lb., and for the United States, previous to 4th October 1913, 10 cents per lb., after that date, by virtue of the passing of the Underwood Tariff Bill, 2 cents per lb. Taking a dollar as 4s. 2d and a cent as $\frac{1}{100}$ dollar, calculate the difference

[1] These figures are taken from a summary kindly lent by the Dorland Agency, and were first published in the *New York Times* of 30th September 1913.

between the duty on chloroform in the United Kingdom and in the United States at the end of 1912 and of 1913.

50. Employing the data of the previous question, express in English money the change in duty on chloroform produced by the passing of the Bill referred to.

51. The statement given below balances when the amount for Commissions is put in in the left-hand column. Find that amount.

TRIAL BALANCE.

	£	s.	d.	£	s.	d.
Preference share capital				25000	0	0
Ordinary share capital				12500	0	0
Cash paid on shares forfeited and since re-issued. (This is to be transferred to Freehold Property Account) . .				15	0	0
Freehold property	21500	0	0			
Goodwill Account	5000	0	0			
Formation expenses	750	0	0			
Wages	3417	10	6			
Purchases, brewing materials . . .	32526	0	0			
Loans on mortgage at 5 per cent. per annum				3750	0	0
Commissions						
Loan interest paid, less tax, to June 30, 1912	88	5	8			
Discounts	2368	0	0			
Reserve for doubtful debts				200	0	0
Travelling expenses	1812	0	0			
Rates and taxes	1400	0	0			
General trade expenses	3659	0	0			
Repairs and renewals	663	10	0			
Salaries	1852	0	0			
Reserve fund				5000	0	0
Stock, January 1, 1912	14511	0	0			
Sales				66796	0	0
Profit and Loss Account, January 1, 1912.	2425	0	0			
Plant and machinery	5614	0	0			
Barrels	777	0	0			
Debtors	20825	0	0			
Creditors				10715	0	0
Directors' fees	1300	0	0			
Furniture and fittings	112	0	0			
Bad debts	215	0	0			
Rents received				279	0	0
Beer duty paid	5763	0	0			
Cash in hand	10	0	0			
Steadfast Bank Limited				3411	0	6
Horses and drays	562	0	0			
	£			£		

SECTION X

ENGLISH MONEY: CALCULATION OF COST

A. (i) By Compound Multiplication and Division

104. *Multiplication and Division by a Single Figure.*

EXAMPLE 1.—**A grocer buys 9 cwt. of sugar at £1 16s. 8½d. per cwt., how much does he pay?**

$$\pounds 1\ 16s.\ 8\tfrac{1}{2}d. \times 9 \doteqdot \pounds 2 \times 9 = \pounds 18.$$

£	s.	d.
1	16	$8\frac{1}{2}$
		9
£16	10	$4\frac{1}{2}$

METHOD.

9 halfpennies =		$4\frac{1}{2}$d.
$8 \times 9 + 4$d.	= 76d. = 6s.	4d.
$16 \times 9 + 6$s.	= 150s. = £7	10s.
$1 \times 9 + \pounds 7$	= **£16.**	

∴ The cost of 9 cwt. of sugar is £16 10s. 4½d.

EXAMPLE 2.—**Europeans employed in the diamond-mining industry at the Cape, in the year 1913, could have earned £6 11s. 6d. by working six days. How much was the average daily wage?**

$$\pounds 6\ 11s.\ 6d. \div 6 \doteqdot \pounds 6 \div 6 = \pounds 1.$$

	£	s.	d.
6	£6	11	6
	£1	1	11

METHOD.

$\pounds 6 \div 6 = \pounds 1.$
$11s. \div 6 = 1s.$ and 5s. over.
$5s. + 6d. \div 6 = 66d. \div 6 = 11d.$

∴ a European could earn 21s. 11d. a day in the Cape diamond mines in 1913.

105. *Multiplication and Division by a Number which can be Factorised.*

EXAMLPE 3.—**The oversea trade of Australia in 1912 was £33 17s. 2d. per inhabitant. How much would this be for ninety-six people?**

$$\pounds 33\ 17s.\ 2d. \times 9 \doteqdot \pounds 34 \times 100 = \pounds 3400,$$

Here £33 17s. 2d. $\times 96 =$ £33 17s. 2d. multiplied by 8, and the product multiplied by 12.

£	*s.*	*d.*
33	17	2
		8
270	17	4
		12
£3250	8	0

METHOD.

$2 \times 8 = 16$d. $= 1$s. 4d.
$17 \times 8 + 1 = 137$s. $=$ £6 17s.
$33 \times 8 + 6 =$ £270.
The multiplication by 12 is performed similarly.

$\therefore$ the oversea trade of Australia was, in 1912, £3250 8s. for every ninety-six inhabitants.[1]

EXAMPLE 4.—**If fifty-four people were to cross from England to Newfoundland in the winter their total saloon fares would be £622 7s. Find the winter fare. (The summer saloon fare is about £13, why should it be greater than the winter fare?)**

£622 7s. $\div 54 \simeq$ £600 $\div 50 =$ £12.

£622 7s. $\div 54$
$=$ £622 7s. $\div 6$, and the result $\div 9$.

	£	*s.*	*d.*
6	622	7	0
9	103	14	6
	£11	10	6

METHOD.

$622 \div 6 =$ £103 and £4 over.
£4 7s. $= 87$s.
and $87 \div 6 = 14$s. and 3s. over.
3s. $= 36$d. and 36d. $\div 6 = 6$d.
Then $103 \div 9 =$ £11 and £4 over.
£4 14s. $= 94$s.
and $94 \div 9 = 10$s. and 4s. or 48d. over.
and 48d. $+$ 6d. $\div 9 = 6$d.

$\therefore$ the saloon fare from England to Newfoundland is £11 10s. 6d. for each adult in winter, and it is lower than the summer fare because the demand for accommodation will probably be less in winter.

106. *Multiplication and Division by any Number (without Decimalising).*

EXAMPLE 5.—**It costs a corporation £156 18s. $10\frac{1}{2}$d. to repair 100 yards of a certain road; determine the cost of repairing 57300 yards at this rate.**

Since 57300 yards $= 573$ hundred yards, it is necessary to find the value of £156 18s. $10\frac{1}{2}$d. $\times 573$.

£156 18s. $10\frac{1}{2}$d. $\times 573 \simeq 157 \times 573 =$ £89961.

[1] The student will find it most advantageous to write down the real import of his result, as we have done here, rather than: "*Ans.* £89928 15s. $4\frac{1}{2}$d."

We really have here four sums, the results of which are to be added together thus:

	£	s.	d.
(1) 573 halfpennies = 286½d. = 23s. 10½d. =	1	3	10½
(2) 573 × 10 pence = 5730d. = 477s. 6d. =	23	17	6
(3) 573 × 18 shillings = 10314s. . =	515	14	0
(4) 573 × 156 pounds . . . =	89388	0	0
∴ The cost required is . . .	£89928	15	4½

∴ the cost of repairing 57300 yards of road at £156 18s. 10½d. per 100 yards is £89928 15s. 4½d.

Example 6.—**A firm spends £1882 1s. 0½d. a year in advertising, what expenditure does this represent per day?**

Reckoning 365 days, since Sundays clearly enter into the matter, we see that £1882 1s. 0½d. ÷ 365 will give the cost per day, which is ≏ 1880 ÷ 365 = £5.

	£	s.	d.
	5	3	1½
365	1882	1	0½
	1825		
	57 . . . (i)		
	20		
	1141		
	1095		
	46 . . . (ii)		
	12		
	552		
	365		
	187 . . . (iii)		
	4		
	750		
	730		
	20		

Method.

£(1882 ÷ 365) = **£5**.
£(1882 − 365 × 5) = £1882 − £1825.
= £57 . . . . (i)
£57 = 57 × 20s.
= 1140s.
1140 + 1s = 1141s.
1141 ÷ 365 = **3s.**
1141 − 365 × 3 = 1141s. − 1095s.
= 46s. . . . . (ii)
46s. = 46 × 12d.
= 552d.
552d. + 0d. = 552d.
552 ÷ 365 = **1d.**
552 − 365 × 1d. = 552d. − 365d.
= 187d. . . . (iii)
187d. = 187 × 4 farthings
= 748 farthings
748 farthings + 2 farthings = 750 farthings
750 ÷ 365 = 2 farthings
= ½d.
To the nearest farthing.

EXAMPLES. XXVI.

1. Find the value of each of the following:

(1) £5 8s. 6d. multiplied by 5.
(2) £7 15s. 8d. multiplied by 7.
(3) £9 18s. 6½d. multiplied by 9.
(4) £12 7s. 5¼d. multiplied by 3.

(5) £18 17s. $9\frac{1}{2}$d. multiplied by 6.
(6) £159 15s. $11\frac{1}{4}$d. multiplied by 8.
(7) £596 19s. $11\frac{3}{4}$d. multiplied by 12.
(8) £875 15s. $0\frac{1}{2}$d. multiplied by 9.
(9) £9876 17s. $4\frac{1}{4}$d. multiplied by 11.
(10) £15864 15s. $7\frac{3}{4}$d. multiplied by 10.

2. Multiply, using factors, the amount given in—

Question 1, part (1), by 25, 36, and 42.
" " (2), by 49, 84, and 63.
" " (3), by 27, 108, and 125.
" " (4), by 96 and 99.
" " (5), by 198 and 144.
" " (6), by 160 and 192.
" " (7), by 595 and 825.
" " (8), by 2380.
" " (9), by 627.
" " (10), by 145.

3. Divide the following:

(1) £27 2s. 6d. by 5.
(2) £54 9s. 8d. by 7.
(3) £89 6s. $10\frac{1}{2}$d. by 9.
(4) £37 2s. $3\frac{3}{4}$d. by 3.
(5) £113 6s. 9d. by 6.
(6) £1278 7s. 6d. by 8.
(7) £7168 19s. 9d. by 12.
(8) £7881 15s. $4\frac{1}{2}$d. by 9.
(9) £108645 10s. $10\frac{3}{4}$d. by 11.
(10) £158647 16s. $5\frac{1}{2}$d. by 10.

4. Divide each of the following, using factors:

(1) £135 12s. 6d. by 25.
(2) £195 6s. by 36.
(3) £227 17s. by 42.
(4) £381 7s. 8d. by 49.
(5) £653 16s. by 84.
(6) £1072 2s. 6d. by 108.
(7) £1240 17s. $8\frac{1}{2}$d. by 125.
(8) £3740 2s. 9d. by 198.
(9) £25567 10s. by 160.
(10) £2084289 19s. 2d. by 2380.
(11) £492524 2s. $9\frac{3}{4}$d. by 825.
(12) £6192796 1s. $0\frac{3}{4}$d. by 627.

5. Evaluate the following, without employing factors:

(1) £356 10s. 6d. × 537 and 685.
(2) £658 12s. $5\frac{1}{2}$d. × 624 and 749.
(3) £859 15s. $8\frac{1}{4}$d. × 585 and 691.
(4) £1764 17s. $9\frac{1}{2}$d. × 796 and 984.
(5) £5692 18s. $7\frac{1}{4}$d. × 975 and 1432.

6. Divide—

(1) £502973 17s. $2\frac{1}{4}$d. by 585.
(2) £594111 0s. $0\frac{3}{4}$d. by 691.
(3) £1404852 2s. 2d. by 796.
(4) £1736651 7s. by 984.
(5) £8152276 11s. 2d. by 1432.

7. From the sum of £56 18s. $6\frac{1}{4}$d. and £91 17s. $3\frac{1}{2}$d. take five times 18s. $9\frac{1}{2}$d.

8. What is a seventeenth of £4 5s. $8\frac{1}{2}$d.? (Correct to the nearest farthing.)

9. Add a sixth of £50 7s. 3d. to three times £38 12s. $0\frac{1}{2}$d.

10. Add eight times £96 17s. $6\frac{1}{2}$d. to the difference between £25 15s. and $6\frac{1}{2}$d.

11. From twelve times 5s. $8\frac{1}{2}$d. take half of 17s. $6\frac{1}{2}$d.

12. What is the value of $\frac{1}{2}$ of 15s. + $\frac{3}{4}$ of £1 10s.?

13. Express $\frac{2}{5}$ of £1 10s. 10d. as a fraction of $\frac{1}{8}$ of £5 10s.

14. What is the difference between a farthing and £$\frac{1}{1000}$? (Answer as the decimal of £1.)

15. Express ($\frac{7}{8}$ of 15s. 4d.) as a fraction of ($\frac{13}{20}$ of £10 0s. 5d.).

16. How many pence are there in $\frac{1}{30}$ of £156 6s. 3d.?

17. Express the result of the last question as the fraction of £70 15s. 9d.

18. Evaluate 0·25 of £50 14s. 6d. + 0·5 of £1 4s. 2d.

19. Express 0·85 of 20 guineas as a fraction of 0·78 of £40 15s. 6d.

20. Decrease £7 16s. 8d. by one-eighth of itself.

21. How many incandescent gas mantles costing $2\frac{1}{2}$d. each can be bought for 12s.?

22. What change would there be in the last question? Why must it be less than $2\frac{1}{2}$d.?

23. How many pounds of inch nails costing $2\frac{1}{4}$d. per lb. can be bought for half a sovereign?

24. What fraction of a shilling would there be in change in the last sum?

25. Permanganate of potash,[1] which when dissolved in water can be used to stain floors, is sold at $7\frac{1}{2}$d. per lb., and a 56-lb. bag costs 2s. $3\frac{1}{2}$d. less than 56 separate pounds. What is the price of 1 lb. at the latter rate, and what is the difference between the price per pound when bought in the 56-lb. bag and the price of one single pound?

[1] This chemical rapidly destroys a brush, and so care should be taken to employ an old one.

26. The average price per ton of tin ore in Cornwall was—

	£	s.	d.
1915, 4th January	86	4	11
„ 18th January	87	13	8
„ 1st February	96	13	8

Find the average price per ton of tin ore during the period given, and by how much it is above or below the average for 2nd February 1914, namely, £104 2s. 7d. per ton.

27. For what amount must a cheque be drawn to pay the following account? (Tabulate the statement.)

INDIARUBBER HOT-WATER BOTTLES.

Best Quality.

(Manufactured in London.)

No. required.				£	s.	d.
6	About 12 × 6 in. .	Each 3s. 11d.	Cover 10d.			
4	„ 12 × 8 „ .	„ 4s. 7d.	„ 10d.			
5	„ 12 × 10 „ .	„ 5s. 3d.	„ 1s. 1d.			
2	„ 14 × 8 „ .	„ 5s. 6d.	„ $10\frac{1}{2}$d.			
2	„ 14 × 10 „ .	„ 5s. 9d.	„ 1s. 1d.			
4	„ 16 × 10 „ .	„ 6s. 9d.	„ 1s. 3d.			
4	„ 16 × 12 „ .	„ 7s.	„ 1s. 6d.			
	Supply cover with each bottle.					

28. The average price of wheat per quarter for the first week in January in each of the following years is given:

1906.	1907.	1908.	1909.	1910.	1911.	1912.	1913.
28s. 6d.	26s.	35s. 3d.	32s. 9d.	33s. 6d.	30s. 6d.	33s. 3d.	30s. 6d.

Find the average price for the eight years given.

29. Seventy-five horses valued at £45 each are sent into the United States, where the duty is 10 per cent. on the value of the animals. What duty will be payable? (10 per cent. $= \frac{10}{100} = \frac{1}{10}$.)

30. If the horses referred to in Question 29 had been sent into Queensland the duty would have been 10s. a head, what would have been the total duty?

31. An Axminster carpet containing 125 square yards costs £30 4s. 2d., what is the price per square yard?

32. If it cost 5d. to travel by a "workman's" train from Wimbledon to Waterloo and back, a distance of 7 miles each way, find the cost per mile as a decimal of a penny.

33. Taking the rate per mile in the previous question, find how much the return ticket to Plymouth should be if the distance of that town from Waterloo be 224 miles. (Answer in shillings and pence.)

34. The actual cost of a return ticket to Plymouth is 37s. 4d. How much does this exceed the cost at the "workman's" rate?

35. A particular Turkey carpet costs 5s. 6d. a square yard, and it contains 124½ square yards. What did it cost?

36. A house furnisher is retiring from business, and he instructs his assistants to mark all his goods at a price found by deducting one-sixth of their present price. Determine the sale price of the following articles:

Article.	Present Price. £	s.	d.	One-sixth of Present Price.	Sale Price.
An oak sideboard	13	19	6		
An antique brass fender	7	7	6		
A marble statue holding electric light fittings	29	15	0		
A pier-glass (mirror)	5	3	0		

37. A family consumed twenty-two loaves a week when the price per 2-lb. loaf was 2¼d., but when the price was raised to 3¼d. the consumption fell to nineteen loaves a week. How much did the bread bill increase under the circumstances given? (The prices are for London in July 1914 and January 1915 respectively.)

38. If a railway company charges 1s. 1½d. return from one station to another, 1568 passengers travel on the average. If, however, the charge is 8½d. return, 2978 passengers travel. Which is the more remunerative arrangement for the company, and by how much?

39. If in the previous question the 2978 passengers had paid the higher rate, how much more would the company have received from these passengers than it actually did receive from them?

40. A dealer bought 52 out of a flock of 108 sheep for £58 10s., and afterwards agreed to take the remainder at the same rate. How much ought he to pay for the second lot?

41. Sherry is sold retail at 13s. 6d. a gallon. What is the cost of a butt of sherry (Portugal) containing 108 gallons?

42. If there are fifty-two dozen bottles to the butt of sherry, what should be the cost per dozen at the rate of a butt?

43. A silversmith buys a Sheffield plate cake-basket for 8 guineas, and adds on $\frac{5}{12}$ths of the cost price to determine his selling price. In order to effect a clearance he marks it at £10 19s. 6d. How much less does he receive than he anticipated?

44. Express the actual amount of money received by the sale of the article mentioned in Question 43 as a fraction of the amount the silversmith expected to receive.

A. (ii) Rapid Methods for Dozens and Scores

107. To find the price per dozen, given the price of one article.

EXAMPLE 1.—**A particular class of pencil costs 2½d. each, what is the price per dozen?**

2½d. × 12 = 2½s. = 2s. 6d.

If the price had been 3½d. or 4½d. the price per dozen would have been

3½d. × 12 = 3½s. = 3s. 6d.
or 4½d. × 12 = 4½s. = 4s. 6d.

The price per dozen is found from the price of a single article by calling the price in pence, shillings.

Thus—

1. Cost of 1 article, 8½d.
   „ per dozen, 8½s., *i.e.* 8s. 6d.
2. Cost of 1 article, 1s. 3½d., *i.e.* 15½d.
   „ „ 12 articles, 15½s., *i.e.* 15s. 6d.
3. Cost of 1 article, £1 15s. 6d. (426d.).
   „ „ 12 articles, 426s., *i.e.* £21 6s.

108. The cost of any number of dozens can readily be found from the cost of one dozen.

EXAMPLE 2.—**What would a gross of slippers cost at 4s. 1½d. each?**

Cost each = 4s. 1½d. = 49½d.
Cost per dozen = 49½s. = 49s. 6d. = 594d.
but 1 gross = 12 dozen
∴ cost of 1 gross = cost of 1 dozen dozen
„ „ 1 dozen = 594d.
„ „ 1 gross = 594s.
= £29 14s.

109. Given the price per dozen, to find that of one. Express price in shillings and call the shillings pence.

Example 3.—**Scallops[1] are sold at 2s. 9d. a dozen, what is the price each?**

One dozen cost 2s. 9d. $= 2\frac{3}{4}$s.

Calling this pence, we have the cost of one $= 2\frac{3}{4}$d.

Example 4.—**Five dozen bottles of gum cost 6s. 3d., what is gained on each bottle by selling it for $1\frac{1}{2}$d.?**

5 dozen bottles of gum cost 6s. 3d.
1 „ „ „ „ 1s. 3d. $= 1\frac{1}{4}$s.

Calling this pence, we have $1\frac{1}{4}$d. as the cost per bottle;
$\therefore \frac{1}{4}$d. is gained on each bottle sold.

110. The cost per *Score* is found, when the price of one article is given, by expressing that price in shillings and calling the shillings pounds.

Example 5.—**The cost of a small gun-metal wristlet watch is 9s. 8d., find the cost of twenty.**

Cost is 9s. 8d. each $= 9\frac{2}{3}$s. each.
Calling this price £'s, we have—
Cost per score $= £9\frac{2}{3}$.
$= £9$ 13s. 4d.

111. Given the price per score, to find the price of one article call the price in pounds, shillings.

Example 6.—**In the process of properly equipping an electrical laboratory twenty accumulators $13\frac{9}{16}'' \times 9\frac{1}{16}'' \times 6\frac{3}{4}''$ (over all), weighing $70\frac{3}{4}$ lb. each, are supplied by a firm for £85 13s. 4d. What is the average cost of an accumulator of the size given?**

20 accumulators cost £85 13s. 4d. $= £85\frac{2}{3}$.
Calling this price shillings, we have the cost of one accumulator
$= 85\frac{2}{3}$s.
$= £4$ 5s. 8d.

[1] An edible shell-fish found in abundance off the coast of Palestine. The shell used to be worn by pilgrims in their hats to suggest, frequently falsely, they had been to the Holy Land.

EXAMPLES. XXVII.

Write down the cost of the following:

1.	1	dozen	pairs of half-hose @ 1s. 1½d. a pair.
2.	1	,,	handkerchiefs @ 7½d. each.
3.	1	,,	of port @ 4s. 8d. each.
4.	1	,,	yards of calico @ 2¾d. a yard.
5.	1	,,	red flannel @ 4s. 3½d. a yard.
6.	1	,,	rugs @ 5s. 9½d. each.
7.	1	,,	watches @ £1 6s. 8d. each.
8.	1	,,	pairs boots @ £1 1s. 6d. each.
9.	1	,,	,, shoes @ 4s. 11½d. each.
10.	1	,,	,, gloves @ 2s. 11¾d. each.
11.	2	,,	yards of tape @ ¾d. a yard.
12.	2	,,	yards of unbleached calico @ 5½d. a yard.
13.	4	,,	fountain pens @ 7s. 6d. each.
14.	4	,,	books @ 3s. 6d. each.
15.	3	,,	penknives @ 9¼d. each.
16.	6	,,	pencils @ 4½d. a dozen.
17.	7	,,	,, @ 4½d. each.
18.	9	,,	bundles of sticks @ ¾d. each.
19.	12	,,	yards of velvet @ 5s. 9½d. a yard.
20.	14½	,,	lb. of tea @ 1s. 6d. per lb.
21.	20	,,	loaves @ 3¼d. each.

Find the cost of—

22. 25 score of pilchards @ 3½d. a score.
23. 18 dozen of mackerel @ 1s. 10d. a dozen.
24. 500 lb. sugar @ 3½d. a lb.
25. 5½ score tongues @ 1s. 10d. each.
26. 7¼ dozen newspapers @ ½d. each.
27. A dozen rose bushes @ 9½d. each.
28. 200 tons of coal @ 18s. 6d. a ton.
29. 5 score magazines @ 4½d. each.
30. 15¼ dozen brandy @ 6s. 3d. each.
31. 18½ dozen California sherry @ 25s. a dozen.
32. 8 score bananas @ two a penny.
33. 22¼ dozen oranges @ three a penny.
34. 3 gross penholders @ 4½d. a dozen.
35. 18½ gross nibs @ 9½d. a gross.

In Questions 36 to 49 calculate the price as required—

36. 20 dozen American bent-wood chairs cost £54 10s., what is the cost of one?

37. What is the price per bottle of Marsala if 1000 cost £87 10s.?

38. What is the price of **borax per pound** if 9½ dozen lb. cost 28s. 6d.?

39. 10½ dozen bottles of cod-liver oil cost 9 guineas, what is the price per bottle?

40. 60 lb. gum-arabic cost £3 10s., what is the price per *quarter* pound?

41. What is the price of cochineal per ounce if 1000 ounces cost £14 11s. 8d.?

42. What does zinc sulphate cost per pound if 100 lb. cost 70s. 10d.?

43. 5½ dozen lb. boric lint cost £4 19s., find the price per lb.

44. The price paid for 150 yards of oiled silk is £13 2s. 6d., what is charged per yard?

45. If 40 dozen tooth-brushes cost £9, what is the price of each one?

46. 440 lb. of Castile soap cost £10 1s. 8d., what is the price per lb.?

47. 15½ dozen lb. Normandy pippins cost £7 7s. 3d., what is the price per lb.?

48. 18½ dozen lb. dripping cost £5 11s., what is the price of 3 lb.?

49. Five 5-lb. boxes of beeswax cost £2 1s. 8d., what is the price per lb.?

B. (i) Complete Decimalisation of Sums of Money

112. EXAMPLE 1.—**Reduce 6½d. to the decimal of 1s., to three places.**

6½d. = 6·5d., and the decimal required

$= \frac{6·5}{12}$ (NOT $\frac{6·5}{1}$, for 6·5 is pence and 1 is a shilling)

= 0·541

= 0·542, to three places.

EXAMPLE 2.—**Reduce 5s. 9d. and 7s. 8½d. to the decimal of £1.**

5s. 9d. = 5·75s.

= £5·75 ÷ 20 = £0·575 ÷ 2

= 0·2875 exactly.

7s. 8½d. = 7s. 8·5d.

$8·5\text{d.} = \frac{8·5}{12}\text{s.} = 0·70833\text{s.}$

∴ 7s. 8½d. = 7·70833s.

= £7·70833 ÷ 20

= £0·385416

= £0·385, to three places.

EXAMPLE 3.—**Express £8 5s. 6d in £'s and the decimal of £1.**

Here 5s. 6d. = 5·5s. = £0·275 (dividing by 20);
∴ £8 5s. 6d. = £8·275.

EXAMPLE 4.—**Reduce £8 5s. 6d. to the decimal of £12 10s. 8d., to three places.**

£8 5s. 6d. = £8·275 (Example 3)
10s. 8d. = 10·666s.
= £0·5333 (dividing by 20);
∴ £12 10s. 8d. = £12·5333;

$$\therefore \text{ decimal required} = \frac{8{\cdot}275}{12{\cdot}5333};$$

and, dividing by contracted methods to four places and correcting the third, we have 0·660, correct to three places. (Note that **the third place must be put in** even though it be zero.)

It will perhaps be well to draw attention to the fact that to reduce the amounts given to farthings and then to divide would be an absurdity, because of the length of the work involved.

113. Taking the reverse process, we have—

EXAMPLE 5.—**Express 8·75s. as a compound quantity (that is, in shillings and pence).**

We note that 0·75s. = $\frac{3}{4}$ of 1s. = 9d.
∴ 8·75s. = 8s. 9d.

EXAMPLE 6.—**Reduce 18·642s. to shillings and pence.**

It is necessary to reduce **the decimal alone,** thus—

0·642s. = **7**·704 pence (multiplying the shillings by 12),
and 0·704d. = **2**·816 farthings („ „ pence „ 4);
∴ 18·642s. = 18s. 7$\frac{3}{4}$d. to the nearest farthing.

EXAMPLE 7.—**If you were to put £1 in a bank paying 3 per cent.[1] for 30 years, it would be worth £2·4273. Express this in pounds, shillings, and pence.**

Here, as before, we reduce the **decimal** to shillings, and then the new **decimal** to pence, and so on.

We repeat that great care must be exercised NOT to multiply the figures in dark type (see over) by 20, 12, or 4.

[1] See Section XX.

Thus:

$$\begin{array}{rl}
£2{\cdot}4273 & \text{Reduce decimal to shillings.} \\
20 & \\
\hline
8{\cdot}5460 & \text{,, ,, pence.} \\
12 & \\
\hline
6{\cdot}5520 & \text{,, ,, farthings.} \\
4 & \\
\hline
2{\cdot}2080 &
\end{array}$$

∴ the value of the £1 would be £2 8s. 7d. to the nearest penny.

EXAMPLES. XXVIII.

1. Express the following as decimals of 1s. to six places where possible:

(1) $1\frac{1}{2}$d.	(2) $2\frac{1}{2}$d.	(3) $3\frac{1}{4}$d.	(4) $2\frac{3}{4}$d.
(5) $\frac{3}{4}$d.	(6) $4\frac{1}{2}$d.	(7) $5\frac{1}{4}$d.	(8) $6\frac{3}{4}$d.
(9) $7\frac{1}{2}$d.	(10) $9\frac{1}{4}$d.	(11) $11\frac{3}{4}$d.	(12) $10\frac{1}{4}$d.

2. Express the amounts in Question 1 as decimals of £1 (correct to four places).

3. Express the following as decimals of £1:

(1) 6s. 8d.; 2s. 6d.; 1s.; 16s. 8d.; 6d.; 1s. 8d.; 18s.
(2) 5s.; 3s. 4d.; 2s.; 13s. 4d.; 3d.; 16s.; 6s.

4. What decimals of 10s are—

(1) 2s. 6d.; 1s. 8d.; $7\frac{1}{2}$d.; 2d.; 4s.; 4d.?
(2) 3s. 4d.; 1s. 3d.; $1\frac{1}{2}$d.; 3d.; 10d.; 5s. 6d.?

5. What decimal of £1 is each of the following sums of money:

(1) 18s. 9d.; 17s. 8d.; 15s. $7\frac{1}{2}$d.; 8s. $9\frac{1}{4}$d.?
(2) 6s. $8\frac{1}{4}$d.; 7s. $8\frac{1}{2}$d.; 19s. $9\frac{1}{4}$d.; 18s. $11\frac{1}{2}$d.?
(3) 5s. $9\frac{1}{2}$d.; 2s. $3\frac{1}{4}$d.; 1s. $11\frac{3}{4}$d.; 12s. $10\frac{1}{4}$d.?
(4) 15s. 11d.; 17s. $4\frac{1}{4}$d.; 3s. $5\frac{1}{2}$d.; 1s. $1\frac{1}{4}$d.?
(5) $\frac{1}{4}$d.; $1\frac{1}{4}$d.; $9\frac{3}{4}$d.; 10s. $10\frac{1}{2}$d.?

6. Express 4 guineas as the decimal of $5\frac{1}{2}$ guineas.
7. What is £7 10s. 8d. as the decimal of £50 15s. $8\frac{1}{2}$d.?
8. Reduce £10 10s. 10d. to the decimal of £12 10s.
9. Decimalise the following sums of money to six places:

(1) £74 13s. $3\frac{1}{2}$d.; £73 19s. $5\frac{1}{2}$d.
(2) £38 14s. $3\frac{3}{4}$d.; £125 16s. $5\frac{3}{4}$d.
(3) £179 19s. $2\frac{1}{2}$d.; £250 8s. $5\frac{3}{4}$d.
(4) £760 6s. $4\frac{3}{4}$d.; £158 13s. $9\frac{3}{4}$d.
(5) £731 5s. $10\frac{1}{2}$d.; £33 15s. $6\frac{3}{4}$d.

10. Convert the following into pounds, shillings, and pence:

(1) £3·5; £25·1; £56·3.
(2) £25·25; £33·75; £29·4.
(3) £29·2; £53·6; £8·6.
(4) £68·8; £291·3; £157·875.
(5) £900·625; £153·375; £964·125.
(6) £295·16; £358·83; £86·083.
(7) £56·416; £290·583; £129·916.
(8) £78·0625; £986·0416; £537·02083.

11. Express in money the following decimals of £1, correct to one farthing:

(1) 0·01; 0·001; 0·025.
(2) 0·0125; 0·03333; 0·0375.
(3) 0·0291666; 0·0208333.
(4) 0·0041666; 0·016666.
(5) 0·008333; 0·0416666.

12. Convert the following decimals of a shilling into pence and farthings, correct to one farthing:

(1) 0·5; 0·25; 0·75; 0·4.
(2) 0·6; 0·22; 0·24; 0·36.
(3) 0·35; 0·65; 0·95; 0·72.
(4) 0·88; 0·93; 0·83; 0·384.
(5) 0·752; 0·855; 0·796; 0·385.

13. Express the following in shillings, pence, and farthings, to the nearest farthing:

(1) 8·5s.; 7·6s.; 9·3s.; 8·4s.
(2) 3·75s.; 7·68s.; 9·83s.; 8·04s.
(3) 5·96s.; 8·84s.; 12·36s.; 15·68s.
(4) 7·35s.; 12·98s.; 15·65s.; 18·62s.
(5) 25·81s.; 36·38s.; 28·32s.; 50·01s.

14. Express the following decimals as shillings, pence, and farthings, correct to the nearest farthing:

(1) £31·0125; £521·05; £81·025.
(2) £22·0375; £48·9625; £207·9875.
(3) £315·14584; £176·3625; £525·87084.
(4) £28·48; £56·266; £24·872.
(5) £158·671; £533·944; £438·588.
(6) £586·9213; £794·7048; £458·5380.
(7) £1358·54688; £2329·74788; £129·5072
(8) £4583·68021; £215·87292; £5801·04271.
(9) £2964·03125; £139·42188; £4586·19479.
(10) £6350·39271; £2240·33646; £3941·47188.

15. Express £8 13s. 7¼d. as the decimal of £20 17s. 5½d.

16. How many times is £15 8s. 8¼d. contained in £209 11s. 7½d., and what amount of money remains over?

17. Express the remainder in the previous question as a decimal of £15 8s. 5¼d.

18. If a clerk insures his life for £100 with profits[1] when his age is twenty-six, he can do so by paying £2 4s. 1d. (average for British life offices) per annum. If, however, he insures when he is twenty-seven he has to pay £2 5s. 2d. By what decimal of £1 is the premium increased for the year increase in age?

19. Continuing the theme of the previous question, the premium at the age of thirty-six is £2 17s. 3d., and at thirty-seven, £2 18s. 11d. By what decimal of £1 do these premiums differ?

20. By what decimal of £1 is the increase of premium from twenty-six to twenty-seven greater or less than the increase from thirty-six to thirty-seven?

21. By what decimal of £1 do the premiums for the ages twenty-six and thirty-six, and twenty-seven and thirty-seven differ, and by how much is the increase in premium from twenty-six to thirty-six less than the increase from twenty-seven to thirty-seven?

22. It costs 2s. 10d. to send a 7-lb. parcel from London to Bulgaria, and 3s. 2d. to send an 11-lb. parcel to the same country. By what decimal of £1 does the cost of transit per pound in the first case exceed the cost per pound in the second?

23. A merchant has two motor-cars, one of 40 H.P., the other 75 H.P. On the former he has to pay a licence of 10 guineas and on the latter (being over 60 H.P.) 40 guineas. By what decimal of £1 is the licence increased for each unit increase in H.P. between 40 and 75 H.P.?

24. The price of silver in the London market in 1903 was $24\frac{3}{4}$d. per troy ounce, and in 1912, $28\frac{1}{16}$d. per troy ounce. Express the increase in price during the period given as the decimal of a shilling.

25. On 31st August 1914 the amount of gold in the Reichsbank was said to be, in thousands of £'s, 77825; and on 31st October of the same year, 92916. Express the increase as a decimal of the smaller amount.

(The student is informed that the value of the issue of Treasury notes in the two months had increased by 35 million £'s and the current account had fallen by 57 million £'s, and so the position was really much worse financially at the end of October than at the end of August.)

26. The value of the imports into Japan from Australia from the latest returns was equivalent to £19 13s. 4d. for every

[1] See § 258 and footnote p. 384.

twenty inhabitants in the latter. Find the value of the imports for every inhabitant in the Commonwealth.

B. (ii) Approximate Decimalisation

114. We have explained the method of decimalising any sum of money completely, but it is often necessary to decimalise to three places only, and our monetary system is suited to this, for the coins of the realm represent £'s, correct to three places of decimals.

Thus, 2s. = £$\frac{1}{10}$ = £0·1, and hence 1s. = £0·05, while 1 farthing = £$\frac{1}{1000}$ very nearly.

115. To decimalise to three places we take first 0·05 for every 1s. in the shillings of the sum to be decimalised.

Thus— £5 2s. = £5·1
£5 9s. = £5·45 (£5 + 9 × 0·05s.)
£8 18s. = £8·90,

and so on.

116. In dealing with pence and farthings it is best to reduce them to farthings, and on dividing by 1000 we shall be very near the true value, but have to allow for the fact that

1 farthing = £$\frac{1}{960}$th and not £$\frac{1}{1000}$th.

This is done by adding 1 farthing if the pence and farthings are 12 farthings or between 12 and 36 farthings, and 2 farthings if they are 36 farthings or above.[1]

The rule then may be stated thus—

To decimalise a sum of money to three places—
(1) Write down the £'s.
(2) Take 0·05 for every 1s. in the amount.

[1] That this rule is true will appear from the following:

3d. = £0·0125 = £0·013, to three places.
4d. = £0 0166 = £0·017, ,, ,,
7d. = £0·02916 = £0·029, ,, ,,
10d. = £0·0416 = £0·042, ,, ,,

and by applying the rule—

3d. = 12 farthings, add 1, 13 farthings, or £0·013, to three places.
4d. = 16 farthings, ,, 1, 17 farthings, or £0·017, ,, ,,
7d. = 28 farthings, ,, 1, 29 farthings, or £0·029, ,, ,,
10d. = 40 farthings, ,, 2, 42 farthings, or £0·042, ,, ,,

The student should prove the rule for 5d., 6d., etc.

(3) Reduce pence and farthings to farthings, add *one* if farthings are either 12 or between 12 and 36; add *two* if 36 or above, and divide by 1000 to get the decimal.

Thus—

EXAMPLE 1.—**£6 8s. $2\frac{1}{2}$d.**

$= £6{\cdot}4 + 2\frac{1}{2}\text{d.}$
$= £6{\cdot}4 + 10$ farthings
$= £6{\cdot}4 + £\frac{10}{1000}$ (nearly)
$= £6{\cdot}4 + 0{\cdot}01$
$= £6{\cdot}41.$

EXAMPLE 2.—**£8 15s. 3d.**

$= £8{\cdot}75 + 12$ farthings $+ 1$ farthing (by rule)
$= £8{\cdot}75 + \frac{13}{1000}$
$= £8{\cdot}75 + 0{\cdot}013$
$= £8{\cdot}763.$

EXAMPLE 3.—**£8 17s. $9\frac{1}{2}$d.**

$= £8{\cdot}85 + 38$ farthings $+ 2$ farthings (by rule)
$= £8{\cdot}85 + 0{\cdot}040$
$= £8{\cdot}890.$

EXAMPLE 4.—**£156 19s. $9\frac{3}{4}$d.**

$= £156{\cdot}95 + 39$ farthings $+ 2$ farthings (by rule)
$= £156{\cdot}95 + 0{\cdot}041$
$= £156{\cdot}991.$

117. It is very important to recognise that **if a sum of money is written correct to three places and it is multiplied by a large number that the result is by no means correct even to a shilling.**

EXAMPLE 5.—**Standard gold is worth £3 17s. $10\frac{1}{2}$d. per troy ounce,[1] calculate the value of the gold in 1000 troy ounces.**

$£3\ 17\text{s.}\ 10\frac{1}{2}\text{d.} = £3{\cdot}89375$
$= £3{\cdot}894$, to three places.

$\therefore$ value of 1000 troy ounces of gold
$= £3{\cdot}894 \times 1000$
$= £3894.$

But $£3\ 17\text{s.}\ 10\frac{1}{2}\text{d.} \times 1000 = £3893\ 15\text{s.}$;

$\therefore$ the error made in working to three places is 5s., and consequently the approximate method would not be used here, for the loss on purchasing a large number of sovereigns would be enormous.

[1] § 140.

EXAMPLES. XXIX. (a)

Express the following sums of money in £'s and the decimal of a £, correct to three places:[1]

1. £36 15s. 2d.; £28 17s. 2¾d.; £45 18s. 1¼d.
2. £74 13s. 3½d.; £85 13s. 4¼d.; £73 19s. 5½d.
3. £38 14s. 3¾d.; £79 12s. 4¾d.; £125 16s. 5¾d.
4. £179 19s. 2½d.; £564 19s. 1½d.; £100 10s. 2¾d.
5. £275 16s. 4½d.; £378 12s. 5¼d.; £250 8s. 5¾d.
6. £868 12s. 3¾d.; £564 15s. 4¼d.; £760 6s. 4¾d.
7. £236 14s. 6½d.; £58 15s. 7¼d.; £158 13s. 9¾d.
8. £33 15s. 6¾d.; £726 16s. 9½d.; £731 5s. 10½d.
9. £58 17s. 10¼d.; £384 18s. 7½d.; £79 7s. 11¼d.
10. £960 3s. 7¾d.; £276 13s. 7¼d.; £158 8s. 6½d.
11. £724 1s. 8¼d.; £158 9s. 8½d.; £396 19s. 5¼d.
12. £356 15s. 9½d.; £73 7s. 9¼d.; £1001 16s. 5¾d.
13. £21 11s. 11¼d.; £196 5s. 10¾d.; £354 17s. 9¼d.
14. £5 10s. 11½d.; £843 3s. 11¾d.; £768 15s. 11¾d.
15. £394 9s. 10½d.; £75 12s. 8¾d.; £596 18s. 4½d.

118. In taking the reverse process the method described in § 113 will serve, or we may apply the rule given in § 116 in the reverse way, thus—

(1) Write the decimal correct to three places.

(2) **Divide** the first two places by 5 and call the quotient shillings.

(3) Put the third place down by the side of the remainder from (2), if any, and call the number farthings; subtract 2 if it is 36 or over, 1 if 12 or between 12 and 36, and convert the remainder into pence and farthings.

Thus—

Example 6.—**Reduce £8·65324 to a compound quantity, correct to the nearest farthing.**

By the method of § 113 we have—

£8·653
= £8 13·06s.
= £8 13s. 0·7d.
= £8 13s. 0¾d.

By the rule of § 118 we have—

£8·653, and
0·65 ÷ 5 = 13s. and 0 over,
while the 3 gives us ¾d.;
∴ £8·653 = £8 13s. 0¾d.

[1] The student should practise decimalisation of money *every day* for himself by writing down, say, twenty different sums of money and decimalising them to-day, and then reconverting the decimal to money to-morrow, and so testing the result against the original amount.

Example 7.—**Reduce £7·93786 to a compound quantity, to the nearest farthing.**

£7·93786 = £7·938 to three places.

1. £7·938
   = £7 18·76s.
   = £7 18s. 9·12d.
   = £7 18s. 9d.

2. £7·938
   and 0·93 ÷ 5
   = 18s. + 3 (over)
   and 38 f. − 2 f. = 36 f. = 9d. ;
   ∴ £7·938 = £7 18s. 9d., as before.

EXAMPLES. XXIX. (b)

Reduce the following to compound quantities, correct to the nearest farthing:

1. £394·494 ; £75·636 ; £596·919.
2. £5·548 ; £843·199 ; £768·799.
3. £21·597 ; £196·295 ; £354·889.
4. £356·790 ; £73·389 ; £1001·824.
5. £724·534 ; £158·485 ; £396·977.
6. £960·182 ; £276·680 ; £158·427.
7. £58·893 ; £384·931 ; £79·397.
8. £33·778 ; £726·840 ; £731·294.
9. £236·727 ; £58·780 ; £158·691.
10. £868·616 ; £564·768 ; £760·320.
11. £275·819 ; £378·622 ; £250·424
12. £179·960 ; £564·956 ; £100·511.
13. £38·716 ; £79·620 ; £125·824.
14. £74·665 ; £85·668 ; £73·973.
15. £36·758 ; £28·861 ; £45·905.

C. Calculation of Cost: (i) By Decimalising the Sum of Money

119. We now apply the principles of decimalisation to find the cost of any number of articles.

Example 1.—**Find the cost of 306 bracket clocks at £8 15s. 8d. each.**

We want the result correct to three places.

£8 15s. 8d. × 306 ≏ £9 × 306 = £2754.

∴ there will be four figures before the decimal point and three after, seven in all; we take one extra for safety, and so need eight figures.

Hence,

Cost of one clock = £8 15s. 8d.

= £8·7833333 (eight figures)

∴ „ „ 306 clocks = £8·7833333 × 306,

which, by contracted methods = £2687·6999,

or £2687·700, to three places,

or £2687 14s.

The student should now decimalise £8 15s. 8d. to three places only, and see what difference this makes to the result.

Example 2.—**Find the cost of 436 electric bells, best quality metal gong $3\frac{1}{2}$ inches diameter, at 3s. $7\frac{1}{2}$d. each.**

3s. $7\frac{1}{2}$d. × 436 ≏ £0·2 × 436 = £87.

We want three places of decimals and £87 gives two figures; so, two figures + three places + one place for safety = six figures.

3s. $7\frac{1}{2}$d. = £0·181250 (six figures)

∴ cost = £0·181250 × 436

= £79·0250

= £79 0s. 6d.

In all such cases as the above ≏ enables us to decide the number of figures necessary, and this often means that the original sum of money has to be written correct to six or seven decimal places **to get three correct in the answer.**

EXAMPLES. XXX. (a)

Find the cost of the following to the nearest penny by decimalising the price:

1. 720 tons of anthracite coal at £4 2s. 6d. per ton.
2. 126 barrels of wine at £5 3s. 4d. per barrel.
3. 321 firkins of soap at £6 6s. 8d. per firkin.
4. 240 chests of tea at £10 1s. 6d. per chest.
5. 486 cwt. of mace at £20 13s. 4d. per cwt.
6. 740 motor-boats at £215 12s. 9d. each.
7. 584 lots of timber at £524 7s. $6\frac{1}{4}$d. each.
8. 370 freehold houses at £431 5s. 6d. each.
9. $246\frac{2}{3}$ lots of mining plant at £646 18s. 3d. each.
10. 1168 water-colours at £262 3s. $9\frac{1}{4}$d. each.
11. 624 lots of machinery at £658 12s. $5\frac{1}{2}$d. each.
12. 585 ground rents at £859 15s. $8\frac{1}{4}$d. each.
13. 796 oil-paintings at £1764 17s. $9\frac{1}{2}$d. each.
14. 975 lots of scrap steel at £5692 18s. $7\frac{1}{4}$d. each.
15. 432 lots of china clay at £630 8s. 6d. each.

C. (ii) By the Method of Nine Multiples

120. A method of calculating cost, known as the "Method of Nine Multiples," is particularly useful for finding the cost of any number of articles at a FIXED PRICE, and also in exchange problems, for determining the value of any number of francs, etc., in English money.

121. Example 1.—**Construct a table of nine multiples for a fixed price of 1s. $0\frac{3}{4}$d., and thence calculate the cost of 986 articles.**

$$1\text{s. } 0\tfrac{3}{4}\text{d.} \times 986 \simeq 1\text{s.} \times 1000 = £50.$$

We express 1s. $0\frac{3}{4}$d. as the decimal of £1.

$$\begin{aligned} 1\text{s. } 0\tfrac{3}{4}\text{d.} &= 1\text{s.} + \tfrac{3}{4}\text{d.} \\ &= £0{\cdot}05 + £0{\cdot}0031249 \\ &= £0{\cdot}0531249. \end{aligned}$$

The table required will give the cost of 1, 2, 3, . . . 9 articles at 1s. $0\frac{3}{4}$d., and is sometimes called a BASE TABLE.

Base Table for 1s. $0\frac{3}{4}$d.

1 . . .	0·05312499
2 . . .	0·10624999
3 . . .	0·1593749
4 . . .	0·2124999
5 . . .	0·2656249
6 . . .	0·3187499
7 . . .	0·3718749
8 . . .	0·4249999
9 . . .	0·4781249

It is now easy to calculate the cost of 986 articles, for $986 = 900 + 80 + 6$, and 900 is merely 9×100, which means that the decimal point in 0·4781249 is moved two places to the right.

$$\begin{aligned} \therefore \text{ Cost of } 900 &= £47{\cdot}812499. \\ \text{Cost of } 80 &= 4{\cdot}249999. \\ \text{Cost of } 6 &= 0{\cdot}318749. \end{aligned}$$

We want the result to three places, so that it is unnecessary to write the decimals beyond the fourth place in this case, although our table would enable us to find the cost of any number of articles up to 10000, correct to three places.

$$\text{Cost of 986 articles} = £52{\cdot}381 = £52\ 7\text{s. } 7\tfrac{1}{2}\text{d.}$$

122. Example 2.—**Make out a table of nine multiples, given that 25·22 francs = £1, and thence calculate the value of 3768 francs in English money.**

Since 25·22 francs = £1
1 franc = £0·39651,

from which the nine multiple table can be constructed at once.

We leave it as an exercise to the student to finish this question and to show that 3768 francs

= £149·405
= £149 8s. 1d.

EXAMPLES. XXX. (b)

Make out Base Tables for the following prices:

1. 3d.	2. 6d.	3. 9d.	4. 1s. 3d.
5. 2s. 6d.	6. 3s. 6d.	7. 5s. 6d.	8. 10s. 6d.
9. $6\frac{1}{2}$d.	10. 1s. $1\frac{1}{2}$d.	11. 1s. $6\frac{1}{2}$d.	12. 2s. $0\frac{1}{2}$d.
13. $11\frac{3}{4}$d.	14. 2s. $6\frac{3}{4}$d.	15. 9s. 11d.	16. 5s. $9\frac{3}{4}$d.

Calculate the cost of—

17. 15, 36, and 24 articles at each of the prices in Nos. 1 to 4.
18. 29, 78, and 89 articles at each of the prices in Nos. 5 to 8.
19. 125 and 356 articles at each of the prices in Nos. 9 to 12.
20. 456, 788, and 1042 articles at each of the prices in Nos. 13 to 16.

C. (iii.) By Practice

123. The student has already learnt that it is possible to calculate the cost of any number of articles by decimalising the price, multiplying, and reconverting to money (§ 119).

If, however, a grocer were reckoning the cost of 356 cases of butter at £1 10s. 6d. each, he would say:

356 @ £1 = £356
356 @ 10s. or £$\frac{1}{2}$ = £178
356 @ 6d. or £$\frac{1}{40}$ = £8 18s.
Total cost = £543 roughly.

124. To the simple process adopted by the grocer, mathematicians have given the name of Aliquotation, which means, in ordinary language, that the merchant reckoned the shillings and pence such fractions of £1 that their **numerators were unity.**[1]

[1] 13s. 4d. = $\frac{2}{3}$ of £1 and is not an aliquot part of £1, as its numerator is not unity.

What we have to do then is "to spot" fractions of £1 in any sum of money, and work just as the grocer did.

Thus, £5 17s. 6d.

$$= £5 + 10s.\ (£\tfrac{1}{2}) + 5s.\ (£\tfrac{1}{4}) + 2s.\ 6d.\ (£\tfrac{1}{8});$$

or £3 18s. 8d.

$$= £3 + 10s.\ (£\tfrac{1}{2}) + 6s.\ 8d.\ (£\tfrac{1}{3}) + 2s.\ (£\tfrac{1}{10});$$

and then the calculation of the cost of any number of articles is easy.

We call this way of calculating cost "Practice."

The aliquot parts of £1 are given, and you can calculate those of 1s. for yourself:

10s.	$= \frac{1}{2}$ of £1.	1s. 4d.	$= \frac{1}{15}$ of £1.
6s. 8d.	$= \frac{1}{3}$,,	1s. 3d.	$= \frac{1}{16}$,,
5s.	$= \frac{1}{4}$,,	1s.	$= \frac{1}{20}$,,
4s.	$= \frac{1}{5}$,,	8d.	$= \frac{1}{30}$,,
3s. 4d.	$= \frac{1}{6}$,,	$7\frac{1}{2}$d.	$= \frac{1}{32}$,,
2s. 6d.	$= \frac{1}{8}$,,	6d.	$= \frac{1}{40}$,,
2s.	$= \frac{1}{10}$,,	4d.	$= \frac{1}{60}$,,
1s. 8d.	$= \frac{1}{12}$,,	3d.	$= \frac{1}{80}$,,

125. Suppose, then, we take the following question:

EXAMPLE 1.—**Calculate the cost of making 564 carpets at an average cost of £3 10s. each.**

Now, 560 @ £3 each = £1680, and 560 @ £4 each = £2240;
∴ 560 @ £3 10s. is half-way between, or, roughly, £1950.

	£	s.	d.	
Again,	564	0	0	= cost of 564 carpets @ £1 each,
and	1692	0	0	= cost of 564 carpets @ £3 each.

Now 10s. = £½;

∴ cost of carpets at 10s. each equals half cost at £1 each, *i.e.* half of £564.

So the sum appears:

	£	s.	d.	
	564	0	0	= cost @ £1 each.
	1692	0	0	= ,, £3 each.
10s. = ½ of £1 ∴	282	0	0	= ,, 10s. each.
	£1974	0	0	= ,, £3 10s. each.

EXAMPLE 2.—**If the price of fine hard Para rubber be £9 3s. 7½d. per half cwt., calculate the selling price of 859 half cwt. lots.**

859 at £9 3s. 7½d. ≏ 860 at £9 each = £7740.

Here we see that 3s. 7½d. must be broken up into aliquot parts of £1 or of 1s.,

e.g. (1) 3s. 7½d. = 2s. 6d. + 1s. + 1½d.;
and 2s. 6d. = £$\frac{1}{8}$; 1s. = £$\frac{1}{20}$; and 1½d. = $\frac{1}{8}$s.;

or (2) 3s. 7½d. = 2s. + 1s. + 6d. + 1½d.;
and 2s. = £$\frac{1}{10}$; 1s. = ½ of 2s.; 6d. = ½ of 1s.; and 1½d. = ¼ of 6d.

Taking the former—

	£	s.	d.		£	s.	d.	
	859	0	0	= cost @	1	0	0	each lot.
	7731	0	0	= ,, @	9	0	0	each lot.
2s. 6d. = $\frac{1}{8}$ of £1	107	7	6	= ,, @	0	2	6	,,
1s. = $\frac{1}{20}$ of £1	42	19	0	= ,, @	0	1	0	,,
1½d. = $\frac{1}{8}$ of 1s.	5	7	4½	= ,, @	0	0	1½	,,
	£7886	13	10½	= ,, @	£9	3	7½	,,

There are two most important points to notice:

1. That the first line, £859 0s. 0d., is *not* added to the others.

2. That is taking 2s. 6d. *and* 1s. we take fractions of the top line, for they are here fractions of £1; and in taking 1½d. we take $\frac{1}{8}$ of the 1s. line, for 1½d. = $\frac{1}{8}$ of 1s., *not* of £1.

126. EXAMPLE 3.—**What should be paid for repairing 586 miles of railway track at an average price of £89 18s. 5½d. per mile?**

586 at £89 18s. 5½d. ≏ 580 at £90 = £52200.

(1) £89 18s. 5½d. is just 1s. 6½d. short of £90, so that we may find the cost at £90 per mile and SUBTRACT the cost at 1s. 6½d. per mile.

	£	s.	d.		£	s.	d.	
	586	0	0	= cost @	1	0	0	per mile.
	52740	0	0	= ,, @	90	0	0	,,
1s. = $\frac{1}{20}$ of £1	29	6	0	= ,, @	0	1	0	,,
6d. = ½ of 1s.	14	13	0	= ,, @	0	0	6	,,
½d. = $\frac{1}{12}$ of 6d.	1	4	5	= ,, @	0	0	0½	,,
	£45	3	5	= ,, @	£0	1	6½	,,

∴ Cost at £89 18s. 5½d. per mile
= £52740 − £45 3s. 5d.
= £52694 16s. 7d.

Or (2) decimalising—

Cost = 586 × £90 − 586 × £0·0770833
= £52740 − £45 3s. 5d.
= £52694 16s. 7d., as before.

The student will probably find the first method the quicker here, but the second can often be used to advantage, *e.g.* in finding the cost of paving 6 miles 4 furlongs 6 chains 18 yards at £3 15s. 6d. a mile (see §§ 148 and 149).

127. Example 4.—**In some cases it is necessary to find the cost of such a number as $538\frac{2}{5}$ things at £30 6s. 8d. each.**

If so, we decimalise the fraction thus:

	£538·4	= cost @ £1 each.
	£16152·0	= „ £30 each.
6s. 8d. = $\frac{1}{3}$ of £1	179·4666	= „ 6s. 8d. each.
	£16331·4666	

= £16331 9s. 4d. to the nearest farthing.

EXAMPLES. XXXI.

Find, by Practice, the cost of the following numbers of articles at the prices given for one:

1. 500 at 10s., 350 at 5s.
2. 600 at 6s. 8d., 1500 at 13s. 4d.
3. 764 at 15s., 1566 at 3s. 4d.
4. 5680 at 2s. 6d., 1534 at 2s.
5. 7128 at 1s. 6d., 5000 at 1s. 3d.
6. 9140 at 6d., 6284 at 3d.
7. 720 at £4 2s. 6d.
8. 126 at £5 3s. 4d.
9. 321 at £6 6s. 8d.
10. 240 at £10 1s. 6d.
11. 486 at £20 13s. 4d.
12. 520 at £24 19s. 6d.
13. 972 at £124 18s. 4d.
14. 105 at £129 18s. 8d.
15. 630 at £219 19s. 8d.
16. 352 at £199 19s. $4\frac{1}{2}$d.
17. 360 at £449 19s. 6d.
18. 128 at £359 18s. 9d.
19. 432 at £630 8s. 6d.
20. 740 at £215 12s. 9d.
21. 584 at £524 7s. $6\frac{1}{2}$d.
22. 370 at £431 5s. 6d.
23. $246\frac{2}{3}$ at £646 18s. 3d.
24. 1168 at £262 3s. $9\frac{1}{4}$d.
25. 624 at £658 12s. $5\frac{1}{2}$d.
26. 585 at £859 15s. $8\frac{1}{4}$d.
27. 796 at £1764 17s. $9\frac{1}{2}$d.
28. 975 at £5692 18s. $7\frac{1}{4}$d.
29. 595 at £596 19s. $11\frac{3}{4}$d.
30. 627 at £9876 17s. $4\frac{1}{4}$d.

EXAMPLES. XXXII.

Miscellaneous Questions

1. A clerk earns £50 a year, how much does he earn per day? (To the nearest farthing.)

2. By paying a registration fee of 8d. a package can be insured for £120, what decimal of £1 must be paid for £120 cover?

3. It costs 6·921 shillings to drive a small motor-car 50 miles, find the cost per mile, to the nearest farthing.

4. A package can be insured for £180 by paying 11d. By what decimal of £1 is the charge to cover every sovereign's worth of goods at the £180 rate less than the charge at the £120 rate? (Question 2.)

5. A corn merchant in London sends two cablegrams, one to Winnipeg (Manitoba) and the other to Calgary (Alberta). The first contains 15 code words and is charged at 1s. 5d. a word, the latter, seventeen words, charged at 1s. 6d. a word. Find the difference in cost and express it as the decimal of £1.

6. The following details are from *Whitaker's Almanack*:

IMMEDIATE ANNUITY[1] FOR EVERY £100 PAID IN.

Office.	Males. Age 55.			Females. Age 55.		
	£	*s.*	*d.*	£	*s.*	*d.*
1 . .	8	2	2	7	7	11
2 . .	7	8	2	6	14	8
3 . .	7	6	0	6	15	8

Determine by what decimal of £1 the amount of the annuity paid to males exceeds that paid to females by each office.

7. Using the data of the last question, determine by what decimal of £1 the first-named office is more remunerative to the annuitant than either the second or third (males only).

8. Explain the reason for the disparity between the annuities paid to males and females, given that the expectation of life for males aged 55 is 15·79 years, and for females 17·24 years.[2]

9. Determine the amount of money received by an annuitant aged 55 from the first life office if he lived for the expected period. (Question 8.)

10. A locomotive consumes $4\frac{1}{2}$ tons of coal costing 17s. $11\frac{1}{2}$d. a ton (on the average) in running 220 miles, find in pence and the decimal of a penny the cost of fuel per mile.

11. A rubber company produced in the year 325700 pounds of rubber, and sold it for £42748. Find to the nearest farthing the average price per pound.

12. In December 1913 it was estimated that there were 2,540,633 acres in the United States upon which irrigation work had been commenced, and that the expenditure thereon amounted to 86,430,997 dollars. Reckoning 4s. 1d. as being the value

[1] Part III. [2] See Section XVII. § 254.

of a dollar, calculate to the nearest penny the expenditure per acre.

13. In 1913, 64,116,000 short tons[1] of hay were produced in the United States, and the value was 797,077,000 dollars. Reckoning 4s. 1d. as the value of a dollar, calculate to the nearest penny the value per short ton of the hay grown in that year.

14. In 1912, 557,983,794 lb. of greasy wool were exported from Australia, and the value of this was £26,332,617. Calculate to the nearest penny the value per pound exported.

15. In the fiscal year 1914 the United Kingdom imported from Canada 1,250,912 cwt. of cheese, the value of which was 19,063,539 dollars (according to the official unrevised statement). Take the Canadian dollar as worth 4s. 1d., and calculate to the nearest penny the value per pound of the cheese imported. (112 lb. = 1 cwt.)

16. In the year 1913 there were 154957 depositors in the Savings Bank of Queensland, according to the year-book issued by the Government of that State, and no less than £8,213,116 stood to their credit. How much is this, per depositor, to the nearest shilling?

17. There is a transporter bridge which crosses the river Usk at Newport (Monmouth), and the cost of maintenance for the year ending 31st March 1914 was £1822 1s. The length from shore to shore is 645 feet. Find to the nearest penny the cost of maintenance per foot.[2]

18. A company has 782 horses, the average value of which is £39 14s. 9d. For what amount must they be insured,[3] supposing that their value is completely covered?

19. The total distance covered by the trains of a railway company in a year was 25,321,434 miles, and the traffic receipts were £5,250,433 8s. 6d. for the same period. Calculate the receipts derived from every mile run.

20. Find the average value of the Haikuan tael for the years 1911–13, given that the Chinese Maritime Customs returned the average exchange in the years named as 2s. $8\frac{5}{16}$d., 3s. $0\frac{5}{8}$d., and 3s. $0\frac{1}{4}$d. respectively.

21. In one week 197150 great hundreds of eggs were imported into the United Kingdom; calculate the retail value of this importation at 9d. per dozen, allowing 200 eggs for breakages.

22. It is frequently possible to buy bananas in the heart of

[1] A short ton is 2000 lb.
[2] By the courtesy of the Borough Engineer of Newport.
[3] Section XVII. B. (5), p. 380.

the West End of London from costers at the rate of two a penny, but 1½d. each is the figure charged in many of the restaurants. Presuming the purchase price is the same in both cases, calculate the difference in profit for every bunch of bananas sold. (A bunch costs 10s., and, for the purposes of the question, can be regarded as containing 320 bananas.)

23. In the year 1912–13 there were 29,618,584 money orders issued in India, to the value of £34,854,327. Calculate to the nearest penny the average value of each money order issued.

24. Under the circumstances of the last question, 15,449,071 paid messages were sent over Government Telegraphs, and the money received for them was £716793. Calculate to the nearest tenth of a penny the cost per message.

25. In 1903 there were 21131 miles of State railway[1] open for traffic in India, and 25125 in 1912. The gross earnings in the former year amounted to £24,005,469, and in the latter to £41,100,467. Calculate, to the nearest pound, the earning capacity of the railways per mile in the two cases, and decide whether it has increased or decreased in ten years, and by how much.

26. 24,489,692 gallons of oil valued at £571945 were produced in British India and exported therefrom by sea in 1913. Calculate the value of the oil per gallon to the nearest penny.

27. The Australian Government paid £2792 as bounty upon 335013 gallons of kerosene in 1912–13. Calculate the bounty paid per gallon to the nearest penny.

28. The value of the 278,518,411 lb. of tea produced in, and exported by sea from, British India in 1913 was £8,862,651. Calculate the value per pound to the nearest penny. What information do you gain from the results of Questions 23 to 26 and 28?

REVISION QUESTIONS. I.[2]

A.

SECTION I.

1. From the data given in Question 27, page 12, write down the eight years in which the loans were greatest, and find by how much the greatest of these eight loans exceeds the smallest.

2. Find the area of the "Open Spaces" of London from the

[1] About five-sevenths of the total length of railway belongs to the State.

[2] The student should work again some of the purely drill questions in the sections referred to before proceeding to these problems.

data given below, after having arranged them in order of size, the largest first:

Name of Open Space.	Area in Acres.	Name of Open Space.	Area in Acres.
Barnes Common . .	120	Hampstead Heath and Parliament Hill . .	551
Wimbledon Common .	1000	Hyde Park . . .	360
Streatham ,, .	69	Richmond Park and Petersham Park . .	2357
Clapham ,, .	205	Kew Gardens . . .	288
Tooting ,, .	218	Kensington Gardens .	274
Greenwich Park . .	185	Primrose Hill . .	62
St. James's ,, . .	93	Victoria Park . .	217
Battersea ,, . .	199	The Green Park . .	58
Regent's ,, . .	408		

3. Has the first six months or the last six of a leap year the greater number of days?[1]

4. The statement below shows in detail the exports of the several products of the cocoa-nut palm from India to foreign countries during 1913–14 (*Indian Trade Journal*):

		Rupees.
Cocoa-nuts	344111	22760
Coir fibre	14812 cwt.	171739
,, manufactures .	772262 ,,	8891113
Cordage and rope . .	60420 ,,	1052837
Copra[2]	763832 ,,	15597397
Coco-nut oil	1091477 galls.	2326099
,, poonac . .	84166 cwt.	404474
	Total .	

Find (1) the total weight, in hundredweights, of those things whose weights are given; (2) the value of the exports in rupees (Rs.); (3) supposing the exports had been (i) coir fibre, 15 thousand cwt.; (ii) cordage and rope, 60 thousand cwt.; (iii) coco-nut poonac, 90 thousand cwt., draw a line half an inch long to represent (i), and similar lines of appropriate length to represent (ii) and (iii).

5. The latitude of Newcastle is 55° N., that of Capetown 35° S. What is the difference in latitude between these two places?

[1] In a leap year February has twenty-nine days, in other years twenty-eight. See § 144.

[2] Copra was chiefly used for soap manufacture, but its value as a food has been established, particularly in the manufacture of nut-butter. The trade in this commodity is steadily increasing.

6. The exports of commodities from Ceylon from Jan. 1 to Sept. 7, 1914, and for the corresponding period in 1913, are given below (from the *Ceylon Observer*):

Countries.	Black Tea.		Desiccated Cocoa nuts.	
	1914. lb.	1913. lb.	1914. lb.	1913. lb.
To United Kingdom .	78752323	79024022	9734332	8199915
,, Austria . . .	226061	182232	566540	341250
,, Belgium . . .	40688	86902	1088510	682816
,, France . . .	602551	493199	329980	156390
,, Germany . . .	386977	402672	3439054	2627300
,, Holland . . .	27022	13029	995342	346773
,, Denmark . . .	14549	12494	21450	48100
,, Italy	37342	21998	1190	...
,, Russia . . .	10536703	12314899	1950	...
,, Spain . . .	30073	37916	459160	268086
,, Norway . . .	5900	11616	37000	7410
,, Sweden . . .	70583	81703	46800	30680
,, Turkey . . .	306111	147124	728	280
,, Western Australia .	616037	544198	57770	51070
,, South Australia . .	1262718	1053538	172972	96707
,, Victoria . . .	5176606	4993506	567279	434438
,, New South Wales .	7409223	7432143	318508	181189
,, Queensland . .	683316	623704	83430	65370
,, Other Australian States	34498	25605	8238	3604
,, New Zealand . .	4148992	3556182	308692	125830
,, United States . .	7076092	5682847	5480931	5212911
,, Canada and Newfoundland	4012925	4305039	561284	426716
,, Other countries in America . . .	100460	100079	84000	36352
,, Egypt	419797	375744	...	...
,, Africa	1613025	1515533	133514	102964
,, Madagascar . .	19416	28909	...	...
,, India	1120895	1023343	...	...
,, Straits . . .	339493	314966	...	...
,, China	4375803	5624630	...	...
,, Philippine Islands .	43099	36974	240	260
,, Japan	32435	36428	...	...
,, Mauritius . . .	57939	50314	...	...
,, Malta	56453	70583	...	...
Total exports from Jan. to Sept. 7 . . .				
Week ended Sept. 7, 1914	2404957	...	689758	...
From Jan. 1 to Aug. 31, 1914		...		...

(1) Fill in the blank spaces in the above table; (2) which countries were the ten best customers of Ceylon, (i) in tea, (ii) in desiccated cocoa-nuts in 1914?

7. Find the increase in the total funds of ordinary Friendly Societies to the nearest £1000 if the funds were £22,043,450 and £22,735,568 in two successive years.

8. Use the figures of Question 27, page 12, to find to the nearest £1,000,000 the amount borrowed by the Government in the five years 1801–1805.

9. The dead-weight debt of the United Kingdom is as follows: 1910-11, £713,245,408; 1911-12, £685,232,459; 1912-13, £674,744,567; 1913-14, £661,473,765. Calculate to the nearest £10000 the annual decrease in the debt from one year to the next, and state when that decrease was greatest. (The student should compare the results of this question and the last one.)

B. (1)

SECTION II.

1. 5942 acres of land in New Zealand produced 307736 bushels of maize last year. Calculate the quantity produced per acre to the nearest bushel.

2. 48,116,377 lb. of cocoa were produced from 336154 acres in Trinidad and Tobago[1] last year. What was the average weight of cocoa produced from one acre of land?

3. If the value of the cocoa referred to in the last question be 1s. per lb., calculate the value of the yield per acre to the nearest shilling.

4. A reaping-machine can mow 60 square yards in five minutes, calculate the number of square yards that can be mown (1) in one minute, and (2) in eight hours of sixty minutes each.

5. If you went to Queensland and resolved to follow the lucrative and exacting profession of a hawker and pedlar, you would have to pay a licence of £10 per annum. How much would you have to earn in a year if you wanted to make 10s. a week to live on, and to save 15s. a week and pay your licence as well? How much would you save in a year? (1 year = 52 weeks.)

6. A motor-bus company employs 736 drivers, and a licence,

[1] Trinidad and Tobago comes second in the list of British possessions in the production of cocoa. The Gold Coast easily comes first, with 113,239,980 lb.

costing 5s., has to be taken out for each of them. How much must be paid for the whole of the licences?

7. In the last fiscal year Ceylon exported 111,646,640 lb. of tea to the United Kingdom, valued at £3,411,921, while its next largest exportation was to Australia, and amounted to 21,015,220 lb., valued at £642225. Was the number of pounds obtainable for £1 greater or less in the United Kingdom than in Australia, and by how many pounds (*i.e.* was the market better in the United Kingdom than in Australia *so far as these figures show*)?

8. By referring to the latest Statistical Abstract we find that the total value of all articles imported into the Federated Malay States last year was 6,462,551 Straits Settlements silver dollars, or £753964. Estimate as nearly as you can the number of such dollars in £1.

9. The most valuable of the exports of British North Borneo last year was estate-grown tobacco, and 20288 bales were exported, valued at 2,919,970 dollars.[1] What was the value of each bale, to the nearest dollar?

10. What was the value per ton of the 25290 tons of nickel which were exported from Canada in 1914 if the total value was £1,104,807? (Give the answer to the nearest £1.)

B. (2)

1. In the most recent edition of a little handbook entitled *The Queensland Sugar Industry* we find that in 1912–13 the population of the Commonwealth was 4,644,852, and the weight of sugar available for consumption was 507,324,160 lb. Calculate the consumption per head of the population to the nearest pound.

2. The distance by rail from Brisbane in Queensland to Sydney in New South Wales is 725 miles, and the fare is 63s. What is the charge per mile (first class), to the nearest penny?

3. If the Indian rupee is worth 1s. 4d. and £1 is worth 240 pence, how many rupees must we reckon to £1?

4. A clerk sees in *The Times of India* an advertisement by a merchant in Calcutta for a young fellow to learn the tea trade, and he offers a salary of 1000 rupees per annum. The clerk is now getting £1 a week. If you were he, would you go to India

[1] In the Straits Settlements, Labuan, the Federated and Protected Malay States, British North Borneo, and Sarawak, the Straits Settlements silver dollar has been made the standard coin by Orders in Council of various dates, and since 1906 the value of the dollar has been fixed at 2s. 4d.

(all other things being equal) if the chief object in going were to better yourself financially?

5. What is the median of the following lengths: 35 feet; 41 feet; 38 feet; 38 feet; 45 feet; 39 feet; 10 feet; 42 feet; 39 feet?

6. Find both average and median of the following readings of the barometer; 30 inches; 31 inches: 28 inches; 26 inches; 29 inches; 31 inches; 29 inches; 25 inches.

7. We observed the time taken on ten evenings on which we travelled between two stations on one of the London tube railways to be as follows (in seconds): 150; 160; 155; 165; 162; 300; 153; 150; 158; 157. Find the average and the median time. Which would you take here as being more relevant if you had no other data to go upon than that supplied?

8. At a sale of cattle and sheep the following prices were realised: 10 cows at £55 each; 8 sheep at £5 each; 6 sheep at £4 each; 12 sheep at £6 each; 15 cows at £40 each; 5 cows at £35 each; 10 sheep at £8 each. What was the average price per cow, per sheep, and per head?

9. The value of £1 in centimes was, for ten successive observations, as follows: 2525, 2522, 2524, 2523, 2521, 2518, 2520, 2522, 2524, 2525. Calculate the average value of £1 in centimes and also in francs, if 1 franc = 100 centimes.

10. It is estimated that 2,108,886,516 journeys were made during a year either by rail, tram, omnibus, or cab by the 7,310,584 inhabitants of Greater London. Find, to the nearest whole number, the average number of journeys per person per week.

11. A salesman took the following sums in shillings per day; 12, 28, 48, 77, 66, 42, 26, 10, 9, 17, 33, 66, 73, 87, 9, 7, 17, 22, 41, 53, 61, 66, 72, 122, 60, 16, 5, 4, 2, 137, 0. Find his average takings, also the median and the mode, explaining what each number represents.

C. (1)

SECTIONS III. AND IV.

1. A tailor, at sale-time, reduces the price of gentlemen's suits from 45s. to 35s. What fraction is the reduced price of the original price?

2. If the tailor in the previous question reduces his original price of 45s. by one-third of itself, what is his sale price?

3. A cutler buys cheese knives for 5s. 8d. a dozen. Under ordinary circumstances he makes one-fourth of his buying price profit; to effect a clearance he contents himself with one-

seventeenth of his buying price as profit. Find his ordinary selling price and his "clearance" price, per dozen.

4. In the fiscal year 1914 Canada imported from Newfoundland 1,841,000 dollars worth of stuffs for consumption, and exported 4,770,000 dollars worth. What fraction was the former of the latter?

5. A truss of old hay weighs 56 lb. and a truss of new hay (to September 1) weighs 60 lb. What fraction is a truss of old hay of a truss of new hay?

6. Which weighs the more, $\frac{3}{14}$ of a truss of old hay or $\frac{4}{15}$ of a truss of new hay?

7. Three men provide some of the capital[1] for a business. If the first subscribes $\frac{13}{110}$, the next $\frac{5}{11}$, and the third $\frac{1}{5}$, what fraction of the whole capital do they provide?

8. A number of houses are to be built on and to occupy $\frac{37}{60}$ of a site; the roads are to take up $\frac{2}{15}$. What will be the total fraction occupied by houses and roads?

9. A commercial traveller earns £560 a year. He pays $\frac{1}{10}$ of his income in rent, $\frac{1}{20}$ in rates and taxes,[2] and spends $\frac{1}{28}$ on his season ticket. How much does he spend on the three items named?

10. A merchant's income is £600 a year, and he pays 1s. in the pound Income-Tax.[3] How much does he pay in this way, presuming that it is charged on his whole income?

11. A public bath is arranged so that a covered swimming bath occupies $\frac{1}{4}$ of the site; an open-air bath, $\frac{3}{20}$; private baths, $\frac{1}{18}$. What fraction of the whole is thus taken up?

C. (2)

1. Taking the data given in Question 12, page 56, draw a diagram similar to that in Question 20, page 59, but taking a height of half an inch to represent a million dollars.

2. The value of wheat and flour imported into the United Kingdom from Canada and from Victoria will be found in the following table, expressed in thousands of £'s:

Imports from		Year.	
		1911.	1912.
Canada	Wheat	$6555\frac{2}{3}$	$4528\frac{2}{3}$
	Flour	$190\frac{4}{5}$	$326\frac{3}{7}$
Victoria	Wheat and flour	$2137\frac{1}{2}$	1220

[1] Section XVII. B. (1).
[2] Section XVII. B. (3).
[3] Section XVII. B. (3), § 241.

Find the value of the wheat and flour imported from Canada and from Victoria respectively in the two years given, and reduce the latter to a fraction of the former after having expressed the values to the nearest £1000.

3. Draw a line 5 inches long to represent a railway line 5 miles long. The company concerned, gradually extend the line and the increments are represented by $\frac{1}{2}''$; $\frac{1}{4}''$; $\frac{5}{8}''$; $\frac{3}{10}''$; $\frac{15}{16}''$. (*Note.*—$\frac{1}{2}$ inch is often written $\frac{1}{2}''$.) Increase the $5''$ line by these amounts, and determine the final length of the railway.

4. A draper decides upon having a spring sale which is to be spread over six days. He disposes of his stock as follows: 1st day, $\frac{1}{12}$; 2nd day, $\frac{1}{10}$; 3rd day, $\frac{3}{20}$; 4th day, $\frac{1}{15}$; 5th day, $\frac{7}{30}$; 6th day, $\frac{1}{45}$. What fraction of his whole stock has he sold at the end of three days, and at the end of the week?

5. Employing the data of Question 15, page 60, find the excess or defect of each year's dividend over the **preceding** one. Place a + sign before the figures when there is an excess, and a − sign when a defect. (Tabulate the result.)

6. Taking 1 inch to represent £1 per cent.[1] dividend (Question 15, page 60), draw rectangles half an inch wide, and of height determined by the amount of the dividend, to represent the latter graphically.

7. Find **from the diagram drawn** in Question 6 the difference between the dividend paid in 1907 and in 1911.

8. The following figures give the number of people employed in the various kinds of mining work in the countries named, for every 100 of the population:

United Kingdom.	United States.	France.	Germany.	Austria-Hungary.	Belgium.	Italy.
$2\frac{1}{5}$	$\frac{4}{5}$	$\frac{9}{10}$	$1\frac{3}{5}$	$1\frac{1}{6}$	$2\frac{8}{9}$	$\frac{13}{35}$

What is the difference between the number of people employed in mining in Belgium and in each of the other countries? Arrange the countries in descending order of the percentage of people engaged in mining.

[1] Section XVIII.

9. From the data given find the increase in length of lines open for each year for the years stated (railways in the United Kingdom):

Year.	Length of Lines Open on 31st December (Thousands of Miles).	Increase.
1906 . .	$22\frac{7}{11}$	
1907 . .	$23\frac{1}{10}$	+
1908 . .	$23\frac{1}{7}$	
1909 . .	$23\frac{7}{25}$	
1910 . .	$23\frac{13}{35}$	
1911 . .	$23\frac{41}{100}$	
1912 . .	$23\frac{4}{9}$	

10. The rateable value of Birmingham in millions of pounds is $4\frac{23}{49}$; of Liverpool, $4\frac{4}{5}$; and of Manchester, $4\frac{39}{56}$. Of these three great provincial cities which has the highest rateable value?

11. In the last question find the amount by which the lowest rateable value of the three cities is less than that of the other two.

12.

ENGLAND AND WALES.

Year.	Population.	Number of Convictions at Superior Courts.
1850	17773324	20537
1912	36539636	11665

What is the population for every one convicted in the two years given above?

13. From the results obtained in Question 12 give, to the nearest whole number, the difference between the number of people for every conviction in 1850 and in 1912, and thence make a statement regarding the increase or decrease of crime in the period given.

C. (3)

SECTION V.

1. On a farm of 480 acres $\frac{3}{5}$ of the land is arable. The farmer plants $\frac{1}{2}$ of this with wheat, allowing 3 bushels to the

acre, $\frac{1}{4}$ of it with barley, allowing $2\frac{1}{2}$ bushels to the acre, and the remainder with other crops. How many bushels of barley and wheat will he require for seed?

2. If $\frac{3}{7}$ of the agricultural land of a district is arable, $\frac{7}{16}$ pasture, and $\frac{2}{5}$ of the remainder woodland, and the remaining 1800 acres are common land, find the number of acres in the whole, and the size of each kind of land.

3. A person has $\frac{7}{9}$ of a property worth £4500. He sells $\frac{1}{5}$ of his share to A and $\frac{2}{3}$ of $\frac{3}{4}$ of the remainder to B. What fraction of the whole property has he left, and what is it worth?

4. If a mile is 1760 yards and there are 3 feet to a yard, what fraction of a mile does a man, whose stride is 33 inches, move in walking a hundred steps?

5. A small tea service consists of a number of articles which cost $6\frac{1}{2}$d. each. The service cost 7s. $0\frac{1}{2}$d. Of how many articles is it composed?

6. The details concerning emigration from the British Isles are as follows:

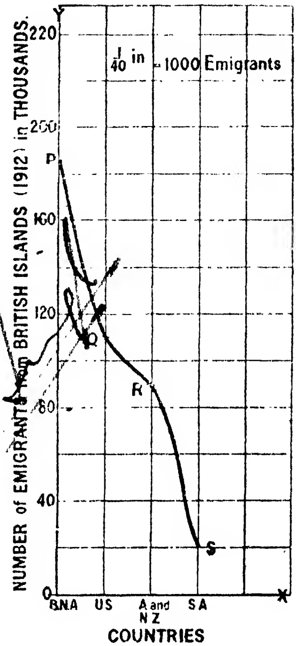
Year.	Destination of Emigrants.				Total.
	To British North America.	To the United States.	To Australia and New Zealand.	To South Africa.	
1912	186150	$\frac{782}{1241}$	$\frac{659}{1241}$	$\frac{188}{1241}$	
		of those emigrating to British North America.			

Find, to the nearest ten, the number of emigrants to the last three countries, and the total number to the nearest hundred.

7. The number of immigrants to the British Isles in 1912 was 340700, to the nearest hundred. Find (1) the excess of the number of emigrants over the number of immigrants; and (2) express the excess of the number of emigrants over the number of immigrants as a fraction of the population of the British Isles, which may be taken as $45\frac{1}{2}$ millions.

8. From the data given in or obtained from Question 6 write the numbers correct to a thousand, and then proceed to represent

the state of emigration graphically thus: Draw two lines, **OX** and **OY** (called axes), at right angles to one another (see figure). At convenient distances ($\frac{1}{2}$ inch) along the horizontal or "X axis" place letters B.N.A. (British North America), U.S., etc. Take $\frac{1}{40}$ inch to represent 1000 emigrants, then 186 thousand requires $\frac{186}{40}$ or $4\frac{13}{20}$ inches. Mark off along the line **OY** this distance, then **P** will represent 186 thousand emigrants to B.N.A. If the number of emigrants to the U.S. were (say) 110 thousand, $2\frac{3}{4}$ inches would be necessary, and **Q** would show the 110 thousand emigrants to the U.S. Taking (by way of illustration) 90 and 20 to represent those to A. and N.Z. and to S.A., the points **R** and **S** represent these numbers. The line **PQRS** shows graphically the emigration to the countries named. The student will put in the proper points required.[1]



9. An auctioneer buys a number of articles and sells them in accordance with the following table: (Bring the prices to shillings.)

Article.	Price the Auctioneer Pays.	The Fraction of his Buying Price (B.P.) that he makes Profit.	His Actual Profit.
1. A clock . . .	84s.	$\frac{5}{14}$	
2. An oil-painting .	£7 10s. 0d.	$\frac{7}{30}$	
3. A water-colour .	£5 5s. 0d.	$\frac{4}{21}$	
4. Three Chippendale chairs . .	£75	$\frac{19}{60}$	
5. A Doulton vase .	18s.	$\frac{5}{12}$	
6. A Crown Derby tea service (part) .	£16 16s. 0d.	$\frac{11}{12}$	
Totals .	C.		P.

[1] This question can be left till a later stage if found too difficult. See Part III. for complete treatment of this important subject.

Find (1) the profit on each article (1–6) in shillings; (2) the article on which his actual profit was greatest; (3) his total profit; (4) the fraction that his total profit (P.) is of the price he gave (C.).

D. (1)

SECTIONS VI.–VIII.

1. The *Victorian Year-Book* for 1912–13 gives the following details for the rainfall in the districts named:

Basin or District.	Third Quarter, 1912.	
	Amount.	Average.
Glenelg and Wannon Rivers . . .	1028 pts.	937 pts.
Cape Otway Forest	1344 ,,	1204 ,,
Mallee	558 ,,	428 ,,

If 100 pts. (points) = 1 inch, tabulate this statement, giving the rainfall in inches correct to $\frac{1}{10}$ inch.

2. In the recent edition of the *Queensland Sugar Industry* we find the following important information:

Year.	To Each Acre Crushed.	
	Tons of Cane.	Tons of Sugar.
1906 . .	17·61	1·88
1908 . .	15·54	1·64
1910 . .	19·45	2·20
1912 . .	12·72	1·45

From the above data find in which year the ratio of the weight of sugar produced to the weight of cane was greatest, and also the yield per ton of cane for that year.

3. If a tierce of West Indian sugar contain 8½ cwt., and it is packed into 40 boxes for sale, what weight, in pounds and the decimal of a pound, will each contain?

4. Write the answer in the last question correct to the nearest pound.

5. A hop-picker in Australia receives 12 shillings for picking 40 bushels of hops, how many pence and the decimal of a penny is that per bushel?

6. In 1912 the weight of fish landed in the United Kingdom and caught by men employed in this country was 24,100,000 cwt., and its value £12,780,000. What did the fish make (in shillings) per cwt., to two decimal places?

7. In March 1913, 9046 cwt. of butter was imported into the United Kingdom from the Argentine Republic, and in March 1914, 11364 cwt. How many times does the latter importation exceed the former, to two places of decimals?

8. The total number of great hundreds (120) of eggs imported into the United Kingdom from all sources in March 1913 was 1,308,521, and in March 1914, 2,095,170. Express the former number as a decimal of the latter, to two places. What information does the result give?

D. (2)

1. According to the statistical abstract of the United States for 1913, the expenditure for the public schools of Alaska from 1902–11 was as follows:

Year ending 30th June.	Total Cost in Thousands of Dollars.
1902 . . .	26·478
1903 . . .	31·487
1904 . . .	41·587
1905 . . .	81·550
1906 . . .	47·706
1907 . . .	56·016
1908 . . .	78·877
1909 . . .	85·135
1910 . . .	112·909
1911 . . .	121·508

Find the total amount expended in the ten years given, correct to 100 dollars.

2. In March 1914 goods to the value shown were sent from the countries named to Canada:

Country (being part of the British Empire).	Value of Goods sent to Canada, in Millions of Dollars.
The United Kingdom . . .	11·1231
British East Indies (*i.e.* India, Ceylon, etc.)	0·8166
British Guiana	0·5793
New Zealand	0·3900
Hong-Kong	0·2354
British West Indies . . .	0·1523
Other parts of the British Empire	

Find (1) the total value in dollars for the figures given; (2) the value sent from the other parts of the British Empire, if the value for the whole Empire was 13·5903 million dollars.

3. A half-sovereign weighs 61·63723 grains when coined, and it is not allowed to remain in circulation after its weight has become 61·125 grains. What weight may be lost before the coin is withdrawn?

4. The price per pound of exported Canadian cheese in 1911 was 5·472 pence, and of Australian cheese 7·5 pence. Find by how much the latter is the dearer.

5. From the data of Question 1 find the average amount spent per annum in £'s sterling, 1902–1911, reckoning 4s. to the dollar.

6. The weight of wheat in millions of hundredweights imported into the United Kingdom during the first week of the years 1909–13 is given below:

1913.	1912.	1911.	1910.	1909.
2·3013	1·8008	1·7999	1·9378	1·7047

Find, in tons, the average weight imported.

D. (3)

1. The value of a mark (silver coin) is 11·7483 pence, and of a franc 9·513 pence. Find the number of francs which are worth 100 marks.

2. A gold florin or gulden of the Netherlands is worth 19·824 pence, how many marks are worth the same as 25 gulden

3. A quantity surveyor charges £2 5s. on every £100 in the value of a contract. If a hospital cost £76585, what amount does he receive, if he allows £0·75 on every £100, by reason of his being a member of the governing committee?

4. What is the cost of 186·85 gallons of lubricating oil at 10·75 pence per pint?

5. How many gallons of petrol at 1s. 6d. a gallon can be bought for £25·86?

6. How many pounds of lump sugar at $3\frac{3}{4}$d. a lb. can be bought for half a guinea?

7. If the price of sugar rises to $4\frac{1}{2}$d. a lb., how many fewer pounds can be bought for half a guinea than in the previous question?

8. In the year 1912–13 no fewer than 891,950,285 letters and post cards were delivered in India. Taking the population of that Empire as 315,156,396, calculate to two places of decimals the average number of letters per head of the population.

9 Matches manufactured in Northern Rhodesia have to pay a duty of 6d. per gross on boxes or packages not containing more than 100 matches in a box or package. Find the duty per single box as the decimal of a penny.

D.[1] (4)

1. Find the value of the following, to four significant figures: (*a*) $5{\cdot}933 \times 3{\cdot}284$; (*b*) $5{\cdot}933 \div 3{\cdot}284$.

2. Compute the value of the following, by contracted methods, to four significant figures · (*a*) $0{\cdot}5584 \times 0{\cdot}4813$; (*b*) $0{\cdot}5584 \div 0{\cdot}4813$.

3. Having given that to five significant figures $\sqrt{2} = 1{\cdot}4142$ and $\sqrt{3} = 1{\cdot}7321$, find by contracted multiplication and division $\sqrt{6}$ and $\sqrt{1{\cdot}5}$ as accurately as the data will allow.

4. Find, correct to three significant digits, the value of $0{\cdot}23045 \times 15{\cdot}8476$.

5. Multiply $40{\cdot}1870378$ by $2{\cdot}149173945$ by a contracted method, correct to the seventh decimal place.

6. Find the product of $246{\cdot}9254$ and $0{\cdot}008729$, to three decimals.

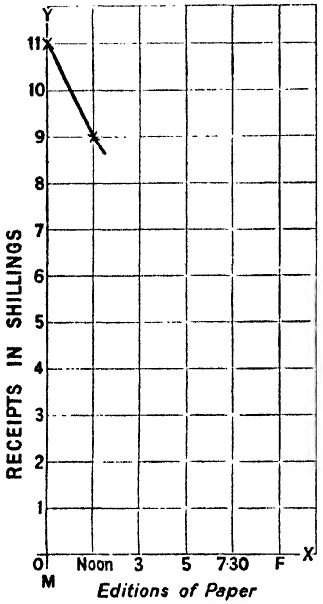
E. (1)

SECTION IX.

1. Employing the data given in Question 17, page 127, write down the amount of the newspaper seller's receipts for Monday, correct to one shilling. Represent graphically (taking $\frac{1}{2}$ inch for

[1] See also Examples XXII. (a) and XXII. (b).

1s.) his day's takings, setting off points along **OX**, half an inch apart, to represent the editions of the paper. (The graph is partly drawn in figure.)



2. The mean price of bread in England and Wales in 1910 was 5·79 pence per 4 lb. loaf, and in 1911, 5·57 pence. Express the difference between these prices as the decimal of a shilling.

3. A patent fire-resisting safe costs £8 16s., and a fire-and-burglar-resisting safe costs £18 3s.; express the difference between these prices as a fraction of the higher price.

4. Employing the data of the last question, find, by reducing the prices to shillings, how many times the higher-priced safe is dearer than the lower-priced one.

5. How much would it cost you to build and equip a greenhouse, given that cost of timber is £8 8s. 6d.; glass, £2 10s. 6d.; screws, nails, etc., 5s. 6d.; paint, 7s. 8d.; cement and sand, 15s. 9d.; heating apparatus, £3 12s. 6d.; door and window fittings, 4s. 9d.; earthenware pots, £4 15s.; plants, £7 18s. 9d.; water cans, syringe, etc., 8s. 9d.; putty and odds and ends, 3s. 8d.; thermometer (maximum and minimum), 4s. 6d.?

6. SEEDS FOR AUTUMN SOWING.

Begonia	3	packets	@ 1s. 3d.	per packet.
Calceolaria	2	,,	@ 1s. 6d.	,, ,,
Carnation	2	,,	@ 1s. 8d.	,, ,,
Cineraria	4	,,	@ 9d.	,, ,,
Gloxinia	3	,,	@ 1s. 6d.	,, ,,
Phlox	5	,,	@ 4d.	,, ,,
Silene pendula	4	,,	@ 3d.	,, ,,
Wallflower	12	,,	@ 3½d.	,, ,,

What is the cost of this collection of seeds after allowing a discount of 1d. in every complete shilling?

7. A draper's assets are £58964 18s. 10d., and his liabilities

are £49764 19s. 11d. What is the excess of the former over the latter? Taking the difference correct to £1, what would the draper have had in hand if he had placed half of it in his bank, given away $\frac{1}{30}$th, and used $\frac{1}{10}$th on his own personal expenses? (Give the answer in £'s and the decimal of a £, to two places.)

8. A grocer's position is as follows:

1915.		Receipts.	Expenditure.
July 1.	Cash in hand, £565		
,, 2.	Received for goods sold, £25 15s. 8d.		
,, 3.	,, from Tom Jones, £96 18s. 6d.		
,, 3.	Allowed him discount,[1] £3 10s. 6d.		
,, 4.	Received from P. Owen, £198 12s. 8d.		
,, 5.	Paid rent due Midsummer, £50 10s.		
,, 6.	Paid for goods received, £105 15s. 8d.		
,, 6.	Discount allowed me, £6 5s. 3d. .		

Fill in the amounts in the proper columns, total them, and find the balance.

9. The duty on wine imported into the United Kingdom, exceeding 30 degrees and not exceeding 42 degrees of proof spirit, is 3s. per gallon. The duty on wine containing 41 per cent. of proof spirit imported into the Australian Commonwealth is 14s. per gallon. By how much is the duty on 36 gallons of wine imported into the United Kingdom less than that on 40½ gallons imported into Australia?

E. (2)

1. The following is an annual account of a small business concern; find the total of the amounts given: wages, £325; wages and overtime work, £88 14s. 5d.; water rate, £10 5s.; gas fittings, £5 12s. 10d.; gas and electric light, £30 15s. 8d.; coal, £12 3s. 8d.; taxes, £1 18s. 9d.; insurance, £8 16s. 7d.; printing, postage, etc., £15 10s.; bank interest on loan, £5 7s. 6d.

2. By what amount does 800 guineas exceed the total in the last question?

[1] See Section XXI.

3. Give some account of what is meant by the Latin Union, and show in what ways it facilitates commerce.

4. Give an account of the chief coins used in France, the United States, and India; what is their approximate value in English money?

5. The income of a society is given below; fill in the spaces and find the total income.

Branch Number.	Entrance Fees.	Current Year Subs. (1912–13).	Arrears Subs. (1911–12).	Subs. Paid in Advance (1913–14).	Total.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
1 . .	1 15 0	51 18 9	15 7 6	1 2 6	
2 . .	3 7 6	54 15 0	17 8 9	1 2 6	
3 . .	...	5 5 0	...	...	
4 . .	1 7 6	40 10 0	2 12 6	0 7 6	
5 . .	2 0 0	32 16 3	3 7 6	0 7 6	
6 . .	0 10 0	13 2 6	0 7 6	...	
7 . .	1 5 0	29 16 3	0 7 6	...	
8 . .	...	2 12 6	0 15 0	...	
9 . .	0 2 6	4 2 6	...	...	
10 . .	1 7 6	13 2 6	0 3 9	0 15 0	
11 . .	1 2 6	12 0 0	...	...	
12 . .	3 0 0	17 17 6	0 15 0	0 7 6	
13 . .	1 10 0	4 10 0	...	...	
14 . .	0 7 6	12 0 0	1 2 6	...	
15 . .	0 15 0	...	...	...	
16 . .	0 10 0	2 5 0	...	...	

6. Find by how much the totals in the columns of the last sum are respectively less than £19 15s. 6d.; 300 guineas; £43 8s. 9d.; and £5.

7. The entrance fees paid to a society were as follows: £1 15s.; £3 17s. 6d.; £1 7s. 6d.; £2; 10s.; £1 5s.; 2s. 6d.; £1 7s. 6d.; £1 2s. 6d.; £3; £1 10s.; 7s. 6d.; 15s.; 10s. What was the total amount paid?

8. In the previous question find by how much the total exceeds the sum of the first seven amounts.

9. From the following data find, to the nearest farthing, the

average amount of the rates levied in the London cities and boroughs in 1912–13 (from *Whitaker's Almanack*):

Cities and Boroughs.	Rates, 1912–13.		Cities and Boroughs.	Rates, 1912–13.	
	s.	*d.*		*s.*	*d.*
Westminster . . .	6	10	Holborn	8	0
Battersea	8	4	Islington	7	10
Bermondsey . . .	9	4½	Kensington . . .	7	0
Bethnal Green . .	8	0	Lambeth	7	11
Camberwell . . .	8	10½	Lewisham	7	10
Chelsea	7	4	Paddington . . .	7	7
Deptford	8	0	Poplar	11	7
Finsbury	7	4	St. Marylebone . .	7	3
Fulham	7	8½	St. Pancras . . .	7	7
Greenwich	8	2	Southwark . . .	7	10¾
Hackney	7	11	Stepney	8	10½
Hammersmith . .	7	10	Stoke Newington .	7	9
Hampstead . . .	7	8	Woolwich	8	11

F. (1)

SECTION X.

1. A clerk earns 18s. 8d. a week, how long will he take to earn £100 16s.?

2. If the clerk (Question 1) saves 1s. 9d. a week, how long will it take him to save £9 12s. 6d.?

3. What decimal of the clerk's earnings is saved?

4. It costs £7 7s. to have the second page of the cover of a particular paper for advertisement purposes, and £3 15s. 6d. for half a page. How many times dearer would it be to have a whole page at the half-page rate than it is to have it at the whole-page rate?

5. Last year 18,186,244 great hundreds of eggs were imported into England at a cost of £7,235,416. How many eggs, on an average, would that be for a shilling? (A great hundred = 10 dozen.)

6. A boilermaker in England can earn 38s. a week, and in New South Wales 1s. 4½d. an hour. If a man in New South Wales works 8½ hours a day for six days a week, does he earn more or less than his fellow-craftsman in England, and how much?

7. The roof and front of a studio have to be glazed with glass costing 4¼d. a square foot. The total area is 198½ square feet, what will be the cost of glazing?

8. Taking the data of the previous question, and allowing $2\frac{3}{4}$ days of eight hours each for the work, what would a glazier earn at $11\frac{1}{2}$d. an hour?

9. Find the total cost of glazing and of labour, adding on 7s. 6d. for putty and odds and ends. (Questions 7 and 8.)

F. (2)

1. How many pairs of half-hose at 1s. $0\frac{1}{2}$d. a pair can be paid for with a sovereign, and what change will there be?

2. A Government department places an order for nine fire-resisting safes (36 × 21 × 18 in. internal measurements) at £25 2s. each (with inside fittings), and for twelve fire-and-burglar-resisting safes (40 × 20 × 16 in.), with fittings at £44 11s. each. What cheque must be drawn to pay the account?

3. A man has held a railway reason ticket from 1st January to 31st March this year,[1] and it cost him 22s. 6d. If he had not a season ticket he would have had to pay 3d. each way each week-day. How much has the season ticket saved him?

4. Potatoes in Sydney (New South Wales) make 7s. 6d. per cwt., and in Melbourne (Victoria) 4s. 5d. per cwt. How much does the cost per pound in the former exceed that in the latter?[2]

5. What is the difference in the cost of a ton of potatoes in Sydney and in Melbourne?

6. If it costs 2s. 6d. a day to feed a soldier on active service, what does it cost per week (7 days) to feed a million and a half men?

7. If a soldier uses 53 rounds of ammunition a day, and 100 rounds cost 7s. $4\frac{1}{2}$d., find the cost per week of 120 men. (Reckon 7 days.)

8. Linoleum for the floor of a hall costs 3s. 11d. a square yard, and the total cost of covering the floor, exclusive of labour and waste, is £126 18s. Find the area of the hall in square yards.

9. 240 14-lb. tins of French coffee cost £192, what is the price per tin?

10. What is the price per pound in the last question?

11. 1 gross reams of emery cloth cost £120. What is the price per quire? (24 sheets = 1 quire, and 20 quires = 1 ream.)

12. 20 dozen lb. Tunis dates cost £7. What is the price per pound?

13. $16\frac{1}{2}$ dozen tins cods' roes cost £6 3s. 9d., what is the price per tin?

[1] Take 1915, and reckon ninety days.
[2] Circular No. 2: *Australia and New Zealand* (Emigrants' Information Offices).

14. 400 bottles of peaches cost £40 16s. 8d., what is the price per bottle?

F. (3)

1. A man buys a halfpenny paper on his way to town every morning (but Sundays). What decimal of £1 does he spend in a week, and in a quarter (thirteen weeks)?

2. The *Daily News* costs $\frac{1}{2}$d. a copy, and the *Strand Magazine* $4\frac{1}{2}$d. a copy. Express the first price as a decimal of the second.

3. The price of candles in Melbourne (New South Wales) (according to official returns for 1914) averaged $6\frac{3}{4}$d. per lb., and in Adelaide (South Australia) $7\frac{1}{2}$d. per lb. Express the cost per pound in the former city as the decimal of that in the latter.

4. How many pounds of candles in Melbourne will cost the same as 36 lb. in Adelaide?

5. Packages containing jewellery and other valuables can be insured against loss in being sent from Great Britain to foreign countries. A payment of 4d. insures a package for £12; a payment of 5s. 10d. insures for £400. Find by what decimal of a shilling the premium necessary to insure for £1 at the former rate exceeds the premium necessary to insure for £1 at the latter rate.

6. The import duty on cocoa (United Kingdom) is 1d. per lb., and on coffee 14s. per cwt.[1] Express the difference between the duty on $1\frac{1}{2}$ cwt. of these articles as the decimal of £1.

7. If £1 were put into the Post Office Savings Bank [2] and left there for fifty years it would have amounted to £3·4371. How much would have been added to the original amount (to the nearest farthing)?

8. 1,430,000 crans of herrings (about 1000 herrings to a cran) were caught, during the season, off the coast of Scotland. Their value is estimated at £3,500,000. At this rate, how many herrings would be worth half a crown?

9. From 1903 to 1912 (both inclusive) the lowest value of silver per Troy ounce [3] in London was $23\frac{11}{16}$d. (in 1909), and the highest, $30\frac{7}{8}$d. (in 1906). If a merchant had bought a quantity of the metal in 1906 and sold it in 1909, what decimal of a shilling would he have lost on every ounce he purchased?

F. (4)

1. The rateable value of Birmingham is £4,463,404, and its population 850945, find to the nearest farthing the rateable value per family of five persons.

[1] Previous to the Budget of 1915.
[2] Section XX.
[3] § 140.

2. Take 147 tons of tin oxide as worth £16823, and find to the nearest farthing the value per pound. (2240 lb. = 1 ton.)

3. In one year we imported 4,257,195 cwt. of butter, of the value £24,109,862. Find, to the nearest farthing, the value per pound.

4. In the year 1913 Austria-Hungary exported to Canada 40189 lb. of sugar, the value of which was £375 10s. in English money. Calculate to the nearest farthing the value per pound.

5. The following abstract is taken from the returns of an important company :

Class of Passenger.	Number.	Receipts.	Average Fare per Passenger.
Ordinary—		£	
1st class . . .	1496304	261281	
2nd ,, . . .	2191477	271581	
3rd ,, . . .	45707047	2019618	
Workmen . . .	9332134	89775	

Fill in the last column in pence and the fraction of a penny.

6. Taking the details of the last question, find the average fare paid per passenger in the four sections taken together.

7. The Government of Russia is proposing to spend 322,500,000 roubles on the development of the Amur Railway, including the construction of a bridge over the river. Express the capital expenditure in that project as a decimal of the capital of an English railway company, namely, £35,000,000, given that a rouble is worth 2s.

8. Leeds to London (St. Pancras) is 196 miles; London to Paris, *via* Victoria, Newhaven, and Dieppe, 224 miles. The fares are 15s. 5d. and 19s. 5d. respectively. Calculate the average cost per mile on the two journeys to the nearest tenth of a penny.

SECTION XI

WEIGHTS AND MEASURES

REDUCTION

A. Historical Introduction: English and Metric Systems

128. In our last section we dealt with money, which may be called a measure of value, and we now proceed to consider the measures of length, area, volume, weight, quantity, and time in succession.

A merchant has taken new premises and wants to know whether his old counter will go into his new shop. It would be awkward to remove the counter to the shop to try whether it would fit, so he takes a stick of convenient length,—say as long as his arm,—finds the counter is 3½ times the length thereof, walks to his new premises and sees whether 3½ times the length of the stick will serve his purpose.

Or again, think of the fourteenth century, of the time of Chaucer, or even of an earlier time still, and you will recall that wool, produced in Yorkshire, led to the manufacture of woollen cloth, and to the development of a large trade with the Continent.

Now suppose an English merchant had a roll of cloth, and a Flemish merchant of the time wished to buy it; he would want to know how much cloth there was. The English merchant might then do just as thousands of housewives do to-day—take the end of the cloth in his right hand, stretch out his arm and, with his left hand, hold the cloth to his chin, and thus measure off an arm's-length of cloth; then the whole bundle might be "25 arm's-lengths," which the Flemish merchant would buy for so much. As a matter of fact the arm's-length is very nearly a yard, although we are told that that measure originated in Egypt. We are not concerned here with debating archæological questions, but this we do say, that, be the yard Egyptian in origin or not, English merchants would never have used it unless it had been convenient to them for measuring their cloth.

We might note that the arrows of the Middle Ages were about a yard long too, clearly because had they been 2 yards long our archers could not have shot them. In a similar way we have a "foot."

129. Fishermen measure the lengths of their boats, ropes, nets, and the like, in fathoms, because a tall man can just reach that distance from tip to tip of his fingers when his arms are stretched out. A cable's length is 100 fathoms, or 600 feet. The height of horses is measured in "hands" (4 inches), and long ago men used their hands to measure off a span, that is, 9 inches.

It is obvious that our English system of measurement is based entirely on convenience and rapidity of commercial working, *as it was regarded by merchants of the past.* It has no scientific basis whatever, and is not a simple decimal system.

130. We now take, for example, the English Lineal measure.

LINEAR MEASURE.

12 inches (in.) = 1	foot (ft.)
3 feet (ft.) = 1	yard (yd.)
22 yards = 1	chain (ch.)
or 100 links = 1	,, ,,
10 chs. = 1	furlong (fur.)
or 40 poles (po.) = 1	,, ,,
8 fur. = 1	STATUTE MILE (mi.)
hence 220 yds. = 1	fur.
1760 yds. = 1	mi.
5280 ft. = 1	,,
63360 in. = 1	,,
but, 6080 ft. = 1	NAUTICAL MILE
and 1 KNOT = 1	,, ,, per hour.

A KNOT is NOT a distance, but a speed, *e.g.* "A T.B.D. makes 30 KNOTS"—that is, a Torpedo-Boat Destroyer steams 30 *nautical miles* per hour. In giving distances from port to port it is necessary to say whether they are expressed in nautical or in statute miles, and it is convenient to remember that

33 KNOTS = 38 *statute* miles per hour (nearly).

131. We have said that our standard of length was chosen for convenience, but we have no guarantee that all merchants will have arms of the same length, and can imagine that considerable defrauding of the public might result, hence the standards for our weights and measures had to be fixed by law.[1]

King Henry I. decided that the length of the yard should be that of *his* arm—a somewhat arbitrary decision perhaps—and a little later King John signed Magna Carta, which decreed that there should be one system of weights and measures throughout the country. In spite of Magna Carta, confirmed several times, we find different standards (of weight particularly) in this country.[2]

[1] Our "Standards" were kept at Winchester before the Conquest, but now they are to be found at the Standards Office, Old Palace Yard, Westminster. The YARD is the distance between the centres of two brass plugs embedded in a bar of bronze, and the POUND Avoirdupois is the weight of a particular piece of platinum.

[2] § 140.

132. Now an ideal system[1] of weights and measures should be—

1. Uniform.
2. Accurate.
3. Easily understood and used.
4. Widely known.
5. Simple in calculation

Our system can certainly not lay claim to any but No. 2, and in a lesser degree to No. 4, but a decimal system satisfies all the conditions, and chief among them, from the student's standpoint, is the last!

THE METRIC SYSTEM

133. The Metric System is a delightfully easy decimal system based upon a standard called the METRE, which has multiples in tens and subdivisions in tenths, the former being denoted by Greek prefixes and the latter by Latin prefixes, thus:

			Length.	*Capacity.*		*Weight.*
Fractions:	$\frac{1}{1000}$	**milli-**	**metre,**	**litre,** (§ 143)	**or**	**gram.** (§ 143)
	$\frac{1}{100}$	**centi-**	,,	**litre,**	**or**	**gram.**
	$\frac{1}{10}$	**deci-**	,,	,,	,,	,,
Multiples:	**10**	**Deca-**	**metres,**	**litres,**	**or**	**grams.**
	100	**Hecto-**	,,	,,	,,	,,
	1000	**Kilo-**	,,	,,	,,	,,

Hence the METRIC TABLE OF LENGTH is—

$\frac{1}{1000}$ or 0·001 metre	= 1 millimetre	(mm.)
$\frac{1}{100}$ or 0·01 ,,	= 1 centi- ,,	(cm.)
$\frac{1}{10}$ or 0·1 ,,	= 1 deci- ,,	(dm.)
1 ,,	= 1 m.	
10 metres	= 1 Decametre	(Dm.)
100 ,,	= 1 Hecto- ,,	(Hm.)
1000 ,,	= 1 Kilo- ,,	(Km.)

Note the contractions used, *e.g.* dm. and Dm., etc. The Decametre and Hectometre are not frequently used. The metre will be employed just as we employ the yard, the cm. as the inch, the mm. particularly in scientific work, and the Km. as miles.

134. In determining the length of the metre the French Government took very great care. They had a standard called

[1] *The Coming of the Kilogram*, by H. O. Arnold-Forster.

the "toise[1] of Peru," which had been used to measure an arc of the meridian[2] in Peru, and was employed to measure an arc between Dunkirk and Barcelona, from which the length of a quadrant of the earth's circumference from the pole to the equator was calculated. This length was divided into 10 million parts, and each was called a metre. It was found subsequently that the original measurement was not quite right, and so the metre is not exactly what it was thought to be. It is now defined as the length of a platinum bar, called the "mètre des Archives," which is kept in Paris and regarded as the standard by all the countries which employ the metric system. The United Kingdom and some British Colonies and Dependencies, the United States, and Russia are the chief countries in which the metric system is NOT employed.[3]

135. It is obvious that the metric system complies with conditions 1–4, § 132, and we now show that it satisfies condition 5 far better than the English system can do.

EXAMPLE 1.—(1) **Reduce 9 mi. 6 fur. to fur.**

9 mi. 6 fur.
8
78 fur.

Since 1 mi. = 8 fur.
9 „ = 72 „
and 72 fur. + 6 fur. = 78 „

Where there is no simple relationship between the numbers 9, 6, and the answer 78.

(2) **Reduce 9 Km. 6 Hm. to Hm.**

9 Km. 6 Hm.
10
96

Where the result 96 contains exactly the same numbers 9, 6, as were given.

EXAMPLE 2.—(1) **Reduce 796 chs. to yds.**

Since 22 yds. = 1 ch.
796 chs. = 796 × 22 yds.
= 17512 yds.

And again 796 bears no obvious relationship to 17512.

(2) **Reduce 796 Dm. to m.**

796 Dm. = 7960 m.
Since 1 Dm = 10 m.

Where 796 does bear a simple and obvious relationship to 7960.

[1] A toise = 6·39459 feet.

[2] If you imagine a circle to be drawn on the surface of the earth, with the centre of the earth as its centre, passing through Dunkirk and Barcelona, then the part of that circle running from one town to the other through France and Spain is called "an arc of the meridian."

[3] The metric system is employed universally for all scientific work, but NOT for all engineering work.

EXAMPLE 3. — (1) **Express 63360 in. in chs.**

$$\begin{array}{r|l} 12 & 63360 \text{ in.} \\ 3 & \;\;5280 \text{ ft.} \\ 22 & \;\;1760 \text{ yds.} \\ & \;\;\;\;\;80 \text{ chs.} \end{array}$$

$\therefore$ 63360 in. = 80 chs.

The method here is

63360 ÷ 12 to bring inches to feet.
5280 ÷ 3 to bring feet to yards.
and 1760 ÷ 22 to bring yards to chains.

(2) **Express 63360 cm. in Dm.**

63360 cm.
= 633·60 m. (dividing by 100)
= 63·360 Dm. (dividing by 10)

And with very little practice we can write the result at once.

The student will observe at once that any reduction in the English system means long and tedious working, but the corresponding reduction in the metric system involves moving the decimal point.

136. We give further illustrations of the metric system first, and then of the English system.

1. THE METRIC SYSTEM

EXAMPLE 4.—(a) **Reduce 8 dm. 5 cm. 9 mm. to mm.**

We write 800 mm. + 50 mm. + 9 mm., or 859 mm. as the answer.

(b) **Express 25 Km. 300 m. in Km.**

$$\text{We write } 25 \text{ Km.} + \frac{300}{1000} \text{ Km.} = 25{\cdot}3 \text{ Km.}$$

(c) **Reduce 8 Hm. 6 m. 5 cm. to mm.**

We write 8 Hm. **0** Dm. 6 m. **0** dm. 5 cm. **0** mm, and then 806050 mm.

Note very carefully that we must here, and in all similar cases, put in the quantities left out in the question, namely, 0 Dm. 0 dm. and 0 mm.

EXAMPLE 5.—**Reduce 158653 cm. to Km., m., etc.**

Here we have—

Km.	Hm.	Dm.	m.	dm.	cm.
1	5	8	6	5	3

and so we write—

1 Km. 5 Hm. 8 Dm. 6 m. 5 dm. 3 cm.

which is the same as 1·58653 Km.; or 15·8653 Hm.; or 158·653 Dm.; or 1586·53 m.; or 15865·3 dm.; or 158653 cm.

2. THE ENGLISH SYSTEM

137. EXAMPLE 6.—**Reduce 15 mi. 3 chs. 2 ft. to inches.**

We have really

15 mi. **0** fur. 3 chs. **0** yds. 2 ft. **0** in.

$$\begin{array}{rl}
15 & \\
8 & \\
\hline
120 \leftarrow & \text{fur. } (i.e.\ 15 \times 8 + 0) \\
10 & \\
\hline
1203 \leftarrow & \text{chs. } (i.e.\ 120 \times 10 + 3) \\
22 & \\
\hline
26466 \leftarrow & \text{yds. } (i.e.\ 1203 \times 22 + 0) \\
3 & \\
\hline
79400 \leftarrow & \text{ft. } (i.e.\ 26466 \times 3 + 2) \\
12 & \\
\hline
\underline{\underline{952800}} \leftarrow & \text{in. } (i.e.\ 79400 \times 12 + 0)
\end{array}$$

EXAMPLE 7.—**Reduce 476400 in. to miles, etc.**

$$\begin{array}{r|l}
12 & 476400 \text{ in.} \\
\hline
3 & 39700 + 0 \text{ in. (over)} \\
\hline
22 & 13233 + 1 \text{ ft. (over)} \\
\hline
10 & 601 + 11 \text{ yds. (over)} \\
\hline
8 & 60 + 1 \text{ ch. (over)} \\
\hline
 & 7 \text{ mi.} + 4 \text{ fur. (over)}
\end{array}$$

$\therefore$ 476400 in. = 7 mi. 4 fur. 1 ch. 11 yds. 1 ft.

DECIMALISATION

138. EXAMPLE 8.—**Reduce 5 mi. 3 fur. 6 chs. 10 yds. 2 ft to miles and the decimal of a mile, to two places.**

$$10 \text{ yds. } 2 \text{ ft.} = 10\tfrac{2}{3} \text{ yds.} = 10{\cdot}667 \text{ yds.}$$

$$6 \text{ chs. } 10{\cdot}667 \text{ yds.} = 6 \text{ chs. } \frac{10{\cdot}667}{22} \text{ ch. (for 22 yds.} = 1 \text{ ch.)}$$

$$= 6{\cdot}485 \text{ chs.}$$

$$3 \text{ fur. } 6{\cdot}485 \text{ chs.} = 3 \text{ fur. } \frac{6{\cdot}485}{10} \text{ ch. (for 10 chs.} = 1 \text{ fur.)}$$

$$= 3{\cdot}649 \text{ fur.}$$

$$5 \text{ mi. } 3{\cdot}649 \text{ fur.} = 5 \text{ mi. } \frac{3{\cdot}649}{8} \text{ fur. (for 8 fur.} = 1 \text{ mi.)}$$

$$= 5{\cdot}456 \text{ mi.}$$

$$= 5{\cdot}46 \text{ mi., to two places.}$$

Note.—On reducing this result back to a compound quantity the student will *not* get the original length, for we have not left enough decimal places, and so the yards and feet would be wrong.

EXAMPLE 9.—**Express 3·251346 miles as a compound quantity.**

(mi.)	**3**·251346	Multiply the **decimal** by 8 to get furlongs.
(fur.)	**2**·010768	„ „ „ „ 10 to get chains.
(ch.)	**0**·107680	„ „ „ „ 22 to get yards.
(yds.)	**2**·368960	„ „ „ „ 3 to get feet.
(ft.)	**1**·106880	„ „ „ „ 12 to get inches.
(in.)	**1**·282560.	

The quantity required is—

3 mi. 2 fur. 0 ch. 2 yds. 1 ft. 1 in., to the nearest inch.

The student might be interested to see there effect of dropping two of the original decimal places, thus—

(mi.)	**3**·2513
(fur.)	**2**·0104
(ch.)	**0**·1040
(yds.)	**2**·2880
(ft.)	**0**·8640
(in.)	**11**·3680

which shows that the "inches" of the result are altered if we take 4, instead of 6, places.

English and Metric Tables of Weights and Measures.

139. The student should master the following tables very thoroughly before proceeding to subsequent sections which will illustrate their application to commerce.[1]

LINEAR MEASURE (see §§ 130 and 133).

SUPERFICIAL MEASURE (AREAS).

English.		*Metric.*	
144 square in.	= 1 sq. ft.	100 sq. mm.	= 1 sq. cm.
9 sq. ft.	= 1 sq. yd.	100 sq. cm.	= 1 sq. dm.
4840 sq. yds.	= 1 acre (ac.)	100 sq. dm.	= 1 sq. m.
or 40 sq. poles	= 1 rood (rd.)	100 sq. m.	= 1 sq. Dm. or 1 ARE.
4 roods	= 1 ac.	100 ARES	= 1 HECTARE (HA.)
640 ac.	= 1 sq. mi.		
also			
10000 sq. links	= 1 sq. ch.		
10 sq. chs.	= 1 ac.		

Note.—Surveyors largely use the chain for measuring land, but the area is often stated as (*e.g.*) 5 acres 3 roods 10 poles when offered for sale. An acre was reckoned to be the area that a horse could plough in one day, and since 10 square chains = 1 acre, a piece of land 10 chains long and 1 chain wide is 1 acre in area; *i.e.* a field 220 yards by 22 yards is an acre. An acre is roughly a square whose sides are 69·5 yards long.

The cost of glazing, whitewashing, etc., paving, slating, carpentry, and joinery is estimated per sq. foot; of plastering, carpeting, and papering rooms, per sq. yard.
1 rod of brickwork = 272 sq. ft.

CUBIC MEASURE (VOLUMES).

(Examples XXXV. and Section XIV.)

English.		*Metric.*	
1728 cub. in.	= 1 cub. ft.	1000 cub. mm.	= 1 cub. cm. (c.c.).
27 cub. ft.	= 1 cub. yd.	1000 cub. cm.	= 1 cub. dm.
		1000 cub. dm.	= 1 cub. m.
		1 cub. m.	= 1 stere, and so on.

[1] Drill examples on weights and measures will be found in Examples XXXIII. to XXXV., simple problems in Examples XXXVI. and XXXVII., and a complete set of more difficult problems of an interesting character in Sections XII. to XVI.

The following frequently occur:

A load of "square" timber .	= 50 cub. ft.
" " "round" " .	= 40 " "
" " earth	= 27 " "
A shipping ton [1] (merchandise)	= 40 " "
" " (timber) .	= 42 " "
" " (registered) .	= 100 " "
A stack of wood . . .	= 108 " "

The "Petrograd Standard Hundred" is becoming the usual method of estimating timber, and it consists of 120 pieces 12 ft. × 11 in. × $1\frac{1}{2}$ in., or of 165 cub. ft. Timber merchants refer to it as the "P.S.H."

WEIGHT.[2]

140. Our forefathers took "a grain of wheat from the middle of the ear," dried it, and used it as a weight of one grain; hence we have

TROY WEIGHT.

24 grains	= 1	pennyweight (dwt.)
20 dwt.	= 1	ounce (oz.)
12 oz.	= 1	pound (lb. T.)
5760 grains	= 1	" " "

In 1420, Henry V. and the King of France signed the Treaty of Troyes at a town of that name in France which had been of some commercial importance for many years. Possibly "Troy" is derived from "Troyes." In 1758 the pound Troy became the Imperial Standard Pound from which all other weights have been made. Troy weight is used exclusively for precious metals and for gems.

Pure gold is of fineness 24 carats, and 22-carat gold means that there are 22 parts pure gold out of 24.

1 gold carat	= 240 grains T.
but, 1 diamond carat	= $3\frac{1}{6}$ " "
and, 1 pearl grain	= $\frac{4}{5}$ " "

[1] Shipowners reserve to themselves the right to ship goods either by the ton "weight" or ton "volume." If they take iron they will quote per ton weight; if large but light things, per ton volume.

[2] It is unnecessary to repeat here the multiples and subdivision of the gram, § 133.

AVOIRDUPOIS WEIGHT.

(*Averia* = coarse goods, and *poid* = weight.)

This weight is used for the heavy articles of everyday life, and 1 lb. means 1 lb. Av. unless otherwise stated.

16 drams	= 1 oz.
16 oz.	= 1 lb.
14 lb.	= 1 stone
8 lb.	= 1 stone (**dead meat**)
28 lb.	= 1 quarter (qr.)
112 lb. or 4 qrs.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton
100 lb.	= 1 cental
2240 lb.	= 1 long ton
2000 lb. or 20 centals	= 1 short ton
7000 grains	= 1 lb. Av.

Since 5760 grains = 1 lb. T. and 7000 grains = 1 lb. Av.

∴ 1 *lb. of gold,* weighed in T. weight, *is lighter than* 1 *lb. of feathers,* weighed in Av. weight.

1 oz. (Av.) = 437·5 grs., and 1 oz. (T.) = 480 grs.,

∴ 1 oz. of gold is heavier than 1 oz. of feathers.

Chemists buy and sell by Avoirdupois weight, but make up prescriptions by Apothecaries' weight, while we use the corresponding fluid measure for photographic work, although, happily, the metric system is largely used.

APOTHECARIES' WEIGHT.

20 grs.	= 1 scruple (scr. or ℈)
3 scr.	= 1 dram (ʒ)
8 drms.	= 1 oz. (Tr.) (℥)
12 oz.	= 1 lb. (Tr.)

APOTHECARIES' FLUID MEASURE.

60 minims (min.)	= 1 dm.
8 drms.	= 1 oz.
20 Oz.	**= 1 PINT.**
8 pts.	= 1 imperial gallon.

141. MEASURE OF QUANTITY OR CAPACITY.

For corn, potatoes, etc. / For liquids	4 gills[1]	= 1 pint (pt.)
	2 pts.	= 1 quart
	4 qts.	= 1 gallon
	2 galls.	= 1 peck
For corn, potatoes, etc.	4 pks.	= 1 bushel
	8 bush.	= 1 quarter
	36 ,,	= 1 chaldron
	40 ,,	= 1 load
	80 ,, or 10 qrs.	= 1 last

[1] Or noggins or quarterns.

142. We have treated reduction from the standpoint of length up to the present in order to keep our ideas fixed, but the student will understand that the METHOD applies to all reduction, and facility in working can be secured by learning the tables by heart. We now illustrate addition and subtraction of weights and measures by reference to other tables.

EXAMPLE 10.—**What is the tare of three trucks which weigh respectively 5 tons 3 cwt. 2 qrs.; 6 tons 3 qrs.; 8 tons 18 cwt. 3 qrs.?**

(1) Tons.	(2) Cwt.	(3) Qrs.
5	3	2
6	0	3
8	18	3
20	3	0

Col. (3) = 8 qrs.
= 2 cwt. **0** qr.

" (2) = 21 cwt.
and 21 cwt. + 2 cwt.
= 23 cwt.
= 1 ton **3** cwt.

" (1) = 19 tons
and 19 tons + 1 ton
= **20** tons.

∴ tare is 20 tons 3 cwt. 0 qr.

EXAMPLE 11.—**What is the difference between 2 pks. 1 gall. 3 qts. 1 pint and 3 pks. 2 qts.?**

The method is precisely the same in principle as subtraction of money, only instead of dealing with 4, 12, and 20, we deal with the number of pints in 1 quart, of quarts in 1 gallon, and of gallons in 1 peck, which are *dotted* for convenience (2 pts. = 1 qt., etc.).

	2	4	2
3 pks.	0 gall.	2 qts.	0 pt.
2 "	1 "	3 "	1 "
0 pk.	**0** gall.	**2** qts.	**1** pt.

1 pt. + **1** pt. = 0 + 2 pts. = 1 qt.

3 qts. + 1 qt. + **2** qts. = 6 qts., *i.e.* 2 qts. + 4 qts. or 1 gall.

1 gall. + 1 gall. + **0** gall. = 2 galls. or 1 pk.

2 pks. + 1 pk. + **0** pk. = 3 pks.

There is an increasing tendency to sell corn, etc., by weight rather than by volume.

A bushel	of	English wheat	weighs	60 lb.
,,	,,	foreign ,,	,,	62 lb.
,,	,,	English barley	,,	50 lb.
,,	,,	French ,,	,,	$52\frac{1}{2}$ lb.
,,	,,	English oats	,,	39 lb.
,,	,,	foreign ,,	,,	38–40 lb.
,,	,,	rye and maize	,,	60 lb.

The Gram.

143. In the metric system the unit of weight, the Gram, is the weight of 1 c.c. of distilled water at 4° C.[1] The temperature has to be specified because the weight which 1 c.c. will hold varies as the temperature changes (see § 247 B. (3), Question 9).

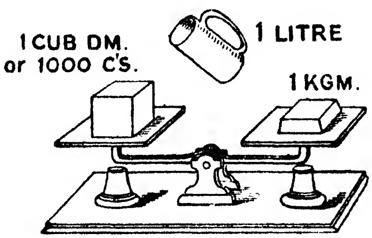
It is clear, then, that **the metric system derives the unit of weight from the unit of length, while the English system does not,** and this fact gives the former an immense advantage over the latter, particularly in calculations involving specific gravity (Section XIV. B.). So we have—

1 centigram (cgm.)	$= \frac{1}{100}$ gram or gramme (gm.).
1 dgm.	$= \frac{1}{10}$ gm.
10 gm.	= 1 Dgm.
1000 gm.	= 1 Kgm.
100 Kgm.	= 1 quintal
1000 Kgm.	= 1 *metric* ton.

Weights are usually expressed in Kgms. or gms. and decimals of those standards, or in metric tons.

The Kgm. is used where we use pounds, the gm. where we use ounces; they are universally used for scientific purposes and increasingly in pharmaceutical work.

Since 1 c.c. of water weighs 1 gm., 1000 c.c. or 1 c.dm. of water weighs 1000 gm.



Now, 1000 c.c. of water make up 1 litre;

∴ **1 litre of distilled water at 4° C. weighs 1 Kgm.,** a fact which is illustrated in the accompanying diagram.

The litre is subdivided just as the other weights and measures (§ 133), and

[1] Section XVII. A. (2).

the Hectolitre is used for measuring corn, etc.; *e.g.* "4,595,414 Hl. of wheat were exported from Roumania, *via* the mouths of the Danube, in 1914" (*Board of Trade Journal*).

144. TIME.

60 seconds (60″)	=	1 minute (1′)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day
7 days	=	1 week
4 weeks	=	1 lunar month
28, 29, 30 or 31 days	=	1 calendar month
$52\frac{1}{7}$ weeks	=	1 year
365 days	=	1 year
366 days	=	1 leap year
10 years	=	a decade
100 years	=	a century.

145. TABLE OF EQUIVALENTS.

(*a*) ENGLISH TO METRIC.

	Measure.		Value in Metric Units.
Lineal.	1 inch	=	2·539954 centimetres
,,	1 yard	=	0·914383 metre
,,	1 mile	=	1·609315 Kilometre
Superficial.	1 square inch	=	6·451367 square centimetres
,,	1 acre	=	0·40467 Hectare
Cubic.	1 cubic yard	=	0·7645134 cubic metre
,,	1 cubic foot	=	28·3153119 litres
Capacity.	1 gallon	=	4·543457 ,,
Weight.	1 lb. (Troy)	=	0·373242 Kilogram
,,	1 lb. (Avoirdupois)	=	0·4535926 ,,
,,	1 ton	=	1·0160475 metric ton

(*b*) METRIC TO ENGLISH.

	Measure.		Value in English Units.
Lineal.	1 centimetre	=	0·3937079 inch
,,	1 metre	=	1·093633 yard
,,	1 ,,	=	39·3707 inches
,,	1 Kilometre	=	0·62138 mile
Superficial.	1 square metre	=	1·196033 square yard
,,	1 Hectare	=	2·4711431 acres
Cubic.	1 cubic metre	=	1·3080215 cubic yard
Capacity.	1 litre	=	1·7607734 pint
,,	1 ,,	=	0·0353166 cubic foot
Weight.	1 gram	=	15·4323488 grains
,,	1 Kilogram	=	2·20462125 lb. Av.
,,	1 metric ton	=	0·984206 ton

146. We shall assume the following approximate relationships:

English to Metric.		*Metric to English.*	
1″	= 2·54 cm.	1 m.	= 39·37 in.
1 yd.	= 0·91 m.	100 Km.	= 62 miles
1 sq. in.	= 6·45 sq. cm.	1 Ha.	= 2½ ac.
1 ac.	= 0·4 Ha.		
		1 litre	= 1¾ pts.
1 cub. in.	= 16·4 c.c.	1 Hl.	= 2·75 imperial bushels
1 gallon	= 4·54 litres		
1 lb.	= 0·454 Kg.	1 Kg.	= 2·2 lb.

1 cub. ft. of water = 1000 oz.
= 6·25 galls.
1 gallon of ,, = 10 lb.
1 metric ton = 2204·6 lb.
1 English ton = 2240 lb.
£1 = 25·22 francs
= 20·43 marks
= 4·87 dollars

For other money values see page 116.

EXAMPLES. XXXIII.

THE METRIC SYSTEM: REDUCTION

Express the following quantities as required:

1. 10 m. 3 dm. 5 cm. in cm. 2. 18 m. 8 cm. in mm.
3. 350 Km. 8 Hm. 6 Dm. 5 m. in mm.

Nos. 4 to 8 as compound quantities—

4. 856 mm. 5. 986 cm. 6. 586 m.
7. 56823 mm. 8. 97684 mm.

Nos. 9 to 11 as square millimetres—

9. 9 sq. dm. 6 sq. cm.
10. 8 sq. m. 9 sq. dm. 6 sq. mm.
11. 6 sq. Dm. 5 sq. m. 3 sq. mm.

Nos. 12 to 15 as compound quantities—

12. 58629 sq. mm. 13. 95362 sq. cm.
14. 86954 sq. m. 15. 9·7562 sq. Km.

Nos. 16 to 19 as cubic millimetres—

16. 5 c.m. 8 c.cm. 17. 6 c.m. 8 c.mm.
18. 9 c.m. 8 c.dm. 19. 5 c.dm. 8 c.cm.

Nos. 20 to 22 as compound quantities—

20. 8964 c.c. 21. 5865 c.dm.
22. 51648 c.m.

Nos. 23 to 25 as milligrammes—

23. 8 gm. 3 dgm. 24. 5 Kgm. 8 gm. 2 cgm.
25. 28 Kgm. 5 dgm.

Nos. 26 to 30 as compound quantities—

26. 856 gm. 27. 53 cgm.
28. 59846 gm. 29. 86543 mgm.
30. 15962 dgm.

Nos. 31 to 33 as litres—

31. 5 l. 6 dl. 32. 8 l. 6 cl. 33. 50 Hl. 8 dl.

Nos. 34 to 36 as compound quantities—

34. 8656 cl. 35. 9562 dl. 36. 683 l.

Nos. 37 and 38 as ares and Hectares—

37. 853 sq. m. 38. 9865 sq. Dm.

39. 56 Ha. 88 ares in sq. m.
40. 56 sq. Km. 86 sq. Dm. in sq. m.

Nos. 41 to 45 as compound quantities—

41. 58·64 sq. m. 42. 88·65 l.
43. 86·9645 Km. 44. 64·8653 Ha.
45. 76·586 c.m.

What is the Ratio of—

46. 10 m. 5 dm. 5 cm. to 18 m. 5 cm.?
47. 8 sq. m. 9 sq. dm. 6 sq. mm. to 6 sq. Dm. 5 sq. m. 3 sq. mm.?
48. 6 c.m. 8 c.mm. to 9 c.m. 8 c.dm.?
49. 5 Kgm. 8 gm. 2 c.gm. to 28 Kgm. 5 dgm.?

What decimal is—

50. 8 l. 6 cl. of 50 Hl. 8 dl.?
51. 50 ares of 25 Ha.?
52. 50·805 sq. m. of 5 sq. Km.?

What is the sum of—

53. 6·865 gm.; 8 gm. 10 cgm.; 3 Kgm. 108 gm.; 8·564 Kgm.?
54. 8·65 Ha. 5·64 ares; 3 ares 8 sq. m.?

By how much does—

55. 3·865 Hl. exceed 10 l. 8 cl.?

56. 5·658 Km. exceed 85·64 m.?

57. What is the length of 50 pieces of chain each 3·54 m. in length?

58. How many pieces of rope 1·3 m. long can be cut off from one which is $\frac{1}{250}$ of 2·5 Km.?

THE METRIC SYSTEM: CALCULATION OF COST

147. Example 1.—**Find the cost of 2 metric tons 55 Kgm. 8 Hgm. of coal at 30·25 francs per metric ton.**

2 m.t. 55 Kgm. 8 Hgm. = 2·0558 metric tons.

∴ the cost = 30·25 × 2·0558 francs
= 62·19 francs (to two places).

Example 2.—**Find the cost of 8 Kl. 5 l. 8 dl. of wine at 7500 francs per Kl.**

8 Kl. 5 l. 8 dl. = 8·0058 Kl.

∴ 8·0058 × 7500 = the cost required
= 60043·5 francs
= 60044 francs, to the nearest franc.

EXAMPLES. XXXIV.

Find the cost of the following:

1. 25 gm. 15 dgm. 8 cgm. of copper at 30 centimes per Kgm.
2. 150 gm. 18 dgm. 10 cgm. of silver at 0·035 francs per gm.
3. 56 Kgm. 530 gm. 89 dgm. of lead at 25 centimes per Kgm.
4. 3 metric tons 98 Kgm. 860 gm. of coal at 35·55 francs per metric ton.
5. 2 metric tons 158 Kgm. 750 gm. of patent fuel at 0·04 francs per Kgm.
6. 6 Kgm. 75 gm. 38 dgm. of an alloy at 3·75 francs per Kgm.
7. 25 Kgm. 84 gm. 15 dgm. of glass tubing at 55 centimes per Kgm.
8. Oil is valued at 2 mk. 10 pf. per litre, what is the value of a consignment of 100 l. 10 dl. 18 cl. at the price given?
9. What is the value of 150 l. 16 dl. 10 cl. of wine at 5 francs 25 centimes per litre?
10. Determine the value of 25 Hl., 186 l., and 300 cl. of wine exported from Bordeaux at 250 francs per Hl.

11. What should be the cost of 36 l., 25 dl., and 580 cl. of naphtha exported from Italy at 3 lira per litre?

12. Find the cost of 6 Hl., 15 l., and 280 cl. of paraffin exported from Rotterdam at 8·5 gulden per Hl.

13. Find the value of a consignment of wine from Bordeaux consisting of 3 Kl. 8 Hl. 15 l., and 520 cl. at 2 francs per litre.

14. What are the French units of length, capacity, and weight? Contrast the French system with the English. What is the English equivalent for a metre, a litre, an are, and a gram?

EXAMPLES. XXXV.

THE ENGLISH SYSTEM: REDUCTION

Reduce—

(1) 8 ft. 6 in.; (2) 5 yds. 2 ft.; (3) 3 chs. 8 ft.; (4) 6 fur. 18 chs.; (5) 1 mile; (6) 5·75 miles; (7) 6 mi. 3 fur. 5 chs. 16 yds. to inches.

Express—

(8) 5280 ft.; (9) 63360 in.; (10) 1760 yds. in miles.

(11) 358 chs.; (12) 20·5 fur.; (13) 726·84 yds. as compound quantities.

(14) 5 sq. ft. 8 sq. in.; and (15) 80 sq. yds. 6 sq. ft. in sq. feet.

(16) 3 ac. 6 sq. chs. 500 sq. lks. in square links.

(17) 5 ac. 3 rds. 10 sq. po. in square poles.

(18) 8 sq. mi. 500 ac. in square yards.

(19) 4840 sq. yds.; (20) 85634 sq. ft.; (21) 586 sq. in.; (22) 5,000,000 sq. lks.; and (23) 8346 sq. po. as compound quantities.

(24) 10 cub. yds. 8 cub. ft. and (25) 15·8 cub. yds. in cubic inches.

(26) 50634 cub. in. and (27) 8654 cub. ft. as compound quantities.

(28) 15 lb. 8 oz.; (29) 3 qrs. 10 oz.; (30) 12 st. 8 lb.; and (31) 6 st. 10 oz. in grains.

(32) 8 st. 6 lb. and (33) 5 st. 12 oz. dead meat in pounds and ounces.

(34) 1 cwt.; (35) 1·86 cwt.; (36) 4 cwt. 2 qrs. 10 lb. in pounds.

(37) 1 ton; (38) 2·86 tons; (39) 3 tons 2 qrs.; (40) 5 tons 3 cwt. 4 lb. in pounds.

(41) 4840 lb.; (42) 86924 oz.; (43) 1434 qrs.; (44) 86·5 cwt. as compound quantities.

(45) 6 galls. 3 qts.; (46) 4 pks. 8 pts. in pints.

(47) 5 qrs. 3 bush, 2 pks. in gallons.

(48) 6254 pts.; (49) 5863 qts.; (50) 8563 galls.; (51) 7624 pts. as compound quantities.

(52) $10\frac{1}{2}$ lb. (Av.) and (53) 6 lb. (Tr.) in grains.

(54) 8 lb. 3 oz. 4 dwt. and (55) 5 lb. 5 oz. in pennyweights.

(56) 3·64 oz. (Tr.) and (57) 3·64 oz. (Av.) in grains.

(58) 865 lb. in centals; and (59) 9864 lb. in stones.

(60) 8 hrs. 5 min. 10 sec.; (61) 8·64 hrs.; (62) 9·5 min. in seconds.

(63) 6 days 5 hrs. and (64) 6·5 days in minutes.

(65) 8654 sec.; (66) 9632 min.; (67) 568 hrs. as compound quantities.

(68) 60° 8′; (69) 80° 56″ in seconds.

(70) 8654 sec.; (71) 783 min.; (72) 8·64 min. as compound quantities.

What fraction is the first quantity of the second (Questions 73 to 80)—

(73) 8 ft. 6 in. of 5 yds. 2 ft.?

(74) 7 fur. 8 chs. of 1 mile?

(75) 5 sq. ft. 8 sq. in. of 80 sq. yds. 6 sq. ft.?

(76) 10 cub. yds. 8 cub. ft. of 15·8 cub. yds.?

(77) 1·86 cwt. of 4 cwt. 2 qrs. 10 lb.?

(78) 1 lb. Av. of 1 lb. Tr.?

(79) 1 lb. Tr. of 1 lb. Av.?

(80) 1 oz. Av. of 1 oz. Tr.?

(81) From the last three questions state the constant multiplier for converting lb. Av. to lb. Tr.; lb. Tr. to lb. Av.; oz. Av. to oz. Tr.; and thence write down 10 lb. Av. in Tr.; 20 lb. Tr. in Av.; 20 oz. Av. in oz. Tr.

Reduce—

(82) 6 ℥ 5 ʒ; (83) 8 ℥ 6 ʒ 2 ℈; (84) 12 ℥ 4 ʒ 15 ℈ 15 grs. to grains.

(85) Express 856 grs. and 324 ʒ as compound quantities.

Find the sum of (Questions 86 to 89)—

(86) 4 mi. 1 fur.; 5 fur. 6 chs. 8 yds.; 2 mi. 3 fur.; 8 chs. 6 yds.

(87) 6 sq. yds. 8 sq. ft. 5 sq. in.; 5 sq. yds. 10 sq. in.; 6 sq. ft. 3 sq. in.; 9 sq. yds.

(88) 5 tons 3 cwt. 2 qrs.; 8 cwt. 2 qrs. 10 lb.; 14 stone; 592 lb.; $48\frac{1}{2}$ cwt.

(89) 6 oz. 8 dwt. 10 grs.; 6·8 dwt.; 3 lb. 5 oz.; 8 lb. 6 oz. 5·6 dwt.

What is the difference between (Questions 90–92)—

(90) 20 tons 16 cwt. 1 qr. 18 lb. and $\frac{5}{8}$ of 64 tons?

(91) 24 cwt. 8 lb. and 0·4 of 15·5 tons?
(92) 3 qts. 1·6 pts. and $\frac{3}{4}$ of 2·4 galls.?

What decimal is—

(93) 5 yds. 2 ft. of 3 chs. 8 ft.?
(94) 6 sq. ft. 8 sq. in. of 3·5 sq. yds.?
(95) 8 fur. 5 po. of 4·4 mi.?
(96) 3 tons 4 cwt. 2 qrs. of 0·75 of 24 tons?
(97) 5 oz. 8 dwt. 10 grs. of $\frac{1}{2}$ of 3 lb. (Tr.)?
(98) 2 hrs. 8 min. of half a day?
(99) 5 mi. 6 fur. 5 chs. of 500 chs.?
(100) 13 ac. 3 rd. 5 sq. chs. of 20 ac.?
(101) 36 cub. in. of 5 cub. ft.?
(102) $2\frac{1}{2}$ pints of 3 galls.?
(103) $\frac{1}{5}$ of 10 galls. 3 qts. of 2·25 bush.?
(104) 3 galls. 2 qts. 1 pt. of 5 galls. 3 qts.?
(105) The number of days from 25th December to 25th March in an ordinary year of the number in a leap year.

EXAMPLES. XXXVI.

Miscellaneous Questions

1. How many c.cm. are there in 1·5 c.dm.?
2. What decimal of 6·4 c.m. is 8·64 c.cm.?
3. What number of c.c. does 4·5 l. contain?
4. Express 8·4 lb. as a fraction of 4·2 cwt.
5. By what must 3 cwt. 2 qrs. be multiplied to give 5 cwt. 14 lb.?
6. What is the excess of 8 ℥ over 6 ℥ 3 ʒ 2 ℈ 12 grs.?
7. What must be added to 6 oz. 15 dwt. 18 grs. of platinum to give 8 oz. 12 dwt. 6 grs.?
8. By how many grains is 6 oz. of tobacco heavier or lighter than 6 oz. of silver?
9. A Winchester quart bottle contains 80 fluid ounces; how many pints will one of them hold?
10. How many fluid ounces are there in a quart?
11. Reduce $\frac{5}{8}$ of 10·4 gm. to the decimal of $\frac{7}{10}$ of 56·02 Kgm.
12. What decimal of a long ton is a short ton?
13. Express 530 short tons in long tons.
14. How many short tons equal one long ton?
15. How many long tons are there in 370 short tons?
16. By what must any number of long tons be multiplied to give the corresponding number of short tons? (In other words, "What is the constant multiplier for converting long tons to short tons"?)

17. What is the constant for converting short tons to long tons?

18. A tank holds 100 gallons of water, what weight does the bottom sustain as the decimal of 1 ton?

19. A milkman gives 19 fluid ounces of milk for a pint; what decimal of a pint is he short? If 1 pint be valued at 2d. and 100 gallons are delivered each morning, how much does he gain by his mistake each week?

20. A wine merchant finds he has sold the following quantities of wine: 5 galls. 3 qts. 1 pt., $6\frac{1}{2}$ gallons, 10 quarts, 18 pints, 28 half-pints. How much has he sold altogether, to the nearest pint?

21. One cask of oil contains 5 galls. 3 qts. 1 pt., and another 7 gallons. What quantity must be added to the first in order that it may contain as much as the second?

22. A chemist finds he has $28\frac{1}{2}$ fluid ounces of camphorated oil in one bottle, and half a pint in another. Can he put the two quantities into a Winchester quart bottle which will hold 80 ounces (fluid)?

23. If a barrel contains $3\frac{1}{2}$ gallons of methylated spirit which is sold at $6\frac{1}{2}$d. a pint, find the retail value of the spirit, allowing half a pint for waste.

B. THE ENGLISH SYSTEM: CALCULATION OF COST

Compound Practice and Nine-Multiple Tables

148. The following examples illustrate the method of calculating the cost of given weights or measures of commodities at stated prices, and also emphasise the use of nine-multiple tables in general commercial work.

Example 1.—**Find the cost of 15 tons 12 cwt. 2 qrs. of coal at 33s. 6d. per ton.**

Method (i)

33s. 6d. = £1 13s. 6d. = £1·675.

£1·675 = cost of 1 ton.

	£25·125	= cost of 15 tons.	
$\frac{1}{2}$ of £1·675 =	0·8375	= ,, ,,	10 cwt.
$\frac{1}{5}$ of £0·8375 =	0·1675	= ,, ,,	2 cwt.
$\frac{1}{4}$ of £0·1675 =	0·041875	= ,, ,,	2 qrs.
	£26·171875	= ,, ,,	15 tons 12 cwt. 2 qrs.

£26·1719 = £26 3s. 5d. (to the nearest penny).

Method (ii)

33s. 6d. = £1·675.
15 tons 12 cwt. 2 qrs. = 15·625 tons.

∴ the cost of 15 tons 12 cwt. 2 qrs. @ 33s. 6d. a ton
= 1·675 × 15·625
= 26·1719 (to four places)
= £26 3s. 5d.

149. Example 2.—**Find the cost of 10 pks. 1 gall. 3 qts. 1 pt. of beans at 43s. 6d. a peck.**

Method (i)

	£2·175	= cost of	1 pk.
	£21·75	= ,, ,,	10 pks.
½ of 2·175 =	1·0875	= ,, ,,	1 gall.
½ of 1·0875 =	0·5438	= ,, ,,	2 qts.
½ of 0·54375 =	0·2719	= ,, ,,	1 qt.
½ of 0·2719 =	0·1359	= ,, ,,	1 pt.
	£23·7891	= ,, ,,	10 pks. 1 gall. 3 qts. 1 pt.

or, £23 15s. 9d., to the nearest penny.

Method (ii)

43s. 6d. = £2 3s. 6d. = £2·175

and 10 pks. 1 gall. 3 qts. 1 pt. = 10 pks. + 0·5 pk. + 0·375 pk. + 0·0625
= 10·9375.

∴ Cost = 2·175 × 10·9375
= £23·7891, to four places
= £23 15s. 9d., to the nearest penny.

150. Let us suppose that a contractor agrees to supply truck loads of coal at a fixed price of 33s. 6d. per ton (see Example 1, § 148). It is necessary to see that he would not calculate the cost of every load by practice, but, having weighed the truck and allowed tare, he would refer to a *permanent nine-multiple table* showing the price per cwt., thus:

1 cwt. costs	£0·08375 (*i.e.* $\frac{1}{20}$ of £1·675)	5 cwt. costs	£0·41875
		6 ,, ,,	0·50250
2 cwt. costs	£0·16750	7 ,, ,,	0·58625
3 ,, ,,	0·25125	8 ,, ,,	0·67000
4 ,, ,,	0·33500	9 ,, ,,	0·75375

Hence—

15 tons 12 cwt. 2 qrs. = 312 cwt. 2 qrs.

300 cwt. cost . .	£25·125	(moving the decimal in the value of 3 cwt.).
10 „ „ . .	0·8375	
2 „ „ . .	0·1675	
and 2 qrs. ($\frac{1}{2}$ cwt.) cost	0·0419	
∴ total cost is	£26·1739	

= £26·174

= £26 3s. 5d. as before.

All the work that the clerk would have to do would be the last lines, much of which might be performed mentally.

151. If the price given should reduce to a non-terminating decimal, then the number of decimal places required in the table must be calculated (cf. § 119).

Thus—

Cost of 1 cwt., £0·022423224. To find the cost of 1000 tons or 20000 cwt. we have: cost ≏ 20000 × £0·02 = £400 (*three figures*), and we want the result to *three places*.

∴ we take six figures + one for safety, *i.e.* seven figures in all.

∴ Cost of 1 cwt. = £0·0224232 (seven figures)

and cost of 1000 tons = £448·464

= £448 9s. 3d., to the nearest penny.

Had we taken the cost per cwt. as £0·02242, then the result would have been £0·00242 × 20000 = £448·4

= £448 8s.

The difference is not great on 1000 tons, but we must remember that our annual coal production is about 265 million tons!

EXAMPLES. XXXVII.

Calculate the cost of paving roads of the lengths given at the price quoted (Questions 1 to 10)—

1. 1000 yds. 2 ft. @ 10s. 6d. per yard.
2. 20 chs. 3 yds. @ 40s. 1d. per furlong.
3. 3 fur. 2 chs. 2 yds. @ £50 per furlong.
4. 4 fur. 5 chs. 4 yds. 2 ft. @ £500 6s. 8d. per mile.
5. 5 fur. 3 chs. 12 yds. 1 ft. @ £15 12s. 6d. per chain.
6. 6 mi. 5 fur. 2 chs. 4 yds. @ £250 10s. per mile.
7. 20 mi. 6 fur. 100 yds. @ £320 15s. per mile.

8. 21 mi. 2 fur. 250 yds. @ £40 10s. per furlong.

9. 57 mi. 4 fur. 8 chs. @ £36 15s. per furlong.

10. 70 mi. 5 fur. 3 chs. @ £450 16s. 8d. per mile.

11. Prepare a nine-multiple table expressing yards as the decimal of a mile.

12. From the table of the last question express the quantities given below in miles and the decimal of a mile: 6 mi. 5 fur. 2 chs. 4 yds.; 15 mi. 6 fur. 3 chs.; 5 mi. 3 fur. 3 chs. 10 yds. 2 ft.

13. Given that 1 inch = 2·539954113 cm., make out a nine-multiple table (*a*) to convert inches to centimetres; (*b*) to convert yards to metres; and then express 125 inches in centimetres and 220 yards in metres, to three places.

14. Given that 6 square yards = 5·0165828992 square metres, make out a nine-multiple table for converting square yards to square metres, and thence find the number of square metres in an acre.

15. If 3 Ha. = 7·4134293 acres, make a nine-multiple table, and thence find the area of an estate of 156 Ha. 56 ares, in acres, roods, and poles.

16. Find the cost of ploughing 5 ac. 3 rds. 15 po. at £2 13s. 3d. per acre.

17. What should it cost to survey 2 sq. mi. 50 ac. 3630 sq. yds. at £1000 15s. 9d. per square mile?

18. What is the value of 59 cub. yds. $26\frac{1}{2}$ cub. ft. of gravel at 5s. 3d. per cubic yard?

19. What is the difference between the area of the rice fields of Japan in the years 1913 and 1914, if their area in the former year was 2,859,812 chō and in the latter 2,869,192? (1 chō = 2·45 acres.)

20. Express the area of the department (Seine) in which Paris is situated as a decimal of the area of Greater London, given that the former is 47360 Hectares and the latter 692 square miles.

21. Make out a nine-multiple table for converting pounds to the decimal of 1 cwt., and thence write down a table giving pounds as the decimal of a ton.

22. From the table of the last question, express in tons and the decimal of a ton: 5 tons 3 cwt. 2 qrs. 8 lb.; 6 tons 9 cwt. 1 qr. 15 lb.; 15 tons 16 cwt. 3 qrs. 19 lb.; 16 tons 5 cwt. 1 qr. 24 lb.

23. Find by practice the cost of each of the weights in the last question at £3 14s. 6d. per ton, and confirm the results by decimalising the shillings and pence and employing the nine-multiple table and contracted multiplication.

An iron merchant purchased metal as given below (Questions 24 to 29). Calculate the price paid for each lot: (1) By practice; (2) by decimalisation, using the table of Question 21 (above)—

24. 25 tons 12 cwt. 2 qrs. 18 lb. at 61s. 3d. per ton.

25. 36 tons 7 cwt. 1 qr. 15 lb. at 61s. 6d. per ton.

26. 42 tons 13 cwt. 3 qrs. 10 lb. at 62s. 9d. per ton.

27. 51 tons 5 cwt. 1 qr. 8 lb. at £3 1s. 3d. per ton.

28. 72 tons 14 cwt. 3 qrs. 2 lb. at £3 1s. 6d. per ton.

29. 21 tons 6 cwt. 3 qrs. 19 lb. at £3 2s. 9d. per ton.

30. With spelter (zinc) at £55 4s. 6d. per ton, how much would you be prepared to accept for 3 tons 17 cwt. 1 qr. 18 lb. of the metal?

31. What should be the price of spelter per lb. at the price quoted in the previous question?

32. Quote spelter in francs per Kgm. from the figures in Question 30.

33. What is the value of 5 tons 14 cwt. 58 lb. of tin at £190 4s. per ton? (Look in your newspaper and find the price to-day.)

34. How much must a butcher pay for 36 stone 4 lb. of beef at 5s. 2d. a stone?

35. What is the wholesale value of beef per lb., and what is the profit by selling the quantity given in the last question at $11\frac{1}{2}$d. per lb.?

36. A provision merchant finds that he has purchased 8 tons 13 cwt. 2 qrs. 15 lb. of bacon at an average price of 75s. 6d. per ton during the year. How much money has been expended on bacon?

37. What is the value of 15 cwt. 3 qrs. $18\frac{1}{2}$ lb. of rice at 18s. 8d. per cwt.?

38. How much should be paid for 5 tons 17 cwt. 1 qr. 18 lb. of rubber at £230 15s. 9d. per ton?

39. What is the price of the rubber in the last question in marks per Kilogram?

40. A butter merchant writes to a Danish firm and asks whether $2\frac{3}{4}$ tons of butter can be supplied for £250. The reply offers the butter at 2 kroner 25 ore per Kgm. What difference is there between the two prices for the whole quantity? Suppose yourself to be the merchant and write a letter asking the question in such a way that a Danish firm can understand it.

41. What is the value of 5 galls. 3 qts. 1 pint of petrol at half a dollar a gallon?

42. If petrol be 1s. 6d. a gallon, convert the price into dollars per gallon for American customers.

Calculate the cost of the following quantities of wine at the prices given (Questions 43 48):

43.	28 galls.	3 qts.	1 pint	@ 5s. 9d.	a gallon.
44.	36 galls.	2 qts.	1 pint	@ 8s. 6d.	"
45.	14 galls.	1 qt.	1½ pint	@ 11s. 6d.	"
46.	18 galls.	1 qt.	½ pint	@ 4s. 3d.	"
47.	86 galls.	2 qts.	1 pint	@ 11s. 6d.	"
48.	24 galls.	3 qts.	1½ pint	@ 4s. 8d.	"

49. Find the value of 56 qrs. 6 bush. 3 pks. of wheat at 61s. 9d. per quarter.

50. What is 80 qrs. 7 bush. 2 pks. of barley worth at 35s. 4d. per quarter?

51. Determine the cost of 28 qrs. 3 bush. 1 pk. of oats at 30s. 10½d. per quarter.

52. With wine at 6·25 francs per litre, find the cost per pint in English money.

53. Vinegar makes 5½d. a pint in England; express this price in lira per litre for Italian customers interested in the Cornish mackerel export trade, in which vinegar is largely used.

54. You want to buy a thousand gallons of oil at 8½d. a gallon, and M. Knovitch of Odessa offers to consign it to you (f.o.b.) at 75 roubles per 1000 litres. Is this price greater or less than you desired, and how much would you gain or lose by accepting the quotation? Write a letter in English or, far better, in Russian to M. Knovitch stating what you want.

C. Invoices

HOME TRADE

Peto (*reads*).

"Item.	A capon	2s. 2d.[1]
Item.	Sauce	4d.
Item.	Sack, two gallons	5s. 8d.
Item.	Anchovies and sack after supper	2s. 6d.
Item.	Bread	½d."

1 King Henry IV., Act II. sc. 4.

152. On going to a grocer's shop and buying a number of articles it is usual for the salesman to take his book in which there is a piece of carbon paper between two leaves and write upon the upper one a list of our purchases and the price to be paid. He then hands us over one leaf, called "the Invoice," and retains the other, "the Dupe," or Duplicate, for the information of his head office.

153. The essential features of an invoice are that it should contain a statement of the nature, quantity, quality, and price

[1] A sixteenth-century invoice.

of the goods sold, and its use is that being sent with the goods it enables the purchaser to check the goods received against the "items" of the invoice, and so see whether they agree.

In the same way, butchers send out weight tickets with the joints, dairymen record the quantity of milk, and bakers the number of loaves delivered, in a suitable book held by the customer.

154. We now give an example of an invoice sent by Messrs. Higson & Sons Ltd. to Mrs. Jacobs.

INVOICE

Mrs Jacobs,
1115, Old Sq York

Bought of Higson & Sons, Ltd.
PROVISION MERCHANTS.

Terms *One Month* April 1 19

			£	s	d
8½	lbs Bacon	1/2		9	11
6	Tins Apricots.	8½d		4	3
2 doz	New Laid Eggs	1/6		3	0
12½	lbs Sugar	3½d		3	8
6	" Tea	2/6		15	0
4	" Raisins	4½d		1	6
		£	1	17	4

155. In many trades it is usual to pay for the goods on delivery by a cash payment. If, however, this is not done and credit is allowed, the vendors will, at the commencement of the next month, send in a " Statement of Account."

STATEMENT OF ACCOUNT

Mr Jacobs,
1115, Old Sq. York.

Bought of Higson & Sons Ltd.

PROVISION MERCHANTS

Terms - One Month December 31 19..

19		£	s	d	£	s	d
Jan. 1	To Balance Brought Forward				3	5	8
Feb. 1	To goods	2	8	6			
Mar. 1	" "	1	15	8			
April 1	" "	2	3	8			
May 1	" "	1	15	6			
June 1	" "	2	5	8			
July 1	" "	3	6	2			
Aug. 1	" "	2	1	4			
Sep. 1	" "	3	5	8			
Oct. 1	" "	2	1	6			
Nov 1	" "	3	0	0			
Dec. 1	" "	2	1	4	26	5	0
					29	10	8
Mar 25	By cash	6	6	0			
June 24	" "	6	6	0			
Sep 30	" "	6	6	0	18	18	0
				£	10	12	8

156. Now let us suppose that Mrs. Jacobs deals regularly with Messrs. Higson & Sons, and that she pays them from time to time, and let us suppose that a year has passed and that the account is still not quite settled; then the final statement will be posted and will appear as on page 211, while each month previously one similar, but obviously not so long, has been sent to her.

COAL MERCHANT'S ACCOUNT
(Receipted)

Telephone, X Y Z City.

Greenwood Collieries Association,
West Street, Preston.

also at YORK, SHEFFIELD, HULL & PORTSMOUTH.

HEAD OFFICE - *10, Anderton St., Birkenhead.*

No. 685. 10/3/1

Mr. Joshua, 84 Bellbury St. Preston

Tons	Cwts		Rate per ton.	£	s	d
2		Wallsend.	28/6	2	17	0
3	10	Selected	28/-	4	18	0
1	.	House Coal,	27/6	1	7	6
5		Kitchen	25/6	6	7	6
10		Stove Coal.	24/6	12	6	0
1		Anthracite	42/6	2	2	6
			£	29	18	6

TERMS. Cash on or before Delivery. Cheques crossed, P & Q's Bank.

Take notice that you are to receive herewith 22 Tons 10 Cwt of - - - Coal in 225 Sacks, containing 224 lbs of Coal in each Sack.

John Duff. *Carman.*

Receipt Stamp

Where Coal is delivered by means of a vehicle, the Seller must deliver, or send by post or otherwise to the Purchaser or his Servant before any part of the Coal is unloaded, a Ticket or Note in this form.

Any Seller of Coal who delivers a less quantity than is stated in this Ticket or Note is liable to a fine.

Any person attending on a vehicle used for the delivery of Coal, who, having received a Ticket or Note for delivery to the Purchaser, refuses or neglects to deliver it to the Purchaser or his Servant is liable to a fine.—52 & 53 Vict. Ch. 21 Weights and Measures Act 1889.

Received on delivery the above amount.

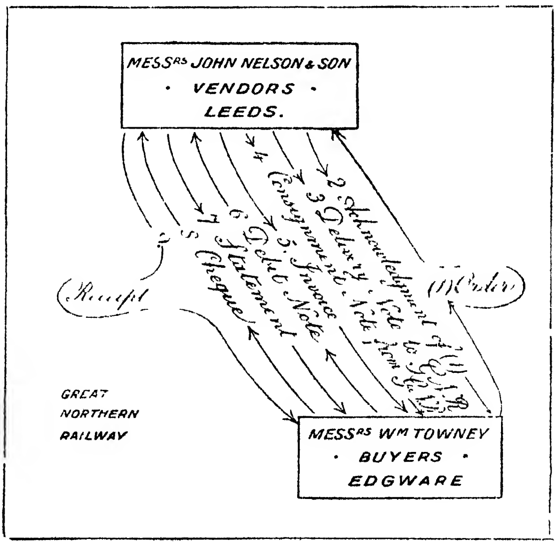
John Duff *Carman.*

157. A pro forma invoice is one which is sometimes prepared by a salesman when he procures for a customer goods which he does not stock, or it may be a statement of the estimated cost of goods.

158. We now give three other types of invoice, which the student should carefully study, as they embody points which the law demands shall be printed on such invoices. The receipt will be understood after § 168 has been read. I. Account sent by a coal company, receipted by carman (shown on opposite page); II. Account sent by a gas company, and receipt for the amount thereof (Plates II. and III.); and III. Invoice from a firm of biscuit manufacturers (Plate IX., p. 226).

159. If now we look at these operations from the point of view of a firm which deals on a much larger scale, we shall find that the procedure is as follows:

Messrs. Towney wish to purchase goods from John Nelson & Sons, but instead of sending a clerk to order them they have a book called an "Order Book," in which the items required are written down; the page is then torn out (leaving a duplicate), and sent either by post or by hand to Messrs. Nelson. Hence we have the following diagrammatic representation of the procedure described in the succeeding pages:



160.

ORDER FORM

Telegrams,
000 WEST.
Phone, OY6.

No. 125

3561, Beenten St.,
Edgware.
July 10th 19

Messrs John Nelson & Sons,
53, Bolton St, Leeds

Please supply goods as under & oblige
Yours faithfully,
Wm Towney

50 yds No 3185 Black Serge @ 1/6.
30 yds No 401 Navy Serge @ 2/6.
25 yds No 356 Black Vicuna @ 2/.

PLEASE QUOTE ON INVOICE THE No OF THIS ORDER

161. On receiving the order, Nelson & Sons will acknowledge receipt of the same and intimate that the goods will be sent by passenger train the next day but one. The cloth is packed in cases and then handed over to the Great Northern Railway, together with a **delivery note** as follows:

DELIVERY NOTE

No 312.
July 12, 19
M The Great Northern Rlwy

Received from John Nelson & Sons Ltd in good order & condition the undermentioned goods viz
3 Cs Cloth per Passenger Train
Messrs Wm Towney
3561 Beenten St
Edgware
Received by P.K.

No 312.

53, Bolton St,
Leeds.
July 12th 19

M The Great Northern Rlwy Co
Kindly receive from John Nelson & Sons, Ltd. in good order and condition the undermentioned Goods –
3 Cs Cloth per Passenger Train
Messrs Wm Towney
3561, Beenten St
Edgware

To the left of the perforation on the delivery note, which is torn from a book, there is the counterfoil upon which are copied the details that appear on the delivery note, and the carman (P. N.) initials that counterfoil when he receives the goods.

162. A consignment note, repeating the details given on the delivery note and adding the weight of the cases, will next be prepared (as shown on Plate IV.).

163. John Nelson & Sons will send on the invoice (p. 216) by post.[1]

164. John Nelson & Sons will next send a Statement[2] to Messrs. Towney, who will thereupon pay the account, unless this has already been done.

[1] It is a common practice for the vendors to allow the purchasers a DISCOUNT (see Section XXI.) for payment within a certain time.

$$5\ \%\ \text{discount} = 1\text{s. in the pound},$$
$$\therefore\ 2\tfrac{1}{2}\ \%\ \text{discount} = 6\text{d. in the pound},$$

and the student can quite easily hit off the proper amount from these facts.

Thus— 5 % on £4 10s. 0d. = 4s. 6d.
$2\frac{1}{2}$ % on £3 15s. 8d. = 6d. × 3 + $\frac{2}{3}$ of 6d.
= 1s. 6d. + $4\frac{1}{2}$d.
= 1s. $10\frac{1}{2}$d.

The following contractions are sometimes used on invoices and in other formal statements in commerce:

E. & O.E. means "Errors and Omissions Excepted."
f.o.b. means (delivered) "free on board ship."
f.o.r. means (delivered) "free on rail."
c.i.f. means "cost (of goods), insurance (by senders), and freight" are included in the quotation.
"per pro." If J. Brown has restricted authority to act for Michs & Co. he must, on signing a receipt (or letter, etc.) for them, write:

J. Brown
per pro
Michs & Co.

and, if he exceeds his authority, the firm may not be held responsible.

[2] If some of the cloth had been returned by Wm. Towney, Nelson & Sons would have sent him a Credit Note (c/n) in red ink with the usual normal heading (see Plate VIII., p. 224), and it would run:

"By return of goods invoiced on 13th. . . £2 10s. 0d."

INVOICE

53, Bolton St.,
Leeds.
July 13th 10

Messrs Wm Towney

Bought of John Nelson & Sons, Ltd
CLOTH WORKERS.

				£	s	d
1 65	50	yds No 3185 Black Serge	1/6	3	15	0
1 64	30	" 401 " Navy	2/6	3	15	0
1 63	25	356 Black Vicuna	2/-	2	10	0
		3 Cases			3	0
		Per Great Northern Rlwy Carriage Forward	£	10	3	0

Messrs. Towney will return the cases and send Nelson & Sons a **Debit Note**, as shown on top of page 217.

165. Now it is clear that they will not pay in cash, for it would cost too much to send the money by registered post, and it would also be very inconvenient. It may, however, be sent—

(1) By Postal Orders.
(2) By Post Office Orders.
(3) By Bank Notes.
(4) By Cheque.

In (1) all that is necessary is to send a clerk to the post office with instructions to buy postal orders to the value of £10 3s., the amount of the invoice. They would cost no less than 1s. 3d. in addition to the £10.

Charges for Inland Money Orders.

Ordinary Orders.

For sums not exceeding	Poundage.
£1	2d.
3	3d.
10	4d.
20	6d.
30	8d.
40	10d.

Telegraph Orders.

1. Poundage as for ordinary Inland Money Orders.

2. A charge for the Telegram of Advice at the ordinary rate for Inland Telegrams. If the remitter wishes it to be repeated, he must also pay an additional half rate.

3. A supplementary fee of 2d. for each order.

A deposit must also be paid in respect of porterage for delivery of the Telegraph Money Order, if porterage is likely to be incurred.

The amount of the Order required, the Poundage and any other charges payable, must be handed in before the Order is issued.

No single Inland Order can be issued for a sum exceeding £40.

The remitters of Orders payable in the United Kingdom can obtain Advices of Payment by paying an additional fee of 2d.

For use of Postmaster, if required.

		Date.
Second Advice sent	....	....
Name of Payee, corrected to	....	....
Duplicate Order Notice No. 50 received from Metropolitan Office.	....	....

[2168] 22352/889 4500m S/15ss 4228 G & S 83

PLATE V.

POST OFFICE INLAND MONEY ORDER.

No. 3-D.

INLAND MONEY ORDER REQUIRED

Office Stamp.

No. of Order.

No. FOR

£	s.	d.
10	3	=

Payable at Leeds

[Payment will be subject to the possession by the Postmaster of sufficient funds.]

	* Prefix	Christian Name (or Initial).	Surname.
to	Messrs	John Nelson & Sons	

* If the payee of a Telegraph Money Order is a woman, the prefix "Mrs." or "Miss," as the case may be, should be given for inclusion in the Telegram of Advice.

† Sent by Wm Towney

† Address 3561, Beenton St, Edgware.

† IMPORTANT.—The sender's name and address are required for official purposes. The person presenting the Order for payment must be prepared to state the name of the sender.

The Space below FOR TELEGRAPH ORDERS ONLY.

If the order is to be telegraphed, write the words "By Telegraph" across the Form, and fill in the further particulars asked for below.

1. Address at which the Telegraph Order is to be delivered. If to be called for at a Post Office write the word "Postoffice." [In the latter case the sender should inform the payee where he must apply for the Order.] ....

2. State here whether the Telegraph Money Order is to be crossed for payment through a bank. ....

PRIVATE MESSAGE.

3. A Private Message to the payee may be added to the Telegram of Advice, for delivery with the Order. **The sender's name (or name and address), if to be communicated to the payee, must form part of the Private Message.**

Write Private Message here (not to exceed twelve words).

....	....	....	....
....	....	....	....
....	....	....	....

DEBIT NOTE

3561, Beonten St.,
Edgware.

Messrs. John Nelson & Sons, July 16th 19

Dr. to Wm. Towney.

			£	s	d
	3	Empty cas retd per G N Rly		3	

Or (2), to tell him to buy a post office order (Plate V.), which would cost 4d., and post that on to Leeds.

If it were "crossed" & Co., as shown (see Plate V.), then it would not be cashed at a post office, but paid into the Bank and credited to John Nelson's account.

(3) Two five, or one ten-pound Bank of England note could be used, but there is certainly a risk in sending notes through the post unregistered,[1] and, indeed, as there are odd shillings and pence in the account, postal orders must be sent as well as notes.

166. The student will recognise at once that all the methods we have described are very awkward and costly.

In (4) we have almost the ideal method of settling an account, namely, by means of a cheque costing just the value of the 1d. stamp which, in this country, it MUST bear.

A cheque can be written in a moment, enclosed with an advice of remittance (a simple statement: "We beg to hand you cheque for £10 in settlement of your account"), and, if crossed, is useless to any one but the drawee (§ 169); again, since it returns through the Banks to the DRAWER (Messrs. Towney in our case),

[1] It is usual to cut notes in half and send one half first, and then, on receiving an acknowledgment, the other half.

it is a receipt in itself. It will appear as shown on Plate VI. (p. 220).

167. When Messrs. Nelson receive the cheque they must sign their name on the back, that is to say, ENDORSE it, and present it to the Power Banking Co. for payment, or, what is much more likely, they will pay it into *their* Bank, and it will ultimately find its way to the proper one *via* the Bankers' Clearing-House, which deals with such matters. (See diagram opposite.)

It is necessary to explain this last statement. Messrs. Towney's bank is at Edgware and is controlled by the Power Banking Co. Messrs. Nelson's bank is at Leeds and is called the Central Bank. Now Messrs. Towney send the cheque to Leeds, but there is no branch of their Bank in the city, and even if there were Messrs. Nelson would still pay the cheque into the Central Bank and would reckon that they had £10 in that Bank. Now in reality the *Power Bank* has to pay the money on Messrs. Towney's behalf, and hence the manager of the Central Bank could, if he wished, send the cheque to Edgware and receive £10 gold per registered post. Magnify this last operation a million times and you have stupendous labour and enormous expense. To obviate all this, we have the Bankers' Clearing-House, and the arrangements are such that the Central Bank sends all cheques drawn upon *other* Banks to that centre, where clerks representing the various Banks assemble and simply exchange cheques with one another, recording the transactions on slips of paper, and taking away with them cheques drawn on the Bank that they represent.

If, then, Jones hands Brown cheques for £50000 and Brown hands Jones cheques for £49980, on the day of settlement £20 passes between them, and if it had not been for the Clearing-House [1] nearly £100000 would have had to pass either in gold or in notes.

We now illustrate the travels of a cheque (see diagram on opposite page).

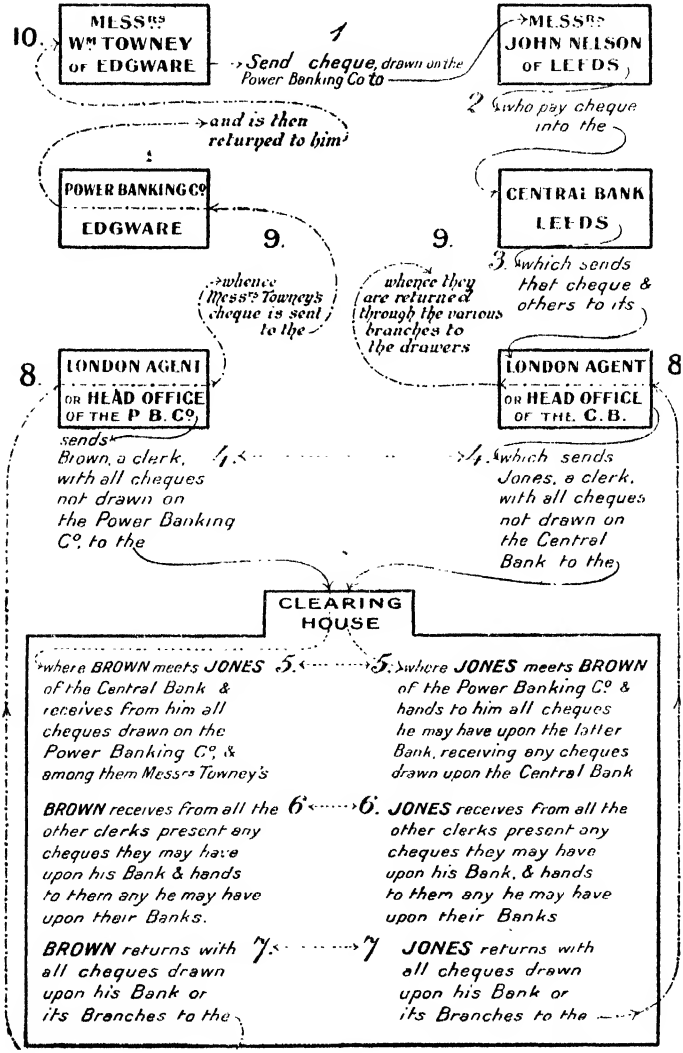
---

[1] The following note is taken from a London daily paper: "In 1868 the total amount dealt with by the Bankers' Clearing-House was £3,425,000,000; it was £16,436,000,000 in 1914; and now, the latest figures available indicate that the work of the Clearing-House is still increasing. Where forty years ago fifty men could get through the day's work, now five hundred are needed, and the accommodation they require is more than a five-story building can provide.

"In the early hours of the day there is hardly breathing-room in the offices in Post Office Court, Lombard Street, and the din created by over two-hundred mechanical reckoners working at high pressure is terrific. Nearly every one to-day has a banking account, and the very good custom of paying everything by cheque is fast becoming established.

"Every cheque contributes one penny to the revenue by way of stamp duty. On many days in the week more than a million articles are dealt with at the Clearing-House."

THE TRAVELS OF A CHEQUE.



168. Very often there is printed on the back of the cheque a **form of receipt** such as the following:

CHEQUE RECEIPT

Received from Messrs. Wm. Towney
the sum named on the face hereof

1D STAMP

John Nelson & Sons
July 19th/19

THE SIGNATURE ON THIS CHEQUE IS INTENDED TO BE AN ENDORSEMENT AS WELL AS A RECEIPT

If the amount is £2 or over, a penny stamp must be placed on the receipt, and the name is usually written over it.

If, however, such a form of receipt is not provided, then Nelson & Sons will send Wm. Towney a formal receipt similar to the above but on a form of their own.

Many firms, who ask for payment on delivery, provide their vanmen with an **adhesive receipt** which is very useful, for it is torn off from the counterfoil, and simply stuck on the invoice; it can be prepared beforehand, and thus any error on the part of the vanman is avoided. Thus—

ADHESIVE RECEIPT

No 25
Thos Jones & Co.,
Nett Cash Received
£ 1 . 10 3
Discount or Deductions
£ - - . -

No 25 15/7/19
Received from Thos Jones & Co.,
the sum of One pounds
ten shillings & three pence
John Brown.
£ 1 10 3

169. The cheque which we have represented is called an "order" cheque, and it must be paid to the person named, and endorsed by him. If the word "Bearer" had replaced the word "Order" then *any one* could have presented it at the Bank and received payment without even endorsing it, although bankers usually ask for an endorsement so that they may have the signature of the payee.[1]

If the cheque is "crossed" as the P.O.O. (Plate V., p. 217), then it can be cashed only if Nelson & Sons have a banking account into which they can pay it.

The cheque is said to be drawn on the Power Banking Company; **Wm. Towney is the DRAWER and Messrs. John Nelson & Sons are the PAYEES.**

170. In connection with business transactions it is important to understand the meaning of "Legal tender," or, in simpler language, "How you may legally pay an account."

Pence may be used up to 1s., silver coins up to 40s.,[2] gold coins, of full weight, or Bank of England notes, NOT country notes, up to any amount, and, at present, 10s. or £1 Treasury notes are also legal tender. From August 1914 to March 1915 postal orders were legal tender, and on going into a post office for six penny stamps and presenting 1s. the clerk kept the money and handed over the stamps and a 6d. postal order for the change.

In normal times, however, postal orders, post office orders, and cheques are used for, and accepted in payment of, accounts. They are far more convenient than coin for general purposes, but whereas no one may refuse gold he might find it very awkward to receive it. For suppose one firm owed another £10000 and sent down a cart-load of sovereigns, it would be a very cumbersome way of dealing with the account, but if a cheque is drawn the gold tends to remain in the vaults of the Bank, where it is safe, and a little slip of paper passes between the parties concerned.

In America and on the Continent cheques are much less in vogue, and paper money much more so, than in England. An American cheque, of which we give a reproduction (Plate VII., p. 222), bears no embossed stamp.

[1] Cheques drawn on English Banks if cashed in Scotland are charged sixpence.
[2] See Questions 77 to 93, Examples LVIII., Section XV.

EXAMPLES. XXXVIII.

Make out invoices[1] in proper form for the following, supplying names, etc., where necessary.

1. 2 lb. butter at 1s. $6\frac{1}{2}$d.; 3 lb. sugar at $4\frac{1}{4}$d.; $1\frac{1}{2}$ lb. syrup at $3\frac{1}{4}$d.; 3 bundles of chips at 6d. a dozen; 1 qt. of paraffin oil at 8d. a gallon.

2. 1 doz. towels at 9d. each; $\frac{1}{2}$ doz. buttons at 9d. a dozen; 4 yds. of braid at $1\frac{1}{2}$d. a yard; 2 doz. yds. of calico at $6\frac{1}{2}$d. a yard.

3. Altering equipment, 5s. 10d.; strap to bottle and two swivels, 3s.; 2 pairs leather boots at 18s. 8d., £1 17s. 4d.; best leather belt, 7s. 9d.; service tunic, £1 3s. 8d.; service lantern, 5s. 10d.; alterations to coat, 7s. 6d.

4. Prepare an invoice and then a statement of the following account, the latter being dated 1st February 1913: 8th Jan. 1913. Messrs. Jones & Co. Ltd., Corporation Street, Manchester, bought of Lewis & Co., 11 Nairn Court, Bolton. 15 pieces grey calico, 900 yds. @ 6d.; 10 ditto, 600 yds. @ $5\frac{1}{2}$d.; 5 ditto, 350 yds. @ $5\frac{3}{4}$d.; 12 doz. gents' handkerchiefs @ 10s. per dozen. Total ( ). Packing charges, 6s. 6d. Discount, 5 per cent. Sent per L. & Y. Ry.

5. Make out an invoice for the following articles, bought by Henry Jones from Thomas Watson, greengrocer and poultry dealer, Chester: 1 turkey, weighing 20 lb. 8 oz., at 10d. per lb.; 3 bushels of potatoes at 1s. 2d. per peck; 20 eggs at 8 for 1s. Receipt the bill, allowing 5 per cent. for cash.

6. Make out in proper form an invoice having the following items: Purchaser, Mrs. Slater; tradesmen, Messrs. William Smith & Co., drapers and outfitters, 15 Corporation Street, Mixham; date of purchase, Feb. 14, 1914; articles bought, $12\frac{1}{2}$ yds. of cloth at 5s. 9d. per yard; 18 yds. of flannel at 1s. 7d. per yard; 20 yds. of silk at 7s. $3\frac{1}{2}$d. per yard; 22 yds. calico at $8\frac{3}{4}$d. per yard; 3 doz. reels of cotton at $2\frac{1}{2}$d. per reel; 3 gross hat-pins at 1s. $1\frac{1}{2}$d. each pin.

7. Make out a complete set of documents for the last transaction from order note to receipt for cheque. The debit note should allow 5s. for returns, and the credit note for the 3 doz. reels of cotton. Render statement on 31st March, presuming £2 2s. was paid on account on 1st March.

8. Prepare an invoice for M. Calm of Rouen, quoting in the

[1] The student will find that questions often run, "Make out A BILL for the following." We shall always use the term "Invoice" and reserve "Bill" for a "Bill of Exchange" (Section XXI. C. and Plate XIII., p. 468).

French system: 5 cwt. iron nails @ 30s. cwt.; 300 ft. iron 1″ tube @ 3d. ft.; 28 ft. brass 1″ tube @ 6d. ft. (Take 1 cwt. = 50·8 Kgm.; 1 ft. = 0·305 metre.)

9. Prepare an invoice for 2 boxes Sultanas, 2 qrs. 10 lb. gross, @ 15s. per qr., tare 10 lb.; 3 chests tea, 36 lb. each net, @ 2s. 6d. lb.; 6 boxes raisins, 1 cwt. 2 qrs., @ £3; 3 bags East Indies rice, 4 cwt. 2 qrs. 10 lb., £8 10s. Receipt the invoice, and allow 5 per cent. discount at a month.

10. Make out an invoice for: 8 drums Turkey figs, 1 cwt. 26 lb., @ £2 10s. per cwt.; 5 bales coffee (Mocha), 2 cwt. each gross, @ £9 per cwt., tare 1 qr.; 3 bales cinnamon, 96 lb. each gross, tare 6 lb. each, @ 1s. 6d. lb.; 6 barrels prunes, 2½ cwt. each gross, 56 lb. tare, @ 6d. lb.; 8 gallons honey, of 12 lb. per gall., gross 140 lb., tare 40 lb., @ 1s. per lb. F.o.b. to Lévi Frères of Calais from Jacob & Co. of Shoreditch. Discount, 2½ per cent. at 2 months.

11. Write a letter as from the Calais firm to the London firm, in French if you can, complaining that the prices are in English money, and then write a letter of apology and render a statement in francs and centimes, at 25 francs to the £1.

12. An amateur can erect a very good wireless installation and make the necessary apparatus at the following cost: Aerial earth-wires and insulators, 5s. 6d.; loose coupled inductance, 3s. 6d.; tubular variable condenser, 2s. 6d.; 5-cup rotary detector, 1s. 6d.; potentiometer, 2s. 6d.; telephone, 3s. 6d.; 5-way earthing switch, 2s.; block condenser, 2s. Make out an invoice for this, supposing that the amateur maker sells each article at a third more than it cost him. (The price in a shop would be about £5, but the above *was good enough* to get time signals from Paris.)

13. Messrs. Lakin are stocktaking. Prepare a notice such as might be sent out to Williams & Co., notifying them of this, and enclose a statement of the following: Aug. 17, £1 7s.; Aug. 26, £1 3s.; Sept. 26, 13s. 10d.; Oct. 13, £2 8s. 4d.; Nov. 13, £2 18s. 3d.; Dec. 3, £1 2s. 11d.; Dec. 14, 1s. 8d.; and a payment of £5 on 1st December.

14. Prepare a statement of the following and allow an item Dec. 5, by returns, £7 0s. 10d: Oct. 3, £3 1s. 11d.; Oct. 5, £2 14s.; Oct. 19, £1 18s. 11d.; Oct. 19, £3 14s. 3d.; Oct. 23, £9 5s. 8d.; Oct. 24, £7 13s.; Nov. 1, 9s. 10d.; Nov. 21, 9s.; Nov. 24, £1 3s. 8d.; Dec. 1, £10 7s. 7d.; Dec. 4, £5 2s. 5d.; Dec. 5, £1 6s. 7d.; Dec. 15, £1 4s. 9d.; Dec. 29, 18s.

15. Invoice the following: R. Johns, Sons, & Brother, importers of tea, coffee, colonial produce, 1596 Fent Street, W.C.

Established in the sixteenth century. No. 8156. June 15, 1915. 2 cwt. soda @ 2s. 9d. cwt.; 56 lb. white salt @ 16s. cwt.; 3 lb. white pepper @ 1s. lb.; 1 cwt. lump sugar, £1 16s.; 1 doz. qts. each, orange, vanilla, and lemon jellies @ 5s. dozen.

16. Invoice the following: 1 case lump sugar, 1 cwt. 18 lb. gross, 18 lb. tare, @ 19s. 3d. cwt.; 1 cwt. castor, 1 cwt. 8 lb. gross, 19s. 6d. cwt., tare 8 lb.; 1 case sardines, 100 tins @ 8s. 6d. dozen; gross, 1 cwt. 2 lb.; 3 × 7's coffee and chicory @ 10d., tare 8 lb.; 76 lb. rolled oats @ £1 1s. cwt., tare 20 lb. Per L.S.W. Ry. @ 2s. 6d. cwt. Carriage paid. Discount, 2½ per cent. at 1 month.

17. Invoice the following: Invoice No. 8561. John Williams, Redcross sanitary wadding mfrs., Bolton-under-Water. 1 only best No. 86 @ 11s. 6d. dozen; 1 only each of No. 89 @ 17s.; No. 91 @ £1 0s. 9d.; No. 94 @ £1 4s. 3d.; No. 96 @ £1 6s.; 1 piece white woven No. 169 @ 1s. 1d.; 40 yds. white mull No. 481 @ 2¾d.

(*Note.*—1 only at 11s. 6d. doz. is charged 11d., the 6d. of the doz. price being usually ignored.)

18. See Plate VIII. opposite.

19. From the following details prepare a complete set of documents, ruled, in proper fashion, beginning with a letter from the biscuit makers soliciting an order, to the receipt, and a polite letter trusting to receive further orders. The student should not do this in a slipshod way, but should give time and trouble to it. "Telegrams, ABC West. 'Phone, 000 Westminster. Works, London and Edinburgh; Black Wharf, West Southwark, Blackdown, N.E. Sellers, Pundit & Co. Buyers, Wendon & Co. Ltd., 6 Tint Street, Hampstead, N.W. 15/8/1 . Terms cash. Sent by van No. 613. Carter, Brown. Statement rendered on 30/9/1 , nothing having been paid. Credit note 28/8/1 allows for 8 tins charged. All empties when returned should be advised by post; no allowance made until actually received. The tins of other manufacturers cannot be credited. 1/8 boudoir, 4 lb., rate 104s., 3s. 9d.; 1/8 cracknel, 3 lb., rate 117s., 3s. 2d.; 1/8 oval digestive, 7¼ lb., rate 72s., 4s. 8d.; 1/8 ½d. gingerbread, 6 lb., rate 48s., 2s. 7d.; 1/8 mow creams, 10¼ lb., rate 58s., 5s. 4d.; 1/8 macaroons, 6½ lb., rate 60s., 3s. 6d.; 2/8 shortbreads, 9½, 9¾, 19¼ lb., rate 58s., 10s.; 8 tins @ 6d. = 4s. (Kindly note we regret we do not make ½d. sponge cakes.) Before signing we should esteem it a favour if customers would please see that all advertising matter entered above is delivered. To avoid trouble and inconvenience to customers through empties accumulating

on their premises, our carters have instructions to ask for empties on all occasions when delivering goods."

(*Note.*—Rate = price per cwt. ; an 8-lb. tin does not always contain 8 lb. of biscuits ; hence the weights given.)

20. The following is an actual copy of a solicitor's charges on winding up an estate. The names are of course imaginary. Prepare a properly arranged account. "The executors of the will of P.Q.R., Esq., deceased, to S.M., solicitors. April 25. Attending executors and giving instructions for Probate, 6s. 8d.; perusing will (fos. 7), 2s. 4d.; copy to keep, 2s. 4d.; oath to executors, 6s. 8d.; marking will as exhibit, 1s.; affidavit for Inland Revenue, 6s. 8d.; making calculation of value of freehold property and drawing a fair copy schedule thereof, 10s. 6d.; making copies of will, 4s. 8d.; making two copies each of two letters, 2s. April 27. Engrossing and collating will, 10s. 6d. May 7. Probate under seal, 3s.; extracting, 6s. 8d.; clerk's fee, 2s.; letters, messengers, etc., 5s.; Disbursements—commissioner's fee, 7s.; estate duty, £2 10s.; Probate Court fees, 15s. 1d. July 10. Preparing conveyance of house and generally winding up the estate, £6 6s.; Disbursements—stamp duty on conveyance, 11s.; Messrs. Y. & Z.'s charges, £1 11s. 6d."

21. Make out an invoice for the following: Mr. P. A. Bardell, physician and surgeon, for professional services to E. L. Jacobs, Esq., 18th January to 27th February, £ . Details of account as they might appear when abstracted from the doctor's record (he merely sends Mr. J. the account for the sum). Jan. 18, 2 visits, 5s. 6d. each; Jan. 19, 2 visits, 5s. 6d. each; Jan. 20, 1 visit, 5s. 6d.; consultation with Dr. M., £1 10s. 6d.; consultation with Dr. N., £1; consultation with Dr. P., 10s. 6d; Jan. 21, 3 visits, 5s. 6d. each; Jan. 22, 2 visits, 5s. 6d. each; 1 night visit, 11s.; Jan. 23, 2 visits, 5s. 6d. each; Jan. 24, 1 visit, 5s. 6d.; Jan. 25, 1 visit, 5s. 6d.; administration of chloroform (Dr. P.), £1 1s.; operation (Dr. M.), 25 guineas; Jan. 25, night visit, 11s.; Jan. 26, 3 visits, 5s. 6d. each; Jan. 27, 2 visits, 5s. 6d. each; Jan. 28, 29, 30, 31, 2 visits on each day, 5s. 6d. each; Feb. 1 to 8, 1 visit each day, 5s. 6d. each; Feb. 10 to 12, 1 visit each day, 5s. 6d. each; Feb. 15 to 21, 1 visit each day, 5s. 6d. each.

22. Make out a complete set of documents for the following: Vendors, A. Benks & Co. Ltd., Uphill Street, Boxtow, E. Buyers, J. P. Dooke Ltd., North Road, Chicker, W. Nett cash. Accounts can be paid to our credit at any branch of the Loan and Wick Bank. 18th January. Tgms., Benks: ABC Code, 4th and 5th edns. 'Phone, Private Branch Exch., 0610 Bank.

Jams, Jellies, Marmalade, Pickles, Sauces, Spices, Rice. Guaranteed pure in accordance with the Food and Drugs Act. 2 lb. nutmegs 80, 9½d. lb., 1s. 7d.; 1 box clnd. Sultanas B., 59s. 6d. cwt., 13s. 3½d.; 1 box clnd. currants B., 36s. 6d. cwt., 13s. 0½d.; 2 × ½ boxes Valencias, 49s. cwt., 12s. 3d.; 7 lb. med. c'nts., 38s. 6d. cwt., 2s. 5d.; 28 lb. amber syrup, 17s. 6d. cwt., 4s. 4½d.; 1 lb. whole cinnamon, 1s. 9d.; 1 lb. whole mace, 2s. 5d.; 9-lb. keg fine mustard, 8¼d. lb., 6s. 2¼d.; 8 × 7 lb. jars red currant jelly, 46s. cwt., £1 3s.; 8 × 7 lb. jars strawberry jelly, 33s. 6d. cwt., 16s. 9d.; 32 × 7 lb. jars marmalade, 23s. cwt., £2 6s. To follow—2 × 7 lb. boxes mixed peel, 48s. cwt. 1 tin ground mixed spice, 7d.; 1 bush. No. 2 pearl peas, 14s. 6d. To follow—1 doz. qts. lemon table jellies; 1 doz. qts. orange table jellies; 1 doz. qts. champagne table jellies. 1 only gall. best B.B. ink, 2s. 9d.; 1 jar, 6d.; ctge., 3d. cwt., 3d.; 1 drum, 3s.; 48 jars, 3d. each, 12s.; 1 bag, 4d.

23. Make a complete set of forms for the following invoice, allowing 2½ % at 1 month. Date 18/12/–; statement 31/1/–. Casks, tins, etc., charged, £1 15s. Debit note for returns 21/12/–, 28s. (This would be for returns on a previous order, of course, for the buyer would not keep the carman waiting while he emptied out castor oil and paint into his own cans.) Accounts paid by order cheque crossed on Alpha Banking Co., dated 8/2/–. Purchasers, The Weatherby Manufacturing Co. Ltd. Vendors, Messrs. Banning & Marsden, drysalters, etc., of Lowndes Square, Blackburn. Amount of invoice paid to John Gee for the vendors. Oct. 1, 14 lb. black paint at 24s. per cwt.; 4 galls. boiled oil at 2s. 6d. per gall.; 1 gall. turpentine at 2s. 6d. per gall. Oct. 16, 42 lb. castor oil at 3½d. per lb. Oct. 28, 1½ cwt. French glue at 50s. per cwt.; ¾ firkin soft soap at 9s. per firkin. Nov. 1, 2½ doz. No. 6 English sash tools at 7s. 6d. per doz. Nov. 5, 3 cwt. Calvert's 15 % disinfecting powders at 13s. per cwt. Nov. 11, 2½ cwt. white lead at 21s. per cwt.; ¾ cwt. red lead at 21s. per cwt. Nov. 15, 2¼ galls. turpentine at 2s. 6d. per gall. Dec. 3, 1 quart colza oil, 10d.; bottle 2d. Dec. 15, 5½ cwt. Cop starch at 24s. cwt. Dec. 16, 32 lb. English tallow at 21s. per cwt.; 3 bottles oxalic acid solution at 1s. 4½d. each.

D. The Subsidiary Books of a Firm

1. THE PURCHASES BOOK

171. In § 160 we saw that Wm. Towney ordered cloth from John Nelson which was duly delivered. In order that the firms concerned may keep a record of this transaction, each of them has—

1. A Purchases Book; and
2. A Sales Book.

Immediately Wm. Towney receives the goods, he makes an entry in his Purchases Book, thus:

WM. TOWNEY'S PURCHASES BOOK.

Number of Invoice.	Date.	Particulars.	Ledger Folio.[1]	Details.			Totals.		
				£	s.	d.	£	s.	d.
125	July 10.	John Nelson	10						
		Cloth .	. .	10	0	0	10	0	0

Suppose, further, that on 15th July he had bought from J. Williams, linings, £5, velvet, £3; from P. Ross, silks, £150; from M. Jones, serge, £50, silks, £35, linen, £62; the entries for the day would have been:

PURCHASES BOOK.

Number of Invoice.	Date.	Particulars.	Ledger Folio.	Details			Totals.		
				£	s.	d.	£	s.	d.
210	July 15.	J. Williams	56						
		Linings .	. .	5	0	0			
		Velvet .	. .	3	0	0	8	0	0
211	,,	P. Ross	40						
		Silks . .	. .	150	0	0	150	0	0
212	,,	M. Jones	30						
		Serge .	. .	50	0	0			
		Silks . .	. .	35	0	0			
		Linen .	. .	62	0	0	147	0	0
				305	0	0	305	0	0

2. THE SALES BOOK

172. In just the same way, John Nelson would make an entry in his Sales Book thus:

JOHN NELSON'S SALES BOOK.

Number of Invoice.	Date.	Particulars.	Ledger Folio.	£	s.	d.	£	s.	d.
125	July 10.	Wm. Towney	45						
		Cloth . .	. .	10	0	0	10	0	0

[1] "Ledger folio" means the page in the ledger in which John Nelson's account will be found. See Section XXIX.

The student will see that **in this case** the two books are exactly the same in FORM and differ only in detail; but often they differ in form, see § 176.

173. Let us now take, as a more difficult illustration, the case of Walter Ralegh, tobacconist, whose purchases are as follows:

Jan. 10. Bought of M. Hawes, tobacco £10, cigars £100.
,, 12. ,, ,, P. Roberts, cigars £56.
,, 13. ,, ,, R. Dunn, cigarettes £100.
,, 14. ,, ,, M. Hawes, tobacco £80, cigarettes £60.
,, 15. ,, ,, R. Dunn, tobacco £50, cigarettes £50.
,, 18. ,, ,, P. Roberts, cigars £30, tobacco £90.

It is necessary to observe that we have here three distinct classes of goods, namely, tobacco, cigarettes, and cigars, and that the working of the business will be facilitated by keeping the three accounts separate both in the Purchases and in the Sales Books. Hence we have:

PURCHASES DAY BOOK[1] OF WALTER RALEGH.

Number of Invoice.	Date of Invoice.	Name (and sometimes Address).	Ledger Folio.	Total.	Tobacco	Cigarettes.	Cigars.
				£ s. d.	£ s. d.	£ s. d.	£ s. d.
1	Jan 10	M. Hawes .	28	110 0 0	10 0 0		100 0 0
2	,, 12	P Roberts.	58	56 0 0			56 0 0
3	,, 13	R Dunn .	10	100 0 0		100 0 0	
4	,, 14	M. Hawes .	28	140 0 0	80 0 0	60 0 0	
5	,, 15	R. Dunn .	10	100 0 0	50 0 0	50 0 0	
6	,, 18	P. Roberts.	58	120 0 0	90 0 0		30 0 0
				£626 0 0	£230 0 0	£210 0 0	£186 0 0

174. From time to time he has SOLD goods and has entered the details in his Sales Book as follows:

SALES BOOK[1] OF WALTER RALEGH.

Number of Invoice.	Date of Invoice.	Name (and sometimes Address) of Customer.	Ledger Folio.	Total.	Tobacco.	Cigarettes.	Cigars.
				£ s. d.	£ s. d.	£ s. d.	£ s. d.
56	Jan. 18	T. Brown .	6	110 0 0	25 0 0		115 0 0
57	,, 25	S. Wilson .	100	130 0 0		130 0 0	
58	,, 26	T. Brown .	6	81 0 0	56 0 0		25 0 0
59	,, 26	S. Wilson .	100	145 0 0	30 0 0	15 0 0	100 0 0
60	,, 30	P. Huntley	35	37 0 0	25 0 0		12 0 0
60	,, 30	P. Huntley	35	160 0 0		150 0 0	10 0 0
				£693 0 0	£136 0 0	£295 0 0	£262 0 0

These books might be called respectively the "Analysed" Purchases and Sales Books.

175. Suppose, now, that on Jan. 1 the value of his stock was £500, we see that in the month he had purchased goods to the value of £626 and sold goods to the value of £693, while his residual stock we will value at £650.

His position is then as follows:

Goods on hand, Jan. 1	£500
„ bought, up to Jan. 31	626
Total value . .	£1126
„ sold, up to Jan. 31	£693
„ on hand on Jan. 31	650
Total value . .	£1343

Difference, £1343 − £1126 = £217.

Hence he is £217 better off, all other things being equal on Jan. 31 than on Jan. 1.

176. We can now see that EVERY FIRM must have a Purchases Book and a Sales Book **ruled in a manner suited to its business.** Thus, a large grocer might keep one set of books for tea, coffee, cocoa, and other similar goods, and another set for lard, butter, hams, cheese, eggs, and so on. A wine merchant would have columns for wines and wine duty, and for spirits and spirit duty in his Purchases Book, while in his Sales Book he would provide for sales (1) to hotels and licensed houses, and (2) to private customers, and also columns showing what was sold in bottles, what in gallons, casks, and so on.

The student should devise some Purchases and Sales Books for himself in such simple cases as those of a dairyman, (see Question 34, p. 460), a butcher, a newspaper agent, a cycle dealer, and so on.

3. THE RETURNS BOOKS

177. Again, in business it is often necessary to return goods because they are not in accordance with the invoice, or are not to sample, or are damaged, and hence we will suppose that Wm. Towney has found the black vicuna (§ 160) damaged and has returned it by rail and sent a debit note (see p. 230).

The cloth would have gone OUT FROM Wm. Towney's warehouse and *in to* John Nelson's;

∴ Wm. Towney's clerk would make an entry in his Returns

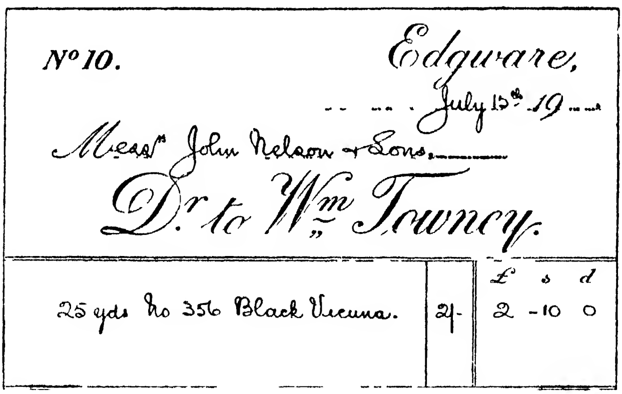
Outwards Book, and John Nelson's in his Returns Inwards Book, and he would also send Wm Towney a **credit note** (Plate VIII., p. 224) for £2 10s., and deduct that amount when sending the next statement to him.

The details are:

WM. TOWNEY'S RETURNS OUTWARDS BOOK.

Number of Credit Note	Date	Particulars (of Goods sent back)	Ledger Folio	£	s.	d
25	July 15	John Nelson, Black Vicuna		2	10	0

Debit note sent by Wm. Towney to John Nelson:



No 10. Edgware,
July 15th 19..

Messrs John Nelson & Sons.
Dr. to Wm Towney.

		£	s	d
25 yds No 356 Black Vicuna.	2/-	2	10	0

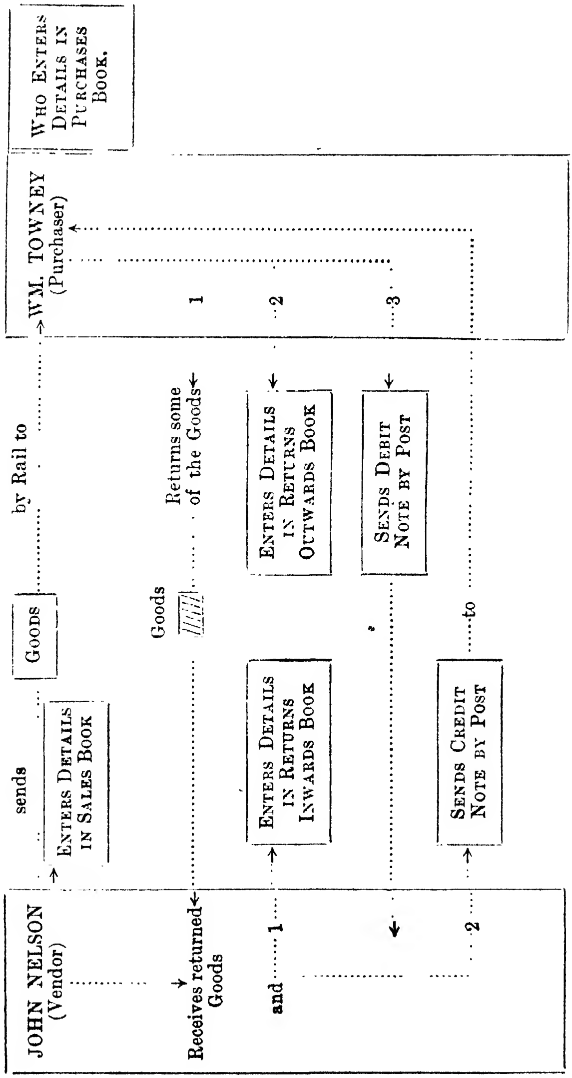
The entry in John Nelson's Returns Inwards Book would run:

"Wm. Towney, Black Vicuna," etc.,

while the credit note would be in red ink, or on red paper, and run:

"Messrs. Wm. Towney Credited By John Nelson & Sons, 25 yds., No. 356," etc. (Cf. Plate VIII., p. 224.)

Diagrammatically we have—



4. THE SIMPLE CASH BOOK

178. A great deal of business is done on credit, a smaller amount is treated on the cash system, and so a Cash Book is necessary.

If the cashier receives £5 for goods supplied, he enters it on the Cash Received or Left Hand or Dr. side of his Cash Book, for he is now a debtor to the business for £5.

If, on the other hand, he pays £3 10s., he enters this on the Cash Paid or Right Hand or Cr. side of the book, thus—

Enter up the following items in the Cash Book:

			£	s.	d.
(1)	Jan. 10.	Paid for carriage of goods .	0	15	0
(2)	„ 11.	Received for goods . .	1	10	0
(3)	„ 11.	John Thomas paid . .	6	0	0
(4)	„ 12.	Tom Brown paid . . .	1	10	0
(5)	„ 12.	Paid wages	0	15	0
(6)	„ 13.	Paid W. Wilkins' % . .	2	18	0
(7)	„ 15.	Received for goods . .	0	10	0

Dr. CASH BOOK. *Cr.*

Date.	Particulars of Receipts.	Ledger Folio.	£	s.	d.	Date.	Particulars of Payments.	Ledger Folio.	£	s.	d.
Jan. 11.	To Cash for goods (2).	7	1	10	0	Jan. 10.	By Carriage of goods (1)	10		15	0
„ 11.	„ John Thomas (3) .	3	6	0	0	„ 12.	„ Wages (5) . .	15		15	0
„ 12.	„ Tom Brown (4) .	6	1	10	0	„ 13.	„ W. Wilkins (6) .	18	2	18	0
„ 15.	„ Cash for goods (7).	7		10	0		„ Balance carried forward . .		5	2	0
			9	10	0				9	10	0
„ 17.	„ Balance brought forward . . (Cash in till)		5	2	0						

179. In the case we have chosen the receipts are greater than the payments,

∴ the cashier has cash in his till;

∴ **a balance on the Dr. side of the cash account means that there is cash in hand**; hence, if the account be made up each day, the cashier can check his receipts against his payments and the difference gives his cash in the till.

5. CASH BOOK WITH CASH AND BANK COLUMNS

180. All important firms have two cashiers, one whose office is on the premises, and the other, the banker, whose offices are elsewhere. The latter deals with larger amounts than the former, but it is convenient to keep a record of the cash receipts and payments and of the bank receipts and payments side by side in the Cash Book. We shall presume that all accounts over £10 are paid by cheque and those under that amount in cash.

Let us take the following:

			£	s.	d.	
Jan	16.	Received from John Thomas	250	0	0	(Bank item)
,,	17.	Paid W. Wilkins	150	0	0	(,, ,,)
,,	17.	,, wages	8	10	0	(Cash ,,)
,,	18.	Received from Tom Brown .	2	0	0	(,, ,,)
,,	18.	,, ,, W. Wilkins .	3	10	0	(,, ,,)
,,	19.	Paid for stationery . .	12	15	0	(Bank ,,)
,,	20.	Received from John Thomas	5	0	0	(Cash ,,)
,,	22.	Paid Tom Brown . . .	10	10	0	(Bank ,,)

Dr. CASH BOOK WITH BANK COLUMNS. *Cr.*

Date.	Particulars of Receipts.	Ledger Folio.	Cash received by Cashier. £	s	d	Cash received by Bank. £	s	d	Date.	Particulars of Payments	Ledger Folio	Cash paid by Cashier £	s	d	Cash paid by Bank. £	s.	d.
Jan. 16.	To John Thomas	3				250	0	0	Jan 17.	By W. Wilkins .	18				150	0	0
,, 18	,, Tom Brown .	6	[illegible]	0	0				,, 17.	,, Wages . .	15	8	10	0			
,, 18.	,, W Wilkins .	18	3	10	0				,, 19.	,, Stationery .	20				12	15	0
,, 20.	,, John Thomas	3	5	0	0				,, 22.	,, Tom Brown	6				10	10	0
										,, Balance carried forward		2	0	0	76	15	0
			10	10	0	250	0	0				10	10	0	250	0	0
,, 27.	,, Balance brought forward		2	0	0	76	15	0									

The result of the week's trading is then: Cash in hand £2, and in Bank £76 15s., and with these amounts the next week's business is begun.

6. CASH BOOK WITH CASH, BANK, AND DISCOUNT COLUMNS

181. Lastly, since discount[1] is often allowed on a cash payment, it is convenient to have discount columns in the Cash Book.

A discount ALLOWED by us to our customer is entered on the Dr. side, while a discount RECEIVED is entered on the Cr. side, in accordance with the maxim of book-keeping, "Debit losses, credit gains." Thus:

[1] See Section XX.

Jan. 1. Cash in hand, £50; cash in Bank, £500. Jan. 2. Paid Tom Brown, £20; discount, £2.[1] Jan. 3. Received from W. Wilkins, £5. Jan. 3. Paid John Thomas' account, £9 5s.; discount, 10s.[1] Jan. 4. Received from John Thomas, £25 discount, £2. Jan. 4. Paid for goods, £3 15s. Jan. 5. Paid for goods, £2 15s. Jan. 5. Received from Tom Brown, £250 discount, £5.

Note that the cash in hand, £50, and cash in Bank, £500, make up £550, which we shall call the "Capital" of the firm.

Dr. CASH BOOK WITH CASH, BANK, AND DISCOUNT COLUMNS. *Cr.*

Date.	Particulars of Receipts.	Ledger Folio.	Discount allowed.			Cash received by Cashier.			Cash paid into Bank.			Date.	Particulars of Payments.	Ledger Folio.	Discounts received.			Cash paid to Cashier.			Cash paid to Bank.		
Jan.			£	s.	d.	£	s.	d.	£	s.	d.	Jan.			£	s.	d.	£	s.	d.	£	s.	d.
1	To Capital a/c .	1				50	0	0	500	0	0	2	By Tom Brown	6	2	0	0				20	0	0
3	,, W. Wilkins	18				5	0	0				3	,, John Thomas	3	0	10	0	8	15	0			
4	,, John Thomas	3	2	0	0				25	0	0	4	,, Goods . .	7				3	15	0			
6	,, Tom Brown	6	5	0	0				250	0	0	5	,, do. . . .	7				2	15	0			
													,, Balance carried forward					39	15	0	755	0	0
			7	0	0	55	0	0	775	0	0				2	10	0	55	0	0	775	0	0
8	,, Balance brought forward . .					39	15	0	755	0	0												

The discount columns are added up at regular intervals and transferred to the Discounts Account in the Ledger. This will be understood after § 310 has been read.

[1] It is important to distinguish between "Paid Tom Brown, £20; discount, £2," and "Paid John Thomas' account, £9 5s.; discount, 10s." In the first case, £20 was actually paid, and £2 allowed you, so that the account was £22. In the second case, the account was £9 5s., and 10s. discount was allowed off that amount; therefore you actually paid £8 15s. cash, and this appears in the cash column, while the 10s. appears in the discount column. In both cases, cash and discount = amount of account.

7. PETTY CASH BOOK ON THE IMPREST SYSTEM

182. It is often convenient to entrust to a junior clerk the payment of certain small items such as stamps, gum, string, etc., and he is furnished with a Petty Cash Book in which to record his transactions. The system employed most largely now is called the Imprest System, in which a small sum of money, say £5, is paid over by the cashier to the junior clerk, who pays the small accounts out of that amount, and then, at the end of the day, or week, the cashier refunds to the junior the amount by which he is short of £5, *i.e.* the amount he has paid, say £3. The great advantage of this is that when the cashier draws a cheque for the £3 he records that on the Cr. side of the cash account, and in doing so records the amount of petty cash spent.

Dr. PETTY CASH BOOK—IMPREST SYSTEM. *Cr.*

Date.	Particulars.	Amount.	Date.	Particulars.	Amount.	Telegrams.	Stamps.	Travelling.	Office Requisites.	Sundries.
		£ *s.* *d.*			£ *s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*	*s.* *d.*
Jan. 1	To Cash (from cashier) .	5 0 0	Jan. 1	By Telegrams	0 10 0	10 0				
			,, 3	,, Fares . .	0 5 6			5 6		
			,, 3	,, Stamps .	0 15 0		15 0			
			,, 5	,, Office Requisites.	0 12 0				12 0	
			,, 5	,, Sundries .	0 6 0					6 0
			,, 6	,, Lunch .	0 2 6					2 6
				,, Balance c/f	2 9 0					
		5 0 0			5 0 0	10 0	15 0	5 6	12 0	8 6
Jan. 8	To Balance brought for.	2 9 0								
	,, cashier .	2 11 0								

EXAMPLES. XXXIX.

Make entries in the Purchases Day Book for the following (Questions 1 to 3), supplying names, dates, etc., as in § 171 :

1. Jan. 1. Bought from P. Ham, cocoa, 10s.; tea from J. Johns, £5; coffee from P. Head, £8. Jan. 3. Paper bags from R. Rate, £1 15s. Jan. 5. Oil, £3 18s. 6d., from L. Mann.

2. Feb. 4. Bought of R. Wilson, silk, £6 10s., and calico, 15s. 8d. Feb. 8. Of P. Wills, linen, £3 12s. 6d.; ladies' hose, 25s. 9d.; and ribbons, 28s. 11d.

3. July 12. Bought from T. Jones, tools, £5 2s. 6d.; B. Wilson, bar iron, £3 15s. July 14. T. Rolls, corrugated iron,

£8 15s.; R. Nett, nails, £1 2s. 6d. July 15. B. Wilson, 8-inch iron bolts, 25s. 9d.; T. Rolls, safety locks, £5 5s.

Enter the following in Sales Book, supplying details for yourself (Questions 4 to 6):

4. Sold D. Williams, jam, £6 15s.; R. Roberts, coffee, £5 10s.; R. Josephs, butter, £8 5s.; P. Nett, lard, £3 15s.

5. Sold P. Jacobs, castor oil, £3 10s.; L. Lyon, paraffin, £5 3s.; R. Rouse, petrol, £8 5s.; Q. Fipp, cyanide, £8 15s.; L. Mole, paint, £6 3s.; V. West, white lead, £6 5s.

6. Sold N. Thomas, flour, £8 15s.; R. Vaule, flour, B quality, £7 10s.; J. Brunt, cakes, £6 3s.; L. Mole, rolls, £1 3s.; T. Lovel, flour, A quality, £6 5s.; S. More, rice, £5 2s.; R. Lane, Patna rice, £3 5s.

Enter in Purchases or Sales Books the following (Questions 7 to 9):

7. Bought tea from R. Parker, £6 3s. on Jan. 5.
   ,, butter from R. Jones, £3 8s. on Jan. 7.
   Sold T. Roberts, tea, £3 10s. on Jan. 9.
   Bought cocoa from Jones & Co., £5 3s. 8d. on Jan. 12.
   Sold rice to Robins & Co., £3 2s. 8d. on Jan. 12.
   ,, cocoa to Jones & Sons, £3 4s. 6d. on Jan. 15.

8. July 8. Bought cycle frames from Brown & Co., £8 15s.
   ,, 10. Sold tyres to Wilson & Johns, £3 15s.
   ,, 11. ,, lamps to Jones & Co., £8 14s.
   ,, 12. Bought bells from T. Webb, £3 4s.
   ,, 14. ,, inner tubes from R. Wells, £5 8s. 6d.
   ,, 15. Sold valves to R. Wells, £3 5s. 8d.

9. Jan. 5. Bought from T. Brown, coal, £3 6s. 8d.
   ,, 5. Sold to R. Menel, coke, £6 2s.
   ,, 8. ,, P. Rolfe, coal tar, £3 5s.
   ,, 8. Bought from P. Wells, paraffin, £6 5s.
   ,, 10. ,, R. Jones, coal, £5 10s.
   ,, 10. Sold to V. Williams, coke (broken), £8 2s. 6d.
   ,, 11. ,, ,, ,, (unbroken), £3 5s. 6d.
   ,, 12 Bought from P. Wells, coal tar, £8 5s.
   ,, 15. ,, M. Winter, turpentine, £3 5s. 6d.
   ,, 15. Sold to R. Jones, Stockholm tar, £5 2s.
   ,, 16. Bought from P. Wells, iron pipes, £2 6s.

Prepare the proper debit and credit notes for the following transactions (Questions 10 to 14), and make the necessary entries in the Returns Books, and also make the Purchases and Sales Books entries:

10. Jacob Jones bought from R. Philp on Jan. 2, tarred rope, 2 in. in diameter, for £38, and returned one piece valued at £15.

11. Roger Head bought from P. Wills on June 30, 24 yards twill @ 6d. a yard; 40 yards silk @ 2s. a yard; 80 yards calico @ 4d. a yard, and returned 40 yards of the calico on July 3 as not to pattern.

12. Mary Rosen sold Jane Austen 20 serge skirts, £25; 30 flannel blouses, £7 10s.; 50 boxes of ribbons, £12 10s.; 18 yards silk, £2 10s., of which 10 blouses and 5 boxes of ribbons were returned.

13. H. Watson sold W. Prince 1000 bricks, 32s. 6d.; cement, 22s.; lime, 56s. 6d.; sand, £1 15s.; stone blocks, £8 15s.; cement blocks, £18 10s., of which the first and last items were returned as imperfect.

14. R. Wells bought from P. Johns one doz. cut-glass jugs, 21s.; one doz. liqueur bottles, 18s.; two doz. vinegar bottles, 30s.; two doz. pickle bottles, 30s.; three doz. fountain pens, 36s.; two copying presses, 60s.; of which one doz. pickle bottles and one and a half doz. pens were returned as not to invoice.

Enter the following items as in a Simple Cash Book, and bring down the balance (Questions 15 to 17):

15. Jan. 1. Paid for sundries, 5s.
,, 2. Paid R. Toms, £8; received from T. Johns, £5.
,, 3. Received from R. Toms, £6; paid R. Stone, £3 10s.
,, 4. K. Wills paid us £8.
,, 6. Paid carriage, 5s. 6d.
,, 6. Sold goods for cash, £3 10s. 6d.

16. Mar. 5. Paid wages, £5; received cash for goods, £3.
,, 6. R. Roberts paid £8.
,, 7. Received from K. Mar, £8 10s.; paid M. Trip, £5 5s.
,, 8. Paid carriage, £3 2s. 6d.; T. Rond paid £6 2s.
,, 10. Cash purchases, £2 5s.

17. June 5. Cash in hand, £10; paid wages, £6.
,, 7. Paid for cycle, £8; received cash for goods, £5 10s.
,, 8. Paid rent, £8 10s.; R. Tall paid £7.
,, 10. Received from J. Nott, £5 10s.
,, 11. Paid for coal, 30s.

Enter the following on sheets of paper ruled as a Cash Book with Bank Columns, treating all amounts of £10 and over as Bank items, and determine the amount of cash in hand and in Bank at the last date given in each question (Questions 18 to 20):

18. July 8. Cash in safe, £150; sold goods for cash, £5.

July 9. Bought coke, 50s.; bought tar, 30s.
„ 10. Received from P. Vail, £5; T. Roma paid £10.
„ 11. T. Pole paid £12 12s.; bought rope, £12 10s.
„ 13. Sold tar, £5 5s.

19. Aug. 10. Cash in hand, £250; in Bank, £280; cash sales, £8.
„ 11. Bought goods for cash, £8; paid R. Jacobs, £15.
„ 12. Sold goods for cash, £5; received from R. Jacobs, £12.
„ 17. Bought sundries, £14; sold goods for cash, £6.
„ 14. Received from B. Hall, £16; bought cigars, £2.

20. Sept. 26. Cash in hand, £500; in Bank, £500; paid T. Rove, £12.
„ 28. Bought goods for cash, £8; sold R. Veal goods, £10 10s.
„ 30. Bought coal, 50s.; paid gas bill, £5; paid rates, £6 6s.
„ 30. Received cash for goods, £17; P. Batey paid £3 15s.

Enter the following on sheets of paper ruled as a Cash Book with Cash, Bank, and Discount Columns, and find the cash in hand and in Bank, and determine whether the discounts allowed are greater than those received and by how much (Questions 21 to 23):

21. Oct. 1. Cash in hand, £500; in Bank, £750.
„ 2. Paid G. Fair, £50; discount, 15s.
„ 3. A. Ball paid £15; discount, £1.
„ 4. Purchases, £8.
„ 5. Received from C. Cox, £12; discount, 10s.
„ 6. Paid for hardware, £5; discount, 5s.

22. Nov. 8. Cash in hand, £250; in Bank, £180.
„ 9. Received from R. Lamb, cheque, £15; discount, 15s.
„ 10. Paid for sundries, 50s.; discount, 1s.
„ 12. Bought goods for cash, £6; discount, 6s.
„ 13. T. Jones paid £25; discount, 10s.

23. Dec. 12. Cash in hand, £800; in Bank, £1000.
„ 13. R. Faith paid £8 10s.; discount, 10s.
„ 14. Received from F. Thomas, cheque, £16 10s.; discount, £1.
„ 15. Paid V. Waters, cash, £26; discount, 10s.
„ 16. „ T. Raven, cheque, £25; received B. Rogers' cheque, £12.
„ 17. „ wages, £8 8s.; sundries, 15s.

Dec. 18. Paid office expenses in cash, £15.
„ 19. „ into Bank, £18 cash.
„ 20. Drew cheque for office expenses, £12.

Draw up a petty cash account on the Imprest System in each of the following cases (Questions 24 to 26), and determine for how much a cheque must be drawn by the cashier to reimburse the clerk the amount spent:

24. Jan. 8. Clerk receives £5 from cashier, and spends 5s. stamps; 3s. gum.
„ 9. 2s. 6d. brown paper; 8s. 6d. envelopes; 3s. 6d. Roberts' fares.
„ 10. 2s. 6d. carbon paper; 4s. 6d. cyclostyle ink.

25. June 15. Clerk receives £6 from cashier, and pays 3s. 6d. telegrams; 8s. sundries.
„ 16. 3s. string; 2s. 6d. stationery; 8d. fares.
„ 17. Office cleaning, 3s. 6d.; teas, 2s. 6d.

26. Aug. 10. Clerk receives £10 from cashier; telegrams, 5s.; 'phone, 6d.; ink, 1s.
„ 11. Stamps, 3s. 6d.; G.W.R. carriage, 15s.
„ 12. Wire to Leicester, 1s. 6d.; fares, 3s.
„ 13. Parcel post, 2s. 6d.; teas, 3s.
„ 14. Repairs to motor, 8s.; order book, 2s. 6d.
„ 15. Foolscap, 4s. 6d.

SECTION XII

LENGTHS

A. Length

EXAMPLES. XL.

1. Measure the length of a sheet of foolscap, (1) in inches, (2) in centimetres, and (3) find how many centimetres there are to an inch, to one place of decimals.

2. Measure the width of the sheet of foolscap used for the previous question (1) in inches, (2) in centimetres, and (3) find the number of centimetres in an inch, to one place of decimals.

3. What is the average of the results of parts 3 in the last two questions?

4. Mark off 4 feet on a table or a counter, and with the help of a metre scale find the number of metres and centimetres in that length and then the number of centimetres in 1 foot.

5. The wheel of a taxi is 2 ft. 7 in. in diameter, how many centimetres is the centre of the axle above the ground?

6. Express the length of this page as the decimal of a metre.

7. A chain has 100 links and is 22 yards long, what is the length of a link in inches and in centimetres?

8. Mark off 20 yards, measure the distance in metres, and find the number of inches in a metre.

9. How many miles are there in 100 Kilometres?

10. How many decimetres are there in 18 inches?

11. How many yards and the decimal of a yard are there in a metre?

12. How many yards are there in a Kilometre and in a mile?

13. How many Kilometres are there in a mile?

14. How many furlongs are there in a Kilometre?

15. Express 100 yards as the decimal of 1 mile.

16. How many metres does a runner sprint in a hundred yards race?

17. What is the difference between 100 yards and 100 metres? Which is the greater?

18. The 220 yards running record for 1913 was $21\frac{3}{5}$ seconds; how many feet per second were covered by the runner (W. R. Applegarth)?

19. In the circumstances of the last question how many metres per second were covered?

20. W. J. Bailey won the mile cycling championship in 2 min. $45\frac{4}{5}$ secs.; how many metres per second did he ride?

21. Express half a mile in metres, and determine what fraction it is of a Kilometre.

22. The radius of the revolving plate of a gramophone is $10\frac{1}{2}$ inches, can a French record 0·23 metre in diameter be used upon it?

23. What is the cost of 600 fathoms of rope at $7\frac{1}{2}$d. a foot?

24. A fleet of 15 pilchard nets costs £60, and each net is 120 yards long, what is the price per yard?

25. A fleet of mackerel nets is $1\frac{1}{2}$ miles long and costs £80, what is the cost per fathom?

26. A skein of cotton is 120 yards long, find the value of 3 dozen skeins if 432 yards are worth $4\frac{1}{2}$d.

27. If a hank of cotton consists of 7 skeins, calculate from the data of the previous question the value of a hank.

28. A wrap of worsted is 80 yards long, find the value of a hank of seven wraps if 28 yards are worth $1\frac{1}{4}$d.

29. The Argentine Ministry of Marine requires sea-going vessels carrying fifty or more people to be provided with a radio-telegraph installation capable of transmitting 500 Kilometres. Express this distance as a decimal of the distance from London

to Edinburgh, 330 miles. (The distance BY RAIL is 392 miles, but radiograms travel as the crow flies.)

30. In the Olympic Games, instead of having a quarter-of-a-mile race, as is usual in English athletic sports, they have one of 400 metres. Find in centimetres the difference between the two distances. (Answer to the nearest centimetre.)

31. Compare the speed of a steamer going 15 knots with that of a runner who does a quarter of a mile in 55 seconds.

32. When buildings are being erected it is usual to put a hoarding about 12 feet high around them in order to protect both them and also the passers-by. If the circuit of such a building be 2 fur. 6 chs. 15 yds., how many 9-inch boards will be required to enclose it properly?

33. Telephone and telegraph wires are placed underground in earthenware pipes, and canes screwed end to end are passed through the pipes, and then the wires are hauled through. If the canes are 6 feet long, how many must be carried to pass from end to end of pipes 1 fur. 3 chs. $10\frac{1}{2}$ yds. in length?

34. Express the height of Mount Everest, 29002 feet, in miles and the decimal of a mile.

35. What is the height of Mount Everest in Kilometres?

36. The greatest depth of the North Sea is 1998 feet, what is this depth as the decimal of a mile?

37. Express the depth of the North Sea in fathoms.

38. The Amazon is 4000 miles long and the Congo 3000 miles. Express the difference between their lengths in Kilometres.

39. Dover to Folkestone is 10 Km.; Dover to London is 105 Km.; by how many miles is the latter distance greater than the former? (To two decimal places.)

40. Berlin to Cologne is 596 Km.; Berlin to Danzig is 490 Km.; express the latter as the decimal of the former.

41. Calais to Paris is 295 Km., and Calais to Plymouth is 430 Km.; express in miles the difference between these distances.

42. (1) London to Paris (*via* Newhaven) is 244 miles; (2) London to Paris (*via* Folkestone) is 254 miles; (3) London to Paris (*via* Dover) is 283 miles. How much farther is it to travel by route (3) than by either of the others? (Give the result in Kilometres.)

43. A number of luggage trucks are of the following lengths: 3 yds. 2 ft. 6 in.; 4 yds. 5 in.; 8 yds. 1 ft. 10 in.; 12 ft. 6 in.; $20\frac{1}{2}$ ft.; $5\frac{1}{4}$ yds. The length of chain connecting one truck with the next is 2 ft. 8 in. What is the length of the train?

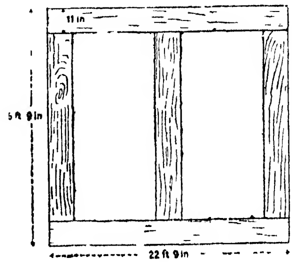
44. If the average length of a railway coach is $42\frac{1}{2}$ feet, how many of them can be placed on a siding $28\frac{1}{4}$ chains in length?

45. Train rails 12 ft. 6 in. long are laid along a track, and

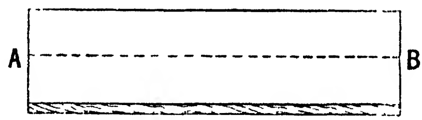
three-eighths of an inch is left between them to allow for expansion; how far will 100 such rails extend?

46. A number of allotments are 8 yds. $2\frac{1}{2}$ ft. wide. How many can be set out on a piece of land 30·25 chains wide?

47. A house-breaker has to erect a scaffold in front of a building which he has contracted to pull down, and it has to support a small crane. If he employs wood 11 inches square, find the total length required from the details given in the diagram, which represents, in elevation, one of the THREE pieces of the supporting framework.

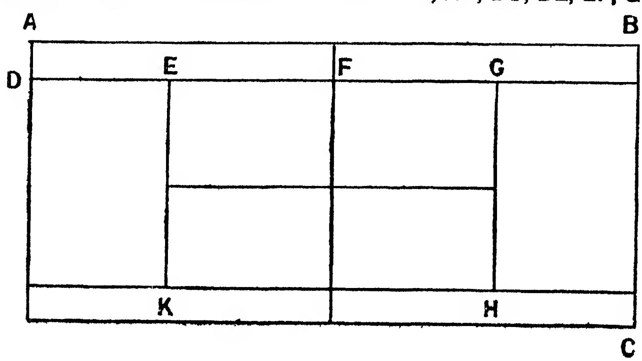


48. The figure represents one-half of the hinge of a door half full size. Find, by measuring, the length and width of the hinge. Place four dots in the diagram along the line AB equi-distant from one another and from the top and bottom, to represent the centres of the screw-holes, and find the distance between the centres.



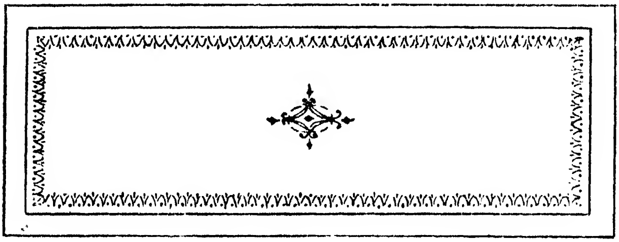
49. What are the length and width of the hinge in Question 48 in centimetres?

50. The figure represents a tennis court to the scale of $\frac{1}{2}$ inch = 12 feet. Find the dimensions of the court, AB, BC, DE, EF, GH.



51. Draw to the scale of 1 inch = 20 yards, a "rugger" ground 110 yards long and 75 yards wide.

52. Figure represents the top of a mahogany table and the inner rectangle a piece of green leather stuck to it.



Given that the scale is $\frac{1}{2}$ inch = 1 foot, find the dimensions of the table and of the leather.

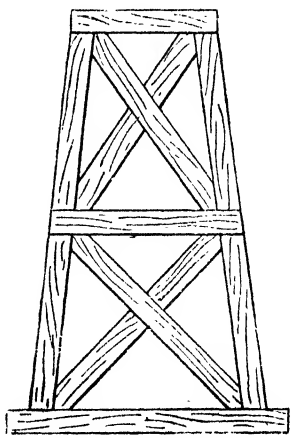
53. What fraction of the table referred to in Question 52 is covered with leather? (Work this after reading Section XIII.)

54. The length of a thermometer tube is 15·76 cm. and its diameter is 0·3 cm. Represent this full size, regarding the bulb as a circle 0·25 cm. in radius.

55. Find, by measuring, the length over all in the last question, expressing the result in inches and the decimal of an inch.

56. What is the perimeter (*i.e.* the distance round) of the tennis court referred to in Question 50 and of the rectangle EGHK?

57. A small footbridge is carried over the railway lines between two suburban stations. The elevation of the supporting framework is shown in the diagram, and there are five such supports. Determine, by measuring, the total length of timber required, ignoring any overlapping.



Scale, $\frac{1}{10}$ in. = 1 ft.

58. The columns which support the architrave of a building in Trafalgar Square are made of seven pieces of stone 4 ft. $9\frac{1}{2}$ in. high. The base of each column is 1 ft. 3 in. and the capital 2 ft. 6 in. high. What is the height of the architrave from the pavement?

59. How many metres does one of the forwards run in a game of "rugger" if he gets the ball on the centre line and scores a try, presuming he runs roughly in a straight line? (Question 51.)

60. A cable's length is 600 feet. Express this length in metres for the benefit of continental captains.

61. A piano is 4 feet high, 2 feet deep, and 5 feet long. Find in metres and the decimal of a metre, the dimensions of a rectangular case for sending the piano abroad, allowing 1 inch clearance all round the instrument for packing material.

62. Convert the following lengths into centimetres. Details for motor cycle made by a leading English firm: Height from base to rocker, $23\frac{3}{4}$ inches; width of crank case, $3\frac{1}{2}$ inches; length between perpendiculars, 12 inches; deepest cooling rib, 1 inch; induction pipe diameter (inside), $1\frac{1}{4}$ inch; carburettor stub diameter, $1\frac{1}{2}$ inch; diameter of fly-wheels, $8\frac{1}{2}$ inches; saddle from ground, 31 inches.

63. The plan given in diagram shown on page 245 represents to the scale of $1'' = 100$ miles the distances by rail from London to the great commercial centres and ports. Express the distances to the nearest mile. (The lines are drawn straight for convenience.)

64. The normal gauge in Spanish railways is 1·67 metre and that on English railways is 4 ft. $8\frac{1}{2}$ in. How many times is the Spanish gauge greater or smaller than the English?

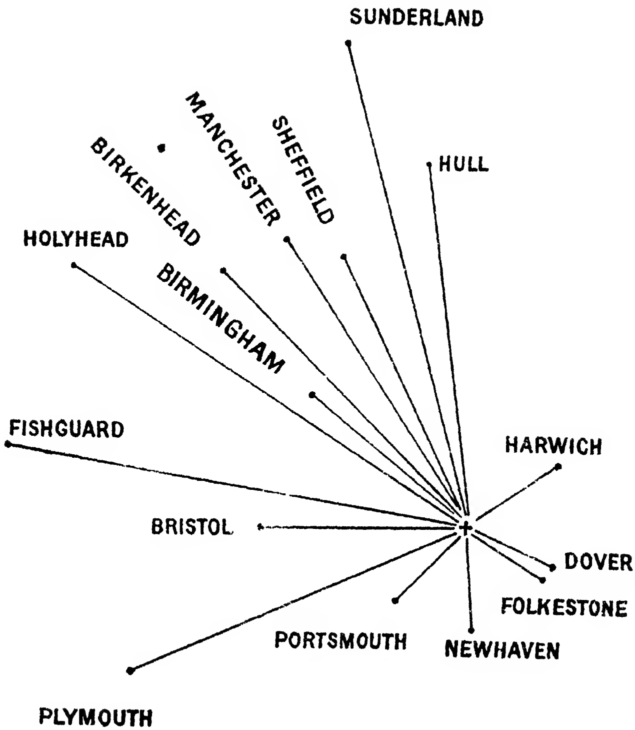
65. In a particular tender, engines have to be supplied with a speed of 50 Kilometres per hour; convert this into miles per hour for the information of a Leeds firm.

66. A map is drawn to the scale of 10 miles to $\frac{1}{4}$ inch, what is the actual distance between two places which are (i) 3 inches, (ii) 8 inches, and (iii) $6\frac{1}{2}$ inches apart?

67. The distance from London to Birmingham is 110 miles, and on a railway map we find it to be 1·32 inches. How many statute miles are represented by 1 inch, and how far are two places apart if they are separated by 4·8 inches on the map?

68. There are three railways which connect London and Liverpool; the run on one is $200\frac{3}{4}$ miles, on another $217\frac{1}{2}$ miles, and on the third $237\frac{1}{2}$ miles. The first-class fare by either route is £1 9s. Find the cost of travelling per mile on each railway, and explain why the fare is the same on all three railways.

69. If on a map 1 inch represents 80 miles, what fraction of the actual distance between two towns is that measured off on it?



Scale, 1″ = 100 miles.

B. Sides of Triangles

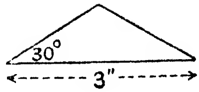
EXAMPLES. XLI.

1. Draw a triangle with two of the sides 1·5 inch and 2·1 inches, and the angle between them 45 degrees. Measure its perimeter (1) in inches, (2) in centimetres.

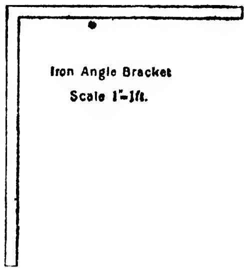
2. From the result of the previous question find what decimal of an inch is equal to a centimetre.

3. Draw a triangle with two sides 6·3 centimetres long and the angle between them 30 degrees. Measure its perimeter in inches and in centimetres, and prove that 1 inch = 2·54 centimetres.

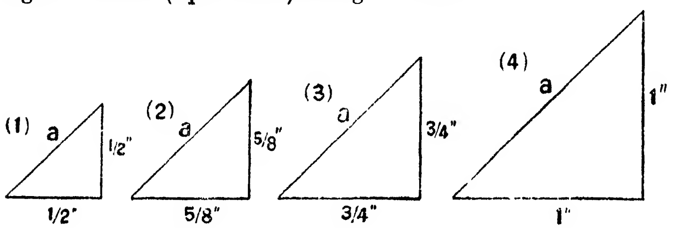
4. Draw a triangle whose base is 3 inches long and whose base angles are each 30 degrees. Find the length of the sides in inches and the perimeter in centimetres.



5. An iron angle bracket is represented in figure (drawn to the scale of 1 inch to 1 foot). What length of iron would be required to execute an order for $8\frac{1}{2}$ dozen brackets?



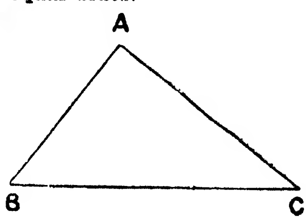
6. Measure the length of the side *a* of each of the right angled isosceles (equal-sided) triangles drawn:



7. Taking the data of the previous question (for each triangle 1–4), divide the length of the side *a* by that of one of the equal sides, and so find how many times either of the equal sides is contained in the longest side or hypotenuse of the right-angled isosceles triangle. (Note $\sqrt{2} = 1{\cdot}41$, to two places.)

8. **Make a general statement concerning the length of the hypotenuse of a right-angled isosceles triangle as compared with the length of either of its equal sides.**

9. ABC represents a piece of land such that AB = AC and the angle A is a right angle. Tabulate values for BC, given the length of either AB or AC:



Length of AB or AC.	Length of BC calculated to two places in accordance with the Statement made in the Last Question.
(1) 80·2 inches (2) 5·7 miles (3) 6·5 metres	

10. How many yards of fencing would be required to go round the land referred to in part (3) of the last question?

11. Write down to two decimal places the length of the diagonal of square tiles of the following sizes: (1) 3 inches; (2) 6 inches; (3) 8·5 centimetres; (4) 7·3 centimetres; (5) 0·18 metre; (6) 2·5 inches; (7) 13·4 centimetres; (8) 5·6 inches; (9) 3·5 inches; (10) 4·5 inches.

12. Draw triangles whose sides are: (1) 3, 4, 5 inches; (2) 3, 4, 5 centimetres; (3) 1·5, 2, 2·5 inches; (4) 1·5, 2, 2·5 centimetres; (5) 4·5, 6, 7·5 inches; (6) 4·5, 6, 7·5 centimetres; (7) 6, 8, 10 inches; (8) 6, 8, 10 centimetres. Measure the angle opposite the longest side in each triangle, correct to one decimal place.

13. In the case of each of the triangles drawn in the last question, (1) square the lengths of the two shorter sides; (2) add together the squares obtained in (1); (3) square the length of the longest side (the hypotenuse); (4) make a statement regarding the results of (2) and (3). Tabulate thus:

Example.	Square on First Side.	Square on Second Side.	Sum of Squares on First and Second Side.	Square on Third Side.
1 . .	$3^2=9$	$4^2=16$	$3^2+4^2=9+16=25$	$5^2=25$
2, etc. .				

Note.—**The results of Question 12 showed that the triangles whose dimensions were given were right-angled triangles and that the longest side—the HYPOTENUSE—was opposite the right angle. Question 13 showed that the SUM of the SQUARES on the sides containing the right angle equalled the SQUARE on the HYPOTENUSE.**

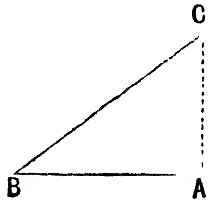
We can say for convenience that if a and b be the lengths of two sides of a right-angled triangle, and c that of the hypotenuse, then

$$a^2+b^2=c^2.$$

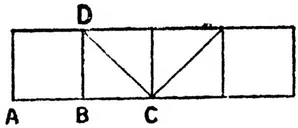
The following Questions 14 to 27 illustrate the application of the facts stated above :

14. Calculate the length of the hypotenuse of triangles whose sides are: (1) 9 and 12 inches; (2) 12 and 16 centimetres; (3) 6 and 8 metres; (4) 45 and 60 feet; (5) 18 and 24 yards; (6) 180 and 240 poles; (7) 30 and 40 furlongs; (8) 90 and 120 metres; (9) 405 and 540 miles; (10) 45 yards and 180 feet; (11) 105 yards and 420 feet; (12) 405 yards and $\frac{27}{88}$ mile.

15. **ABC** represents the slope of a conservatory roof. If the pieces of wood **AB** and **BC** are, respectively, 15 feet and 8 yards 1 foot, find the height of **C** above **A**.

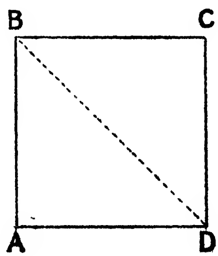


16. The diagram shows part of the leaded lights for a front door. **AB** = 3 inches = **BD**; **DC** = 5 inches. The lines drawn represent the lead. What length of lead would be required for this light?

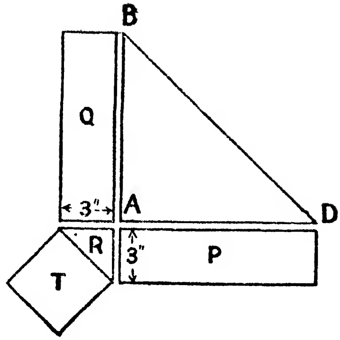


17. **ABCD** represents a square tile of 6 inch edge. It is cut along **BD**. What must be the length of the edge of another tile which will just fit along **BD**?

18. **ABD** represents half a square tile. If **AB** be 7 inches long, what must be the length of tiles which will just fit along **AC** or **BD**?

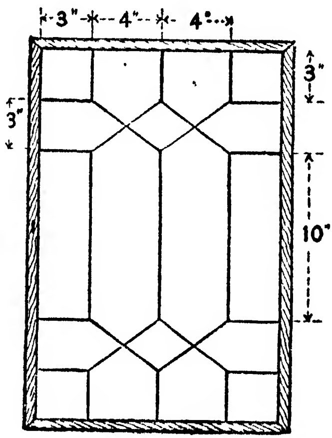


19. Tiles **P** and **Q** are placed along the sides of the tile **ABD** of Question 17. They are each 3 inches wide. Tiles **R** and **T** are fitted in as shown. Find the dimensions of the tiles **P** and **Q** and of the square tile **T**.

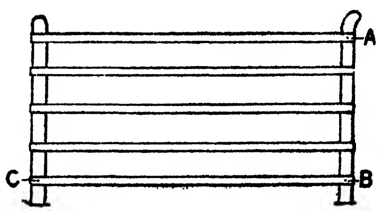


20. What is the length round the outside of the tiles drawn in the last question?

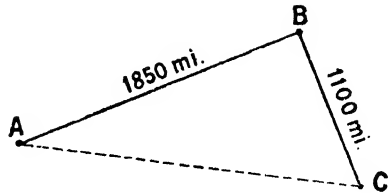
21. The diagram represents the door of a secretaire, and the lines drawn show the strips of mahogany in which the glass is set. What length of wood is required to fit up the door, neglecting the outside framework?



22. A gate has to be strengthened by putting a piece of iron $1\frac{1}{2}$ inch wide and $\frac{3}{8}$ inch thick from **A** to **C**. If **AB** = 6 feet and **BC** = $1\frac{1}{2}$ yard, find the length of iron required.



23. If a steamer sails from Aden (**A**) to Bombay (**B**) it covers 1850 statute miles. If it then continues its trip to Colombo (**C**) it sails 1100 statute miles more, at right angles to its previous course. How far would it sail if it went from Aden to Colombo direct?



24. Draw the lines **AB** and **BC** in the last question to the scale of 100 miles $= \frac{1}{2}$ inch. Measure **AC** and see how nearly this value agrees with that of Question 23.

25. Two air pilots ascend at Paris: one flies to Bordeaux and thence to Berne, the other flies direct to Berne, and consequently at right angles to the line of flight of the first airman to Bordeaux. How much farther does the former fly than the latter, presuming a straight-line flight in both cases, given Paris to Bordeaux $= 320$ miles; and Paris to Berne $= 275$ miles.

26. Draw to scale (1 inch $= 100$ miles) a figure representing the distances given in the last question; measure the distance from Bordeaux to Berne, and compare it with the distance calculated in Question 25.

27. Express the result of Question 25 in Kilometres, reckoning 100 Kilometres to 62 miles.

C. Circles

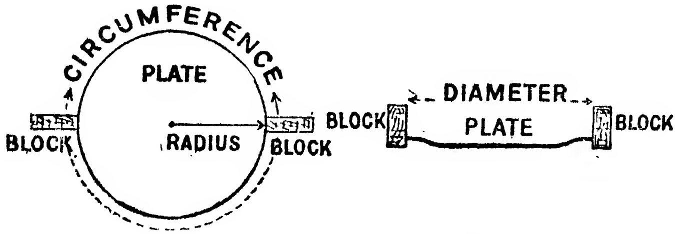
EXAMPLES. XLII.

1. Measure, in inches and the decimal of an inch, (1) the circumference and (2) the diameter of the objects named, then find the value of (circumference ÷ diameter) in each case, and tabulate the results as shown: A half-crown, a dinner plate, a motor wheel, a bicycle wheel, a cart wheel, a watch, a bucket, a plant pot, a saucepan, a ball, a cocoa tin, a soda-water syphon, a tumbler, an electric bell push, the dial of a clock.

Circular Article.	Circumference.*	Diameter.†	Circumference ÷ Diameter.
A Half-Crown			

Note *.—The circumference of an object can best be measured by winding a piece of cotton round it some twenty times, measuring the length of the cotton, and so finding the circumference. If, however, as in the case of half a crown, this is not possible, then a long thin strip of fairly firm paper should be cut and wound round.

Note †.—The diameter can be measured with a pair of callipers or by placing a block at either end of a diameter of the object, and measuring the distance between the sides of the blocks adjacent to the object, thus:



PLAN. ELEVATION.

2. Find the average of the values obtained in column 4 of the last question, and find by how much it differs from 3·1416.

3. Repeat Questions 1 and 2, employing the centimetre as the unit of length, and again find the value of π.

Note.—**Column 4, Question 1, shows that the diameter of a circle will divide into its circumference about $3\frac{1}{7}$ times.**

A more exact relationship is:

$$\text{Circumference} = 3{\cdot}14159 \times \text{Diameter}$$

For all ordinary purposes we call this

$$C = 3{\cdot}1416 \times D$$

or $$C = 3\tfrac{1}{7}D$$

or $$C = \pi D \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The symbol π (pi) is used universally to denote 3·1416 or $3\frac{1}{7}$.

Now $$\text{Diameter} = \text{twice radius}$$

$$\therefore\ D = 2R$$

$$\therefore\ C = 2\pi R \quad . \quad . \quad . \quad . \quad (2)$$

The student should remember equations (1) and (2).

If the radius of a wheel is 7 feet, then C, its circumference, is readily obtained, for

$$C = 2 \times \frac{22}{7} \times 7$$

$$= 44 \text{ feet.}$$

In the same way, if the circumference of a tree is 33 feet, we have $C \div \pi$ for its diameter and $C \div 2\pi$ for its radius. So—

$$D = \frac{33 \times 7}{22}$$

$$= 10\tfrac{1}{2} \text{ feet, and } R = 5\tfrac{1}{4} \text{ feet.}$$

4. Find the circumference of circles whose radii are, respectively: (1) 3 ft.; 6 ft.; 8 metres; 15 metres. (2) $2\frac{1}{2}$ in.; $7\frac{3}{4}$ ft.; $8\frac{1}{2}$ yds.; $9\frac{1}{4}$ cm. (3) $5\frac{1}{8}$ ft.; 6·4 yds.; 9·3 metres; 8·12 cm.

5. Express, in yards, the circumference of circles whose diameters are: (1) 0·312 ft.; 0·406 ft.; 0·732 ft. (2) 9·6 in.; 8·94 in.; 19·73 in. (3) $2\frac{1}{2}$ ft.; 2 ft. $10\frac{1}{2}$ in.; 1 ft. $3\frac{3}{4}$ in.

6. What are the radii of circles whose circumferences are: (1) $22\frac{2}{11}$ ft.; 28 ft. 5 in.; 9 ft. $5\frac{2}{3}$ in. (2) 352 metres; 583 cm.; 7986 mm. (3) 33 poles; 0·8657 mile; 5280 ft.

7. What is the circumference of the wheel of a locomotive if its diameter is 6 feet?

8. The radius of a taxi wheel is 1 ft. 3 in., what is its circumference?

9. Express the result of the last question in millimetres.

10. The diameter of the top of a bucket is 12 inches, what is the length of its circumference?

11. The blade of the propeller of an aeroplane is 8 feet long, measured from the shaft to the extremity of the blade. How far does the end of the blade travel in making one revolution?

12. What length of iron rails will be required to go round a circular lawn 50 feet in diameter?

13. The wheel of a steam roller is 1 ft. 9 in. in radius, and a strip 1 foot wide cut right across the wheel weighs 8·75 cwt.; find the weight of the wheel (neglecting that of the spokes).

14. The wheel of a locomotive is 6 feet in diameter, how many revs.[1] does it make when the engine covers a mile?

15. If the engine referred to in the last question be running at 50 miles an hour, how many revs. will the wheel make per minute?

16. Two cogged wheels run together as in the gears of a motor-car. If one wheel has 21 cogs, how many must the other have in order that it may go round $3\frac{1}{2}$ times while the wheel with the 21 cogs goes round once?

17. The curve on a railway line takes the shape of an eighth of the circumference of a circle of radius 5834 feet; what is the length of the rail forming the curve?

18. If a train may travel round the curve referred to in the

[1] A recognised contraction in mechanical and electrical trades for "revolutions."

last question at 28 miles an hour, how long will it take to go from end to end of the curve?

19. The span of a bridge is 25 feet, and it is surmounted by a semicircular arch; how many bricks 9 inches long will be required to build the first row of brickwork?

EXAMPLES. XLIII.

Miscellaneous Questions

1. An English firm contracts to put an iron staircase outside a building in Lyons which is 37·7 metres high, and the "riser" of each stair is to be 7 inches. How many stairs will be required?

2. What would be the cost in francs of erecting the staircase referred to in Question 1, reckoning 13s. 6d. a stair.

3. The tubes of radiators employed for heating purposes are each $2\frac{7}{8}$ inches wide, and are assembled together in sets of twos, threes, etc., according to the size of the office or store which has to be warmed. What space would be necessary for a radiator consisting of 18 such tubes? (Allow 3 inches on either side for clearance.)

4. Express in metres the length necessary for the 18-tube radiator of the previous question, allowing clearance.

5. The coal-delivery carts in London are furnished with an iron tube by means of which lumps of coal are broken and so knocked into the cellars. A merchant has 340 carts, and each has an iron tube 6 ft. 3 in. long. How many feet of iron are required for the whole of the carts?

6. Convert the result of the last question into metres.

7. An ordinary towel is 38 inches long and $\frac{3}{4}$ inch must be allowed at each end for turning in and hemming when it is being made. How many towels can be made from 125 yards of towelling?

8. What would be the cost per dozen towels if the material (Question 7) cost $8\frac{1}{2}$d. per yard?

9. If an aeroplane can cover, on the average, 48 miles per hour, how long would it take to fly from Dover to Ostend, a distance of 98 Kilometres?

10. A steamship can make 20 knots in crossing the Atlantic. Find its time from Southampton to New York, a distance of 3110 statute miles.

11. The distance from Dieppe to Paris is 201 Kilometres. What would it cost to travel from the former to the latter at a cost of 1d. per mile?

12. What would be the cost in francs of the journey referred to in the previous question?

13. The second-class fare from Dieppe to Paris is 640 pfennige. If 100 pf. = 1 mk. = 11·7 pence, find the fare from Dieppe to Paris in English money.

14. Compare the second-class fare from Dieppe to Paris with the third-class fare that would be charged in England for a journey of the same distance.

15. The distance from London to Doncaster is quoted in a continental guide-book as 251 Kilometres. What is the third-class fare, reckoning 1d per mile?

16. What would be the cost in dollars of the journey referred to in the previous question, if 1 dollar = 4s. 1·3d.?

17. How many threads are there on the shaft of a screw 3·75 inches long, if there are 5 threads to a centimetre?

18. How many $1\frac{1}{2}$-inch nails can be cut from iron wire 33 yds. $1\frac{1}{2}$ ft. long?

19. How many metres of boxwood are required to make $1\frac{1}{2}$ dozen 12-inch rulers, allowing $\frac{1}{4}$ inch over and above the 12 inches at each end of the ruler and 2 inches for waste?

20. If, on the average, $3\frac{3}{4}$ yards of cloth are required to make a suit of clothes for a man, find what length of khaki is required to clothe an army of a million men.

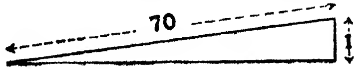
21. There is a rise of 1 in 70[1] on an incline outside one of the great London termini. If a train run a mile and a half up this incline, find the number of feet by which it is above its original level.

22. If the train of the previous question, after having negotiated the first incline, runs down another of 1 in 250 for two miles, find the number of feet it has descended vertically and the difference between its *original* level and its final level.

23. Ostend to Brussels is 122 Kilometres; Brussels to Liége is 100 Kilometres; and Liége to Aachen is 55 Kilometres. Represent to the scale of 1 cm. = 10 Kilometres the distance from Ostend to the junction at Aachen.

24. The second-class fares between the stations named in Question 23 are (1) 640 pf.; (2) 520 pf.; and (3) 290 pf. Find the cost of a second-class ticket from Ostend to Aachen,

[1] An incline of 1 in 70 (illustrated in the diagram) means that there is a rise of 1 foot or 1 inch or 1 mile for every 70 feet, inches, or miles travelled *along the incline.*



presuming it to be the sum of the three fares quoted, expressing the cost in English money so that a traveller may know the fare before leaving England. (1 mk. = 11·75d.)

25. Taking the average speed of the transcontinental mails as 50 miles per hour, calculate the time taken for mails to travel from London to Paris, a distance of 489 miles.

26. How long should the mails take from Paris to Constantinople, a distance of 1900 miles? (See Question 25.)

27. A steam roller is driven at an average speed of 4 miles an hour. A stretch of road 1000 yards long has to be rolled, and this necessitates the roller's travelling from end to end twenty times. If the driver receives 1s. 9½d. an hour, how much does he earn by rolling the road referred to?

28. If a train travels at 60 miles an hour, how many feet does it cover in a second?

29. How long would it take a train travelling at 30 miles an hour to cover 100 yards?

30. Compare the speed of the train in the last question with Applegarth's record (Question 18, p. 240).

31. Which travels more quickly, a train which covers 22 feet a second, or a torpedo boat destroyer (T.B.D.) steaming at 30 knots?

32. A submarine can make 18 knots on the surface and 13 knots submerged. How long does it take to travel 100 nautical miles if it is submerged for two-fifths of its journey?

33. At what average speed in feet per second must an express train travel to run from London to Birmingham in two hours if the distance is 110 miles?

34. Another route to Birmingham from London is 113 miles long, but the express trains cover this distance in two hours. By how many feet per second does the speed on this route exceed that on the one mentioned in Question 33?

35. No less than £13,044,000 has been expended on the Manchester Ship Canal[1] (construction, land purchase, engineering and surveying fees, and general expenses). If the canal is 35½ miles long, calculate the expenditure for every mile, and thence for every 100 yards.

36. New York to Southampton is 3110 statute miles, and Quebec to Liverpool, 2708 statute miles. How much longer would it take a steamer making 18 knots to travel from New York to Southampton than from Quebec to Liverpool?

37. Brindisi to Port Said is 930 miles; Paris to Rome is 910 miles. How much shorter time would it take a train averaging

[1] *The Port of Manchester*, No. 185, February 1915, p. 19.

45 miles an hour to make the second run than a ship steaming 20 knots to make the first?

38. The distance from Liverpool to Quebec is 2708 statute miles and the steerage fare is £6 15s. By how much does the third-class railway fare in England of 1d. per mile exceed the cost per mile by steamer to Quebec?

39. The fare from Quebec to Vancouver is £8 3s. 7d., and the distance is 3080 miles. Find the fare per mile between the cities named.

40. Montreal is 180 miles farther up the St. Lawrence River than Quebec. How long would it take a boat steaming at 10 knots to travel from one city to the other, ignoring the current or tide?[1]

41. The river St. Lawrence flows down at the rate of 4 miles per hour at Montreal. If the steamer referred to in Question 40 were steaming *down*-stream how long would it take to travel from Montreal to Quebec?

42. The distance from Petrograd to Moscow is 375 miles. Express this distance in versts, given that 1 verst = 0·66 mile.

43. The source of the river Nile, which is 3600 miles in length, is 10000 feet above sea-level. Calculate, for the information of French geographers, the average fall in level in metres per Kilometre.

44. There are four classes on some railways on the Continent. Given that the fourth-class fare is 2s. per 100 Kilometres, the third class is 3s. 2d. per 100 Kilometres, the second class is 4s. 9d. per 100 Kilometres, and the first class is 7s. 10d. per 100 Kilometres, find the cost per mile by each class.

45. The fares from London to Lancaster are: first class, 34s.; third class, 19s. 2d. The distance is 230 miles. Find the cost per mile by each class.

46. Compare the results of the last two questions.

47. A cricketer walks from one wicket to the other, a distance of 22 yards, in twenty-four steps. How many such steps would he take in 1 mile? If he walks at 3 miles an hour, how long will it take him to walk from wicket to wicket?

48. A starts walking at 5 miles per hour, keeps up that rate for $1\frac{1}{4}$ hours, then has a rest of 10 minutes, and then goes on at

[1] His Excellency the High Commissioner for Canada most kindly informs me that "the currents of the St. Lawrence River between Quebec and Montreal vary from an extreme of 6 knots to a minimum of 1 knot. In the design of the channel great care has been exercised not to produce cross-currents. For 100 miles below Montreal the stream is always downwards, even though there may be a slight tide. The level of the river is highest in April and May, and falls to its lowest level in October and November."

4 miles an hour. B starts from the same place 5 minutes later than A, and walks at a uniform rate of $4\frac{1}{2}$ miles an hour without stopping. Where will he catch up A?

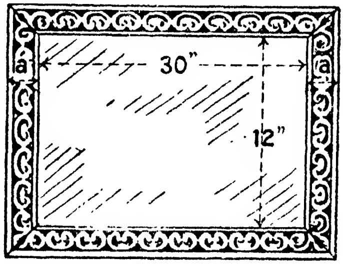
49. What is the cost of putting down the lines, erecting posts, and supplying all the necessary equipment (excluding rolling stock and power station equipment) for a tramway system 7 mi. 6 fur. 5 chs. in length at £7500 per mile? (Overhead trolley wire system.)

50. Find the cost as in the last question, but for a conduit tramway system 8 mi. 3 fur. 2 chs. long at £14040 per mile.

51. The land on which a railway track is to be constructed costs, on the average, £8007 per mile. What would be the cost of land for 56 mi. 5 fur. 2 chs. of track?

52. What length of 3-inch moulding will be required for the frame shown in figure?

Note.—The length of the picture over all is 30″ + twice the width of the moulding *a.a.*, or 30″ + 6″ = 36″; so the width is 12″ + 6″, or 18″, and so total length is readily found.



53. What length of 4-inch figured-oak moulding will be necessary if a space 30 × 20 inches is to be allowed for, and what will be the cost if it is sold in 12-foot lengths costing 3s. 2d. each, and one cannot buy less than half a length?

54. A picture-frame maker agrees to frame a picture 60 × 40 inches for 2s. 6d. a foot, $2\frac{1}{2}$ inch gilt moulding, anything less than 1 foot charged as 1 foot. What is the cost? (If the picture were 60 × 40 inches, the space enclosed by the frame would be about $59\frac{1}{2} \times 39\frac{1}{2}$ inches, but **this would be called** 60 × 40 inches.)

55. What would it cost an emigrant to Queensland to fence round his grazing farm, a distance of 23 mi. 65 chs. 20 yds., at £28 15s. 9d. per mile?

56. In a very richly timbered part of Australia (the Heberton district) it cost £1,257,992 to construct $47\frac{1}{2}$ miles of railway. What was the average cost per chain?

57. In the year 1906 the Japanese Government purchased the railways in Japan for yen 476,320,000, and the total length was 1157 ri. What did the Government pay in £'s per mile? (1 ri = 2·44 miles, and 1 yen = 2s. $0\frac{1}{2}$d.)

SECTION XIII

AREAS

183. Since the yard, metre, etc., are measures of length, the SQUARE yard, SQUARE metre, etc., are measures of area, and the CUBIC yard, CUBIC metre, measures of volume.

The word "SQUARE" must **always** be used before "yard," "metre," etc., in speaking of an AREA, and "CUBIC yard," "CUBIC metre," etc., for a VOLUME.

184. We shall often refer to the plan and to the elevation of an object. By the term PLAN we mean the shape of an object as seen from ABOVE. The plan of a box of oranges, a book, a picture frame resting on the table, the table itself, or a dog's kennel, is a rectangle; that of a football is a circle. The ELEVATION means *for us* the shape of an object looked at from

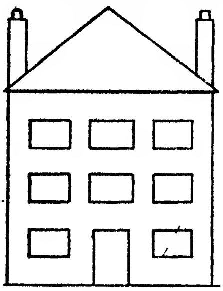


FIG. 1.

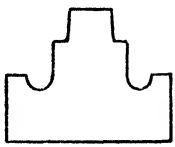


FIG. 2.

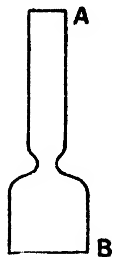


FIG. 3.

THE FRONT or SIDE, but without any regard to perspective. The elevation of a house would be, roughly, as in Fig. 1; of an ink pot as in Fig. 2; of a round lamp-glass as in Fig. 3. The plans would be rectangles in the first two cases and circles in the last. Two circles are necessary to show the top, **A**, smaller than the bottom, **B**, of the lamp-glass.

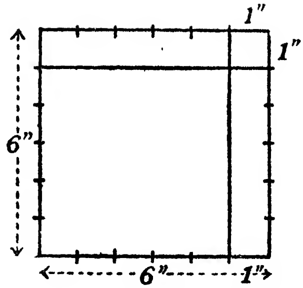
(What we have said is not absolutely satisfactory, but it is near enough for the commercial student.)

A. Rectangles

EXAMPLES. XLIV. (a)

1. Take a piece of inch-squared paper divided into tenths and draw on it a square whose side is 1 inch in length. Count the number of small squares which it contains, and determine what fraction of 1 square inch each small square occupies.

2. Draw on squared paper a square with its sides 6 inches long, divide each of them into inches, draw lines parallel to the sides through each pair of the marked points, and then count



the number of square inches in the whole figure.

3. Draw on squared paper a rectangle whose edges are 5 cm. and 3 cm. respectively. Find graphically the number of square centimetres which it contains.

Note.—From Question 2 we find that, with each side of the square 6 inches long, the area is 36, or 6×6 square inches; and from Question 3, that when the sides are 5 cm. and 3 cm. the area is 15, or 5×3 square centimetres. Hence we conclude that **the area of a square, and so of any rectangle, equals its length times its breadth,** or,

area of rectangle = length × breadth.

Be very careful to have length and breadth in the same units—that is, **BOTH** in inches, feet, or centimetres, and not length in inches and breadth in yards or metres.

4. Divide the rectangle referred to in Queition 3 into square inches as in Question 2, and then determine the number of square centimetres in one square inch.

5. Draw a rectangle $3\frac{1}{2}$ inches by $2\frac{1}{2}$ inches, and find how many squares of side $\frac{1}{2}$ inch are enclosed by it.

6. Measure the length of the sides of the rectangle of the previous question in centimetres, and calculate its area in square centimetres.

7. From the results of the last two questions, calculate the number of square centimetres in 1 square inch.

8. A paving brick is 9 inches long and $4\frac{1}{2}$ inches wide; draw a rectangle to the scale $\frac{1}{2}$ inch = 1 inch, to represent a plan of the largest face of the brick.

9. If a paving brick is $1\frac{3}{4}$ inch deep, draw its elevation to the same scale as in Question 8.

10. Use the diagrams of Questions 8 and 9 and a centimetre scale to determine the dimensions of a paving brick in centimetres. (Remember your diagrams are half full size.)

11. An Empress slate is the largest size in common use, and is 26×16 inches. Draw such a slate a quarter full size.

12. Determine the length and width of an Empress slate by measuring in centimetres the lines drawn in the last question.

13. Find the area of an Empress slate in square centimetres.

14. What decimal of 6 square feet is the area of an Empress slate?

15. Given that 1 inch = 2·54 cm., calculate the number of square centimetres in 1 square inch.

16. Find the area of this page—(1) in square centimetres;

(2) in square inches. What decimal of a square inch is equal to 1 square centimetre?

185. EXAMPLE 1.—**What is the area in square feet of a rectangle of length 3 ft. 4 in. and width 2 yds. 2 ft.?**

$$\text{Area} = 3 \text{ ft. } 4 \text{ in.} \times 8 \text{ ft.} = 3\tfrac{1}{3} \text{ ft.} \times 8 \text{ ft.}$$
$$= \tfrac{10}{3} \times 8 = 26\tfrac{2}{3} \text{ square feet.}$$

EXAMPLE 2.—**Find the area of a rectangular field 2 mi. 5 chs. by 3 mi. 4 chs., in square miles.**

$$\text{Area} = 2 \text{ mi. } 5 \text{ chs.} \times 3 \text{ mi. } 4 \text{ chs.} = 2\tfrac{5}{80} \text{ miles} \times 3\tfrac{4}{80} \text{ miles}$$
$$= 2\tfrac{1}{16} \times 3\tfrac{1}{20} = \tfrac{33}{16} \times \tfrac{61}{20} \text{ square miles}$$
$$= 6{\cdot}29 \text{ square miles.}$$

EXAMPLES. XLIV. (b)

1. Find the areas of rectangles of the following dimensions:
   (1) 3 ft. 6 in. by 2 ft. 4 in. in square feet.
   (2) 1 ft. 9 in. by 1 ft. 2 in. in square feet.
   (3) 8·96 cm. by 0·64 m. in square centimetres.
   (4) 5 cm. 2 mm. by 36 cm. 3 mm. in square centimetres.
   (5) 5 yds. 2 ft. 1 in. by 3 ft. 3 in. in square yards.
   (6) 11 yds. 1 ft. 2 in. by 10 ft. 6 in. in square yards.
   (7) 3 chs. 3 yds. by 5 chs. 4 yds. in square chains.
   (8) 1 mi. 5 chs. by 1 mi. 30 chs. in square miles.

2. What is the area of cross-section[1] of a square bar of edge 0·9 inch?

3. A centimetre scale is 1·5 cm. wide and 0·35 cm. thick, find its area of cross-section.

4. What is the area of cross-section of a square iron bar the side of which is 0·84 metre long?

5. The *Times* newspaper (London) is 62·3 cm. long and 46·8 cm. wide. How many square metres of paper will there be in a paper whose pages are numbered 1–16?

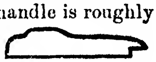
6. What is the area of a rug 4 ft. by 2 ft. 6 in.?

7. By how many square feet does 150 square feet exceed the area of a carpet, 10 ft. 6 in. by 10 ft. 9 in.?

8. A room is 18 ft. by 12 ft. 6 in., and the carpet is 14 ft. 6 in. by 9 ft. 3 in. What area is uncarpeted?

Hint.—Find the area of the room and of the carpet; the difference is the area uncarpeted.

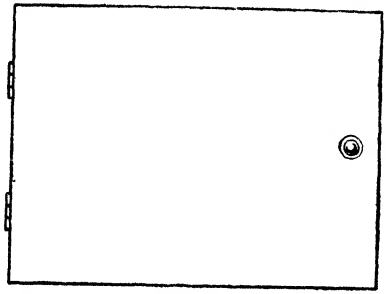
---

[1] If we took a walking-stick and cut it in two, perpendicular to its length, the ends thus made would be circular, and the area of the circle would be the area of cross-section of the stick. The cross-section of one's finger or of a broom handle is roughly a circle, that of a piece of picture-frame moulding might be 

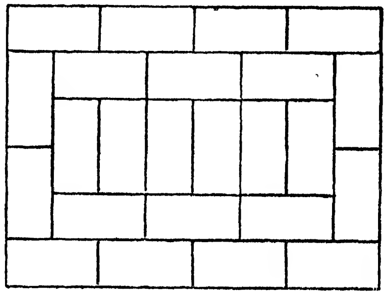
9. A ten-shilling Treasury note is $5 \times 2\frac{1}{2}$ in., and a £1 note is $5\frac{7}{8}$ by $3\frac{1}{4}$ in. Find the area of each note in square inches.

10. How many times is the area of the £1 note greater than that of a 10s. note?

11. The diagram represents a cupboard door to the scale $\frac{1}{2}$ in. = 9 in. Determine its area.

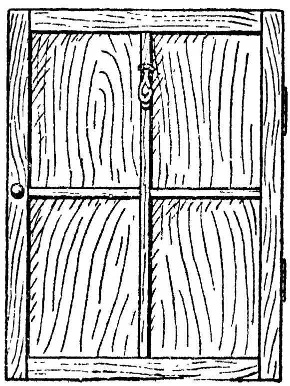


12. Taking each of the bricks in the figure to be $9 \times 4\frac{1}{2}$ in., determine the area of the portion of a pavement which is drawn, neglecting the space occupied by cement.



13. A *Whitaker's Almanack* is 18·8 cm. by 12·7 cm., draw a rectangle half full size to represent a plan of the book. Find the area of the cover of the *Almanack* in square centimetres.

14. The diagram represents a door to the scale $\frac{1}{4}$ in. = 1 ft. Neglecting joints, calculate (1) the area of wood $\frac{1}{8}$ in. wide; (2) the area of wood $\frac{1}{16}$ in. wide; (3) the area of the panels.

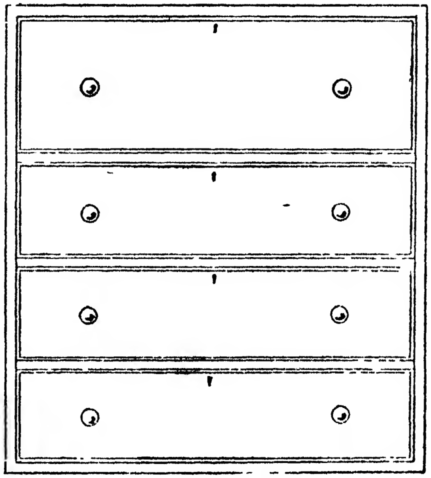


15. What is the value of the door in the last question at $5\frac{1}{2}$d. per square foot?

16. The diagram shows the elevation of a chest of drawers. Will the piece of furniture fit into an alcove 4 feet long and 3 feet high?

17. Find the cost of French polishing the front of the chest of drawers referred to in Question 16, at 1s. 3d. per square foot.

18. Reckoning 30·5 cm. = 1 ft., determine the area of the front of the chest of drawers as the decimal of 1 square metre.



Scale, $\frac{1}{16}''$ = 1 inch.

B. Square Root.

186. Not infrequently one is given the area of a square, and asked to find the length of the side. If the latter be 8 feet, then the area is $8 \times 8 = 64$ square feet, where, clearly, the length of the side is the square root (§ 33 § 35) of the area, and can often be found by factorising (§ 26–§ 32) the number of square yards, etc., in the area.

Let us consider the following:

Example 1.—**A room, in the form of a square, is 1750 square feet in area, what is the length of one side of it?**

We want the square root of 1750—that is, the square root of 1750·0000 . . . . Now, since it is impracticable here to adopt the method of factorisation, as $1750 = 7 \times 5^3 \times 2$ and is not a perfect square, we proceed as follows:

Method.

(1) Mark off the figures in twos, on both sides of the decimal point, **beginning from the point.**

(2) Find the nearest square to 17; $4^2 = 16$; put up the 4.

(3) Square 4; 16; subtract from 17.

(4) Bring down the next pair of figures, 50.

(5) Double 4; 8 into 15, one; put up 1* after the 4 AND after the 8.

(6) $81 \times 1^* = 81$; subtract from 150.

```
        ....double 4.......
                          1750·00|41·83
                          16     |
        81               |150
                         | 81*
   828                   |-----
                         | 6900
   double 41             | 6624
   8363                  |-----
                         |  27600
                         |  25089
   double 418            |  -----
                         |   2511
```

We now come to the decimal point, which we put down, and bring down the next two figures, 00, and then repeat (5) and (6)—that is, double 41 and divide into 690; it goes 8 times, and so on.

Length required is 41·8 feet, to the nearest tenth of a foot.

187. EXAMPLE 2.—**Find the length of the side of a stretch of country roughly square and of area 56723·851 square miles.**

We extract the square root, proceeding just as before.

```
                      56723·851000|238·167
                      4           |
   43                |167
   double 2          |129
   468               | 3823
   double 23         | 3744
   4761              |   7985
   double 238        |   4761
   47626             |   322410
   double 2381       |   285756
   476327            |    3665400
                     |    3334289
   double 23816      |-----------
                          etc.
```

∴ the length of the side is 238·167 miles.

Note.—We insert the point in the quotient only.

EXAMPLES. XLV.

Find the length of the side of square allotments of the areas given, Questions 1 to 25:

1. 4356 square feet.
2. 5041 square metres.
3. 6084 square feet.
4. 7056 square feet.
5. 8281 square metres.
6. 8836 square Kilometres.
7. 9801 square yards.
8. 9604 square yards.
9. 13689 square metres.
10. 20736 square yards.
11. 76176 square Kilometres.
12. 77841 square feet.
13. 144400 square feet.
14. 24·5025 square miles.
15. 96·04 square miles.
16. 24·2064 square miles.
17. 2 square miles.
18. 5 square miles.
19. 7 square chains.
20. 13 square chains.
21. 1·21 square mile.
22. 0·00784 square Kilometre.
23. 0·001681 square mile.
24. 0·00006889 square mile.
25. 6·249 square chains.

26. What is the length of the side of squares whose areas are: (1) 5 square inches; (2) 64516 feet; (3) 1568·16 yards; (4) 44 acres 16 poles; (5) 1 acre 3 roods?

27. A square field is 1 acre in area. What is the length of its sides?

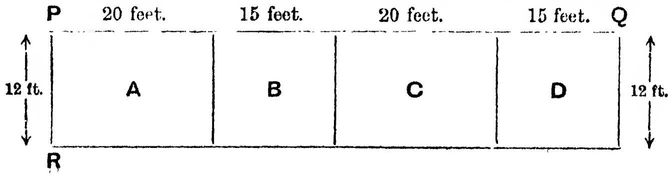
28. What is the cost of fencing the field of the last question at 3s. 6d. per yard?

29. A map is drawn to the scale of 100 miles = 1 inch, and the area of a province is 3·61 square inches. What is its actual area? If it be roughly square, what is the length of each side?

30. What is the length of the side of a square the area of which is 98·01 square yards?

C. Papering Walls, Carpeting and Tiling Floors, etc.

188. In papering a room it is necessary to know the area of the walls upon which the paper is to be hung. Some students find the volume of the room as if the paperhanger's business was to fill it with paper. Most rooms are roughly rectangular in shape, ∴ the area of each wall equals its length times its height. Imagine the walls of a room 20 feet long and 15 feet wide hinged so that they can be opened out into one long wall as in diagram:



Area of **A**, **B**, **C**, **D** = length **PQ** × height **PR** = $(20+15+20+15) \times 12$ square feet. Now $20+15+20+15$ is the **distance round the room,** which is called the PERIMETER of the room;

$\therefore$ **Area of walls** = PERIMETER × **Height**

$= \mathbf{70 \times 12 = 840}$ **square feet.**

Also

Area of floor or ceiling = Length × width (of room)

$= \mathbf{20 \times 15}$ **square feet**

$= \mathbf{300}$ **square feet.**

189. It is useless to regard all the irregularities of a room and measure them very precisely, for it is always necessary, in estimating, to allow at least one piece of paper extra, and a decorator will measure the GREATEST length, the GREATEST width, and the GREATEST height of a room, and calculate from those measurements.

EXAMPLES. XLVI. (a)

1. What length of skirting board will be required for offices or warehouses of the following dimensions? (That is to say, find the perimeter of the offices or warehouses.)

(1) 8 ft. 9 in. wide, 12 ft. 10 in. long.
(2) 15 ft. 8 in. long, 8 ft. 10 in. wide.
(3) 5 m. 58 cm. long, 3 m. 26 cm. wide.
(4) 15 ft. 5 in. long, 12 ft. 10 in. wide.
(5) 6 m. 39 cm. long, 5 m. 6 cm. wide.
(6) 26 ft. 8 in. long, 18 ft. 8 in. wide.
(7) 8 m. 20 cm. long, 6 m. 40 cm. wide.
(8) 80 ft. 6 in. long, 60 ft. 6 in. wide.
(9) 12 m. 59 cm. long, 8 m. 19 cm. wide.
(10) 115 ft. 8 in. long, 28 ft. 3 in. wide.

2. Find the area of the walls of the rooms of the last question if the heights be, for part (1) 10 ft.; (2) 12 ft. 6 in.; (3) 3 m.; (4) 12 ft. 6 in.; (5) 5 m.; (6) 13 ft.; (7) 5 m.; (8) 20 ft. 3 in.; (9) 6 m.; (10) 25 ft.

3. The length of an office is 15 ft. 6 in., the width is two-thirds of the length, find the perimeter.

4. Find the amount of floor space available in the room, the dimensions of which are given in Question 3.

5. A house is 55 feet high and its frontage is 0·5 of its height, find the area of the front of the house; and allowing 0·35 of that area for windows, calculate the cost of painting it at $9\frac{1}{2}$d. per square yard.

6. A warehouse is 40 ft. 6 in. long and its width is two-

thirds its length. The height is 15 ft. 6 in. Find the area of the four walls.

7. What would it cost to distemper the walls of the warehouse referred to in Question 6, reckoning $7\frac{1}{2}$d. per square yard?

8. A decorator agrees to distemper the ceiling of an office, 15 ft. by 9 ft. 6 in., for 3s. 6d. What rate does he charge per square yard for his work?

9. Estimate the cost of painting the front of a house, together with the woodwork of windows and doors, if the length is 38 ft. and the height 25 ft. 6 in., at 1s. 6d. per square yard per coat. Put on two coats of paint.

10. An important London firm quotes as under for Wilton carpets: (1) 9 ft by 12 ft., £7 5s.; (2) 11 ft. 3 in. by 15 ft., £11 5s.; (3) 13 ft. 6 in. by 15 ft., £13 10s. What is the cost per square yard of each of the carpets?

11. If carpet, 26 inches wide, is sold retail at 4s. 6d. per square yard, find the cost of providing 48 linear yards of such a carpet.

Hint.—Area of carpet $= l \times w = 48 \times \frac{26}{36}$ square yards.

12. The "treads" of a flight of eighteen stairs are 9 inches wide, and the "risers" are $7\frac{1}{2}$ inches high. Allowing 6 inches of carpet at the foot and at the top of the flight and half inch per stair extra for the round front of the tread, calculate the length of carpet required for the flight of stairs.

13. What would be the cost of the carpet in the last question at 3s. $9\frac{1}{2}$d. a linear yard?

14. An office is 15 ft. by 12 ft. 6 in. by 12 ft. high, calculate the area of the walls and of the floor and ceiling.

190. EXAMPLE 1.—**(*a*) How many pieces of English wall-paper, 21 inches wide and 12 yards long, will be required to paper the walls of an office 20 feet long, 16 feet wide, 12 feet high?**

(*b*) Find also the cost of papering the room if the paper cost 5s. per piece and we allow one piece extra for waste.

(*a*) Area of walls = perimeter × height $= (20 + 16) \times 2 \times 12 = 36 \times 2 \times 12$ square feet.

$$\text{Area of each piece of paper} = \frac{21}{12} \times 36 = 63 \text{ square feet.}$$

$$\therefore \text{ number of pieces required} = 36 \times 2 \times 12 \div 63.$$

$$= \frac{\overset{4}{\cancel{36}} \times 2 \times 12}{\underset{7}{\cancel{63}}} = \frac{96}{7} = 13\tfrac{5}{7} \text{ pieces.}$$

∴ we shall want 14 pieces and 1 extra, or 15 pieces.

(*b*) The cost is 5s. per piece.

$$=15 \times 5 = 75\text{s.} = £3\ 15\text{s.}$$

The method is precisely the same for questions on the tiling of floors, etc.

EXAMPLES. XLVI. (b)

1. Find the cost of papering rooms from the following details, tabulating the results:

	Length.	Width.	Height.	Price for Paper and for Hanging (per Piece).
1	18 ft.	12 ft. 6 in.	10 ft.	3s. 6d.
2	5 m. 40 cm.	4 m.	3 m. 50 cm.	2 fr. 50 c.
3	25 ft. 6 in.	15 ft.	12 ft.	4s. 6d.
4	6 m. 25 cm.	4 m. 10 cm.	3 m.	3 fr. 10 c.
5	28 ft.	18 ft. 6 in.	15 ft.	3s.
6	7 m. 10 cm.	5 m. 8 cm.	4 m.	4 mks. 50 pf.
7	36 ft. 6 in.	25 ft. 6 in.	15 ft.	2s. 6d.
8	6 m. 5 cm.	5 m.	5 m. 30 cm.	3 mks. 25 pf.
9	45 ft.	36 ft. 3 in.	18 ft.	5s. 9d.
10	8 m. 30 cm.	6 m. 10 cm.	5 m. 40 cm.	7 fr. 10 cents.

The width of the paper is to be taken as 21 in., or 45 cm., as the case may be, and the length 12 yds. or 9 metres.

2. Find the cost of painting the ceilings of the rooms whose dimensions are given in Question 1, at the following rates: (1) 3s. per square yard; (2) 4 fr. per square metre; (3) 3s. 6d. per square yard; (4) 3 fr. per square metre; (5) 2s. 6d. per square yard; (6) 3 mks. 10 pf. per square metre; (7) 2s. per square yard; (8) 2 mks. 50 pf. per square metre; (9) 2s. 6d. per square yard; (10) 3 fr. 10 c. per square metre.

Note.—In all such questions as this, a fraction of a square yard or of a square metre would be reckoned as 1 square yard or 1 square metre.

3. Find the cost of tiling the floors of halls of the dimensions given, with tiles of size and price quoted below. (Tabulate the results.)

	Length.	Width.	Size of Tiles.	Cost of Tiles (per Dozen).
1 . .	18 ft.	12 ft. 6 in.	6×6 in.	5s. 6d.
2 . .	5 m.	3 m. 30 cm.	10×10 cm.	6 fr.
3 . .	16 ft.	14 ft.	4×2 in.	4s. 6d.
4 . .	6 m.	4 m. 50 cm.	15×15 cm.	8 fr. 50 c.
5 . .	24 ft.	20 ft. 6 in.	$3\frac{1}{2} \times 3\frac{1}{2}$ in.	3s.
6 . .	8 m. 40 cm.	5 m.	10×15 cm.	5 mks. 50 pf.
7 . .	30 ft. 6 in.	25 ft. 8 in.	4×4 in.	4s.
8 . .	7 m. 6 cm.	6 m. 40 cm.	10×15 cm.	6 mks. 80 pf.
9 . .	42 ft. 6 in.	30 ft. 6 in.	2×2 in.	2s. 6d.
10 . .	5 m. 50 cm.	4 m.	12×12 cm.	5 mks. 50 pf.

Note.—It is not possible to purchase fewer than half a dozen tiles, so any number less than six is reckoned as being six.

4. What would be the cost of covering the floors of the rooms whose dimensions are given in the last question, with linoleum at the following prices?

1	3s. per square yard.	6	2 mks. 30 pf. per square metre.
2	2 fr. per square metre.	7	2s. per square yard.
3	2s. 6d. per square yard.	8	3 mks. 10 pf. per square metre.
4	2 fr. 50 c. per square metre.	9	2s. 9d. per square yard.
5	2s. 6d. per square yard.	10	2 mks. 50 pf. per square metre.

5. Estimate for a customer at Bordeaux the cost of papering the rooms of his château with French paper, the details being—

Room.	Length.	Width.	Height.	Per Piece.	
				Price.	Cost of Hanging.
1	10·6 m.	3·8 m.	5·6 m.	8 fr.	1 fr.
2	12·9 m.	5·7 m.	6·8 m.	12 fr. 15 c.	2 fr. 50 c.
3	15·6 m.	8·9 m.	9·5 m.	14 fr.	2 fr. 75 c.
4	20·5 m.	15·8 m.	14·6 m.	18 fr. 10 c.	3 fr.

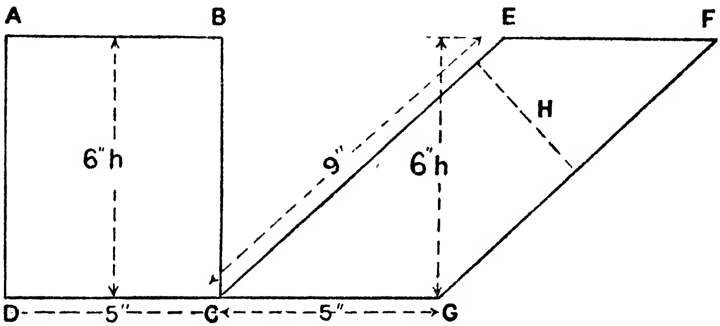
Width of a piece of French paper = 45 centimetres.
Length " " = 9 metres.

Reckon all measurements of room to the nearest metre above those given.

6. Estimate for painting the ceilings of the rooms in the château of your customer at Bordeaux at 8 fr. 50 c. per square metre.

D. (1) Area of Parallelograms

191. Draw the figure shown in diagram to the dimensions given.



Cut out ABCD and EFGC, and then, by cutting the latter up into pieces, show that it exactly fits into ABCD.

This being so, **the parallelogram and the rectangle having equal bases and heights have the same area, namely, CG × *h*.**

Hence, the area of a parallelogram = base × height.

Note.—The height of EFGC is not the length of the side CE, but the length of a line perpendicular to its two opposite sides EF and CG. We could also have said GF × H equals the area.

EXAMPLES. XLVII. (a)

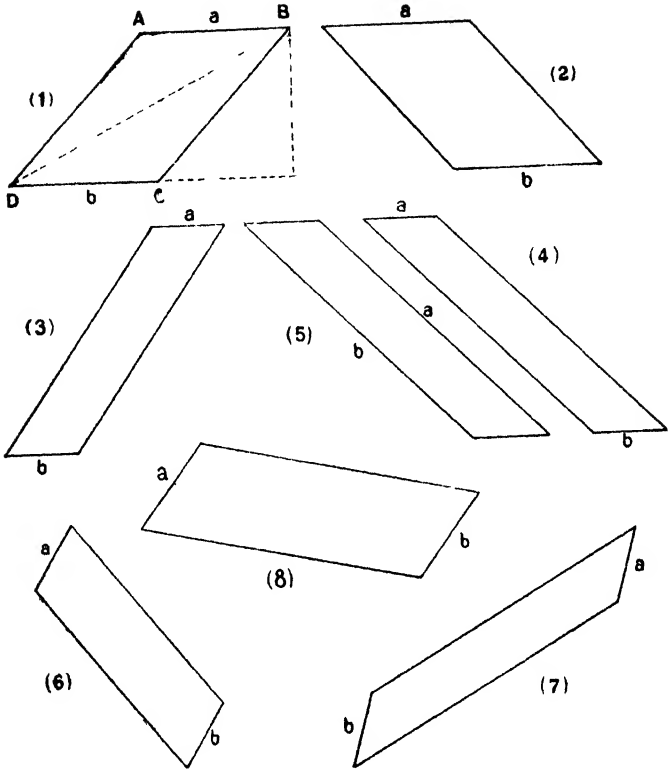
1. Draw lines to show the perpendicular distance p, between the sides a and b of each of the figures shown on page 270.

2. The parallelograms drawn in the last question are full size. Find their areas in square inches.

3. Measure the lengths necessary to find the area of the parallelograms drawn in Question 1, and give the areas in square centimetres.

4. Tabulate as suggested below, using the results of Questions 2 and 3 (for Figures 1–8 of Question 1):

No. of Figure.	Area in Square Inches.	Area in Square Centimetres.	Decimal of Square Inches in 1 Square Centimetre.
1 . . .			
2, etc. . .			

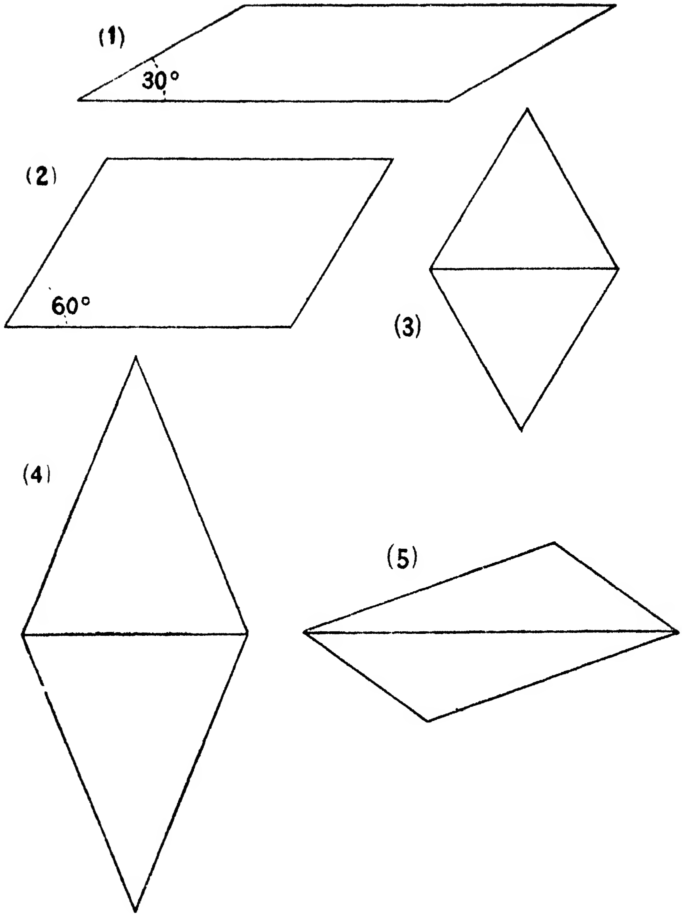


5. In making lights for front doors, church windows, etc., the coloured glass is cut to the shape required and held in position by lead. Find the area of each piece of glass represented in the diagrams shown on page 271, by measuring the length of one side and the distance between it and the opposite side (1, 2, 3 in square inches and 4 and 5 in square centimetres).

D. (2) Area of Triangles

EXAMPLES. XLVII. (b)

1. Draw each of the Figures 1 to 8 in Question 1 (above) on squared paper, double the size shown. Draw one diagonal in



each, cut the figures in two along the diagonal, and record the results as follows:

No. of Figure.	No. of Squares in the Parallelogram. (1)	No. of Squares in the First Triangle. (2)	No. of Squares in the Second Triangle. (3)
6, etc. .			

2. Make (1) a statement concerning the results written down in columns 2 and 3 of Question 1, and (2) a statement concerning the figures in column 1, and those in either column 2 or column 3. Then state a rule for finding the area of a triangle.

It is obvious that in Fig. 1, Question 1, page 270, the diagonal DB divides the parallelogram into two equal triangles;

$$\therefore \text{ area of triangle} = \tfrac{1}{2} \text{ area of parallelogram}$$
$$= \tfrac{1}{2}\,\mathrm{DC} \times \text{height}.$$

3. Draw (on inch-squared paper) two equilateral triangles with sides of lengths given in parts 1–10, arranging each pair so that they form a parallelogram, and so confirm the rule just given: (1) 1 in. long; (2) $2\frac{3}{4}$ in. long; (3) 3·2 in. long; (4) 3·6 cm. long; (5) 5·25 cm. long; (6) 7·75 cm. long; (7) 0·075 m. long; (8) 0·08 m. long; (9) 0·065 m. long; (10) 0·75 ft. long.

We summarise our results so far:

Area of rectangle = length × breadth.
„ parallelogram = base × height.
„ triangle = half the area of the parallelogram with its base and height the same as those of the triangle.

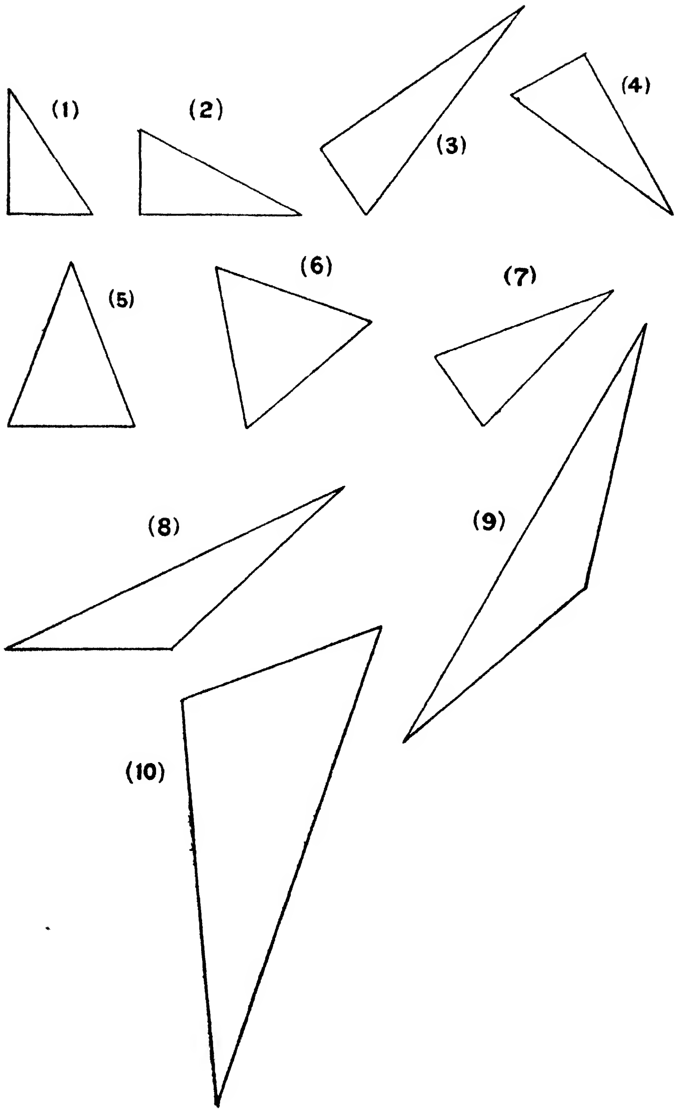
$$= \tfrac{1}{2}\,(\text{base of triangle} \times \text{height})$$
$$= \tfrac{1}{2}\, b \times h.$$

4. Draw on inch-squared paper triangles whose sides are of the lengths given, and use the last statement to find their areas: (1) 3 in., 4 in., 5 in.; (2) 1·5 in., 2 in., 2·5 in.; (3) 3·2 cm., 2·1 cm., 2 cm.; (4) 4·2 in., 3·1 in., 2·5 in.; (5) 0·5 ft., 0·25 ft., 4 in.

5. Find the area of the triangles shown on page 273, measuring the required lengths with an inch scale.

6. Find the area of the following triangles, expressing the results in the units named.

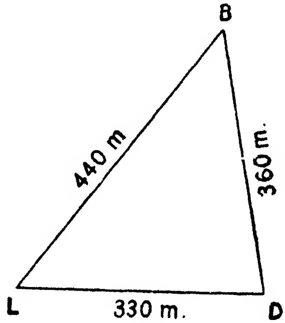
	Height.	Length of Base.	
(1)	6·3 in.	5·9 in.	Area in sq. in.
(2)	18·9 in.	17·82 in.	„ „
(3)	15·9 cm.	19·86 cm.	„ sq. cm.
(4)	7·95 cm.	9·93 cm.	„ „
(5)	2 ft. 6 in.	3 ft. 4 in.	„ sq. in.
(6)	7 ft. 6 in.	6 ft. 8 in.	„ „
(7)	4 m. 29 cm.	8 m. 56 cm.	„ sq. cm.


(1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(9)
(10)

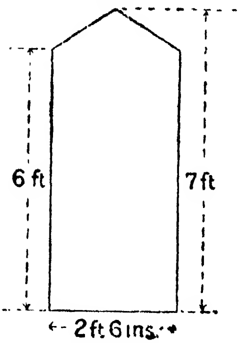
7. Find the area of the following triangles, expressing the results in the units named.

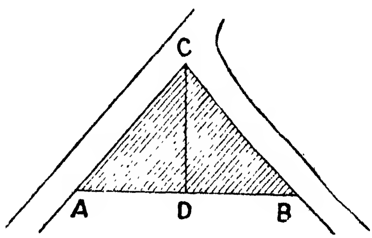
	Height.	Length of Base.	
(1)	8 m. 58 cm.	17 m. 12 cm.	Area in sq. cm.
(2)	15 ft. 10 in.	50 ft. 8 in.	,, sq. ft.
(3)	7 ft. 11 in.	101 ft. 4 in.	,, ,,
(4)	3 yds. 1 ft. 5 in.	6 yds. 2 ft. 3 in.	,, ,,
(5)	13 yds. 1 ft. 6 in.	6 yds. 2 ft. 10 in.	,, ,,
(6)	12 yds. 1 ft. $3\frac{1}{2}$ in.	8 yds. 2 ft. $2\frac{1}{2}$ in.	,, sq. yds.
(7)	17 yds. 1 ft. 5 in.	24 yds. 2 ft. 7 in.	,, ,,

8. England and Wales is shaped roughly like a triangle. **B** is Berwick, **L** is Land's End, and **D** is Dover. Draw the figure to the scale of 2 inches = 100 miles, and calculate (after drawing another necessary line) the area of England and Wales to the nearest 100 square miles.



9. A door is shaped as shown in the diagram below (to left). Find its area from the dimensions given.



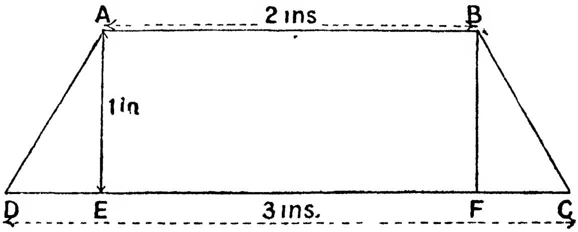


10. A piece of land stands at the fork of a road, shown above (to right). The length **AB** is 40 ft. and **CD** is 28 ft. 6 in. Find the area of the land in square yards.

E. (1) Area of Trapeziums

EXAMPLES. XLVIII.

1. Calculate the area of the triangles **ADE**, **BFC**, and of the rectangle **ABFE**. Add the three areas together and so find that of the whole figure.



2. Find the average length of **AB** and **DC**. Multiply it by the width, **AE**, of the figure, and compare the result with that of Question 1.

The figure drawn in Question 1 is called a trapezium.

3. Collect the results of Questions 1 and 2 as follows:

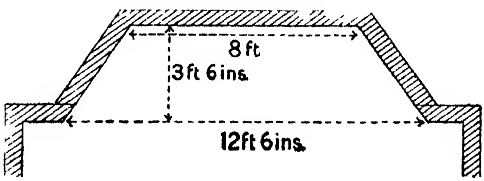
Area of a Trapezium found		
	(1) By Adding Three Areas Together.	(2) By Multiplying the Average Length of Sides by Width.

From the results of the last two questions we have:

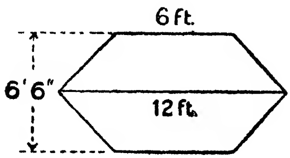
Area of a Trapezium = average length of the two parallel sides × the perpendicular distance between them.

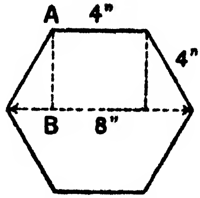
Apply this rule to the following questions (4 to 14):

4. The diagram represents the floor of a room extending into the bay window. What area of carpet will be required for the bay?



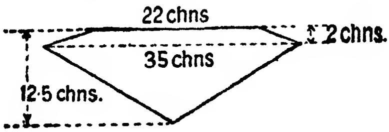
5. A flower bed is of the shape and dimensions shown below (to left). Find its area.



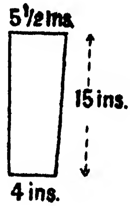


6. Street lamps in some parts of London are shaped as shown in the figure above (to right). Find the area of the base of the lamp, using the dimensions given. (The length AB must be found by the method of Question 13, p. 247.)

7. What is the area of the field represented in the diagram?

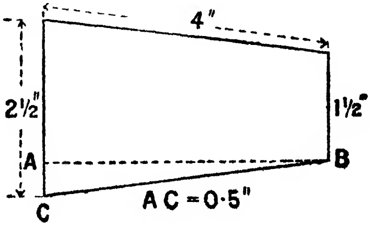


8. Each piece of glass in the lamps referred to in Question 6 is of the shape and size shown. Find the total area of glass required to glaze 150 such lamps.

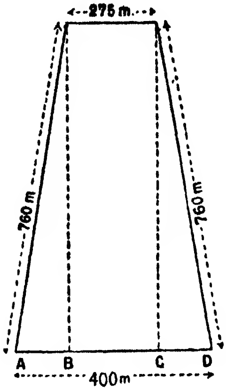


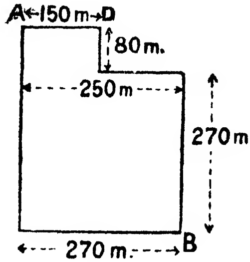
9. Find the cost of glazing the 150 lamps (Question 8) at a cost of $8\frac{1}{4}$d. per square foot.

10. A lock manufacturer has occasion to use pieces of iron of the shape shown, and they are stamped out. What area of sheet iron will be required to make 3 gross? (AB must be found by the method of Question 13, p. 247.)



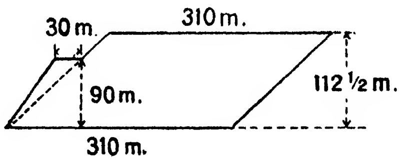
11. Find the area of the Province of Saskatchewan from the details given in the diagram (AB = CD) shown below (to left).





12. The diagram shown above (to right) represents the State of Utah (U.S.A.). The lines at the bottom and top are parallel and the angles at A, B, and D are right angles. Find, from the dimensions given, the area of the State to the nearest 100 square miles.

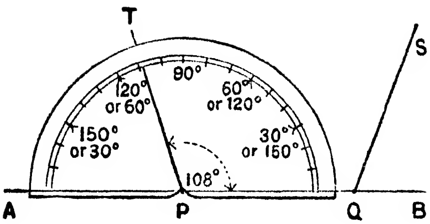
13. In this diagram we represent the State of Tennessee, where the northern and southern boundaries are parallel. Find the area of the State to the nearest 100 square miles.

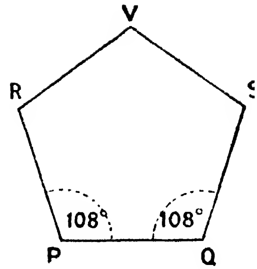


14. Offer some comment upon the shape of the States drawn in the last three questions, and the shape of (say) Yorkshire in England, or Chile in South America.

E. (2) The Area of a Pentagon, Hexagon, or Octagon—The Field-Book

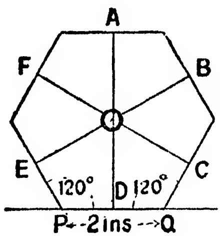
192. A Pentagon.—*Construction.*—Let its sides be 2 inches long. **Take a piece of squared paper** and on it **draw a line AB about 6 inches long.** In it take the point P, put the centre of the **protractor at P,** and draw PT through the centre and the 108° **mark.** Then the angle TPB is 108°. Mark off 2 inches along PT

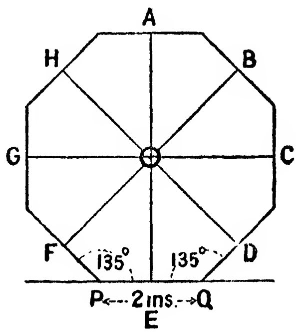




and PB, and you have two sides, PR and PQ, of the pentagon. In the same way make an angle PQS[1] 108°, and QS 2 inches, then draw RV, SV each 2 inches, and the figure is constructed.

A HEXAGON AND AN OCTAGON.—*Construction.*—These are constructed in just the same way, except that the angles are marked off with the protractor as 120° and 135° respectively. Thus—





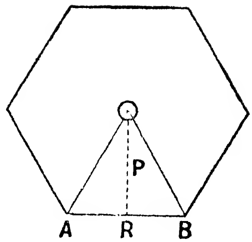
193. To find the Area of a Pentagon, Hexagon, or Octagon.

(We presume the student has drawn the figures for himself.)

1. With a pair of compasses find by trial the centre, O, of each figure, and measure its distance from each of the sides, OA, OB, OC, etc.; tabulate the results for each, and then find the average distance of the centre from the sides.

[1] Most protractors are graduated so that when the centre is placed at Q the angle PQS can be read off as 108° on the right-hand side, just as TPQ was on the left-hand side. If not, then SQB must be read off as 72°, *i.e.* 180°–108°, and thus PQS will be 108°.

2. A pentagon is made up of five triangles, a hexagon of six, and an octagon of seven, and we choose a hexagon, remarking that the method is the same for all three figures.



Now, area of $\mathsf{OAB} = \dfrac{\mathsf{AB} \times \mathsf{OR}}{2}$

$\therefore$ area of hexagon $= 6 \times \dfrac{\mathsf{AB} \times \mathsf{OR}}{2}$

and so the area of the hexagon is known at once, for **AB** is given and **OR** is either given or must be measured (1, above); hence,

$$\text{area of pentagon} = 5 \times \frac{\mathsf{AB} \times \mathsf{OR}}{2},$$

$$\text{and, area of octagon} = 8 \times \frac{\mathsf{AB} \times \mathsf{OR}}{2}.$$

For any such regular rectilineal figure[1] we have this rule:

Area of figure equals length of one side (L) × perpendicular distance (P) of that side from the centre × number of sides (N) ÷ 2.

$$\therefore \textbf{Area} = \frac{\mathsf{L} \times \mathsf{P} \times \mathsf{N}}{2}.$$

194. It is easy to show, but the commercial student should merely remember, that $\mathsf{OR} = 0{\cdot}866\mathsf{AB}$ in a HEXAGON, and $\mathsf{OR} = 1{\cdot}207\mathsf{AB}$ in an OCTAGON.

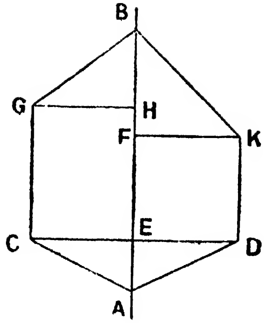
$$\therefore \text{area of hexagon} = \frac{\mathsf{AB} \times \mathsf{AB} \times 0{\cdot}866 \times 6}{2} = 2{\cdot}598\ \mathsf{AB}^2$$

$$\text{,,} \quad \text{octagon} = \frac{\mathsf{AB} \times \mathsf{AB} \times 1{\cdot}207 \times 8}{2} = 4{\cdot}828\ \mathsf{AB}^2.$$

THE FIELD-BOOK

195. In surveying a field of the shape shown in the figure (p. 280) let **AB** be due north. The surveyor makes a cross on the ground at **A**, and with his chain measures from **A** to **E** and finds **AE 3** chains.

[1] By a "regular rectilineal figure" we mean one whose sides are straight lines which are all equal to one another, as also are all its angles.



He then measures EC, and finds C 6 chains west of E, and D 6 chains east.

Also, AF = 8 chains.
FK = 6 chains east.
AH = 10 chains.
HG = 6 chains west.
AB = 14 chains.

He records his measurements in his Field Book thus:

FIELD BOOK.

CHAINS NORTH

	14 (B)	
West to G, 6	10 (H)	
	8 (F)	East to K, 6
West to C, 6	3 (E)	East to D, 6

FROM A.

Note.—All distances measured along AB are taken from A.
i.e. AE = 3 ; AF = 8 ;
not AE = 3 ; EF = 8.

196. The area of the field is found at once, thus:

$$\text{Area of triangle ACD} = \frac{CD \times AE}{2}$$

$$= \frac{12 \times 3}{2} = 18 \text{ square chains.}$$

$$\text{Area of EFKD} = EF \times FK$$

$$= 5 \times 6 = 30 \quad \text{,,} \quad \text{,,}$$

$$\text{,, GHEC} = EH \times GH$$

$$= 7 \times 6 = 42 \quad \text{,,} \quad \text{,,}$$

$$\text{Area of triangle GHB} = \frac{GH \times HB}{2}$$

$$= \frac{6 \times 4}{2} = 12 \quad \text{,,} \quad \text{,,}$$

$$\text{,, ,, BFK} = \frac{FK \times BF}{2}$$

$$= \frac{6 \times 6}{2} = 18 \quad \text{,,} \quad \text{,,}$$

Total area = 120 square chains.

EXAMPLES. XLIX. (a)

1. Find the area of pentagonal tiles the sides of which are 1 inch, $1\frac{1}{2}$ inch, 8 cm., 10 cm.

2. Determine the area of the cross-section of some hexagonal blind rollers if the sides are—(1) $\frac{3}{4}$ inch; (2) $1\frac{1}{4}$ inch; (3) 2·5 cm. long.

3. A firm of artists' colourmen supply wooden models, octagonal in section, with sides, 4 inches, 6 inches, 8 inches, or 20 cm. long. Calculate the area of the surface of the two ends of each of the models.

4. Find the sectional area of regular pentagonal iron pillars to be used in the erection of an hotel if the sides are for one set, 1 foot long; for another, 1 ft. 6 in. long; and for another, 9 inches long.

5. What area must be allowed for hexagonal columns in bath stone, if they are regular in shape and the sides are—(1) 1 ft. 3 in.; (2) 8 inches; (3) 36 cm.; (4) 40 cm.?

6. Find the sectional area of metal used in a column, octagonal in shape both externally and internally, if each of the outer sides is 1 foot long and each of the inner ones 10 inches long.

(Further questions will be found in Examples LII.)

EXAMPLES. XLIX. (b)

Make out the Field-Book entries and find the area of the land concerned from the following:

1. **A** to **B**, south to north: **A** to **C**, 10 chains; **C** east to **D**, 10 chains; **A** to **B**, 120 chains.

2. **A** to **B**, east to west: **A** to **C**, 50 chains; **C** north to **D**, 30 chains; **A** to **B**, 300 chains.

3. **A** to **B**, north to south: **A** to **C**, 10 chains; **C** east to **D**, 15 chains; **C** west to **E**, 15 chains; **A** to **B**, 50 chains.

4. **A** to **B** west to east: **A** to **C**, 60 chains; **C** north to **D**, 30 chains; **A** to **E**, 120 chains; **E** south to **F**, 40 chains; **A** to **B**, 200 chains.

5. **A** to **B** is north to south: **A** to **E**, 5 chains; east to **D**, 4 chains; west to **C**, 5 chains; **E** to **F**, 10 chains; east to **L**, 5 chains; **F** to **H**, 4 chains; west to **G**, 8 chains; **H** to **B**, 4 chains.

6. **A** to **B** east: **A** to **E**, 6 chains; **E** to **G** south, 6 chains; **E** to **F**, 7 chains; **F** to **M** north, 8 chains; **F** to **H**, 5 chains; **H** to **L** south, 10 chains; **H** to **B**, 4 chains.

7. **A** to **B** north-east: **A** to **C**, 6 chains; **C** to **F** north-west, 8 chains; **C** to **D**, 5 chains; **D** to **H** south-east, 6 chains; **D** to **E**, 5 chains; **E** to **G** north-west, 10 chains; **E** to **B**, 5 chains.

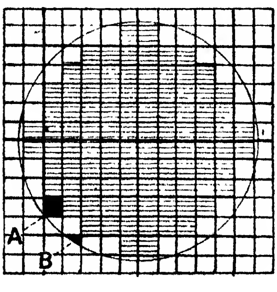
(Further questions will be found in Examples LII.)

F. Area of Circles, Cylinders, Cones

I. CIRCLES

EXAMPLES. L. (a)

1. Put a penny on a piece of inch-squared paper, draw a line round it carefully, count the number of squares contained in the circle,[1] and thence determine the area of the penny in square inches. Use the above method for Questions 2 to 7.



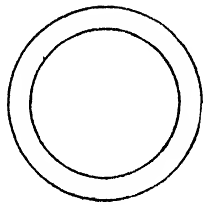
[1] In estimating the area of a figure with the help of squared paper, the student will remember that in Question 1, page 258, he has found that the area of a small square on inch-squared paper divided into tenths is $\frac{1}{100}$ sq. in. Consequently, he must reckon up the number of complete squares in the figure and then roughly estimate the number of squares to which the odd pieces are equal. Thus **A + B** make up one square.

2. Draw a circle 1 inch in radius and find its area in square inches.

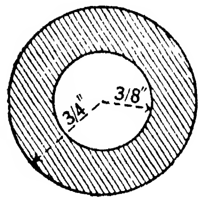
3. Find the area of a circle 2·5 cm. in radius.

4. Draw two concentric circles to represent the boundary of a lawn and of a path round it, and let the radii of the circles be $1\frac{1}{4}$ inch and $1\frac{1}{2}$ inch. Find what area represents the path.

5. The two circles represent in plan the inner and outer surfaces of the glass vacuum vessel which forms the essential part of flasks which keep their liquid contents either hot or cold. Find the sectional area of the evacuated space between the two surfaces, by drawing two circles 2 inches and $2\frac{3}{4}$ inches respectively, in diameter.

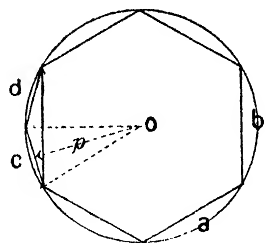


6. The cross-section of an iron tube is shown in the diagram. Find (1) the area of the hole; (2) the area of the iron.



7. The face of an old Venetian watch consists of a circle of enamel $\frac{3}{4}$ inch in diameter, around which is a ring of gold which is $1\frac{1}{4}$ inch across. Find the area of the gold ring.

8. Draw a circle with a penny and place round the figure six lines a, b, etc., each half the length of the diameter of the circle. Draw two lines c, d, equal to one another as shown. Measure c and p very carefully in inches, and then calculate the area of the little triangle. Twelve such triangles can be drawn in the circle. Find the area of the twelve.[1]



9. Calculate for the circle drawn in the previous question the value of $\frac{22}{7}$ times the square of the radius (*i.e.* πr^2), and find by how much the result differs from that of Question 8.

[1] This simple experiment should be performed by drawing at least three triangles and by finding the average area, which must then be multiplied by 12, and the problem can be varied by working in centimetres.

10. Fill in the following table, using the results obtained graphically in Questions **2**, **3**, **4**, **5**, for column α:

Radius of Circle.	Area found Graphically. α		Area Calculated. $A=\pi r^2$. β	Difference between Columns α and β.
1 inch .	Qn. 2.			
2·5 cm. .	Qn. 3.			
$1\frac{1}{2}$ inch .	Qn. 4.			
2 inches .	Qn. 5.			

197. From the results obtained above, the student will observe that there is very little difference between columns α and β.

Now the shorter the lines c and d (Question 8) become, the nearer they coincide with the circumference of the circle, and at the same time p becomes more and more nearly equal to r. At last we have a very large number of small triangles whose area is $\frac{a \times r}{2}$, and if we add together all the short lines a we shall have the circumference of the circle, *i.e.* $2 \times \frac{22}{7} \times r$ (p. 251).

$$\therefore \text{ the area of the circle} = \text{all the } a\text{'s added together} \times \frac{r}{2}$$

$$= \text{the circumference} \times \frac{r}{2}$$

$$= 2 \times \frac{22}{7} \times r \times \frac{r}{2}$$

$$\therefore \mathsf{A} = \frac{22}{7} r^2 \text{ or } \pi r^2,$$

and we shall ask the student *always* to use this expression.

$$\text{Again, } \mathsf{A} = 3{\cdot}1416 r^2 \text{ or } 0{\cdot}7854 d^2, \text{ since } d^2 = \left(\frac{r}{2}\right)^2 = \frac{r^2}{4}$$

and $$r^2 = \frac{\mathsf{A}}{3{\cdot}1416}$$

or $$r = \sqrt{\frac{\mathsf{A}}{3{\cdot}1416}} \qquad \therefore r = \frac{\sqrt{\mathsf{A}}}{1{\cdot}772} = 0{\cdot}564\sqrt{\mathsf{A}};$$

a fact which may sometimes be useful for **checking results.**

EXAMPLES. L. (b)

We shall assume that $\pi = \frac{22}{7}$, $A = \frac{22}{7}r^2$, and $r^2 = \frac{7}{22}A$, unless otherwise stated.

Find the area of circular plots of ground, whose radii are given below, by employing the equation:

area of circle $= \pi r^2$:

1. 4·8 in.	2. 5·3 cm.	3. 5·6 ft.
4. 1·8 m.	5. 5·9 yds.	6. 1·9 mi.
7. 8·5 m.	8. 5·6 mi.	9. 0·5 in.
10. 0·84 cm.	11. 0·54 cm.	12. 0·001 in.
13. 0·01 in.	14. 0·005 m.	15. 0·103 ft.
16. 0·008 mi.		

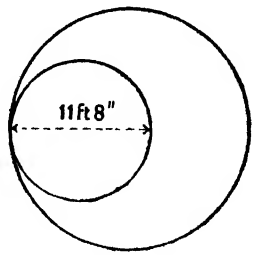
17. What is the radius of a circle whose area is 154 square centimetres?

18. Find the diameter of a circle whose area is 1386 in square inches.

19. What would be the circumference of a circle whose area is 0·2464 square metre?

20. Two circles differ in area by 2·2 square centimetres. The radius of the smaller is 0·3 centimetre, find the radius of the latter.

21. The diameter of the tunnel on one of the London Tube railways is 11 ft. 8 in., but at the stations it is enlarged as shown in the diagram to a tube of diameter 20 feet. By how much does the sectional area of the tube at the stations exceed that of the tunnels?



22. What must be the radius of a circular flower-bed in order that it may enclose 2464 square yards?

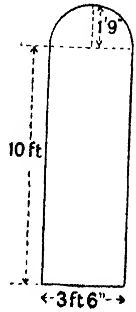
23. How many square yards would there be in a path 4 feet wide made round the outside of the bed in the last question?

Hint.—(i) Find area of bed; (ii) find area of circle which includes bed and path; (iii) find area of path.

24. What must be the length of the side of a square of platinum, that its area may be the same as that of a circle of that metal 2·75 inches in radius.

25. The sectional area of the bore of a gun is 8 square inches and that of the muzzle (over all) is 0·55 square foot. What is the thickness of the metal?

26. A church window is of the form and dimensions shown in the diagram. What would it cost to glaze it at 3s. 6d. per square foot?



27. A Birmingham firm makes the shafting for the propellers of steam vessels. If the radius of one shaft is $3\frac{1}{2}$ inches and of another $4\frac{1}{4}$ inches, find the difference in sectional area between them.

28. Two lead pipes are of the same thickness of metal, namely, 0·7 inch, but the bore of one is 2·6 cm., and of the other 3·5 cm. Find the difference in the sectional area of the metal.

29. A post-office pillar box, such as one sees at the edge of the pavement, has an external diameter of 45 cm. and an internal diameter of 42·5 cm. Find the area of the iron section.

30. What would it cost to provide and fix in a semicircular glass light over a front door, if the diameter of the glass is 2 ft. 6 in. and the glazier charges 1s. 3d. per square foot?

31. The port-holes of a French steamboat are $9\frac{3}{4}$ inches in diameter, and there are thirty-eight on the port side and thirty-two on the starboard side. What would it cost to put in the glass, reckoning $7\frac{1}{2}$d. per square foot, and quoting in francs for the information of the French Steamship Company?

32. A large block of mahogany, 1 foot thick and roughly a circle of 8·5 feet in diameter, is imported from the West Indies into Austria. Its value is 3s. $6\frac{1}{2}$d. per square foot. What must the Austrian timber merchant pay in kroner? (1 krone = 10d.).

33. Some forms of electrical instruments as well as (*e.g.*) clocks are provided with glass shades and wooden bases. How many circular base boards, 8·5 cm. in radius, can be cut from an American whitewood board 8 inches wide and 8 feet long?

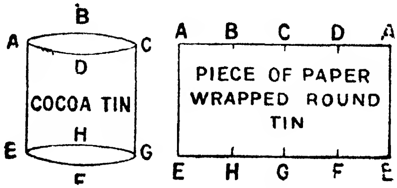
34. What area of white wood would be wasted under the circumstances of the last question?

35. What would be the value of each base board at 1s. $3\frac{1}{2}$d. per square foot, including machine cutting to size required?

36. A "round" scent-bottle is made of thick glass, the inside circumference of which is 8·8 cm. and the outside 11 cm. What is the thickness of the glass?

II. SURFACE AREA OF A CYLINDER AND OF A CONE

198. If a piece of paper were wrapped just round a cylindrical cocoa tin, and then taken off and put flat on the table, the area of the paper would be that of the curved surface of the tin.



AA or **EE** is the circumference of the tin, and **AE** its height.

∴ **area of curved surface of cylinder = perimeter**[1] **or circumference × height.**

$$= 2\pi r + \textbf{height};$$

r and h being in the SAME UNITS.

The total area will be $2\pi r \times h +$ area of *both* ends, if the cylinder is closed, and of one end if open.

199. The area of the curved surface of a *Cone*

$$= \frac{\textbf{circumference of base} \times \textbf{slant height}}{2}$$

$$= \frac{2\pi r \times \mathrm{S}}{2}.$$

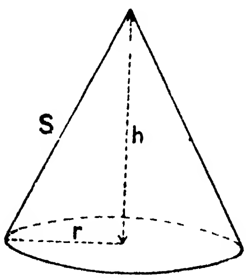


FIG 1

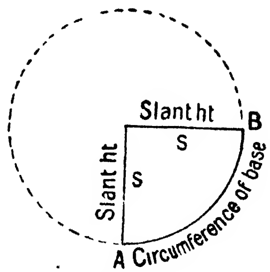


FIG.2.

The truth of this expression can be proved by wrapping a piece of paper round a cone so that it just fits, and then removing

[1] The student will see that the area of (a) the walls of a room, and (b) the curved surface of a cylinder equals (perimeter × height), and so the surface area of any similar object is found in the same way.

it, cutting it down and placing it flat on the table, when its shape will be as in Figure 2.

Now the area of the surface of the cone is $\frac{1}{4}$ of the area of a circle with radius **S**, *i.e.* $\frac{1}{4}\pi S^2$.

Also, circumference of base (*i.e.* AB) $\times S \div 2$

$= \frac{1}{4}$ circumference of circle of radius $S \times S \div 2$

$= \frac{1}{4} \times 2\pi S \times S \div 2 = \frac{1}{4}\pi S^2$

$\therefore$ area of surface cone $=$ circumference of base $\times S \div 2$.

Note.—In all cones with which we shall deal, h, r, and **S** form a right-angled triangle, and so $S^2 = h^2 + r^2$ (p. 247).

$\therefore$ if h and r are given, **S** MUST BE FOUND.

EXAMPLES. LI.

What must be the dimensions of advertisement wrappers for cylindrical tins whose dimensions are given in Questions 1 to 6? (Allow $\frac{1}{4}$ inch or 1 cm., as the case may be, on the circumference for sticking the edges of the wrapper.)

1.	Radius,	1 inch;	height,	3 inches.
2.	„	1·5 inch;	„	3½ inches.
3.	„	2·8 cm.;	„	8 cm.
4.	Diameter,	3·6 inches;	„	6 inches.
5.	„	4·2 inches;	„	0·8 inch.
6.	„	5·6 cm.;	„	0·1 m.

7. What will it cost to paint a cylinder of iron, 7 feet in diameter and 20 feet high, at 5½d. per square foot?

8. What should be charged to hand-paint 3 dozen vases, roughly cylindrical in form, 2 inches in diameter, 8 inches high, at ¾d. per square inch?

9. A chimney-stack 120 feet high is roughly cylindrical in shape. Its internal diameter is 8 feet and external 10 feet. What is the difference between the area of the inside and outside surfaces of the stack?

10. A dust-bin is 2 feet across and 3 feet high; how many square feet of zincked iron are necessary to make it, if the cover contains a half square foot more than the bottom and 1 square foot extra is allowed for all joints?

11. Find the cost of polishing the curved surface of marble ornaments, conical in shape, at 1½d. per square inch, the dimensions being:

Radius	of base,	1 inch;	slant	height,	3 inches.
„	„	1·5 inch;	„	„	3½ inches.
„	„	2·8 cm.;	„	„	8 cm.
„	„	1·8 inch;	**vertical**	„	6 inches.
„	„	2·8 cm.;	„	„	12 cms.

12. On the south-west coast of England there is a large conical stone which is painted white, by which fishermen and others engaged in coast trade can steer. It is 25 feet high and the circumference of the base is 21 feet. What does it cost to whiten this "mark" at 6s. 8d. per square yard?

EXAMPLES. LII.

Miscellaneous Questions

1. What is the cost of tiling a floor 17 ft. long and 14 ft. 2 in. broad with tiles 5 inches square, if they are sold at 7s. 6d. per hundred and we cannot buy less than ten of them?

2. Find the cost of erecting railways along the boundary of a field 5386 yds. 2 ft. long at £3 15s. 3d. per chain.

3. What is the cost of sowing grain on 56 ac. 5 sq. chs. 300 sq. yds. of prepared land at a cost of £56 16s. 6d. per acre?

4. What would it cost an emigrant to Canada to clear two plots of land 150 acres and 350 sq. chs. in area at £2 15s. 6d. per acre?

5. What would it cost an emigrant to buy a good dairy farm of 125 ac. 3 rds. 10 po. in Ontario at a cost of £15 12s. 7d. per acre?

6. What would it cost the emigrant to rent the farm referred to in the last question at a cost of £1 4s. 9d. per acre?

7. What would a piece of land 5 sq. chs. 215 sq. yds. cost at the rate of 3s. 6d. per square foot?

8. Find the cost of repairing a road $\frac{3}{5}$ mile in length and 30 ft. wide at 2s. 8d. per square yard.

9. What can a freehold cleared farm of 150 ac. 6 sq. chs. 250 sq. yds. be bought for in Newfoundland at 75 dollars per acre? (1 dollar = 4s. 2d.)

10. In Newfoundland the Government leases half a square mile of land for 10 dollars down, and 20 dollars for the first year, 30 per annum for the next five years, 50 per annum for the next five, and 100 per annum for the remainder of the term of the lease, namely, ninety-nine years. If a miner keeps his holding for eight years, calculate the amount he pays in that time for every square yard he is renting.

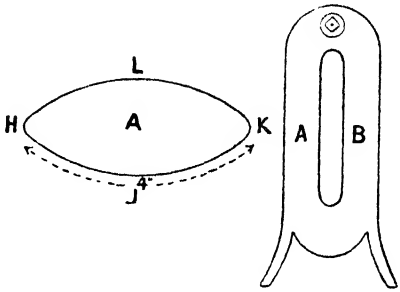
11. The area devoted to agricultural farms in Queensland is 4,849,706 acres, divided into 14465 selections; find the average area of a selection and its value at £2 16s. 8d. per acre.

12. A person who is a citizen of the United States, or who is prepared to become one, can have a piece of land up to 160 acres

in area at $1\frac{1}{4}$ dollar per acre per annum, and his title is perfected by the issue of a patent after five years of actual settlement. If then a man has a settlement of 148 ac. 6 sq. chs., and the fees and commission charges are $26\frac{1}{2}$ dollars, find what he pays during his first five years' occupation.

Note.—The fees, etc. are obviously NOT annual charges. (1 dollar = 4s. $1\frac{1}{2}$d.)

13. A rough sketch of the plan of the tube of a radiator is shown in the diagram, as well as an end view of the radiator. (1) Find the length of the heating surfaces **HJKL** for both **A** and **B** in the plan. (2) Taking the mean height of the tubes to be 26 inches, find the total heating surface per tube.

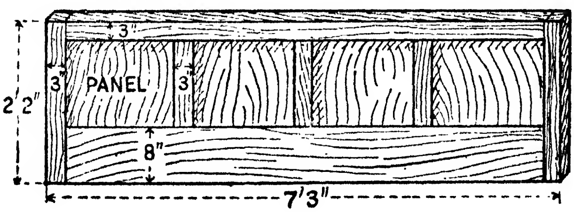


14. Employing the data of the last question, find how many tubes will be necessary to warm a dried-fruit store if the heating surface of the radiators must be 1000 square feet.

15. What is the value of 350 chō 3 tan 150 tsubo of land in Japan at £20 5s. 8d. per acre? (1 chō = 10 tan = 3000 tsubo = 2·45 acres.)

16. The area of a small estate near Tokio is 530 tsubo, and it is to be sold for yen 6000. What is the price per square yard in English money? (1 tsubo = 3·95 square yards.)

17. The diagram represents the elevation of one part of the back of a seat for a lecture theatre executed in dark oak. Calculate (1) the length of wood 3″ wide required; (2) the



length of wood 8″ wide required. In each case allow 1″ to the foot extra for the joints.

18. In the previous question calculate the number of square feet of oak required for the section given, allowing $\frac{1}{8}$ square foot to every square foot of surface for joints.

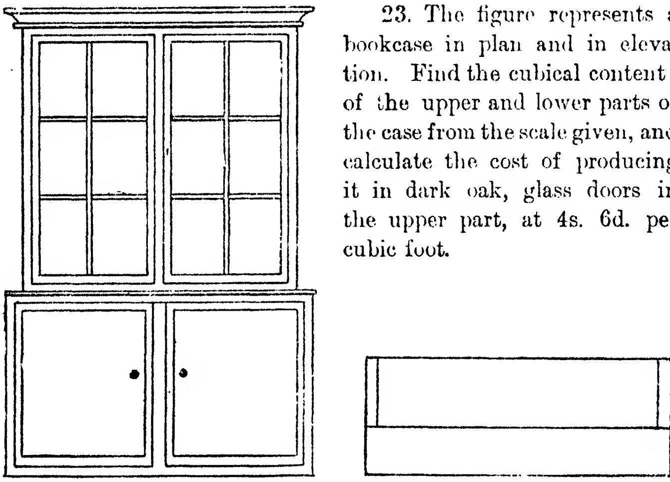
19. The thickness of the wood 3″ and 8″ wide in Question 17 is $1\frac{1}{2}$″. Each foot in the inch [1] of the former costs $3\frac{1}{2}$d., and of the latter $5\frac{3}{4}$d. Calculate the cost of the framework of the section given (allow for joints).

20. If the panels referred to above cost 10d. per square foot, calculate the whole cost of material in making thirty-six such sections (allow for joints).

21. The population of Japan in 1914 was 53,600,000, and the area 24800 square ri [2]; determine the population per square mile for comparison with British statistical returns.

22. A firm estimates at 4s. 9d. per cubic foot for providing and erecting cupboards 7 feet high and 1 ft. 3 in. square, in section suitable for hanging up one's coat and hat, etc. Find the cost of a set of such cupboards which occupy a recess 18 ft. 9 in. long.

23. The figure represents a bookcase in plan and in elevation. Find the cubical content [3] of the upper and lower parts of the case from the scale given, and calculate the cost of producing it in dark oak, glass doors in the upper part, at 4s. 6d. per cubic foot.



ELEVATION. PLAN.

Scale, $\frac{1}{4}$ inch = 1 foot.

[1] Boards which are over 1 inch thick are always quoted at per foot in the inch. Thus, wood $1\frac{1}{2}$ inch in thickness and 6 feet long would be reckoned as $6 \times 1\frac{1}{2}$, or 9 feet "in the inch," the width being also stated.

[2] 1 square ri = 5·95 square miles.

[3] See Section XIV.

24. The top of a sideboard is made of mahogany and is 7 ft. 6 in. long, 2 ft. 6 in. wide, and $1\frac{1}{2}$ inch thick. How many feet super[1] at 1 inch in the foot does it contain, and what is the value of the wood at $4\frac{1}{2}$d. per foot super? (To the nearest shilling.)

25. Determine the area of Japan[2] in square miles, given that it is 24352·25 square ri.[3]

26. The area of Great Britain and Ireland is 121090 square miles, find the ratio of the area of Japan to that of the United Kingdom.

27. In the States of Perak and Selangor there are extensive forests reserved by Government and the trees are used as required. From the bark of the mangrove trees a substance called cutch is made. If an acre of forest yields (among other things) $1\frac{1}{2}$ ton of cutch, what would be the yield of an estate of 250 square miles?

28. If the cutch referred to in Question 27 makes 28s. 6d. per cwt. when sold retail to the English fishermen for "curing" their nets, find the retail value of the yield of the estate mentioned.

29. The British Standard Specification for Portland cement requires that it shall be ground so fine that it will pass through a sieve with 32400 holes per square inch; calculate, as the decimal of a square inch, the average sectional area of a particle of cement.

30. "A square" of flooring contains 100 feet super, and prepared oak, tongued and grooved, and bored for secret nailing, is sold in pieces $4\frac{1}{2}$ inches long and 1 inch wide; how many such pieces go to a square?

31. In the Provinces of Nagano and Fukui (Japan) waterproof paper is made by pasting together two layers of paper, employing a secret waterproof adhesive solution for the purpose.[4] The width of the paper is 140 cm., and it is made up into shirts which are sold at yen 1·25 apiece. If it requires $4\frac{1}{2}$ yards of 44-inch material to make a man's shirt, and it costs 9d. per yard in England, does it cost more to provide a shirt in England or in Japan, if 1s. must be allowed for making in the former case?

32. Take a piece of squared paper and mark off on it a square whose edge is 9 inches long. Cut out the square and weigh it. Describe a circle 6 inches in diameter within the square already drawn. Cut out the circle very carefully. Weigh it and calculate its area[5] by comparison with the weight of the square of known area.

[1] See footnote 1 on page 291.
[2] *Financial and Economic Annual of Japan.*
[3] 1 square ri = 5·95 square miles.
[4] At Fukui one layer of stouter paper is used.
[5] Frequently we find 1 sq. Km., etc., written 1 Km^2.

ANSWERS TO EXAMPLES

PART I.

EXAMPLES. I.

SECTION I. A and B.

ADDITION AND SUBTRACTION

P. 5.—**1.** (*a*) 10, 12, 14, 16, 18, 20, 22. (*b*) 90, 92, 94, 96, 98, 100.
(*c*) 398, 400, 402, 404, 406, 408, 410, 412, 414, 416, 418, 420, 422.
(*d*) 1564, -66, -68, -70, -72, -74, -76, -78, -80, -82, -84, -86, -88, -90, -92, -94, -96.
(*e*) 85986, -88, -90, -92, -94, -96, -98, 86000, -002, -004, -006, -008.

2. (*a*) 9, 11, 13, 15, 17, 19, 21, 23. (*b*) 89, 91, -3, -5, -7, -9, 101.
(*c*) 397, 399, 401, 403, 405, 407, 409, 411, 413, 415, 417, 419, 421, 423.
(*d*) 1563, -65, -67, -69, -71, -73, -75, -77, -79, -81, -83, -85, -87, -89, 91, -93, -95, -97.
(*e*) 85985, -87, -89, -91, -93, -95, -97, -99, 86001, -003, -005, -007, -009.

3. (*a*) 597, 598, 599, 600, 601, 602.
(*b*) 10568, 10569, 10570, 10571, 10572, 10573.
(*c*) 15999, 16000, 16001, 16002, 16003, 16004.
(*d*) 999, 1000, 1001, 1002, 1003, 1004.
(*e*) 10000, 10001, 10002, 10003, 10004, 10005.
(*f*) 101010, 101011, 101012, 101013, 101014, 101015.

4. Odd numbers—
(*a*) 599, 601, 603, 605. (*b*) 10569, 10571, 10573, 10575.
(*c*) 15601, 15603, 15605, 15607.

Even numbers—
(*a*) 598, 600, 602, 604. (*b*) 10570, 10572, 10574, 10576.
(*c*) 16000, 16002, 16004, 16006.

5. 30, 32, 34, 36, 38, sum=170. **6.** 120. **7.** 196; 210; 14.

P. 6.—**8.** 68712. **9.** 223; 126; 270.

10. (*1*) 164198. (*2*) 10697. (*3*) 16894. (*4*) 4,917,613.
(*5*) 11138. (*6*) 54,760,613. (*7*) 13237. (*8*) 46,984,386.
(*9*) 37746. (*10*) 97,077,575.

P. 6.—11. (*1*) 5380. (*2*) 11,214,500. (*3*) 79840. (*4*) 120,877,587.
(*5*) 224008. (*6*) 467571. (*7*) 34385. (*8*) 78,069,662.

12. Row totals—
(*1*) 20,076,528. (*2*) 3,015,847. (*3*) 2,689,705. (*4*) 1,253,466.
(*5*) 14680. (*6*) 154419. (*7*) 759050. (*8*) 212759.
(*9*) 10,128,319. (*10*) 258345. (*11*) 219052. (*12*) 407230.
Total, 39,189,400.

Column totals—
(*1*) 22,101,615. (*2*) 15,853,656. (*3*) 51479. (*4*) 36225.
(*5*) 279160. (*6*) 296144. (*7*) 277026. (*8*) 294095.

P. 7.—13. (*1*) 809146. (*2*) 677933. (*3*) 2,468,387.
(*4*) 3,235,927. (*5*) 504203.

14. (*1*) 454837. (*2*) 337561. (*3*) 1,402,223. (*4*) 1,617,697.
(*5*) 2,245,218. (*6*) 10,077,345. (*7*) 11,884,672.
(*8*) 23,059,439. (*9*) 17,230,517.

P. 8.—15. (*1*) 26,152,262. (*2*) 21,914,323. (*3*) 122,849,915.
(*4*) 120,915,852. (*5*) 137,115,628. (*6*) 135,198,699.

16. Brazil, The Argentine, Bolivia, Peru, Columbia, Venezuela, Chile, Paraguay, Ecuador, British Guiana, Uruguay, Dutch Guiana, French Guiana. Area in square miles = 7,413,000.

P. 9.—17. 2083, 2511, 2523.

18. (*1*) Aug. 5. £206,836,556; Sept. 2. £394,658,913. Difference, £187,822,357.

(*2*) Gold + £20,150,643. Notes - £3,817,660. (The - sign implies a decrease.) Reserve + £20,968,303. Other securities + £56,469,036. Other deposits + £77,069,216. Government securities + £16,982,819.

(*3*) £187,822,357.

(*4*) On September 2nd; for its gold had increased by £20,000,000 and its note circulation fallen by £800000, while the value of the securities has also risen.

19. £62,686,451; £62,403,576. Increase from 1913-14, £282875.
(Increase from 1913-14 marked + ; decrease - .)
- £33100; + £36000; + £63600; + £168000; - £78722; - £23000; + £25000; + £63920; - £79000; - £70490; + £97667; + £32600; + £29200; + £51200.

P. 10.—20. (*a*) Greatest, 1913; smallest, 1901. (*b*) Greatest, 1912; smallest, 1901.

No, because Great Britain has to import a great deal of foodstuffs, etc., for her own consumption.

Totals, £711,800,000 and £464,900,000.

Excess of imports over exports, £246,900,000.

21. Linggi, Merlimau, Ceylon (Para), Mergui Crown, Nagolle, Neboda, Hayoep, Bukit Mertajam, Jong-Landor, Kombok, Bakap, Kepong; 554075 lb. for twelve estates, 609326 lb. for them all.

P. 11.—22. (*1*) Area of Empire, 11,726,000 square miles.

(*2*) Population of Empire, 427,105,757.

In order of size: (1) Canada and Newfoundland, (2) Australia, (3) India, (4) South Africa, (5) East and Central Africa, (6) West Africa, (7) West Indies, etc., (8) United Kingdom, (9) New Zealand, (10) Ceylon, etc., (11) Gibraltar, etc., (12) Eastern Asia.

In order of population (numbered as above), (3), (8), (6), (5), (1), (4), (2), (10), (7), (9), (12), (11).

The area of Canada is 3,750,000 square miles and its population is roughly 7,000,000. The area of the United Kingdom is 121000 square miles and its population 45,000,000. So that the greatest area has by no means the greatest population. Canada is young, England is old; Canada is largely agricultural, vast tracts of her land are untouched by human hand; the United Kingdom is very largely industrial and only a small proportion of her land is not put to use at the present time.

23. It exceeds (1) by 3,759,000 square miles; (2) by 1,952,000 square miles; (3) by 1,292,000 square miles.

(*1*) Population of Asiatic Possessions = 320,055,240.

(*2*) ,, ,, European ,, = 45,666,665.

(*3*) ,, ,, African ,, = 47,300,000.

(*4*) ,, ,, Australasian ,, = 5,499,983.

(1) exceeds (2) by 274,838,575, (3) by 272,755,240, and (4) by 314,555,257.

24. There are 2,756,511 more in the United States than in France; 2,786,307 more than in Belgium; 2,787,505 more than in Switzerland. There are 1198 fewer in Switzerland than in Belgium.

P. 12.—25. Increase, 1811–1861 = 9,901,968. Increase, 1861–1911 = 16,009,045. The increase 1861–1911 is the greater by 6,107,077. The increase from 1811–1911 = 25,911,013.

26. Length of line for 1861 is 4 inches, and for 1911, 8 inches.

27. 1813; £119,749,448. The loan was nearly £4 per head of the population. The greatest loan was raised in 1812–13–14, the three years immediately preceding the Battle of Waterloo.

P. 13.—28. £5,462,452. **29.** £13685. **30.** £27,934,000. The numbers given correct to 1000 are close enough for practical purposes.

31. £553. **32.** £496029. **33.** £18,966,538. **34.** £5419.

35. (*1*) £2,430,207. (*2*) £5,206,000.

P. 14.—36. £980541; £5,000,000.

P. 15.—37. In thousands of pounds:

Estate duty (England and Wales) = £20937. Total (Scotland) = £17061. Total, "Other Sources" = £828.

Customs (Ireland) = £3166.

Line totals in order, customs first: £33517; £38060; £25375; £10104; £687; £1956; £44700; £437. Grand total, £154836.

P. 15.—**38.** Aloe fibre value, 1912, £45465. Raw sugar, 1913, £2,048,721.
Increases or decreases in order—
+11400; +2; −1368; −2916; −7672; +2138; −151; −301758; +935; +65506. Total decrease, 233844.

P. 16.—**39.** Increases and decreases in order—
+5206; −15638; −45198; −47; −2218; +31385; −4573; −82211; −118500; −309176; +172640; +47147; +26924; +36008. Totals, 2,325,933; 2,207,433.

EXAMPLES. II.

SECTION I. C.

APPROXIMATION

P. 19.—**1.** 360; 570; 870; 530; 1060; 3130.
2. 5000; 4000; 8000; 4000; 3000; 3000; 4000.
3. To 10000: 6,590,000; 12,780,000; 295,740,000.
To 100000: 6,600,000; 12,800,000; 295,700,000.
To 1,000,000: 7,000,000; 13,000,000; 296,000,000.
4. 430; 740; 860; 930; 860; 780; 1100; 2800.
5. 8630; 7510; 5030; 6000; 7820; 15600; 1550; 30000.
6. To 5 significant figures: 8,695,400; 86,274,000; 80,000,000.
To 6 significant figures: 8,695,430; 86,274,300; 80,000 000.
To 3 significant figures: 8,700,000; 86,300,000; 80,000,000.
7. 12000. **8.** £53,000,000.
P. 20.—**9.** £6,898,000. **10.** £17,000,000. **11.** £2,049,000.
12. £17,192,000. **13.** 8,300,000 tons. **14.** 2,361,000 tons.
15. 22,749,000 lb. **16.** £188,400,000. **17.** £3,445,330.

EXAMPLES. III.

SECTION II. A and B.

MULTIPLICATION AND DIVISION

P. 34.—**1.** (*1*) 6364; 3096. (*2*) 6174; 12348. (*3*) 12000; 19200.
(*4*) 47136; 329952. (*5*) 166170; 1,495,530. (*6*) 704692; 5,637,536.
(*7*) 54,578,052; 218,312,208. (*8*) 541,517,571; 2,707,587,855.
(*9*) 7,856,496,552. (*10*) 98,869,900; 123,587,375; 97,881,201; 87,994,211; 9,885,012,602.
2 and **3.** Answers as above.
4. (*1*) 17,660,688; 17,747,260; 18,006,976.
(*2*) 56,169,477; 62,959,194; 67,279,923.
(*3*) 24,978,084; 24,157,208. (*4*) 34,288,800; 164,194,368.
(*5*) 4,328,082,036. (*6*) 72,894,452,436.
5. (*1*) 1,994,727. (*2*) 439,144,042. (*3*) 461,672,744.
(*4*) 702,973,089. (*5*) 658,890,793,314.

P. 34. — **6.** (*1*) 21,564,592. (*2*) 43,129,184. (*3*) 129,387,552.
The second result is twice, and the third six times, the first. If the first has been found, the second can be obtained by multiplying it by 2 and the third by multiplying it by 6.

P. 35. — **7.** Divisible by—
2: (1), (3), (5), (6), (8), (9), (10), (11), (13), (14), (17), (18).
3: (2), (4), (5), (8), (9), (10), (11), (13), (15), (18).
4: (9), (10), (14), (17), (18). 5: (2), (10), (15), (16).
8: (10), (14), (17). 9: (2), (5), (10), (11), (13), (18).
10: (10). 11: (7), (9), (12), (15).

8. (*1*) 3×7; $2 \times 2 \times 3 \times 3$; $2 \times 2 \times 2 \times 2 \times 3$. (*2*) 2×43; 2×37; $2 \times 7 \times 7$.
(*3*) $2 \times 3 \times 3 \times 7$; $2 \times 3 \times 5 \times 5$; $5 \times 5 \times 7$.
(*4*) $3 \times 5 \times 5 \times 7$; $2 \times 2 \times 3 \times 3 \times 11$; $3 \times 3 \times 3 \times 37$.
(*5*) $2 \times 5 \times 47$; $2 \times 2 \times 7 \times 13$; $2 \times 2 \times 7 \times 19$.

9. $7 \times 11 \times 13$; 17×29; $3^3 \times 7^2$; $2^2 \times 3^3 \times 13$; 2×11^3; $2 \times 3^2 \times 13^2$; $2 \times 3^4 \times 13 \times 17$; 23×29.

10. 7396; 23104; 9801. **11.** 46656; 493039; 830584; 2,628,072.

12. 59319; 300763; 117649; 1,771,561.

13. 18, 19, 21, 32, 37, 41, 47, 52, 64, 73, 75, 96, 94, 86, 99.

14. 4, 8, 11, 15, 23, 34, 46, 74, 75, 71. 79, 90, 94, 81.

15. (*1*) 14 and 19 over; 7 and 19 over.
(*2*) 160 and 13 over; 80 and 13 over.
(*3*) 121 and 5 over; 75 and 85 over.
(*4*) 269 and 28 over; 89 and 316 over.
(*5*) 236 and 295 over; 219 and 115 over.

16 and **17.** Answers as above.

18. (*1*) 1964 (both parts). (*2*) 2865 (both parts). (*3*) 5683 (both parts)
(*4*) 73954. (*5*) 73954. (*6*) 758642.

P. 36. — **19.** (*1*) 86572. (*2*) 68583. (*3*) 58634. (*4*) 65312.
(*5*) 987. (*6*) 86942. (*7*) 76843. (*8*) 65312.
(*9*) 886473. (*10*) 132564. (*11*) 86954. (*12*) 988699.

20. Answers as above. **21.** 165 pages.

22. The number (from basement) is: 175, 66, 90, 96, 96, 105, 104, 105. Total 837.

23. 186 square yards. **24.** 58 spans.

25. Total length, 1188 feet; width, 147 feet. **26.** £164120.

P. 37. — **27.** 9856 yards. **28.** 4463 tons. **29.** £843.

30. 314692 books. Tabulate thus:

No. of Shelves.	No. of Books.	Total.
756	50	37800
876	93	81468
1576	124	195424

P. 37.—**31.** 730 feet. **32.** 216 (week); 11268 (year).

33. 18 and 939 dozen; 11 and 563 score (to the nearest score); 1½ and 78¼ gross.

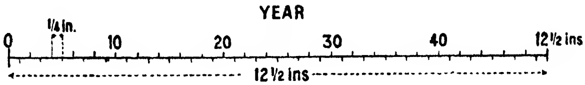
34. 3,324,810 members. **35.** £2,130,925, to the nearest £1.

36. 8 rolls per pile. Height = 10 ft. 8 in., or each pile is 4 ft. 8 in. higher than the man.

37. 16 ounces. **38.** (*1*) 1. (*2*) 3. (*3*) 4. (*4*) 4. (*5*) 3.

P. 38.—**39.** 2669 feet. **40.** 50 pieces. **41.** £2 5s. 10d. £3 19s. 2d.

42. Since ½ inch represents 3 feet, ¼ inch represents 18 inches, and 12½ inches represents 75 feet,



There are 50 spaces of ¼ inch, and so 50 pieces of paper.

43. 175 people. **44.** 2 people.

45. By 173 people. India has been inhabited from the dawn of the world's history, but Canada has been colonised comparatively recently.

46. £3. **47.** £14,428,000. **48.** £2.

P. 39.—**49.** 5d. per lb. **50.** 25s. per ton. **51.** 83s. per cwt.

52. 76s. per cwt., or 7s. per cwt. less than the butter from Jamaica.

53. 73s. per ounce. Cf. Questions 77–96, Examples LVIII.

54. 564. **55.** £142.

56. £37, which is £105 per inhabitant less for New Zealand than for the Falkland Islands. The student will not conclude, therefore, that the latter place is **necessarily** more prosperous than the former.

57. 501. **58.** 9 for Australia and 17 for Canada. **59.** 24 bushels.

P. 40.—**60.** 80135 letters. **61.** 230665 letters.

62. There are far more letters per post office in Japan than in Australia, for the population of the former is ten times greater than the latter; but the number of letters per person in Japan is fewer than in Australia, and the total external trade of Japan in 1912 was 117 million £'s and of Australia 157 million £'s.

EXAMPLES. IV.

SECTION II. C.

AVERAGES

P. 42. — **1.** 24d. **2.** 1,570,000 tons.

P. 43. — **3.** 2,697,861 thousand roubles. **4.** 52802, imports; 7844, exports.

5. 41744. (Note the rapid decrease in the amount exported. The British Government is gradually reducing the sale of this terrible drug.)

P. 44. — **6.** 806313 Haikuan taels.

7. Greatest in first shop, £43; least in third shop, £33.

8. (*1*) 35. (*2*) £38. (*3*) £35. **9.** 168 (nearly). **10.** £214720.

11. Average, 1,121,000 dollars; median, 1,179,000 dollars. The numbers vary so much and are so few that the value of the median over the average cannot readily be seen.

P. 45.—**12.** Averages are 80° and 82°, to the nearest degree; difference, 2°.

13. 95°.

14. Average, 61; median, 60. In finding the *mode* we see that 56, 58, 60, and 61 occur twice each, and so we should have to take more measurements to find which tended to become more frequent. It is probable that the mode will not exceed 61, and the tailor would be very likely to estimate on the highest of the three numbers (average, median, or mode) for his own sake.

15. If he took the average he would get 2s. more per man than if he took the median. If the mode be 61 the cost would be the same as for the average, and 2s. per man greater than the median.

16. 39 minutes (to the nearest minute). **17.** 12 feet.

P. 46.—**18.** Average, £19,392,000; difference, £211000. **19.** £151000.

EXAMPLES. V.

SECTION III.

FRACTIONS

P. 49. — **1.** Employ the method of § 52. **2.** 6 ninths, 24 thirty-sixths.

3. 8 tenths go forwards and 2 tenths backwards.

4, 5, and 6. Just as in § 52.

P. 50. — **7.** (*6*) $\frac{20}{7}$, $\frac{121}{112}$, $\frac{61}{28}$, $\frac{59}{14}$. (*7*) $\frac{76}{3}$, $\frac{77}{9}$, $\frac{13}{6}$. (*8*) $\frac{487}{22}$, $\frac{734}{121}$, $\frac{55}{2}$.

8. 74. **9.** $42\frac{1}{2}$. **10.** $106\frac{1}{3}$.

11. 50 sixteenths. **12.** 6 fourths. **13.** 400.

14. 16. **15.** $\frac{5}{14}$. **16.** $\frac{5}{8}$. **17.** $\frac{1}{25}$.

18. No; for $\frac{16}{96}$ or $\frac{1}{6}$ does not equal $\frac{15}{80}$ or $\frac{3}{16}$. (This can be proved by drawing a long line dividing it into 6 parts and taking 1; and then a line of EQUAL length into 16 parts and taking 3 of them.)

19. $\frac{5}{121}$. **20.** $\frac{7}{366}$.

21. (*1*) $\frac{1}{2}$, $\frac{7}{8}$, $\frac{1}{2}$, $\frac{43}{52}$, $\frac{2}{5}$, $\frac{13}{105}$. (*2*) $\frac{1}{11}$, $\frac{333}{785}$, $\frac{97}{1933}$, $\frac{1}{11}$, $\frac{3}{4}$.
(*3*) $\frac{2}{27}$, $\frac{1}{11}$, $\frac{224}{241}$, $\frac{1}{26}$, $\frac{729}{3136}$. (*4*) $\frac{1}{81}$, $\frac{9}{49}$, $\frac{4}{9}$, $\frac{1}{4}$, $\frac{1}{16}$. (*5*) $\frac{1}{3}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{9}$.

22. $\frac{26}{55}$. **23.** £3500.

P. 51.—**24.**

England and Wales	$\frac{3931}{3581}$;	roughly,	$\frac{6}{7}$.
Scotland	$\frac{3170}{25067}$;	,,	$\frac{3}{25}$.
Ireland	$\frac{960}{25067}$;	,,	$\frac{1}{27}$.

25. $\frac{1}{9}$.

26. The value is less than the true amount by £83000 (to the nearest £1000).

27. $\frac{43}{72}$.

P. 51. — **28.** 7 in. and $4\frac{7}{10}$ in ; 7 in. and 5 in. to nearest inch ; $\frac{5}{7}$.
29. $\frac{37}{112}$. **30.** 6 times. **31.** $\frac{3}{7}$. **32.** $\frac{1}{4}$: $\frac{3}{4}$ is left.
33. 4 tenths fired, 6 tenths left ; 2 fifths fired, 3 fifths left.
34. 73. **35.** (*1*) 50. (*2*) 80. (*3*) 90.
36. $\frac{1}{2}$ of one-sixth ; $\frac{7}{12}$. **37.** $\frac{1}{10}$. **38.** $\frac{19}{275}$.

P. 52. — **39.** 50 lb. ; 40 lb. ; 90 lb.
40. (*1*) 3 feet. (*2*) 1 foot. (*3*) 2 feet. (*4*) 4 feet. (*5*) 5 feet. (*6*) $3\frac{1}{2}$ feet.
41. A scale should be chosen showing inches divided into 8ths, 32nds, and 10ths. One leg of a pair of compasses (or dividers, preferably) should be placed on the inch mark, and the other $3\frac{1}{8}$ inches from it ; then this length should be transferred to the part of the scale giving 16ths, and should then include $3\frac{2}{16}$, and so on.
42. 64. **43.** 24 rashers. **44.** 12 ounces.
45. (*1*) 4 ounces. (*2*) $\frac{1}{8}$. (*3*) $\frac{1}{2}$.

EXAMPLES. VI.

SECTION III.

FRACTIONS—(*continued*)

P. 52. — **1.** $1\frac{1}{2}$ lb. ; 2s. **2.** $3\frac{1}{2}$d. **3.** $2\frac{1}{18}$ lb. **4.** $\frac{5}{8}$.
5. Wardrobe, £30 ; dressing-table, £12 ; rest of suite, £18 ; two chairs and washstand usually.

P. 53. — **6.** $\frac{5}{9}$ absorbed, $\frac{4}{9}$ remained. (Find value of each export, value of total, value of last three.)
7. Gloves, £210000 ; iron, £520000 ; yarn, £1,040,000.
8. 108 million tons. **9.** 98 million tons. **10.** 3 to 4, 18 to 23, 1 to 7.
11. 10. **12.** 6. **13.** £100. **14.** $\frac{9}{70}$.
15. The order is—First quarter, $\frac{28}{70}$; second quarter, $\frac{15}{70}$; third quarter, $\frac{10}{70}$; fourth quarter, $\frac{9}{70}$. The bill is greatest in the first quarter.
16. $\frac{17}{180}$. **17.** £850000. **18.** 3 to 2.
19. Fraction for Merchant Taylors', $\frac{25}{257}$; fraction for Clothworkers', $\frac{30}{257}$; income of Grocers' Company, £38000.

P. 54. — **20.** $\frac{96}{325}$ of total.
21. Law and Justice, £600 ; Police, £140 ; Ports and Pilotage, £160 ; Education, £20 ; Medical, etc., £380.

EXAMPLES. VII.

SECTION IV. A.

ADDITION OF FRACTIONS

P. 55. — **1.** (*1*) $2\frac{1}{8}$. (*2*) $2\frac{9}{16}$. (*3*) $1\frac{11}{16}$. (*4*) $10\frac{29}{64}$. (*5*) $17\frac{7}{60}$.
2. (*1*) $4\frac{1}{40}$. (*2*) $7\frac{1}{16}$. (*3*) $15\frac{49}{144}$. (*4*) $82\frac{23}{28}$. (*5*) $18\frac{9}{41}$.
(*6*) $11\frac{3}{10}$. (*7*) $36\frac{11}{105}$. (*8*) $55\frac{85}{121}$. (*9*) $29\frac{53}{63}$. (*10*) $58\frac{47}{130}$.

P. 55.—**3.** (*1*) 29 lb. (*2*) $23\frac{33}{64}$ lb. (*3*) 27 lb.

4. (*1*) 3s. $7\frac{1}{2}$d. (*2*) 3s. (*3*) 3s. $4\frac{1}{2}$d.

5. They receive respectively £48, £120, £160, £96, £144, £224 ; total £792.

6. Most quickly on Friday : time, $4\frac{3}{110}$ hours.
Most slowly on Thursday : ,, $4\frac{13}{33}$,,

P. 56.—**7.** 4 lb. **8.** .0s. $2\frac{1}{2}$d. **9.** 128 soldiers. **10.** $146\frac{15}{64}$ tons.

11. £10512. **12.** $88\frac{11}{16}$ million dollars.

P. 57.—**13.** (*1*) Total entered and cleared, $883\frac{17}{48}$ million tons.
(*2*) Total British ships entered and cleared, $391\frac{41}{100}$ million tons.
(*3*) Fraction required $=\frac{391}{883}$ or nearly $\frac{400}{900}$, *i.e.* $\frac{4}{9}$;
$\therefore$ $\frac{4}{9}$ of the total tonnage is British.

14. $1887\frac{3}{8}$ thousand tons.

15. (*1*) 15,096,000 dollars. (*2*) £3,019,200.

P. 58.—**16.**

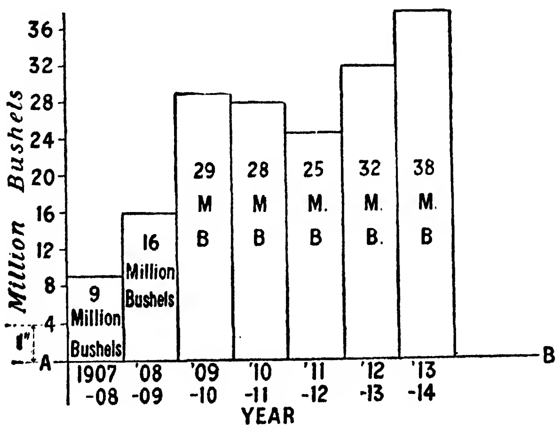
Totals (Thousand Tons).		
1912.	1913.	1914.
$1119\frac{99}{100}$	$1547\frac{5}{48}$	$1345\frac{1}{20}$

17. (*1*) $38\frac{3}{10}$ hundred thousand lbs. (*2*) $16\frac{7}{80}$ thousand £'s.

18. $2446\frac{1}{12}$ thousand £'s.

P. 59.—**19.** $176\frac{97}{150}$ million bushels.

20. Approximate values are : 16, 29, 28, 25, 32, 38 million bushels.



The student should take a piece of paper large enough to reproduce this diagram full size, and should colour it if possible.

EXAMPLES. VIII.

SECTION IV. A.

SUBTRACTION OF FRACTIONS

P. 59. — **1.** $\frac{7}{12}$. **2.** $\frac{7}{20}$. **3.** $7\frac{3}{4}$d. **4.** $\frac{29}{30}$. **5.** $\frac{9}{32}$.

P. 60. — **6.** $4\frac{1}{16}$. **7.** $3\frac{5}{6}$. **8.** (*1*) $\frac{239}{18}$ and $\frac{308}{18}$; (*2*) $\frac{717}{54}$ and $\frac{924}{54}$.

9. (*1*) $\frac{5}{8}$. (*2*) $\frac{9}{16}$. (*3*) $\frac{15}{32}$ remains. **10.** £$20\frac{1}{80}$. **11.** $\frac{4}{12}$ left.

12. $\frac{19}{24}$. **13.** $\frac{47}{75}$. **14.** $\frac{45}{64}$. **15.** 1, $1\frac{3}{4}$, $1\frac{1}{4}$, $\frac{3}{4}$, nil.

16. The first is heaviest; it is $5\frac{31}{64}$ lb. heavier than the second and 2 lb. heavier than the third.

17. $\frac{11}{45}$.

18. $\frac{56}{65}$ delivered, $\frac{9}{65}$ remain.

19. (*1*) See Question 18. (*2*) $\frac{336}{390}$; $\frac{54}{390}$. (*3*) $\frac{11\frac{1}{5}}{13}$; $\frac{1\frac{4}{5}}{13}$.

20. Expenditure is $\frac{9379}{9386}$ of revenue.

P. 61. — **21.** (*1*) Egypt, 31 people per square mile; France, 191 people per square mile.

(*2*) $\frac{31}{191}$ is nearly $\frac{1}{6}$, and it tells us that the density of population in Egypt is only $\frac{1}{6}$ of that in France; or that France is 6 times more crowded than Egypt.

22. 6.

23. Exports to the United Kingdom from Germany exceed those from France by $24\frac{187}{250}$ million £'s; from Norway by $63\frac{37}{250}$ million £'s; from Italy by $61\frac{99}{125}$ million £'s; from Japan by $66\frac{13}{125}$ million £'s.

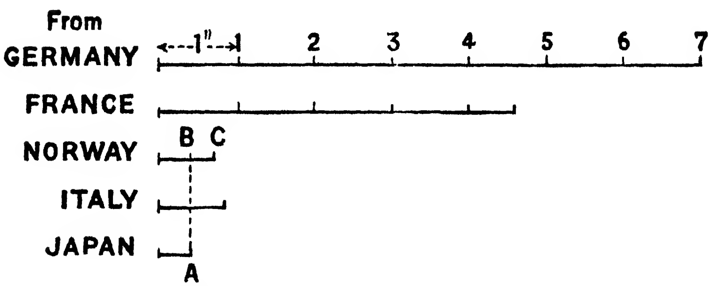
24. (*1*) 70, 46, 7, 8, 4.

(*2*) Length of lines is 7 in.; 4·6 in.; 0·7 in.; 0·8 in.; 0·4 in.

Arrange thus:—

Exports to United Kingdom to nearest million £'s.

Scale, $\frac{1}{10}'' =$ £1,000,000.



25. To read off the difference required draw the dotted line **AB** and measure the distance **BC**, which is 0·3 in. and represents £3,000,000.

EXAMPLES. IX.

SECTION IV. B. (i)

ADDITION OF FRACTIONS: MORE DIFFICULT

P. 63. — **1.** (*1*) $37\frac{131}{240}$. (*2*) $49\frac{53}{210}$. (*3*) $52\frac{281}{360}$. (*4*) $39\frac{313}{840}$. (*5*) $35\frac{13}{140}$.
(*6*) $68\frac{233}{792}$. (*7*) $51\frac{103}{360}$. (*8*) $86\frac{29}{35}$. (*9*) $171\frac{263}{672}$. (*10*) $223\frac{851}{2600}$.
2. (*1*) $63\frac{1303}{1881}$. (*2*) $73\frac{956}{1827}$. (*3*) $44\frac{418}{555}$. (*4*) $224\frac{3053}{3267}$.
(*5*) $37\frac{101}{270}$. (*6*) $44\frac{31}{34}$. (*7*) $166\frac{117}{121}$. (*8*) $93\frac{809}{1020}$.
3. $\frac{3}{14}$.

P. 64. — **4.** (*1*) $\frac{663}{2400}$. (*2*) $\frac{281}{432}$. (*3*) Friday. (*4*) Tuesday.
5. 33 thousand acres. **6.** $\frac{317}{1008}$. **7.** $\frac{137}{780}$.

P. 65. — **8.** $3704\frac{517}{3020}$. **9.** 14,816,000 shillings, or £740800.
10. Exports, £4,400,000; imports, £4,600,000. Imports are $\frac{23}{22}$ of the exports.
11. (*1*) To United Kingdom, $253180\frac{1223}{14700}$; to United States, $2205\frac{761}{900}$; to Canada, $78\frac{83}{100}$. (*2*) Aloes, $23\frac{31}{210}$; diamonds and (raw) gold as to United Kingdom; feathers, $13885\frac{47}{480}$; fish, $285\frac{79}{100}$.
12. The United States and Canada can obtain aloes and gold from markets nearer than South Africa. Canada has no need to import fish, and the United States very little need, because of the enormous quantity obtained from off their coasts. The United Kingdom will re-export some of the other commodities. See the statistical abstracts both for the United States and South Africa.

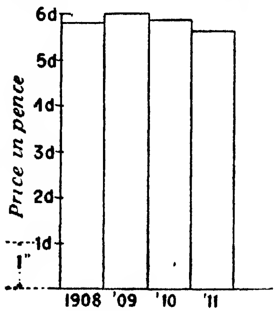
P. 66. —**13.** $21\frac{3367}{125400}$, or $21\frac{1}{37}$ mi. (nearly).

EXAMPLES. X.

SECTION IV. B. (ii)

SUBTRACTION OF FRACTIONS

P. 67. — **1.** (*1*) $\frac{13}{18}$. (*2*) $17\frac{19}{36}$. (*3*) $6\frac{11}{30}$. (*4*) $4\frac{47}{84}$. (*5*) $5\frac{251}{330}$. (*6*) $2\frac{35}{132}$.
(*7*) $1\frac{82}{105}$. (*8*) $70\frac{25}{96}$. (*9*) $22\frac{143}{200}$. (*10*) $132\frac{28}{65}$. (*11*) $46\frac{5}{12}$.
(*12*) $2\frac{54}{77}$. (*13*) $2\frac{59}{300}$. (*14*) $21\frac{25}{153}$. (*15*) $45\frac{223}{528}$. (*16*) $15\frac{193}{484}$.
2. $1\frac{7359}{8712}$. **3.** $237\frac{1759}{8100}$. **4.** $\frac{13}{222}$s. **5.** $\frac{31}{280}$. **6.** $4\frac{1837}{2280}$.
7. $\frac{11}{12}$, $\frac{9}{10}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{15}{13}$, $\frac{28}{87}$. **8.** $2\frac{2327}{5720}$. **9.** $2\frac{49}{50}$d., $\frac{25}{56}$d.
10. 1908–09, rise of $\frac{11}{25}$d.; 1909–10, fall of $\frac{131}{525}$d.; 1910–11, fall of $\frac{403}{2100}$d.
11.



Variation in price of a 4-lb. loaf in Lancashire and Cheshire for years given. Scale, 1″ = 1d.

P. 68. 12. $42\frac{223}{343}$. 13. $\frac{51}{332}$. 14. $2\frac{1024}{1071}$.
15. (*1*) $4\frac{214}{223}$ thousand £'s; $3\frac{101}{103}$ thousand £'s. (*2*) $\frac{43058}{43708}$.
(*3*) The rateable value of Leeds.

EXAMPLES. XI.

SECTION IV.

ADDITION AND SUBTRACTION OF FRACTIONS

P. 68. — 1. $\frac{103}{1200}$. 2. $4\frac{7}{15}°$; $10\frac{3}{5}°$. 3. $16\frac{11}{30}°$.

4. Smallest, $\frac{2}{41}$; largest, $\frac{8}{27}$; difference, $\frac{274}{1107}$; which gives the fraction required, for his total takings are represented by unity.

P. 69. — 5. $519\frac{77}{505}$ over 1700; $493\frac{51}{202}$ over 1800; $488\frac{659}{1010}$ over the other two (all in millions of £'s.). 6. £$467\frac{178}{505}$ millions.

7. England and Wales, $\frac{951}{1477}$; Scotland, $\frac{2003}{2570}$; Ireland, $\frac{33}{58}$.

8. The order is Scotland, England and Wales, Ireland. So that of the total number of students in training there was a greater proportion of women in Scotland than in either of the other countries, and the smallest proportion was in Ireland.

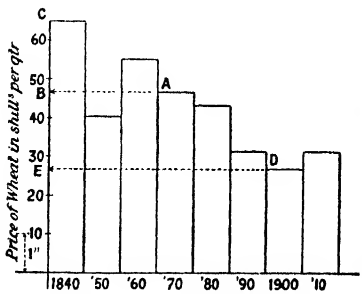
9. It increased by $\frac{137}{208}$, or about $\frac{7}{10}$ thousand dollars.

10. $16\frac{23}{96}$ or about £$16\frac{1}{4}$ millions.

11. £$2\frac{4}{95}$ millions, *i.e.* about £$2\frac{1}{24}$ millions, or £2,041,700.

12. £$18\frac{27}{490}$ millions, or about £$18\frac{1}{18}$ millions, *i.e.* £18,055,660.

13. Prices are 66, 40, 53, 47, 44, 32, 27, 32 shillings.



Variation in the price of British wheat in the United Kingdom for the years given. Scale, $\frac{1}{10}''=1$s.

Duty of 1s. per quarter in the years 1850 and 1860.

From the 1870 level draw **AB**, and read off from **B** 47s. to **C** 66s. This gives 19s. for the difference ($19\frac{6}{12}$s. by subtracting). In like manner draw **DE** where **E** represents 27s. and **C** 66s., giving 39s. as the fall from 1840–1900 ($39\frac{5}{12}$s. by subtracting).

P. 70.—14. The values in the last column should be, in order, from the 1850 line, $-26\frac{1}{12}$, $+13$, $-6\frac{1}{3}$, $-2\frac{7}{12}$, $-12\frac{5}{12}$, -5, $+4\frac{2}{3}$.

EXAMPLES. XII.

SECTION V. A.

MULTIPLICATION OF FRACTIONS

P. 73. — 1. (*1*) $\frac{3}{7}$. (*2*) $\frac{5}{16}$. (*3*) $\frac{15}{41}$. (*4*) $\frac{13}{128}$. (*5*) $\frac{5}{12}$. (*6*) $\frac{1}{13}$. (*7*) $2\frac{2}{5}$.
(*8*) $\frac{5}{12}$. (*9*) $\frac{35}{184}$. (*10*) $\frac{4}{161}$. (*11*) $\frac{11}{192}$. (*12*) $38\frac{3}{4}$. (*13*) $15\frac{19}{30}$. (*14*) 84.
(*15*) $\frac{1}{300}$. (*16*) $\frac{11}{14}$. (*17*) $1\frac{9}{14}$. (*18*) $1\frac{7}{40}$. (*19*) $5\frac{27}{28}$. (*20*) $39\frac{13}{35}$.
(*21*) $\frac{9}{25}$, $\frac{16}{25}$, $\frac{1}{9}$. (*22*) $\frac{53}{300}$. (*23*) $\frac{1}{400}$. (*24*) $1\frac{287}{2700}$. (*25*) $\frac{1}{16}$.

P. 73. — **2.** £157 and £628. **3.** 732 trees. **4.** £3714.
5. £35000. **6.** 2864 acres. **7.** £120000.
8. First, £450 ; second, £1750 ; third, £1400. **9.** $4\frac{1}{2}$ gallons.

P. 74. — **10.** £217. **11.** 23 tons. **12.** $31666\frac{2}{3}$ H.P.
13. $\frac{9}{22}$. **14.** $1\frac{4}{55}$. **15.** $\frac{9}{20}$. **16.** £1 15s.
17. Axminster costs £2 6s. $1\frac{1}{2}$d. ; Wilton costs £2 5s. $4\frac{1}{2}$d.; ∴ the former is dearer by 9d.
18. $\frac{1}{16}$. **19.** $\frac{9}{100}$.
20. The method is that of § 67 and § 68, except that the rectangle is divided instead of a straight line.
21 and **22.** *Vide* 20. **23.** $\frac{189}{400}$.
24. The fraction is $\left(\frac{1}{3}\times\frac{5\frac{1}{8}}{16\frac{2}{7}}\right)+\frac{2}{3}$ of $\frac{3-1\frac{1}{4}}{2\frac{2}{7}}$, which equals $\frac{85}{144}$.

EXAMPLES. XIII.

SECTION V. B.

DIVISION OF FRACTIONS

P. 77. — **1.** (*1*) $1\frac{1}{2}$. (*2*) $\frac{8}{25}$. (*3*) $1\frac{1}{2}$. (*4*) $\frac{10}{63}$.
(*5*) $\frac{5}{8}$. (*6*) 6. (*7*) $1\frac{1}{6}$. (*8*) $\frac{15}{32}$. (*9*) $1\frac{26}{63}$.
(*10*) (*a*) $\frac{1}{10}$, $\frac{1}{56}$, $\frac{1}{2}$, 2, $\frac{8}{7}$, $\frac{10}{9}$, $\frac{99}{56}$, $\frac{8}{103}$, $\frac{16}{351}$; (*b*) $\frac{7}{16}$, $\frac{3}{62}$, $\frac{5}{36}$, $\frac{1}{2}$, $\frac{125}{96}$.
(*11*) $\frac{9}{10}$. (*12*) 2. (*13*) $1\frac{3}{5}$. (*14*) $\frac{25}{33}$. (*15*) 15. (*16*) $2\frac{1}{2}$.
(*17*) $1\frac{7}{8}$. (*18*) $\frac{1}{30}$. (*19*) $3\frac{29}{32}$. (*20*) $1\frac{1}{8}$. (*21*) $8\frac{1}{8}$. (*22*) $1\frac{3}{7}$.
(*23*) $1\frac{1}{9}$. (*24*) $3\frac{7}{8}$. (*25*) $1\frac{7}{9}$. (*26*) 1. (*27*) $1\frac{1}{5}$. (*28*) $28\frac{1}{2}$.

P. 78. — (*29*) The latter is greater. (*30*) (i) $\frac{1}{20}$; (ii) $\frac{1}{5}$; (iii) $\frac{7}{8}$.
2. $3\frac{17}{20}$. **3.** $3\frac{5}{6}$. **4.** $12\frac{1}{30}$. **5.** $12\frac{129}{140}$.
6. $7\frac{1}{8}$. **7.** $38\frac{1413}{16448}$, $2\frac{11157}{16448}$, $361\frac{9}{16}$, $1\frac{34053}{296152}$.
8. (i) $\frac{1}{40}$; (ii) $3\frac{1}{5}$; (iii) $\frac{7}{54}$. **9.** $\frac{653}{1020}$. **10.** $1\frac{127}{255}$.
11 to **14.** As in § 71, only use a rectangle or square instead of a straight line.
15. $\frac{203}{288}$. **16.** $\frac{21}{40}$. **17.** $\frac{1}{8}$.

EXAMPLES. XIV.

SECTION V. C.

MISCELLANEOUS QUESTIONS

P. 78. — **1.** (*1*) $2\frac{1}{2}$. (*2*) $\frac{1}{2}$. (*3*) $4\frac{1}{2}$. (*4*) $\frac{1}{5}$.
(*5*) 9. (*6*) $6\frac{1}{4}$. (*7*) $3\frac{1}{2}$.

P. 79. — (*8*) $\frac{55}{108}$. (*9*) $\frac{589}{800}$. (*10*) $\frac{19}{160}$. (*11*) $\frac{2}{45}$.
(*12*) $\frac{5}{8}$. (*13*) $\frac{5}{8}$. (*14*) $4\frac{1}{2}$. (*15*) $3\frac{6}{13}$.
(*16*) $41\frac{287}{384}$. (*17*) $\frac{17}{20}$. (*18*) $1\frac{26}{27}$. (*19*) $\frac{25}{228}$. (*20*) $\frac{27}{28}$.
2. £315. **3.** 3 quarts. **4.** 9d. ; 9d. **5.** $3\frac{3}{5}$d.
6. 15s. ; £9 7s. **7.** 2s. 6d. **8.** $\frac{1}{30}$. **9.** 1s. 9d. **10.** 12s.

P. 80. — **11.** £38,382,000. **12.** £2 16s. $6\frac{1}{2}$d. (nearly). **13.** $4\frac{1}{7}$d.

P. 80. — **14.**

Year.	Imports.	Exports and Imports.
1810 . . .	£39,938,030	£83,506,790
1912 . . .	£738,718,690	£1,337,679,790

15.

Year.	Imports.	Exports.	Excess of Imports over Exports.
1810 . .	£39,938,030	£42,568,760	− £2,630,730
1912 . .	£738,718,690	£598,961,100	+ £139,757,590

16. $3\frac{1}{2}$d. (nearly). **17.** (*2*) 3s. 6d. (*3*) 8s. 9d. (*4*) 10s. 10d.

P. 81. — **18.** 16s. 6d. **19.** $\frac{1}{10}$d. **20.** 75 bills. **21.** $\frac{7}{100}$, 114.

22. (*1*) $2\frac{10}{41}$. (*2*) $2\frac{26}{75}$. (*3*) $2\frac{46}{65}$. **23.** $5438\frac{3}{5}$ (very nearly).

24. 480 N.D.'s in £100 sterling. **25.** $71\frac{10}{99}$ yards. **26.** 420.

P. 82. — **27.** The prices are very approximate.

Article.	Cost per Week for Two Persons. In London.			In Brisbane.		
	£	s.	d.	£	s.	d.
Fresh meat	0	5	$7\frac{3}{8}$	0	4	1
Bacon	0	2	$11\frac{7}{16}$	0	2	$9\frac{1}{4}$
Butter	0	1	$2\frac{1}{16}$	0	0	$11\frac{1}{4}$
Eggs	0	1	8	0	1	1
Bread	0	1	$9\frac{7}{8}$	0	2	$1\frac{9}{16}$
Flour	0	0	$10\frac{3}{8}$	0	1	$0\frac{1}{8}$
Tea	0	0	10	0	0	$8\frac{3}{4}$
Sugar	0	0	8	0	0	8
Potatoes	0	0	$10\frac{1}{4}$	0	0	$7\frac{3}{4}$
Jam	0	0	$4\frac{3}{8}$	0	0	$7\frac{7}{8}$
Milk	0	3	4	0	4	2
Coal	0	2	9	0	2	0
Gas	0	0	$6\frac{1}{2}$	0	1	0
Rent	0	15	0	0	17	6
	£1	18	$5\frac{1}{2}$	£1	19	$4\frac{1}{2}$

It is 11d. a week cheaper for two people to live in London than in Brisbane; or, the cost of living in London and in Brisbane is about the same.

EXAMPLES. XV.

SECTION VI. I.

DECIMALS: CONVERTING DECIMALS INTO FRACTIONS

P. 84. — **1.** *(1)* $\frac{1}{2}$, $\frac{43}{50}$, $\frac{47}{50}$, $\frac{93}{500}$, $\frac{287}{500}$. *(2)* $\frac{112}{125}$, $\frac{487}{500}$, $\frac{899}{1000}$, $\frac{379}{500}$, $\frac{439}{500}$.
(3) $\frac{1}{20}$, $\frac{43}{500}$, $\frac{47}{500}$, $\frac{93}{5000}$, $\frac{287}{5000}$. *(4)* $\frac{9}{1000}$, $\frac{7}{1000}$, $\frac{1}{125}$, $\frac{3}{500}$, $\frac{9}{1250}$.
(5) $\frac{1}{2500}$, $\frac{1}{2000}$, $\frac{1}{200}$, $\frac{1}{1250}$, $\frac{3}{10000}$. *(6)* $\frac{9}{10}$, $\frac{7}{10}$, $\frac{47}{50}$, $\frac{93}{500}$, $\frac{287}{500}$.
(7) $\frac{909}{1000}$, $\frac{177}{250}$, $\frac{161}{200}$, $\frac{453}{500}$, $\frac{4991}{5000}$. *(8)* $\frac{7003}{10000}$, $\frac{1601}{2000}$, $\frac{453}{500}$, $\frac{6001}{10000}$, $\frac{1126}{1250}$.
(9) $6\frac{4}{5}$, $9\frac{16}{25}$, $12\frac{863}{1000}$, $15\frac{183}{250}$, $10\frac{3}{500}$.
(10) $7\frac{22}{25}$, $8\frac{24}{25}$, $15\frac{3601}{5000}$, $18\frac{9601}{10000}$.
(11) $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{801}{100000}$, $\frac{1126}{1250}$.
(12) $\frac{1}{10000}$, $\frac{1}{100000}$, $3\frac{3}{10000}$, $33\frac{33}{10000}$.
(13) $9\frac{427}{500}$, $73\frac{163}{250}$, $86\frac{3}{200}$, $290\frac{7}{500}$.
(14) $285\frac{92}{125}$, $864\frac{1}{1000}$, $900\frac{1}{250}$, $8500\frac{101}{10000}$.
(15) $9700\frac{13}{50}$, $7501\frac{17}{20}$, $9001\frac{3}{10000}$.

2. *(1)* 10s., 5s., 24s., 15s. *(2)* 3s. 5d. (nearly), 5s., 2s. 6d., 6s.
(3) 2s. 9d., $2\frac{21}{40}$s., $3\frac{1}{10}$s., $4\frac{7}{10}$s. *(4)* $2\frac{2}{5}$s., $3\frac{1}{5}$s., $4\frac{4}{5}$s., $7\frac{7}{25}$s.
(5) $11\frac{3}{5}$s., $14\frac{18}{25}$s., $8\frac{7}{10}$s. **3.** 17 cwt.; $11\frac{1}{5}$s.

4. 84 lb., $1926\frac{2}{5}$ lb. **5.** $253\frac{1}{2}$ lb. **6.** $\frac{75}{344}$.

7. $425\frac{2}{5}$ lb. **8.** $\frac{38}{45}$.

P. 85. — **9.** The alteration from July to November 1914 is, in hundredths of a franc: Paris cheque, fall of 5; Italian cheque, rise of 20; Madrid cheque, fall of 5; hence the changes are, respectively, 5, 20, and 5 centimes.

10. Adelaide, 9 hours 14 minutes; Brussels, 0 hour 17 minutes; Chicago, 6 hours 10 minutes; Edinburgh, 11 hours 47 minutes; Vancouver, 3 hours 38 minutes.

11. $\frac{35}{888}$ gives (4) as a fraction of (1); $\frac{421}{444}$ gives (3) as a fraction of (1); $\frac{134}{555}$ gives (2) as a fraction of (1).

P. 86.—**12.** We are not informed as to the total imports or exports, which are £129000 and £112000 respectively. Coin and bullion is imported and none exported; ∴ if precious metals are found, they are not exported but used. Manufactured goods constitute 43 % of the imports; ∴ there is a considerable demand for such goods, and about $1\frac{1}{2}$ % of either native or re-exported goods (Class 3) find their way out of the island. Some raw material is imported in connection with local trade, but a much greater quantity is exported, namely, $37\frac{1}{2}$ % of the total exports. The largest imports and exports are animals, food, and drink. In point of fact, **arrowroot**, sugar, rum, cocoa, coffee, and spices are its chief products, with Canadian flour and fish as its imports.

13. $\frac{401}{1200}$ foot, $\frac{17}{75}$ foot.

(Note that the October value must be taken from the sum of the other two.)

14. $\frac{109}{240}$ foot and $\frac{823}{1200}$ foot. **15.** $3\frac{16}{25}$, $3\frac{19}{50}$, $3\frac{9}{20}$.

16. 130 lb. 2 oz.; 108 lb. 1 oz.; 109 lb. 4 oz.

EXAMPLES. XVI.

SECTION VI. II. (a)

DECIMALS: CONVERTING FRACTIONS INTO DECIMALS

P. 87. — **1.** (*1*) 0·2, 0·9, 0·03, 0·25, 0·79, 0·86, 0·92, 0·99.
(*2*) 0·6, 0·7, 0·8, 0·9. (*3*) 0·003, 0·05, 0·07, 0·04, 0·3, 0·2.
(*4*) 0·439, 0·53, 0·503, 0·79, 0·709.
(*5*) 0·0056, 0·078, 0·0001, 0·001, 0·0025, 0·8899.
(*6*) 2·7, 8·9, 12·8, 15·09, 17·9. (*7*) 28·909, 56·9, 73·8, 98·8008.
(*8*) 156·001, 574·0001, 742·00001.
(*9*) 963·000001, 856·00034, 9880·0135.

2. One-tenth: 0·2 foot, 0·5 foot, 0·8 yard, 8·0 inches, 20·0 tons. One hundredth: 0·02 foot, 0·05 foot, 0·08 yard, 0·80 inch, 2·00 tons. One-thousandth: 0·002 foot, 0·005 foot; 0·008 yard. 0·08 inch, 0·2 ton.

3. £0·0006, £0·00084, £0·000906, £0·0000085, £0·0001004 £0·00001014.

P. 88. — **4.** 0·087 ton. **5.** 87 tons. **6.** (i) 0·156 lb. (ii) 0·0156 lb.

7. 11·7 tons. **8.** 0·24 oz. silver nitrate, 5 oz. of cyanide.

(The student could make a solution of this strength after using the washed precipitate just dissolved in a weak cyanide solution. The latter is very poisonous.)

9. Length, 7 inches; width, $4\frac{7}{10}$ inches.

10. Length, 12·95 inches; width, 8·06 inches. **11.** 0·4 lb. **12.** 6·48s.

13. 2·24d. per lb.; compare this with the RETAIL price in London, 15d. per lb.!

EXAMPLES. XVII.

SECTION VI. II. (b)

DECIMALS

P. 89. — **1.** (*1*) 0·08, 0·12, 2·24, 21·72, 156·76.
(*2*) 0·16, 21·98, 156·74, 0·02, 20·4.
(*3*) 8·148, 0·148, 28·152, 86·788.
(*4*) 26·0392, 58·15, 78·95, 152·0304.
(*5*) 89·3968, 93·316, 88·0002, 1·00002.

P. 90. — **2.** 6·24d. per lb. **3.** £1·08 per gallon, 2·7s. per pint.

4. 2·88d. per foot. **5.** 0·96d. per sheet.

6. 16·96 tons. **7.** 1·2s. per yard.

8. 0·063s., *i.e.* about $\frac{3}{4}$d. Very often, in the summer, the Cornish fishermen have to sell mackerel to the fish-buyers at the rate of three a penny; the same fish are sold in London for 3d. and often 5d. EACH.

9. Between 7 and 8 (7·8016) on the average.

10. 0·96 dollar per bushel. **11.** 1·832s. per cwt.

EXAMPLES. XVIII.

SECTION VI. II. (c)

DECIMALS

P. 91. — **1.** (*1*) 0·667, 0·600, 0·875, 0·818, 0·833, 0·938, 0·611.
(*2*) 0·840, 0·643, 0·639, 0·387, 0·629, 0·715.
(*3*) 0·108, 0·966, 0·802, 0·796. (*4*) 0·002, 0·000, 0·003.
(*5*) 0·903, 0·893. (*6*) 0·787, 0·980.
(*7*) 3·270, 7·096, 9·036. (*8*) 128·596, 156·582, 379·994.
(*9*) 536·2055, 5964·2671. (*10*) 22·0007, 3600·0004.
2. 0·75, 0·83, 0·86, 0·78, 0·70 ; the order is $\frac{6}{7}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{3}{4}$, $\frac{7}{10}$.
3. 0·769, 0·768 ; and $\frac{10}{13}$ is larger. **4.** 0·991. **5.** 0·809.
6. Russia, 0·199 ; United States, 0·192 ; India, 0·1 ; France, 0·088 ; Canada, 0·06.

P. 92. — **7.** 0·69. **8.** 0·368. **9.** 0·761. **10.** 0·790.
11. 0·952. **12.** 0·536. **13.** 0·96. **14.** £6133·33.
15. 1·03. **16.** 1·026. **17.** 76·67.

P. 93.—**18.** Indian Empire, 166 people per square mile ; United Kingdom, 380 people per square mile ; ∴ the United Kingdom is about two and a half times more densely populated than India.
19. £3·22.

EXAMPLES. XIX.

SECTION VII.

DECIMALS : A. ADDITION AND SUBTRACTION

P. 93. — **1.** (*1*) 29·074. (*2*) 171·5951.
P. 94. — (*3*) 342·72051. (*4*) 62846·11101.
2. (*1*) 180·2827. (*2*) 289·0061. (*3*) 261·4644.
(*4*) 228·4720. (*5*) 191·1094. (*6*) 246·1685.
(*7*) 873·9984. (*8*) 75·6342. (*9*) 1416·3330. Total, 3762·4687.
3. Row totals—
(*1*) 47·84024. (*2*) 612·97544. (*3*) 1333·58094.
(*4*) 1382·58870. (*5*) 824·90444. (*6*) 2002·26642.
(*7*) 384·64464.
Column totals—
(*1*) 2871·13023. (*2*) 1702·73875. (*3*) 1594·6718.
(*4*) 420·26. Grand total, 6588·80078.
4. Differences between columns (1) and (2)—
(*1*) 102·693. (*2*) 207·7837. (*3*) 182·2556.
(*4*) 79·7528. (*5*) 27·8034. (*6*) 4·2795.
(*7*) 63·4616. (*8*) 2·1448. (*9*) 74·623.
Differences betweeen columns (3) and (4)—
(*1*) 57·7786. (*2*) 51·879. (*3*) 8·8752.
(*4*) 129·966. (*5*) 147·784. (*6*) 129·362.
(*7*) 714·265. (*8*) 42·0879. (*9*) 583·67.

P. 94. — **5.** Differences between columns (1) and (2)—
(*1*) 2·878. (*2*) 551·36642. (*3*) 693·72914.
(*4*) 621·2786. (*5*) 448·70256. (*6*) 573·85908.
(*7*) 330·20904.
6. 2·17. **7.** 6·174, 4·139; difference, 2·035.
8. Sum, 1883·009130125; difference, 90·990570125.
9. 1792·01886. **10.** 55·68705. **11.** 1·55721. **12.** 93·54442909.
P. 95.—**13.** 7·80031. **14.** 17·75. **15.** 16065·9893. **16.** 6·5.
17. 6·5. **18.** 10·9894. **19.** 16·1738. **20.** 3·0999.
21. 154·789139. **22.** 3·069. **23.** 13·0708. **24.** −10·9417.
25. 2·4925. **26.** 1912, 25·19; 1913, 20·36; 1914, 20·37.
P. 96.—**27.** 1200 cwt. greater. **28.** £2,936,400. **29.** 555·408.
30. £26,300,200. **31.** £5,061,260. **32.** 907888, 895985.
33. 59514·7. **34.** £37,900,000.
P. 97.—**35.** 5·591d. per 4-lb. loaf (England and Wales).
36. 0·449d.
37. The North and Yorkshire, South-eastern Counties, Lancashire and Cheshire, Eastern and South-western Counties, Wales and Monmouth, London, West Midlands, South Midlands, North Midlands. 1·11d. per 4-lb. loaf.
38. £25,800,000. **39.** £1,247,000. **40.** £4,546,200.
P. 98.—**41.** £29,959,687.

EXAMPLES. XX.

SECTION VIII. A.

DECIMALS: MULTIPLICATION AND DIVISION

P. 101. — **1.** (*1*) 87·32, 98·56, 89·04. (*2*) 956·3, 897·2, 1283·4.
P. 102. — (*3*) 150090, 290010, 180001. (*4*) 18100, 29010, 8·03.
(*5*) 678560, 859730, 9800010·4.
2. (*1*) 8·043, 7·692, 5·834. (*2*) 2·899, 73·884, 83·658.
(*3*) 0·2156, 0·8894, 9·3673. (*4*) 0·00087304, 0·998603, 0·88135.
(*5*) 0·0000004, 0·00000101, 0·00000005.
3. 50·75, 120·86, 10·58, 58·96, 86·29.
4. 5075, 12086, 1058, 5896, 8629.
5. 85·75, 986·38, 35·86, 982·15, 863·25.
6. 8575, 98638, 3586, 98215, 86325.
7. 85·76, 56·38, 963·29, 86·85, 72·13, 19846·03.
8. 8576, 5638, 96329, 8685, 7213, 1984603.
9. 52 francs 52 centimes.
10. (*1*) 11·726, 23·452. (*2*) 26·992, 34·704.
(*3*) 47·346, 78·91. (*4*) 90·2, 36·08.
(*5*) 920·024, 805·021. (*6*) 402·0306, 536·0408.
(*7*) 1·626, 4·065. (*8*) 7·544, 10·373.
(*9*) 0·0065, 0·0104. (*10*) 27·3609, 36·4812.

P. 102. — **11.** (*1*) 2·9315, 1·46575. (*2*) 0·55086, 0·4284
(*3*) 5·26067, 3·1564. (*4*) 3·608, 9·02.
(*5*) 14·3754, 16·429. (*6*) 44·670067, 33·5025.
(*7*) 0·4065, 0·1626. (*8*) 0·117875, 0·08573.
(*9*) 0·00026, 0·0001625. (*10*) 0·3378, 0·25334.

12. (*1*) 3·52, 9·10. (*2*) 17·363, 11·520.
(*3*) 4·094, 8·020. (*4*) 0·0008, 0·7758.

P. 103. — **13.** (*1*) 0·304, 0·1032, 0·485. (*2*) 25·4448, 532·381, 631·2204.
(*3*) 694·6046, 1361·70801, 152·520201.
(*4*) 518·051401, 945·4345226, 0·035600012.
(*5*) 3971·8605608, 179·480028.

14. (*1*) 1389·209. (*2*) 5446·83. (*3*) 2072·206.
(*4*) 1890·49102. (*5*) 26619·1. (*6*) 8579·6.
(*7*) 82084·898. (*8*) 69·5925969. (*9*) 6·54. (*10*) 51019016·4.

15. (*1*) 0·4750, 7·1667, 7·7600. (*2*) 1·8387, 15·5034, 9·7649.
(*3*) 32·8263, 21·2236, 149·5150.
(*4*) 57·1794, 37·7871, 222500·075. (*5*) 689·4442, 448700·07.

16. (*1*) 16·4132. (*2*) 21·2236. (*3*) 0·0700. (*4*) 0·1890.
(*5*) 7·7517. (*6*) 923236·8257. (*7*) 0·3336. (*8*) 0·9206.

17. (*1*) 4·2 (*2*) 2. (*3*) 60·04. (*4*) 25·08.
(*5*) 102·09 (*6*) 57·27. (*7*) 23·04. (*8*) 19·32.
(*9*) 9·5. (*10*) −6·58. (*11*) 0·5. (*12*) 0·456, 0·033.
(*13*) 0·078. (*14*) 0·0703. (*15*) 4.

P. 104. — (*16*) 0·173. (*17*) 3·79. (*18*) 1. (*19*) 0·75.
(*20*) 3·28. (*21*) 2·07. (*22*) −0·69. (*23*) 9·875.
(*24*) 1·56. (*25*) 0·167. (*26*) 9·236 and 85·3.

18. 517·344, 0·2639, 1·426. **19.** 0·01, 0.64, 0·4. **20.** 14·167.

21. 0·03493. **22.** (i) 0·1111, (ii) 1·0133 ; 0·11 (nearly).

23. (*a*) 0·40704, (*b*) 0·00184576, (*c*) 28·63.

24. 1 : 0·1. **25.** 37·13112. **26.** 0·259.

P. 105.—**27.** 80. **28.** (i) 0·6428571, (ii) 1·5, (iii) 0·96429. **29.** $\frac{17}{25}$.

EXAMPLES. XXI.

SECTION VIII. A.

MISCELLANEOUS QUESTIONS

P. 105. — **1.** 3·976s. per bushel. The price of Canadian wheat is 0·456s. per bushel higher than that of the Australian.

2. 5d. (nearly). **3.** £530. **4.** £4 11s. 3d.

5. 18·04s., *i.e.* 18s. **6.** £2 8s. 3d. **7.** £19 9s. 8d.

8. 1·51. **9.** No ; 0·0016, to 4 places.

P. 106. — **10.** 1·11. **11.** Above, by 0·57369 grain. **12.** 23·55.

13. £112,198 18s. 8d. **14.** 157·284.

15. 36·25 bushels ; 14½ times. **16.** 13·5s. **17.** £1·7 million.

18. 0·068d. **19.** £9687 10s.

P. 107. -**20.** *Olympic*, 5441 tons per 100 feet length; *Mauretania*, 4191 tons per 100 feet length.
21. 9·51 millions. **22.** 49·8 millions.
23. 5·24 times greater in 1912 than in 1700.
24. Very nearly 11 times as dense. **25.** 0·00017.
26. 10·7. The Indian Empire, with its enormous native population, is included in the former, while in the latter are large tracts of country where there are very few settlers indeed. Rhodesia and the territory to the North of that province are not yet opened up even. **27.** 2·85s.
P. 108.—**28.** £4 6s. 6d. **29.** 1s. 8·6d.
30. 2s. 9d., very nearly. **31.** 26400 cub. feet; £2·97.

EXAMPLES. XXII. (a)

SECTION VIII. B.

CONTRACTED METHODS: MULTIPLICATION

P. 110.—**1.** 21·500. **2.** 8·458. **3.** 44·789. **4.** 649·569.
5. 1050·36 **6.** 11·2244. **7.** 0·2445.
P. 111.—**8.** 85·189. **9.** 61·29117. **10.** 3·398. **11.** 5575·326.
12. 0·00384 **13.** 90·1966. **14.** 130·572. **15.** 7·9882 grams.
16. 3791·632 francs. **17.** 23·385 bushels. **18.** 5582. **19.** 1389.
20. 1295·5 Kilograms. **21.** £3 6s. 3d. **22.** 37·01 metres.
23. 301·49 c.c. **24.** 10·28 francs. **25.** 3 dwt. 31·3230 grains.

EXAMPLES. XXII. (b)

SECTION VIII. B.

CONTRACTED METHODS: DIVISION

P. 113.—**1.** 3·523. **2.** 5·803. **3.** 111·533. **4.** 156·7.
5. 3·0154. **6.** 32·810. **7.** 28·386. **8.** 101·622.
9. 0 12003. **10.** 0·0431. **11.** 5·30043. **12.** 0·00137.
13. 25·682. **14.** 5·7384. **15.** 19599·6. **16.** 0·0000000318.
17. 5584·4. **18.** 0·0150. **19.** 0·297. **20.** 4975.
21. (*a*) 17·62; (*b*) 19·70. **22.** 196·292. **23.** 171·1.
24. 0·025. **25.** 0·044. **26.** 25465. **27.** 0·012324.
28. 99979. **29.** 0·066554. **30.** 15352·3. **31.** 0·31831.
32. 2·865. **33.** 1036·169. **34.** 0·0666.
P. 114.—**35.** 0·3801. **36.** 15·194. **37.** £0·06275, or 1s. 3·061d.
38. 1·628. **39.** 2·500 bushels. **40.** 23·389 francs.
41. 19·26d. per lb. **42.** 782·90 Kilograms. **43.** 1434·4 acres.
44. £261 14s., correct to 1s. **45.** 3·433 feet per 100 yards.
46. 5·441 thousand tons per 100 feet. **47.** 1·965 cubic foot.
48. 3·9428 Kilometres.
P. 115.—**49.** 0·1500 grain per square inch. **50.** 0·00078 millimetre.

EXAMPLES. XXIII.

SECTION IX. A.

ADDITION AND SUBTRACTION OF MONEY

P. 119. — **1.** £108920 11s. 6d. **2.** £132565 5s. 6d. **3.** £11307 18s. 1d.
4. £4272 7s. 6$\frac{1}{4}$d. **5.** £30236 8s. 1d. **6.** £150732 1s. 6$\frac{1}{4}$d.
7. £86184 18s. 6d. **8.** £67981 9s. 5d. **9.** £53134 16s. 9d.
P. 120. — **10.** £55113 13s. 4d. **11.** £48345 12s. 6d. **12.** £19823 3s. 2$\frac{3}{4}$d.
13. £35308 8s. 1d. **14.** £4272 7s. 11$\frac{1}{2}$d. **15** £34881 1s. 9d.
16. £5757 14s. 4d. **17.** £87933 4s. 1d. **18.** £62243 7s. 9$\frac{1}{4}$d.
P. 121. — **19.** £1701 13s. 5d. **20.** £52992 15s. 5d. **21.** £47301 7s. 5d.
22. £61332 17s. 1d. **23.** (*1*) £412 5s. 4d.
P. 122. — (*2*) £269 4s. 11d. (*3*) £413 17s. (*4*) £337 8s. 2d.
(*5*) £432 16s. 9d. (*6*) £474 15s. 6d. (*7*) £969 16s. 3d.
(*8*) £151 6s. 10d. (*9*) £93 10s. 1d. (*10*) £67 15s. 8d.
(*11*) £111 18s. 2d. (*12*) £190 15s. 6d. (*13*) £43 19s. 5d.
(*14*) £77 19s. 8d.
24. Column totals—
£14 15s. 3d., £3 13s. 6d., £12 7s. 7$\frac{3}{4}$d., £29 3s. 6d.
Line totals—
£22 13s. 6$\frac{3}{4}$d., £13 4s. 3$\frac{1}{2}$d., £5 19s. 4$\frac{1}{2}$d., £18 2s. 8d. Grand total, £59 19s. 10$\frac{3}{4}$d.
25. (*1*) £114 12s. 4d. (*2*) £71 5s. 5d. (*3*) £132 19s. 2d.
(*4*) £103 6s. (*5*) £153 5s. 6d. (*6*) £211 5s. 2d.
(*7*) £173 5s. 3d. (*8*) £30 13s. 10d. (*9*) £15 16s. 5d.
(*10*) £1 2s. 3d. (*11*) £12 0s. 7d. (*12*) £16 4s. 11d.
(*13*) £4 1s. 6d. (*14*) £16 12s. 2d.
26. (*1*) £4 18s. 9d. (*2*) £27 0s. 4d. (*3*) £38 5s. 3d.
(*4*) 2s. 10d. (*5*) £25 1s. 4d. (*6*) £69 0s. 2d.
(*7*) £10 17s. 2d. (*8*) £13 5s. 8d. (*9*) £8 17s. 4d.
(*10*) 12s. 9d. (*11*) £1 15s. 3d (*12*) 10s.
(*13*) 10s. 10d. (*14*) £1 2s. 6d.
27. (*1*) 11$\frac{1}{4}$d. (*2*) 18s. 10$\frac{1}{4}$d. **28.** 16s. 5d.
29. £23 6s. 2d. and £2 18s. 10d.

EXAMPLES. XXIV.

SECTION IX. B.

REDUCTION OF MONEY

P. 124. — **1.** £6 1s. 6$\frac{1}{2}$d. ; £20 9s. 6$\frac{1}{2}$d. **2.** £15 16s. 1d. ; £187 10s.
3. 3805 farthings. **4.** £5 16s. 5d. **5.** $\frac{11}{288}$.
6. £396 12s. **7.** 24 (2d. over). **8.** £33 3s. 8d.
P. 125. — **9.** £1 17s. 4$\frac{3}{4}$d. **10.** £9820 10s.
11. £446 8s. **12.** £1220 10s. 6d. **13.** $\frac{17}{40}$. **14.** $\frac{13}{24}$.

P. 125. — **15.** 0·75. **16.** 0·967, to 3 places. **17.** $\frac{1}{8}$.
18. (i) 6s. 6d. ; (ii) 10s. 5d. ; ∴ (ii) is greater by 3s. 11d.
19. 10. **20.** £1 9s. 8d. **21.** £1 1s. 4d. **22.** £5 16s. 8d.
23. (i) 18617d. ; (ii) 35722d. ; (iii) 1898d. ; (iv) 8382d.
24. (i) 78412f. ; (ii) 284008f. ; (iii) 12164f. ; (iv) 81210f.
25. (i) 3840 ; (ii) 21928 ; (iii) 36660 ; (iv) 10216.
26. 83s. (4d. over). **27.** 219d. **28.** £375. **29.** 144 marks
30. £35 7s. 9d. **31.** 7s. 11d. **32.** 6188d.
33. 27510 half-crowns. **34.** 36 men. **35.** 25 men.
P. 126. — **36.** 100 cheques. **37.** 85 subscribers.
38. 45 inkstands ; 10½d. over. **39.** 14400. **40.** 128 tons.

EXAMPLES. XXV.

SECTION IX.

MISCELLANEOUS QUESTIONS

P. 126. — **1.** £3 3s. 9d. **2.** 0·7096. **3.** £1 5s. 8¾d ; 15s. 9d.
4. £1 5s. 9¼d. ; £2 3s. 3¾d. **5.** 3s. per ton.
6. £53 18s. 7¾d. ; £11 1s. 4¼d.
7. £53 19s. 2½d. ; £54 10s. 7d. ; £65 10s. 7d.
8. £52 2s. 4¾d. ; £52 17s. 7¼d. **9.** 8s. 3d. more ; 4s. 6d. more.
P. 127. — **10.** £2287 2s. 2d. **11.** £786 4s. 2d.
12. £115 2s. 3½d. ; £124 2s. 3½d. ; £9 ; £119 12s. 3½d.
13. (*1*) £22 10s. 8d. ; £34 16s. ; (*2*) £11 ; £1 5s. 4d.
14. £27 12s. 5d. ; £36 16s. 6d. **15.** £43 12s. 3d.
16. £21, £19, £17, £7, £20 17s., £19 8s., £17 9s., £6 16s.
17. (*1*) Monday, £2 9s. 2½d. ; Tuesday, £2 8s. 0½d. ; Wednesday, £2 19s. 9½d. ; Thursday, £3 19s. 1½d. ; Friday, £2 7s. 11d. ; Saturday, £2 5s.
(*2*) Morning, £3 3s. 6d. ; noon, £2 6s. 5d. ; 3 p.m., £2 18s. 3d. ; 5 p.m., £2 16s. 3d. ; 7.30 p.m., £3 0s. 6d. ; final, £2 4s. 2d.
(*3*) £16 9s. 1d.
P. 128. — **18.** £2000 ; total, £132140 7s. 2d. **19.** 2¾d. **20.** 4½d.
21. It is cheaper by 2d. to buy by the dozen.
P. 129. — **22.** 42. **23.** Nothing. **24.** £3 1s. 8d.
25. 2s. 9d., 0·1375. **26.** $\frac{1835}{2202}$.
27. (*1*) (i) £5 2s. 4d. ; (ii) In order of sizes: £5 13s., £7 4s., £10 16s. 8d., £15 16s. 6d., £21 3s.
(*2*) £6 2s., £7 19s. 6d., £11 14s. 3d., £17 1s. 11d., £22 17s. 10d.
The sum of (*1*) (i) and (ii) should equal that of (*2*).
28. 12s. 2d. ; 11s. 9d.
P. 130. — **29.** Fencing, £40 ; cows, £72 ; horses, £30 ; total cost, £214 11s.
30. £185 11s. **31.** £166 9s. **32.** £48 2s.
33. 18·67s. per cwt. (to two places of decimals) ; 5s. 10d.
34. £0·933 per cwt. **35.** 2s. 8·09d.

P. 131. — **36.** £25 13s. **37.** £2 7s. **38.** £4 19s.
39. £8 7s. **40.** £6 11s. 9d.
P. 132. — **41.** £9 13s. 7½d. **42.** £6 6s. **43.** £4 13s. 4d.
44. Cost per 100 at rate of five is: £1 13s. 4d., £4 15s., £5 6s. 8d., £10, which is higher than the 100 rate by 4s. 4d.. 10s.. 8s. 8d., £1.
45. Total, £258 10s.
P. 133. — **46.** £9,264,849 15s. 2d.
P. 134. — **47.** £49 0s. 8d. **48.** £263 17s. 9d.
49. (1912) 3s. 11d. per lb.; (1913) 4s. 3d. per lb.
P. 135. — **50.** 4d. **51.** Total, £127666 0s. 6d. Commissions, £515 14s. 4d.

EXAMPLES. XXVI.

SECTION X. A. (i)

CALCULATION OF COST BY COMPOUND MULTIPLICATION AND DIVISION

P. 138. — **1.** (*1*) £27 2s. 6d. (*2*) £54 9s. 8d. (*3*) £89 6s. 10½d. (*4*) £37 2s. 3¾d.
P. 139. — (*5*) £113 6s. 9d. (*6*) £1278 7s. 6d.
(*7*) £7163 19s. 9d. (*8*) £7881 5s. 4½d.
(*9*) £108645 10s. 10¾d. (*10*) £158647 16s. 5½d.
2. (*1*) £135 12s. 6d., £195 6s., £227 17s.
(*2*) £381 7s. 8d., £653 16s., £490 7s.
(*3*) £268 0s. 7½d., £1072 2s. 6d., £1240 17s. 8½d.
(*4*) £1187 14s., £1224 16s. 3¾d. (*5*) £3740 2s. 9d., £2720 2s.
(*6*) £25567 10s., £30681. (*7*) £355214 7s. 7¼d., £492524 2s. 9¾d.
(*8*) £2,084,289 19s. 2d. (*9*) £6,192,796 1s. 0¾d.
(*10*) £2,300,393 8s. 7¾d.
3. (*1*) £5 8s. 6d. (*2*) £7 15s. 8d. (*3*) £9 18s. 6½d.
(*4*) £12 7s. 5¼d. (*5*) £18 17s. 9½d. (*6*) £159 15s. 11¼d.
(*7*) £596 19s. 11¾d. (*8*) £875 15s. 0½d.
(*9*) £9876 17s. 4¼d. (*10*) £15864 15s. 7¾d.
4. (*1*) £5 8s. 6d. (*2*) £5 8s. 6d. (*3*) £5 8s. 6d.
(*4*) £7 15s. 8d. (*5*) £7 15s. 8d. (*6*) £9 18s. 6½d.
(*7*) £9 18s. 6½d. (*8*) £18 17s. 9½d. (*9*) £159 15s. 11¼d.
(*10*) £875 15s. 0½d. (*11*) £596 19s. 11¾d. (*12*) £9876 17s. 4¼d.
5. (*1*) (i) £191453 18s. 6d.; (ii) £244219 12s. 6d.
(*2*) (i) £410980 14s.; (ii) £493308 11s. 3½d.
(*3*) (i) £502973 17s. 2¼d.; (ii) £594111 0s. 0¾d.
(*4*) (i) £1,404,852 2s. 2d.; (ii) £1,736,651 7s.
(*5*) (i) £5,550,606 19s. 0¾d.; (ii) £8,152,276 11s. 2d.
P. 140. — **6.** (*1*) £859 15s. 8¼d. (*2*) £859 15s. 8¼d. (*3*) £1764 17s. 9½d.
(*4*) £1764 17s. 9½d. (*5*) £5692 18s. 7¼d.
7. £144 1s. 10¼d. **8.** 5s. 0½d. **9.** £124 4s.

P. 140. — **10.** £800 14s. $9\frac{1}{2}$d. **11.** £2 19s. $8\frac{3}{4}$d. **12.** 30s.
13. $\frac{148}{165}$. **14.** £0·000041667. **15.** $\frac{644}{6253}$ **16.** $1250\frac{1}{2}$d.
17. $\frac{25091}{33978}$. **18.** £13 5s. $8\frac{1}{2}$d. **19.** $\frac{35789}{83909}$.
20. £6 17s. 1d.
21. 57 mantles.
22. $1\frac{1}{2}$d. It must be less than $2\frac{1}{2}$d., for if it were $2\frac{1}{2}$d. or more, then another mantle could be bought.
23. 53 lb. **24.** $\frac{1}{16}$s. **25.** $\frac{55}{112}$d. or $\frac{1}{2}$d. nearly.
P. 141. — **26.** £90 4s. 1d. ; the 1915 price is £13 18s. 6d. below the 1914.
27. £8 14s. 3d. **28.** 31s. $3\frac{3}{8}$d. **29.** £337 10s.
30. £37 10s. **31.** 4s. 10d.
P. 142. — **32.** 0·357d. **33.** 13s. 4d. **34.** 24s. **35.** £34 4s. 9d.
36.

One-sixth of Present Price.			Sale Price.		
£	s.	d.	£	s.	d.
2	6	7	11	12	11
1	4	7	6	2	11
4	19	2	24	15	10
0	17	2	4	5	10

37. $6\frac{3}{4}$d. **38.** The latter, by £17 5s. 5d. **39.** £62 0s. 10d.
40. £63. **41.** £72 18s.
P. 143. — **42.** £1 8s. $0\frac{1}{2}$d. **43.** 18s. 6d. **44.** $\frac{438}{478}$.

EXAMPLES. XXVII.

SECTION X. A. (ii)

RAPID METHODS FOR DOZENS AND SCORES

P. 145. — **1.** 13s. 6d. **2.** 7s. 6d. **3.** £2 16s. **4.** 2s. 9d.
5. £2 11s. 6d. **6.** £3 9s. 6d. **7.** £16. **8.** £12 18s.
9. £2 19s. 6d. **10.** £1 15s. 9d. **11.** 1s. 6d. **12.** 11s.
13. £18. **14.** £8 8s. **15.** £1 7s. 9d. **16.** £1 7s.
17. £1 11s. 6d. **18.** 6s. 9d. **19.** £41 14s. **20.** £13 1s.
21. £3 5s. **22.** 7s. $3\frac{1}{2}$d. **23.** £1 13s. **24.** £7 5s. 10d.
25. £10 1s. 8d. **26.** 3s. $7\frac{1}{2}$d. **27.** 9s. 6d. **28.** £185.
29. £1 17s. 6d. **30.** £57 3s. 9d. **31.** £23 2s 6d.
32. 6s. 8d. **33.** 7s. 5d. **34.** 13s. 6d.
35. 14s. $7\frac{3}{4}$d. **36.** 4s. $6\frac{1}{4}$d. **37.** 1s. 9d.
P. 146. — **38.** 3d. **39.** 1s. 6d. **40.** $3\frac{1}{2}$d. **41.** $3\frac{1}{4}$d.
42. $8\frac{1}{2}$d. **43.** 1s. 6d. **44.** 1s. 9d. **45.** $4\frac{1}{2}$d.
46. $5\frac{1}{4}$d. **47.** $9\frac{1}{4}$d. **48.** 1s. 6d. **49.** 1s. 8d.

EXAMPLES. XXVIII.

SECTION X. B. (i)

CALCULATION OF COST: COMPLETE DECIMALISATION OF SUMS OF MONEY

P. 148. — **1.** (*1*) 0·125. (*2*) 0·208333. (*3*) 0·270833. (*4*) 0·229167.
(*5*) 0·0625. (*6*) 0·375. (*7*) 0·4375. (*8*) 0·5625.
(*9*) 0·625. (*10*) 0·770833. (*11*) 0·979167. (*12*) 0·854167.

2. (*1*) 0·0063. (*2*) 0·0104. (*3*) 0·0135. (*4*) 0·0115.
(*5*) 0·0031. (*6*) 0·0188. (*7*) 0·0219. (*8*) 0·0281.
(*9*) 0·0313. (*10*) 0·0385. (*11*) 0·0190. (*12*) 0·0428.

3. (*1*) 0·3333, 0·125, 0·05, 0·8333, 0·025, 0·0833, 0·9.
(*2*) 0·25, 0·1667, 0·1, 0·6667, 0·0125, 0·8, 0·3.

4. (*1*) 0·25, 0·1667, 0·0625, 0·0167, 0·4, 0·0333.
(*2*) 0·3333, 0·125, 0·0125, 0·025, 0·0833, 0·55.

5. (*1*) 0·9375, 0·8833, 0·781, 0·439. (*2*) 0·334, 0·385, 0·989, 0·948.
(*3*) 0·290, 0·114, 0·099, 0·643. (*4*) 0·796, 0·868, 0·173, 0·055.
(*5*) 0·0010417, 0·0052, 0·040625, 0·544.

6. 0·727, to three places. **7.** 0·148, to three places. **8.** 0·843.

9. (*1*) £74·664583, £73·972917. (*2*) £38·715625, £125·823958.
(*3*) £179·960417, £250·423958. (*4*) £760·319792, £158·690625.
(*5*) £731·29375, £33·778125.

P. 149. — **10.** (*1*) £3 10s., £25 2s., £56 6s. (*2*) £25 5s., £33 15s., £29 8s.
(*3*) £29 4s., £53 12s., £8 12s.
(*4*) £68·16s., £291 6s., £157 17s 6d.
(*5*) £900 12s. 6d., £153 7s. 6d., £964 2s. 6d.
(*6*) £295 3s. 2½d., £358 16s. 7¼d., £86 1s. 8d.
(*7*) £56 8s. 3¾d., £290 11s. 8d., £129 18s. 3¾d.
(*8*) £78 1s. 3d., £986 0s. 10d., £537 0s. 5d.

11. (*1*) 2½d., ¼d., 6d. (*2*) 3d., 8d., 9d. (*3*) 7d., 5d.
(*4*) 1d., 4d. (*5*) 2d., 10d.

12. (*1*) 6d., 3d., 9d., 4¾d. (*2*) 7¼d., 2¾d., 3d., 4¼d.
(*3*) 4¼d., 7¾d., 11½d., 8¾d. (*4*) 10½d., 11¼d., 10d., 4½d.
(*5*) 9d., 10¼d., 9½d., 4½d.

13. (*1*) 8s. 6d., 7s. 7¼d., 9s. 3½d., 8s. 4¾d.
(*2*) 3s. 9d., 7s. 8¼d., 9s. 10d., 8s. 0½d.
(*3*) 5s. 11½d., 8s. 10d., 12s. 4¼d., 15s. 8¼d.
(*4*) 7s. 4¼d., 12s. 11¾d., 15s. 7¾d., 18s. 7½d.
(*5*) 25s. 9½d., 36s. 4½d., 28s. 3¾d., 50s.

14. (*1*) £31 0s. 3d., £521 1s., £81 0s. 6d.
(*2*) £22 0s. 9d., £48 19s. 3d., £207 19s. 9d.
(*3*) £315 2s. 11d., £176 7s. 3d., £525 17s. 5d.
(*4*) £28 9s. 7¼d., £56 5s. 3¾d., £24 17s. 5¼d.
(*5*) £158 13s. 5d., £533 18s. 10½d., £438 11s. 9d.

P. 149. — **14.** (*6*) £586 18s. 5d., £794 14s. 1¼d., £458 10s. 9d.
(*7*) £1358 10s. 11¼d., £2329 14s. 11½d., £129 10s. 1¾d.
(*8*) £4583 13s. 7¼d., £215 17s. 5½d., £5801 0s. 10¼d.
(*9*) £2964 0s. 7½d., £139 8s. 5¼d., £4586 3s. 10¾d.
(*10*) £6350 7s. 10¼d., £2240 6s. 8¾d., £3941 9s. 5¼d.
15. 0·416. **16.** 13, £8 18s. 8¼d.

P. 150. — **17.** 0·579. **18.** £0·054167. **19.** £0·0833. **20.** £0·029 less.
21. £0·658, £0·6875, 7d. **22.** £0·0058. **23.** £0·9.
24. 0·276s. **25.** 0·194.
26. £0·9833. Observe that this figure, roughly £1, represents the imports into Japan from Australia only, and not the total trade. The total trade of Japan is £2 per head; of the United Kingdom, £23, and of Australia, £29 per head (see *Whitaker*, p. 107).

EXAMPLES. XXIX. (a)

SECTION X. B. (ii)

DECIMALISATION OF SUMS OF MONEY

P. 153. — **1.** £36·758; £28·861; £45·905. **2.** £74·665; £85·668; £73·973.
3. £38·716; £79·620; £125·824. **4.** £179·960; £564·956; £100·511.
5. £275·819; £378·622; £250·424. **6.** £868·616; £564·768; £760·320.
7. £236·727; £58·780; £158·691. **8.** £33·778; £726·840; £731·294.
9. £58·893; £384·931; £79·397. **10.** £960·182; £276·680; £158·427.
11. £724·534; £158·485; £396·972. **12.** £356·790; £73·389; £1001·824.
13. £21·597; £196·295; £354·889. **14.** £5·548; £843·199; £768·799.
15. £394·494; £75·636; £596·919.

EXAMPLES. XXIX. (b)

SECTION X. B. (ii)

REDUCING DECIMALISED SUMS OF MONEY TO COMPOUND QUANTITIES

P. 154. — **1.** £394 9s. 10½d.; £75 12s. 8¾d.; £596 18s. 4½d.
2. £5 10s. 11½d.; £843 3s. 11¾d.; £768 15s. 11¾d.
3. £21 11s. 11¼d.; £196 5s. 10¾d.; £354 17s. 9¼d.
4. £356 15s. 9½d.; £73 7s. 9¼d.; £1001 16s. 5¾d.
5. £724 1s. 8¼d.; £158 9s. 8½d.; £396 19s. 6½d.
6. £960 3s. 7¾d.; £276 13s. 7¼d.; £158 8s. 6½d.
7. £58 17s. 10¼d.; £384 18s. 7½d.; £79 7s. 11¼d.
8. £33 15s. 6¾d.; £726 16s. 9½d.; £731 5s. 10½d.
9. £236 14s. 6½d.; £58 15s. 7¼d.; £158 13s. 9¾d.
10. £868 12s. 3¾d.; £564 15s. 4¼d.; £760 6s. 4¾d.
11. £275 16s. 4½d.; £378 12s. 5¼d.; £250 8s. 5¾d.
12. £179 19s. 2½d.; £564 19s. 1½d.; £100 10s. 2¾d.
13. £38 14s. 3¾d.; £79 12s. 4¾d.; £125 16s. 5¾d.
14. £74 13s. 3½d.; £85 13s. 4¼d.; £73 19s. 5½d.
15. £36 15s. 2d.; £28 17s. 2¾d.; £45 18s. 1¼d.

EXAMPLES. XXX. (a)

SECTION X. C. (i)

CALCULATION OF COST: BY DECIMALISING THE SUM OF MONEY

P. 155. — **1.** £2970. **2.** £651. **3.** £2033. **4.** £2418. **5.** £10044. **6.** £159571 15s. **7.** £306236 4s. 4d. **8.** £159571 15s. **9.** £159571 15s. **10.** £306236 4s. 4d. **11.** £410980 14s. **12.** £502973 17s. 2d. **13.** £1,404,852 2s. 2d. **14.** £5,550,607. **15.** £272343 12s.

EXAMPLES. XXX. (b)

SECTION X. C. (ii)

CALCULATION OF COST BY THE METHOD OF NINE MULTIPLES

P. 157.—

	1.	2.	3.	4.	5.	6.
(1)	0·0125	0·025	0·0375	0·0625	0·125	0·175
(2)	0·0250	0·050	0·0750	0·1250	0·250	0·350
(3)	0·0375	0·075	0·1125	0·1875	0·375	0·525
(4)	0·0500	0·100	0·1500	0·2500	0·500	0·700
(5)	0·0625	0·125	0·1875	0·3125	0·625	0·875
(6)	0·0750	0·150	0·2250	0·3750	0·750	1·050
(7)	0·0875	0·175	0·2625	0·4375	0·875	1·225
(8)	0·1000	0·200	0·3000	0·5000	1·000	1·400
(9)	0·1125	0·225	0·3375	0·5625	1·125	1·575

	7.	8.	9.	10.	11.
(1)	0·275	0·525	0·02708333	0·05625	0·077083
(2)	0·550	1·050	0·05416667	0·11250	0·154167
(3)	0·825	1·575	0·08125000	0·16875	0·231250
(4)	1·100	2·100	0·10833333	0·22500	0·308333
(5)	1·375	2·625	0·13541667	0·28125	0·385417
(6)	1·650	3·150	0·16250000	0·33750	0·462500
(7)	1·925	3·675	0·18958333	0·39375	0·539583
(8)	2·200	4·200	0·21666667	0·45000	0·616667
(9)	2·475	4·725	0·24375000	0·50625	0·693750

	12.	13.	14.	15.	16.
(1)	0·102083	0·04895833	0·128125	0·49583333	0·290625
(2)	0·204167	0·09791667	0·256250	0·99166667	0·581250
(3)	0·306250	0·14687500	0·384375	1·48750000	0·871875
(4)	0·408333	0·19583333	0·512500	1·98333333	1·162500
(5)	0·510417	0·24479167	0·640625	2·47916667	1·453125
(6)	0·612500	0·29375000	0·768750	2·97500000	1·743750
(7)	0·714583	0·34270833	0·896875	3·47083333	2·034375
(8)	0·816666	0·39166667	1·025000	3·96666667	2·325000
(9)	0·918750	0·44062500	1·153125	4·46250000	2·615625

P. 157. — **17.** (i) 3s. 9d., 9s., 6s.
(ii) 7s. 6d., 18s., 12s.
(iii) 11s. 3d., 27s., 18s.
(iv) 18s. 9d., 45s., 30s.

18. (i) £3 12s. 6d., £9 15s., £11 2s. 6d.
(ii) £5 1s. 6d., £13 13s., £15 11s. 6d.
(iii) £7 19s. 6d., £21 9s., £24 9s. 6d.
(iv) £15 4s. 6d., £40 19s., £46 14s. 6d.

19. (i) £3 7s. 8½d., £9 12s. 10d.
(ii) £7 0s. 7½d., £20 0s. 6d.
(iii) £9 12s. 8½d., £27 8s. 10d.
(iv) £12 15s. 2½d., £36 6s. 10d.

20. (i) £22 6s. 6d., £38 11s. 7d., £51 0s. 3½d.
(ii) £58 8s. 6d., £100 19s. 3d., £133 10s. 1½d.
(iii) £226 2s., £390 14s. 4d., £516 13s. 2d.
(iv) £132 10s. 6d., £229 0s. 3d., £302 16s. 7½d.

EXAMPLES. XXXI.

SECTION X. C. (iii)

CALCULATION OF COST BY PRACTICE

P. 160. — **1.** £250, £87 10s. **2.** £200, £1000. **3.** £573, £261.
4. £710, £153 8s. **5.** £534 12s., £312 10s. **6.** £228 10s., £78 11s.
7. £2970. **8.** £651. **9.** £2033. **10.** £2418.
11. £10044. **12.** £12987. **13.** £121419. **14.** £13643.
15. £138589 10s. **16.** £70389. **17.** £161991 **18.** £46072.
19. £272343 12s. **20.** £159571 15s. **21.** £306236 4s. 4d.
22. £159571 15s. **23.** £159571 15s. **24.** £306236 4s. 4d.
25. £410980 14s. **26.** £502973 17s. 2d. **27.** £1,404,852 2s. 2d.
28. £5,550,607. **29.** £355214 7s. 7d. **30.** £6,192,796 1s.

EXAMPLES. XXXII.

SECTION X.

MISCELLANEOUS QUESTIONS

P. 160. — **1.** 2s. 9d. **2.** £0·08.
P. 161. — **3.** 1¾d. **4.** £0·000028. **5.** 4s. 3d. = £0·2125.
6. (i) £0·7125; (ii) £0·675; (iii) £0·517. **7.** (i) £0·7; (ii) £0·808.
8. Since a woman aged 55 will usually live longer than a man, the offices can afford to pay the latter more during his life than they can the former.
9. £128 (to the nearest £1). **10.** 4·408d.
11. 2s. 7½d. **12.** £6 18s. 11d.
P. 162. — **13.** £2 10s. 9d. per short ton. **14.** 11d. **15.** 7d.
16. £53. **17.** £2 16s. 6d. **18.** £31074 14s. 6d.
19. 4s. 1¾d. **20.** 2s. $11\frac{1}{16}$d. **21.** £73930 12s. 6d.
22. £1 6s. 8d.

P. 163.—**23.** £1 3s. 6d. **24.** 11·1d.

25. (i) £1136; (ii) £1636; (iii) the earning capacity per mile has increased by £499·8 in the decade; (iv) see Question 28.

26. 6d. **27.** 2d.

28. 8d. India: *Conclusions*—1. That about 30 million money orders were issued of average value, £1 4s. 2. That about 15½ million telegrams were sent at a cost of about 11d. each. 3. That the earning capacity of the railways per mile has increased by about £500 a mile, implying more traffic in passengers and goods. 4. That about 280 million lb. of tea were produced, and made 8d. per lb. Why is tea so much dearer in England?

REVISION QUESTIONS. I.

A.

SECTION I.

P. 163.—**1.** 1813, 1814, 1801, 1812, 1815, 1811, 1805, 1802; £25,011,028.

2. 6664 acres.

P. 164.—**3.** The latter is longer by two days.

4. (i) 1,695,492 cwt. (ii) 28,466,419 Rs. (iii) If ½ inch represents 15 thousand cwt., then lines 2 inches and 3 inches represent 60 and 90 thousand cwt. respectively.

5. 90°, not 55° − 35°, for one town is north and the other south of the Equator.

P. 165.—**6.** Total exports—

Black tea:	Jan. 1 to Sept. 7,	1914 =	129,636,110 lb.
,,	,, ,,	1913 =	130,224,121 ,,
,,	Jan. 1 to Aug. 31,	1914 =	127,231,153 ,,
Cocoa-nuts:	Jan. 1 to Sept. 7,	1914 =	24,498,894 ,,
,,	,, ,,	1913 =	19,465,911 ,,
,,	Jan. 1 to Aug. 31,	1914 =	23,809,136 ,,

Four Largest Customers. Tea.	Excess (+), Defect (−) of 1914 Exportation over the 1913.	Four Largest Customers. Desiccated Cocoa-nuts.	Excess (+), Defect (−) of 1914 Exportation over the 1913.
United Kingdom	−271,699	United Kingdom	+1,534,417
Russia	−1,778,196	United States	+268,020
New South Wales	−22,920	Germany	+811,754
United States	+1,393,245	Belgium	+405,694

The best customers of Ceylon are, for tea: the United Kingdom, Russia, New South Wales, the United States, Victoria, China, New Zealand, Canada and Newfoundland, Africa, South

Australia; and for desiccated cocoa-nuts: the United Kingdom, the United States, Germany, Belgium, Holland, Victoria, Austria, Canada and Newfoundland, Spain, France.

P. 166. — 7. £8692000. **8.** £2,000,000.

9. Decrease, 1910–11 to 1911–12, £28,010,000.
,, 1911–12 to 1912–13, £10,490,000.
,, 1912–13 to 1913–14, £13,270,000.
,, greatest, 1910–11 to 1911–12.

B. (1)

SECTION II.

1. 52. **2.** 143. **3.** £7 3s. **4.** 12; 5760.
5. £75. **6.** £184.

P. 167. — **7.** 33 lb. per £1 in both countries; ∴ the markets were the same.
8. $8\frac{1}{2}$ nearly. **9.** 144. **10.** £44.

B. (2)

1. 113 lb. per head.
2. 1d. (The same as the third class fare in England.)
3. 15 Rs. **4.** Yes, and be better off by £14 13s. 4d. per annum.

P. 168. — **5.** Median, 39; average, $36\frac{1}{3}$; showing how inordinately the 10 feet reduces the value of the average and not of the median.
6. Average, $28\frac{5}{8}$ inches; median, 29 inches.
7. Average, 171 seconds; median, $157\frac{1}{2}$ seconds. Better to take the median, so as to prevent the 300 seconds (which means an abnormal delay) exerting too great an influence.
8. Cows, £44·17 per head; sheep, £6 per head; altogether, £23·3 per head.
9. 2522·4 centimes; 25·224 francs. **10.** 288.
11. (*1*) Average, 38·9; (*2*) median, 33; (*3*) mode, 66. The value of (1) is unduly increased by the 122 and 137, and hence (2) is lower than (1); (3) measures the selling power of the salesman, and would be more conclusive if a larger number of observations had been made.

C. (1)

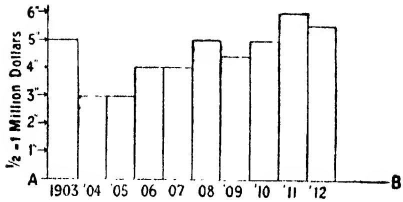
SECTIONS III AND IV.

1. $\frac{7}{9}$. **2.** 30s.
3. Selling price, 7s. 1d.; clearance price, 6s. per dozen.

P. 169. — **4.** The imports are $\frac{1871}{4770}$ of the exports (nearly $\frac{2}{5}$). **5.** $\frac{11}{18}$.
6. $\frac{4}{15}$ truss of new hay. **7.** $\frac{17}{22}$. **8.** $\frac{3}{4}$.
9. £104. **10.** £30. **11.** $\frac{41}{60}$.

C. (2)

P. 169. — 1.



The values are 10, 6, 6, 8, 8, 10, 9, 10, 12, 11 million dollars. (We cannot reproduce the figure full size here, but the student should draw it on good stiff paper, full size, and colour it, when it will be seen to be very effective.)

2.

	1911.	1912.
From Canada . .	$6746\frac{7}{15}$	$4855\frac{2}{21}$
,, Victoria .	$2137\frac{1}{2}$	1120

Fractions are $\frac{1064}{3373}$ and $\frac{244}{1057}$.

P. 170. — 3. The lengths are, in miles—5, $5\frac{1}{2}$, $5\frac{3}{4}$, $6\frac{3}{8}$, $6\frac{27}{40}$, $7\frac{49}{80}$; ∴ final length $=7$ mi. 4 fur. 9 chs.

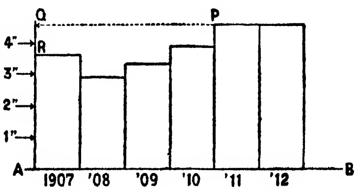
(The student should draw the line for himself.)

4. (i) $\frac{1}{3}$. (ii) $\frac{59}{90}$.

5.

1907–8.	1908–9.	1909–10.	1910–11.	1911–12.
$-\frac{3}{4}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{3}{4}$	*nil.*

6.



7. Draw the dotted line **PQ** ; then **Q** represents $4\frac{1}{2}$, and **R** $3\frac{1}{2}$; ∴ difference $=1$.

P. 170.—8.

United Kingdom.	United States.	France.	Germany.	Austria-Hungary.	Italy.
$\frac{31}{45}$	$2\frac{4}{45}$	$1\frac{89}{90}$	$1\frac{22}{45}$	$1\frac{13}{18}$	$2\frac{41}{90}$

In descending order we have—Italy, United States, France, Austria-Hungary, Germany, United Kingdom.

P. 171.—9. 464, 43, 147, 114, 16, 34 thousand miles.

10. Liverpool. 11. $\frac{81}{245}$, $\frac{89}{392}$.

12. 865 and 3132 people for every conviction in 1850 and 1912 respectively.

13. There were 2267 more people for every conviction in 1912 than in 1850 ; ∴ crime has diminished to about one-fourth.

C. (3)

SECTION V.

1. 432 bushels of wheat, 180 of barley.

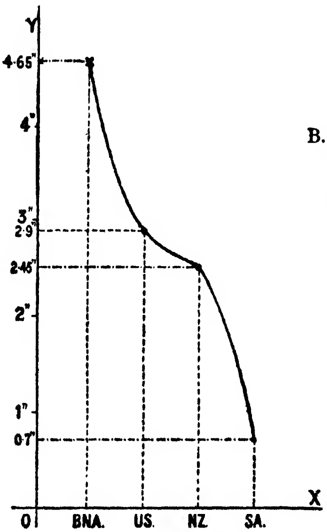
P. 172.—2. 9600 acres arable, 9800 pasture, 1200 woodland ; total, 22400 acres.

3. $\frac{7}{27}$; £1166 13s. 4d. 4. $\frac{5}{66}$ miles. 5. 13 articles.

6. To United States, 117300 ; to New Zealand, 98850 ; to South Africa, 28200. Total, 430500.

7. (*1*) 89800 is net loss by emigration and immigration ; (*2*) $\frac{449}{227500}$ (roughly $\frac{1}{500}$ of the population represents the loss).

P. 173.—8.



B.N.A., 186 thousand = 4·65 inches.
U.S., 117 thousand = 2·9 inches.
N.Z., 99 thousand = 2·48 inches.
S.A., 28 thousand = 0·7 inch.

9.

(*1*) The profit is :

(1) 30s. (2) 35s.
(3) 20s. (4) £23 15s.
(5) 7s. 6d. (6) £4 8s.

(*2*) No. 4.

(*3*) £32 15s. 6d.

(*4*) $\frac{437}{1467}$.

D. (1)

SECTIONS VI.–VIII.

P. 174.—1.

Basin or District.	Third Quarter, 1912.	
	Amount.	Average.
Glenelg and Wannon Rivers . .	10·3	9·4
Cape Otway Forest	13·4	12·0
Mallee	5·6	4·3

2. Greatest in 1912; 0·1148 tons of sugar per ton of cane.
3. 23·8 lb. **4.** 24 lb. **5.** 3·6d. **6.** 10·61s.
P. 175. — **7.** 1·26.
8. 0·62, which means that for every 62 great hundreds imported in 1913, there were 100 great hundreds imported in 1914.

D. (2)

1. 683300 dollars. **2.** (*1*) 13,296,700 dollars. (*2*) 293600 dollars.
P. 176. — **3.** 0·51223 grs. **4.** 2·028d. per lb. **5.** £136660. **6.** 95445 tons.

D. (3)

1. 123·5 francs. **2.** 42·18 marks. **3.** £1148 15s. 6d.
4. £66 19s. 1d. **5.** 344·8 gallons.
P. 177. — **6.** $33\frac{3}{8}$ lb. **7.** $5\frac{3}{8}$ lb. fewer. **8.** 2·83 letters per person.
9. 0·0417, correct to four decimal places.

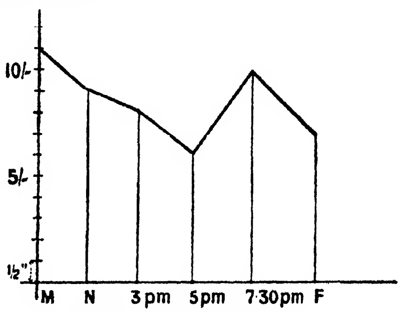
D. (4)

1. (*a*) 19·48; (*b*) 1·807. **2.** (*a*) 0·2688; (*b*) 1·160.
3. (*a*) 2·449; (*b*) 1·225 (last figure doubtful).
4. 3·65. **5.** 86·3689346. **6.** 2·155.

E. (1)

SECTION IX.

1. 11s., 9s., 8s., 6s., 10s., 7s. See graph.
P. 178. — **2.** 0·0183s.
3. $\frac{17}{33}$.
4. $2\frac{1}{16}$ times dearer.
5. £29 15s. 10d.
6. £1 1s. 10d. net (£1 3s. 9d. gross).
7. £9199 18s. 11d.; £3373·333 in hand.



SCALE ½" = 1/- per day.

P. 179. — **8.**

Date.	Receipts.	Expenditure.
	£ s. d.	£ s. d.
July 1 . . .	565 0 0	...
,, 2 . . .	25 15 8	...
,, 3 . . .	96 18 6	3 10 6
,, 4 . . .	198 12 8	...
,, 5 . . .	...	50 10 0
,, 6 . . .	6 5 3	105 15 8
Total . .	892 12 1	159 16 2
Balance .	732 15 11	

9. £22 19s.

E. (2)

1. £504 4s. 5d. **2.** £335 15s. 7d.

P. 180. — **3.** The states composing the Latin Union are, to some extent at any rate, allied in blood and in spirit, while their languages have partly a common original. Although the coins have different names in the different states, yet they have the same value; the face value of the gold coins is that of the fine gold which they contain—a fact which makes them standards of value, and which also facilitates their circulation.

4. A very important question. (See § 97.)

5. Vertical totals in order of columns—£19; £296 13s. 9d.; £42 7s. 6d.; £4 2s. 6d.; £362 3s. 9d.

Horizontal totals (1) to (16)—(1) £70 3s. 9d.; (2) £76 13s. 9d.; (3) £5 5s.; (4) £44 17s. 6d.; (5) £38 11s. 3d.; (6) £14; (7) £31 8s. 9d.; (8) £3 7s. 6d.; (9) £4 5s.; (10) £15 8s. 9d.; (11) £13 2s. 6d.; (12) £22; (13) £6; (14) £13 10s.; (15) 15s.; (16) £2 15s.

6. 15s. 6d.; £18 6s. 3d.; £1 1s. 3d.; 17s. 6d.

7. £19 10s. **8.** £8 12s. 6d.

P. 181. — **9.** 8s. 0¼d.

F. (1)

SECTION X.

1. 108 weeks. **2.** 110 weeks. **3.** 0·09375.

4. 1·027 times. **5.** 15 eggs. **6.** £1 12s. 1¼d.

7. £3 10s. 4d.

P. 182. — **8.** £1 1s. 1d.

9. £4 19s. (correct to 1s.). The builder would charge £5.

F. (2)

P. 182. — **1.** 19 pairs; $2\frac{1}{2}$d. change. **2.** £760 10s. **3.** 16s.
4. 0·33d. **5.** £3 1s. 8d. **6.** £1,312,500. **7.** £164 3s. 4d.
8. 648 square yards. **9.** 16s. **10.** 1s. 2d. **11.** 10d.
12. 7d. **13.** $7\frac{1}{2}$d. **14.** 2s. $0\frac{1}{2}$d.

F. (3)

P. 183. — **1.** £0·0125, £0·1625. **2.** 0·111. **3.** 0·9. **4.** 40.
5. 0·0132s. **6.** £0·35. **7.** £2 8s. 9d.
8. 51 herrings. **9.** 0·599s. per ounce (Troy).

F. (4)

P. 184. — **1.** £26 4s. $6\frac{1}{4}$d. **2.** 1s. $0\frac{1}{4}$d. **3.** 1s. $0\frac{1}{4}$d. **4.** $2\frac{1}{4}$d.
5. Average fares in order are—3s. $5\frac{7}{8}$d., 2s. $6\frac{1}{8}$d., $10\frac{3}{4}$d., $2\frac{1}{4}$d.
6. 10·9d. **7.** 0·9214. **8.** 0·9d., 1·0d.

EXAMPLES. XXXIII.

SECTION XI.

WEIGHTS AND MEASURES

A. THE METRIC SYSTEM: REDUCTION

P. 198. — **1.** 1035 cm. **2.** 18080 mm. **3.** 350,865,000 mm.
4. 8 dm. 5 cm. 6 mm. **5.** 9 m. 8 dm. 6 cm. **6.** 5 Hm. 8 Dm. 6 m.
7. 5 Dm. 6 m. 8 dm. 2 cm. 3 mm.
8. 9 Dm. 7 m. 6 dm. 8 cm. 4 mm. **9.** 90600 sq. mm.
10. 8,090,006 sq. mm. **11.** 605,000,003 sq. mm.
12. 5 sq. dm. 86 sq. cm. 29 sq. mm.
13. 9 sq. m. 53 sq. dm. 62 sq. cm.
14. 8 sq. Hm. 69 sq. Dm. 54 sq. m.
15. 9 sq. Km. 75 sq. Hm. 62 sq. Dm.
P. 199. — **16.** 5,000,008,000 c.mm. **17.** 6,000,000,008 c.mm.
18. 9,008,000,000 c.mm. **19.** 5,008,000 c.mm.
20. 8 c.dm. 964 c.cm. **21.** 5 c.m. 865 c.dm.
22. 51 c.Dm. 648 c.m. **23.** 8300 mgm.
24. 5,008,020 mgm. **25.** 28,000,500 mgm.
26. 8 Hgm. 5 Dgm. 6 gm. **27.** 5 dgm. 3 cgm.
28. 59 Kgm. 8 Hgm. 4 Dgm. 6 gm.
29. 8 Dgm. 6 gm. 5 dgm. 4 cgm. 3 mgm.
30. 1 Kgm. 5 Hgm. 9 Dgm. 6 gm. 2 dgm.
31. 5·6 l. **32.** 8·06 l. **33.** 5000·8 l.
34. 8 Dl. 6 l. 5 dl. 6 cl. **35.** 9 Hl. 5 Dl. 6 l. 2 dl.
36. 6 Hl. 8 Dl. 3 l. **37.** 8·53 ares. **38.** 98 Ha. 65 ares.

P. 199.—39. 568800 sq. m. 40. 56,008,600 sq. m.
41. 58 sq. m. 64 sq. dm. 42. 8 Dl. 8 l. 6 dl. 5 cl.
43. 86 Km. 9 Hm 6 Dm. 4 m. 5 dm. 44. 64 Ha. 86 a. 53 sq. m.
45. 76 c.m. 586 c.dm. 46. 1 : 1·71. 47. 1 : 74·8.
48. 1 : 1·5. 49. 1 : 5·59. 50. 0·00161. 51. 0·02.
52. 0·000010161. 53. 11686·965 gms. 54. 8·7372 Ha.
P. 200.—55. 376·42 l. 56. 5·57236 Km. 57. 177 m.
58. 7 pieces.

EXAMPLES. XXXIV.

SECTION XI.

WEIGHTS AND MEASURES

A. THE METRIC SYSTEM: CALCULATION OF COST

P. 200. — 1. 0·797 centimes. 2. 5·3165 francs. 3. 1413·47 centimes.
4. 110·16 francs. 5. 86·35 francs. 6. 22·796 francs.
7. 13·797 francs. 8. 212·48 marks. 9. 796·425 francs.
10. 6722·5 francs.
P. 201. — 11. 132·9 lira. 12. 52·513 gulden. 13. 7640·4 francs.
14. See §§ 133 and 134; § 143; § 135; §§ 145 and 146.

EXAMPLES. XXXV.

SECTION XI.

WEIGHTS AND MEASURES

B. THE ENGLISH SYSTEM: REDUCTION

P. 201. — 1. 102 inches. 2. 204 inches. 3. 2472 inches.
4. 61776 inches. 5. 63360 inches. 6. 364320 inches.
7. 408456 inches. 8. 1 mile. 9. 1 mile.
10. 1 mile. 11. 4 mi. 3 fur. 8 chs. 12. 2 mi. 4 fur. 5 chs.
13. 3 fur. 3 chs. 2 ft. 6 in. 14. 5·056 sq. ft.
15. 726 sq. ft. 16. 360500 sq. links. 17. 930 sq. poles.
18. 27,200,800 sq. yds. 19. 1 acre.
20. 1 ac. 9 sq. chs. 318 sq. yds. 8 sq. ft. 21. 4 sq. yds. 10 sq. in.
22. 50 acres. 23. 52 ac. 26 sq. pos.
24. 480384 cub. in. 25. 737164·8 cub. in.
26. 1 cub. yd. 2 cub. ft. 522 cub. in. 27. 320 cub. yds. 14 cub.
28. 108500 grs. 29. 592375 grs. 30. 1,232,000 grs.
31. 592375 grs. 32. 70 lb. 33. 40 lb. 12 oz.
34. 112 lb. 35. 208·32 lb. 36. 514 lb.
37. 2240 lb. 38. 6406·4 lb. 39. 6776 lb.
40. 11540 lb. 41. 2 tons 3 cwt. 24 lb.

P. 201.—**42.** 2 tons 8 cwt. 2 qrs. 12 ozs. **43.** 17 tons 18 cwt. 2 qrs.
44. 4 tons 6 cwt. 2 qrs. **45.** 54 pints.
46. 72 pints. **47.** 348 gallons.

P. 202.—**48.** 12 qrs. 1 bush. 2 pks. 1 gall. 3 qts.
49. 22 qrs. 7 bush. 1 gall. 3 qts. **50.** 133 qrs. 6 bush. 1 pk. 1 gall.
51. 14 qrs. 7 bush. 1 gall. **52.** 73500 grs.
53. 34560 grs. **54.** 1984 dwt. **55.** 1300 dwt.
56. 1747·2 grs. **57.** 1592·5 grs. **58.** 8·65 centals.
59. 704·57 stones. **60.** 29110 sec. **61.** 31104 sec.
62. 570 sec. **63.** 8940 min. **64.** 9360 min.
65. 2 hrs. 24 min. 14 sec. **66.** 6 days 16 hrs. 32 min.
67. 3 wks. 2 days 16 hrs. **68.** 216480 sec.
69. 288056 sec. **70.** 2° 24′ 14″. **71.** 13° 3′.
72. 8′ 38″·4. **73.** $\frac{1}{2}$. **74.** $\frac{39}{40}$. **75.** $\frac{91}{13068}$.
76. $\frac{1300}{2133}$ ($\frac{3}{5}$ nearly). **77.** $\frac{2604}{6425}$ ($\frac{2}{5}$ nearly). **78.** $\frac{175}{144}$.
79. $\frac{144}{175}$. **80.** $\frac{175}{152}$.
81. 1·21528, 0·82286, 0·91146; 12·153 lb. (Troy), 16·46 lb. (Av.), 18·23 oz. (Troy). **82.** 3180 grs.
83. 4240 grs. **84.** 6315 grs.
85. (i) 1 ℥ 6 ʒ 0 ℈ 16 grs.; (ii) 40 ℥ 4 ʒ.
86. 7 mi. 2 fur. 4 chs. 14 yds. **87.** 21 sq. yds. 5 sq. ft. 18 sq. in.
88. 8 tons 7 cwt. 2 qrs. 1 st.
89. 12 lb. (Troy) 6 oz. (Troy) 0 dwt. 19·6 grs.
90. 19 tons 3 cwt. 2 qrs. 10 lb.

P. 203. — **91.** 4 tons 19 cwt. 3 qrs. 20 lb. **92.** 3 qts. 0·8 pt.
93. 0·0825. **94.** 0·192. **95.** 0·2308.
96. 0·179. **97.** 0·301. **98.** 0·177.
99. 0·93. **100.** 0·7125. **101.** 0·00417.
102. 0·10417. **103.** 0·1194. **104.** 0·6304. **105.** 0·989.

EXAMPLES. XXXVI.

SECTION XI.

MISCELLANEOUS QUESTIONS

P. 203. —**1.** 1500 c.cm. **2.** 0·00000135.
3. 4500 c.cm. **4.** $\frac{1}{60}$.
5. $1\frac{13}{28}$. **6.** 1 ℥ 4 ʒ 0 8 grs.
7. 1 oz. 16 dwt. 12 grs. **8.** Lighter by 255 grs.
9. 4 pints. **10.** 40 fl. oz.
11. 0·000166. **12.** 0·892857143.
13. 473·2 long tons. **14.** 1·12.
15. 330·36 long tons. **16.** 1·12.

P. 204.—**17.** 0·892857143.
18. 0·446 tons. **19.** £2 6s. 8d. per week of 7 days.
20. 18 galls. 3 qts. 1 pt. **21.** 1 gall. 0 qt. 1 pt.
22. Yes. **23.** 14s. $10\frac{3}{4}$d.

EXAMPLES. XXXVII.

SECTION XI.

WEIGHTS AND MEASURES

B. THE ENGLISH SYSTEM: CALCULATION OF COST

Compound Practice and Nine-Multiple Tables

P. 206.—**1.** £525 7s. **2.** £4 0s. 9d. **3.** £160 9s. 1d.
4. £282 15s. 3d. **5.** £836 17s. 8d. **6.** £1666 7s. 11d.
7. £6673 15s. 9d.

P. 207.—**8.** £6931 0s. 5d.
9. £16934 8s.
10. £31857.

11.

Yards.	Miles.
1	= 0·0005681818
2	= 0·0011363636
3	= 0·0017045455
4	= 0·0022727273
5	= 0·0028409091
6	= 0·0034090909
7	= 0·0039772727
8	= 0·0045454545
9	= 0·0051136364

12. (i) 6·652273 mi.; (ii) 15·7875 mi.: (iii) 5·41857 mi.

13.

(*a*)

Inches.	Centimetres.
1	= 2·539954113
2	= 5·079908226
3	= 7·619862339
4	= 10·159816452
5	= 12·699770566
6	= 15·239724679
7	= 17·779678792
8	= 20·319632905
9	= 22·859587018

(*b*)

Yards.	Metres.
1	= 0·9143834807
2	= 1·8287669615
3	= 2·7431504422
4	= 3·6575339230
5	= 4·5719174037
6	= 5·4863008845
7	= 6·4006843752
8	= 7·3150678570
9	= 8·2294513367

125 in. = 317·494 cm., and 220 yds. = 201·164 metres.

14.

Square Yards.	Square Metres.
1	= 0·8360971498
2	= 1·672194299
3	= 2 508291449
4	= 3·344388599
5	= 4·1804857493
6	= 5·0165828992
7	= 5·852680049
8	= 6·688777198
9	= 7·524874348

The tenth decimal place is uncertain. 1 ac. = 4840 sq. yds. = 4046·71 sq. metres.

15.

Hectares.	Acres.
1	= 2·4711431
2	= 4·9422862
3	= 7·4134293
4	= 9·8845724
5	= 12·3557155
6	= 14·8268585
7	= 17·2980016
8	= 19·7691447
9	= 22·2402878

156 Ha. 56 ares = 386 ac. 3 rds. 21 po.

16. £15 11s. 2d. **17.** £2080 18s. 8d. **18.** £15 14s. 11d.
19. 22981 acres. **20.** 0·264.

P. 207.—21.

Lb.	Cwt.	Tons.
1=	0·008928571	0·0004464286
2=	0·017857142	0·0008928571
3=	0·026785713	0·0013392856
4=	0·035714284	0·0017857142
5=	0·044642856	0·0022321428
6=	0·053571428	0·0026785714
7=	0·062499999	0·0031249999
8=	0·071428571	0·0035714285
9=	0·080357142	0·0040178571

22. (i) 5·17858; (ii) 6·469196; (iii) 15·845982; (iv) 16·273214.

23. (i) £19 5s. 10d.; (ii) £24 2s.; (iii) £59 0s. 6d.; (iv) £60 12s. 4d.

P. 208.—24. £78 10s. **25.** £111 16s. 9d. **26.** £133 18s. 11d.

27. £157. **28.** £223 13s. 5d. **29.** £66 19s. 6d.

30. £213 15s. **31.** 5·9d. per lb. **32.** 1·37 francs per Kgm.

33. £1089 1s. 3d. **34.** £9 8s. 7d. **35.** £4 11s. 3d.

36. £32 15s. 6d. **37.** £14 17s. 1d. **38.** £1354 16s. 11d.

39. 4·64 marks per Kgm. **40.** £103 13s. (correct to 1s.).

56 Old Street, Liverpool,
July 15, 19—.

Messrs. De Hartog & Co., Copenhagen.

Gentlemen,—

We should esteem it a favour if you would let us know whether you can supply butter at 4 kroner per Kgm., in lots of about 3000 Kgm.

Thanking you,
Yours truly,
T. Willambury & Co. Ltd.
(J. S. T.)

41. 2·94 dollars. **42.** 0·365 dollars.

P. 209.—43. £8 6s. **44.** £15 11s. 4d. **45.** £8 6s.

46. £3 17s. 10d. **47.** £49 16s. 2d. **48.** £5 16s. 4d.

49. £175 10s. 1d. **50.** £142 19s. 9d. **51.** £43 17s.

52. 2s. 9¾d. per pint. **53.** 1·02 lira per litre.

54. M. Knovitch's quotation is slightly over 11s., or 5·3 roubles greater.

The letter might be written either in Russian or in French, and the question should not ask for 1000 gallons at 8½d. a gallon, but for 1000 litres at so many roubles. Thus:

130 King Street, Walthamstowe,
England, *Sept.* 18, 19—.

M. Knovitch, Oil Merchants, Odessa, Russia.

Gentlemen,—

We shall be obliged if you will kindly quote for a consignment of about 5000 litres of oil (f.o.b.) to be dispatched to us one week from date of order. A prompt reply will oblige.

Yours truly,
Joseph Briggs & Sons.
(T. N.)

EXAMPLES. XXXVIII.

SECTION XI. C.

INVOICES

(We shall not reproduce here invoice forms, but the student should use a ruled form for EVERY QUESTION.)

P. 222. — **1.** 3s. 1d., 1s. 0¾d., 5d., 1½d., 2d.; total, 4s. 10¼d.

2. 9s., 4½d., 6d., 13s.; total, £1 2s. 10½d. **3.** £4 10s. 11d.

4.

Total . . .	£50 12 8½
Discount . . .	2 10 7
	£48 2 1½
Packing charges . .	0 6 6
	£48 8 7½

5. Total, 33s. 7d.; discount, 1s. 8d.; net, £1 11s. 11d.

6. Cloth, £3 11s. 10½d.; flannel, £1 8s. 6d.; silk, £7 5s. 10d.; calico, 16s. 0½d.; cotton, 7s. 6d.; hat pins, £24 6s.—total, £37 15s. 9d.

7. It is unnecessary to repeat here §§ 155–169.

P. 223. – **8.** Iron nails weigh 254 Kg., and cost 150s.; iron tube, 1″, is 91·5 m., and costs 75s.; brass tube, 1″, is 8·54 m., and costs 14s. Total cost is 239s., or about 300 francs.

9. 30s., £13 10s., £3, £8 10s.; total, £26 10s.; discount, £1 6s. 6d. =£25 3s. 6d. Put stamp on receipt.

10. Refer to Plate IX., Invoice.

	£	s.	d.
8 drums Turkey figs, 1 cwt. 26 lb., @ £2 10s. per cwt. . .	3	1	7
5 bales Mocha coffee, 9 cwt. 3 qrs. net, @ £9 per cwt. . .	87	15	0
3 bales cinnamon, 270 lb. net, @ 1s. 6d. per lb. . . .	20	5	0
6 barrels prunes, 1624 lb. net, @ 6d. per lb. . . .	40	12	0
8 gallons honey, 100 lb. net, @ 1s. per lb. . . .	5	0	0
	156	13	7
Discount . .	3	18	4
	152	15	3

11. 3819 francs. **12.** Total cost, £1 10s. 8d.

13.

15 OLD STREET, SUTTON, DERBY,
15/10/19—.

Messrs. WILLIAMS & Co.,
53 Hugh Road Hendy.

GENTLEMEN,—

We beg to inform you that we are at present stocktaking, and that it would be a great convenience to us if you could let us have your cheque for the amount outstanding on your account a statement of which we have pleasure in enclosing.

Thanking you,
We are,
Yours faithfully,
D. J. LAKIN & Co.
(J. T.)

£4 15s. is outstanding. See § 155.

14. £42 8s. 9d. net.

P. 224.—**15.** 5s. 6d., 8s., 3s., £1 16s., 15s. ; total, £3 7s. 6d.

16. 19s. 3d., 19s. 6d., £3 10s. 10d., 17s. 6d., 14s. 3d. ; total, £7 1s. 4d. Tare is 1 cwt. 18 lb., 1 cwt. 8 lb., 1 cwt. 2 lb., 29 lb., 96 lb. ; total, 4 cwt. 41 lb. Carriage, 10s. 11d.

Cost of goods	£7	1	4
Discount	0	3	6
Net	£6	17	10
Carriage	0	10	11
	£7	8	9

17. £4 19s. 2d. **18.** £1 7s. 6d.

19. See §§ 152–170, and refer to Plate IX., Invoice.

P. 225.—**20.** Total, £15 10s. 7d. **21.** £44 1s. 6d.

22. Total, £8 3s. 1d. (goods sent) ; add to this cost of jars, etc., 16s. 1d.

P. 226.—**23.** The amounts are—3s., 10s., 2s. 6d., 12s. 3d., £3 15s., 6s. 9d., 18s. 9d., £1 19s., £2 12s. 6d., 15s. 9d., 5s. 7½d., 10d., 2d., £6 12s., 6s., 4s. 1½d. ; total, £19 4s. 3d.

EXAMPLES. XXXIX.

SECTION XI. D.

THE SUBSIDIARY BOOKS

P. 235.—**1–3.** For method, see § 171. We give totals—
(*1*) £19 3s. 6d. (*2*) £13 12s. 10d. (*3*) £25 5s. 9d.

P. 236.—**4–6.** See § 172 for method. Totals are—
(*4*) £24 5s. (*5*) £38 1s. (*6*) £38 3s.

7–9. See §§ 171–176. Totals are—
(*7*) Purchases Book, £14 14s. 8d. ; Sales Book, £9 17s. 2d.
(*8*) Purchases Book, £17 7s. 6d. ; Sales Book, £15 14s. 8d.
(*9*) Purchases Book, £28 18s. 2d. ; Sales Book, £25 17s.

P. 237.—**10–14.** For method, see § 177. Returns Books, §§ 171–176 ; Purchases and Sales Books ; debit note, § 177 ; credit note, Plate VIII., p. 224.

(*10*) Purchases Book, £38 ; less returns, £15 ; d/n from J. J. to R. P., £15 ; c/n from R. P. to J. J., £15.

(*11*) Total, £5 18s. 8d. ; less returns, 13s. 4d. ; d/n from buyer to seller, 13s. 4d. ; c/n from seller to buyer, 13s. 4d.

(*12*) Total, £47 10s. ; less returns, £3 15s. ; d/n and c/n as before.

(*13*) Total, £34 11s. ; less returns, £20 2s. 6d ; d/n and c/n as usual.

(*14*) Total, £9 15s. ; less returns, £1 13s.

P. 237.—**15–17.** For method, see §§ 178 and 179.

(*15*) *Dr.* total . £22 10 6
Cr. total . 12 0 6
Cash in hand £10 10 0

(*16*) *Dr.* total . £25 12 0
Cr. total . 15 12 6
Cash in hand £9 19 6

(*17*) *Dr.* total . . £28 0 0
Cr. total . . 24 0 0
Cash in hand £4 0 0

18–20. See § 180 for method.
(*18*) Cash in hand, £161 5s. ; Cash in Bank, £10 2s.
P. 238.— (*19*) Cash in hand, £259 ; Cash in Bank, £279.
(*20*) Cash in hand, £481 19s. ; Cash in Bank, £515 10s.

21–23. See § 181 for method. We give the balances—
(*21*) Cash in hand, £487 ; Cash in Bank, £727 ; Discount (*Debit*), 10s.
(*22*) Cash in hand, £241 10s. ; Cash in Bank, £220 ; Discount (*Debit*), 8s.
(*23*) Cash in hand, £740 7s. ; Cash in Bank, £1009 10s. ; Discount (*Debit*), £1.

P. 239.—**24–26.** See § 182 for method.
(*24*) £1 9s. 6d. (*25*) £1 3s. 8d. (*26*) £2 10s.

EXAMPLES. XL.

SECTION XII. A.

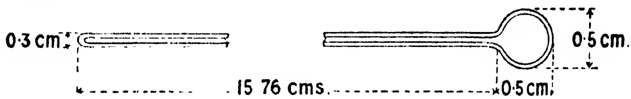
LENGTH

P. 239. — **1.** (*1*) 12·95 in. (*2*) 32·92 cm. (*3*) 2·5 cm.
2. (*1*) 8·06 in. (*2*) 20·5 cm. (*3*) 2·5 cm.
3. 2·5. **4.** 1 m. 22 cm. ; 30·5 cm. **5.** 39·4 cm.
P. 240. — **6.** 0·178 m.
7. 7·92 in., 20·12 cm. **8.** 18·29 m., 39·37 in. **9.** 62·14 mi.
10. 4·57 dm. **11.** 1·0936 yd. **12.** 1093·6 yds., 1760 yds.
13. 1·6 Km. **14.** 4·97 fur. **15.** 0·0568.
16. 91·44 m. **17.** 8·56 m. or 9·36 yds. ; the latter.
18. 30·56 ft. per sec. **19.** 8·47 m. per sec.
20. 9·71 m. per sec. **21.** 804·65 m. ; $\frac{4}{5}$ Km.
22. Yes ; diameter of French record = 9·06 in. ; or diameter of plate = 26·67 cm., which is greater than 0·23 m.
23. £112 10s. **24.** 8d. per yard. **25.** 1s. 2½d. per fathom.
26. 3s. 9d. **27.** 8¾d. **28.** 2s. 1d. **29.** 0·9415.
P. 241.—**30.** 233 cm.
31. Steamer, 25⅓ ft. per sec. ; runner, 24 ft. per sec. ; ∴ the former is 1·056 time quicker than the latter.
32. 2348. **33.** 149 canes. **34.** 5·49 mi.

P. 241.—**35.** 8·84 Km. **36.** 0·378 mi. **37.** 333 fathoms.
38. 1609 Km. **39.** 59·03 mi. **40.** 0·822.
41. 83·9 mi. **42.** 62·76 Km. ; 46·67 Km.
43. 37 yds. 0 ft. 10 in. **44.** 43 trucks. **45.** 1253·09 ft.

P. 242.—**46.** 75. **47.** 261 ft. 9 in.
48. Length, 4 in. ; width, 1 in., neglecting shading ; distance between centres, $\frac{7}{8}$ in.
49. Length, 10 cm. ; width, 2·5 cm.
50. **AB**=78 ft., **BC**=36 ft., **DE**=18 ft., **EF**=21 ft., **GH**=27 ft.

P. 243.—**51.** Length, $5\frac{1}{2}$ in. ; width, $3\frac{3}{4}$ in.
52. Length (table), 6 ft. 6 in. ; width (table), 2 ft. 6 in. ; length (leather), 6 ft. ; width (leather), 2 ft.
53. $\frac{13}{18}$.
54.



55. 6·4 in. **56.** 228 ft. ; perimeter of **EGHK** is 138 ft.
57. $587\frac{1}{2}$ ft. **58.** 37 ft. $3\frac{1}{2}$ in.

P. 244. **59.** 50·3 m. **60.** 182·88 m.
61. 1·27 m., 0·66 m., 1·575 m.
62. 60·325, 8·89, 30·48, 2·54, 3·175, 3·81, 21·59, 78·74 cm.
63. To Dover, 70 ; Folkestone, 70 ; Newhaven, 58 ; Portsmouth, 70 ; Plymouth, 210 ; Bristol, 114 ; Fishguard, 260 ; Holyhead, 260 ; Birmingham, 110 ; Birkenhead, 193 ; Manchester, 185 ; Sheffield, 160 ; Sunderland, 265 ; Hull, 195 ; Harwich, 65 (miles).
64. Spanish gauge is 1·164 time greater than the English.
65. 31·07 mi. per hour. **66.** 120, 320, 260 mi.
67. 1 in. $=83\frac{1}{3}$ mi. ; 400 mi.
68. 1·73d. ; 1·6d. ; 1·46d. **69.** $\frac{1}{506880}$.

EXAMPLES. XLI.

SECTION XII. B.

TRIANGLES

P. 245.—**1.** (*1*) 5·1 in. (*2*) 12·95 cm. **2.** 0·3937 in. = 1 cm.
3. (*1*) 6·325 in. (*2*) 16·1 cm.

P. 246.—**4.** 1·75 in. ; perimeter, 16·6 cm.
5. For one bracket, 2·55 ft. ; for $8\frac{1}{2}$ doz., 260 ft.
6. (*1*) 0·7 in. (*2*) 0·875 in. (*3*) 1·05 in. (*4*) 1·4 in.
7 and **8.** In a right angled isosceles triangle the length of the hypotenuse is 1·414, or $\sqrt{2}$ times the length of either of the equal sides.

P. 246.— **9.** (*1*) 113·08 in. (*2*) 8·037 mi. (*3*) 9·165 m.

P. 247.—**10.** 24 yds. 1 ft. (to nearest foot).

11. (*1*) 4·24 in. (*2*) 8·48 in. (*3*) 12·02 cm. (*4*) 10·32 cm. (*5*) 0·25 m. (*6*) 3·54 in. (*7*) 18·95 cm. (*8*) 7·92 in. (*9*) 4·95 in. (*10*) 6·36 in.

12. All the angles measured should be right angles.

13. (*1*) The last two columns should agree. (*2*) 25. (*3*) and (*4*) 6·25. (*5*) and (*6*) 56·25. (*7*) and (*8*) 100.

P. 248.—**14.** (*1*) 15 in. (*2*) 20 cm. (*3*) 10 m. (*4*) 75 ft. (*5*) 30 yds. (*6*) 300 poles. (*7*) 50 fur. (*8*) 150 m. (*9*) 675 mi. (*10*) 225 ft. (*11*) 175 yds. (*12*) 675 yds.

15. 20 ft. **16.** 53 in. (**DC**=5 in.). **17.** 8·5 in.

18. 9·9 in. (correct to one decimal place).

P. 249.—**19.** **P** and **Q** are 6 in. × 3 in. ; **T** is 4·24 in. × 4·24 in.

20. **BD**=8·5 in. ; perimeter=39·2 in.

21. The crosspieces are 5 in. long ; total, 112 in.

22. 3·98 ft.

P. 250.—**23.** 2152 mi.

24. **AB**=9·25 in. ; **BC**=5·5 in. ; **AC**=10·76 in. or 2152 mi.

25. 467 mi.

26. Question 25 gave 421·9 as the distance by measurement, it should be 422 mi. **27.** 753 Km.

EXAMPLES. XLII.

SECTION XII. C.

CIRCLES

P. 250.—**1.** The value of the NUMBER in the last column should be 3·14, or $3\frac{1}{7}$.

P. 251.—**2.** The difference should be very small.

3. Note that π is a NUMBER; it is not centimetres or inches, but the RATIO of inches to inches, or of centimetres to centimetres. π has ALWAYS the same value, namely, 3·1416.

P. 252.—**4.** (*1*) 18·85 ft., 37·70 ft., 50·265 m., 94·25 m.
(*2*) 15·71 in., 48·69 ft., 53·41 yds., 58·12 cm.
(*3*) 32·2 ft., 40·21 yds., 58·43 m., 51·0195 cm.

5. (*1*) 0·3269 yd., 0·425 yd., 0·7669 yd.
(*2*) 0·8381 yd., 0·7805 yd., 1·7224 yd.
(*3*) 2·619 yds., 3·0119 yds., 1·375 yd.

6. (*1*) 3·529 ft., 4·52 ft., 1·507 ft.
(*2*) 56 m., 92·75 cm., 1·2705 m.
(*3*) 5·25 poles, 0·137725 mi., 840 ft.

7. 18·85 ft. **8.** 7·85 ft. **9.** 2394 mm. **10.** 37·7 in.
11. 50·27 ft. **12.** 157·08 ft. **13.** 4 tons 16 cwt. **14.** 280 revs.
15. 233 revs. **16.** 6 cogs. **17.** 4582 ft. **18.** 1 min. 51·6 sec.

P. 253.—**19.** 53 bricks.

EXAMPLES. XLIII.

LENGTHS

MISCELLANEOUS QUESTIONS

P. 253.—**1.** 212 stairs. **2.** £143 2s. **3.** $57\frac{3}{4}$ in. **4.** 14·67 m.
5. 2125 ft. **6.** 647·68 m. **7.** 113 towels. **8.** 9s. 4d.
9. 1 hr. 16 min. 7 sec. **10.** 5 days 15 hours. **11.** 10s. 5d.

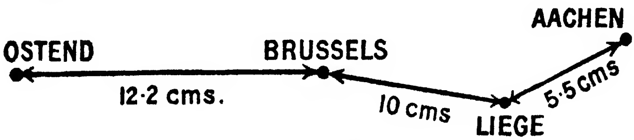
P. 254.—**12.** 13·13 francs. **13.** 6s. 3d.

14. Using the answers to Questions 11 and 13, we find that the second class continental fare is 4s. 2d. cheaper than the third class rate in England.

15. 13s. **16.** 3·164 dollars. **17.** 47·6 threads. **18.** 804 nails.
19. 5·766 m. **20.** 2130·682 mi. **21.** 113·14 ft.

22. Descent is 42·24 ft.; difference of level, 70·9 ft.

23.



Scale, 1 cm. = 10 Km.

24. 14s. $2\frac{1}{2}$d.

P. 255.—**25.** 9 hrs. 47 min. **26.** 38 hours. **27.** 5s. 1d.

28. 88 ft. per sec. **29.** $6\frac{9}{11}$ sec.

30. The train travels at 44 ft. per sec.; Applegarth at 30·56 ft. per sec.; *i.e.* the speed of the former is (rather less than) $1\frac{1}{2}$ time quicker than the latter, who could not, of course, keep up his pace.

31. 22 ft. per sec. = 15 miles per hour, 30 knots = $35\frac{3}{11}$ miles per hour; ∴ the latter is the faster.

32. 6 hrs. 25 min. **33.** 80·67 ft. per sec.

34. The speed is 82·87 ft. per sec., and ∴ 2·2 ft. per second faster.

35. £367437 per mi.; £20877 per 100 yds. (correct to £1).

36. 19 hrs. 24 min.

37. Times are: Brindisi to Port Said, 40 hrs. 23 min.; Paris to Rome, 20 hrs. 13 min.; difference, 20 hrs. 10 min. (taking distances in question as statute miles).

P. 256.—**38.** 0·4018d. per mile.

39. 0·6374d. per mile, or rather more than the rate by steamer. Why should this be so?

40. 15 hrs. 38 min. (180 is statute miles; *vide* question).

41. 11 hrs. 36 min. **42.** 568·18 versts.

43. 0·53 m. per Km. (to two places).

44. Fourth, 0·3862d. per mile; third, 0·6115d. per mile; third, 0·9173d. per mile; first, 1·513d. per mile.

P. 256.—**45.** First class, 1·774d. per mile; third class, 1d. per mile.
46. First class in England is a little more expensive than first on the Continent, and third class in England about the same as second class on the Continent.
47. 1920 steps; 15 seconds. **48.** $8\frac{1}{4}$ miles from starting-point.
P. 257.—**49.** £58593 15s. **50.** £117936. **51.** £453596 11s. **52.** 9 ft.
53. 3s. 2d. **54.** £2 7s. 6d. **55.** £685 16s. 7d. **56.** £331 1s.
57. £17224 per mile, to the nearest £1.

EXAMPLES. XLIV. (a)

SECTION XIII. A.

AREAS: RECTANGLES

P. 258. — **1.** The square contains 100 small squares, ∴ one small square $=\frac{1}{100}$ Sq. in.
2. 36 sq. in.
P. 259. — **3.** 15 sq. cm.
4. Length, 1·97 in.; breadth, 1·18 in.; area, 2·3146 sq. in.; ∴ 15 sq. cm. = 2·3146 sq. in.; and 6·45 sq. cm. = 1 sq. in.
5. 35.
6. Length, 8·88 cm.; breadth, 6·35 cm.; area, 56·4 sq. cm.
7. 6·45 sq. cm. in 1 sq. in.
8. The rectangle should be $4\frac{1}{2}$ in. long and $2\frac{1}{4}$ in. wide.
9. The elevation is a rectangle $4\frac{1}{2}$ in. by $\frac{7}{8}$ in. for front face, and $2\frac{1}{4}$ in. by $\frac{7}{8}$ in. for the end.
10. Length, 22·9 cm.; width, 11·43 cm.; thickness, 4·44 cm.
11. A rectangle 6·5 in. by 4 in. **12.** 66·05 cm.; 40·65 cm.
13. 2684 sq. cm. **14.** 0·5 (nearly). **15.** 6·45.
16. (*1*) 211·6 sq. cm. (*2*) 32·9 sq. in. 0·16.

EXAMPLES. XLIV. (b)

SECTION XIII. A.

RECTANGLES—(*continued*)

P. 260. — **1.** (*1*) $8\frac{1}{8}$ sq. ft. (*2*) 2·04 sq. ft. (*3*) 5·734 sq. cm.
(*4*) 188·76 sq. cm. (*5*) 6·17 sq. yds. (*6*) 39·86 sq. yds.
(*7*) 16·25 sq. chs. (*8*) 1·46 sq. mi.
2. 0·81 sq. in. **3.** 0·525 sq. cm. **4.** 0·7056 sq. m.
5. 2·3325 sq. m. **6.** 10 sq. ft. **7.** By $37\frac{1}{8}$ sq. ft
8. 90·875 sq. ft.
P. 261. — **9.** $12\frac{1}{4}$ sq in.; $19\frac{3}{32}$ sq. in. **10.** 1·5275.
11. 2 in. × $1\frac{1}{2}$ in. in diagram, or 36 in. × 27 in. = 972 sq. in.
12. 972 sq. in. or 6·75 sq. ft.
13. Length, 9·4 cm.; breadth, 6·35 cm.; area, 238·8 sq. cm.
14. (*1*) 26 ft. (*2*) 12 ft. (*3*) 35 sq. ft. **15.** 22s.

P. 262. — **16.** $2\frac{3}{16}$ in. × $2\frac{7}{16}$ in. (high); width, 35 in.; height, 39 in.; ∴ the chest of drawers is too high for the alcove.
17. 11s. 10d., *i.e.* 12s. **18.** 0·88 sq. m.

EXAMPLES. XLV.

SECTION XIII. B.

SQUARE ROOT

P. 264. — **1.** 66 ft. **2.** 71 m. **3.** 78 ft.
4. 84 ft. **5.** 91 m. **6.** 94 Km.
7. 99 yds. **8.** 98 yds. **9.** 117 m.
10. 144 yds. **11.** 276 Km. **12.** 279 ft.
13. 380 ft. **14.** 4·95 mi. **15.** 9·8 mi.
16. 4·92 mi. **17.** 1·414 mi. **18.** 2·236 mi.
19. 2·646 chs. **20.** 3·606 chs. **21.** 1·1 mi.
22. 0·028 Km. **23.** 0·041 mi. **24.** 0·0083 mi.
25. 2·495 chs.
26. (*1*) 2·236 in. (*2*) 254 ft. (*3*) 39·6 yds.
(*4*) 21 chs. (*5*) 92 yds.
27. 69·6 yds. **28.** £48 14s., to the nearest 1s.
29. Area, 36100 sq. mi.; length of side, 190 mi. **30.** 9·9 yds.

EXAMPLES. XLVI. (a)

SECTION XIII. C.

PAPERING WALLS, CARPETING AND TILING FLOORS, ETC.

P. 265. — **1.** (*1*) 43 ft. 2 in. (*2*) 49 ft. (*3*) 17·68 m. (*4*) 56 ft. 6 in.
(*5*) 22·9 m. (*6*) 90 ft. 8 in. (*7*) 29·20 m.
(*8*) 282 ft. (*9*) 41·56 m. (*10*) 287 ft. 10 in.
2. (*1*) $431\frac{3}{8}$ sq. ft. (*2*) $612\frac{1}{2}$ sq. ft. (*3*) 53·04 sq. m.
(*4*) $706\frac{1}{4}$ sq. ft. (*5*) 114·5 sq. m. (*6*) $1178\frac{3}{8}$ sq. ft.
(*7*) 146 sq. m. (*8*) $5710\frac{1}{2}$ sq. ft. (*9*) 249·36 sq. m.
(*10*) $7195\frac{5}{8}$ sq. ft.
3. 51 ft. 8 in. **4.** $160\frac{1}{8}$ sq. ft. **5.** Area, $1512\frac{1}{2}$ sq. ft.; £4 6s.
6. $232\frac{1}{2}$ sq. yds.

P. 266. — **7.** £7 5s. 6d., to the nearest 6d. **8.** 2·66d. **9.** £16 3s.
10. (*1*) 12s. (*2*) 12s. (*3*) 12s. **11.** £7 16s.
12. We take the question to mean 18 treads and 19 risers, for which 28 feet of carpet would be required. **13.** £1 15s. $4\frac{1}{2}$d.
14. Walls, 660 sq. ft.; floor or ceiling, $187\frac{1}{2}$ sq. ft.

EXAMPLES. XLVI. (b)

SECTION XIII. C.

PAPERING WALLS, ETC.

(Fractions of a piece of paper are regarded as one piece in the answers, which are given to the nearest 1s. or 1 franc.)

In working sums such as these we find—(1) Area of walls, (2) area of paper, (3) number of **complete** pieces of paper required, (4) cost.

Thus 1 (*1*) Area of walls = 610 sq. ft.; area of one piece of paper = 63 sq. ft.; number of *complete* pieces required = 10; cost = £1 15s.

Another very common way, but not a satisfactory one from a commercial point of view, is that cost equals $\frac{\text{Area of walls}}{\text{Area of one piece of paper}} \times \text{cost per piece}$;

or $\frac{61 \times 10}{63} \times 3{\cdot}5\text{s.} = \frac{610}{63} \times 3{\cdot}5\text{s.} = £1\ 13\text{s.}\ 9\text{d.}$,—which would be wrong, for the paperhanger must hang long pieces of paper, otherwise the walls would have to be patched. Then the papering could be done for £1 13s. 9d.

P. 267. — **1.** (*1*) £1 15s. (*2*) 43 francs. (*3*) £3 12s. (*4*) 50 francs. (*5*) £3 9s. (*6*) 113 marks. (*7*) £3 15s. (*8*) 94 marks. (*9*) £13 10s. (*10*) 277 francs.

2. Area is taken to nearest square yard or square metre **above**, and then cost calculated.

(*1*) £3 15s. (*2*) 86 francs. (*3*) £7 11s. (*4*) 78 francs. (*5*) £7 5s. (*6*) 115 marks. (*7*) £10 8s. (*8*) 78 marks. (*9*) £22 15s. (*10*) 158 francs.

P. 268. — **3.** Here it is useless, too, to divide area of floor by area of tiles, for you cannot use up the scraps; and, in the trade, the broken pieces which may have to be put in would be reckoned as a whole tile. Hence, we have in (1) number of tiles in a row along length is 36, and in width, 25, or 900 in all, *i.e.* 75 dozen; hence, cost is £20 12s. 6d.)

(*1*) £20 12s. 6d. (*2*) 825 francs. (*3*) £75 12s.
(*4*) 850 francs. (*5*) £73 14s. 6d.
(*6*) 1309 marks or 1284 marks.[1] (*7*) £118 2s.
(*8*) 1731 marks or 1741 marks.[1] (*9*) £486 2s. 6d.
(*10*) 718 marks.

4. (*1*) £3 15s. (*2*) 33 francs. (*3*) £3 2s. 3d.
(*4*) 67 fr. 50 c. (*5*) £6 16s. 8d.
(*6*) 97 marks (to nearest mark). (*7*) £8 14s.
(*8*) 140 marks. (*9*) £19 16s. (to nearest shilling).
(*10*) 55 marks.

[1] According as the tiles are placed with the shorter edge along the length, or width, of the room.

P. 268. — **5.** (*1*) 405 francs. (*2*) 967 francs. (*3*) 2077 francs.
(*4*) 5803 francs.
6. (*1*) 374 francs. (*2*) 663 francs. (*3*) 1224 francs.
(*4*) 2856 francs.

Note.—In Nos. 5 and 6 we have expressly stated that the dimensions of the room are to be taken to the nearest metre above those given. Had we NOT done so, then the student would have proceeded thus—

5. (*1*) Perimeter $= 2\,(10{\cdot}6 + 3{\cdot}8)$ m.

Area of walls $= 28{\cdot}8$ m. $\times 5{\cdot}6 = 161{\cdot}28$ sq. m.

Area of 1 piece of paper $= 9 \times 0{\cdot}45$ sq. m.

$\therefore$ No. of pieces of paper $= \dfrac{161{\cdot}28}{4{\cdot}05} = 40$, to nearest piece (above).

$\therefore$ Cost $= 9 \times 40$ francs
$= 360$ francs.

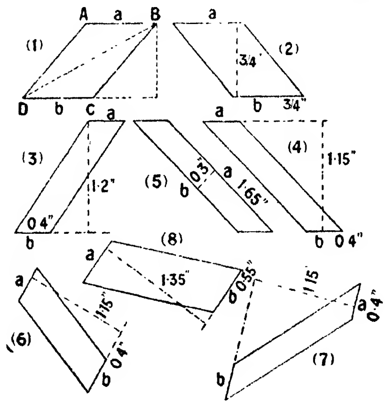
But a decorator would *not* calculate the cost to so close a figure as this.

EXAMPLES. XLVII. (a)

SECTION XIII. D. (1)

AREA OF PARALLELOGRAMS

P. 269. — **1.**



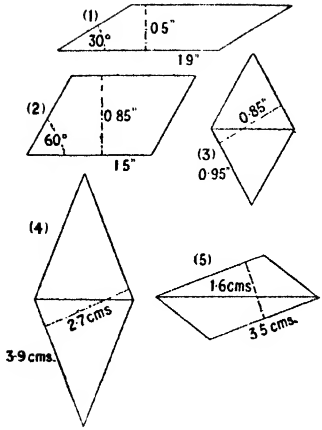
The lines p are dotted in in the diagrams and their lengths given.

2. (*2*) 0·56. (*3*) 0·48. (*4*) 0·46. (*5*) 0·495.
(*6*) 0·46. (*7*) 0·46. (*8*) 0·74 sq. in.

3. The student should reckon 1 sq. in. for 6·45 sq. cm., convert his results into square inches, and compare with Question 2.

4. The results in column 3 should be the same, and equal 0·155.

P. 270. — 5.



Areas are :

(*1*) 0·95 sq. in. (*2*) 1·275 sq. in. (*3*) 0·8 sq. in.
(*4*) 10·53 sq. cm. (*5*) 5·6 sq. cm.

EXAMPLES. XLVII. (b)

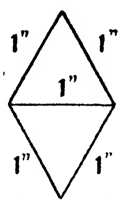
SECTION XIII. D. (2)

AREA OF TRIANGLES

P. 270. — **1.** Should be done very carefully, using a very sharp knife for cutting, and a steel straight-edge if possible, rather than a wooden or cardboard scale.

P. 272. — **2.** Column (1) should be double column (2) or column (3), and column (2) should be just the same as column (3). Learn that the parallelograms formed are double the respective triangles.

3. The arrangement is—



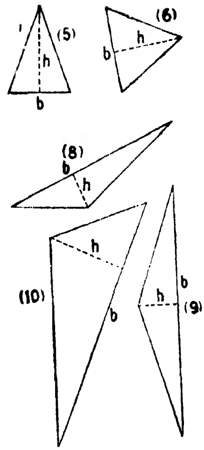
and a similar kind of figure for each part of the question. Master the matter in dark type in Question 3 before you proceed.

4. The areas are—(*1*) 6 sq. in. (*2*) 1·5 sq. in.
(*4*) 3·78 sq. in. (*5*) 5·1 sq. in.

P. 272. — 5.

Measure b and h in each case, and take $bh \div 2$ as the areas, which are

(*1*) 0·15 sq. in.
(*2*) 0·19 sq. in.
(*3*) 0·28 sq. in.
(*4*) 0·225 sq. in.
(*5*) 0·28 sq. in.
(*6*) 0·32 sq. in.
(*7*) 0·225 sq. in.
(*8*) 0·37 sq. in.
(*9*) 5·0 sq. in.
(*10*) 0·114 sq. in.



6. (*1*) 18·585 sq. in. (*2*) 168·399 sq. in. (*3*) 157·887 sq. cm.
(*4*) 39·472 sq. cm. (*5*) 600 sq. in. (*6*) 3600 sq. in.
(*7*) 183612 sq. cm.

P. 274. — **7.** (*1*) 734418 sq. cm. (*2*) 401$\frac{1}{9}$ sq. ft. (*3*) 401$\frac{1}{8}$ sq. ft. (*4*) 105$\frac{13}{32}$ sq. ft.
(*5*) 421$\frac{7}{8}$ sq. ft. (*6*) 54·3 sq. yds. (*7*) 217·2 sq. yds.

8. 58600 sq. miles (base, 330 miles; perpendicular, 355 miles).

9. 16 sq. ft. 36 sq. in. **10.** 570 sq. ft.

EXAMPLES. XLVIII.

SECTION XIII. E. (1)

AREA OF TRAPEZIUMS

P. 274. — **1.** Triangle ADE$=\frac{1}{4}$ sq. in.; triangle BFC$=\frac{1}{4}$ sq. in.; rectangle ABFD$=2$ sq in.; area of figure$=2\frac{1}{2}$ sq. in.

P. 275. — **2.** $2\frac{1}{2}$ sq. in. **3.** Learn by heart the matter in dark print.

4. $35\frac{7}{8}$ sq. ft.

P. 276. — **5.** $58\frac{1}{2}$ sq. ft. **6.** 41·6 sq. in. (correct to $\frac{1}{10}$ sq. in.).

7. 240·75 sq. chs. **8.** 446 sq. ft. (correct to 1 sq. ft.).

9. £15 6s. 8d. (correct to 1d.).

10. 24 sq. ft. nearly.

P. 277. — **11.** 256000 sq. mi. (correct to 100 sq. mi.). **12.** 82200 sq. mi.

13. 36000 sq. mi. (correct to 1000 sq. mi.).

14. Compare the regular boundaries of the former with the irregular ones of the latter.

EXAMPLES. XLIX. (a)

SECTION XIII. E. (2)

AREAS OF PENTAGONS, HEXAGONS, OCTAGONS

P. 281. — 1. The length of OR is, for the tiles, in order, 0·65 in., 1 in., 5·4 cm., 6·8 cm. The areas are 1·6 sq. in., 3·75 sq. in., 108 sq. cm., 170 sq. cm.

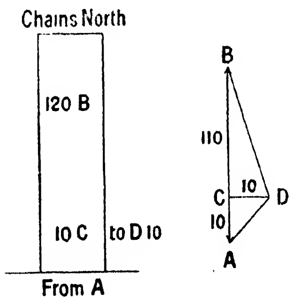
2. 1·46 sq. in., 4·06 sq. in., 16·2 sq. cm.
3. 77·25 sq. in., 173·808 sq. in., 308·992 sq. in., 193·12 sq. cm.
4. OR equals 0·65 ft., 1 ft., 6·1 in. Areas are : 1·6 sq. ft., 3·75 sq. ft., 137·25 sq. in.
5. 4·06 sq. ft., 166·272 sq. in., 3367·008 sq. cm., 4156·8 sq. cm.
6. 1·5 sq. ft., correct to $\frac{1}{10}$ sq. ft.

EXAMPLES. XLIX. (b)

SECTION XIII. E. (2)

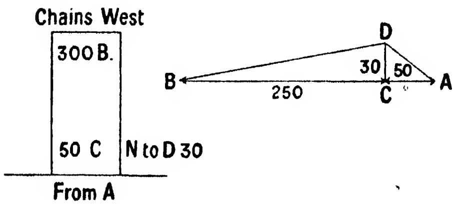
THE FIELD BOOK

P. 282.— 1.



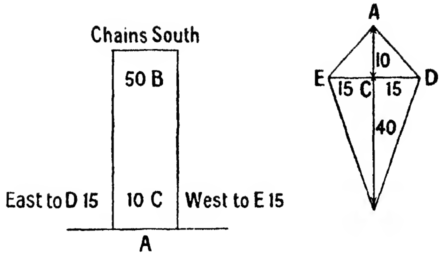
$$\text{Area} = \frac{10 \times 10}{2} + \frac{110 \times 10}{2}$$
$$= 600 \text{ sq. chs.}$$

2.



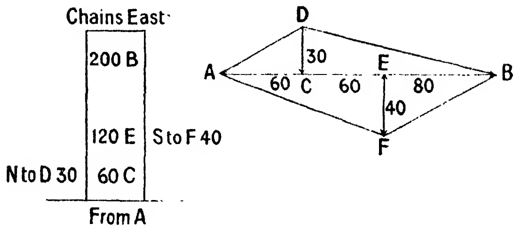
$$\text{Area} = \frac{250 \times 30}{2} + \frac{30 \times 50}{2}$$
$$= 4500 \text{ sq. chs.}$$

P. 282.—3.



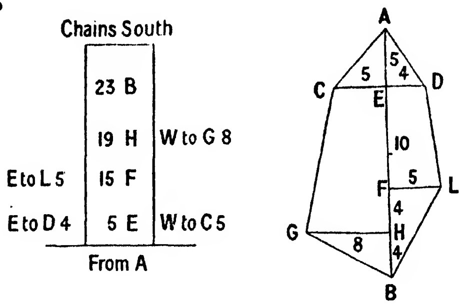
$$\text{Area} = \frac{50 \times 15}{2} + \frac{50 \times 15}{2}$$
$$= 750 \text{ sq. chs.}$$

4.



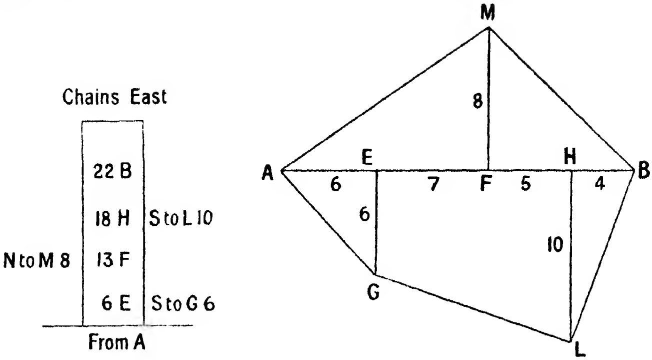
$$\text{Area} = \frac{30 \times 200}{2} + \frac{40 \times 200}{2}$$
$$= 7000 \text{ sq. chs.}$$

5



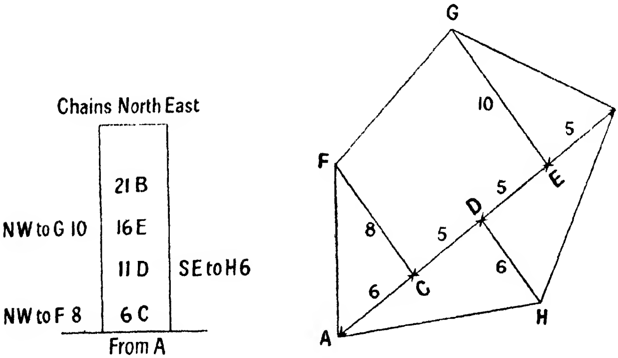
$$\text{Area on left of } \mathbf{AB} = \frac{25}{2} + \frac{14 \times 13}{2} + \frac{32}{2}$$
$$= 119\tfrac{1}{2} \text{ sq. chs.}$$
$$\text{Area on right of } \mathbf{AB} = \frac{20}{2} + \frac{10 \times 9}{2} + \frac{40}{2}$$
$$= 75 \text{ sq. chs.}$$
$$\text{Total} = 194\tfrac{1}{2} \text{ sq. chs.}$$

P. 282.—6.



$$\text{Area} = \frac{22 \times 8}{2} + \frac{36}{2} + 12 \times 8 + \frac{40}{2}$$
$$= 222 \text{ sq. chs.}$$

7.



$$\text{Area} = \frac{48}{2} + 10 \times 9 + \frac{50}{2} + \frac{21 \times 6}{2}$$
$$= 202 \text{ sq. chs.}$$

EXAMPLES. L. (a)

SECTION XIII. F. (1)

AREA OF CIRCLES

P. 282. — **1.** Area is $4\frac{1}{2}$ sq. in.

P. 283. — **2.** See Question 10, part (i). **3.** See Question 10, part (ii).

P. 283. — **4.** Area of outer circle . . $=7{\cdot}07$ sq. in.
,, inner ,, . . $=4{\cdot}91$,,
,, path . . . $=2{\cdot}16$,,

5. Area of outer circle . . $=5{\cdot}94$ sq. in.
,, inner ,, . . $=3{\cdot}14$,,
Sectional area of evacuated space $=2{\cdot}80$,,

6. Area of outer circle . . $=1{\cdot}76$ sq. in.
,, hole . . . $=0{\cdot}44$,,
Sectional area of iron . . $=1{\cdot}32$,,

7. $\frac{11}{14}$ sq. in.

8 and **9.** The areas should be the same.

P. 284. — **10.** (β) (*1*) $3\frac{1}{7}$ sq. in. (*2*) 19·6 sq. cm.
(*3*) 7·07 sq. in. (*4*) 12·57 sq. in.
(*a*) and (β) should differ by very little.

EXAMPLES. L. (b)

SECTION XIII. F. (1)

AREA OF CIRCLES—(*continued*)

P. 285. — **1.** 72·41 sq. in. **2.** 88·28 sq. cm. **3.** 98·56 sq. ft.
4. 10·18 sq. m. **5.** 109·4 sq. yds. **6.** 11·35 sq. mi.
7. 227·07 sq. m. **8.** 98·56 sq. mi. **9.** 0·786 sq. in.
10. 2·2176 sq. cm. **11.** 0·91646 sq cm. **12.** 0·00000314 sq. in.
13. 0·000314 sq. in. **14.** 0·0000786 sq. m. **15.** 0·033343 sq. ft.
16. 0·000201 sq. mi. **17.** 7 cm. **18.** 42 in.
19. 1·76 m. **20.** 0·8831 cm. **21.** 207 sq. ft.
22. 28 yds. **23.** 240 sq. yds. **24.** 4·87 in.

P. 286.—**25.** 3·42 in. **26.** 38·5 sq. ft. (nearly). **27.** 18·3 sq. in.
28. 1·98 sq. in. **29.** 172 sq. cm. (correct to 1 sq. cm.).
30. 3s. 1d. (correct to 1d.). **31.** 28·6 francs.
32. 241·26 kroner. **33.** 14 base boards. **34.** 1776 sq. cm.
35. 4d. (very nearly). **36.** 8·35 cm.

EXAMPLES. LI.

SECTION XIII. F. (2)

SURFACE AREA OF A CYLINDER AND OF A CONE

P. 288. —**1.** 6·53 in. × 3 in. **2.** 9·68 in. × 3·5 in. **3.** 18·6 cm. × 8 cm.
4. 11·56 in. × 6 in. **5.** 13·45 in. × 0·8 in. **6.** 18·6 cm. × 0·1 m.
7. Curved surface, £10 1s. 8d. ; total surface, £11 17s. (very nearly).
8. £5 13s. 2d. **9.** 754 sq. ft. **10.** 26·6 sq. ft.
11. (*1*) 1s. $2\frac{1}{4}$d. (*2*) 2s. $0\frac{3}{4}$d. (*3*) 2s. $8\frac{3}{4}$d.
(*4*) 4s. 5d. (*5*) 2s. $1\frac{1}{4}$d.

P. 289.—**12.** £9 16s. (to the nearest 1s.).

EXAMPLES. LII.

SECTION XIII.

MISCELLANEOUS QUESTIONS

P. 289.—**1.** £5 5s. **2.** £921 4s. 10d. **3.** £3214 2s. 8d.
4. £416 5s. and £97 2s. 6d. **5.** £1966 6s. 11d. **6.** £155 13s. 10d.
7. £4150 2s. 6d. **8.** £1408. **9.** £2354 (nearly).
10. 0·009d. **11.** (i) $335\frac{1}{4}$ ac. (nearly). (ii) £950. **12.** £197.

P. 290.—**13.** 416 sq. in. **14.** 347 tubes (reckoning **A** and **B** as one tube).
15. £17410 (correct to £1). **16.** 5s. 10d.
17. (*1*) 16 ft. 1 in. (*2*) 7 ft. 4 in. (correct to 1 in.).

P. 291.—**18.** 17·7 sq. ft., *i.e.* 18 sq. ft. (correct to 1 sq. ft.). **19.** 12s. 4d.
20. £22 4s. for frame; £12 13s. 2d. for panels, allowing $\frac{1}{8}$ sq. ft. per sq. ft. for joints; total cost, £34 17s. 2d.
21. 363 people per square mile. **22.** £39 (very nearly).
23. Cubical contents are: Upper, 48·72, and lower, 61·44 cub. ft.; cost, £24 15s. 9d.

P. 292.—**24.** 11s. **25.** 144896 sq. mi.
26. Area of Japan is 1·196 times greater than that of Great Britain and Ireland.
27. 240000 tons. **28.** £6,840,000. **29.** 0·0000309 sq. in.
30. 3200. **31.** About 10d. more in England.
32. The area of the circle should be 28·3 sq. in. (nearly).

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