# Integrated MCDM Models for Performance Assessment and Ranking in Public Transport Sector

**THESIS**

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"*Believe you can and you're halfway there.*"

***-Theodore Roosevelt***

## BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

### CERTIFICATE

This is to certify that the thesis titled **"Integrated MCDM Models for Performance Assessment and Ranking in Public Transport Sector "** submitted by **Ms. Swati Goyal**, ID No. **2018PHXF0409P** for the award of Ph.D. of the institute embodies original work done by her under our supervision.

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***Dedicated to***

***My Beloved Family***

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# Abstract

The transport sector is a critical driver of economic growth, employment, and access to essential services like healthcare and education, serving millions daily. Government investments, including initiatives like Bharatmala Pariyojana, aim to enhance road infrastructure and connectivity. Sustainable transportation, encompassing user-friendliness, reliability, fuel efficiency, reduction road congestion, and less environmental impact, is now a top priority. Given the sector's pivotal role, a thorough understanding of its operations is essential. The primary objective is to maximize output while minimizing inputs, achieved through efficient resource utilization. This involves assessing productivity (output-input ratio) and efficiency (observed vs. optimal output or input). Continuous performance evaluation is crucial to gauge the effectiveness of resource allocation.

Passenger transportation is inherently a "service business" and poses unique challenges in assessing productivity and efficiency. In particular, public transport requires advanced analytical tools for evaluation. Multi-criteria decision-making (MCDM) techniques have proven invaluable in optimizing the sector's performance. The thesis explores various MCDM models tailored to investigate the productivity and efficiency of India's public transport system.

The thesis commences with a foundational introduction in the first chapter, providing a conceptual framework for the proposed work. This initial chapter provides a concise overview of the methodologies employed, including MCDM, fuzzy MCDM, data envelopment analysis (DEA), and fuzzy DEA techniques. These sophisticated methods analyze and assess various performance, efficiency, and productivity aspects across diverse contexts. MCDM enables decision-makers to evaluate multiple conflicting criteria, while fuzzy MCDM deals with uncertainties and imprecise information in decision-making. On the other hand, DEA is utilized for assessing the relative efficiency of entities by comparing their inputs and outputs, while fuzzy DEA extends this approach to handle vague or uncertain data, providing a more comprehensive evaluation of productivity and efficiency. These techniques collectively offer a robust framework for comprehensively evaluating and enhancing performance within complex systems or scenarios. This chapter expands on the study's focus on the transportation sector, detailing the specific geographical areas under scrutiny. It explores challenges impeding efficiency and development, defines research objectives, and pinpoints gaps in existing knowledge, setting the stage for this study's contribution and advancement in the field.

Moving to the subsequent chapter, a unique blend of MCDM techniques is discussed. This chapter involves a meticulous process of screening vital criteria, assigning appropriate weight to screened criteria, and subsequently ranking the RSRTC depots based on their performance

scores. This research encompasses a critical step, namely sensitivity analysis, conducted to ascertain the impact of varying criteria weights on depot rankings. This analysis is pivotal in identifying potential outliers and affirming the robustness of the efficiency scores acquired through the hybrid MCDM methods.

In the third chapter, a thorough examination of prior research utilizing DEA models is conducted, which serves as a crucial foundation for developing and applying an approach known as the new slack model (NSM) under the variable returns to scale (VRS) assumption. This innovative model is employed to assess the overall technical, pure technical, and scale efficiencies of RSRTC depots over the years 2005-2022, taking into account the categorization of depots based on topographical considerations. The chapter offers an insightful analysis of the observed growth trends and efficiency patterns and provides valuable recommendations for potential reductions in input quantities. This comprehensive evaluation significantly contributes to a deeper understanding of the performance dynamics of RSRTC depots, facilitating informed decision-making and resource optimization strategies.

The subsequent chapter 4 measures the total factor productivity (TFP) and applies the Malmquist productivity index (MPI) and Luenberger productivity index (LPI) using NSM model over a specified time frame of 11 years (2008–2019). Further, it is evaluated total productivity change in terms of technological change (Frontier shift) and technical efficiency change (Catch-up Effect). The outcomes derived from these models offer a more realistic reflection of real-world scenarios compared to efficiency evaluation. A detailed examination of the progress and shifts in performance is exhibited in RSRTC depots for the consecutive 11 years 2008-2019. Thus, the study demonstrates a significant trend wherein the decline in productivity across several depots predominantly stems from technological changes, emphasizing the pivotal role of technological advancements in shaping and influencing overall productivity within the transportation system. This in-depth analysis provides critical insights into the evolving dynamics of depot performance over the specified period. It is worth noting that the utilization of the study contributes to a refined and accurate assessment of productivity changes, enhancing the practical applicability of the results in real-world scenarios.

Chapter 5 introduces an application of the inverse super-efficiency DEA model. This approach precisely determines the necessary amount of inputs and outputs to achieve a specific efficiency target. This chapter goes on to validate efficient DMU ranking by employing the super-efficiency DEA model and its inverse counterpart. This reverse approach establishes a robust and reliable framework for assessment, ensuring the accuracy of efficiency evaluations. Furthermore, the chapter serves as a valuable guide for decision-makers in optimizing resource allocation and operational strategies. By leveraging the insights gained from this model, stakeholders can make choices to enhance overall efficiency within the system. This study identifies the areas for improvement and implements targeted interventions to streamline operations and resource utilization. The IDEA super-efficiency model thus represents a significant step forward in the quest for operational excellence and efficiency in the transportation sector.

Chapter 6 proposes a novel approach fuzzy cross-efficiency DEA model. This proposed model employs a “credibility approach” to ensure accurate assessments in a fuzzy environment. This study addresses scenarios where data may be incomplete or missing and resolves using kNN approach. This chapter presents a detailed evaluation of STUs performance in India for the financial year 2017-18. Efficiency scores are obtained using a fuzzy cross-efficiency model and compared with those derived from the fuzzy DEA model, offering valuable insights into its effectiveness. This study significantly contributes to the advancement of robust methodologies for evaluating and enhancing the efficiency of STUs and using ensemble ranking to rank the units uniformly, making the evaluation more accurate and reliable.

A summary and conclusions are presented in the final chapter of the thesis which also outlines potential future directions and areas for further research. The thesis investigates and deliberates on the diverse potential applications of further integrated DEA model extensions in the public transport sector.

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# List of Abbreviations

| | |
|---|---|
| DEA: | Data Envelopment Analysis |
| MCDM: | Multi-Criteria Decision Making |
| SFA: | Stochastic Frontier Analysis |
| SD: | Standard Deviation |
| DMU : | Decision-Making Unit |
| DM: | Decision-Maker |
| LPP : | Linear Programming Problem |
| CCR: | Charnes, Cooper & Rhodes |
| BCC: | Banker, Charnes & Cooper |
| NSM: | New Slack Model |
| CRS: | Constant Returns to Scale |
| VRS: | Variable Returns to Scale |
| PPS: | Production Possibility Set |
| PTE: | Pure Technical Efficiency |
| SE: | Scale Efficiency |
| TFP: | Total Factor Productivity |
| MPI: | Malmquist Productivity Index |
| LPI: | Luenberger Productivity Index |
| DDF: | Directional Distance Function |
| kNN: | k-Nearest Neighbors |
| $CO_2$ : | Carbon Dioxide |
| GDP : | Gross Domestic Product |
| RSRTC: | Rajasthan State Road Transport Corporation |
| STUs: | State Transport Undertakings |
| TOPSIS: | Technique for Order of Preference by Similarity to Ideal Solution |
| VIKOR: | VIekriterijumsko KOmpromisno Rangiranje |
| ELECTRE: | ELimination and ChoiceExpressingREality |
| PROMETHEE: | Preference Ranking Organization Method for Enrichment Evaluations |

| | |
|---|---|
| IDEA: | Inverse Data Envelopment Analysis |
| AHP: | Analytic Hierarchy Process |
| TFN : | Triangular Fuzzy Numbers |
| FAHP: | Fuzzy Analytic Hierarchy Process |
| FDM : | Fuzzy Delphi Method |

# Chapter 1

# Introduction and Preliminaries

## 1.1 Overview of Transportation

Throughout history, the innate human desire for mobility has served as a gauge of societal advancement. The evolution of transportation mirrors the progression of civilization itself. Facilitating the movement of people and goods from point of origin to destination, ‘transporters’ perform one of the most important activities in every phase of advanced society. Where roads are viewed as the vital pathways of a nation, the transportation of passengers and goods is likened to the circulation of blood, energizing the economic activity of a country. Consequently, passenger road transport services stand as an indispensable companion to a nation’s economic development.

Transportation is a crucial component of both the industrial and social infrastructure. It plays a significant role in fostering economic development, ensuring national security, and enhancing the population’s overall quality of life. This transportation network is a collaborative endeavor involving private and public vehicle operators, facilitating the movement of millions of people daily. Transportation sustainability has emerged as a paramount concern in the contemporary world, driven by a growing awareness of the interplay between transportation and the environment. This concern is accompanied by various associated issues, including traffic accidents, congestion, noise and air pollution, and global warming. These factors significantly affects the overall performance of transportation systems and impact passengers’ travel experiences, influence mode choices.

In India, the situation is further complicated by inherent problems in the public bus transport sector. It faces severe financial constraints, marked by excessive operating costs, overstaffing, low productivity, and imprudent use of resources. The combination of these financial and operational challenges presents significant institutional barriers. To address these challenges, the public transport goal needs to be improved to 60% of all motorized trips by 2030, and fatalities must be reduced by 50% [1]. The declining passenger satisfaction and adverse effects on daily commuters underscore the pressing need for an improved and reliable public transit system, as many commuters rely heavily on it for their daily travels. These pressing concerns for sustainable transportation encompass the crucial dimensions of social, environmental, and economic progress. The urgency to revamp existing public transit systems has thereby gained renewed attention in recent years [2, 3]. Achieving sustainability and efficiency in transportation systems, whether on a global, local, or urban scale, is essential to realizing these goals.

The significance of an efficient and extensive public transport system cannot be overstated in this context. It serves as a key element of any strategy that seeks to optimally meet the mobility demand that arises from rapid economic growth [4, 5]. Additionally, increasing the share of public transport is essential for reducing sector emissions and improving social cohesion. It is hard-pressed to meet its fleet replenishment needs, augment fleets to cater to growing demand, or introduce efficiency-improving technologies [6]. In the post-liberalization era, it would be difficult for governments to continue to provide financial support to cover the deficits in bus transportation, especially with the growing emphasis on fiscal discipline. Existing financial constraints and the inability to raise resources for investments are also caused in part by uneconomical operations to meet the universal service obligation with tariffs that do not reflect the cost of service delivery [7]. Thus, optimal pricing of public transport services to ensure economic efficiency and cost recovery is critical for ensuring a sustainable public transportation system.

## 1.2 Operations Research (OR)

"Operations research is neither a method nor a technique; it is or is becoming a science and, as such, is defined by a combination of the phenomena it studies." ***Russell L. Ackoff***

India is one of the first nations to use operations research (OR) in practical applications. In Hyderabad, the Regional Research Laboratory, the first OR unit, was established in 1949. According to the Merriam-Webster dictionary, OR is the application of scientific and especially mathematical methods to the study and analysis of problems involving complex systems. It is a discipline that develops and applies advanced analytical methods to improve decision-making[1]. It continues to be a relatively young scientific field despite its rapid advancement. Its methodologies and applications are poised for further expansion, building on the influence of its past achievements. The primary goal of OR is to achieve optimal outcomes within specified constraints. This optimization process demands a thorough analysis and refinement of potential options. When working towards optimization, one invariably encounters real-world limitations. For instance, adhering to labor regulations on maximum work hours per individual becomes

[1] https://www.informs.org/Explore/What-is-O.R.-Analytics/What-is-O.R.

a critical constraint for companies grappling with hiring challenges. In the realm of business problem-solving using OR, a comprehensive evaluation of various options is essential, enabling a balanced consideration of their respective merits and demerits while keeping these constraints firmly in mind.

### 1.2.1 Multi-Criteria Decision Making

Multi-criteria decision-making (MCDM) is a subset of a broader class of OR models that provide solutions for decision assistance and evaluation of complicated issues with competing criteria and significant uncertainty. In other words, MCDM is a technique for determining the best option, ranking, and sorting the alternatives. Since the 1970s, it has been a powerful tool in the fields of decision-making, value judgment, and evaluation. MCDM is a well-known acronym, and Stanley Zionts helped popularize the acronym with his 1979 article "MCDM — If Not a Roman Numeral, Then What?" MCDM is involved with creating and solving multi-criteria decisions and planning problems. The decision-making process in the MCDM approach has five sequential steps: (i) to define the goal or problem, (ii) to generate the alternatives, (iii) to select the criteria and sub-criteria to evaluate the alternatives, (iv) to collect the judgment regarding the importance, the relative importance of criteria, and (v) finally, the ranking of alternatives. Typically, there are multiple optimal solutions for the same situations; hence, the preferences of the decision-maker must distinguish between alternatives. Several methods are available in the literature for MCDM. Some important names are analytic hierarchy process (AHP), analytic network process (ANP), best-worst method (BWM), Brown–Gibson model, data envelopment analysis (DEA), Decision EXpert (DEX), ELimination and ChoiceExpressingREality (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE), VIekriterijumsko KOmpromisno Rangiranje (VIKOR), and Technique for the Order of Prioritisation by Similarity to Ideal Solution (TOPSIS).

Utilizing multi-criteria benchmarking tools, rather than relying solely on standard accounting procedures, can significantly enhance the overall performance of firms. This approach proves especially beneficial for managing and elevating service performance. For instance, how can the number of buses, employees, and fuel consumption be minimized in the transport sector while maintaining high-quality services? How can organizations effectively prioritize depots for sustainable development? While DEA models offer solutions to these diverse challenges, it's worth noting that a single DEA model may not suffice. This thesis explores the application of different types of DEA models and MCDM techniques. Despontin et al. (1983) [8] discussed more than a hundred different MCDM techniques. Moreover, Mardani et al. (2016) [9] systematically reviewed 89 papers regarding transportation system problems from 1993 to 2015 using MCDM techniques. Consequently, it does not matter which MCDM technique is better or worse, as the appropriateness of the methodology depends on the specific decision circumstances [10].

### 1.2.2 Fuzzy MCDM

The day-to-day problems encountered often exhibit a level of uncertainty and vagueness. Traditional MCDM techniques face challenges when dealing with such problems [11, 12]. As a result, fuzzy MCDM techniques prove invaluable in estimating the subjective evaluations made by individuals.

As a result, fuzzy MCDM techniques prove invaluable in estimating the subjective evaluations made by individuals. Over the past few years, significant efforts have been made to address uncertainty, imprecision, and subjectivity, primarily through the application of fuzzy set theory. The integration of fuzzy set theory into multi-criteria evaluation methods within the framework of utility theory has demonstrated practicality.

Unlike conventional MCDM techniques, fuzzy MCDM techniques are adept at evaluating the best alternatives based on predetermined criteria. Moreover, these techniques can be extended beyond individual decision-makers to accommodate a group of decision-makers. The process involves various intermediate steps, including the evaluation of alternatives versus criteria, criteria versus criteria, and so forth. Criteria weights are assessed using linguistic values, represented as fuzzy numbers. These linguistic variables can be articulated through sentences or words in artificial or natural languages. In most fuzzy MCDM techniques, the values obtained in the final steps are in fuzzy form, necessitating the application of proper defuzzification methods to convert them into crisp sets, as discussed in subsequent sections.

Furthermore, in fuzzy MCDM problems, the final evaluation values of alternatives remain fuzzy numbers, requiring an appropriate ranking approach for defuzzification into crisp values for decision-making. Among the array of fuzzy MCDM techniques, fuzzy AHP (FAHP) is widely favored in several domains owing to its effectiveness in addressing decision-making problems involving numerous criteria and alternatives. Other noteworthy fuzzy techniques, such as fuzzy TOPSIS and fuzzy PROMETHEE, have also gained prominence for their ability to handle complex decision-making scenarios. More recently, the integration of fuzzy set theory with type-2 and type-3 fuzzy sets has introduced additional advantages inherited from these advanced fuzzy set concepts.

## 1.3 Efficiency & Productivity

Efficiency analysis seeks to examine the relationship between inputs (resources) and outputs (production) in a system, aiming to assess the ability of decision-making units (DMUs) to maximize output while utilizing a specific quantity of inputs and current technology. Alternatively, it also seeks to minimize inputs while achieving a specific level of output. These functions are especially useful when dealing with multiple inputs and outputs, often represented by a production possibility frontier (PPF). Efficiency encompasses various dimensions, including technical, allocative, economic, operational, environmental, and dynamic aspects [13].

Productivity and efficiency are often used interchangeably but refer to slightly different concepts. Productivity measures the ratio of specific outputs to inputs. However, efficiency evaluates how closely the output levels align with the highest potential yield, considering the available resources and technology. Efficiency serves as a more comprehensive performance evaluation tool compared to productivity since it considers limiting constraints such as scale and aims to evaluate the value of the products in broader terms (see figure 1.1).

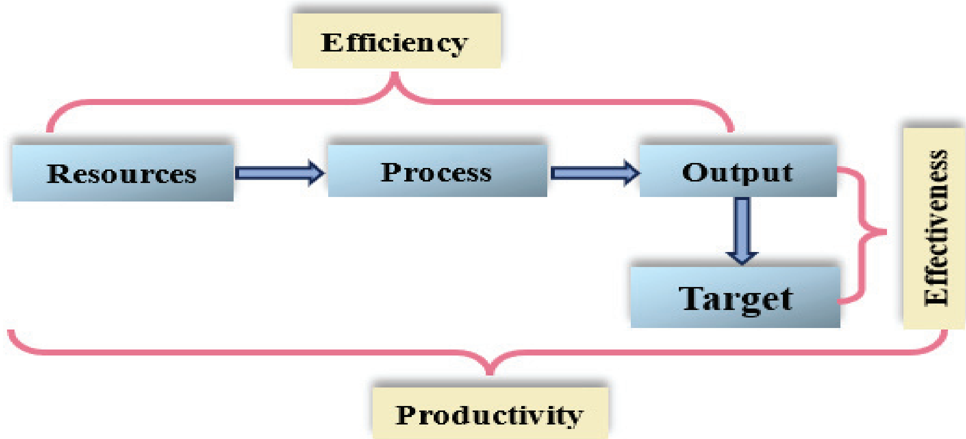


**Fig. 1.1:** Visualization of efficiency and productivity

## 1.4 Data Envelopment Analysis: An Overview

The concept of efficiency measurement through frontier analysis is attributed to the pioneering work of [14], which debuted in 1957. Farrell's seminal contribution offered a "data-oriented" approach to assess the efficiency of homogeneous DMUs in scenarios with single input and output. This early study laid the groundwork for the more comprehensive framework of DEA, which is formally introduced by Charnes et al. [15] in 1978. Contrary to Farrell's method, which is restricted to a single output and input, DEA generalized this concept by allowing for multiple inputs and outputs, even handling different dimensional units, thus enhancing its versatility and applicability across various contexts. At its core, DEA provides a way to compare the performance of similar units—such as organizations, departments, or companies—by assessing how efficiently they convert inputs like labor, capital, and energy into outputs [16]. The mathematical framework for DEA is rooted in linear programming, providing a rigorous yet flexible methodology for efficiency assessment. DEA is originally developed to evaluate non-profitable and government organizations, but it has subsequently applied to the service operations of a variety of private companies [17].

DEA calculates these efficiency scores by comparing the performance of each DMU to a frontier that represents the best possible performance achievable within the dataset. This frontier is constructed based on the most efficient units in the dataset, forming an ideal benchmark. A unique feature of DEA is its scoring system: it assigns efficiency scores ranging from zero to one to each DMU. Units with a score of one are considered efficient. A score of one indicates that the DMU is considered fully efficient according to the evaluated inputs and outputs. It suggests that this particular unit is utilizing its resources optimally and achieving the maximum possible output given the inputs and technology at hand. An efficiency score below one indicates that the unit could potentially improve its performance by optimizing resource usage or output generation relative to its peers. Units achieving a score of one align closely with this benchmark, while scores below one indicate deviation from this optimal performance level. Consider, for example, a simple case where two depots, A and B, evaluated on number of buses and fuel use, with passenger count as the output. Using DEA, Depot A scores 1, indicating full efficiency, while Depot B has an efficiency score of 0.8, which means that Depot B could achieve the same passenger service while using 20% decrease resources like buses and fuel. Therefore, Depot B has an opportunity to become 20% more efficient by making adjustments in resource allocation or management.

Another strength of DEA is the ability to evaluate the comparative efficiency of DMUs without requiring predetermined weights for inputs and outputs. This autonomy allows each DMU to select the most favorable weight for inputs and outputs to calculate efficiency. One of the basic assumptions in DEA is that all DMUs under study should be homogeneous, meaning they should share the same production process to ensure a fair comparison. Apart from efficiency scores, DEA also provides information on slack variables and shadow prices, offering deeper insights into where improvements can be made. Notably, DEA is often employed as one of the tools in MCDM, adding to its multifaceted utility in performance evaluation. Two different methodologies, parametric and non-parametric, are accessible for the production frontier [18]. DEA over parametric methods like stochastic frontier analysis (SFA) lies in its non-parametric nature, thereby not requiring any assumption about the functional form of the production frontier. In contrast, SFA is constrained by its need for a pre-defined functional form and is sensitive to model specification errors. DEA is more flexible in constructing an efficiency frontier based on actual data without assuming a specific mathematical relationship between inputs and outputs. This makes DEA model more robust to specification and allows it to be applied even in complex or poorly understood production settings. Furthermore, unlike parametric methods that often demand large sample sizes for statistical reliability, DEA can produce reliable efficiency estimates with smaller datasets.

Its objective approach, which minimizes the impact of human bias by avoiding subjective weight assignment, further sets it apart from other MCDM techniques. While the basic DEA model assumes constant or variable returns to scale and input-oriented or output-oriented efficiency, various extensions like network DEA and fuzzy DEA models are developed to address their limitations. These extended models accommodate more complex, real-world scenarios, such as hierarchical structures or uncertain data. Over time, DEA has witnessed a remarkable surge in new publications across various domains to evaluate efficiency [19].

### 1.4.1 Basic Concept of DEA

DEA focuses on boundary analysis rather than central tendencies, which distinguishes it from other methodologies like statistical regression [20]. Rather than fitting a regression line through the data's centroid, DEA 'elevates' a segmented linear surface to envelop the set of observed data points. This approach allows DEA to reveal intricate relationships that may remain obscured when using traditional methodologies. As illustrated in figure 1.3, DEA establishes a 'best practice' frontier.

**Theoretical Frontier:** The theoretical frontier represents an idealized or optimal level of performance that could theoretically be achieved under perfect conditions. This is essentially an abstraction, often based on theoretical or mathematical models, that serves as the ultimate standard for efficiency. A DMU that operates on this frontier is considered perfectly efficient. However, it's important to note that the theoretical frontier may not always be practically achievable due to various constraints like technology limitations, budget restrictions, or other real-world factors.

**Best Practice Frontier:** On the other hand, the best practice frontier is an empirical construction based on actual observed data. It is formed by the "best-performing" DMUs in the sample under study. In DEA, these are the units that have an efficiency score of 1. The best practice frontier serves as a more attainable benchmark for other DMUs in the sample, showing what is practically possible given current technology, practices, and resources. Inefficient DMUs are those that operate below this frontier, and they can potentially improve their efficiency by moving towards this best practice frontier.

The main difference between the two is that while the theoretical frontier is often based on a model or idealized assumptions, the best practice frontier is empirically derived from actual data. Both are useful but serve different purposes: the theoretical frontier provides a standard for what is conceptually possible, while the best practice frontier provides a standard for what is practically achievable.

By employing DEA on this data set, units S1, S2, S3, and S4 are identified as efficient, forming a 'best practice frontier' that encapsulates the entire data set. Units falling within this envelope are thus considered inefficient. Such inefficient DMUs are provided with targeted recommendations for enhancement, guiding them toward points (like S1, S2, S3, and S4) on the efficiency frontier. The proximity of a unit to this frontier serves as an empirical measure of its efficiency, or lack thereof.

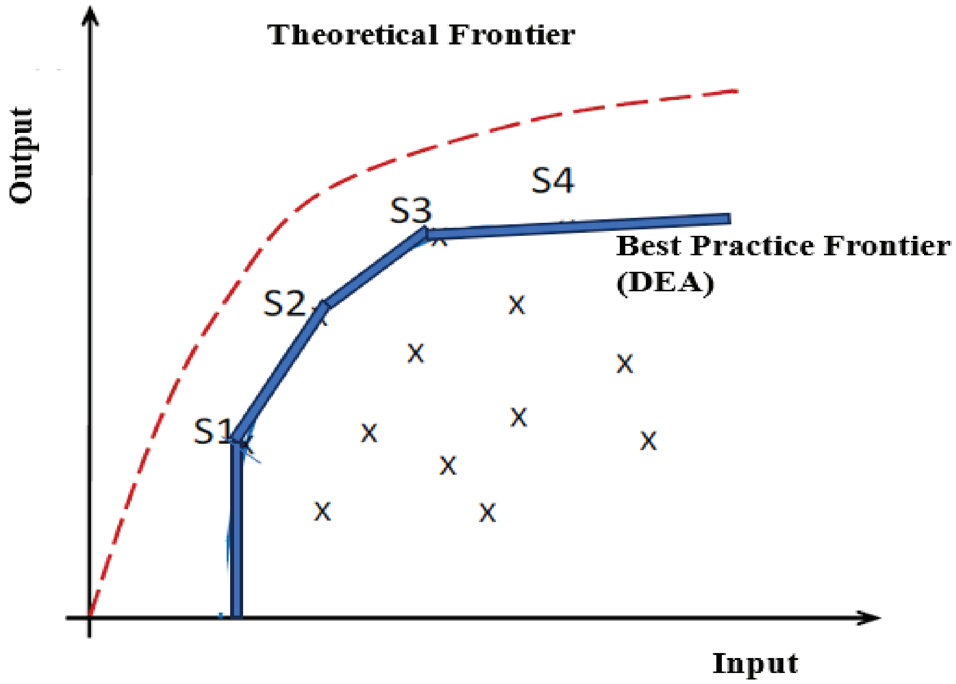


**Fig. 1.2:** Theoretical and best practice frontiers [21].

### 1.4.2 Mathematical Formulation

Charnes et al. (1978) [15] defined DEA as a mathematical programming approach that uses observed data to derive empirical insights into relationships. DEA extended the concept of measuring technical efficiency from a single-input/single-output model to a more complex multiple-input/multiple-output framework. It accomplished this by calculating a relative efficiency score, which is formulated as the ratio of a singular 'virtual' output to a singular 'virtual' input.

Suppose there is $n$ number of DMUs, each producing $s$ outputs from $m$ inputs. Thus, $x_i$ represents the $i^{th}$ input and $y_r$ represents the $r^{th}$ output. In DEA, multiple inputs and outputs are linearly aggregated using weights $u_i$ and $v_r$ $(u_i, v_r \geq 0)$. The resulting efficiency score is expressed as follows:

$$\text{Efficiency} = \frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}} = \frac{\sum_{r=1}^{s} v_r y_r}{\sum_{i=1}^{m} u_i x_i} \tag{1}$$


**Fig. 1.3:** Production process.

### 1.4.3 Multiplier vs. Envelopment in DEA: Input and Output Orientations

The dual of dual is primal. Given this duality, the distinctions between 'primal DEA program' and 'dual DEA program' are inherently relative. Broadly, DEA can be conceptualized from two primary perspectives:

1. **Multiplier DEA Programs:** These primarily involve the weights of inputs and outputs, represented by '$u$' and '$v$'. They focus on the direct measurement of efficiency '$\theta$' by assigning weights to specific inputs and outputs.

2. **Envelopment DEA Programs:** On the other hand, the weights of DMUs, are symbolized by '$\lambda$'. The essence of envelopment is about enveloping or enclosing the data points, suggesting efficiency '$f$' in a broader perspective.

Within the framework of DEA programs, there are two main orientations specific to the handling of inputs and outputs:

1. **Input-Oriented Envelopment DEA:** This approach aims to achieve observed outputs using minimal inputs. Efficiency is a function of input reduction, making it input-oriented. It aligns with the dual of the output maximizing multiplier model.

2. **Output-Oriented Envelopment DEA:** This version emphasizes achieving the maximum output given the existing amount of inputs, making it output-oriented. It corresponds with the dual of the input minimizing multiplier model.

### 1.4.4 Radial and Non-radial Models in DEA

The classification into radial and non-radial models in DEA provides distinct approaches for assessing efficiency, offering varying perspectives on the optimization of inputs and outputs within DMUs [19].

**Radial:** Radial DEA models are primarily concerned with proportional changes in inputs and outputs while maintaining a fixed level of efficiency. Efficiency scores in radial models are determined by how close a DMU is to the efficient frontier, and any radial movement toward the frontier represents an improvement in efficiency. A DMU can reduce inputs and increase outputs in radial models to enhance efficiency. It's important to note that radial DEA models exhibit unit invariance but not necessarily translation invariance.

**Non-radial:** Non-radial DEA models take a different approach by considering non-proportional changes in inputs and outputs. These models assess efficiency while allowing for variations in input and output quantities without adhering to a strict proportionality constraint. Efficiency scores in non-radial models depend on the magnitude and direction of changes in inputs and outputs relative to the efficient frontier. Non-radial models offer a more flexible view of efficiency, acknowledging that real-world organizations may not always achieve proportional adjustments in their operations.

This thesis is centered exclusively on radial models within DEA. Specifically, the models are concentrated on emphasizing the proportional adjustments in inputs and outputs while maintaining a fixed level of efficiency.

### 1.4.5 Radial DEA Models

The analysis predominantly revolved around several specific radial models within DEA.

#### 1.4.5.1 CCR Model

The basic radial DEA model developed by Charnes, Cooper and Rhodes (called CCR model) [15] to evaluate the relative efficiency of DMUs. Specifically, table 1.1 presents all four multiplier and envelopment versions of input- & output-oriented models, with assumptions of constant returns to scale (CRS). These orientations focus on output-maximizing and input-minimizing, respectively. This model is proven to be an effective tool for identifying empirical frontiers and evaluating relative efficiency. The model framework is given below:

**Table 1.1:** LPPs of CCR Model.

| **Multiplier version** | **Envelopment version** |
|---|---|
| Output-maximizing model | Input-oriented CCR model |
| $\max \quad \theta_d = \sum_{r=1}^{s} u_{rd} y_{rd}$<br>subject to $\sum_{i=1}^{m} v_{id} x_{id} = 1$<br>$\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad \forall j$<br>$u_{rd} \geq 0 \quad \forall r = 1, \cdots, s$<br>$v_{id} \geq 0 \quad \forall i = 1, \cdots, m.$ | $\min \quad f_d$<br>subject to $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{id} f_d \quad \forall i$<br>$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rd} \quad \forall r$<br>$\lambda_j \geq 0 \quad \forall j.$ |
| Input-minimizing model | Output-oriented CCR model |
| $\min \quad \theta_d = \sum_{i=1}^{m} v_{id} x_{id}$<br>subject to $\sum_{r=1}^{s} u_{rd} y_{rd} = 1$<br>$\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad \forall j$<br>$u_{rd} \geq 0 \quad \forall r = 1, \cdots, s$<br>$v_{id} \geq 0 \quad \forall i = 1, \cdots, m.$ | $\max \quad f_d$<br>subject to $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{id} \quad \forall i$<br>$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rd} f_d \quad \forall r$<br>$\lambda_j \geq 0 \quad \forall j.$ |

**Definition 1.4.1** *A $DMU_d$ is called CCR efficient in the multiplier version if the optimal value $\theta_d^*$ =1, and there exists at least one optimal pair of weights (v,u) with v > 0, u> 0, otherwise $DMU_d$ is CCR-inefficient. In other words, a $DMU_d$ is inefficient if it is possible to reduce any of its inputs without reducing its output and without expanding some other inputs.*

OR

**Definition 1.4.2** *A $DMU_d$ is said to be CCR efficient in the envelopment version if $f_d^*$ =1 and all input-output slacks are zero, i.e., $(S_i^{+*}, S_r^{-*})$= 0.*

#### 1.4.5.2 BCC Model

Banker, Charnes and Cooper introduced an innovative advancement to the DEA model, known as the BCC model [22]. This model incorporated the variable returns to scale (VRS) assumption through the inclusion of a convexity condition in the existing CCR DEA. The returns to scale assumption, especially VRS, ensures the estimation reflects economic reality. Thereafter, the BCC model emerged as more prevalent in the current practical scenarios and suitable for inferences by the researchers. The BCC model yields a measure of technical efficiency that overlooks the effect of scale by only comparing a DMU to a unit of a similar scale. The BCC model takes into account the variation of efficiency with respect to the scale of operations and hence measures pure technical efficiency (PTE). Thus, CCR model is a specific case of BCC model, and hence, the efficiency score obtained by the BCC model is greater than or equal to the efficiency score obtained by the CCR model.

**Table 1.2:** LPPs of BCC Model.

| **Multiplier version** | **Envelopment version** |
|---|---|
| Output-maximizing model | Input-oriented BCC model |
| $\max \quad \theta_d = \sum_{r=1}^{s} u_{rd} y_{rd} - w_d$ <br> subject to $\sum_{i=1}^{m} v_{id} x_{id} = 1$ <br> $\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} - w_d \leq 0 \quad \forall j$ <br> $u_{rd} \geq 0 \quad \forall r = 1, \cdots, s$ <br> $v_{id} \geq 0 \quad \forall i = 1, \cdots, m$ <br> $w_d$ free. | $\min \quad f_d$ <br> subject to $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{id} f_d \quad \forall i$ <br> $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rd} \quad \forall r$ <br> $\sum_{j=1}^{n} \lambda_j = 1$ <br> $\lambda_j \geq 0 \quad \forall j.$ |
| Input-minimizing model | Output-oriented BCC model |
| $\min \quad \theta_d = \sum_{i=1}^{s} v_{id} x_{id} - w_d$ <br> subject to $\sum_{r=1}^{s} u_{rd} y_{rd} = 1$ <br> $\sum_{r=1}^{s} u_{rd} y_{rj} - \sum_{i=1}^{m} v_{id} x_{ij} - w_d \leq 0 \quad \forall j$ <br> $u_{rd} \geq 0 \quad \forall r = 1, \cdots, s$ <br> $v_{id} \geq 0 \quad \forall i = 1, \cdots, m$ <br> $w_d$ free. | $\max \quad f_d$ <br> subject to $\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{id} \quad \forall i$ <br> $\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rd} f_d \quad \forall r$ <br> $\sum_{j=1}^{n} \lambda_j = 1$ <br> $\lambda_j \geq 0 \quad \forall j.$ |

**Example 1.4.1** *Consider the data set of five DMUs with a single input and a single output. The input-oriented CCR and BCC models is apply for the same data set and report the results.*

*Observe that the CCR efficiency score is less than or equal to the BCC efficiency score for all the DMUs. This implies that a CCR-efficient DMU is also BCC-efficient, but the converse is not necessarily true (units A and C are BCC-efficient but not CCR-efficient). Figure 1.2 exhibits the efficient frontiers. DMUs D, E, and F are CCR as well as BCC inefficient. These DMUs can achieve efficient frontiers only by controlling.*

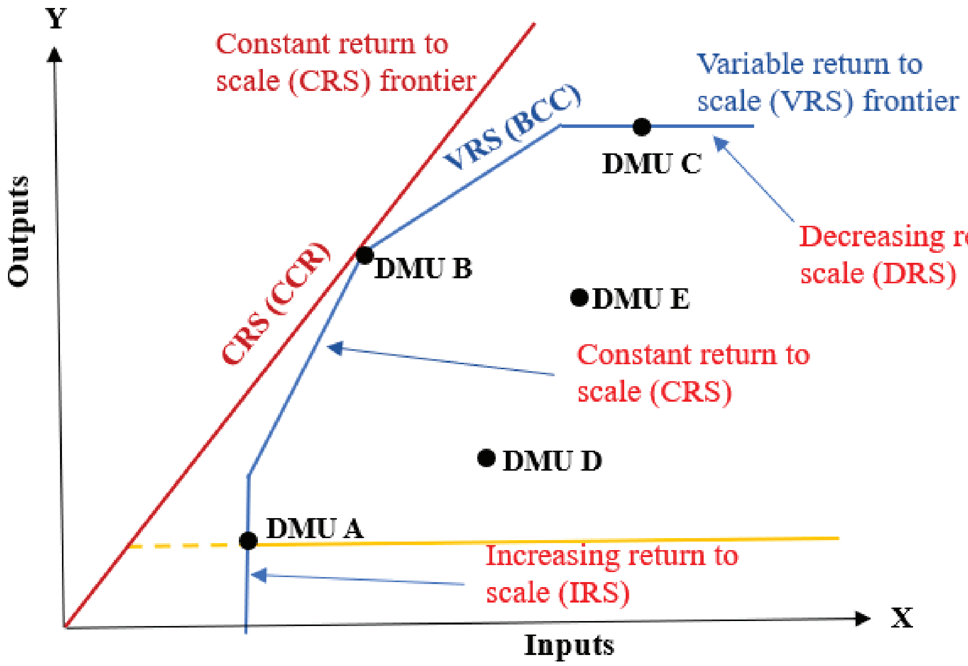


**Fig. 1.4:** CRS and VRS frontiers.

**Definition 1.4.3** *Let $(f_d^*, \lambda^*, S^{-*}, S^{+*})$ be an optimal solution of BCC envelopment model for $DMU_d$. If $f_d^* = 1$ and all the slacks $S^{-*}$ = 0, $S^{+*}$ = 0, then $DMU_d$ is called BCC-efficient (or efficient); otherwise, it is called BCC-inefficient (or inefficient). The reference set of $DMU_d$ is defined by $R_d$ = $\{j|\lambda_j^* > 0,\ j = 1, \cdots, n\}$.*

**Definition 1.4.4** ***Return to scale (RTS)*** *refers to the rate by which output changes if all inputs are changed by the same factor. Traditionally, RTS is conceptualized for single-output scenarios. Contemporary approaches have introduced multifaceted definitions to address diverse production contexts [23]. The two scales generally employed are constant returns to scale (CRS), and variable returns to scale (VRS).*

**Definition 1.4.5** ***CRS*** *renders to the output increases by the same proportion as the increment in inputs. This implies that production scale doesn't impact efficiency.*

**Definition 1.4.6** ***VRS*** *occurs when a change in all the inputs does not result in the same proportional change in the output. VRS further encompasses Increasing Returns to Scale (IRS) and Decreasing Returns to Scale (DRS).*

**Definition 1.4.7** ***IRS*** *is a proportional inputs increase that results in a greater than proportional output increase, suggesting efficiencies at larger scales.*

**Definition 1.4.8** ***DRS*** *is a proportional input increase that leads to a less than proportional output rise, indicating reduced efficiencies at small scales.*

**Definition 1.4.9** ***Production Possibility Set (PPS)*** *is defined as the set of all outputs that can be produced using the available inputs. Mathematically, suppose n DMUs with m inputs and s outputs. The input and output vectors of* $DMU_j$ $(j = 1, \cdots, n)$ *are presented by* $x_{ij} = (x_{1j}, \cdots, x_{mj})$ *and* $y_j = (y_{1j}, \cdots, y_{sj})$*, respectively. The PPS constructed under CRS postulate is defined by:*

$$P_{CRS} = \left\{ (x,y) : x \geq \sum_{j=1} \lambda_j x_j, \quad y \leq \sum_{j=1} \lambda_j y_j, \quad \lambda_j \geq 0,\ j = 1, \ldots, n \right\}$$

*and the PPS constructed under the VRS postulate is defined as:*

$$P_{VRS} = \left\{ (x,y) \mid x \geq \sum_{j=1}^{n} \lambda_j x_j, \quad y \leq \sum_{j=1}^{n} \lambda_j y_j, \quad \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0,\ j = 1, \ldots, n \right\},$$

*where* $\lambda_j$ *is called the intensity scalar for* $DMU_j$

**Definition 1.4.10** *A* $DMU_d$ *with inputs* $x_d$ *and outputs* $y_d$ *is said to be efficient if there does not exist any other* $(x,y) \in PPS$*,* $(x,y) \neq (x_d, y_d)$ *such that* $x \leq x_d$ *and* $y \geq y_d$*.*

### 1.4.6 Concepts & Definitions

Some of the important definitions related to DEA are here:

**Definition 1.4.11** ***Decision-Making Units****: The decision-making units (DMUs) refer to the entities or organizational units under evaluation in DEA. These units can represent various entities such as companies, departments, regions, or any other organizational divisions that are subject to performance assessment. In DEA, DMUs are assessed based on inputs and outputs to determine their relative efficiency and performance compared to other units. The goal is to identify the most efficient units as improvement benchmarks and provide insights for resource allocation and decision-making processes.*

**Definition 1.4.12** ***Efficiency***: *Efficiency refers to how closely the use of resources to produce outputs of a certain quality matches the ideal use of resources for outputs of the same quality. If and only if any improvement in a DMU's inputs or outputs would force some other inputs or outputs to deteriorate, that DMU is said to be fully efficient.*

**Definition 1.4.13** ***Inefficiency***: *The level by which a DMU falls over its cost frontier and below its output and profit frontiers can be viewed as a measure of inefficiency.*

**Definition 1.4.14** ***Input Slacks***: *The excesses in the actual input, known as input slacks, are used to calculate the target input, which is required for the inefficient units to become the efficient units.*

**Definition 1.4.15** ***Output Slacks***: *The shortfalls in the actual output, known as output slacks, are used to calculate the target output, which is required for the inefficient units to become the efficient units.*

**Definition 1.4.16** ***Peer Counts***: *The peers for inefficient DMUs are those efficient DMUs that have a positive value of $\lambda$'s. The total number for which an efficient DMU becomes peers for inefficient DMUs is called peer count.*

**Definition 1.4.17** ***Productivity***: *Productivity refers to the measure of output or results generated from a specific set of inputs or resources within a given period of time. It quantifies the efficiency and effectiveness of a process, system, or organization in converting inputs into valuable outputs.*

### 1.4.7 DEA: A Brief Survey

The groundbreaking study on DEA is written by Charnes et al. (1978) [15], which is also one of the most cited articles in the European Journal of Operational Research. The discipline has grown steadily and quickly. The frequency of adoption of DEA as a method for efficiency analysis can be estimated using [24], which provides a bibliography of more than 472 publications and dissertations published between 1978 and 1995. A bibliography of the DEA from 1978 to 2001 is produced in 2002 by Taveres et al. [25], which included 3203 published publications by 2152 different authors. The theoretical and real-world applications of DEA are covered in a survey and analysis of the first 30 years of literature by Emrouznejad et al. (2008) [26]. Kuah et al. (2010) [27] highlighted the multilevel DEA models, stochastic DEA models, and fuzzy DEA models. Liu et al. (2013) [28] conducted a comprehensive survey of DEA applications; two-thirds of the DEA articles are embedded in application data, while the remaining one-third are solely methodological. Emrouznejad and Marra (2014) [29] concluded that two-stage contextual analysis and network DEA applications are a recent trend. Several research articles related to DEA theoretical and top-5 applications are included: banking, health care, agriculture & farm, transportation, and education. Emrouznejad and Yang (2018) [30] carried out an extensive survey that included 10,300 journal articles with contributions from 11,975 different authors, spanning publications from 1978 to the end of 2016. They recognized important journals, the most well-known of which are the European Journal of Operational Research, the Journal of the Operational Research Society, and others. The report also identified the main application areas for DEA research in 2015 and 2016 as being public policy, supply chain, finance, agriculture, and transportation. DEA authors and 25,137 specific keywords across all DEA-related articles in the database, which increased significantly in recent years. Panwar et al. (2022) [31] studied a detailed literature and cover the development about the DEA in the last five decades.

A considerable amount of DEA literature on theoretical and methodological extensions is reported in [22, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]. There are several DEA models presented in the literature to address different issues.

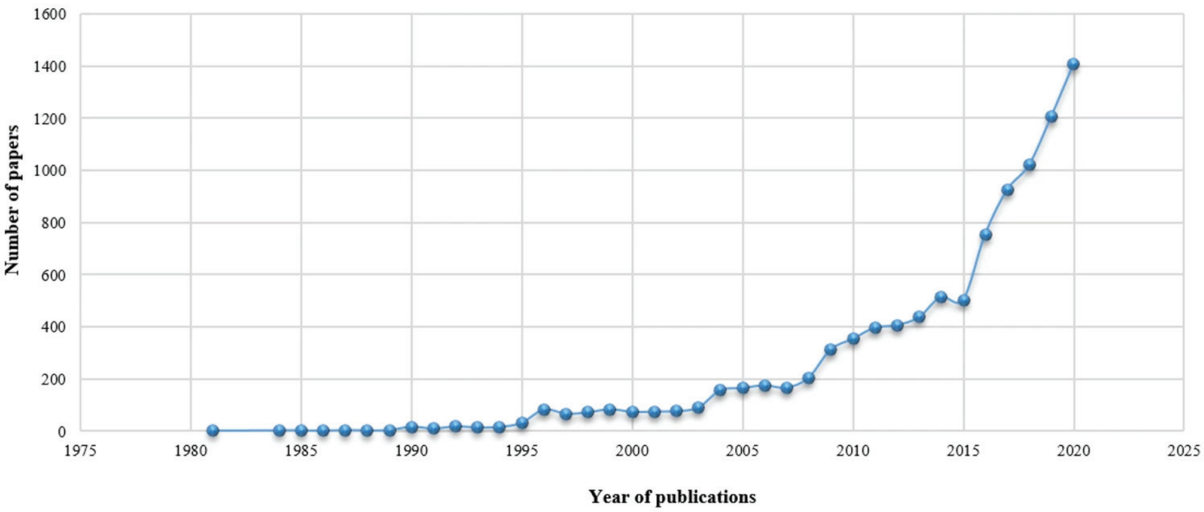


**Fig. 1.5:** Distribution of DEA-related articles by year (1975-2020).

### 1.4.8 Concept of Productivity

Productivity is the relationship between the output per unit generated by a production or a service system and the input, such as labor, capital, or other resources. It is implied in every economic activity and primarily stands for producing more and more outputs from fewer and fewer resources. When the productivity of two firms is compared, the more productivity firm produces more output with the same input or the same output with lesser input.

The two most commonly used types of productivity: partial factor productivity (PFP) and total factor productivity (TFP). PFP measures the ratio of total output to a single or partial input. TFP, or multifactor, is a measure of the ratio of total output and total input. It denotes the combined effect of all resources in generating the total output units. It is defined as the ratio of a weighted sum of output to the weighted sum of inputs. The directional distance function (DDF) provides a framework for understanding how production processes can be analyzed and evaluated in a multidimensional context, considering multiple inputs and outputs. This has practical applications in various industries and economic analyses, enabling a more nuanced assessment of productive efficiency. Generally, two kinds of distance functions are widely used in studies: the Shephard distance function [55] and the DDF [56]. Therefore, the DDF is a generalized form of the Shephard distance function. Economists frequently use specialized indexes created to evaluate TFP in order to have a deeper knowledge of economic performance. These indicators are essential for measuring the whole effect of several inputs, including labor, capital, and technology, on output. DDF can be estimated in at least two different ways: non-parametric DEA approach and parametric approach. The non-parametric DEA approach has become a cornerstone in productivity analysis due to its unique ability to handle diverse input-output technologies without relying on specific functional form assumptions. Analysts evaluate the underlying forces that drive economic growth by using tools like the Fisher index, Paasche index, Törnqvist index, Kendrick index, Solow index, Translog index, Malmquist productivity index, Malmquist index, Hicks-Moorsten index, and Färe-Primont index, as well as additive indexes such as the Luenberger-Hicks-Moorsten total factor productivity index or other TFP indices, giving light on the relative contributions of various causes. These indexes provide us with a strong lens through which to view the intricate dynamics of productivity.

#### 1.4.8.1 Productivity Measurement Approaches

In the non-parametric framework, Malmquist productivity index, characterized by Caves et al. (1982) [57] via distance functions and defined in the form of the geometric mean of two-adjacent-period indexes, is used to measure productivity growth. In connection with the additive nature of the DDF, Champer (1996) [58] introduced the Luenberger indicator. This indicator, which is based on differences, assesses DDF by taking into account both output expansions and input contractions. It marks a significant milestone as the initial endeavor to measure productivity changes over time, using graph measures as the basis. Among those is the Malmquist-Luenberger (M-L) index, introduced by Chung et al. (1997) [59]. The M-L index is a special form of the Malmquist index in that it measures productivity change but in the specific context of producing undesirable outputs. Oh and Heshmati (2010) [60] point out that the standard version of MLPI does not consider the progressive feature of technology. They proposed their own version of MLPI, which they appropriately called sequential MLPI (SMLPI). Although MLPI is an adequate approach for measuring productivity change, it has a few shortcomings, as pointed out by Choi et al. (2015) [61]. Pastor and Lovell (2005) [62] presented a global Malmquist index with all period data. Their index satisfies circularity, generates a single measure for cross-period observations, and is immune to infeasible solutions. This adaptability makes it an indispensable tool for assessing productivity changes across various industries and sectors [63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73]. Taking a closer look at the topic in Chapter 3, entails providing a thorough analysis and engaging in an in-depth discussion of the issues raised.

### 1.4.9 Inverse DEA Model

After evaluating the efficiency of the DMUs, two types of questions can arise. The first type is the resource allocation problem. This determines how much inputs are needed to increase if outputs are increased to some level, given that the efficiency of the DMU remains unchanged. The other type of problem is an investment analysis problem, which determines how much outputs are needed to increase if inputs are increased to some level, given that the efficiency of the DMU remains the same. Generally, the interior technical formation of the DMU does not change drastically in a short period of time. These formations of DEA models ensure that the inverse DEA model can be used for resource allocation and investment analysis problems [74]. The inverse DEA model is first proposed by Wei et al. (2000) [75] for input or output estimation and discussed the solution for the resource allocation and investment analysis problems. Yan et al. (2002) [74] extended this methodology to include the preferences of decision-makers through the implementation of priority cones. In addition, Lertworasirikul et al. (2011) [76] proposed the inverse DEA model assumption. Assume there are $m$-inputs, $s$-outputs, and $n$-number of DMUs and $X_{ij}$= amount of $i^{th}$ input consumed by $j^{th}$ DMU, $Y_{rj}$= amount of $r^{th}$ output produced by $j^{th}$ DMU, and $f^*$ is given efficiency. Consider the $r^{th}$ output in the ratio form, where $1 \leq r \leq s$. Consider that the output values of $DMU_0$ are increased from $Y_0$ to $Y_0 + \Delta Y_0$, $\Delta Y_0 \neq 0$. Then, the minimum change in input $\Delta X_0 \neq 0$ can be computed by the inverse CCR model and inverse DEA model with VRS that are given as follows:

**Table 1.3:** Inverse-DEA models.

| Inverse CCR model | Inverse BCC model |
|---|---|
| $\min \quad \Delta x_0 = (\Delta x_{10}, \cdots, \Delta x_{m0})^T$<br>subject to<br>$\sum_{j=1}^{n} \lambda_j x_{ij} \leq f^*(x_{i0} + \Delta x_{i0}) \quad \forall i$<br>$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \Delta y_{r0} \quad \forall r$<br>$\lambda_j \geq 0, \quad \forall j.$ | $\min \quad \Delta x_0 = (\Delta x_{10}, \cdots, \Delta x_{m0})^T$<br>subject to<br>$\sum_{j=1}^{n} \lambda_j x_{ij} \leq f^*(x_{i0} + \Delta x_{i0}) \quad \forall i$<br>$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0} + \Delta y_{r0} \quad \forall r$<br>$\sum_{j=1}^{n} \lambda_j = 1$<br>$\lambda_j \geq 0, \quad \forall j.$ |

Chapter 4 provided a comprehensive analysis and engage in-depth discussion of the raised concerns.

### 1.4.10 Fuzzy Set Theory

In many practical applications, input and output data cannot be precisely measured. Imprecision or approximation may arise from various sources, such as indirect measurements, model estimations, subjective interpretations, and expert judgments based on available information. Ignoring uncertainty and impreciseness in data sets might diminish decision models' utility and predictive capacity [77]. Consequently, methodologies that enable analysts to handle imprecise or approximate data explicitly are of significant interest. DEA models, being highly sensitive to potential imprecision in the dataset, benefit from such approaches. By formulating the evaluation problem within the framework of fuzzy set theory, analysts can augment the traditional "crisp" DEA to encompass and represent the uncertainty inherent in real-world problems. In 1965, Zadeh [78] developed the fuzzy set theory, in which a grade of membership function is developed, which provided an effective tool for dealing with vagueness and uncertainty. Fuzzy techniques have various benefits compared to crisp methods because they have more flexible decision boundaries, allowing them to be adapted to a certain application area and more accurately reflect its peculiarities. Fuzzy optimization is a method designed to address ambiguity stemming from uncertain parameters, expressed as elements whose membership in a specific set lacks clarity. In contrast to robust optimization, which focuses solely on parameter uncertainty, fuzzy optimization provides a framework to handle a broader spectrum of uncertainties within the problem's structure. These encompass variations in the decision maker's level of ambition concerning the objective, fluctuations in the range of coefficients within the objective function(s), and uncertainties regarding the satisfaction level of constraints. Below are some definitions pertaining to fuzzy set theory.

**Definition 1.4.18** ***Crisp set:*** *A set A is said to be crisp if it is a well-defined collection of distinct objects called elements of A. If an element x is in A, it is written as $x \in A$; otherwise, $x \notin A$. Let X be the universal of discourse. Then, A is defined as $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X;\ \mu_{\tilde{A}}(x) \in \{0,1\}\}$ in which $\mu_{\tilde{A}}(x)$ is the characteristic function defined by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} \tag{1.1}$$

**Definition 1.4.19** ***Fuzzy set :*** *Let X be the universal of discourse. Then a fuzzy set $\tilde{A}$ in X is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X; \mu_{\tilde{A}}(x) \in [0,1]\}$ in which $\mu_{\tilde{A}}(x)$ is called the membership grade function of x the fuzzy set $\tilde{A}$.*

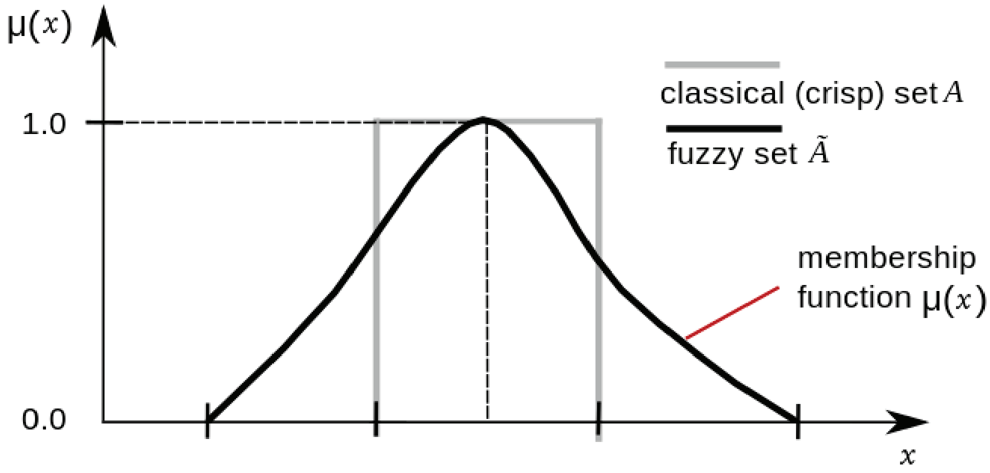


**Fig. 1.6:** Membership function for crisp and fuzzy set.

**Definition 1.4.20** ***Support of a fuzzy set :*** *A support for a fuzzy set $\tilde{A}$ on X denoted by $S(\tilde{A})$ can be defined as,*

$$S(\tilde{A}) = \{x \in X | \mu_{\tilde{A}}(x) > 0\}. \tag{1.2}$$

**Definition 1.4.21** $\alpha-$ ***cut :*** *An $\alpha-$level cut for a fuzzy set $\tilde{A}$ on X denoted by $\mu_{\tilde{A}}(x)$ defined as,*

$$\mu_{\tilde{A}}(x) = \{x \in X | \mu_{\tilde{A}} \geq \alpha\}. \tag{1.3}$$

**Definition 1.4.22** ***Height of a fuzzy set :*** *The supremum of the fuzzy membership function is defined to be the height of a fuzzy set.*

**Definition 1.4.23** ***Normal fuzzy set:*** *If the Height of the membership function is 1, then the fuzzy set is called normal fuzzy set.*

**Definition 1.4.24** ***Crossover point :*** *The crossover point of A is an element that has a membership value of 0.5.*

**Definition 1.4.25** ***Convex fuzzy set :*** *A fuzzy set $\tilde{A}$ of $X$ is said to be convex fuzzy set if,* $\mu_{\tilde{A}}(x) \geq min\{\mu_{\tilde{A}}(b),\ \mu_{\tilde{A}}(a)\}\ \forall\ a \leq x \leq b)$.

**Definition 1.4.26** ***Core of a fuzzy set :*** *The set of all elements whose fuzzy membership grade function has a value of 1 is defined as the core of the fuzzy set. The core of the fuzzy set is crisp in nature and is defined as,*

$$Core(\tilde{A}) = \{x \in X | \mu_{\tilde{A}}(x) = 1\}. \tag{1.4}$$

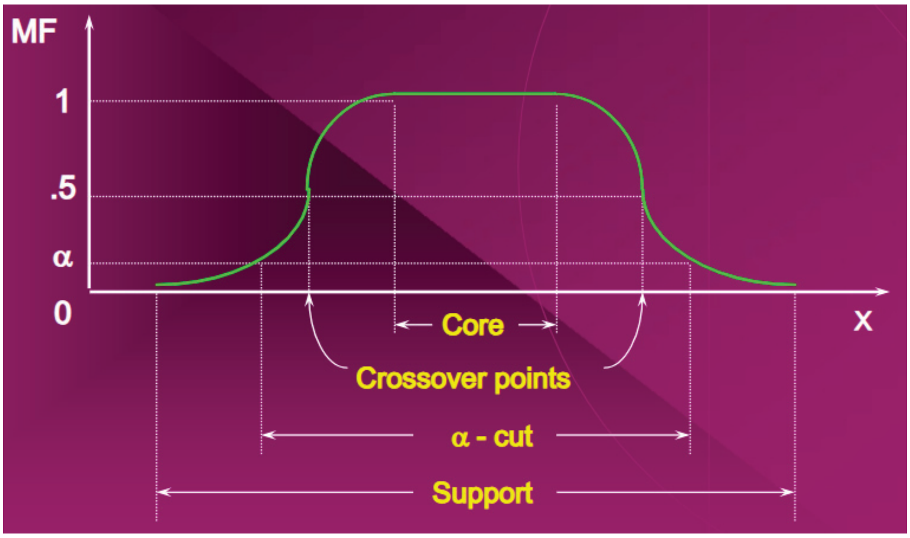


**Fig. 1.7:** Intuitive diagram for basics of fuzzy sets.

### 1.4.11 Fuzzy Numbers

Fuzzy numbers are the generalization of real numbers whose weight function is also known as the membership grade function, where the membership grade is an intermediate binary value between 0 and 1 [79]. Let $\tilde{A}$ be a fuzzy set of a universal set $X$ and such a set distinguishes with the membership grade function $\mu_{\tilde{A}}\{x\} : X \to [0,1]$, which associates with each element $(x = x_1, x_2, \cdots, x_n)$ in X to a real number lies between $[0,1]$. The fuzzy set $\tilde{A}$ can be defined as:

$$A(\tilde{x}) = \{x, \mu_{\tilde{A}}\{x\}, \quad x \in X\} \tag{1.5}$$

A fuzzy set $\tilde{A}$ defined on real numbers $R$ is said to be fuzzy number [80] if it satisfies the following properties:

1. $\tilde{A}$ is normal,
2. $(\tilde{A})_\alpha$ is a closed interval for every $\alpha \in (0,1]$,
3. The support of $\tilde{A}$ is bounded.

There are many fuzzy numbers, depending on the nature of the user and the decision-making process. However, the most commonly used and user-friendly membership grade functions are triangular, gaussian, bell-shaped, and trapezoidal fuzzy numbers.

**Definition 1.4.27** ***Triangular Fuzzy Number :*** *TFNs are frequently favored by analysts due to their computational simplicity. This simplicity makes TFNs an accessible and practical choice for representing uncertainty in various applications. The TFN is implanted because of its widespread acceptance in the literature, which is the most notable and fundamental [81]. The left and right sides of its linear representations are such that its membership function for a TFN $\tilde{A} = (a,b,c)$, $a < b < c$, where a represents the lower bound, b signifies the mean and c denotes the upper bound. Now, the membership function is described below as:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & c \le x. \end{cases}$$

*If $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two TFNs. Then, the basic mathematical operations of these two TFNs are as follows:*

$$(\tilde{A} + \tilde{B}) = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \quad a_1, a_2 \ge 0 \tag{1.6}$$

$$(\tilde{A} - \tilde{B}) = (a_1 - c_2, b_1 - b_2, c_1 - a_2) \quad a_1, a_2 \ge 0 \tag{1.7}$$

$$(\tilde{A} \times \tilde{B}) = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2) \quad a_1, a_2 \geq 0 \tag{1.8}$$

$$(\tilde{A} \div \tilde{B}) = (a_1 \div a_2, b_1 \div b_2, c_1 \div c_2) \quad a_1, a_2 \geq 0 \tag{1.9}$$

$$(\tilde{A}^{-1}) = \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right) \text{Inverse of a triangular fuzzy number.} \tag{1.10}$$

**Definition 1.4.28** ***Trapezoidal fuzzy numbers :*** *Another widely used fuzzy number is the trapezoidal fuzzy number. A trapezoidal fuzzy number $\tilde{M}$ membership function for $(a,b,c,d)$ such that $(a < b < c < d)$, is defined as,*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x \leq a \\ \frac{x-r}{s-r}, & \text{if } a \leq x \leq b \\ 1, & \text{if } b \leq x \leq c \\ \frac{u-x}{u-t}, & \text{if } c \leq x \leq d \\ 0, & \text{if } x \geq d. \end{cases} \tag{1.11}$$

**Definition 1.4.29** ***Gaussian fuzzy number :*** *A fuzzy number is called a Gaussian fuzzy number if it has a membership function,*

$$\mu_{\tilde{A}}(x) = te^{\frac{(x-a)^2}{b}}, \; x \in R, \; b > 0. \tag{1.12}$$

**Definition 1.4.30** ***Cauchy fuzzy number :*** *A fuzzy number is called a Cauchy fuzzy number if it has a membership function,*

$$\mu_{\tilde{A}}(x) = \frac{1}{1 + (\frac{x-b}{a})^2}, \; x \in R, \; a > 0. \tag{1.13}$$

### 1.4.12 Fuzzification

Fuzzification is the process of converting precise decisions within the input space into fuzzy sets facilitated by the fuzzifier. It involves acknowledging that quantities traditionally seen as deterministic actually carry a significant degree of uncertainty. This uncertainty can be expressed using a membership grade function. At the outset of the process, the fuzzification function is applied to each input variable to ascertain the level of uncertainty. It interprets and evaluates input variables, transforming them from crisp real numbers into practical approximations represented as fuzzy numbers. This assignment can be done through intuitive means or based on algorithmic or logical operations. Commonly utilized fuzzification approaches include rank-ordering, inference, neural networks, intuition, genetic algorithms, inductive reasoning, soft-partitioning, meta-rules, fuzzy statistics, and angular fuzzy sets [82].

### 1.4.13 Defuzzification

Defuzzification involves the process of approximating a fuzzy set by determining the nearest vertex in the unit hypercube. Conceptually, one can envision a fuzzy set as an assortment of membership values or a vector of values within the unit interval. Defuzzification condenses this vector into a singular scalar value, typically representing the most typical value. The prevalent methods for converting fuzzy sets to crisp sets include the Max-Membership Principle, Centroid method, Weighted Average method, and Mean-Max Membership. This defuzzification procedure results in the transformation of a fuzzy set into a single-valued crisp quantity or into a crisp set, thereby converting a fuzzy matrix to a crisp matrix or rendering a fuzzy number crisp [83, 84, 85, 86].

## 1.5 Fuzzy DEA Models

The conventional DEA approach necessitates precise measurements of both inputs and outputs. However, in practical scenarios, the recorded input and output data values are rarely exact or entirely accurate, potentially leading to erroneous conclusions. Several scholars have proposed various fuzzy techniques to address this data ambiguity. Since the seminal work by Sengupta et al. (1992) [87, 88], there is sustained interest and expansion in the field of fuzzy DEA (FDEA) research. Ali et al. (2013) [89] conducted a comprehensive review of FDEA methods and categorized them into six primary classifications: tolerance approach, $\alpha-$level based approach, fuzzy ranking approach, possibility approach, fuzzy arithmetic, and fuzzy random/ Type-2 fuzzy sets. They further organized pioneering articles that didn't fit into these six categories. The following subsections provide concise introductions to these six categories and present a brief overview of the relevant literature.

### 1.5.1 Tolerance Approach

In 1992, Sengupta et al. [87] introduced one of the earliest FDEA models employing the tolerance approach. This technique involves incorporating uncertainty into the DEA model by specifying tolerance levels for constraint violations. While it obscures the signs of inequality or equality, it is not directly linked to ambiguous coefficients. The tolerance approach does entail significant complexity, as it entails formulating a DEA model with a fuzzy objective function and ambiguous constraints, which may or may not be met. Nonetheless, ambiguity is inherent in most production processes, encompassing failure to achieve specified objectives and data inaccuracies. The tolerance approach introduces adaptability by relaxing the DEA relationship and treating input and output coefficients as crisp values.

### 1.5.2 $\alpha$-cut Approach

The $\alpha$-cut approach stands out as one of the most widely used FDEA models, as evidenced by its extensive representation in the FDEA literature. This approach revolves around converting the FDEA model into a set of parametric programs that determine the lower and upper bounds of the membership functions at varying values of $\alpha$-level. Girod et al. (1996) [90] introduced a fuzzy BCC and free disposal hull (FDH) model, which is radial efficiency. These models, following the approach presented by Carlsson et al. (1986) [91], allow inputs to vary between risk-free (upper) and impossible (lower) limits, while outputs can oscillate between risk-free (lower) and impossible (upper) limits. Triantis et al. (1998) [92] brought forth the fuzzy LP technique for measuring technical efficiency, establishing precise inputs and outputs within risk-free and impossible limits. Then, three CCR, BCC, and FDH fuzzy models are developed in terms of their risk-free and impossible constraints, along with their membership grade function. Additionally, Girod et al. (1999) [93] provided a detailed discussion of this paper by implementing it on a roadmap. Triantis et al. (2003) [94] extended their earlier work on FDEA to incorporate non-radial DEA metrics of technical efficiency within an integrated performance measurement system. They compared this proposed method to a thorough examination of the radial technical efficiency of the same manufacturing production line. To compute the fuzzy interval efficiency of DMUs, Meida et al. (1998) [95] utilized the $\alpha-$cut. Kao et al. (2000) [96] devised an approach for evaluating the efficiency of DMUs with uncertain observations in the BCC model. They built on the idea of substituting the FDEA model into the family of traditional crisp DEA models. Using the $\alpha-$cut methodology and Zadeh's (1978) [97] extension concept, they estimated the membership functions of the fuzzy efficiency measures. This involved transforming the FDEA model into a pair of parametric mathematical algorithms and employing Chen et al. (2012) [98] ranking fuzzy number method to compute the DMU's performance. The interval is formed by solving this model at a specified value of $\alpha-$cut, and multiple such intervals can be employed to construct the corresponding fuzzy efficiency.

### 1.5.3 Fuzzy Ranking Approach

The fuzzy ranking approach is another significant technique that has garnered attention in the FDEA literature. This method centers around the utilization of a fuzzy linear algorithm to determine the fuzzy efficiency rating of DMUs, necessitating the ranking of the fuzzy set. Guo et al. (2001) [99] pioneered in creating a fuzzy ranking approach for efficiency measurement. They introduced a fuzzy CCR model in which fuzzy constraints are transformed into crisp constraints by establishing a possibility level and applying a rule for comparing fuzzy numbers. To rank a set of DMUs, a fuzzy aggregation method is employed for multiple attribute fuzzy values [100]. In their subsequent work, Guo et al. (2009) [101] proposed a novel FDEA model tailored to address restaurant location problems in China. This demonstrates the versatility and applicability of the fuzzy ranking approach beyond generic efficiency measurements.

### 1.5.4 Possibility Approach

Fuzzy set theory forms the foundational principle of possibility theory. As per Zadeh [97], a fuzzy variable is linked to a possibility distribution, and a random variable is associated with a probability distribution. In a fuzzy LP model, fuzzy coefficients can be viewed as fuzzy variables, and constraints can be seen as fuzzy events. Therefore, possibility theory provides a framework for calculating the probabilities of these fuzzy events. For a comprehensive understanding of possibility theory, one can refer to the work of Dubois et al. [102]. Guo et al. [103] developed FDEA models based on possibility and necessity measures. Subsequently, Lertworasirikul et al. (2003) [104, 105, 106] introduced the "possibility approach" and "credibility approach" for addressing the ranking problem in FDEA models. They introduced the possibility approach by considering uncertainty in fuzzy objectives and constraints through possibility measures from optimistic and pessimistic viewpoints. This transformed the FDEA model into a credibility programming-DEA model, with fuzzy variables replaced by expected credits derived using credibility measures.

### 1.5.5 Fuzzy Arithmetic Approach

A concise investigation into handling fuzzy data through the application of fuzzy arithmetic is presented in [107]. Furthermore, Wang et al. (2009) [108] introduced two FDEA models based on fuzzy arithmetic, incorporating fuzzy inputs and outputs. These models convert the fuzzy CCR model into a three-LP model to assess DMU efficiency. Additionally, they devised a fuzzy ranking system for DMU prioritization. Abdoli et al. (2011) [109] conducted a study evaluating the productivity of a group of knowledge workers using an FDEA-based approach. Jafarian et al. (2012) [110] expanded upon Chiang et al. (2000) [111] multi-objective static DEA model, extending it into a fuzzy dynamic multi-objective DEA model. They simplified the fuzzy multi-objective programming problem into a single-objective programming problem by leveraging Zimmerman's approach [112]. Additionally, Jafarian et al. (2012) [110] utilized triangular fuzzy numbers to represent missing data in their model. They resolved the model using the FDEA approach established by Wang et al. (2009) [108] after formulating the LP model using the former method.

### 1.5.6 Fuzzy DEA: A Brief Survey

One of the challenges in real-world situations is that the available data might be present in an uncertain or qualitative form, or sometimes some data might be missing. Erdogan et al. (2015) [113] concluded that the conventional DEA models are absurd to use with these types of data. Sometimes, when the available data is in qualitative form, it is transformed into numerical data by inviting subject or domain experts to evaluate the degree of confidence in all possible situations. Subsequent evidence from various studies in different settings around the world shows that people routinely inflate small probabilities when answering these types of questions [114]. The fuzzy set theory, which is developed by Zadeh [78], handles this situation more effectively, and it has expanded to deal with the concept of partial fact ranging from correct to incorrect. Fuzzy theory has become the fundamental tool for handling imprecision or vagueness, aiming at tractability, robustness, and low-cost solutions for real-world problems. Many researchers applied DEA models to evaluate the efficiency of DMUs under fuzzy environments [115, 116]. Wen et al. (2009) [117] extended the traditional DEA models to a fuzzy DEA model based on credibility measures. Chen et al. (2013) [118] concluded that the use of the fuzzy SBM DEA model for estimating efficiency values not only represents the characteristic of the uncertainty of the efficiency values, it also presents the potential effect of risk volatility on efficiency values. Hsiao et al. (2011) [119] concluded that linguistic terms could not entirely fit the conventional DEA models. Puri et al. (2013) [120] used fuzzy SBM DEA models to handle the imprecise data and calculate the mix-efficiencies. Wanke et al. (2016) [121] presented an analysis of the efficiency using fuzzy DEA and stochastic DEA models based on the $\alpha-$ level approach and different tail dependence structures, respectively. Recently, Bakhtavar et al. (2019) [122] used a special risk prioritization algorithm by failure mode and effects analysis by SBM DEA model under fuzzy conditions.

Lertworasirikul et al. (2003) [105, 106] studied the fuzzy DEA models built by Guo et al. (2000) [103], which took the possibility criterion and the necessity criterion as a measure and solved the ranking problem with two distinct approaches, namely, the possibility approach and credibility approach. Furthermore, Agarwal et al. (2014) [123] applied possibility measures to solve the fuzzy SBM DEA model. The possibility measure is used but has no self-dual property, which is undoubtedly needed for practice. Liu et al. (2002) [124] proposed a credibility measure that shows the self-dual character. The credibility theory, which manages personal conviction degree numerically, is given by Liu et al. [125] and refined in his next research [126, 127]. Wen et al. (2010) [128] used the credibility measure to solve the CCR DEA model. The model is apt for the constant return to scale, but both primal and dual forms of the CCR model are required to measure the relative efficiency and efficient targets. Many researchers used fuzzy set theory to handle qualitative data and integrate it with different models. Agarwal et al. (2014) [123] extended the conventional DEA model to a fuzzy DEA model and solved it with the help of the $\alpha$-cut approach. Yu et al. (2017) [129] used fuzzy DEA for sustainability and found that suppliers with low carbon footprints exhibited poor efficiency, which may be attributed to the additional effort required. Wanke et al. (2018) [130] used fuzzy DEA and stochastic DEA to analyze the Angolan banks. Other recent applications of fuzzy DEA, such as Peykani et al. (2019) [131] evaluated the efficiency using an adjusted fuzzy DEA approach. Gupta et al.[132] proposed portfolio efficiency evaluation using BCC-DEA and RDM models under fuzzy environments. The application of the model is shown by using superior risk measures of value at risk and conditional value at risk under a credibility measure. The BCC DEA model is extended with a fuzzy environment for evaluating the efficiency of DMUs and solved by credibility measures in Chapter 6.

Upon reviewing the existing literature, several research gaps have identified within the public transport sector.

1. There is still a lack of a more robust approach for accurately determining criteria, criteria weights and effectively evaluating performance in transport sector.

2. The efficiency and productivity evaluation is not analyzed enough for public transport sector in India using the DEA model.

3. The limited exploration of inverse DEA for optimizing resource allocation in the context of the transport sector.

4. The ranking of efficient DMUs in the FDEA problem still has a complex solution that needs to be simplified.

### 1.5.7 Study Region

Rajasthan is one of the original states in India when the country gained independence and reorganized in 1956. Situated in the northwestern region, it stands as the nation's largest state, spanning an area of approximately 342,239 $km^2$, covering 10.4% of the country's geographical area. The area of the state is nearly equivalent to some of the countries in the world, like Italy (3,01,200 $km^2$), Poland (3,12,600 $km^2$), and Norway (3,24,200 $km^2$). The latest census in 2021 estimates Rajasthan had roughly 5.68% of the country's total population. Over the past 70 years, Rajasthan's population has grown nearly fivefold, from about 15.97 million in 1951 to 79 million in 2021. The absolute growth peaked between 1991 and 2001 with an addition of roughly 12.5 million people, as shown in figure 1.8.

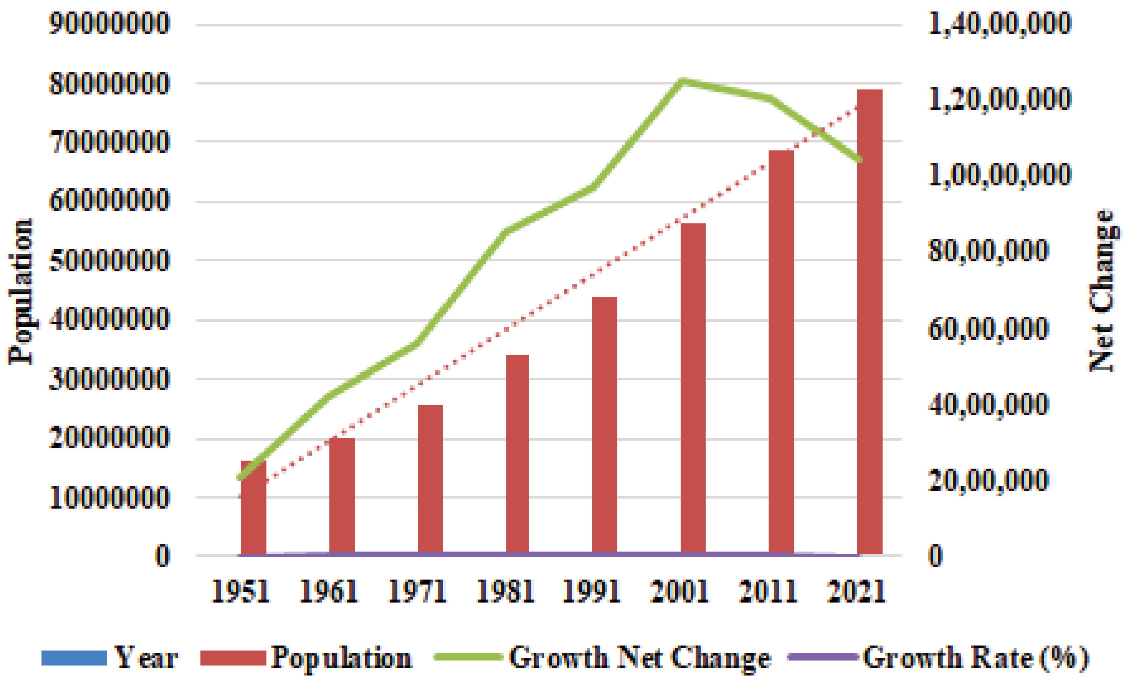


**Fig. 1.8:** Population growth over a period of Rajasthan.

Rajasthan is known for its rich cultural heritage and is also significant from an economic standpoint. Cities like Jaipur, Udaipur, Jaisalmer, and Jodhpur attract domestic and international tourists. There is an average increase of 10.01% in the per capita income of Rajasthan, whereas this increase has only 7.89% at the all-India level. Figure 1.9 provides a snapshot of the economic health of Rajasthan state, as gauged by their per capita output. Rajasthan has the lowest Net State Domestic Product (NSDP) per capita of Rs.1,56,149 for the year 2022-23.

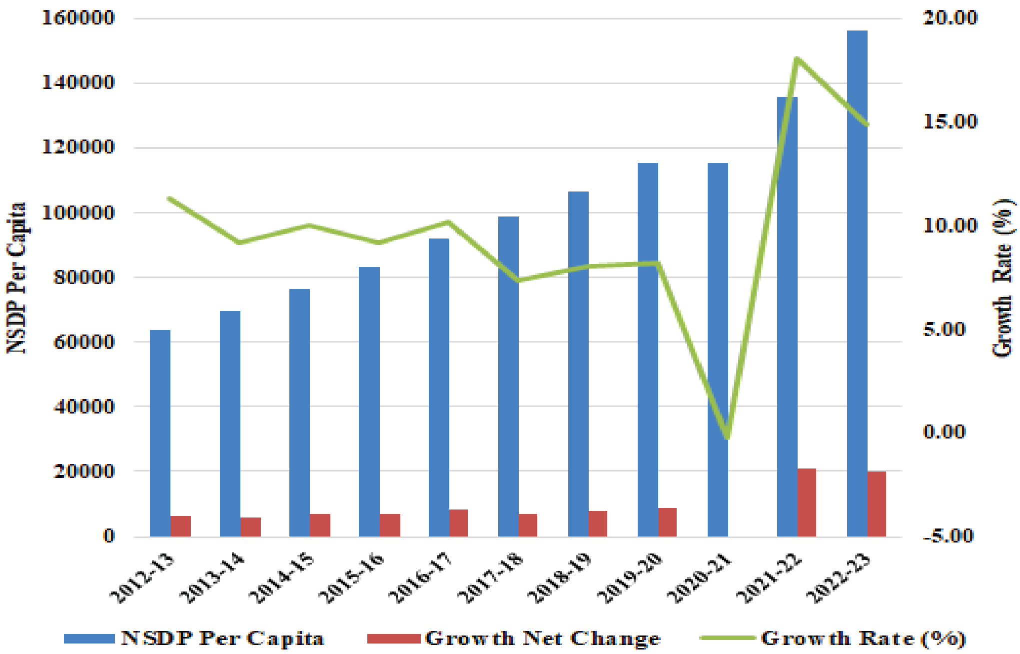


**Fig. 1.9:** Rajasthan NSDP per capita over the years.

Geographically, Rajasthan has huge diversity - a region of rolling sand dunes and lofty hills, vast deserts, freezing cold and scorching heat, fertile plains in the east, and sparsely populated areas in the west. The Aravalli range, stretching diagonally across Rajasthan, serves as a geological divider, bifurcating the state into the western arid plain and the distinct eastern plain. The state's expansion is between 23°3' to 30°12' North latitude and 69°30' to 78°17' East longitude. It has a pretty different topography as compared to other states, as it comprises most of the wide and inhospitable Thar Desert. The Thar Desert encapsulates 12 districts (Barmer, Bikaner, Churu, Ganganagar, Hanumangarh, Jaisalmer, Jhunjhunun, Jodhpur, Jalor, Nagaur, Pali, and Sikar) in the state and spreads in 61.11 % of the total area of the state. The desert area of Rajasthan is among the few tropical deserts of the world which has the highest population density.

#### 1.5.7.1 Public Transport Infrastructure of the State

The transport sector encompasses different models of transport, including roads, railways, airways, inland waterways, and shipping, facilitating efficient transportation of goods and citizens across the country. Economic growth is closely linked to the transportation of goods and passengers. This efficiency contributes significantly to the transport sector in gross state domestic product (GSDP)/ gross state value added (GSVA), which is evident from figure 1.10. The majority of the transport sector's contribution in Rajasthan comes from road transport, highlighting its importance in the state's economy. The road transport is pivotal for socio-economic development, fostering social, regional, and national integration.

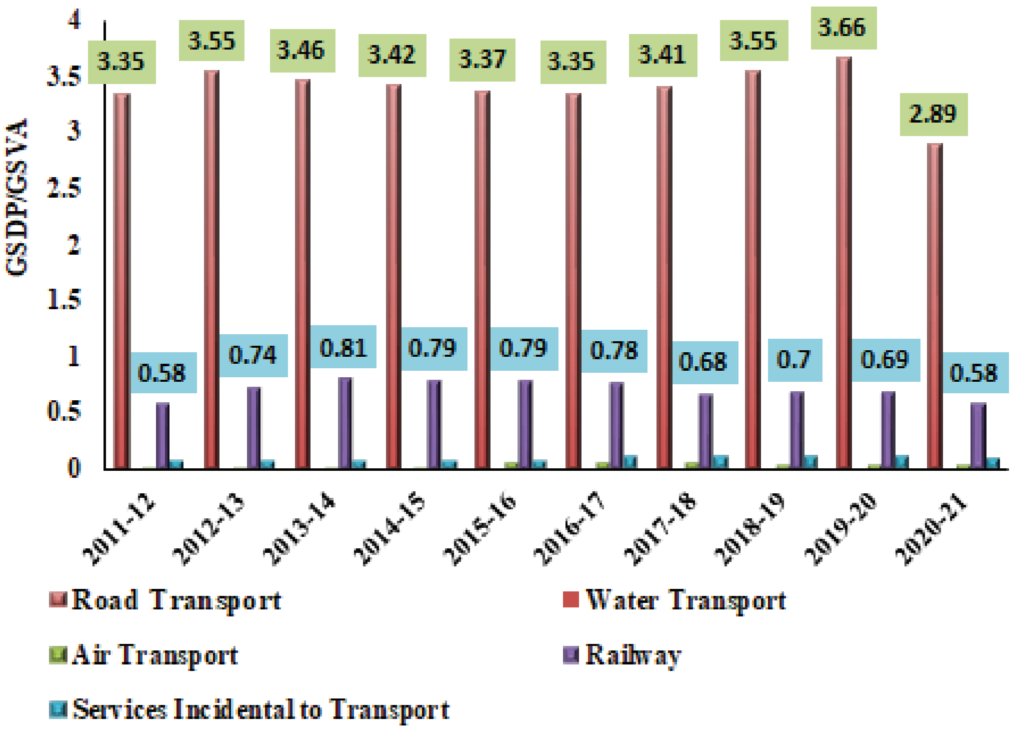


**Fig. 1.10:** Contribution of transport sector in GSDP/ GSVA of Rajasthan.

From March 2012 to March 2021, the number of registered motor vehicles in the state surged from 8,985,568 to 20,223,021, marking a growth of 125.06% over the decade. Two-wheelers, representing 73.78% of the total in 2012, increased to 75.86% by 2021. The proportion of cars, jeeps, and taxis grew marginally from 10.40% in 2012 to 10.90% in 2021. However, the share of buses declined from 0.93% to 0.61%, and goods vehicles saw a reduction from 4.02% to 3.68% during the same period. Road Transport has steadily expanded its scope of operations to its inherent suitability for handling freight and passengers.

Rajasthan state road transport corporation (RSRTC) is established on October 1, 1964, under the provisions of the Road Transport Corporation Act, 1950, with a modest fleet of 421 buses operating across 8 depots and serving 29,000 passengers daily. Its headquarters locateded in Jaipur, RSRTC is a state government initiative that offers both interstate and intercity bus transportation at subsidized fares, making it more accessible and affordable to the general public. This subsidized fare service not only eases travel for daily commuters but also promotes better connectivity across the state. As Rajasthan is the largest intercity bus transport service provider, RSRTC boasts 52 small and large bus depots spread across the 33 districts of the state. RSRTC has witnessed considerable growth. Today, its vast fleet consists of 4,500 buses from 52 depots, covering 1.6 million *km* and catering to the transportation needs of 1 million passengers daily.

Currently, RSRTC provides services beyond Rajasthan, reaching out to states like Gujarat, Haryana, Uttar Pradesh, Delhi, Himachal Pradesh, Uttarakhand, Madhya Pradesh, Jammu & Kashmir, Chandigarh and Maharashtra. At present, RSRTC has ordinary, express, deluxe, semi-deluxe, A.G. (Gandhi Rath and sleeper), air conditioning, volvo (also with LCD and/or pantry), sleeper, and Mercedes buses in its fleet. On RSRTC buses, specific seats are reserved exclusively for female passengers. These seats are usually marked with symbols or colors to indicate the reservation. The reserved seats are meant to ensure that female passengers always have a place to sit, even if the bus is crowded. For the convenience of the passenger at the depot, the corporation generally has a rest room, canteens, lockers, waiting rooms, charging points, CCTV surveillance, a booking office, toilets, drinking water, a parking area, a timetable and fare chart display, inquire counters, a public address system, lights, fans, seats and benches, ATM machines, and PCO, etc.

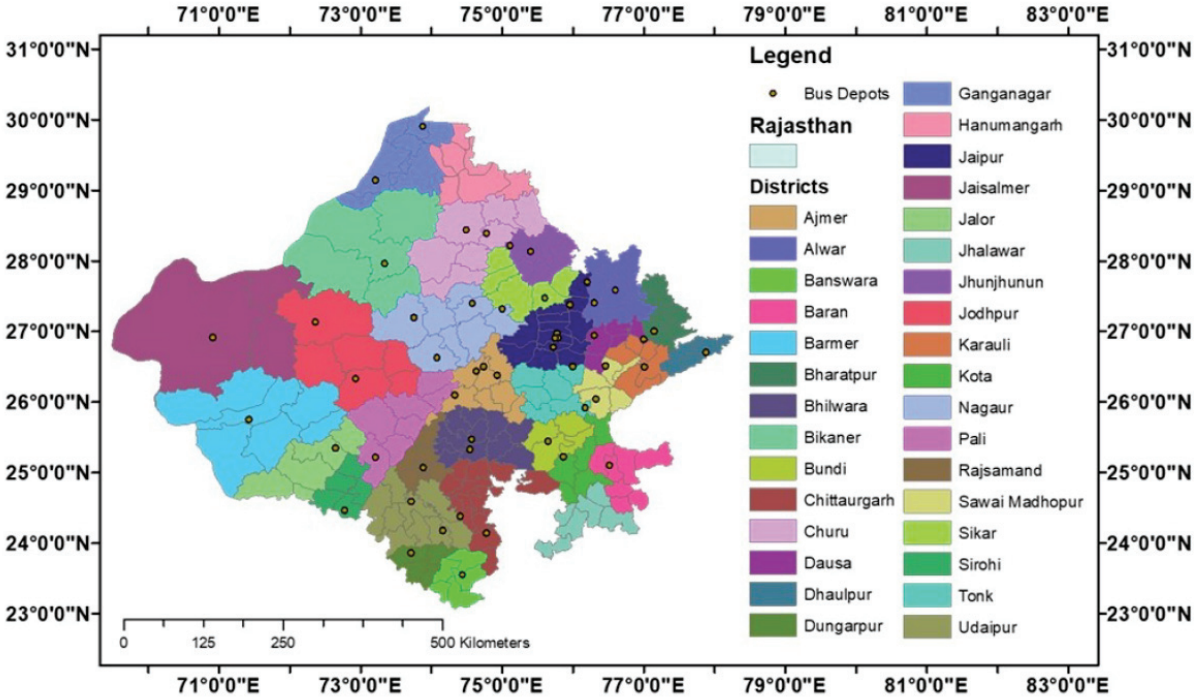


**Fig. 1.11:** Map of Rajasthan.

In 2011, RSRTC introduced the integrated transport management system (ITMS) to modernize its operations. The ITMS comprises an online reservation system (ORS), electronic ticket machines (ETMs) integrated with ORS, a public online reservation system (PORS), an online management information system (MIS), and a vehicle scheduling and dispatch system. A year later, in 2012, RSRTC rolled out an RFID-based smart card concessional pass system (RFID SCPPS). This system facilitates the issuance of RFID smart cards for various purposes, including concessional/free travel, monthly season tickets, and prepaid e-purse travel. It plays a crucial role in providing transportation options to residents, tourists, and businesses, thereby contributing to the state's economic and social development.

#### 1.5.7.2 Challenges in Transportation

According to a report [133], a few years back, due to the different topological conditions (deserts, mountains, planes), the villages of Rajasthan are not well connected with each other through an effective transport system. Nowadays, rural transport services are developed to improve the social and economic status of the people. In spite of the above problem, many steps have taken by the Government for the improvement of road network. The major obstacle in the development of transport networks in the state is its geography, marked by the Thar Desert and Aravalli mountain range, which presents unique challenges and opportunities for transportation. The vast expanse and variable terrain require specialized transport solutions. There are several problems in the construction and maintenance of transport networks in the state.

## 1.6 Thesis Objectives

The management of the public transport sector must implement strategic measures to ensure the optimal utilization of available resources, aiming to maximize output. Efficient and judicious use of resources is imperative for enhancing the productivity of depots. The objective of this thesis is to tackle the aforementioned challenges, with a specific focus on achieving the following objectives:

1. Identify and rank the significant criteria influencing performance using hybrid MCDM approach.

2. To highlight the growth trends and efficiency patterns of all DMUs at an aggregate level using the NSM model with VRS assumptions.

3. To analyze total factor productivity changes, technical efficiency, and technological progress using NSM model over the time of DMUs.

4. To analyze a super-efficiency inverse DEA (IDEA) model for ranking the efficient DMUs.

5. To apply the credibility approach to the fuzzy cross-efficiency DEA model for evaluating efficiency.

The study aims to provide a refined and robust framework for accurately assessing the efficiency of the entities.

## 1.7 Thesis Contribution

The culmination of this research endeavors to shed light on several pivotal aspects within the domain of efficiency analysis and productivity measurement in the transport sector. Through the exploration of innovative methodologies and models, this thesis embarks on a comprehensive journey encompassing various chapters that collectively contribute to the advancement of decision-making processes and operational enhancements in transportation systems. Through the collective exploration and development of these models and methodologies, this thesis stands as a testament to the commitment towards offering practical solutions for improving efficiency, resource allocation, and overall performance enhancement in the dynamic landscape of transportation.

1. **Hybrid MCDM Approach:** In this study, the innovative application of MCDM techniques is substantial practical implications. By effectively identifying and ranking the most critical criteria affecting the performance of transportation depots, this study offers valuable insights for decision-makers and stakeholders. For instance, transportation authorities can utilize these findings to prioritize their focus on key performance criteria, directing resources and efforts towards areas that significantly impact depot efficiency. This prioritization allows for targeted improvements, potentially leading to enhanced operational effectiveness and resource allocation within the transport depots. Moreover, the integration of sensitivity analysis into the assessment process ensures a more robust and reliable model for ranking based on their performance. This method enables a deeper understanding of the stability and consistency of efficiency scores, offering decision-makers confidence in the reliability of ranking systems.

2. **Efficiency Measurement:** The DEA model and subsequent analysis of the RSRTC depots offered significant practical advantages of the transport sector. By identifying efficiency trends and categorizing depots, this research enabled a detailed understanding of the performance variations among RSRTC depots over time. These insights play a pivotal role in optimizing depot operations and resource allocation. The findings aid in pinpointing areas of excellence within certain depots, allowing transport authorities to replicate successful practices across the network and improve overall efficiency. Moreover, determining input targets and assessing returns to scale assists in more efficient resource allocation, leading to enhanced operational effectiveness and efficiency across

the RSRTC depots. Understanding growth trends across different depot categories further supports long-term strategic planning, allowing authorities to adapt proactively to evolving transport demands. Additionally, a comparative analysis of depot efficiency facilitates the identification of top-performing depots, fosters healthy competition, and encourages efficiency-enhancing strategies across the entire transport system.

3. **Productivity Evaluation:** Productivity evaluation is a prominent implications for real-life applications in the transport sector. By employing a NSM model with the VRS assumption and utilizing measures such as MPI and LPI, this research enabled transport authorities to assess productivity changes within RSRTC depots over a specified time frame. This analysis is provided crucial insights into the dynamics of productivity shifts from the period 2008-2019. These insights are instrumental in identifying periods of enhanced productivity, determining efficiency fluctuations, and identifying the factors influencing depot performance changes over time. By understanding these productivity trends, transport authorities can gain valuable knowledge for strategic planning and resource allocation. They can identify and replicate successful practices observed during periods of heightened productivity, implement targeted interventions to address declining efficiency, and optimize resource allocation to improve the overall operational effectiveness within the transport system. Ultimately, this research provides decision-makers with actionable insights to enhance productivity, streamline operations, and improve the quality and efficiency of public transportation services for commuters and stakeholders in the transport sector.

4. **Inverse Super-Efficiency DEA Model:** The implementation of the inverse super-efficiency DEA model is offered tangible benefits for real-world decision-making. By ranking the efficient depots and determining precise input-output quantities for predefined efficiency objectives, this analysis is suggested a clear roadmap for optimizing resources. Transport authorities is utilized these insights to allocate resources more effectively, enhance operational strategies, and streamline depot performance. Ultimately, these methodologies is contributed to improving the overall efficiency and effectiveness of transportation services, benefiting commuters, stakeholders, and broader community.

5. **Cross-Efficiency BCC DEA Model:** Introducing a noval model to assess the efficiency of STUs in a fuzzy environment, this research tackles a crucial challenge faced by decision-makers. The model is addressed the missing data and provides a practical framework for conducting more accurate assessments of STUs' operational effectiveness. In real-life scenarios, this meant that transport authorities and policymakers gain a clearer and more precise understanding of how efficiently STUs operate, despite potential uncertainties

in the data. This improved understanding empowers decision-makers to make informed choices in resource allocation, strategic planning, and policy development. It is allowed for targeted interventions to enhance the efficiency of STUs, leading to better transportation services for commuters and stakeholders.

Overall, the application of these methodologies in the transport sector can assist transportation authorities, managers, and policymakers in decisions making to optimize depot performance, streamline operations, and ultimately improve the quality and efficiency of public transportation services. In essence, this research equips transport authorities with actionable insights to streamline operations, allocate resources effectively, plan strategically, and foster continual improvement within the transportation network.

## 1.8 Thesis Organization

This thesis is dedicated to delving into the mathematical intricacies of modeling and introduces an innovative fuzzy DEA model for the purpose of efficiency analysis and productivity measurement. Comprising a total of seven chapters, this thesis embarks on a comprehensive exploration. Chapter 1 serves as an introduction, encompassing key aspects including transportation, diverse DEA models, essential definitions within the realm of DEA, as well as discussions on productivity, the relevance of fuzzy set theory, and the concept of inverse DEA. Additionally, it outlines the research objectives and sheds light on the specific gaps addressed within this thesis.

Chapters 2-6 deal with the main contribution of research work, Chapters 2 to 5 encompass the core of this research, focusing on the analysis of RSRTC depots. Building upon this foundation, Chapter 6 expands the scope to encompass the entire Indian transport sector, specifically addressing state transport undertakings (STUs). Chapter 7 consolidates the findings and contributions, presenting a comprehensive conclusion along with promising avenues for future research. This progression showcases the evolution of the study from a focused examination of RSRTC depots to a broader exploration of the Indian transport landscape.

Chapter 2 is applied a one-of-a-kind combination of MCDM techniques for this objective. This study is to identify and subsequently rank the key criteria that exert a significant influence on performance within the public transport sector. Performance is achieved by applying a hybrid MCDM approach, which combines multiple methodologies to ensure a comprehensive and accurate assessment. This study is provided a clear understanding of which criteria require focused attention and strategic prioritization for optimal performance enhancement. In this regard, sensitivity analysis detects the outliers and determines the robustness of efficiency scores by varying the criteria weights. This proposed model is useful for ranking all depots.

The second objective of the thesis is addressed in Chapter 3 presents the measures of efficiency by establishing a input-oriented NSM DEA model under VRS assumption considering the 52 depots of RSRTC for multi-period. The depots are categorized into three distinct groups for comparative analysis. The study assesses efficiency patterns, establishes input targets, determines the return to scale, and analyzes growth trends across all three categories of depots.

Chapter 4 measures the total factor productivity (TFP) and incorporates the Malmquist productivity index (MPI) and Luenberger productivity index (LPI) using NSM model over a specified time frame (2008–2019). Further, it is evaluated total productivity change in terms of technological change (Frontier shift) and technical efficiency change (Catch-up Effect). The outcomes derived from these models offer a more realistic reflection of real-world scenarios compared to efficiency evaluation. In this context, this approach offers a more robust framework for studying productivity changes over time. This capability allows for capturing the cumulative impact of evolving efficiencies and the dynamic shift of the efficient frontier, against which efficiencies are measured. This nuanced methodology comprehensively explains productivity dynamics in a changing operational landscape.

Chapter 5 delves into the application of the inverse super-efficiency DEA model. This approach serves the purpose of ascertaining the precise quantity of inputs and outputs required to achieve a predefined efficiency objective. By doing so, the aim to validate the ranking of efficient depots through the utilization of both super efficiency and inverse super efficiency DEA models. This comprehensive analysis provides a robust framework for evaluating and comparing the performance of depots, ensuring a more accurate assessment of their efficiency levels. Additionally, it offers valuable insights for decision-makers in optimizing resource allocation and operational strategies to enhance overall efficiency.

Chapter 6 develops the fuzzy cross-efficiency DEA model, which incorporates both self- and peer evaluations, in fuzzy environments is employing a "credibility approach". The main objective of this chapter is to tackle a real-life issue involving missing data. By applying the proposed model, measure the performance of STUs for the fiscal year 2017-18 and subsequently compare the efficiency scores obtained with those derived from the fuzzy cross-efficiency DEA model. This comprehensive analysis provides a robust framework for evaluating and benchmarking the efficiency of STUs, ensuring a more accurate assessment of their operational effectiveness. Additionally, it offers valuable insights for decision-makers in optimizing resource allocation and strategic planning to enhance overall efficiency.

The final chapter encapsulates the primary conclusions drawn from the extensive study conducted throughout this work. Additionally, it provides valuable insights into potential avenues for future research and exploration in this field. This chapter serves as a comprehensive culmination of the findings and a catalyst for further academic endeavors.

# Chapter 2

# A Comparative Analysis of Hybrid MCDM Methods for Performance Assessment and Ranking

In this chapter, a concise yet comprehensive overview of the key performance assessment criteria is offered, employing hybrid multi-criteria decision-making (MCDM) techniques within the context of the sustainable public road transport sector of Rajasthan state.

## 2.1 Introduction

Sustainable transportation is integral to society's economic and social progress. Often referred to as the "lifeblood" of daily commuters, transportation systems are widely acknowledged for their vital role [134]. With the expansion of cities in the $20^{th}$ century, the growth of transportation networks not only drove urban development but also brought about a series of challenges in the pursuit of sustainability. A range of decision-making methods and tools exist to support the research and development (R&D) of transportation systems. The significance of reaping sustainability benefits and effectively utilizing resources in transport systems is widely recognized, leading to substantial investments in R&D. Nevertheless, a pressing concern now arises—how to translate the promise of sustainable transportation into a competitive advantage. To thrive in today's competitive landscape, many organizations acknowledge the strategic importance of benchmarking to enhance performance and cultivate a commitment to gaining a competitive edge [135, 136]. Moreover, operation management and criteria selection are important issues in transportation system development. In many cases, operational management decisions involve multiple aspects and conflicting criteria, making solutions more challenging to achieve. To implement a successful policy or project in the transport sector, it is critical to involve multiple stakeholders in the decision-making process [137].

Based on the above aspects, this chapter addresses the following questions:

[1]This work has appeared in **Goyal, S., Agarwal, S., Singh, NS., Mathur, T., and Mathur, N., 2022. Hybrid MCDM methods for performance assessment and ranking public transport sector: A case study. Sustainability, 14(22), pp. 15110.**

**(a)** Which criteria could be the most effective to evaluate the performance of a depot?

**(b)** Which screened criteria and aspects have the most significance for performance assessment?

**(c)** How the performance of the depot can be improved by aggregating all the attributes of the collected criteria?

**(d)** How ranking results change due to variations in criteria weight?

Therefore, this study builds a hybrid MCDM hierarchy to rank the depots based on the performance score by simultaneously taking into account the operational, passenger, cost, and quality aspects. A hybrid MCDM technique such as fuzzy Delphi method (FDM), fuzzy Analytic Hierarchy Process (FAHP), TOPSIS-VIKOR-ELECTRE approach is used to quantify the performance. Indeed, a hybrid approach, which combines two or more MCDM methods, holds the potential to address intricate decision-making challenges effectively. The purpose of employing the FDM is to discern the most crucial criteria by gathering expert opinions through structured questionnaires. The weights of the criteria must be assigned after they have established using FDM to reflect their relative relevance. FAHP is a pair-wise comparison approach that compares the significance of two criteria. Group decision-making has emerged as a crucial and highly significant aspect of MCDM [138, 139, 140]. A collective of individuals is often better equipped to navigate the complexity of a problem compared to a single decision-maker (DM) [141]. Finally, the study employs TOPSIS-VIKOR-ELECTRE to evaluate and compare the rankings of depots, utilizing the weights determined through FAHP. Additionally, a sensitivity analysis is conducted across various scenarios, involving the adjustment of criteria weights. This research aims to apply an appropriate MCDM analysis that uses experts' opinions to rank the effectiveness of the available public transport system in attracting users away from using private vehicles. MCDM is a technique for determining the best option, ordering, ranking, and sorting the alternatives. Since the 1970s, it is a powerful tool in the fields of decision-making, value judgment, and evaluation. Furthermore, MCDM techniques are commonly used due to their capacity to address complicated problems with inadequate data systems. A detailed insight into the proposed technique is presented in the following sections. The purpose of this study is to answer these questions so as to help Rajasthan's government come up with appropriate solutions for developing a transport system.

The subsequent sections are organized as follows: Section 2.2 lays the foundation by providing an overview of pertinent literature. Following this, Section 2.3 thoroughly explains all the criteria used in the evaluation. Section 2.4 introduces the methodologies employed in detail, namely the fuzzy Delphi method, fuzzy AHP, and TOPSIS-VIKOR-ELECTRE methods. Section 2.5 presents an evaluation of the comparative TOPSIS-VIKOR-ELECTRE results, shedding light on their implications. Additionally, Section 2.6 focuses on sensitivity analysis regarding weight variation, contributing to the methodology's robustness. Finally, Section 2.7 draws conclusions by synthesizing the key findings and their implications. This structured approach ensures a coherent flow of information throughout the research.

## 2.2 Literature Review

This section aims to review the application of MCDM in public transportation.

### 2.2.1 MCDM Methods in Public Transportation

MCDM is a decision-making approach that combines various techniques to help decision-makers to make decisions based on their preferences among two or more criteria [142], rapidly growing in the public transportation problems. According to Perez et al. (2015) [143], 58 distinct MCDM approaches might be employed in private and public urban passenger transportation systems to make essential judgments in evaluating the design and operation of public transportation systems available between 1982 and 2014. As a result, several researchers now use MCDM as one of their primary decision-making tools when assessing the performance of public transit systems [144, 143]. According to Hassan et al. (2013) [145], the services performance of the transport sector is frequently assessed using MCDM methodologies. Macharis et al. (2015) [137] pointed out that MCDM is mostly used in the transport sector's appraisal of the AHP approach.

There are various decision-making methodologies developed by researchers in the given literature. Among the most commonly used MCDM methods is AHP, 276 publications published during the period 1985-2012 are examined, and it is noted that 33% of research works used AHP technique and developed a variation in the transportation sector. Yedla and Shrestha (2003) [146] employed AHP to assess six sustainable ways of transportation. Sezhian et al. (2011) [147] used fuzzy TOPSIS & AHP and ANOVA systematic algorithms for determining the best-performing depot in India. Cafiso et al. (2013) [148] reported several rounds of the Delphi process to improve consensus among the participants for bus safety and suggested that Kendall's test can be used to assess the level of concordance.

Streimikiene et al. (2013) [149] proposed the interval TOPSIS method on the road transport sector. Aydin and Kahraman (2014) [150] focused on a hybrid fuzzy AHP and VIKOR MCDM techniques for selecting public vehicles instead of trams, metros, commuter trains, and bus rapid transit (BRT) systems. Erdogan and Kaya (2016) [151] advocated the utilization of type-2 fuzzy sets to more accurately account for uncertainties in the decision-making process, incorporating three distinct techniques: Delphi, AHP, and TOPSIS. Hawas et al. (2016) [152] employed the TOPSIS approach and K-mean clustering algorithm to build strategies and reliable guidelines to improve public transit accessibility in urban cities. Nassereddine and Eskandari (2017) [153] applied an integrated MCDM approach, combining Delphi, GAHP, and PROMETHEE, in the evaluation of public transportation systems in Tehran. Avenali et al. (2018) [154] used a hybrid cost model that combines a bottom-up and a top-down method to calculate unit standard costs for the Italian local public bus transportation industry. Demirel et al. (2018) [155] and Dudek et al. (2018) [156] suggested AHP, fuzzy AHP, and ELECTRE to evaluate the selection of public transport. Guner (2018) [157] presented a two-stage AHP-TOPSIS technique. Jamshidi (2018) [158] assessed the efficiency and ranked the criteria that influence passenger satisfaction through the two-stage Delphi method in the road transport industry. Kiciński and Solecka (2018) [159] evaluated various scenarios using multiple criteria for the enhancement of the urban public transportation system in the city of Cracow. Melander et al. (2019) [160] conducted a Delphi survey involving experts from academia, industry, and government, which unveiled a diverse and multi-dimensional vision of future developments in goods transport in Sweden for the year 2050. Jasti and Ram (2019) [161] developed an integrated and sustainable framework by utilizing MCDM methods such as AHP and direct weighting for the urban bus system of Hyderabad. Moslem and Duleba (2019) [162] utilized a fuzzy AHP model in the context of sustainable decision-making for public transport development issues. Sekar and Aydin (2020) [163] integrated two MCDM methods, namely interval-valued intuitionistic fuzzy analytical hierarchy process (IVIF-AHP) and COmbinative Distance-based Assessment (CODAS), to assess the quality of public transportation services. Karam et al. (2021) [164] used a hybrid analytical method that combined meta-synthesis, The fuzzy Delphi method (FDM), and AHP to improve the sustainability of the transportation industry. The performance of the metropolitan public transportation system is analyzed using a fuzzy multi-criteria analysis approach (MCAA) [165]. Bouraima et al. (2023) [166] introduced an enhanced fuzzy step-wise weight assessment ratio analysis (SWARA) method, incorporating both the Bonferroni operator and Measurement of Alternatives and Ranking according to the Compromise Solution (MARCOS) techniques within a unified methodological framework for evaluating the BRT system in Tanzania. No approach is better than the others; each has its own set of benefits and drawbacks as well as application areas. The same multi-criteria decision problem may be solved using multiple techniques to produce more reliable decision information. Each approach has its own set of benefits and drawbacks as well as application regions; no approach is superior to the others. The same multi-criteria decision problem may be solved by more than one technique, resulting in more reliable decision data [167].

The aforementioned literature review highlighted the importance of MCDM techniques in the assessment of the transport sector. However, the suitable criteria for performance assessment depend on the problem statement. Further, studies on proposing a metric to determine performance are limited.

## 2.3 Evaluation Criteria

The selection of appropriate criteria for performance measurement is pivotal as it forms the foundation of the evaluation process and serves as a guide for improvements across various aspects of the sector. Drawing on extensive prior research and aligning with prevalent trends in the transportation sector, 29 critical criteria is meticulously identified. These encompass a broad spectrum of factors and are categorically organized into four key criteria: operational service, service quality, passenger service, and cost effects, as shown in table 2.1. This table offers a comprehensive overview of the essential metrics used in the evaluation.

All criteria are explained briefly as follows:

### 2.3.1 Operational Service: Ensuring Smooth Operations

Operational service focuses on various critical metrics that collectively evaluate the efficiency and effectiveness of the depot's daily operations, ensuring that services run smoothly and on schedule. Efficiently managing the workforce is crucial for overall operational success as it affects service quality, maintenance, and administrative functions. This, in turn, can potentially increase the depot's ability to serve passengers, reduce waiting times, and accommodate peak-hour demands [168]. Monitoring the services helps assess whether the depot is meeting the demand effectively. Similarly, it's important to strike a balance to avoid overcapacity and operational inefficiencies.

- ***Total vehicles***: This criterion quantifies the total number of vehicles in the depot's fleet.
- ***Schedule vehicles***: These are vehicles that are officially assigned to specific routes and schedules. Monitoring the number of scheduled vehicles ensures that the depot has adequate resources to maintain regular service operations.
- ***Operated vehicles***: This criterion assesses the number of vehicles in operation on a given day. It provides insights into the depot's ability to deploy its fleet for service effectively.
- **Off-road vehicles**: These are vehicles that are temporarily out of service due to maintenance, repairs, or other reasons. Tracking the number of off-road vehicles is crucial for maintaining an efficient and reliable fleet.

- ***Operated trips***: This criterion measures the total number of trips successfully completed by the depot's vehicles within a specific period. It provides valuable insights into the depot's fleet's operational efficiency and capacity utilization.

- ***Extra trips***: This refers to additional services that are provided beyond the regular schedule to accommodate unexpected passenger demand or special events. Monitoring extra trips helps assess the depot's flexibility and responsiveness to dynamic passenger needs.

- ***Curtailed trips***: This occurs when a scheduled trip is terminated before reaching its final destination, often due to unforeseen circumstances or operational reasons. Tracking curtailed trips is important for identifying areas where service disruptions may occur and implementing corrective measures.

- ***Total no. of employees***: This criterion quantifies the overall workforce employed by the depot, including both operational and administrative staff. It provides an indication of the depot's organizational size and capacity to manage day-to-day operations.

- ***Number of routes***: This criterion represents the total count of distinct routes served by the depot. It indicates the breadth of the depot's transportation network.

- ***Routes distance***: This criterion measures the combined length of all routes serviced by the depot, reflecting the extent of geographical coverage.

### 2.3.2 Service Quality: Enhancing Passenger Experience

Service quality is the heart of efficient public transportation systems. Transit service quality influences the commuter's choice [169]. Commuter attitudes toward transport services are very important when it comes to improving demand for the services [170]. Therefore, it is highly essential to identify the lagging service parameters to increase commuter satisfaction and revenue for the sustainable transport system. It encompasses factors such as punctuality, reliability, passenger safety, and overall customer satisfaction. Assessing service quality provides insights into the ability of depots to meet the demands and expectations of passengers. Choosing service quality as a criterion is paramount:

- ***Rate of breakdown***: This criterion quantifies the frequency of breakdowns or malfunctions experienced by the depot's vehicles. A lower number of breakdowns indicates a more reliable and well-maintained fleet, ultimately contributing to improved service quality and passenger satisfaction.

- ***Rate of accidents***: Safety is an important concern in public transportation. Monitoring the number of accidents provides crucial insights into the effectiveness of safety measures and the overall security of passengers and vehicles.

- ***Punctuality***: Timeliness is a fundamental aspect of service quality. Punctuality ensures that passengers can rely on the service schedule, leading to increased trust and usage of public transport.

- ***Fleet utilization***: This criterion evaluates the optimal use of the depot's fleet. Efficient fleet utilization indicates a well-managed operation that can cater to passenger demand effectively.

- ***Vehicle utilization***: This criterion focuses on the efficiency of individual vehicles. Maximizing vehicle usage ensures cost-effectiveness and meets passenger demands efficiently.

- ***Tyre efficiency***: The condition and efficiency of tires are crucial for ensuring passenger safety and maintaining operational efficiency. Regular monitoring and maintenance of tires contribute to overall service quality.

### 2.3.3 Passenger Service: Meeting Traveler Needs

Passenger service criteria encompass several essential metrics that collectively gauge the effectiveness of a depot's transportation services with a strong focus on passenger experience. This metric provides a nuanced understanding of service coverage and frequency. Collectively, these criteria provide a comprehensive view of how effectively the depot caters to passenger demands, taking into account both quantity and quality of service, thus they are playing a pivotal role in evaluating the overall performance of the depot.

- ***Number of passengers***: The volume of transportation activity within a specific year stands as a key indicator, mirroring the comprehensive demand and utilization of the depot's services.

- ***Passenger km occupied***: It quantifies the cumulative distance traveled by each passenger, offering insights into the extent of journeys facilitated by the depot.

- ***Description of kilometers***: This criterion is calculated as the total kilometers operated during a specific period, divided by the total number of buses on the road in that period, and then divided by the number of days within the same period.

- ***Load factor***: It represents the percentage of total passenger kilometers relative to the total carrying capacity and assesses the efficiency of passenger occupancy.

### 2.3.4 Cost Effects: Analyzing Financial Aspects

Cost-effect criteria examine the financial aspects of depot operations, considering cost-saving measures and resource allocation strategies. A comprehensive evaluation of these criteria enables a complete understanding of the depot's financial performance. Consequently, this understanding empowers precise, well-informed decision-making and facilitates the implementation of targeted improvements. Key financial criteria include:

- ***Income per seat per km (in lakh)***: This metric is calculated by dividing the total income by the product of the average number of seats in a bus and the average kilometers traveled by buses of the depot. It helps us to understand the income generated per unit of capacity and distance traveled.

- ***Total income per km***: This criterion measures the total income generated by the depot per km traveled. It provides insights into the revenue earned for each unit of distance covered.

- ***Operating income (in lakh)***: This criterion represents the total operating income, a key indicator of the depot's financial health.

- ***Operating income per km***: Operating income, also known as operating earnings, is divided by the kilometers traveled to determine the operating income per km. This metric reflects the financial performance of the depot in relation to its operational activities.

- ***Income per vehicle per day***: This criterion assesses the income generated per bus per day, providing insights into the daily revenue generation efficiency.

- ***Total expenditure per km***: Total expenditure is divided by kilometers traveled to calculate the total expenditure per km. This metric helps us understand the cost implications of the depot's operations on a per-km basis.

- ***Profit/loss per km***: It is calculated as the difference between total income per km and total expenditure per km. This metric indicates whether the depot is operating at a profit or loss on a per km basis.

- ***Consumption rate of diesel and oil***: This set of criteria examines fuel consumption efficiency, including diesel consumption per km per liter and engine oil top-up per km per liter. These metrics help evaluate the cost-effectiveness of fuel usage in depot operations.

**Table 2.1:** Definitions of criteria

| Category | Criteria | Description |
|---|---|---|
| Operational Service | Total vehicles | The number of vehicles held by a depot input. |
| | Scheduled vehicles | Total number of vehicles that are pre-assigned to a depot for that year. |
| | Operated vehicles | Total number of vehicles that actually operated for a depot for that year. |
| | Off-road vehicles | Total vehicles out of operated vehicles that remained away from operation for a depot. |
| | Scheduled trips | Total count of trips scheduled for a depot for that year. |
| | Operating trips | Total trips actually operated in a year. |
| | Extra trips | Unscheduled trips that operated in a year. |
| | Curtailed trips | Total count of cancelled trips . |
| | Total no. of employees | The number of employees in a depot which is indicative of labor input. |
| | No. of routes | The number of routes which is described as network size. |
| | Routes distance | The route distance is described as the total km traveled by a passenger. |
| Service Quality | Rate of break down | This indicator (is a measure of the mechanical reliability of a fleet) expressed in terms of the number of breakdowns per 10,000 kilometers. |
| | Rate of accident | This indicator is defined as the number of accidents per 100,000 kilometers. |
| | Punctuality | Percentage of scheduled trips that departed depot at their scheduled time. |
| | | Percentage of scheduled trips that arrived depot at their scheduled time. |
| | Fleet utilization | Fleet utilization is the percentage of the number of buses on the road to buses held by the depots. |
| | Vehicle utilization | Vehicle utilization is the total kilometers traveled by bus per day. |
| | Tyre efficiency | Ratio of km traveled to maximum km possible tire. |
| Passenger Service | Number of passengers | Total number of passenger traveled in a year. |
| | Passenger km occupied | Passenger km occupied is the cumulative distance traveled by each passenger. |
| | Description of km | Total kl operated during a period, divided by the total number of buses in that particular period and then divided by the number of days in the period . |
| | Load factor | Percentage of total passenger kilometers to total carrying capacity. |
| Cost Effects | Income per seat per km (in lakh) | Total income divided by (average number of seats in a bus * kilometers traveled). |
| | Total income per km | Total income divided by kilometers traveled. |
| | Operating income(in lakh) | Operating income, also referred to as operating earnings. |
| | Operating income per km | Total operating income divided by kilometers traveled . |
| | Income per vehicle per day | Income divided by total buses per day. |
| | Total expenditure per km | Total expenditure divided by kilometers traveled. |
| | Profit/ loss per km | Total income per km-total expenditure per km. |
| | Consumption rate of diesel and oil | Diesel consumption km per liter. |
| | | Engine oil top up km per liter. |
| | | Engine oil consumption per thousand km. |

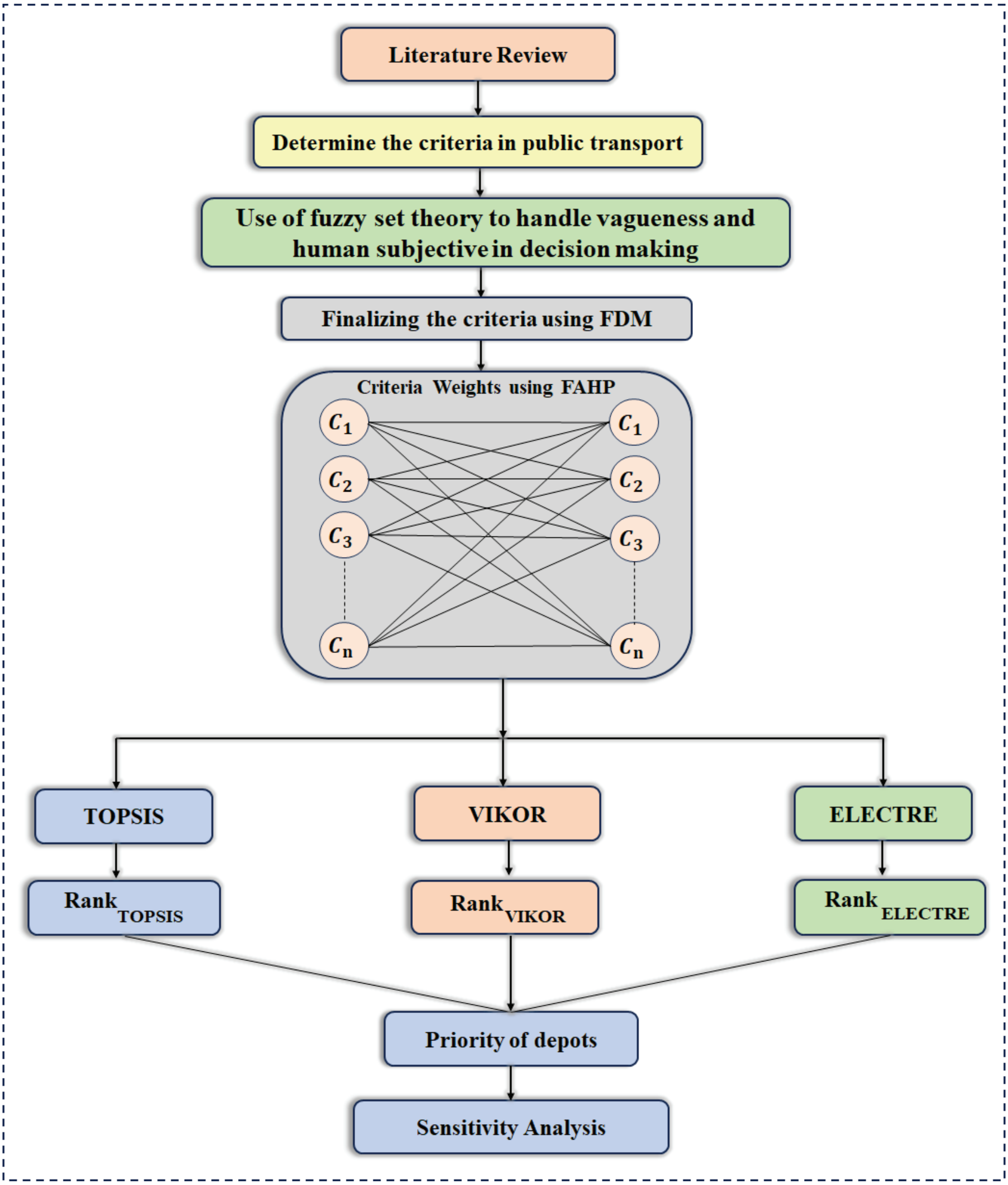


**Fig. 2.1:** Schematic illustration of the proposed framework.

## 2.4 Proposed Methodology

This section outlines a three-phase research process centered around hybrid techniques such as FDM, FAHP, TOPSIS-VIKOR-ELECTRE. Firstly, using FDM method to develop the the decision-making problem's structure and identify significant criteria. Following this, the FAHP method is applied to select the criteria weights. Finally, the TOPSIS-VIKOR-ELECTRE approach is employed to aggregate the criteria, rank the depots under evaluation, and conduct a sensitivity analysis. This section provides a comprehensive overview of the research methodology. Figure 2.1 illustrates the steps involved in the proposed methodology.

### 2.4.1 Determining the Linguistic Terms

Assigning precise numerical values to criteria and alternatives can be challenging in real-world scenarios. Diverse experts may hold varying perspectives and preferences. Fuzzy MCDM addresses this by enabling the integration of multiple opinions and offering a method to consolidate them. To tackle this problem, Zadeh (1965) [78] first proposed the fuzzy set theory to deal with the impreciseness and subjectivity unpredictability of human judgments. Their flexibility in handling uncertainty makes them a crucial mathematical tool in various real-world applications [171]. Fuzzy set theory plays a vital role in translating qualitative linguistic expressions like 'good,' 'very good,' 'poor,' and 'very poor' into quantifiable fuzzy numbers [144]. By leveraging fuzzy set theory, these linguistic terms effectively manage uncertainties and represent imperfect information. A linguistic term conveys information using words or sentences from a natural or artificial language rather than precise numerical values [172]. This approach is indispensable for accurately representing and analyzing information in a fuzzy environment. In addition, this study makes use of seven linguistic scales that are frequently used in MCDM problem-solving. Triangular fuzzy numbers (TFNs) are employed to represent the linguistic term, illustrated in table 2.2.

**Table 2.2:** Linguistic terms and corresponding TFNs for the significance criteria weight.

| **Linguistics term** | **Corresponding TFNs** |
|---|---|
| Very High Importance | (0.9, 1, 1) |
| High Importance | (0.7, 0.9, 1) |
| Medium High Important | (0.5, 0.7, 0.9) |
| Medium Importance | (0.3, 0.5, 0.7) |
| Medium Low Importance | (0.1, 0.3, 0.5) |
| Low Importance | (0, 0.1, 0.3) |
| Very Low Importance | (0, 0, 0.1) |

### 2.4.2 Fuzzy Delphi Method

In 1950s, Dalkey and Helmer (1963) [173] proposed the Delphi method at the Rand Corporation [174]. The Delphi method is widely employed in many management decision-making, prediction, analysis of public policy, and project organizing to acquire the most accurate judgment among a group of experts. Furthermore, this method has proven to be the most effective in detecting the trend of an enduring criterion. When investigating distributed group decisions, there is no apparent solution to a policy issue [175]. On the contrary, Delphi approach allows for the full integration of multiple expert opinions, and it is time-consuming and expensive. Additionally, it's important to acknowledge that even a small group of experts may face limitations in addressing all pertinent issues. Moreover, due to its iterative nature in seeking convergent answers through repeated surveys, the Delphi approach may experience a lower rate of questionnaire return [176].

Murray et al. (1985) [177] introduced the fuzzy Delphi method (FDM) by combining the Delphi technique with fuzzy set theory to address impreciseness and vagueness inherent [178]. By introducing a fuzzy set theory-based approach, the FDM allows for the representation of imprecise or subjective information, providing a more nuanced understanding of expert opinions. The FDM yields results in an objective manner and eliminates the need for multiple survey rounds, allowing for conclusions to be reached in a single round [179]. Hsu et al. (2000) [180] utilized TFN to incorporate expert advice and implement the FDM. It provides a useful extension to help address these issues and improve the accuracy of expert-based decision making. To pick the most effective criteria, this study used the FDM method.

The details of the procedure for the FDM method using these five steps are described below.

**Step 1:** ***Determine criteria***

This study uses a comprehensive literature review and the conceptual framework to determine the 29 criteria. The several literature-based criteria for performance measures in the transport sector are listed in a tabular format (table 2.1).

**Step 2:** ***Collect expert judgements***

A panel of five distinguished experts from academia has carefully selected to participate in the questionnaire. Each expert is asked to provide judgments on the importance of criteria using a linguistic scale, which includes categories such as 'very high importance,' 'high importance,' 'medium high importance, 'medium importance,' 'medium low importance, 'low importance,' and 'very low importance.' To facilitate the evaluation, the criteria are organized into four distinct sections: operational service, passenger service, cost effects, and quality. This structured approach ensures a comprehensive assessment of the various facets of the subject matter.

**Step 3:** ***Establish the expert opinion into triangular fuzzy numbers***

The approach transforms linguistic assessments into TFNs. The TFN is used to consider the fuzziness in the judgments made by experts. In this study, linguistic criteria are chosen to analyze the relevance of each criterion based on the table 2.2. In the instance where an expert indicates "very high importance," this corresponds to the TFN values of (0.9, 1, 1). The current study applies Klir and Yuan (1995) [80] geometric mean model to determine the consensus of a group decision as assessed by the experts.

Assume that, the fuzzy number represents opinion of the $i^{th}$ expert of n experts $\tilde{Z}_{ij} = (a_{ij}, b_{ij}, c_{ij}), \ \forall\, i = 1, 2, \cdots, n, j = 1, 2, \cdots, m$ where m is the number of criteria.

First, computes the fuzzy weights of criteria $\tilde{A}_j = (a_j, b_j, c_j)$, $\forall\ j = 1, 2, \cdots, m$ as defined in given equations,

$$a_j = \min_i a_{ij} \qquad j = 1, 2, \cdots, m \tag{2.1}$$

$$b_j = \left( \prod_{i=1}^{n} b_{ij} \right)^{1/n} \qquad j = 1, 2, \cdots, m \tag{2.2}$$

$$c_j = \max_i c_{ij} \qquad j = 1, 2, \cdots, m \tag{2.3}$$

where, $a_j$ represents lower value, $b_j$ denotes geometric mean and $c_j$ stands for the highest value within the fuzzy numbers. The indices $i$ and $j$ represent the number of experts and criteria, respectively.

**Step 4:** ***Defuzzify the data***

Defuzzification can be accomplished using a variety of complex approaches. The mean approach is one of the simplest and is defined by equation (2.4),

$$M_j = \frac{(a_j + b_j + c_j)}{3}, \qquad j = 1, 2, \cdots, m \tag{2.4}$$

Hence, defuzzified number $M_j$ quantifies the collective judgment of all experts based on the effectiveness of criteria.

**Step 5:** ***Identification of essential criteria***

Finally, by specifying a "$r$" threshold, the appropriate criteria are screened out of a large number of criteria. The screening criteria are as given below:

(a) If $M_j \geq r$, then add $j^{th}$ criteria in evaluation index.

(b) If $M_j < r$, then omit $j^{th}$ criteria from the list.

The threshold of $r = 0.6$ is chosen for consideration as an evaluation criterion. The next round is selected if the total number of criteria is higher than or equal to 0.6. Otherwise, it is discarded.

**Table 2.3:** TFN of linguistic terms used in this study.

| Linguistic Scale | Crisp Scale | FAHP Scale |
|---|---|---|
| Equally important | 1 | (1,1,1) |
| Weakly important | 3 | (2,3,4) |
| Fairly important | 5 | (4,5,6) |
| Strongly important | 7 | (6,7,8) |
| Absolutely important | 9 | (9,9,9) |
| Interpolation scale | 2 | (1,2,3) |
| | 4 | (3,4,5) |
| | 6 | (5,6,7) |
| | 8 | (7,8,9) |

### 2.4.3 FAHP Method

Analytic hierarchy process (AHP), first introduced by Saaty [181] in 1977, is a widely adopted methodology for addressing complex systems and making decisions among multiple options. Professionals and academics heavily rely on AHP across various engineering and management domains [182]. The core objective of the AHP technique is to decompose a problem into smaller sub-problems organized in a hierarchical structure [183]. In conventional AHP, a nine-point scale is employed for conducting pairwise comparisons between categories and criteria. However, this discrete scale can be problematic, particularly in handling uncertainty and vagueness in expert judgments [184].

FAHP adds fuzzy logic theory to the AHP technique to deal with the imprecision of expert assessments. Various authors have presented several modifications to the FAHP approach and applications in the literature. The first study that employed the fuzzy set theory to AHP with fuzzy triangular numbers is suggested by Van and Predryez (1983) [185]. Buckley (1985) [186] pioneered the application of trapezoidal fuzzy numbers to represent the decision maker's evaluation of alternatives for each criterion. Change (1996) [187] introduced a new approach to dealing with FAHP. This study applied fuzzy triangular numbers for the pairwise comparison scale based on FAHP [187], which is easier to compute than other FAHP approaches. Assigning a TFN to each linguistic scale, as summarized in table 2.3.

The FAHP approach consists of the following five steps:

**Step 1:** ***Hierarchy structure of the criteria weight***

The goal is to identify and rank the criteria for improving public transport performance. This investigation encompasses three levels within the hierarchical framework. At the top level lies the overarching goal of the study. In the middle layer, specific categories are identified.

Finally, all the criteria pertinent to the public transport system are housed in the bottom layer. These criteria are derived through the application of the FDM method, a specialized approach tailored to handle imprecise data and expert opinions.

**Step 2:** ***Representation of the relative importance for pairwise comparison***

The study employs the interval consideration approach to evaluate the range of ratings provided by each expert. Within this framework, experts utilize a fuzzy pairwise comparison matrix to express their assessments in linguistic terms, leveraging their expertise and experience to determine the relative value of one criterion in comparison to another. Various methods exist for aggregating expert judgments, including the average method, geometric mean method, interval consideration technique, and others. For this study, the interval consideration approach is specifically chosen to gauge the spectrum of rankings assigned by experts. TFNs are used for consolidating expert rankings. The following expressions elucidate the calculations involved in the ratings provided by different experts.

$$\tilde{x_{ij}} = (l_{ij}, m_{ij}, n_{ij}) \quad \text{where} \quad i = 1,2,\cdots,n, \;\; j = 1,2,\cdots,m, \;\; e = 1,2,\cdots,E \tag{2.5}$$

$$l_{ij} = \min_{e}(l_{ije}),$$

$$m_{ij} = \left(\prod_{e=1} m_{ije}\right)^{\frac{1}{n}}$$

$$n_{ij} = \max_{e}(n_{ije})$$

where $n$ and $m$ represent the number of rows and columns, respectively, and $E$ signifies the number of experts.

**Step 3:** ***Fuzzy weight determination***

$$M^1_{G_i}, M^2_{G_i}, \cdots, M^m_{G_i}, \quad i = 1,2,\cdots,n \tag{2.6}$$

The fuzzy synthetic extent value for the $i^{th}$ object is determined as follows in equation (2.7).

$$S_i = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} n_j)\right) \odot \left(\frac{1}{\sum_{i=1}^{n} n_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i}\right) \tag{2.7}$$

**Step 4:** ***Degree of possibility***

The degree of possibilities of $M_1 = (l_1, m_1, n_1) \leq M_2 = (l_2, m_2, n_2)$ is interpreted as:

$$V(M_1 \leq M_2) = hgt(M_1 \cap M_2) \tag{2.8}$$

$$= \begin{cases} 1, & \text{if } m_1 \leq m_2 \\ \frac{n_1 - l_2}{(n_1 - m_1) + (m_2 - l_2)}, & \text{if } l_2 \geq n_1 \\ 0, & \text{otherwise} \end{cases} \tag{2.9}$$

To compare $M_1$ and $M_2$ both the values of V($M_1 \leq M_2$) and V($M_2 \leq M_1$). The degree of possibility $V(M_1 \geq M_2, M_3, ....., M_e)$ for a convex fuzzy numbers $M$ and $M_i$ $(i = 1, 2, \cdots, e)$ can be defined by:

$$\begin{aligned} V(M \geq M_1, M_2, \cdots, M_e) &= V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \cdots \text{ and } (M \geq M_e)] \\ &= \min[V(M \geq M_i)], \quad i = 1, 2, 3, \cdots, e. \end{aligned} \tag{2.10}$$

Consider that,

$$d^{'}(A_i) = [\ \min[V(M_1 \geq M_e)],\ \min[V(M_2 \geq M_e)], ......., \ \min[V(M_n \geq M_e)]], i = 1, 2, ...., n; e \neq i \tag{2.11}$$

The weight vector for $(i = 1, 2, ...., n)$ object can be calculated as follows:

$$W^{'} = (d^{'}(A_1), d^{'}(A_2), ....., d^{'}(A_n))^T, \tag{2.12}$$

where, normalized weighted vector $W$ is a non-fuzzy number.

$$W = (d(A_1), d(A_2), ....., d(A_n))^T \tag{2.13}$$

**Step 5:** ***Consistency measurement of the judgments***

The significance of priorities lies in their foundation on consistent matrices. Consistency ensures that pairwise comparisons are guided by logic rather than randomness. The consistency index ($CI_k$) is determined using the equation derived from the comprehensive eigenvalue method ($\lambda_{max}$), as introduced by Saaty (2004) [188]. To maintain consistency, the value of the consistency ratio ($CR_k$) should ideally be less than 0.1 for the weights to be considered reliable. If the ratio exceeds this threshold, it is recommended to re-evaluate the corresponding weights to rectify any inconsistencies.

$$CI_k = \frac{\lambda_{max} - n}{n - 1} \tag{2.14}$$

$$CR_k = \frac{CI_k}{RI_k} \tag{2.15}$$

where random index ($RI_k$) differs for each matrix size $n$. Table 2.4 is used to calculate the $n$ size consistency index matrix of a randomly generated pairwise comparison.

**Table 2.4:** Random consistency index.

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| RI | 0 | 0 | 0.58 | 0.89 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

### 2.4.4 Ranking Methods

This section provides a concise overview of three prominent methods widely used for ranking:

- TOPSIS
- VIKOR
- ELECTRE

#### 2.4.4.1 TOPSIS Method

Hwang and Yoon (1981) [189] offered a method technique for ranking the alternative across several criteria, order of preference by resemblance to the ideal solution (TOPSIS). According to Zavadskas et al. (2016) [190], the TOPSIS method is the second most famous MCDM method. The most desirable outcome must be the furthest away from both the positive and negative ideal solutions for the successful TOPSIS strategy [191]. In contrast to the negative ideal solution, which maximizes cost criteria at the expense of benefit criteria, the positive ideal solution maximizes benefit criteria while minimizing cost criteria. The distances to the ideal solutions, both positive and negative, are computed simultaneously. Based on their comparative closeness and the sum of these two distance values, a preference ranking is created by Yue (2011) [192].

The method is applied in this experiment through seven steps, which are as follows:

**Step 1:** ***Normalized decision matrix***

The decision matrix is to be determined, where $i = 1,2,...,n$ denotes the alternatives and $j = 1,2,...,m$ represents the criteria. $x_{ij}$ refers to the $j^{th}$ criterion associated to the $i^{th}$ alternative and represented as follows:

$$X_{ij} = \begin{bmatrix} 1 & \tilde{r}_{12} & \tilde{r}_{13} & \dots & \tilde{r}_{1m} \\ \tilde{r}_{21} & 1 & \tilde{r}_{23} & \dots & \tilde{r}_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{r}_{n1} & \tilde{r}_{n2} & \tilde{r}_{n3} & \dots & 1 \end{bmatrix}$$

**Step 2:** ***Vector normalized decision matrix***
The decision matrix is $x_{ij}$ "normalized" by translating different scales and units among different criteria into a common measurable unit to allow comparisons between the criteria. In order to create a matrix whose element $P = [p_{ij}]_{m\times n}$ with $i = 1,2,\cdots,n; j = 1,2,.....m$ is calculated by,

$$p_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}} \qquad i = 1,2,.....n; j = 1,2,.....m \tag{2.16}$$

The matrix $P$ represents the relative rating of the alternatives.

**Step 3:** ***Weighted normalized decision matrix ($V_{ij}$)***
The normalized matrix is computed by multiplying the columns of the normalized matrix $p_{ij}$ with associated weights $w_j \in [0,1]$ :

$$V_{ij} = (w_i.p_{ij}) \quad \forall\, i, j \tag{2.17}$$

where $\sum_{i=1}^{n} w_i = 1$

**Step 4:** ***Positive and negative ideal solutions***
The best preferable option is represented by the positive ideal solution (PIS), denoted as $[S^+]$, while the worst preferable alternative is indicated by the negative ideal solution (NIS), denoted as $[S^-]$.

$$S^+ = \{V_1^+,\, V_2^+, ..V_j^+ .., V_n^+\} = \{(max_j\, V_{ij} \mid j \in J), (min_j\, V_{ij} \mid j \in J') | j = 1,2,......,m\} \tag{2.18}$$

$$S^- = \{V_1^-,\, V_2^-, ...V_j^- .., V_n^-\} = \{(min_j\, V_{ij} \mid j \in J), (max_j V_{ij} \mid j \in J') |\, j = 1,2,......,m\} \tag{2.19}$$

**Step 5:** ***Euclidean distance measure***
The measure is to calculate Euclidean distance $[D_i^+, D_i^-]$ from PIS $[S^+]$ and NIS $[S^-]$ to each component from the ideal $(V_j^+)$ and non-ideal alternatives $(V_j^-)$, where:

$$D_i^+ = \sqrt{\sum_{i=1}^{n} (V_{ij} - V_i^+)^2}, i = 1,2,......,n;\ 0 < D_i^+ < 1 \tag{2.20}$$

$$D_i^- = \sqrt{\sum_{i=1}^{n} (V_{ij} - V_i^-)^2}, i = 1,2,......,n;\ 0 < D_i^- < 1 \tag{2.21}$$

**Step 6:** ***Relative closeness coefficient to the ideal solution***
The relative closeness coefficient $(\xi_i^*)$ is used to define the $i^{th}$ alternative $(V_i)$ to an ideal solution

$S_i$,

$$\xi_i^* = \frac{D_i^-}{(D_i^+ + D_i^-)}; 0 < \xi < 1 \tag{2.22}$$

where the higher value of $\xi_i^*$ represents the best performance.

**Step 7:** ***Priority Ranking of the alternatives***
The preferences of a group of alternatives can be arranged in descending order of $\xi_i^*$.

#### 2.4.4.2 VIKOR Method

The VIekriterijumsko KOmpromisno Rangiranje (VIKOR) technique is created to address complex MCDM issues, including several attributes with divergent and incompatible criteria (non-commensurable units). The VIKOR method is introduced by Opricovi [193] in 1998. As a planned tool, VIKOR's distinctive structure is employed when decision experts are unable to adequately communicate their preferences during the system design phase. This approach offered the decision-maker a compromise ranking of attributes based on the closest to the "ideal" solution using the initial weights of a problem with competing criteria [194]. Any attribute that is added or removed could affect the results of the VIKOR ranking. For both the opponent and the majority, this tactic preserved minimal personal regret and maximum group usefulness [195].

On the other hand, the TOPSIS technique does not account for the relative distances between ideal solutions. In contrast to TOPSIS, the relative closeness is not always near to the ideal values, VIKOR's aggregate function is always closest to the best solutions. The VIKOR technique is used to get around this restriction. There aren't many research studies in the literature that address the numerous VIKOR application domains. For a number of case studies, the revised VIKOR approach is suggested by Rao (2008) [196].

The five steps that follow are an explanation of the mathematical algorithm VIKOR computations:

**Step1:** ***Normalize decision matrix***
The goal of normalization is to standardize the matrix entry unit. The numerical attributes $j$ on each criterion $i$ are used to determine the normalized values of the attributes $a_{ij}$. The following definition applies to the appropriate normalized value $r_{ij}$.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{n} {a_{ij}}^2}} \quad i = 1, 2, \cdots, n; j = 1, 2, \cdots, m \tag{2.23}$$

**Step2:** ***Ideal solutions***
The positive-ideal solution (PIS) $f_i^+$ and negative ideal solution (NIS) $f_i^-$ values of all criterion

$i = 1,2,\cdots,n$ and $j = 1,2,3,\cdots,m$,

$$f_i^+ = \begin{cases} \max_i f_{ij} & \text{for benefit criterion} \\ \min_i f_{ij} & \text{for cost criterion} \end{cases} \tag{2.24}$$

$$f_i^- = \begin{cases} \min_i f_{ij} & \text{for benefit criterion} \\ \max_i f_{ij} & \text{for cost criterion} \end{cases} \tag{2.25}$$

where $j$ represents the number of alternatives, $n$ represents the number of criteria and $f_{ij}$ represents the rating of the $i^{th}$ criterion.

**Step 3:** ***The values of $D_j$ and $R_j$***

The utility ($D_j$) and regret ($R_j$) measure for each attributes are given below:

$$D_j = \sum_{i=1}^{n} \frac{w_i(f_i^+ - f_{ij})}{(f_i^+ - f_i^-)} \tag{2.26}$$

$$R_j = \max_i \frac{w_i(f_i^+ - f_{ij})}{(f_i^+ - f_i^-)} \tag{2.27}$$

where $w_i$ is the weight of $i^{th}$ criteria.

**Step 4:** ***Compute the value of $Q_j$***

$$Q_j = \frac{w(D_j - D^+)}{(D^- - D^+)} + \frac{(1-w)(R_j - R^+)}{(R^- - R^+)} \tag{2.28}$$

$D^+ = \min_j D_j$, $D^- = \max_j D_j$, $R^+ = \min_j R_j$, $R^- = \max_j R_j$ .

where the solution obtained by $D^+$ and $R^+$ correspond to the maximum group of utility and the opponent's minimum individual loss, respectively, and $w$ = 0.5 is supplied as a weight for the approach of the "majority of criteria". However, $w$ is capable of setting any value between 0 and 1.

**Step 5:** ***Rank the order of preference***

Calculate the rank of the alternatives by the given ranking index ($Q_j$) in decreasing order.

#### 2.4.4.3 ELECTRE-I Method

According to Benayoun (1996) [197], ELECTRE-I (ELimination and ChoiceExpressingREality -I) is one of the most popular approaches for representing a decision-maker's preferences across a variety of areas. In addition to ELECTRE-I, several alternative approaches such as ELECTRE-IV, ELECTRE-II, ELECTRE-TRI, ELECTRE TRI-C, and ELECTRE TRI-N have emerged from the ELECTRE. Bojkovic et al. (2010) [198] used ELECTRE-I to examine the performance of transportation systems in relation to sustainability development challenges. To reduce the subjectivity of the decision-maker, they offered a variation of the ELECTRE approach. Karacasu and Arslan (2010) [199] used the ELECTRE approach to compare two different public bus networks, one run by the local government and the other by private businesses. However, ELECTRE-I is unable to calculate the ranking of attributes. ELECTRE-II is proposed to address the flaw in ELECTRE-I and create a ranking of alternatives. A AHP-based ELECTRE-I to optimal design approach is proposed [200]. This study shows that AHP-based ELECTRE-I models may react effectively when competing criteria are present, and they are particularly useful for making decisions that call for widespread agreement.

This study's methodology is divided into the following eight steps:

**Step 1:*Normalized decision matrix***

The normalization of the assessment matrix is the process of converting various scales and units across several criteria into common measurable units to enable comparisons across the criteria. To achieve this, a variety of normalized processes is employed to construct an element $r_{ij}$ of the normalizing evaluation matrix $R$ if $f_{ij}$ is the evaluation matrix $R$ of alternative $j$ under evaluation criterion $i$.

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{j=1}^{m} a_{ij}^2}} \quad i = 1,2,\cdots,n; j = 1,2,\cdots,m \tag{2.29}$$

**Step 2: *Weighted normalized decision matrix* $(V_{ij})$**

To produce the weighted normalized decision matrix, multiply the normalized matrix $r_{ij}$ with its associated weight $w_i$.

$$V_{ij} = w_i * r_{ij} \quad i = 1,2,3,\cdots,n, \quad j = 1,2,3,\cdots,m. \tag{2.30}$$

where $\sum_i^n w_i = 1^n = 1$

**Step 3: *Ascertainment of concordance ($C_{pq}$) and discordance ($D_{pq}$) sets***

Let $A_i = \{p,q,r,\cdots\}$ indicates a finite set of attributes. In the following formulation, the attribute sets are divided into two different sets: $C_{pq}$ and $D_{pq}$. If the following criteria are satisfied, the concordance set is used to describe the dominance query. After complimenting $C_{pq}$, get a set of discordance intervals ($D_{pq}$):

$$C_{pq} = \{j|a_{pj} \geq a_{qj}\}, \quad D_{pq} = \{j|a_{pj} \leq a_{qj}\} = \{j - C_{pq}\} \tag{2.31}$$

**Step 4: *Concordance set matrix***

The concordance interval matrix ($C_{pq}$) between $A_p$ and $A_q$ can be estimated based on the decision maker's preference for attributes. The concordance index is established by the equation,

$$C_{pq} = \sum_{j=C_{pq}} W_j \tag{2.32}$$

**Step 5:** ***Discordance interval matrix***

The discordance index ($D_{pq}$) can be interpreted as the existence of discontent in the choice of the scheme 'p' as opposed to 'q'. In more detail, the involves

$$D_{pq} = \frac{\max_{j \in D_{pq}} \mid V_{pj} - V_{qj} \mid}{\max_{j \in m,n} \mid V_{mj} - V_{nj} \mid} \tag{2.33}$$

where $D_{pq}$ represents the discordance index and $m$, $n$ is used to compute the weighted normalized value among all target attributes.

**Step 6:** ***Concordance interval matrix***

Below is the concordance index matrix for satisfaction measurement:

$$\bar{c} = \sum_{p=1}^{m} \sum_{q=1}^{m} \frac{c(p,q)}{m(m-1)} \tag{2.34}$$

Hence, $\bar{c}$ is the critical value which is evaluated by the average dominance index. Thus, boolean matrix (F) is

$$F = \begin{cases} f(p,q) = 1 & \text{if } c(p,q) \geq \bar{c} \\ f(p,q) = 0 & \text{if } c(p,q) < \bar{c} \end{cases} \tag{2.35}$$

**Step 7:** ***Discordance interval matrix***

$$\bar{d} = \sum_{p=1}^{m} \sum_{q=1}^{m} \frac{d(p,q)}{m(m-1)} \tag{2.36}$$

Based on the discordance index mentioned above, the discordance index matrix (E) is given by

$$E = \begin{cases} e(p,q) = 1 & \text{if } d(p,q) \leq \bar{d} \\ e(p,q) = 0 & \text{if } d(p,q) > \bar{d} \end{cases} \tag{2.37}$$

**Step 8:** ***Net superior and inferior values***

The net superior ($\bar{c_p}$) is the sum together the number of competitive superiority for all attributes.

$$c_p = \sum_{q=1}^{n} C(p,q) - \sum_{q=1}^{n} C(q,p) \tag{2.38}$$

On the contrary, the inferior values ($\bar{d}_p$) are used to determine the number of inferiority ranking the attributes.

$$d_p = \sum_{q=1}^{n} D(p,q) - \sum_{q=1}^{n} D(q,p) \tag{2.39}$$

**Table 2.5:** Statistics of the criteria.

| Category | Criteria | Mean | SD | Variance |
|---|---|---|---|---|
| Operational Service | Total vehicles | 70.46 | 24.72 | 610.92 |
| | Scheduled vehicles | 80.41 | 27.10 | 734.33 |
| | Operated vehicles | 55.75 | 21.02 | 441.88 |
| | Off-road vehicles | 10.25 | 6.13 | 37.58 |
| | Scheduled trips | 80402.75 | 33344.19 | 1111834768 |
| | Operating trips | 69571.79 | 28248.91 | 798001060.39 |
| | Extra trips | 936.17 | 922.94 | 851826.34 |
| | Curtailed trips | 13632.75 | 8242.58 | 67940444.46 |
| | Total no. of employees | 272.29 | 116.97 | 70.46 |
| | No. of routes | 42.89 | 14.08 | 198.34 |
| | Routes distance | 9064.31 | 2966.59 | 8800646.88 |
| Service Quality | Rate of break down | 0.20 | 0.15 | 0.15 |
| | Rate of accident | 0.05 | 0.03 | 0.00 |
| | Departure time | 98.40 | 4.06 | 16.48 |
| | Arrival time | 99.23 | 2.30 | 5.28 |
| | Fleet utilization | 79.06 | 7.40 | 54.72 |
| | Vehicle utilization | 391.87 | 46.39 | 2151.81 |
| | Tyre efficiency | 91860.48 | 22437.66 | 503448490.20 |
| Passenger Service | Number of passenger | 59.38 | 28.03 | 785.42 |
| | Passenger km occupied | 3.90 | 1.50 | 2.26 |
| | Description of km | 104.57 | 39.51 | 1561.01 |
| | Load factor | 76.08 | 4.99 | 24.90 |
| Cost Effects | Income per seat per km (in lakh) | 66.77 | 9.26 | 9.26 |
| | Total income per km | 3299.40 | 355.74 | 126552.40 |
| | Operating income (in lakh) | 3447.68 | 1483.43 | 2200561.01 |
| | Operating income per km | 3254.65 | 349.20 | 121941.96 |
| | Income per vehicle per day | 12824.60 | 2869.61 | 8234687.30 |
| | Total expenditure per km | 3989.21 | 438.97 | 192691.50 |
| | Profit/ loss per km | 689.77 | 400.46 | 160367.04 |
| | Diesel consumption km per liter | 5.04 | 0.30 | 0.09 |
| | Oil consumption top up km per liter | 0.62 | 0.21 | 0.05 |
| | Engine oil consumption per thousand km | 12824.60 | 2869.62 | 8234687.30 |

## 2.5 Data

The study's demonstration focused on transportation system management, which is visually depicted in figure 2.1. The illustration provides a thorough structure for screening vital criteria, establishing weights for assessment criteria, and subsequent stages for conducting a comparative analysis using three distinct methods: TOPSIS, VIKOR, and ELECTRE. The study is employed data from 52 RSRTC bus depots during the 2017-18 fiscal year, encompassing statistical analyses, including the computation of means, standard deviations (SD) and variance in table 2.5. Further, steps involved in implementing the MCDM process are elucidated in the following subsection:

**Table 2.6:** List of accepted criteria by FDM analysis .

| Category | Criteria | Average Fuzzy Weights $(a_j, b_j, c_j)$ | Defuzzification $(M_j)$ |
|---|---|---|---|
| C1: Operational Service | C11: Operated vehicles | (0.7,0.95,1) | 0.883 |
| | C12: Operating trips | (0,0,0.3) | 0.91 |
| | C13: Total no. of employees | (0.7,0.97,1) | 0.891 |
| | C14: Routes distance | (0.5,0.81,1) | 0.772 |
| C2: Service Quality | C21: Punctuality | (0.5,0.87,1) | 0.789 |
| | C22: Fleet utilization | (0.5,0.89,1) | 0.797 |
| | C23: Vehicle utilization | (0.7,0.97,1) | 0.891 |
| | C24: Rate of break down | (0.3,0.69,1) | 0.662 |
| C3: Passenger Service | C31: Number of passengers | (0.7,0.95,1) | 0.883 |
| | C32: Passenger km occupied | (0.7,0.95,1) | 0.883 |
| | C33: Load factor | (0.7,0.95,1) | 0.883 |
| C4: Cost Effects | C41: Total income per km | (0.3,0.63,1) | 0.643 |
| | C42: Operating income per km | (0.3,0.73,1) | 0.677 |
| | C43: Total expenditure per km | (0.5,0.87,1) | 0.789 |

### 2.5.1 Resuts and Discussion

**Phase1: Identification and Classification of Criteria Using FDM Technique**

The existing literature and available data revealed 29 criteria pertinent to assessing performance in the public transport sector. The FDM technique is applied to address potential ambiguities in determining the crucial criteria particularly due to the extensive range of criteria available. This involved the creation of a questionnaire designed and developed to solicit expert opinions. The experts chosen for the analysis have at least five years of experience in their respective fields. In order to ensure a well-rounded perspective, five experts from academia, renowned for their significant influence on policy decision-making, are included. The collective responses from all experts' questionnaires are combined to formulate comprehensive judgments. The experts are tasked to rate the influence of criteria on performance using a linguistic scale ranging from 0 to 1. A rating of 1 signified a substantial impact on performance, while a rating of 0 denoted a minimal impact. The judgments of experts are captured using a scale shown in table 2.2.

Following the process of defuzzification and filtering, table 2.6 showcases the precise numerical values that represent the consolidated judgments of the experts. A threshold value of $r = 0.60$ is taken based on prior studies and expert consultation to determine whether a given criteria should be included or excluded. The criteria having a threshold value $< 0.60$ are accepted (A); otherwise, they are rejected from the list. 14 vital criteria are accepted, while 15 criteria are not accepted. As a result, the experts collectively ranked "total no. of employees" and "vehicle utilization" as the most vital criteria of operational service and service quality. Conversely, criteria "income per seat per km (in lakh)", "scheduled vehicles" and "no. of routes" failed to reach the threshold level. "curtailed trips" and "profit/ loss per km" emerged as the least relevant criteria. Then, the process led to the exclusion of certain significant criteria typically used to assess the quality of public transportation services, including accident rates, total vehicles, scheduled trips, and distance covered. These findings suggest that preferences and rankings of quality criteria may vary across different regional contexts and evolve over time, influenced by the expansion of transportation services and the growing impact of new modes.

**Phase 2: FAHP Computations for Priority Weights**

The study listed the most significant criteria for assessing the performance of bus depots. The decision hierarchy structure for the process of choosing criteria encompasses three levels, as shown in figure 2.2. The first level's primary objective is to rank the depots based on performance scores. The second level involves the categorization of criteria into four distinct groups, while the third level focuses on assigning significant weights to these categories and criteria. To ascertain the relative weights of the main categories and their respective criteria, an FAHP is employed. Expert responses from a panel of five members are collected using Saaty's 1–9 scale and utilized for pair-wise comparisons within each criterion. These comparisons are carried out using equation (2.5).

The resulting fuzzy comparison judgments for all categories and criteria pertaining to the ultimate objective are detailed in table 2.7. These values are computed using the geometric mean of the evaluation findings. Subsequently, a pair-wise comparison matrix is generated, forming the basis for criteria weight determination. In order to verify the accuracy of the priority rankings within the pair-wise comparison matrix, consistency indices and ratios are calculated. A decision-making group validated the weights at the end of this stage. Table 2.8

derives the criteria's relative weights, consistency index, and consistency ratio.

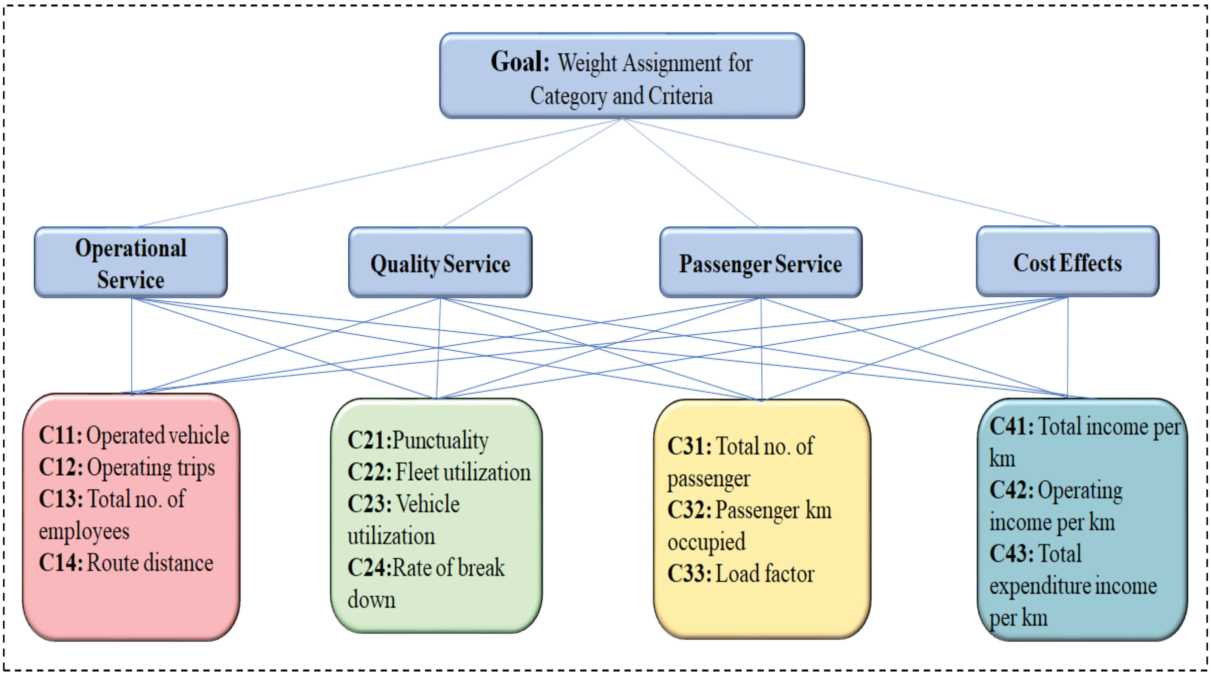


**Fig. 2.2:** Decision hierarchy of criteria.

**Table 2.7:** Fuzzy pairwise comparison matrix corresponding to the criteria.

| | | | | | |
|---|---|---|---|---|---|
| $\lambda_{max} = 4.18$, CR=0.069 | | | | | |
| | Operational Service | Service Quality | Passenger Service | Cost Effects | Local Weight |
| Operational Service | (1.00,1.00,1.00) | (2.00,2.00,2.00) | (2.00,2.91,4.00) | (2.00,3.87,7.00) | 0.501 |
| Service Quality | (0.25,0.34,0.50) | (0.50,0.59,1) | (1.00,1.00,1.00) | (0.33,1.00,3.00) | 0.143 |
| Passenger Service | (0.50,0.50,0.50) | (1.00,1.00,1.00) | (1.00,1.68,2.00) | (2.00,2.21,3.00) | 0.242 |
| Cost Effects | (0.14,0.26,0.50) | (0.33,0.45,0.50) | (0.33,1.00,3.00) | (1.00,1.00,1.00) | 0.113 |
| **Fuzzy pairwise comparison matrix and relative local weights corresponding Operational Service** | | | | | |
| $\lambda_{max} = 4.18$, CR=0.093 | | | | | |
| | Operated vehicles | Operating trips | Total no. of employees | Routes Distance | Local Weight |
| Operated vehicles | (1.00,1.00,1.00) | (0.50,1.73,3.00) | (2.00,2.91,9.00) | (0.33,1.41,3.00) | 0.361 |
| Operating trips | (0.14,0.28,2.00) | (1.00,1.00,1.00) | (2.00,2.21,3.00) | (0.25,1.46,3.00) | 0.303 |
| Total no. of employees | (0.11,0.28,0.50) | (0.33,0.37,0.50) | (1.00,1.00,1.00) | (0.33,0.33,0.33) | 0.078 |
| Routes Distance | (0.14,0.29,2.00) | (0.14,0.27,1.00) | (2.00,2.21,3.00) | (1.00,1.00,1.00) | 0.258 |
| **Fuzzy pairwise comparison matrix and relative local weights corresponding Service Quality** | | | | | |
| $\lambda_{max} = 4.18$, CR=0.083 | | | | | |
| | Punctuality | Vehicle utilization | Fleet utilization | Rate of break down | Local Weight |
| Punctuality | (1.00,1.00,1.00) | (2.00,3.31,5.00) | (2.00,2.21,3.00) | (0.50,0.71,2.00) | 0.367 |
| Vehicle utilization | (0.20,0.30,0.50) | (1.00,1.00,1.00) | (0.50,1.19,2.00) | (0.33,0.45,0.50) | 0.119 |
| Fleet utilization | (0.33,0.45,0.50) | (0.50,0.84,2.00) | (1.00,1.00,1.00) | (0.33,0.58,2.00) | 0.179 |
| Rate of break down | (0.50,1.41,2.00) | (2.00,2.21,3.00) | (0.50,1.73,3.00) | (1.00,1.00,1.00) | 0.335 |
| **Fuzzy pairwise comparison matrix and relative local weights corresponding Passenger Service** | | | | | |
| $\lambda_{max} = 4.18$, CR=0.082 | | | | | |
| | Passenger km Occupied | Number of passengers | Load factor | | Local Weight |
| Passenger km Occupied | (1.00,1.00,1.00) | (0.50,0.84,1.00) | (1.00,1.19,2.00) | | 0.337 |
| Number of passengers | (1.00,1.19,2.00) | (1.00,1.00,1.00) | (1.00,1.68,2.00) | | 0.444 |
| Load factor | (0.50,0.84,1.00) | (0.50,0.59,1.00) | (1.00,1.00,1.00) | | 0.219 |
| **Fuzzy pairwise comparison matrix and relative local weights corresponding Cost Effects** | | | | | |
| $\lambda_{max} = 4.18$, CR=0.092 | | | | | |
| | Total income per km | Total expenditure per km | Operating income per km | | Local Weight |
| Total income per km | (1.00,1.00,1.00) | (0.33,0.76,1.00) | (0.50,0.84,1.00) | | 0.244 |
| Total expenditure per km | (1.00,1.32,3.00) | (1.00,1.00,1.00) | (1.00,1.00,1.00) | | 0.388 |
| Operating income per km | (1.00,1.19,2.00) | (1.00,1.00,1.00) | (1.00,1.00,1.00) | | 0.368 |

The values of random index $(RI_k)$ from table 2.4 vary for different numbers of criteria. $RI_k$ is calculated to be 0.58 for three criteria. Similarly, the $RI_k$ is determined to be 0.89 for four criteria. These values serve as reference points for assessing the consistency of pairwise comparisons in the decision-making process.

**Table 2.8:** The calculation result of weights by FAHP.

| **Category** | **Local Weight using FAHP** | **Criteria** | **Local Weight using FAHP** | **Global Weight using FAHP** | **Rank Rank** |
|---|---|---|---|---|---|
| Operational Service | 0.501 | Operated vehicles | 0.361 | 0.181 | 1 |
| | | Operating trips | 0.303 | **0.152** | 2 |
| | | Total no. of employees | 0.078 | 0.039 | 10 |
| | | Routes distance | 0.258 | 0.129 | 3 |
| Service Quality | 0.143 | Punctuality | 0.367 | 0.053 | 6 |
| | | Vehicle utilization | 0.119 | 0.017 | 13 |
| | | Fleet utilization | 0.179 | 0.026 | 12 |
| | | Rate of break down | 0.335 | 0.048 | 7 |
| Passenger Service | 0.242 | Passenger km occupied | 0.337 | 0.082 | 5 |
| | | Number of passengers | 0.444 | 0.107 | 4 |
| | | Load factor | 0.219 | 0.053 | 6 |
| Cost Effects | 0.113 | Total income per km | 0.244 | 0.028 | 11 |
| | | Total expenditure per km | 0.388 | 0.044 | 8 |
| | | Operating income per km | 0.368 | 0.042 | 9 |

The result is presented in table 2.8, which shows that the “Operational Service” (C1) is the highest weight among the category “Passenger Service” (C3) is ranked as the second most important category. “Service Quality” and “Cost Effects” are ranked third and fourth, respectively.

- **Operational Service (C1)**
  ‘Operated vehicles’ (C11) has the highest priority, followed by ‘Operating trips’ (C12), ‘Routes distance’ (C14), and ‘Total no. of employees’ (C13).

- **Service Quality (C2)**
  ‘Punctuality’ (C21) has the highest priority, followed by ‘Rate of break down’ (C24), ‘Fleet utilization’ (C23), and ‘Vehicle utilization’ (C22).

- **Passenger Service (C3)**
  ‘Number of passengers’ (C32) has the highest priority, followed by ‘Passenger km occupied’ (C31), and ‘Load factor’ (C33)

- **Cost Effects (C4)**
  'Total expenditure per km' (C42) has the highest priority, followed by 'Operating income per km'(C43), and 'Total income per km' (C41).

**Phase 3: Overall Comparison of MCDM Methods for Performance Evaluation**

Using FAHP technique, established the weights for all the screened criteria from FDM method and subsequently conducted the ranking process. To comprehensively evaluate the ranking of depots, employ three distinct MCDM methods, namely TOPSIS, VIKOR, and ELECTRE, all of which are integral in the analysis of this study. The rankings of the depots, based on their final scores obtained from these three MCDM algorithms, are provided in table 2.9.

- **TOPSIS Results**
  The results obtained by TOPSIS are tabulated in table 2.9. Sikar is the best-performing depot, with the highest performance score of 0.78, while Karauli is the worst-performing depot, with the smallest performance score of 0.02.

- **VIKOR Results**
  Table 2.9 illustrates the assigned rank for the depots based on the VIKOR Index values. In VIKOR, Sikar is the best-performing depot, with a performance value of 0.047, while Karauli is the worst-performing depot, with a 0.995 performance value. As an illustration, Sikar is calculated at the top spot with aggregate depots, and the value of the index is 0.954 (1-0.047), which is the closest value to the ideal solution 1.

- **ELECTRE Results**
  The ranking of the RSRTC bus depots is determined using the inferior and superior values of ELECTRE. The obtained rankings are tabulated in table 2.9. Ranking by ELECTRE, Alwar is the best-performing depot (33.12), while Jaisalmer is the worst-performing depot (-40.74) out of 52 depots according to ELECTRE.
  A decision matrix serves as the beginning point for evaluating the rank of the depots. Using the MCDM technique, the efficient depot is evaluated in this study, considering various competing criteria. The results evinced that Sikar is the best-performing depot, while Jaisalmer and Karauli are the worst-performing depot in all methods. Abu Road and Dungarpur have the same rank in all three methods. These three methods are ranked in a comparable order, though not identically. TOPSIS and VIKOR produced 97.6% similar rankings of the depots. VIKOR and ELECTRE produced 95.74% similar rankings of the depots.

**Table 2.9:** Scores and ranking of each method for depot.

| **Methods** | **TOPSIS** | | **VIKOR** | | **ELECTRE** | | **Methods** | **TOPSIS** | | **VIKOR** | | **ELECTRE** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Depots | Score | Rank | Score | Rank | Score | Rank | Depots | Score | Rank | Score | Rank | Score | Rank |
| Abu Road | 0.09 | 46 | 0.68 | 46 | -25.98 | 46 | Jalore | 0.08 | 40 | 0.53 | 37 | -11.24 | 36 |
| Ajaymeru | 0.21 | 7 | 0.21 | 6 | 17.62 | 11 | Jhalawar | 0.18 | 15 | 0.31 | 15 | 8.19 | 22 |
| Ajmer | 0.6 | 5 | 0.19 | 5 | 21.78 | 6 | Jhunjhunu | 0.25 | 8 | 0.28 | 14 | 21.89 | 5 |
| Alwar | 0.31 | 3 | 0.08 | 2 | 33.12 | 1 | Jodhpur | 0.15 | 12 | 0.27 | 10 | 13.69 | 12 |
| Anoopgarh | 0.08 | 37 | 0.58 | 38 | -6.08 | 35 | Karauli | 0.02 | 52 | 0.96 | 52 | -33.05 | 49 |
| Banswara | 0.1 | 35 | 0.48 | 29 | -2.88 | 28 | Khetri | 0.08 | 47 | 0.69 | 47 | -16.85 | 42 |
| Baran | 0.15 | 26 | 0.48 | 28 | -4.08 | 33 | Kota | 0.14 | 20 | 0.33 | 18 | 7.68 | 20 |
| Barmer | 0.14 | 33 | 0.58 | 39 | -7.66 | 37 | Kotputli | 0.21 | 29 | 0.52 | 31 | -1.87 | 31 |
| Beawar | 0.22 | 25 | 0.56 | 36 | 3.75 | 24 | Lohagarh | 0.17 | 14 | 0.32 | 16 | 15.5 | 13 |
| Bharatpur | 0.18 | 16 | 0.33 | 17 | 18.7 | 8 | Matsya Nagar | 0.2 | 18 | 0.38 | 20 | 11.33 | 15 |
| Bhilwara | 0.13 | 17 | 0.3 | 13 | 10.65 | 14 | Nagaur | 0.1 | 30 | 0.47 | 27 | -6.6 | 32 |
| Bikaner | 0.13 | 10 | 0.28 | 12 | 10.15 | 16 | Pali | 0.08 | 44 | 0.68 | 45 | -18.43 | 43 |
| Bundi | 0.15 | 31 | 0.45 | 26 | -1.45 | 27 | Phalaudi | 0.06 | 42 | 0.66 | 42 | -18.38 | 45 |
| Chittorgarh | 0.17 | 13 | 0.22 | 7 | 14.79 | 10 | Partapgarh | 0.04 | 51 | 0.91 | 50 | -40.07 | 51 |
| Churu | 0.14 | 41 | 0.55 | 40 | -12.81 | 38 | Rajasamand | 0.06 | 45 | 0.68 | 44 | -18.72 | 44 |
| Dausa | 0.14 | 36 | 0.54 | 32 | -7.27 | 41 | Sardaarshahar | 0.14 | 24 | 0.5 | 30 | -4.41 | 34 |
| Deluxe | 0.09 | 32 | 0.47 | 25 | 2.24 | 25 | Sawaimodhopur | 0.06 | 48 | 0.81 | 48 | -31.55 | 50 |
| Dhaulpur | 0.15 | 27 | 0.47 | 24 | 1.4 | 30 | Shapur | 0.16 | 39 | 0.61 | 41 | -16.25 | 40 |
| Didwana | 0.1 | 38 | 0.54 | 33 | -11.25 | 39 | Sikar | 0.78 | 1 | 0.05 | 1 | 30.34 | 3 |
| Dungarpur | 0.17 | 23 | 0.43 | 23 | 3.54 | 23 | Sirohi | 0.08 | 43 | 0.68 | 43 | -17.04 | 47 |
| Falna | 0.05 | 49 | 0.79 | 49 | -26.75 | 48 | Srimadhopur | 0.22 | 19 | 0.38 | 22 | 9.42 | 17 |
| Ganga Nagar | 0.25 | 11 | 0.29 | 11 | 9.36 | 19 | Tijara | 0.19 | 28 | 0.55 | 34 | 0.9 | 29 |
| Hanumangarh | 0.3 | 2 | 0.16 | 4 | 21.46 | 7 | Tonk | 0.16 | 21 | 0.36 | 19 | 10.6 | 18 |
| Hindaun | 0.2 | 22 | 0.4 | 21 | 9.42 | 21 | Udaipur | 0.13 | 9 | 0.21 | 9 | 21.37 | 4 |
| Jaipur | 0.13 | 6 | 0.27 | 8 | 17.77 | 9 | Vaishali Nagar | 0.19 | 4 | 0.13 | 3 | 31.5 | 2 |
| Jaisalmer | 0.01 | 50 | 0.91 | 51 | -40.74 | 52 | Vidhyadhar Nagar | 0.23 | 34 | 0.49 | 35 | 3.24 | 26 |

## 2.6 Sensitivity Analysis

According to Chang et al. (2007) [201], minor relative weight variations may lead to significant variations in the overall ranking. Although such weights frequently depend totally on subjective assessments, looking into the ranking's consistency across different criteria is important. FAHP technique is utilized to derive the category, and criteria weights are examined in determining the dominance of each scenario. As perceptions of decision-makers vary, this method can be validated using sensitivity analysis. To highlight the stability of ranking, sensitivity analysis is essential. In this regard, eight different cases are analyzed by changing the weights of the main criteria. Initially, the weight value is modified by increasing or decreasing the criterion weight by 5%, 10%, 20%, or 50%, respectively. When a criterion weight increases or decreases by 5%, 10%, 20%, or 50%, the remaining criteria must be proportionally adjusted to keep the criterion weight at 1. Considering these sets of weights, MCDM methods TOPSIS, VIKOR, and ELECTRE are used to examine each criterion.

**Table 2.10:** Sensitivity analysis of criteria weights.

| Change Criterion Weights | TOPSIS | | | VIKOR | | | ELECTRE | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 2 | >2 | 0 | 2 | >2 | 0 | 2 | >2 |
| 0.05 | 10 | 2 | 3 | 0 | 5 | 10 | 3 | 2 | 10 |
| -0.05 | 8 | 2 | 5 | 0 | 6 | 9 | 0 | 2 | 13 |
| 0.1 | 7 | 2 | 6 | 0 | 0 | 15 | 2 | 2 | 11 |
| -0.1 | 8 | 1 | 6 | 0 | 2 | 13 | 0 | 0 | 15 |
| 0.2 | 7 | 1 | 6 | 0 | 0 | 15 | 1 | 0 | 14 |
| -0.2 | 7 | 1 | 6 | 0 | 0 | 15 | 0 | 0 | 15 |
| 0.5 | 6 | 2 | 7 | 0 | 0 | 15 | 0 | 0 | 15 |
| -0.5 | 6 | 3 | 6 | 0 | 0 | 15 | 0 | 0 | 15 |

This analysis helps in checking the consistency of results regarding whether the model or system works in the most favorable or unfavorable conditions. Sensitivity analysis is used in this study to see how the ranking of depots varied as the weights of the criteria are changed. The sensitivity coefficient indicated that increasing or decreasing the criterion weight by 5%, 10%, 20%, or 50% resulted in single, double, or multiple changes in the rankings of alternatives. The sensitivity coefficient is equal to 0 if the rank is the same as the original rank. When the rank of one depot increased, the rank of another fell, resulting in a sensitivity coefficient of 2.

The number of criteria that affected the rankings after adjusting the weight of one criterion is displayed in table 2.10. The results demonstrated that, when weights are increased (decreased) by 5%, the ranking of depots has some impact on the ranking with the VIKOR and ELECTRE techniques but no impact on TOPSIS. When the criterion weights are increased or decreased by 50%, the ranking of the TOPSIS technique is the least affected, whereas ELECTRE showed the most significant change (46% and 69% change) and VIKOR exhibited a change (39% and 72% change). Only the TOPSIS ranking results remained nearly unchanged when the weight varied drastically (50%), whereas the remaining two methods are altered by roughly 39% to 72%. In VIKOR, the top-valued weights are much more affected when weights are decreased compared to when they are increased. For example, when the weight of the operated vehicles (C11) criterion is increased by 10%, observed 21 places of ranking changes, but when it is decreased by 10%, observed almost twice as many ranking changes, i.e., 39 places. On the other hand, in the ELECTRE method, the operated vehicles (C11) criterion ranking is not affected, even though it is affected in all other models. The change in ranking is expected to be high, but this is not the case for ELECTRE. "operational income per km" (C43) is the least affected criterion in all the models. Among the considered scenarios, observed extreme deviations in punctuality (C21), vehicle utilization (C23), fleet utilization (C22), rate of breakdown (C24), passenger-km occupied (C32), number of passengers (C31), and load factor (C32). This through sensitivity analysis is achieved, which can be based on scenarios that represent potential future developments or various viewpoints on the relative relevance of the criteria.

## 2.7 Conclusions

Promoting sustainable transportation is a cornerstone of society's progress, contributing to economic growth and improved social well-being. With a surge in personal vehicle usage, encouraging individuals to shift towards public transit is pivotal. However, apprehensions about service quality remain a concern for potential users, underscoring the need for public transit providers to enhance their offerings. Effective selection of evaluation criteria plays a critical role in minimizing depot corrosion and failures. This chapter adds to the existing body of knowledge by introducing a novel hybrid analytical approach that integrates fuzzy Delphi, fuzzy AHP, and TOPSIS-VIKOR-ELECTRE methods. Leveraging fuzzy set theory to address the inherent ambiguity associated with human subjectivity in assessing crucial criteria, resulting in more robust and reliable outcomes. Drawing from an extensive literature review, identified 29 distinct criteria, which are subsequently grouped into four categories. Through the collaborative effort of experts, the FDM narrowed down this list to 14 criteria deemed pivotal for the assessment process. Following this, the FAHP is employed to ascertain the relative significance of both the four categories and the 14 selected criteria.

The outcomes of this evaluation shed light on the prioritization of these categories and criteria. The weights derived from the FAHP process are subsequently utilized in TOPSIS, VIKOR, and ELECTRE for ranking the 52 RSRTC depots for the year 2017-18. These MCDM techniques yielded substantial results, bridging gaps in previous research within the public transport sector, particularly in the context of criteria selection. The proposed models stand out for their simplicity, convenience, precision, and efficiency, offering valuable support to decision-makers in their criterion selection process. This innovative hybrid MCDM method exhibits versatile applicability to address similar criterion selection challenges in the transportation industry and other decision-making problems.

# Chapter 3

# Assessing the Efficiency of Bus Transport Sector Using Data Envelopment Analysis

## 3.1 Introduction

Rajasthan state road transport corporation (RSRTC) plays a crucial role in connecting cities, towns, and villages via intercity public bus transportation service. However, despite the rising travel demand due to rapid urbanization and motorization, passenger traffic of RSRTC decreased by an average rate of approximately 0.157 million every year for the period 2007-17. The decline in the use of public transport is attributed to several issues, including unorganized and inconvenient bus services, which are often characterized by inefficiency and subpar service quality. These problems plague the system, pushing many commuters to opt for alternative means of transportation such as private vehicles or app-based taxi services. This shift demonstrates a lack of confidence in public transport and highlights the urgent need for comprehensive reforms and strategic planning within this sector [202]. Compounding this problem is that the RSRTC operates at a loss, and the challenges posed by inefficient depot operations escalate its financial difficulties. Such issues lead to rising costs, which affect sustainability across various economic tiers. This situation underscores the urgent need to thoroughly review and optimize operational strategies. The recommendation based on the findings of the study would also help the government to bridge the gap between actual performance and feasible or achievable performance, and to frame suitable policies for the development of the passenger road transport sector in Rajasthan.

In this chapter, a sustainable benchmarking analysis is utilized to enhance the efficiency of RSRTC with the ultimate aim of elevating organizational performance and ensuring cost-effective services. There is no doubt that benchmarking analysis, which involves an analysis of

[1]This work has appeared in **Goyal, S., Agarwal, S., Mathur, T., and Mathur, N., 2021. Assessing the Radial Efficiency Performance of Bus Transport Sector Using Data Envelopment Analysis. InHandbook of Machine Learning for Computational Optimization, 2021, 2, pp.71-87. CRC Press.**

the sources and determinants of operational efficiency, would be a valuable aid to policy formulators in designing appropriate policies aiming to improve the overall health and competitiveness of RSRTC. Performance of the overall system can be increased by correctly identifying subunit inefficiencies and then improving subunit performance with changes consistent with system structures, goals, and constraints. In fact, internal benchmarking would help to identify the criterion for taking corrective actions based on certain priority considerations.

Turning attention to the production frontier, there are two distinct methodologies: parametric and non-parametric [18]. Parametric methods require a specific functional form for the relationship between criteria, whereas no such assumptions are required in a functional form for non-parametric methods. Non-parametric methods are valuable in a range of applications, including assessing the performance of non-profit organizations such as public transport sector. Data envelopment analysis (DEA) is a non-parametric linear programming-based empirical approach for estimating the efficiency of a group of similar decison-making units (DMUs). Efficiency analysis within the DEA framework relied on traditional models such as CCR [15] and BCC [22]. While these models offered valuable insights into efficiency analysis, they fundamentally distinguished efficient DMUs from their inefficient counterparts. A DMU is considered fully efficient when its efficiency score equals one. Some may exhibit efficiency, a condition in which they are efficient with a reference point but are not fully efficient because of the presence of slacks in their input and output. It is important to consider other DEA models in order to address specific challenges and provide nuanced perspectives.

Inefficiencies, represented by deviations from optimal resource use, can significantly affect the determination of the efficiency scores. Basic models frequently present zero values for several multipliers, suggesting that certain factors such as inputs or outputs are not fully considered when calculating efficiency scores. Additionally, significant discrepancies can arise in the input and output weighting of various items, where certain components might have exceptionally high or low values. Effectively, this can lead to oversight of some inputs or outputs, which is often untenable in practical applications. Addressing these issues, NSM model introduced by Agarwal et al. (2011) [203] is a noteworthy development in efficiency assessment models. NSM directly integrates input and output slack for efficiency assessment. This approach offers a more accurate depiction of the performance. NSM maintains the key properties of the radial DEA model. Notably, the dual form of NSM indicates that all multipliers are positive. This ensures that every input and output has a substantial impact on the performance evaluation of DMUs. Consequently, the NSM delivers a holistic assessment of efficiency, considering the influence of slack on the overall performance of DMUs. The NSM efficiency score does not exceed the CCR efficiency score. In addition, if a DMU is NSM-efficient, then it is CCR-efficient. Thereby presenting a more comprehensive assessment tool for managers seeking to improve their overall technical efficiency (OTE), with the direct impact of input slack on the efficiency of DMUs. The NSM-DEA model is carried out using MATLAB. These results are cross-checked using Python.

This chapter is organized into four sections. Section 3.2 provides a comprehensive overview of the existing literature and studies on the evaluation of bus transport efficiency. Section 3.3 presents the nuances and specifics of the NSM for efficiency assessment by DEA, including data, classification of depots, input and output criteria, and input targets for inefficient depots of the RSRTC. The chapter culminates in the last section, 3.4, summarizing the key findings and insights of the study.

## 3.2 Review of Research and Development Trends in Bus Transport Sector

This section reviews some DEA-based studies on the transport sector worldwide. This review is bifurcated into international and national contexts, providing an overview of the advancements and key findings in the domain.

### 3.2.1 International Status

The ideal expression of efficiency measures the value of the produced outputs to consumers or society, suggesting that it is a comprehensive and normative tool for evaluation [204, 205]. Several seminal works have paved the way for contemporary research in the domain of efficiency in the transportation sector. In 1996, Kerstens [206] took a deep dive into the technical efficiency of French urban transit companies, employing DEA methodologies and Tobit regression and illuminating the critical determinants of efficiency, ranging from vehicle age to population density. Preceding this, Viton (1997) [207] broke new ground by offering a comprehensive production frontier analysis for multimode bus transit in the USA, expanding the discourse to encompass both motor-bus and demand-responsive services, thereby broadening the scope of efficiency measurement in transit systems. Similarly, Cowie et al. (1999) [208] addressed scale and technical efficiencies using DEA and the Mann-Whitney U-Test of the British bus industry. Their analysis pinpointed that while there is a surge in the number of operating companies immediately after deregulation, efficiencies varied significantly across regions due to differences in competition and market density. They suggested that privately owned companies are more technically efficient than publicly owned companies because of their significantly higher organizational efficiency. Husain et al. (2000) [209] analyzed the efficiency of 46 service units within Malaysia's road transport department (RTD) using the CCR model. Their findings highlighted notable inefficiencies in several units, particularly concerning the overutilization of labor and associated elevated costs. They recommended reductions in labor and costs for these units to improve operational efficiency. The study emphasized the significance of consistent efficiency assessments in the public sector for optimal resource allocation and enhanced service delivery. Nolan et al. (2002) [210] developed a method for evaluating the efficiency of public organizations in meeting the social objectives specified by the intermodal surface transportation efficiency act (ISTEA), focusing on urban transit agencies in the USA. The research pointed out the challenges of evaluating and measuring social policy objectives and emphasized the need for appropriate accountability measures and data collection procedures in regulatory public policies. Karlaftis (2003, 2004) [211, 212] is a pioneer in transportation efficiency research. Studies, focusing on USA transit systems, utilized traditional DEA models with VRS assumptions, dividing 256 bus transits into six groups, and assessing their technical efficiency and effectiveness. Odeck (2006) [213] evaluated the impact of inputs on the relative efficiency of bus operators in the Norwegian bus industry. This study provided insights into the potential for input saving in the sector, suggesting a potential of approximately 21%, and found no significant differences in efficiency scores between urban and rural bus operators and no performance differences with respect to ownership.

Karlaftis (2009) [214] expanded the research to European transit systems, assessing the impact of ownership and competition. Garcia (2009) [215] proposed a comparative technical and scale efficiency analysis of public bus transport in Spain using DEA. Use of principal component analysis (PCA) to reduce potential measures of supply-and demand-side and quality outputs. Tobit regression analysis showed negative efficiency levels in relation to population density and peak-to-base ratio. No relationship is found between efficiency levels and form of ownership (public versus private). Barnum et al. (2011) [216] investigated the complexities of public transit systems within a metropolitan area in the USA Using the BCC model, they meticulously evaluated the technical efficiency (TE) of various transit types from buses to trams. Their research revealed specific regions in which resource utilization could be optimized. More than just a diagnostic tool, their study provided actionable insights into buses to trams. They recommended a strategic method for redistributing resources, ensuring that, while costs are reduced, the overall quality and quantity of transit services remain consistent. Baležentis & Baležentis (2011) [217] employed a combination of multi-objective optimization by ratio analysis (MULTIMOORA) and DEA, both of which are well-recognized for the efficient assessment of passenger and freight transport. The DEA method is used to evaluate the technical and scale efficiency of the Lithuanian transport sector, providing estimates of both actual and potential efficiency. In a comprehensive study, Karlaftis and Tsamboulas (2012) [218] evaluated the efficiency of public bus systems using multiple methodologies, including DEA, SFA, and Neural Networks (NN), with a keen interest in the specification sensitivity of the findings. Carvalho et al. (2015) [219] employed an optimized super-efficiency DEA model to evaluate the performance of Brazilian municipalities over a six-year span. Their analysis highlighted divergent strategies among cities: while Curitiba and Betim prioritized effectiveness, São Bernardo and Salvador leaned towards efficiency. The study underscored that, although the methodology is straightforward, the choice of inputs and outputs profoundly impacted the outcomes.

Wu et al. (2016) [220] suggested that DEA is a more efficient technique than SFA for parallel systems, which is passenger and freight transportation of China's 30 provincial-level regions for year 2012. Passenger transportation is relatively more efficient than freight transportation. Another contribution is the exploration of alternative weight choices for the efficiency evaluation of the transportation system to enhance the accuracy and applicability of the findings. Holmgren (2018) [221] proposed a model for evaluating the efficiency of public transport operations and compared its results with those obtained from competing models estimated using the same data from 27 Swedish counties from 1986 to 2015. The study emphasized the importance of utilizing both demand-oriented measures (e.g., the number of trips or passenger kilometers) and supply-oriented measures (e.g., vehicle kilometers or seat kilometers). These dual measurements effectively captured not only the quantity consumed, but also the inherent quality of the consumed quantity. A key takeaway from this research is that smaller models that rely solely on one output measure might inadvertently underestimate the true efficiency of the public transport sector. Chen et al. (2019) [222] introduced a novel methodology by integrated cumulative opportunity measure and DEA method. This approach is developed to evaluate the accessibility-based service effectiveness (ABSEV) of bus transit systems. Their study specifically analyzed urban bus transit in Edmonton, Canada, focusing on the dual objectives of enhancing transportation accessibility and addressing concerns related to social equity. This study offered valuable insights for urban transportation planners and policymakers aiming to strike a balance between service efficiency and equitable access. Karim et al. (2019) [205] analyzed the efficiency and effectiveness of bus services across six public bus firms in various Moroccan cities in 2013. Using the CRS DEA model, they identified potential areas for performance enhancement. To account for external factors beyond the control of these transport companies, which could impact the efficiency scores, they further employed Tobit regression in their study.

Li et al. (2020) [223] proposed an approach for road transportation, considering carbon emission intensity and fairness, leading to increased allocation efficiency. The proposed approach is applied to the empirical study of emission quota allocation in the road transportation sector of 30 provinces in China, providing practical findings and implications for achieving emission reduction targets and improving the efficiency of industrial development. Shen et al. (2020) [224] proposed a benchmarking approach to assess sustainable road transport among the 28 EU countries, considering both desirable achievements and undesirable costs. This study underscored the importance of this approach to sustainable development and emphasized that performing well in only one aspect is insufficient. Sweden is identified as the best-performing country for both factors.

### 3.2.2 National Status

After delving into the numerous global studies on efficiency, it's pivotal to shift the focus towards India. A limited number of researchers have delved into this area, pointing to a significant gap in the existing literature. Using the DEA model, Ramanathan (1999) [225] analyzed the efficiency of 29 state transport undertakings (STUs) from 1993 to 1994. The study found that operators managed fuel efficiently but observed significant inefficiencies in fleet and staff usage. Regression analysis also indicated that older fleets and hilly operation areas negatively impact efficiency. Jha and Singh (2001) [226] embarked on an exploration of technical efficiency, with a spotlight on the cost-inefficiencies plaguing major Indian road transport enterprises. Their findings revealed pronounced discrepancies in cost inefficiency across STUs of varying sizes. Notably, STUs of smaller to medium scale demonstrated heightened levels of inefficiency compared to their larger counterparts. Moreover, th study offered invaluable perspectives on the operational efficiency of road transport entities. Such insights hold significant potential for guiding policy frameworks, especially in considerations related to the consolidation or segmentation of STUs. Karne et al. (2003) [202] examined the possibility of splitting Maharashtra state road transport corporation (MSRTC) into a large organization and smaller corporations to improve its financial recovery using CRS DEA model for the period 2000-2002. This study emphasized the need for policymakers and managers of MSRTC to measure and treat the problems facing MSRTC as an emergency. It identified falling load factors due to competition from private bus and taxicab operators as one of the problems adversely affecting the financial profitability of the MSRTC. Bhagavath (2006) [13] employed the BCC model to analyze the technical efficiency of 44 STUs. This study revealed that only eight of these STUs operated at the optimal scale efficiency (SE). One of the interesting findings of this study is that STUs operating as companies are relatively more technically efficient than others, indicating a potential relationship between the organizational structure and efficiency levels of STUs. Overall, the study highlighted the importance of using DEA to measure technical efficiency in transportation and suggested avenues for further research to improve the measurement methodology. Badami and Haider (2007) [227] analyzed the financial and operational performance of public bus transit services in Indian cities during the 1990s. Despite increasing fares and declining ridership, it highlighted persistent losses in the public bus transit system. This study contributes to the understanding of the challenges faced by Indian cities in maintaining and enhancing public transit services, particularly for the urban poor. This study suggested a disaggregated approach based on the needs and motivations of different groups in relation to public transit, along with improved operating conditions and policies to address the challenge of providing financially viable and affordable public bus transit services.

Agarwal et al. (2010) [50] utilized an input-oriented basic DEA model, considering four input variables (fleet size, total staff, fuel consumption, and accidents per lakh km) and three output variables (bus utilization, passenger kilometers, and load factor), to evaluate the efficiencies of 35 STUs spanning the period from 2004 to 2008. Their findings indicated an average efficiency of 83.26%. Intriguingly, even when inputs are reduced by 16.74% from their current levels, there is no observed change in output. This study also identified 18 STUs as pure technical efficient, implying that they have no scope for further input reduction while maintaining the same output level. The remaining 17 STUs are relatively inefficient. Furthermore, Nagadevara et al. (2010) [228] introduced four inter-temporal variations in DEA efficiency across subunits of the 25 Karnataka state road transport corporation (KSRTC) from 2004 to 2009. Their findings noted consistent efficiency in 11 depots throughout the five-year period. Notably, the kolar depot experienced a significant decline in efficiency. Kumar (2011) [229] employed an array of DEA models encompassing CCR, BCC, and Andersen and Petersen's super-efficiency. Computed various technical efficiency values for 31 individual SRTUs. Subsequently, a Tobit regression analysis is conducted to delve into the variations in efficiency across different SRTUs. A salient outcome of this study is the impact of various input and output factors on efficiency. The results showcased that five SRTUs established the efficiency frontier, implying optimal operations. In contrast, the other 26 SRTUs are identified as inefficient, indicating substantial room for input minimization. This study suggested that these SRTUs are squander approximately a quarter of their resources in their production endeavors. A central takeaway from this study is identifying the occupancy ratio as the pivotal determinant across all efficiency metrics. Hanumappa et al. (2015) [230] applied DEA to measure both long-term and short-term efficiencies of premium bus services operated by Bangalore metropolitan transport corporation (BMTC) and focused explicitly on Volvo buses only, which may not provide a comprehensive picture of the overall performance of BMTC's entire bus fleet. They identified opportunities for improvement at the bus depot and route level.

Venkatesh and Kushwaha (2017) [231] utilized a non-radial input-oriented DEA model to measure technical efficiency, while mitigating the influence of slack on the efficiency score of STUs. They offered a thorough insight into efficiency trends over the twelve-year span from 2002 to 2013. A notable observation is declined in technical and labor efficiencies for STUs during this period. This study also noted that larger STUs, in terms of fleet strength, perform better than smaller STUs and suggested potential areas for further improvement, such as examining the drivers of efficiency through regression analysis and computing efficiency at disaggregated levels, such as division, depot, or route. Venkatesh and Kushwaha (2018) [232] addressed the need to study the efficiency levels of STUs, considering their operation under high levels of government-imposed regulatory constraints. Investigated the minimum cost efficiency using VRS assumption for both short and long runs in STUs. This study highlighted the importance of obtaining efficiency in the short run, where some inputs are fixed, and as well as in the long run, where all inputs are variable. Their findings revealed that through efficiency-enhancing measures, STUs could potentially reduce costs by up to 9123.35 million dollars. This study also indicated a trend of increasing reluctance to minimize costs over time. Additionally, they pinpoint that certain STUs function with an inadequately small fleet size in the short term, leading to sub-optimal operations. On a related note, Saxena (2019) [233] explored the technical, scale, and managerial efficiencies of Delhi transport corporations (DTC) interstate buses by utilizing the DEA technique and regression analysis. The study has data from central institute of road transport (CIRT) publications, focused on 25 government-run transport undertakings, and intentionally omitted private operators from the analysis. The findings indicated that DTC ranks among the least efficient STUs, which showed a technical inefficiency rate of 50.94% and operated under decreasing returns to scale. The evaluation contributed to possible enhancements in the services provided by public transport. Gulati (2022) [234] made significant advancements in recent studies on the efficiency of the public bus transit system in India. This study intuitively incorporated the number of accidents as an undesirable output and offered a more comprehensive assessment of system performance. The slack-based measure (SBM) undesirable window analysis approach assessed the dynamic efficiency of 8 public bus operators across major metropolitan cities in India. The findings present a reliable efficiency estimate for these public bus companies, which is invaluable for policymakers. By understanding these efficiency performances, policymakers can devise effective strategies and interventions to enhance both the technical and scale efficiency of bus companies in metropolitan cities. This research contributed notably to the literature, filling a crucial gap and setting the stage for further exploration in the domain of bus transport efficiency. Aneja et al. (2022) [235] aimed to measure the efficiency of 20 major depots of Haryana roadways for the year 2017-2018 using DEA and to determine the overall and depot-level efficiency of Haryana roadways. The study used fleet size, total number of staff, and fuel consumption by buses as inputs and bus utilization as output to assess the efficiency of the depots. The findings revealed that the five depots (D1 Gurugram, D3 Chandigarh, D10 Bhiwani, D11 Sirsa, and D15 Delhi) are overall technical efficient. The performance of the depots is not at par with the optimum level, with an overall mean technical efficiency of 91%. Suggested that the depots can produce the same output level by reducing 9% of inputs. Additionally, replacing old buses with new ones can improve fuel efficiency, and capacity-building programs and training should be provided to workers to enhance performance.

Despite the extensive literature review presented above, it is evident that the realm of public bus transport efficiency studies, particularly in the Indian context, exhibits glaring oversight concerning the Rajasthan public road transport sector. No study in the existing literature has holistically approached or shed light on the intricacies of RSRTC depots' efficiency. This under-researched area represents a significant knowledge gap, particularly given Rajasthan's geographical vastness, demographic diversity, and unique transport challenges, which are distinctive from other Indian states.

Consequently, the principal aim of this research is not only to introduce this overlooked dimension but also to rigorously establish a benchmarking analysis model that critically assesses the performance levels of RSRTC depots. This endeavor sought to gauge the overall efficiency and compare region-wise efficiencies across various depots. Such a comprehensive approach provides an unparalleled understanding of resource allocation, key determinants, and pivotal factors that influence and shape operational efficiency within the RSRTC. By addressing this research lacuna, the aim is to contribute a seminal work that enriches the academic discourse and offers actionable insights for policymakers and government transport authorities.

## 3.3 Mathematical Description: NSM Model

Having discussed the merits of NSM in the context of addressing the limitations of the basic CCR model, this section discusses the mathematical specifics underlying NSM. Through a detailed exploration of its formulas and components, the intricate mechanics of the NSM model became evident, offering a clearer understanding of its applications in efficiency assessment.

Each $j^{th}$ DMU $(j = 1, 2, \ldots d, \ldots, n)$ consumes $m$ inputs to produce $s$ outputs define by $x_{ij}(i = 1, 2, \ldots, m)$ and $y_{rj}(r = 1, 2, \ldots, s)$ respectively. The underlying mathematical formulation of the input-oriented NSM DEA model for $DMU_d$ is given below:

$$
\begin{aligned}
&\text{Min } \psi_d^* = \psi_d - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S_{id}^-}{x_{id}} + \sum_{r=1}^{s}\frac{S_{rd}^+}{y_{rd}}\right] \\
&\text{subject to} \\
&\sum_{j=1}^{n}\lambda_j y_{rj} - S_{rd}^+ = y_{rd} \qquad \forall r = 1,\cdots,s \\
&\sum_{j=1}^{n}\lambda_j x_{ij} + S_{id}^- = \psi_d x_{id} \qquad \forall i = 1,\cdots,m \\
&\sum_{j=1}^{n}\lambda_j = 1 \qquad \forall j = 1,\cdots,n \\
&\lambda_j \geq 0 \qquad \forall j = 1,\cdots,n \\
&\psi_d \quad \text{is unrestricted in sign} \\
&S_{id}^- \geq 0,\ S_{rd}^+ \geq 0.
\end{aligned}
\tag{3.1}
$$

where, $\psi_d^*$ : Total input-oriented efficiency.
$\psi_d$: Reduction applied to all inputs of DMU to improve efficiency.
$S_{rd}^+$ : The amount of deficiency for $r^{th}$ output.
$S_{id}^-$ : The amount of excess resources used for $i^{th}$ input.
$\lambda_j$ : Intensity variables for $d^{th}$ DMU.

In the objective function, the term $(S_{rd}^+/y_{rd})$ denotes the inefficiency in the $d^{th}$ DMU due to the output shortfall in the $r^{th}$ output. Similarly, $(S_{id}^-/x_{id})$ defines the inefficiency owing to the existence of slack in the $i^{th}$ input of the $d^{th}$ DMU. Therefore, the expression $\frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S_{id}^-}{x_{id}} + \sum_{r=1}^{s}\frac{S_{rd}^+}{y_{rd}}\right]$ quantifies the average efficiency affected by slack across all the inputs and outputs. It can also calculate the average reduction rate of all $m$ inputs and augmentation rate of all $s$ outputs. Thus, the total output produced efficiency due to the radial and slack parts of all inputs and outputs is given an objective function. Variable $\lambda_j$ plays a significant role. In the mathematical model, if the optimal value $\lambda_j^*$ of $\lambda_j$ is non-zero, then the $j^{th}$ DMU is part of the reference set (peer) for the $d^{th}$ DMU. In practical terms, this implies that the $j^{th}$ DMU operates efficiently in areas where the $d^{th}$ DMU is falling short.

**Definition 3.3.1** *When $d^{th}$ DMU fulfills both the conditions, where $\psi_d^* = 1$ and all the input and output slacks $(S_{id}^+, S_{rd}^-)$ are equal to zero, it is called as total potential technical efficient DMU, Conversely, if the $d^{th}$ DMU has $\psi_d^* \leq 1$ and/or non-zero values for slacks $(S_{id}^+, S_{rd}^- \neq 0)$, it signifies either an excess or deficiency in resources, indicating inefficiency in the performance of DMU.*

**Definition 3.3.2** *The collection of indices corresponding to positive $\lambda_j$ values defines the reference set for the $d^{th}$ inefficient DMU. This reference set, denoted by $R_d$, comprises indices that represent other DMUs that are considered efficient and are used as benchmarks or references to assess the performance of the $d^{th}$ inefficient DMU. Reference set $R_d$ is explained as follows:*

$$R_d = \{DMU_j : \lambda_j > 0, \forall \ j = 1, 2, \ldots, n\} \tag{3.2}$$

**Theorem 3.3.1** *The optimal value $\psi_d^*$ of the NSM model is not greater than the optimal value of $f_d^*$ of the CCR model, i.e., $\psi_d^* \leq f_d^*$.*

**Theorem 3.3.2** *If a $DMU_d$ is efficient in the NSM model, then it is CCR efficient.*

## 3.3.1 Input Targets

The structure of the input-oriented NSM model indicates the ability to manage and regulate excessive resource quantities after implementation, consequently minimizing input redundancies, as demonstrated by the model. When a DMU is inefficient, DEA allows specific targets to be set for the inputs of these inefficient DMUs. This process aims to improve their performance, thereby enabling them to transition from inefficiency to efficiency. Each non-zero value in the slacks provided valuable insights into the depots that are not meeting their performance potential. These insights are valuable for input criteria for the "development" stage of the proposed performance enhancement strategy. The derivation of input targets is accomplished through the following equation (3.3):

$$\textit{Input Target } (t^{*-}) = \textit{Actual Input} * \textit{Overall Technical Efficiency (OTE)} - \textit{Input Slacks } (S^{*-}) \tag{3.3}$$

## 3.3.2 Efficiency Evaluation in DEA

This chapter frames benchmarks for RSRTC depots by analyzing time-series data from 2005 to 2022 concerning 52 bus depots (DMUs). This broader perspective enables observation of evolving trends, pattern identification, and comprehension of efficiency trajectories over an extended period. The aim is to pave the way for enhanced organizational performance and more effective services. However, there are differences in the scope of data and methodological nuances employed. Additionally, the analysis expands by utilizing both CRS and VRS scales, offering a dual perspective for a more comprehensive view. The CRS scale reveals insights into technical efficiency, while the VRS provides an understanding of scale efficiency, considering the impact of depots' operating sizes on their performance.

For certain depots, such as Jaisalmer, Karauli, Partapgarh, Rajasamand, Sawaimodhopur, and Shapur, there are data availability gaps for 2005-2013. To maintain consistency and reliability in the analysis, these depots are omitted from the timeframe. However, as data became consistently available from 2014 onward, these depots are included in the analysis for the years 2014-2022. This approach ensures that the efficiency calculations are based on complete datasets, thereby avoiding potential inaccuracies from earlier incomplete records.

### 3.3.3 Region-Wise Classification of Depots

Understanding the operational dynamics region-wise and recognizing the areas that require strategic intervention is crucial. This understanding is particularly pertinent in the context of the Rajasthan region, characterized by diverse geographical features such as varying rocky terrains, rolling sand dunes, wetlands, barren tracts, and river-drained plains. The topography of the region can be partitioned into the following regions:

- The Aravalli or Hilly regions
- Thar Desert and
- Other arid regions

Considering this varied topography, conducting a region-wise comparison becomes crucial for comprehending the performance disparities among various bus depots. As a result, the depots are categorized based on their operational conditions into three distinct classifications: low, medium, and high advantageous conditions. This categorization helps in assessing and understanding the performance levels of these depots within the context of their operational environments.

- **Low Advantageous Condition:** This category, characterized by the inhospitable thar desert topography, inferior road connectivity, and the absence of facilities such as schools, hospitals, and businesses, includes 25 depots. They are: Anoopgarh, Banswara, Baran, Barmer, Beawar, Bundi, Churu, Dhaulpur, Didwana, Dungarpur, Falna, Hindaun, Jaisalmer, Jalore, Karauli, Khetri, Lohagarh, Pali, Phalaudi, Partapgarh, Sardaarshahar, Shapur, Sirohi, Srimadhopur, and Tijara.
- **Medium Advantageous Condition:** Depots falling under this category operate in regions that combine both plain and Thar Desert topography and have decent road connectivity. The 14 depots in this category are Abu Road, Bharatpur, Bhilwara, Bikaner, Chittorgarh, Dausa, Hanumangarh, Jhalawar, Jhunjhunu, Kotputli, Nagaur, Rajasamand, Sawaimodhopur, and Tonk.
- **High Advantageous Condition:** This category represents regions with well-connected roads, high population density, and ample facilities, including schools, universities, hospitals, and

more. It is termed the “high advantageous” due to these favorable conditions. The 13 depots within this classification are Ajaymeru, Ajmer, Alwar, Deluxe, Ganga Nagar, Jaipur, Jodhpur, Kota, Matsya Nagar, Sikar, Udaipur, Vaishali Nagar, and Vidhyadhar Nagar.

**Table 3.1:** Classification of depots by advantageous condition.

| Category | Depots |
|---|---|
| Low Advantageous Condition | Anoopgarh, Banswara, Baran, Barmer, Beawar, Bundi, Churu, Dhaulpur, Didwana, Dungarpur, Falna, Hindaun, Jaisalmer, Jalore, Karauli, Khetri, Lohagarh, Pali, Phalaudi, Parapgarh, Sardaarshahar, Shahoura, Sirohi, Srimadhopur, Tijara |
| Medium Advantageous Condition | Abu Road, Bharatpur, Bhilwara, Bikaner, Chittorgarh, Dausa, Hanumangarh, Jhalawar Jhunjhunu, Kotputli, Nagaur, Rajasamand, Sawaimadhopur, Tonk |
| High Advantageous Condition | Ajaymeru, Ajmer, Alwar, Deluxe, Ganga Nagar, Jaipur, Jodhpur, Kota, Matsya Nagar Sikar, Udaipur, Vaishali Nagar, Vidhyadhar Nagar |

### 3.3.4 Selection of Inputs and Output

After defining the model and DMUs (bus depots), a crucial step involves determining the relevant criteria for inclusion in the model. The decision on which criteria to incorporate holds significant weight in the DEA methodology, as these criteria play a foundational role in shaping the interpretation of results. Previous literature sources [50, 236, 230, 235] have informed the choice of criteria; however, relying solely on past literature isn’t enough.

The primary objective is to conduct a comprehensive review of depot performance. This necessitates ensuring that the selected criteria are sufficiently comprehensive to provide an accurate representation of a depot’s efficiency. For instance, criteria such as the number of vehicles, employees, fuel consumption, and number of routes offer insights into resource utilization. The number of vehicles and employees can highlight the scale and capacity of operations, whereas fuel consumption provides a direct measure of operational efficiency and environmental impact. Road connectivity serves as a measure of external factors affecting a depot’s performance. In this methodology, four inputs, along with a single output criterion (passenger km occupied), are chosen to assess efficiency. The output criterion measures the total distance traveled by all passengers, representing the service provided by the depot. This rigorous approach to criteria selection is intended to provide a clear perspective on the depot operational dynamics. By adopting this method, this research offers invaluable insights for policymakers, highlighting key performance determinants and suggesting directions for legislative measures to enhance efficiency and service quality.

**Inputs Comprised:**

$I_1$**:** The number of buses is indicative of capital input.

$I_2$**:** The number of employees is indicative of labor input.

$I_3$**:** Fuel consumption, which is indicative of energy input, is calculated as
Fuel consumption (100 kl) = Description of km / average diesel consumption

$I_4$**:** The number of routes, which is described as network (connectivity) size.

**Output Comprised:**

$O_1$**:** Passenger km occupied is the cumulative distance traveled by each passenger, which is defined as below:

Passenger km occupied (lakh) = Average no. of buses $\times$ description of km $\times$ load factor

**Table 3.2:** Descriptive statistics summary of inputs and output over the period 2005–2022.

| | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $O_1$ |
|---|---|---|---|---|---|
| **Max** | 161.000 | 918.000 | 54.053 | 122.000 | 10.276 |
| **Min** | 2.000 | 24.000 | 0.518 | 3.000 | 0.069 |
| **Average** | 78.060 | 321.376 | 21.256 | 46.454 | 3.733 |
| **SD** | 7.261 | 23.661 | 1.785 | 4.409 | 0.345 |

A keen analysis suggests notable variations among the chosen inputs and outputs for depots. To gain more insights into the relationships among these variables for depots during the period 2005–2022, descriptive statistics is employed in table 3.2. The number of buses in the depots ranged from 2 to 161, while the number of employees ranged from 24 to 918. Differences in fuel consumption, from 0.518 to 54.053 kl, suggest varying operational intensities. The number of routes between 3 and 122 highlights the breadth of service coverage. The output, ranging from 0.069 to 10.276 lakhs, indicates diverse service utilization across depots and years. This data provides a picture of the varied scales and nature of RSRTC operations. Then, the collected data is structured into a table format, organizing the values of each variable corresponding to the respective years. Pearson correlation coefficient test matrix revealed strong positive relationships between the number of buses, employees, fuel consumption, and passenger km occupied, as shown in table 3.3. For instance, depots with more buses tend to have more employees, and increased fuel consumption correlates with higher passenger km. Regression analysis explained approximately 90.8% of the variance in the dependent variable. This analysis highlights the significance of the number of buses and fuel consumption as predictors, indicating that an increase in these inputs is associated with a rise in passenger km occupied.

**Table 3.3:** Correlation coefficients between input and output (period 2005-2022).

| | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $O_1$ |
|---|---|---|---|---|---|
| $I_1$ | **1** | | | | |
| $I_2$ | 0.837 | **1** | | | |
| $I_3$ | 0.750 | 0.842 | **1** | | |
| $I_4$ | 0.679 | 0.683 | 0.499 | **1** | |
| $O_1$ | 0.773 | 0.822 | 0.948 | 0.504 | **1** |

The correlation analysis is employed in table 3.2. The regression statistics depict a compelling narrative, and the coefficient of determination, $R^2$, stands at 90.08%, indicating that the regression model explains a vast majority of the variability in the dependent variable. Further underscoring this is the multiple R-value of 0.953, which points towards a robust linear relationship among the variables. Delving into individual predictors, the ANOVA test signifies the model's statistical significance, with the F-statistic being a considerable 2065.907 and an almost negligible significance F value. When examining coefficients, the number of buses (p-value close to zero), the number of employees (p-value: 0.038), and fuel consumption (p-value close to zero) emerge as substantial predictors.

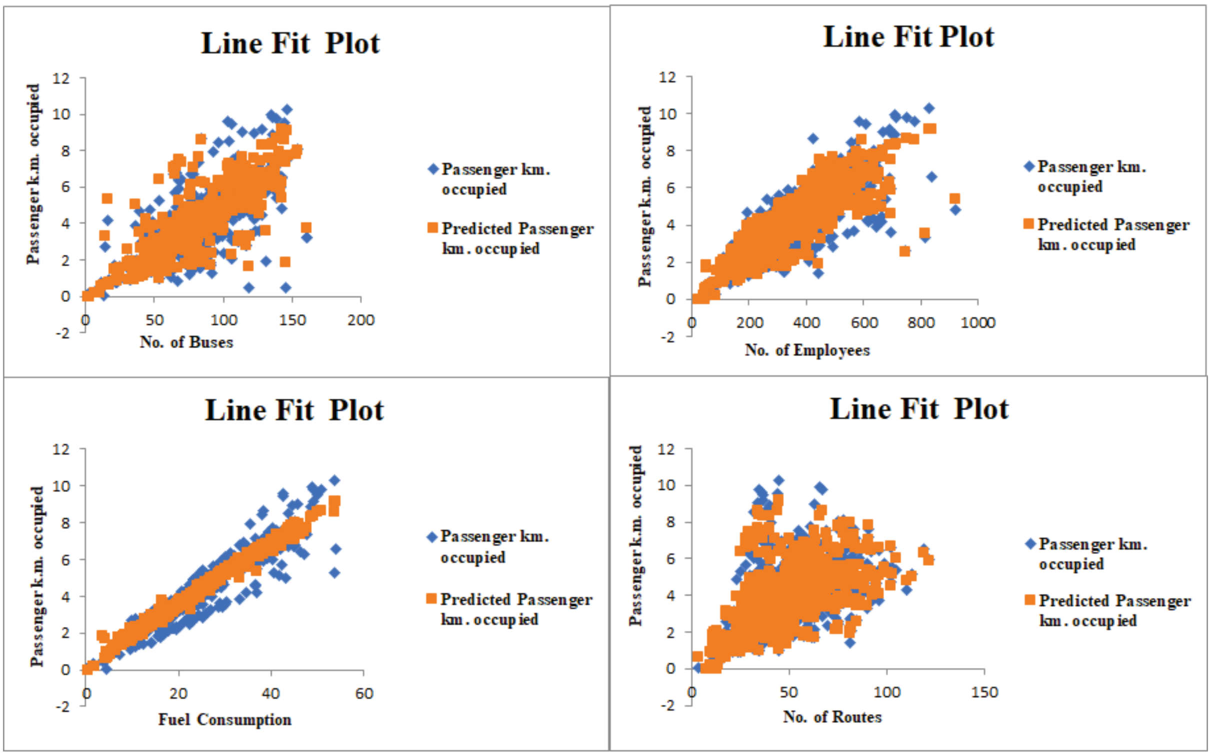


**Fig. 3.1:** Regression relation between inputs and output.

Interestingly, the number of routes doesn't hold significant weight, indicated by its p-value of 0.717. As a supplementary validation step, a cursory glance over residual analysis may be advisable to trace any patterns or anomalies. This depth of analysis reinforces the potency of the chosen variables and the statistical significance of the model.

#### 3.3.4.1 Empirical Results & Discussion

The efficiencies of depots are estimated for time series data from 2005 to 2022. OTE, PTE, and SE scores are obtained from input-oriented NSM model under CRS and VRS assumptions with categorical depots. The categorization of all depots is rooted in the varied region topography challenges and advantages that each depot faces. The classifications – low, medium, and high advantageous conditions – offer a comparative analysis of the depots' operational conditions. Table 3.4 presents the average OTE, PTE, and SE values across the depots for all three categories over the years. The depots are segmented into three primary performance categories: low advantageous condition, medium advantageous condition, and high advantageous condition. This categorization aims to distinguish between the operational capacities and efficiencies of the various depots.

#### 3.3.4.2 Overall Technical Efficiency (OTE) Scores

OTE is a measure of the overall efficiency of a depot in terms of how optimally it uses its resources to achieve the desired outputs. OTE is typically a decomposition of pure technical efficiency (PTE) and scale efficiency (SE).

**Table 3.4:** Average OTE, PTE, and SE values for the period 2005-06 to 2021-22.

| Depots | OTE | PTE | SE |
|---|---|---|---|
| **Low Advantageous Condition** | | | |
| Anoopgarh | 0.896 | 0.976 | 0.919 |
| Banswara | 0.893 | 0.931 | 0.960 |
| Baran | 0.813 | 0.852 | 0.954 |
| Barmer | 0.972 | 0.978 | 0.994 |
| Beawar | 0.896 | 0.997 | 0.898 |
| Bundi | 0.818 | 0.877 | 0.932 |
| Churu | 0.936 | 0.941 | 0.995 |
| Dhaulpur | 0.810 | 0.931 | 0.872 |
| Didwana | 0.795 | 0.918 | 0.868 |
| Dungarpur | 0.823 | 0.883 | 0.932 |
| Falna | 0.766 | 0.978 | 0.784 |
| Hindaun | 0.807 | 0.997 | 0.810 |
| Jaisalmer | 0.944 | 0.951 | 0.992 |
| Jalore | 0.911 | 0.940 | 0.969 |
| Karauli | 0.779 | 1.000 | 0.779 |
| Khetri | 0.828 | 0.894 | 0.927 |
| Lohagarh | 0.843 | 0.998 | 0.844 |
| Pali | 0.855 | 0.904 | 0.947 |
| Phalaudi | 0.901 | 0.913 | 0.987 |
| Partapgarh | 0.703 | 0.789 | 0.896 |
| Sardaarshahar | 0.872 | 0.926 | 0.942 |
| Shapur | 0.770 | 0.974 | 0.793 |
| Sirohi | 0.822 | 0.920 | 0.898 |
| Srimadhopur | 0.797 | 0.875 | 0.910 |
| Tijara | 0.942 | 0.995 | 0.948 |
| **Average** | 0.848 | 0.934 | 0.910 |
| **SD** | 0.067 | 0.053 | 0.066 |
| **Medium Advantageous Condition** | | | |
| Abu Road | 0.815 | 0.969 | 0.842 |
| Bharatpur | 0.811 | 0.991 | 0.818 |
| Bhilwara | 0.924 | 0.963 | 0.959 |
| Bikaner | 0.988 | 0.998 | 0.990 |
| Chittorgarh | 0.790 | 0.935 | 0.847 |
| Dausa | 0.747 | 1.000 | 0.747 |
| Hanumangarh | 0.943 | 1.000 | 0.943 |
| Jhalawar | 0.760 | 0.977 | 0.779 |
| Jhunjhunu | 0.902 | 0.931 | 0.968 |
| Kotputli | 0.891 | 0.935 | 0.953 |
| Nagaur | 0.922 | 0.951 | 0.970 |
| Rajasamand | 0.789 | 0.970 | 0.816 |
| Sawaimodhopur | 0.849 | 0.920 | 0.922 |
| Tonk | 0.854 | 0.954 | 0.898 |
| **Average** | 0.856 | 0.964 | 0.890 |
| **SD** | 0.074 | 0.027 | 0.080 |
| **High Advantageous Condition** | | | |
| Ajaymeru | 0.723 | 0.980 | 0.737 |
| Ajmer | 0.724 | 0.973 | 0.745 |
| Alwar | 0.733 | 0.974 | 0.752 |
| Deluxe | 0.942 | 0.974 | 0.961 |
| Ganga Nagar | 0.854 | 0.951 | 0.895 |
| Jaipur | 0.927 | 0.973 | 0.951 |
| Jodhpur | 0.789 | 0.897 | 0.875 |
| Matsya Nagar | 0.866 | 0.931 | 0.925 |
| Kota | 0.746 | 0.870 | 0.855 |
| Sikar | 0.737 | 0.969 | 0.761 |
| Udaipur | 0.771 | 0.912 | 0.843 |
| Vaishali Nagar | 0.706 | 0.989 | 0.714 |
| Vidhyadhar Nagar | 0.600 | 0.897 | 0.678 |
| **Average** | 0.778 | 0.945 | 0.822 |
| **SD** | 0.096 | 0.039 | 0.096 |

- **Low Advantageous Condition:** Within the examined dataset, efficiency values span a spectrum from 0.636 to 1 across all years. Barmer and Churu depots stand out for their consistently high metrics and incorporate their robust operational efficiency. However, an intriguing observation is a discernible decline in Churu's performance after the year 2014-15, suggesting the possibility of external challenges or strategic shifts influencing its operational dynamics. On the other hand, depots such as Dungarpur and Falna exhibited a more fluctuating efficiency trajectory across the years, indicating potential inconsistencies in their operations impacting their performance. A pattern that emerges from the data is a perceptible dip in efficiency for several depots around the years 2018-19 and 2019-20. Adding to the narrative, Pratapgarh registered the lowest average efficiency, clocking in at 0.703.

- **Medium Advantageous Condition:** The efficiency values for the depots span from 0.658 to 1 across all years. Abu Road, Bharatpur, Bhilwara, and Bikaner have maintained relatively efficient scores throughout the years, demonstrating robust performance efficiency. Chittorgarh and Dausa, while having commendable efficiency scores, display noticeable fluctuations over the years, suggesting variations in their performance. Hanumangarh stands out for its consistently high scores, especially from 2009-10 onwards. Jhalawar and Jhunjhunu registered an evident decrease in efficiency in the middle years, specifically around 2012-13, but both have shown resilience by improving their scores in subsequent years. An overall observation of the depots shows a dip in efficiency values around 2018-19 and 2019-20, a trend consistent with the previous dataset. Still, no depot maintains peak efficiency throughout the entire timeline.
- **High Advantageous Condition:** The efficiency values across the depots span from 0.448 to 1. Deluxe has the highest overall average performance, with a value of 0.942. Its performance consistently remained at the top for several years, with only a slight decrease in 2018-19, 2019-20, and 2020-21. Vidhyadhar Nagar has the lowest overall average performance, with a value of 0.600. This depot's performance seems to be consistently lower compared to the other depots across the years. Ganga Nagar showed steady and consistent growth throughout, whereas Matsya Nagar experienced peak performances in the latter years. It's also worth noting that some depots consistently perform at efficiency levels, like Jaipur, maintaining scores close to or at 1 over the years. Based on the average values for each year, 2013-14 seems to have the highest overall performance, with a value of 0.86. The year 2008-09 seems to have the lowest overall performance, with an average value of 0.647. Notably, a few depots exhibited a decline in scores between the years 2018-19 and 2020-21.

#### 3.3.4.3 Pure Technical Efficiency (PTE) Scores

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In discussions about efficiency, it's crucial to understand the assumptions of the NSM model. Under the CRS assumption, efficiency scores are presumed not to be influenced by the scale or size of an operation. Essentially, under CRS, if inputs are doubled, the outputs would ideally double as well, regardless of the operational size of a depot. However, real-world scenarios often deviate from this ideal scenario. Depots might not always operate at an optimal scale, resulting in inefficiencies. To distinguish such inefficiencies from operational challenges, the VRS assumption is considered within the NSM model. VRS plays a pivotal role in distinguishing between scale inefficiencies and those arising due to converting inputs to outputs. The efficiency score determined under the VRS assumption is termed pure technical efficiency (PTE). As the name suggests, PTE solely evaluates the transformational efficiency of inputs into outputs, abstracting away from scale considerations. It's essential to note that PTE always holds a value greater than or equal to OTE. This is because, when assessed under VRS, there are generally more depots found on the efficiency frontier compared to when evaluated under the CRS framework. This distinction emphasizes the importance of understanding the nature and sources of inefficiencies when devising strategies for improvement.

- **Low Advantageous Condition:** Beawar, Hindaun, Karauli, Lohagarh, and Tijara have consistently achieved better efficiency scores, which are close to or at 1. The year 2020-21 saw a generally increased efficiency across the depots compared to previous years, suggesting possible improvements in practices or favorable conditions. The years 2018-19 and 2019-20 witnessed a decline in average efficiency, which may indicate external challenges or operational inefficiencies that affected multiple depots. Depots like Didwana, Dungarpur, Khetri, and Srimadhopur have shown inconsistency over the years. The overall average PTE efficiency score over the years is 0.934, depots are operating at 93.4% of their potential efficiency under the VRS assumption, is still a margin of 6.6% left for improvement. Identifying specific areas of concern, challenges, and implementing targeted interventions can further boost the operational efficiency of these depots.
- **Medium Advantageous Condition:** Many depots, including Abu Road, Bharatpur, Bikaner, Dausa, and Hanumangarh, consistently operate near maximum efficiency. However, depots such as Chittorgarh, Jhunjhunu, and Kotputli exhibit fluctuating performances, suggesting areas of improvement. Rajasamand, a recent addition since 2014-15, has largely shown commendable scores near 1, but with a dip in 2020-21. The overall average efficiency remains high, but there is a noticeable dip in 2012-13, and in recent years, 2020-21 and 2021-22, the scores are slightly below the total average. Focusing on the reasons behind these fluctuations could lead to enhanced operational efficiency across all depots.
- **High Advantageous Condition:** Depots such as Ajaymeru, Ajmer, and Alwar have maintained consistently good performance scores, nearly approaching 1 across all the years, with averages nearing 0.97. Despite an overall average of 0.974, Deluxe showed a decline in scores for 2020-21 and 2021-22. Ganga Nagar and Jaipur have remained steady, with averages above 0.95. Jodhpur and Matsya Nagar, with averages around 0.9, reveal some variability but generally stay above the overall average. Kota's scores fluctuate, suggesting areas for improvement, with a 0.87 average value. Sikar is nearly impeccable, with an average of 0.969, while Udaipur's

performance is a bit more varied, holding an average of 0.912. Vaishali Nagar and Vidhyadhar Nagar showcase commendable scores, with the latter exhibiting significant improvement since its lower scores in earlier years. The overall average efficiency across all depots has remained strong, with a noticeable increase in performance from 2005-06 to 2018-19, followed by a slight decrease in the recent years, 2019-20 to 2021-22.

#### 3.3.4.4 Scale Efficiency (SE) Scores

Scale efficiency in DEA refers to the extent to which a DMU operates at its most productive scale size (MPSS). At its core, it emphasizes the significance of scale in operational efficiency. Whether a unit is operating on a large scale with expansive resources or on a small scale with limited capacity, the efficiency of its operations often hinges on its scale. A DMU that isn't operating at its MPSS might be incurring unnecessary costs or not capitalizing on potential economies of scale. A comparison of the results obtained under the CRS and VRS technology assumptions assesses whether the size of a depot influences its OTE. SE is the ratio of OTE to PTE scores. A score of one signifies optimal scale efficiency, meaning the DMU operates at its MPSS. Scores less than one, however, indicate deviations from the optimal scale. Such a DMU may either be under-scaled (too small) or over-scaled (too large) relative to its ideal operational size. The comprehensive scale efficiency evaluation from 2005 to 2022 provides a broad overview of depot performance over an extended period. As illustrated in Table 3.4, no single depot maintained consistent full efficiency throughout the examined period. Scale efficiency is a diagnostic tool that highlights scale-related inefficiencies and prompts remedial actions. For managers and decision-makers, understanding SE is instrumental in strategic planning, resource allocation, and capacity management. By continuously monitoring and adjusting scale, organizations can ensure that they are well-positioned to maximize output, minimize costs, and achieve sustainable growth.

- **Low Advantageous Condition:** Barmer, Beawar, Falna, Hindaun, and Karauli consistently show a value of 1 across most years, indicating a consistent performance or achievement of a certain benchmark. On average, depots seem to operate at an efficiency rate above 0.800, indicating a good level of scale efficiency. The depot Barmer consistently showcases high efficiency, reaching a perfect score of unity multiple times. On the contrary, Didwana has the lowest efficiency in 2005-06 with a score of 0.651, though it improved over the years. On average, most depots seem to operate at an efficiency rate above 0.800, indicating a good level of scale efficiency.
- **Medium Advantageous Condition:** Examining the overall average SE 0.889 for all the years provided, while depots maintain good scale efficiency, there is still room for improvement, especially in some specific years. However, some depots like Dausa and Jhalawar experienced

dips in their SE scores in a few specific years. The lowest SE value is 0.673, observed for the depot Dausa in 2012-13, and presents a highly significant opportunity for enhancement with a potential of 32.69%. Multiple depots have achieved a perfect SE score of 1 in various years. For instance, Bikaner achieved this score consecutively from 2005–06 to 2008–09 and then in several other years, indicating optimal operations. Similarly, depots like Bhilwara, Jhunjhunu, Kotputli, Nagaur, and Tonk also reached the perfect SE score in different years. It would be beneficial for stakeholders to delve deeper and understand the underlying factors contributing to the lower efficiencies and address them.

- **High Advantageous Condition:** The depots operated under this condition seemed to perform pretty consistent on an average evaluation, with a general average scale efficiency of 0.822. This implies that, on average, there's a potential for a 17.8% improvement in scale efficiency across all depots over the years under consideration. Deluxe and Jaipur are the standout depots in terms of scale efficiency. Over many years, they consistently achieved perfect scores of 1, indicating optimal performance. Matsya Nagar and Udaipur also showed commendable performance, occasionally hitting the maximum efficiency score. Notably, in the year 2013-14, Udaipur scored a perfect 1. Ajaymeru, Ajmer, Alwar, and Jodhpur demonstrated considerable stability throughout the years, hovering around a score of 0.74 to 0.87. The lowest SE score in the results is 0.460, which is observed for the Vidhyadhar Nagar depot in the year 2009–10.

### 3.3.5 Growth Trends Over Years

This subsection analyzed the growth trends over the years for all categories: low, medium, and high advantageous conditions. The figures 3.2, 3.3 and 3.4 provide a visual representation of the average overall, pure technical, and scale efficiency scores for each category from 2005 to 2022.

- **Low Advantageous Condition:** The average OTE fluctuated in its early years after starting in 2005–06 at a value of 0.660, dropping to its lowest level in 2009–10 at 0.639. The value increased to 0.816 in 2010–2011, a noticeable increase. This rising trend continued, reaching a high of 0.842 in 2011–2012. Following that, the OTE generally remained over the 0.8 threshold despite some ups and downs, showing continuous technical efficiency in recent years.
  The average PTE received a comparatively excellent 2005–2006 start at 0.833. From then on, it showed a rising tendency, with a little decline in 2008–2009, but shortly after, it increased steadily. From 2010 onward, the PTE numbers remained over 0.950, demonstrating excellent pure technical efficiency. It's important to note that the period from 2014–15 to 2020–21 has extraordinarily high PTE values, which are almost equal to 1, indicating nearly flawless efficiency.

The average efficiency for the years provided seems to fluctuate, but generally, there's a slight increase in scale efficiency over the years, starting from 0.892 in 2005-06 to 0.910 in 2021-22. In 2005–2006, the SE started at 0.792. The following years has a slight drop, peaking in 2009–2010 at 0.718. However, the years that followed showed a noticeable improvement, especially from 2010 to 2011, when values continuously remained over 0.800. This suggests that scale efficiency is quite consistent and durable over the past few years.

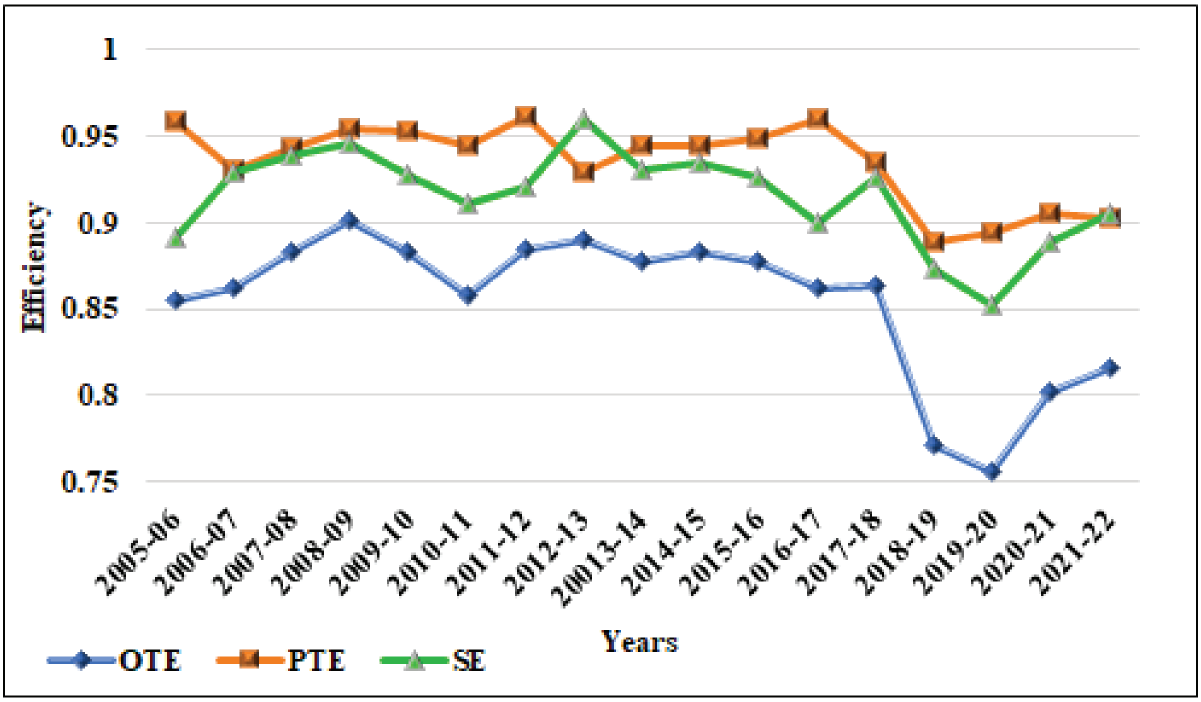


**Fig. 3.2:** Overall, pure technical, and scale efficiency scores for low advantageous category (2005-2022).

- **Medium Advantageous Condition:** OTE started out with a value of 0.855 in 2005-2006. OTE levels showed a steady rise, reaching their maximum in 2008–09 at 0.901. After that, a slight change is noted, although the efficiency remained over 0.850 until 2018–19. However, there is a considerable decline in 2018–19 and 2019–20, falling to 0.771 and 0.755, respectively. Fortunately, there is a renewal in the years that follow, pointing to a recovery at the end of the period.
  PTE, which started at a better efficiency 0.959 in 2005–2006, fluctuated a little initially but usually stayed over 0.930, suggesting incredible pure technical efficiency. Efficiency experienced decreases, most notably in 2018–19 at 0.888 and a little recovery thereafter. Overall, PTE values show that, with just a few exceptions, PTE is continuously high.
  SE began in 2005–2006 at 0.892. The trend for SE has typically upward throughout time, with values mostly rising. In 2012–2013, a substantial increase peaked at 0.960. Minor oscillations

occurred after this peak, but overall efficiency remained high. 2018–19 saw a substantial decline to 0.873, while the following years saw a rebound trend.

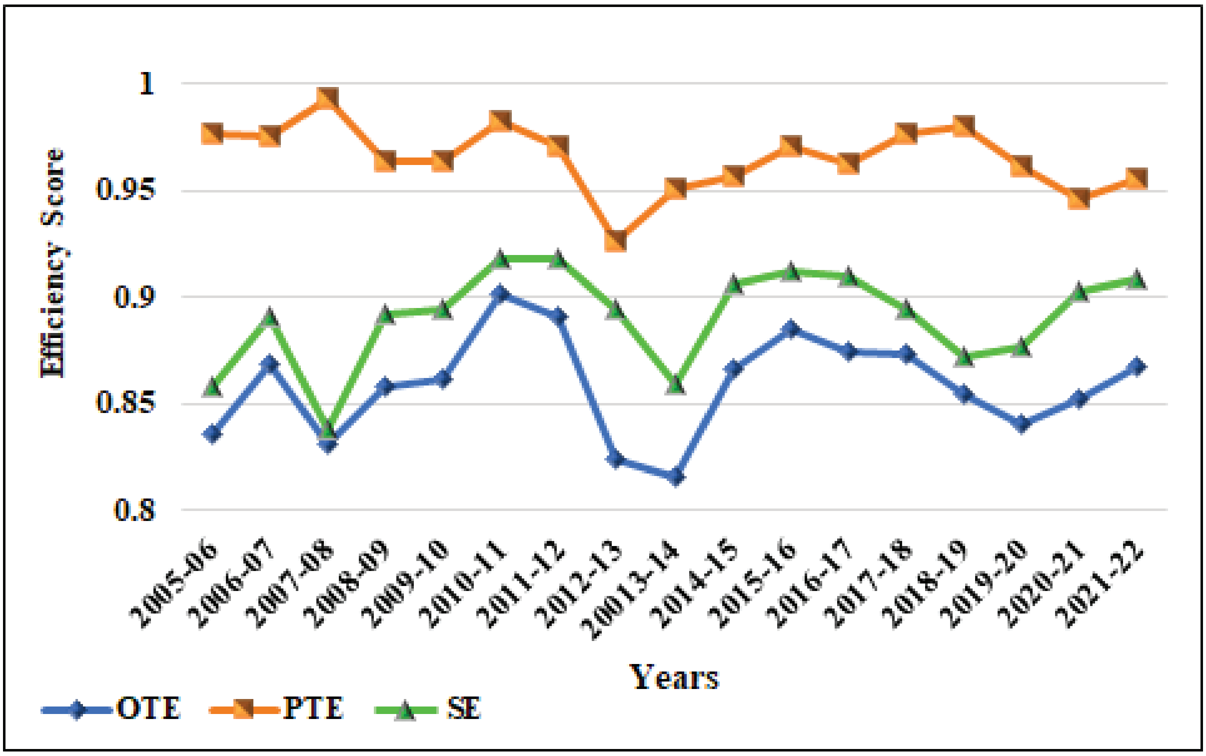


**Fig. 3.3:** Overall, pure technical, and scale efficiency scores for medium advantageous category (2005-2022).

The data shows a typically steady and high-efficiency trend across the analyzed period for all three categories. However, it's crucial to note a trend of decline in all three efficiencies, especially in the years 2018-19 and 2019-20. This declining trend highlights potential problem areas and necessitates a detailed examination of the operational elements during these years to comprehend and address the underlying causes.

- **High Advantageous Condition:** OTE score has minor fluctuations in the middle years. However, a significant decline is observed in 2018-19 and 2019-20, with a slight recovery afterward. PTE started notably high efficiency in 2005-06 and maintained good scores throughout. A drop is noticeable in 2018-19, but it managed to hover near its initial values by the end of the period. SE indicates the high performance until the year 2012–13. There are changes after this high, with a noticeable fall in 2018–19 that is consistent.

  The data indicates consistent performance across OTE, PTE, and SE over the years. Notably, all three categories experienced a decline around 2018-19. By the end of the period in 2021-22, there is a slight recovery, but not to their former peak values. This trend, especially the decline in recent years, should be a point of focus for further investigation.

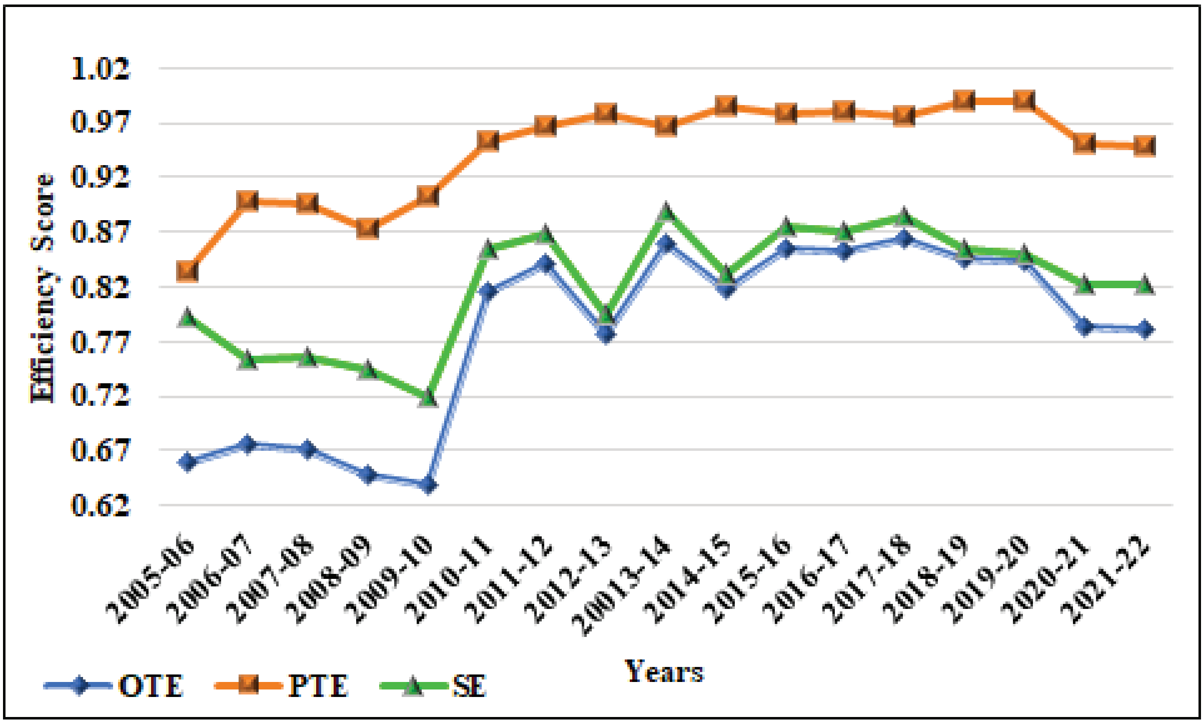


**Fig. 3.4:** Overall, pure technical, and scale efficiency scores for high advantageous category (2005-2022).

#### 3.3.5.1 Returns to Scale

Returns to scale (RTS) is a core concept in production theory for the long run and refers to how the quantity of output responds to a proportionate change in all inputs. It helps to understand how the scale of production affects overall productivity. Several factors can influence the returns to scale, including technological advancements, managerial practices, and input flexibility. RTS can be visually represented on a production function graph. Increasing returns to scale (IRS) would be a concave curve, decreasing returns to scale (DRS) would be a convex curve, and constant returns to scale (CRS) would be a straight line. Baran, Didwana, Falna, Khetri, Phalaudi, Shapur, Sirohi, and Tijara are among the 8 depots out of 25 that have consistently shown IRS despite being in low advantageous conditions from the period 2005-06 to 2021-22. This indicates a consistent pattern in these areas, where output increases in a larger proportion than the increase in inputs year after year. Abu Road, Bharatpur, Dausa, Jhalawar, Rajasamand, and Sawaimodhopur consistently exhibited IRS throughout the entire period under consideration in medium advantageous condition. Lastly, in the high advantageous conditions, Ajaymeru, Ajmer, Kota, and Vidhyadhar Nagar have consistently demonstrated IRS over the years. No depot has consistently displayed CRS or DRS during the span of the period, with the exception of those displaying IRS.

#### 3.3.5.2 Returns to Scale Trends: A Three-Year Analysis (2019-2022)

Focusing on the last three years, from 2019 to 2022, offers a more detailed, current perspective, highlighting recent behavioral shifts in depots. This narrow time frame serves as a magnifying lens, pinpointing recent advancements or challenges.

In low advantageous conditions, the following depots consistently demonstrated IRS over the three-year period: Banswara, Baran, Bundi, Churu, Dhaulpur, Didwana, Dungarpur, Falna, Hindaun, Jalore, Karauli, Khetri, Lohagarh, Pali, Phalaudi, Partapgarh, Sardaarshahar, Shapur, Sirohi, Srimadhopur, and Tijara. These depots have efficiently increased their output more than the input over these years. None of the depots consistently showed CRS over the three-year span. Anoopgarh, Barmer, Beawar, and Jaisalmer all displayed a combination of IRS, CRS, or DRS throughout that time. Anoopgarh showed DRS for the years 2020–21 and 2021-22 after starting with IRS in 2019-20. Beawar shifted from CRS in 2019-20 to IRS in the subsequent years, indicating an improvement in operational efficiency. Barmer started with IRS in 2019-20 and then stabilized at CRS in the subsequent years. Jaisalmer is operating at IRS for the years 2019-20 and 2020-21 and then shifted to CRS in 2021-22. This suggests they have transitioned to a balanced efficiency state.

In medium advantageous condition, 12 depots (Abu Road, Bharatpur, Bhilwara, Chittorgarh, Dausa, Hanumangarh, Jhalawar, Jhunjhunu, Kotputli, Nagaur, Rajasamand, and Sawaimodhopur) exhibited a consistent IRS, only 1 Bikaner depot is CRS. No depot showed a mix of IRS, CRS, or DRS over the period, implying a certain level of stability in their operations. In conclusion, the depots' performance, with a majority showcasing IRS, indicates a positive trajectory in terms of operational efficiency and scalability. Bikaner might benefit from an operational review to identify opportunities for improvement.

In high advantageous condition, the majority of the depots are either showing IRS or DRS. There are fewer depots with CRS. The depots Ajaymeru, Ajmer, Deluxe, Kota, and Vidhyadhar Nagar consistently demonstrated IRS. Jaipur and Matsya Nagar showed consistent CRS during the three years, and Alwar, Ganga Nagar, Jodhpur, Sikar, Udaipur, and Vaishali Nagar consistently pointed out as DRS depots.

The operational efficiency data across various depots over the span of three years indicates that a significant majority of the depots have consistently demonstrated IRS. This consistent performance showcases the operational prowess and effective utilization of resources within these depots. Based on the three-year analysis from 2019 to 2022, valuable insights is garnered that will pave the way for targeted improvements in the upcoming years. By understanding the trends and patterns of returns to scale during this period to make better decisions, ensuring enhanced efficiency and performance for the depot in the future.

**Table 3.5:** Distribution of depots by returns to scale (2019-22).

| | IRS | CRS | DRS |
|---|---|---|---|
| **Low Advantageous Condition** | Banswara, Baran, Bundi, Churu, Dhaulpur, Didwana, Dungarpur, Falna, Hindaun, Jalore, Karauli, Khetri, Lohagarh, Pali, Phalaudi, Partapgarh, Sardaarshahar, Shapur, Sirohi, Srimadhopur, Tijara | | |
| **Medium Advantageous Condition** | Abu Road, Bharatpur, Bhilwara, Chittorgarh, Dausa, Hanumangarh, Jhalawar, Jhunjhunu ,Kotputli, Nagaur, Rajasamand ,Sawaimodhopur, Tonk | Bikaner | |
| **High Advantageous Condition** | Ajaymeru, Ajmer, Deluxe, Kota, Vidhyadhar Nagar | Jaipur, Matsya Nagar | Alwar, Ganga Nagar, Jodhpur Sikar, Udaipur, Vaishali Nagar |

#### 3.3.5.3 Input Targets

For the upcoming target suggestions, primarily focus on the data from the current year, 2021-22. This approach ensures that the recommendations are grounded in the most recent trends and performance metrics, allowing for a more accurate and timely estimate for the forthcoming period. Leveraging this year's data enables us to make informed decisions that are relevant and responsive to current dynamics and challenges.

- **Low Advantageous Condition:**
  Some depots like Anoopgarh, Barmer, Jaisalmer, Karauli, Lohagarh, Shapur, Sirohi, and Tijara have exactly met their targets with no slacks, as evidenced by the identical values in the target and the slack columns. On the other hand, depots like Banswara, Bundi, Dhaulpur, Didwana, Dungarpur, Hindaun, Jalore, Khetri, Phalaudi, Partapgarh, Sardaarshahar, and Srimadhopur have exhibited significant deviations from their targets. For instance, Srimadhopur shows a notable difference with a slack of 19.49% in $I_1$ and a significant 49.60% in $I_2$. Such discrepancies highlight areas where more attention might be required to meet the target values. The average slack values are as follows: $I_1$: 5.58%, $I_2$: 17.31%, $I_3$: 0.74%, and $I_4$: 2.75% for each of the four inputs are noteworthy, providing additional insights into the relative performance of different depots. $I_2$ exhibits the highest slack among the four inputs. The slack values represent the gap

between the expected target and the achieved value, with higher slack indicating a greater deviation from the goal. Overall, the data provides crucial insights into how different locations are performing relative to their set targets and where improvements may be necessary. Table 3.6 provides a detailed breakdown of the actual & target inputs and associated slacks. Additionally, for a visual representation, refer to figure 3.5 for graphical visualization.

**Table 3.6:** Actual, target, and slack in inputs of low advantageous condition depots (2021-22).

| | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $Target_1$ | $Target_2$ | $Target_3$ | $Target_4$ | $Slack_1$ | $Slack_2$ | $Slack_3$ | $Slack_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Anoopgarh | 46.00 | 171.00 | 10.89 | 42.00 | 46.00 | 171.00 | 10.89 | 42.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Banswara | 61.00 | 198.00 | 10.16 | 27.00 | 50.71 | 164.60 | 8.44 | 22.45 | 10.29 | 33.40 | 1.71 | 4.55 |
| Baran | 57.00 | 181.00 | 8.63 | 27.00 | 47.32 | 150.26 | 7.17 | 22.41 | 9.68 | 30.74 | 1.47 | 4.59 |
| Barmer | 45.00 | 182.00 | 10.18 | 27.00 | 45.00 | 182.00 | 10.18 | 27.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Beawar | 43.00 | 193.00 | 10.01 | 30.00 | 42.01 | 188.55 | 9.77 | 29.31 | 0.99 | 4.45 | 0.23 | 0.69 |
| Bundi | 64.00 | 187.00 | 7.30 | 27.00 | 51.54 | 150.60 | 5.88 | 21.74 | 12.46 | 36.40 | 1.42 | 5.26 |
| Churu | 58.00 | 187.00 | 7.78 | 19.00 | 49.97 | 161.13 | 6.70 | 16.37 | 8.03 | 25.87 | 1.08 | 2.63 |
| Dhaulpur | 60.00 | 250.00 | 8.56 | 38.00 | 49.65 | 206.88 | 7.08 | 31.45 | 10.35 | 43.12 | 1.48 | 6.55 |
| Didwana | 53.00 | 156.00 | 5.93 | 25.00 | 42.92 | 126.33 | 4.80 | 20.24 | 10.08 | 29.67 | 1.13 | 4.76 |
| Dungarpur | 83.00 | 227.00 | 8.16 | 33.00 | 62.17 | 170.03 | 6.11 | 24.72 | 20.83 | 56.97 | 2.05 | 8.28 |
| Falna | 31.00 | 92.00 | 5.09 | 25.00 | 26.80 | 79.55 | 4.40 | 21.62 | 4.20 | 12.45 | 0.69 | 3.38 |
| Hindaun | 59.00 | 211.00 | 6.95 | 43.00 | 57.61 | 206.03 | 6.79 | 41.99 | 1.39 | 4.97 | 0.16 | 1.01 |
| Jaisalmer | 23.00 | 82.00 | 5.44 | 10.00 | 23.00 | 82.00 | 5.44 | 10.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Jalore | 51.00 | 159.00 | 9.09 | 23.00 | 48.15 | 150.12 | 8.58 | 21.72 | 2.85 | 8.88 | 0.51 | 1.28 |
| Karauli | 10.00 | 42.00 | 1.71 | 14.00 | 10.00 | 42.00 | 1.71 | 14.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Khetri | 48.00 | 173.00 | 6.98 | 30.00 | 37.90 | 136.61 | 5.51 | 23.69 | 10.10 | 36.39 | 1.47 | 6.31 |
| Lohagarh | 73.00 | 293.00 | 8.38 | 58.00 | 73.00 | 293.00 | 8.38 | 58.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Pali | 49.00 | 169.00 | 9.79 | 26.00 | 48.62 | 167.70 | 9.71 | 25.80 | 0.38 | 1.30 | 0.08 | 0.20 |
| Phalaudi | 41.00 | 123.00 | 6.74 | 17.00 | 39.39 | 118.16 | 6.48 | 16.33 | 1.61 | 4.84 | 0.27 | 0.67 |
| Partapgarh | 32.00 | 92.00 | 4.99 | 21.00 | 24.98 | 71.82 | 3.89 | 16.39 | 7.02 | 20.18 | 1.09 | 4.61 |
| Sardaarshahar | 58.00 | 198.00 | 10.31 | 36.00 | 48.18 | 164.49 | 8.57 | 29.91 | 9.82 | 33.51 | 1.75 | 6.09 |
| Shapur | 53.00 | 159.00 | 4.36 | 35.00 | 53.00 | 159.00 | 4.36 | 35.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Sirohi | 43.00 | 122.00 | 6.96 | 32.00 | 43.00 | 122.00 | 6.96 | 32.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Srimadhopur | 77.00 | 196.00 | 7.38 | 31.00 | 57.51 | 146.40 | 5.52 | 23.15 | 19.49 | 49.60 | 1.87 | 7.85 |
| Tijara | 42.00 | 170.00 | 5.83 | 29.00 | 42.00 | 170.00 | 5.83 | 29.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Average** | 50.40 | 168.52 | 7.50 | 29.00 | 44.82 | 151.21 | 6.77 | 26.25 | 15.03 | 13.87 | 12.67 | 11.74 |

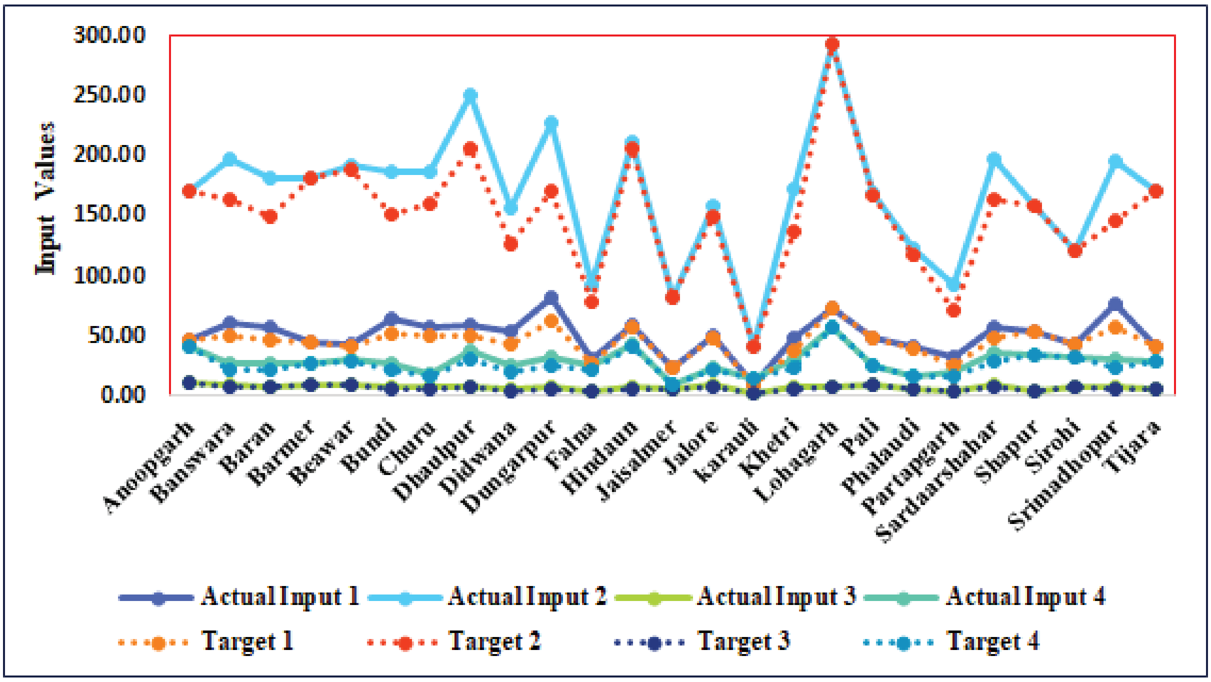


**Fig. 3.5:** Input target pattern of the low advantageous condition depots (2021-22).

**Table 3.7:** Actual, target, and slack in inputs of medium advantageous condition depots (2021-22).

| | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $Target_1$ | $Target_2$ | $Target_3$ | $Target_4$ | $Slack_1$ | $Slack_2$ | $Slack_3$ | $Slack_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Abu Road | 39.00 | 115.00 | 5.54 | 18.00 | 36.75 | 108.38 | 5.22 | 16.96 | 2.25 | 6.62 | 0.32 | 1.04 |
| Bharatpur | 59.00 | 333.00 | 9.59 | 50.00 | 59.00 | 333.00 | 9.59 | 50.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Bhilwara | 80.00 | 243.00 | 12.82 | 38.00 | 78.12 | 237.29 | 12.52 | 37.11 | 1.88 | 5.71 | 0.30 | 0.89 |
| Bikaner | 71.00 | 327.00 | 16.92 | 44.00 | 71.00 | 327.00 | 16.92 | 44.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Chittorgarh | 87.00 | 258.00 | 11.33 | 46.00 | 76.24 | 226.10 | 9.93 | 40.31 | 10.76 | 31.90 | 1.40 | 5.69 |
| Dausa | 59.00 | 230.00 | 6.57 | 49.00 | 59.00 | 230.00 | 6.57 | 49.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Hanumangarh | 76.00 | 276.00 | 15.10 | 51.00 | 76.00 | 276.00 | 15.10 | 51.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Jhalawar | 75.00 | 183.00 | 9.20 | 48.00 | 75.00 | 183.00 | 9.20 | 48.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Jhunjhunu | 83.00 | 340.00 | 12.06 | 38.00 | 69.29 | 283.85 | 10.07 | 31.72 | 13.71 | 56.15 | 1.99 | 6.28 |
| Kotputli | 49.00 | 199.00 | 7.55 | 26.00 | 41.32 | 167.80 | 6.36 | 21.92 | 7.68 | 31.20 | 1.18 | 4.08 |
| Nagaur | 61.00 | 287.00 | 13.31 | 30.00 | 59.94 | 281.99 | 13.08 | 29.48 | 1.06 | 5.01 | 0.23 | 0.52 |
| Rajasamand | 35.00 | 111.00 | 4.80 | 25.00 | 32.02 | 101.56 | 4.39 | 22.87 | 2.98 | 9.44 | 0.41 | 2.13 |
| Sawaimodhopur | 34.00 | 102.00 | 4.97 | 15.00 | 33.95 | 101.85 | 4.96 | 14.98 | 0.05 | 0.15 | 0.01 | 0.02 |
| Tonk | 75.00 | 256.00 | 9.95 | 60.00 | 75.00 | 256.00 | 9.95 | 60.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Average** | 63.07 | 232.86 | 9.98 | 38.43 | 60.19 | 222.42 | 9.56 | 36.95 | 2.88 | 10.44 | 0.42 | 1.47 |

- **Medium Advantageous Condition:**

  Bharatpur, Bikane, Dausa, Hanumangarh, Jhalawar, Tonk depots have exactly the same values for the “Actual” and “Target” inputs. This indicates that these depots have perfectly met their target values without any surplus or deficiency. The slack values for these depots are also zero across the board. Chittorgarh has significant positive slack values, especially for $I_2$ (31.90%) and $I_4$ (5.69%). Jhunjhunu also shows high positive slack values, especially for $I_2$ (56.15%) and $I_4$ (6.28%). Similarly, Kotputli and Nagaur have moderate positive slack values across the year. Rajasamand has positive slack values for all input, indicating that the depot surpassed its targets. $I_2$ has the highest variability, with some depots overshooting their targets by significant margins. The average slack values for each of the four inputs ($I_1$, $I_2$, $I_3$, and $I_4$) are notable: $I_1$ has a slack of 2.88%, $I_2$ has a slack of 10.44%, $I_3$ has a slack of 0.42%, and $I_4$ has a slack of 1.48%. These values offer further insights into the relative performance of various depots. $I_2$ exhibits the highest slack among the four inputs. The comprehensive breakdown of actual and target inputs, along with their respective slacks, can be found in table 3.7. Please refer to figure 3.6 for a graphical illustration.

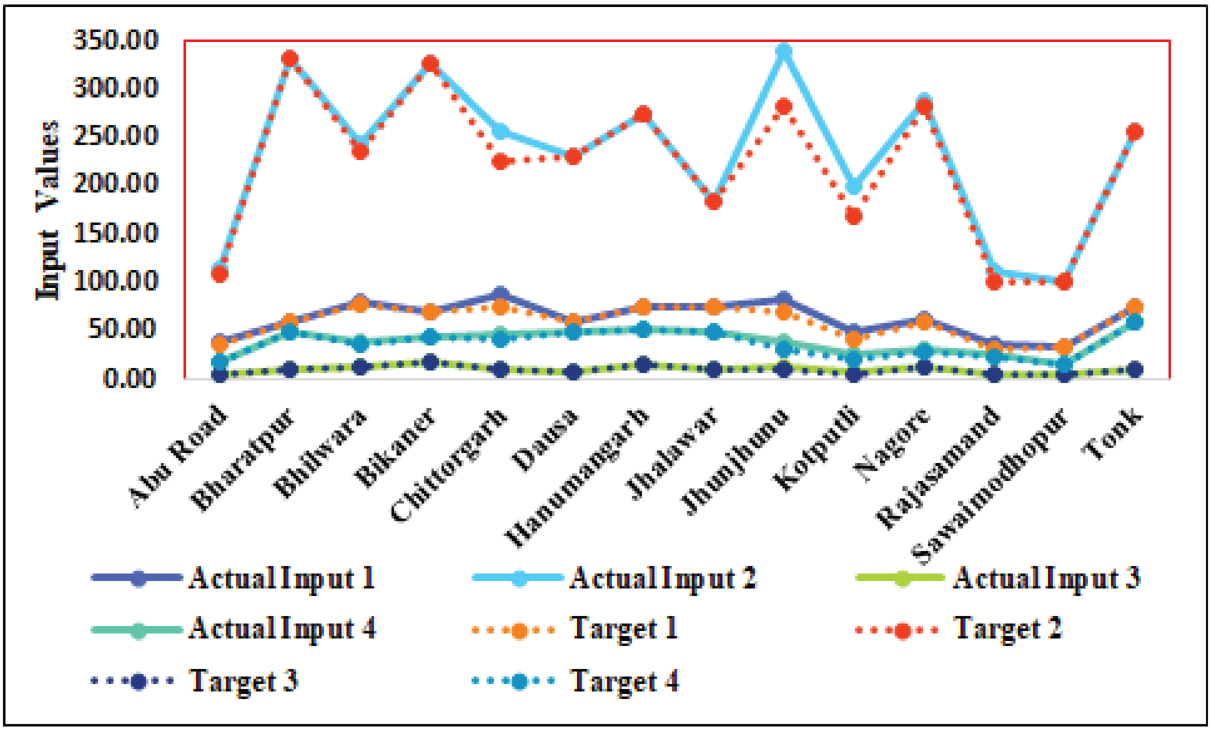


**Fig. 3.6:** Input target pattern of the medium advantageous condition depots (2021-22).

- **High Advantageous Condition:**
  Eight depots, including Ajaymeru, Alwar, Ganga Nagar, Jaipur, Jodhpur, Matsya Nagar, Udaipur, and Vidhyadhar Nagar, have achieved a perfect alignment with their targets. This can be seen as an indication of efficient operations. The depot "Sikar" stands out, having the highest positive slack values in $I_2$ (21.04%) and $I_4$ (8.10%), suggesting potential oversupply or inefficiencies in these areas.

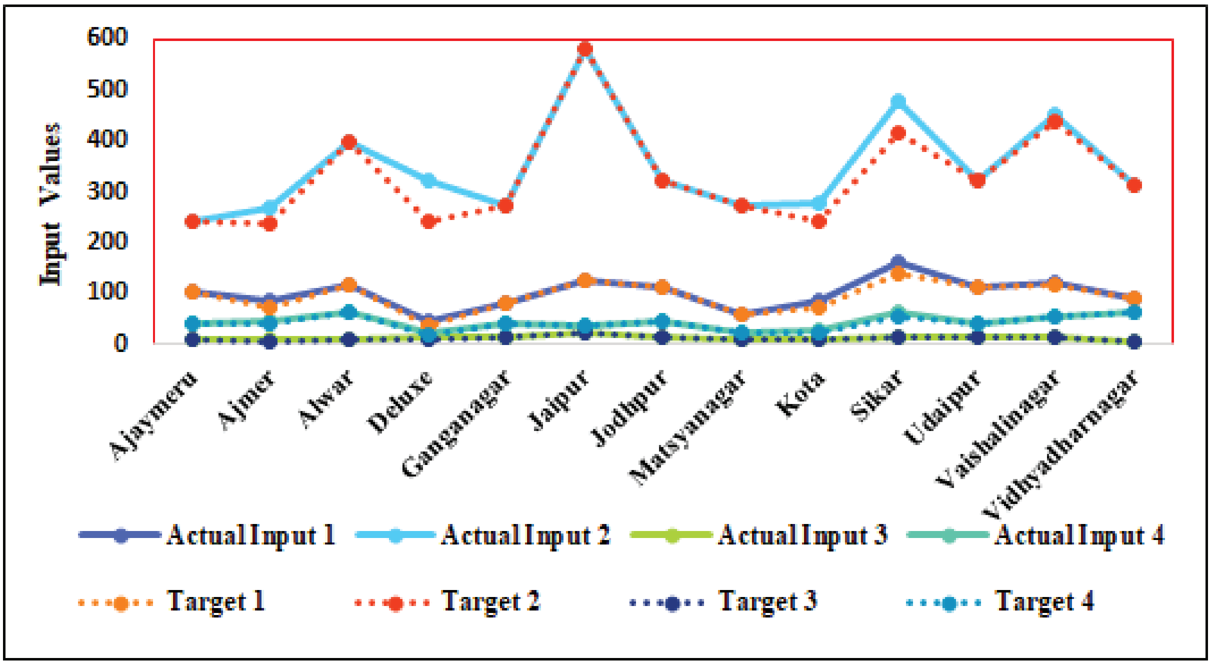


**Fig. 3.7:** Input target pattern of the high advantageous condition depots (2021-22).

**Table 3.8:** Actual, target, and slack in inputs of high advantageous condition depots (2021-22).

| | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $Target_1$ | $Target_2$ | $Target_3$ | $Target_4$ | $Slack_1$ | $Slack_2$ | $Slack_3$ | $Slack_4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ajaymeru | 106.00 | 241.00 | 9.69 | 42.00 | 106.00 | 241.00 | 9.69 | 42.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Ajmer | 84.00 | 268.00 | 8.52 | 45.00 | 74.64 | 238.14 | 7.57 | 39.99 | 9.36 | 29.86 | 0.95 | 5.01 |
| Alwar | 117.00 | 399.00 | 12.65 | 63.00 | 117.00 | 399.00 | 12.65 | 63.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Deluxe | 47.00 | 324.00 | 14.23 | 26.00 | 35.26 | 243.08 | 10.68 | 19.51 | 11.74 | 80.92 | 3.55 | 6.49 |
| Ganga Nagar | 81.00 | 273.00 | 14.35 | 40.00 | 81.00 | 273.00 | 14.35 | 40.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Jaipur | 126.00 | 579.00 | 24.00 | 37.00 | 126.00 | 579.00 | 24.00 | 37.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Jodhpur | 111.00 | 324.00 | 14.11 | 48.00 | 111.00 | 324.00 | 14.11 | 48.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Matsya Nagar | 61.00 | 271.00 | 11.86 | 26.00 | 61.00 | 271.00 | 11.86 | 26.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Kota | 84.00 | 279.00 | 11.30 | 30.00 | 72.69 | 241.45 | 9.78 | 25.96 | 11.31 | 37.55 | 1.52 | 4.04 |
| Sikar | 161.00 | 477.00 | 16.33 | 62.00 | 139.96 | 414.67 | 14.20 | 53.90 | 21.04 | 62.33 | 2.13 | 8.10 |
| Udaipur | 112.00 | 321.00 | 15.98 | 40.00 | 112.00 | 321.00 | 15.98 | 40.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Vaishali Nagar | 120.00 | 453.00 | 15.72 | 55.00 | 116.01 | 437.95 | 15.20 | 53.17 | 3.99 | 15.05 | 0.52 | 1.83 |
| Vidhyadhar Nagar | 91.00 | 314.00 | 7.15 | 63.00 | 91.00 | 314.00 | 7.15 | 63.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Average** | 100.08 | 347.92 | 13.53 | 44.38 | 95.66 | 330.56 | 12.86 | 42.43 | 4.42 | 17.36 | 0.67 | 1.96 |

On the other hand, "Deluxe" depot showcased the most significant discrepancy between actual input and targets, having the highest slack for $I_2$ (80.92%) and $I_3$ (3.55%). This situation at Deluxe merits a thorough investigation into its operations. Vaishali Nagar's operations appear relatively efficient, though they have minor positive slack values, hinting at the potential need for a slight buffer in their operations.

The average slack values for each input ($I_1$, $I_2$, $I_3$, and $I_4$) are significant: $I_1$ shows a slack of 4.42%, $I_2$ exhibits a slack of 17.36%, $I_3$ demonstrates a slack of 0.67%, and $I_4$ displays a slack of 1.96%. These figures provide additional insights into the relative performance of different depots. Notably, $I_2$ stands out with the highest slack among the four inputs. The detailed breakdown of actual and target inputs, along with their respective slacks, is provided in table 3.8 and graphical visualization in figure 3.7.

It is crucial to note that in all three categories, $I_2$ consistently exhibits the highest values. This observation emphasizes the need to allocate special attention and resources towards improving efficiency in this particular input for the current year, 2021-22. This approach allows us to make informed decisions that are relevant and responsive to current dynamics and challenges, with special consideration for the optimization of $I_2$.

## 3.4 Conclusions & Future Scope

This chapter is dedicated to employing an input-oriented NSM model under VRS assumption to measure the efficiency of RSRTC depots over the period 2005-2022. Across the 52 depots, the spectrum of efficiency under low, medium, and high advantageous conditions has showcased varied trajectories. These variations offer a fresh perspective on enhancing service quality, especially when considering the specific inherent characteristics that affect the performance of underperforming RSRTC depots. The transitional year from 2018-19 to 2019-20 stands out as crucial, necessitating further reflection to identify the reasons underlying the declines in efficiency in all categories. Future tactics that are more effective may be enabled by recognizing and addressing the difficulties of this phase. Then, the year 2021-22, can be viewed as a significant inflection point. While there is a recovery observed from the downturn of the preceding years, the depots did not necessarily reach their past performance level, indicating the conclusion of earlier instructions and the beginning of new strategies. The study calculated the input target and slacks for inefficient depots. It found a substantial disparity between actual and achievable performance under current operating conditions in the majority of depots. The continuous improvement and stability in the numbers suggest effective operational strategies and resource utilization over the years. This revamp holds the potential to bridge the observed performance gap, optimize resource utilization, and propel RSRTC toward greater efficiency.

**Table 3.9:** List of all depots of Rajasthan.

| Depot Name | Operating District | Depot Name | Operating District |
|---|---|---|---|
| Abu Road | Sirohi | Jalore | Jalore |
| Ajaymeru | Ajmer | Jhalawar | Jhalawar |
| Ajmer | Ajmer | Jhunjhunu | Jhunjhunu |
| Alwar | Alwar | Jodhpur | Jodhpur |
| Anoopgarh | Sri Ganga Nagar | Karauli | Karauli |
| Banswara | Banswara | Khetri | Jhunjhunu |
| Baran | Baran | Kota | Kota |
| Barmer | Barmer | Kotputli | Jaipur |
| Beawar | Ajmer | Lohagarh | Bharatpur |
| Bharatpur | Bharatpur | Matsya Nagar | Alwar |
| Bhilwara | Bhilwara | Nagaur | Nagaur |
| Bikaner | Bikaner | Pali | Pali |
| Bundi | Bundi | Phalaudi | Jodhpur |
| Chittorgarh | Chittorgarh | Partapgarh | Partapgarh |
| Churu | Churu | Rajasamand | Rajasamand |
| Dausa | Dausa | Sardaarshahar | Churu |
| Deluxe | Jaipur | Sawaimodhopur | Sawaimodhopur |
| Dhaulpur | Dhaulpur | Shapur | Jaipur |
| Didwana | Nagaur | Sikar | Sikar |
| Dungarpur | Dungarpur | Sirohi | Sirohi |
| Falna | Pali | Srimadhopur | Sikar |
| Ganganagar | Sri Ganganagar | Tijara | Alwar |
| Hanumangarh | Hanumangarh | Tonk | Tonk |
| Hindaun | Karauli | Udaipur | Udaipur |
| Jaipur | Jaipur | Vaishali Nagar | Jaipur |
| Jaisalmer | Jaisalmer | Vidhyadhar Nagar | Jaipur |

# Chapter 4

# An Evaluation of Productivity Change in Public Transport Sector Using DEA

## 4.1 Motivation and Objective

In the previous chapter, the purpose of efficiency analysis is to identify inefficient RSRTC bus depots and proposed target values on the efficient frontier for these depots to facilitate future improvements. However, delving deeper into the exploration of the transport sector reveals that efficiency, while crucial, presents only part of the narrative. It's important to acknowledge that when data spans several period, efficiency values alone may not sufficiently depict units' overall performance over the time. Efficiency is measured relative to an efficient frontier that changes over period, rendering efficiency analysis comparative rather than absolute. In such scenarios, studying productivity change over time becomes more reasonable as it reflects the cumulative impact of shifting efficiencies and changes in the efficient frontier. Integrating productivity analysis alongside efficiency aims to offer a more comprehensive view of the transport sector's dynamics and potential avenues for improvement. This chapter serves as a bridge, connecting the efficiency-focused findings with a broader perspective that encompasses productivity and its role in advancing the transport sector.

The non-parametric DEA approach has gained widespread recognition and popularity in the field of productivity analysis due to its versatility in handling various input-output technologies. This adaptability makes it a valuable tool for assessing productivity change in diverse industries and assisting policymakers in decision-making and planning. Abundant literature seeks to decompose productivity growth into several components, including technological change, efficiency change, and scale efficiency change. This approach ensures that resources are allocated effectively to foster sustainable economic growth and competitiveness. Productivity assessment has a pivotal role across a multitude of sectors, exemplified by its application in diverse fields

[1] This work has appeared in **Goyal, S., Agarwal, S. and Mathur T., 2022. An evaluation of the productivity change in public transport sector using DEA-based model. Management Science Letters, 12(2), pp.125-36.**

including airport operations [237], rail industry [238], auto manufacturers [239], hotel management [240], solar still technology [241], and water utility management [242].

O'Donnell (2010) [243] made a significant contribution by showcasing the possibility of decomposing all theoretically meaningful productivity indexes into common underlying factors. In the realm of productivity analysis, various indexes are developed to capture different facets of performance. Among these are ratio-based indexes like the Malmquist index, Hicks-Moorsten index, and Färe-Primont index, as well as additive indexes such as the Luenberger-HicksMoorsten index. This chapter focuses on two prominent productivity indexes: Malmquist productivity index (MPI) and Luenberger productivity index (LPI). These indexes provide valuable insights into productivity dynamics and will be instrumental in the analysis.

Conceptually introduced by Malmquist (1953) [244], the index evaluates the productivity change over time. In non-parametric framework, this index characterized by Caves et al. (1982) [57], defined the distance function as a geometric mean of two adjacent-period indexes. MPI is a widely spread approach to measure productivity change. Fare et al. (1992) [245] further developed the productivity index. MPI is a ratio-based technique that uses distance functions [246], to decomposed total productivity growth into technical efficiency change (TEC) and technological change (TC) components [247, 248]. The limitation of MPI is that one has to choose between orientation (input or output), under one corresponding consideration, either cost minimization or revenue maximization. Additionally, MPI tends to exaggerate both productivity growth and decline. To address these limitations, Chambers et al. (1996) [249] and Chamber et al. (1996) [56] introduced Luenberger productivity index (LPI), which is a difference-based index directional distance function (DDF) and has extensively used as a counterpart of the MPI. LPI adopts a different approach by considering the simultaneous contraction of inputs and expansion of outputs [250]. LPI is known for providing a more conservative estimate of productivity change. It doesn't exaggerate improvements or declines in productivity. MPI often tends to overestimate productivity changes. The study aims to calculate the potential gap between MPI and LPI using both productivity indexes with the DEA technique.

This study observes and analyzes the efficiency changes and productivity for 46 out of 52 RSRTC depots due to the non-availability of data over the period 2008-19. As per the review of literature no such research is done on the productivity of the Rajasthan public transport sector.

The rest of the chapter is organized as follows: Section 4.2 summarizes study on the productivity of the transport sector using the DEA technique. Section 4.3 contains a methodology framework. Section 4.4 discusses an empirical analysis that measures the productivity of RSRTC bus depots, providing policy implications and suggestions. The conclusions and future work are presented in the last section.

## 4.2 Literature Review

Efficiency and productivity assessments are integral to evaluating the performance of transportation systems. Odeck (2018) [251] thoroughly reviewed 11 research papers that employed various methodologies to measure efficiency and productivity within the transportation sector. Hensher and Daniels (1995) [252] evaluated the gross total factor productivity-MPI (GTMPI) of public bus operators in Australia for the financial year 1991-92. Viton (1998) [253] computed the multi-modal MPI production frontier for U.S. bus transit between 1988 and 1992. Karne et al. (2003) [202] conducted a comprehensive analysis spanning from 1996 to 2002, concentrating on the efficiency, financial performance, and productivity of the state transport systems in the Indian state of Maharashtra. Furthermore, this study subdivided the area into six distinct regions for a more detailed examination. Cho and Fan (2007) [254] developed the MPI index for the Guo Gwang bus companies. Odeck (2008) [251] used MPI methodology to assess the productivity of Norwegian bus industry data (1995-2002) for pre-mergers (1995- 1998) and post-mergers (1999-2002) years. Wang et al. (2008) [255] suggested the Malmquist DEA approach for evaluating the productivity of China's transportation over the period 1980-2005. They used the bootstrap method to estimate the confidence interval for technical efficiency. Yu (2008) [256] identified MPI to locate the source of productivity growth for the Taiwan bus transit system (TBTS). Agarwal et al. (2009) [257] examined the productivity of 34 state road transport undertakings (SRTUs) in India using the DEA-based MPI approach for the period 1989-1990 to 2000-2001. Also, multiple regression analysis is assessed to determine the impact of several background and uncontrollable variables on the productivity of SRTUs. Wu and Cathy (2009) [258] studied productivity shifts in Taipei bus transit companies from 2004 to 2007. Findings revealed a decline in average efficiency, measured by MPI, attributed to decreases in pure technical efficiency change and scale efficiency change. Meanwhile, an upswing in the average MPI due to increased technological change. Barros (2010) [259] applied MPI and LPI indexes to 122 urban transport Portugal bus companies. In a study done by Oh (2011) [260], overall productivity assessed using the Malmquist Luenberger Productivity Index (MLPI) for the Seoul 52 bus industry during the years 2003 to 2005. Notably, 2003 represented the pre-reform period, while 2005 marked the post-reform phase. Arman et al. (2013) [261] employed MPI to gauge the productivity shifts over an eight-year span (2002-2009) within Indian public transit agencies. Yu et al. (2017) [262] analyzed the meta-frontier efficiency-change (MEC), technology-change (MTC) and technology-gap-change indexes (TGC) of the transport sector in 30 Chinese provinces from 2000 to 2012 by applying the contemporaneous meta-frontier Malmquist-Luenberger carbon emission performance index (CMML) that included the non-radial DDF. Recently, Gulati (2021) [263] used the sequential Malmquist-Luenberger productivity index (SMLPI) approach to estimate the unbiased TFP of 8 passenger bus companies that operating in big metropolitan cities of India over the period 2011-2016. Moreover, Liu et al. (2021) [264] empirically addressed the green productivity growth rate and stability of China's road transportation using DEA, DDF, and global Malmquist Luenberger index (GMLI) model.

There are limited studies about the productivity change in the road transport sector globally. This work assists in productivity change in terms of technical efficiency change (TEC) and technological change (TC) analysis of RSRTC depots over the consecutive period of 2008-2019. The primary goal of this research is to help policymakers formulate effective policies to enhance the overall health and competitiveness of the RSRTC depots.

## 4.3 Mathematical Description

### 4.3.1 Technical Background of Productivity Change

Let's assume two time periods, denoted as $t$ and $t+1$. For period $t$, an input vector is defined as $x^t_{mj} \in R^m_+$ and an output vector as $y^t_{sj} \in R^s_+$. For each period, $n$ DMUs with different inputs and outputs are observed, denoted for period $t$ as $(x^t_{mj}, y^t_{sj})$, stemming from a reference production technology:

$$T_o = min\Big\{(x^t_{mj}, y^t_{sj}) \in \left(R^m_+ \times R^s_+\right) : x^t_{mj} \text{ produces } y^t_{sj}\Big\} \tag{4.1}$$

Input and output vectors are projected by DDF in a predetermined direction from themselves to the technological frontier. The production technology can be received in terms of the input Shepard distance function (SDF) [55]. The traditional SDF occurs when the direction is outside the origin. The definition of the DDF is as follows:

$$D^t_d(x^t_d, y^t_d) = inf\Big\{\psi_d : \left(x^t_{ij}, \frac{y^t_{rj}}{\psi_d}\right) \in T^t_d,\ \psi_d > 0\Big\} \tag{4.2}$$

This function returns the minimum value of '$\psi_d$' that allows the output to be divided and still remain within the production set defined by technology $T^t_d$. Since $\psi_d \leq 1$, reducing the output by the smallest factor possible yields the greatest proportional expansion of the output vector $y^t$ while considering input $x^t$ and technology $T^t_d$.

Similarly, it is possible to define the following distance functions:

$$D_d^t(x_d^{t+1}, y_d^{t+1}) = inf\left\{\psi_d : \left(x_{ij}^{t+1}, \frac{y_{rj}^{t+1}}{\psi_d}\right) \in T_d^t,\ \psi_d > 0\right\} \tag{4.3}$$

$$D_d^{t+1}(x_d^t, y_d^t) = inf\left\{\psi_d : \left(x_{ij}^t, \frac{y_{rj}^t}{\psi_d}\right) \in T_d^{t+1},\ \psi_d > 0\right\} \tag{4.4}$$

$$D_d^{t+1}(x_d^{t+1}, y_d^{t+1}) = inf\left\{\psi_d : \left(x_{ij}^{t+1}, \frac{y_{rj}^{t+1}}{\psi_d}\right) \in T_d^{t+1},\ \psi_d > 0\right\} \tag{4.5}$$

Similarly, the input distance function at time period $t$ under the production technology $T^t$ can be defined as:

$$D_d^t(x_d^t, y_d^t) = sup\left\{\psi_d : \left(\frac{x_{ij}^t}{\psi_d}, y_{rj}^t\right) \in T_d^t,\ \psi_d > 0\right\} \tag{4.6}$$

### 4.3.2 Malmquist Productivity Index

This field of study traces back to the foundational work of Solow (1996) [265], who introduced a growth accounting framework to measure productivity growth. One notable recent development in this area is the MPI, which has gained popularity due to its practical applications. A key advantage of the MPI is its versatility, allowing for easy calculations using parametric methods like SFA or a non-parametric approach such as DEA for efficiency measurement. The MPI enables the decomposition of productivity change into two distinct components: technical efficiency change (TEC) and technological change (TC). The idea of estimating productivity is an index for DMUs at consecutive periods [57], calculates productivity index using distance functions, and is defined as a geometric mean of indices from two adjacent periods. To compute MPI, it is essential to define input and output indexes accurately. This calculation assumes that the DMUs are efficient and that the production function is known in advance.

Färe et al. (1992) [245] proposed the modifications in MPI, utilizing the non-parametric DEA technique and allowing for consideration of DMUs that may not be efficient. This approach calculates either input or output-oriented DEA models. The input-oriented Malmquist index is as follows equation (4.7) for the time periods $t$ and $t+1$:

$$IMPI_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \left[ \frac{D_{d,vrs}^t(x_d^t, y_d^t)}{D_{d,crs}^t(x_d^{t+1}, y_d^{t+1})} \times \frac{D_{d,crs}^{t+1}(x_d^t, y_d^t)}{D_{d,vrs}^{t+1}(x_d^{t+1}, y_d^{t+1})} \right]^{0.5} \tag{4.7}$$

This indicates the productivity of the production point $(x^{t+1}, y^{t+1})$ in comparison to the production point $(x^t, y^t)$, where $D^t(x^t, y^t)$, $D^{t+1}(x^t, y^t)$, $D^{t+1}(x^{t+1}, y^{t+1})$ and $D^t(x^{t+1}, y^{t+1})$ represent the distance functions in the time periods $t$ and $t+1$.

If $IMPI^t$ is greater than one $(IMPI^t > 1)$, it signifies positive TFP growth, indicating a gain in productivity from period $t$ to time period $t+1$; Conversely, if it is less than one $(IMPI^t < 1)$, it indicates negative TFP growth, implying a loss in productivity. A value of $(IMPI^t = 1)$ signifies no change in productivity between $t$ to time period $t+1$.

Thus, the geometric decomposition of productivity changes into two different components, i.e., the Malmquist technical efficiency change (MTEC) and technological change (MTC) for the period $t$ (first year) and period $t+1$ (second year) [247]. The IMPI can defined as:

$$IMPI_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \frac{D_{d,vrs}^{t+1}(x_d^{t+1}, y_d^{t+1})}{D_{d,vrs}^t(x_d^t, y_d^t)} \left[ \frac{D_{d,crs}^t(x_d^{t+1}, y_d^{t+1})}{D_{d,vrs}^{t+1}(x_d^{t+1}, y_d^{t+1})} \times \frac{D_{d,vrs}^t(x_d^t, y_d^t)}{D_{d,crs}^{t+1}(x_d^t, y_d^t)} \right]^{0.5} \tag{4.8}$$

$$= MTEC_d(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) \times MTC_d(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1})$$

where $D^t(x^t, y^t)$ is defined as the input distance function for period $t$, which is given by $m$ number of input vectors to produce $s$ number of output vectors. The total productivity progresses if $IMPI^{t,t+1}(x^t, y^t)$ greater than 1, the value of $IMPI^{t,t+1}(x^t, y^t)$ less than 1 implies TFP decline and $IMPI^{t,t+1}(x^t, y^t)$ equals to 1 means constant (no change) TFP for the period $t$ to $t+1$. Currently, the first component of equation (4.8) is interpreted as the change in efficiency from period $t$ to period $t+1$; this element is known as efficiency change or catch-up impact. The second component, known as the change in technology, depicts the transition from the old to the new frontier in technology between period $t$ and $t+1$. Technology change (TC) and technical efficiency change $(TEC) > 1 (< 1, 1)$ denote growth (decline, no change) over the period, respectively. MPI can be applied on both constant return to scale (CRS) and variable return to scale (VRS) assumptions to obtain pure technical efficiency change (PTEC) and scale efficiency change (SEC), as mentioned in equations (4.10), (4.11) and (4.12).

$$IMPI_d^{t,t+1}(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) = \frac{D_{d,vrs}^{t+1}(x_d^{t+1},y_d^{t+1})}{D_{d,vrs}^{t}(x_d^{t},y_d^{t})} \times \left[\frac{D_{d,crs}^{t+1}(x_d^{t+1},y_d^{t+1})}{D_{d,crs}^{t}(x_d^{t},y_d^{t})} \times \frac{D_{d,vrs}^{t}(x_d^{t},y_d^{t})}{D_{d,vrs}^{t+1}(x_d^{t+1},y_d^{t+1})}\right] \times \left[\frac{D_{d,crs}^{t}(x_d^{t},y_d^{t})}{D_{d,crs}^{t}(x_d^{t+1},y_d^{t+1})} \times \frac{D_{d,crs}^{t+1}(x_d^{t},y_d^{t})}{D_{d,crs}^{t+1}(x_d^{t+1},y_d^{t+1})}\right]^{0.5} \tag{4.9}$$

$$= PTEC_d(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) \times SEC_d(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) \times TC_d(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1})$$

Although, as shown in equation (4.9), this index consists of three components. PTEC and SEC are measured in the first and second components, respectively, while TC is measured in the third expression. The PTEC and SEC components are the decomposition of TEC index. PTEC is defined as below:

$$PTEC_d^{t,t+1}(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) = \left[\frac{D_{d,crs}^{t+1}(x_d^{t+1},y_d^{t+1})}{D_{d,crs}^{t}(x_d^{t},y_d^{t})} \times \frac{D_{d,vrs}^{t}(x_d^{t},y_d^{t})}{D_{d,vrs}^{t+1}(x_d^{t+1},y_d^{t+1})}\right] \tag{4.10}$$

$$SEC_d^{t,t+1}(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) = \left[\frac{D_{d,crs}^{t+1}(x_d^{t+1},y_d^{t+1})}{D_{d,crs}^{t}(x_d^{t},y_d^{t})} \times \frac{D_{d,vrs}^{t}(x_d^{t},y_d^{t})}{D_{d,vrs}^{t+1}(x_d^{t+1},y_d^{t+1})}\right] \tag{4.11}$$

$$TC_d^{t,t+1}(x_d^t,y_d^t,x_d^{t+1},y_d^{t+1}) = \left[\frac{D_{d,crs}^{t}(x_d^{t},y_d^{t})}{D_{d,crs}^{t}(x_d^{t+1},y_d^{t+1})} \times \frac{D_{d,crs}^{t+1}(x_d^{t},y_d^{t})}{D_{d,crs}^{t+1}(x_d^{t+1},y_d^{t+1})}\right]^{0.5} \tag{4.12}$$

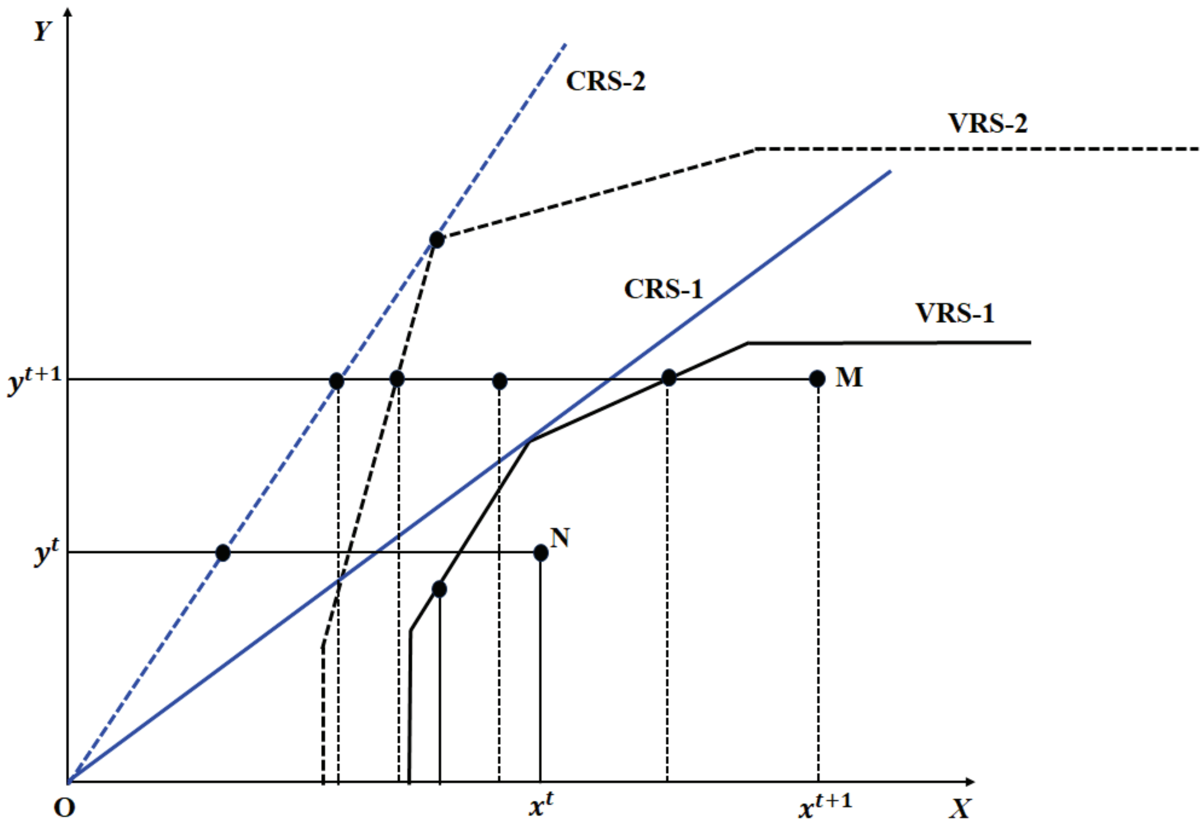


**Fig. 4.1:** MPI under CRS and VRS technology over the period.

Figure 4.1 illustrates the partitioning of technical efficiency change into PTEC and SEC, where the single input 'X' scenario and output 'Y' under CRS and VRS technology assumptions are investigated. The production function is initially exhibited increasing returns to scale, followed by constant returns, and eventually decreasing returns. Let's consider a DMU is operated at point N in time period $t$ and then moved to point M in period $t+1$. In the figure, CRS-1 and CRS-2 denote the frontiers under the CRS assumption for time periods $t$ and $t+1$, respectively. Similarly, VRS-1 and VRS-2 represent the frontiers under the VRS assumption for the same respective time periods.

Intuitively, this model is computing the input-oriented NSM-MPI based model (4.13) for calculating the productivity changes over a consecutive period. $x^t_{id}$ is $i^{th}$ input and $y^t_{rd}$ is $r^{th}$ output of $d^{th}$ DMU for the time period $t$. Now, $D^t(x^t_d, y^t_d)$ and $\psi^*$ represent the OTE score indicating the input reduction required to produce the given output level. The following models (4.13), (4.14), (4.15) and (4.16) presents a summary of the new index model.

$$
\begin{aligned}
& D^t_{d,crs}(x^t_d, y^t_d) = \psi_d - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S^-_{id}}{x^t_{id}} + \sum_{r=1}^{s}\frac{S^+_{rd}}{y^t_{rd}}\right] \\
& \text{subject to} \\
& \sum_{j=1}^{n}\lambda_{jd}y^t_{rj} - S^+_{rd} = y^t_{rd} \qquad \forall(\ r = 1,\cdots,s) \\
& \sum_{j=1}^{n}\lambda_{jd}x^t_{ij} + S^-_{id} = \psi_d x^t_{id} \qquad \forall(\ i = 1,\cdots,m) \\
& \lambda_{jd} \geq 0 \qquad \forall(\ j = 1,\cdots,n) \\
& \psi_d \quad \text{is unrestricted in sign} \\
& S^-_{id} \geq 0,\ S^+_{rd} \geq 0.
\end{aligned}
\tag{4.13}
$$

In the same manner, $D^{t+1}_d(x^{t+1}_d, y^{t+1}_d)$ can be obtained by using the inputs and outputs of the period $t+1$ instead of period $t$,

$$
\begin{aligned}
& D^{t+1}_{d,crs}(x^{t+1}_d, y^{t+1}_d) = \psi_d - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S^-_{id}}{x^{t+1}_{id}} + \sum_{r=1}^{s}\frac{S^+_{rd}}{y^{t+1}_{rd}}\right] \\
& \text{subject to} \\
& \sum_{j=1}^{n}\lambda_{jd}y^{t+1}_{rj} - S^+_{rd} = y^{t+1}_{rd} \qquad \forall(\ r = 1,\cdots,s) \\
& \sum_{j=1}^{n}\lambda_{jd}x^{t+1}_{ij} + S^-_{id} = \psi_d x^{t+1}_{id} \qquad \forall(\ i = 1,\cdots,m) \\
& \lambda_{jd} \geq 0 \qquad \forall(\ j = 1,\cdots,n) \\
& \psi_d \quad \text{is unrestricted in sign} \\
& S^-_{id} \geq 0,\ S^+_{rd} \geq 0.
\end{aligned}
\tag{4.14}
$$

where, $D^t_d(x^{t+1}_d, y^{t+1}_d)$ is the OTE for the first mixed period $t$ and $t+1$ obtained by solving,

$$
\begin{aligned}
& D^t_{d,crs}(x^{t+1}_d, y^{t+1}_d) = \psi_d - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S^-_{id}}{x^{t+1}_{id}} + \sum_{r=1}^{s}\frac{S^+_{rd}}{y^{t+1}_{rd}}\right] \\
& \text{subject to} \\
& \sum_{j=1}^{n}\lambda_{jd}y^t_{rj} - S^+_{rd} = y^{t+1}_{rd} \quad \forall(\ r = 1,\cdots,s) \\
& \sum_{j=1}^{n}\lambda_{jd}x^t_{ij} + S^-_{id} = \psi_d x^{t+1}_{id} \quad \forall(\ i = 1,\cdots,m) \\
& \lambda_{jd} \geq 0 \quad \forall(\ j = 1,\cdots,n) \\
& \psi_d \quad \text{is unrestricted in sign} \\
& S^-_{id} \geq 0,\ S^+_{rd} \geq 0.
\end{aligned}
\tag{4.15}
$$

where, $D^{t+1}_d(x^t_d, y^t_d)$ is second mixed period OTE as shown below,

$$
\begin{aligned}
& D^{t+1}_{d,crs}(x^t_d, y^t_d) = \psi_d - \frac{1}{m+s}\left[\sum_{i=1}^{m}\frac{S^-_{id}}{x^t_{id}} + \sum_{r=1}^{s}\frac{S^+_{rd}}{y^t_{rd}}\right] \\
& \text{subject to} \\
& \sum_{j=1}^{n}\lambda_{jd}y^{t+1}_{rj} - S^+_{rd} = y^t_{rd} \quad \forall(\ r = 1,\cdots,s) \\
& \sum_{j=1}^{n}\lambda_{jd}x^{t+1}_{ij} + S^-_{id} = \psi_d x^t_{id} \quad \forall(\ i = 1,\cdots,m) \\
& \lambda_{jd} \geq 0 \quad \forall(\ j = 1,\cdots,n) \\
& \psi_d \quad \text{is unrestricted in sign} \\
& S^-_{id} \geq 0,\ S^+_{rd} \geq 0.
\end{aligned}
\tag{4.16}
$$

### 4.3.3 Luenberger Productivity Index

The MPI relies on radial DEA models, specifically input- and output-oriented ones. This implies that one can either reduce inputs while keeping outputs constant or expand outputs while maintaining inputs at a fixed level. It is not possible to simultaneously improve both inputs and outputs. In contrast, LPI represents the shortage distance function that takes into consideration both input reductions and output progress [249, 266], for time periods $t$ and $t+1$ based on the production technology of time period $t$, defined as follows in equation (4.17):

$$LPI_d^t(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \left(D_d^t(x_d^t, y_d^t) - D_d^t(x_d^{t+1}, y_d^{t+1})\right) \tag{4.17}$$

Similarly, LPI for periods $t$ and $t+1$ consuming the production technology of period $t+1$ is defined as:

$$LPI_d^{t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \left(D_d^{t+1}(x_d^t, y_d^t) - D_d^{t+1}(x_d^{t+1}, y_d^{t+1})\right) \tag{4.18}$$

The arithmetic average of the two LPIs (4.17) and (4.18) for two periods $t$ and $t+1$ proposed by Chambers et al. (1996) [249] is as follows:

$$LPI_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \frac{1}{2}\left[\left(D_d^t(x_d^t, y_d^t) - D_d^t(x_d^{t+1}, y_d^{t+1})\right) + \left(D_d^{t+1}(x_d^t, y_d^t) - D_d^{t+1}(x_d^{t+1}, y_d^{t+1})\right)\right] \tag{4.19}$$

On the contrary, the LPI index is an arithmetic mean of the DDF indices for $t$ and $t+1$ period. Similarly, LPI can be separated into two parts:

$$\begin{aligned} LPI_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = {} & (D_d^t(x_d^t, y_d^t) - D_d^{t+1}(x_d^{t+1}, y_d^{t+1}) + \\ & \frac{1}{2}\left[\left(D_d^{t+1}(x_d^{t+1}, y_d^{t+1}) - D_d^t(x_d^{t+1}, y_d^{t+1}) + (D_d^{t+1}(x_d^t, y_d^t) - D_d^t(x_d^t, y_d^t)\right)\right] \end{aligned} \tag{4.20}$$

Here, $LPI^{t,t+1} > (<)0$ implies progress (regress) in productivity from period $t$ to $t+1$ due to its components efficiency change and technological change. $LPI^{t,t+1} = 0$ implies the productivity remains same over the two periods $t$ and $t+1$.

The LPI index's Luenberger technological change (LTC) and technical efficiency change (LTEC) components. This decomposition is motivated by the MPI, which is defined by:

$$LTC_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = (D_d^t(x_d^t, y_d^t) - D_d^{t+1}(x_d^{t+1}, y_d^{t+1}) \tag{4.21}$$

$$LTEC_d^{t,t+1}(x_d^t, y_d^t, x_d^{t+1}, y_d^{t+1}) = \frac{1}{2}\left[\left(D_d^{t+1}(x_d^{t+1}, y_d^{t+1}) - D_d^t(x_d^{t+1}, y_d^{t+1}) + (D_d^{t+1}(x_d^t, y_d^t) - D_d^t(x_d^t, y_d^t)\right)\right] \tag{4.22}$$

**Table 4.1:** Statistics summary of RSRTC depots for the period 2008-19.

| Years | | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $O_1$ | $O_2$ |
|---|---|---|---|---|---|---|---|
| 2008-09 | Mean | 88.48 | 372.61 | 25.01 | 11151.6 | 4.36 | 394.83 |
| | Max | 134 | 624 | 42.79 | 19529 | 9.61 | 572 |
| | Min | 45 | 160 | 11.61 | 3151 | 1.86 | 277 |
| | SD | 23.84 | 129.67 | 7.62 | 3504.77 | 1.51 | 50.36 |
| 2009-10 | Mean | 88.22 | 368.09 | 25.56 | 11169.1 | 4.52 | 399 |
| | Max | 135 | 696 | 44.59 | 20489 | 8.95 | 578 |
| | Min | 45 | 156 | 11.81 | 3979 | 1.86 | 280 |
| | SD | 23.82 | 127.66 | 7.75 | 3746.12 | 1.55 | 49.58 |
| 2010-11 | Mean | 90.35 | 376.87 | 25.57 | 11074 | 4.67 | 395.8 |
| | Max | 143 | 918 | 48.96 | 21800 | 9.94 | 586 |
| | Min | 43 | 150 | 11.79 | 3713 | 2.01 | 272 |
| | SD | 26.34 | 150.18 | 8.19 | 3930.38 | 1.58 | 50.9 |
| 2011-12 | Mean | 91.41 | 395.57 | 26.48 | 12236 | 4.7 | 403.46 |
| | Max | 142 | 815 | 50.84 | 30810 | 9.79 | 607 |
| | Min | 43 | 165 | 12.4 | 3506 | 2.11 | 267 |
| | SD | 25.49 | 141.88 | 8.57 | 4843.67 | 1.54 | 52.01 |
| 2012-13 | Mean | 88.63 | 367.54 | 25.97 | 10172.7 | 4.5 | 394.93 |
| | Max | 145 | 744 | 49.09 | 20243 | 9.79 | 614 |
| | Min | 44 | 147 | 12.25 | 3490 | 2.14 | 259 |
| | SD | 24.63 | 134.77 | 8.86 | 3365.72 | 1.62 | 52.66 |
| 2013-14 | Mean | 88.63 | 405.04 | 25.5 | 10351 | 4.59 | 393.8 |
| | Max | 145 | 836 | 54.05 | 18535 | 10.49 | 614 |
| | Min | 44 | 176 | 11.58 | 3301 | 1.96 | 246 |
| | SD | 24.63 | 142.89 | 9.56 | 3369.77 | 1.71 | 53.32 |
| 2014-15 | Mean | 90.26 | 390.48 | 26 | 10234.7 | 4.66 | 394.96 |
| | Max | 154 | 826 | 53.72 | 18552 | 10.28 | 615 |
| | Min | 49 | 172 | 12.81 | 3530 | 2.08 | 287 |
| | SD | 25.25 | 133.8 | 8.91 | 3103.05 | 1.64 | 50.67 |
| 2015-16 | Mean | 86.65 | 365.91 | 24.53 | 9911.37 | 4.54 | 401.63 |
| | Max | 153 | 776 | 50.09 | 17782 | 9.55 | 650 |
| | Min | 47 | 176 | 12.25 | 3189 | 2.19 | 337 |
| | SD | 25 | 122.89 | 8.52 | 3014.77 | 1.54 | 50.55 |
| 2016-17 | Mean | 83.11 | 339.5 | 23.81 | 9234.76 | 4.2 | 391.72 |
| | Max | 147 | 709 | 48.57 | 17368 | 8.89 | 608 |
| | Min | 50 | 148 | 11.33 | 2703 | 1.98 | 323 |
| | SD | 22 | 115.4 | 8.34 | 3153.92 | 1.4 | 46.65 |
| 2017-18 | Mean | 81.28 | 315.35 | 25.1 | 10065.3 | 4.45 | 387.33 |
| | Max | 141 | 692 | 49.32 | 17794 | 9.17 | 596 |
| | Min | 45 | 144 | 10.86 | 4338 | 2.16 | 306 |
| | SD | 22.13 | 112.56 | 8.04 | 2804.66 | 1.36 | 46.25 |
| 2018-19 | Mean | 74.5 | 290.98 | 22.47 | 9543.28 | 4.19 | 391.93 |
| | Max | 140 | 665 | 45.75 | 16769 | 9.02 | 586 |
| | Min | 41 | 123 | 9.86 | 4586 | 1.97 | 311 |
| | SD | 22.05 | 108.18 | 7.41 | 2682.35 | 1.34 | 45.38 |

### 4.3.4 Data, Inputs and Outputs Selection

Improving the performance and delivering the effective service of the transport sector is a vital goal of this proposed study. This study is using secondary data of 52 depots from the annual report of RSRTC. The research is hampered by a lack of data points for a few years, excluding the six depots (Jaisalmer, Karauli, Partapgarh, Rajasamand, Sawaimodhopur and Shapur). Data is taken for the period 2008-2019 of 46 RSRTC depots. The following section first describes four inputs and two outputs that contribute to evaluating efficiency in the transport sector identified from the literature survey [236, 203, 230]. Specifically, the number of buses $(I_1)$, the number of employees $(I_2)$, fuel consumption $(I_3)$ and routes distance $(I_4)$ are inputs. The outputs are passenger-kilometer occupied $(O_1)$ and vehicle utilization $(O_2)$. Table 4.1 represents the summary of statistics of all the variables for 46 depots from 2008 to 2019.

## 4.4 Empirical Results and Discussion

Initially, operational technical efficiency (OTE) is computed using the input-oriented NSM DEA model. The best and worst depots are investigated based on efficiency values across all the depots. These results are given in table 4.2. In addition, the extended analysis applying the NSM model along with IMPI and LPI to determine the productivity changes of 46 depots refined and deteriorated over the period 2008-2019 in the following subsection. If the efficiency score is 1, then DMU is referred to as efficient; otherwise inefficient (<1). It is noticed that half of the number of depots during the study period is relatively less than the average efficiency score of 0.888. The efficiency scores show Deluxe and Falna are the most efficient $(\psi^* = 1)$ depots in the study period. The remaining depots are inefficient for at least one year during the study period. On the other hand, Jhalawar has the lowest average inefficiency score of 0.748 during the entire period. Jaipur is not efficient only for the year 2008-09. Similarly, Kotputli and Phalaudi are not efficient only for 2010-17. All the depots are performing better in the year 2017-18, with the highest average efficiency value is 91.597%. The average minimum efficiency value in the year 2012-13 is 78.612%. The following subsections describe the IMPI, LPI, MITEC, MTC, LTEC, and LTC values during the study period by table and figures.

**Table 4.2:** Efficiency scores of 46 depots for the period 2008-19.

| Depots | Years | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2008-09 | 2009-10 | 2010-11 | 2011-12 | 2012-13 | 2013-14 | 2014-15 | 2015-16 | 2016-17 | 2017-18 | 2018-19 | Mean |
| Abu Road | 0.855 | 0.760 | 0.858 | 0.885 | 0.933 | 0.981 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.934 |
| Ajaymeru | 0.691 | 0.699 | 0.812 | 0.853 | 0.838 | 0.845 | 0.831 | 0.830 | 0.792 | 0.833 | 0.768 | 0.799 |
| Ajmer | 0.828 | 0.805 | 0.884 | 0.866 | 0.875 | 0.851 | 0.885 | 0.862 | 0.855 | 0.862 | 0.758 | 0.848 |
| Alwar | 0.711 | 0.705 | 0.905 | 0.870 | 0.829 | 0.805 | 0.829 | 0.853 | 0.866 | 0.901 | 0.827 | 0.827 |
| Anoopgarh | 0.869 | 0.865 | 0.919 | 0.954 | 1.000 | 1.000 | 0.995 | 0.916 | 0.869 | 1.000 | 0.956 | 0.940 |
| Banswara | 0.854 | 0.873 | 0.886 | 0.956 | 0.954 | 0.915 | 0.859 | 0.884 | 0.879 | 0.839 | 0.860 | 0.887 |
| Baran | 0.798 | 0.890 | 0.908 | 0.910 | 0.942 | 0.869 | 0.843 | 0.874 | 0.873 | 0.887 | 0.846 | 0.877 |
| Barmer | 0.935 | 0.904 | 0.958 | 1.000 | 1.000 | 0.947 | 0.976 | 1.000 | 1.000 | 1.000 | 1.000 | 0.975 |
| Beawar | 0.709 | 0.736 | 0.836 | 0.835 | 0.848 | 0.897 | 0.949 | 0.877 | 0.925 | 0.987 | 1.000 | 0.873 |
| Bharatpur | 0.758 | 0.768 | 0.777 | 0.821 | 0.893 | 0.848 | 0.877 | 0.854 | 0.849 | 0.866 | 0.801 | 0.828 |
| Bhilwara | 0.739 | 0.784 | 0.912 | 0.888 | 0.891 | 0.920 | 0.935 | 0.914 | 0.965 | 1.000 | 0.931 | 0.898 |
| Bikaner | 0.782 | 0.705 | 0.900 | 0.914 | 0.824 | 0.850 | 0.902 | 0.872 | 0.860 | 0.914 | 0.860 | 0.853 |
| Bundi | 0.761 | 0.780 | 0.791 | 0.841 | 0.905 | 0.841 | 0.892 | 0.882 | 0.858 | 0.808 | 0.812 | 0.834 |
| Chittorgarh | 0.669 | 0.665 | 0.779 | 0.801 | 0.800 | 0.752 | 0.774 | 0.778 | 0.805 | 0.861 | 0.748 | 0.766 |
| Churu | 0.971 | 0.903 | 0.919 | 0.923 | 1.000 | 1.000 | 1.000 | 0.966 | 0.917 | 0.987 | 0.953 | 0.958 |
| Dausa | 0.700 | 0.736 | 0.758 | 0.795 | 0.854 | 0.780 | 0.834 | 0.864 | 0.869 | 0.951 | 0.894 | 0.821 |
| Deluxe | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Dhaulpur | 0.757 | 0.788 | 0.794 | 0.918 | 0.925 | 0.861 | 0.884 | 0.861 | 0.865 | 0.858 | 0.856 | 0.852 |
| Didwana | 0.946 | 0.918 | 0.843 | 0.820 | 0.943 | 0.903 | 0.928 | 0.963 | 0.907 | 0.958 | 0.917 | 0.913 |
| Dungarpur | 0.732 | 0.722 | 0.858 | 0.916 | 0.851 | 0.837 | 0.890 | 0.863 | 0.811 | 0.749 | 0.727 | 0.814 |
| Falna | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Ganga Nagar | 0.738 | 0.761 | 0.945 | 0.997 | 0.970 | 1.000 | 0.949 | 0.942 | 0.904 | 0.927 | 0.902 | 0.912 |
| Hanumangarh | 0.789 | 0.758 | 0.898 | 0.996 | 1.000 | 1.000 | 0.944 | 0.971 | 0.914 | 0.888 | 0.854 | 0.91 |
| Hindaun | 0.776 | 0.763 | 0.778 | 0.843 | 0.865 | 0.855 | 0.885 | 0.901 | 0.897 | 0.881 | 0.856 | 0.845 |
| Jaipur | 0.841 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.986 |
| Jalore | 0.932 | 0.892 | 0.991 | 0.868 | 0.859 | 0.839 | 0.889 | 0.872 | 1.000 | 0.858 | 0.900 | 0.900 |
| Jhalawar | 0.671 | 0.713 | 0.768 | 0.744 | 0.825 | 0.729 | 0.728 | 0.746 | 0.763 | 0.767 | 0.773 | 0.748 |
| Jhunjhunu | 0.827 | 0.973 | 0.989 | 1.000 | 0.882 | 0.894 | 0.887 | 0.909 | 0.819 | 0.939 | 0.879 | 0.909 |
| Jodhpur | 0.673 | 0.665 | 0.869 | 0.932 | 0.887 | 0.814 | 0.859 | 0.869 | 1.000 | 0.970 | 0.890 | 0.857 |
| Khetri | 0.955 | 0.847 | 0.874 | 0.866 | 0.918 | 0.907 | 0.834 | 0.865 | 0.795 | 0.915 | 0.985 | 0.887 |
| Kota | 0.694 | 0.708 | 0.839 | 0.865 | 0.808 | 0.790 | 0.823 | 0.811 | 0.993 | 0.796 | 0.776 | 0.809 |
| Kotputli | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.887 | 1.000 | 1.000 | 0.99 |
| Lohagarh | 0.790 | 0.796 | 0.840 | 0.932 | 0.914 | 0.838 | 0.846 | 0.864 | 0.858 | 0.826 | 0.819 | 0.848 |
| Matsya Nagar | 0.824 | 0.876 | 0.963 | 0.990 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.968 |
| Nagaur | 0.855 | 0.839 | 0.838 | 0.920 | 0.893 | 0.922 | 0.875 | 0.893 | 0.888 | 0.926 | 0.886 | 0.885 |
| Pali | 0.984 | 0.982 | 0.957 | 0.994 | 1.000 | 0.947 | 0.987 | 0.998 | 1.000 | 1.000 | 0.974 | 0.984 |
| Phalaudi | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.985 | 1.000 | 1.000 | 0.999 |
| Sardaarshahar | 0.915 | 0.847 | 0.873 | 0.911 | 0.900 | 0.958 | 0.918 | 0.897 | 0.916 | 0.909 | 0.946 | 0.908 |
| Sikar | 0.697 | 0.669 | 0.828 | 0.888 | 0.831 | 0.820 | 0.875 | 0.915 | 1.000 | 1.000 | 0.941 | 0.860 |
| Sirohi | 0.895 | 0.963 | 0.953 | 0.986 | 0.957 | 0.999 | 0.968 | 0.974 | 1.000 | 1.000 | 0.982 | 0.971 |
| Srimadhopur | 0.861 | 0.809 | 0.781 | 0.835 | 0.943 | 0.816 | 0.791 | 0.768 | 0.737 | 0.790 | 0.811 | 0.813 |
| Tijara | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 0.955 | 1.000 | 1.000 | 0.996 |
| Tonk | 0.814 | 0.808 | 0.871 | 0.885 | 0.866 | 0.843 | 0.841 | 0.838 | 1.000 | 0.853 | 0.826 | 0.859 |
| Udaipur | 0.644 | 0.625 | 0.800 | 0.810 | 0.848 | 0.737 | 0.834 | 0.802 | 1.000 | 0.791 | 0.858 | 0.795 |
| Vaishali Nagar | 0.703 | 0.688 | 0.763 | 0.768 | 0.679 | 0.781 | 0.821 | 0.813 | 0.833 | 0.839 | 0.810 | 0.773 |
| Vidhyadhar Nagar | 0.918 | 0.690 | 0.984 | 0.988 | 1.000 | 0.963 | 0.936 | 1.000 | 1.000 | 1.000 | 1.000 | 0.948 |
| **Mean** | 0.823 | 0.819 | 0.885 | 0.908 | 0.912 | 0.895 | 0.904 | 0.903 | 0.911 | 0.914 | 0.895 | 0.888 |

### 4.4.1 IMPI & LPI Results

As mentioned earlier, IMPI is a combination of MTEC and MTC. These two components are individually enumerated and then analyzed. Furthermore, the changes in MTEC can be decomposed into two main components: PTEC and SEC. Figure 4.2 represents the annual average values of TFP using IMPI and LP1 applied to measure the productivity changes with an input-oriented NSM model for all depots during the study period.

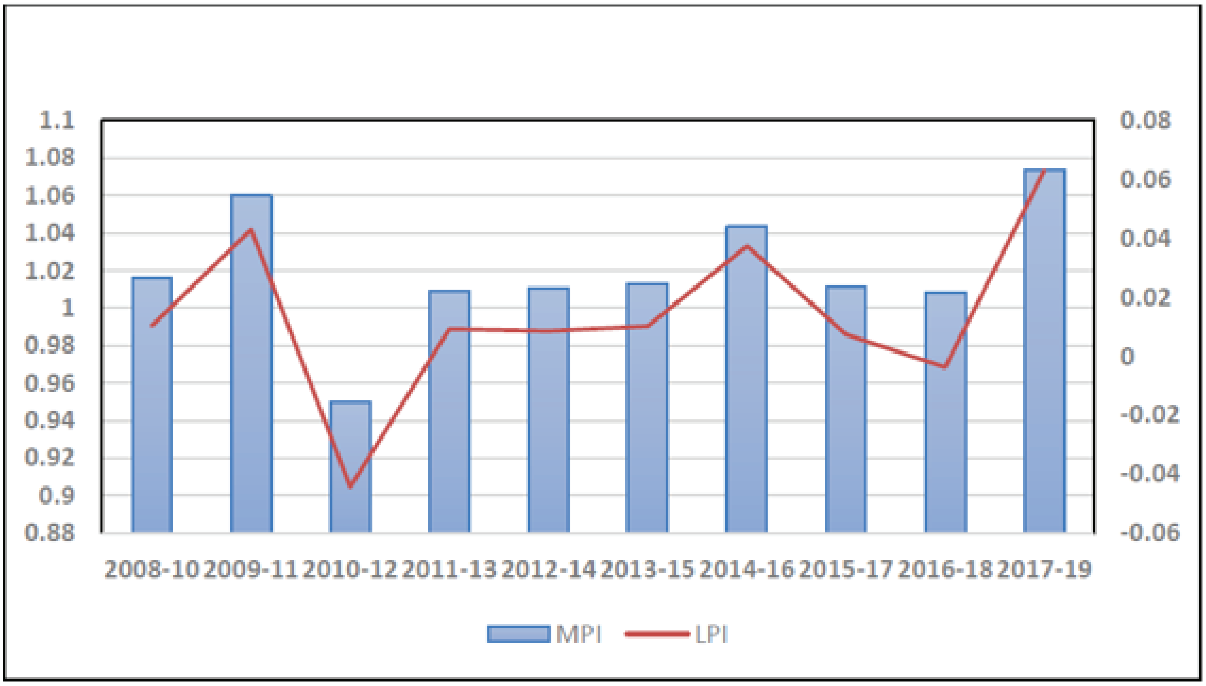


**Fig. 4.2:** Trends of IMPI and LPI values for all depots over the period 2008-19.

The positive average IMPI growth is 1.957% depot-wise, while MTEC has progressed by 1.289% and MTC has declined by -0.772% throughout the period. This growth is due mainly to the positive value of MTEC and not because of MTC. Similarly, LPI gained about 1.41% and LTEC increased by 3.383%, and LTC declined by -0.564% for each depot. This progressed mainly due to LTEC, while LTC is negative most of the time. The highest average IMPI progress value is 7.343% in 2017-19, and the decrease in TFP is -5.022% in 2010-12. Moreover, the highest average IMPI value is 5.898% for Beawar, whereas Tijara has the lowest average IMPI value -0.0261% over the entire period. Further, the highest average TFP value of LPI increased by 6.306% in 2017-19 and decreased by -4.432% in 2010-12. It is noted that Beawar has the highest average LPI value 5.05%, and Tonk has declined -0.638% of productivity for the entire study period. The following sub-subsections describe two components of productivity. Moreover, all average values of productivity indices shown in table 4.3.

**Table 4.3:** Productivity scores of 46 depots using MPI and LPI.

| Depots | MPI | LPI | MTEC | LTEC | MTC | LTC | MSEC | MPTEC |
|---|---|---|---|---|---|---|---|---|
| Abu Road | 1.035 | 0.034 | 1.025 | 0.031 | 1.014 | 0.036 | 1.004 | 1.021 |
| Ajaymeru | 1.108 | 0.013 | 1.015 | 0.043 | 1.005 | -0.016 | 1.001 | 1.019 |
| Ajmer | 1.183 | 0.001 | 0.993 | 0.025 | 1.01 | -0.023 | 1.011 | 0.986 |
| Alwar | 1.019 | 0.012 | 1.020 | 0.085 | 1.001 | -0.061 | 1.007 | 1.014 |
| Anoopgarh | 1.018 | 0.015 | 1.010 | 0.029 | 1.010 | 0.002 | 1.005 | 1.011 |
| Banswara | 1.007 | 0.005 | 1.002 | 0.016 | 1.005 | -0.007 | 0.985 | 1.015 |
| Baran | 1.006 | 0.004 | 1.004 | 0.011 | 1.003 | -0.004 | 0.997 | 1.010 |
| Barmer | 1.023 | 0.022 | 1.009 | 0.025 | 1.014 | 0.018 | 1.014 | 1.003 |
| Beawar | 1.059 | 0.051 | 1.036 | 0.063 | 1.023 | 0.038 | 1.013 | 1.025 |
| Bharatpur | 1.014 | 0.011 | 1.009 | 0.019 | 1.007 | 0.003 | 1.006 | 1.002 |
| Bhilwara | 1.036 | 0.030 | 1.028 | 0.051 | 1.009 | 0.008 | 1.000 | 1.025 |
| Bikaner | 1.018 | 0.013 | 1.018 | 0.064 | 1.006 | -0.038 | 1.001 | 1.006 |
| Bundi | 1.013 | 0.009 | 1.007 | 0.016 | 1.007 | 0.002 | 1.012 | 0.995 |
| Chittorgarh | 1.024 | 0.016 | 1.016 | 0.044 | 1.010 | -0.011 | 1.009 | 1.010 |
| Churu | 1.008 | 0.007 | 1.002 | 0.014 | 1.008 | 0.001 | 0.998 | 1.002 |
| Dausa | 1.034 | 0.028 | 1.027 | 0.037 | 1.009 | 0.018 | 1.015 | 1.014 |
| Deluxe | 1.007 | 0.003 | 1.000 | -0.034 | 1.007 | 0.040 | 0.991 | 1.000 |
| Dhaulpur | 1.018 | 0.014 | 1.017 | 0.026 | 1.004 | 0.003 | 1.001 | 1.011 |
| Didwana | 1.012 | 0.009 | 1.004 | 0.012 | 1.009 | 0.007 | 1.006 | 0.998 |
| Dungarpur | 1.011 | 0.006 | 1.007 | 0.018 | 1.006 | -0.005 | 1.013 | 0.994 |
| Falna | 1.020 | 0.018 | 1.000 | -0.008 | 1.020 | 0.044 | 1.000 | 1.000 |
| Ganga Nagar | 1.032 | 0.025 | 1.023 | 0.088 | 1.012 | -0.039 | 0.988 | 1.033 |
| Hanumangarh | 1.017 | 0.013 | 1.011 | 0.091 | 1.010 | -0.065 | 0.985 | 1.023 |
| Hindaun | 1.014 | 0.012 | 1.013 | 0.030 | 1.002 | -0.006 | 1.006 | 1.006 |
| Jaipur | 1.016 | 0.011 | 1.019 | 0.129 | 0.998 | -0.107 | 0.982 | 1.019 |
| Jalore | 1.019 | 0.012 | 1.008 | 0.024 | 1.012 | 0.001 | 1.011 | 0.996 |
| Jhalawar | 1.023 | 0.015 | 1.017 | 0.025 | 1.007 | 0.006 | 0.999 | 1.017 |
| Jhunjhunu | 1.018 | 0.012 | 1.009 | 0.045 | 1.010 | -0.022 | 0.995 | 1.010 |
| Jodhpur | 1.032 | 0.022 | 1.037 | 0.061 | 1.000 | -0.017 | 1.006 | 1.026 |
| Khetri | 1.020 | 0.015 | 1.008 | 0.022 | 1.012 | 0.007 | 0.982 | 1.025 |
| Kota | 1.015 | -0.003 | 1.015 | 0.000 | 0.996 | -0.006 | 1.013 | 1.007 |
| Kotputli | 1.023 | 0.021 | 1.001 | -0.015 | 1.021 | 0.056 | 1.003 | 1.001 |
| Lohagarh | 1.012 | 0.008 | 1.007 | 0.023 | 1.006 | -0.006 | 1.002 | 1.005 |
| Matsya Nagar | 1.026 | 0.019 | 1.019 | -0.005 | 1.007 | 0.044 | 1.002 | 1.019 |
| Nagaur | 1.012 | 0.010 | 1.005 | 0.018 | 1.009 | 0.001 | 0.984 | 1.015 |
| Pali | 1.008 | 0.006 | 0.999 | 0.004 | 1.010 | 0.008 | 1.013 | 0.990 |
| Phalaudi | 1.015 | 0.014 | 1.000 | 0.001 | 1.015 | 0.027 | 0.996 | 1.000 |
| Sardaarshahar | 1.018 | 0.015 | 1.009 | 0.034 | 1.009 | -0.003 | 0.990 | 1.018 |
| Sikar | 1.023 | 0.017 | 1.036 | 0.140 | 0.991 | -0.105 | 0.999 | 1.023 |
| Sirohi | 1.017 | 0.015 | 1.010 | 0.011 | 1.008 | 0.018 | 0.994 | 1.016 |
| Srimadhopur | 1.036 | 0.031 | 1.018 | 0.026 | 1.018 | 0.036 | 0.993 | 1.024 |
| Tijara | 1.000 | -0.004 | 1.000 | -0.034 | 0.999 | 0.027 | 0.999 | 1.000 |
| Tonk | 1.007 | -0.006 | 1.006 | -0.026 | 0.998 | 0.014 | 1.000 | 1.007 |
| Udaipur | 1.034 | 0.012 | 1.041 | 0.077 | 0.992 | -0.052 | 1.024 | 1.016 |
| Vaishali Nagar | 1.018 | 0.013 | 1.018 | 0.089 | 1.001 | -0.064 | 1.008 | 1.008 |
| Vidhyadhar Nagar | 1.040 | 0.022 | 1.016 | 0.111 | 1.024 | -0.066 | 0.997 | 1.022 |
| **Mean** | 1.025 | 0.014 | 1.013 | 0.034 | 1.008 | -0.006 | 1.001 | 1.011 |

### 4.4.2 MTEC & LTEC Results

The technical efficiency change (TEC) shows the gap in two frontiers for 2008 and 2019. Usually, the change in productivity using proper technology and efficient utilization of inputs of the depot can be related to the performance of the technical experience throughout the study period for reforming the management of depot services. MTEC and LTEC consist of the change in efficiency of all depots between 2008 and 2019. Often, MTEC is an important attribute for accumulated TFP progress. The average of MTEC is 1.290% per year 2008-19. The highest average of MTEC in 2009-11 is 9.094%. In 2017-19, the minimum average MTEC value is 0.978%, which means that the average MTEC value of depots decreased by 2.200% during the period. Udaipur experienced the greatest growth in MTEC 4.088% between 2008 and 2019. The technical efficiency of Deluxe and Falna neither increased nor decreased (MTEC= 1), indicating that they are on the efficiency frontier.

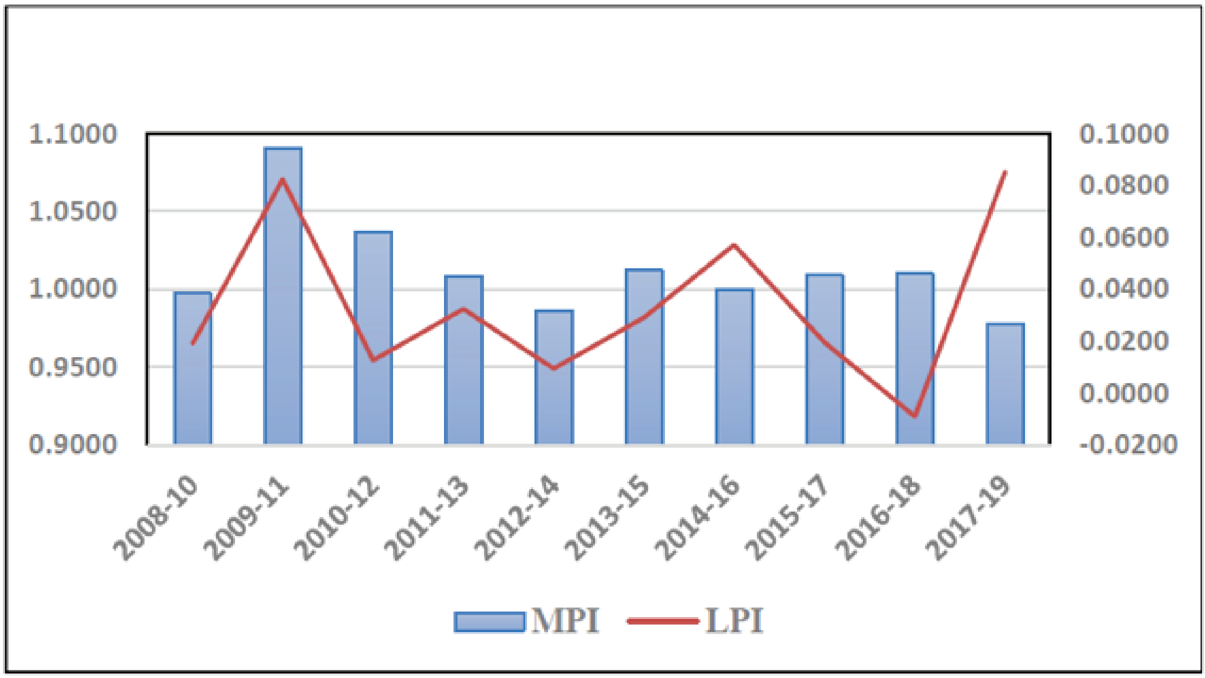


**Fig. 4.3:** MTEC and LTEC over the period 2008-19.

There are eight depots (Abu Road, Beawar, Bhilwara, Dausa, Ganga Nagar, Jodhpur, Sikar, and Udaipur) progress (20-40) % in the MTEC, whereas Ajmer and Pali showed a decline in average MTEC by 6% and 12% over the period 2008-19. Figure 4.3 shows the MTEC and LTEC values for 46 depots over the study period. The LTEC results show that the average efficiency growth over the entire period is 3.380%. The highest average LTEC value in 2017-19 is 8.525%, while -0.892% decreases the average LTEC in 2016-18. Similarly, in 2008-19, 7 bus depots offered declining LTEC values over the study period, and deluxe has the highest drop in LTEC -3.441%. LTEC observed an almost positive value over the period for 39 depots. As per the result, each RSRTC bus depot is growing at different rates of LTEC.

### 4.4.3 MTC & LTC Results

TC calculates the impact of change (shift) in the bus depot of productivity growth range, which helps explain the impact of technological change on productivity and the use of production functions. Figure 4.4 presents the MTC and LTC values for 46 depots over the study period. On the contrary, the maximum average MTC score of bus depots in 2017-19 is 9.835%, while in 2010-12, there is a declined of -8.38%. Jaipur, Kota, Sikar, Tijara, Tonk, and Udaipur depots showed a decline in average MTC with values -0.245%, -0.359%, -0.859%, -0.074%, -0.245% and -0.781%, respectively over the study period. Jhunjhunu showed a decrease in MTC for the years 2008-10, 2009-11, and 2010-12 while increasing for the other study periods. In 2008-10, 2009-11, 2012-14, 2014-16, 2016-18 and 2017-19, average LTC increased by 0.15%, 0.35%, 0.75%, 1.75%, 0.14% and 4.08% for all depots, respectively. During the period 2010-12, the maximum average decline in LTC is -10.135%. Further, Kotputli progressed in the average LTC by 5.593%, and Jaipur regressed in the average LTC by -10.54% between 2008-19. Twenty-one depots showed a decline in average LTC growth from 2008-10 to 2017-19.

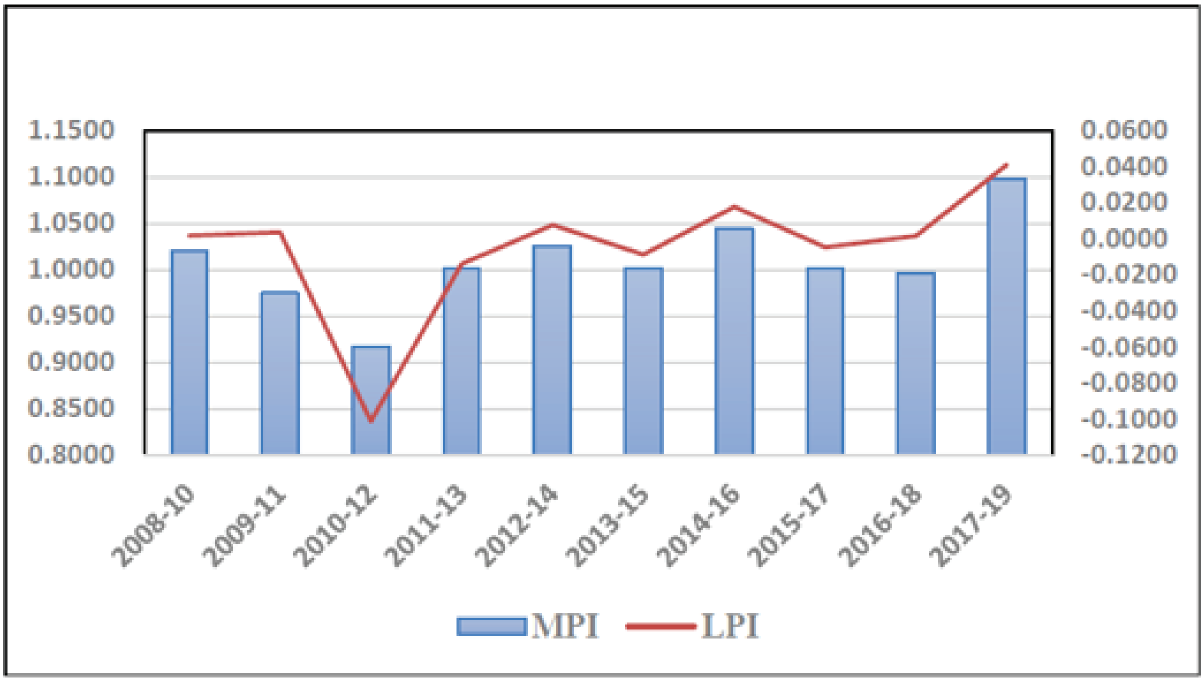


**Fig. 4.4:** MTC and LTC over the period 2008-19.

## 4.5 Conclusions

Various DEA models are commonly utilized in the transport sector to calculate productivity. This study, however, employed an input-oriented NSM DEA model to measure overall productivity growth using MPI and LPI. Further, it evaluated total productivity change into technological change (Frontier Shift) and technical efficiency change (Catch-up Effect). This study expanded the TFP change utilizing the NSM model, meeting radial properties, unit invariance, and translation invariance criteria. The comparison of IMPI and LPI trends for 46 RSRTC depots during 2008-2019 revealed that average productivity progressed by 1.957% and 1.41%, respectively. The study tested the approximation techniques of IMPI using LPI and found them to be less accurate than in prior studies. Setting a benchmark for these 46 depots, the study developed the concept of self-productivity for RSRTC depots in 2008 and 2019 using IMPI and LPI. The results suggest implications for policymakers in legislative frameworks, advocating financial development for appropriate management strategies and performance enhancement.

# Chapter 5

# Ranking of Efficient DMUs Using Super-Efficiency Inverse DEA Model

## 5.1 Introduction

DEA is a mathematical programming-based technique for measuring the efficiency of a group of DMUs based on observed inputs and outputs. The interest lies in using less input to produce more output in the DEA literature. Incorporating prior information into the traditional DEA model helps decision-makers fit their desires into the efficiency analysis process. An appealing topic is how to maintain the efficiency score of an examined DMU if its fundamental structure changes marginally. The inverse DEA (IDEA) methodology estimates feasible output levels while scaling up inputs and maintaining efficiency scores within the DEA framework. The concept of IDEA is initially introduced by Zhang and Cui [267] in 1999. However, it's important to note that IDEA studies operate under the assumption that the efficiency levels of observed DMUs remain constant over the period of interest. This assumption primarily restricts the application of IDEA and allows exploration of what alternative combinations of inputs and/or outputs could have led to the same efficiency score. As inverse optimization technique for managerial decisions, IDEA models offer valuable insights into resource allocation to achieve a specific level of competitiveness. Additionally, these models empower the decision-makers to explore improvements in efficiency scores. In recent years, advanced versions of IDEA models have emerged and found applications in various fields, including the banking sector [268], transportation management [269], financial management [270], and many more.

One of the main issues discussed in traditional DEA models is their inability to efficiently rank all the efficient DMUs. In practical applications, decision-makers are interested in ranking all DMUs for resource allocation and performance reviews. DEA experts have thoroughly investigated this issue, and a number of ranking methods are introduced based on different

[1]This work has appeared in **Goyal, S. Talwar, M. Agarwal, S. and Mathur T., 2021. Ranking of efficient DMUs using super-efficiency inverse DEA model. In Soft Computing for Problem Solving: Proceedings of the SocProS 2022. Singapore: Springer Nature Singapore. 2, pp 615-626, 2023.**

techniques and approaches. Most of these techniques can be viewed as offering supplementary post-analysis to conventional DEA models, aiming to enhance the ultimate ranking. The super-efficiency IDEA method is the futuristic approach for ranking DMUs because of its ability to estimate the input changes based on any increments of the outputs, and hence is named the growth potential method. Ranking from this method is more reliable because

- This method ranks DMUs based on their growth potential,
- The wasted inputs (values of slack variables) are low, and
- The variation in outputs is controllable.

Hence, this refers to the fact that the decision-maker can select different increments in output to gain better insight regarding input variables, giving them more flexibility. In other words, efficient DMUs have no scope for improvement in terms of efficiency, but ranking is vital for preserving service quality. This research proposes a ranking for efficient depots of RSRTC using the super-efficiency IDEA model. This study ranked the efficient depots based on the highest IDEA efficiency score. This model finds utility in conducting production analysis and optimizing resource allocation to enhance system efficiency in the future. Furthermore, they serve as a tool to examine DMs' preferences and policies, aiding in future decision-making, particularly in estimating inputs and outputs during the analysis of production and resource allocation, ultimately contributing to increased resource efficiency.

The remaining part of the chapter is written as follows: section 5.2 briefly introduces the IDEA model extensions and their applications. The new methodology is presented in section 5.3 and applies to the specific context in section 5.4 along with a detailed discussion of the results. In the last section, concluding remarks are mentioned.

## 5.2 Literature Review

This section reviewed the extended versions of IDEA models and applications in various fields. Furthermore, the features of the developed IDEA model in this chapter are compared to other proposed models for the transport sector to highlight the existing gap in this field.

### 5.2.1 Ranking Methods

Adler et al. (2002) [271] reviewed ranking methods of DEA and, for instance, specifically emphasized the advancement of the research field from 1986 to 2000. Aldamak and Zolfaghari (2017) [272] studied four more categories of maturely developed papers published up to 2015 that focus on resolving the DEA ranking issue in order to increase the discrimination power of this analytical technique. Nevertheless, a subset of research studies have concentrated on devising ranking approaches tailored specifically for DEA, such as cross-efficiency, super efficiency, benchmarking ranking, and virtual frontier DEA (VFDEA) for ranking efficient DMUs. Sexton et al. (1986) [273] proposed a cross-efficiency method for ranking DMUs based on the idea that both units are self- and peer-evaluated. Andersen and Petersen (1993) [274] first introduced the super-efficiency model by removing the DMUs under evaluation in order to improve the ranking and construct a new frontier with the remaining ones. Mehrabian et al. (1999) [275] offered a comprehensive ranking of efficient units after addressing issues of infeasibility and stability. Tone (2001) [52] presented super-efficiency on the SBM model, which is always feasible & stable and has the benefits of non-radial models. Sueyoshi et al. (1999) [276] proposed a “benchmark approach” with the use of a slack-adjusted DEA model and OERA (Offensive earned-run average) to overcome the shortcomings of multiple efficient units identified through the DEA Model. Jahanshahloo et al. (2004) [277] showed a ranking method of efficient DMUs using leave one out and L1 norm. Jahanshahloo et al. (2007) [278] proposed a new model with the idea that efficient units can be the target DMU for inefficient DMUs. Bian and Xu (2013) [279] proposed a virtual frontier DEA, which is further developed by Li et al. (2015) [280]. A new optimal frontier is constructed, called the virtual frontier. Cui and Li (2014) [281] proposed that the reference DMU set remains unchanged during the evaluation process. Wanke and Barros (2016) [121] applied virtual frontier dynamic range adjusted model DEA (VDRAM-DEA) to assess Latin American airlines’ efficiencies. Also, Barros et al. (2017) [282] used VDRAM-DEA to evaluate Angolan hydroelectric power stations, which caused higher efficiency score discrimination. Soleimani et al. (2020) [283] introduced the single-objective linear programming (SOLP) IDEA model to rank the efficient DMUs by removing one DMU in the PPS. All ranking methods evaluate units from a particular perspective, and each has advantages and hindrances compared to others.

### 5.2.2 Applications of IDEA models

The idea of IDEA was introduced by Zhang and Cui (1999) [267] in the context of project evaluation in public sectors. Subsequently formalized by Wei et al. (2000) [75] using the multiple-objective linear programming (MOLP) techniques to solve the problems of resource allocation and investment analysis. Yan et al. (2002) [74] further modified the IDEA method by adding a cone constraint to resource allocation. In an inverse problem, the efficiency score of a DMU is given, and the objective is to determine the optimal quantity of inputs and/or outputs required to reach the efficiency target. This problem is studied in many theoretical and applied publications. Hadi and Foroughi (2006) [284] extended the work of [75] by allowing arbitrary changes in input and output levels. Hadi et al. (2008) [285] introduced another IDEA to estimate the inputs under given increasing outputs while preserving the efficiency score. Lin (2010) [286] tackled efficiency measurement and revenue-setting challenges in Taiwan chain stores by applying imprecise DEA and IDEA models. Lertworasirikul et al. (2011) [76] introduced an inverse BCC model for resource allocation without altering efficiency scores. Gattoufi et al. (2014) [287] introduced a novel application of IDEA in mergers and acquisitions to estimate the optimal level of inputs and outputs from pre-merger DMUs, which is needed to be kept by the post-merger entity to reach a given efficiency target. Ghobadi et al. (2014) [288] applied the IDEA concept in a dynamic environment with fuzzy data. Jahanshahloo et al. [289] proposed the IDEA model with a non-radial enhanced Russell model. Jahanshahloo et al. [290] presented the time based IDEA model per an inter-temporal dependence assumption and developed a MOLP to solve the IDEA problem for weak Pareto solutions. Hadi-Vencheh et al. [291] improved the IDEA models for interval data instead of crisp values in measuring the efficiency score. Lim (2016) [292] proposed the IDEA model to establish new product targets based on anticipated shifts in the production frontier. Zhang and Cui (2016) [293] integrated IDEA models into 12 scenarios. Amin et al. (2017) [294] extended the idea of mergers and acquisitions using IDEA. They introduced a more comprehensive restructuring approach, aiming to enhance efficiency in post-restructuring firms through either consolidation using synergy or splitting using reverse synergy. In this regard, Çakır (2017) [295] proposed a resource allocation solution in a fuzzy setting, introducing an integrated Shannon's entropy-IDEA model using a two-phase approach. Eyni et al. (2017) [296] suggested IDEA for sensitivity analysis of DMUs with undesirable inputs and outputs. Ghiyasi (2017) [297] developed IDEA models utilizing cost and revenue efficiency, incorporating price information. Recently, Amin and Al-Muharrami (2018) [268] formulated a novel method with integrated goal programming (GP) and IDEA for handling mergers and acquisitions involving firms with negative data in the banking industry. Also, Emrouznejad et al. (2018) [298] developed an IDEA model to determine the optimal allocation of $CO_2$ emissions quota in the Chinese manufacturing industries. Wegener and Amin (2019) [299] addressed the novel IDEA model to minimize GHG emissions with an application in the oil and gas industry. Hosseininia and Saen (2020) [300] transformed the inverse SBM model into a linear programming format, maintaining DMUs' efficiency scores while estimating other parameters. They also proposed an optimal combination of outputs and inputs within the production possibility set (PPS). Guijarro et al. (2020) [301] suggested a model that merges IDEA for calculating input savings from a merger and employs a genetic algorithm to address the combinatorial aspects of the problem. Hu et al. (2020) [302] noted that radial-based IDEA models may be unreliable due to their disregard for slacks. They emphasized the drawbacks through a counterexample and subsequently introduced a modified model to address this issue. Soleimani et al. (2020) [283] introduced a SOLP IDEA model to rank the efficient DMUs by removing the itself DMU in the PPS. Zhang and Cui (2020) [303] highlighted that the conventional IDEA models are radial-based. They pointed out that inverse radial DEA models may overlook slacks. Consequently, they introduced an integrated framework termed non-radial IDEA models employing multi-objective programming. In another study, Aslani et al. (2021) [304] addressed supply chain structures and intermediate products in an IDEA model to quantify the Bullwhip effect in SCs with uncertain demands. Here, uncertain demands and forecasted data are treated as inputs and outputs. Chen and Wang (2021) [269] identified limitations in the IDEA model under VRS and introduced constraints for inputs and outputs to address this issue. They also factored in efficiency and technological changes when optimizing inputs and outputs in IDEA models. Numerical results on instances presented by Le et al. (2021) [305] assessed the efficiency and effectiveness using the inverse frontier-based benchmarking DEA model with a mixed integer linear program in the education system. Sayar et al. (2021) [270] introduced an enhanced IDEA model accounting for income and budget limitations in planning and budgeting. This model enables decision-makers to determine input values and output income shares while considering income constraints in assessing supermarket efficiency within a chain.

The IDEA has the potential to solve challenging decision-making problems. However, there are various applications of IDEA in the real world. The literature review shows that there are limited studies on the public transport sector to rank efficient depots. To fill this gap, this chapter introduces a model to minimize the input with a super-efficiency IDEA model with an application in public transportation.

## 5.3 Description of the Methodology

The super-efficiency, IDEA, and super-efficiency IDEA models are described in the following subsections.

### 5.3.1 Super-efficiency DEA Model

In the CCR DEA model, two types of DMUs exist: efficient and inefficient. When DMUs are on the production frontier at the same time, the standard CCR model has poor discrimination, making further evaluation and comparison between these DMUs problematic. To address the lack of a CCR model to evaluate the DMU's efficiency effectively and accurately, Andersen and Petersen [274] suggested the super-efficiency DEA model, a complete efficiency measure. The super-efficiency CCR model is a method that combines super-efficiency and the CCR model. It can further distinguish efficiency among DMUs and then sort the relative efficiency of these DMUs. Super-efficiency DEA has a similar function formula to the CCR model used for determining the most productive scale size in the traditional DEA framework. For measuring the efficiency of $n$ $DMU_d$, whereas each $DMU_d \in 1, 2, \cdots, n$, considering $m$ inputs to produce $s$

outputs represented by $x_{id}(i = 1, \cdots, m)$ and $y_{rd}(r = 1, \cdots, s)$ respectively, the linear form of the input-oriented super-efficiency model for measuring the efficiency of $d^{th}$ DMU is defined as follows:

$$
\begin{aligned}
&\min\ \theta_d \\
&\text{Subject to} \\
&\sum_{j=1, j\neq d}^{n} y_{rj}\lambda_{jd} \geq y_{rd}, \quad \forall\ r = 1, \cdots, s \\
&\sum_{j=1, j\neq d}^{n} x_{ij}\lambda_{jd} \leq \theta_d x_{id}, \quad \forall\ i = 1, \cdots, m \\
&\lambda_{jd} \geq 0, \quad \forall\ j = 1, \cdots, n
\end{aligned}
\tag{5.1}
$$

In model (5.1), the first and second constraints are linear inequality involving the output and input values $y_{rj}$ and $x_{ij}$ their corresponding multipliers $\lambda_{jd}$ for all $j$ except $d^{th}$. In the super-efficiency DEA model, inefficient DMUs receive the same efficiency score as in the CCR model. However, its inputs can be proportionally increased for an efficient DMU while maintaining the same efficiency score. The ratio of this input increase is termed its super-efficiency value.

### 5.3.2 IDEA Model

In the following scenario if the efficiency score remains the same, how much input should increase when the outputs increase? As a result, evaluate the increase in outputs and inputs to answer this question considering $y_d$ to $y_d + \Delta y_d$ and $x_d$ to $x_d + \Delta x_d$, $y_d, x_d > 0$ and $\Delta y_d \in R_+^{s\times n}$, $\Delta x_d \in R_+^{m\times n}$, respectively. The updated inputs $x_d + \Delta x_d$ are represented by $\alpha_d$ and outputs $y_d + \Delta y_d$ are denoted by $\beta_d$ for $DMU_d$. The following MOLP model is used to calculate the $\alpha_d$ [306]:

$$
\begin{aligned}
&\min\ \alpha_d = (\alpha_{1d}, \alpha_{2d}, \cdots, \alpha_{md}) \\
&\text{Subject to} \\
&\sum_{j=1}^{n} y_{rj}\lambda_{jd} \geq \beta_{rd}, \quad \forall\ r = 1, \cdots, s \\
&\sum_{j=1}^{n} x_{ij}\lambda_{jd} \leq \theta_d \alpha_{id}, \quad \forall\ i = 1, \cdots, m \\
&\alpha_{id} \geq x_{id}, \quad \forall\ i = 1, \cdots, m \\
&\lambda_{jd} \geq 0, \quad \forall\ j = 1, \cdots, n.
\end{aligned}
\tag{5.2}
$$

### 5.3.3 Super-efficiency IDEA Model

The fundamental IDEA models produce a new DMU. The production possibility set (PPS) in these models is built by prior DMUs. It is built by $n$ DMUs $(DMU_1,\cdots,DMU_d,\cdots,DMU_n)$, while among them changed the $DMU_d$ with $DMU_d^{'}$ by creating a new technology set that is $(DMU_1,\cdots,DMU_d^{'},\cdots,DMU_n)$. Hadi-Vencheh et al. (2015) [291] proposed a new approach by removing $DMU_d$ that new PPS constructed as $(DMU_1,\cdots,DMU_{(d-1)},DMU_{(d+1)},\cdots,DMU_n)$. Generally, PPS is built by replacing the 'perturbed DMU' with a new unit that includes the updated inputs and outputs. The super-efficiency IDEA model is a valuable tool for decision-makers when determining the actual ranking levels of efficient DMUs after changing the PPS. After eliminating $d^{th}$ DMU, the PPS is not going to change $(DMU_1,DMU_2,\cdots,DMU_{(d-1)},DMU_d^{'},$ $DMU_{(d+1)},\cdots,DMU_n)$. The model is given as follows:

$$
\begin{aligned}
&\min\ \alpha_d = (\alpha_{1d},\alpha_{2d},\cdots,\alpha_{md})\\
&\text{Subject to}\\
&\sum_{j=1,j\neq d}^{n} y_{rj}\lambda_{jd} \geq \beta_{rd}, \quad \forall\ r=1,\cdots,s\\
&\sum_{j=1,j\neq d}^{n} x_{ij}\lambda_{jd} \leq \theta_d\alpha_{id}, \quad \forall\ i=1,\cdots,m\\
&\alpha_{id} \geq x_{id}, \quad \forall\ i=1,\cdots,m\\
&\lambda_{jd} \geq 0, \quad \forall\ j=1,\cdots,n
\end{aligned}
\tag{5.3}
$$

### 5.3.4 Ranking with IDEA

In this section, utilization of IDEA models using CRS technology for the following reasons: (i) In one of the models, there is a uniform increase in all outputs of a unit, necessitating a model that remains stable with changes in dimensions. (ii) The VRS model renders some units infeasible, whereas the CCR model proves feasible.

#### 5.3.4.1 Single-Objective IDEA Model

The main problem in the super-efficiency IDEA model is that it is MOLP, which implies that it does not deliver an optimum solution of the model while still maintaining a non-exclusive Pareto solution. This study employs an inverse DEA-based single-objective IDEA model to overcome this limitation. The objective function is constructed by increasing the inputs. Accordingly, the input increment should be the same to produce a single-objective function for all inputs. As a result, all outputs of DMU have the same increase.

Sometimes, the increasing quantity of a unit of them is not always the same. For example, in the road transport sector, one input is the fleet size (number of buses), and another is fuel consumption (1000 kl). Therefore, all the criteria are normalized using the described equation,

$$\left.\begin{aligned} \hat{x}_{ij} &= \frac{x_{ij}}{max\ x_{ij}}, \\ \hat{y}_{ij} &= \frac{y_{ij}}{max\ y_{ij}}. \end{aligned}\right\} \tag{5.4}$$

Hence, the increment of inputs are the same for all inputs to achieve a single-objective function. Thus, outputs are increased by the same amount in all DMUs. All outputs of efficient DMU are increased by (y+ $\beta$%), and later the growing percentage of inputs (x+ $\alpha$%) is calculated. As a result, the single-objective linear programming IDEA model for efficient DMUs is given as follows:

$$\begin{aligned} &\alpha_d = \min\ \alpha \\ &\text{Subject to} \\ &\sum_{j=1, j\neq d}^{n} \hat{y}_{rj}\lambda_{jd} \geq \hat{y}_{rd} + \beta, \quad \forall\ r = 1, \cdots, s \\ &\sum_{j=1, j\neq d}^{n} \hat{x}_{ij}\lambda_{jd} \leq \hat{x}_{id} + \alpha \quad \forall\ i = 1, \cdots, m \\ &\hat{x}_{ij} + \alpha \geq \hat{x}_{id}, \quad \forall\ i = 1, \cdots, m \\ &\lambda_{jd} \geq 0, \quad \forall\ j = 1, \cdots, n \end{aligned} \tag{5.5}$$

All $DMU_d$ outputs are enlarged by a comparable amount $\beta$ in the model (5.5). All inputs and outputs are dimensionless.

**Definition 5.3.1** *The rank of $DMU_j$ is better than $DMU_s$, when $\alpha_j^* \geq \alpha_d^*$.*

## 5.4 Numerical Illustration

Passenger transportation is a "service business", and measuring the efficiency of a service business is complicated. This study aims to overcome the complex nature of the organizational operation of passenger transportation at 52 RSRTC bus depots for the year 2018-19. In the present work, DEA is utilized to sweep up the changes in efficiency. The process involves two main steps: Step 1 focuses on calculating the efficiency values of bus depots using the CCR model (1.1), aiming to identify the efficient units among them. The next step involves applying the proposed super-efficiency IDEA model, represented by equation (5.5). This super-efficiency

IDEA model is used specifically for ranking the bus depots that are identified as efficient in Step 1.

### 5.4.1 Data

This study used a secondary data set of 52 bus depots provided by the annual reports of RSRTC for 2018-19. Various criteria affect the transport system. Six significant criteria are selected to assess the transport system, incorporating four input and two output criteria for the evaluation of efficiency among the 52 bus depots. The descriptive statistics of inputs and outputs are given in table 5.1.

**Table 5.1:** Descriptive statistics of inputs and outputs.

| | Criteria | Max | Min | Average | SD |
|---|---|---|---|---|---|
| Inputs | Fleet size | 140.00 | 14.00 | 70.08 | 24.64 |
| | Labor | 665.00 | 54.00 | 271.08 | 116.88 |
| | Fuel consumption | 45.75 | 4.12 | 20.91 | 8.26 |
| | Routes distance | 16769.00 | 3496.00 | 9064.31 | 2966.59 |
| Outputs | Passenger km occupied | 9.02 | 0.77 | 3.90 | 1.50 |
| | Vehicle utilization | 586.00 | 311.00 | 391.87 | 46.39 |
| | **Efficiency** | **1.00** | **0.80** | **0.91** | **0.05** |

### 5.4.2 Empirical Results

Initially, the CCR model (1.1) is employed to compute the efficiency scores for 52 RSRTC depots. Among these, 7 depots—Beawar, Jaisalmer, Karauli, Matsya Nagar, Shapur, Tijara, and Vidhyadhar Nagar—demonstrated optimal efficiency, each achieving a score of 1 (as shown in Table 5.2). On the other hand, Dungarpur exhibited the lowest efficiency score, which is 0.801, compared to the other depots. However, the lack of uniformity in efficiency scores among the seven top-performing depots complicates the process of establishing a definitive ranking.

**Table 5.2:** Data and efficiency scores of efficient depots.

| Depots | I1 | I2 | I3 | I4 | O1 | O2 | CCR Score |
|---|---|---|---|---|---|---|---|
| Beawar | 47.00 | 238.00 | 23.40 | 7285.00 | 4.75 | 392.00 | 1.00 |
| Jaisalmer | 25.00 | 91.00 | 9.61 | 4620.00 | 1.63 | 491.00 | 1.00 |
| Karauli | 14.00 | 54.00 | 4.12 | 3629.00 | 0.77 | 382.00 | 1.00 |
| Matsya Nagar | 73.00 | 306.00 | 23.32 | 6088.00 | 4.92 | 387.00 | 1.00 |
| Shapur | 61.00 | 196.00 | 9.45 | 3505.00 | 2.26 | 335.00 | 1.00 |
| Tijara | 50.00 | 199.00 | 16.75 | 7238.00 | 3.49 | 393.00 | 1.00 |
| Vidhyadhar Nagar | 84.00 | 349.00 | 9.86 | 4871.00 | 2.70 | 311.00 | 1.00 |

Subsequently, the ranking of 7 efficient depots is evaluated using the model (5.5) under the assumption of CRS, and outcomes are detailed in table 5.3. To measure the input growth rate, increase the amount of outputs of all efficient depots by '0.15'. The depot with the greater objective function is ranked higher by the given definition, 5.3.1. The change in outputs for the efficient Beawar are 4.898 and 392.15, respectively, so The obtained value of $\alpha_1^*$=0.234 for the Beawar, hence, Beawar secured the top rank with the most significant value of $\alpha_k^*$, and Jaisalmer secured rank 7 with the smallest value of $\alpha_k^* = 0.132$. The exact process is repeated for the remaining depots to determine their rankings.

**Table 5.3:** Ranking of efficient depots.

| Depots | Beawar | Jaisalmer | Karauli | Matsya Nagar | Shapur | Tijara | Vidhyadhar Nagar |
|---|---|---|---|---|---|---|---|
| min ($\alpha_k^*$) | 0.234 | 0.132 | 0.156 | 0.187 | 0.147 | 0.132 | 0.157 |
| Rank | 1 | 7 | 4 | 2 | 5 | 6 | 3 |

The rank of efficient bus depots of super-efficiency IDEA and super-efficiency methods are tabulated in table 5.4. In most cases, the outcomes obtained using the proposed methodology and the super-efficiency method are comparable. Note that the results in this section are more reliable due to the fact that this method ranks depots based on their growth.

**Table 5.4:** Ranking of efficient depots using super-efficiency IDEA and super-efficiency model.

| Depots | Super-efficiency IDEA model | Super-efficiency model |
|---|---|---|
| Beawar | 1 | 2 |
| Jaisalmer | 7 | 6 |
| Karauli | 4 | 1 |
| Matsya Nagar | 2 | 3 |
| Shapur | 5 | 4 |
| Tijara | 6 | 7 |
| Vidhyadha Nagar | 3 | 5 |

## 5.5 Conclusions

DEA models assess the relative efficiency of depots and effectively identify inefficient units, it is difficult to differentiate between efficient ones. The IDEA approach has gained popularity in DEA literature for re-evaluating depot efficiency when input or output values change. The IDEA method primarily analyzes how the relationship between inputs and outputs in production units changes while maintaining efficiency level. This serves as a valuable tool for managerial decision-making, providing insights into resource allocation for achieving specific level of competitiveness. Additionally, decision-makers should consider preferences for future planning and evaluate potential system changes. This study applied a ranking system for efficient depots using the super-efficiency IDEA method alongside a single-objective LP model. The research assesses the relative efficiency of 52 RSRTC bus depots for the 2018-19 period. The proposed model identifies 7 RSRTC depots as efficient and 45 as inefficient, with efficiency scores below 1. Subsequently, the efficient depots are ranked using the super-efficiency IDEA model. This application also provides a novel analytical framework for addressing quality service decisions in efficient depots, including optimal resource allocation among bus depots.

# Chapter 6

# A Novel Fuzzy Cross-Efficiency Evaluation and Ranking in DEA

## 6.1 Introduction

The most notable aspect of DEA is self-evaluation. The essence of self-evaluation lies in computing the ratio of weighted outputs to weighted inputs for each DMU, which fundamentally serves to quantify how effectively the DMU is converting its inputs into outputs. DEA allows each DMU to select or assign input and output weights during self-evaluation to optimize its maximum efficiency score. DEA has some deficiencies in performance evaluation [307]. A drawback arising from the weights assigned during self-evaluation introduces variability and uncertainty into the efficiency assessment, i.e., the weights may be unrealistic, resulting in a false impression of DMU efficiency [308, 309]. In addition, multiple DMUs achieve an efficiency score of 1, leading to challenges in ranking them effectively, especially when DMs requiresa a clear hierarchy for practical applications. These issues plague applying the traditional DEA method to real-world scenarios. DEA approach may need modifications or additional considerations to cater to the requirements of DMs and improve its applicability in practice, such as employing alternative evaluation models or introducing certain constraints in weight selection.

Given the identified drawbacks of the traditional DEA models, especially in handling self-evaluation biases and challenges in ranking DMUs, a promising avenue that has emerged in the literature is the concept of cross-efficiency evaluation, originally presented by Sexton et al. (1986) [273]. Doyle and Green (1994) [310] extended the evaluation of efficiency assessment of the DMUs paradigm by incorporating the secondary goal approach in both self- and peer-evaluations. Cross-efficiency evaluation offers the benefit of establishing a distinct ranking for DMUs in a unique order. Additionally, Boussofiane et al. (1991) [311] proved that this approach is effective in distinguishing between various efficient DMUs. DEA cross-efficiency is explicitly designed to determine weights for each DMU, resulting in $n$ sets of weights to acquire $n$ efficiencies. The average $n$ efficiency scores provides a final efficiency score for each DMU. The cross-efficiency model assesses efficiency from a global optimal viewpoint, allowing for DMU ranking while preventing unrealistic multipliers. As a result, cross-efficiency assessment boasts enhanced distinguishing power of DMUs and yields results that are more indicative [312]. The competence of cross-efficiency evaluation to handle multiple solutions has led to its widespread adoption in several fields, for example, portfolio selection [313], supply chain management [314], public resource management [315], resource allocation [316], power plant performance [317], and healthcare service [318].

Decision-making is a routine aspect of human life, as individuals constantly make choices every day. A notable obstacle faced by those making decisions is the existence of uncertainty or ambiguity. Uncertainty is prevalent due to incomplete, missing, and imprecise data, which significantly affects the feasibility and reliability of the results by DEA models [89]. An integrated approach is employed to address such data challenges and enhance the accuracy of the DEA results. Initially, the k nearest-neighbor (kNN) algorithm, a machine-learning technique, is employed to impute missing data. This technique operates by classifying the nearest neighbors based on Euclidean distance and then leveraging these neighbors for imputation. While kNN imputation significantly reduces uncertainty, a degree of ambiguity or vagueness may still persist. Panchal et al. (2018) [319] concluded that fuzzy methodology is a powerful tool for handling uncertainty and imprecision in the data. To address this remaining ambiguity, fuzzy set theory is applied. This theoretical framework allows for the representation of vague or imprecise information. Expanding on this, researchers have adeptly incorporated membership functions from fuzzy set theory to refine the imputation process further [320, 112]. This integration enhances the precision of the filled missing values and provides a means to quantify and manage the inherent vagueness in the data.

This study introduces a novel fuzzy cross-efficiency evaluation approach under the VRS assumption to provide a more reliable and comprehensive evaluation of DMUs in a fuzzy environment. Wu et al. (2009) [258] suggested augmenting the VRS model with an extra constraint to ensure non-negative cross-efficiencies. However, this adjustment may not effectively tackle the underlying issue of negative VRS cross-efficiency. The proposed method explores all possible weights for each DMU, eliminating the need for subjective weight choices. The negative efficiency scores can arise with the VRS assumption, which can pose difficulties in accurately measuring the overall cross-efficiency and is not a reasonable outcome. De et al. (2013) [321] proposed a method to address this challenge. This method restricts the multiplier values employed in the model to ensure that all DMUs exhibit positive efficiency scores. The proposed model incorporates secondary objectives to handle multiple optimal weights with positive efficiency scores. This study is based on the fuzzy DEA model as detailed in the work of Lertworasirikul et al. (2003) [322]. The method is designed to ensure that the cross-efficiency assessments maintain consistency with the original fuzzy efficiency measurements. This suggests a rigorous and internally coherent approach to evaluating the performance of decision-making units under uncertainty. It also transforms the fuzzy DEA model into a crisp linear programming model using credibility measures at different credibility levels. This transformation simplifies the evaluation process and explicitly addresses the lower and upper bounds of fuzzy efficiency, resulting in more accurate rankings of DMUs.

The subsequent sections are organized in the following manner: section 6.2 provides an overview of relevant literature, providing a contextual foundation. Section 6.3 delves into the fuzzy cross-efficiency evaluation method using the credibility-level approach. Section 6.4 demonstrates the real-world utility of this approach in public transport sector. Subsequently, it includes discussions on the empirical outcomes and introduces the ensemble ranking method to enhance the robustness of the evaluation. Finally, the study draws its conclusions in the section 6.5.

## 6.2 Literature Review

This section is divided into two subsections: the first focuses on reviewing cross-efficiency evaluation and extended ranking methods. The second delves into fuzzy cross-efficiency evaluation. This organization aims to offer a concise overview of the current literature in this area.

### 6.2.1 Cross Efficiency: A Brief Survey

The cross-efficiency evaluation method has emerged as the widely applied approach for assessing the discriminating power of a DEA model [323]. However, it is crucial to acknowledge that, despite its extensive adoption with the DEA framework. The cross-efficiency concept in DEA allows the overall efficiency of a DMU to be evaluated through self-evaluation or peer-evaluation. In contrast to conventional DEA, where a DMU is evaluated by its optimal weights (self-evaluation), cross-efficiency appraises a DMU by a set of weights that are optimally obtained in favor of all other DMUs (peer-evaluation) [273]. Doyle and Green (1994) [310] incorporated a secondary goal to determine unique optimal weights. The two most popular secondary goal approaches, aggressive and benevolent formulations, involve an additional criterion to choose weights that ensure optimal efficiency. These approaches are often utilized in weight selection, considering specific additional objectives when determining the weights. Various methods are proposed to select suitable weight sets for computing cross-evaluation further to enhance the robustness and reliability of cross-efficiency evaluation. For instance, Liang et al. (2008) [324] introduced the game cross-efficiency DEA method, which can generate a set of cross-efficiency scores representing a Nash equilibrium point for DMUs. Different techniques, including super-efficiency and mixed-integer linear programming [325], are proposed to select suitable weight sets. Wang and Chin (2010) [326] extended the aggressive and benevolent models by proposing four alternative models. Jahanshahloo et al. (2011) [327] introduced a symmetrical technique to enhance the DEA cross-efficiency evaluation. They proposed a secondary goal model capable of selecting symmetric weights for DMUs. Researchers have also addressed the issue of significant variations among weights chosen from different solutions by implementing techniques such as setting lower bounds [328, 329, 330], implementing ordered weight averaging operators [331], applying minimax and maximin formulations [332] and assessing the proposed methodologies. Alternatively, some studies considered all possible weights in the weight space to estimate the interval efficiency of the DMUs [333, 334]. Orkcu et al. (2015) [335] and Wu et al. (2016) [323] introduced positive feedback in the secondary goal evaluation strategies, which involved using target identification methodologies with desirable/undesirable targets. This addition is aimed at enhancing the reliability of cross-efficiency evaluation. Olyaie (2019) [336] adopted a criterion that maximizes efficiency scores while minimizing the number of satisfied units.

A classic way to aggregate cross-efficiencies into final efficiencies is to take averages (e.g., Sexton et al. (1986) [273], Doyle and Green [310], and Sun et al. (2020) [337]). Despite being simple and widely used, the average method fails to take into account the relative importance of the evaluators. The reference value of the opinions of different evaluators is different, and ignoring this difference will lead to bias in the results [338]. Wu et al. (2011) [307] introduced Shannon entropy into cross-efficiency evaluation and assigned different weights to each evaluator DMU. Yang et al. (2012) [333] defined an alternative strategy for ranking DMUs using minimal and maximal game cross-efficiency scores, which considers all possible weight sets in weight space, eliminating the need for the decision maker to choose between aggressive or benevolent strategies according to the idea of [339]. Moreover, Lahdelma and Salminen (2001) [340] proposed the Holistic Acceptability Index (HAI) to gauge overall cross-efficiency scores. This index is subsequently employed by Yang et al. (2012) [333] for ranking DMUs. Alcaraz et al. (2013) [341] accounted for all possible weight choices that all DMUs can make, resulting in a range of possible rankings for each DMU instead of a singular ranking position. Oukil and Amin (2015) [342] combined cross-evaluation, preference voting, and ordered weighted averaging (OWA) to discriminate among DMUs. An et al. (2018) [343] presented a combined DEA/AHP method for full ranking for all DMUs by combining DEA and analytic hierarchy process (AHP) techniques. This method evaluates every possible cross-efficiency of a DMU in relation to all other DMUs, determining the interval cross-efficiency for each DMU. Liu (2018) [344] introduced the concept of the signal-to-noise (SN) ratio as a numerical index for ranking DMUs. This study emphasized the importance of considering the ranges and variances of cross-efficiencies as alternative ranking factors. Yu et al. (2019) [345] proposed a method that combines interval data, cross-efficiency evaluation, and stochastic multi-criteria acceptability analysis 2 (SMAA2) approaches to rank DMUs based on interval cross-efficiencies.

These different cross-efficiency evaluation formulations lead to different ranking results for the DMUs. This research aims to fill this gap by proposing an ensemble ranking method that combines the various techniques. This approach contributes to a more thorough and reliable full ranking of DMUs. As a result, an unbiased average ensemble efficiency score is derived for each DMU's ranking.

### 6.2.2 Fuzzy cross-efficiency

Fuzzy DEA models have limitations in ranking efficient DMUs, as they rely only on self-evaluation and lack adequate discriminative power. However, incorporating peer evaluation through cross-efficiency evaluation can overcome this weakness and effectively rank the DMUs in fuzzy environments. It is crucial to acknowledge that different approaches exist for calculating the efficiency score in fuzzy cross-efficiency DEA models. The method of fuzzy cross-efficiency evaluation employed depends on the model's specific features and underlying assumptions.

Sirvent and León (2014) [346] proposed a fuzzy cross-efficiency assessment based on the fuzzy DEA model introduced by Guo and Tanaka (2001) [99]. Subsequently, in 2015, Dotoli et al. [347] presented a solution for the DEA cross-efficiency in case of uncertainty in inputs and/or outputs. These uncertainties are represented as triangular fuzzy numbers. They applied the secondary goals approach [310]. Then, the fuzzy cross-efficiency ranked the DMUs using a center-of-area method. In the same year, Han et al. (2015) [348] constructed the following concepts regarding fuzzy DMUs: Firstly, a fuzzy DMU operating at minimal input and maximal output is considered optimal. Secondly, a fuzzy DMU at its maximal input and minimal output represents poor efficiency. Lastly, a fuzzy DMU with average input and output signifies daily operational production. Chen (2016) [349] utilized the concept which is introduced by [350] for self-evaluation and the weight selection methodology for computing the fuzzy peer-evaluated efficiency of DMUs. The final cross-efficiency of a DMU is calculated by averaging self-evaluated and peer-evaluated efficiency values. Ruiz and Sirvent (2017) [351] generated a fuzzy cross-efficiency assessment founded on the fuzzy DEA model highlighted in [322]. In the same year, Hatami-Marbini et al. [352] introduced a possibilistic programming challenge transformed into an interval-programming problem utilizing the credibility level approach. Chen et al. (2020) [353] studied a comprehensive model for fuzzy multi-objective portfolio selection by incorporating the fuzzy mean-semivariance model and DEA cross-efficiency model. Liu (2021) [354] extended the model proposed by Ramon et al. [334] with the concept of $\alpha$-cut approach. This adaptation facilitated the evaluation of DMUs' efficiency through fuzzy cross-efficiency analysis. Their proposed method compared to existing methods for evaluating DMUs efficiency in a fuzzy environment, and the results indicated that their method is more accurate and efficient. Existing studies have mainly focused on CRS assumptions to avoid negative efficiency scores in fuzzy cross-efficiency. There is a limited exploration of production technology with VRS [355]. To address the fuzzy model, they employed the $\alpha$-cut technique. This model presented challenges to solving the large number of constraints and time-taking. Furthermore, there seems to be a gap in the literature regarding the model's application or discussion in real-world contexts.

# 6.3 Preliminaries

## 6.3.1 Credibility Measure

The traditional DEA models, designed for precise data, face limitations in real-world scenarios. Various fuzzy approaches, such as tolerance, alpha-cut, ranking, possibility, and credibility methods, are developed to address these challenges. Sengupta (1992) [87] introduced the first fuzzy model, employing tolerance levels to handle uncertainty. Kao and Liu (2000) [96] used the alpha-cut method to convert fuzzy inputs and outputs into intervals. Emrouznejad (2013) [356] introduced the 'local alpha-cut level' for multi-objective modeling. Guo and Tanaka (2001) [99] proposed a fuzzy ranking approach through bi-level linear programming. Building on Zadeh's [97] fundamental principles of possibility theory for fuzzy sets, Lertworasirikul et al. (2002) [357] and Lertworasirikul et al. (2002) [358] introduced "possibility" approaches within the DEA-CRS model to address ranking challenges. Subsequently, Lertworasirikul et al. (2003) [322] extended this possibility-based methodology to encompass a fuzzy DEA-VRS model. However, it is noted that this "possibility" approach lacks the self-duality property, an essential characteristic both theoretically and practically. These methodologies, while innovative, exhibit limitations that may restrict their applicability in specific contexts [320].

Lertworasirikul et al. (2003) [322] introduced a "credibility approach", which transforms fuzzy variables into expected credits. On the other hand, Liu and Liu (2002) [124] outlined a credibility principle to suggest self-duality measures. This approach is particularly noteworthy because it addresses some of the limitations of other fuzzy methods. Methods like tolerance levels, $\alpha$-cut and possibility, and the credibility approach offer more robust and reliable results in a broader range of scenarios. For instance, Fasanghar et al. (2015) [359] used fuzzy credibility-constrained programming. Amini et al. (2019) [360] applied it to evaluate railway safety. Mahla and Agarwal (2021) [361] combined credibility with fuzzy SBM in Indian oil sector. This study examined the potential of the credibility measure when dealing with the fuzzy DEA model. The applicability of this approach to real-world problems has not been extensively explored.

Let $\xi$ be a nonempty set with $P\{\xi\}$ be the power set of $\xi$. Liu (2006) [362] defined the credibility set function $\mathrm{Cr}\{\cdot\}$ as the credibility measure if and only if the following five axioms are satisfied:

1. $Cr\ \{\emptyset\} = 1$,
2. $Cr$ is increasing, i.e., $Cr\ \{y\} \leq Cr\{x\}$ whenever $y \subset x \in \xi$,
3. The credibility measure is self-dual, i.e., $Cr\ \{y\}$ + $Cr\ \{y^C\}$=1 for any event $y \in P\{\xi\}$,
4. $Cr\ \{\cup_i y_i\} \wedge 0.5 = Sup_i Cr\{y_i\}$ for any events $y_i$ with $Sup_i Cr\{y_i\}$<0.5.
5. $Cr_k$ satisfy the first four axioms on the sets $\xi_k$, $k = 1, 2, \ldots, n$, respectively, and let $\xi = \xi_1 \times \xi_2 \times \cdots, \times \xi_n$. Then for each $(\xi_1, \xi_2, \ldots, \xi_n) \in \xi$, $Cr\{\xi_1, \xi_2, \ldots, \xi_n\} = Cr_1\{\xi_1\} \wedge Cr_2\{\xi_2\} \wedge \cdots \wedge Cr_n\{\xi_n\}$.

The triplet $(\xi, P(\xi), Cr)$ is called the credibility space.

**Theorem 6.3.1** *The two fuzzy variables $\psi_1$ and $\psi_2$ are specified on the credibility space $(\xi, P(\xi), Cr)$. If both $Cr\{\psi_1 = z\}$ and $Cr\{\psi_2 = z\}$ are quasi concave, then for any specific $0.5 \leq \alpha \leq 1$.*

*1. $Cr\{\psi_1 + \psi_2 \leq a\} \geq \alpha$ if and only if $(\psi_1)^U_{2(1-\alpha)} + (\psi_2)^U_{2(1-\alpha)} \leq a$,*
*2. $Cr\{\psi_1 + \psi_2 \leq a\} \leq \alpha$ if and only if $(\psi_1)^U_{2(1-\alpha)} + (\psi_2)^U_{2(1-\alpha)} \geq a$.*

**Definition 6.3.1** *The credibility distribution of triangular fuzzy number $\tilde{M} = (r, s, u)$ such that $(r < s < u)$ is defined as,*

$$\mathrm{Cr}(\tilde{M} \leq b) = \begin{cases} 0, & \text{if } r < b \\ \frac{b-r}{2(s-r)}, & \text{if } r \leq b < s \\ \frac{b+u-2s}{2(u-s)}, & \text{if } s \leq b < u \\ 1, & \text{if } r \geq b. \end{cases} \tag{6.1}$$

$$\mathrm{Cr}(\tilde{M} \geq b) = \begin{cases} 1, & \text{if } r < b \\ \frac{2s-r-b}{2(s-r)}, & \text{if } r \leq b < s \\ \frac{u-b}{2(u-s)}, & \text{if } s \leq b < u \\ 0, & \text{if } r \geq b. \end{cases} \tag{6.2}$$

Where b is a scalar value used to compute the credibility measure depending on its position relative to $r$, $s$, and $u$ .The credibility measure is used to convert fuzzy-chance constraints into their equivalent crisp ones for a given confidence level $\alpha \geq 0.5$ as equation (6.3):

$$\begin{aligned} \mathrm{Cr}(\tilde{M} \leq b) \geq \alpha &\iff (2-2\alpha)s + (2\alpha-1)u \\ \mathrm{Cr}(\tilde{M} \geq b) \geq \alpha &\iff (2-2\alpha)s + (2\alpha-1)r \end{aligned} \tag{6.3}$$

### 6.3.2 Cross-Efficiency Evaluation in DEA

For calculating the efficiency of $n$ DMUs, where each DMU has $i$ inputs and $r$ outputs, consider $i^{th}$ inputs is indicated by $x_{ij}$ $(i = 1, \cdots, m)$ to produce $r^{th}$ outputs is denoted by $y_{rj}$ $(r = 1, \cdots, s)$ of $\mathrm{DMU}_j$, $(j = 1, \cdots, n)$, respectively. A more flexible and adaptable approach is the input-oriented BCC DEA model. This model is used to assess the efficiency of a specific $DMU_d$ under the assumption of variable returns to scale (VRS).

$$\begin{aligned} \text{Max } E_{dd} = \ & \sum_{r=1}^{s} u_{rd} y_{rd} - w_d \\ \text{s.t. } & \sum_{i=1}^{m} v_{id} x_{id} = 1 \\ & \sum_{r=1}^{s} u_{rd} y_{rj} - w_d - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad j = 1, \cdots, n \\ & u_{rd} \geq 0 \quad r = 1, \cdots, s \\ & v_{id} \geq 0 \quad i = 1, \cdots, m \\ & w_d \ \text{free}. \end{aligned} \tag{6.4}$$

Linear programming model (6.4) is solved to determine the self-evaluation efficiency value $E_{dd}$ of $DMU_d$, where $v_{id}^*$ $(i = 1, \cdots, m)$ represents the input weights, and $u_{rd}^*$ $(r = 1, \cdots, s)$ represents the output weights.

**Definition 6.3.2** *$E_{dd}$ is based on self-evaluation framework for $DMU_d$. If $E_{dd}^* = 1$ and all weight values $v_{id}^*, u_{rd}^* > 0$ $\forall$ i, r, then $DMU_d$ is deemed efficient. Conversely, if $E_{dd}^* < 1$, then $DMU_d$ is considered as inefficient.*

The free variable $w_d$ in the objective function model (6.4) can generate the often negative efficiency when $\sum_{r=1}^{s} u_{rd} y_{rd} - w_d$ becomes negative due to $w_d$ assuming significant negative value [321]. A slight intuitive modification to the multiplier model is proposed to address this issue by adding a constraint in a model (6.5).

$$
\begin{aligned}
\text{Max } E_{dd} = & \sum_{r=1}^{s} u_{rd} y_{rd} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id} x_{id} = 1 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad j = 1, \cdots, n \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d \geq 0, \quad j = 1, \cdots, n \\
& u_{rd} \geq 0 \quad r = 1, \cdots, s \\
& v_{id} \geq 0 \quad i = 1, \cdots, m \\
& w_d \text{ free.}
\end{aligned}
\tag{6.5}
$$

Each DMU selects optimal weights $u_{rd}^*$ and $v_{id}^*$ to maximize its efficiency score [363]. However, this approach may introduce a bias in self-evaluation. Cross-efficiency offers a solution by considering the weights chosen by all $n$ DMUs, incorporating both self- and peer-evaluations. To obtain the optimal solution ($u_{rd}^*$, $v_{id}^*$) for a particular $DMU_d$ use the model (6.5). The efficiency of $DMU_j$, denoted as $E_{dj}$, is then determined based on the weights chosen by $DMU_d$. This process yields a peer-evaluation cross-efficiency, which is calculated as follows:

$$
E_{dj} = \frac{\sum_{r=1}^{s} u_{rd}^* y_{rj} - w_d^*}{\sum_{i=1}^{m} v_{id}^* x_{ij}} \tag{6.6}
$$

The solution of model (6.6) generates an $n \times n$ matrix of cross-efficiency values. To calculate the final cross-efficiency score for a specific $DMU_j$, the average of all the cross-efficiency scores corresponding to $d^{th}$ from the matrix be computed. This calculation can be performed using the following formula:

$$
\bar{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_{jd}, \qquad j = 1, 2, \cdots, n \tag{6.7}
$$

DMUs may not always have unique optimal weights generated from the model (6.5). Consequently, the traditional cross-efficiency assessment may lack consistency. To address this issue, several cross-efficiency evaluation models are developed. Two prominent approaches are the “aggressive” and “benevolent” models, these models help to establish the lower and upper bounds of cross-efficiency by considering all possible weight combinations for each DMU.

$$
\begin{aligned}
\text{Min } E_{dj}^{A} = \; & \sum_{r=1}^{s} u_{rd} y_{rj} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id} x_{ij} = 1 \\
& \sum_{r=1}^{s} u_{rd} y_{rd} - w_d - E_{dd}^{*} \sum_{i=1}^{m} v_{id} x_{id} = 0 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad j = 1, \cdots, n, j \neq d \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d \geq 0 \quad j = 1, \cdots, n \\
& u_{rd} \geq 0 \quad r = 1, \cdots, s \\
& v_{id} \geq 0 \quad i = 1, \cdots, m \\
& w_d \;\; \text{free in sign}
\end{aligned}
\tag{6.8}
$$

$$
\begin{aligned}
\text{Max } E_{dj}^{B} = \; & \sum_{r=1}^{s} u_{rd} y_{rj} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id} x_{ij} = 1 \\
& \sum_{r=1}^{s} u_{rd} y_{rd} - w_d - E_{dd}^{*} \sum_{i=1}^{m} v_{id} x_{id} = 0 \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d - \sum_{i=1}^{m} v_{id} x_{ij} \leq 0 \quad j = 1, \cdots, n, j \neq d \\
& \sum_{r=1}^{s} u_{rd} y_{rj} - w_d \geq 0 \quad j = 1, \cdots, n \\
& u_{rd} \geq 0 \quad r = 1, \cdots, s \\
& v_{id} \geq 0 \quad i = 1, \cdots, m \\
& w_d \;\; \text{free in sign}
\end{aligned}
\tag{6.9}
$$

Models (6.8) and (6.9) are designed to prevent the occurrence of negative efficiency values. In these models, $E_{dd}^{*}$ represents the positive efficiency score of $DMU_d$ as computed from the model (6.5).

The aggressive model (6.8) is designed to minimize the cross-efficiency of all other DMUs, while the benevolent model (6.9) seeks to maximize them to some extent. The weights acquired from these two models might differ from those obtained from the model (6.5). By maintaining the efficiency of $DMU_d$ at its current level of $E^*_{dd}$, models (6.8) and (6.9) produce the smallest and largest cross-efficiency for $DMU_d$. Only precise input and output can be used in these models to calculate the cross-efficiency. Fuzzy DEA (FDEA) via fuzzy set theory is proposed to handle efficiency evaluation in the presence of inaccuracies in the data. These models (6.8) and (6.9) are modified into fuzzy cross-efficiency models to accommodate this fuzziness.

### 6.3.3 Fuzzy Cross-Efficiency Model

Assume that a set of DMUs has fuzzy numbers representing imprecise inputs $\tilde{x}_{ij}$ and outputs $\tilde{y}_{rj}$. These inputs and outputs describe the efficiency of each DMU within a production process characterized by $m$ inputs and $s$ outputs. In this context, the fuzzy BCC DEA model (6.5) is defined as follows:

$$\begin{aligned}
\text{Max } \tilde{E}_{dd} = \ & \sum_{r=1}^{s} u_{rd}\tilde{y}_{rd} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id}\tilde{x}_{id} = 1 \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d - \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} \leq 0 \quad j = 1,\cdots,n \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d \geq 0 \quad j = 1,\cdots,n \\
& u_{rd} \geq 0 \quad r = 1,\cdots,s \\
& v_{id} \geq 0 \quad i = 1,\cdots,m \\
& w_d \ \text{ free in sign}
\end{aligned} \tag{6.10}$$

The models (6.8) and (6.9), which are originally designed for crisp data, can be adapted to handle fuzzy data. This modification allows for the application of aggressive and benevolent models to scenarios where the observed values are represented as fuzzy numbers, ensuring that the cross-efficiency evaluation considers the fuzziness in the data.

$$\begin{aligned}
\text{Min } \tilde{E}^{A}_{dj} = \ & \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} = 1 \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rd} - w_d - \tilde{E}^{*}_{dd}\sum_{i=1}^{m} v_{id}\tilde{X}_{id} = 0 \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d - \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} \leq 0 \quad j = 1,\cdots,n, j \neq d \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d \geq 0 \quad j = 1,\cdots,n \\
& u_{rd} \geq 0 \quad r = 1,\cdots,s \\
& v_{id} \geq 0 \quad i = 1,\cdots,m \\
& w_d \ \text{ free in sign}
\end{aligned} \tag{6.11}$$

$$\begin{aligned}
\text{Max } \tilde{E^{B}_{dj}} = \ & \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d \\
\text{s.t. } & \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} = 1 \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rd} - w_d - \tilde{E}^{*}_{dd}\sum_{i=1}^{m} v_{id}\tilde{x}_{id} = 0 \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d - \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} \leq 0 \quad j = 1,\cdots,n, j \neq d \\
& \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - w_d \geq 0 \quad j = 1,\cdots,n \\
& u_{rd} \geq 0 \quad r = 1,\cdots,s \\
& v_{id} \geq 0 \quad i = 1,\cdots,m \\
& w_d \ \text{ free in sign}
\end{aligned} \tag{6.12}$$

### 6.3.4 Fuzzy DEA Model with Credibility Approach

In the context of solving fuzzy cross-efficiency DEA problems, the $\alpha$-cut method, as demonstrated by Liu and Lee (2021) [354], is a widely known approach. However, it comes with the drawback of generating a significant number of equations, specifically $4n+(m+s+1)n^2$ equations for each model, even when dealing with relatively small problems. As an alternative, Lertworasirikul et al. (2003) [322] introduced a different method that employs a credibility approach to solve fuzzy DEA models. In the upcoming section, the focus will be on exploring a fuzzy BCC DEA model, complemented by a credibility approach that incorporates a non-negativity constraint. This approach aims to make the concept clearer and more understandable while addressing the challenges associated with solving fuzzy DEA problems:

$$
\begin{aligned}
\text{Max } \tilde{E}_d = \; & \left\{ \sum_{r=1}^{s} u_{rd}\tilde{y}_{rd} \right\}^{U}_{2(1-\alpha)} - w_d \\
\text{s.t. } & \left\{ \sum_{i=1}^{m} v_{id}\tilde{x}_{id} \right\}^{L}_{2(1-\alpha)} = 1 \quad k = 1, \cdots, n \\
& \left\{ \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} - \sum_{i=1}^{m} v_{id}\tilde{x}_{ij} \right\}^{U}_{2(1-\alpha)} - w_d \leq 0 \quad j = 1, \cdots, n \\
& \left\{ \sum_{r=1}^{s} u_{rd}\tilde{y}_{rj} \right\}^{U}_{2(1-\alpha)} - w_d \geq 0 \quad j = 1, \cdots, n \\
& u_{rd} \geq 0 \quad r = 1, \cdots, s \\
& v_{id} \geq 0 \quad i = 1, \cdots, m \\
& w_d \text{ free in sign.}
\end{aligned}
\tag{6.13}
$$

**Definition 6.3.3** *$DMU_d$ is efficient only if the optimal value of model (6.13), denoted as $\tilde{E}^*_d$, is greater than or equal to 1, at a credibility level $\alpha$ within the range [0.5, 1]. Conversely, if this condition is not met, $DMU_d$ is deemed inefficient at the same credibility level.*

**Lemma 6.3.2** *Let $(E^*_d, u^*_{rd}, v^*_{id})$ be an optimal solution of model (6.13) for a given $DMU_d$, then $\{v^*_{id}\tilde{x}_{id}\}^{L}_{2(\alpha-1)} = 1$ holds.*

**Definition 6.3.4** *Assuming $(E^*_d, u^*_{rd}, v^*_{id})$ represents an optimal solution of model (6.13) for a specific $DMU_d$ at a credibility level within the range [0.5,1], then the credibility cross-efficiency of $DMU_j$, $(j = 1, \cdots, n)$, evaluated at the same credibility level, obtained the weights of $DMU_j$, is defined as the ratio:*

$$
E^{Cr}_{dj} = \frac{\{u^*_{rd}\tilde{y}_{rd}\}^{U}_{\alpha} - w^*_d}{\{v^*_{id}\tilde{x}_{id}\}^{L}_{\alpha}} \tag{6.14}
$$

**Definition 6.3.5** *Let $E^{Cr}_{dj}$, $(j = 1, \cdots, n)$ represent the cross-efficiency for a specific $DMU_d$ at credibility level $\alpha$ within the range [0.5,1], the credibility cross-efficiency value of $DMU_j$, $(j = 1, \cdots, n)$, evaluated at the same credibility level, expressed as:*

$$\bar{E}_j^{Cr} = \frac{1}{n}\sum_{d=1}^{n} E_{dj}^{Cr}, \quad j = 1, \cdots, n \tag{6.15}$$

Based upon the previous lemma and the fundamental definitions of the credibility approach, the aggressive and benevolent models for fuzzy BCC DEA cross-efficiency are transformed into the following models:

$$\begin{aligned}
\text{Min } E_{dj}^{Cr,A} = \ & \left(\sum_{r=1}^{s} u_{rd} \sum_{j=1, j\neq d}^{n} \tilde{y}_{rj}\right)_{2(1-\alpha)}^{L} - w_d \\
\text{subject to } & (\sum_{i=1}^{m} v_{ik} \sum_{j=1, j\neq d}^{n} \tilde{x}_{ij})_{2(1-\alpha)}^{U} = 1, \\
& (\sum_{r=1}^{s} u_{rd}\tilde{y}_{rd})_{2(1-\alpha)}^{U} - E_{dd}(\sum_{i=1}^{m} v_{id}\tilde{x}_{ij})_{2(1-\alpha)}^{L} - w_d \le 0, \\
& (\sum_{r=1}^{s} u_{rd}\tilde{y}_{rj})_{2(1-\alpha)}^{U} - (\sum_{i=1}^{m} v_{id}\tilde{x}_{ij})_{2(1-\alpha)}^{L} - w_d \le 0, \quad j = 1, \cdots, n, j \neq d \\
& (\sum_{r=1}^{s} u_{rd}\tilde{y}_{rj})_{2(1-\alpha)}^{U} - w_d \le 0, \quad j = 1, \cdots, n, j \neq d \\
& u_{rd} \ge 0 \quad r = 1, \cdots, s \\
& v_{id} \ge 0 \quad i = 1, \cdots, m \\
& w_d \ \text{ free in sign.}
\end{aligned} \tag{6.16}$$

$$
\begin{aligned}
\text{Max } E_{dj}^{Cr,B} = \; & \left( \sum_{r=1}^{s} u_{rd} \sum_{j=1, j \neq d}^{n} \tilde{y}_{rj} \right)_{2(1-\alpha)}^{U} - w_d \\
\text{subject to } & (\sum_{i=1}^{m} v_{ik} \sum_{j=1, j \neq d}^{n} \tilde{x}_{ij})_{2(1-\alpha)}^{L} = 1, \\
& (\sum_{r=1}^{s} u_{rd} \tilde{y}_{rd})_{2(1-\alpha)}^{U} - E_{dd} (\sum_{i=1}^{m} v_{id} \tilde{x}_{ij})_{2(1-\alpha)}^{L} - w_d \leq 0, \\
& (\sum_{r=1}^{s} u_{rd} \tilde{y}_{rj})_{2(1-\alpha)}^{U} - (\sum_{i=1}^{m} v_{id} \tilde{x}_{ij})_{2(1-\alpha)}^{L} - w_d \leq 0, \quad j = 1, \cdots, n, j \neq d \\
& (\sum_{r=1}^{s} u_{rd} \tilde{y}_{rj})_{2(1-\alpha)}^{U} - w_d \leq 0, \quad j = 1, \cdots, n, j \neq d \\
& u_{rd} \geq 0 \quad r = 1, \cdots, s \\
& v_{id} \geq 0 \quad i = 1, \cdots, m \\
& w_d \text{ free in sign.}
\end{aligned}
\tag{6.17}
$$

The lower and upper bounds of efficiency are determined by the models (6.16) and (6.17), denoted as $(E_k^{Cr,A})_\alpha^L$ and $(E_k^{Cr,B})_\alpha^U$ respectively, for each DMU across different $\alpha$ levels.

### 6.3.5 Fuzzy Correlation Coefficient of Input $\tilde{x}$ and Output $\tilde{y}$

In the vast landscape of statistical analysis, the correlation coefficient has traditionally stood as a pivotal tool, consistently employed to discern the relationship between variables across various scientific and engineering fields. While these conventional techniques are effective for precise data, the increasing challenges in dealing with ambiguous and imprecise information necessitate the exploration of more advanced methodologies. This led to the convergence of traditional correlation with fuzzy logic—a mathematical approach adept at analyzing vague or uncertain data. Pioneering this convergence, Puri and Yadav [120] introduced a groundbreaking method in 2013 and concluded that the correlation coefficient is a reliable measuring operator for the applicability of generalized fuzzy sets in decision-making.

For two precise data sets, represented as $r(x,y)$ with $x = x_1, x_2, ..., x_n$ and $y = y_1, y_2, ..., y_n$, the correlation coefficient is calculated using the following formula:

$$
r_L(x,y) = \frac{n \sum_{i=1}^{n} x_L^i y_L^i - \sum_{i=1}^{n} x_L^i \sum_{i=1}^{n} y_L^i}{\sqrt{n \sum_{i=1}^{n} \left(x_L^i\right)^2 - \left(\sum_{i=1}^{n} x_L^i\right)^2} \sqrt{n \sum_{i=1}^{n} \left(y_L^i\right)^2 - \left(\sum_{i=1}^{n} y_L^i\right)^2}}
\tag{6.18}
$$

However, precise data might not always be available in real-life situations due to imprecisions or ambiguities. Hence, for two fuzzy data sets, $\tilde{x}$ and $\tilde{y}$, depicted as $\tilde{x_!}, \tilde{x_2}, \tilde{x_3}, \cdots, \tilde{x_n}$ and $\tilde{y_!}, \tilde{y_2}, \tilde{y_3}, \cdots, \tilde{y_n}$, the fuzzy correlation coefficient, denoted as $\tilde{r}(\tilde{x}, \tilde{y})$, is calculated using the following formula:

$$r_R(\tilde{x},\tilde{y}) = \frac{n\sum_{i=1}^{n} x_R^i y_R^i - \sum_{i=1}^{n} x_R^i \sum_{i=1}^{n} y_R^i}{\sqrt{n\sum_{i=1}^{n}\left(x_R^i\right)^2 - \left(\sum_{i=1}^{n} x_R^i\right)^2}\sqrt{n\sum_{i=1}^{n}\left(y_R^i\right)^2 - \left(\sum_{i=1}^{n} y_R^i\right)^2}} \tag{6.19}$$

An expected value method is employed to derive the fuzzy correlation coefficient between these fuzzy data sets. The initial step involves determining the expected intervals (EI) of the fuzzy datasets, namely find $EI(\tilde{x}_i) = [\tilde{x_L^i}, \tilde{x_R^i}]$, $EI(\tilde{y}_i) = [\tilde{y_L^i}, \tilde{y_R^i}]$, $i = 1,2,\cdots,n$. Using these intervals, the expected interval of the fuzzy correlation coefficient,

$\tilde{r}(\tilde{x},\tilde{y})$, is identified as $r^{EI}(\tilde{x},\tilde{y}) = [r_L(\tilde{x},\tilde{y}), r_R(\tilde{x},\tilde{y})]$, where:

- Both $r_L(\tilde{x},\tilde{y})$ and $r_R(\tilde{x},\tilde{y})$ lie within the range [-1,1].
- $r_L(\tilde{x},\tilde{y})$ is equal to $r_L(\tilde{y},\tilde{x})$ and similarly for $r_R(\tilde{x},\tilde{y}) = r_R(\tilde{y},\tilde{x})$.
- $r_L(\tilde{x},\tilde{y})$ and $r_R(\tilde{x},\tilde{y})$ both equals to 1 if $\tilde{x} = \tilde{y}$.

The average value of the fuzzy correlation coefficient is symbolized as $r^{EV}(\tilde{x},\tilde{y})$ and is expressed by:

$$r^{EV}(\tilde{x},\tilde{y}) = \frac{r_L(\tilde{x},\tilde{y}) + r_R(\tilde{x},\tilde{y})}{2} \tag{6.20}$$

Additionally, this average value adheres to the properties:

- $r^{EV}(\tilde{x},\tilde{y})$ is confirmed to the range [-1, 1].
- $r^{EV}(\tilde{x},\tilde{y})$ is symmetrical, i.e.,

$$r^{EV}(\tilde{x},\tilde{y}) = r^{EV}(\tilde{y},\tilde{x}) \tag{6.21}$$

- $r^{EV}(\tilde{x},\tilde{y}) = 1$ if $\tilde{x} = \tilde{y}$.

When the value of $r^{EV}(\tilde{x},\tilde{y})$ is positive for every $\tilde{x}$ and $\tilde{y}$, it indicates that the FDEA model maintains consistency, and the incorporation of fuzzy inputs and outputs is appropriate.

**Remark 6.3.1** *If the data is crisp, the proposed approach gives similar results as given by the conventional correlation coefficient formula defined in equation (6.20). In the case of crisp data,*

$$r_L(\tilde{x}, \tilde{y}) = r_R(\tilde{x}, \tilde{y}) = r^{EV}(\tilde{x}, \tilde{y}) = r(x, y). \tag{6.22}$$

## 6.4 Fuzzy Cross-Efficiency of Transport Sector

Evaluating the efficiency of the transport sector is a crucial concern for policymakers. In this section, the proposed method is discussed to assess the efficiency of the transport system amid ongoing improvements, taking into account the inherent uncertainties. However, evaluating transport efficiency under uncertainty is essential, particularly in assessing the success of upcoming reforms and verifying the sustainability of long-term reforms. The proposed fuzzy cross-efficiency DEA approach is applied to address the uncertainty in evaluating the efficiency of the transport sector. Applying this methodology is straightforward and beneficial for validating and planning targeted transportation reforms.

The proposed framework consists of the following three main phases as shown in figure 6.1:

**Input-output data collection phase:** The selection of the appropriate criteria necessary for an effective performance evaluation serves as the beginning of this phase. The chosen input-output variables are then the subject of a systematic collection of quantitative data, which forms the basis for the ensuing analytical steps. This data is organized and preprocessed with great care to ensure its accuracy and relevancy.

**Fuzzy DEA-efficiency phase:** This phase commences with the application of fuzzy DEA methodologies to the gathered data. To refine the precision of the results, efficiencies are assessed using a credibility measure with multiple credibility levels. The subsequent analysis of these findings provides a clearer understanding of the relative efficiencies of the units under study, highlighting their comparative strengths and areas of potential improvement. Furthermore, the fuzzy cross-efficiency technique is implemented, encompassing credibility level and considering both aggressive and benevolent perspectives, reflecting the multifaceted nature of the evaluation process.

**Ranking phase:** This crucial phase starts with the thorough creation of a decision matrix that combines cross-efficiencies obtained from both (aggressive and benevolent) approaches. The Ensemble ranking technique is used to ensure a multi-dimensional and thorough evaluation of the STUs, improving the robustness and integrity of the ranking process.

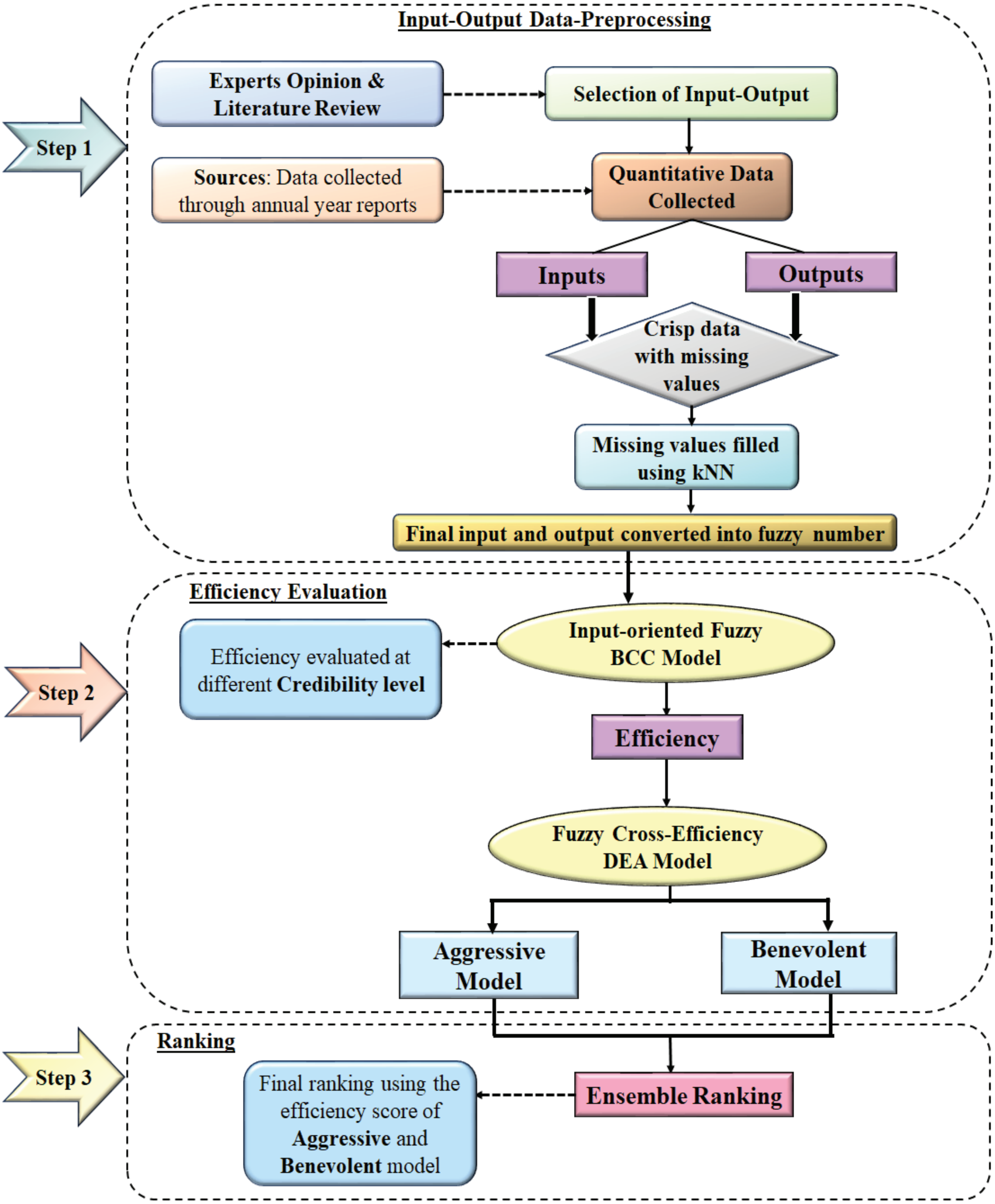


**Fig. 6.1:** Schematic view of the proposed approach.

### 6.4.1 Data Collection

This study delves into an extensive analysis of the financial and operational performance of public bus transit systems in India, focusing on data compiled from the financial year 2017-2018. The database utilized for this in-depth examination originates from a comprehensive report published by the Central Institute of Road Transport (CIRT). The dataset specifically pertains to publicly owned and operated bus transit corporations operating in various states. These corporations, commonly known as "state transport undertakings" (STUs), serve as the cornerstone of public road transportation across India. STUs, in their role as critical providers of mobility services, are indispensable components of India's public transportation landscape. They fulfill a crucial function by connecting urban and rural areas, ensuring that passengers nationwide have access to reliable transportation services. STUs are distinctively characterized by their commitment to non-profit objectives, with each state government playing a pivotal role in overseeing and guiding their operations. This unique characteristic aligns with their mission to prioritize public service and accessibility over profit generation. As such, the sustainability and effectiveness of STUs are paramount to the nation's continued growth and development. Due to not availability of data, some STUs from the data set are excluded, including SMTU, NGST, MZST, ARPST, TNSTC(TNV), CHNTU, PRTC, TRPTC, BSRTC, and CSTC from the 47 STUs, resulting in a final sample of 37 STUs. These exclusions are made primarily due to the unavailability of complete and reliable data for these specific STUs. In pursuing rigorous analysis, ensuring data accuracy and consistency is paramount. During this process, it's worth noting that some missing values persist within the dataset, which may be attributed to various real-life factors, such as data recording discrepancies or specific operational challenges faced by certain STUs. These real-life constraints emphasize the importance of meticulous data handling and analysis to derive meaningful insights into the performance of public transport sector.

### 6.4.2 Data Pre-processing

Pre-processing is the preliminary stage of any data analysis process, involving the preparation and transformation of raw data into a refined and usable format, including addressing missing values. The presence of missing data in annual reports can decrease the statistical power of the analytical model and adversely affect decision-making processes. This problem is common and unavoidable in data-driven transport systems. Several factors contribute to this significant problem, including imperfect manual data entry procedures, inaccurate measurements and equipment errors.

Imputation techniques are frequently used to address these challenges. Among these techniques, the k nearest neighbour (kNN) method proves to be typically successful for imputing numerical variables. This method is based on Euclidean distance or similar measures, making it increasingly favored in empirical studies for its accuracy and simplicity. The main advantage of kNN imputation is that it doesn't require any parametric assumption, allowing it to adapt to distinct types of variables or features that are important in estimation. In comparison, mean imputation (MEI) is another approach, but still lacking in popularity for real-world problems.

Some degree of ambiguity or vagueness may still exist after predicting and filling the values that are missing. The fuzzy set theory is employed to overcome this remaining ambiguity. Furthermore, the utilization of fuzzy numbers plays a pivotal role in reducing uncertainty when predicting missing values. Employing membership functions from fuzzy set theory to express uncertain values provides a robust strategy for handling erroneous data directly [112]. Through the fuzzification process, predicted missing values are transformed into triangular fuzzy numbers. The application of Saaty's scale [181] to fuzzify these values in triangular fuzzy numbers enhances their suitability for representing uncertainty.

### 6.4.3 Inputs & Outputs

The development of an evaluation framework requires that the various inputs and outputs impacts of the transport service be organized in a meaningful manner [364]. As calculation of efficiency using DEA is sensitive in the selection of inputs and outputs, as the literature evidences varied approaches [365]. Subjective evaluation grounded in expert opinion remains a common method, yet others applied objective techniques based on mathematics and statistics. Number of vehicles ($I_1$), number of drivers ($I_2$), staff ratio per bus and total traffic staff ($I_3$), administration ($I_4$) & conductors ($I_5$) and fuel consumption ($I_6$) are the most considered variables since they represent the main inputs in the production process. On the other hand, the most frequently used output measures are such as vehicles by traveled kilometers ($O_1$) and the number of passengers carried ($O_2$).

Lower and upper correlations are calculated using the equations (6.18) and (6.19). The upper diagonal entries indicate the upper correlation values, while the lower diagonal entries section represents the lower correlation values in the table 6.1. These values are derived from correlation analysis and provide an upper or lower estimate of the relationship between criteria. Given that the final correlation is the average of both the lower and upper correlations, it provides a more comprehensive perspective on the relationships between these criteria in table 6.2.

**Table 6.1:** Upper & lower correlation between fuzzy inputs-outputs for the year (2017-18).

| Variables | Inputs | | | | | | Outputs | |
|---|---|---|---|---|---|---|---|---|
| | $(I_1)$ | $(I_2)$ | $(I_3)$ | $(I_4)$ | $(I_5)$ | $(I_6)$ | $(O_1)$ | $(O_2)$ |
| $(I_1)$ | **1** | 0.919 | 0.825 | 0.901 | 0.804 | 0.819 | 0.976 | 0.798 |
| $(I_2)$ | 0.896 | **1** | 0.848 | 0.777 | 0.801 | 0.812 | 0.94 | 0.741 |
| $(I_3)$ | 0.797 | 0.839 | **1** | 0.693 | 0.836 | 0.786 | 0.799 | 0.797 |
| $(I_4)$ | 0.859 | 0.755 | 0.688 | **1** | 0.687 | 0.729 | 0.874 | 0.645 |
| $(I_5)$ | 0.795 | 0.784 | 0.820 | 0.663 | **1** | 0.845 | 0.799 | 0.663 |
| $(I_6)$ | 0.793 | 0.786 | 0.738 | 0.697 | 0.825 | **1** | 0.823 | 0.859 |
| $(O_1)$ | 0.967 | 0.913 | 0.775 | 0.838 | 0.787 | 0.797 | **1** | 0.758 |
| $(O_2)$ | 0.781 | 0.722 | 0.758 | 0.599 | 0.639 | 0.820 | 0.742 | **1** |

In table 6.1, the correlation coefficients of input and output variables are presented. Since all values are positive, there are no inverse relationships observed among the provided criteria. The relationships between input and output are predominantly strong. Most of the correlation coefficients are close to 1. The correlation coefficients among the input and output criteria are checked to ensure that the dataset satisfies the isotonicity property and to ensure that the input and output variables are important and relevant.

**Table 6.2:** Final correlation between fuzzy inputs-outputs for the year (2017-18).

| Variables | Inputs | | | | | | Outputs | |
|---|---|---|---|---|---|---|---|---|
| | $(I_1)$ | $(I_2)$ | $(I_3)$ | $(I_4)$ | $(I_5)$ | $(I_6)$ | $(O_1)$ | $(O_2)$ |
| $(I_1)$ | 1 | | | | | | | |
| $(I_2)$ | 0.907 | 1 | | | | | | |
| $(I_3)$ | 0.811 | 0.844 | 1 | | | | | |
| $(I_4)$ | 0.88 | 0.766 | 0.69 | 1 | | | | |
| $(I_5)$ | 0.799 | 0.792 | 0.828 | 0.675 | 1 | | | |
| $(I_6)$ | 0.806 | 0.799 | 0.762 | 0.713 | 0.835 | 1 | | |
| $(O_1)$ | 0.971 | 0.926 | 0.787 | 0.856 | 0.793 | 0.81 | 1 | |
| $(O_2)$ | 0.789 | 0.731 | 0.778 | 0.622 | 0.651 | 0.839 | 0.75 | 1 |

### 6.4.4 Fuzzy DEA Efficiency Results

In order to evaluate the optimal efficiency scores of each STU, fuzzy DEA model (6.13) is applied at different credibility level-$\alpha$ for each STU, respectively, and the obtained results are presented in table 6.3. The values seem to range between 0 and 1 for each STU at different credibility levels, with 1 indicating the best performance. According to the results, 11

STUs, namely, GSRTC, UPSRTC, KSRTC, RSRTC, TNSTC (MDU), TNSTC (KUM), TNSTC (VPM), TNSTC (CBE), SETC (TN), NMMT, and PMPML with an efficiency score of 1 have the best efficiency in converting inputs to outputs at all credibility levels [0.5, 0.9] with 0.1 steps computed. Whereas APSRTC, KnSRTC, TNSTC (SLM), ASMSTC, and SKNT show a declining trend as $\alpha$ increases. This suggests that these STUs may be sensitive to the changing conditions or parameters associated with increasing $\alpha$ values. STUs such as TSRTC, HRTC, UTC, BEST, DTC, BMTC, MTC (CNI), TMTU, KMTU, KADMTU, and OSRTC have generally lower scores across the $\alpha$ values, especially as $\alpha$ increases, which could be areas of concern. Out of 37 STUs, 26 have efficiency scores below 1, indicating that they are inefficient. The scores of the inefficient STUs range between 0.426 and 0.986. The weakest performance is related to the MTC (CNI) STU, which belongs to Chennai. The findings suggested that the fuzzy DEA model lacks the ability to rank the efficient DMUs, and its low discriminatory power is a limitation when it comes to evaluating efficiency scores. The standard deviation (SD) increases as $\alpha$ increases, indicating that the variance in scores among STUs grows with increasing $\alpha$ values. This suggests that the STUs are more diverse in their performance or response to the conditions represented by higher $\alpha$ values. Table 6.3 reveals that the performance of many STUs is sensitive to the value of $\alpha$. An increase in $\alpha$ seems to affect the performance of several STUs negatively.

**Table 6.3:** Fuzzy BCC efficiency scores of STUs for the year 2017-18.

| STUs | $\alpha = 0.5$ | $\alpha = 0.6$ | $\alpha = 0.7$ | $\alpha = 0.8$ | $\alpha = 0.9$ | Rank |
|---|---|---|---|---|---|---|
| APSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 0.878 | 14 |
| MSRTC | 0.898 | 0.918 | 0.940 | 0.838 | 0.653 | 24 |
| GSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| UPSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| KnSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 0.874 | 15 |
| KSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| RSRTC | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| NWKnRTC | 0.852 | 0.938 | 1.000 | 1.000 | 1.000 | 17 |
| TNSTC (MDU) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| STHAR | 0.964 | 0.964 | 0.919 | 0.830 | 0.723 | 22 |
| TNSTC (KUM) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| TNSTC (VPM) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| TNSTC (CBE) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| NEKnRTC | 0.878 | 0.878 | 0.819 | 0.709 | 0.591 | 26 |
| TNSTC (SLM) | 1.000 | 1.000 | 1.000 | 1.000 | 0.986 | 12 |
| TSRTC | 0.762 | 0.760 | 0.743 | 0.736 | 0.738 | 28 |
| SETC (TN) | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| NBSTC | 0.951 | 0.947 | 0.918 | 0.881 | 0.819 | 21 |
| SBSTC | 0.970 | 0.970 | 0.970 | 0.960 | 0.896 | 18 |
| KDTC | 0.957 | 0.956 | 0.933 | 0.893 | 0.795 | 20 |
| ASMSTC | 1.000 | 1.000 | 0.905 | 0.768 | 0.661 | 23 |
| ANST | 0.892 | 0.892 | 0.802 | 0.658 | 0.530 | 27 |
| SKNT | 1.000 | 1.000 | 1.000 | 0.929 | 0.837 | 18 |
| HRTC | 0.868 | 0.865 | 0.758 | 0.646 | 0.540 | 29 |
| UTC | 0.867 | 0.867 | 0.867 | 0.826 | 0.713 | 25 |
| MEGTC | 0.971 | 0.976 | 0.972 | 0.970 | 1.000 | 13 |
| NMMT | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| BEST | 0.789 | 0.789 | 0.702 | 0.604 | 0.508 | 30 |
| DTC* | 0.810 | 0.788 | 0.663 | 0.539 | 0.431 | 35 |
| BMTC | 0.822 | 0.802 | 0.668 | 0.542 | 0.433 | 33 |
| MTC (CNI) | 0.801 | 0.798 | 0.656 | 0.533 | 0.426 | 36 |
| PMPML | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1 |
| AMTS | 0.860 | 1.000 | 1.000 | 1.000 | 1.000 | 16 |
| TMTU | 0.755 | 0.753 | 0.663 | 0.548 | 0.448 | 37 |
| KMTU | 0.839 | 0.811 | 0.667 | 0.544 | 0.437 | 31 |
| KADMTU | 0.839 | 0.807 | 0.668 | 0.545 | 0.437 | 32 |
| OSRTC | 0.813 | 0.796 | 0.670 | 0.544 | 0.436 | 34 |
| **Average** | 0.923 | 0.926 | 0.889 | 0.839 | 0.778 | |
| **SD** | 0.085 | 0.089 | 0.136 | 0.185 | 0.227 | |

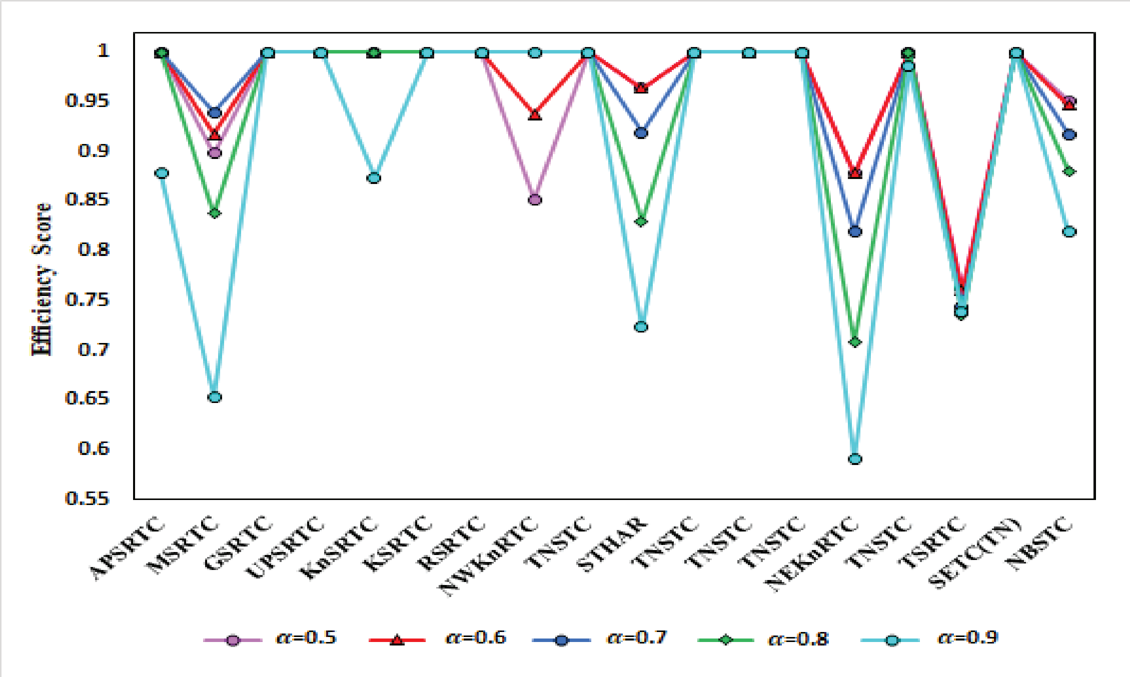


**Fig. 6.2:** Fuzzy BCC efficiency scores across (1-18) STUs (2017-2018).

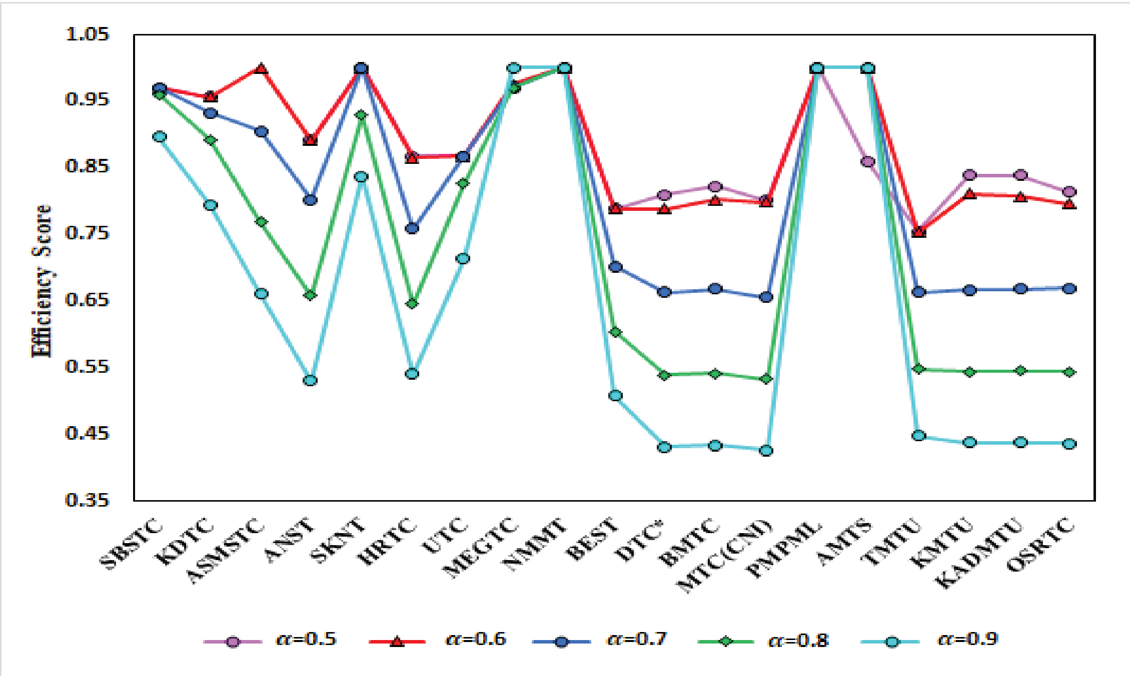


**Fig. 6.3:** Fuzzy BCC efficiency scores across (19-37) STUs (2017-2018).

### 6.4.5 Fuzzy Cross-Efficiency Evaluation

The present section examines the fuzzy cross-efficiency scores of 37 STUs. The cross-evaluation method is more effective than the self-evaluation method, as it can differentiate between non-dominated STUs where the latter method cannot. Then, use the results of table 6.3 into models (6.17) and (6.16) to compute the interval cross-efficiency of each STU. The average fuzzy interval cross-efficiency evaluation results are presented in table 6.4 under the same condition of maintaining STU's self-evaluation efficiency value unchanged. The results also include average and standard deviation values at the end. In this study, the cross-efficiency values are denoted as $(E_k^{Cr,A})_\alpha^L$ and $(E_k^{Cr,B})_\alpha^U$, respectively. The aggressive evaluation approach aims to minimize the cross-efficiency values, while the benevolent approach seeks to maximize them. Consequently, the resulting values represent the lower bounds of efficiency for the aggressive evaluation and the upper bounds for the benevolent evaluation. This dual assessment provides a comprehensive understanding of the performance spectrum of various STUs across five different $\alpha$ measures, which are shown as 'min' and 'max' efficiency scores for each $\alpha$. The gap between the min and max values is monotonically decreased until the iteration process converges to an optimal solution of the fuzzy cross-efficiency DEA model. For aggressive and benevolent evaluation, the mean efficiency score assigned by STUs is in the range [0.639, 0.878] for all credibility levels [0.5, 0.9], respectively. The results provided, that UPSRTC has the highest average in some of the measures, nearing 0.998 in one of them. While TMTU and MTC (CNI) appear to be the lowest performing STUs in certain measures, with values as low as 0.353 for MTC (CNI). AMTS has values that range from 0.673 to 1.000, indicating significant variability in their performance. The standard deviation for $\alpha$=0.9 measures is 0.190, which is relatively high. This means that the performance of the STUs varies significantly for these particular measures. In other words, some STUs might perform exceptionally well, while others might be performing poorly in these specific measures. The wider range suggests inconsistency among the STUs for these performance metrics. The effectiveness of the proposed methodology in ranking between efficient STUs is demonstrated in a complex problem. Interestingly, the best performer is changed under different credibility measure level. However, the fact that TMTU always stays almost at the bottom of the list does not change the rank.

**Table 6.4:** Average fuzzy cross-efficiency scores of 37 STUs (2017-18).

| | $\alpha = 0.5$ | | $\alpha = 0.6$ | | $\alpha = 0.7$ | | $\alpha = 0.8$ | | $\alpha = 0.9$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| STUs | min | max | min | max | min | max | min | max | min | max |
| APSRTC | 0.804 | 0.984 | 0.839 | 0.981 | 0.810 | 0.927 | 0.711 | 0.780 | 0.642 | 0.667 |
| MSRTC | 0.619 | 0.785 | 0.659 | 0.793 | 0.586 | 0.676 | 0.499 | 0.550 | 0.447 | 0.463 |
| GSRTC | 0.713 | 0.954 | 0.746 | 0.932 | 0.764 | 0.926 | 0.744 | 0.860 | 0.860 | 0.905 |
| UPSRTC | 0.844 | 0.998 | 0.869 | 0.994 | 0.861 | 0.971 | 0.827 | 0.909 | 0.838 | 0.879 |
| KnSRTC | 0.710 | 0.914 | 0.768 | 0.919 | 0.741 | 0.861 | 0.662 | 0.744 | 0.648 | 0.677 |
| KSRTC | 0.788 | 0.976 | 0.803 | 0.946 | 0.820 | 0.945 | 0.782 | 0.870 | 0.889 | 0.927 |
| RSRTC | 0.751 | 0.939 | 0.768 | 0.917 | 0.799 | 0.924 | 0.767 | 0.866 | 0.778 | 0.818 |
| NWKnRTC | 0.684 | 0.835 | 0.707 | 0.914 | 0.703 | 0.931 | 0.642 | 0.889 | 0.727 | 0.886 |
| TNSTC (MDU) | 0.732 | 0.940 | 0.781 | 0.935 | 0.793 | 0.925 | 0.768 | 0.861 | 0.875 | 0.912 |
| STHAR | 0.769 | 0.943 | 0.788 | 0.922 | 0.771 | 0.877 | 0.689 | 0.756 | 0.661 | 0.687 |
| TNSTC (KUM) | 0.722 | 0.947 | 0.794 | 0.968 | 0.785 | 0.931 | 0.736 | 0.830 | 0.821 | 0.846 |
| TNSTC (VPM) | 0.843 | 0.993 | 0.883 | 0.992 | 0.876 | 0.975 | 0.843 | 0.921 | 0.861 | 0.896 |
| TNSTC (CBE) | 0.780 | 0.982 | 0.823 | 0.978 | 0.798 | 0.931 | 0.767 | 0.857 | 0.846 | 0.883 |
| NEKnRTC | 0.673 | 0.847 | 0.704 | 0.838 | 0.668 | 0.768 | 0.584 | 0.642 | 0.546 | 0.567 |
| TNSTC (SLM) | 0.633 | 0.815 | 0.696 | 0.836 | 0.705 | 0.812 | 0.665 | 0.744 | 0.720 | 0.749 |
| TSRTC | 0.590 | 0.714 | 0.603 | 0.711 | 0.590 | 0.684 | 0.555 | 0.634 | 0.569 | 0.601 |
| SETC (TN) | 0.824 | 0.964 | 0.847 | 0.966 | 0.852 | 0.958 | 0.829 | 0.913 | 0.849 | 0.886 |
| NBSTC | 0.751 | 0.936 | 0.787 | 0.929 | 0.749 | 0.866 | 0.702 | 0.777 | 0.721 | 0.751 |
| SBSTC | 0.727 | 0.923 | 0.750 | 0.896 | 0.773 | 0.898 | 0.755 | 0.856 | 0.775 | 0.818 |
| KDTC | 0.796 | 0.945 | 0.831 | 0.943 | 0.813 | 0.908 | 0.761 | 0.829 | 0.722 | 0.749 |
| ASMSTC | 0.694 | 0.937 | 0.732 | 0.912 | 0.688 | 0.822 | 0.585 | 0.664 | 0.583 | 0.610 |
| ANST | 0.632 | 0.822 | 0.665 | 0.816 | 0.643 | 0.749 | 0.546 | 0.608 | 0.502 | 0.524 |
| SKNT | 0.764 | 0.930 | 0.766 | 0.921 | 0.797 | 0.930 | 0.740 | 0.840 | 0.726 | 0.761 |
| HRTC | 0.634 | 0.818 | 0.671 | 0.812 | 0.615 | 0.717 | 0.533 | 0.593 | 0.494 | 0.513 |
| UTC | 0.680 | 0.827 | 0.716 | 0.834 | 0.703 | 0.801 | 0.650 | 0.717 | 0.608 | 0.633 |
| MEGTC | 0.748 | 0.928 | 0.722 | 0.917 | 0.719 | 0.910 | 0.673 | 0.873 | 0.788 | 0.868 |
| NMMT | 0.812 | 0.983 | 0.797 | 0.975 | 0.777 | 0.953 | 0.724 | 0.908 | 0.801 | 0.871 |
| BEST | 0.550 | 0.716 | 0.586 | 0.738 | 0.521 | 0.653 | 0.433 | 0.565 | 0.412 | 0.496 |
| DTC | 0.639 | 0.790 | 0.658 | 0.772 | 0.584 | 0.659 | 0.497 | 0.537 | 0.423 | 0.435 |
| BMTC | 0.649 | 0.800 | 0.665 | 0.781 | 0.589 | 0.664 | 0.503 | 0.543 | 0.426 | 0.439 |
| MTC (CNI) | 0.520 | 0.708 | 0.576 | 0.738 | 0.497 | 0.616 | 0.407 | 0.502 | 0.353 | 0.419 |
| PMPML | 0.696 | 0.933 | 0.660 | 0.968 | 0.655 | 0.981 | 0.588 | 0.939 | 0.739 | 0.952 |
| AMTS | 0.673 | 0.835 | 0.684 | 0.981 | 0.664 | 0.998 | 0.595 | 1.000 | 0.730 | 1.000 |
| TMTU | 0.468 | 0.667 | 0.494 | 0.659 | 0.467 | 0.593 | 0.376 | 0.473 | 0.358 | 0.405 |
| KMTU | 0.659 | 0.816 | 0.676 | 0.793 | 0.592 | 0.667 | 0.506 | 0.546 | 0.428 | 0.440 |
| KADMTU | 0.659 | 0.815 | 0.674 | 0.791 | 0.593 | 0.667 | 0.507 | 0.547 | 0.429 | 0.441 |
| OSRTC | 0.635 | 0.790 | 0.655 | 0.773 | 0.583 | 0.660 | 0.495 | 0.537 | 0.426 | 0.4396 |
| **Average** | 0.699 | 0.877 | 0.725 | 0.878 | 0.701 | 0.831 | 0.639 | 0.743 | 0.649 | 0.698 |
| **SD** | 0.088 | 0.092 | 0.087 | 0.092 | 0.108 | 0.127 | 0.127 | 0.153 | 0.170 | 0.190 |

**Table 6.5:** Aggressive and Benevolent ranking for all credibility level of 37 STUs (2017-18).

| | $\alpha = 0.5$ | | $\alpha = 0.6$ | | $\alpha = 0.7$ | | $\alpha = 0.8$ | | $\alpha = 0.9$ | |
|---|---|---|---|---|---|---|---|---|---|---|
| STUs | min | max | min | max | min | max | min | max | min | max |
| APSRTC | 5 | 3 | 4 | 3 | 6 | 12 | 14 | 18 | 22 | 22 |
| MSRTC | 33 | 33 | 31 | 28 | 32 | 29 | 32 | 30 | 29 | 30 |
| GSRTC | 17 | 8 | 17 | 13 | 15 | 13 | 10 | 12 | 4 | 5 |
| UPSRTC | 1 | 1 | 2 | 1 | 2 | 4 | 3 | 5 | 7 | 10 |
| KnSRTC | 18 | 20 | 13 | 17 | 17 | 21 | 19 | 21 | 21 | 21 |
| KSRTC | 7 | 6 | 7 | 10 | 4 | 7 | 4 | 9 | 1 | 3 |
| RSRTC | 11 | 13 | 13 | 18 | 7 | 15 | 6 | 10 | 11 | 14 |
| NWKnRTC | 21 | 22 | 21 | 20 | 20 | 8 | 21 | 7 | 15 | 7 |
| TNSTC (MDU) | 14 | 12 | 12 | 12 | 10 | 14 | 5 | 11 | 2 | 4 |
| STHAR | 9 | 11 | 10 | 15 | 14 | 19 | 16 | 20 | 20 | 20 |
| TNSTC (KUM) | 16 | 9 | 9 | 7 | 11 | 8 | 12 | 16 | 8 | 13 |
| TNSTC (VPM) | 2 | 2 | 1 | 2 | 1 | 3 | 1 | 3 | 3 | 6 |
| TNSTC (CBE) | 8 | 5 | 6 | 5 | 8 | 8 | 6 | 13 | 6 | 9 |
| NEKnRTC | 23 | 21 | 22 | 23 | 23 | 25 | 25 | 25 | 26 | 26 |
| TNSTC (SLM) | 31 | 28 | 23 | 24 | 19 | 23 | 18 | 21 | 19 | 18 |
| TSRTC | 34 | 35 | 34 | 36 | 30 | 28 | 26 | 26 | 25 | 25 |
| SETC (TN) | 3 | 7 | 3 | 9 | 3 | 5 | 2 | 4 | 5 | 7 |
| NBSTC | 11 | 15 | 11 | 14 | 16 | 20 | 15 | 19 | 18 | 17 |
| SBSTC | 15 | 19 | 16 | 22 | 13 | 18 | 9 | 14 | 12 | 14 |
| KDTC | 6 | 10 | 5 | 11 | 5 | 17 | 8 | 17 | 17 | 18 |
| ASMSTC | 20 | 14 | 18 | 21 | 22 | 22 | 24 | 24 | 24 | 24 |
| ANST | 32 | 25 | 28 | 26 | 26 | 26 | 27 | 27 | 27 | 27 |
| SKNT | 10 | 17 | 15 | 16 | 9 | 11 | 11 | 15 | 16 | 16 |
| HRTC | 30 | 26 | 27 | 27 | 27 | 27 | 28 | 28 | 28 | 28 |
| UTC | 22 | 24 | 20 | 25 | 20 | 24 | 20 | 23 | 23 | 23 |
| MEGTC | 13 | 18 | 19 | 18 | 18 | 16 | 17 | 8 | 10 | 12 |
| NMMT | 4 | 4 | 8 | 6 | 12 | 6 | 13 | 6 | 9 | 11 |
| BEST | 35 | 34 | 35 | 34 | 35 | 35 | 35 | 29 | 35 | 29 |
| DTC | 28 | 31 | 32 | 33 | 33 | 34 | 33 | 34 | 34 | 35 |
| BMTC | 27 | 30 | 28 | 31 | 31 | 32 | 31 | 33 | 32 | 34 |
| MTC (CNI) | 36 | 36 | 36 | 34 | 36 | 36 | 36 | 36 | 37 | 36 |
| PMPML | 19 | 16 | 30 | 7 | 25 | 2 | 23 | 2 | 13 | 2 |
| AMTS | 23 | 22 | 24 | 3 | 24 | 1 | 22 | 1 | 14 | 1 |
| TMTU | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 37 | 36 | 37 |
| KMTU | 25 | 27 | 25 | 28 | 29 | 30 | 30 | 32 | 31 | 32 |
| KADMTU | 25 | 28 | 26 | 30 | 28 | 30 | 29 | 31 | 30 | 31 |
| OSRTC | 29 | 31 | 33 | 32 | 34 | 33 | 34 | 34 | 32 | 33 |

### 6.4.6 Ensemble Ranking Method

In accordance with Mohammadi and Rezaei [366], this study introduces a half-quadratic programming approach to calculate an ensemble ranking of alternative sites. The ensemble ranking is aggregated by computing the weighted sum of the rankings under all sets of ranked dimension weights, in which the set-wise weights are calculated using the minimizer function. Apparently, the weights with respect to the rankings under all sets of ranked dimension weights conform to the non-zero and unit-sum properties. This approach is particularly valuable for mitigating decision bias. This methodology aims to calculate a unified ranking system by combining various individual ranking systems, thereby enhancing the consensus and validity of the rankings. Consider '$n$' distinct techniques, each assigning a rank $(R_1, R_2, R_3, \ldots, R_n)$ to a particular alternative according to their respective methods. Simultaneously, there exists a consolidated ranking denoted as $R^*$ for the same alternative. The primary objective of the ensemble ranking technique is to minimize the Euclidean distance between each individual rank and the consolidated ranking, $R^*$. To achieve this goal, a quadratic minimization function is formulated as follows:

$$\min \frac{1}{2} \sum_{j=1}^{n} ||R_j - R^*||_2.$$

Here, $R_j$ represents the individual computed rank, and $R^*$ is the assumed consolidated ranking. This function aims to enhance the consensus with the consolidated ranking by reducing the distance from it. An optimal weighted ensemble ranking procedure is devised by allocating distinct weights $w_1, w_2, \ldots, w_n$ to each procedure and iteratively adjusting these weights until they converge to a conclusive solution. Each auxiliary variable, signifying an individual ranking method, is denoted by $\alpha_j$, where

$$\alpha_j = \delta(||R_j - R^*||_2^2).$$

The weights are calculated by normalizing the auxiliary variables using the following formula:

$$w_j = \frac{\alpha_j}{\sum_1^n \alpha_j}.$$

The consolidated ranking is derived by taking the sum of each rank, each of which is multiplied by its corresponding weight,

$$R^*(\text{optimal}) = \sum_j w_j R_j.$$

The consensus index, denoted as $C$, provides a measure of the degree of agreement among the various rankings and is used to form the ensemble ranking $R^*$. The calculation of the index $C$

for an ensemble ranking $R^*$, considering the rankings $R_j$ where $j = 1, 2, \ldots, n$, can be expressed as follows

$$C(R^*) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{N_\sigma(R_i^* - R_i^j)}{N_\sigma(0)},$$

where $N_\sigma(\cdot)$ is the probability density function of the Gaussian distribution with a standard deviation of $\sigma$ and a mean of zero, and $m$ is the number of alternatives.

The trust level, represented as $T$, signifies the extent to which one can have confidence in the ensemble ranking. It reflects the reliability of this ranking. When an individual ranking significantly differs from the majority of rankings, it receives a lower weight, consequently exerting less influence on the ensemble ranking. In such cases, the trust level is less affected. The calculation of the trust level can be summarized as follows

$$T(R^*) = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} W_j \frac{N_\sigma(R_i^* - R_i^j)}{N_\sigma(0)}.$$

**Table 6.6:** Ensemble ranking results.

| STUs | R* | Aggressive Ranking | R* | Benevolent Ranking | Ensemble Ranking |
|---|---|---|---|---|---|
| APSRTC | 9.731 | 10 | 11.812 | 12 | 17 |
| MSRTC | 31.503 | 32 | 29.902 | 30 | 31 |
| GSRTC | 12.927 | 13 | 10.333 | 9 | 8 |
| UPSRTC | 2.835 | 2 | 4.259 | 2 | 2 |
| KnSRTC | 17.464 | 18 | 20.011 | 21 | 21 |
| KSRTC | 4.710 | 4 | 7.056 | 7 | 4 |
| RSRTC | 9.443 | 9 | 14.029 | 14 | 10 |
| NWKnRTC | 19.801 | 19 | 12.489 | 13 | 18 |
| TNSTC (MDU) | 8.808 | 7 | 10.643 | 11 | 7 |
| STHAR | 13.615 | 15 | 17.189 | 19 | 20 |
| TNSTC (KUM) | 11.298 | 11 | 10.614 | 10 | 9 |
| TNSTC (VPM) | 1.522 | 1 | 3.216 | 1 | 1 |
| TNSTC (CBE) | 6.834 | 5 | 8.095 | 8 | 5 |
| NEKnRTC | 23.717 | 25 | 24.090 | 25 | 25 |
| TNSTC (SLM) | 21.950 | 23 | 22.676 | 23 | 22 |
| TSRTC | 29.921 | 31 | 29.846 | 28 | 28 |
| SETC (TN) | 3.110 | 3 | 6.353 | 5 | 3 |
| NBSTC | 14.111 | 16 | 17.103 | 18 | 19 |
| SBSTC | 12.977 | 14 | 17.371 | 20 | 14 |
| KDTC | 7.800 | 6 | 14.751 | 16 | 12 |
| ASMSTC | 21.550 | 21 | 21.204 | 22 | 24 |
| ANST | 27.962 | 28 | 26.229 | 26 | 26 |
| SKNT | 11.992 | 12 | 14.884 | 17 | 14 |
| HRTC | 27.966 | 29 | 27.229 | 27 | 27 |
| UTC | 20.881 | 20 | 23.799 | 24 | 24 |
| MEGTC | 15.705 | 17 | 14.313 | 15 | 15 |
| NMMT | 9.344 | 8 | 6.640 | 6 | 6 |
| BEST | 35.000 | 35 | 32.194 | 33 | 35 |
| DTC | 31.980 | 33 | 33.469 | 35 | 34 |
| BMTC | 29.768 | 30 | 32.049 | 32 | 32 |
| MTC (CNI) | 36.163 | 36 | 35.593 | 36 | 36 |
| PMPML | 22.457 | 24 | 5.470 | 4 | 19 |
| AMTS | 21.742 | 22 | 5.087 | 3 | 17 |
| TMTU | 36.837 | 37 | 37.000 | 37 | 37 |
| KMTU | 27.943 | 27 | 29.877 | 29 | 30 |
| KADMTU | 27.546 | 26 | 30.054 | 31 | 29 |
| OSRTC | 32.491 | 34 | 32.653 | 34 | 33 |
| **Confidence Index** | 0.996 | | 0.997 | | |
| **Trust Level** | 0.996 | | 0.997 | | |

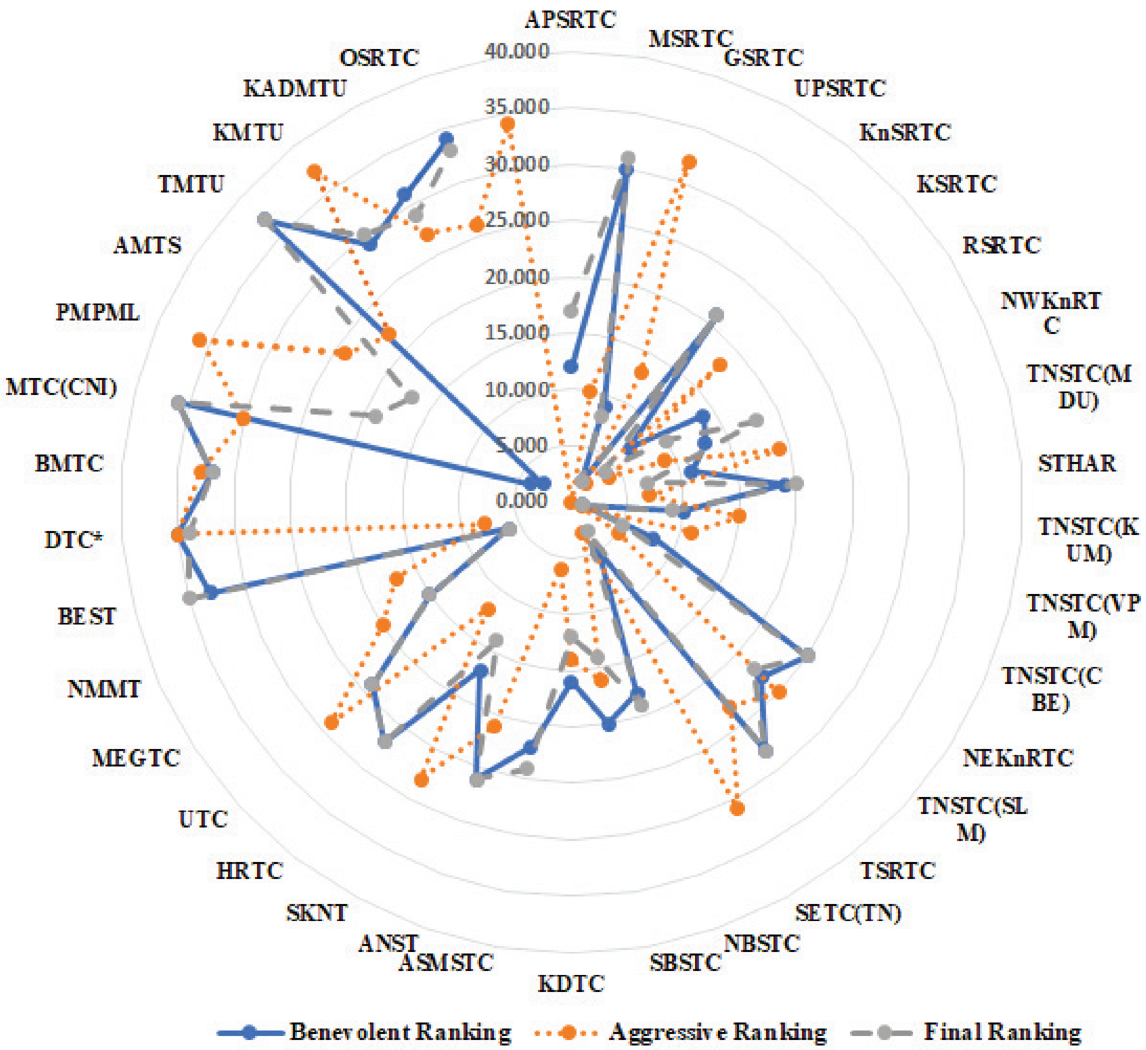


**Fig. 6.4:** Ranking comparisons.

Table 6.5 tabulates the ranking of maximum and minimum efficiency values, which are used as the ranking inputs in the half-quadratic programming approach. As depicted in table 6.6, the consolidated ranking, denoted as $R^*$, and ensemble ranking are presented. The results show that TNSTC (VPM) is the most promising STU since it obtains a lower score of $R^*$ at 3.316. In contrast, TMTU ranks as the least efficient STU, recording the highest score of 37 STUs. Moreover, These weight values imply a preference relationship among credibility levels for aggressive and benevolent fuzzy cross-efficiency models. For the aggressive model, the weight values are distributed as follows: $\alpha(=0.5)$ holds a weight of 0.196, $\alpha(=0.6)$ is 0.204, and $\alpha(=0.7)$ comes in at 0.221, $\alpha(=0.8)$ is 0.216, and $\alpha(=0.9)$ stands at 0.163. On the other hand, for the benevolent model, the respective weight values are: $\alpha(=0.5)$ at 0.175, $\alpha(=0.6)$ at 0.203, $\alpha(=0.7)$ is 0.2171, $\alpha(=0.8)$ at 0.206 and $\alpha(=0.9)$ stands at 0.198. Highlighting the robustness of the ranking, the confidence indices and trust levels are recorded at (0.996, 0.997) for both metrics, respectively. This shows that the reliability with regard to the final ensemble assessment and the agreement among all ranking inputs are sufficiently relevant. Figure 6.4 highlights the differences between the ensemble ranking and the rankings obtained from both aggressive and benevolent models. A noteworthy observation is that out of 37, 33 exhibit varied rankings. However, the positions of specific STUs, namely TNSTC (VPM) at rank 1, NEKnRTC at 25, MTC (CNI) at 36, and TMTU at 37, remain constant across the models. Such consistency suggests that the performance evaluations for these units are notably stable and robust.

## 6.5 Conclusions

This study introduces a novel method, the fuzzy cross-efficiency BCC DEA model, to assess the performance of STUs in terms of efficiency, which takes into account the uncertainty factor with missing values in a dataset. This approach integrated kNN and the fuzzy logic framework, is allowing the model to maintain its discriminatory ability while also being able to handle uncertainty using fuzzy logic. This approach considered all possible weights of all DMUs simultaneously and eliminated the need to select a specific set of weights. This model has a significant advantage in not generating negative efficiencies, making it suitable for situations with VRS assumptions. Two mathematical programs (aggressive and benevolent) for each credibility measure at different credibility levels determine the fuzzy cross-efficiency. By solving these mathematical programs, it is possible to obtain the min and max bounds of the fuzzy cross-efficiency with ease. However, it is demonstrated that the use of two different formulations - aggressive and benevolent - can lead to two distinct efficiency rankings. This inconsistency poses a challenge to the reliability of the cross-efficiency evaluation process. To address this, the ensemble ranking method is applied, offering a unified ranking for all STUs and ensuring a more robust and dependable assessment of their performance.

# Chapter 7

# Conclusions and Future scope

## 7.1 Results and Findings

This thesis addressed the MCDM techniques, DEA, IDEA, and fuzzy DEA models to evaluate the efficiency and productivity in the public transport sector. Additionally, various integrated techniques guide criteria selection and weights, rank all DMUs, and identify the most efficient ones. The primary contributions of this research are broadly categorized into two vital areas: (i) the practical implementation of the proposed methodologies and models in the public transport sector; and (ii) the development of innovative fuzzy DEA techniques adept at handling uncertain, missing, and ambiguous data environments. An in-depth analysis of the RSRTC depots forms the cornerstone of this research, serving as the basis for a comprehensive investigation. This study extends its scope beyond the RSRTC to encompass the broader study region of India's state transport undertakings (STUs).

The conclusions derived from each of these techniques are meticulously presented in their respective chapters. In summary, these methods extend beyond theoretical applications within the public transport sector, demonstrating their relevance and applicability in a wider array of real-world scenarios. Furthermore, the research addressed critical limitations often encountered in traditional DEA models. For instance, fuzzy DEA models demonstrate an ability to handle uncertainty and ambiguity in data. To bolster this capability, an integrated fuzzy DEA model with kNN method is deployed to impute missing values in the transport sector. This approach is demonstrated to reduce uncertainties in the dataset. A application of the each methods are shown in each chapter with reference to the public transport sector. From the results perspective, the following are the study's specific findings:

- Selection of criteria is vital in preventing depot corrosion and failures. Chapter 2 applied a novel hybrid approach that integrates fuzzy Delphi, fuzzy AHP, and TOPSIS-VIKOR-ELECTRE methods. Leveraging fuzzy set theory addressed subjectivity and ambiguity in criteria assessment, leading to more robust outcomes. After identifying 29 distinct criteria, expert collaboration and fuzzy Delphi Method narrowed the list to 14 pivotal criteria. The Fuzzy AHP method determined the relative significance of the four categories and 14 screened criteria. The resulting weights are used in TOPSIS, VIKOR, and ELECTRE approaches to rank 52 RSRTC depots for the year 2017-18. These MCDM techniques provided substantial results, addressing gaps in previous research within the public transport sector, especially in criteria selection. The proposed models are characterized by their simplicity, convenience, precision, and efficiency, offering valuable support to decision-makers. This innovative hybrid MCDM method is adaptable and address significant criterion selection challenges in transportation and other decision-making contexts.

- The chapter 3 evaluates the efficiency of RSRTC depot using the input-oriented NSM model with VRS assumption from 2005 to 2022. Across the 52 depots spanning from low, medium and high advantageous conditions, identified a range of efficiency levels. These nuanced variations offer a valuable opportunity for tailored interventions to enhance service quality. The results of the time series analysis revealed that the RSRTC has 84.8%, 85.6%, and 77.8% average OTE for the period 2005-2022. Many years have shown a decline in efficiency across all categories. Average PTE (93.4%, 96.4%, and 94.5%) and SE (91%, 89%, and 82.2%) are estimated in all three categories. This reveals that the inefficient use of input resources and average inputs slack is about 5.23 %, and the average due scale size is about 12.6%. Addressing this challenges can lead to more effective strategies. In 2021-22, while there is a recovery, depots didn't reach past performance levels, signifying a shift in strategies. The study identified significant performance gaps in depots, emphasizing the need for optimization which marks a vital step in enhancing RSRTC depot efficiency.

- Chapter 4 measures the total factor productivity (TFP) and incorporates the Malmquist productivity index (MPI) and Luenberger productivity index (LPI) using input-oriented NSM model over a specified time frame (2008–2019). Additionally, the total productivity change decomposed into two key components: technical change (Frontier Shift) and technical efficiency change (Catch-up Effect). The MPI and LPI indexes applied to the panel data of 46 depots reveal that, on average, the total factor productivity (TFP) in the depots has increased by a rate of 1.95% and 1.41%. MTC has declined by -0.772%, and LTC declined by -0.564% in technical change throughout the period. Twenty-one depots showed a decline in average LTC growth from 2008-10 to 2017-19. Thus, the study demonstrates a significant trend wherein the decline in productivity across several depots predominantly stems from technological changes, emphasizing the pivotal role of technological advancements in shaping and influencing overall productivity within the transportation system.

- DEA models are effective in evaluating the relative efficiency of DMUs. However, distinguishing between efficient units can be challenging, discussed in the chapter 5. The inverse DEA (IDEA) approach has emerged as a valuable tool for re-evaluating DMU efficiency when input or output values change. It analyzed the relationship between inputs and outputs in production units while maintaining efficiency levels, offering valuable insights for resource allocation and competitiveness. For managerial decision-making, it is crucial to consider preferences and evaluate potential system changes. In this study, we applied a super-efficiency IDEA method and a single-objective LP model to rank efficient depots among 52 RSRTC bus depots for 2018-19. The model identified 7 RSRTC depots as efficient and 45 as inefficient, with efficiency scores below 1. The super-efficiency IDEA model is used to rank the efficient depots. This application provides a novel framework for making quality service decisions in efficient depots, including optimal resource allocation among bus depots.
- Chapter 6 introduced the fuzzy cross-efficiency BCC DEA model to evaluate STUs' efficiency, addressing uncertainty with missing data. The approach integrates k-NN and fuzzy logic, maintaining discrimination ability. Considering all possible weights simultaneously eliminates the need for specific weight selection. This model's advantage is its ability to avoid generating negative efficiencies, which is particularly useful for situations with VRS assumptions. To determine fuzzy cross-efficiency, mathematical programs (aggressive and benevolent) for each credibility measure at different $\alpha$ levels, providing access to the minimum and maximum bounds, are used. However, applying two different formulations could lead to distinct efficiency rankings, challenging the evaluation process's reliability. To address this, the ensemble ranking method ensures a unified ranking for all STUs and enhances performance assessment robustness. This advancement significantly contributes to the field, providing a more accurate evaluation of STUs' efficiency and performance.

## 7.2 Recommendation

The following recommendations are made on the basis of the findings of the study:

- In depots for which the value of efficiency scores is less than unity, improved performance could result from the diffusion of new technology knowledge, improved managerial practices, and more effective utilization of resources.
- To mitigate input slack in under-performing depots, it is advisable to consider reducing employee strength. Inefficient depots have the opportunity to adopt practices akin to those of the most efficient depots, resulting in substantial cost savings.
- Drivers of new technological buses should receive ongoing training to ensure proficiency. Additionally, incentives to consistently high-performing employees can be a motivational tool.

Public recognition of their achievements can further inspire them. A systematic evaluation of their work performance over time should be considered when considering promotions.

- Most depots are found to be operated at increasing returns to scale. Expanding their capacity and fleet utilization can increase their productivity and efficiency.
- Efforts are required to be made toward replacing old buses and induction of modern buses to raise efficiency.
- The RSRTC has to operate its services on socially obligatory routes even if it does not meet the targets of performance indicators. Nevertheless, the corporation should consolidate its operation on nationalized routes.
- The use of more advanced fuel-efficient models of buses, better maintenance of fleets and roads, and replacement of worn-out buses are likely to help in improving the performance of the corporation.
- Promoting fuel conservation plays a pivotal role in enhancing the efficiency and productivity of transport services. To achieve improved fuel efficiency, it is advised to streamline the number of stops, ensure regular fleet maintenance, monitor proper tyre pressures, and maintain a consistent travel speed. Raising awareness among operating staff about the importance of fuel conservation is crucial. To this end, implementing reward and promotion schemes is strongly recommended. These incentives can effectively motivate staff to actively participate in fuel-saving efforts.
- The analysis reveals that the TFP decline in the depots is primarily attributed to a decrease in technical efficiency. To enhance technical efficiency, depots should focus on optimizing input utilization to increase output.

## 7.3 Future Scope

- The utilization of DEA in conjunction with artificial neural network (ANN) models is another aspect to consider; we can potentially enhance the accuracy and depth of efficiency predictions. This integration allows for a more comprehensive assessment of performance by leveraging the strengths of both methodologies.
- While the study emphasizes the importance of enhancing productivity for the sustained success and prosperity of DMUs, it's worth noting a potential limitation. Introducing an advanced model like the Sequential Malmquist Luenberger productivity index (SMLPI) could be a valuable avenue for future research and could further enhance the assessment of productivity in competitive environments. This potential enhancement could serve as a valuable addition to the existing framework.

- Explore advanced data imputation methods to help fill in missing values more accurately. Techniques like multiple imputation or machine learning-based imputation models may be considered.
- As an extension of this study, it's important to acknowledge a potential limitation of DEA, which arises from its reliance on favorable weights for assessing the efficiency of a DMU. This can sometimes lead to a lack of differentiation among many DMUs. To address this issue, a potential avenue for future research could involve the application of the cross-efficiency method, specifically in the context of a neutral cross-efficiency model. This model could incorporate the most favorable weights for both input and output within the framework of the DEA model. This extension holds promise for refining the assessment of DMU efficiency.

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# List of Publications

**Published/ Accepted**

- **Goyal S**, Talwar MS, Agarwal S, Mathur T, “Ranking of Efficient DMUs Using Super-Efficiency Inverse DEA Model,” In Soft Computing for Problem Solving: Proceedings of the SocProS 2022. Singapore: Springer Nature Singapore, 2, pp 615-626, 2023 (Scopus).
- **Goyal, S**, Agarwal S, Singh NS, Mathur T, Mathur N, “Analysis of Hybrid MCDM Methods for the Performance Assessment and Ranking Public Transport Sector: A Case Study,” Sustainability, 14(22), pp 15110, 2022 (SCI).
- **Goyal S**, Agarwal S, Mathur T, “An evaluation of the productivity change in public transport sector using DEA-based model,” Management Science Letters, 12(2), pp. 125-36, 2022.
- **Goyal S**, Agarwal S, Mathur T, Mathur N, “Assessing the Radial Efficiency Performance of Bus Transport Sector Using Data Envelopment Analysis,” InHandbook of Machine Learning for Computational Optimization, pp. 71-87, 2021, (CRC Press).
- **Goyal S**, Agarwal S, Mathur T, “A Novel Fuzzy Cross-Efficiency Evaluation Methodology and Ranking,” *(Communicated)*.

# Workshops/ Conferences

**Presented Work in International Conferences**

- Presented paper entitled **"A Comparative Framework to Investigate the Resource Allocation of Public Transport Sector Based on Inverse Data Envelopment Analysis"** in International Conference Mathematical and Statistical Computation (ICMSC 2022), organized by the Department of Mathematics, SKIT Jaipur (Rajasthan), India, held from March 03–05, 2022.
- Presented paper entitled **"Total Factor Productivity by NSM DEA-Based MPI Approach"** in International Conference and $22^{nd}$ Annual Convention of Vijnana Parishad of India on Advances in Operations Research, Statistics, and Mathematics (AOSM 2019), organized by the Department of Mathematics, BITS Pilani, Pilani Campus, Rajasthan, held from December 28–30, 2019.
- Presented paper entitled **"Measure the Efficiency of Depots of RSRTC Using DEA"** in International Conference on Business Analytics and Operations Research (ICBAOR 2019), organized by Manipal Academy of Higher Education (MAHE), and held from June 14–16, 2019.

**Attended Workshop**

- Participated in the **"International Workshop on recent Advancement in Data Envelopment Analysis and Applications (IWRADEAA 2021)"**, Department of Mathematics, BITS Pilani, Pilani Campus, Rajasthan, India, held from July 10-11, 2021.
- Attended the online short-term course entitled **"Optimization Theory, Methods, and Applications"**, organized by Department of Mathematics at IIT Roorkee, India, held from August 18–20, 2020.
- Participated in the **"International Workshop on Data Envelopment Analysis for Performance Evaluation and Benchmarking"**, organized by the Faculty of Economics, South Asian University (SAU), New Delhi, India, held from January 12–14, 2020.
- Seven days workshop on **"Academic Writing"** organized by the Department of Humanities and Social Sciences of BITS Pilani held at BITS Pilani, Pilani Campus, India, held from April 5-11, 2019.
- Participated in the **"Attended the 51st Annual Convention of the Operational Research Society of India & International Conference (ORSI 2018)"**, organized by IIT Bombay, Coimbatore, India, held from December 16–19, 2018.
- Participated in the **"Academic Writing Competition"**, organized by Birla Institute of Technology & Science (BITS) Pilani, Pilani Campus, India, on October $25^{th}$ 2018.

- Participated in the **"Research Methodology & Latex"**, organized by B K Birla Institute of Engineering & Technology, Pilani, India, held from September 7-8, 2018.

# Brief Biography of the Supervisor

Prof. Shivi Agarwal is an Associate Professor in the Department of Mathematics at BITS-Pilani, and she joined the institution in August 2007. She completed her Ph.D. from the Department of Mathematics at IIT Roorkee in 2008. Her research area is Operations Research, especially Efficiency and Productivity Analysis, DEA models, Fuzzy Logic and its Applications, and Artificial Neural Networks. She has published more than 25 research papers in esteemed journals and international/national conference proceedings. She is a member of many international and national bodies like FIM, IDEAS, ORSI, etc., signifying her active involvement in the academic community. She has presented her research work at numerous conferences in India and abroad. She visited Temple University, Philadelphia, USA, in 2009, Xian University, Xian, China, in 2012, Yeildz Technical University, Istanbul, Turkey, in 2013, Technical University Braunscwick, Germany, Leshan Normal University, Leshan, China, in 2015, and Waseda University, Tokyo, Japan in 2023 to present her research work. Moreover, she engaged in collaborative research at VSB-Technical University of Ostrava, Ostrava, Czech Republic, from May 29 to July 17, 2016. Her academic achievements include receiving the "Best Paper Presentation Award" at $20^{th}$ International Conference on Mathematical, Computational and Statistical Sciences in Stockholm, Sweden, from July 11–12, 2016. Additionally, she has organized the International Workshop on Recent Advancements in Data Envelopment Analysis and Applications (IWRADEAA 2021) at BITS Pilani, Pilani Campus, Rajasthan, held from July $10^{th}$-$11^{th}$, 2021. She has supervised and co-supervised doctoral candidates Dr. Deepak and Dr. K S Pritam. Currently, she is guiding three Ph.D. students: Jyoti, Shantnu, and Ishu. More on her research contributions can be found at https://www.researchgate.net/profile/Shivi-Agarwal. Contact her at shivi@pilani.bits-pilani.ac.in.

# Brief Biography of the Co-supervisor

Prof. Trilok Mathur is an Associate Professor in the Department of Mathematics at BITS-Pilani, having joined the institution in July 2008. He worked as a lecturer at Banasthali University in India from 2005 to 2008 before joining BITS-Pilani. He pursued his Ph.D. from the Department of Mathematics at the University of Rajasthan, Jaipur, India, in 2005. His research contributions are notable, with a substantial publication record of over 40 research papers in esteemed journals and international/national conference proceedings. This demonstrates his active engagement and significant contributions to the field of Mathematics. His primary research areas include the theoretical and application of Fractional Calculus, Complex Analysis, MCDM Techniques, and Fuzzy Logic. His extensive engagement in research is evident from his visits to various internationally renowned universities, such as the University of Malaya, Malaysia; Imperial College, London, UK; Ivan Franko National University, Lviv, Ukraine; Leshan Normal University, China; and VSB Technical University of Ostrava, Czech Republic, where he presented his research findings. He is a member of prestigious organizations such as IAENG, the Indian Mathematical Society (IMS), SSFA, Rajasthan Ganita Parishad (RGP), and many international and national bodies. More on his research contributions can be found at https://www.researchgate.net/profile/Trilok-Mathur. Contact him at tmathur@pilani.bits-pilani.ac.in.

# Brief Biography of the Candidate

Ms. Swati Goyal graduated with a B.Sc. degree from the Choudhary Charan Singh University (C.C.S.U), Meerut, in 2014 and post-graduated with an M.Sc. degree in Mathematics from the Department of Mathematics, C.C.S.U, Meerut in 2016. She is a dedicated scholar pursuing her Doctor of Philosophy (Ph.D.) degree in the Department of Mathematics at Birla Institute of Technology and Science (BITS) Pilani, Pilani Campus. Under the supervision of Dr. Shivi Agarwal and Dr. Trilok Mathur, her research focuses on modeling for performance analysis, resource utilization and ranking within the public transport sector, showcasing her interest in the application of mathematical concepts to real-world problems. She has made progress in her academic journey, with four research publications in peer-reviewed journals and conference proceedings. Additionally, her active participation in academic conferences (four international conferences attended) and workshops (five international workshops attended) demonstrates her commitment to staying updated with the latest developments in her research field. Her academic achievements include receiving the “Best Paper Award” in the Research Scholar Category at the ICBAOR conference, Manipal Academy of Higher Education (MAHE), in 2019. Moreover, she gained valuable research experience through a research internship at the prestigious Indian Institute of Science (IISc) Bangalore in 2023, indicating her proactive approach to enhance her skills and knowledge.