CHAPTER

# 3 Basic Demand and EOQ Models for Two-Successive Generation Short Life Cycle Products

Frequent innovations in the hi-technology market have impacted the lives of consumers significantly. But at the same time, it has also increased the expectations of end-users to manifold. As a result, due to continuous innovation and competition, the life-cycle of hi-technology products has shortened considerably in recent years. This doesn't imply that the newer technology will substitute the older technology immediately on its arrival in the market; rather it competes with old technologies till it completely cannibalizes the market share of old technologies (Pae and Lehmann 2003). Also, the risk associated with maintaining inventory for hi-technology products is higher than the other consumable

Goldman (1982) suggested that hi-technology products are often typified as a short life cycle product as these products have a steep waning phase just after reaching the maturity phase. Juneau and Coats (2001) argued that often the underlying of a product evolves with time and hence exponential time demand function is suitable to study optimal EOQ policies.

Several extensions of EOQ models were proposed by relaxing the constant rate assumptions of the demand function, viz considering substitution effect (Paul et al. 2014; Teshakkor et al. 2016; Qin et al. 2014; Rabbani et al. 2016; Mahmoodi 2016; Liu et al. 2015), nonstationary demand (Arkan and Hejazi, 2012; Chakraborty et al. 2013; Chanda and Kumar, 2011a; Kumar and Chanda, 2017) and varied consumer purchase behavior (Chanda and Kumar, 2011a; Kumar et al. 2013; Chanda and Agarwal, 2014; Kumar and Chanda, 2017; Kumar and Chanda, 2018; Chanda and Kumar; 2019), etc. Zhou et al. (2014) used geometric programming to determine the optimal price, quality level, and lot size for the retailer. Chen et al. (2016) developed a model which is a blend of EOQ (for a manufacturer) and EPQ (for retailer) model and also incorporated the influence of quality loss function to simultaneously determine the optimal process characteristic. Moon and Lee (2000) discussed an EOQ model considering the life cycle dynamics and approximated it using a normal and exponential distribution. Similarly, Wu et al. (2017) considered the product life cycle in a trapezoidal pattern.

Some of the well-known inventory models in the last decades that used variable demand patterns have been presented in Table3.1. Such a survey is expected to guide in the choice of an appropriate demand rate function for capturing the substitution effect of technology generational products on EOQ policies.

**Table3.1.** Some of the important inventory researches in the area of non-constant demand or the demand substitution

| Author (Year) | Issues addressed through variable demand rate function | Dissimilarity from the current study |
|---|---|---|
| Paul et al. (2014); Teshakkor et al. (2016); Qin et al. (2014); Rabbani et al. (2016); Mahmoodi (2016); Liu et al. (2015) | These studies worked upon the joint inventory optimization or replenishment of multiple products. | These studies were for functional products and not for innovative products. Also, these studies did not consider the effect of demand substitution. |
| Pentico (1974); Pentico (1988); Chand et al. (1994); Van Ryzin and Mahajan (1999); Agrawal and Smith (2003); Maddah and Bish (2007); Yucel et al. (2009); Honhon and Seshadri (2013); Yu et al. (2017); Aouad et al. (2018); Farahat and Lee (2018); Aouad et al. (2019); Fu et al. (2019); Geunes and Su (2019); Chan et al. (2020); Jing and Mu (2020); Majumder et al. (2020); Rasouli et al. (2020); Hsieh and Lai (2020) | These studies deal in the assortment based substitution | These studies limit themselves to the substitution only for non-engineered products, which is different from the technological products with short product life cycle |
| McGillivray and Silver (1978); Parlar (1988); Pasternack and Drezner (1991); Bitran and Dasu (1992); Lippman and McCardle (1997); Balakrishnan and Geunes (2000); Mahajan and van Ryzin (2001a); Netessine and Rudi (2003); Bayindir et al. (2007); Nagarajan and Rajagopalan (2008); Deflem and Nieuwenhuyse (2013); Burnetas et al. (2018); Schlapp and Fleischmann (2018); Geunes and Su (2019) | These studies deal with the stock-based substitution | Stock-based substitution is different from demand substitution due to technological up-gradation and changing consumer preferences. |
| McGuire and Staelin (1983); Birge et al. (1998); Tang and Yin (2007); Karakul (2008); Bish and Suwandechochai (2010); Burkart et al. (2012); Cosgun et al. (2017); Surti et al. (2018) | These studies deal with the price based substitution | Price based substitution is dissimilar from the technological substitution experienced in the case of multigenerational products. |
| Arkan and Hejazi (2012); Herbon and Khmelnitsk (2017); Chakraborty et al. (2013) | These studies appreciated that the time may vary with time. | The nature of variation considered here is not as per the product life cycle dynamics. |
| Chanda and Kumar (2011a); Chanda and Kumar (2019) | This study considered the innovation diffusion dependent demand | The study is only for a single generation product and did not consider the substitution |

| | | |
|---|---|---|
| Ke et al (2013) | This study considered the product life extensions of innovative products | The study did not take into account the ordering cost dynamics; and also considered the joint interaction effect to be proportional to the adoption rate rather than the cumulative adoption of the second generation. |
| Chanda and Agarwal (2014); Kumar and Chanda (2018) | This study considered the innovation diffusion dependent demand under substitution | The study only considered the innovation effect and ignored the imitation effect. |
| Kumar and Chanda (2017) | This study worked upon innovation diffusion dependent demand | The study didn't discuss the relationship between the innovation, imitation, and inventory, focusing more on price, advertising |

From Table3.1, it can be observed that the integration of product life cycle dynamics with the inventory models for technology generations has not been adequately covered in the existing literature. However, apart from Nagarajan and Rajagopalan (2008), and Chanda and Agarwal (2014), none of the models discussed substitution from the product generations' perspective, which is a general trend in the technology market. Chanda and Agarwal (2014) used the successive-generation innovation-diffusion demand framework to discuss inventory policies for two substitutable technology generations. One of the limitations of the Chanda and Agarwal (2014) model was it doesn't capture the full life-cycle dynamics of technology products due to the exponential nature of demand function.

To understand the influence of substitution on EOQ policies, a new demand model for two successive-generation technology products is proposed in the next section The proposed demand model relaxes the assumption of the exponential demand rate function of Chanda and Agarwal (2014). The proposed demand model can capture the entire life-cycle dynamics of successive-generations technology products under the innovation-substitution effect.

### 3.1. Demand Modelling Framework and Development

In the following subsections, a detailed discussion on the development of the two-generation demand model is presented, which will be further used for inventory modeling and cost modeling in the upcoming sections.

#### 3.1.1. Demand Model Assumptions

a. Demand rate follows the successive generation adoption process
b. A potential buyer can buy only a single unit of product from the same generation.
c. The potential market size of each generation product shall remain constant.

d. The diffusion of a technology product in the social system takes place due to the innovation effect (mass media) and imitation effect (word of mouth).
e. On purchasing a new technology product, the purchaser will not revert to the old technology product

### 3.1.2. Demand Model Notations

$\lambda_j(t)$ demand rate at time $'t'$ of $j^{th}$ generation product ($j$ = 1,2)

$p_j$ innovation coefficient for $j^{th}$ generation product ($j$ = 1,2)

$q_j$ imitation coefficient for $j^{th}$ generation product ($j$ = 1,2)

$\tau$ introduction time of the second generation product

$\varepsilon_1(t)$ is the hazard rate that gives the conditional probability of purchase in a small interval of time ($t$, $t+\Delta t$), if the purchase has not occurred until time $t$.

$F_j(t)$ is the cumulative adoption function till time $t$ for $j^{th}$ generation product ($j$ = 1,2)

$f_j(t)$ is the adoption function at time $t$ for $j^{th}$ generation product ($j$ = 1,2)

In this subsection, the demand model for two successive technology generation products shall be discussed. To begin with, a single generation model framework shall be discussed, and later it will be extended for two-generation products.

### 3.1.3. Demand model for the single generation scenario

To model the single generation demand, the Bass innovation diffusion model (Bass 1969) is considered in this chapter. Bass model is based on the assumption that the size of potential purchasers is constant and all the potential purchasers will buy the product over time. The probability of a potential adopter (who has not adopted the product until time $t$) to adopt the product at time $t$ is given by innovation effect and by imitation effect. Adoption rate can be given as

$$\varepsilon_1(t) = \frac{f_1(t)}{1-F_1(t)} = p_1 + q_1 F_1(t) \qquad (3.1)$$

Here, $F_1(t) = \int_0^t f_1(t)$ is the cumulative probability of buying the product until time $t$ (3.2)

Solving (3.1), (3.2) with the initial condition $F_1(0) = 0$, the following can be obtained:

$$F_1(t) = \frac{\{1-\exp(-b_1 t)\}}{\{1+a_1.\exp(-b_1 t)\}} \qquad (3.3)$$

Where $a_1 = \frac{q_1}{p_1}$ $and$ $b_1 = (p_1 + q_1)$ (3.4)

Differentiation equation (3.3), $f_1(t) = \frac{b_1^2 \exp(-b_1 t)}{[p_1\{1+a_1.\exp(-b_1 t)\}^2]}$ (3.5)

Thus, the cumulative purchasers until time $t$ can be given by $M_1F_1(t)$ and the number of buyers at time $t$ will be $M_1f_1(t)$.

Thus, $\lambda_1(t) = M_1f_1(t)\ for\ t < \tau$ (3.6)

### 3.1.4. Demand model for two generations scenario $(t \geq \tau)$

Once the next generation product is introduced, a sharp competition between the two generations of products can be observed in the market. As a result, demand for the first generation product may get reduced as some of the potential adopters can skip and directly purchase the second generation. Thus the demand function of the first generation product at any time '$t$' can be given as

$$\lambda_1'(t) = M_1f_1(t) - M_1f_1(t)F_2(t)\ for\ t > \tau \quad (3.7)$$

The demand for the second-generation product consists of two components:

a) the normal purchase of the second generation product, given by $M_2f_2(t)$

b) the likely purchasers of the first generation product who skip the first generation instead go for the next generation, given by $M_1f_1(t)F_2(t)$

Thus,

$$\lambda_1(t) = M_1f_1(t) \qquad for\ t < \tau$$
$$\lambda_1'(t) = M_1f_1(t) - M_1f_1(t)F_2(t) \qquad for\ t > \tau$$
$$\lambda_2(t) = M_2f_2(t) + M_1f_1(t)F_2(t) \quad (3.8)$$

Where, $F_2(t) = \frac{1-\exp(-b_2(t-\tau))}{[1+a_2.\exp(-b_2(t-\tau))]}$ (3.9)

On the lines of (3.5), the adoption rate for second generation can be stated as

$$f_2(t) = \frac{b_2^2\exp(-b_2(t-\tau))}{[p_2\{1+a_2.\exp(-b_2(t-\tau))\}^2]} \quad (3.10)$$

Where $a_2 = \frac{q_2}{p_2}$ $and$ $b_2 = (p_2 + q_2)$

In the next subsection, a detailed discussion of the validation of the proposed model is presented. To explore the descriptive and predictive ability of the proposed demand model, the output of the model is compared with with the Norton and Bass (Norton & Bass 1987). The Norton and Bass model is based on the assumption that the coefficient of innovation $(p)$ and coefficient of imitation $(q)$ remains the same across generations.

A similar approach to model demand for technology generation product was also considered by Jiang and Jain (2012). But Jiang and Jain (2012) didn't consider explicitly the inter-generational repeat purchase behavior of consumers in their model.

### 3.1.5. Validation of the proposed demand model

To check the descriptive and predictive ability of the model, the two generations of IBM system datasets[4] (Gen1 (vacuum tubes) and Gen2 (transistors)) from the Mainframe Industry (USA) as reported by Phister (Phister 1976) have been used. The software package SAS is used to execute the ordinary least square (OLS) estimation technique to jointly estimate the parameters of the simultaneous nonlinear equations. To compare the descriptive and predictive ability of the models, the parameters of the Norton and Bass model (equation 3.3, 3.5, 3.9 and 3.10) have been estimated and the proposed model (equation 3.7 and 3.8) using 16 years of data of the Gen1 family (1955 to 1970) and 12 years of data of the Gen2 family (1959 to 1970). Parameter estimates and the fit of both the models are presented in Table3.2. The fit of the models was evaluated through Error Sum of Squares (SSE) and Adjusted $R^2$ as given in Table3.2.

Summary of the Estimation for the proposed model and Norton-Bass model

| Model | No. of Para | Para | Estimate | $t-value$ | Approx $Pr > \|t\|$ | SSE | | Adjusted $R^2$ Gen1/ Gen2 |
|---|---|---|---|---|---|---|---|---|
| Norton-Bass | 4 | $m_1$ | 2597.87 | 10.35 | <0.0001 | 2701851 | 98535699 | 0.7837/ 0.3701 |
| | | $m_2$ | 6306.962 | 5.15 | 0.0009 | | | |
| | | $p$ | 0.014781 | 0.66 | 0.5228 | | | |
| | | $q$ | 1.649168 | 2.45 | 0.0325 | | | |
| Proposed | 6 | $m_1$ | 16869.32 | 29.05 | <0.0001 | 249371 | 1561782 | 0.9786 / 0.9887 |
| | | $m_2$ | 79970.28 | 46.79 | <0.0001 | | | |
| | | $p_1$ | 0.011793 | 7.38 | <0.0001 | | | |
| | | $q_1$ | 0.6176 | 20.28 | <0.0001 | | | |
| | | $p_2$ | 0.00956 | 10.57 | <0.0001 | | | |
| | | $q_2$ | 0.6418 | 33.34 | <0.0001 | | | |

From Table3.2, it can be concluded that the proposed demand model gives a better fit to the two generations of IBM system datasets in comparison to the Norton-Bass model. From Table3.1, it can also be observed that all the estimates of the proposed model are highly significant. It is also observed that value of the coefficient of innovation for the second generation is lesser than the first generation whereas the coefficient of imitation for the second generation is higher than the first generation, indicating that word-of-mouth plays an important role in the diffusion of advanced generations' product and the role of the coefficient of innovation reduces across generations (Chanda and Bardhan 2008). In

[4] www.bassbasement.org/F/N/BBDL/IBM%20SIU%20Phister.xls

the next stage, the parameter values as presented in Table3.1 have been used to predict sales of the Gen1 and Gen2 family from 1971-1974 and compare it with the original sales values. In Figure3.1, the number of users of the Gen1 and Gen2 family along with the estimated/forecasted sales values as obtained through the proposed model and Norton-Bass model are presented.

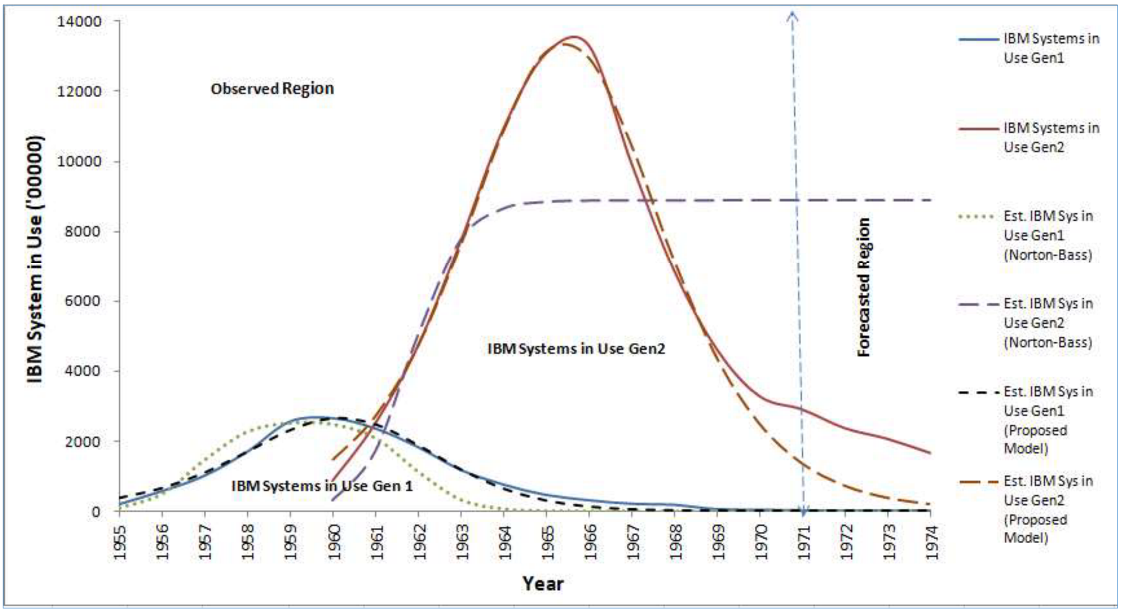


**Figure 3.1:** Actual users of the Gen1 and Gen2 family along with the estimated/forecasted sales values

The Mean Error, Mean absolute deviation, Root-mean-squared-error, and Mean absolute percentage error with four-step-ahead forecasts (1971-1974) for Gen1 and Gen2 family is reported in Table3.3. The finding is interesting as it indicates that the proposed model demonstrates better predictive capability than the Norton-Bass model.

| Model | IBM Systems in Use | Mean Error | Mean absolute deviation | Root mean square error | Mean absolute percentage error |
|---|---|---|---|---|---|
| **Norton-Bass** | Gen 1 | 6.9998 | 6.9998 | 8.1237 | 99.99 |
| | Gen 2 | -6641.08 | 6641.08 | 6656.49 | 309.64 |
| **Proposed** | Gen 1 | 6.8822 | 6.8822 | 7.9554 | 98.77 |
| | Gen 2 | 1591.14 | 1591.14 | 1593.48 | 72.91 |

Though the Norton-Bass model is remaining one of the important models to describe multiple generation product scenarios, however, it can't be used to all business scenarios (Kim et al, 2000). Norton-Bass model is an installed base successive generation model and only includes first-time sales and intervallic renewals along with substitution effect. The model doesn't consider up-gradation sales from an earlier generation and can be fitted only on units-in-use data (Jiang & Jain, 2012). Thus the use of the Norton-Bass model won't be very successful to make inventory-related business decisions, as these decisions are mostly based on sales data. The proposed multi-generation demand model can predict the sales data efficiently as presented in Table3.2. The above discussions along with the forecasting ability of the proposed model give additional impetus to use it for modeling inventory policies.

In this chapter,[5] two new Economic Order Quantity (EOQ) models for a successive generation of hi-technology products have been proposed for both single-period and multi-period inventory planning using the demand model as proposed in section 3.1. The major focus of this chapter is to understand the interaction effect of technology generations on the consumers' buying behavior and subsequently on inventory policies by using the innovation diffusion framework.

### 3.2. Optimal Single Period Inventory Model for short life cycle successive generations' hi-technology products

Extensive research has been done to understand the impact of marketing-mix variables on EOQ policy. Unfortunately, little attention has been paid to explore the influence of life cycle dynamics on optimal inventory policies of technology products. Rogers (1983) defined *diffusion of innovation* as the process through which an innovation is accepted among the members in a social system over time. Moon and Lee (2000) discussed an EOQ model considering the life cycle dynamics and approximated it using a normal and exponential distribution. Ke et al (2013) suggested that often the inventory cost for generational products goes too high to influence the time-to-market of next-generation products. Chanda and Aggarwal (2014) argued that to develop inventory policies for successive generations' technology products, a technically superior demand model should be used to capture the substitution effect across the technology generations. Undeniably, these models were important in inventory literature as they explored the different aspects of EOQ policies. However, these models didn't consider one of the important characteristics of technology adoption i.e. substitution-diffusion of generational products, a common phenomenon in the technology market.

Goldman (1982) suggested that hi-technology products are often typified as a short life cycle product as these products have a steep waning phase just after reaching the maturity phase. The objective of the section is to examine how the technology substitution in combination with other model parameters influence the optimal inventory policies for the hi-technology product. Therefore, the inventory model

[5] This Chapter is based on two research papers:

a. Nagpal, Gaurav and Chanda, Udayan. "Optimal inventory policies for short life cycle successive generations' technology products". *Journal of Management Analytics,* Accepted for publication
b. Nagpal, Gaurav and Chanda, Udayan. "Multi-period Q-type Inventory Model for multi-generation technology products with short product life cycle". *International Journal of e-adoption* (Under Review)

for technology products needs to be designed to adapt to demand uncertainty due to frequent changes in the behavioral aspect of consumers. However, very limited researches are available in supply chain literature that has studied the combined effect of market and technology competition on the firms' inventory policies. Often it has been observed that uncertainty in customer expectations in the hi-technology market, leads to a miscalculation of the right quantity of stock which needs to be stored by the retailers to meet the buyer's requirements. This section is focused on a single-period inventory model in this paper due to the short product life cycle of hi-technology products. The importance of understanding the mechanism of the diffusion-substitution process for technology generation products and its effect on the EOQ policy has also been elaborated.

In this section, a single-period EOQ model for two successive generation hi-technology products is proposed using the demand model as discussed in section 3.1. Assumptions and notations used to develop the single period EOQ model is discussed in the following subsections

### 3.2.1. Assumptions of the Model

- The supply of the inventory is instantaneous
- Lead time in the supply of the products is not considered
- Shortages are not permitted in the supply chain
- Product sales are limited to a single geography
- The overall market potential of the respective generation product shall remain constant

### 3.2.2. Notations of the Model

$A_j$ ordering cost for $j^{th}$ generation product ($j$ = 1,2)

$C_j$ purchase cost for $j^{th}$ generation product ($j$ = 1,2)

$I_j$ inventory carrying charge (% of cost) for $j^{th}$ generation product ($j$ = 1,2)

$M_j$ potential market for $j^{th}$ generation product ($j$ = 1,2)

$T_j$ length of the planning horizon for $j^{th}$ generation product ($j$ = 1,2)

$Q_j$ number of items received at the beginning of the period for $j^{th}$ generation product ($j$ = 1,2)

A detailed inventory modeling framework under a two-generation product situation is discussed in section 3.2.3.

### 3.2.3. Inventory Modeling for the single generation scenario

In the absence of any normal deterioration, the decrease in inventory takes place due to the demand usage. Thus $\lambda_j(t)$ for $0 \le t \le T$ can be given as

$$\lambda_j(t) = -\frac{d\left(I_j(t)\right)}{dt}\ ;\ (j = 1,2) \tag{3.11}$$

Several cost components for the first generation and second generation can be defined in the following sections.

$$I_1(t) = \begin{cases} \int_0^t -\lambda_1(t)dt & for\ t < \tau \\ \int_\tau^t -\lambda_1(t)dt & for\ t \geq \tau \end{cases} \tag{3.12}$$

Solving and using no shortages condition, the following are obtained:

$$I_1(t) = \begin{cases} Q_1 - M_1F_1(t) & for\ t < \tau \\ Q_1 - M_1F_1(t) + M_1\int_\tau^t f_1(t)F_2(t)dt & for\ t > \tau \end{cases} \tag{3.13}$$

$\int_\tau^t f_1(t)F_2(t-\tau)dt$ can be defined as: $\int_\tau^t f_1(t)F_2(t)dt = J(t) - J(\tau)$ (3.14)

From (3.13) and (3.14), it can be inferred that

$$I_1(t) = \begin{cases} M_1\left(F_1(T_1) - J(T_1) + J(\tau) - F_1(t)\right) & for\ t < \tau \\ Q_1 - M_1F_1(t) + M_1J(t) - M_1J(\tau) & for\ t \geq \tau \end{cases} \tag{3.15}$$

Since the total initial stock gets exhausted in the time horizon $T_1$, it can be said

$$Q_1 = \int_0^\tau \lambda_1(t)dt + \int_\tau^{T_1} \lambda_1(t)dt = M_1[F_1(T_1) - J(T_1) + J(\tau)] \tag{3.16}$$

From (3.15) and (3.16), the following are obtained:

$$I_1(t) = \begin{cases} M_1\left(F_1(T_1) - J(T_1) + J(\tau) - F_1(t)\right) & for\ t < \tau \\ M_1\left(F_1(T_1) - J(T_1) + J(t) - F_1(t)\right) & for\ t \geq \tau \end{cases} \tag{3.17}$$

The total cost $\left(K_j(T_1)\right)$ is the sum of ordering cost, basic purchase cost, and the inventory carrying cost, interest charged, and interest earned. Thus $K_1(T_1)$ can be given as:

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}\left[\int_0^\tau I_1(t)dt + \int_\tau^{T_1} I_1(t)dt\right] \tag{3.18}$$

From equations (3.16), (3.17), and (3.18) the following can be inferred:

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{M_1C_1[F_1(T_1) - J(T_1) + J(\tau)]}{T_1} + \frac{M_1C_1I_1}{T_1}[[F_1(T_1) - J(T_1)]T_1 + J(\tau)\tau] - \frac{M_1C_1I_1}{T_1}\int_0^{T_1} F_1(t)dt +$$
$$+ \frac{M_1C_1I_1}{T_1}\int_\tau^{T_1} J(t)dt \tag{3.19}$$

### 3.2.4. Inventory Modeling for the two generations scenario $(t \geq \tau)$

Integrating equation (3.11) for the second generation, it can be said that

$$I_2(t) = -\int_\tau^t \lambda_2(t)dt + Constant \tag{3.20}$$

Since Inventory at time $\tau$ is $Q_2$, $\Longrightarrow Constant = Q_2$

Thus, (3.22) $\Rightarrow I_2(t) = Q_2 - \int_{\tau}^{t} \lambda_2(t)dt$ (3.21)

Substituting $\lambda_2(t)$ (as given in equation (3.8)) in (3.21) gives the following equation:

$$I_2(t) = Q_2 - M_2F_2(t) - M_1 \int_{\tau}^{t} f_1(t)F_2(t)dt \quad (3.22)$$

From (3.14) and (3.22), it can be derived that

$$I_2(t) = Q_2 - M_2F_2(t) - M_1J(t) + M_1J(\tau) \quad (3.23)$$

The planning horizon for the second generation $T_2$ shall always be lesser than the planning horizon for the first generation $T_1$ by $\tau$.

Thus, $\boldsymbol{T_2 = T_1 - \tau}$ (3.24)

Since initial inventory will get exhausted by the time $T_2$, thus the initial inventory is the total demand for the second-generation product during the period $(\tau, t)$

So, $Q_2 = M_2F_2(T_1) + M_1J(T_1) - M_1J(\tau)$ (3.25)

Substituting the value of $Q_2$ from (3.25) in (3.23), it can be said that

$$I_2(t) = M_2F_2(T_1) + M_1J(T_1) - M_2F_2(t) - M_1J(t) \quad (3.26)$$

Thus the total cost per unit time for the second generation product $\left(K_2(T_1)\right)$ can be given

$$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)} \int_{\tau}^{T_1} I_2(t)\, dt = \frac{A_2}{(T_1-\tau)} + \frac{C_2[M_2F_2(T_1-\tau)+M_1J(T_1)-M_1J(\tau)]}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[M_2F_2(T_1-\tau) + M_1J(T_1)](T_1-\tau) - \frac{I_2C_2}{(T_1-\tau)} \int_{\tau}^{T_1} [M_2F_2(t-\tau) + M_1J(t)]\, dt \quad (3.27)$$

The total cost can be given as:

$$K(T_1) = K_1(T_1) + K_2(T_1) = \frac{A_1}{T_1} + \frac{A_2}{(T_1-\tau)} + \frac{M_1C_1[F_1(T_1)-J(T_1)+J(\tau)]}{T_1} + \frac{C_2[M_2F_2(T_1-\tau)+M_1J(T_1)-M_1J(\tau)]}{(T_1-\tau)} + \frac{M_1C_1I_1}{T_1}[[F_1(T_1) - J(T_1)]T_1 + J(\tau)\tau] - \frac{M_1C_1I_1}{T_1}\int_0^{T_1}[F_1(t)]dt + \frac{M_1C_1I_1}{T_1}\int_{\tau}^{T_1} J(t)dt + \frac{I_2C_2}{(T_1-\tau)}[M_2F_2(T_1) + M_1J(T_1)](T_1-\tau) - \frac{M_2I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1}[F_2(T_1)]\, dt] - \frac{M_1I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1}[J(t)]\, dt]] \quad (3.28)$$

Since our objective is to minimize the cost function $K(T_1)$, hence the necessary conditions for minimizing $K(T_1)$ are $\frac{\partial K(T_1)}{\partial T_1} = 0$ with the sufficient condition $\frac{\partial^2 K(T_1)}{\partial {T_1}^2} > 0$ (3.29)

Depending on the path of ordering costs per unit time, purchase costs per unit time, and inventory holding costs per unit time, the following theorem can be developed:

***Theorem 1:*** *The total cost curve per unit time is a convex curve to the origin and hence, has a unique point of minima.*
Proof: See Appendix 3.B.

***Theorem 2:*** *The optimal supply quantity for the first generation increases while that for the second generation decreases with the increase in the introduction timing of the second generation.*
Proof: See Appendix 3.C.

In the next subsection, the solution procedure to find the optimal solution for $T_1^*$ is discussed. Since cost function as defined in equation (3.30) is highly non-linear, hence finding an analytical solution for the problem is difficult. The problem is solved numerically under given parameter values. Once the value of $T_1^*$ is known, values of optimum ordered quantities and optimum costs can easily be identified using equation (3.18), (3.21), (3.27), and (3.29).

### 3.2.5. Solution procedure

The solution procedure to find the optimal solutions can be summarized in the following algorithm

Step 1: Enter the base values of all model parameters such as per-unit costs, coefficients of innovation and imitation, potential market sizes, time to the introduction of second-generation products, etc. for each generation independently.

Step 2: Compute all possible values of $T_1$ for the given value of $\tau$ using equation (3.29) as the case may be.

Step 3: Select the appropriate value of $T_1$ using equation (3.29) that satisfies the sufficiency condition $\frac{\partial^2 K(T_1)}{\partial {T_1}^2} > 0$

Step 4: Compute the total optimal cost $K(T_1)$ from equation (3.28) and $K_1(T_1)$ and $K_2(T_1)$ using equations (3.19) and (3.27).

Step 5: Finally, compute the value of $Q_1^*$ and $Q_2^*$ using equation (3.16) and (3.25) respectively.

### 3.2.6. Numerical illustration

The nature and behavior of the proposed model can be understood through the numerical example. In this subsection, the sensitivity analysis on different model parameters has been performed and the results have been represented in different numerical tables. To perform the sensitivity analysis the following parameter values have been considered. Since the single period model consists of the supply for the entire life cycle of a product at the single time, it can be used only for the products with small one-time market potential.

$A_1 = INR\ 25000, A_2 = INR\ 15000, C_1 = INR\ 1500, C_2 = INR\ 2100, M_1 = 515, M_2 = 1140, p_1 = 0.09, p_2 = 0.05, q_1 = 0.9, q_2 = 1.5, I_1 = 0.1, I_2 = 0.12, \tau = 0.6$ years

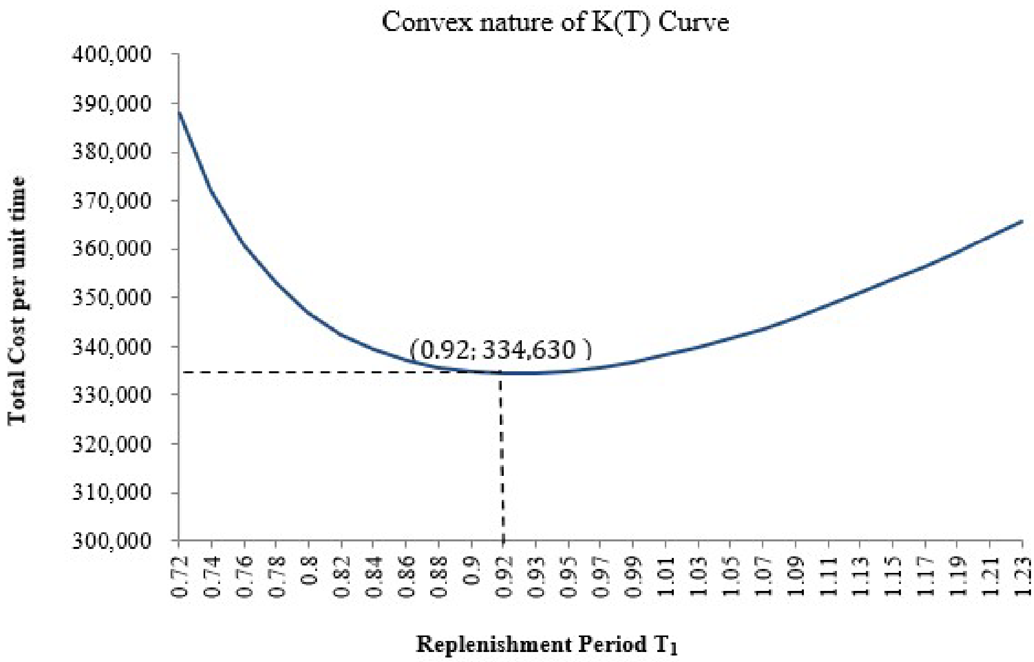


**Figure3.2:** Convex curve of $K(T)$

As shown in Figure3.2, the cost curve is convex to the origin when plotted as a function of the planning horizon. Upon minimizing the total cost per unit time, as proposed in the model, the optimal length of the planning horizon can be found. At $T_1^* = 0.92$, and $T_2^* = 0.32$, the optimal cost, and optimal ordered quantity values are obtained as follows: $K(T_1^* = 0.92) = 334{,}630$, $Q_1(T_1^* = 0.92) = 61.36$ and $Q_2(T_2^* = 0.32) = 23.08$. Further, Table3.4 below suggests that when the other parameters remain constant and the value of $\tau$ changes, the optimal length of the planning horizon and optimal ordering quantity reduces with the increase in $\tau$. This is because the greater $\tau$ decreases the fluctuation of demand and brings more predictability into the system, allowing the time-space for more optimal planning. However, it is not possible to increase the $\tau$ beyond a limit, because then, the competition will take over the non-innovating players.

**Table3.4:** Sensitivity analysis on

| | $\tau$ =.6 | $\tau$ =.8 | $\tau = 1$ |
|---|---|---|---|
| $K(T^*)$ | 334,630 | 337,884 | 343,630 |
| $T_1^*$ | 0.92 | 1.12 | 1.30 |
| $T_2^*$ | 0.32 | 0.52 | 0.70 |

| | | | |
|---|---|---|---|
| $Q_1(T_1^*)$ | 61.36 | 80.35 | 99.26 |
| $Q_2(T_2^*)$ | 23.08 | 23.03 | 21.23 |

Next, the sensitivity analysis is conducted with regards to innovation and diffusion coefficients. Similar parameter values are considered as used for the base case and kept increasing the values of coefficients of innovation and diffusion by 20% each one at a time. The results are shown in Table 3.5.

**Table 3.5.** Sensitivity analysis on innovation and imitation coefficient of $i^{th}$ generation $(i = 1,2)$

| Parameter | Base Value | $K(T^*)$ | $T_1^*$ | $T_2^*$ | $Q_1(T_1^*)$ | $Q_2(T_2^*)$ |
|---|---|---|---|---|---|---|
| $p_1$ | 0.09 | 334,630 | 0.92 | 0.32 | 61.36 | 23.08 |
| | | | | | | |
| | 0.126 | 371,122 | 0.91 | 0.31 | 82.16 | 22.16 |
| $q_1$ | 0.90 | 334,630 | 0.92 | 0.32 | 61.36 | 23.08 |
| | 1.08 | 343,070 | 0.91 | 0.31 | 65.72 | 22.19 |
| | | | | | | |
| $p_2$ | 0.05 | 334,630 | 0.92 | 0.32 | 61.36 | 23.08 |
| | | | | | | |
| | 0.07 | 393,770 | 0.89 | 0.29 | 58.73 | 28.49 |
| $q_2$ | 1.50 | 334,630 | 0.92 | 0.32 | 61.36 | 23.08 |
| | 1.80 | 341,564 | 0.89 | 0.29 | 58.70 | 21.41 |
| | | | | | | |

From Table 3.4, it can be observed that with the increase in the coefficients of innovation and imitation, the value of $K(T^*)$ increases while the optimal $T^*$ is decreases. $K(T^*)$ increases because of faster adoption of the next generation product, resulting in a higher quantum of procurement, and hence the increase in overall costs. The optimal $T^*$ decreases because it makes carrying costs per unit time rise faster, and thus expedites the point of minima. With the faster diffusion of the first generation product, the optimal supply quantity of first-generation rises at the expense of the second generation. With the faster diffusion of the second generation product, the optimal supply quantity of the first generation falls. If that quicker diffusion is due to innovation, the optimal supply quantity of second-generation increases; while if that is due to imitation, the optimal supply quantity for the second-generation declines.

The above EOQ model tried to discuss a very basic foundation on the inventory planning and optimization for sequential-generation products under the innovation diffusion framework. It considered the imitation effect that has not been considered by the earlier works. In the analysis, it has been observed that with the increase in the coefficient of innovation and imitation, the optimal planning horizon reduces because of the greater fluctuation in demand. It is also observed that with an increase in the values of the coefficients of innovation and imitation, the value of optimal cost increases, and the optimal ordered quantity increases due to an increase in the demand. However, a lot of realistic scenarios need to be studied. The other possible extension can be to test the adaptability of the proposed framework in multi-period replenishment settings. In the next section, the above framework has been extended and a new multi-period inventory optimization model has been proposed for short life cycle hi-technology product.

### 3.3. Optimal Multi-Period Inventory Model for short life cycle successive generations' hi-technology products

In this section, a multi-period EOQ model is developed for the technology products that have multiple generations with a short product life cycle. Two types of multi-period inventory models are available in the literature. One is the Periodic Review model (P-type model) and the other one is a Continuous Review (Q-type model). In the periodic review model, the inventory level is reviewed at a fixed interval of time periodically regardless of the existing inventory levels. Whereas in the continuous review system inventory is reviewed continuously and once the inventory drops to a predecided level, immediately a fixed quantity of items are reordered. As most of the hi-technology products are highly valued and have a short lifecycle, hence multi-period inventory strategies especially continuous review can be considered to reduce cost. Inventory literature on replenishment policies for substitutable items is rich. But most of the above researches was confined within the boundary of item-level substitution demand pattern.

Kim et al. (2000) suggested that the demand for a technology product is not only linked with the dynamics of successive generations but also by the complementarities and competition with the other product categories. In this section, a continuous review (Q-type) inventory model for two-successive generation technology products is been discussed. To develop the EOQ model, the same demand function as discussed in section 3.1 is used. The objective of this section is to study the influence of the adoption-substitution effect of two successive generation hi-technology products on inventory policies under a continuous review system. The results obtained in this section may help inventory managers to achieve the desire customer satisfaction level and simultaneously optimize the inventory cost.

In this section, a multi-period Q-type EOQ model for two successive generation hi-technology products is proposed using the demand model as discussed in section 3.1. The proposed multi-period inventory model is based on the same assumptions and notations as considered in subsections 3.2.2 and 3.2.3. Besides, other notations are also used as given in the next subsections.

### 3.3.1. Additional Notations

The following are the notations used while creation of the Model:

$A$ : common ordering cost for $j^{th}$ generation product ($j$ = 1,2)

$\rho$: the length of each planning horizon

$m$: the sequence of planning horizon before the launch of the second generation product

$m'$: the sequence of planning horizon after the launch of the second generation product

$D_{j,m}$ is the demand of the $jth$ generation product in the mth planning horizon

$G_j(t)$ is the cumulative demand of the $j^{th}$ generation till time $t$ ($j$ = 1,2)

$Q_{j,m}$ : Order Quantity in the $mth$ planning horizon for $j^{th}$ generation product ($j$ = 1,2)

$Q_{j,m'}$ : Order Quantity in the $m'th$ planning horizon for $j^{th}$ generation product ($j$ = 1,2)

$Q_{3,m'}$ : Combined Order Quantity for both the generations' products in the $m'th$ planning horizon

$pr_j$ : selling price per unit for $j^{th}$ generation product ($j$ = 1,2)

$EOQ_{j,m}$ : Economic Order Quantity in the $mth$ planning horizon for $j^{th}$ generation product ($j$ = 1,2)

$EOQ_{j,m'}$ : Economic Order Quantity in the $m'th$ planning horizon for $j^{th}$ generation product ($j$ = 1,2)

Inventory decisions are the intermediate-term decisions that are taken for a specific period and then reviewed at the end of that period for the next period. Therefore, it is plausible to divide the entire product life cycle into many planning horizons and then find out the optimal EOQ for each of those planning horizons. A detailed multi-period EOQ modeling framework for the two-successive generation hi-technology product situation is discussed in section 3.3.2 and 3.3.3.

### 3.3.2. Inventory Model for the single generation scenario

In the absence of second-generation products, when there is no competition let it be assumed that $m$th planning horizon begins at time $t = (m-1)\rho$ and ends at time $t = m\rho$. This scenario is depicted in Figure3.3. If $Q_1$ be the EOQ in this planning horizon, then the number of replenishments in $m^{th}$ planning horizon $'n'$ can be given as $n = [G_1(m\rho) - G_1(m-1)\rho\,]/Q_1$ (3.30)

The time at which the $i^{th}$ replenishment cycle starts is given by

$$t_{i,m} = G_1^{-1}[G_1(m-1)\rho + (i-1)Q_1] \tag{3.31}$$

The time at which the $i^{th}$ replenishment cycle ends is given by

$$t_{(i+1),m} = G_1^{-1}[G_1(m-1)\rho + (i)Q_1] \tag{3.32}$$

The inventory at time t in the $i^{th}$ replenishment cycle is the demand between that time and the end of the corresponding replenishment cycle. It can be written as

$$I_1(t) = \int_{u=t}^{u=t_{(i+1),m}} \lambda_1(u)du \tag{3.33}$$

The inventory carrying cost in the $i^{th}$ replenishment cycle of the $m^{th}$ planning horizon can be written as

$$I_1C_1 \int_{t=t_{i,m}}^{t=t_{(i+1),m}} I_1(t)dt = I_1C_1 \int_{t=t_{i,m}}^{t=t_{(i+1),m}} [\int_{u=t}^{u=t_{(i+1),m}} \lambda_1(u)]\ dt \tag{3.34}$$

The total inventory varying cost across all the replenishment cycles within the planning horizon can be written as $HC_{Q_1} = I_1 C_1 \sum_{i=1}^{i=n} \int_{t=t_{i,m}}^{t=t_{(i+1),m}} [\int_{u=t}^{u=t_{(i+1),m}} \lambda_1(u)\ ]\, dt$ (3.35)

The Ordering Cost in the $mth$ planning horizon with $Q_1$ the order quantity for the first generation product is given by $OC_{Q_1} = \sum_{i=1}^{i=n}(A + A_1)$ (3.36)

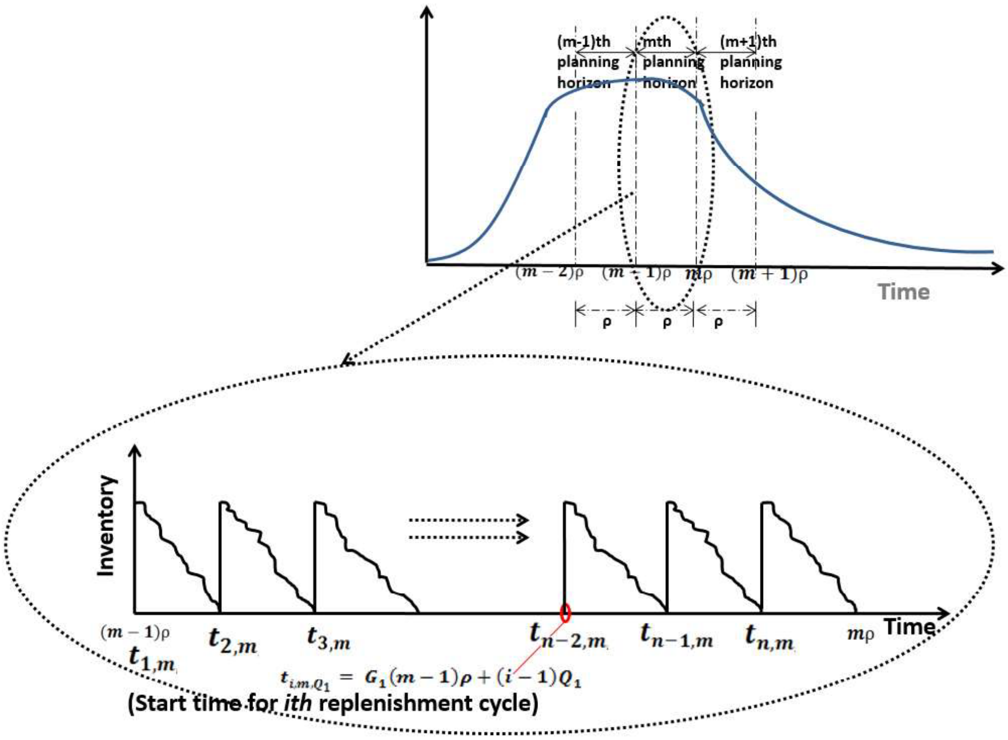


**Figure3.3.** Inventory behavior of technology products in case of single generation product

The total replenishment costs in $m^{th}$ planning horizon is given by:

$$TRC_m = \sum_{i=1}^{i=n}(A + A_1) + I_1 C_1 \sum_{i=1}^{i=n} \int_{t=t_{i,m}}^{t=t_{(i+1),m}} [\int_{u=t}^{u=t_{(i+1),m}} \lambda_1(u)]\ dt \quad (3.37)$$

The revenue of the $m^{th}$ planning horizon is

$$Rev_{1,m} = pr_1 \sum_{i=1}^{i=n} \int_{t=t_{i,m}}^{t=t_{(i+1),m}} \lambda_1(t) dt \quad (3.38)$$

The basic purchase cost in $m^{th}$ planning horizon is

$$BPC_{1,m} = C1 \sum_{i=1}^{i=n} \int_{t=t_{i,m}}^{t=t_{(i+1),m}} \lambda_1(t) dt \quad (3.39)$$

The contribution margin in the $m^{th}$ replenishment cycle is given by

$$TCM_{1,m} = Rev_{1,m} - BPC_{1,m} \quad (3.40)$$

The total profit in the $m^{th}$replenishment cycle is given by

$$TP_m = Rev_{1,m} - BPC_{1,m} - TRC_m \quad (3.41)$$

### 3.3.3. Inventory Model for the two generations scenario

When the second-generation product is launched, the sales of the first generation product start declining fast. As a result, it will be taking a long time to drain off the inventory. Let the $m'$th planning horizon begins at time $t = \tau + (m' - 1)\rho$ and ends at time $t = \tau + m'\rho$. This scenario is depicted in Figure3.4.

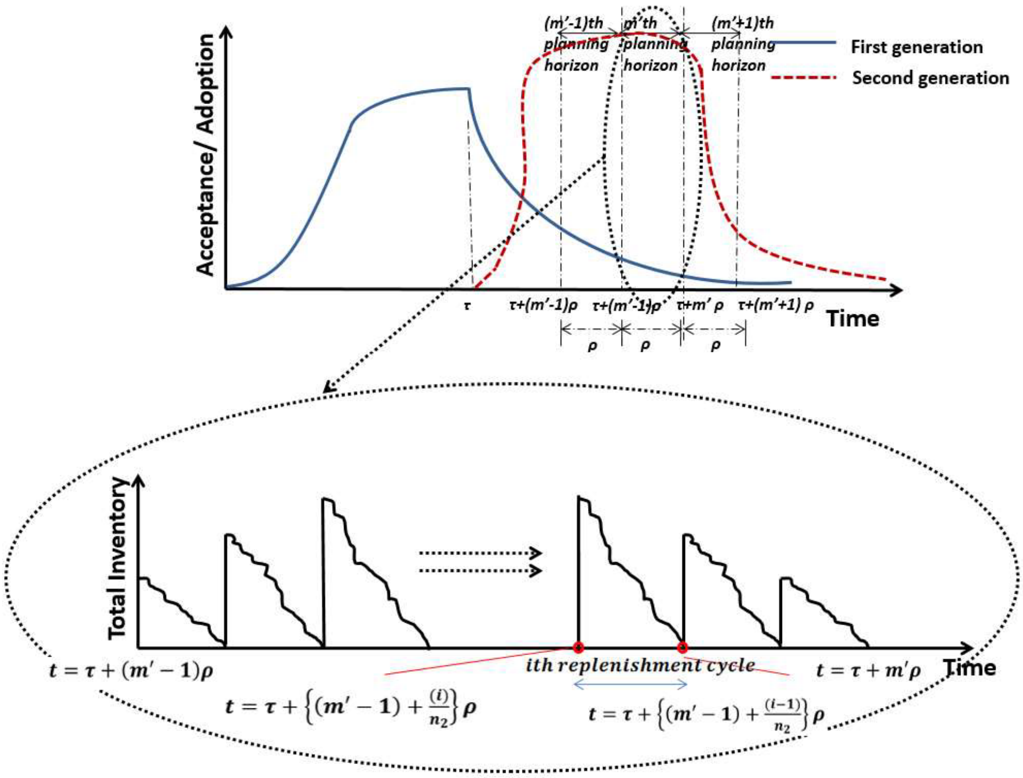


**Figure3.4.** Inventory behavior of technology products in case of two successive generations

If $Q_1'$ and $Q_2$ be the EOQ in this planning horizon, then the number of replenishments in $m'$th planning horizon is

$$n_1' = [G_1'(m'\rho) - G_1'((m' - 1)\rho)\,]/Q_1' \quad (3.42)$$

$$n_2 = [G_2(m'\rho) - G_2((m' - 1)\rho)\,]/Q_2 \quad (3.43)$$

The time at which the $ith$ replenishment cycle of $m'$th planning horizon starts for the first generation product is given by $t_{i,m'} = G_1'^{-1}[G_1'((m' - 1)\rho) + (i - 1)Q_1']$ (3.44)

The time at which the $ith$ replenishment cycle of $m'$th planning horizon ends for the first generation product is given by $t_{(i+1),m'} = G_1'^{-1}[G_1'((m' - 1)\rho) + (i)Q_1']$ (3.45)

The time at which the $ith$ replenishment cycle starts for the second generation product is given by

$$t_{i,m'} = G_2^{-1}[G_2((m' - 1)\rho) + (i - 1)Q_2] \quad (3.46)$$

The time at which the $ith$ replenishment cycle ends for the second generation product is given by

$$t_{(i+1),m'} = G_2'^{-1}[G_2((m'-1)\rho) + (i)Q_2] \quad (3.47)$$

Since the case of pooled replenishment for both the generations of products is considered hence $t_{i,m'}$ can be considered as same for both the generations as well as the $t_{(i+1),m'}$

Also in the case of pooled replenishment, it can be considered that $n_1' = n_2$ (3.48)

On the similar lines as above, the value of the holding costs, the ordering costs, and the total replenishment costs for both the generations in $m'th$ planning horizon is given by equations (3.49), (3.50), and (3.51).

$$HC_{m'} = I_1 C_1 \sum_{i=1}^{i=n_1} \int_{t=t_{i,m'}}^{t=t_{(i+1),m'}} [\int_{u=t}^{u=t_{(i+1),m'}} \lambda_1'(u)du]\ dt + I_2 C_2 \sum_{i=1}^{i=n_2} \int_{t=t_{i,m'}}^{t=t_{(i+1),m'}} [\int_{u=t}^{u=t_{(i+1),m'}} \lambda_2(u)du]\ dt \quad (3.49)$$

$$OC_{m}' = n_1'(A + A_1 + A_2) \quad (3.50)$$

$$TRC_{m'} = n_1'(A + A_1 + A_2) + HC_{m'} \quad (3.51)$$

Let $BPC_{j,m'}$ , $Rev_{j,m'}$ and $TCM_{j,m'}$ be the Basic purchase cost, Revenue, and Contribution margin respectively for the $jth$ generation product in the $m'th$ planning horizon. Let $TP_{m'}$ be the total profit for both the generation products in case of consolidated logistics in the $m'th$ planning horizon.

$$BPC_{m'} = BPC_{1,m'} + BPC_{2,m'} = C_1 \sum_{i=1}^{i=n_1} \int_{t_{i,m'}}^{t_{(i+1),m'}} \lambda_1'(t)dt + C_2 \sum_{i=1}^{i=n_2} \int_{t_{i,m'}}^{t_{(i+1),m'}} \lambda_2(t)dt \quad (3.52)$$

$$Rev_{m'} = Rev_{1,m'} + Rev_{2,m'} = pr_1 \sum_{i=1}^{i=n_1} \int_{t_{i,m'}}^{t_{(i+1),m'}} \lambda_1'(t)dt + pr_2 \sum_{i=1}^{i=n_2} \int_{t_{i,m'}}^{t_{(i+1),m'}} \lambda_2(t)dt \quad (3.53)$$

$$TCM_{m'} = TCM_{1,m'} + TCM_{2,m'} = Rev_{m'} - BPC_{m'} \quad (3.54)$$

$$TP_{m'} = TCM_{m'} - TRC_{m'} \quad (3.55)$$

Subject to the cost path of ordering costs per unit time, purchase costs per unit time, and inventory holding costs per unit time of the product of each generation, the propositions and special cases can be discussed in the upcoming part of this section.

### 3.3.4. Theorems and Special Cases

**Theroem 1:** The higher coefficients of innovation and imitation for the second generation lead to lower EOQ for the first generation product.

Proof: Refer Appendix 3.D.

**Theorem 2:** In case of pooled logistics for the two generations' products, the optimal EOQ for the first generation falls with the increase in the innovation and imitation coefficients for the second-generation product. However, in the case of un-pooled logistics for the two generations' products, with the increase in the innovation and imitation coefficients for the second-generation product, the optimal EOQ for the first generation falls.

Proof: Refer Appendix 3.E

### 3.3.5. Numerical Illustrations

The following parameters are taken for the numerical illustrations:

$A = INR\ 500000,\ A_1 = INR\ 50000,\ A_2 = INR\ 50000,\ I_1 = .15,\ I_2 = .15,\ M_1 = 100000,$ $M_2 = 120000,\ p_1 = .5,\ q_1 = 2.5,\ p_2 = .6,\ q_2 = 4.0,\ pr_1 = INR\ 3500,\ pr_2 = INR\ 4200,\ C_1 = INR\ 1500,\ C_2 = INR\ 2200,\ \tau = 0.5$ years

First, the model is run for a single generation scenario and the results are obtained as shown in Table3.6.

**Table3.6:** Results of the Economic Order Quantity derived by running the model for single generation product (All the financial figures of Revenue, Contribution Margin, Holding Cost, Ordering Cost and Total Profit in Mn USD, and the EOQ in absolute units)

| *First Planning Horizon* ($m = 1$) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $EOQ_{1,m}$ | $HC_m$ | $OC_m$ | $TRC_m$ | $Rev_{1,m}$ | $TCM_{1,m}$ | $TCM_{1,m}\%$ | $TP_m$ | $TP_m\%$ |
| **18,271** | **0.96** | **1.20** | **2.16** | **127.9** | **73.08** | **57%** | **70.92** | **56.1%** |
| *Second Planning Horizon* ($m = 2$) | | | | | | | | |
| $EOQ_{1,m}$ | $HC_m$ | $OC_m$ | $TRC_m$ | $Rev_{1,m}$ | $TCM_{1,m}$ | $TCM_{1,m}\%$ | $TP_m$ | $TP_m\%$ |
| **19,552** | **1.15** | **1.20** | **2.35** | **136.87** | **78.21** | **57%** | **75.86** | **55.42%** |
| *Third Planning Horizon* ($m = 3$) | | | | | | | | |
| $EOQ_{1,m}$ | $HC_m$ | $OC_m$ | $TRC_m$ | $Rev_{1,m}$ | $TCM_{1,m}$ | $TCM_{1,m}\%$ | $TP_m$ | $TP_m\%$ |
| **17,727** | **1.06** | **0.6** | **1.66** | **62.04** | **35.45** | **57%** | **33.79** | **54.46%** |
| *Fourth Planning Horizon* ($m = 4$) | | | | | | | | |
| $EOQ_{1,m}$ | $HC_m$ | $OC_m$ | $TRC_m$ | $Rev_{1,m}$ | $TCM_{1,m}$ | $TCM_{1,m}\%$ | $TP_m$ | $TP_m\%$ |
| **4918** | **0.30** | **0.6** | **0.90** | **17.21** | **9.83** | **57%** | **7.38** | **42.88%** |

Figure3.5 shows the plot of the figures in Table3.6 in graphical representation.

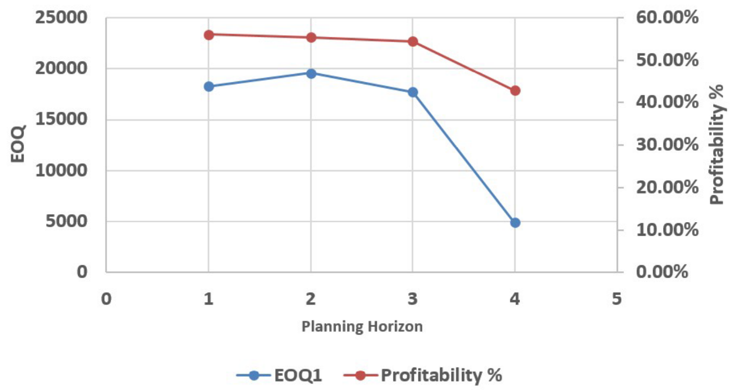


**Figure3.5.** Behavior of $EOQ_1$ and $TP_m\%$ with the change in $m$

**Observation 1:** (From Table3.6 and Figure3.5):

Since these are the technology products with the short product life cycle, the demand reaches a peak in the initial stages of the product itself, and therefore the EOQ is highest in the initial stages.

**Observation 2:** (From Table3.6 and Figure3.5):

As the planning horizon becomes bigger in size, the overall profitability reduces due to reducing economies of scale in the replenishment with the reduction in the demand rate.

**Observation 3:** (From Table3.6 and Figure3.5):

During the later stages of the product lifecycle, the volumes decline to a level where the ordering costs become a much more significant portion of the overall cost structure, jeopardizing the profitability and necessitating an exit of the product from the market.

Now, this model is run for the two successive generation products after assigning the suitable values to the parameters. The following results are obtained as shown in the Table3.7

**Table3.7:** Results of the Economic Order Quantity derived by running the model for multiple generations (All the financial figures of Revenue, Contribution Margin, Holding Cost, Ordering Cost and Total Profit in Mn USD, and the EOQ in absolute units)

| ***First Planning Horizon*** $(m' = 1)$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $EOQ_{1,m'}$ | $EOQ_{2,m'}$ | $HC_{1,m'}$ | $HC_{2,m'}$ | $OC_{m'}$ | $TRC_{m'}$ | $Rev_{1,m'}$ | $Rev_{2,m'}$ | $TCM_{1,m'}$ | $TCM_{2,m'}$ | $TP_{m'}$ | $TP_{m'}\%$ |
| **10,216** | **24,106** | **0.66** | **1.18** | **1.80** | **3.63** | **107.27** | **303.73** | **61.30** | **144.63** | **202.3** | **49.2%** |
| ***Second Planning Horizon*** $(m' = 2)$ | | | | | | | | | | | |
| $EOQ_{1,m'}$ | $EOQ_{2,m'}$ | $HC_{1,m'}$ | $HC_{2,m'}$ | $OC_{m'}$ | $TRC_{m'}$ | $Rev_{1,m'}$ | $Rev_{2,m'}$ | $TCM_{1,m'}$ | $TCM_{2,m'}$ | $TP_{m'}$ | $TP_{m'}\%$ |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1,607** | **19,865** | **0.118** | **1.875** | **1.8** | **3.79** | **16.88** | **250.31** | **9.64** | **119.20** | **125.0** | **46.8%** |
| ***Third Planning Horizon ($m' = 3$)*** | | | | | | | | | | | |
| $EOQ_{1,m'}$ | $EOQ_{2,m'}$ | $HC_{1,m'}$ | $HC_{2,m'}$ | $OC_{m'}$ | $TRC_{m'}$ | $Rev_{1,m'}$ | $Rev_{2,m'}$ | $TCM_{1,m'}$ | $TCM_{2,m'}$ | $TP_{m'}$ | $TP_{m'}\%$ |
| **192** | **12,738** | **0.01** | **1.15** | **0.6** | **1.76** | **0.67** | **53.50** | **0.38** | **25.48** | **24.1** | **44.5%** |

**Observation 4:** (From the comparison of Table3.6 and Table3.7, and Figure3.6):

Since the demand for the first generation product gets cannibalized by the first generation product, its EOQ is much lesser after the launch of the second-generation product, than in the single generation scenario.

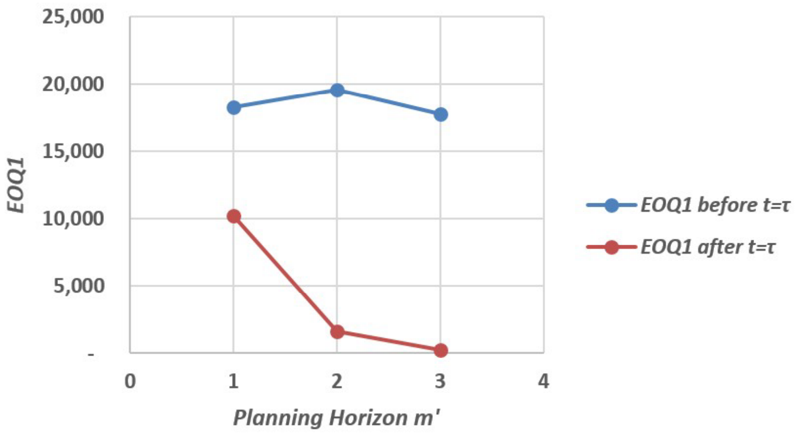


**Figure3.6.** Influence of the launch of the second generation product on $EOQ_1$

### 3.3.6. Sensitivity Analysis

In this subsection, the sensitivity analysis of the proposed model is performed for different values of the introduction time of the second generation product. The results are represented in the Table3.8.

**Table3.8: Sensitivity Analysis of the Proposed Inventory Model with different values of $\tau$** (All monetary figures in INR ('0000), and the EOQ figures in the number of units) in the first planning horizon

| τ | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| ***TP%*** | 49.11% | 49.17% | 49.16% | 49.22% | 48.93% | 48.73% | 48.49% | 48.31% | 47.94% |
| ***TP*** | 207.55 | 208.69 | 206.66 | 202.30 | 194.50 | 185.85 | 176.65 | 167.97 | 159.38 |
| ***$EOQ_1$*** | 10,510 | 19,837 | 19,650 | 10,216 | 14,332 | 12,704 | 10,905 | 9,097 | 7,405 |
| ***$EOQ_2$*** | 24,785 | 54,275 | 53,753 | 24,106 | 43,501 | 42,788 | 42,107 | 41,497 | 40,972 |
| ***$Rev_1$*** | 110.35 | 113.61 | 112.54 | 107.27 | 98.50 | 87.31 | 74.95 | 62.52 | 50.89 |
| ***$Rev_2$*** | 312.29 | 310.85 | 307.86 | 303.73 | 298.97 | 294.07 | 289.39 | 285.20 | 281.59 |
| ***TRC*** | 4.22 | 4.25 | 4.25 | 3.63 | 4.16 | 4.07 | 3.98 | 3.56 | 3.79 |
| ***TP*** | 207.55 | 208.69 | 206.66 | 202.30 | 194.50 | 185.85 | 176.65 | 167.97 | 159.38 |
| ***TP%*** | 49.11% | 49.17% | 49.16% | 49.22% | 48.93% | 48.73% | 48.49% | 48.31% | 47.94% |

The Table3.8 helps the readers observe that the profitability as % of the revenues is optimal at a certain value of $\tau$. This phenomenon can also be justified by the business rationale of the demand fulfillment models. If the second generation product gets launched too early or too late, the chances for the consolidation of the demand fulfillment activities for the two generations reduce, thereby, resulting in higher replenishment costs.

The Figure3.7 captures graphically the trend of the variation in EOQ with the change in the launch timing of the second generation product.

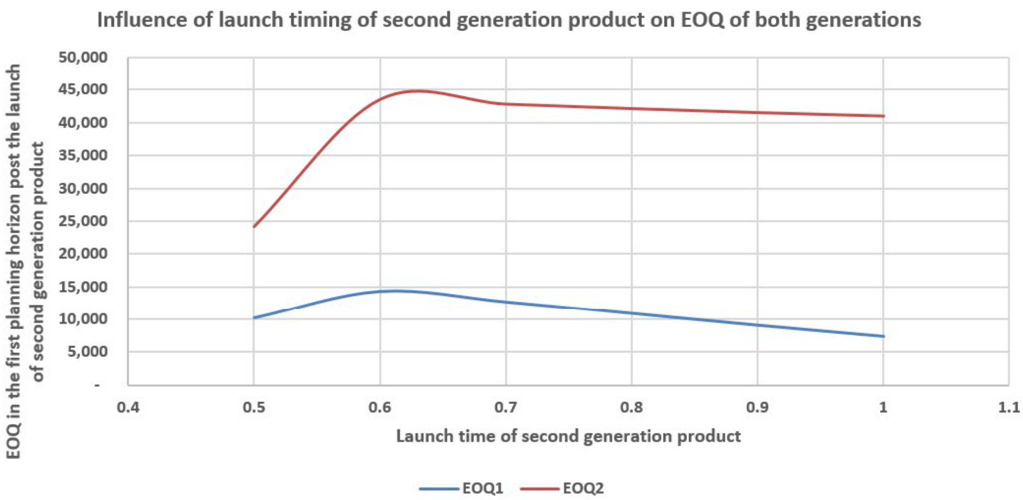


**Figure3.7. The pictorial representation of the relationship between the EOQs and $\tau$**

As shown in the Figure3.8, the absolute profits and the profitability are optimal at a particular value of $\tau$ due to loss in the potential of replenishment synergies from demand pooling for very low or very high values of $\tau$.

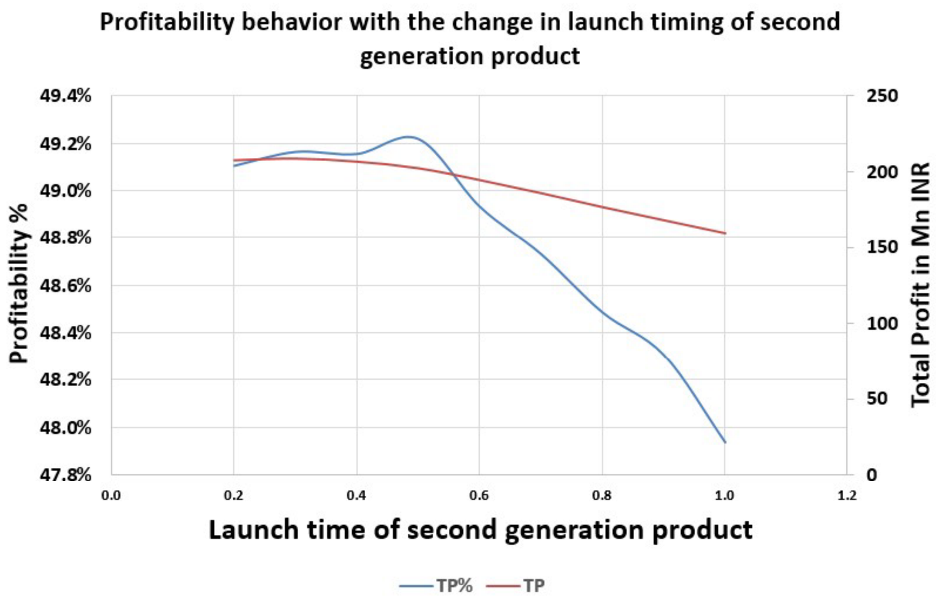


**Figure3.8. Impact of $\tau$ on the absolute profits and the profitability**

For the first planning period post the launch of the second-generation product, the EOQ of both the generations of products falls with the increase in $\tau$. This is because the delay in the launch of the second generation product shifts the first planning period to a later period by when the first generation product has declined largely on account of the shorter product life cycle, and the resulting fall in demand rate leading to lesser EOQ. Similarly, the lesser original demand rate of the first generation product in the later periods leads to a lesser scope of cannibalization (in absolute terms) by the second generation product. This, in turn, leads to a lower demand rate and hence, lower demand for the second-generation product.

## 3.4. Major academic and business implications of the proposed EOQ models

The inventory replenishment models discussed in this chapter has many important insights that can be used by inventory managers and practitioners dealing with hi-technology products. The most important point of learning that this chapter explains the demand substitution between the two successive generations of the hi-technology product. A new substitution-adoption model is also discussed for two-generation technology products. During the literature survey, it is observed that most of the inventory models were mainly concerned about accounting for demand substitution due to stock-outs or pricing or assortment decisions and completely ignored the technology-driven demand substitution. These models make the supply chain managers realize how important it is to consider the lifecycle dynamics while planning for the inventories of high technology products. The study makes it easier for them to acknowledge that inventory management would simply fail if the demand dynamics are not adequately

The proposed models also reflect the convex nature of the total cost per unit time curve for high technology products, indicating that the same can be optimized to ensure the proper utilization of scarce resources. The managers from the functions other than the supply chain can also derive benefits from this study as they can use the validated model for developing demand forecasts for the technology products for the sales planning as well as for financial planning. Some of the major findings of this chapter can be summarized as follows:

- With the introduction of the second-generation product, the revenues of the first generation product start declining and get exhausted by those of the second generation, which take some time to reach the peak, before beginning to decline.
- Even for the demand governed by innovation diffusion (which is time-varying and highly non-linear), for any generation of product, the EOQ tends to increase with the demand rate, while tends to fall with the decline in the demand rate.
- As the first generation product declines to a level where its contribution margin is inadequate to cover the sum of holding cost and product-specific ordering cost, it may make sense to discontinue the first-generation product in the market.
- The faster the diffusion of the latest generation product in the market, the lower (*the higher*) the EOQ that needs to procure by the managers in each lot in case of unpooled logistics (*the pooled logistics*)
- It makes sense to go for unpooled logistics for the multiple generations when the product non-specific ordering costs are lower than the potential savings in the replenishment costs with the differential replenishment cycles for the multiple generations.

The chapter draws insights on the influence of innovation and imitation effect on the diffusion of technology and the implications for inventory management. The managers can also understand the impact of the cost of capital on the inventory norms. The firms that have higher WACC (Weighted Average Cost of Capital) should have lower optimal EOQ as compared to the firms that lower WACC. The managers can also realize the impact of the diffusion rate on the inventory norms for multi-generational products. This chapter tried to discuss a very basic foundation on the inventory planning and optimization for sequential-generation products under the innovation diffusion framework. However, a lot of realistic scenarios need to be studied in future research. One of the most prominent ones among them is the influence of the trade credits on the inventory decisions for technology generations. In the next chapter new EOQ models under the trade-credit benefit are discussed in detail.

## Appendix 3

**A. Nature of the cost components for Single Period Model**

From Equation (3.28), it can be observed that

$$K(T_1) = O_1(T_1) + O_2(T_1) + B_1(T_1) + B_2(T_1) + H_1(T_1) + H_2(T_1) \qquad (3.A.1)$$

$O_i(T_1)$ = Ordering cost for ith generation per unit time

$B_i(T_1)$ = Basic Purchase cost for ith generation per unit time

$H_i(T_1)$ = Inventory Holding cost for ith generation per unit time

Thus, the optimal planning horizon is arrived at by minimizing the sum of the three costs per unit time: ordering cost per unit time, basic procurement cost per unit time, and the inventory holding cost per unit time.

Using the above notations, the following equations can be derived:

$$O_1(T_1) = \frac{A_1}{T_1} \quad (3.A.2)$$

$$O_2(T_1) = \frac{A_2}{(T_1-\tau)} \quad (3.A.3)$$

$$B_1(T_1) = \frac{M_1C_1[F_1(T_1)-J(T_1)+J(\tau)]}{T_1} \quad (3.A.4)$$

$$B_2(T_1) = \frac{C_2[M_2F_2(T_1-\tau)+M_1J(T_1)-M_1J(\tau)]}{(T_1-\tau)} \quad (3.A.5)$$

$$H_1(T_1) = \frac{M_1C_1I_1}{T_1}[[F_1(T_1)-J(T_1)]T_1 + J(\tau)\tau] - \frac{M_1C_1I_1}{T_1}\int_0^{T_1}[F_1(t)]dt + \frac{M_1C_1I_1}{T_1}\int_\tau^{T_1} J(t)dt \quad (3.A.6)$$

$$H_2(T_1) = \frac{I_2C_2}{(T_1-\tau)}[M_2F_2(T_1) + M_1J(T_1)](T_1-\tau) - \frac{M_2I_2C_2}{(T_1-\tau)}\int_\tau^{T_1}[F_2(T_1)]\,dt] - \frac{M_1I_2C_2}{(T_1-\tau)}\int_\tau^{T_1}[J(t)] \quad (3.A.7)$$

Let $O_1'(T_1), O_2'(T_1), B_1'(T_1), B_2'(T_1), H_1'(T_1), H_2'(T_1)$ denote the first derivative of the above expressions with respect to $T_1$ and $O_1''(T_1), O_2''(T_1), B_1''(T_1), B_2''(T_1), H_1''(T_1), H_2''(T_1)$ denote the second derivative of the above expressions with respect to $T_1$. $B_1'(T_1)$ can be defined as:

$$B_1'(T_1) = \frac{M_1C_1[F_1'(T_1)-J'(T_1)]}{T_1} - \frac{M_1C_1[F_1(T_1)-J(T_1)+J(\tau)]}{{T_1}^2} = \frac{M_1C_1[f_1(T_1)-f_1(T_1)F_2(T_1)]}{T_1} - \frac{M_1C_1[F_1(T_1)-J(T_1)+J(\tau)]}{{T_1}^2} \quad (3.A.8)$$

The nature of the above expression depends on the difference between the value of $f_1(T_1)[1-F_2(T_1)]$ at time $T_1$ and its cumulative average till time $T_1$. For small values of $T_1$, where the first generation product is in a high growth stage and the second generation product has just been introduced with lower innovation coefficient than the first-generation product; the expression $f_1(T_1)[1-F_2(T_1)]$ shall be increasing with $T_1$. However, as the value of $T_1$ increases further to a value $\gg \tau$, this expression starts declining because, after the advent of maturity stage for 1st generation product and the launch of the second-generation product, the rate of growth in the adoption rate of the first-generation product is minimal, and is lesser than the rate of increase in the interaction effect. However, since $B_1(T_1)$ is a function of demand rate, and the market potential is constant, the value of $B_1(T_1)$ will average out in the long run, and the variance in $B_1(T_1)$ will be much lesser than the variance in $H_1(T_1)$ and $O_1(T_1)$. Therefore, the effect of $B_1(T_1)$ can be ignored.

The similar logic applies to $B_2(T_1)$ also, and therefore, the effect of $B_2(T_1)$ can also be ignored. Similarly, the holding costs path per unit time for both the generations can be explained as follows:

$$H_1(T_1) = \frac{M_1C_1I_1}{T_1}[[F_1(T_1) - J(T_1)]T_1 + J(\tau)\tau] - \frac{M_1C_1I_1}{T_1}\int_0^{T_1}[F_1(t)]dt + \frac{M_1C_1I_1}{T_1}\int_\tau^{T_1}J(t)dt \quad (3.A.9)$$

$$\Rightarrow H_1'(T_1) = \{M_1C_1I_1[f_1(T_1) - f_1(T_1)F_2(T_1)] - \frac{M_1C_1I_1}{T_1}[F_1(T_1) - J(T_1)]\} - \frac{M_1C_1I_1}{{T_1}^2}\int_\tau^{T_1}J(t)dt + \frac{M_1C_1I_1J(\tau)\tau}{T_1} \quad (3.A.10)$$

In general, new generation products are introduced when the existing generation product has passed its growth stage or is in the late growth stage. Hence, it can be concluded that the first term in equation (3.A.10) is positive and increases with $T_1$ unless $T_1 \gg \tau$. This is because $f_1(T_1) - J'(T_1)$ is monotonically increasing with $T_1$ unless $T_1 \gg \tau$. This happens due to the initial growth of the second-generation product at its early stage of the lifecycle can be less than the market share of the existing generation (Chanda and Bardhan 2008). Therefore, the first part of the above expression is the difference of an increasing function from its cumulative average and is always positive. The second part of the above expression boils down to $J(\tau)\tau - \frac{1}{T_1}\int_\tau^{T_1}J(t)dt$. This is the difference between the cumulative interaction effect lost due to the time gap between the introduction of two generations and the average interaction effect per unit time after the launch of the second-generation product. Since $J(t)$ is a curve that is nearly constant for small values of $T_1$ near to $\tau$, and increases faster as $T_1 \gg \tau$. It can be argued that the cumulative interaction effect lost due to the introduction time gap will be greater than the average interaction effect per unit time for small values of $T_1$, and hence the above expression is positive. Thus, it can be inferred that $H_1'(T_1)$ is always positive. Using a similar analogy, it can also be argued that $H_2'(T_1)$ is always positive. The second-order derivative of $H_2(T_1)$ can be given as:

$$H_2''(T_1) = I_2C_2[M_2f_2'(T_1) + M_1J''(T_1)] - \frac{M_2I_2C_2}{(T_1-\tau)}[f_2(T_1)] - \frac{M_1I_2C_2}{(T_1-\tau)}J'(T_1) + \frac{M_2I_2C_2}{(T_1-\tau)^2}[F_2(T_1)] + \frac{M_1I_2C_2}{(T_1-\tau)^2}J(T_1) + \frac{M_2I_2C_2F_2(T_1)}{(T_1-\tau)^2} + \frac{M_1I_2C_2}{(T_1-\tau)^2}J(T_1) - \frac{2M_2I_2C_2}{(T_1-\tau)^3}\int_\tau^{T_1}[F_2(T_1)]\,dt - \frac{2M_1I_2C_2}{(T_1-\tau)^3}\int_\tau^{T_1}[J(t)]\,dt] \quad (3.A.11)$$

***B. Theorem 3.1:*** *The total cost curve per unit time is a convex curve to the origin and hence, has a unique point of minima.*

Proof: If there are two functions $f(x)$ and $g(x)$ over the range $[a, b]$ such that $f(x)$ decreases with $x$, $g(x)$ increases with $x$, $f(a) > g(a)$; $f(b) < g(a)$; then there exists a unique point of change in curvature of $f(x) + g(x)$. And if $|f'(x)| > g'(x)$, then the point of change in curvature is a point of minima.

In the $K(T_1)$ curve, the two major influencing components are $O(T_1)$ and $H(T_1)$. This is because the basic purchase cost per unit time $B(T_1)$ remains almost constant because of the constant market potential. Since $O'(T_1)$ is always negative and $H'(T_1)$ is always positive;

Also, as $T_1 \to \tau, O(T_1) \gg 0$ and $H(T_1) \to 0$ (3.B.1)

Also, as $T_1 \to \infty, O(T_1) \to 0$ and $H(T_1) \to \infty$ (3.B.2)

Therefore, it can be inferred that there is only one point of change in curvature in the $K(T_1)$ curve, which is the point of minima. Hence, the curve is convex to the origin.

***C. Theorem 3.2:*** *The optimal supply quantity for the first generation increases while that for the second generation decreases with the increase in the introduction timing of the second generation.*

Proof:

Reproducing the equations (3.16) and (3.25) here:

$$Q_1 = M_1[F_1(T_1) - J(T_1) + J(\tau) \quad (3.C.1)$$

$$Q_2 = M_2F_2(T_1) + M_1J(T_1) - M_1J(\tau) \quad (3.C.2)$$

Since $J(\tau)$ denotes the potential interaction effect till time $\tau$, had the second generation been introduced in the market simultaneously with the first generation product. Thus, it is a positive quantity and increases with an increase in $\tau$. Hence, it can be suggested that the optimal supply quantity for the first generation increases while that for the second generation decreases with the increase in the introduction timing of the second generation.

***D. Theorem 3.3:*** *The higher coefficients of innovation and imitation for the second generation lead to lower EOQ for the first generation product.*

Proof: With the faster diffusion of the second-generation product, the sales of the first generation decline faster, leading to lower volumes. This phenomenon is illustrated in figure3.D.1. Therefore, the inventory carrying costs increase for the same lot size. This pushes the EOQ for the first generation product to higher levels, while pulling down the EOQ for the second-generation product.

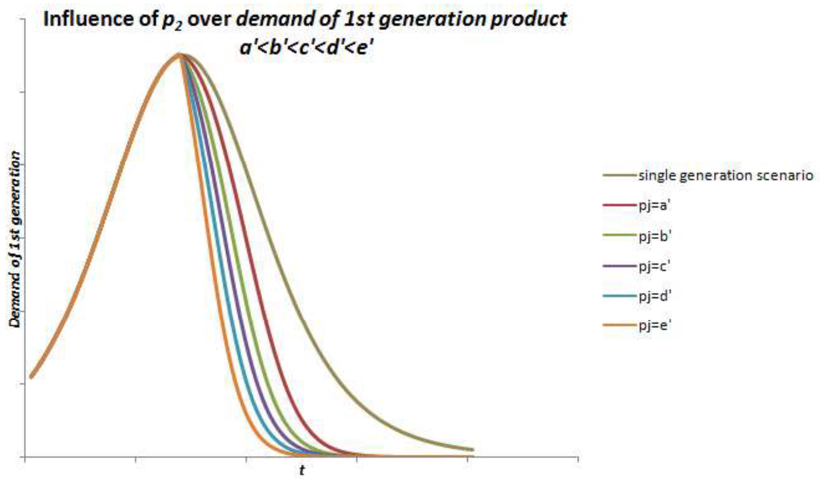


**Figure 3.D.1.** Influence of innovation and imitation coefficients of the second generation on the demand of the first-generation product

$$F_2(t) = \frac{1 - \exp(-b_2(t-\tau))}{[1 + a_2.\exp(-b_2(t-\tau))]} = \frac{b_2(t-\tau)}{[1 + a_2(1 - b_2(t-\tau))]}$$

$$= \frac{\{(p_2 + q_2)(t-\tau)\}}{\left[1 + \left(\frac{q_2}{p_2}\right)(1 - (p_2 + q_2)(t-\tau))\right]} \; for\ small\ values\ of\ (t-\tau)$$

$$\frac{\partial(F_2(t))}{\partial(p_2)} = \frac{(t-\tau)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]} + \frac{\{(p_2+q_2)(t-\tau)\}}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]^2} * \left[\left(\frac{q_2}{(p_2)^2}\right)(1-(p_2+q_2)(t-\tau)) + \left(\frac{q_2}{p_2}\right)(t-\tau)\right] = \frac{(t-\tau)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]} * \left[1 + \frac{(p_2+q_2)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]}\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\left(\frac{1}{p_2} + (t-\tau)\right)\right] \quad (3.D.1)$$

$$\frac{\partial(F_2(t))}{\partial(q_2)} = \frac{(t-\tau)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]} - \frac{\{(p_2+q_2)(t-\tau)\}}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]^2} * \left[\left(\frac{1}{p_2}\right)(1-(p_2+q_2)(t-\tau)) - \left(\frac{q_2}{p_2}\right)(t-\tau)\right] = \frac{(t-\tau)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]} * \left[1 - \frac{(p_2+q_2)}{\left[1+\left(\frac{q_2}{p_2}\right)(1-(p_2+q_2)(t-\tau))\right]}\left(\frac{1}{p_2}\right)(1-(p_2+2q_2)(t-\tau))\right] \quad (3.D.2)$$

The expressions (3.D.1) and (3.D.2) converge to “0” for smaller values of $(t-\tau)$, while converges to “1” for larger values of $(t-\tau)$. The Figure3.D.2 and Figure3.D.3 show the variation of cumulative adoption fraction for each generation with its innovation and imitation coefficients.

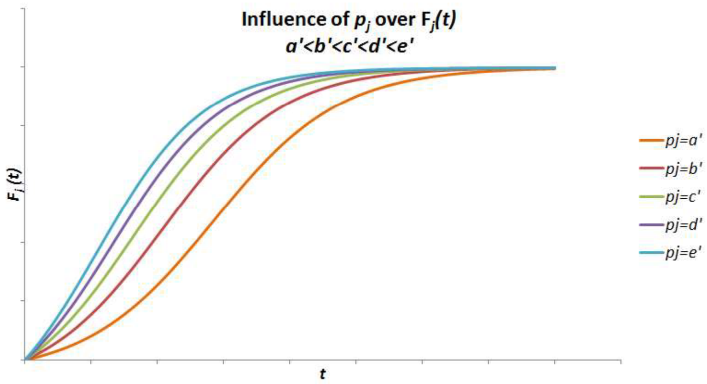


**Figure3.D.2.** Influence of innovation coefficients on the cumulative adoption function

It is visually evident from the above expressions that

$$\frac{\partial(F_2(t))}{\partial(p_2)} > 0\ and\ \frac{\partial(F_2(t))}{\partial(q_2)} > 0 \quad (3.D.3)$$

$$\frac{\partial(\lambda_1(t))}{\partial(F_2(t))} = -M_1 f_1(t) < 0 \quad (3.D.4)$$

From the two equations (3D.3) and (3D.4), it can be inferred that:

$$\frac{\partial(\lambda_1(t))}{\partial(p_2)} < 0\ and\ \frac{\partial(\lambda_1(t))}{\partial(q_2)} < 0 \quad (3.D.5)$$

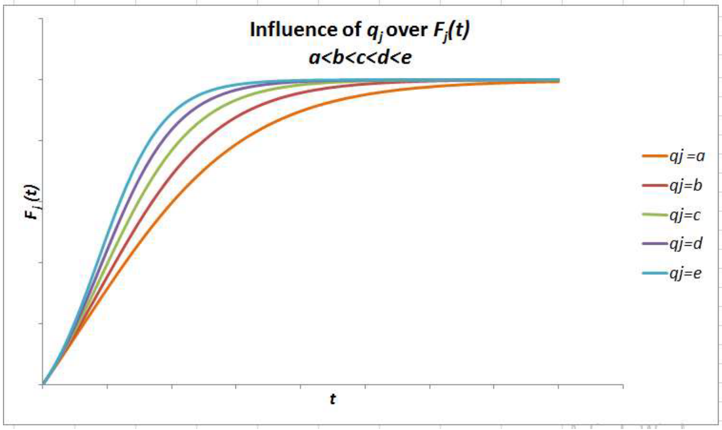


**Figure3.D.3.** Influence of imitation coefficient on cumulative adoption fraction

Higher innovation and imitation coefficients of the second generation result in the faster diffusion of the second generation product, lowering the demand for the first generation due to cannibalization. As a result, the time spent by the inventory of first-generation product in the system increases for a fixed lot size, increasing the holding costs, which tends to shift the lot size of the first generation to a lower value as shown in Figure3.D.4.

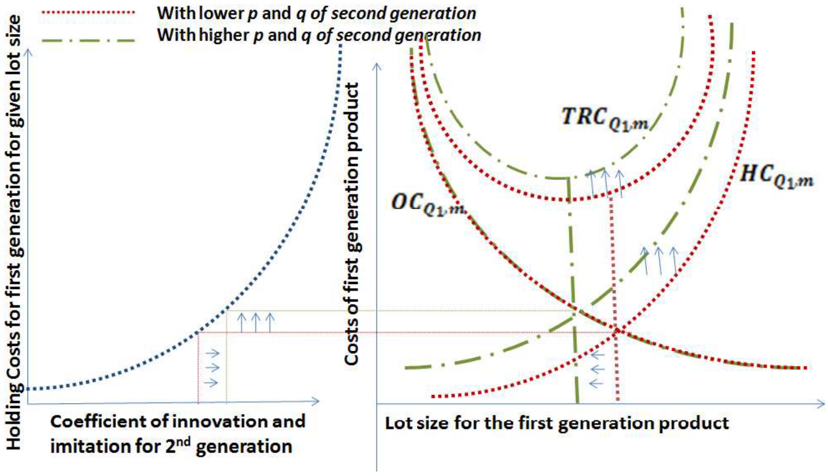


**Figure3.D.4.** Influence of innovation and imitation coefficients of second-generation products on the optimal replenishment lot size of the first generation product

***E. Theorem 3.4.*** *In case of pooled logistics for the two generations' products, the optimal EOQ for the first generation increases with the increase in the innovation and imitation coefficients for the second-generation product. In the case of un-pooled logistics for the two generations' products, with the increase in the innovation and imitation coefficients for the second-generation product, the optimal EOQ for the first generation falls, and that for second generation rises.*

Proof: In the case of pooled logistics, the sales volumes of both the generations of the products need to be consolidated. Since the second-generation product has much higher market potential as compared to the first generation product, the increase in the demand of the second generation product far outweighs the fall in the demand of the first-generation product.

$$\lambda(t) = \lambda_1'(t) + \lambda_2(t) = M_1 f_1(t) + M_2 f_2(t) \; for \; t > \tau \quad (3.E.6)$$

$$\frac{\partial(\lambda(t))}{\partial(t)} = M_1 \frac{\partial(f_1(t))}{\partial(t)} + M_2 \frac{\partial(f_2(t-\tau))}{\partial(t)} = \frac{M_1 b_1^2}{p_1} \frac{\partial\left(\frac{(1-b_1 t)}{\{1+a_1.(1-b_1 t)\}^2}\right)}{\partial(t)} + \frac{M_2 b_2^2}{p_2} \frac{\partial\left(\frac{(1-b_2(t-\tau))}{\{1+a_2.(1-b_2(t-\tau))\}^2}\right)}{\partial(t)} =$$

$$\frac{M_1 b_1^2}{p_1} \left[\frac{2.a_1 b_1(1-b_1 t)}{\{1+a_1.(1-b_1 t)\}^3} - \frac{b_1}{\{1+a_1.(1-b_1 t)\}^2}\right] + \frac{M_2 b_2^2}{p_2} \left[\frac{2.a_2 b_2(1-b_2(t-\tau))}{\{1+a_2.(1-b_2(t-\tau))\}^3} - \frac{b_2}{\{1+a_2.(1-b_2(t-\tau))\}^2}\right] =$$

$$\frac{M_1 b_1^3}{p_1\{1+a_1.(1-b_1 t)\}^2} \left[\frac{2.a_1(1-b_1 t)}{\{1+a_2.(1-b_2(t-\tau))\}} - 1\right] + \frac{M_2 b_2^3}{p_2\{1+a_2.(1-b_2(t-\tau))\}^2} \left[\frac{2.a_2(1-b_2(t-\tau))}{\{1+a_2.(1-b_2(t-\tau))\}} - 1\right] \quad (3.E.7)$$

It can be visually observed that the expression (3.E.7) is always positive for smaller values of $t$ since $a_1 \; and \; a_2 \gg 1$. Hence, it can be argued that the total demand rate increases with time in the initial stages of the launch of the second-generation product. Consequently, the combined sales volumes of both the products increase with higher innovation and imitation coefficients in the initial stages. This higher sales volume results in a faster turnover of inventories and thus reduces the inventory carrying costs, and therefore increasing the EOQ for both the generations.

However, in case of un-pooled logistics, the trade-off between the holding costs and ordering costs is looked at separately for both the generations of products. The rising sales of the second generation product and the falling sales for the first generation product makes the EOQ increase and decrease for the second generation product and the first generation product respectively.