Studies on RGA Analysis for Control Configuration Selection of Decentralized Multivariable Nonlinear Chemical Processes

THESIS

Submitted in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

By

AMIT JAIN

Under the supervision of

Prof. B. V. Babu

BITS Pilani
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BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

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Dedicated

To

Almighty God,

My Family Members

&

Friends

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE
PILANI, RAJASTHAN, INDIA

CERTIFICATE

This is to certify that the thesis entitled **"Studies on RGA Analysis for Control Configuration Selection of Decentralized Multivariable Nonlinear Chemical Processes"** submitted by **Amit Jain** ID. No. **2006PHXF424P** for the award of Ph.D. Degree of the Institute, embodies the original work done by him under my supervision.

Signature in full of the Supervisor ____________________

Name in capital block letters **Prof. B V BABU**

Designation Vice-Chancellor,
Galgotias University
Greater Noida, Uttar Pradesh, India

Date: 15/5/15

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ABSTRACT

With the advent of technological advancements in chemical process industries, more and more processes are getting interconnected with the objectives of minimizing energy consumption, maximizing heat recovery, effective utilization of resources and/or to minimize the cost. These interconnections, often result in plants that are too large or too complex to control in a conventional manner. The control of such plants necessitates the need for disintegrating the whole control system design into independent controller subsystems, which will require the minimal plant information and should be capable of handling plants nonlinearities or uncertainties in a robust manner. The disintegrated or independent controller subsystem is termed as decentralized control system and is often a preferred choice than its centralized counterpart.

A decentralized control system consists of multiple input-output control loops. These control-loops often interact with each other to the extent that their control becomes a daunting task. Thus, inputs and outputs are paired in such a way that the control-loop interactions be minimized. The problem is often termed as an input-output pairing problem or the control configuration selection problem. The performance of a multi-loop control system depends strongly on proper selection of control configuration. The selected control configuration must correspond to the control-loops having minimum closed-loop interaction. For the analysis of control-loop interaction various tools are available in the literature and one of the most popular of them is the steady-state "relative gain array (RGA)". The RGA has many advantages over other interaction measures: (i) it requires the minimal process information, i.e. the steady-state gains only, (ii) it is independent of the input-output scaling, (iii) it represents all the process information in a single matrix. It has many algebraic properties and has also found to be linked to many closed-loop system properties such as: robustness, integrity, failure tolerance, decentralized integral controllability (DIC), etc.

Despite its numerous advantages the RGA also suffers from certain limitations which restricts its use as a reliable tool. Inconsideration of process dynamics is one such limitation of the RGA. Many attempts have been made to extend the applicability of RGA to dynamic systems and various tools are available in the literature. In the present study the most popular of them have been analyzed for accuracy. A new measure of process interaction the "relative response array (RRA)" has also been proposed. The proposed measure has four different versions based on: (i) controller-independent/ controller-dependent approach, and (ii) time-average/time-varying approach. The controller dependent versions are useful from the viewpoint of practical realizability, whereas time varying versions are particularly important for detailed dynamic analysis of the process. The capability of the proposed measure and other popular methods as a potential tool of dynamic interaction measurement are tested on four case studies (two benchmark test problems and two industrial problems) adopted from the literature. It has been observed that in one of the case studies the steady-state RGA fails to identify the best pairings, whereas in another case study, few popular dynamic measures of interaction were unsuccessful in identifying the best pairings. However, the proposed interaction measure was successful in identifying the best control configuration for all the case studies. In addition to identifying the best pairings, the time-varying versions of the proposed measure provided much more detailed information about the process dynamic behavior. The resulted pairings are then analyzed for stability and performance based on closed-loop simulation behavior obtained from the Internal Model Control (IMC) / Integrated Time Average Error (ITAE) controller design and tuning. The Niederlinksi Index (NI) has also been used as a measure of closed-loop stability.

In most studies on RGA analysis, the process model is frequently assumed. However, a mathematical model for the real process can never be perfect and there is always some uncertainty associated with the model. The uncertainty may arise due to process nonlinearities, external disturbances, change in operating conditions, etc. Still, a very few attempts have been made towards the sensitivity of the RGA analysis to model uncertainty and is mainly limited to the steady-state systems only. The objective of the present study is to gain insights into how the process dynamics can affect control configuration decision based on RGA analysis in the face of model uncertainty. For this study, parametric uncertainty in the gain and residence time of the process has been considered. Analytical expressions for worst-case bounds of uncertainty in steady-state and dynamic RGA are derived for two-input, two-output (TITO) plant models. Three of the four case studies as mentioned above are adopted from the literature to demonstrate the applicability of the proposed approach. Since the proposed approach is currently limited to 2×2 process models only, it could not be extended for one of the case study of 3×3 plant model. The obtained bounds of uncertainty in RGA provide valuable information pertaining to the maximum tolerable uncertainty for which control loop pairing remain unchanged. A further increase in uncertainty beyond the limiting point results in ambiguity in pairing decision and may also lead to the change in input-output pairing during the time range of interest. The results obtained in the case studies show that the tolerable uncertainty increase if the uncertainty analysis is carried out under dynamic framework, particularly for the uncertainty in steady-state gains, whereas the tolerable uncertainty decreases if the uncertainty is considered in both the process gains and the residence times. Thus, the sensitivity analysis carried out for control-loop pairing for inaccuracies in model parameters of the considered case studies, ensures the selection of robust control configuration.

Keywords—*Decentralized Control, Interaction Measure, Relative Gain Array, Relative Response Array, Input-Output Pairing, Control Configuration Selection, Parametric Uncertainty, Worst-Case Bounds, Sensitivity Analysis, Residence Time, Robustness.*

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NOMENCLATURE

Main Notation

$\lvert\bullet\rvert$	Absolute value of $\bullet$
$(\bullet)!$	Factorial, $n! = \prod_{i=1}^{n} i$
$(\bullet)_{ij}$	ij^{th} element of matrix
$(\bullet)^{-T}$	Inverse transpose
$(\bullet)_{i}$	i^{th} element of vector, i^{th} column of matrix
$\lVert \bullet \rVert_{p}$	p-norm of vector, matrix or transfer matrix
$(\bullet)\times(\bullet)$	$(\bullet)$ - dimensional matrix
$\det(\bullet)$	Determinant
$\otimes$	Schur or Hadamard product (element by element multiplication)
$C_i(s),\ g_{Ci}(s)$	Controller transfer function in the i^{th} loop
D_i	i^{th} disturbance
e_{ij}^{*}	Effective energy between y_i and u_j
e_{ij}	Effective gain between y_i and u_j with other input-output loop open
$\hat{e}_{ij}$	Effective gain between y_i and u_j with other input-output loop closed
E	Effective gain matrix
E^{*}	Effective energy matrix
$g_{ij}(s)$	ij^{th} element of process transfer function matrix $G(s)$
$G^{ij}(s)$	Matrix $G(s)$ obtained on eliminating i^{th} row and j^{th} column
$G(0)$	Steady-state gain matrix
$G(s)$	Transfer function matrix of the plant model
$\hat{k}^{h}$ and $\hat{K}^{h}$	Highest value of interaction quotient
$\hat{k}^{l}$ and $\hat{K}^{l}$	Lowest value of interaction quotient

$\hat{k}_{uc,dy}$	Dynamic interaction quotient under model uncertainty (based on $\hat{K}_N$)
$\hat{k}$	Steady-state interaction quotient
$\hat{k}_{uc}$	Steady-state interaction quotient under model uncertainty (based on $\hat{k}$)
k_{ij}	Steady-state gain between y_i and u_j
$K_{N,ij}$	ij^{th} element of normalized gain matrix, K_N (normalized gain, $k_{ij}/\tau_{ar,ij}$)
$\hat{K}_N$	Normalized interaction quotient
K	Steady-state gain matrix
s	Laplace variable
u_j	j^{th} manipulated (input) of the process
y_i	i^{th} controlled (output) of the process
$y_{i,sp}$	Set point of output y_i

Greek Symbols

α and β	Uncertainty associated with gain and residence time, respectively
ω	Frequency of the system
ω_B	Bandwidth
ω_c	Critical frequency
Ω	Bandwidth matrix
ζ	Damping coefficient
λ_{ij} or Λ_{ij}	Relative gain between y_i and u_j
$\Lambda(G)$	RGA matrix for plant G
θ_{ij}	Time delay between y_i and u_j
$\theta_{\max}$	Largest dead time of the process
ϕ_{ij}	Process gain in an isolated loop (open loop)
$\tau_{ar,ij}$	Average residence time between y_i and u_j
τ_{ij}	Time constant between y_i and u_j

τ_D	Dominant time constant of the process
ρ_{ij}	ij^{th} element of relative response array

Abbreviations

ARG	Average Relative Gain
BRG	Block Relative Gain
CD-TA-RRA	Controller Dependent Time Average Relative Response Array
CD-TV-RRA	Controller Dependent Time Varying Relative Gain Array
CI-TA-RRA	Controller Independent Time Average Relative Response Array
CI-TV-RRA	Controller Independent Time Varying Relative Response Array
CV	Controlled Variable
DIC	Decentralized Integral Controllable
DIX	Dynamic Interaction Index
DRGA	Dynamic Relative Gain Array
EREA	Effective Relative Energy Array
ERGA	Effective Relative Gain Array
FCC	Fluid Catalytic Cracker
FOPTD	First Order Plus Time Delay
FDRGA	Frequency Dependent Relative Gain Array
GRDG	Generalized Relative Disturbance Gain
HIIA	Hankel Interaction Index Array
IC	Integral Controllable/Controllability
ICI	Integral Controllable With Integrity
IE	Integral Error
IMC	Internal Model Control
IS	Integral Stabilizability
ITAE	Integrated Time Average Error
MILP	Mixed Integer Linear Programming
MIMO	Multiple Input Multiple Output
MINLP	Mixed Integer Nonlinear Programming

MV	Manipulated Variable
NI	Niederlinski Index
NRG	Non-Square Relative Gain
NRGA	Normalized Relative Gain Array
PI	Proportional Integral
PM	Participation Matrix
PRG	Partial Relative Gain
RDA	Relative Dynamic Array
RDG	Relative Disturbance Gain
REA	Relative Exergy Array
REM	Relative Error Matrix
RGA	Relative Gain Array
RHP	Right Half Plane
RIA	Relative Interaction Array
RNGA	Relative Normalized Gain Array
RRA	Relative Response Array
SISO	Single Input Single Output
SOPTD	Second Order Plus Time Delay
SVD	Singular Value Decomposition
TITO	Two-Input Two-Output

CHAPTER - 1
INTRODUCTION

1.1 The Choice for Decentralized Control System

In the past few decades, advancements in industrial processes resulted in complex, large scale plants with multiple-input and multi-output (MIMO) that obligates high demands on control configuration. Such challenging problems arise in the control of interconnected process systems with strong interaction. Often, the interconnection is sought in order to employ resources effectively and minimize costs. Sometimes the integration of processes are carried out through interconnection with an objective of efficient energy utilization. The term large scale plants came into existence when it has been realized that such plants cannot be controlled in a straight forward manner. This is because the systems to be controlled are too large and the problems to be solved are too complex, i.e., they cannot simply be solved by using faster computer with larger memory (Bakule, 2008). For the control of such large scale MIMO plants the control system community has suggested two kinds of controllers, viz., a multivariable centralized controller and a set of decentralized controllers.

Multivariable centralized controllers may always outperform decentralized controllers, but this performance gain must be traded-off against the cost of obtaining and maintaining a sufficiently detailed plant model (Skogestad and Postlethwaite., 2001). In spite of the centralized controllers having the capability to extract "optimal" performance, the use of decentralized controllers for multivariable plants is the rule in industrial process control applications (Campo and Morari, 1994). In addition to its inherent simplicity, a decentralized control system exhibits several advantages over a fully centralized design. In the ideal case, these advantages include:

i. Flexibility of bringing individual subsystems in and out of service during process operation in order to handle changing control objectives;
ii. In case of the failure of another component in the individual subsystem, the independent controller will still try to drive its associated variable to its set point;
iii. Flexibility of tuning individual subsystem, in case of slowly varying process conditions;
iv. Ease of designing individual SISO subsystem compare to designing a full multivariable system, i.e., fewer design parameters need to be specified for SISO subsystems.

Decentralized control tries to divide the whole plant into various subsystems and designing independent controllers for each subsystem as a way to handle the control of overall plant (Figure 1.1). This division may sometimes correspond to a real plant or is just a "conceptual" model for control system design (Bakule, 2008). Formally a decentralized controller can be defined (Skogestad and Postlethwaite, 2001) as:

Decentralized controller is a control system consisting of non-interacting feedback controllers, which interconnect a set of output measurements/commands with a subset of manipulated inputs. These subsets should not be used by any other controller.

The task of designing a decentralized control system can be split into two separate sub-problems:

1. Input-output pairing selection, which is often termed as the control configuration selection,
2. Controller design, i.e., designing individual controllers for independent SISO subsystems.

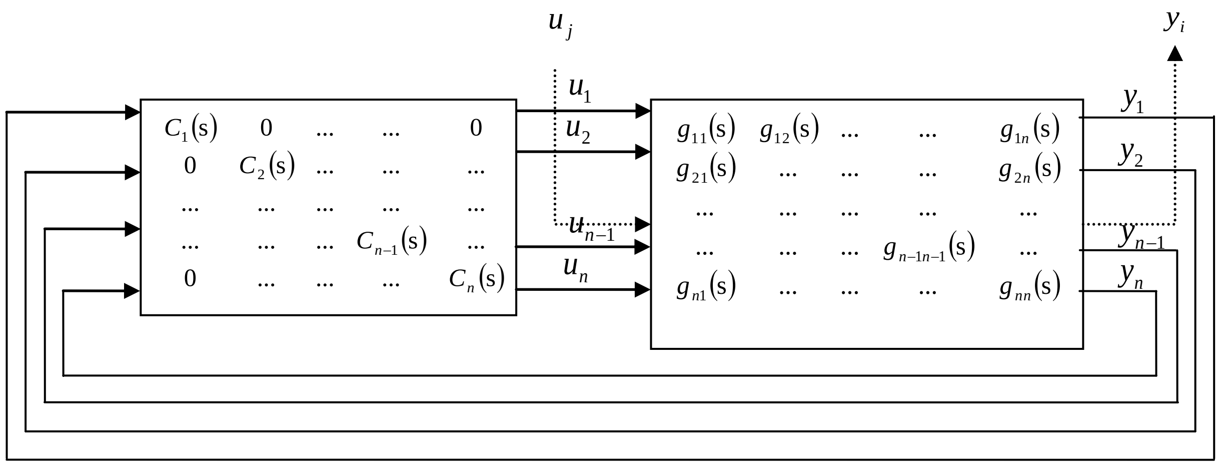


Figure 1.1. Decentralized multivariable control system with a diagonal controller and a detached input-output pair.

The former problem is often concerned with defining the control system structure, i.e., which of the available plant inputs is to be used to control each of the plant outputs. For large scale systems, in particular, the decision about the control system's structure is fundamental to the achievable performance. A poor choice of structure will impose control performance limitations that cannot be overcome by any advanced controller. This can be quite complex, since for a given plant there are many alternate decentralized control structures possible. For example, for a plant with '*n*' manipulated inputs and '*n*' controlled outputs, there are *n*! different multi-loop SISO decentralized control systems possible (Niederlinski, 1971). Thus, efficient screening techniques are needed for eliminating the undesirable control structures and determining the best controller pairing from among the large number of possible pairing. Foss (1973) encouraged the researchers in the area of control structure design to close the gap between theory and applications. But the gap still exists up to some extent even today. This study deals with one specific issue in the control structure design, namely that of input-output pairing or control configuration selection.

1.2 Interactions and Input-Output Pairing Problem

As discussed in Section 1.1, the decentralized control system divides a larger set of variables grouped into several smaller sets of MIMO subsystem (Figure 1.2). In comparison to single-input, single-output (SISO) systems, the MIMO systems are more difficult to control due to the interactions among manipulated inputs and controlled outputs. The most serious problems occur when transmission interaction is present, in which the effects of one control loop "loop back" through interaction and affects the action of other controllers (McAvoy, 1983).

To demonstrate the effect of interactions, let us consider a block diagram of

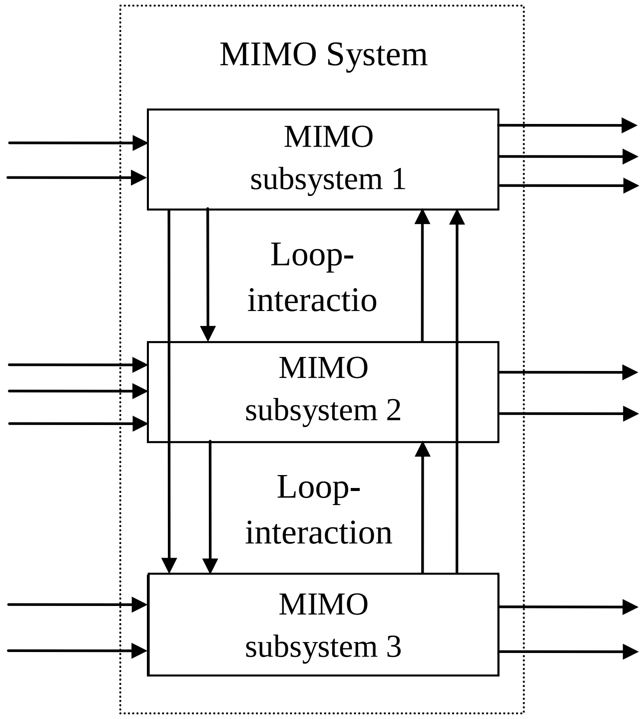


Figure 1.2. MIMO system with interaction.

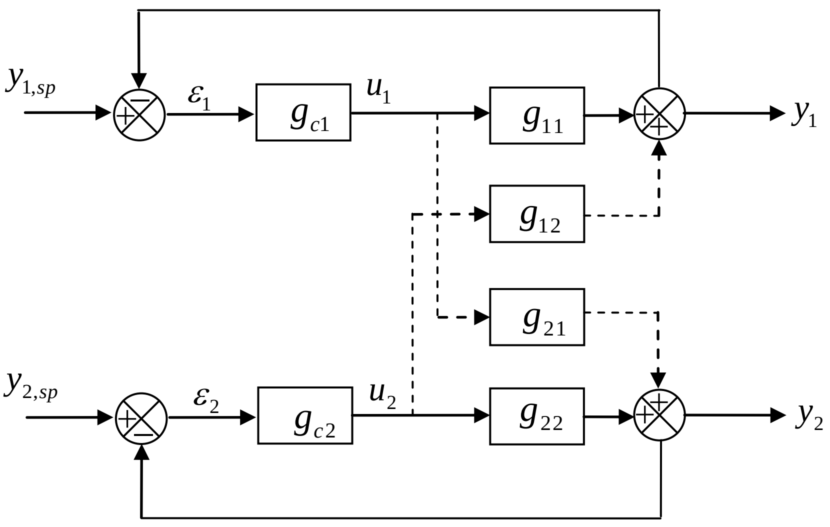
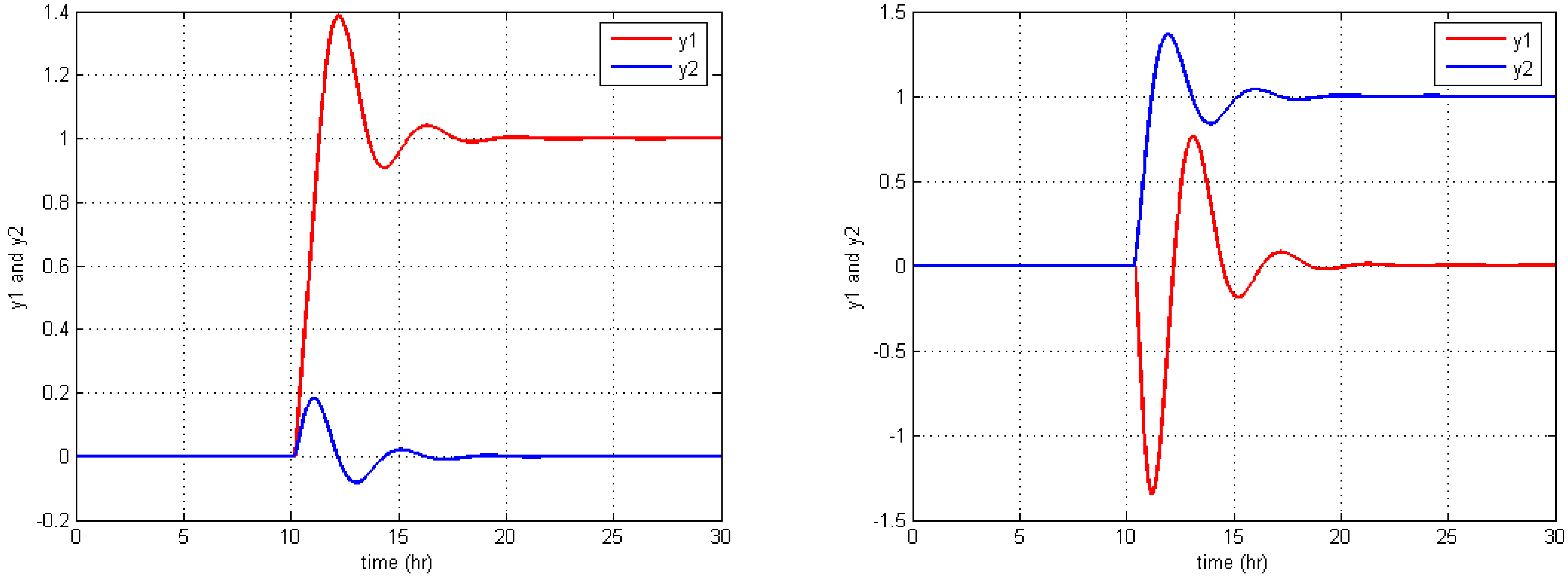


Figure 1.3. Block diagram representation of 2×2 multi-loop control system with $(1-1)/(2-2)$ configuration.

2×2 plant model, configuring $(1-1)/(2-2)$ (diagonal) pairing (Figure 1.3). The pairing is said to be diagonal when input u_1 is used as manipulated variable to control output y_1 via process transfer function g_{11}; similarly u_2 and y_2 forms the pair via process transfer function g_{22}. Any change in input u_1 and u_2 affects output y_2 and y_1 via the process transfer functions g_{21} and g_{12} respectively. Thus, each manipulated variable affects both controlled variables. Now consider a case where y_1 deviates from its set point $y_{1,sp}$ which causes controller g_{c1} to take corrective action by adjusting the manipulated variable u_1. The change in u_1 propagates to loop 2 via g_{21} and makes y_2 to change from its set point $y_{2,sp}$. To bring y_2 to its set point, the controller g_{c2} manipulates u_2. The change in u_2 not only affects y_2 but y_1 as well. The process interaction thus causes control loop interactions, which sometimes can be very severe and drastically reduces control system performance.

Figure 1.4 illustrates the performance degradation of an industrial polymerization reactor system (Chien et al., 1999) which may result due to loop interactions. The figure shows the closed-loop response of output signals y_1 and y_2 for a unit step change in their set point. For control system design, a decentralized PI controller is employed in Simulink (Figure 1.5) using IMC based tuning parameters (Seborg et al., 2010). Ideally, if there are no closed-loop interactions present, then y_1 should not be affected when a step change is given in y_2 and vice versa. From Figure 1.4 it is evident that the outputs are coupled since the step change in y_1 and y_2 affects y_2 and y_1 respectively, significantly.

For industrial processes, the considered MIMO system could be somewhat complex. It is quite common for a chemical industry to have complexity of several



(a) **(b)**

Figure 1.4. Decentralized control output responses of an industrial polymerization reactor (Chien et al., 1999) with a step change in the set point of the first output, y_1 [Figure 1.4 (a)] and the second output, y_2 [Figure 1.4 (b)].

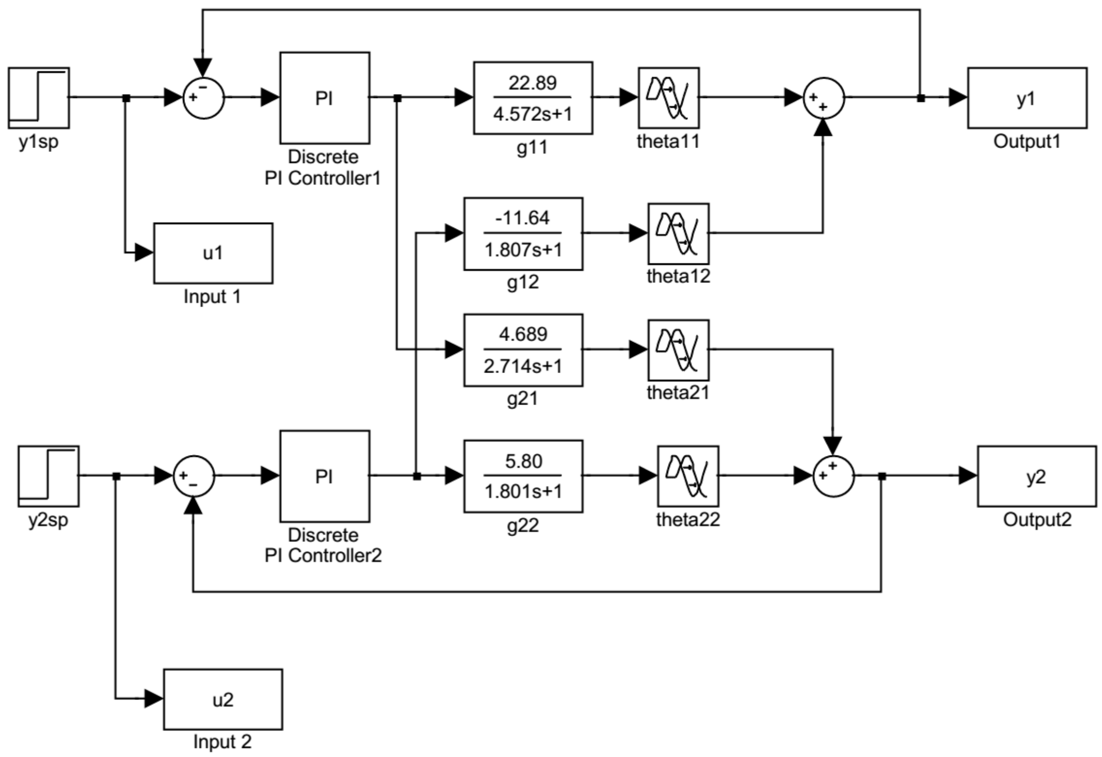


Figure 1.5. Simulink Model of an Industrial Polymerization Reactor (Chien et al., 1999).

hundreds of control loops. Even a simple distillation column with five controlled and five manipulated variables will have 120 possible control loop pairings. The choice for best pairing selection is often not at all obvious. Also, the proper selection of pairing is important for the stability of the overall system since a bad pairing may result in stable individual loops but the unstable overall system. By appropriate pairing of input-output variables the effect of interactions can be minimized and improved control system performance can be achieved. Generally, the stronger the interactions between the loops, the more difficult would be to obtain a satisfactory performance. Evidently, there is a strong need for a measure that not only can select the best pairing or reject the worst pairings but also can give some information on the extent of interaction present.

The control system design procedure for a multivariable plant involves two major steps. The first step involves the selection of suitable input and output which depends on the type of plant model G, e.g., linear or nonlinear, time-invariant or time-varying, physical principles based or black box. The detailed survey of input-output selection methods are reported in the literature (van de Wal and de Jager, 2001). The second step involves the selection of control configuration, where it is decided which outputs are to be connected to which inputs. van de Wal and de Jager (2001) has defined the input-output pairing problem for decentralized control system as:

> *Establishing which outputs in 'y' are used to determine each input in 'u' is often called partitioning or pairing in case of a diagonal controller.*

Analysis and development of tools for input-output pairing problem is the focus of this thesis. The aim is to select the control configuration that can generate acceptable performance of the system. Objectives to be taken into account include stability, set point tracking, and robustness considering performance specifications for subsystems or for the overall system. The aim is also to validate the most commonly used tools for this problem. In addition, a number of new tools and corrections to the existing ones are proposed in this thesis that can be used to select control configurations which allow achieving a specified performance. The tools are also tested on several case studies taken from the process industry. The discussion is limited to linear quadratic (square) plant models, i.e., models with as many inputs as outputs. It is further assumed that the selection of appropriate inputs and outputs has already been done and our focus would be on the pairing of those selected variables.

1.3 Interaction Measures and Tools for Control Configuration Selection

The seminal work of Bristol (1966) on the relative gain array is the first formal approach towards measuring the interaction between the various loops of a decentralized multi-loop control system. It is one of the most favored and widely used tool for the control configuration selection. The best part of RGA is its simplicity, i.e., it requires only the crude process model information particularly the steady-state gains. However, few researchers have discarded its use, as it does not give good results for processes with dominant dynamics. Subsequently, many extensions were proposed to overcome its limitations and for wider applicability. McAvoy (1983) and Shinskey (1967) presented many successful industrial applications of RGA.

The literature is also replete with the interaction measures which do not follow RGA based approach and subsequent extension methodologies. The RGA based approaches mostly use transfer function models. The Niederlinski index (1971) is one such method, often being used, as a complementary tool to the RGA analysis, as a measure of closed-loop stability. It provides sufficient condition for instability of *n*-dimensional closed-loop system under decentralized control. The eigen value based methods such as singular value decomposition (SVD) (Morari, 1983) and Jacobi iteration matrix (Mijares et al., 1986) are also available. Another common approach is the state space based methods using the concept of observability and controllability gramian, e.g., the participation matrix (PM) (Conley and Salgado, 2000), and the Hankel interaction index array (HIIA) (Wittenmark and Salgado, 2002). As an alternative to HIIA, Birk and Medvedev (2003) suggested the use of the system H_2 norm as a base for an interaction measure.

1.3.1 Relative Gain Array

The relative gain array proposed by Bristol (1966) is the foremost tool for control configuration selection. Originally, it was proposed as an empirical tool, and later supported by strong theoretical justifications (Tung and Edgar, 1981). It is one of the most popular tool for the selection of control configuration (van de Wal and de Jager, 1995). RGA considers steady-state properties of the plant and gives suggestion on the possible solution to the pairing problem for a decentralized (diagonal) control structure. RGA is also a good indicator of worst pairings that should be avoided due to possible stability and performance issues. Many different ways of defining RGA are available in the literature. Bristol in his seminal work (Bristol, 1966) defined RGA as: "The ratio of two gains representing first the process gain in an isolated loop (ϕ_{ij}) and, second, the apparent process gain in that same loop when all other control loops are closed ($1/\phi_{ji}^{-1}$). The ratio of these gains defines an array (RGA) M with elements,

$$\mu_{ij} = \phi_{ij}\phi_{ji}^{-1}\text{".} \tag{1.1}$$

Eq. (1.1) can be generalized for the overall plant as given in Eq. (1.2),

$$\Lambda(G) = G(0) \otimes G(0)^{-T} \tag{1.2}$$

where, $\Lambda(G)$ is the RGA matrix for plant G, $\otimes$ denotes element-by-element multiplication, $G(0)$ and $G(0)^{-T}$ are the steady-state gain matrix under open-loop and closed-loop, respectively. The Eqs. (1.1) and (1.2) are limited to steady-state systems only.

Later, many extensions and generalizations of RGA have been proposed in the literature to overcome its limitations. The most important of them is its dynamic extension to analyze a given plant at non-zero frequencies. Since the pairing decision based on steady-state (zero frequency) information alone may sometimes leads to faultier variable pairings and thus, underperforming loops sometimes are unstable. The dynamic extension of RGA can be obtained by replacing $G(0)$ with $G(s = j\omega)$ in Eq. (1.2), i.e., as a function of frequency and will be termed as dynamic relative gain array (DRGA) as given by Eq. (1.3):

$$\Lambda(G) = G(s) \otimes G(s)^{-T} \tag{1.3}$$

Hence, the dynamic RGA has precisely the same form as the steady-state RGA. Noteworthy here is the fact that the dynamic RGA assumes "perfect control", which may not be an appropriate assumption, especially at high frequencies.

In the past few decades several tools resembling RGA have been developed. Few important among them are the dynamic relative gain array (DRGA) proposed by Tung and Edgar (1981), Witcher and McAvoy's (1977) relative dynamic array (RDA), performance relative gain array (PRGA) as proposed by Hovd and Skogestad (1992), partial relative gain (PRG) suggested by Haggblom (1997), relative interaction array (RIA) proposed by Zhu (1996), block relative gain (BRG) introduced by Manousiouthakis et al. (1986) , disturbance relative gain (DRG) proposed by Stanley et al. (1985), effective relative gain array (ERGA) given by Xiong et al. (2005), effective relative energy array (EREA) proposed by Monshizadeh-Naini et al. (2009) , relative normalized gain array (RNGA) introduced by He et al. (2009). The detailed review of methods are reported in the literature (Kinnaert, 1995 and van de Wal and de Jager, 1995). Further extensions are also available for RGA applications to non-square systems (Chang and Yu, 1990), processes with integrators/differentiators (Arkun and Downs, 1990; Hu et al., 2010), non-linear processes (Moaveni and Khaki-Sedigh, 2007), unstable or non-minimum phase processes (Hovd and Skogestad, 1994). The soft computing based techniques for RGA analysis also started to gain emphasis, the most applied of them are the approaches based on neural network, fuzzy-logic and mixed integer linear and non-linear optimization (French et al., 1995; Mollov et al., 2001; Paramasivan and Kienle, 2010; Xu and Shin, 2007).

1.4 Motivation

The selection of suitable control configuration which can minimize the closed-loop interaction is vital to the design of a decentralized control system. For a set of wrongly paired variables, the control system behaves poorly, even with highly sophisticated controllers. Thus, it is essential to carry out an input-output pairing analysis for the robust performance of the control system.

The most popular tool for this purpose is the steady-state gain based relative gain array (RGA). However, for processes with dominant dynamics it is completely inappropriate to carry out the variable pairing selection based on steady-state gain information alone. Thus, it is important to extend the steady-state RGA to processes with dominant dynamics, retaining its inherent properties such as the ease of calculation and simplicity of analysis.

Further, there is no process or parameter that does not involve a certain amount of uncertainty in measurements. Therefore, it is essential to consider the uncertainty in the measurement of process gain, process time constant and time delay terms of the plant model. However, so far only the uncertainty in process gains has been considered in most of the work reported in the literature on control configuration selection for uncertain process models. This motivates us to further extend the work on uncertain process models with both steady-state and dynamic frameworks, considering uncertainty not only in steady-state gains but also in time constants and time delays.

1.5 Objectives

This thesis focuses on the selection of suitable control configuration for decentralized control of multivariable systems, in order to reduce the interaction between individual control loops. Accordingly, the goal is to analyze and further develop the well known tool for control configuration selection the "Relative Gain Array", and further extend it to dynamic systems.

To achieve this larger goal the following objectives are set:

1. Analyze the strength of the relative gain array as a tool for the selection of control configuration of processes with dominant dynamics & uncertainty and its usefulness as an interaction measure.
2. Critically review and analyze the strengths and weaknesses of existing dynamic RGA (DRGA) methods and suggest possible improvements and/or new measures of loop interaction and control configuration selection for dealing with the weaknesses identified.

3. Extend the work on RGA analysis of uncertain steady-state process models to dynamic process models considering uncertainty not only in gain (steady-state approach) but also in time constant and time delays of the process model.

1.6 Thesis Structure

After having introduced the basic concepts of decentralized control together with discussions on the need of a suitable tool for determining best control configuration, **Chapter 1** enlists various objectives set for this study.

In **Chapter 2,** the detailed literature review of the work on the applicability of notable tool of control configuration selection, the relative gain array to steady-state and dynamic process systems is presented. The chapter also discusses the work on the applicability of RGA to uncertain process models. Though the thesis focuses on the studies on the relative gain array, a brief review of other available tools is also presented. At the end of the chapter, the existing gaps in the literature are discussed together with the scope of the work carried out in this thesis.

Chapter 3 presents various tools in line with the definition of the relative gain array and its extensions that are used in this thesis. The new method developed for the control configuration selection of dynamic systems, "the relative response array" is also presented. For uncertain process models, newer approaches for the selection of control configuration considering uncertainty in process gains, time constant and time delays are proposed and discussed in this chapter.

Chapter 4 introduces various case studies used for the testing of methods, existing as well as the proposed ones. The case studies comprise of two benchmark test problems (Grosdidier and Morari, 1986; McAvoy, 1983) and two industrial problems: (i) Shell oil fractionators problem (Adusumilli et al., 1998), and (ii) Distillation column control problem (Grosdidier and Morari, 1986).

Chapter 5 presents the results and discussion on the case studies. The chapter also focuses on the applicability of tools developed in Chapter 3 and its comparison with the other popular methods of control configuration selection on various case studies.

Chapter 6 gives the summary of the thesis. The conclusions drawn on the basis of the results obtained in Chapter 5 are also listed in this chapter together with the major contributions made through this thesis.

CHAPTER - 2
LITERATURE REVIEW

Various studies reported in literature for the selection of control configuration based on steady-state RGA, its extension to dynamic systems, and to plants with uncertain process models are discussed in Section 2.1 and other methods of control configuration selection, i.e., non-RGA based approaches are discussed in section 2.2.

2.1 RGA Analysis for Control Configuration Selection

A commonly used tool for solving the control configuration selection problem for multi-loop (decentralized) SISO controllers is the relative gain array (RGA). It has found widespread applicability as a measure of closed-loop interaction supported with sound theoretical justification and mathematical proofs (Tung and Edgar, 1981). RGA has a number of algebraic properties even at high frequencies, which makes it a potential tool for control configuration selection (Grosidier et al., 1985).

2.1.1 Steady-State Approach

The first formal tool for the selection of control configuration and to measure the process interaction was proposed by the Bristol (1966). His definition of the measure, "Relative Gain Array" was applicable to linearized, time invariant, multivariable processes described by a square gain matrix of steady-state gains between manipulated inputs and controlled outputs. The various algebraic properties of measure were also discussed together with pairing rules. It is recommended to use the steady-state based RGA for determining the initial design structure first and subsequently the performance be analyzed considering process dynamics and nonlinearity.

For the stability analysis of the control-loop pairings, a very useful measure called Niederlinski Index (NI) is proposed as a theorem (Niederlinski, 1971). It is defined for an interacting linear multivariable control system with integrating controllers using steady-state process gain matrix as basis. The values of NI need to be positive for stable control system. A negative value of NI implies that the system is structurally monotonic unstable. The NI can be used to eliminate those input-output pairings which results in a structurally monotonic unstable system. This theorem has been found to be very useful and later been used as complementary tool with Bristol's relative gain array for analyzing the stability of closed-loop pairings.

Jafarey et al. (1979) presented the applicability of RGA for dual composition control of binary distillation column. Their approach involves deriving an approximate expression for RGA based on the Smoker equation for various control configurations. The possible extension of the approach for multi-component distillation column was also discussed.

The RGA as originally proposed by Bristol (1966) suffers from many disadvantages, one of which is its inability to deal with process disturbances. Stanley et al. (1985) were the first to propose a dimensionless steady-state measure, the relative disturbance gain (RDG) which permits disturbances to be included in the operability analysis. The RDG mainly helps in analyzing whether the interactions obtained are favorable or unfavorable. It has been suggested that the control configuration should be chosen based on RGA from the schemes with best RDGs for the known disturbances.

Grosdidier et al. (1985) presented novel techniques for extracting closed-loop system properties based on steady-state gain information. The properties taken up for investigation comprise of closed-loop stability, sensor and actuator failure tolerance, feasibility of decentralized control structures, and robustness with respect to modeling errors. The said properties were developed assuming integral based controller for open-loop stable systems. The relationship of RGA with the important system performance measures such as RHP zeros, Robustness and Condition number were also shown.

Luyben (1987) showed that the closed-loop interaction measure RGA is sensitive to the changes in the steady-state gain if an integral action based controller is used. The robustness of RGA was analyzed in terms of the control system property integral controllability (IC). The analysis was conducted on various distillation configurations. The result showed that the plants with large RGA elements go unstable even for small changes in gains. The limiting condition (permissible change in gain) for stability of closed-loop had also been generated. Later, Luyben (1987) observed similar findings for the sensitivity analysis of RGA to the size of manipulated variable increment for determining steady-state plant gains. For a conventional R-V distillation column (MV_s: reflux and vapor boil-up), the studies were carried out for small increments in reflux and vapor boil-up (1% to 0.0001%). Based on the gains obtained, RGA was found to vary from plus infinity to minus infinity. It was found that the steady-state gain data obtained from plant test for a distillation column is not reliable for finding RGA, because of their approximate values and high sensitivity RGA to small variations in gain.

Papadourakis et al. (1987) extended the RGA to a system with recycle loops. Prior to this work, RGA had been applied mainly to individual process units only. It had been clearly demonstrated that the RGA for plants gets significantly affected if the plant has recycle loops in place. It was strongly recommended that the RGA calculated for individual subunits are not reliable and the other units of the plant must be considered in conjunction with the unit under study.

For the applicability of RGA to integrating process (eg. liquid level tank system), Arkun and Downs (1990) presented a generalized method. Their method involves determining steady-state gains for integrating (non-self-regulating) processes based on state space and singular value decomposition (SVD) approach. The gains obtained had been utilized for finding the RGA matrix. It is then used to systematically synthesize control structures for large process flow sheets which contained a subset of integrators. The method of finding the RGA for such processes under various conditions (such as zero dynamics) had also been shown. The RGA for an integrating process is defined as the ratio of the open-loop integrator gain to that of open-loop integrator gain modified by closure of other loops The applicability of the approach was shown with the help of a reactor-splitter example.

Chang and Yu (1990) were the first to extend the RGA to non-square systems (i.e., the systems with unequal number of inputs and outputs). The proposed measure, the non-square relative gain (NRG) array, had been defined in the same way as the square RGA, i.e., the ratio of open loop gains to that of the closed-loop gain but in least square sense. The closed-loop gain under least square sense offers advantage of minimizing the steady-state errors. The various properties of NRG array had also been derived. The applicability of NRG array had been shown with the help of two distillation column examples. In a recent study, Sobana and Panda (2015) performed control configuration selection analysis for a non-square (2×3) reverse osmosis desalination process for both servo and regulatory problems based on realtive gain array.

Chiu and Arkun (1990) presented a new necessary condition for decentralized closed-loop integrity, i.e., the chosen control structure maintains its nominal stability for any combination of control-loop failures. Based on the developed condition, the new pairing rules were also defined for which the decentralized closed-loop system remains integral controllable. The theoretical results obtained were compared with the existing ones available in the literature. The proposed necessary condition is appreciable because it depends only on the steady-state gain information of the plant. In the subsequent work (Chiu and Arkun, 1991), the stability condition for a two-input-two-output (TITO) decentralized control system, where each loop is a single-input-single-output (SISO) control loop, was developed. For deriving the generalized stability conditions the assumption of stabilization of each individual loop (keeping the other loop open) was relaxed here, as was the case with previous work on the relative gain array and the Niederlinski index. The results obtained were limited to only 2×2 plants under MIMO decentralized control.

Wolff et al. (1992) discussed the procedure for the controllability analysis of linear systems, with the review of the tools available for the same. The applicability of RGA as a controllability measure was shown with its application to Fluid Catalaytic Converter (FCC) reactor. The study was carried out under the light of performance limitation measures such as right half plane (RHP) poles and zeros, time delays etc. It was recommended to observe the pairing performance in the frequency region around the closed-loop bandwidth of the system.

Chang and Yu (1992) proposed a new measure relative disturbance gain (RDG) array for analyzing the disturbance rejection capability of all possible control structures and consequently selecting the best controller structure. The control structures considered in the study are diagonal, block diagonal, triangular or full controller structure. A generalized version of relative disturbance gain "generalized relative disturbance gain (GRDG)" was also proposed and its application for systems with considerable process dynamics was shown.

Alkaya et al. (1992) used RGA together with other interaction measures for selecting the best control configuration for dual composition control of an industrial, multi-component, high-purity ethyl-benzene distillation column. In another study (Ariburnu et al., 1993) for the same industrial multi-component distillation system the RGA was utilized and the results were verified with IMC based PI control. Similarly, the applicability of RGA in finding the non-realistic combination of manipulated and controlled variable for the middle vessel column dual composition problem was presented by Farschman and Diwekar (1998). For the analysis, a time varying analytical expression for the RGA was also derived.

Zeghal and Palazoglu (1993) developed automated pairing procedure for a decentralized control system using relative gain array and block relative gain. The procedure was then implemented as portable software and was tested in solving a heat exchanger network problem. The step testing method was used to identify the 7×7 transfer function matrix. Individual controller design for fully decentralized system was then performed using IMC design procedure for disturbance and set point changes.

Campo and Morari (1994) investigated the open loop stable multivariable systems such that there exists a decentralized controller which can provide desired closed-loop system properties such as unconditional stability, integrity with respect to actuator and sensor failure, and decentralized unconditional stability. Necessary, sufficient, and necessary and sufficient conditions were derived based on open loop steady-state gain. The connection between D-stability and DIC had also been demonstrated. Their major contribution is towards essentially determining the availability of an integral controller based on steady-state gain information. The chosen controller should the closed-loop system to exhibit required flexibility characteristics and any one of the three properties from unconditional stability, decentralized stability, or integrity.

Hovd and Skogestad (1994) were the first to generalize the Niederlinski index (Niederlinski, 1971) and steady-state relative gain array (Bristol, 1966) based pairing criteria for integrity of open-loop unstable systems. The generalization was done on the basis of odd and even number of unstable poles in plant transfer function matrix and modified (diagonal) transfer function matrix. Two theorems were proposed for relative gain array and Niederlinski index. It was found that the RGA is a special case of Niederlinski index. The uniqueness of RGA was highlighted in obtaining a good set of algebraic properties and its efficacy in deciding appropriate set of pairings in one "glance".

A detailed survey on control configuration selection tools for linear multi-loop control system was presented by Kinnaert (1995). The methods for analyzing the suitability of multi-loop control strategy was also presented. The use of dynamic relative gain array was also emphasized in cases where steady-state based RGA fails to identify the suitable pairings. For dealing with the stability issue of multi-loop control schemes, the use of Niederlinski index (Niederlinski, 1971) is recommended. The results on control structure design starting from input-output selection to controller design via control configuration selection were reported in the literature (van de Wal and de Jager, 1995; Wal, 1994). The popular tools for control configuration selection such as relative gain array, block relative gain, performance relative gain and relative sensitivity were discussed in brief. The unresolved challenges, some of which had been still unexplored were presented.

Piette et al. (1995) presented a graphical tool for visualizing interactions among the control loops. The interpretation of popular interaction measure such as relative gain array and singular value decomposition were described graphically for linear and nonlinear processes. The interaction measures were interpreted in terms of constant output contours in the input space and the gradients of these outputs. In particular, the interactions by RGA analysis are summarized in terms of “relative gain surface plot”. These plots can then be used to find the regions of significant interaction and changes in the interaction structure.

Hwang (1995) presented a new technique for geometric interpretation of RGA, evolved from the analysis of the stability boundaries in the proportional gain space. The technique was applicable not only for selecting best control configuration but also for the controller design. It was reported that their new measure of interaction, the dynamic interaction index (DIX) gives quantitative reliable interaction information in comparison to RGA, which may mislead in some cases.

Zhu (1996) presented a new steady-state interaction measure relative interaction array (RIA). The properties of RIA are explored in analogy to RGA. It’s applicability in analysis of system stability, integrity and robustness was discovered. To overcome the dependence of best pairing through RIA over individual loops, a new measure based on overall interaction was also proposed. Though the proposed method showed promising results for systems at steady-state, its applicability for dynamic systems was not explored.

Zhu et al. (1997) presented a novel approach for qualitative and quantitative analysis of dynamic and steady-state interaction based on closed-loop structural analysis. The new steady-state interaction measure, relative interaction array (RIA) (Zhu, 1996), defined analogously to RGA and can give information about the size and direction of the interaction and also the one way interaction. The properties and pairing rules based on RIA had also been defined. Further, the closed-loop system performance and stability were investigated in a classical unified framework and it was recommended that the variable pairing corresponding to dominant negative interaction should be avoided. The guidelines for ensuring stability under positive feedback by suitable controller design were given.

Haggblom (1997) developed various analytical relations for partial control of a distillation column keeping inventory loops closed. The frequency dependent relative gains for different control structures were found by generating transfer function model based analytical expression for all control structures. For the same problem, another approach based on a new measure of process control structure selection was presented by Haggblom (1997a; 1997b). The new measure partial relative gain (PRG) for a subsystem was defined as the RGA for the subsystem with the rest of the system under integral feedback control, i.e., a partially controlled system. A necessary condition for integral controllability with integrity was derived for overall open loop system based on RGA and for relevant subsystem based on PRG. For systems with higher dimensions (3×3) and larger), PRG offers an advantage over RGA, in that it can be used for explicitly comparing feasible control structures with each other.

An optimization based approach for the control structure selection having minimum interaction is presented by Kookos et al. (1999). Their approach utilizes the two popular interaction measures RGA and RIA, for formulating mixed integer nonlinear programming (MINLP) problem and mixed integer linear programming (MILP) problem, respectively. The requirement of integrity and stability in control loops had been used as constraints to the optimization problem. Their method is more suitable for systems with arbitrary large dimensions, which is difficult to analyze using steady-state RGA, RIA and other interaction measures.

Johansson (2000; 2002) discussed a multivariable level control problem for a quadruple tank system proposed by Johansson and Nunes (1998). The laboratory experiments for quadruple tank discusses in particular the performance limitations imposed by right half plane zeros (RHP Zeros). The control system performance had been studied for both the minimum and non-minimum phase systems. As a part of their study, the RGA was generated for quadruple tank system and a condition was developed upon linearization, for which the quadruple tank process prefers decentralized control structure. The developed laboratory based quadruple tank system had later been adopted for many multivariable control system studies.

Jørgensen and Jørgensen (2000) proposed an optimization based method for automated selection of control structure. Their method involves formulation of control structure selection problem as a mixed integer linear programming (MILP) problem based on Parseval's theorem utilizing the concept of relative gain array (RGA) and internal model control (IMC). The proposed method was applied for plant wide control of a hydro de-alkylation plant.

The RGA has long been considered as an interaction measure suitable only for linear systems. The possible extension of RGA to nonlinear systems was proposed by Glad (2000), which involves the formation of an array of nonlinear functions.

Kariwala et al. (2003) derived new algebraic properties of block relative gain (BRG) proposed originally by Manousiouthakis et al. (1986). Further, the new algebraic properties had been shown to be related to closed-loop system properties such as closed-loop stability, controllability, block diagonal dominance, and interactions. It was shown that the BRG for strongly interacting systems is nearly unity. Most of the results presented were based on steady-state gain information only and are useful for controllability analysis and pairing selection at design stage.

The application of RGA to waste water treatment has been addressed by Samuelsson et al. (2005) and recently by Vu et al. (2014). Samuelsson et al. (2005) studied the degree of channel interaction for nitrate removal in an activated sludge process (ASP). The study comprised of decentralized control configuration selection for different operating points in a bioreactor model. The part of work also includes the centralized control structure study and its comparison with the decentralized control strategy whereas Vu et al. (2005) focused on biological neutrient removal process by developing a continuous recycle system based on sequencing batch reactor. The comparision of proportional intergral controller and generic model control strategies are also presened.

Fatehi and Shariati (2007) introduced normalized relative gain array (NRGA) matrix obtained through a combination of the original RGA matrix and its selection rules. The Hungarian algorithm (Cooper and Steinberg, 1974) used to solve the pairing problem which can be interpreted as an assignment problem. The major advantage of their method is that the pairing can be performed automatically without a need of human decision. Using this method, it is possible to pair adaptively the inputs and outputs in a nonlinear and/or time variable process, where the optimal pairing may change from time to time.

Cheng and Li (2010) proposed a new method of analyzing interactions by means of relative gain table and relative gain graph. Their method suggests that there are more than one feasible way of pairing with relatively weak interaction if we analyze the interactions between any loops by gradually decreasing the other closed-loops. However their method is limited only to the steady-state systems with perfect control.

Monshizadeh-Naini et al. (2011) introduced a vector which describes popular interaction measures [RGA, NI and relative error matrix (REM)] in a parametric form. The vector is called descriptive vector whose each element is a certain pairing choice. This vector can be used for gaining more insight of plants with interaction, for which the Jury Algorithm (Ogata, 1995) was used.

Montelongo-Luna et al. (2011) presented a new measure of control loop interaction, "relative exergy array (REA)" as a means to compare thermodynamic efficiency for control structures of MIMO systems. It was found that the use of exergy in RGA calculation gives more detailed information about the interactions in process control structures. REA is defined as "the ratio of the gain change in the steady-state exergy of the stream of given controlled variable with respect to that of the stream of given manipulated variable, when all loops are open to the gain change in the steady-state exergy of the stream of given controlled variable with respect to that of the stream of given manipulated variable when all other loops are closed in 'perfect control'" (Montelongo-Luna et al., 2011). The applicability of the approach is demonstrated for a nonlinear MIMO system with the help of a high-purity dual-composition-control distillation column problem.

Liao et al. (2012) presented a fuzzy logic based approach for the selection of control configuration. They have developed an overall T-S Fuzzy model as a collection of individual loop T-S Fuzzy models which on linearizing around operating point gives steady-state gain and normalized integrated error. The pairing selection was done based on minimum interaction offered and applying the rules of RNGA-RGA-NI. Their approach is suitable only for centralized controllers.

2.1.2 Dynamic Approach

The RGA has been well accepted as a tool for control configuration selection. The reason behind its popularity is the large number of advantages it offers over other tools of interaction measure. However, RGA also has certain limitations which made some of the researchers to discard its use as a potential tool. RGA, being dependent on the steady-state gain information alone, does not give due emphasis to the process dynamics. It may lead to the incorrect variable pairing particularly for processes with dominant process dynamics. The attempts of extending the steady-state RGA to consider the process dynamics in control configuration selection can be divided in two broad categories: (i) the controller independent approach, and (ii) the controller dependent approach. The literature review based on these two categories is presented in this section.

Bristol (1977) was the first to generalize steady-state RGA (Bristol, 1966) to cover the non-zero frequencies by replacing steady-state gains in the definition of RGA with the corresponding process transfer function matrices. To consider the dynamics, the open and closed-loop gains were calculated individually and the ratio of these gains were used to define the dynamic RGA. Various properties of the measure have also been discussed in the light of process dynamics.

Witcher and McAvoy (1977) proposed a model independent approach for defining dynamic RGA. Their approach is based on the calculation of "dynamic potential" defined as integral of the open-loop step response to a unit step change in input. The interaction measure "relative dynamic array (RDA)" can easily be obtained by replacing steady-state gain in the definition of RGA with the "dynamic potential". Since the RDA is defined analogous to the RGA, it follows most of the properties of the RGA. To demonstrate the applicability of RDA, distillation column and fluid catalytic cracker examples are used.

Tung and Edgar (1981) were the first to provide a rigorous convergence proof of the Bristol's steady-state RGA (Bristol, 1966). Before this theoretical ground the RGA was considered more or less to be an empirical tool. Tung and Edgar (1981) also extended the RGA to dynamic process systems in both the state space and frequency domain. The applicability of the approach had been shown with the help of distillation column control example.

Gagnepain and Seborg (1982) presented a new measure of process interaction called average relative gains matrix (ARG). Their approach is similar to that presented by Witcher and McAvoy (1977), defining relative dynamic array (RDA). However, the approach of Gagnepain and Seborg (1982) differs from Witcher and McAvoy (1977) in that the open-loop step responses were utilized in quite a different manner such as to avoid disadvantages associated with RDA. The simulation studies were carried out for three different control schemes: conventional multi-loop control, partial decoupling, and complete decoupling. It was found that the multi-loop control strategy gives best control in most cases.

The need for the dynamic interaction analysis, particularly in the cases where steady-state RGA is insufficient, had been emphasized by McAvoy (1983). The dynamic RGA is enforced for the cases where the steady-state RGA varies largely with frequency or is widely different from the RGA obtained at natural loop frequency. The strategy to analyze interaction in systems having pure integrating element had also been presented in this study. The studies were conducted on distillation column control problem with level control being the integrating element. This study also emphasized the need for considering the control aspects during the design stage itself.

Manousiouthakis et al. (1986) defined a new measure of process interaction for decentralized control system called block relative gain (BRG). The new measure generalized the RGA for block pairing of inputs and outputs rather than essentially considering a single input, single output case. Unlike RGA, the BRG can directly be used for analyzing interactions under dynamic framework. Like RGA, the BRG is also scaling independent. The authors modeled the properties of BRG and successfully applied to two industrial examples: (i) boiler furnace with four burners and four heating coils; and (ii) heat integrated stirred tank reactor.

Meeuse and Huesman (2002) presented an extension of the popular interaction measure RGA to compare open- and closed-loop step responses in the entire time domain of interest. The closed-loop responses were all based on the internal model control with best achievable control performance. The open and closed-loop responses are represented parametrically for each selected pairing and are shown graphically as an array. The usefulness of the results were shown for two examples of 2×2 plant model for which the static RGA failed to identify the correct pairings. McAvoy et al. (2003) presented a novel approach to defining a dynamic RGA (DRGA). Using the available dynamic process model, a proportional output optimal controller was designed based on the state space approach and the resulting controller gain matrix was used to define a DRGA. Several examples in which the normal RGA gives the inaccurate interaction measure and wrong pairings were studied, and in all cases the new DRGA method found to give more accurate interaction assessment and the best pairings.

Xiong et al. (2005) presented a simple, new dynamic loop pairing criterion for decentralized control of multivariable processes utilizing both the steady-state gain and bandwidth information of the process open loop transfer function elements. The new measure effective relative gain array (ERGA) is proposed based on defining effective gain matrix and is found to be a useful tool not only for interaction analysis but also for designing decentralized control systems.

Xiong and Cai (2006) proposed a novel approach for design of a decentralized control system for multivariable processes. The measure proposed in the study, effective relative gain array (ERGA), quantifies the interaction effect for a particular loop from all other closed-loops through both steady-state gain and critical frequency variations. The detuning factor for the controller design was derived and applied to both the normal processes and process with unstable zeros has been showed.

He et al. (2009) proposed another measure of process interaction, the relative normalized gain array (RNGA), based on both the steady-state and transient information of the process transfer functions. It was reported that the RNGA gives unique and optimal pairing decision. Various properties of the measure were stated by taking analogy to the RGA. The integral error (IE) criteria was utilized to evaluate the process dynamic properties. However, RNGA had not been applied to higher dimensional systems.

Monshizadeh-Naini et al. (2009) proposed an extended criteria compatible with the definition of energy for the input-output pairing of MIMO systems. Their approach was the extension of the concept of ERGA (Xiong et al., 2005) with reinforced weight on the steady-state gain over bandwidth as oppose to equal weighting in ERGA. In defining the measure "effective relative energy array (EREA)", the system H_2 norm which is considered to be the energy of the system impulse response was exploited. The applicability of the measure was shown with the help of examples adopted from the literature. An important criteria, the bandwidth of the closed-loop system, was analyzed for its effect on selected pairings.

Sendjaja and Kariwala (2011) designed a decentralized control system for selecting the control structure of nonlinear dynamic model of solid oxide fuel cell system (SOFC). The performance of designed controller is then compared with advanced control strategies. The design of decentralized control involves three steps (i) input-output variable selection, (ii) input-output pairing selection, and (iii) controller tuning. For the problem of variable selection, a new method had been proposed. The method was based on keeping all the controlled variables (CVs) at set point and choosing only those CVs which facilitates constraint satisfaction for all key variables. The pairing selection was done based on well established RGA approach. Finally for controller tuning, simple internal model control (SIMC) tuning method was used. They concluded that even under the presence of disturbances the proposed control structure closely meets the operational objectives.

Recently, a novel approach to control configuration selection and to calculate the control-loop interactions has been proposed (Sujatha and Panda, 2013), which is based on the comparison of areas of the undesirable responses or the interactive loops. To demonstrate the implementation of this approach, various distillation columns have been considered by the authors.

2.1.3 Uncertain Process Models

In most of the studies on the analysis of RGA and its properties, the process model is frequently assumed. It is well known that the quality of the control achievable for a given control system is strongly depends on the model uncertainty (Skogestad and Morari, 1985). However, in practice, the models of real systems always have some uncertainty associated with them. This uncertainty may be caused by the linearization of non-linear real process model, impulsion in physical parameters, and/or unavailablity of acurate process model at high frequencies (Skogestad and Postlethwaite., 2001). Thus, process models can never be perfect (Chen and Seborg, 2002). Still, the sensitivity of the RGA analysis to model uncertainty has not been given due attention. A system is said to be sensitive if it becomes unstable for a small mismatch (uncertainty) in plant and model (Grosdidier et al., 1985). For plants with uncertain process models, an incorrect pairing decision may result if the RGA analysis is carried out based only on the nominal model of the process.

A decentralized multi-loop control system may have independent controller in each loop, and each controller may be tuned such that the individual loops give satisfactory performance and are stable. However, they may result in unstable closed-loop performance due to loop interactions when both loops are closed together. For such systems (open-loop stable), Grosdidier et al. (1985) suggested techniques for analyzing closed-loop system (feedback controller with integral action) properties: integral stabilizability (IS), integral controllabililty (IC), failuare sensitivity, and robustness with respect to modeling errors. It is noteworthy that all the results were developed with regard to the RGA.

Skogestad and Morari (1987b) carried out rigorous analysis of control system stability and performance for large RGA elements in the context of model uncertainty. The emphasis of the study was on correlating the size of RGA elements to model uncertainty. Both independent relative element uncertainty and uncertainty on each manipulated input was considered. It was shown that the plants with large RGA elements are always ill-conditioned and easily become singular even for small relative errors in transfer function matrix elements. It was recommended that the inverse based controller for plants with large RGA elements should not be used. Similarly, Yu and Luyben (1987) showed that there exists a quantitative relation between the allowable perturbation in each element of the steady-state gain matrix and its corresponding RGA matrix, beyond which the closed-loop system becomes unstable (loses integral controllability).

Chen et al. (1992) studied the effect of model uncertainties on open-loop system properties which ultimately affects the closed-loop system properties. The diagonal structured uncertainty was considered for analyzing robustness difficulties associated with closed-loop system both for scaled and unscaled plants. Several estimates for the worst case deviations were given in terms of scaled plant condition number and the RGA. The previous conjecture that the plant with large RGA elements and condition number are potentially difficult to control was reinforced. They concluded that the results obtained from the RGA and block relative gains under certain cases may be misleading or are overly optimistic. Later, Chen et al. (1994), extended the effect of model uncertainties on open-loop system properties. Considering the diagonal structured uncertainty in the plant input the deviations of the open loop transfer function from its nominal value were analyzed for robustness difficulties. For the given uncertainty, the worst case bounds of open-loop transfer function were given in terms of structured singular value, condition number, relative gain array and block relative gains. It was concluded that the plant with large relative gain, block relative gain and structured singular value results in large deviation in open loop transfer function w.r.t their nominal values.

Chen and Seborg (2002) presented lower and upper bounds on the RGA elements for systems under simultaneous additive perturbations in all the elements of steady-state gain matrix. The analytical bounds are derived for 2×2 control problems and for general $n\times n$ control problems. For the given uncertainty the worst-case bounds of open-loop transfer function were given in terms of relative gain array. The worst-case bounds helps in determining the lower and upper limit on the allowable uncertainty before the RGA element changes sign and system loses integrity.

Kariwala et al. (2006) extended RGA to norm-bounded uncertain systems. They presented a method for calculating a tight bound on the worst-case relative gain and derived necessary and sufficient conditions for the sign change of the relative gain over the uncertainty set. A signal based approach for the representation of relative gain to uncertain systems was also proposed.

Agustriyanto and Zhang (2007) proposed an optimization based method for calculating the worst case lower and upper bounds of relative gain array (RGA) and relative disturbance gain array (RDGA) for uncertain process models. The range of RGA and RDGA obtained provides information regarding sensitivity to gain uncertainties which is important from determining control structure and robustness analysis.

Haggblom (2008) presented a method for analyzing integral controllability (IC) and integrity of uncertain systems under model uncertainty for decentralized control system. Their method was beneficial particularly in significantly reducing the worst-case combinations of model uncertainty. By the use of the proposed method a tighter bound on the allowable uncertainty can be obtained. The more elaborative work on closed-loop system properties under uncertainty was presented by Firouzbahrami and Nobakhti (2011; 2012). First, a thorough review of work on the analysis of uncertain systems (Firouzbahrami and Nobakhti, 2011) was presented. It was then followed by the integrity analysis of closed-loop system based on the derived closed-loop system properties such as integral controllable (IC), integral controllable with integrity (ICI), and decentralized integral controllable (DIC), using a model of both unstructured and structured independent uncertainties. In their subsequent study, Firouzbahrami and Nobakhti (2012) extended the integrity properties such as integral stabilizable (IS), integral controllable (IC), integral controllable with integrity (ICI) and decentralized integral controllable (DIC) to uncertain systems considering the norm bounded additive uncertainties. In tune with the previous work (Grosdidier et al., 1985) on integrity of uncertain systems, the necessary and sufficient conditions are derived for IS, IC and ICI, whereas only sufficient condition was provided for DIC. Though the conditions are derived considering unstructured uncertainties, it was found that the results can easily be extended to structured uncertainties, and may lead to the increased degree of conservatism.

2.2 Other (Non-RGA based) Methods for Control Configuration Selection

Relative gain array (RGA) (Bristol, 1966) is the first formal tool for the selection of control configuration. It is the most applied method for analyzing interactions and is based on steady-state gain information of MIMO processes. It is independent of process model and its dynamics, scaling of inputs and outputs, disturbances and includes all possible pairing information in a single matrix. As a rule, the input-output pairing must corresponds to the positive relative gains that have values as close to unity as possible.

Skogestad and Morari (1987a) used singular value decomposition (SVD) as a useful tool to determine if a system is prone to control loop interactions resulting in sensitivity problems that arise from model mismatch in process gains. SVD considers directional changes in the disturbances. Though SVD has good geometric interpretation in terms of selection of measurement and pairing of variables, it depends on input–output scaling. Moreover, with weak interactions and with larger dimensional systems, more criteria are to be considered for selection of pairs.

Mijores et al. (1986) suggested a criteria for measuring process interaction based on the steady-state gain matrix which was later extended to the dynamic systems (Mijores et al., 1986). It is based on the difficulty of inverting the process transfer function matrix at zero frequency or for any other frequency. This is done by defining a matrix, said to be as Jacobi iteration matrix, whose element is a normalized process transfer function element, i.e., each element of a row is divided by its corresponding diagonal element. The spectral radius of the Jacobi iteration matrix (i.e., the largest magnitude eigen value of Jacobi iteration matrix) serves as a convergence criteria (i.e., the spectral radius should be less than one) for an iterative method of solving a system of equation $G(s)u(s) = y(s)$. The Jacobi eigen value criteria not only provides best pairing but also gives information on the stability of a multi-loop control system.

Conley and Salgado (2000) and Salgado and Conley (2004) suggested a completely different approach based on the state space representation of process, for the analysis of closed-loop interaction by considering observability and controllability Gramians in so called Participation Matrices (PM). Later, following a similar approach, Wittenmark and Salgado (2002) introduced the Hankel Interaction Index Array (HIIA). One key property of these methods is that the whole frequency range is taken into account in one single measure. Furthermore, these measures seem to give appropriate suggestions for controller structures selection. The use of the system H_2 norm as a base for an interaction measure was proposed by Birk and Medvedev (2003) as an alternative to the HIIA. Recently, based on the notion of Gramians a new measure of process interaction and control configuration selection is proposed for MIMO stochastic systems (Shaker and Shaker, 2014). These interaction measures though seem to overcome most of the disadvantages of the RGA, they are computationally more demanding and depend on the input-output scaling unlike RGA.

The summary of all RGA based and non-RGA based methods is shown in Table 2.1, listing interaction measures with reference, definition and important properties.

2.3 Existing Gaps of Research

In spite of wide acceptance of the RGA as a tool for control configuration selection and as a measure of closed-loop interaction, its applicability to processes with considerable dynamics is quite limited. The RGA analysis, if carried out using steady-state gain alone, may result in incorrect interaction measures and consequently leads to wrong loop pairing decisions.

To overcome this limitation of RGA and in generalizing its applicability to processes with dominant dynamics, several extensions (based on the very definition of RGA) as a measure of control configuration selection have later been proposed. In the absence of comparison of these various RGA based methods in terms of merits, limitations and applicability, a rigorous analysis of these methods is required for the purpose of choosing the appropriate one suitable for a given circumstance, preferably the one independent of controller design and tuning. Since, one of the properties of RGA is that it is independent of the process disturbances, i.e., it does not give any insight into the

Table 2.1. RGA based and Non-RGA based Interaction measures for MIMO processes.

S. N.	Interaction Measure (References)	Definition	Important properties
		RGA based Methods	
1.	Relative Gain Array (RGA) (Bristol, 1966)	The RGA is given as, $\Lambda = G(0) \otimes G(0)^{-T}$; where, $G(0)$ is the steady-state gain matrix.	• Independent of process model, scaling and disturbances. • It fails to identify the existence of non-diagonal elements.
2.	Relative Dynamic Array (RDA) (Witcher and McAvoy, 1977)	The RDA is given as, $\lambda_{ij}(\theta) = \phi_{ij}(\theta)[\phi_{ij}(\theta)]^{-T}$; where, $\lambda_{ij}(\theta)$ is the ij^{th} element of RDA such that the dynamic potential ϕ_{ij} is given by $\phi_{ij}(\theta) = \int_0^{\theta} y_i(t)\,\mathrm{d}t$.	• Defined analogous to RGA. • Independent of process model, scaling and disturbances. • Assumes perfect control of closed-loops.
3.	Average Relative Gain (ARG) Matrix (Gagnepain and Seborg, 1982)	For process transfer function matrix, $G_{ij}(s) = \dfrac{k_{ij} e^{-d_{ij} s}}{T_{ij} s + 1}$ The ARG Matrix is given as, $\mu_{ij}^{*} = \dfrac{1}{\theta - \theta_1} \int_{\theta_1}^{\theta} \mu_{ij}(t) dt;$ where, θ_1 is the least time for which $D^{-1}(t)$ exist, $\theta = \theta_1 + \text{largest time constant}$, $\mu_{ij}(t) = D_{ij}(t)[D_{ij}(t)]^{-T}$,	• Defined based on open-loop step response. • Assumes perfect control of closed-loops. • Time limits for which averaging was conducted is injudicious.

S. N.	Interaction Measure (References)	Definition	Important properties
		$D_{ij}(t)=0 \quad \forall t<d_{ij}$ $=D_{ij}^{*}=\dfrac{\text{average change in } y_i}{\text{change in } u_j} \quad \forall d_{ij}\le t\le\theta$	
4.	Relative Disturbance Gain (RDG) (Stanley et al., 1985)	For a 2×2 system given by, $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}=\begin{bmatrix} K_{11} & K_{12} & K_{F1} \\ K_{21} & K_{22} & K_{F2} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ d \end{bmatrix}$ The RDG, β_1 is given by, $\beta_1=\dfrac{(\partial u_1/\partial d)_{y_1 y_2} \quad (\text{perfect } y_2 \text{ control})}{(\partial u_1/\partial d)_{y_1 u_2} \quad (\text{no control over } y_2)}$	• Defined similar to RGA. • Dimensionless. • Independent of scaling and controller design. • Requires steady-state information alone.
5.	Block Relative Gain (BRG) (Manousiouthakis et al., 1986)	The *m*-dimensional BRG (left and right) is defined as, $BRG_l=\left[\left.\dfrac{\partial y_1}{\partial u_1}\right\|_{\substack{u_2=0\\F=0}}\right]\cdot\left[\left.\dfrac{\partial y_1}{\partial u_1}\right\|_{\substack{y_2=0\\F_1=0\\F_2=1}}\right]^{-1}=G_{11}.G_{11}^{-1}$ $BRG_r=\left[\left.\dfrac{\partial y_1}{\partial u_1}\right\|_{\substack{y_2=0\\F_1=0\\F_2=1}}\right]^{-1}\cdot\left[\left.\dfrac{\partial y_1}{\partial u_1}\right\|_{\substack{u_2=0\\F=0}}\right]=G_{11}^{-1}.G_{11}$ Here, $F_1=0$ indicates that the first *m* loops are open, and $F_2=1$ means, the last *n-m* loops are closed and are under perfect control i.e. $y_2=0$.	• Requires enormous computions, particularly for an *n* dimensional system a total of $(n!)^2$ BRGs needs to be computed. • Generalizes RGA to the block pairing of inputs and outputs. • Defined based on the assumption of perfect control. • Linked to manhy closed-loop system properties.

S. N.	Interaction Measure (References)	Definition	Important properties
6.	Non-square Relative Gain (NRG) (Chang and Yu, 1990)	The NRG for a non-square system is defined as, $$\Lambda^N = G \otimes \left(G^+\right)^T$$ Where, G^+ is the pseudo-inverse obtained from the steady-state closed loop relation, under least square perfect control, $\bar{u} = G^+ \bar{y}^{set}$.	• The sum of NRG elements along the longer side of G matrix equals unity. • The sum of the elements in each row of NRG falls between zero and unity.
7.	Relative Interaction Array (RIA) (Zhu, 1996)	The RIA for the plant G is given as, $$RIA = \frac{1}{G \otimes \left(G^{-1}\right)^T} - 1$$	• It is an equivalent interaction measure to RGA. • the pairing should correspond to RIA elements close to zero.
8.	Partial Relative Gain (PRG) Array (Haggblom, 1997b)	The PRG for the subsystem $G_m(s)$ is given by, $$\Lambda_m^P(G) = \Lambda\left(\bar{G}_m\right) = \bar{G}_m \otimes \bar{G}_m^{-T}$$ Where, the subsystem $G_m(s)$ is the part of the system $G(s)$, and $\bar{G}_m$ denotes the transfer function matrix of the subsystem $G_m(s)$ when remaining system is under integral feedback control.	• The PRG is defined analogous to RGA. • It provides necessary condition for integrity and integral controllability. • It may also be used as a tool for analyzing systems requiring block decentralized control.
9.	Effective Relative Gain Array (ERGA) (Xiong et al., 2005)	The ERGA is given as, $$\Phi = E \otimes E^{-T}$$ Where, $E = G(0) \otimes \Omega$	• Defined analogous to the RGA. • Gives equal weight to steady-state gain and bandwidth of the plant.
10.	Effective Relative Energy Array (EREA) (Monshizadeh-Naini et al., 2009)	The EREA is given by, $$\mathrm{EREA} = E^* \otimes (E^*)^{-\mathrm{T}}$$ Where, $E^* = \lvert G(0) \rvert \times G(0) \times \Omega$	• Defined analogous to the RGA. • Gives more weight to the steady-state gain than the bandwidth information.
11.	Normalized Relative Gain Array (NRGA) (He et al., 2009)	The NRGA is defined as, $$\Phi = K_N \otimes K_N^{-T}$$	• Integrated error (IE) criterion was used for the analysis of process dynamic properties.

S. N.	Interaction Measure (References)	Definition	Important properties
		Where, $k_{N,ij} = \dfrac{g_{ij}(j0)}{\tau_{ar,ij}}$, and $\tau_{ar,ij}$ is the process residence time.	• Defined analogous to RGA. • All the properties of RGA are directly extendible to NRGA.
12.	Frequency-dependent Dynamic Relative Gain Array (DRGA) (Seider et al., 2009)	The frequency dependent DRGA is given by, $$DRGA_{ij}(\omega) = sign(\lambda_{ij}\{0\}) \bullet \lvert \lambda_{ij}\{j\omega\} \rvert$$	• The sign of RGA element was considered explicitly because it carries important information about system stability.
Non-RGA based Methods			
1.	Singular Value Decomposition (Skogestad and Morari, 1987a)	The SVD is defined as, $$K = U\Sigma V^T;$$ Where, U and V are orthonormal unitary matrices ($UU^T = I, VV^T = I$), and Σ is diagonal matrix of "singular values".	• Gives information about the directional sensitivity of the process. • Depends on the input-output scaling.
2.	Jacobi Eigen value criteria (Mijores et al., 1986)	Spectral radius of $A(j\omega)$, $\rho(A(j\omega))=$ largest magnitude eigen value of $A(j\omega)$. Jacobi iteration matrix: $$A(j\omega)=\begin{bmatrix} 0 & a_{12}(j\omega) & . & . & a_{1m}(j\omega) \\ a_{21}(j\omega) & 0 & . & . & . \\ . & . & . & . & . \\ . & . & . & . & a_{21}(j\omega) \\ a_{m1}(j\omega) & . & . & a_{m(n-1)}(j\omega) & 0 \end{bmatrix}$$ Where, $a_{ij}(j\omega) = g_{ij}(j\omega)/g_{ii}(j\omega)$.	• Pairing should corresponds to the smallest spectral radius. • $\rho(A(j\omega))<1$ensures stability. • Suggests unique best pairing.

S. N.	Interaction Measure (References)	Definition	Important properties
3.	Hankel Interaction Index Array (HIIA) (Wittenmark and Salgado, 2002)	The HIIA is given by, $$[\Sigma_H]_{ij} = \frac{\| G_{ij} \|_H}{\Sigma_{kl} \| G_{kl} \|_H};$$ Where, $\| G_{ij} \|_H$ represents the Hankel norm of process transfer function element G_{ij}.	• The pairing corresponds to the largest element in each row of HIIA. • Each row of HIIA sums to unity. • It is difficult to decide whether an entry in the HIIA matrix is large enough to be relevant.
4.	Participation Matrix (PM) (Conley and Salgado, 2000; Salgado and Conley, 2004)	The PM is defined as, $$\phi_{ij} = \frac{\mathrm{trace}[P_j Q_i]}{\mathrm{trace}[PQ]};$$ Where, P_j and Q_i are the controllability Gramian and observability Gramian, respectively; $\mathrm{trace}[P_j Q_i]$ is the sum of the squared HSVs of the subsystem with input u_j and output y_i. Note that $\mathrm{trace}[PQ]$ equals the sum of all $\mathrm{trace}[P_j Q_i]$.	• Sensitive to time delays. • The input-output pairing corresponds to the unit sum of ϕ_{ij}.
5.	Σ_2 (H_2-norm) (Birk and Medvedev, 2003)	The Σ_2 is defined as, $$[\Sigma_2]_{ij} = \frac{\| G_{ij} \|_2}{\Sigma_{kl} \| G_{kl} \|_2};$$ Where, $\| G_{ij} \|_2$ indicates the H_2-norm of G_{ij}.	• Independent of the selected state space realization. • The structure of the plant is preserved. • Independent of frequency scaling. • Unaffected by time delays.

effect of such disturbances on control configuration selection problem. However, it is clearly shown in the literature that the nature of disturbances has a profound effect on the quality of control and how the RGA should be interpreted. The literature on addressing this issue is scarce and hence more efforts are required in this direction.

Although the effect of model uncertainty on RGA analysis has received considerable attention, its extension to the dynamic systems is still in the nascent stage. The majority of the work on model uncertainty is limited to the uncertainty in steady-state gains only, and no significant effort has been made towards uncertainty consideration in measurement of process time constant and time delays.

Furthermore, other unresolved issues in selection of control configuration based on RGA analysis are: (i) From steady-state analysis, only necessary (not sufficient) stability conditions are available for closed-loop stability. Not enough work is reported towards the development of sufficient stability conditions, (ii) The development of effective methods, especially for decentralized control of MIMO systems containing integrators, is still in the early stages, (iii) The extension of the concept of dynamic RGA and model uncertainty for non-square systems has not yet been addressed.

2.4 Scope of Work

Control configuration selection problem for decentralized control of multi-loop plant system is addressed. For the selection of best control configuration and analysis of the extent of interaction present, the widely accepted and probably the most reported tool the "RGA" has been adopted. The focus of the work is on the development of this well known tool, and to extend its application by overcoming its limitations.

Various extensions of the RGA, reported in the literature to dynamic systems have been compared and critically analyzed. In order to overcome the shortcomings of the existing reported measures of process interaction, a new measure, namely, "relative response array (RRA)" is proposed in this study. The properties of the RRA have also been developed. Further, the proposed RRA approach is successfully applied to two benchmark problems (Grosdidier and Morari, 1986; McAvoy, 1983) and two industrial problems (Adusumilli et al., 1998; Grosdidier and Morari, 1986).

The aim of this work is to gain insights into how process dynamics can affect control configuration decision based on RGA analysis in the face of structured model uncertainty. Parametric uncertainty in gain and residence time (includes both time constant and dead time) of the process has been considered. Analytical expressions for worst-case bounds of uncertainty in steady-state and dynamic RGA are derived for TITO plant models. Various benchmark problems and a few real industrial problems are considered to demonstrate the results obtained using proposed approach for uncertainty analysis. The obtained bounds of uncertainty in RGA elements provide valuable information pertaining to the necessity of robustness and accuracy in the model of decentralized multivariable systems.

Throughout the work it has been assumed that the transfer function based model of the plant under study is available and that the relevant input and output for the model are also available. The work here is focused on suitable pairing of these inputs and outputs, which ensure robust performance.

CHAPTER - 3

RELATIVE GAIN ARRAY: THEORY AND METHODS

In this chapter, various theoretical concepts and methods of control configuration selection are discussed. The chapter is divided into three major sections. Section 3.1 discusses the steady-state RGA based approach for control configuration selection, algebraic properties of RGA, its pairing rules and stability considerations. Section 3.2 gives the details on the RGA analysis under dynamic framework, and various methods utilized in this study. Also, a new measure of process interaction under dynamic framework is proposed. Section 3.3 focuses on the sensitivity analysis of RGA elements to parametric uncertainty. The research in the area is limited to the uncertainty in process gain only. A new approach in dynamic framework for control configuration selection that is being proposed in this study considering uncertainty in process gains, time constants and time-delays is also discussed.

3.1 Steady-State Approach

The Relative Gain Array (RGA) (Bristol, 1966) is the first systematic method proposed for the analysis of closed-loop interaction and input-output pairing in linear multivariable plants. It was originally defined (Bristol, 1966) based on the steady-state gain information of the plant, which can easily be obtained from the open-loop step test response of the plant. This empirical method is the most widely used control configuration selection strategy in the practical designs of process control systems.

In order to express RGA mathematically, a linear, multivariable square plant as given by Eq. (3.1) is considered**,**

$$G(s) = \left[g_{ij}(s)\right] = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & \dots & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & \dots & \dots & g_{nn}(s) \end{bmatrix} \tag{3.1}$$

In general, the output $y(s)$ is related to input $u(s)$ by the expression,

$$y(s) = G(s)u(s) \tag{3.2}$$

Each term in the matrix $G(s)$, i.e., $g_{ij}(s)$ $(i, j = 1, 2, ..., n)$ represents open-loop gain from the input u_j to the output y_i with all other inputs except u_j being constant. Rewriting Eq. (3.2) as,

$$u(s) = G^{-1}(s)y(s) \tag{3.3}$$

Eq. (3.3) interprets that the gain from u_j to y_i is $1/\left[G^{-1}(s)\right]_{ji}$ when all output variables except y_i are under tight control and kept at their nominal values (no off-set). The relative gain is the ratio of these gains, i.e., "open-loop" to the "closed-loop" gains. The matrix combining all the relative gains is termed as the RGA matrix, and can be computed as given by Eq. (3.4),

$$\Lambda(s) = G(s) \otimes \left(G^{-1}(s)\right)^T \tag{3.4}$$

where $\otimes$ denotes element-by-element product (known as Hadamard or Schur product). The steady-state form of Eq. (3.4) can be obtained by replacing the transfer function matrix $G(s)$ with the corresponding steady-state gain matrix K or $G(0)$. The inverse $G^{-1}(s)$ may be non-proper and non-casual, and the assumption of perfect control may not be meaningful except at steady state (Skogestad and Hovd, 1990).

3.1.1 Important Algebraic Properties of RGA

The RGA possesses several useful algebraic properties. Some of the most important of them are listed below (Bristol, 1966; Kinnaert, 1995):

Property 1: Any permutations in the rows and columns of the plant transfer function matrix G results in the same permutations in RGA matrix.

Property 2: It is independent of input-output scaling (, i.e., independent of the units of inputs and outputs). Mathematically,

$$\Lambda(G) = \Lambda(D_1 G D_2) \tag{3.5}$$

where D_1 and D_2 are diagonal scaling matrices of the same dimension as G.

Property 3: The division of $G(s)$ by $1/[G^{-1}(s)]$ [Eq. (3.4)] normalizes the RGA in such a way that the sum of each row (and each column) of RGA is 1, i.e., for a $n \times n$ RGA:

$$\sum_{i=1}^{n} \Lambda_{ij} = \sum_{j=1}^{n} \Lambda_{ij} = 1 \tag{3.6}$$

Property 4: If the transfer function matrix, G, is diagonal or lower or upper triangular (permuted to have non-zero diagonal elements), then the RGA will be identity matrix.

Property 5: The RGA is sensitive to relative element-by-element uncertainty in the process transfer function matrix, G. The mathematical relation between the two is given by Eq. (3.7):

$$\frac{d\lambda_{ij}}{\lambda_{ij}} = \left(1-\lambda_{ij}\right)\frac{dg_{ij}}{g_{ij}} \qquad (3.7).$$

3.1.2 RGA Pairing Rules & Recommendations

The following rules are recommended for input-output variable pairing (Kinnaert, 1995; Skogestad and Postlethwaite., 2001):

1. $\Lambda_{ij} = 1$ indicates that the open and closed-loop gain between output y_i and input u_j is identical, i.e., there is no interaction of the (*i-j*)th loop with other control loops. Thus, the pairing of output y_i with input u_j should be a preferred choice.

2. $\Lambda_{ij} = 0$ indicates that the manipulated input u_j, does not affect the control output y_i. Thus, (*i-j*)th pairing should not be a preferred choice.

3. $\Lambda_{ij} = 0.5$ indicates high degree of interaction. The other control loops have the same effect on the output y_i, as the manipulated input u_j.

4. $0.5 < \Lambda_{ij} < 1$ indicates that there is an interaction between the control loops. However, the open loop gain will dominate and it is preferable to pair input u_j with output y_i, as it would minimize the interactions;

5. $\Lambda_{ij} > 1$ indicates that the interaction from other loops will reduce the gain of the (*i-j*)th control loop. As Λ_{ij} increases, the degree of interaction becomes more severe.

6. $\Lambda_{ij} < 0$ indicates conflicting gains, i.e., the open and closed-loop gains will have different signs. Thus, the opening and closing of other loops will have severe undesirable effect on the $(i\text{-}j)^{th}$ control loop.

The concise rule for input-output variable pairing as stated by Bristol (1966) is as follows:

> *The measure corresponding to the paired variables should be positive and as close to one as possible. Numbers negative or much larger than one are to be avoided and large negative numbers are particularly undesirable.*

In conclusion, the variable pairing should be corresponding to the positive RGA elements with values greater than 0.5 and close to 1.

3.1.3 RGA and Stability

The RGA is one of the most popular tool for the selection of best control configuration. However, the selected configuration must be analyzed for closed-loop integrity. The integrity of a closed-loop system is defined (Macfarlane, 1972) as "The ability of a closed-loop system to remain stable under sensor/actuator failure". The Niederlinski theorem (Niederlinski, 1971) often used as a measure of closed-loop integrity and is used as complementary tool to RGA. It was originally proposed to eliminate the worst pairings that results in structurally monotonic unstable systems. The Niederlinski theorem is described below:

A linear closed-loop multivariable or a multi-loop control system with integrating controllers (I, PI, PID) are structurally monotonic unstable (i.e., unstable for all controller constants (if and only if)

$$\frac{\det[G(0)]}{\prod_{i=1}^{n} g_{ii}(0)} < 0 \tag{3.8}$$

The theorem stated as per Eq. (3.8) has been derived on the basis of two assumptions: (i) Any closed-loop in a set of multi-loop control system with feedback controllers can be tuned to remain stable even if other *n*-1 feedback loops are opened, (ii) All the elements $g_{ij}(s)$ of the plant transfer function matrix $G(s)$ are stable rational functions. The condition expressed by Eq. (3.8) is necessary and sufficient for plant systems of order 2×2 and smaller, and is only sufficient otherwise (Grosdidier et al., 1985). The theorem can be generalized in terms of relative gain array for $n\times n$ systems and followed as pairing rule (Gagnepain and Seborg, 1982): "The pairing corresponding to the negative RGA elements produces a low degree of integrity and hence, should be avoided". However, the negative RGA elements are sufficient but not necessary for instability (Grosdidier et al., 1985).

3.2 Existing Dynamic Approaches

The RGA has long been used as a measure of closed-loop interaction in multi-loop SISO systems. Despite its widespread popularity, its usefulness as a measure of variable pairing for systems with significant process dynamics as RGA fails to provide an accurate configuration. In has also been observed that in extreme situations, the pairing decision based on steady state RGA may lead to underperforming control configuration. In such situations, it is necessary to consider the effect of process dynamics so as to avoid incorrect loop pairings. McAvoy (1983) found that considering the process dynamics at the design stage leads to loop identification with adverse interaction, and the information obtained can be utilized to improve control decisions. Thus, to incorporate the effect of process dynamics on variable pairing selection, various extensions of the steady-state RGA have been proposed. These methods can be categories as: (i) that involves detailed controller design and tuning; and (ii) that is independent of controller design and tuning. The methods independent of controller tuning obviously have an edge over controller dependent methods as they are simple to calculate, less prone to error and are less expensive in terms of computational cost. The most widely used methods under second category as mentioned above are discussed in the following sub-sections 3.2.1 through 3.2.5 (i.e., RDA, ERGA, EREA, Frequency dependent DRGA, RNGA).

3.2.1 Relative Dynamic Array (RDA)

The relative dynamic array (RDA) as proposed by Witcher and McAvoy (1977) was based on the determination of "dynamic potential" ϕ_{ij}, which is the integral of open loop response $y_i(t)$ of controlled variables to a step change of unit magnitude in manipulated variable u_j at time $t=0$, as expressed by Eq. (3.9):

$$\phi_{ij}(\theta)=\int_0^{\theta} y_i(t)\,\mathrm{d}t \tag{3.9}$$

For a $n \times n$ square matrix, $i, j = 1, 2......n$, Witcher and McAvoy (1977) suggested to use the time period 'θ ' to be 20% to 100% of the dominant time constant of the process. The dynamic potential ϕ_{ij} can then be used to obtain the element $\lambda_{ij}(\theta)$ of the relative dynamic array as given by Eq. (3.10):

$$\lambda_{ij}(\theta) = \phi_{ij}(\theta)[\phi_{ij}(\theta)]^{-T} \tag{3.10}$$

Consequently, the best pairing corresponds to the largest positive element in the RDA.

3.2.2 Effective Relative Gain Array (ERGA)

In order to consider the advantages associated with RGA, and to overcome its limitations, the concept of effective relative gain array (ERGA) has been introduced (Xiong et al., 2005). The assumption of "perfect control" underlying in the definition of RGA limits its applicability to multi-loop systems at low frequency range only. The ERGA method thus considers the finite bandwidth control, i.e., the region around the critical frequency of the transfer function. The bandwidth of a transfer function can loosely be defined as the frequency range in which the control is effective (Skogestad and Postlethwaite., 2001). The ERGA method utilizes both the steady-state gain information and bandwidth information of the open-loop transfer function elements to define a dynamic loop pairing criterion for decentralized control of multivariable process. It provides a comprehensive description of dynamic interaction among individual loops without requiring the specification of the controller type and with much less computation. The major advantages associated with the ERGA method are: simple calculation, independent of controller design and tuning, and easy to implement and understand.

Considering a $n \times n$ multivariable plant as given by Eq. (3.1), with $s = j\omega$. Let,

$$g_{ij}(j\omega) = g_{ij}(0)g_{ij}^{0}(j\omega) \quad i, j = 1,2,...,n \tag{3.11}$$

where, $g_{ij}(0)$ and $g_{ij}^{0}(j\omega)$ are respectively the steady state gain and the normalized transfer function of $g_{ij}(\omega)$, i.e., $g_{ij}^{0}(0)=1$ (Xiong et al., 2005). A new gain is defined based on the steady state gain and the dynamic information of plant, and will be termed as the effective gain, e_{ij}, as given by Eq. (3.11):

$$e_{ij} = g_{ij}(0)\int_{0}^{\omega_{B,ij}} \left|g_{ij}^{0}(j\omega)\right| d\omega \tag{3.12}$$

where $\omega_{B,ij}$ for $i,\ j=1,2,...,n$ are the bandwidths of the transfer function $g_{ij}^{0}(j\omega)$. The (closed-loop) bandwidth $\omega_{B,ij}$ is the frequency, where $g_{ij}(\omega)=0.707g_{ij}(0)$ (Skogestad and Postlethwaite., 2001). Based on approximating the frequency response of $g_{ij}(\omega)$ for bandwidth up to $\omega_{B,ij}$ by rectangular area (Fig 3.1), the Eq. (3.12) yields the effective gain e_{ij} as given by Eq. (3.13):

$$e_{ij} \approx g_{ij}(0)\omega_{B.ij} \qquad \forall\, i,\ j=1,2,...,n \tag{3.13}$$

Therefore, the effective gain matrix is defined as given by Eq. (3.14):

$$E = \begin{bmatrix} g_{11}(0) & g_{12}(0) & \cdots & g_{1n}(0) \\ g_{21}(0) & \cdots & \cdots & g_{2n}(0) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(0) & \cdots & \cdots & g_{nn}(0) \end{bmatrix} \otimes \begin{bmatrix} \omega_{B,11} & \omega_{B,12} & \cdots & \omega_{B,1n} \\ \omega_{B,21} & \cdots & \cdots & \omega_{B,2n} \\ \cdots & \cdots & \cdots & \cdots \\ \omega_{B,n1} & \cdots & \cdots & \omega_{B,nn} \end{bmatrix} \tag{3.14}$$

or in a vector form as given by Eq. (3.15):

$$E = G(0) \otimes \Omega \tag{3.15}$$

where, $G(0)$ is the steady-state gain matrix, operator $\otimes$ is the Schur product or

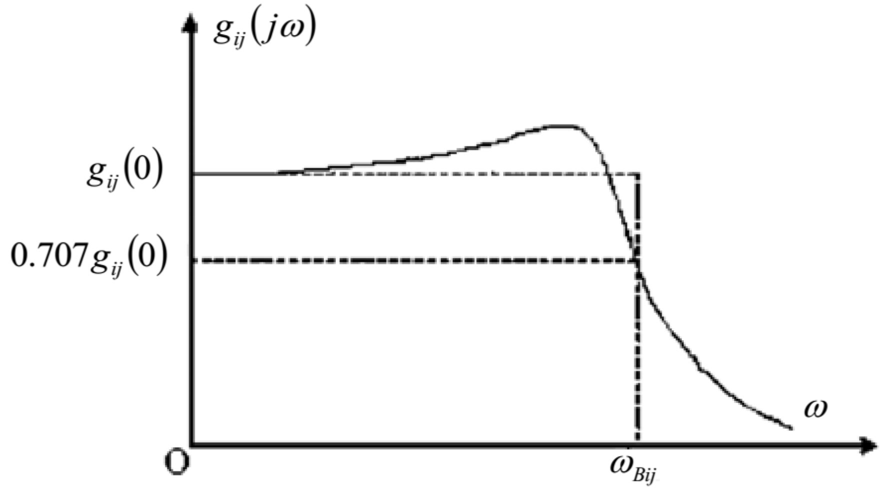


Figure 3.1. Response curve and effective energy of $g_{ij}(\omega)$. [Source: Xiong, Q., Cai, W. J., and He, M. J., 2005. A Practical Loop Pairing Criterion for Multivariable Processes. Journal of Process Control, 15 (7), 741-747.]

element-by-element multiplication (Hadamard or Schur product) and Ω is the bandwidth matrix. In the ideal case, when the phase margin equals 90°, the. For most practical cases, the phase margin is less than 90°, thus $\omega_{B,ij} < \omega_{C,ij}$, and the performance improvement is possible under effective control. Beyond a phase margin of 90°, no further performance improvement can be achieved (Skogestad and Postlethwaite., 2001).

Let $\hat{e}_{ij}$ be the effective gain between i^{th} output variable and j^{th} input variable with all other input-output loops closed. Then, the effective relative gain is defined as given by Eq. (3.16):

$$\phi_{ij} = \frac{e_{ij}}{\hat{e}_{ij}} \tag{3.16}$$

Similarly, calculating the effective relative gains for all input/output combinations and arranging them in an array result in the matrix that has the same form as the RGA and will be called the effective relative gain array (ERGA), as given by Eq. (3.17):

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \dots & \phi_{1n} \\ \phi_{21} & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \phi_{n1} & \dots & \dots & \phi_{n(n-1)} & \phi_{nn} \end{bmatrix} \tag{3.17}$$

Alternately, the ERGA can also be calculated from the effective gain matrix as given by Eq. (3.15). Then Eq. (3.17) can be expressed as Eq. (3.18):

$$\Phi = E \otimes E^{-T} \tag{3.18}$$

Since, by definition the ERGA has the same form as the RGA (i.e., both are defined on the basis of relative gains), all the properties defined for the RGA are directly extendible to the ERGA.

3.2.3 Effective Relative Energy Array (EREA)

The energy of the system impulse response (H_2 norm) have been used to define an interaction measure, the effective relative energy array (Monshizadeh-Naini et al., 2009) which overcomes the limitation associated with the ERGA (Xiong et al., 2005). It was found that in the ERGA method the tradeoff between the steady-state gain and bandwidth of the plant has not been considered appropriately and thus it fails to suggest correct pairing decision.

For the given transfer function the H_2 norm is described as given by Eq. (3.19):

$$\|h(s)\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |h(j\omega)|^2 \,\mathrm{d}\omega \tag{3.19}$$

Approximating the integrator in Eq. (3.19) with a rectangular area, we obtain Eq. (3.20)

$$\|h(s)\|_2^2 = \frac{1}{\pi}(h(0))^2 \bullet \omega_c \tag{3.20}$$

ω_c in Eq. (3.20) denotes the critical frequency or the bandwidth of the system. Eq. (3.20) reflects the effective energy of a subsystem and for the ij^{th} transfer function element as given by Eq. (3.21):

$$e_{ij} = g_{ij}(0)^2 \bullet \omega_{c,ij} \tag{3.21}$$

The effective energy from Eq. (3.21) is a function of square of the steady-state gain and bandwidth of the ij^{th} transfer function element. However, squaring the $g(0)$ results in the loss of essential information and may lead to the wrong input-output pairing (Monshizadeh-Naini et al., 2009). Therefore, the definition of effective energy needs to be modified as given by Eq. (3.22):

$$e_{ij}^{*} = sign\{g_{ij}(0)\} \bullet g_{ij}(0)^2 \bullet \omega_{c.ij} = \frac{|g_{ij}(0)|}{g_{ij}(0)} \bullet g_{ij}(0)^2 \bullet \omega_{c,ij} \tag{3.22}$$

on simplifying Eq. (3.19) to Eq. (3.20):

$$e_{ij}^{*} = |g_{ij}(0)| \bullet g_{ij}(0) \bullet \omega_{c,ij} \tag{3.23}$$

The above definition carries both the steady-state gain and response speed information. Based on the definition of effective energy, the effective energy matrix is defined as given by Eq. (3.24):

$$E^{*} = |G(0)| \times G(0) \times \Omega \tag{3.24}$$

where $G(0)$ is expressed by Eq. (3.25):

$$|G(0)| = \begin{vmatrix} |g_{11}(0)| & |g_{12}(0)| & .... & |g_{1n}(0)| \\ |g_{21}(0)| & |g_{22}(0)| & .... & |g_{2n}(0)| \\ .... & .... & .... & .... \\ |g_{n1}(0)| & |g_{n2}(0)| & .... & |g_{nn}(0)| \end{vmatrix}; \tag{3.25}$$

$G(0)$ and Ω have their usual meaning as defined in Eq. (3.15). Now, the effective relative energy array can be defined analogous to that of the RGA in Eq. (3.4), as given by Eq. (3.26):

$$\text{EREA} = E^* \otimes (E^*)^{-\text{T}} \tag{3.26}$$

Since the EREA is also defined in the same way as the RGA and the ERGA, it follows similar properties and pairing rules as that of the RGA and the ERGA.

3.2.4 Frequency Dependent Relative Gain Array (FDRGA)

A frequency dependent measure of loop-interaction for dynamic processes is presented by Seider et al. (2009). It gives the relative gain array as a function of frequency. For a multivariable plant given by Eq. (3.1), the RGA matrix is defined as in Eq. (3.4). Each element of this RGA matrix, $\Lambda_{ij}(s)$ is given by Eq. (3.27):

$$\Lambda_{ij}(s) = \frac{(-1)^{i+j} g_{ij}(s) \det\left(G^{ij}(s)\right)}{\det(G(s))} \quad i, j = 1, 2, \ldots.. n \tag{3.27}$$

where, $G^{ij}(s)$ is the matrix $G(s)$ obtained on eliminating i^{th} row and j^{th} column. The frequency dependent form of the RGA element given by Eq. (3.28) can be obtained by substituting $s = j\omega$ in Eq. (3.27). Thus, each element of frequency dependent DRGA can be defined as given by Eq. (3.28):

$$DRGA_{ij}(\omega) = sign(\lambda_{ij}\{0\}) \bullet |\lambda_{ij}\{j\omega\}| \tag{3.28}$$

where, $|\bullet|$ is the absolute value of $\bullet$. The sign associated with RGA element carries important information concerning the stability of the system, which has been explicitly considered in the definition.

3.2.5 Relative Normalized Gain Array (RNGA)

The effective control of decentralized control system depends strongly on the input-output pairing decision. The variables paired are often corresponds to the dominant transfer functions of the model. He et al. (2009) developed a new measure of process interaction the relative normalized gain array (RNGA), considering both the steady-state and transient information of the process. For the analysis of process dynamic properties, the integrated error (IE) criterion have been used.

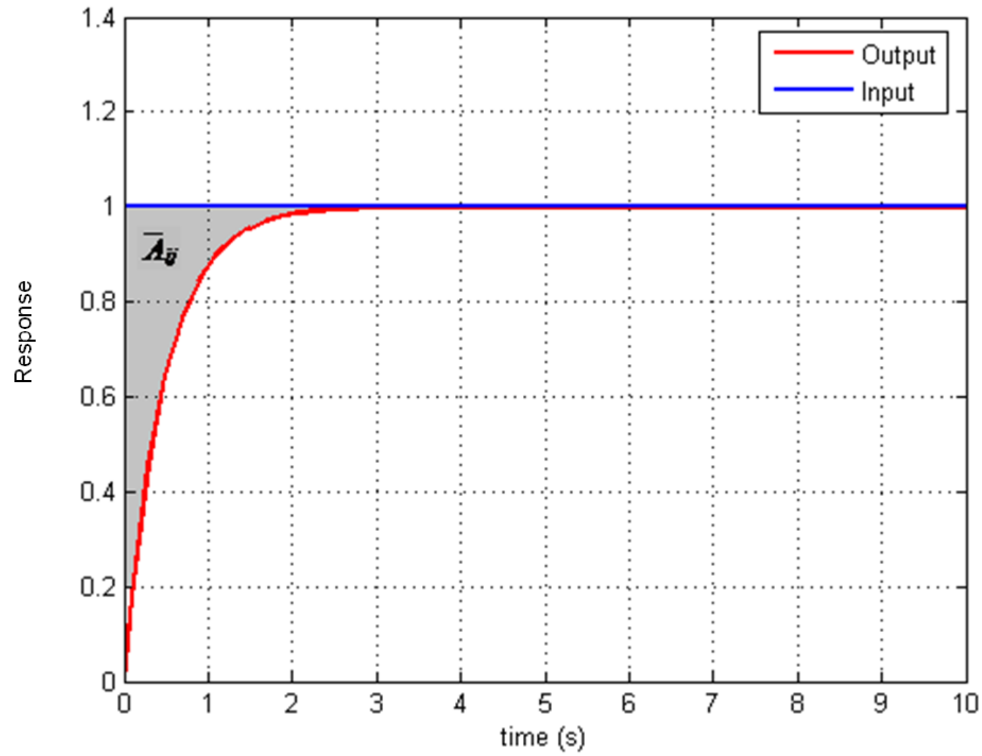
Let the *ij*th element of the process transfer function matrix $G(s)$ be expressed as given by Eq. (3.29):

$$g_{ij}(s) = g_{ij}(j0) \otimes \bar{g}_{ij}(s) \tag{3.29}$$

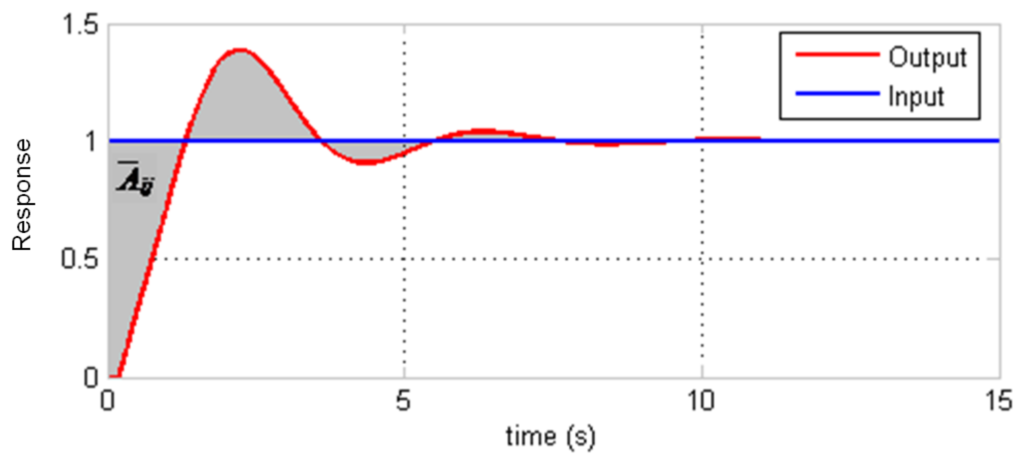
where, $g_{ij}(j0)$ denotes the steady-state gain and $\bar{g}_{ij}(s)$ is the normalized gain such that the $\bar{g}_{ij}(j0) = 1$. Assume, for a unit step disturbance in input u_j the output $\bar{y}_i$ corresponding to the normalized transfer function element $\bar{g}_{ij}(s)$ goes to unity from its resting position at zero. From Figure 3.2 [response curve generated using simulink for a simple first order process (Figure 3.2a) and an industrial-scale polymerization reactor (Chien et al., 1999) (Figure 3.2b)], the shaded area $\bar{A}_{ij}$ can be given by Eq. (3.30):

$$\bar{A}_{ij} = \int_0^\infty \left(\bar{y}_i(\infty) - \bar{y}_i(t)\right) dt \tag{3.30}$$

The $\bar{A}_{ij}$ is termed as the average residence time, $\tau_{ar,ij}$ of $\bar{g}_{ij}(s)$, in essence, is an accumulation of the difference between the expected and real outputs of the process. The larger and smaller values of $\tau_{ar,ij}$ are indicative of the slower and faster process dynamics respectively.



(a)



(b)

Figure 3.2. Typical response of non-oscillatory (a) and oscillatory (b) processes with shaded area representing $\overline{A}_{ij}$.

Since, the input-output pairing decision depends on both the steady-state gain and process dynamics information ($\tau_{ar,ij}$), to assimilate both these parameter's information, the normalized gain (NG) $k_{N,ij}$ is defined as given by Eq. (3.31):

$$k_{N,ij} = \frac{g_{ij}(j0)}{\tau_{ar,ij}} \tag{3.31}$$

For obvious reason of effective control and faster response speed, the pairing corresponding to the large normalized gain should be preferred. Based on Eq. (3.31), the normalized gain matrix would be $K_N = \lfloor k_{N,ij} \rfloor_{n\times n}$. In tune with the definition of relative gain (Bristol, 1966), the relative normalized gain is defined as given by Eq. (3.32):

$$\phi_{ij} = \frac{k_{N,ij}}{\hat{k}_{N,ij}} = \frac{\text{normalized gain with all other loops open}}{\text{normalized gain with all other loops closed}} \tag{3.32}$$

Based on the definition of relative normalized gain, the relative normalized gain array (RNGA) for all possible combinations of input and output of the plant model can be expressed as given by Eq. (3.33):

$$\Phi = K_N \otimes K_N^{-T} \tag{3.33}$$

Since, the RNGA is also defined analogous to the definition of RGA (Bristol, 1966), all the properties applicable for RGA are directly extendible to the RNGA. The pairing of the variable based on RNGA analysis should be corresponding to the positive RNGA elements with values close to unity. The pairing corresponding to the large RNGA elements should be avoided.

Due to various limitations associated with the existing approaches as discussed above, a new measure of dynamic interaction, the relative response array (RRA) is proposed in this study which is discussed in detail in section 3.3.

3.3 New Proposed Dynamic Approach: Relative Response Array (RRA)

A new measure of process interaction "relative response array" (RRA) is introduced in this study. The proposed measure is defined based on the approaches presented in Witcher and McAvoy (1977), Gagnepain and Seborg (1982), and Meeuse and Huesman (2002). Both Witcher and McAvoy's (1977) and Gagnepain and Seborg's (1982) approach was based only on the open-loop response of the plant model and assumes perfect control of the closed-loop which is impossible to attain for any real process. Meeuse and Huesman's (2002) approach being graphical in nature has limited applicability. Further, for finding the pairing in given time range of interest, the dynamic RGA needs to be calculated at every time instant which is quite cumbersome. In addition, normally as per the definition of Laplace inverse, the responses are calculated from zeroth time instant to infinite time period which is effectively a very long time duration and involves higher computational costs. This may not be required from the practical point of view, since the pairing decision is based not only on the stable response but also on the fast response. Thus, we are combining these approaches with a new defined time period of interest of not more than a combined time including the largest dead time and dominant time constant. The details of this proposed new measure is discussed in the following sections:

RRA is defined as a ratio of integral responses, i.e., the integration of open-loop step response to that of the integration of closed-loop step response based on IMC controller. It is assumed that the transfer function model of dynamic process is available. Considering a multivariable square plant as given by Eq. (3.34):

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & . & . & g_{1n} \\ g_{21} & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ g_{n1} & . & . & . & g_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ . \\ . \\ u_n \end{bmatrix} \tag{3.34}$$

where, $Y = \lfloor y_1 \; y_2 \ldots y_n \rfloor^T$ and $U = \lfloor u_1 \; u_2 \ldots u_n \rfloor$ are vectors of outputs and inputs respectively, for a plant model $G(s) = \lfloor g_{ij}(s) \rfloor$. The output response is calculated by taking the inverse Laplace transform of the corresponding plant element.

3.3.1 Integral Open-loop Response

The open-loop response of the controlled output y_i can be obtained by keeping all other manipulated inputs constant (i.e., under manual control) except u_j which is under step change. Since there will be no response during the time interval [0, θ_{ij}], the lower limit of time interval should be the corresponding dead time (θ_{ij}) of the element. The decision on control loop pairing can be taken only after the response is achieved for the element with dominant time constant (τ_D) of the process. At the most, it may be possible that the largest dead time ($\theta_{\max}$) of the process belongs to the element having dominant time constant. In order to consider the extreme case, the upper limit of the time interval should be the sum of dominant time constant and the largest dead time of the process. The output response is thus calculated in time interval $\lfloor \theta_{ij}, (\tau_D + \theta_{\max}) \rfloor$. Thus, the integral response

ϕ_{ij} for open loop unit step change in input u_j is given by Eq. (3.35):

$$\phi_{ij}(t) = \int_{\theta_{ij}}^{\tau_D+\theta_{\max}} y_{ij,ol}(t)dt \tag{3.35}$$

where, $y_{ij,ol}(t) = \mathcal{L}^{-1}\left\{g_{ij}(s).\frac{1}{s}\right\}$ and $g_{ij}(s)$ is the ij^{th} element of plant $G(s)$. It is recommended to observe the responses for 20% to 100% of the time period, $(\tau_D + \theta_{\max})$, over which the integration is being performed (Witcher and McAvoy, 1977). The idea is to observe all the responses under the limitations of process dead times and time constants.

3.3.2 Integral Closed-loop Response

The closed loop response of a given controlled output can be obtained by keeping all other output variables at their set point (i.e., under automatic control). Thus, the integral response $\tilde{\phi}_{ij}(t)$ of a closed loop system for a unit step change in input u_j is given by Eq. (3.36):

$$\tilde{\phi}_{ij}(t) = \int_{\theta_{ij}}^{\tau_D+\theta_{\max}} y_{ij,cl}(t)dt \tag{3.36}$$

where

$$y_{ij,cl}(t) = \mathcal{L}^{-1}\left\{\left(g_{ij}(s) - g_{i,l\neq j}(s)Q_{k\neq i,l\neq j}\,g_{k\neq i,j}(s)\right).\frac{1}{s}\right\} \tag{3.37}$$

The controller $'Q'$ is designed based on IMC principle (Morari and Zafiriou, 1989). The IMC control structure for a 2×2 plant model is shown in Figure 3.3. Typically, an IMC controller design consists of following three steps (Seborg et al., 2010):

i.) The plant model $\tilde{G}$ is to be factorized into two parts as given by Eq. (3.38):

$$\tilde{G}(s) = \tilde{G}_{+}(s)\tilde{G}_{-}(s) \tag{3.38}$$

where $\tilde{G}_{-}$ contains the invertible part of the plant; $\tilde{G}_{+}$ contains non-invertible part (R.H.P zeros, Dead times etc.) such that the $\tilde{G}_{-}$ has a stable and realizable inverse $\left(\tilde{G}_{-}^{-1}\right)$.

ii.) In the presence of a model/plant mismatch, a filter as represented by Eq. (3.39) is introduced into the IMC structure in order to ensure stability:

$$F(s) = \frac{1}{(\gamma s + 1)^{r}} \tag{3.39}$$

and *r* is chosen to make the controller proper (or semi proper). Filter tuning parameter γ is a direct expression of the trade-off between robustness and performance. The smaller value of γ results in faster response whereas large γ indicates more robust closed loop system. It is recommended to use γ to be half of the process time constant.

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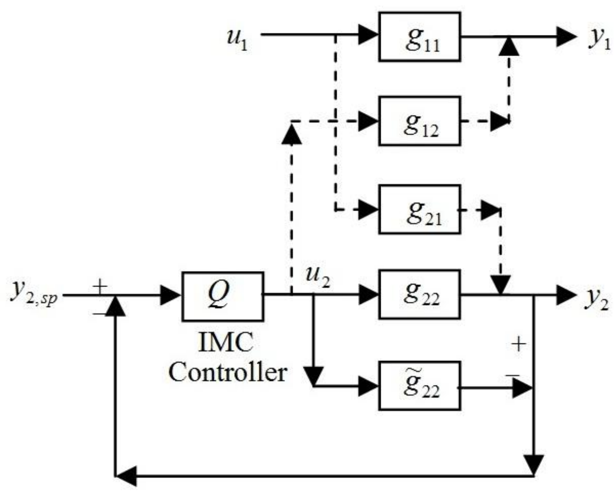


Figure 3.3. Block diagram of IMC control structure for 2×2 plant model.

(iii) Finally, invert the invertible part of the plant model and cascade it with a filter that makes the controller proper as given by Eq. (3.40):

$$Q(s) = \tilde{G}_{-}^{-1}(s)F(s) \tag{3.40}$$

The Laplace inverse of open and closed loop systems with dead time result in the Heaviside step function [$h(t-\theta)$]. For open loop system, the value of θ in Heaviside step function is same as the dead time of the element whereas for the closed loop system, it may be much higher because of the combination of various dead times.

The assumption of 'perfect control' underlying in the definition RGA has been relaxed here with a practically realizable controller, i.e., of internal model control (IMC).

Based on the above expressions a new interaction measure Relative Response Array (RRA) denoted by, ρ_{ij} is given by Eq. (3.41):

$$\rho_{ij} = \frac{\phi_{ij}(t)}{\tilde{\phi}_{ij}(t)} = \frac{\int_{\theta_{ij}}^{\tau_D+\theta_{\max}} y_{ij,ol}(t)dt}{\int_{\theta_{ij}}^{\tau_D+\theta_{\max}} y_{ij,cl}(t)dt} \tag{3.41}$$

By defining RRA in the form of Eq. (3.41), one of the property of RGA (Property 3) is lost which was based on the assumption of perfect control. In our approach we have considered practically realizable controller, thus losing this general property does not affects the pairing decision.

The RRA is found to have the following properties:

i.) the preferred variable pairings is associated with positive RRA elements closest to '1';

ii.) pairing the variables with negative RRA elements and much greater than 1 should be avoided;

iii.) the RRA asymptotically approaches steady-state RGA when the responses are averaged for infinite time period.

3.3.3 Various variants of RRA:

The RRA defined in Eq. (3.41) consists of open-loop and closed loop responses of the process elements. Further, the closed-loop response is calculated based on the assumption of IMC controller. The design of IMC controller is required in order to relax the assumption of "perfect control", since it is impossible to achieve perfect control in real process plants. However, with the assumption of perfect control a good approximation of the actual closed-loop response could be achieved.

The RRA obtained using closed-loop response under perfect control will be independent of controller (IMC) design and is termed as controller-independent relative response array (CI-RRA). For the closed-loop response based on IMC controller design the corresponding RRA will be called controller-dependent relative response array (CD-RRA). In certain cases, the change in input-output pairing occurs during the time range of interest. In such cases, it is important to analyze the RRA value at different points of time, i.e., the RRA is required to be a function of time. For this purpose, a time varying RRA is defined for both controller-dependent and controller-independent systems and the corresponding RRA will be termed as "controller-dependent time-varying relative response array (CD-TV-RRA)" and "controller-independent time-varying relative response array (CI-TV-RRA)" respectively. However, a time-varying RRA matrix may pose difficulty in interpretation. Since both the pairing decision and the extent of interaction will vary with time. Thus, for general purposes a time-average relative response array for both controller-dependent and controller-independent systems are defined, and are termed as "controller-dependent time-average relatieve response array (CD-TA-RRA)" and "controller-independent time-average relative response array (CI-TA-RRA)" respectively.

In order to extend the applicability of the RRA, the newly proposed four different variants of the RRA are defined mathematically in Section 3.3.3.1 through Section 3.3.3.4.

3.3.3.1 Controller-independent time-average RRA (CI-TA-RRA)

The CI-TA-RRA is a controller independent measure of closed-loop interaction and is based on the concept of open-loop step response discussed in Section 3.3.1. For defining CI-TA-RRA, we first need to define the time-average open-loop response based on Eq. (3.35) as given by Eq. (3.42):

$$\phi_{ij,ol} = \frac{1}{(\tau_D + \theta_{\max}) - \theta_{ij}} \int_{\theta_{ij}}^{\tau_D + \theta_{\max}} y_{ij,ol}(t)dt \tag{3.42}$$

Since the controller-independent approach assumes "perfect control" of closed loops, Eq. (3.4) of RGA is directly extendible to define CI-TA-RRA as given by Eq. (3.43):

$$\rho_{ij} = \phi_{ij,ol} \otimes (\phi_{ij,ol})^{-T} \tag{3.43}$$

where, ρ_{ij} is the ij^{th} element of the matrix CI-TA-RRA. The CI-TA-RRA is defined analogously to the RGA, therefore, all the properties and pairing rules of RGA are directly applicable to CI-TA-RRA.

3.3.3.2 Controller-independent time-varying RRA (CI-TV-RRA)

The CI-TV-RRA has the same form as Eqs. (3.42) and (3.43), except that the open-loop response is not averaged, rather generated as a function of time. Therefore, the controller-independent time-varying open-loop response is as given by Eq. (3.44):

$$\phi_{ij,ol}(t) = \int_{\theta_{ij}}^{\tau_D+\theta_{\max}} y_{ij,ol}(t)dt \tag{3.44}$$

and the corresponding ij^{th} element of the CI-TV-RRA can be expressed mathematically as in Eq. (3.43). The time-varying RRA considers the response in various time fragments such as 20%, 30% and so on till 100% of $(\tau_D + \theta_{\max})$.

3.3.3.3 Controller-dependent time-average RRA (CD-TA-RRA)

The assumption of "perfect control" underlying in the definition of controller-independent approaches has been relaxed here and is substituted with a practically realizable IMC controller. The use of IMC controller has advantage of achieving best possible, faster closed-loop response. The time-average open-loop response for the CD-TA-RRA is same as that in Eq. (3.42). Since the open-loop response are always independent of any controller. The effect of controller-dependence appears in the calculations of closed-loop response. Therefore, the controller-dependent time-average closed-loop response based on Eq. (3.36), is given by Eq. (3.45):

$$\tilde{\phi}_{ij} = \frac{1}{(\tau_D + \theta_{\max}) - \theta_{ij}} \int_{\theta_{ij}}^{\tau_D + \theta_{\max}} y_{ij,cl}(t)\,dt \tag{3.45}$$

where, $y_{ij,cl}(t)$ has usual meaning as is Eq. (3.37). Based on these definitions of time-average open and closed-loop responses, the CD-TA-RRA can be defined based on Eq. (3.41) as given by Eq. (3.46):

$$\rho_{ij} = \frac{\text{time - average open - loop response}}{\text{time - average closed - loop response}} = \frac{\phi_{ij}}{\tilde{\phi}_{ij}} \tag{3.46}$$

3.3.3.4 Controller-dependent time-varying RRA (CD-TV-RRA)

The RRA defined in Eq. (3.41), is nothing but the CD-TV-RRA. Thus, the Eq. (3.41) is directly applicable as CD-TV-RRA.

3.4 Uncertainty Consideration

Although for years, in most studies on the analysis of RGA and its properties, the availability of a process model is frequently assumed, the sensitivity of the RGA analysis to model uncertainty is in nascent stage. However, in practice, the models of real systems always have some uncertainty associated with them. Thus, process models can never be perfect. For plants with uncertain process models, an incorrect pairing decision may result if the RGA analysis is carried out based only on a nominal model of the process. The problem further aggravates when a sensitivity analysis of RGA elements to model uncertainty is carried out based on steady-state process model alone.

In this Section, analytical expressions are derived for worst-case bounds on uncertainty in two-input two-output (TITO) plant models under steady-state and dynamic framework. The parametric model uncertainty is considered and is presented in this study with the aim to identify the possible input-output selection changes resulting from the parameter changes.

3.4.1 Sensitivity Analysis of RGA Elements for Parametric Uncertainty

Let a multivariable system $G(s)$ with n controlled variables and n manipulated variables given by Eq. (3.47):

$$G(s) = [g_{ij}(s)]_{n\times n} = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \dots & g_{1n}(s) \\ g_{21}(s) & \dots & \dots & g_{2n}(s) \\ \dots & \dots & \dots & \dots \\ g_{n1}(s) & \dots & \dots & g_{nn}(s) \end{bmatrix} \tag{3.47}$$

The normalized gain $K_{N,ij}$ for the transfer function element $g_{ij}(s)$ as reported in (He et al., 2009) are defined as given below:

i.) For FOPTD process given by, $g_{ij}(s) = \dfrac{k_{ij}}{\tau_{ij}s+1}e^{-\theta_{ij}s}$; $K_{N,ij}$ is defined as given by Eq. (3.48):

$$K_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}}; \quad \text{where, } \tau_{ar,ij} = \tau_{ij} + \theta_{ij} \tag{3.48}$$

ii.) For SOPTD process given by, $g_{ij}(s) = \dfrac{k_{ij}}{\tau_{ij}^2 s^2 + 2\zeta_{ij}\tau_{ij}s+1}e^{-\theta_{ij}s}$; $K_{N,ij}$ is defined as given by Eq. (3.49):

$$K_{N,ij} = \frac{k_{ij}}{\tau_{ar,ij}}; \quad \text{where, } \tau_{ar,ij} = 2\zeta_{ij}\tau_{ij} + \theta_{ij} \quad \forall \quad 0 < \zeta < \infty \tag{3.49}$$

The process transfer function matrix of a 2×2 plant model in terms of normalized gain can be given as Eq. (3.50):

$$K_N = \begin{bmatrix} K_{N,11} & K_{N,12} \\ K_{N,21} & K_{N,22} \end{bmatrix} \tag{3.50}$$

The corresponding matrix of RGA elements in terms λ_{11} can obtained using property 3 (Section 3.1.1) and will be as given by Eq. (3.51):

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{bmatrix} \tag{3.51}$$

here, λ_{11} is the relative gain between output y_1 and input u_1, i.e., as given by Eqs. (3.52) and (3.53):

$$\lambda_{11} = \frac{(\partial y_1/\partial u_1)_{u_2}}{(\partial y_1/\partial u_1)_{y_2}} = \frac{1}{1-\hat{K}_N} \tag{3.52}$$

where,

$$\hat{K}_N = (-1)^n \frac{|K_{N,12}||K_{N,21}|}{|K_{N,11}||K_{N,22}|} \tag{3.53}$$

$\hat{K}_N$, here is referred to as the normalized interaction quotient and n, is the number of negative elements in $G(s)$. For steady-state systems $G(s=0)$, the normalized interaction quotient would be referred to as steady-state interaction quotient $\hat{k}$ and would be defined as in Eq. (3.50) replacing normalized gains with corresponding steady-state gains. If $\hat{K}_N = 1$ the RGA matrix would be a singular point. For odd number of negative elements (i.e., $n = \text{odd number}$) the RGA matrix can never be singular as $\hat{K}_N \neq 1$.

3.4.1.1 For Uncertain Process Gains

Steady-state Approach

A steady-state approach involves only the steady-state gain information of the plant model and does not consider the effect of process dynamics (such as residence time). Thus, the interaction quotient under steady-state approach can be obtained by substituting, $K_N = K$, i.e., the normalized process gain matrix will be substituted with the steady-state gain matrix. Further, it is assumed that all the elements of the steady-state gain matrix, K have symmetric upper and lower bounds. Thus, the nominal steady-state gain element, k_{ij} varies within $k_{ij} \pm \Delta k_{ij}$. Let $\left|\Delta k_{ij}\right| \le \alpha_{ij}\left|k_{ij}\right|$ represents the uncertainty bound to the every element of K, then from Eq. (3.53), the steady-state interaction quotient under model uncertainty, $\hat{k}_{uc}$ is given by Eq. (3.54),

$$\hat{k}_{uc} = (-1)^n \frac{\left(|k_{12}| \pm \alpha_{12} |k_{12}|\right)}{\left(|k_{11}| \pm \alpha_{11} |k_{11}|\right)} \times \frac{\left(|k_{21}| \pm \alpha_{21} |k_{21}|\right)}{\left(|k_{22}| \pm \alpha_{22} |k_{22}|\right)} \tag{3.54}$$

simplifying Eq. (3.54) gives Eq. (3.55),

$$\hat{k}_{uc} = (-1)^n \frac{|k_{12}||k_{21}|}{|k_{11}||k_{22}|} \times \left[\frac{1 \pm (\alpha_{12} + \alpha_{21})}{1 \mp (\alpha_{11} + \alpha_{22})}\right] \tag{3.55}$$

the substitution of steady-state based interaction quotient, $\hat{k}$ in Eq. (3.55) leads to Eq. (3.56):

$$\hat{k}_{uc} = \hat{k}\left[\frac{1 \pm (\alpha_{12} + \alpha_{21})}{1 \mp (\alpha_{11} + \alpha_{22})}\right] \tag{3.56}$$

The respective limits of the $\hat{k}_{uc}$ is represented as given by Eq. (3.57):

$$\hat{k}^l \le \hat{k}_{uc} \le \hat{k}^h \tag{3.57}$$

where,

$$\hat{k}^{l}\left(=\min\left(\hat{k}_{uc}\right)\right)=\min\left\{\hat{k}\left[\frac{1\pm\left(\alpha_{12}+\alpha_{21}\right)}{1\mp\left(\alpha_{11}+\alpha_{22}\right)}\right]\right\} \tag{3.58}$$

and
$$\hat{k}^{h}\left(=\max\left(\hat{k}_{uc}\right)\right)=\max\left\{\hat{k}\left[\frac{1\pm\left(\alpha_{12}+\alpha_{21}\right)}{1\mp\left(\alpha_{11}+\alpha_{22}\right)}\right]\right\} \tag{3.59}$$

For $\hat{k}_{uc}\neq 1$, based on (3.54) the RGA element λ_{11} changes within the uncertainty limits as given by Eq. (3.60):

$$\frac{1}{1-\hat{k}^{l}}\leq\lambda_{11}\leq\frac{1}{1-\hat{k}^{h}} \tag{3.60}$$

For $\hat{k}_{uc}=1$(singular point), the RGA element λ_{11} passes through discontinuity. Thus the λ_{11} changes within the uncertainty limits as given by Eq. (3.61):

$$-\infty\leq\lambda_{11}\leq\frac{1}{1-\hat{k}^{h}} \quad\text{and}\quad \frac{1}{1-\hat{k}^{l}}\leq\lambda_{11}\leq\infty \tag{3.61}$$

<u>Dynamic Approach</u>

In order to understand the effect of process dynamics on input-output pairing decision for the processes with uncertain gains, Eq. (3.54) can be used to define a new interaction quotient referred to as "dynamic interaction quotient" under model uncertainty, $\hat{k}_{uc}$ and is given by Eq. (3.62):

$$\hat{k}_{uc,dy}=\left(-1\right)^{n}\frac{\left(|k_{12}|\pm\alpha_{12}|k_{12}|\right)/\tau_{ar,12}}{\left(|k_{11}|\pm\alpha_{11}|k_{11}|\right)/\tau_{ar,11}}\times\frac{\left(|k_{21}|\pm\alpha_{21}|k_{21}|\right)/\tau_{ar,21}}{\left(|k_{22}|\pm\alpha_{22}|k_{22}|\right)/\tau_{ar,22}} \tag{3.62}$$

simplifying Eq. (3.62) gives Eq. (3.63):

$$\hat{k}_{uc,dy}=\left(-1\right)^{n}\frac{|k_{12}||k_{21}|}{|k_{11}||k_{22}|}\times\frac{\left(\tau_{ar,11}\right)\left(\tau_{ar,22}\right)}{\left(\tau_{ar,12}\right)\left(\tau_{ar,21}\right)}\times\left[\frac{1\pm\left(\alpha_{12}+\alpha_{21}\right)}{1\mp\left(\alpha_{11}+\alpha_{22}\right)}\right] \tag{3.63}$$

the substitution of $\hat{K}_N$ and $\hat{\tau}_{ar}$ in Eq. (3.63) gives Eq. (3.64):

$$\hat{k}_{uc,dy} = \hat{K}_N \left[\frac{1 \pm (\alpha_{12} + \alpha_{21})}{1 \mp (\alpha_{11} + \alpha_{22})} \right] \tag{3.64}$$

where,

$$\hat{K}_N = \frac{\hat{k}}{\hat{\tau}_{ar}} \tag{3.65}$$

while

$$\hat{\tau}_{ar} = \frac{(\tau_{ar,12})(\tau_{ar,21})}{(\tau_{ar,11})(\tau_{ar,22})} \tag{3.66}$$

Eqns. (3.57) to (3.61) are now directly applicable by replacing $\hat{k}_{uc}$ by $\hat{k}_{uc,dy}$ and $\hat{k}$ by $\hat{K}_N$.

3.4.1.2 For Uncertain Process Gains and Residence Time (*Time Constants and Time Delays*)

Assuming lower bounds of uncertainty in all the elements of the steady-state gain, K and average residence time, τ_{ar} to be symmetrical with that of higher bounds. Thus, the nominal steady-state gain element, k_{ij} varies within $k_{ij} \pm \Delta k_{ij}$ and average residence time, τ_{ar} within $\tau_{ar} \pm \Delta\tau_{ar}$. Let $|\Delta k_{ij}| \leq \alpha_{ij}|k_{ij}|$ and $\Delta\tau_{ar} \leq \beta_{ij}\tau_{ar}$ represent the uncertainty bound to the every element of k and τ_{ar} respectively, then from Eq. (3.53) the dynamic interaction quotient under model uncertainty, $\hat{k}_{uc,dy}$ is given by Eq. (3.66):

$$\hat{k}_{uc,dy} = (-1)^n \frac{(|k_{12}| \pm \alpha_{12}|k_{12}|)/(\tau_{ar,12} \pm \beta_{12}\tau_{ar,12})}{(|k_{11}| \pm \alpha_{11}|k_{11}|)/(\tau_{ar,11} \pm \beta_{11}\tau_{ar,11})} \times \frac{(|k_{21}| \pm \alpha_{21}|k_{21}|)/(\tau_{ar,21} \pm \beta_{21}\tau_{ar,21})}{(|k_{22}| \pm \alpha_{22}|k_{22}|)/(\tau_{ar,22} \pm \beta_{22}\tau_{ar,22})} \tag{3.66}$$

on simplifying Eq. (3.66) gives Eq. (3.67):

$$\hat{k}_{uc,dy} = (-1)^n \frac{(|k_{12}|/\tau_{ar,12})(|k_{21}|/\tau_{ar,21})}{(|k_{11}|/\tau_{ar,11})(|k_{22}|/\tau_{ar,22})} \times \frac{[1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})]}{[1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})]} \quad (3.67)$$

substitution of interaction quotient in Eq. (3.64) gives Eq. (3.68):

$$\hat{k}_{uc,dy} = \hat{K}_N \times \left[\frac{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})}{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})} \right] \quad (3.68)$$

The corresponding lower and upper limits on the $\hat{k}_{uc,dy}$ will be as given by Eq. (3.69) and (3.70) respectively,

$$\hat{k}^l \left(= \min\left(\hat{k}_{uc,dy}\right)\right) = \min\left\{ \hat{K}_N \times \left[\frac{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})}{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})} \right] \right\} \quad (3.69)$$

$$\hat{k}^h \left(= \max\left(\hat{k}_{uc,dy}\right)\right) = \max\left\{ \hat{K}_N \times \left[\frac{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})}{1 \pm (\alpha_{12} + \alpha_{21} + \beta_{11} + \beta_{22})} \right] \right\} \quad (3.70)$$

The worst case bounds for the RGA elements can be obtained by substituting $\hat{k}^l$ and $\hat{k}^h$ from Eqs. (3.69) and (3.70) in Eq. (3.60) and (3.61), as the case may be.

These newly proposed variants of various measures are applied to four case studies (2-benchmark test problem and 2-industrial problems) and compared with other conventional measures. The case studies considered are presented in Chapter 4 and the results obtained are discussed in Chapter 5.

CHAPTER - 4

CASE STUDIES: SELECTED NONLINEAR CHEMICAL PROCESSES

Four different case studies are considered for demonstrating the applicability and limitations of various RGA based, non-RGA based and newly proposed interaction measures which are described in Chapter 3. In this chapter, the considered case studies are introduced and described. The case studies comprise of two benchmark test problems and two industrial problems reported in the literature (Adusumilli et al., 1998; Grosdidier and Morari, 1986; McAvoy, 1983).

The benchmark test problems considered for the study are 2×2 control problems having typical process dynamics. The industrial problems adopted for the study are: (i) Shell oil fractionators problem reported in the literature (Adusumilli et al., 1998) as a 2×2 control problem (ii) Doukas and Luyben distillation column reported in literature (Grosdidier and Morari, 1986) as a 3×3 control problem. The process elements involved in the four case studies considered are of first order, first order plus time delay, second order and second order plus time delay.

4.1 Benchmark Test Problems

4.1.1 Case Study-1: 2×2 Process Model with a Second Order Element

This case study which describes 2×2 process model with a second order element was introduced as a benchmark test problem by Grosdidier and Morari (1986). The process transfer function model is defined as:

$$G(s)=\begin{bmatrix} \dfrac{5}{4s+1} & \dfrac{2.5e^{-5s}}{(2s+1)(15s+1)} \\ \dfrac{-4e^{-6s}}{20s+1} & \dfrac{1}{3s+1} \end{bmatrix} \tag{4.1}$$

The process elements involved in this benchmark problem are of first order, first order plus time delay and second order plus time delay. The problem is typical in the sense that it has slow off-diagonal dynamics due to the presence of relatively large time constants and dead times than its diagonal counterpart. When RGA was applied as closed-loop interaction measure it was found to give off-diagonal pairing. However, the off-diagonal elements response is sluggish in comparison to the diagonal elements. Thus, it would be of interest to analyze the pairing recommendation based on dynamic RGA methods for this case study.

4.1.2 Case Study 2: 2×2 Process Model with typical Process Dynamics

This case study describes a 2×2 process model with typical process dynamics which was developed by McAvoy (1983) based on material balance for a distillation tower.

The assumption made in the process model are: (i) analyzer dynamics were modeled as a pure dead-time, and (ii) the level of reflux accumulator and reboiler are under perfect control. Based on these assumptions and the process dynamics considered following plant model is generated [Eq. (4.2)]:

$$G(s)=\begin{bmatrix} \dfrac{-0.805}{(18.3s+1)(5.6s+1)} & \dfrac{0.055}{(5.76s+1)(1.25s+1)} \\ \dfrac{-0.465e^{-0.3s}}{(28.3s+1)(0.62s+1)} & \dfrac{-0.055}{3.3s+1} \end{bmatrix} \tag{4.2}$$

The transfer function elements in this benchmark test problem are of first order, second order and second order plus time delay. This case study is quite an important benchmark test problem because of its uniqueness in the sense that its dynamics is such that the two popular controller-independent measures (ERGA and EREA) failed to identify the correct control loop pairings. So, it will be of great interest to test the proposed RRA approach on this benchmark test problem.

4.2 Industrial Case Studies

4.2.1 Case Study-3: Shell Heavy Oil Fractionator Problem

Shell Heavy Oil Fractionator whose schematic is shown in Figure 4.1 has been considered as a case study for the control configuration selection problem. The characteristic features of heavy oil fractionator are as follows:

i.) The fractionator has three product draws (i.e., top draw, side draw and bottoms). The compositions of top and side draw product streams are specified on the basis of market demands and economics. However, bottoms draw compositions are not controlled.

ii.) The three reflux loops (i.e., upper reflux, intermediate reflux and bottoms reflux) plays crucial role in maintaining the product streams at desired specifications and also helps in their separation by removing the excess heat.

iii.) The heat removed by heat exchangers from reflux loops is used to heat other columns of the plant. Thus, they cannot be controlled independently, and results in reduced degree of freedom for control.

iv.) Out of three reflux loops, the bottom reflux loop has an enthalpy controller that regulates the heat requirement via steam make, whereas the heat duty requirements of the top and intermediate reflux loops acts as a disturbance to the fractionator column.

v.) The feed to the fractionator column is in gas phase and fulfills the heat requirement of the column, and also maintains the temperature on the base of column below a certain specified value.

Thus, for the Shell oil fractionator column the top and side draw endpoint compositions are the controlled variables, whereas top draw, side draw and bottoms reflux flow rates are available as manipulated variables.

A simplified 2×2 model for the Shell heavy oil fractionator problem with top end point and side end point as controlled variables, and top draw and side draw as manipulated variable was introduced by Adusumuli et al. (1998) and is given by Eq. (4.3):

$$G(s) = \begin{bmatrix} \dfrac{4.05e^{-27s}}{50s+1} & \dfrac{1.77e^{-28s}}{60s+1} \\ \dfrac{5.39e^{-18s}}{50s+1} & \dfrac{5.72e^{-14s}}{60s+1} \end{bmatrix} \tag{4.3}$$

The transfer function elements in this simplified model are all first order plus time delay. Further, the process has large time constant and time delay associated with each element of the process transfer function matrix [Eq. (4.3)].

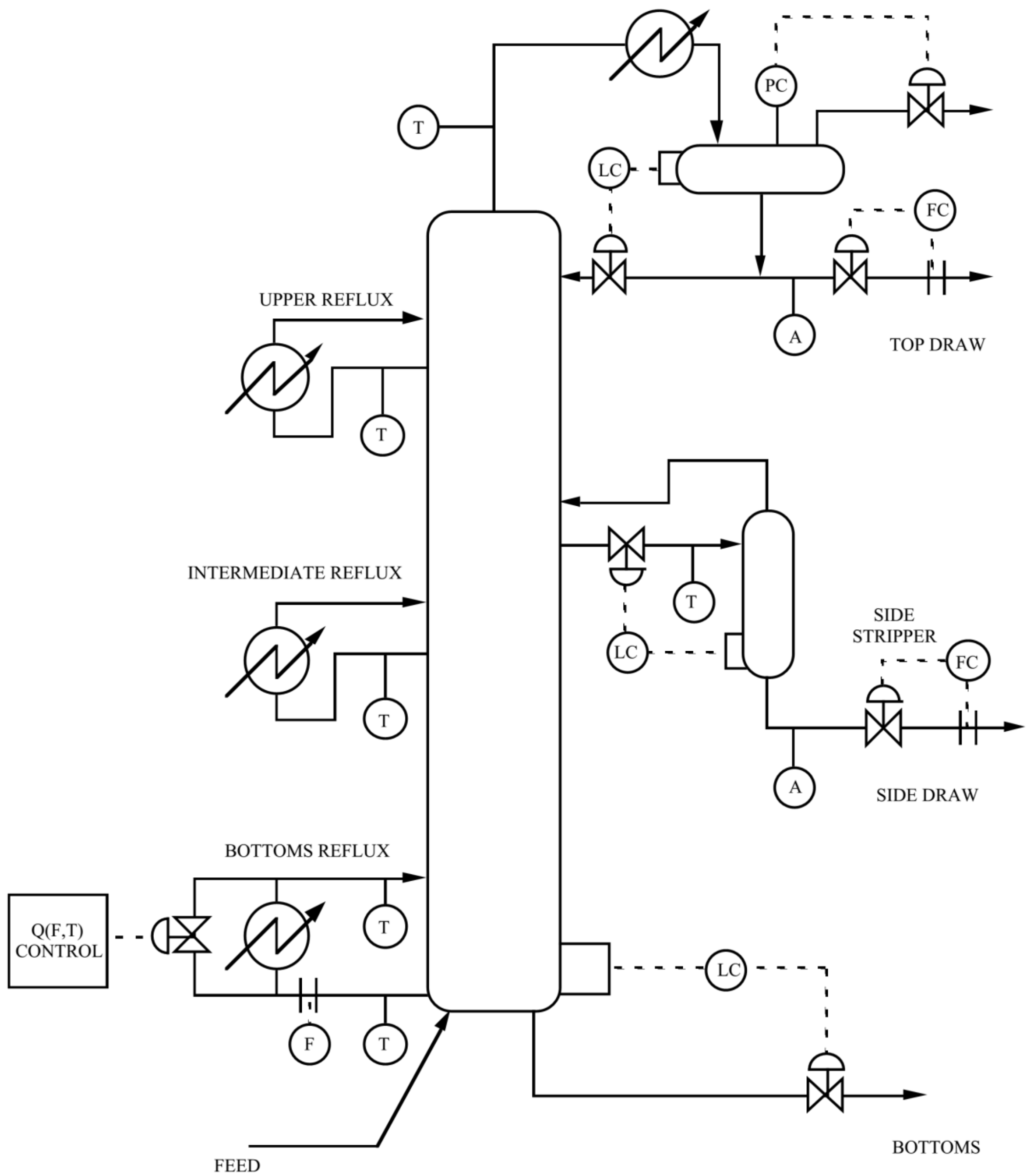


Figure 4.1. Shell Heavy Oil Fractionator Control Problem.

(Adopted from: Prett and Garcia, Fundamental Process Control, Butterworths, 1988.)

4.2.2 Case Study-4: Distillation Column Control Problem

A Doukas and Luyben (DL) distillation column producing a liquid side stream product, shown in Figure 4.2 has been considered in this study for the control configuration selection problem. The column dynamics were originally studied by Doukas and Luyben (1978). The objective was to separate and maintain the composition of the three product streams (i.e., benzene, toluene, and xylene) at their pre-specified values. The concentration of impurities in three product streams are controlled by adjusting three process streams (i.e., side stream flow rate, reflux ratio and reboiler heat duty) as manipulated variables.

The transfer function model of the column expressing the relation between controlled and manipulated variables is shown in Table 4.1.

The transfer function model $G(s)$ of the DL distillation column in simplified form (Grosdidier and Morari, 1986) is given by Eq. (4.4):

$$G(s)=\begin{bmatrix} \dfrac{0.374e^{-7.75s}}{(22.2s+1)^2} & \dfrac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \dfrac{-9.811e^{-1.59s}}{(11.36s+1)} \\ \dfrac{-1.986e^{-0.71s}}{(66.67s+1)^2} & \dfrac{5.24e^{-60s}}{(400s+1)} & \dfrac{5.984e^{-2.24s}}{(14.29s+1)} \\ \dfrac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \dfrac{-0.33e^{-0.68s}}{(2.38s+1)^2} & \dfrac{2.38e^{-0.42s}}{(1.43s+1)^2} \end{bmatrix} \tag{4.4}$$

In the considered DL distillation control problem the transfer function elements are of first order plus time delay and second order plus time delay. Interestingly, the process has large variations in the time constants and time delays from element to element, i.e., the time constants are varying in the range 1 to 400 and time delays are varying in the range 1 to 60 min. Such variations in time constants and time delays will make the process dynamics typical particularly under closed loop interaction.

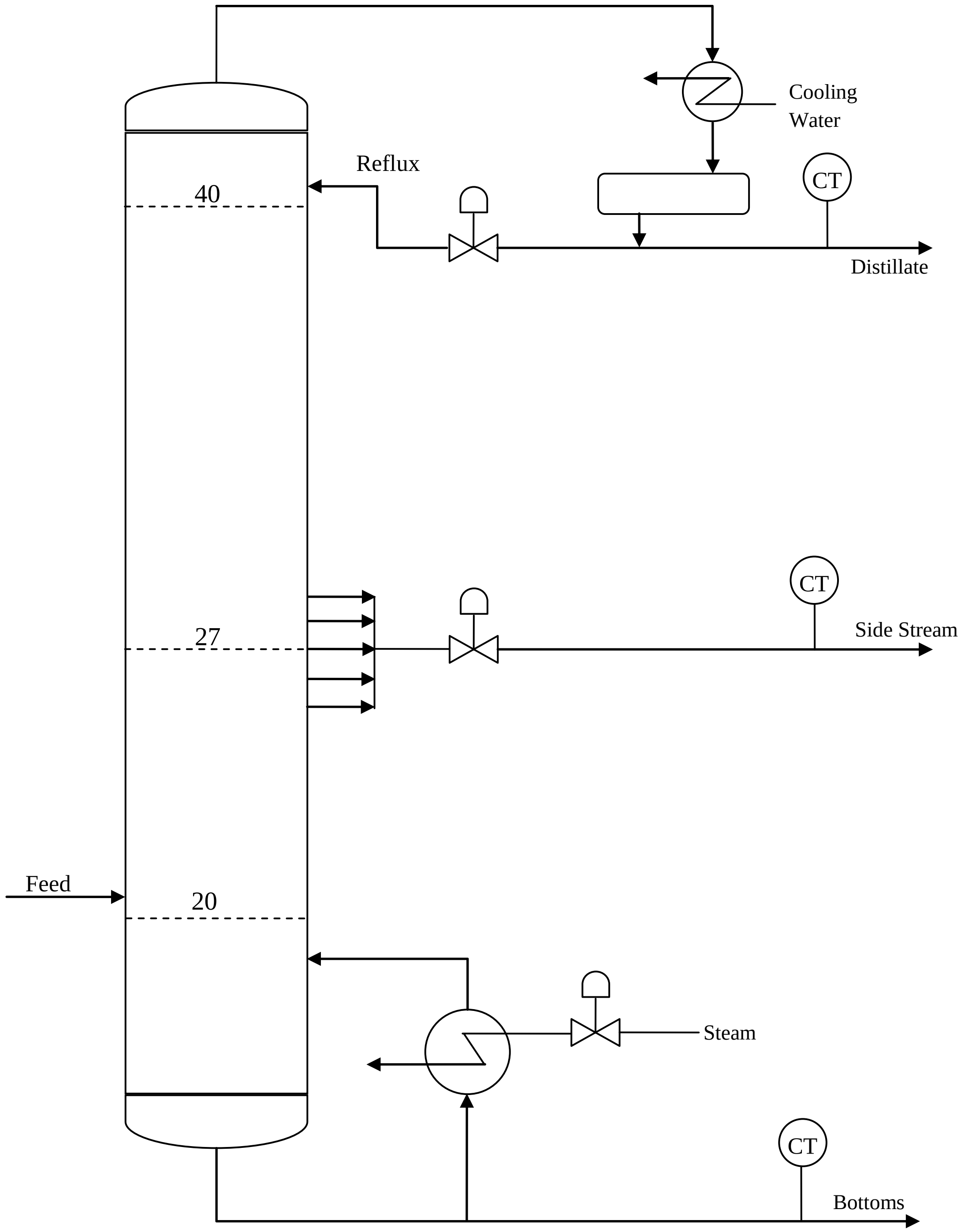


Figure 4.2. Doukas Luyben Distillation Column.

Table 4.1. Process Transfer Function Matrix Elements of DL Distillation Column (case study-4).

Controlled Variables \ Manipulated Variables	**Reflux Ratio (u_1)**	**Side stream flow (u_2)**	**Reboiler heat duty (u_3)**
Toluene impurity in the bottom (y_1)	$\dfrac{0.374e^{-7.75s}}{(22.2s+1)^2}$	$\dfrac{-11.3e^{-3.79s}}{(21.74s+1)^2}$	$\dfrac{-9.811e^{-1.59s}}{(11.36s+1)}$
Toluene impurity in the distillate (y_2)	$\dfrac{-1.986e^{-0.71s}}{(66.67s+1)^2}$	$\dfrac{5.24e^{-60s}}{(400s+1)}$	$\dfrac{5.984e^{-2.24s}}{(14.29s+1)}$
Benzene impurity in the side stream (y_3)	$\dfrac{0.0204e^{-0.59s}}{(7.14s+1)^2}$	$\dfrac{-0.33e^{-0.68s}}{(2.38s+1)^2}$	$\dfrac{2.38e^{-0.42s}}{(1.43s+1)^2}$

CHAPTER - 5

RESULTS AND DISCUSSION: COMPARISON AND IMPLEMENTATION OF PROPOSED TECHNIQUES

In this Chapter, case studies (plant models) introduced in Chapter 4 are analyzed for the best control configuration. First, the input-output pairing analysis is conducted based on conventional steady-state RGA approach. The recommended pairing results are then compared with the input-output pairing obtained from dynamic RGA methods such as: frequency dependent RGA (FDRGA), effective relative gain array (ERGA), effective relative energy array (EREA), and the proposed method "relative response array (RRA)". For verifying the correctness of the results (i.e., verifying the chosen pairing for stability and response speed), the response curve are generated based on IMC/ITAE controller tuning rules for all possible control configurations. All the calculations are performed in Matlab/Simulink. The Matlab codes corresponding to each case study are given in Appendix I - IV, respectively.

The Chapter also presents the sensitivity of RGA analysis (i.e., input-output pairing decision) to parametric uncertainty in each element of the plant model considered in various case studies. The objective is to observe the effect of model uncertainty on pairing decision, i.e., whether the input-output pairing changes within the considered uncertainty range. The objective is also to find the limiting point or maximum tolerable uncertainty for which the input-output pairing does not change. The model uncertainty in the study comprises of uncertainty in process gain, process time constant and time delay. The uncertainty in time constant and time delay are taken care of by a collective term of residence time as defined in Chapter 3.

5.1 Benchmark Test Problems

5.1.1 Case Study-1: 2×2 Process Model with a Second Order Element

Considering the 2×2 process transfer function model introduced in Chapter 4, Section 4.1.1 and given by Eq. (4.1). The steady-state and dynamic RGA analysis of the test problem is conducted in Section 5.1.1.1 and 5.1.1.2, respectively, followed by uncertainty analysis in Section 5.1.1.3.

5.1.1.1 RGA Analysis: Steady-State Approach

For the plant model [Eq. (4.1)], the steady-state gain matrix is given by Eq. (5.1):

$$K = \begin{bmatrix} 5 & 2.5 \\ -4 & 1 \end{bmatrix} \tag{5.1}$$

The steady-state RGA matrix corresponding to the steady-state gain matrix [Eq. (5.1)], is obtained based on Eq. (3.4) as:

$$\Lambda = \begin{bmatrix} 0.3333 & 0.6667 \\ 0.6667 & 0.3333 \end{bmatrix} \tag{5.2}$$

The off-diagonal elements of RGA matrix [Eq. (5.2)] are greater than 0.5 and close to 1. Therefore, the recommended pairing based on steady-state RGA analysis is off-diagonal, i.e., (1-2)/(2-1) output-input pairing.

5.1.1.2 RGA Analysis: Dynamic Approach

<u>Frequency Dependent RGA (FDRGA)</u>

Figure 5.1 shows the frequency dependent RGA values for elements Λ_{11} and Λ_{12} as a function of frequency for the plant model [Eq. (4.1)] based on Eq. (3.28). The diagonal

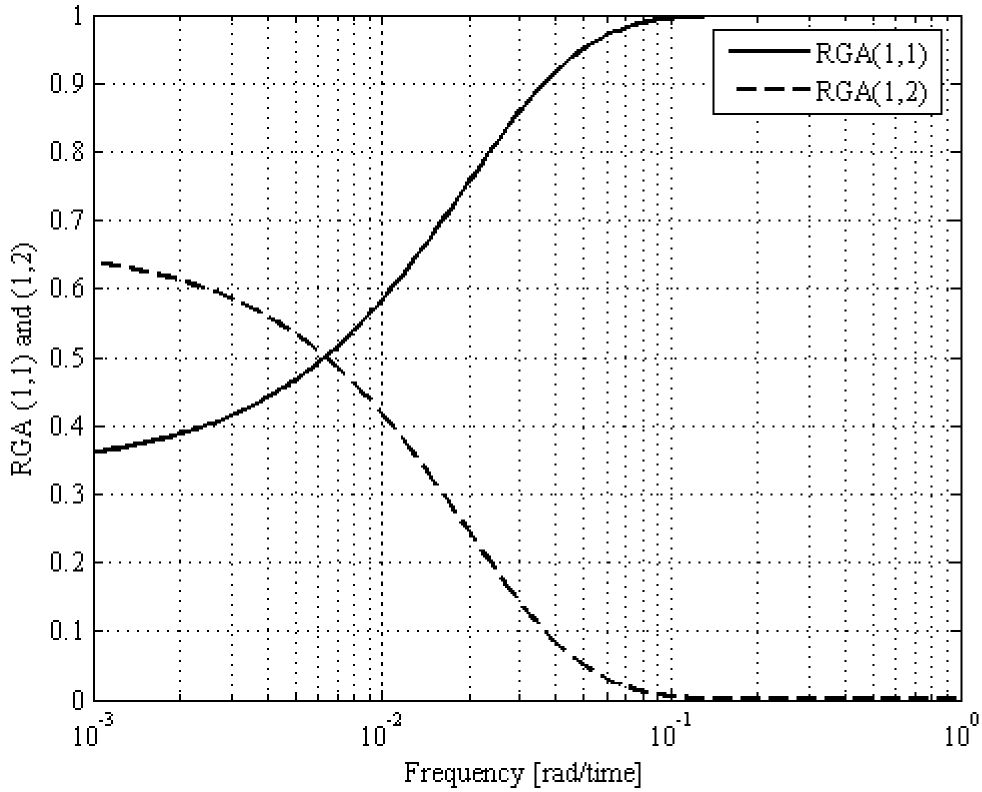


Figure 5.1. Frequency dependent RGA for the diagonal (solid line) and off-diagonal (dashed line) pairing for case study-1 (benchmark test problem 4.1.1).

RGA element, Λ_{11} [i.e., element RGA(1,1) in Figure 5.1] is less than 0.5 at low frequencies and increases to a value of 1 at moderate and high frequencies, whereas off-diagonal RGA element, Λ_{12} [i.e., element RGA(1,2) in Figure 5.1], is close to 0.6 at low frequencies and decreases to 0 at high frequencies. Thus, off-diagonal pairing is favored at low frequencies, but at moderate and high frequencies the diagonal pairing is more favorable. For the given plant model [Eq. (4.1)], most time constants are of the order of 10, which corresponds to the frequency around 0.1 rad/time and in this frequency range, i.e., in the frequency of interest the diagonal RGA elements are greater than 0.5 and close to 1. Therefore, the recommended pairing for the given benchmark test problem is "diagonal". The results pertaining to the frequency dependent RGA are simulated using Matlab. The Matlab code used for the same is given in Appendix-I.

Relative Response Array (RRA) **(Proposed Approach)**

The RRA analysis is conducted based on all four variants for the plant model [Eq. (4.1)] as follows:

i.) Controller-independent time-average RRA (CI-TA-RRA)

The CI-TA-RRA obtained based on Eqs. (3.42) and (3.43) is given by Eq. (5.3):

$$\rho_{CI-TA} = \begin{bmatrix} 0.721 & 0.279 \\ 0.279 & 0.721 \end{bmatrix} \tag{5.3}$$

The diagonal elements of CI-TA-RRA [Eq. (5.3)] are greater than 0.5 and close to 1. Therefore, the recommended variable pairing is "diagonal", i.e., (1-1)/(2-2).

ii.) Controller-independent time-varying RRA (CI-TV-RRA)

Table (5.1) shows the values of CI-TV-RRA elements ρ_{11} and ρ_{12} based on Eqs. (3.44) and (3.45) corresponding to 10 to 100 percentage of the maximum time of response observation, i.e., the summation of dominant time constant and maximum process dead time as discussed in Chapter 3, Section 3.3.3.2. In the whole time range of interest the diagonal elements of CI-TV-RRA are greater than 0.5 and close to 1. Thus, as per RRA pairing rules (Section 3.3), the pairing recommended using CI-TV-RRA analysis is "diagonal".

iii.) Controller-dependent time-average RRA (CD-TA-RRA)

The CD-TA-RRA for the considered benchmark test problem [Eq. (4.1)] is obtained based on Eqs. (3.42), (3.45) and (3.46) is given by Eq. (5.4):

$$\rho_{CD-TA} = \begin{bmatrix} 0.888 & 0.357 \\ 0.357 & 0.881 \end{bmatrix} \tag{5.4}$$

The analysis of CD-TA-RRA [Eq. (5.4)] shows that the diagonal elements are greater than 0.5 and close to 1. Therefore, as per RRA pairing rules (Section 3.3), the recommended input-output variable pairing is "diagonal", i.e., (1-1)/(2-2).

iv.) Controller-dependent time-varying RRA (CD-TV-RRA)

The Table (5.2) shows the values of CD-TV-RRA elements for the given plant model [Eq. (4.1)] based on Eq. (3.41). All the elements of the CD-TV-RRA, i.e., ρ_{11}, ρ_{12}, ρ_{21} and ρ_{22} are calculated in the range of 10 to 100 percentage of the maximum time of response observation, i.e., the sum of dominant time constant and maximum process dead

Table 5.1. Controller-independent time-varying relative response array (CI-TV-RRA) elements for case study-1 (benchmark test problem 4.1.1).

Percentage of $\left(\boldsymbol{\tau}_{\mathbf{D}} + \boldsymbol{\theta}_{\mathbf{max}}\right)$	$\boldsymbol{\rho}_{11}$	$\boldsymbol{\rho}_{12}$
10	1.000	0.000
20	1.000	0.000
30	0.999	0.001
40	0.987	0.013
50	0.958	0.042
60	0.916	0.084
70	0.867	0.133
80	0.816	0.184
90	0.767	0.233
100	0.721	0.279

Table 5.2. Controller-dependent time-varying relative response array (CD-TV-RRA) elements for case study-1 (benchmark test problem 4.1.1).

Percentage of $(\tau_D + \theta_{\max})$	ρ_{11}	ρ_{12}	ρ_{21}	ρ_{22}
10	1.000	0.000	0.000	1.000
20	1.000	0.000	0.000	1.000
30	1.000	0.026	0.052	1.000
40	1.000	0.076	0.122	1.000
50	0.999	0.133	0.184	0.999
60	0.990	0.188	0.238	0.989
70	0.972	0.239	0.285	0.969
80	0.948	0.284	0.324	0.943
90	0.919	0.324	0.358	0.913
100	0.888	0.357	0.386	0.881

time. In the whole time range of interest the diagonal elements ρ_{11} and ρ_{22} of CD-TV-RRA [Table (5.2)] are greater than 0.5 and close to 1, whereas off-diagonal elements ρ_{12} and ρ_{21} are both less than 0.5. Thus, the recommended pairing based on CD-TV-RRA is "diagonal".

For both the CD-TA-RRA and CD-TV-RRA, the closed-loop responses are calculated based on IMC controller "Q", defined in Eq. (3.40). The filter "F" used to make the controller proper is $1/(0.1s+1)^r$, where, "r" is taken to be 1 and 2 for first and second order process elements, respectively. The Matlab codes for the calculation of all versions of RRA are given in Appendix-I.

For the same problem, i.e., for the plant model given by Eq. (4.1), Monshizadeh-Naini et al. (2009) based on the effective relative gain array (ERGA) and effective relative energy array (EREA) analysis, have also obtained "diagonal" pairing.

Closed-loop Performance

Table (5.3) shows the IMC based PI controller settings in terms of function block parameters for Simulink model (Figure 5.2) of the given plant [Eq. (4.1)].

The simulation results in Figure 5.3 indicates the response of output y_1 and y_2 for unit step change in set point $y_{1,sp}$ and $y_{2,sp}$. In order to compare the diagonal and off-diagonal control structures the step change in set point of y_1 and y_2 is given at 5 and 50 time units respectively. As is evident from Figure 5.3, both y_1 and y_2 show the best set point tracking for diagonal pairing and result in stable and faster response in comarison to

Table 5.3. IMC based PI controller settings (function block parameters) for Simulink model of case study-1 (benchmark test problem 4.1.1).

Plant Element	Desired Closed-loop Time Constant, τ_c	Controller Gain, K_c	Integral mode gain, K_c/τ_I
$\frac{5}{4s+1}$	0.6	1.6667	0.4167
$\frac{2.5e^{-5s}}{(2s+1)(15s+1)}$	6	0.4615	0.0308
$\frac{-4e^{-6s}}{20s+1}$	5	– 0.7273	– 0.0364
$\frac{1}{3s+1}$	0.5	6	2

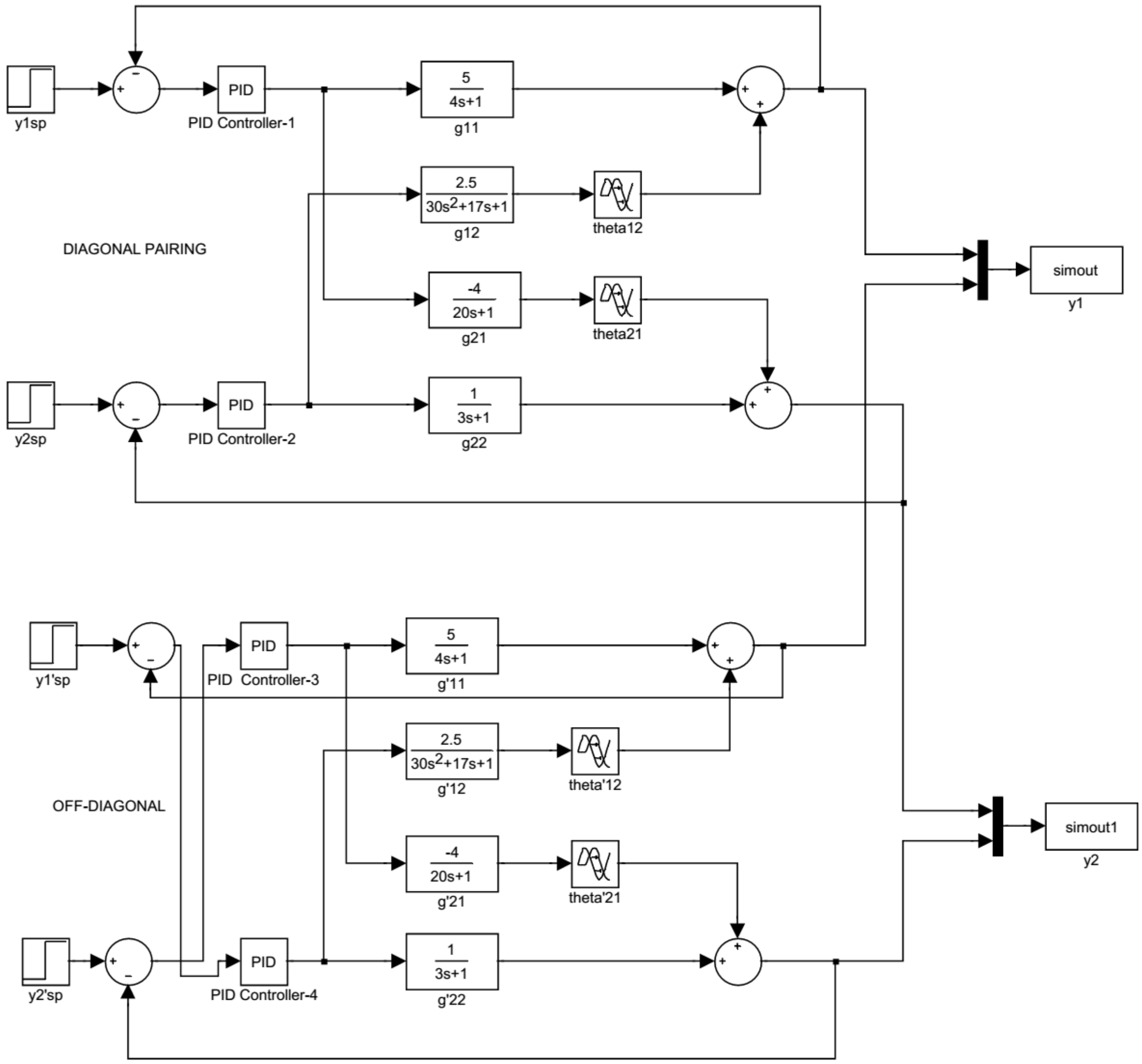
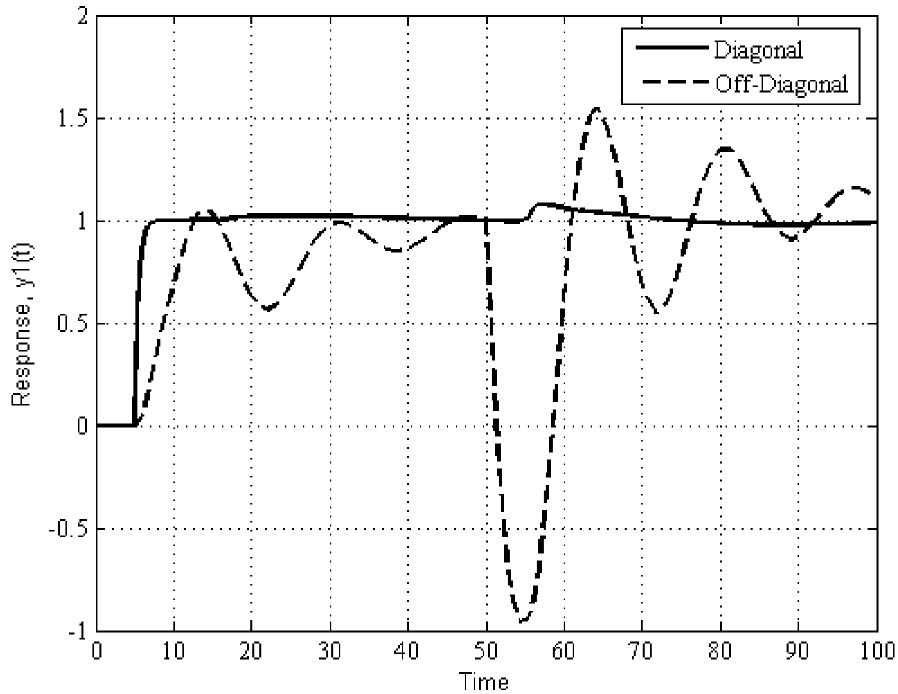
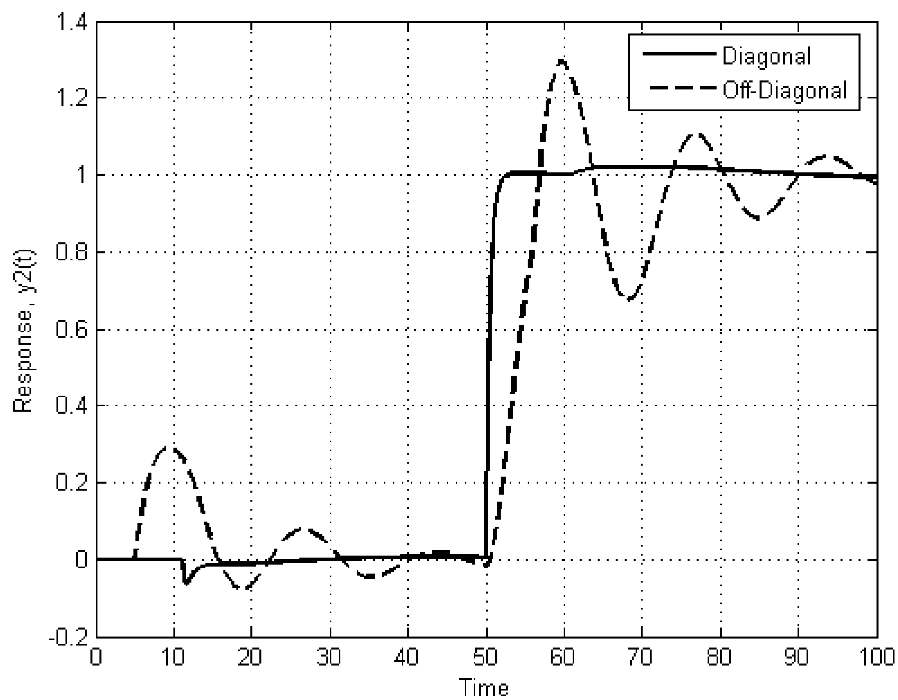


Figure 5.2. Simulink model of diagonal and off-diagonal pairing for case study-1 (benchmark test problem 4.1.1).



(a)



(b)

Figure 5.3. Comparison of diagonal and off-diagonal pairing results for output (a) y_1 and (b) $\boldsymbol{y}_2$ in case study-1 (benchmark test problem 4.1.1).

off-diagonal pairing, which keeps oscillating and has large settling time.

For the benchmark test problem 4.1.1 [Eq. (4.1)], the steady-state RGA analysis [Eq. (5.2)] recommends "off-diagonal" pairing, whereas all the dynamic RGA methods suggest "diagonal" pairing. The closed-loop performance analysis of diagonal and off-diagonal pairing clearly shows (Figure 5.3) the diagonal pairing as the best pairing. The similar conclusion is drawn by Grosdidier and Morari (1986) based on magnitude and phase characteristics of the model. Thus, it can be concluded that the steady-state RGA fails to identify the correct control configuration. However, dynamic RGA methods and proposed RRA method are successful in finding the best control configuration. Also, for the recommended "diagonal" pairing, the Niederlinski index, NI (Niederlinski, 1971) given by Eq. (3.8) comes out to be 3, which indicates that the recommended (diagonal) pairing is stable. The summary of the input-output pairing results obtained using steady-state and dynamic RGA methods is presented in Table (5.4).

5.1.1.3 Uncertainty Analysis: Steady-state and Dynamic Approach

For the plant model given by Eq. (4.1), there is only one negative element. Therefore, exponent n in Eq. (3.53) is 1. The steady-state gain matrix and its corresponding RGA matrix for the model [Eq. (4.1)] are given in Eqs. (5.1) and (5.2) respectively. The steady-state interaction quotient $\hat{k} = -2$ (for $n = 1$) is obtained based on Eq. (3.53) by replacing the normalized gain with the steady-state gain.

The average residence time corresponding to each element of the plant model [Eq. (4.1)] is given in Eq. (5.6):

Table 5.4. Comparison of pairing recommendation by various methods for case study-1 (benchmark test problem 4.1.1).

<table>
<tr><th>Plant Model</th><th colspan="2">Pairing Method</th><th>Resulting Array</th><th>Recommended Pairing</th></tr>
<tr><td rowspan="9">$G(s)=\begin{bmatrix} \dfrac{5}{4s+1} & \dfrac{2.5e^{-5s}}{(2s+1)(15s+1)} \\ \dfrac{-4e^{-6s}}{20s+1} & \dfrac{1}{3s+1} \end{bmatrix}$</td><td colspan="2">RGA</td><td>$\Lambda=\begin{bmatrix}0.3333 & 0.6667\\0.6667 & 0.3333\end{bmatrix}$</td><td>Off-diagonal, i.e., (1-2)/(2-1)</td></tr>
<tr><td colspan="2">ERGA</td><td>$\Lambda=\begin{bmatrix}0.9265 & 0.0735\\0.0735 & 0.9265\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">EREA</td><td>$EREA=\begin{bmatrix}0.8631 & 0.1369\\0.1369 & 0.8631\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">Frequency-dependent DRGA</td><td>Figure 5.1</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td rowspan="4">RRA (Proposed Method)</td><td>CI-TA-RRA</td><td>$\rho_{CI-TA}=\begin{bmatrix}0.721 & 0.279\\0.279 & 0.721\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CI-TV-RRA</td><td>Table 5.1</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TA-RRA</td><td>$\rho_{CD-TA}=\begin{bmatrix}0.888 & 0.357\\0.357 & 0.881\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TV-RRA</td><td>Table 5.2</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">Closed-loop response analysis</td><td>Figure 5.3</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
</table>

$$\tau_{ar,11} = 4;\ \ \tau_{ar,12} = 22;\ \ \tau_{ar,21} = 26;\ \ \tau_{ar,22} = 3 \tag{5.6}$$

Based on steady-state gain given in Eq. (5.1) and their corresponding average residence times from Eq. (5.6), the normalized gain matrix is calculated using Eqs. (3.48) and (3.49) for first and second order process elements, respectively, as given in Eq. (5.7):

$$K_N = \begin{bmatrix} 1.2500 & 0.1136 \\ 0.1538 & 0.3333 \end{bmatrix} \tag{5.7}$$

The normalized interaction quotient based on normalized gain information from Eq. (5.7) is obtained using Eq. (3.53) as $\hat{K}_N = -0.0420$.

The uncertainty in parameters is assumed to vary in the range $\pm 1\%$ to $\pm 50\%$. For simplicity of understanding the extent of uncertainty in gain (α) and residence time (β), of the process have been considered to be same, i.e., $\alpha = \beta$.

Table (5.5) shows the lower and upper bounds on interaction quotient for three different cases: (i) Considering uncertainty only in process gains, under steady-state framework; (ii) Considering uncertainty only in process gains, under dynamic framework; and (iii) Considering uncertainty in both process gains and residence times of the process.

Now, corresponding to the upper and lower bound on interaction quotient given in Table (5.5), the upper and lower bound on RGA element λ_{11} for the above three cases is given in Table (5.6), and the following conclusions can be drawn:

i.) The RGA element λ_{11} for the specific case of uncertainty in steady-state gains only, under steady-state framework remains less than 0.5 up to an uncertainty of 16.7%. This suggests that the pairing upto an uncertainty of

Table 5.5. Lower and upper bounds on interaction quotient under uncertainty for case study-1 (benchmark test problem 4.1.1).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	**Interaction Quotient (For uncertain gain under Steady-state framework)**		**Interaction Quotient (For uncertain gain under dynamic framework)**		**Interaction Quotient (For uncertain gain and residence time under dynamic framework)**	
	Lower bound $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound** $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound** $\hat{k}^l$	**Upper bound** $\hat{k}^h$
1	-2.0816	-1.9216	-0.0437	-0.0403	-0.0455	-0.0387
5	-2.4444	-1.6364	-0.0513	-0.0343	-0.0629	-0.0280
10	-3.0000	-1.3333	-0.0629	-0.0280	-0.0979	-0.0180
15	-3.7143	-1.0769	-0.0779	-0.0226	-0.1678	-0.0105
20	-4.6667	-0.8571	-0.0979	-0.0180	-0.3776	-0.0047
25	-6.0000	-0.6667	-0.1259	-0.0140	$-\infty$	0
30	-8.0000	-0.5000	-0.1678	-0.0105	0.0038	0.4615
35	-11.3333	-0.3529	-0.2378	-0.0074	0.0070	0.2517
40	-18.0000	-0.2222	-0.3776	-0.0047	0.0097	0.1818
45	-38.0000	-0.1053	-0.7972	-0.0022	0.0120	0.1469
50	$-\infty$	0	$-\infty$	0	0.0140	0.1259

Table 5.6. Lower and upper bound on RGA element λ_{11} under uncertainty for case study-1 (benchmark test problem 4.1.1).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	**Lower and Upper bound on RGA element, λ_{11}**		
	For uncertain gain under Steady-state (Chen and Seborg, 2002)	**For uncertain gain under dynamic state (Proposed Approach)**	**For uncertain gain and residence time (Proposed Approach)**
1	$0.3245 \le \lambda_{11} \le 0.3423$	$0.9582 \le \lambda_{11} \le 0.9612$	$0.9565 \le \lambda_{11} \le 0.9627$
5	$0.2903 \le \lambda_{11} \le 0.3793$	$0.9512 \le \lambda_{11} \le 0.9668$	$0.9408 \le \lambda_{11} \le 0.9728$
10	$0.2500 \le \lambda_{11} \le 0.4286$	$0.9408 \le \lambda_{11} \le 0.9728$	$0.9108 \le \lambda_{11} \le 0.9823$
15	$0.2121 \le \lambda_{11} \le 0.4815$	$0.9277 \le \lambda_{11} \le 0.9779$	$0.8563 \le \lambda_{11} \le 0.9896$
20	$0.1765 \le \lambda_{11} \le 0.5385$	$0.9108 \le \lambda_{11} \le 0.9823$	$0.7259 \le \lambda_{11} \le 0.9954$
25	$0.1429 \le \lambda_{11} \le 0.6000$	$0.8882 \le \lambda_{11} \le 0.9862$	$0 \le \lambda_{11} \le 1$
30	$0.1111 \le \lambda_{11} \le 0.6667$	$0.8563 \le \lambda_{11} \le 0.9896$	$1.0038 \le \lambda_{11} \le 1.8571$
35	$0.0811 \le \lambda_{11} \le 0.7391$	$0.8079 \le \lambda_{11} \le 0.9927$	$-\infty \le \lambda_{11} \le 1.0070$ & $1.3364 \le \lambda_{11} \le \infty$
40	$0.0526 \le \lambda_{11} \le 0.8182$	$0.7259 \le \lambda_{11} \le 0.9954$	$-\infty \le \lambda_{11} \le 1.0098$ & $1.2222 \le \lambda_{11} \le \infty$
45	$0.0256 \le \lambda_{11} \le 0.9048$	$0.5564 \le \lambda_{11} \le 0.9978$	$-\infty \le \lambda_{11} \le 1.0121$ & $1.1721 \le \lambda_{11} \le \infty$
50	$0 \le \lambda_{11} \le 1$	$0 \le \lambda_{11} \le 1$	$1.0142 \le \lambda_{11} \le 1.1440$

16.7% should correspond to the off-diagonal elements λ_{12} and λ_{21}. However, for the uncertainty above 16.7% the upper range of λ_{11} begins to exceed 0.5 and varies in the range 0-1 at 50% uncertainty. This indicates the increase in process interaction and the off-diagonal pairing may not be able to perfectly control the process.

ii.) For case-ii, i.e., considering uncertainty in steady-state gains only under dynamic framework, the RGA element λ_{11} found to be greater than 0.5 and less than 1 for uncertainty up to 46%. Above this limiting uncertainty the lower limit of λ_{11} goes below 0.5 and the interaction effects begins to increase and the recommended pairing may not be able to control the process at its best.

iii.) For uncertainty in both the steady-state gain and residence time (i.e., time constant and time delay) under dynamic framework (case-iii), the RGA element λ_{11} remains greater than 0.5 upto an uncertainty of 23%. Beyond which the interaction effects increases and at an uncertainty of 35% the diagonal element λ_{11} passes through discountinuity.

It can be concluded based on uncertainty analysis for above three cases that the pairing analysis must be conducted under dynamic framework because the steady-state analysis lead to incorrect variable pairing. Further, the tolerable uncertainty in process gain increases from 16.7% in steady-state analysis to 46% in dynamic. However, with uncertainty in all the model paramters (viz. process gain, time constant and time delay) the tolerable uncertainty for chosen control configuration to remain invariant decreases to 23%. Thus, sensitivity analysis of control configuration can be misleading if uncertainty is considered in process gain alone.

5.1.2 Case Study-2: 2×2 Process Model with Typical Process Dynamics

Considering a 2×2 process model developed based on material balance for a distillation tower, as discussed in Chapter 4, Section 4.1.2 and given by Eq. (4.2). For the considered model [Eq. (4.2)], the steady-state and dynamic RGA analysis is conducted, followed by the uncertainty analysis.

5.1.2.1 RGA Analysis: Steady-state Approach

For the process model [Eq. (4.2)], the steady-state gain matrix is given by Eq. (5.8):

$$K = \begin{bmatrix} -0.805 & 0.055 \\ -0.465 & -0.055 \end{bmatrix} \tag{5.8}$$

The corresponding steady-state RGA matrix (Bristol, 1966) is obtained based on Eq. (3.4) and is given by Eq. (5.9):

$$\Lambda = \begin{bmatrix} 0.6339 & 0.3661 \\ 0.3661 & 0.6339 \end{bmatrix} \tag{5.9}$$

The diagonal elements of the RGA matrix [Eq. (5.9)] are greater than 0.5 and close to 1. Thus, the recommended pairing based on steady-state RGA analysis is "diagonal", i.e., $(1-1)/(2-2)$ variable pairing. In the following section, the RGA analysis is conducted based on various dynamic RGA methods discussed in Chapter 3.

5.1.2.2 RGA Analysis: Dynamic Approach

<u>Effective Relative Gain Array (ERGA)</u>

The ERGA method (Xiong et al., 2005) utilizes both the steady-state gain and bandwidth information of the plant model for selecting the best control configuration. The bandwidth matrix as defined in Eq. (3.15), for the considered plant model [Eq. (4.2)] is given in Eq. (5.10):

$$\Omega = \begin{bmatrix} 0.0988 & 0.3727 \\ 0.2387 & 0.3030 \end{bmatrix} \tag{5.10}$$

The corresponding effective gain matrix is obtained by substituting steady-state gain and bandwidth information from Eqs. (5.8) and (5.10) respectively, into Eq. (3.14) as:

$$E = \begin{bmatrix} -0.0795 & 0.0205 \\ -0.1110 & -0.0167 \end{bmatrix} \tag{5.11}$$

Based on effective gain matrix [Eq. (5.11)], the ERGA matrix is calculated using Eq. (3.15), and is given by Eq. (5.12):

$$\Phi = \begin{bmatrix} 0.3681 & 0.6319 \\ 0.6319 & 0.3681 \end{bmatrix} \tag{5.12}$$

The ERGA follows the same pairing rules as the RGA, therefore the control loop variable pairing will correspond to the positive ERGA elements greater than 0.5. For model under consideration [Eq. (4.2)] the best pairing corresponds to "off-diagonal" elements, i.e., $(1-2)/(2-1)$.

Effective Relative Energy Array (EREA)

The effective gain matrix for EREA analysis (Monshizadeh-Naini et al., 2009) is calculated based on Eq. (3.24), and is given in Eq. (5.13):

$$E^* = \begin{bmatrix} -0.0640 & 0.0011 \\ -0.0516 & -0.0009 \end{bmatrix} \tag{5.13}$$

Based on the effective gain matrix given by Eq. (5.13), the EREA matrix is obtained using Eq. (3.26) as given by Eq. (5.14):

$$EREA = \begin{bmatrix} 0.5021 & 0.4979 \\ 0.4979 & 0.5021 \end{bmatrix} \tag{5.14}$$

The EREA matrix defined by Eq. (3.26) has same form as that of the RGA defined by Eq. (3.4), it thus follows same pairing rules as applicable to RGA. The EREA matrix [Eq. (5.14)] shows that, both the diagonal and off-diagonal elements are close to 0.5, i.e., for both the possible control configurations, the closed-loop system will suffer from high degree of interaction. In such a situation, no precise pairing could be concluded.

Frequency dependent RGA (FDRGA)

Figure 5.4 shows the frequency dependent RGA (Seider et al., 2009) values for elements Λ_{11} and Λ_{12} as a function of frequency for the given plant model [Eq. (4.2)] based on Eq. (3.28).

It is clearly observable from Figure 5.4 that the diagonal element Λ_{11} $(=\Lambda_{22})$ is greater than 0.5 in the whole frequency range, whereas its off-diagonal counterpart remains less than 0.5. Therefore, it can be concluded that as per the frequency dependent

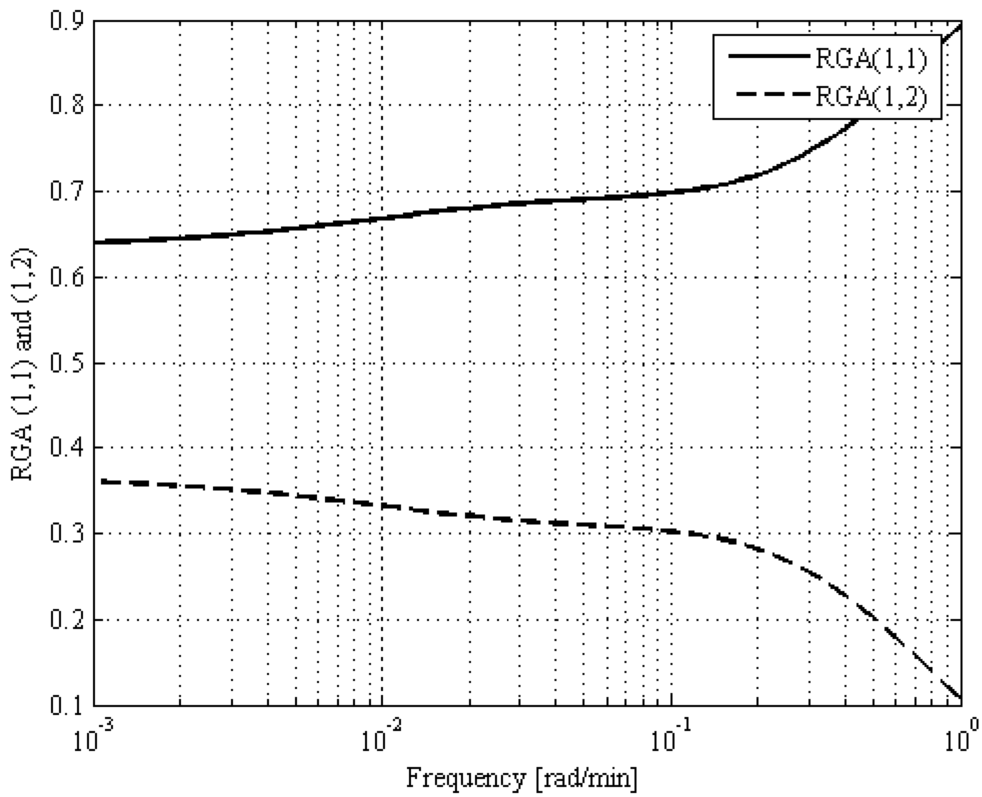


Figure 5.4. Dynamic RGA for the diagonal (solid line) and off-diagonal (dashed line) pairing for case study-2 (benchmark test problem 4.1.2).

RGA the "diagonal" pairing is best suited for this model [Eq. (4.2)]. All the calculations of frequency dependent RGA are conducted in Matlab and the corresponding Matlab code is given in Appendix-II.

Relative Response Array (RRA)

The pairing analysis would now be conducted based on all four variants of the proposed RRA method:

i.) Controller-independent time-average RRA (CI-TA-RRA)

The CI-TA-RRA for the distillation column benchmark test problem [Eq. (5.8)] based on Eqs. (3.42) and (3.43) is given in Eq. (5.15):

$$\rho_{CI-TA} = \begin{bmatrix} 0.661 & 0.339 \\ 0.339 & 0.661 \end{bmatrix} \tag{5.15}$$

The diagonal elements of CI-TA-RRA [Eq. (5.15)] are greater than 0.5 and close to 1, whereas both off-diagonal elements are less than 0.5. Therefore, the recommended variable pairing is (1-1)/(2-2), i.e., "diagonal".

ii.) Controller-independent time-varying RRA (CI-TV-RRA)

Table (5.7) shows the values of CI-TV-RRA elements ρ_{11} and ρ_{12} based on Eqs. (3.43) and (3.44) corresponding to 10 to 100 percentage of the maximum time period of response observation, i.e., the sum of dominant time constant and maximum process dead time. In the whole time range of interest, the diagonal elements of CI-TV-RRA are greater than 0.5 and close to 1, thus the recommended pairing is "diagonal".

Table 5.7. Controller-independent time-varying relative response array (CI-TV-RRA) for case study-2 (benchmark test problem 4.1.2).

Percentage of $(\tau_D + \theta_{\max})$	ρ_{11}	ρ_{12}
10	0.878	0.122
20	0.753	0.247
30	0.703	0.297
40	0.681	0.319
50	0.670	0.330
60	0.665	0.335
70	0.662	0.338
80	0.661	0.339
90	0.661	0.339
100	0.661	0.339

iii.) Controller-dependent time-average RRA (CD-TA-RRA)

For the distillation column benchmark problem given by Eq. (4.2), the CD-TA-RRA is obtained using Eqs. (3.42), (3.45) and (3.46) as:

$$\rho_{CD-TA} = \begin{bmatrix} 0.697 & 0.329 \\ 0.318 & 0.696 \end{bmatrix} \tag{5.16}$$

The analysis of CD-TA-RRA [Eq. (5.17)] shows that the pairing corresponding to the elements greater than 0.5 and close to 1lies with diagonal elements, i.e., (1-1)/(2-2) variable pairing is recommended.

iv.) Controller-dependent time-varying RRA (CD-TV-RRA)

Table (5.8) shows the values of CD-TV-RRA elements calculated for the distillation column benchmark test problem [Eq. (5.8)] based on Eq. (3.41). All the elements of the CD-TV-RRA, i.e., ρ_{11}, ρ_{12}, ρ_{21} and ρ_{22} are calculated in the range of 10 to 100 percentage of the maximum time of response observation, i.e., the summation of dominant time constant and maximum process dead time. In the whole time range of interest the diagonal elements ρ_{11} and ρ_{22} of CD-TV-RRA are greater than 0.5 and close to 1. Thus, as per RRA pairing rules (Section 3.3), the recommended pairing is "diagonal".

For both the CD-TA-RRA and CD-TV-RRA, the closed-loop response are calculated based on IMC controller "Q" defined in Eq. (3.40). The filter "F" used to make the controller proper is $1/(0.1s+1)^r$, where, "r" is taken to be 1 and 2 for first and second order process elements respectively.

Table 5.8. Controller-dependent time-varying relative response array (CD-TV-RRA) for distillation column case study-2 (benchmark test problem 4.1.2).

Percentage of $(\tau_D + \theta_{\max})$	ρ_{11}	ρ_{12}	ρ_{21}	ρ_{22}
10	0.951	0.328	0.168	0.935
20	0.836	0.348	0.268	0.799
30	0.772	0.353	0.301	0.739
40	0.739	0.351	0.314	0.714
50	0.721	0.348	0.319	0.703
60	0.710	0.344	0.320	0.699
70	0.704	0.340	0.320	0.697
80	0.701	0.336	0.320	0.696
90	0.698	0.332	0.319	0.696
100	0.697	0.329	0.318	0.696

Closed-loop Performance

Table (5.9) shows the IMC based PI controller settings in terms of function block parameters for Simulink model (Figure 5.5) of the distillation column benchmark test problem [Eq. (4.2)].

The simulation results in Figure 5.6 indicates the response of output y_1 and y_2 for unit step change in set point $y_{1,sp}$ and $y_{2,sp}$. In order to compare the diagonal and off-diagonal control structures the step change in set point of y_1 and y_2 is given at 10 and 150 time units respectively. It can precisely be concluded from Figure 5.6, that the diagonal pairing gives the superior closed-loop performance, i.e., oscillation free better set point tracking in comparison to the off-diagonal pairing which has larger overshoot and higher settling time. Thus, the variable pairing corresponding to the diagonal elements should be a preferred choice, i.e., (1-1)/(2-2) variable pairing is the best pairing for the given plant model [Eq. (4.2)].

For the benchmark test problem 4.1.2 [Eq. (4.2)], the steady-state RGA analysis and frequency dependent RGA both recommends "diagonal" pairing, whereas the

Table 5.9. IMC based PI controller settings (function block parameters) for Simulink model of case study-2 (benchmark test problem 4.1.2).

Plant Element	Desired Closed-loop Time Constant, τ_c	Controller Gain, K_c	Integral mode gain, K_c/τ_I
$\dfrac{-0.805}{(18.3s+1)(5.6s+1)}$	6	-1.9597	-0.1071
$\dfrac{0.055}{(5.76s+1)(1.25s+1)}$	1.5	38.0826	6.6116
$\dfrac{-0.465e^{-0.3s}}{(28.3s+1)(0.62s+1)}$	3	-15.5256	-0.5486
$\dfrac{-0.055}{3.3s+1}$	0.5	-120	-36.3636

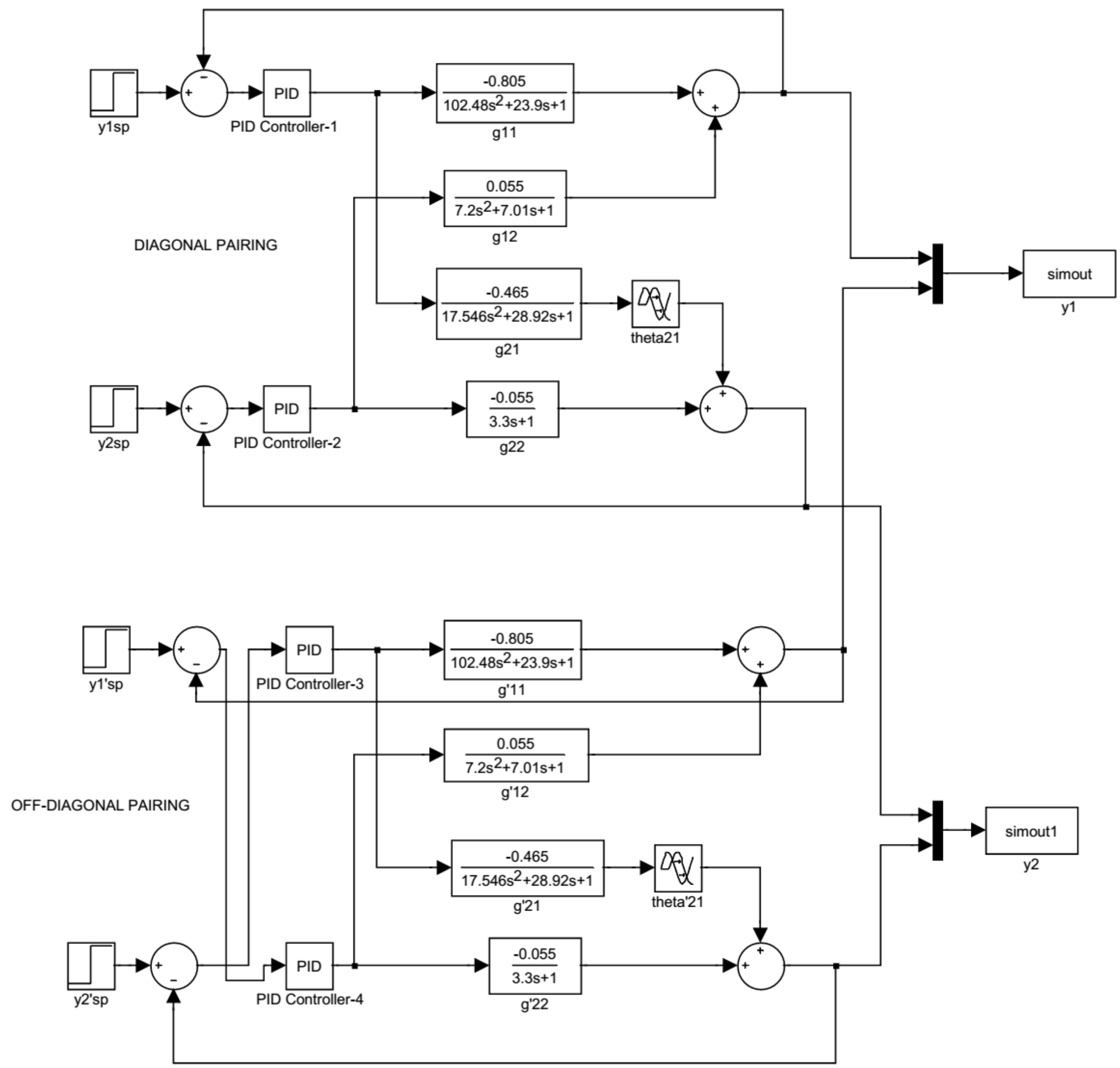


Figure 5.5. Simulink model of diagonal and off-diagonal pairing for case study-2 (benchmark test problem 4.1.2).

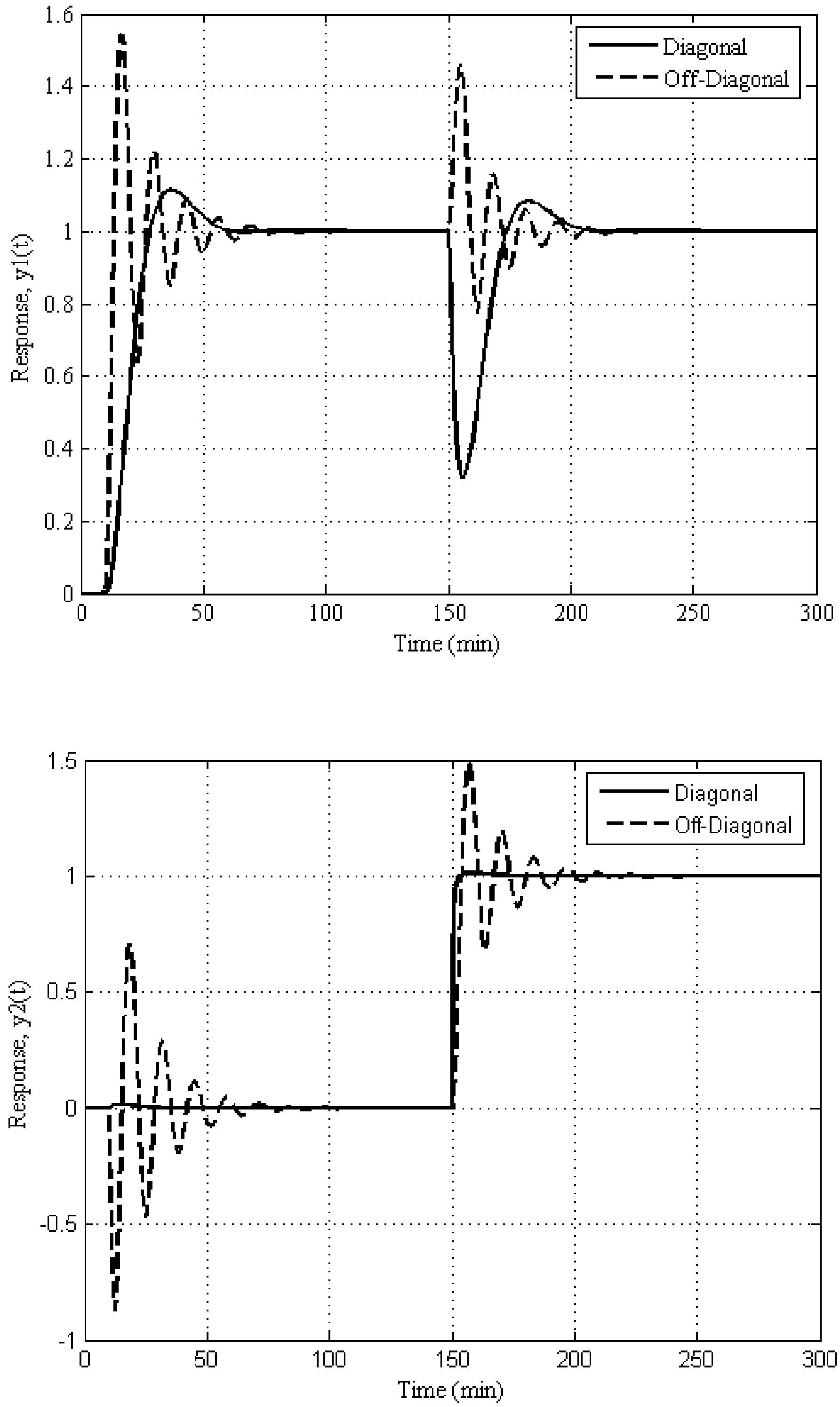


Figure 5.6. Comparison of diagonal and off-diagonal pairing results for output (a) y_1 and (b) $\boldsymbol{y_2}$ in distillation column case study-2 (benchmark test problem 4.1.2).

bandwidth dependent ERGA method gives incorrect pairing results, i.e., recommends "off-diagonal" pairing and effective energy based EREA method could not conclude to any suitable pairing as both its diagonal and off-diagonal elements are close to 0.5. However, the proposed dynamic method relative response array (RRA) and all its versions successfully identifies the high performing "diagonal" pairing. The results are verified based on the closed-loop performance analysis of diagonal and off-diagonal pairing which clearly shows the superiority of diagonal pairing over its off-diagonal counterpart. Also, for the recommended "diagonal" pairing the Niederlinski index, NI (Niederlinski, 1971) given by Eq. (3.5) comes out to be 0.187 (positive), that indicates the stability of recommended (diagonal) pairing.

The summary of pairing analysis results for distillation column benchmark test problem 4.1.2 [Eq. (4.2)] by steady-state and dynamic RGA methods are given in Table (5.10).

5.1.2.3 Uncertainty Analysis: Steady-state and Dynamic Approach

For the distillation column benchmark problem given by Eq. (4.2), there are three negative element, therefore, exponent n in Eq. (3.53) is 3. The steady-state gain matrix and its corresponding RGA matrix for the distillation column model [Eq. (4.2)] are given in Eqs. (5.8) and (5.9) respectively. The steady-state interaction quotient $\hat{k} = -0.5776$ (for $n = 3$) can be obtained based on Eq. (3.53) by replacing normalized gain with the steady-state gain. The average residence time corresponding to each element of the distillation column benchmark problem [Eq. (4.2)] are:

Table 5.10. Comparison of pairing recommendation by various methods for case study-2 (benchmark test problem 4.1.2).

<table>
<tr><th>Plant Model</th><th colspan="2">Pairing Method</th><th>Resulting Array</th><th>Recommended Pairing</th></tr>
<tr><td rowspan="9">$$G(s)=\begin{bmatrix} \dfrac{-0.805}{(18.3s+1)(5.6s+1)} & \dfrac{0.055}{(5.76s+1)(1.25s+1)} \\ \dfrac{-0.465e^{-0.3s}}{(28.3s+1)(0.62s+1)} & \dfrac{-0.055}{3.3s+1} \end{bmatrix}$$</td><td colspan="2">RGA</td><td>$\Lambda=\begin{bmatrix}0.6339 & 0.3661\\0.3661 & 0.6339\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">ERGA</td><td>$\Phi=\begin{bmatrix}0.3681 & 0.6319\\0.6319 & 0.3681\end{bmatrix}$</td><td>Off-Diagonal, i.e., (1-2)/(2-1)</td></tr>
<tr><td colspan="2">EREA</td><td>$EREA=\begin{bmatrix}0.5021 & 0.4979\\0.4979 & 0.5021\end{bmatrix}$</td><td>No unique pairing</td></tr>
<tr><td colspan="2">Frequency-dependent DRGA</td><td>Figure 5.4</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td rowspan="4">RRA
(Proposed Method)</td><td>CI-TA-RRA</td><td>$\rho_{CI-TA}=\begin{bmatrix}0.661 & 0.339\\0.339 & 0.661\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CI-TV-RRA</td><td>Table 5.7</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TA-RRA</td><td>$\rho_{CD-TA}=\begin{bmatrix}0.697 & 0.329\\0.318 & 0.696\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TV-RRA</td><td>Table 5.8</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">Closed-loop response analysis</td><td>Figure 5.6</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
</table>

$$\tau_{ar,11} = 23.9;\ \ \tau_{ar,12} = 7.01;\ \ \tau_{ar,21} = 29.22;\ \ \tau_{ar,22} = 3.3 \tag{5.17}$$

Based on steady-state gain given in Eq. (5.9) and their corresponding average residence times from Eq. (5.17), the normalized gain matrix is calculated using Eqs. (3.48) and (3.49) for first and second order process elements respectively as given in Eq. (5.18):

$$K_N = \begin{bmatrix} 0.0337 & 0.0078 \\ 0.0159 & 0.0167 \end{bmatrix} \tag{5.18}$$

The normalized interaction quotient based on normalized gain information from Eq. (5.18) is obtained using Eq. (3.53) as $\hat{K}_N = -0.2224$.

The uncertainty in parameters is assumed to vary in the range $\pm 1\%$ to $\pm 50\%$. For simplicity of understanding the extent of uncertainty in gain and residence time of the process have been considered to be same, i.e., $\alpha = \beta$.

Table (5.11) shows the lower and upper bounds on interaction quotient for three different cases:

i.) Considering uncertainty only in process gains, under steady-state framework;

ii.) Considering uncertainty only in process gains, under dynamic framework;

iii.) Considering uncertainty in both process gains and residence times of the process.

Now, corresponding to the upper and lower bound on interaction quotient, the upper and lower bound on RGA element λ_{11} for the above three cases is given shown in Table (5.12).

Table (5.12) compares the outcome of uncertainty analysis of steady-state and dynamic RGA for distillation column benchmark problem 4.1.2. The following observations can be made:

Table 5.11. Lower and upper bounds on interaction quotient under uncertainty case study-2 (benchmark test problem 4.1.2).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	**Interaction Quotient (For uncertain gain under Steady-state framework)**		**Interaction Quotient (For uncertain gain under dynamic framework)**		**Interaction Quotient (For uncertain gain and residence time under dynamic framework)**	
	Lower bound $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound,** $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound** $\hat{k}^l$	**Upper bound** $\hat{k}^h$
1	-0.6012	-0.5550	-0.2315	-0.2137	-0.2410	-0.2053
5	-0.7060	-0.4726	-0.2718	-0.1820	-0.3336	-0.1483
10	-0.8665	-0.3851	-0.3336	-0.1483	-0.5190	-0.0953
15	-1.0728	-0.3110	-0.4131	-0.1198	-0.8897	-0.0556
20	-1.3478	-0.2476	-0.5190	-0.0953	-2.0018	-0.0247
25	-1.7329	-0.1925	-0.6673	-0.0741	-∞	0
30	-2.3106	-0.1444	-0.8897	-0.0556	0.0202	2.4466
50	-∞	0	-∞	0	0.0741	0.6673

Table 5.12. Lower and upper bound on RGA element λ_{11} under uncertainty for case study-2 (benchmark test problem 4.1.2).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	**Lower and Upper bound on RGA element λ_{11}**		
	For uncertain gain under Steady-state (Chen and Seborg, 2002)	**For uncertain gain under dynamic state (Proposed Approach)**	**For uncertain gain and residence time (Proposed Approach)**
1	$0.6245 \le \lambda_{11} \le 0.6431$	$0.8120 \le \lambda_{11} \le 0.8239$	$0.8058 \le \lambda_{11} \le 0.8297$
5	$0.5862 \le \lambda_{11} \le 0.6791$	$0.7863 \le \lambda_{11} \le 0.8460$	$0.7498 \le \lambda_{11} \le 0.8709$
10	$0.5358 \le \lambda_{11} \le 0.7220$	$0.7498 \le \lambda_{11} \le 0.8709$	$0.6583 \le \lambda_{11} \le 0.9130$
15	$0.4824 \le \lambda_{11} \le 0.7628$	$0.7077 \le \lambda_{11} \le 0.8930$	$0.5292 \le \lambda_{11} \le 0.9473$
20	$0.4259 \le \lambda_{11} \le 0.8016$	$0.6583 \le \lambda_{11} \le 0.9130$	$0.3331 \le \lambda_{11} \le 0.9759$
25	$0.3659 \le \lambda_{11} \le 0.8385$	$0.5998 \le \lambda_{11} \le 0.9310$	$0 \le \lambda_{11} \le 1$
30	$0.3021 \le \lambda_{11} \le 0.8738$	$0.5292 \le \lambda_{11} \le 0.9473$	$-\infty \le \lambda_{11} \le -0.6913$ & $1.0206 \le \lambda_{11} \le \infty$
50	$0 \le \lambda_{11} \le 1$	$0 \le \lambda_{11} \le 1$	$-\infty \le \lambda_{11} \le 1.0801$ & $3.0053 \le \lambda_{11} \le \infty$

i.) The RGA element λ_{11} for the specific case of uncertainty in steady-state gains only under steady-state framework, remains greater than 0.5 up to an uncertainty of 13%, which suggests the pairing should corresponds to the diagonal elements λ_{11} and λ_{22}. However, for the uncertainty above 13% the lower limit of λ_{11} goes below 0.5 and varies in the range 0-1 at uncertainty of 50%. This indicates the increase in closed-loop interaction and the possibility of under-performance of chosen pairing.

ii.) When considering uncertainty in steady-state gains only under dynamic framework, the RGA element λ_{11} is found to be greater than 0.5 and less than 1 for uncertainty up to 31%, beyond which its lower limit goes below 0.5. At an uncertainty of 50%, the RGA element λ_{11} may take any value in the range 0-1 and the chosen pairing performance may deteriorate.

iii.) For uncertainty in both the steady-state gain and residence time (i.e., time constant and time delay) under dynamic framework, the RGA element λ_{11} remains greater than 0.5 for uncertainty up to 15%. Beyond 15% uncertainty, the lower limit of λ_{11} goes below 0.5 and the performance of the chosen pairing may deteriorate due to increase in closed-loop interaction. Further, the RGA element λ_{11} changes sign at uncertainty of 26% and may cause the system to become unstable. Thus, for the pairing to remain diagonal the tolerable uncertainty is 15%.

It can be concluded from the above discussion that the uncertainty tolerance capability of recommended diagonal pairing [Table (5.10)] can truly be estimated only on the dynamic analysis. Further, the consideration of uncertainty in process gain only may lead to overly optimistic results on the extent of tolerable uncertainty. Thus, it is important to consider the uncertainty in all model parameters in order to find the maximum tolerable plant uncertainty for robust performance.

5.2 Industrial Case Studies

5.2.1 Case Study-3: Shell Heavy Oil Fractionator Problem

The Shell heavy oil fractionator control problem introduced in Chapter 4, Section 4.2.1 and given by Eq. (4.3) is analyzed for the selection of best control configuration using steady-state and dynamic RGA based methods. The uncertainty analysis is also conducted, so as to find the tolerable uncertainty for which the chosen input-output pairing remains unchanged.

5.2.1.1 RGA Analysis: Steady-state Approach

For the Shell heavy oil fractionator problem [Eq. (4.3)], the steady-state gain matrix is given in Eq. (5.19):

$$K = \begin{bmatrix} 4.05 & 1.77 \\ 5.39 & 5.72 \end{bmatrix} \tag{5.19}$$

The corresponding steady-state RGA matrix obtained based on Eqs. (5.19) and (3.4) is given in Eq. (5.20):

$$\Lambda = \begin{bmatrix} 1.7002 & -0.7002 \\ -0.7002 & 1.7002 \end{bmatrix} \tag{5.20}$$

As per RGA pairing rules, the pairing should correspond to positive RGA elements greater than 0.5 and close to 1. Therefore, the recommended pairing based on steady-state RGA analysis is "diagonal", i.e., (1-1)/(2-2).

5.2.1.2 RGA Analysis: Dynamic Approach

Now, we will analyze the input-output pairing decision based on dynamic RGA approaches:

Effective Relative Gain Array (ERGA)

The bandwidth matrix for the Shell heavy oil fractionator model [Eq. (4.3)] is given in Eq. (5.21):

$$\Omega = \begin{bmatrix} 0.020 & 0.017 \\ 0.020 & 0.017 \end{bmatrix} \tag{5.21}$$

The corresponding effective gain matrix is obtained by substituting steady-state gain and bandwidth information from Eqs. (5.19) and (5.21) respectively, into Eq. (3.15) and is given by Eq. (5.22):

$$E = \begin{bmatrix} 0.0810 & 0.0301 \\ 0.1078 & 0.0972 \end{bmatrix} \tag{5.22}$$

Based on effective gain matrix [Eq. (5.22)], the ERGA matrix is calculated using Eq. (3.18) and is given in Eq. (5.23):

$$\Phi = \begin{bmatrix} 1.7002 & -0.7002 \\ -0.7002 & 1.7002 \end{bmatrix} \tag{5.23}$$

Since the ERGA defined in Eq. (3.18) has the same form as that of the RGA [Eq. (3.4)], it follows the same pairing rules as RGA. Therefore, the pairing will correspond to the positive ERGA elements greater than 0.5, which lies with the "diagonal" elements of the Shell oil fractionator model.

Effective Relative Energy Array (EREA)

The effective gain matrix for EREA analysis is calculated based on Eq. (3.24) and is given in Eq. (5.24):

$$E^* = \begin{bmatrix} 0.3281 & 0.0533 \\ 0.5810 & 0.5562 \end{bmatrix} \tag{5.24}$$

For effective gain matrix [Eq. (5.24)], the EREA is obtained based on Eq. (3.26), and is given in Eq. (5.25):

$$EREA = \begin{bmatrix} 1.2042 & -0.2042 \\ -0.2042 & 1.2042 \end{bmatrix} \tag{5.25}$$

Since the EREA defined by Eq. (3.26) has same form as that of RGA defined by Eq. (3.4), it follows same pairing rules as applicable to RGA. Therefore, the input-output pairing analysis based on EREA matrix [Eq. (5.25)] suggests "diagonal" pairing, i.e., (1-1)/(2-2) pairing.

Frequency Dependent RGA (FDRGA)

The frequency response of RGA elements λ_{11} and λ_{12} for the Shell heavy oil fractionator model [Eq. (4.3)] under dynamic framework is shown in Figure 5.7.

The analysis of Figure 5.7 shows the dominance of diagonal pairing in the entire frequency range. At low frequencies (below 10^{-2} rad/min) the diagonal RGA elements are positive and close to 1.6 whereas off-diagonal elements are negative, and the pairing corresponding to the negative elements should always be avoided. However, at higher frequencies the diagonal RGA elements are slightly greater than 0.5 whereas off-diagonal elements remain less than 0.5. Thus, the best input-output variable pairing for the entire frequency range is "diagonal", i.e., (1-1)/(2-2).

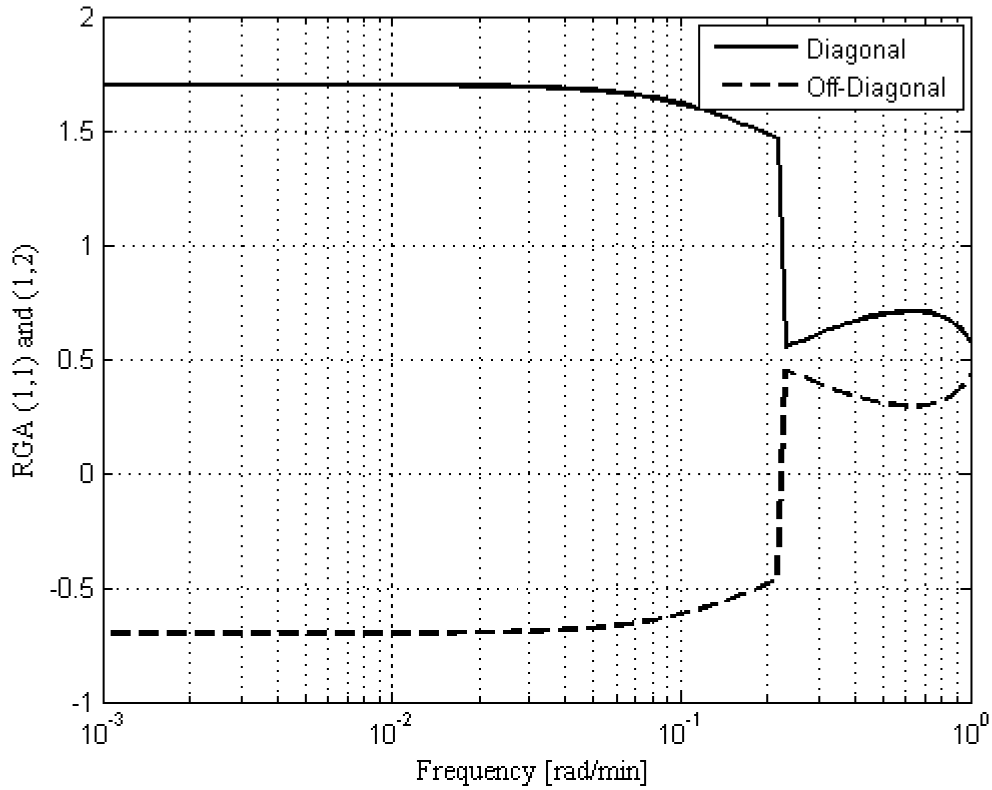


Figure 5.7. Dynamic RGA for the diagonal (solid line) and off-diagonal (dashed line) pairing for case study-3 (Shell heavy oil fractionator problem 4.2.1).

Relative Response Array (RRA)

The RRA analysis for the Shell heavy oil fractionator problem [Eq. (4.3)] is conducted using its all four variants discussed in Chapter 3, Section 3.3. The RRA analysis is as follows:

i.) Controller-independent time-average RRA (CI-TA-RRA)

The CI-TA-RRA is obtained based on Eqs. (3.42) and (3.43), and is given by Eq. (5.26):

$$\rho_{CI-TA} = \begin{bmatrix} 1.638 & -0.638 \\ -0.638 & 1.638 \end{bmatrix} \tag{5.26}$$

The diagonal elements of CI-TA-RRA are positive and close to 1, whereas off-diagonal elements are negative. It is strongly recommended to avoid the pairing of variables corresponding to the negative RRA elements. Therefore, the recommended pairing based on CI-TA-RRA analysis is "diagonal", i.e., (1-1)/(2-2).

ii.) Controller-independent time-varying RRA (CI-TV-RRA)

Table (5.13) shows the values of CI-TV-RRA elements ρ_{11} and ρ_{12} based on Eq. (3.43) and (3.44) corresponding to 10 to 100 percentage of the maximum time of response observation, i.e., the sum of dominant time constant and maximum process dead time. It is observed that up to 30% of the maximum response period of observation no responses are obtained, due to singularity of RGA matrix. This is due to the fact that the 30% of the maximum response period of observation is less than the largest time delay of the fractionator model [Eq. (4.3)]. Therefore, once the largest dead time is over (corresponding to the 32% of maximum response period of observation), the RGA matrix becomes non-singular and the interaction effects are observed. Thereafter, in the whole

Table 5.13. Controller-independent time-varying RRAs (CI-TV-RRA) for case study-3 (Shell heavy oil fractionator problem 4.2.1).

Percentage of $\left(\tau_D + \theta_{\max}\right)$	ρ_{11}	ρ_{12}
10	Undefined	Undefined
20	Undefined	Undefined
30	Undefined	Undefined
40	1.296	-0.296
50	1.428	-0.428
60	1.503	-0.503
70	1.553	-0.553
80	1.589	-0.589
90	1.617	-0.617
100	1.638	-0.638

period of response observation, the diagonal elements remain positive and greater than 0.5 whereas off-diagonal elements remain negative throughout. It is recommended to avoid pairing corresponding to the negative RRA elements for the stability of closed-loop system. Therefore, the obvious input-output pairing choice is "diagonal" pairing, i.e., (1-1)/(2-2) variable pairing.

iii.) Controller-dependent time-average RRA (CD-TA-RRA)

The CD-TA-RRA for the Shell heavy oil fractionator model [Eq. (4.3)] is obtained based on Eqs. (3.42), (3.45) and (3.46) and is given in Eq. (5.27):

$$\rho_{CD-TA} = \begin{bmatrix} 1.276 & -1.725 \\ -4.218 & 1.182 \end{bmatrix} \tag{5.27}$$

Since the off-diagonal elements are negative and it is strongly recommended to avoid the variable pairing corresponding to negative RRA elements, the recommended variable pairing is "diagonal", i.e., (1-1)/(2-2).

iv.) Controller-dependent time-varying RRA (CD-TV-RRA)

The CD-TV-RRA for the given Shell heavy oil fractionator model [Eq. (4.3)] based on Eq. (3.41) is shown in Table (5.14).

The RRA elements in the Table (5.14) are defined as the ratio of integral open and closed-loop step responses [Eq. (3.41)]. For a particular transfer function element there will be no open-loop response available until the process dead time is over. Similarly, the closed-loop response for a particular transfer function element [Eq. (3.36)] is also a function of various combination of process dead times. Due to these combinations of time delay terms the effect of interactions is observable only after the largest combined dead

Table 5.14. Controller-dependent time-varying relative response array (CD-TV-RRA) for case study-3 (Shell heavy oil fractionator problem 4.2.1).

Percentage of $(\tau_D + \theta_{\max})$	ρ_{11}	ρ_{12}	ρ_{21}	ρ_{22}
10	Undefined	Undefined	Undefined	Undefined
20	Undefined	Undefined	Undefined	1.000
30	Undefined	Undefined	1.000	1.000
40	1.000	1.000	1.000	1.000
50	1.000	1.094	1.036	1.000
60	1.032	2.376	1.463	1.015
70	1.102	30.318	2.607	1.054
80	1.169	-4.193	7.594	1.098
90	1.227	-2.305	-13.865	1.142
100	1.276	-1.725	-4.218	1.182

time (sum of process dead time) corresponding to diagonal or off-diagonal pairing is passed.

For the given example [Eq. (4.3)] the largest possible time delay combination for diagonal and off-diagonal pairing are 41 and 46 min, respectively. Therefore, the process interaction effects of significance for the Shell oil model [Eq. (5.20)] is observable only after 46 minutes, i.e., nearly 53 percentage of maximum response period of observation. In Table (5.14), the significant response period of observation is 60 to 100 percentage of maximum response period of observation. During this period the CD-TV-RRA clearly indicates that the diagonal elements ρ_{11} and ρ_{22} are greater than 0.5 and close to 1. Also, the off-diagonal elements of CD-TV-RRA during the considered period of observation change sign and pass through discontinuity which are the characteristics of an unstable loop. Thus, it is not favorable to pair variables corresponding to off-diagonal elements. Therefore, the obvious choice for pairing is the "diagonal", i.e., (1-1)/(2-2) variable pairing.

Closed-loop Performance

Table (5.15) shows the ITAE controller settings in terms of function block parameters for Simulink model (Figure 5.8) of the Shell heavy oil fractionator model [Eq. (4.3)].

The simulation results in Figure 5.9 indicates the response of output y_1 and y_2 for unit step change in set point $y_{1,sp}$. In order to compare the diagonal and off-diagonal control structures the step change in set point of y_1 is given at time 10 min. It can precisely be concluded from Figure 5.9, that the diagonal pairing, i.e., pairing variables $y_1 - u_1$ and $y_2 - u_2$ gives the stable closed-loop response whereas off-diagonal pairing

Table 5.15. ITAE based PI controller settings (function block parameters) for Simulink model of case study-3 (Shell heavy oil fractionator problem 4.2.1).

Plant Element	**Desired Closed-loop Time Constant, τ_c**	**Controller Gain, K_c**	**Integral mode gain, K_c/τ_I**
$\frac{4.05e^{-27s}}{50s+1}$	25	0.2544	0.0048
$\frac{1.77e^{-28s}}{60s+1}$	25	0.6654	0.0106
$\frac{5.39e^{-18s}}{50s+1}$	16	0.2772	0.0054
$\frac{5.72e^{-14s}}{60s+1}$	15	0.3885	0.0064

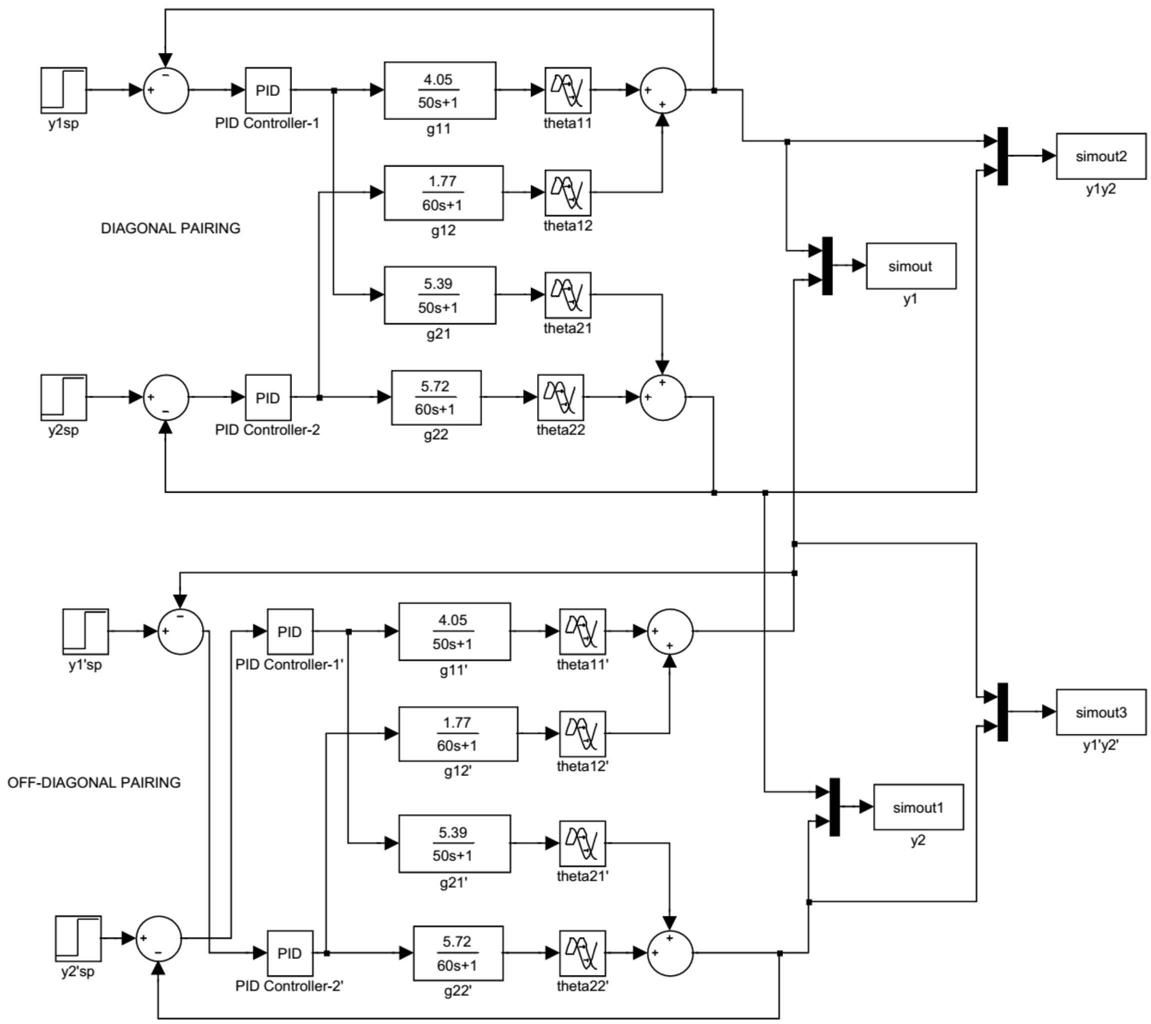
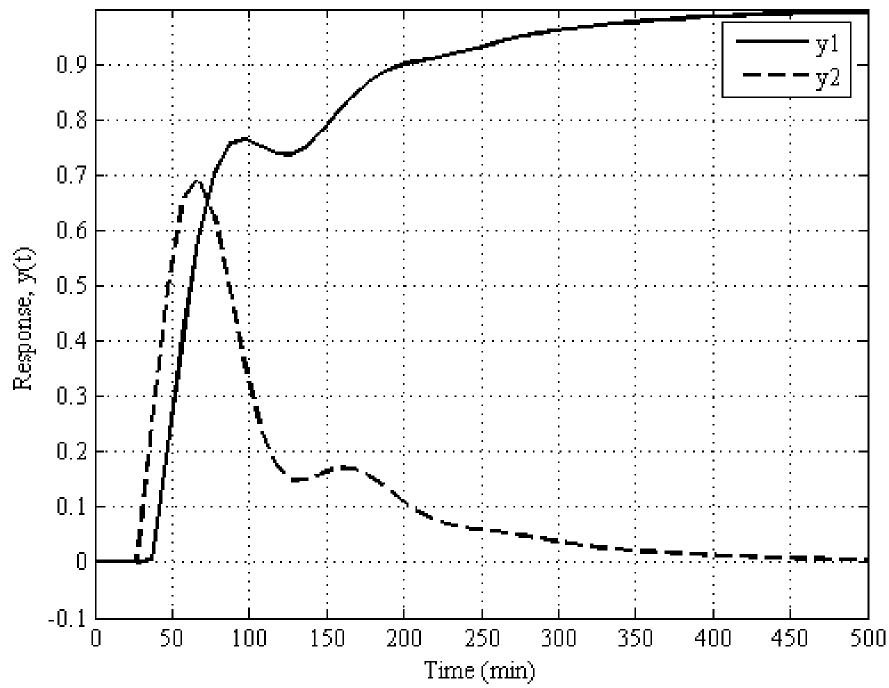
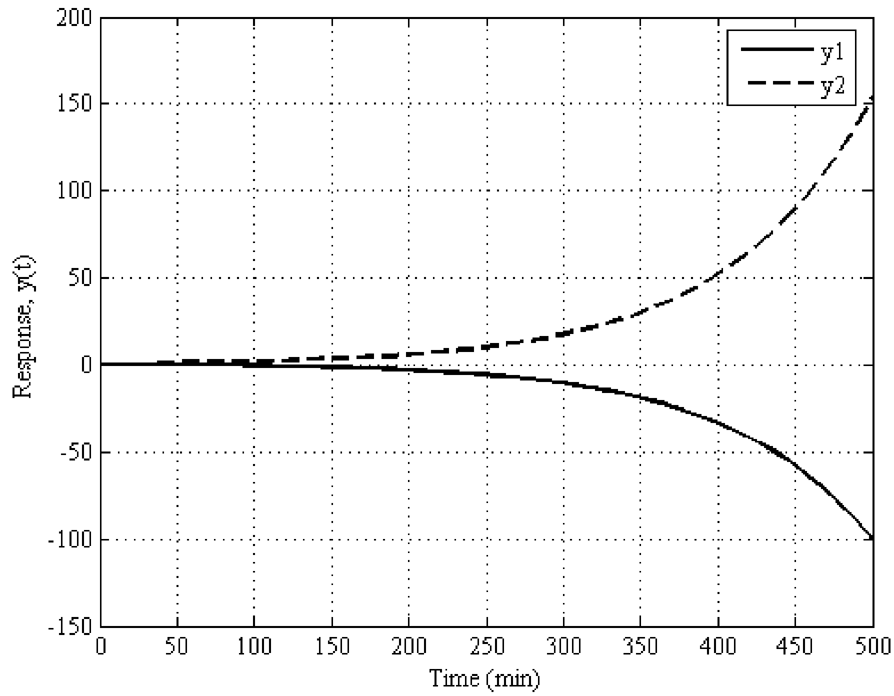


Figure 5.8. Simulink model of diagonal and off-diagonal pairing for case study-3 (Shell heavy oil fractionator problem 4.2.1).



(a) Diagonal Pairing



(b) Off-diagonal Pairing

Figure 5.9. Comparison of diagonal (a) and off-diagonal (b) pairing results in case study-3 (Shell heavy oil fractionator problem 4.2.1).

results in unbounded closed-loop response. Thus, the variable pairing corresponding to the diagonal elements should be a preferred choice, i.e., (1-1)/(2-2) variable pairing is the best pairing for the Shell heavy oil fractionator model [Eq. (4.3)].

For Shell heavy oil fractionator problem 4.2.1 [Eq. (4.3)], the steady-state and dynamic methods of RGA analysis all concludes to "diagonal" pairing as shown in Table (5.16). In addition, the proposed relative response array (RRA) method gives much more information about the robustness of chosen pairing with change in process dynamics [Table (5.13) and (5.14)].

For the recommended "diagonal" pairing the Niederlinski index, NI (Niederlinski, 1971) given by Eq. (3.5) comes out to be positive, whereas for the off-diagonal pairing it comes out to be negative. This indicates the stability of recommended (diagonal) pairing.

5.2.1.3 Uncertainty Analysis: Steady-state and Dynamic Approach

For the Shell heavy oil fractionator model given by Eq. (4.3), there are no negative element, therefore, exponent n in equation (3.53) is 0. The steady-state interaction quotient $\hat{k}$ from Eq. (3.53), for $n = 0$ is 0.4118. The average residence time corresponding to each element of the fractionator model are given in Eq. (5.28):

$$\tau_{ar,11} = 77; \;\; \tau_{ar,12} = 88; \;\; \tau_{ar,21} = 68; \;\; \tau_{ar,22} = 74 \tag{5.28}$$

Based on steady-state gain and average residence time information from Eqs. (5.19) and (5.30) respectively, the normalized gain matrix is calculated using Eqs. (3.48) and (3.49) for first and second order process elements respectively and is given in Eq. (5.29):

Table 5.16. Comparison of pairing recommendation by various methods for case study-3 (Shell heavy oil fractionator problem 4.2.1).

<table>
<tr><th>Plant Model</th><th colspan="2">Pairing Method</th><th>Resulting Array</th><th>Recommended Pairing</th></tr>
<tr><td rowspan="9">$$\boldsymbol{G(s)=\begin{bmatrix}\frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1}\\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1}\end{bmatrix}}$$</td><td colspan="2">RGA</td><td>$\Lambda=\begin{bmatrix}1.7002 & -0.7002\\ -0.7002 & 1.7002\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">ERGA</td><td>$\Phi=\begin{bmatrix}1.7002 & -0.7002\\ -0.7002 & 1.7002\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">EREA</td><td>$EREA=\begin{bmatrix}1.2042 & -0.2042\\ -0.2042 & 1.2042\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">Frequency-dependent DRGA</td><td>Figure 5.7</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td rowspan="4">RRA
(Proposed Method)</td><td>CI-TA-RRA</td><td>$\rho_{CI-TA}=\begin{bmatrix}1.638 & -0.638\\ -0.638 & 1.638\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CI-TV-RRA</td><td>Table 5.13</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TA-RRA</td><td>$\rho_{CD-TA}=\begin{bmatrix}1.276 & -1.725\\ -4.218 & 1.182\end{bmatrix}$</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td>CD-TV-RRA</td><td>Table 5.14</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
<tr><td colspan="2">Closed-loop response analysis</td><td>Figure 5.9</td><td>Diagonal, i.e., (1-1)/(2-2)</td></tr>
</table>

$$K_N = \begin{bmatrix} 0.0526 & 0.0201 \\ 0.0793 & 0.0773 \end{bmatrix} \quad (5.29)$$

The normalized dynamic interaction quotient based on normalized gain information [Eq. (5.29)] is obtained using Eq. (3.53) as $\hat{K}_N = 0.3921$.

The uncertainty in parameters is assumed to vary in the range $\pm 1\%$ to $\pm 50\%$. For simplicity of understanding the extent of uncertainty in gain and residence time of the process have been considered to be same, i.e., $\alpha = \beta$.

Table (5.17) shows the lower and upper bounds on interaction quotient for three different cases:

i.) Considering uncertainty only in process gains, under steady-state framework;

ii.) Considering uncertainty only in process gains, under dynamic framework;

iii.) Considering uncertainty in both process gains and residence times of the process.

Now, corresponding to the upper and lower bound on interaction quotient, the upper and lower bound on RGA element λ_{11} for the above three cases can be given as is shown in Table (5.18).

The following conclusions can be drawn based on Table (5.18):

(i) The RGA element λ_{11} for the specific case of uncertainty in steady-state gains only under steady-state framework, remains greater than 0.5 up to an uncertainty of 20.83%, which suggests the pairing should corresponds to the diagonal elements λ_{11} and λ_{22}. However, for uncertainty above 20.83% the λ_{11} changes sign that makes the pairing decision ambiguous or no unique pairing can be obtained.

Table 5.17. Lower and upper bounds on interaction quotient under uncertainty for case study-3 (Shell heavy oil fractionator problem 4.2.1).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	**Interaction Quotient**					
	(For uncertain gain under Steady-state framework)		**(For uncertain gain under dynamic framework)**		**(For uncertain gain and residence time under dynamic framework)**	
	Lower bound $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound** $\hat{k}^l$	**Upper bound** $\hat{k}^h$	**Lower bound** $\hat{k}^l$	**Upper bound** $\hat{k}^h$
1	0.3957	0.4286	0.3768	0.4081	0.3620	0.4248
5	0.3369	0.5033	0.3208	0.4793	0.2614	0.5882
10	0.2745	0.6177	0.2614	0.5882	0.1681	0.9150
20	0.1765	0.9609	0.1681	0.9150	0.0436	3.5293
30	0.1030	1.6473	0.0980	1.5686	-4.3135	-0.0356
50	0	∞	0	∞	-1.1764	-0.1307

Table 5.18. Lower and upper bound on RGA element λ_{11} under uncertainty for case study-3 (Shell oil fractionator problem 4.2.1).

Percentage of uncertainty $\alpha\%$ or $\beta\%$	Lower and Upper bound on RGA element λ_{11}		
	For uncertain gain under Steady-state (Chen and Seborg, 2002)	For uncertain gain under dynamic state (Proposed Approach)	For uncertain gain and residence time (Proposed Approach)
1	$1.6547 \le \lambda_{11} \le 1.7502$	$1.6045 \le \lambda_{11} \le 1.6896$	$0.8058 \le \lambda_{11} \le 0.8297$
5	$1.5082 \le \lambda_{11} \le 2.0134$	$1.4724 \le \lambda_{11} \le 1.9204$	$0.7498 \le \lambda_{11} \le 0.8709$
10	$1.3785 \le \lambda_{11} \le 2.6160$	$1.3540 \le \lambda_{11} \le 2.4284$	$0.6583 \le \lambda_{11} \le 0.9130$
20	$1.2143 \le \lambda_{11} \le 25.8593$	$1.2020 \le \lambda_{11} \le 11.7640$	$0.3331 \le \lambda_{11} \le 0.9759$
30	$-\infty \le \lambda_{11} \le -1.5449$ & $1.1148 \le \lambda_{11} \le \infty$	$-\infty \le \lambda_{11} \le -1.7588$ & $1.1087 \le \lambda_{11} \le \infty$	$-\infty \le \lambda_{11} \le -0.6913$ & $1.0206 \le \lambda_{11} \le \infty$
50	$-\infty \le \lambda_{11} \le 0$ & $1 \le \lambda_{11} \le \infty$	$-\infty \le \lambda_{11} \le 0$ & $1 \le \lambda_{11} \le \infty$	$-\infty \le \lambda_{11} \le 1.0801$ & $3.0053 \le \lambda_{11} \le \infty$

(ii) When considering uncertainty in steady-state gains only under dynamic framework, the RGA element λ_{11} found to be greater than 0.5 for uncertainty up to 21.83%, beyond which the λ_{11} changes sign and the chosen control configuration may not be good enough to control the process at the set point. In such a situation no unique pairing could be recommended.

(iii) For uncertainty in both the steady-state gain gains and residence time (i.e. time constant and time delay) under dynamic framework the RGA element λ_{11} remains greater than 0.5 only up to an uncertainty of 10.92%, beyond which the λ_{11}changes sign and no unique pairing can be recommended.

It can be concluded from Table (5.18) that for all three cases of uncertainty the recommended pairing remained diagonal only. However, the tolerance to uncertainty in steady-state gains increases under dynamic framework from its steady-state counterpart. But, for uncertainty in both the gain and residence time the tolerable uncertainty reduces drastically. Thus, it is strongly recommended to carryout uncertainty analysis under dynamic framework considering uncertainty in all model parameters.

5.2.2 Case Study-4: Distillation Column Control Problem

Consideringthe Doukas Luyben distillation column discussed in Chapter 4, Section 4.2.2 given by Eq. (4.4).

For the given distillation column plant model [Eq. (4.4)] the selection of best control configuration are carried out based on steady-state and various dynamic RGA approaches. Since the given distillation column model is a 3×3 control problem, the controller-dependent RRA methods and uncertainty analysis would not be applicable as they are limited to 2×2 systems only.

5.2.2.1 RGA Analysis: Steady-state Approach

The steady-state gain matrix for the distillation column model [Eq. (4.4)] is given by Eq. (5.30):

$$K = \begin{bmatrix} 0.374 & -11.3 & -9.811 \\ -1.986 & 5.24 & 5.984 \\ 0.0204 & -0.33 & 2.38 \end{bmatrix} \tag{5.30}$$

The steady-state RGA matrix corresponding to Eq. (5.30) is given in Eq. (5.31):

$$\Lambda = \begin{bmatrix} -0.0986 & 1.0004 & 0.0983 \\ 1.0926 & -0.1043 & 0.0117 \\ 0.0060 & 0.1039 & 0.8900 \end{bmatrix} \tag{5.31}$$

As per pairing rules, the pairing will correspond to those elements which are greater than 0.5 and non-negative. Therefore, the pairing based on steady-state RGA analysis for the given distillation model [Eq. (4.4)] is: (1-2)/(2-1)/(3-3).

5.2.2.2 RGA Analysis: Dynamic Approach

The dynamic RGA analysis would now be conducted based on methods discussed in Chapter 3 (Section 3.2) as follows:

***<u>Effective Relative Gain Array</u>* (*<u>ERGA</u>*)**

The bandwidth matrix [defined in Eq. (3.15)] for the considered distillation column problem [Eq. (4.4)] is given in Eq. (5.32) :

$$\Omega = \begin{bmatrix} 0.045 & 0.046 & 0.088 \\ 0.015 & 0.003 & 0.070 \\ 0.140 & 0.420 & 0.699 \end{bmatrix} \tag{5.32}$$

Based on the steady-state gain matrix [Eq. (5.30)] and bandwidth matrix [Eq. (5.32)], the effective gain matrix is obtained as per Eq. (3.14) and is given in Eq. (5.33):

$$E = K \otimes \Omega = \begin{bmatrix} 0.0168 & -0.5198 & -0.8634 \\ -0.0298 & 0.0157 & 0.4189 \\ 0.0029 & -0.1386 & 1.6636 \end{bmatrix} \tag{5.33}$$

Using effective gain matrix [Eq. (5.33)], the effective relative gain array is calculated based on Eq. (3.18) and is given in Eq. (5.34):

$$\Phi = E \otimes E^{-T} = \begin{bmatrix} -0.0497 & 0.9260 & 0.1238 \\ 1.0293 & -0.0168 & -0.0125 \\ 0.0205 & 0.0908 & 0.8887 \end{bmatrix} \tag{5.34}$$

Analysis of ERGA [Eq. (5.34)] shows that the elements (1-2), (2-1) and (3-3) are close to 1 and positive. Therefore, based on ERGA matrix the recommended pairing is (1-2)/(2-1)/(3-3).

Effective Relative Energy Array (EREA)

For generating EREA matrix we first need to determine the effective energy array, which is obtained using steady-state gain and bandwidth information from Eqs. Eq. (5.30) and (5.32) respectively of the DL distillation column [Eq. (4.4)] as given in Eq. (5.35):

$$E^* = |K| \otimes K \otimes \Omega = \begin{bmatrix} 0.0063 & -5.8737 & -8.4705 \\ -0.0592 & 0.0824 & 2.5066 \\ 0.0001 & -0.0457 & 3.9594 \end{bmatrix} \tag{5.35}$$

Based on the effective energy matrix [Eq. (5.35)], the effective relative energy array is calculated as per Eq. (3.26) and is given in Eq. (5.36):

$$EREA = E \otimes E^{-T} = \begin{bmatrix} -0.0020 & 0.9586 & 0.0164 \\ 1.0014 & -0.0015 & 0.0001 \\ 0.0006 & 0.0159 & 0.9835 \end{bmatrix} \quad (5.36)$$

Since the EREA is defined in the same way as the RGA [Eq. (3.4)], it follows the same pairing rules as RGA. The analysis of EREA clearly shows that the element corresponding to variable pair $y_1 - u_2$, $y_2 - u_1$ and $y_3 - u_3$ are close to 1 and non-negative. Therefore, recommended pairing is (1-2)/(2-1)/(3-3).

Frequency Dependent RGA (FDRGA)

The variation of frequency dependent RGA for all the RGA elements of DL distillation column model [Eq. (4.4)] is obtained based on Eq. (3.28) using Matlab, and is shown in Figure (5.10). The Matlab code for the same is given in Appendix-IV.

For the given distillation column model [Eq. (4.4)] the frequency range of interest varies within 0.001 to 1. From Figure 5.10 (a) it can be observed that for output y_1, the RGA element (1,1) and (1,3) are close to zero or negative under most frequency range of interests, whereas only the RGA element (1,2) remains positive and greater than 0.5. Similarly for Figure 5.10 (b) the RGA element (2,1) is positive and close to 1 in the most frequency range of interest whereas elements (2,2) and (2,3) are close to zero and less than 0.5. Therefore, variable $y_2 - u_1$ should be paired. And, for output y_3 from Figure 5.10 (c), the RGA element (3,3) remains positive and close to 1 in the most frequency range of interest. Therefore, variable $y_3 - u_3$ should be paired. Thus, overall recommended variable pairing is: (1-2)/(2-1)/(3-3) and their variations with frequency are shown in Figure 5.10 (d).

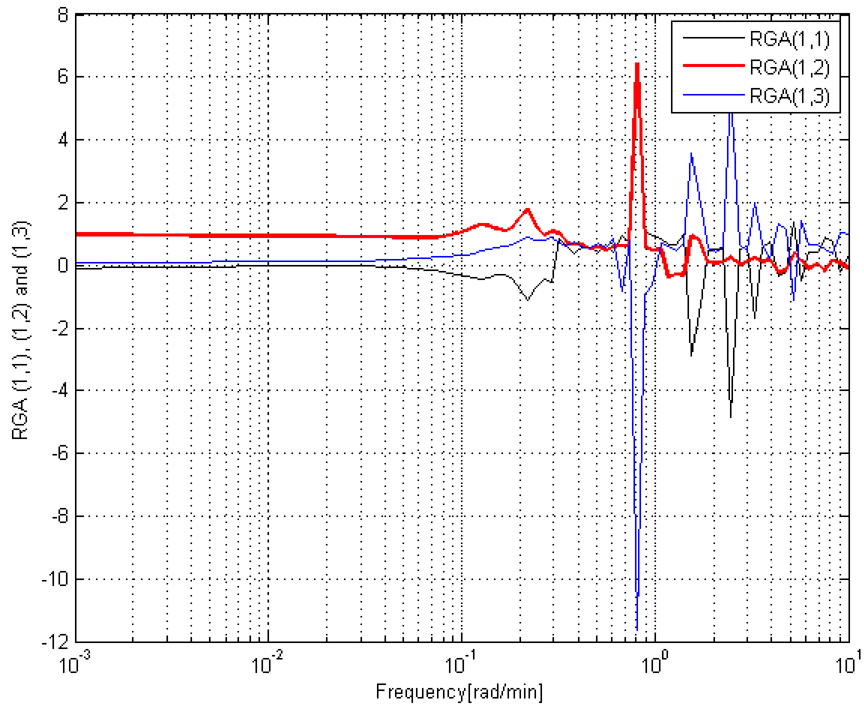


Figure 5.10 (a) RGA elements (1-1)/(1-2)/(1-3) response for various frequencies for case study-4 (DL distillation column problem 4.2.2).

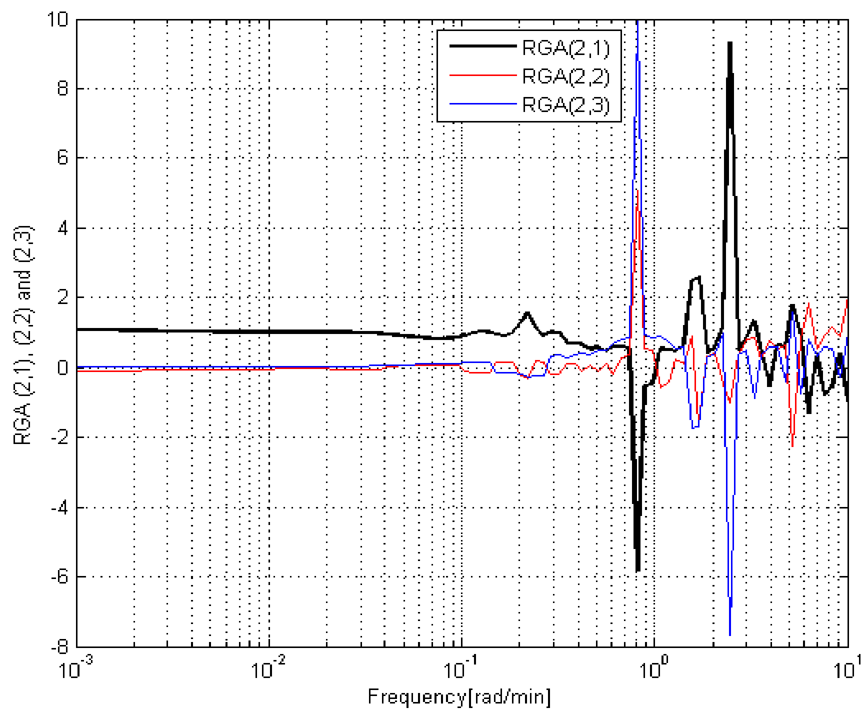


Figure 5.10 (b) RGA elements (2-1)/(2-2)/(2-3) response for various frequencies for case study-4 (DL distillation column problem 4.2.2).

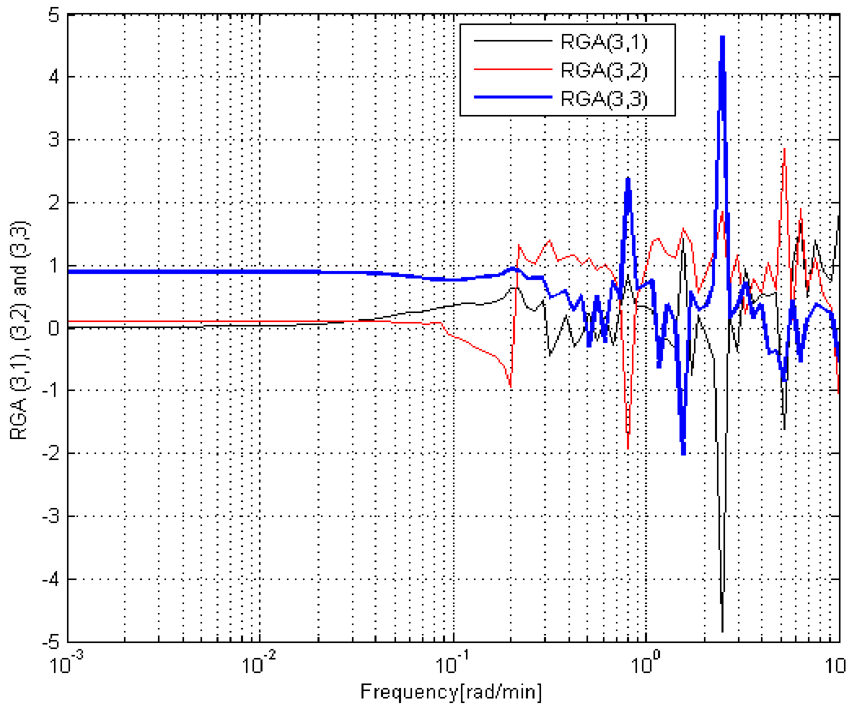


Figure 5.10 (c) RGA elements (3-1)/(3-2)/(3-3) response for various frequencies for case study-4 (DL distillation column problem 4.2.2).

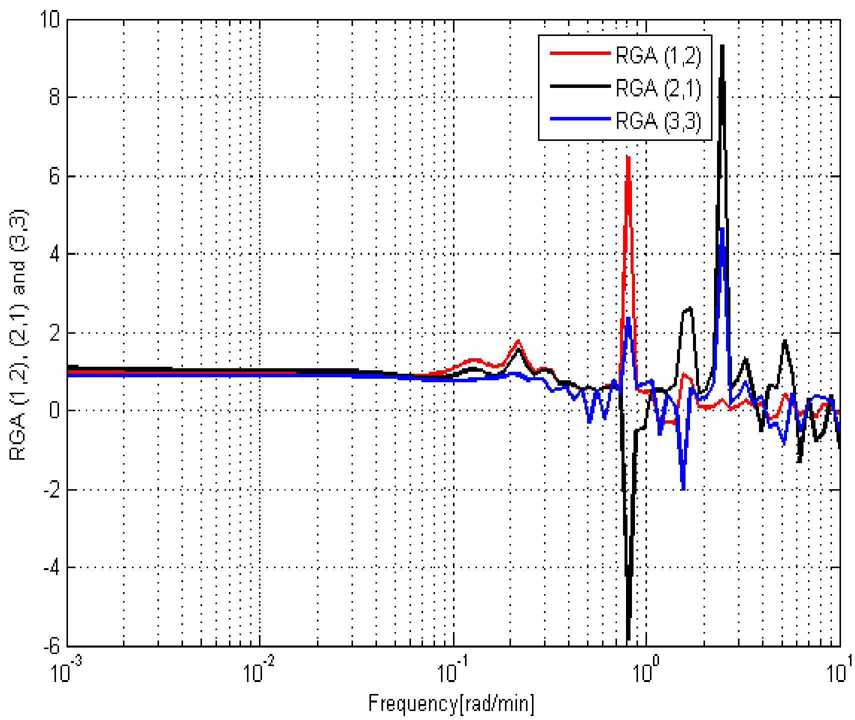


Figure 5.10 (d) RGA elements (1-2)/(2-1)/(3-3)] response for various frequencies for case study-4 (DL distillation column problem 4.2.2).

Relative Response Array (RRA)

The given distillation column model [Eq. (4.4)] is 3×3 control problem. As discussed in Chapter 3 (Section 3.3) that the controller-dependent versions of relative response array could not be defined for 3×3 systems and are currently limited to 2×2 systems only. Therefore, in the following section we will discuss only the controller-independent versions of RRA:

i.) Controller-independent time-average RRA (CI-TA-RRA)

The CI-TA-RRA is obtained based on Eqs. (3.42) and (3.43), and is given in Eq. (3.37) as:

$$\rho_{CI-TA} = \begin{bmatrix} -0.059 & 0.950 & 0.109 \\ 1.037 & -0.051 & 0.014 \\ 0.022 & 0.101 & 0.877 \end{bmatrix} \qquad (5.37)$$

The elements corresponding to variable pair $y_1 - u_2$, $y_2 - u_1$ and $y_3 - u_3$ of CI-TA-RRA are greater than 0.5 and close to 1, all other elements are either less than 0.5 or are negative. It is recommended to avoid the pairing of variables corresponding to the negative and less than 0.5 RGA elements. Therefore, the recommended pairing based on CI-TA-RRA analysis is (1-2)/(2-1)/(3-3).

ii.) Controller-independent time-varying RRA (CI-TV-RRA)

Table (5.19) shows the values of CI-TV-RRA elements based on Eqs. (3.43) and (3.44) corresponding to 10 to 100 percentage of the maximum time of response observation, i.e., the sum of dominant time constant and maximum process dead time. It can be

Table 5.19. Controller-independent time-varying relative response array (CI-TV-RRA) for case study-4 (DL distillation column problem 4.2.2).

Percentage of $\left(\tau_D + \theta_{\max}\right)$	ρ_{11}	ρ_{12}	ρ_{13}	ρ_{21}	ρ_{22}	ρ_{23}	ρ_{31}	ρ_{32}	ρ_{33}
10	0.005	1.009	-0.014	0.809	0.000	0.191	0.186	-0.009	0.822
20	-0.003	1.015	-0.012	0.902	-0.008	0.106	0.100	-0.007	0.906
30	-0.012	1.026	-0.013	0.946	-0.019	0.073	0.067	-0.007	0.940
40	-0.019	1.033	-0.014	0.968	-0.026	0.058	0.051	-0.007	0.956
50	-0.023	1.038	-0.015	0.981	-0.031	0.050	0.042	-0.007	0.965
60	-0.027	1.042	-0.015	0.991	-0.035	0.045	0.036	-0.007	0.971
70	-0.030	1.046	-0.016	0.998	-0.039	0.041	0.032	-0.007	0.975
80	-0.033	1.050	-1.016	1.004	-0.043	0.039	0.029	-0.007	0.978
90	-0.036	1.053	-0.017	1.009	-0.046	0.037	0.027	-0.007	0.980
100	-0.039	1.056	-0.018	1.013	-0.049	0.036	0.025	-0.007	0.982

observed that in the whole period of response observation the elements ρ_{12}, ρ_{21} and ρ_{33} remains positive and greater than 0.5, whereas all other elements remained either negative or less than 0.5 throughout. Thus, the recommended pairing based on CI-TV-RRA is: (1-2)/(2-1)/(3-3).

Closed-loop Performance

Table (5.20) shows the IMC controller settings in terms of function block parameters for Simulink model (Figure 5.11) of the given DL distillation column [Eq. (4.4)].

The simulation results in Figure 5.12 indicates the response of outputs y_1, y_2 and y_3 for a unit step change in set point. In order to compare all the possible control configurations (pairing) the step change in set point of y_3 is given at 10 minutes. It can precisely be concluded from Figure 5.12 (a) - (f), that the pairing (1-2)/(2-1)/(3-3) [Figure 5.12 (c)] and (1-3)/(2-1)/(3-2) [Figure 5.12 (e)] gives the stable closed-loop response, whereas all other pairing results in unbounded closed-loop response. Further compairison of the pairing (1-2)/(2-1)/(3-3) [Figure 5.12 (c)] and (1-3)/(2-1)/(3-2) [Figure 5.12 (e)] shows: (i) the former pairing is having better set point tracking ability (i.e., in variable y_3), (ii) the output y_1 is having lower settling time in latter [(1-3)/(2-1)/(3-2)] pairing but is having higher overshoot for load rejection, (iii) for output y_2 the latter pairing is higher positive and negative overshoot in comparison to the former and will demand high control action. Thus, the the variable pairing corresponding to (1-2)/(2-1)/(3-3) elements should be a preferred choice, i.e., (1-2)/(2-1)/(3-3) variable pairing is the best pairing for the given distillation column model [Eq. (4.4)].

Table 5.20. IMC based PI controller settings (function block parameters) for Simulink model of case study-4 (DL distillation column problem 4.2.2).

Plant Element	Desired Closed-loop Time Constant, τ_c	Controller Gain, K_c	Integral mode gain, K_c/τ_I	Derivative mode gain, $K_c\tau_D$
$\dfrac{0.374e^{-7.75s}}{(22.2s+1)^2}$	7	8.0486	0.1813	97.7311
$\dfrac{-11.3e^{-3.79s}}{(21.74s+1)^2}$	4	-0.4936	-0.0114	-5.3654
$\dfrac{-9.811e^{-1.59s}}{(11.36s+1)}$	2	-0.4433	-0.0365	-0.3294
$\dfrac{-1.986e^{-0.71s}}{(66.67s+1)^2}$	7	-8.7082	-0.0653	-0.2612
$\dfrac{5.24e^{-60s}}{(400s+1)}$	50	1.0258	0.0024	28.6270
$\dfrac{5.984e^{-2.24s}}{(14.29s+1)}$	2	0.8254	0.0536	0.8573
$\dfrac{0.0204e^{-0.59s}}{(7.14s+1)^2}$	1	440.2516	30.8299	1571.6982
$\dfrac{-0.33e^{-0.68s}}{(2.38s+1)^2}$	0.7	-8.5859	-1.8038	-10.2172
$\dfrac{2.38e^{-0.42s}}{(1.43s+1)^2}$	0.5	1.3062	0.4567	0.9339

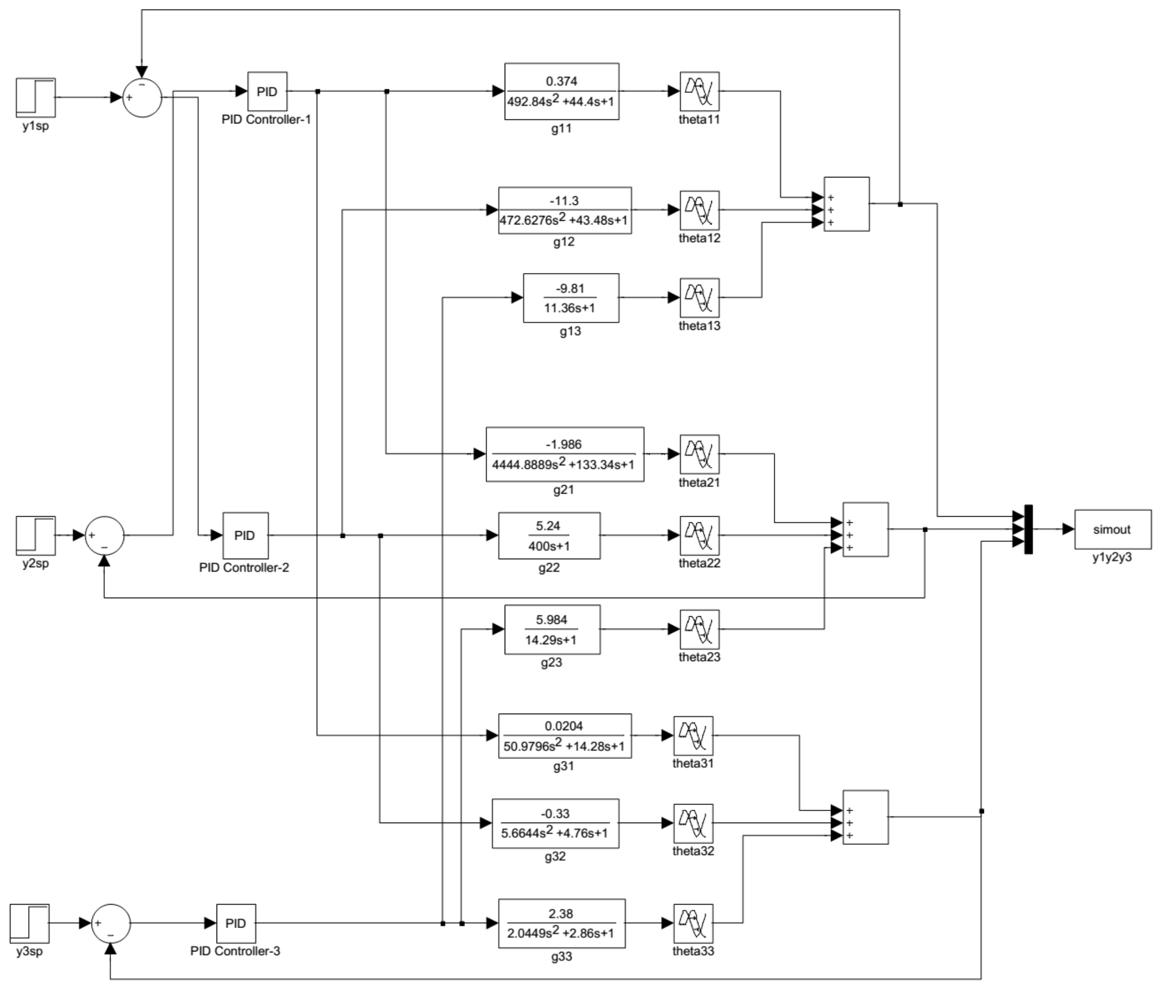


Figure 5.11. Simulink model of (1-2)/(2-1)/(3-3) pairing for case study-4 (DL distillation column problem 4.2.2).

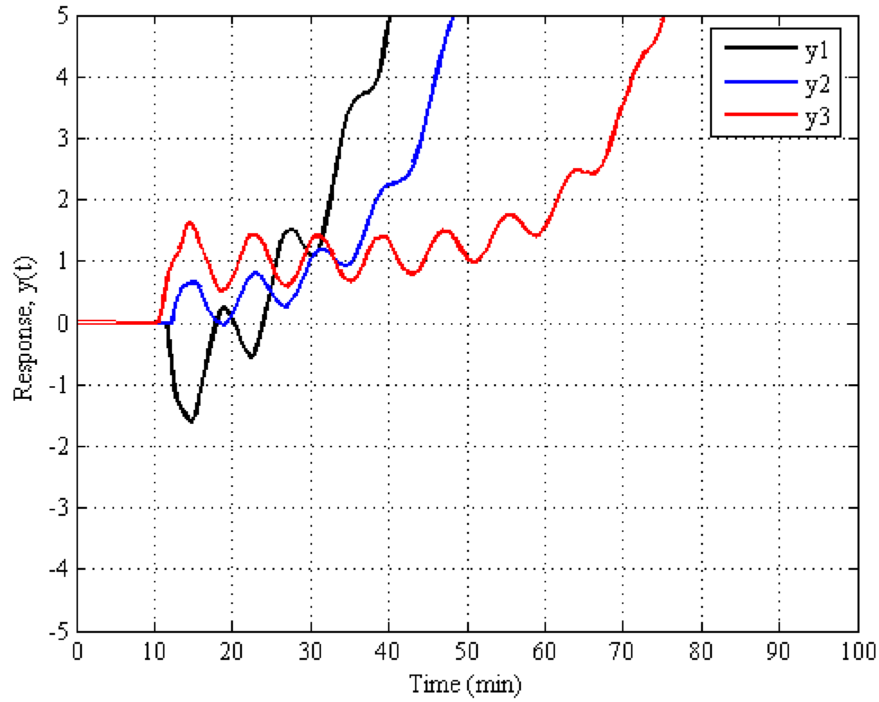


Figure 5.12 (a). Comparison of output responses for (1-1)/(2-2)/(3-3) pairing for case study-4 (DL distillation column problem 4.2.2).

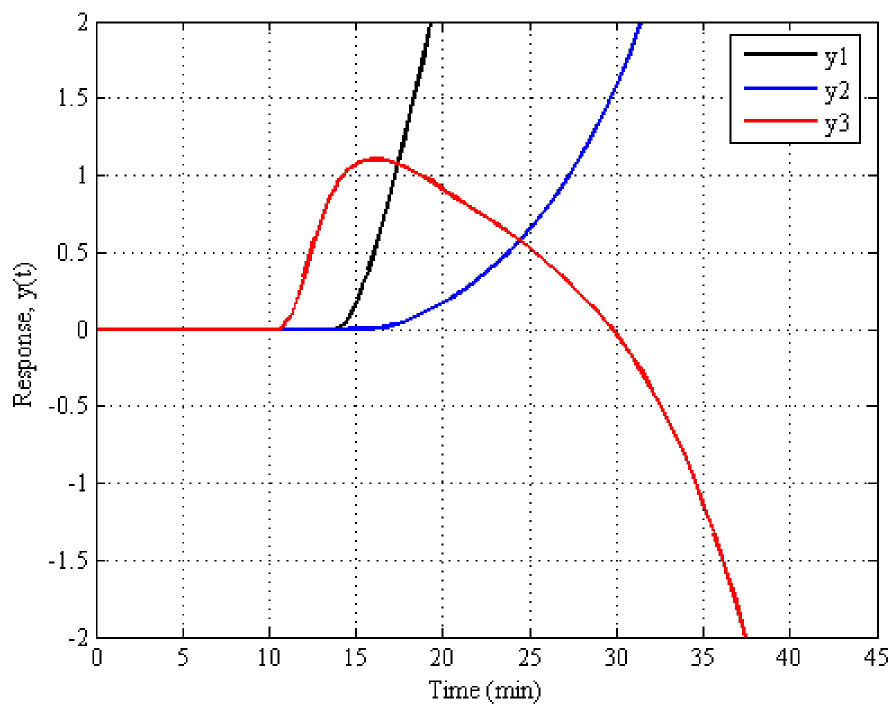


Figure 5.12 (b). Comparison of output responses for (1-1)/(2-3)/(3-2) pairing for case study-4 (DL distillation column problem 4.2.2).

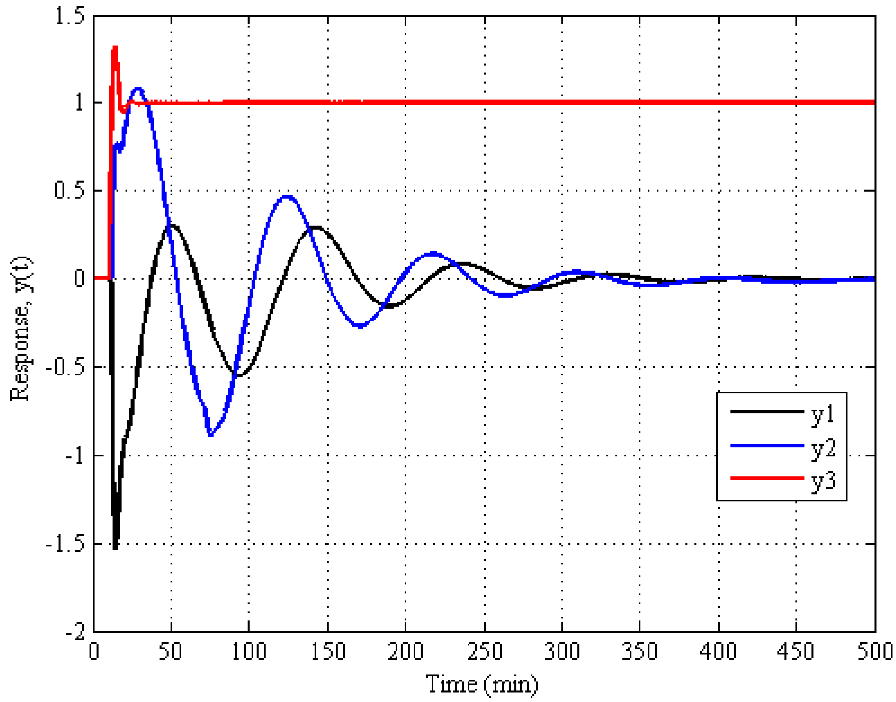


Figure 5.12 (c). Comparison of output responses for (1-2)/(2-1)/(3-3) pairing for case study-4 (DL distillation column problem 4.2.2).

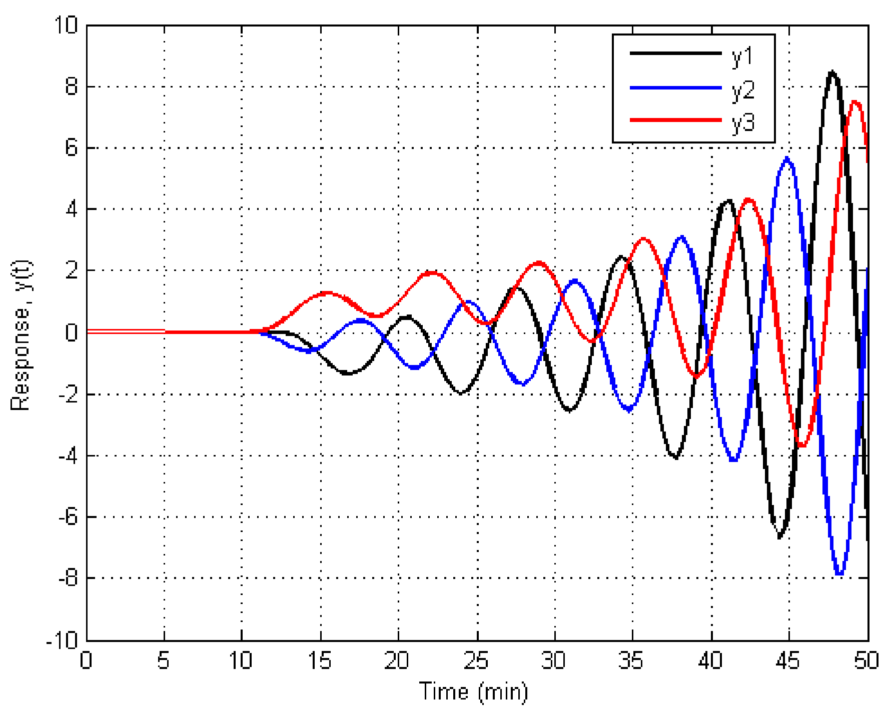


Figure 5.12 (d). Comparison of output responses for (1-2)/(2-3)/(3-1) pairing for case study-4 (DL distillation column problem 4.2.2).

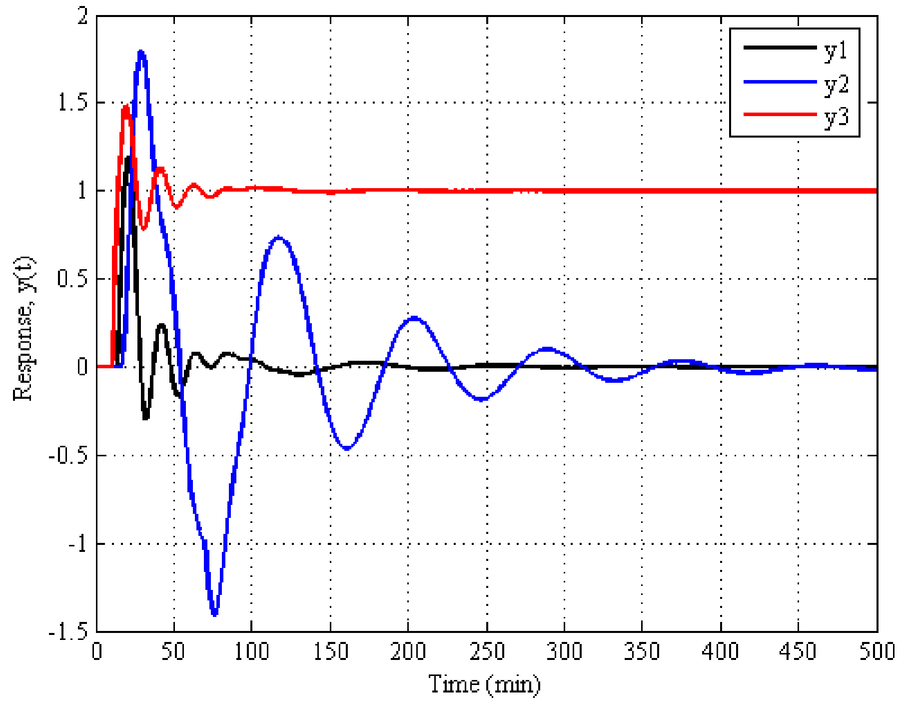


Figure 5.12 (e). Comparison of output responses for (1-3)/(2-1)/(3-2) pairing for case study-4 (DL distillation column problem 4.2.2).

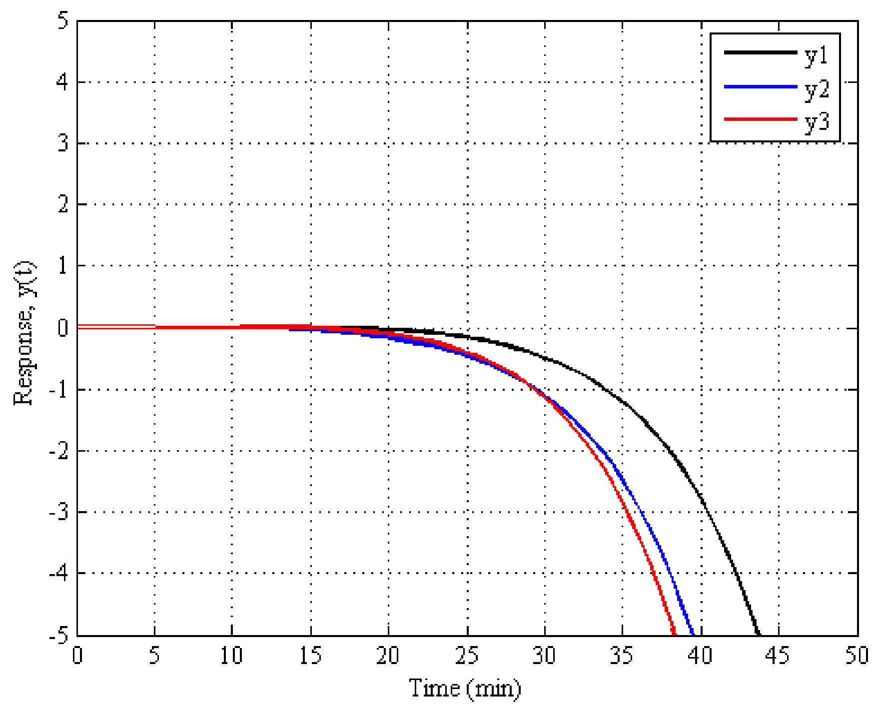


Figure 5.12 (f). Comparison of output responses for (1-3)/(2-2)/(3-1) pairing for case study-4 (DL distillation column problem 4.2.2).

Table 5.21. Comparison of pairing recommendation by various methods for case study-4 (DL distillation column problem 4.2.2).

<table>
<tr><th>Plant Model</th><th colspan="2">Pairing Method</th><th>Resulting Array</th><th>Recommended Pairing</th></tr>
<tr><td rowspan="9">$$G(s)=\begin{bmatrix} \dfrac{0.374e^{-7.75s}}{(22.2s+1)^2} & \dfrac{-11.3e^{-3.79s}}{(21.74s+1)^2} & \dfrac{-9.811e^{-1.59s}}{(11.36s+1)} \\ \dfrac{-1.986e^{-0.71s}}{(66.67s+1)^2} & \dfrac{5.24e^{-60s}}{(400s+1)} & \dfrac{5.984e^{-2.24s}}{(14.29s+1)} \\ \dfrac{0.0204e^{-0.59s}}{(7.14s+1)^2} & \dfrac{-0.33e^{-0.68s}}{(2.38s+1)^2} & \dfrac{2.38e^{-0.42s}}{(1.43s+1)^2} \end{bmatrix}$$</td><td colspan="2">RGA</td><td>$$\Lambda=\begin{bmatrix} -0.0986 & 1.0004 & 0.0983 \\ 1.0926 & -0.1043 & 0.0117 \\ 0.0060 & 0.1039 & 0.8900 \end{bmatrix}$$</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td colspan="2">ERGA</td><td>$$\Phi=\begin{bmatrix} -0.0497 & 0.9260 & 0.1238 \\ 1.0293 & -0.0168 & -0.0125 \\ 0.0205 & 0.0908 & 0.8887 \end{bmatrix}$$</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td colspan="2">EREA</td><td>$$EREA=\begin{bmatrix} -0.0020 & 0.9586 & 0.0164 \\ 1.0014 & -0.0015 & 0.0001 \\ 0.0006 & 0.0159 & 0.9835 \end{bmatrix}$$</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td colspan="2">Frequency-dependent DRGA</td><td>Figure 5.10</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td rowspan="4">RRA (Proposed Method)</td><td>CI-TA-RRA</td><td>$$\rho_{CI-TA}=\begin{bmatrix} -0.059 & 0.950 & 0.109 \\ 1.037 & -0.051 & 0.014 \\ 0.022 & 0.101 & 0.877 \end{bmatrix}$$</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td>CI-TV-RRA</td><td>Table 5.19</td><td>(1-2)/(2-1)/(3-3)</td></tr>
<tr><td>CD-TA-RRA</td><td colspan="2">---</td></tr>
<tr><td>CD-TV-RRA</td><td colspan="2">---</td></tr>
<tr><td colspan="2">Closed-loop response analysis</td><td>Figure 5.12</td><td>(1-2)/(2-1)/(3-3)</td></tr>
</table>

For the DL distillation column control problem 4.2.2 [Eq. (4.4)], the steady-state and dynamic methods of RGA analysis concludes to "diagonal" pairing. In addition, the proposed relative response array (RRA) methods gives much more information regarding the process dynamics. The CI-TV-RRA [Table (5.19)] clearly indicates the dominance of (1-2)/(2-1)/(3-3) control configuration over all other possible configurations. For the recommended pairing the Niederlinski index, NI (Niederlinski, 1971) given by Eq. (3.5) comes out to be 1.025 (positive) which shows the high possibility of the recommended pairing to remain stable. However, the conclusion can only be drawn from negative NI for system of dimensions 3×3 and higher.

The summary of input-output pairing results based on steady-state and dynamic methods of RGA analysis for DL distillation column control problem is shown in Table (5.21). For the considered industrial problem all the pairing methods both steady-state and dynamic results in (1-2)/(2-1)/(3-3) variable pairing. The controller dependent forms of RRA are under development for process 3×3 and higher.

CHAPTER - 6
CONCLUDING REMARKS

In the present study, the most popular tool for control configuration selection, the steady-state "relative gain array (RGA)" is extended to consider the effect of process dynamics. Performing the interaction analysis and taking decision for best control configuration based on steady-state information alone may lead to wrong pairings of variables, i.e., the resulting pairings may lead to underperformance of overall system. Thus, in the present study we have shown with the help of various case studies that it is important to consider the effect of process dynamics in control configuration selection. For the purpose of extending the steady-state RGA to dynamic systems, a new measure of process interaction is proposed in this study.

It is more than four decades since RGA has been introduced as a measure of closed-loop interaction. Very few attempts have been made towards the sensitivity of RGA analysis to model uncertainty and the work is majorly limited to the steady-state systems alone. The extension of RGA approach to uncertain process models under dynamic framework is proposed in this work. The aim is to gain insights into how process dynamics can affect control configuration decision based on RGA analysis in the face of model uncertainty.

The results obtained from proposed approaches are validated with that available in the literature. Also the results are analyzed based on IMC/ITAE controller design and tuning. This chapter presents a brief summary of the present work followed by conclusions, major contributions and future scope for research in this area.

6.1 Summary

6.1.1 Introduction

The advancements in chemical process technology have resulted in large scale multivariable plants. The control of such plants are carried out either by a centralized or a set of decentralized control system. In most industries the use of decentralized control is often preferred for handling a multivariable problem, over its centralized control counterpart. The reasons attributed to the choice are: ease of handling, independent tuning, maintenance flexibility and robustness against failures of individual loops. In designing a decentralized multi-loop control system, one has to first deal with the problem of obtaining suitable input-output pairings. The pairing should be such that the resulting loops are stable individually and together. For an *n*-dimensional square plant, i.e., a plant with *n*-inputs and *n*-outputs, there are *n*! possible pairings. It is a computationally challenging task to evaluate the performance of all the pairings. For wrongly paired variables the control system performance may get worse even for a highly sophisticated controller. Thus, any tool capable of screening out the worst pairings or selecting out the best pairing with minimum closed-loop interaction, will greatly reduce the computational efforts. For this purpose, a widely accepted tool is the Bristol's "relative gain array (RGA)" defined as:

> "The ratio of two gains representing first the process gain in an isolated loop (ϕ_{ij}) and, second, the apparent process gain in that same loop when all other control loops are closed ($1/\phi_{ji}^{-1}$). The ratio of these gains defines an array (RGA) M with elements, $\mu_{ij} = \phi_{ij}\phi_{ji}^{-1}$".

The RGA has many advantages over other interaction measures (Birk and Medvedev, 2003; Conley and Salgado, 2000; Mijores et al., 1986; Skogestad and Morari, 1987; Wittenmark and Salgado, 2002), such as: (i) it requires only the steady-state gain information of the plant model, (ii) it is independent of the 'units' of input-output variables, (iii) it is independent of controller design and tuning, (iv) it contains all the process information in one single matrix. It has also found to be associated with many closed-loop properties such as robustness, stability, tolerance to the failure of actuators and sensors, decentralized integral controllability (DIC) etc. Although the advantages offered by RGA are fascinating, it has certain practical limitations that restricts its use as a reliable tool. It is a steady-state measure of process interaction and hence does not consider the effect of process dynamics in control configuration selection. This may lead to wrong variable pairings with inferior performance particularly for processes with dominant dynamics. Also it is defined on the assumption of "perfect control" which is significant under steady-state conditions only. In the present study, it is our attempt to overcome some of these limitations of RGA.

6.1.2 Gaps in Literature

The past few decades have witnessed numerous attempts of extending the applicability of RGA to dynamic processes. These approaches can be categorized either as controller-independent approach or the one which requires rigorous controller design and tuning. The former is often based on the assumption of perfect control which is not realizable practically and has no significance particularly for dynamic systems. No serious attempts have been made in generating an interaction measure with the simplicity attributes of RGA and also considers the physical realizability of the controller.

In most of the studies on the analysis of RGA and its properties, the availability of a process model is frequently assumed. However, in practice, the models of real systems always have some uncertainty associated with them. Still, the issue of RGA analysis sensitivity to model uncertainty has not been adequately addressed. For plants with uncertain process models, an incorrect pairing decision may result, if the RGA analysis is carried out based only on the nominal model of the process. The problem further aggravates when a sensitivity analysis of RGA elements to structured/parametric model uncertainty is carried out based on steady-state process model alone. Further, the majority of the work is limited to the uncertainty in steady-state gains only, and no significant effort has been made towards uncertainty consideration in measurement of process time constant and time delays.

6.1.3 Scope of Work

In the present study, control configuration selection problem for decentralized control of multi-loop plant system is addressed. For the selection of best control configuration and analysis of the extent of interaction present, the widely accepted and probably the most reported tool the "RGA" has been adopted. The focus of the work is on the development of this well known tool, and to extend its application by overcoming its limitations. For this purpose various extensions of the RGA, reported in the literature to dynamic systems have been critically analyzed and compared. A new measure of process interaction the "relative response array (RRA)" is proposed. The properties of the RRA have also been developed. The applicability of the approach is shown with the help of two benchmark test problems and two industrial problems.

Another major area of focus of the present study is on gaining insights into how process dynamics can affect control configuration decision based on RGA analysis in the face of structured model uncertainty. Parametric uncertainty in gain and residence time (includes both time constant and dead time) of the process has been considered. Analytical expressions for worst-case bounds of uncertainty in steady-state and dynamic RGA are derived for two input two output (TITO) plant models. Two benchmark test problems together with two industrial problems which have been used in several prior studies is considered here to demonstrate the effectiveness of the approaches. The obtained bounds of uncertainty in RGA provide valuable information pertaining to the necessity of robustness and accuracy in the model of decentralized multivariable systems.

Throughout the work, it has been assumed that the transfer function based model of the plant under study is available and that the relevant input and output for the model are also available. The work here is focused on the suitable pairing of these inputs and outputs, which can ensure robust performance.

6.1.4 Relative Gain Array: Theory and Methods

An approach of extending the steady-state relative gain array to consider the effect of process dynamics on input-output pairing decision is developed. The proposed measure of interaction the relative response array (RRA) is defined by following (i) a controller-independent approach, and (ii) a controller-dependent approach. The controller-independent approach is a conventional approach developed on the assumption of "perfect control" of output variables in a feedback control scheme. Though the assumption of perfect control is valid only for steady-state systems but can provide a good approximation for dynamic systems. In the controller-dependent approach the assumption of perfect control is relaxed with a practically implementable controller. For this purpose an IMC based controller is designed in a feedback control scheme.

On the basis of controller-independent approach two different variants of relative response array are proposed: (i) The controller-independent time-average relative response array (CI-TA-RRA), and (ii) The controller-independent time-varying relative response array (CI-TV-RRA). The controller-independent measures of RRA are defined on the basis of integral open-loop response of controlled outputs for a unit step change in their corresponding set-point. In particular, the CI-TA-RRA is based on the time weighted average of the response and CI-TV-RRA is defined as a function of time, therefore CI-TV-RRA generates RRA matrix at each time instant and provides a better representation of the dynamic behavior of the system. Similarly, on the basis of controller-dependent approach the version of relative response array proposed are: (i) The controller-dependent time-average relative response array (CD-TA-RRA), and (ii) The controller-dependent time-varying relative response array (CD-TV-RRA). The controller-dependent measures of RRA (CD-TA-RRA and CD-TV-RRA) has same meaning as that of the corresponding controller-independent measures (CI-TA-RRA and CI-TV-RRA). However, for producing controller-dependent measures of RRA the open and closed-loop step responses are calculated separately. Further the closed-loop system response is obtained considering a IMC controller in the feedback loop. The RRA follows the same basic definition as the relative gain array (RGA) i.e. defined as the ratio of open-loop gain with all other loops under manual control to that of closed-loop gain with all other loops under automatic control. The various properties of RRA are also stated in this thesis.

In this study a novel approach is suggested to analyze the effect of process dynamics on the sensitivity of RGA elements. For incorporating the effect of process dynamics, a dynamic RGA approach available in the literature have been utilized in a distinctive manner. Also, the analytical expressions are derived for worst-case bounds on uncertainty in steady-state and dynamic RGA elements for TITO plant models. For this purpose, the parametric model uncertainty in gain and residence time (combined term for time constant and dead time) of the plant has been considered. The work has been carried out with the objective of identifying the limiting value of uncertainty for which possible pairing changes may occur.

6.1.5 Case Studies: Selected Nonlinear Chemical Processes

In this work four different case studies are considered for showing the applicability of the methods studied and developed. The examples considered comprise of two benchmark test problems and two industrial problems reported in the literature. The benchmark test problems considered for the study are control problems having typical process dynamics. The industrial problems adopted for the study are: (i) Shell oil fractionator problem reported in the literature as a control problem (ii) Doukas and Luyben distillation column reported in literature as a control problem. The process elements involved in the study of both set of problems are of first order, first order plus time delay, second order and second order plus time delay.

6.1.6 Results and Discussion: Comparison and Implementation of Proposed Techniques

6.1.6.1 RGA Analysis: Steady-state and Dynamic Approach

Each of the plant model introduced above have been analyzed for the selection of best control configuration. First the steady-state RGA analysis is conducted for obtaining suitable control configuration. The pairing decision is then analyzed based on dynamic RGA methods such as: frequency dependent dynamic RGA, effective relative gain array (ERGA), effective relative energy array (EREA), and relative response array (RRA). For verifying the correctness of the results (i.e. verifying the chosen pairing for stability and response speed) the response curve is generated based on rigorous IMC/ITAE controller tuning rules for the pairing decision obtained from steady-state RGA and dynamic RGA methods. All the calculations are performed in Matlab/Simulink.

It has been observed that in one of the case studies based on benchmark test problem, the steady-state RGA fails to identify the best pairings and recommends wrong pairings, whereas all the dynamic RGA approaches including all the variants of the proposed RRA approach were successful in identifying the best pairing. In another case study of benchmark test problem, few of the popular dynamic RGA (ERGA and EREA) methods could not conclude to any suitable pairing whereas all the variants of the proposed RRA approach successfully found the best pairing with no ambiguity. However, for both the real life control problems all the methods successfully identifies the best pairings.

6.1.6.2 RGA Elements Sensitivity under Model Uncertainty

The case studies considered have also been analyzed for the sensitivity of RGA elements under parametric uncertainty in each element of the plant model. The objective is to observe the effect of model uncertainty on pairing decision, i.e., whether the input-output pairing changes for the considered uncertainty range. The objective is also to find the limiting point or maximum tolerable uncertainty for which the input-output pairing does not change. The model uncertainty in the study comprises of uncertainty in process gain, process time constant and time delay. The uncertainty in time constant and time delay are included in a combined term of residence time.

The results of sensitivity analysis on RGA elements clearly show that the input-output pairing may change depending on the extent of uncertainty present in the model parameters. Further it has been observed that for uncertainty in steady-state gains, the tolerable uncertainty for the input-output pairing to remain unchanged increases if the RGA analysis is carried out under dynamic framework rather than the steady-state framework. However, the tolerable uncertainty is found to be decreasing if the uncertainty is considered in all the model parameters (steady-state gain, time constant and dead time). Though for the purpose of sensitivity in RGA elements the uncertainty in model parameters are considered up to a value of around 50%, the results can easily be extended for higher level of uncertainty.

6.2 Conclusions

Based on the results obtained in the present study, the following conclusions are drawn:

1. The steady-state RGA does not always gives the correct pairing suggestion about the best control configuration.

2. It is essential to perform RGA analysis under dynamic framework particularly for plants with dominant dynamics.
3. The popular dynamic methods (ERGA and EREA) available in the literature are shown to be true for the set of hypothetically framed examples. However, for one of the case studies they completely failed to identify the best control configuration.
4. Proposed measure of control configuration selection the relative response array (RRA) is a promising tool. It successfully identifies the best control configuration for all the four case studies, even in cases where steady-state RGA and controller independent dynamic RGA methods (ERGA and EREA) failed.
5. The time-varying variants of RRA provides detailed information (eg. information regarding passing of RRA elements through discontinuity, sign change of RRA elements etc.) about the process dynamic behavior in comparison to other RGA methods.
6. The obtained RGA uncertainty bounds found to be providing valuable information about the variation in extent of loop interaction with the change in process parameters.
7. The RGA sensitivity analysis for uncertainty in process gain under steady-state framework gives undervalued tolerable uncertainty, whereas the actual plant (works under dynamic framework) can tolerate much higher levels of uncertainty in plant model, provided the uncertainty in process time constant and dead time are tightly controlled.

8. The presence of uncertainty in time constant and dead time measurement together with the uncertainty in process gain can significantly reduce the tolerable uncertainty levels. In such cases, the pairing decision needs to be critically analyzed.
9. The worst-case bounds on RGA elements can provide good estimates of tolerable uncertainty in plant model for which the recommended input-output variable pairing remain unchanged.
10. The lower values of tolerable uncertainty emphasizes the need for high accuracy in plant model determination.

6.3 Major Contributions

1. A state-of-the-art review is carried out on the control configuration selection methods emphasizing relative gain array as a tool.
2. A new measure of process interaction the "relative response array (RRA)" and its four different variants (CI-TA-RRA, CI-TV-RRA, CD-TA-RRA and CD-TV-RRA) are proposed. The proposed measure can provide the best control configuration for dynamic process systems.
3. The precision of the proposed measures are tested on four case studies consisting of two benchmark test problems with typical process dynamics and two industrial control problems.
4. Simulink models were developed for testing the stability and performance of chosen control configuration based on closed-loop response analysis using IMC/ITAE controller design and tuning.

5. The most popular methods of control configuration selection based on RGA approach under steady-state and dynamic framework are studied and compared with the proposed RRA approach.
6. A new approach for the sensitivity analysis of RGA elements for parametric uncertainty is proposed.
7. For sensitivity analysis of RGA elements under dynamic framework the uncertainty in process gain, process time constant and dead time have been considered together for the first time. The previous studies considered the uncertainty in steady-state gain only and the sensitivity analysis is conducted under steady-state framework.
8. A new interaction parameter "dynamic interaction quotient" has also been introduced. The dynamic interaction quotient helps in determining the worst-case upper and lower bounds on RGA elements under uncertainty.
9. Analytical expressions are derived for the worst-case bound on RGA elements for parametric uncertainty in 2×2 plant models.
10. Tolerable uncertainty on plant models (case studies) were determined using worst-case bounds of RGA elements under parametric uncertainty, for which the control-loop pairing remains unchanged.

6.4 Future Scope of Research

The future scope of this work is enumerated below:

1. The controller-dependent variants of the proposed measure "relative response array" have been developed for 2×2 plant models in this study. The approach

can be extended for system having 3×3 dimension or higher.

2. The RRA approach can also be extended for plants with integrator and differentiators.
3. The closed-loop response of controller-dependent variants of RRA are defined assuming IMC controller. Other model based controllers can also be tried and tested.
4. In the present study, the analytical expression are derived for worst-case bound on RGA elements for parametric uncertainty in 2×2 plant models. The current study can be extended to 3×3 plant models. In addition the other forms of uncertainty (say unstructured uncertainty) in plant model can also be considered.
5. For more rigorous sensitivity analysis of RGA with different uncertainty range for each model parameter the optimization approach (conventional, evolutionary and/or hybrid methods of optimization) can be developed.

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LIST OF PUBLICATIONS

International Journals

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BIOGRAPHIES

Biography of the Candidate

Amit Jain completed his Bachelor of Engineering (B.E.) in Chemical Engineering from Institute of Technology and Management, Gwalior in the year 2001. He obtained Master of Engineering in Chemical Engineering from Birla Institute of Technology and Science (BITS), Pilani with practice school at Grasim Industries Limited, Nagda in June 2003.

He served in various positions: Assistant Lecturer, Chemical Engineering Department, Nirma Institute of Technology, Ahmedabad from Jun., 2003 - Oct., 2003; Lecturer, Chemical Engineering Department, Nirma University, Ahmedabad from Oct., 2003 - Oct., 2006; Assistant Lecturer, Department of Chemical Engineering, Birla Institute of Technology and Science (BITS), Pilani from Oct., 2006 - Dec., 2006; and is currently working as Lecturer, Department of Chemical Engineering, Birla Institute of Technology and Science (BITS), Pilani, since Jan., 2006. He has 12 years of teaching experience and has guided 2 M.E. Dissertations, 4 B.E. Thesis and around 10 project students. He taught courses such as Process Dynamics and Control, Fluid Mechanics, Process Plant Safety, Biochemical Engineering, Environmental Pollution Control, Chemical Process Calculations, Selected Chemical Engineering Operations, Mathematical Methods in Chemical Engineering, Optimization, Thermodynamics, Structure and Properties of Materials, Measurement Techniques-II. His research interests include process dynamics and control, modeling and simulation, and environmental biotechnology.

Biography of the Supervisor

An acknowledged researcher and renowned academician, **Dr B V Babu** has 30 years of teaching, research, consultancy, and administrative experience. He did his PhD from IIT Bombay. He is currently the Vice Chancellor at Galgotia University, Greater Noida (since May 2014). Earlier, he was the Pro Vice Chancellor of DIT University, Dehradun (2013-14) and founding Director of Institute of Engineering and Technology (IET) at JK Lakshmipat University, Jaipur (2011-2013). Prior to that, Dr Babu served at BITS Pilani for 15 years (1996-2011), and held various administrative positions as Dean - Student Welfare Division, Dean - Faculty Division I, Dean - Educational Hardware Division and Head - Chemical Engineering Department, to name a few. Before that he worked at Gujarat University for 11 years (1985-1996).

He is a member of various national and international academic and administrative committees. He was the Chairman-Diploma Board, Member of National Council and also an Executive Member of Indian Society for Training and Development (ISTD), New Delhi. He is external expert member of Board of Studies at Banasthali University and MNIT Jaipur. He is the Peer Team Member of National Assessment and Accreditation Council (NAAC), a statutory body of UGC, New Delhi and served as Chairman, Member Coordinator and Member of various visiting Peer Teams for many Universities and Institutions. He is also member of Peer Review Committee for The Natural Sciences and Engineering Research Council of Canada (NSERC), Canada; National Research Foundation (NRF), International Research Grants, South Africa; Council of Scientific & Industrial Research, New Delhi; and member of Research Project Review Committee, Ministry of Steel, Government of India, New Delhi. He is Honorary Board Advisor for many Educational Endeavors and Consulting Firms.

He is the recipient of CSIR's 'National Technology Day Award', obtained in recognition of the research work done. He received 'Kuloor Memorial Award-2006' awarded for the Best Technical Paper published in the Institute's Journal "Indian Chemical Engineer" in its issues for 2005. He is also the recipient of 'Best Paper Awards' in CHEMCON-2000 and CHEMCON-2001 for the papers he presented at the International Symposia & Annual Sessions of Indian Institute of Chemical Engineers (IIChE). He is the Life Member/Fellow of many professional bodies such as IIChE, IE (I), ISTE, ICCE, IEA, SOM, ISSMO, IIIS, IAENG, SPE, ISTD, etc. He is Editor-in-Chief and Editorial Board Member of several International and National Scientific Journals.

Professor Babu is a distinguished academician and an acknowledged researcher with an h-index of 34 & i10-index of 79 and over 4527 citations (as in May 2015). He is well known, internationally, for his algorithm, MODE (Multi Objective Differential Evolution) and its improved variants. Besides several highly-cited publications in International journals, he also has two well accepted textbooks (one published by Springer, Germany). He has wide ranging research interests in the areas of Evolutionary Computation (population-based nature-inspired search algorithms for the optimization of complex engineering problems for both single- and multiobjective formulations), Energy Engineering (biomass gasification, pyrolysis and process intensification) and Environmental Engineering (low cost adsorbents for removing heavy metals and bio-filtration for removing volatile organic compounds). He supervised 8 PhD candidates and PhD Examiner for 20 students of IITs, NITs and Universities, and currently guiding 2 PhD candidates.

Overall he has over 230 research publications to his credit. He completed three consultancy projects successfully. He has published six books and has written several chapters, invited editorials and articles in various books, lecture notes and international journals. He organized several International and National Conferences, Workshops & Seminars and also chaired several Technical Sessions. He has been Invited Speaker and delivered Keynote Addresses at various International Conferences, Seminars and Workshops.

Dr Babu's hobbies include singing, music compositions, playing keyboard, reading spiritual books, walking, and practicing Yoga and Pranayama.

Google Scholar: http://scholar.google.co.in/citations?user=1yVdKQ8AAAAJ&hl=en&oi=ao

APPENDIX - I

MATLAB CODE FOR THE CASE STUDY – 1

2×2 **Process Model with a Second Order Element**

The Matlab codes used in generating the results for various methods used in this study are as follows:

1. Frequency Dependent RGA

```
%Script File for the Computing Dynamic RGA
%Define a Vector of Frequency Values on a Log Scale
w=logspace(-3,0,200);
s=i*w;

%Compute the Frequency Response for each Element of gij
g11=5./(4*s+1);
g12=2.5*exp(-5*s)./((2*s+1).*(15*s+1));
g21=-4*exp(-6*s)./(20*s+1);
g22=1./(3*s+1);

%Compute Lambda(i,j) as a Function of Frequency
L12=-g12.*g21./(g11.*g22-g12.*g21);
lam12=sign(real(L12)).*abs(L12);

%Using RGA Property of Unity Sum
lam11=1-lam12;

%Plot the Results
semilogx(w,lam11,'-k',w,lam12,'--k')
xlabel('Frequency[rad/min]')
ylabel('RGA (1,1) and (1,2)')
```

2. Controller-Independent Time-Average RRA (CI-TA-RRA)

```
%Variable Initialization and Declaration
clear all
```

```
clc
syms s t;

%Process Transfer Function Elements
g11=5./(4*s+1);
g12=2.5*exp(-5*s)./((2*s+1).*(15*s+1));
g21=-4*exp(-6*s)./(20*s+1);
g22=1./(3*s+1);

%Average Integral Open-loop Response
y11ol=(1/(26-0))*int(ilaplace(g11*(1/s)),t,0,26);
y12ol=(1/(26-5))*int(ilaplace(g12*(1/s)),t,5,26);
y21ol=(1/(26-6))*int(ilaplace(g21*(1/s)),t,6,26);
y22ol=(1/(26-0))*int(ilaplace(g22*(1/s)),t,0,26);

%Overall Open-loop Response Matrix
yol=[y11ol y12ol; y21ol y22ol];

%Obtaning CI-TA-RRA
Roe=yol.*inv(yol)';

%For-loop for Results Printing
for i=1:1:2
    for j=1:1:2
        fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
    end
    fprintf('\n');
end
```

3. Controller-Independent Time-Varying RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=5./(4*s+1);
g12=2.5*exp(-5*s)./((2*s+1).*(15*s+1));
g21=-4*exp(-6*s)./(20*s+1);
```

```
g22=1./(3*s+1);

% For-loop for Open-loop Response
for p=0.1:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=(1/(26-0))*int(ilaplace(g11*(1/s)),t,0,p*26);
   y12ol=(1/(26-5))*int(ilaplace(g12*(1/s)),t,5,p*26);
   y21ol=(1/(26-6))*int(ilaplace(g21*(1/s)),t,6,p*26);
   y22ol=(1/(26-0))*int(ilaplace(g22*(1/s)),t,0,p*26);

   %Overall Open-loop Response Matrix
   yol=[y11ol y12ol; y21ol y22ol];

   %Obtaining CI-TV-RRA
   Roe=yol.*inv(yol)';

   %For-loop for Results Printing
   for i=1:1:2
      for j=1:1:2
         fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
      end
      fprintf('\n');
   end
end
```

4. Controller-Dependent Time-Average RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=5./(4*s+1);
g12=2.5*exp(-5*s)./((2*s+1).*(15*s+1));
g21=-4*exp(-6*s)./(20*s+1);
g22=1./(3*s+1);
```

```
%Average Open-loop Response of Transfer Function Elements
y11ol=(1/(26-0))*int(ilaplace(g11*(1/s)),t,0,26);
y12ol=(1/(26-5))*int(ilaplace(g12*(1/s)),t,5,26);
y21ol=(1/(26-6))*int(ilaplace(g21*(1/s)),t,6,26);
y22ol=(1/(26-0))*int(ilaplace(g22*(1/s)),t,0,26);
%IMC Filter Desgin for Closed-loop System
F11=1/(0.1*s+1);
P11=g11;
Q11=F11/P11;

F12=1/(0.1*s+1).^2;
P12=g12/exp(-5*s);
Q12=F12/P12;

F21=1/(0.1*s+1);
P21=g21/exp(-6*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22;
Q22=F22/P22;

%Average Closed-loop Response of Transfer Function Elements
y11cl=(1/(26-0))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,0,26);
y12cl=(1/(26-5))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,5,26);
y21cl=(1/(26-6))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,6,26);
y22cl=(1/(26-0))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,0,26);

% Obtaining CD-TA-RRA
Roe11=y11ol/y11cl;
Roe12=y12ol/y12cl;
Roe21=y21ol/y21cl;
Roe22=y22ol/y22cl;

%Printing CD-TA-RRA Results
fprintf('Roe11=%6.3f \t Roe12=%6.3f \t Roe21=%6.3f \t Roe22=%6.3f \n',
        double(Roe11),double(Roe12),double(Roe21),double(Roe22));
```

5. Controller-Dependent Time-Varying RRA (CD-TV-RRA)

```
%Variable Initialization and Declaration
```

```
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=5./(4*s+1);
g12=2.5*exp(-5*s)./((2*s+1).*(15*s+1));
g21=-4*exp(-6*s)./(20*s+1);
g22=1./(3*s+1);

%IMC Filter Desgin for Controller Dependent Open and Closed-loop System
F11=1/(0.1*s+1);
P11=g11;
Q11=F11/P11;

F12=1/(0.1*s+1).^2;
P12=g12/exp(-5*s);
Q12=F12/P12;

F21=1/(0.1*s+1);
P21=g21/exp(-6*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22;
Q22=F22/P22;

%For-loop for Open and Closed-loop Response
for p=0.0:0.1:1
    fprintf('For p=%2.2f \t',p);

    %Average Open-loop Response of Transfer Function Elements
    y11ol=(1/(26-0))*int(ilaplace(g11*(1/s)),t,0,p*26);
    y12ol=(1/(26-5))*int(ilaplace(g12*(1/s)),t,5,p*26);
    y21ol=(1/(26-6))*int(ilaplace(g21*(1/s)),t,6,p*26);
    y22ol=(1/(26-0))*int(ilaplace(g22*(1/s)),t,0,p*26);

    %Average Closed-loop Response of Transfer Function Elements
    y11cl=(1/(26-0))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,0,p*26);
    y12cl=(1/(26-5))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,5,p*26);
```

```
    y21cl=(1/(26-6))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,6,p*26);
    y22cl=(1/(26-0))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,0,p*26);

    %Obtaining CD-TV-RRA
    Roe11=y11ol/y11cl;
    Roe12=y12ol/y12cl;
    Roe21=y21ol/y21cl;
    Roe22=y22ol/y22cl;

    %Printing CD-TV-RRA Results
    fprintf('Roe11=%6.3f \t Roe12=%6.3f \t Roe21=%6.3f \t Roe22=%6.3f \n',
        double(Roe11),double(Roe12),double(Roe21),double(Roe22));
end
```

6. Uncertainty Analysis: Steady-State and Dynamic Approach

```
%Variable Declaration and Initialization
clc
k11=5;
k12=2.5;
k21=4;
k22=1;
tauar11=4;
tauar12=22;
tauar21=26;
tauar22=3;
n=1;

KN11=k11/tauar11;
KN12=k12/tauar12;
KN21=k21/tauar21;
KN22=k22/tauar22;

%Finding Steady-state and Dynamic Interaction Quotient
Kcap=((-1)^n)*((k12*k21)/(k11*k22));
KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));

%For-loop for Finding Kh, Kl and Uncertainty Bounds
for i=1:1:3
```

```
if i==1
   fprintf('Steady-State Analysis:\n');
   KNcap=Kcap;
   factor=2;

elseif i==2
   fprintf('Dynamic Analysis:\n');
   KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
   factor=2;

elseif i==3
   fprintf('Uncertain Gain-Residence Time:\n');
   KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
   factor=4;
end

if n==1|3
   fprintf('alpha=0.01\t');
   alpha=0.01;
   klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
   khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
   fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

   % RGA elements variation range
   if klmin<khmax
      upperlimit=(1/(1-khmax));
      lowerlimit=(1/(1-klmin));
      fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
   else
      upperlimit=(1/(1-khmax));
      lowerlimit=(1/(1-klmin));
      fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
      lowerlimit);
   end


   fprintf('alpha=0.05\t');
   alpha=0.05;
   klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
   khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
```

```
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.1\t');
alpha=0.1;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

   % RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.15\t');
alpha=0.15;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
```

```
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.2\t');
alpha=0.2;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.25\t');
alpha=0.25;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
```

```
end

fprintf('alpha=0.3\t');
alpha=0.3;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end


fprintf('alpha=0.35\t');
alpha=0.35;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end
```

```
fprintf('alpha=0.4\t');
alpha=0.4;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.45\t');
alpha=0.45;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.5\t');
alpha=0.5;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
```

```
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

    % RGA elements variation range
    if klmin<khmax
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
       lowerlimit);
    end

    fprintf('alpha=1.0\t');
    alpha=1;
    klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
    khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

    % RGA elements variation range
    if klmin<khmax
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
       lowerlimit);
    end


else
    fprintf('alpha=0.01\t');
    alpha=0.01;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
```

```
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.05\t');
alpha=0.05;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.1\t');
alpha=0.1;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
```

```
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=0.15\t');
alpha=0.15;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=0.2\t');
alpha=0.2;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
```

```
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=0.25\t');
alpha=0.25;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=0.3\t');
alpha=0.3;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end
```

```
fprintf('alpha=0.35\t');
alpha=0.35;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.4\t');
alpha=0.4;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
   lowerlimit);
end

fprintf('alpha=0.45\t');
alpha=0.45;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
```

```
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=0.5\t');
alpha=0.5;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
    upperlimit=(1/(1-khmax));
    lowerlimit=(1/(1-klmin));
    fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
    lowerlimit);
end

fprintf('alpha=1\t');
alpha=1;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
```

```
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
      else
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
        lowerlimit);
      end

    end
  end
```

APPENDIX - II

MATLAB CODE FOR THE CASE STUDY – 2

2×2 Process Model with typical Process Dynamics

The Matlab codes used in generating the results for various methods used in this study are as follows:

1. Frequency Dependent RGA

```
%Script File for the Computing Dynamic RGA
%Define a Vector of Frequency Values on a Log Scale
w=logspace(-3,0,100);
s=i*w;

%Compute the Frequency Response for each Element of gij
g11=-0.805./((18.3*s+1).*(5.6*s+1));
g12=0.055./((5.76*s+1).*(1.25*s+1));
g21=-0.465*exp(-0.3*s)./((28.3*s+1).*(0.62*s+1));
g22=-0.055./(3.3*s+1);

%Compute Lambda(i,j) as a Function of Frequency
L12=-g12.*g21./(g11.*g22-g12.*g21);
lam12=sign(real(L12)).*abs(L12);

%Using RGA Property of Unity Sum
 lam11=1-lam12;

%Plot the Results
semilogx(w,lam11,'-k',w,lam12,'--k')
xlabel('Frequency[rad/min]')
ylabel('RGA (1,1) and (1,2)')
```

2. Controller-Independent Time-Average RRA (CI-TA-RRA)

```
%Variable Initialization and Declaration
clear all
```

```
clc
syms s t;

%Process Transfer Function Elements
g11=-0.805./((18.3*s+1).*(5.6*s+1));
g12=0.055./((5.76*s+1).*(1.25*s+1));
g21=-0.465*exp(-0.3*s)./((28.3*s+1).*(0.62*s+1));
g22=-0.055./(3.3*s+1);

%Average Integral Open-loop Response
y11ol=(1/(10.42-0))*int(ilaplace(g11*(1/s)),t,0,10.42);
y12ol=(1/(10.42-0))*int(ilaplace(g12*(1/s)),t,0,10.42);
y21ol=(1/(10.42-0.3))*int(ilaplace(g21*(1/s)),t,0.3,10.42);
y22ol=(1/(10.42-0))*int(ilaplace(g22*(1/s)),t,0,10.42);

%Overall Open-loop Response Matrix
yol=[y11ol y12ol; y21ol y22ol];

%Obtaining CI-TA-RRA
Roe=yol.*inv(yol)';

%For-loop for Results Printing
for i=1:1:2
    for j=1:1:2
        fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
    end
    fprintf('\n');
end
```

3. Controller-Independent Time-Varying RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=-0.805./((18.3*s+1).*(5.6*s+1));
g12=0.055./((5.76*s+1).*(1.25*s+1));
g21=-0.465*exp(-0.3*s)./((28.3*s+1).*(0.62*s+1));
```

```
g22=-0.055./(3.3*s+1);

% For-loop for Open-loop Response
for p=0.1:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=(1/(10.42-0))*int(ilaplace(g11*(1/s)),t,0,p*10.42);
   y12ol=(1/(10.42-0))*int(ilaplace(g12*(1/s)),t,0,p*10.42);
   y21ol=(1/(10.42-0.3))*int(ilaplace(g21*(1/s)),t,0.3,p*10.42);
   y22ol=(1/(10.42-0))*int(ilaplace(g22*(1/s)),t,0,p*10.42);

   %Overall Open-loop Response Matrix
   yol=[y11ol y12ol; y21ol y22ol];

   %Obtaining CI-TV-RRA
   Roe=yol.*inv(yol)';

   %For-loop for Results Printing
   for i=1:1:2
      for j=1:1:2
         fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
      end
      fprintf('\n');
   end
end
```

4. Controller-Dependent Time-Average RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=-0.805./((18.3*s+1).*(5.6*s+1));
g12=0.055./((5.76*s+1).*(1.25*s+1));
g21=-0.465*exp(-0.3*s)./((28.3*s+1).*(0.62*s+1));
g22=-0.055./(3.3*s+1);
```

```
%Average Open-loop Response of Transfer Function Elements
y11ol=(1/(10.42-0))*int(ilaplace(g11*(1/s)),t,0,10.42);
y12ol=(1/(10.42-0))*int(ilaplace(g12*(1/s)),t,0,10.42);
y21ol=(1/(10.42-0.3))*int(ilaplace(g21*(1/s)),t,0.3,10.42);
y22ol=(1/(10.42-0))*int(ilaplace(g22*(1/s)),t,0,10.42);

%IMC Filter Desgin for Closed-loop System
F11=1/(0.1*s+1)^2;
P11=g11;
Q11=F11/P11;

F12=1/(0.1*s+1).^2;
P12=g12;
Q12=F12/P12;

F21=1/(0.1*s+1)^2;
P21=g21/exp(-0.3*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22;
Q22=F22/P22;

%Average Closed-loop Response of Transfer Function Elements
y11cl=(1/(10.42-0))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,0,10.42);
y12cl=(1/(10.42-0))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,0,10.42);
y21cl=(1/(10.42-0.3))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,0.3,10.42);
y22cl=(1/(10.42-0))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,0,10.42);

% Obtaining CD-TA-RRA
Roe11=y11ol/y11cl;
Roe12=y12ol/y12cl;
Roe21=y21ol/y21cl;
Roe22=y22ol/y22cl;

%Printing CD-TA-RRA Results
fprintf('Roe11=%6.3f \t Roe12=%6.3f \t Roe21=%6.3f \t Roe22=%6.3f \n',
        double(Roe11),double(Roe12),double(Roe21),double(Roe22));
```

5. Controller-Dependent Time-Varying RRA (CD-TV-RRA)

```
%Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=-0.805./((18.3*s+1).*(5.6*s+1));
g12=0.055./((5.76*s+1).*(1.25*s+1));
g21=-0.465*exp(-0.3*s)./((28.3*s+1).*(0.62*s+1));
g22=-0.055./(3.3*s+1);

%IMC Filter Design for Controller Dependent Open and Closed-loop System
F11=1/(0.1*s+1)^2;
P11=g11;
Q11=F11/P11;

F12=1/(0.1*s+1).^2;
P12=g12;
Q12=F12/P12;

F21=1/(0.1*s+1)^2;
P21=g21/exp(-0.3*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22;
Q22=F22/P22;

% For-loop for Open and Closed-loop Response
for p=0.1:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=(1/(10.42-0))*int(ilaplace(g11*(1/s)),t,0,p*10.42);
   y12ol=(1/(10.42-0))*int(ilaplace(g12*(1/s)),t,0,p*10.42);
   y21ol=(1/(10.42-0.3))*int(ilaplace(g21*(1/s)),t,0.3,p*10.42);
   y22ol=(1/(10.42-0))*int(ilaplace(g22*(1/s)),t,0,p*10.42);

   %Average Closed-loop Response of Transfer Function Elements
```

```
    y11cl=(1/(10.42-0))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,0,p*10.42);
    y12cl=(1/(10.42-0))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,0,p*10.42);
    y21cl=(1/(10.42-0.3))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,0.3,p*10.42);
    y22cl=(1/(10.42-0))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,0,p*10.42);
    %Obtaining CD-TV-RRA
    Roe11=y11ol/y11cl;
    Roe12=y12ol/y12cl;
    Roe21=y21ol/y21cl;
    Roe22=y22ol/y22cl;

    %Printing CD-TV-RRA Results
    fprintf('Roe11=%6.3f \t Roe12=%6.3f \t Roe21=%6.3f \t Roe22=%6.3f
        \n', double(Roe11),double(Roe12),double(Roe21),double(Roe22));
end
```

6. Uncertainty Analysis: Steady-State and Dynamic Approach

```
%Variable Initialization and Declaration
clc
k11=0.805;
k12=0.055;
k21=0.465;
k22=0.055;
tauar11=23.9;
tauar12=7.01;
tauar21=29.22;
tauar22=3.3;
n=3;

KN11=k11/tauar11;
KN12=k12/tauar12;
KN21=k21/tauar21;
KN22=k22/tauar22;

%Finding Steady-state and Dynamic Interaction Quotient
Kcap=((-1)^n)*((k12*k21)/(k11*k22));
KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));

%For-loop for Finding Kh, Kl and Uncertainty Bounds
for i=1:1:3

    if i==1
        fprintf('Steady-State Analysis:\n');
        KNcap=Kcap;
```

```
        factor=2;

    elseif i==2
        fprintf('Dynamic Analysis:\n');
        KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
        factor=2;

    elseif i==3
        fprintf('Uncertain Gain-Residence Time:\n');
        KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
        factor=4;
    end

    if n==1|3
        fprintf('alpha=0.01\t');
        alpha=0.01;
        klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
        khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

        fprintf('alpha=0.05\t');
        alpha=0.05;
        klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
        khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
```

```
end

fprintf('alpha=0.1\t');
alpha=0.1;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
end

fprintf('alpha=0.15\t');
alpha=0.15;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
end

fprintf('alpha=0.2\t');
alpha=0.2;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
```

```
    else
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.25\t');
    alpha=0.25;
    klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
    khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
    % RGA elements variation range
    if klmin<khmax
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.3\t');
    alpha=0.3;
    klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
    khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
    % RGA elements variation range
    if klmin<khmax
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
       upperlimit=(1/(1-khmax));
       lowerlimit=(1/(1-klmin));
       fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.5\t');
    alpha=0.5;
    klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
    khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
```

```
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

    else
        fprintf('alpha=0.01\t');
        alpha=0.01;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

        fprintf('alpha=0.05\t');
        alpha=0.05;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end
```

```
        fprintf('alpha=0.1\t');
        alpha=0.1;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

        fprintf('alpha=0.15\t');
        alpha=0.15;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

        fprintf('alpha=0.2\t');
        alpha=0.2;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
        % RGA elements variation range
        if klmin<khmax
           upperlimit=(1/(1-khmax));
           lowerlimit=(1/(1-klmin));
           fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
```

```
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.25\t');
    alpha=0.25;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
    % RGA elements variation range
    if klmin<khmax
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.3\t');
    alpha=0.3;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
    % RGA elements variation range
    if klmin<khmax
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
    end

    fprintf('alpha=0.5\t');
    alpha=0.5;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
    % RGA elements variation range
```

```
        if klmin<khmax
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<= inf\n',upperlimit,
lowerlimit);
        end

    end
end
```

APPENDIX - III

MATLAB CODE FOR THE CASE STUDY – 3

Shell Heavy Oil Fractionator Problem

The Matlab codes used in generating the results for various methods used in this study are as follows:

1. Frequency Dependent RGA

```
%Script File for the Computing Dynamic RGA
%Define a Vector of Frequency Values on a Log Scale
w=logspace(-3,0,100);
s=i*w;

%Compute the Frequency Response for each Element of gij
g11=4.05*exp(-27*s)./(50*s+1);
g12=1.77*exp(-28*s)./(60*s+1);
g21=5.39*exp(-18*s)./(50*s+1);
g22=5.72*exp(-14*s)./(60*s+1);

%Compute Lambda(i,j) as a Function of Frequency
L12=-g12.*g21./(g11.*g22-g12.*g21);
lam12=sign(real(L12)).*abs(L12);

%Using RGA Property of Unity Sum
lam11=1-lam12;

%Plot the Results
semilogx(w,lam11,'-k',w,lam12,'--k')
xlabel('Frequency[rad/min]')
ylabel('RGA (1,1) and (1,2)')
```

2. Controller-Independent Time-Average RRA (CI-TA-RRA)

```
%Variable Initialization and Declaration
clear all
```

```
clc
syms s t;

%Process Transfer Function Elements
g11=4.05*exp(-27*s)./(50*s+1);
g12=1.77*exp(-28*s)./(60*s+1);
g21=5.39*exp(-18*s)./(50*s+1);
g22=5.72*exp(-14*s)./(60*s+1);

%Average Integral Open-loop Response
y11ol=(1/(88-27))*int(ilaplace(g11*(1/s)),t,27,88);
y12ol=(1/(88-28))*int(ilaplace(g12*(1/s)),t,28,88);
y21ol=(1/(88-18))*int(ilaplace(g21*(1/s)),t,18,88);
y22ol=(1/(88-14))*int(ilaplace(g22*(1/s)),t,14,88);

%Overall Open-loop Response Matrix
yol=[y11ol y12ol; y21ol y22ol];

%Obtaining CI-TA-RRA
Roe=yol.*inv(yol)';

%For-loop for Results Printing
for i=1:1:2
   for j=1:1:2
      fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
   end
   fprintf('\n');
end
```

3. Controller-Independent Time-Varying RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=4.05*exp(-27*s)./(50*s+1);
g12=1.77*exp(-28*s)./(60*s+1);
g21=5.39*exp(-18*s)./(50*s+1);
```

```
g22=5.72*exp(-14*s)./(60*s+1);

% For-loop for Open-loop Response
for p=0.32:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=(1/(88-27))*int(ilaplace(g11*(1/s)),t,27,p*88);
   y12ol=(1/(88-28))*int(ilaplace(g12*(1/s)),t,28,p*88);
   y21ol=(1/(88-18))*int(ilaplace(g21*(1/s)),t,18,p*88);
   y22ol=(1/(88-14))*int(ilaplace(g22*(1/s)),t,14,p*88);

   %Overall Open-loop Response Matrix
   yol=[y11ol y12ol; y21ol y22ol];

   %Obtaining CI-TV-RRA
   Roe=yol.*inv(yol)';

   %For-loop for Results Printing
   for i=1:1:2
      for j=1:1:2
         fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
      end
      fprintf('\n');
   end
end
```

4. Controller-Dependent Time-Average RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=4.05*exp(-27*s)./(50*s+1);
g12=1.77*exp(-28*s)./(60*s+1);
g21=5.39*exp(-18*s)./(50*s+1);
g22=5.72*exp(-14*s)./(60*s+1);
```

```
%Average Open-loop Response of Transfer Function Elements
y11ol=(1/(88-27))*int(ilaplace(g11*(1/s)),t,27,88);
y12ol=(1/(88-28))*int(ilaplace(g12*(1/s)),t,28,88);
y21ol=(1/(88-18))*int(ilaplace(g21*(1/s)),t,18,88);
y22ol=(1/(88-14))*int(ilaplace(g22*(1/s)),t,14,88);

%IMC Filter Desgin for Closed-loop System
F11=1/(0.1*s+1);
P11=g11/exp(-27*s);
Q11=F11/P11;

F12=1/(0.1*s+1);
P12=g12/exp(-28*s);
Q12=F12/P12;

F21=1/(0.1*s+1);
P21=g21/exp(-18*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22/exp(-14*s);
Q22=F22/P22;

%Average Closed-loop Response of Transfer Function Elements
y11cl=(1/(88-27))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,27,88);
y12cl=(1/(88-28))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,28,88);
y21cl=(1/(88-18))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,18,88);
y22cl=(1/(88-14))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,14,88);

% Obtaining CD-TA-RRA
Roe11=y11ol/y11cl;
Roe12=y12ol/y12cl;
Roe21=y21ol/y21cl;
Roe22=y22ol/y22cl;

%Printing CD-TA-RRA Results
fprintf('Roe11=%6.3f \t Roe12=%6.3f \t Roe21=%6.3f \t Roe22=%6.3f \n',
          double(Roe11),double(Roe12),double(Roe21),double(Roe22));
```

5. Controller-Dependent Time-Varying RRA (CD-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=4.05*exp(-27*s)./(50*s+1);
g12=1.77*exp(-28*s)./(60*s+1);
g21=5.39*exp(-18*s)./(50*s+1);
g22=5.72*exp(-14*s)./(60*s+1);

%IMC Filter Desgin for Controller Dependent Open and Closed-loop System
F11=1/(0.1*s+1);
P11=g11/exp(-27*s);
Q11=F11/P11;

F12=1/(0.1*s+1);
P12=g12/exp(-28*s);
Q12=F12/P12;

F21=1/(0.1*s+1);
P21=g21/exp(-18*s);
Q21=F21/P21;

F22=1/(0.1*s+1);
P22=g22/exp(-14*s);
Q22=F22/P22;

% For-loop for Open and Closed-loop Response
for p=0.1:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=(1/(88-27))*int(ilaplace(g11*(1/s)),t,27,p*88);
   y12ol=(1/(88-28))*int(ilaplace(g12*(1/s)),t,28,p*88);
   y21ol=(1/(88-18))*int(ilaplace(g21*(1/s)),t,18,p*88);
   y22ol=(1/(88-14))*int(ilaplace(g22*(1/s)),t,14,p*88);

   %Average Closed-loop Response of Transfer Function Elements
```

```
    y11cl=(1/(88-27))*int(ilaplace((g11-g12*Q22*g21)*(1/s)),t,27,p*88);
    y12cl=(1/(88-28))*int(ilaplace((g12-g11*Q21*g22)*(1/s)),t,28,p*88);
    y21cl=(1/(88-18))*int(ilaplace((g21-g22*Q12*g11)*(1/s)),t,18,p*88);
    y22cl=(1/(88-14))*int(ilaplace((g22-g21*Q11*g12)*(1/s)),t,14,p*88);
   %Obtaining CD-TV-RRA
    Roe11=y11ol/y11cl;
    Roe12=y12ol/y12cl;
    Roe21=y21ol/y21cl;
    Roe22=y22ol/y22cl;

    %Printing CD-TV-RRA Results
    fprintf('Roe11=%6.3f\tRoe12=%6.3f\tRoe21=%6.3f\tRoe22=%6.3f\n',
          double(y11ol),double(y11cl),double(y12ol),double(y12cl));
end
```

6. Uncertainty Analysis: Steady-state and Dynamic Approach

```
%Variable Declaration and Initialization
clc
k11=4.05;
k12=1.77;
k21=5.39;
k22=5.72;

tauar11=77;
tauar12=88;
tauar21=68;
tauar22=74;

n=0;

% Normalized Gain Calculation
KN11=k11/tauar11;
KN12=k12/tauar12;
KN21=k21/tauar21;
KN22=k22/tauar22;

%Finding Steady-state and Dynamic Interaction Quotient
Kcap=((-1)^n)*((k12*k21)/(k11*k22));
KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
```

```
%For-loop for Finding Kh, Kl and Uncertainty Bounds
for i=1:1:3,

   if i==1
      fprintf('Steady-State Analysis:\n');
      KNcap=Kcap;
      factor=2;

   elseif i==2
      fprintf('Dynamic Analysis:\n');
      KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
      factor=2;

   else
      fprintf('Uncertain Gain-Residence Time:\n');
      KNcap=((-1)^n)*((KN12*KN21)/(KN11*KN22));
      factor=4;
   end

   if n==1||n==3
      fprintf('No. of negative elements: %f \t alpha=0.01\t',n);
      alpha=0.11;
      klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
      khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
      fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
      % RGA elements variation range
      if klmin<khmax
         upperlimit=(1/(1-khmax));
         lowerlimit=(1/(1-klmin));
         fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
      else
         upperlimit=(1/(1-khmax));
         lowerlimit=(1/(1-klmin));
         fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                  inf\n',upperlimit, lowerlimit);
      end


      fprintf('alpha=0.05\t');
```

```
alpha=0.05;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.1\t');
alpha=0.1;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.2\t');
alpha=0.2;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);
```

```
% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.3\t');
alpha=0.3;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.5\t');
alpha=0.5;
klmin=KNcap*((1+factor*alpha)/(1-factor*alpha));
khmax=KNcap*((1-factor*alpha)/(1+factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
```

```
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                inf\n',upperlimit, lowerlimit);
    end
end

if n==0
    fprintf('Hello 2 alpha=0.01\t');
    alpha=0.11;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

    % RGA elements variation range
    if klmin<khmax
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
    else
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                inf\n',upperlimit, lowerlimit);
    end

    fprintf('alpha=0.05\t');
    alpha=0.05;
    klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
    khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
    fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

    % RGA elements variation range
    if klmin<khmax
        upperlimit=(1/(1-khmax));
        lowerlimit=(1/(1-klmin));
        fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
```

```
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.1\t');
alpha=0.1;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f \n',klmin, khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
            inf\n',upperlimit, lowerlimit);
end

fprintf('alpha=0.2\t');
alpha=0.2;
klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

% RGA elements variation range
if klmin<khmax
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
   fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
else
   upperlimit=(1/(1-khmax));
   lowerlimit=(1/(1-klmin));
```

```
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                  inf\n',upperlimit, lowerlimit);
        end

        fprintf('alpha=0.3\t');
        alpha=0.3;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

        % RGA elements variation range
        if klmin<khmax
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                  inf\n',upperlimit, lowerlimit);
        end
        fprintf('alpha=0.5\t');
        alpha=0.5;
        klmin=KNcap*((1-factor*alpha)/(1+factor*alpha));
        khmax=KNcap*((1+factor*alpha)/(1-factor*alpha));
        fprintf('klmin=%2.4f \t khmax=%2.4f\n',klmin,khmax);

        % RGA elements variation range
        if klmin<khmax
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('%2.4f <=lemda11<= %2.4f\n',lowerlimit,upperlimit);
        else
            upperlimit=(1/(1-khmax));
            lowerlimit=(1/(1-klmin));
            fprintf('-inf <=lemda11<= %2.4f & %2.4f <=lemda11<=
                  inf\n',upperlimit, lowerlimit);
        end
    end
end
```

APPENDIX - IV

MATLAB CODE FOR THE CASE STUDY – 4

DL Distillation Column Control Problem

The Matlab codes used in generating the results for various methods used in this study are as follows:

1. Frequency Dependent RGA

```
%Script File for the Computing Dynamic RGA
%Define a Vector of Frequency Values on a Log Scale
w=logspace(-3,1,100);
s=i*w;

%Compute the Frequency Response for each Element of gij
g11=0.374*exp(-7.75*s)./((22.2*s+1).^2);
g12=-11.3*exp(-3.79*s)./((21.74*s+1).*(21.74*s+1));
g13=-9.811*exp(-1.59*s)./(11.36*s+1);
g21=-1.986*exp(-0.71*s)./((66.67*s+1).*(66.67*s+1));
g22=5.24*exp(-60*s)./(400*s+1);
g23=5.984*exp(-2.24*s)./(14.29*s+1);
g31=0.0204*exp(-0.59*s)./((7.14*s+1).*(7.14*s+1));
g32=-0.33*exp(-0.68*s)./((2.38*s+1).*(2.38*s+1));
g33=2.38*exp(-0.42*s)./((1.43*s+1).*(1.43*s+1));
G=(g11.*((g22.*g33)-(g32.*g23)))-(g12.*((g21.*g33)-(g31.*g23)))
                        +(g13.*((g21.*g32)-(g31.*g22)));

%Compute Lambda(i,j) as a Function of Frequency
L11= (g11.*((g22.*g33)-(g23.*g32)))./(G);
L12=-(g12.*((g21.*g33)-(g31.*g23)))./(G);
L13= (g13.*((g21.*g32)-(g31.*g22)))./(G);
L21=-(g21.*((g12.*g33)-(g13.*g32)))./(G);
L22= (g22.*((g11.*g33)-(g31.*g13)))./(G);
L23=-(g23.*((g11.*g32)-(g31.*g12)))./(G);
L31= (g31.*((g12.*g23)-(g13.*g22)))./(G);
L32=-(g32.*((g11.*g23)-(g21.*g13)))./(G);
L33= (g33.*((g11.*g22)-(g12.*g21)))./(G);

lam11=sign(real(L11)).*abs(L11);
lam12=sign(real(L12)).*abs(L12);
```

```
lam13=sign(real(L13)).*abs(L13);
lam21=sign(real(L21)).*abs(L21);
lam22=sign(real(L22)).*abs(L22);
lam23=sign(real(L23)).*abs(L23);
lam31=sign(real(L31)).*abs(L31);
lam32=sign(real(L32)).*abs(L32);
lam33=sign(real(L33)).*abs(L33);

%Plot the Results

semilogx(w,lam11,'k',w,lam12,'r',w,lam13,'b')
xlabel('Frequency[rad/min]')
ylabel('RGA (1,1), (1,2) and (1,3)')

semilogx(w,lam21,'k',w,lam22,'r',w,lam23,'b')
xlabel('Frequency[rad/min]')
ylabel('RGA (2,1), (2,2) and (2,3)')

semilogx(w,lam31,'k',w,lam32,'r',w,lam33,'b')
xlabel('Frequency[rad/min]')
ylabel('RGA (3,1), (3,2) and (3,3)')

%Plotting the Result in the Same Graph
% subplot(3,1,1),semilogx(w,lam11,'k',w,lam12,'r',w,lam13,'b')
% subplot(3,1,2),semilogx(w,lam21,'k',w,lam22,'r',w,lam23,'b')
% subplot(3,1,3),semilogx(w,lam31,'k',w,lam32,'r',w,lam33,'b')
%
% subplot(3,1,1),xlabel('Frequency[rad/min]')
% subplot(3,1,1),ylabel('RGA (1,1), (1,2) and (1,3)')
%
% subplot(3,1,2),xlabel('Frequency[rad/min]')
% subplot(3,1,2),ylabel('RGA (2,1), (2,2) and (2,3)')
%
% subplot(3,1,3),xlabel('Frequency[rad/min]')
% subplot(3,1,3),ylabel('RGA (3,1), (3,2) and (3,3)')
```

2. Controller-Independent Time-Average RRA (CI-TA-RRA)

```
%Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=0.374*exp(-7.75*s)./(22.2*s+1).^2;
g12=-11.3*exp(-3.79*s)./(21.74*s+1).^2;
```

```
g13=-9.811*exp(-1.59*s)./(11.36*s+1);
g21=-1.986*exp(-0.71*s)./(66.67*s+1).^2;
g22=5.24*exp(-60*s)./(400*s+1);
g23=5.984*exp(-2.24*s)./(14.29*s+1);
g31=0.0204*exp(-0.59*s)./(7.14*s+1).^2;
g32=-0.33*exp(-0.68*s)./(2.38*s+1).^2;
g33=2.38*exp(-0.42*s)./(1.43*s+1).^2;

%Average Integral Open-loop Response
y11ol=(1/(460-7.75))*int(ilaplace(g11*(1/s)),t,7.75,460);
y12ol=(1/(460-3.79))*int(ilaplace(g12*(1/s)),t,3.79,460);
y13ol=(1/(460-1.59))*int(ilaplace(g13*(1/s)),t,1.59,460);
y21ol=(1/(460-0.71))*int(ilaplace(g21*(1/s)),t,0.71,460);
y22ol=(1/(460-60))*int(ilaplace(g22*(1/s)),t,60,460);
y23ol=(1/(460-2.24))*int(ilaplace(g23*(1/s)),t,2.24,460);
y31ol=(1/(460-0.59))*int(ilaplace(g31*(1/s)),t,0.59,460);
y32ol=(1/(460-0.68))*int(ilaplace(g32*(1/s)),t,0.68,460);
y33ol=(1/(460-0.42))*int(ilaplace(g33*(1/s)),t,0.42,460);

%Overall Open-loop Response Matrix
yol=[y11ol y12ol y13ol; y21ol y22ol y23ol; y31ol y32ol y33ol];

%Obtaning CI-TA-RRA
Roe=yol.*inv(yol)';

%For-loop for Results Printing
for i=1:1:3
   for j=1:1:3
      fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
   end
   fprintf('\n');
end
```

3. Controller-Independent Time-Varying RRA (CI-TV-RRA)

```
% Variable Initialization and Declaration
clear all
clc
syms s t;

%Process Transfer Function Elements
g11=0.374*exp(-7.75*s)./(22.2*s+1).^2;
g12=-11.3*exp(-3.79*s)./(21.74*s+1).^2;
g13=-9.811*exp(-1.59*s)./(11.36*s+1);
g21=-1.986*exp(-0.71*s)./(66.67*s+1).^2;
g22=5.24*exp(-60*s)./(400*s+1);
```

```
g23=5.984*exp(-2.24*s)./(14.29*s+1);
g31=0.0204*exp(-0.59*s)./(7.14*s+1).^2;
g32=-0.33*exp(-0.68*s)./(2.38*s+1).^2;
g33=2.38*exp(-0.42*s)./(1.43*s+1).^2;

% For-loop for Open-loop Response
for p=0.0:0.1:1
   fprintf('For p=%2.2f \t',p);

   %Average Open-loop Response of Transfer Function Elements
   y11ol=double(int(ilaplace(g11*(1/s)),t,7.75,p*460));
   y12ol=double(int(ilaplace(g12*(1/s)),t,3.79,p*460));
   y13ol=double(int(ilaplace(g13*(1/s)),t,1.59,p*460));
   y21ol=double(int(ilaplace(g21*(1/s)),t,0.71,p*460));
   y22ol=double(int(ilaplace(g22*(1/s)),t,60,p*460));
   y23ol=double(int(ilaplace(g23*(1/s)),t,2.24,p*460));
   y31ol=double(int(ilaplace(g31*(1/s)),t,0.59,p*460));
   y32ol=double(int(ilaplace(g31*(1/s)),t,0.68,p*460));
   y33ol=double(int(ilaplace(g33*(1/s)),t,0.42,p*460));

   %Overall Open-loop Response Matrix
   yol=[y11ol y12ol y13ol; y21ol y22ol y23ol; y31ol y32ol y33ol];

   %Obtaining CI-TV-RRA
   Roe=yol.*inv(yol)';

   %For-loop for Results Printing
   for i=1:1:3
      for j=1:1:3
         fprintf('\t\tRoe%1.0f%1.0f=%3.3f',i,j,double(Roe(i,j)));
      end
      fprintf('\n');
   end
end
```