CHAPTER

5 Inventory Replenishment Policies for Two Successive Generations' Price-Sensitive Technology Products

The selling price is a very essential component of the marketing mix variables and overall value proposition to the customer. Price has always been an important aspect of the overall value proposition and one of the four Ps of marketing. The firms use selling price as a strategic tool to influence the demand (DeGraba, 1995; Gallego et al. 2008; Levin et al. 2009). In today's retail context, price markdowns have become a regular modality (Cachon & Swinney, 2009; Adida & Ozer, 2019). Due to shorter product life cycles and faster obsolescence of technology, the players in the technology domains experience faster price erosion. If they make the right pricing decisions, it can be their key to growth and profitability. The importance of pricing in the case of technology products is higher because of the need to cover up the research and innovation development expenses. A lot of caution is supposed to be exercised in the pricing of the product since that affects the demand for the product. The original inventory models that did not consider the pricing dynamics were limited to cost minimization. The incorporation of the price variable extended their scope to a larger objective of profit maximization. Keeping this into consideration, this chapter has used the Generalized Bass Model (1994) using the selling price of the product as one of the decision variables to develop the demand model for a two-successive generation technology product.

In general, the inventory replenishment policies in an organization are neither very long term in nature, not too short term. Generally, the norms related to replenishment cycle length are fixed for a certain period of intermediate length and then reviewed again at the end of the period to fix them at another appropriate level for another fixed period, referred to as the planning horizon here. The conventional multi-period inventory model is based upon the assumption that the demand does not vary with time.

Since the demand for technology products is not constant as explained above, it makes sense to change the conventional p-model for the technology generations. The objective of this work is to develop a model that maximizes the total profit in different planning horizons for the successive generations of technology products under the demand that is governed by innovation diffusion and is also price-dependent.

It is a fact that the price of a product has a predominant influence over its demand. Some of the pioneering works in this regard have been by Simon (1979), Parker (1992), and Tam and Hui (1999). There also exist a good number of research studies that have been done on inventory optimization for price-dependent demand. While some research studies have taken demand as a linear function of price, others have taken it as a polynomial function or as an exponential function of price. Nascimento and Vanhonacker (2008) concluded that in the absence of protection, the skimming price strategies prove to be the most optimal ones. Yang et al. (2013) developed a model to determine the optimal special order quantity and retail price to maximize profit and established an algorithm to find the optimal solution. Tayal et al. (2014) incorporated the time to expiry and also allowable shortages to formulate the inventory model for multiple items. Paul et al. (2014) studied how the multiple imperfect items under price discounts can be replenished jointly. Ghoreishi et al. (2015) developed the EOQ Model under inflation-induced and price dependent demand while allowing the customer returns and gradual deterioration. Giri et al. (2016) worked on pricing decisions for substitutable and complementary products. Fang (2016) also worked on joint pricing and replenishment for substitutable products with uncertain demand rates. Jana and Das (2017) studied the inventory model for multiple items in a two-warehouse system under nested discount on the costs. Taleizadeh et al. (2020) proposed the inventory model for multiple serviceable items under contrasting inventory replenishment norms. Table5.1 summarizes the work of the research studies that have been done on the inventory optimization for the price-dependent demand for multiple items.

Table5.1. Tabular review of existing literature on multi-item inventory modeling under price-dependent demand

Research Study	Multiple items inventory	Price dependent demand	Demand Substitution	Joint replenishment	Innovation Diffusion demand	Multi-generation
Chakraborty et al., (2013)	√	√	√	X	X	X
Duran and Luis, (2013)	√	√	X	√	X	X
Otrodi et al., (2016)	√	√	√	√	X	X
Fang, (2016)	√	√	√	√	X	X
Giri et al., (2016)	√	√	√	X	X	X
Jain et al., (2012)	√	√	√	X	X	X

Kar et al., (2001)	√	√	√	X	X	X
Kuo and Huang, (2012)	√	√	X	X	√	√
Lee and Lee, (2018)	√	√	√	X	X	X
Liu et al., (2015)	√	√	√	√	X	X
Mahmoodi, (2016)	√	√	√	√	X	X
Mousavi et al., (2014)	√	√	X	X	X	X
Neda et al., (2016)	√	√	√	√	X	X
Panda and Maiti, (2009)	√	√	X	X	X	X
Paul et al. (2014)	√	√	√	√	X	X
Talebian et al., (2013)	√	√	√	X	X	X
Tayal et al., (2014)	√	√	X	X	X	X
Tsao and Sheen, (2012)	√	√	X	X	X	X

Thus, The work that has considered the influence of price on the inventories of multi-generational products is very rare. Most of the studies cited above have considered conventional products only. Also, as shown in Table5.1, there does not exist much work on inventory modeling for multiple generation products and in fact, no EOQ model for the multi-generation model is available that considers the influence of price. Nagarajan and Rajagopalan (2008) suggested that the pattern of adoption-substitution can play an important role in inventory control for technology generations product. Kreng and Wang (2013) argued that most of the classical multi-generation models are of little use for policy decisions; as they do not consider explicitly the effect of marketing variables, such as pricing or advertising to understand the demand dynamics. Chanda and Aggarwal (2014) discussed optimal replenishment policies for two-generation technology products.

In this chapter[7], the impact of parallel diffusion of two successive generations' products with price elastic demand on inventory policies of the monopolist has been studied. This chapter considers the supply chain scenario of technology products where the manufacturer needs to determine the optimal pricing and inventory replenishment policies to maximize the supply chain surplus. To model the p-type replenishment cycles for two consecutive generations of technology products, price is used as one of the important adoption variables to understand the pattern of substitution. Some of the examples of such products are cellular handsets or IT routers or home electronics whose price softens after the launch

[7] This chapter is based on the following research paper:
Nagpal, G. & Chanda, U. (2021). Inventory Replenishment Policies for Two Successive Generations' Price-Sensitive Technology Products. *Journal of Industrial Management and Optimization.* Accepted for publication.

of the product. The supply chain practitioners in such a business often find themselves in trouble when faced with the pricing and inventory issues. For example, if a manufacturer has to price the different generations of a technology product to meet specific objectives around the selective targeting and operational excellence, this research study can be of immense help. The proposed framework has been characterized to give marketing-operational insights to optimize supply chain efficiency. The demand model is also characterized by considering the life-cycle dynamics for a P-type inventory system. The model has been solved by using a genetic algorithm technique. The impact of yearly price drop with the technology maturation and the price sensitivity of demand on the profit margins vis-à-vis on replenishment policies has also been studied. The chapter also brings forward the dynamics of the launch of newer generations and the pricing strategies on optimal inventory replenishment policies. Numerical illustrations have also been covered in the chapter.

5.1. Inventory Modelling Framework

The purpose of this model is to help the supply chain managers and practitioners of technology products in making sound business decisions related to pricing and replenishment. It is very important to maximize the supply chain surplus of the overall chain since a greater surplus increases the size of the overall pie to be shared among the various supply chain entities. Therefore, the agenda of this model is to maximize the overall surplus, and hence the profit, of the supply chain. In the following subsections, the model assumptions and notations are presented. Going forward, a detailed discussion on the development of the two-generation demand model is presented, which will be further used for inventory modeling and cost modeling in the upcoming sections.

5.1.1. Notations of the Model

A fixed cost of ordering per order for any shipment exclusive of the product-specific costs (j = 1,2)

A_j fixed product-specific ordering cost per order for j^{th} generation product (j = 1,2)

C_j inventory holding expense (as a % of basic purchase cost) for j^{th} generation product (j = 1,2)

M_j the size of market potential for j^{th} generation product (j = 1,2)

$\lambda_1(t)$ and $\lambda_1'(t)$ demand rate at time $'t'$ of 1^{st} generation product for $t \leq \tau$ and $t \geq \tau$ respectively

$\lambda_2(t)$ demand rate at time $'t'$ of 2^{nd} generation product

p_j coefficient of innovation for j^{th} generation product (j = 1,2)

q_j coefficient of imitation for j^{th} generation product (j = 1,2)

τ be the time instant at which the second-generation product is introduced in the market

$\emptyset(t)$ is the conditional probability of a prospective adopter (who has not yet adopted the product till time t) adopting the product in the time interval(t, $t+\Delta t$)

$F_j(t)$ is the cumulative fraction of adopters that have adopted the jth generation product until time t.

$f_j(t)$ is the fraction of adopters who adopt the jth generation product at time t.

$x_j(t)$ is the influence of the price changes of the *j*th generation product at time t.

$X_j(t)$ is the cumulative influence of the price changes of the *j*th generation product till time *t*

ϒ is a factor proportional to the decline in selling price per unit time due to technology adoption

P_{tj} is the selling price of jth generation product at the time instant t

P_{oj} is the selling price of jth generation product at the time of launch

ρ is the length of the planning time horizon for which the inter-replenishment time interval is fixed

n is the number of inventory replenishment cycles in the *mth* planning horizon in a single generation scenario

HC is the inventory holding cost in any planning horizon

OC is the inventory ordering cost in any planning horizon

m and *m'* denote the sequence of the planning horizon before and after the launch of second-generation respectively

TR_m is the total revenue in the *mth* planning horizon in the single generation scenario

TCM_m is the total contribution margin, i.e. Revenue, net off material cost for the goods sold in the m_{th} planning horizon in the single generation scenario

$TC_{m,n}$ is the total cost of ordering and holding in mth planning horizon with n replenishment cycles in the single generation scenario

$TP_{m,n}$ is the total profit incurred from the goods sold in the mth planning horizon with n replenishment cycles in the single generation scenario

$TR_{1m}{}'$ is the total revenue in the *m'th* planning horizon from the first generation product

$TR_{2m}{}'$ is the total revenue in the *m'th* planning horizon from the second generation product

n_1 *and* n_2 are the number of replenishment cycles for the first generation and second generation product respectively in the *m'th* planning horizon post the launching of the next generation product

$$y_m' = \begin{cases} 0; if\ first\ first\ generation\ product\ is\ discontinued\ by\ the\ m'th\ planning\ horizon \\ 1; if\ the\ first\ generation\ product\ is\ still\ in\ business\ in\ the\ m'th\ planning\ horizon \end{cases}$$

$TCM_{m'}$ is the total contribution margin, i.e. Revenue, net off material cost for the goods sold in the $m'th$ planning horizon in the single generation scenario

TC_{m',n_1,n_2} is the total cost of ordering and holding in $m'th$ planning horizon with n_1 and n_2 replenishment cycles of the first and the second generation product respectively

TP_{m',n_1,n_2} is the total profit incurred from the goods sold in the $m'th$ planning horizon with n_1 and n_2 replenishment cycles of the first and the second generation product respectively

β is the influence of the change in price on the adoption rate and is analogous to price elasticity of demand

$$z = \begin{cases} 0; if\ the\ logistics\ is\ not\ pooled\ for\ the\ two\ generations \\ 1; if\ the\ logistics\ is\ pooled\ for\ the\ two\ generations \end{cases}$$

5.1.2. **Assumptions of the Model**

a) The supply gets replenished instantaneously
b) Shortages are not allowed
c) The selling price of the product is dependent upon the stage of the life cycle.
d) The rate of demand is influenced by the innovation diffusion process and follow the assumptions as discussed in section 3.1.3 and can be given as follows:
$\lambda_1(t) = M_1 f_1(t)\ for\ t < \tau$
$\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t) F_2(t)\ for\ t > \tau$
$\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t) F_2(t)$
e) Logistics for both the generations of the product may be pooled or un-pooled, depending upon the nature of ordering costs
f) The fixed ordering cost can be broken into two components-one being product-specific, and the other being product-non-specific.
g) The number of replenishment cycles, and thus, the length of each replenishment cycle is fixed for each planning horizon at the beginning of that period.

In the upcoming sections, the inventory model for two successive technology generation products shall be discussed. First, the single generation model shall be discussed, and later it will be extended for two-generation products.

5.1.2.1. Price Model and Demand Variation for Price elastic demand

Since the demand is dependent upon price as well as the innovation diffusion dependent, the basic demand model needs to be appended to incorporate the influence of the selling price, and the selling price also needs to be modeled.

In the case of innovative products, the firms follow the price skimming strategy in the introduction stages since the huge research and development costs get amortized over the smaller volumes. The target segment in the initial stages is also the adopter category of the consumers, who are ready to pay a premium for that. Also, the lack of economies of scale in production and distribution, coupled with the lack of competition from the other players enables the product to be sold at a higher price. However, gradually, the price of the product starts falling with time as the firm adopts the penetration pricing strategy, and has a learning curve on cutting down the costs. Since the rate of price drop due to technology maturation falls with time, it can best be expressed as a logarithmic function of time instant t.

$$P_1(t) = \begin{cases} P_{01}\ for\ t \le 1 \\ P_{01}(1 - \Upsilon \ln(t - \delta t))\ for\ t > 1 \end{cases} \quad (5.1)$$

Where δt is very small

Thus, the price of the technology product remains constant for a small period post the launch after which it starts reducing due to multiple reasons such as the learning curve, increasing economies of scale, increasing competition, recovery of initial costs, etc. The behavior of the price with time is thus shown in Figure5.1.

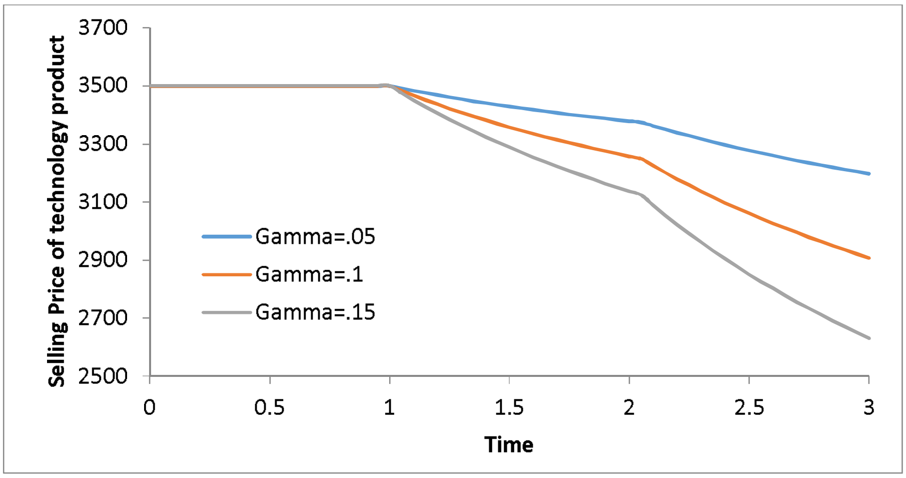

Figure5.1. The behavior of the price of the technology products with the time elapsed after the launch for different values of annual % price drop "Υ"

The adaptation of the Demand Model to incorporate the influence of the price has been discussed below.

Bass et al. (1994) proposed the following hazard-rate based adoption model (also called as Generalized Bass Model or GBM) by incorporating a variable $x(t)$ as follows:

$$\frac{f(t)}{[1-F(t)]} = [p + qF(t)]x(t); \quad (5.2)$$

The solution of the GBM (5.2) for a single-generation can be stated as:

$$F_1(t) = (1 - \exp(-(X_1(t) - X_1(0))b_1)/(a_1 * \exp(-(X_1(t) - X_1(0))b_1) + 1) \quad (5.3)$$

$$f_1(t) = x_1(t) * \left[\frac{{b_1}^2}{p_1}\right] \exp(-(X_1(t) - X_1(0))b_1)/(a_1 * \exp(-(X_1(t) - X_1(0))b_1) + 1)^2 \quad (5.4)$$

where $a_1 = \frac{q_1}{p_1}$ *and* $b_1 = (p_1 + q_1)$;

Considering the GBM, the influence of price $(x(t))$ is given by

$$x_1(t) = 1 + \beta[\frac{P_1(t)}{P_{01}} - 1] \quad (5.5)$$

Where β is the sensitivity of $x_j(t)$ for the jth generation product in the GBM with the change in the price of the product $P_j(t)$.

Thus, the cumulative influence of price changes at time t can be derived by integrating equation (5.5) w.r.t. t and written as:

$$X_1(t) = t + \beta[-\Upsilon(1+t)(\ln(1+t) - 1)] \quad (5.6)$$

Similarly, in the case of the second-generation product that gets launched after time τ, the fraction of adopters at any time t (without considering any interaction effect with the potential buyers of the first generation) can be written as:

$$f_2(t) = x_2(t)\left[\frac{b_2{}^2}{p_2}\right]\exp(-(X_2(t) - X_2(\tau))b_2) / (a_2\exp(-(X_2(t) - X_2(\tau))b_2) + 1)^2 \quad (5.7)$$

And the cumulative adopters till time t can be written as:

$$F_2(t) = (1 - \exp(-(X_2(t) - X_2(\tau))b_2) / (a_2\exp(-(X_2(t) - X_2(\tau))b_2) + 1 \quad (5.8)$$

and $a_2 = \frac{q_2}{p_2} and b_2 = (p_2 + q_2)$

Also, $P_2(t) = P_{02}(1 - \Upsilon\ln(t - \tau + 1))$ (5.9)

and $x_2(t) = 1 + \beta[P_2(t)/P_{02} - 1]$ (5.10)

Thus, $X_2(t) = t + \beta[-\Upsilon(1 + t - \tau)(\ln(1 + t - \tau) - 1) - t]$ (5.11)

5.1.3. Inventory Modeling for the single generation product scenario

If there is no deterioration of the product, the consumption of inventory takes place on account of the demand usage only. Therefore, it can be said that

$$\lambda_j(t) = -\frac{d(I_j(t))}{dt}\ ;\ (j = 1,2) \quad (5.12)$$

Figure5.2 shows the inventory replenishment cycles in the P-Model. If time t lies in the ith replenishment cycle of the mth planning horizon, in which a total of n replenishments are done. Then inventory at time t (as given in Figure 2) is the demand of the product from time t till the end of the corresponding replenishment cycle. Since the time point corresponding to the end of the ith replenishment cycle in such case is given by $\{(m-1) + \frac{i}{n}\}\rho$, it can be said that inventory at time t is:

$$I_1(t) = \int_{u=t}^{u=\{(m-1)+\frac{(i)}{n}\}\rho} \lambda_1(u)\, du \quad (5.13)$$

The Total Cost in the mth planning horizon with n replenishment cycles can be stated as the sum of ordering costs and the inventory carrying costs

$$TC_{m,n} = n(A + A_1) + I_1C_1\left[\int_{(m-1)\rho}^{m\rho} I_1(t)dt\right] \quad (5.14)$$

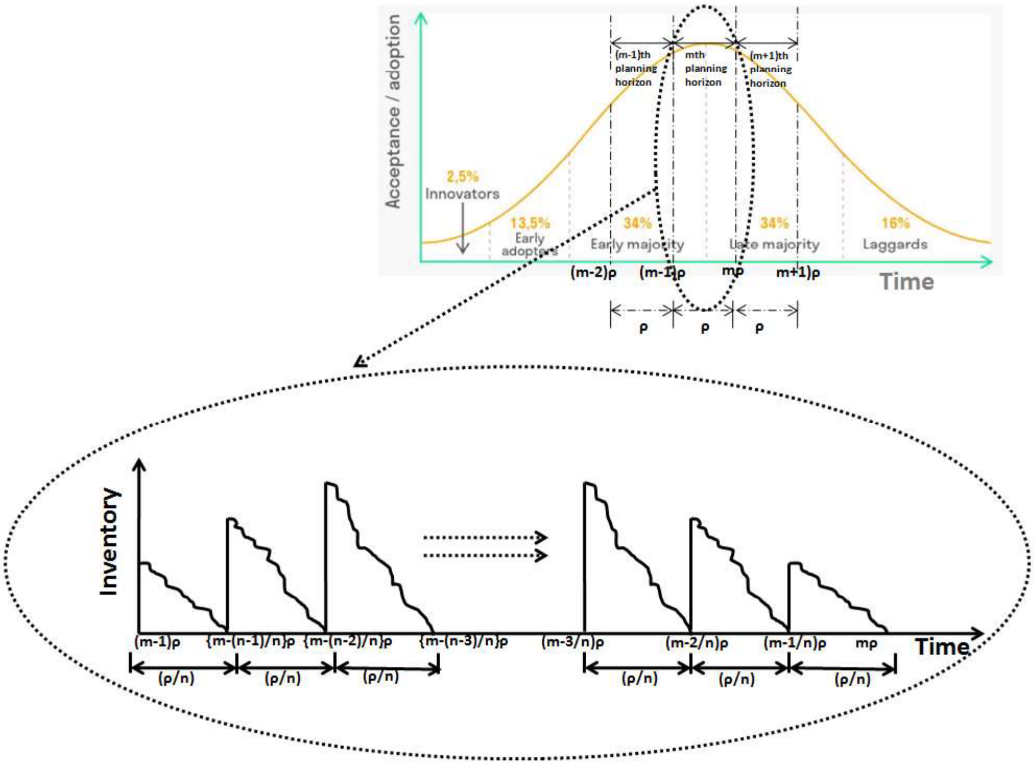

Figure5.2. The P Model of inventory management before the launch of the second generation

$$TC_{m,n} = n(A + A_1) + I_1C_1 \left[\sum_{i=1}^{i=n} \int_{t=\{(m-1)+ (i-1)/n\}\rho}^{t=\{(m-1)+\frac{(i)}{n}\}\rho} I_1(t)dt \right] \tag{5.15}$$

From equations (5.17), (5.18), and (5.19), the following is obtained:

$$TC_{m,n} = n(A + A_1) + I_1C_1 \left[\sum_{i=1}^{i=n} \int_{t=\{(m-1)+(i-1)/n\}\rho}^{t=\{(m-1)+\frac{(i)}{n}\}\rho} [\int_{u=t}^{u=\{(m-1)+\frac{(i)}{n}\}\rho} \lambda_1(u)\, du]\, dt \right] \tag{5.16}$$

The contribution margin and total profit in the mth replenishment cycle are written as:

$$TCM_m = \int_{t=(m-1)\rho}^{t=m\rho} \{P_1(t) - C_1\}\, \lambda_1(t)\, dt \tag{5.17}$$

$$TP_{m,n} = TCM_m - TC_{m,n} \tag{5.18}$$

The objective function is to find the value of n for which the Total Profit $TP_{m,n}$ in the m^{th} planning horizon is maximum and can be given as:

$$\begin{cases} Max.\ \ TP_{m,n} \\ Subject\ to\ the\ constraints\ n\ is\ a\ positive\ integer \end{cases}$$

In the above formulation, n is the only decision variable.

5.1.4. Inventory Modeling after the launch of the second generation product

After the second generation product is introduced, let the new policies be worked out in terms of several replenishment cycles for each subsequent planning horizon. For simplicity, it can be assumed that the next-generation product is launched at a time when the first generation product has just got replenished by the vendor. The inventories of the products can now be stated as:

$$I_1(t) = \int_{t=t}^{t=\tau+\left\{(m\prime-1)+\frac{(i)}{n_1}\right\}\rho} \lambda_1'(t)\,dt \qquad \text{for } t \geq \tau \qquad (5.19)$$

$$I_2(t) = \int_{t=t}^{t=\tau+\left\{(m\prime-1)+\frac{(i)}{n_2}\right\}\rho} \lambda_2(t)\,dt \qquad (5.20)$$

The $m'th$ planning horizon post the introduction of the second generation product starts at $t = \tau + (m' - 1)\rho$ and ends at $t = \tau + m'\rho$. The contribution margin during this period is the area under the contribution margin graph drawn on the time axis, between these two instants of time.

As illustrated in Figure5.3, the period corresponding to the ith cycle of the first-generation product in the $m'th$ period post the introduction of the second generation product starts at $t = \tau + \left\{(m' - 1) + \frac{(i-1)}{n_1}\right\}\rho$ and ends at $t = \tau + \left\{(m' - 1) + \frac{(i)}{n_1}\right\}\rho$

Similarly, the period corresponding to the ith cycle of the second generation product in the $m'th$ period after the launch of the second generation starts at $t = \tau + \left\{(m' - 1) + \frac{(i-1)}{n_2}\right\}\rho$ and ends at $t = \tau + \left\{(m' - 1) + \frac{(i)}{n_2}\right\}\rho$

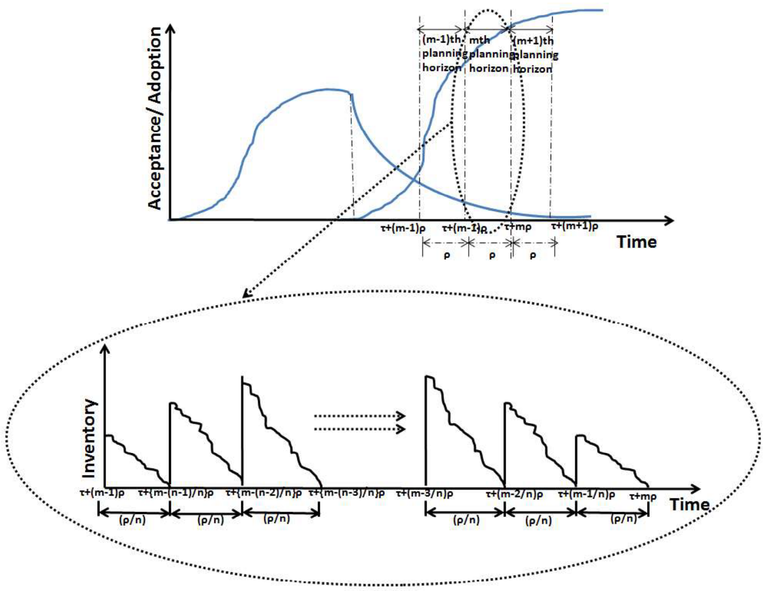

Figure5.3. The P Model of inventory management in the mth planning horizon with n replenishments before the launch of first-generation

The contribution margin in the $m'th$ planning horizon is given by

$$TCM_{m'} = y_m{}' \int_{t=\tau+(m'-1)\rho}^{t=\tau+m'\rho} [P_1(t) - C_1]\lambda_1'(t)\,dt \; + \; \int_{t=\tau+(m'-1)\rho}^{t=\tau+m'\rho} [P_2(t) - C_2]\,\lambda_2(t)\,dt \qquad (5.21)$$

$$TC_{m',n_1,n_2} = n_1(A + A_1) + n_2(A + A_2) - z.n_1.A +$$

$$I_1C_1\{\sum_{i=1}^{i=n_1} \int_{t=\tau+\{(m\prime-1)+\frac{(i-1)}{n_1}\}\rho}^{t=\tau+\{(m\prime-1)+\frac{(i)}{n_1}\}\rho} [\int_{u=t}^{u=\tau+\{(m'-1)+\frac{(i)}{n_1}\}\rho} \lambda_1'(u)\,du]\,dt\} +$$

$$I_2C_2\{\sum_{i=1}^{i=n_2} \int_{t=\tau+\{(m\prime-1)+\frac{(i-1)}{n_2}\}\rho}^{t=\tau+\{(m\prime-1)+\frac{(i)}{n_2}\}\rho} [\int_{u=t}^{u=\tau+\{(m\prime-1)+\frac{(i)}{n_2}\}\rho} \lambda_2(u)\,du]\,dt\} \qquad (5.22)$$

Where $z = 1\; if\; n_1 = n_2,\; else\; zero$

$$TP_{m',n_1,n_2} = TCM_{m'} - TC_{m',n_1,n_2} \qquad (5.23)$$

The objective function is to minimize TP_{m',n_1,n_2}

Subject to the constraints n is a positive integer

In the above formulation, there are three decision variables n_1, n_2, $z$and $y_m{}'$. Our objective is to maximize the profit function TP_{m',n_1,n_2}.

Also, after considering the % price drop per unit time Υas the decision variable, and the price sensitivity of demand β as another decision variable, the optimization problem becomes maximization of profit, which can be written as

Max. TP_{m',n_1,n_2} = Max. $[TCM_{m'} - TC_{m',n_1,n_2}]$.

Where n_1 and n_2 are positive integers.

Depending on the path of ordering costs per unit time, purchase costs per unit time, and inventory holding costs per unit time, the following theorems can be developed:

Theorem 5.1: The higher the market potential and the innovation and imitation coefficients of the second generation, the earlier the phase-out of the first generation.

Proof: See Appendix 5.A.

Theorem 5.2: If the newer generation product is sold at a premium, the optimal exit time of the earlier generation product gets stretched.

Proof: See Appendix 5.B.

Theorem 5.3: When the second generation product caters to a niche segment with very little market potential, the n_m^* will increase with its launch unless it is sold at a substantial premium.

Proof: See Appendix 5.C.

Theorem 5.4: Penetration pricing is a better strategy than price skimming when price elasticity of demand is very high. On the other hand, if the price elasticity of the products is high, it makes better sense to follow the price skimming for the second generation.
Proof: See Appendix 5.D.

In the next subsection, the solution procedure to find the optimal solution is discussed. Since the profit function as defined in equation (5.18) and (5.23) is highly non-linear, hence finding an analytical solution for the problem is difficult. The problem is solved numerically under given parameter values. The genetic algorithm has been used to find the optimal replenishment frequency in each planning horizon.

5.1.5. Solution procedure

The optimization problem formulated above is a Mixed Integer Non-Linear Programming Problem (MINLP). This chapter proposes the use of genetic algorithms (GAs) in determining the optimal number of replenishments. The use of GA proves to be appropriate in solving the model because of its highly non-linear demand pattern and several discontinuities in the objective function. The fitness function used has been formulated in a way that covers the revenues net of the basic purchase costs, inventory carrying costs, and the ordering costs. The results of GA have been validated with the results obtained from a greedy search throughout the solution domain.

A flowchart illustrating the working of GA has been shown in Figure5.4. At first, a population of chromosomes (one-dimensional vectors of size four, corresponding to each decision variable) is randomly generated and ranked based on their fitness values. The fittest chromosomes are selected to create a mating pool. Genetic operations, i.e. the crossover and mutation, are performed on these chromosomes in the mating pool and a set of child chromosomes are generated, completing one generation. In this example, a single-point crossover (at the center) has been used with a crossover fraction of 0.6 and a uniform mutation function with a mutation rate of 2%. A different set of these functions or rates may lead to faster or slower convergence. Generally, the child chromosomes obtained after performing the genetic operations on parent chromosomes must have a better fitness value. This results in the eventual convergence of GA to an approximate optimal solution after a few generations. With the presented formulation and given parameter settings, the numerical illustration considered here, converged to the optimal solution in as few as twenty generations, proving GA to be extremely efficient.

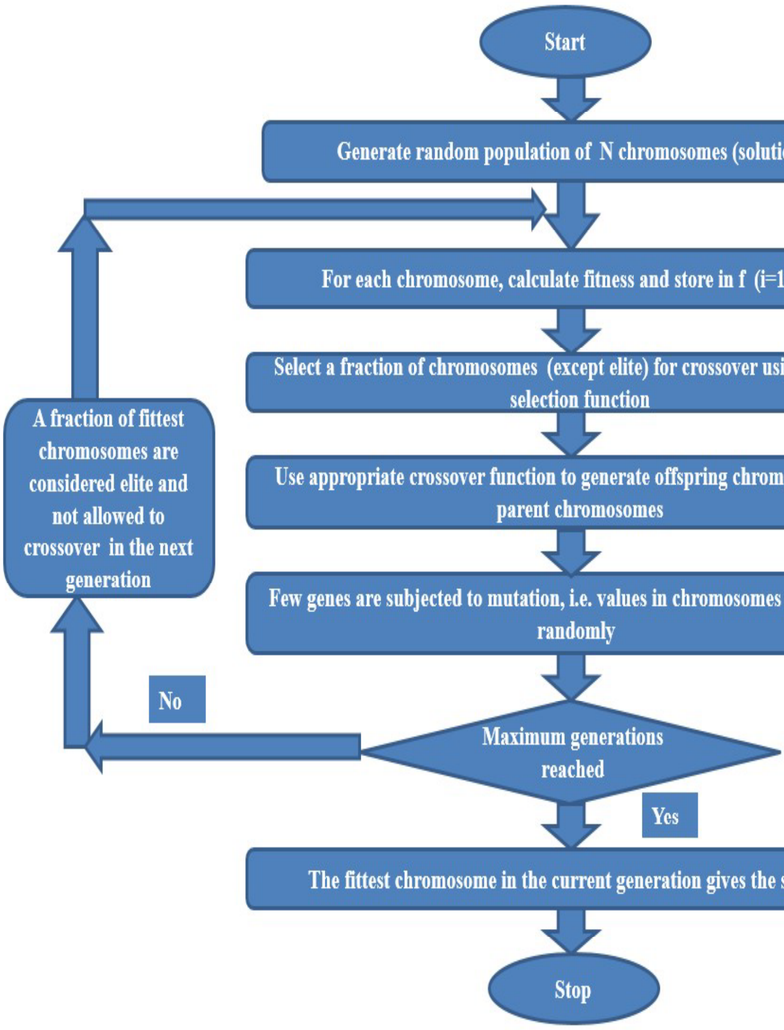

Figure5.4. The steps performed in the optimization of inventory policies using a genetic algorithm

5.1.6. Numerical illustration

These are the data points that have been considered consider in the numerical illustration.

$\rho = 0.5,\ p_1 = .5,\ q_1 = 2.5,\ p_2 = .6,\ q_2 = 3.5,\ M_1 = 100000,\ M_2\ 120000, A = INR\ 1500000,\ \ A_1 = A_2 = INR\ 250000, C_1 = INR\ 1500,\ C_2 = INR\ 1700,\ \ \tau = 0.5,\ \Upsilon = .15,\ \beta = -1,\ I_1 = I_2 = .15,\ P_{01} = INR\ 3500,\ P_{02} = INR\ 4200$

Here, the multi-generational scenario has been considered in which the second-generation product has been launched at time instant = 0.5. And it is found that the optimal replenishments as well as the continuity of the business of the first-generation product in each planning horizon. The behavior of revenue and profitability metrics with time in Table 5.3 can also be observed. The summary of the results is shown in Table5.2. The cells with the lowest replenishment costs have been shaded green in

Table5.2. The non-specific ordering cost per order is taken as a large number so that the model will tend to yield the same replenishment frequency for the products of both the generations.

Table5.2. Optimal n_1 and n_2 in pooled logistics determined by minimizing the sum of holding cost and carrying costs for each planning horizon *(All the costs are in Mn INR)*

	$m' = 1, y_m{}' = 1$			$m' = 2, y_m{}' = 1$			$m' = 3, y_m{}' = 1$			$m' = 4, y_m{}' = 0$		
$n_1 = n_2$	HC	OC	TC	HC	OC	TC	HC	OC	TC	HC	OC	TC
1	5.96	2.00	7.96	4.54	2.00	6.54	1.12	2.00	***3.12***	0.19	2.00	***2.19***
2	2.92	4.00	***6.92***	2.4	4.00	***6.40***	.60	4.00	4.60	0.10	4.00	4.10
3	1.4	6.00	7.40	1.5	6.00	7.50	.66	6.00	6.66	0.2	6.00	6.20
4	1.0	8.00	9.00	1.1	8.00	9.10	.50	8.00	8.50	0.1	8.00	8.10

The behavior of the ordering costs and holding costs has been illustrated for different replenishment frequencies in the different planning horizons in Table5.2. For the initial planning horizons, the overall demand for both the products is increasing, leading to increased volumes and more number of optimal replenishment cycles. In the later planning periods, the volumes start declining and therefore, the optimal number of replenishment cycles within these planning horizons falls.

Table5.3. Total Revenue, Profits and Opportunity Loss for two generations scenario *(The Revenue and Profit figure are in Mn INR)*

					$for\ y_m{}' = 1$	$for\ y_m{}' = 0$	
m'	$n_1 = n_2$	$y_m{}'$	$TR_{1m'}$	$TR_{2m'}$	TP^*_{m',n_1n_2}	TP^*_{m',n_1n_2}	OL^*_{m',n_1n_2}
1	2	1	107.93	189.71	171.67	110.25	61.42
2	2	1	86.36	267.84	206.37	157.27	49.09
3	1	1	62.83	284.81	204.31	168.66	35.65
4	1	1	49.95	249.17	176.66	148.37	28.29

The upcoming part of the chapter justifies the findings and draws insights from them. Here, $TR_{1m}{}'$ and $TR_{2m}{}'$ denote the revenues from the first and the second generation product respectively in the m'th planning horizon.

Also,

(OL^*_{m',n_1n_2}) is the opportunity loss of discontinuing the first generation product which is

$$\left(OL^*_{m',n_1n_2}\right) = TP^*_{m',n_1n_2 for\ y_{m'}=1} - TP^*_{m',n_1n_2 for\ y_{m'}=0}$$

5.1.7. Discussions on Replenishment Policies and Pricing Policies

It can be observed from the numerical illustration section that the total Revenue from the first generation keeps declining after the launch of the second generation, while that of the second generation initially increases, reaches a peak, and then, starts declining. The contribution margin % which is simply the revenue net of the basic product cost keeps falling as the technology becomes more popular. After the launch of the second generation, the optimal number of replenishment cycles increases till the second generation product reaches the maturity stage, because of large volumes, and therefore, the over-play of the inventory holding costs. After the second generation product reaches the maturity stage, the optimal number of replenishment cycles again starts falling due to lower volumes and therefore, the downplay of the inventory holding costs. Also, when the first generation product starts giving a negative contribution to the overall profit under the reduced scale, then it can be optimal to discontinue this product. It is also evident that the profit margins reduce over the years. This is in line with the established construct that with the technology becoming more common and more adopted by the masses, the profit margins start declining.

If the replenishment policies are examined concerning the pooling of logistics, there is a mixed effect. The benefits of pooling can be in terms of reduced ordering costs. So, the pooling will not give any benefit if the ordering costs are a very small portion of the overall replenishment costs. As shown in Table5.4, when the non-specific ordering cost "*O*" is made nil, the optimal number of replenishments for both the generations may not be equal due to missing economies of pooling. On the contrary, when the non-specific ordering cost is a significant proportion of the overall ordering costs, the model will always suggest the pooled logistics solution where $n_1 = n_2$ to contain the ordering costs through pooled replenishments.

Table5.4. Comparison of the replenishment dynamics: Joint vs Dis-joint for both the generations of products (*The Profit figures are in Mn INR)*

	Υ= 1.5, β= 1.5					
	with order synergies (*O* is large)			**without order synergies (*O*= 0)**		
Planning Horizon	n_1	n_2	TP^*_{m',n_1n_2}	n_1	n_2	TP^*_{m',n_1n_2}
1st	2	2	164.72	3	6	168.72
2nd	2	2	202.72	1	7	207.29
3rd	1	1	208.85	1	3	210.97
4th	1	1	182.00	1	1	183.55
5th	1	1	156.20	1	1	157.74
	Υ= 1.5, β= 10					
	with order synergies (*O* is large)			**without order synergies (*O*= 0)**		
Planning Horizon	n_1	n_2	TP^*_{m',n_1n_2}	n_1	n_2	TP^*_{m',n_1n_2}

1st	1	1	20.87	1	1	19.37
2nd	1	1	4.40	2	1	2.88
3rd	1	1	4.04	2	2	5.93
4th	1	1	12.11	2	3	14.84
5th	1	1	22.63	2	7	2.62

Now, the chapter considers the price drop % per unit time Υ as well as the price sensitivity β as variables. So, with the change in these variables, our revenues and hence profits also get impacted. The genetic algorithms have been used to study the behavior of total profit since there are multiple variables in the optimization problem and the global optimum needs to be found.

Table5.5. Total Profit values $TP^*_{m',n_1n_2}$for different values of yearly price drop % with the change in β *and with the change in* Υ *(The Profit figures are in Mn INR)*

	β =10	**β =20**	**β =100**
Υ=0.05	118.5	34.6	98.5
Υ=0.10	33.2	50.3	155.3
Υ=0.15	20.9	74.0	218.2
Υ=0.20	49.2	86.2	285.7
Υ=0.25	63.1	97.2	356.0

From Table5.5, it is evident that the total profit initially falls with the increase in % drop in the price until a point at which the increase in volumes due to price drop nullifies the negative effect of the price drop on the revenues; and the profits start increasing. The Table5.4 also shows that for smaller values of β, the total profit either get reduced or witnesses a mild increase with the increase in the Υ (the % drop in selling price); while for larger values of β , the increase in the profits is much higher for the same % drop in price. This is in line with the expectation since the larger price sensitivity leads to a higher increase in sales volumes for a given decline in the selling price.

Table5.6. Total Profit values $TP^*_{m',n_1n_2}$for different planning horizons with different values of price sensitivity β *(The Profit figures are in Mn INR)*

Planning Horizon	**β=10**	**β=20**	**β=30**	**β=100**
1st	20.87	74.10	92.26	218.15
2nd	4.40	25.89	20.26	233.27
3rd	4.04	20.35	18.60	404.46
4th	12.19	19.30	20.59	59.30
5th	32.00	20.92	23.82	97.45

Table5.6 shows how the % drop in price over time is accompanied by an increase in demand, and therefore, an increase in profit. The higher the value of β, the more is the spike in the demand and therefore the spike in the total profit.

5.2. Major academic and business implications of the proposed EOQ model

This research has demonstrated how to find the optimal inventory policies over multiple periods for a multi-generation technology product whose demand is dependent on price as well as lifecycle dynamics. The managers can understand the complexity of making inventory optimization decisions in case of innovative products that get diffused in a non-linear fashion and also undergo price softening over some time, resulting in faster-changing demand. The study has also illustrated the effect of pricing phenomenon on the inventory replenishment norms for the multi-generational technology products. The study shows that the changes in the price of one generation of technology products have a bearing on the demand rate of the other substitutable generations of the product.

Important learning that this work has to offer to the practicing managers is that the higher replenishment efficiencies can be achieved with the launch of newer and more advanced generations products, on the account of higher demand rate leading to faster movement of inventories. This research study also illustrates that if the demand for the product is highly price-sensitive then the penetration pricing can be preferred to the price skimming in the initial product life cycle stages. This is because the positive effect of the increased demand rate far outweighs the negative effect of the price drop on the revenues. However, if its demand is relatively insensitive to the selling price, it makes sense to use price skimming until a possible later point in the product life cycle. This is because the positive effect of the increased price on the revenues is much higher than the negative effect of the fall in the demand rate.

On similar lines, the managers will not find it hard to infer that the perfect quality is not always of the best quality. Since quality has a trade-off with price, it may make business sense to limit the quality to an optimal extent in case of price elastic products. The study also sheds light upon certain factors that can expedite the phase-out of the first generation product. Such factors can be mentioned as the higher market potential of the second generation product, the higher innovation and imitation coefficients of the second generation product, the earlier launch of the second generation product, and the higher product-specific ordering costs of the first generation product.

The practitioners also need to acknowledge that when the non-specific ordering costs (the ones that yield synergies in joint replenishments) are a significant portion of the overall ordering costs, it makes sense to go for the consolidated replenishment of the multiple generations products. Another important observation is the behavior of the optimal inventory policy with the price softening. With the price softening of the technology, the profits initially decline before starting to rise as the price softening exceeds a threshold limit. However, if the demand is highly price-sensitive, the profits increase throughout with the price softening.

In the earlier and the current chapters, the inventory modeling for the multi-generational technology products has been explored well in the context of trade credits and price elasticity. While both these variables are part of the marketing mix, it is worthwhile to understand that the environmental constraints such as the constraint of the storage space need no less consideration in the formulation of inventory policies. It is in this context that the next chapter is going to witness the development of the inventory model under limited storage space for the multigenerational technology products.

Appendix 5

A. Theorem 5.1: *The higher the market potential and the innovation and imitation coefficients of the second generation, the earlier the phase-out of the first generation.*

Proof: As shown in Figure5.A.1, the higher market potential and the faster spread of the next generation product bring the sales of the first generation to an unsustainable level (when the contribution margin is not sufficient enough to cover the holding costs and product-specific ordering costs).

Considering $\exp(-x) \approx 1 - x,$ the following equations can be obtained

$$F_2(t) = (X_2(t) - X_2(\tau))b_2/(a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1)$$

$$\frac{\partial(\lambda_1(t))}{\partial(p_2)} = \frac{\partial(\lambda_1(t))}{\partial(F_2(t))}\frac{\partial(F_2(t))}{\partial(p_2)} = -M_1 f_1(t)[\frac{\partial(F_2(t))}{\partial(a_2)}\frac{\partial(a_2)}{\partial(p_2)} + \frac{\partial(F_2(t))}{\partial(b_2)}\frac{\partial(b_2)}{\partial(p_2)}]$$

$$= -M_1 f_1(t)\left[\frac{\partial(F_2(t))}{\partial(a_2)}\frac{(-q_2)}{(p_2)^2} + \frac{\partial(F_2(t))}{\partial(b_2)}\right] = -M_1 f_1(t)\left[\frac{(X_2(t)-X_2(\tau))b_2}{(a_2(1-(X_2(t)-X_2(\tau))b_2)+1)^2} + \frac{(X_2(t)-X_2(\tau))}{a_2(1-(X_2(t)-X_2(\tau))b_2)+1} - \frac{(X_2(t)-X_2(\tau))^2 a_2 b_2}{(a_2(1-(X_2(t)-X_2(\tau))b_2)+1)^2}\right] = -\frac{M_1 f_1(t)(X_2(t)-X_2(\tau))}{a_2(1-(X_2(t)-X_2(\tau))b_2)+1} * \left[1 + \frac{b_2}{(a_2(1-(X_2(t)-X_2(\tau))b_2)+1)}\left(1 - \frac{(X_2(t)-X_2(\tau))a_2}{(a_2(1-(X_2(t)-X_2(\tau))b_2)+1)}\right)\right]$$

Since $(X_2(t) - X_2(\tau))b_2 < 1$, and since $(X_2(t) - X_2(\tau))a_2(1 + b_2) < 1 + a_2$ and it can be inferred that $\frac{\partial(\lambda_1(t))}{\partial(p_2)} < 0$

$$\frac{\partial(\lambda_1(t))}{\partial(q_2)} = \frac{\partial(\lambda_1(t))}{\partial(F_2(t))}\frac{\partial(F_2(t))}{\partial(q_2)} = -M_1 f_1(t)\left[\frac{\partial(F_2(t))}{\partial(a_2)}\frac{(1)}{(p_2)} + \frac{\partial(F_2(t))}{\partial(b_2)}\right]$$

$$= -M_1 f_1(t)\left[-\frac{F_2(t)}{(a_2)^2}\frac{(1)}{(p_2)} + \frac{F_2(t)}{b_2} + \frac{(X_2(t) - X_2(\tau))^2 a_2 b_2}{(a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1)^2}\right]$$

$$= -M_1 f_1(t)\left[F_2(t)\left(\frac{1}{b_2} - \frac{1}{p_2(a_2)^2}\right) + \frac{(X_2(t)-X_2(\tau))^2 a_2 b_2}{(a_2(1-(X_2(t)-X_2(\tau))b_2)+1)^2}\right]$$

$$= -M_1 f_1(t)\,[F_2(t)[\frac{1}{(q_2+p_2)} - \frac{1}{\frac{(q_2)^2}{p_2}}] + \frac{\left(X_2(t)-X_2(\tau)\right)^2 a_2 b_2}{\left(a_2\left(1-\left(X_2(t)-X_2(\tau)\right)b_2\right)+1\right)^2}]$$

Since $\frac{(q_2)^2}{p_2} \gg (q_2+p_2)$, it can be inferred that $\frac{\partial(\lambda_1(t))}{\partial(q_2)} < 0$

While the first part of the above expression is always <0, the second part is >0, and therefore, the demand rate for the first generation product falls with the higher coefficients of innovation and imitation of the second generation.

This theorem can help understand that the earlier generation product will have to be phased out faster if the newer generation product is diffusing faster.

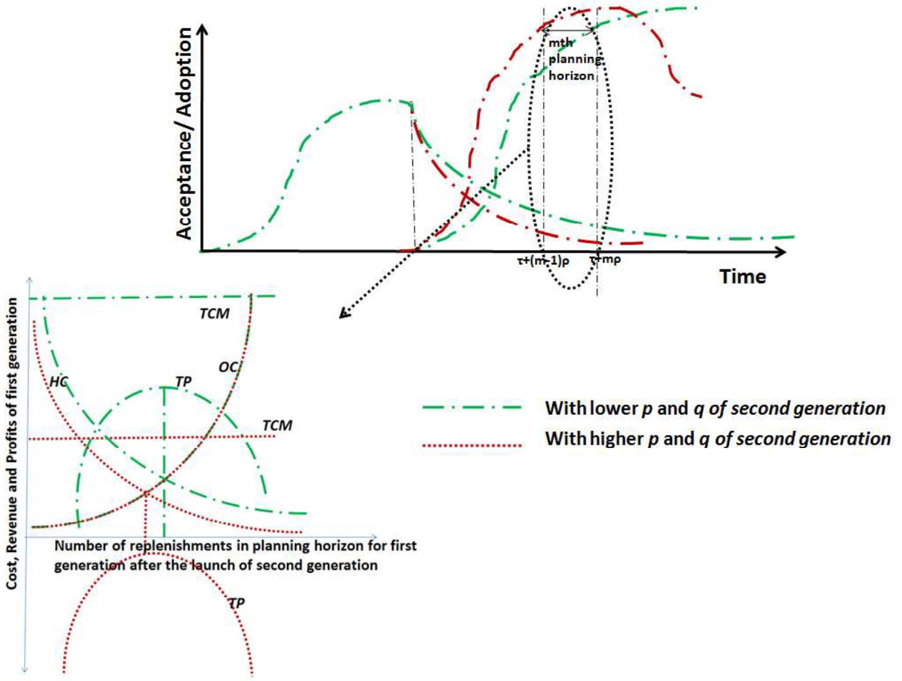

Figure5.A.1. Influence of diffusion rate of 2nd generation product on phase-out timing of 1st generation

B. Theorem 5.2: *If the newer generation product is sold at a premium, the optimal exit time of the earlier generation product gets stretched.*

Proof: The higher the price of the newer generation product, the lesser the cannibalization of older generation products due to the price elasticity of the demand. Therefore, it may make a business sense to continue with the older generation product for a longer length of time.

$$\frac{\partial[\lambda_1'(t)]}{\partial[P_2(t)]} = -M_1 f_1(t)\,\partial[F_2(t)]/\,\partial(P_2(t)) > 0$$

$$\frac{\partial[F_2(t)]}{\partial(P_2(t))} = \frac{\partial[F_2(t)]}{\partial(X_2(t))}\frac{\partial[X_2(t)]}{\partial(P_2(t))}$$

Considering $\exp(-x) \approx 1 - x$, the following can be obtained:

$$F_2(t) = (X_2(t) - X_2(\tau))b_2/(a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1)$$

$$\frac{\partial[F_2(t)]}{\partial(X_2(t))} = \frac{b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1} - \frac{(X_2(t) - X_2(\tau))a_2(b_2)^2}{(a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1)^2}$$

$$= \frac{b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1}[1 - \frac{(X_2(t) - X_2(\tau))a_2 b_2}{a_2(1 - (X_2(t) - X_2(\tau))b_2) + 1}]$$

$$\frac{\partial[X_2(t)]}{\partial(P_2(t))} = \beta_2[-\Upsilon(1 + t - \tau)(\ln(1 + t - \tau) - 1)]$$

While $\frac{\partial[F_2(t)]}{\partial(X_2(t))} > 0$, $\frac{\partial[X_2(t)]}{\partial(P_2(t))} < 0$, and therefore, $\frac{\partial[F_2(t)]}{\partial(P_2(t))} < 0$

Since $\frac{\partial[F_2(t)]}{\partial(P_2(t))} < 0$, $\frac{\partial[\lambda_1'(t)]}{\partial[P_2(t)]} > 0$ since it becomes the product of two negative quantities.

Thus, it can be said that the higher selling price of the advanced generation product enhances the sales volumes of the first generation product.

This theorem can help the managers use price skimming from the newer generation product and treat the older generation product as a cash cow. There is no point in penetration pricing of the second generation product since that will lead to loss of opportunity sales for the earlier generation product and overall loss of consumer surplus.

C. Theorem 5.3: *When the second generation product caters to a niche segment with very little market potential, it needs to be sold at a substantial premium.*

Proof: If the second-generation product is a niche product, the ordering cost per unit is high due to lower volumes. In that case, the revenue from the product may not be sufficient enough to cover all the costs.

Total Profit per unit quantity is given by $[P - C - \frac{n(A+A_1)}{\int \lambda(t)dt}]$

In a bid to make it profitable, the inventory holding costs need to be reduced by decreasing the frequency of inventory replenishment in the planning horizon, or the selling price of the product needs to be increased.

$$\lim_{\lambda(t)\to 0+} \left(n(A + A_1)/\int \lambda(t)dt\right) = M, where\ M\ is\ a\ very\ large\ number$$

Therefore for the profit to be retained at normal levels, the overall value of $P - C$ should be large enough, and therefore, the product needs to be sold at a premium.

This theorem helps the managers understand that they need to strike a fine balance between the profitability and cash velocity in the supply chain.

D. Theorem 5.4: *Penetration pricing is a better strategy than price skimming when price elasticity of demand is very high. On the other hand, if the price elasticity of the products is high, it makes better sense to follow the price skimming for the second generation.*

Proof: There are two conflicting effects of the technology price softening, the reduction in the price per unit, and the increase in the sales quantity due to negative price elasticity of demand. For the large values of the coefficient β, the positive impact of the price softening on the volumes outweighs the negative effect of the same on the drop in price per product. As a result of this, the overall revenues tend to increase. Also, an increase in volumes will tend to bring in replenishment efficiencies through more frequent replenishments. Figure5.D.1 exhibits the variation in revenue with the price elasticity of demand.

If the demand is highly elastic, a fall in price leads to a much larger % of the increase in the volumes as compared to the % drop in the selling price per unit. Thus, a fall in price leads to an increase in revenues, making the penetration pricing a more suitable proposition. On the other hand, for the larger price elasticity of demand, a fall in price results in a much smaller % of the increase in the volumes as compared to the % drop in the selling price per unit; and therefore, the price skimming would be a better strategy. This is also substantiated with the results of numerical illustration depicted in Table5.5 and Table5.6.

$$\frac{\partial[\lambda_1'(t)]}{\partial[\mathrm{Y}]} = \frac{\partial[\lambda_1'(t)]}{\partial[x_1(t)]}.\frac{\partial[x_1(t)]}{\partial[\mathrm{Y}]}$$

While $\frac{\partial[\lambda_1'(t)]}{\partial[x_1(t)]}$ is always positive, the $\frac{\partial[x_1(t)]}{\partial[\mathrm{Y}]}$ is also positive, implying that $\frac{\partial[\lambda_1'(t)]}{\partial[\mathrm{Y}]}$ is positive.

Also, it can be seen that $\frac{\partial[P_1(t)]}{\partial[\mathrm{Y}]}$ is always negative since $\frac{\partial[P_1(t)]}{\partial[\mathrm{Y}]} = -P_{01}\ln(t+1)$

$$\frac{\partial[TR_1(t)]}{\partial[\mathrm{Y}]} = \lambda_1'(t).\frac{\partial[P_1(t)]}{\partial[\mathrm{Y}]} + P_1(t).\ \frac{\partial[\lambda_1'(t)]}{\partial[\mathrm{Y}]}$$

Where $\frac{\partial[\lambda_1'(t)]}{\partial[\mathrm{Y}]}$ is proportional to β.

In the above expression, the first part is negative and the second part is positive.

For larger values of β, the second part of the above expression is bigger than the first part, making the overall $\frac{\partial[TR_1(t)]}{\partial[\mathrm{Y}]}$ positive.

$$\frac{\partial[TR_1(t)]}{\partial[\beta]} = P_1(t).\ \frac{\partial[\lambda_1'(t)]}{\partial[\beta]} > 0$$

Thus, with the increase in β, the total revenue increases.

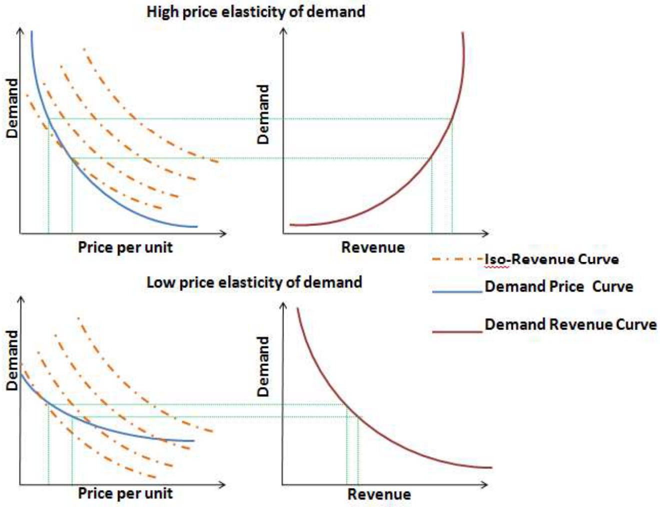

Figure5.D.1. Influence of price elasticity of demand on the diffusion pattern and revenues

This theorem helps the managers appreciate that penetration pricing would be more optimal for the more price elastic products, and the skim pricing would be more optimal for the inelastic products.