CHAPTER

7 Economic Order Quantity Model for Two Generation Consecutive Technology Products under Imprecise Business Environment

Due to the uncertainty in the hi-technology industry, it is not easy to have access to perfect information and that makes the traditional deterministic EOQ models irrelevant for this type of market. An effective way to overcome this challenge is the use of fuzzy set theory postulated by Zadeh (1965). Zadeh defined the fuzzy set and said that it is associated with a membership function that assigns a grade to each object. Also, at times, the trade credits can be imprecise or uncertain. Even though the credit terms are generally pre-agreed upon between the buyer and seller still there can be the cases where the actual credit terms are much different from the agreed-upon credit terms. It happens due to the working capital availability with the buyer and the relative bargaining power between the buyer and the seller. Similarly, one can see that the procurement cost also fluctuates with the changing equilibrium between the market forces of demand and supply. It is with this consideration the present chapter[9] has discussed an inventory model for the technology products under the uncertain trade credit terms. The model has been solved to illustrate its utility with the help of suitable numerical values of parameters.

One of the earliest works on inventory optimization using fuzzy set theory was by Park (1987) who considered that the cost parameters in the EOQ model are imprecise, and re-examined the traditional model from the perspective of fuzzy systems theory. Roy and Maiti (1997) developed a fuzzy EOQ model with warehouse capacity constraint under the price-dependent demand and quantity dependent setup cost. Yao and Lee (1999) expressed the fuzzy lot size as a normal trapezoidal fuzzy number. Ghomi and Rezaei (2003) took the consumption as a crisp number while considering the holding cost, ordering cost and selling price to be fuzzy trapezoidal numbers. Mahata and Goswami (2007) took the

[9]This chapter is based on the following research paper:
Nagpal, G. and Chanda, U. (2021). Inventory Modelling for technology generation products with demand influenced by innovation diffusion and uncertain trade credit terms under imprecise procurement costs" submitted in International Journal of Advanced Operations Management (Under Review)

demand rate, ordering cost, holding cost and purchase cost as fuzzy numbers and de-fuzzified the total cost using graded mean integration representation method. Mahata and Goswami (2010) developed the inventory model for imprecise inventory costs under trade credit financing for deteriorating items within the EPQ framework. Mahata (2011) proposed an EOQ model for gradual non-instantaneous replenishment under trade credits for deteriorating items. Shah et al. (2012), and Parvathi and Gajalakshmi (2013) also took ordering cost, holding cost and lot size as triangular fuzzy numbers and used graded mean integration representation method for de-fuzzification. Taleizadeh et al. (2013) used meta-heuristic algorithms like bees colony optimization to solve the fuzzy EOQ Model under pre-payment and quantity discounts.

Guchhait et al. (2014) considered the demand to be fuzzy and dependent upon the selling price and trade credit. Chakraborty et al. (2015) considered holding cost, procurement cost, ordering cost as well as the selling price in the imprecise environment, and solved the model using a genetic algorithm to derive the optimal credit period and length of procurement cycle under space and budget constraints. Yadav et al. (2015) developed optimal inventory policies for the retailer by taking the opportunity cost and interest rates as fuzzy triangular numbers. Fuzzy profit functions were defined and de-fuzzified using signed distance method. Chanda and Kumar (2017) developed an EOQ model under fuzzy selling price and advertising expense. Garai et al. (2019) used trapezoidal fuzzy numbers to define the time-varying inventory holding cost and the price dependent demand and developed a fully fuzzy inventory model, treating all the input parameters and decision variables as imprecise.

The next section summarizes the recent studies done on the inventory modelling of multiple items under imprecise conditions, with the use of fuzzy logic. Recently, Maiti (2020) formulated the fuzzy inventory model for multiple substitutable items being sold at multiple outlets managed by a single entity. Adak and Mahapatra (2020) developed an inventory model for items whose demand and deterioration are dependent upon reliability as well as time. For substitutable demand, there is plenty of existing literature on demand modelling and the inventory policies of such products (Kumar and Chanda (2017) and Chanda and Kumar (2019)). But when it comes to the products with successive technology generations under the fuzzy logic, the literature is very limited. Apart from the ones discussed above, the Table7.1 describes the review of many more studies done on inventory modelling of multiple items situations using the fuzzy logic.

Table7.1. Review of the studies on multi-item inventory modelling using fuzzy logic

Work	EOQ/ EPQ	Membershi p function	Technique used	Decision Variable	Innovation diffusion demand or technology generations?

Adak and Mahapatra (2020)	EOQ	Trapezoidal	FNLP	Replenishment quantity	No
Baykasoglu and Goken (2007)	EOQ	Triangular	Fuzzy Ranking Methods, Signed distance method	Replenishment quantity, sales price, discount on backlogged sales	No
Baykasoglu and Goken (2011)	EOQ	Triangular	Conventional Derivation Method	Lot size, shortage amount, Promotional outlays	No
Bjork (2012)	EPQ	Triangular	Conventional Derivation Method	Length of the production run	No
Chakraborty et al. (2013)	EOQ	Triangular	GA	Lot size	No
Chakraborty et al. (2015)	EOQ	Parabolic	GA	Lot size	No
Das and Maiti (2013)	EPQ	Triangular	Conventional Derivation Method	Planning Time horizon	No
Das et al. (2000)	EOQ	Linear	GP Method	selling period	No
Das et al. (2004)	EOQ	Linear	FNLP	time of placing the order	No
Garai et al. (2019)	EOQ	Trapezoidal	FNLP	Inventory carrying cost	No
Huang (2011)	EOQ	Triangular	Software	Lot size	No
Islam and Roy (2007)	EPQ	Triangular	GP technique	Production quantity	No
Jana et al. (2013)	EPQ	Parabolic	GRG, GA	production quantity, backlog %	No
Jana et al. (2014)	EOQ	Triangular	GA	Lot size, Re-order point, Item-specific area allocation	No
Maiti (2008)	EOQ	Parabolic Fuzzy number, Linear fuzzy number	GA	lead time, Replenishment quantity	No
Maiti and Maiti (2007)	EOQ	Triangular	Conventional Derivation Method	Sales price, promotional outlays	No
Maity (2011a)	EPQ	Trapezoidal	KKT Theorem, GRG	Production rate	No
Maity (2011b)	EPQ	Triangular	KKT Theorem, GRG	Production amount, cycle time	No
Maity and Maiti (2005)	EPQ	Trapezoidal	KKT Theorem, GRG	Length of the production run	No
Maity and Maiti (2007)	EPQ	Triangular	KKT Theorem	Production amount, service level	No

Maity and Maiti (2008)	EPQ	Interval	Conventional Derivation Method	Lot size	No
Mandal and Roy (2006)	EPQ	Exponential	GP Method	Production quantity, Maximum back-ordering percentage	No
Mandal and Roy (2006)	EPQ	Triangular	GP Method	Length of the production run	No
Mandal et al. (2005)	EPQ	Triangular	GPM	Production rate	No
Mandal et al. (2010)	EPQ	Triangular	KKT Theorem, GRG	Production amount	No
Mandal et al. (2011)	EPQ	Triangular	GA	Length of the production run	No
Mezei and Bjork (2015)	EPQ	Triangular	Signed distance method	Production quantity, reorder point	No
Mondal and Maiti (2002)	EOQ	Linear	FNLP, GA	Replenishment quantity, sales price, marketing expense	No
Mousavi et al. (2014)	EOQ	Trapezoidal	PSO	Lot size, lead time	No
Nia et al. (2014)	EPQ	Triangular	GA	Production amount, service level	No
Panda and Kar (2008)	EOQ	Triangular	GP technique	Lot size, shortage amount	No
Panda and Maiti (2009)	EPQ	Triangular	GP technique	Length of the production run	No
Panda et al. (2008)	EPQ	Triangular	GRG	Production lot size, optimal reliability	No
Pappis and Karacapilidis (1995)	EPQ	Triangular	Developing algorithm	Coefficients of the production function	No
Roy and Maiti (1997)	EOQ	Linear	GP technique	Lot size, Re-order point	No
Roy and Maiti (1998)	EOQ	Linear	FNLP	Lot size, Demand rate	No
Roy et al. (2008)	EOQ	Triangular	FNLP	Lot size, Re-order point	No
Saha et al. (2010)	EOQ	Trapezoidal	GRG	advertising frequency, inventory level, item-specific storage area allocation, item-specific storage area allocation	No
Taleizadeh et al. (2011)	EOQ	L-R	PSO, GA	Lot size	No
Taleizadeh et al. (2013)	EOQ	Triangular	PSO, GA	Lot size	No

Taleizadeh et al. (2013)	EOQ	L-R	FNLP	the order quantity for the buyer and manufacturer	No
Wang et al. (2013)	EOQ	Trapezoidal	FNLP	Lot size, lead time, re-order point, safety stock	No
Wee et al. (2009)	EOQ	Linear	FNLP	Lot size, lead time	No
Xie et al. (2006)	EOQ	Triangular	2-level SC coordination algorithm	Retailer's lot size	No
Xu and Liu (2008)	EOQ	Triangular	Conventional Derivation Method	Lot size, lead time, Re-order point	No
Xu and Zhao (2008)	EPQ	Fuzzy rough variable	FNLP	Production lot size, optimal reliability	No
Xu and Zhao (2010)	EPQ	Fuzzy rough variable	FNLP	optimal reliability	No
Yadavalli et al. (2005)	EOQ	Parabolic Fuzzy number, Linear fuzzy number	Langrange Multiplier technique with KKT Conditions	Replenishment quantity, sales price, marketing expense, shortage amount, service expense	No
Yao and Ouyang (2003)	EOQ	Triangular	FNLP	binary ordering variable	No

It is visible from the Table7.1 that the research work on the use of fuzzy logic for inventory modelling of multi-generational products is missing. In this chapter, a new EOQ model is proposed under imprecise business conditions using the fuzzy set theory.

7.1. Inventory Modeling

The demand modelling framework used in this chapter is similar to the one used in the multi-period model as discussed in Chapter 4. The notations and assumptions for the proposed model have been outlined in sections 7.1.1 and 7.1.2 respectively.

7.1.1. Notations for the Model

The notations behind the proposed model are stated below:

τ is the point of time at which the product of the second-generation product is launched in the market

PD_i is the period for which the trade credit is offered by the distribution channel to the customer of the ith generation product

$\phi_i(t)$ is the fraction of the normal market potential of the ith generation product that adopts it at the time $(t + \Delta t)$ when the trade credits are not offered

$f_i(t)$ is the fraction of the normal market potential of the ith generation product that adopts it at the time $(t + \Delta t)$ when trade credits are offered

$F_i(t)$ is the cumulative fraction of the normal market potential of the product of the ith generation that has adopted it till time instant t

p_i and q_i are the coefficient of innovation and imitation respectively of the product of the ith generation

$\lambda_1(t)$ and $\lambda'_1(t)$ demand rate at time $'t'$ of 1^{st} generation product for $t \leq \tau$ and $t \geq \tau$ respectively

$\lambda_2(t)$ demand rate at time $'t'$ of 2^{nd} generation product

$CD_1(t)$ is the cumulative demand of the first generation product until time t before the introduction of the second generation

$CD_2(t)$ is the cumulative demand of the second generation product till time t

M_i is the market potential of the ith generation product when the trade credits are not offered

I_r is the opportunity cost of credit offered to the consumer (in % terms) by the distribution channel

I_i is the inventory holding cost as % of the basic purchase cost for the product of ith generation

C_i be the basic purchase cost per unit for the product of the ith generation

pr_i is the selling price per unit for the product of the ith generation

η denotes the sequence of the planning time horizon

RC_i is the replenishment cost (i.e. ordering cost + inventory carrying cost) for the ith generation product

TCM_i are the contribution margin per unit (selling price net off basic purchase cost) for the product of the ith generation

Rev_i is the total revenue for the product of ith generation

OC_i is the total ordering cost for the product of ith generation

BC_i is the total basic purchase cost for the product of the ith generation

HC_i be the total inventory holding cost for the ith generation product

TP_i is the total profit for the product of ith generation

A is the fixed non-product-specific ordering cost per order, while A_i is the fixed product-specific ordering cost per order of the ith generation product irrespective of the order volumes

$\xi_{1\eta}$ is the quantity of the first generation product ordered in each lot in the time horizon η

$\xi_{2\eta}$ is the quantity of the second generation product ordered in each lot in the time horizon η

$\xi_{1\eta'}$ is the quantity of the first generation product ordered in each lot in the time horizon η' after the introduction of the advanced generation product

ζ is the length of the time horizon for which the inventory norms are fixed

7.1.2. Assumptions for the Model

The assumptions behind the proposed model are stated below:

- There are two generations of technology with the first generation being launched at time $t = 0$ and the second generation being launched at time $t = \tau$.
- The demand for the products follow the innovation diffusion and is also influenced by the trade credits and the degree of demand dependent discount on the selling price. The rate of demand is influenced by the innovation diffusion process and follow the assumptions as discussed in section 3.1.3 and can be given as follows:

  $\lambda_1(t) = M_1 f_1(t)\ for\ t < \tau$

  $\lambda_1'(t) = M_1 f_1(t) - M_1 f_1(t) F_2(t)\ for\ t > \tau$

  $\lambda_2(t) = M_2 f_2(t) + M_1 f_1(t) F_2(t)$
- The credit period and the procurement cost are imprecise
- The backlogging of the demand is not allowed, and nor are the shortages.
- The replenishment of both the generations of the product happens jointly

7.1.3. Modelling for Single generation scenario

If the replenishment of lot size in this planning time horizon is $\xi_{\eta 1}$, the number of shipments to be delivered in one such period is

$$j = \left(\frac{1}{\xi_{\eta 1}}\right)[CD_1(\eta.\zeta) - CD_1((\eta - 1).\zeta)] \tag{7.1}$$

The starting time of the *kth* replenishment cycle is given by

$$t_{k,\eta,\xi_{\eta 1}} = \left(\eta - 1 + \frac{(k-1)}{j}\right)\zeta \tag{7.2}$$

The ending time of the *kth* replenishment cycle is given by

$$t_{(k+1),\eta,\xi_{\eta 1}} = \left(\eta - 1 + \frac{k}{j}\right)\zeta \tag{7.3}$$

$$TCM_1 = Rev_1 - BC_1 \tag{7.4}$$

$$TP_1 = TCM_1 - RC_1 - CC_1 \tag{7.5}$$

Detailed expressions for the revenue and cost elements have been mentioned in Appendix 7A.

Fuzzy decision variables: By the model assumptions, basic purchase cost per unit (C_1) and the credit period (PD_1) are imprecise, and thus treated as fuzzy variables and represented as $\widetilde{C_1}$ and $\widetilde{PD}_1$. Therefore, the total Profit in the planning horizon TP_1 can be stated in the form of fuzzy sets. The fuzzy profit per unit time $\widetilde{TP}_1$ can be defined as follows:

$$\widetilde{TP}_1 = \widetilde{Rev_1} - \widetilde{BC}_1 - \widetilde{RC}_1 - \widetilde{CC}_1 \tag{7.6}$$

The Trapezoidal membership function has been deployed for fuzzy variables $\widetilde{C_1}$ and $\widetilde{PD}_1$. Let $\widetilde{C_1}$ is a trapezoidal fuzzy number, $\widetilde{C_1} = (c, a, b, d)$, where a, b, c, d are real numbers and $c \leq a \leq b \leq d$.

The Trapezoidal membership function $\mu_{\tilde{A}}(x)$ for the Fuzzy numbers can be described as follows and is graphically defined in Figure7.1.

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{x-c}{a-c}; c \le x \le a \\ w; a \le x \le b \\ w\frac{x-d}{b-c}; b \le x \le d \\ 0;\ otherwise \end{cases}$$

Where, $0 < w \le 1$

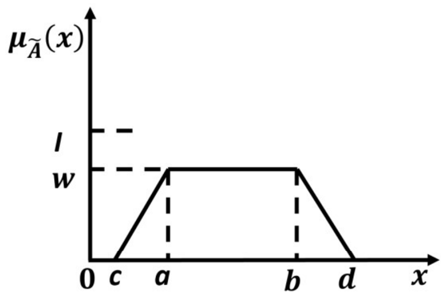

Figure7.1. Visual Representation of the fuzzy number

The fuzzy variables $\widetilde{C_1}$ and $\widetilde{PD_1}$ are defined as follows:

$\widetilde{C_1} = (C_{11}, C_{12}, C_{13}, C_{14});\quad (C_{11} \ge C_{12} \ge C_{13} \ge C_{14})$

$\widetilde{PD_1} = (PD_{11}, PD_{12}, PD_{13}, PD_{14});\ (PD_{11} \le PD_{12} \le PD_{13} \le PD_{14})$

The Chen's Function Principle (1985) has been used for the operations on fuzzy numbers, and to compute the fuzzy total profit. Thus, the membership function of $\widetilde{TP_1}$ can be defined as

$\widetilde{TP_1} = (TP_{11}, TP_{12}, TP_{13}, TP_{14})$.

Where $\widetilde{TP_{1\iota}} = \widetilde{TCM_{1\iota}} - \widetilde{RC_{1\iota}} - \widetilde{CC_{1\iota}}$ (7.7)

Using Median rule of defuzzification method, the total profit in the planning horizon *m*, i.e. "TP_{1m}" can be given as:

$$TP_{1m} = \left(\frac{1}{4}\right)[\sum_{\iota=1}^{4} \widetilde{TP_{1\iota}}]$$

The objective function is stated as under:

$$\boldsymbol{Max. TP_{1m}} = \left(\frac{1}{4}\right)[\sum_{\iota=1}^{4} \widetilde{TP_{1\iota}}] \qquad (7.8)$$

Subject to all $\widetilde{J_\iota}$ being positive integers

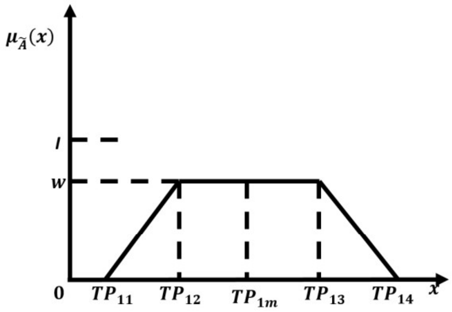

Figure7.2. De-fuzzifying the Total Profit Function by Median Rule

The Figure7.2 represents the median rule of defuzzification for the total profit function.

7.1.4. Inventory Model for the two generations scenario

After the second generation product has been launched, let the planning time horizon η' start at time $t = \tau + (\eta' - 1)(\zeta)$ and end at time $t = \tau + (\eta')(\zeta))$. This has been already illustrated in the Figure4.7.

If $\xi_{1\eta 1}{}'$ and $\xi_{2\eta 1}$ be the order quantities (under credit periods PD_{11} and PD_{21} of the products of the first generation and the second generation respectively, the number of orders to be placed for the products is:

$j_1' = [CD_1'(\tau + (\eta)(\zeta))\ ) - CD_1'(\tau + (\eta - 1)(\zeta)))\ ]/\xi_{1\eta}{}'$ (7.9)

$j_2 = [CD_2(\tau + (\eta)(\zeta))\ ) - CD_2(\tau + (\eta - 1)(\zeta)))\ ]/\xi_{2\eta}$ (7.10)

where $CD_1'(t)$ is the cumulative demand of the first generation product until time instant t after the second generation product is launched in the market

The starting time for the *kth* replenishment cycle for the first generation is $t_{(k-1),\xi_{1\eta}'} = \tau + (\eta - 1 + (k-1)/j_1')\zeta)$ and the ending time is $t_{k,\xi_{1\eta}'} = \tau + (\eta - 1 + k/j_1')\zeta)$

Similarly, the starting time for the *kth* replenishment cycle for the second generation is $t_{(k-1),\xi_{2\eta}} = \tau + (\eta - 1 + (k-1)/j_2)\zeta)$ and the ending time is $t_{k,\xi_{2\eta}} = \tau + (\eta - 1 + k/j_2)\zeta)$

When the logistics for both the generations of products are pooled to reap the operational synergies, it can be said that $t_{(k-1),\xi_{1\eta}'} = t_{(k-1),\xi_{2\eta}}$; and $t_{k,\xi_{1\eta}'} = t_{k,\xi_{2\eta}}$

The inventory economics can be stated as under:

Let RC', $CC', BC', OC', HC', Rev', TCM', TP'$ be the total replenishment cost (ordering + holding), credit cost, basic purchase cost, ordering cost, holding cost, Revenue, Total contribution margin and Total Profit respectively for both the generations of products combined in a planning time horizon post the launch of the second generation product

$TP' = Rev' - BC' - RC' - CC'$ (7.11)

Detailed expressions for the revenue and cost elements have been mentioned in Appendix 7B.

The optimization problem is the maximization of the total profit and can be stated as

Max. $TP' = Rev' - BC' - RC'$
Subject to the constraints
$j_1', \ j_2$ are positive integers
$j_1' = \ j_2$

Fuzzy decision variables: Since the basic purchase cost per unit C_2 and the credit period PD_2, being imprecise by model assumptions, can be represented fuzzy variables as $\widetilde{C_2}$ and $\widetilde{PD}_2$. Therefore, the fuzzy total Profit in the planning horizon TP' can be defined as follows:

$$\widetilde{TP}' = \widetilde{Rev'} - \widetilde{BC}' - \widetilde{RC}' - \widetilde{CC}' \quad (7.12)$$

Let $\widetilde{C_1}$, $\widetilde{C_2}$, $\widetilde{PD}_1$ and $\widetilde{PD}_2$ are the trapezoidal fuzzy numbers with the known relative order of magnitude. The new fuzzy variables introduced here $\widetilde{C_2}$ and $\widetilde{PD}_2$ are defined as follows:

$\widetilde{C_2} = (C_{21}, C_{22}, C_{23}, C_{24}); \quad (C_{21} \geq C_{22} \geq C_{23} \geq C_{24})$

$\widetilde{PD}_2 = (PD_{21}, PD_{22}, PD_{23}, PD_{24}); \ (PD_{21} \leq PD_{22} \leq PD_{23} \leq PD_{24})$

Using Chen's Function Principle (1985) (as in the case of single generations scenario earlier in the chapter), the membership function of $\widetilde{TP}'$ is defined as

$\widetilde{TP}' = (TP_1', TP_2', TP_3', TP_4')$.

Where $\widetilde{TP_i}' = \widetilde{TCM_i}' - \widetilde{RC_i}' - \widetilde{CC_i}'$ (7.13)

And $\widetilde{TCM_i}' = \widetilde{Rev_i}' - \widetilde{BC_i}'$

Using Median rule of defuzzification method, the total profit in the planning horizon η, i.e. "TP_{1m}" can be given as:

$$TP_m' = \left(\frac{1}{4}\right)[\sum_{i=1}^{4} \widetilde{TP_i'}]$$

The objective function is stated as under:

$$Max\ TP_m' = \left(\frac{1}{4}\right)[\sum_{i=1}^{4} \widetilde{TP_i'}] \quad (7.14)$$

Subject to all $\widetilde{j_{1i}}'$ and $\widetilde{j_{2i}}$ being positive integers

And $\widetilde{j_{1i}}' = \widetilde{j_{2i}}$

7.1.5. Theorems and Special Cases

Depending on the path of ordering costs per unit time, purchase costs per unit time, and inventory holding costs per unit time, the following theorems are being proposed:

Theorem 1: *The uncertain nature of the basic purchase cost results in an increase in the EOQ*

Proof: See Appendix 7.C.

Theorem 2: *With the incorporation of repeat purchase in the EOQ Model, the EOQ of the second generation product increases.*
Proof: See Appendix 7.D.

Theorem 3: *As the tendency of the customer to innovate and imitate increases, the EOQ of the technology products increases in the earlier phases of time and falls in the later phases.*
Proof: See Appendix 7.E.

In the next subsection, the solution procedure to find the optimal value of replenishment frequency is discussed. Since cost function as defined in equation (7.13) and (7.23) is highly non-linear, hence finding an analytical solution for the problem is difficult. The problem is solved numerically under given parameter values. Once the value of optimal replenishment frequency in each planning horizon is known, the replenishment quantities can also be worked out using the demand equations.

7.1.6. Solution procedure

The solution procedure to find the optimal solutions can be summarized in the following algorithm
Step 1: Enter the base values of all model parameters such as per-unit costs, coefficients of innovation and imitation, potential market sizes, time to the introduction of second-generation products, etc. for each generation independently.
Step 2: Compute the total profit at all possible values of replenishment frequency for the given value of τ, with the help of equations (7.8) and (7.14)
Step 3: Select the appropriate value of replenishment frequency where the first derivative of total profit is zero and the second derivative is negative.
Step 4: Finally, compute the value of replenishment lot sizes using the equation (7.1), (7.9) and (7.10) respectively.

7.1.7. Numerical illustration

The numerical illustrations have been performed by assigning certain values to the parameters as follows:
$A = INR\ 1000000,\ A_1 = INR\ 200000,\ A_2 = 200000,\ \ I_1 = .15,\ I_2 = .15,\ M_1 = 100000,\ M_2 = 200000,\ p_1 = .5,\ q_1 = 2.5,\ p_2 = .6,\ q_2 = 4.0,\ I_r = .18,\ pr_1 = INR\ 3500,\ pr_2 = INR\ 4500,\ C_{11} = INR\ 1500,\ C_{11} = INR\ 1600,\ C_{13} = INR\ 1700,\ C_{14} = INR\ 1800,\ C_{21} = INR\ 1700,\ C_{22} = INR\ 1800,\ C_{23} = INR\ 1900,\ C_{24} = INR\ 2000,\ \alpha = .5,\ \tau = 0.5,\ \varepsilon = 0.5,$
$PD_{11} = .5,\ PD_{12} = .4,\ PD_{13} = .3,\ PD_{14} = .2,\ PD_{21} = .5,\ PD_{22} = .4,\ PD_{23} = .3,\ PD_{24} = .2$
First, the model for the single generation scenario has been run considering the absence of the second generation product. The following results have been obtained as captured in Table7.2.

Table7.2. The optimal EOQ under single generation scenario

Period	EOQ_1 (‘000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = 0\ to\ 0.5$	435.7	1525	712
$t = .5\ to\ 1.0$	466.2	1631	767
$t = 1\ to\ 1.5$	211.7	741	343

As it can be observed, the optimal EOQ increases initially due to the rising sales volumes, and then starts declining. This is because of the higher demand initially caused by the innovation effect of the advertising, which leads to the faster attainment of the maturity stage in the product life cycle.

Then, the two generations model is run. On creating an optimization code in Matlab and running the same, the following results can be obtained:

Table7.3. The optimal EOQ post the launch of the second-generation product with repeat purchase of the second generation by the existing adopters of first-generation

Period	EOQ_1 (‘000 units)	EOQ_2 (‘000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = .5\ to\ 1$	81.4	838.7	16235	8867
$t = .1\ to\ 1.5$	5.6	459.3	6259	3420
$t = .15\ to\ 2$	0.8	128.0	1158	611

On comparing the Table7.2 and Table7.3 (as shown in Figure7.3 and Figure7.4), it was discovered that with the launch of the second-generation product, the EOQ of the first generation product falls due to cannibalization of a fraction of its demand by the second generation product.

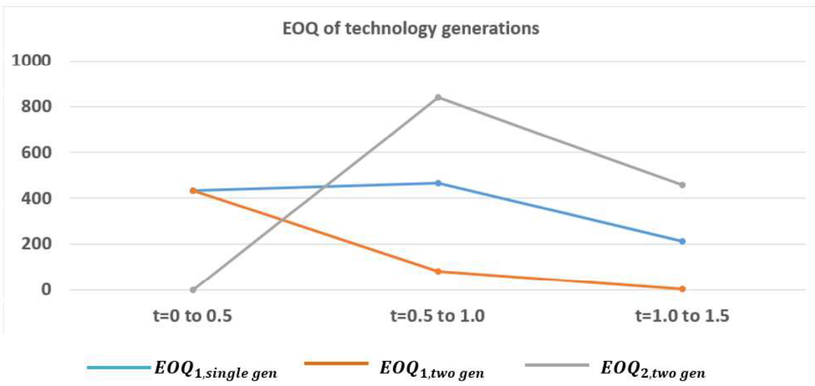

Figure7.3. The EOQ behaviour of the technology generations over the time horizons

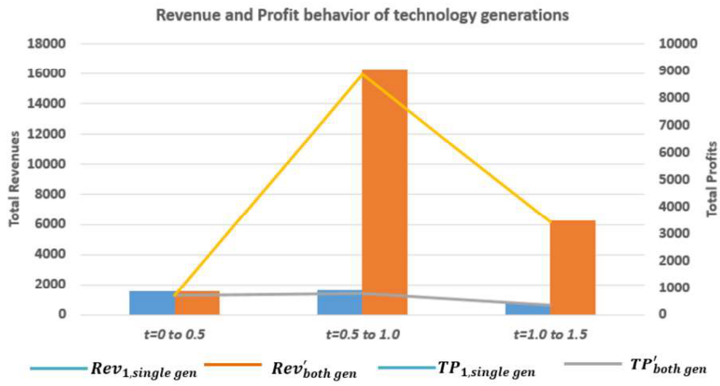

Figure7.4. The revenues and profit behaviour of the technology generations over the time horizons

On comparing the Table7.3 and Table7.4, it can be found that the EOQ of the second-generation product is much lesser when the repeat purchase by the existing adopters of the earlier generation product is not allowed. The reduction in demand induced by this leads to the fall in EOQ.

Table7.4. The optimal EOQ post the launch of the second generation product without repeat purchase of the second generation by the existing adopters of the first generation

Period	EOQ_1 (‘000 units)	EOQ_2 (‘000 units)	Rev' (Mn INR)	TP' (Mn INR)
$t = .5\ to\ 1$	81.4	421.3	8723	4442
$t = .1\ to\ 1.5$	5.6	435.3	5935	3229

Now, the sensitivity analysis of the model is performed with the change in the innovation and imitation coefficients. The results are displayed in Table7.5.

Table7.5. The optimal EOQ with the increase in the innovation and imitation coefficients by 20%, with and without allowing the repeat purchase

Repeat purchase?	EOQ_1 (‘000 units)	EOQ_2 (‘000 units)	Rev' (Mn INR)	TP' (Mn INR)
Yes	78.6	1017.3	19410	10629
No	78.6	498.5	10073	5458

On comparing the Table7.5 with Table7.3, It can be observed that the optimal EOQ of the second generation product rises with the increase in innovation and imitation effect with the faster diffusion caused by an increase in coefficients.

7.2. Major academic and business implications of the proposed EOQ models

This chapter has many important implications for managerial decision making. First and foremost, it lays down how the imprecise situations of the real world can be converted into decision-making problems using the fuzzy logic. It becomes very important for the managers of today's dynamic and unpredictable business situations to handle the uncertainty by quantifying the same, for which the fuzzy set theory proves to be of great help. Second, it is observed that there is a significant jump in the overall volumes with the launch of the advanced generation of a product, contingent to the additional utility brought by the newer product to the consumer in terms of function and features. The revenue and the total profit increase considerably with the launch of the second generation product since that has higher market potential and faster diffusion as compared to that of the first generation by having more features. This leads to an increase in the overall market, and hence, better inventory replenishment efficiencies. The industry practitioners and product planners need to leverage this learning and keep coming up with innovative features in the products to improve the overall volumes and operational productivity.

Third, the EOQ of the technology products is higher in the initial periods and then falls in the later periods. This is because the volumes in case of technology products are very high in the initial time zones, which lead to higher EOQ. Fourth, the EOQ for the second generation product is higher than that of the earlier generation products. This is because the second-generation product is a better version of the earlier generation product, and hence, enjoys higher demand rate, leading to increase the EOQ. Fifth, the managers can make out how to incorporate the influence of the repeat purchase tendency of the consumer into the inventory optimization policies and decisions. Fourth, this work can also help the sellers of high technology products, who have a buyer with a history of imprecise payment timings, in determining the length of the credit period that they can offer to the buyer. Finally, when the procurement costs are uncertain in the dynamic demand-supply equilibrium, this model will help the inventory practitioners to make a sound business decision and strike the right balance between the pessimistic situation and the optimistic situation.

This chapter has used the fuzzy set theory to model and optimize the inventory decisions under imprecise procurement cost and trade credits. There are some important implications of the chapter for the managers and inventory practitioners of technology products, which have been discussed in the chapter. There are a few extensions in which further work can happen in the future. First, the full or partial backlogging of the demand and the product shortages can be allowed. This is because, at times, it may be more economical for the seller to make the buyer wait for some time and get the product later. Second, the demand pattern can be considered to be stochastic, and the safety stock provisioning for a certain service level can be done in the model. This is because it is not economical to meet 100% demand in uncertain demand conditions. Third, the environmental cost of the e-waste produced due to repeat purchase of electronic products can be quantified and considered in the model. This becomes very important in today's context of increasing thrust on business sustainability. Fourth, future research can consider the influence of the financial derivatives that can be used to hedge the increase in procurement costs or credit rate of interest. Fifth, the influence of the receivables trading in making the credit period decisions can be considered as an extension of this work, since the trading of receivables is a very common practice in the high growth organizations that have plenty of lucrative investment opportunities and would like to rotate their funds faster.

In the past five chapters including this chapter, the different models for inventory optimization of multi-generational products under different scenarios have been developed and proposed with the numerical illustrations and implications for managers and academicians. It is also important to summarise this work, elaborate its achievements in terms of the insights generated from this work, its limitations and the future possible extensions of this research work. All this has been discussed in the upcoming chapter of the book.

Appendix 7

A. Cost Equations for Single generation case

The inventory carrying cost in the *kth* replenishment cycle in the planning time horizon η are given by

$$HC_1 = I_1 C_1 \int_{t_{k,\eta,\xi_{\eta 1}}}^{t_{(k+1),\eta,\xi_{\eta 1}}} I_1(u)du = I_1 C_1 \int_{t=t_{k,\eta,\xi_{\eta 1}}}^{t=t_{(k+1),\eta,\xi_{\eta 1}}} [\int_{u=t}^{u=t_{(k+1),\eta,\xi_{\eta 1}}} \lambda_1(u)du]\ dt \quad (7.A.1)$$

The trade-credit cost in the *kth* ordering cycle of the planning time horizon η is given as

$$CC_1 = I_r . C_1 . PD_1 . \int_{t_{k,\eta,\xi_{\eta 1}}}^{t_{(k+1),\eta,\xi_{\eta 1}}} \lambda_1(t)dt \quad (7.A.2)$$

The replenishment costs (sum of ordering and inventory carrying costs) for the η time horizon are given as

$$RC_1 = j(A + A_1) + \sum_{k=1}^{k=j} HC_1 \quad (7.A.3)$$

The revenue is given by

$$Rev_1 = pr_1 . \sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta 1}}}^{t=t_{(k+1),\eta,\xi_{\eta 1}}} \lambda_1(t).dt \quad (7.A.4)$$

Basic Purchase cost in the planning time horizon η is given as

$$BC_1 = C_1 . \sum_{k=1}^{k=j} \int_{t=t_{k,\eta,\xi_{\eta 1}}}^{t=t_{(k+1),\eta,\xi_{\eta 1}}} \lambda_1(t).dt \quad (7.A.5)$$

$$\widetilde{TCM_{1\iota}} = \widetilde{Rev_{1\iota}} - \widetilde{BC_{1\iota}}$$

$$\widetilde{Rev_{1\iota}} = pr_1 . \sum_{k=1}^{k=\widetilde{j_\iota}} \int_{t=\widetilde{t_{k,\eta,\iota}}}^{t=t_{\widetilde{(k+1),\eta,\xi_{\eta\iota}}}} \widetilde{\lambda_{1\iota}}(t).dt$$

$$\widetilde{BC_{1\iota}} = \widetilde{C_{1\iota}} . \sum_{k=1}^{k=\widetilde{j_\iota}} \int_{t=t_{\widetilde{k,\eta,\xi_{\eta 1\iota}}}}^{t=t_{\widetilde{(k+1),\eta,\xi_{\eta 1\iota}}}} \widetilde{\lambda_{1\iota}}(t).dt$$

$$\widetilde{RC_{1\iota}} = \widetilde{J_\iota}(A + A_1) + \sum_{k=1}^{k=j_\iota} \widetilde{HC_{1\iota}}$$

$$\widetilde{CC_{1\iota}} = I_r.\widetilde{C_{1\iota}}.\widetilde{PD_{1\iota}}.\int_{t=t_{k,\eta,\xi_{\eta 1 i}}}^{t=t_{(k+1),\eta,\xi_{\eta 1 i}}} \widetilde{\lambda_{1\iota}}(t)dt$$

$$\widetilde{HC_{1\iota}} = I_1\widetilde{C_{1\iota}}\int_{t=\widetilde{t_{k,\eta,\xi_{\eta 1\iota}}}}^{t=\widetilde{t_{(k+1),\eta,\xi_{\eta 1\iota}}}} [\int_{u=t}^{u=\widetilde{t_{(k+1),\eta,\xi_{\eta 1\iota}}}} \widetilde{\lambda_{1\iota}}(u)du\,]\,dt$$

$$\widetilde{t_{k,\eta,\xi_{\eta 1\iota}}} = \left(\eta - 1 + \frac{k-1}{\widetilde{J_\iota}}\right)\zeta$$

$$\widetilde{t_{(k+1),\eta,\xi_{\eta\iota}}} = \left(\eta - 1 + \frac{k}{\widetilde{J_\iota}}\right)\zeta$$

$$\widetilde{J_\iota} = \left(\frac{1}{\xi_{\eta 1 i}}\right)[\widetilde{CD_{1\iota}}(\eta.\zeta) - \widetilde{CD_{1\iota}}((\eta-1).\zeta)]$$

$$\widetilde{\lambda_{1\iota}}(t) = M_1\widetilde{f_{1\iota}}(t) \quad for\ t \le \tau$$

$$\widetilde{\lambda_{1\iota}'}(t) = M_1\widetilde{f_{1\iota}}(t) - M_1\widetilde{f_{1\iota}}(t)\widetilde{F_{2\iota}}(t-\tau) \quad for\ t \ge \tau$$

$$\widetilde{\lambda_{2\iota}}(t) = M_2\widetilde{f_{2\iota}}(t) + M_1\widetilde{f_{1\iota}}(t)\widetilde{F_{2\iota}}(t-\tau) + M_1\widetilde{F_{1\iota}}(t)\widetilde{f_{2\iota}}(t-\tau) \quad for\ t \ge \tau$$

$$\widetilde{f_{1\iota}}(t) = \frac{\widetilde{b_{1\iota}^2}\exp(-\widetilde{b_{1\iota}}t)[\exp(\alpha.\widetilde{PD_{1\iota}})]^2}{\left[p_1\{1+\widetilde{a_{1\iota}}.\exp(-\widetilde{b_{1\iota}}t)\}^2\right]}$$

$$\widetilde{F_{1\iota}}(t) = \frac{[1-\exp(-\widetilde{b_{1\iota}}t)]\exp(\alpha.\widetilde{PD_{1\iota}})}{[1+\widetilde{a_{1\iota}}.\exp(-\widetilde{b_{1\iota}}t)]}$$

$$\widetilde{f_{2\iota}}(t) = \frac{\widetilde{b_{2\iota}^2}\exp(-\widetilde{b_{2\iota}}(t-\tau))[\exp(\alpha.\widetilde{PD_{2\iota}})]^2}{\left[p_2\{1+\widetilde{a_{2\iota}}.\exp(-\widetilde{b_{2\iota}}(t-\tau))\}^2\right]}$$

$$\widetilde{F_{2\iota}}(t) = \frac{[1-\exp(-\widetilde{b_{2\iota}}(t-\tau))]\exp(\alpha.\widetilde{PD_{2\iota}})}{[1+\widetilde{a_{2\iota}}.\exp(-\widetilde{b_{2\iota}}(t-\tau))]}$$

$$\widetilde{b_{1\iota}} = p_1/\exp(\alpha.\widetilde{PD_{1\iota}}) + q_1$$

$$\widetilde{a_{1\iota}} = q_1\exp(\alpha.\widetilde{PD_{1\iota}})/p_1$$

B. Cost Equations for Two generations Case

The revenue, basic purchase cost, replenishment costs, and credit costs for both the product generations can be stated as

$$\widetilde{Rev_\iota}' = pr_1.\sum_{k=1}^{k=\widetilde{J_{1\iota}}'}\int_{t=\widetilde{t_{k,\eta,\xi_{1\eta\iota}}}'}^{t=\widetilde{t_{(k+1),\eta,\xi_{1\eta\iota}}}'} \widetilde{\lambda_{1\iota}}'(t).dt + pr_2.\sum_{k=1}^{k=\widetilde{J_{2\iota}}}\int_{t=\widetilde{t_{k,\eta,\xi_{2\eta\iota}}}}^{t=\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}}} \widetilde{\lambda_{2\iota}}(t).dt \qquad (7.B.1)$$

$$\widetilde{BC_\iota}' = \widetilde{C_{1\iota}}.\sum_{k=1}^{k=\widetilde{J_{1\iota}}'}\int_{t=\widetilde{t_{k,\eta,\xi_{1\eta\iota}}}'}^{t=\widetilde{t_{(k+1),\eta,\xi_{1\eta\iota}}}'} \widetilde{\lambda_{1\iota}}'(t).dt + \widetilde{C_{2\iota}}.\sum_{k=1}^{k=\widetilde{J_{2\iota}}}\int_{t=\widetilde{t_{k,\eta,\xi_{2\eta\iota}}}}^{t=\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}}} \widetilde{\lambda_{2\iota}}(t).dt \qquad (7.B.2)$$

$$\widetilde{RC_\iota}' = \widetilde{J_{1\iota}}'(A + A_1) + \sum_{k=1}^{k=\widetilde{J_{1\iota}}'}\widetilde{HC_\iota}' \qquad (7.B.3)$$

$$\widetilde{CC_\iota}' = I_r.\widetilde{C_{1\iota}}.\widetilde{PD_{1\iota}}.\int_{t=t_{k,\eta,\xi_{1\eta i}}'}^{t=t_{(k+1),\eta,\xi_{1\eta i}}'} \widetilde{\lambda_{1\iota}}'(t)dt + I_r.\widetilde{C_{2\iota}}.\widetilde{PD_{2\iota}}.\int_{t=\widetilde{t_{k,\eta,\xi_{2\eta\iota}}}'}^{t=\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}}'} \widetilde{\lambda_{2\iota}}(t).dt \qquad (7.B.4)$$

$$\widetilde{HC_\iota}' = I_1\widetilde{C_{1\iota}}\int_{t=\widetilde{t_{k,\eta,\xi_{1\eta\iota}}}'}^{t=\widetilde{t_{(k+1),\eta,\xi_{1\eta\iota}}}'}[\int_{u=t}^{u=\widetilde{t_{(k+1),\eta,\xi_{1\eta\iota}}}'}\widetilde{\lambda_{1\iota}}'(u)du]dt \; +$$

$$I_2C_{2\iota}\int_{t=\widetilde{t_{k,\eta,\xi_{2\eta\iota}}}'}^{t=\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}}'}[\int_{u=t}^{u=\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}}'}\widetilde{\lambda_{2\iota}}(u)du]\,dt \qquad (7.B.5)$$

$$\widetilde{t_{k,\eta,\xi_{1\eta\iota}}}' = \left(\eta - 1 + \frac{k-1}{\widetilde{J_\iota}'}\right)\zeta$$

$$\widetilde{t_{(k+1),\eta,\xi_{1\eta\iota}}}' = \left(\eta - 1 + \frac{k}{\widetilde{J_\iota}'}\right)\zeta$$

$$\widetilde{t_{k,\eta,\xi_{2\eta\iota}}} = \left(\eta - 1 + \frac{k-1}{\widetilde{J_2}}\right)\zeta$$

$$\widetilde{t_{(k+1),\eta,\xi_{2\eta\iota}}} = \left(\eta - 1 + \frac{k}{\widetilde{J_2}}\right)\zeta$$

$$\widetilde{J_{\iota\iota}}' = \left(\frac{1}{\xi_{1\eta i}'}\right)[\widetilde{CD_{1\iota}}'(\eta.\zeta) - \widetilde{CD_{1\iota}}'((\eta - 1).\zeta)]$$

$$\widetilde{J_{2\iota}} = \left(\frac{1}{\xi_{2\eta i}}\right)[\widetilde{CD_{2\iota}}(\eta.\zeta) - \widetilde{CD_{2\iota}}((\eta - 1).\zeta)]$$

$$\widetilde{\lambda_{1\iota}^{\prime}}(t) = M_1\widetilde{f_{1\iota}}(t) - M_1\widetilde{f_{1\iota}}(t)\widetilde{F_{2\iota}}(t-\tau) \;\; for\; t \geq \tau$$

$$\widetilde{\lambda_{2\iota}}(t) = M_2\widetilde{f_{2\iota}}(t) + M_1\widetilde{f_{1\iota}}(t)\widetilde{F_{2\iota}}(t-\tau) + M_1\widetilde{F_{1\iota}}(t)\widetilde{f_{2\iota}}(t-\tau) \;\; for\; t \geq \tau$$

C. Theorem 7.1: *The uncertain nature of the basic purchase cost results in an increase in the EOQ.*

Proof: For a given value of trade credit in any planning horizon, the demand is constant. Let it be called as *D*. So, the EOQ can be stated as $EOQ = \sqrt{(\frac{2(O+O_1)D}{H})} = \sqrt{(\frac{2(O+O_1)D}{(I_1\zeta+I_r.PD_1)\,C_1})}$

$\frac{\partial(EOQ_m)}{\partial C_1} = -(.5)\sqrt{\left(\frac{2(O+O_1)D}{(I_1\zeta+I_r.PD_1\zeta)}\right)}\,(C_1)^{-1.5}$ which is always negative

$\frac{\partial^2\,(EOQ_m)}{\partial^2 C_1} = (.75)\sqrt{\left(\frac{2(O+O_1)D}{(I_1\zeta+I_r.PD_1)}\right)}\,(C_1)^{-2.5}$ which is always positive

That means the fall in EOQ with an increase in basic purchase cost by x% is lower than the rise in EOQ by a decrease in basic purchase cost by x%. Thus, the variability of the basic purchase cost results in a fall in the EOQ.

D. Theorem 7.2: *With the incorporation of repeat purchase in the EOQ Model, the EOQ of the second generation product increases.*

Proof: The repeat purchase leads to increase in the demand rate of the second generation product, ensuring faster depletion of inventories for a given lot size and therefore, lesser holding costs in comparison to the ordering costs. This phenomenon is illustrated in Figure7.B.1.

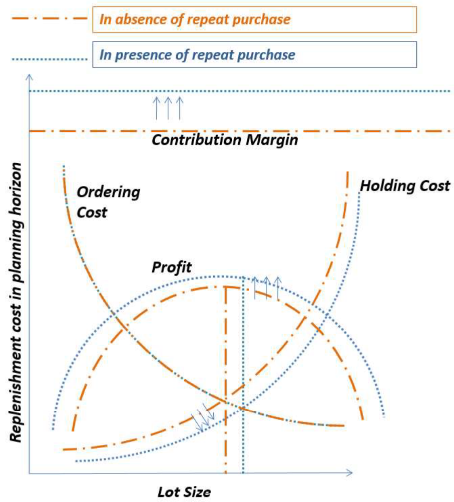

Figure7.D.1. Influence of the repeat purchase (of the advanced product by the existing adopters of older product) on the economic lot size of the advanced product

This tends to increase the EOQ for the second generation product, and shifting of the point of maxima in the total cost curve to the right.

E. Theorem 7.3: *As the tendency of the customer to innovate and imitate increases, the EOQ of the technology products increases in the earlier phases of time and falls in the later phases.*

Proof: With the increase in innovation and imitation, the adoption rate increases further, leading to higher demand in the initial phase and lower demand in the later phase of time. This results in faster (and lower) depletion of inventories in the earlier (and later) phase of time, leading to an increase (and decrease) in EOQ in the initial (and later) period. This is illustrated in Figure7.C.1.

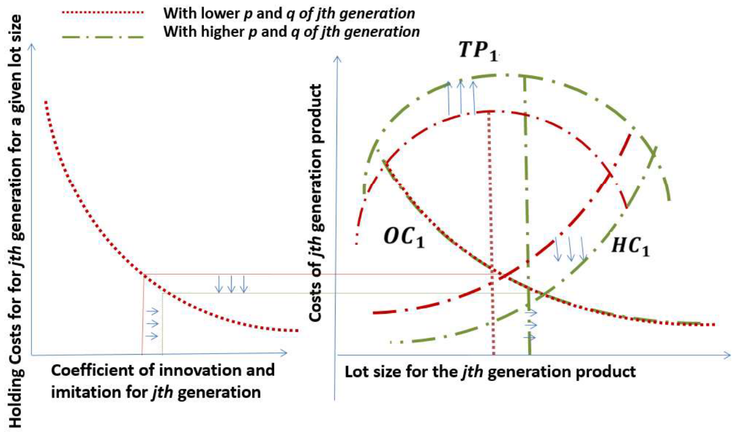

Figure7.E.1. Influence of innovating and imitating tendency of the consumer on EOQ for technology products