Chapter 3 Optimal Investment Strategy in Equity Markets using Fuzzy MCDM's

3.1 Introduction

The stock market is well known as the equity market is considered as an interface between buyers and sellers to trade-off. In the equity market, investors affirm ownership within a company. It is considered as one of the most vital components of a free-market economy. For any major economy, the stock or equity market is a crucial part of measuring the country's gross domestic product nominally or through purchasing power parity. It plays an influential role in the growth of the economy and hence in HDI (Human development index), which keeps government, nationalized banks and industries busy observing the trends of markets closely. The equity market is important from industry and investor's sentiments. The stock market is a collection of companies where capital can be traded by an investor in the form of shares and becomes owner to a certain portion of the company and also assure the warrant of settlement. The primary functioning of the stock market is to collect funds and issue shares to investors and act as a common platform for buyers and sellers. The global total market capitalization is about USD 90 trillion in 2019, end out of which the United States and China possess major share and with a large gap Japan with about 6% share comes second closely followed by United Kingdom with 5% (Ding and Zhong 2020). Capital investments in the stock market are associated with greater amounts of risk. In case of financial difficulties, the price of the stock may slash down drastically and it may also reach its zenith in case of market growth. This uncertainty associated with stocks made it to be renowned as a volatile market. Therefore, to invest in a potentially profitable stock, there is a great need to understand the trends in the rise and fall of stocks corresponding to a particular sector or an industry to invest profitably. Because of this, it is planned to identify the best sectors in BSE SENSEX and spend accordingly for strategic management. Nine crucial parameters such as Return on assets (ROA), Earnings per share (EPS), Price to sales ratio (P/S), Price to cash flow ratio (P/CF), Dividend yield (DY), Price to earn ratio (P/E), Book value per share (BVPS), Return on equity (ROE), Price to book ratio (P/B) which impact investing strategies are considered and are classified into cost and effect groups using the Fuzzy DEMATEL MCDM technique. Subsequently, Fuzzy AHP and Fuzzy TOPSIS are used to evaluate and study the dominance of various sectors in BSE SENSEX, which includes Automotive, Finance, Information

Technology, Oil, Pharmaceuticals and Power. Four cause criteria's namely ROE, BVPS, PE ratio, PB ratio is considered, which are obtained after the analysis of Fuzzy DEMATEL to study the dominance of each sector. The results obtained will help in prioritizing the sectors for future investments. In addition, the present chapter analyzes each sector by scrutinizing the normality using the Anderson- Darling normality test. Anderson- Darling normality test, along with other statistical measures, are used to scrutinize the behavior of a stock market. The mean, standard deviation, variance, skewness, kurtosis along the 1st quartile, median and 3rd quartile are well known (Cleary 2006). These measures unveil the probable risk in the equity market. Further, A-squared and p-value are evaluated by using the Anderson- Darling normality test (Pettitt 1977; Hadi *et al.* 2009). All the measures defined below are evaluated based on the performance of sectors.

Mean

Mean can be used as one of the company's performance indicators and is evaluated over a period of time. The mean can be applied to any of the criteria like stock price, ROE, BVP, PE ratio, PB ratio, etc. and is calculated by taking their average.

Standard deviation

Standard deviation is one of the most common measures used by many traders to evaluate possible risks in case if the share is estimated using mean and plays a vital role in calculating volatility in the market. It is derived by applying to the investment rate generally taken annually. Standard deviation is proportionate to the variance of stock value and mean. The normality of data sets is inversely proportional to the value of standard deviation. The square of standard deviation is defined to be variance.

Skewness

Most economists consider skewness as a significant measure to calculate the risk of the market. This indeed is used to calculate the asymmetricity of the equity market from normally distributed data sets. The majority of data sets in any of the stock markets possess negative or positive skew compared to rarely normally distributed markets where zero skewness is observed. The left-biased skew indicates negative skew and right indicates positive.

Kurtosis

Kurtosis calculates the relation between the tails of distribution to the overall shape of the distribution. Kurtosis is used to derive the volatility of the market. Similar to skewness, kurtosis can be classified by negative or positive kurtosis in share market. Low or negative kurtosis probably unveils minimal peak and high or positive kurtosis unveils probable elevated peak as compared to the normally distributed market. Further, the categorization of kurtosis explores the similarity of any distribution with the normal distribution.

Anderson Darling Normality Test

This test estimates whether or not the data follow a particular trend. Further, it analyzes different volatile markets. A-squared defines the volatility, the larger the value more is the volatility. The p-value in this test concludes the data sets follow a specific distribution. The less the p-value, the fewer data points follow a certain distribution generally (0.05 or 0.1) (Sánchez-Espigares *et al.* 2019).

First quartile (Q_1)

This quartile in the equity market is defined as the value or asset at the boundary under 25% of the data sets.

Third quartile (Q_3)

The third quartile can be defined as the value or asset at the boundary exceeding 75% of the data sets in the equity market.

Median

This is one of the quartiles which is intermediate to the first and third quartiles.

3.2 BSE SENSEX

There are 16 stock exchanges with a market capitalization of more than 1 trillion USD and among them, BSE and NSE are from India. Many stock exchanges are located in India, but two are dominant due to their significant contribution to the Indian economy, which are BSE (Bombay stock exchange) and NSE (National stock exchange) located at Mumbai. The present study emphasizes on BSE due to multiple reasons. The establishment of BSE dates back to the year 1875 and is the oldest among all the stock exchanges in Asia. By market capitalization, BSE is 11th largest exchange around the globe and with 6 microseconds median trade speed, it claims to be the world's fastest stock exchange (Poshakwale 1996; Adholiya and Chouhan 2019). There are five indices in BSE which are BSE SENSEX, S&P BSE Smallcap, S&P BSE Midcap, S&P BSE Large Cap and BSE 500. Among them, the most prominent by capitalization is BSE SENSEX. Hence sectors in BSE SENSEX are given prominence.

Thirty prominent companies listed in the BSE SENSEX (BSE30) are determined on free-float capitalization. BSE SENSEX base value has defaulted at 100. The Index attained its historic high of 42,273.87 in February 2020 and marked its least value of 113.28 in December 1979, which can be observed from Figure 3.1.

Figure 3.1. Historical SENSEX from Jan. 1986 - May. 2020

The number of investors has increased in the past decade significantly and reaches to 32.3 million investors in India (Adholiya and Chouhan 2019). These act as auction between seller and buyer continuously complying transaction at a location. Protecting the investor, determining a realistic price, financing industry, creating new ventures, attracting foreign investments and delivering financial needs to the government are some of the objectives of stock exchanges. The growth rate of BSE SENSEX is shown in Table 3.1.

Table 3.1: Growth rate of BSE SENSEX over 2012-2018

Year	**Growth Rate (%)**
2012-13	7.58
2013-14	11.63
2014-15	39.02
2015-16	-19.01
2016-17	21.89
2017-18	17.46

Due to mammoth volatility in the equity market, investing in the right company is a challenging task. An average investor ends up with losses by trading in the market. Long term investment of stocks in well-diversified index funds like BSE SENSEX and NIFTY50 surpassed returns gained from debt funds for decades (Modigliani and Miller 1958). Investor’s portfolio must be diversified and hence choosing the right company from the sectors should be one's priority.

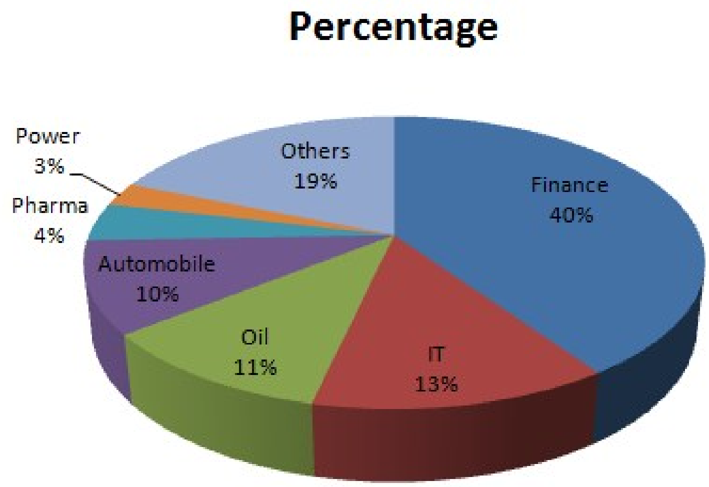

Figure 3.2. Sector wise breakup of BSE SENSEX

Hence, understanding the hierarchy of sectors and capital investment in the right proportions is a must for a healthy diversified portfolio. There is immense potential growth investing in the stock market, especially in developing countries such as India and investing in the appropriate sector is a healthy way out. The current chapter's objective is to a trade-off between profits and reduces the risk. Six sectors are appraised in the present study by their market capitalization and impact to Indian Economy. Figure 3.2 shows the sector wise breakup of SENSEX.

It is evident from the statistics of the past decade that the volume of investments and the number of investors in BSE has increased tremendously. This is due to the high returns on investments of around 13 percent annually. Since the foundation of BSE SENSEX, there is a large variation in the behaviour of stock markets with respect to normal distribution. By using the Anderson Darling normality test, high fluctuations can be observed in the market interpreted by p-value and A-squared. In subsequent subsections, the values of the above mentioned parameters are evaluated for considered sectors in BSE SENSEX and all performance indicators considered to estimate BSE SENSEX sectors are shown in Table 3.3.

3.2.1 Finance Sector (S_1)

Among all the sectors in BSE SENSEX, financial services contribute a major share. Since last five years, tremendous growth has been taking place in the Indian financial sector of current firms as well as in new firms entering the market.

This sector comprises many companies that can be categorized in various entities like mutual funds, insurance companies, commercial and private banks, pension funds, etc. Significant share is held by commercial banks with more than 63 percent of financial sector. Many economists correlate overall behavior of economy with the performance of financial industry.

Table 3.2: BSE SENSEX companies along with sector classification.

BSE Companies	Sector classification
Hero MotoCorp Ltd.	Automobiles
Bajaj Auto Ltd.	Automobiles
Mahindra & Mahindra Ltd.	Automobiles
Maruti Suzuki India Ltd.	Automobiles
Tata Motors Ltd.	Automobiles
Tata Motors – DVR Ordinary	Finance
HDFC Bank Ltd.	Finance
Kotak Mahindra Bank Ltd.	Finance
ICICI Bank Ltd.	Finance
Axis Bank Ltd.	Finance
State Bank Of India	Finance
Housing Development Finance Corporation Ltd.	Finance
ITC Ltd.	Cigarettes/Tobacco
Larsen & Toubro Ltd.	Engineering
Hindustan Unilever Ltd.	Household & Personal Products
Infosys Ltd.	IT
Wipro Ltd.	IT
Tata Consultancy Services Ltd.	IT
Asian Paints Ltd.	Paints
Cipla	Pharma
Dr. Reddys Laboratories Ltd.	Pharma
Lupin	Pharma
Sun Pharmaceutical Industries Ltd.	Pharma
Adani Ports and Special Economic Zone Ltd.	Port
NTPC Ltd.	Power
Power Grid Corporation Of India Ltd.	Power
Coal India Ltd.	Power
Oil & Natural Gas Corporation Ltd.	Oil
Reliance Industries Ltd.	Oil
Tata Steel Ltd.	Steel & Iron Products

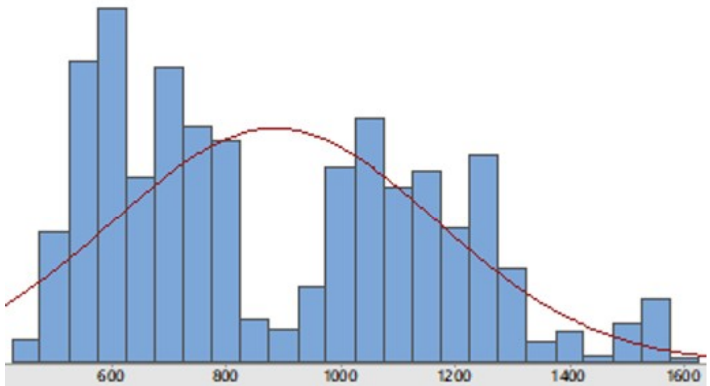

Figure 3.3. Report on finance sector

Analyzing the data extracted in the past 5 years, it is observed that 25% of the period the stock values is observed to be less than 627.42 and for 75% of the period, the price is found to be less than 1115.23. The median is found to be 806.23 which is 28.5% more than Q_1 and 38.33% less than Q_3 which indicates that the median is more biased towards the first quartile, which can be seen from Figure 3.3.

3.2.2 Automobile Sector (S_2)

The automobile sector deals with the economic and financial performances of manufactures related to automobiles, auto maintenance, dealerships, etc. The influence of the sector will also affect other industries like food, oil, transportation, etc. From the past five year's statistics shown in Table 3.3, it is evident that the growth of this sector is significant.

This industry is further expected to grow in the near future due to the increase of automobiles around the world and especially in emerging markets like India and China. The vehicle number is projected to grow to 2.9 billion by the end of 2050. (Williams *et al.* 2012). The report of the automobile sector in the past five years can be depicted in Figure 3.4.

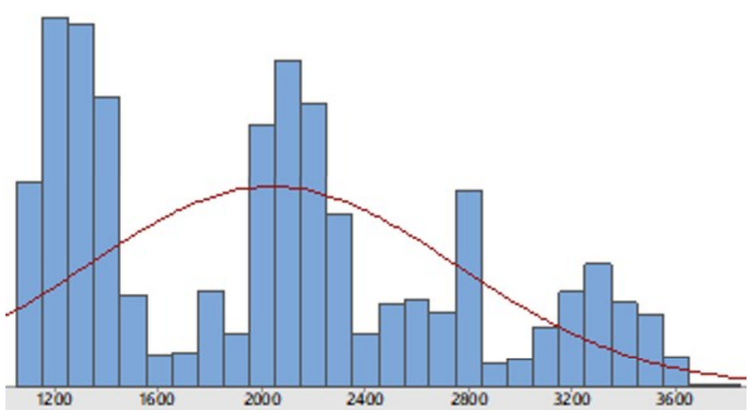

Figure 3.4. Report on the Automobile Sector

Table 3.3: Statistical measures of six sectors

	Mean	**St.Dev**	**Variance**	**Min**	**Max**	Q_1	**Median**	Q_3	**Range**	**Skew**	**Kurt**	**A-sq**
Finance	885.0	280.9	78942.1	453.8	1581.1	627.4	806.2	1115.2	1127.3	0.42	-0.89	32.72
Automobile	2038.2	715.5	511914.9	1052.9	3759.0	1331.4	2050.2	2526.1	2706.0	0.45	-0.86	34.01
IT	1551.5	318.3	101334.1	1156.1	2545.5	1324.8	1407.8	1758.2	1389.4	1.10	0.13	86.85
Oil	611.7	81.9	6711.2	462.1	908.7	556.7	596.0	641.7	446.7	1.13	1.43	33.04
Pharma	1268.6	317.0	100461.1	738.5	1999.5	1043.1	1199.7	1510.8	1261.1	0.22	-0.92	17.32
Power	146.8	23.7	565.2	103.0	201.1	130.8	140.1	165.0	98.1	0.64	-0.68	51.04

Note 1*: St. Dev=Standard Deviation; Min= Minimum; Max= Maximum; Skew=Skewness; Kurt= Kurtosis; A-sq= A-squared.*

Note 2*: The p-values of all the sectors came to be less than 0.005.*

Over the past five years, the mean of the stock price of the automobile sector is found to be 2038.2 and the standard deviation to be 715.5. The asymmetricity can be derived from the skewness, which is found to be around 0.45 derived from the past five years. The automobile sector exhibits positive skewness, which is a positive trend for an investor. The negative kurtosis suggests that the unpredictable and has high variance with respect to normal distribution.

From the past stock trends, which is shown in Figure 3.4, it can be inferred that the average stock over the period of 5 years is 2038.2, with minimum and maximum shares as 1052.9 and 3759, respectively, with an optimal gain of more than 257% for an investor. From Q_1, it is evident that, for 25% of the period, the stock value is observed to 1331.4, whereas the stock is observed to be 2526.1 for 75 % of the period. These cumulative statistics, as discussed in the aforementioned sections, aid investors to analyze the probable stock for the coming years.

By the Anderson Darling normality test, it is observed that all sectors are mostly unpredictable since the p-value is less than 0.005 and doesn't follow a normal distribution. Comparatively, the automobile sector is almost as volatile as the finance or oil sectors.

3.2.3 Information Technology Sector (S_3)

The future of this sector is optimistic with the emergence of fields such as artificial intelligence, machine learning, cloud computing, internet of things, business analytics, and neural networks. Government and private company's dependence on software is increasing in day to day life. According to a survey conducted by the leading IT service Tata Consultancy Services (TCS), the impact of the sector in business is going to be highly volatile in the years 2020 and 2021. The plot of the IT sector over the past five years is shown in the following Figure 3.5.

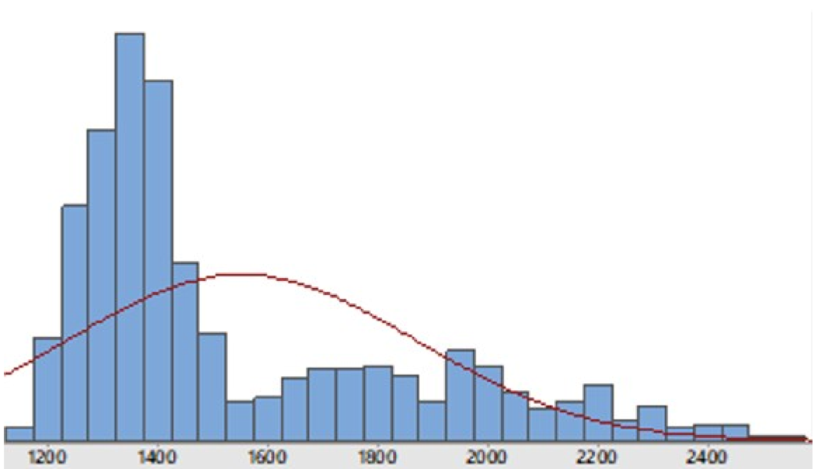

Figure 3.5. Report on Information Technology sector

With the shift of increasing stock price and high returns towards the years succeeding the median, the IT sector in SENSEX is one of the top priorities for the investors. This is due to the positive skewness 1.096 and positive kurtosis 0.12, which can be observed in Figure 3.5. With a mean of 15551.5 and a standard deviation of 318.3, an investor can experience a maximum profit of more than 120% since the maximum and minimum values of the last 5 years of this sector is found to be 1156.1 and 2545.5 respectively. From Table 3.3, it is evident that the Q_1 of IT sector is 1324.8 and Q_3 is 1758.2. With median of 1407.8, this sector experiences a highly biased towards the Q_3. Like other sectors, the IT sector is also unpredictable and is even more uncertain as the A-squared is highest among the six sectors obtained from the Anderson Darling normality test.

3.2.4 Oil Sector (S_4)

The oil sector is a stock category related to providing oil and gas. Integrated power companies, refining, drilling, oil and gas reserves are some areas which are included in this sector. From the future perspective, alternative and renewable sources of energy can play a predominant role. The acceptance of electric cars is booming in many parts of the world, which can impact this industry. Countries with the highest oil reserves constitute Venezuela, middle east countries, Canada and the USA impact Oil prices in different countries. The growth rate of oil sector can be derived from the subsequent Figure 3.6.

The oil sector is the most volatile sector among the six sectors due to the high kurtosis value 1.43. But in recent years, the stock price is increased due to various factors like increasing barrel price of crude oil and swift towards alternate sources of energy, which had altered the skewness value of 1.13 can be seen from Figure 3.6.

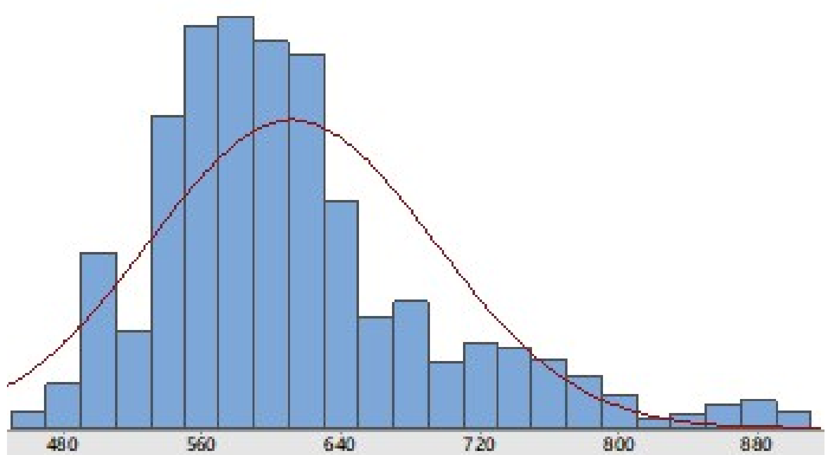

Figure 3.6. Report on Oil sector

The average price of this sector can be observed as 611.7, with a standard deviation of 81.92 and variance 6711.16. The returns are among the least in the six sectors with minimum and maximum values of 462.05 and 908.73. The Q_1 and Q_3 values of this sector are found to be 556.71 and 641.76 respectively and unlike other sectors, the median 595.99 is least biased to any quartile.

3.2.5 Pharma Sector (S_5)

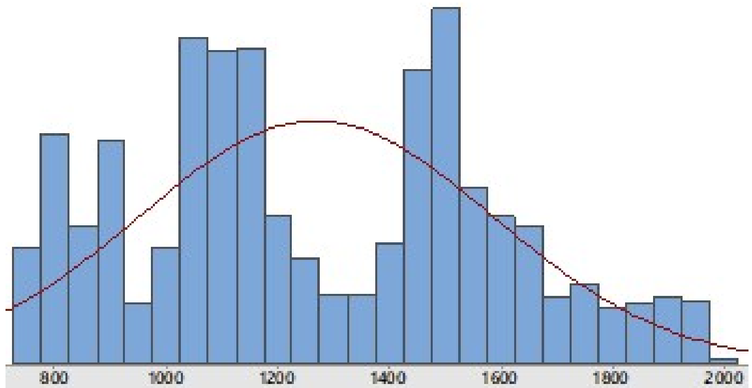

Figure 3.7. Report on Pharma sector

The Pharma sector is one of the rapidly growing sectors, with an estimated sale of $1.3 billion around the globe in 2018 (Lidstone and MacLennan 1999). Recently the interdisciplinary research is increasing, mostly overlapping with the biotechnology sector. The Pharma industry growth chart can be observed in Figure 3.7.

Among the six sectors, the Pharma sector is the least volatile sector as the kurtosis value being 0.92, which can be observed from Table 3.3. Even the skewness is among the least at 0.218. The mean of this sector stands at 1268.6, with a standard deviation of 316.9 and variance 100461.1. The range of stock price is 1261.1, with a minimum value of 738.5 and a maximum of 1999.5.

The median with 1199.7 is slightly biased towards Q_1 which is 1043.1 than Q_3 of 1510.8. Like all other sectors, this sector is also unpredictable as the p-value is less than 0.005 and the A-squared value is 17.32 obtained from the Anderson Darling test.

3.2.6 Power Sector (S_6)

The power sector majorly consists of stocks that are intact with produce and supply energy. Various renewable and non-renewable resources belong to this sector and have a great opportunity for growth in the future due to the extinction of many sources. The distribution chart can be observed in Figure 3.8.

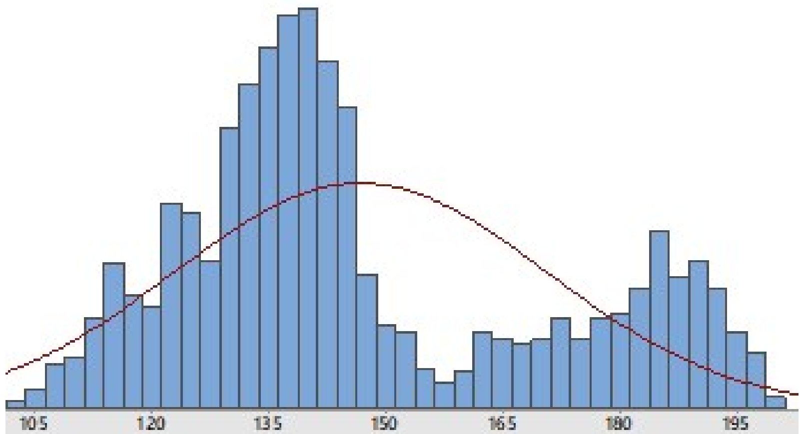

Figure 3.8. Report on Power sector

The power sector experiences best skewness value with 0.64 after IT and Oil sectors, which evidences asymmetricity towards the right and has a negative kurtosis value of 0.68, which is peak indicator conclusive from Figure 3.8. Usually, the power sector is among the least average stock price with 146.8 with a minimum 103 and a maximum 201.05. This sector registered the least growth value of less than 96% in the last 5 years. The first quartile value is 130.81 and the third quartile with 165 has a median of 140.15 which has a tendency towards Q_1.

Similar to other sectors, the power sector is highly inconsistent with any of the distribution and is highly unpredictable with A- squared value of 51.04 which is a high score after the IT sector.

3.3 Evaluation Criteria's to Evaluate Performance Index of Sectors in BSE SENSEX

Evaluating and forecasting the behavior of equity sectors are determined by the assessment of certain evaluative criteria (synonymously used for financial derivatives). This study considers the following four criteria for analyzing BSE SENSEX sectors.

3.3.1 Return on assets (ROA)

ROA is a financial derivative that helps to identify the profit percentage of a company with consideration to the overall resources and is derived by calculating the net income of a company to its total assets. This ratio can be mathematically given by (3.1)

$$ROA = \frac{Total\ net\ income\ after\ taxes}{Total\ asset\ of\ a\ company} \tag{3.1}$$

3.3.2 Earnings per share (EPS)

EPS serves as one of the indicators to evaluate the company's profitability. It is measured by taking the ratio of the company's profit by outstanding shares obtained from common stocks given by (3.2).

$$EPS = \frac{Net\ income\ after\ taking\ preffered\ dividends}{Outstanding\ shares} \tag{3.2}$$

3.3.3 Price to sales ratio (PS ratio)

This ratio estimates the value of a market of the company's sales. The premature growth stocks can be valued using this ratio. PS ratio can be obtained using (3.3)

$$PS\,ratio = \frac{Company's\ total\ market\ captilization}{Company's\ total\ shares} \tag{3.3}$$

3.3.4 Price to cash flow ratio (PCF ratio)

The ratio of share price and cash flow for every share can be defined as the PCF ratio. This ratio acts as a stock indicator which is used to measure stock price with respect to cash flow and can be mathematically given by (3.4)

$$PCF\ ratio = \frac{Price\ of\ Share}{Cash\ flow\ per\ share} \tag{3.4}$$

3.3.5 Dividend yield (DY)

DY is the amount of value a company pays to equity shareholders with respect to the current price of the stock. It is the ratio of the annual dividend to the price of a share and is given by (3.5)

$$DY = \frac{Annual\ dividend}{Price\ of\ share} \tag{3.5}$$

3.3.6 Price to earn ratio (PE ratio)

It is a predominant financial derivative to assess a company's valuation. This ratio evaluates cost of earnings gained per share. The mathematical equation of PE ratio can be given by (3.6)

$$PE\ ratio = \frac{Worth\ of\ a\ company\ per\ share}{Earnings\ (in\ total)\ per\ share} \tag{3.6}$$

Generally, investors endeavor to estimate the growth or predict if a company is undervalued or overvalued based on erstwhile trends by using the PE ratio (Peter D. Easton 2004).

3.3.7 Book value per share (BVPS)

This derivative may be employed as a tool to govern sector equity with respect to the current value of a sector (stock price). BVPS can be mathematically represented by (3.7).

$$BVPS = \frac{Total\ equity\ held\ by\ the\ investor}{Total\ number\ of\ shares} \tag{3.7}$$

3.3.8 Return on equity (ROE)

ROE is the crucial financial measure indicating the net productivity by evaluating the return rate of interest produced by net assets of the company. The return rate of interest in percentages termed as "ROE" is the ratio of total income (followed by dividends of preferred stocks) to total worth of the company and governed by (3.8).

$$ROE = \frac{Net\ companies\ equity - \ Equity\ preferred}{Total\ companies\ worth} \times 100 \tag{3.8}$$

3.3.9 Price to book ratio (PB ratio)

PB ratio is a convenient tool for evaluating sectors or companies which obey homogeneous valuations of an asset. Investors consider historical data to predict a rise in asset prices. It can be defined as the ratio between current asset cost in the market and the value of assets (net). This ratio can be explicitly given by (3.9).

$$PB\ ratio = \frac{Current\ asset\ cost\ in\ the\ market}{Assets - \ Liabilities\ in\ total} * Number\ of\ outstanding\ shares \tag{3.9}$$

3.4 Identification and Classification of Criteria using Fuzzy DEMATEL

Step 1: Selection of crucial parameters.

The crucial criteria which help in the construction of a financial portfolio are identified from previous studies. The defined criteria include ROA, EPS, PS ratio, PCF ratio, DY, PE ratio, BVPS, ROE, PB ratio. The aforementioned criteria are classified into cost and effect groups by using fuzzy DEMATEL is used in this study to perform this activity. Three senior academicians, two portfolio managers, and a financial analyst were chosen as expert panel and responses were garnered in linguistic form.

Step 2: To choose subject proficient and connoisseurs.

The intermediate 5-scale approach (0 indicating negligible or no influence and four indicating prominent influence) has been used to transform individual subject proficient and connoisseur's linguistic opinions into the numerical scale and then into an equivalent trapezoidal fuzzy number

(TrFN) by the transformation function $TrFN: X \rightarrow [0,1]^4$ where $X = \{0,1,2,3,4\}$, which can be depicted from Table 3.4. The TrFN's are represented by fuzzy criteria evaluation matrices for each individual subject proficient and connoisseurs. The 5-scale approach used for transformation of opinion along with transformation function can be derived from Table 3.5. The fuzzy scores corresponding to Return on Assets and Return on equity are shown in Table 3.6. It is noted that $x \in X$.

Step 3: To obtain the fuzzy criteria relation matrix.

To obtain the fuzzy criteria relation matrix (R), the fuzzy numbers are transformed to crisp numbers by using the bisection of area defuzzification method (Bobyr *et al.* 2017), as shown in Table 3.7. Further, the fuzzy average criteria matrix (A) is obtained by taking the average of all fuzzy criteria relation matrices of order $m \times m$ shown in Table 3.7.

Step 4: To evaluate the normalized criteria relation matrix.

The normalized criteria relation matrix (N) is derived using the equations (3.10) and (3.11) The matrix obtained by these equations enables to identify key financial ratios of BSE SENSEX as shown in Table 3.8.

$$k = \min\left[(max\sum_{h=1}^{m}|a_{gh}|)^{-1}, (max\sum_{g=1}^{m}|a_{gh}|)^{-1}\right] \tag{3.11}$$

with $N = k \times A$

Step 5: To calculate effective criteria relation matrix.

Effective criteria relation matrix (E) is constructed by equation (3.12) and the obtained matrix is shown in Table 3.9.

$$E = N(I - N)^{-1} \tag{3.12}$$

where I represents the identity matrix.

Table 3.4: Aggregate linguistic scale used of subject proficient and connoisseurs

Linguistic opinions	$\boldsymbol{X}$	$\boldsymbol{TrFN(x)}$
Negligible influence (No)	0	(0,0,0.1,0.2)
Very low influence (VL)	1	(0.1,0.2,0.3,0.4)
Low influence (L)	2	(0.3,0.4,0.5,0.6)
High influence (H)	3	(0.5,0.6,0.7,0.8)
Very high influence (VH)	4	(0.7,0.8,0.9,1)

Table 3.5: Linguistic opinions to trapezoidal fuzzy conversion scale

	ROA	EPS	PS ratio	PCF ratio	DY	PE ratio	BVPS	ROE	PB ratio
ROA	No	H	L	L	VL	L	L	H	VL
EPS	H	No	L	VL	L	H	L	H	VL
PS ratio	L	VL	No	L	L	H	L	L	VL
PCF ratio	VL	VL	H	No	VL	L	VL	L	L
DY	VL	VL	H	VL	No	VL	L	L	L
PE ratio	VH	L	VH	VH	H	No	H	VH	VH
BVPS	H	VH	VH	H	H	L	No	H	VH
ROE	VH	H	L	VH	H	VH	VH	No	H
PB ratio	H	H	VH	H	VL	VH	VH	VH	No

Table 3.6: Fuzzy scores of corresponding to ROA and ROE over other criteria

	ROA			ROE		
ROA	No	0	(0,0,0.1,0.2)	H	3	(0.5.0.6,0.7,0.8)
ROE	H	3	(0.5.0.6,0.7,0.8)	No	0	(0,0,0.1,0.2)
BVPS	L	2	(0.3,0.4,0.5,0.6)	VL	1	(0.1,0.2,0.3,0.4)
EPS	VL	1	(0.1,0.2,0.3,0.4)	VL	1	(0.1,0.2,0.3,0.4)
DY	VL	1	(0.1,0.2,0.3,0.4)	VL	1	(0.1,0.2,0.3,0.4)
PE ratio	VH	4	(0.7,0.8,0.9,1)	L	2	(0.3,0.4,0.5,0.6)
PB ratio	H	3	(0.5.0.6,0.7,0.8)	VH	4	(0.7,0.8,0.9,1)
PS ratio	VH	4	(0.7,0.8,0.9,1)	H	3	(0.5.0.6,0.7,0.8)
PCF ratio	H	3	(0.5.0.6,0.7,0.8)	H	3	(0.5.0.6,0.7,0.8)

Table 3.7: Fuzzy criteria relation matrix

	ROA	ROE	BVPS	EPS	DY	PE ratio	PB ratio	PS ratio	PCF ratio
ROA	0.08	0.65	0.45	0.45	0.25	0.45	0.45	0.65	0.25
ROE	0.65	0.075	0.45	0.25	0.45	0.65	0.45	0.65	0.25
BVPS	0.45	0.25	0.075	0.45	0.45	0.65	0.45	0.45	0.25
EPS	0.25	0.25	0.65	0.075	0.25	0.45	0.25	0.45	0.45
DY	0.25	0.25	0.65	0.25	0.075	0.25	0.45	0.45	0.45
PE ratio	0.85	0.45	0.85	0.85	0.65	0.075	0.65	0.85	0.85
PB ratio	0.65	0.85	0.85	0.65	0.65	0.45	0.075	0.65	0.85
PS ratio	0.85	0.65	0.45	0.85	0.65	0.85	0.85	0.075	0.65
PCF ratio	0.65	0.65	0.85	0.65	0.25	0.85	0.85	0.85	0.075

Table 3.8: Normalized criteria relation matrix

	ROA	ROE	BVPS	EPS	DY	PE ratio	PB ratio	PS ratio	PCF ratio
ROA	0.23	0.28	0.30	0.28	0.21	0.28	0.27	0.32	0.23
ROE	0.33	0.21	0.32	0.26	0.25	0.32	0.29	0.34	0.24
BVPS	0.27	0.22	0.23	0.27	0.23	0.30	0.26	0.28	0.22
EPS	0.22	0.20	0.30	0.19	0.18	0.25	0.21	0.26	0.23
DY	0.22	0.20	0.30	0.21	0.16	0.22	0.24	0.26	0.23
PE ratio	0.46	0.36	0.50	0.45	0.36	0.35	0.42	0.48	0.42
PB ratio	0.41	0.40	0.47	0.40	0.35	0.38	0.31	0.43	0.40
PS ratio	0.45	0.39	0.43	0.44	0.36	0.45	0.44	0.36	0.39
PCF ratio	0.43	0.38	0.48	0.42	0.30	0.45	0.44	0.47	0.30

Table 3.9: Effective criteria relation matrix

	ROA	ROE	BVPS	EPS	DY	PE ratio	PB ratio	PS ratio	PCF ratio
ROA	0.23	0.28	0.30	0.28	0.21	0.28	0.27	0.32	0.23
ROE	0.33	0.21	0.32	0.26	0.25	0.32	0.29	0.34	0.24
BVPS	0.27	0.22	0.23	0.27	0.23	0.30	0.26	0.28	0.22
EPS	0.22	0.20	0.30	0.19	0.18	0.25	0.21	0.26	0.23
DY	0.22	0.20	0.30	0.21	0.16	0.22	0.24	0.26	0.23
PE ratio	0.46	0.36	0.50	0.45	0.36	0.35	0.42	0.48	0.42
PB ratio	0.41	0.40	0.47	0.40	0.35	0.38	0.31	0.43	0.40
PS ratio	0.45	0.39	0.43	0.44	0.36	0.45	0.44	0.36	0.39
PCF ratio	0.43	0.38	0.48	0.42	0.30	0.45	0.44	0.47	0.30

Table 3.10: Datasets of D+R and D-R for sector criteria in BSE SENSEX

	D	R	$D+R$	$D-R$	Nature	Enablers
ROA	2.40	3.02	5.42	-0.61	effect	E_1
EPS	2.55	2.63	5.18	-0.09	effect	E_2
PS ratio	2.28	3.33	5.61	-1.05	effect	E_3
PCF ratio	2.04	2.91	4.95	-0.87	effect	E_4
DY	2.03	2.40	4.43	-0.37	effect	E_5
ROE	3.71	3.22	6.93	0.5	cause	C_1
BVPS	3.54	2.88	6.42	0.66	cause	C_2
PE ratio	3.81	3.01	6.82	0.79	cause	C_3
PB ratio	3.68	2.63	6.31	1.04	cause	C_4

Step 6: To calculate effective criteria relation matrix.

The row and column aggregates R and D respectively of effective criteria relation matrix is calculated using equations (3.13) and (3.14).

$$R = \left\{ \sum_{h=1}^{m} e_{gh} \right\}_{m \times 1} \tag{3.13}$$

$$D = \left\{ \sum_{g=1}^{m} e_{gh} \right\}_{1 \times m} \tag{3.14}$$

where 'e_{gh}' are elements of Effective criteria relation matrix E along with aggregates R and D gages the overall impact of g^{th} criteria over h^{th} criteria and overall impact of h^{th} criteria over g^{th} criteria respectively.

Step 7: Plot effect-cause graph from the data set.

The effect-cause graph is plotted by the data set ($D - R$; $D + R$)

The prominent criteria in the SENSEX can be derived from the dataset of $D + R$, whereas the magnitude of $D - R$ represents the effect of each criterion. The threshold value of $D - R$ with '0' distinguishes the criteria into cause group and effect group. The criteria with magnitude less than the threshold promote the variable fall under effect group and rest falls under the category of cause group. The effect-cause plot can be depicted from the Ishikawa diagram Figure 3.9, which is obtained from Table 3.10.

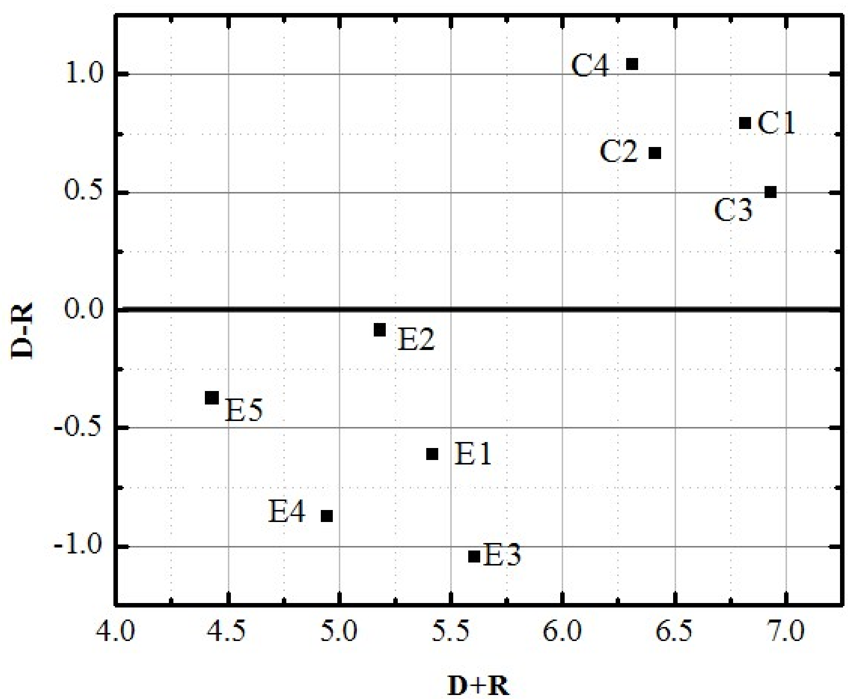

Figure 3.9. Effect-cause Ishikawa diagram of criteria in BSE SENSEX

3.5 Philosophy of Fuzzy AHP and Fuzzy TOPSIS

Even though the statistical measures have significant advantages, there are many disadvantages to their account. In the late 1929's, the global stock market parameters were showing positive trends such as positive skewness, and right kurtosis, but the market crashed (Campbell and Hentschel 1991). To improve the precision in projection, both Fuzzy AHP and Fuzzy TOPSIS are used for a greater understanding of the market.

For the past several years, multiple criteria decision making under uncertain environment became a choice in making an appropriate decision. Numerous techniques such as AHP, TOPSIS, ELECTREE, extent analysis have evolved and been used for many applications(Jha and Puppala 2017; Yanık and Eren 2017). Fuzzy AHP and Fuzzy TOPSIS are derived based on AHP and TOPSIS, respectively, in an uncertain environment and decision making outcomes. The crisp scale proposed by Saaty is used for evaluation using AHP, and rating scale on an 11 scale explained qualitatively is used in TOPSIS. Further, fuzzy AHP redeems a crisp Saaty scale to fuzzy scale by using a triangular membership function which is utilized in analyzing sectors in the study. The dominance of each sector over the other is measured using a fuzzy Saaty scale under multiple evaluative criteria's.

3.6 Fuzzy Analytical Hierarchy Process (Fuzzy AHP)

Even though multiple MCDM's were introduced in the early 1960s, many of these methods are proved to end with undesirable results. To overcome these drawbacks and to provide reasonable and logical methods, Fuzzy AHP is introduced (Kalban 2004; Dağdeviren and Yüksel 2008; Chatzimouratidis and Pilavachi 2008). The dominance of each sector over the other is measured using a fuzzy Saaty scale under multiple evaluative criteria. The schematic representation of the present study is shown in Figure 3.10.

Four cost criteria obtained from fuzzy DEMATEL, namely ROE, BVPS, PE ratio and PB ratio were considered and the objective can be depicted Figure 3.9 and Figure 3.10. From the obtained data shown, it is evident that the attributes are crisp and hence are ineffective for dealing real life applications. Since a wide range of criteria is considered in evaluating the sector, weightage for each of the criteria is considered.

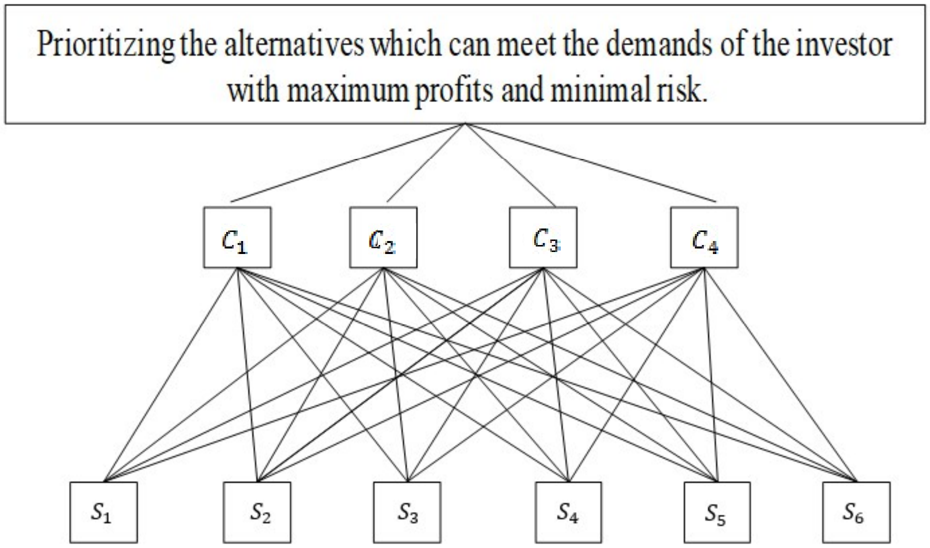

Figure 3.10. Pictorial representation of impact of sectors by taking 4 criteria

In the current chapter, each sector's cumulative score is calculated by using Fuzzy AHP. The advantage is not only restricted to the order of priority but also conveys the optimal investment percentage an individual can invest in a particular sector. Evaluation of the performance index is discussed in subsequent sections.

3.7 Analysis and Discussions of Fuzzy AHP

The Analysis of Fuzzy AHP for identifying the best sector can be classified into five steps explained below. By using Fuzzy AHP, the outcome of the defuzzified score of each sector is analyzed against the four criteria, and then the performance index is executed. These five steps are explained in subsequent subsections.

Step 1: Fuzzification of the crisp Saaty scale.

The range of the Saaty scale, which is fuzzified, varies from 1 to 9 is shown in Table 3.11. For fuzzifing the crisp values, triangular membership function is utilized to represent the generalized fuzzy triangular membership function, which is well known from the literature (Ertuğrul and Karakaşoğlu 2009). By analyzing the data from Table 3.12, we conclude that a crisp rating does not deviate significantly. Based on this analysis, a span of 1 is preferred for fuzzification. (1, 1, 2) along with (8, 8, 9) are considered for the corresponding fuzzification of crisp ratings of border values, respectively. The Saaty scale, which is fuzzified related to the prevailing crisp values, can be derived from taking the value of ɳ as 1 where ɳ is the offset distance ranging from 0.5 to 2.(Srdjevic and Medeiros 2008).

Table 3.11. Saaty's crisp and fuzzified scale (Pishchulov et al. 2019)

Saaty's crisp values (x)	Definition for Judgment	Fuzzified Saaty's value
1	Negligible dominance	(1, 1, 1+η)
3	Dominance is week	(3-η, 3, 3+η)
5	Dominance is Strong	(5-η, 5, 5+η)
7	Dominance is Demonstrative	(7-η, 7, 7+η)
9	Dominance is Absolute	(9-η, 9, 9)
2,4,6,8	Intermediate values	(x-1, x, x+1), x=2, 4, 6, 8

where η is called as offset distance; (0.5 < η < 2)

Step 2: Analyzing each individual criterion over the other

Analyzing the domination of an individual criterion over others is exercised initially to prioritize sectors that are determined by the decision maker, as shown in Table 3.12. Saaty's crisp scale is utilized to decide the significance of every individual criterion over other (i.e., range of weights are from 1 to 9) and then assigned the crisp ratings, which can be shown in Table 3.11 by triangular fuzzy numbers. The generalized form of the matrix concerning criteria evaluation can be given mathematically by (3.15).

$$C = \begin{pmatrix} C_{11} & \cdots & C_{1N} \\ \vdots & C_{ij} & \vdots \\ C_{N1} & \cdots & C_{NN} \end{pmatrix} \tag{3.15}$$

where C_{ij} can be interpreted as the dominance of i^{th} evaluative criteria over j^{th} criteria which imply C_{ij}=1 for $i = j$; (where $i = j = 1, 2, 3 \ldots\ldots\ldots, N$) which can be observed to be diagonal entries in C matrix.

The weight of evaluative criteria can be derived by a fuzzy synthetic approach expressed mathematically by (3.16).

$$f_i = \sum_{j=1}^{N} s_{ij} \otimes \left[\sum_{k=1}^{N} \sum_{l=1}^{N} s_{ij} \right]^{-1}, i = 1, 2, \ldots N \tag{3.16}$$

here f_i is a fuzzy number which is normalized with unity as its medium with $i = 1 \ldots N$ (number of criteria's).

It has to be noted that the fuzzy extent can be replaced by the result that came from the extension principle or fuzzy arithmetic (Pishchulov *et al.* 2019). It is complicated to compute the extension principle rather than fuzzy arithmetic but is certainly helpful in the reduction of uncertainty. The evaluated criteria are calculated by attaching fuzzy weights to them using Table 3.12 and Table 3.13.

Table 3.12: Criteria evaluation (Crisp rating)

	ROE (%)	**Book Value per Share**	**PE Ratio**	**PB Ratio**
ROE (%)	1.00	2.00	0.14	0.17
Book Value per Share	9.00	1.00	0.13	0.14
PE Ratio	8.00	0.50	1.00	1.00
PB Ratio	6.00	9.00	0.50	1.00

Table 3.13: Fuzzified weights of criteria's

	C_1	C_2	C_3	C_4
C_1	(1,1,2)	(1,1,2)	(0.12,0.14,0.17)	(0.14,0.17,0.2)
C_2	(8,9,9)	(1,1,2)	(0.11,0.13,0.14)	(0.12,0.14,0.17)
C_3	(7,8,9)	(0.33,0.5,1)	(1,1,2)	(1,1,2)
C_4	(5,6,7)	(8,9,9)	(0.33,0.5,1)	(1,1,2)

Step 3: Analyzing each individual sector over the other.

Six sectors, namely Automobiles, IT, Oil, Finance, Pharma and Power, are considered, and the scores are evaluated over each other by using four criteria. The mathematical representation can be given by the matrix W_k (3.17).

$$W_k = \begin{bmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & b_{ij} & \vdots \\ b_{B1} & \cdots & b_{BN} \end{bmatrix}, i = 1,2,\ldots,B; j = 1,2,\ldots,N \tag{3.17}$$

here N denotes the number of criteria's and B represents the number of alternatives evaluated over criteria.

Values in Table 3.14 are taken into consideration to correlate all sectors against each of the evaluative criteria, which are to be normalized initially. Then by using these normalized values, a pairwise comparative matrix is prepared by taking all the sectors and evaluated criteria into consideration. The matrices of all the sectors over criterion are shown in Table 3.14 – Table 3.17.

Table 3.14: Sector wise analysis over Criteria 1

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1.00	0.13	0.11	1.00	0.25	0.33
S_2	8.00	1.00	0.50	7.00	5.00	5.00
S_3	9.00	2.00	1.00	9.00	6.00	7.00
S_4	1.00	0.14	0.11	1.00	0.33	0.50
S_5	4.00	0.20	0.17	3.00	1.00	1.00
S_6	3.00	0.20	0.14	2.00	1.00	1.00

Table 3.15: Sector wise analysis over Criteria 2

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1.00	1.00	0.11	0.25	1.00	0.33
S_2	1.00	1.00	0.11	0.20	1.00	0.25
S_3	9.00	9.00	1.00	5.00	9.00	6.00
S_4	4.00	5.00	0.20	1.00	4.00	2.00
S_5	1.00	1.00	0.11	0.25	1.00	0.33
S_6	3.00	4.00	0.17	0.50	3.00	1.00

Table 3.16: Sector wise analysis over Criteria 3

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1.00	7.00	9.00	9.00	4.00	9.00
S_2	0.14	1.00	2.00	3.00	0.33	3.00
S_3	0.11	0.50	1.00	1.00	0.20	1.00
S_4	0.11	0.33	1.00	1.00	0.20	1.00
S_5	0.25	3.00	5.00	5.00	1.00	5.00
S_6	0.11	0.33	1.00	1.00	0.20	1.00

Table 3.17: Sector wise analysis over Criteria 4

	S_1	S_2	S_3	S_4	S_5	S_6
S_1	1.00	0.33	0.25	5.00	0.50	4.00
S_2	3.00	1.00	2.00	7.00	4.00	8.00
S_3	4.00	0.50	1.00	8.00	2.00	7.00
S_4	0.20	0.14	0.13	1.00	0.17	0.50
S_5	2.00	0.25	0.50	6.00	1.00	5.00
S_6	0.25	0.12	0.14	2.00	0.20	1.00

Table 3.18: Weights of Sectors over Criteria 1

S_1	(0.025, 0.034, 0.073)
S_2	(0.211, 0.319, 0.461)
S_3	(0.274, 0.409, 0.547)
S_4	(0.027, 0.037, 0.083)
S_5	(0.069, 0.113, 0.194)
S_6	(0.050, 0.088, 0.164)

Furthermore, by fuzzy division operation, each sector against all the criteria are evaluated. The weights obtained against C_1 can be observed in Table 3.18. The evaluated weights for the other three criteria i.e. $C_2, C_3$and C_4 are shown in Table 3.19 – Table 3.21.

Table 3.19: Weights of Sectors over Criteria 2

C_2	
S_1	(0.036, 0.047, 0.107)
S_2	(0.036, 0.046, 0.103)
S_3	(0.340, 0.501, 0.644)
S_4	(0.122, 0.208, 0.326)
S_5	(0.036, 0.047, 0.107)
S_6	(0.085, 0.150, 0.248)

Table 3.20: Weights of Sectors over Criteria 3

C_3	
S_1	(0.336, 0.494, 0.634)
S_2	(0.063, 0.120, 0.206)
S_3	(0.036, 0.048, 0.111)
S_4	(0.035, 0.046, 0.104)
S_5	(0.150, 0.244, 0.367)
S_6	(0.035, 0.046, 0.104)

Table 3.21: Weights of Sectors over Criteria 4

C_4	
S_1	(0.085, 0.140, 0.235)
S_2	(0.195, 0.316, 0.491)
S_3	(0.178, 0.284, 0.444)
S_4	(0.018, 0.027, 0.060)
S_5	(0.112, 0.186, 0.306)
S_6	(0.025, 0.047, 0.093)

After the formation of pair wise comparative matrices and by using the scale mentioned in Table 3.11, the conversion of crisp ratings to a triangular fuzzy number is followed.

Step 4: Computing performance matrix and determining the fuzzy score

Fuzzy interval arithmetic is applied for the sector wise performance of each of the 6 sectors over throughout the four criteria's which can be illustrated through the performance matrix (Seçme *et al.* 2009) given by (3.18).

$$z = \begin{bmatrix} y_{11} \otimes w_1 & \dots & y_{1k} \otimes w_k \\ . & . & . \\ y_{N1} \otimes w_1 & \dots & y_{NK} \otimes w_k \end{bmatrix} \tag{3.18}$$

where $N = 1,2,\dots,6$ and $k = 1,2,3,4$

The set $\{y_{11},\dots,y_{NK}\}$ is a set of priority vectors and $\{w_1,\dots,w_k\}$ represents the weights of each criterion. Evaluation of performance matrix z corresponding to each of the sectors over C_1, C_2, C_3 and C_4 are calculated. Table 3.22 indicates the performance matrix of S_1 against the criteria. The performance matrix of remaining sectors (P_i's) are shown in Table 3.23 – Table 3.27.

Table 3.22: Performance matrix for Sector 1 (Finance)

P_1	a
C_1	(0.025, 0.034, 0.073)
C_2	(0.036, 0.047, 0.107)
C_3	(0.336, 0.494, 0.634)
C_4	(0.085, 0.140, 0.235)

Table 3.23: Performance matrix for Sector 2 (Automobile)

P_2	a
C_1	(0.211, 0.319, 0.461)
C_2	(0.035, 0.046, 0.103)
C_3	(0.063, 0.120, 0.206)
C_4	(0.195, 0.316, 0.0491)

Table 3.24: Performance matrix for Sector 3 (IT)

P_3	a
C_1	(0.274, 0.409, 0.547)
C_2	(0.340, 0.501, 0.644)
C_3	(0.036, 0.048, 0.111)
C_4	(0.178, 0.284, 0.444)

Table 3.25: Performance matrix for Sector 4 (Oil)

$\boldsymbol{P_4}$	**a**
$\boldsymbol{C_1}$	(0.027, 0.037, 0.083)
$\boldsymbol{C_2}$	(0.122, 0.208, 0.326)
$\boldsymbol{C_3}$	(0.035, 0.046, 0.104)
$\boldsymbol{C_4}$	(0.018, 0.027, 0.060)

Table 3.26: Performance matrix for Sector 5 (Pharma)

$\boldsymbol{P_5}$	**a**
$\boldsymbol{C_1}$	(0.069, 0.113, 0.194)
$\boldsymbol{C_2}$	(0.036, 0.047, 0.107)
$\boldsymbol{C_3}$	(0.150, 0.244, 0.367)
$\boldsymbol{C_4}$	(0.112, 0.186, 0.306)

Table 3.27: Performance matrix for Sector 6 (Power)

$\boldsymbol{P_6}$	**a**
$\boldsymbol{C_1}$	(0.050, 0.088, 0.164)
$\boldsymbol{C_2}$	(0.085, 0.150, 0.248)
$\boldsymbol{C_3}$	(0.035, 0.046, 0.104)
$\boldsymbol{C_4}$	(0.025, 0.047, 0.093)

The summation of all the assessments maximum, minimum, median, mean, decision maker's inputs and mixed operators are analyzed. Additive synthesis is used to analyze the performance for each sector to reduce risks or negative impact and to maximize returns. The sector wise performance can be represented by (3.19).

In (3.19), F_i represents the fuzzy score of i^{th} sector; y_{ij} represents priority vectors of i^{th} sector with j^{th} criteria. The weight w_j represents the evaluated criteria's fuzzy weight. Here $i = 1,2,\dots,6$ and $j = 1,2,3,4$. Table 3.28 represents the fuzzy weights of all sectors.

$$F_i = \sum_{j=1}^{C} y_{ij} \otimes w_j \tag{3.19}$$

Step 5: Defuzzifiying the cumulative score of each Sector and computing the values.

The aggregated score for all the sectors is considered for the sector wise prioritization will be discussed in subsequent sections. Each score for the sectors is in triangular fuzzy number. Hence for converting this fuzzy number to a crisp value, defuzzification of scores for each sector is done.

Table 3.28: Overall Sector performance evaluated against 4 criteria's

S_1	(0.425, 0.200, 0.096)
S_2	(0.450, 0.197, 0.084)
S_3	(0.574, 0.288, 0.134)
S_4	(0.191, 0.079, 0.036)
S_5	(0.376, 0.160, 0.070)
S_6	(0.197, 0.076, 0.032)

There are many methods to determine the defuzzified score by the level of optimism. The method of total integral value is considered for a wide range of benefits (Liou and Wang 1992). The estimation of a fuzzy number by using the method of total integral value can be mathematically given by (3.20).

$$I_T^A = 0.5[\lambda b_3 + (1-\lambda)b_1 + b_2],\ \lambda \in [0,1] \tag{3.20}$$

where, λ is the optimism index, which indicates the decision maker's level of risk. The value λ is directly proportional to an optimism degree. Generally, $\lambda = 0, 0.5, 1$ corresponds to pessimist, moderate and optimist respectively. Then the total integral values for each sectors are calculated by using (3.10) for getting deffuzified score corresponding to λ, which is shown in Table 3.29.

Table 3.29: Scores of Sectors which are defuzzified concerning specified λ

	λ = 0	**λ = 0.5**	**λ = 1**	**Ranking**
S_1	0.239305	0.174449	0.478611	3
S_2	0.248428	0.176461	0.496855	2
S_3	0.336948	0.247883	0.673896	1
S_4	0.106747	0.07501	0.213495	5
S_5	0.205555	0.145593	0.411109	4
S_6	0.107185	0.07398	0.214371	6

3.8 Analysis by using Fuzzy TOPSIS

The analysis of Fuzzy TOPSIS can be classified in five steps:

Note: Throughout this section, $i = 1,2,\dots,m$ and $j = 1,2,\dots,n$ notations are adopted for generalized matrix

Step 1: Criteria weight evaluation. Criteria weights are determined by using Table 3.12.

Step 2: Fuzzifying decision matrix**.**

For m alternatives which are sectors S_i in our study along with n evaluative criteria C_j decision matrix is constructed, followed by its fuzzification to get a generalized fuzzified decision matrix, which is given by (3.21).

$$\widetilde{D} = \begin{pmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \tilde{x}_{ij} & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{pmatrix} \tag{3.21}$$

where $\tilde{x}_{ij}$ represents the triangular fuzzy number of j^{th} criteria and i^{th} alternative which can be calculated by $\tilde{x}_{ij} = (p_{ij}, q_{ij}, r_{ij})$. The fuzzified scale from crisp values can be evaluated in Table 3.30.

Table 3.30: Crisp values (CR) and corresponding triangular fuzzy number (TFN)

CR	**Definition of judgment**	**TFN**	**CR**	**Definition of judgment**	**TFN**
1	Negligible dominance	(0, 0, 1)	7	Medium good dominance	(5, 6, 7)
2	Strongly weak dominance	(0, 1, 2)	8	Good dominance	(6, 7, 8)
3	Very weak dominance	(1, 2, 3)	9	Demonstrative dominance	(7, 8, 9)
4	Weak dominance	(2, 3, 4)	10	Strongly good dominance	(8, 9, 10)
5	Medium weak dominance	(3, 4, 5)	11	Absolute dominance	(9, 10, 10)
6	Fair dominance	(4, 5, 6)			

Generally depending upon higher or lower values of criteria, benefit or cost criteria are classified. Since the present study criteria are benefit criteria, the evaluated fuzzy decision matrix $\widetilde{D}$ is given in Table 3.31.

Table 3.31: Fuzzy decision-making matrix

Sectors	C_1	C_2	C_3	C_4
S_1	(0,0,1)	(0,1,2)	(9,10,10)	(4,5,6)
S_2	(7,8,9)	(0,0,1)	(2,3,4)	(9,10,10)
S_3	(9,10,10)	(9,10,10)	(0,0,1)	(7,8,9)
S_4	(0,0,1)	(4,5,6)	(0,0,1)	(0,0,1)
S_5	(2,3,4)	(0,1,2)	(5,6,7)	(5,6,7)
S_6	(1,2,3)	(3,4,5)	(0,0,1)	(0,1,2)

Since there is a particular amount of uncertainty associated with criteria weights of alternatives, the generalized version of TOPSIS, fuzzy TOPSIS is maneuvered in this study. Initially, the criteria weights and criteria of each alternative are fuzzified on a scale of 11. Further, through the generalized mean defuzzification method, the fuzzified values are transformed into crisp values.

Step 3: Evaluating the weighted normalized fuzzy decision matrix.

Since the data of $\widetilde{D}$ can be obtained from different sources, there is a great need to normalize the matrix to acquire a dimensionless matrix to compare various criteria. Further, in this study, $\tilde{G} = [\tilde{g}_{ij}]$ represents the normalized fuzzy decision matrix where $\tilde{g}_{ij}$ is the normalized fuzzy value which can be obtained by the fuzzy operations given by (3.22).

$$\tilde{g}_{ij} = \begin{cases} \left(\frac{p_{ij}}{z_j^+}, \frac{q_{ij}}{z_j^+}, \frac{r_{ij}}{z_j^+}\right); \; if\ x_j\ is\ benefit\ criteria \\ \left(\frac{z_j^-}{p_{ij}}, \frac{z_j^-}{q_{ij}}, \frac{z_j^-}{r_{ij}}\right); \; if\ x_j\ is\ cost\ criteria \end{cases} \tag{3.22}$$

where z_j^+ and z_j^- represents the greatest and the least values of j^{th} criteria respectively.

Normalizing fuzzy decision matrix by (3.22), the evaluated normalized fuzzy decision matrix is given to be Table 3.32.

Table 3.32: Normalized fuzzy decision-making matrix

Sectors	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$	$\boldsymbol{C_4}$
$\boldsymbol{S_1}$	(0,0,0.1)	(0,0.1,0.2)	(0.9,1,1)	(0.4,0.5,0.6)
$\boldsymbol{S_2}$	(0.7,0.8,0.9)	(0,0,0.1)	(0.2,0.3,0.4)	(0.9,1,1)
$\boldsymbol{S_3}$	(0.9,1,1)	(0.9,1,1)	(0,0,0.1)	(0.7,0.8,0.9)
$\boldsymbol{S_4}$	(0,0,0.1)	(0.4,0.5,0.6)	(0,0,0.1)	(0,0,0.1)
$\boldsymbol{S_5}$	(0.2,0.3,0.4)	(0,0.1,0.2)	(0.5,0.6,0.7)	(0.5,0.6,0.7)
$\boldsymbol{S_6}$	(0.1,0.2,0.3)	(0.3,0.4,0.5)	(0,0,0.1)	(0,0.1,0.2)

Let $\tilde{O} = [\tilde{o}_{ij}]$ represents the weighted normalized decision matrix obtained by the tensor product $\tilde{o}_{ij} = \tilde{g}_{ij} \otimes B_j$, where B_j is the final weighted value of j^{th} criteria. Table 3.33 represents the final weighted normalized fuzzy decision matrix.

Table 3.33: The final weighted normalized fuzzy decision matrix

Sectors	C_1	C_2	C_3	C_4
S_1	(0,0,0.032)	(0,0.009,0.039)	(0.105,0.0179,0.416)	(0.135,0.281,0.463)
S_2	(0.055,0.135,0.287)	(0,0,0.019)	(0.023,0.054,0.166)	(0.303,0.562,0.772)
S_3	(0.071,0.169,0.319)	(0.049,0.090,0.197)	(0,0,0.042)	(0.236,0.449,0.695)
S_4	(0,0,0.032)	(0.021,0.045,0.118)	(0,0,0.042)	(0,0,0.077)
S_5	(0.016,0.051,0.128)	(0,0.009,0.039)	(0.058,0.107,0.291)	(0.169,0.337,0.540)
S_6	(0.008,0.034,0.096)	(0.016,0.036,0.098)	(0,0,0.042)	(0,0.056,0.154)

Step 4: Formulating a fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS).

Let I_+ and I_- denotes FPIS and FNIS respectively, then by weighted normalized fuzzy decision matrix, FPIS, and FNIS can be given by (3.23).

$$I_+ = (\widetilde{o_1}^+, \quad \widetilde{o_2}^+, \dots, \widetilde{o_n}^+) \tag{3.23}$$

$$I_- = (\widetilde{o_1}^-, \quad \widetilde{o_2}^-, \dots, \widetilde{o_n}^-)$$

where $\widetilde{o_i}^+$ and $\widetilde{o_i}^-$ represent the greatest and smallest generalized mean fuzzy numbers, respectively.

Table 3.34: Generalized mean

Sectors	C_1	C_2	C_3	C_4
S_1	0.010625	0.016116	0.233043	0.293006
S_2	0.159165	0.006553	0.081059	0.545776
S_3	0.186334	0.112048	0.013858	0.46011
S_4	0.010625	0.061665	0.013858	0.025736
S_5	0.06468	0.016116	0.152134	0.348708
S_6	0.045783	0.050278	0.013858	0.070201

For any fuzzy number $\tilde{x}_{ij} = (p_{ij}, q_{ij}, r_{ij})$, the generalized mean is obtained by (3.24). (Wang and Chan 2013)

$$M(\widetilde{o}_{ij}) = \frac{r_{ij}^2 - p_{ij}^2 - p_{ij}q_{ij} + q_{ij}r_{ij}}{[3(r_{ij} - p_{ij})]} \tag{3.24}$$

and a generalized mean of the six evaluated sectors over the criteria can be observed from Table 3.34.

Step 5: Obtain a hierarchy of sectors by computing distance from FPIS or FNIS**.**

After obtaining I_+ and I_- , the alternative distances ($d^+$and d^-) are evaluated by the method of area compensation as by (3.25).

$$\tilde{d}_k^+ = \sum_{j=1}^{m} d(\tilde{o}_{ij}, \tilde{o}_j^{\,+}) \\ \tilde{d}_k^- = \sum_{j=1}^{m} d(\tilde{o}_{ij}, \tilde{o}_j^{\,-}) \tag{3.25}$$

Table 3.35: Alternative distance and relative closeness index

Sectors	d^+	d^-	$\tilde{R}_k$	$1-\tilde{R}_k$	**Ranking**
S_1	0.593653	0.655367	0.475295	0.524705	4
S_2	0.358884	0.997969	0.264497	0.735503	1
S_3	0.453915	0.892788	0.337057	0.662943	2
S_4	1.09472	0.110361	0.90842	0.09158	6
S_5	0.482272	0.667982	0.419274	0.580726	3
S_6	0.998963	0.148815	0.870345	0.129655	5

A relative index $\tilde{R}_k$ based on closeness is calculated by combining $d^+$and d^- as (3.26).

$$\tilde{R}_k = \frac{\tilde{d}_k^-}{\tilde{d}_k^+ + \tilde{d}_k^-} \tag{3.26}$$

Table 3.35 represents the relative closeness index of alternatives along with the final ranking.

3.9 Summary and Conclusions

BSE SENSEX is significantly growing year by year and is expected to grow at a much faster pace due to the exceptional GDP growth rate of India and hence its suitable time for investors to build a well-diversified sectored portfolio. Prioritizing of sectors in the right proportions is a must, for a healthy portfolio. A careful assessment of parameters is made by choosing two of widely used fuzzy Multi-criteria decision making (MCDM) technique’s which are fuzzy AHP and fuzzy TOPSIS. This chapter highlights the significance of important parameters that can give more returns and an opportunity to have a healthy portfolio. It has to be noted that only six major sectors are considered based on market capitalization and impact on GDP during the years 2012 to 2018.

The findings of the study infer that all sectors have positive skewness. It is noted that IT sector possesses a high value of skewness and low positive kurtosis whereas the Oil sector with high skewness and high kurtosis. Besides these, the rest four sectors have negative kurtosis with the Pharma sector attaining the least. Further, it is observed that all the sectors don't follow a particular trend and are highly volatile. From the findings, it is evident that IT sector outpaces other sectors in terms of volatility. Taking optimistic, pessimistic, and moderate views, the order of priorities is calculated. The orders of hierarchy obtained from both of the considered MCDM techniques vary even though similar input and weights are assigned for each of the sectors. In fuzzy AHP, the IT sector is found to be dominant than other sectors by a huge margin followed by the automobile and finance sector. The remaining sectors are lagging far behind in terms of performance. IT is observed to be dominant, which is followed by Automobile, Finance, Pharma, Oil, and Power. Furthermore, the hierarchy is drawn, and the relative dominance helps for optimal investment and can aid proportional investments in the future.

On the contrary to fuzzy AHP, the order of priorities is also observed by fuzzy TOPSIS in terms of cumulative scores. It is found that the Automobile sector to be dominant than other sectors by a huge margin followed by IT and Pharma sectors. The remaining sectors are lagging far behind in performance. Hence there is a great necessity to construct a portfolio which is in line with both techniques and manages according to the perception of users.