CHAPTER

4 Economic Order Quantity Model for two generations consecutive technology products under permissible delay in payments

A trade-credit scheme is often offered by retailers to consumers to increase the diffusion of the technology products in the social system. Also, trade credits play an important role in business transactions related to these products. The importance of the trade credits is on account of two reasons: first, these products are high-value products that need support on the working capital constraints; and second, the distribution channel can find pushing these products more lucrative in the presence of the trade credits. Trade financing among firms is one of the most popular sources of financing globally. There are many benefits of inter-firm credit in a supply chain. To trade off liquidity risk and profitability, a credit policy is not only desirable but also essential for an organization's success (Kehinde et al. 2017). A combination of bank financing and supplier financing gives a retailer the best of both the Worlds (Chod, 2017). Also, the firms receiving purchase orders from creditworthy firms can borrow money to enhance sales (Yamanaka, 2016). Trade credits help the buyer in making strategic investments also, and thus, are an important mechanism of collaboration among the supply chain partners (Baiman & Rajan, 2002). Therefore, the supply chain is no more considered as the flow of material only. Rather, it is now understood to comprise of the flow of funds also (Timme & Timme, 2000). The importance of trade credit services has also increased with the increased focus of the manufacturers on their core competency (Maloni & Benton, 1997).

The integration of material and information flows within the supply chain with the flow of financial resources has become very important in recent times (Pfohl et al. 2003). Sometimes, the suppliers with weak bargaining power with their customers, sell a larger share of goods on credit, and offer a longer payment period before charging penalties (Fabbri and Klapper, 2016). The value of interest earned during the trade credit period improves the overall value proposition of the product to the buyer, incentivizing him to procure more volumes. The trade credits are also a risk mitigation mechanism for the buyer in case of uncertain demand. Sometimes, the firms offer differential trade credits on the different products to promote cross-selling and to promote the sales of one product at the expense of another one. At times, it also happens that the firms offer higher credit period on the higher profit margin products while acting conservative on the low margin products. The trade credits not only influence the sales volumes but also influence the overall profitability dynamics of the products. Trade credit offered by the supplier to the buyers is a crucial tool to enhance the sales. Table 4.1 mentions the patterns of the credit-linked demand that have been considered by the earlier studies.

Table4.1. Demand patterns considered by the existing studies on credit-linked demand

Demand Pattern	Studies
$D(PD) = K\ exp(\alpha.PD)$ where PD is the permissible delay in payments , or the credit period, K and α are constants >0	Chern et al. (2013), Chern et al. (2014), Wang et al. (2014), Wu et al. (2017), Su et al. (2007), Chung (2012a)
$D(N,p) = D_0\ N^{\alpha}\ p^{\beta}$ where N is the credit period, p is the selling price; D_0 , α and β are constants >0	Ho et al. (2011)
$D(s,N) = \alpha(s) - [\alpha(s) - \beta(s)]\exp(-rN)$ where $\alpha(s)$ is the maximum demand at the selling price of s, N is the credit period, p is the selling price; $0 \leq r < 1$ is the rate of demand saturation	Thangam and Uthayakumar (2009), Jaggi et al. (2008)
$D(t) = D_0\exp(b_1 N(M - N)t)$ where N is the credit offered by retailer to the customer, and M is the credit period offered to the retailer by the supplier, b_1 is a constant	Banu and Mondal (2016)

Therefore, it becomes highly imperative for the supply chain managers to consider the effect of trade credits on the optimal replenishment norms for these products. This influence needs to be considered by the supply chain practitioners involved in the procurement of these products. The cost efficiencies can be achieved if the perfect balance between the two conflicting costs- one time fixed cost of ordering and the recurring inventory carrying cost is achieved. This is because while the former of these costs rise with the increase in replenishment frequency, the later falls with the same. Panda et al. (2005) used non-linear goal programming for determining the EOQ for multi-item supply chains. Tsao and Sheen (2007) developed the inventory models to incorporate the purchase costs that were dependent upon time and lot size for multiple items under the permissible delay of payments. Tsao (2010) extended the scope of existing research by considering multiple echelons with trade allowances under the credit period. However, none of the works on inventory modeling under credit financing has covered the multiple generation products.

When technology products are considered, their demand is governed by the diffusion of innovations theory. And it is not just these products, but the usage of newer functions on these products that follows the innovation diffusion theory (Lim et al. 2019). Kim et al. (2017) also proposed that even the demand for the low involvement products gets influenced by electronic word-of-mouth suggesting an imitation effect. When it comes to the inventory modeling for the technology generations under the trade credits mechanism, the work is hard to find in the existing literature. Chanda and Kumar (2017) formulated the EOQ model for technology products under the trade credits while considering dynamic pricing and advertising. Chanda and Kumar (2019) developed a similar model under trade credits for dynamic market potential. None of the works on inventory modeling under credit financing has covered the multiple generation products.

In this chapter[6], the optimal replenishment policies for two succeeding generations' technology products under partial trade credit financing are being discussed. Life cycle dynamics are used to track demand rates of technology generations. For precise estimation of demands of technology generations product, it is important to correctly identify the introduction time of the advanced generation product. As it is often been seen that advanced generation product reduces the sales of existing generation product. Thus for a multi-generation product, it is important to incorporate the interaction effect among generational products in replenishment policies. In the first model, the total cost function has been formulated for five different situations depending upon the new generation introduction timing and the length of the trade credit period. A detailed sensitivity analysis is been performed to explore the efficacy of the model in a given situation. In the second model, the multi-period inventory replenishment decisions under trade credits and credit dependent demand have been considered.

Single period Inventory Model under Trade Credits

The objective of this model is to minimize the total cost and develop the EOQ model for successive generations of high-technology products (that get diffused through the innovation diffusion process) under the trade credit mechanism. In this section, the same demand model is used as formulated in section 3.1 to derive the EOQ policies. The results of this research are expected to help the practitioners in making inventory policies for such products while considering the impact of credit terms.

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[6] This chapter is based on two research papers:

a. Nagpal, G. & Chanda, U. (2021). "Economic Order Quantity Model for Two-Generation Consecutive Technology Products under Permissible Delay in Payments", *International Journal of Procurement Management. 14*(1), 193-225.
b. Nagpal, G. & Chanda, U. (2021). "Inventory Replenishment Policies for Two Successive Generations of Technology Products under Permissible Delay in Payments", *International Journal of Information Systems and Supply Chain Management.* (Under Review)

4.1.1. Notations of the Model

A_j fixed cost of ordering per order for j^{th} generation product (j = 1,2)

C_j basic procurement cost per unit for j^{th} generation product (j = 1,2)

I_j inventory holding expense (as a % of cost) for j^{th} generation product (j = 1,2)

M_j the size of market potential for j^{th} generation product (j = 1,2)

T_j the duration of the planning horizon for j^{th} generation product (j = 1,2)

Q_j number of units received at the beginning of planning horizon for j^{th} generation product (j = 1,2)

p_j coefficient of innovation for j^{th} generation product (j = 1,2)

q_j coefficient of imitation for j^{th} generation product (j = 1,2)

τ be the time at which the second-generation product is introduced

$\lambda_1(t)$ and $\lambda_1'(t)$ demand rate at time $'t'$ of 1^{st} generation product for $t \leq \tau$ and $t \geq \tau$ respectively

$\lambda_2(t)$ demand rate at time $'t'$ of 2^{nd} generation product

$\pi_1(t)$ is the conditional probability of a prospective adopter (who has not yet adopted the product till time t) adopting the product in time (t, $t+\Delta t$)

$F_1(t)$ is the cumulative fraction of adopters till time t.

$f_1(t)$ is the fraction of adopters at time t.

P_1 is the selling price per unit of the first-generation product

P_2 is the selling price per unit of the second-generation product

I_c is the annual interest rate paid on the sales accomplished post the payment to the supplier

I_e is the annual interest rate earned on the sales accomplished before the payment to the supplier

PD is the credit period offered to the retailer by the supplier

4.1.2. Assumptions of the Model

- The supply gets replenished instantaneously
- There is zero lead time in the procurement of inventories
- The shortages in meeting the demand at any time instant are not allowed
- The rate of demand is influenced by the innovation diffusion process and follow the assumptions as discussed in section 3.1.3 and can be given as follows:

  $\lambda_1(t) = M_1 f_1(t) \qquad for\, t < \tau$

  $\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t) F_2(t) \qquad for\, t > \tau$

  $\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t) F_2(t)$
- The interest rates are the same across the generations
- There is a finite credit period being offered by the supplier.

If there is no deterioration of the product, the consumption of inventory takes place on account of the demand usage only. Therefore, for $0 \leq t \leq T$

$$\lambda_j(t) = -\frac{d\left(I_j(t)\right)}{dt} \; ; \; (j = 1,2) \tag{4.1}$$

4.1.3. Inventory Modeling for the single generation scenario

$$I_1(t) = \begin{cases} \int_0^t -\lambda_1(t)dt & for\ t < \tau \\ \int_\tau^t -\lambda_1'(t)dt & for\ t \geq \tau \end{cases} \tag{4.2}$$

Solving equation (4.2) and considering that inventory is zero at the time "0", the following equation can be obtained:

$$I_1(t) = \begin{cases} Q_1 - M_1F_1(t) & for\ t < \tau \\ Q_1 - M_1F_1(t) + M_1\int_\tau^t f_1(t)F_2(t)dt & for\ t > \tau \end{cases} \tag{4.3}$$

Let $\int_\tau^t f_1(t)F_2(t-\tau)dt$ be written as: $L(t) - L(\tau)$ (4.4)

By substituting (4.4) into (4.3), the following can be said:

$$\text{So, } I_1(t) = \begin{cases} M_1\left(F_1(T_1) - L(T_1) + L(\tau) - F_1(t)\right) & for\ t < \tau \\ Q_1 - M_1F_1(t) + M_1L(t) - M_1L(\tau) & for\ t \geq \tau \end{cases} \tag{4.5}$$

Since the initial supply gets consumed in the planning horizon, it can be inferred that

$$Q_1 = \int_0^\tau \lambda_1(t)dt + \int_\tau^{T_1} \lambda_1'(t)dt = M_1[F_1(T_1) - L(T_1) + L(\tau)] \tag{4.6}$$

Incorporating (4.6) into (4.5) gives the following:

$$I_1(t) = \begin{cases} M_1\left(F_1(T_1) - L(T_1) + L(\tau) - F_1(t)\right) & for\ t < \tau \\ M_1\left(F_1(T_1) - L(T_1) + L(t) - F_1(t)\right) & for\ t \geq \tau \end{cases} \tag{4.7}$$

The total cost per unit time $\left(K_j(T_1)\right)$ is the sum of ordering cost, basic procurement cost, the inventory holding cost, the interest charged, and the interest earned. Thus, $K_1(T_1)$ can be given as:

Case 1: When $PD \in (0, \tau]$

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}\left[\int_0^\tau I_1(t)dt + \int_\tau^{T_1} I_1^*(t)dt\right] + \frac{I_cC_1}{T_1}\left[\int_{PD}^\tau I_1(t)dt + \int_\tau^{T_1} I_1^*(t)dt\right] - \frac{I_eP_1}{T_1}\int_0^{PD} t\lambda_1(t)dt$$

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\alpha_{1\tau} - \alpha_{1PD} + \beta_{1T1} - \beta_{1\tau}] - \frac{I_eP_1}{T_1}[\Upsilon_{1PD} - \Upsilon_{10}] \tag{4.8}$$

Where

$\alpha_{1t} = \int_0^t I_1(t)dt$

$\beta_{1t} = \int_0^t I_1^*(t)dt$

$\Upsilon_{1t} = \int_0^t t\lambda_1(t)dt$

Case 2: When $PD \in (\tau, T_1)$

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\int_0^{\tau} I_1(t)dt + \int_{\tau}^{T_1} I_1^*(t)dt] + \frac{I_cC_1}{T_1}\int_{PD}^{T_1} I_1^*(t)dt - \frac{I_eP_1}{T_1}[\int_0^{\tau} t\lambda_1(t)dt + \int_{\tau}^{PD} t\lambda_1'(t)dt] = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\beta_{1T1} - \beta_{1P}] - \frac{I_eP_1}{T_1}[\Upsilon_{1\tau} - \Upsilon_{10} + \xi_{1PD} - \xi_{1\tau}] \quad (4.9)$$

Where, $\xi_{1t} = \int_0^t t\lambda_1'(t)dt$

Case3: When $PD \in (T_1, \infty)$;

$$K_1(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\int_0^{\tau} I_1(t)dt + \int_{\tau}^{T_1} I_1^*(t)dt] - \frac{I_eP_1}{T_1}[\int_0^{\tau} t\lambda_1(t)dt + \int_{\tau}^{T_1} t\lambda_1'(t)dt + \int_{T_1}^{PD}[\int_0^{\tau}(\lambda_1(t)\,dt + \int_{\tau}^{T_1}\lambda_1'(t)\,dt]dt] = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] - \frac{I_eP_1}{T_1}[\Upsilon_{1\tau} - \Upsilon_{10} + \xi_{1T1} - \xi_{1\tau} + [PD - T_1]Q_1 = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\int_0^{\tau} I_1(t)dt + \int_{\tau}^{T_1} I_1(t)dt] \quad (4.10)$$

4.1.4. Inventory Modeling for the two generations scenario $(t \geq \tau)$

Integration of equation (4.1) for the second generation gives:

$$I_2(t) = -\int_{\tau}^{t} \lambda_2(t)dt + Constant \quad (4.11)$$

Since Inventory at time τ is Q_2, $\Longrightarrow Constant = Q_2$

Thus, the equation (4.11) states that $I_2(t) = Q_2 - \int_{\tau}^{t} \lambda_2(t)dt$ (4.12)

Incorporating the demand equation from the assumptions in (4.12), the following can be obtained:

$$I_2(t) = Q_2 - M_2F_2(t) - M_1\int_{\tau}^{t} f_1(t)F_2(t)dt \quad (4.13)$$

Incorporating (4.4) in (4.13) gives:

$$I_2(t) = Q_2 - M_2F_2(t) - M_1L(t) + M_1L(\tau) \quad (4.14)$$

Since the planning horizon for both the generations is assumed to be the same, the inventory stocking period for the second generation T_2 shall be lesser than that for the first generation T_1 by τ.

Thus, $\boldsymbol{T_2 = T_1 - \tau}$ (4.15)

Since initial inventory will get consumed by time T_2, thus the initial inventory is the total demand for the second-generation product during the period (τ, t)

So, $Q_2 = M_2F_2(T_1) + M_1L(T_1) - M_1L(\tau)$ (4.16)

Incorporating (4.16) in (4.14) gives:

$$I_2(t) = M_2F_2(T_1) + M_1L(T_1) - M_2F_2(t) - M_1L(t) \quad (4.17)$$

Thus, the total cost per unit time for the second generation product $(K_2(T))$ can be given

Case 1: When $PD \in (0, \tau]$

Case 1a: When $PD \in (0, \tau]$ and $PD \in (0, T_1 - \tau]$

$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1} I_2(t)\,dt + \frac{I_cC_2}{(T_1-\tau)}\int_{PD+\tau}^{T_1} I_2(t)dt - \frac{I_eP_1}{(T_1-\tau)}\int_{\tau}^{PD+\tau}(PD + \tau -$
$t)\lambda_2(t)dt = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] + \frac{I_cC_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2(PD+\tau)}] - \frac{I_eP_1}{(T_1-\tau)}[(PD +$
$\tau)(\vartheta_{2(PD+\tau)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau})]$ (4.18)

Where

$\alpha_{2t} = \int_0^t I_2(t)\,dt$

$\vartheta_{2t} = \int_0^t \lambda_2(t)dt$

and $\phi_{2t} = \int_0^t t\lambda_2(t)dt$

Case 1b: When $PD \in (0,\tau]$ and $PD \in (T_1-\tau,\tau]$ or when $PD \in (T_1-\tau,\tau]$ and $T_1 \in (\tau,2\tau)$

$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1} I_2(t)\,dt - \frac{I_eP_2}{(T_1-\tau)}\int_{\tau}^{T_1}(T_1 - t)\lambda_2(t)dt - \frac{I_eP_2}{(T_1-\tau)}(PD - T_1 +$
$\tau)Q_2 = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau}) + (PD -$
$T_1 + \tau)Q_2]$ (4.19)

Case2: When $PD \in (\tau, T_1)$

Case 2a: When $PD \in (\tau, T_1)$ and PD $\in (T_1 - \tau, T_1)$

$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1} I_2(t)\,dt - \frac{I_eP_2}{(T_1-\tau)}\int_{\tau}^{T_1}(T_1 - t)\lambda_2(t)dt - \frac{I_eP_2}{(T_1-\tau)}(PD - T_1 +$
$\tau)Q_2 = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(T1)} - \phi_{2\tau}) + (PD -$
$T_1 + \tau)Q_2]$ (4.20)

Case 2b: When $PD \in (\tau, T_1)$ and PD $\in (\tau, T_1 - \tau)$

$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1} I_2(t)\,dt + \frac{I_cC_2}{(T_1-\tau)}\int_{PD+\tau}^{T_1} I_2(t)dt - \frac{I_eP_2}{(T_1-\tau)}\int_{\tau}^{PD+\tau}(PD + \tau -$
$t)\lambda_2(t)dt$
$= \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2(T1)} - \alpha_{2\tau}] + \frac{I_cC_2}{(T_1-\tau)}[\alpha_{2(T1)} - \alpha_{2(PD+\tau)}] - \frac{I_eP_2}{(T_1-\tau)}[(PD + \tau)(\vartheta_{2(PD+\tau)} -$
$\vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau})]$ (4.21)

Case3: When $PD \in (T_1, \infty)$;

$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_{\tau}^{T_1} I_2(t)\,dt - \frac{I_eP_2}{(T_1-\tau)}\int_{\tau}^{T_1}(T_1 - t)\lambda_2(t)dt - \frac{I_eP_2}{(T_1-\tau)}(PD - T_1 +$
$\tau)Q_2 = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(T1)} - \phi_{2\tau}) + (PD -$
$T_1 + \tau)Q_2]$ (4.22)

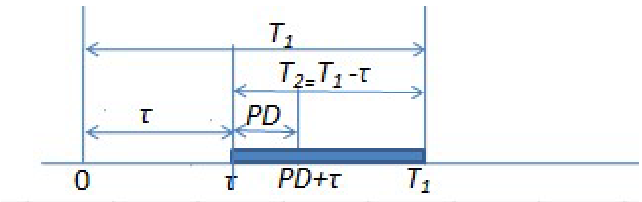
Using the above equations, the total cost can be given as follows:

Total cost per unit period $(K(T_1))$ can be given as:

$K(T_1) = K_1(T_1) + K_2(T_1)$

Since the objective is to minimize the cost function $K(T_1)$, hence the necessary conditions for minimizing $K(T)$ are $\frac{\partial K(T_1)}{\partial T} = 0$ with the sufficient condition $\frac{\partial^2 K(T_1)}{\partial T^2} > 0$

Case 1a: <u>When $PD \in (0,\tau]$ and $PD \in (0, T_1 - \tau]$</u>



$$K(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\alpha_{1\tau} - \alpha_{1PD} + \beta_{1T1} - \beta_{1\tau}] - \frac{I_eP_1}{T_1}[\Upsilon_{1PD} - \Upsilon_{10}] + \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] + \frac{I_cC_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2(PD+\tau)}] - \frac{I_eP_1}{(T_1-\tau)}[(PD + \tau)(\vartheta_{2(PD+\tau)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau})] \quad (4.23)$$

$$K(T_1) = \frac{A_1}{T_1} - \frac{I_eP_1}{T_1}[\Upsilon_{1P} - \Upsilon_{10}] + \frac{A_2}{(T_1-\tau)} - \frac{I_eP_1}{(T_1-\tau)}[(PD + \tau)(\vartheta_{2(PD+\tau)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau})] + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\alpha_{1\tau} - \alpha_{1PD} + \beta_{1T1} - \beta_{1\tau}] + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] + \frac{I_cC_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2(PD+\tau)}] \quad (4.24)$$

In the above expression, the following two terms $\frac{A_1}{T_1}$ and $\frac{A_2}{(T_1-\tau)}$ are decreasing with T_1 at an increasing rate. This is because of the first derivative w.r.t. T_1 being lesser than zero, and the second derivative w.r.t T_1 being positive for each of these. Also, the two expressions of interest earned are decreasing with T_1 at an increasing rate, based on a similar agreement. This interest earned is getting deducted to give the overall cost. Since for small values of PD< τ, interest earned is very small, and is too minimal to nullify the decline in the ordering cost with the planning horizon. Thus, the ordering cost net off the interest earnings per unit time falls with the increase in the length of planning horizon.

It is well-known that if there are two functions in x, $f(x)$ and $g(x)$ over the interval (a, b) such that $f'(x) < 0$ and $g'(x) > 0$ in the interval (a, b); $f''(a) > g''(a)$ and $f''(b) < g''(b)$, then the curve $h(x) = f(x) + g(x)$ is convex to the origin.

During the early stage of introduction, the demand rate will follow the increasing pattern and because of which the basic purchase costs per unit time increase. So, the first derivative as well as the second derivative of basic purchase cost per unit time with respect to planning horizon is positive. However, the second derivative of the declining ordering cost is much higher than the second derivative of the rising basic purchase costs and holding costs, as a result of which the overall cost declines with the increase in the length of planning horizon. Gradually, as the demand picks up, the second derivative of the rising basic purchase costs ($\frac{Q_1C_1}{T_1}$ and $\frac{Q_2C_2}{(T_1-\tau)}$) and that of holding costs exceeds the second derivative of the falling ordering costs; and therefore, the overall costs start rising with the planning horizon. Since the overall costs per unit time initially fall and then rise with the planning horizon, they are convex to the origin and have a point of minima. The Figure4.1 shows the convex nature of the total cost curve for the case 1a.

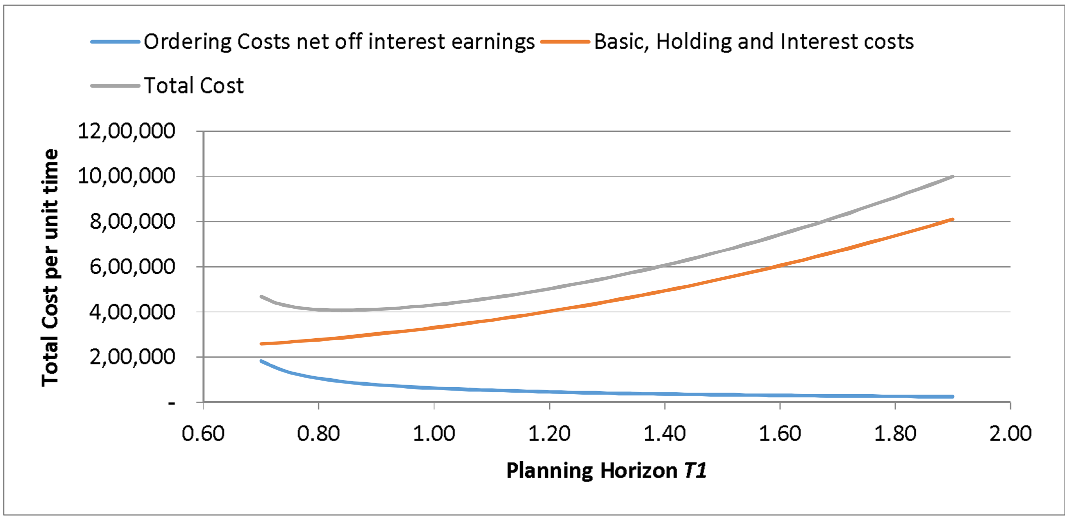
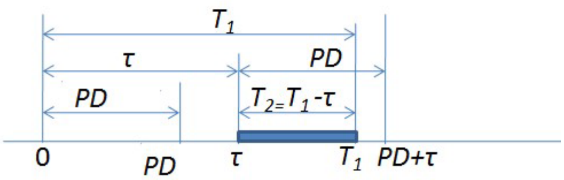


Figure4.1. Total Cost Curve for Case 1a

Case 1b: When $PD \in (0,\tau]$ and $PD \in (T_1-\tau,\tau]$ or when $PD \in (T_1-\tau,\tau]$ and $T_1 \in (\tau,2\tau)$



$$K(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\alpha_{1\tau} - \alpha_{1PD} + \beta_{1T1} - \beta_{1\tau}] - \frac{I_eP_1}{T_1}[\Upsilon_{1PD} - \Upsilon_{10}] + \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau}) + (PD - T_1 + \tau)Q_2] \quad (4.25)$$

$$K(T_1) = \frac{A_1}{T_1} - \frac{I_eP_1}{T_1}[\Upsilon_{1PD} - \Upsilon_{10}] + \frac{A_2}{(T_1-\tau)} - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau}) + (PD - T_1\tau)Q_2] + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\alpha_{1\tau} - \alpha_{1P} + \beta_{1T1} - \beta_{1\tau}] + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] \tag{4.26}$$

Now, consider the above total cost per unit time expression. The ordering costs (net off the interest earnings) per unit time fall with the increase in the length of planning horizon. While the basic purchase costs and the interest charges per unit time increase with the increase in the length of the planning horizon, since a larger planning horizon indicates the larger lot of supply, and these costs are volume dependent. For the small values of the planning horizon T_1, it pertains to the initial stages of the product life cycle, when the volumes are very low. Therefore, the basic purchase costs, and the interest charges per unit time will rise at a lower rate as compared to the rate of increase in the ordering costs with the increase in the length of planning horizon. Thus, the total cost per unit time declines with the planning horizon for the smaller values of the planning horizon. As the value of the planning horizon increases, this pattern reverses, and the total cost per unit time starts increasing with the planning horizon; thereby, creating a point of minima and the convexity of the total cost curve to the origin.

Since the length of the planning horizon in the case 1b less than 2τ, it corresponds to the initial stages of the product life cycle, in which the demand-dependent costs (like basic purchase cost, holding cost, and interest charges) per unit time increase at a very slow rate. Therefore, while the total cost per unit time of the first generation product achieves a point of minima by then, the cost per unit time of the second generation product is less prone to achieving the point of minima. However, the overall costs of the first generation product during this stage are higher than those of the second generation stage. The Figure4.2 shows the convex nature of the total cost curve for the case 1b.

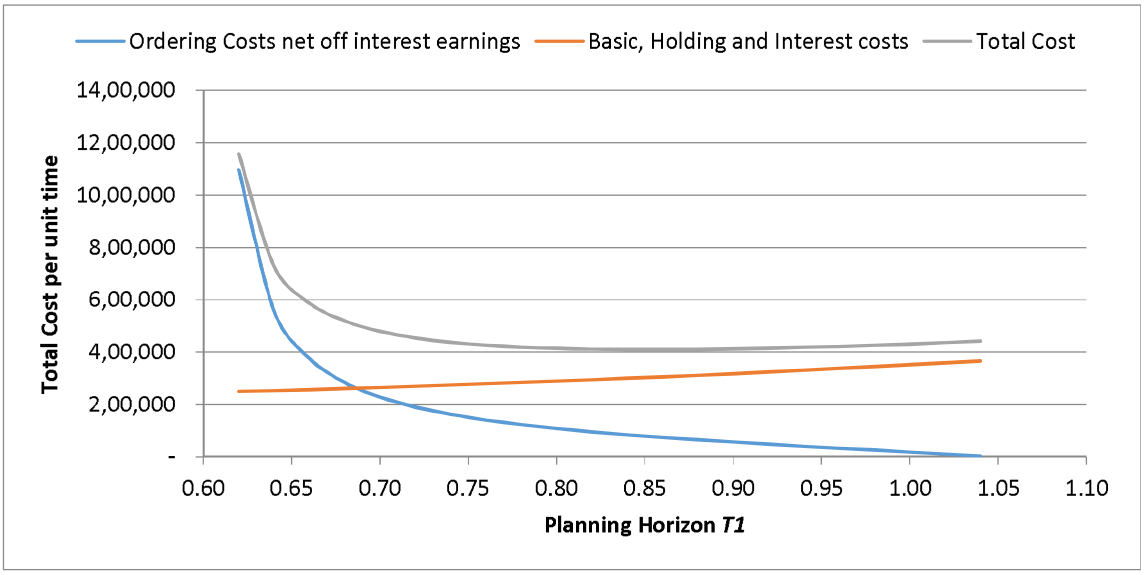
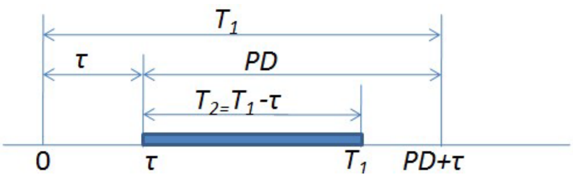


Figure4.2. Total Cost Curve for Case 1b

Case 2a: When $PD \in (\tau, T_1)$ and PD $\in (T_1 - \tau, T_1)$



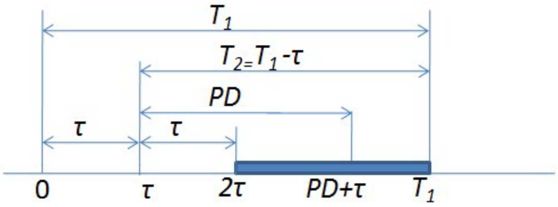
$$K(T_1) = \frac{A_1}{T_1} + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\beta_{1T1} - \beta_{1PD}] - \frac{I_eP_1}{T_1}[\Upsilon_{1\tau} - \Upsilon_{10} + \xi_{1P} - \xi_{1\tau}] + \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(T1)} - \phi_{2\tau}) + (PD - T_1 + \tau)Q_2] \quad (4.27)$$

Re-arranging the terms, the following expression is obtained:

$$K(T_1) = \frac{A_1}{T_1} - \frac{I_eP_1}{T_1}[\Upsilon_{1\tau} - \Upsilon_{10} + \xi_{1PD} - \xi_{1\tau}] + \frac{A_2}{(T_1-\tau)} - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(T1)} - \phi_{2\tau}) + (PD - T_1 + \tau)Q_2] + \frac{Q_1C_1}{T_1} + \frac{I_1C_1}{T_1}[\alpha_{1\tau} - \alpha_{10} + \beta_{1T1} - \beta_{1\tau}] + \frac{I_cC_1}{T_1}[\beta_{1T1} - \beta_{1PD}] + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] \quad (4.28)$$

From the above expression, the first part is the ordering costs net off interest benefits from trade credit mechanism per unit time. This part falls with the increase in the length of the planning horizon. The second part of the expression is the inventory carrying cost and interest cost per unit time which increases with the length of the planning horizon since these costs are incurred on a larger lot of supply. However, the small values of planning horizon correspond to the initial stages of productlife cycle when the volumes are quite small, and hence, rise in he second ortion of the expressions is much lesser than the fall in the first portion, which leads to the falling cost per unit time. For small values of planning horizon, the total cost falls for the short terms of the planning horizon. But as the length of the planning horizon increases, the total cost per unit time starts increasing due to the sharper rise in the product holding costs and interest charges as compared to the fall in ordering costs. Thus, the total cost curve is convex to the origin.

Case 2b: When $PD \in (\tau, T_1)$ and PD $\in (\tau, T_1 - \tau)$



$$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2(T1)} - \alpha_{2\tau}] + \frac{I_cC_2}{(T_1-\tau)}[\alpha_{2(T1)} - \alpha_{2(PD+\tau)}] - \frac{I_eP_2}{(T_1-\tau)}[(PD + \tau)(\vartheta_{2(PD+\tau)} - \vartheta_{2\tau}) - (\phi_{2(PD+\tau)} - \phi_{2\tau})] \quad (4.29)$$

The Figure4.3 shows the convex nature of the total cost curve for the case 2a.

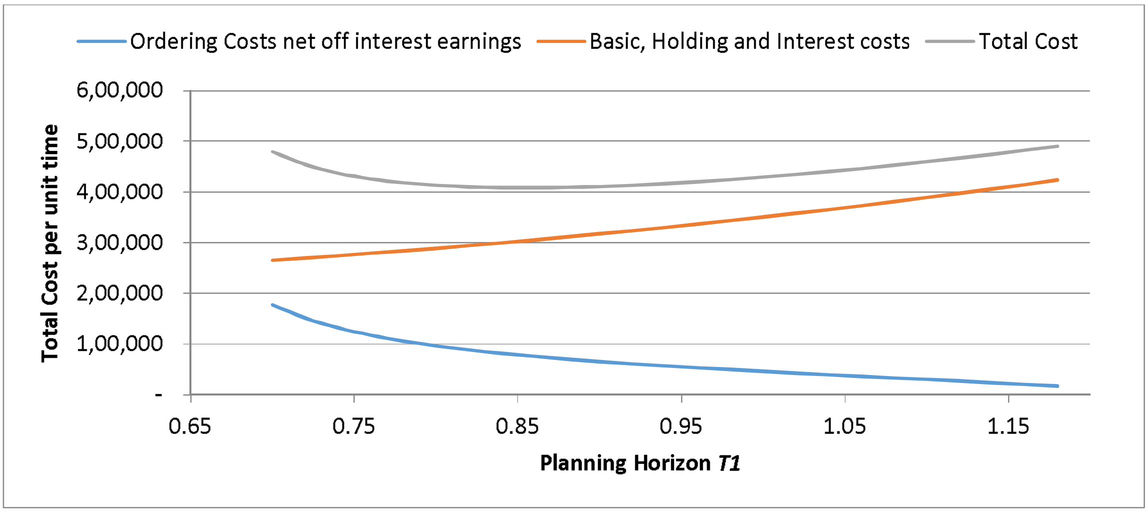


Figure4.3. Total Cost Curve for Case 2a

This scenario corresponds to relatively larger values of planning horizon since it has to be greater than the credit period, which in turn has to be greater than the introduction timing τ. By this time, the product lifecycle has reached a stage where the rising demand dependent costs (i.e., the inventory holding cost and interest cost per unit time) have nullified the declining ordering costs. Hence, the total cost per unit time curve is convex to the origin with minima at the lowest possible value of the planning horizon.

Case3: When $PD \in (T_1, \infty)$:

$$K_2(T_1) = \frac{A_2}{(T_1-\tau)} + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}\int_\tau^{T_1} I_2(t)\,dt - \frac{I_eP_2}{(T_1-\tau)}\int_\tau^{T_1}(T_1-t)D_2(t)dt - \frac{I_eP_2}{(T_1-\tau)}(PD - T_1 + \tau)Q_2 = \frac{A_2}{(T_1-\tau)} - \frac{I_eP_2}{(T_1-\tau)}[T_1(\vartheta_{2(T1)} - \vartheta_{2\tau}) - (\phi_{2(T1)} - \phi_{2\tau}) + (PD - T_1 + \tau)Q_2] + \frac{Q_2C_2}{(T_1-\tau)} + \frac{I_2C_2}{(T_1-\tau)}[\alpha_{2T1} - \alpha_{2\tau}] \quad (4.30)$$

The total cost per-unit-time curve for this situation is also convex to the origin on similar lines as per the above arguments.

The Figure4.4 shows the convex nature of the total cost curve for the case 2b.

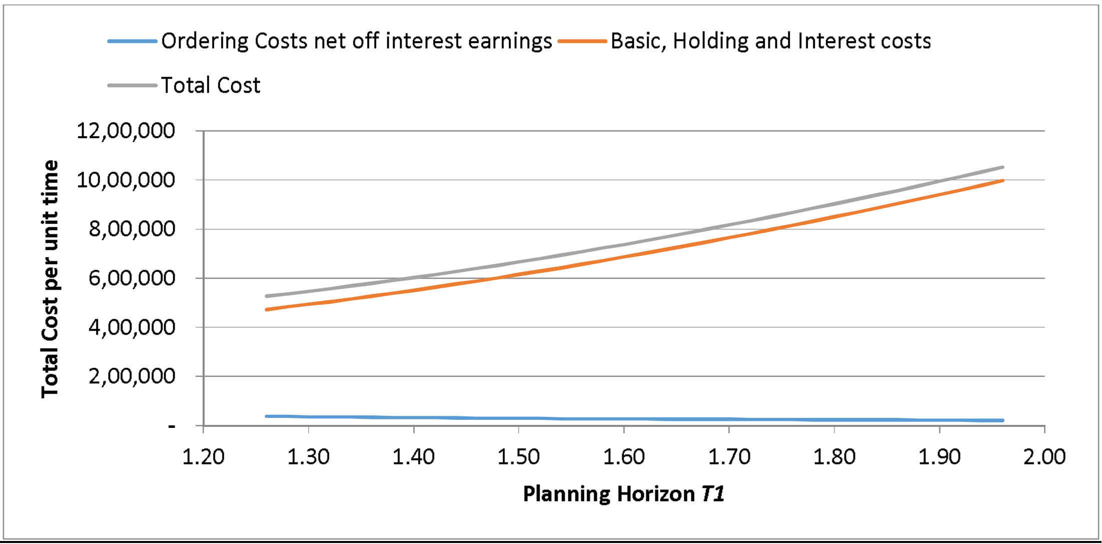


Figure4.4. Total Cost Curve for Case 2b

The Figure4.5 shows the convex nature of the total cost curve for case 3.

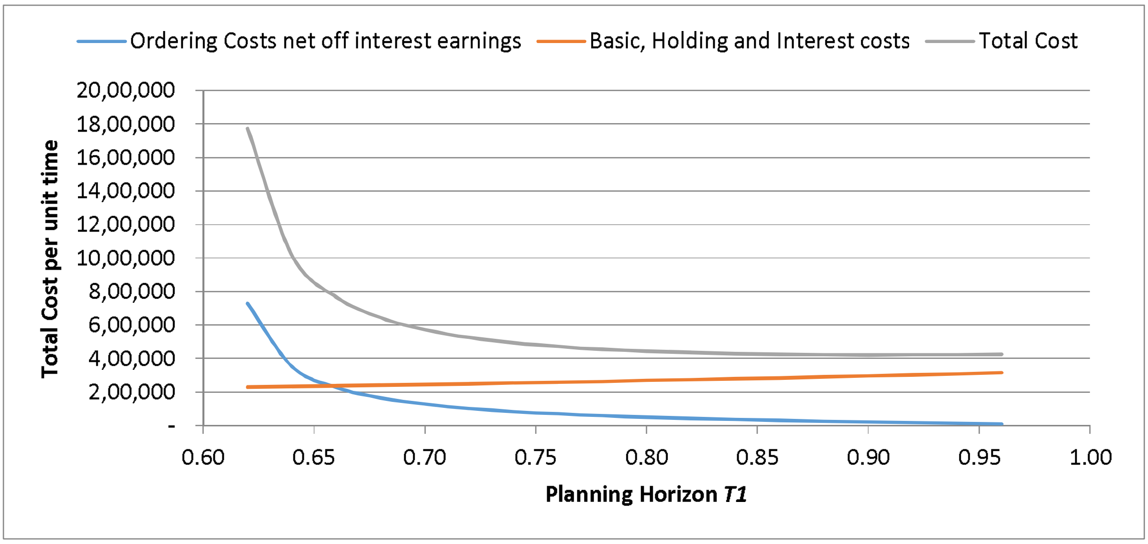


Figure4.5. Total Cost Curve for Case 3

4.1.5. Theorems and Special Cases

In the subsection above, it has been shown that the nature of the total cost curve is convex to the origin. In the current subsection, some of the important theorems have been proposed and discussed.

Theorem 4.1: With the increase in the credit terms, the total cost per unit time does not change for very small values of the planning horizon, but decreases for the larger values of the planning horizon.
Proof: Please refer to Annexure 4A.

Theorem 4.2: The second generation gets more benefit from the increase in credit term as compared to the first generation
Proof: Please refer to Annexure 4B.

Theorem 4.3: With the increase in the introduction timing of the second generation product, the optimal cost reduces and the optimal length of the planning horizon increases
Proof: Please refer to Annexure 4C.

Theorem 4.4: With the increase in the interest rates, the optimal planning horizon falls for lesser values of credit periods and increases for higher values of the credit period
Proof: Please refer to Annexure 4D.

4.1.6. Numerical illustration

The behavior of the model proposed in the earlier section can be illustrated with the help of the numerical example. In this section, first, the sensitivity of the optimal planning horizon is checked with the coefficient of innovation, the coefficient of imitation and the introduction timing of the second generation. At a later stage, the sensitivity with the credit period and the interest rate is checked. Since the single period model consists of the supply for the entire life cycle of a product at the single time, it can be used only for the products with small one-time market potential. To perform the former range of sensitivity analysis, the following parameter values have been considered:

$A_1 = INR\ 30000, A_2 = INR\ 15000, C_1 = INR\ 1500, C_2 = INR\ 2200, M_1 = 500, M_2 = 1000, p_1 = 0.1, p_2 = 0.06, q_1 = 1, q_2 = 1.6, I_1 = 0.15, I_2 = 0.15, \tau = 0.6, I_e = .06, I_c = .08, PD = 0.1$

At $T^* = 0.86$, it can be observed that the optimal cost and optimal ordered quantity are: $K(T^* = 0.86) = 409{,}843$, $Q_1(T^* = 0.86) = 54.18$ and $Q_2(T^* = 0.84) = 17.76$. Furthermore, Table4.2 below depicts that when the value of τ is changed while keeping other parameters constant, the optimal length of the planning horizon and optimal ordering quantity reduces with the increase in τ. This is because the greater τ helps us pool the inventories over a larger period and brings more predictability of demand into the system. However, the inventory managers do not have the luxury to increase the introduction time of the second generation beyond a limit, because then, the competition will take over the non-innovating players.

Table4.2. Sensitivity analysis on τ

	τ =.6	τ =.7	τ =.8
$K(T^*)$	409,843	417,859	427,716
T^*	0.86	0.96	1.08
$Q_1(T^*)$	62.42	72.57	85.57
$Q_2(T^*)$	19.35	19.37	21.23

Next, a sensitivity analysis was conducted with regards to innovation and diffusion coefficients together. The parameter values were considered as used for the scenario; i.e., τ =.6 as the base case and taking the different values of coefficients innovation and diffusion. The results are shown in the tables below.

Table4.3. Sensitivity analysis of innovation coefficients

	Innovation coefficient of 1st generation			**Innovation coefficient of 2nd generation**		
	0.1	**0.13**	**0.17**	**0.06**	**0.075**	**0.09**
$K(T^*)$	409,843	444,852	488,795	409,843	470,784	500,629
T^*	0.86	0.86	0.86	0.86	0.84	0.82

From Table4.3, it can be observed that with the increase in the coefficients of Innovation, the value of $K(T^*)$ also increases. This is because the increase in the innovation coefficient leads to the increase in demand, and hence the overall cost curve moves up. While the optimal T^* does not change (with the increase in innovation effect) for first-generation, it decreases for the second generation. The optimal T^* does not change much for the first generation because the increase in the demand (caused by higher innovation coefficient) has the conflicting effects of increasing inventory carrying cost per unit time and decreasing the ordering cost per unit time; both of which strike a fine balance. For the second generation, the ordering costs per unit time show a sharper fall as compared to the rise in holding costs per unit time. This in turn is because of lower fixed costs of ordering in case of second-generation products on account of the learning curve.

Table4.4. Sensitivity analysis of imitation coefficients

	Imitation coefficient of 1st generation			Imitation coefficient of 2nd generation		
	1.0	1.3	1.5	1.6	1.8	2.0
$K(T^*)$	409,843	428,503	442,109	409,843	414,949	419,600
T^*	0.86	0.84	0.82	0.86	0.85	0.84

From Table4.4, it can be found that with the increase in the coefficient of imitation for first-generation, the value of $K(T^*)$ rises while the optimal T^* falls. $K(T^*)$ increases because of the increase in demand due to more imitation, and therefore, an upward shift in the total cost per unit time curve. The optimal T^* increases for both the generations because the greater consumption rate of inventories caused by the higher imitation coefficients, results in a faster increase in the inventory carrying costs and early achievement of cost minima; and hence, the optimal T^* decreases.

The above research work tried to lay a foundation for consideration of credit period in the inventory planning and optimization for high technology multi-generation products under the innovation diffusion framework. The framework is also is expected to help the inventory managers and practitioners in evaluating the influence of trade credit mechanism on the overall cost structure of successive generation products over the short-range and long-range period. The framework can be further generalized by considering credit-dependent demand for optimizing multiperiod inventory control problems. In the next section, a new multiperiod EOQ model is proposed using credit-dependent demand.

4.2. Multi-period Inventory Model using Credit Linked demand Model

As discussed, trade credit offered by the supplier to the buyers is a crucial tool to enhance the sales. Sometimes, the suppliers with weak bargaining power with their customers, sell a larger share of goods on credit, and offer a longer payment period before charging penalties (Fabbri and Klapper, 2016). Trade financing among firms is one of the most popular sources of financing globally. Kehinde et al. (2017) suggested that to balance the liquidity risk and profitability, a credit policy is not only desirable but also essential for an organization's success. Thus, from the hi-technology market perspective, it becomes important for the supply chain managers to consider the effect of trade credits on the optimal replenishment policies. The proposed EOQ model in this section discusses the importance of trade credit strategy on procurement policies in the different stages of the product life cycle. Apart from the notations and assumptions as used in section 4.1.1, some additional notations and assumptions are also used in this section as given below.

4.2.1. Additional Notations

M_j' is the Market Potential of the jth generation product after the incorporation of the trade credits

$CD_1(t)$ is the cumulative demand of the first generation product before the launch of the second generation till time t

$CD_2(t)$ is the cumulative demand of the second generation product before the launch of the second generation till time t

$CD_1'(t)$ is the cumulative demand of the first generation product post the launch of the second generation till time t

η denotes the sequence of the planning horizon

Rev_i is the total turnover for the ith generation product

Rev' is the total turnover in a planning horizon post the launch of the second generation product

BC_i is the total basic purchase cost for the ith generation product

BC' is the total basic purchase cost in a planning horizon post the launch of the second generation product

TCM_i is the contribution margin for the ith generation product

TCM' is the total contribution margin in a planning horizon post the launch of the second generation product

A_i is the fixed product-specific ordering cost of the ith generation product irrespective of the order volumes, while A is the fixed generic ordering cost

OC_i is the total ordering cost specific to the ith generation product

OC is the total generic ordering cost which is not specific to the products being ordered

HC_i is the total inventory holding cost for the ith generation product

HC' is the total inventory holding cost in a planning horizon post the launch of the second generation product

I_r is the interest cost on the credit offered

z is a binary variable which equals 1 for joint replenishment and 0 for disjoint replenishment

RC_i is the replenishment cost (including inventory ordering, inventory carrying and interest on credit) for the ith generation product

RC' is the total replenishment cost (including inventory ordering, inventory carrying and interest on credit) in a planning horizon post the launch of the second generation product

TP_i is the total profit for the ith generation product

TP' is the total profit in a planning horizon post the launch of the second generation product

$\xi_{i\eta}$ is the quantity of the ith generation product ordered in each lot in the horizon η

$\xi_{i\eta'}$ is the quantity of the ith generation product ordered in each lot in the horizon η′ post the launch of the second generation product

ζ is the length of the horizon for which the inventory norms are fixed

4.2.2. Additional Assumptions

- The credit period is offered by the retailer to the customer and it tends to increase the market potential of the product. Thus mathematically, the Market Potential for the first generation product gets increased from M_1 to M_1'; and similarly, that for the second generation product from M_2 to M_2' in the presence of the credit period.
  $M_1' = M_1.\exp(\alpha.PD_1)$
  $M_2' = M_2.\exp(\alpha.PD_2)$
  where α is a positive constant
- The rate of demand is influenced by the innovation diffusion process and follow the assumptions as discussed in section 3.1.3 and can be given as follows:
  $\lambda_1(t) = M_1' f_1(t) \quad for\ t \leq \tau$
  $\lambda_1'(t) = M_1' f_1(t) - M_1' f_1(t) F_2(t)\ for\ t \geq \tau$
  $\lambda_2(t) = M_2' f_2(t) + M_1' f_1(t) F_2(t)$
- The interest rates are the same across the generations
- There is a finite credit period being offered by the supplier.

The models of credit linked demand as discussed in Table 4.1 have not been applied to the technology generations. Since the Demand Model $D(PD) = K\ exp(\alpha.PD)$ has been the most popular one among the ones mentioned above, the same model shall be integrated with the Norton and Bass Model (1987) in this research for the technology generations.
The adoption functions have been specified in equations of section (3.1.3).

6.2.3. Inventory Model in case of single generation scenario

The business decisions related to the procurement of inventories are tactical by nature. For a particular planning horizon, the retailers decide their EOQ and then, review it at regular periodic intervals, with the life cycle dynamics of the product changing in each of these horizons. Thus, the retailers try to determine their inventory replenishment strategies based on the stage of the life cycle of the product. If the length of each of those horizons is ζ, and there is ξ_η amount of order quantity in the η horizon, the value of ξ_η for which the total profit is maximum in the horizon η needs to be determined.
The time at which the horizon η starts is given by $(\eta - 1)\zeta$, and the time at which it ends is $(\eta)\zeta$ as shown in Figure4.6.

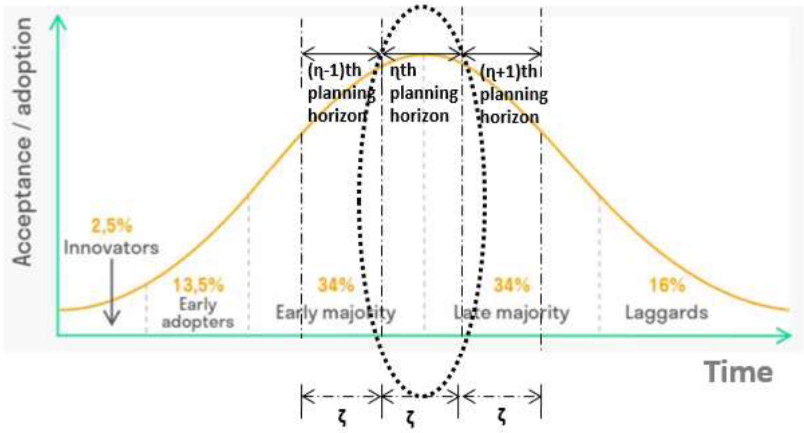


Figure4.6. Horizons into which product life cycle is divided in the single generation case

Since the EOQ in this horizon is ξ_{η}, the number of orders to be placed in this period is given by

$$j = \left(\frac{1}{\xi_{\eta}}\right)[CD_1(\eta.\zeta) - CD_1((\eta-1).\zeta)] \quad (4.31)$$

The time of the start of the *kth* ordering cycle is given by $t_{k,\eta,\xi_{\eta}} = (\eta - 1 + (k-1)/j)\zeta$ (4.32)

The time of the end of the *kth* ordering cycle is given by $t_{(k+1),\eta,\xi_{\eta}} = (\eta - 1 + k/j)\zeta$ (4.33)

The holding costs of inventory in the *kth* ordering cycle in the horizon η are given by

$$I_1 C_1 \int_{t_{k,\eta,\xi_{\eta}}}^{t_{(k+1),\eta,\xi_{\eta}}} I_1(t)dt = I_1 C_1 \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} [\int_{u=t}^{u=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(u)du]\ dt$$

(4.34)

Net interest costs on the credit period in *kth* replenishment cycle of the η horizon are given as =

$$I_r.C_1.PD.\int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(t)dt \quad (4.35)$$

The replenishment costs for the η horizon are given as

$$RC_1 = j(A + A_1) + I_1 C_1 \sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} [\int_{u=t}^{u=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(t)dt]\ dt +$$

$$I_r.C_1.PD.\sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(t)dt \quad (4.36)$$

The revenue is given by

$$Rev_1 = pr_1.\sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(t).dt \quad (4.37)$$

$$BC_1 = C_1.\sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{(k+1),\eta,\xi_{\eta}}} \lambda_1(t).dt \quad (4.38)$$

$$TCM_1 = Rev_1 - BC_1 \quad (4.39)$$

$$TP_1 = TCM_1 - RC_1 \quad (4.40)$$

6.2.4. Inventory Model for the two generations scenario

Let us say that planning horizon η' begins at time $t = \tau + (\eta' - 1)(\zeta)$ and ends at time $t = \tau + (\eta')(\zeta))$.

The time at which the horizon η starts is given by (η'-1) ζ, and the time at which it ends is (η) ζ as shown in Figure4.7.

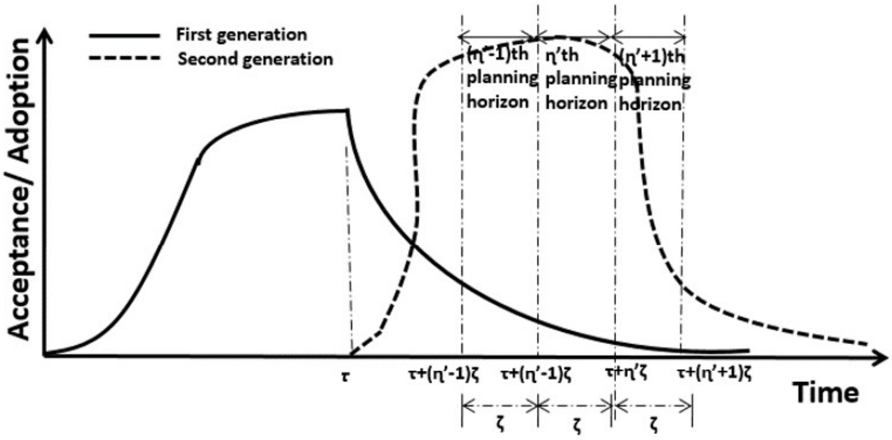


Figure4.7. Horizons into which product life cycle is divided after the launch of the second generation product

If $\xi_{1\eta}'$ and $\xi_{2\eta}$ be the order quantities of the products of the first generation and the second generation respectively, the number of orders to be placed for the products are:

$$j_1' = [CD_1'(\tau + (\eta)(\zeta))) - CD_1'(\tau + (\eta - 1)(\zeta)))]/\xi_{1\eta}' \quad (4.41)$$

$$j_2 = [CD_2(\tau + (\eta)(\zeta))) - CD_2(\tau + (\eta - 1)(\zeta)))]/\xi_{2\eta} \quad (4.42)$$

The *kth* replenishment cycle for the first generation starts at the time $t_{(k-1),\xi_{1\eta}'} = \tau + (\eta - 1 + (k-1)/j_1')\zeta)$ and ends at the time $t_{k,\xi_{1\eta}'} = \tau + (\eta - 1 + k/j_1')\zeta)$

Similarly, the *kth* replenishment cycle for the second generation starts at the time $t_{(k-1),\xi_{2\eta}} = \tau + (\eta - 1 + (k-1)/j_2)\zeta)$ and ends at the time $t_{k,\xi_{2\eta}} = \tau + (\eta - 1 + k/j_2)\zeta)$

In case of consolidated logistics for both the generations of products, the clubbed quantity to be transported in the *kth* replenishment cycle is $t_{(k-1),\xi_{\eta 1}'} = t_{(k-1),\xi_{\eta 2}}$; and $t_{k,\xi_{\eta 1}'} = t_{k,\xi_{\eta 2}}$

Similar to the way the economics in the single generation case has been worked out, the economics of the two generations situation can be captured as below:

$$RC' = RC_1 + RC_2 = j_1'(A_1 + A) + j_2(A_2 + A) - z.(A) +$$
$$H_1C_1 \sum_{k=1}^{k=j_1'} \int_{t=t_{(k-1),\xi_{1\eta}'}}^{t=t_{k,\xi_{1\eta}'}} [\int_{u=t}^{u=t_{k,\xi_{1\eta}'}} \lambda_1'(u)du]\, dt \; +$$
$$H_2C_2 \sum_{k=1}^{k=j_2} \int_{t=t_{(k-1),\xi_{2\eta}}}^{t=t_{k,\xi_{2\eta}}} [\int_{u=t}^{u=t_{k,\xi_{2\eta}}} \lambda_2(u)du]\; dt \quad + I_r.C_1.PD_1.\sum_{k=1}^{k=j_1'} \int_{t=t_{k,\eta,\xi_{1\eta}}}^{t=t_{k,\xi_{1\eta}'}} \lambda_1'(t)dt \; +$$
$$I_r.C_1.PD.\sum_{k=1}^{k=j_1'} \int_{t=t_{k,\eta,\xi_{\eta}}}^{t=t_{k,\xi_{2\eta}}} \lambda_2(t)dt \quad (4.43)$$

Where $z = 1\; if\;\; j_1' = j_2, and\; 0\; otherwise$

$$Rev' = pr_1 . \sum_{k=1}^{k=j_1'} \int_{t=t_{k,\eta,\xi_{\eta 1}'}}^{t=t_{(k+1),\eta,\xi_{\eta 1}'}} \lambda_1'(t).dt \; + \; pr_2 . \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta,\xi_{\eta 2}}}^{t=t_{(k+1),\eta,\xi_{\eta 2}}} \lambda_2(t).dt \qquad (4.44)$$

$$BC' = BC_1 + BC_2 = C_1 . \sum_{k=1}^{k=j_1'} \int_{t=t_{k,\eta,\xi_{\eta 1}'}}^{t=t_{(k+1),\eta,\xi_{\eta 1}'}} \lambda_1'(t).dt \; + \; C_2 . \sum_{k=1}^{k=j_2} \int_{t=t_{k,\eta,\xi_{\eta 2}}}^{t=t_{(k+1),\eta,\xi_{\eta 2}}} \lambda_2(t).dt \qquad (4.45)$$

$$TP' = Rev' - BC' - RC' \qquad (4.46)$$

The optimization problem is:

Max $TP' = Rev' - BC' - RC'$

Subject to the constraints

$j_1' = j_2$

j_1' and j_2 are positive integers

6.2.5. Theorems and Special Cases

Theorem 4.5: With the increase in the trade credits, the holding costs tend to fall given other factors constant.

Proof: Please refer Annexure 4E

Theorem 4.6: With the increase in trade credits, the total contribution margin tends to increase.

Proof: Please refer Annexure 4F

Theorem 4.7: Offering the higher trade credits on the newer generation product expedites the phase-out timing of the first generation product.

Proof: Please refer Annexure 4G

Theorem 4.8: A threshold level of credit sensitivity is required for the trade credits to increase the profits in a supply chain.

Proof: Please refer Annexure 4H

Theorem 4.9: For the fast-moving popular products with lower per-unit contribution margins, the retailers should offer a negative or lesser credit period, while for the slow-moving and higher per-unit contribution margin products, it makes sense to offer a higher credit period.

Proof: Please refer Annexure 4I

Special Case 4.1: When the credit period is two-sided, i.e. from supplier to retailer PD_{sr} and from retailer to customer PD_{rc} , the demand for the products increases with PD_{rc} as long as $PD_{sr} < \left(\frac{1}{2}\right) PD_{rc}$, Also, there shall be no influence on demand when $PD_{sr} = PD_{rc}$ or $PD_{sr} = 0$

Proof: Please refer Annexure 4J

Special Case 4.2: Under the capital constraints of credit, it is better to offer credits on the newer generation product rather than the earlier generation product.

Proof: Please refer Annexure 4K

6.2.6. Solution procedure

The solution procedure to find the optimal solutions can be summarized in the following algorithm

Step 1: Enter the base values of all model parameters such as per-unit costs, coefficients of innovation and imitation, potential market sizes, time to the introduction of second-generation products, etc. for each generation independently.

Step 2: Compute all possible values of profit for the given value of τ using equation (4.40) and (4.46) as the case may be.

Step 3: Select the appropriate value of replenishment frequency that satisfies the sufficiency condition that the second derivative of profit has to be negative, with the first derivative being positive.

Step 4: Compute the EOQ from equation (4.31), (4.41), and (4.42).

6.2.7. Numerical Illustrations

Let us use the following values of the parameters to illustrate the model proposed above.

$H_1 = .15, \quad H_2 = .15, \quad p_1 = .5, \quad q_1 = 2.5, \quad p_2 = .06, \quad q_2 = .4, \quad I_r = .18,$
$pr_1 = INR\,3500, \quad pr_2 = INR\,4200, \quad C_1 = INR\,1500, \quad C_2 = INR\,1700,$
$PD_1 = .25, \quad PD_2 = .25, \quad \alpha = .5, \quad \tau = 0.5, \quad \zeta = 0.5,$
$M_1 = 100000, \quad M_2 = 120000, \quad A = INR\,500000, \quad A_1 = INR\,50000,$
$A_2 = INR\,50000$

The values of innovation and imitation coefficients have been taken different in the different models to get a wide variety of illustrations. With the help of the proposed model, the results as tabulated in Table4.5 were obtained. The number of replenishments at the optimal EOQ in the first two planning horizons have been delivered. The optimal EOQ refers to the lot size that delivers the highest profit. Since the technology products can have very short product life cycles in the light of changing consumer preferences and ever-up-gradation of technologies and business models, it can be observed how the share of the first generation product reaches minuscule levels over a short period.

Table4.5. The optimal EOQ corresponding to the pooled logistics scenario (All figures in Mn INR unless stated otherwise)

Planning Horizon	*Technology Generation*	*EOQ (Absolute Units)*	*Revenue*	*Contribution Margin*	*Replenishment Cost (Ordering + Holding)*	*Credit Cost*	*Total Profit*
$m' = 1$	1st	11,577	121.56	69.46	2.47	4.56	254.40
	2nd	27,315	344.18	204.87		12.91	

$m' = 2$	1st	1,821.14	19.12	10.93	1.96	0.72	166.45
	2nd	22,511.11	283.64	168.83		10.64	
$m' = 3$	1st	72.39	0.76	0.43	0.55	0.03	33.67
	2nd	4,811.51	60.63	36.09		2.27	

The cross elasticity of demand for any generation product with the credit terms of the other generation can also be checked. On changing the credit terms of the second generation, the demand for the first generation increases, and vice versa.

The Table4.6 shows the influence of the credit terms of the products on the demand of each other. It can be seen that the cumulative adoption of a product is dependent upon the credit term of that product as well as the substitutable products. The increase (or decrease) in the credit terms of the substitutable generation product leads to a fall (or rise) in the demand for a product.

Table4.6. Influence on the adoption of the first-generation product by changing the relative credit terms with the second generation product

Cumulative Number of adopters (Mn)						
	PD_1=PD_2=.25		*PD_1=0, PD_2=.5*		*PD_1=.5, PD_2=0*	
t	*1st Gen*	*2nd Gen*	*1st Gen*	*2nd Gen*	*1st Gen*	*2nd Gen*
0.5	0.04	0.00	0.04	0.00	0.36	0.00
1	0.04	0.15	0.04	0.15	0.04	0.15
1.5	0.08	0.23	0.09	0.21	0.07	0.25
2	0.10	0.23	0.12	0.22	0.09	0.26
2.5	0.11	0.24	0.13	0.22	0.10	0.26

The Table4.7 shows that credit sensitivity has an important influence on the total profit. If the credit sensitivity (the influence of trade credits on the sales volumes) as denoted by α is low, the total profit falls with the increase in trade credits. On the other hand, for the products that have high credit sensitivity, the total profit increases with the trade credits. This is also illustrated in Figure4.8.

Table4.7. The behavior of total profit (in Mn INR) for different values of credit sensitivity

$PD_1 = PD_2$	α=.05	α=.25	α=.5
0	239.63	239.63	239.63
0.05	236.49	239.39	241.57
0.1	233.35	239.20	243.66
0.15	230.22	239.04	245.91
0.2	227.08	238.92	248.33
0.25	223.95	238.84	250.91
0.3	220.82	238.80	253.66
0.35	217.68	238.80	256.59
0.4	214.56	238.84	259.70

0.45	211.43	238.92	262.99

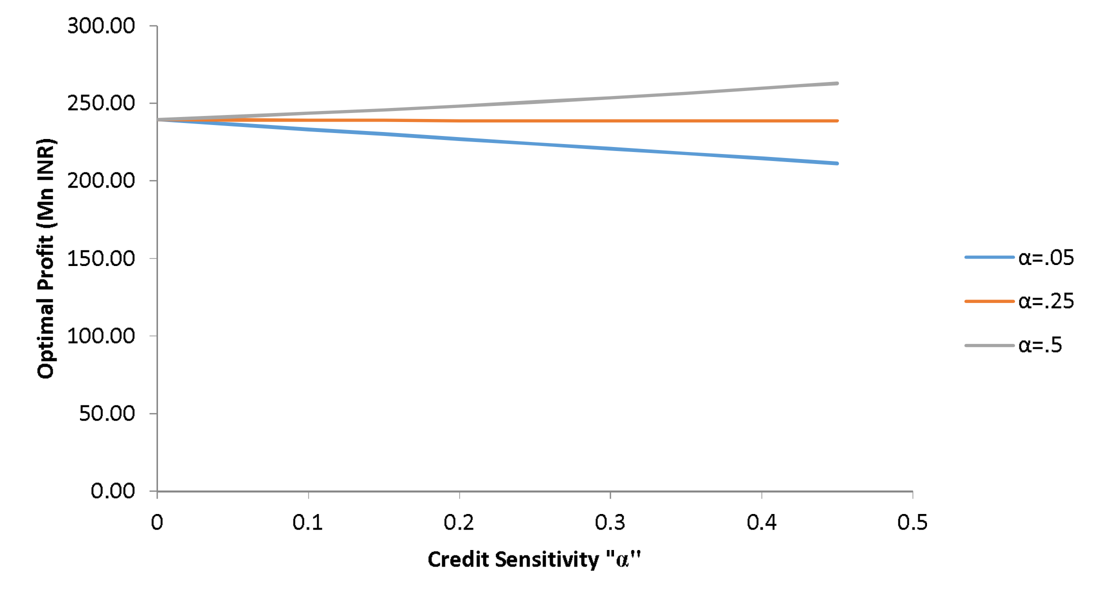


Figure4.8. The trend of the optimal profit for varying levels of credit sensitivity

4.3. Academic and Business Implications of the proposed models

This research study can guide the industry practitioners in appreciating the cannibalization among the different generations of the technology products. It can also guide them in determining the optimal credit terms in business. The optimal credit terms would be the one at which the total profit is maximized. With the increase in the credit period, the contribution margin increase and the holding costs fall, but at the expense of a rise in the credit costs. It makes business sense to offer incremental credit as long as the rise in credit costs is overweighed by the rise in the contribution margin and fall in holding costs. This research has illustrated the importance of the credit period and the interest rates in formulating the inventory policies for the multi-generation products. The key insights generated for the managers are:

- The higher the credit period, the lesser shall be the overall costs
- The savings per unit time in the interest costs due to the credit period offering are higher in the longer planning horizons than in the shorter planning horizons
- It makes sense for a manager to negotiate a better credit term for the later generation product, as the expense of that for an older generation product. The credit period offerings give more benefit to the later generations' product than to the earlier generations' product.
- The value of the credit offer is more in the cycles of higher interest rate than in the cycles of lesser interest rate

- The value of the credit period offering is more in the growing economies as compared to that in the stagnant economies

This chapter also helps the managers understand the interplay between the credit periods and the demand rates of multiple generations' substitutable products. Another important implication for the managers is that the credit period offers become more beneficial in the times of recessionary business cycles as compared to the growing business cycles. Also, the credit terms provide more value for the low-volume high-margin products, in contrast to the high-volume low-margin products. The inventory practitioners in the supply chains of the technological products can make sound business decisions related to the inventory replenishment frequency, economic order lot size, and credit terms. The differential service levels can be explored in a capital-constrained supply chain, and generally, it is better to achieve higher service levels for the later generation product at the expense of that for the earlier generation product.

It also came out that the higher credit period offered on any product has a negative influence on the demand of the substitutable products. It is also important to note that the in case of multi-echelon supply chains, the credit periods offered by the intermediate echelons benefit the total profit only till a certain point.

While the influence of the trade credits has been covered in detail in this chapter, the next possible dimension that comes to the mind is the price of the product. The price elasticity of demand is a phenomenon that cannot be escaped from in today's hyper-competitive markets for high technology products. Therefore, one of the important extensions of this work is to consider the impact of the price variations on the inventory decisions for technology generation products.

Appendix 4

A. Theorem 4.1: *With the increase in the credit terms, the total cost per unit time does not change for very small values of the planning horizon, but decreases for the larger values of the planning horizon.*

Proof: This is because the interest earned during the credit period per unit time is proportional to the consumption in that unit time. It can be seen that interest earnings per unit time are given by $\frac{I_e P}{(T)} \int_0^T tD(t)dt$. For very small values of the planning horizon, the product is into the introduction and growth stage when the demand is very less, and hence, the interest benefit is lesser. As the planning horizon increases, the demand rate $D(t)$ increases significantly, resulting in higher interest earnings per unit time.

For small values of the planning horizon, $D(t)$ the relationship between credit period and interest earnings is typically more sloped, or convex, rather than a straight line. Therefore, convexity is a better measure for assessing the impact on interest earnings when there are large fluctuations in credit periods.

B. Theorem 4.2: *The second generation gets more benefit from the increase in credit term as compared to the first generation*

Proof: This is because the second-generation product, by possessing advanced features, has higher demand in the market, and therefore, has greater consumption per unit time as compared to the first-generation product. Thus, the interest earnings per unit time are given by $\frac{I_eP}{(T)}\int_0^T t\lambda(t)dt$. The term interest earnings per unit time are given by $(\frac{1}{T})\int_0^T t\lambda(t)dt$. Since the $q_2 > q_1, p_2 < p_1, q_2 \gg p_2, q_1 \gg p_1$, the demand rate for the second generation is much higher than that of the first generation. And hence, it gets a higher benefit of the credit period than the first-generation.

C. Theorem 4.3: *With the increase in the introduction timing of the second generation product, the optimal cost reduces and the optimal length of the planning horizon increases*

Proof: With the increase in the introduction timing of the second generation product, the increase in the overall demand gets delayed, and hence the holding costs (which increase with the demand rate) take more time to nullify the fall in the ordering costs.

It is known that if there are two functions in x: $f(x)$ *and* $g(x)$ over the interval (a, b) such that $f'(x) < 0$ *and* $g'(x) > 0$ in the interval (a, b); $f''(a) > g''(a)$ *and* $f''(b) < g''(b)$, then the curve $h(x) = f(x) + g(x)$ is convex to the origin and there exists a point c in thee interval (a, b) such that $f'(c) + g'(c) = 0$. As the $g'(x)$ falls, then the point of minima c, $(where\ f'(c) + g'(c) = 0)$ shifts towards the right.

Also, it can be observed that the high time gap between the launch of successive generations enables us to leverage the economics of pooling across different times, and therefore allows us to have the longer planning horizon for the minimum cost.

D. Theorem 4.4: *With the increase in the interest rates, the optimal planning horizon falls for lesser values of credit periods and increases for higher values of the credit period*

Proof: For the smaller values of the credit period, interest earnings per unit time are much lesser than the interest charges per unit time. Therefore, an increase in the interest rates will lead to a faster increase in g(x) and will facilitate the early reaching of the minima. Hence, the optimal planning horizon will reduce with the increase in interest rates for smaller values of the credit period.

E. Theorem 4.5: *With the increase in trade credits, the total contribution margin tends to increase.*

Proof: $\frac{\partial(HC_1)}{\partial(PD_1)} = \left[\frac{\partial(HC_1)}{\partial(\lambda_1(t))}\right]\left[\frac{\partial(\lambda_1(t))}{\partial(f_1(t))}\right]\left[\frac{\partial(f_1(t))}{\partial(PD_1)}\right]$

All three terms in the expression above are positive.

It is known that holding costs tend to fall with the increase in demand rate due to lesser time spent by the inventories in the system. Hence, $\left[\frac{\partial(HC_1)}{\partial(\lambda_1(t))}\right]$ is negative.

$$\left[\frac{\partial(\lambda_1(t))}{\partial(f_1(t))}\right] = M_1 > 0$$

$$\frac{\partial\big(f_1(t)\big)}{\partial(PD_1)} = \frac{2\alpha b_1^2 \exp(-b_1 t)\,[\exp(\alpha.PD_1)]^2}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]} - \frac{2b_1 p_1 \alpha \exp(-b_1 t)\exp(\alpha.PD_1)}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]} + \frac{b_1^2\,\mathrm{t}.\alpha.\exp(-b_1 t)\exp(\alpha.PD_1)\,p_1}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]}$$

$$= \frac{2\alpha b_1^2 \exp(-b_1 t)\,[\exp(\alpha.PD_1)]^2}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]} - \frac{2b_1 p_1 \alpha \exp(-b_1 t)\exp(\alpha.PD_1)}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]} + \frac{b_1^2\,\mathrm{t}.\alpha.\exp(-b_1 t)\exp(\alpha.PD_1)\,p_1}{[p_1\{1 + a_1.\exp(-b_1 t)\}^2]}$$

$$\frac{\partial\big(f_1(t)\big)}{\partial(PD_1)} = \alpha.\exp(-b_1 t)\exp(\alpha.PD_1)\,/[p_1\{1 + a_1.\exp(-b_1 t)\}^2])\,[2b_1^2 \exp(\alpha.PD_1) - 2b_1 p_1 + b_1^2\mathrm{t}.p_1]$$

Since $b_1 \exp(\alpha.PD_1) > p_1$, it can be concluded that $\frac{\partial(f_1(t))}{\partial(PD_1)}$ is always positive

The Figure4.E.1. illustrates the influence of the credit period on the adoption rate for any product.

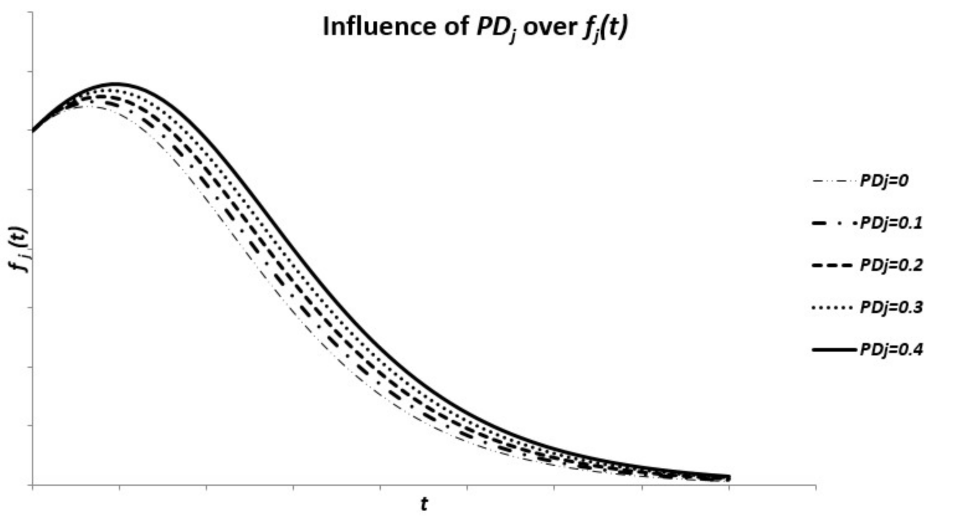


Figure4.E.1. Influence of the credit period PD_1 on the adoption rate $f_1(t)$

The Figure4.E.2. shows the influence of the credit period on the cumulative adoption rate of any product.

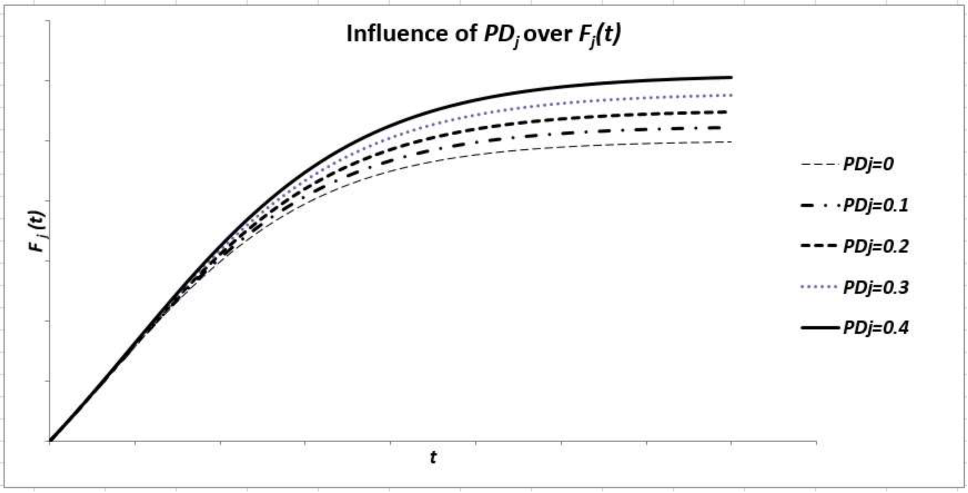


Figure4.E.2. Influence of the credit period PD_1 on the cumulative adoption rate $F_1(t)$

So, $\frac{\partial(HC_1)}{\partial(PD_1)} < 0$

F. Theorem 4.6: With the increase in trade credits, the total contribution margin tends to increase.

Proof: If the partial derivative of the total contribution margin is taken w.r.t. credit terms, the following expression can be obtained:

$$\frac{\partial(TCM_1)}{\partial(PD_1)} = \left[\frac{\partial(TCM)}{\partial(\lambda_1)}\right]\left[\frac{\partial(\lambda_1(t))}{\partial(f_1(t))}\right]\left[\frac{\partial(f_1(t))}{\partial(PD_1)}\right]$$

$$\left[\frac{\partial(TCM_1)}{\partial(\lambda_1)}\right] = pr_1 - C_1 > 0$$

Since the later two terms in the expression above have already been proven to be positive,

$$\left[\frac{\partial(TCM_1)}{\partial(\lambda_1)}\right] > 0$$

G. Theorem 4.7: *Offering the higher trade credits on the newer generation product expedites the phase-out timing of the first generation product.*

Proof: The higher trade credits on the second generation product leads to an increase in its demand rate, at the expense of cannibalization of the earlier generation product. The lower demand rate of the first generation product, thus caused, results in the replenishment costs from being recovered from the contribution margin, thus making the first-generation product a loss proposition.

$$\frac{\partial(\lambda_1)}{\partial(PD_2)} = \left[\frac{\partial(\lambda_1(t))}{\partial(f_2(t))}\right]\left[\frac{\partial(\lambda_1(t))}{\partial(f_2(t))}\right] = -M_1 \left[\frac{\partial(f_2(t))}{\partial(PD_2))}\right]$$

$$\frac{\partial(f_2(t))}{\partial(PD_2))} = \frac{2\alpha b_2^2 \exp(-b_2(t-\tau))\,[\exp(\alpha.PD_2)]^2}{[p_1\{1 + a_1.\exp(-b_1(t-\tau))\}^2]} > 0$$

Thus, it can be said that

$$\frac{\partial(\lambda_1)}{\partial(PD_2)} < 0$$

The Figure4.G.1 sums up the phenomenon explained above.

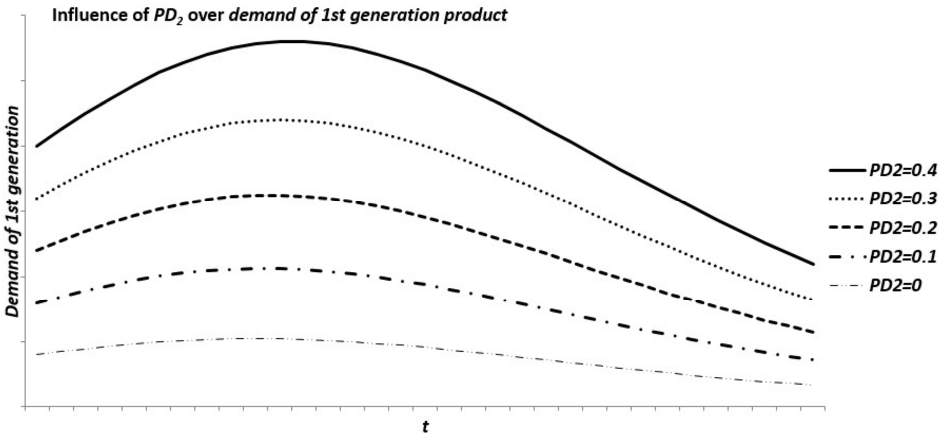


Figure4.G.1. Influence of the credit period of second product generation on the demand of the first generation product

H. Theorem 4.8: A threshold level of credit sensitivity is required for the trade credits to increase the profits in a supply chain

Proof: As the credit term increases, the contribution margin of the product increases due to a rise in the demand, while the credit costs also increase. It makes sense to increase the credit terms until the point where the increase in contribution margin is more than the rise in credit costs. Let D be the demand in the absence of the credit period, and ΔD be the rise in demand with the credit period PD_1, then

$(\Delta D)(pr_1 - C_1) > (D)pr_1.(I_r)(PD_1)$

$\frac{\Delta D}{D} > I_r.(PD_1)/(1 - C_1/pr_1)$

Or, $\exp(\alpha.PD_1) - 1 > I_r.(PD_1)/(1 - C_1/pr_1)$

Or, $\exp(\alpha.PD_1) > I_r.\frac{(PD_1)}{\left(1-\frac{C_1}{pr_1}\right)} + 1$

Or, $\alpha > \left(\frac{1}{PD_1}\right).\ln(I_r.\frac{(PD_1)}{\left(1-\frac{C_1}{pr_1}\right)} + 1)$

From the above expression, it is evident that if the interest rates are higher, the trade credits are useful only with higher credit sensitivity. Also, if the contribution margins are lower, the trade credits are useful only with higher credit sensitivity.

I. Theorem 4.9: *For the fast-moving popular products with lower per-unit contribution margins, the retailers should offer a negative or lesser credit period, while for the slow-moving and higher per-unit contribution margin products, it makes sense to offer a higher credit period.*

Proof: The credit period has three effects on the profit margin:

a) the increase in contribution margin due to higher volumes, as explained below
b) the reduction in inventory carrying costs due to faster movement of inventories caused by higher demand rate, and
c) the higher credit costs, as explained below

$\frac{\partial(\lambda_1)}{\partial(PD_1)} = M_1 \frac{\partial(f_1)}{\partial(PD_1)} > 0$ as proved above in Theorem 1

$\frac{\partial(TCM_1)}{\partial(PD_1)} = (pr_1 - C_1).\frac{\partial\left(\sum_{k=1}^{k=j}\int_{t=t_{k,\eta,\xi_\eta}}^{t=t_{(k+1),\eta,\xi_\eta}} \lambda_1(t)dt\right)}{\partial(PD_1)} > 0$ since it is cumulative of a positive function

$$\frac{\partial(RC_1)}{\partial(PD_1)} = I_r.C_1.\sum_{k=1}^{k=j}\int_{t=t_{k,\eta,\xi_\eta}}^{t=t_{(k+1),\eta,\xi_\eta}} \lambda_1(t)dt \quad > 0$$

While the first two have a positive influence on the profit, the third effect has a negative influence on the profit. For the mass market technology products that enjoy smaller contribution margins per unit, and faster inventory turnover rates, the positive effect on the retailer's profit will be lesser than the negative effect on credit costs. Therefore, it makes sense for the retailers to offer lesser credit periods (or sometimes, negative credit periods, i.e. insisting on advance collection from customers) on the popular technology products. While in the case of the technology products with higher contribution margin per unit and slower inventory turnover rates, the positive effect of the first two influences is higher than the negative effect of the third influence, making it an attractive proposition to offer a higher credit period.

J. Special Case 4.1: *When the credit period is two-sided, i.e. from supplier to retailer PD_{sr} and from retailer to customer PD_{rc} , the demand for the products increases with PD_{rc} as long as $PD_{sr} < \left(\frac{1}{2}\right)PD_{rc}$, Also, there shall be no influence on demand when $PD_{sr} = PD_{rc}$ or $PD_{sr} = 0$*

Proof: As proposed by Banu and Mondal (2016), the demand in such a case is proportional to $\exp(b_1 PD_{sr}(PD_{rc} - PD_{sr}))$

$$\frac{\partial([\exp(b_1 PD_{sr}(PD_{rc} - PD_{sr}))])}{\partial(PD_{sr})} = [b_1.\exp(b_1 PD_{sr}(PD_{rc} - PD_{sr}))]((PD_{rc} - 2.PD_{sr})$$

This is positive $PD_{rc} - 2.PD_{sr} > 0$. Therefore, the demand for the product rises with the increase in $2.PD_{sr}$ till the point where $PD_{sr} < \left(\frac{1}{2}\right)PD_{rc}$

Also, the expression $\exp(b_1 PD_{sr}(PD_{rc} - PD_{sr}))$ reaches the value of 1 when $PD_{sr} = PD_{rc}$ or $PD_{sr} = 0$, and therefore, loses its multiplier effect on the demand.

K. Special Case 4.2: *Under the capital constraints of credit, it is better to offer credits on the newer generation product rather than the earlier generation product.*

Proof: This is a special case of constraints on the credit capital. Let us consider that SL_i be the service level of the *ith* generation product, D_i be the demand of the *ith* generation product during the credit cycle and $TVPPU_i$ be the total variable profit per unit for the *ith* generation product. The problem becomes approximately a linear program with the following formulation.

Max. $Z = SL_1.D_1.TVPPU_1 + + SL_2.D_2.TVPPU_2$

Subject to the constraints:

$SL_1.D_1 + SL_2.D_2 - CC <= 0$

$z \leq M.SL_1$

$z \leq 1$

SL_1 *and* $SL_2 \geq 0$ *and* ≤ 1

M is a very large number

The newer generation products have higher contribution margins as compared to the older generation products on account of price skimming for the advanced features.

Therefore, $TVPPU_1 < TVPPU_2$

Since these are technology products, within a very short span of the launch of next-generation, the significant cannibalization happens, making $D_1 < D_2$

Hence, $TVPPU_1.D_1 < TVPPU_2.D_2$. Therefore, it makes sense to increase the SL_2 at the expense of SL_1, This is illustrated in the figure4.L.1.

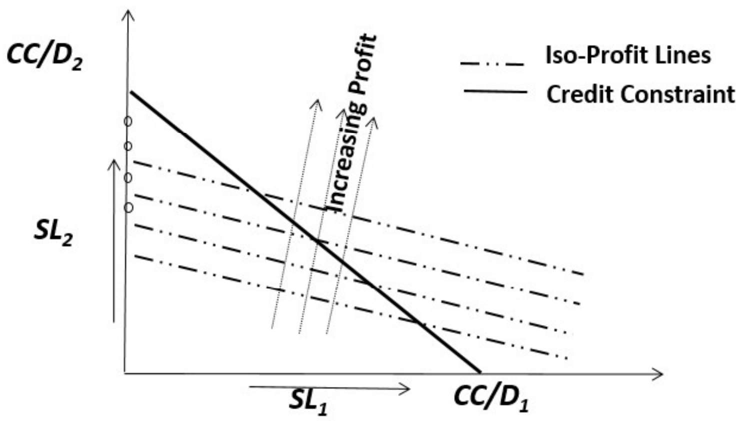


Figure4.K.1. The service level determination for Profit Maximization in a capital-constrained supply chain