Appendix I

```
clear
clc
n = 50;            % Size of the swarm " no of birds "
bird_setp  =50;    % Maximum number of "birds steps"
dim = 2;           % Dimension of the problem

c2 =2.0;           % PSO parameter c1
c1 =2.0;           % PSO parameter c2
w =0.9;            % pso initial weights
fitness=0*ones(n,bird_setp);

          %------------------------------%
          %    initialize the parameter %
          %------------------------------%

r1 = rand(dim, n);
r2 = rand(dim, n);
current_fitness =0*ones(n,1);

          %-------------------------------------------------%
          %   Initial swarm and velocities positions        %
          %-------------------------------------------------%

current_position = 10*(rand(dim, n)-.5);
velocity = .3*randn(dim, n) ;
local_best_position  = current_position ;


          %-----------------------------------------%
          %     Evaluate initial population         %
          %-----------------------------------------%

for i = 1:n
    current_fitness(i) = tracklsq(current_position(:,i));
end


local_best_fitness  = current_fitness ;
[global_best_fitness,g] = min(local_best_fitness) ;

for i=1:n
    globl_best_position(:,i) = local_best_position(:,g) ;
end
                          %-------------------%
                          % UPDATED VELOCITY  %
                          %-------------------%

velocity = w *velocity + c1*(r1.*(local_best_position-current_position)) +
c2*(r2.*(globl_best_position-current_position));
```

```
                        %------------------%
                        %   SWARM UPDATE   %
                        %------------------%

current_position = current_position + velocity ;

                      %-------------------------%
                      %  evaluate new swarm     %
                      %-------------------------%
```

Main Loop

```
% Iterations'counter %

iter = 0 ;
while  ( iter < bird_setp )
iter = iter + 1;

for i = 1:n,
current_fitness(i) = tracklsq(current_position(:,i)) ;
end


for i = 1 : n
        if current_fitness(i) < local_best_fitness(i)
           local_best_fitness(i)  = current_fitness(i);
           local_best_position(:,i) = current_position(:,i)    ;
        end
 end


 [current_global_best_fitness,g] = min(local_best_fitness);


if current_global_best_fitness < global_best_fitness
   global_best_fitness = current_global_best_fitness;

    for i=1:n
        globl_best_position(:,i) = local_best_position(:,g);
    end

end


 velocity = w *velocity + c1*(r1.*(local_best_position-current_position)) +
c2*(r2.*(globl_best_position-current_position));
 current position = current position + velocity;

 sprintf('The value of interation iter %3.0f ', iter );


end % end of while loop its mean the end of all step that the birds move it
```

```
xx=fitness(:,50);
[Y,I] = min(xx);
current_position(:,I)
```

Appendix II

Fractional Lion Algorithm Pseudo-code

Input: Data points S_M, S_F, S_N

Output: Optimal centroid

Procedure

Start

Read initial points, S_M, S_F

Evaluate fitness function

Calculate Fertility

If $R^{ref} \leq f(S_M)$

$R^{ref} \leftarrow f(S_M)$

End

If $f(S_l^{F+}) \leq f(S_l^F)$

$S_l^{F+} \leftarrow S_l^F$

End

Reset L_r *and* S_r

Cross-over and mutation S^{new} *and* S^C

Gender clustering S_{M_C} *and* S_{F_C}

Set A_c

Generation based on fractional calculus

$S^{lion} = \alpha S_l^M + 0.5\alpha S_{l-1}^M$

Provincial defense

If S_N *wins*

$S_M \leftarrow S_N$

End

Territorial occupancy

If $f(S_M) \leq f(S_{M_C})$

$S_M = S_{M_C}$

else if $f(S_F) \leq f(S_{F_C})$

$S_F = S_{F_C}$

End

Clear S_r

Until then iterate

$N_f^{max} < N_f$, *Optimal centroid obtained*

End

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List of Publications

The research works included in the thesis have been published in following peer reviewed articles.

1. Pritam, K. S., Mathur, T., Agarwal, S., and Mathur, H. D. (2020). New fractional PID-controller to mitigate frequency variations in power systems. *Journal MESA*, *11*(2), 333-346.

2. Pritam, K. S., Mathur, T., and Agarwal, S. (2020). Hierarchy of Sectors in BSE SENSEX for Optimal Equity Investments Using Fuzzy AHP. In *Intelligent Communication, Control and Devices*, 393-404. Springer, Singapore.

3. Pritam, K. S., Mathur, T., and Agarwal, S. (2019). An Efficient Portfolio Management for Trading Under Uncertain Environment. *Int. J Sup. Chain. Mgt Vol*, *8*(2), 277.

4. Pritam, K. S., Mathur, T., Agarwal, S., and Paul S.K. A Novel Methodology for Perception-based Portfolio Management. *Annals of Operations Research* (Under Review).

5. Pritam, K. S., Sugandha, Mathur, T., and Agarwal, S. Underlying Dynamics of Crime Transmission with Memory. *Chaos, Solitons & Fractals* (Under Review).

List of Paper Presentations in International/National Conferences

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The following works have been presented in International/National conferences

1. "Fractional Bioheat using Finite Element Method", *International Conference on Special Functions and Applications*, Society of Special function & their Applications, Department of Mathematics, University College of Engineering and Technology, Bikaner, Rajasthan, 21-23, October 2019.

2. "P-index: Fuzzy Fractional Indicator for the Management of Portfolio", *International Conference on Business Analytics and Operations Research*, Manipal University, Manipal, Karnataka, June 14-16, 2019.

3. "An Efficient Framework for the Portfolio Management using Multi-Criteria Techniques" *International Conference on Management, Sciences, Engineering and Applications*, Baba Institute of Technology and Sciences, Visakhapatnam, 20, December 2018.

4. "A new efficient fractional order PID controller to minimize transient response" *International Conference on Special Functions and Applications*, Society of Special function & their Applications, Department of Mathematics, University College of Engineering and Technology, Bikaner, Rajasthan, 02-04, November 2017.

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