# HYDRAULICS APPLIED TO SEWER DESIGN

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# PREFACE

THE following pages are founded on notes of lectures delivered to third-year students sitting for the B.Sc. Tech. degree in Municipal and Sanitary Engineering in the Victoria University of Manchester, and also on notes collected by the author from his own practice as a municipal engineer for the past twenty-five years.

Owing to the very favourable way in which these notes have been received, and the frequent requests from former students and practising engineers for the data referred to, the writer has decided to publish them in book form, in the hope that they may be of help to students of other universities and technical colleges, and to his brother municipal engineers.

The writer has freely drawn on many of the standard textbooks relating to the subject, and has attempted to acknowledge these in the text. Besides a very large number of American, French, and German authorities, he would express his indebtedness to:—Professor Lea's "Hydraulics," Professor Unwin's "Hydraulics," Professor Gibson's "Hydraulics," Professor Merriman's "Hydraulics," Metcalf & Eddy's "American Sewerage Practice," the late Albert Wolheim's "Sewerage Engineer's Notebook," the *Engineering News*, the *Engineering Record*, the *Transactions of the American Society of Civil Engineers*, and many others.

It is usual, in books intended for practical use, to state that either no so-called "advanced" mathematics has been used, or to make some abject apology for so doing. In the present instance such mathematics has been relegated to appendices, where those who care to may consult them. The writer trusts that before very many more years have passed such a humiliating procedure may have disappeared from the textbooks of the nation which founded engineering science, and is the last to bring its tutorial work up to the level of its practical experience—at all events in textbooks intended for daily use in the office.

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# INTRODUCTION

**Attributes of a Fluid.**—A considerable amount of confusion exists, even in the minds of practising engineers, as to the fundamental attributes which underlie the action of fluids in motion. While the theories which enable the pressures of fluids at rest to be calculated are now commonly taught in the elementary schools, and form part of the necessary knowledge for the entrance examinations of the universities, the laws of fluids in motion are almost always left to the technical textbooks, and are, even then, very vaguely defined and explained. It is, for instance, frequently stated without any qualification that there is no internal friction in fluids, whereas this would only be true in the case of perfect fluids. All fluids possess in some small degree what is known as "viscosity," or resistance to shearing. This viscosity may only be neglected when the velocities are small.

**Viscosity.**—A full description of viscosity would be long and out of place in this treatise, besides involving mathematics beyond the power of elementary students. We will therefore summarize them as much as possible, and only so far as to enable the reader to obtain some idea as to how they affect the problem of flow in channels. As water is by far the commonest liquid with which the engineer has to deal, all our remarks will apply to it, unless otherwise stated.

It must first be realized that water has the power of adhesion and also cohesion. The former enables it to cling to the surface of a containing solid, and the second enables its component molecules to cling to each other, and so come under the action of gravity. Both of these powers are due to molecular attraction, and are treated of in books of physics. The only resistance which is of importance to the engineer is that due to viscosity, and commonly termed "frictional" resistance, through its apparent resemblance to the frictional resistance between solids. This resistance is only apparent when the liquid is in motion, and has the appearance of a shear. The force is found to involve the velocity and the position of the film of liquid in the cross-section of the flow, and decreases with a rise in temperature. It is stated that the cohesion largely affects the viscosity, and that, at low temperatures, the greater cohesion more than outweighs the diminished diffusibility, and hence increases the viscosity.

The resistances to motion due to viscosity always take the form of eddies, and these will be visible at the boundary between the liquid and the containing solid.

It must be now realized that there are two types of motion, namely, (*a*) steady, and (*b*) unsteady, sinuous, or eddying motion. The first implies motion in which all the particles move in parallel lines, and never occurs in practical hydraulics. In unsteady, or sinuous, motion in a viscous fluid, the direction of the forces at any particular point are indeterminate, and general equations of motions are impossible of application. The elaborate mathematical analyses of steady motion which form the introduction to many textbooks are therefore unnecessary, and, even if mastered, would be quite misleading. It must, however, be fully understood that viscosity, even if small in effect, cannot be entirely ignored, and is the real reason why a constant has to be always introduced to make the theoretical result agree with experiment.

It is usually assumed that the pressure at any point in a liquid is due to the hydrostatic pressure at that point, due to the depth below the free surface. It is also assumed that the acceleration of the particles is due to external forces and not to the pressure due to surrounding particles.

According to experiments made by Professor Gibson (*see* his "Hydraulics"), it appears as if, in steady motion, the resistance were independent of the nature of the containing surface. With sinuous motion, however, there is interchange of the particles adjoining the solid surface, and such interchange will be the greater the rougher such surface, and hence will vary with the material of the surface.

From the mathematical analysis for flow in a cylindrical pipe running full we find that the mean velocity occurs at 0·707 of the radius from the centre, the curve of velocities being parabolic. The researches of Bazin (Men. Ac. des Sc. 1897) and Williams, Hubbell, and Fenkell (Proc. Am. Soc. C.E. 1901) show that the curve is more nearly elliptical and tangential to the sides of the pipe. The mean velocity becomes 0·855 of the maximum, and occurs at 0·74 of the radius. From later experiments, notably those of Cole (Trans. Am. Soc. C.E. 1902) and Morrow (Proc. Roy. Soc. 1905), it is concluded that the mean velocity is in the neighbourhood of 0·69 of the radius in small pipes, and 0·75 in large ditto, and that if observations be made at these points, and further multiplied by a constant varying between 0·79 and 0·86 (average value 0·84) the mean velocity will be obtained with fair accuracy.

**Density of Water.**—The density of water at ordinary temperatures may be taken as 62·424 lbs. per cubic foot (or roughly 62·4), and that of sewage is practically the same. Salt water weighs about 64 lbs. per cubic foot.

**Head.**—The head producing pressure at any point in a liquid is the vertical height from the point in question to the free surface of the water. Since the weight of a cubic foot of water is 62·4 lbs., the pressure per sq. in. due to a column of water 1 ft. high is $\frac{62 \cdot 4}{144} = 0 \cdot 434$ lb.

Let $p =$ pressure at point in lbs. per sq. in.
$h =$ head in feet.

Then $p = 0 \cdot 43h$, or $\frac{p}{0 \cdot 43} = h$.

Atmospheric pressure is the pressure due to the atmosphere on a surface at sea-level, and is roughly equal to the pressure produced by a column of water 32·9 ft. high.

Pressures registered by a gauge are over and above atmospheric pressures. If these latter are to be included the pressures are termed " absolute."

**Average Velocity.**—As explained above, all motions in practical work in fluids are unsteady, the water moving in swathes or bands. Each band has a different velocity, and the bands and their component particles move quite erratically. If, therefore, a quantity $Q$ pass through a cross-sectional area $A$ in $t$ seconds, the average velocity is said to be $\frac{Q}{At}$ feet per second.

This is really the average of all the components of the various velocities normal to the cross-section. It may not actually be the velocity of one single particle, but for the purposes of calculation we are compelled to assume such a value.

# HYDRAULICS APPLIED TO SEWER DESIGN

## CHAPTER I

### DATA FOR USE IN CALCULATIONS

**Quantity of Sewage per Head.**—The quantity usually allowed in this country per diem is 30 gallons per unit of the population. Formerly, textbooks gave elaborate tables showing the amount to be consumed for various domestic and public uses, but nowadays the water supply is considered a more accurate guide. In designing a sewerage scheme the discharge from the existing culverts should be accurately gauged for as long a period as practicable, and only taking account of dry-weather flows. Such records will, however, include the amount due to infiltration to leaky sewers. Once the latter is ascertained, the amount may be added to the estimated future water supply to obtain the dry-weather flow to be allowed for. In a system where both sewage and rain-water are taken by the one sewer, the former is usually comparatively insignificant in proportion to the latter.

**Population.**—Various methods have formerly been suggested for estimating the future value of the population contributing to a sewerage system, but few of them are of any practical use. Such are (*a*) the estimation of the increase by geometrical progression, (*b*) ditto by arithmetical progression. Both are entirely useless. The first would probably give far too high results, and we have no justification for considering that the increase will follow any mathematical law. Increase in population is governed by many things which are quite impossible to foresee. Such are (*a*) the inclusion of an outside suburban area, (*b*) the change of the centre of a town from residential to office occupation, (*c*) the building of an important factory in a certain part (which will generally mean a number of workmen's dwellings), (*d*) the moving of a factory from the centre of the city to the outskirts due to want of room, (*e*) the changing of good property to slum ditto, (*f*) the decline of a staple industry or change from one to another which may or may not mean a considerable alteration in the sewage flow, (*g*) the unforeseen rise of popularity of some suburb for residential purposes, and so on.

To a great extent the engineer is helpless, and all that he can do is to divide up the drainage district into what he considers are, or will be, areas of distinct type, and consider each area on its own merits.

When the present writer was engaged on such an estimate, the areas were divided as follows :—

(1) Best class of residential houses, detached and in their own grounds of two or more acres in extent.

(2) Smaller houses, semi-detached, with fairly large gardens, back and front.

(3) Smaller houses still, practically no front garden, and small back ditto, semi-detached or in terraces.

(4) Workmen's cottages and poor-class dwellings, no garden in front and only a back yard.

(5) Areas composed almost entirely of blocks of offices.

(6) Agricultural land, or land which will probably be unbuilt on to any extent.

Typical areas were selected for each of the above and a careful house-to-house census taken. The whole drainage area was then mapped out and a probable population assigned according to which class the engineer considered each sub-area would eventually fall into. Nothing better than this method can, in the present writer's opinion, be devised.

**Variation in Daily Flow.**—While the total dry-weather flow is fairly constant, the rate differs considerably throughout the day. No definite laws can be laid down, each town having its own peculiarities due to the character and occupations of its inhabitants. In small towns, moreover, the sudden discharge from large water-using factories or mills will have a controlling effect on the discharge. It will be usually correct to say that the minimum rate of flow is somewhere about 3 a.m. and, in a purely residential district, the maximum is about noon. In many cases the maximum rate is about twice the mean. Careful gaugings should therefore be taken to ascertain the present variation, and then the question considered as to whether the future rate is likely to be in the same proportions.

**Minimum Gradients of Sewers.**—It is usual to estimate the invert gradient on the basis of a mean velocity of not less than 3 ft. per second for the pipe sewers, and not less than 1½ ft. per second for the largest brick or concrete sewers. The question, however, is settled by the least velocity which will prevent deposition of silt and the highest velocity consistent with erosion of the lining.

This latter usually limits the velocity to 3 ft. per second. It must always be remembered that the hydraulic, and not the invert gradient, determines the discharging capacity of a sewer (*see* "Invert and Hydraulic Gradients"). In the case of inverted syphons the velocity may be much greater, as they are usually composed of iron pipes, and must be kept clean.

**Discharge of Storm-Water from Large Areas.** — It frequently happens that the engineer, in designing a new sewerage scheme, has to consider the storm-water run-off from large tracts of uninhabited or purely agricultural areas. The tendency of towns is to be built on the banks of a stream or on the foreshore of the sea, and hence the uninhabited areas are usually on higher ground at the back of the town. Such areas will usually run to several thousand acres, and the run-off will be considerable. While streams will always exist into which such water will gravitate, the problem usually confronts the engineer as to how he is to get the flow through the congested area of the town in safety to the sea or river. In the past, ill-considered schemes for confining the flow have frequently resulted in periodic flooding of the town, with heavy damage to property. In many cases the stream has been covered in and converted into a sewer, and has been running at something approaching its maximum capacity at the very time it should have been free to deal with the upland waters.

The earliest formulæ for estimating rain-water run-off applied to large areas, whether such areas were uninhabited or not. Later a large number of investigations and consequent formulæ were developed in connection with irrigation problems in tropical countries.

Probably the first investigator of the problem was the famous engineer Hawksley, who was called in as a consultant on the first London Main Drainage Scheme, in 1853 to 1856. He took the experiments of John Roe, one of the London district surveyors, as his basis. As the symbols now to be used will constantly recur throughout this discussion, they will be tabulated :—

$Q$ = rate of run-off in cubic feet per second.
$p$ = coefficient of impermeability.
$A$ = area drained in acres.
$S$ = slope of ground in feet per 1000.
$R$ = intensity of rainfall in inches per hour.

Then Hawksley's formula may be written in the form :—

$$Q = p.A.R^n \times \sqrt{\frac{S^m}{A}}$$

$n$ and $m$ being experimental coefficients given by Hawksley as 4 and 1. Hawksley, from Roe's gaugings, assumed that 1 in. per hour was a maximum rate of rainfall to be allowed for, and he took $p$ as 0·7.

Gregory (Trans. Am. Soc. C.E., Vol. 58) has collected the various formulæ founded on Hawksley's and published the following table, giving the suggested values of the unknowns. It must be remembered that all these values, except the first, are from foreign experiments, and hence liable to be excessive for this country.

| Authority. | $p$ | $R$ | $n$ | $m$ |
|---|---|---|---|---|
| Hawksley .. .. | 0·7 | 1 | 4 | 1 |
| Bürkli-Zeigler .. .. | ·7–·9 | 1–3 | 4 | 1 |
| Adams .. .. .. | 1·837 | 1 | 6 | ½ |
| McMath .. .. | ·75 | 2·75 | 5 | 1 |
| Parmley .. .. | 0–1 | 4 | 6 | 1½ |

The authorities are all widely known, and have many supporters, Adams and McMath in the United States, and Bürkli-Zeigler on the Continent. Gregory himself suggests the formula :—

$$Q=\frac{pRAS^{0\cdot 186}}{A^{0\cdot 14}}$$

and makes $pR=2\cdot 8$ for impermeable surfaces. This, however, will only apply to the United States and Canada.

Hering, another well-known authority in the States, suggests :—

$$Q=pRA^{0\cdot 85}S^{0\cdot 27}$$

where $pR$ varies from 1·02 to 1·64. He also offers an alternative formula :—

$$Q=pRA^{0\cdot 833}S^{0\cdot 27}$$

Parker ("Control of Water") suggests :—

$$Q=cA^{0\cdot 75 \text{ to } 0\cdot 86}S^{0\cdot 16 \text{ to } 0\cdot 25}$$

$c$ being a constant depending on $p$, and $R$ the total quantity of rain falling ; but this formula probably refers to tropical or semi-tropical countries. All the above formulæ give widely different results. According to Parker, the formulæ must not be used for areas of more than 1000 acres. They must not, moreover, be used for areas which differ in character from the areas from which they were derived.

For much larger areas a different type of formula is generally given.

Kuichling (Report on N. Y. State Barge Canal, 1901) suggests :—

$$Q=\frac{a}{M+b}+c$$

Where $a$, $b$, and $c$ are constants as given below, $M=$ area drained in square miles.

| *a* | *b* | *c* | Remarks. |
|---|---|---|---|
| 44,000 | 170 | 20 | Rate of discharge occasionally exceeded. |
| 127,000 | 370 | 7·4 | Rate of discharge rarely exceeded. |
| 35,000 | 32 | 10 | Rate of discharge for areas less than 100 acres. |
| 25,000 | 125 | 15 | Rate of discharge for frequent floods. |

The formulæ apply to the mountainous regions of New England.

Metcalf and Eddy (Am. Sew. Prac., Vol. **1**) suggest

$$Q=\frac{440}{M^{0\cdot27}}$$

$Q$ being the run-off in cubic feet per second per square mile.

The present writer has records of a very large number of other formulæ, but will not quote them as they all are only true for a particular district, and are all foreign. All that can be said is that the ultimate formula will probably be found to be of the type :—

$$Q=\frac{c}{M^n}$$

but that at present $c$ and $n$ are unknown for this country, and the engineer must determine them by experiment. The engineer must also decide as to what frequency of recurrence he will allow, and choose the corresponding storm from past records.

**Intensity of Rainfall on Small Areas.**—This branch of the sewerage problem is of the greatest importance to municipal engineers, partly because the rate of run-off will be higher than on the larger areas. It has, however, received very little scientific attention in this country. The only attempt, as far as the present writer is aware, is due to Lloyd Davies (Proc. Inst. C.E., Vol. **174**), and this is based on the earlier work carried out in the United States by Emil Kuichling.

Let $R$ now =total rainfall in inches for the period $t$.

$t$=duration of storm in minutes.

$I$=average intensity of fall in inches per hour.

Then $\frac{t}{60}$= period of storm in hours and $R\Big/\frac{t}{90}=I_1$ or $R=\frac{It}{60}$.

The values of $R$ and $I$ must be obtained if possible from systematic and unbroken automatic records extending over many years. In the ideal system a series of automatic gauges should be placed at intervals throughout the area so that an accurate estimate may be derived from them. These records are then collected and the intensities $I$ worked out. As the intensity will naturally vary inversely as the length of time used to calculate the same, the curve for any chosen storm will approximate to a rectangular hyperbola, as in Fig. 1. The engineer will take a large number of storms and plot the intensities for varying periods, and will then draw his curve to either include all storm rates, or to exclude all the extraordinary storms, or to represent the mean values. The second method is preferable. Lloyd Davies (as before) gives curves for the City of Birmingham, and a curve for Manchester is given in *The Surveyor* for February, 1914.

If possible, the records should cover twenty years, but at present this is probably impossible. Five years should be looked upon as an absolute minimum. Unfortunately, the use of automatic rainfall recorders is rare in this country, and those actually installed are mostly unreliable and give records on far too small a scale. The present writer would suggest that the installation of such automatic gauges ought to be undertaken by the Government, as the information would prove of inestimable value to the public health, and municipal authorities appear to consider them a luxury.

The curve decided upon need not necessarily be a geometrical one, and no equation need be found for it.

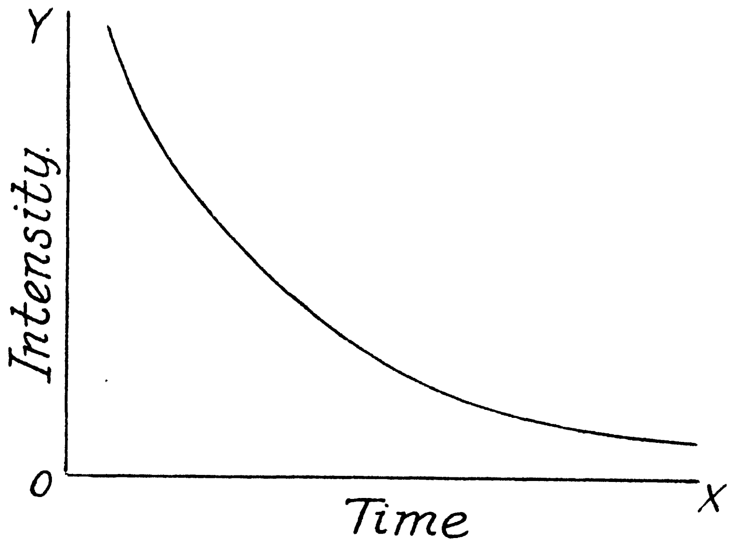


FIG. 1.

It is, however, interesting to establish one in order that comparison may be made with other results.

Such equations will usually take one of two forms :—

$$I = \frac{c_1}{t + c_2} \text{ or } I = \frac{c_3}{t^n}$$

The first is the commonest, but the second will probably prove more exact for any particular curve.

The following table has been compiled by the present writer from various authorities in the hope that it may prove of use :—

| Locality. | $c_1$ | $c_2$ | $c_3$ | $n$ | Authority. | Conditions. |
|---|---|---|---|---|---|---|
| Gt. Britain .. | 240 | 30 | — | — | Parker (after Mills) | " very rare " curve. |
| ,, | 168 | 30 | — | — | ,, | " remarkable " curve. |
| ,, | 84 | 30 | — | — | ,, | " too numerous to mention " curve. |
| ,, | 63 | 30 | — | — | Parker (after Lloyd Davies) | Birmingham "working" curve. |
| Berlin .. | 36 | 10 | — | — | Parker | covers storms of once a year. |
| East U.S.A. | 360 | 30 | — | — | Talbot | maximum curve. |
| ,, | 105 | 15 | — | — | ,, | " ordinary " curve. |
| ,, | 420 | 30 | — | — | ,, | " maximum authentic " curve. |
| Boston U.S.A. | 120 | 20 | — | — | Kuichling | covers " about once in 15 years." |
| ,, | 105 | 20 | — | — | (unknown) | covers " about once in 10 years." |
| ,, | — | — | 38·64 | ·687 | Sherman | maximum curve. |
| ,, | — | — | 25·12 | ·687 | ,, | maximum basis of design |
| ,, | — | — | 15·5 | ·5 | Metcalf and Eddy | ditto |
| ,, | 105 | 30 | — | — | Dorr | ordinary basis of design. |
| Baltimore .. | 300 | 25 | — | — | Hendrick | maximum curve. |
| ,, | 105 | 10 | — | — | ,, | " bases of design " curve. |
| Philadelphia | — | — | 27 | ·5 | Bureau of Surveys | maximum curve. |
| ,, | — | — | 18 | ·5 | ,, | " high " curve. |
| ,, | — | — | 9 | ·5 | ,, | " ordinary " curve. |
| Savannah .. | 191 | 19 | — | — | de Bryn Kops | " maximum " curve. |
| ,, | 163 | 27 | — | — | ,, | " once in two years " curve |
| ,, | 141 | 27 | — | — | ,, | " once a year " curve. |
| Chicago .. | 120 | 15 | — | — | Hill | not stated. |
| Louisville .. | — | — | 14 | ·5 | Metcalf and Eddy | ,, |
| New Orleans.. | — | — | 19 | ·5 | ,, | ,, |
| S. Francisco .. | — | — | 5 | ·5 | ,, | ,, |
| General (U.S.A.) | — | — | 32 | ·8 | Gregory | ,, |
| ,, | — | — | 8 to 15 | ·5 | Metcalf and Eddy | ,, |
| Manchester (England) | 50 | 30 | 7·36 | ·67 | Coleman | " working " curve (includes all but rarest storms). |

From the above it appears safe to conclude that, for temperate climes, the formula may be generally written :—

$$I = \frac{c_1}{t+30}$$

According to Parker (" Control of Water ") the same formula may be used for the tropics. He states that for Manilla $c_1$ may be taken as 220 for ordinary storms, and 290 for maximum storms. If the general formula just given applies to ordinary storms, according to Parker the maximum curve will be something like :—

$$I = \frac{2 \cdot 5c_1}{t+30} \text{ to } \frac{3c_1}{t+30}$$

Parker states that the value of $c_1$ is closely equal to 25 times the maximum fall in one day which occurs in the local records extending over 20 years, and that the above formulæ are probably correct up to a period of 4 hours. The present writer obtained his formula for Manchester from records continuous for 14 years. The curve was not geometric, and the following table would more exactly fit it for subdivisions of the whole period :—

| $c_1$ | Period. |
|---|---|
| 60 | Periods not longer than 15 minutes. |
| 50 | Periods between ¼ hour and 1 hour. |
| 48 | Periods over 1 hour up to 4 hours. |

Localized thunderstorms give the heaviest intensities over small areas, but for large areas heavy continuous rain of from 6 to 24 hours will give the highest results.

According to Metcalf and Eddy (Am. Sew. Prac., Vol. 1), the equation need not apply to more than 2 hours. It will be noted that the various authorities adopt a widely differing estimate of what curve should be used.

In estimating intensities from the automatic records, as small an interval of time as practicable should be adopted. This will, of course, depend on the scale of the record, but most types will allow of 10 minutes' intervals being read, and the best will allow 5 minutes. The above curves are used in connection with Kuichling's method of estimating run-off (which see).

**Coefficient of Impermeability.**—In discussing Hawksley's formula for run-off (*see* "Discharge of Storm-Water from Large Areas") we introduced a term $p$ to represent the degree to which an area would throw off the rain-water falling upon it. Theoretically, $p$ represents the ratio between the run-off and the total rainfall, but practically it is not quite so simple. If $p$ is known, then the run-off from the area may be found as follows :—

Let $Q$ = run-off in cubic feet per minute at the outlet to the area.
,, $A$ = area of catchment surface in acres.
,, $r$ = total maximum rainfall in inches which falls in a given time $t$ (in minutes).

$r$ will be taken from a diagram similar to Fig. 1, p. 15.

$$\text{Then } Q = (43560A) \times \frac{r}{12t} \times p = 3630 \frac{Apr}{t}$$

Instead of $v$ it is usually more convenient to use $R$, being the rate of fall in inches per hour.

$$\text{Then } r = \frac{Rt}{60} \text{ or } R = \frac{60r}{t}$$

$$\text{Then } Q = 3630 \times \frac{Ap}{t} \times \frac{Rt}{60} = 60 \cdot 5\ ApR \text{ cub. ft. per min.}$$

We will now return to the consideration of $p$.

It will be noted in the above value of $Q$ that $p$ is necessarily assumed as of constant value throughout the storm. Actually this is not so. Any surface, however impermeable, has a certain degree of absorptive power, and $p$ is never 100 per cent. A great deal depends on the climatic conditions immediately preceding the storm. If the storm occurs just after a long period of drought the surface, especially if suburban or rural, will soak up and hold a large amount of water before it begins to throw it off. By the time saturation is reached and the absorbed water begins to reach the streams, the maximum intensity of the storm will most probably be past. Even in towns, where the bulk of the surface is paved, or composed of roofs, the surface still hold an appreciable amount for the first few minutes of the storm, which period is usually the more critical. If, on the other hand, there has been a long period of wet weather, the effect is immediately felt in the sewer. Again, if several storms follow one after another, the time interval between the storms will affect the run-off—in short, the intensity is the important point, and not the total fall.

In order to be on the safe side it is, however, usual to take $p$ from the proportion of impermeable to total area, care being taken to make all allowances for the future character of the surface. The ratio of impermeable to permeable area will always be on the increase.

In taking out the area, due care must be taken to see whether the geographical district coincides with the area as defined by the contours. Although a certain area may be outside the boundaries as regards administration, it is possible that the rain-water may eventually be taken by the latter. Another point which has to be allowed for is the fact that water off the impermeable area may soak into the permeable area, and so increase its nominal run-off. It must, however, be remembered that impermeable areas are generally inhabited, and hence are well provided with means of rapidly transmitting the water to the sewers.

We will now consider the method of finding an average value for $p$ for a composite area $A$.

Let $A$ be composed of subsidiary areas $A_1$, $A_2$, etc., of which the value of $p$ will be $p_1$, $p_2$, etc.

$$\text{Then } Ap = A_1p_1 + A_1p_2 + \text{ etc.}$$

$$\text{Therefore } p = \frac{A_1}{A}p_1 + \frac{A_2}{K}p_2 + \text{ etc.}$$

The great authority on this branch of the subject is E. Kuichling, and he prepared the following table in connection with the intercepting sewers at Boston, U.S.A. (Report of Kuichling and Bryant, Boston, 1911) :—

| Type of Surface. | $p$ |
|---|---|
| Waterproof roof surfaces .. .. .. .. .. | ·7 –·95 |
| Asphalte surfaces in good order .. .. .. .. | ·85–·9 |
| Stone, brick, and wooden pavements with tight cement joints | ·75–·85 |
| Ditto, but with open or uncemented joints .. .. .. | ·5 –·7 |
| Ditto inferior and with open joints .. .. .. .. | ·4 –·5 |
| Macadamized roads .. .. .. .. .. .. | ·25–·6 |
| Gravel roads and footpaths .. .. .. .. .. | ·15–·3 |
| Unpaved surfaces, railway yards, and vacant lots .. | ·1 –·3 |
| Parks, gardens, lawns, and meadows, varying according to the surface, slope, and character of subsoil .. .. | ·05–·25 |
| Wooded areas, depending on surface, slope, and character of subsoil .. .. .. .. .. .. .. | ·01–·2 |

The above table is built up on the character of the surface, and not on the density of the population. Kuichling also gives the following table :—

| Class of District. | $p$ |
|---|---|
| Most densely built up portion of district .. .. .. | 0·7 –0·9 |
| Adjoining well built up portions .. .. .. .. | 0·5 –0·7 |
| Residential portions, detached houses .. .. .. | 0·25–0·5 |
| Suburban areas with few buildings .. .. .. .. | 0·1 –0·25 |

The method of the first table is preferable to that of the second, which is, at times, apt to be misleading. For instance, the present writer is aware of an entirely impervious area which forms the centre of an industrial city, which at night-time was found to contain only 4·7 residents to the acre, practically all caretakers. Also a town, built practically on the solid rock, may have a very small population per acre. The engineer must therefore trust ultimately to his local knowledge of the district under consideration. Gregory (Trans. Am. Soc. C.E., Vol. 58) suggests the formula :—

$$p = 0{\cdot}175 t^{\frac{1}{3}}$$

where $t$ is the "time of concentration" (*see* "Time of Concentrations"); but this only applies to totally impervious surfaces.

Professor Ogden ("Sewer Design") has prepared the following table from Kuichling's studies of the City of Rochester (N.Y.) :—

| Average number of persons per acre. | Percentage of totally impervious surface. | | | Total per cent. of fully impervious surface per acre. |
|---|---|---|---|---|
| | Roofs. | Improved streets. | Unimproved streets and yards. | |
| 15 | 8·4 | 3·3 | 3·0 | 14·7 |
| 25 | 14·0 | 7·0 | 4·3 | 25·3 |
| 32 | 18·0 | 10·2 | 5·0 | 33·2 |
| 40 | 22·5 | 14·7 | 5·4 | 42·6 |
| 50 | 28·0 | 19·0 | 5·6 | 52·6 |

Lloyd Davies (Proc. Inst. C.E., Vol. 174) gives the following rule for the City of Birmingham :—

$$p = 0{\cdot}0105P$$

where $P$ is the population per acre.

The following table was used for the new main drainage scheme for Manchester (*see* Supplement to *The Surveyor*, April, 1914). To obtain the table, typical areas of fifty acres in extent were taken for all except the last, and the population obtained by counting the inhabited houses and multiplying by a flat rate of 4·7 persons per house :—

| Population per acre. | Percentage area of buildings and impermeable area. | Percentage area of back-yards, gardens, and open spaces. | | Percentage area of streets and passages impermeable. | Total percentage impermeable area. |
|---|---|---|---|---|---|
| | | Impermeable. | Permeable. | | |
| 179 | 42 | 15·67 | 0·33 | 42 | 99·67 |
| 164 | 36 | 22 | 2·0 | 40 | 98 |
| 146 | 29 | 22 | 11 | 38 | 89 |
| 136 | 26 | 32 | 11 | 31 | 89 |
| 103 | 33 | 22 | 8 | 37 | 92 |
| 85 | 26 | 16 | 26 | 32 | 74 |
| 66 | 19 | 13 | 47 | 21 | 53 |
| 52 | 18 | 19 | 40 | 23 | 60 |
| 24 | 12 | 6 | 58 | 24 | 42 |
| 4·7 | 7 | 3 | 76 | 14 | 24 |

Metcalf and Eddy (Am. Sew. Prac., Vol. 1) found that for American cities there was a somewhat definite relation between the density of population and the street surface. The following table is compiled from their curve :—

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Percentage of area . | 3 | 5·7 | 8·2 | 10·7 | 13 | 15 | 16·6 | 18·1 | 19·2 | 19·8 |
| Population per acre . | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

K. Imhoff gives the following table as generally applicable to German cities. The quantity of house sewage is based on 100 litres per head per day, all passing off in 12 hours, and the storm-water is assumed at a rate of 1·69 ins. per hour :—

| Condition of surface. | Population per acre. | Quantity of house sewage in c.f.s. per acre. | Storm-water. | |
|---|---|---|---|---|
| | | | Coefficient of run-off. | Rate of run-off per acre in c.f.s. |
| Very thickly built up .. | 141 | 0·0116 | 0·80 | 1·37 |
| Closely built up .. | 101 | 0·0083 | 0·60 | 1·03 |
| Well built up .. .. | 61 | 0·005 | ·0·25 | 0·43 |
| Suburban .. .. | 40 | 0·0033 | 0·15 | 0·26 |
| Unsettled .. .. | 0 | 0 | 0·05 | 0·086 |

It is usual to assume that the " coefficient of run-off " is the same thing as the coefficient of impermeability, but this is not necessarily so (*see* " Coefficient of Retention "). In cases where no data exist, some such method as the following (Metcalf and Eddy) must be used :—

| | | | | |
|---|---|---|---|---|
| Let | 15 per cent. of area be estimated as | impermeable roof | | $p_1 = 0{\cdot}95$ |
| ,, | 30 ,, ,, ,, ,, | pavements | .. | $p_2 = 0{\cdot}90$ |
| ,, | 40 ,, ,, ,, ,, | lawns .. | .. | $p_3 = 0{\cdot}15$ |
| ,, | 15 ,, ,, ,, ,, | gardens | .. | $p_4 = 0{\cdot}10$ |

Let $p$ = average coefficient for whole area.

$$p = \frac{A_1}{A}p_1 + \frac{A_2}{A}p_2 + \text{etc.}$$ (*see* " Coefficient of Impermeability ")

$$\therefore\ p = \left(\frac{0{\cdot}15A}{A} \times 0{\cdot}95\right) + \left(\frac{0{\cdot}3A}{A} \times 0{\cdot}9\right) + \text{etc.}$$

$$= 0{\cdot}4875 \text{ say } 0{\cdot}5$$

**Kuichling's Law and the " Time of Concentration."**— The time of concentration is defined as the time taken for rain falling at the farthest point in the area to reach the outlet to that area. Such time is made up of two periods, the first the time taken for the rainwater to reach the sewer from the place where it touches the surface, and secondly the time taken to travel along the sewer to the outlet from the area. The second portion of the time is an ordinary hydraulic problem, but the first is very uncertain owing to a variety of causes which cannot be accurately estimated.

Metcalf and Eddy (Am. Sew. Prac., Vol. I) give as their experience that the time of travel over the surface, usually termed the " time of entry," is seldom less than three or more than ten minutes, but it must be remembered that this applies to American cities.

W. W. Horner (Eng. News. 1910) found that the rainfall from streets, footpaths, and roofs, reaches the sewer in from 2 to 5 minutes with streets of gradients from 1 in

200 to 1 in 20, but that the velocity over grass-plots was very slow, and that, even in heavy rains, from 10 to 20 minutes are required to flow 100 feet. Such period allowed should never be long, and should be made up by taking a low coefficient of impermeability.

C. E. Gregory (Trans. Am. Soc. C.E., Vol. 65) considers that an allowance of 5 minutes for streets may be correct, but that, for a maximun rate, the time is longer, and depends entirely on local circumstances.

Kuichling was the first to consider the question of the relation of the time of concentration to the discharging capacity of the culvert (Report to the Common Council of Rochester, 1889), and the following is the law he enunciated, a law which is now generally accepted by drainage engineers.

"The discharge from a given surface will become a maximum for the condition that the duration of such rate is equal to the time required for the water which falls on the more distant points to reach the place of observation, or, in other words, that the entire area is contributing to such discharge. . . . The conclusion is accordingly irresistible that the rates of rainfall adopted in computing the dimensions of a main sewer must correspond to the time required for the concentration of the drainage waters from the whole tributary area when small, or from such as will produce an absolute maximum discharge when an area is very large."

This law was adopted in the consideration of flood discharges at Birmingham (*see* Lloyd Davies, Proc. Inst. C.E., Vol. 164), and such diagrams as Fig. 1 are then necessary, the time co-ordinate including the time of entry.

With regard to the time of entry, the present writer had occasion to go to considerable trouble to obtain values. The area was divided into three typical sub-divisions as follows :—

(*a*) Best class of property composed of large mansions in their own extensive grounds.

(*b*) Second class of property principally composed of semi-detached houses with smaller gardens.

(*c*) Cottage property and working-class areas where the houses are in terraces, and have little or no space between them and the street line.

It was found, however, that no strict law could be given. In class (*a*) the time varied from 1½ to 4½ minutes, depending on the distance of the house from the road. The probable average was about 2 minutes.

In class (*b*) the time averaged from 1 to 5 minutes, with an average of 1 minute.

In class (*c*), when the houses abutted on the street, the time was half a minute or less, but many cottages are a long way from the street line, and the time reached 7 minutes or more.

As a general rule, the present writer would recommend from ½ minute to 10 minutes, but rarely more than 5 minutes.

In estimating the time taken to travel down the sewer, as the latter is generally not yet in existence, the velocity has to be roughly guessed at. This may be done by taking the ground contours and plotting a rough level section of the proposed route, and taking the hydraulic gradient as parallel to the surface ditto. If this cannot be done, the only thing is to assume an average velocity of say 3 feet per second, and divide the length of the longest route of sewer in the area by this value, to obtain the time of concentration.

Having obtained the value of the time of concentration, we obtain from a diagram corresponding to Fig. 1, p. 15, the maximum rate of rainfall to which, by Kuichling's law, such area can ever be subjected, and this will be the value to insert in the formula :—

$$Q = 60{\cdot}5ApR$$

given on p. 18.

While most drainage engineers, including the present writer, agree with the law in its general form, a rigid adherence to it may lead to errors in certain cases, and it must be clearly understood that Kuichling himself stated that the value obtained would be the absolute maximum, and that circumstances might possibly modify the result. Unfortunately, in the present writer's opinion, this point has been subsequently overlooked.

**Distribution of the Intensity of Downpour throughout a Storm.**—Most engineers assume that all parts of the surface under a storm are equally affected. Frühling, however ("Entwässerung der Städte"), assumes that the maximum rate of fall is at the geometrical centre of the area as shown in Fig. 2, and falls off to nothing at the boundaries. Such an assumption implies that the storm is always circular on plan, a fact not proved by the present writer, and is apparently founded on an isolated experiment at Breslau, where Frühling considered that he found that the rate of fall, at 10,000 feet from the centre of the storm, was half that at the centre. He derived the formula :—

$$D = 1 - 0{\cdot}005\sqrt{L}$$

where $D$ is the intensity at distance $L$ from the centre. In English measures this becomes :—

$$D = 1 - 0{\cdot}0028\sqrt{L}\ (L \text{ in feet}).$$

With this law the fall vanishes at a radius of 7½ miles.

Storms, however, are rarely either circular on plan

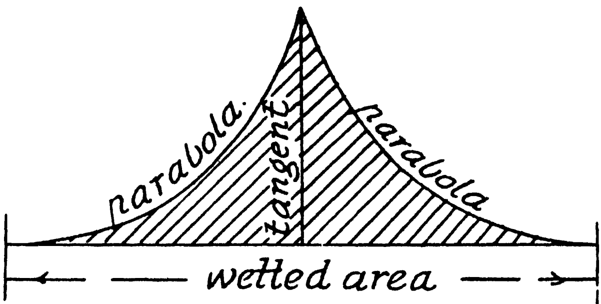


FIG. 2.

or of any regular geometric shape. The above law requires a great deal more experimental work on it before it should have the slightest authority. All that can be done at present is to take the downfall as uniform throughout the area affected.

**Coefficient of Retention.**—German, and certain American, engineers are accustomed to increase the time of concentration by allowing for the time of wetting the surface before run-off commences, also the time during which the fall is evaporated during commencement of the storm due to latent heat in the surface, especially in summer, also the time taken to collect and fill up surface depressions, and time due to being held up by vegetation or porous earth.

Frühling ("Entwässerung der Städte") has published a table of "coefficients of retention," but it is not reproduced here as the present writer has doubts as to its correctness; in fact, no reliable information at present exists as far as he is aware.

**Coefficient of Retardation.**—Under this name German engineers are accustomed to calculate the ratio between the area affected by the storm and the whole drainage area. In most cases, in this country at any rate, this ratio will be unity. If *t* is the period of the downpour, the portion of the area to be taken will be the largest fraction of the whole area which can be included between two "contours," which contours will be a distance apart equal to the distance travelled during the period of the downpour. The German engineers first find the period of the storm from the time of concentration, and then assume a retardation coefficient which will further modify the time just found. The present writer sees no valid argument for this, and does not recommend the method. The time of concentration is not, as pointed out by Metcalf and Eddy, a constant value, but is greater in light storms, when the culverts are only partially filled, than in heavy storms, when the higher velocities are reached in the culverts. It is, therefore, the custom to assume a uniform velocity in all cases, calculated from the velocity when running half full. Such a coefficient as that suggested could only be of use when calculating run-off from an area from gaugings taken in the existing sewers.

**Storage Capacity of a Sewer.**—At the commencement of a storm there will be a considerable space available for storage, depending on the quantity of domestic and trade sewage present at the moment. If the branch sewers can always discharge freely into the main sewer, such storage will be valuable in easing the first rush on the main. If, however, the branches discharge into the side of the main, it is probable that the latter will soon be running full, and so choking the discharge of the branches. These branches will therefore have to act under pressure to get rid of their water, and will rapidly lose any storage capacity. It would be far wiser, therefore, to make no allowance for storage in design, but look upon it merely as a factor of safety.

**"Rational" Method of Estimating Storm-Water Discharge.**—The present writer believes this method is due to W. W. Horner (Eng. News. 1910). It is now universally used in the United States and on the Continent, but is not generally known in Great Britain. The procedure is as follows :—

From a careful contoured survey of the area, the various subsidiary drainage areas are defined and their areas, general slope, and average value of the coefficient of impermeability are ascertained and tabulated. The time of concentration is then worked out, and, following Kuichling's law, the maximum possible rate of rainfall is ascertained from a diagram for the district corresponding to Fig. 1, p. 15. From this the capacity of the main sewer of the subsidiary area is calculated, allowing for the maximun dry-weather flow of sewage proper.

In estimating the rainfall run-off for St. Louis, Horner took an average built-up block, containing 40 persons per acre, and found the impervious area to be 45·7 per cent. He then constructed the following table, allowing 60 per cent. run-off from the impervious portion, and 20 per cent. from the pervious remainder. $p$ is therefore a run-off coefficient :—

| Duration of storm in minutes. | Percentage run-off. | | $p$. |
|---|---|---|---|
| | Impervious portion. | Pervious portion. | |
| 10 | 60 | 20 | 0·4 |
| 15 | 70 | 30 | 0·5 |
| 20 | 80 | 35 | 0·55 |
| 20 | 85 | 40 | 0·6 |
| 60 | 95 | 50 | 0·7 |
| 120 | 95 | 60 | 0·75 |

The values of $p$ are arrived at as follows :—

$$45{\cdot}7 \times 0{\cdot}60 + 54{\cdot}3 \times 0{\cdot}20 = p \times 1{\cdot}00.$$

Hence $p = 0{\cdot}3828$, say 0·4, and so on.

The above table is quoted by Metcalf and Eddy (Am. Sew. Prac., Vol. 1), who suggest taking $p$ as 0·6 instead of 0·4, or, in other words, taking the full run-off from the impervious portion and ignoring the pervious, and this has been the present writer's custom. In this latter method, therefore, the coefficient of run-off is the same as the coefficient of impermeability.

On p. 18 we derived the formula :—

$$Q = 60{\cdot}5ApR$$

where $Q$ = run-off in cubic feet per minute.
$A$ = area of surface in acres.
$p$ = coefficient of run-off or impermeability.
$R$ = rate of rainfall in inches per hour.

In designing a sewerage scheme it will be useful to have a diagram, in which the co-ordinates are $Q$ and $pR$, or, in other words, a diagram giving $Q$ per acre for given values of $p$ and $R$. $p$ will, of course, have a value derived from the area when fully developed in the future, and not at the time of designing the scheme. Having, by the above process, determined the sizes of the principal sewer in each sub-area, the size of the main sewer for the whole area may be determined.

C. H. Nordell (Eng. News. 1909) proposed a variant on the above method which, however, as far as the present writer is aware, has not been largely adopted.

Having selected a typical storm of maximum intensity, he proposed to plot a corresponding intensity diagram

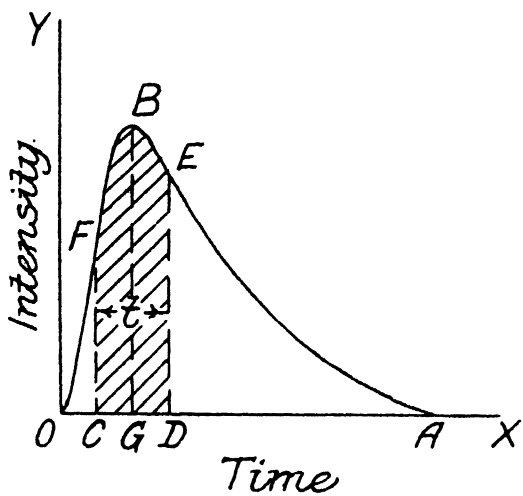


FIG. 3.

*OBA*, as shown in Fig. 3. *OA* is the duration of the storm and *BG* the maximum intensity, which, according to Nordell, occurs near the commencement of the storm. He then calculates the time of concentration *t* in the usual manner and plots two vertical lines *CF* and *DE*, at the given time interval *t* apart, on tracing-paper. He then suggests moving the tracing-paper to and fro over the intensity diagram until the verticals *CF* and *DE* intercept the maximum area on the intensity diagram.

The average ordinate to the area *FBEDC* is then to be taken as the intensity $R$ in the formula :—

$$Q = 60{\cdot}5ApR$$

The method has been severely criticized by American engineers, principally from the point of view that the ordinary intensity diagram is not as shown, but is flatter, or without a high intensity for a very short period.

**Author's Method.**—The present writer suggests the following method as approaching more nearly to the ideal, and as making the most practical use of the various methods previously offered. It is, moreover, adaptable to any local conditions and to the experience of the designer.

In the same way as under Horner's method, prepare an up-to-date map showing all the subsidiary drainage areas, and tabulate the areas in acres, future populations, future coefficients of impermeability, and times of concentration (future).

Mark on the plan all points where the subsidiary areas will discharge into the main trunk sewer, and differentiate between combined sewers and what are either open or covered watercourses for the reception of storm-water only.

It is now necessary to collect all the available automatic rainstorm records for the given area. These should cover a period of at least 10 years, so that the rarest storms may be included. Select all the most notable storms, paying more attention to those which are short in duration and great intensity, than to those which may give a greater total downpour but which are spread over a longer period of time. It must be always remembered that it is the sudden heavy downpours which last a short time which do the damage, a sewer of very much less capacity being able to deal with much more rain if only it is more evenly spread over a long period.

Several courses are now open to the engineer as to choosing a storm on which he may base his calculations. The downfall curves finally selected by him will cover all sorts of periods, and all kinds of different rates and different distributions of the intensity. He may therefore decide on selecting one storm out of the lot which he will consider as both typical and also of reasonably high value, or he may plot all the selected curves one over the other, and draw an enveloping curve for absolute maximum, or a mean curve, or a curve which is about two-thirds the absolute maximum. Whatever curve he finally selects, whether hypothetical or actual, becomes his "working" curve under this method.

From the working curve he calculates the corresponding diagram of intensities of fall throughout the storm, and plots them on a time base after the manner of Fig. 4, p. 34.

Such intensity diagram may be after the same type as Fig. 4 or quite different in shape. In Figs. 5 to 8, pp. 34 to 36, are shown intensity diagrams for Manchester, Newcastle-on-Tyne, and Kew, the latter having been calculated from records kindly supplied by the Meteorological Office. In the diagram as first plotted, a large number of sudden drops and rises will appear, as shown in Fig. 4, but the diagram to be used is shown by the smooth curve $OAB$.

The next step is to draw a diagram showing the rate of run-off, the average value for $p$ having been determined as shown on p. 20.

Before, however, doing this, it is necessary to decide whether the storm selected must be considered as being

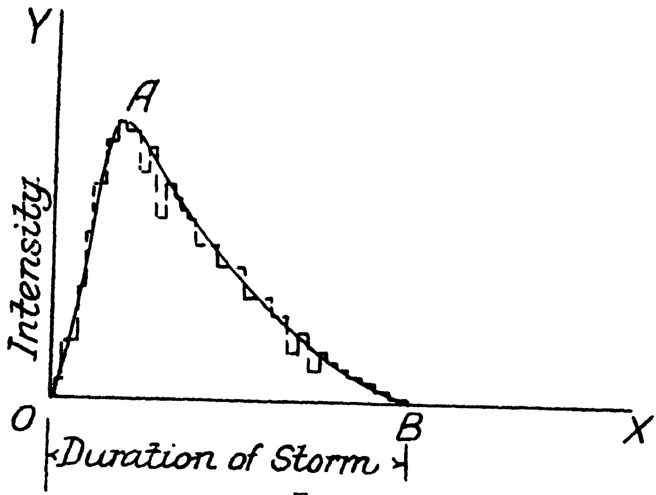


FIG. 4.

stationary or moveable. The writer spent a considerable amount of time trying to find whether there was any natural law connecting the intensity of the downpour with the motion of the storm. He finally came to the conclusion that there was no such law, the heaviest storms being included amongst both the stationary and moving classes. While some appeared to start and finish over a well-defined stationary area, others travelled at a usual value of about 12 miles per hour.

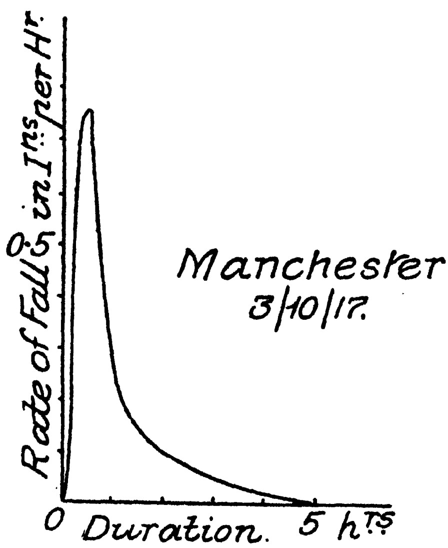


FIG. 5.

As regards direction of travel, it appears in this

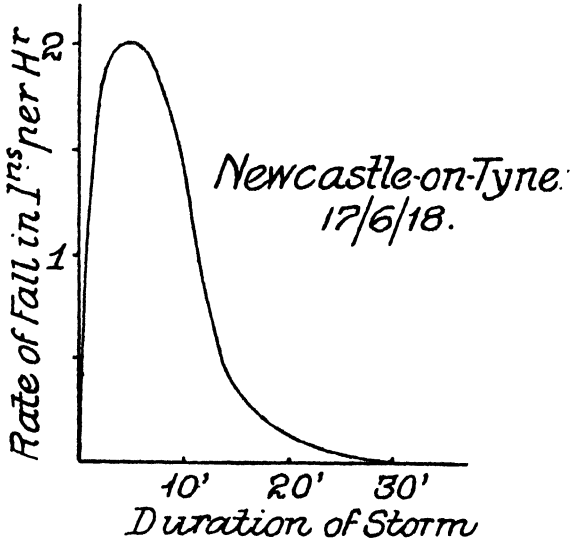


FIG. 6.

country to be practically always from the west or south-

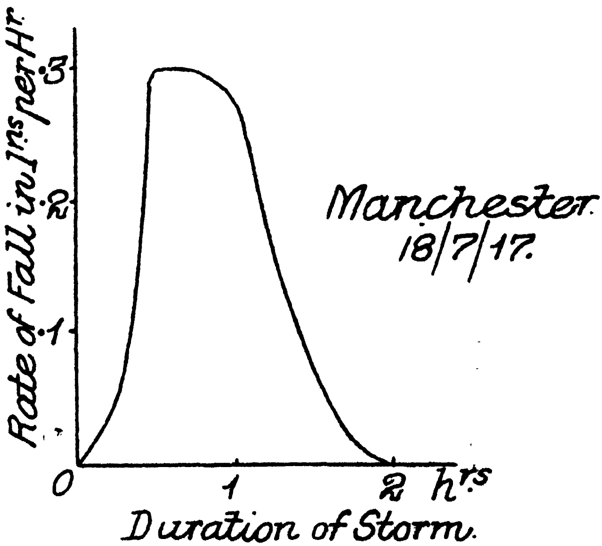


FIG. 7.

west, and to gradually turn northwards. It will be seen later that, if the storms are taken as moving, it may be necessary to make an allowance for the direction of travel under certain circumstances.

When the storm is stationary, the differences in intensities recorded by the rain-gauge must be due to differences in the rate of precipitation of the rain. This may be due to variations which are uniform throughout the storm area or not, but at present there is no information

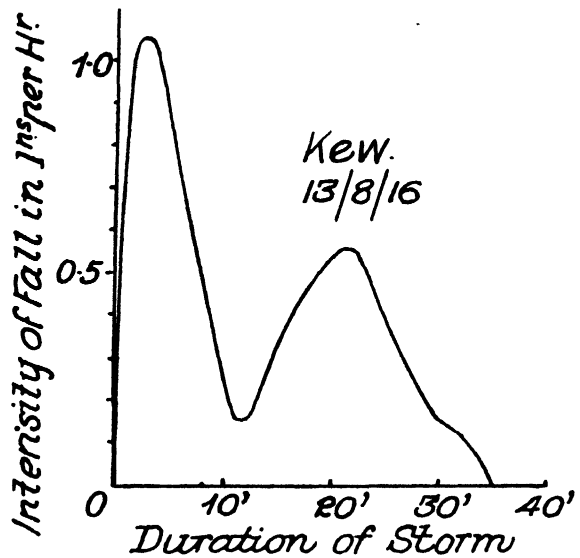


FIG. 8.

on this point. When, however, the storm is moving, the variation may be due to different intensities which occur in parts of the storm, affecting the gauge in turn. If Fig. 4 is due to this, it will mean that the fringes of the storm contain the smaller intensities, and that the maximum intensity is nearer the front end of the storm.

Let $T$ = time of duration of storm, or length $OB$ of Fig. 4.

Let $t$ = time of concentration for any area.

Reproduce Fig. 4 as in Fig. 9, and extend *OB* to *C* so that *BC* represents *t*. Then *OC* will represent the duration of the effect at the outlet to the area, supposing the storm stationary.

Let Fig. 10, p. 38, represent the area, taken for simplicity

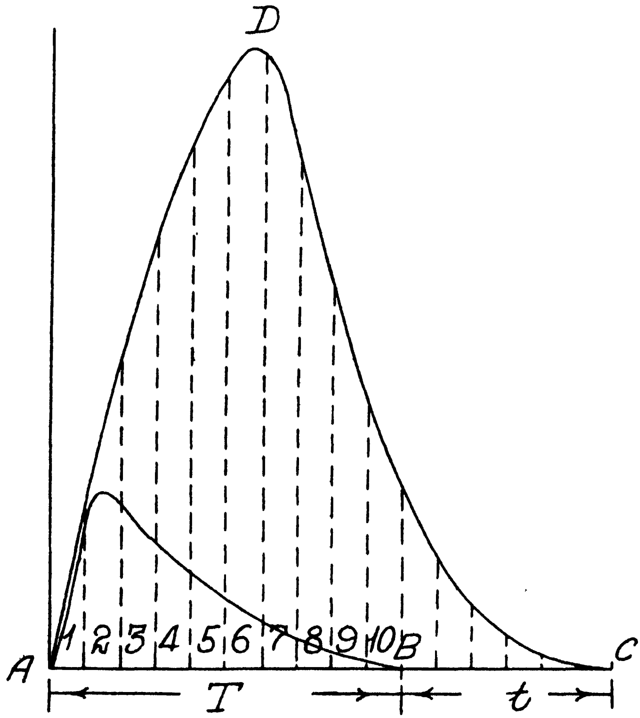


FIG. 9.

as rectangular. Let *ZW* be the main sewer of the area discharging into the intercepting sewer *WK* at *W*.

Let *t* be six-tenths of *T* or 0·6*T*.

Suppose the area *XY* were also divided into six equal sub-areas as shown by dotted lines.

Divide *AB* of Fig. 9 into ten equal divisions, then *BC* can be divided into six equal divisions of width the same as those of *AB*. Number the divisions of *AB* (1), (2), (3), etc., as shown. Draw the verticals through the points of division of $AB$ and $BC$. The mean ordinates of the strips of the intensity diagram will represent the mean rate of intensity for successive intervals of time $T/10$, $2T/10$, $3T/10$, etc.

When the storm bursts, each of the sub-areas of Fig. 10 will receive a rain of mean intensity equal to the mean ordinate of strip (1) of Fig. 9 for a time-period $T/10$.

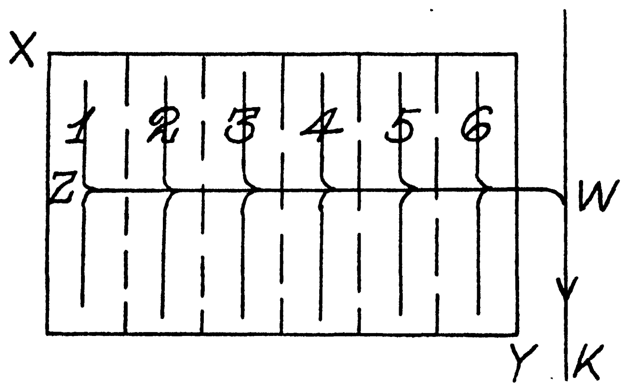


FIG. 10.

The rate of run-off from each sub-area will be :—

$$60{\cdot}5 \times \frac{A}{6} \times p \times i_1$$

where $i_1$ is the mean ordinate of the first strip of Fig. 9.

As the factor $60{\cdot}5\frac{A}{6}p$ will constantly recur, let us denote it by $c$.

At the end of time-interval $2T/10$ the rate of run-off from sub-area (2) is $ci_1$ and from sub-area (1) is $ci_2$.

Hence the total rate of run-off after time $2T/10$ is :—

$$c(i_1+i_2).$$

Similarly after time $3T/10$ it is :—

$$c(i_1+i_2+i_3)$$

and so on.

After time-interval $6T/10$ it will be :—

$$c(i_1+i_2+i_3+i_4+i_5+i_6),$$

and the whole area is contributing.

From now onward no sub-area will contribute with intensity $i_1$ and after time-interval $7T/10$ the rate of run-off will be :—

$$c(i_7+i_6+i_5+i_4+i_3+i_2).$$

After time-interval $8T/10$ it will be :—

$$c(i_8+i_7+i_6+i_5+i_4+i_3),$$

and so on.

After time-interval $10T/10$ it will be :—

$$c(i_{10}+i_9+i_8+i_7+i_6+i_5),$$

and the storm ceases.

After time-interval $11T/10$ the rate of run-off will be :—

$$c(i_{10}+i_9+i_8+i_7+i_6).$$

After time-interval $12T/10$ it will be :—

$$c(i_{10}+i_9+i_8+i_7),$$

and so on.

The final rate of run-off will become zero at $16T/10$ or $(T+t)$.

These rates can be very conveniently found and shown graphically. In Fig. 11 set off a base line equal to $ABC$ of Fig. 9. Reproduce the intensity diagram of Fig. 4 six times as shown, making them overlap by a time-interval $T/10$. Then each individual curve represents the rate of run-off from a sub-area if multiplied by $c$, and the sum of the overlapping strips represents the total rate of run-off if multiplied by $c$.

Let this summation be carried out and plotted on Fig. 9, as shown. We thus get the curve $ADC$. A series of such curves will have to be obtained for each sub-area such as $XY$, Fig. 10.

There is one small error in the above method. The actual run-off from sub-area (6) will commence immediately the storm breaks, whereas we have had to assume time-intervals of $T/10$. Actually, therefore, we have to allow a slight addition as shown dotted in Fig. 11, but the effect is insignificant.

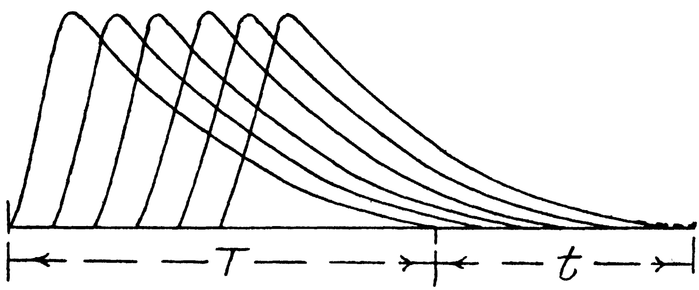


FIG. 11.

The method can be very easily applied to a moving storm. Let the storm, for instance, be supposed to enter the area $XY$ at $Z$ and travel parallel to $ZW$ and in that direction. Let us take the same storm as before, so that the intensity-of-fall diagram is the same.

The interval at which the run-off from each sub-area arrives at the outlet $W$ depends on the rate of travel of the storm. As a first case let the storm travel at the same rate as the flow down the sewer $ZW$.

The intensity of run-off (1) from sub-area (1) will pass through sub-area (2) at the same time as the latter sub-area is receiving its intensity (1). The same will occur with sub-area (3), so that by the time the storm has reached $W$ the rate of run-off will be $c$ times $6i_1$.

After another time-interval $T/10$ it will be $6ci_2$, and so on. The Fig. corresponding to Fig. 12 will therefore be the Fig. 4 with its ordinates all increased in the proportion of 6 to 1, or, in other words, with all the curves

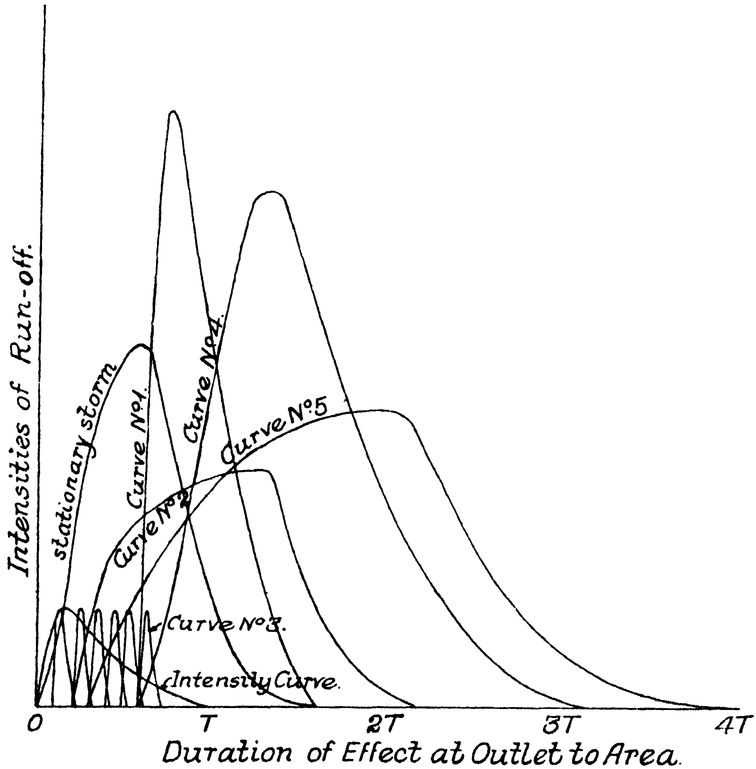


FIG. 12.

of Fig. 11 coinciding. We should then obtain curve No. 1, Fig. 12, which has a greater momentary intensity than the maximum of Fig. 9, but the duration of the run-off is only $T$ in place of $(T+t)$.

Let the storm now travel up the area from $W$ to $Z$ and at the same rate as before.

Sub-area (6) receives intensity (1), a time-interval $T/10$ before sub-area (5), so that the time-interval between the two run-offs is $T/10+T/10$, or $2T/10$ when they reach the outlet. The Fig. corresponding to Fig. 11 has therefore to have the ordinates $T/10$ apart, but the diagrams will overlap an amount $2T/10$.

The total rate of run-off is shown by curve No. 2.

If the storm be supposed to travel at 12 miles an hour, the usual rate assumed for the flow in the sewer is about 2 miles an hour. Let us first suppose the storm passing from $Z$ to $W$ and that these rates obtain.

Sub-area (6) receives the intensity (1) at a time $t$ or $6T/10$ after the storm enters the area. Sub-area (1) receives its intensity (1) a time $t$ before sub-area (6), but the storm is so rapid in travel in comparison to the flow in the sewer that the run-off from sub-area (6) will pass off first.

The storm travels at 6 times the rate of the sewer flow, hence sub-area (6) receives intensity (1) a time-interval $t/6$ or $T/10$ after sub-area (1), and passes it off in time $T/10$.

Sub-area (5) receives its intensity (1) a time $t/36$ or $T/60$ before sub-area (6). Hence, intensity (1) from sub-area (5) reaches the outlet $T/60$ before plus $T/10$ after sub-area (6), or $5T/60$ after sub-area (6). The sub-areas receive the different intensities at time-intervals of $T/60$, hence they will arrive at the outlet at the same intervals.

To draw the figure corresponding to Fig. 11, therefore, we contract the strips to $T/60$ from $T/10$, and make the overlap $5T/60$. The resulting total rate of run-off curve is shown by curve No. 3, and is negligible.

Finally, let the storm travel from $Z$ to $W$ at half the rate of flow down the sewer.

Each sub-area receives its intensities of the same value at time-intervals of $2T/10$ instead of $T/10$, as in the first case.

The first rate of run-off from sub-area (1) reaches the outlet in time $6T/10$. The first rate of run-off is received by sub-area (2) a time $2T/10$ after sub-area (1), but only takes a time 5/6ths of $6T/10$ or $5T/10$ to reach the outlet. Hence, the first intensity reaches the outlet from sub-area (2), a time-interval $(2T/10+5T/10)$ after, plus $6T/10$ before or $T/10$ after the same intensity from sub-area (1).

The figure corresponding to Fig. 11 is therefore found by making the strips $2T/10$ wide and placing the individual diagrams with an overlap of $T/10$. The curve of total rate of run-off is No. 4 of Fig. 12.

If the storm had travelled up the area with the same speed, the total rate of run-off would have been curve No. 5.

The writer also investigated the question of the storm crossing the area at right angles to the length, but found the results negligible.

He trusts therefore that the above examples will show the flexibility of the method.

The idea of supposing the sub-areas as all of equal area, width, and permeability, may be objected to, and is not theoretically necessary. It, however, much simplifies the work and obtains sufficiently accurate diagrams of total rate of run-off.

The student must carefully avoid the error of supposing that $t=0{\cdot}6T$ in every case. It was so taken here because it happened to be so in an actual example worked by the writer, but $t$ may be any fractional value of $T$, proper or improper. $T$ is most conveniently divided into 10 parts, and the area into the same number of imaginary divisions as $t$ is, but such areas are not found on the plan or otherwise used, except to find the value of $c$.

Finally, it may be remarked that it is easier to first ignore the value of $c$ altogether, and find the curve $ADC$ of Fig. 9 as if $c=1$. Such a curve will then apply to any area in which $t$ bears the given proportion (0·6 in the present case) to $T$, and a whole series of such curves may be drawn giving results for when $t=0{\cdot}5T$, $0{\cdot}6T$, $0{\cdot}7T$, and so on, before the actual results of the run-off from each area are gone into. This will considerably expedite the work, as one assistant can be engaged on this part of the work while another is engaged on envaluating $p$, $A$, etc.

It should be noted that the case where the storm is supposed to follow approximately the route of the main sewer of the area and to move in the same direction of flow and at the same speed, will probably give the maximum rate of run-off for that main sewer. As, however, the time of run-off is limited to $T$, the effect on the intercepting sewer $WK$ of Fig. 10 may not be so severe as in the other cases quoted, as the whole effect may have been received by $WK$ and passed forward before the arrival of the flow from the next area higher upstream. This question will now be investigated.

To take the simplest case, let us suppose there are only two areas, as shown in Fig. 13. $AB$ is the main intercepting sewer receiving the run-off from (1) at $A$ and from (2) at $B$. $C$ is the outlet from (2).

Let it take a time $t_3$ for the run-off from (1) to go from $A$ to $B$.

Let us suppose, further, that the time of travel from $C$ to $B$ is negligible.

In Fig. 14, p. 46, take a vertical line $OY$, and at any point $P$ on $OY$ draw a horizontal $PG$. On $PG$ mark off $PF = t_3$, and $FG$ the time of duration of effect at the outlet to the area (1). Draw the diagram $FLG$ representing the total rate of run-off diagram for area (1).

Take any point $H$ on $OY$ above (or below) diagram $FLG$ and, starting from $H$, draw on the horizontal $HK$ the rate of run-off diagram for sub-area (2). From any point $O$ on $OY$ draw a horizontal line $OC$, and draw on

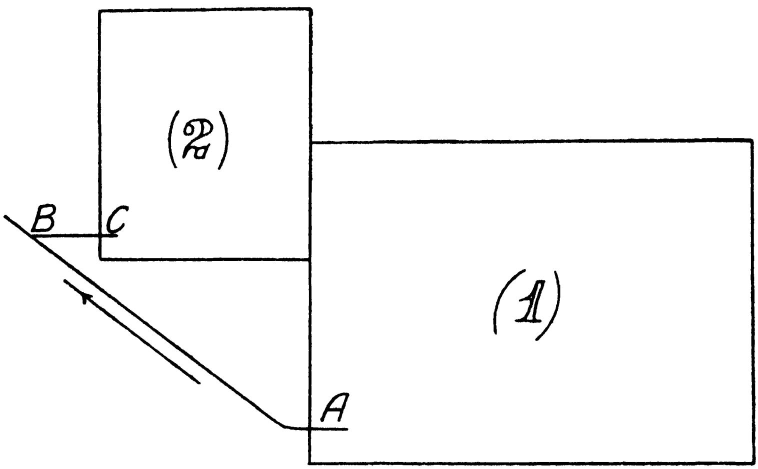


FIG. 13.

$OC$ a diagram $OEDC$, compounded of diagrams $FLG$ and $HMK$.

Diagram $OEDC$ will represent the rate of run-off diagram for the point $B$ in Fig. 13. From $O$ up to time $OA$ we merely have the run-off from area (2). From time $OA$ up to time $OB$ we have a run-off compounded of runs-off from both areas. From time $OB$ to time $OC$ we have only the run-off from area (1).

Great care must be taken to get the time-intervals $t_3$ in the proper relative positions. The maximum ordinate *ND* will govern the size of the main sewer from *B* (Fig. 13) onwards until the next connection is reached.

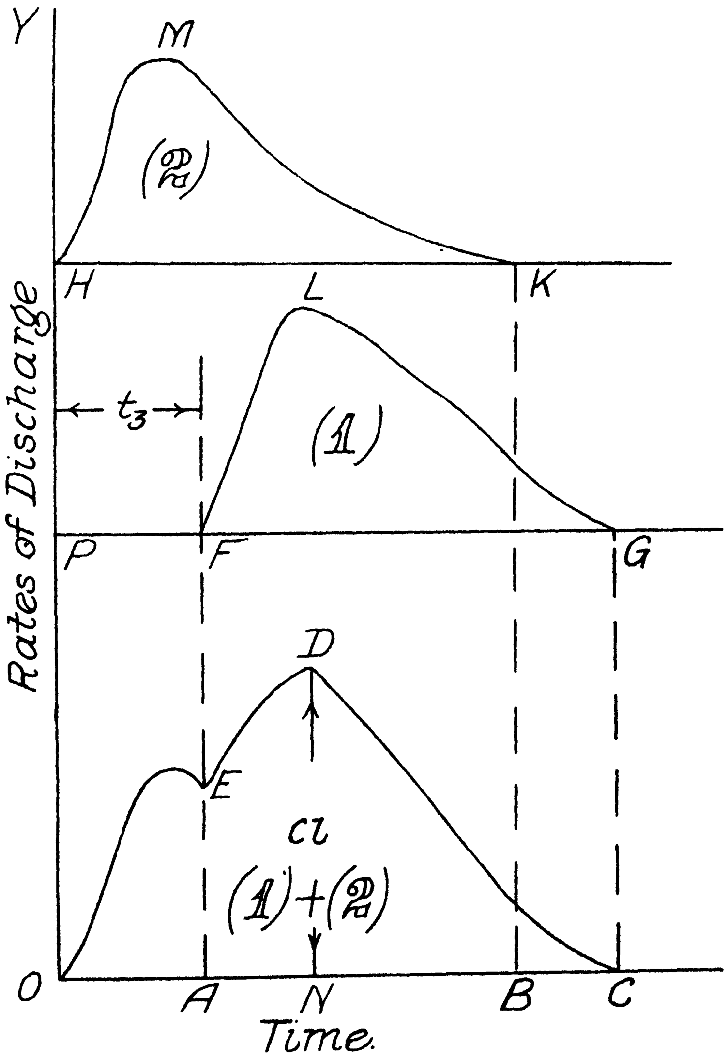


FIG. 14.

Objection may be raised that the rate *ND* is only momentary, but the bursting strength of the sewer, or the liability to flooding adjacent property, is not dependent on time. It is to be noted that the proposed method is absolutely general.

It is interesting to see the effect of varying the ratio of $T$ to $t$. This is shown in Fig. 15, where diagrams of total run-off from an area are shown for values of $t = 0{\cdot}6T$, $T$, and $2T$. We see at once that the greater $t$ is relatively to $T$, the less the maximum intensity, the area $A$ and the

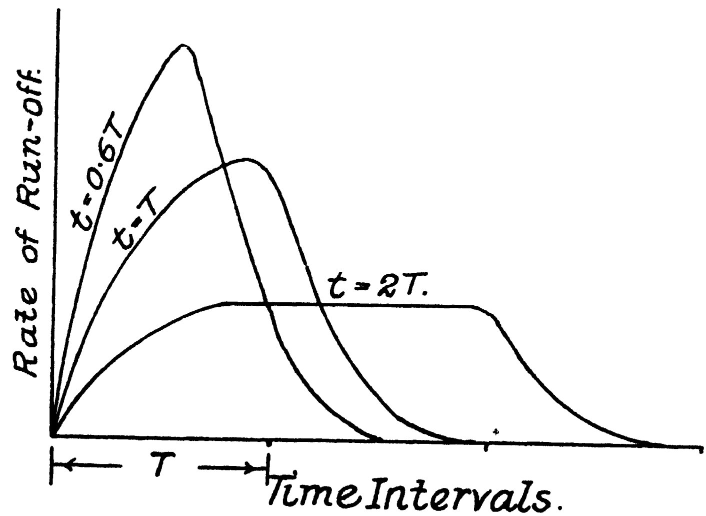


FIG. 15.

intensity diagram for the storm being the same for all three. This furnishes an independent proof of Kuichling's law.

As the capacity of the main sewer is calculated on the instantaneous value $DN$ (Fig. 14), it might be considered that this is the only value of the diagram $OEDC$ which is useful. It must, however, be remembered that the value $DN$ will only control the size (provided the gradient of the main sewer remains constant) until the next connection is reached. A further diagram, similar to *OEDC*, will then be obtained and have to be combined with the first, and it may easily turn out that the maximum ordinate is made up from some other ordinate of *OEDC* besides *DN*, and hence it is necessary to have the whole diagram.

It is, further, interesting to note, in connection with Fig. 15, that the effect of increasing the ratio of *t* to *T* is to make the peak intensity move forward in point of time.

In the above discussion we have supposed that all the water entering the sewers is passed forward without any

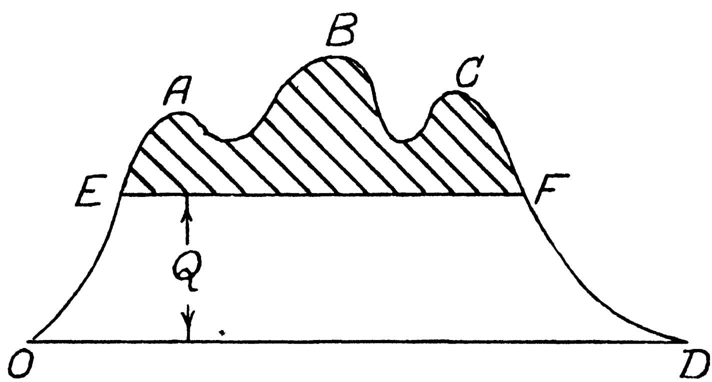


FIG. 16.

loss, or, in other words, that there are no storm overflows on the system. It is, however, only practical engineering to install such overflows where reasonable, and hence we will now consider how they will affect the diagram of run-off.

Let Fig. 16 represent the diagram of rate of run-off similar to *OEDC* of Fig. 14, and representing the flow just above the storm overflow. Let *Q* be the rate of flow which we intend the main sewer to carry after the overflow has come into action. *Q* will be taken as so many times the average dry-weather flow, usually about 6. Draw $EF$ parallel to $OD$ and at a height above it to represent $Q$ on the discharge scale. Then the portion of the diagram shown hatched will be removed by the overflow sill to the storm-water culvert, and the diagram for the main sewer, until the next connection is reached, will be $OEFD$.

More often than not the storm overflow will be situated inside the area, whereas the diagram $OEABCFD$ refers to the effect on the main sewer at the outlet to the area. The question becomes of more importance when the area is large. We therefore proceed as in Fig. 17, p. 50.

Let $ABC$ be the diagram for the area not allowing for any storm overflows. Let the whole area be $A$ in extent, and the part *not* overflowed be $A_1$. The ordinates of $ABC$ are to be reduced in the proportion of $A_1/A$ and so obtain the diagram $AB'C$, which will apply to the part of the whole area which is not overflowed. This reduction can be easily done graphically. At $C$ draw a vertical, and on it, to any convenient scale, mark off $CD$ and $CE$ to represent $A_1$ and $A$. Join $EA$ and $DA$. From $B$ draw $BF$ parallel to $AC$ to cut $AE$ in $F$. Draw $FG$ vertical to cut $AD$ in $G$. Draw $GB'$ parallel to $AC$ to cut the vertical through $B$ in $B'$. All other points on curve $AB'C$ are found in the same manner.

Reproduce the hatched area on a horizontal base, as shown at $A'B''C'$. Treat this in the same way as we did in Fig. 16. The hatched portion of this diagram will be the quantity passed to the overflow. Now combine the unhatched parts of diagrams $AB'C$ and $A'B''C'$, and so get $A''B'''C''$. This will be the corrected diagram for the run-off to the main sewer $A'B''C'$ referring to area $(A-A_1)$ which is overflowed.

The present writer would again emphasize the fact that the method is entirely independent of the shape of

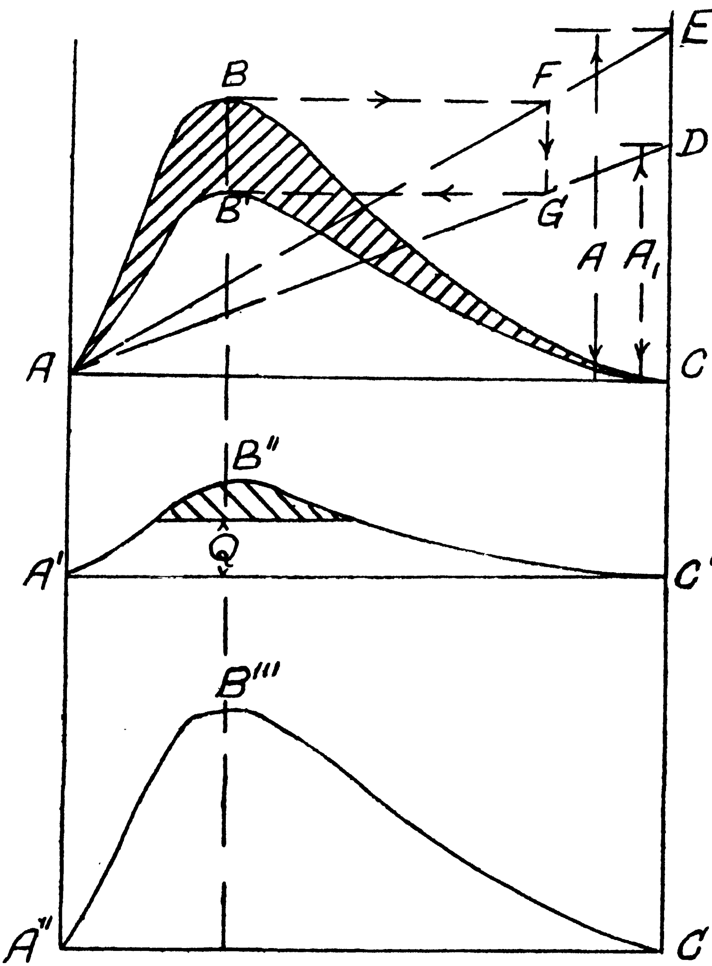


FIG. 17.

the storm-intensity diagram. He, however, considers, after long study of all the diagrams he has been able to examine, that these are more usually of the type shown in Fig. 4. This must only be considered to apply to a single storm, and not to a series of storms following closely on each other.

Finally, of course, it must be urged that the method be used without rigidity, as no method which can be devised can ever give the result with complete accuracy. A vast amount of analysis of records remains to be done before any dogmatic theory can be laid down, but, in the present author's opinion, some such variation of the rational method will be universally utilized in the future.

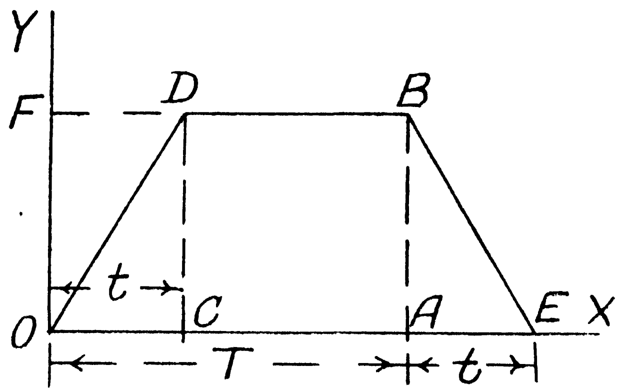


FIG. 18.

The writer would close this discussion with a note that the rational method was originally due to Frühling ("Entwässerung der Städte"), but his diagram was as shown in Fig. 18, that is to say, he took the original storm-intensity diagram as a rectangle, whereas the present writer takes the actual diagram, which he considers is nearer the facts, and hence more scientific. Many American engineers adopt the German method.

Thus, the intensity diagram is *OFBA*, *OF* being the average rate of fall taken from Kuichling's law. $OC = AE = t$. Hence, the intensity diagram, allowing for the time of concentration effects at the beginning and end of storm, is *ODBE*.

It rests with the designer which, or how much of each, of the above methods he will utilize.

The storm-water problem as outlined in this chapter is given more in detail in a paper read by the author before the Manchester Branch of the Institution of Civil Engineers, in November, 1921.

# CHAPTER II

## DISCHARGE FORMULÆ FOR CHANNELS

**Laws of Fluid Resistance.**—The laws governing the resistance to flow in fluids are, roughly speaking, directly opposite to those relating to frictional resistance between solids. These laws may be summarized as follows :—

(1) Resistance to flow varies directly as the area of contact between the fluid and the containing surfaces. Let this area be $A$.

(2) The resistance is independent of the pressure.

(3) The resistance varies as some power of the mean velocity of flow (*see* Introduction). Let this velocity be $v$.

(4) The resistance varies directly as the density of the liquid. Let this density be $w$.

We have already stated (*see* Introduction) that the effect of variations in the density are practically negligible.

Let $F$ = resistance per unit area of surface of the containing channel.

Then $FA$ = total resistance.

Let $\mu$ = resistance per unit area of surface at unit velocity.

Then, from the above four laws, we have :—

$$FA \text{ varies as } w.A.v^n$$
$$\text{or } FA = \mu w A v^n$$
$$\text{or } F = \mu w v^n$$

where $n$ is, at present, some unknown power of $v$.

$\mu$ is what is usually, but somewhat erroneously, termed the "coefficient of friction," but we have already (*see* Introduction) shown that the resistance is due to eddying at the boundaries of flow, and implies a loss of energy. As this loss is loss of kinetic energy, it affects the value of $v$. It might be wondered as to why we use the mean velocity of flow when the velocity affected by the eddies is that at the boundaries. It is, however, extremely difficult to determine this latter velocity, and hence we have to proceed to find the mean velocity purely by experiment.

**The Chezy Formula.**—Had we previously, in the discussion of viscosity, gone into the mathematical side of the

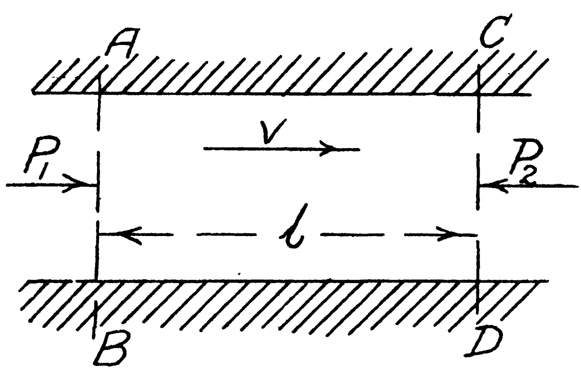


FIG. 19.

question, we should have arrived at a formula for the mean velocity, closely resembling that now known as the Chezy formula. The following, however, is the so-called proof usually given, although it is not strictly analytical.

Let Fig. 19 represent a length $l$ of a pipe of uniform bore, running full, the mean velocity of flow being $v$.

Let the cross-sectional area of the pipe be $A$.

Let $P =$ length of the circumference of the cross-section of the pipe. This is usually termed the "wetted perimeter."

Then $Pl =$ wetted area of pipe in length $l$.

Let $F =$ resistance to flow per unit area of pipe surface.

$F.P.l =$ total resistance in length $l$.

Let $P_1 =$ force acting on cross-section $AB$, and $P_2$ at $CD$.

Then $P_1 = P_2 + FPl$, or $FPl = P_1 - P_2$.

In the cases which we shall consider in the present volume, the moving force is gravity, and the loss is one of head of position.

Let this lost head be $h$.

Then $\frac{(P_1 - P_2)}{Aw} = h$ (*See* " Head " in Introduction.)

$$\text{or } (P_1 - P_2) = wAh$$
$$= F.P.l.$$

But in the article on the Laws of Fluid Resistance we proved that $F = \mu w v^n$.

Where $\mu =$ fluid resistance per unit area at unit velocity.

$w =$ density of fluid.

$$\therefore wAh = \mu w.P.l.v^n$$

In the cases with which the civil engineer has to deal, the value of $n$ is approximately 2.

$$\therefore h = \mu \frac{P}{A}.l.v.^2$$

$\frac{A}{P}$ is usually denoted by $R$, and $R$ is termed the " hydraulic mean radius," or " depth."

$$\therefore \frac{h}{l} = \frac{\mu v^2}{R}$$

$\frac{h}{l}$ is the sine of the angle of slope of the hydraulic gradient, oı of the free surface, if the pipe does not run full. It is usually denoted by $S$.

Substituting, and solving out for $v$, we get :—

$$v=\sqrt{\frac{1}{\mu}}\sqrt{RS}=c\sqrt{RS}$$

$c$ being an experimental constant. $\mu$ is often denoted by $f$ to distinguish it from the $\mu$ used for solid friction.

The hydraulic mean depth (H.M.D.) of a circular pipe running full, of diameter $d$, is :—

$$\frac{\pi d^2}{4}\Big/\pi d=\frac{d}{4}$$

The value of $c$ will be discussed later.

**Formulæ for Velocity of Flow in Channels.**—We have, in the previous article, shown how the Chezy formula may be built up; it now merely remains to discuss the various modern forms of the equation, which have superseded all those of the early nineteenth century.

Writing the formula in the general way it becomes

$$v=cR^nS^m$$

There are now three types of formulæ in common use :—

(1) Those which make $n$ and $m$ constant and vary $c$ to cover all varieties of channels.

(2) Those which keep $c$ constant and vary the values of $n$ and $m$.

(3) Those which partly combine the methods of (1) and (2).

The older and more popular method is the first, usually referred to as the Chezy formula. In this the values of $n$ and $m$ are each taken at $\frac{1}{2}$, and $c$ varies in innumerable ways.

By far the best known formula is that of Kutter and

Ganguillet, usually known as Kutter's formula. In this the value of $c$ is :—

$$\frac{a+\frac{l}{n}+\frac{m}{s}}{1+\left(a+\frac{m}{s}\right)\frac{n}{R}}$$

In this expression $a$, $l$, $m$ are constants respectively equal to 41·66, 1·811, and 0·00281. $n$ is a variable depending on the "roughness" of the channel surface. $R$ and $S$ have the same meaning as in the Chezy formula.

The value of $c$ is cumbrous, and the present writer's opinion that it is unnecessarily so is upheld by many eminent hydraulicians. The formula is old (1872), and was originally designed for estimating the discharge of rivers, but is now applied to every kind of channel, artificial or natural. It is especially well known in the United States, where an enormous amount of work has been carried out to find the values of $n$ corresponding to different classes of lining. This is always termed determining new values of $n$, but what is actually found from the experiments are values of $c$, from which $n$ is subsequently derived. Elaborate labour-saving tables have been devised, notably those of Moore and Silcock ("Sanitary Engineering"), and Woolheim ("Sewerage Engineer's Handbook"), and it is therefore doubtful whether the use of the formula will die out for a long time to come. We will, therefore, herewith summarize some of these values of $n$ as stated by various authorities.

*Kutter and Ganguillet* :—

| | |
|---|---|
| Channels lined carefully with planed boards or smooth cement | 0·01 |
| „ „ with common boards .. .. .. .. | 0·012 |
| „ „ „ ashlar or neatly-jointed brickwork .. .. | 0·013 |
| „ „ „ rubble masonry .. .. .. .. | 0·017 |
| „ „ „ earth in brooks or rivers .. .. .. | 0·025 |
| Streams with detritus or aquatic plants .. .. .. .. | 0·03 |

*Louis d'A. Jackson* (much used in U.S.A.) :—

| | |
|---|---|
| Well-planed timber | 0·009 |
| Plaster in pure cement | 0·01 |
| Plaster in cement of one-third sand | 0·011 |
| Unplaned timber | 0·012 |
| Ashlar and brickwork | 0·013 |
| Canvas lining on frames | 0·015 |
| Rubble | 0·017 |
| Canals in very firm gravel | 0·02 |
| Rivers and canals in perfect order | 0·025 |
| Ditto, but with occasional weeds and stones | 0·03 |
| Ditto, in bad order with detritus | 0·035 |

In 1877–78 he made further experiments on behalf of the Indian Government, and found the following values for earth-lined canals :—

| | |
|---|---|
| Firm, regular, well-trimmed soil | 0·02 |
| Firm earth above the average | 0·0225 |
| Ordinary earth in average condition | 0·025 |
| Rather soft, friable earth, below average | 0·0275 |
| Damaged canals, in defective condition | 0·03 |
| Smooth cement, worked plaster, planed wood, and glazed surfaces, all in perfect order | 0·01 |
| Ditto, but inferior condition, also brickwork, ashlar, unglazed stoneware in good condition | 0·013 |
| Brickwork, ashlar, and stoneware in inferior condition, rubble in cement, plaster in good order | 0·017 |
| Inferior rubble in cement, coarse rubble rough cut, in normal condition | 0·02 |
| Coarse dry rubble in bad condition | 0·0225 |

*C. D. Smith* (Am. Soc. Irrn. Eng., 1894) states that he has carefully checked the following values :—

| | |
|---|---|
| Firm soil, trimmed with shovel | 0·02 |
| Ditto, banks worn tolerably smooth, the soft dirt worn off, leaving bank surface rather uneven | 0·021 |
| Clay with some loose gravel | 0·022 |
| Clay, where velocity not great enough to wear banks smooth | 0·023 |
| New ditch in loam or clay as first made | 0·025 |
| Ditto, banks sloping, and with occasional weeds | 0·026 |
| Pure sand, uniform cross-section, recently put in order | 0·0275 |
| Ditto, grown up, with weeds not reaching surface at centre | 0·04 |
| Ditto, badly grown up | 0·045 |

*J. B. Lippencott* (Eng. News. 1907) :—

| | |
|---|---|
| Tunnels and covered concrete conduits with plastered surfaces | 0·012 |
| Open concrete work, plastered or not, with vegetation | 0·016–0·018 |
| Ditto, where silting occurs, at least .. .. .. .. | 0·02 |

*Freeman, Stearns, and Schuyler* (Eng. News. 1907) :—

| | |
|---|---|
| Open masonry-cement conduits, smoothly plastered masonry .. | 0·018 |
| Concrete-lined tunnels, covered masonry conduits .. .. | 0·014 |
| Steel pipe with rivet heads and seams projecting .. .. | 0·016 |
| Earth canals with bottom as left by dredger .. .. .. | 0·0275 |

*Bureau of Surveys, Philadelphia* (1909) :—

| | |
|---|---|
| Old sewers, brick bottom, not clean .. .. .. .. | 0·017 |
| New sewers, stone block bottom, clean .. .. .. .. | 0·016 |
| „ clean brick bottom .. .. .. .. | 0·015 |
| Concrete or brick sewer, vitrified brick invert, clean .. | 0·012–0·013 |
| Concrete sewers, granolithic lining to invert .. .. .. | 0·011 |
| Open-box channel, planed planks .. .. .. .. | 0·011 |
| Old sewers bad condition (material not stated) .. .. | 0·017–0·020 |

*Metcalf and Eddy* (Am. Sewerage Practice, Vol. 1) :—

| | |
|---|---|
| Vitrified pipe sewers .. .. .. .. .. .. | 0·015 |
| Large concrete sewers (good condition, $v = 3$ f.s. or more) .. | 0·012 |
| Concrete sewers, ordinary good conditions .. .. .. | 0·013 |
| Brick sewers with close joints .. .. .. .. .. .. | 0·014 |
| Ordinary brick sewers .. .. .. .. .. .. | 0·015 |
| Ditto, on flat gradients and roughly laid .. .. | 0·017–0·02 |

They consider that 0·013 must not be used for vitrified pipe sewers unless great care is taken during construction.

*A. Woolheim* (Sewerage Eng. Notebook) :—

| | Perfect. | Good. | Fair. | Bad. |
|---|---|---|---|---|
| Glazed stoneware .. | 0·01 | 0·011 | 0·013 | 0·015 |
| Ordinary brickwork .. | 0·012 | 0·013 | 0·015 | 0·017 |
| Glazed brickwork .. | 0·011 | 0·012 | 0·013 | 0·014 |
| Cement mortar rendering | 0·011 | 0·012 | 0·013 | 0·015 |
| Neat ditto .. .. | 0·01 | 0·011 | 0·012 | 0·013 |
| Dressed ashlar .. .. | 0·013 | 0·014 | 0·015 | 0 017 |
| C. Iron uncoated .. | 0·012 | 0·013 | 0·014 | 0·015 |
| W. Iron and steel .. | 0·011 | 0·012 | 0·013 | 0·014 |

*T. H. Nevitt* (Eng. News. Rec. 1919) :—

| | |
|---|---|
| 12-in. cast iron sludge main, 5 years old .. .. | 0·017–0·018 |

He considers the differences found due to variations in the densities of the sludge.

The present writer has been accustomed to use the following values in sewerage schemes :—

| | |
|---|---|
| Vitrified pipe sewers .. .. .. .. .. .. | 0·013 |
| Brick sewers laid in cement .. .. .. .. .. | 0·015 |

There are a large number of other results in print, but the above should enable the engineer to make a suitable choice. Fresh values are continually being suggested, especially in foreign periodicals. Care must always be used in adopting such values that the cases always correspond in all details.

**Flynn's Modification.**—It is doubtful whether Kutter's formula would have become so popular if it had not been for the discovery of P. J. Flynn that the results are scarcely affected by taking the value of $S$ in $c$ as constant and equal to 0·001. Under this arrangement the numerator of the fraction denoting $c$ becomes dependent only on $n$, and may be termed $K$. The expression may now be written

$$c=\frac{K}{1+\frac{x}{\sqrt{R}}}$$

where $K$ and $x$ have the values given in the following table :—

| $n$ | $k$ | $x$ | $n$ | $k$ | $x$ |
|---|---|---|---|---|---|
| 0·01 | 225·6 | 0·445 | 0·016 | 157·6 | 0·710 |
| 0·011 | 209·1 | 0·489 | 0·017 | 150·9 | 0·756 |
| 0·012 | 195·4 | 0·533 | 0·02 | 134·9 | 0·888 |
| 0·013 | 183·8 | 0·577 | 0·025 | 116·9 | 1·110 |
| 0·014 | 173·8 | 0·621 | 0·03 | 104·8 | 1·332 |
| 0·015 | 165·2 | 0·667 | | | |

It is stated by Woolheim that this modification will give results within 0·5 of 1 per cent., and, seeing that the value of $n$ is always uncertain, this will be within the limits of accuracy of the formula.

The present writer has therefore prepared the alignment chart, Fig. 20, on this basis, which will enable the value of $c$ to be found once $\sqrt{R}$ and $n$ are determined. It is used by joining the chosen values of $n$ and $\sqrt{R}$, and the value of $c$ is intercepted on the intervening line. Such values will be quite near enough at any rate for preliminary calculations.

We will now consider formulæ of the second class, in which the value of $c$ is kept constant and the powers of $R$ and $S$ varied.

Possibly the best known of these in this country is that of Crimp and Bruges, which is as follows :—

$$v = 124\sqrt[3]{R^2}\sqrt{S}$$

The present writer is unaware as to how this formula has been obtained, but believes that it is a variant of the Chezy formula made to fit the experimental work of the authors.

Thrupp's formula is one which was much advocated at one time, but does not appear to have come into common use. In this we have :—

$$v = R^x C^{\frac{1}{n}} S^{\frac{1}{2}}$$

The values of the powers are as follows :—

| | | | |
|---|---|---|---|
| Good brick sewers .. | $x$ = 0·61 | $n$ = 2·0 | $c$ = 0·007746 |
| Cement rendering .. | 0·67 | 1·74 | 0·004 |
| New cast-iron .. | 0·67 | 1·85 | 0·005347 |
| Old ditto .. .. | 0·66 | 2 0 | 0·017115 |

There are a large variety of other formulæ, but as they are not generally in use, and are principally intended for

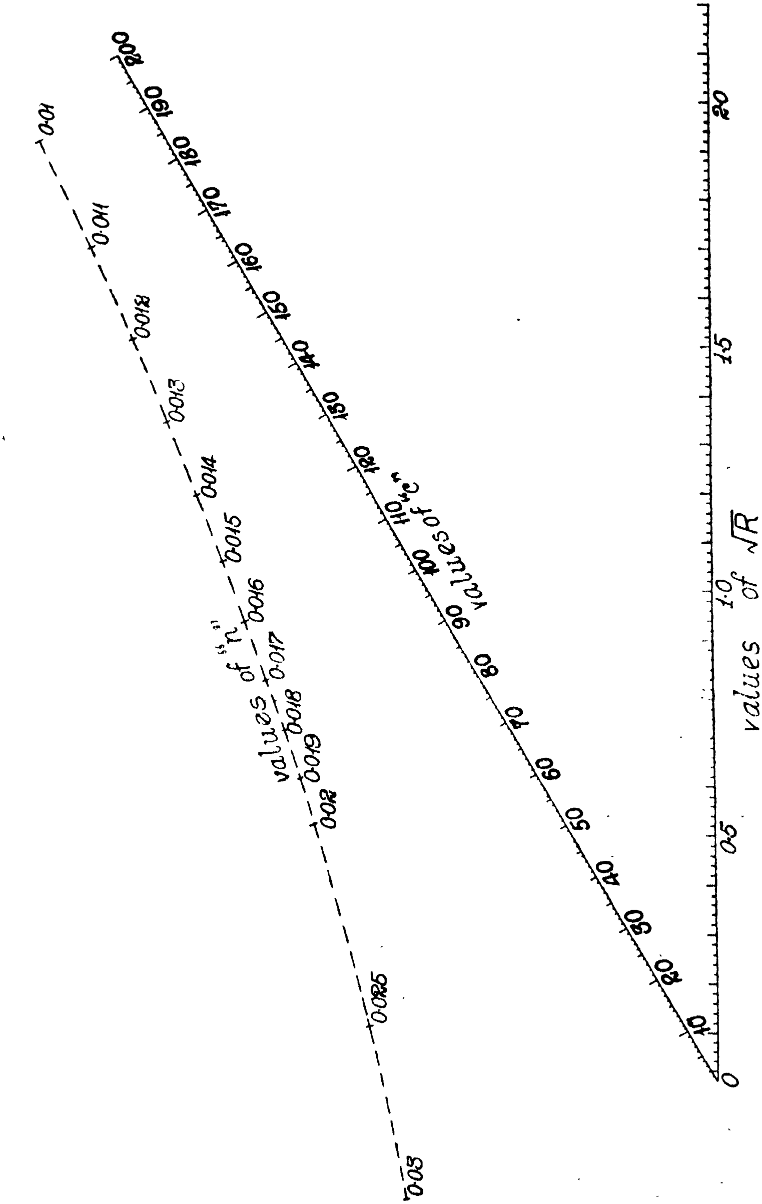
values of "n"
0·01
0·011
0·012
0·013
0·014
0·015
0·016
0·017
0·018
0·019
0·02
0·025
0·03
values of "c"
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
values of √R
0
0·5
1·0
1·5
2·0

FIG. 20.

water-supply mains under pressure, they will not be given here.

The third type of formula varies both the value of $c$ and also the powers of $R$ and $S$.

**Hazen and Williams' Formula.**—This is well known in the United States, but was originally intended for water-supply mains. According to Metcalf and Eddy ("Am. Sew. Prac.") it is also applicable to sewers. The formula is as follows :—

$$v = a.c.R^n S^m$$

$a$ is a fixed constant and equal to $0{\cdot}001^{-0{\cdot}04}$.

$c$ is a variable dependent on the material.

$n = 0{\cdot}63$, and $m = 0{\cdot}54$.

The following values of $c$ are recommended by the authors for use :—

| | |
|---|---|
| New cast-iron pipe, straight and smooth .. .. .. .. | 140 |
| Ordinary new cast-iron pipe .. .. .. .. .. .. | 130 |
| Ordinary old cast-iron pipe, and for calculations for future capacity .. .. .. .. .. .. .. .. | 100 |
| New rivetted-steel pipe .. .. .. .. .. .. | 110 |
| Steel pipe under future conditions .. .. .. .. .. | 95 |
| Masonry, concrete culverts, or clean plaster with very smooth surface .. .. .. .. .. .. .. .. | 140 |
| Ditto after a moderate time when slime-covered .. .. .. | 130 |
| Ditto under ordinary conditions .. .. .. .. .. | 120 |
| Cement-lined pipe (Metcalf) .. .. .. .. .. .. | 110 |
| Brick sewers in good condition .. .. .. .. .. | 100 |
| Vitrified pipe sewers in good condition .. .. .. .. | 110 |

**Manning's Formula.**—This is comparatively a new formula (*see* Manning, "Flow of Water in Channels and Pipes"). It is as follows :—

$$v = \frac{1{\cdot}49}{n} R^{0{\cdot}67} S^{0{\cdot}5}$$

$n$ is the same as in Kutter's formula. Parker ("Control of Water") considers this to be as accurate a formula as Kutter's.

**Bazin's Formula.**—This famous French engineer proposed the formula :—

$$v = \frac{157 \cdot 6}{1 + \frac{\gamma}{\sqrt{R}}} \sqrt{RS}$$

He proposed the following values for $\gamma$ :—

| | |
|---|---|
| Smooth cement or planed wood .. .. .. .. .. | 0·109 |
| Planks, bricks, cut stone .. .. .. .. .. .. | 0·29 |
| Rubble masonry .. .. .. .. .. .. .. | 0·833 |
| Earth channels, very smooth and revetted with stone .. .. | 1·54 |
| Ordinary earth channels .. .. .. .. .. .. | 2·35 |
| Ditto, bed covered with boulders, or weeds on sides .. .. | 3·17 |

**Claxton Fidler's Formula.** — This is considered by Professor Gibson (" Hydraulics ") to be the probable type of formula in the future when the sections of the channels are uniform :—

$$s = \frac{fv^n}{R^m}$$

The following values of the variables are given by Gibson :—

| Section. | Surface material. | *n* | *m* | *f* |
|---|---|---|---|---|
| Circular .. | Smooth neat cement .. | 1·75 | 1·167 | 0·0000676 |
| Rectangular .. | " " .. | " | " | 0·0000787 |
| Circular .. | Cement and sand .. | " | " | " |
| " .. | Smooth brick .. .. | " | " | " |
| Rectangular .. | Smooth ashlar .. .. | " | " | 0·0000904 |
| Circular .. | Base metal with rivetted joints .. .. .. | 1·77 | 1·18 | 0·0000871 |
| " .. | Rough brickwork .. | 1·80 | 1·20 | 0·0000977 |
| Rectangular .. | Rough brickwork or ashlar .. .. .. | " | " | 0·0001122 |
| " .. | Rubble masonry .. | 2·10 | 1·50 | 0·0002240 |

**"Short" Pipes.**—A point which is frequently overlooked by sewerage engineers is the fact that all formulæ for the velocity in sewers are only applicable to lengths in which the flow has become steady. Professor Merriman ("Hydraulics," 1916) defines a very short pipe as one under 50 diameters in length, and a short pipe as one about 500 diameters long. He gives a method by which the minimum length may be ascertained by choosing a percentage of error obtained by disregarding all losses except that due to friction, as against the value obtained when all losses are allowed for.

For instance, let $v_1$ = velocity of flow, allowing for loss due to friction only.

Let $v$ = velocity, making all possible allowances.

Then Merriman takes $\frac{v_1}{v} = 1{\cdot}01$, and taking $\mu$ (*see* Chezy's formula) as 0·005 the value of $l$ will be $3750d$, where $d$ is the diameter of the circular culvert. This value appears to the present writer to be excessive. If $\mu$ be taken as 0·006 and $\frac{v_1}{v} = 1{\cdot}05$ (maximum allowable value), then $l = 750d$. Professor Gibson gives $100d$ as the minimum value of $l$, but gives no proof. The present writer is inclined to suggest $500d$ as an absolute minimum in drainage work. Parker ("Control of Water") states that the disturbing effect of the entry of the pipe was found by him, by experiments on a model, to extend from $50d$ to $100d$ from the entrance.

**Inverted Syphon.**—This term has come into use to denote the case shown in Fig. 21. The case frequently occurs when a valley has to be crossed by a sewer, and the outlet *B* is lower than the inlet *A*. It is obvious that the pipe will now be under pressure. Using Bernouilli's Theorem (*see* Appendix III.), we state the following equation :—

$$H = \frac{v^2}{2g} + \frac{fl}{R} \cdot \frac{v^2}{2g} + \frac{cv^2}{2g}$$

where $H$ is the loss in head of position between $A$ and $B$, $f$ is the coefficient of friction, $l$ is the length of $ACB$, $R$ is the hydraulic mean depth of the pipe when running full, $v$ is

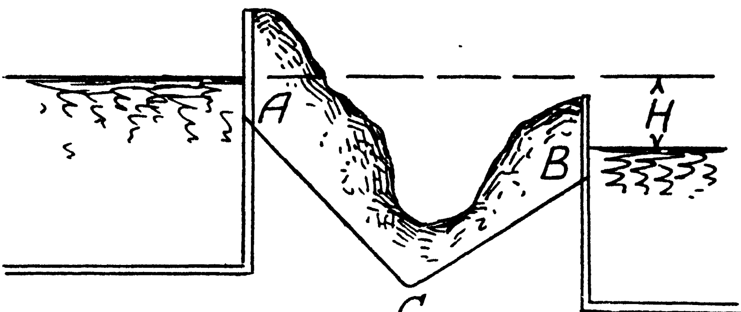


FIG. 21.

the mean velocity of flow in the pipe, $c$ is a constant referring to the loss at entry at $A$. $c$ may be safely neglected in sewerage problems. Hence, solving out for $v$ we get :—

$$v = \sqrt{\frac{2gH}{1 + \frac{fl}{R}}}$$

$R$ will $= \frac{d}{4}$ for circular pipes, where $d$ is the diameter of the pipe.

An automatic cut-off valve should always be placed at $A$, in case of accidents such as bursts.

**Submerged Outfalls.**—Such outfalls occur in the case of sewers entering rivers or the sea, and a common error must be guarded against in calculating the discharging capacity.

Let Fig. 22 represent such an outfall diagrammatically. Then the true lost head in length $AB$ is $h$ and not $H$ when

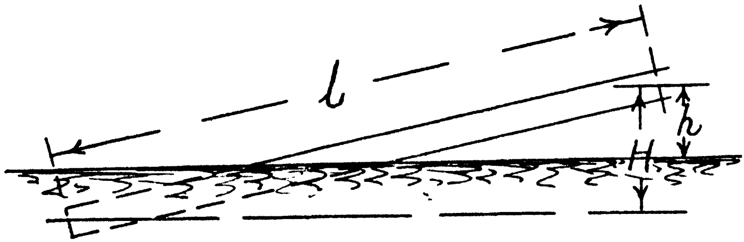


FIG. 22.

the surface of the river or sea is as shown. The full length $l$, however, must be used in calculating the velocity of flow.

**Invert and Hydraulic Gradients.**—It is, as far as the present writer is aware, the almost invariable custom in this country to calculate the velocity and discharge of a sewer from the invert gradient. This gradient is usually fixed by the maximum velocity allowable to avoid erosion of the surface of the invert, whereas it is, in all cases, the hydraulic gradient which fixes the velocity, and hence the discharge, whether the sewer runs under pressure or not. Neglect of this obvious fact has wrecked what were otherwise well-thought-out schemes. Metcalf and Eddy ("Am. Sew. Prac.," Vol. 1) quote the case of the City of Brooklyn, where flooding ensued and was specifically referred to this omission by the Metropolitan (U.S.A.) Sewerage Commission of 1910.

The point to be emphasized may be best explained by an example. In Fig. 23 suppose $AF$ were a 4-ft. dia. brick sewer laid at a uniform invert gradient of 1 in 1600. Let us suppose that there are six branches as at $F$, $E$, $D$, $C$, $B$, and $A$, each contributing 5 cubic feet per second to the main sewer. For simplicity, let us suppose the junctions are at even distances of 500 feet apart. The quantities in the sewer at *A*, *B*, etc. will be 5, 10, 15, etc. cubic feet per second.

By means of such a diagram as that given later we may, theoretically, calculate the corresponding depth of flow at *A*, *B*, *C*, etc. Let us suppose this done. Then we get a longitudinal section after the style of Fig. 24. At *A*

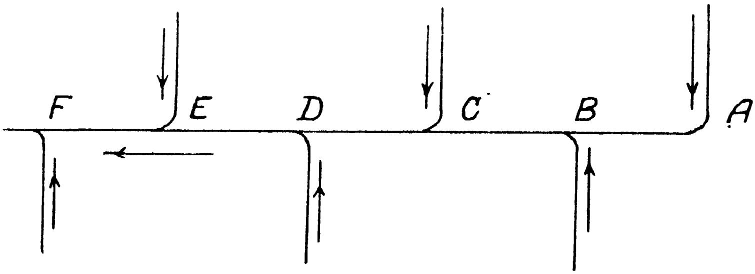


FIG. 23.

the depth of flow corresponds to 5 cubic feet per second, and at *F* to 30 cubic feet per second. At each connection the flow will rise up the sewer, until the average hydraulic gradient is much flatter than the invert gradient.

In the present case the average gradient of the water surface (which forms the hydraulic gradient) will be about

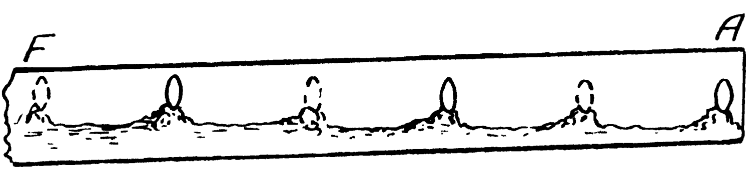


FIG. 24.

1 in 2417, so that the discharging capacity is about 29·8 cubic feet per second instead of a nominal 36·6 cubic feet per second, calculated on the invert gradient. This will mean that the size of the sewer will have to be enlarged.

The only way we could make the hydraulic gradient parallel to the invert gradient would be to have the whole

30 cubic feet per second inserted at $A$, and to have no further connections between $A$ and $F$. Usually, of course, the size of the sewer would be gradually increased from $A$ to $F$, the variations occurring at the junctions with the laterals, but the error is liable to be overlooked in such cases as ordinary street sewers receiving the discharge of successive houses and surface gullies. The solution of the problem becomes one of trial and error, but it should not involve much extra labour.

It is usual to allow a limiting average velocity of 3 ft. per second in vitrified pipe sewers and from 2 to 1 ft. per second

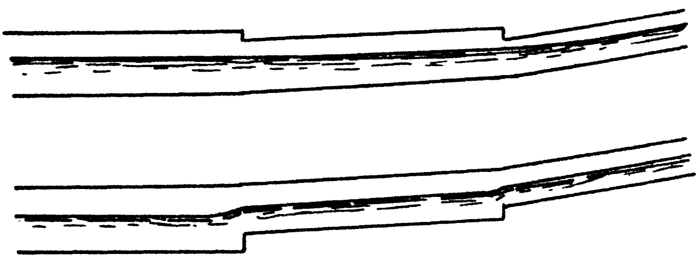

FIG. 25.

in large sewers, the smaller value applying to the largest sizes. Sewers should never be put under pressure unless constructed of metal pipes. This is not always preventable, especially in times of storm, but probably this will cause no harm if it does not last too long and the pressure is small. As the size of the sewer increases the invert gradient usually diminishes, as shown in Fig. 25. The upper part of the figure shows the usual method of keeping the inverts flush at the change of gradient, and raising the crown to get the increased size. The better way, theoretically, would be to lower the invert, as shown in the lower figure, but this is often not possible on account of the reduced level of the final outfall.

A common rule in England for finding the minimum gradient for the smaller laterals has been to make it equal to :—

$$1 \text{ in } 5d + 50$$

where $d$ is the diameter of the sewer in inches.

Another common rule for pipe sewers was to make the gradient 1 in $10d$, but both these rules must be used with caution.

**Self-Cleansing Velocities.**—While it is necessary to limit the mean velocity in sewers in order that the surface of the channel, and more especially the invert, may not be eroded, it is also necessary to fix a minimum limit to the velocity in order that the liquid may not deposit its solids, and so cause obstructions and lessen the discharging capacity. This fact has been realized from the commencement of scientific sanitary work, but has only been properly investigated in recent years.

Bazalgette (Proc. Inst. C.E., Vol. 25) considered the question when designing the old main drainage scheme for London. He estimated that the following velocities were, at least, necessary to move the material described. The present writer is, however, unaware as to whether these are mean velocities or bottom velocities, but probably they are the former. They are in feet per second. Fireclay 0·25, sand 0·5, coarse sand 0·66, fine gravel 1·00, 1 in. dia. pebbles 2·0, stones (egg-size) 3·0. These values are of little use.

Blackwell (Brit. Metro. Drainage Commission) pointed out that the specific gravity had a marked effect on the velocities required.

Coal (sp. gr. 1·26) $v = 1{\cdot}25$–$1{\cdot}50$, coal (sp. gr. 1·33) $v = 1{\cdot}5$–$1{\cdot}75$. Brickbat (sp. gr. 2·0) $v = 1{\cdot}75$–$2{\cdot}0$, brickbat (sp. gr. 2·12) $v =$

2·0–2·25, brickbat (sp. gr. 2·18) $v = 2{\cdot}25–2{\cdot}50$. Flint (sp. gr. 2·66) $v = 2{\cdot}5–2{\cdot}75$.

These results are also almost useless as they do not state whether mean or bottom velocities are referred to, nor do they state the size or shape of the objects moved. The velocity required to move a flat piece of slate will be very much greater than a fairly round pebble of the same volume or weight. In the case of the slate, the pressure of the water tends to squeeze out the liquid underneath the fragment and so create a vacuum, increasing the pressure. The water also, of course, has not nearly so much surface to act on as in the case of a more or less cubical specimen. As regards the question of specific gravity it must be remembered that any solid will lose a certain proportion of its weight when immersed, due to buoyancy effect. Again, a round specimen will be much more easily moved than a reactangular piece of the same material of the same size.

The present writer had occasion to investigate the problem with respect to brick-lined culverts, and came to the following conclusions :—

(*a*) The bottom velocity is the deciding factor with respect to rolling solids along the invert, provided there is no retardation due to silt, etc.

(*b*) The depth of flow has no connection with the problem.

(*c*) The shape of the solid is most important. Round smooth bodies need a much smaller velocity than rough, flat, or irregular bodies.

(*d*) That the velocity varied according to the density of the solid.

(*e*) That a material composed of loose fine grains tended to be spread over, and stick to the bottom and sides, and that the force required to move it bore no relation to the character or weight of the particles.

He also determined the following table of bottom velocities :—

| Material moved. | Minimum velocity in ft. per sec. |
|---|---|
| Fine cinders | 1·42 |
| Grit screenings | 1·57 |
| Large cinders | 1·60 |
| ¾-in. dia. pebbles | 1·67 |
| 1¼-in. dia. pebbles | 1·74 |
| ¾-in. grit stone cubes | 1·90 |
| 1½-in. grit stone cubes | 2·50 |
| 1¼-in. granite cubes | 2·55 |

The last three items are broken stone as passed through an ordinary sieve. The materials were chosen so as to be representative of the ordinary mineral solids found in a sewer.

The best-known experiments are those of Deacon (Proc. Inst. C.E., Vol. 118). He experimented with sand in a trough with glass sides. The first movement took place when the surface velocity was 1·3 ft. per second, and was confined to a few of the finest grains. As the velocity rose, more and more of the sand was moved, until when 2·9 ft. per second was reached the sand ripples were merged in a confused moving mass which was carried along in suspension.

*Lichelas* ("Annals des Ponts et Chaussées," 1871) is considered one of the authorities on this subject, but his results are more applicable to natural channels and canals. A very complete summary of his results, applicable to irrigation problems, is given in Parker's "Control of Water." Parker also deduces the following table from Thrupp's experiments (Proc. Inst. C.E., Vol. 171) for a depth of flow of over 0·4 ft.

| | |
|---|---|
| Mud and silt not moved if $v$ less than .. .. .. | $0{\cdot}40d^{0{\cdot}5}$ |
| Fine silt moved if $v$ more than .. .. .. .. .. | $0{\cdot}40d^{0{\cdot}5}$ |
| Fine sand moved if $v$ more than .. .. .. .. | $1{\cdot}50d^{0{\cdot}35}$ |
| Coarse sand moved if $v$ more than .. .. .. .. | $0{\cdot}50d^{0{\cdot}3}$ |
| Pea-sized pebbles moved if $v$ more than .. .. .. | $2{\cdot}2d^{0{\cdot}3}$ |
| Egg-sized pebbles moved if $v$ more than .. .. .. | $5{\cdot}0d^{0{\cdot}25}$ |
| Large stones moved if $v$ more than .. .. .. .. | $17{\cdot}0d^{0{\cdot}15}$ |

These results only apply when $v = 0{\cdot}4$ ft. per second, but Parker considers even this doubtful. The above results apply to natural-lined or earth channels.

Parker gives the following table from his own experiments for minima bottom velocities :—

| | | |
|---|---|---|
| Soft earth or fine clay .. .. .. .. .. | 0·25 | ft. per sec. |
| Soft clay (very variable, depends on adhesion of particles) .. .. .. .. .. .. | 0·5 | ,, |
| Finest sand (as left after erosion of clay) .. .. | 0·5 | ,, |
| Fine sand (usual river sand) .. .. .. .. | 0·7 | ,, |
| Coarser ditto .. .. .. .. .. .. | 0·8 | ,, |
| Gravel or coarse sand (depends on size of grains) .. | 1·0 | ,, |
| 1-in. dia. pebbles .. .. .. .. .. | 2·0 | ,, |
| Egg-sized pebbles .. .. .. .. .. | 3–3·3 | ,, |
| 3-in. dia. stones (uncertain) .. .. .. .. | 5·0 | ,, |
| 6–8-in. dia. boulders .. .. .. .. .. | 6·6 | ,, |
| 12–18-in. dia. boulders .. .. .. .. .. | 10·0 | ,, |

The present writer believes that the above apply to natural channels.

The following results are given in the *Deutsche Bauzeitung*, 1883, from experiments made in the Rhone. They refer to gravel :—

| | | |
|---|---|---|
| Pea size $v$ (if disturbed) .. .. | 2·46 f.s. | (not disturbed) 3·87 f.s. |
| Bean size $v$ (if disturbed) .. .. | 2·95 ,, | (not disturbed) 4·30 ,, |
| Hazel to walnut size $v$ (if disturbed) .. | 3·48 ,, | (not disturbed) 4·92 ,, |
| Pigeon-egg size .. .. .. .. | 3·67 ,, | |
| Pigeon-egg (¼ lb.) .. .. .. | 4·92 ,, | |
| Pigeon-egg (5 lbs.) .. .. .. | 5·90 ,, | |
| 0·6-ft. dia. .. .. .. .. | 6·56 ,, | |

Parker makes the following remark :—

" If the water carries silt, some of the finer materials (*i.e.* forming bed) will not occur until the tabulated velocities (*i.e.* these last quoted) have been exceeded in a ratio which depends on the quantity of silt already present in the water. A river which carries a large quantity of sandy silt, is able to roll along gravel and boulders at smaller velocities than those indicated in the table. The action appears to be due to the fact that the lower layers of the river, being heavily charged with silt, in reality form a fluid which has a density greater than that of pure water. The extra flotation thus obtained renders the stones more easily moved. Exact figures cannot be given."

The above remarks, as far as they relate to silting of natural watercourses, are extremely summary and incomplete, and students of irrigation are referred to the authorities quoted, especially to Kennedy (Proc. Inst. C.E., Vol. 119).

*J. R. Freeman* (Report on the Charles River Dam, 1903) quotes observations of Mills and Hale on the Essex canal on the Merrimac River at Lawrance, and deduces that a bottom velocity of 0·8 ft. per second will not disturb fine soft sand, and that the velocity necessary to prevent silting was from 1·3 to 1·5 ft. per second.

*Hering and Trautwine* give the following table in their translation of Kutter's "Flow of Water":—

| Nature of bed. | Minimum bottom velocity. |
|---|---|
| River mud and clay (sp. gr. 2·64) | 0·25 ft. per sec. |
| Sand, size of aniseed (sp. gr. 2·55) | 0·35 ,, |
| Clay, loam, fine sand | 0·5 ,, |
| Sand, size of peas (sp. gr. 2·55) | 0·6 ,, |
| Common river sand (sp. gr. 3·36) | 0·7 ,, |
| Sand, size of beans (sp. gr. 2·55) | 1·07 ,, |
| Gravel | 2·0 ,, |
| Round pebbles 1-in. dia. (sp. gr. 2·61) | 2·13 ,, |
| Coarse gravel and small cobbles | 3·0 ,, |
| Angular stones and egg-sized flints (sp. gr. 2·25) | 3·23 ,, |
| Angular broken stone | 4·0 ,, |
| Soft slate and shingle | 5·0 ,, |
| Stratified rock | 6·0 ,, |
| Hard rock | 10·0 ,, |

These values appear to be those above which the solids will be transported and erosion take place. They apply only to natural channels.

The Metropolitan Sewerage Commission of New York, 1910, adopted the following table of minimum velocities to move solids :—

| | |
|---|---|
| Fine clay and silt .. .. .. .. | $v =$ 0·25 ft. per sec. |
| Fine sand .. .. .. .. .. .. | „ 0·5 „ |
| ½-in. dia. pebbles .. .. .. .. .. | „ 1·0 „ |
| 1-in. dia. pebbles .. .. .. .. .. | „ 2·0 „ |

It was further stated that a mean velocity of about 1 ft. per second would prevent serious deposit of sewage (fœcal matter?) on the tidal flats if the sewage is reasonably broken up. The present writer cannot say whether the tabular velocities are mean or bottom, or whether they refer to natural or artificial beds, but probably to the former in each case.

**Depth of Flow for Maximum Velocity and Discharge.**—It may be of interest at times to know the depth of flow at which the maximum velocity and discharge occur, and these may be found from mathematical analysis. Such analysis will not, however, be given here, but only the results, as the reasoning is somewhat advanced for ordinary students. We will take the case of a circular culvert of internal radius $r$.

Let $A$, $P$, $R$ be the cross-sectional area, wetted perimeter, and hydraulic mean depth at any given rate of discharge for a circular sewer.

Then for the maximum mean velocity $A = 2·738r^2$, $P = 4·494r$, and $R = 0·608r$. The maximum mean velocity, therefore, occurs at a depth of flow of $1·626r$.

For a maximum rate of discharge $A = 3·044r^2$, $P = 5·30r$, and $R = 0·573r$. Hence the depth of flow for maximum discharge is $1·699r$.

As tables of velocity and discharge of channels are almost invariably calculated for when running full, it is useful to have diagrams which will give the proportionate velocity

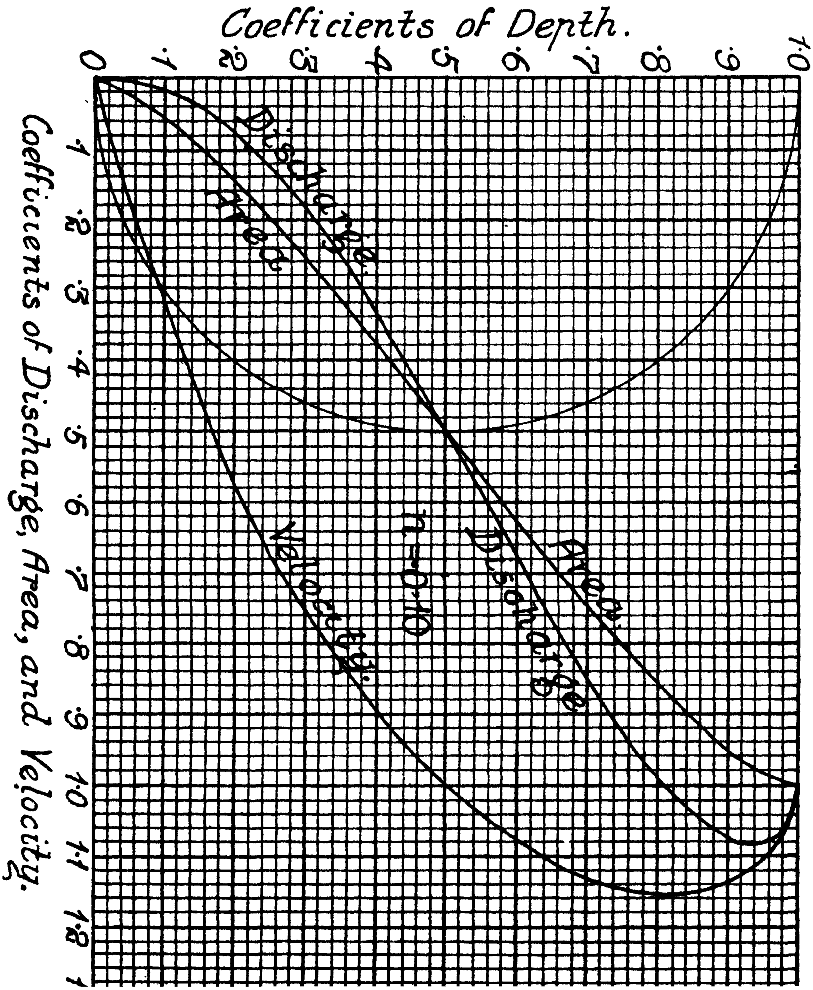


FIG. 26.

and discharge at any fraction of the full depth, the values when running full being taken as unity in each case. Figs. 26 to 36 are such diagrams, which need little explanation.

It may, however, be stated that they have been calculated from Kutter's formula. If calculated for any other value of $n$ the curves would be very slightly modified, but not sufficient to make any practical error. In the case of the

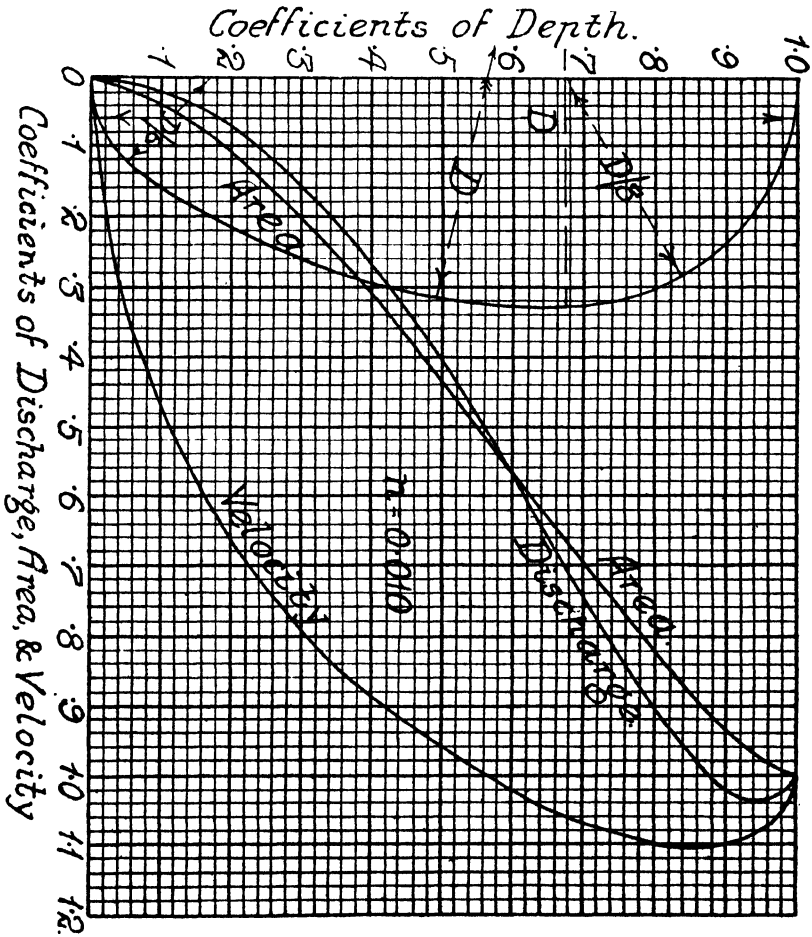


FIG. 27.

rectangular channel the full flow has been taken when the depth of flow equals the width of the channel, which latter is taken as open. Such a diagram is useful for the channels which usually occur in a sewage disposal works.

In case the reader desires to be more exact, the values for a circular section are given in the following table :—

| $h/l$ | $A = d^2 \times$ | $\sqrt{R} = \sqrt{d} \times$ | $A\sqrt{R} = d^2\sqrt{d} \times$ |
|---|---|---|---|
| 0·05 | 0·0146 | 0·180 | 0·0026 |
| 0·1 | 0·041 | 0·252 | 0·0103 |
| 0·2 | 0·112 | 0·347 | 0·0388 |
| 0·3 | 0·198 | 0·413 | 0·0816 |
| 0·4 | 0·293 | 0·463 | 0·1355 |
| 0·5 | 0·392 | 0·5 | 0·196 |
| 0 6 | 0·492 | 0·527 | 0·259 |
| 0·7 | 0·587 | 0·544 | 0·319 |
| 0·8 | 0·673 | 0·551 | 0·371 |
| 0·9 | 0·744 | 0·546 | 0·406 |
| 0·95 | 0·771 | 0·535 | 0·412 |
| 1·0 | 0·785 | 0·5 | 0·392 |

*d* is the internal diameter.

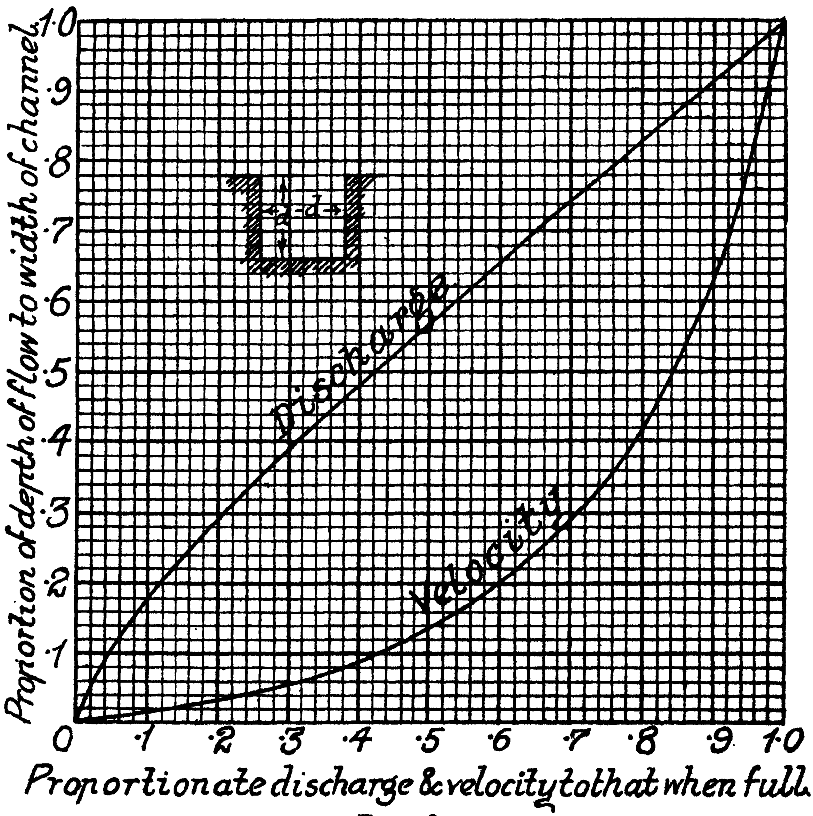


FIG. 28.

It may also be required to calculate the discharge for a different value of $n$ to those in the tables. This may be done as follows :—

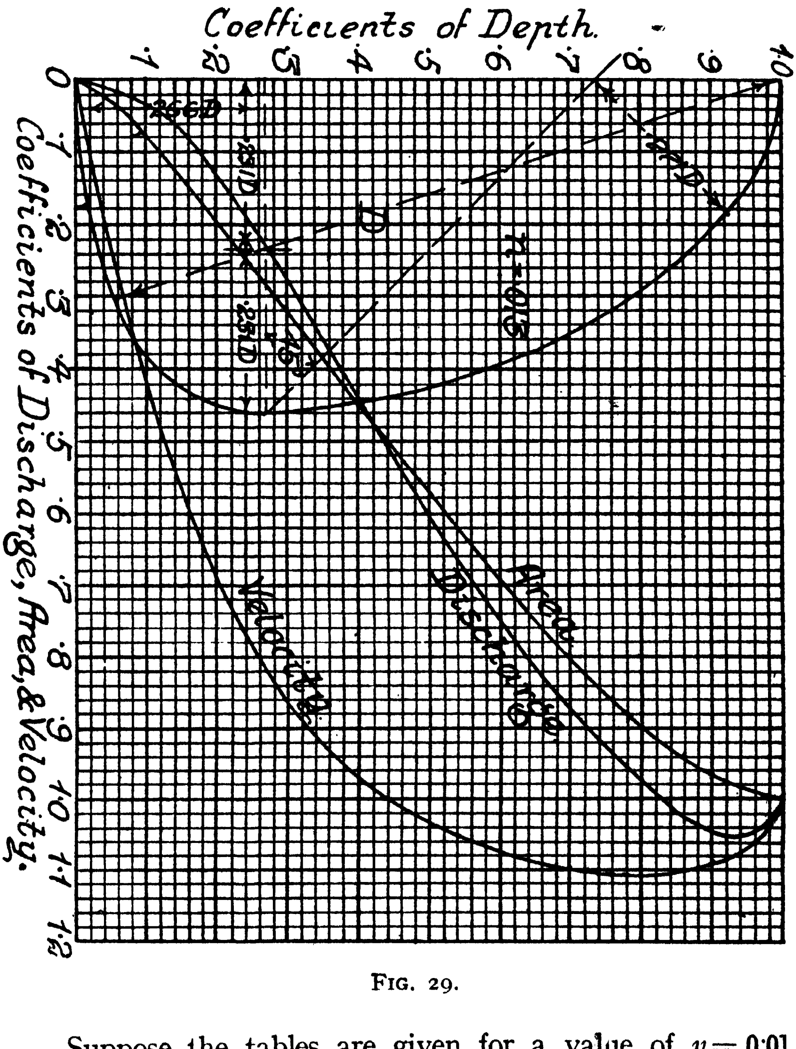


FIG. 29.

Suppose the tables are given for a value of $n = 0{\cdot}01$. This is too high for any but the very smoothest linings, and we may wish to obtain the rate of discharge when the value of $n$ is $0{\cdot}013$.

Let $Ac_1\sqrt{RS}$ = rate of discharge when $n = 0{\cdot}01, = Q_1$

Let $Ac_2\sqrt{RS}$ = ,, ,, ,, $n = 0{\cdot}03, = Q_2$

Then $$\frac{Q_2}{Q_2} = \frac{Ac_2\sqrt{RS}}{Ac_1\sqrt{RS}} = \frac{c_2}{c_1} \text{ and } Q_2 = \frac{c_2}{c_1} Q_1$$

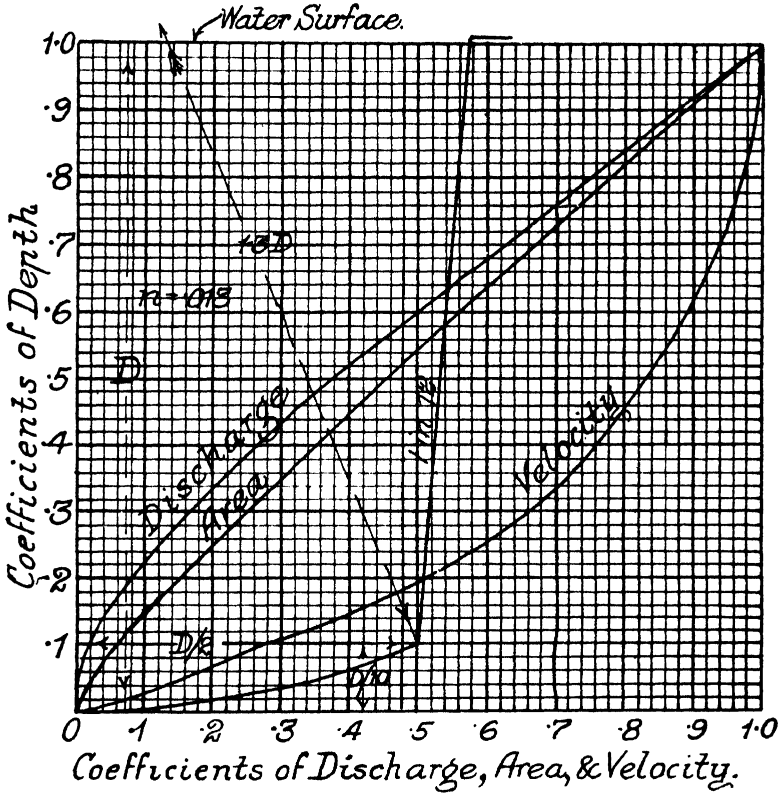


FIG. 30.

The use of the coefficients of discharge and velocity as found from such diagrams as Fig. 26 is not always fully appreciated.

Take, for instance, the method for calculating the necessary height at which the sill of a storm overflow requires to be placed. Such a height is usually settled by the consideration of the dilution of the normal dry-weather flow necessary to render the overflowed water innocuous. A

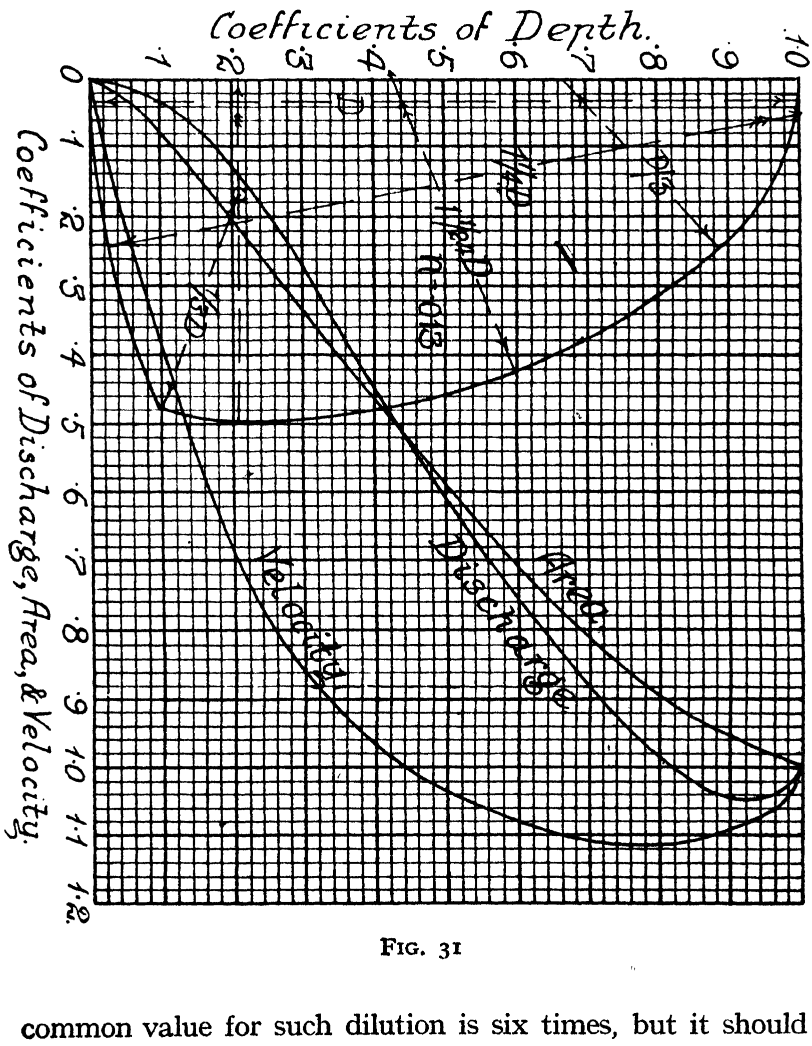


FIG. 31

common value for such dilution is six times, but it should entirely depend upon the tendency to putrescibility of the sewage before dilution.

For instance, let the average dry-weather flow in a 10-ft. diameter sewer be at 3 feet, or 0·3 of the total depth.

From Fig. 26 we find that the coefficient of comparative

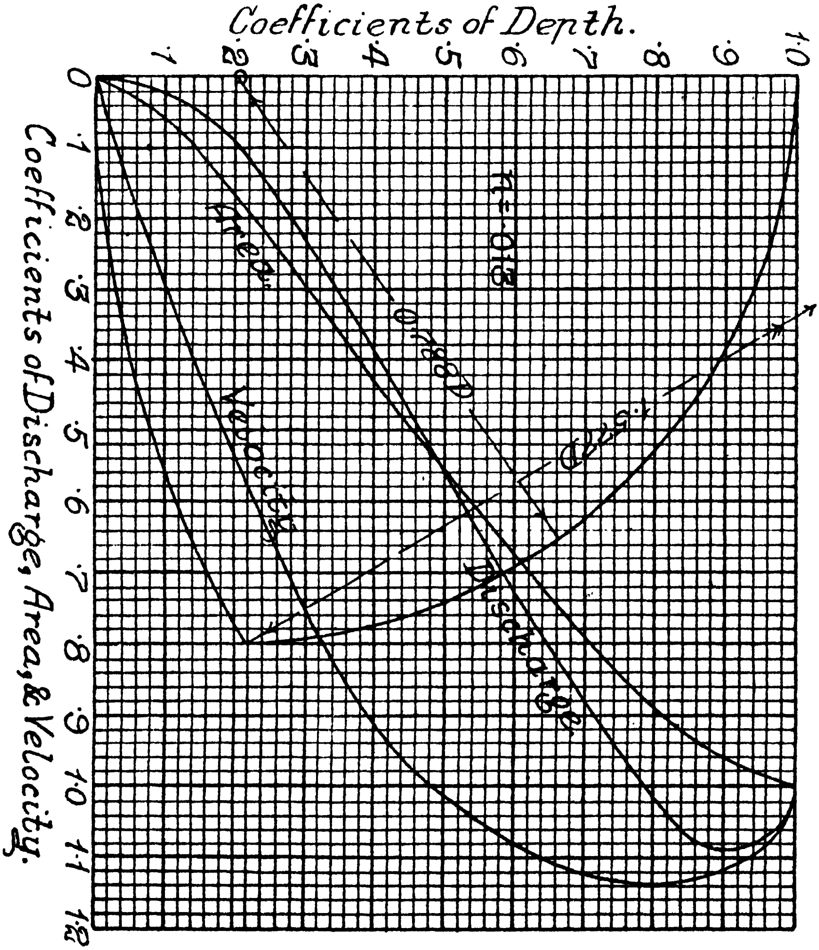


FIG. 32.

discharge corresponding to 0·3 of the depth is 0·18, and 6 × 0·18 = 1·08 when the required rate of dilution is 6 times the normal flow.

1·08 of the discharge running full corresponds on the same diagram to about 0·9 of the depth.

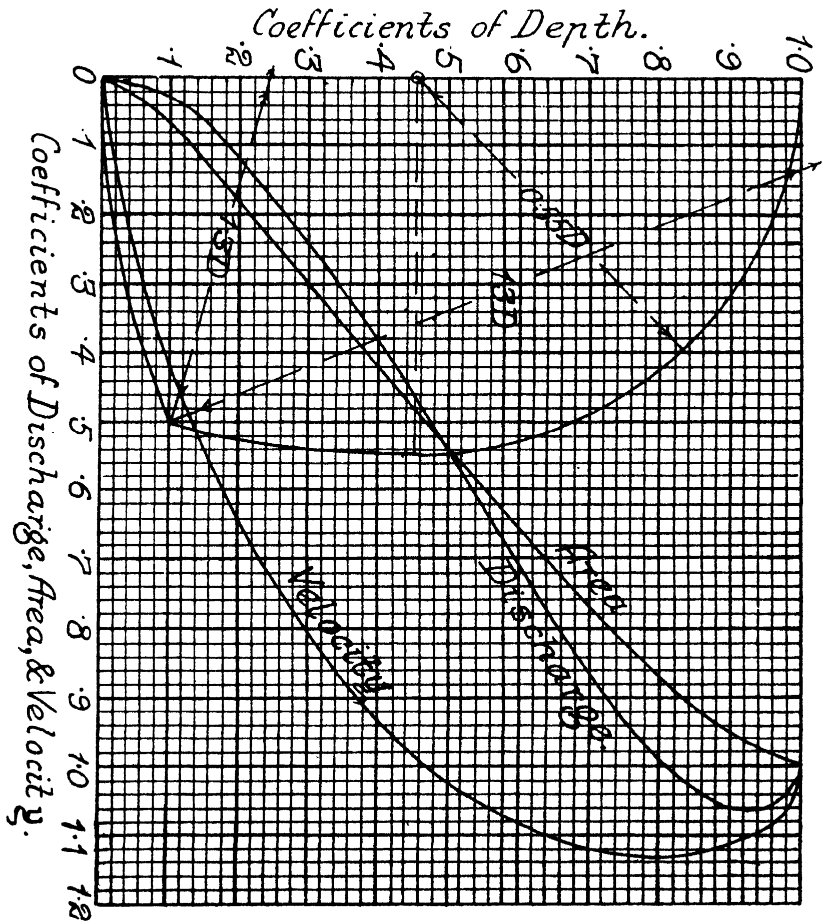


FIG. 33.

Hence, the height of the overflow sill should be 0·9 × 10 feet, or 9 feet above the invert.

We are thus able to obtain a solution without calculating the hydraulic mean depth, or using the gradient, and the present writer was able to use this method in one case where over 200 storm overflows had to be analysed. It must, however, be noted that the method fails when

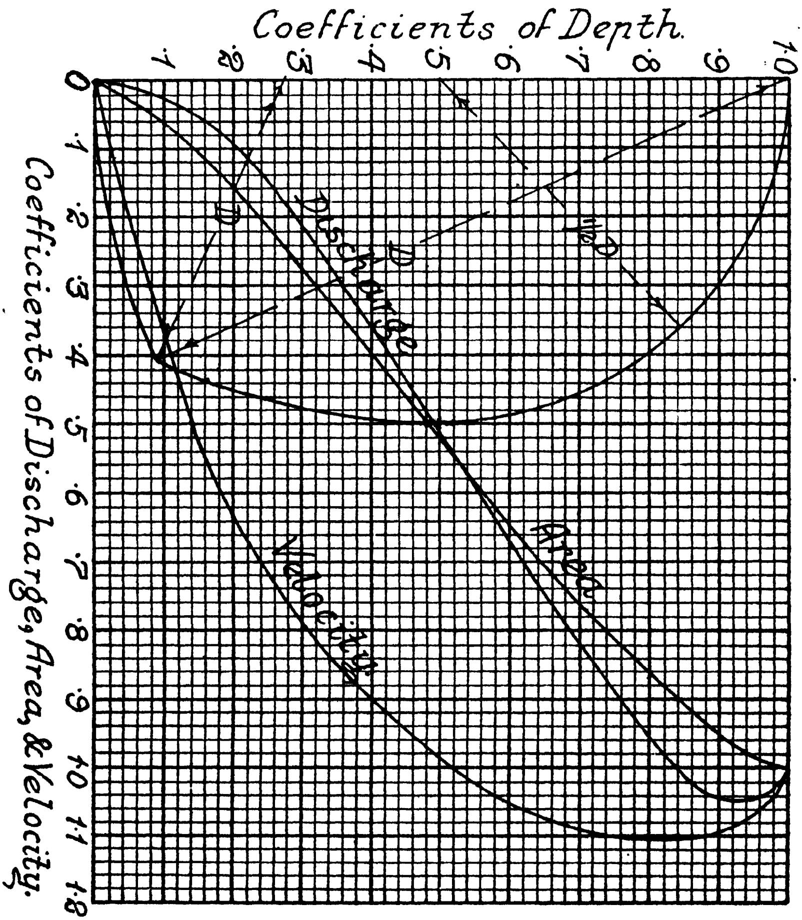


FIG. 34.

the average dry-weather flow is at more than 0·3 of the total depth.

Before leaving this branch of the subject, the writer would call attention to what he considers an error in calculating such sill heights. It is his experience that where the ordinary sewage flow is strong, the maximum dry-

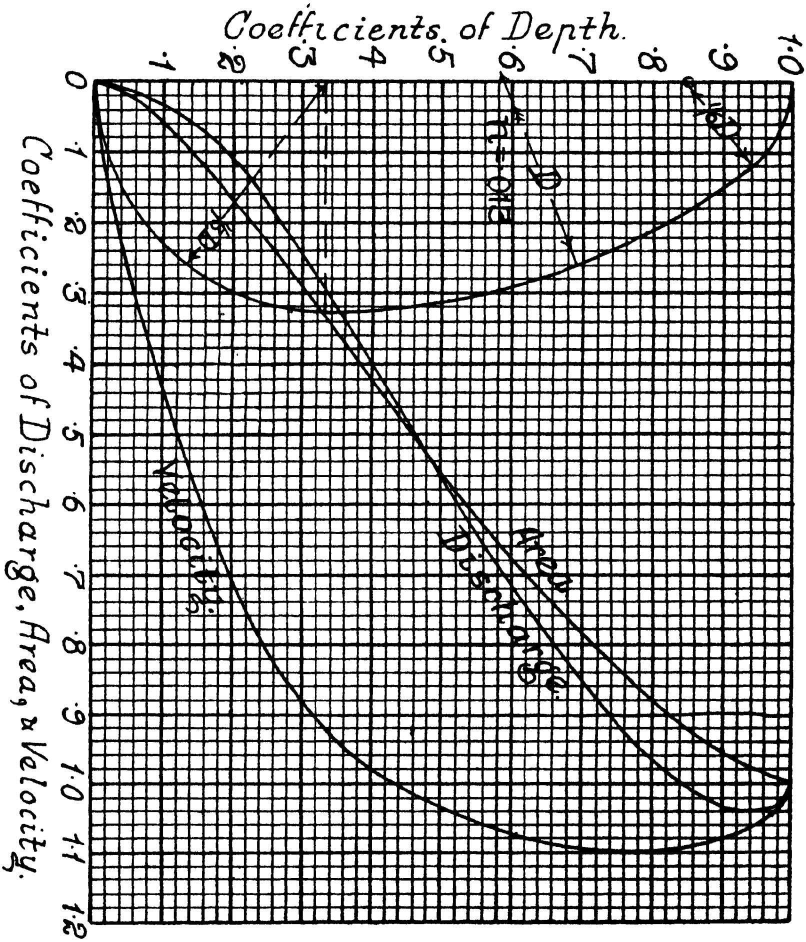


FIG. 35.

weather flow is quite as putrescible as the average ditto, and hence he considers that such calculations should be made on the maximum value. It is a fact that the first water crossing the sill is always exceptionally bad, owing probably to the stirring-up of sediment by the sudden increase of flow.

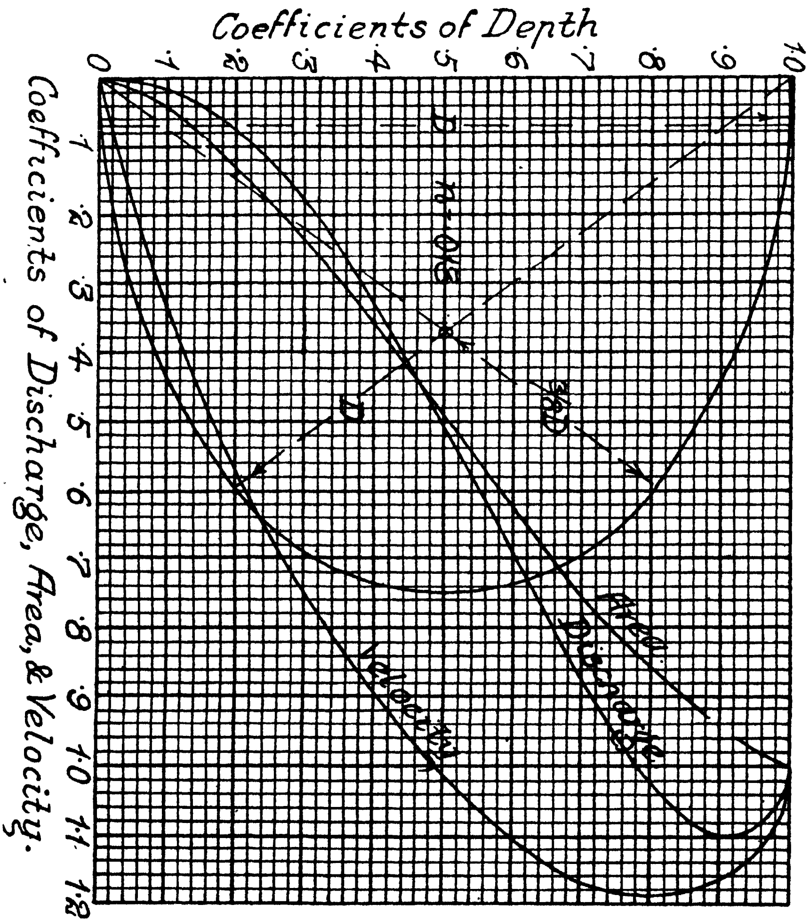


FIG. 36.

**Form of Channel for Maximum Discharge.**—It may at times be possible to vary the shape of the cross-section of an open channel so as to get a maximum discharge.

We will therefore give the results in two cases; the analysis, however, involves advanced mathematics. Provided the cross-sectional area of flow and the gradient are fixed, the discharge depends on the ratio of the cross-sectional area to the wetted perimeter.

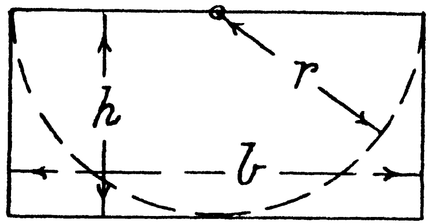

FIG. 37.

The figure which gives this ratio a maximum value is a semicircle, but such a section may not always be convenient. Open channels are usually rectangular or flat-bottomed with sloping sides. Let Fig. 37 represent such a rectangular channel. Then it may be shown that the ratio must have the value of $\frac{h}{2}$. Hence, a circle of radius $h$ will touch the sides of the required channel, which will be $h$ high and $2h$ wide.

Now take the case when the sides are sloping. It is first necessary to define the slope of the sides. Let it be

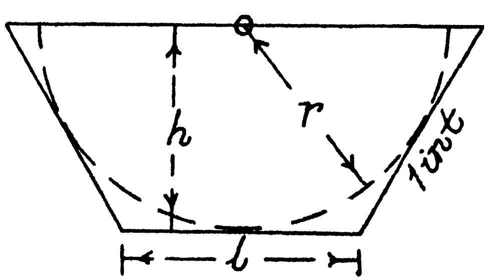

FIG. 38.

one vertical to $t$ horizontal. Then we have from geometry that the length of the slope is :—

$$h\sqrt{t^2 + 1}$$

And it may be proved that the maximum discharge is obtained when $\frac{l}{2}+th=h\sqrt{t^2+1}$.

If this is done, a circle whose centre bisects the water-line and has a radius $h$ will touch the three sides:

$$\text{also } h=\frac{l}{2}\Big/(\sqrt{t^2+1}-t)$$

When this design has been worked out, it may be found that the value of $h$ is unsuitable for the given value of the discharge. It will then be necessary to alter $t$, as this is the independent variable, and this may entail a fresh value for $c$ in the Chezy formula, owing to the necessity of altering the material forming the surface of the slopes.

**Flow round Bends in Open Channels.**—Usually such bends will be arcs of circles, and the theory governing the flow was first investigated by Professor J. Thompson (Proc. Roy. Soc. 1877).

Let $R_1$, $R_2$ be the radii of the inside and outside of the channel respectively.

Then it may be proved (*see* Lea's "Hydraulics," or Gibson's "Hydraulics") that the velocity on any stream-line is inversely as the radius of curvature of that stream-line.

Let $p_1$ and $p_2$ be the pressures at $R_1$ and $R_2$. Then we may obtain the equation:—

$$\frac{p_2-p_1}{w}=\frac{c^2}{2g}\left(\frac{1}{R_1^2}-\frac{1}{R_2^2}\right)$$

and the stream-lines form what is known as a "free vortex." $c$ is the constant value of $vr$, *i.e.* if $V$ is the velocity at $R_1$, $VR_1=c$.

$\frac{p_2-p_1}{w}$ is the difference in "pressure head" from one side of the stream to the other. From the above result we see that the water tends to heap up at the outer bank, but, contrary to what might have been expected, the velocity is least at the outer bank. In natural or earth-lined channels this does not, however, imply increased scour at the inner bank. As a matter of fact, the bottom velocities, which affect the scour, are less at the inner side. The increase of the pressure head at the outer side is not balanced by the centrifugal force, and a rotary motion is set up on the cross-section as shown in Fig. 39. These cause scouring at the outer bank and deposition

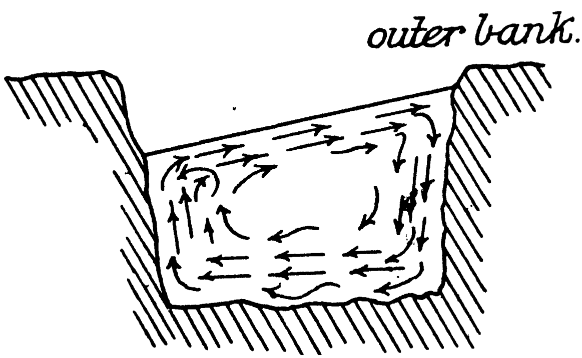


FIG. 39.

at the inner, and hence eventually accentuate the sharpness of the bend.

Professor Gibson ("Hydraulics") states that while the above theory accounts for part of the erosion, part, especially in flood-times, is due to the impact of the rushing water on the outer bank, and causes most of the effect. Under such circumstances the surface velocity is at a maximum near the outer, and not the inner bank.

Very little has been done to determine the head lost through bends.

Professor Gibson ("Hydraulics") has analysed the experiments of Hobson (Eng. Rec. 1911) on a sewer 9·8 diameter, which contained straight end lengths of 640 and 1075 feet each. Between these lengths, and next to the 1075-ft. length, were two 50-ft. radius reverse curves 220 ft. long, and next to the 640-ft. length a curve of 100 ft. radius and 120 ft. long. From the results Gibson suggests $c = 128$ for straight channels in the Chezy formula, 118 in the case of the curve of 100-ft. radius, and 88 in the case of the reverse curve of 50-ft. radius.

It must be remembered that the disturbing effect of the curves is felt a long way down-stream in the straight

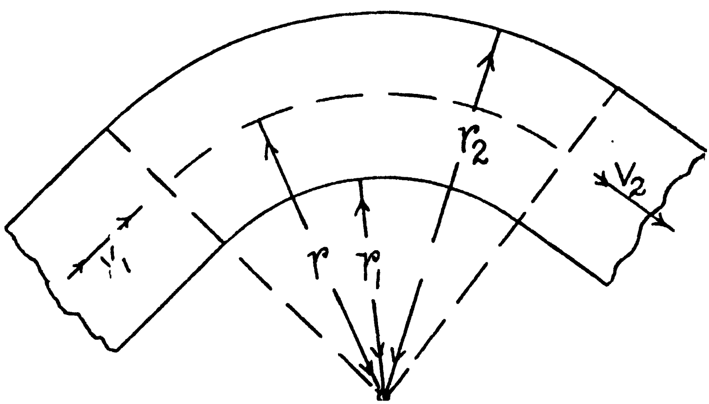

FIG. 40.

length. Hence it is difficult, and probably unnecessary to try and localize the losses and variations in the value of $c$.

The above is probably on right lines, but the present writer offers the following suggested method as a means of discussion. Take Fig. 40. By Thomson's theory $vr$ is a constant.

Let $v_1$ be the average velocity in the straight length at the commencement of the curve, and let us suppose $v_1$ remains the velocity round the inner side of the curve.

Let $v_2 =$ velocity at outer side of curve.

Then $v_1 r_1 = v_1 r_2$.

velocity at mean radius $r = \frac{v_1 + v_2}{2} = v$ say

$$\therefore v = \tfrac{1}{2}\left(v_1 + \frac{v_1 r_1}{r_2}\right) = \frac{v_1 r_2 + v_1 r_1}{2r_2} = \frac{v_1 r}{r_2}$$

Using the Chezy formula we should have $v_1 = c_1\sqrt{RS_1}$ and $v = c\sqrt{RS} = v_1\frac{r}{r_2}$ and taking $R$ as constant, and $S_1 = S$.

$$\therefore c\sqrt{RS} = \frac{r}{r_2} c_1\sqrt{RS} \text{ or } c = c_1 \times \frac{r}{r_2}$$

Hence, for a sewer 10 ft. diameter and 100 ft. mean radius, with $c_1$ equal to 100, we should have :—

$$c = 100 \times \frac{100}{105} = 95$$

This is hardly enough according to Gibson's figures, but the above method is open to several objections and the whole matter is very obscure.

Another way would be to keep $c$ constant and vary the gradient for the length of the curve.

Let $v = c\sqrt{RS}$ as before, and let $v_1 = c\sqrt{RS_1}$.

Then $v = v_1\frac{r_1}{r} = c\sqrt{RS}$.

$$\therefore c\sqrt{RS} = c\sqrt{RS_1} \times \frac{r_1}{r} \text{ or } S = S_1 \times \left(\frac{r_1}{r}\right)^2$$

$$\text{But if } r = \frac{r_1 + r_2}{2}\ S = S_1\left(\frac{2r_1}{r_1 + r_2}\right)^2$$

In the example this will work out to :—

$$S = S_1 \times \frac{361}{441} = 0{\cdot}8S_1$$

Markmann (Eng. News. 1910) suggests the formula :—

$$S = S_1 \times \frac{v_1^2}{2gr^2}$$

but the present writer does not consider it accurate.

It must be remembered that as $v_1$ itself is dependent on the losses at any bends, once steady flow is established, the question cannot be solved directly, and the present writer is inclined to think that it is doubtful whether any allowance made will not introduce a greater error than making no allowance at all.

**Government Requirements as to Sewerage.**—According to Moore and Silcock (San. Eng. 1909) the former Local Government Board were considered to desire the following requirements in schemes submitted for their sanction.

The quantity of sewage for domestic dry-weather flow is to be estimated at 30 gallons per head per day.

All sewage diluted over 6 times the average dry-weather flow may be overflowed in combined systems.

The dilution is to be calculated on the average rate of flow per 24 hours.

Separate system only to be adopted where no nuisance possible and where combined system would be too costly.

Storm-water overflows to be avoided wherever possible, and then only with consent of river authority.

Points to which special care is to be paid in drawing out schemes are: sufficient gradients, proper position of outfall when into tidal waters, proper position of overflows, danger of contamination of water supplies.

**Gauging.**—In designing a new sewerage scheme it is invariably necessary to take gaugings of the flow in the existing sewers to form a basis for the calculations. The following notes will outline the methods as briefly as possible, but students should carefully study the authorities referred to. The most accurate method of all is to run the flow into a tank whose capacity can be accurately determined at any depth. Choosing a period when the volume represents the dry-weather flow, one assistant should make an accurate record of the depths of flow throughout a given time in the sewer and another assistant do the same at the tank. From this the true discharging capacity of the sewer can be determined. By either fixing $S$ or $c$ in the Chezy formula (*see also* article on "Hydraulic Gradient *versus* Invert Gradient") a correct formula for the discharging capacity of this sewer is obtained.

Another method is to put colouring matter into the head of the sewer and finding the time taken for all such colouring matter to reach the outlet. Parker ("Control of Water") considers it a very accurate method, but the objection to it for sewers is the difficulty in distinguishing the colour in foul water. Eocin (Beuzenburg, Trans. Am. Soc. C.E., Vol. 30) or fluorescin (Parker) are recommended. To make the colour show up, a white tile may be placed in the invert. The coloured liquid must be inserted at one go. Parker recommends a glass bottle which is smashed against the bottom. A pint is said by him to be suitable for a 4-ft. pipe. The method is not accurate unless the pipe is less than 500 ft. long. If the coloured streak at the outlet is more than 1 per cent. of the length of the pipe there is probably some obstruction. Parker considers the velocity will be within 1 per cent. of the true value.

Stromeyer (Proc. Inst. C.E., Vol. 160) has used a similar method with chemicals. Samples are taken just above the point of insertion and at a given distance down-stream, and the results analysed. The results are said to be within 1 per cent., but it requires an expert operator. A special apparatus is used for inserting the chemical.

The method of timing floats is extremely antiquated, and liable to gross errors. If used at all they should be of the rod type and float upright the full depth of flow. If surface floats are used, an allowance must be made for the ratio between the surface and mean velocities. Floats are extremely liable to drift to the sides of the channel, where the velocity is low. There they may spin round and round in the eddies without moving forward. All floats are liable to move from side to side in some particular stream-line, instead of keeping to the centre line of flow. They are also at the mercy of a breeze.

Current meters are more satisfactory. If used to read one point only, they should be fitted with a telephone arrangement so that the number of revolutions in a given time may be accurately measured. One method of using is to raise the instrument slowly up the vertical centre line of the section and take the average reading as the mean value. A factor of reduction has to be used for various velocities, and a rating chart should be supplied by the makers. The writer would warn purchasers of these instruments that a chart showing a straight line must be treated with suspicion, and has probably been drawn from one or, at the most, two isolated readings.

For large sections of flow a single spot reading in the cross-section is insufficient owing to the uncertainty of the exact position of the velocity having the same value as the mean. The following method was devised by the writer, although he has subsequently found that it had already been suggested by Professor Unwin.

Let Fig. 41 show the cross-section at which the mean velocity is required. It is necessary that the water surface should remain constant for a considerable time, and the cross-section must be square to the current. At the level *AC* stretch a fine wire across the section, at a convenient height above the water surface. This can only be done if the stream is comparatively narrow, and such line is intended to serve as a base line. On *AC* mark the points 1, 2, 3, etc. by knots or tabs of some kind. These

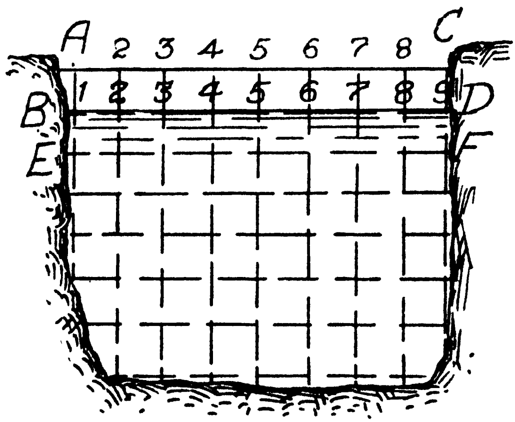


FIG. 41.

points will mark the positions of the imaginary verticals, and need not necessarily be at equal distances apart. If *AC* forms the floor of a bridge this makes a convenient base. An accurate cross-section of the channel, if not already in existence, must be obtained. The readings are now taken by a current meter or Pitot tube, as quickly as possible, at the intersections of the horizontal and vertical lines, each reading being so booked that its position on the cross-section can be immediately identified. Having worked out the corresponding velocities, they are marked on a copy of the cross-section, and a series of " contour " lines, exactly corresponding to the contours on a survey, are drawn in, so that the section now has the appearance of Fig. 42.

If necessary, mark on a new set of horizontal and vertical section lines, and draw a series of sections to

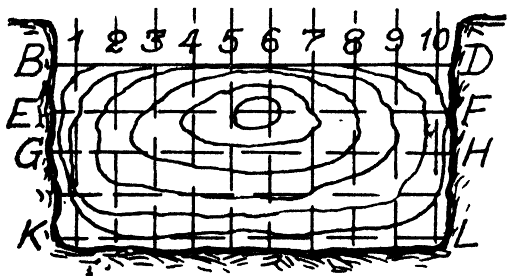


FIG. 42.

represent the velocities on lines *BD*, *EF*, etc. Fig. 43 will, for instance, represent velocities on line *BD*. Find the areas of each of these and, dividing it by the length of *BD*, find the average ordinate, and hence the average velocity on the horizontal *BD*. If the area *BB′D* is found in square inches, the length *BD* must be measured in full-size inches.

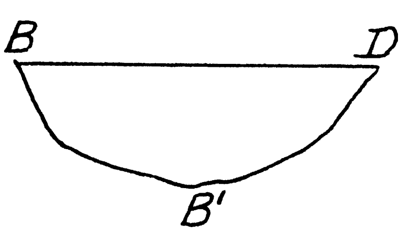


FIG. 43.

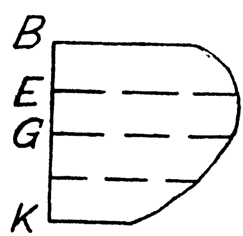


FIG. 44.

Now draw a vertical line *BK*, as in Fig. 44, the points *B*, *E*, *G*, etc. representing the positions of the horizontal sections in Fig. 42.

Through the points *B*, *E*, *G*, etc. draw horizontals to represent the mean velocities found from such diagrams as Fig. 43. Draw a smooth curve through the ends of such ordinates, and find the average horizontal ordinate of the figure. This latter ordinate will give the value of the mean velocity of flow, and also its position.

In the case of rivers it will not be possible to use the wire *AC*, Fig. 42, and it will be necessary to use a boat which fixes its position from poles marking *A* and *C* on the shore. In order to ensure that the velocities are taken at the spots intended, the writer has been accustomed to fix the recording instrument on a pole, which latter was marked in feet and quarters. Care must be taken that the pole is not so fixed as to interfere with the currents round the meter. It is very advisable to run one or more check sets of readings on other days to confirm, or otherwise, the first results obtained.

In the case of a large outfall sewer, a temporary wooden staging may be rigged up inside, at such a height as not to be washed away, and provided there is sufficient head room.

In the smallest streams and small sewers a weir may be placed across the flow (*see* "Weirs"). The method has, however, several disadvantages. The capacity of a weir is relatively small and the structure backs up the water a long way upstream. Weirs require very accurate setting and reading to get reliable results, and the coefficients of discharge for the larger sizes are not well established. In the case of a sudden rise in flow they would probably be wrecked, and the operator's life endangered unless an exit is handy.

In the case of current meters, Parker states that a three-minutes' run at one point will be sufficient.

In rating meters or Pitot tubes the method of towing them at a known speed through still water has been found inaccurate, and the only way is to test them against tank measurements.

The method of averaging a series of spot velocities given above may at times be abbreviated, especially if time is limited, but care must be taken that accuracy is not sacrificed. Readers are referred to Parker's "Control of Water," where more detailed notes on gauging are given.

In floods and similar emergencies we must use surface floats, and hence the question of the ratio between the surface and the mean velocity must be known or assumed. As a float tends to work into dead water at the sides, or is affected by currents and wind, it is necessary to follow it down an open channel, and keep it as much as possible in the centre of the stream. This may be done with a pole or a boat, but, in any case, a good number of trials should be made so that a reliable average result may be determined. Any obviously unreliable runs must be left out. Parker recommends globular floats and flat circular discs. The writer has used oranges, as they float just below the surface, are easily seen, and are unaffected by winds, and Parker also mentions their use.

As regards the estimation of the mean velocity, Harlacher (Proc. Inst. C.E., Vol. 91) is considered an authority. He states that, in natural streams, the mean velocity is about 0·85 the surface ditto. The ratio is greatest for sandy beds and least for beds of gravel the size of the fist. In the rivers of the Punjaub, where the beds are of very fine sand, the mean is 0·93 of the surface. Hoyt and Grover ("River Discharge") also give the average value as 0·85, and a maximum of 0·89 for small streams with rough beds. They state that the deeper the stream the higher the coefficient. In his observations, Harlacher took a reading every 2 sq. ft. in the case of small streams, up to 20 sq. ft. in the case of rivers. Parker states that even with the best results with surface floats, there is a possibility of from 10 to 15 per cent. error.

When only a very few observations can be taken on a cross-section, there are three standard methods in use: (1) the one-point method; (2) the two-point method; (3) the three-point method.

In the first, the average reading is considered to be at 0·6 of the depth. According to Hoyt and Grover, there is a variation of from −6 to +4 per cent. with a mean of zero. The method is stated by Parker to be inappropriate to very shallow or very deep streams. Barrows (Trans. Am. Soc. C.E., Vol. 66) gives the coefficient the value of 0·88, and Hoyt and Grover confirm this. Parker gives 0·7.

In the second method the readings are taken at 0·2 and 0·8 of the depth, and the mean velocity is taken as half the sum of the velocities thus determined.

The mean of the surface and bottom velocities, even if the latter could be accurately found, does not give the true mean, nor does the reading at half the depth.

In the third method the following formula is used:

$$v_{\text{mean}} = (v_{0\cdot2} + 2v_{0\cdot5} + v_{0\cdot8})/4$$

It is apparently a mean of the first and second methods, and hence cannot be correct if either of the others is.

Parker states that the old rule

$$v_{\text{mean}} = (v_0 + 2v_{0\cdot5} + v_1)/4$$

is also incorrect.

Some very famous rules were made by Francis in the Lowell hydraulic experiments.

Let $Q$ = rate of discharge formed by a weir.
$Q$ = ditto as found by rod floats.
$D$ = proportion of total depth not covered by rod.
$Q=Q_v\{-0{\cdot}116(\sqrt{D}-0{\cdot}1)\}$.

If the rod is length $l$, and $d$ is the total depth of the stream,

$$D=\frac{d-l}{d}$$

Parker recommends the following variant in the case of earth channels :—

$$Q_w=Q_v\{1-0{\cdot}2(\sqrt{D}-0{\cdot}1)\}$$

as he considers Francis' value only applicable to smooth artificial channels. His own results should not be used for $D$ greater than 0·1. The value 0·2 corresponds to Kutter's $n=0{\cdot}2$, and may be varied accordingly.

The rods are usually from ¾ in. to 1 in. square, and weighted so as to remain upright with about 1 in. or 1½ ins. exposed. Parker suggests as a useful set for rivers, a series starting with 1-ft. immersion and increasing by 3 ins. to 4 ft., then by variations of 6 ins. to 8 ft. Long lengths may be made of metal tubes, filled partly with shot. Champagne bottles make good floats of the first type.

In using meters in a sewer great care and patience are required to get accurate readings owing to the quantity of floating matter, such as paper or rags. The present writer would prefer to use a Pitot tube, as there are no moving parts, but the orifices are continually getting choked with soap, etc. Fortunately, this is immediately shown on the gauge. We will now give an outline description of this instrument as used by the present writer. This is shown diagrammatically in Fig. 45. The Pitot tubes proper are shown at the base of the instrument. They are formed of two copper tubes $\frac{1}{8}$-in. bore, one curved gently to face the flow, with an orifice chamfered internally, the other is soldered to the first at the back, and the only outlet to the stream is by two holes about $\frac{1}{16}$ in. diameter at *K*, one on either side of the tube. The Pitot tube is fitted to two rubber tubes at the upper end, and these fit on to metal pipes at *GG*. These pipes have a cross connecting piece containing a valve *F*, by which they may be disconnected. At *G* and *G* are also two valves worked simultaneously by one bar. The upper ends of the two pipes connect to two gauge-glasses *A* and *B* with a gauge *E* between them. This gauge is best marked in inches and tenths. The gauge-glasses are connected at the top and lead to a suction tube *D*, fitted with a valve *C*. The mode of action is as follows:

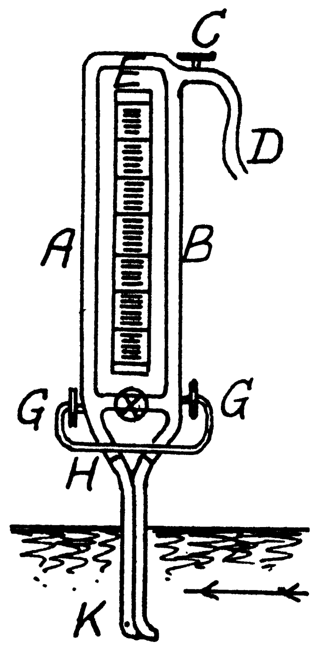

FIG. 45.

Open all valves and immerse the nozzles *K* in clean water. By means of *D* draw up water into the gauge-glasses and force it out again. Do this several times to clean out all the passages. Finally, close all valves, leaving the water standing about half-way up in the gauge-glasses. Open valve *F*, and so allow the water in both gauge-glasses to take a common level, and then close. The instrument is now ready for action.

Immerse the nozzles to the given depth, the open-ended, or pressure nozzle, pointing direct upstream. Open the valves *G*, *G* by the common handle *H*. The water will now rise in gauge-glass *B* and fall in *A*. Let the difference of the levels when steady be *h*. Let *v* be the velocity of the water at *K*.

$$\text{Then } h = c\frac{v^2}{g}.$$

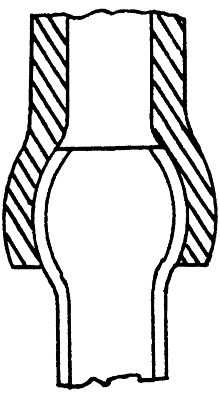

FIG. 46.

Once *h* is determined, shut down handle *H*, and the reading is fixed until the valve in the cross-piece is opened and the water in the glasses once more becomes level.

It is not advisable to have the glasses more than about 18 ins. above the water-level, or otherwise the pressure will force out the air in the liquid in the instrument, and this will appear in bubbles, keeping the water in a continual state of agitation. The rubber connecting tubes may be any length desired. These tubes are preferably made of "steam pressure" rubber piping, as anything else rapidly corrodes in the sewage. Such piping has a canvas layer inserted. Pure rubber tubing will also kink and so break the pressure.

Much work with such tubes has been done in the United States, to endeavour to standardize the Pitot tubes and fix a value for *c*.

Such are Gardner Williams (Trans. Am. Soc. C.E., Vols. 27, 47); Gregory (*ibid.*, Vol. 57); Murphy (*ibid.*, Vol. 47) ; E. S. Cole (*ibid.*, Vol. 27) ; Freeman (*ibid.*, Vol. 21) ; *also* Stanton (Proc. Inst. C.E., Vol. 156) ; Smith (Proc. Vic. Inst. E. 1909) ; and many others. The various values found for $c$ depend on the types of instruments (*see* Unwin's "Hydraulics," Parker's "Control of Water," Lea's "Hydraulics"), but in all cases it was in the neighbourhood of 0·5, as in the present writer's apparatus.

Every instrument should be carefully calibrated before being first used. The two Pitot tubes are preferably about ¾ in. apart, so as not to affect each other, but in some designs one tube is inside the other. In the writer's case the horizontal length of the impact tube was about 1½ ins., but in Professor Unwin's and others it is very much longer. The tubes vary in bore from ⅛ in. up to ½ in. The instrument is no good for velocities below 4 ins. a second, and uncertain for velocities of about 1 ft. per sec.

Parker states that the great drawback in the instrument is the difficulty in ensuring that the two nozzles do not interfere with each other. According to White (Journ. Assoc. of Eng. Soc. 1901) the shape of the impact nozzle is immaterial. All the tubes must be as smooth as possible.

**"Backwater Function" in Sewers.**—Any sudden obstruction in a sewer, such as a sunk weir, causes the water to back up behind it, and the expression determining the curve of the water surface is known as the "backwater function."

The mathematical analysis covers a variety of possible cases and is long and involved. Only one case is of interest to the sewerage engineer, and it is usual to obtain a solution by an approximation.

We first calculate the discharge of the channel without any obstruction from any chosen formula, which formula will, however, be probably of the type $Q = Ac\sqrt{RS}$, where the symbols have the meanings as given for the Chezy formula, pp. 55 and 56.

Let us now suppose a sunk weir inserted at any chosen spot in the stream. The effect will be to make the water surface above the weir (*i.e.* the hydraulic gradient) take the form of a very flat curve bowed downwards, and this effect will extend a considerable way upstream.

We are supposed to know the invert gradient, the

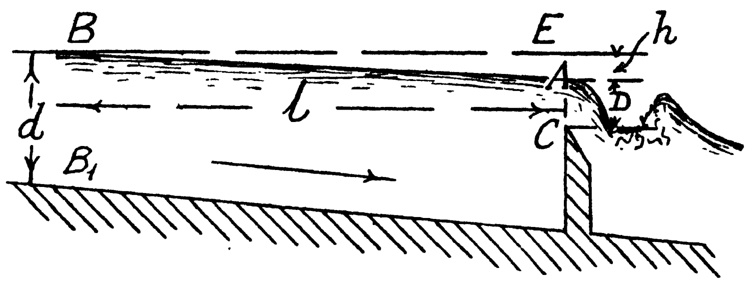


FIG. 47.

depth of flow, the cross-sectional area of flow $A$, and the hydraulic mean depth $R$, the two latter as before the introduction of the weir.

Fix the level desired for the sill $C$, and then calculate, by any favourite weir formula, the depth $D$ of flow over the sill.

Now take a convenient short length $l$, Fig. 47, and assume some probable value $S$ for the surface (hydraulic) gradient in this length.

Let $h$ = drop in water surface in length $l$.

$s = h/l$ or $h = Sl$.

From the assumed value of $S$ calculate $h$ and plot it up to $E$ from $A$, or $AE = h$.

Draw a horizontal through $E$ to cut a vertical $BB_1$ at $l$ from $C$, and thus measure the depth $BB_1 = d$.

Now assuming $d$, work out $Q$, and find whether it will equal the $Q$ giving height $D$ over sill, that is to say, find whether we get the same value of $h$ as before. If not, alter value of $S$ until we do.

Taking the section $BB_1$ as the bottom end of another length $l$, proceed as before, and continue the process until the effect has evidently vanished. In choosing the first trial value for $S$ remember that it will always be less than the invert gradient, and each successive length upstream will have a sharper hydraulic gradient than the last.

The above method may appear tedious and may be quickened by a device founded on the method of H. R. Leach (Eng. News Rec. 1919), but using Kutter's formula.

$$Q = Ac\sqrt{RS}$$

If we take the "Flynn modification," the value of $Ac\sqrt{R}$ is independent of $S$, and depends solely on the depth of flow $d$ and the chosen value of $n$.

Hence $Q$ may be written $Q = k\sqrt{S}$, and $k$ varies as $d$.

For the given channel plot a diagram as Fig. 48, in which values of $d$ are the ordinates and $k$ are the abscissæ, and so get the curve shown.

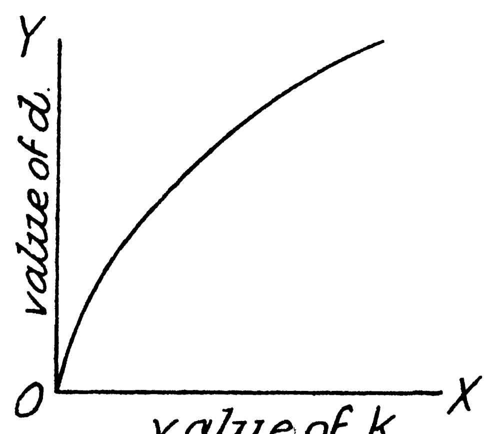

FIG. 48.

Now take the equation $Q = k\sqrt{S}$ and write it in its logarithmic equivalent, $\log S = 2(\log Q - \log k)$.

$Q$ in any given case under consideration will be known, hence the value of $\log Q$ is known. We can now make a diagram connecting the values of $\log Q$, $\log S$ and $\log k$.

Taking the given value of $\log Q$, we have :—

$$\log S = 2 \log Q \text{ when } \log k = 0$$
$$\log k = \log Q \text{ when } \log S = 0$$

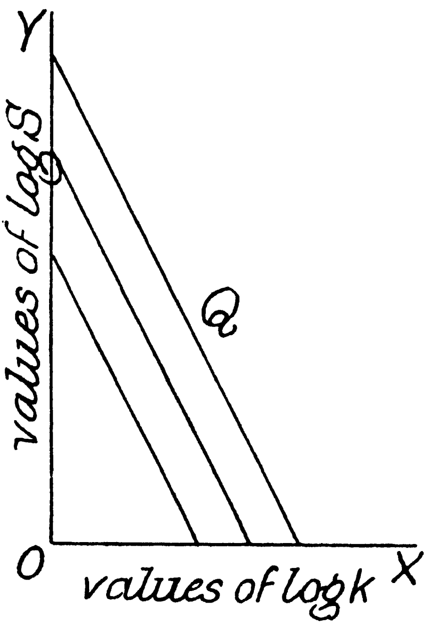


FIG. 49.

That is to say, the intercepts on the axes of rectangular co-ordinates are $2 \log Q$ and $\log Q$.

Take the axes $OY$, $OX$ (Fig. 49) for the values of $\log S$ and $\log k$. Find the values of $2 \log Q$ and $\log Q$ and plot them up $OY$ and $OX$. This will give us the line for $Q$, which will therefore have a slope of 2 to 1. There will be a series of $Q$ lines for different values of $Q$. To use the diagram, take the given value of $Q$ and calculate $\log k$. Find the intercept for $\log S$ to correspond to $\log k$ found by the given $Q$ line. This will make the work very rapid.

**Loss of Head due to Sluices.**—The general hydraulic gradient in a sewer is affected by any kind of obstruction, and one of these has already been considered for the case of a sunk weir (*see* " Backwater Function "). Probably the commonest cause of obstruction is a penstock or sluice valve.

Take, for instance, a rectangular channel whose full depth is $d$, as shown in Fig. 50. The only well-known experiments are those due to Weisbach ("Mechanics of Engineering") and are old, and fresh experiments are badly needed. Let the valve be closed until a depth $= nd$ is left open. Then Weisbach gives the following table :—

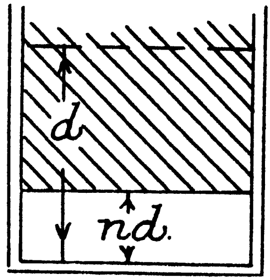


FIG. 50.

| $n$ | 0·1 | 0·2 | 0·3 | 0·4 | 0·5 | 0·6 | 0·7 | 0·8 | 0·9 | 1·0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_k$ | 193 | 44·5 | 17·8 | 8·12 | 4·02 | 2·08 | 0·95 | 0·39 | 0·09 | 0·0 |

where $c_k$ is the coefficient of resistance in the formula :—

$$\text{lost head} = c\,\frac{v^2}{2g}$$

$v$ being the average velocity of flow in the channel, running full. The experiments were made on a pipe 0·96 in. high by 1·98 ins. wide, and it is doubtful if they are correct for large channels, where the losses will probably be relatively smaller.

Fig. 51 shows the corresponding figure for a circular pipe and the following tables the values :—

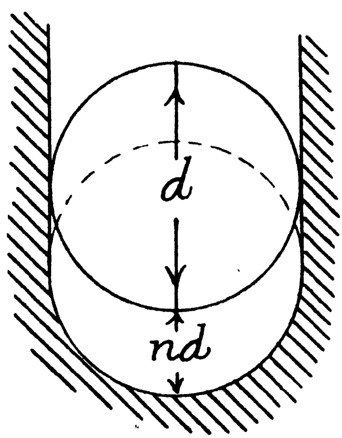


FIG. 51.

| $n$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{3}{4}$ | $\frac{7}{8}$ |
|---|---|---|---|---|---|---|---|
| $c_k$ | 97·8 | 17·0 | 5·52 | 2·06 | 0·81 | 0·26 | 0·07 |

These are also due to Weisbach and are for a 1·57-in. diameter pipe running full. Kuichling (Trans. Am. Soc. C.E., Vol. 26) experimented on a 24-in. pipe and obtained the table :—

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0·1 | 0·125 | 0·2 | 0·25 | 0·3 | 0·375 | 0·4 | 0·5 | 0·6 | 0·625 | 0·7 |
| $c_k$ | 92·6 | 69·4 | 34·6 | 25·0 | 15·4 | 8·77 | 7·3 | 3·4 | 1·43 | 1·13 | 0·59 |

All these values are for pipes running under pressure, and hence are unsuitable for most cases of sewers. Smith (Trans. Am. Soc. C.E., Vol. 34) experimented on a 30-in. diameter pipe, and, according to Parker ("Control of Water") obtained values similar to those of Kuichling. Parker, however, does not consider the results reliable, and hence they are not quoted here.

Duane (Trans. Am. Soc. C.E., Vol. 26) obtained the values given in the following table, the values of $m$ and $n$ being found thus :—

Suppose the area of opening $= nA$, where $A$ is the area when valve fully open.

Let $Q =$ rate of discharge of opening under a head $h_0$.

$$Q = c_D m A \sqrt{2gh_0} \text{ and } v = c_D m \sqrt{2gh_0}$$

$c_D$ being a coefficient of discharge.

But if we take $c_k$ as the coefficient of resistance :—

$$\frac{v^2}{2g} \cdot c_k = h_0$$

$$\therefore v = \sqrt{2gh_0} \times \frac{1}{\sqrt{c_k}} \cdot \text{ or } c_D = \frac{1}{m\sqrt{c_k}}$$

The values of $m$ and $n$ as quoted by Parker are as follows :—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $n$ | 0·125 | 0·25 | 0·375 | 0·5 | 0·625 | 0·75 | 0·875 |
| $m$ | 0·125 | 0·287 | 0·443 | 0·593 | 0·729 | 0·851 | 0·946 |

Graeff ("Traité d'Hydraulique") experimented on a

16-in. main under heads from 52 ft. to 131 ft. and obtained values of $c_D$ for all values of $n$ from 0·795 to 0·82.

Parker considers that, for all practical work, $c_D$ may be taken between 0·75 and 0·80.

The sluice may also be considered as a large drowned orifice.

Let $h_1$, $h_2$ be the heads over upper edge of orifice on upstream and down-stream sides.

Then $(h_1 - h_2) =$ effective head on orifice.

$$Q = c_D \,.\, A \,.\, \sqrt{2g}\sqrt{(h_1 - h_2)}$$

$A$ being the area of the orifice.

Usually there will be a large velocity of approach.

Let this be $v_1$, and let $\frac{v_1^2}{2g} = h$.

$$Q \text{ now} = c_D A \sqrt{2g}\sqrt{h_1 - h_2 + h}$$

Gibson ("Hydraulics") states that $c$ is about 1 per cent. less than the corresponding value for an orifice discharging freely under the same effective head.

Hanbury Brown ("Irrigation") gives the following approximate values for $c_D$ in the case of sluices :—

| Description of opening. | $c_D$ |
|---|---|
| Ordinary lock sluice .. .. .. .. .. .. .. | 0·62 |
| Small regulator openings with shallow water .. .. .. | 0·57 |
| Regulator openings up to 6 ft. wide with recesses in the piers .. | 0·62 |
| Ditto between 6 ft. and 13 ft. wide .. .. .. .. .. | 0·72 |
| Ditto over 13 ft. wide .. .. .. .. .. .. | 0·82 |
| Ditto up to 6 ft. wide with straight and continuous piers .. .. | 0·72 |
| Ditto between 6 ft. and 13 ft. wide .. .. .. .. | 0·82 |
| Ditto over 13 ft. wide .. .. .. .. .. .. | 0·92 |

The value of $c_D$ depends on the amount the sluice is open, but the values of $h_1$ and $h_2$ are difficult to measure accurately.

It is hardly worth while giving any further extracts on experiments on sluices, as the subject is not generally of such relative importance in sewerage work. Readers who wish to obtain more detailed information should consult: Parker ("Control of Water"); Bornemann ("Civil ingenieur," Vol. 26); Benton (Punj. Irr. Branch, Paper No. 8); Chatterton ("Hyd. Exp. in the Kistna Delta").

Molesworth's "Pocket Book" gives $c_D = 0{\cdot}6$ for lock sluices.

**Flow at Bridge Piers.**—It may, at some time, be

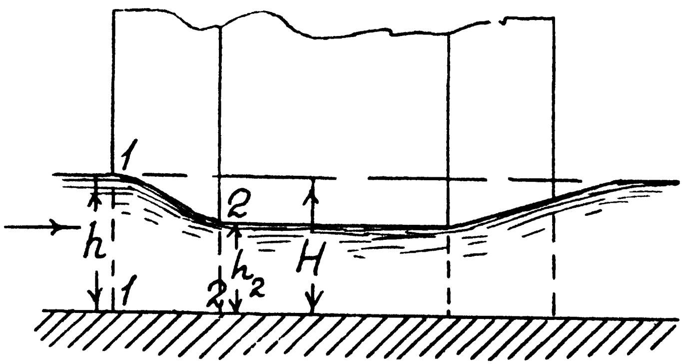


FIG. 52.

necessary to obtain an idea of the effect of bridge piers on the flow of an open channel.

The problem is theoretically an example of the "backwater function," but the results thus found are of no practical use. There are therefore several empirical formulæ published, but several of these, again, are unreliable. The following is used by the present writer and refers to Figs. 52, 53. The effect of the pier is to raise the surface of the water at the upstream nose (section 1–1) and then to depress it below the original level shown dotted. The effect is, of course, dependent on the relation of $b_1$ to $b_2$. As the length of the pier is relatively short, the values of any experimental constants used are very uncertain, especially that for friction, which latter will, therefore, be neglected.

Let $v_1$, $v_2$ be the average velocities at sections 1–1 and 2–2.

By Bernouilli's Theorem (Appendix III.)

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g}$$

Hence $v_2^2 = v_1^2 + 2g(h_1 - h_2)$.

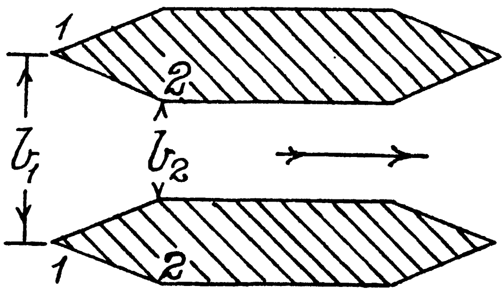


FIG. 53.

Let $Q$ = rate of discharge, and let the channels be taken as approximately rectangular.

$$Q = c_D b_2 h_2 v_2 \text{ and } v_2 = Q/c_D b_2 h_2,\ v_1 = Q/b_1 h_1.$$

Hence $Q^2/(c_D b_2 h_2)^2 = Q^2/b_1^2 h_1^2 + 2g(h_1 - h_2)$

Let $(h_1 - h_2) = x$.

$$\text{Then } x = \frac{Q^2}{2g}\left(\frac{1}{c_D^2 b_2^2 h_2^2} - \frac{1}{b_1^2 h_1^2}\right)$$

$c_D$ depends on the type of cutwater and varies from 0·95 (Eytelwein) for pointed ones to 0·85 for flat ends.

Professor Merriman ("Hydraulics," 1916) gives a different result. The first assumes the flow to take the shape shown in Fig. 54.

He then deduces the formula :—

$$d=\frac{v^2}{2g}\left\{\left(\frac{B}{bc_D}\right)^2-\left(\frac{D}{D+d}\right)^2\right\}$$

As $d$ appears on both sides of the equation, the value of $d$ is indeterminate. He therefore proposes to first take $d$ as zero in the bracket and then insert the value of $d$

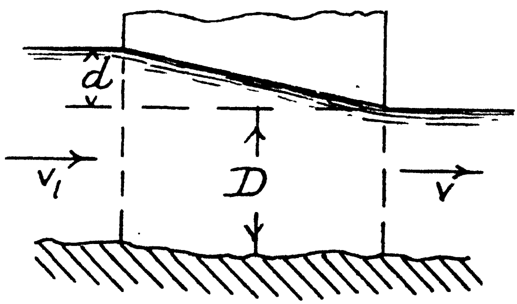


FIG. 54.

thus found, in the bracket. He states also that the value of $c_D$ is never less than $\frac{3}{4}$ or more than unity. The present writer considers the formula is not firmly established, but it is given here as it is apparently well known in the United States under the name of Hulton's Formula (*see* Trans. Am. Soc. C.E. 1882), and has apparently received general approval in that country.

## CHAPTER III

### DISCHARGE OF WEIRS

As already mentioned under the heading of "Gauging," weirs are constantly used in sewerage work to check discharge formulæ, and hence some notes on the various types used will now be given.

An enormous amount of work has been done to obtain accurate values of the experimental constants, and hence

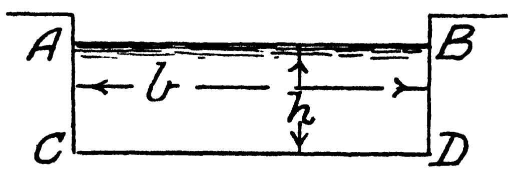


FIG. 55.

it will only be possible to summarize the results in the briefest possible manner.

Fig. 55 represents diagrammatically the common type of " square notch " weir. $AB$ is the level of the water in the supply reservoir, and $CD$ is the sill of the weir.

In the theoretical treatment of the problem of finding the rate of discharge through the notch, the velocity in any horizontal film is assumed to be due to the head above such film, as measured from $AB$. Under this assumption the velocity at level $AB$ is zero, and increases regularly to a maximum at $CD$.

It may hence be mathematically proved that the average velocity is $\frac{2}{3}$rds the maximum, or $\frac{2}{3}\sqrt{2gh}$.

Hence, if $Q$ = rate of discharge :—

$$Q = bh \times \tfrac{2}{3}\sqrt{2gh} \text{ theoretically}$$

$$\text{or } Q = \tfrac{2}{3} c_D b \sqrt{2g} h^{\frac{3}{2}} \text{ actually}$$

where $c_D$ is a coefficient of discharge less than unity, and allowing for the effect of the sill $CD$, and any other causes of resistance.

It will be obvious to any thoughtful reader that the above theoretical treatment is quite opposite to facts. The film at the crest has the maximum velocity, and that at $CD$ the least. If, however, we obtain the formula under Boussinesq's Theory (Comptes Rendus, 1887, 1889), we obtain a similar result. Under this method the water is supposed to fall in a vertical vortex.

It is here necessary to issue a note of warning to the effect that values of $c_D$ may only be used when the weir is exactly similar to the one from which the coefficient was originally obtained. Further to this, the sides $AC$ and $BD$, and the sill $CD$ must be well away from the sides and bottom of the reservoir. Usually the notch is placed in an open channel and, as there will be no " still head," an allowance must be made for the " velocity of approach."

Let $v$ = velocity of approach.

Then the increase of head over the crest to allow for the velocity of approach, or in other words, the head to still water, is usually taken as approximately

$$h + \frac{v^2}{2g}$$

The value of $c_D$ is usually about 0·423, so that the value of $c_D\sqrt{2g}$ is about 3·39.

The notch must be formed in a thin sheet of metal bevelled on the down-stream side to a sharp edge, and perfectly vertical, to give this value. The ends must be truly vertical and the sill horizontal.

The U.S.A. Dept. of Agriculture (Bulletin, No. 813) have recently made experiments on larger weirs of this type and suggest the formula :—

$$Q = 3{\cdot}247H^{1{\cdot}48} - \frac{0{\cdot}566b^{1{\cdot}8}}{1 + 2b^{1{\cdot}8}} \times h^{1{\cdot}9}$$

It frequently happens that the velocity of approach $v$ is not known, and a method of trial and error has to be used to determine it.

Let $A$ = cross-sectional area of approach channel.

Let $Q$ = discharge of weir not allowing for $v$.

Then $v = Q/A$ approximately.

This value of $v$ is then to be inserted in the value for the total head on the weir and $Q$ again calculated. If the new value of $Q$ is practically the same as the first, the assumed value of $v$ is correct, but if not recalculate $v$ from the new value of $Q$ and proceed as before. It is rarely necessary to repeat this operation more than twice.

If the weir is in the side of a practically unlimited stretch of water, such as a reservoir, the value of $A$ is indeterminate. It will, however, be possible to then obtain $h$ from the unaffected water surface.

The value of $Q = cbh^{\frac{3}{2}}$ takes no account of the effect of the vertical ends of the notch. To allow for this the well-known Francis' formula (Lowell "Hydraulic Experiments," 1883) is almost always used. In this :—

$$Q = c\left\{b - \frac{nh}{10}\right\}h^{\frac{3}{2}}$$

where $c$ is usually taken as $\frac{10}{3}$ and $n$ is the number of "end contractions," *i.e.* two in Fig. 56.

The expression $\frac{n}{10}$ is by no means constant, and depends on the size of the weir. The idea on which the formula is founded is that there is a central length of the notch equal to $(b - 2mh)$ say, where the flow is unaffected by the ends. The effective width of the weir is therefore expressed by some such value as $(b - nkh)$ where $k$ is a coefficient assumed by Francis to have an average value of $\frac{1}{10}$ and $n$ is the number of end contractions. Theoretically the expression becomes more and more unreal as $nkh$ approaches the value of $b$, and hence it is usually assumed that $b$ should never be less than 6 ins. or more than $\frac{1}{3}b$.

According to some experiments at Cornell University (Proc. Am. Soc. C.E., Vol. 44) the formula may be used for values of $h$ up to 5 feet on a weir 6·56 ft. wide.

Francis specified that the sill should be at least $3h$ above the channel bed, and the ends of the notch should be at least $2h$ from the channel sides.

If the sides of the channel are rectangular and coincide with the ends of the notch $n = 0$, and we obtain the formula first quoted.

Bazin ("An. Ponts. et Chaussées," 1898) has done much work on weirs, and his experiments and formulæ are considered to be very exact. This formula contains a term involving the depth from the sill to the bed, but is cumbrous to use, and is said to be only applicable exactly to the weir he worked with. Details of this weir are given in Parker ("Control of Water").

The experiments of Fteley and Stearns (Trans. Am. Soc. C.E. 1883) should also be consulted by those who wish to go fully into the problem, also Hamilton Smith (" Hydraulics ").

All the above notes refer to weirs placed accurately at right angles to the flow, but experiments have been carried out by Aichel (Zeitschrift Deutsche I.V. 1908) on the effect of making the weir oblique.

Let $Q$ = discharge of weir placed normal to flow.
$Q_1$ = ditto on oblique weir.
$\theta$ = acute angle made by oblique weir with side of channel.
$P$ = depth of bed of channel below sill.

$$\text{Then } Q_1 = Q\left(1 - \frac{250h}{rP}\right)$$

$h$ is here total head (*i.e.* allowing for velocity of approach), and $r$ is an experimental value as per following table. No end contractions are allowed for.

| $\theta°$ | Channel 10 inches wide. | Channel 20 inches wide. |
|---|---|---|
| 15 | 305 | 362 |
| 30 | 532 | 700 |
| 45 | 893 | 1250 |
| 60 | 1923 | 2275 |
| 75 | 6579 | 6597 |

Parker (" Control of Water ") observes that when $P$ is not small (it was 10 ins. in the experiments) compared to $h$, the effect of the obliquity is small.

The effect of sloping the weir down-stream is to increase the flow, and if sloped upstream to decrease the flow. Figures are not given here as they will probably be rarely, if ever, required, but may be obtained from Bazin (" Ecoulement en Déversoir," ii.).

**Crenelated Weirs.**—Such weirs have been constructed in America as shown in Fig. 56 with the idea of increasing the length of sill, and hence the rate of discharge. They are said to give a rate of discharge equal to that of a straight weir of the same total sill length, but the present writer

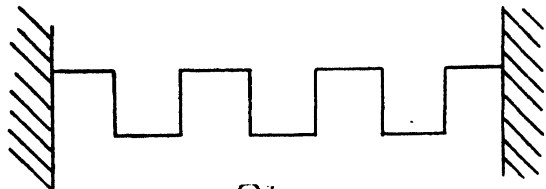


FIG. 56.

doubts this. Parker ("Control of Water") suggests a value of 0·9*b*, and says that the sill should be well above the bed, and the escape channel should preferably be deep and narrow.

**Broad-Crested Weirs.**—Weirs of any size will generally have a sill of appreciable thickness, and the formulæ previously quoted will not accurately apply. If we have a sharp-cornered rectangular sill such as that shown in Fig. 57, the flow is theoretically of the type there shown, and it may be proved, mathematically, that:—

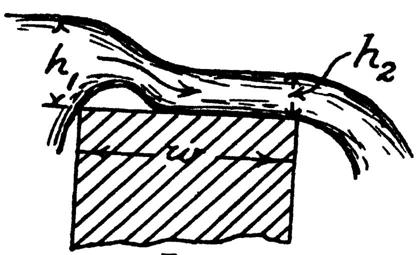


FIG. 57.

$$Q = 3·08bh_1^{\frac{3}{2}}$$

and that $h_2 = \frac{2}{3}h_1$.

In all weir formulæ the "nappe," or falling sheet of water, is supposed to spring free of the weir. This occurs with a thin sheet, but in broad sills $h$ must be greater than $1\frac{1}{2}w$ for this to occur.

Bazin gives :—

$$Q_1 = Q\left(0{\cdot}7 + 0{\cdot}185\frac{h}{w}\right)$$

where $Q$ is the discharge of the corresponding sharp-edged weir in a thin plate.

Horton ("Weir Experiments") suggests :—

$$Q_1 = 2{\cdot}64bh^{\frac{3}{2}}$$

$h$ now including any velocity of approach.

Gibson ("Hydraulics") considers that Bazin's formula is too low when $h$ is greater than $1\frac{1}{2}$ ft.

It is considered that as the ratio of $\frac{h}{w}$ affects $Q_1$ it will be better to write the formula as :—

$$Q_1 = 3{\cdot}08cbh^{\frac{3}{2}}$$

and deduct values for $c$. These are said to be between 0·82 and 0·87, and to agree very closely with the Cornell experiments.

If the upstream edge of the sill be rounded off, the discharge is materially affected.

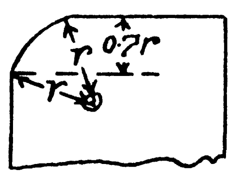


FIG. 58.

Fteley and Stearns (Trans. Am. Soc. C.E., Vol. 12) experimented with a sill as shown in Fig. 58, where $r$ was various values up to 1 in.

The effect was to raise $h$ to $h + 0{\cdot}7r$, and this is confirmed by Parker ("Control of Water"). This result assumes a free nappe.

**Curved Weirs.**—Weirs in a large river or swift-flowing stream are frequently curved upstream on plan for the sake of strength. If the curvature is flat the length of the sill may be measured along the arc, and an allowance of 1 per cent. be allowed off a similar straight weir.

An extreme case of sharp curvature will be the mouth of a vertical pipe, such as is used to draw off the supernatant water in settling tanks.

Experiments on such pipes were carried out at the Cornell University (Proc. Am. Soc. C.E. 1906), the internal diameters varying from 2 ins. up to 12 ins., but strange to say the flow was outward.

When $h$ was less than $0{\cdot}028d^{1{\cdot}04}$ ($d$ being the diameter of the pipe in feet) the formulæ for ordinary sharp-crested weirs were found to apply, but when $h$ was greater than $0{\cdot}107d^{1{\cdot}03}$ the flow became similar to a jet.

For the first case $Q = 8{\cdot}8d^{1{\cdot}29}h^{1{\cdot}29}$ cub. ft. per sec.
For the second case $Q = 5{\cdot}7d^{2}h^{0{\cdot}53}$ cub. ft. per sec.

Gourley (Proc. Am. Soc. C.E., Vol. 184) experimented on similar pipes for inward flow, and obtained the formula :—

$$Q = c.l.h^{1{\cdot}42}$$

$l$ being the length of the outer circumference in feet, and $c$ has the following values :—

| | | | | | |
|---|---|---|---|---|---|
| $d$ (in inches) .. .. | 6·9 | 10·1 | 13·7 | 19·4 | 25·9 |
| $c$ " .. .. | 2·93 | 2·94 | 2·97 | 2·99 | 3·03 |

$d$ is here the external diameter.

**The Cipoletti Weir.**—Such a weir is shown in Fig. 59, and is due to an attempt to combine the accuracy of the $V$-notch weir with the capacity of the square-notch weir.

Theoretically :—

$$Q = \sqrt{2g}\{\tfrac{2}{3}c_{D}bh^{\frac{3}{2}} + \tfrac{8}{15}c'_{D}\tan\theta h^{\frac{5}{2}}\}$$

where $c_D$ refers to the central rectangle and $c'_D$ to the $V$

notch made up of the two sloping ends. $c_D$ is usually assumed equal to $c'_D$, and if we assume the modification of Francis, and that the sloping of the sides counterbalances the end contraction, we have :—

$$Q = 3{\cdot}367bh^{\frac{3}{2}}$$

This is the formula of Cipoletti (Gior. del Gen. Civ. 1886). Flinn and Dyer (Trans. Am. Soc. C.E., Vol. 32) obtained 3·283 instead of 3·367 for heads from 0·3 ft. to 1·25 ft., and $b$ from 3 ft. to 9 ft., using Hamilton Smith's correction for the velocity of approach $\left(h' = h + 1{\cdot}4\frac{v^2}{2g}\right)$.

All weirs with end contractions are very liable to

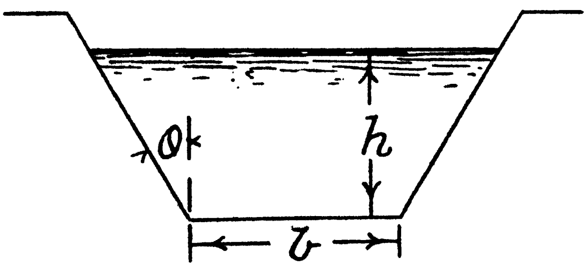


FIG. 59.

accumulate silt on the upstream side. This gradually affects the rate of discharge by altering the height of the sill above the stream bed. The result is not so marked when the end contractions are absent.

**Large Weirs.**—With large weirs the above formulæ must be modified for accuracy.

Carpenter (Trans. Am. Soc. C.E., Vol. 19) suggests the formula :—

$$Q = c(b - 0{\cdot}2D)\left\{(D+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\right\}$$

where $D$ = actual head at crest over sill.

$h$ = head due to velocity of approach.

If $H = D + h$, $c$ is obtained from the following table ($H$ is in feet) :—

| $H$ | $c$ | $H$ | $c$ |
|---|---|---|---|
| 1·9–0·9 | 3·53 | 0·6–0·5 | 3·58 |
| 0·9–0·8 | 3·54 | 0·5–0·4 | 3·61 |
| 0·8–0·7 | 3·55 | 0·4–0·3 | 3·66 |
| 0·7–0·6 | 3·56 | | |

Experiments on large weirs were carried out at Cornell (Horton, " Weir Discharges ") and the following formula obtained :—

$$Q = 3{\cdot}278b\{(D+h)^{\frac{3}{2}} - h^{\frac{3}{2}}\}$$

where $D$ ranged from 2 to 4·88 feet. If $D$ below 3 ft. Parker says that 3·278 becomes 3·321.

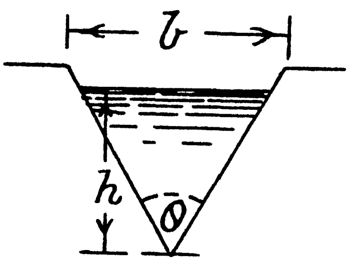

Fig 60.

Parker considers that a good formula for large flat-topped weirs is :—

$$Q = 3{\cdot}50bD^{\frac{3}{2}}$$

and Chatterton (" Hyd. Exp. in the Kistna Delta ") made experiments from which Parker deduces the formula :—

$$Q = 3{\cdot}09bD^{\frac{3}{2}}$$

If there is an apron sloping down-stream, the last equation becomes :—

$$Q = 3{\cdot}5bD^{3}$$

**V-notch Weir.**—Such a weir is shown in Fig. 60, and is not often of use in sewerage work owing to its relatively small capacity.

It may be proved by mathematical analysis that, theoretically, the rate of discharge is given by the formula :—

$$Q = \frac{8}{15} c_D \sqrt{2g} \tan \frac{\theta}{2} h^{\frac{5}{2}}$$

In Thomson's notch (*see* Brit. Assoc. Report, 1861) the angle $\theta$ is taken as 90°, so that $\tan \frac{\theta}{2} = 1$, and the formula further simplifies to :—

$$Q = 2{\cdot}635 h^{\frac{5}{2}}$$

if $c_D$ be taken as 0·62.

According to the experiments of Barr (Eng. 1910), the influence of the height of the sill above the channel floor was negligible at values of $3h$ and over.

To allow for velocities of approach add a head equivalent to $\frac{1{\cdot}4v^2}{2g}$. The notch must be very carefully made and fixed, and the slope of the sides must be exactly the same.

Parker states that the velocity of approach is ⅔rds the above value.

**Storm-Water Overflow Weirs.**—The most important weir to a municipal and drainage engineer is the overflow weir, and this is usually of one of three types.

These types are shown diagrammatically in Figs. 61, 62, 63, pp. 124, 125. In Fig. 61 the main channel is diverted through a sharp angle, and a weir placed across the original direction of the flow. As regards calculations, the length of such weir must be taken from the projection in the original direction of flow. Such a weir is suitable when the storm-overflow culvert can proceed in the original direction, but becomes somewhat awkward if the main channel has to be again turned into its previous line.

Fig. 62 shows the method of inserting a horizontal cut-water *B*, usually in the form of a metal plate. This plate is curved on plan at the down-stream end, and also turned upwards so as to throw the excess flow into the

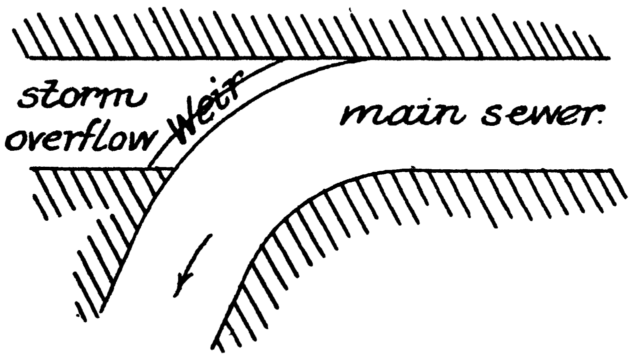


FIG. 61.

storm-water channel *C*. The design is very compact, and the only objection, in the present writer's opinion, is the fact that solids collect on the plate and cause a nuisance, and that the cutting edge is liable to be chipped by collid-ing floating bodies. Such edge should, therefore, be made

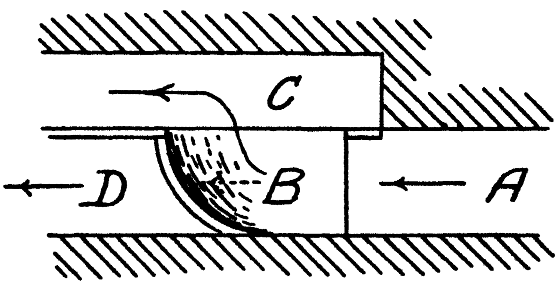


FIG. 62.

removable. It has also been stated that the plate causes an obstruction to the main flow by increasing the wetted perimeter, and hence the friction, but this cannot be very serious.

Fig. 63 shows by far the commonest method, and merely consists of cutting a rectangular notch in the side of the main channel, the sill being at the level to which the flow is to be reduced. Such weirs have, however, a very low efficiency, and hence have to be a considerable length to draw down the water to any appreciable extent. *C* in the figure is the overflow channel.

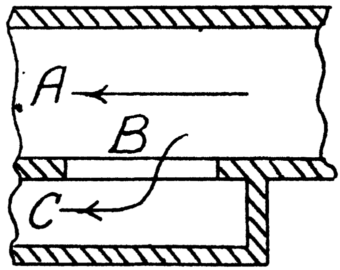

FIG. 63.

The ordinary weir formulæ are quite unsuitable for such a construction. The water passes over the sill at an angle depending on the velocity of flow in the main channel, and is heaped up at the upstream end of the weir. Very little information at present exists as to the discharging capacity of such weirs, but what there is will now be quoted.

Frühling ("Die Entwässerung der Städte") proposed to use the following formula :—

$$Q = 4bh^{\frac{3}{2}}$$

where *h* is the head over sill in the main channel before the effect of the weir is felt. Such a formula will obviously give excessive values, as it is greater than that for an ordinary weir normal to the flow. There is considerable contraction at the upstream end of the notch, and when there is any depth of flow at the down-stream end, that end acts as a contractor and considerably impedes the flow. The above formula makes no allowance for any of this.

The only other published result, as far as the present writer is aware, is that of Parmley (Trans. Am. Soc. C.E., Vol. 55).

Fig. 64 shows the cross-section of an ordinary overflow for a circular sewer.

Parmley takes the centre $O$ of the circular section as

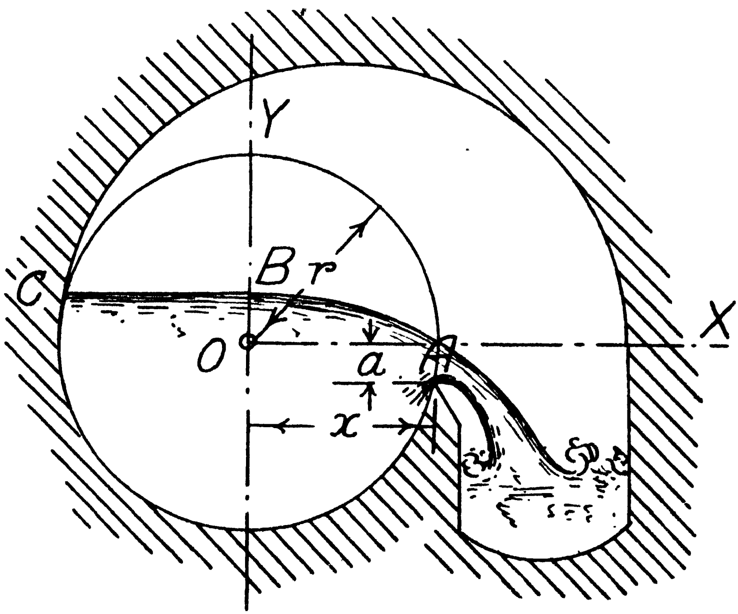


FIG. 64.

the origin of co-ordinates, and first obtains an expression :—

$$t = \Sigma \left( \frac{0 \cdot 6 \sqrt{r^2 - y^2}}{(a+y)^{\frac{3}{2}}} \delta y \right)_{y_2}^{y_1}$$

where $t$ is the time taken for main flow to traverse the length of the sill.

Such an expression is difficult to integrate, so he proceeds to assume an "equivalent" rectangular main channel as Fig. 65, where $w$ is the width of the channel and $y$ is now measured from sill-level. He then calculates the discharge as if it were normal to the sill and composed of unit lengths of uncontracted weir, each with a uniform but different value of $y$. He is, therefore, able to obtain the formula :—

$$t^{y_1}_{y_2} = \frac{w}{1\cdot67}\left\{\frac{1}{\sqrt{y_2}} - \frac{1}{\sqrt{y_1}}\right\}$$

$y_1$ and $y_2$ being the values of $y$ at the upstream and downstream ends of the weir.

He then assumes that the average velocity of flow in the main channel remains constant throughout the length of the weir. Let this velocity be $v$.

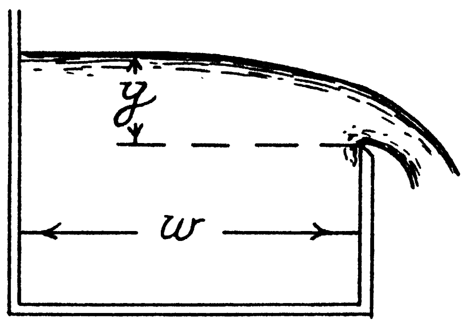


FIG. 65.

Let $b$ = length of weir. Then $b = vt$.

Hence, knowing $v$ and $t$, the value of $b$ may be found for given values of $y_2$ and $y_1$.

There are a number of objections which may be urged against the above method of analysis. Theoretically, when $y_2 = 0$, $t$ is infinite, which is not reasonable. Actually, however, the formula is rational, as the curve of the crest of the flow is continually approaching the sill but never meeting it. By choosing a small value for $y_2$ we can make the formula definite. To obtain the result, Parmley takes Francis' formula for the unit lengths, namely :—

$$q = 3\cdot33y^{\frac{3}{2}}$$

and hence makes no allowance for end contraction.

The greater the length of sill the more nearly will this assumption be correct. Parmley realizes the error by stating that the value of *b* thus found should be increased, but gives no data.

The value of *v* is not constant, but must vary as the value of *y* diminishes. The present writer made some 4000 experiments on a model in which the sill length could be varied, and obtained a purely empirical formula and also a theoretical one, which, however, he is not, at present, in a position to make public. Parmley's result may be proved by the theory of dimensional units.

It is interesting to note that the above formula takes into account the fact that the length of the sill varies directly as the width of the channel, a result independently arrived at by the present writer. *v* will have to be taken as the average velocity in the main channel for the length of the sill.

Owing to the number of debatable assumptions and approximations made in the above analysis, the result must be considered only as a rough guide, and a considerable amount added to the value of *b* so found to allow for contingencies.

For drawings and detailed discussion on such weirs *see* Lloyd Davies (Proc. Inst. C.E., Vol. 174).

Wherever possible weirs should be provided, in order to keep down the expense of construction, although it has been stated that the Government officials are against their free use. The connection to the relief sewer should be as near the river or watercourse as practicable, and the height of the sill will depend on the amount of dilution of the average dry-weather flow at which the water may be safely discharged.

Often an old, diverted watercourse can be made into a storm overflow culvert.

In calculating the sill heights care must be taken that they are well above the flood-level of the stream into which the water is discharged. Flap valves on the outlets are of very little use, even when they are periodically examined and kept oiled.

**Leaping Weirs.**—Such a weir is shown, diagrammatically in Fig. 66, and can only be used where plenty of fall

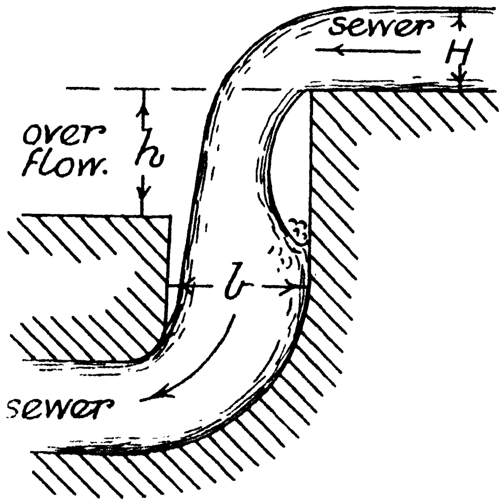


FIG. 66.

is available. It is variously attributed to Sir Alexander Binnie and J. F. Bateman. The idea is that, when the flow increases over a certain amount, a portion of the water leaps the gap, but when the flow is small (*i.e.* in dry weather) it all falls down into the sewer.

The mean horizontal velocity at the crest is assumed to be $v = 0{\cdot}67\sqrt{2gH}$, $h = \frac{gt^2}{2}$, and $b = vt$ where $t$ = time of travel of a drop from the upper sewer to the edge of the overflow.

$$\therefore\ b = 0{\cdot}67t\sqrt{2gH} \text{ and } h = \frac{0{\cdot}56b^2}{H}$$

The value 0·67 is usually attributed to Unwin. $H$ must allow for the velocity of approach, or $\frac{v^2}{2g}$ may be added to the measured value of $H$. The type of leaping weir shown in Fig. 67 has been suggested for use in the

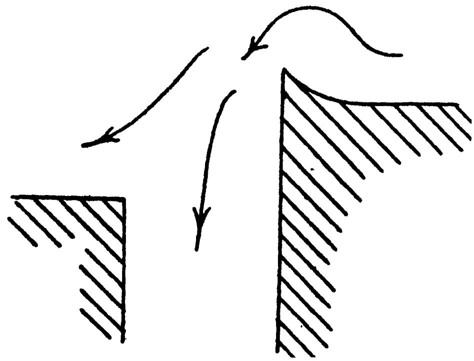

FIG. 67.

United States. It will be difficult to calculate, and the projecting crest on the upper edge will cause silting, especially with low flows and flat gradients.

**Sunk Weirs.**—Such a weir is shown in Fig. 68. The

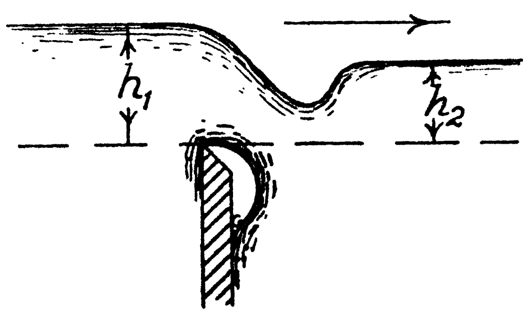


FIG. 68.

usual theoretical solution of the discharging capacity is old, being due to Dubuat, and is not satisfactory.

It is founded on the supposition that the discharge is in two parts: ($a$) due to a free notch under a head $(h_1 - h_2)$, and $b$) an orifice under head $h_2$.

Let $h$ = head due to velocity of approach.

Then the theoretical formula is :—

$$Q = cb\left\{\sqrt{2g(h_1 - h_2 + h)}\right\}\left(\tfrac{2}{3}(h_1 - h_2 + h) + h_2 - \frac{\tfrac{2}{3}h^{\frac{3}{2}}}{(h_1 - h_2 + h)^{\frac{1}{2}}}\right)$$

The value of $c$ is uncertain, and it is difficult to accurately measure $h_2$.

Gibson ("Hydraulics") states that the following expression gives a close approximation :—

$$Q = cb\sqrt{2g(h_1 - h_2 + h)} \times \{\tfrac{2}{3}(h_1 + h) + \tfrac{1}{3}h_2\}$$

Francis (Trans. Am. Soc. C.E., Vol. 12) has experimented on this type of weir, and it appears that the value of $c$ depends on the ratio of $h_1$ to $h_2$. He gives the following table :—

| | | | | | | |
|---|---|---|---|---|---|---|
| $h_2/(h_1+h)$ .. | 0 | 0·1 | 0·3 | 0·5 | 0·7 | 0·9 |
| $c$ .. .. | 0·623 | 0·625 | 0·606 | 0·594 | 0·594 | 0·596 |

The end contractions were suppressed and the nappe not aerated.

Fteley and Stearns (*ibid.*) give the following :—

| | | | | | |
|---|---|---|---|---|---|
| $h_2/(h_1 + h)$ .. | 0·1 | 0·3 | 0·5 | 0·7 | 0·9 |
| $c$ .. .. | 0·630 | 0·605 | 0·590 | 0·585 | 0·595 |

Francis' results are for $h_1$ between 0·85 ft. and 2·3 ft. Parker ("Control of Water") considers these results agree within 1 per cent.

Herschel (Trans. Am. Soc. C.E., Vol. 14) has examined the experiments of Francis and also of Fteley and Stearns, and considers that they may both be expresssed by the formula :—

$$Q = 3{\cdot}33cbh_2^{\frac{3}{2}}$$

The values of $c$ depend on the ratio of $h_2$ to $h_1$, and the following table is given by Parker, who considers the results to be correct within 1 per cent. Bazin has also made experiments, but the resultant formula is elaborate.

| $h_2/h_1$ | $c$ | $h_2/h_1$ | $c$ |
|---|---|---|---|
| 0·0 | 1·0 | | |
| 0·01 | 1·006 | 0·12 | 1·003 |
| 0·02 | 1·009 | 0·13 | 1·0 |
| 0·03 | 1·009 | 0·14 | 0·997 |
| 0·04 | 1·010 | 0·15 | 0·994 |
| 0·05 | 1·011 | 0·16 | 0·991 |
| 0·06 | 1·010 | 0·17 | 0·988 |
| 0·07 | 1·009 | 0·18 | 0·984 |
| 0·08 | 1·009 | 0·19 | 0·981 |
| 0·09 | 1·008 | 0·20 | 0·978 |
| 0·10 | 1·007 | 0·21 | 0·973 |
| 0·11 | 1·005 | 0·22 | 0·970 |

A sunk weir such as that just considered causes a raising of the water surface upstream (*see* " Backwater Function "). We may obtain an approximate expression for this.

Let $H$ = head on sill.

$b$ = length of weir.

$h$ = height of weir sill above the bed of channel.

Let $h_3$ = depth of flow at weir site supposing weir were absent.

Rate of discharge of weir $= \frac{2}{3}c_D b\sqrt{2g}H^{\frac{3}{2}} = cbH^{\frac{3}{2}} = Q$ say.

$$\text{Then } H = \left\{\frac{Q}{cb}\right\}^{\frac{2}{3}}.$$

Rise in water surface due to weir $= H + h - h_3$

$$= \left\{\frac{Q}{bc}\right\}^{\frac{2}{3}} + h - h_3$$

There are a large number of other types of weir sections illustrated in Lea's " Hydraulics " and Gibson's " Hydraulics."

# APPENDIX I

## USEFUL CONVERSION UNITS, Etc.

| | | |
|---|---|---|
| To convert | cubic feet into Imperial gallons multiply by .. | 6·24 |
| ,, | Imperial into U.S.A. gallons multiply by .. .. | 1·20 |
| ,, | feet head of water into lbs. per square inch pressure multiply by .. .. .. .. .. | 0·434 |
| ,, | kilogrammes into lbs. multiply by .. .. .. | 2·2046 |
| ,, | metres into feet multiply by .. .. .. | 3·2808 |
| ,, | square metres into square feet multiply by .. | 10·764 |
| ,, | cubic metres into cubic feet multiply by .. .. | 35·315 |
| ,, | Imperial gallons into litres multiply by .. .. | 4·544 |

1 inch of rain per hour is equivalent (nearly) to a rate run-off of 1 cubic foot per second per acre.

1 inch of rain per square mile= $(2{\cdot}32 \times 10^6)$ cubic feet.

100,000 Imperial gallons per acre = 4·4 ins. deep per acre.

1 cubic foot of water weighs 62·424 lbs. and contains 6·24 gallons. Hence a gallon of water weighs 10 lbs.

# APPENDIX II

## VISCOSITY AND THE DISCHARGE FORMULA

WE have already noted in the text the fact that viscosity is the cause of resistance to flow in enclosed channels. Although similar to, it is not a shear resistance, but has all the appearances of such.

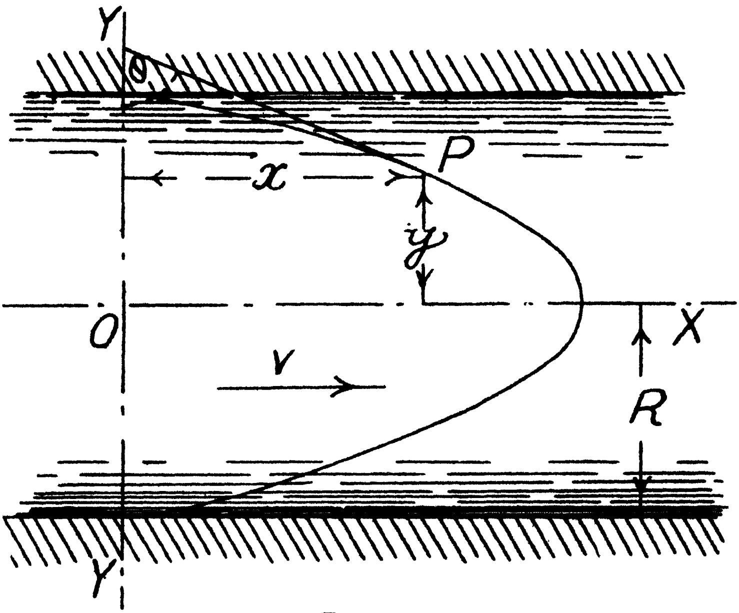


FIG. 69.

The magnitude of this stress is proportional to the rate of distortion over any chosen plane area, and is hence proportional to a rate of change of velocity on a space base.

Let Fig. 69 (after Gibson) represent such a closed channel filled with running water. Let $P$ be a particle having a velocity $v$, and be at distances $x$ and $y$ from the axes $OY$ and $OX$. Let $p$ be the distortive stress accompanying relative motion of adjacent layers in a direction parallel to $OX$.

Let us assume that there is no velocity normal to the plane $YOX$.

Suppose the ordinates to the curve shown, drawn from $YOY$, represent the velocities at any distance $y$ from $OX$.

At $P$ draw a tangent to the curve making an angle $\theta$ with the line $YOY$.

Then since $\frac{dv}{dy} = \tan\theta$, $p$ varies as $\tan\theta$.

Also, since $p$ varies as $\frac{dv}{dy}$ let $p = \mu \cdot \frac{dv}{dy}$

$\mu$ is what is termed the coefficient of viscosity.

According to Poiseuille the value of $\mu$ decreases as the temperature increases and the viscosity slightly increases as the pressure increases. Gibson states that this is borne out by the more recent experiments of Barnes and Coker at McGill. It is probable that cohesion largely enters into the question, and that the greater cohesion at the lower temperatures more than counterbalances the diminished diffusibility, and hence increases the viscosity.

Since the interchange of molecules varies as the area over which it takes place, the value of $p$ will vary accordingly. Hence, therefore, if eddies are formed in the course of a stream, the area over which interchange of momentum may take place is greatly increased, and hence the viscous resistance to motion is increased. It will always be found that resistance to motion is accompanied by eddies, and hence increased loss of head.

Viscosity is independent of the velocity of translation of the particles, being merely a physical property of the liquid. If motion in parallel straight lines is assumed, the resistance is directly inferred from the viscosity. Actually the motions are extremely complex and interwoven, and a direct solution, based on physical properties, cannot be made. The energy absorbed in overcoming viscous resistance finally appears as heat.

With unsteady motion in a viscous fluid the direction of the forces at any particular point are indeterminate, and the general equations of motion become impossible of application. It is usual to state in textbooks that the effect of viscosity is neglected, and they then proceed to obtain formulæ which have to be modified by a constant to make them fit the experimental results. As such a constant must actually include the effect of viscosity the general laws have been gone into.

It is usually assumed, and is proved in the general equations for stream-line motion, that the pressure at any point in the motion is equivalent to the hydrostatic pressure. A further assumption made is that the acceleration of the particles is due to external forces only, and not to the pressure exerted by surrounding particles, or, in other words, that the pressure throughout the cross-section of flow is uniform.

It must further be pointed out that the resistance to flow is not due to any slip between the liquid and the containing surface if the force of adhesion is sufficient to overcome the shear in the fluid at these points. With unsteady motion there is an interchange of the particles by the breaking down of the adhesion of the molecules adjoining the containing surface. Any such interchange will be the greater the rougher the containing surface, and hence will vary with the material of the pipe.

Let us now consider Fig. 70 as representing the cross-section of a horizontal circular pipe of radius $R$. By making the pipe horizontal we eliminate gravity and hence assume that the flow is produced by a uniform change of pressure head along the length of the pipe.

We have already shown that the tangential stress, or tractive force per unit area, of a particle $P$ is :—

$$\mu \cdot \frac{dv}{dy}$$

Let us now consider a cylindrical shell concentric with the pipe, of length $\delta x$, and having an inner radius $y$ and an outer $y + \delta y$.

The rate of change in the tangential force between the inside and outside radius is :—

$$\frac{d}{dy}\left\{2\pi y \,.\, \delta x \,.\, \mu \,.\, \frac{dv}{dy}\right\}$$

Hence the difference between the tangential forces on the inside and outside surfaces is :—

$$\frac{d}{dy}\left\{2\pi y \,.\, \delta x \,.\, \mu \,.\, \frac{dv}{dy}\right\}\delta y$$

The rate of change in the pressures on the ends of the imaginary cylinder is :—

$$\frac{dp}{dx}$$

Hence the change in length $\delta x$ per unit area is :—

$$\frac{dp}{dx} \,.\, \delta x$$

The area of the ends of the cylinder (neglecting $(\delta y)^2$) is :—

$$2\pi y \,.\, \delta y$$

Hence the difference in pressures between the two ends of the cylinder is :—

$$2\pi y \,.\, \delta y \,.\, \frac{dp}{dx} \,.\, \delta x$$

Therefore :—

$$2\pi y \,.\, \delta y \,.\, \frac{dp}{dx} = \frac{d}{dy}\left\{2\pi y \,.\, \delta x \,.\, \mu \,.\, \frac{dv}{dy}\right\} \,.\, \delta y$$

Hence we have :—

$$2\pi y \,.\, \delta y \,.\, \frac{dp}{dx} = 2\pi \,.\, \delta x \,.\, \mu \,.\, \delta y \,.\, d\left(y \,.\, \frac{dv}{dy}\right)$$

or

$$\frac{y}{\mu} \,.\, \frac{dp}{dx} = d\left(y \,.\, \frac{dv}{dy}\right)$$

Integrating for $y$ we get :—

$$\frac{dv}{dy} = \frac{y}{\mu} \,.\, \frac{dp}{dx}$$

Integrating again for $v$, and remembering that $v=0$ when $y=R$. If we suppose that there is no slip at the boundaries, we get :—

$$v=-\frac{dp}{dx}\cdot\frac{1}{4\mu}\left(R^2-y^2\right)$$

The flow through any cross-section of the whole pipe is :—

$$\int_0^R v\,.\,2\pi y\,.\,dy=Q \text{ say}$$

Hence, remembering that $Q=0$ when $y=0$, we have :—

$$Q=\frac{\pi\,.\,dp\,.\,R^4}{8\mu\,.\,dx}$$

In taking finite quantities we must substitute $\frac{p_1-p_2}{l}$ for $\frac{dp}{dx}$, where $p_1$ and $p_2$ are the intensities of pressure at the ends of a length $l$.

$v$ is maximum when $y=0$.

$$\therefore\ v_{\max}=\frac{(p_1-p_2)\,R^2}{4\mu l}$$

and
$$Q=\frac{\pi R^4\,(p_1-p_2)}{8\mu l}$$

But $Q=(\pi R^2)v_{av}$ where $v_{av}$ is the average velocity.

$$\therefore\ v_{\max}=2v_{av.}$$

From the above we find that the average velocity occurs at 0·707 $R$ and the equation to the curve of velocities is that of a parabola. This is the result obtained by Poiseuille.

If we allow for slip at the containing surface we proceed as follows :—

To allow for slip we must assume some value of the magnitude of the slip.

Gibson ("Hydraulics") takes $v$ at $R$ to vary as the tangential stress $\mu\left(\frac{dv}{dy}\right)_{y=R}$

Let $v' =$ velocity at the bounding surface.

$$v' \text{ varies as } \mu\left(\frac{dv}{dy}\right)_{y=R^2} = -k\left(\frac{dv}{dy}\right)_{y=R} \text{ say.}$$

But we already have :— $$\frac{dv}{dy} = \frac{y}{2\mu} \cdot \frac{dp}{dx}$$

$$\therefore v' = \frac{-ky}{2\mu} \cdot \frac{dp}{dx}$$

Going through the same procedure as before we get :—

$$Q = \frac{\pi R^4}{8\mu} \left\{ 1 + \frac{4k}{R} \right\} \left( \frac{p_1 - p_2}{l} \right)$$

According to Darcy $k = 20{\cdot}4$ (in English measures). The mean velocity may be found to occur at $0{\cdot}689R$. Later work (*see* Bazin, Mem. Ac. des Sc. 1897, and Williams, Hubbell, and Fenkell, Proc. Am. Soc. C.E. 1901) is considered to show that the curve of velocities is elliptical. If this is so we have :—

$$v_{\text{mean}} = v' + \tfrac{2}{3}(v_{\text{max}} - v')$$

In all these cases the maximum $v$ is taken to be at the axis of the pipe. The last-mentioned mean velocity will be found to occur at $0{\cdot}75R$.

Cole (Trans. Am. Soc. C.E. 1902) and Morrow (Proc. Roy. Soc. 1905) have also experimented on the problem, and according to Gibson ("Hydraulics") the conclusion is that the mean velocity is in the neighbourhood of $0{\cdot}69R$ in small pipes, and $0{\cdot}75R$ in large ditto, and if the observations be made at these points and further multiplied by a constant varying from $0{\cdot}79$ to $0{\cdot}86$ (average $0{\cdot}84$) the mean velocity will be obtained with a fair degree of accuracy.

# APPENDIX III

## BERNOUILLI'S THEOREM

THIS theorem is the most important in hydraulics, and the following proof (founded on that given in Lea's "Hydraulics") may prove of interest.

Let *AB*, Fig. 70, represent any length of a pipe of either

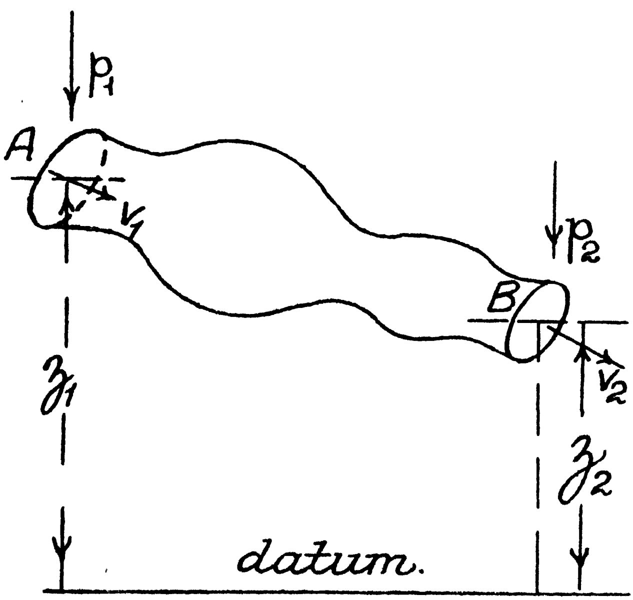


FIG. 70.

uniform or irregular cross-section. The only stipulation is that it must always run full with steady motion. Let the pressure, velocity, and head of position be $p_1$, $v_1$, and $z_1$ at *A*, and similarly be $p_2$, $v_2$, and $z_2$ at *B*, above a given datum.

Let a quantity $\delta Q$ enter $A$ in time $\delta t$. Then, in the same time a quantity $\delta Q$ will lean at $B$, since water is assumed as incompressible.

Let $a_1$ = cross-sectional area of pipe at $A$.

Then $v_1 = \frac{\delta Q}{a_1 . \delta t}$ = average velocity at $A$.

Similarly, $v_2 = \frac{\delta Q}{a_2 . \delta t}$ where $a_2$ refers to $B$.

The kinetic energy of mass $\delta Q$ at entry at A $= (w . \delta Q) \frac{v_1^2}{2g}$ where $w$ is the density of the water, and at exit at

$$B = (w . \delta Q) \frac{v_2^2}{2g}$$

The total kinetic energy of the water which has neither entered nor left remains the same, since the motion is steady. Hence the change in kinetic energy is :—

$$\frac{w . \delta Q}{2g}(v_2^2 - v_1^2)$$

The work done by gravity $= w . \delta Q(z_1 - z_2)$

The total pressures at $A$ and $B$ are : $p_1 a_1$ and $p_2 a_2$.

The work done at $A$ by the pressure there during the time $\delta t$ is :— $p_1 a_1 v_1 . \delta t = p_1 . \delta Q$

Similarly at $B$ it is : $-p_2 a_2 v_2 . \delta t = -p_2 . \delta Q$.

But the gain in kinetic energy equals the work done.

Therefore : $\frac{w . \delta Q}{2g}(v_2^2 - v_1^2) = w . \delta Q(z_1 - z_2) + (p_1 - p_2)\delta Q$

Which reduces to $\frac{v_2^2}{2g} - \frac{v_1^2}{2g} = z_1 + \frac{p_1}{w} - z_2 - \frac{p_2}{w}$

Hence $\frac{v_1^2}{2y} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2y} + \frac{p_2}{w} + z_2$ = a constant.

The above is the original theorem and takes no account of resistances between $A$ and $B$. In practice a head $h$, which includes all head lost through friction, bends, sudden alterations in section, etc., has to be added to the right side of the equation to make it true.

In all cases the values $p_2$ and $p_1$ are absolute pressures, that is include atmospheric pressure. Usually this does not affect the result, but cases may occur in which otherwise one of the p's might become negative.

# APPENDIX IV

Table of values of $A$ and $\sqrt{R}$ for use in formula $Q = Ac\sqrt{RS}$ for circular sewers running full.

| dia. | $A$ | $\sqrt{R}$ | dia. | $A$ | $\sqrt{R}$ | dia. | $A$ | $\sqrt{R}$ |
|---|---|---|---|---|---|---|---|---|
| 6″ | 0·196 | 0·353 | 7′ 3″ | 41·282 | 1·346 | 13′ 9″ | 148·49 | 1·854 |
| 9″ | 0·441 | 0·433 | 7′ 6″ | 44·179 | 1·369 | 14′ 0″ | 153·94 | 1·870 |
| 1′ 0″ | 0·785 | 0·50 | 7′ 9″ | 47·173 | 1·397 | 14′ 3″ | 159·48 | 1·887 |
| 1′ 3″ | 1·227 | 0·559 | 8′ 0″ | 50·265 | 1·414 | 14′ 6″ | 165·13 | 1·904 |
| 1′ 6″ | 1·767 | 0·612 | 8′ 3″ | 53·456 | 1·436 | 14′ 9″ | 170·87 | 1·920 |
| 1′ 9″ | 2·405 | 0·661 | 8′ 6″ | 56·745 | 1·457 | 15′ 0″ | 176·71 | 1·936 |
| 2′ 0″ | 3·141 | 0·707 | 8′ 9″ | 60·132 | 1·479 | 15′ 3″ | 182·65 | 1·952 |
| 2′ 3″ | 3·976 | 0·750 | 9′ 0″ | 63·617 | 1·50 | 15′ 6″ | 188·69 | 1·968 |
| 2′ 6″ | 4·908 | 0·790 | 9′ 3″ | 67·201 | 1·520 | 15′ 9″ | 194·83 | 1·984 |
| 2′ 9″ | 5·939 | 0·829 | 9′ 6″ | 70·882 | 1·541 | 16′ 0″ | 201·06 | 2·0 |
| 3′ 0″ | 7·068 | 0·866 | 9′ 9″ | 74·662 | 1·561 | 16′ 3″ | 207·39 | 2·015 |
| 3′ 3″ | 8·296 | 0·901 | 10′ 0″ | 78·540 | 1·581 | 16′ 6″ | 213·82 | 2·031 |
| 3′ 6″ | 9·621 | 0·935 | 10′ 3″ | 82·516 | 1·605 | 16′ 9″ | 220·35 | 2·046 |
| 3′ 9″ | 11·045 | 0·968 | 10′ 6″ | 86·590 | 1·620 | 17′ 0″ | 226·98 | 2·061 |
| 4′ 0″ | 12·566 | 1·0 | 10′ 9″ | 90·763 | 1·639 | 17′ 3″ | 233·71 | 2·076 |
| 4′ 3″ | 14·186 | 1·031 | 11′ 0″ | 95·033 | 1·658 | 17′ 6″ | 240·53 | 2·091 |
| 4′ 6″ | 15·904 | 1·060 | 11′ 3″ | 99·402 | 1·677 | 17′ 9″ | 247·45 | 2·106 |
| 4′ 9″ | 17·721 | 1·089 | 11′ 6″ | 103·87 | 1·695 | 18′ 0″ | 254·47 | 2·121 |
| 5′ 0″ | 19·635 | 1·118 | 11′ 9″ | 108·43 | 1·714 | 18′ 3″ | 261·59 | 2·136 |
| 5′ 3″ | 21·648 | 1·145 | 12′ 0″ | 113·10 | 1·732 | 18′ 6″ | 268·80 | 2·150 |
| 5′ 6″ | 23·758 | 1·172 | 12′ 3″ | 117·86 | 1·750 | 18′ 9″ | 276·12 | 2·165 |
| 5′ 9″ | 25·967 | 1·199 | 12′ 6″ | 122·72 | 1·768 | 19′ 0″ | 283·53 | 2·179 |
| 6′ 0″ | 28·274 | 1·224 | 12′ 9″ | 127·68 | 1·785 | 19′ 3″ | 291·04 | 2·193 |
| 6′ 3″ | 30·680 | 1·25 | 13′ 0″ | 132·73 | 1·802 | 19′ 6″ | 298·65 | 2·208 |
| 6 6″ | 33·183 | 1·275 | 13′ 3″ | 137·89 | 1·820 | 19′ 9″ | 306·35 | 2·221 |
| 6′ 9″ | 35·785 | 1·299 | 13′ 6″ | 143·14 | 1·837 | 20′ 0″ | 314·16 | 2·236 |
| 7′ 0″ | 38·485 | 1·322 | | | | | | |

The values of $A$ and $\sqrt{R}$ are in feet units.

# APPENDIX V

Values of $\sqrt{S}$ for use in formula $Q = Ac\sqrt{RS}$. The values are correct to the nearest fourth decimal place.

| Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| 10 | ·3162 | 44 | ·1507 | 78 | ·1133 | 112 | ·0945 | 146 | ·0827 |
| 11 | ·3015 | 45 | ·1491 | 79 | ·1125 | 113 | ·0941 | 147 | ·0824 |
| 12 | ·2887 | 46 | ·1474 | 80 | ·1118 | 114 | ·0936 | 148 | ·0821 |
| 13 | ·2773 | 47 | ·1458 | 81 | ·1111 | 115 | ·0932 | 149 | ·0819 |
| 14 | ·2672 | 48 | ·1443 | 82 | ·1104 | 116 | ·0928 | 150 | ·0816 |
| 15 | ·2582 | 49 | ·1428 | 83 | ·1097 | 117 | ·0924 | 151 | ·0813 |
| 16 | ·25 | 50 | ·1414 | 84 | ·1091 | 118 | ·0920 | 152 | ·0811 |
| 17 | ·2425 | 51 | ·1400 | 85 | ·1084 | 119 | ·0916 | 153 | ·0808 |
| 18 | ·2357 | 52 | ·1387 | 86 | ·1078 | 120 | ·0913 | 154 | ·0805 |
| 19 | 2294 | 53 | ·1374 | 87 | ·1072 | 121 | ·0909 | 155 | ·0803 |
| 20 | ·2236 | 54 | ·1361 | 88 | ·1066 | 122 | ·0905 | 156 | ·0800 |
| 21 | ·2182 | 55 | ·1348 | 89 | ·1060 | 123 | ·0901 | 157 | ·0798 |
| 22 | .2132 | 56 | ·1336 | 90 | ·1054 | 124 | ·0898 | 158 | ·0795 |
| 23 | ·2085 | 57 | ·1324 | 91 | ·1048 | 125 | ·0894 | 159 | ·0793 |
| 24 | ·2041 | 58 | ·1313 | 92 | ·1042 | 126 | ·0890 | 160 | ·0790 |
| 25 | ·2 | 59 | ·1302 | 93 | ·1037 | 127 | ·0887 | 161 | ·0788 |
| 26 | ·1961 | 60 | ·1291 | 94 | ·1031 | 128 | ·0883 | 162 | ·0785 |
| 27 | ·1924 | 61 | ·1280 | 95 | ·1026 | 129 | ·0880 | 163 | ·0783 |
| 28 | ·1890 | 62 | ·1270 | 96 | ·1020 | 130 | ·0877 | 164 | ·0781 |
| 29 | ·1857 | 63 | ·1270 | 97 | ·1015 | 131 | ·0874 | 165 | ·0778 |
| 30 | ·1826 | 64 | ·1250 | 98 | ·1010 | 132 | ·0870 | 166 | ·0776 |
| 31 | ·1796 | 65 | ·1240 | 99 | ·1005 | 133 | ·0867 | 167 | ·0774 |
| 32 | ·1768 | 66 | ·1231 | 100 | ·1 | 134 | ·0864 | 168 | ·0771 |
| 33 | ·1741 | 67 | ·1222 | 101 | ·0995 | 135 | ·0861 | 169 | ·0769 |
| 34 | ·1715 | 68 | ·1213 | 102 | ·0990 | 136 | ·0857 | 170 | ·0768 |
| 35 | ·1690 | 69 | ·1204 | 103 | ·0985 | 137 | ·0854 | 171 | ·0765 |
| 36 | ·1666 | 70 | ·1195 | 104 | ·0980 | 138 | ·0851 | 172 | ·0762 |
| 37 | ·1644 | 71 | ·1187 | 105 | ·0975 | 139 | ·0848 | 173 | ·0760 |
| 38 | ·1622 | 72 | ·1178 | 106 | ·0971 | 140 | ·0845 | 174 | ·0758 |
| 39 | ·1601 | 73 | ·1170 | 107 | ·0966 | 141 | ·0842 | 175 | ·0756 |
| 40 | ·1581 | 74 | ·1162 | 108 | ·0962 | 142 | ·0839 | 176 | ·0754 |
| 41 | ·1561 | 75 | ·1155 | 109 | ·0957 | 143 | ·0836 | 177 | ·0752 |
| 42 | ·1543 | 76 | ·1147 | 110 | ·0950 | 144 | ·0833 | 178 | ·0749 |
| 43 | ·1525 | 77 | ·1139 | 111 | ·0949 | 145 | ·0830 | 179 | ·0747 |

| Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| 180 | ·0745 | 230 | ·0659 | 280 | ·0598 | 330 | ·0550 | 380 | ·0513 |
| 181 | ·0743 | 231 | ·0658 | 281 | ·0596 | 331 | ·0549 | 381 | ·0512 |
| 182 | ·0741 | 232 | ·0656 | 282 | ·0595 | 332 | ·0549 | 382 | ·0512 |
| 183 | ·0739 | 233 | ·0655 | 283 | ·0594 | 333 | ·0548 | 383 | ·0511 |
| 184 | ·0737 | 234 | ·0654 | 284 | ·0593 | 334 | ·0547 | 384 | ·0510 |
| 185 | ·0735 | 235 | ·0652 | 285 | ·0592 | 335 | ·0546 | 385 | ·0510 |
| 186 | ·0733 | 236 | ·0651 | 286 | ·0591 | 336 | ·0545 | 386 | ·0509 |
| 187 | ·0731 | 237 | ·0649 | 287 | ·0590 | 337 | ·0545 | 387 | ·0508 |
| 188 | ·0729 | 238 | ·0648 | 288 | ·0589 | 338 | ·0544 | 388 | ·0507 |
| 189 | ·0727 | 239 | ·0647 | 289 | ·0588 | 339 | ·0543 | 389 | ·0507 |
| 190 | ·0725 | 240 | ·0645 | 290 | ·0587 | 340 | ·0542 | 390 | ·0506 |
| 191 | ·0723 | 241 | ·0644 | 291 | ·0586 | 341 | ·0541 | 391 | ·0506 |
| 192 | ·0721 | 242 | ·0643 | 292 | ·0585 | 342 | ·0541 | 392 | ·0505 |
| 193 | ·0720 | 243 | ·0641 | 293 | ·0584 | 343 | ·0540 | 393 | ·0504 |
| 194 | ·0718 | 244 | ·0640 | 294 | ·0583 | 344 | ·0539 | 394 | ·0504 |
| 195 | ·0716 | 245 | ·0639 | 295 | ·0582 | 345 | ·0538 | 395 | ·0503 |
| 196 | ·0714 | 246 | ·0637 | 296 | ·0581 | 346 | ·0538 | 396 | ·0502 |
| 197 | ·0712 | 247 | ·0636 | 297 | ·0580 | 347 | ·0537 | 397 | ·0502 |
| 198 | ·0711 | 248 | ·0635 | 298 | ·0579 | 348 | ·0536 | 398 | ·0501 |
| 199 | ·0709 | 249 | ·0634 | 299 | ·0578 | 349 | ·0535 | 399 | ·0501 |
| 200 | ·0707 | 250 | ·0632 | 300 | ·0577 | 350 | ·0534 | 400 | ·05 |
| 201 | ·0705 | 251 | ·0631 | 301 | ·0576 | 351 | ·0534 | 401 | ·0499 |
| 202 | ·0703 | 252 | ·0630 | 302 | ·0575 | 352 | ·0533 | 402 | ·0499 |
| 203 | ·0702 | 253 | ·0629 | 303 | ·0574 | 353 | ·0532 | 403 | ·0498 |
| 204 | ·0700 | 254 | ·0627 | 304 | ·0573 | 354 | ·0531 | 404 | ·0497 |
| 205 | ·0698 | 255 | ·0626 | 305 | ·0572 | 355 | ·0531 | 405 | ·0497 |
| 206 | ·0697 | 256 | ·0625 | 306 | ·0572 | 356 | ·0530 | 406 | ·0496 |
| 207 | ·0695 | 257 | ·0624 | 307 | ·0571 | 357 | ·0529 | 407 | ·0496 |
| 208 | ·0693 | 258 | ·0622 | 308 | ·0570 | 358 | ·0528 | 408 | ·0495 |
| 209 | ·0692 | 259 | ·0621 | 309 | ·0569 | 359 | ·0528 | 409 | ·0495 |
| 210 | ·0690 | 260 | ·0620 | 310 | ·0568 | 360 | ·0527 | 410 | ·0494 |
| 211 | ·0688 | 261 | ·0619 | 311 | ·0567 | 361 | ·0526 | 411 | ·0493 |
| 212 | ·0686 | 262 | ·0618 | 312 | ·0566 | 362 | ·0525 | 412 | ·0493 |
| 213 | ·0685 | 263 | ·0617 | 313 | ·0565 | 363 | ·0525 | 413 | ·0492 |
| 214 | ·0683 | 264 | ·0615 | 314 | ·0564 | 364 | ·0524 | 414 | ·0491 |
| 215 | ·0682 | 265 | ·0614 | 315 | ·0563 | 365 | ·0523 | 415 | ·0491 |
| 216 | ·0680 | 266 | ·0613 | 316 | ·0562 | 366 | ·0523 | 416 | ·0490 |
| 217 | ·0679 | 267 | ·0612 | 317 | ·0562 | 367 | ·0522 | 417 | ·0490 |
| 218 | ·0677 | 268 | ·0610 | 318 | ·0560 | 368 | ·0521 | 418 | ·0489 |
| 219 | ·0676 | 269 | ·0609 | 319 | ·0560 | 369 | ·0521 | 419 | ·0488 |
| 220 | ·0674 | 270 | ·0608 | 320 | ·0559 | 370 | ·0520 | 420 | ·0488 |
| 221 | ·0672 | 271 | ·0607 | 321 | ·0558 | 371 | ·0519 | 421 | ·0487 |
| 222 | ·0671 | 272 | ·0606 | 322 | ·0557 | 372 | ·0518 | 422 | ·0487 |
| 223 | ·0670 | 273 | ·0605 | 323 | ·0556 | 373 | ·0518 | 423 | ·0486 |
| 224 | ·0668 | 274 | ·0604 | 324 | ·0555 | 374 | ·0517 | 424 | ·0486 |
| 225 | ·0666 | 275 | ·0603 | 325 | ·0555 | 375 | ·0516 | 425 | ·0485 |
| 226 | ·0665 | 276 | ·0602 | 326 | ·0554 | 376 | ·0516 | 426 | ·0484 |
| 227 | ·0663 | 277 | ·0601 | 327 | ·0553 | 377 | ·0515 | 427 | ·0484 |
| 228 | ·0662 | 278 | ·0600 | 328 | ·0552 | 378 | ·0514 | 428 | ·0483 |
| 229 | ·0661 | 279 | ·0599 | 329 | ·0551 | 379 | ·0514 | 429 | ·0483 |

| Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| 430 | ·0482 | 480 | ·0456 | 530 | ·0434 | 580 | ·0415 | 630 | ·0398 |
| 431 | ·0481 | 481 | ·0456 | 531 | ·0434 | 581 | ·0415 | 631 | ·0398 |
| 432 | ·0481 | 482 | ·0455 | 532 | ·0433 | 582 | ·0414 | 632 | ·0398 |
| 433 | ·0480 | 483 | ·0455 | 533 | ·0433 | 583 | ·0414 | 633 | ·0397 |
| 434 | ·0480 | 484 | ·0454 | 534 | ·0433 | 584 | ·0414 | 634 | ·0397 |
| 435 | ·0479 | 485 | ·0454 | 535 | ·0432 | 585 | ·0413 | 635 | ·0397 |
| 436 | ·0479 | 486 | ·0454 | 536 | ·0432 | 586 | ·0413 | 636 | ·0396 |
| 437 | ·0478 | 487 | ·0453 | 537 | ·0431 | 587 | ·0413 | 637 | ·0396 |
| 438 | ·0478 | 488 | ·0453 | 538 | ·0431 | 588 | ·0412 | 638 | ·0396 |
| 439 | ·0477 | 489 | ·0452 | 539 | ·0431 | 589 | ·0412 | 639 | ·0395 |
| 440 | ·0477 | 490 | ·0452 | 540 | ·0430 | 590 | ·0412 | 640 | ·0395 |
| 441 | ·0476 | 491 | ·0451 | 541 | ·0430 | 591 | ·0411 | 641 | ·0395 |
| 442 | ·0476 | 492 | ·0451 | 542 | ·0429 | 592 | ·0411 | 642 | ·0395 |
| 443 | ·0475 | 493 | ·0450 | 543 | ·0429 | 593 | ·0411 | 643 | ·0394 |
| 444 | ·0474 | 494 | ·0450 | 544 | ·0429 | 594 | ·0410 | 644 | ·0394 |
| 445 | ·0474 | 495 | ·0449 | 545 | ·0428 | 595 | ·0410 | 645 | ·0394 |
| 446 | ·0473 | 496 | ·0449 | 546 | ·0428 | 596 | ·0410 | 646 | ·0393 |
| 447 | ·0473 | 497 | ·0448 | 547 | ·0427 | 597 | ·0409 | 647 | ·0393 |
| 448 | ·0472 | 498 | ·0448 | 548 | ·0427 | 598 | ·0409 | 648 | ·0393 |
| 449 | ·0472 | 499 | ·0447 | 549 | ·0427 | 599 | ·0408 | 649 | ·0392 |
| 450 | ·0471 | 500 | ·0447 | 550 | ·0426 | 600 | ·0408 | 650 | ·0392 |
| 451 | ·0471 | 501 | ·0446 | 551 | ·0426 | 601 | ·0408 | 651 | ·0392 |
| 452 | ·0470 | 502 | ·0446 | 552 | ·0426 | 602 | ·0407 | 652 | ·0392 |
| 453 | ·0470 | 503 | ·0446 | 553 | ·0425 | 603 | ·0407 | 653 | ·0391 |
| 454 | ·0469 | 504 | ·0445 | 554 | ·0425 | 604 | ·0407 | 654 | ·0391 |
| 455 | ·0469 | 505 | ·0445 | 555 | ·0424 | 605 | ·0406 | 655 | ·0391 |
| 456 | ·0468 | 506 | ·0444 | 556 | ·0424 | 606 | ·0406 | 656 | ·0390 |
| 457 | ·0468 | 507 | ·0444 | 557 | ·0424 | 607 | ·0406 | 657 | ·0390 |
| 458 | ·0467 | 508 | ·0444 | 558 | ·0423 | 608 | ·0405 | 658 | ·0390 |
| 459 | ·0467 | 509 | ·0443 | 559 | ·0423 | 609 | ·0405 | 659 | ·0389 |
| 460 | ·0466 | 510 | ·0443 | 560 | ·0422 | 610 | ·0405 | 660 | ·0389 |
| 461 | ·0466 | 511 | ·0442 | 561 | ·0422 | 611 | ·0404 | 661 | ·0389 |
| 462 | ·0465 | 512 | ·0442 | 562 | ·0422 | 612 | ·0404 | 662 | ·0389 |
| 463 | ·0464 | 513 | ·0441 | 563 | ·0421 | 613 | ·0404 | 663 | ·0388 |
| 464 | ·0464 | 514 | ·0441 | 564 | ·0421 | 614 | ·0403 | 664 | ·0388 |
| 465 | ·0464 | 515 | ·0441 | 565 | ·0421 | 615 | ·0403 | 665 | ·0388 |
| 466 | ·0463 | 516 | ·0440 | 566 | .0420 | 616 | ·0403 | 666 | ·0387 |
| 467 | ·0463 | 517 | ·0440 | 567 | ·0420 | 617 | ·0402 | 667 | ·0387 |
| 468 | ·0462 | 518 | ·0439 | 568 | ·0420 | 618 | ·0402 | 668 | ·0387 |
| 469 | ·0462 | 519 | ·0439 | 569 | ·0419 | 619 | ·0402 | 669 | ·0387 |
| 470 | ·0461 | 520 | ·0438 | 570 | ·0419 | 620 | ·0402 | 670 | ·0386 |
| 471 | ·0461 | 521 | ·0438 | 571 | ·0418 | 621 | ·0401 | 671 | ·0386 |
| 472 | ·0460 | 522 | ·0438 | 572 | ·0418 | 622 | ·0401 | 672 | ·0386 |
| 473 | ·0460 | 523 | ·0437 | 573 | ·0418 | 623 | ·0401 | 673 | ·0385 |
| 474 | ·0459 | 524 | ·0437 | 574 | ·0417 | 624 | ·04 | 674 | ·0385 |
| 475 | ·0459 | 525 | ·0436 | 575 | ·0417 | 625 | ·04 | 675 | ·0385 |
| 476 | ·0458 | 526 | ·0436 | 576 | ·0417 | 626 | ·0399 | 676 | ·0385 |
| 477 | ·0458 | 527 | ·0436 | 577 | ·0416 | 627 | ·0399 | 677 | ·0384 |
| 478 | ·0457 | 528 | ·0435 | 578 | ·0416 | 628 | ·0399 | 678 | ·0384 |
| 479 | ·0457 | 529 | ·0435 | 579 | ·0415 | 629 | ·0399 | 679 | ·0384 |

| Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| 680 | ·0383 | 730 | ·0370 | 780 | ·0358 | 830 | ·0347 | 880 | ·0337 |
| 681 | ·0383 | 731 | ·0370 | 781 | ·0358 | 831 | ·0347 | 881 | ·0337 |
| 682 | ·0383 | 732 | ·0370 | 782 | ·0358 | 832 | ·0347 | 882 | ·0337 |
| 683 | ·0383 | 733 | ·0369 | 783 | ·0357 | 833 | ·0346 | 883 | ·0336 |
| 684 | ·0382 | 734 | ·0369 | 784 | ·0357 | 834 | ·0346 | 884 | ·0336 |
| 685 | ·0382 | 735 | ·0369 | 785 | ·0357 | 835 | ·0346 | 885 | ·0336 |
| 686 | ·0382 | 736 | ·0369 | 786 | ·0357 | 836 | ·0346 | 886 | ·0336 |
| 687 | ·0381 | 737 | ·0368 | 787 | ·0356 | 837 | ·0346 | 887 | ·0336 |
| 688 | ·0381 | 738 | ·0368 | 788 | ·0356 | 838 | ·0345 | 888 | ·0335 |
| 689 | ·0381 | 739 | ·0368 | 789 | ·0356 | 839 | ·0345 | 889 | ·0335 |
| 690 | ·0381 | 740 | ·0368 | 790 | ·0356 | 840 | ·0345 | 890 | ·0335 |
| 691 | ·0380 | 741 | ·0367 | 791 | ·0355 | 841 | ·0345 | 891 | ·0335 |
| 692 | ·0380 | 742 | ·0367 | 792 | ·0355 | 842 | ·0345 | 892 | ·0335 |
| 693 | ·0380 | 743 | ·0367 | 793 | ·0355 | 843 | ·0344 | 893 | ·0335 |
| 694 | ·0379 | 744 | ·0367 | 794 | ·0355 | 844 | ·0344 | 894 | ·0334 |
| 695 | ·0379 | 745 | ·0366 | 795 | ·0355 | 845 | ·0344 | 895 | ·0334 |
| 696 | ·0379 | 746 | ·0366 | 796 | ·0354 | 846 | ·0344 | 896 | ·0334 |
| 697 | ·0379 | 747 | ·0366 | 797 | ·0354 | 847 | ·0344 | 897 | ·0334 |
| 698 | ·0378 | 748 | ·0366 | 798 | ·0354 | 848 | ·0343 | 898 | ·0334 |
| 699 | ·0378 | 749 | ·0365 | 799 | ·0354 | 849 | ·0343 | 899 | ·0333 |
| 700 | ·0378 | 750 | ·0365 | 800 | ·0353 | 850 | ·0343 | 900 | ·0333 |
| 701 | ·0378 | 751 | ·0365 | 801 | ·0353 | 851 | ·0343 | 905 | ·0332 |
| 702 | ·0377 | 752 | ·0365 | 802 | ·0353 | 852 | ·0342 | 910 | ·0331 |
| 703 | ·0377 | 753 | ·0364 | 803 | ·0353 | 853 | ·0342 | 915 | ·0330 |
| 704 | ·0377 | 754 | ·0364 | 804 | ·0353 | 854 | ·0342 | 920 | ·0329 |
| 705 | ·0376 | 755 | ·0364 | 805 | ·0352 | 855 | ·0342 | 925 | ·0329 |
| 706 | ·0376 | 756 | ·0364 | 806 | ·0352 | 856 | ·0342 | 930 | ·0328 |
| 707 | ·0376 | 757 | ·0363 | 807 | ·0352 | 857 | ·0341 | 935 | ·0327 |
| 708 | ·0376 | 758 | ·0363 | 808 | ·0352 | 858 | ·0341 | 940 | ·0326 |
| 709 | ·0375 | 759 | ·0363 | 809 | ·0351 | 859 | ·0341 | 945 | ·0325 |
| 710 | ·0375 | 760 | ·0362 | 810 | ·0351 | 860 | ·0341 | 950 | ·0324 |
| 711 | ·0375 | 761 | ·0362 | 811 | ·0351 | 861 | ·0341 | 955 | ·0323 |
| 712 | ·0375 | 762 | ·0362 | 812 | ·0351 | 862 | ·0341 | 960 | ·0323 |
| 713 | ·0374 | 763 | ·0362 | 813 | ·0351 | 863 | ·0340 | 965 | ·0322 |
| 714 | ·0374 | 764 | ·0362 | 814 | ·0350 | 864 | ·0340 | 970 | ·0321 |
| 715 | ·0374 | 765 | ·0361 | 815 | ·0350 | 865 | ·0340 | 975 | ·0320 |
| 716 | ·0374 | 766 | ·0361 | 816 | ·0350 | 866 | ·0340 | 980 | ·0319 |
| 717 | ·0373 | 767 | ·0361 | 817 | ·0350 | 867 | ·0340 | 985 | ·0319 |
| 718 | ·0373 | 768 | ·0361 | 818 | ·0350 | 868 | ·0339 | 990 | ·0318 |
| 719 | ·0373 | 769 | ·0361 | 819 | ·0349 | 869 | ·0339 | 995 | ·0317 |
| 720 | ·0372 | 770 | ·0360 | 820 | ·0349 | 870 | ·0339 | 1000 | ·0316 |
| 721 | ·0372 | 771 | ·0360 | 821 | ·0349 | 871 | ·0339 | 1010 | ·0315 |
| 722 | ·0372 | 772 | ·0360 | 822 | ·0349 | 872 | ·0339 | 1020 | ·0313 |
| 723 | ·0372 | 773 | ·0360 | 823 | ·0348 | 873 | ·0339 | 1030 | ·0311 |
| 724 | ·0372 | 774 | ·0359 | 824 | ·0348 | 874 | ·0338 | 1040 | ·0310 |
| 725 | ·0371 | 775 | ·0359 | 825 | ·0348 | 875 | ·0338 | 1050 | ·0309 |
| 726 | ·0371 | 776 | ·0359 | 826 | ·0348 | 876 | ·0338 | 1060 | ·0307 |
| 727 | ·0371 | 777 | ·0359 | 827 | ·0348 | 877 | ·0338 | 1070 | ·0306 |
| 728 | ·0371 | 778 | ·0358 | 828 | ·0347 | 878 | ·0337 | 1080 | ·0304 |
| 729 | ·0370 | 779 | ·0358 | 829 | ·0347 | 879 | ·0337 | 1090 | ·0303 |

| Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ | Slope = 1 in. | $\sqrt{S}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1100 | ·0301 | 1240 | ·0284 | 1370 | ·0270 | 1500 | ·0258 | 1760 | ·0238 |
| 1110 | ·0300 | 1250 | ·0283 | 1380 | ·0269 | 1520 | ·0256 | 1780 | ·0237 |
| 1120 | ·0300 | 1260 | ·0282 | 1390 | ·0268 | 1540 | ·0255 | 1800 | ·0236 |
| 1130 | ·0297 | 1270 | ·0281 | 1400 | ·0267 | 1560 | ·0253 | 1820 | ·0234 |
| 1140 | ·0296 | 1280 | ·0279 | 1410 | ·0266 | 1580 | ·0251 | 1840 | ·0233 |
| 1150 | ·0295 | 1290 | ·0278 | 1420 | ·0265 | 1600 | ·025 | 1860 | ·0232 |
| 1160 | ·0294 | 1300 | ·0277 | 1430 | ·0264 | 1620 | ·0248 | 1880 | ·0231 |
| 1170 | ·0292 | 1310 | ·0276 | 1440 | ·0263 | 1640 | ·0247 | 1900 | ·0230 |
| 1180 | ·0291 | 1320 | ·0275 | 1450 | ·0263 | 1660 | ·0245 | 1920 | ·0228 |
| 1190 | ·0290 | 1330 | ·0274 | 1460 | ·0262 | 1680 | ·0244 | 1940 | ·0227 |
| 1200 | ·0289 | 1340 | ·0273 | 1470 | ·0261 | 1700 | ·0242 | 1960 | ·0226 |
| 1210 | ·0287 | 1350 | ·0272 | 1480 | ·0260 | 1720 | ·0241 | 1980 | ·0225 |
| 1220 | ·0286 | 1360 | ·0271 | 1490 | ·0259 | 1740 | ·0240 | 2000 | ·0224 |
| 1230 | ·0285 | | | | | | | | |

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