GRAPHIC STATICS

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PREFACE

This text represents the development of notes during a period of fifteen years in a course offered to engineering students at Purdue University.

Though intended primarily for the undergraduate student in mechanical engineering, it offers certain features that should be of value also to students in other departments. For students in civil engineering, the chapter on Trusses and Bents should be of use as an introduction to the graphical analysis of structures. This chapter is designed to be of value also in instruction in aeronautical design. To students in machine design, the chapters on Cranes and Machines should be helpful. The text assumes a general knowledge of statics.

A special feature, distinguishing this text from other works on graphic statics, is the grouping in one volume of the three important topics—Trusses and Bents, Cranes, and Machines. Another feature is the liberal supply of illustrated examples which have been solved in detail and fully explained. Numerous problems have been furnished for solution by the student. Answers have been given for checking the solutions. Instructors preferring problems without answers may make changes in the data given.

The authors wish to express their thanks to R. G. Dukes, Dean of the Graduate School and Head of the Department of Applied Mechanics at Purdue University, for permission to use the available material of the department and to their colleague, A. P. Poorman, Professor of Applied Mechanics at Purdue University, for his thorough reading of the manuscript and suggestions as to form and content.

PURDUE UNIVERSITY,
January, 1932.

SEIBERT FAIRMAN.
CHESTER S. CUTSHALL.

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GRAPHIC STATICS

CHAPTER I

INTRODUCTION

1. Definitions.

Although a knowledge of statics is presupposed in this text, a brief presentation of some of its principles will be given in this and the succeeding chapter.

Force is an action between two bodies that causes, or tends to cause, a change in their state of rest or motion. A force has magnitude, direction, line of action and point of application, all of which must be considered in determining its effect upon a body.

A *rigid body* is a solid which may be considered as undergoing no change in form when subjected to the action of forces. Although no body is absolutely without change of form when under the action of forces, the amount of this change in many cases is so small that it can be neglected with no appreciable error.

Static equilibrium exists when a body under the action of forces is at rest or in uniform motion.

Statics is that branch of applied mechanics which treats of the effects of forces on bodies in equilibrium. Such bodies are either at rest or in uniform motion.

Graphic statics consists of the solution of problems of statics by means of accurately constructed geometrical figures. The unknown quantities required are obtained directly from the figures by scaling lines and angles. The methods used are fundamentally simple because the magnitude, direction and line of action of a force may be represented by the length, inclination and position of a straight line. On this account many problems which otherwise would require tedious computations become fairly simple when they are solved by the use of graphical principles.

2. Instruments.

For the proper solution of problems in graphic statics the following instruments are necessary:

1. Drawing board and thumb tacks.
2. Set of drawing instruments.
3. T-square.
4. 45° and 30°–60° triangles (large size).
5. Engineers scale (1-foot triangular).
6. Protractor.
7. Lettering pencil (3H or 4H).
8. Hard drawing pencil (5H or 6H).

3. The Drawing Plate.

The drawing plate suitable for most of the problems in this text is 18 inches by 24 inches. The following arrangement of border, title, coordinates, scales, etc., is suggested: The plate will be laid out as shown in Fig. 1. There will be a $\frac{1}{2}$-inch border all around the plate. The lettering may be vertical or inclined. The problem number and title shall consist of $\frac{1}{4}$-inch upper case letters. All other information on the plate shall consist of $\frac{5}{32}$-inch (or 0.15 inch) upper case letters and $\frac{3}{32}$-inch (or 0.10 inch) lower case letters.

4. Classification of Forces.

Forces may be classified as distributed forces and concentrated forces. A distributed force is one whose place of application is an area. A concentrated force is one whose place of application is so small that it can be considered as a point.

When two or more forces form a force system the following classification may be used:

- Force Systems
  - Coplanar
    - Concurrent
      - Colinear
      - Noncolinear
    - Nonconcurrent
      - Parallel
      - Nonparallel
  - Noncoplanar
    - Concurrent
    - Nonconcurrent
      - Parallel
      - Nonparallel

Coplanar forces lie in one plane; noncoplanar forces lie in different planes; concurrent forces intersect in a

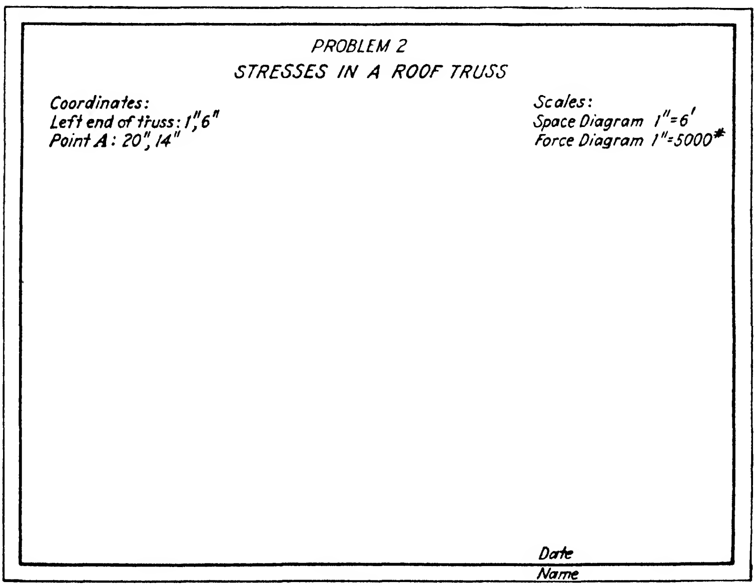


FIG. 1.

point; nonconcurrent forces do not intersect in a point; forces are parallel if their lines of action are parallel, and nonparallel if their lines of action are not parallel; forces are colinear if they have a common line of action.

5. Vector Quantities.

Quantities which possess both magnitude and direction are vector quantities and may be represented by vectors. Vectors are lines which have definite lengths, inclinations and directions. The length of the vector to some convenient scale represents the magnitude of the quantity, the inclination shows its position and an arrowhead shows the direction.

6. Graphical Representation of a Force.

Since a force has magnitude and direction it may be represented by a vector. Figure 2 shows how two forces P and Q with their resultant R may be represented.

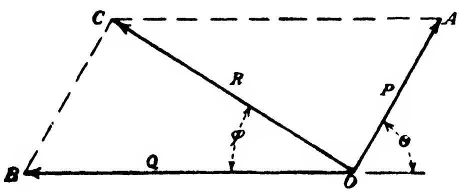


Fig. 2.

To some convenient scale OA represents the magnitude of force P, angle θ shows its inclination with the horizontal and the arrowhead shows its direction. In a similar manner, OB represents the magnitude of the horizontal force Q and its direction is to the left. The line OC shows the resultant R in amount, inclination and direction.

7. Transmissibility of Forces.

By the principle of transmissibility of forces, the external effect of a force upon a rigid body is the same for all points of application along its line of action. This is a principle of experience and applies to the external effects only. The principle of transmissibility of forces is used constantly in graphical solutions.

8. The Free Body.

In analyzing problems the free-body method will be used. By this method the whole structure or some part of it is considered to be separated from its surrounding parts. This free body, drawn accurately to some convenient scale, constitutes the space diagram. Upon this free body are drawn vectors representing all of the external loads, and the actions of the removed parts. Figure 3 shows a free body taken from a cantilever roof

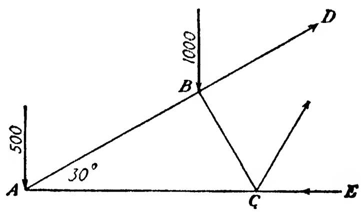


FIG. 3.

truss. The loads of 500 pounds and 1000 pounds are the known loads and are shown in their positions by vectors. The unknown stresses in the members *BD*, *CD* and *CE* are also shown by their vectors. Upon this free body the stresses in members *BD*, *CD* and *CE* are the only external unknown quantities, and solution may be made by applying the principles of statics. The stresses in members *AB*, *AC* and *BC* are internal forces and require other free bodies for their solution.

9. Two-force and Three-force Members.

The two-force member is a rigid body subjected to the action of forces at only *two points*. For equilibrium the resultant of the forces at one point must be colinear with the resultant of those at the other point. Therefore, such a member carries an axial stress or force of direct tension or direct compression. In solving for the stress in a two-force member, the member is considered to be cut and the joint at one end or the other is taken as the free body. Vectors representing the actions of the removed parts are drawn axial to the member. In Fig. 4, *AC* is a two-force member, and in Fig. 3, all members are two-force members.

Three-force members are subjected to the action of forces at *three or more points.* A member of this type is subjected to bending and shearing forces, so the stresses in such a member are in general not axial. In Fig. 4,

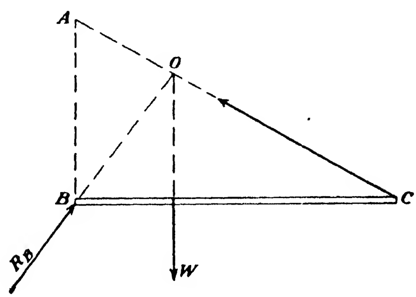


FIG. 4.

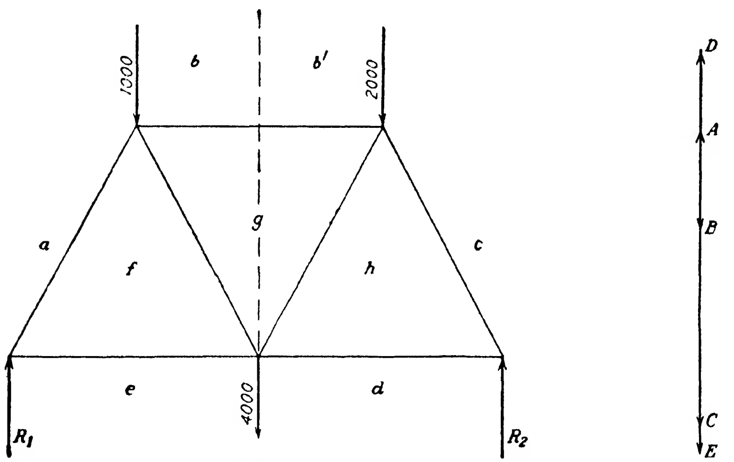
BC is a three-force member. Three-force members cannot be cut as are two-force members, because the stress is not axial. The force at *B* becomes a pin reaction and acts on the member at the hinged or fixed point. In Fig. 4, this reaction is R_B and its line of action is *OB*.

10. Bow's Notation.

Bow's notation will be used in all problems in Trusses and Bents (Chap. III). This notation furnishes a handy means of designating forces. The spaces between forces are lettered with lower case letters. Any force is designated by the letters of the spaces it separates. The clockwise direction around the free body will be used in determining the order of letters. Figure 5(*a*) shows a truss so lettered. Reaction R_1 is known as force *ea*, the 1000-pound load as *ab*, the 2000-pound load as *bc* (or *b′c*), the reaction R_2 as *cd* and the 4000-pound load as *bb′* when considering the entire truss as the free body and as *de* when considering the middle joint as the free body. Upon the pin at the left end of the truss as the free body, the forces are known as *ea*, *af* and *fe*.

11. Space Diagrams and Force Diagrams.

Figure 5(*a*) represents what is known as a space diagram. It is an accurate drawing of the free body



(a) Space Diagram (b) Force Diagram

FIG. 5.

to be used in the solution of a problem. On this diagram are shown all of the external forces that act upon the free body in their proper positions.

Figure 5(*b*) shows the force diagram for the same free body. In applying Bow's notation to the force diagram, upper case letters are used: thus vector *AB* shows the amount and direction of the load *ab*; *BC* that of *bc* and so on.

CHAPTER II

FUNDAMENTAL PRINCIPLES

12. The Parallelogram of Forces.

The parallelogram law: If two concurrent forces are represented by their vectors, both of which are directed either toward or away from their point of intersection, the diagonal of the completed parallelogram drawn through their point of intersection completely represents their resultant.

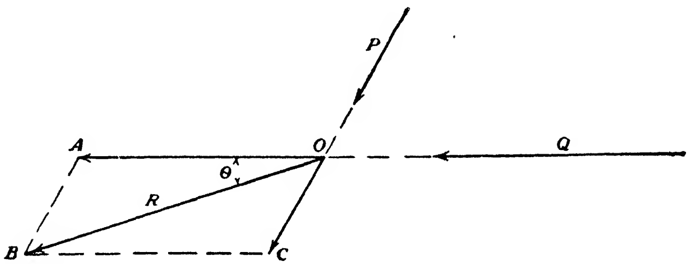


FIG. 6.

In Fig. 6, vectors P and Q represent two forces which intersect at O. By the principle of transmissibility of forces, vectors P and Q may be transmitted to the positions of OC and OA respectively. The diagonal OB of the parallelogram $OABC$ gives the resultant R of the two forces P and Q in amount, direction and position.

The single force to hold these two forces in equilibrium will be a force equal to R and colinear with it, but opposite in direction. Therefore, *if three nonparallel forces are in equilibrium they must meet in a common point.* This is a very important principle in the solution of problems by graphics.

13. Resolution of a Force into Two Components.

Any force, such as R in Fig. 6, may be resolved into two components by constructing a parallelogram upon the force vector as a diagonal. The two sides, such as P and Q, that intersect the line of action of force R will be the required components. The construction of the paralellogram can be made anywhere along the line of action of force R.

14. The Force Triangle.

If force Q of Fig. 6 is placed along the line BC, as shown in Fig. 7 (a), P and Q form two sides of the triangle OBC. The third side of the triangle is the resultant R.

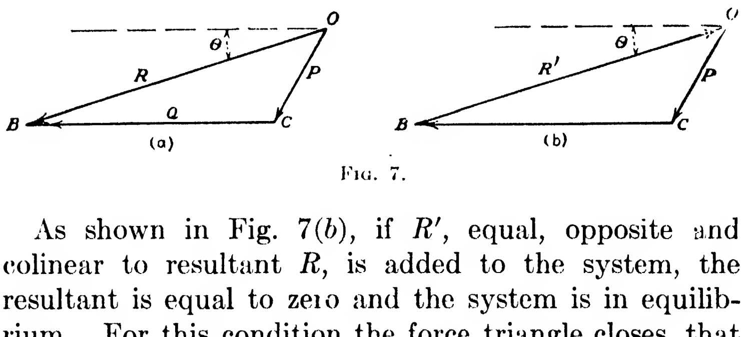


FIG. 7.

As shown in Fig. 7(b), if R', equal, opposite and colinear to resultant R, is added to the system, the resultant is equal to zero and the system is in equilibrium. For this condition the force triangle closes, that is, the vectors follow each other in regular order. *If three forces are in equilibrium their resultant is equal to zero and the force triangle or any projection of it must close.*

15. The Force Polygon.

The principle explained in Art. 14 can be extended to include three or more forces. In Fig. 8(a) four force vectors, M, N, P and Q, form four sides of an incomplete polygon. The closing vector OD as shown gives the resultant R. This is obvious from the figure which is seen to be made up of a series of force triangles, OAB, OBC and OCD.

If force R', equal, opposite and colinear to resultant R, is added to the system as shown in Fig. 8(b), the resultant is equal to zero and the system is in equilibrium. *If*

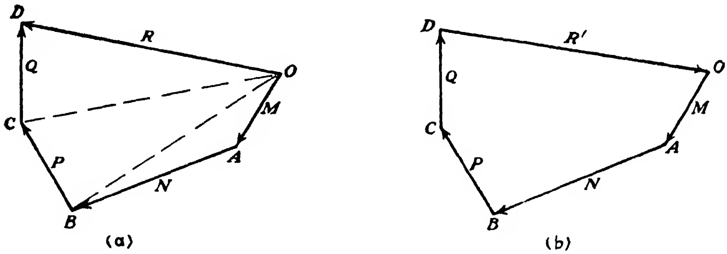


Fig. 8.

a system of three or more forces is in equilibrium their resultant is equal to zero and the force polygon or any projection of it must close.

16. Parallel Forces: Composition by Resolution.

The resultant of two parallel forces may be obtained by first resolving one of the forces into two com-

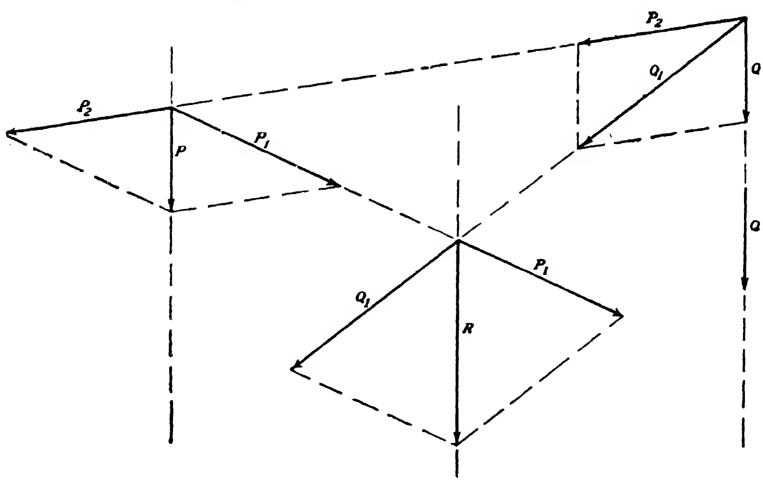


Fig. 9.

ponents and then recombining the three forces into one resultant.

In Fig. 9, P and Q, two parallel forces, are to be combined into their resultant R. Force P is resolved into components P_1 and P_2. Component P_2 is combined by the parallelogram law with force Q to obtain resultant Q_1. Force Q_1 is then combined by the same law with component P_1 to obtain the final resultant R. This resultant is completely obtained in amount, direction and position.

The resultant of two parallel forces which act in the same direction is equal to the sum of the two forces and lies between them, nearer the larger force.

The resultant of two parallel forces which act in opposite directions is equal to the difference of the two forces and lies outside of the two forces, on the side of the larger force.

17. Parallel Forces: Composition by Inverse Proportion.

Let it be required to find the resultant of the two parallel forces P and Q, shown in Fig. 10(a), by inverse proportion.

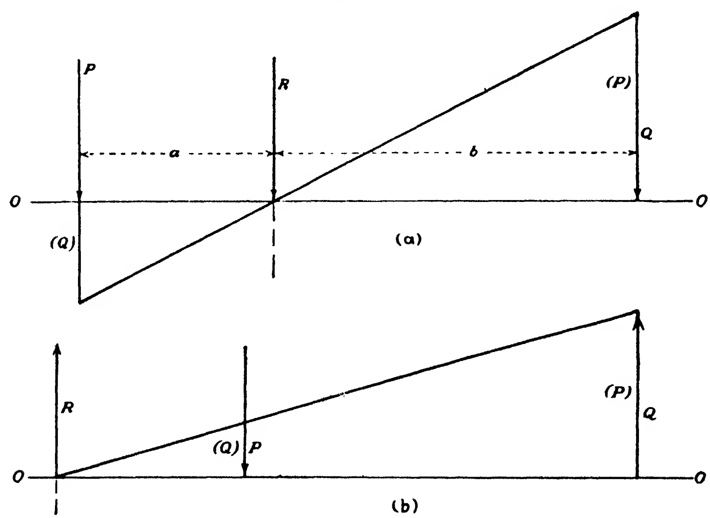


FIG. 10

Any convenient base line OO is drawn. This base line is not necessarily perpendicular to the forces.

Along the line of action of force Q, force P is laid off from the base line OO; and on the line of action of force P, force Q is laid off in the opposite direction from the same base line OO. The line connecting the ends of these two scaled vectors will intersect OO at the point where the resultant R acts. $R = P + Q$.

Proof: The equation of moments with respect to any point on the line of action of R gives $Pa = Qb$ or $P/Q = b/a$. The construction of Fig. 10(a) conforms to this equation; that is, the line of the resultant of the two parallel forces divides any line drawn between them into two segments inversely proportional to the two forces.

Figure 10(b) shows the application of this principle when the two forces are acting in opposite directions. As before, any convenient base line OO is drawn. Since the forces are acting in opposite directions they are both laid off either above or below OO, each one on the line of action of the other. The line connecting the ends intersects the base line OO at the point where the resultant R acts. $R = P - Q$. The proof for this construction is similar to the preceding one and is left as an exercise for the student.

18. Parallel Forces: Composition by the Funicular Polygon.

The resultant of the two forces ab and bc in Fig. 11(a) is required.

The force diagram consisting of vectors AB and BC is laid off as shown in Fig. 11(b). AB is the scaled value of force ab and BC is the scaled value of force bc. AC is therefore the scaled value of the resultant ac. Any convenient pole point O is selected and the rays AO, BO and CO are drawn. In the space diagram, Fig. 11(a), string ao is drawn parallel to ray AO through any convenient point on the line of action of force ab. Through this point on ab, the string ob is drawn parallel to ray

OB until it intersects the line of action of force bc, and from this intersection string oc is drawn parallel to ray OC. The resultant R acts through the point of intersection of strings ao and oc.

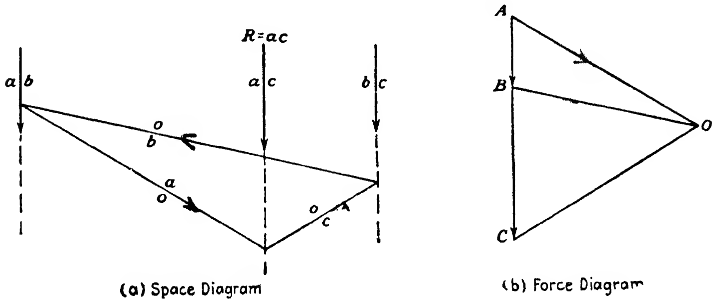


FIG. 11.

Proof: In Fig. 11(b), AB can be resolved into two components, AO and OB in amount. In the space diagram, Fig. 11(a), these components are acting in the positions ao and bo. The resultant of forces ob and bc lies on line oc and is parallel to OC. The resultant R of ao and oc, and therefore of the original two forces, is AC, acting at the point where ao and oc intersect, as explained in Art. 16.

19. Resolution of a Force into Two Parallel Components.

Inverse Proportion.—Let it be required to resolve force R of Fig. 12 into two parallel components, one along

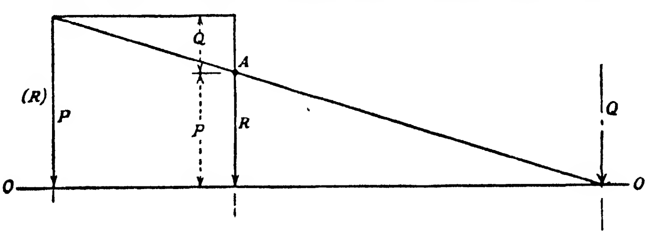


FIG. 12.

P and the other along Q. Any convenient base line OO is drawn. Vector R is transferred to the line of action of either required component, say P, and is laid down from the base line OO as R. The line drawn from the end of vector R to the point where the line of action of component Q intersects the base line OO, intersects vector R at point A and divides it into the required components P and Q.

If a line is drawn through point A parallel to OO, the figure becomes the same as Fig. 10(a) and the same proportionality of the sides of the triangles is seen to be true.

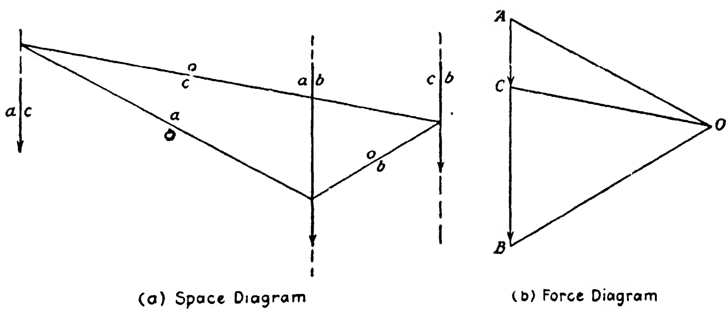


Fig. 13.

Funicular Polygon.—Force ab of Fig. 13(a) is to be resolved into two parallel components, one along the line ac and the other along the line cb.

In the force diagram, vector AB is laid off, as shown in Fig. 13(b). Vector AB is the scaled value of force ab. Any pole point O is selected and the rays AO and BO drawn. In the space diagram shown in Fig. 13(a), string ao is drawn parallel to ray AO through any convenient point on the line of action of force ab. Through this point string ob is drawn parallel to ray OB. Through the points where these strings, ao and ob, intersect their respective component lines ac and cb, the string oc is drawn. Ray OC in the force diagram is drawn parallel to the string oc in the space diagram and divides AB into the two required components AC and CB. The proof for this construction is identical with that of Art. 18.

The force system shown in Figs. 12 and 13 will be in equilibrium if forces equal and opposite to P and Q of Fig. 12 or ac and cb of Fig. 13 are added to the original systems.

20. Concurrent Forces: Composition by Resolution.

If two concurrent forces do not meet within the limits of the drawing, they can be combined into their resultant in the same manner as explained in Art. 16.

In Fig. 14, the two forces P and Q which are concurrent, but not within the limits of the drawing, are to be combined into their resultant R.

Force P is resolved into two components P_1 and P_2.

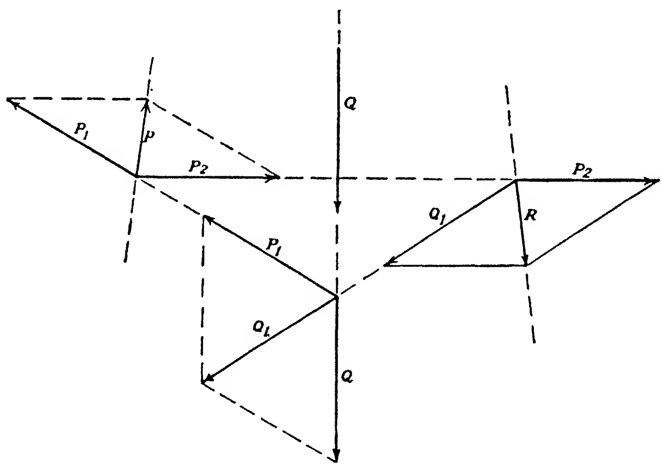


FIG. 14.

Component P_1 is combined by the parallelogram law with force Q to obtain resultant Q_1. Q_1 is then combined in the same manner with component P_2 to obtain the final resultant R in amount, direction and line of action. In solutions of this type care must be used in selecting workable components of P or Q.

21. Nonconcurrent Forces: Composition by the Funicular Polygon.

The resultant of two or more nonconcurrent forces is obtained readily by the funicular polygon.

The three forces *ab*, *bc* and *cd*, shown in Fig. 15(*a*), are to be combined into their resultant $R = ad$.

In the force diagram, vectors AB, BC and CD are laid down as shown in Fig. 15(*b*). Any pole point O is selected and the rays AO, BO, CO and DO are drawn.

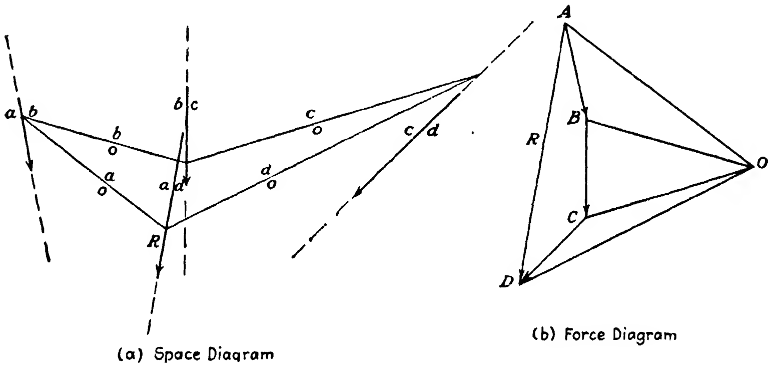


FIG. 15.

On the space diagram, Fig. 15(*a*), string *ao* is drawn parallel to ray AO intersecting force *ab* at any convenient point on its line of action. Through this point, string *bo* is drawn parallel to ray BO. At the point of intersection of string *bo* with the line of action of force *bc*, string *co* is drawn parallel to ray CO and, as before, at the point of intersection of string *co* with the line of action of force *cd*, string *do* is drawn. Strings *ac* and *do* intersect at a point on the line of action of the resultant force R. The magnitude of the resultant R is obtained by scaling vector AD of the force diagram. This solution is similar to the one explained in Art. 18.

22. Resolution of a Force into Two Components, One Fixed in Direction.

Inverse Proportion.—Force R of Fig. 16 is to be resolved into two components: P at point N acting in a given direction, and Q at point M.

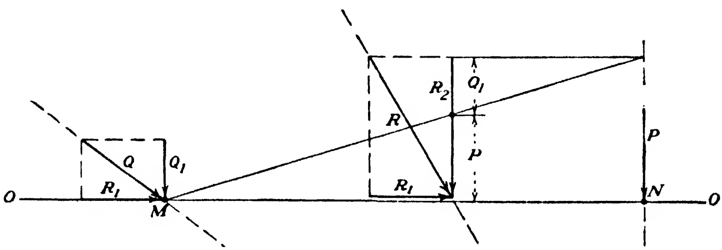


Fig. 16.

Force R is resolved into two components: R_2 parallel to the line of action of the required component at point N; and R_1 perpendicular to R_2. By the method of inverse proportion, as explained in Art. 19, component R_2 is resolved into two forces P and Q_1. Component P acts at point N and is one of the required components. Component R_1 acts at point M and is recombined with force Q_1 to obtain the other required component Q.

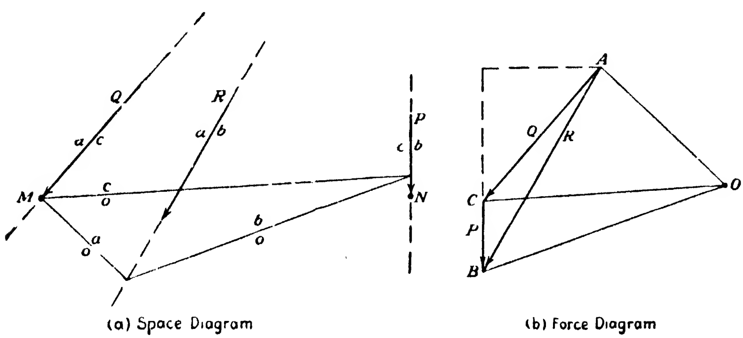


Fig. 17

Funicular Polygon.—In Fig. 17, force R is to be resolved into two components: P acting vertically through point N; and Q acting through point M.

In the force diagram, vector AB is laid down as shown in Fig. 17(b). From any pole point O the rays AO and BO are drawn. In the space diagram, string ao is drawn parallel to ray AO through point M. String bo is drawn parallel to ray BO at the point of intersection of string ao and the line of action of force R. String co is drawn from point M to the intersection of string bo and the line of action of force P. Ray OC in the force diagram, parallel to string co, intersects the vertical line through B at point C. Vectors CB and AC are then the required components P and Q.

In a force system of this type equilibrium is established if forces equal and opposite to P and Q are added to the original force.

23. Resolution of a Force in Space into Three Components.

It is sometimes required to resolve a force into three mutually rectangular components.

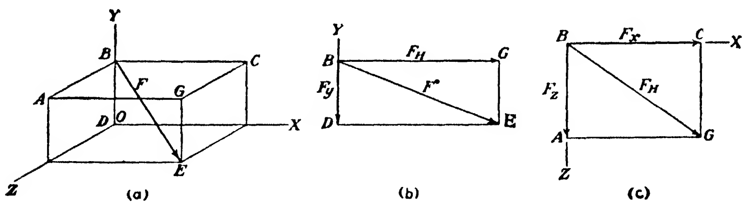


Fig. 18.

In Fig. 18(a), force F is a force in space and it is required to resolve it into three components F_x, F_y and F_z. Plane $BDEG$ is passed through force F and the Y-axis, as shown in Fig. 18(b). The length of BG is obtained from the diagonal of the parallelogram $BCGA$, Fig. 18(c). In Fig. 18(b), force F is resolved into two components F_y and F_H. In Fig. 18(c), vector F_H is resolved into components F_x and F_z. The required components F_x; F_y and F_z are thus obtained.

24. The Four-force System.

In the solution of many problems in graphic statics there occurs a system of four forces in equilibrium all of whose directions are known and one of which is known in both amount and direction.

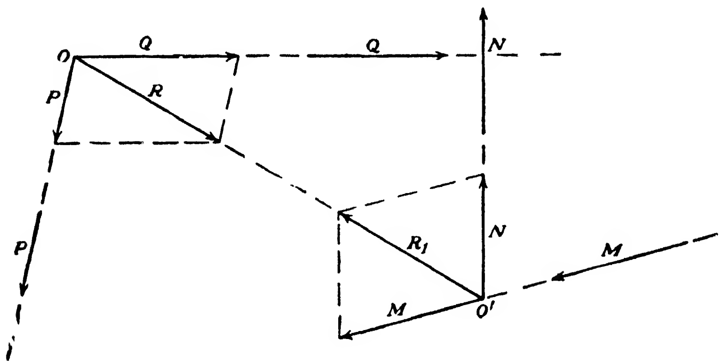


Fig. 19.

In Fig. 19, the four forces P, Q, M and N are known to be in equilibrium. Force Q is known in amount and direction, but the others are known in direction only. The amounts of the unknown forces are required. The forces are assumed to be combined by pairs. Forces Q and P intersect at O, and forces N and M intersect at O'. Since these forces are in equilibrium the resultant of one pair must be equal, opposite and colinear with the resultant of the other pair. This resultant therefore must lie on the closing line OO'. At point O, the resultant R must be the diagonal of the parallelogram whose sides are P and Q, from which one of the required forces, P, is obtained. Resultant R_1 of M and N must be equal and opposite to resultant R of P and Q. At point O', vector R_1 is resolved into components in the known directions from which the two remaining forces M and N are obtained.

CHAPTER III

TRUSSES AND BENTS

25. Trusses and Their Construction.

Jointed frames such as those illustrated in Fig. 20 are known as trusses. The trusses considered in this chapter consist of straight members all of which may be assumed to lie in the same plane. The members of such structures may be riveted together or joined by pins through their ends. In the following work, all trusses will be treated as if pin connected with any single member extending only from one joint to the next, but not to a third. It will also be assumed that the loads are applied to the truss at the joints only. The weight of a member may be assumed to be negligible in comparison

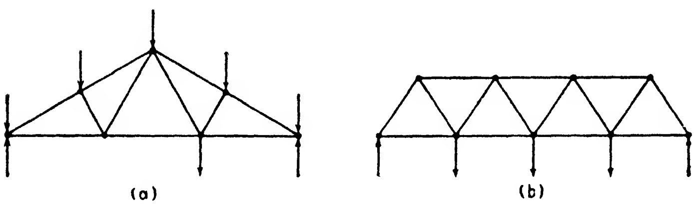


FIG. 20.

with the other forces acting upon it or, if not negligible, to be equally divided between the two ends.

Under these conditions, forces will act at only two points, the ends of the member. Each member thus conforms to the definition of a two-force member as given in Art. 9 and the stresses are therefore axial; that is, simple tension or compression. In the analysis of stresses in such a truss, it is permissible to take a section through any of the members since the stresses, thus made external to the free body, are known to be axial.

Many trusses are riveted at the joints and have some members extending through several joints. In the strict sense, certain members of a truss constructed in this way would be three-force members as defined in Art. 9 and the stresses would not be axial alone, but would involve shearing and bending. However, it is customary to consider such a truss as consisting of two-force members unless the ability to take bending is necessary for stability.

Graphical methods are especially well adapted to the determination of reactions and stresses in trusses and similar frames. Bow's notation, as explained in Art. 10, will be used in the following graphical analyses of trusses.

26. Resultant of Loads.

The loads on a truss are often combined into their resultant before the external reactions are found. The various principles and methods required for combining forces were explained in Chapter II.

The roof truss in Fig. 21 is acted upon by vertical dead loads and oblique wind loads as shown. The resultant of these loads is to be determined.

Since the vertical roof loads constitute a symmetrical system of parallel forces, their resultant R_1 will, by inspection, be 16,000 pounds acting vertically downward through the middle of the truss. By the inverse-proportion method of Art. 17, the 6000-pound load and R_1 are combined into their resultant R_2 = 22,000 pounds.

By inspection, the resultant R_3 of the diagonal wind loads is 10,000 pounds, acting perpendicular to the upper chord at its middle point. By the parallelogram method of Art. 12, the forces R_2 and R_3 are combined into their resultant R_4 = 31,500 pounds. Since all loads have now been taken into account, R_4 must be the resultant of all of the loads.

27. Reactions.

If the truss shown in Fig. 21 rests upon supports at each end, each of these supports must exert a vertical reaction, and one or both must resist the horizontal thrust due to the wind load. If both ends are hinged,

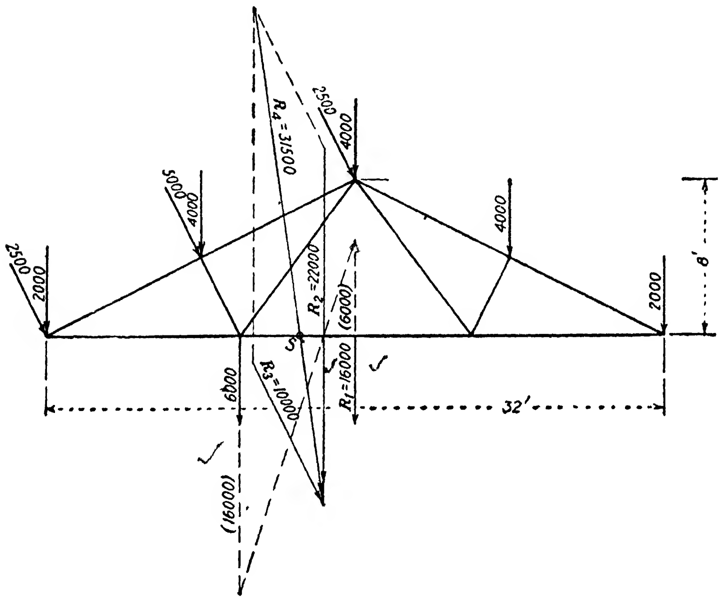


Fig. 21.

the division of the horizontal thrust between the two supports is indeterminate. It is sometimes assumed in such a case that the two ends share this force equally, but it is safer in design to assume that either end support can take all of the force. In order to allow for expansion and contraction of the truss, one end is often supported on rollers and a bed plate, and the other is hinged. In this case, the reaction at the rollers must be vertical and the hinged end must take all of the horizontal thrust.

If the resultant of all loads has been found, the process of determining the reactions consists, briefly, of resolving the resultant load into vertical (V) and horizontal (H) components. The V component in turn may be resolved by the inverse-proportion method of Art. 19 into vertical

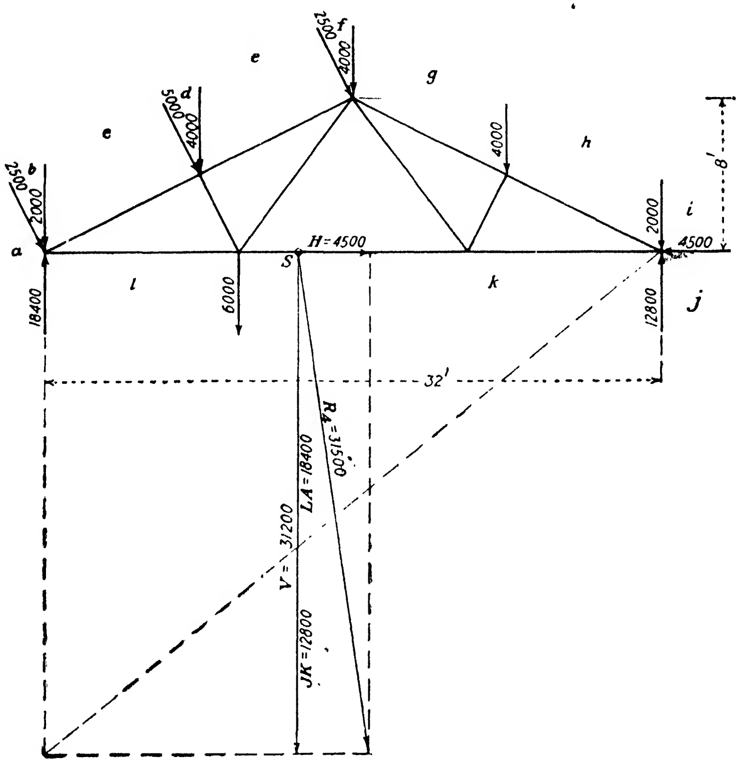


Fig. 22.

components at the supports, and the H component may be divided between the supports or all transferred to one support as explained in the preceding paragraph. The reactions of the supports are equal and opposite to these components of the load.

The truss of Art. 26 will be used to illustrate the procedure and is reproduced in Fig. 22. In order to simplify the figure, all of the construction for obtaining the resultant R_4 has been omitted. The truss is supported by rollers at the left end and a hinge at the right end. The spaces have been lettered in accordance with Bow's notation.

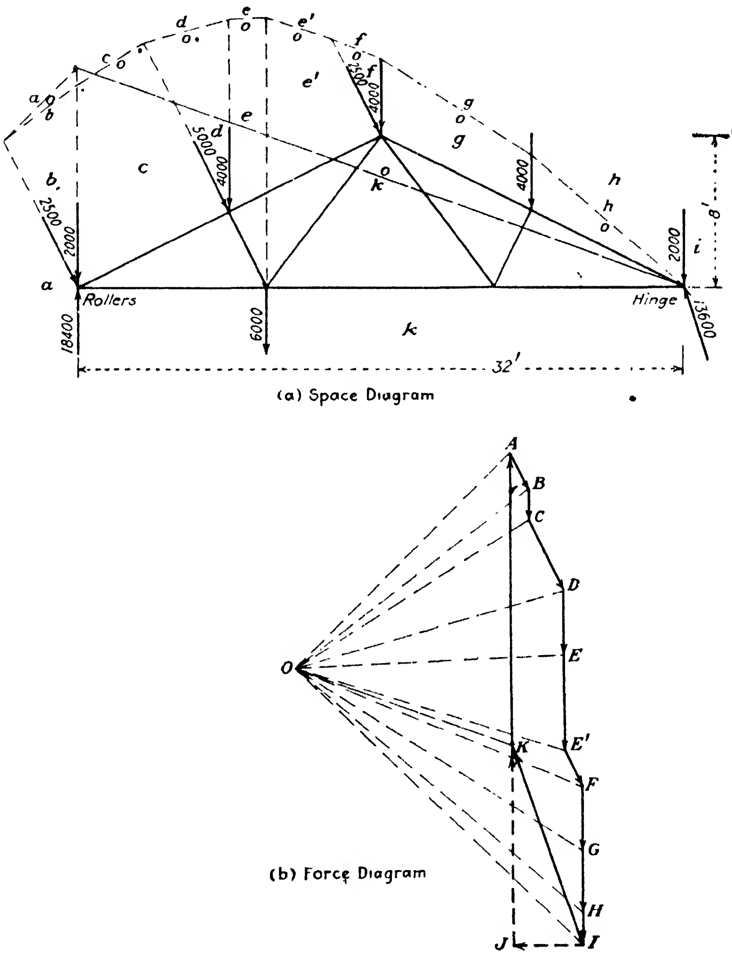
Since the resultant of the reactions must be colinear with R_4, it is necessary that the horizontal reaction IJ intersect the resultant of the vertical reactions LA and JK on the line of action of resultant R_4. This intersection is point S. Therefore, at point S, the resultant R_4 is resolved into V and H components by the parallelogram method. These vectors scale 31,200 pounds and 4500 pounds respectively. By the inverse-proportion method of Art. 19, the value of V is laid off on the line of action of LA and the diagonal is drawn to the opposite corner of the truss thus dividing V into its components acting at the ends of the truss. The reactions JK and LA are equal and opposite to these components which scale 12,800 pounds and 18,400 pounds respectively. The entire amount of force H must be supported at the hinged end and thus determines the reaction IJ = 4500 pounds.

If desired, the vertical and horizontal reactions at the hinged end may be combined by the parallelogram method into the resultant reaction at the hinge. (This is not shown in the diagram.)

Instead of combining all of the loads into their resultant, and from this determining the reactions as was just explained, it is also possible to determine separately the *partial* reactions due to each particular load or combination of loads, and then combine these partial reactions algebraically to obtain the total reactions.

The reactions may also be obtained by the funicular or equilibrium polygon which was discussed in Arts. 18, 19 and 21. Figure 23(a) shows the same truss used in the preceding paragraphs, but with a slight change of lettering. The line of action of the 6000-

pound load has been extended upwards to divide the original space e into two parts, e and e'. The entire space below the truss will now be known as k. The



(a) Space Diagram

(b) Force Diagram

Fig. 23.

reaction at the left end, unknown in amount but known to be vertical, will be called KA. The reaction at the right end, unknown both in amount and direction, will be called IK.

In the force diagram, Fig. 23(*b*), all of the known forces are laid off to scale as indicated, *AB*, *BC*, . . . *HI*. It is known that reactions *IK* and *KA* must close the force polygon, but the location of point *K* on the line *KA* is unknown. Any convenient pole point *O* is chosen and the rays *OA*, *OB*, etc., are drawn. In the space diagram, the funicular polygon is drawn, beginning at the hinge at the right end since it is the only point known on the reaction *IK*. The string *oi* is zero length since it is drawn between *hi* and *ik*. The strings *oh*, *og*, *of*, *oe′*, *oe*, *od*, *oc*, *ob* and *oa* are drawn, in the order named, parallel to *OH*, *OG*, etc., in the ray diagram. Since the system of forces is in equilibrium, the funicular polygon must close. String *ok* is accordingly drawn as the closing side. The ray *OK* in the force diagram parallel to string *ok* in the funicular polygon, locates point *K* and thus determines the amount of the reaction *KA* and the amount and direction of reaction *IK*. *KA* scales 18,400 pounds. *IK* may be resolved, as indicated, into *H* and *V* components which scale 4500 pounds and 12,800 pounds respectively. By referring to Fig. 22, it will be seen that these results check the values given for *LA*, *IJ* and *JK* respectively, as determined by the previous methods.

The funicular-polygon method possesses certain advantages, only a few of which will be discussed here. It will be noted that, when this method is used, it is unnecessary to determine the resultant of the loads. Also, in the case of trusses having a large number of unsymmetrical parallel loads, this method saves much work in combining forces. It is especially useful, also, in the case of loads which are nonparallel but so nearly parallel that they do not meet within the limits of the drawing.

28. Internal Stresses: Method of Joints.

If all of the members meeting at a given joint in a truss are two-force members, the joint may be taken as a free body acted upon by a system of concurrent forces. If all but two of the forces are known, the two unknown forces may be determined. The method based on the polygon law, Art. 15, is usually employed. Successive joints may be used, in turn, as free bodies until all of the stresses have been determined. If more than two of the forces intersecting at a given joint are unknown, the joint cannot be solved. Some other free body should then be selected, as will be discussed later.

The truss used in the preceding articles will again be taken as an example. Figure 24 shows this truss with

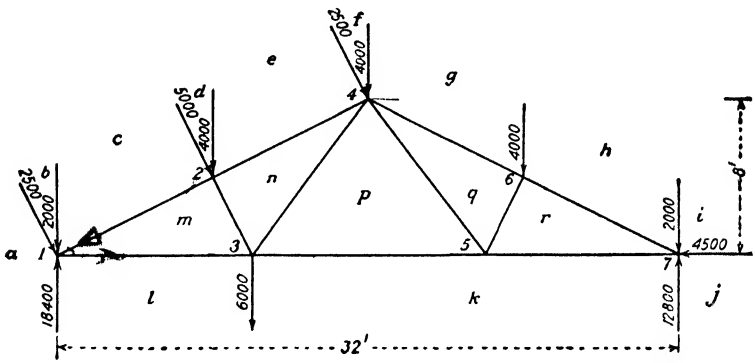


FIG. 24.

its loads and reactions. The steps employed in the determination of the reactions have previously been explained and are therefore omitted in the figure. The various joints are numbered and, in the discussion which follows, will be referred to by their respective numbers.

It will be noted that the solution for the internal stresses may be begun at either end of the truss, since either of these joints has but two unknown forces acting upon it. All other joints have at least three unknown forces. Joint 1 which will be used here as the first free body is acted upon by five forces, of which three are known and two are unknown. The polygon of these forces, Fig. 25, may be laid out in accordance with instructions given in connection with Bow's notation; that is, the forces are taken in clockwise order around the joint, starting with the first known force which is *LA*. Then *AB* is added, followed by *BC*. The next force is *CM* which is known in direction, so a line is drawn through point *C* parallel to the direction of *cm* in the space diagram, but indefinite in length. Similarly, *ML* is known in direction and must close the polygon at point *L*. Therefore a line is drawn through point *L* parallel to the direction of *ml* and indefinite in length. The intersection of these lines locates point *M* and determines the values of vectors *CM* and *ML* which scale 31,500 pounds and 27,000 pounds respectively.

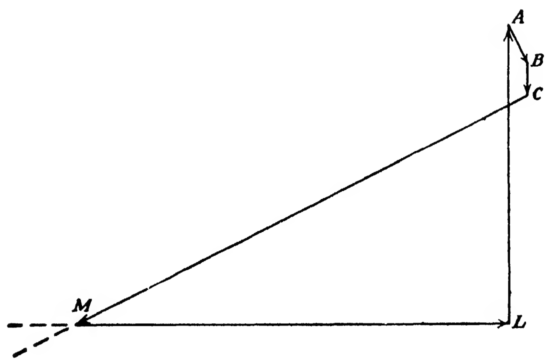


Fig. 25.

It should be noted that forces *LA*, *AB* and *BC* are known in direction, arrowheads being used in the force polygon to indicate these directions. By following the direction of these arrows around the remainder of the polygon it is seen that force *CM*, acting down to the left, must *push* on the joint, and the stress is therefore compression. Force *ML*, acting horizontally to the right, must *pull* on the joint, and therefore the stress is tension. The directions of these vectors must be reversed when the adjacent joints are considered as the free bodies.

The direction in which a member acts upon a joint may also be determined very quickly and easily in the following manner: The letters by which a stress is designated will be taken in clockwise order around the joint. The direction obtained by taking these letters in the same order in the force polygon will give the direction of the stress action on the joint. For example, in the joint just considered, the horizontal member would be designated as *ML* when taking the spaces in clockwise order. By taking the letters in the same order in the force polygon, Fig. 25, the direction from *M* to *L* is

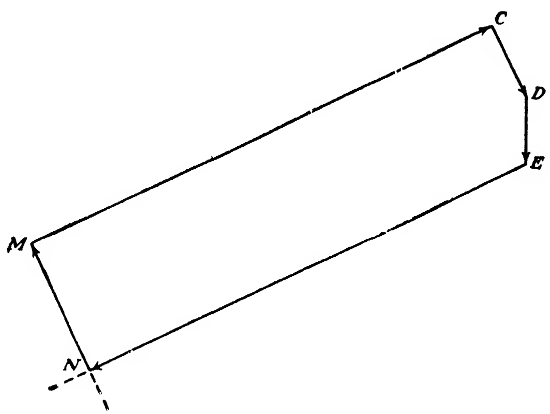


Fig. 26.

toward the right. Thus force *ML* acts to the right on this joint and must be tension. This is another important feature of Bow's notation.

The next joint having only two unknown forces is joint 2. The polygon of forces for this joint is constructed in the same way as the preceding one and is shown in Fig. 26. The known vectors *MC*, *CD* and *DE* are laid out in order first, followed by the unknown vectors EN and NM. These are found to scale 29,500 pounds and 8700 pounds respectively. Stresses EN and NM are both compressive because the arrowheads in the force polygon show each to be pushing toward the joint.

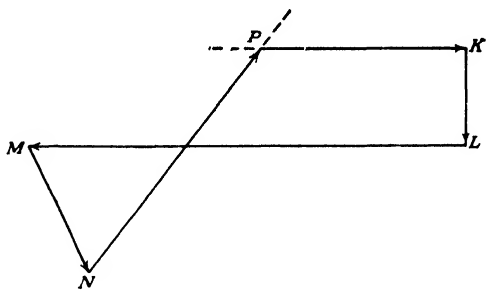


Fig. 27.

In a similar manner, the polygon for joint 3 can be constructed as shown in Fig. 27. The values as scaled from this polygon are NP = 17,100 pounds tension and PK = 12,800 pounds tension.

The polygons for the remaining joints in order are shown in Fig. 28. From them the values are scaled as follows:

GQ = 21,900 pounds compression
QP = 4000 pounds tension
QR = 3600 pounds compression
RK = 16,600 pounds tension
HR = 23,600 pounds compression.

It is not necessary to construct the polygon for the last joint since the stresses acting there have already been found from preceding joints.

Since the several separate polygons which have thus been constructed contain sides which are common to one or more of the other polygons, it is convenient to link the polygons together during the course of their construction. The force diagram thus obtained is shown in Fig. 29.

By thus linking the separate polygons together, duplication of lines is avoided and a considerable saving of time, space and labor is effected.

The part of Fig. 29 consisting of lines representing the loads and reactions is reproduced in Fig. 30 and is

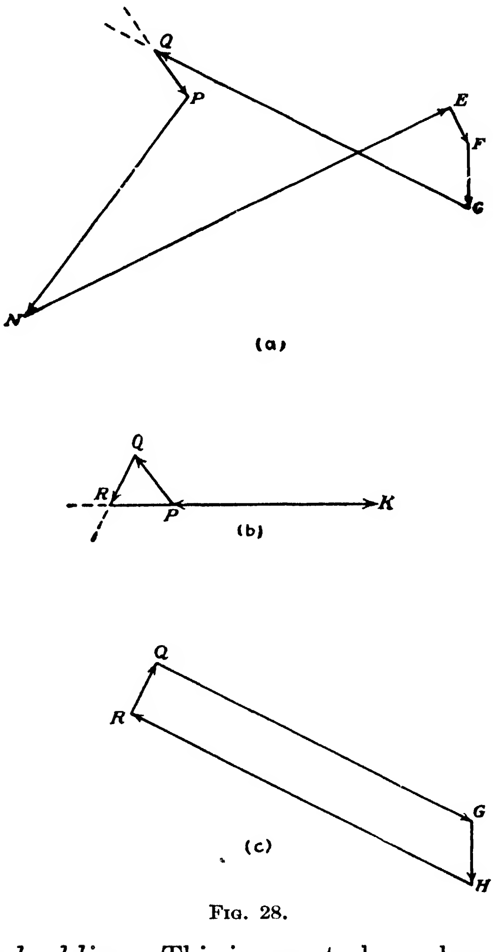


FIG. 28.

called the *load line.* This is seen to be a closed polygon and actually is the polygon of forces for the entire truss considered as a free body. The directions of all of the forces composing the load line are fixed and should be indicated by arrowheads. The arrowheads have been omitted from the other polygons in the force diagram of

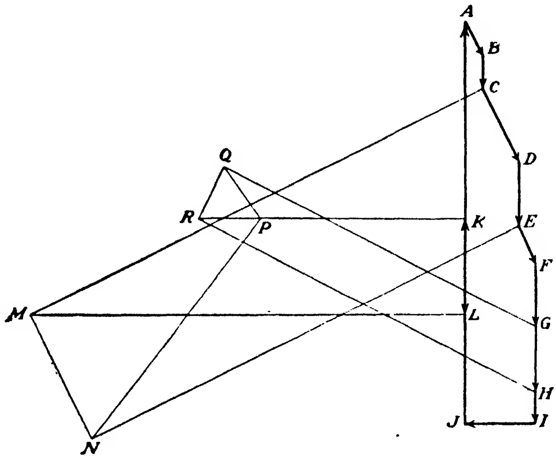


Fig. 29.

Fig. 29 because it has already been shown that the vectors representing the internal stresses must be reversed in going from one joint to the next.

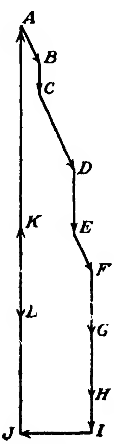


Fig. 30.

It will be found convenient in subsequent problems to lay out the load line completely, before proceeding with the determination of internal stresses. Not only will this serve as a useful frame upon which to build the remaining polygons, but failure of the load line to close, when laid out properly, indicates an error in determining the reactions. It should be noted, however, that closing of the load line is not positive proof that the reactions are correct. A failure of the final force polygon to close properly indicates an error made in some part of the solution.

The graphical solution of any truss may now be said to consist of two or three figures, depending on the method by which the reactions are obtained.

If the first method given in Art. 27 for determining reactions is used, two figures will suffice. The first, called the *space diagram*, will consist of the truss, drawn to some convenient scale, properly dimensioned and lettered, and showing all of the loads. The constructions by which the reactions are found should be made on the space diagram, using some convenient scale for laying out the forces. The second figure, called the *force diagram*, will consist of the load line and linked polygons, drawn to some convenient scale (usually the same as that employed for forces in the space diagram). The scales chosen should be such that both figures may be placed on the same drawing sheet.

If the second or funicular-polygon method for determining reactions is used, three figures may be necessary. It will be seen that the load line of Figs. 29 and 30 is not the same as that of Fig. 23(*b*). In solving for the unknown reactions in Fig. 23(*b*), it was necessary to have all of the vectors of the known forces joined together; so the line of action of the 6000-pound load was projected upward and became *ee′*. In constructing the frame to which the force polygons for the joints are linked, it is necessary that this load shall be acting at its proper place on joint 3, and it thus becomes separated from the other known vectors in the load line, as in Figs. 29 and 30. When the reactions have been found, this can be done. It follows, therefore, that three figures may be necessary, the first of which will be the space diagram drawn as directed in the preceding paragraph. The funicular or string polygon will be drawn in this space diagram as shown in Fig. 23(*a*). The second figure, called the *ray diagram*, will consist of a load line and rays as shown in Fig. 23(*b*). The third figure will consist of another load line and the linked polygons for the internal stresses as shown in Fig. 29. The second and third figures will usually be drawn to the same scale for forces. In case there are no loads between the reactions, the load lines of Figs. 23(*b*) and 29 become identical and the second and third figures, discussed above, may be united into a single diagram, as is done in Example 1 of Art. 33. Otherwise, as in the case of the truss of this article, changes required in the order in which forces are taken and the consequent modifications in lettering may make it necessary to separate the ray diagram from the linked polygon diagram. All figures required should be constructed on the same drawing sheet.

In Art. 27, it was pointed out that the total reactions may be determined by adding algebraically the partial reactions obtained by considering separately each particular load or group of loads. This procedure applies likewise to the internal stresses. For instance, the reactions and stresses, due to wind loads only, may be determined separately. Then the reactions and stresses due to dead loads may be determined. The resultant stress in any given member is the algebraic sum of the several partial stresses thus obtained.

29. Internal Stresses: Method of Sections.

In the analysis of some trusses the method used in the preceding article must be supplemented by other processes in order to complete the solution. The truss shown in Fig. 31 is an example.

The reactions, LA = 24,400 pounds and JK = 19,600 pounds, may be found by the methods given in Art. 27. If the first method is used, the loads on the upper chords are combined into their resultant R_1 = 32,000 pounds, acting at the center, Fig. 31. The 12,000-pound load is then combined by inverse proportion with R_1 to obtain R_2 = 44,000 pounds, which is the resultant of all of the loads. R_2 is then resolved by inverse proportion into its components acting at the ends of the truss. The reactions JK and LA are equal and opposite to these components.

If the funicular-polygon method is used to obtain the reactions, a slight change in the lettering is necessary as

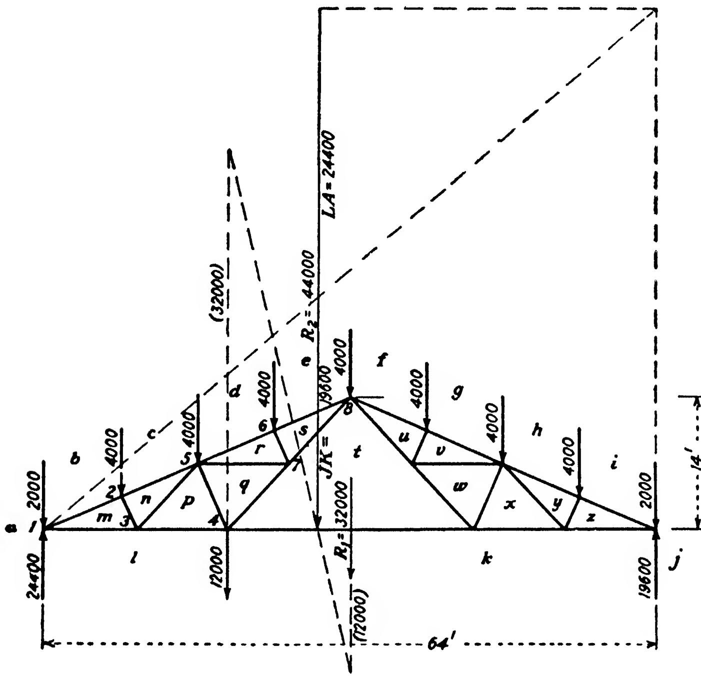


FIG. 31.

indicated in Fig. 32. The line of action of the 12,000-pound load is extended upward to divide space d into two parts, d and d'. This load is now known as DD' and the entire space between the reactions is designated as k. The left reaction then becomes KA instead of LA. The load line is laid out from point A to J as in Fig. 32(b). Any convenient pole point O is selected and the rays AO, BO, etc., are drawn. In Fig. 32(a), the funicular polygon is constructed with the left end of the truss as the initial point. String ao is of zero length since it is drawn between *ka* and *ab*. Strings *bo*, *co*, . . . *io* are drawn parallel to rays *BO*, *CO*, etc. String *jo* is of zero

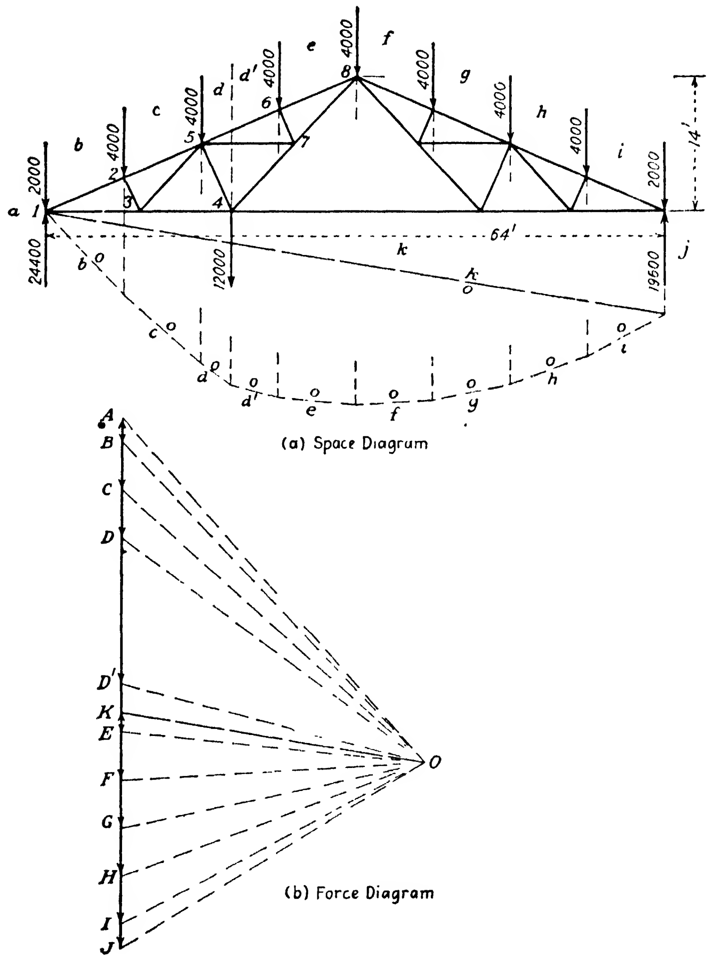


Fig. 32.

length because it is drawn between *ij* and *jk*. Since the force system is in equilibrium, the funicular polygon must close; so *ko* is necessarily the closing string. The ray *KO* in Fig. 32(*b*), drawn parallel to string *ko*, locates point K and thus determines the reactions JK and KA which scale 19,600 pounds and 24,400 pounds as before.

The procedure for determining internal stresses will now be considered. The load line and the force polygons for the first three joints are constructed in the usual manner as shown in Fig. 33. An inspection of the next joints 4 and 5 shows that each is acted upon by three unknown forces and thus cannot be solved. The same situation arises if the solution is made by starting at the

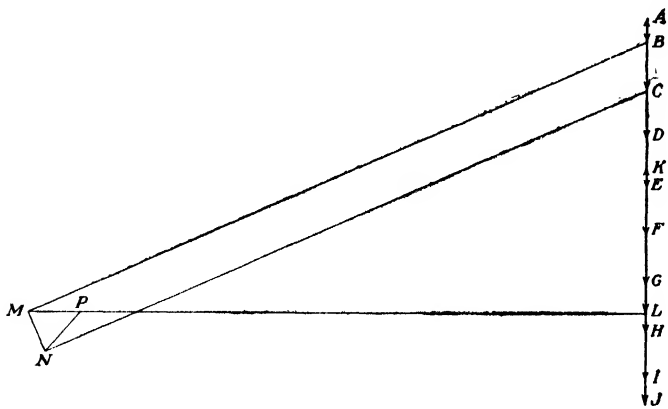


Fig. 33.

right end of the truss. The method of joints can therefore be carried no further unless one of the unknown stresses is determined by other methods. If the stress TK, for instance, can be obtained, joint 4 will have only two unknown forces and can be solved. The solution of the remaining joints can then readily be made.

The stress TK can be found by using half of the truss as a free body. If the left half is used, the known forces will consist of the loads and the reaction LA. The unknown forces will be the stress TK and the hinge reaction at the apex of the truss. If the loads and reactions can be combined into their resultant, the number of forces on the free body is reduced to three. These, being nonparallel, must intersect at a common point, as explained in Art. 12. In this way the direction of the unknown hinge reaction at the apex is determined. The force triangle can then be constructed and the value of stress *TK* scaled off.

In this example, however, the resultant of the loads and the reaction is outside of the limits of the drawing; so the stress *TK* must be determined by parts. The loads are temporarily neglected and the *partial* stress

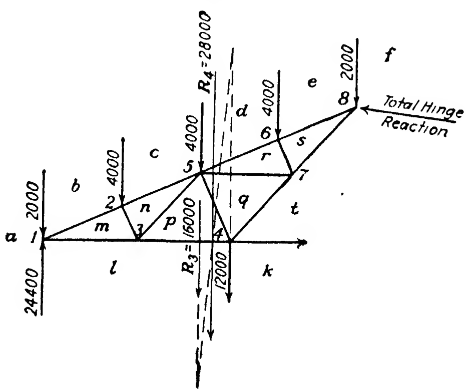


Fig. 34.

TK, due to reaction *LA* alone, is found by use of the force triangle as explained above. Next, the reaction *LA* is neglected and the *partial* stress *TK*, due to the loads, is found. The two partial stresses are then combined algebraically to obtain the resultant stress *TK*.

In Fig. 31, a section is passed through the hinge at the top, cutting the member *tk*; the portion of the truss on the left of this section is taken as the free body, shown in Fig. 34. Since the 4000-pound load *EF* passes through the hinge, it has no moment about this point and therefore cannot have any effect on the stress *TK*. Consequently, in the solution it does not matter how much of the load *EF* is made to act on this free body. It is convenient in this case to use 2000 pounds, or half of the load EF, because the system of loads on the upper chord thus becomes symmetrical, and the resultant R_3 of these loads is seen by inspection to be 16,000 pounds, acting through joint 5. The 12,000-pound load KL is now combined with R_3 by inverse proportion to obtain R_4, which is the resultant of all of the loads on the free body and scales 28,000 pounds.

If the reaction LA is temporarily neglected, the remaining three forces on the free body are R_4, the partial stress TK and the partial hinge reaction at the top, as shown in Fig. 35. These forces must be concurrent

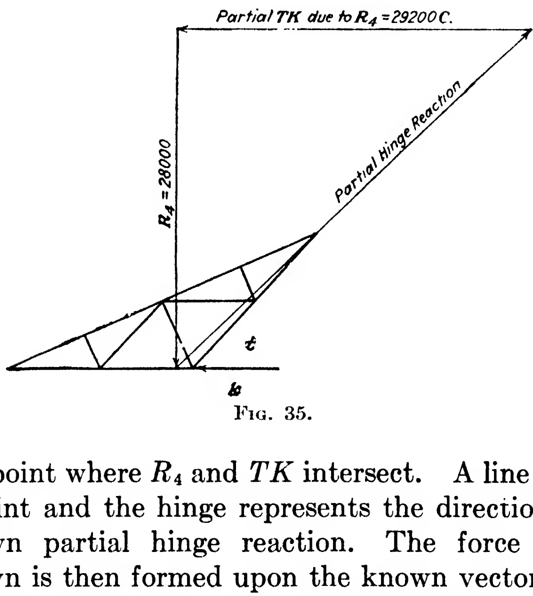


FIG. 35.

at the point where R_4 and TK intersect. A line through this point and the hinge represents the direction of the unknown partial hinge reaction. The force triangle as shown is then formed upon the known vector R_4 and the partial stress TK, due to the loads, is found to scale 29,200 pounds compression.

Next, the loads are neglected, leaving LA and the other partial values for the stress TK and the hinge reaction as forces on the free body, as shown in Fig. 36. These forces must be concurrent at the point where LA and TK intersect, namely, the left end of the truss.

The direction of the partial hinge reaction in this case coincides with the upper chord of the truss. The force triangle, as shown, is formed upon the known reaction *LA* and the partial stress *TK*, due to the reaction, is found to scale 55,800 pounds tension.

The resultant stress *TK* is found by adding algebraically the partial stresses thus obtained. Hence $TK =$ 29,200 pounds compression + 55,800 pounds tension =

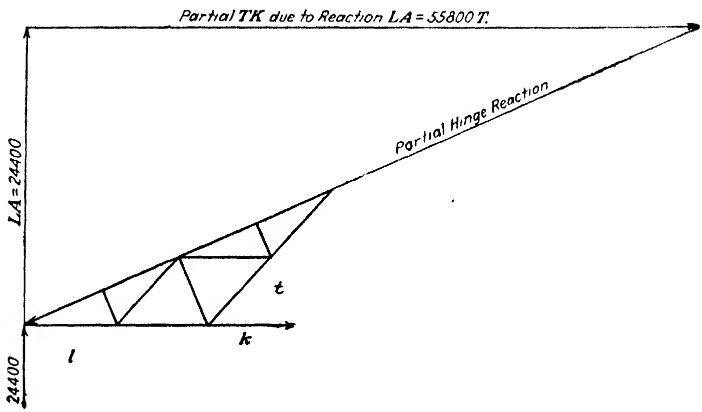


FIG. 36.

26,600 pounds tension. The above constructions for determining stress *TK* should be made directly on the space diagram of the truss.

Joint 4 now has only two unknown forces, *PQ* and *QT*, acting upon it and the polygon may be completed. Vector *TK* acts to the right, so must end at *K*, and the initial point *T* of the force polygon is thus located. The remaining joints offer no further difficulty since there are no more than two unknown forces on each successive free body.

The complete solution of the truss is shown in Fig. 37. The values of the internal stresses as scaled from the force diagram are given on page 42.

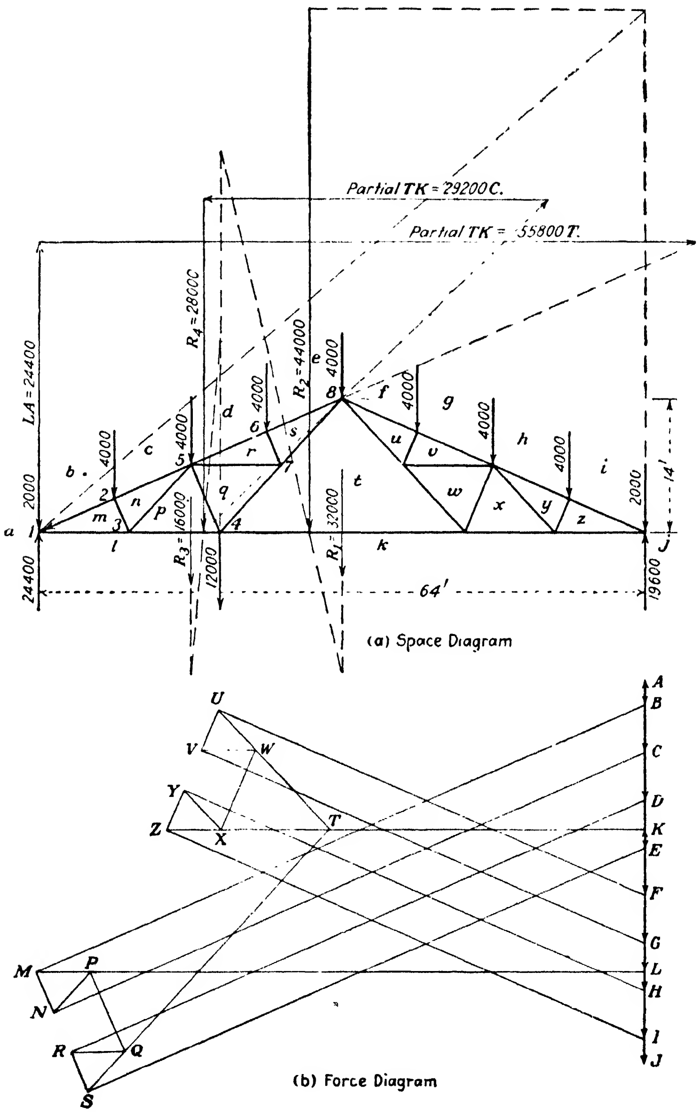

Partial TK = 29200 C.
Partial TK = 55800 T.
LA = 24400
R_4 = 2800 C
R_2 = 44000
R_3 = 16000
R_1 = 32000
4000
2000
12000
24400
19600
64′
14′
(a) Space Diagram
(b) Force Diagram

FIG. 37.

BM = 56,000 lb. *C*	*KX* = 35,700 lb. *T*	*RS* = 3,600 lb. *C*
CN = 54,400 lb. *C*	*KZ* = 40,400 lb. *T*	*ST* = 30,000 lb. *T*
DR = 52,800 lb. *C*	*LM* = 51,300 lb. *T*	*TU* = 13,700 lb. *T*
ES = 51,200 lb. *C*	*LP* = 46,800 lb. *T*	*TW* = 9,200 lb. *T*
FU = 39,200 lb. *C*	*MN* = 3,600 lb. *C*	*UV* = 3,600 lb. *C*
GV = 40,800 lb. *C*	*NP* = 4,500 lb. *T*	*VW* = 4,500 lb. *T*
HY = 42,400 lb. *C*	*PQ* = 7,200 lb. *C*	*WX* = 7,200 lb. *C*
IZ = 44,000 lb. *C*	*QR* = 4,500 lb. *T*	*XY* = 4,500 lb. *T*
KT = 26,600 lb. *T*	*QT* = 24,500 lb. *T*	*YZ* = 3,600 lb. *C*

30. Internal Stresses: Method of Substitution.

The difficulty encountered in the analysis of the truss in the preceding article may be overcome in another manner. Let a section be taken through the truss of Fig. 31, cutting the members *es*, *st* and *tk*. The free-body diagram of the part of the truss on the left of the section is shown in Fig. 38(*a*).

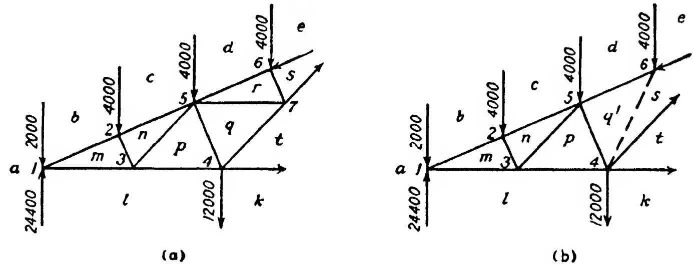


Fig. 38.

From a study of the free body it is evident that the stresses *ES* and *ST* are independent of the form of the framework in the other part of the free body. Therefore, let the members *qr* and *rs* be removed and replaced by member *q′s* as shown in the modified free-body diagram, Fig. 38(*b*). The solution of the truss thus modified does not involve the difficulty previously mentioned and is easily accomplished by the method of joints. The stresses *ES* and *ST* thus obtained must be identical with the stresses *ES* and *ST* in the original truss. The values of *ES* and *ST* may then be used in the solu-

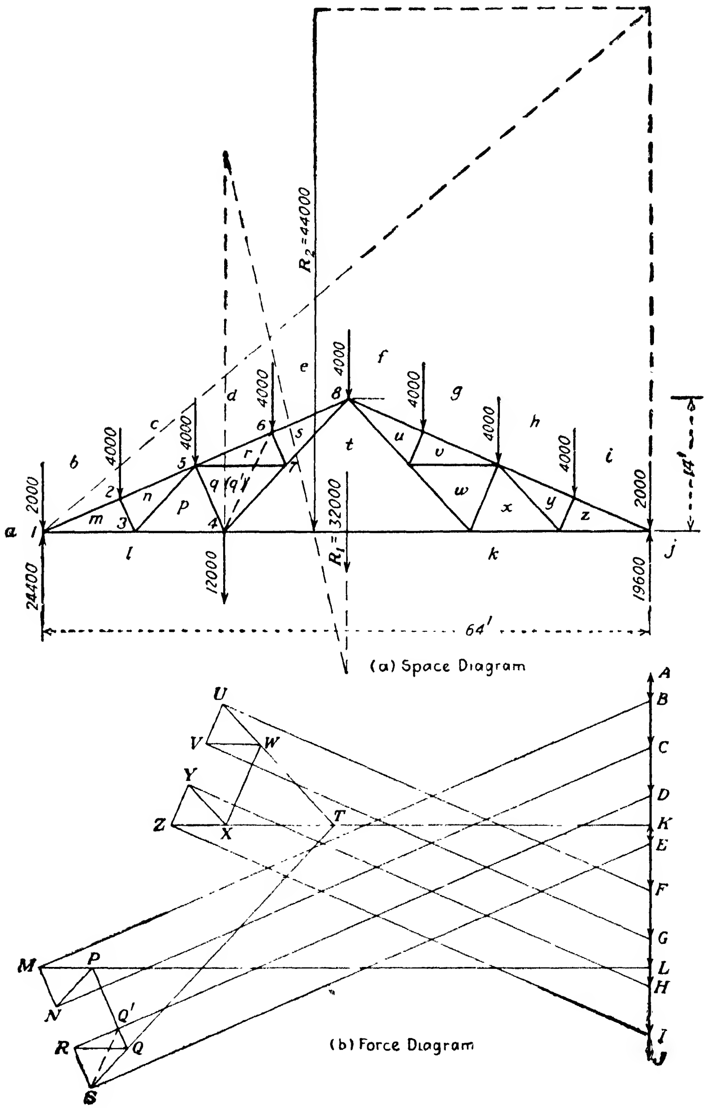

R2 = 44000
4000
4000
4000
4000
4000
4000
4000
2000
2000
24400
12000
R1 = 32000
19600
14'
64'
(a) Space Diagram
(b) Force Diagram

FIG. 39.

tion of the preceding joints and the stresses in the other members of the original truss obtained. The construction employed in this substitution method should be made directly upon the force diagram.

The complete solution of the truss, using the *false member* $q's$, is shown in Fig. 39. The force diagram, Fig. 39(b), is constructed in the following order. The load line is laid down, followed by the polygons for the

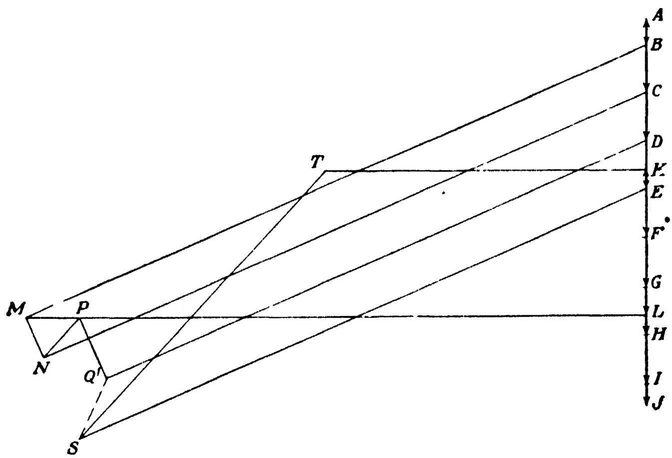


FIG. 40.

first three joints. After the members qr and rs have been replaced by false member $q's$, the polygon for joint 5 is constructed, thus locating point Q' in the force diagram. The force polygon for joint 6 is now added, which locates point S. Vectors ST and TK can then be drawn, locating point T. The diagram, as it would appear at this point, is shown in Fig. 40. The false member $q's$ is now replaced by the original members qr and rs and the remainder of the solution is easily completed. (The values of the internal stresses, as scaled from the completed force diagram, were given in Art. 29).

31. Reactions and Stresses in Bents.

A bent consists of a truss and supporting columns. In Fig. 41(*a*), *UVW* and *XYZ* are columns, connected to the truss by pins at *U* and *X* and by knee-braces attached at *V* and *Y*. The free-body diagram of the column *UVW*, Fig. 41(*b*), shows that forces are acting at more than two points on this member. This is also true of column *XYZ*. Each of these members is thus a three-force member as defined in Art. 9. Such mem-

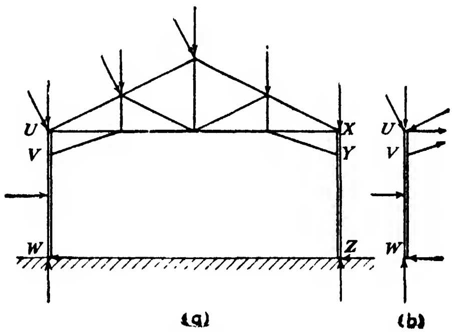


FIG. 41

bers are subjected to shearing and bending as well as axial stresses. A section should not be taken through a three-force member, but the entire member should be included in the free body. No attempt will be made here to determine the internal stresses in the columns.

The columns may be either hinged or fixed at the bases. In Fig. 41, they are shown hinged. If the columns are fixed at the bases, a lateral load will deflect the bent in the direction of the load, but portions *XY* and *UV* may be assumed to remain vertical. It likewise may be assumed that the tangents at the fixed ends *Z* and *W* will remain vertical. Under these conditions there will be points of counterflexure midway between *Y* and *Z* and midway between *V* and *W*. These points of counterflexure are equivalent to hinged ends; hence a column with its bases fixed may be treated as if it were hinged at a point midway between the base and the point of attachment of the knee-brace. (The solution of a bent with columns fixed at the bases is shown in Example 2, Art. 33.)

If the bases of the columns are hinged, the distribution of the horizontal component of the reactions between the two hinges is indeterminate and some assumption must be made. It may be assumed that each hinge takes one-half; or that one hinge takes all, the other none. The latter assumption is safer in design.

If the bases of the columns are fixed, the assumption that points of counterflexure are located midway between the bases and the knee-braces must necessarily be accompanied by the assumption that each column takes half of the horizontal reaction.

The graphical analysis of a bent consists of the following steps:

(*a*) The determination of the reactions on the columns, using the entire bent as a free body.

(*b*) The determination of the stresses in the members attached to the columns, using the columns as free bodies.

(*c*) The determination of the stresses in the remaining members of the truss, using the method of joints supplemented when necessary by the auxiliary method of Art. 29 or Art. 30.

The simple bent shown in Fig. 42 will be used as an illustration. The reactions and stresses are to be determined on the basis that each column is hinged at the base and that the horizontal components of the hinge reactions are equal.

In accordance with the procedure outlined above, the reactions at the bases of the columns will first be obtained. The three wind loads acting upon the upper chord are combined by inspection into their resultant

R_1 = 6000 pounds. The two horizontal wind loads are combined algebraically into their resultant R_2 = 10,000 pounds. R_1 and R_2 are then combined by the parallelogram method to obtain R_3, the resultant of all wind loads, which scales 13,750 pounds. The vertical

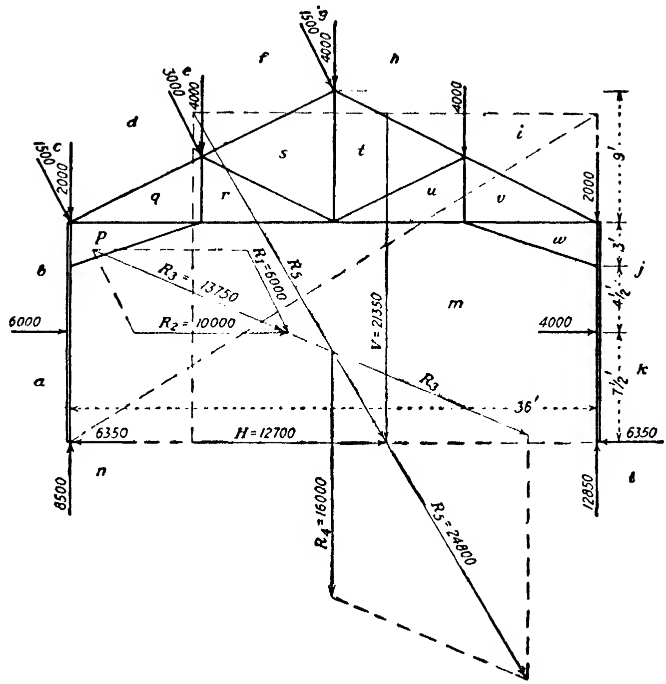


FIG. 42.

dead loads are combined by inspection into their resultant R_4 = 16,000 pounds, which is then combined with R_3 by the parallelogram method to obtain R_5 which scales 24,800 pounds. This is the resultant of all of the loads. The resolution of R_5 into V and H components must be made at the point where R_5 crosses the horizontal line through the bases of the columns, since the horizontal reactions must act along this line. The V component thus obtained scales 21,350 pounds and is resolved by inverse proportion into components at the two hinges. The vertical reactions LM and NA are equal and opposite to these components, which scale 12,850

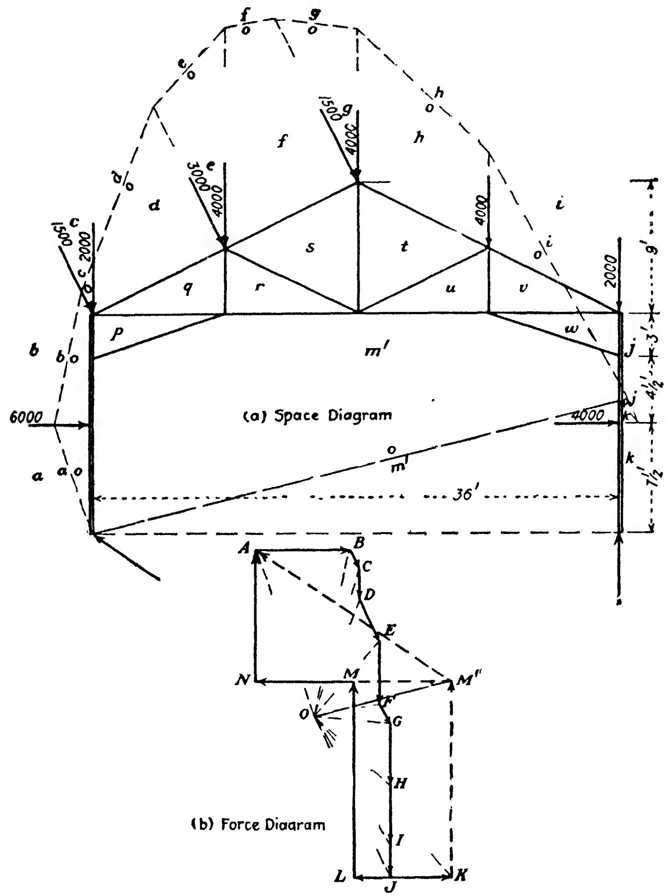


FIG. 43.

and 8500 pounds respectively. The H component of R_5 scales 12,700 pounds and, according to the initial assumption, each of the horizontal reactions KL and MN will be one-half of H or 6350 pounds.

The funicular-polygon method for determining the reactions requires a slight modification in the diagram, as indicated in Fig. 43. It must be assumed temporarily that only one column is hinged, say that at the left end, and the other supported by rollers. This assumption is made because it is necessary to fix the direction of one reaction. The right reaction is now vertical and is known as KM', the left reaction is unknown in direction and becomes $M'A$, and the entire space between the reactions is designated as m'. The loads AB, BC, . . . JK are laid off to scale in the load line, Fig. 43(b), the pole point O is chosen, and rays OA, OB, . . . OK are drawn. The reaction line KM' is known to be vertical, but the location of point M' is unknown. Starting at the base of the left column in the space diagram, since this is the only point known to lie on the line of action of the reaction $M'A$, the strings oa, ob, . . . ok are drawn parallel to the corresponding rays OA, OB, etc. Since the loads and reactions on the truss are in equilibrium, the funicular polygon must close, and om' is thus drawn as the closing string. The ray OM' in the force diagram, parallel to the string om', locates point M' and determines the reactions KM' and $M'A$. To obtain the true reactions, based on the original conditions of a hinge at the end of each column, the reaction $M'A$ is resolved into the H and V components $M'N$ and NA respectively. The H component $M'N$ is then divided into two equal parts, $M'M$ and MN. The half portion $M'M$ is transferred to the other column and becomes the horizontal reaction KL. This resolution is shown as a part of Fig. 43(b) and finally gives KL, LM, MN and NA as the component reactions at the bases of the columns as obtained before in Fig. 42.

The next step in the solution will be to determine the stresses DQ, QP and PM. The left column is the free body, as shown in Fig. 44(a). The known forces MN, NA and AB are combined into their resultant $R_7 = 8500$ pounds. If the four forces at the top of the column are replaced by their resultant R_8, the number of forces on the free body is reduced to three, namely R_7, PM and R_8, as shown in Fig. 44(b). These three forces must be

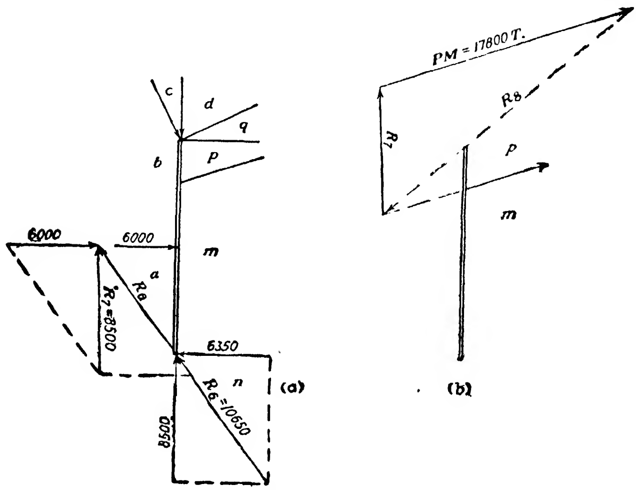


FIG. 44.

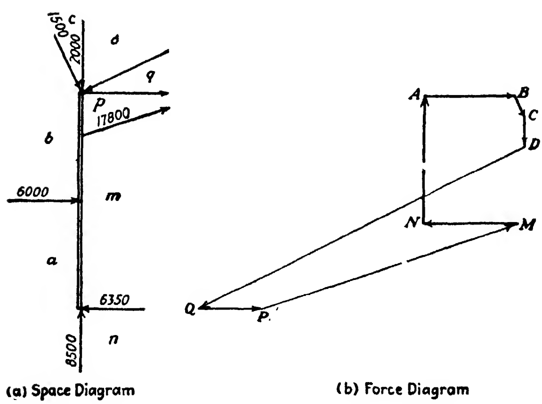


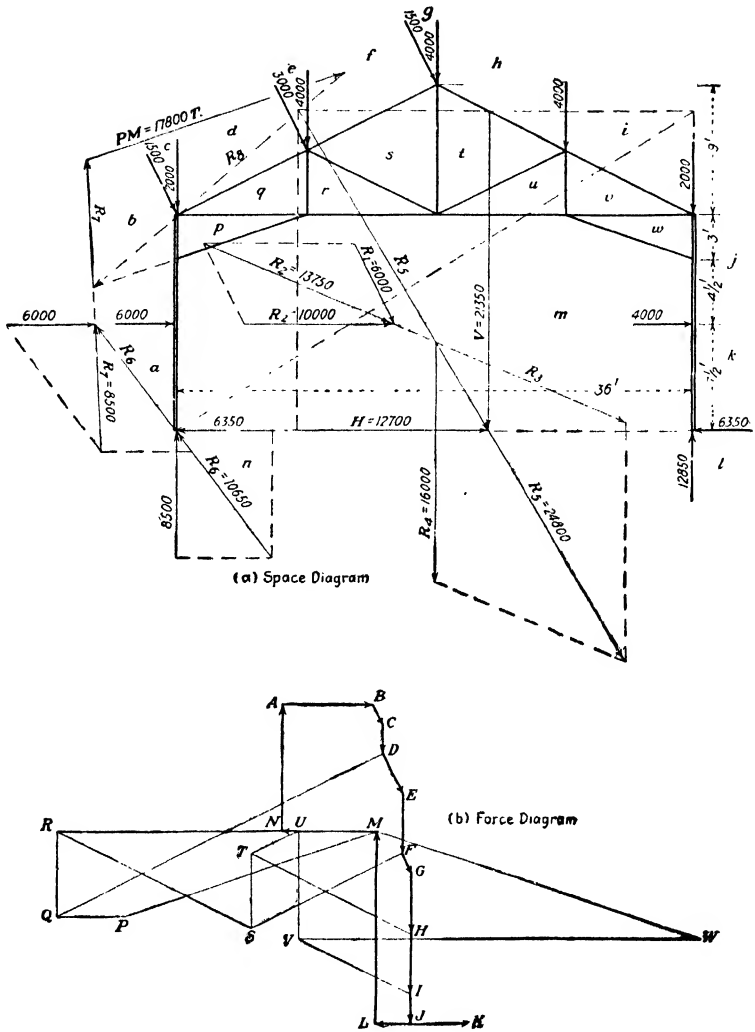
FIG. 45.

concurrent at the point where R_7 and pm intersect. A line through this point and the top of the column must therefore be the line of action of R_8. The triangle of forces may then be constructed upon the known force R_7, and the value of PM is found to be 17,800 pounds tension. The preceding constructions should all be made directly upon the space diagram of the bent.

The stresses DQ and QP are now to be determined by using the column again as a free body, with R_8 replaced by the original four forces at the top and with R_7 replaced by its original components. The free body is shown in Fig. 45(*a*). All of the forces on the free body are now known except DQ and QP. These stresses may be obtained by drawing the force polygon as shown in Fig. 45(*b*). This polygon should be constructed as a part of the force diagram for the entire bent. If the load line for the bent has already been laid out, it may be used as the frame upon which to build the above polygon, since the sides MN, NA, AB, BC and CD of the polygon are also a part of the load line.

After the stresses DQ, QP and PM have been determined, the remaining internal stresses may be found by the method of joints. The complete solution of the bent is shown in Fig. 46. As already pointed out in previous examples, the closing of the load line in this solution serves as a partial check on the reactions. There is, however, no similar check available for the internal stresses unless the right-hand column is solved independently as a free body to determine the stress VW. The values of the internal stresses, as scaled from Fig. 46(*b*), are as follows:

DQ = 24,300 lb. C	MR = 21,450 lb. T	RS = 14,700 lb. C
FS = 11,100 lb. C	MU = 5,100 lb. T	ST = 4,900 lb. T
HT = 11,850 lb. C	MW = 23,100 lb. C	TU = 3,600 lb. T
IV = 8,200 lb. C	PQ = 4,500 lb. T	UV = 7,200 lb. C
MP = 17,800 lb. T	QR = 5,700 lb. T	VW = 27,100 lb. C



(a) Space Diagram

(b) Force Diagram

FIG. 46.

32. Redundant Members.

A study of the trusses illustrated in the preceding articles shows that stability would be destroyed by the removal of any single member. Trusses which have sufficient members for stability, but no more, are often called complete or perfect trusses. If additional members not required for stability are added, the truss in general will be statically indeterminate. In some cases of the latter type, analysis will show that under certain

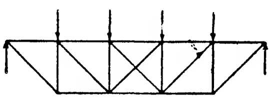


Fig. 47.

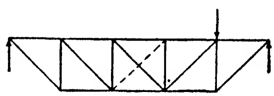


Fig. 48.

conditions of loading some member is not acting and the truss becomes statically determinate. Such a member is called a *redundant* member. In Fig. 47, stability would not be destroyed upon the removal of either of the two diagonal members in the middle panel, provided that the remaining member is capable of carrying either tension or compression. If the two diagonals are designed to carry tension only, then both will not be stressed simultaneously. For instance, if the above truss is loaded as shown in Fig. 48, the dotted member, if stressed at all, would be in compression. If designed to carry tension only, such as a cable or slender rod, the conclusion would be that this member does not act and is therefore redundant. The truss is then statically determinate and the internal stresses may be found by methods already explained.

In case both diagonals act simultaneously, the solution becomes statically indeterminate and beyond the scope of this text. For further analysis of such trusses, reference should be made to standard texts on Structures.

33. Examples.

As a review and further illustration of the principles and methods taken up in the preceding articles, the following examples are given together with their complete graphic solutions. A statement of the steps taken, together with a brief explanation, accompanies each solution. In cases where more than one method is offered for obtaining a particular result, both solutions are placed on the same diagram.

Example 1. A nacelle located on the lower wing of a bombing plane carries an engine and machine gunner. The dimensions and loads, as distributed to the various joints on

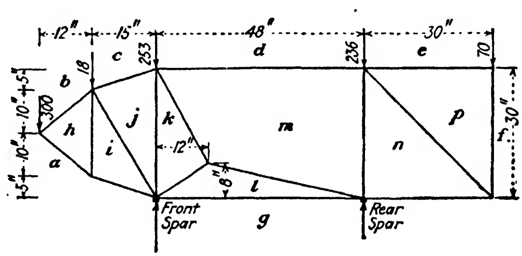


Fig. 49.

one of the trusses, are shown in Fig. 49. The truss is supported by the front and rear wing spars as shown. Determine the reactions and the internal stresses in the members.

Solution: The truss is laid out to scale, dimensioned and lettered as shown in Fig. 50(a).

The reactions FG and GA are found by the funicular-polygon method, the ray diagram with O as the pole point being drawn as part of the force diagram, Fig. 50(b); and the funicular polygon as part of the space diagram, Fig. 50(a). The ray OG in the force diagram, drawn parallel to the closing string og in the funicular polygon, determines the values of reactions FG and GA, which scale 175 lb. and 702 lb. respectively.

The solution for the internal stresses is now made, successive joints being used as free bodies. If the solution is begun at the left end of the truss as is usually done, only three joints

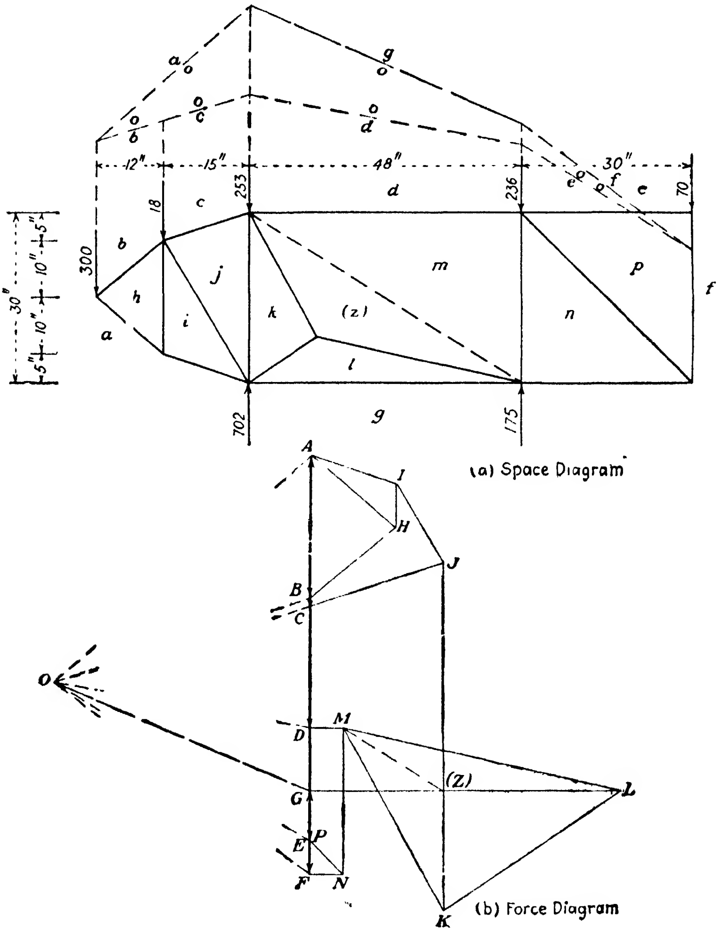


FIG. 50.

can be solved. Each of the next two joints has three unknown forces acting upon it. If the solution is begun at the right end, however, this difficulty is avoided. If the same condition should occur regardless of which end is used as a starting point, then the method of sections of Art. 29 or the method of substitution of Art. 30 must be employed. When the latter is used, a false member *mz* is substituted temporarily for the three actual members in the panel. This makes it possible to continue with the method of joints, thus locating point *M*, after which the solution is readily completed.

The first and third of these possible solutions are shown in Fig. 50. The values of the internal stresses, as scaled from the force diagram, Fig. 50(*b*), are as follows:

AH = 235 lb. *C*	*FP* = 70 lb. *C*	*KL* = 450 lb. *T*
AI = 190 lb. *C*	*FN* = 70 lb. *C*	*KM* = 430 lb. *T*
BH = 235 lb. *T*	*GL* = 655 lb. *C*	*LM* = 595 lb. *T*
CJ = 297 lb. *T*	*HI* = 89 lb. *T*	*MN* = 306 lb. *C*
DM = 70 lb. *T*	*IJ* = 191 lb. *C*	*NP* = 100 lb. *T*
EP = 0	*JK* = 722 lb. *C*	

Example 2. The columns of the bent shown in Fig. 51 are fixed at the bases. Assuming that the points of counterflexure will be located halfway between the bases and the

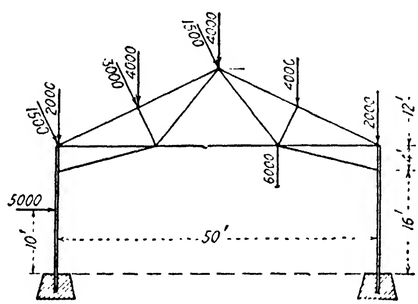


FIG. 51.

points of attachment of the knee-braces, solve for the reactions and all of the internal stresses.

Solution: Since the points of counterflexure in the columns are equivalent to hinged ends, the bent will be treated as if hinged at these points. The effect of this assumption is to reduce the length of the columns from 20 ft. to 12 ft., and to decrease the horizontal wind load, in the same proportion, from 5000 lb. to 3000 lb., acting at a point 6 ft. above the hinge. The bent is accordingly laid out to scale, dimensioned and lettered as shown in Fig. 52(*a*). The solution, from this point on, will be similar to that of the example in Art. 31.

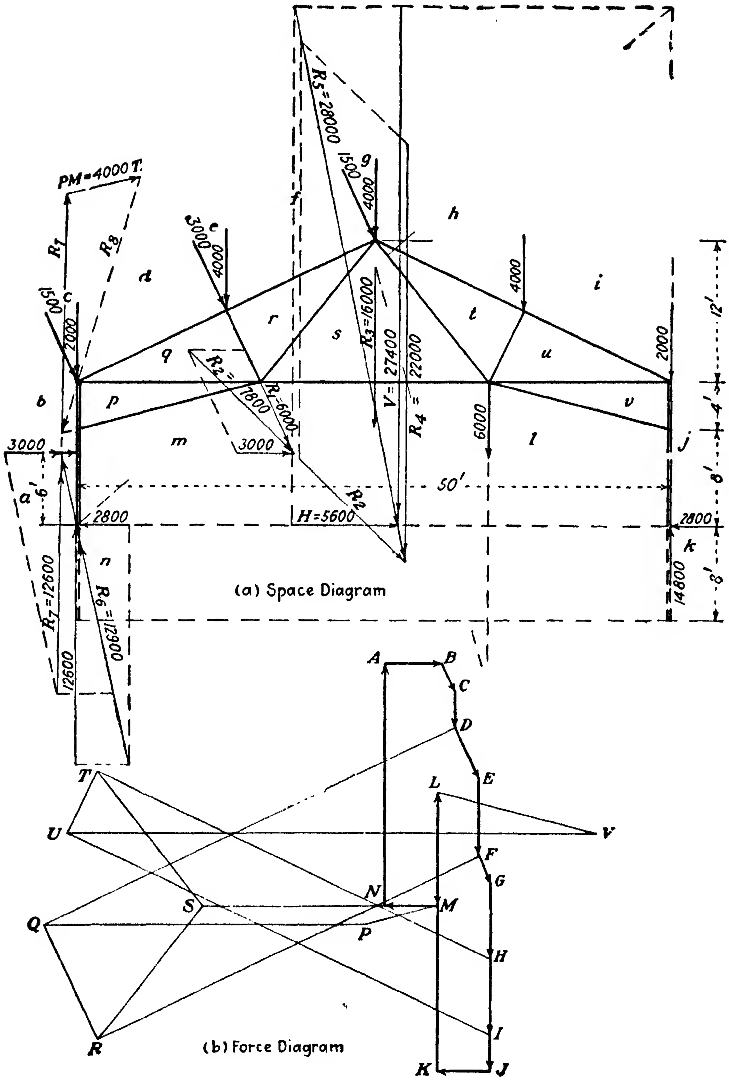


Fig. 52.

The first step is to determine the reactions, with the entire bent as the free body. The wind loads are combined into their resultant R_2 which scales 7800 lb. The dead loads are combined into their resultant $R_4 = 22{,}000$ lb. By combining R_2 and R_4, the resultant R_5 of all the loads is obtained and scales 28,000 lb. At the point where R_5 intersects the horizontal line through the hinges, it is resolved into H and V components which scale 5600 lb. and 27,400 lb. respectively. The V component is now resolved by inverse proportion into its two components at the hinges, thus determining the vertical reactions KL and NA which scale 14,800 lb. and 12,600 lb. respectively. As stated in Art. 31, the location assumed for the points of counterflexure requires that the horizontal reactions upon the columns shall be equal. Each of the reactions JK and NA must therefore be half of the H component of the resultant load, or 2800 lb.

The left column is now used as a free body to determine the stress PM in the knee-brace. The reactions MN and NA and the 3000-lb. wind load are combined into their resultant $R_7 =$ 12,600 lb. The four forces acting at the top of the column are considered as combined into a single unknown force R_8. This reduces the number of forces on the free body to three, the known force R_7, the unknown stress PM and the unknown resultant force R_8. These forces must be concurrent at the point where the line mp intersects the line of action of R_7. The direction of the resultant force R_8 is thereby established and the force triangle is drawn as shown in Fig. 52(a), from which PM scales 4000 lb. T. The entire left column is again considered as a free body with the resultants R_7 and R_8 replaced by their original components, and the force polygon is laid out in Fig. 52(b) to determine the stresses DQ and QP.

The force polygons for the remaining joints are then added, as shown in Fig. 52(b), to complete the solution. The values of the internal stresses as scaled from the force diagram are as follows:

DQ = 23,800 lb. C	MP = 4,000 lb. T	ST = 8,900 lb. T
FR = 22,100 lb. C	MS = 12,300 lb. T	TU = 3,600 lb. C
HT = 22,800 lb. C	PQ = 16,800 lb. T	UV = 27,800 lb. T
IU = 24,600 lb. C	QR = 6,600 lb. C	
LV = 8.650 lb. C	RS = 9,000 lb. T	

Example 3. A bent with a flat top has dimensions and loads as shown in Fig. 53. The columns are hinged at their bases and it is to be assumed that the left column takes all of the horizontal reaction. Determine the reactions on the columns and the internal stresses in the remaining members.

Solution: The bent is laid out, dimensioned and lettered as shown in Fig. 54(*a*).

The first step in the solution is to determine the reactions, with the entire bent as the free body. The various loads are combined into their resultant R_2 which scales 36,900 lb. At the point where R_2 intersects the horizontal line through the bases of the columns it is resolved into H and V components

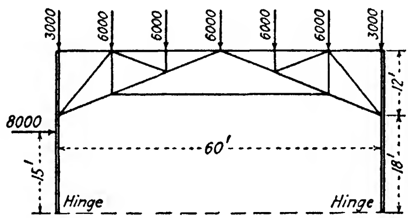


FIG. 53.

which scale 8000 lb. and 36,000 lb. respectively. The V component is resolved by inverse proportion into its two components at the hinges, thus determining the vertical reactions IJ and KA which are 20,000 lb. and 16,000 lb. respectively. The horizontal reaction JK at the base of the left column must support all of the H component and is thus 8000 lb.

As an alternative method, the reactions may be found by the use of the funicular polygon. The line of loads is laid out in Fig. 54(*b*) and the rays are drawn from the pole point O. The funicular polygon is then drawn in Fig. 54(*a*) in which oj is the closing string. The ray OJ in the force diagram then determines the reactions.

Before the force diagram for the internal stresses can be completed it is necessary to determine the stresses JR and CL (or HW). Either the right or the left half of the bent may be used as a free body in solving for stress JR. Since the right half is somewhat simpler, it is used in this case, as shown in

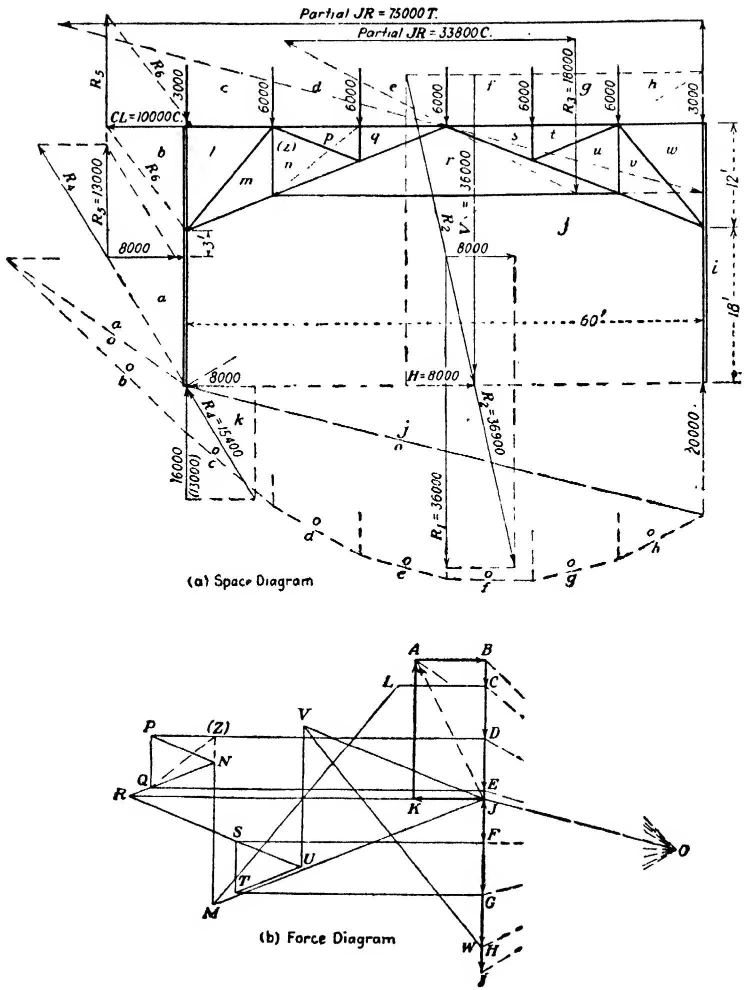

Partial JR = 75000 T.
Partial JR = 33800 C.
CL = 10000 C.
R₅ = 13000
R₄ = 15400
R₂ = 36900
R₁ = 36000
R₃ = 18000
H = 8000
60'
12'
18'
(a) Space Diagram
(b) Force Diagram

FIG. 54.

Fig. 54(a). The loads on the right half of the bent are temporarily neglected and the partial stress JR, due to the reaction IJ alone, is obtained and found to scale 75,000 lb. tension. The loads on the right half of the bent are now combined into their resultant R_3 = 18,000 lb. The reaction IJ is then temporarily neglected while the partial stress JR due to R_3 is obtained and found to scale 33,800 lb. compression. The resultant stress JR is the algebraic sum of the partial stresses and is therefore 41,200 lb. tension. The substitution method, replacing members np and pq by the false member zq, may be used instead of the foregoing solution and the constructions for it are also shown in Fig. 54.

The left column is used as a free body to determine the stress CL. The four known forces on the free body, AB, BC, JK and KA, are combined into their resultant R_5 which scales 13,000 lb. The two unknown forces LM and MJ, acting at the same point on the column, are considered as combined into a single unknown force R_6, acting at this point. The number of forces on the free body is thus reduced to three: the known resultant force R_5, the unknown stress CL and the unknown resultant force R_6. These forces must be concurrent at the point where line cl intersects R_5 and the direction of R_6 is thus determined. The triangle of the forces is then drawn from which CL scales 10,000 lb. compression. The entire left column is again considered as a free body with the resultants R_5 and R_6 replaced by their original components. The only unknown forces acting on the column now are LM and MJ, which are determined by laying out the force polygon for the column in the force diagram, Fig. 54(b).

The force polygons for the remaining joints, taken in order from left to right, are then added to the force diagram, as shown in Fig. 54(b), to complete the solution. The stress HW may be determined by using the right column as a free body. The value thus obtained may be used as a check on the solution already made, or may be used, instead of CL, in beginning the layout of the force polygons. In this case the joints are solved in order from right to left. (It will be seen, by inspection of the right column, that stress HW is zero.)

The values of a few of the internal stresses, as scaled from Fig. 54(*b*), are as follows:

$$LM = 31{,}100 \text{ lb. } C$$
$$MJ = 31{,}500 \text{ lb. } T$$
$$DP = EQ = 38{,}700 \text{ lb. } C$$
$$VJ = 22{,}700 \text{ lb. } T$$
$$VW = 31{,}000 \text{ lb. } C$$

PROBLEMS

NOTE: In the following problems, the scales and coordinates given are based on the standard-size drawing plate as recommended in Art. 3. The coordinates are to be measured from the left and lower borders respectively. A few of the answers are given by which to check the solutions.

Problem 1. Determine the reactions and stresses in the Fink roof truss, shown in Fig. 55. Scales: 1 in. = 6 ft.; 1 in.

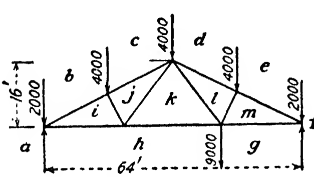


FIG. 55.

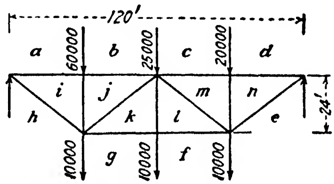


FIG. 56.

= 3000 lb. Coordinates: Left end of truss—1 in., 10 in.; point A in force diagram—21 in., 10 in.

Ans. HA = 10,800 lb.; BI = 19,650 lb. C; HK = 13,550 lb. T; DL = 25,450 lb. C.

Problem 2. Determine the reactions and stresses in the bridge truss, shown in Fig. 56. Scales: 1 in. = 10 ft.; 1 in. = 10,000 lb. Coordinates: Left end of truss—$\frac{1}{2}$ in., 10 in.; point H in force diagram—$22\frac{1}{2}$ in., 6 in.

Ans. HA = 77,500 lb.; AI = 96,875 lb. C; JK = 12,000 lb. C; LF = 106,250 lb. T.

Problem 3. Determine the reactions and stresses in the Warren bridge truss, shown in Fig. 57. Scales: 1 in. = 5 ft.; 1 in. = 2000 lb. Coordinates: Left end of truss—2 in., 12 in.; point G in force diagram—21 in., 7 in.

Ans. GA = 12,330 lb.; AH = 15,800 lb. C; HG = 9800 lb. T; CK = 13,800 lb. C.

Problem 4. Determine the reactions and stresses in the cantilever bridge truss, shown in Fig. 58. Scales: 1 in. = 20

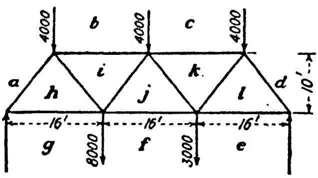


Fig. 57.

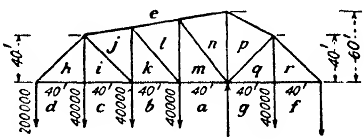


Fig. 58.

ft.; 1 in. = 100,000 lb. Coordinates: Left end of truss—1 in., 8 in.; point A in force diagram—14 in., 12 in.

Ans. GA = 860,000 lb.; AM = 540,000 lb. C; HE = 282,800 lb. T; EN = 705,000 lb. T; KJ = 217,000 lb. T.

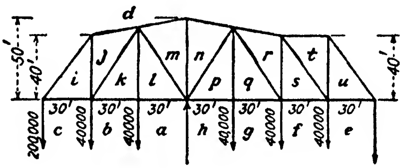


Fig. 59.

Problem 5. Determine the reactions and stresses in the cantilever bridge truss, shown in Fig. 59. Scales: 1 in. = 20

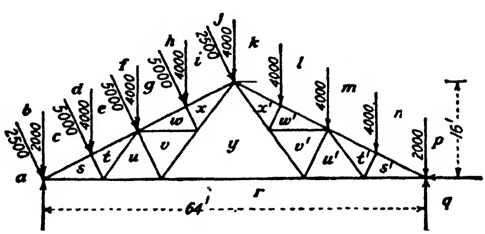


Fig. 60

ft.; 1 in. = 50,000 lb. Coordinates: Left end of truss—1 in., 9 in.; point A in force diagram—13 in., 14 in.

Ans. HA = 520,000 lb.; LA = 295,000 lb. C; ID = 250,000 lb. T; DN = 440,000 lb. T; NP = 204,000 lb. C.

Problem 6. Determine the reactions and stresses in the Fink roof truss, shown in Fig. 60. Scales: 1 in. = 6 ft.; 1 in. = 5000 lb. Coordinates: Left end of truss—1 in., 4 in.; point A in force diagram—20 in., 15 in.

Ans. RA = 28,300 lb., PQ = 9000 lb.; QR = 21,600 lb.; CS = 54,300 lb. C; TU = 9700 lb. T; RY = 18,200 lb. T; GW = 50,600 lb. C; YX' = 11,800 lb. T; MT' = 41,800 lb. C.

Problem 7. Determine the reactions and stresses in the cambered Fink truss, shown in Fig. 61. Scales: 1 in. = 6 ft.;

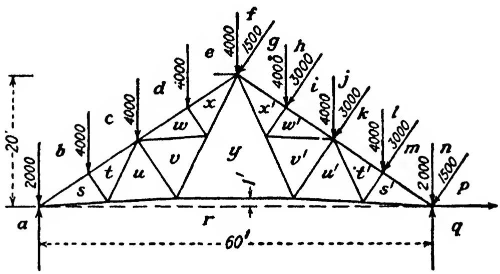


Fig. 61.

1 in. = 4000 lb. Coordinates: Left end of truss—2 in., 5 in.; point A in force diagram—22 in., 14 in.

Ans. RA = 19,600 lb.; BS = 34,100 lb. C; TU = 3200 lb. T; RY = 18,400 lb. T; IW' = 33,300 lb. C.

Problem 8. Determine the reactions and stresses in the airplane truss, shown in Fig. 62. Scales: 1 in. = 10 in.;

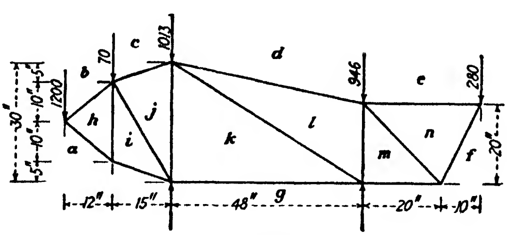


Fig. 62.

1 in. = 300 lb. Coordinates: Left end of truss—1 in., 10 in.; point A in force diagram—16 in., 13 in.

Ans. GA = 2805 lb.; JK = 1910 lb. C; ML = 1140 lb. C; LK = 810 lb. T.

Problem 9. Determine the reactions and stresses in the tower, shown in Fig. 63. The internal diagonals can take tension only. Scales: 1 in. = 6 ft.; 1 in. = 1000 lb. Coordinates: Top of tower—5 in., 13 in.; point A in force diagram—19 in., 12 in.

Ans. EF = 6890 lb.; AL = 4030 lb. C; JK = 1270 lb. T; HA = 6170 lb. C.

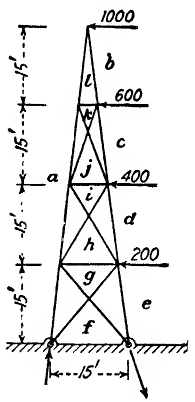


FIG. 63.

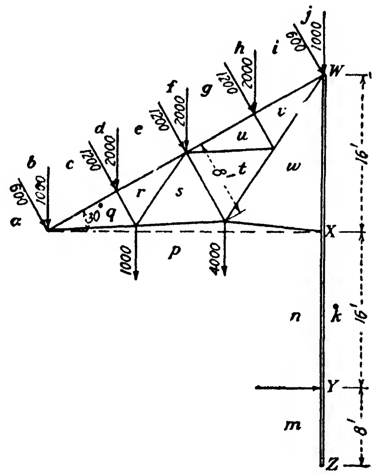


FIG. 64.

Problem 10. The cantilever truss, shown in Fig. 64, is supported by a column which is hinged at the bottom and supported horizontally at point Y as shown. Determine the reactions on the column at Y and Z and the internal stresses in the members of the truss. Scales: 1 in. = 4 ft.; 1 in. = 2000 lb. Coordinates: Left end of truss—1 in., 11 in.; point A in force diagram—14 in., 11 in.

Ans. MN = 18,700 lb.; NW = 15,480 lb. C; CQ = 3450 lb. T; VW = 12,150 lb. T.

Problem 11. The railway-platform roof truss, shown in Fig. 65, is supported by a column which is hinged at the bottom and supported horizontally at point Y as shown. Determine the reactions on the column at Y and Z and the internal stresses in the members of the truss. Scales: 1 in. = 5 ft.; 1 in. = 5000 lb. Coordinates: Left end of truss—2 in., 10 in.; point A in force diagram—14 in., 12 in.

Ans. KL = 5900 lb.; CM = 19,800 lb. T; NP = 7300 lb. T; IQ = 10,300 lb. C.

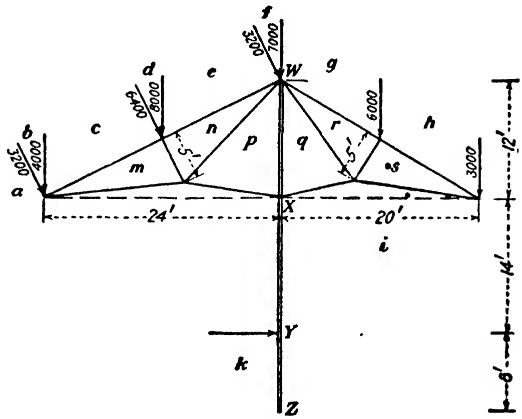


Fig. 65.

Problem 12. The columns of the bent, shown in Fig. 66, are hinged at the bases. Assuming the left column to carry all of the horizontal reaction, determine the reactions on the columns and the stresses in the members of the truss. Scales:

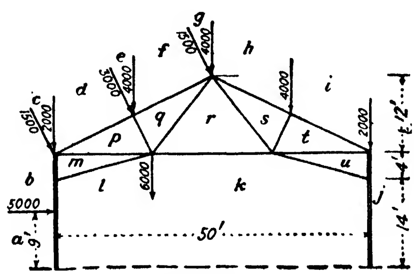


Fig. 66.

1 in. = 6 ft.; 1 in. = 4000 lb. Coordinates: Base of left column—4 in., 9 in.; point A in force diagram—18 in., 9 in.

Ans. JK = 13,400 lb.; LM = 23,800 lb. T; DP = 38,500 lb. C; KR = 19,400 lb. T; HS = 24,300 lb. C.

Problem 13. The columns of the flat top bent, shown in Fig. 67, are hinged at the bases. Assuming the right column to carry all of the horizontal reaction, determine the reactions on the columns and the stresses in the members of the truss.

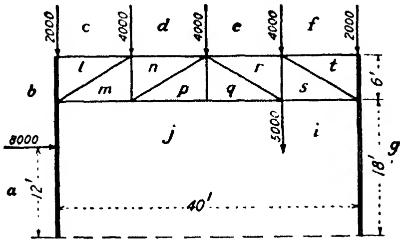


FIG. 67.

Scales: 1 in. = 5 ft.; 1 in. = 3000 lb. Coordinates: Base of left column—3 in., 8 in.; point A in force diagram—12 in., 13 in.

Ans. JA = 6900 lb.; CL = 8000 lb. T; JM = 7840 lb. C; ST = 23,500 lb. C.

Problem 14. The columns of the bent, shown in Fig. 68, are hinged at the bases. Assuming the left column to take all

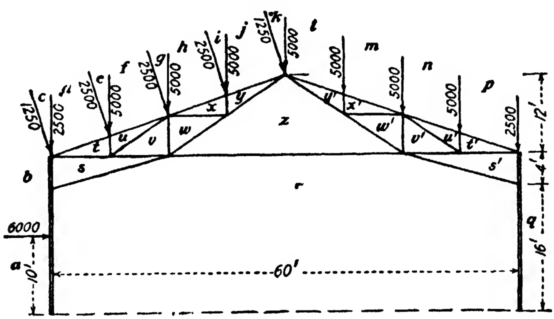


FIG. 68.

of the horizontal reaction, determine the reactions on the columns and the stresses in the members of the truss. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb. Coordinates: Base of left column—3 in., 8 in.; point A in force diagram—20 in., 12 in.

Ans. QR = 24,900 lb.; RZ = 38,000 lb. T; RS = 34,800 lb. T; DT = 80,800 lb. C; UV = 12,400 lb. T; YZ = 50,800 lb. T; RS' = 0.

Problem 15. The columns of the flat top bent, shown in Fig. 69, are hinged at the bases. Assuming half of the horizontal reaction to be carried at each column, determine the reactions on the columns and the stresses in the members of the truss. Scales: 1 in. = 6 ft.; 1 in. = 5000 lb. Coordi-

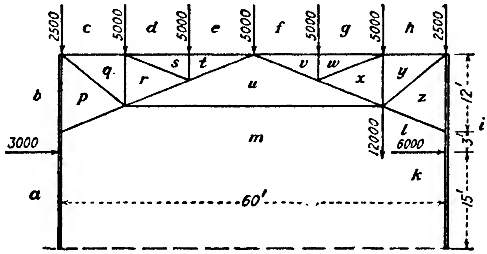


Fig. 69.

nates: Base of left column—2 in., 6 in.; point A in force diagram—20 in., 10 in.

Ans. NA = 14,750 lb.; MU = 38,400 lb. T; MP = 8200 lb. T.

Problem 16. The columns of the bent, shown in Fig. 70, are hinged at the bases. Assuming the right column to take

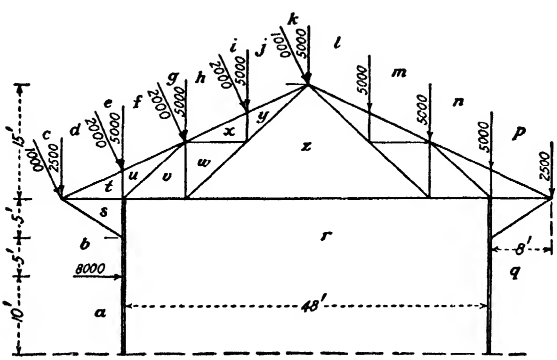


Fig. 70.

all of the horizontal reaction, determine the reactions on the columns and the stresses in the members of the truss. Scales: 1 in. = 6 ft.; 1 in. = 5000 lb. Coordinates: Base of left column—3 in., 3 in.; point A in force diagram—14 in., 12 in.

Ans. RA = 22,600 lb.; RZ = 7900 lb. C; BS = 19,200 lb. T; UV = 37,400 lb. C; HX = 8600 lb. C.

Problem 17. The columns of the bent, shown in Fig. 71, are hinged at the bases. Assuming the right column to take all of the horizontal reaction, determine the reactions on the columns and the stresses in the members of the truss. Scales: 1 in. = 6 ft.; 1 in. = 5000 lb. Coordinates: Base of left column—2 in., 5 in.; point A in force diagram—18 in., 12 in.

Ans. QA = 26,400 lb.; WP = 1600 lb. T; DR = 16,400 lb. T; TW = 12,800 lb. T.

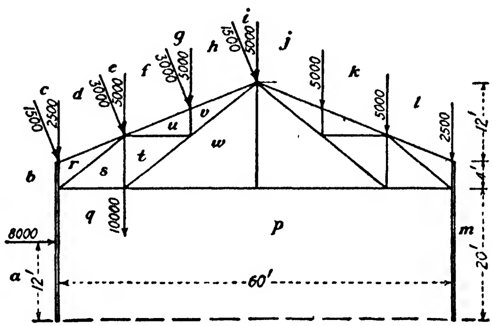


Fig. 71.

Problem 18. The columns of the bent, shown in Fig. 71, are hinged at the bases. Assuming the left column to take all of the horizontal reaction, solve for the reactions on the columns and the stresses in the members of the truss. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb. Coordinates: Base of left column—3 in., 8 in.; point A in force diagram—$20\frac{1}{2}$ in., 8 in.

Ans. WP = 27,200 lb. T; DR = 42,000 lb. C; TW = 28,300 lb. T.

Problem 19. The columns of the flat top bent, shown in Fig. 67, are fixed at the bases. Solve for the stresses in the members of the truss. Use scales and coordinates as given in Problem 13.

Ans. JK = 2500 lb.; KA = 8300 lb.; CL = 2500 lb. C; JM = 10,550 lb. T; ST = 20,900 lb. C.

Problem 20. A truss of an airplane fuselage is shown in Fig. 72. The diagonals in the last four panels can take tension only. The loading given is for a three-point landing.

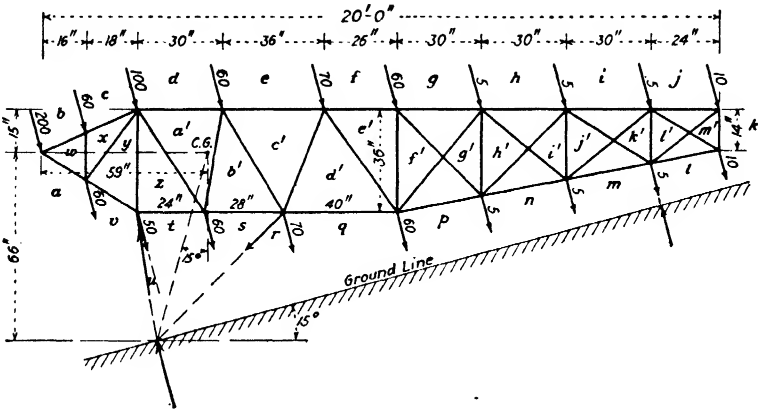


FIG. 72.

Determine the reactions and all internal stresses. Scales: 1 in. = 20 in.; 1 in. = 60 lb. Coordinates: Left end of truss—4 in., 12 in.; point V in force diagram—17 in., 14 in.

Ans. (Factor omitted) Reaction LM = 165 lb.; RS = 98 lb.; UV = 788 lb.; EC' = 233 lb. C; $H'I'$ = 140 lb. T.

CHAPTER IV

CRANES, DERRICKS AND DREDGES

34. Cranes, Derricks and Dredges.

Cranes, derricks and dredges are structures used to raise, transfer and lower heavy loads. The loads can be moved horizontally as well as vertically. The two principal classes of cranes are rotating and translating

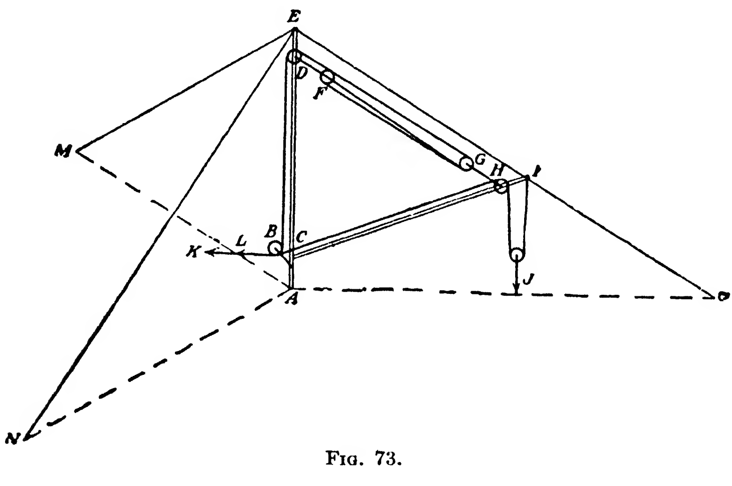


Fig. 73.

cranes. The main feature of the rotating crane is the mast with a boom hinged or fixed to it, as shown in Fig. 73. The boom generally is capable of motion in a vertical plane and can rotate about the mast. The mast is supported by a suitable pivot at the base and by guy wires or stiff legs at the top. The motions of the boom and its load are controlled by cables and pulleys.

The rotating crane will be considered in this chapter. This class of cranes has several important types: the pure crane which has a fixed boom, the derrick which has a movable boom and the dredge which is so designed that it operates below the ground level. The derrick type is shown in Fig. 73. *AE* is the mast, and *CI* is the boom hinged to the mast at point *C*. In most structures of this type the mast and boom are three-force members, as defined in Art. 9. *IJHBL* and *FGFDBK* are cables which lead from their supports at *I* and *F* to suitable power drums near *L* and *K*. *EM*, *EN* and *EO* are supporting guy wires which are capable of carrying tensile stress only. If only two supports are used, such as *EM* and *EN*, they must be stiff members capable of carrying either tension or compression. If, instead of the cable, a single tie *DH* is used to support the boom at a fixed level, the structure becomes the pure crane type. The dredge is shown in Fig. 77. In addition to the mast and boom the dredge has the handle which supports the load by means of a bucket or dipper. The bucket is not shown in Fig. 77.

35. Pulley Alignments.

In the problems of this chapter the stress carried by any cable will be considered as constant throughout its entire working length. The alignment of pulleys will be made with this assumption and any slight angularity of the cables will be neglected. One arrangement of pulleys and cables is shown in Fig. 74. The line of the load *L* bisects the distance *MG*.

Figure 75 shows another arrangement of pulleys and cables. The cable is fastened at *A* and the angularity of strand *AC* will be neglected. With two strands on the right side and one on the left, the line of action of load *Q* will pass $r/3$ from *B*, *r* being the radius of the pulley. This can be proved by using pulley *A* as a free body and writing the equation of moments with respect to point *B*.

A third arrangement of pulleys and cables is shown in Fig. 76(*a*). The length of *CD* must be known. To

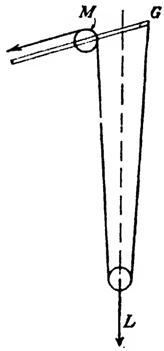


Fig. 74.

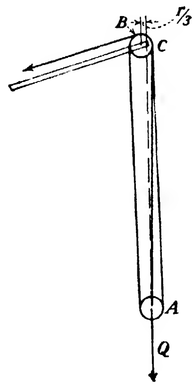


Fig. 75.

locate the direction of *CD* the parallelogram in Fig. 76(*b*) is laid off in proper stress proportion, two to one in this case. *AD* and *DE* of Fig. 76(*b*) are parallel to

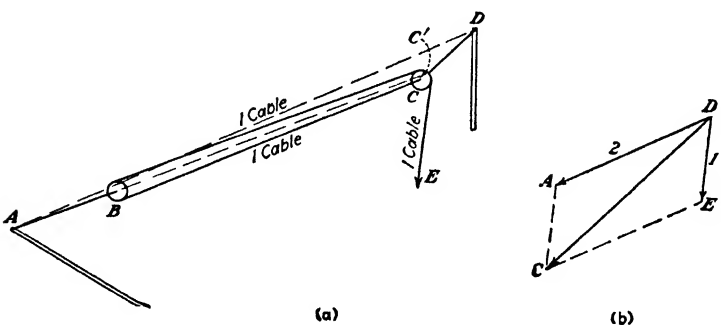


Fig. 76.

AD and the cable at *E*, respectively, of Fig. 76(*a*). The diagonal of this parallelogram represents the approximate line of action of the force in member *CD*. If the length *CD* is large, another parallelogram similar to the one shown in Fig. 76(*b*) must be drawn, using *AD* parallel to *BC′* of Fig. 76(*a*). The diagonal of the resulting parallelogram will give a closer approximation to the true line of action of the force in member *CD*. As explained earlier in this article, *C′* is distant $r/2$ from *C*. *ABC′* is a straight line and is the line of action of the resultant stress in the two strands of cable. If *CD* is small and *AD* very large, the error in assuming that *ABCD* lies on *AD* is very small. Often, instead of the short link *CD*, the pulley is fastened to the mast by a bracket. In this case no alignment problem of importance exists.

36. Pin Reactions and Cable Stresses.

The dredge shown in Fig. 77 consists of the platform *AC*; handle *DF*, weight 10,000 pounds, acting at its

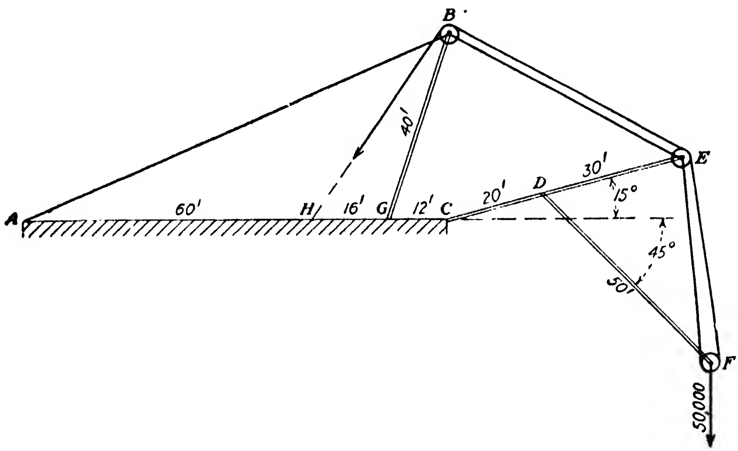


FIG. 77.

middle point; boom *CE*, weight 20,000 pounds, acting at its middle point; tie *BE*; mast *BG*; cable *EFEBH* and two backstays at *AB*.

In solving for the pin reaction at *D* and the cable stress *EF*, member *DF* is used as a free body. It is a three-force member with five forces acting upon it as follows: reaction *D*, two cable pulls *EF*, its own weight of 10,000 pounds and the load of 50,000 pounds at F. The cables are two-force members, so the directions of their stresses are known. Figure 78(a) shows this free body with its loads, known and unknown, in place. By inverse proportion, as explained in Art. 17, the load of 50,000 pounds and the weight of 10,000 pounds are combined into their resultant $R_1 = 60{,}000$ pounds.

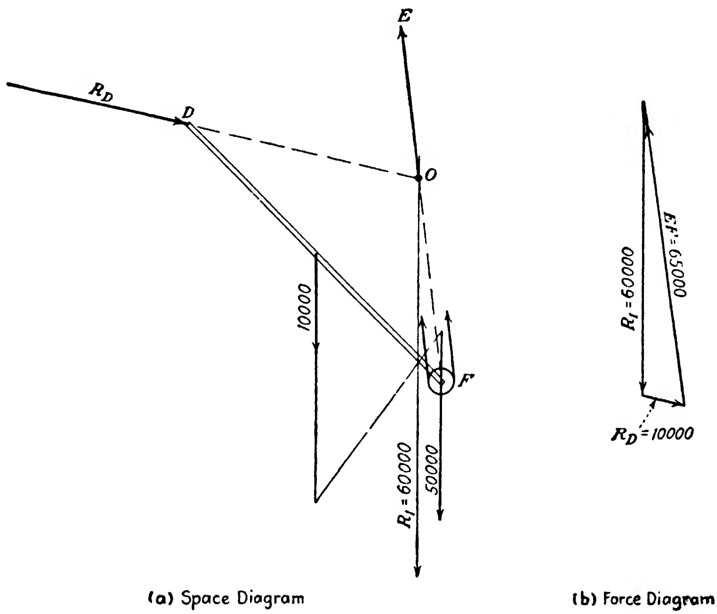


Fig. 78.

This resultant intersects the line of action of the resultant cable stress at point O. Since three forces in equilibrium must meet in a common point, the reaction at D must act through points D and O. The triangle of forces for this free body is shown in Fig. 78(b) and from it can be scaled the reaction R_D as 10,000 pounds, and the cable stress as half of FE or 32,500 pounds tension.

For a second illustration, Fig. 77 is again considered. Let it be required to find the pin reaction at C and the tie-rod stress EB. In making this solution, the free body consists of *CDEF*, the boom, handle and load, as shown in Fig. 79(*a*). The known forces are the three weights, 10,000 pounds, 50,000 pounds and 20,000 pounds, and the tensile stress in the cable of 32,500 pounds. The unknown external forces are the two required forces, the pin reaction at *C* and the tie-rod stress *EB*. By inverse proportion, the 10,000-pound load and the 50,000-pound load are combined into

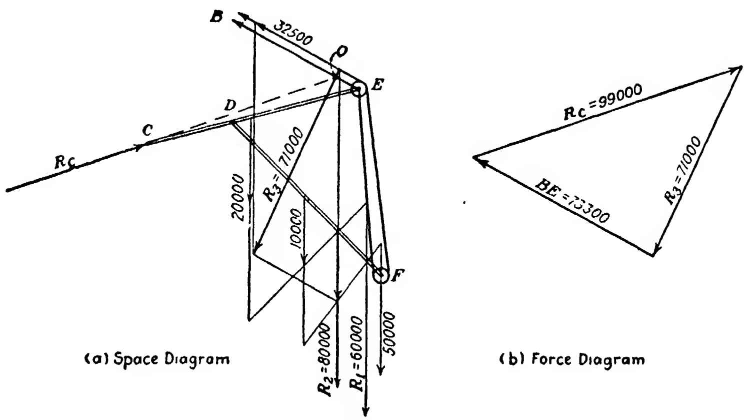


FIG. 79.

their resultant $R_1 = 60{,}000$ pounds. This resultant, by the same method, is combined with the 20,000-pound load into the resultant $R_2 = 80{,}000$ pounds. Force R_2 and the cable stress *EB′* of 32,500 pounds are then combined by the parallelogram method into the final resultant $R_3 = 71{,}000$ pounds. (The cable stress *EB′* was obtained in the last paragraph.) R_3 intersects the line of action of the unknown tie-rod stress at point *O*. The pin reaction must therefore act through points *C* and *O*. The force triangle for this free body is shown in Fig. 79(*b*) and from it the tie-rod stress *EB* scales 73,300 pounds tension, and the pin reaction at *C* scales 99,000 pounds.

A still different case occurs in the derrick shown in Fig. 80. In this derrick the cable is fastened to the boom at L, and the resultant of the stresses in CE and LE must be considered. In solving for this cable stress and the pin reaction at G, the three-force member BG is used as a free body. Upon this free body, as shown in Fig. 81(a), there are six external forces acting: two weights, 12,000 pounds and 4000 pounds; the cable

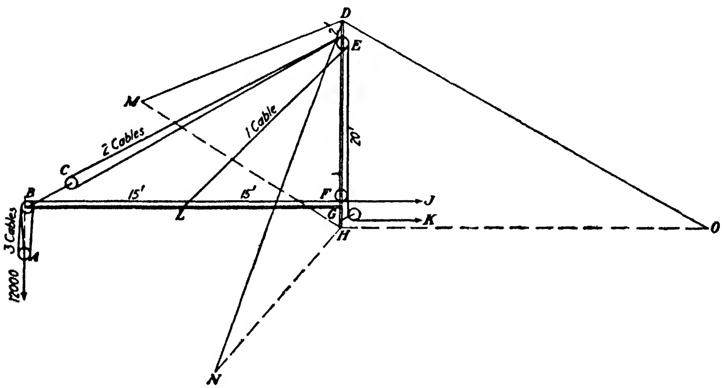


Fig. 80.

stress BL which is one-third of 12,000 pounds; the unknown pin reaction at G; and the stresses in the two strands of cable EC and the one strand of cable EL. The resultant of the three known loads can be determined by the methods already explained. This resultant is 16,500 pounds. $LECEGK$ is one cable and the stress in it is assumed to be constant throughout its entire length. Before a solution can be made of this free body, the line of action of the resultant cable stress must be determined. There are two strands of cable EC and one strand EL; therefore, with any convenient scale, a length of two units is laid off from O on OC to represent the stress in the two strands of cable EC, and from O on OL a length of one unit is laid off to represent the stress in the one strand of cable EL.

On the diagonal of the parallelogram thus formed will lie the line of action of the resultant stress in the cable.

The loads on this free body are thus reduced to three, namely: the resultant of the known loads which is 16,500 pounds, the unknown pin reaction at G and the unknown resultant stress in the cable. These three forces must

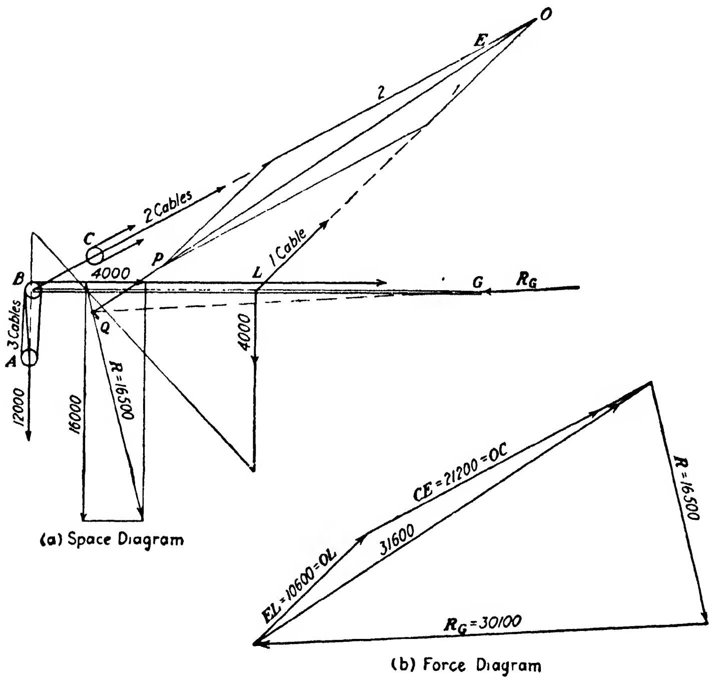


FIG. 81.

intersect at one point. The known resultant intersects the line of action of the resultant cable stress at point Q. The line of action of the pin reaction at G therefore lies on GQ. The force triangle for this free body is shown in Fig. 81(b) and from it the pin reaction at G scales 30,100 pounds and the resultant stress in the cable scales 31,600 pounds. This resultant stress in the cable is resolved into its two components: one parallel to

OC; the other parallel to *OL*. The component *OC* gives the stress in *CE* and scales 21,200 pounds tension, and the component *OL* gives the stress in *EL* and scales 10,600 pounds tension. The stress in one strand of cable is of course 10,600 pounds tension.

37. Mast and Backstay Stresses.

Cranes, derricks and dredges in which the mast and backstays may be considered as two-force members may be solved in the following manner.

Figure 82 shows a dipper dredge. The mast *EF* and the backstays *AF* and *CF* will be considered as two-

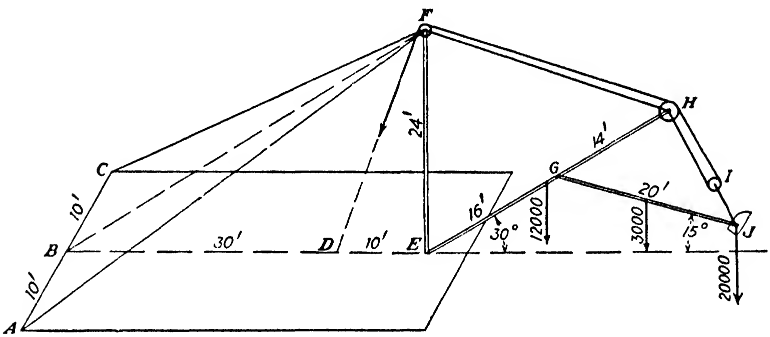
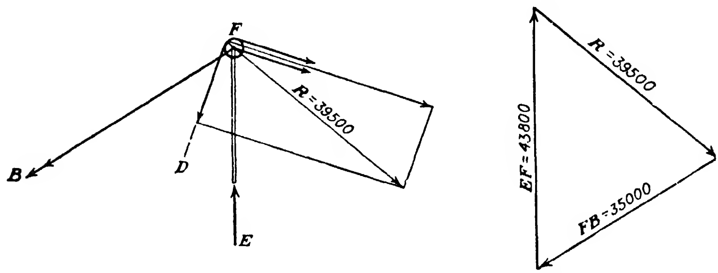


Fig. 82.

force members. The cable is *HIHFD*, and *FH* is a tie rod.

As explained in Art. 36 the handle must be the first free body, and from it the pin reaction at *G* and the tension in the cable *HI* are found to be 14,000 pounds and 14,300 pounds respectively. Next, the boom *EH*, or the boom and handle *EGHJ*, is used as a free body from which the pin reaction at *E* and the tension in the tie rod *FH* are found to be 42,400 pounds and 23,300 pounds respectively. The next free body is the joint at *F*, as shown in Fig. 83. The known cable stresses *FD* and *FH* and the tie-rod stress *FH* are combined into their resultant *R* which is 39,500 pounds. With the boom and handle in the plane of BEF, the stress in the mast and the resultant of the stresses in AF and CF are obtained. The force triangle for this free body is shown in Fig. 83(b). The stress in the mast EF scales

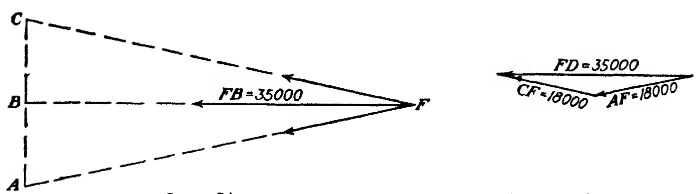


(a) Space Diagram (b) Force Diagram

FIG. 83.

43,800 pounds compression and the resultant FB scales 35,000 pounds.

In solving for the component stresses in AF and CF, a plane is passed through $ABCF$, as shown in Fig. 84(a).



(a) Space Diagram (b) Force Diagram

FIG. 84.

The solution is shown in Fig. 84(b) from which AF and CF each scale 18,000 pounds tension.

38. Tipping Forces.

In solving for backstay or guy-wire stresses for cranes and derricks of the type shown in Fig. 85, another method will be explained.

The two cables $NDJDJDGH$ and $KLKEGH$ enter the mast at H and thence pass to power drums. These cable stresses and the pin reactions at I and F must be obtained by the method of Art. 36.

The *tipping force* is the horizontal force at O produced by all of the loads on the mast, boom, handle and bucket, and is the force that produces a stress in the backstays or guy wires.

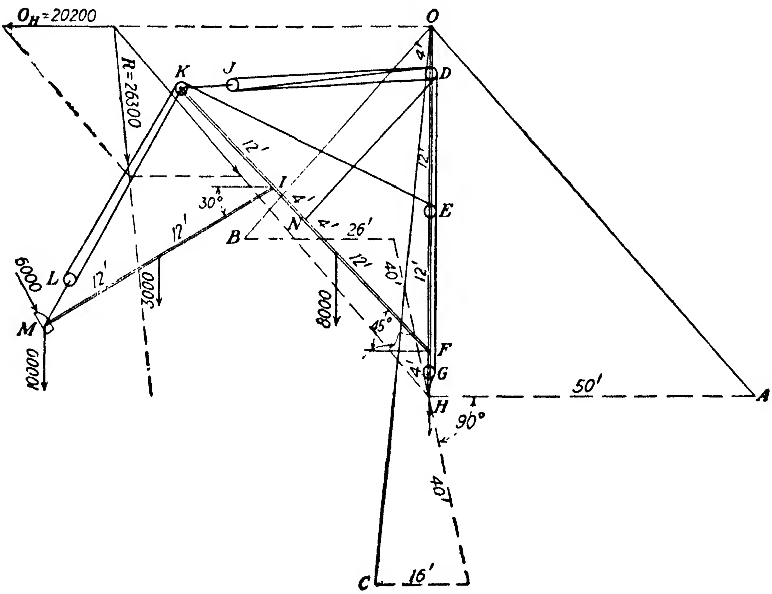


FIG. 85.

To obtain this force the mast, boom, handle and bucket together are considered as the free body, as shown in Fig. 85, and all of the known external loads are combined into their resultant R = 26,300 pounds. (The solution for R is not shown on the figure.) This resultant should really contain the cable stresses at H, but since in this case they act at the base of the mast and have no effect on the tipping force at O, they have been omitted. This resultant R is resolved into two components: *one horizontal through point O*; the other acting through the base of the mast at H. The tipping force O_H from this solution scales 20,200 pounds. The reason that O_H is the true tipping force becomes obvious if the equation of moments with respect to point H is considered.

39. Maximum Stresses.

For illustration let it be required to find the maximum tensile stress in the guy wire OA of the derrick dredge shown in Fig. 85. The horizontal projection of the guy wires is shown in Fig. 86(a). The three supporting

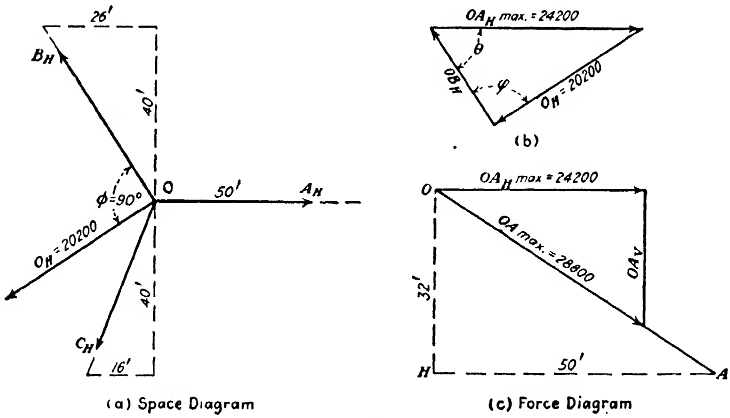


FIG. 86.

members OA, OB and OC are guy wires and therefore can carry tension only. The tipping force O_H of 20,200 pounds as obtained in the preceding article acts at O and may swing through the entire circle. As shown below, the position of O_H to produce the maximum tension in OA will be at an angle of 90° from OB_H. OC, being a guy wire, cannot carry compression and therefore has zero stress. From the triangle of forces in Fig. 86(b), the value of OA_H may be scaled as 24,200 pounds. The true stress in OA is now obtained from Fig. 86(c) where the true length and inclination of OA are shown. From point O the value of OA_H is laid off horizontally. The vertical component OA_V is laid off from the end of OA_H. The diagonal thus formed, which lies on the line OA, Fig. 86(*c*), represents to scale the true maximum tension in OA and scales 28,800 pounds.

The proof that $\varphi = 90°$ for maximum stress is as follows: From Fig. 86(*b*), by the law of sines, $OA_H = O_H \sin \varphi / \sin \theta$. O_H and θ are constant, but φ may vary. To give the maximum value of OA_H the sine of φ must be 1.00 and, therefore, $\varphi = 90°$.

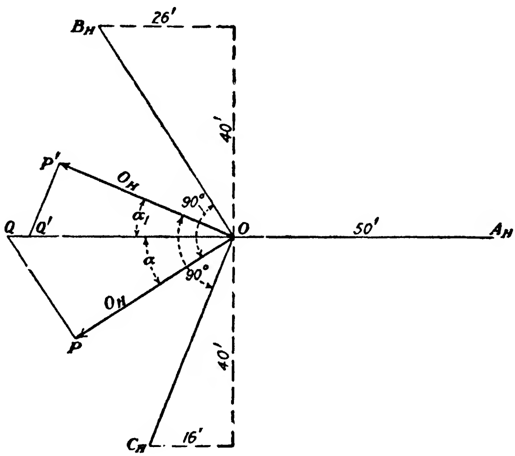


FIG. 87.

In this illustration the tipping force O_H might have been placed at an angle of 90° with OC_H, but in this position OA_H is not as large as when vector O_H is placed at an angle of 90° with OB_H. To determine which of these two possible positions governs, the procedure is as follows. Figure 86(*a*) is reproduced in Fig. 87. Vector O_H is placed at an angle of 90° with OB_H and the force triangle OPQ is drawn (stress $OC = O$). Next, vector O_H is placed at an angle of 90° with OC_H and its force triangle $OP'Q'$ is drawn (stress $OB = O$). From the figure it is seen that OQ, and not OQ', gives the greatest value of OA_H. The governing position for the boom

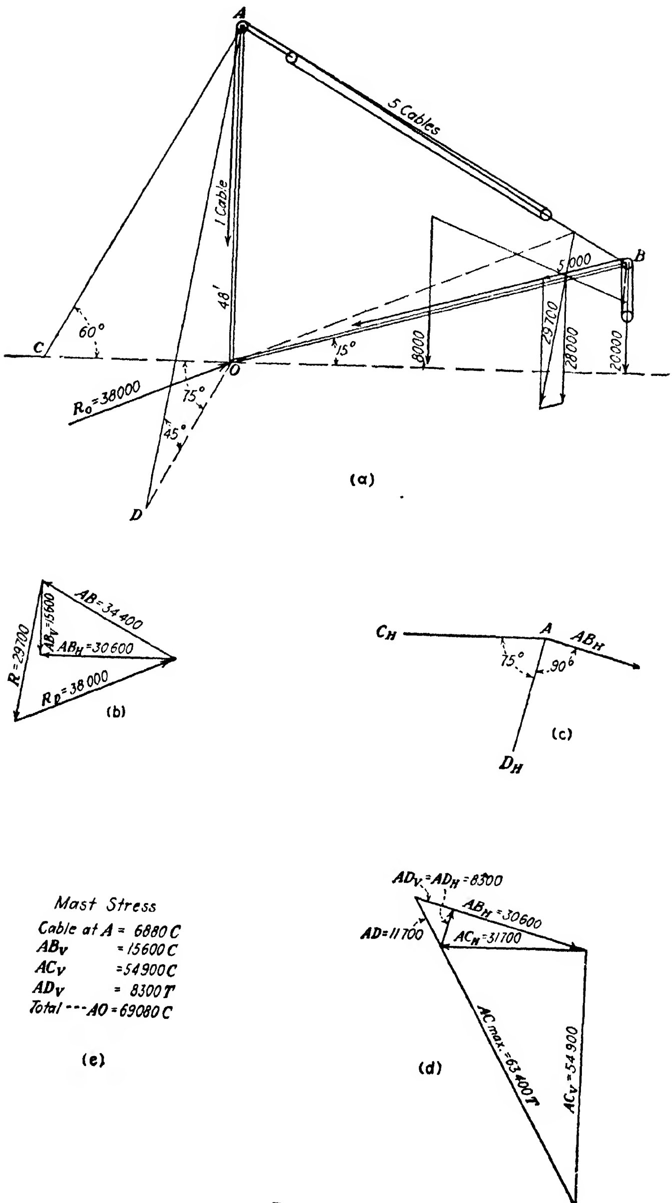

A
5 Cables
1 Cable
48'
B
5000
60°
C
15°
8000
29700
28000
20000
O
R_o=38000
75°
45°
D
(a)
AB=34400
AB_V=15600
AB_H=30600
R=29700
R_o=38000
(b)
C_H
A
AB_H
75°
90°
D_H
(c)
AD_V=AD_H=8300
AB_H=30600
AD=11700
AC_H=31700
AC max.=63400T
AC_V=54900
(d)
Mast Stress
Cable at A = 6880 C
AB_V = 15600 C
AC_V = 54900 C
AD_V = 8300 T
Total ---AO = 69080 C
(e)

Fig. 88.

or tipping force O_H is that position in which α, the angle which O_H makes with the line of OA_H produced, is the larger.

40. Examples.

As a review and further illustration of the principles and methods taken up in this chapter, the following examples are given with their complete graphic solutions. A statement of the steps taken, together with a brief explanation, accompanies each solution.

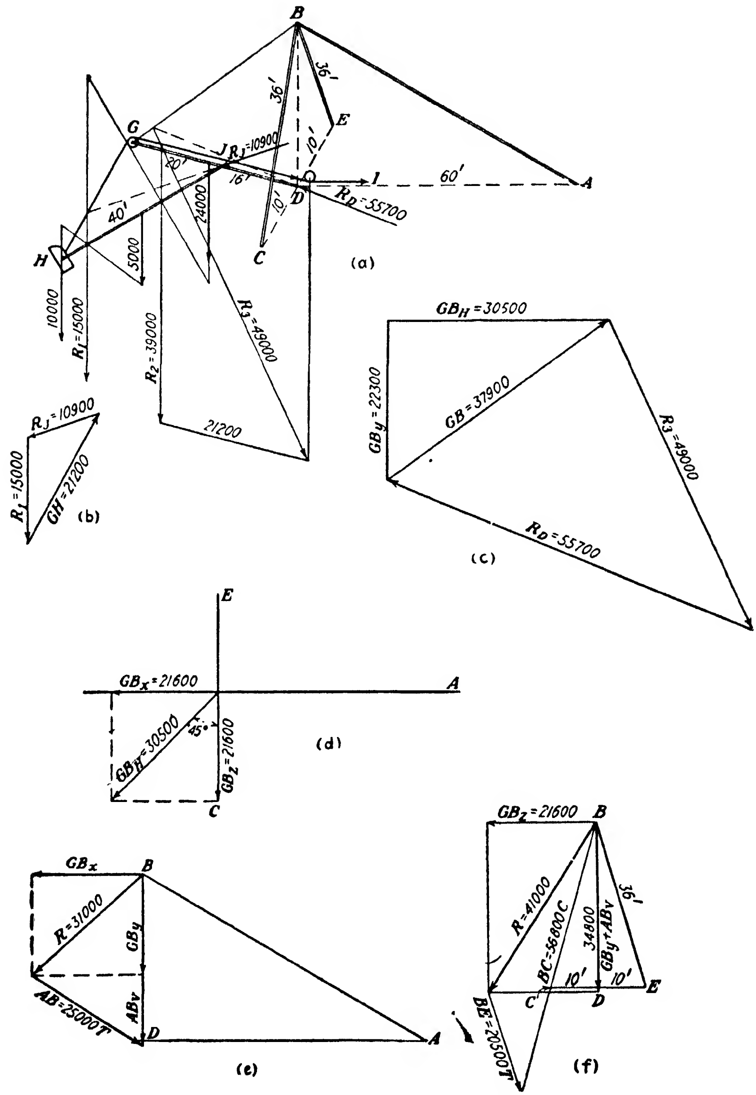
Example 1. The derrick crane shown in Fig. 88(a) has a mast OA, 48 ft. long; and boom OB, 64 ft. long. The boom weighs 8000 lb. and its center of gravity is 30 ft. from O. All pulleys are 2 ft. in diameter. Angle AOB may vary between 15° and 75°. The boom may swing from OC to OD. Solve for the reaction at O and the stress in cable AB. Place the boom in the position for maximum tension in AC and solve for this tension and the corresponding compression in the mast.

Solution: Since the 20,000-lb. weight is held by four strands of cable, the stress in each strand is 5000 lb.

The boom is the first free body. The loads of 20,000 lb. and 8000 lb. and the cable stress of 5000 lb. are combined into their resultant which scales 29,700 lb. The pin reaction at O acts through O and the point where the 29,700-lb. resultant intersects the line of action of the resultant cable stress AB. The solution for this free body is shown in Fig. 88(b) from which R_O scales 38,000 lb., and AB scales 34,400 lb. The stress in one strand of cable along AB is therefore 6880 lb. tension.

The tipping force at A is obtained as the horizontal component of AB and is 30,600 lb. (This is true only when the mast is a two-force member, otherwise the method described in Art. 38 is to be used.)

Figure 88(c) shows the plan view and the position of the boom for maximum tension in AC. The force triangle for this projection is shown in Fig. 88(d), from which AC_H scales



B
36'
36'
E
G
J
R_J=10900
10'
20'
16'
60'
I
A
D
R_D=55700
40'
24000
10'
C
(a)
H
5000
10000
R_1=15000
R_2=39000
R_3=49000
21200
R_J=10900
R_1=15000
GH=21200
(b)
GB_H=30500
GB_y=22300
GB=37900
R_3=49000
R_D=55700
(c)
E
GB_x=21600
A
GB_H=30500
45°
GB_z=21600
(d)
C
GB_x
B
R=31000
GB_y
AB=25000T
AB_V
D
A
(e)
GB_z=21600
B
R=41000
BC=56800C
34800
GB_y+AB_V
36'
10'
10'
C
D
E
BE=20500T
(f)

FIG. 89.

31,700 lb. and AD_H scales 8300 lb. On this force triangle are erected the triangles from which the true stresses and the vertical components of these stresses are obtained. AC_V is perpendicular to AC_H, and AC is drawn at an angle of 60° with AC_H. AC (maximum) scales 63,400 lb. tension. Similarly AD_V is perpendicular to AD_H, and AD is drawn at an angle of 45° with AD_H. AD scales 11,700 lb. compression.

To get the compression in the mast, the vertical components of the forces at A are added algebraically. This is shown in tabular form in Fig. 88(*e*). The compression in the mast is 69,080 lb.

Example 2. Figure 89 shows a dipper dredge. The A-frame is vertical, and line AD is normal to the plane BCE. The pulleys are 2 ft. in diameter. Cable HGI extends to a hoisting engine. Boom GD weighs 24,000 lb. with its center of gravity at its middle point. Handle JH weighs 5000 lb. with its center of gravity at its middle point. The bucket and load weigh 10,000 lb. In the dumping position the boom is at an angle of 15° with the horizontal, handle JH is at an angle of 30° with the horizontal and the boom and handle are swung toward C tnrough an angle of 45°. Solve for the pin reactions at J and D, the stress in the cable and the stresses in the three supporting stays when in this dumping position.

Solution: The first free body consists of the bucket, load and handle. The 10,000-lb. and 5000-lb. loads are combined into their resultant which is 15,000 lb. The pin reaction at J acts through J and the point where the 15,000-lb. resultant intersects the line of action of the unknown cable stress GH. The force triangle for this free body is shown in Fig. 89(*b*). Vector R_J scales 10,900 lb., and the tension in the cable GH scales 21,200 lb.

The next free body consists of the boom, handle, bucket and load. The resultant of all of the known loads, 10,000 lb., 5000 lb., 24,000 lb. and 21,200 lb., is 49,000 lb., as shown in Fig. 89(*a*). The pin reaction at D acts through D and the point where the 49,000-lb. resultant intersects the line of action of the unknown tie-rod stress GB. The force triangle for this free body is shown in Fig. 89(*c*) from which R_D scales 55,700 lb. and GB scales 37,900 lb. tension.

The stress in GB is next resolved into three rectangular components as explained in Art. 23. First, as shown in Fig. 89(c), it is resolved into GB_y, 22,300 lb.; and GB_H, 30,500 lb.

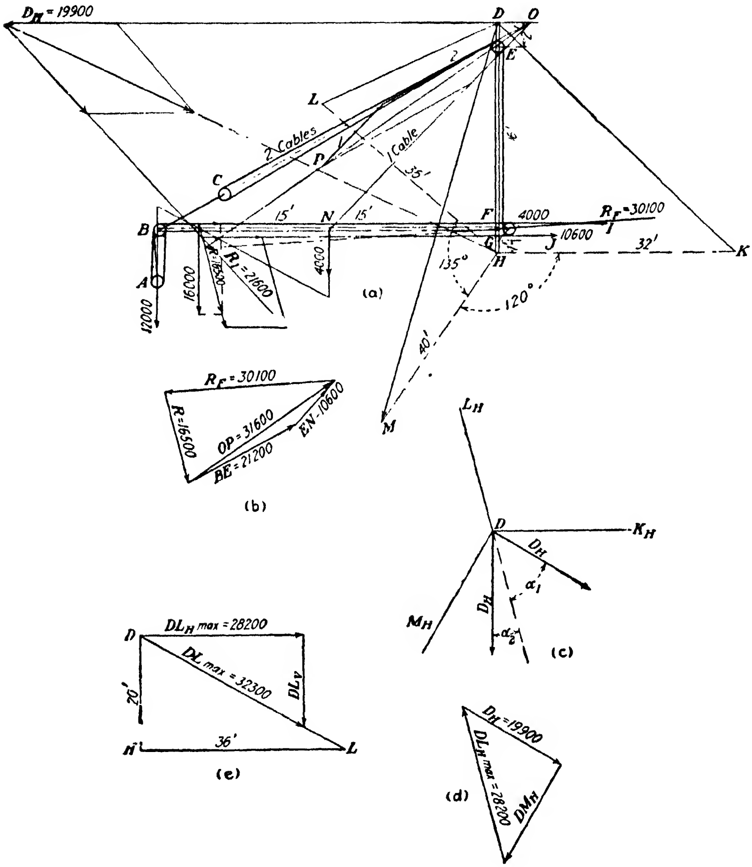


Fig. 90.

GB_H is then resolved, as shown in Fig. 89(d), into GB_x and GB_z, each 21,600 lb.

Stresses GB_x and GB_y act in the plane of ABD and in this plane the value of the stress in the backstay AB can be obtained. Its force triangle is shown in Fig. 89(e) from which AB scales 25,000 lb. tension.

Figure 89(*f*) shows the projection of the forces at B on the plane of BCE, and also shows the solution for the stresses in members BC and BE. GB_y, AB_V and GB_z are combined into their resultant, 41,000 lb. The triangle of forces of this resultant, BC and BE, gives the required stresses. BC scales 56,800 lb. compression, and BE scales 20,500 lb. tension.

Example 3. Figure 90 shows a derrick. The boom is 30 ft. long and weighs 4000 lb. with its center of gravity at N. The mast HD is 20 ft. high. There are two cables, $ABABI$ and $NECEGJ$. The boom is horizontal and may swing in the complete circle. Solve for the pin reaction at F, the cable stresses and the maximum tension in DL.

Solution: The first free body is the pulley at A from which the cable stress of 4000 lb. is obtained by inspection. The next free body consists of the 12,000-lb. load and the boom BF. The known loads of 12,000 lb., 4000 lb. and the 4000-lb. cable stress are combined into one resultant, 16,500 lb. The point of intersection of this resultant and the line of action of the resultant stress in the cable $NECEGJ$ is a point on the line of action of the pin reaction at F. The line of action of this resultant cable stress, OP, must be determined by the method of Art. 36. The force triangle is shown in Fig. 90(*b*) and from it the stress in the cable scales 10,600 lb. tension, and the pin reaction at F scales 30,100 lb.

The boom and mast together constitute the next free body. The resultant of all the external loads, 12,000 lb., 4000 lb., the 4000-lb. cable stress and the 10,600-lb. cable stress, is 21,600 lb. By the method of Art. 38 this resultant is resolved into two components, the tipping force at D, 19,900 lb., and a component through the base of the mast. This construction is shown in Fig. 90(*a*).

The plan view of the guy wires is shown in Fig. 90(*c*). The two possible positions for maximum tension in DL are shown, and, as α_1 is larger than α_2, the tipping force must be placed at an angle of 90° with DM (stress $DK = O$). The solution for DL_H maximum is shown in Fig. 90(*d*) and its vector scales 28,200 lb. The final part of the solution is shown in Fig. 90(*e*) where the true length and inclination of DL are shown to scale. Vector DL_H maximum is laid off and vector DL_V completes the triangle from which DL maximum scales 32,300 lb. tension.

PROBLEMS

NOTE: In the following problems, the scales given are based on the standard-size drawing plate as recommended in Art. 3.

Problem 1. Figure 91 shows a smokestack. Consider the resultant wind pressure of 3000 lb. as acting at the center of the stack. With this wind pressure in the position to produce maximum tension in AD, solve for this tension. Scales: 1 in. = 10 ft.; 1 in. = 600 lb.

Ans. AD = 6190 lb. T

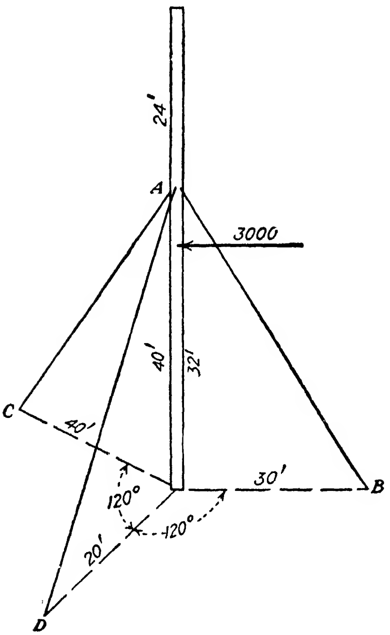


FIG. 91.

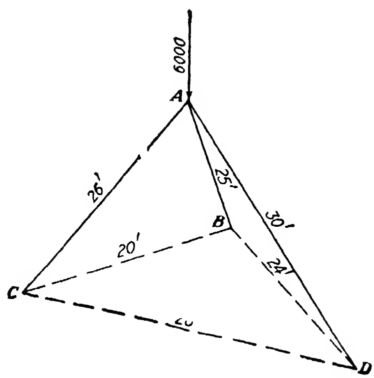


FIG. 92.

Problem 2. The tripod, shown in Fig. 92, has a load of 6000 lb. acting vertically at A. Solve for the stresses in AB, AC and AD. Scales: 1 in. = 5 ft.; 1 in. = 1000 lb.

Ans. AB = 2720 lb. C; AC = 2650 lb. C; AD = 1550 lb. C.

Problem 3. The tripod, shown in Fig. 93, supports a load of 4000 lb. by means of a cable over a pulley at A. The cable T passes thence to a pulley at E. Solve for the stresses in

AB, AC and AD. Scales: 1 in. = 5 ft.; 1 in. = 1000 lb.

Ans. AB = 3480 lb. C; AC = 700 lb. T; AD = 5250 lb. C

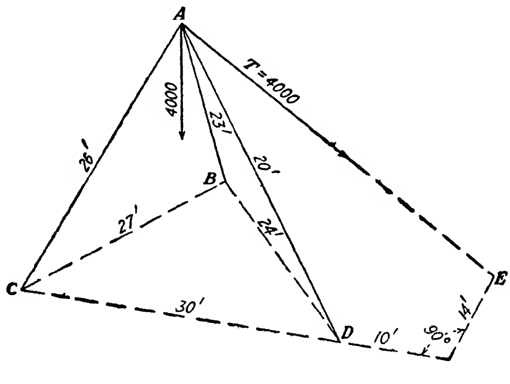


FIG. 93.

Problem 4. Figure 94 shows a derrick. The boom weighs 8000 lb. with its center of gravity 10 ft. from E, and is placed

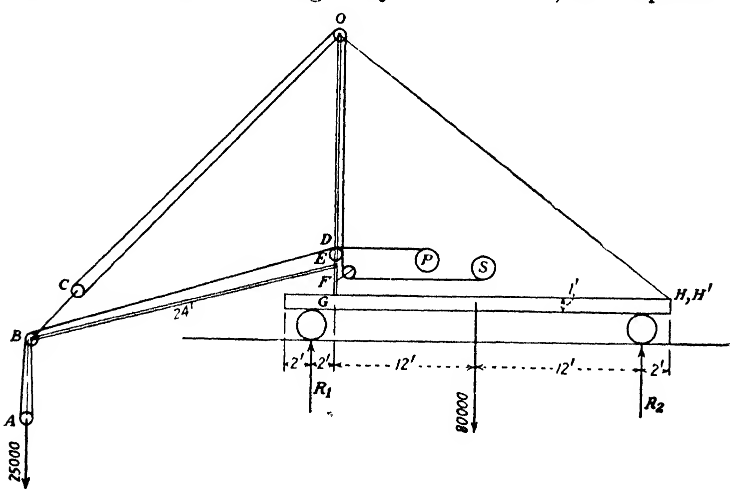


FIG. 94.

15° below the horizontal. Mast OG weighs 10,000 lb. Distances GF, FE and ED are each 1 ft. and height of mast OG is 20 ft. The backstays OH and OH' form an isosceles triangle with the vertex at O and base HH' 10 ft. All pulleys are 1 ft. in diameter and the wheels are 2 ft. in diameter. There are two cables, $OCOFS$ and $BABDP$. Solve for the cable stresses, the pin reactions at E and G, reactions R_1 and R_2 and the stresses in OH and OH'. Scales: 1 in. = 4 ft.; 1 in. = 10,000 lb.

Ans. R_2 = 20,700 lb.; OH = OH' = 18,500 lb. T.

Problem 5. Figure 95 shows an excavator. The boom weighs 4000 lb. with its center of gravity at its middle point and is placed at an angle of 30° above the horizontal. The mast, weight neglected, is 60° with the horizontal. The two

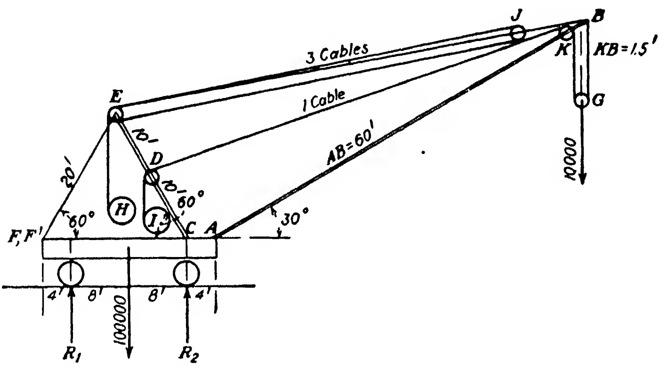


FIG. 95.

backstays form an isosceles triangle with its vertex at E and base FF' 12 ft. long. Tie JB is 4 ft. long. All pulleys are 1 ft. in diameter and the wheels are 2 ft. in diameter. Solve for the cable stresses, the pin reactions at A and C, the stresses in EF and EF' and the reactions R_1 and R_2. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb.

Ans. EF = 14,000 lb. T; R_2 = 106,000 lb.

Problem 6. The structure, shown in Fig. 96, is an extension boom derrick. EG may vary from 4 ft. to 20 ft. The platform weighs 100,000 lb. with its center of gravity midway between wheels A and B. The boom weighs 20,000 lb. with its center of gravity, when fully extended as shown, 20 ft. from E. The pulley near E is on a bracket fastened to the boom, and, for the position shown, DE is a straight line. All pulleys are 1.5 ft. in diameter and the wheels are 3 ft. in diameter. The pulley near E is 2 ft. from the boom. Solve for the stresses in the cables, the pin reaction at H and the reactions at A and B when the load at I is 5000 lb. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb.

Ans. R_H = 51,400 lb.; R_B = 102,000 lb.

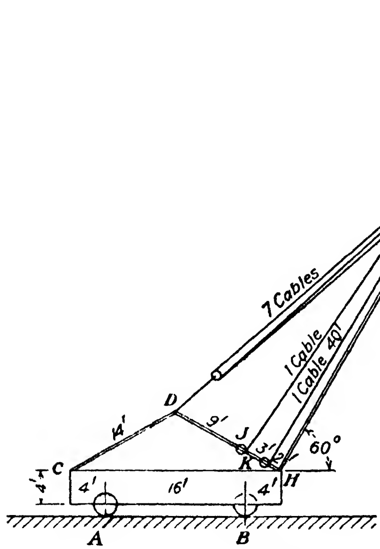


FIG. 96.

Problem 7. With the same general data as in Example 1, place the boom in the position to produce: (*a*) maximum tension in AD; (*b*) maximum compression in AC. Solve for these maximum stresses and the corresponding stresses in the mast. Scales: 1 in. = 10 ft.; 1 in. = 6000 lb.

Ans. (*a*) AD = 44,900 lb. T; AO = 39,780 lb. C. (*b*) AC = 63,400 lb. C; AO = 24,100 lb. T.

Problem 8. Figure 97 shows a dredge. The A-frame BCE is vertical. Line AD is normal to the plane BCE. All pulleys are 2 ft. in diameter. The boom weighs 24,000 lb. with its center of gravity at its middle point. Handle JFH weighs 5000 lb. with its center of gravity at its middle point. The bucket and load weigh 10,000 lb. with the center of gravity at H. In the filling position, boom GD is at an angle of 30°

with the horizontal; handle JFH is at an angle of 75° with the horizontal; and the pressure normal to the handle at the bucket is 6000 lb. The boom and handle are in the plane of ABD. Solve for the stress in the cable, the pin reactions at F and D

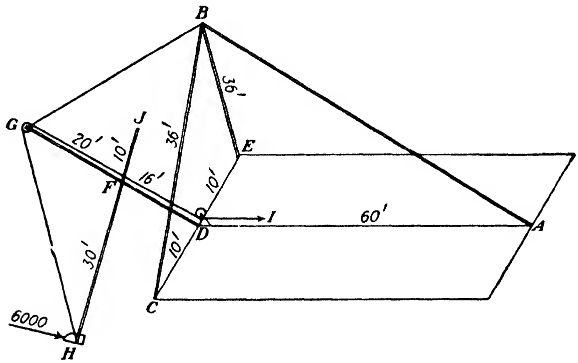


FIG. 97.

and the stresses in AB, BC and BE. Scales: 1 in. = 10 ft.; 1 in. = 6000 lb.

Ans. AB = 27,200 lb. T; BC = BE = 13,700 lb. C.

Problem 9. Figure 98 shows a dredge with a sloping A-frame. The boom weighs 16,000 lb. with its center of

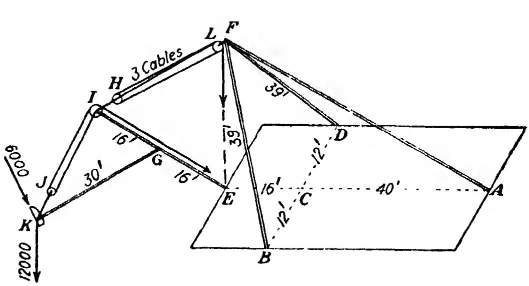


FIG. 98.

gravity 14 ft. from E, and is at an angle of 15° with the horizontal. The handle weighs 4000 lb. with its center of gravity at its middle point and is at an angle of 30° with the horizontal. Pulley I is 2 ft. in diameter; all other pulleys are 1 ft. in diameter. Tie rod FL is 1 ft. long. The boom is rotated toward D through an angle of 60° from the plane of AEF. With the boom in this position, solve for all cable stresses, pin reactions, and the stresses in AF, BF and DF. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb.

Ans. AF = 56,400 lb. T; BF = 4900 lb. T; DF = 83,600 lb. C.

Problem 10. With the same general data as in Example 3, place the boom in the position to produce maximum tension in DM, and solve for this maximum tension. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb. All pulleys are 1 ft. diameter.

Ans. DM = 31,500 lb. T.

Problem 11. Figure 99 shows a derrick. The boom AG is 60 ft. long and weighs 4000 lb. with its center of gravity

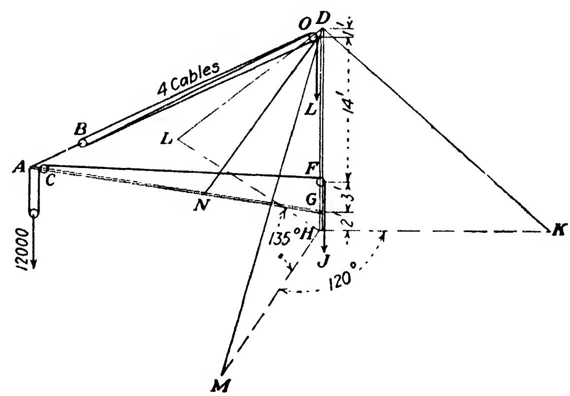


FIG. 99.

at its middle point. Pulley O is on a bracket with its center 1 ft. from the axis of the mast. Angle AGD may vary from 15° to 90°. The boom may swing through the complete circle horizontally. AB = 4 ft.; AC = 1.5 ft.; NG = 24 ft. All pulleys are 1 ft. in diameter. DK, DL and DM are guy wires. HK = 40 ft.; HL = 50 ft.; HM = 65 ft. Solve for all of the cable stresses, the pin reaction at G and the maximum tension in DK. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb.

Ans. DK = 54,100 lb. T.

Problem 12. Figure 100 shows a derrick. The boom is 50 ft. long and weighs 4000 lb. with its center of gravity at its middle point. Pulley E is on a bracket with its center 1 ft. from the axis of the mast. Angle AGD may vary from 15° to 75°. The boom may swing through the complete circle

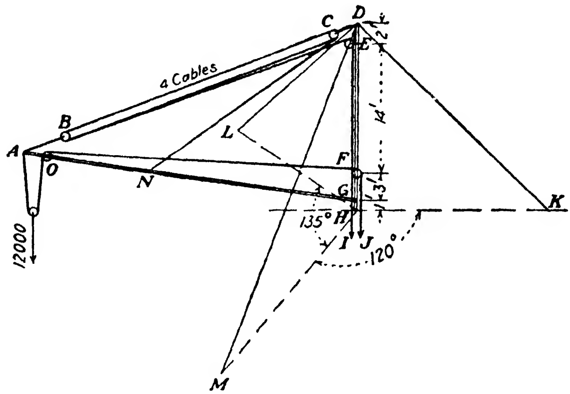


Fig. 100.

horizontally. DK, DL and DM are guy cables. HK = 30 ft.; HL = 56 ft.; HM = 65 ft.; AO = 2 ft.; AN = 20 ft.; GN = 30 ft.; AB = 3 ft.; CD = 3 ft. All pulleys are 1 ft. diameter. Solve for all cable stresses, the pin reaction at G and the maximum tension in DL. Scales: 1 in. = 6 ft.; 1 in. = 6000 lb.

Ans. DL = 50,500 lb. T.

CHAPTER V

MACHINES

41. Machines.

The computation of the various stresses and reactions set up in a machine often becomes a very tedious process. This is especially true when there are many moving parts and friction is considered. Such problems may often be solved much more readily by the graphical method than by any other.

The principal types of reactions involved in machines are reactions at sliding surfaces, reactions between journals and bearings, reactions between gears and reactions at rolling surfaces. These will be taken up in order.

42. Reactions at Sliding Surfaces.

In general the resultant reaction between two sliding surfaces will consist of two components, one normal to the surfaces and the other tangent to the surfaces, the latter being due to friction. When slipping is occurring, the ratio of the tangential component to the normal component is called the *coefficient of kinetic friction*. Within certain limits of temperature, speed, time, etc., this coefficient is practically a constant, depending only on the materials.

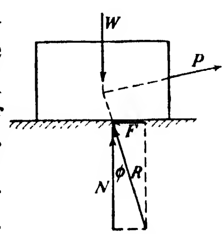


FIG. 101.

In Fig. 101, which represents a block being pulled along a horizontal surface by force P, the reaction R of the surface upon the block is the resultant of the normal pressure N and the frictional resistance F.

By the definition above, the coefficient of friction f is the ratio of F to N. Thus, $f = F/N$ or $F = fN$. The angle φ between the resultant R and the normal to the surface is called the *angle of kinetic friction* or, more simply, the *angle of friction*. From the figure, tan $\varphi = F/N$, or $\varphi = \tan^{-1} f$. This shows that the resultant reaction at sliding surfaces is inclined to the normal at an angle whose tangent is the coefficient of kinetic friction. It should be noted in particular that the resultant reaction is inclined to the normal in such a direction as to oppose the motion of the free body, relative to the surface with which it is in contact. If the friction is so small as to be negligible, the tangential component F disappears and the resultant reaction becomes normal to the surfaces.

The wedge and block, shown in Fig. 102(a), will be used as an example. Let it be required to determine the force P necessary to raise the block A, if the coefficient of friction is 0.2 at all surfaces.

The free-body diagram for the block A is shown in Fig. 102(b). The resultant reactions, R_1 between A and C, and R_2 between A and B, are each inclined to their respective normals at the angle φ whose tangent is 0.2. The angle φ is easily laid off in the following way. With any convenient scale, ten units are laid off from the surface in the normal direction and from this point two units are laid off in the tangential direction. The diagonal is then drawn. The tangent of the angle between the diagonal and the normal is thus 0.2. The reactions R_1 and R_2 must each be inclined so as to oppose the motion of the block A. Since the block moves upward relative to the wall C, the reaction R_1 must have a component downward and thus lies on the upper side of the normal. Since the block A moves to the left relative to the wedge B, the reaction R_2 must have a component acting to the right and therefore lies on the left side of the corresponding normal. The force triangle of Fig. 102(c) is then laid off to some convenient scale and the values of R_1 and R_2 are found to scale 449 pounds and 956 pounds respectively.

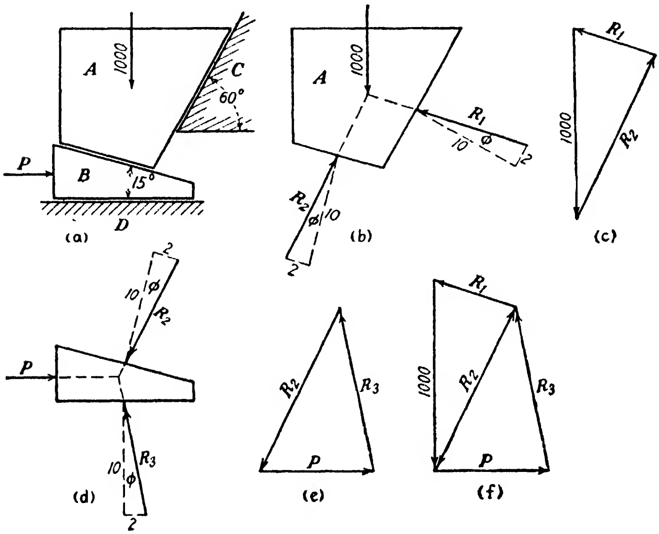


Fig. 102.

The reaction R_2 thus obtained now becomes the known force on the wedge B whose free-body diagram is shown in Fig. 102(d). The triangle of forces for this free body is shown in Fig. 102(e) from which P scales 595 pounds and R_3 scales 875 pounds. The two force triangles may be combined into a single force diagram as shown in Fig. 102(f).

If friction were negligible, each of the reactions would be normal to the surface on which it acts. The subsequent procedure involved in the solution would be the same as that in the above example with friction, and the results obtained would be: $R_1 = 268$ pounds, $R_2 = 899$ pounds, $R_3 = 866$ pounds and $P = 232$ pounds.

43. Journal Reactions: the Friction Circle.

For obvious reasons a bearing must be slightly larger than the journal which rotates within it and this difference in size is gradually increased by wear in service. It is seen, therefore, that a journal is in contact with its bearing only along a narrow arc instead of around the complete circumference. For all practical purposes, the pressure may be considered to be acting along a line of

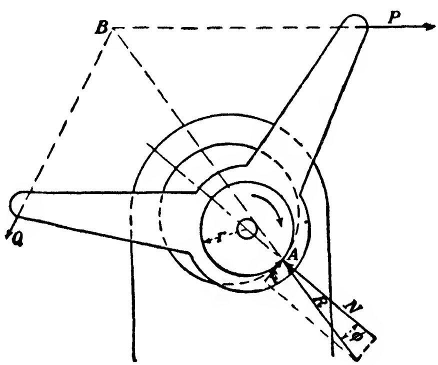


Fig. 103.

contact at the middle of the arc of pressure, which in the plane projection of the bearing becomes a point. If the friction is so small as to be negligible, this point of contact is not changed in position when the journal rotates. The reaction between the journal and its bearing must pass through this point and must be normal to the surfaces. It must therefore pass through the center of the journal and the center of curvature of the bearing. If the friction is appreciable, however, the journal will roll from its position of rest until the resultant reaction of the bearing acts at the angle of friction φ with the normal or radius at the point of contact, when slipping of the journal on the bearing will take place. If the coefficient of friction remains constant, the journal will remain in this position as it rotates. This condition is shown in Fig. 103. The driving force P causes the crank to rotate clockwise against the resistance Q. The journal of the crank rolls up the right side of the bearing until it reaches point A where it begins to slip. At this point, the resultant reaction R is inclined at the angle of friction φ with the normal and in such a direction as to oppose the slipping. It will be seen that the reaction R does not pass through the center of the journal and bearing but is tangent to a small circle, concentric with the journal whose radius is $r \sin \varphi$. This circle is called the *friction circle.* If the coefficient of friction is small, $\sin \varphi$ may be taken as equal to $\tan \varphi$ with no appreciable error. Then the radius of the friction circle becomes $r \tan \varphi$ or fr.

When the radius r and the coefficient f are known, the friction circle can be drawn and then used to locate the point of contact of the journal and its bearing. In Fig. 103, P and Q are extended to intersect at point B. The resultant reaction R must pass through point B and must also be tangent to the friction circle. The point of contact A is thus determined. In order to determine on which side of the friction circle the reaction must be tangent, any one of the following rules may be used:

1. *The reaction is tangent to the friction circle on that side of the bearing toward which the journal rolls.* In Fig. 103, the journal is seen to roll upward on the right side of the bearing. Therefore, the reaction R is tangent to the friction circle on the right side.

2. *The reaction is tangent to the friction circle on that side which reduces the mechanical advantage.* Friction always opposes the driving force and aids the resistance. In Fig. 103, the reaction R must be tangent on the right side of the friction circle, so as to reduce the lever arm

of the driving force P and increase the lever arm of the resistance Q.

3. *The reaction is tangent to the friction circle on that side where the pressure exerted by the free body is in the same direction as its relative rotation.* The resultant action of forces P and Q upon the crank causes the latter to press downward toward the right upon the bearing. The direction of rotation of the crank is clockwise. It will be seen, from Fig. 104, that the direction of the pressure exerted by the crank upon the bearing and the direction of rotation of the crank coincide on the right side of the friction circle and are opposite on the left side. This rule is perhaps somewhat mechanical in its manner of application, but, when once understood, it furnishes a very quick way for determining the side of tangency.

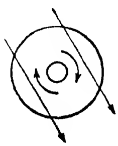


FIG. 104.

The above rules apply equally well if the journal is stationary and the bearing turns. Therefore, in the

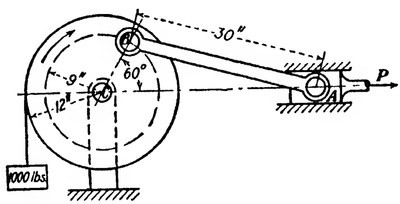


FIG. 105.

analysis of a given free body, the journal may be made a part of the free body with the bearing external, or the bearing may be considered as a part of the free body with the journal external. In order to illustrate further their manner of application, each of the above rules will be used in turn in the analysis of subsequent problems.

The hoisting mechanism, shown in Fig. 105, will be used as the first example. The mechanism consists of a drum C from which the load is suspended by means of a flexible cable, a connecting rod AB and a crosshead A which is acted upon by the steam pressure P, transmitted through the piston rod. The force P, required to raise the load, is to be determined when the crank pin B is in the position shown. All bearings are 3 inches in diameter. The coefficient of friction is 0.15 for the bearings and 0.3 for the crosshead A and guides.

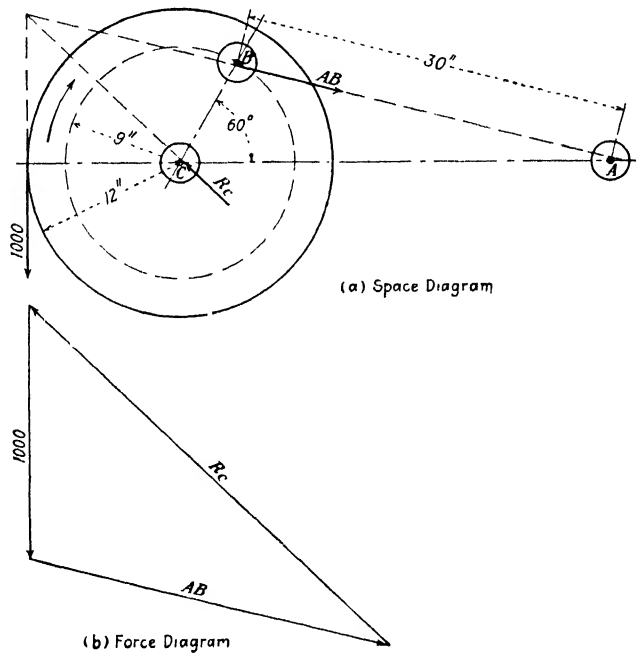


FIG. 106.

The first free body is the rotating drum, Fig. 106(a), which is acted upon by three forces, the 1000-pound load, the tension in the connecting rod AB and the bearing reaction at C. The line of action of the stress AB must be tangent to the friction circles at A and B. The diameter of these circles, as previously explained, will be the product of the diameter of the journal and the coefficient of friction or 3 inches $\times$ 0.15 = 0.45

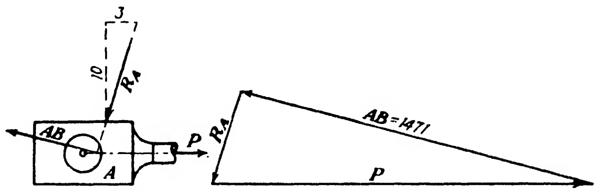
inch. By applying the first rule to the friction circle at A, assuming the connecting rod to carry the journal and the bearing to be a part of the crosshead, it will be seen that the tension in the rod AB pulls the journal at A to the left side of the bearing. The journal at A is rotating counterclockwise in this bearing and therefore, due to friction, tends to roll up. Hence the line of action of the stress AB is tangent on the upper side as shown.

By applying the second rule to the friction circle at B, it is seen that in order to diminish the mechanical advantage of the driving force AB, its lever arm with respect to C must be shortened by making its line of action tangent to the friction circle on the lower side. The direction of stress AB is thus determined by the common tangent to the friction circles at A and B.

The reaction R_C must pass through the intersection of the force AB and the line of action of the load, and at the same time must be tangent to the friction circle at C. The resultant effect of force AB and of the load upon the drum is to cause the journal to press downward and to the right upon the bearing. The drum rotates clockwise. By applying the third rule, it is seen that the reaction R_C must be tangent on the right side of the friction circle because the pressure is in the direction of rotation on this side.

The directions of the forces having been thus determined, the force triangle, Fig. 106(b), can be drawn, from which R_C and AB scale 1975 pounds and 1471 pounds respectively.

The crosshead A is now taken as the free body, the external forces being AB, P and the guide reaction R_A. If a small clearance is assumed between the crosshead and the guides, there will be contact on one side only. Since the connecting rod AB is in tension, it will tend to lift the crosshead up against the upper guide. Thus the guide reaction will be exerted downward on the upper side of the crosshead. This reaction must pass through the intersection of forces AB and P and must be inclined to the normal at the angle φ whose tangent



(a) Space Diagram (b) Force Diagram

FIG. 107.

is 0.3, and in such direction as to oppose the motion of the crosshead. This is shown in Fig. 107(a). From the force triangle, as drawn in Fig. 107(b), forces P and R_A scale 1539 pounds and 389 pounds respectively.

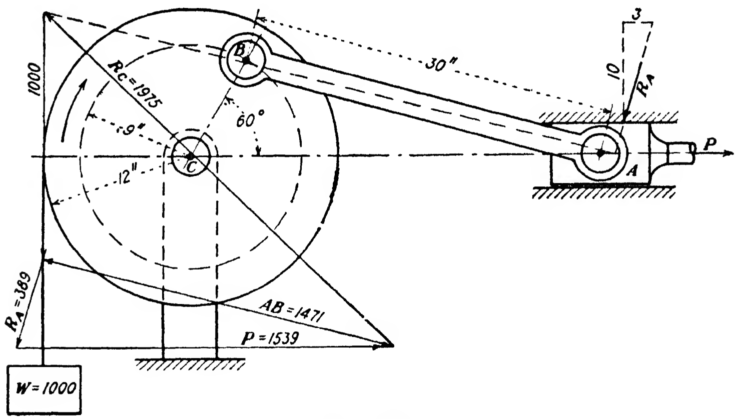


FIG. 108.

The various parts of the solution of the above problem are shown united into a single diagram in Fig. 108.

If friction were so small as to be negligible, the forces AB and R_C in the above example would pass through the centers of their respective bearings, and force R_A

would be normal to the guide. From the solution of the problem under such conditions, the following results would be obtained: $R_C = 1900$ pounds, $AB = 1378$ pounds tension, $R_A = 366$ pounds and $P = 1328$ pounds.

44. Gear Pressures.

It is shown in standard texts in Mechanism that, for involute gear teeth, the line of action of the pressure is constant in direction at some angle α with the common tangent to the pitch circles. The angle of obliquity α is usually 15° and will be so taken in all of the examples and problems to follow. Figure 109(a) shows two

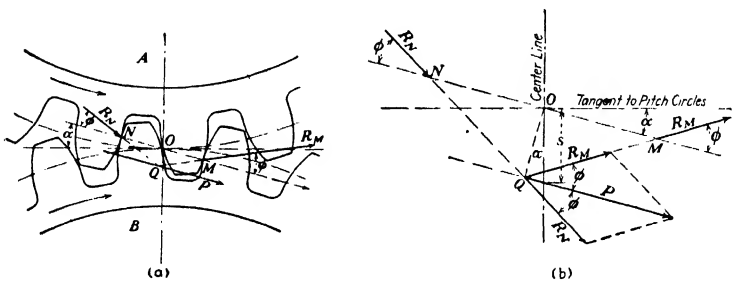


Fig. 109.

involute gears in mesh, A being the driver and B the follower. Pressure will occur at points M and N. If friction were negligible, the pressure line MN would be normal to the surfaces of the teeth and would pass through the pitch point O, at an angle of 15° with the common tangent to the pitch circles or 75° with the line of centers on the entering side of the driver. The arrowhead indicates the direction of the pressure exerted by the driver A upon the follower B; the reaction or resistance of the follower B upon the driver A would be in the reverse direction.

The effect of friction will now be determined by an approximate method which is sufficiently accurate for all practical purposes. Since sliding is occurring at points M and N of Fig. 109(a), the resultant pressure lines, considering the effects of friction, will be R_M and R_N, each making an angle φ with the common normal MN. The resultant force exerted by A upon B will be P, the resultant of R_M and R_N, as shown in Fig. 109(b). If it is assumed that the tooth spacing is so perfect that the pressure is equally divided between the two contact points, then R_M and R_N are equal. P will consequently bisect the angle between R_M and R_N, and therefore must be parallel to the line MN which is the pressure line when friction is neglected. If these conditions are assumed to be true throughout the cycle of tooth action, it follows that the effect of friction is to displace the pressure line parallel to itself. From Fig. 109(b), which represents the conditions of Fig. 109(a) on an enlarged scale, the perpendicular displacement OQ is equal to $\frac{1}{2}MN \tan \varphi$ or $\frac{1}{2}f \cdot MN$. Also, $MN = p \cos \alpha$, where p is the circular pitch. Hence, $OQ = \frac{1}{2}fp \cos \alpha$. The radial displacement s along the line of centers will be $OQ/\cos \alpha = \frac{1}{2}fp$. In subsequent problems the value of the distance s will be stated in preference to assigning values to f and p.

It should be noted that this shifting of the pressure line is in the direction from the driver to the follower, thus increasing the lever arm of the resistance acting upon the driver and decreasing the lever arm of the driving force upon the follower.

In the case of the less commonly used cycloidal teeth, the pressure line is not constant in direction but its average will not differ much from that of the involute system. An approximate solution may be made on this basis.

In the graphic solution of problems involving gears, it is unnecessary to draw the outlines of the teeth since the direction of the pressure and its point of application are all that are required. This is shown in the simplified diagram, Fig. 110. Here, the two pitch circles are shown tangent to each other at O, the pitch point. The line MN, drawn through the point O at an angle of 75° with the line of centers on the entering side of the driver, would be the pressure line if friction were neg-

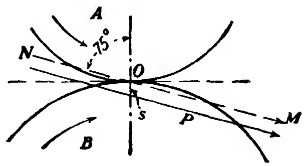


Fig. 110.

lected. With friction considered, the vector P, parallel to MN but displaced by the distance s from O along the center line, shows the line of action of the pressure.

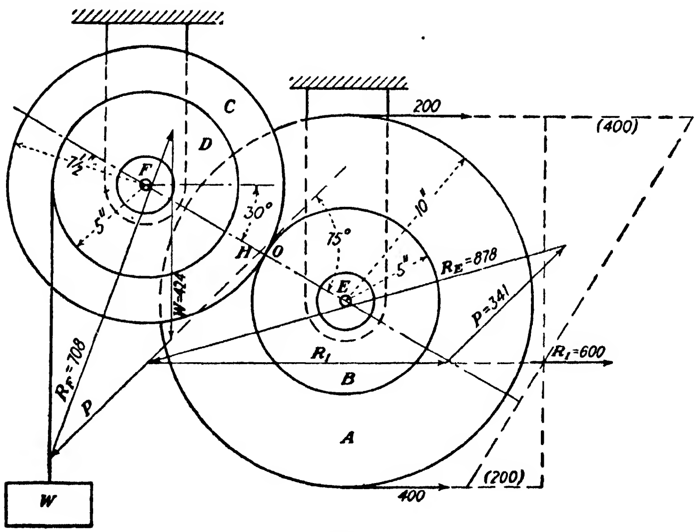


Fig. 111.

The hoisting mechanism shown in Fig. 111 will be used as an illustration of the solution of a simple problem involving gear pressure. It consists of a belt-driven pulley A, rigidly attached to a gear wheel B which meshes with another gear C rigidly attached to a drum D, from which the load W is suspended by a flexible cable. The bearings are 3 inches in diameter, $f = 0.2$ and $s = 0.3$ inch. The load W that can be raised by the given belt pulls is to be determined.

Pulley A and gear B are taken as the first free body. There are four forces acting, the two known belt tensions, the gear pressure P and the bearing reaction at E. In accordance with the explanation previously given, the gear pressure P is laid out at an angle of 75° with the line of centers on the entering side of the driver B and passes through point H which is 0.3 inch from O, the point of tangency of the pitch circles. The displacement OH is from the driver B toward the follower C, as shown. The parallel belt pulls are combined by the method of inverse proportion into their resultant R_1. The number of forces is thereby reduced to three, R_1, P and the reaction at bearing E. These forces must be concurrent at the point where R_1 and P intersect. The bearing reaction R_E must therefore pass through this point and must also be tangent to the friction circle at E. The resultant of forces R_1 and P tends to lift the journal at E to the upper right side of the bearing. The rotation of the journal is counterclockwise and so the journal tends to roll downward on the right side of the bearing. Thus, R_E is shown tangent on the lower side of the friction circle. Since the directions of the unknown forces have now been determined, the force triangle can be constructed as indicated. From it P scales 341 pounds, and R_E scales 878 pounds.

The gear C and the drum D together constitute the other free body, the external forces being P, W and the bearing reaction at F. The latter force must pass through the intersection of P and W, and also must be tangent to the friction circle at F. The resultant of forces P and W upon the free body causes the journal at F to press downward upon the bearing. The rotation of the journal is clockwise. These directions coincide on the right side of the friction circle, and so R_F is drawn tangent to this side. Since the directions of all of the forces are now known, the force triangle is constructed as shown, and from it R_F and W scale 708 pounds and 424 pounds respectively.

If friction were neglected in the above example, the gear pressure P would be drawn through the pitch point O and the two bearing reactions would pass through the centers of the corresponding bearings. The following results would then be obtained: $R_E = 943$ pounds, $P = 417$ pounds, $R_F = 938$ pounds and $W = 600$ pounds.

45. Reactions at Rolling Surfaces.

If a wheel or roller and the surface upon which it moves were perfectly rigid so that no deformation could take place, the surface of contact would be reduced to a line which would appear as a point in the plane projection. The reaction between the rolling body and the surface would necessarily pass through this point of contact. However, since all materials possess some elasticity, there will be a flattening of the rolling body at the surface of contact and a depression in the surface upon which the rolling takes place. The effect on the moving body then is the same as if there were a small obstruction constantly in its path. This condition is shown in Fig. 112 which is drawn with all of the deformation shown in the plane. The resultant reaction R acts through the point B at the distance a in front of the normal radius. Experiments show that the distance a is practically independent of the size of the rolling body or of the load, and depends only on the materials. It is called the *coefficient of rolling resistance.*

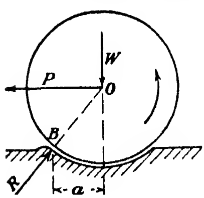


Fig. 112.

In the graphical solution of problems in which rolling resistance is involved, the reaction of a guide or track upon a simple roller will therefore act through a point at distance a ahead of the normal radius, as shown in Fig. 113(a). If, instead of being a simple roller, the rolling body consists of a wheel supported by a bearing, the reaction must not only conform to the above requirements, but must also be tangent to the friction circle at the journal, as shown in Fig. 113(b).

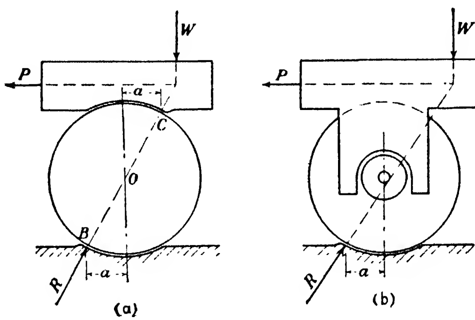


FIG. 113.

The elevator shown in Fig. 114 is an example of a mechanism involving rolling resistance. It consists of a cage $ABCD$, with a wheel attached at each of the four corners. The wheels roll on the vertical guides M and N. The cage is loaded as indicated and raised by means of the tension T in the supporting cable. The wheels are 12 inches in diameter, the bearings are 4 inches in diameter, $f = 0.3$ and $a = 0.5$ inch. The tension T necessary to raise the elevator with uniform speed is to be determined.

The cage and the attached wheels constitute the free body. If a small clearance between the guides is assumed, the rotational effect of forces T and W upon the cage causes the wheels B and D to swing free of the guides, and these are therefore not acting. There are now four external forces on the free body, consisting of the tension T, the load W and the reactions upon the wheels A and C. The reaction R_A must pass through a point 0.5 inch above the normal radius of the wheel A and must also be tangent to the friction circle. The journal of the wheel A presses to the left on the bearing and its rotation is clockwise. The directions of the pressure and the rotation coincide on the lower side; hence, the reaction R_A must be tangent on the lower

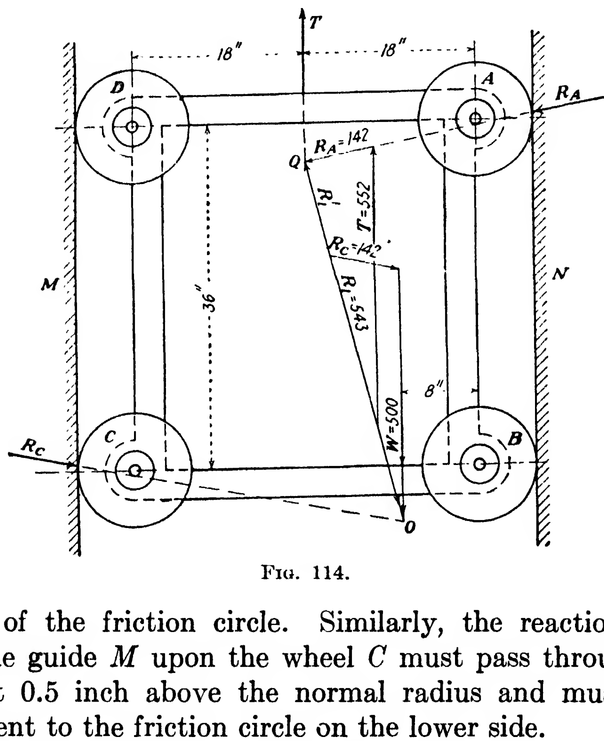


FIG. 114.

side of the friction circle. Similarly, the reaction R_C of the guide M upon the wheel C must pass through a point 0.5 inch above the normal radius and must be tangent to the friction circle on the lower side.

All the forces acting on the free body are now known in direction and one, the load W, is also known in magnitude. This is therefore an example of the four-force system as explained in Art. 24. Forces W and R_C are extended to intersect at point O, and forces T and R_A at Q. Since, for equilibrium, the resultant of one of these pairs must be colinear with the resultant of the other pair, the line OQ must be the line of action of these resultants or the closing line. At point O, a triangle is laid out of forces W, R_C and their resultant R_1, from which R_C scales 142 pounds and R_1 scales 543 pounds. At point Q, the resultant R_1' of the other pair of forces is laid out, equal to R_1 but opposite in direction, and another triangle is constructed from which R_A scales 142 pounds and T scales 552 pounds.

If friction and rolling resistance in the above example were negligible, each of the two guide reactions would be normal to the guides and would pass through the center of the corresponding wheel. The results obtained under these conditions would be: $R_A = 139$ pounds, $R_C = 139$ pounds and $T = 500$ pounds.

46. Efficiency.

The efficiency of a machine is defined as the ratio of the energy output to the energy input. For a machine with friction, the efficiency must necessarily be less than 100 per cent. It may easily be shown that, when the driving force is the same for both cases, the efficiency is the ratio of the resisting force that can be overcome with friction to the resisting force that could be overcome without friction. Or, if the resistance is the same in each case, the efficiency is the ratio of the driving force which would be required without friction to the driving force required with friction.

For example, the efficiency of the wedge and block of Art. 42 is the ratio of the driving force P without friction to the force P with friction, or $\frac{232}{595} = 0.39$, or 39 per cent. The efficiency of the hoisting mechanism of Art. 44 is the ratio of the resistance W with friction to that without friction, or $\frac{424}{600} = 0.707$, or 70.7 per cent. The low efficiencies obtained in these two examples, and in many examples and problems that follow, are due to the relatively high values assumed for the coefficients of friction. These high values, approximately those for dry or poorly lubricated surfaces, have been used in order that the friction circles might be of appreciable size on the small-scale drawings necessary.

47. Examples.

The following examples serve to illustrate further the application of the principles and methods taken up in this chapter and to indicate how certain special situations may be handled that often arise in the graphic solution of machines. In these examples, as well as in the problems which follow, only a rough outline of the framework of the machine is given since a detail drawing is not essential.

Example 1. A steam hoist, consisting of an elevator and hoisting mechanism driven by a reciprocating engine, is shown in Fig. 115. The elevator carries a 2200-lb. load and is raised or lowered by the flexible cable T. The wheels A and B are attached to the elevator and roll upon the fixed vertical guide MN. The cable T winds on a drum C which is rigidly attached to a gear D. The latter meshes with another gear F which is driven by the connecting rod GH. The crosshead H moves horizontally between fixed guides under the action of the steam pressure P, communicated through the piston rod. All bearings are 3 in. in diameter, wheels A and B are 8 in. in diameter, $f = \frac{1}{3}$, $a = \frac{1}{4}$ in. and $s = \frac{1}{4}$ in. Solve for force P necessary to raise the load when the connecting rod and the crank pin are in the position shown.

Solution: The first free body is the elevator which is acted upon by four forces: the load W, the cable tension T and the reactions R_A and R_B. On account of rolling resistance, the reaction R_A must pass through a point $\frac{1}{4}$ in. above the normal radius of wheel A and, since there is friction at the bearing, it must be tangent to the friction circle. The journal of the wheel A presses to the right on its bearing and rotates counterclockwise. Since pressure and rotation coincide in direction on the lower side, the reaction must be tangent on the lower side of the friction circle and the direction of the reaction R_A is thus determined. Similarly, the reaction R_B must pass through a point $\frac{1}{4}$ in. above the normal radius of the wheel B and must also be tangent to the friction circle on the lower side. It is now seen that the directions of all of the four

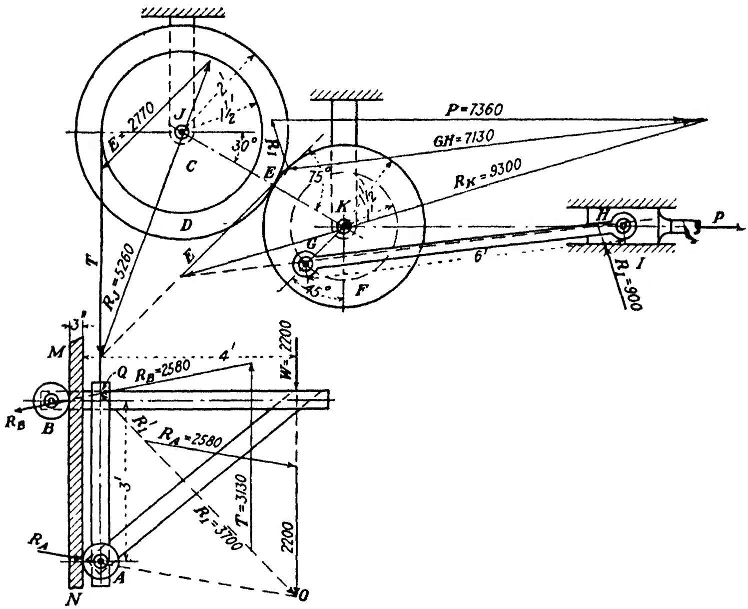


FIG. 115.

forces are known and the magnitude of one, W, is known. The elevator is thus a four-force body and is solved in the same way as the elevator in Art. 45. The forces W and R_A are extended to intersect at point O, and the forces T and R_B intersect at point Q. The line OQ is the closing line or line of action of the resultant of each pair. At point O, a triangle is laid out of the forces W, R_A and their resultant R_1, from which R_A and R_1 scale 2580 lb. and 3700 lb. respectively. At point Q, a triangle is laid out of forces T, R_B and their resultant R_1' which is equal and opposite to R_1. From this triangle, R_B and T scale 2580 lb. and 3130 lb. respectively.

The second free body, consisting of drum C and gear D together, is acted upon by three forces: the cable tension T, the gear pressure E and the reaction R_J at the bearing. The gear pressure E is shown acting at a distance of $\frac{1}{4}$ in. from the point of tangency of the pitch circles, this distance being laid off toward point J so as to reduce the lever arm of the driving force E upon the follower D. The bearing reaction R_J passes through the intersection of forces T and E and is tangent to the friction circle at J on the right side, because the journal tends to roll up the right side of this bearing. The force triangle of these forces is shown drawn on the space diagram and from it R_J scales 5260 lb. and the gear pressure E scales 2770 lb.

The third free body is the gear F which is acted upon by the known gear pressure E, the tension GH in the connecting rod and the bearing reaction at K. The line of action of the stress GH must be tangent to the friction circle at each end of the connecting rod GH. The reaction R_K must pass through the intersection of forces E and GH and must also be tangent to the friction circle at K. (It is left to the student to check the positions of these forces, relative to the friction circles.) The force triangle for this free body gives $R_K = 9300$ lb. and tension $GH = 7130$ lb.

The fourth free body is the crosshead H which is acted upon by tension GH, the guide reaction R_I and the piston-rod tension P. The force triangle is shown in Fig. 115, from which P and R_I scale 7360 lb. and 900 lb. respectively.

To obtain the efficiency, it is necessary to solve for P with friction neglected. The resulting solution gives $P = 4030$ lb. Hence the efficiency is $\frac{4030}{7360} = 0.547$, or 54.7 per cent.

Example 2. The mechanism shown in Fig. 116 is known as the Evans straight line motion. All bearings are 3.25 in. in diameter, $f = \frac{1}{4}$ and $s = 0.3$ in. Assuming $T_2 = 2T_1$, determine the belt tensions necessary to raise the load Q when the crank pin D is in the position shown.

Solution: The first free body is the walking-beam BEFH which is acted upon by four forces, all of which may be determined in direction. One of the forces, Q, is also known in magnitude. The known load Q acts vertically and must be tangent to the friction circle at B. The member DE is in compression, and the line of action of this stress must be tangent to the friction circle at each end. The link AF is in tension, and the line of action of this stress must likewise be tangent to the friction circles at the ends. The block H is sliding to the left and presses against the upper side of guide

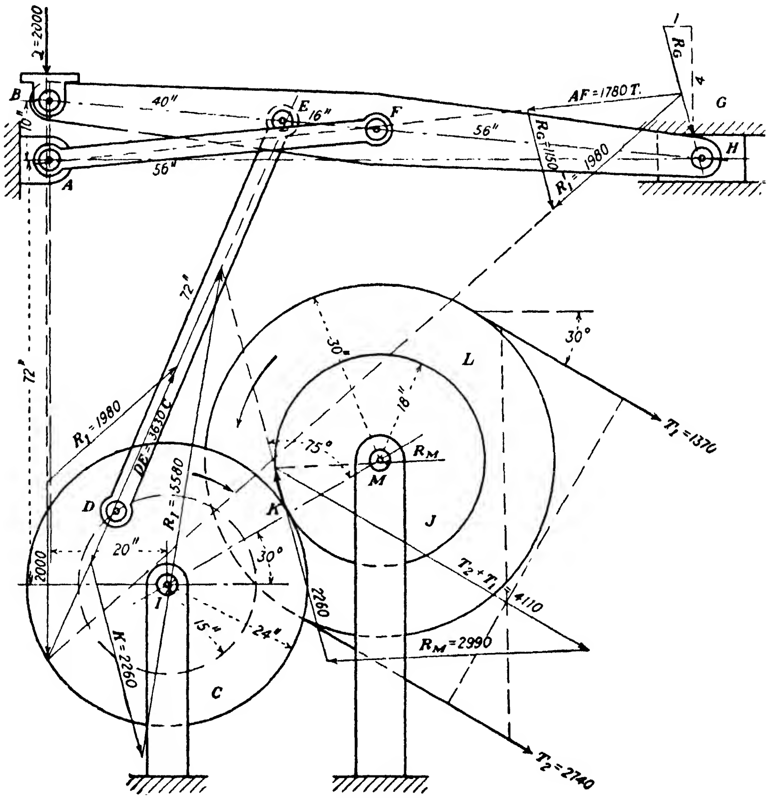


FIG. 116.

G. The latter fact may be shown by considering moments with respect to the intersection of AF and ED. The load Q tends to rotate the free body counterclockwise about this point and thus to lift the block H upward. The guide reaction R_G must therefore be downward and to the right, and at the same time tangent to the friction circle. The directions of all of the forces are now established.

As shown previously, the forces are paired: Q with DE, and R_G with AF. The line connecting the two intersections must be the line of action of the resultant of each pair. The triangle of the forces Q, DE and their resultant R_1 is drawn as shown, giving $R_1 = 1980$ lb. and $DE = 3630$ lb. compression. The resultant R_1' of forces R_G and AF is equal and opposite to the resultant just obtained, so another force triangle at their intersection gives the values of R_G and AF which scale 1150 lb. and 1780 lb. respectively.

The second free body is the gear C which is acted upon by three forces: the known stress DE, the gear pressure K and the reaction at the bearing I. The solution of this free body involves no new procedure and is shown in Fig. 116. Reaction R_I scales 5580 lb. and gear pressure K scales 2260 lb.

The third free body consists of gear J and pulley L, which are rigidly fastened together. There are four external forces: the known gear pressure K, the belt pulls T_1 and T_2 and the bearing reaction R_M. Although the belt tensions are as yet unknown, the relationship $T_2 = 2T_1$ makes it possible to locate their resultant by inverse proportion, using any convenient scale. This reduces the system to three forces and the solution is readily completed as shown in Fig. 116. After the resultant belt pull $T_1 + T_2$ has been obtained, it may be resolved into its components T_1 and T_2. From the force diagram, $R_M = 2290$ lb., $T_1 = 1370$ lb. and $T_2 = 2740$ lb.

Example 3. A lifting table operated by hydraulic pressure is shown in Fig. 117. The table L moves vertically between fixed guides U and V. The bearings A and F of the two gear sectors are fixed in position. The wheels D and E are 10 in. in diameter; all bearings are 4 in. in diameter; $f = 0.2$, $s = 0.2$ in. and $a = 0.2$ in. With a 12,000-lb. resistance Q acting against the table, determine the driving force P necessary to raise the table when it is in the position shown.

Solution: The table L is the first free body. It is acted upon by four forces: the load Q, the stresses in links BC and GH and a guide reaction upon the wheel at D or E. To determine whether D or E is acting, the position of the load Q must be considered. Since Q is nearer the supporting link GH, it will cause the stress in link GH to be larger than that in link

BC. The horizontal component of the thrust of *GH* upon the table will consequently be greater than that of *BC*; hence, the resultant horizontal action upon the table *L* is toward the left and brings the roller *D* into action. The reaction R_D of

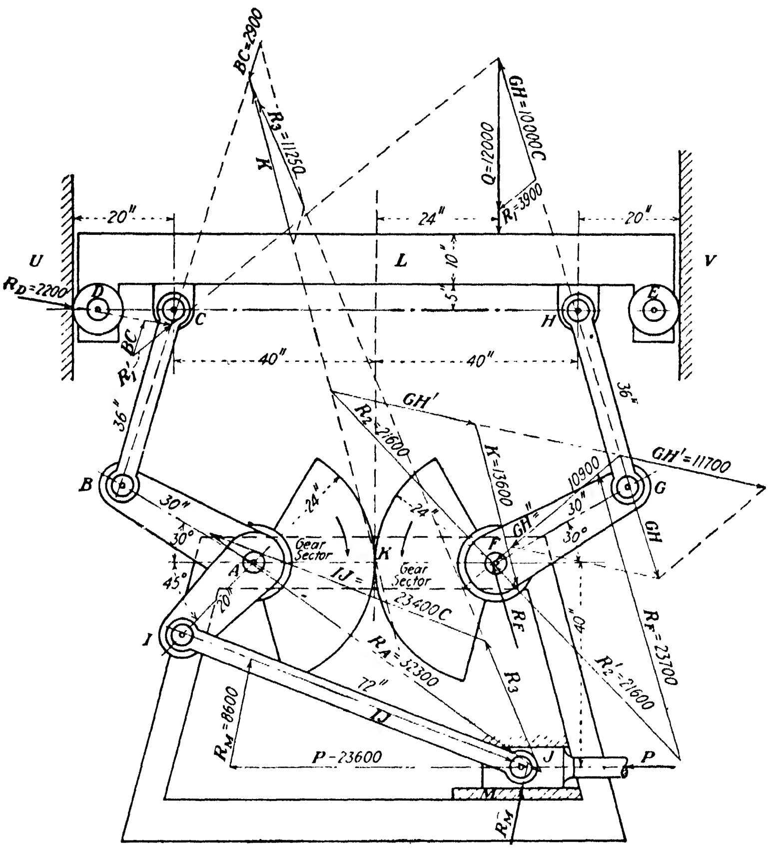


FIG. 117.

the guide upon the wheel *D*, passes through a point at a distance *a* above the normal radius and is also tangent to the friction circle. The lines of action of the stresses *BC* and *GH* must be tangent to the friction circles as shown. The directions of all of the forces are thus determined and the magnitude of one is known. In the resulting four-force solution, shown in Fig. 117, the force *BC* is paired with reaction R_D, and force

GH with load Q. The closing line is drawn connecting the two intersections and thus determines the direction of the resultant of each pair. The triangle of the forces Q, GH and their resultant R_1 is drawn, from which the compression GH scales 10,100 lb. and R_1 scales 3900 lb. The resultant R_1' of the forces BC and R_D is equal and opposite to the resultant R_1 just obtained, so another force triangle at their intersection gives the values of R_D and BC, which scale 2200 lb. and 2900 lb. respectively.

The second free body is the gear sector F and its attached arm FG. It is acted upon by only three forces: the known stress GH, the gear pressure K and the reaction R_F at the bearing. A difficulty arises here, however, in the fact that the stress GH and the pressure K are so nearly parallel that they do not intersect on the drawing sheet. To overcome this difficulty, the force GH is resolved at any convenient point on its line of action into two components. One of these, GH', must intersect the line of action of force K; the other, GH'', must pass through some point on the line of action of the reaction R_F. Since the latter is unknown in direction but must be tangent to the friction circle at F, there is only one point known to be on its line of action; this will be the point of tangency on the left side of the friction circle. The component GH'' is therefore drawn through this point. There are now four forces instead of three, and the four-force method of solution is used. The force GH' is paired with gear pressure K, and the force GH'' with the reaction R_F. The line of resultants or closing line is then drawn. The triangle of the forces GH', K and their resultant R_2 is drawn as shown in Fig. 117; from it, the gear pressure K scales 13,600 lb. and the resultant R_2 scales 21,600 lb. The resultant R_2' of the forces GH'' and R_F is equal and opposite to the resultant R_2 just obtained. Therefore, the triangle of forces R_F, GH'' and their resultant R_2' gives both the amount and direction of the reaction R_F which scales 23,700 lb.

The third free body is the gear sector A and its attached arms AB and AI. It is acted upon by four forces: the stress BC, the gear pressure K, the connecting-rod stress IJ and the reaction R_A at the bearing. Since the forces BC and K are known and intersect on the drawing, they may be readily combined into their resultant R_3 which scales 11,250 lb. This reduces the system to three forces and the remainder of the solution of this free body is completed in the usual way, as shown in Fig. 117. The stress IJ scales 23,400 lb. compression, and the reaction R_A scales 32,300 lb.

The fourth free body is the sliding block J which is acted upon by three forces: the known stress IJ, the hydraulic pressure P and the guide reaction R_M. The solution of this free body is also shown as part of Fig. 117 and gives R_M = 8600 lb. and P = 23,600 lb.

PROBLEMS

NOTE: In the following problems, the scales given are based on the standard-size drawing plate as recommended in Art. 3.

Problem 1. The wedge shown in Fig. 118 is being forced under the block. Solve for force P and all reactions (1)

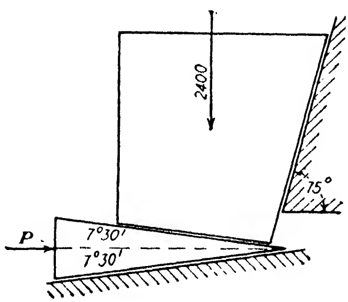


FIG. 118.

without friction; (2) with friction, if f = 0.4 at all sliding surfaces. Scales: 1 in. = 600 lb.

Ans. (1) P = 600 lb. (2) P = 2860 lb.

Problem 2. In the steam hoist, shown in Fig. 119, all bearings are 3 in. in diameter and f = 0.3 for all moving surfaces. With crank pin B in the position shown, solve for force P, guide reaction D, stress in the connecting rod BC and bearing reaction A, both without friction and with friction. Scales: 1 in. = 1 ft.; 1 in. = 5000 lb.

Ans. Without friction: BC = 25,300 lb. C; P = 25,200 lb. With friction: BC = 28,700 lb. C; P = 29,500 lb.

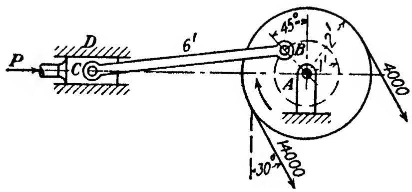


FIG. 119.

Problem 3. In the toggle-joint press, shown in Fig. 120, all bearings are 1 in. in diameter and f = 0.2 at all moving surfaces. Determine the resistance Q and all reactions and stresses both without friction and with friction. Compute the efficiency. Scales: 1 in. = 3 in.; 1 in. = 400 lb. Coordinates: point A—9 in., 5 in.

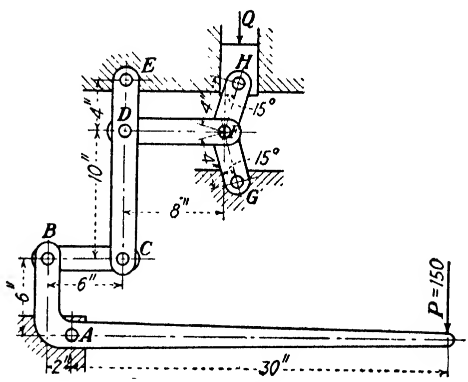


FIG. 120.

Ans. Without friction: BC = 750 lb. C; DF = 2630 lb. C; Q = 4900 lb. With friction: BC = 740 lb. C; DF = 2480 lb. C; Q = 3550 lb.

Problem 4. In the interlocking signal levers and compensator, shown in Fig. 121, all pins are 1 in. in diameter and f = 0.2. Solve for the force at O, (1) without friction; (2) with friction. Tabulate the values of all reactions and stresses and compute the efficiency. Scales: 1 in. = 4 in.; 1 in. = 10 lb.

Ans. Without friction: $R_E = 64.4$ lb.; $FG = 40$ lb. C; $R_K = 59.4$ lb.; $O = 37.5$ lb. With friction: $R_E = 61.2$ lb.; $FG = 37.5$ lb. C; $R_K = 53$ lb.; $O = 32$ lb.

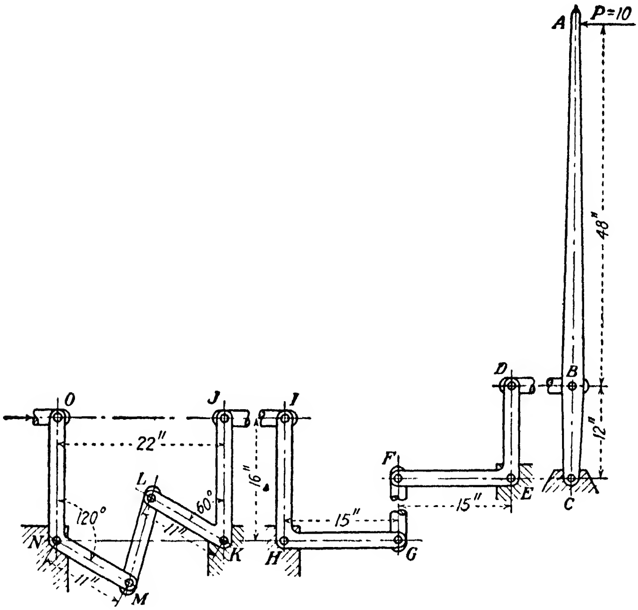


FIG. 121.

Problem 5. In the vertical piston pump, shown in Fig. 122, the center of the eccentric B is 3 in. from the center of the gear wheel A to which it is fastened. All bearings are 2 in. in diameter, $f = 0.15$ and $s = 0.25$ in. Assuming $T_2 = 2T_1$, determine the belt pulls required, with friction only, to overcome the resistance Q when the eccentric is in the position shown. Scales: 1 in. = 4 in.; 1 in. = 300 lb. Coordinates: point A—12 in., 4 in.

Ans. $BC = 2070$ lb. C; $F = 590$ lb.; $T_2 = 353$ lb.

Problem 6. Figure 123 shows an ore crusher with belt drive. All bearings are 3 in. in diameter, $f = 0.2$ and $s = 0.25$ in. Determine all reactions and stresses and the resistance K for the position shown, first without friction, then with friction. Scales: 1 in. = 6 in.; 1 in. = 3000 lb. Coordinates: point C—14 in., 11 in.

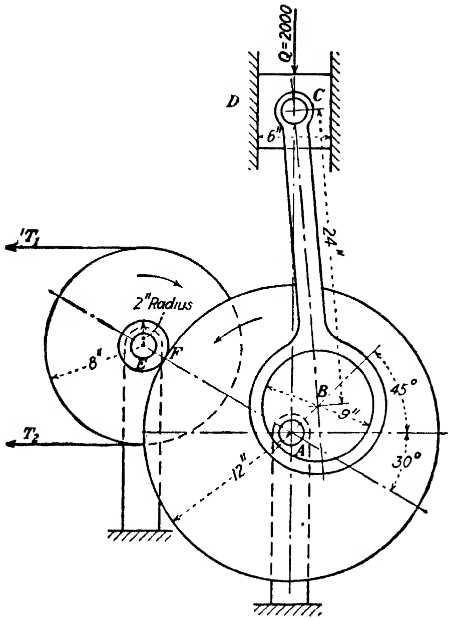


FIG. 122.

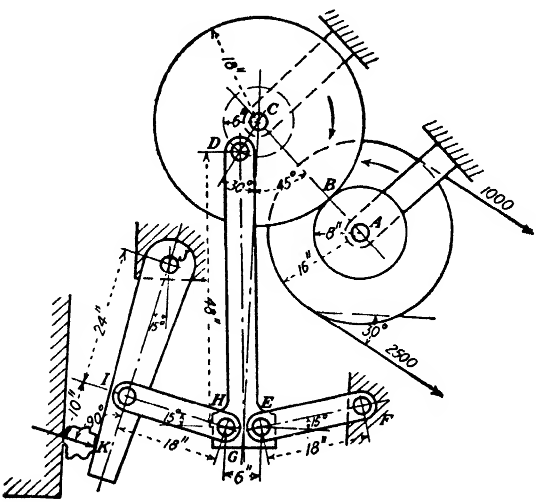


FIG. 123.

Ans. Without friction: B = 3150 lb.; DG = 16,700 lb. T; R_J = 9600 lb.; K = 23,100 lb. With friction: B = 2850 lb.; DG = 12,400 lb. T; R_J = 6700 lb.; K = 15,200 lb.

Problem 7. With crank pin B in the position indicated, solve for the driving force P and all reactions and stresses in the steam hoist shown in Fig. 124, first without friction, then with friction. All bearings are 3 in. in diameter, $f = \frac{1}{3}$, and s = 0.25 in. Scales: 1 in. = 10 in.; 1 in. = 300 lb. Coordinates: point A—12 in., 13 in.

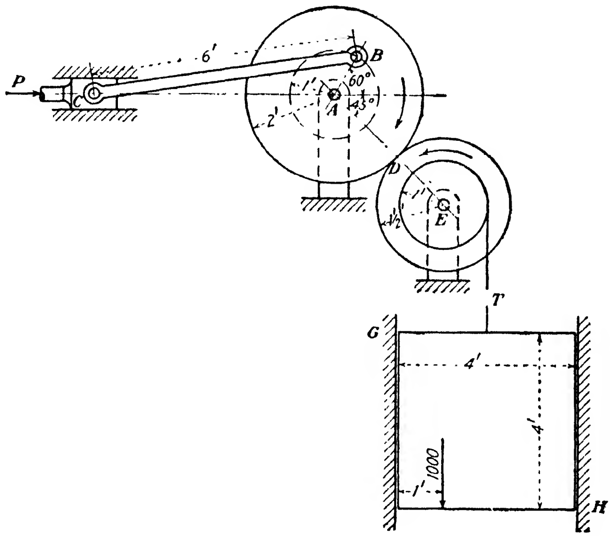


Fig. 124.

Ans. Without friction: $G = H$ = 250 lb.; D = 700 lb.; P = 1720 lb. With friction: $G = H$ = 260 lb.; D = 880 lb.; P = 2560 lb.

Problem 8. Assuming $T_2 = 2T_1$, solve for the belt tensions required to drive the power water pump in Fig. 125 when in the position shown, first without friction, then with friction. All bearings are 1.5 in. in diameter, f = 0.1, and s = 0.25 in. Scales: 1 in. = 2 in.; 1 in. = 200 lb. Coordinates: point A —9 in., 11 in.

Ans. Without friction: $H = 490$ lb.; $R_A = 704$ lb.; $T_2 =$ 556 lb. With friction: $H = 535$ lb.; $R_A = 773$ lb.; $T_2 =$ 676 lb.

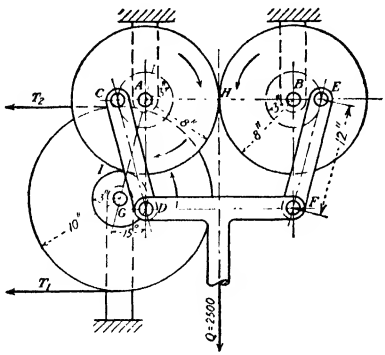


FIG. 125.

Problem 9. Solve Example 3 if the load Q acts at a point 24 in. on the *left* of the center of the table. Scales: 1 in. = 10 in.; 1 in. = 3000 lb. Coordinates: point A—10 in., 6 in.

Ans. $I = 2300$ lb.; $BC = 10{,}000$ lb. C; $K = 3700$ lb.; $R_A = 24{,}600$ lb.; $P = 22{,}100$ lb.

Problem 10. A lifting table for steel ingots is shown in Fig. 126. Journals A and E are 6 in. in diameter; all other

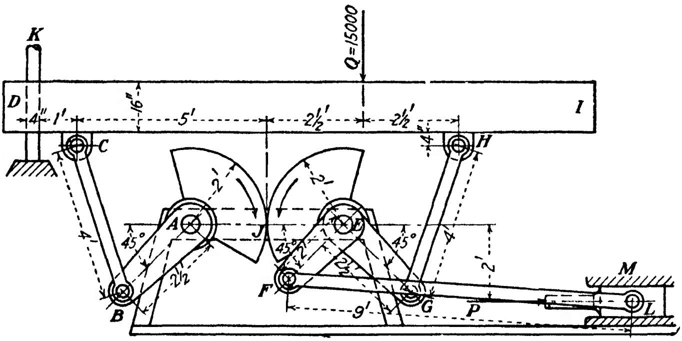


FIG. 126.

journals are 4 in. in diameter; $f = 0.2$; and $s = 0.5$ in. Arm AB and the gear sector are both keyed fast to journal A. Arms EF and EG and the corresponding gear sector are all keyed fast to journal E. K is a fixed vertical guide. Solve for the pressure P and all reactions and stresses as the table is raised against the resistance Q: (1) without friction, (2) with friction. Scales: 1 in. = 10 in.; 1 in. = 3000 lb. Coordinates: point A—6 in., 10 in.

Ans. Without friction: R_D = 2300 lb.; GH = 11,600 lb. C; J = 4840 lb.; R_E = 22,750 lb.; P = 23,650 lb. With friction; R_D = 2170 lb.; GH = 11,530 lb. C; J = 5950 lb.; R_E = 25,650 lb.; P = 27,000 lb.

Problem 11. The block D in the quick-return shaper, shown in Fig. 127, is pinned to the gear A and slides in the

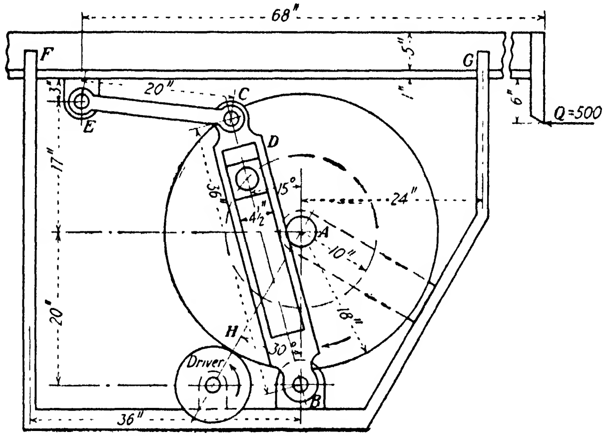


FIG. 127.

slotted crank BC. Journal A is 4 in. in diameter; journal D is 3 in. in diameter; journals B, C and E are each 2 in. in diameter; f = 0.2; and s = 0.25 in. Solve for all forces, both without friction and with friction. Scales: 1 in. = 6 in.; 1 in. = 100 lb. Coordinates: point A—10 in., 6 in.

Ans. Without friction: R_F = 26 lb.; R_D = 600 lb.; H = 300 lb. With friction: R_F = 21 lb.; R_D = 628 lb.; H = 384 lb.

Problem 12. Figure 128 shows an embossing press in which P is the driving force which presses the table upward against the resisting force Q. Rollers J and N are 4 in. in diameter; all bearings are 2 in. in diameter; f = 0.2; a = 0.1

in. and $s = 0.2$ in. With the mechanism in the position shown, solve for all reactions and stresses and the driving force P, with friction only. Scales: 1 in. = 5 in.; 1 in. = 4000 lb. Coordinates: point B—12 in., 5 in.

Ans. $R_J = 6700$ lb.; $KL = 24{,}700$ lb. C; $KE = 33{,}700$ lb. C. Reaction of guide on sliding block = 20,000 lb.; $R_B = 13{,}500$ lb.; $P = 3100$ lb.

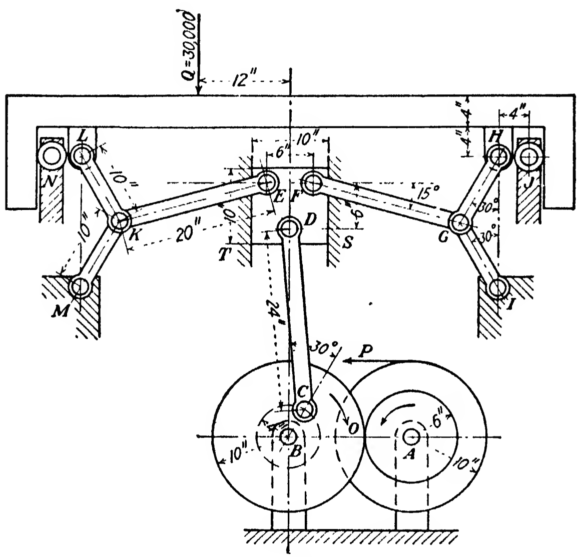


FIG. 128.

Problem 13. Solve Problem 12 if the load Q acts at a point 12 in. on the *right* of the table center.

Ans. $P = 2900$ lb.

Problem 14. In the press, shown in Fig. 129, D consists of two independent wheels and a journal. The outer wheel, 8 in. in diameter, rolls upon the cam C; the inner wheel, 6 in. in diameter, rolls between the fixed vertical guides. The circular cam C, 8 in. in diameter, is keyed fast to the gear B, 6 in. in diameter, which is driven by the gear A whose diameter is 4 in. The common line of centers of cam C and gear B is 30° with the vertical, the center of C being 1 in. from the center of B. Rollers K and L are 4 in. in diameter; all bearings are 2 in. in diameter; $f = 0.2$; $a = 0.1$ in.: and $s = 0.25$ in.

Assuming $T_1 = 3T_2$, solve for these tensions and all reactions and stresses, with friction only, when the mechanism is in the position shown. Scales: 1 in. = 4 in.; 1 in. = 4000 lb. for all forces up to and including D, and 1 in. = 600 lb. for all remaining forces. Coordinates: point A—12 in., 3 in.

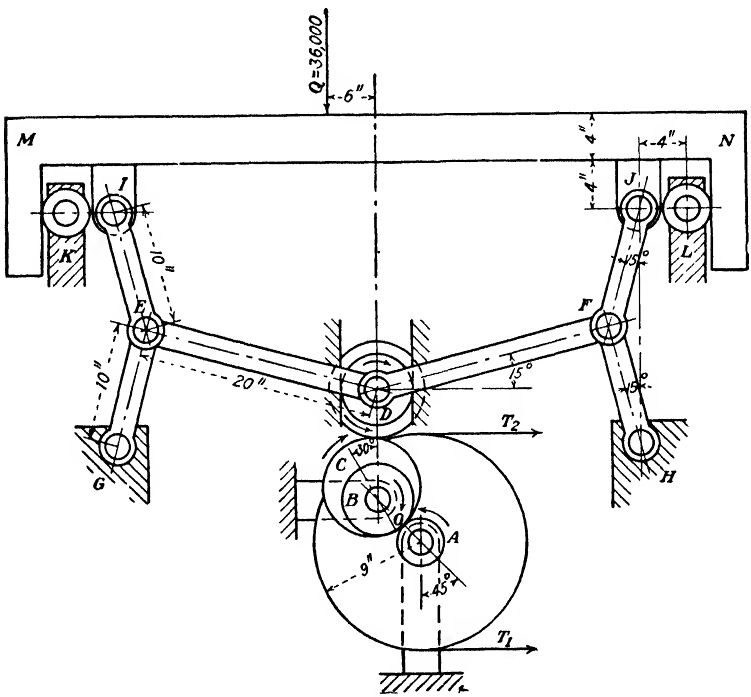


FIG. 129.

Ans. R_L = 2750 lb.; IE = 23,800 lb. C; DE = 12,400 lb. C. Reaction of guide on roller D = 6500 lb. Reaction between cam and roller = 6500 lb.; O = 4590 lb.; T_1 = 1840 lb.

Problem 15. The ore car, shown in Fig. 130, is drawn up the incline by the counterweight W. All bearings are 3 in. in diameter, f for bearings = 0.25 and a = 0.6 in. Solve for the counterweight necessary to draw the empty car weighing 1000 lb. up the incline with uniform velocity with the brakes off. Scales: 1 in. = 1 ft.; 1 in. = 400 lb. Coordinates: point A—14 in., 8 in.

Ans. R_K = 475 lb.; D = 590 lb.; T = 630 lb.; W = 1320 lb.

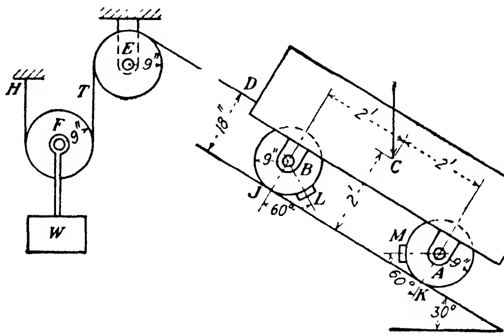


FIG. 130.

Problem 16. With the same counterweight W, acting as resistance on the ore car described in Problem 15, solve for the maximum weight ot loaded car that can be lowered down the incline at a uniform speed with brakes set so as to cause skidding to impend. Solve also for the normal pressure necessary on each brake shoe. f between wheel and rail = 0.4. f for brakes = 0.3.

Ans. T = 690 lb.; D = 750 lb.; R_K = 3200 lb. Weight of loaded car = 4840 lb. Normal pressure on M = 2810 lb.

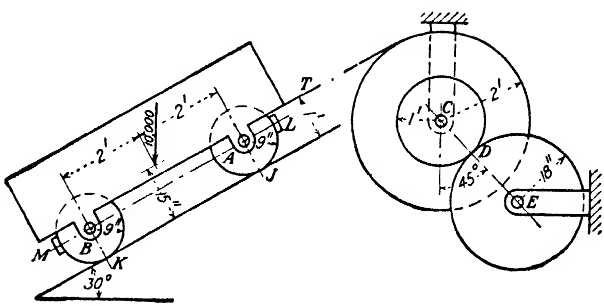


FIG. 131.

Problem 17. In Fig. 131, the car is moving down the incline against the gear resistance D. All bearings are 3 in.

diameter. The various coefficients are: f for wheel on rail $= 0.25$; f for brake on wheel $= 0.2$; f for bearings $= 0.2$; $a = 0.6$ in. and $s = 0.6$ in. The brake on wheel A is not acting, but the brake on wheel B is set so as to cause skidding to impend. Solve for the gear resistance D necessary for equilibrium and for all other reactions and stresses, including the normal brake-shoe pressure M. Scales: 1 in. = 1 ft.; 1 in. = 2000 lb. Coordinates: point A—6 in., 8 in.

Ans. $R_K = 5400$ lb.; $R_A = 3400$ lb.; $T = 3380$ lb.; $D = 6300$ lb. Normal pressure on $M = 3120$ lb.

APPENDIX

48. Purpose.

The purpose of this appendix is to present the graphical solution for some parts of Mechanics not considered in Chapters III, IV or V. These solutions, while not commonly used, are to be preferred in many cases where an approximation is not advisable and the algebraic solution is too tedious, if not almost impossible.

49. Centroid of an Area.[1]

The centroid of an area can be obtained graphically. In many very irregular areas the graphical solution is to be preferred.

If an area A is made up of several finite component areas whose centroids are known, the equation from which the centroid of the entire area can be obtained is:

$$A\bar{x} = A_1x_1 + A_2x_2 + \cdot \cdot \cdot A_nx_n. \tag{1}$$

Area A is the sum of A_1, A_2, etc. Each value of x is the distance measured from the Y-axis of reference to the centroid of the area it multiplies.

If area A cannot be divided into convenient finite-component areas, as above, it may be divided into differential-component areas. The equation which must then be used is:

$$A\bar{x} = \int x dA. \tag{2}$$

A is the sum of the differential areas dA. The values of $\bar{x}$ and x are, as before, centroidal distances for their

[1] For preliminary and further references see Poorman's "Applied Mechanics," Chap. IX, McGraw-Hill Book Company, Inc., New York.

respective areas. It should be noticed that these equations can be written in terms of y and z, as well as x.

Figure 132 shows an irregular area $A = BCDE$, of which the centroid is required. Two parallel axes Y and Y_1 are drawn at a distance b apart and placed so that area A lies between them. Any line BC is then drawn parallel to axis Y and the bounding points B and C projected upon axis Y_1 at points F and G. From

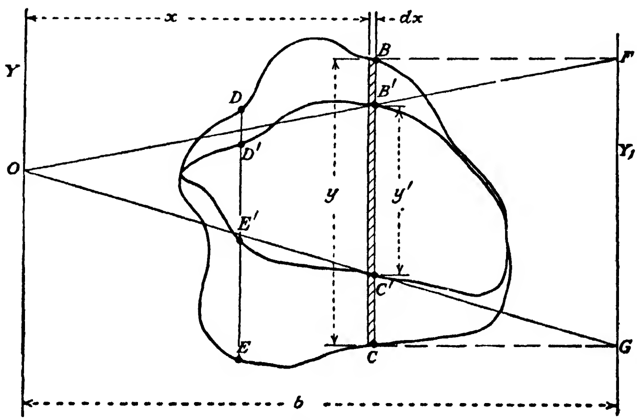


FIG. 132.

these points F and G, straight lines are drawn intersecting axis Y at any convenient point O. On these lines, points B' and C' on the boundary of a new area A' are obtained. This process is continued using the same point O until the new area $B'C'D'E'$ can be accurately formed. The values of areas A and A' may be measured with a planimeter.

The centroidal distance $\bar{x} = bA'/A$.

Proof: Equation (2), $A\bar{x} = \int x dA$.

From the similar triangles FOG and $B'OC'$,

$$y'/x = y/b,$$

or

$$xy = by'.$$

Substituting in Eq. (2)

$$A\bar{x} = \int xdA = \int xydx = b\int y'dx = bA',$$

$$\therefore \bar{x} = bA'/A.$$

Often the centroid of an irregular area can be obtained easily by experiment. The desired area must be cut out on cardboard or some other suitable material and suspended from a point near its boundary and a vertical line drawn through this point. Next, some other point of suspension is selected and another vertical line drawn. These two vertical lines intersect at the centroid of the area. The proof for this method lies in the fact that the weight of the cardboard must act through its center of gravity and be colinear with the reaction at the point of suspension.

PROBLEMS

Problem 1. Using the graphical method, show that the centroid of a triangle is one-third of the altitude measured from the base.

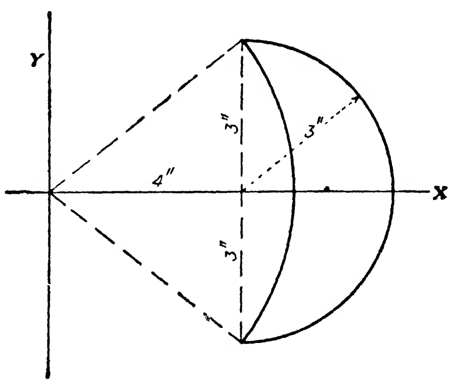


Fig. 133.

Problem 2. Locate the centroid of the crescent shown in Fig. 133.

Ans. $\bar{x} = 5.66$ in.

Problem 3. Locate the centroid of the rail section shown in Fig. 134.

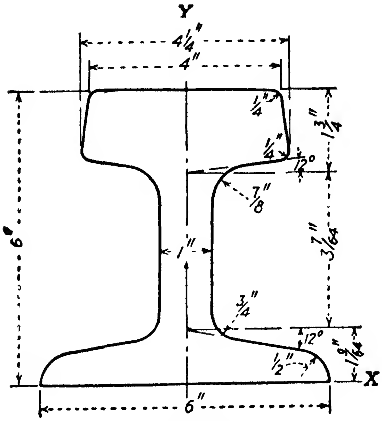


Fig. 134.

Ans. $\bar{y} = 3$ in.

50. Moment of Inertia of Area.[1]

The moment of inertia of an area may be defined as an algebraic expression of the form

$$\int_A^B x^2 dA, \int_A^B y^2 dA \text{ and } \int_A^B \rho^2 dA.$$

The quantities x, y and ρ are measured perpendicularly from their axes y, x and z to the area dA.

The moment of inertia of the very irregular area $A = BCDE$, shown in Fig. 135, will be obtained with respect to the Y-axis. Two parallel axes Y and Y_1 are drawn at a distance b apart and placed so that area A lies between them. Any line BC parallel to axis Y is then drawn and the bounding points B and C projected on axis Y_1 at F and G. From points F and G straight lines are drawn to any convenient point O on axis Y.

[1] For preliminary and further references see Poorman's "Applied Mechanics," Chap. X, McGraw-Hill Book Company, Inc., New York

From these two lines new points B' and C' are obtained, which are projected on axis Y_1 at points H and I. From these new points H and I, lines are drawn to point O. On these lines B'' and C'' are obtained. These points are on the boundary of a new area A''. This process is continued, using the same point O until the boundary of the new area $B''C''D''E''$ can be accurately formed. The value of this area A'' may be measured with a planimeter.

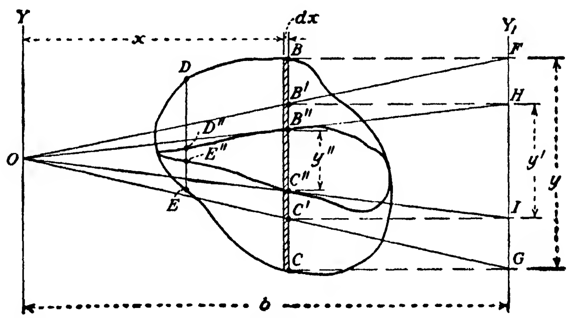


FIG. 135.

The moment of inertia $I_Y = b^2A''$.

Proof: The fundamental form of the expression for the moment of inertia of area A with respect to the Y-axis is:

$$I_Y = \int x^2dA = \int x^2ydx. \tag{1}$$

From the similar triangles FOG and $B'OC'$,

$$y/b = y'/x, \tag{2}$$

and from the similar triangles HOI and $B''OC''$,

$$y'/b = y''/x. \tag{3}$$

By eliminating y' between Eqs. (2) and (3),

$$x^2y = b^2y''.$$

Substituting this value of x^2y in Eq. (1)

$$I_Y = \int x^2 y dx = b^2 \int y'' dx = b^2 A''.$$

Having obtained I_Y, the moment of inertia with respect to the centroidal Y-axis can be obtained by the well-known transfer formula:

$$I_Y = I_{Y_0} + d^2 A.$$

I_Y is the moment of inertia of area A with respect to the Y-axis. I_{Y_0} is the moment of inertia of area A with respect to the centroidal Y-axis and d is the distance, measured perpendicularly, between axes Y and Y_0.

This transfer formula, as well as the other expressions of moment of inertia of areas, can be written in terms of X and Z as well as Y.

PROBLEMS

Problem 1. Using the graphical method, show that the moment of inertia of a rectangle with respect to the base is $bh^3/3$.

Problem 2. Determine the moment of inertia with respect to the Y-axis of the area shown in Fig. 133.

Ans. $I_Y = 322$ in.^{4}

Problem 3. Solve for the moment of inertia of the area, shown in Fig. 134, with respect to the centroidal X-axis.

Ans. $I_{X_0} = 69.3$ in.^{4}

51. Bending Moments in Different Planes.[1]

The beam shown in Fig. 136 has loads applied in two different planes. Let it be required to determine the amount and location of the maximum resultant bending moment.

By writing the equations of moments with respect to R_2 and then R_1, Figs. 136(a) and 136(b), the following reactions are obtained: $R_{1V} = 229$ pounds, $R_{1H} = 112$

[1] For preliminary and further references see Poorman's "Strength of Materials," Chaps. V and VI, McGraw-Hill Book Company, Inc., New York.

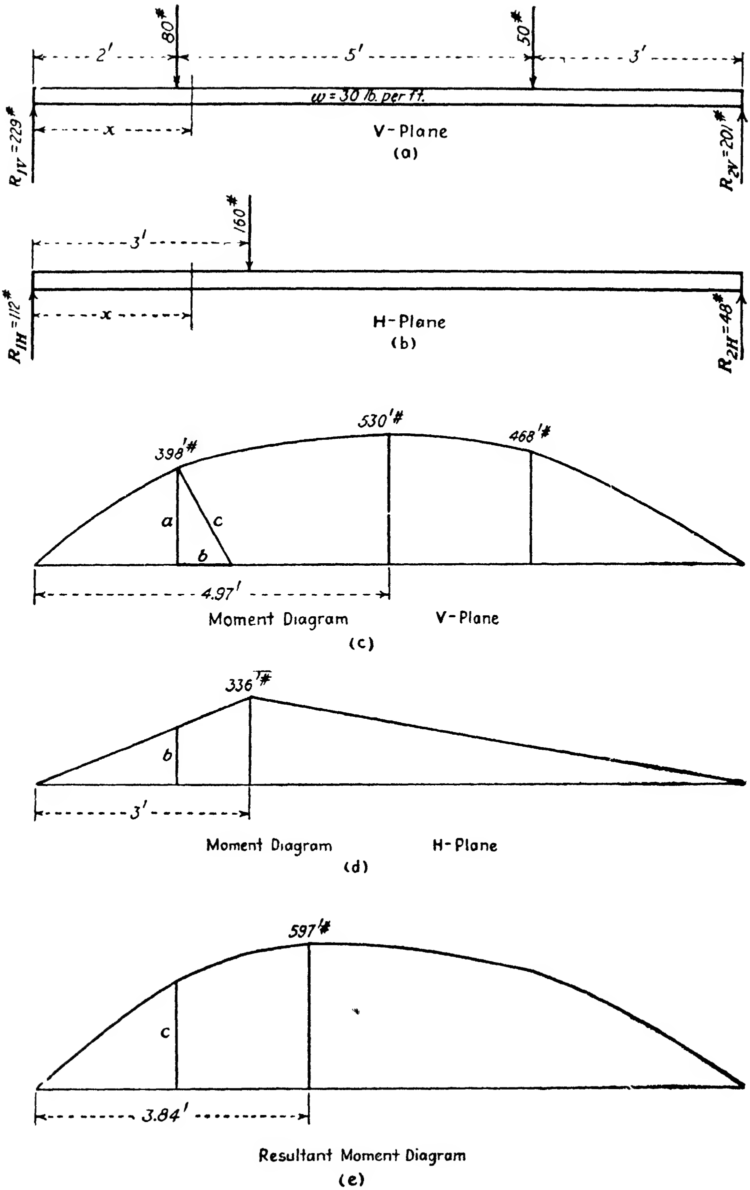

80#
50#
2'
5'
3'
w = 30 lb. per ft.
R1V = 229#
x
V-Plane
(a)
R2V = 201#
160#
3'
R1H = 112#
x
H-Plane
(b)
R2H = 48#
530'#
398'#
468'#
a
c
b
4.97'
Moment Diagram
V-Plane
(c)
336'#
b
3'
Moment Diagram
H-Plane
(d)
597'#
c
3.84'
Resultant Moment Diagram
(e)

FIG. 136.

pounds, $R_{2V} = 201$ pounds and $R_{2H} = 48$ pounds. In the vertical plane, Fig. 136(a), the moment equations are:

$$M_{(0-2)} = 229x - 15x^2.$$
$$M_{(2-7)} = 149x - 15x^2 + 160.$$
$$M_{(7-10)} = 99x - 15x^2 + 510.$$

In the horizontal plane, Fig. 136(b), the moment equations are:

$$M_{(1-3)} = 112x.$$
$$M_{(3-10)} = 480 - 48x.$$

In each of these five moment equations, x is measured from the left end of the beam and its limits are shown in parentheses.

From these equations, the data necessary to plot the moment diagrams in Figs. 136(c) and 136(d) can be obtained. These moment diagrams must be plotted accurately to some convenient scale.

In Fig. 136(c), the maximum bending moment in the vertical plane occurs at a point 4.97 feet from the left end of the beam and is 530 foot-pounds. In Fig. 136(d), the maximum bending moment in the horizontal plane occurs at a point 3 feet from the left end of the beam and is 336 foot-pounds. The point of maximum resultant bending moment may occur at the point of maximum bending moment in the vertical plane or at the point of maximum bending moment in the horizontal plane, or at some point between these two limits. This maximum resultant bending moment and its location are obtained by plotting the resultant moment diagram shown in Fig. 136(e). The ordinates for this diagram are equal to the square root of the sum of the squares of the corresponding values from Figs. 136(c) and 136(d). Thus ordinate c in Fig. 136(e) is equal to the vector sum of ordinate a, Fig. 136(c), and ordinate b, Fig. 136(d).

A convenient graphical method from which c can be obtained is shown by the triangle in Fig. 136(c).

The maximum resultant bending moment obtained from the diagram shown in Fig. 136(e) occurs at a point 3.84 feet from the left end of the beam and is equal to 597 foot-pounds.

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