Abstract

The relationship between predator and its prey is one of the dominant area in ecology and applied mathematics. Mathematical models are powerful tool to analyze the dynamical complexity of such relationships. Formulation of a model with appropriate characteristics and its dynamical analysis would provide the clear view about the interaction among species. There are numerous factors involved with every prey-predator system such as carrying capacity, searching and attack rate, handling time, conversion efficiency, etc. In addition, several ecological aspects can also be found in the evolution of models describing the prey-predator interaction in different situations. Complex habitat, feeding switching, fear of enemy, providing additional food, and time delay are some of them. In the classical ecological models, the effect of these aspects is neglected.

This thesis aims at constructing and analyzing the dynamics of prey-predator interaction models with different characteristics and to observe how significant they are. The growing population need more energy and resources to consume but the environment has a finite carrying capacity for each natural resource. It can lead to over exploitation of these resources. To get rid of this, we need to develop some optimal policies for conservation of resources. Refuge is also a good idea for conservation of a species where a fraction of total population is kept away from consumption. In the ecological models, functional response is the intake rate of a consumer as a function of food density. In classical models, the intake rate has been taken as law of mass action which is directly proportional to the available resource. In the case of abundance of resources, use of law of mass action is not suitable to address the interaction. In this situation nonlinear consumption rate gives better insight due to its saturated nature.

Time delay occurs so often in almost every situation, that to ignore them is to ignore reality. Models with delay are much more realistic then non-delayed models. There are several time delays that represent lag in response of a specific process or action such as hunting, digestion of food, defence in prey population, maturation, negative feedback, etc. In our thesis, we incorporate these discrete delays in some models and observed that delay is capable to change the dynamics of the system.

Group hunting is a vital characteristic among predators that occurs when more than one predator individual hunt on a common victim. Cooperation in hunting increases the success rate of predators. Theoretical ecologists have shown that spotted hyena succeeded in 15% of solo hunt of wildebeest calves but it increases up to 74% during group hunts. Even lions are not an exception to group hunting, they hunt on Thomson's gazelle succeeded in 15% of solitary attack but in 32% of communal attack. In classical models, the effect of fear induced by predators was

ignored because at that time it was not observable in the field and no such evidence existed. The separation of fear effect from direct killing was being possible just because of the tremendous experimental work by several theoretical ecologists in recent years.

Acknowledging these assertions, we have proposed and analyzed some prey-predator models in the form of system of nonlinear ordinary differential equations (ODEs) and delay differential equations (DDEs) incorporating various parameters. These models have been analyzed using the stability theory and bifurcation and chaos theory. Computer simulation have also been carried out to validate the obtained results. This thesis is divided into eight separate chapters whose abstracts are given below.

In Chapter 1, a brief introduction about the problem with literature survey is presented. We provide background of the problem, objectives of the thesis, some definitions and mathematical tools that are frequently used in the upcoming chapters. Some basic population models, functional response and its type, several common bifurcations with their typical examples are also given in this chapter.

Recently, some field experiments and studies show that predators affect prey not only by direct killing, they induce fear in prey which reduces the reproduction rate of prey species. Considering this fact, chapter 2 of the thesis is devoted to a mathematical model to study the fear effect and prey refuge in prey–predator system with gestation time delay. It is assumed that prey population grows logistically in the absence of predators and the interaction between prey and predator is followed by Crowley–Martin type functional response. We obtained the equilibrium points and studied the local and global asymptotic behaviors of nondelayed system around them. It is observed from our analysis that the fear effect in the prey induces Hopf-bifurcation in the system. It is concluded that the refuge of prey population under a threshold level is lucrative for both the species. Further, we incorporate gestation delay of the predator population in the model. Local and global asymptotic stabilities for delayed model are carried out. The existence of stable limit cycle via Hopf-bifurcation with respect to delay parameter is established. Chaotic oscillations are also observed and confirmed by drawing the bifurcation diagram and evaluating maximum Lyapunov exponent for large values of delay parameter.

In chapter 3, we consider a two-dimensional prey-predator system with two delays. One delay is for negative feedback of the prey population and another is for gestation delay of the predator population. The predator is partially dependent on the prey followed by Holling type-II functional response. Due to habitat complexity and prey refuge, the Holling type-II functional response is modified in this work. We discuss the boundedness, permanence, local and global asymptotic behavior of the non-delayed and delayed models. The existence of periodic solutions via Hopf-bifurcation with respect to both the delays is established. The stability and direction of Hopf-bifurcation is also analyzed by using Normal form theory and Centre manifold theory. Lastly, numerical simulations have been carried out to confirm the analytical findings. The main objective of this work is to balance the prey-predator relationship in the presence of habitat complexity, prey refuge and delays.

The rate of reproduction and survival of species largely depend upon their age. A juvenile predator cannot involve in predation and reproduction of new progeny. It depends on parents for food. In the chapter 4 of the thesis, a three-dimensional dynamical model consisting of a prey, a mature predator and an immature predator is proposed and analyzed. The interaction between prey and mature predator is assumed to be of the Crowley–Martin type, and both the prey and mature predator are harvested according to catch-per-unit-effort (CPUE) hypothesis. Steady state of the system is obtained, stability analysis (local and global both) are discussed to explore the longtime behaviour of the system. The optimal harvesting policy is also discussed with the help of Pontryagin's maximum principle. The harvesting effort is taken as an effective control instrument to preserve prey and predator and to maintain them at an optimal level.

On the other side, if prey is immature then predator get more attraction towards it. The growth of immature prey depends upon the population size of mature prey. Chapter 5 proposes a three-dimensional prey–predator model with stage structure in prey (immature and mature) including maturation delay in prey population and gestation delay in predator population. It is assumed that the immature prey population is consumed by predators with Holling type I functional response and the interaction between mature prey and predator species is followed by Crowley–Martin type functional response. We analyzed the equilibrium points, local and global asymptotic behavior of interior equilibrium point for the non-delayed system. Hopf-bifurcation with respect to different parameters has also been studied for the system. Further, the existence of periodic solutions through Hopf-bifurcation is shown with respect to both the delays. Our model analysis shows that time delay plays a vital role in governing the dynamics of the system. It changes the stability behavior of the system into instability, even with the switching of stability. The direction and stability of Hopf-bifurcation are also studied by using normal form method and center manifold theorem. Finally, computer simulation and graphical illustrations have been carried out to support our theoretical investigations.

In chapter 6, we propose two prey–predator models with strong and weak Allee effects in prey population with Crowley–Martin functional response. Further, gestation delay of the predator population is introduced in both the models. We discussed the boundedness, local stability and Hopf-bifurcation of both non-delayed and delayed systems. The stability and direction of Hopf-bifurcation is also analyzed by using Normal form theory and Center manifold theory. It is shown that species in the model with strong Allee effect become extinct beyond a threshold value of Allee parameter at low density of prey population, whereas species never become extinct in weak Allee effect if they are initially present. It is also shown that gestation delay is unable to avoiding the status of extinction. Lastly, numerical simulation is conducted to verify the theoretical findings.

In chapter 7, the impact of additional food and two discrete delays on the dynamics of a prey-predator model is investigated. The interaction between prey and predator is considered as Holling Type-II functional response. The additional food is provided to the predator to reduce its dependency on the prey. One delay is the gestation delay in predator while the other delay is the delay in supplying the additional food to predators. The positivity, boundedness and persistence of the solutions of the system are studied to show the system as biologically well-behaved. The existence of steady states, their local and global asymptotic behavior for the non-delayed system are investigated. It is shown that (i) predator's dependency factor on additional food induces a periodic solution in the system, and (ii) the two delays considered in the system are capable to change the status of the stability behavior of the system. The existence of periodic solutions via Hopf-bifurcation is shown with respect to both the delays. Our analysis shows that both delay parameters play an important role in governing the dynamics of the system. The direction and stability of Hopf-bifurcation are also investigated through the normal form theory and the center manifold theorem. Numerical experiments are also conducted to validate the theoretical results.

Chapter 8 is devoted to the analysis of the dynamics of a prey-predator model with fear effect in prey and team hunting by predator. Moreover, effects of anti-predation response delay and cooperation delay are also investigated. The persistence, stability (local and global) and several types of bifurcations (saddle-node, transcritical, Hopf-bifurcation) are discussed. We have examined the effect of cost of fear, attack rate and delay parameters on the dynamics of the model by taking them as bifurcation parameter. We carried out the dynamics of the model in various bi-parameter planes to examine the change in bifurcation value on varying the parameters. It is noticed that the system attains chaotic nature via Hopf-bifurcation and period doubling bifurcation for large values of cooperation delay and verified by evaluating maximum Lyapunov exponent and Poincare sections. Further the chaotic oscillations can be controlled from the system by introducing anti-predation response delay. Our simulation results exhibit that group hunting guarantees the persistence of predators while their survival is threatened in absence of cooperation in hunting. In contrast to the above, the predator may extinct at large strength of cooperation.