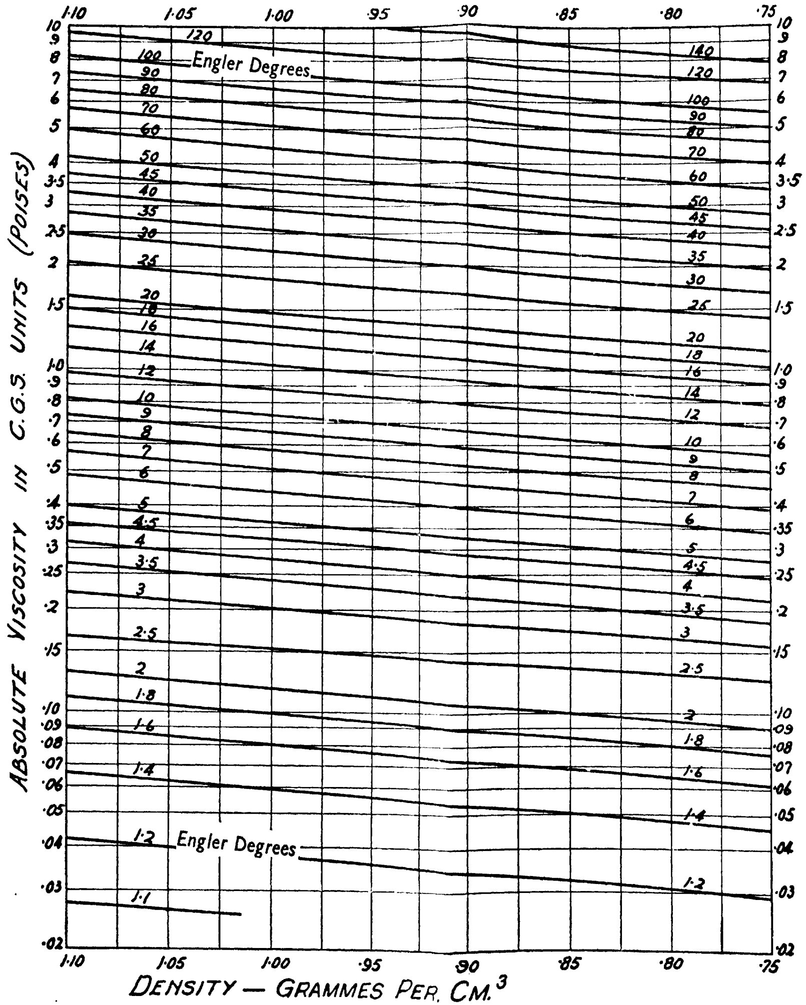
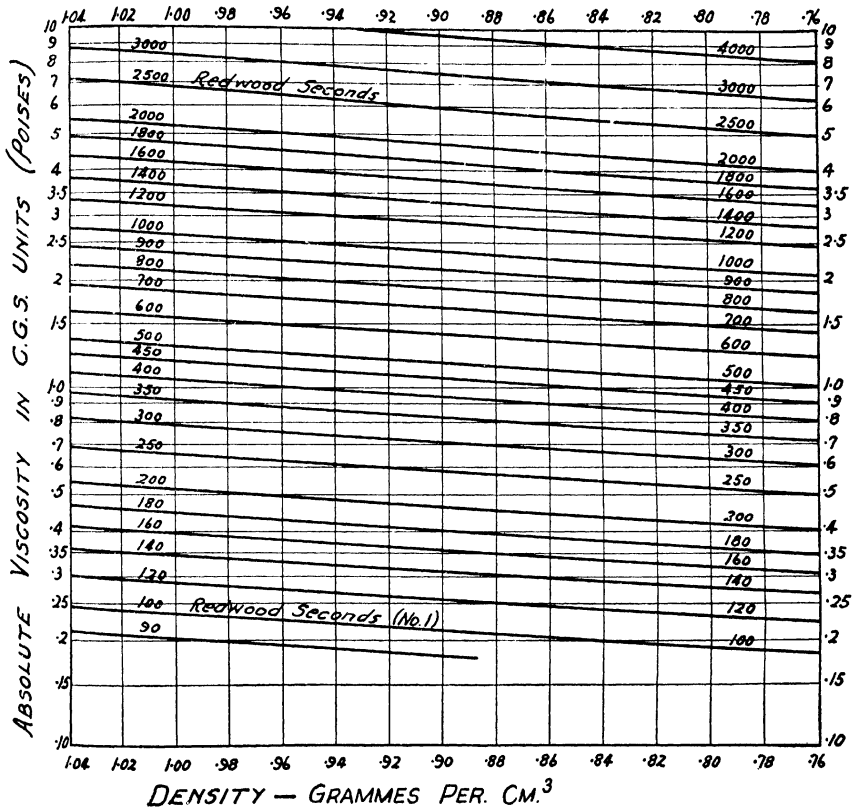
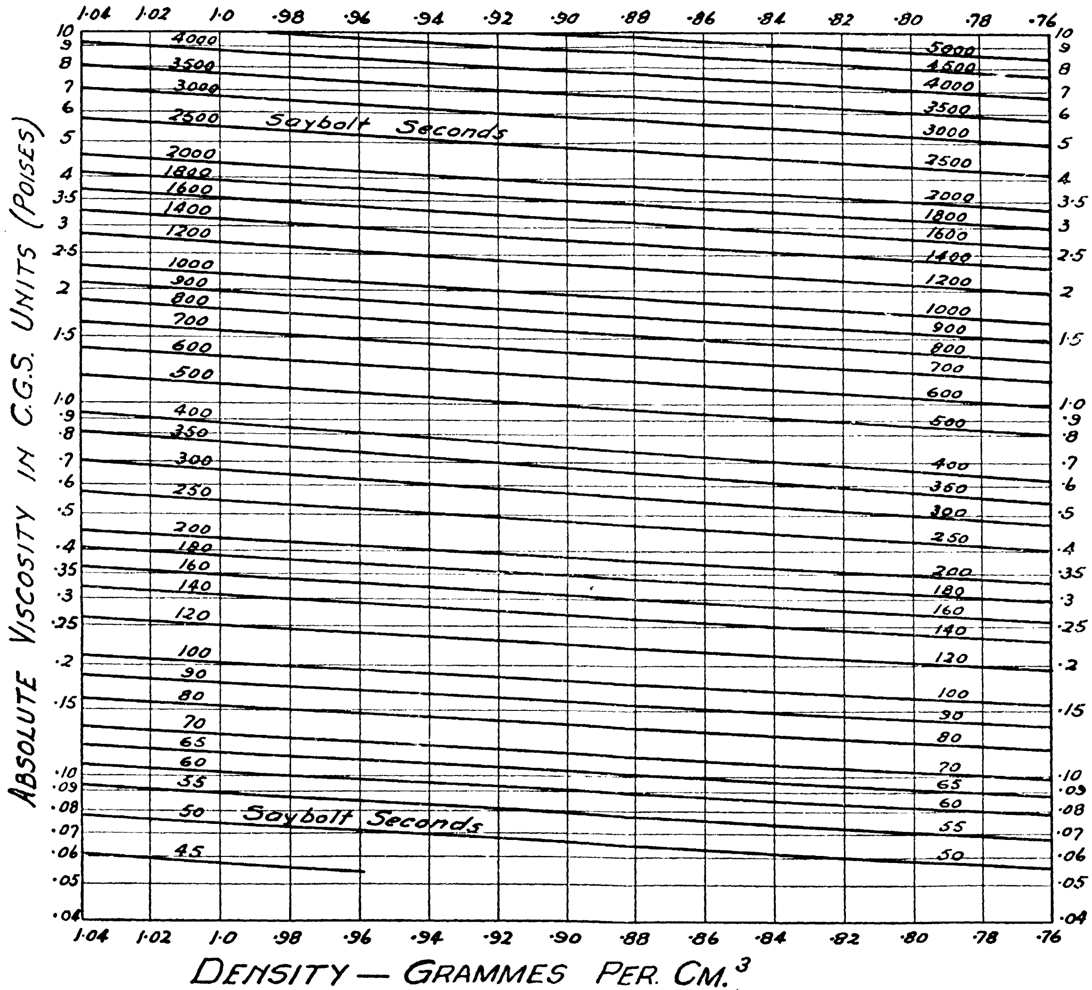


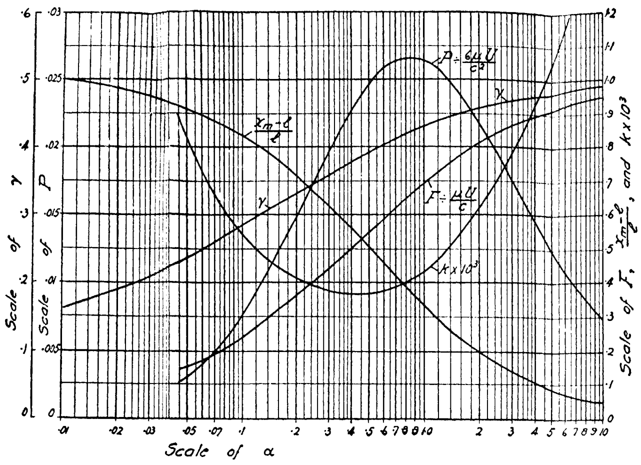
Folder III, *a*.—Diagram for Engler viscometer. For explanation, see Sect. III, 2, pp. 40, 43.



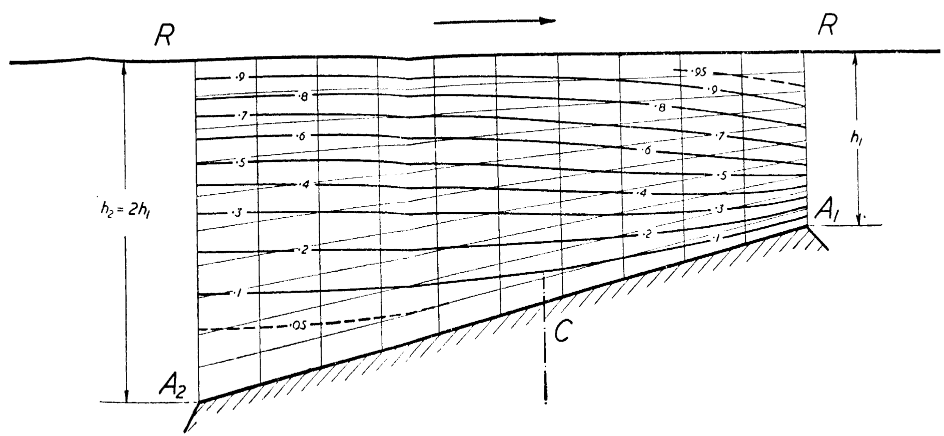
Folder III, *b*.—Diagram for Redwood viscometer. For explanation, see pp. 41, 43.



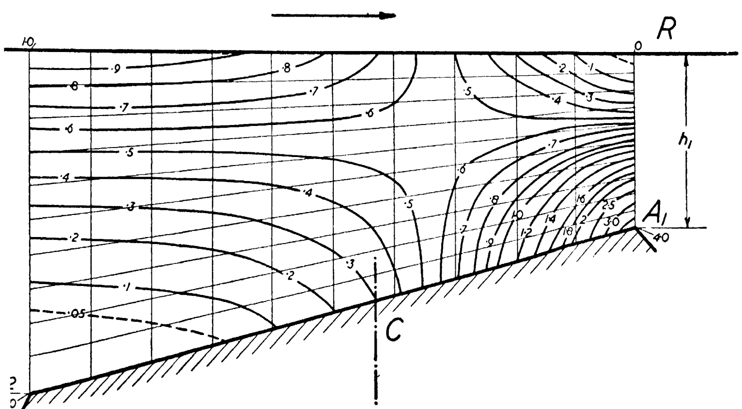
Folder III, c. — Diagram for Saybolt viscometer. For explanation, see pp. 42, 43.



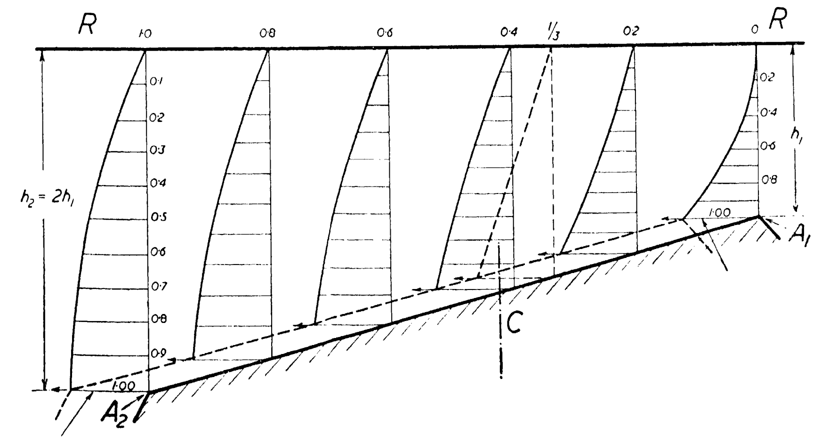
Folder IV, A. — Characteristics of pad of unlimited width. See Sect. IV, 5. p. 81



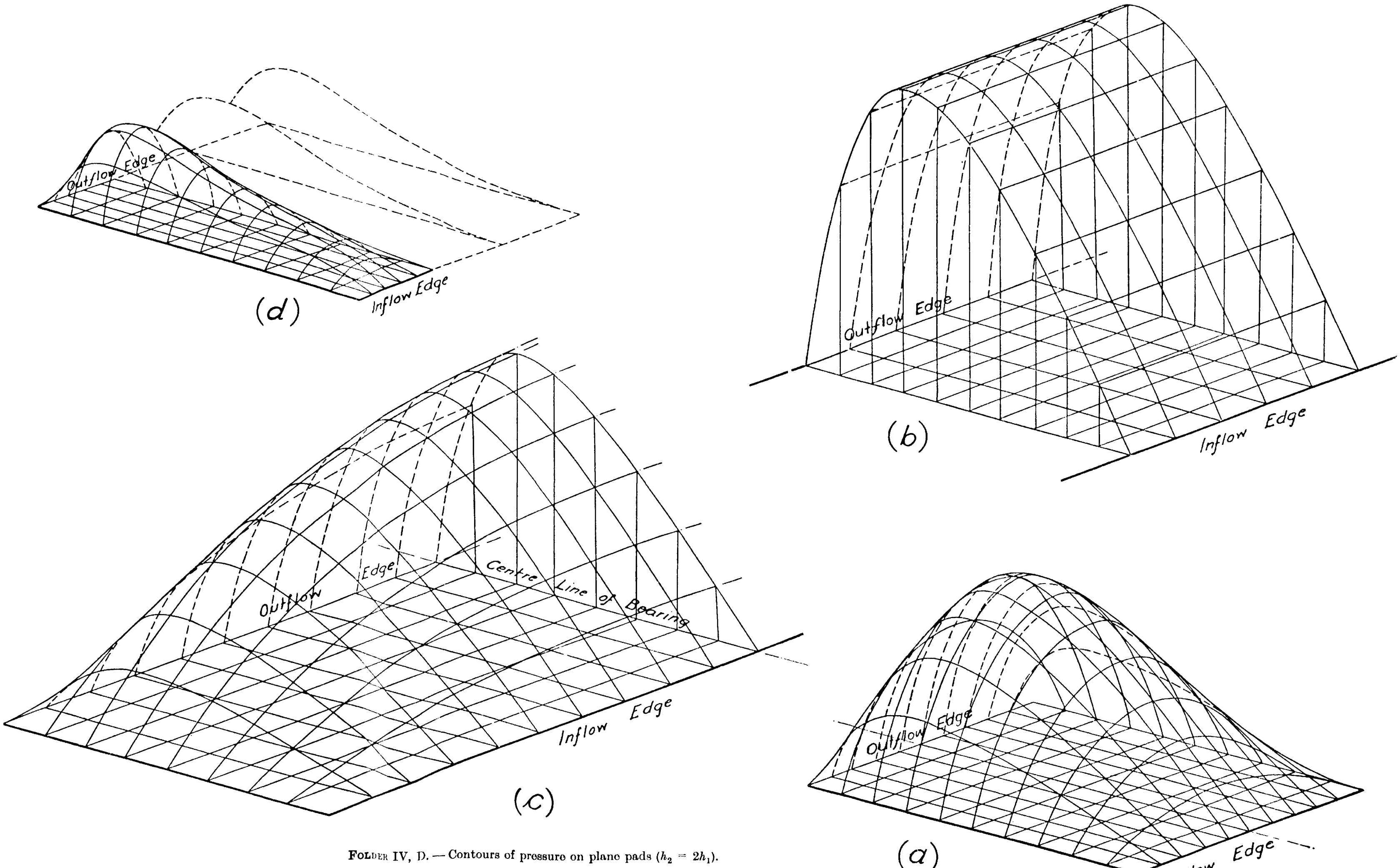
Folder IV, B. — Flow of lubricant between pad of unlimited width ($h_2 = 2h_1$) and runner: Lines of equal velocity relative to pad. See p. 85.



IV, C. — Rates of generation of heat in lubricant between pad of unlimited width ($h_2 = 2h_1$) and runner. See p. 86.

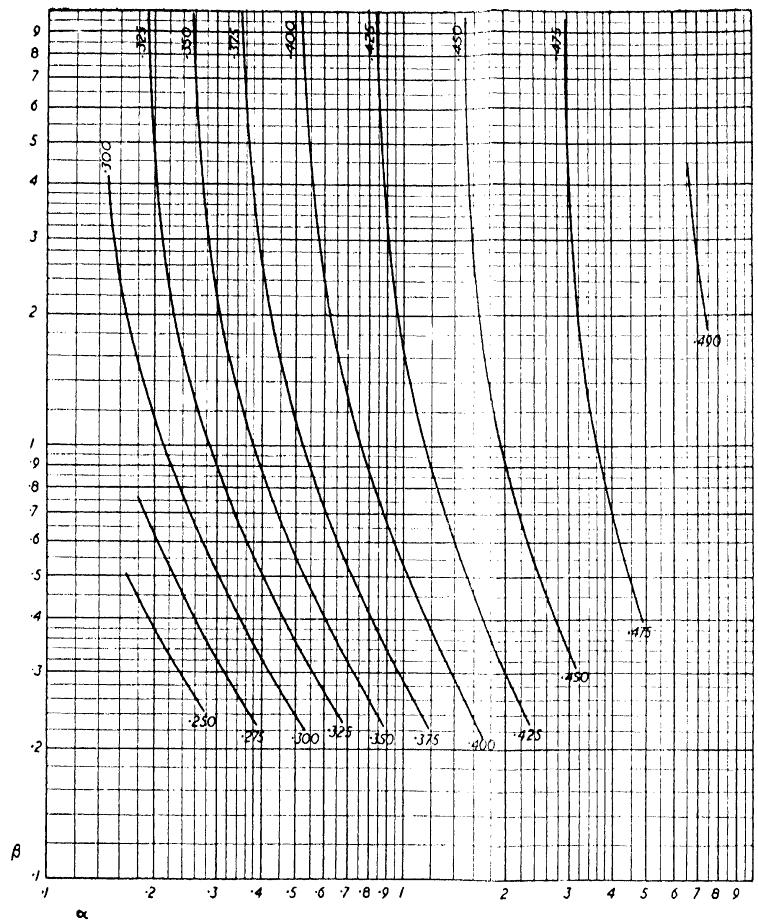


Folder IV, B′. — Flow of lubricant between pad of unlimited width ($h_2 = 2h_1$) and runner: Horizontal lines show velocities relative to runner. See p. 85.

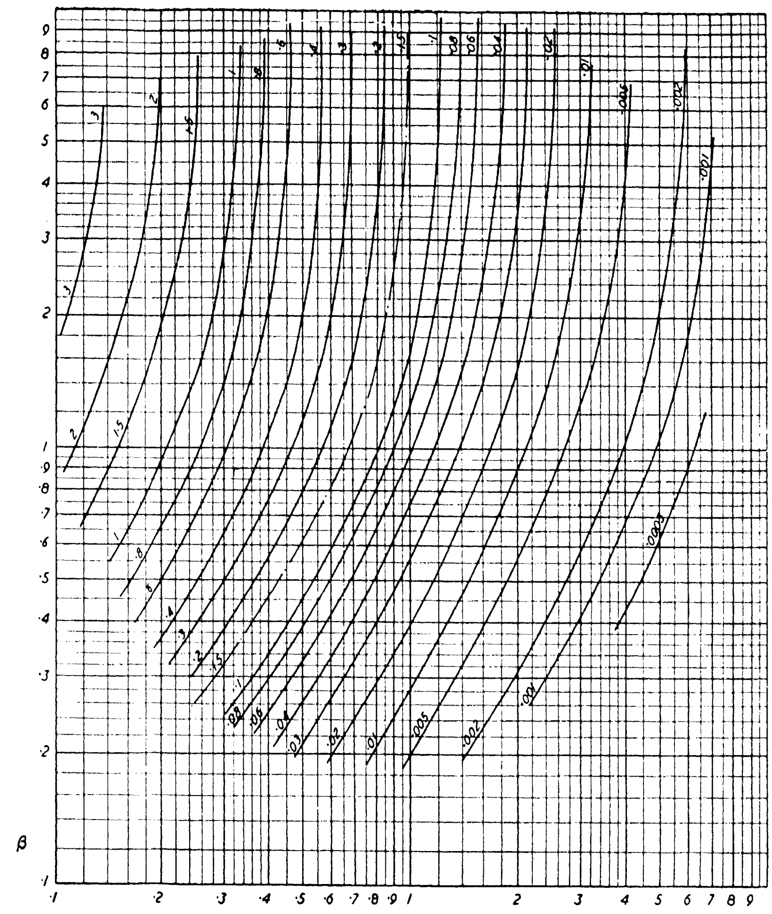


FOLDER IV, D. — Contours of pressure on plane pads ($h_2 = 2h_1$).

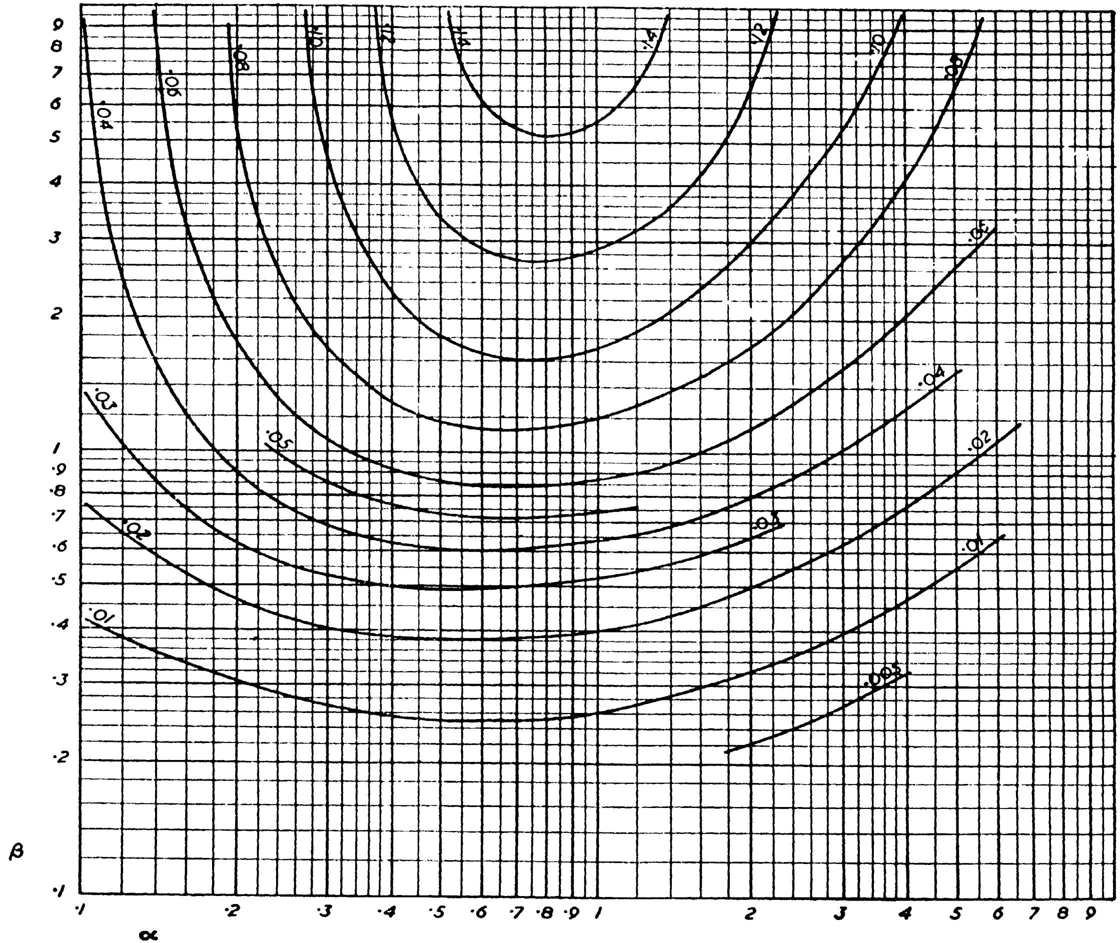
(a) Square pad. (b) Portion $b = l$ of pad of unlimited width. (c) Pad of width 4 times length. (d) Pad of width $\frac{1}{5}$ of length. See p. 98.



FOLDER IV, E. — Positions of centres of pressure of plane pads, having varying values of $\beta = b/l$ and of $\alpha = a/l$. See pp. 99–100.



FOLDER IV, F. — Charges per unit width of plane pads, having varying values of $\beta = b/l$ and of $\alpha = a_1/l$ (the diagram shows the numerical coefficients of $\mu U/c^2$, c being uniform throughout the series). See Sect. IV, 13, pp. 99–100.



Folder IV, G. — Charges per unit width of plane pads, having varying values of $\beta = b/l$ and of $\alpha = a_1/l$ (the diagram shows the numerical coefficients of $\mu U/c_2$, h_1 being uniform throughout the series). See Sect. IV, 13, pp. 99–101.

FOLDING DIAGRAMS

The diagrams hinged to the front and back covers fold outwards clear of the text pages. By this means each diagram can be studied in conjunction with the text section to which it primarily refers, or equally easily in conjunction with any other text section. Roman numerals indicate **chapter** to which each folder refers.

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LUBRICATION

Its Principles and Practice

BY

A. G. M. MICHELL
M.C.E.(Melb.), F.R.S.

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"Theory is the captain, practice the soldiers."

—*Leonardo da Vinci.*

PREFACE

The primary aim of this book is to assist practice.

It is intended for the use of both the practising engineer and the student. I have endeavoured, as far as may be possible within its comparatively small compass, to provide not only a general account of the subject of Lubrication, but also a collection of such essential data and factual information relating to the subject as is needed from day to day by the practician (whether operating engineer or designer), and which may not be readily found elsewhere in a single volume.

The criterion which has decided the inclusion or omission of any particular matter has been its practical utility, and this test has been applied both to the material used in the book itself, and to most of the wider range of subject matter which I have endeavoured to make available to the reader by means of the references. It is to be understood in this connexion, however, that I include, as being of "practical utility", not only matter immediately applicable to the current needs of the practician, but also whatever leads directly to improvement and advance in the art of lubrication. It is hoped that the relatively small amount of original matter, hitherto unpublished, which appears in the book will be found to answer to this description. Such matter is contained mainly in certain sections of Chapters IV, VI, VII and VIII, and in Appendices I and III.

While only so much of theory has been given in the text as is immediately applicable to practice, I have sought to provide in it a sufficient basis and justification for all the methods and constructions which are described or mentioned as being representative of sound practice.

It will be seen that I have not attempted, except incidentally and in single instances, to deal with the means of lubrication employed in automobiles and aircraft, these being the products of specialist firms who supply detailed directions for the operation and maintenance of their machines along with the machines themselves.

The preparation of a large part of the book has been made practicable only by the friendly assistance and co-operation of a considerable number of prominent firms who have made available to the author for his information and study, and for inclusion in the book at his option, a mass of material describing their machines and methods. I have endeavoured to make due acknowledgment of this valued help in each passage in which such material has been specifically used; but I desire to take this opportunity of expressing my thanks to all who have proffered this kind of assistance, whether or not I have found myself able to make explicit use of it.

Other matter—theoretical, descriptive, and of the nature of technical data—which I have consciously borrowed, is acknowledged by means of the references.

My thanks are due to my friend, Miss Winifred Waddell, for checking the manuscript of the mathematics of Appendix I, and to my friend and associate, Mr. A. J. Seggel, for extending to me throughout this undertaking the aid and support which I have owed to him during almost the whole of my professional life.

To the publishers and printers of the volume I offer cordial acknowledgment of their care and skill in its production, and their consideration for the wishes of the author. Their criticism, advice and suggestions have often been invaluable.

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TABLE OF EQUIVALENT UNITS

The Table on pp. xv–xvii gives the comparative magnitudes of the chief units used for the measurement of the physical quantities which appear in the present volume. In the text all important numerical results have been stated both in the engineering units current in English-speaking countries and in metric units; the Table will enable data obtained from other sources to be converted into whichever of these systems the reader prefers.

A few lines of explanation may facilitate the use of the Table.

Each of the numbers given in the body of the Table is the factor by which the unit defined above it has to be multiplied in order to be equal in magnitude to the unit which has the factor unity in the same horizontal line. Thus the centimetre must be increased in length 2·54 times to be equal in length to an inch, and the inch multiplied by the factor $3{\cdot}937 \times 10^{-1}$ equals a centimetre.

The numerical expressions for any given length, or other physical quantity, in terms of any two alternative units, being in inverse proportion to the magnitudes of those units, are therefore in *direct* proportion to the factors attached to them in a horizontal line of the Table. Thus a length which measures **10** on an inch scale measures $10 \times 2{\cdot}54 =$ **25·4** on a centimetre scale.

Similarly (from Section 15 of the Table), a certain source of power may be alternatively rated at **1** horse-power, **0·7067** B.Th.U. per second, or **178·1** gramme-calories per second.

In addition to " absolute " units based on the three primary metric units (gramme, centimetre and second), and on the corresponding units of the Anglo-Saxon " absolute " systems (pound, foot or inch, and second or minute), factors are also given in the Table for numerous units in common use which are not derivable from either of these " absolute " systems. Such are, for example, the pound weight, the horse-power and the calorie. These are non-absolute either because their magnitudes depend on the acceleration of gravity at the locality and altitude where the measurement is made, or because they involve the properties of particular kinds of matter, examples of the latter class being the B.Th.U., calorie and watt. In the Table the acceleration due to gravity is taken as 980·665 in centimetre-second units (being its value at latitude 45° N. or S.), according to the compromise value accepted by international agreement. (The equivalent value in foot-second units is 32·174, and in inch-second units 386·088.)

The factors given in the Table for interconversions between all absolute units may be taken as being correct to the order of 1 part in 100,000. To this order the accepted British and American primary units are identical. A much lower degree of accuracy, of course, suffices for most engineering calculations, and the tabular factors can be advantageously abbreviated according to the nature of each application. The factors for non-absolute units (pounds weight, gallons, etc.) and their derivatives, have an order of accuracy of about one part in a thousand, at least.

Throughout the book the rule laid down for scientific and technical publications by the Scientific Congress convened by the Royal Society in June, 1946, has been followed, all numerical results of importance being given in C.G.S. metric units, as well as in the units used in the papers from which the results are derived, and commonly employed by engineers. In the case of coefficients of viscosity, in accordance with the resolutions of the World Petroleum Congress held in 1933, no unit other than the C.G.S. unit, the *poise*, or its decimal derivative, the *centipoise*, has been employed. Factors are, however, given in the Table, which, together with formulæ in Chapter III, will enable measurements stated in other units to be referred to the poise.

TABLE OF EQUIVALENT UNITS

Abbreviations: cm.—centimetre; kg.—kilogramme; sec.—second; cal.—calorie (centigrade); B.Th.U.—British Thermal Unit; wt.—weight.

UNITS OF	UNITS AND FACTORS			
1. Mass.	Gramme	Pound		
	1	$2{\cdot}20462 \times 10^{-3}$		
M	$4{\cdot}53592 \times 10^{2}$	1		
2. Length.	Cm.	Inch	Foot	
	1	$3{\cdot}937 \times 10^{-1}$	$3{\cdot}28084 \times 10^{-2}$	
	2·54	1	$8{\cdot}3333 \times 10^{-2}$	
L	$3{\cdot}048 \times 10$	$1{\cdot}2 \times 10$	1	
3. Time.	Sec.	Minute	Hour	
	1	$1{\cdot}66667 \times 10^{-2}$	$2{\cdot}77778 \times 10^{-4}$	
	$6{\cdot}0 \times 10$	1	$1{\cdot}66667 \times 10^{-2}$	
T	$3{\cdot}6 \times 10^{3}$	$6{\cdot}0 \times 10$	1	
4. Area.	Square cm.	Square inch	Square foot	
	1	$1{\cdot}5500 \times 10^{-1}$	$1{\cdot}07639 \times 10^{-3}$	
	6·4516	1	$6{\cdot}9444 \times 10^{-3}$	
L^2	$9{\cdot}2903 \times 10^{2}$	$1{\cdot}44 \times 10^{2}$	1	
5. Volume.	Cubic cm.	Cubic inch	Cubic foot	
	1	$6{\cdot}1024 \times 10^{-2}$	$3{\cdot}53146 \times 10^{-5}$	
	$1{\cdot}63871 \times 10$	1	$5{\cdot}7870 \times 10^{-4}$	
L^3	$2{\cdot}83169 \times 10^{4}$	$1{\cdot}728 \times 10^{3}$	1	
5*a*. Liquid Measures of Volume.	Cubic cm.	Imperial gallon	U.S.A. "fluid gallon"	Litre = (1×10^{3} millilitres)
	1	$2{\cdot}19970 \times 10^{-4}$	$2{\cdot}6416 \times 10^{-4}$	$9{\cdot}9997 \times 10^{-4}$
	$4{\cdot}54609 \times 10^{3}$	1	1·20095	4·54596
	$3{\cdot}78542 \times 10^{3}$	$8{\cdot}3267 \times 10^{-1}$	1	3·78530
L^3	$1{\cdot}00003 \times 10^{3}$	$2{\cdot}19975 \times 10^{-1}$	$2{\cdot}64165 \times 10^{-1}$	1
6. Moment of a Plane Area.	$Cm.^4$	$Inch^4$	$Foot^4$	
	1	$2{\cdot}40251 \times 10^{-2}$	$1{\cdot}15861 \times 10^{-6}$	
	$4{\cdot}16232 \times 10$	1	$4{\cdot}8225 \times 10^{-5}$	
L^4	$8{\cdot}6310 \times 10^{5}$	$2{\cdot}0736 \times 10^{4}$	1	
7. Moment of a Volume.	$Cm.^5$	$Inch^5$	$Foot^5$	
	1	$9{\cdot}4587 \times 10^{-3}$	$3{\cdot}80122 \times 10^{-8}$	
	$1{\cdot}05723 \times 10^{2}$	1	$4{\cdot}01875 \times 10^{-6}$	
L^5	$2{\cdot}63073 \times 10^{7}$	$2{\cdot}48832 \times 10^{5}$	1	

[*Contd.*

UNITS OF	UNITS AND FACTORS			
8. Density.	Gramme per cubic cm.	Pound per cubic inch	Pound per cubic foot	
	1	$3{\cdot}61273 \times 10^{-2}$	$6{\cdot}2428 \times 10$	
	$2{\cdot}76799 \times 10$	1	$1{\cdot}728 \times 10^{3}$	
ML^{-3}	$1{\cdot}60185 \times 10^{-2}$	$5{\cdot}7870 \times 10^{-4}$	1	
9. Angular Velocity.	Radian per sec.	Revolution per sec.	Revolution per minute	
	1	$1{\cdot}59154 \times 10^{-1}$	$9{\cdot}5492$	
	$6{\cdot}2832$	1	$6{\cdot}0 \times 10$	
T^{-1}	$1{\cdot}0472 \times 10^{-1}$	$1{\cdot}66667 \times 10^{-2}$	1	
10. Linear Velocity.	Cm. per sec.	Inch per sec.	Foot per sec.	Foot per min.
	1	$3{\cdot}937 \times 10^{-1}$	$3{\cdot}28084 \times 10^{-2}$	$1{\cdot}9685$
	$2{\cdot}54$	1	$8{\cdot}3333 \times 10^{-2}$	$5{\cdot}0$
	$3{\cdot}048 \times 10$	$1{\cdot}2 \times 10$	1	$6{\cdot}0 \times 10$
LT^{-1}	$5{\cdot}08 \times 10^{-1}$	$2{\cdot}0 \times 10^{-1}$	$1{\cdot}66667 \times 10^{-2}$	1
11. Force.	Dyne (gramme, cm., sec.$^{-2}$)	Pound, inch, sec.$^{-2}$	Poundal (pound, foot, sec.$^{-2}$)	
	1	$8{\cdot}6796 \times 10^{-4}$	$7{\cdot}2330 \times 10^{-5}$	
	$1{\cdot}15212 \times 10^{3}$	1	$8{\cdot}3333 \times 10^{-2}$	
MLT^{-2}	$1{\cdot}38255 \times 10^{4}$	$1{\cdot}2 \times 10$	1	
11*a*. Force. Weight.	Dyne	Pound wt. at Latitude 45°	Kg. wt. at Latitude 45°	
	1	$2{\cdot}24809 \times 10^{-6}$	$1{\cdot}01972 \times 10^{-6}$	
	$4{\cdot}44822 \times 10^{5}$	1	$4{\cdot}53592 \times 10^{-1}$	
MLT^{-2}	$9{\cdot}80665 \times 10^{5}$	$2{\cdot}20462$	1	
12. Force (or wt.) per Unit of Length.	Dyne per cm. (gramme, sec.$^{-2}$)	Poundal per ft. (pound, sec.$^{-2}$)	Pound wt. per foot	Kg. wt. per cm.
	1	$2{\cdot}20462 \times 10^{-3}$	$6{\cdot}85218 \times 10^{-5}$	$1{\cdot}01972 \times 10^{-6}$
	$4{\cdot}53592 \times 10^{2}$	1	$3{\cdot}10810 \times 10^{-2}$	$4{\cdot}62541 \times 10^{-4}$
Surface-Tension.	$1{\cdot}45939 \times 10^{4}$	$3{\cdot}2174 \times 10$	1	$1{\cdot}48818 \times 10^{-2}$
MT^{-2}.	$9{\cdot}80665 \times 10^{5}$	$2{\cdot}16199 \times 10^{3}$	$6{\cdot}71968 \times 10$	1
13. Pressure. Stress.	Dyne per square cm.	Pound wt. per square inch	Pound wt. per square foot	Metric Atmosphere (Kg. wt. per sq. cm.)
(Force or wt.	1	$1{\cdot}45038 \times 10^{-5}$	$2{\cdot}08855 \times 10^{-3}$	$1{\cdot}01972 \times 10^{-6}$
per unit	$6{\cdot}89474 \times 10^{4}$	1	$1{\cdot}44 \times 10^{2}$	$7{\cdot}03071 \times 10^{-2}$
area.)	$4{\cdot}78802 \times 10^{2}$	$6{\cdot}94444 \times 10^{-3}$	1	$4{\cdot}88243 \times 10^{-4}$
$ML^{-1}T^{-2}$	$9{\cdot}80665 \times 10^{5}$	$1{\cdot}42233 \times 10$	$2{\cdot}04816 \times 10^{3}$	1
14. Work. Moment of Force.	Erg (= Dyne cm.)	Pound wt. inch	Pound wt. foot	Kg. wt. cm.
	1	$8{\cdot}85073 \times 10^{-7}$	$7{\cdot}37561 \times 10^{-8}$	$1{\cdot}01972 \times 10^{-6}$
	$1{\cdot}12985 \times 10^{6}$	1	$8{\cdot}3333 \times 10^{-2}$	$1{\cdot}15213$
	$1{\cdot}35582 \times 10^{7}$	$1{\cdot}2 \times 10$	1	$1{\cdot}38255 \times 10$
$ML^{2}T^{-2}$.	$9{\cdot}80665 \times 10^{5}$	$8{\cdot}6796 \times 10^{-1}$	$7{\cdot}2330 \times 10^{-2}$	1

UNITS OF	UNITS AND FACTORS			
14a. Work. Energy. Heat.	Joule (= 1 watt sec. = 1·0005 × 10⁷ × erg)	Horse-power hour	B.Th.U.	Gramme cal.
	1	$3{\cdot}727 \times 10^{-7}$	$9{\cdot}485 \times 10^{-4}$	$2{\cdot}3900 \times 10^{-1}$
	$2{\cdot}683 \times 10^{6}$	1	$2{\cdot}545 \times 10^{3}$	$6{\cdot}413 \times 10^{5}$
	$1{\cdot}0544 \times 10^{3}$	$3{\cdot}930 \times 10^{-4}$	1	$2{\cdot}51996 \times 10^{2}$
ML^2T^{-2}.	4·184	$1{\cdot}5595 \times 10^{-6}$	$3{\cdot}96830 \times 10^{-3}$	1
15. Rate of Work. Power.	Watt (= 1·0005 × 10⁷ × erg sec.⁻¹)	Horse-power	B.Th.U. per sec.	Gramme cal. per sec.
	1	$1{\cdot}342 \times 10^{-3}$	$9{\cdot}485 \times 10^{-4}$	$2{\cdot}390 \times 10^{-1}$
	$7{\cdot}455 \times 10^{2}$	1	$7{\cdot}067 \times 10^{-1}$	$1{\cdot}781 \times 10^{2}$
	$1{\cdot}0544 \times 10^{3}$	1·415	1	$2{\cdot}51996 \times 10^{2}$
ML^2T^{-3}	4·184	$5{\cdot}6148 \times 10^{-3}$	$3{\cdot}9683 \times 10^{-3}$	1
16. Viscosity.	Poise (Gramme cm.⁻¹ sec.⁻¹)	Centipoise	Pound, inch⁻¹ sec.⁻¹	Pound, foot⁻¹ sec.⁻¹
	1	$1{\cdot}0 \times 10^{2}$	$5{\cdot}599 \times 10^{-3}$	$6{\cdot}719 \times 10^{-2}$
	$1{\cdot}0 \times 10^{-2}$	1	$5{\cdot}599 \times 10^{-5}$	$6{\cdot}719 \times 10^{-4}$
	$1{\cdot}786 \times 10^{2}$	$1{\cdot}786 \times 10^{4}$	1	$1{\cdot}2 \times 10$
$ML^{-1}T^{-1}$	$1{\cdot}488 \times 10$	$1{\cdot}488 \times 10^{3}$	$8{\cdot}333 \times 10^{-2}$	1

GREEK ALPHABET

Α	α	alpha	Ν	ν	nu
Β	β	beta	Ξ	ξ	ksi
Γ	γ	gamma	Ο	ο	omicron
Δ	δ	delta	Π	π, ϖ	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ, ς	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	Υ	υ	upsilon
Ι	ι	iota	Φ	φ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

LIST OF ALPHABETICAL SYMBOLS

(Column headed " Section " gives number of the Section of Text in which the symbol is introduced or defined; the column " Dimensions ", the magnitude of the unit of measurement in terms of the units of mass, length and time. For the mode of numbering the Sections, see the Note facing p. 1.)

Symbol	Signification	Section in which defined	Dimensions
A	(and other Roman capitals) designate objects and points in figures.	II, 2	—
$\mathfrak{A}$	An area.	II, 4	L^2
a	A linear dimension; radius, regarded as constant.	II, 13	L
a	A numerical constant.	III, 2	—
a_1, a_2	Linear dimensions of a bearing pad in the direction of its length.	IV, 3	L
$\bar{a}$	x co-ordinate of a centre of pressure.	IV, 3	L
b	A linear dimension, usually a width.	II, 7	L
b	A numerical constant.	II, 10	—
C, C'	Constants of integration.	II, 6	—
°C.	Temperature Centigrade.		—
c	Small inclination between plane bearing surfaces.	IV, 3	—
D	A diameter.	III, 11	L
d	Diagonal length of a quadrilateral figure.	V, 3	L
E	Electric potential.	A, II	—
E	Young's modulus of elasticity.	V, 3	$ML^{-1}T^{-2}$
e	Base of Naperian (" natural ") logarithms.		—
F, F'	A tangential force on an area.	II, 4	MLT^{-2}, MLT^{-2}
f	Tangential force per unit area.	II, 2	$ML^{-1}T^{-2}$
f	Shearing stress.	II, 5	$ML^{-1}T^{-2}$
G	Moment of forces (couple).	IV, 9	ML^2T^{-2}
g	Acceleration due to gravity.	II, 12	LT^{-2}
H	Linear dimension in direction of z; thickness.	IV, 15	L
h	Linear dimension in direction of z; thickness.	II, 14	L
h_m	Thickness of oil film at point of max. pressure.	VI, 2	L
i	Numerical constant; hydraulic slope.	V, 7; X, 1	—
I	Moment of inertia of an area.	V, 3	L^4
I	Electric current.	A, II	—

Symbol	Signification	Section in which defined	Dimensions
I_m	Moment of inertia of a volume.	VII, 7	L^5
I_1	Bessel function.	IV, 12	—
J	Joule's mechanical equivalent of heat.	IV, 8	—
j	Thickness of a capillary film.	VII, 10	L
K	A numerical constant.	III, 8	—
K_1	Bessel function.	IV, 12	—
k	A numerical ratio.	III, 11	—
k	Coefficient of resistance.	VI, 2	—
L	Magnitude of unit of length.	II, 2	L
L	Length (of a rolling element, etc.).	VII, 8	L
l	A length, usually constant.	II, 5	L
M	Magnitude of unit of mass.	II, 2	M
M	A tractional couple.	VI, 2	ML^2T^{-2}
$\mathfrak{M}$	Mass (of a ball).	VII, 8	M
m	Density, i.e. mass per unit volume.	II, 12	ML^{-3}
m	Generalized numerical coefficient.	IV, 12	—
N	Poisson's ratio.	V, 3	—
N	An arbitrary number.	VI, 9	—
n	A numerical constant.	II, 10	—
n	Distance normal to a surface.	II, 9	L
P	Total pressure on a unit width (or length).	IV, 3	MT^{-2}
$\bar{P}$	Total force on an area.	II, 17	MLT^{-2}
p	Fluid pressure per unit area.	II, 6	$ML^{-1}T^{-2}$
p_0, p_1	Fluid pressure per unit area.	II, 7	$ML^{-1}T^{-2}$
p_m	mth term in an expanded expression for p.	IV, 12	—
p_e	An intensity of pressure between solids.	VIII, 3	$ML^{-1}T^{-2}$
Q	Rate of volume flow.	II, 7	L^3T^{-1}
Q_x	Rate of volume flow, in direction of subscript.	II, 14	L^3T^{-1}
$\mathfrak{Q}$	A volume of liquid.	IX, 2	L^3
q	A numerical index.	V, 7	—
R	A radial length, usually constant.		L
R	A thermodynamic constant.	II, 13	L^2T^{-2}
R_a	Axial load in a thrust bearing.	VII, 7	MLT^{-2}
R_r	Radial load on a bearing.	VII, 8	MLT^{-2}
r	A radial length, usually variable.	II, 12	L
r	Electric resistivity.	A, II	—
r_a	Radius of a ball or roller.	VII, 7	L
r_m	Radial distance from the axis of a bearing to the centre of a ball or roller.	VII, 7	L
r_m	Corresponding distance to point of maximum pressure on a pad.	A, II	L
Sc	Surface stress.	VIII, 1	—

[Contd

Symbol	Signification	Section in which defined	Dimensions
s	A shearing stress.	II, 13	$ML^{-1}T^{-2}$
s	A length measured along a curve.	II, 10	L
s_{yz}	A shearing stress in direction defined by subscript.	II, 13	$ML^{-1}T^{-2}$
T	Magnitude of unit of time.	II, 2	T
T	Temperature, in deg. C., unless noted otherwise.	II, 10	—
T_a	Absolute temperature by centigrade scale.	II, 13	—
T_w	Time of discharge of viscometer.	III, 2	T
t	A period of time.	II, 5	T
t	A linear dimension (thickness).	III, 11	L
t	Clearance space of piston-rings.	VIII, 3	L
U	A velocity.	II, 2	LT^{-1}
U'	A resultant fluid velocity.	II, 14	LT^{-1}
u, v, w	Components of velocity in the respective directions of co-ordinates x, y, and z.	II, 8	LT^{-1}
V	Velocity (in the direction of y co-ordinate).	IV, 2	LT^{-1}
v_c	Critical velocity of flow.	X, 2	LT^{-1}
V_m	Molecular volume.	II, 10	L^3
W	Velocity in the direction of z co-ordinate.	II, 17	LT^{-1}
W	Energy.	IV, 8	ML^2T^{-2}
W_f	Rate of loss of energy by friction.	VIII, 3	ML^2T^{-3}
W_i	Rate of work indicated.	VIII, 3	ML^2T^{-3}
W_m	Molecular weight.	II, 13	M
w	Velocity in direction of z co-ordinate	II, 8	LT^{-1}
$\overline{w}$	Mean velocity in a tube.	II, 13	LT^{-1}
X, Y, Z	Axes of rectangular co-ordinates.		—
x, y, z	Linear dimensions in rectangular co-ordinates.		L
X_2, X_3	Definite integrals.	VI, 2	—
x_m	Value of x at which pressure is maximum.	IV, 3	L
Z	Specific viscosity.	III, 2	—
z	A number, as of elements in a bearing.	VII, 12	—
α	An angle, constant.		—
α	$= a_1/l$, ratio determining location of a pad.	IV, 3	—
β	$= b/l$ width-length ratio of a pad.	IV, 13	—
Γ	Centrifugal force (of a ball).	VII, 7	MLT^{-2}

Symbol	Signification	Section in which defined	Dimensions
γ	Ratio determining pivoting-point of a pad.	IV, 5	—
Δ	A finite difference, as Δp, a fall of pressure.	VIII, 3	—
δ	Sign of an infinitesimally small quantity.		—
δ	A difference of radii; clearance.	VI, 2	L
δ'	A difference of radii; negative clearance.	VI, 6	L
δ_x	Displacement (axial) due to deformation.	VIII, 8	L
δ_r	Displacement (radial) due to deformation.	VIII, 8	L
∂	Sign of partial differentiation.		—
ϵ	Distance between centres; eccentricity.	VI, 2	L
ζ	A variable, e.g. $= mx$.	IV, 12	
θ	An angle, usually variable.	II, 4	—
κ	Ratio of eccentricity (ϵ) to clearance (δ).	VI, 2	—
λ	Length dimension in pad $= (1 - \gamma)l$.	V, 3	L
μ	Coefficient of viscosity.	II, 2	$ML^{-1}T^{-1}$
μ_0	Coefficient of viscosity at 0° C.	III, 8	$ML^{-1}T^{-1}$
μ_0	Coefficient of viscosity; regarded as constant.	IV, 14	$ML^{-1}T^{-1}$
ν	Coefficient of friction.	VII, 7	—
ν	Index of an empirical formula.	IV, 14	—
ξ	Co-ordinate in length-direction of pad.	V, 3	L
Π	Atmospheric pressure.	II, 16	$ML^{-1}T^{-2}$
π	Ratio of circumference to diameter of a circle $= 3{\cdot}14159.\ .\ .\ .$		—
ϖ, ϖ_m	A factor in the terms of expansion of p, containing variable x only.	IV, 12	—
ρ	Curvature, reciprocal of radius.	II, 10	L^{-1}
ρ_1, ρ_2	Principal curvatures of a surface.	II, 10	L^{-1}
Σ	Sign of summation.		—
σ	Rate of shear (rate of change of an angle).	II, 5	T^{-1}
σ_{yz} etc.	Shear in directions defined by subscripts.	II, 8	T^{-1}
τ	Surface tension.	II, 10	MT^{-2}
τ_{12} etc.	Surface tensions between fluids designated by subscripts.	II, 11	MT^{-2}
Φ_m	A function of Bessel functions.	IV, 12	—
Φ	An unknown function.	IV, 14	—
ϕ	An angle, variable.	VI, 2	—
ψ	Angular velocity (of spin of a ball).	VII, 7	T^{-1}
Ω	Angular velocity.	VII, 7	T^{-1}
ω	Angular velocity.		T^{-1}
ω_A, ω_B	Angular velocities.	III, 11	T^{-1}

IMPORTANT NOTE

All numbering (i.e. of *text-sections*, *figures*, *tables*, and *equations*) starts afresh at the beginning of each chapter. In each case a Roman numeral indicates the chapter to which the section, figure, table, or equation belongs. For example:—

Sect. II, 16, *means* Section 16 of Chapter II.
Fig. II, 4, *means* Figure 4 of Chapter II.
Equation IV, 14, *means* Equation 14 of Chapter IV. etc.

Bibliographical and various other notes are collected at the end of the book in a *List of References* which appears on pages 302 to 308. Thus 'Ref. VIII, 2', in the text, indicates that the reader should look up entry number 2 in group VIII in the *List of References.*

Folders attached to the front and back covers allow easy examination of large diagrams and graphs in conjunction with any page of the text. A *Roman numeral* displayed on each folder indicates the chapter to which it chiefly refers. Individual diagrams are identified by a *letter or Arabic numeral.*

CHAPTER I

Introductory

Lubrication indispensable to mechanism—Conditions essential to its action—Effective lubrication a main objective in machine design—Clarification of some technical terms.

Discovery of the beneficial effects of lubrication must have followed closely upon the making of the most primitive contrivances of a mechanical kind, and it would have been quickly recognized that the lubricant not only lessened the muscular effort of using the contrivance, but also diminished the wear and tear of its working parts. At a later period of mechanical history, when machines of considerable size and power were constructed, it would be found that lubrication served still another purpose, insomuch as it prevented the local heating which, in its absence, must have often threatened ignition and combustion of the early wooden machines.

These primary services rendered to mechanism by lubrication—increased mechanical efficiency, diminution of wear, elimination of destructive heating—are, all three, of vital importance to the existence of machines to-day. It is not, at a first approach, obvious that there is any causal relation between these various effects of a lubricant; when investigated, however, all three functions are found to have the same origin, and to be capable of a single explanation, which, broadly stated, is that the lubricant, in so far as it approaches its ideal action, interposes a partially, or entirely, continuous stratum of fluid between the mutually opposed parts of the machine to which it is applied; it thus prevents, partly or wholly, direct contact of the solid members with one another. When the application of the lubricant attains the ideal, and the interposed stratum of fluid is complete and continuous, solid contacts and solid friction are of course completely obviated, together with the possibility of abrasion or seizing.

The ideal is most readily attained in those members of machines called *bearings*, which (without any attempt to frame a strict definition) may be described as those organs whose sole function in a machine is to transmit mutually opposed forces between relatively moving parts, without performing any work. In other machine parts, where mechanical energy is generated or transmitted, it is usually more difficult to provide the conditions necessary for the formation of continuous lubricating films, and an incomplete attainment of the ideal is often all that can be realized.

The beneficial effects of lubrication are not to be ascribed to special virtues possessed by particular substances, such as oils and greases which are unctuous or oleaginous to the touch. Every fluid is capable of acting as a lubricant under appropriate conditions, the property on which the lubricative action depends, viscosity, being possessed in varying degrees by all fluids.

The objective of rational machine design is to contrive that the construction of the bearings and other parts of the machine in working contact, as well as the quality and quantity of the lubricant, shall be such that the nearest practicable approach to continuous-film lubrication is maintained at all points and at all times. It will be seen hereinafter that there are types of bearings available for most purposes, in which complete lubrication is the automatic result of the relative motions within the bearings themselves, depending only on their precise form and on the supply to them of a sufficient quantity of a suitable, uncontaminated lubricant.

While histories of the theory and practice of lubrication are not within the intended scope of this book (except in so far as they may be traced from its bibliographic references by any reader interested in those subjects), it may be remarked that the hydrodynamic theory of viscous fluids is a science of comparatively recent development. Its application to lubrication is still more recent, and it was not until 1886 that the automatic formation of continuous films of lubricant between the surfaces of bearings was explained by Osborne Reynolds (Ref. 1), in connexion with forms of journal bearings then in existence. More recent still is the idea that new forms and types of bearings might be contrived and designed with deliberate intention to comply with the requirements of the theory.

Formerly the art was wholly empirical and, as was inevitable in the absence of a sound basic theory, many rules were laid down for the construction of bearings, and for modes of application of lubricants, which the theory and subsequent experience have shown to be irrelevant and often erroneous. Such delay as there has been in full appreciation by engineers of the validity of the theory has been mainly due, in the earlier stages, to difficulties in ensuring the high degree of precision in the formation of bearing surfaces which the theory shows to be necessary. It may be added that even to-day, conventional standards of workmanship are often inadequate for the best results, and greater refinements will be necessary in the future for the realization of advances which the theory shows to be both possible and desirable.

The importance of the part which the provision of efficient bearings plays, or might play, in the economy of machines of all kinds, is not always recognized. Marine propulsion by means of high-speed turbines geared to the propellers, and the enormously powerful and highly efficient hydroelectric units of the present day, are examples of new forms of machinery which have become practicable only by the adoption of new types of bearings. It may be reasonably inferred that in many other instances continued adherence to conventional modes of bearing construction has involved retention of types of machines which are unnecessarily large, cumbrous, slow of action and expensive.

The subject matter of the hydrodynamic theory of bearings is the action of the thin, but continuous, film of lubricant interposed between the bearing surfaces. It was shown by Reynolds that the essential condition for the stability of such a film is that it shall be automatically caused to flow in a direction in which the distance between the surfaces is contracting. It has been usual to designate such a lubricating film as a "wedge film" or "tapered film", but these terms do not bring out the essential feature of the action, which is that the film is in motion in the direction of convergence of the wedge. In this book such a film is designated alternatively as a "convergent film", or as a "fluent film", the latter term being introduced for the purpose of emphasizing at once both the essential motion, and the continuity, of the layer of fluid. With the same object, the mode of lubrication which depends on the existence of such a film (this being the only known means of preventing solid-to-solid contacts) is called "fluent lubrication".

Another verbal novelty which has been introduced into the text in the interests of clarity, is the use of the term "charge" to denote the force applied to any single element of a bearing by the fluid pressures existing in the lubricant, or the equal force which is exerted between the element and the remainder of the machine. The term "load" has been reserved for the total force imposed on the bearing as a whole. In some cases, as for instance in a journal, the "charge" and the "load" are equal; in other cases it is not necessary to make any pedantic distinction between them, the context serving to prevent confusion. On similar grounds the expression "coefficient of resistance" has been adopted to denote the ratio of the total frictional or viscous force, or couple, resisting the movement of a bearing member, to the effective load on the bearing; the term "coefficient of friction" is used only in its established sense of the ratio of the frictional force to the normal force at a defined point or surface of contact.

Although the Reynolds theory of lubrication and its principal later developments and applications have been generally accepted, there still survive in technical literature and parlance various notions and expressions belonging to the earlier, empirical period. The mark which these survivals, and neologisms of the same kind, have in common, and which is a token at once of their origin and of their inutility, is their non-quantitative character. At least in the manner in which it is commonly used (that is to say, with an implication that there exists some mode of lubrication which is not referable to the viscosity of the lubricant or to the known laws of the variation of viscosities with pressure and temperature), the expression "boundary lubrication" belongs to this class.

Originally employed by Hardy (see Sect. IV, 18) to denote the change brought about in the coefficient of static friction between two solid bodies by the presence of traces of fluid, the expression has been vaguely extended to include effects of which the majority, if not the whole, are readily explicable as being due to the simultaneous occurrence of solid-contact friction and viscous-fluid action. In one or two sections of Chapter IV and in Appendix I this explanation has been elaborated, and to some extent made quantitative, for conditions in which actual solid contacts do not occur. Viewed in this manner, it is evident that transitional effects between "dry" solid friction and fluent-film lubrication may be expected to occur as a continuous series, and such effects are accordingly designated in the text by such terms as "modified solid contact" and "mitigated solid friction". The condition which arises when the irregularities or "rugosities" of relatively sliding surfaces are sufficiently large to modify the form of flow of the lubricant, but not to cause discontinuity of its substance, is called "rugulose lubrication". Until proof has been given of the existence of effects which are independent of the viscosity of the lubricant, and of the known laws of viscous flow, it seems rational to regard the observed phenomena as transitional effects, dependent, on the one hand, on the irregularities unavoidably present in the co-acting surfaces, and on the other hand, on the known variability of the viscosities of lubricants with varying temperatures and pressures.

This view of the subject provides, at any rate, guidance for design, so that it may be in all cases quantitative in spirit, though it may not be always numerically precise; at the same time a basis is found for (what is of little less practical importance to a machine than sound design) the correct selection, mode of supply, and preservative treatment of the lubricant.

CHAPTER II

Physical Properties of Lubricants: Typical Modes of Flow

1. Introductory.

The property of fluids to which is due their utility as lubricants is the frictional resistance within their substance which they oppose to any internal movement, and to any tangential motion of their bounding surfaces. This kind of internal friction is called *viscosity*. Although it is possessed by all fluids, both liquid and gaseous, it varies greatly in degree from one liquid to another, and from one gas to another. As a class liquids are much more viscous than gases.

In almost all fluids the viscosity changes rapidly with change of temperature, decreasing in most liquids, but increasing in gases, with rise of temperature; within the range of everyday experience, viscosity changes little with variation of any other physical condition of the fluid.

The viscous resistance which liquids offer to motion is often sufficiently great to be apparent to ordinary observation, as when, for instance, the liquid is poured from one vessel to another. A liquid which flows under its own weight very slowly, or which is moved with comparative difficulty, is commonly said to be " thick " or " heavy ", while one which flows readily is described as " thin " or " light ". There is however no necessary, or general, correspondence between the specific gravity of a liquid and its viscosity; mercury, the densest of liquids at ordinary temperatures, is one of the least viscous. The use of the terms " heavy " and " light " in this connexion is therefore to be avoided, but there is no similar objection to the use of " thin " and " thick " as non-technical, descriptive and, indeed, apt terms for slightly viscous and highly viscous liquids respectively.

The relations between the viscosity of a fluid and its other physical properties, and the explanations of viscosity that have been given in terms of the molecular constitution of bodies, are matters outside the scope of this book.

The viscosity of gases can be explained as being due to inter-diffusion and mutual collisions of the molecules of the gas, and its laws are found to be in agreement with the generally accepted molecular, or kinetic, theory of gases. A full exposition of this theory, and one of the most recent, is given in Ref. II, 1.

In liquids, viscosity seems to arise in a different way, and the precise manner of its origin is not so well understood. Several hypotheses, differing rather widely, have been advanced. Ref. II, 2 will give the reader a sketch of one of the most recent and far-reaching of these theories, together with a bibliography of the various other theories that have been developed to explain liquid viscosity, as well as the other physical properties of liquids, on a molecular basis.

2. Quantitative Definition and Law of Viscosity.

A quantitative definition of viscosity can be given in terms of the simplest mode of internal motion and change of shape of which a fluid is susceptible.

Imagine a body of fluid contained between the opposed plane and parallel surfaces, A and B, fig. II, 1, of two rigid plates. These plates may be assumed to be horizontal, and are to be regarded as being of unlimited extent. The fluid is assumed to be uniform throughout in all its properties, including its temperature, and to be either free from any hydrostatic pressure, or at least to be subject only to the normal downward increase of pressure due to gravity, so that there is no variation of pressure in a horizontal direction. Assume that the upper plate, B, is moving horizontally from left to right with velocity U; that the lower plate A is stationary; and that this relative motion of the plates has continued for so long a period of time that the condition of motion of the fluid has become steady.

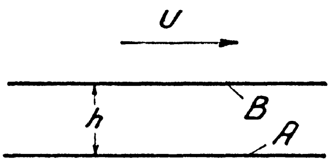


Fig. II, 1

It is a conclusion drawn from all concordant experimental investigations of fluid motion that under these conditions, and provided that the velocity U is within certain limiting values, the plates A and B will be subjected to tangential forces f, for each unit of area of either of the opposed faces, of a magnitude

$$f = \frac{U}{h}\mu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{II, 1}$$

in which h is the perpendicular distance between the faces of the plates, and μ is a constant, known as the "*coefficient of viscosity*", which is characteristic of each particular fluid at a given temperature. The tangential force on the stationary plate A acts in the direction of the velocity U. The force on the moving plate B acts in the directly opposite direction to its motion.

The unit value of the coefficient μ in the C.G.S. ("metric") system is called the "poise". (See Sect. II, 13, p. 27.) Decimal multiples and fractions may be derived from it and used as secondary units, as from other units of the system, but the only one that is frequently employed is the "centipoise", i.e. one-hundredth part of the poise. As is apparent from equation II, 1, the unit, being directly proportional to a force per unit area and to a length, and inversely proportional to a velocity, has (in terms of the fundamental units of mass, length and time) dimensions

$$\frac{MLT^{-2} . L^{-2} . L}{LT^{-1}}, \text{ or } ML^{-1}T^{-1}.$$

It is also established as the result of many varied experiments on all kinds of fluids that the particles of the fluid which are in immediate contact with a solid body have no appreciable motion relatively to the surface of the solid. Thus, in fig. II, 1, the layer of fluid in contact with the plane A is to be regarded as being at rest, and the layer in contact with the plane B as moving with velocity U. Equation II, 1 can be regarded both as a definition of the "coefficient of viscosity" μ and as a statement of the experimentally demonstrated law that, in the circumstances illustrated in fig. II, 1, the resistance to the motion of the plate B is expressed by an equation of the form given, with a constant coefficient μ. The law is sometimes called "Newton's law of viscosity" (Ref. II, 3), though it was enounced by Newton rather as an hypothesis than as an established law.

3. Limitations of the Law.

The conditions represented by fig. II, 1 are of course purely ideal, and equation II, 1 is only applicable to any fluid within a certain range of the velocity U. Within wide ranges, however, the law is found to hold for a great many ordinary fluids with a very high degree of accuracy. Such fluids may be called, within that range, *true fluids* in distinction from a varied class of substances, commonly regarded as fluids, for which it is found that equation II, 1 only holds when the velocity U exceeds a certain lower limit, below which the substance behaves as a solid (and even then only approximately). Ordinary oil-paint, and many other substances consisting of finely divided solid particles suspended in a liquid medium, belong to this class. Another class of bodies includes wax and pitch and many other similar substances which, at ordinary temperatures, appear on casual observation to be solids, but which are found on investigation to be true fluids obeying the law of equation II, 1 with very considerable accuracy, and over wide ranges of velocities, though with high values of their viscosity constants.

In all fluids equation II, 1 is only applicable so long as the velocity U does not exceed a certain limit which is dependent on the values of μ and the dimension h, or, in circumstances of motion different from those of fig. II, 1, on some corresponding dimension of length. Reference to these "critical velocities" will be made in various cases throughout the book, and reference may here be made particularly to Sect. 13 of this chapter, and to Sects. 1 and 2 of Chapter X.

A "true fluid", as defined above, having a definite constant of viscosity over a range of velocities or rates of deformation, is to be clearly distinguished from the "perfect fluid" of hydrodynamical theory, the latter being a purely imaginary substance offering no internal resistance to relative motions of its parts. This conception is appropriate to the discussion of fluids in bulk, when the momentum and inertia of the mass of the fluid is the main consideration; but it has only a minor place in the discussion of fluids as lubricants, which is mainly concerned with layers or films of liquid which are very thin relatively to their longitudinal and lateral extent.

4. Laminar Motion in Parallel Planes.

In order to examine more fully the state of motion and the forces existing within the body of the fluid represented in fig. II, 1, consider the condition of a very thin plane layer C of the fluid, parallel to the planes A and B, and situated at a distance z from A (see fig. II, 2). Since the fluid is moving steadily, without acceleration, the fluid above the layer C must, by Newton's First Law of Motion, exert a forward tangential force on the upper face of the layer C equal to the retarding force which acts, according to equation II, 1, on the face of the moving plate. On any area $\mathcal{A}$ which we may take as being large in length and width in comparison with h, each of these forces is of magnitude

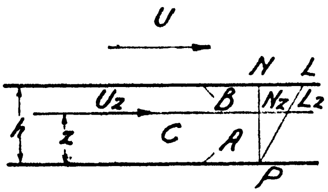


Fig. II, 2

$$F = f \times \mathcal{A}.$$

The same force, for the like reason, acts forwardly on the lower face of the layer C, and by the same rule as gave equation II, 1 the magnitude of this force is, per unit area,

$$f_z = \frac{U_z}{z} \cdot \mu,$$

if U_z is the forward velocity of the layer C.

Consequently $$\frac{U_z}{z} \cdot \mu = f_z = f = \frac{U}{h} \cdot \mu,$$

and $$\frac{U_z}{U} = \frac{z}{h}. \qquad \text{II, 2}$$

That is to say, each horizontal layer has a forward velocity which is proportional to its distance from the stationary plate A. Its velocity may consequently be represented by the length of the portion N_zL_z of the layer at z, which is intercepted between the vertical line PN and the inclined straight line PL, to the same scale as the length NL represents the velocity U of the plane B.

The condition of internal motion of the fluid between the planes A and B can thus be regarded as the same as that of a great number of plane and indefinitely thin but rigid plates (laminæ), each representing a thin layer of the fluid, such as the layer C, and together making up the total thickness h. Each lamina can be regarded as sliding over the one below it with velocity $\frac{dU}{dz} . \delta z$, where δz is the thickness of the lamina. Motion of this kind is called *laminar motion.*

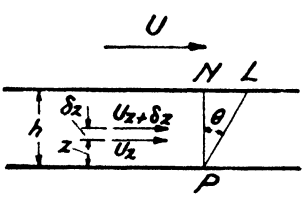


Fig. II, 3

Proceeding to the limit of small quantities, it will be seen from fig. II, 3 that

$$U_{z+\delta z} - U_z = U\frac{z + \delta z}{h} - U\frac{z}{h} = U\frac{\delta z}{h}$$

or $$\frac{dU}{dz} = \frac{U}{h}. \qquad \text{II, 3}$$

Geometrically, $$\frac{dU}{dz} = \frac{\text{NL}}{\text{PN}} = \tan\theta,$$

and $$U_z = z\tan\theta, \qquad \text{II, 4}$$

the variable length $z \tan\theta$ representing a velocity on the same scale as NL represents the velocity U.

5. Continuous Shear.

From another, slightly different, point of view the simple mode of fluid motion which has been described with reference to figs. II, 1, 2 and 3, is described as a motion of continuous shear. The movement of any of the laminæ which takes place in a very short interval of time δt is $U_z\,\delta t$, and can be represented, as in fig. II, 2, by an intercept N_zL_z in a triangle such as PNL, or $z\tan\theta$ in fig. II, 3.

Consider a square prism of the fluid extending lengthways at right angles to the plane of fig. II, 4, and represented on that plane by its trace, the square DEFG. The lengths l of the sides of this square, having regard to the use which will be made later of the argument in this section, should be assumed to be small relatively to the distance h between planes A and B, although this assumption is unnecessary in connexion with the motion illustrated in figs. II, 1, 2 and 3. By the small movement which takes place in the time δt, the particles of fluid on the lines DE and GF move to corresponding points on the lines DE′ and GF′, which make equal angles $\delta\theta$ with DE and GF, the square prism DEFG having thus been "sheared" with all the fluid which it contained into the parallelogram DE′F′G. The angle $\delta\theta$ may be called the "*angle of shear in time* δt".

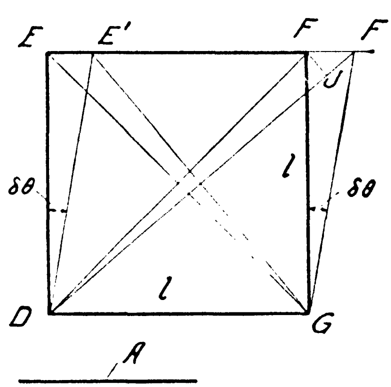


Fig. II, 4

If the diagonals of the original square, DF and GE, as well as the diagonals DF′ and GE′ of the parallelogram, are drawn, and a perpendicular FJ dropped on the diagonal DF′, it is seen, since the angle DFE is $\frac{1}{4}\pi$ and the angle DF′F differs from it only very slightly, that to the first order DJ = DF and

$$\mathrm{DF}' = \mathrm{DF} + \mathrm{JF}' = \mathrm{DF} + \mathrm{FF}' \times 1/\sqrt{2}$$

$$= \sqrt{2}\,.\,l + \frac{1}{\sqrt{2}}\,l\,\delta\theta = \sqrt{2}\,.\,l(1 + \tfrac{1}{2}\delta\theta);$$

and similarly $\qquad \mathrm{GE}' = \sqrt{2}\,.\,l(1 - \tfrac{1}{2}\delta\theta),$

that is to say, the two diagonal planes of the square prism are respectively lengthened and shortened by the fraction $\frac{1}{2}\delta\theta$ of their original lengths; or, in other words, the rate of shear $d\theta/dt$ represents the same motion as a combination of pure extension directed 45 degrees upwards and towards the right hand, relatively to the base plane DG, combined with a pure contraction in a direction at right angles to that extension, both extension and contraction taking place at the rate $\frac{1}{2}\frac{d\theta}{dt}$.

Reverting to equation II, 1, it is seen that the experimental law which is stated therein can be also expressed as

$$f = \mu \frac{dU}{dz}, \qquad \text{II, 5}$$

or, alternatively, with reference to fig. II, 3, as

$$f = \mu \frac{d \,.\, \tan\theta}{dt}, \text{ or } f = \mu\sigma, \qquad \text{II, 6}$$

in which $\sigma = d \,.\, \tan\theta/dt$ is the time-rate of change of the tangent of the angle of shear or, more briefly, the *rate of shear*, f being from this point of view called the *shearing stress.*

6. Laminar Flow in one direction between parallel and stationary planes.

In the simple example of pure shear discussed in Sects. II, 2–5, it has been postulated that there is no variation of hydrostatic pressure in the direction of the motion of the fluid. The question now to be examined is: What would be the motion of the fluid if its pressure varied longitudinally? Fig. II, 5 relates to a body of viscous fluid contained between two unlimited parallel plates, as in figs. II, 1, 2 and 3, with the differences that both plates are now stationary, and that a pressure p_1 is maintained on a plane section DD′ normal to the plates at the left-hand side, greater than the pressure p_0 similarly maintained at EE′ on the right-hand side. In this instance, the fluid is assumed to be a liquid and to be incompressible. The distance l between DD′ and EE′ is assumed to be large relatively to h. In this case the liquid will still flow in horizontal laminæ, as in the earlier figures, and in a steady condition of pure shear, but neither the rate of shear σ, nor the shearing stress f, will now be uniform from layer to layer.

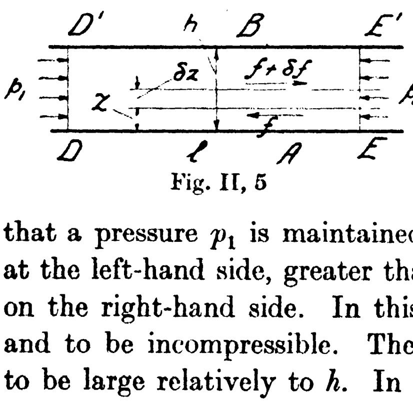


Fig. II, 5

Consider the lamina between the ordinates z and $z + \delta z$ in fig. II, 5. Since its motion is steady, the difference between the shearing forces on its upper and lower surfaces must balance the difference between the pressures on its ends; thus, regarding the shearing stress as being directed forwardly on the upper surface of the lamina,

$$(p_1 - p_0)\delta z + \frac{l\,df}{dz}\delta z = 0,$$

or

$$\frac{df}{dz} = -\frac{p_1 - p_0}{l}; \qquad \text{II, 7}$$

and since, from equation II, 5,

$$f = \mu \frac{dU}{dz},$$

$$\frac{d^2U}{dz^2} = -\frac{p_1 - p_0}{\mu l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 8}$$

and by simple integration, with constants C and C',

$$U = -\frac{p_1 - p_0}{\mu l}\left(\tfrac{1}{2}z^2 + Cz + C'\right).$$

The plates being stationary, $U = 0$ both when $z = 0$ and when $z = h$, so that

$$C' = 0$$

and $$\tfrac{1}{2}h^2 + Ch = 0,$$

or $$C = -\tfrac{1}{2}h.$$

Thus, finally, $$U = -\frac{p_1 - p_0}{\mu l}\left(\tfrac{1}{2}z^2 - \tfrac{1}{2}zh\right)$$

$$= \frac{p_1 - p_0}{2\mu l} z(h - z); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 9}$$

so that the value of U at any ordinate z is represented by the intercept at that ordinate between the normal line PN (fig. II, 6) and a parabola, such as PQN, having its axis parallel to, and midway between, the planes A and B.

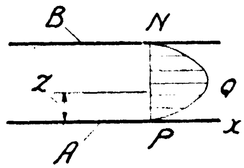


Fig. II, 6

The shearing stress f, being $\mu \frac{dU}{dz}$, is given from equation II, 9, as

$$f = \frac{p_1 - p_0}{2l}(h - 2z), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 10}$$

vanishing on the midway plane $z = \tfrac{1}{2}h$, and having its greatest values

$$f_0 = -f_h = (p_1 - p_0)\frac{h}{2l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 11}$$

on the planes A and B bounding the liquid at $z = 0$ and $z = h$.

7. Volume-rates of Flow between stationary parallel planes.

The total flow of the liquid across any transverse section, such as PN, of fig. II, 6, taken over a width b measured in the direction perpendicular to the figure, is

$$Q = b\int_0^h U\,dz$$

$$= \frac{b}{2\mu l}(p_1 - p_0)\int_0^h (zh - z^2)\,dz$$

$$= \frac{bh^3}{12\mu l}(p_1 - p_0). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 12}$$

Since there is no variation of the conditions in the direction of the flow of the liquid from DD′ to EE′ (fig. II, 5) it can be concluded that the pressure diminishes uniformly in that direction, which may be taken as the co-ordinate direction of x.

Thus
$$\frac{\partial p}{\partial x} = -\frac{p_1 - p_0}{l},$$

so that
$$Q = -\frac{bh^3}{12\mu}\frac{\partial p}{\partial x}, \qquad \text{II, 13}$$

which remains true whether $\partial p/\partial x$ is uniform or varying with x.

The flow of fluids in tubes of cylindrical forms is closely analogous to the flow between planes as dealt with in this section, but of course requires consideration of the conditions of flow varying in two directions transverse to the lines of flow. This case will be discussed in Sect. II, 13.

The following sections deal with viscous fluids as existing in three dimensions. Incidentally it will be shown that, when no discontinuities exist in the fluid, there is in it only one kind of internal resistance, and one coefficient of viscosity, viz. that already defined and designated as μ.

For numerical values of the coefficient of viscosity of the fluids which are utilized for lubrication, and for an account of the methods and apparatus which are used for determining those values, the reader is referred to Chap. III, wherein there will also be given a fuller account of the dependence of the values of μ on pressure, temperature, and other physical conditions, than is contained in the summary statements in Sect. II, 3.

8. Relative Velocities and Rates of Deformation in three dimensions.

If u, v, w (fig. II, 7) are the components parallel to the axes OX, OY, OZ of the velocity of a particle of fluid at the point $O' \equiv (x, y, z)$, the corresponding components for the neighbouring point $O'' \equiv (x + \delta x, y + \delta y, z + \delta z)$ are

$$\left.\begin{aligned} u' &= u + \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z, \\ v' &= v + \frac{\partial v}{\partial x}\delta x + \frac{\partial v}{\partial y}\delta y + \frac{\partial v}{\partial z}\delta z, \\ u' &= w + \frac{\partial w}{\partial x}\delta x + \frac{\partial w}{\partial y}\delta y + \frac{\partial w}{\partial z}\delta z, \end{aligned}\right\}$$

and the component velocities of the second point relatively to the first are

$$\left.\begin{aligned} u' - u &= \frac{\partial u}{\partial x}\delta x + \frac{\partial u}{\partial y}\delta y + \frac{\partial u}{\partial z}\delta z, \\ v' - v &= \frac{\partial v}{\partial x}\delta x + \frac{\partial v}{\partial y}\delta y + \frac{\partial v}{\partial z}\delta z, \\ v' - w &= \frac{\partial w}{\partial x}\delta x + \frac{\partial w}{\partial y}\delta y + \frac{\partial w}{\partial z}\delta z. \end{aligned}\right\}$$

It is clear on inspection of fig. II, 7, that the derivatives $\partial u/\partial x$, $\partial v/\partial y$, $\partial w/\partial z$ represent, respectively, the rates of elongation or stretching of the element in the directions of X, Y, and Z, while the pairs of sums of derivatives

$$\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\quad \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x},\quad \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y},$$

represent respectively the rates of change of the angles between the edges δy and δz, δz and δx, and δx and δy of the originally cubical element.

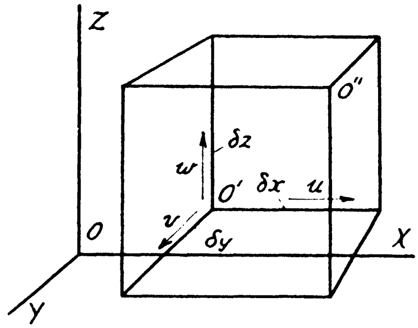


Fig. II, 7

Thus, by means of these three derivatives and three sums of derivatives, any deformation of the element can be expressed. In the applications which follow in the present and succeeding chapters the axes X, Y, and Z will be so chosen with respect to the boundaries of the body of fluid that the rates of elongation such as $\partial u/\partial x$ are everywhere small compared with the rates of shear represented by the three sums of derivatives $\partial w/\partial y + \partial v/\partial z$, etc. Now in any homogeneous liquid or gas, such as will be discussed, there is no physical difference of its properties depending on the direction of the co-ordinates, consequently, the shearing stresses, or frictional forces, being linear functions of the rates of shear

$$\sigma_{yz}=\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\ \text{etc.},$$

may be expressed as

$$\left.\begin{aligned} s_{yz}&=\mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right),\\ s_{zx}&=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right),\\ s_{xy}&=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right),\end{aligned}\right\}\quad \text{II, 14}$$

the notation of the shearing forces s_{yz}, etc. being such that the first subscript indicates the direction of the normal to the plane on which the force acts, and the second subscript the direction of the force itself; thus s_{yz} is the shearing stress on a plane normal to OY, and it acts in the direction of OZ.

It will be seen that all the three shearing stresses depend on the single coefficient of viscosity μ. By comparison of, say, the second of the equations II, 14, and fig. II, 7 with equation II, 5, viz.

$$f=\mu\frac{dU}{dx},$$

and fig. II, 3 in which $\partial w/\partial x$ was, by assumption, zero, it will be seen that this constant μ is the same as the coefficient denoted by the same letter in the discussions on the special case of laminar motion.

9. Conditions at the Bounding Surfaces of Fluids.

Before the laws of fluid friction can be applied to actual cases, account must be taken of the behaviour of the fluid where it is in contact with its boundaries. In the case of liquids the conditions at a free upper surface (usually a surface of contact with air at atmospheric pressure) have also to be considered.

It is clear in the first place that the presence of a solid boundary necessitates that on the bounding surface the velocity of the fluid normal to, and relative to, that surface must be zero. The normal velocity must furthermore be very small at all points *near* the solid surface. For let A (fig. II, 8) be the fixed bounding surface and A′ a surface parallel to, and very near, the surface A. For simplicity the surfaces A and A′ may be regarded as plane, or at any rate as having radii of curvature very large compared with the normal distance between A and A′. Let the average velocity of the fluid towards A, taken over a circle of radius R, be v, the normal distance between A and A′ being δn. Then a volume $\pi R^2 v$ of the fluid flows through the circle R in unit time.

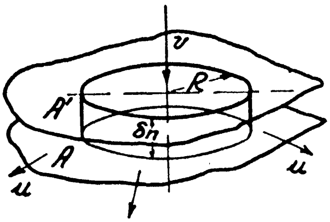


Fig. II, 8

In the same time a volume $2\pi R \,.\, \delta n \,.\, u$ flows outward between the surfaces past the circumference of the circle, u being the average outward radial velocity. These volumes must be equal,

$$\therefore\ v = 2u\frac{\delta n}{R}, \qquad \text{II, 15}$$

showing that the normal velocity is very small in comparison with the velocities of the fluid parallel to, and very near, the surface of the solid.

It has already been stated in Sect. II, 2, and will be more fully explained in Sect. II, 10, that the tangential velocity of a fluid in contact with a solid surface is always zero. At a distance δn from the surface, u is therefore $\sigma\delta n$, which (since σ is finite) is a small quantity of the first order in δn. It follows, from equation II, 15, that v, the normal velocity, is a small quantity of the *second* order in δn.

Some of the experimental evidence for the conclusion that there is never any discontinuity of motion between a fluid and a solid with which it is in contact, will be quoted in Sect. II, 13 in which the flow of fluids in tubes is discussed. In liquids, even when the molecular attraction between the liquid and a solid appears to be comparatively small, so that the liquid does not spread over or "wet" the surface of the solid, there is no observable discontinuity of motion at the common surface. The same is true of the contact of a liquid with another fluid with which it does not mix.

In gases the same rule is observed under all ordinary circumstances, at least as an approximation so close that the rule can be considered exact in all questions which arise in connexion with lubrication. When, however, a gas is at a pressure so low that its molecules are at distances apart which are comparable with the dimensions of the containing vessel, phenomena are observed which can be regarded as arising from a discontinuity of tangential motion at the solid surface. According to Maxwell (Ref. II, 4) the motion of the gas is then very nearly the same as if a stratum of depth equal to twice the mean free path of the gas molecules had been removed from the solid and filled with the gas, there being no slipping between the gas and the imagined new surface.

At free surfaces, which of course can exist only in liquids, not in gases, the normal velocity relative to the surface is again zero, the argument given above in connexion with fig. II, 8 applying without any modification. The liquid surface may however have a tangential velocity, which may vary from point to point of the surface. When, as is usual, the free surface is a surface of contact between the liquid and the atmosphere, it may accordingly be assumed that the law of viscous shear holds in both fluids up to their common surface, and that there is no discontinuity of tangential motion at that surface. The rate of shear of each body at their common surface can then be determined by equating the shearing stress in either of the bodies with that in the other, having regard to their respective coefficients of viscosity, and expressing the condition that their *relative* tangential velocity is zero. Experiment has shown (Ref. II, 5) that, at least in the cases of water with an uncontaminated surface, and of oils and other liquids which dissolve films of grease or similar foreign substances, there are no frictional resistances inherent in the surface film to be taken account of in such a calculation.

10. Surface Tension.

It can be inferred from ordinary observation, and is confirmed by experiment, that the surfaces of liquids can be regarded as being in a state of tension and as seats of energy. The fact that raindrops and other isolated drops of water or oil, and small globules of mercury, form themselves into approximately spherical shapes suggests that this apparent tension, which is named *surface tension,* is uniform in all directions on the surfaces. Precise investigation confirms these inferences and establishes that the measure of the quasi-tension of the surface is a characteristic of each liquid, varying with its temperature, but nearly independent of the curvature of its surface provided that the radius of curvature does not approach molecular dimensions.

The conception of a surface film in a state of constant tension, analogous to an elastic skin containing the fluid body, but unlike any actual elastic skin in being capable of indefinite extension or contraction, is no doubt somewhat artificial and abstract. The tendency of the surfaces of liquids to form spheres, or other shapes of minimum area under the conditions imposed on them, is to be regarded as due to the mutual attraction of the molecules throughout the substance of the liquid tending to draw the mass as closely together as possible. This mutual attraction is regarded as being, in liquids, the means which prevents the molecules at the surface from escaping immediately into the atmosphere and, in solids, the originating cause of their strength and rigidity. A surface layer, being attracted from one side only by such molecular forces, is necessarily in a different condition from the interior parts of the substance, whether the substance is a liquid or a solid. The surface common to a solid and a fluid in contact with it, like the meeting surface of two liquids which do not mix, or of a liquid and a gas, can also be regarded, from similar considerations, as a surface film having a specific " surface tension ".

It can be shown that the work done by the internal attractions of a body of fluid in changing from any given external shape to one that is more compact, is proportional to the resulting reduction of the area of the surface. It can therefore be represented quantitatively by the work done by the hypothetical surface-skin under constant tension. Let it be imagined, then, that a section of small depth and length along the surface is drawn in the superficial layer of a liquid, and that a tension is exerted across this section, of amount $\tau \cdot \delta s$, proportional to the length δs, τ being a constant. This specific tension τ, characteristic of the liquid and known as its " surface tension ", is measured in C.G.S. units as dynes per centimetre, i.e. as (gm. cm. sec.^{-2}) per centimetre, the dimensions of its unit of measurement being accordingly $MLT^{-2}L^{-1}$, or MT^{-2}.

The surface tension of a liquid, so defined, is usually understood to be the tension of its surface in contact with atmospheric air, in other words the tension of the common surface of the liquid and air. In contact with other gases, or with its own vapour, the surface tension of the liquid will differ more or less from this standard. The variation of the constant τ with temperature T (which is rather rapid in most liquids) can generally be expressed by a formula having the form

$$\tau = \tau_0(1 - bT)^n,$$

τ_0 being the value of τ at 0° C., and b and n numerical constants whose values for a few liquids are given in the table opposite. Their values are however not well determined for lubricating oils, nor indeed are the values of τ for any one temperature at all accurately known for oils. This is probably due, in the main, to the readiness with which oils dissolve other organic compounds with resulting contamination of their surfaces.

For many liquids what is known as " Eötvös' Rule " holds (Ref. II, 6). This rule states that (V_m being the molecular volume of a liquid, i.e. its " molecular weight " divided by its density),

$$\frac{d}{dT}(\tau V_m{}^{2/3}) = -2{\cdot}1,$$

a numerical constant. For fuller information the reader may consult Ref. II, 6, which contains a clear general account of the subject of Surface Tension and Capillarity.

Table II, 1 gives values of τ, collected from various sources, at the temperatures stated:

TABLE II, 1

SURFACE TENSIONS OF VARIOUS LIQUIDS
(in dynes per centimetre)

Liquid	Temperature deg. C.	Supernatant gas	Surface tension, τ
Water	15	A	74·3
,,	15	V	71·4
,,	50	A	68
,,	100	A	59
Olive oil	20	A	32
Paraffin (m = 0·84)	25	A	26
Glycerol	18	—	65
Hexane	8	V	19
Mercury	18	A	547
Molten tin	fusion	—	352
,, copper	,,	—	580
,, iron	,,	—	950

The letters "A" and "V" in the third column indicate respectively that the surface of the liquid is in contact with air, or with the vapour of the liquid itself.

m in column 1 is the density in C.G.S. units.

Some additional figures for commercial lubricants are given in Sects. III, 6 and 7.

The following short table (Table II, 2) gives for a few pairs of liquids the surface tensions of their common surfaces at 20° C. (Refs. II, 7 and 8.)

TABLE II, 2

SURFACE TENSIONS OF COMMON BOUNDING SURFACES OF VARIOUS PAIRS OF LIQUIDS AT APPROXIMATELY 20° C.
(in dynes per centimetre)

Liquids	Surface tension, τ
Water—Mercury	418
Olive oil—Water	20·5
,, Mercury	335
Petroleum—Water	28
,, Mercury	284

The subject of surface tension is of interest in connexion with lubrication for several reasons. The first and perhaps most important of these reasons is that the quantity of lubricant which can be supplied by any particular means to a bearing, or other kind of machine element requiring lubrication, often depends on the relations between the surface tension of a film of the lubricant and the other forces acting upon the film. Instances will appear in later chapters (particularly Sects. VI, 9, VI, 10 and VII, 10). A second reason is that the automatic spreading of a liquid lubricant over the solid surfaces on which its presence is required, depends both on the surface tension of a surface of the liquid in contact with air, and on the molecular attractions between the liquid and the solid, or solids, which (as already pointed out in this section) can be regarded as resulting in a kind of surface tension, and which causes in the liquid a tendency to spread on the solids. A third consideration of some importance is that the viscosities of liquid lubricants are usually measured by methods which require some account to be taken of the surface tension of the liquid.

All three of these subjects are most conveniently dealt with under the heading of the next section (Sect. II, 11, *Capillarity*). There is however one principle of general application which may be stated as a preliminary.

The existence of tension in any *curved* surface of a liquid implies, in general, that there is a greater hydrostatic pressure on one side of the surface than on the other. Imagine a portion of a liquid bounded by a surface which is, over a certain part of its extent, an arc of a circular cylinder. The axis of the cylinder (fig. II, 9) is assumed to be normal to the plane of the figure, and AB is the trace on this plane of the cylindrical arc which bounds on its concave side a liquid L. It can be supposed that on the convex side of the arc there is a gas, or another liquid M, with which the liquid L does not mix; between L and M there will be a surface having a tension τ per unit length in the axial direction, acting tangentially at all points such as C, D, on the cylindrical interface.

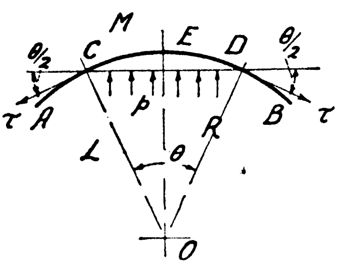


Fig. II, 9

Let it be assumed for the moment that the hydrostatic pressure in the outer liquid M is zero, and that the inner liquid L is at a uniform pressure p, the effect of gravity being ignored for the same reasons as in Sect. II, 2.

By resolving in the vertical direction the forces acting on the cylindrical segment of liquid contained between the arc CD and the chord CED, it is seen that

$$2\tau \sin \tfrac{1}{2}\theta = p \times \mathrm{CD} = p \, . \, 2R \sin \tfrac{1}{2}\theta,$$

so that

$$\tau = pR, \qquad \text{II, 16}$$

in which R is the radius of the cylinder.

Equation II, 16 can be written alternatively

$$p = \frac{\tau}{R} = \rho\tau, \qquad \text{II, 17}$$

in which ρ is the curvature of the cylinder in the plane of the figure, the curvature in the plane at right angles thereto being zero.

It can be readily seen that in the case of a surface of double curvature the corresponding equation connecting the surface tension with the pressure of the liquid must be

$$p = (\rho_1 + \rho_2)\tau, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 18}$$

ρ_1, ρ_2 being the principal curvatures, which may be of the same or opposite signs.

It is further evident that equations II, 16–18 must still hold if the fluid M on the outerside of the surface-film AB has a positive uniform pressure p' less than p, the pressure p in these equations being now replaced by $p - p'$, the *excess* of the pressure on the inner side of the film over the pressure on the outer side.

In the important case of a spherical surface, the principal curvatures are of the same sign and equal to one another, so that equation II, 18 becomes

$$p = 2\rho\tau = \frac{2}{R}\tau, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 19}$$

R being the radius of the sphere.

11. Capillarity.

Of the same kind as the superficial forces which are involved in the formation of the bounding surfaces of liquids, and in the spreading of films of fluid on solid surfaces, are those which are concerned in the rise of a liquid in a vertical capillary tube, or between closely adjacent solid bodies partially submerged in the liquid; the formulæ of Sects. II, 9 and 10 are immediately applicable to the calculation of these effects.

The height to which a liquid rises in a tube of given size is one of the most convenient data for determining its surface tension, and of thereby evaluating the corrections which may have to be made when the coefficient of viscosity of the liquid is ascertained by measuring its rate of flow in a similar tube.

When a liquid rises in a narrow tube by the action of its surface tension, or as is said, " by capillarity ", its surface generally makes a finite angle with the wall of the tube. This angle is determined by the conditions of equilibrium of the three surface tensions which are involved, namely, the tension of the surface between the liquid and the tube, that of the surface between the liquid and the atmosphere, or whatever other non-mixing fluid may be above it in the tube, and lastly, the tension of the surface between the air, or other upper fluid, and the tube.

The angle (measured in the liquid) between the surface of the liquid and the tube-wall may be either *acute* as in fig. II, 10 (p. 20), or *obtuse* as in fig. II, 11, the former corresponding to a *rise*, and the latter to a *lowering*, of the surface of the liquid inside the tube relatively to its level externally, as indicated at EE in the figures. Taking the former of these cases (fig. II, 10), consider the equilibrium of the tensions of the three contact-surfaces in the immediate neighbourhood of the solid wall of the tube, as shown in fig. II, 12. If, for brevity, the liquid in the tube, the air, or other fluid above the liquid, and the solid matter of the tube are denoted respectively by the numerals 1, 2, and 3, the three surface tensions can be called τ_{12}, τ_{23}, and τ_{31} as shown in fig. II, 12, in which the curved surface 1, 2 between the two fluids is represented by its tangent plane which makes the angle θ with the wall of the tube, which is similarly regarded as being represented by its tangent plane.

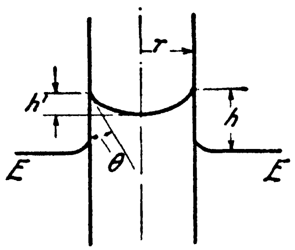


Fig. II, 10

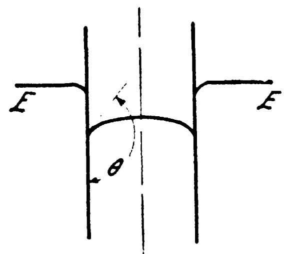


Fig. II, 11

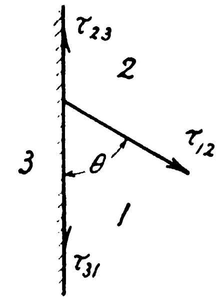


Fig. II, 12

Resolving the tensions vertically downwards

$$\tau_{12} \cos \theta + \tau_{31} - \tau_{23} = 0, \qquad \text{II, 20}$$

from which $\cos \theta$ and θ are determined when the three surface tensions are known.

It is more usual, however, to make use of equation II, 20 for the purpose of determining one of the three surface tensions when the other two are already known, and the angle θ has been ascertained by experimental observation.

The somewhat intangible and difficult conception of a surface tension as a physical reality appertaining to the common surface of a fluid and a solid may be clarified by detailed consideration of the meeting surfaces of three *fluid* bodies in the special case illustrated in fig. II, 13.

Imagine three fluids which do not mix with one another and of which we suppose one, M, being the one of highest specific gravity, to rest in the bottom of a wide vessel A, the two others, of which one, R, lighter than M, is also a liquid, while the third, L, is a still lighter liquid, or a gas, both resting on the heavy liquid M and separated from one another by their common surface film. Suppose further that the upper surfaces of R and L are in contact with a solid horizontal plate B, and that a pressure p is maintained in the fluid L greater than that at corresponding levels in R, so that the surface of the liquid M in contact with R is raised against gravity to a higher general level than the meeting surface of M and L.

The three surfaces MR, ML and LR will meet on a common horizontal line whose trace on the plane of the figure is the point C. All these surfaces will be curved in the neighbourhood of C, and the surface LR will be curved throughout so that its tension balances the difference of the pressures on its two sides, this difference varying from one level to another on account of the different specific gravities of R and L, according at each level to equation II, 16, which thus determines the curvature at each level of the surface RL.

When the three surface tensions are known, the angles which the three surfaces make with one another at C can be determined by resolving the surface tensions at C both vertically and horizontally, and expressing the necessary condition that the sum of the three angles is 2π.

Now there is every reason to believe that the surface conditions at the mutual boundary of a fluid and a solid surface are of a similar nature and a like order of magnitude to those between two fluids of chemical constitution similar respectively to the fluid and solid in question. Suppose then that after being placed in the bottom of the vessel A (fig. II, 13) the liquid M had been frozen to form a horizontal slab, before the fluids R and L had been placed above it. The conditions would then be as represented in fig. II, 14. There would now be no visible evidence of any surface film MR or ML. The film RL, curved as in fig. II, 13, will now abut directly on the horizontal surface of the solid M, and the angle which that film makes with M can be obtained by resolving the surface tensions at C′ horizontally, with the same result as was obtained by resolving vertically the tensions shown in fig. II, 12.

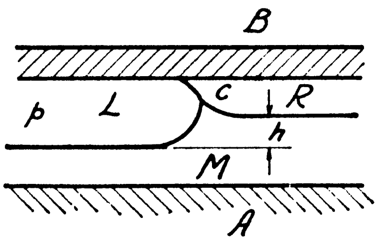


Fig. II, 13

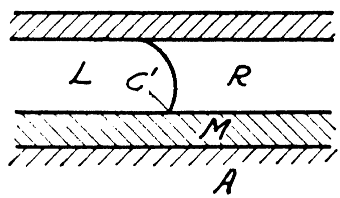


Fig. II, 14

If we attempt, in the case of fig. II, 14, to establish another equation between the forces at C′ by resolving them vertically as in the case of fig. II, 13, we are apparently met by a paradox, since $\tau(MR)$ and $\tau(ML)$ have no component in the vertical direction to balance that of $\tau(RL)$. This paradox is, however, removed if we can suppose that a part of the surface of the solid M, of molecular dimensions, is capable of withstanding the component normal to the solid surface of the tension $\tau(RL)$. There is no reason why we should not regard this as being the case, since the postulated tension in the surface RL, which is to be regarded as distributed over a film thickness of about 10^{-8} centimetre, is of the same order as the molecular forces of cohesion in ordinary solids, as known from their strength characteristics. Both in the mutual contact of three fluids illustrated in fig. II, 13, and in the contact of two fluids with a solid, as in fig. II, 14, it has, however, to be realized that in the immediate neighbourhood of the line C, or C′, there must be some degree of mixing of the molecules of the three substances, and in the case of fig. II, 14, there will be a small (molecular) region of transition from the solid state of the substance M to the fluid state of the other two substances, so that the surface of M will not remain strictly plane at C′.

12. Rise of Liquids in Capillary Spaces.

The angle of contact between the surface of a liquid and a solid material having been determined by equation II, 20, or by direct observation and measurement, the height to which the fluid will rise in a narrow tube, or other interspace of capillary dimensions between walls of solid material, can be calculated approximately without difficulty. For an approximation, which is sufficiently accurate for the solution of all problems which are likely to arise in connexion with lubrication, it is permissible to regard the liquid surface within a tube (this surface being known as a *meniscus*) as being a segment of a sphere intersecting the wall of the tube at a small angle θ. In the case of a capillary interspace between two plane and parallel walls, the meniscus may, with similar accuracy, be taken as a segment of a circular cylinder having a horizontal axis and cutting the walls at the same angle θ, which is small for oils.

Assuming, in the case of a tube, that the depth of the meniscus at its centre below the circle of intersection with the tube wall (see fig. II, 10) is

$$h' = r(\sec\theta - \tan\theta),$$

r being the internal radius of the tube, the volume of fluid above the lowest point of the meniscus is the difference between the volume of a circular disc of radius r and height h' and the volume of a segment of a sphere the radius of whose basal circle is r and whose height is h'. This latter volume is approximately $\frac{2}{3}\pi r^2 h'$, and that of the disc is $\pi r^2 h'$, so the required volume above the lowest point of the meniscus is $\frac{1}{3}\pi r^2 h'$.

The vertical component of the surface tension of the meniscus, taken all round the circumference of the tube, is $\tau \,.\, 2\pi r \,.\, \cos\theta$, and this tension supports the weight of both the liquid above the level of the bottom of the meniscus, and of the cylinder of the liquid which is inside the tube below that level and above the level of the liquid outside at a distance from the tube. The volume of this cylinder is

$$\pi r^2(h - h').$$

Hence if m be the density of the liquid and g the constant acceleration due to gravity,

$$\tau \,.\, 2\pi r\cos\theta = \{\tfrac{1}{3}\pi r^2 h' + \pi r^2(h - h')\}mg$$
$$= \pi m g r^2\{h - \tfrac{2}{3}h'\},$$

or, since

$$h' = r(\sec\theta - \tan\theta),$$

$$h - \tfrac{2}{3}r(\sec\theta - \tan\theta) = 2\tau\frac{\cos\theta}{mgr}$$

and

$$h = \tfrac{2}{3}r(\sec\theta - \tan\theta) + \frac{2\tau}{mgr}\cos\theta. \qquad \text{II, 21}$$

If the angle θ is zero, the fluid being then said to "wet the tube",

$$h = \frac{2r}{3} + \frac{2\tau}{mgr}. \qquad \text{II, 22}$$

If the angle θ is "negative", as defined in connexion with fig. II, 11 (i.e. if θ shown in fig. II, 12 is greater than a right angle), and there is consequently a *lowering*, instead of a rise, of the circle of contact of the meniscus below the level of the liquid outside the tube, the amount of this lowering can be determined by the same process as resulted in equation II, 21. The meniscus will in this case be convex upwards. Mercury is the only substance liquid at ordinary temperatures which shows such a lowering when in contact with glass, or with iron, or its alloys.

The height of ascent by capillarity of a liquid which is contained between two closely adjacent solids presenting parallel plane surfaces to each other, can be similarly calculated on the approximate assumption that the upper surface of the liquid meniscus is, in this case, a segment of a circular cylinder. If the plane surfaces are vertical, and at a distance $2b$ apart, the approximate equation, analogous to equation II, 21, for the rise to the edges of the meniscus is

$$h = \frac{\pi}{4} b \frac{1 - \sin\theta}{\cos\theta} + \frac{2\tau}{mgb} \cos\theta. \quad . \quad . \quad . \quad . \quad . \quad \text{II, 23}$$

For accurate solutions of the problems treated approximately in this section, with detailed calculations of the forms of the menisci, as well as an account of other aspects of the subject of Capillarity, the reader may consult Ref. II, 9, and the bibliography therein. For a somewhat less elaborate account of the subject, see Ref. II, 6.

13. Flow of Viscous Fluids in Tubes.

On the principles which have been explained in the preceding sections of this chapter, we can proceed to calculate the flow of viscous fluids in various cases of practical interest. Take first the case of a viscous fluid flowing through a tube of circular section, of which the length l is assumed to be large compared to the diameter. The fluid flows through the tube in consequence of a difference of pressure $p_1 - p_2$ maintained between its two ends. The effect of gravity is disregarded, or, if it is included, $p_1 - p_2 + gmh$ is to be written instead of $p_1 - p_2$ throughout, h being the difference of level between the ends of the tube, and m the mean density of the fluid taken over the whole length l. The fluid may be either a liquid or a gas.

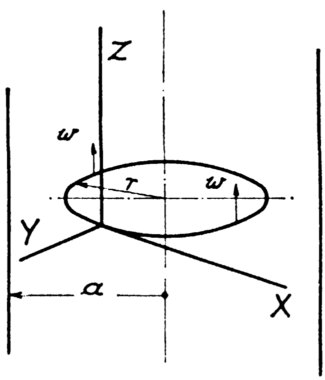


Fig. II, 15

The motion at a distance from either end of the tube will be sensibly parallel to the axis, and the pressure (and consequently the density) of the fluid will be sensibly uniform over each normal section. Also, from symmetry, at any one section the velocity must be the same at all points at a given radius r from the axis of the tube. Let w be the velocity at this radius, p being the pressure at the section, and a the radius of the tube (see fig. II, 15).

The mass discharged per unit time, which must be the same for all sections, is

$$\int_0^a mw \,.\, 2\pi r \,.\, dr = Qm. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 24}$$

The tube is supposed to be so nearly straight that effects due to its curvature can be neglected, and the tangent line to its axis, at the section considered, is taken as the co-ordinate axis Z. In the first instance it is to be supposed that the motion is so slow that the kinetic energy of the fluid is inappreciable, or alternatively, that the fluid is incompressible.

Let the co-ordinates x and y be taken, at any point of the fluid, tangential to the circumference, and radially outwards, respectively, then, since the velocity w does not vary in the circumferential direction, and since there are no circumferential or radial velocities u or v, the differentials in the second and third of equations II, 14 all vanish, and the stresses s_{zx} and s_{xy} are both shown to be zero, these being respectively the stresses in any plane normal to the axis and in any radial plane. The first of equations II, 14 is the only one of the three that remains, viz.

$$s_{yz} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right),$$

and this is reduced to $$s_{yz} = \mu \frac{\partial w}{\partial y}$$

since v is zero throughout. This equation may be written as

$$s_{rz} = \mu \frac{\partial w}{\partial r},$$

since y is in the direction of the radius, and it expresses that there is an upward traction on each unit of area of the cylindrical body of fluid inside the cylindrical surface of radius r, which is proportional to the rate of increase of the velocity outward along the radius. Considering a disc of this cylinder, of length δz, the total traction on its peripheral surface, whose area is $2\pi r\,\delta z$, must be equal to the difference of the pressures on its plane ends, due allowance being made for gravity. Thus

$$2\pi r s_{rz}\,\delta z = 2\pi r\mu \frac{\partial w}{\partial r}\,\delta z = \pi r^2 \frac{\partial p}{\partial z}\,\delta z,$$

or $$\frac{\partial w}{\partial r} = \frac{r}{2\mu}\frac{\partial p}{\partial z},$$

and therefore, integrating, $$w = \frac{1}{2\mu}\left(\tfrac{1}{2}r^2 - C\right)\frac{\partial p}{\partial z}.$$

Since $w = 0$ when $r = a$, $C = \frac{1}{2}a^2$

and $$w = -\frac{1}{4\mu}\frac{\partial p}{\partial z}(a^2 - r^2), \qquad \text{II, 25}$$

the negative sign of the right-hand member of the equation indicating that the flow is in the opposite direction to the increase of pressure.

From equations II, 24 and 25, the total mass flow Qm through the tube is

$$Qm = \int_0^a mw \,.\, 2\pi r\,dr = -\int_0^a \frac{m}{4\mu} 2\pi r(a^2 - r^2)\frac{\partial p}{\partial z}\,dr$$

$$= -\frac{2\pi m}{4\mu}\left[\tfrac{1}{2}a^2r^2 - \tfrac{1}{4}r^4\right]_0^a \frac{\partial p}{\partial z}$$

$$= -\frac{\pi m a^4}{8\mu}\frac{\partial p}{\partial z}. \qquad \text{II, 26}$$

In the case of a liquid m and μ may usually be taken as uniform along the length of the tube, so that $\partial p/\partial z$ is also uniform along the length and equal to $(p_2 - p_1)/l$, where p_1, p_2 are the pressures at the inlet and outlet ends, respectively, of the tube of length l, then

$$Qm = \frac{\pi m a^4}{8\mu}\frac{p_1 - p_2}{l}. \quad \text{II, 27}$$

In the case of a gas, $m = \frac{p}{RT_a}$,

T_a being the absolute temperature and R a constant known as the "gas constant". Then, from II, 26

$$Qm\,dz = -\frac{\pi a^4}{8\mu RT_a} p\,dp, \quad \text{II, 28}$$

and if T_a and μ can be regarded as constant throughout the length of the tube, equation II, 28 when integrated gives

$$Qml = \frac{\pi a^4}{16\mu RT_a}(p_1^2 - p_2^2),$$

or

$$Qm = \frac{\pi a^4}{16\mu l RT_a}(p_1^2 - p_2^2), \quad \text{II, 29}$$

as the mass of gas flowing through the tube.

For any perfect gas the value of R is $83{\cdot}15 \times 10^6$ ergs per gramme molecule; the value of R for 1 gramme of the gas is obtained by dividing this number by the molecular weight W_m of the gas, e.g. in the case of oxygen by $W_m = 16$ (Ref. II, 10).

In a tube of elliptical internal section whose major and minor axes are of lengths $2a$ and $2b$ respectively, the contour being defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(with the origin of co-ordinates at the centre), the velocity at any point in the tube is given by

$$w = -\frac{1}{2\mu}\frac{dp}{dz}\frac{a^2b^2}{a^2+b^2}\left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right), \quad \text{II, 30}$$

being zero at all points of the perimeter, since the last factor on the right-hand side of the equation is zero there.

The mass flow of a liquid through the tube is

$$Qm = m\int_{-a}^{a}\int_{-b}^{b} w\,dx\,dy = \frac{\pi m}{4\mu}\frac{a^3b^3}{a^2+b^2}\frac{p_1 - p_2}{l}. \quad \text{II, 31}$$

The value of w and Qm given by equations II, 30 and 31, reduce to those given by equations II, 25 and II, 27 respectively for the velocity and mass-flow in a circular tube when b is made equal to a. It is easily shown from these four equations that, both in the circular and the elliptical tubes, the maximum velocity of flow, which occurs at the centre of the section in each case, is double the mean velocity. For the complete solution of the elliptical tube see Ref. II, 11.

It has already been remarked in Sect. II, 3 that the resistance of fluids to pure shear is only directly proportional to the rate of shear so long as the velocity does not exceed certain limits depending on the viscosity of the fluid and the linear scale of the body of the fluid in which the shearing motion takes place. Since the viscous motions in tubes which have been discussed in this section are essentially motions of shearing strain and stress, it is to be expected, and is confirmed by experiment, that they are subject to similar limitations.

Above a critical velocity characteristic of the fluid and the diameter of the tube which are concerned, the velocity and volume of discharge are no longer directly proportional to the rate of fall of pressure along the tube (with a constant coefficient μ), but more nearly proportional to the square root of the fall of pressure. The laws of flow and the values of the constants involved will be given, together with tables and diagrams for viscous flow, in a later chapter (Sects. X, 1 and 2) dealing with the supply and distribution of fluids for the purpose of lubrication.

In deriving the equations of this section, the kinetic energy of the fluid has been treated as negligible. In the case of a liquid, however, all the formulæ remain correct even when its kinetic energy is appreciable, provided that they are not applied to the end portions of the tubes where the flow is modified by the conditions of entry or discharge. At the ends the variation of velocity across the sectional area of the tube will not, in general, follow the same law as in the middle portions, where it is associated with a uniformly varying pressure along the length of the tube. There are in consequence resistances to the flow arising from accelerations and retardations near the inlet and outlet, and from disturbances there to the uniform rates of shear in the middle portions of the tube. In the case of a tube which is square-ended, that is to say, is cut off at each end on a plane transverse section, and is not provided with any "bell-mouth" or other modification of its form to facilitate entry of the fluid, the flow is reduced to approximately the same extent as if there were a diminution of the fall of pressure between the ends of amount $1{\cdot}12\bar{w}^2/(2g)$, where $\bar{w}$ is the mean velocity in the tube, as calculated from the formulæ given in this section or as found by observation.

Equation II, 27 given above for the flow of a liquid in a tube of circular section expresses three laws which were discovered, in the first instance for water, by Poiseuille (Ref. II, 12) as the result of a long series of very accurate experiments, before they had been deduced mathematically from "Newton's Law of Viscosity". These three laws are:

The volume of flow in a given time is proportional to:

1. The difference of the pressures at the two ends of the tube (Law of Pressures);
2. The fourth power of the diameter of the tube (Law of Diameters);
3. The reciprocal of the length of the tube (Law of Lengths).

It is on account of the fundamental importance of these experiments of Poiseuille that the unit of viscosity, μ, in the C.G.S. system, has been called the *poise*. It is to be noted, however, that Poiseuille did not employ that unit, or any other "coefficient of viscosity" based on the conception of a condition of shear in the fluid. From the circumstance that the tubes upon which he made his principal measurements were all of very small diameter (of the order of, and for the greater part less than, one millimetre), he did not detect any limitation to the velocities of flow for which the laws he discovered are true. Such limitation seems to have been first recognized, and was to some extent investigated, by Darcy (Ref. II, 13).

14. Flow in Any Direction between Parallel Planes.

In Sects. II, 4 to 7, the motion of a fluid moving in *one* direction between a pair of parallel planes has been dealt with. In many applications of the theory of viscous fluids to the problems of lubrication, the fluid motions must be treated as two-dimensional, taking place in *any* direction parallel to the planes.

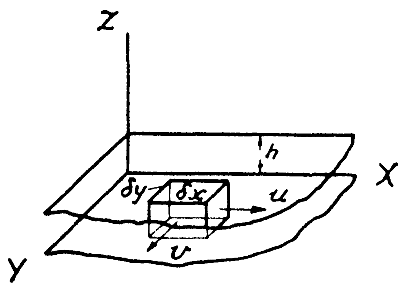


Fig. II, 16

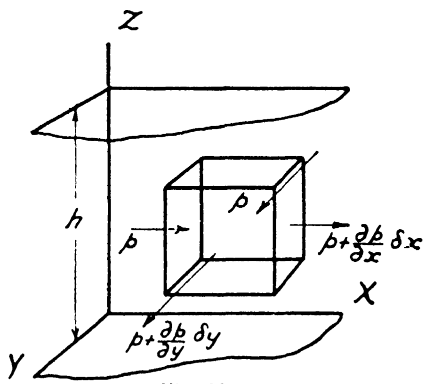


Fig. II, 17

As before, the axis of Z is taken normal to the planes, one of which is defined by $z = 0$, and the other by $z = h$, h being small in comparison with the extent of the planes in the X and Y directions, as indicated in fig. II, 16. The planes are here assumed to be fixed. For the reasons explained in Sect. II, 9, the components of velocity normal to the planes may be regarded as being everywhere negligible. In other words, the rates of shear, and the momentum, of the fluid in the Z direction can be neglected, and the fluid pressure p can be considered not to vary in that direction but to be a function of x and y only. On the other hand, the rates of change of the components of velocity parallel to the planes, i.e. the components of u and v, are, in general, rapid in the direction of Z compared with their rates of change in directions parallel to the planes. On the planes, at $z = 0$ and $z = h$, u and v are everywhere zero.

Consider a rectangular element (fig. II, 17) anywhere between the planes $z = 0$ and $z = h$; the viscous tractions on its lower face, in the directions of X and Y, are, from equation II, 5, respectively,

$$-\mu \frac{\partial u}{\partial z} \delta x \delta y \text{ and } -\mu \frac{\partial v}{\partial z} \delta x \delta y.$$

The corresponding tractions on the upper face are

$$\mu\left(\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial z^2} \delta z\right) \delta x \delta y \text{ and } \mu\left(\frac{\partial v}{\partial z} + \frac{\partial^2 v}{\partial z^2} \delta z\right) \delta x \delta y.$$

The algebraical sums of these pairs of tractions, added to the differences of the fluid pressures on the pairs of faces parallel to YZ and ZX, respectively, are equal to any increases that there may be in the momenta of the rectangular element in the X and Y directions respectively, thus

$$\left(\mu \frac{\partial^2 u}{\partial z^2} - \frac{\partial p}{\partial x}\right) \delta x \, \delta y \, \delta z = m \, \delta x \, \delta y \, \delta z \frac{du}{dt}$$

or

$$\left.\begin{aligned} \mu \frac{\partial^2 u}{\partial z^2} &= \frac{\partial p}{\partial x} + m \frac{du}{dt} \\ \text{and similarly,} \quad \mu \frac{\partial^2 v}{\partial z^2} &= \frac{\partial p}{\partial y} + m \frac{dv}{dt}. \end{aligned}\right\} \quad \text{II, 32}$$

In the applications which will be made of the present section to problems of lubrication the momentum terms $m\,du/dt$ and $m\,dv/dt$ are relatively small, and will therefore be neglected in the paragraphs which follow.

The equations II, 32 then reduce to

$$\left.\begin{aligned} \mu \frac{\partial^2 u}{\partial z^2} &= \frac{\partial p}{\partial x} \\ \text{and} \quad \mu \frac{\partial^2 v}{\partial z^2} &= \frac{\partial p}{\partial y}. \end{aligned}\right\} \quad \text{II, 33}$$

Since p is independent of z, these equations can be directly integrated and give

$$\frac{\partial u}{\partial z} = \frac{1}{\mu} \frac{\partial p}{\partial x} (z + C_1),$$

$$\therefore \; u = \frac{1}{\mu} \frac{\partial p}{\partial x} (\tfrac{1}{2} z^2 + C_1 z + C_2),$$

and similarly

$$v = \frac{1}{\mu} \frac{\partial p}{\partial y} (\tfrac{1}{2} z^2 + C_1 z + C_2),$$

the constants of integration, C_1 and C_2, being obviously the same in both of these last equations. Since u and v are both zero when $z = 0$, or $z = h$, it is at once seen by putting these values of z in succession into either of the equations that $C_2 = 0$, and that $C_1 = -\frac{1}{2}h$.

Therefore

$$\left.\begin{aligned} u &= \frac{1}{\mu} \frac{\partial p}{\partial x} \frac{z(z-h)}{2} \\ \text{and} \quad v &= \frac{1}{\mu} \frac{\partial p}{\partial y} \frac{z(z-h)}{2}. \end{aligned}\right\} \quad \text{II, 34}$$

The resultant velocity of the fluid at any point (x, y, z) is

$$U' = u^2 + v^2 = \frac{1}{\mu}\left\{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2\right\}^{\frac{1}{2}} \frac{z(z-h)}{2}, \quad . \quad . \quad \text{II, 35}$$

being in the direction of, and proportional to, the most rapid fall of pressure, and varying along each normal to the two planes according to a parabolic law, being zero on each plane and at a maximum midway between them.

If the volume of flow from plane $z = 0$, to plane $z = h$, is Q_x per unit width in the direction of Y (see fig. II, 16, p. 27), then since

$$Q_x \, \delta y = \delta y \int_0^h u \, dz$$

(from equation II, 34)

$$= \frac{\delta y}{2\mu} \frac{\partial p}{\partial x} \int_0^h (z^2 - zh) \, dx$$

$$= \frac{\delta y}{2\mu} \left[\tfrac{1}{3}z^3 - \tfrac{1}{2}z^2 h\right]_0^h \frac{\partial p}{\partial x} = -\frac{\delta y}{12\mu} h^3 \frac{\partial p}{\partial x},$$

$$\therefore \; Q_x = -\frac{h^3}{12\mu}\frac{\partial p}{\partial x}$$

and, similarly,

$$Q_y = -\frac{h^3}{12\mu}\frac{\partial p}{\partial y}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 36}$$

Thus the total flow in any direction across a unit width perpendicular to that direction is equal to the rate of *decrease* of the pressure in that direction multiplied by the constant $h^3/(12\mu)$.

The same relation evidently holds for the flow of a viscous liquid in the space between two fixed concentric cylinders, in either the axial, circumferential, or any oblique direction, provided that the radii of the cylinders are so nearly equal that their difference can be neglected in comparison with each of them.

In all these cases, it is evident, considering any small rectangular element δx, δy, of the interspace, that since the same amount of the fluid must flow out of the element in unit time as flows into it,

$$\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y}\right) \delta x \, \delta y = 0,$$

and therefore, from equation II, 36,

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 37}$$

it being noted that the surfaces $z = 0$ and $z = h$ are fixed relatively to each other.

15. Flow between Parallel Planes having Relative Motion Parallel to each other.

If the plane $z = h$ (fig. II, 16, p. 27), while remaining at distance h from the plane $z = 0$, is moving parallel to that plane with components of velocity u_1 and v_1 in the X and Y directions, uniform rates of shear u_1/h and v_1/h in these two directions will be superimposed on the varying velocities u and v determined by the equations II, 34.

The components of velocity at the point (x, y, z) will then become

$$\left.\begin{aligned} u' &= \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{z(z-h)}{2} + u_1\frac{z}{h} \\ \text{and} \qquad v' &= \frac{1}{\mu}\frac{\partial p}{\partial y}\frac{z(z-h)}{2} + v_1\frac{z}{h}, \end{aligned}\right\} \quad \text{II, 38}$$

but neither the pressures nor the relation $\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0$ will be affected.

16. Flow between Parallel Planes having Relative Motion Normal to each other.

If the plane $z = h$, instead of being moved in its own plane, as in Sect. II, 15, is caused to move normally away from the plane $z = 0$, with velocity dh/dt, so that the distance between the planes increases at this rate, it is evident that, while the volume of the element $h\,\delta x\,\delta y$ (see fig. II, 16) is increased by this movement at the rate $(dh/dt)\,\delta x\,\delta y$, there must be an excess of inflow over outflow through the vertical sides of the element to fill its additional volume. Expressing this in symbols,

$$-\delta y\frac{\partial Q_x}{\partial x}\delta x - \delta x\frac{\partial Q_y}{\partial y}\delta y = \frac{dh}{dt}\delta x\,\delta y$$

or

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -\frac{dh}{dt},$$

and consequently, from equations II, 36,

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{h^3}\frac{dh}{dt}. \quad \text{II, 39}$$

As an important special case, having several applications in connexion with lubrication, consider the case of two parallel circular discs of radius a, and suppose that they are in communication all round their circumferences with a large body of fluid at a uniform pressure Π, while the upper plane is moved normally upwards with respect to the lower. It is evident from the symmetry that the inflow between the planes will be everywhere radial and that the pressure will diminish symmetrically from Π at radius a to a minimum at the centre. Taking, instead of a rectangular element as in figs. II, 16 and II, 17, an element (see fig. II, 18) extending through the whole height between the planes and contained between two radial planes at a small angle $\delta\alpha$ apart and between radii r and $r + \delta r$, its rate of increase of volume as the upper plane rises at the rate dh/dt will be $(dh/dt)\,\delta r\,.\,r\,.\,\delta\alpha$. This must be equal to the

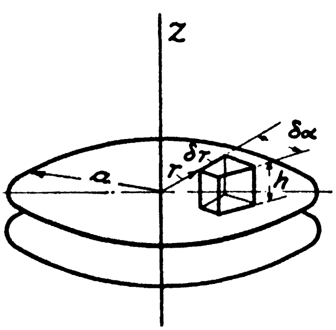


Fig. II, 18

rate of increase of the inward radial flow from r to $r + \delta r$, and therefore, as in either of the equations II, 36,

$$\frac{dh}{dt} \cdot \delta r \cdot r \cdot \delta\alpha = \frac{\partial}{\partial r}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial r} r \cdot \delta\alpha\right)\delta r,$$

or

$$\frac{12\mu}{h^3}\frac{dh}{dt} r = \frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right).$$

Integrating,

$$\frac{6\mu}{h^3}\frac{dh}{dt} r^2 = r\frac{\partial p}{\partial r} + C.$$

Since the radial velocity is zero at the centre, from II, 36, $\partial p/\partial r = 0$ when $r = 0$, therefore $C = 0$, and everywhere

$$\frac{\partial p}{\partial r} = \frac{6\mu}{h^3}\frac{dh}{dt} r.$$

Integrating again,

$$p = \frac{3\mu}{h^3}\frac{dh}{dt} r^2 + D;$$

but when

$$r = a, \ p = \Pi,$$

$$\therefore \ \Pi = \frac{3\mu}{h^3}\frac{dh}{dt} a^2 + D,$$

so that

$$p = \Pi - \frac{3\mu}{h^3}\frac{dh}{dt}(a^2 - r^2). \qquad \text{II, 40}$$

By integrating $p - \Pi$ over the area of the disc, the total force P necessary to maintain the motion of the disc against the viscous resistance is obtained. Thus

$$\begin{aligned} \bar{P} &= \int_0^a 2\pi r \frac{3\mu}{h^3}\frac{dh}{dt}(a^2 - r^2)\,dr \\ &= \frac{6\pi\mu}{h^3}\frac{dh}{dt}\int_0^a (a^2 r - r^3)\,dr \\ &= \frac{6\pi\mu}{h^3}\frac{dh}{dt}\left(\tfrac{1}{2}a^4 - \tfrac{1}{4}a^4\right) \\ &= \frac{3\pi\mu}{2h^3} a^4 \frac{dh}{dt}. \end{aligned} \qquad \text{II, 41}$$

With change of sign, the same expression gives the downward force which must be applied to the upper disc in order to cause it to approach the lower disc with velocity dh/dt, the flow between the discs being then outwards, and the pressures between them greater than Π.

17. Motion of a Sphere in a Viscous Fluid.

Another solution of general interest in connexion with lubrication, and especially as giving a direct (if, in practice, only approximate) means of determining the coefficient of viscosity of the fluid in absolute measure, is that of the steady motion of a sphere through the fluid.

The solution, given originally by Stokes (Ref. II, 14) and known by his name, is derived on the assumptions that the sphere has been acted on by a force, constant in magnitude and direction, for a sufficiently long period of time for the conditions of velocity and pressure to have become steady throughout the fluid, which is assumed to extend to distances very great in all directions compared with the radius of the sphere. At such great distances all the parts of the fluid are supposed to be at rest relatively to one another, and the pressure to be zero. The origin of rectangular co-ordinates is taken at the centre of the sphere with the Z axis in the direction of its velocity W, which is constant relatively to the fluid at a distance. The X and Y axes are taken in any two directions perpendicular to each other and to OZ, the component velocities at any point (x, y, z) of the fluid, being u, v, and w relative to the frame of co-ordinates which moves with the sphere. Stokes proved that

$$\left.\begin{aligned} u &= \frac{3WR}{4}\frac{xz}{r^3}\left(1-\frac{R^2}{r^2}\right), \\ v &= \frac{3WR}{4}\frac{zy}{r^3}\left(1-\frac{R^2}{r^2}\right), \\ w &= \frac{3WR}{4}\frac{z^2}{r^3}\left(1-\frac{R^2}{r^2}\right) - W\left(1-\frac{3R}{4r}-\frac{R^3}{4r^3}\right), \end{aligned}\right\} \quad . \quad . \quad \text{II, 42}$$

in which R is the radius of the sphere, and r the distance to its centre from the point (x, y, z), so that $r^2 = x^2 + y^2 + z^2$.

It is immediately evident from equations II, 42, that the component velocities u, v, and w are all zero when $r = R$, thus complying with the necessary condition that there is no relative motion, either normal or tangential, of the fluid relatively to the solid sphere. It is likewise readily seen that u and v both vanish when r is very large compared with R, so that there is no motion in any radial direction at a great distance from the line of motion of the sphere. The component w, however, when r is taken as being very large compared with R, becomes $-W$, expressing merely the fact that, at a great distance in the direction of motion, the velocity of the fluid with respect to the sphere is equal and opposite to that of the sphere through the fluid as a whole.

It also appears from Stokes' analysis that the pressure in the fluid at any point is given by

$$p = \tfrac{3}{2}\mu WR\,\frac{z}{r^3};$$

it is therefore, according to the initial assumption, zero at all points at a great distance. It is also zero at all points of the equatorial plane $z = 0$, and has equal positive and negative values at corresponding points of the two sides of that plane.

The total force on the sphere due to the viscous resistance of the fluid acts in the direction opposite to its motion and is of magnitude

$$P = 6\pi\mu RW, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{II, 43}$$

being proportional to the radius of the sphere.

In the case of a sphere of density m falling by its own weight steadily through an unlimited fluid of density m', from equation II, 43,

$$P = \frac{4\pi}{3}\left(m - m'\right)gR^3 = 6\pi\mu RW,$$

and therefore

$$W = \tfrac{2}{9}\,\frac{m - m'}{\mu}\,gR^2, \qquad \text{II, 44}$$

and the time of falling through a given height H is

$$t = \frac{9\mu H}{2(m - m')gR^2} \qquad \text{II, 45}$$

Equation II, 43 can be applied to form approximate estimates of the rate of motion of small globules of liquid, shaped by surface tension into nearly the form of spheres, through another fluid, e.g. raindrops falling through air, or globules of oil rising under gravity through water. In such applications of the formula it has however to be remembered, not only that the globule is subject to reactions from the fluid through which it moves, these reactions tending (unless it is extremely small) to deform it appreciably from the assumed spherical shape, but also that, when it is a question of the flow of one fluid over another, it is no longer true (as is the case when one of the bodies is a solid) that there is no tangential motion at the common surface. (See Sect. II, 9, especially its later paragraphs.)

A close approximation to fact can, for this reason, only be obtained by the application of equation II, 44 if the liquid of the sphere has a much higher coefficient of viscosity than that of the fluid through which it moves.

Another limitation to direct applications of equation II, 44 arises from the necessarily limited extent of the fluid in practical cases. Modifications of, and additions to, the theoretical treatment for the purpose of taking account of the presence of bounding walls of various usual shapes and locations have been made by various authors. For discussions of these, and the whole subject of the present section, the reader may consult Refs. II, 15 and II, 16.

CHAPTER III

Numerical Constants of Lubricating Fluids

1. Apparatus for Measuring Viscosities in Absolute Units.

The application of the laws of fluids which have been discussed in Chapter II to the practical problems of lubrication involves, of course, quantitative knowledge of their properties, and especially of their coefficients of viscosity.

In principle, the determination of the viscosity of a fluid is very simple, since it involves, as shown by (e.g.) equation II, 5, only simultaneous measurement, in standard units, of a shearing force and a rate of shear on an element of the fluid. When these two measurements have been made the coefficient of viscosity is at once obtained in absolute measure as the quotient of the one figure by the other. In the carrying out of this direct method of measurement, however, difficulties arise due, on the one hand, to the very small thicknesses of the laminæ of ordinary fluids with which it is necessary to work to obtain shearing forces of readily measurable magnitude, and on the other hand, to the difficulty of securing uniformity of shear and shearing force over an area sufficiently large for convenient measurements.

In practice, therefore, most of the accurate determinations which have been made of fluid viscosities have been less direct, being based on the Law of Poiseuille for the viscous flow of fluid in tubes of capillary dimensions, as expressed, for liquids and gases respectively, in equations II, 27 and 29. The essential features of the apparatus used for the purpose, with minor variations in the hands of Poiseuille and other investigators, are shown more or less diagrammatically in fig. III, 1. It consists primarily of a capillary tube C, as nearly uniform as may be in bore, and of length preferably a hundred or more times as great as the diameter of the bore. This tube is inserted between the two limbs, A_1 and A_2, formed of glass tubing, each limb comprising a central measuring vessel, B_1, B_2, connected by narrow necks to smaller bulbs above and below it. The necks are engraved with marks m_1, m_2 on the one limb, and m_1', m_2' on the other, so that the volume of the vessel B_1, between the marks m_1 and m_2, is approximately equal to that of the vessel B_2 between the marks m_1' and m_2'. The capillary tube and limbs are enclosed in a glass-walled water-bath D, which is in circuit, as shown in the figure, with another vessel E containing a heating coil (or, according to requirements, a cooling coil), and a propeller P by means of which the water is circulated through the vessel E and the bath D, each of which is suitably lagged with non-conducting materials and provided with a thermometer.

The remainder of the apparatus consists of a source of compressed air, with reservoir R and manometer M, and cocks for admitting the compressed air alternately to either of the measuring limbs while the other is switched to the atmosphere.

In the use of the apparatus the fluid to be tested, if a liquid, is filled into the capillary tube and limbs up to the mid-height of each measuring vessel.

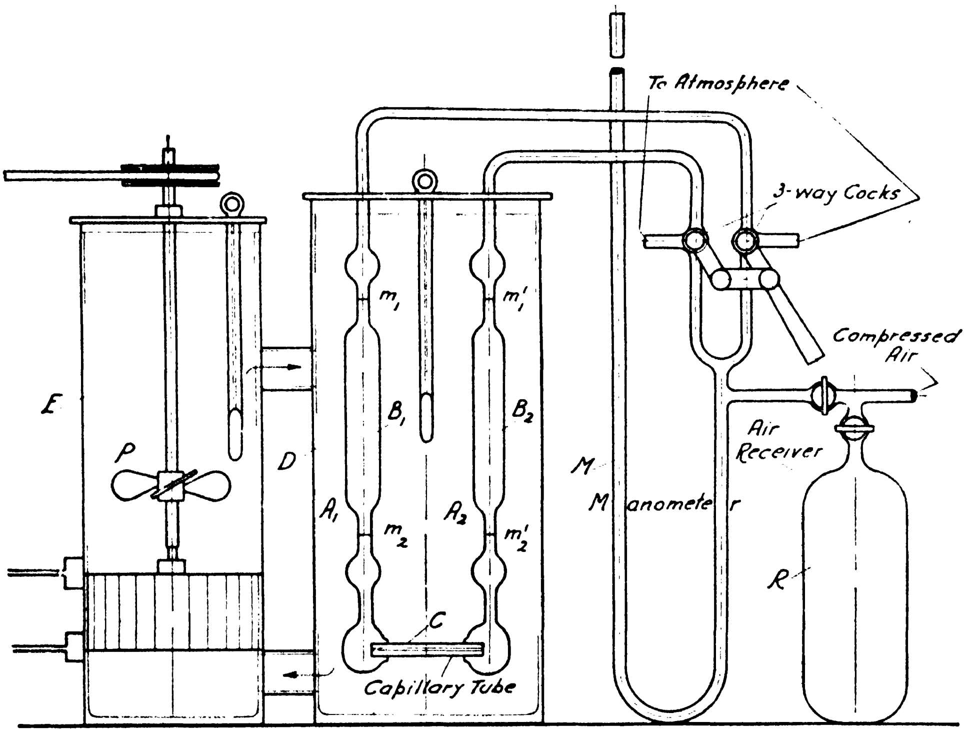


Fig. III, 1.—Apparatus for the determination of the absolute viscosity of liquids

Compressed air is first admitted to one of the limbs, say A_2, until the liquid surface stands in the lower bulb on that side and the upper bulb on the other, the air cocks are then reversed so as to cause the liquid to flow from the A_1 to the A_2 limb under a difference of pressure Δp measured by the manometer, and the time interval between the passage of the liquid surface on the A_1 side from the mark m_1 to the mark m_2 is observed, the flow being thereafter allowed to continue until the two surfaces stand in the lower bulb on that side and the upper bulb on the other. The air cocks are then switched over, admitting air to the A_2 limb, and the time of passage of the surface on that side from the mark m_1' to the mark m_2' observed; this double reading is repeated a sufficient number of times to give a reliable average. During each displacement of the fluid from one limb to the other, the effective pressure varies on account of the gravitational effect of the varying heights of the column in each limb, and a correction must be made on this account, although the mean effective pressure does not usually differ to more than a small extent from the pressure difference when the liquid is at the same height in both limbs.

Another correction has to be applied to take account of the loss of kinetic energy of the fluid in its entrance into, and discharge from, the capillary tube. The proportion which this loss bears to the work done in overcoming the viscous resistance increases in direct ratio to the speed of the flow, and accordingly forms the limiting condition to the rapidity with which the measurements can be carried out. It is generally accepted that the correction to be applied in order to take account of this effect, when both the entry and discharge ends of the capillary tube are accurately square-edged, is a deduction from the observed effective pressure of 1·12 times the kinetic energy corresponding to the mean velocity in the tube. (For a full discussion of the question, see Ref. III, 1.)

For accurate results it is essential that the diameter of the bore of the capillary tube at all points of its length shall be very exactly determined, since, according to Poiseuille's law, the value determined for the coefficient of viscosity varies proportionately to the fourth power of the diameter. The diameters at the ends of the tube can be found by direct microscopic measurement; those at other points of its length are usually determined by introducing a known weight of mercury into the tube and measuring the length of the column which it forms at different locations throughout the length. Full accounts of the details of the apparatus used and results obtained for various liquids will be found in Refs. II, 11 and II, 12, and also III, 2–5. The last two of these contain accounts of the use of the method for oils of the range of viscosity appropriate to lubricants; Ref. III, 2 deals especially with the determination of the viscosity of water.

An apparatus of the same kind can be applied to the determination of the viscosity of a gas, the volume of gas to be dealt with being usually confined between two volumes of mercury, or other liquid in which the gas is not soluble. How this may be effected is illustrated in fig. III, 2, which shows the capillary tube and measuring limbs of fig. III, 1 as being inverted and in communication with vessels of mercury G_1, G_2. Initially the mercury stands at the same level in the four vessels B_1, B_2, G_1, G_2, at mid-height of the former two, and the gas under test is confined above the mercury in the upper halves of the two measuring vessels and in the capillary tube C. The fluids are displaced from the one set of vessels to the other by compressed air acting above the columns of mercury in the vessels G_1 and G_2, and the processes of measurement are, in principle, the same as in the measurement of liquid viscosities. The relatively low values of the coefficients of viscosity of gases (roughly of the order of one-hundredth of that of water) necessitates the use of capillary tubes of much smaller bore, and of greater length, than those used for liquids.

Another kind of apparatus, used for determining the viscosities of highly viscous liquids and semi-solids, in absolute units, is the co-axial cylinder machine shown in fig. III, 3, and known as the Couette-Hatschek apparatus (Ref. III, 6). This consists essentially of an outer cylinder E which can be rotated at constant speed about its axis, as by the toothed wheel shown, and of an inner co-axial cylinder A which is suspended by the torsion wire W, carrying near its lower end a small mirror M. The fluid, whose viscosity is to be measured, fills the space between the two cylinders, which is narrow relatively to the radius of the inner, stationary, cylinder, so that when the outer cylinder is rotated

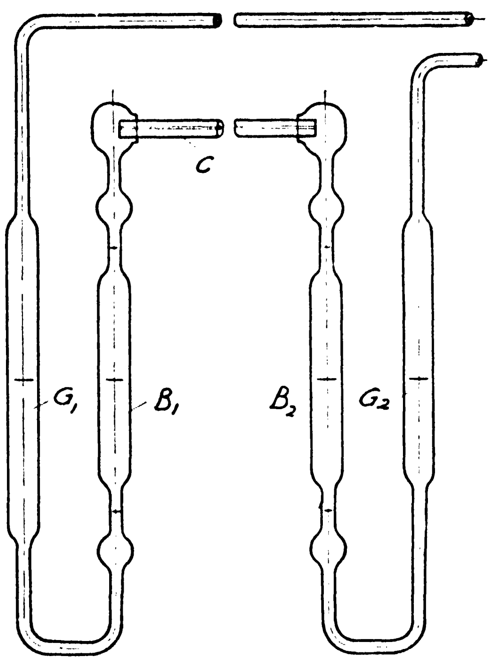


Fig. III, 2.—Apparatus for the determination of the absolute viscosity of gases

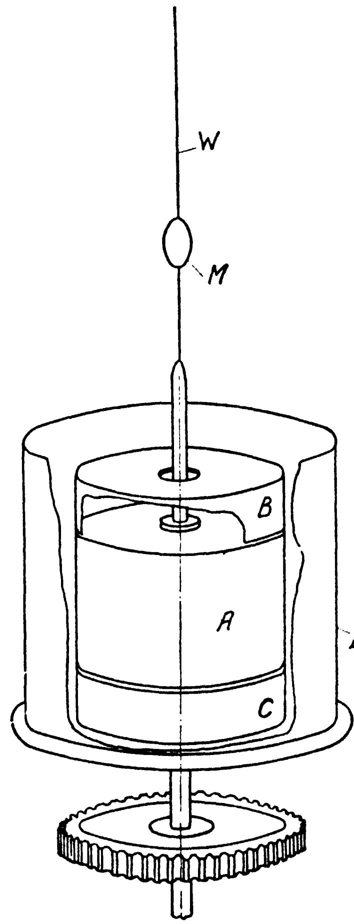


Fig. III, 3.—Couette-Hatschek viscometer

the motion of the fluid approximates to pure laminar shear between the two walls. The shearing force is measured by the torsional resistance of the previously calibrated suspending wire W, as indicated by deflection of a beam of light by the mirror M. In order to prevent disturbance of the laminar motion by the resistance of the ends of the cylinder A, guard-rings B and C are fixed co-axially, and of the same diameter with it, above and below. Corrections for the limited length of the cylinder A are deduced by varying its length and simultaneously changing the lengths of the rings B and C, so as to keep the total length of the assemblage constant. Formulæ for the effects of the departure of the conditions of the fluid between two cylinders, with finite difference of radii, from the conditions between two planes, have been worked out by Margules, Ref. III, 7.

A simpler apparatus which is available for the direct determination of the viscosities of fluids in absolute measures derived from the dimensions of the apparatus itself, is the falling sphere of Stokes, already discussed in Sect. II, 17 (Ref. II, 14). In practice, equations II, 44 and II, 45, given in that section, have to be corrected to take account of the necessary presence of containing walls for the fluid at distances from the sphere comparable with its own radius, instead of at a relatively very great distance as postulated in the derivation of those equations. These corrections, which are best arrived at by making comparative tests with liquids of known viscosity, do not detract from the method of measurement being in principle an absolute one, and it is in fact more adaptable, in suitable cases, for making direct and approximately correct determinations of the viscosities of lubricating fluids than the other two methods which have been described previously in this chapter. The greatest practical difficulty is in the determination of the time of fall of the sphere with sufficient accuracy. This is best done either photographically (using transparent vessels for tests at low pressures, or in some cases Roentgen rays), or by electrical methods, which, however, are only applicable to the testing of non-conducting fluids. By the latter means of observation, Bridgman has determined the viscosities of various fluids, especially water, compressed in steel vessels, up to pressures as high as 12,000 atmospheres (Ref. III, 8), and various other investigators have determined the viscosities of lubricating liquids at less extreme pressures. To the latter work, references will be given when some of the results are stated in Sect. III, 9.

2. Secondary Viscometers of the Jet Type.

As already stated, it is usually necessary for making practical measurements of the viscosities of lubricating fluids to use a secondary viscometer, i.e. one whose constant, or constants, cannot be directly derived from its own measurements, but must be determined by comparison of its results with those of an absolute instrument, usually one of the capillary-tube type.

The best known, and most generally used of these secondary viscometers are the Engler, Redwood, and Saybolt instruments, which are respectively regarded as standard instruments in the Continent of Europe, in British countries, and in the United States. All three are of broadly similar construction, the measurement depending on the flow of the lubricant under gravity through a tube or " jet " which is too short in comparison with its diameter to give an " absolute " measurement, or even a measurement which can be related to an absolute value by a single constant. The instruments have therefore to be calibrated with an absolute viscometer over their whole range before their readings can be translated into absolute units. Furthermore, since the flow through the jet is determined by the gravitational head of the liquid under test, the quantity measured is not the true viscosity μ, but the "kinematic viscosity" μ/m, m being the density of the fluid, which must be known before the true viscosity can be deduced from the measurement.

The three instruments named are illustrated in outline in figs. III, 4–6 respectively, from which their general similarity will be evident.

The *Engler viscometer*, first described by C. Engler in 1885 (Ref. III, 9), is illustrated in figs. III, 4*a* and *b*, which show respectively a diagrammatic vertical section of the instrument, and a dimensioned detail section of the jet and its accessory parts.

The instrument is provided with a separate measuring vessel which is graduated to receive 200 cm.^{3} of the fluid, and has a total capacity of about 240 cm.^{3} The cup, into which the liquid is placed for test, is comparatively wide and shallow, being 10·6 cm. in diameter, and 3·2 cm. in effective depth from the gauge-points to which the fluid is brought at the start of a test, to the upper end of the jet. The cup is provided with a double-walled cover through which a thermometer enters directly into the liquid to be tested. It is also provided with a water-jacket for hot or cold water, with thermometer and a stirrer not shown in the figures. The double-walled cover is of marked service in preventing condensation of atmospheric moisture on the lower side of the cover and the upper surface of the test-fluid itself.

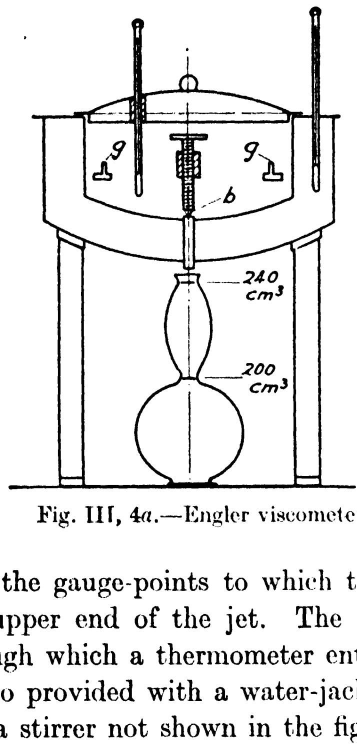


Fig. III, 4*a*.—Engler viscometer

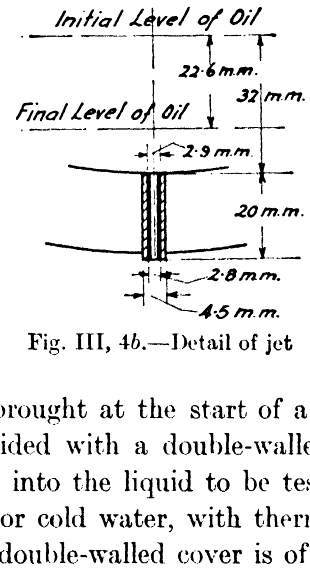


Fig. III, 4*b*.—Detail of jet

The jet of the Engler instrument, in its standard form, is lined with platinum to guard against corrosion. The jet is 2·0 cm. in length, 0·29 cm. in diameter at its upper end, and 0·28 cm. at the lower end, each diameter having a tolerance of 0·002 cm. In the use of the instrument, the valve b being closed, slightly more than 200 cm.^{3} of the oil to be tested is placed in the cup and brought to the temperature at which it is to be tested by means of the water-jacket. The surface of the oil is then slowly lowered by partially opening the stop-valve, the whole instrument being simultaneously levelled by bringing the surface of the oil precisely to the level of each of the three gauge-points g, which are spaced 120 degrees apart around the inner circumference of the cup.

The stop-tap is then closed and the empty 240-cm.^{3} measuring vessel is placed under the jet. The tap being again opened, the time of outflow of the oil to the 200-cm.^{3} mark on the neck of the container is observed by stop-watch, the temperature of the oil in the cup being noted immediately before, and during the outflow. A separate determination of the density of the oil at this temperature must be made, if the true viscosity of the oil is to be determined.

The time of discharge of 200 cm.^{3} of water at 20° C. of an Engler viscometer of standard dimensions is given as 51 seconds. An approximation to the absolute viscosity of the oil tested can be derived from this constant T_w, and the actual time of outflow of 200 cm.^{3} of the oil T, by the empirical formula

$$\frac{Z}{m} = 4{\cdot}072\,\frac{T}{T_w} - 3{\cdot}513\,\frac{T_w}{T}, \qquad \text{III, 1}$$

m being the density and Z the so-called "specific viscosity" of the oil, i.e. the ratio of its absolute viscosity at the temperature of the measurement to that of water at 0° C. which is 0·0179 poises. The absolute viscosity of the oil, as determined in this way, is therefore

$$\mu = Z \times 0{\cdot}0179 \text{ poises.}$$

This process of measurement and calculation is obviously only applicable, even as an approximation, to fluids whose viscosity is much greater than that of water. The crude ratio $T : T_w$ in which T_w may either be taken at the conventional value, 51 seconds, given above, or observed on a particular instrument, is sometimes used as a measure of the viscosity of the fluid, under the designation "Engler degrees", without reference to the density of the fluid, and without making any correction for the large inertia effects, varying from one fluid to another, at the outflow from the jet. An approximate rational formula giving the absolute viscosity μ in terms of the flow Q cm.^{3} per second, from a standard Engler instrument, or other viscometer of the same type, is

$$\mu = \frac{\pi}{8}\,\frac{p_0 - p_1}{l}\,\frac{a^4}{Q}\left(1 - \frac{mQ^2}{\pi^2 a^4 (p_0 - p_1)}\right), \qquad \text{III, 2}$$

in which a is the internal radius, l the length of the jet, and $p_0 - p_1$ the mean head producing the flow through the jet, all in absolute units (Ref. III, 10). The mean pressure $p_0 - p_1$ is usually calculated from the pressure differences at the beginning and the end of the test, viz. $(p_0 - p_1)_a$, and $(p_0 - p_1)_b$ by the approximate formula

$$(p_0 - p_1)_{\text{mean}} = \frac{(p_0 - p_1)_a - (p_0 - p_1)_b}{\log_e (p_0 - p_1)_a - \log_e (p_0 - p_1)_b} \qquad \text{III, 3}$$

The *Redwood viscometer*, of which there are two forms, intended respectively for testing fluids of high and low viscosities, was first described by the designer, Boverton Redwood, in 1886 (Ref. III, 11). The instrument of the No. 1 type, which is the form suitable for liquids of low or moderate viscosities, and the one in general use, is illustrated in outline in fig. III, 5. It consists of a cylindrical cup, fitted like the Engler instrument inside a water-bath, with thermometers, stirrer, and means for stopping the entry to the jet. It has, however, only a single gauge-mark for the starting level, and the normal capacity is only 50 cm.³ The length of the jet, which is of agate, is only 1·0 cm. and its diameter 0·15 cm. These dimensions, however, presumably on account of the difficulty of drilling a hole in agate accurately to a given size, are not rigidly standardized. The instrument is calibrated by adjusting the height of the gauge-mark so that 50 cm.³ of rape oil at 60° F. are discharged in 535 seconds.

The method of use of the instrument is similar to that of the Engler viscometer, the time of discharge of 50 cm.³ into the measuring vessel being recorded with simultaneous observation of the temperature of the liquid in the cup. The time of outflow is used as a measure of the viscosity of the liquid, under the name of "Redwood seconds". Tests (Ref. III, 12) have shown that the absolute viscosity of a liquid can be approximately deduced from the number of "Redwood seconds" T by means of the formula

$$\mu = m\left\{aT - \frac{b}{T}\right\} \quad . \quad . \quad \text{III, 4}$$

for values of T greater than 200 sec., m being the density of the fluid in grammes per cubic centimetre, and the constants a and b respectively $2{\cdot}60 \times 10^{-3}$ and 1·715 in C.G.S. units.

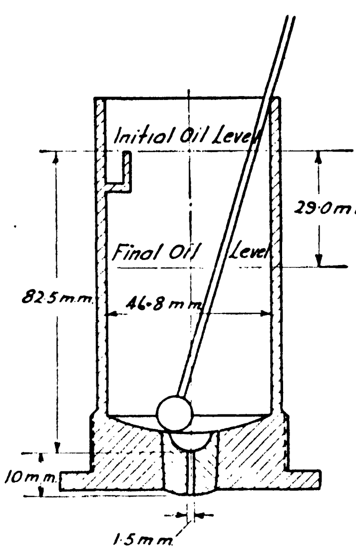


Fig. III, 5.—Outline section of the cup of a Redwood viscometer

The No. 1 Redwood Viscometer is commonly used for lubricating oils of average viscosity at normal machine-room temperatures. For testing oils which have to be used at low temperatures, or pumped through pipe-lines under winter conditions, the "No. 2" form (called sometimes the "Admiralty Fuel-oil Viscometer") has been introduced. In this instrument the bore diameter of the jet has been increased so that the time of outflow of an oil of high viscosity is only about one-tenth of that for the "No. 1" type. For these "No. 2" instruments the constants a and b in equation III, 4 are given in the Reference last quoted as being respectively $2{\cdot}70 \times 10^{-2}$ and 11·2.

Of the various forms of the *Saybolt viscometer*, the one in general use, known as the "Saybolt Universal", is illustrated in outline in fig. III, 6. The cup C in this instrument has the form of a wide tube surmounted at the top by a gallery G, and it is surrounded by a water-bath B which can be heated by a ring gas-burner D, or by an electric heating coil. The jet J at the bottom of the cup is closed, not by a valve inside the cup but, except when hot or corrosive liquids are tested, by a stopper inserted from below into an adjutage K which projects below the jet. The more important of the dimensions of the normal Saybolt Universal Viscometer, as accepted by the U.S. Bureau of Standards, are

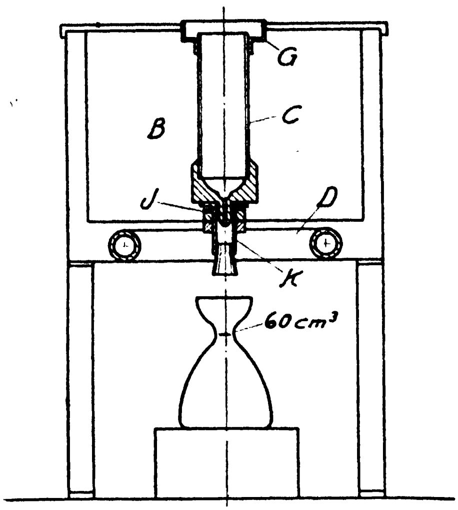


Fig. III, 6.—The Saybolt Universal Viscometer: outline section

	cm.
Length of outlet tube or "jet"	1·225
Diameter of bore of outlet tube	0·1765
Diameter of cup	2·975
Average head	7·36

(Refs. III, 13 and 14).

The standard quantity of fluid passed through the jet in a test, or "run", is 60 cm.^{3}, the measuring flask having a capacity of about 70 cm.^{3} The procedure in making a test, after the water-bath has been brought to the prescribed temperature, is as follows:

The oil to be tested is brought to approximately the correct temperature in a separate vessel; the jet of the viscometer is then cleaned with some of the oil to be tested, and the stopper inserted in the adjutage. The oil is then poured into the cup of the viscometer until it just begins to overflow into the upper gallery, this constituting the starting gauge-mark, and the temperatures of the water in the bath and the oil in the cup are finally adjusted. The measuring flask is then placed under the adjutage and the stopper quickly withdrawn, the stop-watch being started at the same moment. The stop-watch is stopped when the oil in the container flask reaches the 60 cm.^{3} mark.

The time in seconds required for the " run " of 60 cm.^{3} is taken as the measure of the viscosity of the oil, designated as " Saybolt seconds ".

It is, of course, as are the corresponding measurements of the Engler and Redwood viscometers, only related to the true viscosity of the liquid tested through the density of the latter. The approximate formula of equation III, 4 which enables the absolute viscosity μ to be calculated when the density m, and time of efflux T are known, can be used for the Saybolt Universal instrument with the values

$$a = 2{\cdot}20 \times 10^{-3}, \text{ and } b = 1{\cdot}80,$$

and the approximate formula of equation III, 3 for the mean head, can be used for the Saybolt as for the Redwood instrument.

It has to be emphasized that the direct readings of all viscometers of the short-capillary jet type (such as Saybolt or Redwood "seconds", or Engler "degrees") are only to be regarded as numbers roughly classifying the liquids to which they are applied, and are useless as quantitative measures of viscosity for the calculation of the motion of the liquid under given conditions. Since these crude numbers are, however, in general use in the literature, and in the commercial handling of lubricants, diagrams are given in Front Folders III, *a*, *b*, and *c*, which enable the viscosity of one of the more viscous lubricants to be determined with reasonable accuracy, when its "number", according to one or other of these instruments, and its density are known. These diagrams have been calculated in the main from equations such as III, 1 and III, 4 by inserting the accepted values of their constants for each instrument. They will enable values of the true viscosities of many lubricants, sufficiently accurate for most practical purposes, to be derived from the readings, otherwise quantitatively meaningless, of the conventional short-tube viscometers.

Of these commercial viscometers it has been well said by Dr. Hersey (Ref. III, 15):

> "The commercial viscometers so widely used in the petroleum industry are fitted with outlet tubes that are too short to be described as capillaries. For very high viscosities the efflux times are inconveniently long; for low viscosities they are complicated by kinetic energy and turbulent motion. In either case the flow takes place under a falling head; and the effective temperature of the sample is uncertain. The standardization of such instruments has brought about a reasonable degree of uniformity, but at the risk of perpetuating the complications. . . . It will be noted that the reading" [of the Saybolt instrument] "changes less than 10 per cent (i.e. from 32 to 35 seconds), for 100 per cent increase in viscosity in the range from 1 to 2 centipoises. Thus the commercial instruments are insensitive to differences of viscosity in the low-viscosity range; and at best give only the ratio of viscosity to density (kinematic viscosity), requiring a knowledge of the specific gravity before the viscosity can be computed."

The World Petroleum Congress at its meeting in London in 1933 adopted the C.G.S. unit (poise) as its standard measurement of viscosity. The industry, unfortunately, has failed as yet to adhere consistently to that decision.

3. Secondary Viscometers of other than the Jet Type.

A practical viscometer giving measurements which are proportional to absolute viscosities and readily convertible thereto, is the MacMichael viscometer or "Viscosimeter" which is illustrated, to some extent diagrammatically, in fig. III, 7 (p. 44).

Based on the same general principle as the Couette-Hatschek apparatus shown in fig. III, 3, this instrument consists essentially of a horizontal metal disc D suspended by a torsion-wire W so that it hangs near the flat bottom of a cylindrical cup C containing the liquid whose viscosity is to be measured. The cup is immersed in oil, or water, contained in an outer bath B, both the bath and the cup being carried upon, and partaking of the motion of, the rotatable table T.

A stiff hollow rod R, surrounding the wire W co-axially, is rigidly attached at its lower end both to the wire W and to the disc D; and at its upper end the hollow rod has a liner K through which the fixed upper end of the wire

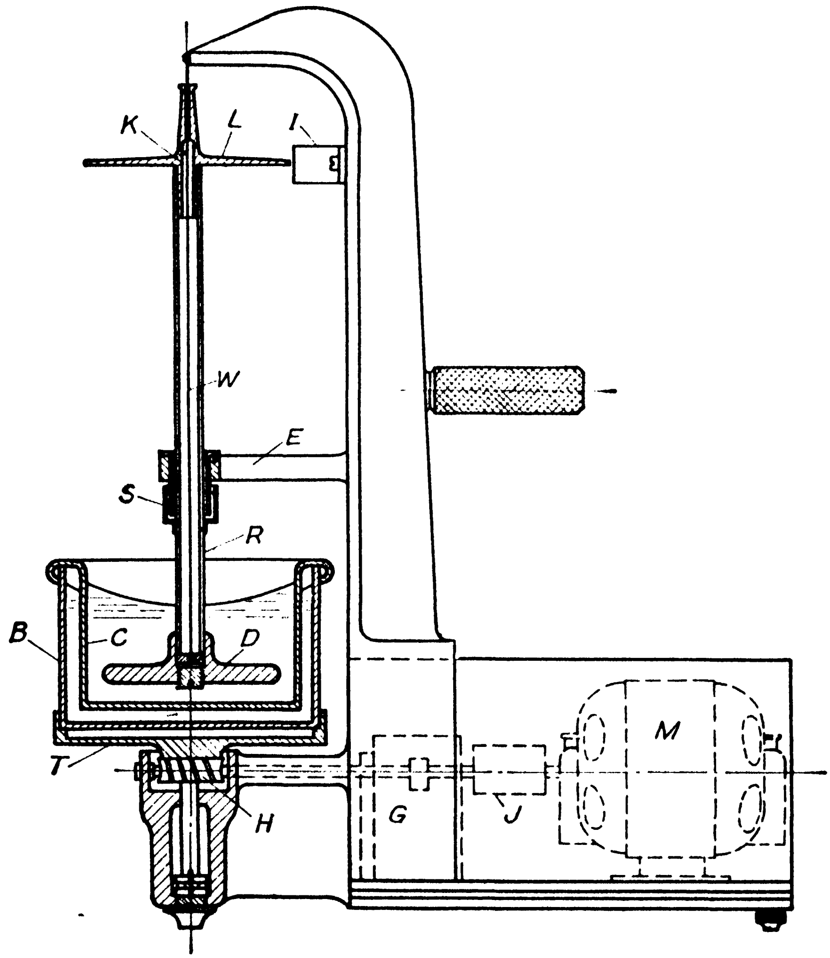


Fig. III, 7.—The MacMichael viscometer

passes with sufficient clearance to allow the tube to rotate freely. A circular dial-plate L, engraved with a circumferential scale, is rigidly attached to both liner K and tube R so as to rotate with the latter, and consequently with the disc D, a fixed index I showing the extent of its movement. A dashpot S, of which one member is fixed to the tube R, and the other is supported by the bracket E, and which contains a viscous oil, damps out rotary oscillations of the tube and disc.

In the use of the instrument, the table T is rotated at constant speed by the small electric motor M, the motion being transmitted through a reduction gear at G, and the worm and worm wheel shown at H. The speed of rotation is controlled by a rheostat, not shown, and by a governor on the motor spindle at J. Electrical heating coils and another rheostat, neither of which appears in the figure, are provided for maintaining the liquids in the bath B and the cup C at the temperatures required for the test.

When the table T is rotated, carrying with it the bath B and the cup C, the liquid in the latter imposes on the disc D a turning-moment which is very nearly proportional over a wide range to the viscosity of the liquid and to the speed of rotation, this speed being observed by means of a counter and a stop-watch, or by an electrically driven chronograph. The turning-moment on the disc, being resisted by the torsional stiffness of the wire W, is measured by the angle through which the dial L rotates relatively to the fixed index I. A reading on the dial is thus obtained which, at a given speed, is proportional to the viscosity of the fluid under test; this reading can be expressed in absolute units of viscosity when a constant has been determined for the apparatus, with the particular wire in use, by testing in it a liquid of known viscosity. Several wires, of different torsional rigidities, are provided with each instrument, thus enabling liquids having a wide range of viscosities to be tested.

It is a noteworthy characteristic of the MacMichael instrument that it can be applied to mixtures and colloidal solutions, such as paints, clay, glue, etc., as well as to oils and other liquids containing small quantities of fine solid particles in suspension. On the other hand, it is sufficiently sensitive to giv measurements of fluids with viscosities of the same order as that of water, by the use of a suitable torsion wire.

A simple form of secondary viscometer convenient for many applications is the Flowers instrument. This consists essentially of a glass tube containing a spherical ball of diameter considerably smaller than the bore of the tube. In use, the tube is filled with the liquid to be tested, and placed in a sloping position at a definite angle. The ball, being released after being brought to the upper end of the tube, traverses it from top to bottom with a quasi-rolling motion (which is unfortunately too complex for mathematical computation), and the time occupied by the ball in reaching a defined point near the bottom of the tube is recorded. The instrument must be calibrated by testing with it liquids of known viscosity, and in most cases corrections must be applied for inertia effects. A description of the apparatus by its originator will be found in Ref. III, 16.

This instrument has been successfully applied to the measurement of the viscosities of liquids at very high pressures, such as the measurements of Hersey and Shore referred to in Sect. III, 9. Fig. III, 8 shows a longitudinal section of the Flowers viscometer as applied in this investigation. For pressures over about 7000 lb. in.$^{-2}$ (500 atmospheres), a thick-walled viscometer tube of chrome-vanadium steel, and of the proportions shown in the figure, was used, the bore being $\frac{27}{64}$ in. (1·07 cm.), and the rolling steel ball $\frac{1}{4}$ in. (0·635 cm.) in diameter. The tube was connected to the pressure-generating and measuring apparatus by a long length of flexible steel tubing, the arrangement being such that, on starting a measurement, the tube was automatically raised to the standard angle of 15 degrees from the horizontal. The ball, initially at the left-hand end of the tube, then starts to roll down the inclined bore and, on reaching the lower end, operates a signal by making an electric contact, its free rolling path being $10\frac{3}{16}$ in. (25·9 cm.) long.

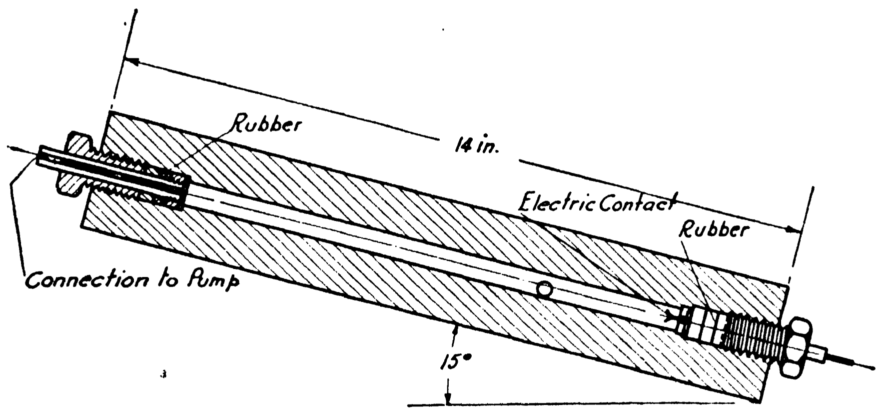


Fig. III, 8.—The Flowers viscometer, as used for liquids under high pressures

Another secondary viscometer, which is designed on a principle different from all of those previously described, is the Michell "Workshop Viscometer". This instrument makes use of the type of viscous flow which occurs between parallel surfaces when one of the surfaces is moved in a direction normal to the other, this action having been discussed mathematically for the case of two plane surfaces in Sect. II, 16. In the Michell viscometer the surfaces are not planes, but parallel segments of spheres; however, the solution for plane surfaces provides a sufficient approximation to the actual case for practical purposes.

The commercial instrument consists of a one-inch stainless-steel ball fitting into a shallow spherical cup of cast iron. Three very small projections, spaced 120 degrees apart on the spherical surface of the cup, maintain a minimum clearance of the order of 1×10^{-3} cm. between the surfaces of the ball and the cup. The cup is provided with a hollow handle of non-conducting material projecting from its reverse side, into which a short thermometer is inserted so as to have its bulb in thermal contact with the metal of the cup (see fig. III, 9).

In the use of the instrument a few drops of the oil to be tested are placed in the cup, held with its spherical recess upwards, and the ball is then dropped into the recess, displacing most of the oil into the small gallery which extends circumferentially around the cup, and forming a meniscus in the oil surface, as shown at G. The whole instrument is then inverted, the spherical distal end B only of the handle being held between the fingers of the operator, so that the ball hangs vertically, suspended by the negative pressures developed in the lamina of oil. After a period of 20 seconds or more, depending on the viscosity of the oil, the lamina, withdrawing oil from the gallery G, becomes so thick that it can no longer maintain negative pressures sufficiently great to support the ball, which then falls. The time elapsed from the inversion of the ball, as determined by a stop-watch, is proportional to the absolute viscosity of the oil and, in this sense, the instrument is an "absolute" viscometer. Each instrument must however be calibrated, and a constant factor assigned to it, by testing with it a liquid of known viscosity, since the interspace between the ball and the cup is too narrow to be measured with sufficient accuracy for a constant to be assigned to the instrument by calculation. This "workshop viscometer" has the advantages of being easily cleaned, and of requiring only a very small sample of the oil for a test. Like the MacMichael and Flowers instruments it is adapted for use on colloidal solutions and suspensions, as well as on homogeneous liquids such as oils. On the other hand, it is rather sensitive to the presence of small foreign particles, or bubbles of air, in the liquid and on this account its readings should always be repeated until concordant figures are obtained.

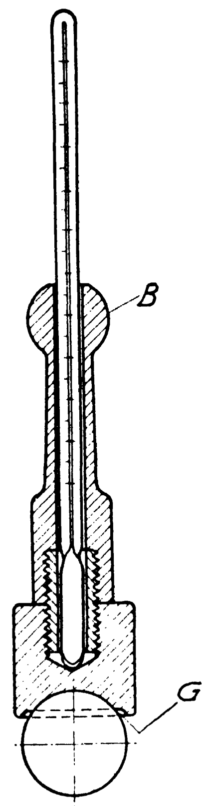


Fig. III, 9.—The Michell viscometer

There is evident need for the development of some new type of viscometer suited to the requirements of engineers concerned with lubrication. It is only by the personal use of a reliable viscometer that the plant engineer can assure himself that the lubricants which he purchases and uses conform, in the most essential of their properties (their viscosity), to the characteristics of the bearings of his machines. He also requires to know that they retain this property unchanged during their use. In order that habitual use may be made of the instrument it must obviously be convenient in operation, not easily damaged or deranged, portable, and easily cleaned. In addition, to fulfil the ideal, it should give its measurements in absolute units by means of its own internal constants, which should be readily checked, if not by the user, at least by any Standards Testing Laboratory.

4. Calibration of Secondary Viscometers.

The choice of liquids suitable for the calibration of secondary viscometers over the range required for testing usual lubricants is rather limited, and none is entirely satisfactory for the purpose. Evidently the liquid should be readily available in a pure, and chemically determinate, condition and it should have a coefficient of viscosity comparable with the liquids for which the secondary viscometer is to be used, especially lubricating oils. It must, of course, be non-corrosive and of low volatility.

Water, mercury, many solutions of alcohols and their derivatives in water, as well as solutions of many salts, would conform to these ideals were it not that their viscosities are too low for direct comparison with the oils most used for lubrication. Oils are usually of varying and indeterminate composition, and consequently of viscosities too uncertain to allow them to be adopted as standards always available for comparison. Another objection to the use of oils is that absolute viscometers of the capillary-tube class in which oils have been used can only be cleaned for further use with great difficulty.

On the whole, solutions of glycerol, and of sucrose, in water, appear to fulfil the requirements better than any other liquids which have been suggested. By mixture with water in suitable proportions, glycerol will give practically the whole range of viscosities required for comparison with lubricants, as shown by Table III, 1. The figures given in this table have been derived mainly from a much fuller table contained in Ref. III, 18.

TABLE III, 1

C.G.S. VISCOSITY OF SOLUTIONS OF GLYCEROL IN WATER AT 20° C.

Density	Percentage of glycerol in solution	C.G.S. viscosity, μ	Specific gravity referred to water at 20° C.
1·0981	—	0·0375	1·100
1·0988	40·0	·0379	1·1007
1·1080	—	·0437	1·11
1·1180	—	·0515	1·12
1·1280	—	·0615	1·13
1·1380	—	·0746	1·14
1·1480	—	·0924	1·15
1·1580	—	·1170	1·16
1·1679	—	·1520	1·17
1·1779	—	·203	1·18
1·1828	71·04	·236	1·1848
1·1879	—	·280	1·19
1·1979	—	·400	1·20
1·2037	78·78	·498	1·2057
1·2079	—	·593	1·21
1·2134	82·41	·750	1·2155
1·2179	—	·920	1·22
1·2218	85·55	1·111	1·2240
1·2278	—	1·511	1·23
1·2378	—	2·69	1·24
1·2441	93·96	4·08	1·2463
1·2478	—	5·26	1·25
1·2546	98·00	8·69	1·2568
1·2578	—	11·16	1·26

Sucrose, on the other hand, in water solutions at ordinary temperatures, affords only a range of viscosities up to about $\mu = 0{\cdot}05$, which is not sufficiently high for direct comparisons with lubricating oils as a class. Table III, 2 gives the viscosities of the solutions containing 10, 20, and 40 per cent of sucrose, as determined by Hosking [Ref. III, 2 (1900)].

TABLE III, 2

C.G.S. VISCOSITIES OF SUGAR SOLUTIONS (μ)

°C.	Percentage of sucrose		
	10 per cent	20 per cent	40 per cent
0	·02436	·03720	·1476
5	·02050	·03042	·1133
10	·01754	·02578	·0895
15	·01518	·02212	·0730
20	·01328	·01910	·0607
25	·01173	·01674	·0508
30	·01041	·01485	·04233
35	·00933	·01319	·03618
40	·00843	·01180	·03132
45	·00763	·01059	·02728
50	·00699	·00961	·02410

The "percentage of sucrose" is the ratio of the weight of sucrose to the total weight of solution.

Clean samples of commercial cane sugar, refined for human consumption, are sufficiently pure sucrose to be used for the purpose. Both the sucrose and the glycerol solutions are apt to cause trouble by "gumming" when used in narrow tubes or interspaces, if allowed to evaporate partially, or to be exposed long to high temperatures.

5. Classes of Liquids commonly used as Lubricants.

Although every liquid, being more or less viscous and perfectly adhesive to solids, is capable of acting as a lubricant under suitable conditions, comparatively few possess combinations of qualities which render them desirable as lubricants for machinery. Apart from a degree of viscosity suitable to the particular application, it is essential that a lubricant shall be chemically stable and inert towards metals. In practice, choice is almost limited to liquids of the following classes:

Oils, either mineral, vegetable, or animal;
Water, sometimes with dissolved or suspended matter designedly added;
Greases and soaps, the latter mixed with water or oil; and (of recent introduction)
Silicones. (Lubricants of this class, which have found application as yet only under special circumstances, are discussed briefly in the last section of the present chapter, Section III, 12.)

Oils, greases and soaps are, wholly or in part, carbon compounds. A strict classification should be, of course, upon a chemical basis, but, both on account of the complexity of the chemical facts involved and because most of the lubricants in practical use are mixtures, either natural or artificial, of substances of different chemical classes, such a classification would have little practical value. A few chemical facts may, however, be usefully recalled.

Mineral oils are essentially hydrocarbons. A broad distinction is made between them according as they consist mainly of hydrocarbons of the paraffin series, defined by the general formula C_nH_{2n+2}, or mainly of the naphthene series with the formula C_nH_{2n}. All commercial lubricating oils of mineral origin were formerly obtained by distillation of the natural products; most of them were mixtures of several different distillates; recently, however, both in Germany and in the United States, so called "synthetic lubricants" have been produced from natural, as well as other, hydrocarbon gases. Although not at present of great commercial importance, a brief summary of the properties of some typical oils of this class is given in Table III, 5, p. 57 (Ref. III, 17).

The special characters of mineral oils, which result in their general use as lubricants, are their chemical inertness (implied in the case of the paraffin series by its name) and their very wide range of viscosities, corresponding to the wide range of the constants n of the formulæ of the various constituents of the oils which occur naturally in abundance. They have little tendency to become oxidized, or to develop acids, so as to become corrosive, and if properly distilled, so as to be free from volatile constituents, they are subject to little loss or change of properties during use. Therefore, it is possible for them to be used over and over again, after occasional filtration, provided that the bearings and lubrication systems of the machines to which they are applied are so designed (in the manner explained in later chapters) that the working parts of the bearings are kept apart by continuous films of the oil, which, furthermore, is not allowed to be contaminated by combustion products or other foreign substances capable of combining chemically with mineral oils. Vegetable and animal oils and greases do not admit of being similarly restored to condition by filtration, or by any other known process of purification.

Mineral oils are not liable to spontaneous ignition, though, if raised to moderately high temperatures, they will burn if ignited by a flame or spark. Mineral oils may usually be distinguished from vegetable or animal oils by their fluorescence, a bluish or greenish lustre which is best seen by dipping a glass rod in the oil and viewing it, when withdrawn, in a good light against a dark background. A distinct fluorescence proves the presence of hydrocarbons, but they can be deprived of it by chemical treatment, so that non-fluorescence does not prove their absence.

The organic oils and fats, the former being sometimes called "fixed" oils because they cannot be volatilized without chemical decomposition, are either vegetable or animal, and are found ready formed in various tissues of plants and animals. All these oils become fats when they solidify at low temperatures; and all the fats become oils when they are melted, as they can be at temperatures of 65° C. (150° F.) or lower. Each of the various oil-yielding plants and animals produces a different oil or fat from the others, and a great variety is found in commerce. They are all non-fluorescent, but some vegetable oils are green from the presence of chlorophyll.

Unlike mineral oils, the fixed oils combine readily with oxygen which they derive from the atmosphere. In doing so, they become acidic, and in some cases change from a liquid to an elastic solid form, when they are termed "drying oils". This property is, of course, adverse to their use as lubricants, and in consequence of the absorption of oxygen they may, under certain conditions, spontaneously ignite and cause fires. They are also, in varying degrees, capable of saponification, not only by combination with bases, but by contact with hot water, when they form glycerol and organic acids which are liable to attack metals and form metallic soaps. From this cause fixed oils are, except as additives in small fractions to mineral oils, unsuitable for steam-cylinder lubrication.

The chief vegetable oils used as lubricants are castor, rape, palm, and olive oils, the former two being obtained from the seeds, and the latter two from the fruits, of the respective plants. The most important of the lubricating oils of animal origin are lard oil, tallow oil, neatsfoot oil, and various oils obtained from marine animals, as "whale", "sperm", and "dolphin-jaw" oils, which are used for special purposes such as lubricating light spindles, and traditionally for clockwork.

The special merits of the fixed oils, as a class, for use as lubricants are, first, that they retain their viscosities at high temperatures much better than do mineral oils, and, secondly, that their non-volatility prevents them from being expelled from between solids in intimate contact by the heat developed by friction. It is probable also that they form more stable molecular films on the surfaces of metals than do mineral oils, and are thus more effective in preventing the welding together of opposed metallic prominences.

Soaps are used in lubricants chiefly for the making of compounded greases, and for the thickening of mineral oils for special purposes. The bases most commonly used for compounding greases are lime, soda, lead, and aluminium. Some details of these are given in Sect. VII, 16. Large quantities of compounded greases are used for lubricating the axle-bearings of vehicles; the greases for this purpose usually consisting of soda neutralized by the fatty acids of palm oil and tallow, with an excess of the latter. One half or more of the mixture is water.

Liquid lubricants, consisting still more largely of water with soap, of which a portion is usually potash ("soft") soap, are used for lubricating and cooling the cutting-tools of lathes and other machine tools. Water itself is used for the bearings (generally formed of wood or other organic materials) of various kinds of hydraulic machinery, rolling machinery, etc. For these applications see Sect. VI, 10.

Rational selection of one or other of the various lubricants above mentioned depends upon quantitative knowledge of their properties; of these the most important are, as a rule, as indicated in Chapter II, their coefficients of viscosity, especially at the highest and lowest of the temperatures to which they are liable to be subjected in use; their "pour-points", i.e. the lowest temperatures at which they remain liquid; and (if exposed to high temperatures) their "flash-points", or ignition temperatures. When special calculations of the behaviour of a lubricant have to be made, it may also be necessary to know its density, heat capacity, specific conductivity for heat, its surface tension in the presence of air, water, and metals, and, in some cases, the variations of its viscosity and density at high pressures.

Numerical data on these properties, for a selection of the chief lubricants in current general use, are given in the following sections of this chapter.

6. Physical Constants of Commercial Lubricants: Mineral Oils.

The accompanying Tables III, 3 and 4 contain a digest of the physical characters of a series of typical mineral oils which are in general use. In Table III, 3 the oils are broadly classified according to the applications for which they are especially suitable, but they are, of course, not restricted to these particular uses; it may often happen that an oil which is listed under one class of application may be the most suitable oil of the whole series for some particular purpose in another class. For the information contained in these tables the author is indebted to the courtesy of the Vacuum Oil Company Pty. Ltd., Australia; Shell Mex and B.P. Ltd., London; and Caltex Oil (Australia) Pty. Ltd.; these firms are respectively designated under the table-heading " Suppliers " by the letters V., S., and C.

For greater convenience of tabulation, viscosities are stated in the table in terms of the " centipoise "—one-hundredth of the poise.

It will be seen that, in each of the principal classes in which the oils are arranged in this table, there is given, in addition to a series of oils having special names or designations and definite physical constants, a statement of the " range " of properties of oils suitable, and commercially available, for applications within the class. Thus, under the class heading " High-speed Spindles ", the ranges of properties of the oils generally considered suitable for " high speed ", and for " very high speed " spindles are listed.

It is to be understood, of course, that Table III, 3 represents usual, or conventional, practice, and is based on the assumption that the various oils are used in bearings of the types normally used at the present time for the various purposes. The selection of lubricants for bearings of unusual forms, or more

Table III, 3

Characteristics of Lubricating Oils for Various Applications

Application	Commercial designation	Suppliers	Specific gravity at 15° C. (60° F.)	Absolute viscosity, centipoises					Flash-point (closed test)	Pour-point
				20° C. (68° F.)	38° C. (100° F.)	50° C. (122° F.)	60° C. (140° F.)	100° C. (212° F.)		
Refrigeration: Industrial units	Normal range	V	·900–·920	110–216	37–66	21·5–34·5	14·0–21·5	4·8–5·8		−23° C. (−10° F.)
	Gg. Arctic oil, C. Hy.	V	·915	140	47	27·5	16·5	4·8		−26° C. (−15° F.)
	Low temp. range	V	·880–·890	20–25	9·0–12·9	6·3–8·0	4·7–6·0	2·0–2·4		−37° C. (−35° F.)
	Gg. Arctic oil, light	V	·885	25	12	7·8	5·8	2·4		−45° C. (−50° F.)
	A.B. 11	S	·875	26	13	—	6·2	—	150° C. (30(° F.)	−35° C. (−30° F.)
	A. 11	S	·890	61	25	—	11·0	3·7	170° C. (34(° F.)	−40° C. (−40° F.)
	A.D. 11	S	·900	195	65	—	22	6·9	188' C. (37(° F.)	−35° C. (−30° F.)
Refrigeration: Domestic units	—	V	·860–·880	32·5–62	16–27	10·5–17·0	8·0–12·5	3·4–4·7		
	"White oil" S. 1041	V	·870	43·5	20·5	13·3	9·8	3·8		−12° C. (10° F.)
	"White oil"	V	·875–·890	65–133	27–52	17–29	12–20	4·5–6·5		
	"White oil" S. 1043	V	·881	105	42	25	17	5·6		−18° C. (0° F.)
High-speed spindles	"High speed"	V	·860–·890	39·5–76	18·0–30·0	11·2–17·0	8·4–12·0	3·1–4·1		
	Gg. Vacuoline oil "C."	V	·880	56	24·0	14·3	10·1	3·73		
	"Very high-speed"	V	·845–·880	11·2–39	6·4–17·6	4·8–10·9	3·8–8·0	1·90–3·0		
	Velocite oil "E."	V	·860	16·9	8·85	5·94	4·45	2·00		
	Tellus 11	S	·835	7·6	4·8	—	2·5	—	63° C. (145 F.)	−40° C. (−40° F.)
	J.Y. 1	S	·890	25·7	12·0	—	5·7	2·3	155° C. (310° F.)	−40° C. (−40° F.)
	Regal oil "A."	C	·865–·882	60	26	—	—	—	190° C. (375° F.)	−5° C. (25° F.)
Electric motors and generators	—	V	·885–·905	126–196	48·2–67·5	26·0–36·7	17·3–23·8	5·3–7·2		
	Gg. D.T.E. Heavy Medium	V	·900	185	62·3	33·2	20·9	6·1		
	J.Y. 4	S	·887	154	55	—	20	6·6	205° C. (400° F.)	−15° C. (5° F.)
	J.Y. 5	S	·890	185	[illegible]	—	23	6·7	205° C. (400° F.)	−15° C. (5° F.)

TABLE III, 3—(*continued*)

Application	Commercial designation	Suppliers	Specific gravity at 15° C. (60° F.)	Absolute viscosity, centipoises					Flash-point (closed test)	Pour-point
				20° C. (68° F.)	38° C. (100° F.)	50° C. (122° F.)	60° C. (140° F.)	100° C. (212° F.)		
Steam turbines:	—	V	·885–·895	67–160	28·0–53	16·5–29	11·5–18·4	4·0–5·6		
Direct-coupled	Gg. D.T.E. 797	V	·855	67	28·0	16·7	11·9	4·3		
	Gg. D.T.E. Light	V	·885	73	29·0	16·4	11·8	4·02		
	Gg. D.T.E. Medium	V	·890	120	42·5	24·1	16·1	5·0		
Geared	—	V	·865–·905	142–300	44·7–90	27·4–46	18·3–29	5·1–7·4		
	Gg. D.T.E. Special	V	·870	140	52·8	31·6	20·8	6·7		
D.C. and Geared	B. 8	S	·870	68	28	—	12	4·45	205° C. (400° F.)	−15° C. (5° F.)
	B.C. 8	S	·876	130	51	—	20	6·4	205° C. (400° F.)	−15° C. (5° F.)
	B.D. 9	S	·881	228	82	—	29	8·3	205° C. (400° F.)	−15° C. (5° F.)
Automobile engines	Mobiloil Arctic	V	·880	156	59·5	33·5	23·0	7·0		
	,, "A."	V	·880	325	105	57	36	9·8		
	,, "A.F."	V	·880	430	135	71	45·5	11·6		
	,, "B.B."	V	·885	635	186	95	59	14·3		
An aero-engine lubricant	Aero-shell 100	S	·889	725	230	—	70	17·5	(Open test) 260° C. (500° F.)	−15° C. (5° F.)
Enclosed gears	High-speed, Light-load	V	·885–·905	110–250	39–80	21–40	15–26	4·9–7·0		
	Gg. D.T.E. Heavy Medium	V	·900	185	62·5	33·2	21·0	6·1		
	Medium-speed, Medium-load	V	·885–·905	296–475	105–145	50–78	32–46	8·6–11·6		
	Gg. D.T.E. Extra-heavy	V	·895	377	114	59	38	9·6		
	Low-speed, Heavy-load	V	·885–·905	670–1425	195–390	98–190	63–110	14·8–23·0		
	Gg. D.T.E. A.A.	V	·895	1070	268	165	87	20·7	(Closed test)	
	B.A. 8	S	·875	85	38	—	15·5	5·3	205° C. (400° F.)	−15° C. (5° F.)
	B.D. 9	S	·881	225	82	—	29	8·3	205° C. (400° F.)	−15° C. (5° F.)
	C.G. 2	S	·890	550	184	—	58	15·0	220° C. (430° F.)	0° C. (32° F.)
	C.Y. 6	S	·898	1160	330	—	89	19·5	230° C. (450° F.)	−12° C. (10° F.)
	B.C. 12	S	·898	1520	440	—	126	27	260° C. (500° F.)	0° C. (32° F.)
	"Thuban 90"	C	·910–·928	—	[illegible]60	—	—	13–14	230° C. (450° F.)	−12° C. (10° F.)
	"Thuban 140"	C	·907–·925	—	—	—	—	37	300° C. (575° F.)	−4° C. (25° F.)

"Extreme Pressure" (E.P.) Lubricants	S.A.E. 90	V	·890–·920	480–1190	153–295	78–140	46–80	11·3–16·5		
	Mobiloil G.X.	V	·910	510	160	83	53	13·5		
	S.A.E. 140	V	·900–·940	1390–5900	355–1200	175–495	100–255	20·5–37		
	Mobiloil G.X.H.	V	·935	2900	690	310	178	33·3		
	Spirax E.P. 140	S	·945	1880	575	—	165	36	260° C. (500° F.)	–10° C. (15° F.)
Steam cylinders	Up to 340° F.	V	·895–·910	980–1495	275–395	130–186	80–110	18·5–23·0		
	Gg. Sup. Cyl. Oil Rarus	V	·905	1210	320	154	92	19·3		
	340°–500° F.	V	·895–·910	1470–2270	385–600	185–275	108–162	22·5–32·5		
	Gg. Sup. Cyl. Oil 600 W.	V	·900	1795	475	226	135	27·5		
	Over 500° F.	V	·895–·910	2230–5080	590–1165	270–505	160–290	31–49		
	Gg. Sup. Cyl. Oil Ex Hecla	V	·905	2880	730	340	195	37		
	B.E. 4 Sup. Steam	S	·955	6200	1175	—	225	34	260° C. (500° F.)	10° C. (50° F.)
	B. 6 Sup. Steam	S	·910	3200	850	—	215	42	285° C. (545° F.)	7° C. (45° F.)
Non-condensing engines: Wet steam to 8 atmos.	Pinnacle Cylinder Oil	C	·910–·927 (Compound 8 %).	—	—	—	—	24	260° C. (500° F.)	7° C. (45° F.)
,, ,, ,, 14 ,,	Honor Cylinder Oil	C	·904–·921 (Compound 5 %).	—	—	—	—	26	280° C. (530° F.)	7° C. (45° F.)
Dry steam to 18 ,, / Sup. heat to 15 ,, and 350° C.	Leader Cylinder Oil	C	·910–·921 (Compound 5 %).	—	—	—	—	32	300° C. (575° F.)	7° C. (45° F.)
Condensing engines, enclosed, with oil circulation: Wet steam to 10 atmos.	Vanguard Mineral Cylinder Oil	C	·901–·903	—	410	—	—	22	240° C. (470° F.)	–8° C. (20° F.)
Dry steam to 18 atmos.	Pinnacle Mineral Cylinder Oil	C	·910–·927	—	—	—	—	25	255° C. (490° F.)	24° C. (75° F.)
Sup. heat to 18 ,, and 260° C.	Leader Mineral Cylinder Oil	C	·904–·921	—	—	—	—	36	300° C. (575° F.)	7° C. (45° F.)
Sup. heat to 18 ,, and 350° C.	Cavis Mineral Cylinder Oil	C	·910–·924 (Compound 4 %).	—	—	—	—	42	310° C. (590° F.)	24° C. (75° F.)

Of the seven last oils in the table, those which are marked "Compound 8%", etc., are compounded to the extent of the percentages stated, the others being purely mineral oils.

accurately and closely dimensioned than usual, should be based on the physical constants shown in the table, without regard to its classification of purposes.

Table III, 4 gives the specific heats of a number of mineral oils of the classes listed in Table III, 3 and some others. As will be seen from the table, the specific heats increase markedly at the higher temperatures to which lubricants are subjected in use. When known, the surface tensions of the oils are also given.

TABLE III, 4

Description of oil	Temperature, deg. C.	Specific heat, C.G.S.	Surface tension, dynes cm.$^{-1}$ at 12° C.
Paraffin oils	15	0·448	—
,, ,,	100	0·51	—
Heavy refined oils	40–90	0·50	—
,, ,, ,,	90–150	0·55	—
,, ,, ,,	150–200	0·61	—
Light refined oils	40–90	0·48	—
,, ,, ,,	90–150	0·53	—
,, ,, ,,	150–200	0·585	—
" FFF " cylinder oil	20	0·476	36·7
" Victory Red " oil	20	0·423	38·5
" Bayonne " oil	20	0·460	36·1
Mobiloil "A."	20	—	34·3
Mobiloil " BB."	20	—	35·5
Naphthenic oils	15	0·428	—

Comparatively little investigation has been made into the heat conductivities of mineral lubricating oils but, in spite of the difficulty of such investigations, the results obtained by various experimenters seem to be fairly accordant, and to show that the conductivity does not vary greatly with temperature, or from one oil to another, within the range of lubricating oils. The conductivity is of course a very important constant, not only as affecting the behaviour of the oil as a lubricant, but as being necessary for the calculation of heat transmission in oil circulation systems embodying heating or cooling apparatus.

The following values of the conductivity in C.G.S. units have been observed:

Paraffin-base oils at 15° C.	$3{\cdot}5 \times 10^{-4}$
The same oils at 100° C.	$3{\cdot}0 \times 10^{-4}$
" Bayonne " oil at 20° C.	$3{\cdot}5 \times 10^{-4}$
Vaseline at 25° C.	$4{\cdot}4 \times 10^{-4}$

In addition to the mineral oils listed in Table III, 3, which are obtained mainly by distillation of naturally occurring crude oils, a series of lubricating oils is now produced (though not yet in general commercial use, so far as is known to the author) which are produced by synthetic methods from hydrocarbon gases as the raw materials (Ref. III, 17).

TABLE III, 5

CHARACTERISTICS OF SOME SYNTHETIC MINERAL OILS

Solubility	Designation	Specific gravity at 15° C. (60° F.)	Absolute viscosity, centipoises					Flash-point	Pour-point
			−45° C. (−50° F.)	−35° C. (−30° F.)	−18° C. (0° F.)	44° C. (110° F.)	99° C. (210° F.)		
Insoluble in water, soluble in petrol and kerosene	"UCON" L B 140	0·983	~50,000	~8000	~1000	28	6·0	225° C. (440° F.)	−45° C. (−50° F.)
	" L B 440	1·001	—	~55,000	~5500	85	14	250° C. (485° F.)	−37° C. (−35° F.)
	" L B 650	1·004	—	—	~10,000	140	22	275° C. (525° F.)	−32° C. (−25° F.)
			−40° C. (−40° F.)	−30° C. (−20° F.)	−18° C. (0° F.)	38° C. (100° F.)	99° C. (210° F.)		
Soluble in water, less soluble than "L.B." oils in petroleum	"UCON" 50 H B 55	0·993	~1500	460	150	8·5	—	125° C. (255° F.)	−62° C. (−80° F.)
	" 50 H B 400	1·047	~70,000	~13,000	~3000	82	17	250° C. (450° F.)	−50° C. (−55° F.)
	" 50 H B 5100	1·059	—	—	~55,000	1020	170	245° C. (470° F.)	−37° C. (−35° F.)

Representative examples of lubricants of this kind, with their chief physical constants, are listed in Table III, 5 (p. 57); it is to be understood that a great variety of other compounds having, as a rule, properties intermediate between those that are listed, can be obtained.

Particularly to be noted are the relatively high densities of these products, and the high values of the viscosities at high temperatures of some of them which have comparatively low pour-points. They are said to be, in general, without appreciable action on india-rubber, either natural or synthetic, a characteristic which suggests their use for lubrication of bearings formed of that material, as for instance those mentioned in Sect. VI, 10.

7. Physical Constants of Liquid Lubricants other than Mineral Oils.

On account of the natural variability of the organic oils obtained from vegetable and animal sources, their physical constants are, in general, not so well defined as those of mineral oils; it is probably for this reason mainly that comparatively few systematic investigations have been made, or at any rate published, on the lubricating properties of oils of these classes.

The data in the following brief tables have been collected from various sources, especially Refs. III, 18 and 19.

TABLE III, 6

DENSITIES OF FIXED OILS

Temperature, deg. C.	Density, gm. cm.$^{-3}$, at atmospheric pressure			
	Castor oil	Rape oil	Sperm oil	Lard oil
0	0·97	0·915	0·890	0·922
20	0·955	0·903	0·878	0·910
40	0·94	0·890	0·866	0·898
60	0·928	0·878	0·854	0·886
80	0·915	0·866	0·842	0·875
100	0·900	0·856	0·830	0·864

TABLE III, 7

VISCOSITIES OF FIXED OILS AT VARIOUS TEMPERATURES

Temperature, deg. C.	Viscosity, poises				
	Castor oil	Rape oil	Olive oil	Sperm oil	Trotter oil
0	—	—	3·26	—	—
20	7·30	0·90	—	0·33	0·88
40	2·24–2·73	0·40	0·38	0·17–0·19	0·37
60	0·60–0·67	0·20	0·15	0·08	0·17
100	0·17	0·08	0·07	0·046	—

The approximate solidifying temperatures ("setting points") of these oils are:

Castor Oil	−10° to −18° C.
Rape Oil	−2° to −12° C.
Sperm Oil	about 0° C.
Olive Oil	4° to −6° C.

The specific heats and surface tensions measured at 20° C. and 12° C. respectively of the oils listed in Table III, 7 are shown in Table III, 8.

TABLE III, 8

Oil	Specific heat, C.G.S. at 20° C.	Surface tension, dynes cm.$^{-1}$ at about 12° C.
Castor	0·598	37·6
Rape	0·488	36·6
Sperm	0·493	38·3
Trotter	0·483	38·3

Further information on the variation of the viscosity of fixed lubricating oils with varying temperature is given in Sect. III, 8 and data on the variations of both density and viscosity with varying pressure in Sect. III, 9, together with similar information for mineral oils.

Not so much for their direct importance in application to lubrication as for their wide utility in associated problems, the following statement of the viscosities of water and of mercury at varying temperatures given in Table III, 9 will be of interest.

TABLE III, 9

VISCOSITIES IN POISES

Temperature, deg. C.	Water at atmospheric pressure	Water at 1000 atmospheres	Mercury at atmospheric pressure
−20	—	—	1·88 × 10^{-2}
0	1·785 × 10^{-2}	1·644 × 10^{-2}	1·68 ,,
10	1·306 ,,	1·326 ,,	1·64 ,,
20	1·001 ,,	—	1·55 ,,
30	0·799 ,,	0·918 ,,	—
40	0·655 ,,	—	—
60	0·468 ,,	—	—
75	—	0·427 ,,	—
80	0·356 ,,	—	—
100	0·283 ,,	—	1·22 ,,
200	—	—	1·01 ,,

The values for water at 1000 atmospheres were determined by Bridgman (Ref. III, 8) together with values for a series of higher pressures up to 10,000 atmospheres.

Little attention appears to have been given to the determination of the heat conductivities of fixed oils. At atmospheric temperatures the conductivity in C.G.S. units of castor oil is approximately $4{\cdot}25 \times 10^{-4}$, and that of olive oil $3{\cdot}95 \times 10^{-4}$.

The heat conductivity of water is notably higher than that of oils, being approximately $1{\cdot}4 \times 10^{-3}$ C.G.S. units at temperatures between that of its maximum density, 4° C., and 25° C. This fact, considered in connexion with the relatively small thickness of lubricating films of water, is of obvious importance in its use as a lubricant.

8. Variations of Viscosity with Changes of Temperature.

In Tables III, 3, 5 and 7, the coefficients of viscosity of representative series of lubricating liquids have been given for the range of temperatures to which the lubricants are usually subjected in practice. In all cases the viscosities fall with rising temperatures, but the mineral oils lose their viscosities much more rapidly than animal and vegetable oils, and these more rapidly than water. An oil in which viscosity falls with rise of temperature less rapidly than in another oil, is sometimes said to have a higher "viscosity index" than the latter; it is evident that, other things being equal, a high viscosity index is an advantage in a lubricant, since the thickness of a bearing film increases with the coefficient of viscosity, and the oil with the higher index will retain a greater thickness, and consequently give greater protection against metal-to-metal contact in a bearing. Other considerations, however, as will be seen later, detract from the importance of the viscosity index *per se.*

Although the tables already referred to give the variations of viscosity of representative lubricants at ordinary working temperatures, it is desirable in many applications of the theory of lubrication to have formal expressions for the laws of variation in viscosity of lubricants of the different classes, and some of these will now be given.

For air, the viscosity in poises is given for temperature T in degrees centigrade by the approximate empirical formula

$$\mu = \mu_0(1 + {\cdot}273 \times 10^{-3}T),$$

where μ_0 is the viscosity at 0° C., and has the numerical value $1{\cdot}71 \times 10^{-4}$ C.G.S. units.

A more exact formula, which is however not very convenient for practical use, was derived by Sutherland (Ref. III, 20) from the molecular theory of gases, viz. the absolute temperature,

$$T_a = \frac{KT_a^{\frac{3}{2}}}{\mu} - C,$$

in which, for air, K has the value $1{\cdot}5 \times 10^{-5}$, and $C = 1{\cdot}24 \times 10^{2}$ in C.G.S. units. When the value of the viscosity of air is required for high temperatures, as will probably be increasingly the case with the advent of gas turbines, this formula should be used in preference to empirical formulæ. No similar rational formula has been obtained for the viscosity of liquids, and recourse must be had, when a continuous expression for μ in terms of T is desired, to empirical formulæ having constants adjusted to conform to the results obtained experimentally for each particular liquid.

Thus for olive oil, Reynolds found that the formula

$$\mu = ae^{-bT}$$

was in agreement with his experimental determinations (Ref. III, 21), but the results obtained from that formula, with the numerical constants published by Reynolds, are not in accord with the values for olive oil found by other experimenters.

In general it has been found necessary to employ formulæ having at least three disposable constants in order to express with reasonable accuracy the data obtained for any given fluid, as in the formula known as Slotte's (Ref. III, 22) which has the form

$$\mu = \frac{a}{(T - c)^b},$$

in which the constants a, b, and c are determined by experiment for any particular fluid.

Another similar formula is Vogel's, viz.

$$\log \mu = \log a + \frac{b}{T - c},$$

to which the same remarks as to the constants are applicable. It may be remarked that by both of these formulæ, μ has finite values when $T = 0$, but that $\mu = \infty$ when $T = c$, which is therefore implied to be the temperature of solidification of the lubricant.

As a rule, when the coefficient of viscosity of an oil is required to be known at some temperature between those for which its value has been determined by experiment and recorded in available tables, it can be obtained, with sufficient accuracy for practical purposes, by plotting a smooth curve through the three points nearest to the required point for which the viscosity is known.

9. Variations of Density and Viscosity of Liquid Lubricants with Changes of Pressure.

The viscosity of most liquids increases with pressure, though in most cases not to a sufficient extent to be of importance for lubrication until the pressure becomes equal to many atmospheres. Water is exceptional in its behaviour, inasmuch as its viscosity at low temperatures diminishes at first with increasing pressure, as shown by Table III, 9 in Sect. III, 7. At pressures above about 1000 atmospheres, at 0° C., and at all pressures when the temperature exceeds about 20° C., however, it follows the same rule as the majority of liquids, its absolute viscosity increasing with pressure.

Experimental results for the increase of density of various lubricants with increasing pressure are given in fig. III, 10, and corresponding data for the

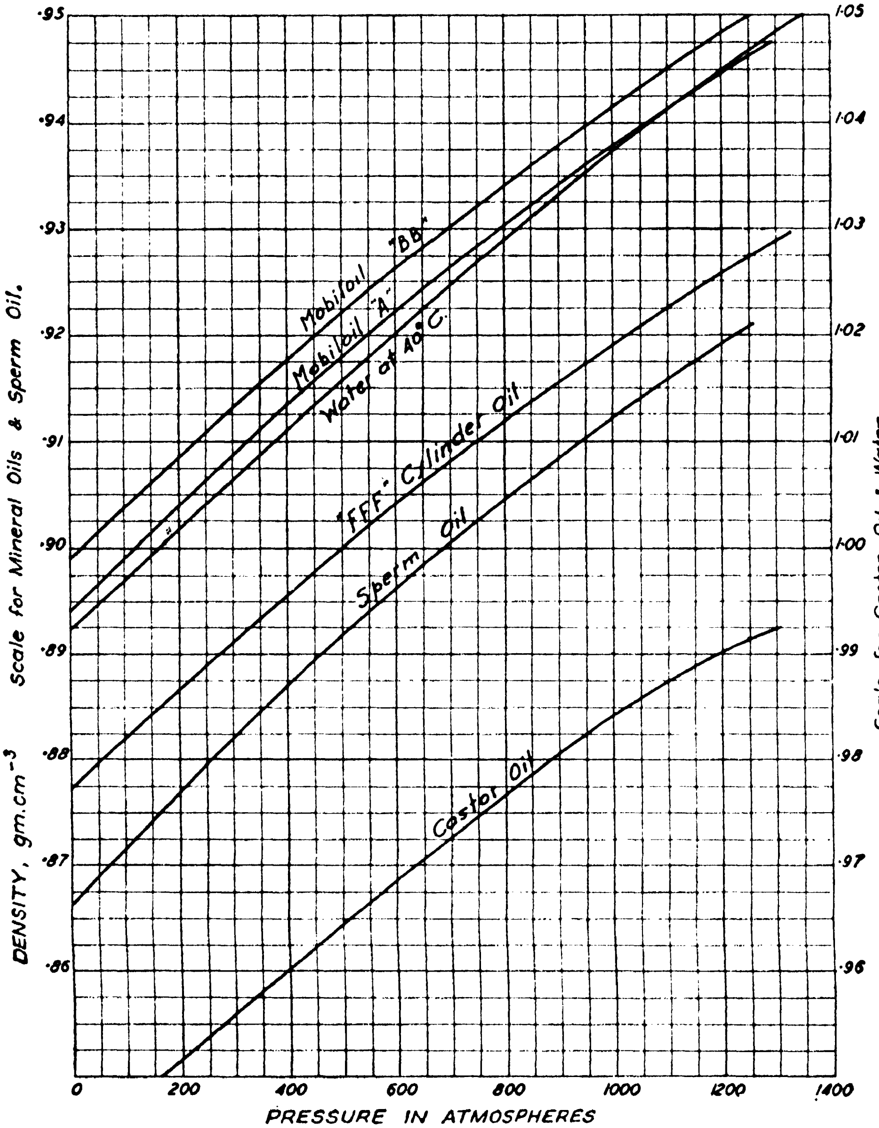


Fig. III, 10.—Variations of densities of lubricants with varying pressures

increase of viscosity with pressure in fig. III, 11. The data embodied in these figures have been collected from various sources, but chiefly from the published reports of Hyde (Ref. III, 19); Hersey and Shore; and of Kleinschmidt, on investigations undertaken for the Committee on Lubrication of the American Society of Mechanical Engineers (Ref. III, 23 and 24). It will be seen from the diagrams that the rate of increase of density of all the usual types of liquid lubricant is of the same order as that of water, being roughly about 1 per cent for each 200 atmospheres of pressure up to pressures of at least 1000 atmospheres. Increase of density of lubricants due to the working pressures in bearings or, at any rate, in fluent-film bearings, is therefore of negligible

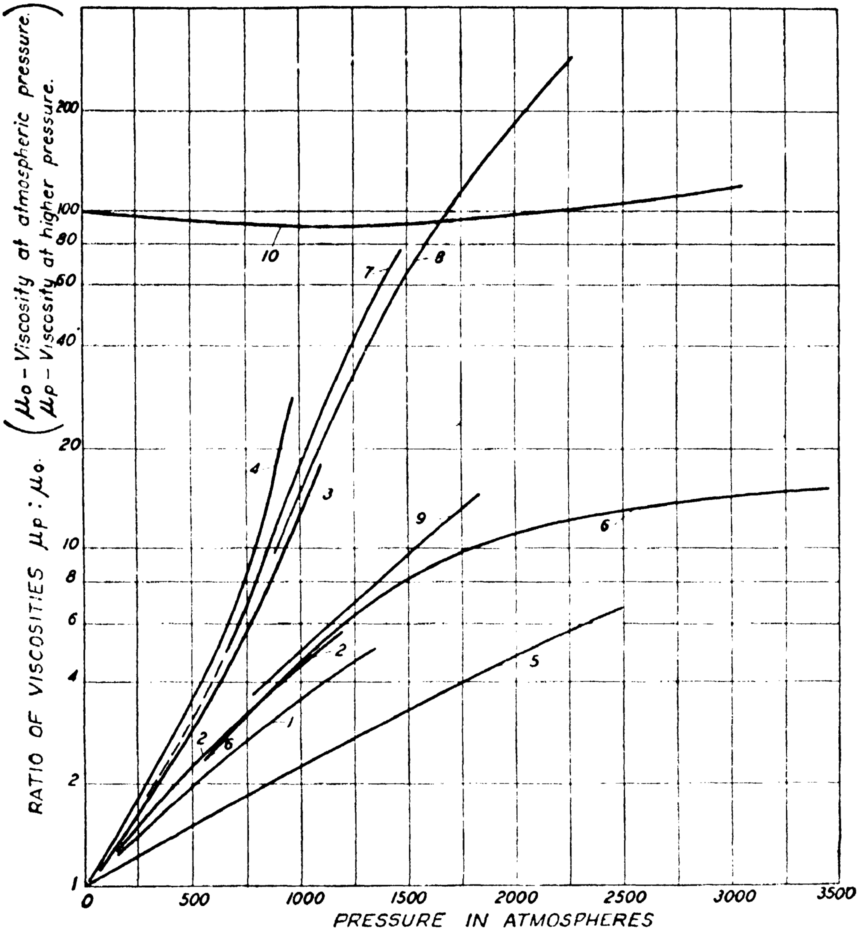


Fig. III, 11.—Variations of viscosities of lubricants with varying pressures

Curve	Liquid	Curve	Liquid
1	Sperm oil at 40° C.	6	Castor oil at 30° C.
2	Castor oil at 40° C.	7	Mobiloil "A." at 35° C.
3	Mobiloil "A." at 40° C.	8	Mobiloil "A." at 95° C.
4	Mobiloil "BB." at 40° C.	9	Lard oil at 25° C.
5	Castor oil at 100° C.	10	Water at 0° C.

importance. It can only come into consideration under conditions of point or line contact, where the intensity of loading approaches or exceeds the crushing strength of the metal members of the bearing, as in the bearings of the rolling type dealt with in Chap. VII. In any case, any effect that the change of density may have, will be masked by the much greater change which takes place in the viscosities of all lubricating liquids with the same changes of pressure.

The increase of viscosity of liquid lubricants with pressure, as shown in fig. III, 11, being of the order of 10 per cent in fixed oils and 20 per cent in most mineral oils for the first 100 atmospheres, or 1500 lb. per sq. inch, is not of much practical importance in bearings operating with fluent films at the pressures usual in such bearings at the present time. It must however greatly affect the conditions under which the lubricant operates in cases of line contact, and point contact, as in the rolling bearings described in Chap. VII, and in the action of the teeth of gear wheels discussed in Sect. VIII, 2.

10. Viscosities of Oils under rapidly varying Rates of Shear.

It has been usual to assume that the coefficients of viscosity of liquids are dependent only on the rate of the shearing motion which is taking place at any instant of time, and are independent of the rate of increase or decrease of the velocity of shearing. That such is the case for gases may be inferred from the molecular theory of gaseous viscosity, but molecular theories of liquids are not sufficiently established to enable a similar conclusion to be drawn from them, and the question can only be decided by experiment. The matter has been recently investigated in the Bell Telephone laboratories by Mason (Ref. III, 25).

In these experiments a rapidly vibrating crystal was arranged to generate a viscous wave in the liquid, and the viscosity was determined by measuring the changes in the properties of the crystal caused by the torsional stresses set up in it. The frequency of the oscillation was of the order of 10^4 to 10^5 cycles per second. Amongst other liquids of viscosities from 10^{-2} to 10 poises, a number of mineral oils were tested, both by means of the crystal and by the usual flow method. The following Table III, 10 gives the results in poises obtained for a number of lubricating oils by each method, and the percentage variation of the result with varying shear from that for constant shear.

TABLE III, 10

Designation of oil	Temperature deg. C.	Viscosity, poises, under		% Variation from constant shear
		constant shear	varying shear	
Bayol "B"	27·1	0·01669	0·0155	−7
Terress 43	26·7	0·415	0·379	−9
Aviation Lubricant	27·2	0·1881	0·166	−11
Castor oil	27·2	6·31	6·15	−2·5
Refrigerator lubricant	26·9	0·595	0·579	−2·5
Transil oil 10	27·1	0·115	0·123	+8

It will be seen that the changes in the coefficients of viscosity resulting from the very rapidly varying rates of shear imposed on the oils were not very large. It may be concluded that, at the much less rapid rates of change of shearing velocity to which oils are subjected in their use as lubricants, their viscosities may be taken as being for practical purposes unaffected.

11. Relation of the Viscosity of Lubricants to the sizes of different machines of the same type.

It is well known that in two machines which are "scale models" of each other (i.e. which are geometrically similar in all respects, but of different dimensions) the various stresses on the structures due to imposed forces of equal intensity, and also those due to the inertia forces arising from equal lineal speeds, will be the same in both machines.

Conversely, if the materials of the machines are to be fully stressed, as is in general necessary for economical construction, machines of similar design, and performing the same functions, should be scale models of each other, and their cyclic speeds should be inversely proportional to their dimensions so that their linear speeds at corresponding points may be equal. All deflections and displacements will then be proportional, at corresponding points and instants of time, to the dimensions of the machine.

Thus, for example, if the parts of two such machines, A and B, are related in size as $1:k$, their rotative, or other cyclic speeds, ω_A, ω_B, are to be related so that

$$\frac{\omega_B}{\omega_A} = \frac{1}{k}.$$

The inertia forces of any two corresponding elements of volume are then in the proportion of

$$\omega_A^2 : \omega_B^2 \times k \times k^3, \text{ or } 1 : k^2,$$

and since the sectional areas of material by which these forces are carried are also in the proportion of $1 : k^2$, the stresses are the same in each case.

Similarly, if in two heat engines operating on the same cycle, but of different sizes, the diameters of the respective cylinders are D and kD, and the thicknesses of the cylinder walls are t and $k't$, the circumferential tensions are respectively $\frac{1}{2}pD$ and $\frac{1}{2}pk\mathrm{D}$, while the tensile stresses are

$$\frac{pD}{2t} \text{ and } \frac{pkD}{2k't}.$$

In order that these stresses may be equal, k must be equal to k', according to the stated rule. The same rule applies to all other stresses self-developed in the machine. It does not apply to gravitational forces, but these are of minor, and usually negligible, importance in machine design, at any rate with respect to the parts which are related to lubrication.

Applying the same condition (viz. that the intensities of the pressures and other stresses are to be independent of the size of the machines) to the bearings of two similar machines of different sizes, it can be shown that the viscosity of the lubricant should not be invariable, but should be suitably related to the size of the machine. Taking any of the fundamental equations for the viscous motion of the lubricant, e.g. equation II, 40 for the velocity and the intensity of pressure p between two mutually approaching planes, viz.

$$p = \Pi - \frac{3\mu}{h^3}\frac{dh}{dt}(a^2 - r^2),$$

it will be seen that the intensity of stress is always proportional to μ, the coefficient of viscosity, and to a velocity (in the instance cited dh/dt), and is inversely proportional to a linear dimension, in this case $h^3/(a^2 - r^2)$.

Since the velocity is supposed to be the same in both machines, it is necessary, in order that the intensity of stress p shall also be the same, that the viscosity shall be proportional to the linear dimension.

This rule is general, and is in fact involved in the basic equation for viscous motion,

$$f = \frac{U}{h}\mu.$$

f the shearing stress, and U the velocity being invariables, μ must be proportional to h, the linear dimension.

It may be stated, then, that for logical machine design:

In similar machines the coefficient of viscosity of the lubricant should be directly proportional to the size of the machine, and the cyclic speed should be inversely proportional to the size. All stresses, including the pressures and shearing stresses of the lubricants, then become equal at corresponding points of all machines of the series, and all displacements and elastic deflections proportional to the size of the machine. To some considerable extent this rule as to coefficients of viscosity, although it does not seem to have been previously enunciated, is observed in practice as the result of experience. Thus it will be seen from Table III, 3, that the lubricants used for "high-speed spindles" are generally of much lower viscosity than those applied to the journals of "motors and generators", the latter being of larger size but operating with surface velocities of the same order, and under conditions otherwise similar. In the same way, the lubricants used for large diesel engines are much more viscous than those used for automobile units.

To carry out the rule strictly, extending it to the smallest machines, would involve that the bearing clearances and film thickness of the lubricant should be reduced proportionately to the diameter, or other ruling dimension, of the bearing. This may perhaps be impracticable in very small bearings, or at any rate may be inconsistent with manufacturing practices under commercial conditions. There would seem, however, to be no good reason for departing from the rule (at least in the better classes of machine production) unless clearances of the order of 10^{-4} cm. or less are involved.

Recognition of the rule as a convention would evidently clarify the subject, and also greatly simplify the processes involved in the design of series of similar machines of different sizes.

It may be pointed out that the rule is fully consistent with necessary provision for thermal expansion between closely fitting parts, whether the expansions in question arise from varying air temperatures during machining operations, or from ranges of temperatures of working fluids in operation; in either case the variations of temperature are practically independent of the size of the machine, and the linear expansions consequently proportional to its size.

In the comparatively few discussions that have been published on the principles involved in the design of a series of machines of different sizes, it has been usual to assume arbitrarily that the viscosity of the lubricant has a fixed value (see, for instance, Ref. III, 26). Any practical considerations, such as difficulty in obtaining lubricants of a sufficient range of viscosity, and otherwise suitable for the purpose of the machines, which may formerly have been of weight, seem now, in view of such data as are contained in Tables III, 3 and III, 5 to have little cogency.

12. Silicones and Fluorocarbons.

The well-known fact that silicon can replace carbon in many of the simpler compounds of carbon with hydrogen, with retention of the chief chemical characters of the latter substances, has suggested from time to time that the silicon compounds might find applications as lubricants. Recently some of the substances of this kind which are liquids at ordinary temperatures have received attention for the purpose, especially those known as " dimethyl " and " methylphenyl " " silicone fluids " (Ref. III, 27). These fluids have not as yet been used as lubricants to a sufficient extent to enable their permanence, and satisfactory behaviour in other respects, under normal conditions of machine operation, to be established; nor do the results of rigorous examinations of their relevant physical properties appear to have been published. It would seem, however, in general, that their viscosities, and in particular those of the dimethyl fluids, vary less rapidly with temperature than do those of hydrocarbon oils. Thus the viscosity of a dimethyl silicone-fluid designated as " DC 500 " appears, from the reference given above, to fall only in the ratio of about 10 to 1 with a rise of temperature from 0° to 210° F. (—18° to 99° C.), and not more than about 50 to 1, from 0° to 500° F. (—18° to 260° C.). Few, if any, mineral oils (see Tables III, 3 and III, 5) have so low a ratio of variation as from 10 to 1 in the range from the freezing-point to the boiling-point of water, while all of them volatilize to a considerable extent, and become unavailable as lubricants below 500° F. or 260° C.

The use of the silicone fluids is thus indicated for the lubrication of bearings or other machine parts which have unavoidably to operate at temperatures much above the boiling-point. A minor difficulty in their use arises from the low values of their surface tensions, that of the " DC 500 " fluid being reported to be about 20 dynes-cm.$^{-1}$ In consequence, these lubricants have a marked tendency to escape out of bearing-housings and to seep out of joints in piping. Another inconvenience arises from the apparent incompatibility of the silicone lubricants with bearing surfaces of iron or steel; thus journals and collars of steel require to be electroplated, preferably with chromium, and stationary bearing parts are best made of bronze.

Data on other experimental uses of silicone lubricants, especially in their application to hydraulic transmission units, may be found in Ref. III, 28.

Another class of substances, analogous to the hydrocarbons, which may in the future become of importance as lubricants, are the fluorocarbons. These form an extensive series of compounds, the simplest of which is CF_4 (corresponding to CH_4), in which fluorine atoms replace the hydrogen atoms of the series of hydrocarbons; their boiling-points range with those of the latter, but are in general considerably lower for corresponding numbers of carbon atoms. They are more inert chemically than the hydrocarbons, especially with respect to oxidation, and many of them are comparatively non-volatile oils or greases. They are thus in various ways available, and desirable, as high-temperature lubricants. Further information, and a bibliography of papers on these compounds, may be found in Ref. III, 29.

It does not appear that any effort has been made in the tests which have hitherto been carried out on bearings lubricated with silicone fluids to provide the conditions necessary for fluent-film lubrication. Doubtless the deformations of the members of the bearings at high temperatures, which are of chief interest in these tests, present difficulties in establishing and maintaining ideal conditions, but until the behaviour of the silicone fluids as fluent films has been investigated, their true value as lubricants can hardly be assessed. Hitherto the main objective in their application appears to have been the carrying of high specific loads at temperatures too high for the use of mineral oils. It appears that some of the silicone fluids, such as " DC 500 ", will carry at 400° F. (205° C.) intensities of loading of the same order as the mineral oils best adapted to work at high temperatures will endure at 210° F. or 100° C.; this is indeed to be expected from the fact that the viscosities of the two liquids, at those respective temperatures, are of the same order, viz. about 5 or 10 centipoises.

At the higher temperatures and loads the silicone lubricants tend to become more viscous rather rapidly, and to coagulate into jelly-like substances so as to obstruct the pipes and passages through which the fluid is supplied or circulated, and to interfere with the action of the bearing. It appears that for this reason the dimethyl silicone cannot be used continuously at temperatures of 450° to 500° F. (230° to 260° C.) for more than about 30 hours, or for correspondingly shorter periods at still higher temperatures, without removal of the gel-like substances that are produced. For some applications, however, such as that to gas-turbines for aircraft, this want of permanence of the properties of the lubricants may not be prohibitive of their use.

CHAPTER IV

Sliding Bearings

1. Preliminary.

The distinguishing characteristic of the bearings which are included under this title, is that one of the two mutally opposed members of the bearing has a motion of pure tangential sliding relatively to the surface of the other. A typical example, and the most common, is that of a cylindrical journal of a rotating shaft supported on a semi-cylindrical stationary member variously called the " brass ", or " pad ", or simply the " bearing ". Such a pair of bearing members is shown in outline perspective view in fig. IV, 1. The motion of the brass relative to the journal is, at all points, tangential to the surface of the journal.

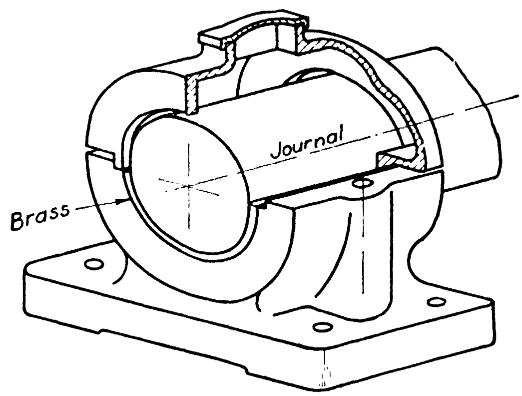


Fig. IV, 1

The surface of the journal is continuous and unlimited in the direction of the relative motion, but that of the brass is limited. In this case, as in all cases, the dimensions of the bearing members in the direction of the motion will be called their " lengths ", their dimensions in the direction at right angles to the motion their " widths ". Similarly in all sliding bearings one of the co-acting bearing surfaces is of unlimited length, in the sense that it always presents an unbroken length of surface to the other, which is usually of limited length. The curvature of the unlimited surface is uniform throughout its length, that is to say the surface is either a cylinder, or some other surface of revolution, or a plane. The curvature of the limited surface may vary along its length.

Lubrication is essential to the operation of all sliding bearings, the continued presence of a lubricant between the opposed bearing surfaces being necessary to prevent excessive localized contact pressures in the solid members, with consequent friction, heating, and attrition. In successful examples the solid surfaces are held apart by the automatic action of the fluid or semi-fluid lubricant, so that no direct contact exists between the solid members of the bearing, and no attrition or friction between them can take place. The mutual pressure between the relatively moving members is then transmitted through, and borne by, the lubricant and distributed by it over the surfaces of the solids as hydrostatic pressure, varying from point to point, of which the resultant is sufficient to equilibrate the load on the bearing.

In what may be regarded as the ideal mode of this action, the space between the bearing surfaces varies in the direction of the relative motion so that the thickness of the interposed layer of lubricant diminishes progressively from one end of the interspace to the other in the manner shown diagrammatically in fig. IV, 2.

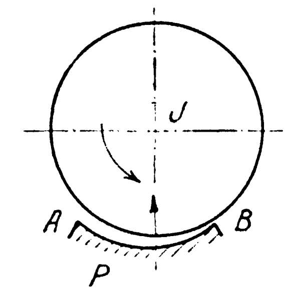


Fig. IV, 2

The direction of rotation of the journal J, as shown in the figure being anti-clockwise, and the brass or pad P being stationary, the interspace diminishes from its left-hand end A, where the surface of the journal is approaching that of the brass, to the right-hand end B, where it is receding from the brass.

These respective ends of the interspace are often called the "entering" and "leaving" ends, or the "inflow" and "outflow" ends, from the circumstance that the lubricant which is supplied to the bearing tends to flow through the interspace from A to B, i.e. in the same general direction as the motion of the surface of the journal.

The interspace between journal and brass is, of course, greatly exaggerated relatively to the diameter of the journal in fig. IV, 2. In practice the layer of lubricant between the surfaces is so thin that it is usually referred to as a "film", its thickness being rarely much more than one ten-thousandth part of its length or width. Nevertheless it is to be regarded as being a body of fluid subject to the same laws of hydromechanics, and particularly of viscosity, as any three-dimensional bulk of viscous fluid, with the exceptions that, on account of its extremely small thickness relatively to its other dimensions, any variations of fluid pressure in the direction of its thickness can be neglected, and that the directions of its flow at every point may be regarded as being, to the first order, in a plane parallel to the direction of motion of the moving solid surface.

The mode in which automatic lubrication takes place in journal bearings under the conditions sketched in fig. IV, 2, was first discovered by Osborne Reynolds (Ref. III, 21 and IV, 1). In his work on the subject he pointed out that the same kind of action may take place between two *plane* surfaces, one of which is continuous lengthwise and the other of limited length, but although he established the principles and the leading formulæ for this case, he made no practical application of his mathematical treatment of it. In the theoretical discussion of the subject it is always assumed, when cylindrical or other curved surfaces are in question, that their radii of curvature are large in comparison with the thickness of the viscous film, and the inertia forces in the fluid are usually neglected in comparison with those arising from its viscosity. The only geometrical features which come into the question are the thickness of the film at the various points of the developed surface of the bearing, and the rates of change of that thickness having regard to the direction of relative motion. It is therefore merely a matter of mathematical convenience whether the continuous surface is regarded as plane, and the action treated by means of rectangular co-ordinates, or as a surface of revolution to be handled by curvilinear ones. The former case, being the simpler, will be taken first in this chapter, after establishing in the next section the differential equation of the viscous motion which applies to all cases.

2. General Equation for Viscous Flow between relatively moving Non-parallel Surfaces.

Fig. IV, 3 is a generalized representation of any pair of relatively sliding members of a bearing, one of which (the "limited" member) is represented

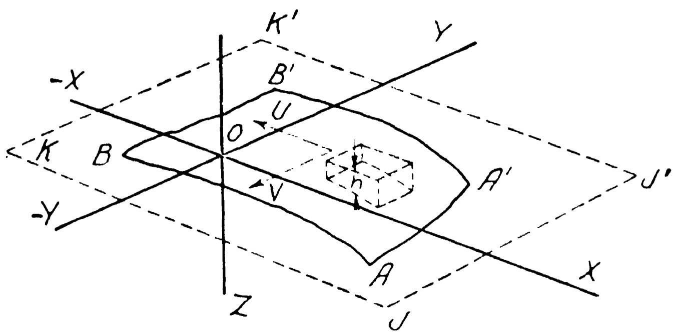


Fig. IV, 3

by the curved surface ABB'A' and is regarded as being stationary while the other (the "continuous", or "unlimited" surface, indicated by the dotted rectangle JKK'J') is, in accordance with the last paragraph of Sect. IV, 1, treated as a plane coincident with the plane $z = 0$. In the figure the surface ABB'A' is regarded as being below the plane $z = 0$ and the positive direction of z is downwards.

The plane surface JKK'J' is assumed to be moving in its own plane $z = 0$, in the directions of X and Y respectively, with components of velocity U and V in the directions of those axes but of negative signs. (The reason for thus taking the positive directions of U and V in the opposite directions to the directions of increase of x and y will be shown in Sect. IV, 3 in connexion with the application of the theory which will be given in that section.)

As in Sect. II, 14 (fig. II, 17, p. 27), consider a rectangular element at (x, y) extending from the surface $z = 0$ to the fixed surface ABB'A', where $z = h$, it being remembered that h is variable and is to be treated as being small relatively to the radii of curvature of the surface. The interspace between the two surfaces JKK′J′ and ABB′A′ being assumed to be filled with a viscous fluid, the motions of the fluid which result from the motion of the surface JKK′J′ in its own plane can be considered as taking place in two stages: first, that of the motions of the fluid element at (x, y) due to rates of variation of the pressure and of the thickness of the film around that element, assuming both surfaces to remain stationary; and secondly, the further changes of position due to the assumed motion of the surface JKK′J′.

In both of the imagined stages into which the action is thus divided, it is postulated, in accordance with the experimental fact emphasized in Chap. II, that no slipping of the fluid relatively to solid boundaries can take place at any point.

As to the first of these imagined stages of the action, it is seen from equation II, 36 that the surpluses of the volumes of flow into the element at (x, y) over the volumes flowing out of it, in the directions X and Y respectively, are

$$\left.\begin{aligned} -\delta x\,\delta y\,\frac{\partial Q_x}{\partial x} &= \frac{\partial}{\partial x}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right)\delta x\,\delta y, \\ \text{and}\qquad -\delta x\,\delta y\,\frac{\partial Q_y}{\partial y} &= \frac{\partial}{\partial y}\left(\frac{h^3}{12\mu}\frac{\partial p}{\partial y}\right)\delta x\,\delta y. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad \text{IV, 1}$$

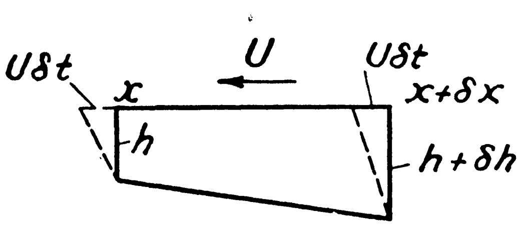


Fig. IV, 4

In the second stage, by the movements of the plane JKK′J′ in the directions of X and Y (see fig. IV, 4 for the motion in the X plane), the volume of the element decreases at the rate

$$\tfrac{1}{2}\left\{\left(h + \frac{\partial h}{\partial x}\,\delta x\right) - h\right\}U\,\delta y + \tfrac{1}{2}\left\{\left(h + \frac{\partial h}{\partial y}\,\delta y\right) - h\right\}V\,\delta x,$$

or

$$\left\{\tfrac{1}{2}U\frac{\partial h}{\partial x} + \tfrac{1}{2}V\frac{\partial h}{\partial y}\right\}\delta x\,\delta y. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{IV, 2}$$

The sum of the right-hand members of equations IV, 1 and the expression IV, 2 must be zero if the fluid is incompressible, so that if μ can be regarded as constant, as is usual,

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) + 6\mu\left(U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y}\right) = 0, \quad . \quad . \quad \text{IV, 3}$$

or in the more general case, where μ is variable, owing to different temperatures existing at different parts of the film, or other causes of variation which have to be taken into account,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial y}\right) + 6\left(U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y}\right) = 0. \quad . \quad . \quad \text{IV, 4}$$

This is the general differential equation which determines the fluid pressure p at every point of the fluid between the relatively moving surfaces, when the values of h and μ are assigned for each point, and when account is taken of the values of p around the periphery of the limited surface. Usually this boundary-value of p is uniform all round, and it can in most cases be taken as equal to Π, the atmospheric pressure.

The complete solution of the differential equation IV, 4 is often impracticable, but exact or approximate solutions can be obtained in a number of idealized cases which are sufficiently close to actual conditions for practical purposes. The simplest of these assumptions, which, while not exact, often gives useful results, is that both of the relatively moving surfaces are plane and consequently h is in direct linear relation to x and y.

This assumption is accordingly made in the next following sections (Sects. IV, 3–5) to obtain solutions of equation IV, 4 which are applicable both as exact solutions of the problems arising in plane bearings, and as useful approximations in many other cases. It will appear that the general condition necessary to the automatic generation of pressures in the lubricating fluid (and consequently to the maintenance of the fluid in position between the opposed surfaces), is that the surfaces converge towards one another in the direction of the relative motion of the continuous surface. The proper design of sliding bearings is always directed to bringing this condition about. Whenever, by wrong design of the bearing, or under abnormal conditions of operation, the surfaces fail to have the proper mutual convergence, their self-lubricating action ceases or, at best, changes its character to such an extent that the resistance to the relative movement is enormously increased, and mutual abrasion of the opposed surfaces sets in.

Before entering on the detailed discussion of the theory of convergent surfaces and its numerical applications to practical cases, it is desirable to emphasize that the realization of the theory in practice is dependent on the construction of the bearings being in strict accordance with the postulated conditions, especially as to the exact forms and relative position of the co-acting surfaces. This presumes degrees of accuracy of manufacture which, probably, were rarely realized at the time when Osborne Reynolds made his discovery, and which are by no means always to be counted upon in bearings produced at the present time. It will be found, when numerical deductions from the theory are being drawn, as in later sections of this chapter, that the thickness of the film of lubricant which has to be maintained without discontinuity between the bearing surfaces is rarely as great as one-thousandth of an inch ($2{\cdot}5 \times 10^{-3}$ cm.), and is commonly much less than this. The variations of the thickness over the whole extent of the surfaces must closely follow those determined by the theory if the bearing is to operate, even approximately, as the theory indicates.

It has been common in the past to attribute discrepancies between results deduced from the theory of viscous lubrication, and those of test or practical experience, to various causes, such as hypothetical and occult properties of the lubricants, when those discrepancies might have been more naturally and correctly explained as being due to differences between the actual and the postulated forms of the bearing surfaces.

3. Plane Sliding Bearings of Unlimited Widths.

In the discussion of sliding bearings in which both the co-acting surfaces are plane, it will be assumed, in the first instance, that these planes are of unlimited width (i.e. that they are of indefinite extent in the direction at right angles to the motion), and that neither the distance h between the planes, nor the coefficient of viscosity of the lubricant μ, varies in that direction. As in

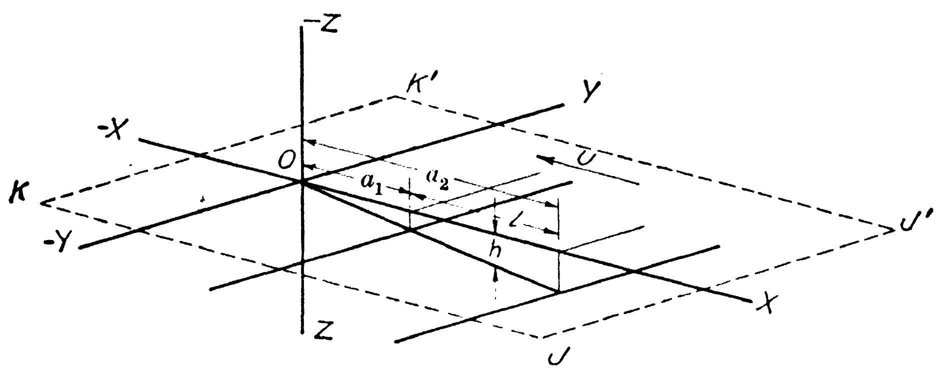


Fig. IV, 5

the last section, the moving surface is taken to be of unlimited length in the direction of motion, which is taken as parallel to the X co-ordinates; the stationary plane, on the contrary, is of limited length in this direction, extending only from $x = a_1$ to $x = a_2$. The distance h measured normally between the planes is assumed to be everywhere small compared with $a_2 - a_1$; the inclination of the stationary plane is independent of x, so that $h = cx$, c being a constant, very small compared with unity.

In fig. IV, 5 which illustrates the conditions assumed, as well as in figs. IV, 3 and 4, the positive direction of OZ is downwards, and the unlimited plane JKK'J' (assumed as in fig. IV, 4 to coincide with the plane $z = 0$) moves in the negative direction of x with velocity U relatively to the fixed plane.

The reason for taking U as positive when it is in the opposite direction to the direction of increase of x, both in this case and in the more general case to be discussed in Sect. IV, 12, is that the continuous plane must move in the direction of its convergence with the limited plane in order that the pressures generated in the fluid in the interspace between the planes may be of positive sign.

None of the conditions vary in the direction of Y, so that $\partial h/\partial y$ and $\partial p/\partial y$, as well as V, are zero. The pressure of the fluid is assumed to be atmospheric ($p = \Pi$) at the entry as well as the leaving edges of the limited plane, i.e. at $x = a_1$, and $x = a_2$, for all values of y.

Equation IV, 3 thus becomes

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + 6\mu U \frac{\partial h}{\partial x} = 0, \quad \text{IV, 5}$$

which can be directly integrated with respect to x and gives

$$\frac{\partial p}{\partial x} + 6\mu U \frac{h - h_m}{h^3} = 0, \quad \text{IV, 6}$$

h_m being the value of h where $\partial p/\partial x = 0$, that is to say where p has a maximum or minimum value, x at that point being denoted by x_m.

Thus
$$\frac{\partial p}{\partial x} = -6\mu U\left(\frac{1}{h^2} - \frac{h_m}{h^3}\right) \quad \text{IV, 7}$$

or, since
$$h = cx, \text{ and } h_m = cx_m,$$

$$\frac{\partial p}{\partial x} = -\frac{6\mu U}{c^2}\left(\frac{1}{x^2} - \frac{x_m}{x^3}\right). \quad \text{IV, 8}$$

Integrating,
$$p = \frac{6\mu U}{c^2}\left(\frac{1}{x} - \frac{x_m}{2x^2} - C\right). \quad \text{IV, 9}$$

But since $p = \Pi$, both when $x = a_1$, and when $x = a_2$,

$$\frac{6\mu U}{c^2}\left(\frac{1}{a_1} - \frac{x_m}{2a_1^2} - C\right) = \frac{6\mu U}{c^2}\left(\frac{1}{a_2} - \frac{x_m}{2a_2^2} - C\right) = \Pi,$$

so that
$$\frac{x_m}{2}\left(\frac{1}{a_2^2} - \frac{1}{a_1^2}\right) = \frac{1}{a_2} - \frac{1}{a_1},$$

and
$$x_m = \frac{2a_1a_2}{a_1 + a_2}, \quad \text{IV, 10}$$

which, a_2 being greater than a_1, is less than their arithmetic mean, or in other words, x_m is nearer to a_1 than to a_2.

It results also from the alternative values of Π that

$$-\frac{6\mu U}{c^2} \times 2C = 2\Pi - \frac{6\mu U}{c^2}\left\{\frac{1}{a_1} + \frac{1}{a_2} - \frac{x_m}{2}\left(\frac{1}{a_1^2} + \frac{1}{a_2^2}\right)\right\}$$

$$= 2\Pi - \frac{6\mu U}{c^2}\left\{\frac{a_1 + a_2}{a_1a_2} - \frac{a_1^2 + a_2^2}{(a_1 + a_2)a_1a_2}\right\}$$

$$= 2\Pi - \frac{6\mu U}{c^2} \cdot \frac{2}{a_1 + a_2},$$

or
$$-\frac{6\mu U}{c^2} C = \Pi - \frac{6\mu U}{c^2(a_1 + a_2)};$$

or, by substitution for C and x_m in equation IV, 9,

$$p = \Pi + \frac{6\mu U}{c^2(a_1 + a_2)}\left(\frac{a_1 + a_2}{x} - \frac{a_1 a_2}{x^2} - 1\right). \quad . \quad . \quad \text{IV, 11}$$

This equation determines the pressure at all points between the two co-acting planes.

From equation IV, 11, putting $x = x_m$ and giving x_m the value shown by IV, 10, the pressure at that point is found to be

$$p_m = \Pi + \frac{3\mu U}{2c^2}\,\frac{(a_1 - a_2)^2}{a_1 a_2(a_1 + a_2)},$$

being positive and a maximum, as shown by equation IV, 8.

It is to be noted that the sign of the second term on the right-hand side of equation IV, 11 is reversed when the direction of U is reversed, the pressures p then becoming lower than atmospheric throughout the interspace.

The total excess of the pressures p over the atmospheric pressure Π, that is to say, the total of the fluid pressures acting on each unit of width of each of the two planes and tending to force them apart, is

$$P = \int_{a_1}^{a_2}(p - \Pi)\,dx = \frac{6\mu U}{c^2(a_1 + a_2)}\int_{a_1}^{a_2}\left(\frac{a_1 + a_2}{x} - \frac{a_1 a_2}{x^2} - 1\right)dx$$

$$= \frac{6\mu U}{c^2(a_1 + a_2)}\left[(a_1 + a_2)\log x + \frac{a_1 a_2}{x} - x\right]_{a_1}^{a_2}$$

$$= \frac{6\mu U}{c^2}\left\{\log\frac{a_2}{a_1} - 2\,\frac{a_2 - a_1}{a_2 + a_1}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{IV, 12}$$

the logarithm being to the base e.

In the particular case in which $a_2 = 2a_1$ (which will be frequently referred to hereinafter as a representative, or standard, case) the numerical value of the expression in brackets on the right-hand side of equation IV, 12 is 0·02648 . . . , and that of the complete numerical factor of $\mu U/c^2$ is

$$6 \times 0{\cdot}02648\ldots = 0{\cdot}1589 \text{ approximately.}$$

Denoting the length of the limited plane $a_2 - a_1$ by l, it is seen that the mean pressure, taken along its length, is

$$\frac{P}{l} = \frac{6\mu U}{c^2}\left\{\frac{1}{l}\log\frac{a_2}{a_1} - \frac{2}{a_2 + a_1}\right\}. \quad . \quad . \quad . \quad . \quad . \quad \text{IV, 13}$$

From equations IV, 11 and 12, the position of the centre of pressure on the limited plane is obtained by taking moments about the centre of co-ordinates, as follows:

$$\bar{a} = \frac{1}{P}\int_{a_1}^{a_2}(p - \Pi)x\,dx = \frac{6\mu U}{Pc^2(a_1 + a_2)}\int_{a_1}^{a_2}\left\{(a_1 + a_2) - \frac{a_1 a_2}{x} - x\right\}dx$$

$$= \frac{6\mu U}{Pc^2(a_1 + a_2)}\left\{\frac{a_2^2 - a_1^2}{2} - a_1a_2 \log \frac{a_2}{a_1}\right\}$$

$$= \frac{1}{2}\,\frac{a_2^2 - a_1^2 - 2a_1a_2 \log \frac{a_2}{a_1}}{(a_1 + a_2)\log \frac{a_2}{a_1} - 2(a_2 - a_1)}, \quad \text{IV, 14}$$

or putting

$$\frac{a_1}{l} = \frac{a_1}{a_2 - a_1} = \alpha,$$

and

$$\frac{a_2}{l} = \frac{l + a_1}{l} = 1 + \alpha,$$

$$\bar{a} = \frac{l}{2}\cdot\frac{1 + 2\alpha - 2\alpha(1+\alpha)\log\frac{1+\alpha}{\alpha}}{(1+2\alpha)\log\frac{1+\alpha}{\alpha} - 2}, \quad \text{IV, 15}$$

from which it is seen that the distance of the centre of pressure on the limited plane from the origin of co-ordinates is dependent only on the length of the pad and the ratio α, or of a_2 to a_1, and is independent of the values of P, μ and c.

Fig. IV, 6 shows for the particular case in which $\alpha = 1$ (i.e. $a_2/a_1 = 2$), the manner in which the pressure $p - \Pi$ varies along the length of the pad, as well

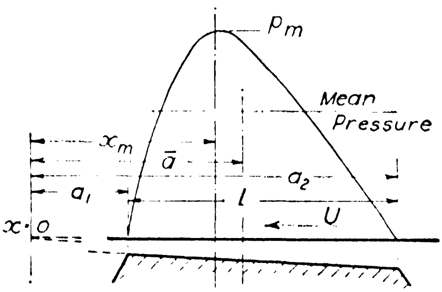


Fig. IV, 6

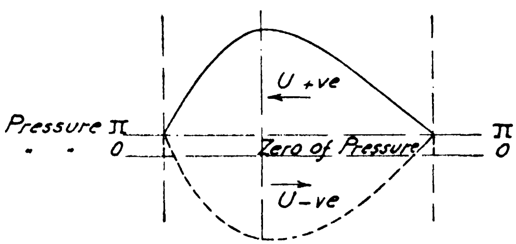


Fig. IV, 6a

as the position of the centre of pressure at $x = \bar{a}$, and that of the maximum pressure p_m at $x = x_m$. It also shows the mean value of the pressure calculated from equation IV, 13.

Fig. IV, 6a shows the relation of the pressures generated in a tapered film of fixed convergence, relatively to atmospheric pressure and to the zero of pressure, both when the continuous surface moves in the direction of convergence (pressures above the atmospheric line), and in the direction of divergence of the planes (pressures below the atmospheric line).

From the principles of viscous motion explained in Sect. II, 4 the shearing resistance to the motion of the unlimited plane (fig. IV, 3, p. 71) per unit of its width, and in the direction opposite to U, is

$$F = \int_{a_1}^{a_2} \mu \frac{U}{h} dx = \int_{a_1}^{a_2} \frac{\mu U}{cx} dx$$

$$= \frac{\mu U}{c} \log \frac{a_2}{a_1}. \quad \text{IV, 16}$$

To obtain the tractional force in the direction of U on the limited surface or "pad", there must be added to F the longitudinal component of the fluid pressures acting on the surface of the pad inclined at the angle $\tan^{-1} c$ to the direction of U, viz. Pc. The total tractive force on the pad thus becomes $F + Pc$. But the distinction between the longitudinal tractive forces thus ascribed as acting on the unlimited and the limited members of the bearing is an artificial one, being due to the shear in the film of fluid being treated in the calculation as exactly in the plane $z = \text{const.}$, instead of being so only to the same order of approximation as the pad is parallel to the plane $z = 0$.

In practical applications it is convenient to regard the tractions on the unlimited and limited members of the bearing as being equal and each denoted (without regard to sign) by

$$F' = F + \tfrac{1}{2}Pc = \frac{\mu U}{c} \log \frac{a_2}{a_1} + \tfrac{1}{2}Pc. \quad \text{IV, 16}a$$

The values of both F and F' are thus dependent, like that of the total pressure P, only on U, μ, c and the ratio of a_2 to a_1, i.e. on α.

In all ordinary cases the second term on the right-hand side of equation IV, 16*a* is small compared with the first term.

From equations IV, 12 and IV, 16*a*, the ratio of the tractive force to the total pressure on this moving plane, which ratio may be regarded as the coefficient of resistance of the pair of planes, considered as members of a lubricated bearing, is found to be

$$k = \frac{F'}{P} = \frac{c}{6} \cdot \frac{\log \frac{a_2}{a_1}}{\log \frac{a_2}{a_1} - 2\frac{a_2 - a_1}{a_2 + a_1}} + \frac{c}{2}. \quad \text{IV, 17}$$

It is thus, likewise, dependent only on the inclination c of the plates to one another, to which it is directly proportional, and on the ratio of a_2 to a_1, that is on α.

It appears immediately from equations IV, 12 and IV, 16 given above for the total pressure P between the plates (this representing the load carried by them when regarded as the members of a bearing), and the tractive force F, that both are directly proportional to the coefficient of viscosity μ of the fluid, and also to the velocity U with which each plate moves with respect to the other.

It is also seen that the load P varies inversely as the square of the inclination c between the planes, i.e. inversely as the square of the distances between them, if their other dimensions and relations remain unchanged. The tractive force F, however, varies inversely as the first power only of c, or of the distances between the surfaces.

It thus results, as already stated with reference to equation IV, 17, that the coefficient of resistance is in direct proportion, other conditions remaining the same, to the distance between the plates. This fact is the governing factor in the design of efficient sliding bearings, but its application is of course limited by the consideration that reduction of the distance between the surfaces is restricted by the degree of accuracy with which it is practicable to form them as true planes free from any irregularities or rugosities which would produce irregularity or discontinuity of the fluid film when they were brought very close together. There are also other considerations, as will be noted later in this chapter, which limit the extent to which c may be reduced in the apparent interests of mechanical efficiency. The mere difficulties of execution in the workshop are to a considerable extent obviated by the means described in the next section.

4. Pivoting Axis of the Plane of Limited Length.

It follows from the remark below equation IV, 15 in the last section that the pad of limited length shown in fig. IV, 6, instead of being supported along the length of its lower side, as there represented, would still be in equilibrium under the fluid pressures if it were supported only at the single point defined by $x = \bar{a}$, as shown in fig. IV, 7, without its condition of equilibrium being disturbed by any variations of the velocity U, or total pressure P, or of the coefficient of viscosity μ (provided that it remained uniform throughout the film), since the existence of the pivoting point would allow the inclination c to vary to maintain the relation between these three quantities which is required by equation IV, 12. The principle of pivoting (Ref. IV, 2) is in fact widely adopted in practice, not only for plane pads but also for journal and other bearings in various forms, and it is found that, with due precautions in construction, the pivoted pad is stable under varying conditions of operation. It has also other advantages of a practical nature.

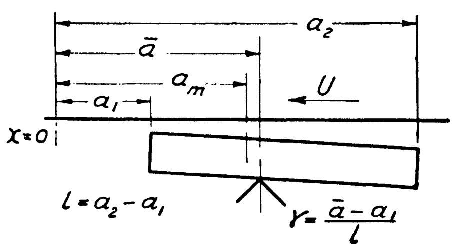


Fig. IV, 7

The chief of these advantages is that pivoting of the pad largely overcomes the difficulty of forming the bearing surfaces with the extremely small clearances and inclinations which have been mentioned in Sect. IV, 2, as being found unavoidable in practice, and are shown by the equations of Sect. IV, 3 to arise of necessity in convergent-film bearings. By the use of the pivot, the production of the extremely slight inclination presents no difficulty, since it takes place automatically, provided of course that the surfaces are accurately plane.

The importance of the correct determination of the position of the pivoting point, whose abscissa $\bar{a}$ is given by equation IV, 15, will be evident on consideration of the numerical and graphical data given in the following sections.

The constants adopted for defining the position of the pivot, in terms of the dimensional constants of the pad, are shown, for convenience of reference, in fig. IV, 7.

5. Numerical Data on Plane Bearings of Unlimited Widths.

The principal results of the formulæ obtained in Sect. IV, 3 are given numerically in Table IV, 1 for a series of different cases defined severally by the parameter α (Col. I) or, alternatively (Col. III), by $h_2 : h_1$, the ratio of the greatest to the least distance between the planes. The table shows in Col. II the position of the pivoting point, expressed in the form of γ, the ratio between its distance from the point of least clearance and the length l of the limited plane; it also shows in Cols. IV and V the total load carried P, and the total tractive force F per unit width of the bearing, both of these being expressed in C.G.S. units, viz. as dynes per centimetre of width. The next two columns, VI and VII, show the same two quantities, expressed as P_L and F_L respectively, in pounds weight per inch. The final column (VIII) gives the coefficient of resistance k, which of course is independent of units, being merely the ratio of F to P, or F_L to P_L.

In order to put the comparison of this series of idealized bearings on a practical basis it is drawn up on the basis of leading dimensions and conditions within the range of usual practice. It is assumed that the length of the limited plane l is 10 centimetres in all cases (i.e. 3·94 inches). The relative velocity U is taken to be 1000 centimetres per second (32·8 ft. per sec.), and the C.G.S. viscosity of the lubricant 5×10^{-2} poises. It is also assumed that in all cases the smallest clearance h_1 between the planes is the same, viz. 10^{-3} centimetre (3·94 ten-thousandths of an inch). Thus the constant c used in the equations of this section is replaced for the purposes of the table by h_1/a_1, c having no longer the same value throughout the series. In other words, the position of the origin of x co-ordinates relatively to the plane of limited and constant length (10 centimetres) varies from one bearing of the series to another, as is shown (e.g.) by the variation of the ratio $a_2 : a_1$.

Values of P, F, etc. for any values of h_1 and l different from 10^{-3} and 10 centimetres respectively can be readily derived from those given in Table IV, 1 by substitution of the appropriate constants.

The same information as is contained in Table IV, 1 is given, with some

TABLE IV, 1

CONSTANTS OF PLANE BEARINGS OF UNLIMITED WIDTH

I	II	III	IV	V	VI	VII	VIII
$\alpha = \frac{a_1}{l}$	$\gamma = \frac{\bar{a} - a_1}{l}$	$\frac{h_2}{h_1} = \frac{a_2}{a_1}$	P = load per unit width, C.G.S.	F = traction per unit width, C.G.S.	P_L = load per unit width, lb./in.	F_L = traction per unit width, lb./in.	k = coefficient of resistance
∞	·500	1·0	0	$5·00 \times 10^5$	0	2·86	—
10	·491	1·1	$2·16 \times 10^8$	4·71 "	$1·234 \times 10^3$	2·69	$2·21 \times 10^{-3}$
5	·482	1·2	3·78 "	4·56 "	2·16 "	2·60	1·21 "
2	·459	1·5	6·56 "	4·05 "	3·75 "	2·31	$6·17 \times 10^{-4}$
1·5	·449	1·67	7·31 "	3·83 "	4·18 "	2·19	5·24 "
1	·431	2·0	7·94 "	3·47 "	4·54 "	1·98	4·36 "
0·5	·393	3·0	7·40 "	2·75 "	4·23 "	1·57	3·71 "
0·4	·379	3·5	6·80 "	2·51 "	3·88 "	1·43	3·69 "
0·3	·359	4·33	5·84 "	2·20 "	3·34 "	1·26	3·77 "
0·2	·331	6·0	4·36 "	1·79 "	2·49 "	1·02	4·11 "
0·14	·306	8·14	3·21 "	1·47 "	1·83 "	0·84	4·58 "
0·10	·283	11	2·19 "	1·20 "	1·25 "	0·69	5·48 "
0·05	·239	21	$9·2 \times 10^7$	$7·6 \times 10^4$	$5·26 \times 10^2$	0·43	8·26 "
0	—	∞	0	0	0	0	—

The values of P, F and k in this Table are calculated for the case of $\mu U = 50$ C.G.S. units. It is also assumed that the length of the limited plane is 10 cm. and that the minimum clearance between the planes h_1 is 10^{-3} cm. in all cases.

additions, in diagrammatic form in Front Folder IV, A. The information given as to the values of the load and the tractive force is expressed more generally than in the table, in so far that the variables plotted are not the actual values of these quantities but the quotients obtained by dividing the values of P and F, given in equations IV, 12 and IV, 16, by $6\mu U/c^2$ and $\mu U/c$ respectively. Thus the values of P and F for any values of these factors can be readily obtained from the diagram.

The diagram, like the table, is prepared on the basis of a constant value of the least clearance, viz. 10^{-3} centimetre, and a length of bearing of 10 centimetres. The curve marked $(x_m - l)/l$ in the diagram gives the position of the point of maximum intensity of fluid pressure, in relation to the length of the bearing, for the various values of α. It is to be noted that the position of this point may differ widely from that of the centre of pressure or "pivoting point", which is shown by the curve marked "γ".

As is seen immediately on inspection of either Front Folder IV, A or Table IV, 1, the total pressure P generated in the bearing is at its maximum, and its coefficient of resistance k at its minimum, within the range of values of the parameter α extending from 0·3 to 1·0, which corresponds to a range of positions of the pivoting point, as shown by the values of γ, of from 0·36 to 0·43 of the length of the bearing from the end of smallest clearance. The value 0·4 of γ, corresponding to the division by the pivoting point of the bearing into two parts of relative lengths 2 to 3, can be adopted as usually giving a satisfactory compromise between the two desiderata of maximum load capacity and minimum frictional resistance which, fortunately, only conflict with one another to a small extent. It will be seen from the diagram that neither desideratum is thus sacrificed by more than 2 or 3 per cent of its optimum value.

It is, of course, always to be remembered in drawing conclusions for application to actual bearings from Table IV, 1, or Front Folder IV, A, or from the equations of Sect. IV, 3 on which they are based, that they apply only to bearings which can be regarded as wide relatively to their length. The load-carrying capacities per unit area of surface are higher, and the coefficients of resistance lower, in wide than in narrower bearings constructed in other respects of similar proportions, and operated under similar conditions.

The results for bearings of normal proportions follow, however, the same general trends as those that are shown by the theory for infinitely wide bearings, and the numerical results derived from it can often be applied, with suitable correction factors, as fair approximations to the performance of actual bearings for which exact calculations may be difficult or unduly onerous. Examples of such modes of approximation will be given in later sections.

It is hardly necessary to point out that if the sign of the velocity U is changed from positive to negative in the analysis of Sect. IV, 3, the differences between p and Π, as calculated by equation IV, 11, and the value of P obtained from equation IV, 12, will be merely changed in sign, not in numerical amount, showing that motion of the unlimited plane relatively to the plane of limited length in the direction of their divergence, produces diminutions of fluid pressure of equal amount to the increases which are generated by movement in the opposite direction. The extent to which this negative action can be actually realized, necessarily depends upon the relation of the absolute, or resultant, pressure at the point where it is a minimum, to the vapour pressure at the existing temperature of the lubricant; apart from that condition, it may depend upon whether the construction of the bearing admits, or excludes, the possibility of the entrance of air in place of the lubricant, at points where the pressure tends to become lower than atmospheric.

6. Volume Flow of Lubricant in Wide Plane Bearings.

From equations IV, 8 and IV, 10 the rate of change of pressure in the direction of the motion at the discharge end of the plane of limited length, i.e. at $x = a_1$, is

$$\frac{\partial p}{\partial x} = -\frac{6\mu U}{c^2}\left\{\frac{1}{a_1^2} - \frac{2a_2}{a_1^2(a_1 + a_2)}\right\}$$

$$= -\frac{6\mu U}{c^2 a_1^2} \cdot \frac{a_1 - a_2}{a_1 + a_2}.$$

Therefore from equations II, 36 the volume of flow due to the variation of pressure (in the direction of x increasing) is

$$Q = -\frac{h_1^3}{12\mu} \cdot \frac{6\mu U(a_2 - a_1)}{c^2 a_1^2(a_1 + a_2)} = -\frac{(ca_1)^3 U(a_2 - a_1)}{2c^2 a_1^2(a_1 + a_2)}$$

$$= -\frac{Uca_1(a_2 - a_1)}{2(a_1 + a_2)}.$$

To this must be added the volume due to the direct shearing action, viz.

$$Q' = -\tfrac{1}{2}Uca_1 \text{ (U being negative).}$$

The total flow Q_1 per unit width of the bearing is therefore

$$Q_1 = \frac{Uca_1}{2}\left(-1 - \frac{a_2 - a_1}{a_1 + a_2}\right) = -Uc\frac{a_1 a_2}{a_1 + a_2}$$

$$= -Uh_1\frac{a_2}{a_1 + a_2}. \quad \ldots \ldots \ldots \ldots \quad \text{IV, 18}$$

The same result would be obtained by calculating in a similar manner the volume flow at the inflow end of the bearing, since the fluid is incompressible, and it can also be obtained, and still more simply, by considering the conditions at the point x_m where $\partial p/\partial x$ is zero, and consequently there is no flow due to rate of change of pressure. The mean velocity of the fluid is there $-\tfrac{1}{2}U$, and since $h_m = cx_m$, from equation IV, 10,

$$Q_1 = -\tfrac{1}{2}Ucx_m = -\frac{U}{2}c\frac{2a_1a_2}{a_1 + a_2} = -Uc\frac{a_1a_2}{a_1 + a_2} = -Uh_1\frac{a_2}{a_1 + a_2},$$

as already determined in equation IV, 18. The negative sign of the result indicates, of course, that the flow is in the direction of x diminishing.

As a numerical illustration, a bearing whose dimensions and characteristics are at about the middle of the range of Table IV, 1 may be taken, e.g. one for which $a_1 = 0{\cdot}5l$; $a_2 = 3a_1 = 1{\cdot}5l$; $l = 10$ cm.; $h_1 = 10^{-3}$ cm. and U is assumed to be 1000 cm. per sec. (32·8 feet per sec.).

Then from equation IV, 18, numerically $Q = 10^3 \times 10^{-3} \times \frac{3}{4} = 0{\cdot}75$ cm.^{3} per sec. per cm. of width of the bearing, or approximately 1·5 imperial gallons per hour per inch of its width.

7. Velocities of Flow in the Lubricating Film.

The flow of lubricant which is determined by equation IV, 18 of the preceding section, takes place, of course, with velocities varying at different points throughout the film, both lengthwise and transversely to the surfaces. The velocity relative to the fixed, limited plane is zero over its surface, and on the moving plane $-U$ (in the direction of increasing x).

The velocity u, taken in the positive direction of x, is easily determined for any point of the film.

From equation II, 34, the velocity (measured in the direction of convergence, i.e. in the negative direction of x) due to grade of pressure is

$$u_p = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{z(h-z)}{2},$$

and from IV, 8 and IV, 10

$$\frac{\partial p}{\partial x} = -\frac{6\mu U}{c^2x^3}\left(x - \frac{2a_1a_2}{a_1+a_2}\right).$$

$$\therefore\ u_p = -\frac{6U}{c^2x^3}\left(x - \frac{2a_1a_2}{a_1+a_2}\right)\frac{z(h-z)}{2}.$$

To this must be added the velocity in the same direction due to shear, or

$$u_s = U\frac{h-z}{h},$$

so that

$$u = u_p + u_s = -\frac{6U}{c^2x^3}\left(x - \frac{2a_1a_2}{a_1+a_2}\right)\frac{z(h-z)}{2}\cdot + U\frac{h-z}{h}$$

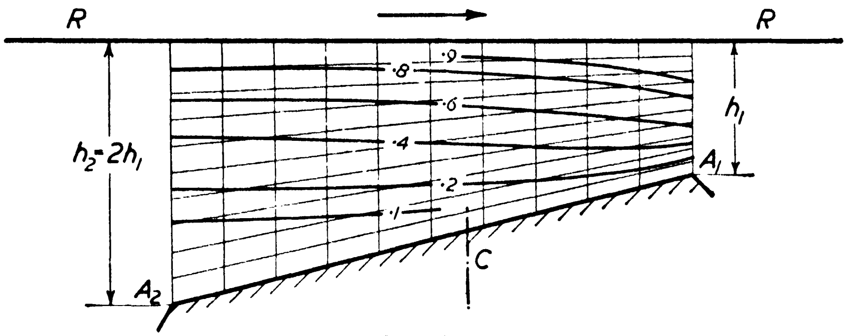
$$= -U(h-z)\left\{\frac{3}{c^2x^3}\left(x - \frac{2a_1a_2}{a_1+a_2}\right)z - \frac{1}{h}\right\}, \quad \ldots\ldots \quad \text{IV, 19}$$

being thus zero on the fixed plane where $z = h$, and U on the moving plane where $z = 0$, as is necessary to comply with the condition there can be nowhere any slipping of the fluid relatively to solid surfaces. It will also be observed that at the point of maximum pressure, where $x = x_m = 2a_1a_2/(a_1 + a_2)$, the velocity varies uniformly from zero on the fixed plane to U at the moving plane.

In fig. IV, 8 the velocities are shown for all points of the film by means of lines of equal velocity relative to the *fixed* plane, expressed as decimals of the velocity of the moving plane, these velocities being calculated for the particular case of a bearing in which $a_2 = 2a_1$, and consequently $x_m = \frac{4}{3}a_1$, and the point of pure shear is at one-third of the length of the film measured from the outflow end. The same diagram is given on an enlarged scale in Front Folder IV, B.

Front Folder IV, B′ contains what is essentially the same information, but transformed to show lines of equal velocity relative to the *moving* plate. In this case the velocity at $z = 0$ is zero, and at $z = h$ (i.e. on the fixed plate) it is U.

It will be observed from the former of these diagrams that, in this case, the velocity of the lubricant relative to the pad nowhere exceeds the velocity of the moving plate, and that it has stationary values in the direction of z, both



Fig, IV, 8

at the outflow end where it is in contact with the moving plate, and at the inflow end where its velocity is zero at its contact with the fixed plate. These characteristics, however, do not apply generally. When $a_2 < 2a_1$, the velocities are everywhere less than U, except at the actual surface, and are nowhere stationary in the direction of z. If $a_2 > 2a_1$ there are always maximum velocities in the film at the outflow end which are greater than U, and at the inflow end there are minimum velocities which are in fact negative, that is to say opposite in direction to U. An extreme example of the latter kind is discussed from a different point of view in Sect. IV, 9.

8. Losses of Energy by Viscous Friction in Lubricating Film: Heat Generated.

The rate of loss of mechanical energy arising from the internal viscosity of the lubricant in any plane bearing, whether of limited or unlimited dimensions, is given immediately when U, the velocity of the moving element, and F', the total tractive force resisting its motion, are known, being simply

$$W_f = F'U.$$

For the class of bearings of unlimited width which have been discussed in the preceding sections it has been noted with respect to equation IV, 16*a* that

$$F' = \frac{\mu U}{c} \log \frac{a_2}{a_1} \quad \text{approximately.}$$

Hence

$$W_f = \frac{\mu}{c} U^2 \log \frac{a_2}{a_1}. \qquad \text{IV, 20}$$

Corresponding to the loss of mechanical energy an amount of heat is produced which, in gramme calories, is W_f/J, J being Joule's Equivalent in ergs (or dyne-centimetres) of the gramme calorie, which is approximately $4{\cdot}18 \times 10^7$ (Ref. IV, 3).

The heat is of course developed in the first place within the film, whose temperature immediately rises at each point at a rate proportionate to the mechanical energy lost at that point, and it is carried away, after the bearing has been in operation for a sufficient length of time for the temperatures to become steady at all points, partly in the fluid itself to the outlet end of the bearing, and partly by conduction, firstly from the interior of the fluid to the metal parts of the bearing, and then through these metal parts to the atmosphere or to some cooling medium applied to the bearing for the purpose.

Equation IV, 20 gives the heat developed in the whole bearing. To determine the amount developed at each point of the film (which is for any cube unit of volume sheared in one direction only, the shearing force in the unit multiplied by the rate of shear) it is seen, referring to equation II, 5, that

$$\delta W = \mu\left(\frac{\partial u}{\partial z}\right)^2,$$

or in heat units (gm. cal.)

$$\delta W = \frac{\mu}{J}\left(\frac{\partial u}{\partial z}\right)^2, \quad \ldots \ldots \ldots \quad \text{IV, 21}$$

u being given, for the plane bearings under consideration, by equation IV, 19.

By this process, which need not be developed in detail, a calculation has been made of the rate of heat loss at all points of the same infinitely wide plane

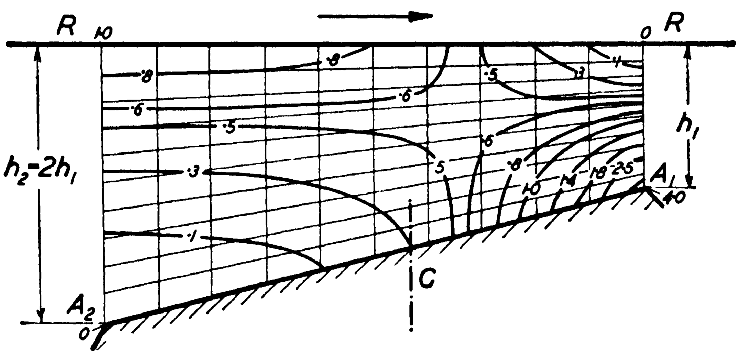


Fig. IV, 9

bearing for which the local velocities of the film are shown in fig. IV, 8 and Front Folders IV, B and IV, B′. The results are given in fig. IV, 9, and, in fuller detail, in Folder IV, C.

Further discussion of these diagrams, and of the whole subject of this section, is deferred to later sections, after the features of various types of sliding bearings have been examined; the subject is introduced at this stage because the comparative simplicity of plane bearings of unlimited width, from the point of view of calculation, allows a clear representation of the main features of the phenomena to be presented.

It is particularly to be remarked from the diagrams that, in the bearing selected for illustration, the generation of heat is far from being uniformly distributed throughout the film. It is on the contrary very markedly localized at two spots, the one, which is by far the more concentrated, in the immediate neighbourhood of the stationary plane at its outflow end, and the other extending along about one-third of the portion of the moving plane which is, at the moment considered, opposite to the inflow end of the stationary member.

The details of the action vary with each particular bearing, but the effects here noted are of general application, and are of much practical significance. It is to be remarked, however, that all locally intensified generation of heat must be accompanied by local rises of temperature, involving in any usual liquid lubricant a corresponding local lowering of the coefficient of viscosity, which has so far been assumed to be uniform.

9. A Special Case of the Infinitely Wide Plane Bearing.

It has been said above (at the end of Sect. IV, 7) that when, in the infinitely wide plane bearing, a_1 is less than $\frac{1}{2}a_2$ there are maximum velocities of the fluid at the outflow end which are greater than U, the velocity of the unlimited plane, and negative velocities in some parts of the film at the inflow end. It is of interest to consider the case which arises when a_1 is made indefinitely small in comparison with a_2, $h_1 = ca_1$ being also reduced to zero, so that a vanishingly small clearance remains between the plates at the "outflow" end.

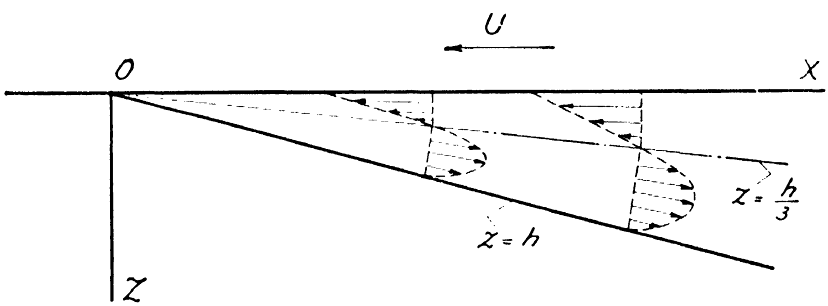


Fig. IV, 10

This extreme case is illustrated in fig. IV, 10. Equation IV, 11 which gives the pressure at all points within the lubricant, with the postulate that this pressure is uniform across the thickness of the film, becomes, when a_1 is made zero,

$$p = \Pi + \frac{6\mu U}{c^2}\left\{\frac{1}{x} - \frac{1}{a_2}\right\}. \qquad \text{IV, 22}$$

The same substitution in equation IV, 19 gives the velocity of the fluid at all points within the film as

$$u = -U(h-z)\left\{\frac{3z}{c^2x^2} - \frac{1}{h}\right\} = -U(h-z)\left(\frac{3z}{h^2} - \frac{1}{h}\right) \qquad \text{IV, 23}$$

$$= U\left\{3\left(\frac{z}{h}\right)^2 - 4\frac{z}{h} + 1\right\}, \qquad \text{IV, 23a}$$

so that $u = u_h = 0$ on the stationary plane where $z = h$ and $u = u_0 = U$ on the surface of the moving plane where $z = 0$. It is also seen from equation IV, 23*a* that the ratio of u to the constant velocity U depends only on the ratio of z to h and, since $u = 0$ when the second factor on the right-hand side of equation IV, 23 vanishes, that there is zero velocity in the fluid when $z = \frac{1}{3}h$ for all values of x.

At all points between the lines $z = 0$ and $z = \frac{1}{3}h$ the flow takes place in straight lines towards the meeting-line of the planes, and at all points between $z = \frac{1}{3}h$ and $z = h$ in straight lines radiating away from that line of intersection. On each of the radiating lines the velocity is uniform along the length.

Strictly interpreted this solution involves a breach of continuity of flow since the areas across which the inward and outward flows take place diminish progressively as the line of intersection is approached. It is to be remembered, however, that the formulæ are based on the assumption that the angle between the planes is indefinitely small and, as a consequence, that the velocity w of the fluid in the direction of z is everywhere negligible. Actually w is, in general, of the same order with respect to u, as δh with respect to δx, and it is admissible to regard continuity of flow as being brought about by flow of this small order of velocity across the plane $z = \frac{1}{3}h$, so as to transfer fluid from the triangle above that plane to the triangle below it, as indicated in fig. IV, 11.

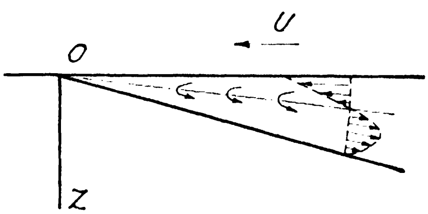


Fig. IV, 11

It results from the constancy of the velocity u along the lines of flow that, when x is very small, the rates of shear increase without limit, as does also the intensity of pressure, as shown by equation IV, 22, as well as the total pressure between the planes, as is seen immediately by integration of the last term of that equation. The moment of the pressures on the stationary plane about the origin of x remains, however, finite, being per unit width

$$G = \int_0^{a_2} px\,dx = \int_0^{a_2} \Pi x\,dx + \frac{6\mu U}{c^2}\int_0^{a_2}\left(\frac{1}{x} - \frac{1}{a_2}\right)x\,dx$$

$$= \tfrac{1}{2}\Pi a_2^2 + \frac{6\mu U}{c^2}\int_0^{a_2}\left(1 - \frac{x}{a_2}\right)dx$$

$$= \tfrac{1}{2}\Pi a_2^2 + \frac{6\mu U}{c^2}\left[x - \frac{x^2}{2a_2}\right]_0^{a_2}$$

$$= \left(\Pi a_2 + \frac{6\mu U}{c^2}\right)\frac{a_2}{2}. \qquad \text{IV, 24}$$

The results obtained in this section apply to the limiting conditions which arise when the stationary member of a plane bearing has a *fixed* inclination to the moving member, i.e. when c is maintained constant while the load or speed, or the viscosity constant, is subjected to variation. If, for instance, the inclination c is held constant while the viscosity constant is continuously lowered by rise of its temperature, the clearance h_1 progressively diminishes with diminution of the quantity of oil which passes through that clearance, and with a consequent reduction of the rate of removal of heat from the bearing. Ultimately, when h_1 approaches zero, the intensity of pressure becomes so high that either there is breakdown of the solid material or, at least, further lubrication of the kind assumed ceases. When, instead of being rigidly held, the stationary member is pivoted in the manner mentioned in Sect. IV, 4, upon increase of load, or diminution of the viscosity, the clearances diminish proportionately along the length of the bearing, the inclination c changing automatically so as to retain the centre of pressure at the point a determined by the position of the pivot.

Ultimately, if the load is increased, or the viscosity diminished sufficiently, the two members of the bearing come into contact but, in this case, it will be contact over the whole surface of the stationary plane (c becoming zero) and the intensity of pressure will remain finite.

Other applications of the results obtained in this section will be made in various sections of the following chapters. What is essentially the same problem analytically has been treated as a matter of elastic-solid theory, and for a fuller discussion and details the reader is referred to Ref. IV, 4.

10. Analogy between the Convergent Lubricating Film and a Solid Wedge.

There is an obvious superficial resemblance between the form and function of a convergent lubricating film, and the form and function of a solid tapering wedge, and it is not unnatural that the tapering lubricating film should be often called a "wedge film". The analogy can indeed be useful as assisting in the formation of a mental picture of the action of the lubricant, but it is important that the essential differences of the two phenomena be recognized in order that the analogy should not mislead.

A solid wedge is usually forced into the convergent interspace by forces applied to its end, or to its free sides, not by a motion of one of the surfaces with which it is in contact relatively to the other. This latter relative motion is essential to the action of the lubricating wedge, while it is non-existent or, at any rate, immaterial in the case of the solid wedge. It is also essential to the fluid wedge, as already pointed out, that one of the lubricated surfaces shall be of unlimited length (so far as the lubricating action is concerned); that the other be discontinuous; and that the former should be the member moving in the direction of convergence. No such condition applies to the action of a solid wedge.

11. Bearings of Finite Width: Experimental Validation of the Theory.

The theoretical examination in the preceding sections of the present chapter, being limited to viscous films of widths which are unlimited in comparison with their lengths, cannot of course be put to direct test by experiment. Before the discussion of bearings of finite width is taken up in the following sections it may be well to premise that the main results which are obtained therein have been confirmed by direct experimental tests, and that the preceding discussion of infinitely wide bearings is confirmed by such tests as giving useful approximations. The theory of bearings of finite width is based on the same general assumptions as those which have already been given for plane bearings of infinite width; but it is much more complicated in mathematical development, and the algebraic results can only be applied numerically to actual bearings by lengthy calculations. Of the comparatively few serious attempts which have been made to confirm or disprove these results, and thereby the underlying physical postulates, by precise laboratory methods (as distinguished from practical trials of actual bearings designed on the indications of the theory), two may be specially mentioned as having led to positive conclusions. These are, in order of their dates, a series of laboratory trials carried out by Frössel in collaboration with Professor L. Prandtl of Göttingen, and published by the latter in 1937 (Ref. IV, 5), and an investigation made by Morgan, Muskat, and Reed (Ref. IV, 6) reported in 1940.

The first of these investigations was carried out by means of curved bearing pads, co-acting with a revolving cylinder of relatively large radius, but realizing, with a degree of approximation within the range of experimental errors, the conditions of action of plane pads. The pads were pivoted in the manner described in Sect. IV, 4, the ratio of the widths to the lengths of the pads being 3 : 1, and the apparatus was completely submerged in the oil. Professor Prandtl has expressed in the following terms the general conclusion drawn from these experiments.

" The pressures recorded not only give the exact picture which would have been expected from theory, but were quite repeatable. The conclusion is drawn that the hydrodynamic theory is correct in every respect."

The second series of experiments above referred to was carried out with plane pads of widths equal to their lengths, and were consequently directly applicable to test numerically the theoretical results which are given for that case in detail in Sects. IV, 12 and 13. Tests were made both with pivoted pads, and with pads fixed at small inclinations to the continuous plane surface which was the moving member of the apparatus. It was found that the latter form, while giving results in general agreement with the theory, and confirmatory of its basic assumptions in all respects, did not afford exact numerical comparisons with the calculated results on account of the impracticability of measuring the very small angle of inclination with sufficient accuracy.

From the experiments with pivoted pads (the position of the pivot relative to the length of the pad being varied in the individual experiments), the conclusion was expressed that the experimental data

" coincide with the predictions in absolute magnitudes without the use of any adjustable or arbitrary constants. The only parameters entering into the experiments are the directly measurable dimensions of the sliders (pads) and the distance of the pivot-line from the trailing edge. With these predetermined, no further adjustment is possible for bringing the experimental data and theoretical predictions into agreement. Yet without such adjustment the measurements confirm in every detail the predictions of the theory."

Incidentally, it was proved by this series of experiments that the frictional resistances to the motion were of the same order as the resistances calculated for a bearing of infinite width acting under corresponding conditions. The validity of these calculations as approximations, subject to corrections such as are hereinafter explained (Sects. IV, 14 and 15), applicable to bearings of finite width, is thus confirmed.

12. Theory of Plane Bearings of Finite Width.

The theory of plane bearings of finite width which is given in Sects. IV, 12 and 13 is based on the same principles as have been given in the preceding sections for the plane bearing of unlimited width. The present problem, being three-dimensional, is however more difficult than the latter two-dimensional problem, and its solution cannot in general be made use of as a simple algebraic formula but only in the forms of series which have to be reduced to numerical shape by rather laborious methods. The theory was originally published and applied (Refs. IV, 2, 7, and 8) in 1905, but has not been replaced by any simpler or essentially modified method, and it will be given in the following sections in what was practically its original substance.

In fig. IV, 12 (as in fig. IV, 5 illustrating the plane bearing of infinite width), the moving plane surface JKK'J' of unlimited extent is assumed to coincide

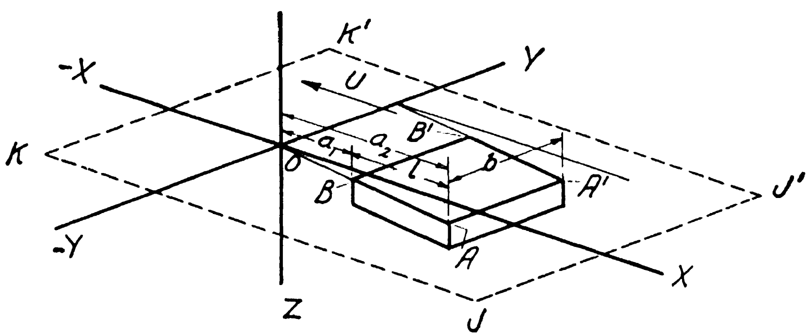


Fig. IV, 12

with the plane $z = 0$, and to move with respect to the fixed and limited plane surface ABB'A', which is coincident with the plane $z = cx$, with constant velocity U in the negative direction of x. The length of the fixed pad is taken as l, and its width as b, both measurements being of the same order. The outflow edge of the pad is at $x = a_1$, and its inflow edge at $x = a_2$, as in the former case. As before also, the inclination of the two planes to one another, c, is taken to be small, and the interspace between them filled with fluid of uniform viscosity μ. One lateral edge of the fixed pad is assumed to be in the plane $y = 0$, so that y is positive at all other points of its width. To simplify the expressions the pressure of the fluid is taken to be zero along all four edges of the pad. When, as is usual, the actual pressure is atmospheric there, the values which are derived by calculation for the pressure inside the film must of course be increased at all points by Π.

The general differential equation for determining the pressures and velocities of the fluid between the planes is derived from the equation IV, 4 after putting, as in the case of the infinitely wide planes, the velocity V in the direction of Y zero.

The equation then becomes

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial y}\right)+6U\frac{\partial h}{\partial x}=0, \quad . \quad . \quad . \quad \text{IV, 25}$$

or, with the further postulate that μ and $\partial h/\partial x = c$ are constant,

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)+6\mu Uc=0. \quad . \quad . \quad . \quad \text{IV, 26}$$

The term $\frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right)$, which was rejected in Sect. IV, 6, must now be retained since p and its derivatives may now vary in the direction of y, and components v of the velocity of the fluid will also exist at all points of the fluid, except on the median line of the pad.

With the further simplification that the two planes of the bearing intersect on the axis of Y, so that $h = cx$ and is invariable in the direction of y, equation IV, 26 becomes

$$h^3\frac{\partial^2 p}{\partial x^2}+3h^2\frac{\partial h}{\partial x}\frac{\partial p}{\partial x}+h^3\frac{\partial^2 p}{\partial y^2}+6\mu Uc=0,$$

or

$$\frac{\partial^2 p}{\partial x^2}+\frac{3}{x}\frac{\partial p}{\partial x}+\frac{\partial^2 p}{\partial y^2}+\frac{6\mu U}{c^2x^3}=0, \quad . \quad . \quad . \quad \text{IV, 27}$$

with the boundary conditions that $p = 0$ when $x = a_1$ and when $x = a_2$, and also when $y = 0$ and when $y = b$. These conditions are obviously symmetrical with respect to the plane $y = \frac{1}{2}b$.

For the solution of the differential equation IV, 27 it is necessary to express p in the form of the sum of a series of terms, such as

$$p = p_1 + p_3 + p_5 + \ldots + p_m + \ldots,$$

each of these terms being an expression containing both x and y in the form

$$p_m = \varpi_m \frac{\sin my}{mx},$$

in which the factor ϖ_m contains x only, and m is any one of the series of odd integers extending from 1 to infinity.

The boundary condition at each of the lateral edges of the pad, and the condition of symmetry about its middle plane $y = \frac{1}{2}b$, will be automatically complied with by the expansion of p into a series of terms of this kind, if y is measured in terms of a fictitious unit of length b/π times the unit in which the breadth was measured as b, this new unit being temporarily introduced for the purpose. The breadth then becomes $\pi b/b = \pi$, and at the same time the length of the pad becomes $\pi l/b$, and the x co-ordinates of its outflow and inflow edges $\pi a_1/b$ and $\pi a_2/b$.

Only odd integers being included in the series of terms for the expansion of p, each of the terms of the expansion will be of the same sign, as well as of the same magnitude, on each side of the middle plane $y = \frac{1}{2}\pi$.

To carry out the process of obtaining a solution of equation IV, 27 by this method of expansion of p in series, it is necessary that the last left-hand term of that equation shall be expressed as the sum of a similar series of sine terms in y. The sums of the corresponding mth terms are then equated individually to zero, and the sine factors eliminated so as to leave a series of functions ϖ_m of x, each of which can be calculated. Since $p = 0$ both when $x = \pi a_1/b$ and when $x = \pi a_2/b$, irrespective of the values of y, each of the ϖ functions of x must be zero when x has either of these values.

The sine series which is appropriate for expressing the last term of equation IV, 27 is

$$\sin y + \tfrac{1}{3}\sin 3y + \ldots + \frac{1}{m}\sin my + \ldots,$$

of which the sum to infinity is known to be $\frac{1}{4}\pi$. Equation IV, 27 can then be written as

$$\frac{\partial^2 p}{\partial x^2} + \frac{3}{x}\frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial y^2} + \frac{6\mu U}{c^2 x^3}\cdot\frac{4}{\pi}\Sigma\frac{\sin my}{m} = 0. \quad . \quad . \quad \text{IV, 28}$$

Taking then any one term $p_m = \dfrac{\varpi_m \sin my}{mx}$

of the expression for p, and writing for simplicity $mx = \zeta$,

$$\frac{\partial p_m}{\partial x} = m\frac{\partial p_m}{\partial \zeta} = m\left\{\frac{1}{\zeta}\frac{\partial \varpi_m}{\partial \zeta} - \frac{\varpi_m}{\zeta^2}\right\}\sin my.$$

$$\therefore \frac{\partial^2 p_m}{\partial x^2} = m^2\frac{\partial^2 p_m}{\partial \zeta^2} = m^2\left\{\frac{1}{\zeta}\frac{\partial^2 \varpi_m}{\partial \zeta^2} - \frac{2}{\zeta^2}\frac{\partial \varpi_m}{\partial \zeta} + \frac{2\varpi_m}{\zeta^3}\right\}\sin my;$$

also
$$\frac{\partial^2 p_m}{\partial y^2} = -m^2\frac{\varpi_m}{\zeta}\sin my;$$

so that the coefficient of $\sin my$ in equation IV, 28 is

$$\frac{m^2}{\zeta}\left\{\frac{\partial^2 \varpi_m}{\partial \zeta^2} + \frac{1}{\zeta}\frac{\partial \varpi_m}{\partial \zeta} - \left(1 + \frac{1}{\zeta^2}\right)\varpi_m + \frac{24\mu U}{\pi c^2}\cdot\frac{1}{\zeta^2}\right\} = 0. \quad . \quad \text{IV, 29}$$

This equation is known to be satisfied by alternative forms which must be combined to give the complete solution. These are

$$\left.\begin{aligned}
\varpi_m &= A_m I_1(\zeta) + B_m K_1(\zeta) \\
&\qquad + \frac{24\mu U}{\pi c^2}\left(1 + \frac{\zeta^2}{3} + \frac{\zeta^4}{3^2 . 5} + \frac{\zeta^6}{3^2 . 5^2 . 7} + \ldots\right), \\
\text{and}\quad \varpi_m &= A_m' I_1(\zeta) + B_m' K_1(\zeta) \\
&\qquad + \frac{24\mu U}{\pi c^2}(\zeta^{-2} + 3\zeta^{-4} + 3^2 . 5\zeta^{-6} + 3^2 . 5^2 . 7\zeta^{-8} + \ldots),
\end{aligned}\right\} \quad \text{IV, 30}$$

I_1 and K_1 being different functions of the variable ζ, and known as species of Bessel functions, each of which can be expressed as expansion series of the variable, and has been tabulated over considerable ranges of its values (Refs. IV, 9–12).

The equations IV, 30 give, on restoring the value $\zeta = mx$, corresponding equations for the general term p_m, viz.:

$$\left.\begin{aligned} p_m &= \frac{\sin my}{mx}\left\{A_m I_1(mx) + B_m K_1(mx) \right. \\ &\qquad\left. + \frac{24\mu U}{\pi c^2}\left[1 + \frac{(mx)^2}{3} + \frac{(mx)^4}{3^2 . 5} + \ldots\right]\right\} \\ \text{and}\quad p_m &= \frac{\sin my}{mx}\left\{A_m' I_1(mx) + B_m' K_1(mx) \right. \\ &\qquad\left. + \frac{24\mu U}{\pi c^2}\left[(mx)^{-2} + 3(mx)^{-4} + 3^2 . 5(mx)^{-6} + \ldots\right]\right\}, \end{aligned}\right\} \quad \text{IV, 31}$$

the first of these forms being suitable for calculation when mx is small, the second when it is large.

The coefficients A_m, B_m, A_m', B_m', etc., must be determined to make p_m vanish when $x = \pi a_1/b$, and when $x = \pi a_2/b$ for all values of y. They will of course be multiples of $24\mu U/(\pi c^2)$, which will be a factor in the final expression for

$$p = \Sigma\, \frac{p_m \sin my}{mx}.$$

Finally, the values of p which have been thus determined for the whole extent of the pad in terms of the temporary unit of length, in terms of which its length was $\pi l/b$ and its width π, are to be assigned to corresponding positions over its area measured in the original unit in which its length was l and its width b.

Various tables, essential to the carrying out of the mathematical processes which have been outlined, are rendered accessible through Refs. IV, 9–12 already cited. Elaborate tables of Bessel Functions are in course of being prepared by the Computation Laboratory of Harvard University (Ref. IV, 13).

The intensity of pressure p having been determined for all points (x, y), the total pressure

$$\overline{P} = \int_0^b \int_a^{a_2} p\, dx\, dy$$

may be found by arithmetical or graphical summation. For examples of simple methods of arithmetical calculation, of sufficient accuracy for the purpose, the reader may consult Ref. IV, 14.

In broad outline the variation of pressure in the direction of motion presents the same features both when the ratio of the width of the pad to its length is limited, and when it is unlimited, at least on longitudinal sections near the central section.

Proceeding from this point of view, it has been proposed (Ref. IV, 15) as a means of abbreviating the arithmetical work, to treat the determination of the distribution of pressure in a plane bearing of finite width as a correction to be applied to the simpler and known distribution of pressure in an infinitely wide bearing, given in equation IV, 11. The actual pressure at every point of the limited plane is thus to be expressed as

$$p = p' + \frac{6\mu U}{c^2(a_1 + a_2)}\left(\frac{a_1 + a_2}{x} - \frac{a_1 a_2}{x^2} - 1\right),$$

in which p' is the correction, or modification, which requires to be determined and applied to the known pressure for the same point in the pad of unlimited width.

The solution of the equation for p' is to be sought in the form

$$p' = \frac{6\mu U}{c^2} \cdot \Sigma \frac{C_m \Phi_m x}{x} \frac{\cosh \alpha_m y}{\cosh \alpha_m(\frac{1}{2}b)}, \quad . \quad . \quad . \quad . \quad . \quad \text{IV, 32}$$

in which Φ_m is a simple function of Bessel functions, and C_m and α_m are coefficients to be chosen so that the resulting series of equations IV, 32 shall satisfy the conditions laid down, viz. that p' must vanish at the inflow and outflow edges of the pad, along with the pressures found for the infinitely wide pad, and that on the lateral edges p' must be everywhere equal and opposite to the pressure found for the wide pad.

This process, though it may seem rather complicated in principle, has advantages as facilitating the numerical computations required for the calculation of some of the features.

Still another mode of solution of equation IV, 26 has been given recently by Mrs. W. L. Wood (Ref. IV, 26). In this solution the equations IV, 30 are replaced by

$$\varpi_m = A_m I_1(\zeta) + B_m K_1(\zeta) + \tfrac{1}{2}\pi L_{-1}(\zeta),$$

in which $I_1(\zeta)$ and $K_1(\zeta)$ are Bessel functions as in equations IV, 30, and $L_{-1}(\zeta)$ is the Struve function for imaginary argument, of order —1, of which tables are in existence (Ref. IV, 27).

Although the numerical calculation of the total pressure on a plane pad of any given length and width involves lengthy algebraic and arithmetical work, the total tractive force required to move the pad is readily obtainable in finite terms. In accordance with equation II, 5, the resistance in the direction of x on any unit area of the moving plane, at the point (x, y), may be taken as

$$f = \mu \frac{U}{h} - \frac{\partial p}{\partial x} \cdot \frac{h}{2},$$

and consequently the total resistance on its rectangular area extending in length from a_1 to a_2, and in width from 0 to b, is

$$F' = \int_0^b \int_{a_1}^{a_2} \left(\mu \frac{U}{h} - \frac{\partial p}{\partial x} \cdot \frac{h}{2}\right) dx\, dy$$

$$= b\int_{a_1}^{a_2} \mu \frac{U}{cx}\, dx - \tfrac{1}{2}c\int_0^b \int_{a_1}^{a_2} \frac{\partial p}{\partial x}\, x\, dx\, dy$$

$$= b\frac{\mu U}{c}\int_{a_1}^{a_2}\frac{dx}{x} - \tfrac{1}{2}c\int_0^b\left\{\Big[px\Big]_{x=a_1}^{x=a_2} - \int_{a_1}^{a_2} p\,dx\right\}dy$$

$$= \frac{\mu U}{c}\,b\log\frac{a_2}{a_1} + \tfrac{1}{2}cb\bar{P}. \qquad \text{IV, 33}$$

This result agrees, when divided by b, with the formula given by equation IV, 16*a* for the resistance per unit width of the plane of unlimited width, showing that the resistance *per unit width* is the same in both cases.

13. Numerical and Graphical Data for Plane Bearings of Finite Width.

By application of the methods described in Sect. IV, 12 a large amount of precise information has been derived and tabulated by various authors to show the results to be obtained from plane sliding bearings of various proportions of width to length, and the behaviour of any particular example can now be stated with considerable accuracy over a wide range of operating conditions. The most important data requiring to be known for purposes of design are of course, in the first place, the total load or charge $\bar{P}$ which can be carried by the pad, and, secondly, the resistance F' which it will offer to the motion.

The total load $\bar{P}$ on a pad or other single element of a bearing, will hereinafter be called its " charge ", the introduction of this term being desirable, not only for the sake of brevity, but also to distinguish the " charge " as the reaction of the element to the pressures acting on it within the bearing, as distinguished from the load imposed on the bearing from without, especially in cases when that load is distributed between several bearing elements.

When the pad of the bearing is to be pivoted, another characteristic of vital importance to be derived from the calculation is the position of the pivot relatively to the length of the bearing, that is to say, the value of $(\bar{a} - a_1)/l = \gamma$, the practical admissibility of which again will depend upon the minimum working clearance which can be permitted in the bearing under consideration.

It will have been observed, from the form of all the equations involving p and $\bar{P}$ in the preceding section, that the group factor $\mu U/c^2$ appears as a multiplying factor in all determinations of pressures in the plane bearing of finite width, as well as in the infinitely wide bearing dealt with in Sects. IV, 3–5. Similarly, the value determined for F' (equation IV, 33), viz.

$$\frac{\mu U}{c}\,b\log\frac{a_2}{a_1} + \tfrac{1}{2}cb\bar{P},$$

is seen (when $\bar{P}$ is shown as the product of $\mu U/c^2$ and a numerical factor) to be always a product of $\mu U/c$ and a numerical factor dependent on the dimensions and proportions of the bearing.

These facts enable the characteristics of bearings of different proportions and sizes to be directly compared, without regard to external factors.

Of special interest is the plane bearing having what is called a "square pad" (i.e. a pad whose width b is equal to its length l) as being, not only a natural standard of comparison with bearings of other proportions, but also as having been found in practice to approximate, in a great many individual and usual instances, to the most advantageous shape. It will be shown hereinafter (Chapter VI) that such is the case for journal bearings as well as for plane bearings.

For these reasons a summary of the leading characteristics of the square bearing will be given as an introduction to a more comprehensive account of the constants of rectangular plane bearings in general.

It is convenient, in order to facilitate comparison between pad bearings of different length-width ratios, to state the charge $\overline{P}$ on a pad as the average charge P per unit of its width, its length l being taken as constant. This is the same basis as that on which the total pressure P on an infinitely wide pad was stated in Sect. IV, 3, e.g. equation IV, 12.

The particular case of the square pad which is taken for numerical illustration is that in which the distance from the outflow edge of the pad to the line of intersection of its plane with the co-acting plane is equal to the length of the pad, that is to say,

$$a_1 = l, \text{ or } \alpha = 1.$$

In this case the charge on the pad is given by

$$\overline{P} = \frac{0{\cdot}0669\mu U l}{c^2},$$

or since the pad is square, alternatively by

$$\overline{P} = \frac{0{\cdot}0669\mu U \mathfrak{A}^{\frac{1}{2}}}{c^2},$$

where $\mathfrak{A}$ is the area of the pad, $l \times b = l^2$.

The charge per unit width is $P = 0{\cdot}0669\mu U/c^2$ in dynes cm.$^{-1}$ (or any other units consistent with those in which μ and U are measured).

The coefficient of resistance is

$$h = \frac{F}{P} = 10{\cdot}8 \ldots \times c.$$

The position of the centre of pressure (pivoting point) is given by

$$\gamma = \frac{\bar{a} - a_1}{l} = 0{\cdot}42 \ldots,$$

or

$$\bar{a} = l \times 1{\cdot}42. \ldots$$

It will be seen by comparison with the value of γ given in Table IV, 1 (p. 81) for the infinitely wide slipper in which $\alpha = 1$ (viz. $\gamma = 0{\cdot}431$) that the position of the pivoting point varies little with the width of the pad so long as the ratio $\alpha = a_1/l$ is unity. It will be seen later that this rule holds approximately for every other assigned value of the ratio, at least when the ratio of width to length is not much below unity.

Fig. IV, 13 (taken from Ref. IV, 7) shows the series of lines of equal pressure (isobars), as multiples, in steps of 10^{-3}, of the constant $6\mu U/c^2$ over the surface of the square pad. Fig. IV, 14 gives the same information in the form of a three-dimensional graph, which conveys an immediate impression to the eye of the form of the isobars in this standard case.

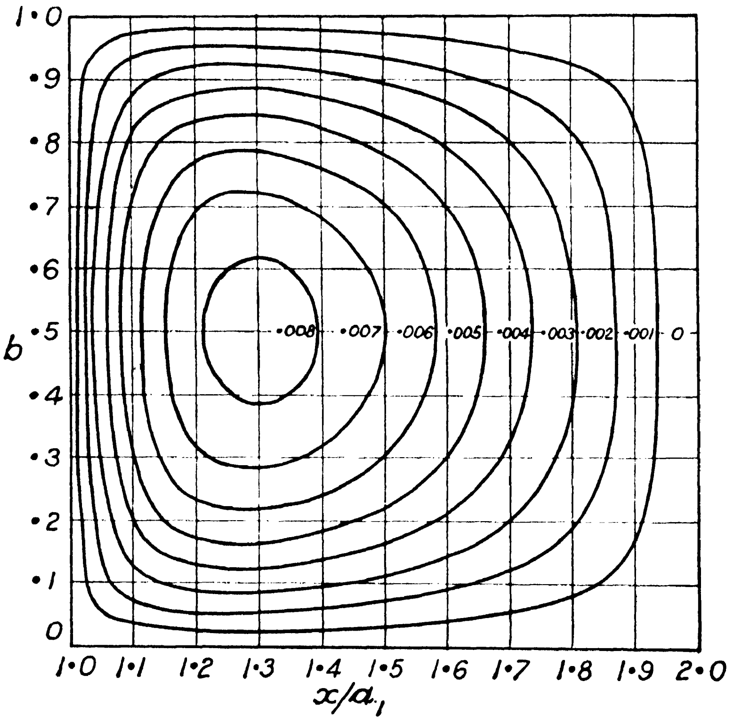


Fig. IV, 13

On Front Folder IV, D there are also shown, on the same scale of dimensions, (a) a replica of fig. IV, 14, and corresponding lines for, (b) a square segment of a pad of unlimited width, (c) a pad of width approximately 4 times greater than its length, and (d) a pad whose length is 3 times greater than its width. The scale of pressures is also the same in each of these diagrams, except (d) in which it is exaggerated 5-fold relatively to the other three figures. A comparison of these figures will enable a clear impression to be formed of the extent to which a diminution of the width of a bearing pad reduces the intensities of pressure which are produced in the different instances under otherwise equal conditions, i.e. with equal lengths; the same sliding velocity; the same viscosity of lubricant; the same inclination, c; and the same position of the resultant of the pressures, the last being defined in all cases by $\alpha = a_1/l = 1$, which corresponds approximately in all the cases illustrated to $\gamma = 0{\cdot}40$ as the constant defining the position of the centre of pressure or pivoting point. Since the length and inclination of the pads, and the value of α, are all uniform throughout this series, the minimum clearance h_L is also the same, and this being the case, the tractional resistance per unit width F will vary from one to another of them only to the extent of the variations incident to the comparatively small second term of the expression in equation IV, 33. In other words, the coefficients of resistance of the four pads will vary approximately in the inverse ratio of their charges.

It will be noted that Folder IV, D (*d*) gives, in addition to the pressures on a pad of length 3 times greater than its width (which are shown by full lines), the outline of the figure (in dotted lines) for two other pads of the same form parallel

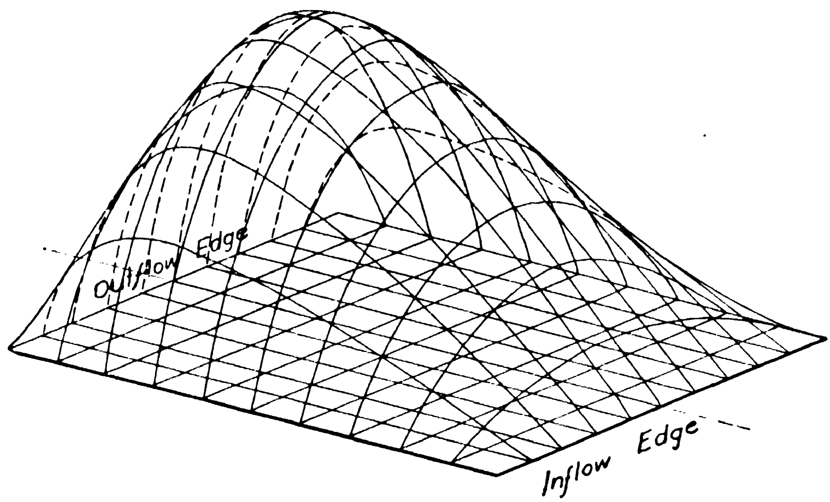


Fig. IV, 14

and adjacent to the first. The whole thus represents a square plane pad in which the pressure of the lubricant is maintained at zero value, not only around the four sides of the square, but also along two other straight lines extending through the length of its surface and dividing it into three equal rectangles. This is the condition which would actually exist in a square plane pad which had two longitudinal grooves in its surface. Grooves of this kind were often provided in bearing surfaces, under the name of "oil grooves", in the days before the principles of film lubrication were known, and they are still too frequently met with in various forms. The effect on the total oil-pressure, and consequently on the charge which the pad can support, may be appreciated by comparing fig. (*a*) of Folder IV, D with fig. (*d*), especially when it is remembered that the vertical scale of fig. (*d*) has been increased 5 times relatively to that of the other figures of the folder.

The principal numerical constants for rectangular plane pads having length-width ratios differing through the widest range required by any usual practical requirements are given in graphical form in Folders IV, E, F, and G.

Of these diagrams the most important, as being in a sense a key to the others is the diagram of Folder IV, E which is reproduced to a smaller scale in fig. IV, 15. It shows for varying values of the ratio of the width of the pad to its length, i.e. $\beta = b/l$ as ordinates, and varying values of the characteristic α as abscissae, the curves of equal values of the constant $\gamma = (\bar{a} - a_1)/l$ which determines the position of the centre of pressure, or pivoting point. It has been already noted that this position is independent of the inclination c of the pad.

It will be observed that in this, as in the other diagrams of the series, the horizontal line $\beta = 1$ designates the square pad; the intersection of that line

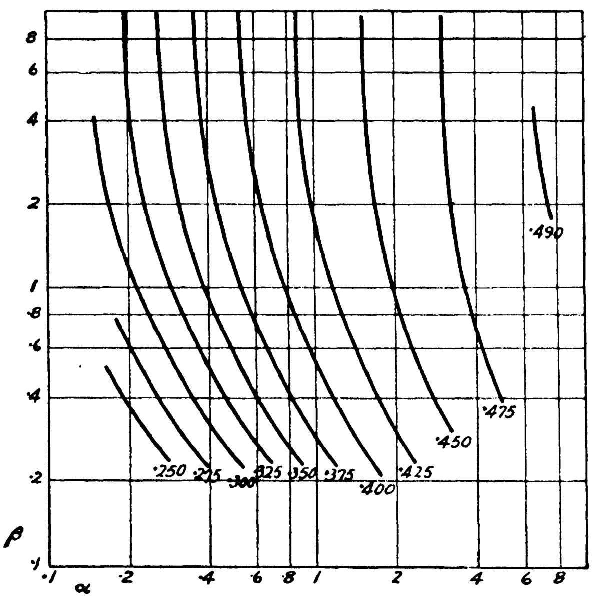


Fig. IV, 15

with the vertical line $\alpha = 1$ designates a square pad subject to the special condition $h_2 = 2h_1$ assumed in the particular example already chosen as a standard, and illustrated in figs. IV, 13 and 14.

It will also be observed that, in the pad of greatest width-length ratio ($\beta = 10$) covered by the diagram of Folder IV, E, the values of γ for various values of α already approach somewhat closely to those given in Table IV, 1 (p. 81) for the pad of unlimited width.

Folder IV, F shows, for the same ranges of values of α and β as the diagram of Folder IV, E, the charge per unit width of bearing of pads of fixed length, but of various width-length ratios, in which the inclination c is maintained invariable, and of the amount which is assumed in the diagrams of figs. IV, 13 and IV, 14. The constants shown on the lines of the diagram are the numerical values of the quotient of P when divided by $\mu U/c^2$.

The curves of Folder IV, G show on a similar scale to the preceding folders the charges per unit width of pads of varying width-length ratios in which the inclination c is varied so as to maintain the clearance h_1 between the co-acting points of the two planes at the same fixed value as that of the square pad of figs. IV, 13, IV, 14 and Folder IV, D and the numerical values shown on the curves are again those of P divided by $\mu U/c^2$.

In the preparation of the data presented in these various diagrams of the constants of plane bearings of varying width-length ratios, direct calculation has been largely supplemented by collation of results obtained from various sources, especially Ref. IV, 15 already cited, and Refs. IV, 16 and IV, 17.

The reader will find in these sources many other results presented in different ways and from various points of view.

14. Conditions not taken into account in the preceding Theoretical Sections.

In the preceding theoretical discussion of the processes of lubrication of plane bearings, no account has been taken of various complicating factors, which in practice are always present in greater or less degree. It has been postulated, for instance, that the sliding surfaces are truly plane and smooth, not only initially when assembled in the bearing, but also when subjected to the forces and heating which exist in the operating bearing. It has also been assumed that the other essential participant in the functioning of the bearing, the viscous lubricant, remains unchanged in its properties when the bearing is operating or, at any rate, that its viscosity remains uniform in all parts of the lubricating film.

Actually, in all practical cases, the solid members of the bearing are deformed when in operation, both by the stresses and by the heat to which they are exposed, to a sufficient extent to modify appreciably the results of the simplified theory in which these influences are ignored. The viscosity of the liquid lubricant, also, is changed considerably by the same agencies, being raised by increases of pressure and reduced by rises of temperature; it consequently varies from place to place in the operating film. The specific volume of lubricant also increases with its temperature, but this effect is comparatively small, and usually of minor importance. Numerical data on these changes of properties have already been given for the lubricants in general use, in Chapter III.

Though it is not impossible to introduce into the mathematical theory additional conditions and assumptions taking account, at least as rough approximations, of the conditions above-mentioned, and to obtain formal solutions of the equations, it is doubtful whether, for practical purposes, the complications thereby introduced are warranted. For the most part, the conditions which bring about considerable changes in the forms of the members of a bearing, or in the viscosity of its lubricant, are conditions the occurrence of which is to be prevented by good design and workmanship. The several disturbing factors vary in relative importance from one case to another, and when one of them assumes special prominence it is usually best dealt with by approximate methods adapted to the particular case. The direct effects of rises of pressure and temperature on the solid members of bearings are discussed in the course of descriptions of particular examples in the next chapter, Sects. V, 2 and V, 3, and occasion is there taken to point out how these effects have been minimized by good design. In this section only complications arising from variations of the coefficient of viscosity of the lubricant will be further considered.

A fairly good approximation to the experimentally determined relations between the viscosity and pressure in many lubricating oils is given (Sect. III, 9) by the empirical formula

$$\mu = \mu_0 e^{\nu p},$$

where μ_0 is the coefficient of viscosity at atmospheric pressure and μ that at pressure p, ν being a viscosity-pressure exponent characteristic of the particular oil. A modification of the general equation for the flow of lubricant in the plane bearing of unlimited width, utilizing this empirical relation, has been developed by Muskat and Evinger (Ref. IV, 18) and may be summarized as follows:

Since the pressure is very nearly uniform across the thickness of the film, the viscosity, in so far as it is dependent on the pressure, will be likewise uniform and, in a bearing of unlimited width, μ will be a function of x only.

If equation IV, 5 be written as

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = -6U\frac{\partial h}{\partial x}, \quad \ldots \ldots \quad \text{IV, 34}$$

and a new variable

$$\Phi = \int_0^p \frac{dp}{\mu}$$

be introduced, equation IV, 34 becomes

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial \Phi}{\partial x}\right) = -6U\frac{dh}{dx}, \quad \ldots \ldots \quad \text{IV, 35}$$

which is of the same form as equation IV, 34 with a fixed value of μ, and consequently has the same form of solution, with Φ replacing p.

In this way, using the relation $\mu = \mu_0 e^{\nu p}$, Muskat and Evinger find that (in the notation used here) the relation between the coefficient of resistance, and the ratio of charge to velocity, is represented for different values of the viscosity-pressure exponent by the curves shown in fig. IV, 16.

In this diagram values of f/c (the ratio of the coefficient of resistance to the inclination of the pad) as ordinates are plotted for different values of the viscosity-pressure exponent against values of $\mu_0 U/(c^2 P)$ as abscissæ. The constant defining each curve of the diagram is not, however, the viscosity-pressure exponent ν itself, but the product of ν by the mean intensity of pressure on the pad, i.e. $\nu' = \nu P/L$, L being the length of the pad in the direction of motion, and the pad being of infinite width.

It will be noted that the curve for $\nu' = 0$ (or $\nu = 0$) is the curve for viscosity μ_0, having no variation with pressure.

Variations of the viscosity of lubricants with varying temperature are much more difficult to take into account, even approximately, than variations

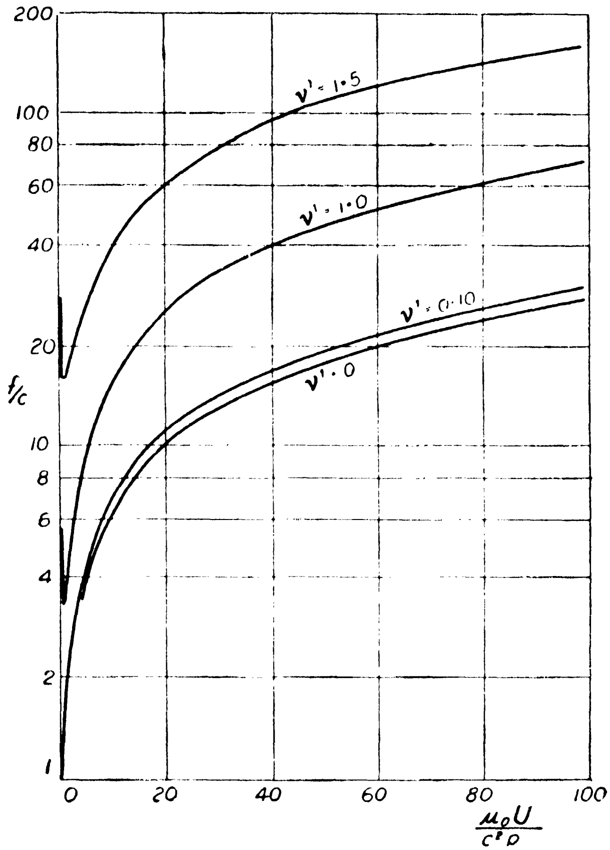


Fig. IV, 16

with pressure. This is mainly for the reason that the temperature, unlike the pressure, cannot be legitimately regarded as being uniform across the thickness of the film. Heat is generated throughout the thickness by the viscous resistance as shown, e.g. by fig. IV, 9 (p. 86), and its conduction to the solid surfaces (which is the chief means by which it is disposed of, as will be shown in Sect. V, 4) involves a fall of temperature through the film. Consequently $\partial\mu/\partial z$ must enter into the equations as well as $\partial\mu/\partial x$.

Several authors, however, have dealt with the problem on the hypothesis that the variation of temperature in the film normally to its surfaces can be ignored, modifying the equations of flow for μ constant by introducing a variation of μ in the direction of x only. For example Duffing (Ref. IV, 19) treats μ as varying along the length of the bearing according to the formula $\mu = \mu_0 h/h_0$, and applies this assumption to bearings of finite, as well as infinite width-length ratios. He finds, however, no significant difference between the charge calculated as carried by the bearing on this assumption and that calculated from the assumption of uniform viscosity, other conditions being unchanged. To this extent, Duffing's investigation confirms the validity of the usual practice of practical workers, which is to take a mean of the estimated temperatures of the various parts of the lubricating film as determining the viscosity of the lubricant, which is then treated as being uniform throughout.

Theoretical discussions have also been published of the effects which may arise from changes with temperature of the specific volume of the lubricant in the various parts of the bearing. Since, in all ordinary lubricants, this effect is small compared with the variations of viscosity which accompany the same changes of temperature and, in some circumstances, is also smaller than changes of viscosity due to increase of pressure, the matter is of theoretical rather than practical interest. Further references to the subject will be found in the last paragraphs of Sect. V, 2.

More important, in most cases, as resulting in discrepancies between the results of experimental investigations of the operation of bearings and established theory, are effects which arise from departures of the actual forms given to the bearing surfaces from those postulated in the theory. These departures may be either defects in the geometrical forms of the surfaces, such as may be originated by the imperfections of the machine tools by which they are produced, or may be merely small irregularities and rugosities on a form which is in general correct. Small rugosities are unavoidable, being in part due to the necessarily finite progressions of the tools or grinding wheels by which the surfaces are produced, and in part to inherent irregularities or granules in the materials themselves; and the thickness of lubricating films is in many cases so small that irregularities of the surfaces which for ordinary purposes can be regarded as negligible may assume importance in the behaviour of the bearing when examined critically. Although obvious, and in a broad sense generally recognized, the far-reaching influence of defects in the geometrical forms and the finish of bearing surfaces is not always perceived, and more or less fanciful or occult causes are sometimes invoked to explain unexpected results.

A fuller examination of this subject, considered quantitatively, is given in the next section and in Appendix I.

Another cause of departure of experimental from theoretical results, equally obvious with that last-mentioned, and equally apt to be disregarded, is an insufficiency of the volume of lubricant supplied to the bearing to enable it to form a complete film according to the theory. The effect of such a deficiency, in the case of a pivoted pad, is that the fluent film will not completely fill the interspace between pad and slider at the inflow end. Since the position of the pivot is fixed relatively to the length of the pad, its position relatively to the film actually formed will be shifted rearward, that is to say, nearer to the inflow end of the effective film, and the inclination of the pad will consequently be reduced from that which would occur with a complete film. With unchanged charge and viscosity the thickness of the film must then decrease, and the tractional resistance increase.

In brief, the pad will behave as if it had been shortened at the inflow end. If the supply of oil is restricted to a sufficient extent, the pad may become in effect a centrally pivoted pad with its surface parallel to, and in direct contact with, that of the slider, and, if it continues to operate in that condition, its mode of operation will be substantially the same as that of the parallel plane bearings discussed in Sect. IV, 16, with the difference that the pressures between the solid surfaces will probably not be statistically uniform over the area of the pad, but may increase from its inflow to its outflow end.

In non-pivoted plane bearings in which the pad surface is formed with a fixed convergence relatively to the slider, the effect of a deficiency in the supply of lubricant is somewhat different. In this case also the primary effect will be that the interspace will not be filled with the fluent film at the inflow end, and that the interspace will be everywhere diminished. The ultimate effect in this case, however, is that the bearing assumes the condition described in Sect. IV, 9, that is to say a condition of theoretically infinite fluid pressure at the outflow end of the pad, where it makes contact with the surface of the slider. In actuality, this condition resolves itself into solid contact and abrasion at that edge.

Another source of anomalous results, similar in some respects to those caused by deficiency in the oil supply, is the entrance of air into the interspace between the surfaces. The air may enter, either mixed with the oil in the form of froth or, in some cases (Ref. IV, 5), as a film of air separating two films of oil. In the latter case it may result in the bearing operating with a lower coefficient of resistance than it would show with an oil film correctly formed according to theory, but the action will be erratic. The means to be adopted to prevent the occurrence of this disturbance will be discussed in Chapter IX, which deals with methods of oil supply and distribution.

15. Limitations shown by Experiment to the Applicability of the Theory of Plane Bearings.

It has been stated (Sect. IV, 11) that the main theoretical results which are given in this chapter for the behaviour of plane bearings of finite width are confirmed by the results of practical tests. While this is true over a wide range of the conditions met with in engineering practice, the theory has, of course, like all such theories, limits to the range of its practical application.

It is shown in Sect. IV, 13 that in any given plane bearing of finite width the charge carried by the pad is proportional to the speed of sliding and to the viscosity of the lubricant, and inversely proportional to the square of the inclination of the sliding surfaces; also that the coefficient of resistance of the bearing is directly proportional to the same inclination. Both the efficiency and the effectiveness of the bearing are therefore enhanced by making the angle of inclination as small as possible. Evidently the unavoidable errors and irregularities of workmanship and of materials impose practical limitations in this respect, and the questions arise:

At what stage does the limit occur in practice? and, what departures from the theoretical results appear as the limit is approached?

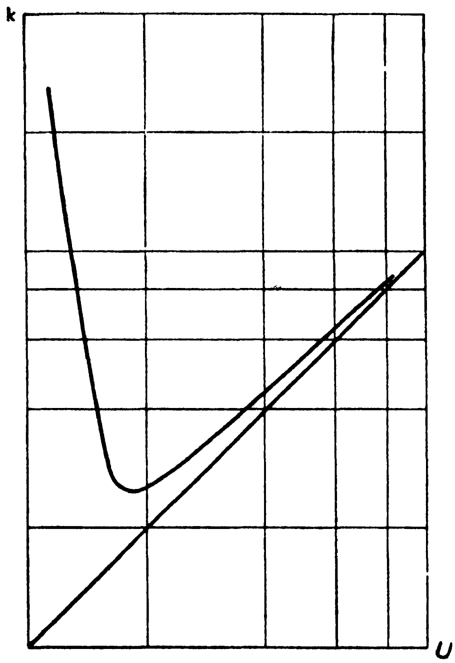


Fig. IV, 17

The diagram of fig. IV, 17 is taken from a paper by Stribeck (Ref. IV, 20) containing many other curves of the same general form, which embody the results of a very accurate series of tests on the limiting conditions of operation of bearings. The bearings actually tested were journal bearings, but the conditions involved are (as will be seen in Chap. VI) essentially the same in them as in plane bearings. The diagram shows the relation of the coefficient of resistance of a particular bearing to the speed of sliding, the load on the bearing and the viscosity of the lubricant being maintained as nearly as practicable constant, so that the inclination and distance apart of the surfaces depended upon the only remaining variable, the sliding velocity.

In order to bring out their relation to one another more clearly, Stribeck's curve has been replotted in fig. IV, 17 to logarithmic scales. It will be seen that, at the higher values of the velocity U, the coefficient of resistance k follows nearly the straight line of proportionality required by theory. It shows, however, a tendency, which increases as the velocity is reduced, to run above a course of strict linearity, and at a velocity of about 5 cm. sec.$^{-1}$ reaches a minimum value from which it rises rapidly as the velocity is diminished still further. The curve is well-defined and quite smooth throughout, suggesting that viscous friction, not solid friction, is operating, although the rising of the latter part of the curve appears to contradict directly the theory of fluid lubrication.

Fig. IV, 18 is a similar diagram showing experimental results obtained by Professor Tenot (Ref. IV, 21) from a tilting-pad bearing which will be again referred to in Sect. VI, 7.

Many other published experiments show similar results. It has been usual to attribute the change in the relation between the velocity and the resistance which is thus found to take place at low velocities, or with high intensities of load, in a given bearing, to the incidence of direct contacts and solid friction between the pad and the slider when the mean distance between their surfaces becomes of the same order as the small irregularities and projections upon them. Muskat and Evinger, however, in the paper already cited (Ref. IV, 18) suggest that it may result from increase of viscosity with pressure, at least in cases in which the approach of the surfaces is brought about by high intensities of pressure. Such, however, was not the case in Stribeck's tests, nor were the highest pressures employed by him sufficient to produce any appreciable changes of viscosity in oils of the kind which he used. Other experiments showing the same phenomena have been made with lubricating fluids (including water) in which there is little or no known pressure-viscosity effect.

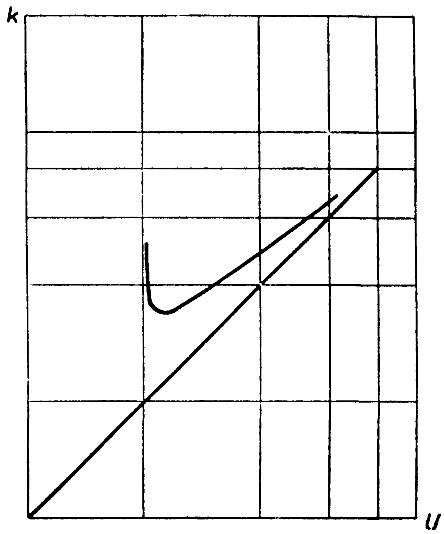


Fig. IV, 18

Professor Heidebroek has maintained (Ref. IV, 22) that the increase in the coefficient of sliding resistance at low sliding velocities is not necessarily due to "dry", or solid, friction between the surfaces. In his view, it may be consistent with the continuance of fluent lubrication at velocities below that corresponding to minimum resistance. He has suggested that, if regard is paid to the existence of rugosities on the bearing surfaces, convergent films may be conceived to exist between them, though these films will be much thinner and exert much greater viscous resistance than would a film of the thickness corresponding to the mean basal plane surfaces.

So far as is known to the author, Professor Heidebroek has not put his suggestion to quantitative proof, nor treated the subject mathematically on the basis of the established theory of the convergent film. In Appendix I, a mathematical treatment is given on the basis of a certain simplifying assumption as to the form of the rugosities on the surfaces, and the conclusion is reached that the recorded experimental results can be reasonably explained on this basis. In this investigation one of the surfaces (that of the pad) is assumed to be uniformly covered with corrugations of sine-wave form, the other surface being truly plane. On this assumption fluent lubrication may continue indefinitely as the inclination of the pad is reduced until the crests of the corrugations on its surface touch the plane surface of the slider, when the inclination becomes zero. It is found however that, before this happens, the coefficient of resistance reaches a minimum value and thereafter rapidly rises as the surfaces approach more closely to one another (in the same way as shown by Stribeck's and Tenot's experiments). It is found that this rise of the coefficient of resistance is due in part to increase of the tractional resistance to sliding, but also, in part, to reduction of the mean fluid pressure, and consequently of the charge which the pad can carry, when the inclination becomes very small. In Back Folder A, 2 the coefficient of resistance k and the charge on the pad, P, are plotted on logarithmic scales as ordinates, against a variable h/H as abscissæ, h/H being the ratio of the clearance between the crests of the corrugations of the pads and the plane surface of the slider to the half-depth H of the corrugations, the latter dimension being regarded as constant. The diagram shows that the coefficient of resistance reaches its minimum when h is rather less than twice as great, but still somewhat greater than H. The charge P carried by the pad, on the other hand, increases until h drops to about $\frac{1}{2}H$.

In actual bearings of course the rugosities will not be regular corrugations, nor of equal height, as assumed in this calculation, and they will occur on both surfaces, not on the pad only. The more prominent of the irregularities on the opposed surfaces will come into contact with one another, while the less prominent are still separated by comparatively thick films of lubricant. The fluent film will then become broken by discontinuities progressively increasing in number and extent as h is reduced, and the resistance to sliding, instead of being solely the resistance of a viscous film of diminishing thickness, will be partly of that nature and will consist partly of frictional and abrasive resistances at the solid contacts, the latter finally predominating. This is the series of conditions which is regarded as taking place during the slowing down and final stoppage of sliding motion, and which is to be regarded as normal in the operation of pivoted-pad bearings. The same series of conditions will occur in reverse order when a loaded pivoted-pad bearing is started up from rest.

Some intermediate condition of the series will exist in the steady running of bearings (such as are dealt with in the next section) which have merely parallel plane surfaces, these being incapable of forming continuous fluent films, either by being pivoted or by other means.

16. Thrust Bearings with Parallel Plane Surfaces.

Although now obsolete, except for rough or trivial purposes, thrust bearings having the opposed sliding surfaces plane and parallel, and consequently incapable of forming convergent films of lubricant, were formerly the only kind of thrust bearings known, being used, for example, for carrying the thrusts of marine propeller shafts up to the largest sizes and highest powers. The very wide experience of their operation for this purpose, though transmitted to the present day by tradition rather than by publication, has provided us with information which can be usefully applied in considering the behaviour of bearings of more rational design, when required to operate under abnormal conditions which do not permit the formation of fluent films, as in starting and stopping.

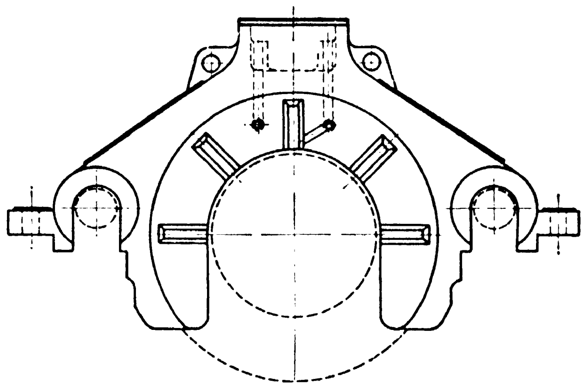


Fig. IV, 19

In the propeller thrust bearings of this formerly standard construction, it was found to be impracticable to carry the whole thrust load on a single pair of sliding surfaces. The propeller shaft was therefore provided with a series of $(n + 1)$ collars, plane-faced on both sides, and with a corresponding series of n double-faced plane stationary members, or shoes, one of which was inserted between each pair of adjacent collars. These shoes, which were usually (as shown in fig. IV, 19) of the general shape of the shoes worn by horses, were supported on a pair of longitudinal bolts (fig. IV, 20) threaded throughout their lengths, and each provided with n pairs of nuts, one pair holding each shoe. Each shoe could thus be adjusted and held against movement in the longitudinal direction with the object of apportioning to it its share of the total thrust load. The shoes were usually faced with a soft bearing-metal, but were sometimes of bronze unmetalled. In either case their surfaces were hand-scraped to fit the collars.

The arrangement for lubrication consisted usually of an oil-box formed in the upper part of each shoe, from which oil holes led to grooves cut in the operative faces. Many forms of these oil grooves were used, and fanatic claims were often made for the superiority of one or other of these forms, but none of them appears to have been durably accepted as being superior to the others.

In spite of many differences in details of construction which were tried during the long period of use of these bearings, there is remarkable unanimity in the records as to their maximum load capacity and their coefficients of resistance, the former being placed at 3, or at the most 4 atmospheres on the shoe surface (40 to 55 lb. in.$^{-2}$), and the latter as 0·03, or at the lowest 0·025. Wear, or abrasion, of the surfaces was continuous, but remained within limits which were regarded as tolerable as long as conditions of operation were not varied. Changes of propeller speed and load, producing differential extension, both thermal and elastic, between the thrust shaft and the longitudinal bolts, and consequent inequality between the charges on the individual shoes, invariably caused heating and excessive abrasion at one or other end of the series.

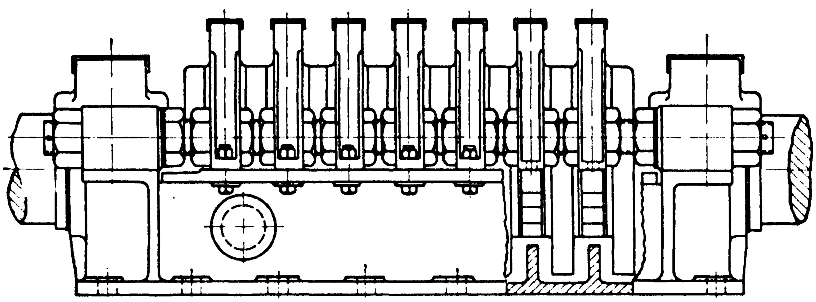


Fig. IV, 20

Since the intensity of load was approximately one-tenth, and the coefficient of resistance from 10 to 20 times as great as those which are conventionally allowed in marine pivoted-pad thrust bearings at the present time, and as sliding speeds are now probably not on an average more than double those often used in the horse-shoe bearings, it would appear that the means of heat disposal from the working surfaces used in present marine practice are not very much more efficient than those of older times. In pivoted-pad thrust units designed for water and steam turbines, the usual intensities of load and the coefficient of resistance are of the same order as in marine practice, but the sliding speeds are from 5 to 30 times as great. It would appear therefore that the means of cooling by oil circulation used in these latter bearings, such as those described in the next chapter (Sect. V, 4), are much more effective than those currently adopted in marine bearings.

17. Conditions occurring in the Starting and Stopping of Sliding Plane Bearings.

The conditions of direct contact, and of approaching contact, between the members of plane bearings, which are discussed in the two preceding sections and in Appendix I, only occur in convergent-film bearings at starting and stopping, or in the event of failure of the supply of lubricant to the bearing.

Putting aside the latter possibility, and the case of bearings which are only subjected to load when running at speed, the case must still be considered of bearings which are required to start or stop when under load.

The most important of these are the thrust bearings of vertical turbines and electric machines which have to carry the weight of the rotors and shafting when starting and stopping. In such units, and particularly in vertical water-turbines such as those described in the next chapter (Sect. V, 4), the slow-motion resistances of the thrust bearings have to be carefully considered, not only on account of the possibility of damage to the surfaces of the bearings, but also for the reason that the starting torque required by the loaded bearing is often comparable with the full-load torque of the machine, and its sudden decrease, when the machine commences to rotate, is likely to involve hydraulic or electric complications. For these reasons the conditions existing in the bearings at starting have been carefully examined by constructors from the practical standpoint, and the data obtained are of great interest both for their own sake and for the light which they throw on the physical actions involved.

The bearings in question carry in some instances gravitational loads of the order of 1000 tons, and may have bearing surfaces several square metres in area. The pivoted pads are usually of steel, or cast iron, coated with a tin base bearing-metal, the collar or slider being of hard alloy-steel or chilled cast iron, very carefully finished to a truly plane and smooth surface. The surface of the bearing metal of the pads is finished by hand-scraping, preferably alternately to two mutually fitting surface plates. Lubrication is effected with a mineral oil having a coefficient of viscosity of the order of 1 poise at atmospheric temperatures, and of about 0·25 poise at 50° C. (120° F.), which is a usual average working temperature of the bearings.

It is found that when such a bearing, with its surfaces oiled before assembly, is started from rest, it momentarily opposes a coefficient of resistance to motion of from 0·20 to 0·40, according to the degree of fineness of the finish given to the surfaces, the smoother the finish the lower being the resistance. The resistance, however, diminishes very rapidly after motion begins, and fluent-film lubrication is usually fully established in the first few revolutions of the shaft, with a coefficient of resistance of the order of 0·001.

The conditions at stopping differ from those at starting in that at starting only enough oil is present to fill the minute depressions between the saliences of the surfaces on which the members rest in mutual contact, and this quantity of oil must be largely augmented before a continuous fluent-film can be formed. At stopping, on the other hand, the fluent-film is initially present and a surplus of oil has to be displaced from between the surfaces before they can make contact on their saliences. In other words, there is at stopping a superfluity, and at starting a deficiency, of oil during the transitional period between full fluid-film lubrication and solid-to-solid contact. At starting, the additional oil required for running conditions must be carried by the slider into the interspace between slider and pad from the inflow end of the pad, so that an effective film cannot be established, nor can tilting of the pad take place, until the slider has moved through approximately the length of the pad. Up to that instant, solid contact between the surfaces must continue to take place.

A very instructive investigation carried out by the Westinghouse Electric Corporation gives detailed particulars of the effects at starting (Ref. IV, 23). Field tests were made on the thrust bearing of a vertical water-turbine carrying a static load of $4{\cdot}6 \times 10^5$ to $5{\cdot}5 \times 10^5$ kg. ($1{\cdot}0 \times 10^6$ to $1{\cdot}2 \times 10^6$ lb.), the diameter of the bearing being 208 cm. (82 in.). The initial surface roughness of the runner surface was measured by a diamond-pointed profile gauge and the root-mean-square of the rugosities is recorded as having been $1{\cdot}1 \times 10^{-5}$ and $1{\cdot}5 \times 10^{-5}$ inch ($2{\cdot}8 \times 10^{-5}$ to $3{\cdot}8 \times 10^{-5}$ cm.) respectively in two runners which were used in the tests. It was found that the high coefficient of resistance which existed momentarily at starting ("breakaway resistance"), which was from 0·25 to 0·30, dropped to a small fraction of that value as soon as the runner had made one-tenth of a revolution, and to the low values characteristic of fluent-film lubrication in about 10 revolutions.

Laboratory tests were carried out, as part of the same investigation, on a thrust bearing having only two sectorial pads of about 3 in.^{2} (20 cm.^{2}) of area each, these being coated with bearing metal and finished with similar accuracy to the large pads, the "high-spots" marked by a surface-plate covering from 3 to 5 per cent of the surface of the pad. Five chilled cast-iron runners which were used in the various tests of this series were finished to give r.m.s. roughness gaugings ranging from 5×10^{-6} to $4{\cdot}5 \times 10^{-5}$ inch ($1{\cdot}3 \times 10^{-5}$ to $1{\cdot}1 \times 10^{-4}$ cm.). All the tests were made at 30° C. (86° F.) with a "steam-turbine oil" having a viscosity of approximately 0·6 poise at 40° C. (104° F.).

After being oiled and loaded to 28 atmospheres (400 lb. in.$^{-2}$) on the pad surfaces, each bearing was left at rest for 5 minutes before being started. The starts of each were then repeated about 40 times, after which its starting-resistance became constant, falling in the case of the most coarsely finished runner from 0·45 or 0·50 initially to 0·40, and rising with all the other runners by factors varying from 30 to 50 per cent. In the case of the smoothest runner the initial value was as low as 0·15.

As the runners accelerated the normal values of fluent-film resistance (of the order of 0·001) were attained at sliding speeds which were lower, the finer the finish of the surface, as shown by Table IV, 2.

TABLE IV, 2

Roughness, r.m.s. in.	Speed, cm. sec.$^{-1}$
5×10^{-6}	5
$1{\cdot}0 \times 10^{-5}$	7·5
$1{\cdot}5 \times 10^{-5}$	10
$2{\cdot}3 \times 10^{-5}$	12

In the runner having the roughest finish ($4{\cdot}5 \times 10^{-5}$ r.m.s. in.) the coefficient of resistance was still as high as 0·004 at 12·5 cm. sec.$^{-1}$

The coefficients of starting resistance fell in each case with increase of the load from 7

to 56 atmospheres (100 to 800 lb. in.$^{-2}$) by about 10 per cent, and increased with increasing temperature, as for instance in the case of the smoothest runner according to Table IV, 3.

TABLE IV, 3

Coefficient of starting resistance		
Temp. of oil	(Pressure 14 atmos.)	(Pressure 56 atmos.)
30° C.	0·20	0·17
60° C.	0·23	0·22
80° C.	0·27	0·27

An "oiliness agent" (of unspecified chemical nature) added to the oil reduced the starting friction from 0·25 to 0·18, and the resistance over the speed range 0·05 to 0·33 ft. sec.$^{-1}$ (1·5 to 10 cm. sec.$^{-1}$), by about 20 per cent; above the latter speed it had no apparent effect.

In these tests there was no appreciable wear or abrasion of the surfaces except with the most coarsely finished runners. The runner with $2{\cdot}3 \times 10^{-5}$ r.m.s. inch finish showed a slight tendency to improve in finish (that is, to wear to a smoother surface) after its first few starts. The coarsest runner alone ($4{\cdot}5 \times 10^{-5}$ r.m.s. in.) visibly abraded the soft metal of the pads, picking up small particles of the metal on to its own surface.

In the discussion following the paper which has been cited as reporting this investigation, S. J. Needs pointed out that some of the field tests showed that the coefficient of resistance fell to about 8 per cent of its starting value after the runner had moved through only one-half of the length of a pad, that is to say, from an average starting value of about 0·33 to about 0·025. At this stage there could be no possibility of the pad tilting as a whole to form a convergent film, and its surface must be regarded as having been parallel with that of the runner. Such being the case, the practical identity of the coefficient resistance (0·025) with that characteristic of the old type of parallel-surface marine thrust bearings (which operated permanently in the parallel condition) is very striking, the coefficient in their case having been commonly within the range 0·025 to 0·030 (as stated in Sect. IV, 16).

18. A Summary of the Various Conditions and Effects of Lubrication in Sliding Bearings.

Various and apparently discordant as are the experimental facts given in the three preceding sections as to the operation of plane bearings under differing conditions, it is possible to give an interpretation of them on a common basis. A continuous series of transitional conditions can be seen as connecting the various modes of behaviour which result in coefficients of resistance so diverse as the 0·20 or 0·40 observed in the starting of thrust bearings from rest and the 0·001, or thereabouts, which is characteristic of the fluent lubrication of pivoted pads in normal operation.

The interpretation which will be given in outline in this section depends, on the one side, on the theory developed in Appendix I (of which the results are summarized in Sect. IV, 15) and, on the other, on the results of laboratory experiments which have been made by various investigators. Of the latter, it will be sufficient for the present purpose to refer to the investigations of W. B. Hardy, a pioneer in the subject, who has given a full summary of his results and conclusions in Ref. IV, 24, and described the details of his apparatus and procedure in Ref. IV, 25. Hardy's measurements of the static resistance to sliding motion were made with a sliding-element, or slipper, of glass or metal, having a smooth spherical contact-surface of approximately 2·5 cm. (1·0 in.) spherical radius, which was loaded with weights of the order of about 100 gm. ($\frac{1}{4}$ lb.), and placed on a horizontal plane surface of the same material having an equally smooth surface. An increasing horizontal force was applied to the slipper until it commenced to move, the highest value of the force observed *before* motion started being taken as the value determining the static coefficient of resistance.

Before each test was made, both surfaces were cleaned by stringent chemical methods and then various chemically pure liquid "lubricants" were applied to them in an enclosure filled with dry air, free from dust. In some cases only extremely thin films of the liquid were applied to the surfaces, in others they were flooded with the liquid in relatively large bulk. Generally speaking, the quantity supplied made comparatively little difference in the results found for any given liquid. The area of the circle of contact between the slipper and the plane was found by observation (as would be predicted by elastic theory) to be less than 1 mm.^{2} The mean intensity of contact pressure therefore exceeded 10 atmospheres, and undoubtedly was locally much greater, exceeding the compressive strength of the materials (see Sect. VII, 11). Abrasion was always observed to take place when the slipper moved.

The static friction of the dry, chemically cleaned, solid materials being of the order of unity, it was found that the application of paraffin oil (medicinal), and of other hydrocarbons of the classes of, or analogous to, those which are used as lubricants, reduced the coefficient by about 70 per cent, that is to say to about 0·3. Other liquids of similar viscosities, and water, had little effect on the coefficient of dry friction, but castor oil, and oleic acid, reduced it to about 0·1.

From the apparent absence of any law of dependence of the results upon the viscosities of the liquids, and for other concordant reasons, Hardy concluded that no matter existed in a fluid state within the contact areas, but that layers of single molecules of the substances applied as liquids (or at most layers a few molecules thick) became bound to the surfaces of the solids, and modified their superficial properties, including their static resistances to sliding. This view has been generally accepted by other investigators, the molecular layer, or layers, being regarded as virtually forming part of the solid body, and being attached to it and to one another by molecular attractions similar to those which exist between the adjacent particles of a homogeneous solid.

The layer of molecules was referred to by Hardy as a " boundary layer ", and the modification which it produces in the sliding resistance of chemically clean solids, as " boundary lubrication ". The latter term has persisted, with a rather indefinitely extended meaning, but must be regarded as having been unfortunately chosen to describe phenomena arising between what were considered to be virtual solids, and subjected to measurement only by means of *static* friction. The condition would seem to be better described as one of " modified ", or " mitigated " solid friction.

It will have been noted that the coefficients of static friction found by Hardy in the case of mitigation by paraffin and other liquids analogous to usual lubricants, viz. approximately 0·30, were of the same order as those found in the large-scale tests of Westinghouse turbine bearings described in Sect. IV, 17, in which the " breakaway " coefficient was given as 0·25 to 0·30, as well as those determined by the laboratory tests made during that series of investigations. This concordance of results between experiments made with a contact area of a fraction of a square millimetre and those made on plane bearing surfaces ranging up to several square metres in area is certainly striking, but is readily explained on the ground that even the most finely finished pair of plane surfaces which can be produced can only make contact, when brought together, on prominences extending over only a very small fraction of the surface areas, and that the mutual pressures on these contacts are of the same order of intensity, and the contacting surfaces are affected by the presence of liquids in the same way as in Hardy's experiments.

The experimental fact, noted in the account of the Westinghouse experiments, and generally recognized by constructors, that the starting resistance is the lower the more accurate and finely finished the surfaces, points to the probability that with very closely fitting surfaces the lubricant is trapped between them over considerable areas as they come together during the slowing-down of the machine, and remains so enclosed and under pressure until it is again started. To a greater or less extent such inclusions of lubricant would of course support a portion of the load on the bearing and, to that extent, relieve the saliences in solid contact from load. The coefficient of friction at starting would then be intermediate between that of modified solid contacts, as determined by Hardy, and the relatively small coefficient due to the shearing of the liquid inclusions. Approximate calculations, similar to that of Sect. II, 16, based upon probable assumptions as to distances between the surfaces and their deflections, under practical conditions, show that effective fluid pressures may be retained in this way for considerable periods of time.

All these considerations, however, apply only to the static condition and to the momentary effects at starting. Immediately after sliding motion begins between the surfaces a quite different set of conditions comes into action.

Thus, in the thrust-bearing tests, when the runner, supported by the solid contacts of small saliences from its surface upon similar projections from the surface of the pad, moves so far that the mutual contacts are lost by these pairs of projections, fresh contacts will be made between other pairs of saliences, such contacts lasting for periods which will diminish in length as the speed of sliding increases. These fresh contacts, however, will take place under conditions different from those of the static contacts, since the newly meeting prominences will be covered with films of liquid lubricant which will not be displaced for at least some fraction of the periods of the contacts, and these fractions will increase progressively as the velocity becomes higher and the periods of contact shorter. The frictional resistances of the contacts will therefore be less than that of starting, and will progressively diminish as the speed increases.

At the same time fluent lubrication will take place between limited and varying areas of the surfaces of the runner and pad in which mutual convergence exists. Even before the pad commences to tilt about its pivot, therefore, the coefficient of resistance to the motion may be expected to fall rapidly, as the experiments show that it actually does.

The condition which exists in this starting phase of the action of a pivoted pad, and which exists permanently in the action of unpivoted, parallel plane bearings, such as the marine thrust bearings of the obsolete type described in Sect. IV, 16, being one in which contacts of saliences necessarily occur, and in which the formation of a continuous and fluent lubricating film is impossible, may be designated as *partial* or *discontinuous* lubrication. It is invariably accompanied by wear and abrasion of the surfaces, and a large mass of concordant experience shows that it is incompatible with a coefficient of resistance lower than 0·03 or 0·025.

After the thrust-bearing runner has moved through a distance greater than the length of the pad, and consequently has carried fresh oil into all parts of the interspace between the surfaces, the pad is enabled to begin to tilt and to form a continuously convergent film. While its inclination is still small the rugosities of the surfaces, though they no longer make direct contacts, will affect the lubricating action in the manner discussed in Appendix I, and the coefficient of resistance will be greater, due to their influence, than it would be between truly plane surfaces under conditions otherwise the same. This condition may be described as one of *rugulose fluent lubrication*, and may be considered to continue under increasing speeds so long as the coefficient of resistance is higher than the minimum indicated in the diagram (Back Folder A, 2). Its beginning as well as its termination, with rising velocity, will naturally depend on the form, as well as the magnitude, of the rugosities occurring in any particular case. In the Westinghouse experiments it appears to have terminated for all but the most coarsely finished surface when the speed attained about 12 cm. sec.$^{-1}$

When the condition of "rugulose" lubrication has terminated, that of normal fluent lubrication with resistance increasing nearly in proportion to the speed sets in.

The sequence of conditions implicit in the interpretation of the experimental facts given in the last preceding sections is set out in Table IV, 4; merely transitional stages between the four main conditions of the sequence being ignored.

TABLE IV, 4

CONDITIONS OF LUBRICATION OF PIVOTED-PAD BEARINGS AT SLIDING VELOCITIES INCREASING FROM ZERO TO NORMAL

Mode of action of pad	Condition of lubrication	Inclination of pad	Characteristic values of coefficient of resistance
1. Static (starting from rest)	No lubrication (mitigated solid friction)	Nil	About 0·30
2. Parallel-plane sliding	Partial and discontinuous	Nil	Falling with increasing movement from about 0·15 to 0·03
3. Inclined-plane sliding	Rugulose fluent-film	Least clearance greater or less than, but of the same order as, the average height of rugosities	Depending on inclination; of the order of 0·002 to 0·010; increases with lowered velocity
4. Inclined-plane sliding	Normal fluent-film	Least clearance large compared with the heights of rugosities	Of the order of 0·001; increasing with higher velocity, or lower intensity of pressure

CHAPTER V

Thrust Bearings and some other Plane Sliding Bearings

1. Uses of Plane Sliding Bearings.

Plane bearings find many varied uses in mechanism, and the theory of their lubrication given in the preceding chapter is applicable to many kinds of machine elements in which rectilinear sliding motions occur. Several of these will be separately dealt with in the various sections of this chapter.

In the most important class of plane sliding bearings, however, namely those known as thrust bearings, the sliding motion is not rectilinear but rotary, the function of these bearings being to prevent longitudinal motion of rotating shafts which are subjected to forces in the axial direction. In most cases the required support is given by forming, or fitting rigidly, on the shaft a collar having a plane surface which abuts on one or more plane surfaces formed on the stationary bearing member. An alternative construction, less frequently used, is to form the continuous plane surface on the stationary member and the separated abutting surfaces on a collar attached to the shaft. In either case, the member having the continuous surface will be called, in accordance with an American usage, the " runner ". The separate supporting surfaces will be called " pads ".

Effective automatic lubrication, according to the theory set out in Chap. IV, can only occur if the divided plane surfaces are individually inclined to the continuous surface of the runner, and if the motion of the runner relative to the divided surfaces is in the direction of the convergence of their mutual inclination. Exceptions, or rather some apparent exceptions, to this established rule will be noticed hereinafter.

The practical conditions necessary for the actual realization of automatic film lubrication, of which the first is the formation of the surfaces accurately to the theoretical shapes, are more readily attained, and their attainment more readily verified, in plane bearings and especially in thrust bearings, than in bearings having curved surfaces. For this reason the more important of these conditions will be somewhat fully analysed in connexion with descriptions of thrust bearings in the present chapter, their applications to other bearings being left to inference from analogy. The circumstance that the relative sliding motion which occurs in thrust bearings is not strictly one of rectilinear translation but is rotary, so that the sliding velocity is not equal at all points of the opposed surfaces, does not in most practical cases affect the application of the theoretical results derived from the assumption of uniform relative velocity to so great an extent as might, at first approach, be expected. Usually it can be assumed, with sufficient accuracy, that the mean relative velocity is uniform throughout. An estimate of the amount of the errors that are likely to be introduced by this assumption, and a sketch of the types of corrections to the calculations, which have been proposed by various authors, are given in Sect. V, 7.

In almost all of the examples of thrust bearings which will be described in this chapter the necessary slight inclination of the pad members to the plane surface of the runner is brought about by pivoting the pads individually in the manner explained in Sect. IV, 4. Only in special instances is it effected by the more difficult and less efficacious method of construction which consists in shaping separately a series of inclined planes on a rigidly supported bearing member. Thrust bearings which are not provided, by one or other of these methods, with a series of inclined surfaces are now obsolete, except for the most trivial or roughest purposes in which simplicity of construction, or reduction of cost, outweighs all other considerations. The former use, however, of the obsolete forms, which was very extensive, has provided very useful data for the valuation of the inclined-plane fluent-film type, and for estimation of the performance of the latter under conditions of deficient lubrication. Reference to this aspect has already been made in the preceding chapter, especially Sect. IV, 16.

Although the principles of action of fluent-film lubrication and the pivoted bearing pad are now widely understood, and almost exclusively applied in thrust bearings, it is only exceptionally that their construction is such as to realize in any full degree the advantages of which they admit. Presumably the circumstance that, even as ordinarily constructed, pivoted-pad bearings carry loads more than ten times greater in intensity than can be placed on parallel surfaced bearings, with greatly reduced resistance, and have a speed range practically unlimited, has minimized incentives to the attainment of more refined constructions or closer attention to the other theoretical conditions of high efficiency. The benefits to be derived consist, however, not only in reduction of mechanical losses almost to the point of elimination, but in removing all sources of wear and deterioration in the bearings, and all causes for stoppages of machines on account of them.

2. Thrust Bearings: General Description and Typical Examples.

The largest single field of application of thrust bearings has hitherto been the carrying of the propeller thrust of self-propelled ships and vessels of all kinds and sizes. Other important applications, in which the pivoted thrust bearing has proved to be indispensable, are to steam and water turbines, especially water turbines constructed with vertical shafts. It is usual, in these latter machines, to allow the thrust bearing to carry not only the weight of the water wheel, and that of the electric generator which it drives, together with the connecting shaft, but also the hydraulic thrust pressures, often uncompensated. Thrust bearings for these turbine units are the largest and most heavily loaded in existence, or at least the largest constructed for continuous operation.

Of similar design and construction are the thrust bearings applied to centrifugal pumps, especially those having vertical or inclined shafts. An early, but still representative, example is illustrated in fig. V, 1 (Ref. V, 1),

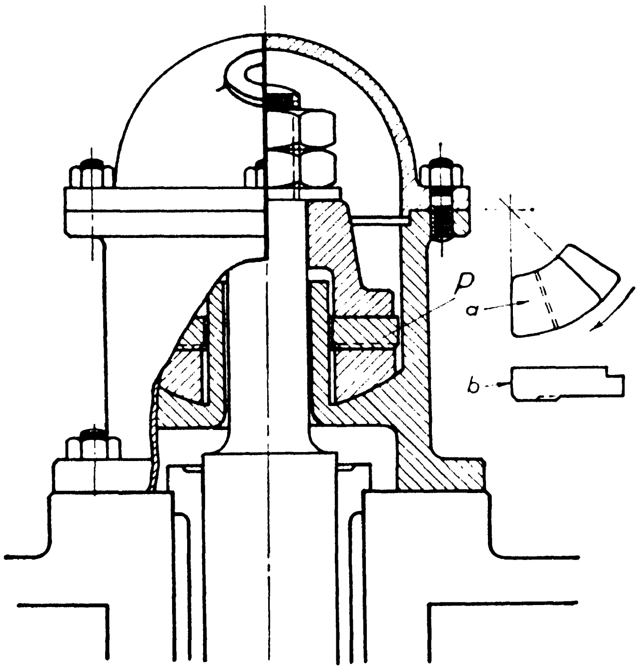


Fig. V, 1.—Pivoted-pad thrust bearing for vertical shaft

views of the sliding surface of one of the pads P and of its outer circumferential edge being shown at (*a*) and (*b*) respectively.

Amongst other applications may be mentioned particularly those to worm and worm-wheel shafts in reduction gearing, as well as to shafts fitted with bevel wheels, and to vertical shafts for all purposes.

In its simplest form, which is very efficient and convenient for application to horizontal shafts of small or moderate size and loading, the stationary member consists of a single thrust pad which is immersed in an oil bath contained in the lower part of the housing of the bearing. An example is shown in fig. V, 2. The runner takes the form of a collar dipping into the oil bath which contains the pad. The pad P is pivoted on a radial rib R placed at the optimum position of about two-fifths of the pad length from the leaving end, and is mounted in a holder H which is itself pivoted on a tangential rib *r*, abutting on the flat

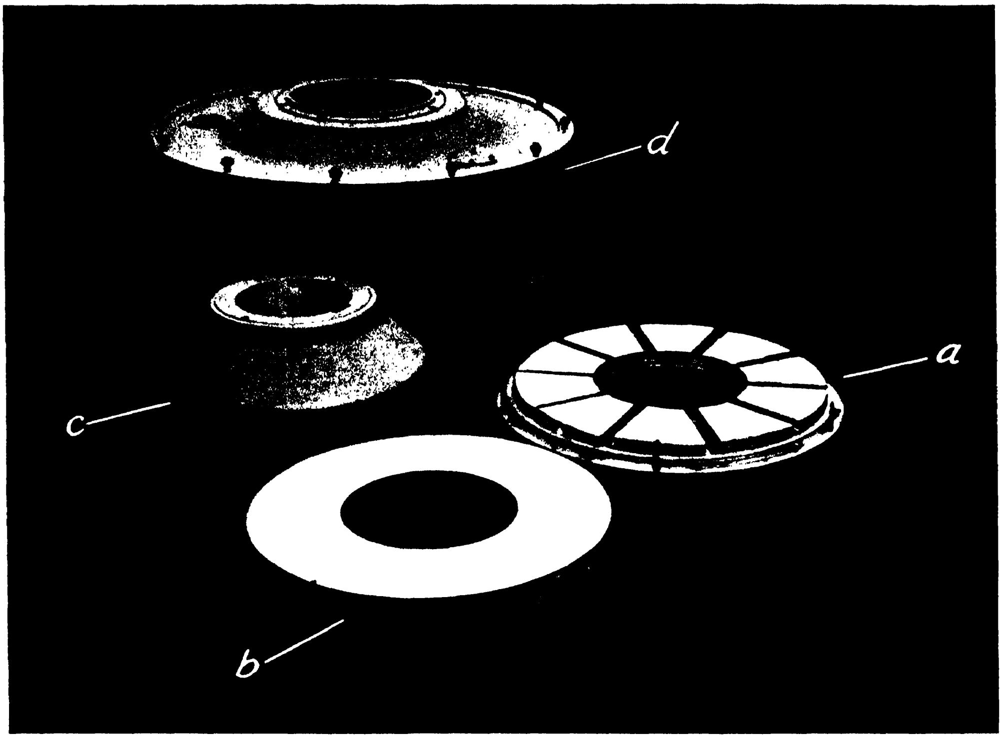


Fig. V, 3.—Component parts of a vertical pivoted-pad thrust bearing by Messrs. Escher-Wyss, Zurich

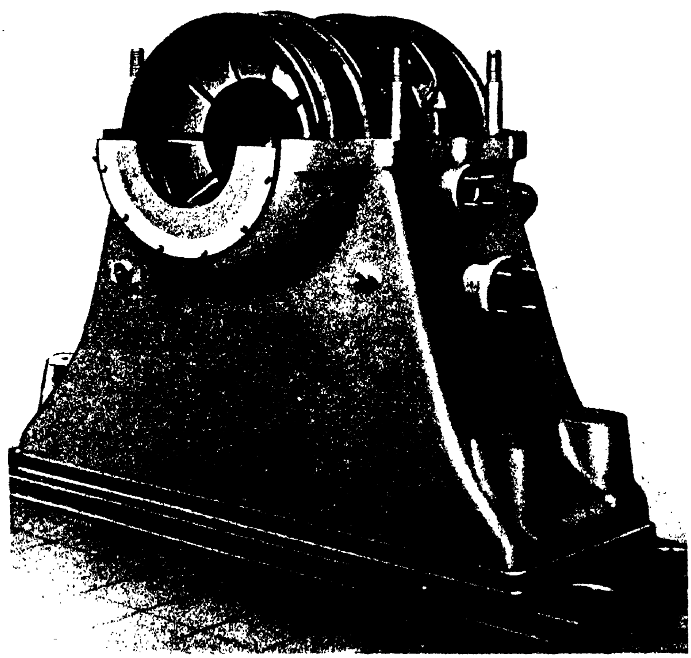


Fig. V, 4.—Double pivoted-pad thrust bearing combined with journal bearing by Messrs. Escher-Wyss, Zurich internal surface of the part of the housing which forms the oil bath. By means of this double pivoting in two directions at right angles, the pad is enabled to present its sliding surface in uniform relation radially, as well as in correctly inclined circumferential relation, to the surface of the collar, without the necessity of any accurate machining of its radial rib or of the surface of the housing on which it is supported. Small bearings of this type are suitable for carrying loads up to 1000 pounds per square inch of pad surface (70 atmospheres), with peripheral runner speeds of 30 feet (1000 centimetres) per second without oil circulation or cooling.

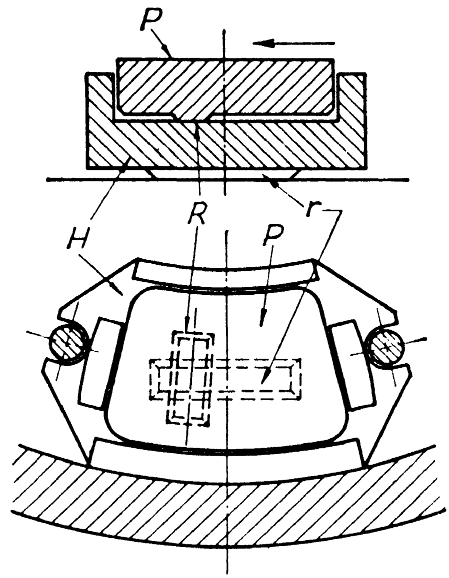


Fig. V, 2.—Single-pad thrust bearing for horizontal shaft

For the illustration (fig. V, 3) of a modern example of a thrust bearing for a vertical water turbine, the author is indebted to the courtesy of Messrs. Escher, Wyss of Zurich. This illustration shows (*a*) the annulus of 10 pivoted segmental pads; (*b*) the runner, consisting of a steel ring machined accurately plane on both sides; and (*c*) the so-called "shoe", or mounting block which is the element directly mounted as a collar on the shaft of the turbine and from which the runner proper is driven by pins engaging in holes in its reverse side, not seen in the picture. The housing (*d*) of the whole bearing forms an oil bath in which the runner and the pads are submerged. This bearing is one of a number of similar bearing units supporting the shafts of vertical hydroelectric generating units at the Etzel Power Plant, Switzerland.

Another type of bearing produced by the same firm is that illustrated in fig. V, 4. This is a horizontal bearing in which a pair of pivoted-pad thrust bearings, each mounted at one end of the pedestal housing, is combined with a journal bearing located between them. The horizontal shaft has two collars one of which runs on each of the two thrust bearings which face in opposite directions and are arranged for opposite directions of rotation. The bearing is thus of the type which is required in a self-propelled vessel which travels alternately in one direction and then back without turning. The actual bearing shown in the illustration was constructed, however, for a storage pump unit.

In a typical pivoted-pad thrust bearing the pads are arranged in annular series surrounding the shaft in the manner shown in the last two illustrations. They are usually spaced uniformly in the series, and often cover the greater part of the surface of the runner with which they make contact, even when a much smaller bearing area would be sufficient to carry the load which they are intended to carry, and would do so more efficiently. In an experimental investigation by Dr. von Freudenreich (Ref. V, 2) into the effects of varying the number of pads in a bearing from 1 to 10 (all the pads being alike, and all conditions being unchanged, except as to the unavoidable change in the oil distribution on the face of the runner due to the changes in the spacing of the pads), it was found that the bearing became overloaded, and abrasion of the surfaces commenced, with total loads proportional to

$$\Sigma P = 475n - 37{\cdot}5n^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 1}$$

n being the number of the pads.

Otherwise expressed, the bearing failed when the charge on each individual pad, in atmospheres, was

$$P = 475 - 37{\cdot}5n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 2}$$

From equation V, 1 it is seen by differentiation with respect to n that the total load on the bearing was greatest, i.e. ΣP a maximum, when

$$n = \tfrac{475}{75},$$

that is to say, with 6 or 7 pads. The result may be attributed with probability to one or other of two factors, or to a combination of both; namely to a less favourable supply of lubricant to the individual pads when a large number was used, or to inequality between the charges on the individual pads, in the case of larger numbers, in spite of precautions taken to minimize such a condition. Whatever the precise cause in this particular investigation, the general result, as to the adverse effect of close spacing of individual pads in an annular series, is in accord with all practical experience, and, in large bearings at least, has a cogent explanation, which will be given at length in Sect. V, 3.

The same investigator and his associates have also found that the most favourable form of pad does not conform exactly to a sector of a circular annulus, but is relatively shorter in the circumferential direction at the outer than at the inner radius. A form of this kind may be said to have become standard practice amongst the recognized makers of pivoted thrust bearings, most of whom also agree in giving four rounded corners to the bearing surfaces of the pads, in place of the abrupt angles of the strict sectorial form. That the small areas within abrupt angles are useless, is immediately apparent from the consideration that, since the increment of pressure is zero along the two limbs of the angle, it must also be zero in any direction between them, and consequently, to the first order, there can be no fluid pressure immediately within the angle, although the frictional resistance per unit area is of the same order there as elsewhere.

The radial width of the pads is usually made at least equal to, and is preferably rather greater than, the mean circumferential length.

Various forms of pivot and modes of pivoting of the pads are in use. One of the simplest, and perhaps the best, at least when accuracy of finish and extreme rigidity of the housing can be relied upon, is to form on the reverse side of the pad a plane supporting surface accurately parallel to its sliding surface, this supporting surface extending from the outflow end of the pad to a radial line which constitutes the nominal pivoting axis. In the actual bearing, however, the compressive strength of the material is always exceeded at this line, and the effective line of the pivot lies nearer to the outflow edge of the pad to an extent which is determined in practice more frequently by observation of the effects than by calculation. The remainder of the reverse side of the pad is relieved to a depth of about a millimetre (0·04 inch), or rather more in large pads. A pad of this type is shown in front and rear view and as viewed radially from the outer side in fig. V, 5. The plane supporting surface or "land" so formed may with advantage be limited in its radial extent to about the middle half of the radial width of the pad, and, to take account of the crushing above mentioned, may be continued, in heavily charged pads, in the circumferential direction somewhat beyond the theoretical axis of pivoting towards the centre of the pad, but so as still to terminate on a radial line. The mean intensity of the pressure between the "land" and the supporting surface will then be about four times greater than the mean fluid pressure on the sliding surface, but will of course be greatly exceeded in the neighbourhood of the pivoting edge where the pressure, when the pad tilts, will locally exceed the elastic strength of the materials.

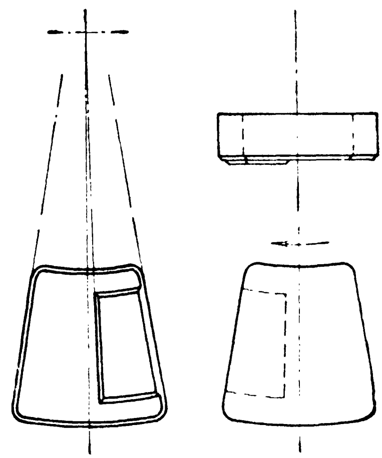


Fig. V, 5

The optimum position of the pivoting axis of the plane pad, being, as shown by fig. IV, 6 (p. 77), a "stationary" condition, may for that reason be afforded a fairly wide latitude without affecting considerably the allowable charge of the pad or its coefficient of tractional resistance. Still wider latitude may be considered to be allowable in view of the circumstance that the temperature, and consequently the viscosity of the lubricating film, vary over its area in a manner which cannot, in practice, be exactly known, but which consists broadly in a diminution of viscosity from the inflow to the outflow end of the pad. The centre of the developed pressure is thus automatically shifted forwards (i.e. in the direction opposite to the flow), and the pivot should be advanced to the same position. Careful investigation shows that the effect is not large in any normal case, but the effect of incorrectly locating the pivoting point too near to the centre of the pad, is thus partially neutralized. Error of location is to a large extent corrected by the automatic self-adjustment of the coefficients α and c to the coefficient γ (see fig. IV, 6). Even a central position of the pivot has been used without total loss of the benefits of pivoting, though always with great (and needless) loss of efficiency.

The thickness of the pads, when of rectangular section, is usually made about one-third of their length, or width, whichever of these dimensions is the greater. A rational basis for the calculation of this thickness will be given in the next section. In order to secure equality in the thickness of the several pads used in the same bearing, when they are fitted with parallel sliding and supporting surfaces as shown in fig. V, 5, the ring from which they are to be afterwards cut should be turned on both surfaces simultaneously, as the last machining operation on their sliding surfaces, and, if they are afterwards hand-scraped, as is usual, the distances at 3 or 4 distributed points between surface plates applied to the two faces of each pad should be accurately gauged and brought to equality.

When this simple method, described above, of pivoting the pad on the forward edge of a " land " on its reverse side parallel to its sliding face is not adopted, a usual practice is to insert a hardened pivot into the back of the pad, matching with a similar hardened stud inserted into the supporting member. This form of construction provides good definition of the virtual position of the pivoting point but presents some difficulty in bringing the sliding surfaces of the several pads of a bearing accurately to one transverse plane. An alternative system, which is aimed to overcome this difficulty is to use unhardened steel studs, or even pins of a still softer metal, such as copper, with the expectation of their being plastically compressed under the bearing load to such extent as to equalize the charges on the several pads, in spite of unavoidable inequalities in their original heights or tightness of fitting. For use in thrust bearings in which, either on account of unavoidable division of the supporting member by a horizontal joint, or from some other circumstance, it is impracticable to provide an exactly plane and rigid support for the pads, Messrs. Kingsbury Machine Works have developed a system of using what are called " levelling plates ", consisting of an annular series of equalizing levers on which the pads are supported. The arrangement will be readily understood from figs. V, 6 and V, 7, of which the former shows the two halves of the complete bearing, which is split into halves on a joint in an axial plane, while the latter view is a developed circumferential section through the pads (called " shoes " in the figure) and levelling plates. (The author is indebted to Messrs. Kingsbury Machine Works for their courteous permission to reproduce these illustrations and the one next following.) The series of levers may extend in an endless chain around the whole bearing, or may be divided into two or more sectors, according to requirements.

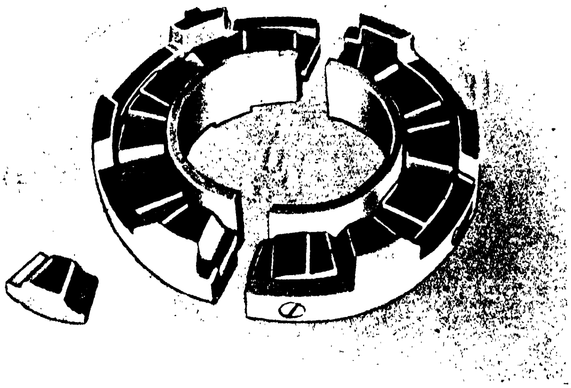


Fig. V, 6.—Thrust bearing with "levelling plates" for pads by Kingsbury Machine Works Inc., Philadelphia

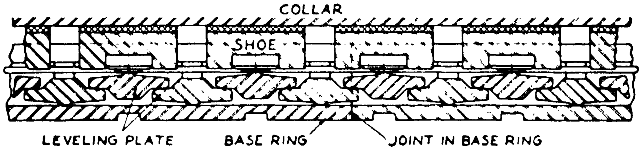


Fig. V, 7.—Levelling plates for pads

Somewhat similar systems of equalization, making use of bearing balls, have been used, or proposed, by other constructors (Refs. V, 3 and V, 4).

A method of pad support which serves to equalize the charges between the pads when they are only 2 or 3 in number, but only partially does so when they are more numerous, is shown in fig. V, 8. It consists in fitting in the bearing housing (not shown in the figure) a pair of heavy foundation rings, mutually

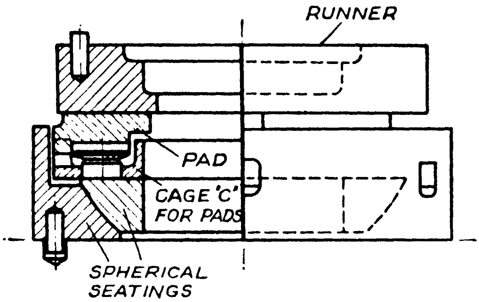


Fig. V, 8

fitted on a spherical surface whose centre is on the axis of the bearing and preferably near to its intersection with the plane of the runner. For the successful use of this construction it is essential that the inclination of the spherical bearing surface to the axis shall be nowhere greater than about 60 degrees, and that the sliding surface of the runner shall be very exactly normal to the axis of rotation; otherwise the intended self-aligning action will be prevented by the friction of the solid-to-solid contact of the two spherical surfaces which do not admit of lubrication; in any case, if there is movement, "galling" or contact corrosion of the spherical seatings will occur.

The pads in this type of construction are supported on the convex spherical ring either directly by means of plane seatings of the kind shown in figs. V, 1 and V, 5, or by means of inset studs as in fig. V, 8. In the latter case a retaining plate, or "cage", marked "C", is used to prevent rotation of the pads relatively to the housing.

Another method of pad support which has been applied by various makers for the purpose of equalizing the charges on the individual slippers of the bearing, and at the same time obtaining the effect of pivoting while dispensing with the use of actual pivots, consists in supporting each pad on springs. This system has been applied mainly to the thrust bearings of large, vertical, water turbines in which the surfaces of the pads may be measured in square feet, or even in square metres, and in which, in consequence, accurate machining of a plane supporting surface in the housing casting may present difficulty. A number of helical steel springs are fitted between each pad and the supporting surface, these being all ground to a common length but either graduated in elasticity, or so located with respect to the length of the pad, that they simulate the effect of an offset pivoting axis. An example of this mode of pad support (fig. V, 11, p. 134) is described and illustrated in Sect. V, 4, in which the advantages and disadvantages of the type are discussed.

Spring-supported thrust bearings are obviously unsuitable for use when thrust loads have to be carried alternately in both axial directions, as well as in other cases when axial movements of the shaft caused by varying thrust loads are not permissible, as in steam turbo-generators and usually in marine propeller-shaft installations.

Of thrust bearings designed on the principle of the inclined sliding plane, or converging film, but not employing pivoted or tilting pads, there appears to be only one form in use, which is known as the "tapered-land" thrust bearing. This consists of essentially the same parts as a pivoted-pad thrust bearing with the exception that the pads, instead of being pivoted, are bedded rigidly on their supporting plate, and are machined to slightly tapering thicknesses in the circumferential direction, so as to present slightly inclined surfaces to the sliding face of the runner. A diagrammatic section and plan view of a portion of such a bearing is given in fig. V, 9. From 6 to 12 "lands" are usually arranged in circumference with interspaces for the discharge of the lubricating oil which is introduced through, or beneath, the supporting plate to the inner circumference of the runner. The inclination of the "tapered lands" is made of the same order as that assumed by pivoted pads of similar dimensions, operating under the same conditions of load, speed, and viscosity of lubricant, but is of course invariable and not self-adapting to variations of these conditions. A small area of each land, at its outflow end, is finished without taper, or in other words is formed parallel to the runner, presumably to form a seating for the collar when the shaft is stationary and the collar makes contact with the bearing. In steam turbines with horizontal shafts, and in other machines in which the thrust load is of vanishing magnitude when the machine comes to rest, this provision may be sufficient. The "tapered-land" bearing is however not well adapted to meet the conditions which arise when a machine is required to start or stop under a thrust load comparable with the operative load, nor is its efficiency under varying operative conditions equal to that of a pivoted bearing designed for the same duty.

A variant form of the "tapered-land" bearing is the "parallel-land" bearing, in which the construction is the same except that the lands are not given a tapered form, or inclination, but are left in one and the same plane. This form does not appear to be in actual use except for bearings intended merely to limit end-play in a shaft, and consequently carrying only light or momentary loads. The superiority claimed for its operation as compared with that of continuous, or undivided, flat bearing plates arises, no doubt, from differential thermal expansion. As the runner, carrying oil in its rugosities, revolves in contact with a flat land, the "leaving" end of the land will be raised to a higher temperature than the "approach" end, and by expansion of the metal will rise to a slightly higher level from the base plate. A quasi-wedge-shaped interspace between "land" and runner being thus formed, the quantity of oil carried in will increase, and the effect will be cumulative up to a point when an equilibrium gradient of temperature is attained. Assuming the use of "lands" each 2 centimetres square and 1 centimetre thick, then for every degree centigrade by which the temperature of the lands is higher at their leaving than their approach ends, the surfaces of the steel lands will assume

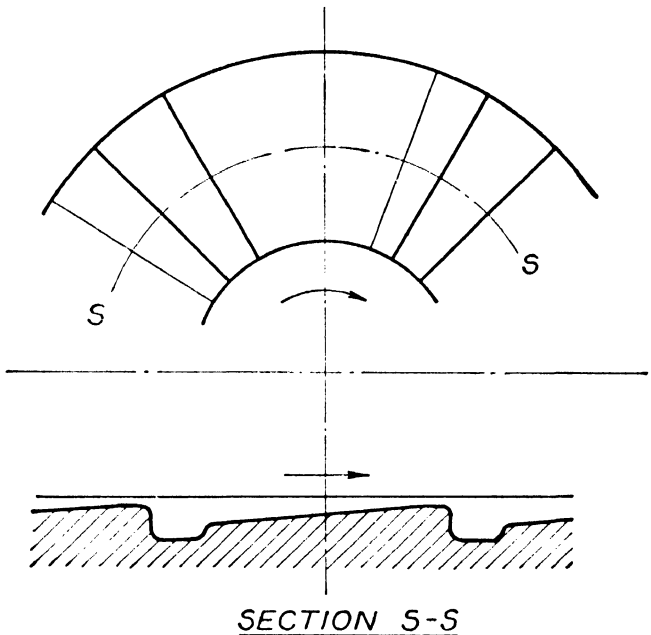


Fig. V, 9

approximate slopes of 6×10^{-6} in the direction enabling them to form and maintain films under pressure. Lubrication films in which the generation of pressure is dependent on thermal dilatation may be called "dilatational pressure-films" or, less precisely, "thermal pressure-films".

With accurately formed smooth surfaces, fluent-film lubrication may reasonably be expected to set in to a certain extent under such conditions.

In practical service of any severity, the abrasion which takes place at starting and stopping, acting mainly on the slightly salient outflow end of the pad, would ultimately abolish its capacity to establish any convergence. The Lubricants and Lubrication Inquiry Committee of the British Department of Scientific and Industrial Research, in its report of 1920, recommended investigation of "the possibility of obtaining satisfactory lubrication between parallel plane surfaces without the use of tilting devices as in the Michell block" (Ref.

V, 5). No report of such investigations appears to have been issued. See, however, Ref. V, 12, which gives a mathematical treatment of dilatational pressure-films, and Ref. V, 13 for a stepped form of bearing pad suggested by Rayleigh.

3. Conditions necessary for Safe Operation and Efficiency in Thrust Bearings.

Apart from adequate mechanical strength, the conditions of safety of operation in a pivoted thrust bearing are solely those which are necessary to the maintenance of a fluent lubricating film between the bearing surfaces, i.e. the continuance during the use of the bearing of a sufficiently close approximation to its ideal plane forms, and a continuance of the supply to the bearing of a sufficient quantity of lubricant of a viscosity suitable to the load and speed. The latter condition implies in practice that the viscosity of the lubricant must be maintained by the use of cooling appliances if necessary. The agents which may jeopardize the operation of the bearing by deforming the parts from their correct form are, in the first place, loads which, though they may be well within the elastic strength of the parts, may be so great as to cause deformations dangerous to the correct continued operation of the fluid film, and secondly, local differences of temperature due to generation of heat within the bearing, which may have the same effect.

Consider first the elastic deformations to which a pivoted pad is subjected by its charge and the reactions of its supporting surfaces. In so far as these deformations are of material consequence to the formation of the designed fluent film, regard need only be paid to the flexural deformations due to the difference between the distribution of the fluid pressures on the sliding surface of the pad and the distribution of the counterbalancing forces on its reverse side. As one of the simplest cases for calculation, take that of a pad formed and supported as in fig. V, 5 (p. 123). The flexural deflections of the portion of the pad (which is only about two-fifths of its length) on the outflow side of the line at which it is held in contact with the supporting plate on which it rests, will evidently be much smaller than those on the inflow side and may, for an approximate treatment, be disregarded. The remainder of the pad, extending for the three-fifths of its length at its inflow end, is subjected to the known fluid pressures on its sliding surface and is unsupported on its opposite side, so that it may be treated as a cantilever subjected to bending by those pressures. The distribution of the pressures, averaged across the width of the pad, may be taken, with sufficient accuracy for the present purpose, in any case in which the length-breadth ratio does not much exceed or fall below unity, as following the same law as in a square pad. The curve in fig. V, 10 gives, for the square pad, the mean pressure per unit width at each point of the length of the pad, together with the position of the resultant of these pressures, which is, of course, the location of the pivoting edge and which occurs at 0·42 of the length of the pad measured from the outflow end A_1.

It will be seen that the pressures shown by the curve at all points between the pivoting edge and the unsupported end of the pad are represented very closely by the ordinates of a straight line whose ordinate p is given by

$$p = p_m\left(1 - \frac{\xi}{OA_2}\right) = p_m\left\{1 - \frac{\xi}{(1-\gamma)l}\right\},$$

where p_m is the maximum of the curve of pressure, ξ the distance from the pivoting edge, and γ, as in Sect. IV, 13, is the ratio of the distance of the pivoting edge from the outflow edge to the total length l of the pad. In the square pad $\gamma = 0{\cdot}42$.

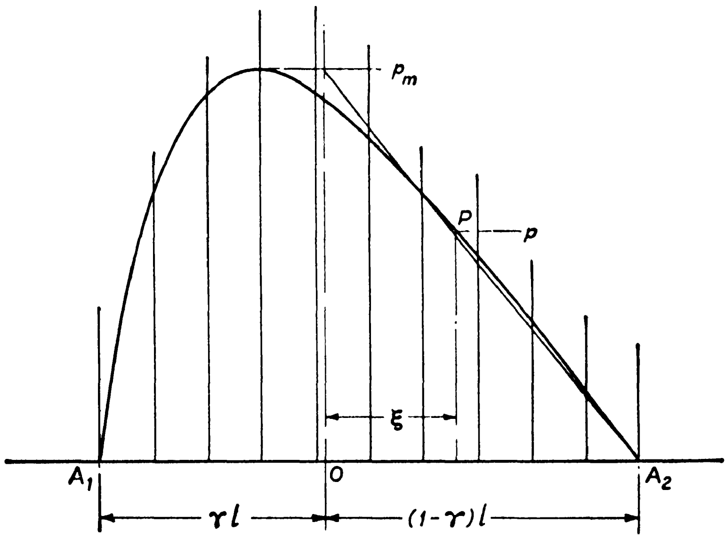


Fig. V, 10

The bending moment at any point ξ of the unsupported portion of the pad due to the forces represented by the straight line A_2P which act between that point and the free end is

$$G = \frac{p_m}{6}\left\{1 - \frac{\xi}{(1-\gamma)l}\right\}\{(1-\gamma)l - \xi\}^2,$$

or writing λ for $(1-\gamma)l$,

$$G = \frac{p_m}{6\lambda}(\lambda - \xi)^3,$$

and the curvature of the pad at ξ, bent by the couple G, will be to the first order,

$$\frac{d^2z}{d\xi^2} = \frac{p_m(1-N^2)}{6EI\lambda}(\lambda - \xi)^3, \quad \ldots \ldots \quad \text{V, 3}$$

z being the deflection at ξ of the pad from the initial plane, I the moment of inertia of its section per unit width, E Young's modulus, and N Poisson's ratio for the material.

From equation V, 3, by integration, observing that both z and $dz/d\xi$ become zero when $\xi = 0$, it results that

$$z = \frac{p_m(1 - N^2)}{6EI\lambda}\{\tfrac{1}{20}(\lambda - \xi)^5 + \tfrac{1}{4}\lambda^4\xi - \tfrac{1}{20}\lambda^5\}. \quad . \quad . \quad . \quad \text{V, 4}$$

At the free, or inflow, end of the pad, where $\xi = \lambda = (1 - \gamma)l$, and where the deflection is greatest, its amount is therefore

$$z_m = \frac{p_m(1 - N^2)}{30EI}(1 - \gamma)^4 l^4. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 5}$$

Taking the case of a steel pad of uniform thickness h, $I = \frac{1}{12}h^3$, $E = 2{\cdot}0 \times 10^6$ kg. wt. cm.$^{-2}$, $N = 0{\cdot}3$,

$$z_m = \frac{p_m}{h^3} \times 1{\cdot}8 \times 10^{-7} \times (1 - \gamma)^4 l^4.$$

It being most convenient for purposes of practical calculation, and for comparison of the results in various cases, to express the thickness h of the pad, and the greatest deflection z_m, as fractions of the length l, the expression for z_m may be written as

$$\frac{z_m}{l} = \frac{p_m}{(h/l)^3} \times 1{\cdot}8 \times 10^{-7} \times (1 - \gamma)^4,$$

or, alternatively, the least allowable thickness h/l, as a fraction of the length of the pad given by

$$\frac{h}{l} = \left\{\frac{p_m}{(z_m/l)} \times 1{\cdot}8 \times 10^{-7} \times (1 - \gamma)^4\right\}^{1/3}, \quad . \quad . \quad . \quad \text{V, 6}$$

an expression which may be regarded as applicable to all steel pads having a breadth to length ratio not greatly different from unity, and as valid for a wide range of values of γ and of the ratio of minimum to maximum film thickness.

The permissible maximum value of the ratio z_m/l will depend upon the mutual inclination c of the sliding surfaces, i.e. in any given case, upon the coefficient of resistance which it is desired to attain. Assuming the latter to be of the order of 1×10^{-3} and, consequently, the inclination c to be about 1×10^{-4}, z_m/l may be reasonably allowed to be about $2{\cdot}5 \times 10^{-5}$.

With these assumptions, and taking $p_m = 70$ atmospheres (which corresponds in the case of a square pad to a mean pressure of $70 \times {\cdot}618 = 43$ atmospheres, or 610 lb. in.$^{-2}$), together with $\gamma = 0{\cdot}4$ (as representing an average ratio of inflow to outflow film thickness), equation V, 6 gives $h/l = 0{\cdot}40$ as the least allowable ratio of the thickness to the circumferential length of the pad. A higher value of this ratio must be chosen if a higher efficiency than that represented by $k = 0{\cdot}001$ is aimed at; in the contrary case a somewhat thinner pad may be used, but h/l can hardly be allowed in any case to be less than 0·30. Since, according to equation V, 6, h/l for a given value of z_m/l varies with the cube root of the imposed pressure, it is only to a minor extent dependent on the charge for which the pad is designed, at any rate within usual limits.

When the pivot of the pad does not extend along its whole width but is localized near the middle of its radius, as when a hardened stud is inserted in the pad to form the pivot, the flexure will no longer be uniform across the width of the pad, but will take place in both the lengthwise and crosswise directions. The deflections at the corners of the pads will then exceed that given by equation V, 6, if the distribution of the fluid pressure were unchanged, but the crosswise deflections will of themselves tend to diminish the pressures generated at the lateral parts of the pad, and, consequently, the deflections. In critical cases an estimate of these effects should be made.

With other modes of support of the pads of the various kinds mentioned in Sect. V, 2, similar calculations require to be made, with the help of simplifying assumptions appropriate to each case, and it is always to be remembered that the runner is subject to elastic deformations of the same kind and order as those of the pads, though usually its deflections are considerably smaller if its thickness is as great as theirs.

It is further to be observed that the elastic deformations of the pad, causing its sliding surface to be no longer plane, will be apt to bring about differential abrasion of the surface during starting and stopping, in the manner discussed in Sect. IV, 17. Further reference to this aspect of the question will be made in Sect. V, 4 after the deformations due to temperature variations have been considered.

Deformations of the plane surface of a thrust bearing pad caused by local variations of temperature are still more difficult to estimate with accuracy than those due to stress under the bearing load. For practical applications, and for comparison of results in different cases, there is the further complication that the linear deformations due to a given rise of temperature, or a given rate of heat generation, are not, like the elastic deformations due to a given loading, proportional to the linear dimensions of the pad (so that similar pads of different dimensions remain geometrically similar after deformations by loadings of equal intensities), but are proportional to the squares of the linear dimensions. Consequently temperature effects which may be negligible in small pads are apt to become considerable in large ones, and an upper limit may be imposed on the scale on which a particular type of pad can be successfully constructed by the thermal expansions which take place in it when running under charges of normal intensity.

Usually the only temperature deformations of a pad which require to be considered are those which arise from the temperature gradient normal to the sliding surface due to the generation of heat by the fluid friction at the surface and its dissipation into the metallic support of the pad, or into oil in contact with its reverse side. If such dissipation of heat is minimized, as by the use of a poor conductor of heat for the pad support, or by the avoidance of any cooling by means of oil circulating over the back of the pad, the gradient of temperature, and consequently the thermal deformations, will be correspondingly reduced. The factors which come into consideration as determining the amounts of deformation in the pad are, therefore, the rate at which heat is generated at its sliding surface, and the fraction of this heat which is conducted through the pad thickness. As representative of conventional practice in the design and operation of large thrust bearings (such as those for large marine propeller shafts, or for the vertical shafts of heavy water-turbines), it may be assumed that the mean pressure on the pad is 25 atmospheres (or nearly 2·5 $\times$ 10^7 dynes per square centimetre), and that the mean sliding speed is 1000 centimetres per second (33 feet per second). It will also be assumed that the coefficient of resistance of the bearing is $1{\cdot}0 \times 10^{-3}$. The mean rate per unit area at which heat is generated at the sliding surface is then

$$\frac{1}{J} \times 2{\cdot}5 \times 10^7 \times 10^3 \times 10^{-3} = \frac{2{\cdot}5 \times 10^7}{4{\cdot}2 \times 10^7} = 6{\cdot}0 \times 10^{-1} \text{ gm. cal. sec.}^{-1},$$

the value of J (Joule's equivalent) being taken as $4{\cdot}2 \times 10^7$ in C.G.S. units.

If $1/n$th of this heat is conducted through the thickness of the pad, and if the pad is of steel having a heat conductivity of $1{\cdot}1 \times 10^{-1}$ C.G.S. unit, the rate of fall of temperature through the pad will be

$$\frac{6 \times 10^{-1}}{1{\cdot}1 \times 10^{-1}} \times \frac{1}{n} = \frac{5{\cdot}5}{n} \text{ degrees centigrade per centimetre.}$$

The coefficient of linear expansion of steel being taken as $1{\cdot}1 \times 10^{-5}$ per degree C., the differential expansion in each centimetre of thickness will then be

$$\frac{1}{n} \times 5{\cdot}5 \times 1{\cdot}1 \times 10^{-5} = \frac{6 \times 10^{-5}}{n},$$

which will also be the measure of the curvature ρ in centimetre units, of the initially plane surface.

If d be the diagonal length of the pad, the deflection of its corners from the tangent plane at its centre (which may be taken as coincident with the plane of its sliding surface before deformation) will be

$$z_m = \tfrac{1}{2}\rho(\tfrac{1}{2}d)^2 = \frac{3 \times 10^{-5}}{4n} d^2. \qquad \text{V, 7}$$

Upon the same grounds as in the case of elastic deformations, the greatest allowable deflection may be taken as $2{\cdot}5 \times 10^{-5}d$, so that

$$\frac{d}{n} \not> 2{\cdot}5 \times \tfrac{4}{3} \not> 2;$$

that is to say, the fraction $1/n$ of the total heat generated at the sliding surfaces which may be conducted away through the pad must not exceed $2/d$, the diagonal length d being measured in centimetres. Conversely, it may be concluded that if the heat generated is dissipated equally by conduction through the runner and through the pad (as may be approximately true when an uncooled collar revolves between two similar sets of pads for the purpose of carrying a thrust load in either direction, and $1/n$ consequently has the value $\frac{1}{2}$), the diagonal lengths of the pads d ought not to be greater than $2 \times n = 4$ centimetres, if the bearing is to be designed to have a coefficient of resistance as low as 0·001.

The best practice in the construction of large thrust bearings is in accordance with the conclusion to be drawn from the above discussion, namely, that no large proportion of the cooling applied to the bearing should be effected by conduction through the pads.

4. Thrust Bearings for Vertical Shafts.

General descriptions of two representative examples of thrust bearings for vertical shafts have already been given in Sect. V, 2 with reference to figs. V, 1 and V, 3 (pp. 120–1). On account of the large dimensions and importance of many of the bearings of this class, and of the onerous conditions under which they usually operate, they have probably been studied with greater care than bearings of any other class. The results of this study and of the experience gained from the operation of the bearings are in large measure applicable to the design of bearings of other kinds but, it must be said, for the most part still await application, particularly to journal bearings.

An example of a recent and well-considered design for a large thrust bearing for a vertical hydro-turbine generating unit is shown in figs. V, 11 (p. 134) and V, 12 (p. 135). The arrangement of the internal cooling system of this bearing is especially noteworthy in connexion with the discussion of temperature gradients in bearing parts which has been given in the preceding section. The bearing illustrated is by the Swedish General Electric Company (Allmänna Svenska Elektriska A.B.) of Västerås, Sweden, and the illustrations and description are derived from the firm's "Asea Journal" of April–June 1947 (Ref. V, 6).

Fig. V, 11 shows an axial half-section of the bearing and a plan view of the whole of one of the twelve pads and a portion of another pad. In bearings of this class the sliding surface of each pad is usually from 30 to 50 centimetres (12 to 20 inches) in mean circumferential length and, in recent examples, the ratio of width to length is from 1·1 to 1·3. An average intensity of loading is 25 atmospheres (350 lb. wt. in.$^{-2}$), with a mean sliding speed varying in different examples from 5 to 15 metres (16 to 50 feet) per second. The oil film thickness at the outflow edge of the pad is normally about 4×10^{-3} centimetre (1·6 thousandths of an inch), and the coefficient of resistance, as determined by test, varies from $1{\cdot}0 \times 10^{-3}$ to $1{\cdot}5 \times 10^{-3}$.

The sectional view (fig. V, 11) shows the massive runner (1), through which are drilled radial holes (12), mounted on the shaft (2). The pads (3) are supported

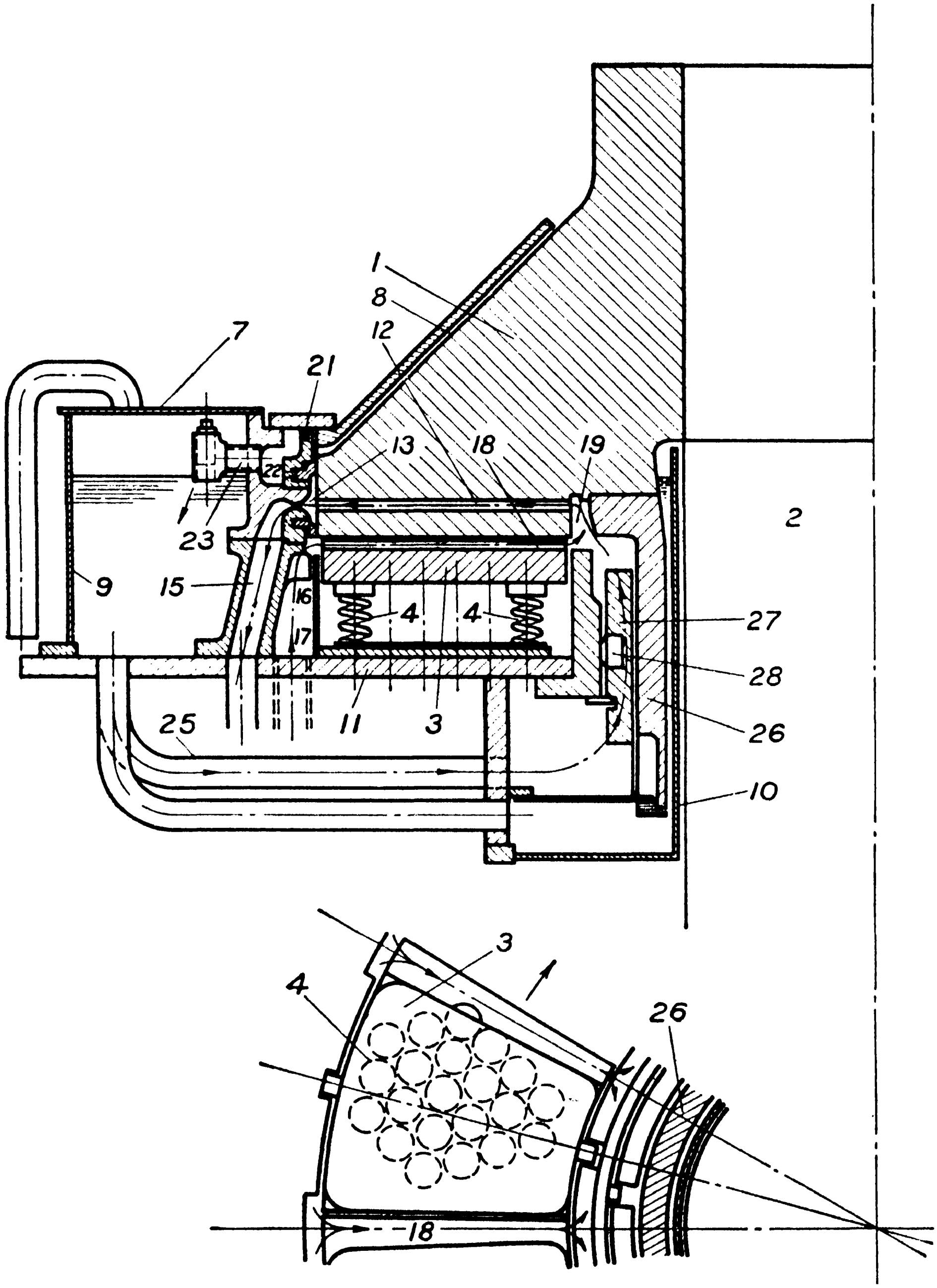


Fig. V, 11.—Thrust bearing for hydraulic turbine and generator, by Allmänna Svenska Elektriska, A.B., Sweden

on a group of helical springs (4), resting on the base plate (11) of the housing of the bearing. The helical springs are so located that the resultant of their reactions to the load acts at the point which is the centre of the fluid pressures, being at 42 per cent of the mean length of the pad from its outflow edge, and near its mean radius. The effect of the spring support is therefore virtually to pivot the pad at this point. The arrangement of the springs about the centre of the group is such that the forces which they exert are distributed over the projected area of the pad in approximately the same manner as the fluid pressures of the oil film. The bending moments, and resulting elastic deflections of the pad from its initially plane surface, are thus minimized, and it is found safe to use a pad thickness which is only one-fifth or one-quarter of the mean pad length. The springs, however, occupy a space greater than that which would be needed to accommodate pivoted pads of the greatest thickness which could be called for by reasons of elastic rigidity.

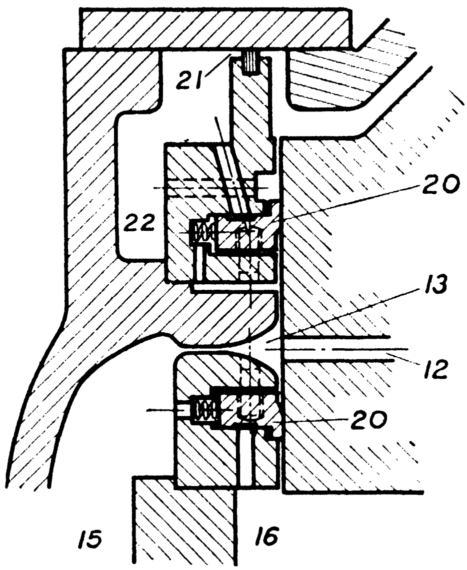


Fig. V, 12.—Enlarged detail of part of fig. V, 11

Thermal deformations of the pads are reduced to negligible importance by the makers' special system of oil circulation which is described below. This serves not only the thrust bearing itself but also the guide, or journal, bearing (27) which supports the hollow journal (26) attached to the runner. A fixed sleeve (10), rising from the base of the housing between the shaft (2) and the inner surface of the hollow journal (26), together with the outer wall (9) of the housing and the housing cover (7), enables the oil in the housing to be maintained at a sufficiently high level to submerge all the operative parts.

The pumping means which energize the circulation of oil, and the means which control and direct it, will now be described briefly with reference to the sectional view of fig. V, 11, and the enlarged detail section, fig. V, 12. When the runner (1) revolves, the ducts (12) already mentioned act as a centrifugal pump and discharge the circulating oil into the chamber (13), from which the greater part of the flow passes through outlets (15) to a cooler outside the bearing. The cooled oil returns to the housing through the inlet chamber (16) which is separated from the spaces below the pads in which the supporting springs are located by the cylindrical partition (17). The cooled oil is thus directed into the open radial grooves (18) formed in the pads at their outflow ends beneath the runner, and flows radially inwards through them to the chamber (19) from which it again enters the pumping ducts (12).

The pressure chamber (13) is sealed, above and below, between the stationary and revolving parts by the sealing members (20), which consist of a number of segments fitting fairly closely to one another and prevented from rotating by special lugs while having a certain degree of freedom radially. At the back of each segment there is a spring of sufficient strength to force the segment against the rotating member when the machine is not running. When the machine is running, pressure is built up in the pressure chamber, causing a certain distribution of oil pressure around the segment by means of suitably arranged oil ducts shown in fig. V, 12. On account of the shape given to the segment the oil pressure is balanced in the radial direction, and only the small spring force acts in this direction. In the axial direction, however, the oil pressure presses the segment against the fixed supporting face with so much force that the friction between the segment and the supporting face is sufficient to hold the segment fixed on that face in spite of the action of the spring. Thus, as the rotating member revolves with some unavoidable, though small, vibrations and eccentricity, it pushes the segments radially outwards to this small distance where they are held fast by the friction with a very slight clearance from the revolving body. Friction and wear are thus prevented, except to the extent which may be caused by occasional and slight contacts.

The oil seals formed by the members (20) are thus not perfectly tight. As seen from fig. V, 11, the oil which leaks past the lower seal joins the main flow just below the seal. The oil passing the upper seal flows through the holes (21) to the chamber (22) and thence through the pipes (23) to the oil container (9), but the pipes (23) can be partially throttled so as to force oil up to any desired level in the air-seal space (8). By this means aeration of the oil is prevented. From the oil container (9) the leakage oil is returned to the suction chamber (19) through the pipe (25), and before reaching the suction chamber passes and lubricates the journal bearing (27), which is also a pivoted-pad bearing, the pads being pivoted on the inset studs (28).

It will be apparent that the mode of oil circulation which has been described constitutes an effective means of removing the heat generated in the lubricating film without allowing it to be conducted to any large extent either into the pads or into the main body of the runner so as to cause detrimental deformations of their sliding surfaces. Since the oil in which the lower portions of the pads are immersed takes no part in the circulation and, being in contact with surfaces at higher temperatures above than below, will not develop any convection currents in its own volume, it will act as an effective heat insulation for the lower surfaces of the pads, which will, in consequence, have no appreciable temperature gradient in the direction of their thickness. Thus thermal deformation of the kind expressed by equation V, 5 may be expected to be negligible.

The passage of oil, heated by resistance at the sliding surfaces, along the grooves (18) and through the ducts (12), involves, of course, the raising of the temperature of the lower part of the runner above the general temperature of the body, but it can be readily understood that deformations of the plane face of the runner are kept within quite safe limits by the rigidity of the exceptionally massive runner which is a feature of the design.

The general design of vertical water-turbines driving electric generators conveniently admits of the placing of the thrust bearing at the upper end of the unit and, the thrust load being always downward, the use of a massive collar or runner having only one sliding surface presents no difficulty. In most other applications the runner can only have a limited axial dimension, either on account of structural reasons or because it must have two sliding surfaces, operative in a single housing, in order to provide for reversal of the thrust.

In such cases, the runner being relatively thin and having some degree of flexibility, it is necessary, in order to prevent thermal deformation, to avoid creating any appreciable temperature gradient normal to the sliding surface. Circulation of the oil through ducts in the runner is then inadvisable, especially in runners of large diameter, and it is preferable that all cooling should be effected by direct transmission of heat to the circulated oil from the portions of the sliding surface of the runner which are exposed between the pads, these portions being made as large as other conditions permit. At the same time, of course, all cooling of the other parts of the runner, either by exposure to contact with circulating oil, or in other ways, is to be avoided.

In the design of any bearing incorporating a system of cooling by oil circulation, it is essential to give consideration to the precise localities in the bearing at which the heat is generated, and to the course of the lines of conduction by which it is to be removed from those localities. In this connexion reference is made to Sect. IV, 8, in which the distribution of the heat generation in plane bearings of unlimited width was discussed, and illustrated by fig. IV, 9 (p. 86) and Front Folder IV, C.

A thrust bearing of the same class as that described in the preceding paragraphs is illustrated in fig. V, 13 (p. 138). This bearing is by Messrs. Kingsbury Machine Works Inc. of Philadelphia. In its arrangement full reliance is placed on the direct contact of the sliding surface of the runner with the circulating oil for the propulsion of the oil through the spaces between the pads and for the cooling of the bearing surfaces, the runner having no ducts of the kind illustrated in fig. V, 11 (p. 134). The oil flowing outwardly from the periphery of the runner (1) is deflected away from the outer surfaces of the pads (2) (called by the makers " shoes ") and from those of the supporting plate (3), and is compelled to flow through the cooling coils (4) through which water is passed, before returning through ports (5) in the lower part of the base of the plate (3) to the inner peripheries of the pads and runner. An oil-retaining sleeve (6) attached to the foundation casting (7) surrounds the shaft and forms the inner wall of the housing which contains the whole bearing and the cooling coils, and which is filled with oil to the level shown in the figure.

An insulating sub-base (8) is inserted between the foundation casting (7) and the supporting plate (3) to prevent stray electric currents from causing electrolytic corrosion of the bearing parts. The bearing being designed for high rotative speeds, a sealing ring (9) surrounds the runner at, and for some distance below, the oil level in order to prevent the throwing of spray from the periphery of the runner and consequent entrainment of air in the oil.

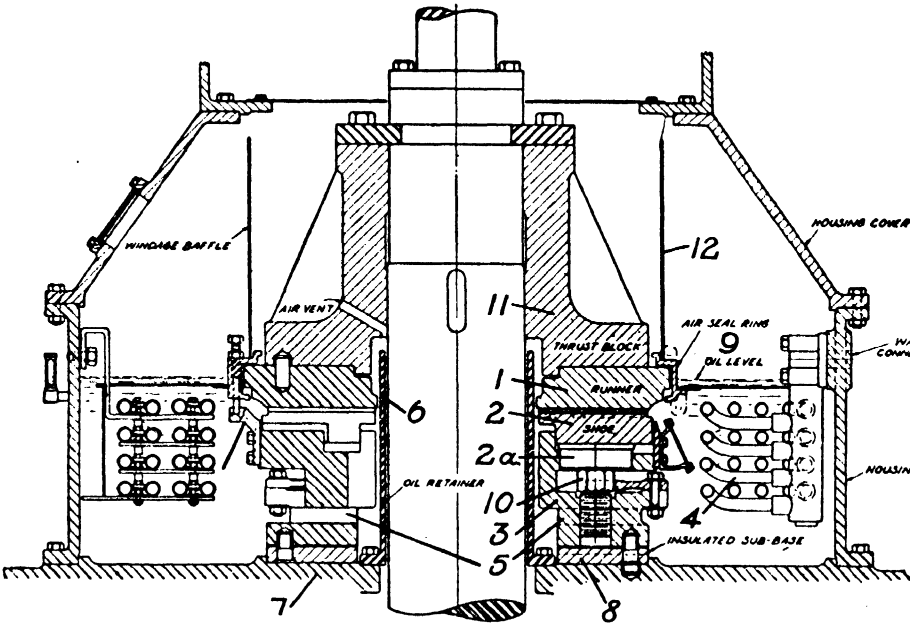


Fig. V, 13.—Thrust bearing for hydraulic turbine and generator, by Kingsbury Machine Works, Inc., Philadelphia

The pads (2) are arranged for point support on the inserted discs (2*a*) which are of hardened steel, and are individually adjustable as to their level by the locked screws (10) which form their pivotal supports. The runner proper (1), which is a highly finished hardened ring, is carried by a massive collar (11) attached to the shaft, the bearing surface for the runner being plane and accurately fitted with a spigot joint. It will be seen that in this example, as in that shown in figs. V, 11 and V, 12, the axial length and the massiveness of the collar are so great that thermal deformations of its lower surface (and in this instance deformations of the runner which is bedded upon it) will be to a large extent suppressed, provided that no considerable thermal gradients exist in the collar itself. The collar being wholly above oil level, and spray from the oil bath being prevented from reaching it by the sealing ring (9) already mentioned, and by the baffle (12), can receive heat of only negligible amount except through the surface in contact with the runner, and can lose heat only by conduction into the shaft; its temperature gradients are therefore likely to be low. Added security against thermal deformations could, however, be given by the insertion of a plate of a suitable layer of non-conducting material between the collar and the runner.

In this design, the pads besides being screened from direct contact with streams of circulating oil, which might establish downward gradients of temperature, are of a thickness of the order necessary to prevent harmful elastic deformations according to the calculations which have been set out in Sect. V, 3.

It is to be remarked that deformations of the pad and runner surfaces due to temperature effects are, in all such cases, most likely to result in abrasion of the surfaces, not during running at full load and speed, but during the slowing down of the machine.

Since the usual modes of deformation result in the working surfaces of the pads becoming convex, the formation of convergent and fluent films of lubricant during running at high speeds will not be prevented, although there will be undue concentrations of pressure on some areas and deficiencies of pressure, or negative pressures, at other places, with resulting losses of efficiency. On slowing down to low speeds, however, while the heat and deformations still remain in the pads, the film is likely to become too thin to prevent metallic contact on the convexity if any considerable load is retained at low speeds.

When the thrust bearing of a vertical shaft is located at or near its lower end (being known, in the former case, as a " footstep bearing "), it is frequently difficult to arrange for draining fouled oil from the bearing by gravity, for the purpose of purification or cooling. In such cases it is usual to provide a sump at a level below that of the bearing and to lift the oil from the sump by a special pump, which is commonly worm-driven or gear-driven from the shaft carrying the thrust bearing. The pump lifts the oil from the sump and circulates it, according to circumstances, either directly back to the bearing or through a reservoir from which it flows to the bearing by gravity. In either case a portion of the flow can be diverted for circulation through a cooling-coil, or withdrawn for purification.

5. Thrust Bearings for Horizontal Shafts, particularly in Steam and Gas Turbines.

Applications of pivoted-pad thrust bearings, and other convergent-film thrust bearings, to horizontal shafts have already been described briefly in Sect. V, 2, as illustrations of general modes of construction of these bearings.

One of the most important fields of application of horizontal thrust bearings is that of turbine heat engines. In steam turbines thrust bearings with pivoted pads are almost invariably used. This field is of much practical importance but presents no special or peculiar conditions affecting design except that rotative speeds are generally higher than in most other large-scale applications, and that the temperatures at which the bearings are called upon to operate are apt to be high, owing to conductance of heat from the working fluid of the turbine: this is especially likely to be the case in gas turbines.

Conventional practice in the design of pivoted-pad thrust bearings in steam turbines is to use a maximum average intensity of loading of about 30 atmospheres (450 lb. in.$^{-2}$) in large bearings, and 20 to 25 atmospheres (300 to 400 lb. in.$^{-2}$) in smaller ones. The surface speed of the runner at its mean radius is usually limited to about 7000 cm. sec.$^{-1}$ (14,000 ft. min.$^{-1}$). Although, usually, any considerable axial loading is exerted only in one direction, it is a common practice to fit bearing pads on both sides of a parallel-faced collar, the second series of pads being provided to carry only accidental loads, and to restrain undue axial movement of the turbine rotor. In such cases a total axial clearance on the two sides of the runner of about 0·05 centimetre (0·01 to 0·02 inch) is commonly allowed.

The pads are usually coated with a tin-base bearing metal, about 0·15 centimetre (0·06 inch) thick, the final turning of the pads being effected while they are still in the form of a ring, after which they are parted radially and hand-scraped on the working surfaces with precautions such as have been described in Sect. V, 2. The total thickness of the pad is usually from one-third to one-half of its length.

It is usually necessary in steam turbines to provide means for the adjustment from time to time of the axial position of the turbine rotor with respect to the fixed blades and casing. For this purpose the thrust bearing is often formed as part of an inner case which is enclosed in an outer fixed housing, and which can be adjusted axially within the housing.

By this construction it is possible to fill the inner casing entirely with oil, in which the collar and thrust pads are immersed, any leakage of oil from the inner casing being caught in the outer casing and returned therefrom to the circulation system. On account of the high rotative speeds the oil should be supplied to the bearing at the inner periphery of the ring of pads, the pressure of the supply being usually about 2 atmospheres absolute pressure, or 15 pounds per square inch by gauge.

Fig. V, 14 (supplied by the courtesy of Brush Electrical Engineering Co., Ltd.) shows an example of such an arrangement. In this construction the inner casing I is in halves and is supported in the housing H on a circumferential seating C. The inner casing supports both of the two series of thrust pads, T_1 and T_2, as well as the journal bearing J. Oil is admitted to the bearing through the circumferential seating, in the first instance to the chamber OC in the inner casing, and thence passes through holes OS to the inner periphery of the thrust bearing T_1; after filling the whole of the space in which the thrust collar TC is enclosed, it passes radially outwards between the pads of this bearing and radially inwards between the pads of the bearing T_2.

It is then delivered past the oil restriction ring R to the circumferential chamber P, from which it is taken off, near the top of the bearing sleeve, by the oil outlet pipe OO and passes thence to the oil return OR.

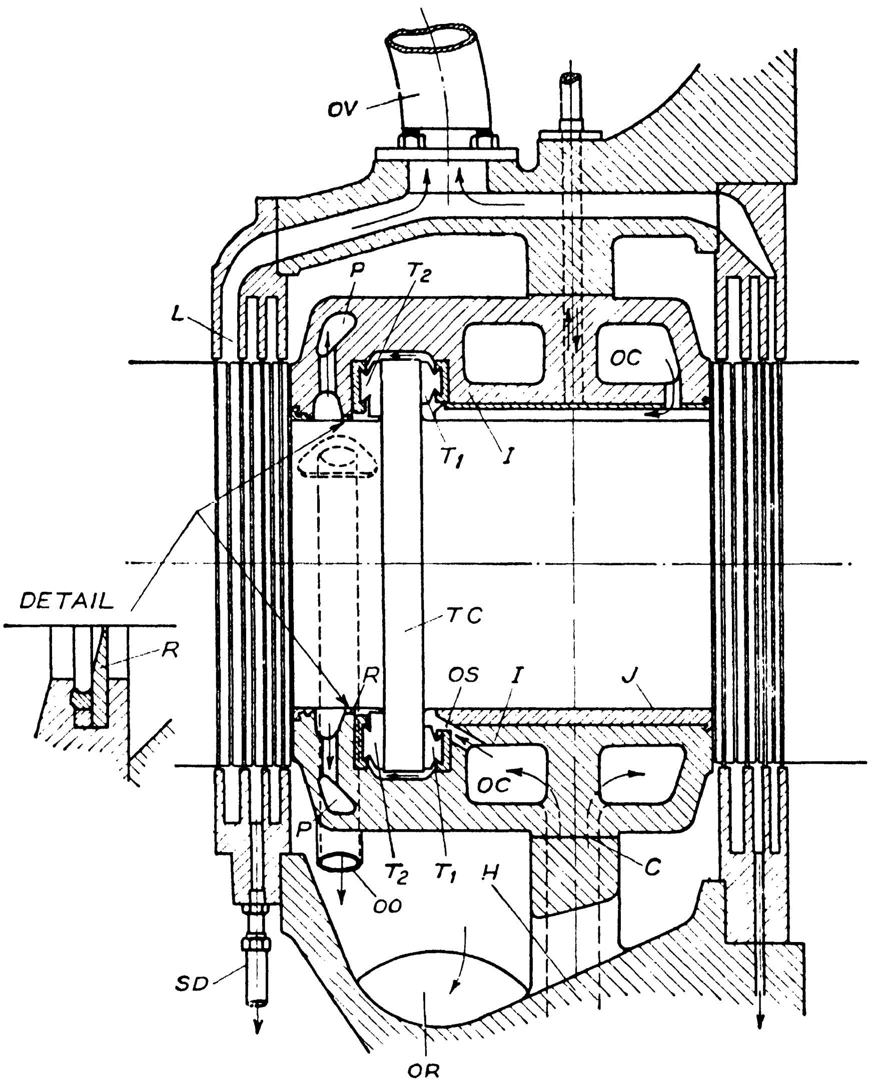


Fig. V, 14.—Thrust bearing for steam turbine, by Brush Electrical Engineering Co., Ltd., Loughborough

Any oil which escapes along the shaft and is caught in the labyrinth L is removed by the drain pipe SD, while oil vapour and spray are taken off by the vent pipe OV.

On account of the relatively high peripheral speeds in steam turbine bearings, the heat generated by viscous resistances is relatively of greater amount than in most other applications and, in addition, the oil often absorbs heat conducted through the shaft and housing from adjacent hot parts of the turbine. The flow of oil must accordingly be large relatively to the dimensions of the bearings. The frictional heat is, of course, to be calculated by theoretical formulæ of the kind given in Sect. IV, 8. The heat conducted from the steam-heated parts of the turbine may be determined approximately from the data for heat transmission referred to in Chap. X. In practice, however, the pipes and passages are usually designed for a large excess of oil flow, and means are provided for reducing the flow to the amount which is found by trial to be actually required. The characteristics of the oils generally used are given in Table III, 3 (p. 53).

In order to avoid excessive absorption of heat, and also to reduce frictional losses, the thrust bearings of steam turbines are often placed at the outer end of the turbine shaft, where the shaft is free of bending loads and its diameter, together with that of the thrust collar, can be kept small. Fig. V, 15 shows a simple arrangement of a thrust bearing (also by the Brush Company) in this position. As in fig. V, 14, the oil is delivered to the inner periphery of the runner, in the present case to both of its sides, passing out radially between the pivoted pads to be discharged at the top of the closed housing, where its flow is controlled by the screw S.

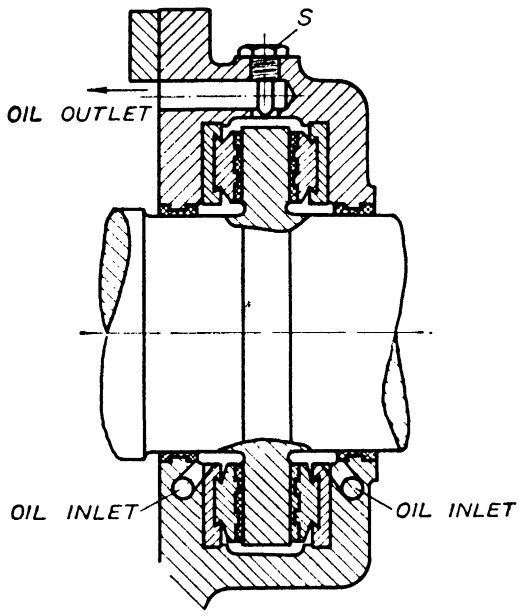


Fig. V, 15

The conditions to be met in the application of thrust bearings to gas turbines may be said to be the same as those in steam turbines, somewhat exaggerated, however, with regard to both rotative speed and temperatures. In gas turbines for stationary operation it will usually be possible, as in the steam turbine bearing last mentioned, to place the thrust bearing on an end of the shaft projecting outside the main casing of the turbine. An example is shown in fig. V, 16 (Ref. V, 7). Normal practice can then be followed with regard to both the design of the bearing and its oil supply, though a larger flow of oil may be required for cooling purposes than would be provided for a bearing in a similar position on a steam turbine.

It is sometimes impracticable in thrust bearings fitted to horizontal shafts to immerse the runner and the pads in oil. In such cases the oil must be supplied in adequate quantity around the inner circumference of the working face of the runner, which distributes it outwards by centrifugal force. The following calculation will show that under normal conditions the viscous film so formed and maintained will be sufficiently thick to feed the interspaces between runner and pads, and to carry away the heat generated. (Fig. V, 17 shows a radial section of such a film, of which the thickness is, of course, greatly exaggerated.)

It is assumed that the radial velocity of the fluid at each point is small compared with the circumferential velocity which (also by assumption) it

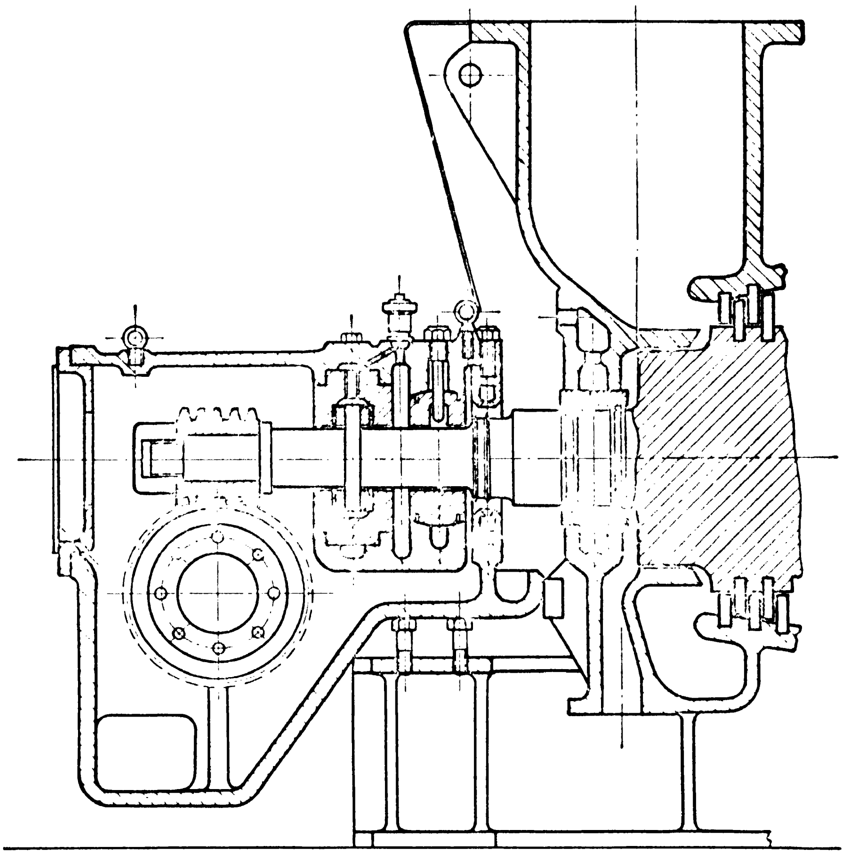


Fig. V, 16.—Thrust bearing for gas turbine, by C. A. Parsons & Co., Ltd.; see Ref. V, 7

partakes of with the runner. In doubtful cases these assumptions should be checked from the numerical results obtained.

Neglecting the effects of gravity, and regarding the thickness of the film as being everywhere small, and consequently the fluid pressure in it as being atmospheric throughout, consider the equilibrium of an element of the fluid of superficial area $\delta\mathfrak{A}$ parallel to the plane of the runner, and included between the free surface of the film and another adjacent surface parallel to it.

If h is the thickness of the film at radius r, and the other face of the element is at distance z from the face OA in contact with the runner, the centrifugal force on the element, equated to the shearing force on the face of the element at z, gives the equation

$$m\omega^2 r \,.\, \delta\mathfrak{A}(h - z) = \mu \frac{\partial u}{\partial z} \delta\mathfrak{A}, \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 8}$$

in which m is the density of the liquid, u its radial velocity outwards (varying with z) and ω the angular speed of the runner.

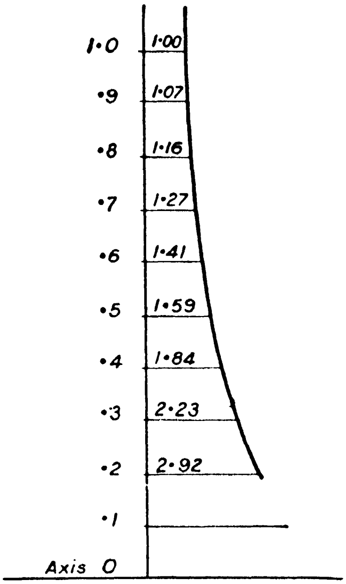


Fig. V, 17

From equation V, 8 $du = \frac{m}{\mu} \omega^2 r(h - z)dz$

from which, by integration, $u = \frac{m}{\mu} \omega^2 r \int_0^z (h - z)dz$

$$= \frac{m}{\mu} \omega^2 rz(h - \tfrac{1}{2}z), \quad . \quad . \quad . \quad \text{V, 9}$$

being zero on the face of the runner, where $z = 0$.

The total outward flow past any circumferential section at radius r is thus

$$Q = \int_0^h 2\pi r u \, dz = \frac{2\pi m\omega^2 r^2}{\mu} \int_0^h (hz - \tfrac{1}{2}z^2)dz$$

$$= \tfrac{2}{3} \frac{\pi m\omega^2 r^2}{\mu} h^3;$$

and since this quantity is independent of r,

$$h = \left(\frac{3\mu Q}{2\pi m\omega^2}\right)^{1/3} \frac{1}{r^{2/3}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 10}$$

or the thickness of the film varies inversely as the two-thirds power of the radius, and has its smallest value at the periphery of the disc, where its value is

$$h' = \left(\frac{3\mu Q}{2\pi m}\right)^{1/3} \frac{1}{U'^{2/3}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{V, 11}$$

U' being the peripheral speed of the runner.

For example, taking Q as 1×10^3 cm.^{3} sec.$^{-1}$, $\mu = 0{\cdot}1$, $m = 1$, $U' = 8 \times 10^3$, as usual values, it is found that $h = 9 \times 10^{-3}$ cm., showing that a sufficient thickness for supplying pads of any usual dimensions will be obtained with the flow assumed.

In a construction developed by Kingsbury Machine Works Inc., the oil pumped through the bearing by the runner is immediately discharged at the top of the inner casing. This is effected by means of what is called an " oil control ring ". The construction is illustrated in figs. V, 18*a* and *b*. The oil is delivered to the bearing, as usual, at the inner periphery of

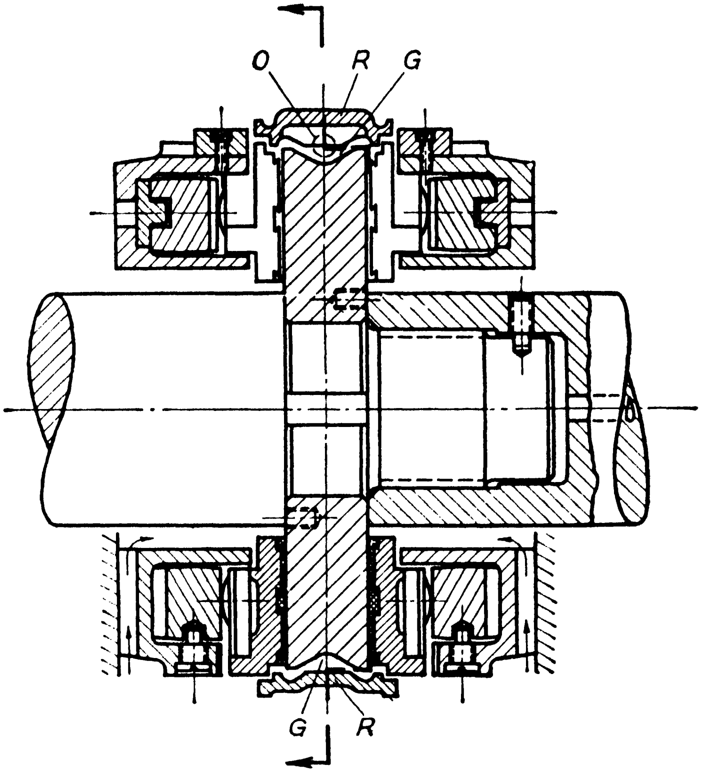


Fig. V, 18a.—Thrust bearing designed for very high rotative speeds, by Kingsbury Machine Works, Inc.

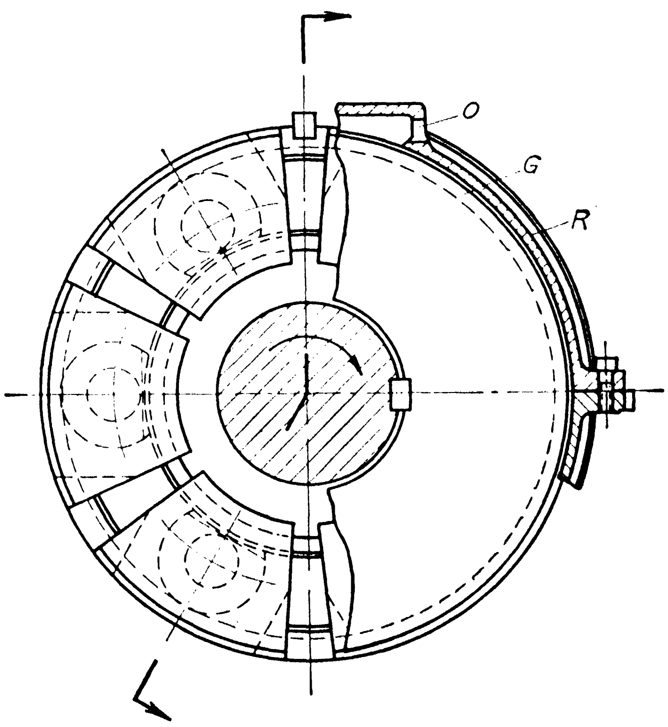


Fig. V, 18b.—Thrust bearing designed for very high rotative speeds, by Kingsbury Machine Works, Inc.

the runner (in this case on both sides of it), the amount flowing being regulated by restriction of the inlet. The periphery of the collar is deeply grooved, as shown at G, so that both its plane faces terminate radially in angular edges from which the oil is thrown off tangentially. Surrounding these edges is the oil-control ring R which forms a circumferential channel in which the oil flows, as in the helix of a centrifugal pump, to an outlet (or sometimes two outlets) situated near the highest point of the ring, as shown in fig. V, 18*b*.

The construction minimizes churning and heating of the oil, which, at very high speeds, may be the chief source of loss of energy in the bearing. An application in which the device is of special advantage on this account is that to turbine-driven boiler feed-pumps.

6. Marine Thrust Bearings.

The application of pivoted thrust bearings to the propeller shafts of ships presents no special peculiarities or difficulties.

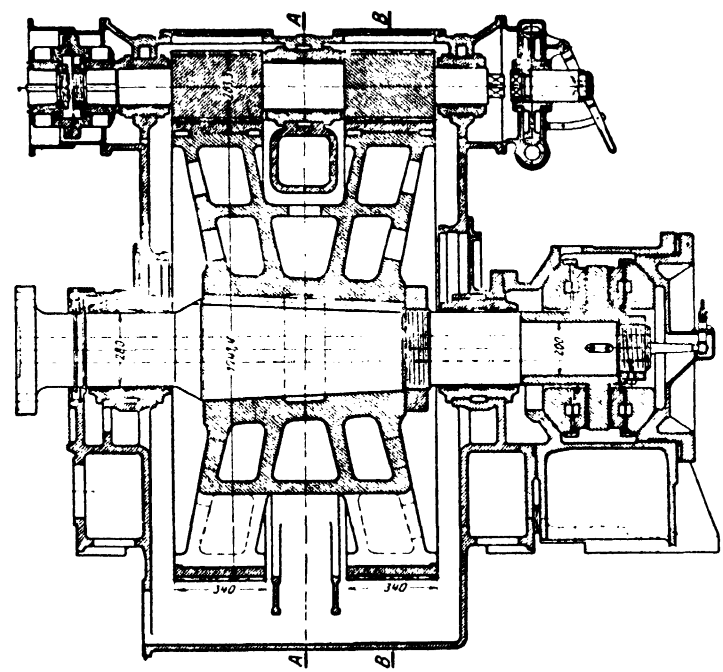


Fig. V, 19.—Thrust bearing and turbine reduction-gearing for marine propeller shaft, by Messrs. Brown, Boveri & Cie., Winterthur

In the case of gear-driven shafts, the thrust bearing is sometimes placed at the extreme forward end of the shaft, ahead of the gear by which the shaft is driven. The shaft can then be reduced in diameter to that which is necessary to carry the thrust load, and the bearing is of minimum size. Fig. V, 19 shows such an arrangement of turbine gearing and pivoted thrust bearing constructed by Messrs. Brown, Boveri & Cie (Ref. V, 8).

More usually, however, even on gear-driven shafts, and necessarily on directly driven shafts, the thrust bearing is placed aft of all the propulsive machinery. The collar is then forged on a special length of the propeller shaft (called the thrust shaft, which is of similar diameter to the rest of the propeller shaft), around which the thrust pads are arranged. It is probably, in part, by reason of the location of the pads on a circle of large diameter in this way, and in part from the difficulty of obtaining accurate finish on a runner which is integral with a heavy piece of shafting, that propeller thrust bearings are usually loaded comparatively lightly. From considerations of propeller efficiency the rotative speeds of propeller shafts are also low. In consequence, provided that the bearing surfaces are sufficiently good to ensure fluent-film lubrication, the rate of energy loss in the bearings is low relatively to their dimensions, and their cooling presents little difficulty.

In small units, radiation and air-convection from the bearing casings usually suffice for cooling.

Details of a typical marine thrust bearing of medium size are shown in Front Folder V, 1, by the courtesy of Messrs. John Brown & Company Limited. Bearings according to this design are fitted on a twin-screw mercantile vessel commissioned in 1947. Each shaft transmits 7500 horse-power at 120 revolutions per minute; the thrust load on each bearing is 84,000 pounds (38,000 kg. wt.), the mean intensity of pressure on the pads having the low value of 200 pounds per square inch (14 atmospheres). The mean sliding speed of the runner is 14·8 feet (450 centimetres) per second.

The bearings are served by a continuous oil circulation, at about 5 pounds per square inch above atmospheric pressure. A simple calculation on the basis of equations IV, 13 and 16, with the assumptions that the location of the pad pivots is normal, and the viscosity of the oil about 0·5 poises, shows that the inclination c of the pad to the runner will be of the order 9×10^{-5}, and the coefficient of resistance of the bearing not greater than $1{\cdot}0 \times 10^{-3}$. This resistance corresponds to a loss of energy in the bearing of $2\frac{1}{4}$ horse-power only, corresponding to 1·60 British Thermal Units or 400 gramme-calories per second, so that it would seem that no means of cooling, other than air-dissipation from the bearing itself, should be necessary under normal conditions of operation.

Fig. V, 20 (p. 148) shows an example of a propeller shaft bearing embodying the more elaborate arrangements for controlling and observing the circulation of oil, and for eliminating all possibility of oil leakage, which are called for by the special conditions of naval service. This illustration is owed to the courtesy of Messrs. Michell Bearings Ltd., who supply bearings of this type for the vessels of the British and many other navies. The thrust pads A of this bearing, like those of the bearing illustrated by Folder V, 1, are supported on circumferentially extending " shoes " B, and engage both sides of the thrust collar C. On each side of the collar is provided a journal bearing, with journal D supported by the stationary bearing-members E, which are pivoted in a similar manner to the pads A of the thrust bearing. (Journal bearings of this kind are more fully described and discussed in Chap. VI.) Oil is circulated through the bearing under pressure, entering through an inlet I on each side of the thrust collar and being discharged from an outlet O. Its flow is so controlled by sealing rings R as to ensure that the central chamber of the bearing, containing the collar C, journals DD, thrust pads A and journal pads E, is com-

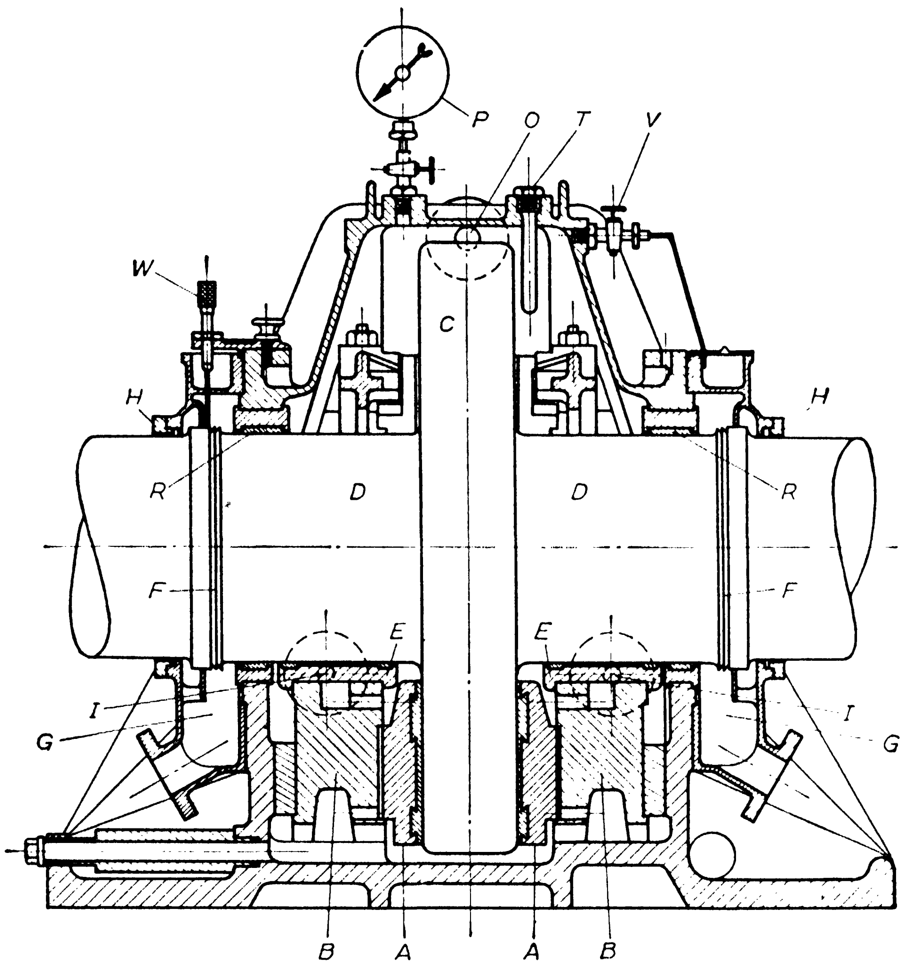


Fig. V, 20.—Marine thrust bearing, by Michell Bearings, Limited

pletely filled with oil, while the smaller chambers G, fore and aft of the central chamber, are relieved of pressure. Flingers F, with stuffing boxes H, prevent any escape of oil along the shaft, and consequent fire risk from this source when the vessel is in action. An oil-pressure gauge P, and thermometer pocket T, with oil-test cock V, allow of verification of the condition of the oil; and the micrometer W enables any wear of the journals D, or journal pads E, with resulting misalignment, to be detected.

7. The Sectorial Feature in Thrust Bearings.

In the applications which have been made so far in this chapter of the general theory of plane bearings to thrust bearings, the rotary character of the motion and in particular variations of the sliding velocity from the inner to the outer radius of the bearing, have been left out of consideration. The justification of this simplification, stated in advance in Sect. V, 1, is that a sufficiently accurate approximation is usually given by assuming that the linear velocity is uniform throughout, with the value which is correct for the mean radius. The results from such an assumption, however, obviously can only be first approximations.

Consideration may be given to the question from two different standpoints: the objective in one case being to obtain more accurate results for the plane and parallel-faced sectorial pad of the conventional form of bearing; and in the other, to replace the conventionally placed surfaces by some modified form or location of the parts more amenable to accurate calculation, or more efficient in operation.

The second line of approach has been the more usual. Thus Boswall (Ref. V, 9) treats the pad surface, not as a plane, but as a helix defined by the relation $h = h_0(1 + b\theta)$ in which h_0 is the film thickness at the outflow edge of the pad (taken as independent of r), and b a non-dimensional constant. He is then able to obtain a solution for p in terms of r and θ in the form of Bessel functions. Applying the solution to a pad having an angular length of 40 degrees, with an outer radius double the inner, and with a thickness of film at its radial inflow edge double that at the outflow edge (also radial), Boswall has given a table of the fluid pressures at 10 equal intervals in both the radial and circumferential directions. This table shows the maximum fluid pressure to occur at a point whose radius is 0·77 of the outer radius of the pad, and which divides the circumferential length of the pad in the ratio 0·70 to 0·30.

The length of the pad at its mean radius being in this case 1·05 times its width, the corresponding figures given by the first rough approximation which ignores the circumferential curvature are 0·75, and the ratio 0·68 to 0·32.

Another investigator, S. M. Skinner (Ref. V, 10) offers the criticism of Boswall's approximation that, in a plane pad of finite length relatively to the radii, h necessarily varies in the direction of the radius, as well as in that of the circumference. He proposes (in the cited reference, which treats of several other extensions of the theory of bearings of finite width-length ratio) to represent the surface of a sectorial bearing pad by the expression

$$h = Ar\theta,$$

so as to take into account the variation of the film thickness in the radial as well as the circumferential direction. This assumption again leads to a solution in Bessel functions, as does also the more general assumption also suggested by Skinner, viz.

$$h = Ar^i\theta^q,$$

i and q being arbitrary numbers. So far as is known to the author, however, neither of these solutions has been completed by numerical calculations.

Instead of postulating a modification of the geometrical form of the pad surface, it seems to be a preferable procedure to seek an alteration in the location of the plane surface of the pad relative to the plane surface of the runner, such that the basic first approximation to the motion may more nearly represent the actual motion in the modified bearing than it does in the bearing hitherto regarded as normally located.

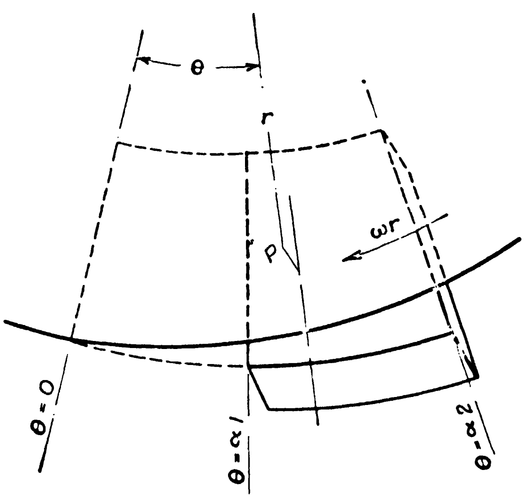


Fig. V, 21

In the first-order approximation so far adopted in the text, it is assumed that the outflow edge of the pad is parallel to the plane surface of the runner, and is approximately radial. Along this edge, therefore, the rate of shear in the lubricating film, being proportional to the circumferential velocity of the runner, has been tacitly assumed to increase in the same proportion as the radius. If that were the case, higher pressures would be produced at the outside of the pad than at the inner side, which is inconsistent with the assumption of uniformity which has been made.

To avoid, or at least to reduce the extent of this inconsistency, the relative location of the pad and runner may be modified as shown in fig. V, 21. The plane in which the surface of the pad lies is assumed to intersect the plane of the runner on the line $\theta = 0$. At any point P, at radius r and angular distance θ from the line of intersection, the distance h between the surfaces, assumed to be plane, is given by

$$h = c'r \sin \theta,$$

in which c' is the inclination of the planes to one another in the direction $\theta = \frac{1}{2}\pi$. The inclination in the circumferential direction, which is the direction of relative motion, is, however,

$$c = \frac{\partial h}{r\,\partial\theta} = c' \cos \theta,$$

and the effective mean inclination of the pad can be taken as being approximately

$$c_p = c' \cos \theta_p,$$

where θ_p is the value of θ at the pivoting point of the pad.

If the outflow edge is radial, at $\theta = \alpha_1$, the thickness of the interspace will vary along its length from $c'r_1 \sin \alpha_1$ to $c'r_2 \sin \alpha_1$, i.e. proportionally to the circumferential velocity of the runner at each point of the radius; consequently, the rate of shear in the film (neglecting flow in the radial direction) will be uniform. The same condition will be fulfilled at the inflow edge if it also is radial ($\theta = \alpha_2$), as well as at every other radial line.

The flow in the sectorial film can thus be treated as being the same as in a rectangular bearing, with linear sliding motion parallel to one edge of the rectangle, if the difference in the cross-flows occurring in the two cases be disregarded; and what may be regarded as an approximation of the second order to the motion is thus obtained. In Appendix II, p. 292, a comparison is made between typical results obtained by this approximation and results derived by Kingsbury experimentally.

8. Approximate Treatment of Different Width-Length Ratios in Plane Bearings.

The labour involved in obtaining the numerical constants for bearings of different width-length ratios from the theoretical equations (even with the aid of tables and diagrams, when they are available) is often considerable; it is consequently worth while to look for means of abbreviating the work, even at some sacrifice of accuracy in the results.

Especially is this the case with sectorial thrust bearings, in which the ratio of the circumferential length of the pad to its radial width is one of the disposable constants which has to be fixed by the designer. A simple method of determining the effect on the pressure generated which will arise from a variation of the constant in either direction is a material help in the process of design.

Upon examination of the variation of the charge P per unit width of pads of different finite widths, all having the same pivot location (i.e. the same values of α or γ), a comparatively simple relation is found to hold approximately for values of the width-length ratio β greater than unity. This relation is expressed by the equation

$$P \approx P_\infty \frac{b - kl}{b} \approx P_\infty \left(1 - \frac{k}{\beta}\right), \quad \ldots \ldots \quad \text{V, 12}$$

l being the length, and b the width of the pad, k a constant, P_∞ the charge per unit width on the pad of infinite width, and $\beta = b/l$.

When put in the form

$$Pb \approx P_\infty (b - kl), \quad \ldots \ldots \ldots \quad \text{V, 13}$$

it is at once seen that this equation states that the total charge on the pad of width b is approximately equal to that which it would carry if its width were reduced by a fixed fraction k of its length, and it were loaded to the same extent per unit width as is the pad of infinite width. The possibility of such a relation being even approximately true is doubtless dependent on the cross-flow in the bearing film being negligible except at points within a distance $\frac{1}{2}kl$ from each of the sides of the pad; consequently, the formula cannot be expected to be applicable when b is smaller than l, or $\beta < 1$.

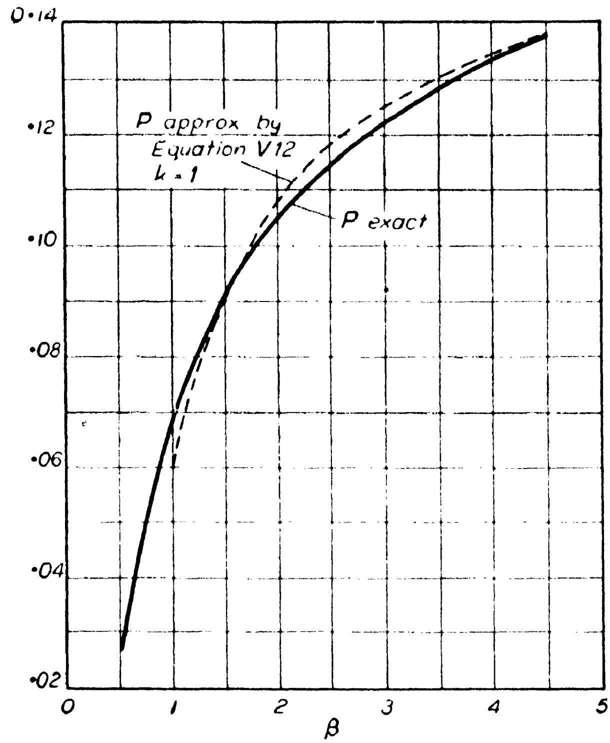


Fig. V, 22

In fig. V, 22 the dotted line shows the results of the equation V, 12 when applied to plane pads of varying width, l and α constant, α being in this case 1·0. The full line gives the more accurate values of P derived from the diagram of Front Folder IV, G. The dotted line is plotted on the assumption that k is 1·0; a closer approximation to the full-line curve in the neighbourhood of any particular value of β can be obtained by adopting a suitably modified value of k. In all cases $P_\infty = 0{\cdot}1589$ (as in the note to equation IV, 12); the true value of P (for β unity) being 0·0669 (as in Sect. IV, 13).

The same formula can be applied, if necessary with adjustment of the constant k in any particular case, for values of α other than unity, with a range from about $\alpha = 0{\cdot}5$ to $\alpha = 3{\cdot}0$.

9. Examples of Plane Sliding Bearings other than Thrust Bearings.

Amongst the most familiar applications of plane bearings having linear motion are the slide blocks, or "shoes", of the crossheads of reciprocating engines, compressors and pumps; the slide-valves of steam locomotives and other steam engines of older types; and the many varied applications to the linearly sliding parts of machine tools. In all these cases of reciprocating motion, of course, the motion takes place alternately in opposite senses, but it often happens that the direction of the component force normal to the direction of sliding is reversed simultaneously with the direction of motion, or that the normal component force is small when the sliding member is moving in one of the directions. In both of these cases pivoted sliding blocks, with the pivot offset from their centres according to the direction of sliding, can frequently be applied, with, in the former of the two cases, two parallel faces co-acting with two parallel bars, one for each of the directions of load.

It is, however, comparatively rarely that pivoted slide blocks or pads are used for the sliding parts of machine tools. The advantage to be derived from their use consists, not so much in reduction of the coefficient of resistance to sliding, as in virtual elimination of wear of the parts and, consequently, permanent retention of the accuracy of the machine. The heat generated by the high frictional resistance of non-pivoted slide blocks may also impair to an appreciable extent the precision of machines of the finer classes. As a conspicuous instance of the advantages to be derived by using pivoted members, may be mentioned the sliding tables of planing machines; in this case, to provide for the reversals of the motion, pivoted pads can readily be fitted to the table in pairs, one pad of each pair being offset as required for motion in one direction, and the other for the return motion.

When pivoted blocks or pads are not employed, the mode of sliding contact is at best that of mitigated solid friction, with a coefficient of resistance not less than 0·03, and more often approximating to 0·10.

When, as in planing machines, the motion is comparatively rapid, and the facilities for lubrication good, effective fluent-film lubrication can be readily secured with pivoted pads, and the coefficient of resistance (including the resistance of the pads which are idle during each stroke) should not exceed 2×10^{-3}.

Crosshead shoes for reciprocating engines, compressors, etc., are usually pivoted centrally, with a view to their surfaces being centrally loaded in both directions of motion. Fig. V, 23 (p. 154) shows, however, that a single slide block can be made to function as a shoe correctly offset for each direction of motion in turn, when the normal component of the load is reversed at the same time. The figure shows the construction as applied to a two-stroke internal-combustion engine. In this construction (Ref. V, 11) the crosshead pin (1) is formed with a cylindrical bearing surface (2) for the small end of the connecting rod, this surface being somewhat eccentric to the other parts of the pin. The portions (3), (3*a*) serve for securing the pin rigidly to the end of the trunk piston; on the remaining portions (4), (4*a*) are pivotally mounted the two crosshead shoes, one on each side of the plane of motion of the connecting rod; only one, however, is shown in the figure. The shoes are thus enabled to act as pivoted bearing

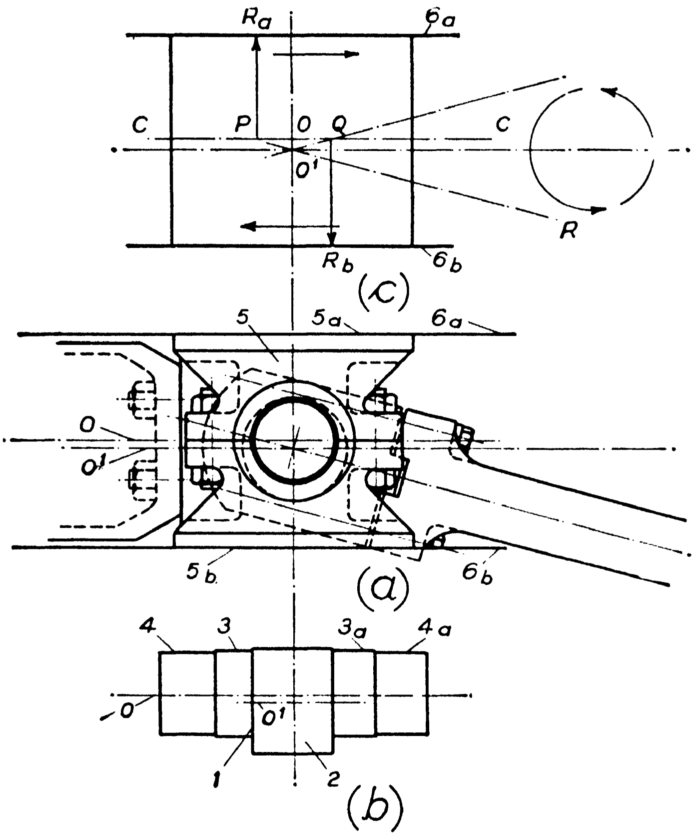


Fig. V, 23

members; and the opposite plane bearing surfaces (5*a*), (5*b*) of each shoe slide in lubricated contact with the plane guide surfaces (6*a*), (6*b*).

The diagrammatic view (fig. V, 23*c*) shows the effect of the eccentricity of the pin (1). The crankshaft is assumed to revolve anticlockwise, and the connecting rod and the piston to be always in compression, as is normally the case in a two-stroke engine. The crosshead will, therefore, be moving to the right when the connecting rod, as shown in fig. V, 23*c*, is below the horizontal centre line, the vertical component of its thrust then acting upwardly on the crosshead. Owing to the eccentricity of the pin the resultant thrusts of the connecting rod and piston, acting along their respective centre lines CO and O′R, intersect at the point P to the left (or rear during this stroke) of the middle point of the crosshead. Consequently the centre of pressure between the crosshead and guide (6a) is at R_a, on the normal line through P, and to the rear of the centre point of the crosshead surface, as required for convergent-film formation. On the return stroke, on the other hand, the connecting rod being above the horizontal centre line, the respective centre lines of action of the connecting rod and piston meet at Q, and the centre of pressure on the guide is at R_b, being again behind the middle point of the crosshead on this, its return stroke.

Suitable methods of supply of lubricants to the bearing surfaces of this kind of mechanism will be described in Chapter IX.

CHAPTER VI

Journal Bearings

1. Classification.

Although it was in connexion with journal bearings that the principle of convergent-film lubrication was first discovered by Osborne Reynolds, general appreciation of the conditions necessary to bring it about, and of the possibility and desirability of eliminating forms of bearings in which it does not take place, has been slower to develop in this field than in that of thrust bearings.

From the standpoint of lubricant theory it is convenient to divide journal bearings into several classes, all capable of forming fluent films, viz.:

(*a*) *Continuous-sleeve bearings*, in which the outer of the pair of relatively-sliding surfaces is an uninterrupted cylinder of slightly larger diameter than the inner (journal) surface, and in which the lubricant forms a film between the two surfaces. Continuous-sleeve bearings are usually of comparatively small size, but from their very simplicity have considerable interest, and, for certain applications, some practical importance. A typical example is sketched in fig. VI, 1.

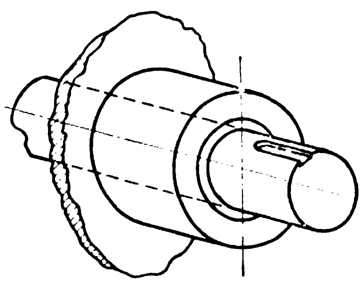


Fig. VI, 1

(*b*) *Divided-sleeve and "partial" bearings.*—In these the outer of the two bearing surfaces approximates to a cylindrical surface, but it is interrupted or incomplete, so that no continuous lubricating film can be formed around the circumference. An example is shown in fig. VI, 2. Journal bearings of this class are probably the most familiar of all forms of machine bearings.

Forming a sub-class of Class (*b*), and its most important section, are what are known as "partial" bearings, in which the effective portion of the bearing surface is less than a half-cylinder.

(*c*) *Multiple-pad bearings.*—In these the outer of the two bearing surfaces is divided into a series of elementary surfaces surrounding, or partially surrounding, the journal. In principle, they are closely related to the multiple-pad thrust bearings described in Chap. V. Like thrust bearing pads, the pads of multiple-pad journal bearings may be individually pivoted, and preferably are so. An example of a bearing of this kind is shown in fig. VI, 3.

(*d*) *Bearings for rolling axles.*—These may belong to any one of the preceding types, but require separate consideration in connexion with their lubrication, mainly on account of their inverted position as compared with other journal bearings.

The choice between the first three of these types for application to any particular case depends on various considerations, both theoretical and practical. A primary consideration is whether the direction of the load varies, or remains nearly constant with respect to the outer member of the bearing. If the load acts invariably in one direction, the effective portion of the outer bearing surface needs, of course, to be only a portion (often merely a small portion) of a complete cylinder, or, it may be in some circumstances only a single pivoted pad. In other cases, as for instance in the bearing of a wheel revolving around a fixed journal, the cylindrical sliding surface, or alternatively an annular series of pads, must extend around the whole circumference of the bearing. In all cases efficient action requires that, by appropriate design, such a condition shall be brought about that a continuous film of lubricant is auto-

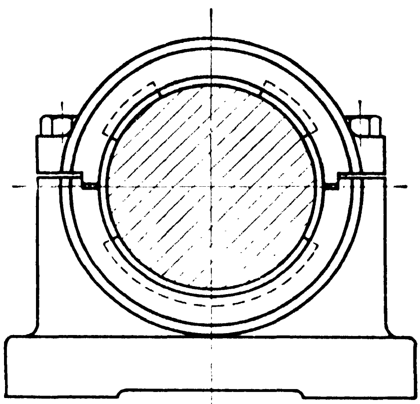


Fig. VI, 2

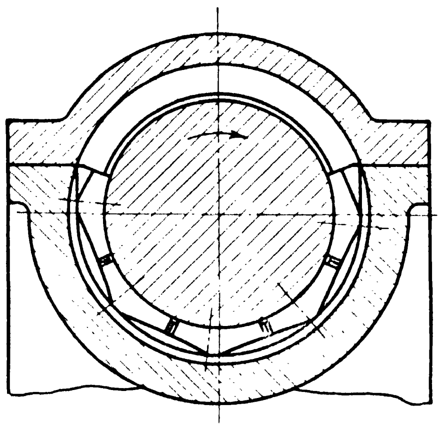


Fig. VI, 3

matically maintained under pressure between the bearing surfaces, and metallic contact and wear thereby prevented. As in plane bearings, the essential condition to be provided, in order that such pressure-film lubrication may exist, is that the interspace between the co-acting members shall converge in the direction of motion.

2. Continuous-sleeve Journal Bearings: Theoretical.

Journal bearings of the continuous-sleeve type [Class (*a*) of the preceding section] are, for intrinsic reasons, limited practically in their proper range of application to small sizes and comparatively light loadings. On the other hand, they are eminently fitted for operation at the highest speeds. They are interesting both on the latter account and because their examination affords a useful and convenient key to the theory of all journal bearings. By the help of the theory which they suggested to him, Sommerfeld (Ref. VI, 1) was able to simplify and clarify, to a notable extent, Osborne Reynolds' pioneer discoveries and calculations on journal bearings in general. The following account follows Sommerfeld's methods, with some modifications and further developments intended to facilitate its numerical and graphical applications to practice.

Fig. VI, 4 is an outline cross-section of a continuous-sleeve bearing, in which the journal J is revolving in the clockwise direction and carries a load P acting in a fixed direction transverse to the axis O. The whole interspace between journal and sleeve is assumed to be filled with the lubricant, which is regarded as being incompressible.

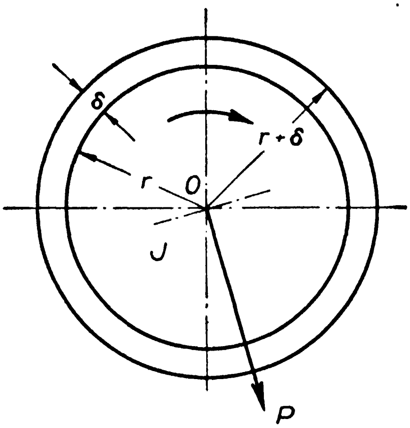


Fig. VI, 4

The mathematical treatment will show that, under these conditions, the behaviour of the bearing will be as follows:

At extremely high rotative speeds the journal will assume a position sensibly concentric with the sleeve. The radius of the journal being r, and that of the sleeve $r + \delta$ (as shown in fig. VI, 4), the thickness of the lubricant film will be nearly equal to δ everywhere around the circumference, as in the figure.

If the speed diminishes to a lower angular velocity ω, the load remaining unchanged in amount and direction, the journal will shift within the sleeve so that its axis occupies an eccentric position O′ (see fig. VI, 5), such that the line of eccentricity OO′ is oriented at right angles to the line of action O′P of the load, and is turned in the direction of rotation from it, i.e. in this instance

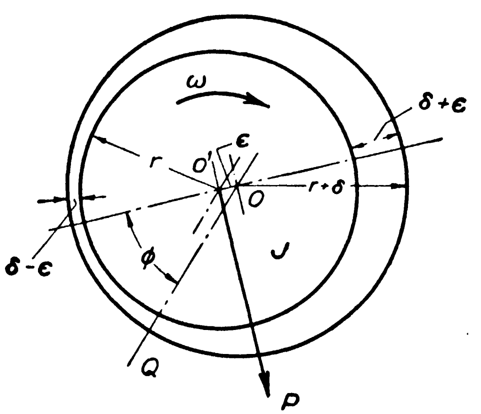


Fig. VI, 5

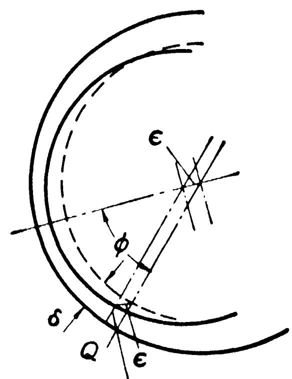


Fig. VI, 5a

clockwise. If ω diminishes further, while the load P remains constant in direction and amount, the eccentricity OO′ will progressively increase as ω diminishes until, at vanishing speed, the surface of the journal comes into contact with that of the sleeve at the point C′ in line with OO′ (fig. VI, 6).

This ideal, and at first sight rather paradoxical, behaviour is realized practically over a wide range of conditions, but is subject to limitations which will be discussed in a later section.

The difference between the radii of the sleeve and journal (called the "radial clearance"), being denoted by δ, is regarded in the present section as a constant; the eccentricity $OO' = \epsilon$, as a variable dependent on the speed and load. The width of the interspace (i.e. the thickness of the lubricating film) at any point of the circumference is denoted by h, as in the preceding chapters. Its value at any point Q of the circumference is given (see fig. VI, 5*a*) by

$$h = \delta - \epsilon \cos \phi = \delta(1 - \kappa \cos \phi), \quad . \quad . \quad . \quad . \quad \text{VI, 1}$$

where ϕ is the angle O′OQ (see fig. VI, 5) and $\kappa = \epsilon/\delta$.

The film of lubricant being everywhere very thin compared with the radii of curvature of the surfaces, the rates of variation of its pressure will depend only upon the linear variation of h along its length, and the same general relations between p, h and $x = r\phi$ will hold as in the case of plane bearings. For the present the sleeve bearing is supposed to be of unlimited width in the direction along its axis, so that the motion of the lubricant is two-dimensional, being everywhere in the plane of the figures.

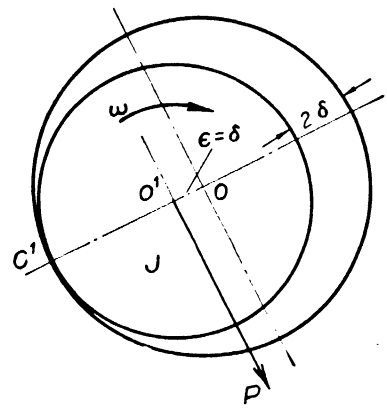


Fig. VI, 6

If the circumferential speed of the journal ωr is denoted by U, then, as in plane bearings (equation IV, 7),

$$\frac{dp}{dx} + 6\mu U \frac{h - h_m}{h^3} = 0,$$

i.e.

$$\frac{dp}{r\,d\phi} = -6\mu U \frac{h - h_m}{h^3},$$

or

$$\frac{dp}{d\phi} = -6\mu U r \frac{h - h_m}{h^3}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{VI, 2}$$

in which h_m is the thickness of the film at a point $(\phi = \phi_m)$, where $dp/d\phi$ is zero, that is to say, at a point where the pressure p has either a maximum or a minimum value.

In the continuous-sleeve bearing, however, h instead of increasing according to a linear relation with x or $r\phi$, as in the plane bearing, is a cyclic function of ϕ, having the same value, for instance, at $\phi = 2\pi$, as at $\phi = 0$. Proceeding from this consideration, the thickness h_m of the film at the point where the maximum pressure occurs can be determined.

Thus from equation VI, 2, since $p_{2\pi} - p_0 = 0$,

$$\int_0^{2\pi} 6\mu U r \frac{h - h_m}{h^3}\, d\phi = 0,$$

or
$$\int_0^{2\pi}\frac{d\phi}{h^2}-\int_0^{2\pi}\frac{h_m}{h^3}\,d\phi=0;$$

so that, from equation VI, 1,
$$\frac{h_m}{\delta}\int_0^{2\pi}\frac{d\phi}{(1-\kappa\cos\phi)^3}=\int_0^{2\pi}\frac{d\phi}{(1-\kappa\cos\phi)^2}.$$

For brevity, call the right-hand and left-hand definite integrals in this equation respectively X_2 and X_3, then
$$h_m=\delta\frac{X_2}{X_3}. \qquad \text{VI, 3}$$

Again from equation VI, 2,
$$\frac{dp}{d\phi}=-6\mu Ur\left\{\frac{1}{h^2}-\frac{h_m}{h^3}\right\}$$
$$=-6\mu Ur\left\{\frac{1}{\delta^2}\cdot\frac{1}{(1-\kappa\cos\phi)^2}-\frac{h_m}{\delta^3}\cdot\frac{1}{(1-\kappa\cos\phi)^3}\right\},$$
and integrating,
$$p=p_0-\frac{6\mu Ur}{\delta^2}\left\{\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^2}-\frac{h_m}{\delta}\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^3}\right\},$$
or, from equation VI, 3,
$$p-p_0=-\frac{6\mu Ur}{\delta^2}\left\{\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^2}-\frac{X_2}{X_3}\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^3}\right\}. \qquad \text{VI, 4}$$

p_0, a constant, representing the atmospheric, or other uniform, pressure at the boundaries of the lubricating film, i.e. at the ends of the sleeve. However distant these may be, they may be regarded as controlling the absolute pressures throughout. In order to determine p as a function of ϕ and κ (δ the radial clearance being regarded as a fixed quantity), the separate integrals in equation VI, 4 are first found by usual methods to be as follows:
$$\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^2}=\frac{\kappa\sin\phi}{(1-\kappa^2)(1-\kappa\cos\phi)}$$
$$+\frac{2}{(1-\kappa^2)^{\frac{3}{2}}}\arctan\left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1}{2}}\tan\tfrac{1}{2}\phi, \qquad \text{VI, 5}$$
and
$$\int_0^{\phi}\frac{d\phi}{(1-\kappa\cos\phi)^3}=\frac{\kappa\sin\phi}{2(1-\kappa^2)(1-\kappa\cos\phi)^2}+\frac{3\kappa}{2(1-\kappa^2)^2}\cdot\frac{\sin\phi}{1-\kappa\cos\phi}$$
$$+\frac{2+\kappa^2}{(1-\kappa^2)^{\frac{5}{2}}}\arctan\left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1}{2}}\tan\tfrac{1}{2}\phi. \qquad \text{VI, 6}$$

On putting $\phi=2\pi$ and $\phi=0$ into the right-hand sides of equations VI, 5 and 6, the values of the definite integrals X_2, X_3 are found to be

$$X_2 = \frac{2\pi}{(1-\kappa^2)^{\frac{3}{2}}}, \quad X_3 = \frac{(2+\kappa^2)\pi}{(1-\kappa^2)^{\frac{5}{2}}},$$

so that

$$\frac{h_m}{\delta} = \frac{X_2}{X_3} = \frac{2(1-\kappa^2)}{2+\kappa^2}. \qquad \text{VI, 7}$$

Substituting this value in equation VI, 4, together with the values found in equations VI, 5 and 6,

$$-\frac{p-p_0}{6\mu Ur/\delta^2} = \frac{\kappa \sin\phi}{(1-\kappa^2)(1-\kappa\cos\phi)} + \frac{2}{(1-\kappa^2)^{\frac{3}{2}}} \cdot \arctan\left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1}{2}} \tan\tfrac{1}{2}\phi$$

$$-\frac{2(1-\kappa^2)}{2+\kappa^2}\left\{\frac{\kappa\sin\phi}{2(1-\kappa^2)(1-\kappa\cos\phi)^2} + \frac{3\kappa\sin\phi}{2(1-\kappa^2)^2(1-\kappa\cos\phi)} + \frac{2+\kappa^2}{(1-\kappa^2)^{\frac{5}{2}}} \arctan\left(\frac{1+\kappa}{1-\kappa}\right)^{\frac{1}{2}}\tan\tfrac{1}{2}\phi\right\}$$

$$= \frac{\kappa}{1-\kappa^2}\left(1-\frac{3}{2+\kappa^2}\right)\frac{\sin\phi}{1-\kappa\cos\phi} - \frac{\kappa}{2+\kappa^2}\cdot\frac{\sin\phi}{(1-\kappa\cos\phi)^2}$$

$$= -\frac{\kappa\sin\phi}{2+\kappa^2}\left\{\frac{1}{1-\kappa\cos\phi} + \frac{1}{(1-\kappa\cos\phi)^2}\right\},$$

or

$$p - p_0 = \frac{6\mu Ur}{\delta^2}\cdot\frac{\kappa}{2+\kappa^2}\cdot\frac{\sin\phi(2-\kappa\cos\phi)}{(1-\kappa\cos\phi)^2}. \qquad \text{VI, 8}$$

this equation determining the pressure at all points of the film. The point at which the pressure is a maximum is readily found by substituting for h in equation VI, 1 the value of h_m given by equation VI, 7, i.e.

$$h_m = \delta\cdot\frac{2(1-\kappa^2)}{2+\kappa^2} = \delta(1-\kappa\cos\phi_m),$$

in which it is to be noted that h_m is always smaller than δ.

Thus

$$\cos\phi_m = \frac{3\kappa}{2+\kappa^2}, \qquad \text{VI, 9}$$

and

$$\sin\phi_m = \frac{(4-\kappa^2)^{\frac{1}{2}}(1-\kappa^2)^{\frac{1}{2}}}{2+\kappa^2}.$$

By inserting these values in equation VI, 8, the maximum pressure is found to be

$$p_m = p_0 + \frac{6\mu Ur}{\delta^2}\cdot\frac{\kappa}{4(2+\kappa^2)}\cdot\left(\frac{4-\kappa^2}{1-\kappa^2}\right)^{\frac{3}{2}}, \qquad \text{VI, 10}$$

and there is, of course, a point of numerically equal negative, and minimum, pressure difference, $p_m - p_0$, at $-\phi_m$.

The accompanying Table VI, 1 gives, for values of κ varying by 0·1 throughout its possible range from 0 to 1, the values of h_m/δ and ϕ_m, as well as those of the ratio $h_{min} : h_{max}$, of the least to the greatest thickness of the lubricating film, these values occurring, of course, at the ends of the diametral line OO′.

TABLE VI, 1

κ	h_m/δ	ϕ_m, deg.	$\frac{h_{min}}{h_{max}}$
0	1	90	1
·1	0·985	81·4	0·818
·2	·941	72·9	·667
·3	·871	64·5	·538
·4	·778	56·3	·429
·5	·667	48·2	·333
·6	·542	40·3	·250
·7	·410	32·5	·176
·8	·273	24·6	·111
·9	·135	16·1	·053
1·0	0	0	0

By similar integrations to those given earlier in this section the resultant force exerted on the whole journal by the pressures p is found to be

$$P = \frac{12\pi\mu U r^2}{\delta^2} \cdot \frac{\kappa}{(1-\kappa^2)^{\frac{1}{2}}} \cdot \frac{1}{(2+\kappa^2)}, \quad . \quad . \quad . \quad . \quad \text{VI, 11}$$

this resultant being taken, as usual, as the load on unit width of the bearing.

Likewise the total tractional couple M necessary to rotate a unit width of the journal at the assumed angular velocity $\omega = U/r$ is given by

$$M = \frac{4\pi\mu U r^2}{\delta} \cdot \frac{1+2\kappa^2}{(1-\kappa^2)^{\frac{1}{2}}\,(2+\kappa^2)}. \quad . \quad . \quad . \quad . \quad \text{VI, 12}$$

Thus k, the coefficient of resistance of the bearing, is found to be

$$k = \frac{M}{Pr} = \frac{\delta}{3r} \cdot \frac{1+2\kappa^2}{\kappa}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{VI, 13}$$

being a minimum, for δ constant, if $\kappa = 1/\sqrt{2}$, when

$$k = \frac{2\sqrt{2}}{3} \cdot \frac{\delta}{r}.$$

If the load capacity of the bearing be expressed according to a usual custom, as the quotient of the resultant load divided by the diametral area, it is found from equation VI, 11 to be

$$\frac{P}{2r} = \frac{6\pi\mu U r}{\delta^2} \cdot \frac{\kappa}{(1-\kappa^2)^{\frac{1}{2}}} \cdot \frac{1}{2+\kappa^2},$$

P in this case being the resultant of all the fluid pressures, both positive and negative, which support the load.

The maximum positive or negative intensity of fluid pressure at any point, from equation VI, 10, is given by

$$\frac{6\mu Ur}{\delta^2} \cdot \frac{\kappa}{4(2+\kappa^2)} \left(\frac{4-\kappa^2}{1-\kappa^2}\right)^{\frac{3}{2}} = p_m - p_0.$$

The ratio of these two pressure intensities is

$$\frac{P}{2r(p-p_0)} = \frac{4\pi(1-\kappa^2)}{(4-\kappa^2)^{\frac{3}{2}}},$$

which, in the typical case when $\kappa = 0{\cdot}5$, has the numerical value 1·30, thus showing that in a continuous-sleeve bearing of unlimited width, the mean effective pressure estimated on the diametral area may be, if full account be taken of the negative pressures, greater than the maximum of the fluid pressure at any point of the circumference.

It is usually unnecessary for purposes of design to calculate the resultant pressure from equation VI, 11, since the magnitude and direction of this resultant are readily determined in the course of the graphical process of calculation which is described later in this section, and which is required to determine other results.

The volume of lubricant flowing circumferentially between journal and sleeve, per unit width of the bearing, is of course everywhere the same, the fluid being assumed to be incompressible, and the width of the bearing unlimited. (The latter limitation is, however, unnecessary, if it can be assumed that the interspace is everywhere filled with lubricant.) As in the case of the plane bearing (Sect. IV, 6), the flow is easily found from consideration of the cross-section of the film at which the pressure is a maximum or a minimum. Since, at either of such points, there is no circumferential change of pressure, the motion is there one of pure shear, and the volume of flow per unit width is given by

$$Q = \tfrac{1}{2}Uh_m = U\delta\,\frac{1-\kappa^2}{2+\kappa^2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{VI, 14}$$

from equation VI, 7.

The same result is applicable to divided-sleeve bearings and the calculation of the flow in them is of greater practical importance than in continuous-sleeve bearings, the circulation of lubricant in the latter being entirely self-contained and automatic.

Fig. VI, 7 (p. 164) shows, for the particular case in which $\kappa = 0{\cdot}5$, the pressures around the whole circumference of the film in a continuous-sleeve bearing by means of straight lines of lengths proportional to $p - p_0$ drawn at intervals of 15 degrees radially outward from the circle representing the journal. The ends of these lines are joined so as to give a locus for the determination, by scaling, of $p - p_0$ at intermediate points of the circle, and the areas within this locus are shaded to distinguish the regions of positive and negative pressure. (These areas, of course, do not quantitatively represent totals or resultants of the pressures.)

A convenient geometrical construction for determining the vertical and horizontal components of the fluid pressure on the journal, and hence the magnitude and direction of the resultant force, is given in fig. VI, 8. As in fig. VI, 7, the line of eccentricity C′O′OC is taken as horizontal and a series of points is set off at equal intervals (15 degrees in the figure) from this line around

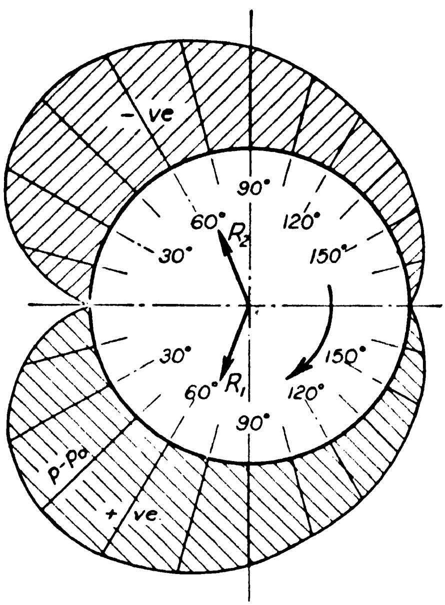


Fig. VI, 7

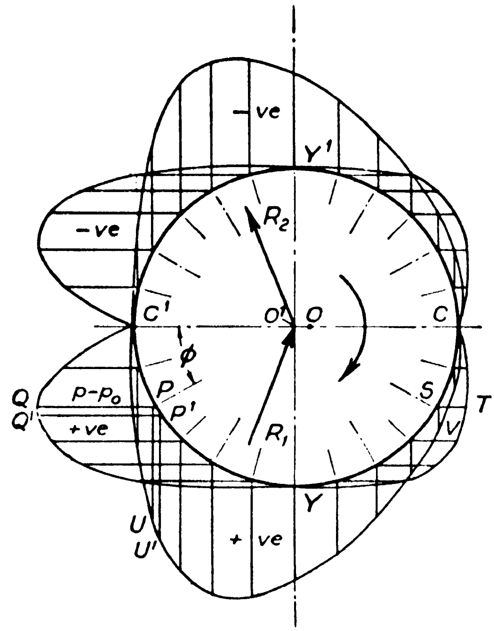


Fig. VI, 8

the circle representing the journal. From every point a vertical and a horizontal line are drawn, and on each of these lines is set off from the circle a distance proportional to the pressure at that point, that is to say, proportional to the value of the $p_1 - p_0$ given by equation VI, 8 for the corresponding angle ϕ. If, at any of these points P at which the pressure is represented in this way by PU in the vertical and PQ in the horizontal direction, an adjacent point P′ is taken at a distance $PP' = r\,\delta\phi$, and lines P′U′, P′Q′ are drawn parallel and equal to PU and PQ, it is seen that the width of the quadrilateral PP′U′U is $r \sin\phi\,\delta\phi$, and that of the quadrilateral PP′Q′Q is $r \cos\phi\,\delta\phi$. The areas of these quadrilaterals, therefore, respectively represent to scale $(p - p_0)r\,\delta\phi \sin\phi$ and $(p - p_0)r\,\delta\phi \cos\phi$, i.e. the vertical and horizontal components of the fluid pressure on the element $r\,\delta\phi$ of the surface of the journal at P.

Completing the construction of lines corresponding to PU and PQ for the other points of the circle in the same way as for P, it is seen that, taking for example the left-hand lower quadrant, the area C′QYPC′ between the locus of points such as Q and the circle of the journal, is made up of quadrilateral strips such as PP′Q′Q, and consequently its area represents to scale the total horizontal force acting on the journal quadrant C′PY. Similarly in the right-hand quadrant the area CTYSC represents the horizontal force which acts on this quadrant in the opposite direction to the force represented by C′PYQC′; so that the resultant horizontal force on the lower semi-cylinder of the journal is given by the difference of these two areas.

Considering the vertical lines in the same way, it is seen that the area C′UVCYC′ represents the total of the vertical components of the fluid pressures which act on the same semi-cylinder, these components being at all points directed upwards.

The resultant of the fluid forces which act on the semi-cylinder is found immediately, in magnitude and direction, as the resultant of its vertical and horizontal components, and is represented in fig. VI, 8 by the line R_1O', which passes, of course, through the centre O′ of the journal.

On the upper half of the diagram, every feature is repeated as the mirror image of the lower half, but the pressures which are represented (i.e. the values of $p - p_0$) are all negative, so that the forces represented by corresponding areas act in opposite directions with respect to the journal. Thus the vertical forces act again upwards, and the total horizontal force acts towards the left, instead of towards the right. The resultant force on the upper semi-cylinder, which is equal in amount to that on the lower, consequently acts in the direction shown by the line $O'R_2$. The resultant of R_1 and R_2 acts vertically upward through the centre of the journal and represents twice the area C′UVCYC′.

The diagram of fig. VI, 8, like that of fig. VI, 7, is drawn approximately to scale for the particular case in which the coefficient of eccentricity κ is 0·5. In Back Folders VI, 1 *a* and *b* are given with greater accuracy the horizontal and vertical force diagrams on a lower half-cylinder for the whole series of values of κ, 0·1, 0·2, . . . , 0·9. It is to be observed that all possible cases of continuous-sleeve bearings are covered by the range of values of κ from 0 to 1, since all possible relations of journal to sleeve necessitate a positive value of δ and an eccentricity $\epsilon = \kappa\delta$, less than δ. It is readily seen, and will be emphasized in succeeding sections of this chapter, that the same restricted postulate does not suffice for divided-sleeve bearings.

3. Practical Characteristics and Uses of Continuous-sleeve Bearings.

The theory and methods of calculation of the behaviour of the lubricating film in continuous-sleeve bearings have been discussed somewhat fully in the preceding section, not so much on account of their practical importance in direct applications to bearings of that class, as with a view to applications in following sections to the more important class of divided-sleeve and, more especially, of "partial" journal bearings.

Continuous-sleeve bearings have, however, special and interesting fields of their own, as well as a wide range of applications in which their simplicity is their main recommendation as compared with bearings of other forms which may be used for the same purposes. It has been shown in the preceding section that, so long as the interspace between journal and sleeve in a continuous-sleeve bearing is maintained full of a lubricant, equal positive and negative pressures are developed at corresponding points on opposite sides of the plane through the two axes. It remains to be considered: What are the conditions necessary to the lubricant remaining in all parts of the interspace? The same general considerations on the matter as have been raised in connexion with plane bearings (in Sects. IV, 14 and 15), with regard to the relations between the negative pressures generated in the film, the atmospheric pressure and the vapour pressure of the lubricating fluid, are of course applicable. The conditions, however, which favour the selection of the continuous-sleeve type of bearing in preference to other types, usually put it into a special position. In its typical and most advantageous applications it is of small dimensions and lightly loaded and, almost necessarily, manufactured with a high degree of accuracy. Since the lubricant circulates only through the bearing itself, no openings into it, except those through which the shaft protrudes, are necessary, or, if they are provided for occasional make-up of lubricant, they need not be left open during operation. The clearance between journal and sleeve may be, and preferably is, made very small, the coefficient of resistance being maintained at a low level, even at high operative speeds, by the use of lubricants of low viscosity.

For a journal of diameter one centimetre, δ the mean radial clearance need not be greater than 5×10^{-5} centimetre, and with the use of suitable materials this clearance will not be greatly changed by variations of temperature during operation. The higher negative pressures, and consequently the principal tendency of air to enter, occur only where the clearance is less than the mean. Before any part of the lubricating film can give way to incoming atmospheric air, a capillary film of lubricant must retreat into the interspace, and the radius of curvature of the meniscus of this film will be of the order of $\frac{1}{2} \times 5 \times 10^{-5}$, or $2{\cdot}5 \times 10^{-5}$ centimetre. A film of an average oil, of this curvature, having a surface tension of 30 dynes cm.$^{-1}$, is capable of sustaining an excess of pressure on its concave side (see equation II, 16) of $30 \div 2{\cdot}5 \times 10^{-5} = 1{\cdot}2 \times 10^{6}$ dynes cm.$^{-2}$, or rather more than one atmosphere. Thus it may be inferred that, in such a case, the film will not break down to admit air from the atmosphere at least for so long as the maximum negative pressure which is generated by the rotation of the journal does not approach the zero of absolute pressure. At that stage the danger of vaporization of the lubricating fluid assumes equal importance to the risk of entry of air.

It has been seen (p. 163) that the figure which represents the resultant load on a typical continuous-sleeve bearing averaged over the diametral area of the journal, is 1·3 times the maximum intensity of positive or negative pressure generated at any one point. It appears, therefore, that an accurately fitted bearing of this class should be able to carry a load of at least one atmosphere on its diametral area without breakdown of the film by defect of pressure.

The circumstance that the lubricant may circulate, in bearings of this class, solely in the bearing interspace and without loss at the sides of the sleeve, greatly favours their successful operation as small bearings of refined construction, since it almost eliminates the risk of introduction of abrasive dust between the surfaces, provided that the lubricant is filtered before being applied. Labyrinths and dust-seals attached to the sleeve and surrounding the shaft

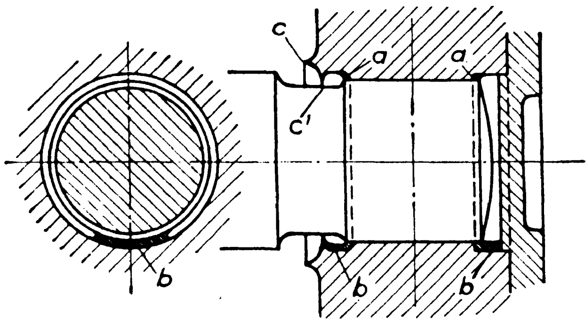


Fig. VI, 9

immediately beyond each end of the journal are, of course, desirable precautions to the same end of dust exclusion. A particular form of construction that is traditional in clock and instrument making is shown in fig. VI, 9, at many times actual size. The recess at *a* carries a meniscus of oil which communicates with a small drop at *b*. The volume of oil contained within the meniscus and drop, though usually not more than one cubic millimetre, is very many times greater than the volume in the interspace between journal and sleeve. The sharp arrises at *c* and *c′* assist in preventing loss of oil by "creep" since they attenuate, almost to zero, the thickness of any capillary film which might reach them. The spindles of such bearings will in most cases be stressed only lightly, and the journals may be short relatively to their diameters; otherwise their deflections require examination. The oil is replenished when necessary, by adding to the drop in the end recess of the sleeve.

Constructed on the same principle, but somewhat larger and more heavily loaded, are the bearings of high-speed polishing wheels and light portable grinding machines. The extremely high rotative speeds called for in this application involve the use of lubricating oils of very low viscosity, and oils suitable for the purpose are now regularly produced (see Sect. III, 5 and Table III, 3, p. 53). The use of oils of more usual grades involves not only excessive resistance and heating of the bearing, with loss of power and speed of the tool, but also break of continuity of the lubricating film owing to the development of excessive negative pressures, and consequent entry of air into the interspace. It is said that in such cases the oil supply, which is commonly by drop-feed, is sometimes deliberately restricted to enable the spindle to run more freely, the oil then forming only local areas of lubricating film, and probably a foam of mixed oil and air, in the interspace (Ref. VI, 2).

Continuous-sleeve bearings are, however, by no means always lightly loaded, their simplicity and the convenience of their production and assembly leading to their use in cases where the maintenance of a continuous lubricating film is quite impossible. As a typical example may be taken the rollers of transmission chains of the roller type (fig. VI, 10). These rollers R, which usually

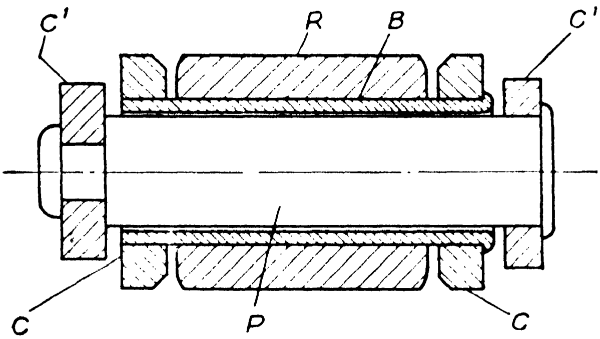


Fig. VI, 10

revolve on hollow journals or bushes B, which themselves are rotatably mounted on hardened and ground pins P fixed rigidly in the side chains CC, are formed, like the journals, of hardened steel and are accurately machined and ground, but the conditions of operation, involving loads of the order of 1000 pounds per square inch of diametral area of the bearings (70 atmospheres) or more, and the slow and intermittent rotation of the rollers on the journals, entirely preclude the maintenance of continuous films or of any fluent films. In practice a thick mineral oil is used as lubricant, of viscosity about 2·0 C.G.S. units at working temperatures. Reference to the figure will show the difficulty of introducing fresh oil to the bearing surfaces to replace the inevitable losses. The surfaces most difficult to reach are, of course, those between the bushes and the pins. In some cases the pins are drilled axially and transversely, and fitted with oil cups at their ends. More usually, however, the chain is run in an enclosed oil bath, or at least within a splash-guard in which it is sprayed with oil from its inner side.

The continuous-sleeve bearing offers a peculiarly favourable field for air lubrication. In this, as in other applications of air lubrication, two conditions in particular have to be complied with. The air must be free from any dust having particle dimensions comparable with the least film thickness; and the materials of the journal and sleeve must be such as to endure temporary rubbing contact at low speeds without detrimental abrasion, especially abrasion of the journal. Since the pressure will necessarily fall below atmospheric in some parts of the film, there will be a continuous exchange of air between these parts of the film and the outer atmosphere, with consequent entrance into the bearing of any dust that may be present in the surroundings. The loads being necessarily light, the air may in this case be regarded as being an incompressible fluid in the film.

4. Divided-sleeve Journal Bearings.

A familar form of divided-sleeve journal bearing is that which is merely a continuous-sleeve bearing of which the sleeve has been manufactured in two parts, and rejoined on a common plane surface before boring to the internal cylindrical form of a continuous sleeve. This mode of construction is usually adopted solely for convenience of assembly of the bearing on a journal which cannot be threaded end-wise into a sleeve formed of a single part. When clamped rigidly and accurately together in a bearing-housing, such split-sleeve bearings have, of course, the same characteristics as the continuous-sleeve bearings discussed in the two preceding sections. The same calculations and formulæ apply to them, and they are subject to the same limitations in practical application as have been therein noted.

More usually, however, the divided sleeve, though it may retain the general form of a continuous sleeve, has its two parts considerably differentiated from one another. If the load to be carried is directed in a substantially constant direction, it is the usual practice to furnish one half only of the bearing with an accurately formed bearing surface, the other being retained merely as a "keeper" to restrict abnormal movements of the journal, or as a means of retaining the lubricant required by the effective half of the bearing. If, in such a half-bearing, the bearing surface is accurately formed over the whole of the effective semi-cylinder it is still, as in the continuous cylinder, a geometrical necessity that it shall have a positive radial clearance δ, with respect to the journal. Then, as before, all possible relations between the bearing surfaces can be represented by appropriate values of δ and an eccentricity coefficient κ between zero and unity. A large, divided-sleeve, journal bearing of this kind is shown in figs VI, 11 (p. 170) and VI, 11*a* (facing p. 181), for which the author is indebted to the courtesy of Messrs. Metropolitan-Vickers Electrical Co., Ltd., of Manchester. In this bearing the divided sleeve is formed as a massive two-part liner for the pedestal block, and both halves of the liner, which are shown in the photographic view (fig. VI, 11*a*), are themselves internally lined with bearing metal. The lower, load-carrying half of the liner is that on the right-hand side of the figure; the upper half, which is primarily a "keep", also serves as an oil-distributing member, having a central hole through its crown through which oil is supplied to the bearing at low pressure. This upper half-sleeve has also two chambers to receive oil rings which encircle the journal and lower half-sleeve, and which are intended to ensure the safe running of the unit for a limited period in the event of a failure of the pressure supply.

The pedestal by which the divided sleeve is supported is shown in the line drawing fig. VI, 11, the pedestal cap being omitted, but the lower half-sleeve being shown in position. On the left-hand side of this view is shown the low-pressure oil pipe which connects through the cap with the hole in the upper half-sleeve already mentioned. Entering through the right-hand side of the pedestal is seen a high-pressure oil pipe which connects with the hole H which

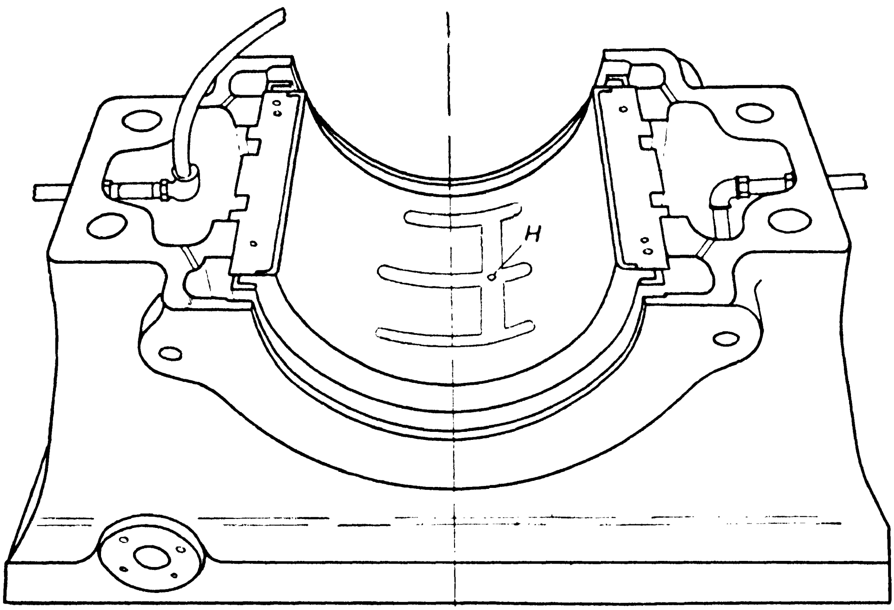


Fig. VI, 11.—Divided-sleeve journal bearing, by Metropolitan-Vickers Electrical Co., Ltd.

appears near the centre of the lower half-sleeve both in this view and in the photographic view. This hole enters a shallow axial groove which is cut in the bearing surface for about three-fourths of its whole width, and which communicates with three similar grooves cut circumferentially in the surface. The purpose of the high-pressure pipe and the system of grooves is to admit oil to the loaded area of the bearing at starting and stopping of the unit. The bearing illustrated being one of four in line, carrying the shafting of a 25,000-kilowatt frequency changer, the adoption of means for reducing the usual static starting resistance of such bearings was obviously desirable. The construction described, however, is open to the criticism that the formation, across so large a portion of the bearing surface, of the grooves for distributing the high-pressure oil, must to a large extent reduce the capacity of the bearing to produce a fluent film during normal running. The objection might have been obviated by the adoption of another form of grooving. Further reference to this point, and to the general subject of supplies of oil under pressure for starting bearings under load, will be made in Sect. VI, 10.

In divided-sleeve semi-cylindrical bearings the flow of lubricant is no longer a two-dimensional circulation as in a continuous sleeve, since, under pressure developed in the film, oil will be discharged at the sides of the sleeve; but, provided that the width-length ratio of the bearing surface is considerably greater than unity, the theory and calculations given in Sect. VI, 2 remain valid as good approximations. The formulæ for the position of the point of maximum pressure ϕ_m, and its amount p_m (equations VI, 9 and 10), and for the thicknesses of the lubricant at all points, remain applicable over the effective semicircle without modification. The formulæ for the resultant of the fluid pressures P, and the tractional couple M, given for the complete cylinder by equations VI, 11 and VI, 12, are of course halved for the half-bearing, but the value given by equation VI, 13 for the coefficient of resistance $k = M/Pr$ remains unaltered.

These formulæ, however, are only applicable to the divided sleeve when the plane of division has the correct inclination to the resultant R of the fluid pressures as determined for the half-cylinder, either by the graphical process described with reference to fig. VI, 8 (p. 164) or otherwise. If, for instance, R is solely the reaction of the bearing to a gravity load of the same magnitude acting on the journal, and consequently acts vertically upwards on the journal, and vertically downwards on the bearing, the limiting plane of the half-bearing must be set at the inclination shown (fig. VI, 8) by the angle between the direction of the force R and the axis OY of the diagram.

When, however, as in the last instance of a gravitational load, the direction of the resultant force on the journal is constant, or nearly so, whether vertical or inclined at any fixed angle, the construction of a divided-sleeve bearing as a semi-cylinder is disadvantageous from the point of view of its efficiency in carrying its load. This is for the reason that (as is immediately seen from the diagrams of fig. VI, 8, or Back Folder VI, 1) the pressures generated in the portions of the film where ϕ is less than, say, 30 degrees, or greater than about 135 degrees, contribute only small components in the direction of the resultant force, while they share largely (especially the former portion) in creating resistance to the rotation of the journal. For this reason it is usual, even when the general form of a half-cylindrical sleeve is retained, to restrict the portion of its surface which is effective, to an arc considerably smaller than π and often not greater than $\frac{1}{2}\pi$ (Ref. VI, 3).

A bearing having a reduced arc of cylindrical bearing surface of this kind is conveniently distinguished by the name of a "partial-sleeve bearing", or "partial journal bearing", reserving the term divided-sleeve bearing for the semi-cylindrical form.

5. Partial Journal Bearings.

The bearing surface of a partial journal bearing arranged for a vertical load is shown in the small perspective view (fig. VI, 12), and a diagrammatic cross-section of a divided-sleeve bearing with a partial bearing surface, also for a vertically directed load, in fig. VI, 13. In the latter figure the upper half-sleeve is merely a keeper, being machined internally with a large clearance from the journal. The effective bearing surface in the lower section of the bearing extends from $\phi = 50°$ to $\phi = 160°$ from the horizontal axial plane, i.e. through 110 degrees of arc.

The graphical method of calculation explained in the last section is readily extended to treat the problems of the partial bearing. In fig. VI, 14, which is a replica on a different scale of the lower half of fig. VI, 7 (i.e. of the pressure locus for $\kappa = 0{\cdot}5$ of Back Folder VI, 1*a*), the curve CBC′ is the pressure locus for a semi-cylinder. If, having regard to

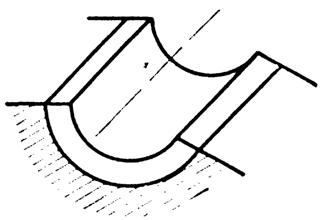


Fig. VI, 12

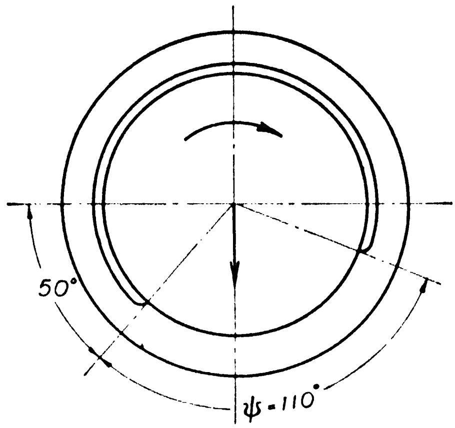


Fig. VI, 13

the degree of variability of direction of a load which is approximately vertical, it is desired to have a partial bearing arc extending through 110 degrees, two points DD′ are found on the curve CBC′ 110 degrees apart from one another, and at equal distances from the circle CYC′ representing the surface of the journal. The fluid pressures at D and D′ are thus equal. It is found by trial that the required points D and D′ are approximately at $\phi = 12°$ and $\phi = 122°$.

The equal pressures indicated by the points D and D′ may be regarded as pressures imposed as boundary conditions at the inflow and outflow edges of a reduced bearing surface extending only through the 110 degrees of arc from D to D′. Deducting from the pressures shown by the curve CDBD′C′ within that arc, the uniform pressure indicated at D and D′, a new pressure locus is given by the radial distances of points on the curve DBD′ from the circle through D and D′. By setting each radial intercept back along its own radial line to the circle of the journal, as in fig. VI, 15, a new pressure locus $D_1B_1D_1'$ is established from which the vertical and horizontal components of the total fluid pressure acting on the 110 degree arc can be derived by the same process as was explained in Section VI, 2 with reference to figs, VI, 7 and 8, and to Back Folder VI, 1.

By the same means the resultant fluid pressure is determined in magnitude and direction. If the direction relative to the vertical line which is so found for the resultant is not that which is desired (indicating an incorrect choice of the line of eccentricity OO′C, in setting out the diagram), adjustment must be made of that line, and consequently of the assumed angular position of the bearing arc, with respect to the vertical.

The diagrams for the various values of κ which are given on Folder VI, 1 enable the calculation to be repeated for conditions differing in that respect, thus showing what extent of variation in any desired direction can be obtained in the various features within the limitations of the problems.

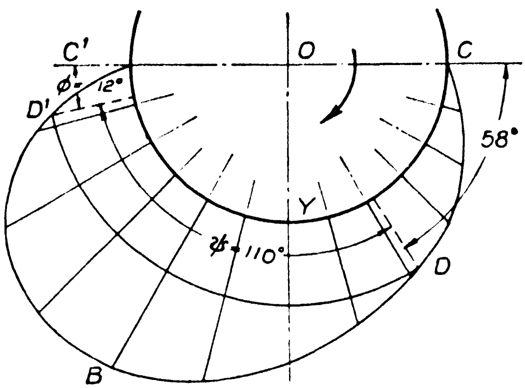


Fig. VI, 14

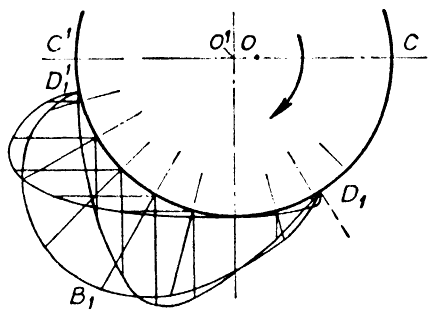


Fig. VI, 15

It will have been observed that, at all stages of the carrying out of the graphical calculations outlined above in this section and in Sect. VI, 2, close contact is kept with the analytical processes on which they are based, so that at any point use can be made of either the one or the other method as may be desired. Thus, for instance, the thicknesses of the films are determined more easily, as well as more accurately, from the analytical formulæ than graphically, as is also the position of the point of maximum pressure and, therefrom, the volume of flow of the lubricant. Determination of the thickness of the film at the outflow edge, where (if the scheme is consistently followed) it is always least, is important as being, just as in plane bearings, a datum for comparison of the relative advantages of different forms on a basis of equal safety of operation. It follows from the nature of the process by which they are obtained, that all forms of partial bearings arrived at by the process are convergent-film bearings from end to end, the possibility of generation of negative pressures being, so to speak, automatically excluded. In general the tendency to generate negative pressures (cancelling positive pressures, with resulting inefficiency or, alternatively, involving local breakdown of the film by vapour generation or entrance of air) is the besetting fault of divided-sleeve bearings designed on any system which does not exclude them.

Any such graphical method as the one described above for the examination of a series of journal bearings, may be applied either for the purpose of selecting one or other of them for a particular practical duty, or for study of their comparative characters and behaviours under a varying range of conditions. In selecting a bearing for any particular application a certain group of dimensions or other numerical features has usually to be accepted as having been fixed by the requirements or by the design of the machine of which the bearing is to form part. Commonly these data include (1) the diameter and width of the journal (the latter at least as an upper limit), (2) the rotative speed, and (3) the amount and approximate direction (or directions, the direction being usually variable) of the resultant load. The total load and the width together determine P (the load per unit width) as it appears in the equations; the diameter and the rotative speed together determine U. There remain at disposal, as variables in the discussion, the machining clearance δ of the bearing with respect to the journal; the eccentricity coefficient κ; the circumferential length of the partial bearing β; and, finally, the coefficient of viscosity of the lubricant μ. Of these, latitude of choice of β is usually restricted, and sometimes eliminated by variability in the direction of the load. Between the other three variables at disposal, viz. δ, κ, μ, one relation is established by the value of P and another by the necessity of compliance with some standard of safety of operation, for which a certain fixed least-thickness of the lubricating film at any point of the bearing is commonly adopted. A third, and the last, relation may be framed to establish some optimum condition as a criterion of merit between the bearings, such as a minimum coefficient of resistance, or a maximum load-carrying capacity, subject to compliance with the other conditions.

Incidentally it may be remarked that the adoption of μ as a variable at disposal is as fully warranted as that of either of the other factors of the bearing design, since lubricants of an almost unlimited range of viscosities are available, any of which can be utilized without difficulties being involved, and, at need, a desired value of μ can often be obtained merely by specification of a change of the operating temperature.

Kingsbury in a well-known paper (Ref. VI, 4) has discussed optima of performance of divided-sleeve journal bearings from a standpoint somewhat different from that of the preceding sections. Proceeding from equations which are essentially the same as those of Sommerfeld, but of a form less convenient for graphical calculation, he discussed them very fully by mainly arithmetical methods. Regarding both the amount and the direction of the load as fixed, together with the journal radius, he treated the circumferential arc subtended by the bearing (ψ of fig. VI, 13), the machining clearance, the eccentricity coefficient, and the parameter μUr (i.e. ψ, δ, μ and μUr of this chapter) as being variables at his disposal. Having thus four variables at his disposal, instead of three, he was led to regard the optimum which he sought as belonging to a class, rather than to an individual, of the complex of bearings considered (as, for instance, to the class having a certain common machining clearance, or that characterized by a certain common position of the load within the arc of the bearing). Since full latitude was allowed in the range of ψ, negative pressures occurred in groups of instances when ψ was large. Kingsbury, recognizing that actual conditions do not in general admit of the existence of negative, or at any rate of sub-atmospheric, and still less of sub-zero pressures, was obliged to make a separate examination to eliminate, or to reclassify, these cases. A somewhat similar elimination of many bearings having low values of ψ would be necessary, to accord with the usual practical condition that the direction of the load is variable.

A summary of Kingsbury's and related work on the subject, both theoretical and experimental, may be found in Ref. VI, 5.

6. Coincident or "Fitted" Journal Bearings.

In the whole of the discussion of sleeve and partial journal bearings in the preceding sections it has been assumed that the surface of the bearing member is of greater radius than the journal, so that in the expression for the film thickness, $h = \delta - \epsilon \cos \phi$ (equation VI, 1), δ is always positive and finite. In complete sleeve bearings and in semi-cylindrical bearings this is necessarily the case, but in partial bearings, a zero, or even a negative value, of δ is admissible, and in either of the latter cases ϵ becomes necessarily greater than δ, instead of less than it. The general formula for the film thickness then becomes $h = \epsilon \cos \phi - \delta'$, where δ' is the excess of the radius of the journal over the radius of the partial bearing surface. It is then clear, both from this formula and from inspection of fig. VI, 16, that the greatest angular length of bearing which is consistent with the formation of an interspace converging throughout its length is $\frac{1}{2}\pi$, and that this is only possible in the case of $\delta' = 0$, when $h = \epsilon \cos \phi$, or, as it may be more conveniently expressed, changing the origin of the angular measurement by $\frac{1}{2}\pi$, $h = \epsilon \sin \xi$. If δ' is finite, the greatest possible length of a continuously converging film is less than $\frac{1}{2}\pi$, and diminishes as the numerical value of δ' increases.

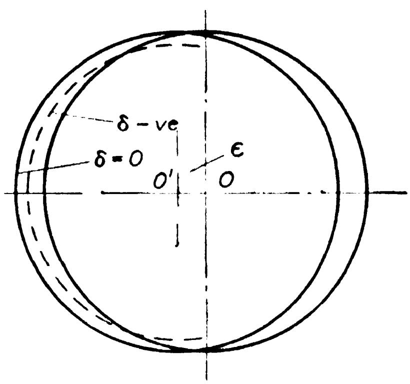


Fig. VI, 16

A partial bearing in which δ' (or δ) is zero, i.e. one in which the radii of the journal and the bearing surface are equal, may be called a "coincident bearing". Sometimes the term "fitted bearing" is used. A bearing of this kind presents the obvious, and very considerable, practical advantage in manufacture, that verification of the correctness of the radius of curvature of the bearing member does not depend on the accuracy of the difficult measurement of the small difference between the radii of a convex shaft and a concave bearing, but can be very readily checked by the direct application of the one part to the other, and, if necessary, corrected by means which are available in every workshop.

The coincident journal bearing also shares with the plane thrust bearing, and other plane bearings, the valuable characteristic that the mutual contact of the bearing members in the absence of lubricant (e.g. during assembly, or transport) is a surface contact, and not, as in other journal bearings, a line contact. Risks of damage to the surfaces are thus obviated. Another important consequence of the same characteristic is that a brief period of running with deficient lubrication (as during the starting and stopping of the rotation of the shaft), does not (provided a correct choice is made of materials) injure the surfaces, but rather tends to improve their condition. Evidence of this self-correcting behaviour in the case of thrust bearings has been quoted in Section V, 4. In journal bearings not of the coincident class, it is, on the other hand, well known that while "running in" with a given location of load usually improves the condition of the bearing for running with the load in that particular location, the bearing will no longer run satisfactorily if the position is changed, or the direction of rotation reversed.

Partial journal bearings in which the radius of the bearing member is smaller than that of the journal appear to offer no advantages either of a theoretical or practical kind, and no further reference will be made to such a possible type.

Although coincident journal bearings can be subjected to calculation by the same mathematical method as bearings in which δ is finite and positive, a simpler method of dealing with them is available which is sufficiently accurate for most practical calculations. The possibility of using this method depends on the fact that the coincident bearing is limited in its application to arc lengths of less than 90 degrees. Within this range of bearing lengths the same method can be used equally for other partial bearings instead of the exact analytical formulæ.

TABLE VI, 2

VARIATION OF FILM THICKNESS ALONG THE LENGTH OF A COINCIDENT, OR "FITTED", JOURNAL BEARING
(Outflow Thickness One-Half of Inflow Thickness)

Fraction of length of bearing	Length of bearing arc			Linear variation
	60°	45°	30°	
0	·500	·500	·500	·500
$\frac{1}{8}$	·609	·587	·572	·563
$\frac{1}{4}$	·707	·667	·643	·625
$\frac{3}{8}$	·793	·742	·710	·688
$\frac{1}{2}$	·866	·810	·774	·750
$\frac{5}{8}$	·924	·872	·837	·813
$\frac{3}{4}$	·966	·923	·894	·875
$\frac{7}{8}$	·991	·965	·946	·938
1·0	1	1	1	1

The approximate method in question consists in treating the film thickness in a bearing of small angular length as varying linearly along the length, i.e. in replacing $\sin \xi$ by ξ in the formula $h = \epsilon \sin \xi$. The attached Table VI, 2 shows in parallel columns the comparative values of the film thickness at 8 intervals in the lengths of coincident surfaces, extending respectively through 60, 45, and 30 degrees of arc, and at corresponding intervals in the length of a bearing in which the thickness varies uniformly. In each instance, the case is taken of a bearing in which the thickness at the outflow end is one-half of that at the inflow end, the latter thickness being the unit in each column.

The departures from the linear law are hardly large enough even in the case of the 60-degree bearing to introduce errors of material importance, and are probably of less consequence in most cases than the departures from actual

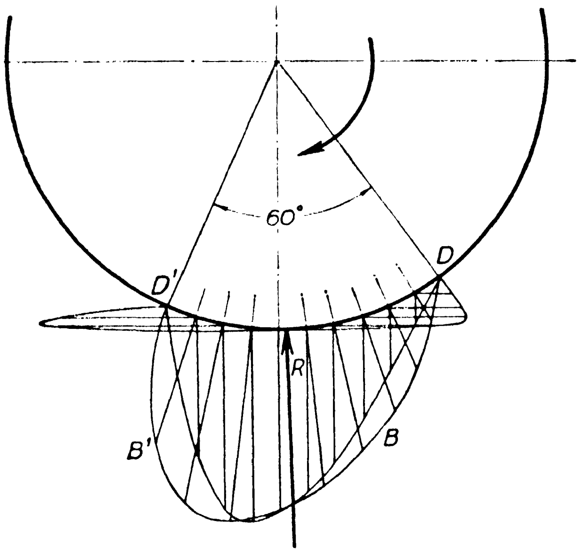


Fig. VI, 17

conditions involved in the assumption that the bearing is of indefinitely great width.

The lubricating film, having been assumed to increase in thickness linearly along its length, is then indistinguishable for purposes of calculation from a plane wedge-film of unlimited width having the same rate of convergence, and the same ratio of inflow to outflow thickness.

All the theoretical results obtained for plane bearings (both of unlimited and of finite width) in Chapters IV and V are therefore applicable, as approximations, to the coincident partial bearing, provided that account is taken of the curvature of the surfaces in dealing with the resultant forces on the journal and bearing, which can readily be done by the graphical methods explained in Sects. VI, 4 and VI, 5. Thus in fig. VI, 17 the total pressures acting on each tenth of the length of a square plane bearing (these total pressures having been taken from the three-dimensional graph in fig. IV, 14, p. 99) have been transferred to the corresponding segments of a 60-degree partial journal bearing to give the curve of radial pressures DBB'D'. These pressures, having been resolved by the same graphical process as was explained in connexion with fig. VI, 8, give the position and magnitude of the resultant R as indicated by the arrow.

7. Multiple-pad and Pivoted-pad Journal Bearings.

A natural development from the partial journal bearing, described at the end of the last section, is the provision of more than one such bearing surface arranged within a common bearing housing or pedestal. Such a construction indeed becomes necessary when the direction of the loading is variable through any considerable angle, and is definitely advantageous, as compared with a single partial bearing of greater circumferential length, or with a semi-cylindrical bearing, when the load alternates between two directions for each of which a suitable pad surface can be located. Such a condition occurs, for instance, in the bearings of a 3-high rolling train which may be engaged from either side, as diagrammatically indicated in fig. VI, 18, each of the rollers rotating continuously in one direction. Obvious difficulties of execution, however, limit the applications of this type of construction, and it may be said that four is the greatest number of fixed pads that can be combined as a rigid combination in a single bearing housing. When, as is commonly the case, the direction of a varying load changes continuously from one position to another, the construction is plainly unsuitable.

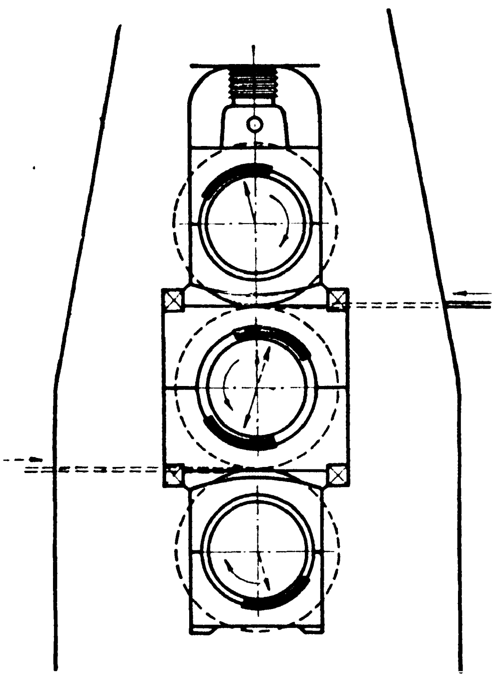


Fig. VI, 18

In this situation the analogy of the thrust bearing immediately suggests the use of pivoted, partial, bearing pads, separately mounted in the bearing housing, so enabling any desired number of bearing surfaces to be provided in practically any required positions, and at the same time ensuring that the charge on each of the surfaces shall be so located with respect to its length as to obtain the optimum degree of convergence of the lubricating film. Early examples of this type of construction (Refs. VI, 6–8), as illustrated in fig. VI, 19, had usually only a small number of pivoted, or tilting, pad members; the tendency has been to increase the number and to reduce the circumferential lengths of the individual pads. The correct position of the axis of pivoting for any desired rate of convergence of the film, or ratio of outflow to inflow film thickness, is of course given in any particular case by a diagram corresponding to that of fig. VI, 17, but when the angular length of the pad is not greater than 45 degrees it is sufficiently accurate to adopt the same position, relatively to the length of the pad, as in a plane pivoted bearing of the same convergence.

The multiple-pad type of bearing is particularly suitable for use for the propeller shafts and tunnel shafts of marine vessels. On account of the unavoidable straining of a ship in rough waters, exact alinement of a series of bearings cannot be preserved, and the line of shafting, in conforming to the varying

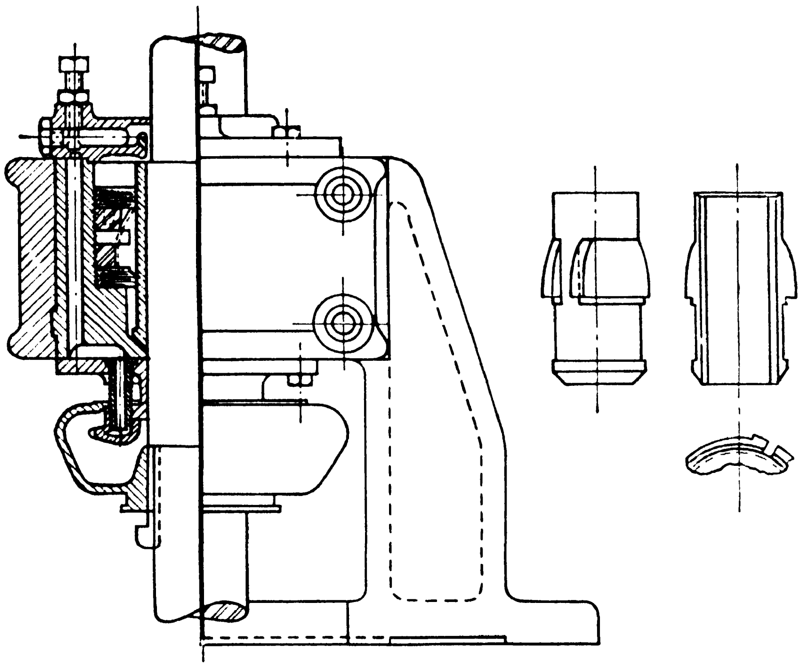


Fig. VI, 19

positions of the axes of the bearings, imposes loads upon them which may act in any radial direction. In practice, all marine multiple-pad journal bearings are constructed with pivoted pads of the coincident surface type.

Fig. VI, 20 shows a propeller shaft, or tunnel shaft, journal bearing of the standard form of construction adopted by Michell Bearings Limited (to whom the author is indebted for permission to reproduce the illustration), for fitting in merchant vessels, in which journal bearings of this type are almost exclusively used. The pivots of the tilting pads are offset to the extent required to form normal, convergent, lubricating films when the propeller and shaft are revolving in the " ahead " direction, the pivots being formed integrally with the pads and directly supported on a cylindrical seating turned in the housing of the bearing. " Astern " rotation of the shaft, which, of course, usually takes place only for comparatively short periods of time, is found to cause no appreciable abrasion of the pads or journal, although it is accompanied by a relatively high coefficient of resistance.

The larger bearings of this type, which is used for marine propeller shafts up to 24 inches (60 centimetres) in diameter, are provided with water circulation for cooling, the water being passed, as indicated in the right-hand view of fig. VI, 20, through a chamber cored out of the lower half of the bearing housing.

In the same half of the housing are also formed two intercommunicating oil chambers, one on each side of the water chamber. The oil is lifted from one of these oil chambers by a circular oil-lifter revolving with the shaft, and is caught on an oil distributing tray in the upper half of the housing from which it is

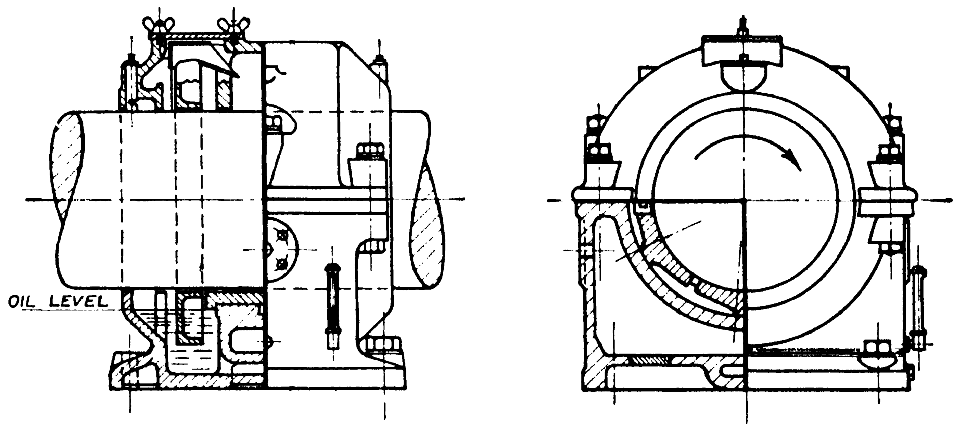


Fig. VI, 20.—Pivoted-pad marine journal bearing, by Michell Bearings Limited

fed by gravity on to the journal at the mid-width of the bearing, and, after supplying the pads, falls back into the oil chambers. An oil deflector at each end of the journal prevents outflow of oil along the shaft.

When fitted at locations along the length of the propeller shafting where no upward deflection of the shaft can occur when the ship is strained in rough water, the bearing pads are omitted from the upper semi-circumference of the bearing, only the three lower ones indicated in the right-hand view of the figure being then used.

In marine bearings of this class conventional practice limits the intensity of loading on the bearing surfaces to relatively low figures. Crushing of the pivoting surfaces of the pads, and of the corresponding seatings of the housings, is therefore not usually of serious extent. When higher intensities of loading are adopted, the construction shown in fig. VI, 21, in which the pivoted pads are supported on accurately turned liners, fitted in the bearing housing, has the advantage that both bearing pivots and liners may then be formed of hard, wear-resisting metals.

The same condition of unpredictability of the direction of the load on the

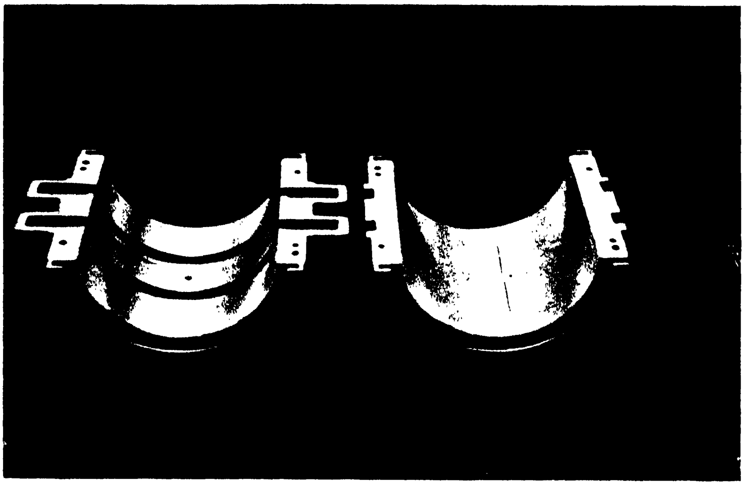


Fig. VI, 11*a*.—Divided-sleeve journal bearing by Metropolitan-Vickers Electrical Co., Ltd. (See p. 169)

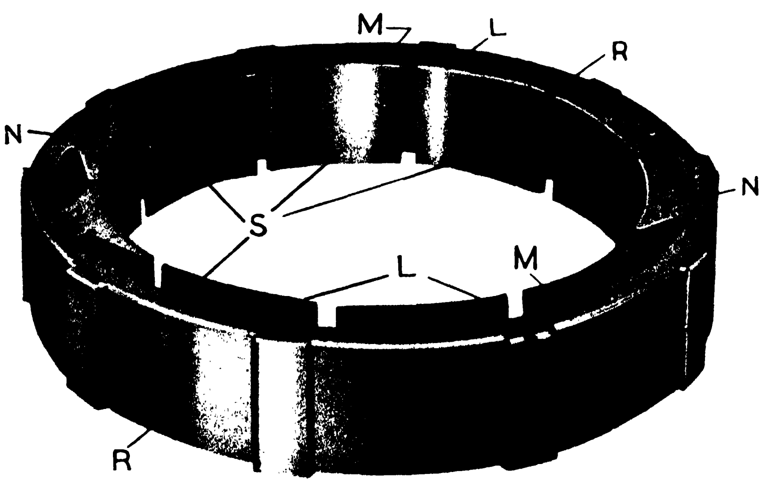


Fig. VI, 22

journals as is brought about by straining in a sea-way often occurs in vertical shafting, especially when it is in long lengths. In this case the loading is usually light, and the conditions may often be satisfactorily met by multiple-pad bearings in which the pads are elastically pivoted on an annular member of which they form integral parts. An example of such a ring is shown in fig. VI, 22 in which the individual pads S, each having a longer limb L and a shorter limb M, are seen to be attached to the ring R by the flexible necks N, the whole being cast in one piece. The pad bearing surfaces, being coated with a soft bearing metal, can be finished with great accuracy by boring with a diamond tool to a common cylindrical form of very slightly larger radius than the journal. Professor Tenot (Ref. IV, 21) has published tests of a bearing of this kind having six 45-degree pads each 8 cm. × 6 cm. of cast iron, uncoated. Loads up to 5 metric tons were carried, with a surface speed of about 2×10^3 centimetres per second, a typical result at this speed showing a specific mean pressure of 111 kg. weight per cm.^{2}, with a coefficient of resistance of $1{\cdot}87 \times 10^{-3}$. The tests were run without cooling. The results are fully analysed and referred to theory in the report cited.

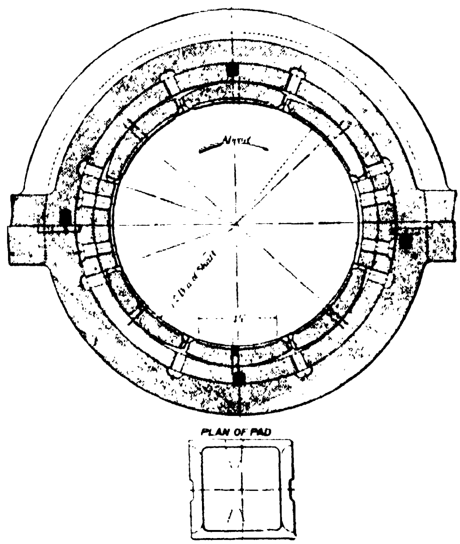


Fig. VI, 21

Pivoted multiple-pad bearings are especially appropriate for use when displacements of the journal from its central position have to be minimized, and lateral vibration or oscillation of the journal within the bearings prevented. Thus they are very advantageous for application to the spindles of high-speed machine tools.

As the journal revolves within the annular series of pads of such a bearing, pressures are, of course, generated not only between it and the pad. or pads, which may be in the line of the loading, but between the journal and each of the other pads. If, as in the case of the spindle of a machine tool which is running idle, or of an accurately alined vertical shaft, or the propeller shaft of a ship running in smooth waters (when the bearings carry only the comparatively light loading due to the weight of the shaft itself), there is only a negligible resultant load on the journal, the fluid pressures on all the pads are nearly equal and the journal is in equilibrium when running almost concentrically within them, as shown diagrammatically in fig. VI, 23a. When the shaft becomes loaded, the journal is displaced laterally within the ring to a small extent, the film thicknesses becoming smaller on the pads in one half-circle, and larger on those in the other semicircle, as indicated in fig. VI, 23*b* (Ref. VI, 9). The resultant of the fluid pressures increases very rapidly with displacement of the journal from the central position according to the formula

$$P = C\mu U \frac{r^2}{\delta^2} \cdot \frac{\kappa}{(1-\kappa^2)^{\frac{1}{2}}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{VI, 15}$$

in which δ is the initially uniform radial clearance between the journal and the bearing surfaces; κ is the ratio of the eccentricity ϵ to δ; and C is a constant for the particular bearing, involving the number and characteristics of the pads, which are assumed to be eight or more in number. The analogy of form between the right-hand side of this equation and equation VI, 11, which gives the resultant fluid force in a continuous-sleeve bearing will be immediately apparent.

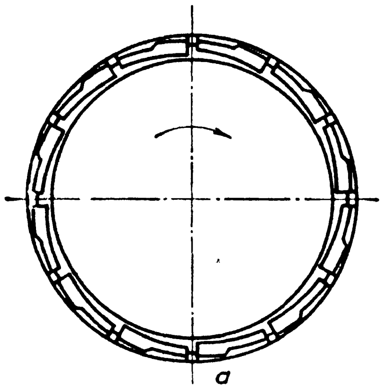


Fig. VI, 23*a*

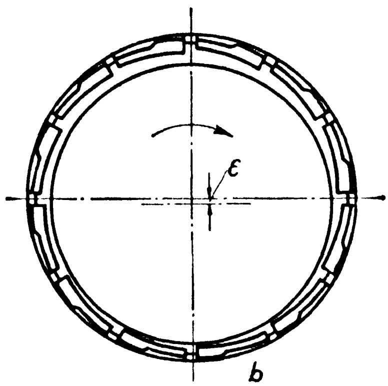


Fig. VI, 23*b*

An illustration of an example of the use of multiple pivoted pads in the bearings of high-speed grinding spindles, which is owed to the courtesy of Messrs. Cincinnati Milling and Grinding Machines, Incorporated, is given in fig. VI, 24. The pivoted shoes, or pads, are of steel lined with a surface of lead bronze, the spindle being of chrome-nickel steel. Of the five pads used in this instance, two are pivoted on the ends of pins rigidly fixed in the solid housing of the bearing and the other three on pins which are adjustable in the housing in a radial direction, and capable of being locked in the positions necessary to give any desired clearances between the working surfaces of the pads and the journal. It is stated that by means of this adjustment, and the automatically self-centring property of the multiple convergent-film pads, the movement of the spindle under a change of load is only a minute fraction of that which occurs in other types of bearings used for the purpose. The distinguishing feature of the firm's construction is the means by which the chamber containing the bearings of the spindle is maintained full of oil at a controlled pressure and free from entrained air.

Pivoted multi-pad bearings offer an advantage over other forms of journal bearings of the sliding type when a shaft is required to rotate, under load, in either direction at will. It has been already pointed out that sleeve bearings and partial bearings which have been " run in " for one direction of loading become incapable of maintaining a fluent film when run in the opposite direction, unless the direction of the load is widely different in the two cases. If in a multi-pad bearing having a sufficient number of pads, individually of relatively small circumferential length, alternate pads are of reversed formation with respect to the positions of their pivots, one half of the number will form fluent films with either direction of rotation, the others remaining inert, generating no fluid pressures and offering comparatively little resistance to the rotation of the journal. A diagrammatic illustration is given in fig. VI, 25.

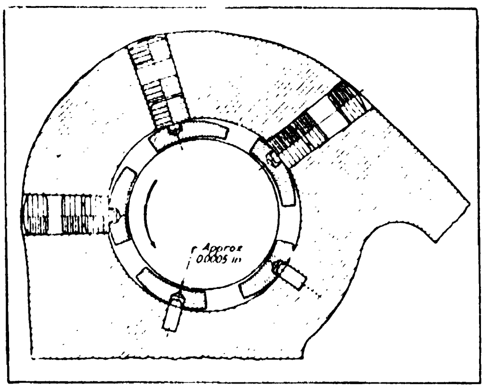


Fig. VI, 24

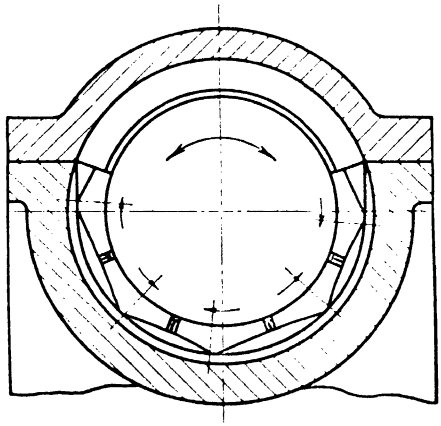


Fig. VI, 25

A distinct type of multiple-pad bearing recently introduced by the author and his associate Mr. A. J. Seggel, is known as the " floating pad bearing ". An example is shown in the photographic illustration fig. VI, 26 (facing p. 184), and details of the characteristic pad in figs. VI, 27*a*, *b*, and *c* (Ref. VI, 16).

8. Internal Storage and Circulation of Lubricants in Journal Bearings.

With the exception of continuous-sleeve bearings (q.v., Sect. VI, 3), all fluent-film journal bearings require to be provided with means for replenishing and circulating the lubricant which enables them to operate. When the machine of which the bearing forms a part runs only intermittently, as is the case, for example, with most machine tools, the bearing may be fed during the period in which the machine is at work from a reservoir fixed at a higher level than the bearing itself, from which the lubricant is allowed to flow under gravity to an unloaded sector of the journal surface, either through a tube, or, if the reservoir is constructed as a part of the machine, through a cored or drilled passage. Many familiar forms of oil reservoirs are in use, with an equal number of devices for controlling the flow from them. Since, after passing through the bearing by flow past its lateral edges, the lubricant runs to waste, the practice of this method of supply is to be avoided unless the resulting waste and grime are trivial. Both may, of course, be avoided by the provision of a suitable receptacle below the bearing, with suitable ducts leading to it; but such provision is rarely made. When a fluent-film bearing runs continuously

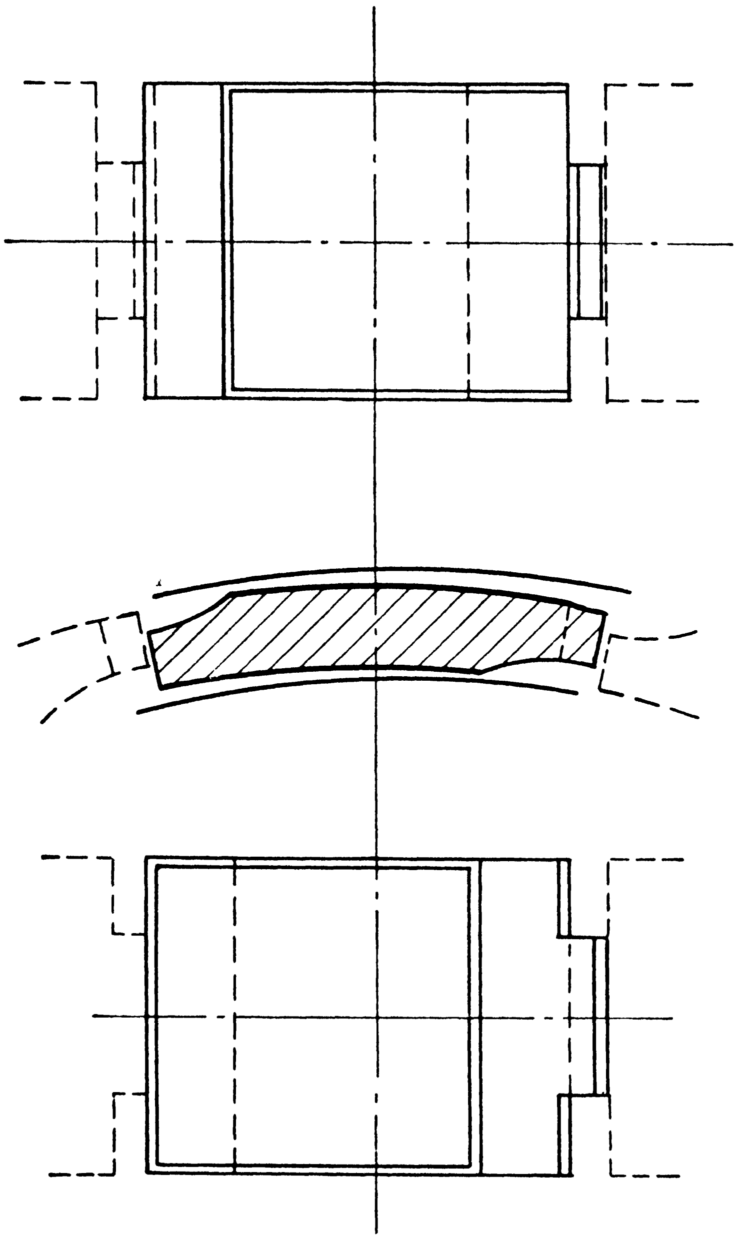


Fig. VI, 27.—Details of floating pads of bearing shown in fig. VI, 26

for a period of several hours, the volume of oil which it requires is so great that storage in a feeding reservoir, or in a waste receptacle, is usually impracticable and, consequently, some means of circulation must be provided. A rough calculation of the quantities involved will make the situation apparent.

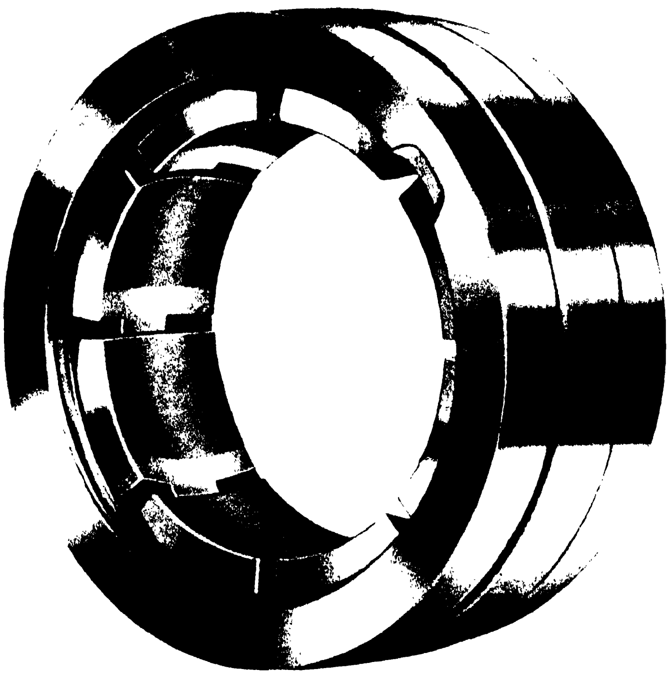


Fig. VI, 26.—Floating-pad journal bearing

Taking a journal of radius r and a width-radius ratio of 2 (the "width" being in the case of a multiple-pad bearing, the sum of the widths of the pads), running at angular speed ω with a film thickness at the point, or points, of pure shear having the lower than ordinary value of $h_m = r \times 10^{-4}$, the volume-rate of inflow under the pads of the bearing, if all are fully supplied, will be approximately

$$Q = \omega r \times 2r \times \tfrac{1}{2}h_m = \omega r^3 \times 10^{-4}.$$

Of this quantity in ordinary cases about one-fifth will be lost by discharge from the sides of the bearing, so that (taking the rotative speed as 1500 r.p.m., or roughly ω as 5×10^5 per hour) the loss will be of the order of $\frac{1}{5} \times 50 \times r^3$, or $10r^3$ per hour, in cubes of whatever unit of length the measurement be made in.

In the case of a bearing of 3 inches diameter this corresponds to a flow of approximately 0·15 gallons per hour, which evidently represents quantities inconveniently large for storage in the adjuncts of an ordinary bearing.

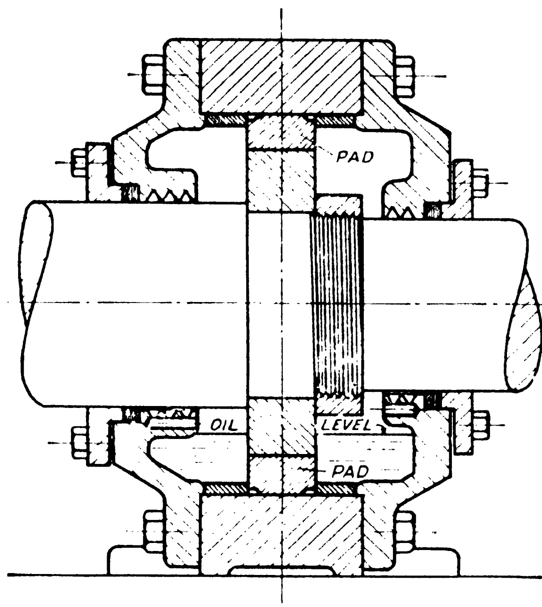


Fig. VI, 28

Submergence of the journal itself in a bath of lubricant is usually impracticable on account of the difficulties attaching to the making of effective seals where the shaft issues from a bearing housing. This difficulty can, however, be overcome by forming the journal surface on an enlargement of the shaft, and this is a satisfactory and efficient arrangement when a multi-pad bearing is in question, the separate pads leaving the portions of the journal between them exposed to contact with the lubricant, as seen in fig. VI, 28. A similar arrangement is usually not practicable in the case of a partial bearing, on account of the great enlargement of diameter which it would be necessary to give to the journal in order to expose any portion of its surface to the lubricant, and it would evidently be quite impracticable in the case of a semi-cylindrical bearing.

Failing submergence of the journal, the simplest and most familiar means for circulating oil between a reservoir and the operating surfaces of a bearing is an oil ring hung from the journal and dipping below the bearing, which it surrounds, into the oil in the reservoir. The ring is caused to revolve, and in doing so to lift the oil from the reservoir to the bearing by its frictional contact with the rotating journal.

A typical example is illustrated in fig. VI, 29. At low speeds the oiling ring, by the effect of its own weight, makes such intimate contact with the journal surface that it is driven as if by solid-to-solid friction, without any relative slip that can be observed. It collects from the reservoir an adhering film of oil which it lifts on its rising semi-circumference, and which flows back along the ring only slowly on account of its viscosity. Calculation of the quantity lifted to the level of the journal can be made on the same principles as, for instance, the calculation in Sect. V, 5 of the quantity of oil raised on a thrust collar rotating in a vertical plane. Of the oil which reaches the journal a large portion will be deposited on its upper surface by direct drainage from the inner surface and sides of the oil ring, but usually a still greater quantity will be carried over on the surface of the ring back to the oil reservoir. The quantity of oil lifted can be increased by grooving the inner surface of the ring, and by this means also the upper limit of the speed at which the ring is driven by virtual solid-to-solid contact with the journal, without appreciable slip, is also increased.

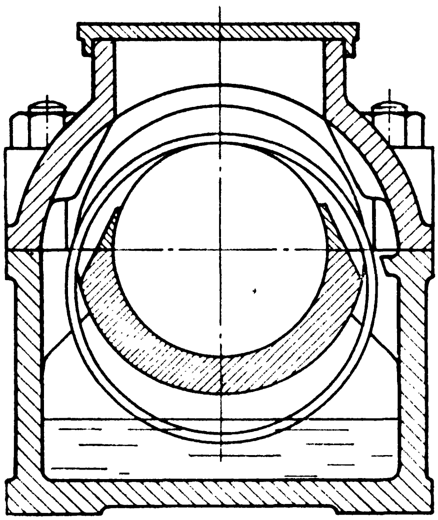


Fig. VI, 29

If the journal is run at higher speeds the quantity of oil raised on the ring, and the thickness of the layer on its inner surface, ultimately becomes so great that the rugosities of the surfaces no longer make contact in the reduced time that is available for them to penetrate the oil layer. The ring is then driven solely by viscous traction, and its speed tends to fall below that due to solid contact. Over a certain range of speeds the motion of the ring is apt to be unstable, alternating between conditions of solid propulsion with increased work done in lifting the oil, and fluid propulsion with lowered speed and less useful effect. Since the viscous traction increases with journal speed that regime, however, can usually be made stable by the adoption of suitable dimensions and weight of the ring, relatively to the diameter and speed of the journal and to the viscosity and desired quantity of the oil.

The resistance to the rotation of the ring which arises from the fluid friction of the body of oil through which it passes in the reservoir, will probably change from pure viscous friction, proportional to the ring speed, to turbulent resistance, roughly proportional to the square of the speed, at some stage of the increase of journal velocity. The ratio of ring speed to journal speed will therefore probably pass, during the acceleration of the machine, through all the stages shown in the diagram of fig. VI, 30. From the starting condition at O the ring is driven at increasing speed by solid-to-solid friction, and therefore at a fixed ratio to the speed of the journal, up to the point A at which solid contact fails and is replaced intermittently by viscous friction, with a varying speed ratio until a journal speed corresponding to the point B is attained; from B to C the ring speed increases with viscous propulsion, but the speed ratio falls progressively; at C turbulent resistance is encountered by the portion of the ring immersed in the reservoir, and the ring speed then hardly rises any further, increasing slip taking place relatively to the journal.

At high speeds the greater part of the quantity of oil lifted by the ring is thrown off from its outer edges near the highest part of its circumference in the forms of streams of spray. In order to take full advantage of the lifting capacity of the ring, this oil should be caught by suitable deflectors attached to the cover of the bearing and caused to flow downwards on to an exposed part of the journal surface.

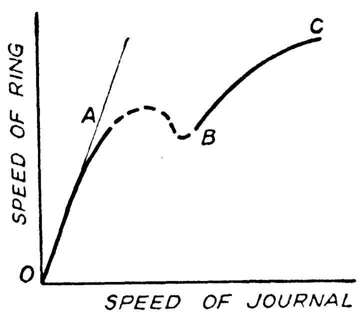


Fig. VI, 30

Usually it is possible to lift by means of one or two rings a greater volume of oil than is required for fluent lubrication of the bearing, and, when external cooling of a bearing is necessary, often sufficient to transfer the heat developed in the bearing to cooling coils suitably arranged in the oil reservoir, with only a small rise of the temperature of the oil above that of the cooling water.

The chief objection to the use of oiling rings is that their action is liable to interruption by the accidental introduction of small obstructions into the ring chamber. The form of the gap in the bearing in which the ring runs requires careful attention, and it is sometimes necessary to fit guiding pieces for the ring to keep it in its correct plane of rotation. Such guides are most effective if they make contact with the ring on its rising side. The ring should be in one piece of metal and accurately turned. Steel, hard brass, and hard bronze are the materials usually preferred. A fuller discussion of the subject, with examples of calculations and tests of the running of oil rings, may be found in Ref. VI, 10 and the papers cited therein.

Sometimes, especially in large bearings, chains of the transmission type are used in preference to rings, being propelled in the same way by frictional contact with the journal. Their advantages consist in their raising, at a given journal speed, larger quantities of oil than rings, and in their requiring a smaller width in the housing in the horizontal direction transverse to the axis of the shaft. A conspicuous instance, however, of the use of oil rings in bearings of large dimensions has been mentioned in Sect. VI, 4 (figs. VI, 11, p. 170 and 11*a*).

When more positive means than a friction-driven ring or chain are desired for circulating the lubricant between an oil reservoir and a journal bearing, it is usual to have recourse to a worm-driven pump. The pump, which may be of either the gear-wheel type, or of centrifugal type, according to the speed conditions, is preferably submerged in the oil reservoir, the impelling wheel being fixed on a vertical or inclined spindle which is fitted at its upper end with a worm-wheel engaging with a worm fixed on the machine shaft at one or other side of the journal. Except for the method of driving, the arrangement is the same in principle as that described in Sect. V, 4, as being used for lubricating thrust bearings, and need not to be further discussed. Reference may also be made to Section VII, 16.

9. Journal Bearings for Rotating Axles of Vehicles.

The lubrication of the journal bearings of the rotating axles of vehicles presents several peculiarities and is subject to special difficulties in achieving the formation of fluent films. The most obvious of these difficulties arises from the fact that the load imposed on the bearing by the journal acts upwardly. Consequently, unless the aperture by which the journal issues from the axle-box can be effectively sealed against loss of oil standing higher than the lower surface of the journal (and sealing in this way is found to be almost impracticable under the usual conditions of operation), special means must be employed to raise the lubricant carried in the axle box to the journal.

A second peculiarity of the application consists in the bearing being usually, and almost necessarily, spring-connected to the body of the vehicle. In ordinary practice this condition involves that the bearing-box itself must contain the whole quantity of lubricant required for at least one journey, together with the means for supplying the lubricant to the journal surface.

Of still greater importance is the difficulty, inherent in the application, of dealing with the frictional conditions of bearings of the sliding type at starting and stopping, this characteristic being of more consequence in vehicular bearings than in others, for the reason that the motive power employed in a traction enterprise is mainly devoted to overcoming tractive resistances. Consequently, a factor which increases the maximum tractive effort (as does starting friction), increases also the size, power, and cost of the propelling units, and, through them, those of the whole system. The traction industry is perhaps the only industry in which the capital cost is determined mainly by bearing friction.

The bearings of railway and tramway vehicles present also another feature which, while not wholly peculiar to them, is not very commonly met with in other applications, namely that the journal is required to run with equal frequency, and equally well, in both directions. Still another circumstance which increases the practical difficulties of the problem is that the bearings are required to do much more than to support a steady vertical load; accelerations and retardations of the vehicle cause varying forces in a horizontal direction, not to speak of the irregularities of tracks, such as points and crossings, which produce shock-loads at all angular positions between the horizontal and vertical. All these conditions are further complicated in the driving axles of locomotives by the horizontal loading due to the action of connecting rods and coupling rods (Ref. VI, 11).

Fig. VI, 31 illustrates by means of a form of bogie axle-box, of a construction formerly general but now becoming obsolete, how the various difficulties above enumerated have been met in conventional practice. The journal J is carried by the " brass " B, which is fixed rigidly in the axle-box. The brass B carries the load when acting in an approximately vertical direction, as in normal running, but when the load is deflected widely from the vertical, as in passing over points and rail joints, the journal makes direct contact with surfaces D and D′, provided on the bearing box itself, beyond the ends of the " brass ". The lubricant, whether grease or oil, is contained in a pan C, which is arranged below the journal in such a way that it can be easily removed for cleaning and refilling. The means for raising the lubricant from the pan to the journal surface consists of a pad P of porous material, together with a wick W, attached to the pad and partially or wholly submerged in the lubricant. Both pad and wick are supported, and the pad held in contact with the lower surface of the journal, by springs S.

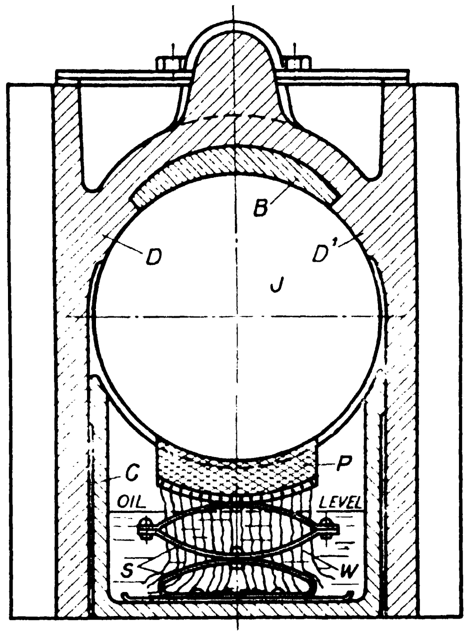


Fig. VI, 31

In the following discussion of the action of a bearing of this kind, it will be assumed that the lubricant is in liquid form, being either oil, or grease melted by the heat generated in the running of the bearing. When grease is used, the same kind of deliquescence takes place while the bearing is standing as is described in Sect. VII, 16 as occurring in grease-lubricated rolling bearings, so that a small amount of lubricant is present on the journal surface at starting. Under running conditions, the lubricant, whether melted grease or oil, is lifted to the journal continuously by the capillary action of the wick and pad. It is readily seen, however, that the quantity of lubricant which can be supplied by this means must always be inadequate for effective fluent lubrication.

An approximate calculation, on the same lines as that given in Sect. VI, 8 (p. 185) for a machine-tool bearing, will show that, even in small railway bearings, such as those used for the bogies of passenger cars, or for small freight trucks, the flow of lubricant necessary to make up the loss at the sides of the journal, under fluent-film conditions, is of the order of one cubic centimetre per second. The following approximate calculations show that a porous pad, or similar device depending on capillary action, and sufficiently compact to be contained in a railway axle-box, is incapable of raising viscous liquids in quantities of such an order.

By equation II, 27, the flow of liquid through a tube of bore-radius a, and length l, lifting to a height h, is given by

$$Q = \frac{\pi a^4}{8\mu l}(p_1 - p_2 - gmh),$$

in which $p_1 - p_2$ is the fall of pressure in the length of the tube, and g, m and μ have their usual significations.

Also, from equation II, 19, the fall of pressure corresponding to a capillary tension τ at the upper end of the tube, of which the lower end is submerged, is given by

$$p_1 - p_2 = \frac{2\tau}{a},$$

so that

$$Q = \frac{\pi a^4}{8\mu l}\left(\frac{2\tau}{a} - gmh\right). \quad \ldots\ldots \quad \text{VI, 16}$$

For the purposes of the calculation a porous pad may be regarded as being made up of a very large number of capillary tubes conveying the fluid, and it is evident that the most favourable assumptions that can be made as to the effectiveness of a bundle of such tubes of a given gross cross-sectional area $\mathfrak{A}$ (including the walls of the tubes and ineffective spaces between them in the section), are that the tubes are all vertical, and thus of the minimum length required for their action, and all of equal circular section.

Suppose that there are N tubes in the bundle, and that the total area of their bores $N\pi a^2$ is a certain fraction of the horizontal sectional area of the pad, so that

$$N\pi a^2 = k\mathfrak{A},$$

and, from equation VI, 16, the total flow

$$\Sigma Q = NQ = \frac{N\pi a^4}{8\mu l}\left(\frac{2\tau}{a} - gmh\right)$$

$$= \frac{k\mathfrak{A}}{8\mu l}(2\tau a - gmha^2). \quad \ldots\ldots \quad \text{VI, 17}$$

Differentiation of this expression with respect to a determines the bore of the tubes which will give the maximum discharge for given values of k, Ω and the other fixed quantities, the radius of this bore being thus found to be

$$a = \frac{\tau}{gmh}.$$

Inserting this value of a in equation VI, 17, the maximum possible flow is

$$\Sigma Q = \frac{k\Omega\tau^2}{8\mu l}\left(\frac{2}{gmh} - \frac{1}{gmh}\right)$$

$$= \frac{k\Omega\tau^2}{8\mu lgmh}. \qquad \text{VI, 18}$$

It is hard to imagine even an artificial construction (if such were practicable) in which k (the ratio of the total area of the effective capillary pores to the sectional area of the bundle), could be made greater than about one-tenth; in the natural materials which are commonly used, the equivalent constant must be much smaller still. Taking then $k = 0{\cdot}1$, l and h as 10 cm. and 5 cm. respectively, Ω as 100 cm.^{2}, τ as 30 dyne cm.$^{-1}$, m as 0·9, and μ as 0·5 poises, the maximum quantity of lubricant that can be lifted per second is found to be, by equation VI, 18,

$$\Sigma Q = \frac{10^{-1} \times 10^2 \times (30)^2}{8 \times 5 \times 10^{-1} \times 10 \times 9{\cdot}8 \times 10^2 \times 9 \times 10^{-1} \times 5}$$

$$= \frac{1}{19{\cdot}6} \text{ cm.}^3 \text{ sec.}^{-1},$$

or about 1/20th of the flow required for full fluent lubrication.

Various designs and proposals have been put forward and tested with the object of attaining fluent lubrication under the conditions which obtain in the bearings of vehicles, but most of these leave much room for further advance, especially with respect to simplicity of construction. One of the most successful hitherto is the Peyinghaus axle-box illustrated in fig. VI, 32 (p. 192). According to this construction an oil lifter is attached to the end of the journal, and, revolving with the journal, lifts oil from the reservoir in the bottom of the box into channels formed in the upper side of the bearing. From these channels the oil drips through ports P on to the surfaces of the journal exposed at each end of the brass. A supply of oil amply sufficient to form a fluent lubricating film is thus provided, in whichever direction the axle may be revolving.

In addition to the usual upper brass, which carries the load, an under brass U is fitted, normally carrying no load and standing clear of the journal, but so close to it as to retain a reserve of oil, held by capillarity between the two surfaces, while the vehicle is stationary. Oil from this source is carried immediately to the upper brass during the first revolution of the axle. The under brass, being supported by contact with the box on both sides, also serves to resist horizontal forces due to impact of the wheels with rail joints or crossing points, or to shocks during shunting operations. A seal is provided around the journal at its entry into the axle-box, which, though not capable of retaining fluid which is under pressure, or standing at any appreciable static head above the under surface of the journal, prevents loss of oil splashed inside the box, and is even said to be effective in retaining the oil in the axle-boxes when wagons are inverted in tipplers.

As will be seen from fig. VI, 32, the under brass U is provided with means of adjustment which enables its inner surface to be brought as close as may be desired to the surface of the journal. Its use, in the manner mentioned,

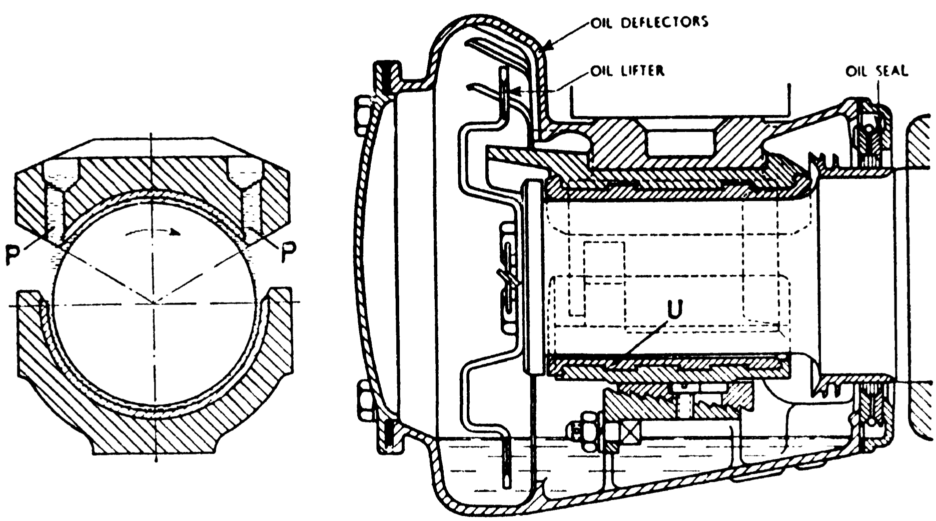


Fig. VI, 32.—Peyinghaus railway axle-box

for carrying horizontal shock-loads necessitates its being of equally robust construction as the main brass, and also involves the making of the lower half of the axle-box of similar thickness and strength to the upper half. The mass of the whole box thus becomes considerable, and being unsprung from the axle, adds to the severity of impacts between wheels and rails. (For a fuller description of the Peyinghaus bearing, see Ref. VI, 12.)

Although the Peyinghaus, and other improved forms of axle bearings, enable fluent-film lubrication of the journal to be established within a very short time after the beginning of rotation of the journal, they do not reduce the high starting friction which is the chief defect of the sliding bearing as applied to vehicle bearings. The only means which has been successfully used for overcoming this difficulty is the direct supply of oil at high pressure to the area of contact between journal and pad at the moment of starting. To facilitate the introduction of the oil between the brass and the journal, the surface of the former may be grooved somewhat in the same manner as in the large journal bearings of the electric phase-changer described in Sect. VI, 4. Preferably, however, only a single groove should be cut, this having, in a bearing running in only one direction, as nearly as possible the shape of one of the isobars of the convergent film formed when the bearing is in normal operation. The groove has then no disturbing effect on the normal formation of the pressure film. In bearings whose journals rotate alternately in each direction, the series of isobars on the bearing surfaces for each direction being reversed at the same time, the groove must be cut approximately midway between a pair of isobars of equal pressures for each direction. In either case an injection of oil at any point of such a groove, when the journal is stationary, puts the central portion of the bearing surface, surrounded by the groove, at the pressure of the supply, and the remainder of the bearing surface at approximately half that pressure.

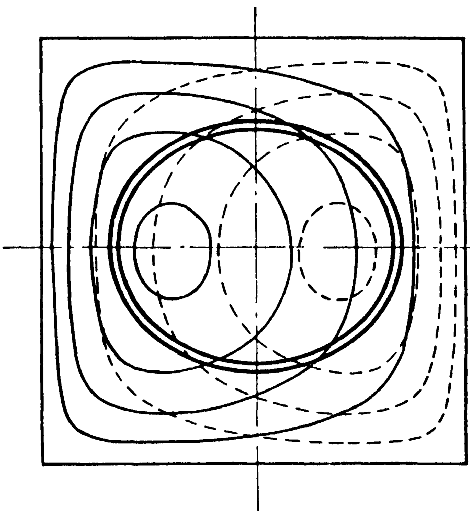


Fig. VI, 33

Fig. VI, 33 shows the contour for a groove in a square bearing, compromising in the manner suggested above for the change of position of the isobars corresponding to change in direction of running, it being assumed in plotting these isobars that the location of the axle with respect to the fixed brass changes with the direction of running to the same extent as if the brass were pivoted on a pivot normally situated for each direction.

The beneficial effect of high-pressure injection of oil at starting has been proved to be very marked (Ref. VI, 13). The coefficient of resistance at starting may be reduced to less than one per cent of that which is usual when no such device is employed. In the tests of rail vehicles published in the reference above, the high-pressure supply of oil was given by means of a pump delivering at from 100 to 200 atmospheres. A much lower pressure is sufficient for the purpose, provided that the delivery takes place into an isobaric groove correctly located in the manner already explained.

As an alternative to the use of a mechanically driven pump, a high-pressure supply of oil for starting may be given by an automatic lubricating apparatus of the kind illustrated in fig. VI, 34, which shows all the parts of this automatic apparatus on a scale much larger than that of the bearing. The brass of the bearing is formed with a small port (5), communicating with an isobaric groove, in the same way as for injection from a high-pressure pump. The port (5) is in constant communication by the tube (7) with the smaller chamber (8) of an accumulator located at a lower level than the bearing, and having also a large chamber (15). This larger chamber is closed by a diaphragm which is loaded by the spring (16). The chamber (8) is fitted with a plunger (10) attached at its outer end to the centre of the diaphragm. A rotary valve (20) alternately opens or closes communication between a by-pass from the port (5) to the large chamber of the accumulator, the by-pass from the port being fitted with a non-

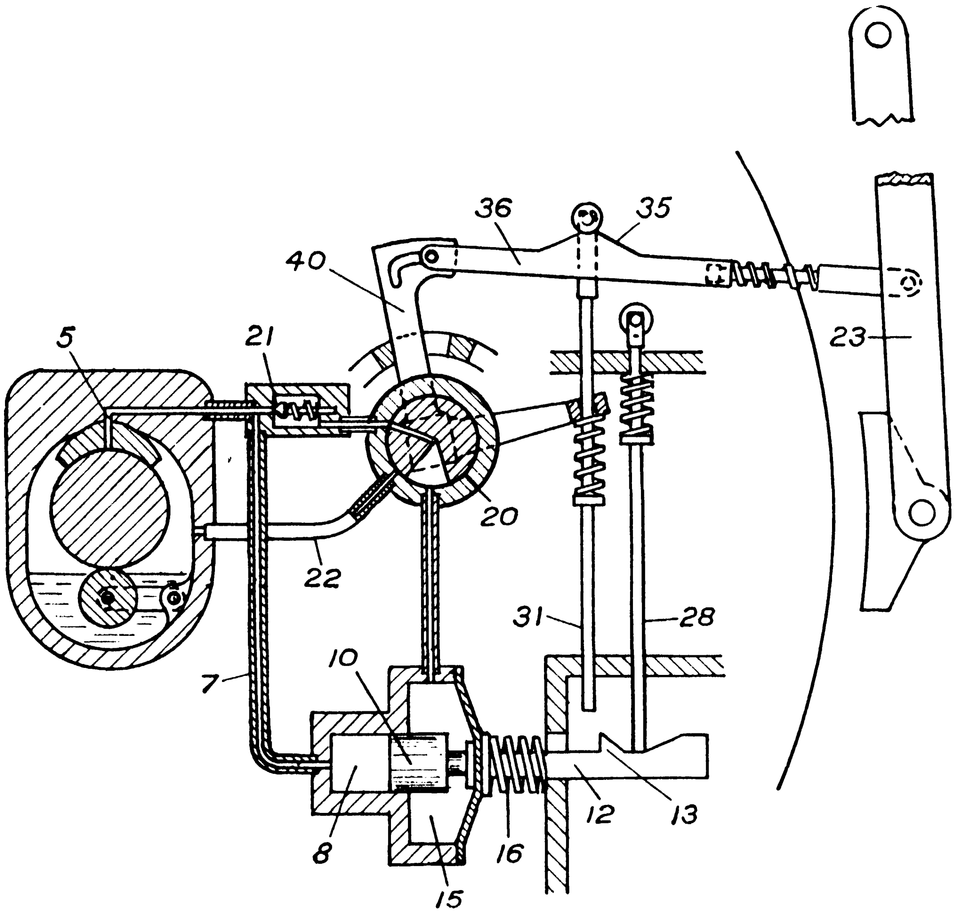


Fig. VI, 34

return valve (21). The rotary valve (20) is actuated through a lever (40) and rod (36) by the brake lever (23) of the vehicle, which also controls, by means of tappet rods (28) and (31) co-acting with cam faces on the plunger rod (12), the movements of the accumulator plunger (10).

The figure shows the relative positions of the parts during the normal running of the vehicle, the brake being raised off the wheel and the valve (20) in a position which allows oil, flowing at high pressure through the port (5) from the fluent lubricating film in the bearing, to pass into the large chamber (15) of the accumulator. The accumulator is thus fully charged during the running of the vehicle, and the spring (16) compressed to its limit. When the brake is applied to stop the vehicle, the rod (36) moves inward, and its ramp (35) allows the tappet (31) to drop and engage tooth (13) on the plunger of the accumulator, which is thus retained in the charged position while the vehicle slows down and is stationary. Pressure is then, of course, no longer generated at the port (5), and the pressure in both chambers of the accumulator falls to the atmospheric value.

On the brake being again raised, preparatory to a fresh start of the vehicle, the ramp (35) lifts the rod (31) clear of the tooth (13), and at the same time moves the valve (20) into such a position that the port leading from it to the non-return valve (21) is closed, and the oil contents of the large chamber (15) of the accumulator are free to be discharged through the pipe (22) to the reservoir of the axle-box. The plunger (10) of the accumulator then carries the full force of the spring (16) and, moving inward, delivers the oil contained in the small chamber (8) into the port (5) and isobaric groove at high pressure, thus forcing the journal clear of the brass by the interposition of a film of oil between the surfaces.

By the use of flexible piping a single set of the apparatus, consisting of accumulator, rotary valve, cam rods and tappets, can be arranged to serve all the bearings of a vehicle; a non-return valve, such as 21, being fitted on the pipe connecting each bearing with the common rotary valve of the set. (For a fuller description of the apparatus, and alternative forms for various purposes, see Ref. VI, 14.)

10. Journal Bearings Lubricated with Water.

The use of water as lubricant has been traditional for a long period in small and simple hydraulic machines, especially those having vertical spindles with footstep bearings and guide bearings most conveniently located below water level. The ease with which the water could be applied to bearings in such positions, or rather the difficulty of excluding it, no doubt prompted its use in the first instance, but, provided it was free from grit and sediment, it proved to be a very efficient, as well as inexpensive, lubricant.

Since water has a coefficient of viscosity roughly only one-hundredth of that of ordinary lubricating oils, and is the best of cooling agents, its use as the lubricant of accurately formed bearings of suitable materials is likely to become more extensive in the future than in the past, especially with high speeds. Although it is equally advantageous in thrust bearings and in journal bearings, present practice, and recent advances in its use have been chiefly in connexion with the latter.

The bearing material most commonly used in the older applications of water lubrication was wood, but in some cases fine-grained stone; the journal or pivot was usually of either bronze or cast iron. In one of the older applications which has survived, and the only one of them which has been developed on a large scale, the same materials are still employed with notable simplicity and efficiency.

Back Folder VI, 2, reduced from drawings for which the author is indebted to the courtesy of Messrs. John Brown & Co., Ltd., shows the stern-tube bearing of one of the 18-inch propeller shafts of a large vessel commissioned in 1942. The shaft, which revolves at 120 r.p.m., is fitted for the whole length (99 in. or 250 cm.) with a bronze tubular liner $\frac{15}{16}$ in. or 2·4 cm. thick; and the fixed stern-tube, within which the shaft revolves, is lined with strips of lignum vitæ, $\frac{7}{8}$ in. (2·3 cm.) in radial thickness and about $3\frac{1}{4}$ in. (7·25 cm.) long in the circumferential direction. In the lower 120 degrees of the circumference of the bearing (within which the load is normally carried), the wooden blocks are arranged with their grain radial, presenting sections transverse to their axes of growth as the bearing surfaces, but in the remainder of the circumference of the bearing the direction of the grain of the blocks is circumferential. The reason for this difference is that the dimensions of the wood in the direction of the grain, being much less variable with changes of moisture content and temperature than its dimensions in other directions, the blocks maintain more nearly constant forms of surface towards the shaft if the grain is radial than they otherwise would; while in the usually unloaded portion of the bearing, they are less likely to cause trouble by deforming the sleeve in which they are held, if the grain is circumferential.

The water which is the lubricating fluid of the bearing is taken from an inboard supply in order to ensure its clarity. After passing through the bearing, in the clearance space between the liner of the shaft and the lignum vitæ strips, and between the chamfered edges of the adjacent blocks shown in the figures, the water is discharged into the sea, at the aft end of the tube. In smaller examples of the same type of construction, sea-water may be admitted directly into the bearing at its outer end, but this practice is subject to the danger that, in shallow waters, fine sand may enter with the water.

When sea-water, or other clean water containing only saline ingredients in appreciable quantities, is used for bearing lubrication, one of the bearing members must, in practice, be of non-metallic material. The films being thin, due to the low viscosity of water, are unavoidably subject to penetration by the small protuberances of the bearing surfaces, and the small local weldings that would result in metals would be liable to extend on account of the complete volatility of the very small mass of the adjacent water films. With wood, or like materials, similar contacts may occur, with local carbonization, but the disintegration of the particular protuberance affected prevents damage to the journal.

Of all the materials which have been generally used with water lubrication, wood appears to admit of the lowest coefficient of friction. It is probable that the mode of lubricated contact is usually, in practice, at the best, that of rugulose film formation, except when the wood or other non-metallic material is used as the lining of a pivoted pad. Present standards of workshop accuracy are scarcely high enough to provide, for a lubricant of so low viscosity as water, the necessary fine, continuously convergent interspaces in fixed pads. It seems probable that the apparent superiority of wood may be due to the alternation in its structure of harder and softer layers adapted to present a succession of small convergent-divergent interspaces admitting of partial film action. If so, it should be possible to obtain similar action by appropriate modes of lamination in synthetic materials.

For most present-day applications of water lubrication, some bearing material which, unlike wood, is capable of being moulded into approximate shape in the metal housing which forms its support, is generally preferred. Such an application, which has become widely established, is that to the journals (" necks ") of heavily loaded rolling-mills. The bearing material in this case is usually a synthetic resin bonded with a cotton fabric, the journal being of non-corrodible steel. The plastic material is most frequently used in the form of a liner in a metal bearing-shell, in somewhat the same manner as a bearing metal is applied to a shell for oil lubrication, except that the plastic liner is very much thicker. The part of the bearing surface which carries the load (see fig. VI, 17, p. 177) is formed as a " partial " bearing having a total angle ψ (see Sect. VI, 5) of 60 to 120 degrees, according to circumstances.

As in metal bearings applied to rolling-mills using oil or grease lubrication, the intensities of pressure commonly used are high, being frequently of the order of 200 atmospheres (3000 lb. in.$^{-2}$), and sometimes much higher still. The surface speed may be 500 cm. sec.$^{-1}$ (1000 ft. min.$^{-1}$) or more, but varies greatly in different cases according to the kind of product for which the rolls are used. The coefficient of resistance appears to be usually of the order of 0·005, but sometimes as low as 0·0025. In spite of this moderately low coefficient, the rate at which heat is generated at the bearing surface is high, being of the order of 10 gramme calories per second per square centimetre (15 British Thermal Units per square inch per minute), and the flow of water must be in great excess over the quantity which is required to form a lubricating film, in order to hold the temperature of the roll necks within tolerable limits. The water is commonly applied in the form of jets impinging on the necks where their surfaces are exposed at each end of the partial bearings, at the empirically determined rate of flow of about 30 cubic centimetres per second per square centimetre of surface, or 3 gallons per minute per square inch. The plastic material being a poor conductor, practically all the heat must be removed from the journal surfaces.

A factor which, in the use of plastic bearing materials, has a quite different order of importance from that which holds in metallic bearings, is the deformation of the material under load. The value of Young's modulus, which in tin and usual bearing alloys is of the order of 5×10^{11} C.G.S. units (7×10^6 lb. in.$^{-2}$), is in an average sample of reinforced synthetic resin only about $3{\cdot}5 \times 10^{10}$ C.G.S. units (5×10^5 lb. in.$^{-2}$). Under bearing loads of the intensity mentioned above as being usual, a plastic liner 2 centimetres ($\frac{3}{4}$ inch) thick must therefore be expected to be directly compressed by the load by more than 10^{-2} centimetre (4 one-thousandths of an inch).

From this consideration alone it is evident that plastic bearings cannot be given the forms necessary for the formation of fluent film with a lubricant of so low a viscosity as water by any machining operations. While the coefficients of resistance actually found in the use of these materials, including lignum vitæ and rubber, are low compared with those associated with mitigated solid friction in metals, they are of higher order (viz. from 10 to 100 times as high) as would be formed with fluent, convergent, water lubrication at the loads and speeds in question.

Realization of the full advantages of water lubrication with such high intensities of loading as are unavoidable in the bearings of rolling mills, evidently involves the attainment of a precision of manufacture which is hardly practicable at present, as well as the use of materials less easily deformed than synthetic resin; it also calls for the strictest observance of the conditions of operation which have been discussed in connexion with thrust bearings in Sect. V, 4.

A liquid near to water in the values of its constants of viscosity and surface tension and, in fact, consisting mainly of water, has been used, experimentally at least, for the lubrication of metal bearings in steam turbines. The purpose in this case has been, not so much higher efficiency, or improved mechanical performance, as avoidance of the fire risk which attaches to the use of oils in close juxtaposition to pipes or casings containing steam at high pressure, and especially superheated steam.

In experiments reported by Samuelson (Ref. VI, 15), the lubricant was formed by diluting one part of a mixture of refined, mineral, lubricating oil (50 per cent), dry soap (28 per cent), free fatty acids (6 per cent), with an antiseptic and contained moisture (16 per cent) with 50 parts of water. The viscosity of the resulting fluid was approximately 0·02 poises at 15° C., at which temperature the viscosity of water is 0·0114 poises.

Tests were carried out with this lubricant on a bearing having a journal 15 cm. (6 in.) in diameter, the partial bearing-bush being 23 cm. (9 in.) wide, and subtending an angle of 100 degrees. The bearing was lined with a "white metal", and bored to a radius 0·0115 cm. (0·0045 in.) greater than the radius of the journal. The loads were light, varying from 4 to 10 atmospheres (60 to 150 lb. in.$^{-2}$), and the journal was run at speeds varying from 3000 to 6000 r.p.m., corresponding to sliding speeds of 2350 to 4700 cm. sec.$^{-1}$

The lubricant was circulated solely by the viscous-film action of the bearing itself. A load of 8 atmospheres (118 lb. in.$^{-2}$) could be carried for an indefinite period with a surface speed of 3200 cm. sec.$^{-1}$ (6300 ft. min.$^{-1}$), the surfaces remaining in good condition and the coefficient of resistance being about one-half of that found in comparative tests of the bearing with oil lubrication.

In this case, again, it is seen that the frictional loss, though low, was much higher than would be the case if it had been possible to construct the bearing accurately to the dimensions required for the formation of a correctly converging film in a fluid of viscosity approximating to that of water.

CHAPTER VII

Rolling Bearings

1. General.

The beneficial effects, of several kinds, which have been enumerated in Chapter I as resulting from the use of lubricants in machines in general, are obtained in the various types of rolling bearings, though by no means so obviously, nor to the same extent, nor in the same manner, as in sliding bearings. All practical experience of rolling bearings proves that lubrication is necessary in them, not so much for the sake of mechanical efficiency, as for reliable operation and reasonable durability. In these respects lubrication is essential to the existence of rolling bearings. While, however, in sliding bearings the lubricant appears as the very life-blood of the organism, in rolling bearings it can be looked upon rather as a kind of medicament, extraneous though vitally necessary.

Several years before Osborne Reynolds discovered the principle of the action of lubricants in sliding bearings, he had investigated the nature of rolling motion and rolling resistances (Ref. VII, 1). His principal experiments were made on a heavy roller of cast iron, ground accurately circular in section and polished. It was arranged to roll on the equally true and smooth plane surface of a cast iron plate which could be laid horizontal or slightly inclined. This roller was found to roll under gravity continuously over the plate when the latter was given an inclination of as little as 1 or 2 parts in 5000, that is with a coefficient of resistance of 2×10^{-4} to 4×10^{-4}. Under these conditions the application of none of the usual kinds of lubricant made any apparent change in the resistance to rolling. The conditions were varied in many respects, but without any conclusive evidence being obtained as to the origin of the small residual resistance, or the reason for its independence of lubrication. Reynolds, however, considered that the resistance arose from the friction between small portions of the roller and the plane having relative sliding motions in the immediate neighbourhood of the ideal line of their mutual contact, where, owing to the difference of their curvatures, the roller and the plane were differently deformed by their mutual pressure.

Later experimenters (Ref. VII, 2) have investigated the rolling together of two cylinders of equal diameters and of the same material. It would seem that, in this latter case, the deformations of the two bodies should be exactly alike and that there should be no sliding, relatively to one another, of the elements in mutual contact. Nevertheless, it is found that a definite resistance to rolling still occurs, and that the application of a lubricant is not directly effective in eliminating the rolling resistance, nor in preventing entirely the abrasion which invariably takes place on the surfaces of such rollers when their mutual pressure is at all considerable. Further reference to these experiments will be made in Sect. VII, 13.

It is not the purpose of this chapter to describe or discuss in detail the many various forms of rolling bearings, but a brief description of a few typical forms, emphasizing the features of each which are most significant from the point of view of lubrication, is necessary in order to clarify further discussion. For descriptions of many other types, and an account of the whole subject of rolling bearings and their uses, the reader may consult Ref. VII, 3.

2. Some Typical Forms of Rolling Bearings.

The types selected for illustration and brief description as an introduction to this chapter are:

A. The roller bearing with cylindrical rollers.

B. The so-called *spherical roller bearing* (the roller path on the outer race being a zone of a sphere), of the type having two rows of rollers.

C. The *deep-groove* type of ball radial-bearing, with a single row of balls.

D. The *thrust bearing*, i.e. an axially loaded bearing.

3. Cylindrical Roller Bearings.

The type of radial roller bearing (Type "A") which is illustrated in fig. VII, 1 is the simplest of all forms of rolling bearings. It consists essentially of an inner member or "race" (which may be either a hardened-steel ring fitted on the shaft of the machine, or, in some cases, may be the shaft journal itself), an annular series of cylindrical rollers, and an outer race whose inner cylindrical surface forms the outer path of the rollers. The inner and outer races are provided with lateral flanges which restrain the rollers, and through them the inner race and the shaft, from being displaced in the axial direction relatively to the outer race. These flanges, which have only a small clearance from the plane ends of the rollers, also serve to prevent any serious degree of axial misalinement of the rollers with respect to the axis of the shaft. The rollers are also guided, and spaced from one another around the circumference, by being fitted into an annular "cage" of simple and light construction.

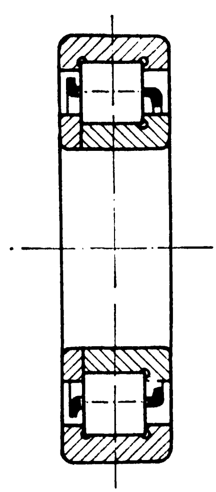


Fig. VII, 1

The surfaces of the inner and outer races which form the rolling-paths of the rollers are ground accurately cylindrical, as are also the surfaces of the rollers, except when as a refinement the end portions of the profiles of the rollers are slightly reduced in diameter in order to concentrate the loads between the rollers and races on their middle portions rather than on their relatively weak ends. Care has to be taken in the assembly of the individual bearings that all the rollers of the bearing are accurately of the same diameter, and that the inner and outer races are matched together so that the total radial movement possible between the inner and the outer race is held between very narrow limits. The separate parts of the bearings are therefore not interchangeable between one bearing and another; and this remark applies generally to all types of rolling bearings which are designed and manufactured to the higher standards.

It will be seen that the conditions of operation of this simple type of rolling bearing conform very closely to those of Osborne Reynolds' experimental roller described in Sect. VII, 1, with the exception that, in the bearing, the paths on which the roller runs are not plane but cylindrical, one concave and the other convex. Apart from the contacts between the rollers and their cages, and occasional rubbing contacts between the flat ends of the rollers and the flanges of the races (these contacts all involving only secondary and minor forces), the motion of the rollers is one of pure rolling. The stresses involved in the regions of rolling contact, and the functions of lubrication in such cases, will be considered in detail in Sects. VII, 8–13.

4. Spherical Roller Bearings.

The roller bearings known under the name "spherical roller bearings" (Type "B" of Sect. VII, 2) partake to some extent of the characters of both roller bearings and ball bearings. The rollers, of which, in the form selected for illustration in fig. VII, 2, there are two annular series or rows, are short frusta of conoids having circular generating lines convex outwards, the diameter of the generating circle being much larger than the diameter of the roller. The rollers are barrel-shaped, with one end of larger diameter than the other. The outer race of the bearing has a spherical inner surface of slightly greater radius than the generating circle of the rollers, so that as the rollers run on the race they make contact with it only on linear circumferential tracks, on either side of which there is a clearance gradually increasing in each axial direction. The inner race has two roller paths each of which is a conoidal surface formed by a circular generating line, concave outwards, of the same radius as the spherical surface of the outer race.

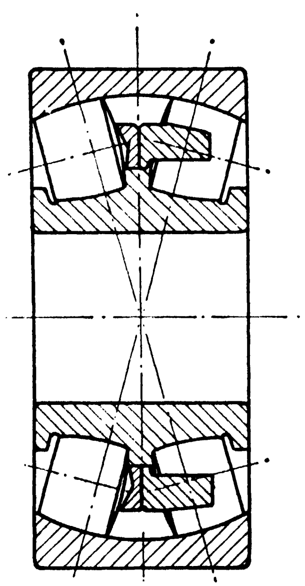


Fig. VII, 2

A central peripheral flange is formed around the inner race between its two roller paths. Both sides of this flange and the inner end surfaces of the rollers are spherical and of the same radius, the centres of the spheres being at the points where the axes of the rollers intersect the axis of the bearing. Consequently, the end surfaces of the rollers and the lateral surfaces of the flange are adapted to make full surface contact together; and the location and inclination of the axes of the rollers are such that both the bearing loads and the centrifugal forces tend to keep the rollers in forcible contact with the flange. At the outer ends of the inner race it is provided with two other flanges, but the only purpose of these is to retain the rollers approximately in place while the bearing is being assembled or dismantled.

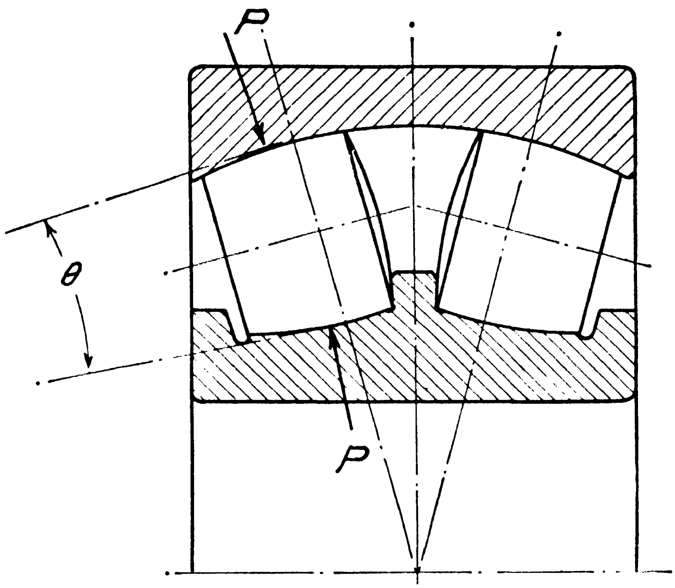


Fig. VII, 3

In this form of roller bearing, unlike that described in the last section, a component of the load on the bearing is transmitted from the rollers to surfaces (viz. those of the central flange) where the mutual contact is of the nature of sliding, or rubbing, rather than of rolling. The amount of the force involved (see fig. VII, 3) for each roller in this rubbing contact is $P \tan \theta$, where P is the resultant load on the roller at each of its points of rolling contact, and θ is the angle at the vertex of the cone tangential to the curved surface of the roller at its circle of rolling contact.

In many other forms of roller bearings besides that shown in figs. VII, 2 and 3, similar means of guiding and controlling the rollers by surface contacts carrying components of the major bearing loads are employed. In some of these cases it has been suggested that it is possible to secure, in some degree at least, lubrication of the fluent-film type, by giving slightly different radii of curvature to the parts in mutual sliding contact. In most cases, however, the contacting surfaces are parallel to each other, and only partial film action, such as that discussed in Sects. IV, 16–18, can be attained. This question is more fully considered in Sect. VII, 14.

5. Deep-Grooved Ball Bearings.

The "deep-grooved" ball bearing (Type "C" of Sect. VII, 2) may have one, or more than one, series of balls, the bearing shown in figs. VII, 4 and 5 having one series only. In this form of ball bearing the race-ways which form the rolling-paths of the balls on the inner and outer races are of circular axial sections, the radii of the circular profiles being each only about 3 per cent greater than the spherical radius of the ball. These race-ways are of depths approximating to one-half of the radius of the ball. In consequence of this form, and of these bearings being usually assembled with an appreciable amount of radial clearance, they are capable of withstanding end-wise loading of amounts comparable with their safe radial loading. When such loads are carried the actual paths of contact of the balls and races are no longer in the middle plane of the bearing, but on each race the path is displaced slightly, and in opposite directions, from the central position assumed with purely radial loading. In the latter case only is the action one of pure rolling; when any axially directed load is carried, the ball has a spinning motion about the radius through its point of contact with each race, compounded with its rolling motion. As in other rolling bearings, the balls of a bearing of this type are fitted in a cage, which partakes of their revolution and keeps them duly spaced apart.

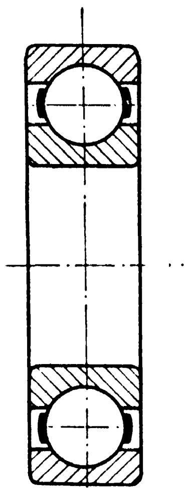


Fig. VII, 4

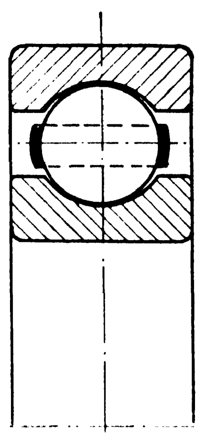


Fig. VII, 5

6. Ball Thrust Bearings.

The simplest form of rolling bearing designed for carrying loading in an axial direction, commonly referred to as "thrust-loading", is the one-direction ball bearing shown in fig. VII, 6. In this type of bearing a single row of balls, set in a cage, runs in two similar grooves formed in the stationary race S and the revolving race T, the latter being fixed on the shaft of the machine. The grooves are usually of less depth than those in the deep-groove type of radial ball bearing but, like them, are formed with a radius of curvature in the direction transverse to the motion of the balls, only slightly greater than the radii of the balls. These bearings are not adapted to carry any appreciable radial loading, but radial loading of each individual ball arises from its own centrifugal force when revolving in its track; at high speeds these forces may be considerable, relatively to the thrust load.

It is evident from the form of the bearing that it is especially important in this class of bearing that all the balls in the row be very accurately equal in diameter, since, assuming the bearing to be accurately constructed in other respects, and apart from deformations under the load, any ball which was larger than the others would carry the total load during the whole revolution.

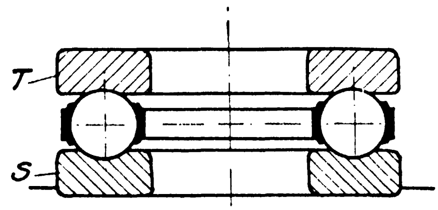


Fig. VII, 6

7. Spinning and Slipping of Balls.

Examination in greater detail of the kinematic operation of this type of ball-thrust bearing brings to light in an extreme form conditions which may arise in all rolling bearings except purely radial types, such as the radial type described in Sect. VII, 3. From figs. VII, 6 and 7, it is apparent that the ball can only have a pure rolling motion if the tangent lines at the two points of contact of the ball with the races intersect on the axis of the bearing as shown in fig. VII, 7. This condition, however, is not one of equilibrium, and it would necessarily revert to the position of lower potential energy of the thrust load shown in fig. VII, 6, which is not one of pure rolling but involves spinning of the ball around the line joining the points of contact, in addition to its rolling motion. From consideration of the different circumferential velocities of a point on the ball and a point on the race (both subtending at the centre of the ball the same small angular distance from the point of contact), the angular velocity of spinning of the ball with respect to each race is easily shown to be $\frac{1}{2}\Omega$ (where Ω is the angular velocity of the shaft), on the assumption that the relative spinning is the same at both points of contact. In this particular case of diametrically opposite contact points, the actual velocity of spinning is, however, indeterminate, since *any* velocity of spin about this diameter is evidently consistent with the other kinematic conditions.

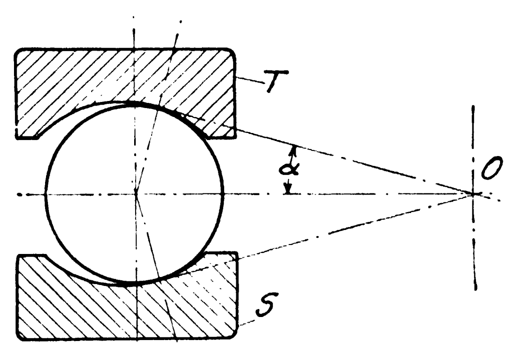


Fig. VII, 7

When the centrifugal forces of the balls, due to their revolution around the axis of the bearing, are comparable with the load, they take up a position still further removed from the axis of the bearing (see fig. VII, 8), and consequently still less consistent with pure rolling, than the position shown in fig. VII, 6. The tangent planes at the two points of contact now intersect on the side of the ball remote from the axis of rotation, making an angle 2β with each other, β being the angle between the line through the ball joining the points of contact and each of its radii to the same points.

The angle β is given by the equation

$$\tan \beta = \frac{\Sigma\Gamma}{2R_a},$$

where $\Sigma\Gamma$ is the sum of the centrifugal forces of all the balls in the bearing, and R_a is its axial load. The angle β being known, the angular velocity with which the ball spins about its radii through the contact points, can be determined by the same considerations as in the case of fig. VII, 7 and is found to be

$$\psi = \frac{\Omega}{2} \cdot \frac{r_m \sin \beta + r_a}{r_a \cos \beta},$$

r_a being the radius of the ball, and r_m the distance of its centre from the axis of the bearing.

The conditions, as regards lubrication, which arise at the points of contact from the spinning of the balls may be compared with those at the spherically ended pivot of a small spindle discussed in Sect. VIII, 4. It is to be remembered, however, that the points of contact of a bearing ball are continually changing their positions on its surface, as well as on the surfaces of the races.

There is still another mode of relative motion possible between the ball and the races at their points of contact, which may arise in many types of ball bearing, but to which the ball-thrust bearing is especially subject, namely, tangential slipping. Such slipping arises

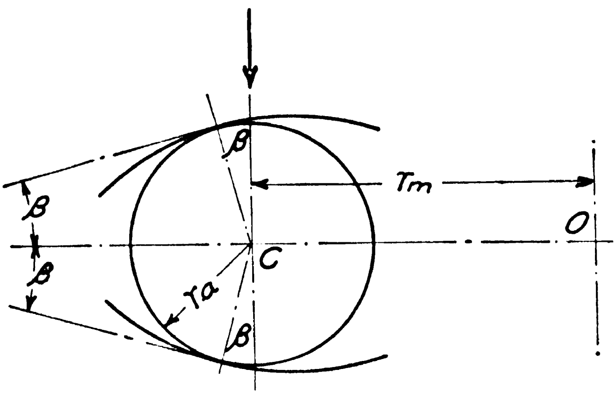


Fig. VII, 8

from the gyrostatic forces on the ball due to its combined circumferential revolution in the bearing, and its rolling and spinning motions. In the following brief discussion the ball is assumed to spin equally with respect to both races.

The change in the direction of the horizontal axis of rotation of the ball (OC in fig. VII, 8) which takes place as its centre C revolves about the vertical axis through O with angular velocity $\frac{1}{2}\Omega$, involves, according to the well-known formula of gyrostatic couples, the existence of a couple whose axis is at right angles, both to the momentary axis of rotation and to the axis of revolution; that is to say, the axis of the couple at each instant is the tangent line through the centre of the ball to its circle of revolution. In the case of fig. VII, 8, in which the line joining the two contact-points of the ball is parallel to the axis of revolution, the moment of the couple is simply the product of the angular velocities of rotation and of revolution of the ball, and its moment of inertia about its diameter, or

$$G = I_m \omega (\tfrac{1}{2}\Omega),$$

I_m being the moment of inertia, and ω the angular velocity of rotation of the ball, given by

$$\omega = \frac{\Omega}{2} \cdot \frac{r_m + r_a \sin \beta}{r_a \cdot \cos \beta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{VII, 1}$$

Thus

$$G = I_m \cdot \frac{\Omega^2}{4} \cdot \frac{r_m + r_a \sin \beta}{r_a \cos \beta} \quad . \; . \; . \; . \; . \; . \quad \text{VII, 1}a.$$

If the ball commences to slip under the action of the couple it must evidently do so at both of its contact-points simultaneously. The condition that the slipping may take place is therefore that the moment of the couple shall be greater than the sum of the normal contact forces on the ball, multiplied by its radius and by a coefficient of friction ν, dependent on the condition of the contact surfaces as influenced by their lubrication. In this connexion (as in the case of spinning of the ball), it has to be recognized that the positions of the points of contact are continuously changing both on the ball and on the races. The resistance to slipping is therefore not a mere matter of static friction. There appear to be no experimental data particularly applicable to the case, but it may be reasonably assumed from analogous cases which are discussed in Sects. IV, 16, 18; V, 9; and VIII, 1, that the coefficient of friction will lie between the limits 0·03 and 0·10.

Taking for a numerical illustration a thrust bearing having a pitch radius to the centre of the balls of 4 cm., the shaft revolving at 3000 r.p.m., with 8 balls of 1 cm. radius, we have, approximately, $\frac{1}{2}\Omega = 157$.

$$I_m = \frac{8\pi}{15} m r_a^5 = 1{\cdot}68 \times 7{\cdot}8 \times 1^5 = 13 \text{ gm. cm.}^2;$$

so that from equation VII, 1*a*, the angle β being regarded as small,

$$G = 13 \times 157^2 \times \frac{4}{1} = 1{\cdot}28 \times 10^6 \text{ dyne-cm.}$$

If the thrust load on the bearing is R_a, the double-contact charge on each ball is $2P = 2 \,.\, \frac{1}{8}R_a = \frac{1}{4}R_a$, so that with a minimum coefficient of friction of 0·03, and a maximum of 0·10, slipping is to be regarded as certain if

$$\frac{R_a}{4} < \frac{G}{\cdot 10} < \frac{1{\cdot}28 \times 10^6}{\cdot 10},$$

or $R_a < 5{\cdot}1 \times 10^7$ dynes, or say 50 kg. weight; and is possible if

$$R_a < \frac{1{\cdot}28 \times 4 \times 10^6}{\cdot 03} < 17 \times 10^7 \text{ dynes, or 170 kg. weight.}$$

Even the larger of these forces, 170 kilograms weight, is very considerably below the usual load of such a bearing as that taken for the example, but it has, of course, to be remembered that most bearings run at times without load, or at least with loads much below their full capacities.

When it occurs, even with light loads, and consequently low contact pressures, slipping causes a special kind of damage to one or other of the bearing surfaces, usually that of the race. This type of abrasion, known as "smearing", is in fact the only physical evidence of the actual occurrence of the phenomenon of slipping as indicated by the gyrostatic theory.

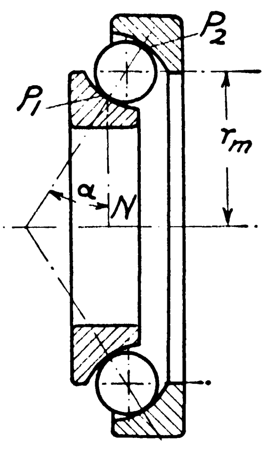


Fig. VII, 9

In cases where it is liable to occur, as in the ball-thrust bearing discussed in the present and the preceding sections, and in the form of ball bearing commonly used for carrying combined thrust and radial loads (shown in axial section in fig. VII, 9), the designer of the machine concerned may sometimes guard against it by arranging that the load on the bearing shall never fall below a safe minimum. Where this cannot be done, it is a common practice to provide a second bearing with an arrangement of springs loading both bearings so as to ensure that the actual loading on each of them is always above its danger point.

As the above example clearly indicates, spinning, with consequent "smear", is a trouble incident to the operation of bearings under exceptionally low loads, or at high speeds. Only thrust bearings and bearings arranged for combined axial and radial loads in which the axial component is greater than, or comparable with, the radial component are subject to it. In bearings of those classes, however, slipping sets a limit to the permissible speed which is distinct from, and often lower than, the limits imposed by considerations of life, which are dealt with in Sects. VII, 8 and 11–13).

8. Conditions of Loading and Deformation which control the nature of Lubrication in Rolling Bearings.

In all forms of rolling bearings the areas of contact or close approach between the rolling elements and the races are very much smaller than corresponding areas in sliding bearings; so small indeed that they are commonly referred to as "lines of contact", or "points of contact", as they would be in fact if the contacting members were actually rigid and unyielding under pressure. Necessarily, therefore, the intensities of pressure on these small areas are much greater than those which occur in sliding bearings, and which in them allow the formation of relatively thick lubricating films.

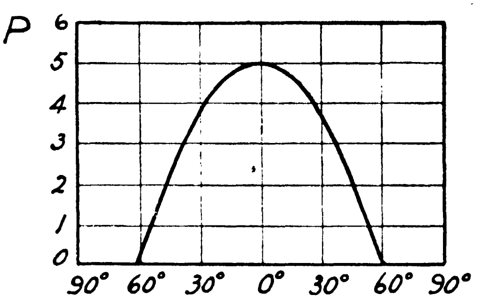


Fig. VII, 10

In all practical rolling bearings the load carried by the bearing is distributed between a number of rollers or balls; in radial bearings between roughly one-half of the total number in the bearing (viz. those which at any moment are situated in the semi-circumference symmetrically opposed to the load); but in a thrust bearing between the whole number. Radial rolling bearings are usually so designed that when fitted in the machine in operating condition but unloaded, there is no appreciable clearance between the rolling elements and the races. It has been shown, both by calculation and experiment (Ref. VII, 4) that when a radial load is applied to a bearing so fitted, the rolling elements in the loaded semi-circumference of the bearing carry charges which vary with their circumferential distances from the line of action of the load, approximately in accordance with the ordinates of a parabolic curve as shown in fig. VII, 10, and that the maximum charge on any element (which it carries when its centre is on the line of the load) is approximately

$$P_{max} = \frac{5}{n} R_r, \qquad \text{VII, 2}$$

where R_r is the load on the bearing, and n is the total number of rolling elements in a bearing of usual design and proportions. The varying charge on the element, shown by the parabolic curve, is represented with sufficient accuracy for practical applications by the equation

$$P = P_{max} \cos\left(\frac{\theta}{60} \times \frac{\pi}{2}\right), \quad . \quad . \quad . \quad . \quad . \quad \text{VII, 3}$$

in which θ is the angle (in degrees) subtended between the line of the load R_r and the radius through the centre of the rolling element. The condition which is implied in the formula, that all the rolling elements in the bearing are so nearly equal in diameter that the load is divided between several of them when passing through the loaded arc, can only be realized in practice by selective assembly. It is to be noted that, if R_r is constant in direction and the outer race is stationary, this race carries no load except on the arc for which θ in equation VII, 3 is less than 60°. All parts of the track of the inner race, on the other hand, come in succession into the arc of loading, and are in turn subjected to the maximum charge of the rolling element.

When the centrifugal forces of the rolling elements are comparable in magnitude with the load imposed on the bearing, the contact force between each element and the outer race is increased by the amount of the centrifugal force of the element. The contact forces on the inner race remain, on the contrary, of the magnitudes due to the imposed load alone. In the generalized form of rolling bearing shown in fig. VII, 9, in which the line P_1P_2 joining the contact points of the ball makes an angle α with the plane of its revolution, and consequently with the direction of its centrifugal force, the peripheral velocity of the centre of the ball can be taken with sufficient accuracy in the present connexion as

$$U = \tfrac{1}{2}\Omega \times P_1N_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{VII, 4}$$

where P_1N_1 is the perpendicular from the point of contact on the inner race to the axis of revolution, the inner race being assumed to be rotating with angular velocity Ω, and the outer race to be stationary. The formula is not exact because, owing to the centrifugal force of the ball, its centre is not on the line P_1P_2 unless the angle α is zero, and for the same reason the radial pitch of the bearing, i.e. the distance from its axis to the centre of the ball, is only approximately given by $r_m = \frac{1}{2}(P_1N_1 + P_2N_2)$. To this sufficient degree of approximation, however, the angular speed of revolution of the ball is, from equation VII, 4, $\frac{1}{2}\Omega P_1N_1/r_m$, and the centrifugal force of the ball, of mass $\mathfrak{M}$, is

$$\mathfrak{M} \times \frac{\Omega^2}{4} \times \left(\frac{P_1N_1}{r_m}\right)^2 \times r_m = \mathfrak{M}\,\frac{\Omega^2}{4}\,\frac{(P_1N_1)^2}{r_m}.$$

Thus the increment, due to centrifugal force, of the charge of the ball on the outer race is

$$\frac{\mathfrak{M}\Omega^2}{4}\,\frac{(P_1N_1)^2}{r_m}\sec\alpha, \; \mathfrak{M} \text{ being } \frac{4\pi}{3} m r_a{}^3,$$

and from equation VII, 3 the maximum total charge, carried once in each revolution, is

$$P' = \left\{\frac{5}{n} R_r + \mathfrak{M} \frac{\Omega^2}{4} \frac{(P_1N_1)^2}{r_m}\right\} \sec \alpha, \quad . \quad . \quad . \quad . \qquad \text{VII, 5}$$

the maximum charge of the ball on the inner race being given by the first member of the right-hand side of this equation, taken alone.

On account of the smallness of the areas of the contact surfaces, idealized as lines and points, on which the charges of the rolling elements are carried, the intensities of pressure in rolling bearings are very high and in all practical cases exceed the yield points of the material. Elastic theory was applied to the problem in the first instance by H. Hertz, and a complete solution obtained on the assumption that the strains are within the elastic limit (Ref. VII, 5), and that the contact forces are normal to the surfaces of the two bodies. A more general solution, taking into account tangential reactions, such as those which arise from frictional resistances between the bodies at the common surface, has been given by M'Ewen, Ref. VII, 10. Such frictional forces are, however, small compared with the normal reactions in the usual operation of rolling bearings, and will be neglected in the discussion which follows. The method of solution (as well as the results, for all except the simplest cases), is complex, but the principal conclusions to which it leads may be stated broadly as follows.

The small areas on which the opposed members come into contact are in general elliptical, the lengths of the axes of the ellipse being functions of the principal radii of curvature of both surfaces at the central point of contact, and of the azimuthal relation of the planes of principal curvature of one surface with respect to the other. Each axis of the ellipse increases proportionately to the cube root of the contact force; the area of contact is consequently proportional to the power $\frac{2}{3}$, and the mean intensity of stress over the contact area proportional to the cube root of the force (Ref. VII, 6).

In the case of a roller, making initially line-contact with its race, the ellipse of contact degenerates into the area between two generating lines, and its width (the distance between the lines) and area are proportional to the square root of the contact force. The maximum intensity of pressure in the case of elliptical contact is one-and-a-half times the mean normal pressure on the area, and in the case of line contact $4/\pi$ times the mean. Over the whole surface of contact and for a certain depth beneath it in each body, the stress is compressive in all directions (see fig. VII, 11, taken from Ref. VII, 7, by courteous permission of the publishers, Cambridge University Press). At a greater depth (of the order of the width of the area of contact), the stresses indicated by dotted lines in the figure become tensile in the tangential direction, while remaining compressive throughout the bodies in the normal direction. The tensile stresses rise to the surface a little outside the ellipse of contact, forming there, in combination with compressive normal stresses, regions of shear. It is in this region, or in the central region of tensile stress at a depth below the surface, that fracture or yielding of the material is most likely to occur.

These conditions, correct only for the very light forces which do not stress the material beyond its yield point, can be imagined to hold for forces of greater magnitudes, and hence arises the conception of an elastic deformation of the bodies, following the same pattern even for the relatively great forces which exist in actual bearings. The stresses do in fact remain within the elastic limits in all parts of the bodies at a relatively considerable distance from the area of contact, and, for this reason and because the region of plastic strain is comparatively small, it is convenient (for example in calculating the total amount of compression of the rolling element along the line between its points of

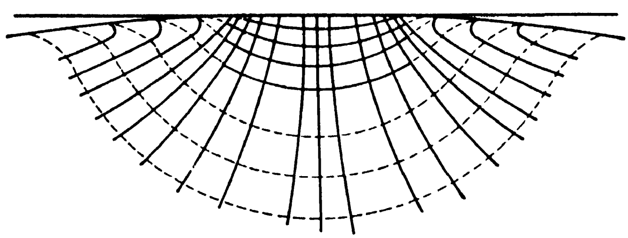


Fig. VII, 11

contact) to regard the excess of the local deformation which the body actually undergoes over that which is found for the same region from the elastic theory, as an empirical correction to be applied to the latter.

In order to facilitate the calculation of the results of the elastic theory, tables have been prepared by various authors of the functions of the curvatures which enter into the solutions in practical cases (Ref. VII, 6).

According to Palmgren (Ref. VII, 3, p. 47), all standard rolling bearings of any one type (such as one of the types listed in Sect. VII, 2) are so nearly scale models of one another that the relevant radii of curvature of the bearing surfaces can be regarded as being, for practical purposes, in fixed proportion to one another and to the diameter of the rolling element, and this author has given (op. cit.) simple formulæ for the total radial displacements arising from elastic compression in the members of the various types. If δ_r and δ_a are the total displacements between the races in the radial and axial directions respectively, due to the assumed elastic deformations, when converted into C.G.S. units these formulæ are, for the four types listed in Sect. VII, 2, as follows:

TYPE A. Cylindrical roller bearing.

$$\delta_r = \frac{P^{0\cdot 9}}{L^{0\cdot 8}} \times 3{\cdot}85 \times 10^{-11} \text{ cm.}, \qquad \text{VII, } 6a$$

P being the maximum charge on one roller in dynes, and L the length of the roller in centimetres.

TYPE B. Spherical roller bearing.

$$\delta_r = \frac{P^{\frac{3}{4}}}{L^{\frac{1}{2}}} \times 1{\cdot}1 \times 10^{-9}. \qquad \text{VII, } 6b$$

TYPE C. Deep-groove ball bearing.

$$\delta_r = \frac{P^{\frac{2}{3}}}{r^{\frac{1}{3}}} \times 1{\cdot}9 \times 10^{-8}, \qquad \text{VII, } 6c$$

r being the radius of the ball in centimetres.

TYPE D. Ball-thrust bearing.

$$\delta_a = \frac{P^{\frac{2}{3}}}{r^{\frac{1}{3}}} \times 3{\cdot}0 \times 10^{-8}. \qquad \text{VII, } 6d$$

These formulæ apply to steel bearings in which Young's modulus of elasticity is always approximately $2{\cdot}0 \times 10^{12}$ dynes-cm.$^{-2}$ ($2{\cdot}9 \times 10^7$ lb. per sq. in.), and Poisson's ratio about 0·30.

For the corresponding displacements due to plastic deformation (called the *permanent* displacement), Palmgren (loc. cit.) has given empirical formulæ applicable to steel rolling members having a hardness of 63·5 to 65·5 Rockwell C.

These formulæ are for a single point contact,

$$\delta_r' \,(\text{or } \delta_a') = \frac{P^2}{r}(\rho_1 + \rho_1')(\rho_2 + \rho_2') \times 6{\cdot}0 \times 10^{-18} \text{ cm.,} \qquad \text{VII, } 7a$$

and for a single line contact,

$$\delta_r' \,(\text{or } \delta_a') = \frac{P^3}{r^{\frac{1}{2}}L^3}(\rho_1 + \rho_2)^{\frac{3}{2}} \times 1{\cdot}4 \times 10^{-11} \text{ cm.,} \qquad \text{VII, } 7b$$

in which ρ_1, ρ_1', etc. are the principal curvatures of the two bodies in the region of contact, and the other symbols have the same meanings as in the corresponding cases of elastic displacement.

Of these permanent deformations one-third may be taken as occurring in the rolling element and two-thirds in the race. The volume of material which is stressed beyond the elastic limit in each revolution of the rolling element will evidently be of the order of the product of δ_r and the area of contact; and this area is of the order of a^2, so that the volume of overstrained material is of the order of $\delta_r a^2$, in which a is the mean radius of the elliptical area of contact. Since the length of the working life of a bearing is limited by cumulative effects of overstrain, its life will diminish rapidly as δ_r increases. The rule has been laid down (Ref. VII, 3, p. 79), that to obtain satisfactory operation, even for a brief length of life, δ_r should not in ordinary cases exceed $r \times 2 \times 10^{-4}$, r being the radius of the rolling element, that is to say, the total deformation of rolling element and race at *one* contact should not be greater than $2r \times 10^{-4}$, of which approximately one-third takes place in the rolling element and two-thirds in the race.

This general rule may be accepted as a practical basis for determining the working conditions of lubrication in rolling bearings, since commercial considerations prevent any factors of safety much greater than those implied in the rule from being adopted.

9. Action and Functions of Lubricants in Rolling Bearings.

In the preceding sections of this chapter only incidental reference has been made to the presence of lubricants in rolling bearings, and little or no account has been taken of the action of a lubricant, although it has been mentioned that its presence is always to be assumed and regarded as being necessary. It remains to be considered how far the conditions and effects which have been discussed must be modified when the presence of a lubricant is taken into account.

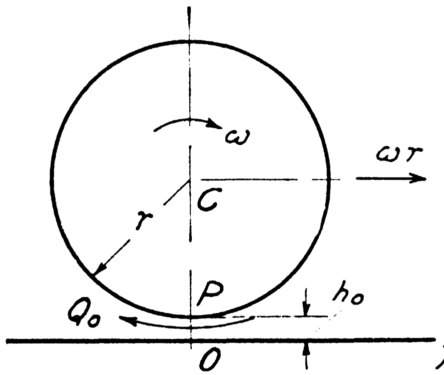


Fig. VII, 12

To illustrate the essentials of the fluid motions and reactions when a roller or ball rolls upon a race in the presence of a lubricant, the example of a cylinder rolling on a plane may be taken as representative; it will be recalled that this was the type of rolling motion selected by Osborne Reynolds in the investigation briefly described in Sect. VII, 1.

The broad result of the following mathematical treatment will be found to be in accordance with that of the experimental investigation, namely that the lubricant has little or no direct effect on the resistance to the rolling motion when the conditions of rolling-bearing practice are complied with.

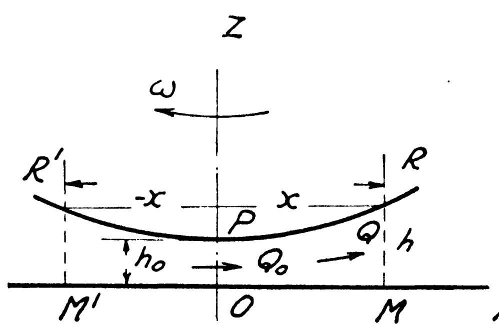


Fig. VII, 13

Figs. VII, 12 and 13 (the latter being merely an enlarged view of a part of fig. VII, 12) show an instantaneous view of a cylinder of radius r, having a rotational velocity ω about its axis, combined with a translational velocity of its centre of ωr parallel to the fixed horizontal plane M′OM. As the result of this combined motion the lowest point P of the roller is momentarily at rest relatively to the plane, and the minimum distance h_0 between the two surfaces remains constant. A fluid lubricant is assumed to be supplied on the advancing side of the roller in sufficient quantity to keep the plane, and at least the portion of the roller shown in fig. VII, 13 by the arc RPR′, continually flooded. As the roller advances it will evidently tend to displace fluid in the direction of OX, and at the same time it causes a certain flow in the negative direction through the space OP.

Let Q be the volume of fluid displaced per unit of width of the roller and plane in the forward direction past the line QM at x, and Q_0 the flow (also considered positive in the forward direction, but actually negative in amount) past the line PO at $x = 0$. It being obvious that only small arcs of the cylinder on each side of the point P take substantial part in the action, quantities involving the second, or higher, powers of the inclination of the surface of the cylinder to the plane may be neglected, and for the same reason the mean direction of the flow Q is taken as being, to the first order, parallel to the plane.

Now R being any point on the periphery of the cylinder at a horizontal distance x to the right of P, and the cylinder being assumed to rotate clockwise with respect to the plane with angular velocity ω around P, it is seen that the area OPRM is decreasing at the rate $\frac{1}{2}\omega x^2$. The total flow out of the prism of this area and unit thickness is accordingly given by

$$\tfrac{1}{2}\omega x^2 = Q - Q_0.$$

Thus
$$Q = Q_0 + \tfrac{1}{2}\omega x^2. \qquad \text{VII, 8}$$

From equation II, 13, the change of fluid pressure p with x is given by

$$Q = -\frac{bh^3}{12\mu}\frac{\partial p}{\partial x},$$

or, b in this case being taken as unity,

$$\frac{\partial p}{\partial x} = -\frac{12\mu Q}{h^3},$$

and from equation VII, 8,

$$\frac{\partial p}{\partial x} = \frac{-12\mu}{h^3}(\tfrac{1}{2}\omega x^2 + Q_0). \qquad \text{VII, 9}$$

From the geometry
$$h = h_0 + \frac{x^2}{2r},$$

and, since the point P is advancing with respect to the fluid at a distance with velocity ωr, by continuity

$$Q_0 = -\omega r h_0. \qquad \text{VII, 10}$$

Inserting these values in equation VII, 9, it becomes

$$\frac{\partial p}{\partial x} = -48\mu\omega r^3\frac{x^2 - 2rh_0}{(2rh_0 + x^2)^3}. \qquad \text{VII, 11}$$

From equation VII, 11, the pressure is found by integration to be

$$p = \frac{6\mu\omega r^2}{h_0}\left\{\frac{x^3 + 6rh_0x}{(2rh_0 + x^2)^2} + \frac{1}{\sqrt{(2rh_0)}} \text{ arc tan } \frac{x}{\sqrt{(2rh_0)}}\right\} + \Pi. \quad \text{VII, 12}$$

The atmospheric pressure Π is derived as the constant of integration by the consideration that, from the symmetry, the pressure must be atmospheric at $x = 0$.

From the form of equation VII, 12, it is immediately seen that the pressure in the lubricant exceeds, or falls below, the atmospheric pressure by equal amounts for equal values of x, positive and negative.

The resultant forces on the roller and plane are consequently zero, assuming that the fluid is capable of sustaining pressures below atmospheric, or tensions, and also assuming that sufficient lubricant is supplied to maintain the flow Q at all points. The ground for the former of these assumptions is the extremely brief period during which the tension occurs at any one point.

From equations VII, 8 and 10, it is seen that $Q = 0$

when $$x^2 = -2\frac{Q_0}{\omega} = 2rh_0,$$

or $$x = \pm\sqrt{(2rh_0)}, \quad \ldots\ldots\ldots \quad \text{VII, 13}$$

reversals of the direction of flow taking place at these two points, which are determined geometrically as the points S and S′ shown in fig. VII, 14, PO′ being taken equal to PO $= h_0$, and the directions of flow being indicated by arrows.

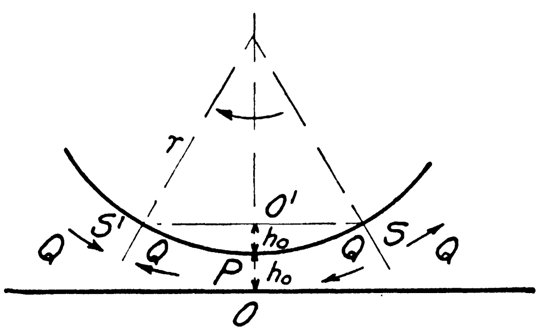


Fig. VII, 14

It is to be observed that although, as already stated, the resultant of the pressures generated in the fluid in the neighbourhood of the point P is zero, the positive pressures relative to Π on the right-hand side of P (x positive), and the negative pressures on the left-hand side (x negative), exert equal and opposite moments around P, both of them resisting the instantaneous motion about that point. There is thus a definite fluid resistance to rolling so long as PO is of finite magnitude.

The condition which has been considered so far in this section, namely, that of a roller in lubricated contact with a single race, only occurs in actual bearings when, in radial bearings, the roller or ball is in an unloaded arc of the bearing and there is an appreciable clearance in that arc.

When the roller or ball enters into the loaded arc of the bearing, it is subjected to equal, or nearly equal, and opposite forces due to the load on the bearing, which act at opposite ends of a diameter of the roller, or very nearly so. In radial bearings the angular speeds of the rolling element relatively to the two races are not, it is true, in general equal, nor are the conditions as to the curvatures the same at both areas of contact, but there is usually approximate equality in these respects, and in thrust bearings the equality can be taken as exact.

In order to avoid interruption of the general course of this discussion, investigations of the actions which take place when a rolling element, surrounded by a film of lubricant, leaves the unloaded arc of a bearing and enters the loaded arc, have been relegated to Appendix III. Only the main results of the analysis need be given at this stage.

It is shown therein that almost immediately upon the ball entering the loaded arc, and being subjected to the charges due to its position there, it is driven through the film of lubricant with which it was covered and the con-

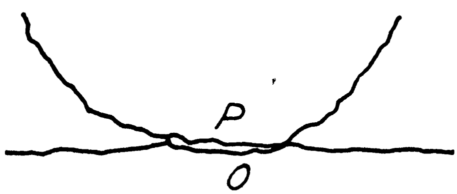


Fig. VII, 15

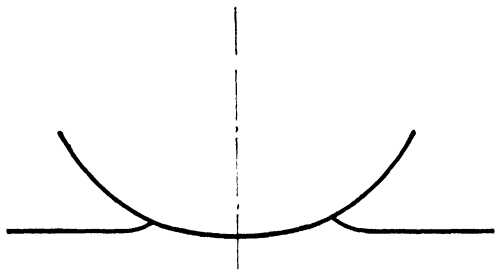


Fig. VII, 16

ditions represented in figs. VII, 12–14 cease to apply. The minimum thickness of lubricating film, shown as h_0 in those figures, diminishes almost instantaneously until the rugosities of the opposed surfaces begin to come into contact as shown in an exaggerated form in fig. VII, 15. Flow of the fluid past the points P and O from the advancing to the retreating side, as in figs. VII, 12–14, is then prevented, and lubricating action ceases on the latter side. The charge on the rolling element is carried partly by direct reactions at the points of contact of the rugosities of the element and those of the plane, and partly, it may be, by hydrostatic pressures in small volumes of fluid trapped between those rugosities. In either case the reactions between roller and plane are confined to the small extent of their partial contact, which is of the same order of area as would exist if no lubricating fluid were present.

In all practical cases, however, the charges on the elements of rolling bearings are too great to be borne by mere projections from their surfaces, or even to be consistent with any deformations of the surfaces (however smooth and closely approximating to their ideal geometrical forms) within the elastic limits of the materials. Plastic deformations, therefore, always take place at the contact areas in the manner roughly indicated in fig. VII, 16.

Further discussion of these conditions is best postponed (to Sect. VII, 13) to follow the next two sections in which the magnitudes of the loads and stresses in rolling bearings (Sect. VII, 11), and the limitations to the supply of lubricant to them (Sect. VII, 10), have been considered.

10. Lubricating Films in Rolling Bearings.

In rolling bearings of all types the volume of lubricant available for lubricating the surfaces of contact is very small compared with that which is both possible and desirable in sliding bearings. Any general flooding of the bearing elements is inadmissible both because of the resistance which a bulk of fluid would offer to the revolution of the rolling elements and the cages, and also because the rolling elements would reduce any lubricant of the usual kinds to a condition of foam, and thereby render the sealing of the bearing against leakage very difficult, if not impossible.

Usually the lubricant is supplied to the housing of the bearing in limited quantity, and only reaches the contact surfaces of the rolling elements and races by means of thin films held on the rotating surfaces by surface tension, in the manner discussed in Sect. II, 10.

Referring to equation II, 16, the force per unit area of the film, which was therein regarded as a pressure tending to displace the film radially outwards from its cylindrical or spherical form, is now replaced by a centrifugal force, which may be designated as a force p per unit area, exerted by the film itself in its rotation in common with the rolling member, or rotating race, on which it is situated.

Considering in the first place a rotating element of radius r, its angular velocity can be taken, as in equation VII, 1, as

$$\omega = \tfrac{1}{2}\Omega \frac{r_m + r \sin\beta}{r \cos\beta},$$

and its surface velocity, due to its rotation alone, is

$$U = \omega r = \tfrac{1}{2}\Omega r_m\left(\sec\beta + \frac{r}{r_m}\tan\beta\right).$$

If j is the thickness of the film, and its density m, the centrifugal force per unit area is

$$mj\frac{U^2}{r} = \frac{mj}{r}\frac{\Omega^2}{4}r_m^2\left(\sec\beta + \frac{r}{r_m}\tan\beta\right)^2. \qquad \text{VII, 14}$$

In the case of a cylindrical roller, from equation II, 17, in which τ is the surface tension of the film,

$$p = \frac{\tau}{r},$$

so that $$mjU^2 = \tau$$

and $$j = \frac{\tau}{mU^2}, \qquad \text{VII, } 14a$$

being independent of the radius of the roller.

In the case of a ball of the same radius r, the retaining force as shown by equation II, 19 is twice as great, but the centrifugal force is the same as for a cylinder rotating at the same surface speed; the greatest thickness of film that can be retained on the surface by the tension τ is therefore

$$j' = 2j = \frac{2\tau}{mU^2}. \qquad \text{VII, } 14b$$

Turning now to the films of lubricant which may exist on the races, it is seen that the outer race of a radial bearing and the lower race of a thrust bearing, if they are stationary, are, of course, subject to no limitation to the thickness of a film upon them on account of centrifugal force, but on the other hand tend to retain small accumulations of oil of more than capillary thickness.

This oil is available for lubrication of the rotating elements as they pass through them, but cannot be carried by the elements to serve for lubrication of their contacts on the inner, or upper race, except in the form of films of the maximum thickness already determined.

The inner race of radial ball bearings, and the revolving race of thrust ball bearings, are usually incapable of carrying any film of more than molecular thickness, since their total curvatures are in almost all cases negative, the negative curvature in the axial plane being greater than the positive curvature in the plane tangential to the track of the balls. In the case of roller bearings, the total curvature is usually positive, so that surface tension tends to retain a film against the action of centrifugal force in the same way as on rolling elements, and the thickness of the film can be calculated from equation VII, 14*a* from which, since the surface velocity is the same for both race and roller, the numerical result will be the same.

It thus appears that, for both rollers and balls, the total thickness of film $\bar{j}$ available for lubrication of their contacts with the inner race is the same for any given surface speed, viz.

$$\bar{j} = \frac{2\tau}{mU^2}. \qquad \text{VII, } 14c$$

The numerical factor 2 is due, in the case of the roller, to the fact that the race brings to the point of contact as great a thickness as the roller, and in the case of a ball, to its being able to carry twice as thick a film as a roller, while the inner race contributes nothing.

Fig. VII, 17 and Back Folder VII, A show the total thickness of film as calculated from equation VII, 14*c* for various surface velocities up to 3500 cm. per sec., on the assumption that the surface tension τ of the lubricant is 25 dynes per cm., and that the density is 0·9 gm. cm.$^{-3}$, these being normal values for lubricating oils.

It is immediately apparent on inspection of these diagrams that the thickness of the lubricating film corresponding to even moderately high surface speeds is much less than is necessary for the lubrication of sliding bearings. At the higher speeds, indeed, the thickness of the film is less than the average height and depth of the rugosities of even the surfaces prepared by the most refined commercial practices. It appears, therefore, that although at these higher speeds the solid surfaces of balls, or roller and race, will at first come into contact on the summits of their rugosities without materially affecting the passage of the fluid film through the interstices between them—in other words, without any considerable generation of pressure in the fluid in the area of contact — yet, when the contact becomes closer, there may be increasing resistance to the passage of fluid (when in figs. VII, 12–14 h_0 becomes zero), resulting possibly in the generation of pressure, and also in an accumulation of fluid immediately in front of the contact area. These conditions will be discussed more fully in Sect. VII, 13.

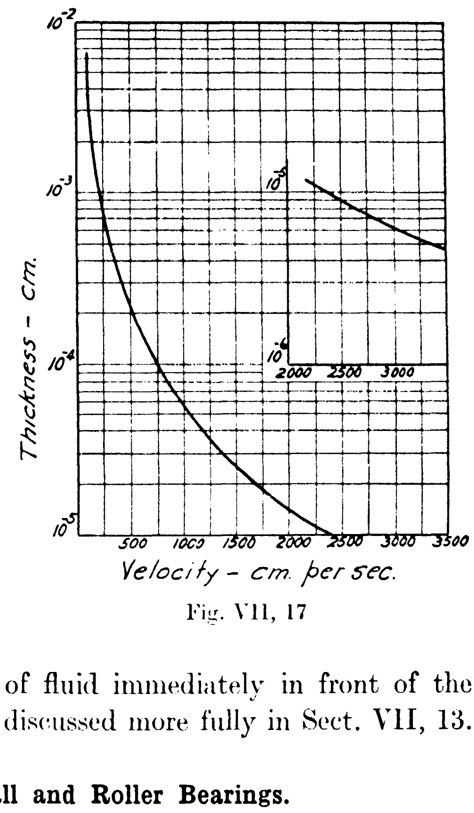


Fig. VII, 17

11. Magnitudes of Contact Forces in Ball and Roller Bearings.

The maximum charges to be carried by the rolling elements of ball and roller bearings are mainly determined by the consideration, stated at the end of Sect. VII, 8, that the permanent deformation at any one contact cannot be safely allowed to exceed a certain multiple of the radius, there given as $r \times 2 \times 10^{-4}$.

The charge which corresponds to a deformation of that magnitude can be found from equation VII, 7*a* or *b* according as the element in question is a roller or a ball.

Having regard to the rule (stated in Sect. VII, 8) that in any one type of bearing the radii of curvature are, in the various examples of the type, proportional to the radii of the rolling elements, it immediately results from these equations that

$$\text{in ball bearings, } P_{\max} = k_1 \times r^2,$$
$$\text{and in roller bearings, } P_{\max} = k_2 r L,$$

P_{max} in each case being the greatest allowable charge which can be carried under conditions in which durability is not of importance, and k_1 and k_2 constants dependent on the properties of the materials and on the numerical relations of the radii of curvature of the rolling elements to r. When these latter factors have been introduced to determine k_1 and k_2 the formulæ respectively become for ball bearings,

$$P_{max} = 2{\cdot}4 \times 10^9 \times r^2 \quad . \quad . \quad . \quad . \quad . \quad \text{VII, 15}a$$

and for roller bearings,

$$P_{max} = 2{\cdot}2 \times 10^9 \times rL. \quad . \quad . \quad . \quad . \quad . \quad \text{VII, 15}b$$

In normal practice, when regard is paid to the necessity of obtaining a reasonable length of life for the bearings, the allowable charges at average surface speeds are of the order of one-half of those given by these formulæ, and diminish rapidly for values of U (the surface speed of rotation) greater than about 2000 cm. sec.$^{-1}$

12. Losses of Energy and Coefficients of Resistance in Rolling Bearings.

The loads permissible in a bearing having been determined according to the preceding section, together with the permanent plastic deformations corresponding to them, it is easy to calculate approximately the loss of energy involved, and the resulting resistance to the motion of the rotating race. As an example, it will be assumed that, in order to obtain efficient operation and a normal working life, the charge P on the rolling elements is limited to one-half of the safe static charge P_{max} as indicated in the preceding section.

Taking the case of a ball bearing, it is shown by equation VII, 7a that the deformation at a point of contact is proportional to the square of the charge P. When, therefore, P is limited to one-half of the static load P_{max}, the permanent deformation at a point of contact will be one-fourth of the amount which is regarded as allowable for static conditions (see end of Sect. VII, 8), viz. $r \times 2 \times 10^{-4}$,

so that $$\delta = 5 \times 10^{-5} \times r.$$

It can easily be shown that the circumferential length of the path traversed by the contact point of a ball with each of the races of a radial ball bearing, during one revolution of the inner race, is $\pi(r_m^2 - r_a^2)/r_m$, in which, as usual, r_a is the radius of the ball, and r_m the radial distance from the axis of the bearing to the centre of the ball. Actual contact pressures and deformations, however, take place, in any one passage of the ball around the circumference, only upon so much of these paths as lies within the loaded arc of the bearing, i.e. approximately upon one-third of the paths (see Sect. VII, 8 and fig. VII, 10, p. 208), or upon a total length, for both races, of $2\pi(r_m^2 - r_a^2)/3r_m$.

At any moment the area of contact of ball and race is elliptical, the geometric mean of the major and minor semi-axes of the ellipse being $a = \sqrt{(2r_a\delta)}$, in which δ is the depth of the plastic deformation. It has already been seen that δ is proportional to the square of the contact force P. If P and δ were constant over the whole of the loaded arc, the total area deformed by the passage of all the balls of the bearing, z in number, during one revolution of the inner race, would be consequently

$$2\pi z \frac{r_m^2 - r_a^2}{3r_m} \times 2a, \quad \text{or} \quad \frac{4\sqrt{2}\,.\,\pi z}{3}\,.\,\frac{r_m^2 - r_a^2}{r_m}\,.\,r_a^{\frac{1}{2}}\delta^{\frac{1}{2}}.$$

Now, from the postulate that the depth of deformation δ is proportional to the square of the charge P, the energy absorbed at a single, stationary, contact can be taken as $\frac{2}{3}P\delta$, and the area of the deformed surface is $\pi a^2 = 2\pi r_a\delta$. Thus the work of deformation per unit area is

$$\frac{2P\delta}{3 \times 2\pi r_a\delta} = \frac{P}{3\pi r_a},$$

and, if P were constant over the whole of the arc of loading, the energy lost during one revolution would be

$$\frac{4\sqrt{2}\,.\,\pi z}{3}\,.\,\frac{P}{3\pi r_a}\,.\,\frac{r_m^2 - r_a^2}{r_m}\,.\,r_a^{\frac{1}{2}}\delta^{\frac{1}{2}}$$

$$= \frac{4\sqrt{2}}{9}\,.\,zP\,.\,\frac{r_m^2 - r_a^2}{r_m}\,.\,\frac{\delta^{\frac{1}{2}}}{r_a^{\frac{1}{2}}}. \quad . \quad . \quad \text{VII, 16}$$

Since, however, P is not constant over the loaded arc, but diminishes according to a parabolic law from a maximum at the middle point to zero at each of the ends of the arc, and since the work of deformation at any point is proportional to $P\delta$, i.e. to P^3, it follows that the total work of deformation is only $\frac{16}{35}$ of what it would be if P were constant at its maximum value. Also, it has been seen that when P has the maximum value consistent with satisfactory operation and a reasonably long life of the bearing, δ/r_a has the value 5×10^{-5}, or $\delta^{\frac{1}{2}}/r_a^{\frac{1}{2}} = \sqrt{(0{\cdot}5)} \times 10^{-2}$.

Thus, if P has this maximum value at the middle point of the arc, the total work of deformation during one revolution will be

$$W = \frac{16}{35} \times \sqrt{(0{\cdot}5)} \times 10^{-2} \times \frac{4\sqrt{2}}{9}\,.\,zP\,.\,\frac{r_m^2 - r_a^2}{r_m}.$$

Now, from equation VII, 2, $Pz = 5R_r$ where R_r is the resultant load on the bearing, so that

$$W = \frac{16 \times 4 \times 10^{-2} \times 5}{35 \times 9}\,.\,R_r\,.\,\frac{r_m^2 - r_a^2}{r_m}$$

$$= \frac{64}{63} \times 10^{-2} \times R_r \times \frac{r_m^2 - r_a^2}{r_m},$$

and the coefficient of resistance k of the bearing, referred to the circumference of the inner race, which is of radius $r_m - r_a$, is

$$k = \frac{W}{2\pi(r_m - r_a) \times R_r} = \frac{32 \times 10^{-2}}{63 \times \pi} \cdot \frac{r_m + r_a}{r_m}. \quad . \quad . \quad \text{VII, 17}$$

In a bearing of this class of usual proportions, $r_m/r_a = 5$ approximately. In that case the value of k, according to equation VII, 17, would be $32 \times 6 \times 10^{-2} \div (63\pi \times 5)$, or $k = 1{\cdot}94 \times 10^{-3}$ approximately.

This result accords so nearly with those obtained experimentally by tests of bearings of the same class operating under the conditions assumed in the calculation (e.g. see Ref. VII, 3, p. 39), as to leave little room for doubt that the basic assumption of the calculation is correct, viz. that the resistance to revolution of well-made ball bearings, as normally operated, arises, at least mainly, from the loss of energy incurred in deformation of the rolling elements and races at their points of mutual contact.

It will be noted that the calculated coefficient of resistance is independent of the speed of the bearing, and that it does not involve either solid friction or any viscous resistance of the lubricant. In all these respects the result is consistent with experience.

A similar calculation may be made for roller bearings by using equation VII, 7*b* to determine the relation between the amount of deformation and the load. The results are numerically similar to those for the ball bearing.

It has been noted that in these calculations no account is taken of any possible effects dependent on speed. Since plastic deformations in general increase with increase of the time of application of a given load, it is to be expected that there will be some lowering of the coefficient of resistance with increase of the speed of the bearing, other conditions remaining unchanged. This expectation appears to be borne out to a certain extent by experiment, but the effect of speed under ordinary conditions appears to be small. Some experimental studies of roller bearings, however, show a marked rise of the coefficient of resistance with increasing speed (see e.g. Ref. VII, 8).

The approximate agreement between the results of the above calculations, in which any possible effects of lubrication are ignored, and the results of actual tests, in which lubrication is always employed, indicates that lubrication has little or no direct influence on the energy losses in rolling bearings of the purely radial type. This conclusion is also in accordance with the observations of Osborne Reynolds mentioned in Sect. VII, 1, and is further borne out by the experiments on more heavily loaded rollers which will be discussed in Sects. VII, 13 and 14. When more than the small amount of lubricant, which experience shows to be necessary to their prolonged operation, is used in rolling bearings, it is found that the coefficient of resistance rises rather rapidly with increase of speed, and that overheating is liable to occur. This is, doubtless, due to the kinetic resistances which any considerable mass of oil in the bearing opposes to the motion of the rolling elements and cages, and in some cases to the secondary viscous resistances discussed in Sect. VII, 14.

Owing to this limitation of the quantity of lubricating oil which it is permissible to apply to a rolling bearing, it is impracticable to carry away any considerable amount of heat generated in the bearing by the means which are effective in sliding bearings, i.e. by convection in the circulated lubricant. Since the total amounts of heat due to frictional resistances are of the same order in both classes of bearings under similar conditions of load and speed, conduction through the shaft and the casing of the bearing must be utilized to a greater extent in rolling than in sliding bearings, if similar limits to bearing temperatures are to be maintained. In rolling bearings which operate at such loads and speeds that they would attain excessive temperatures if solid conduction alone were relied on, the housings of the bearings may be "finned" and fans mounted on the shafts so as to propel cool air against the fins. When these means are insufficient, the bearing casings must be formed with cooling jackets through which cooling water is pumped in series with suitable cooling coils.

13. Effects of the Contact Stresses in Rolling Bearings as influenced by Lubrication.

It has just been shown that lubrication has but little influence in reducing the intensity of the contact forces between the rolling elements and the races of rolling bearings. These forces, invariably involving stresses in the material beyond its elastic limits, necessarily effect its breakdown by fatigue after a sufficient number of repetitions, the number for any particular bearing depending mainly on the load, the material and the greater or less degree of perfection of the surface finish, but being also influenced by the lubrication.

The mode of ultimate breakdown in normal cases is by small flakes of steel becoming detached from the surface affected, which is most commonly one of the race-ways, these being usually less hard than the rollers or balls. The inner surface of the flake is no doubt formed where transition takes place from the outer zone of compression to the zone of tangentially directed tension, as described in Sect. VII, 8 with reference to fig. VII, 7 (p. 205). Before a flake is detached, circumferential cracks crossed by shorter axial cracks appear on the surface. The first flakes are usually small, but if use of the bearing is continued they rapidly become larger and more numerous until the whole race-way is affected.

The broken fragments of the small early flakes doubtless expedite the formation of the later and larger ones, and they can also cause other damage to the bearing surfaces in the shape of indentations and cavities formed where the fragments are caught between the opposed bearing surfaces. Lubrication, if sufficient in quantity, is found to extend the lives of bearings subject to deterioration of this kind, probably by assisting in removal of the fragments from the paths of contact.

In some cases, arising probably in some instances from the fortuitous occurrence of an integral or approximately integral relation between the rotation of the cage relatively to one or other of the races and the number of rolling elements in the bearing, the race becomes marked with a series of small indentations matching with the contour of the rolling elements. Indentations of the same kind may occur as the result of the machine, of which the bearing forms part, being subjected for a length of time to vibration, especially if the machine is not in operation.

It is evident that lubrication cannot prevent, and can do little to mitigate, effects of these kinds which are the inevitable results of repeated stressing of the materials beyond their elastic limits in the areas of contact. Lubrication, however, does usually prevent any general attrition, or "wear" along the paths of contact of the members of rolling bearings, such as would always occur in its absence. When a pair of steel elements are run under load without lubrication, oxides are formed on the surface almost immediately, and are usually detached in flakes. They consist of nearly equal parts of ferric and ferrous oxides. The beneficial effect of lubrication in this respect is probably due to its checking the development of high local temperatures, such as always occur with dry contacts of metals (see e.g. Ref. VII, 9), as well as hindering the access of air to heated spots, and so preventing both softening and oxidation of the surface material.

In some cases a special form of surface damage known as "pitting" may occur. This consists in the formation of small crater-like cavities in the surface of one or both of the rolling elements having diameters of usually less than a millimetre and sometimes of equal depth. The same type of damage occurs (see Sect. VIII, 1), and is often more marked, on the surfaces of gear-wheel teeth (but only on the lines of mutual rolling contact), showing that it arises from the special conditions of lubricated contact there existing. In rolling bearings its presence is generally attributed to the passage of electric currents, or electric sparking, at voltages as low as 1 volt, but since the effect only appears on the areas of actual rolling contact, its originating cause is evidently the overstress in this region. It is apparently a fatigue phenomenon, usually showing itself only after some millions of cycles of stress. It is also said to occur only when a lubricant is present, and for this reason the phenomenon is of special importance for the present discussion.

A very thorough experimental examination of the question has been made by S. Way (Ref. VII, 2) using an apparatus designed for the purpose. Two steel rollers of approximately 1½ inches (3·8 cm.) in diameter and ½ inch (1·3 cm.) in width were rolled together under varied loads of the order of 1000 kg. wt. The diameters of the two rollers differed from one another by about 5 per cent, so that repetition at each revolution of the mutual contact between any two points of the surfaces was prevented. The rollers were of high-carbon, low-manganese steel, hardened and in some tests nitrided. The rotative speed was 400 r.p.m. It was found that small cracks usually began to be visible on the surfaces after less than a million cycles.

In the absence of lubrication these cracks increased in size but no different type of damage appeared during several million additional cycles.

If, after running dry until cracks appeared, or from the start, a lubricant of C.G.S. viscosity about unity at the operating temperature was applied to the rollers, pitting usually appeared within from 100,000 to 1,000,000 revolutions, the effect, however, being absent, or at least less marked, on the nitrided rollers. It was also prevented by the use of a lubricant of high viscosity (about 20 C.G.S. units). In order to test whether an oil film existed between the surfaces, an electromotive force was applied between them; it was found that as soon as load was applied, a considerable current passed, showing that metallic contact existed. By giving an extremely high polish to the bearing surface the formation of pits was prevented, or at least greatly retarded, this observation being consistent with the rarity of the occurrence of pitting in rolling bearings of good quality, except when electrolytic action can be suspected.

The investigator has put forward the theory that the formation of pits is due to the lubricant entering (under the pressure which exists at the approach-side of the region of contact, as explained in Sect. VII, 9) the previously formed cracks in the surface, and so tending to lift a fragment of metal; and he attributes the absence of pitting when a highly viscous lubricant is used to its inability to enter a fine crack. It would seem, however, that since the pressure generated in the oil is proportional to the viscosity (see equation VII, 12), the thicker oil under its higher pressure would enter any crack as readily as a thinner oil under its correspondingly lower pressure. It would appear more probable that the observed effect may be due to the lubricant of higher viscosity resisting extrusion from the contact area under the circumstances illustrated in fig. VII, 15 (p. 216), and so preventing a welding together of the surfaces with a subsequent tearing out of a pellet from one of them when the surfaces separate. The absence of pitting when no lubricant is used may be explained by the absence of welding in conditions which induce oxidation.

Another type of surface damage, known as "smearing" (already mentioned in Sect. VII, 7), occurs when under light loads and at high speeds of rotation the balls of thrust bearings are caused by gyratory forces to slide in a direction transverse to the direction of rolling. It can also occur when rolling elements, after being by some cause retarded in a clearance arc of a bearing, are suddenly accelerated on entering an arc of loading in the manner already described in Sect. VII, 9. The resulting damage to the surface is of the kind which always occurs to the surfaces of mutually sliding solids in the absence of fluent-film lubrication.

14. Secondary Forces on Rolling Elements in Rolling Bearings: Cages.

In addition to the charges which are imposed on the balls and rollers of rolling bearings in the direct lines of reaction to the load on the bearing, they are often subjected to secondary forces of smaller magnitude, associated, as a rule, with rubbing or sliding as well as rolling contacts. Instances have already been noted in the descriptions of the classes of roller bearings (Classes " A " and " B ") described in Sects. VII, 3 and VII, 4 respectively.

In the former class (fig. VII, 1, p. 201) the ends of the rollers are plane, and abut against flanges on the rollers which also have plane surfaces. The contact between these pairs of surfaces are, normally, merely accidental and the mutual pressures are small and indeterminate. The relative motion, being the same as that which takes place under definite forces in the case considered in the next paragraph, need not be separately dealt with here.

In the spherical bearings of Class " B " (see figs. VII, 2 and 3) the ends of the rollers and the faces of the flanges on the inner race are zones of spherical

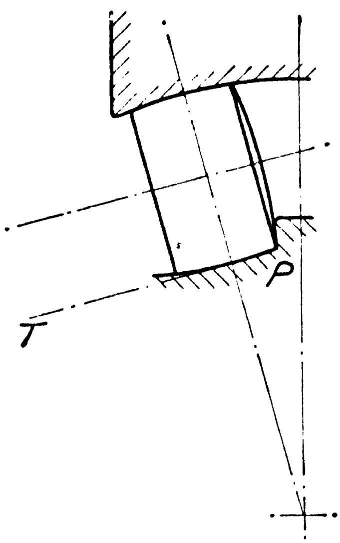


Fig. VII, 18

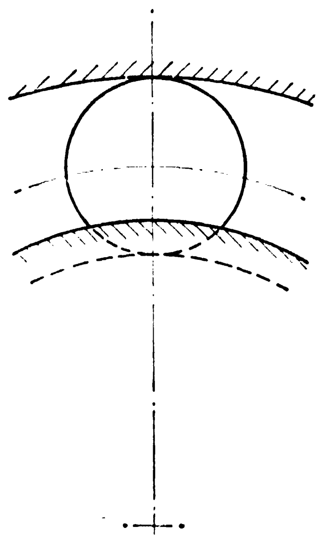


Fig. VII, 18a

surfaces, respectively convex and concave, and having, within the limits of manufacturing accuracy, the same radius and the same axis. The surfaces are thus parallel, and if any lubricating film exists between them it is of uniform thickness. The relative motion of the two surfaces is one of rotation about the radius of the spheres which passes through their innermost point of contact, i.e. about a point corresponding to the point P in figs. VII, 18 and 18*a*. The force acting between the surfaces is approximately $P \tan \alpha$, where α is the vertex angle of the cone which envelopes the roller at the middle of its length, and P is the charge at each of the contact points of the roller. In practice α is about 10 degrees, and the area of spherical surfaces in contact about 0·05 of the diametral area of the roller. The mean intensity of pressure between the parallel rubbing surfaces is consequently greater than the diametral compressive stress in the roller due to its charge, and consequently is of the order of 1000 kg. wt. per sq. cm. which, as shown in Chap. IV, is much more than a lubricant film between parallel rubbing surfaces can carry without solid-to-solid contact. The action is therefore, at best, necessarily one of partial rugulose or alternatively modified surface contact, with consequently high coefficients of friction.

It has been suggested that fluent-film lubrication might be secured in this case by making the radius of curvature of the surface of the flange of the race slightly greater than that of the end of the roller, as shown in an exaggerated extent in fig. VII, 19. That arc of the roller end-surface, which was approaching P during its rolling relatively to the flange surface, would then tend to draw lubricant into a contracting space and generate positive fluid pressures. The idea is, however, illusory, since equal negative pressures would be generated between the mutually receding arcs, as in the case of the approaching and receding arcs of a roller and race discussed in Sect. VII, 9 with reference to figs. VII, 12–14, pp. 212, 215.

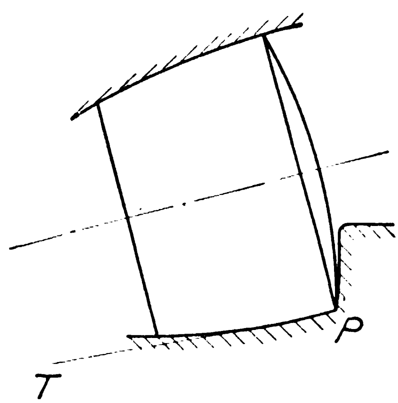


Fig. VII, 19

It is doubtless due to the losses at these secondary contacts that the coefficients of resistance of spherical roller bearings are, as shown by comparative tests, about 70 per cent greater than those of cylindrical roller bearings, or ball bearings, operated under similar conditions.

Secondary forces on the rolling elements of both roller and ball bearings arise also from the operation of their cages. The cages used in rolling bearings are of many different types and modes of construction, depending on the type, and also on the size, of the bearing, and on the speed and loading for which it is intended. Cages never take part in carrying the useful load of the bearing, but they are often subject to considerable loading arising from the centrifugal forces, or other inertia effects of the rolling members. They are usually supported, and located, only by means of their contacts with the rolling elements, but sometimes by one or other of the races of the bearing. In the former case the cage is said to be ball-riding, or roller-riding; in the latter case land-riding.

In small bearings the cages are often formed of pieces of steel strip or brass strip, pressed into shape and so connected together as to form a separate compartment or "pocket" for each of the balls or rollers. In the larger and heavier bearings they are commonly machined from solid metal blanks. Cages are also manufactured of plastic materials, especially phenolic resins laminated with a textile fabric. In all cases the design should be such as to cause no interference with the entry of the lubricant, or its proper distribution within the bearing. The contacts between the rolling elements and the cages, whereby the spacing of the former is controlled, are usually intermittent and involve only minor forces. It might seem at first sight, therefore, that lubrication of these contacts, where there is relative sliding between the convex surface of the rolling element and the concave surface of the pocket, would take place under fluent-film conditions if the interior of the pocket were machined to suitable forms and fitted the rolling element with sufficient accuracy.

Fig. VII, 20 is a generalized and diagrammatic view, being a section through a ball and the adjacent parts of the cage and races, taken in the plane of rotation of the ball, which shows one possible form of construction for achieving this object. In this hypothetical construction the surfaces S and S′ are segments of spheres of the same radius as the ball, but having their centres e and e' offset on either side of its centre C, thereby forming spaces which converge in the direction of the sliding of the ball relatively to the surface of the pocket. Apart, however, from obvious difficulties of manufacture, which might possibly be overcome, there is the more intractable difficulty that the precise position of the ball in a radial direction cannot, in many cases, be defined with any great degree of accuracy. An instance is the varying radial position under varying centrifugal forces of the balls in thrust bearings of the type discussed in Sect. VII, 5. If in such a case the cage were formed with pockets accurately fitted to the balls, it would have to carry, if the speed were other than one for which the bearing was specially designed, components both of the centrifugal forces of the balls and of the charges on them due to the load on the bearing. It is usual, therefore, to make the pockets of the cages so much larger than the balls that radial contact between them is prevented when the balls change their radial positions. For example, in a bearing having a bore fitting a one-inch spindle, the diameters of the pockets may be from 5 to 6 thousandths of an inch (0·015 cm.) greater than that of the balls. It is evident that with such differences in the radii of curvature of their surfaces, there is no possibility of fluent-film lubrication at the points of contact.

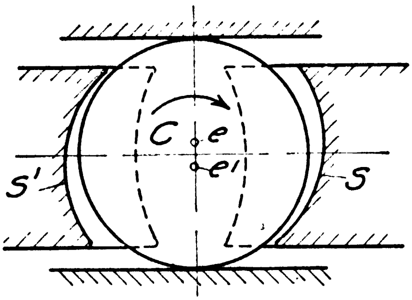


Fig. VII, 20

15. Lives and Rates of Mortality of Rolling Bearings.

It is shown by equation VII, 16 that the energy lost in permanently deforming the members of a rolling bearing during one revolution is proportional to the product of the charge P and the square root of the depth of deformation δ at the points of contact, δ being itself proportional to the square of P. The loss of energy is therefore proportional to P^2, i.e. to the square of the load imposed on the bearing. If it could be assumed that the progressive destruction of the bearing by the accumulated effects of the plastic deformations was measured by the total energy absorbed in them, it would follow that total destruction would result after a number of revolutions varying inversely with the square of the charge. In practice, however, it is found that the limiting number of revolutions, that is to say the " life " of the bearing, measured in terms of its revolutions under load, is more nearly in inverse proportion to the third power of P. In a series of bearings of the same make, all run under conditions equal in all respects except as to their load, the lives measured in hours or days will, of course, vary in the same proportion, i.e. as $1/P^3$. The law is obviously only one of averages, and the individual lives of the members of the series will differ widely in length—in fact from zero up to many times the average life.

It is said (Ref. VII, 3, p. 68) that if a large number of similar bearings are tested under equal conditions, the longest life of any of them will not usually exceed four times the average life, and about 90 per cent of the number may be expected to live longer than one-fifth of the average period.

As in the actuarial treatment of other questions of a similar statistical kind, it is frequently convenient to deal with the subject in terms of " rates of mortality " rather than in terms of average lives, the rate of mortality being defined as the number of lives which terminate in a given period, for example, one year. The law, derived from experience, which governs the average lives of similar bearings of the rolling type operated under similar regimes, but carrying different loads, is then expressed as:

The rate of mortality is proportional to the cube of the load.

Since the rate of mortality, measured by time, is obviously proportional to the speed of the bearing, it follows that to keep that rate at a fixed value (determined, for instance, by economic consideration of the costs of replacements as compared with the cost of bearings of greater load-capacity), the loads on the bearings must be assigned, relatively to a bearing run at some standard speed, so as to be inversely as the cube roots of the speeds.

This rule agrees fairly closely with the published recommendations of manufacturers as to the relation between the maximum working loads, and the speeds at which their bearings can be operated with a given length of life.

Attention to due cleanliness and lubrication are practically the only means within the control of the user of reducing the rate of mortality of bearings, or rather of assuring that the rate of mortality is the lowest of which the construction of the bearings admits. Since the costs involved by interruption of service are often the heaviest part of the total cost of replacement, users of bearings often find it economical to replace them at times when they are not in operation, at fixed periods considerably shorter than their probable lives. Although the gross rate of mortality is increased by such a system, the rate of mortality during operation, which is economically more important, may be reduced considerably by it.

16. Methods of Application and Distribution of Lubricants in Rolling Bearings.

Although lubrication is recognized as being indispensable in the operation of rolling bearings, the universal experience of users confirms the conclusions to be derived from the preceding sections of this chapter, namely, that a very small quantity of lubricant suffices for the requirements of any rolling bearing, and that larger quantities are positively detrimental to its mechanical efficiency. The ideal to be aimed at in the distribution of the lubricant in the bearing is that it should be applied to the rolling elements continuously in the form of a mist. Apart from its mechanical effects in increasing the durability of the bearing, and, in some cases, to a lesser extent reducing frictional resistances, a suitable lubricant, supplied even in minimal quantities, protects the members of the bearing from moisture and other corrosive agents, and from the erosive effects of dust.

Both oil and semi-solid greases are commonly used for the lubrication of rolling bearings. When oil is used, it should be entirely of mineral origin, since all organic oils, whether animal or vegetable, contain, or are liable to form, organic acids capable of corroding steel. For the same reason the various chemical "additives" which are sometimes compounded with mineral lubricating oils, for the sake of the organic acids which they contain, are generally regarded as inadmissible in rolling bearings, although it might seem that the beneficial effects attributed to them, as promoting the adhesion of a lubricating film under conditions of intense pressure between solid bodies in contact, would be especially marked in rolling bearings.

Given chemical purity, as established by the usual tests, the only qualities that need particular attention in the application of a mineral oil to a rolling bearing are its viscosity and its vaporization constants, both taken over the whole range of temperatures to which the bearing is subjected, and especially at its normal temperature of operation. Generally speaking, the viscosity should be as low as is consistent with retention of the oil in the bearing by the means provided for its sealing against waste of oil in the form of spray. Oil is used in rolling bearings when the speed of rotation is high, or when the bearing is exposed to high temperatures, or again when it is desirable to use the same lubricant in the bearing as in other parts of the machine to which the bearing is attached. Greases are preferred, especially in small and low-speed bearings when they are exposed to dust or condensing vapour or to spraying with water, since grease more readily lends itself to the formation of effective seals than does oil.

The greases most commonly used are compounded of mineral oil and soap, the soap having usually either a lime or a soda base. Lime soaps melt at about 100° C. (212° F.), while soda soaps have a melting range extending from about 150° to 180° C. (300–350° F.). Both have a tendency to separate into soap and oil, which is considered to be a necessary feature as providing a small and slow supply of free oil for distribution over the contact surfaces of the bearing. The rapidity of the separation must, however, not be so great as to lead to loss of oil from the bearing-seals, or to the accumulation of a stiff residue of soap in the housing. Roughly comparative estimates of the rate of automatic separation of oil and soap from a grease can be made by allowing it to stand for a month or two in a container, and observing the film of free oil which comes to the surface. It should be readily visible but not in sufficient quantity to flow freely when the vessel is tilted.

The use of lime-base greases is usually limited to bearings in which the temperature with continuous operation does not exceed 45° C. (110–115° F.). Both lime and soda greases can be used at temperatures as low as −20° C. (0° F.). Some soda greases are said to give satisfactory results at temperatures up to 105° C. (220° F.), but usually they are not used above 70° C. or 160° F. At higher temperatures than this rolling bearings are usually lubricated with oil.

When grease is used the bearing should have the whole interior of its housing filled with as much grease as can be retained in it with the covers removed. When the covers are replaced and the bearing put into operation, the bulk of the grease placed between the rolling elements is thrown out into the spaces left empty inside and around the covers. Until this displacement has taken place there will be abnormal resistance to rotation, and considerable heating of the bearing. If its temperature remains unduly high, showing that there is insufficient space inside the covers or elsewhere to receive the excess of grease, some of the grease must be removed, but a sufficient quantity must be left in the bearing to ensure that the cages will run in contact with walls of grease abutting against each of their lateral surfaces. In normal running small quantities of oil separate slowly from these walls and are wiped off by the cages, which transfer the oil to the rolling elements. The condition to be attained as ideal is that of a mist of oil particles diffused throughout the interior of the bearing so as to be caught by, and deposited on, the advancing sides of the rolling elements, but it is doubtful in most cases whether more than the thinnest of capillary films of oil can be delivered by this means to the areas of rolling contact.

"Make-up" and replacement of the grease may be effected in the same manner as the original filling, being usually only necessary at intervals of several months. Since all soaps, and consequently all lubricating greases, contain water, the grease in a bearing may, especially at high temperatures, become hard by loss of water, even before its content of free oil is exhausted. It then requires to be replaced by fresh grease.

"Make-up" can also be effected by means of a grease pump, but this is inadvisable, except for small bearings, because of the risk of insufficiently filling, or of overfilling, the bearing. Some indication of the need for filling can usually be obtained from observation of the temperature of the bearing, a noticeable fall below the normal operating temperature being evidence that the body of grease is no longer in contact with the rotating members of the bearing, and that consequently lubrication of the internal parts is no longer taking place—as well as being a proof that the use of grease as a lubricant does not lower the coefficient of resistance of an unlubricated bearing.

In large and important bearings in which it is undesirable to incur the excessive resistance at starting that results from overfilling, or to shut down for the purpose of adjusting the quantity of grease, a grease valve is sometimes fitted which automatically allows any excess of grease that may be present to be forced out into a suitable receptacle.

When oil is used as the lubricant for a rolling bearing, it is usually distributed to the rotating parts from a reservoir formed in the bottom of the housing (of the same general construction as in sliding bearings), the reservoir being sufficiently large to maintain the oil at nearly constant level. Distribution of the oil within the bearing is then effected, either by the cages being allowed to dip into the oil to a slight depth (so that the rolling elements are submerged for a fraction only of their diameters), or by special means, such as a thrower-ring fixed on the shaft, and similarly arranged to dip slightly into the oil. The oil is then distributed through the interior of the bearing in the form of a light spray thrown off by the rolling elements or by the thrower-ring.

At high speeds, however, too much oil for cool running is distributed by such means, and the rolling elements especially cannot be allowed to dip into the oil. Recourse may then be had, in the case of a horizontal bearing, to an oil flinger on the shaft arranged so that it cannot deliver oil directly from the reservoir to the working parts, but lifts it to an oil pocket in the upper part of the housing. From the oil pocket the oil is led by gravity to a point from which it drips on the cages, and is delivered by them to the rotating elements. Sometimes a wick-feed is arranged instead of a gravity feed, or, in small bearings, as a substitute for both the oil flinger and the pocket. In vertical footstep bearings a small centrifugal impeller fitted on the shaft below the bearing may be arranged to lift oil from a reservoir formed below the end of the shaft and discharge it into a second reservoir at a higher level than the bearing. This, and other constructions that may be used, do not differ in any essential from corresponding constructions which have been described and illustrated in Chaps. IV, V, and VI as being used for the circulation of oil for sliding bearings except with respect to the comparatively small flow of oil with which they are required to deal.

In all rolling bearings which are sufficiently important to warrant the costs involved, a system of oil circulation by means of compressed air should be applied. In this system one or more compressed-air injectors both lift the oil from a low-level reservoir and distribute it in the form of mist into the bearing housing, from which it flows under gravity, or is discharged by a small pump, back to the reservoir through suitable coolers and filters. The compressed air escapes through the clearances of the oil-retaining devices on the bearing, and in so doing prevents the entry, against its current, of dust or water-spray. The filter removes from the oil the fine metallic particles produced by the fatigue-failure and attrition of the working surface, and it is found by experience that the life of the bearing is thereby lengthened considerably.

Cooling the circulated lubricating oil assists in cooling the bearing, but, as pointed out in Sect. VII, 12, the volume rate of oil circulation is too small to assist materially in the necessary cooling of large, high-speed bearings. Other means employed for their cooling are of the same kinds as those already described in Chaps. IV–VI as being used for the cooling of large sliding bearings, and need no further comment.

The means employed for preventing escape of oil from the housings of rolling bearings are also the same as those which have been described and illustrated in Chaps. V and VI, to which the reader is referred.

It will be evident from the descriptions which have been given in this section of the methods which are preferred for the distribution of grease or oil to the co-acting parts of rolling bearings, that the quantities of lubricant which are necessary, and which give the best results in their operation, are extremely small, and probably smaller in almost all cases than the quantities which have been calculated in Sect. VII, 10 as being capable of being retained on the surfaces of the rotating elements.

The assumptions which have been made in Sect. VII, 9 as to the quantities of lubricant that are available for lubrication in the cases illustrated in figs. VII, 12–16, also appear to be fully warranted by the same facts of operational experience.

CHAPTER VIII

Lubrication of Various Mechanisms other than Bearings

1. Lubrication of Gear-wheels and Worm-wheels: Manner of Action.

The conditions of contact between the teeth of a pair of gear-wheels, and between the teeth of a worm-wheel and the thread of a worm, partake of both sliding and rolling. In the action of gear-wheels the rolling motion usually predominates and the conditions controlling lubrication approximate to those which exist between the rollers and races of rolling bearings as discussed in Chap. VII. In worm-wheels the sliding action predominates. In both cases (therein differing from the case of rolling bearings) the relative sliding motion plays an essential part in the distribution and operation of the lubricant.

The features of the kinematic action of the teeth of gear-wheels which are most closely relevant, to the action of the lubricant are illustrated in figs. VIII, 1–3, which represent portions of a pair of involute-toothed gear-wheels in mutual engagement, the lower being the driver, and the motion being from left to right.

On the left-hand side of fig. VIII, 1 the driver-tooth D_1 and the driven-tooth D_2 are just beginning to come into mutual action at the point A (which, of course, represents the projection on the plane of the figure of a line of contact on each tooth parallel to the axes of the wheels), with relative sliding, in the direction indicated by the pair of arrows, combined with mutual rolling. In fig. VIII, 2 the teeth D_1 and D_2 make contact at the pitch line B of the wheels, and the motion is instantaneously one of pure rolling. In fig. VIII, 3 contact between the teeth D_1 and D_2 is about to terminate at C, the relative sliding between them, still accompanied by rolling, being in the direction shown by the arrows. The line ABC is the path of the point of contact, which in this case of involute teeth is a straight line.

It is to be observed that in the "arc of approach", A to B, the line of contact traverses the whole surface of the driven-tooth D_2 from its outer tip at A to the pitch point B, but only a relatively small portion of the driving tooth D_1, this portion being on the inner side of its pitch circle. In other words, the surface of the driven-tooth slides faster past the line of contact than does the surface of the driving-tooth; this difference is shown by the difference of length of the arrows on the two teeth in fig. VIII, 1. In the "arc of recess", B to C, on the contrary, the driving tooth D_1 traverses the contact point by the whole length of its surface from its pitch radius to its outer tip, while the driven-tooth D_2 in this case moves only a small distance relatively to the contact point. In both cases, therefore, the surface which is moving most rapidly towards the line of contact, and which, consequently, can be most effective in bringing lubricant to it, is moving from the dedendum spaces of the driven wheel. For this reason the lubricant should be fed into the dedendum spaces of the driven gear-wheel rather than into those of the driver, as may be done by directing a jet of oil under pressure into these spaces, i.e. from the approach side of the engaging gears and into the tooth-spaces of the driven gear.

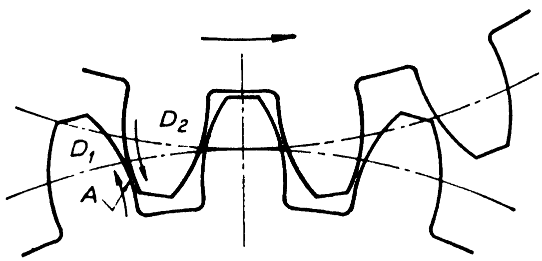


Fig. VIII, 1

In all gear-wheels and worm-wheels the kinematic conditions necessitate that the mutual contacts are, at the best, merely "line contacts", and they may be only "point contacts".

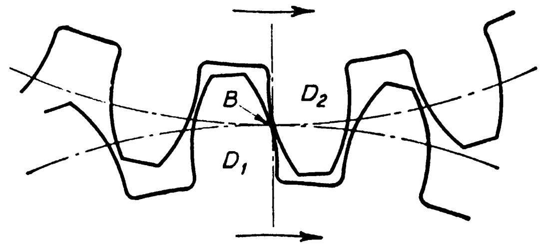


Fig. VIII, 2

From what has been said in preceding chapters, therefore, it will be apparent that true fluent lubrication is never attainable, and metal-to-metal contacts must be expected to occur. The conditions under which these contacts take place are, however, in one respect more favourable than those of the pure rolling contacts of rolling bearings, as described in Chap. VII, inasmuch as the lubricant is continuously brought to the contact lines, or points, by the mutual sliding action described above, while in pure rolling contacts its presence depends on its having been on the surfaces prior to their making contact and its remaining there while the contact is in progress. In another respect the conditions of action in gear-wheel teeth are usually less favourable than in rolling bearings, in so far as it is hardly possible to give to the surfaces of gear teeth either the accuracy of form and smoothness, or the extreme hardness, which are usual in the elements of rolling bearings.

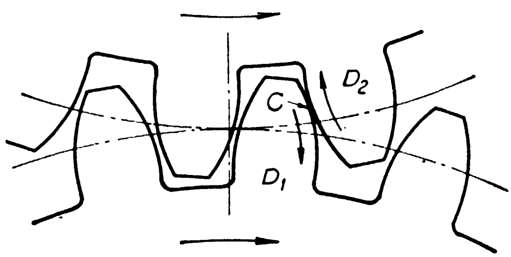


Fig. VIII, 3

On account of the complexity of the actions involved, and of the relatively large size of the unavoidable irregularities of the mating surfaces as compared with the thickness of the films of lubricant which can exist between them, investigations of the action of lubricants in gears by purely theoretical calculations must be expected to be nugatory, and recourse must be had to experiment guided and interpreted by theory. Many such series of experiments, directed from various points of view, have been carried out.

The conditions of stress which exist at the lines of contact, being the same at the pitch lines of the teeth of gear-wheels as at the lines of contact of a pair of cylinders of corresponding radii of curvature when rolling together, have already been discussed (Sect. VII, 13) in connexion with the investigations of Way, and his main results are borne out by direct experience with gear-wheels of materials similar to those which he employed. The phenomenon of " pitting " found by Way, as characteristic of lubricated contact of steel cylinders under heavy loads, and noted in Sect. VII, 13 as occurring in ball and roller bearings, is found to occur also in gear-wheel teeth at or near their lines of pure rolling. Its occurrence in the spur-gearing of automobile change-gears appears to have been first noted and discussed by Lanchester (Ref. VIII, 1) as the incipient stage of the breakdown of the surface of the teeth in the manner which he designated as " erosion ", but which is now generally called " scuffing " or " scoring ". Lanchester noted that pitting occurred only at, or close to, the pitch lines of the teeth, and he inferred that the absence of sliding at those lines deprived the contact lines of lubricant. He suggested that the pits were formed at points where prominences of the opposed surfaces came into contact, and, in the absence of lubricant, welded themselves together, with the result that one of the prominences was torn out by the other when the surfaces separated, the two together thus forming a prominence of larger dimensions and effecting increased abrasion at each new contact. Ultimately the whole of the surfaces of the teeth outside the pitch line can be supposed to become scored or " scuffed " by this means. These opinions appear to be largely confirmed by later experimenters, and to be now generally accepted.

It is, however, apparent that investigations by means of direct experiment with gear-wheels present great difficulties with respect to control and observation of the conditions of lubrication, and must be, furthermore, very onerous and expensive, both in the cost of the apparatus and in its operation. An extensive series of experimental investigations into the behaviour of gear-wheels, employing, like those of Way (Sects. VII, 1 and 13), pairs of contacting cylinders, but in which relative sliding as well as rolling was provided, has been carried out in the research laboratory of Messrs. David Brown & Sons (Huddersfield) Ltd., and the author is privileged by that firm to make use of some of the reports and memoranda which have been drawn up on these tests.

The apparatus employed, of which an earlier form has been described in publications of 1935 (Ref. VIII, 2), consists essentially of two cylindrical discs revolving on parallel axes and making contact along generating lines of their surfaces. They are forced into mutual contact by mounting the smaller of the discs in bearings in a swinging frame, which is pressed by a loaded lever in the direction of the other, larger, disc, which revolves on fixed bearings. This larger disc is driven by a variable-speed motor, and the smaller disc is driven from the shaft of the larger by either gear-wheels or sprocket-chains which can be varied to give any desired ratio of speed to the two discs, either in the same or in opposite directions of rotation. The apparatus is so arranged that both the force with which the discs are pressed together and the driving torque can be measured with accuracy.

Lubrication of the discs is effected either by supplying oil from a pressure pump through nozzles directed between the discs on their approach sides, or alternatively, by allowing the larger of the discs to dip into a bath of oil.

It will be seen that, by these arrangements, the conditions of action of a gear-wheel tooth, either in its approach to a pitch line, or in its recession therefrom, can be simulated over the whole surface of one of the discs, by selecting the driving gears of the pair so that the relative sliding takes place either forward or rearward with respect to the line of contact. To represent the materials used in practice for heavily loaded spur, helical and bevel gears, the discs were made of 4½ per cent nickel-chromium steel, heat-treated to give a tensile strength of 100 tons in.$^{-2}$ ($1{\cdot}6 \times 10^4$ kg. wt. cm.$^{-2}$). The force per unit length of the line of contact, with which the discs are forced together, is converted to a unit, standardized under the name of "surface stress", by a formula in which its measure Sc is

$$Sc = P\,(\rho + \rho')^{0{\cdot}8},$$

P being the load in pounds per unit length of the line of contact, and ρ and ρ' the reciprocals of the radii of the two discs in inches.

In tests made specifically to determine the comparative utilities of different oils for lubricating gear-wheels, the figure of merit adopted for each oil was the Sc stress which a pair of discs would withstand, when lubricated by that oil, before scuffing took place. For these tests the larger of the pair of discs was 4 inches, and the smaller 2 inches in diameter, and their active faces 0·187 inch wide, both being ground to show a surface roughness of 40 to 60 micro-inches (H_{max}) by the Topograph surface-finish instrument. The discs were lubricated by means of a pump discharging directly into the zone of pressure between them, while at the same time the larger disc dipped ⅛ inch into the oil bath.

For the standard tests the larger disc was driven at 212 revolutions per minute, and the smaller at 70·7 rev. per min. in the opposite direction, or one-third of the rotative speed of the larger disc.

In the report of the tests the speed of rolling is defined as that with which a point on either surface moves with respect to the point of contact, which is fixed in space; the speed of sliding as that with which it moves with respect to a point on the other disc.

Thus, by this definition, while the speed of sliding is numerically the same for both discs, their speeds of rolling are different.

In the following digest of the results, and in the accompanying diagrams, both the sliding and the rolling speeds are regarded as being the same for both discs, being defined, as illustrated by fig. VIII, 4, as being respectively

$$\text{speed of sliding} = \omega_1 r_1 - \omega_2 r_2,$$
$$\text{speed of rolling} = \tfrac{1}{2}(\omega_1 r_1 + \omega_2 r_2),$$

in which r_1 and r_2 are the radii of the two gear-wheels having angular velocities ω_1 and ω_2 respectively, the latter being considered to have the same sign when the two wheels are geared together, and opposite signs when they are connected by sprockets and chains. Thus, if in the former case both the two discs and the two connecting gear-wheels are of equal size, the speed of sliding of both discs is zero, and the speed of rolling of each of them is $\omega_1 r_1 = \omega_2 r_2$.

If the discs are connected by sprocket chains so that $r_2 = r_1$, $\omega_2 = -\omega_1$; on the other hand, the speed of sliding is $2\omega_1 r_1$, and their speed of rolling zero.

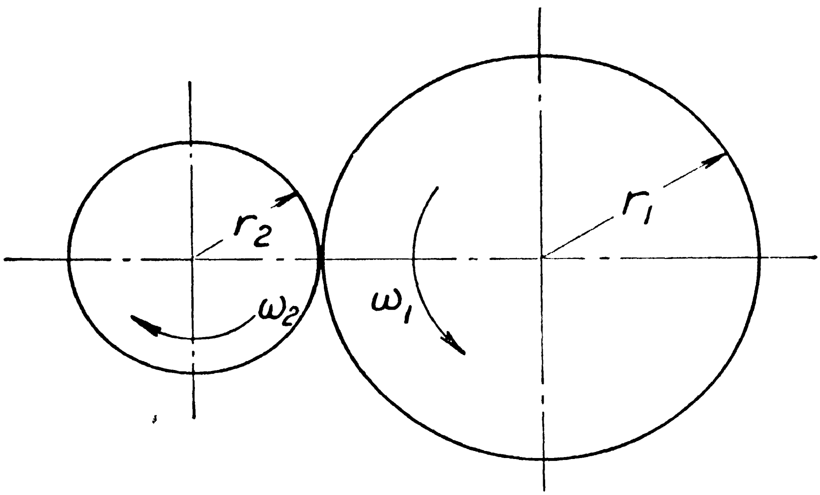


Fig. VIII, 4

Under the conditions of operation described above the load was gradually increased in stages of 1000 *Sc* at a time until failure occurred by scuffing. The lubricants used in the principal series of tests were some 20 different mineral oils, both standard and blended; 10 or 12 mineral oils containing lead naphthenate; and a similar number of proprietary, so-called "extreme-pressure" lubricants.

The apparatus has been used not only for investigations of the modes of lubrication of gear-wheels and worm-wheels, and for the determination of the comparative merits of various lubricants, but also for study of the various modes in which the surfaces break down when they are overloaded or improperly lubricated. These various modes may be classified broadly as follows:

Smooth abrasion, sometimes referred to as "normal wear".

Pitting, already briefly described.

Scuffing, also sometimes called *smearing* or *scoring*, and in America *galling*, consists of a torn or scraped appearance of the surfaces in the direction of sliding.

Ridging consists of the formation of a groove along the pitch line of one tooth, and the building-up of a ridge of material in a corresponding position on the opposed tooth or teeth.

In the mode of wear called "smooth abrasion", which occurs in gear-wheels, but not in worm-wheels, the surfaces remain uniformly smooth, and even polished, and the wear may go on consistently with continuous use of the gears for a long period. The rate of wear can often be reduced by the use of a more viscous lubricant.

Of "pitting" it has been said by H. E. Merritt, one of the investigators using Messrs. David Brown & Sons' apparatus (Ref. VIII, 3), that it may occur with any gear material, and in any gear. It is said to be associated with the formation of cracks which develop at the surface, spread beneath it to a depth related to the depth of penetration of intense stresses, and again emerge at the surface, usually forming a circuit surrounding a flake of metal which becomes detached.

"It is not uncommon for pitting to develop early in the life of a gear, reach a certain point, and then cease to spread. Sometimes it disappears as the surface layers of metal are worn off. Arrested pitting is nearly always found in traction worm-gears and turbine reduction gears, and is usually attributable to local high spots (due to errors in either the cutting or the running position of the gears) which result in stresses sufficient to produce pitting before the irregularities have had time to be removed by surface abrasion."

"Scuffing" is a condition of wear more severe than surface abrasion, and its onset in any marked degree is generally regarded as setting a limit to the continued use of a pair of gears, at any rate with the same lubricant. It is found to occur at both high and low velocities of sliding relatively to the rolling speed. The general conclusions reached in the investigations as to the onset of scuffing under conditions of action of the disc similar to the conditions in spur gearing (including helical and bevel gears) were:

(*a*) In any series of oils, either from the same crude or by blending the same two oils in different proportions, the protection afforded against failure by scuffing is increased with viscosity.

(*b*) Approximately 10 to 20 per cent improvement in the scuffing load of an oil is brought about by the addition of lead naphthenate or other lead soaps to the oil.

(*c*) The load-carrying capacity of an oil can be more than doubled by the addition of certain "extreme-pressure" agents to the oil.

(*d*) Certain additives will prevent failure by scuffing, but promote failure by smooth abrasion, even at loads not greatly in excess of the ultimate load-capacity of the base oil.

(*e*) The load which can be safely carried by a lubricant without fear of failure by scuffing is dependent to a large extent on the mating disc materials, the slide-roll ratio, rubbing-speed and temperature being the same in each case.

(*f*) For a given slide-roll ratio, combination of materials, and lubrication, the surface-stress Sc and the sliding velocity V at failure by scuffing can be related according to the formula $ScV^n = K$ (n and K being constants) within a range of sliding velocities usually encountered in gearing practice.

(*g*) For a given slide-roll ratio, combination of materials, lubricant and mode of lubrication, there is probably a certain sliding velocity which affords the greatest protection from failure by scuffing.

(*h*) Certain additives in oil tend to reduce the index n in the expression ScV^n for a given base oil, thus making the scuffing speed less dependent on the sliding speed. Other additives increase the value of the constant K but have no effect on the index n.

With the speeds stated above (viz. 212 r.p.m. for the 4-inch disc and —70·7 r.p.m. for the 2-inch disc), most "straight" mineral oils of viscosities of the order of 0·25 to 1·0 poise at the test temperature carried safely a load of the order of 10,000 *Sc.*; similar oils containing lead methenate, of somewhat higher viscosities, about 11,000 or 12,000 *Sc*, and oils to which certain "extreme-pressure" agents were added (with resulting viscosities exceeding 1·0 poise), carried loads of from 15,000 to 20,000 *Sc.*

For investigation of the effects of combined sliding and rolling on materials of the kinds which are used in worm and worm-wheel gearing, the same apparatus was used but with different discs. In this case the disc representing the worm-wheel was of phosphor-bronze and was 8 inches in diameter (this disc being on the spindle carried in the fixed bracket of the apparatus), while the disc representing the worm was of 2 per cent nickel-molybdenum steel, 4 inches in diameter, and was carried on the swinging bracket. This steel disc had a minimum hardness of 763 V.P.N. and was given a very fine surface-finish of 5 to 10 micro-inches (H_{max}). The bronze disc had a hardness of 100 Brinell and a surface finish of 10 to 20 micro-inches. The two disc spindles were fitted with sprocket wheels and sprocket chains so as to give the disc surfaces determinate slide-roll ratios.

It might appear at first sight that there is a wide difference between the mode of action of the teeth of gear-wheels, in which both the sliding and rolling are in planes normal to the axes, and the slide-roll contact in a worm-gear, in which the main component of the relative sliding at any point is parallel to the axis of the worm-wheel, with only small components in its radial planes, while the mutual rolling is almost wholly in the latter of these directions. It becomes evident, however, on closer consideration that this difference is only relevant to the issue under investigation in so far that, in the case of a pair of gear-wheels, the rolling and sliding are both transverse to the line of contact of the teeth, while in the worm and worm-wheel, the relative sliding is mainly parallel to the line of contact of tooth and thread, but the rolling is transverse thereto.

Figs. VIII, 5, 6, and 7, will make this point clear, these figures representing the working face of a tooth of a worm-wheel. In fig. VIII, 5 are shown the successive positions of the line of contact of a worm-wheel tooth with the mating worm-thread, the precise form and arrangement of these depending, of course, on the particular design of the gears. As the engagement of the wheel tooth and worm-thread progresses, the line of contact proceeds from the position near the tip to that near the root of the tooth. Fig. VIII, 6 shows the components of sliding at one of these lines of contact, due to the rotation of the gear-wheel, the worm being imagined to slide along its axis without rotating. Fig. VIII, 7 shows the directions of resulting sliding, along the same line of contact, due to the rotation of both wheel and worm.

The difference which is thus illustrated between the conditions in gear-wheels and worm-wheels, with respect to the relation between the directions of rolling and of sliding, is, however, in actuality more apparent than real. In both forms of gearing, and also in the experimental discs designed to represent them, the so-called line of contact is merely an imaginary line passing through

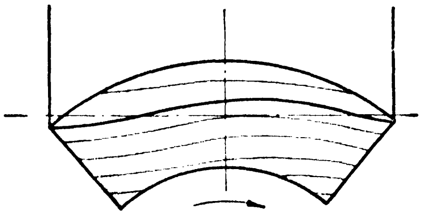


Fig. VIII, 5

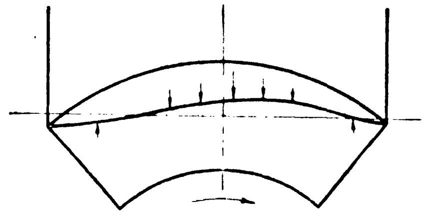


Fig. VIII, 6

a series of protuberances on which the actual contact takes place. The contact forces are borne by a series of meeting pairs of these prominences, and the action will not be affected appreciably by the directions in which the rolling and sliding respectively take place relatively to these individual prominences. The only distinctive feature between one case and another, so far as the lubricating action is concerned, is not the relative directions of the sliding and rolling speeds, but only their relative magnitudes.

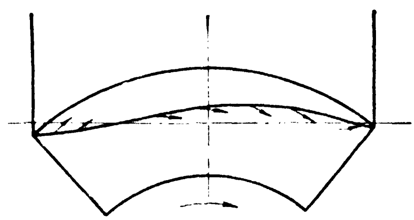


Fig. VIII, 7

In the investigations described, the discs representing worm-gearing were tested for determination of the frictional resistance for about 100 different oils; in addition, tests of wear and abrasion were made with several oils, both standard and blended, and compounded with lead naphthenate and other "extreme-pressure" additives. The loads were increased in stages of 600 *Sc* until a rapid increase of motor torque was observed, the oil temperature being maintained at 155–165° F. (70–75° C.).

The main results of the observations are summarized as follows:

(*a*) The coefficient of friction for any given oil, with any slide-roll ratio, falls progressively as the rubbing speed increases.

(*b*) The coefficient of friction given by any oil under worm-gear conditions is more dependent on the composition and hardness of the bronze member (large disc) than upon those properties of the case-hardened steel member (small disc), other factors, such as surface-finish, being constant.

(*c*) Of all the worm-gear lubricants tested, castor-base oils gave the lowest coefficient of friction.

(*d*) "Additives" in the oil may either increase or lower the coefficient of friction.

(*e*) Oils exhibiting a mean coefficient of friction greater than about 0·03 permit rapid abrasion of the bronze member.

(*f*) Certain additions in base oils increase the load-carrying capacity of the oil under worm-gear conditions though they reduce it under gear-wheel (steel-to-steel) conditions.

The conclusions (*a*), (*d*), and (*e*), stated on p. 239, for gear-wheel conditions, apply also to worm-gear conditions.

The observed variations of the coefficients of resistance under worm-wheel conditions with varying sliding speeds, and different slide-roll ratios, are shown in figs. VIII, 8 and 9. In the tests therein recorded the contact load was 600 *Sc* and the oil temperature 155–165° F. (70–75° C.) in all cases.

The viscosities of the oils in each class varied from about 0·40 to 1·5 or 2·0 poises at 140° F. (60° C.).

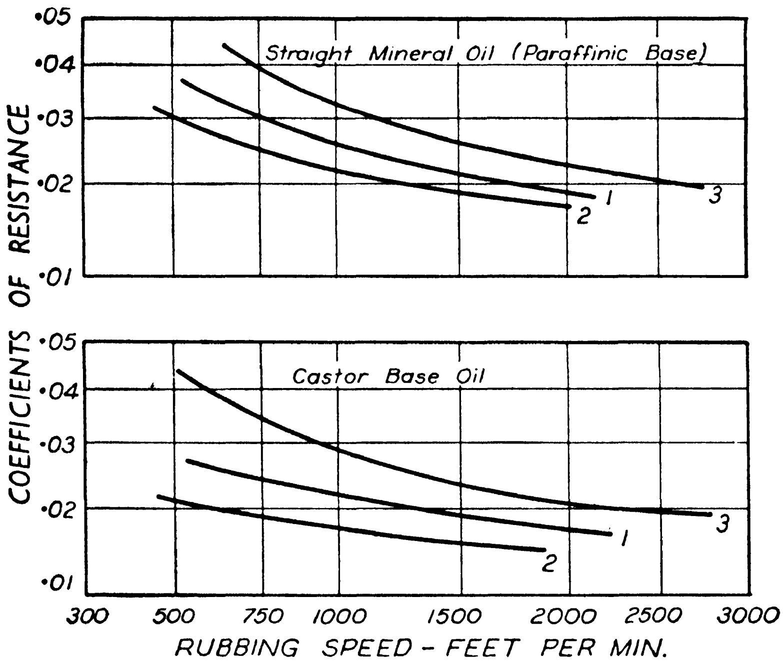


Fig. VIII, 8

It will be seen from the curves of figs. VIII, 8 and 9 that, at any given sliding speed, the coefficient of resistance is greater the higher the ratio of sliding to rolling speed, being approximately 50 per cent greater for the curves "3" in which the sliding to rolling ratio is 5·9 (calculated by the formulæ given on p. 237) than for the curves "2" in which that ratio is only 0·92, and of intermediate value for the curve "1" in which the ratio is 1·55. This rule is seen to hold for all the four classes of lubricant employed. In the curves "3" in which the slide-roll ratio (5·9 to 1) is so high that the motion approximates to pure sliding, comparison is naturally suggested with the results that have been found in bearings with similar materials and lubricants.

It is at once seen that frictional resistance in worm-gears is of a higher order (at least 10 times greater), than that found in bearings in which fluent-film action is obtained. It is indeed a striking fact that at the lower sliding speeds of 500 to 1000 feet per minute (250 to 500 cm. sec.$^{-1}$), the coefficients of resistance are of practically the same magnitude (approximately 0·03) as those which have been noted in previous chapters (e.g. Sect. IV, 16) as characteristic of plane bearings running at similar speeds, in which no convergence can be obtained, and in which one member is of steel and the other of bronze, as in the worm-gears, or one member of steel and the other of soft bearing metal.

All the quantitative results of these investigations, as well as the reported observations on the conditions of the surfaces after operation under various intensities of loading, appear to be consistent with the interpretation offered

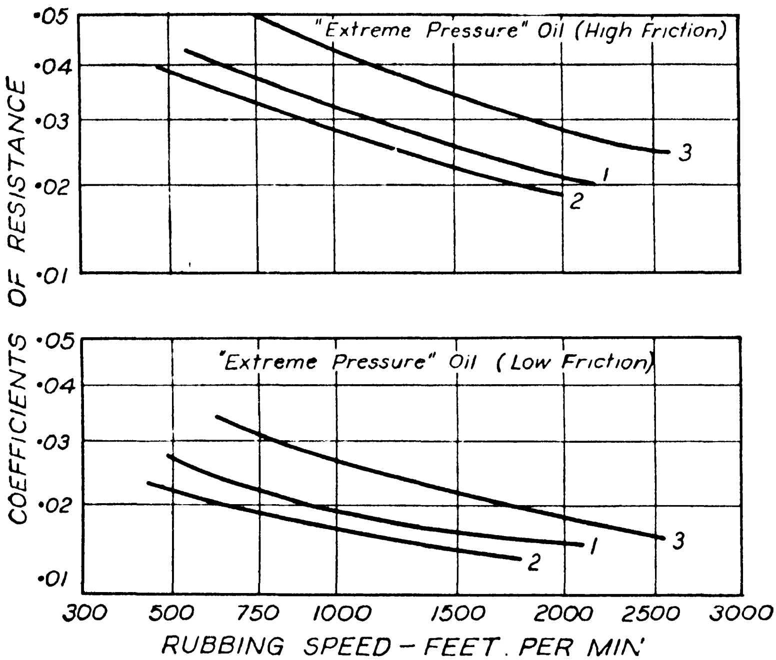


Fig. VIII, 9

in Sects. IV, 15 and 16 of the behaviour of plane sliding bearings without fluent-film lubrication.

In these tests the observed coefficients of resistance show little relation to the coefficients of viscosity of the lubricants, consistently with the conclusion that the resistance is mainly due to modified solid friction at the points of contact of protuberances of the surfaces.

It is to be further noted in fig. VIII, 9 that the oils containing " extreme-pressure " additives fall into two groups, one of which gives coefficients of resistance even lower than those of the " castor-base " oils (fig. VIII, 8), while the other group shows the highest resistances of the whole series. Presumably such additives consisted of metallic soaps, especially lead-soaps, or of compounds of sulphur or chlorine. In so far as these compounds prevent the onset of scuffing, it is presumably by their chemical actions on the metallic protuberances preventing the welding which would take place between clean metallic contact points. It is to be expected that, at the same time, the sliding resistances of the contacts would be changed, but not necessarily reduced.

2. Supply of Lubricant to Gear-wheels and Worm-gears.

The mode of application of the lubricants for the teeth and threads of gear-wheels and worm-gears depends mainly on the maximum peripheral speeds of the wheels, which also in general determine whether the gears are to be enclosed (as they usually should be) or merely shielded. The quantities of lubricant actually required at the lines of contact are small compared with those necessary for the formation of fluent films, but the necessary total flow for the cooling of the teeth or threads of the gears may be large, owing to the relatively high coefficients of resistance.

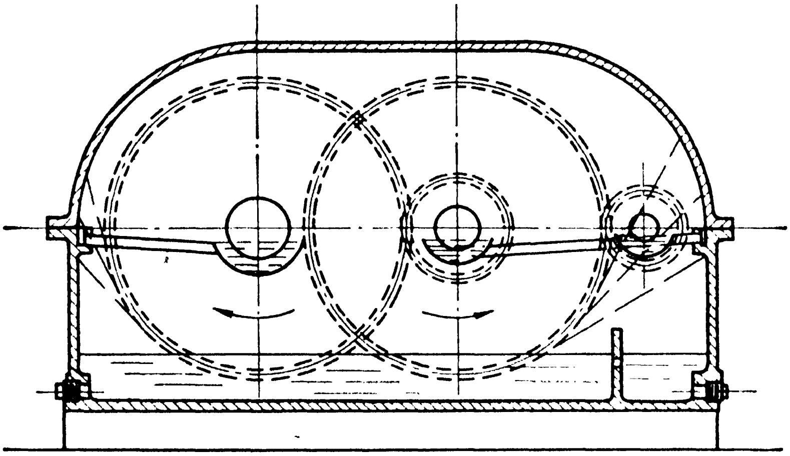


Fig. VIII, 10

It has been noted in the preceding section that, for the purpose of lubrication, the lubricant must be delivered to the inner faces, or roots, of the driven teeth; too large a quantity, however, particularly in the case of very wide, or unusually close-fitting teeth, may result in oil being trapped in such positions, and produce increased resistance with excessive vibration and noise.

For the purpose of cooling, it is usually not important to what particular portion of a tooth or thread the oil is applied so long as a sufficient area of its surface is flooded. It is therefore often desirable to provide two sources of supply to a heavily loaded gear, one of limited amount, for lubrication, and the other in larger quantity, for cooling.

When a gear-set is totally enclosed and the peripheral speed is not higher than 50 feet (1500 centimetres) per second, it is usual to arrange for one or more of the wheels to be partially submerged in a bath of oil contained in the base of the gear casing, the teeth of such wheel, or wheels, then splashing large quantities of oil to the upper parts of the case, where it is caught in suitable trays or ledges, and distributed from them to the higher and smaller gears and the bearings of the set by gravitation (see fig. VIII, 10). With this arrangement, however, the supply of lubricant to the contact lines of the teeth is often left to chance, and may easily be insufficient or even totally wanting. Oil merely dropped on the teeth will not reach their roots unless the inward component of the acceleration of gravity is greater than the centrifugal acceleration. In other cases oil may be pumped from a reservoir in the base of the casing to the points where it is required, but the previously mentioned splash system is equally satisfactory, within the limits of speed to which it is applicable, provided that care is taken to ensure that oil is actually delivered to the inner recesses of the driven teeth.

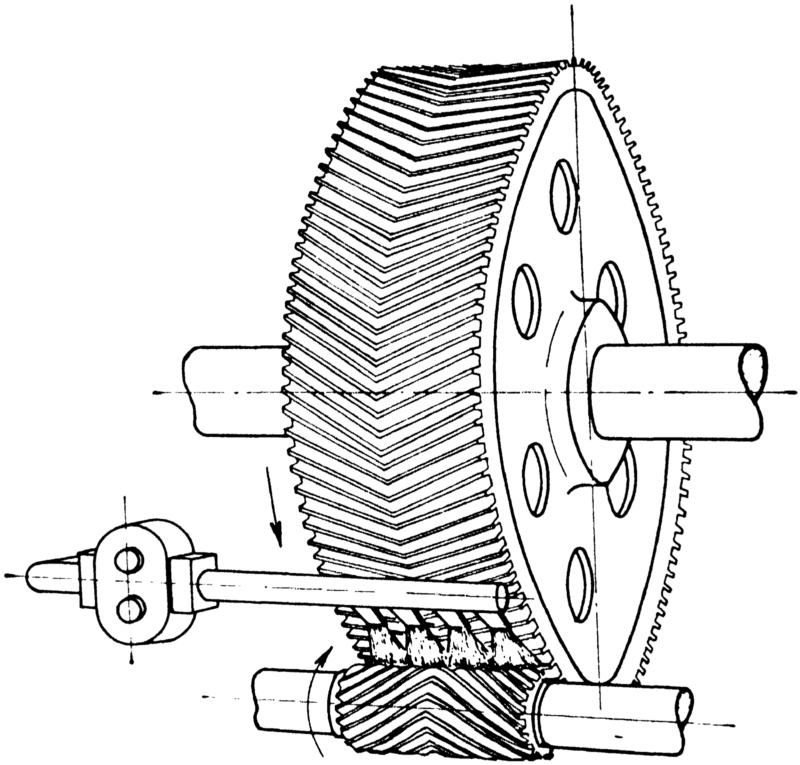


Fig. VIII, 11

Gear-sets running at peripheral speeds above about 40 feet per second (1200 cm. sec.$^{-1}$) are usually lubricated by pressure-jets directed (as shown in fig. VIII, 11) to the meshing-line of the pair of gears from their "approach" side. It is not necessary that the velocity of the jet should exceed the peripheral speed of the gears; it is in fact preferable that it should be of a slightly lower speed, as the oil will then be caught on the forward sides of the teeth and be thus more certain to be carried into the region of contact. Oil jets for wide teeth should either be multiple, or else designed to project a flat stream of oil nearly as wide as the teeth. This may be effected by causing a jet of circular section to strike an inclined, plane, deflector plate, or causing two circular jets to impinge on one another at a slight angle.

When the circulation of lubricant takes place wholly within the casing of the gear-set (any cooling of the circulated oil being effected by finning the case, circulating water through coils in the sump of the case, or similar means), it is important that an effective strainer shall be in the oil circuit.

Systems of forced-feed lubrication in which the lubricant is circulated externally to the gear-casing are dealt with in Chapter IX.

3. Lubrication of Piston-rings.

In the design and operation of reciprocating engines and compressors, questions relating to the lubrication and friction of pistons and their sealing rings are of the highest importance. Not only are the conditions under which pistons and piston-rings operate, especially in internal-combustion engines, often adverse to effective lubrication on account of the effects of heat and of the action of the working fluid in displacing the lubricant, but even the kinematics of the problem are often rendered uncertain by small deviations and displacements of the parts from the intended construction. In consequence, there are great differences in the performance in various cases, and there is much discordance in the results of the various experimental investigations that have been made into the frictional resistances of pistons and piston-rings.

Before discussing the facts of experiment and practice, it is desirable to consider the normal and essential features of the action of a piston-ring. In fig. VIII, 12 a single ring of the usual rectangular section is shown in a radial sectional view of a portion of the piston and cylinder.

The ring, being made of slightly larger diameter than the cylinder bore, is subjected, when placed in the cylinder, to inward radial pressures p_e per unit of area of its circumferential surface, these pressures being equilibrated by elastic stresses within its own substance. It will be assumed that p_e is uniform around the circumference. The ring is of rectangular section and is slightly smaller in axial width, as well as considerably smaller in radial depth, than the piston groove into which it is fitted. The piston is also necessarily smaller in diameter than the cylinder, so that on the greater part of the circumference, at least, a small clearance t exists between the piston and cylinder, and a larger clearance T between the inner face of the ring and the bottom of the ring-groove.

When a fluid pressure p_1 is applied in the cylinder above the piston, a pressure p is transmitted down to the ring by the clearance-space t, and the ring is forced downwards on to the lower face of the ring-groove, thus closing communication from the space T to the lower clearance t below the ring, except for the leakage which takes place due to the unavoidable imperfection of fit between the lower faces of the ring and ring-groove. By this downward movement of the ring, a small clearance t' is formed between its upper face and the upper face of the groove, and the fluid at pressure p is admitted to this space t' and to the space T.

On the outer face of the ring, where it is in lubricated contact with the cylinder wall, there will be normally some downward flow of the lubricant, and also some downward leakage of the working fluid, both fluids in the inter-space between the two surfaces being under pressure varying from p at the top of the ring to $p - \Delta p$, the pressure existing in the clearance t below the ring. A similar gradient of pressure will exist along the lower face of the ring, from p at the inner radius to $p - \Delta p$ at the outer.

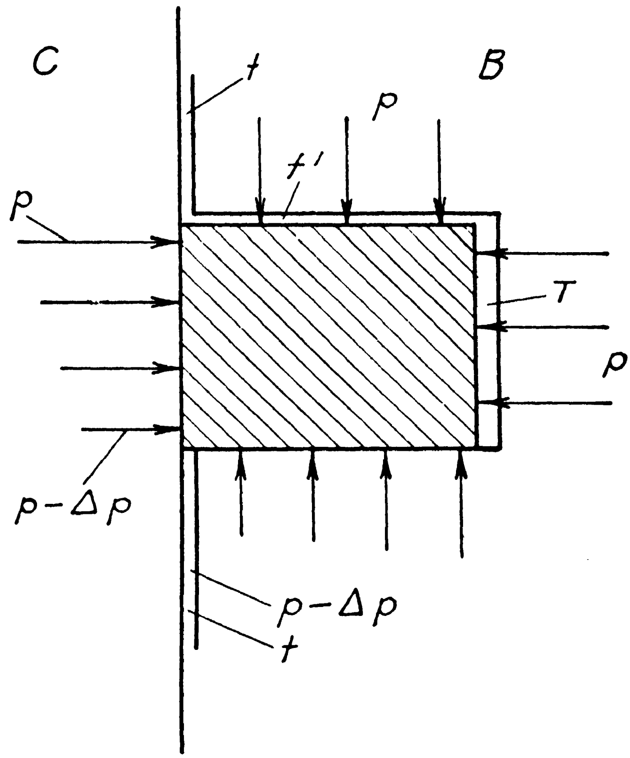


Fig. VIII, 12

The total outward force on the ring (it being remembered that the pressure p acts on the ends of the ring at its circumferential joint, as well as on its inner circumference, and account being taken of the elastic stresses above-mentioned) is

$$2\pi rl(p + p_e),$$

l being the thickness of the ring measured in the axial direction.

The total of the fluid pressures acting inwards is

$$2\pi rl(p - \tfrac{1}{2}\Delta p).$$

The difference between these forces, viz.

$$P = \pi rl(\Delta p + 2p_e), \quad . \; . \; . \; . \; . \; . \; . \quad \text{VIII, 1}$$

must be supported by contacts between the ring and the cylinder wall, these contacts, of course, taking place over small areas of the summits of rugosities distributed over the surfaces of the two members.

Now consider the joint action of the complete assemblage of n rings with which a piston is fitted, as shown in fig. VIII, 13. (In this figure $n = 4$ and the rings are all alike, any distinction between "compression" rings and "scraper" rings being, for the present purpose, ignored.) If p_1 is the pressure of the working fluid in the cylinder above the piston, and p_0 the pressure in the space below the piston, the sum of the falls of pressure at all the n rings is

$$\Sigma\Delta p = p_1 - p_0,$$

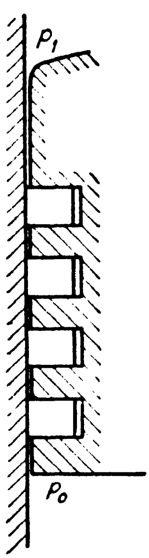


Fig. VIII, 13

and if the forces P for all n rings (equation VIII, 1) are also summed, there results

$$\Sigma P = \pi r l(\Sigma\Delta p + 2np_e) \quad . \quad . \quad . \quad . \quad \text{VIII, 2}$$

$$= \pi r l(p_1 - p_0 + 2np_e) \quad . \quad . \quad . \quad \text{VIII, 3}$$

If p_m is the mean effective pressure in the cylinder during the working stroke of length L, and p_m' that during the return stroke, the "indicated" work of the piston is

$$W_i = \pi r^2 L(p_m - p_m'), \quad . \quad . \quad . \quad \text{VIII, 4}$$

and the frictional loss of energy due to the rings is, from equation VIII, 3,

$$W_f = k\pi r l L(p_m + p_m' + 4np_e), \quad . \quad \text{VIII, 5}$$

k being the coefficient of resistance of the "mitigated-solid" type of sliding contact between rings and cylinder wall under the particular condition of lubrication, temperature and gas-leakage which exist in the case under investigation.

From equations VIII, 4 and 5, the ratio of piston-ring loss to indicated power is

$$k\frac{l}{r}\frac{p_m + p_m' + 4np_e}{p_m - p_m'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{VIII, 6}$$

On account of the large numerical factor attached to the pressure p_e it is evidently important, from the point of view of efficiency, to reduce this intrinsic ring-pressure as far as practicable. Its lower limit in internal-combustion engines and compressors appears to depend mainly on the necessity of obtaining an approximation to gas-tightness at the end of the outstroke and beginning of the instroke of the piston, in order that compression may effectively begin. Especially is this the case in engines which are not provided with means for distributing oil above and between the lower piston-rings prior to starting the engine.

The value of p_e varies in practice very widely; an average value would appear to be about 0·5 atmosphere, or 7 pounds per square inch.

It is very difficult to devise satisfactory experiments for the determination of piston-ring friction in actual engines. To measure the friction losses in an engine complete with its pistons, "motoring" it alternately with and without piston rings, with the object of determining from the difference of the total losses in the two cases the frictional loss due to the rings, is an illusory procedure, since the pumping losses arising from flow of compressed gases past the piston will be of quite different magnitude, being much smaller when the rings are in place than when they are absent. It is far preferable to carry out the investigation by means of special apparatus designed for the purpose. Reference may be made especially to a long series of experiments carried out by Hawkes and Hardy (Ref. VIII, 4). The apparatus used in these experiments consisted essentially of a cylinder of Perlit cast-iron reciprocated by a crank-connecting-rod mechanism and fitted with a double piston of cast aluminium alloy which was restricted in its axial position by springs.

The piston was furnished with rings, and, being arranged vertically, the friction between the piston unit and the cylinder was almost entirely due to the rings. The compression of the springs, being a measure of the frictional resistance, and the pressure of air applied between the two halves of the piston, were recorded by indicator. Several different forms of piston were tested. In one, on which the principal series of experiments was carried out, the number of rings was varied from 4 to 10, and with each of these numbers of rings the cylinder pressure was varied from 1 up to 7 atmospheres (90 lb. in.$^{-2}$ above atmosphere), thus providing favourable data for determining the dependence of the frictional resistance W_f on the number of rings n, as in the equation VIII, 5.

In Table VIII, 1 are given the ratios of the frictional resistances for 4 and for 10 rings as found (*a*) by these experiments, and (*b*) by calculation from equation VIII, 5, on the assumption in the latter case that $p_e = 0{\cdot}5$ atmosphere, or 7 pounds per square inch.

Table VIII, 1

Air pressure, lb. per in.^{2}	Frictional resistance, lb.		Ratio of resistance	
	4 rings	10 rings	(*a*) by experiment	(*b*) by equation
0	19	46·3	2·44	2·50
25	27·5	54·3	1·97	2·05
50	32·5	59·4	1·83	1·79
70	36·4	64	1·76	1·67
90	39·8	67·6	1·70	1·57

From another series of experiments carried out with a piston in which 12 rings were fitted, and which was run both without pressure and with air at 50 pounds per square inch between the halves of the piston, the supply of oil being varied from 0·01 to 0·04 cubic inches per minute (0·0025 to 0·0100 cm.^{3} sec.$^{-1}$), the following values of the coefficient k of equations VIII, 5 and 6 have been calculated.

The piston-speed (mean) in these latter experiments was 8·5 feet (260 cm.) per second, and the temperature 240° F. (115° C.).

The mean of these values of k, viz. 0·039, is somewhat higher than that which is generally found where there is sliding contact between metals without fluent lubrication but with a continuous supply of lubricants, viz. about 0·030. The higher value may be attributed, with some probability, either to the different materials (in this case cast iron on aluminium alloy, as contrasted with steel on bronze or on bearing-metal, in the non-convergent bearings, gear-wheels, etc., for which the value 0·030 has been specially noted in preceding sections), or to the very meagre supply of lubricant allowed in the experimental piston in conformity with invariable practice in the use of prime-mover and compressor pistons.

TABLE VIII, 2

Flow of oil, in.^{3} per min.	Coefficient k	
	Air pressure, atmospheric	Air pressure, 50 lb. in.$^{-2}$ above atmo.
0·01	0·047	0·038
0·04	0·039	0·034

4. Lubrication of Small Pivot Bearings.

An application of lubrication which is of much interest as a limiting case, as well as of great practical importance, is found in the end-pivots of small clockworks and watches, and those of other small mechanisms, such as electricity meters. These bearings usually consist of a ball-ended steel pivot rotating in a cup-like recess of a highly polished jewel or artificial stone. Sapphire is very commonly used, and sometimes diamond. The lubricant traditionally used is one obtained from a marine mammal, but mineral oils of high viscosity are now often employed.

The conditions of use, as to intensity of bearing pressure and speed, are such that even with the highest attainable degree of polish of the surface, there can be no possibility of forming continuous films of lubricant.

The conclusions as to solid-to-solid contact which have been reached with respect to ball bearings in Chapter VII and Appendix III, rest in this case on the additional circumstance that there is no rolling motion, the full load being constantly carried on a single minute area.

Very little quantitative information has been published on the behaviour of bearings of this class, but two investigations carried out respectively by G. F. Shotter and V. Stott, of which summaries have been published in Refs. VIII, 5 and 6, are illuminating, and in general, concordant.

A series of experiments of the former of these authors was made on a pivot bearing in which the ball-end of the hardened steel-wire spindle had a radius of 0·025 in. (0·0635 cm.), and the sapphire cup a radius of 0·0725 in. (0·184 cm.), the load being varied from 17 to 62 gm. wt. The intensity of pressure between the surfaces would, of course, be determined by the crushing strength of the materials, and was estimated by the author as $1{\cdot}68 \times 10^5$ lb. in.$^{-2}$, or $1{\cdot}12 \times 10^4$ atmospheres.

The means used to determine the torque required to rotate the spindle enabled it to be measured approximately down to a minimum of 0·25 dyne-cm. It was found that, with a given load, the frictional resistance did not differ materially when the pivot was lubricated with either of three different mineral oils, of viscosities varying over a range of about 1 : 10, from the resistance when it was run without any lubricant.

On the assumption that the mean intensity of pressure was that estimated by the experimenter, the area of contact, and the mean radius at which the frictional torque was exerted, can be calculated, and hence a value derived for the coefficient of friction. Table VIII, 3 shows the imposed loads, the observed frictional torque in dyne-cm., and the coefficient of friction so calculated, the lubricant being "Pennsylvania Meter Oil No. 2" having a viscosity of the order of 15 poises.

TABLE VIII, 3

Load on pivot, gm. wt.	Frictional torque, dyne-cm.	Coefficient of friction
17	1·25	0·136
32	2·45	0·103
47	3·85	0·090
62	5·3	0·082

It will be seen that the calculated coefficient of friction diminishes when the load is increased, although the frictional torque increases more rapidly than the load. The tests made without lubricant show the same effect.

It will also be seen that the coefficients of friction at the lighter loads are of the same order as those found from the starting resistance of the thrust bearings of water-turbines in the tests quoted in Sect. IV, 17, in the cases when those bearings were very finely finished. The fact that the resistance is nearly independent of lubrication also, is in accord with the observations of Hardy (quoted in Sect. IV, 18), and suggests that, in the tests in which no lubricant was intentionally supplied, atmospheric contamination served in effect (as in Hardy's experiments) as the lubricant, the small size of the part presumably facilitating the access of the contaminating matter.

In one of the series of experiments reported by Stott (Ref. VIII, 6), the pivot of hardened steel wire, which was hemispherical and 0·04 cm. in radius, was run in a cup of synthetic sapphire of 0·175 cm. radius of curvature, the load being 20 gm. wt. and the speed 150 r.p.m. Running "dry", the spindle had a torque resistance of 1·4 dyne-cm., but when lubricated with "clock-oil" 2·2 dyne-cm., which it maintained during a run of about $1·5 \times 10^7$ revolutions extending over a period of six months.

In the test without lubricant copious rust formed around the pivot, and a "pronounced" flat was worn on the pivot after about a million revolutions. Very little rust appeared on the lubricated pivot during the first 3½ million revolutions, but it appeared later, being distributed through the oil; in this case no appreciable change took place in the profile of the pivot.

Calculation of the coefficient of friction, made on the same basis as in the case of Shotter's experiments, gives as the coefficient of friction of the "dry" pivot 0·14, and that of the pivot lubricated with clock-oil 0·23; these figures again suggest that, when no lubricant was supplied, air contamination took its place. It may be supposed that the earlier and more copious rust-formation in the former case was due to the "contaminant" not being present in sufficient quantity to protect the steel pivot from the oxidation due to solid-to-solid friction and local heat production; the observed wear would be the necessary consequence of the oxidation, and would consequently be greater than that of the lubricated pivot, although the frictional resistance was less.

In other tests by the same experimenter a steel bearing-ball, 0·04 cm. radius, was used as the pivot, the cup bearing being of diamond with a radius of curvature of 0·275 cm., and the load 25 gm. wt. In this case the bearing was run without lubrication for 5×10^7 revolutions during a period of 3½ years. The torque was 2 ± 1 dyne-cm., the mean of this range corresponding to a coefficient of friction of 0·135. A ring of debris 0·025 cm. in internal diameter, and 0·033 cm. external diameter, formed around the pivot, but there was no apparent damage to either the steel or the diamond surface.

It will be noted that in none of these experiments did the running resistance differ greatly from what would be expected to be the starting resistance with mollified solid-to-solid contact, that is to say, a frictional coefficient of about 0·15. It is also to be noted that the running resistance was much greater than the minimum which might be easily attained by fitting the spindles with small bearings of either sliding or rolling type; these, however, were inadmissible for the purposes in view, on account of the high starting torques corresponding to their relatively great radii of contact.

CHAPTER IX

Distribution and Treatment of Lubricants in Service

1. Modes of Deterioration of Oils during Use.

Immunity of the bearings of a machine from wear, and consequent immunity of its lubricating oil from contamination by abraded metal, are ideal conditions which, as pointed out in preceding chapters, are brought within practical reach by exclusive use of fluent-film lubrication. Given these conditions, and due care in the handling of the lubricant, it is not impossible for the charge of lubricating oil in a machine, as well as the machine itself, to give an indefinitely extended period of service. For the attainment of that end, however, it is necessary that film lubrication shall be effective in *all* the bearings of the machine, that the oil shall be guarded from *all* avoidable sources of contamination, and also that it shall be freed from the effects of unavoidable contamination by use of such means as will be described briefly in this chapter.

Prior to the placing of a charge of lubricating oil in a machine, a precaution whose importance can hardly be exaggerated is to remove from the interior of the machine all foreign matter, such as traces of foundry sand, metallic chips and filings, and other abrasive dust. Although this is an obvious requirement, and is always attended to by reputable machine makers at their works, complete removal of such matter is extremely difficult to effect, and repeated cleansing of the machine during, and after, erection are essential to avoid contamination of the charge of oil, and consequent damage to the surfaces of bearings. Frequently the interior of the machine is painted during erection with a material insoluble in the lubricating oil, but this should be regarded only as a final precaution to be added to all other means of ensuring cleanliness, and the possibility that the painted coating may become detached by vibration or erosion is not to be overlooked.

After any machine of importance has been put in operation it is usual to subject its charge of lubricating oil either to periodical purification as a whole, or to continuous treatment in a circulation through suitable apparatus provided for the purpose. The former method is generally called " batch " purification; the latter " continuous " purification or treatment.

Apart from contamination by metallic or other particles insoluble in oil, all lubricants are more or less liable to deterioration by various chemical actions which cannot always be prevented in service. The desirable qualities of lubricating oils are, of course, much more readily destroyed in some classes of machinery than in others. Electric generators and motors, water turbines and centrifugal pumps, should cause a minimum of damage if their bearings are of fluent-film types. Steam turbines are in the next class, but are liable to subject the lubricant to damage by high temperatures. In all machines, the oil will require more frequent and drastic treatment if it is allowed to collect the products of abrasion of gear-wheels, rolling bearings, pistons or other working parts in metallic contact. In machine tools, locomotives, and internal-combustion engines, permanent maintenance of the quality of a charge of lubricant is hardly practicable.

The principal modes of deterioration which occur in lubricating oils in service may be classed as:

1. Contamination by solid particles, either of the kinds resulting from abrasion within the machine, as referred to above, or dust carried into the machine from the surrounding atmosphere, or again carbon particles produced in combustion engines.
2. Contamination by liquids, most commonly water, derived from leakage in hydraulic machines, condensation in steam engines, or from atmospheric vapour; emulsification of the oil usually results.
3. Oxidation of the oil by atmospheric oxygen forming "sludge", imperfectly liquid.
4. Partial vaporization, or distillation, changing the constitution of the lubricant.

Considering these various modes of contamination a little more fully:

1. Solid metallic particles in lubricating oil may consist wholly of abraded metal, or in part of oxidized products of such metallic dust. Oxides may float on the oil, forming a scum. Unoxidized metallic particles can usually be removed to a large extent by settling in tanks, assisted by magnetic separators, but the finer particles only by centrifugal separation. The lighter and finer-grained oxides can only be dealt with by filtration.

2. Contamination of oil by long continued contact with water or steam almost always results in the formation of an emulsion. A well-refined mineral oil, such as is generally used for the bearings of steam turbines, will separate readily from a water emulsion when no other kind of contamination is involved. The rate at which this separation takes place under given conditions is a fairly good test of the freedom of the oil from oxidation or other common forms of chemical contamination. Two types of oil-water emulsion occur, the one containing minute drops of water disseminated in a continuous body of oil, the other similar drops of oil in a water aggregate. The latter separates into its constituents much more readily than the former.

3. Oxidation always takes place in oil to a greater or less extent when it is exposed to atmospheric air at high temperatures, especially if it is sprayed through the air or its surface is much agitated. In the presence of water and products of oxidation, a sludge is formed which destroys the fluidity of the oil and its capacity to form fluent viscous films; it also renders the oil liable

to collect without movement on cooling coils, or the walls of the pipes of the circulating system, and thus interferes with the flow and the transmission of heat. It may also clog oilways and filters. On account of its loss of fluidity, oil containing sludge formed by oxidation tends to retain in suspension abraded metallic particles, and other solid impurities, to a greater extent than oil in good condition; thus any abrasion which may be taking place is accelerated.

Other forms of chemical deterioration of oil may occur, and are particularly liable to do so, when metallic soaps, or compounds of sulphur, chlorine or phosphorus are introduced as "additives" to mineral oil for the purpose of preventing seizing, or welding together of the rugosities of gear-wheels, or of bearings in which solid contacts take place.

4. Vaporization, or distillation, and loss of the more volatile constituents of mineral oils, seldom occur to a serious extent, except when, for special reasons, an oil rich in the lighter hydrocarbons has been used. Since it is a function of the volatility of these constituents, and of the temperatures in the system, the remedies are obvious.

2. Methods of Treatment and Purification of Oils in Service.

It is only under exceptionally favourable circumstances that a charge of oil will remain in efficient condition in a machine or a circulating system for a long period without purification. Normally, systematic treatment is required for the removal of suspended solids, water, and sludge.

Three systems of purification are in use, one or other of them being the most appropriate according to the circumstances of each case, viz.:

1. Batch purification.
2. Continuous purification, carried out simultaneously on the whole flow of oil in circulation.
3. By-pass purification.

Batch purification implies the removal of the whole of the oil which is in a machine or a system to a settling tank, and therefore involves the stoppage of the machine or machines, until the oil has either been treated or replaced by a fresh charge of oil. It is thus usually an impracticable method of treatment if the charge of oil is common to two or more machines, and in any case it involves the holding in stock of a quantity of oil additional to that which is required for actual use at any time.

In carrying out the method, the oil withdrawn from use is first run into a settling tank and allowed to cool. Most of the solid particles, and water, and other heavy contaminants which the oil contains will then gradually settle to the bottom of the tank, and can be discharged. The remaining bulk of oil is then passed through a filter or centrifugal separator, or both in succession, and in special cases through special apparatus in which chemical cleaning with suitable reagents is effected.

It is sometimes advisable, when lubricating oil is liable to be badly contaminated, to resort occasionally to batch purification even when a system of continuous treatment is in use, as the latter may permit some portion of the contaminants to remain at all times in the system until its parts can be opened up and cleaned.

Continuous Purification.—In the use of this system, since all the oil in circulation is passed through a purifying apparatus in series with the machine which is being served, any foreign matter or contaminant can only pass once through a bearing before being subjected to the treatment. By the use of suitable filters or centrifuges, practically complete and immediate removal of any suspended impurity can be thus assured, without interruption to the operation of the machine. Continuous purification, however, cannot usually deal effectively with soluble impurities. Sometimes the oil is heated in order to thin it before it reaches the purifying apparatus, and re-cooled to its working temperature before being returned to the bearings.

A very important component of any continuous circulating and purifying system which serves more than one machine is a metering system for each return branch of the circulation, arranged to insure that each machine (and preferably each important bearing) receives its proper proportion of the flow.

By-pass Purification.—By this method only a portion of the main circulation of lubricating oil is passed through the purifying apparatus, which is thus placed in parallel with the machine or machines in service. The fraction diverted for treatment is usually delivered from a main sump to the purifying apparatus by a service pump. If the sump is sufficiently large, a considerable portion of the suspended solids and other heavy contaminants may be deposited in it, and, as in the batch method, removed from the system without going to the purifiers. Heating of the oil on its way to the purifiers, with re-cooling after passing them, is more readily effected than in the total flow system, on account of the smaller flow to be dealt with.

An obvious objection that may be raised against by-pass purification, as compared with continuous purification, is that, in the former, individual solid particles may remain in the system for an indefinite time before being taken into the by-pass stream and intercepted by the filters or centrifuge. In practice, however, the objection has little weight provided that the by-pass is properly arranged so as to sample fairly the whole of the oil in circulation, or, if any selection is made, to select for the by-pass treatment the portion most contaminated with solid particles. The probability of any particle being circulated many times through a bearing then becomes, statistically, very remote. The criterion to be observed is the relation of the rate at which abraded particles are passed through the bearing to the rate at which they are produced in, or introduced into, the system.

Thus if the rates of circulation through the machine bearing and through the filter are respectively Q_m and Q_f while n particles are produced in unit time, and a total number $\bar{n}$ are present at any one time in the system, they will be removed by the filter at the rate

$$\frac{Q_f}{\mathfrak{Q}} \times \bar{n},$$

where $\mathfrak{Q}$ is the total volume of oil in the system. In the steady state, this rate of removal must be equal to the rate of production n, so that the number present is

$$\bar{n} = n \frac{\mathfrak{Q}}{Q_f}, \qquad \text{IX, 1}$$

and the rate at which they are passed through the bearing is

$$\bar{n} \times \frac{Q_m}{\mathfrak{Q}} = n \frac{Q_m}{Q_f}. \qquad \text{IX, 2}$$

In full-flow purification $Q_f = Q_m$ and the particles are passed through the bearings only as fast as they are produced. In by-pass filtration the ratio $Q_f : Q_m$ may be of the order of $1:5$, and the particles will be passed through the bearings five times as fast as they are produced. As compared with the batch system, this difference between the other two systems is in practice of little importance. It is apparent from equation IX, 1, that in the use of both the full-flow and by-pass systems the number of particles present can be at most only the number produced in a period of hours, while in the batch system the accumulation continues for weeks or months, and in each system the number passed through a bearing in unit time is proportional to the number present and the rate of circulation through that bearing.

3. Apparatus for Purification of Oils.

The appliances used for the purification and reclamation of lubricating oils consist mainly of settling tanks, filters and centrifugal separators, together with suitable heating and cooling apparatus and pumps, being each in general construction the same, whichever of the three systems of treatment discussed in the preceding sections is employed.

These pieces of apparatus are built with many variations of detail, and only representative examples can be described here; for illustrations of these the author is indebted especially to The De Laval Separator Company and The Vacuum Oil Company.

4. Settling Tanks.

The Settling Tank (often supplemented by a "Treating Tank", or "Processing Tank"), though the simplest, is perhaps the most important single element of a purification system, especially when this is operated by the batch method. For use by this method, the tank usually consists, as illustrated in fig. IX, 1, of a vertical cylindrical vat having a conical bottom. It is provided with outlets for clean oil at various levels, and with an outlet for the waste oil at the bottom of the cone. It is also preferably furnished with a steam pipe (if steam is available), the open end of which is at about the level of the top of the cone—the purposes of the steam being both to heat and to agitate the oil. Whether treated initially with steam, or not, the charge of fouled oil in the tank must be allowed to stand at rest for a sufficiently long period to allow the impurities to separate out under gravity; this period may be one of days or weeks. The oil is then drawn off from the various outlets in downward succession as fast as it is shown (by withdrawing samples) to have become clear at each level. If the percentage of heavy impurities is large, foul oil may have also to be drawn off progressively from the cock at the bottom.

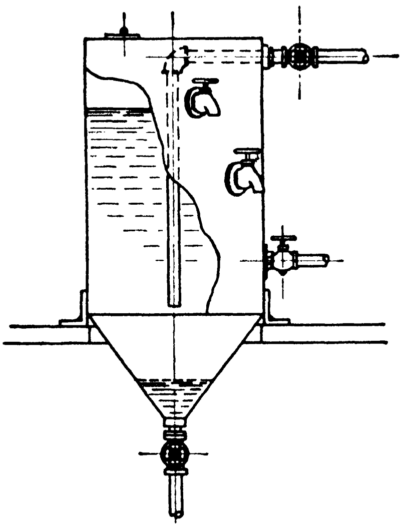


Fig. IX, 1

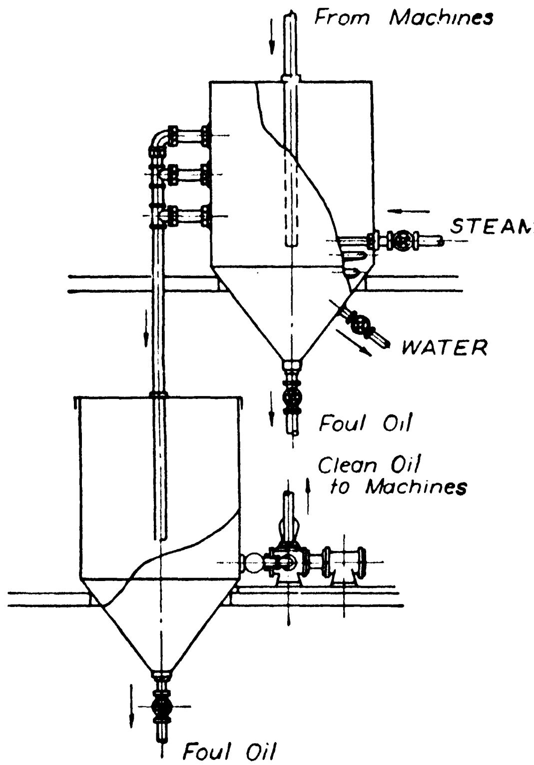


Fig. IX, 2

Two such settling tanks arranged at different levels, as shown in fig. IX, 2, may act in series, steam being used only in the upper one, from the bottom cock of which the greater part of the impurities will be drawn off, though both are provided with foul-oil outlets.

Alternatively to the use of live steam, a single settling tank, or the upper one of a pair, may be fitted with heating coils (fig. IX, 3) through which hot water or steam is passed; or, when more convenient, electric heating coils may be used. Whichever mode of heating is used, the temperature of the surface of the coils must not exceed, at any point, about 170° C. (340° F.) to avoid destructive distillation of the oil. The oil in the tank may be allowed to reach a general temperature of 90° C. (195° F.) before the heat is turned off and the oil is left at rest to be defecated.

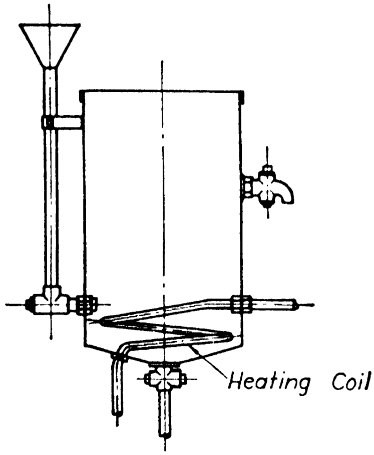


Fig. IX, 3

In a continuous or a by-pass purification system the Settling and Treatment Tank is usually of a simple rectangular form with baffles and partitions as shown diagrammatically in fig. IX, 4, which represents a portion of an installation arranged for carrying out the De Laval-Funk Process of purification. In this case the oil passed through the tank is brought into contact with condensed steam, and agitated therewith for the purpose of extracting water-soluble oxidation products, prior to passing it through a centrifugal separator in which the soluble products are extracted together with the water, while the purified oil is separated for return to the machine which is served.

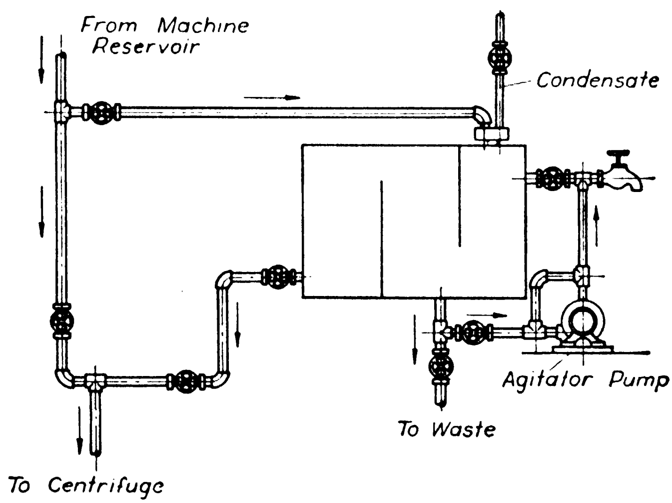


Fig. IX, 4

5. Filters.

Filtration is an effective means of purifying lubricating oil only when the pores or interstices of the filtering medium are extremely fine; when this condition is fulfilled no other appliance is so effective as a filter for removing some of the contaminants of bearing oils, especially oxidized metallic particles and siliceous dust. Usually, however, a filtering medium does not attain its full effectiveness until its inflow-surface has become coated to a considerable extent with the impurities which it is intended to collect. Its action is then slowed down, but it should not be cleaned or replaced as long as it is capable of passing the necessary flow.

Assuming that all the internal parts of a turbine or other machine have been thoroughly cleaned before being assembled and put to work, the chief solid matters which may be expected to contaminate the lubricating oil in service, and which may be dealt with by filtration, are:

(*a*) particles of metal and oxidized metal arising from wear of the bearings at starting and stopping, and from abrasion of gear-wheels or other parts which make solid-to-solid contacts, if these are lubricated by the same circulated oil as serves the bearings;

(*b*) siliceous and other non-metallic particles entering the machine from the atmosphere, or gradually detaching themselves in small quantities from the walls of castings, however carefully these may have been cleaned before assembly;

(*c*) (in mills of various kinds) industrial dusts differing in chemical nature according to the industry;

(*d*) (in internal-cómbustion engines only) carbon, varying in composition from the extremely fine " colloidal " carbon which is produced by incomplete combustion, especially in compression-ignition engines, to disintegrated products of the carbon crusts which are formed on the heads of cylinders and pistons;

(*e*) fibrous material, mainly derived from cloths and rags used for cleaning and wiping the parts of the machine during stoppages.

Removal of these various materials by filtration is in practice sufficiently difficult. Some of them, particularly fine carbon and oxidized metallic dust, require very fine-pored filters to retain them; such filters commonly pass oil only at a rate of the order of one gallon per square foot per hour (5 $cm.^3$ per sq. cm. per hour), whereas the flow of oil in a circulating system usually amounts to many gallons per minute (litres per minute). Very large areas of filtering surface are consequently required. In practice, it is usually necessary to make a choice between the use of comparatively coarse filtration, passing the whole flow of lubricant on the full-flow continuous system, and finer filtration with operation on the by-pass system. In batch purification the filters are commonly of an intermediate fineness.

The materials most commonly used for filters (omitting those which are to be designated as being merely " strainers ") may be roughly classified, in order of fineness, as: felt, cloth, edge-filters of metal and edge-filters of paper. Metal edge-filters usually have interspaces of the order of 0·005 to 0·010 centimetre (0·002 to 0·004 inch). Edge-filters (sometimes called " stream-line " filters) form one of the most effective means of filtration. They consist of series of closely adjacent sheets of thin materials, such as copperfoil, celluloid or specially compressed paper, through which the oil is caused to flow edge-wise. Fig. IX, 5 shows, somewhat diagrammatically, the construction of one form of "stream-line" filter. The cylindrical columns C, C′ (the latter being in section) of compressed-paper discs receive the oil at their peripheral surfaces, where the impurities are retained, and allow the cleaned oil to pass upwards through their axial cores D to the upper chamber of the filter, from which it is withdrawn. The lower end of the cylinder is arranged to receive hot water surrounding the

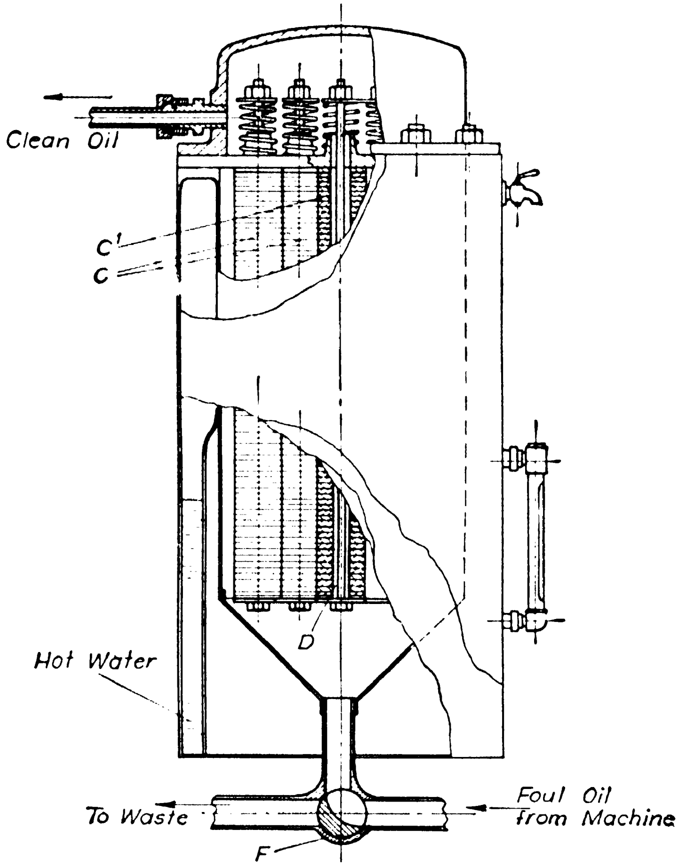


Fig. IX, 5

nest of filter columns, so as to reduce the viscosity of the oil and enable it to pass more rapidly through the narrow interspaces between the discs. The columns are cleaned periodically by supplying compressed air to their cores, thus blowing the collected impurities out of the assembly through the two-way cock F at the bottom, the clean-oil delivery pipe being temporarily closed for this operation.

A combination of a filter with a precipitating tank, which can be used for either batch or continuous filtration, is shown in fig. IX, 6. The fouled oil enters through a heating pan, and then passes to the bottom of the settling-chamber of the apparatus where the separated water and some of the solid impurities are passed off from the overflow shown on the left-hand of the side figure. The oil, rising in, and overflowing from, this settling-chamber, passes to the filtering compartment and into rectangular filtering frames covered with fabric, which hold back the solid contaminants. The cleaned oil issues from the tops of these frames and fills the clean-oil compartment, from which it is withdrawn for use. Eventually, when the filter cloths become so fouled as to be inoperative, they are removed for cleaning or replacement.

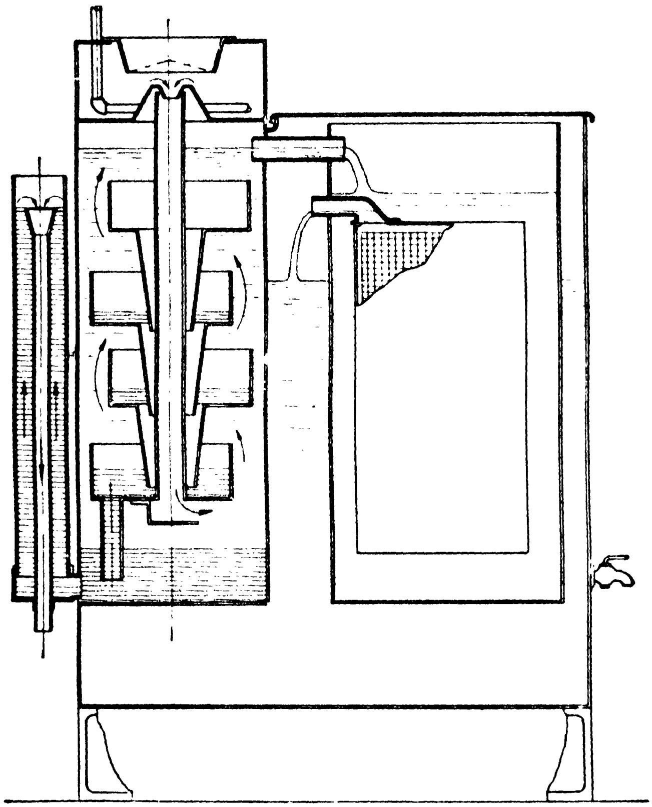


Fig. IX, 6

Since the solid particles in lubricating oils usually consist largely of finely abraded steel and iron, the employment of a magnetic field to form a magnetic trap, or, as it is usually called, a " magnetic filter ", is an obvious recourse. Many forms are in use, most of them involving the passing of the oil in thin layers over magnetic cores or armatures to which the particles are attracted and adhere, and from which they are periodically removed. Since most of the particles are of the order of 10^{-4} cm. in their greatest dimension, their velocity through viscous oil is very low, and, in order that the trap may be effective, it is necessary that the oil containing the iron dust shall traverse a narrow and relatively long passage across which a strong magnetic field is arranged. The narrowness and length of the passage involve a high resistance to the flow of

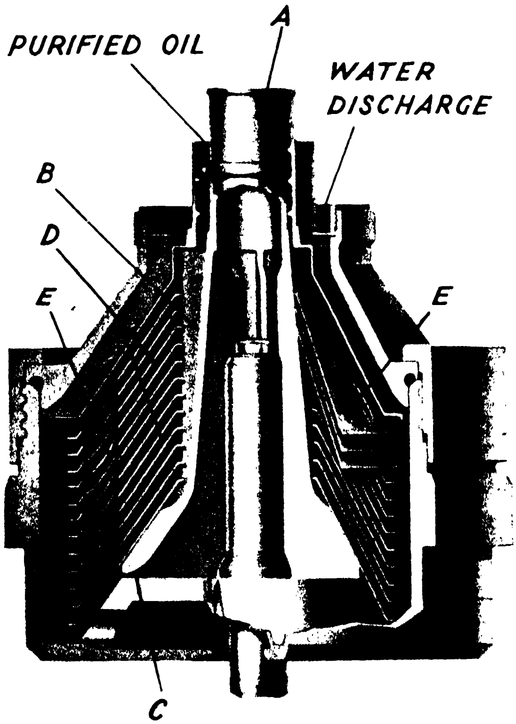


Fig. IX, 7.—Sectional view of bowl and discs of De Laval centrifuge for oil purification viscous oil. On that account a magnetic trap is most suitably located in a by-pass purification line. Even in that case it is probable that a given particle would make many passages through the magnetic trap (and therefore many passages through the machine bearings) before it happened to pass so near to the magnet core as to be attracted to its surface and held there.

The use of magnetic filters will be mentioned again in Sect. IX, 9 in connexion with the lubrication systems of Sulzer diesel engines.

6. Centrifugal Separators.

Centrifugal separators or "centrifuges" are adapted to deal with larger flows of contaminated lubricants than any other apparatus occupying a similarly small amount of space in an engine room or workshop, and are indispensable when large volumes have to be dealt with. When filters and centrifuges are both employed in a circulating system, the centrifuge usually precedes the filter units, or a relatively small part of the flow which passes through the centrifuge may be by-passed through a filter.

The construction of a typical centrifugal separator, comprises a bowl mounted on the upper end of a vertical shaft and rotated at high speed. Various forms of bowl are in use, but that which is commonly employed for purifying oil contains a number of conical plates, or "discs", nesting one inside another coaxially with the bowl proper. The bowl illustrated in fig. IX. 7 is of the type constructed by The De Laval Separator Company. The oil to be treated enters by the feed-tube at the top A, and passes down through the shaft B, from which it is delivered by ports C to the action of the discs D. Through the holes E in the discs it is distributed to each of the series, traversing them upwards. Owing to the differences of density, the impurities are impelled by centrifugal force against the inner (and under) side of each disc, and are then thrown off against the wall of the bowl, from which the fluid portions are discharged at the "water discharge" so marked in the figure. The oil, being lighter, flows along the outer (and upper) surfaces of the discs to the top of the machine and flows off by the "purified oil" pipe.

A considerable space is left between the outer edges of the discs and the inner periphery of the bowl, and in this space the more solid portions of the impurities are retained and impacted by the centrifugal force against the wall of the bowl, until the machine is stopped for their removal. The proper separation of the streams of water and oil is dependent on a balance being maintained between the heights of the columns of water and oil, according to their specific gravities. The heights of the columns can be adjusted in this respect by fitting rings of various diameters in the bowl top. In the use of the apparatus for cleaning the lubricating oil of steam turbines, and in many other cases, a large proportion of the solid impurities is discharged with the water with which the oil is contaminated. In some cases, however, as for instance when transformer oil is being treated, the water content being small, the water passages in the upper part of the bowl are eliminated and what water is extracted is retained in the bowl to be removed periodically with the solid impurities.

A simple arrangement of a purifying unit, including a centrifuge, is shown in Back Folder IX, A, fig. 1. By this arrangement, which is suitable for the service of a single machine such as a steam turbine, either continuous or batch purification can be effected. When the continuous system is being operated, the valve V is kept closed and the level of the oil in the oil reservoir of the machine is maintained at such a height that the dirty oil from the bottom of the reservoir overflows through the sight-glass G to the centrifuge, from which the clean oil is pumped back continuously to the reservoir. When it is desired to effect a batch purification, the valve V is opened, thus short-circuiting the sight-glass overflow G, and the whole of the contents of the reservoir are put through the centrifuge. If required, a treatment tank, such as the De Laval-Funk "Processing Tank", or heater, can be inserted in the supply pipe going to the centrifuge.

Fig. 2 of Back Folder IX, A shows an arrangement of a single centrifuge connected to serve several small turbines, or machines of other kinds, in which only relatively small amounts of solids or carbonized products are likely to be present in the oil. This plan provides for continuous purification of the oil from all the machines to be carried on, or for draining the reservoir of any machine completely from time to time and treating the oil so drained off by the batch system. As in fig. 1 of the plate, the oil reservoir of each machine is provided with an overflow sight-glass through which the foulest of the oil in the reservoir can be fed continuously to the centrifuge, or which can be by-passed so that the whole of the contents of the reservoir can be run off. In the latter case the contents are emptied into a storage tank, in which they can be allowed to remain for a sufficient time for the impurities to settle out from the oil, after which they are passed through the centrifuge and then pumped back to the reservoir of the machine.

7. Circulation combined with Batch Purification.

A simple arrangement of oil circulation for a number of small prime-movers or other machines is shown in fig. IX, 8. Each machine is provided with an oil reservoir or sump, which may be either integral with the machine or separate, and each of these reservoirs can be emptied in turn by a pump which delivers the oil through a strainer, or rough filter, to one or other of a pair of overhead tanks. From the tank the oil is returned by gravitation to the bearings of the machines. A cooler may be interposed in the pipe-line leading to any, or each, of the machines.

When increasing quantities of impurities are found to be present in the oil, the contents of each of the reservoirs in turn is treated by batch purification.

The use of such a system requires care to ensure that each machine receives its due supply of oil, this being preferably taken through a sight-feed overflow device. It is also essential to guard against leakage of water into the oil from the cooling appliances, and to provide means for observing the condition of the oil in the system.

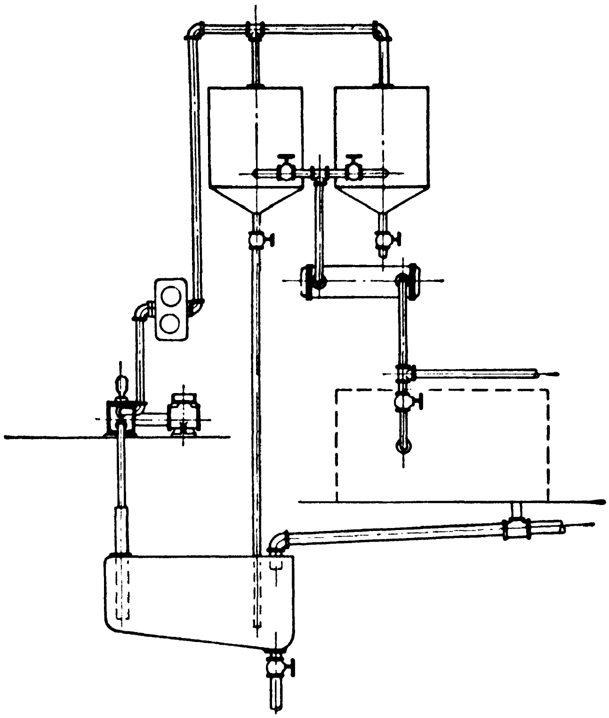


Fig. IX, 8

8. Oil Distribution in Reciprocating Steam Engines.

The arrangements for the circulation and distribution of the lubricant within reciprocating engines vary so much in detail that it is only possible to give illustrative examples of approved practice. In high-speed, totally enclosed engines, circulation is always effected by a pressure-pump, drawing the oil from the crankcase and supplying it to the main bearings, and other points, from which it returns by gravitation to the crankcase sump.

Fig. IX, 9 shows the main features of such a circulation by a cross-sectional view of a high-speed engine by Belliss & Morcom Limited, pioneers of the forced-lubrication system, by whose courtesy the figure is reproduced. It will be seen that the pressure-pump is of valveless, oscillating construction, its plunger being directly operated by a pin in the eccentric strap which actuates the valve of the engine. The necessity for complete reliability in this pump makes the adoption of the simplest possible forms of construction essential.

The pump, which is placed below oil level in the crankpit, draws oil therefrom through a cylindrical strainer covered with wire gauze. The delivery of the pump is connected to the pressure-main, from which branches go to nipples on the caps of the main bearings of the engine. Circumferential grooves turned around the bearings, and longitudinal and radial holes drilled through the journals, crankshaft and crankpins provide for the lubrication of the crankpin bearing; a similar circumferential grooving of this bearing feeds a pipe attached to the connecting-rod whereby oil is supplied to the crosshead pin bearing and to the crosshead guides. Other pipes from the pressure-pump delivery lead to the stuffing boxes, eccentric-rod guides and other points, and to a pressure-gauge which indicates whether the intended pressure of the supply, which is from 10 to 30 lb. per sq. in. (1·7 to 3·0 atmospheres absolute), is being maintained, this pressure being subject to adjustment by a control-valve.

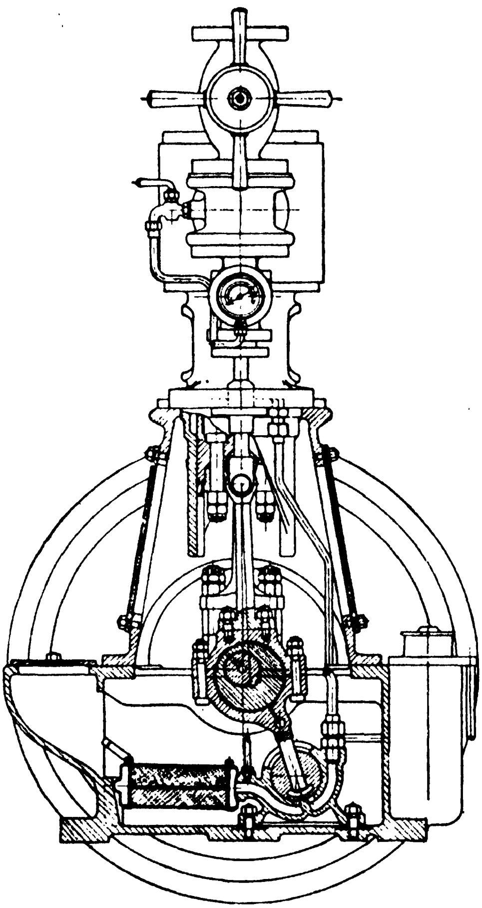


Fig. IX, 9.—Oil circulation system of totally-enclosed steam engine by Messrs. Belliss-Morcom Ltd.

At the right-hand side of the engine bedplate shown in fig. IX, 9 is seen a side pocket separated from the main oil sump. Any oil and water from the steam cylinders which may collect on top of the crankcasing is led down into this pocket by the drain-pipe shown; the oil is there automatically separated from the water and returned to the main sump, the water being run to waste through an external pipe.

An entirely different arrangement is shown in fig. IX, 10, which has been prepared from working drawings obligingly supplied by Messrs. Swan Hunter and Wigham Richardson Ltd., and shows the method of lubrication of the main and crankpin bearings of a triple-expansion marine steam engine of 39-inch stroke running at 120 r.p.m. In this construction one of the crank webs of each of the three cranks of the engine is fitted with a centrifugal lubricator, as shown in the figure. Oil is fed into the annular portion of the lubricator, near its slowest point, by the nozzle of a ½-inch oil-pipe attached to the side of one of the main crankshaft bearings adjacent to the crank carrying the lubricator. The supply is by gravitation from an overhead tank. The radial portion of the lubricator delivers the oil to the crankpin which is drilled longitudinally for half its length and also radially, thus making communication to the interior of the crankpin bearing.

It will be seen that by this construction drilling of the crankshaft and crank webs is avoided, which is of especial advantage when, as in this case, the crank-

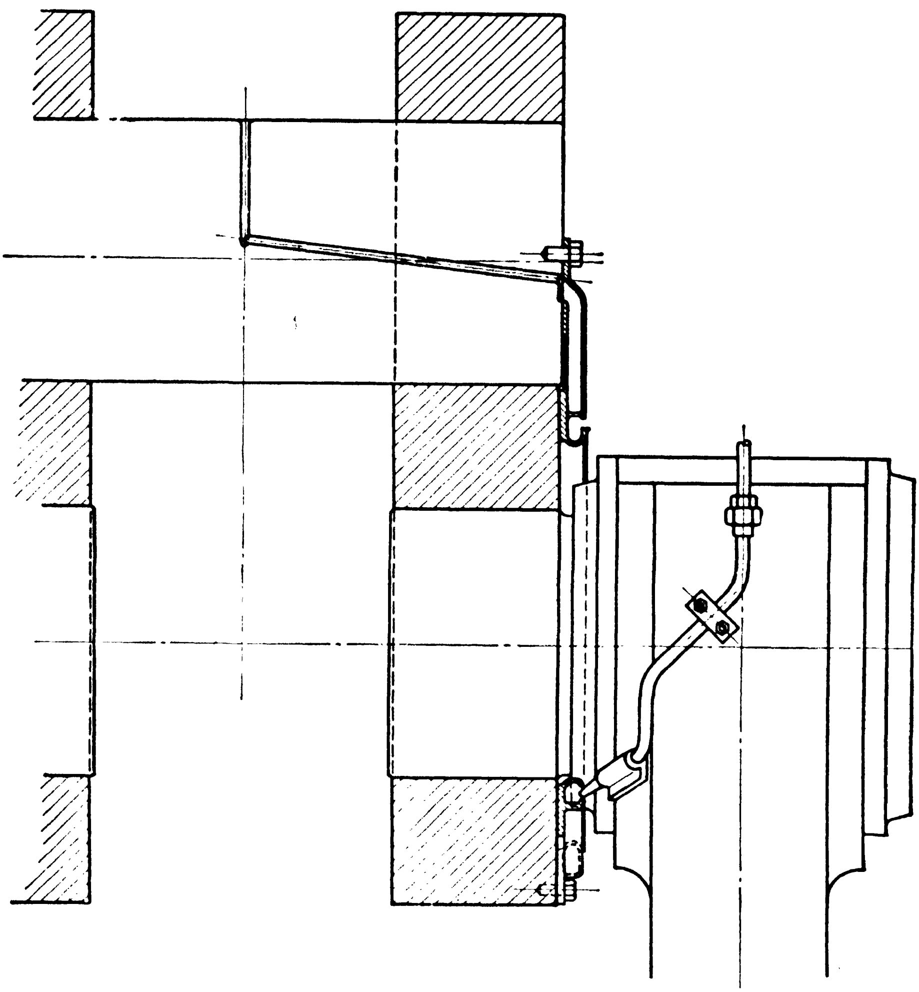


Fig. IX, 10.—Crankshaft lubrication of marine steam engine by Messrs. Swan Hunter and Wigham Richardson Ltd.

shaft is of a built-up type, the crank webs being shrunk both to the main journals and to the crankpins. The disadvantage, unavoidable when the main shaft is drilled with oil-ways, of forming circumferential grooves around the main bearings, and so dividing the bearing surface into two surfaces each of less than one-half of the available width, is also eliminated from the main bearings (though not from the crankpin bearings) by this construction.

9. Diesel Engine Lubrication.

The methods of lubrication of stationary diesel engines are generally similar to those of totally enclosed steam engines; such differences as exist are mainly due to the different conditions of cylinder operation, involving in the diesel engine higher working temperatures and the presence of solid products of combustion deleterious to the lubricating oil.

The system of lubrication adopted by Messrs. Sulzer Brothers of Winterthur, for their high-powered two-stroke diesel engines is shown diagrammatically in fig. IX, 11, redrawn from a diagram courteously supplied, with descriptive and explanatory matter, by that firm. The engine selected as an example is a six-cylinder, single-acting, vertical engine of the crosshead type. It develops 3500 horse-power at 184 r.p.m.; the cylinder bore and stroke are 720 mm. and 900 mm. respectively. The diagram shows the one cylinder-line only, but the oiling system is similar in all six cylinder-lines. The engine has two distinct lubricating systems, one of which deals only with the oil for the cylinders, while the other circulates and purifies the oil which serves all the bearings of the mechanism, and also effects the cooling of the pistons. Different grades of oil, each suitable for its own purpose, are used in the two systems. The bearing oil is subject only to a maximum temperature of 60° C. (140° F.), but the cylinder oil must withstand cylinder-wall temperatures up to 200° C. (390° F.), and is subject to contamination by the products of combustion. In order to keep the two oils separate the engine is constructed with an oil-tight diaphragm between the lower ends of the cylinders and the crankcase, and the lower portions of the trunk pistons pass through suitable sealing and scraping rings which retain all oil dripping from the cylinder walls. The cylinder oil is thus retained on the upper side of the diaphragm from which it can be periodically, or continuously, removed, together with the carbon residues and the metallic particles worn from the piston, piston-rings and cylinder walls, which it contains. This waste oil is not used again, being too much contaminated to be purified or reclaimed by any means except actual refining. The oiling of the cylinders is effected by Bosch positive-type lubricators, shown in the upper right-hand side of the figure.

The oil which lubricates the mechanism is circulated by two pumps of gear-wheel type, which draw the oil from the end of the engine bedplate and deliver it at a pressure of 5 atmospheres gauge (70 lb. per sq. in.), and at the rate of 2750 litres (600 gallons) per minute. The oil is passed first through triple "Auto-clean" filters of 0·2 mm. mesh, and then through two oil coolers capable of extracting $4{\cdot}5 \times 10^8$ gm. cal. ($1{\cdot}8 \times 10^6$ B.Th.U.) per hour. In emergency, either of the coolers can be shut off for cleaning, the other maintaining the service for a short time without unsafe rise of temperature of the oil. After leaving the coolers, a part of the flow is taken off at a pressure of 1½ atmospheres gauge (22 lb. in.$^{-2}$), for supplying the main bearings, crosshead guides, and other parts of the mechanism, but the greater part of the supply goes to the cooling of the pistons, viz. 2000 litres (450 gallons) per minute. For this purpose,

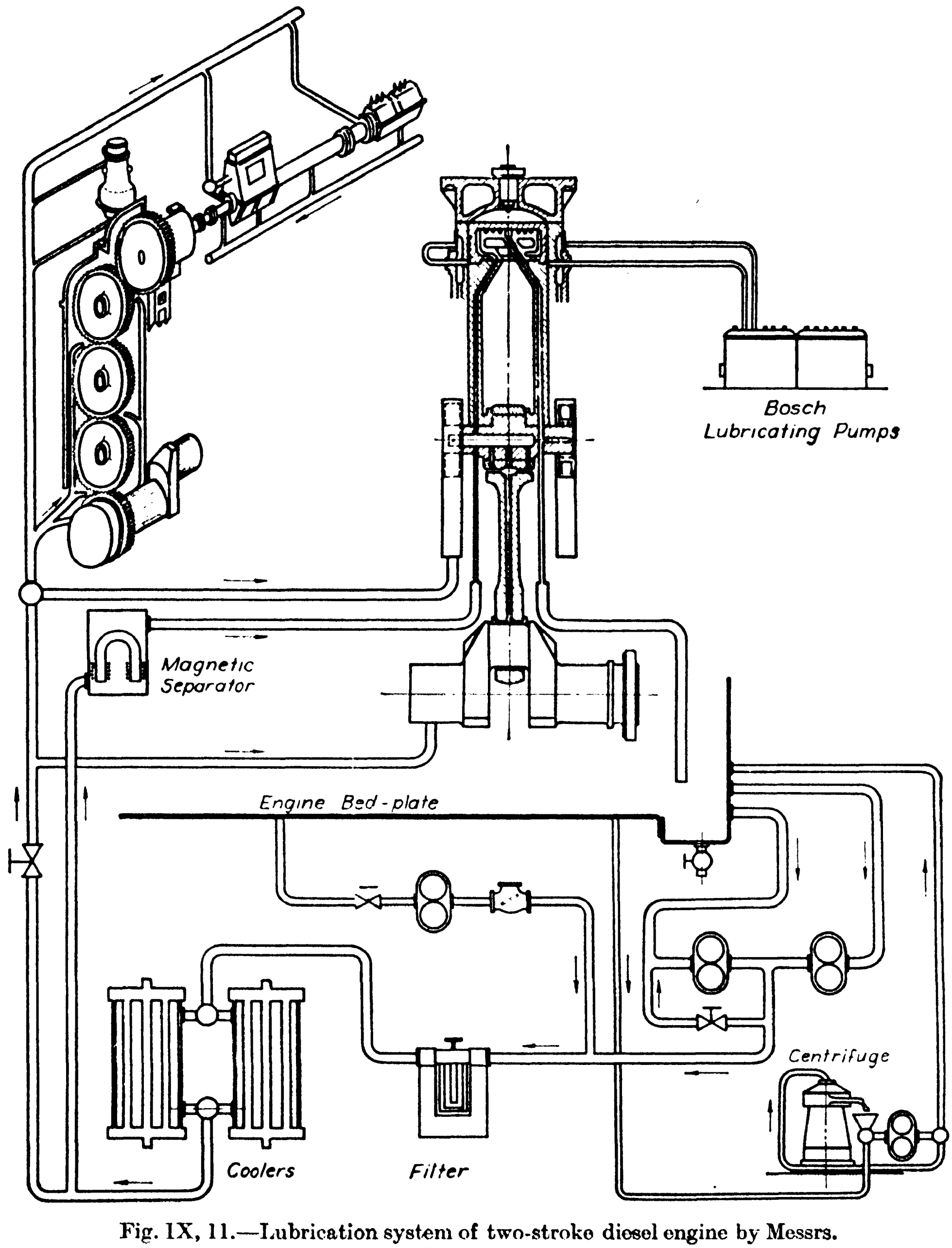


Fig. IX, 11.—Lubrication system of two-stroke diesel engine by Messrs. Sulzer Bros., Winterthur

it is led first through a battery of magnetic filters and then into a pair of vertical hollow and stationary plungers, which enter through glands into vertical cylinders attached to, and reciprocating with, the crosshead. From the crosshead, after lubricating and cooling the crosshead pin, the oil passes through longitudinal drillings in the piston-shell to the piston-head, around which it is circulated so as to cool it effectively.

At full load, the oil extracts from the pistons about $3{\cdot}15 \times 10^8$ gm. cal. ($1{\cdot}26 \times 10^6$ B.Th.U.) of heat per hour. The oil for lubricating the connecting-rod bearings as well as that for the crosshead guides is taken off by a branch from the inlet side of the piston-cooling system, the discharge from these dropping into the crankcase and passing to the sump of the base-plate. The oil which has served to cool the pistons returns through a collecting pipe which opens into a deep end compartment of the bedplate only, and in the neighbourhood of the intake of the circulating pumps; thus any sludge resulting from the relatively high temperatures to which the oil is exposed on the piston walls is prevented from being distributed in the bedplate.

The oil used in this circulation through the mechanism has a viscosity of about 0·6 poises at 50° C. (122° F.). On account of its use for piston-cooling it must have a good resistance to oxidation at temperatures up to about 120° C. (250° F.). The main treatment which it receives is removal of sludge by a centrifuge, which, as shown in fig. IX, 11, is arranged on a by-pass having its own pump for returning the treated oil to the engine bedplate.

Apart from this continuous treatment by a by-pass circulation, the oil in the engine bedplate is usually subjected to batch treatment before it is totally discarded. It is found as a rule that approximately 50 per cent of good oil can be recovered, and even if the use of this recovered oil is not considered advisable in the engine, other uses for it in the plant can usually be found.

As already stated, the oil used in the cylinders is injected by Bosch pumps. These are shown in outline at the top right-hand side of the figure. They are not synchronized with the engine revolutions, so that the actual injections into the cylinder occur at all phases of the piston stroke. The quantity of oil is kept at the minimum consistent with effective lubrication of all parts of the wall of the cylinder. The oil enters by six holes in the wall, three being on the exhaust side and three on the scavenge side; it is spread into a fan shape on the wall from each hole by the action of the piston-rings.

During the expansion phase the oil on the cylinder wall is exposed to combustion gases at temperatures above 1000° C. (1800° F.), after which air (that is to say, oxygen) is supplied. It may be said that the problem is not so much to provide lubrication for the piston and the cylinder liner, since this can be easily effected if a sufficient quantity of oil is supplied, as to do so with a minimum quantity of oil, in order to avoid fouling the piston and its rings by oxidation products of the oil. The quantity must be only of the order of 0·5 cm.^{3} per brake horse-power per hour. It is found that the quantity of oxidized products tends to diminish with the quantity of oil supplied. Their composition varies with the class of oil, and with the conditions of operation; products of a resinous nature especially increase the friction and impair the movement of the piston-rings in their grooves, so that these no longer form effective seals against passage of the combustion gases; then the oil also is blown past the rings, and wear and erosion result.

On a balance of advantages between thicker and thinner oils for the cylinder lubrication, it is found that an oil having a viscosity of from 0·70 to 0·90 poises at 50° C. (122° F.) gives the best overall results.

CHAPTER X

Conveyance of Lubricants in Systems of Distribution and Circulation

1. Flow of Viscous Liquids: Laminar and Turbulent Motion.

In designing systems of piping and other apparatus for the conveyance of liquid lubricants, or in calculating the performance of existing systems, the engineer is met, owing to the relatively high viscosity of the liquids, with conditions which do not arise when he is dealing with ordinary problems of the flow of water. In a given case the mode of motion of the lubricant may be either laminar or turbulent, and often it will be laminar in one part of a system, and turbulent in another part, the two parts having to be separately calculated by formulæ appropriate to each.

Some account of laminar flow in pipes has already been given in Chapter II. In particular, the equation has been given, which connects the volume flowing in a straight tube of circular section with the pressure head required to overcome the frictional resistance proportional to the length of the tube, viz.

$$Q = \frac{\pi a^4}{8\mu} \cdot \frac{p_1 - p_2}{l}, \qquad \text{II, 27}$$

where Q is the rate of volume-flow, a the radius of the section of the tube, μ the coefficient of viscosity, and $p_1 - p_2$ the fall of pressure in length l of the tube, all expressed in some self-consistent system of units, preferably C.G.S.

It is seen from this equation that, in laminar motion, the volume of flow, so far as it is controlled by frictional resistance only, is directly proportional to the rate of fall of pressure along the tube, and inversely proportional to the coefficient of viscosity of the liquid, but is independent of the density of the liquid and its other characters, as well as of all the features of the tube except its diameter. Within ordinary practical variations, the internal smoothness or roughness of the tube has no appreciable effect on the laminar flow.

If p_1 and p_2 be expressed in terms of gravitational head of the liquid which is being dealt with,

$$\frac{p_1 - p_2}{l} = gm\frac{h_1 - h_2}{l},$$

in which m is the density of the liquid, g the constant acceleration due to gravity (taken as 980 cm. sec.$^{-2}$), and h_1 and h_2 the heights of static columns of the liquid which give, at a common base level, the pressures p_1 and p_2; then equation II, 27 becomes

$$Q = \frac{\pi a^4}{8\mu} \cdot \frac{gm(h_1 - h_2)}{l}$$

$$= \frac{\pi a^4}{8\mu} gmi, \qquad \text{X, 1}$$

where i is the "hydraulic slope", or gradient, being the difference of the heights of the columns h_1 and h_2 divided by the length of the tube l.

Conversely, the equation can be written

$$i = \frac{8\mu Q}{\pi g m a^4}. \qquad \text{X, 2}$$

Equation II, 27, however, and its derivatives X, 1 and X, 2, are only applicable within a range of flow extending from zero up to a certain maximum or "critical" flow, which depends on the dimensions of the tube, and on the coefficient μ, to which it is directly proportional, as well as on the density to which it is inversely proportional. At higher velocities than the critical the flow ceases to be laminar and becomes turbulent, and the frictional resistance follows different laws, which rest on experiment rather than on theory.

In the turbulent mode of motion the frictional resistance of any fluid in a given length of the tube is proportional to the surface area in that length, to the density of the fluid, and to a power of the mean velocity of flow somewhat lower than the second power. Thus, if U is the mean velocity,

$$(p_1 - p_2)\pi a^2 = kmU^n \,.\, 2\pi a l,$$

and

$$i = \frac{h_1 - h_2}{l} = \frac{p_1 - p_2}{gml} = \frac{2kU^n}{ga}$$

or, in terms of the volume of flow Q, since $U = Q/(\pi a^2)$,

$$i = \frac{2k(Q/\pi a^2)^n}{ga} = \frac{2kQ^n}{g\pi^n a^{2n+1}}$$

$$= K\frac{Q^n}{a^{2n+1}}, \qquad \text{X, 3}$$

in which K $(= 2k/(g\pi^n))$ and the index n are constants requiring to be determined by experiment for tubes of different sizes and materials, or different degrees of roughness of their internal surfaces. It is found that in pipes having rough or irregular surfaces, the index n approximates nearly to 2, but that in smoother-surfaced pipes it tends to diminish to a limit of about 1·7 for very smooth surfaces. The value of n in any case, appears, however, to depend on the magnitude of the rugosities relative to the diameter of the pipe, rather than on their actual dimensions.

The factor K is found to be nearly constant over a wide range of flow in any given pipe (n being regarded as strictly constant for that range), but varies widely between pipes of different sizes, or of different degrees of roughness.

When $n = 2$, equation X, 3 takes the simple form

$$i = K\frac{Q^2}{a^5}, \qquad \text{X, 4}$$

so that the hydraulic gradient with a given flow is inversely proportional to the fifth power of the diameter of the pipe (neglecting the variations of K from one pipe to another), and with a given hydraulic gradient, the flow is proportional to the $2\frac{1}{2}$th power of the diameter, whereas with laminar flow it is proportional (by equation X, 1), to the 4th power.

The relations between volume flow and hydraulic gradient for both laminar flow (equation X, 2) and turbulent flow (equation X, 3) for a hypothetical tube and liquid (illustrating typical conditions) are shown diagrammatically in fig. X, 1, the value of n in equation X, 3 being taken as 1·75. The curve for the turbulent flow, as in all cases, is concave upwards; the laminar flow, of course, is represented by a straight line. Both lines pass through the origin of co-ordinates, where the "turbulent" curve is tangential to the axis of Q. They therefore necessarily intersect one another, and the point of intersection represents the critical point where the mode of motion is indeterminate as between laminar and turbulent flow. In the diagram the portion of turbulent curve below the point of intersection (or *critical point*) C, and the portion of the laminar line above that point, are shown as dotted lines, to indicate that these portions of the lines are not normally realized; experience shows that, if the velocity of flow in a straight tube is gradually increased from zero, the relation of hydraulic grade to flow follows the straight laminar line up to the point of intersection, and thereafter the turbulent curve. In other words, the experimental value of i lies on that line which, for a given value of Q, shows the greater resistance.

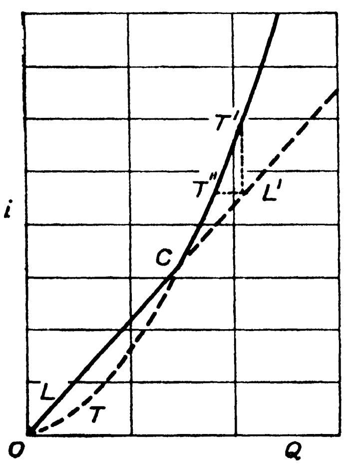


Fig. X, 1

In the case of very smooth tubes, however, if care be taken to avoid imparting any disturbance to the motion of the fluid at, and before, its entry into the tube, the straight line of laminar flow may be followed for a certain distance beyond this critical point. In such a case, on any further increase of flow, the hydraulic slope may suddenly rise from a point L′ on the dotted portion of the laminar line to a point T′ on the turbulent line. Alternatively, if the conditions are such that the hydraulic grade is held constant, the flow may suddenly diminish from that represented by the point L′ to a point T″ at the same height on the turbulent curve.

Thus any condition of flow represented by a point within the triangle CT′L′ (L′ representing the greatest flow which can be obtained with a laminar motion) is to be regarded as being generally unstable and unutilizable in practice, though it may occur temporarily under exceptional conditions. Calculations in which it is assumed that the relation of hydraulic grade to flow follows the course shown by the full lines of the diagram will err, if they err at all, on the side of safety, in the sense that the flow for a given grade will be at least as great as, and the grade for a given flow will not be greater than, that given by the calculations.

The subject of the incidence of turbulent motion is complex, and many features of it which enter, even into the limited problem of the flow in pipes of circular section, have not yet been elucidated. The reader who is interested in developments beyond the scope of this chapter may be referred to Ref. X, 1 (especially Chapters V–VIII), and to the comprehensive and detailed work, Ref. X, 8, as well as to the references therein.

2. Logarithmic Formulæ and Calculation of Flow by Logarithmic Charts.

For the purposes of calculations based on the equations of laminar and turbulent flow given in the preceding section, it is usually more convenient to use the logarithms of the members of the equations than the expressions themselves.

Thus, for laminar flow, from equation X, 2,

$$\log i = \log Q + \log \mu + \log \frac{8}{\pi g m} - 4 \log a, \quad . \quad . \quad . \quad \text{X, 5}$$

and for turbulent flow, from equation X, 3,

$$\log i = n \log Q + \log K - (2n + 1) \log a. \quad . \quad . \quad . \quad \text{X, 6}$$

It will be seen that, for both types of flow, the relation between log Q and log i is linear, and is therefore represented graphically by a straight line. The form of the combined logarithmic diagram (corresponding to fig. X, 1) is as shown in fig. X, 2.

The line for laminar flow LL′, and the line for turbulent flow TT′ intersect at the critical point C, the portion of the former line above C, and of the latter line below C, being dotted with the same significance as in fig. X, 1. As will be seen from equations X, 5 and X, 6, the line for laminar flow is inclined at 45 degrees to the axes, and that for turbulent flow at an angle $\tan^{-1} n$ to the horizontal axis, n being taken for the purpose of the diagram as 1·75.

For another tube (which is assumed to be one of a larger diameter), represented for the same fluid as before, by another pair of lines L_1L_1'; T_1T_1'

(of which the former will again be inclined at 45 degrees, but the latter may have a value of n different from 1·75), the new critical point C_1, at the intersection of the two lines, will occur at a lower value of i, but a greater flow Q. Thus for a series of tubes of different sizes the critical point C will have a locus, which is actually, to a sufficiently close approximation, a straight line.

The position of the critical point, regarded in the preceding paragraphs as being determined by the intersection of the lines of laminar and turbulent flow of a fluid of a given viscosity in a tube of a certain sectional radius a, can be located alternatively by application of the formula discovered by Osborne Reynolds (Ref. X, 2), which connects the critical mean velocity v_c in the tube of radius a with the viscosity μ, and the density m of the fluid; this formula is,

$$v_c = R_e \times \frac{\mu}{ma}.$$

The value of the dimensionless number R_e for smooth and straight circular tubes, such as are dealt with in this section, is, as found by Reynolds and confirmed by other experimenters, approximately 1000.

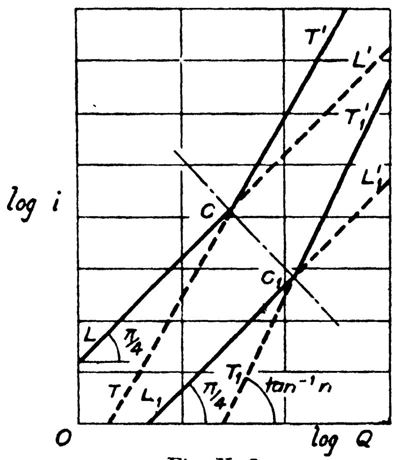


Fig. X, 2

Logarithmic charts of the kind illustrated in fig. X, 2 are probably the most suitable means for tabulating the frictional resistances of viscous liquids, such as lubricants, in systems of piping. For practical use, it is most convenient to present the same data in separate charts for the laminar and turbulent conditions of flow, rather than in a combined diagram of the type indicated in fig. X, 2. A pair of such separate charts are given in Back Folders X, A and X, B for tubes of the commercial sizes likely to be used in lubrication systems, viz. from ⅛ inch to 4 inches in internal diameter in the former chart, and from ¼ inch to 6 inches in the latter.

In the case of laminar motion (Folder X, A), since, according to equation X, 2, the hydraulic gradient i is proportional to the product $Q \times \mu$, the gradient line for each size of tube is plotted against values of this product as abscissæ, and not as a function of Q taken alone. By this means a single chart is made capable of covering all values of both Q and μ, whereas if Q alone were taken as the independent variable a very numerous series of charts, each applicable to only a limited range of values of μ, would be necessary.

For the use of the chart (Folder X, A), therefore, the volume-flow Q is to be expressed in cubic centimetres per second, and μ, the coefficient of viscosity of the lubricant, at the temperature under the actual conditions of flow, in poises. Their product being taken as abscissa, the hydraulic gradient i is immediately found vertically above it in the chart for any particular size of pipe, i being the quotient of the difference of the pressure-heads at the two ends of the tube, expressed as heights of columns of the liquid, divided by the length of the tube.

Since, according to equation X, 2, the gradient is inversely proportional to the density m of the liquid, two gradient lines are shown in the chart for each size of tube, the thicker line of the two being for liquids of density 1·0 and the lighter line for those of density 0·9. The value of i for any other density, within the comparatively small range of the variation of densities of the usual liquid lubricants, can easily be estimated from these two values by inspection.

The "critical points" of transition from laminar to turbulent motion in tubes of the various sizes are given in the chart by oblique straight lines corresponding to the line in fig. X, 2, for a series of values of μ from 0·002 to 1·0 poise. In accordance with the explanations given in connexion with fig. X, 2, it will be understood that the applicability of the gradient line for each size of tube is limited to the part of the line which is below, and on the left-hand side of the critical line having the relevant value of μ. For values of i lying above, and values of $Q \, . \, \mu$ on the right-hand side of this critical line, Folder X, B for turbulent flow is to be used.

For values below the critical value (i.e. in cases of laminar flow), Folder X, A can be used for curved, as well as for straight portions of the pipe-system, provided, of course, that the curved portions have the same sectional form and size as the straight portions. Similarly, in this type of flow, full-way elbows, tees, valves, etc., may be taken as causing only the same loss of head per unit length of the fluid path as straight portions of the conduit. Allowance must, of course, be made for losses of kinetic energy which may occur when the liquid is discharged from tubes into larger vessels, or when its velocity is suddenly reduced in passing from a small to a large section of the conduit. When accuracy in estimating such losses is of importance, it is to be observed that, on account of the non-uniformity of the velocity of a liquid in laminar flow in a tube, its kinetic energy is twice as great as that corresponding to the mean velocity.

The chart of Folder X, B gives for fluids in turbulent motion, the relation between a volume flow and hydraulic grade in a similar manner to that shown by Folder X, A for laminar motion, except that in this case the abscissæ represent the flow Q itself, and not the product $Q\mu$. The information contained in the chart is, however, in this case based essentially on results of experiment, and not entirely on direct calculation from known laws, as in Folder X, A for laminar flow. The results embodied in the chart are derived from experiments on smooth and clean tubes, such as may be expected to be used in circulating and distributing systems for lubricants. The gradient lines shown in the chart for the tubes of various diameters represent smoothed results derived chiefly from the experiments of Darcy (Ref. II, 13), Reynolds (Ref. X, 2), and Saph and Schoder (Ref. X, 3) for tubes of brass, lead, glass, and drawn or welded sheet-iron and steel. In turbulent flow the gradient lines are independent of both the density and the viscosity of the fluid; the critical velocities of transition to laminar flow are given (on the same system as that of Folder X, A) for a range of viscosities from $\mu = 4 \times 10^{-5}$ to $\mu = 8{\cdot}0$ poises. The lower values are of interest, because the chart is applicable to the flow of gases, as well as of liquids, provided that changes of pressure in the gas are not large; and the chart is thus available for calculations of the conveyance of air or hydrogen for cooling purposes.

In the case of the turbulent flow it is not possible, as in laminar flow, to regard the losses of head at bends and other fittings of the pipe-lines, at which the velocities or directions of the flow are changed, as merely additions to the lengths of the tubes. In turbulent flow, changes of direction and variations of section are additional sources of turbulence as well as direct causes of loss of energy, and these losses are often considerable. Rough approximations to the losses of energy which occur in various fittings (treated as factors, or multiples, of the kinetic energy corresponding to the mean velocity of flow in a tube of the diameter for which the fitting is intended) may be made according to Table X, 1.

TABLE X, 1

Kind of fitting	Factor of energy loss
90-degree elbow of average radius.	0·75
,, ,, ,, short radius.	1·00
45-degree elbow of average radius.	0·40
Tee with 90° deflection of stream.	1·80
Tee without ,, ,, ,,	0·70
180-degree short bend.	2·2
Right-angle stop-valve, open.	4 to 6
Globe-valve, open.	8 to 12
Flush entrance to pipe from large vessel.	0·4
Borda ,, ,, ,, ,, ,,	0·8

In turbulent flow, the variation of velocity along the greater part of the tube-radius is much less than in laminar flow, and the total kinetic energy, which may be regarded as being lost when the outlet end of the tube discharges into a large reservoir, is approximately 1·1 times the kinetic energy corresponding to the average velocity of flow, instead of being twice as great, as stated above with respect to laminar motion.

On account of the considerable degree of uncertainty attached to calculations of the losses of head in conduits in which turbulent motion prevails, especially with volumes of flow near to the critical volumes, it is a usual course of discretion in designing to adopt the highest probable estimate of the total losses of head. Not uncommonly a considerable percentage is added even to this estimate as a " factor of safety ", and in order to provide against the results of an over-estimate, it is not unusual to insert reducing valves in the various branches of the pipe-system, the openings of these valves being adjusted after trial, so as to give only the required flow at each point of utilization of the lubricant. It is preferable, however, that the calculations should be made with all the accuracy of which the data permit, and that the allowances for " safety " should be no greater than the unavoidable uncertainties make necessary; otherwise a double economic waste results, on the one hand from needless expenditure on unnecessarily large tubes and fittings, and, on the other, from waste of power in propulsion of the lubricant through the controlling valves.

3. Transfer of Heat to and from Flowing Lubricants.

Heating and cooling of the lubricant is often essential in distributing or circulating systems, as shown in various examples given in Chapter IX. The heat is usually supplied to, or abstracted from, the lubricant while it is being passed through a coil, or nest of tubing, the agent for heating or cooling (usually either water or air) being passed around the outer surfaces of the tubes in the body of the transfer apparatus.

For efficient operation it is essential that both the lubricant and the heating or cooling agent shall be in a state of turbulent motion. When a fluid flowing in a tube is in laminar motion, heat is transmitted into it only by conduction through its substance, at the same rate at each point as if it were at rest, and, as shown by the data given in Sects. III, 6 and 7, the conductivities of the usual liquid lubricants are low. When, on the other hand, a liquid flowing in a tube is in a state of turbulence, heat is transmitted radially through the stream, not only by conduction, but, much more effectively, by the convection of the turbulent motion. Only a very thin layer adjacent to the wall is in a state of laminar, or quasi-laminar flow, and heat is transmitted through this thin layer with a relatively small drop of temperature. An approximation to the thickness of the layer can be calculated by equating the frictional resistance to the turbulent flow in the tube (as given by the diagram of Folder X, B) with the shearing resistance of a film of viscous liquid having zero velocity at its outer surface, and the velocity of the turbulent stream at its inner surface.

By means of an approximate general calculation of this kind, it is found that the thickness of such a film of a liquid, of viscosity μ and density m, is approximately

$$= \frac{400\mu}{mv} \text{ centimetres.}$$

From this result and the known coefficient of conductivity of the liquid, an approximation to the rate of flow of heat, with a given difference of temperature between the two fluids, can be readily found.

More accurate calculations involve taking into account many factors, varying in detail with each form of heat-transfer apparatus. Consideration of these is beyond the scope of this book. For fuller information and data on the subject, existing knowledge of which is mainly empirical, the reader may consult Refs. X, 4, 5 and 6.

An informative theoretical and practical account of cooling systems and appliances, including tubular and other surface coolers employing both water and air as cooling agents, together with vaporization coolers, fans, and other accessory parts of cooling systems, by Dr. G. Lomonossoff, may be found in Part III of his treatise on diesel locomotives (Ref. X, 7).

APPENDIX I

An Approximate Treatment of the Effects of the Rugosities of the Surfaces of Sliding Bearings

The theory of plane bearings given in the text (Chapter IV), and generally accepted, leads to the conclusion that the ratio of tractive force to load diminishes continuously as the inclination of the pad of the bearing to the runner is reduced. Thus, with a given speed and a given viscosity of the lubricant, the coefficient of resistance of a pivoted-pad bearing should decrease continuously and indefinitely as the inclination of the pad is reduced by increasing the load; or, if the load is maintained constant, by diminishing the speed.

The theory assumes that both the pad and the runner have perfectly plane bearing surfaces, which are perfectly smooth in the sense that all projections and depressions from the ideal plane surfaces are of heights or depths which are negligibly small compared with the thickness of the interposed film of lubricant. With these assumptions the coefficient of resistance $k = F/P$ (the ratio of tractive force to load) is found to be directly proportional to c, the inclination of the surface of the pad to that of the runner.

It is found, however, by *ad hoc* experiment, and as a conclusion from practical experience, that, although the tractive coefficient falls in agreement with the theory while the inclination is being reduced down to a certain value, it ultimately reaches a minimum, and thereafter rises with further increase of load or reduction of speed. It has been usual to attribute this apparent disagreement between the results of theory and the facts of experience, to a change in the mode of action of the lubricant taking place when the interspace between the surfaces becomes very small. According to this view, it is supposed that when the minimum of the coefficient of friction is reached the thickness of the lubricating film has become so small that its molecules are subject to the molecular attractions of the solid surfaces, when it no longer forms a fluent film of a determinate viscosity. The term " boundary lubrication " has been attached to the mode of action of the lubricant believed to occur according to this hypothesis (see Sect. IV, 18). Other explanations depending on assumed changes in the viscous properties of the lubricant have also been offered. None of them, however, appears to accord with the experimental facts.

On the other hand, it has been suggested, particularly by Professor E. Heidebroek (Ref. IV, 22), that the apparent anomaly is the effect of the always present rugosities of the surfaces of the runner and pad, and is not inconsistent with the maintenance in the lubricating film of its normal viscosity, or with the laws of viscous motion. According to this view the distribution of the fluid pressures, and the local velocities of flow, together with the total pressure and tractive force, are changed by the presence of the rugosities for a given inclination and velocity of the pad, and a given coefficient of viscosity of the lubricant, but the general mode of action remains that of a fluent film.

This theory does not appear to have been put hitherto to the test of a quantitative examination. It is the object of this Appendix to make such an examination, on the basis of certain simplifying assumptions as to the form of the rugosities, these assumptions being necessary to render the problem amenable to mathematical treatment. It is assumed in the first instance that the surface of either the pad only, or the runner only, is affected by the assumed irregularities, the other of the two surfaces being perfectly plane and smooth. It is further postulated that the rugosities are uniform, and uniformly distributed over the surface of the member affected by them, and that they consist of a series of projections and equal depressions symmetrically arranged above and below a plane; in other words, that the mean surface is a true plane. In order to simplify the algebra, and also to represent the types of irregularity which are most likely to occur on bearing surfaces in practice, the rugosities are assumed to consist of a series of small parallel corrugations, such as result from the usual machining operations of turning and milling. The grooves and ridges of the corrugations may be supposed to be either parallel, transverse, or oblique, to the length of the bearing; for definiteness of illustration, they will be regarded as parallel to it. It is further assumed that the width or "pitch" of the corrugations is very small compared with the length of the pad, which is treated as being of unlimited width, so that the flow of the lubricant can be regarded as being everywhere parallel to the length of the pad and to the direction of sliding. In actual bearings the form of the profiles or cross-sections of the corrugations depends, of course, on the particular shape given to the cutting-edge of the tool by which they are formed. As the simplest hypothetical form, it will be assumed that they have sine-wave profiles.

The general form and arrangement of the bearing surfaces dealt with in the following mathematical discussions are the same as in the plane bearing of unlimited width represented in fig. IV, 5 (p. 74), and the symbols in that figure apply equally to the present case. Fig. A, 1, which agrees generally with fig. IV, 5, shows on an exaggerated scale the form of the corrugations with which the surface of the pad is covered according to hypothesis, the surface of the runner being plane and smooth. This plane and the plane tangential to the crests of the corrugations of the pad intersect one another along the co-ordinate axis OY. The space between the two bearing-members is maintained full of lubricant of viscosity μ. Pressures of intensity p, varying along the length of the pad, are generated in the lubricant when the runner moves over the pad with velocity U in the negative direction of x, the resultant of the pressures p per unit width of the bearing being denoted by P. If the pad is pivoted, the axis of pivoting must be, of course, in the plane of the resultants P.

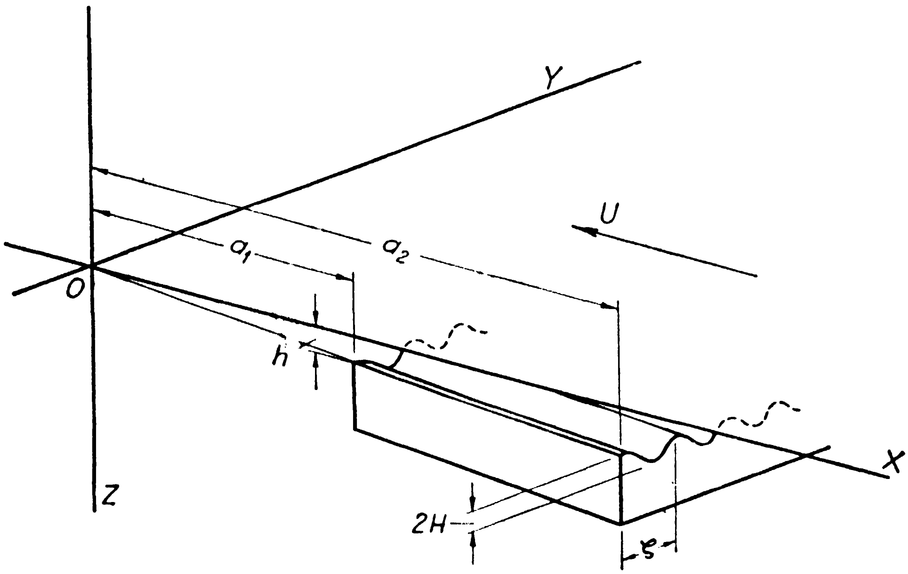


Fig. A, 1

The problem to be considered is then the same as that of Sects. IV, 3–5 of the text, with the difference that the surface of the pad there defined by

$$z = cx, \ \frac{\partial z}{\partial y} = 0,$$

may now be represented by $z = h + H\left(1 + \cos\frac{2\pi y}{\zeta}\right)$, ... A, 1

wherein, as before, $h = cx$, while ζ, the pitch of the corrugations, and H, their half-amplitude in height, are constants.

The equation of flow between the plane runner and the plane pad, both being of infinite width, and p being independent of y, viz.

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + 6\mu U\frac{\partial h}{\partial x} = 0,$$

then becomes $\frac{\partial}{\partial x}\left(\overline{z^3}\frac{\partial p}{\partial x}\right) + 6\mu U\frac{\partial \bar{z}}{\partial x} = 0,$ . . . . A, 2

wherein $\bar{z}$ and $\overline{z^3}$ represent respectively the integral of z (the thickness of the fluid film), and the integral of z^3, both taken over unit width of the bearing at right angles to the corrugations, of which a large number are assumed to be included in the unit width, this being the justification for treating p as dependent only on x.

The width of the bearing being unlimited, the flow is everywhere parallel to OX. In the immediate neighbourhood of the ends of the pad at a_1 and a_2, $p = 0$.

Since the integrals $\bar{z}$ and $\overline{z^3}$ vary only in the direction of x, equation A, 2 can be integrated directly to give

$$\overline{z^3}\frac{\partial p}{\partial x} + 6\mu U(\bar{z} - \bar{z}_0) = 0, \quad \text{A, 3}$$

in which $\bar{z}_0$ is the value of $\bar{z}$ at which $\partial p/\partial x = 0$, that is to say, at which p is at its maximum.

Since $$z = h + H\left(1 + \cos\frac{2\pi y}{\zeta}\right),$$

which may be written $z = h + H(1 + \cos\theta)$,

where $$\theta = \frac{2\pi y}{\zeta},$$

$$\bar{z} = \frac{1}{2\pi}\int_0^{2\pi}\{h + H(1 + \cos\theta)\}d\theta,$$

$$= \frac{1}{2\pi}\Big[(h + H)\theta\Big]_0^{2\pi} = h + H. \quad \text{A, 4}$$

In particular $\bar{z}_0 = h_0 + H$, where h_0 is the value of h at which $\partial p/\partial x = 0$.

Also, $$\overline{z^3} = \frac{1}{2\pi}\int_0^{2\pi}\{(h + H) + H\cos\theta\}^3 d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\{(h + H)^3 + 3(h + H)^2 H\cos\theta + 3(h + H)H^2\cos^2\theta + H^3\cos^3\theta\}d\theta$$

$$= \frac{1}{2\pi}\Big[(h + H)^3\theta + 3(h + H)^2 H\sin\theta + 3(h + H)H^2(\tfrac{1}{2}\sin\theta\cos\theta + \tfrac{1}{2}\theta) + H^3(\tfrac{1}{3}\cos^2\theta + \tfrac{2}{3})\sin\theta\Big]_0^{2\pi}$$

$$= \frac{1}{2\pi}\{(h + H)^3 . 2\pi + 3(h + H)H^2\pi\}$$

$$= h^3 + 3h^2H + \tfrac{9}{2}hH^2 + \tfrac{5}{2}H^3. \quad \text{A, 5}$$

From A, 3 and A, 4,

$$\overline{z^3}\frac{\partial p}{\partial x} + 6\mu U(h - h_0) = 0,$$

and therefore, in view of A, 5,

$$\frac{\partial p}{\partial x} = -12\mu U\frac{h - h_0}{2h^3 + 6h^2H + 9hH^2 + 5H^3}, \quad \text{A, 6}$$

h standing for cx.

The complete solution of the problem in terms of the exact equation A, 6 being intractable, its further handling will be by means of two approximate solutions, for the first of which h will be assumed to be large compared with H, and for the second, H to be large relatively to h.

The first of these assumptions (h large relatively to H), representing the case in which the corrugations of the pád are of small height compared with the thickness of all parts of the lubricating film, passes over in the limit when H is made vanishingly small, to the case of the truly plane and smooth surfaces treated in Sects. IV, 3–5. From the practical standpoint it represents the condition which arises when the load on a bearing is increased (other conditions remaining constant) to such an extent that, with diminution of the inclination of the pad, the effect of the rugosities of the surfaces become appreciable, and of increasing effect as the ratio of H to h becomes larger. In accordance with the assumption, terms containing the second and higher powers of H/h will be neglected in comparison with those of the first power. Equation A, 6 then becomes

$$\frac{\partial p}{\partial x} = -6\mu U \frac{h - h_0}{h^3 + 3h^2H}$$

$$= -6\mu U\left\{\frac{1}{h^2 + 3hH} - \frac{h_0}{h^3 + 3h^2H}\right\}, \quad . \quad . \quad . \quad . \quad \text{A, 7}$$

or putting $h = cx$ and $h_0 = c\bar{a}$,

$$-\frac{c}{6\mu U}\cdot\frac{\partial p}{\partial x} = \frac{1}{x(cx + 3H)} - \frac{\bar{a}}{x^2(cx + 3H)},$$

and, by integration,

$$-\frac{c}{6\mu U}p + C \text{ (const. of integration)}$$

$$= -\frac{1}{3H}\{\log(3H + cx) - \log x\}$$

$$-\bar{a}\left\{-\frac{1}{3Hx} + \frac{c}{9H^2}[\log(3H + cx) - \log x]\right\},$$

or

$$-\frac{3Hc}{6\mu U}p + 3HC$$

$$= \frac{\bar{a}}{x} - \left(1 + \frac{c\bar{a}}{3H}\right)\{\log(3H + cx) - \log x\}. \quad . \quad . \quad \text{A, 8}$$

Since $p = 0$, both when $x = a_1$ and when $x = a_2$,

$$3HC = \frac{\bar{a}}{a_1} - \left(1 + \frac{c\bar{a}}{3H}\right)\{\log(3H + ca_1) - \log a_1\}$$

and

$$3HC = \frac{\bar{a}}{a_2} - \left(1 + \frac{c\bar{a}}{3H}\right)\{\log(3H + ca_2) - \log a_2\},$$

from which two equations are determined, viz.

$$\bar{a} = \frac{\log(3H + ca_1) - \log(3H + ca_2) - (\log a_1 - \log a_2)}{\frac{a_2 - a_1}{a_1 a_2} - \frac{c}{3H}\{\log(3H + ca_1) - \log(3H + ca_2) - (\log a_1 - \log a_2)\}};$$

and

$$6HC = \frac{6Ha_1\{\log(3H + ca_1) - \log a_1\} - 6Ha_2\{\log(3H + ca_2) - \log a_2\}}{-3H(a_1 - a_2) - ca_1 a_2\{\log(3H + ca_1) - \log(3H + ca_2) - (\log a_1 - \log a_2)\}}.$$

For brevity put

$$\log(3H + ca_1) = \kappa_1, \quad \log(3H + ca_2) = \kappa_2,$$

$$\log a_1 = \lambda_1, \quad \log a_2 = \lambda_2,$$

and $$-3H(a_1 - a_2) - ca_1 a_2\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\} = D.$$

Then $$\bar{a} = \frac{3Ha_1 a_2\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\}}{D}$$

and $$3HC = \frac{3Ha_1(\kappa_1 - \lambda_1) - 3Ha_2(\kappa_2 - \lambda_2)}{D}.$$

From A, 8

$$\begin{aligned} -\frac{DHc}{2\mu U} p = &\{-3Ha_1(\kappa_1 - \lambda_1) + 3Ha_2(\kappa_2 - \lambda_2)\} \\ &+ 3Ha_1 a_2\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\} \cdot \frac{1}{x} \\ &+ 3H(a_1 - a_2)\{\log(3H + cx)\} - 3H(a_1 - a_2)\log x, \end{aligned} \qquad \text{A, 9}$$

which gives, on integrating with respect to x between the limits $x = a_1$ and $x = a_2$,

$$\begin{aligned} -\frac{DHc}{2\mu U} P = &\{-3Ha_1(\kappa_1 - \lambda_1) + 3Ha_2(\kappa_2 - \lambda_2)\}(a_2 - a_1) \\ &+ 3Ha_1 a_2\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\}(\lambda_2 - \lambda_1) \\ &+ 3H(a_1 - a_2)\left\{(a_2\kappa_2 - a_1\kappa_1) - (a_2 - a_1) + \frac{3H}{c}(\kappa_2 - \kappa_1)\right\} \\ &- 3H(a_1 - a_2)\{(a_2\lambda_2 - a_1\lambda_1) - (a_2 - a_1)\}. \end{aligned}$$

This reduces to

$$\frac{Pc^2}{6\mu U} = \frac{ca_1 a_2(\lambda_1 - \lambda_2)\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\} + 3H(a_1 - a_2)(\kappa_1 - \kappa_2)}{ca_1 a_2\{(\kappa_1 - \lambda_1) - (\kappa_2 - \lambda_2)\} + 3H(a_1 - a_2)},$$

and, on restoring the values of κ_1, κ_2, λ_1 and λ_2, the equation

$$\frac{Pc^2}{6\mu U} = \frac{\log\frac{a_1}{a_2}\left\{\log\frac{3H + ca_1}{a_1} - \log\frac{3H + ca_2}{a_2}\right\} + \frac{3H}{c}\frac{a_1 - a_2}{a_1 a_2}\log\frac{3H + ca_1}{3H + ca_2}}{\log\frac{3H + ca_1}{a_1} - \log\frac{3H + ca_2}{a_2} + \frac{3H}{c}\frac{a_1 - a_2}{a_1 a_2}} \qquad \text{A, 10}$$

is obtained as the equation giving P, the resultant of the pressures on the pad, per each unit of its width. (The logarithms throughout are, of course, natural logarithms.)

A similar but less involved calculation determines the tractive force, the average intensity of which, at any point x of the length of the bearing, is found by integration across a single corrugation to be

$$f = \frac{\mu U}{2\pi}\int_0^{2\pi}\frac{1}{z}\,d\theta = \frac{\mu U}{2\pi}\int_0^{2\pi}\frac{d\theta}{h + H(1+\cos\theta)}$$

$$= \frac{\mu U}{2\pi}\left[\frac{2}{\{(h+H)^2 - H^2\}^{\frac{1}{2}}}\arc\tan\frac{h^{\frac{1}{2}}\tan\frac{1}{2}\theta}{\{(h+H)^2 - H^2\}^{\frac{1}{2}}}\right]_0^{2\pi}$$

$$= \frac{\mu U}{\{h^2 + 2hH\}^{\frac{1}{2}}} = \frac{\mu U}{\{c^2x^2 + 2cxH\}^{\frac{1}{2}}}. \qquad \text{A, 11}$$

Hence the viscous traction on the whole surface, per unit of its width, is

$$F = \int_{a_1}^{a_2} f\,dx = \frac{\mu U}{c^{\frac{1}{2}}}\int_{a_1}^{a_2}\frac{dx}{(cx^2 + 2Hx)^{\frac{1}{2}}}$$

$$= \frac{\mu U}{c^{\frac{1}{2}}}\left[\frac{1}{c^{\frac{1}{2}}}\log\{2cx + 2H + 2c^{\frac{1}{2}}(cx^2 + 2Hx)^{\frac{1}{2}}\}\right]_{a_1}^{a_2}$$

$$= \frac{\mu U}{c}\log\frac{ca_2 + H + c^{\frac{1}{2}}(ca_2^2 + 2Ha_2)^{\frac{1}{2}}}{ca_1 + H + c^{\frac{1}{2}}(ca_1^2 + 2Ha_1)^{\frac{1}{2}}}. \qquad \text{A, 12}$$

To obtain the total traction per unit width on the pad, there is to be added to this value of F, the resolved part of the charge per unit width (viz. $\frac{1}{2}Pc$) as in Sect. IV, 3 of the text.

Equations A, 11 and A, 12 are not restricted, as is equation A, 10, to values of H which are small in comparison with h, that is to say, with ca_1 or ca_2.

When H becomes vanishingly small in comparison with ca_1 and ca_2, the value of tractive force given by A, 12 reduces to that given for the smooth-surfaced bearing by equation IV, 16, viz.

$$F = \frac{\mu U}{c}\log\frac{a_2}{a_1}.$$

The value of the coefficient of resistance k of the bearing is, of course, given by

$$k = \frac{F}{P}, \qquad \text{A, 13}$$

the value of F being taken from A, 12 and that of P from A, 10. For the purposes of the present discussion the longitudinal component of the charge $\frac{1}{2}Pc$ may be neglected in comparison with F.

Instead of the simplifying assumption hitherto made, that the amplitude of the corrugations H is small in comparison with the thickness h of the lubricating film, the other extreme condition, viz. that H is large compared with h, may similarly be investigated by means of equation A, 6. This is, of course, the condition which arises when a pivoted pad, having a corrugated surface of the postulated form, is subjected to increasing load, so that its inclination c becomes ultimately so small that ca_1 is smaller than the fixed half-amplitude H of the corrugations.

In this case the terms in the denominator of the right-hand expression of equation A, 6, which contain as factors H^2 and H^3, become large relatively to those which contain h^2 and h^3, so that the latter terms can be neglected and the equation becomes

$$\frac{\partial p}{\partial x} = -12\mu U \frac{h - h_0}{9hH^2 + 5H^3}, \quad \ldots \ldots \quad \text{A, 14}$$

which, putting $h = cx$, may be written as

$$\frac{\partial p}{\partial x} = \frac{-12\mu U}{9H^2} \cdot \frac{cx - h_0}{cx + 5H/9}$$

$$= -\frac{4\mu U}{3H^2} \cdot \frac{x - h_0/c}{x + 5H/9c}$$

$$\text{or } \frac{\partial p}{\partial x} = \frac{-4\mu U}{3H^2}\left\{\frac{x}{x + 5H/9c} - \frac{\bar{a}}{x + 5H/9c}\right\}, \quad \ldots \quad \text{A, 15}$$

$\bar{a}$ being, as before, the value of x at which $\partial p/\partial x = 0$.

For brevity, put m for $4\mu U/3H^2$, and n for $5H/9c$, then

$$-\frac{\partial p}{\partial x} = m\left\{\frac{x}{x + n} - \frac{\bar{a}}{x + n}\right\} = m\left\{\frac{x + n}{x + n} - \frac{\bar{a} + n}{x + n}\right\}$$

$$= m\left\{1 - \frac{n + \bar{a}}{x + n}\right\},$$

and, by integration,

$$-p = m\{x - (n + \bar{a}) \log (x + n)\} + C \text{ (const.)}.$$

Since $p = 0$ when $x = a_1$ and when $x = a_2$,

$$m\{a_1 - (n + \bar{a}) \log (a_1 + n)\} + C = 0$$

and

$$m\{a_2 - (n + \bar{a}) \log (a_2 + n)\} + C = 0.$$

By addition,

$$C = -\tfrac{1}{2}m\{(a_2 + a_1) - (n + \bar{a})[\log (a_2 + n) + \log (a_1 + n)]\},$$

and by subtraction,

$$m\{(a_2 - a_1) - (n + \bar{a})[\log (a_2 + n) - \log (a_1 + n)]\} = 0;$$

whence

$$n + \bar{a} = \frac{a_2 - a_1}{\log (a_2 + n) - \log (a_1 + n)},$$

$$\text{and } -p = m\left\{x - \frac{(a_2 - a_1)\log(x+n)}{\log(a_2+n) - \log(a_1+n)}\right\}$$
$$-\frac{m}{2}\left\{(a_2+a_1) - (a_2-a_1)\frac{\log(a_2+n)+\log(a_1+n)}{\log(a_2+n)-\log(a_1+n)}\right\}. \qquad \text{A, 16}$$

Another integration gives the charge, or total of the pressures, on each unit width of the pad, viz.

$$P = \int_{a_1}^{a_2} p\,dx.$$

From A, 16,

$$-\frac{P}{m} = \int_{a_1}^{a_2}\left(-\frac{p}{m}\right)dx$$
$$= \left[\tfrac{1}{2}x^2\right]_{a_1}^{a_2} - \tfrac{1}{2}(a_2+a_1)\left[x\right]_{a_1}^{a_2}$$
$$-(a_2-a_1)\frac{\left[x\log(x+n) - x + n\log(x+n)\right]_{a_1}^{a_2}}{\log(a_2+n)-\log(a_1+n)}$$
$$+\tfrac{1}{2}(a_2-a_1)\frac{\{\log(a_2+n)+\log(a_1+n)\}\left[x\right]_{a_1}^{a_2}}{\log(a_2+n)-\log(a_1+n)}$$
$$= 0 - (a_2-a_1).\left[a_2\log(a_2+n) - a_1\log(a_1+n) - (a_2-a_1)\right.$$
$$+ n\log(a_2+n) - n\log(a_1+n) - \tfrac{1}{2}(a_2-a_1)\{\log(a_2+n)$$
$$\left.+\log(a_1+n)\}\right]\Big/\{\log(a_2+n)-\log(a_1+n)\}$$
$$= \frac{-\{\tfrac{1}{2}(a_2^2-a_1^2)+(a_2-a_1)n\}\{\log(a_2+n)-\log(a_1+n)\}+(a_2-a_1)^2}{\log(a_2+n)-\log(a_1+n)},$$

and

$$P = m\left\{\tfrac{1}{2}(a_2^2-a_1^2)+(a_2-a_1)n - \frac{(a_2-a_1)^2}{\log(a_2+n)-\log(a_1+n)}\right\},$$

or, restoring the expressions for which m and n were substituted,

$$P = \frac{4\mu U}{3H^2}\left\{\frac{a_2^2-a_1^2}{2} + (a_2-a_1)\frac{5H}{9c} - \frac{(a_2-a_1)^2}{\log(a_2+5H/9c)-\log(a_1+5H/9c)}\right\}. \qquad \text{A, 17}$$

The tractive force F is given by equation A, 12, which, as already stated, is valid for all ratios of H to h. For very large values of that ratio, the equation, however, reduces to

$$F = \frac{\sqrt{2}\,.\,\mu U}{c^{\frac{1}{2}}H^{\frac{1}{2}}}\{a_2^{\frac{1}{2}} - a_1^{\frac{1}{2}}\}, \quad \ldots\ldots \quad \text{A, 18}$$

which can be conveniently used for purposes of approximation, and which shows that when the inclination c is so small that ca_1 is much smaller than

H, F no longer varies inversely as c, as in the plane and smooth bearing, but inversely as the square root of c.

In Back Folder A, 2 some of the chief results deducible from these equations have been plotted against the ratio of $h_1 = ca_1$ to H as the independent variable of the diagram, this variable being denoted by q. On account of the large range of values covered by the curves, the diagram is plotted to logarithmic scales.

It gives the values of P, the charge carried by the pad, and of k, the coefficient of resistance, both for a pad having corrugations of half-amplitude H, and for a truly plane pad. In the latter case the independent variable q, being the ratio between ca_1 and the fixed linear magnitude H, is of course proportional to c, the mutual inclination of the two plane surfaces of the bearing.

The values of P and k shown in the diagram have been calculated for the range of q from 2·5 to 30 (i.e. for $q = h_1/H$ large), from equations A, 10 and A, 12; and those for the range of q from $\frac{1}{3}$ to $\frac{1}{30}$ (h_1/H small), from equations A, 17 and A, 18. The intermediate portions of the curves for P and k (i.e. the portions for values between $q = \frac{1}{3}$ and $q = 2{\cdot}5$) have been filled in merely as smooth curves connecting the calculated portions and are to be regarded as rough approximations only. They are consequently shown as dotted lines.

The straight lines marked P_0 and k_0 in the diagram show the values of the charge and the tractive coefficient of a plane and smooth pad of the same length, operating under the same conditions as the corrugated pad, except for the absence of any rugosities on its surface.

Equations IV, 13 and IV, 17 of the text have shown that P_0 varies inversely as the square of c, the inclination, and k_0 directly as its first power. Accordingly their inclinations appear in the logarithmic diagram as being respectively 2 : 1 and 1 : 1 relatively to the horizontal axis, and they are seen to be respectively asymptotic to the curves P and k when the ratio of H to h is vanishingly small.

In accordance with equations A, 10, 12, and 13, the diagram shows the value of k as increasing indefinitely, and that of P as decreasing continuously as the ratio of h to H becomes larger and larger. These results of the calculations are obviously dependent on the idealized form assumed for the corrugations, and especially to the feature that the crests of the corrugations are all, according to the assumptions, tangential to one plane. With rugosities of much less regular forms, such as occur in practice, a limit to the continuity of the curves will exist (owing to the irregular and fortuitous nature of the rugosities of actual surfaces), which must result in solid contacts occurring when the ratio h to H falls to a certain value, and thereafter increasing in number as the ratio is further diminished, until the viscous resistance becomes wholly masked by solid friction.

The similarity of the curve for k (the coefficient of resistance), which is derived from the calculations, to that which has been found by experiment is commented on in Sect. IV, 15 of the text. The minimum value of k occurs, according to the theory, when the minimum clearance is about equal to the height of the corrugations, and this result appears to be roughly correct for the rugosities met with in actual practice. On account of the differences of form between the assumed and the actual irregularities, and also in view of the assumption that the rugosities affect only one of the surfaces, no great significance is to be attached to the precise values of the ratio of h to H at which such effects occur.

APPENDIX II

Investigation of Bearing Problems by means of Physical Analogies

It is well known that the laws of viscous flow in fluids are closely analogous to those of the flow of electricity in conducting substances, and that the equations which express them are in their general forms the same—fluid pressures and velocities in the one case, corresponding to potential and current in the other. Still more nearly related to the velocities and pressures which occur in viscous fluids are the strains and stresses, respectively, of elastic solids.

In some respects it is more convenient to investigate the behaviour of the fluid film of a bearing by experiments on its physical analogue of one or other of these kinds, than by experiments direct on the fluid body itself.

The application of the *electrical analogy* to quantitative examination of the action of fluid films in bearings was originated and developed in detail by Kingsbury, who described his technique and the rationale of his methods in Ref. A, II, 1, with exemplifications giving the results of chief practical interest for particular forms of sectorial thrust-bearing pads, and for other bearings in which exact mathematical treatment is impracticable. Kingsbury also, as a check on the correctness and accuracy of his methods, re-determined by the electrical analogy some of the numerical results, as for instance those for the square, plane bearing-pad, which are known by means of direct calculation, and found very satisfactory agreement.

The general equation for the flow in a viscous film which has been given in the text of Chap. IV (equation IV, 4), viz.

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial y}\right) + 6\left(U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y}\right) = 0,$$

in which μ as well as h may be variable, is paralleled by the equation for the flow of electricity in a sheet of conducting material of varying thickness H and resistivity r, viz.

$$\frac{\partial}{\partial x}\left(\frac{H}{r}\frac{\partial E}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{H}{r}\frac{\partial E}{\partial y}\right) = \frac{I}{\delta \mathfrak{A}},$$

in which E is the electrical potential, and I the amount of current fed into the surface of the sheet over a small area $\delta\mathfrak{A}$.

To make use of this analogy for the determination of the pressures developed in a particular bearing-film, a bath of electrolyte of varying depth H is prepared which has the same shape in plan (but usually on an enlarged scale) as the bearing-pad to be investigated, and in which the values of H/r at all points are proportional to those of h^3/μ at the corresponding points of the bearing area, current I being then fed in over small areas δA, systematically arranged over the surface of the bath, so as to represent the last term on the left-hand side of the equation for viscous flow in proper proportion. This process is repeated for different positions of the elementary area δA, until the whole surface of the bath has been covered, and at the same time, for each location of δA, the potential of the electrolyte is observed at a sufficient number of points of its surface to enable the potential of the whole surface to be mapped. The results are then summed for all the positions of δA.

It will be seen that when the values of the potential E over the whole area of the bath have been determined in this way, those of p at corresponding points of the bearing film will be known, being in direct proportion to the totalized values of E.

Figs. A, 3 and A, 4 (p. 294), taken from the paper to which reference has been made, illustrate in outline the apparatus used. Fig. A, 3 is a plan view of the test-bath, as arranged for experiments on a rectangular pad, together with the electrical circuits. In fig. A, 4 the bath is shown in longitudinal section, L representing the inflow and O the outflow end of the bearing-pad. The fluid F, of low electric conductivity, with a free upper surface, is of a varying depth, at each point proportional to the quotient h^3/μ at the corresponding point of the pad, the resistivity r being regarded as constant. Metal plates, P_1, P_0, inserted for the full depth of the bath at the inflow and outflow ends, establish the equal potentials which correspond to the equal pressures at the ends of the bearing. The bath shown is arranged to represent a portion of a pad of unlimited width, being contained between two longitudinal planes, with no flow in the direction of Y. The sides of the bath are therefore formed of non-conductors. For investigation of a pad of limited width, equipotential plates would be placed at the sides, as well as the ends, of the bath.

Metal-rod electrodes I and E, inserted through the cover of the bath and extending downwards to the full depth of the fluid, provide means, respectively, for supplying current and measuring potential, at points of the area defined by a rectangular grid ruled on the cover (each electrode being located at the centre of a square of the grid). The experimental procedure consists in supplying measured current successively at the centres of all the squares, by electrodes I, and observing the voltages thereby produced at the other grid centres such as E. The sum of the observed values of the voltage produced at a given grid centre by the separate currents gives, after applying the appropriate conversion factors, the pressure in the bearing film at the point corresponding to that grid centre.

It was found by Kingsbury that alternating current gave more satisfactory results than direct current. The supply was taken from 60-cycle mains at 110 volts, the current being controlled by wound rheostats. The bath preferred,

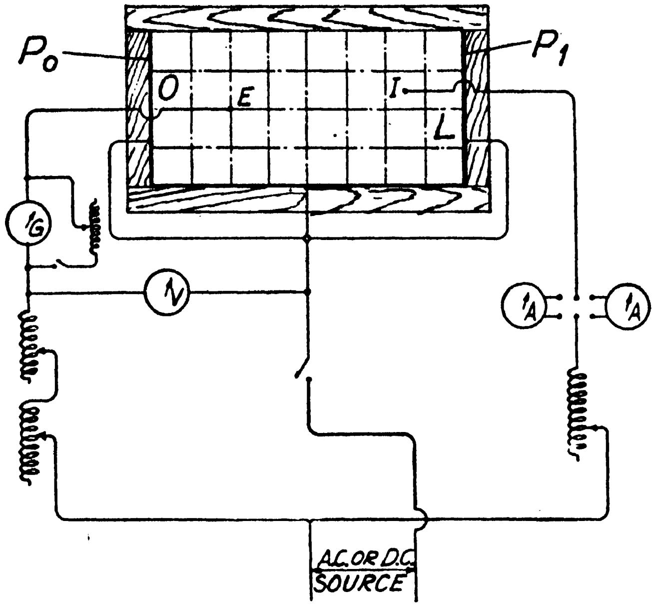


Fig. A, 3

after trial of various electrolytes, was a solution of potassium dichromate in distilled water, having a resistivity of at least 500 in ohm-inch units. Chromium-plated copper electrodes were found to remain clean and unpolarized for long periods. The minimum voltages to be measured at the electrodes E were from

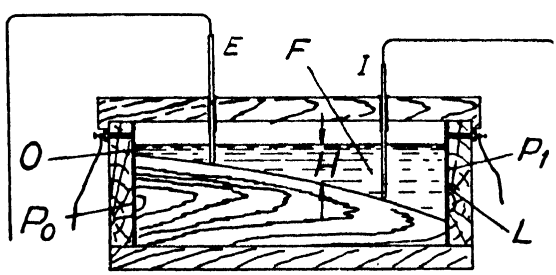


Fig. A, 4

0·05 to 0·06 volt, and were thus within the range of commercial voltmeters. As will be seen on tracing the circuits shown in fig. A, 3, the potentials were measured by a null method; if current were taken from the electrode to operate the voltmeter the distribution of current in the bath would be thereby altered, and the true potential would not be shown.

Fig. A, 5 shows the isobars of a sectorial thrust bearing-pad as determined by Kingsbury using such an apparatus, the location of the pad in relation to the runner of the bearing being given by fig. V, 21 (p. 150). As will be seen from this figure, the line of intersection of the plane bearing surfaces of the pad and runner is radial, and the angles between this radial line and the outflow and inflow edges of the pad, α_1 and α_2 respectively, are such that $\sin \alpha_2 = 2 \sin \alpha_1$. Therefore, in this respect, the location of the sectorial pad is a particular case of the mode of location described in Sect. V, 7 of the text as being adapted to approximate calculation by means of its simulation of a straight rectangular pad. The particular pad shown in fig. A, 5 having, however, the rather low width-length ratio of 0·75, is not of a form favourable to obtaining a very good approximation by that method. Kingsbury found (fig. A, 5) the centre of pressure to be at $\gamma = 0{\cdot}39$, and $r_m = 13{\cdot}37$ inches. From Front Folder IV, E, the location for the corresponding rectangular pad is found to be given by $\gamma = 0{\cdot}41$ and $r_m = 13{\cdot}90$ inches. In a sectorial pad of more nearly normal proportions, having a width-length ratio greater than unity, a closer agreement between the results of the two methods would be expected.

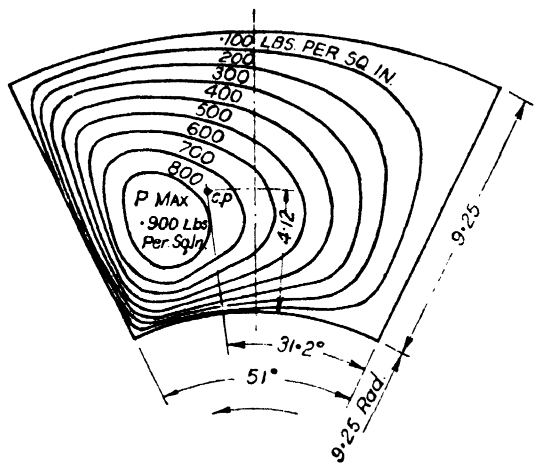


Fig. A, 5

The intensity of pressure having been determined by the electrical method at a sufficient number of points distributed over the area of the pad, the total pressure (charge) on the pad, and its centre of pressure (pivoting-point), are found by arithmetical summation.

The application of this method was extended by Kingsbury to journal bearings, and it was also applied by him to the investigation of some general questions which arise in the application of the theory of viscous flow to lubrication problems, and particularly to the examination of the possibility of considerable kinetic effects arising in the films of bearings running at very high speeds. In such applications, however, it has always to be remembered that the application of both the equations of viscous flow and their electric analogues are limited to effects which are essentially additive, and therefore cannot be applied strictly to kinetic conditions.

The exercise of this experimental method, except in the simplest cases, is, as will have been inferred from the described processes of measurement, somewhat laborious, even the simple and symmetrical square plane pad (for which the isobars, determined by direct calculation, have been given in figs. IV, 13, p. 98, and IV, 14, p. 99) requiring about 900 observations of current and voltage.

The arithmetical work involved in the conversion from the units of the one set of physical phenomena to the other, and in the collation of the results, is also very considerable.

ELASTIC-SOLID ANALOGY.

Investigation of the behaviour of lubricating films by means of experiments on the deformations of elastic solids of corresponding forms is based on the analogy, or rather, identity, which exists between the equations which connect viscous flow and fluid pressures on the one hand, and elastic strains and stresses on the other.

The basic equations relating to the shearing motion of a viscous fluid, which have been given in Chapter II of the text, e.g. equations II, 14,

$$s_{yz} = \mu\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right), \text{ etc.,}$$

for the shearing stresses on the sides of a rectangular element $\delta x\, \delta y\, \delta z$, are equally applicable to the motion of an incompressible viscous fluid and for the deformation of any elastic solid which can likewise be regarded as incompressible. In the first case u, v, w are the velocity components in the directions x, y, z, and μ the coefficient of viscosity; in the second case u, v, w represent the shearing deformations and μ the shear modulus of the solid.

For an interpretation of the physical meaning to be attached to this analogy reference may be made to a passage in Clerk Maxwell's paper on the "Dynamic Theory of Gases" (Ref. A, II, 2). The practical application of the analogy which is given here was first described in Ref. IV, 7 as having been applied as an approximate means of checking the results of the theory of plane bearings of finite width given in Chap. IV, 11–13 of the text.

Unlike the electric method, the elastic-solid method is adapted to give immediately the integrated effects of the action rather than the details. Thus it shows at once the total charge carried by a pad subjected to given conditions, and the position of the centre of pressure, but it is not well adapted to give the intensity of pressure at the various points of the film.

The arrangement of the apparatus is shown in fig. A, 6. A and B are two solid metal plates of uniform thickness; C is a wedge-shaped plate also of metal, or other rigid material, slightly smaller in length and width than the plates A and B, the angle between its upper and lower faces being of the order of 5 degrees. Between A and C, and C and B is placed one of the two precisely similar tapered slabs of material D, D, each of which has one-half of the taper of the plate C, so that, when all five of the above-mentioned parts are in position, the plates A and B are parallel. The slabs D, D are subject to the deformations and stresses which the apparatus is arranged to measure, and they are both formed of some elastic, but practically incompressible, solid material of which india-rubber is a well-known example. The material which has been found most convenient for use is, however, gelatine containing a high percentage of water, any desired shear modulus within reasonable limits being readily obtainable by varying the proportion of water.

For carrying out a test, the plate B is rigidly fastened on the work-bench, to which is also fixed the standard E, whose upper end is provided with a fulcrum for the lever H, carrying at its other, free, end the scale-plate J, with the variable test-weights W. A block K is slidable along the lever H which it supports by means of the rollers R interposed between the upper surface of the plate A and the equally plane and smooth under-surface of the lever H. The plate A is prevented from moving horizontally by a pair of links L, L which attach it to the standard E, but allow it complete freedom to move vertically. The wedge-shaped plate C is provided at the middle of its thickness and width with a link M which passes through an aperture in the standard E and through a coiled spring S. At the distal end of the link M it is screwed and fitted with a large milled-edge nut N abutting against the end of the spring S, which can by these means be compressed between the standard E and the nut N. The spring is calibrated so that the force which it exerts, and the con-

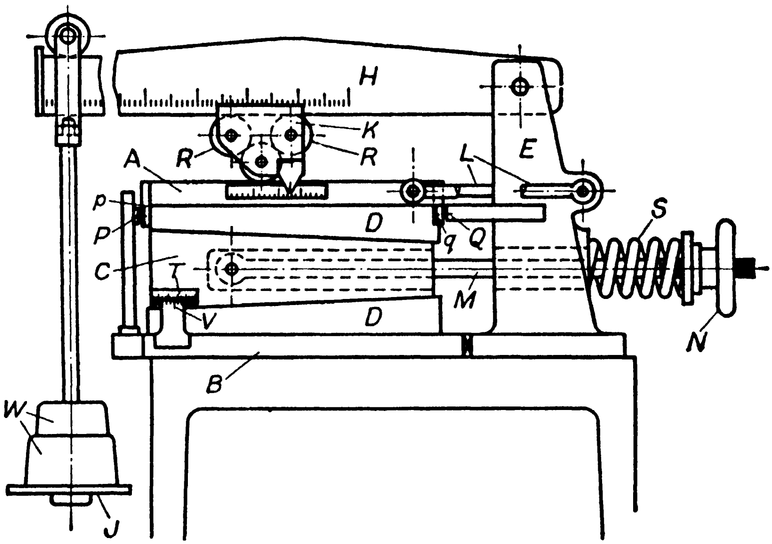


Fig. A, 6

sequent tractive force exerted by the link M on the plate C, are known when the compressed length of the spring is measured. Pointers P, Q, projecting respectively from the base-plate B and from the standard E, in conjunction with short vertical scales *p* and *q* attached to the plate A at its ends, enable any small vertical movements of these ends to be observed. Scales marked on the lever H, and on the upper surface of the plate A, define the positions of the scale-platform J and the lower roller R. The vernier index V attached to the plate B, in conjunction with the scale T on the plate C, measures the horizontal movement of the latter plate.

The procedure in using the apparatus is as follows:

All the parts being in place, and the lever H and block K having been temporarily raised out of engagement with plate A, a suitable small weight is placed on the latter, and its position, both lengthwise and sidewise, adjusted until the plate A, whose precise position is dependent on elastic deformations of the slabs D, D, is truly parallel to plate B, as determined by gauging the vertical distances between their four pairs of corners. The small weight is then fixed in this position.

The lever H, scale platform J, and test-weights W are then placed in position and the bearing block and roller adjusted until the plates A and B are again parallel, the tapered plate C being displaced horizontally through a distance U. Then the compressed length of the spring S is observed.

The displacement U represents the velocity of the pad; the force exerted by the lower roller R on the plate A, its charge; and the tension of the link M, its tractive resistance. The position of the lower roller R determines the position of the resultant pressure.

If results are required in terms of the coefficient of viscosity μ of the lubricant of the bearing, the value of the corresponding constant μ' of the elastic solid is determined by slight modifications of the apparatus and procedure, a parallel-faced plate being substituted for the tapered plate C. Modified arrangements of the apparatus adapt it to be used for experiments on sectorial thrust plates, partial journal bearings, or other machine parts.

Unlike the electric analogue of viscous motion, the elastic-solid analogue gives a direct and concrete image of the action to be examined, and is thus suitable for making rapid qualitative observations and tests of postulated modes of viscous motion, as well as for making rapid quantitative determinations on a series of bearings of a single type, for the purpose of finding the optimum form for some particular application. It can be applied to the investigation of the effects, in any particular bearing, of variations of the viscosity from place to place in the lubricating film. It is not, however, practicable to make corresponding variations in the shear modulus of the gelatine slabs by varying the water content from point to point, since diffusion of water would take place so rapidly as to change the distribution to an appreciable extent towards uniformity before an experiment could be carried through. The same object can, however, be attained by making the product $h^3\mu'$ (with μ' constant) proportional, at all points of the model, to the product $h^3\mu$ (with μ varying as desired) at the corresponding points of the bearing film.

APPENDIX III

Motions of Lubricated Rolling Elements under Rapid Increase of Charge

In Sect. VII, 9, and other passages of the text, it is stated that a ball or roller of a rolling bearing, when subject to a sudden increase of its charge, displaces almost instantly the lubricating film existing between itself and the race of the bearing, so that virtual solid contact immediately takes place. It is the purpose of this Appendix to show that such must be the case.

Considering in the first place the conditions which exist between a ball and an inner race, since one of the principal curvatures of the race surface is positive and the other negative in sign, and both are usually of the same order of magnitude, the race surface may be replaced, with a sufficient degree of approximation for the present discussion, by a plane surface. The thickness of the film of lubricant will be assumed to be small compared with the radius of the ball, but large relatively to the distance between the ball and plane at their points of closest approach. If, as shown in fig. A, 7, this distance is h at any instant of time t during the motion consequent on the sudden application of the charge P, and if the thickness of the film of lubricant (supposed for simplicity to be on the race only) is z_0, above which the pressure is zero, then, as the sphere moves towards the plane, it displaces the liquid radially outwards with velocity u, varying both with the radius r and with the distance z from the plane, z being less than z_0.

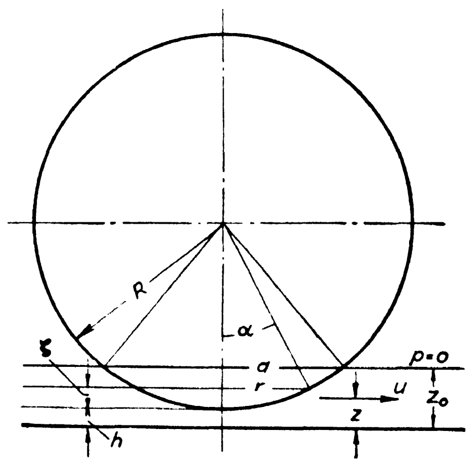


Fig. A, 7

Since z_0 is assumed to be large compared with h at all times, the velocity u will be negligibly small, both where the film thickness is z_0 and on the sphere where $z = z_1$. Thus, as in equation II, 32,

$$\mu \frac{\partial^2 u}{\partial z^2} \cdot dz = \frac{\partial p}{\partial r} \cdot dz.$$

$$\therefore \mu \frac{\partial u}{\partial z} = \frac{\partial p}{\partial r} (z + C),$$

and
$$\mu u = \frac{\partial p}{\partial r}\left(\tfrac{1}{2}z^2 + Cz + D\right).$$

Since $u = 0$, both when $z = 0$ and when $z = z_1$,
$$D = 0 \text{ and } C = -\tfrac{1}{2}z_1,$$

so that
$$u = \frac{1}{2\mu}\frac{\partial p}{\partial r}(z^2 - zz_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{A, III, 1}$$

and the total outward flow taken around the whole circumference at r, from $z = 0$ to $z = z_1$, will be
$$2\pi r\int_0^{z_1} u\,dz = \frac{\pi r}{\mu}\cdot\frac{\partial p}{\partial r}\int_0^{z_1}(z^2 - zz_1)\,dz$$
$$= -\frac{\pi r}{\mu}\cdot\frac{\partial p}{\partial r}\cdot\frac{z_1^3}{6}.$$

If for z_1 be written $h + \zeta$, where $\zeta = r^2/2R$, the total flow is
$$-\frac{\pi r}{6\mu}\cdot\frac{\partial p}{\partial r}(h + \zeta)^3 = -\frac{\pi}{6\mu}\cdot\frac{\partial p}{\partial r}\,r\left(h + \frac{r^2}{2R}\right)^3$$
$$= -\frac{\pi r}{48\mu R^3}\cdot\frac{\partial p}{\partial r}(2Rh + r^2)^3 = -\pi r^2\frac{\partial h}{\partial t},$$

the right-hand member of the equation being the volume of fluid displaced from beneath the ball, from the normal line through its centre to the radius r.

Hence
$$\frac{\partial p}{\partial r} = 48\mu R^3\frac{\partial h}{\partial t}\frac{r}{(2Rh + r^2)^3};$$

or, putting K for $48\mu R^3$,
$$\frac{\partial p}{\partial r} = K\frac{\partial h}{\partial t}\frac{r}{(2Rh + r^2)^3},$$

and, integrating,
$$p = -K\frac{\partial h}{\partial t}\frac{1}{4(2Rh + r^2)^2} + L.$$

But
$$p = 0, \text{ when } r = a;$$
$$\therefore L = K\frac{\partial h}{\partial t}\frac{1}{4(2Rh + a^2)^2},$$

and
$$p = -\frac{K}{4}\frac{\partial h}{\partial t}\left\{\frac{1}{(2Rh + r^2)^2} - \frac{1}{(2Rh + a^2)^2}\right\}. \quad . \quad . \quad . \quad \text{A, III, 2}$$

By a further integration is found the total fluid pressure to the radius $r = a$, by which the charge in the ball is supported, viz.
$$P = \int_0^a 2\pi r p\,dr.$$

Beyond $r = a$, since the velocity is negligibly small, the pressure, being zero at $z = 0$, is negligibly small throughout the film;

thus $$P = -\frac{\pi K}{2}\frac{\partial h}{\partial t}\int_0^a \left\{\frac{r}{(2Rh + r^2)^2} - \frac{r}{(2Rh + a^2)^2}\right\} dr,$$

from which is found by usual processes, restoring the value of K,

$$P = -6\pi\mu R^2 a^4 \frac{1}{h(2Rh + a^2)^2}\cdot\frac{dh}{dt}. \quad . \quad . \quad . \quad \text{A, III, 3}$$

Thus $$dt = -\frac{6\pi\mu R^2 a^4}{P}\cdot\frac{dh}{h(a^2 + 2Rh)^2},$$

and $$t = -M\int_{h_0}^{h_1}\frac{dh}{h(a^2 + 2Rh)^2},$$

where $$M = \frac{6\pi\mu R^2 a^4}{P}.$$

The integral is

$$\frac{1}{a^2}\frac{2R(h_0 - h_1)}{(a^2 + 2Rh_1)(a^2 + 2Rh_0)} - \frac{1}{a^4}\log\frac{(a^2 + 2Rh_1)h_0}{(a^2 + 2Rh_0)h_1},$$

so that

$$t = -\frac{6\pi\mu R^2 a^2}{P}\left\{\frac{2R(h_0 - h_1)}{(a^2 + 2Rh_1)(a^2 + 2Rh_0)} - \frac{1}{a^2}\log\frac{(a^2 + 2Rh_1)h_0}{(a^2 + 2Rh_0)h_1}\right\}, \quad \text{A, III, 4}$$

or, if a^2 is large in comparison with Rh_0 and Rh_1,

$$t = -\frac{6\pi\mu R^2}{P}\left\{\frac{2R(h_0 - h_1)}{a^2} - \log\frac{h_0}{h_1}\right\}. \quad . \quad . \quad . \quad \text{A, III, 5}$$

Thus, if the radius of the ball R is 1·0 cm.; the charge P, 1000 kg. weight or approximately $1{\cdot}0 \times 10^9$ dynes; the viscosity μ, 1·0 poise; and if the initial thickness of the lubricating film is of the order of 10^{-3} cm.; and the initial distance between ball and race 10^{-4} cm.; this distance will diminish to $1{\cdot}0 \times 10^{-5}$ cm. in about 5×10^{-8} sec., and to $1{\cdot}0 \times 10^{-6}$ cm. in about 1×10^{-7} sec. Consequently, within a period of time extremely short compared with the usual period during which a bearing ball is in loaded contact with its race, the thickness of the lubricating film will have been so much reduced that solid contact of the rugosities of the surfaces must take place.

Similar conclusions must be drawn with respect to the brief duration, under load, of lubricating films between rollers and races; the formula for t, corresponding to equation A, III, 5 for a cylindrical roller of radius R and length l, running on a plane, being found by similar methods to be

$$t = \frac{6\sqrt{2}\cdot\pi\mu R^{\frac{3}{2}} l}{P}\cdot\frac{h_1^{\frac{1}{2}} - h_0^{\frac{1}{2}}}{(h_1 h_0)^{\frac{1}{2}}}. \quad . \quad . \quad . \quad . \quad \text{A, III, 6}$$

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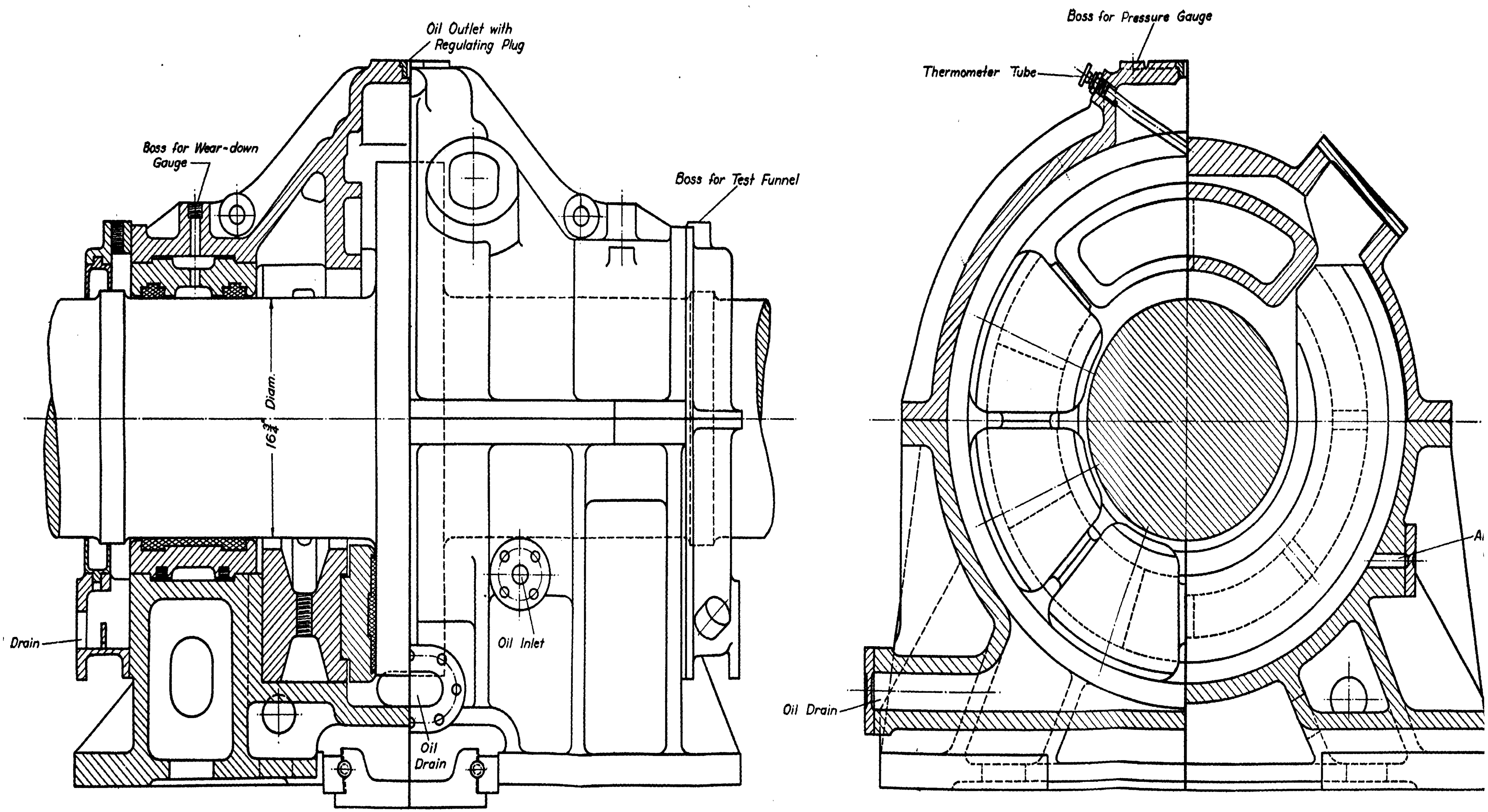

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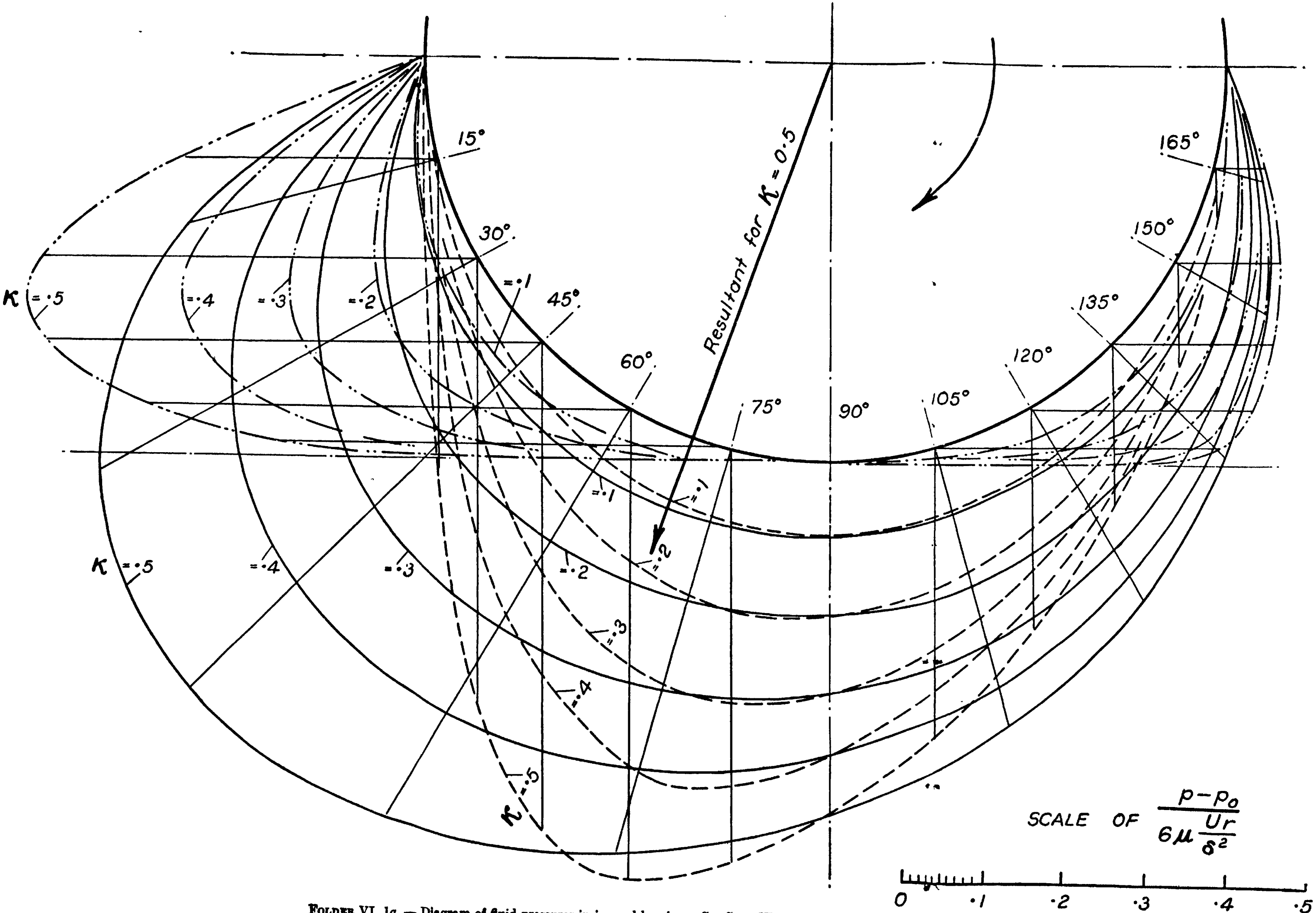

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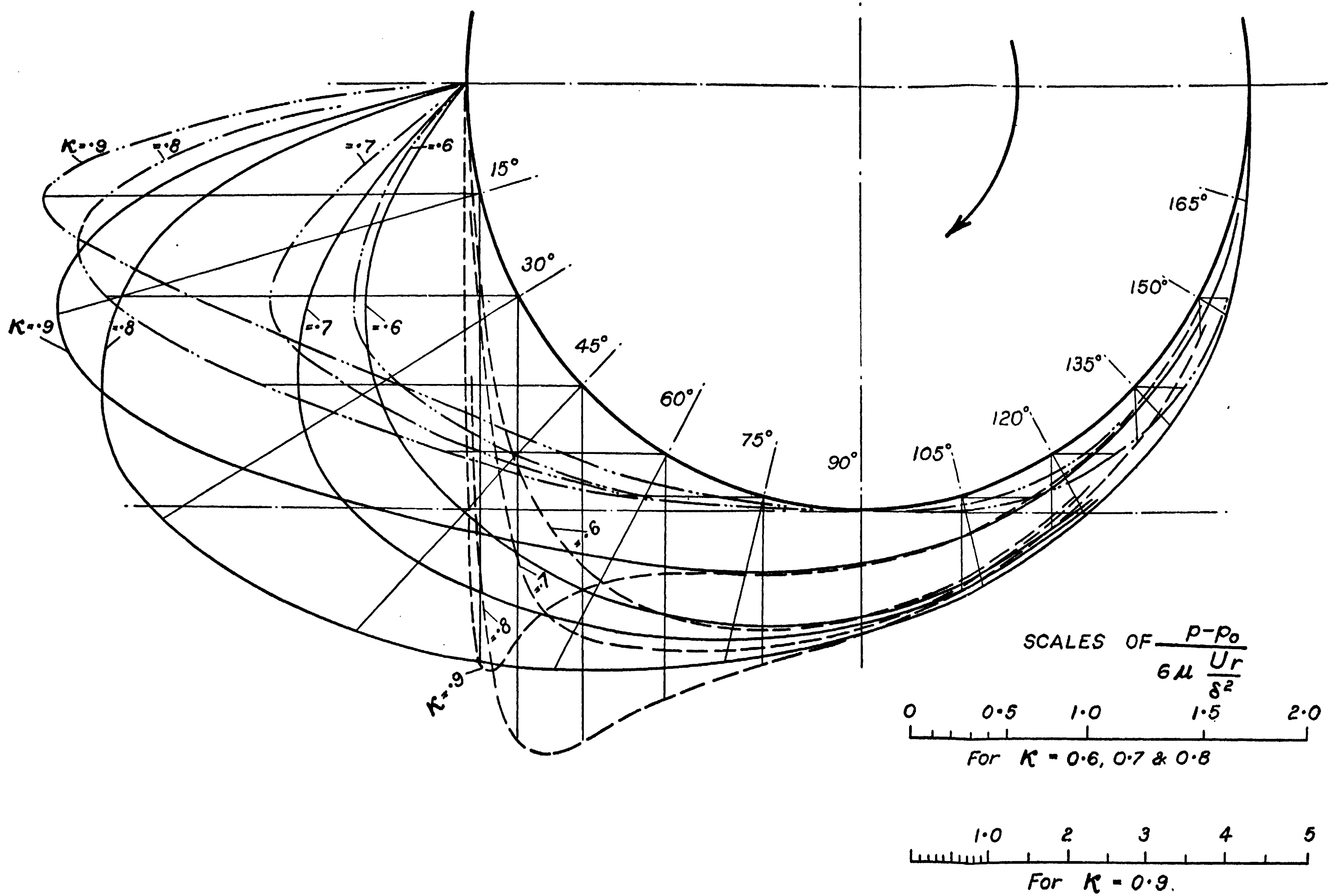

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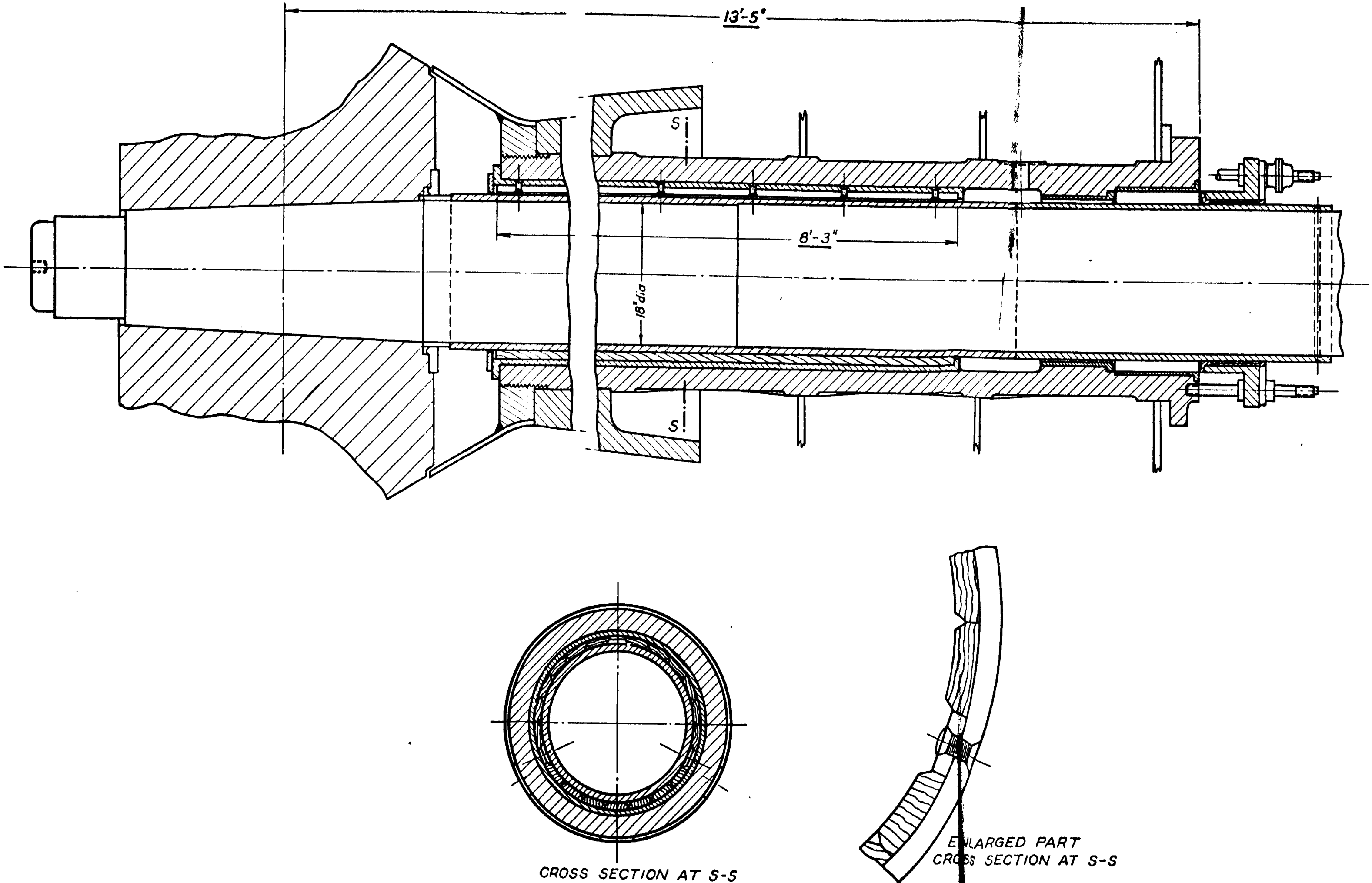

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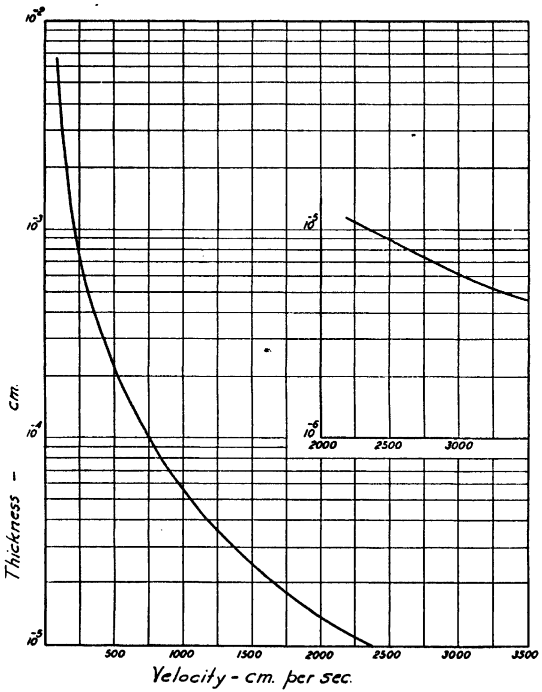

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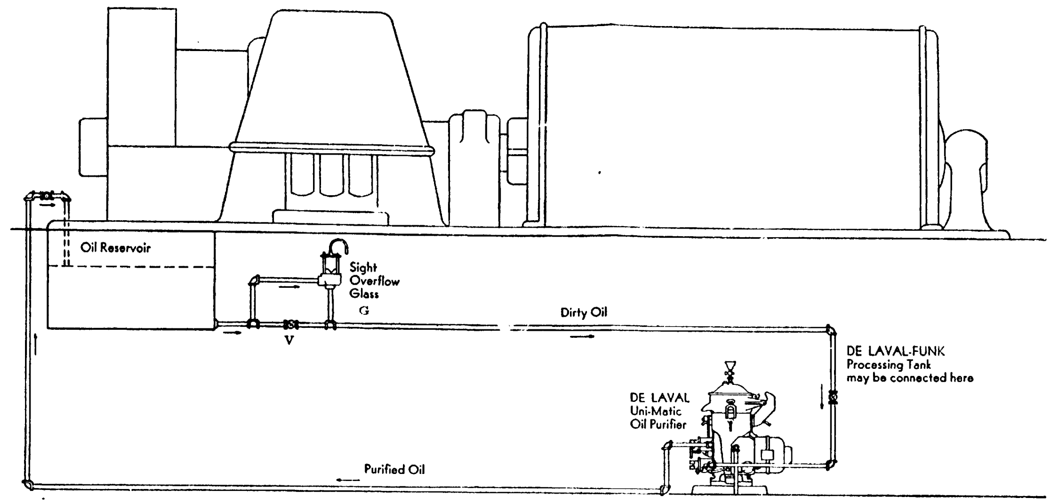
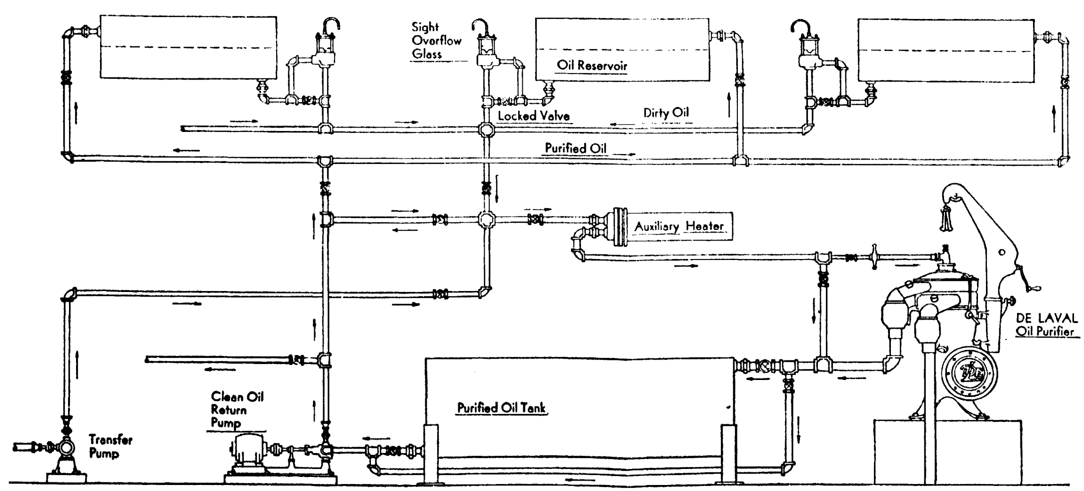


Fig. 1. — For a single turbine



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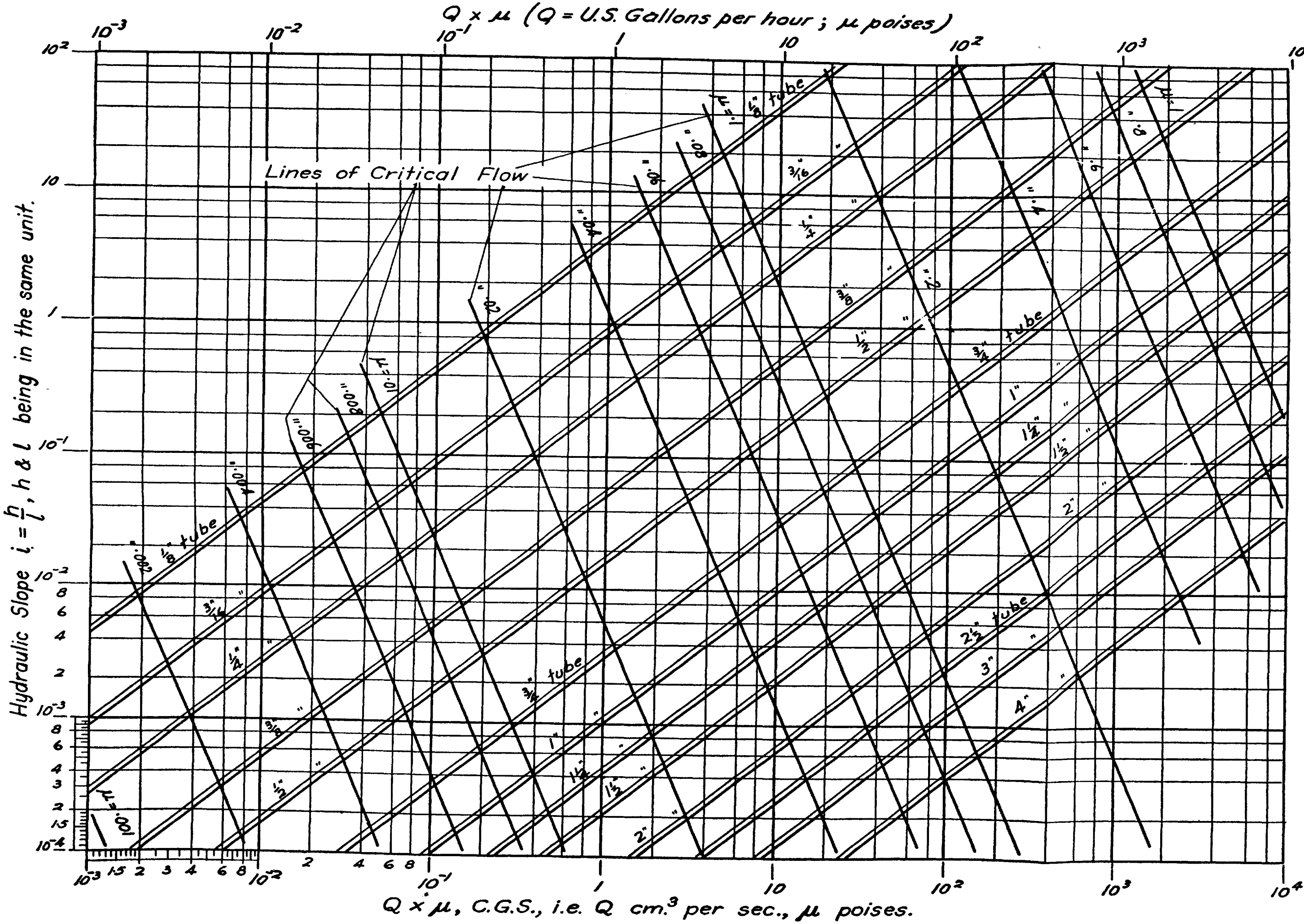
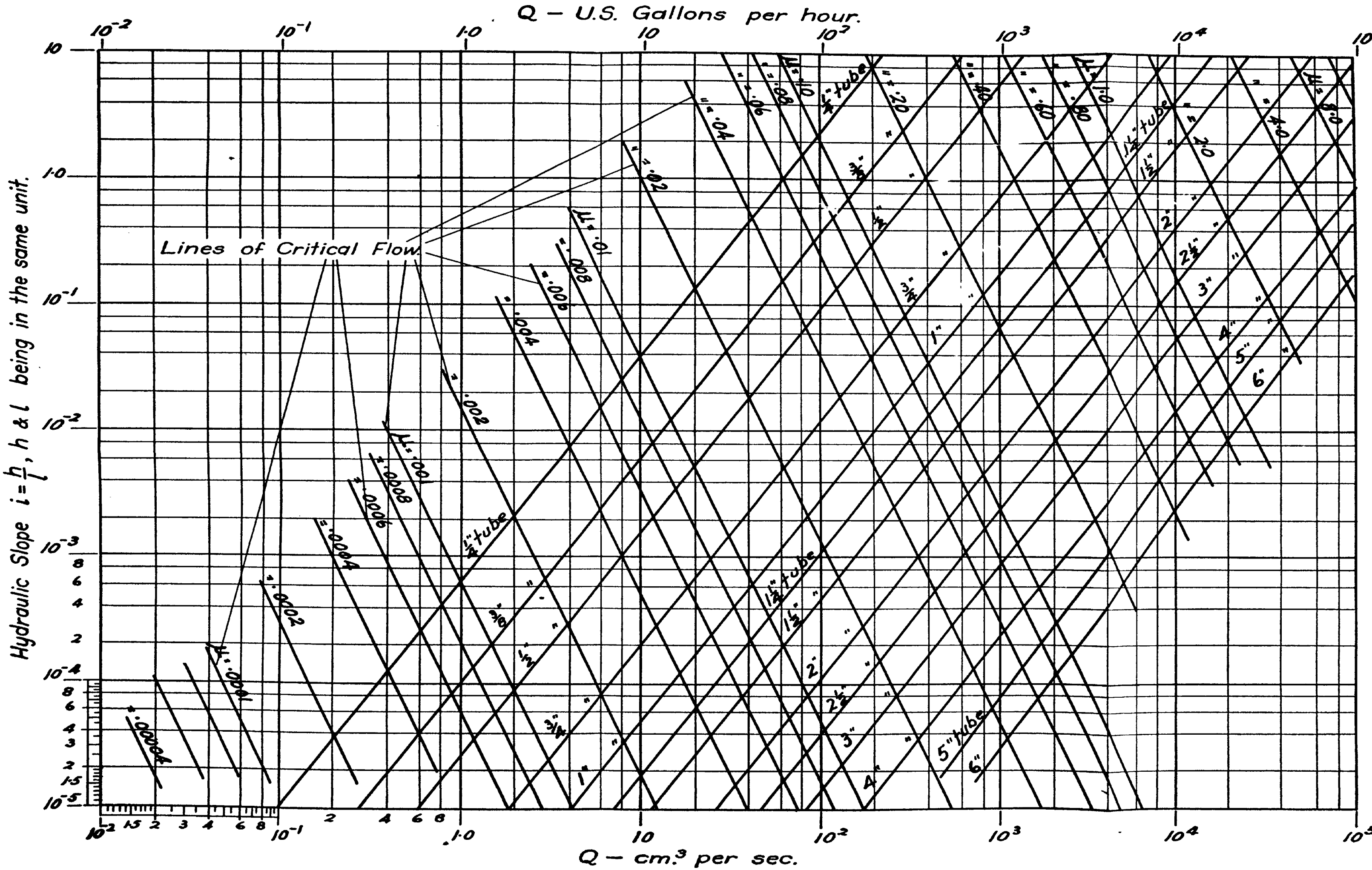


Folder X, A. — Laminar flow in tubes of circular sections. See Sect. X, 2, p. 276.

For the use of this chart the volume of flow, Q, is preferably expressed in cubic centimetres per second, as shown below the bottom line. Alternatively, however, it may be expressed in U.S. Fluid Gallons per Hour, the difference between these two units of flow (approximately 5 per cent) being allowed for by regarding all the vertical grid-lines of the chart as being shifted slightly to the right as indicated on the top line.

For rough calculations the Imperial Gallon per Hour may be used as the unit of measurement of Q in the same way, the required shift towards the right in this case being approximately 1 centimetre.



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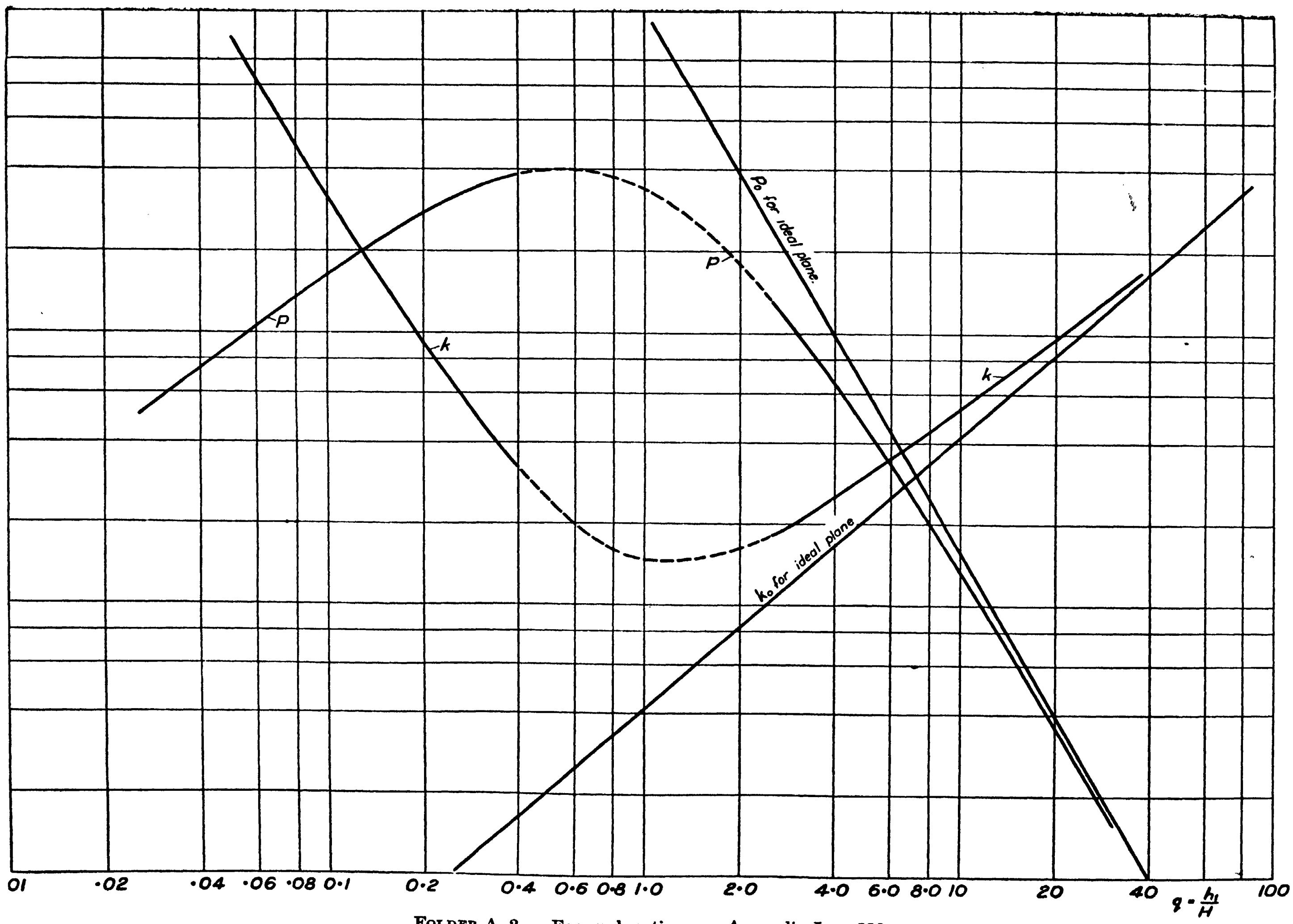

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