Conclusions and future directions

Conclusions

Studies of interacting populations through mathematical modelling is being an issue of attention among ecologists for last few decades. Dealing with mathematical models is a challenging problem due to involvement of lots of environmental and ecological factors that affect the dynamics of the system. Analyzing these models mathematically is not easy because of their nonlinearity and complex behavior. In this thesis, we made an attempt to model various factors in the form of a system of ODEs and DDEs, showing the growth of populations with time.

In each model, positivity, boundedness and persistence of solutions are confirmed to show it is well-behaved. Local stability, instability and global stability all possible equilibrium points are discussed. We have analyzed various bifurcation, specially Hopf-bifurcation with respect to different parameters. We found a range of these parameters where the species fluctuate. Different thresholds are obtained where a bifurcation occurs in the system. Further, direction and stability of Hopf-bifurcation has also been investigated that informs the type of Hopf-bifurcation and the nature of raised periodic solutions. We have also analyzed the bistability phenomenon in which two equilibrium points get stable together, having different region of attraction.

In chapter 2 and 3, a fraction of the prey population is kept away from predation. We studied the influence of prey refuge and shown that conservation of prey individuals below a threshold is lucrative for both the populations but over conservation drops negative effect on predators. Complexity of the habitat helps prey to get rid of enemy. It is shown that prey population increases while predator population decreases with habitat complexity. In chapter 4, we established a stage structured model with harvesting. Optimal harvesting policy has been discussed in detail using the Pontryagin's maximal principle. From the point of view of ecological management, in order to plan harvesting strategies and keep sustainable development of ecosystem, we have used harvesting effort as a control parameter and obtained its optimal level E_δ. If applied effort is less that E_δ, all the species will co-exist at an optimal level and ecological balance can be maintained. If applied effort is larger than E_δ, then it represents over-exploitation and the prey-predator system will be in the danger of extinction.

In chapter 5, the stage structure is taken in prey species with maturation delay. In non-delayed system, the multiplicity of stability switching for the different range of attack rate with respect to intrinsic rate is obtained. Due to incorporation of maturation delay, system undergoes Hopf-bifurcation multiple times while gestation delay destabilize the system only once. In chapter 6, we studied prey-predator system with strong and weak Allee effect. It is shown that system with strong Allee effect can collapse but in weak Allee case, prey population always survives. We observed Hopf-bifurcation with respect to Allee parameter and gestation delay. Role of additional food provided to the predator is investigated in chapter 7. A new parameter ‘dependency factor’is defined that indicates how the predator depends on provided additional food. It is obtained that oscillations can be controlled by increasing the dependency factor. We have also concluded that the delay in the supply of additional food makes the system unstable.

It is known that predators do not remove prey only by direct killing while the also induce fear of being victimized in prey, consequently, the reproduction rate of prey falls off. In chapter 2 and 8, we examined the effect of fear in two different models. We observed that cost of fear plays an important role in stabilization of the system. Uniform oscillations can be removed from the system by increasing the cost of fear. Further, we studied effect of cooperative hunting among predator individuals and delay in cooperation. Interestingly, for some chosen parameters, predators die out if they do not cooperate while hunting. Again, higher cooperative strength of predators also lead to their extinction. This is because of the fact that higher cooperation lead to over-exploitation of prey species. The anti-predation response delay makes the system unstable whereas system becomes chaotic through multiple stability switching in the presence of cooperation delay. We remarked that the chaotic nature can be controlled by introducing the anti-predation response delay. Both the delays make the dynamics of model more extensive and rich.

In this thesis, we proposed 7 deterministic prey-predator models and analyzed them extensively. We hope that our outcomes are useful for understanding of mechanism of the biological organisms that play very significant role to make the nature stable and ecologically balanced.

Future directions

In this thesis, we have analyzed some ecological models involving numerous parameters. We have examined the complex dynamics of these models and obtained important results. In future, our work can be extended in the following directions:

- In the last chapter of this thesis, we have studied the effect of cooperative hunting in a prey-predator model. In future, this model can be extended to stage structure in predator, where immature predators are incapable to hunt prey while mature predators hunt in pack.
- Stochasticity may be involved in interaction of some organisms. Some stochastic models can be developed that will show more realistic dynamics of prey-predator interactions.

- All the models examined in this thesis are only time dependent. The work can also be extended in the direction of spatio-temporal modelling.
- Apart from this, we are interested to deal with epidemic and eco-epidemic models. Further, these models can be explored by incorporating time delay.