Modeling and Simulation of Synchronous Motor with Emphasis on Aspects of Stability

THESIS

Submitted in partial fulfilment
of the requirements for the degree of
DOCTOR OF PHILOSOPHY

by

SUNIL THOMAS

Under the Supervision of
Dr. Adhir Baran Chattopadhyay

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
2015

Modeling and Simulation of Synchronous Motor with Emphasis on Aspects of Stability

THESIS

Submitted in partial fulfilment
of the requirements for the degree of
DOCTOR OF PHILOSOPHY

by

SUNIL THOMAS

Under the Supervision of
Dr. Adhir Baran Chattopadhyay

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
2015

BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

CERTIFICATE

This is to certify that the thesis entitled, **‘Modeling and Simulation of Synchronous Motor with Emphasis on Aspects of Stability’** and submitted by **Sunil Thomas,** ID No **2010PHXF025U** for award of Ph.D. of the Institute embodies original work done by him under my supervision.

Signature of the Supervisor
Dr. ADHIR BARAN CHATTOPADHYAY,
Associate Professor,
Department of Electrical and Electronics Engineering,
BITS Pilani, Dubai Campus.

Date:

ACKNOWLEDGEMENTS

Firstly, I would like to express my heartfelt gratitude to Prof. R. N. Saha, Director, BITS Pilani, Dubai Campus, who has given an opportunity to carry out my ambition of doing a PhD study in my life.

I also express my sincere thanks to Dr. Neeru Sood, Associate Dean, Academic Research Division, BITS Pilani, Dubai Campus, for her immense support and encouragement.

Words are not just enough to express my sincere gratitude to Dr. Adhir Baran Chattopadhyay, my supervisor, for his continuous support during my PhD study and research. He is a 'GURU' and not a mere supervisor to me. His patience, motivation, enthusiasm, and immense knowledge have helped me to gain confidence during the study. His guidance helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and supervisor for my PhD study.

Besides my supervisor, I would like to thank, Dr. R Gomathi Bhavani and Dr. V Santhosh Kumar, my Doctoral Advisory Committee members, and all the Doctoral Research Committee members for their encouragement, insightful comments, and constant motivation.

I thank my fellow colleagues at BPDC for offering me the best of support during my hard times.

Last but not the least, more than anyone else, I would like to thank Mrs. Neethu Sara Kuruvilla, my adorable wife, without her continuous support and sacrifices; nothing would be even possible for me today.

SUNIL THOMAS
2010PHXF025U

ABSTRACT

Steady state stability criteria of A.C. drives play a dominant role for making the drive system practically successful. Generally such analysis is done using small perturbation model, but the complexity of the mathematical formulation depends on the nature of the drive system used. This thesis, at the first stage presents a detailed analysis of steady state stability criterion based on small perturbation model of a current source inverter fed synchronous motor drive system taking *direct*-axis and *quadrature*-axis damper winding into account using generalized theory of electrical machines. The modeling also clearly shows that even at no load the system satisfies steady state stability criterion. Routh-Herwitz criterion is used to finalize the result. The methodology of the proposed research work can be stated as follows: The synchronous motor has been treated as a five coil primitive machine model using the concept of generalized theory of electrical machines. Using the concept of Park's transformation, the armature current in *d*-*q* model has been represented by suitable equations as a function of armature current magnitude in phase model (I_s) and the field angle (β).As the system under consideration is basically a current source inverter fed system, I_s has been considered as a constant and as a consequence the field angle (β) finally appears as a control variable. Furthermore voltage balance equations of the five coils have been expressed in time domain and those equations has been transformed accordingly after applying Laplace Transform technique. As the analysis is focusing on the steady state stability criteria, the author feels that small perturbation technique will be appropriate to apply on the transformed equations. Finally the transfer function $\Delta\beta(s)/\Delta T_L(s)$ have been formulated; where $\Delta\beta(s)$ and $\Delta T_L(s)$ represent small change in transformed field angle load torque respectively. The analysis concludes that the absence of damper winding leads to instability of the machine system.

As synchronous motor has no inherent starting torque, generally it is started as an induction motor with the help of a damper winding and it pulls into synchronism under certain conditions. The second stage of this thesis, exactly concentrates on this particular zone of transition from induction motor mode to synchronous motor mode for a current source inverter fed synchronous motor drive system. Due to complexity of synchronous motor in terms of number of windings and finite amount of air gap saliency, direct modeling of such transition zone in time domain becomes cumbersome at the first instance of modeling. That is why the modeling in complex frequency domain (s-domain) has been taken up using small perturbation model. Such a model clearly shows the role of induction motor as noise function or disturbance function with respect to the open loop block diagram of synchronous motor. Such finding can be quantized in terms of important results and that is presented in a manner such that the results can help the designer for the successful design of a synchronous motor drive system.

Even though direct modeling of such transition zone in time domain becomes cumbersome at the first instance of modeling, ultimately it becomes very much necessary to develop and to have a deeper view of the dynamics of the asynchronous mode of the synchronous motor. That is why firstly the modeling is presented in complex frequency domain and then the time domain modeling is obtained by applying inverse Laplace Transform technique. Furthermore the time domain response of the disturbance function may help a designer to fix up the time instant when the pull in phenomenon will be imposed by throwing the field winding to a DC supply. This is the third stage of the thesis.

It is well known that the transfer function plays a dominant role in the behavior of a three phase synchronous motor drive system from the view point of steady state stability analysis.

Simultaneously it is also true that the transfer function with respect to those machine design parameters of the synchronous motor should be developed in a way such that the magnitude of the sensitivity of the transfer function may lie within a tolerable range. This particular aspect of research problem is taken up as the formulation and the concerned solution constitutes the final stage of the thesis. Such view point of the fabrication process can lead to the development of a new prototype of synchronous motor to become successful in the open market competition. In the present thesis, those machine design parameters have been selected from the view point of experience and practical applications and accordingly the sensitivity analysis have been brought into a particular shape for ease of fabrication process.

As a whole, the total thesis can be looked upon as a research work on the modeling aspects of a three phase current source inverter fed salient pole synchronous motor drive system starting from the modeling viewed with respect to steady state stability phenomenon and ending at the sensitivity analysis of a particular objective function of the motor through the path of the mathematical modeling of the transition zone(in complex frequency domain and also in time domain) of the synchronous motor started as an induction motor with the help of damper windings.

TABLE OF CONTENTS

Page

CHAPTERS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS/SYMBOLS

i_d= current in the D coil in p.u.

i_q=current in the Q coil in p.u.

i_f=current in the F coil in p.u.

i_{kd}=current in the d axis damper coil (KD) in p.u.

i_{kq}=current in the q axis damper coil (KQ) in p.u.

L_d=self-inductance of D coil in p.u.

L_q=self-inductance of Q coil in p.u.

L_{md}=mutual inductance along d axis in p.u.

L_{mq}=mutual inductance along q axis in p.u.

L_{ff}= self-inductance of F (field) coil in p.u

R_f= resistance of the F (field) coil in p.u.

L_{kd}=self-inductance of the KD coil in p.u.

R_{kd}= resistance of the KD coil in p.u

L_{kq}=self-inductance of the KQ coil in p.u.

R_{kq}= resistance of the KQ coil in p.u

β= angle between the field (rotor) m.m.f. axis and armature (stator) m.m.f. axis

J= Polar Moment of Inertia of the machine in Kg-m^2

C.S.I= Current Source Inverter

PMSM= Permanent Magnet Synchronous Motor

Chapter 1: Introduction

It is well known that alternating current (AC) motors like three phase induction motors and three phase synchronous motors play a dominant role as a component of the AC drive system. Even though the three phase induction motor plays the role of workhorse in industrial applications, one inherent problem with this machine is that, decoupling of load component current and magnetizing component current becomes a tedious job. This is the natural difficulty in the case of an electrical motor based on singly excited magnetic field system. Seeing such lacuna, a good number of applications supported by three phase synchronous motor have come into industrial market during last five to ten years. It is not that before ten years synchronous motor was not able to work as a drive component, but the point to be noted is that advent of power electronics in last ten years has naturally compelled synchronous motor to appear in the application area as a stronger driving force.

Based on the above said paragraph, it reveals that the horizon of research problems has opened automatically in the area of synchronous motor drive systems. This opening can be considered as a natural phenomenon because synchronous motor, from the constructional view point has one main complexity which is the air gap saliency and the additional power electronic devices have their own complexities involving topology of the converter/inverter circuits, terminal characteristics of individual power devices and reliability of the power devices during the long period of operation. It is obvious that variation of air gap length along the armature periphery leads to the concept of two extreme air gap permeances. Hence from the view point of mathematical modeling any aspect of synchronous motor and power electronics combination activity cannot be modeled so easily. The reason behind such statement lies in the fact that even though the power electronic components are not subjected to any saliency effect, but it is a must that the combined model must be mathematically treated taking air gap saliency, as a major parameter into account.

Based on the above said fact, it reveals that some considerable amount of analytical investigations are needed in the area of different aspects of modeling of synchronous motor drive system starting from the aspect of steady state stability and ending at the sensitivity analysis aspect, travelling through different operating zones like induction motor mode to synchronous motor mode transition zone and pure synchronous motor zone covering the pull-in phenomenon. Such analytical investigations become the subject matter of the present thesis.

1.1.Academic Aspects of the Research Problem

a) Based on the literature survey it has been observed that a good number of industrial applications are based on current source inverter fed three phase synchronous motor drive system. In the present thesis, steady state stability aspects of such systems incorporating the influence of damper windings (both direct and quadrature axes) has been taken up as a first research problem of the present research work. The major academic aspect from this modeling work comes out to express the machine voltage and torque balance equations, not as space dependent equations rather as space invariant equations. The beauty of such academic interest lies in the fact that a researcher can question himself(or herself), what is the meaning of space invariance. The answer is quite logical. Here the space invariance concept comes in context to variation of air gap permeance when the observer jumps from direct-axis to quadrature-axis. Hence the terminology “space invariancy”, automatically indicates that the armature winding m.m.f can be resolved along '*d*-*q* axes' and this resolved m.m.f can represent two lumped coils placed on '*d*-*q* axes' separately. Taking the effect of field winding and direct-quadrature windings, as an overall combination, all windings can be placed on two axes (d and q). This is the concept of axes model which is the theoretical representation of space invariance.

b) As this whole thesis is devoted to the analytical investigations on different behavioral aspects of a synchronous motor drive system, we can turn our eyes to the starting phenomenon of a synchronous motor. The classical concept of “George’s phenomenon” is automatically involved whenever a researcher concentrates on the modeling of process of synchronous motor. Till now the “George’s phenomenon” has been looked upon as a qualitative fact, but it is very rare to observe the exact mathematical inclusion of “George’s phenomenon” in the time domain or complex frequency domain model of synchronous motor. Hence the next important academic aspect inherently comes out to express “George’s phenomenon” as a part of the whole mathematical model of synchronous motor during the period of transition from induction motor mode to synchronous motor mode.

c) It is academically well known that the concept of “slip” plays a major role when the machine works as an induction motor and this particular parameter conceptually vanishes in the case of the same machine when it works as a synchronous motor due to constancy of its speed at synchronous value. Therefore if someone models a synchronous motor as a set of equations involving axis currents and axis voltages, then it becomes a challenge how to include the transition zone from induction motor mode to synchronous motor mode in the mathematical model of the concerned research work. Such inclusions ultimately boils down to the academic problem on how to bring 'slip' into picture such that the future researcher community will have a clear concept on how a synchronous motor mode can be thought of jumping from induction motor mode. Such aspect is looked upon as a major academic interest.

d) When a synchronous motor is started as an induction motor with the help of damper winding ultimately it jumps to synchronous motor mode, but a considerable amount of time elapses during which the machine behaves as an induction motor. Hence to model such process, it is obvious that the influence of induction motor must appear as an additional function or disturbance function. Now it is the responsibility of the researcher whether to express this disturbance function as a time domain function directly or going through the ‘s’ domain modeling. Such aspects form the subject matter of other major academic interest.

e) The success of giving conclusion in the above said point depends on researchers convenience about the choice of the mathematical transformation technique to be applied on the time differential equation of the said research problem. The choice of a particular mathematical theory of transformation technique, logically depend on the nature of the physical system for which the differential equation(in time frame) have been formulated. A current source inverter fed three phase synchronous motor is switched at time, t=0 second, and the whole drive system has a finite amount of energy to be dissipated during the running of the system. The combination of energy dissipating element and energy storing element in such system leads to the introduction of the complex frequency, 's', which is used as a kernel of Laplace Transform operator. Furthermore, once the concept of t=0 second is a must to incorporate in the mathematical modeling to justify the actual physical problem, the concept of initial value(or energy trapped in the period before switching) must appear in the transformed equation obtained through the application of the suitable mathematical transform operator. To satisfy this need, Laplace Transform operator comes out to be the only choice because in contrast, Fourier Transform operator once applied to a time differential equation does not need the necessity of appearance of initial value in the formulation. Furthermore to have a clear cut view of the time domain behavior of such disturbance function, application of Inverse Laplace Transform is a must and it increases the beauty of the research work. Such discussion backed by specific analysis also forms the subject matter of another academic interest of this research problem.

1.2. Application Aspects of the Research Problem

a) There are certain case studies or specific examples where synchronous motor is used for greater commercial success. Such examples are furnace blowers(driven by synchronous motor) in steel industry(M/s TATA STEEL, India). Another specific example of synchronous motor used as a single phase synchronous motor is to control the position of the guiding shaft in three phase induction regulator. Such induction regulators are used in some many industries(e.g., M/s Crompton Greaves Ltd, India) to control the bus voltage. In this particular application the synchronous motor is generally named as Auto Synchronous Motor revolving at constant speed such that phase change of the rotating magnetic field in induction regulator is created.

b) It is well known that in an electrical machine based on the principle of multiply excited magnetic field excitation system, the process of armature reaction is complicated and as a result the torque angle or load angle appears as a major parameter to decide the performance index of that particular machine. Exactly such statement is also valid for a three phase synchronous motor but at this particular stage we are over simplifying the phenomenon because we are forgetting to indicate that along with torque angle, the magnitude of armature current also becomes a dominating factor to quantify(signify) an armature reaction process. Hence so far as the application of synchronous motor to a specific load is concerned the armature reaction process can be looked up as a function of multi variables. Here the term “Multi” means two and the variables are magnitude of armature current and phase of armature current (which is near to the torque angle). Now if such specific application is considered where both the magnitude and phase are to be controlled then practically or experimentally it becomes a very difficult or challenging job. To come out from this practical difficulty, help of power electronics is a must. As a result in commercial sectors, application of synchronous motor fed by a Current Source Inverter (C.S.I) has made the industrial life more smooth and easy.

The main theme of the above said discussion is that in a C.S.I fed synchronous motor, armature current becomes substantially constant and the practical control circuit will only have to take care of the variation in the torque angle or load angle. That is why in the present thesis analysis of C.S.I fed synchronous motor from different technical viewpoints has been taken up.

c) In any engineering problem generally analysis becomes the first step and when certain finite conclusions from this analysis are drawn then only one work remains. This remaining work must be the design of the particular equipment or machine or part of the machine. It is well known that “sensitivity” plays a dominant role in designing a system or sub-system. In this particular research problem also, sensitivity has a greater role. The job of the researcher is to find the specific machine design parameters, tuning by which the objective function can be controlled. This constitutes a major application on machine design aspect of the current research problem presented in this thesis.

1.3. Explanation/Discussions on Backgrounds Needed to Carry Out the Research Problem

To carry out this complete research work certain specific academic backgrounds are necessary and each of the backgrounds is being outlined in the following subsections for a greater clarity of the thesis:

1.3.1. Generalized Theory of Electrical Machines

The Generalized theory of electrical machines is based on the concept of primitive machine model which comprises of windings placed on two axes(d-q-axes).In such model the position of the actual winding is not important. But its resolved components (which are generally pseudo windings) are more important such that the expressions for voltage balance and torque balance equations (differential equations) do not include any non-constant co-efficient. For the present research problem the primitive machine model becomes a five member model and the parameters of these members (windings) are subjected to be included in the voltage balance or torque balance equations.

1.3.2. Small Perturbation Model

To the best of our understanding, the literature on synchronous motors and its applications have not put much light on the effect of damper windings on the aspects of steady state stability of a three phase C.S.I fed synchronous motor drive system. To carry out the analysis of such effect created by damper windings, small perturbation model is a must because this is the model which can successfully analyze the steady state stability phenomenon. Even though it is technically named as a small perturbation model, it is basically a mathematical treatment based on Taylor’s series expansion and thereafter truncating certain terms. This particular analysis is also known as linearization technique. The beauty of this particular model(linearization technique) is not only to give conclusions on steady state stability aspects but also it can be smoothly extended in consecutive small differential time elements such that it can be used also for analyzing transient stability problems.

1.3.3. Application of Laplace Transforms

As the motivation behind the thesis was initiated with the investigations on the steady state stability aspects, development of a suitable transfer function in Laplace domain became an immediate job to start the research work. As a natural consequence, Laplace Transform was needed to convert the time differential equations of the primitive machine model of synchronous motor, into algebraic equations in "s" domain. Now the question comes before a researcher that out of the tools available for Laplace transforms, i) Normal or single sided Laplace transform and ii) Double sided Laplace transform, which one is to be applied for the present research problem. Generally experience tells that whenever differential equations in space are developed in specific applications, the disturbance at origin or zero space point is not guaranteed. In such cases double sided Laplace Transform, which is an integral transform based on the integration under minus infinity to plus infinity limits, is applied.

But the present research problem does not come under the above said category and it is falling under time differential category. As it is a time differential equation naturally from the practical view point of starting the machine at zero time origin is automatically guaranteed. Hence application of normal Laplace Transform (Single sided) to the time differential equations of the present research problem comes out to be a correct mathematical decision.

1.3.4. Sensitivity Analysis

If ‘u’ is a function of any engineering parameter ‘y’, and if the parameter is tuned, it becomes necessary to observe what are the corresponding changes happening to that parameter. If a small change in ‘y’ leads to a major change in ‘u’, then the system is said to be sensitive with respect to that parameter. This check becomes a necessity in designing practical engineering problems. The same concept can be applied to the current research work carried out in this research. The transfer function obtained in ‘s’ domain for the determination of the steady state stability of a current source inverter fed synchronous motor drive system generally becomes a frequency dependent function. For specific cases like determination of frequency responses etc., ‘$s=j\omega$’ can be used(where ‘s’ is the complex frequency and it is also the kernel of the Laplace Transform operator and 'ω' is the frequency of a sustained oscillating signal and also the imaginary part of the complex frequency) and the corresponding inferences on frequency responses can be drawn. But to treat this current research problem as a machine design problem the variation of parameters with respect to the transfer function is to be incorporated and hence sensitivity analysis study becomes an integral background work of this thesis.

1.4. Objectives of the Thesis

Following are the objectives of the whole research work:

a) To develop a detailed mathematical modeling in time domain and in complex frequency domain taking the effect of damper winding into account for getting an overall picture and also to establish the steady state stability criteria.

b) The exact role of induction motor during the starting process of a three phase synchronous motor is not generally quantitatively known. The second objective is to develop a mathematical model in complex frequency domain and to look upon the synchronous motor during starting as equivalent to the combination of a pure synchronous motor plus an asynchronous motor in the form of a disturbance function.

c) As a consequence of establishment of the second objective , the next one automatically comes out to be the modeling of the same machine in time domain as an extension of objective (b), during the transition zone from induction motor mode to synchronous motor mode.

d) Once the pre decided modeling aspects are bought into certain convenient shapes, then the approach is to fabricate a machine in the direction of objectives (a), (b), (c) turns out to be a great responsibility. The form of essential tool for such activity is to develop the mathematical model of sensitivity analysis. This becomes a major need and its formulation and necessary computation comes out to be the last objective.

1.5. Scope and Limitations of the Thesis

a) Using the concepts of Generalized theory of electrical machines and the established theory of control engineering, steady state stability analysis of a three phase current source inverter fed synchronous motor taking the effect of air gap saliency and damper windings into account was successfully completed.

b) If the above said machine is fed directly from a three phase bus , instead of being fed from a current source inverter, then the analysis of armature reaction and the consequent analysis for steady state stability can be done and it is kept as a future work , even though partially it can be treated as a limitation.

c) It becomes a general interest to know that exactly what is happening during the transition zone from induction motor mode to synchronous motor mode when the above said drive system is started. As a consequence of this type of investigation some scope remains to develop the mathematical model of such transition phenomenon in complex frequency and time domain separately. These responsibilities were successfully executed.

d) As an extension work of Laplace domain part of the above point, some investigations could have been done to represent the whole machine as a filter. Originally this filter could have been modeled in 's' domain and later it could have been converted to a digital filter. However as the direction of the research is taking a clear cut turn if the above said modeling is taken up. Based on this observation the above said aspect is not taken up as a part of the present thesis; rather it can be treated as a limitation or future scope.

e) Magnetic saturation has not been taken into consideration in the whole modeling carried out in Chapters 3,4,5 and 6. Such aspect may be looked upon as a limitation, because to maintain accuracy in the modeling taking the effect of magnetic saturation, obviously circuit theory becomes inadequate and electromagnetic field theory becomes the perfect choice.

1.6. Organization of the Thesis

The thesis is comprised of seven chapters. It is always advantageous to clarify the philosophy involved in each chapter separately. This particular objective is jotted down in the form of the following subsequent paragraphs.

Chapter-1 of this thesis basically presents the introduction of the whole research work. This chapter is again subdivided into seven different sections. This chapter's sections are in a logical way as per the flow of the thinking process. The first part gives up an introductory light about this particular chapter. It starts from the broader spectrum of activities of AC motors and drive systems and slowly jumps into the particular activity of synchronous motor as a component of AC drive system.

Furthermore this particular section indicates clearly the motivation behind the modeling of three phase synchronous motor. The first section of this chapter gives a view about the academic aspects of the current research work involved in this thesis. The second section indicates clearly the application aspects of this particular synchronous motor drive problem investigated in this thesis. The decoupling of the aspects into academic and application has been done intentionally with a view that future researchers can design new experimental setups as an extension to this current research problem, based on some solid theoretical foundation. The third section of this chapter illustrates the academic backgrounds behind the development of this thesis. The academic backgrounds can be categorized in two parts as engineering background and mathematical background. The engineering background constitutes of the generalized theory of electrical machines and the theory of sensitivity analysis. The mathematical background constitutes of application of integral transforms (Laplace Transforms) and small perturbation model (Linearization Technique) based on Taylors series expansion. The fifth section is about the organization of the thesis. The necessity of this section lies in the fact that any reader can make up his mind set to follow the mathematical and application philosophy presented in this thesis. Fourth and fifth sections of Chapter-1 clearly defines the objectives, scope and limitations of the thesis respectively. Section 6 of this chapter outlines the structure of this thesis and the last section of this chapter gives a concluding remark on the salient points presented in this chapter.

Chapter-2 is fully devoted to the discussions on the works carried out by the researcher in the direction of modeling and simulation of synchronous motor drive system and their applications. In other words, this particular chapter gives the portrait of the literature review. Even though the current research problem is based on the axes model of synchronous motor, in the past and recent past there have been a good number of works reported based on the phase model of synchronous motor. That is why to collect the information about various directions of research on synchronous motor; the literature review should have a wide horizon. This is the main reason behind devoting a chapter on literature review.

The steady state stability aspects of a current source inverter fed synchronous motor drive system taking the influence of direct and quadrature axis damper windings into account is investigated in Chapter-3 of this thesis. Such investigations are planned logically in different sections. Those sections include development of the primitive machine model; voltage balance equations of the members of the primitive machine model, in time domain; small perturbation model of those time domain equations; conversion of the time domain perturbed equations in ‘s’ domain after applying Laplace transform; compilation of torque balance equations in time an Laplace domains and finally expression for transfer function. In this particular chapter, the transfer function for such applications has been defined as the ratio of transformed small change in field angle to the transformed small change in load torque, with initial conditions relaxed. Generally in the academic curriculum the very important parameter of a three phase synchronous motor comes under continuous discussion and that parameter is torque angle or load angle. The concept of load angle basically comes out from the armature reaction process in a synchronous machine. The space angle between resultant flux and the field winding flux (i.e. direct axis) is generally known as load angle or torque angle. The whole dynamics of a synchronous motor under different operating conditions (like sudden load change, source unbalance, abrupt variations in parameter value due to temperature or site condition change etc.)depend on the time variation of the torque angle. This is the general procedure of performing any research on dynamics on synchronous machine. However in the current research problem exactly torque angle does not become a dominating factor rather field angle (which is the position of armature m.m.f in space with respect to direct axis) appears as a major variable to be included in the whole analysis. This particular transfer function directly shows how to predict the stability criteria. In the theory of stability from the view point of control system, a particular system may go to unstable region from stable region due to so many factors like irregularity in the supply system, sudden change in the parameter during the operation of the system, unhealthy closing of the system etc. But as the present problem comes into the category of electrical drives, it is quite natural that change in load torque becomes a very major influential factor to dictate that the motor will remain in stable or jump into unstable region. The inner meaning of this statement lies in the fact that a small change in load torque may lead to a considerable amount of change in field angle. The field angle is basically defined as the space angle between the armature current and direct axis. Generally armature current is not considered as a space vector while considered for analysis in electrical machines, rather armature m.m.f is considered as a space vector. However for the clear cut representation of analytical method, the position of armature current in space is used without sacrificing any accuracy in the analysis because m.m.f is the product of number of turns and current and number of turns is a given machine design parameter. Furthermore in this context it may be reminded that , in a synchronous motor ,change in load torque influences the torque angle or the load angle which is generally symbolized as 'δ'. However the estimation of 'δ' can be easily related to field angle (β) by simple geometrical consideration of the space phasor diagram of synchronous motor pertaining to armature reaction phenomenon. The resultant change in field angle, if becomes uncontrollable (or does not come to the state of convergence) forces the motor (or the drive system) to become unstable. This is, in a nut shell the reasoning behind the selection of this particular transfer function in Chapter 3

As synchronous motor has no inherent starting torque generally it is started as an induction motor with the help of damper winding and it pulls into synchronism under certain conditions. Chapter-4 exactly concentrates on this particular zone of transition from induction motor mode to synchronous motor mode for a current source inverter fed synchronous motor drive system. Due to complexity of synchronous motor in terms of number of windings and finite amount of air gap saliency, direct modeling of such transition zone in time domain generally becomes cumbersome at the first instance of modeling. That is why the modeling in complex frequency domain (s-domain) has been taken up using the small perturbation model. Such a model clearly shows the role of an induction motor as noise function or disturbance function with respect to the open loop block diagram of synchronous motor in Laplace domain(s-domain). Such findings can be quantized in terms of important results and that is done in the present chapter such that the result can help the designer for the successful design of a synchronous motor drive system.

Chapter-5 is basically a mathematical consequence of the treatment done in Chapter -4. Once the Chapter-4 enlighten itself as a modeling of noise function (or, disturbance function) in complex frequency domain, to know the behavior of that particular function in time domain becomes a necessity from the view point of the easy assessment of the exact time instant of pull in phenomenon of synchronous motor.Chapter-5 involves the time domain modeling (of the noise function) through the application of Inverse Laplace technique. Apparently it seems to be a straight forward mathematical treatment but involvement of convolution integral for converting the formulation from s- domain to time domain becomes a matter of interest and it may draw the attention of various researchers working in this area. Furthermore the time domain response of the disturbance function may help a designer to fix up the time instant when the phenomenon of pulling the motor into synchronism will be imposed by throwing the field winding to a D.C supply.

To fabricate a practical synchronous motor maintaining the functional aspects as described in Chapter 3, 4 and 5, the approach of machine design must be used. As a stronger tool in machine design sensitivity plays a dominant role. This particular modeling and computation of sensitivity of a synchronous motor under different or particular operating conditions form a subject matter of Chapter 6.The concept of sensitivity, in general, physically signifies that a particular objective function is responsive or immune to the tuning of a particular parameter involved in that function. To convert such philosophy to a suitable mathematical formulation, “partial differentiation" must be used. As a consequence in context to synchronous motor drive system, sensitivity analysis ultimately boils down to a complicated algebraic process. The complicacy involved in such mathematical treatment comes out from the fact that the particular parameters of a particular member of synchronous motor are coupled with other parameters, mathematically through algebraic equations. Furthermore the treatment of synchronous motor based on axis equations makes the sensitivity analysis complicated because we cannot avoid the magnetic couplings between the windings placed on a particular axis(d- or q-). That is why it reveals at this stage that the successful completion of chapter-6 requires a good amount of precaution. To avoid algebraic error, such precaution refers to the step by step follow up in performing the partial differentiation process and finally application of proper quiescent(or operating point) values of the concerned variables at proper stage of calculation.

The last chapter (Chapter-7) shows the end of the thesis with highlighting the exact contributions, the concluding remarks and the future scope of the research work.

1.7. Conclusions

At the present stage (i.e., at the end of chapter-1), it can be concluded that the thesis is expected to give contributions in the following directions (in context to a CSI fed synchronous motor drive system, taking air gap saliency into account):

- A detailed mathematical modeling showing the effect of damper winding parameters on the steady state stability aspects.
- Development of a mathematical model of the transition zone (from induction motor mode to synchronous motor mode) in complex frequency(s-domain).
- Development of a mathematical model of the transition zone (from induction motor mode to synchronous motor mode) in time domain.
- Adding design aspects (with analysis aspects) through the formulation for sensitivity analysis of the transfer function by tuning suitable machine design parameters.

Chapter 2: Literature Review

2.1. Introduction

Any research work in the area of three phase salient pole synchronous motor and the associated drive system faces a challenge of formulating the problem with greater accuracy and achieving the solution with a feasible interpretation. The nature of the problem formulation and the subsequent solution methodology depend on the type of synchronous motor and also on the nature of the converter/inverter involved in the drive system. Therefore to validate the results of the modeling of the current source inverter fed synchronous motor drive system presented in this thesis, a considerable amount of literature review is needed such that any reader of the present thesis can feel the originality of the thinking procedure involved in the mindset of the author of the present thesis. However the investigations carried out in different type of research works on synchronous motor and associated drive systems needs categorization under different subheadings such that the tendency of the researchers from time to time can be felt by a researcher who is trying to enter this area.

Based on the above said two philosophies this separate chapter (chapter 2) is being devoted to the aspects of literature and it is presented in the following sections.

2.2. Discussions on literature review

2.2.1. Transformation from 3-phase to 2-phase model and 3-phase to axis model

The paper entitled “Mathematical model of a salient pole synchronous motor supplied by a frequency converter” [1], is not directly related to stability studies of synchronous motors, but the beauty of this paper is that it can be considered as a foundation literature to start any type of analysis related to stability studies of synchronous motor.

The important point in favor of this work is that air gap saliency is not neglected and all the possible reference frames i.e., three phases (a, b, c), 2 phases (α,β) and two axes (d, q) have been taken up to carry on the analysis. Furthermore from α,β frame the flux linkage has been transformed using well defined algebraic equations. The well-established generalized torque equations have been used to calculate the electromagnetic torque.

The interesting part of work in this paper involves the method of simulation involving small changes or small oscillations. Such equations can easily be used to observe whether a small change in torque angle dies away in time or not. In other words, such work appears to be a strong tool of stability analysis of synchronous motor. Another important observation in this literature is that the author has done detailed analysis on synchronous motor associated with feedback control devices. The simulation results presented in this paper are clear and show the validity of the models used. However it is not clear from the paper that why the author has considered the synchronous motor supplied by a current source. The author of the present thesis feels that the reasoning behind considering the source as a current source can be looked upon in the following lines. If a voltage source is used then the armature reaction variables will be both armature current magnitude and torque angle; on the other side if a current source is used only the torque angle remains as a variable, because the fluctuations in the current delivered by a current source are assumed negligible. Hence the analysis and simulation of stability of synchronous motor fed by a current source becomes much easier.

2.2.2. Research work on permanent magnet synchronous motor

The paper entitled “Effect of Short Circuit Voltage profile on the transient Performance of Saturated permanent magnet synchronous motors”[2], is only taking care of permanent magnet synchronous motors and not normal synchronous motors. The insight philosophy of the above statement lies in the fact that if it is a normal synchronous motor then armature terminal voltage (input supply) becomes zero and such situation has not been analyzed by the researcher so far. Furthermore the paper has only dealt with the condition in which synchronous motor is rotating with synchronous speed.

The literature survey in the area of stability analysis of three phase synchronous motor indicates one important trend that a major amount of work deals with permanent magnet synchronous motor for a deep research investigation. The possible reason is that absence of field winding in any machine assembly structure makes the machine maintenance simpler and easier. That is why possibly various applications like railway traction take the advantage of such feature of permanent magnet synchronous motor. However, after a good amount of literature survey it reveals that to control the electromagnetic torque of synchronous motor if at all the field current has to be controlled, such feature cannot be offered by a permanent magnet synchronous motor.

In the paper titled “Stability analysis of a permanent magnet synchronous motors for railway vehicle traction in a sudden line voltage change” [3], the important observation in reference figure-1 of the above paper represents the block diagram of a current regulation system of permanent magnet synchronous motor taken up for the analysis. Obviously, it is natural that such block diagram indicates that field oriented or vector control of permanent magnet synchronous motor and here arise the question of why vector control is suggested for permanent magnet synchronous motor. The answer lies in the fact that even through a normal three phase synchronous motor is strictly considered as a device based on multiply excited magnetic field system, permanent magnet synchronous motor cannot be considered as this type of device because field current is not available as a separate variable to control the torque. Hence the permanent magnet synchronous motor is considered as a device based on singly excited magnetic field system similar to a three phase induction motor and that is why it needs a vector control system for a smooth operation.

With reference to this research paper the author of the present thesis feels that as it is a permanent magnet synchronous motor, the speed e.m.f contributed by the permanent magnet flux must be treated as a constant or fixed quantity for a particular drive speed. Hence the voltage balance equations of such machines in primitive machine model form is written in time domain then the quantity $\omega\Phi_f$ must appear as a dc voltage in the expression for Vq. Such concept in time domain must lead to a transformed voltage Vq(s) consisting of the term $\omega\Phi_{f}/s$ as a transformed speed voltage. The observation made by the author of the present thesis is that the above said term $\omega\Phi_f/s$ is absent in the matrix equation [1] of this paper, even though the kernel 's' is associated with the 'd' and 'q' axes inductances L_d and L_q separately. This particular point appears as a lacuna of the said research paper and that is why the modified equation along with other equations (2), (3) and (4) of this paper are presented below along with the concerned circuit diagram as shown in figure 2.1.

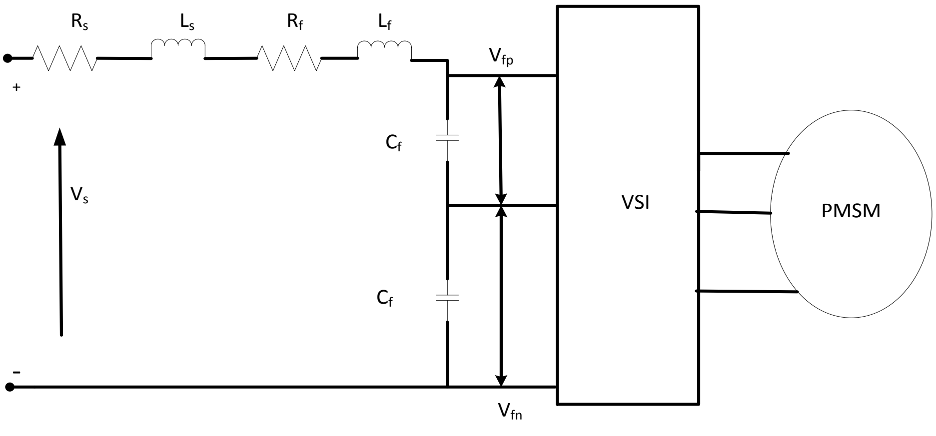


Figure 2.1.The traction circuit for analysis

$$c_f \frac{dv_{fp}}{dt} = i_{ip} - i_s \tag{2.1}$$

$$c_f \frac{dv_{fn}}{dt} = i_s - i_n \tag{2.2}$$

$$V_s = (r_s + r_f)i_s + (L_s + L_f)\frac{di_s}{dt} + (v_{fp} - v_{fn}) \tag{2.3}$$

Axis Model:

$$V_d = r_m i_d + L_d p i_d - \omega L_q i_q \tag{2.4}$$

$$V_q = r_m i_q + L_q p i_q + \omega L_d i_d + \omega \phi_f \tag{2.5}$$

Applying Laplace Transform to the time domain equations (2.4) and (2.5), it yields;

$$V_d(s) = \{r_m + L_d(s)\} i_d(s) - \omega L_q i_q(s) \tag{2.6}$$

$$V_q(s) = \omega L_d i_d(s) + \{r_m + L_q(s)\} i_q(s) + \frac{\omega \phi_f}{s} \tag{2.7}$$

The symbols used in eqns. (2.1) to (2.7) have the meanings as listed below:

V_d and V_q are the d-axis and q-axis voltages respectively.

id and iq, are the d-axis and q-axis currents respectively.

ω stands for the inverter angular frequency.

r_m, is the resistance of the amateur windings.

L_d, and L_q, are the d-axis and q-axis inductance respectively.

ϕ_f is the field flux by permanent magnet.

Equations (2.1) to (2.7) make the building blocks to find out the i_d and i_q time responses, such that the transient components can be identified more vividly.

From the earlier discussions it reveals that permanent magnet synchronous motor is similar to a three phase induction motor. The need of vector control looks upon three phase induction motor and permanent magnet synchronous motor as exactly the same machine because in both cases the excitation is only one. But the insight of the machine clearly indicates that the equivalent circuit of three phase induction motor consists of a shunt branch where as that of permanent magnet synchronous motor still remains as a series circuit like the normal synchronous motor.

Similar type of research in the area of permanent magnet synchronous motor has been dealt in the paper titled “Modeling and stability analysis of a permanent magnet synchronous machine taking into account the effect of cage bars” [4].The additional aspect to be noted in this work is that cage bar modeling is done successfully and the concept of operational impedances along ‘d‘ and ‘q’ axes $X_d(s)$ and $X_q(s)$ respectively are also incorporated in the above said work.

2.2.3. Nonlinear control aspect of synchronous motor

Another interesting research paper titled “Non Linear Control of an Inverter Motor Drive System with Input Filter–Large Signal Analysis of the DC-Link Voltage Stability” [5], deals with an analytical method allowing the large signal stability analysis of an electric system constituted by an input filter connected to an actuator. In this paper, Figure 2 shows the DC model of the voltage source. This figure is basically used in reference to Section B of the paper entitled “Application to the large Signal Stability Study of the dc-link Capacitance”. With reference to this section the expression for the operating point current and voltages are presented in the paper but the method of derivation is not presented. The author of the present thesis feel that if the process of derivation for quiescent variable, i_o and V_{so} are known clearly then in future to apply the “Tagaki Sugeno” method of fuzzy model will be very easy. From this view point, the detailed analysis for expression for i_o and V_{so} along with figure 2.2 of the above mentioned paper are presented as follows:

$$\dot{x} = Ax + Bu \tag{2.8}$$

‘A’ is not a matrix independent of state variable and hence the reason for non linearity of the state equation (2.8) is indicated by the fact that elements of [A] are functions of the state variables and this equation indicates a state equation in standard form.

[A] is defined as

$$\begin{pmatrix} \frac{-r_f}{L_f} & \frac{-1}{L_f} \\ \frac{1}{c} & f(x_2) \end{pmatrix} \tag{2.9}$$

$$f(x_2) = \frac{P}{CV_{so}(x_2 + V_{so})} \tag{2.10}$$

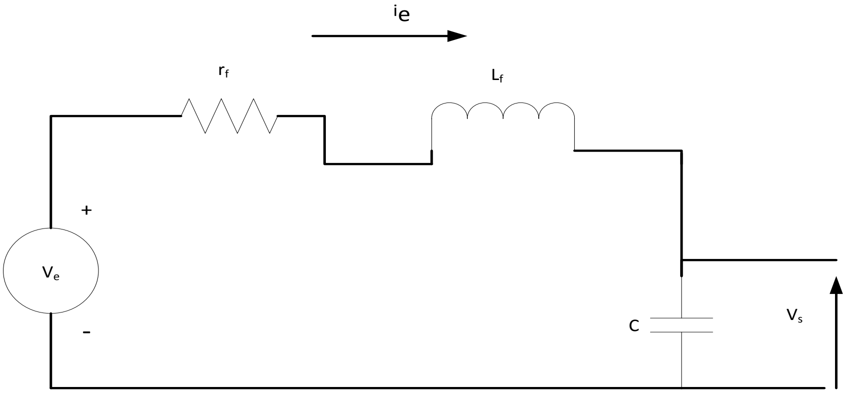


Figure 2.2. DC model of the voltage source.

$$\frac{di_f}{dt} = \frac{-r_f}{L_f} i_e + \frac{1}{L_f} V_s + u \tag{2.11}$$

$$i_o = \frac{v_e - v_s}{r_f} = \frac{P}{V_s} \tag{2.12}$$

and

$$x_1 = i_e \tag{2.13}$$

$$x_2 = \mathrm{v}_s \tag{2.14}$$

$$x_1 = i_e - i_o = i_e - \frac{v_e - v_s}{r_f} \tag{2.15}$$

$$P = i_o v_s = (\frac{v_e - v_s}{r_f}) v_s \tag{2.16}$$

$$P = \frac{(v_s v_e - v_s^2)}{r_f} \tag{2.17}$$

$$-v_s^2 + v_s v_e - p_{rf} = 0 \tag{2.18}$$

$$v_s = \frac{-v_e \pm \sqrt{v_e^2 - 4Pr_f}}{2} = V_{so} \tag{2.19}$$

The symbols used in equations (2.9) to(2.19) have the meanings as listed below:

Vs is the DC link voltage.

Ve is the applied DC Voltage.

r_f and L_f are the equivalent serial resistance and inductance.

P is the load power.

C is the capacitance.

i_e is the current from the DC source Ve.

i_s is the DC link current.

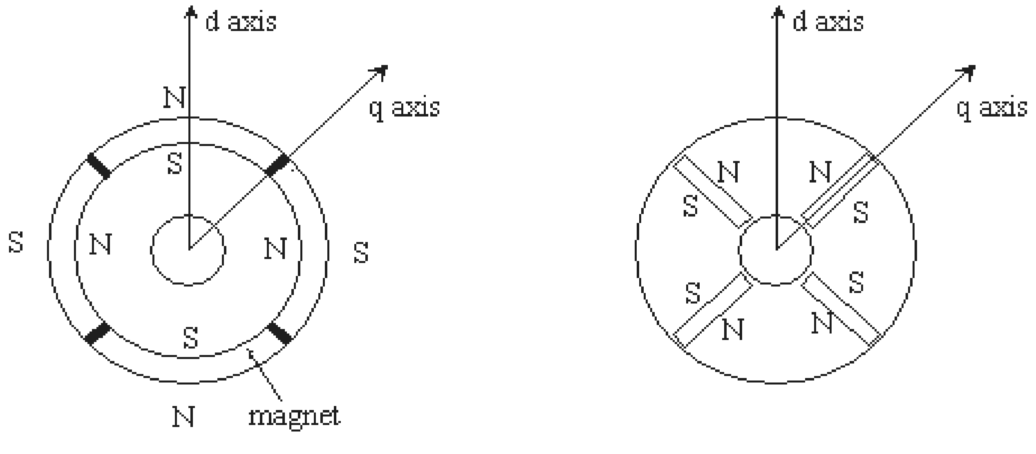
2.2.4. Effect of saturation on permanent magnet synchronous motor

The research paper entitled " Modeling and analysis of permanent magnet synchronous motor by taking saturation and core loss into account" [6], may be considered as one of the most important research output in ninety decades. It basically deals with modeling and simulation of permanent magnet synchronous motor. PMSM is of two types:

(a) Surface-Mounted Permanent Magnet (SPM) type

(b) Interior Permanent Magnet (IPM) type.

Both the machines are represented in figure 2.3.The first configuration basically leads to a uniform air gap configuration where as the second leads to a machine having a particular air gap saliency. For a SPM type configuration as the air gap is approximately uniform throughout the armature periphery, the term ‘L_d-L_q’ is almost zero. However, the scenario of flux paths is not straight forward in an IPM type configuration like that in SPM type. The fact is that for IPM type configuration the magnets are buried inside the machine and as a result air gap saliency in the rotor magnetic circuit is developed. In this context figure 2.3(a) and 2.3(b) will illustrate the constructional difference between SPM type and IPM type.



(a) (b)

Figure 2.3.Permanent magnet excited rotor (4 pole machine). (a) Surface-mounted-magnet type rotor. (b) Interior magnet type rotor.

Furthermore the permeability of the magnets placed on ‘d’ and ‘q’ axis are intentionally made different to have a control on flux distribution around the cylindrical position. Thus compared to the ‘d’ axis flux path the ‘q’ axis flux path is made to have a lower reluctance and in turn it will create a higher permeance. As a result armature reaction flux component along ‘q’ axis may tend to a value of saturation level in the rotor pole tip and the stator. Such phenomenon becomes serious when the load current is maximum.

Therefore with the variation of load current (Ia), a strong variation of ‘q’ axis inductance is expected in IPM type construction. The immediate question arises on what will be the approach of modeling such phenomenon in IPM type machine. The authors of this paper [6], did not put light in such direction, but the author of this thesis proposes some suggestion on the following lines: As the saturation phenomenon is not much serious along ‘d’ axis for a positive value of ‘d’ axis current(I_d>0), concentration of modeling should be given on ‘q’ axis phenomenon. By performing a suitable experiment the plot of Lq vs Iq can be obtained, where Lq and Iq stand for q-axis armature winding self-inductance and q-axis armature current of the synchronous motor, respectively.

2.2.5. Transient stability aspect of large synchronous motor

The paper entitled “Transient stability study of the large synchronous motors starting and operating for the isolated integrated steel-making facility” [7], is an interesting research work targeting the power system, so far as the transient stability is concerned. The strong side of this research work is that a good amount of online power system data such as average, peak loads and reactive power have been reported by the author and it gives the reader an approximate idea about the status of load flow. However the author of this thesis try to go in depth about two aspects which are explained as follows:

CASE A of section IV of the paper(Motor Starting and Operating Analysis)

With reference to this particular CASE A, the author of the paper has put the following statement, “Since an exciter will not be applied during the motor starting; it is therefore necessary to model the synchronous motor as an induction motor with equivalent parameter that match the motor starting characteristics”. The author of the present thesis look this particular statement in the direction of George’s phenomenon which is considered to be very important aspect of synchronous motor starting dynamics. Such explanation is presented in figure 2.4 in the form of a flow chart.

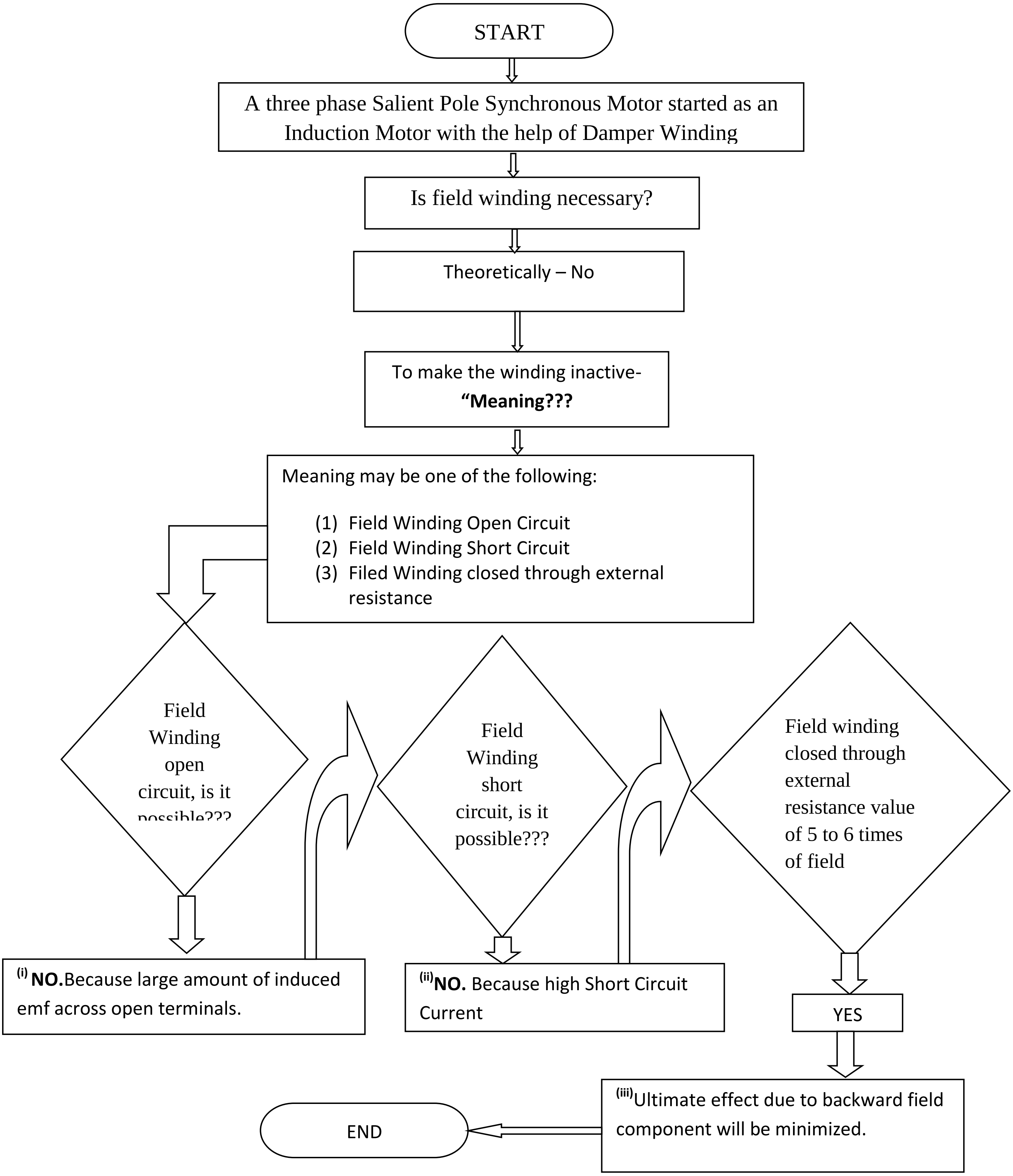


Figure 2.4. Flow Chart Explaining the George’s Phenomenon in Three-Phase synchronous motor.

With reference to figure.2.4, the technical points in box numbers (i), (ii) and (iii) are necessary to be explained in detail:

(i) The reason is that, the machine is already running with three phase supply as input to the armature terminals. Then good amount of transformer e.m.f is induced in open terminals of the filed windings because generally number of turns of field winding per phase is more than the number of turns of armature windings per phase

(ii) The reason is that, least impedance path will lead to high value of current through field winding. As a result the resolved backward rotating magnetic field component of pulsating field m.m.f will have large strength. So there is a chance that the speed of the motor may come down leading to the stage of stalling

The reason is that due to the high value of the external resistance, the strength of the pulsating field (m.m.f in the field winding) will be drastically reduced such that the stage of stalling of the machine may not be developed.

2.2.6. Some aspects of open loop control of permanent magnet synchronous motor drive system:

The paper entitled “Stable and highly efficient operation of open-loop controlled permanent magnet synchronous motor drive” [8], basically deals with the open loop control of permanent magnet synchronous motor(PMSM) drive system. As any control algorithm of a drive system must involve stability criteria, this paper also uses a suitable algorithm based on Lyaponov method of stability. Another important aspect of this paper is that the controller strategy is capable to maintain the input power factor of the drive system close to unity. The detailed model of the drive system starts with the presentation of four first order differential equations and they have been finally represented as a state model. Initially the model appears as a nonlinear state space model but after suitable application of small perturbation technique, the ‘A’ matrix(dimension 4x4) has been obtained in linearized form. It is well known that the Lyapunov criterion of stability assessment is fundamentally based on the selection of a possible energy function, V(x) for the drive system, where ‘x’ stands for the state variable. It is also well known that the criteria for a strictly stable Linear Time Invariant system may be presented as the unique matrix ‘P’ of the well-known equations:

$$A^TP+ PA = Q \tag{2.20}$$

$$V=X^TPX \tag{2.21}$$

must be symmetric positive definite. The authors have successfully applied these criteria and accordingly assured the stability. The authors of this paper have presented one important aspect which is the efficiency optimization algorithm of the drive system. In other words the inverter input voltage (V_{dc}) being constant, the DC link current (I_{dc}) is being minimized such that the efficiency can be optimized. The author is interested to indicate one important aspect that the DC link current is not directly being fed to permanent magnet synchronous motor rather through the PWM inverter. Hence in such case it is very necessary to ensure that corresponding to a minimum value of DC link current (I_{dc}), the output current of the PWM inverter also will be minimum. Such direction of research has not been explored by the author of the above said paper.

There exists one interesting research work entitled “Open Loop Stability Characteristics of Synchronous Drive Incorporating High Field permanent magnet Motor” [9], relating analysis of stability of synchronous motor drive system. The specialty of such motor is that it incorporates permanent magnet rotor of high magnetic pole strength. The construction of the rotor of this motor involves buried magnet as a part of permanent construction. The sectional view of such machine clearly indicates that the air gap saliency is automatically involved. That is why the phasor diagram of a salient pole synchronous machine has been used to involve load (torque) angle in the 'd' and 'q' axes voltage balance equations. The advantage of such machine is that unlike the traditional method of development of primitive machine model before analysis to be started, in this case no such model is needed. However the torque equation is based on the generalized theory of electric machine. Another important point is that the state variable model in terms of small change in signal has been developed. The analysis of the whole paper is straight forward. However in equation (7) of this reference paper [9], the proper symbol for the variable at the operating point could have been used.

The paper titled, “Boost mode test of a current-source-inverter-fed permanent magnet synchronous motor drive for automotive applications” [10], deals with the CSI fed permanent magnet synchronous motor drive used in automotive application. Along with the traditional topology of a CSI fed permanent magnet synchronous motor system a DC-DC Chopper has been used for certain experimental advantages like regulating the current through external large inductor connected with CSI. In the CSI model the authors have used the switching functions for the three phase currents. In the section titled "Compensation of Capacitor Currents”, the total ’d’ and ‘q’ axis currents from the CSI have been expressed as follows.

$$I_{qst} = \omega_e(cr_s + \frac{L_{ds}}{R_c})I_{ds} + [1 - \omega_e{}^2 CL_{qs}(1 + \frac{r_s}{R_c})]I_{qs} + \frac{\omega_e \lambda_m}{R_c} \tag{2.22}$$

and

$$I_{dst} = [1 - \omega_e{}^2 CL_{qs}(1 + \frac{r_s}{R_c})]I_{ds} - \omega_e(cr_s + \frac{L_{ds}}{R_c})I_{qs} - \omega_e{}^2 C\lambda_m(1 + \frac{r_s}{R_c}) \tag{2.23}$$

where the symbols used in the equations 2.22 and 2.23 have the meanings as listed below:

'ω_e' is the rotor electric rotating speed,

'R_c' is the stator core loss resistor,

'r_s' is the stator resistor,

'C' is the filter capacitance per phase,

'L_{ds}' and 'L_{qs}' are the d- and q-axis inductance respectively ,

'λ_m' is the rotor flux generated by the permanent magnet,

'I_{ds}' and 'I_{qs}' are the d- and q-axis torque and flux producing currents drawn by the PMSM,

'I_{dst}' and 'I_{qst}' are the total d- and q-axis currents from the CSI

The important point to be noticed is that the authors have used the rotor flux generated by the permanent magnet (λ_m) with respect to equations (2.22) and (2.23) as a given data. The author of the present thesis here by suggest a point which may open up a new horizon of the research. This particular point is explained as follows:

A permanent magnet field structure system can be represented by an equivalent amount of magnetic vector potential using electromagnetic field theory approach. In this context, it is relevant to note that the vector potential can be expressed as follows:

$$\vec{A} = \frac{\mu}{4\pi} \int \frac{\vec{J}}{r} dv \tag{2.24}$$

where ‘dv’ is the elemental volume, corresponding to full space of volume 'v'.

‘J’ is the actual current density vector in A/m^2

‘r’ is the magnitude of the distance between the source point and the field point and

‘μ’ is the permeability of magnetic material.

Hence such approach will positively help a researcher to design a separate structure of conventional field winding system after knowing the equivalent permanent magnet structural data. In other words such approach will link up ‘Field Theory’ and ‘Circuit Theory’.

2.3. Some more reviews on synchronous motor based on Generalized Theory of Electrical Machines

The paper entitled: "New Digital Simulation Technique for a Current Source Inverter fed synchronous motor-Part 1” [11], basically deals with the digital simulation technique for a CSI fed synchronous motor using some novel approach. Using this approach, prediction of both transient and steady state performance of salient pole synchronous machine becomes easy. Furthermore this paper has investigated the effect of presence of damper winding on performance of the machine. Even though it does not lead to stability criteria studies, digital simulation technique based on ‘d-q’ model for current source inverter fed synchronous motor drive system may help as a tool for stability assessment. Firstly the five coil primitive machine model of a CSI fed system has been presented in a clear manner and the concerned power electronics inverter has been selected as a naturally commutated one. The advantage of this scheme is that it is a closed loop system which is based on the idea that the thyristor gate firing signals are derived from the rotor position sensor. The important aspect of a CSI fed system is that phase voltage to axis voltage transformation has to be expressed via a connecting matrix. The authors of this paper also have used such idea. The simulated/experimental value of electromagnetic torque as a function of time can be easily applied as input data to a transient stability program of the drive system. Even though the overall presentation of this paper is very lucid, the authors could have explained the physical significance of electromagnetic torque due to a permanent magnet structure and the electromagnetic torque contributed by the variation of air gap reluctance. This particular aspect is covered by the author of the present thesis based on a generalized 4-coil primitive machine model [12], in figure 2.5. The concerned analysis ending up with a local conclusion is presented as follows:

A ‘d-q’ model of the synchronous motor has been developed and it consists of four coils.

The electromagnetic torque, Te can be expressed as

$$Te = \left[I^T \right][G][I] \tag{2.25}$$

where,[I] is a Current Matrix (Colum Vector),[G] is a matrix involving self and mutual inductances of and between the coils placed on the same axis respectively.

Furthermore transformation from phase model to axis model has been executed using a suitable connection matrix for both voltages and currents. Basically this particular research work deals with the power electronics phenomenon associated with the synchronous motor drive system whereas the author of the present thesis mainly concentrates in the modeling part of the synchronous motor as a component of the drive system. Based on this philosophy the following machine equations are expanded:

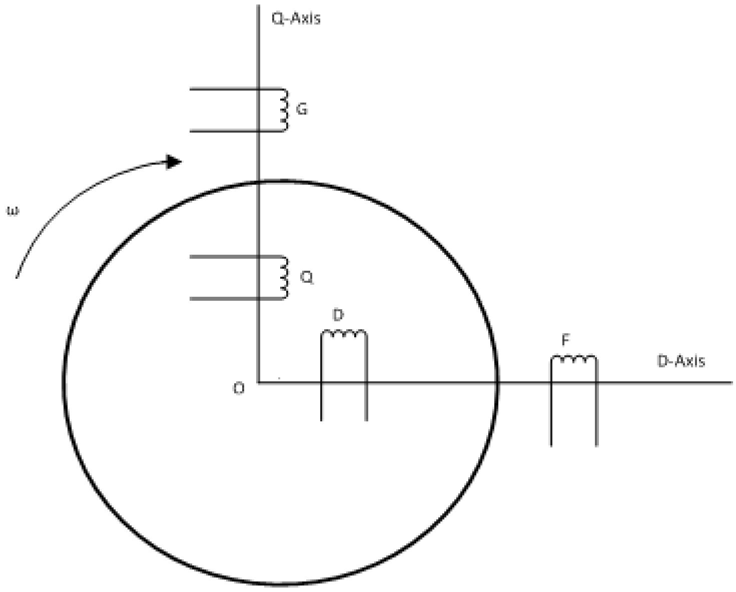


Figure 2.5.Generalized 4-coil primitive machine model

'ω' is positive for Generator and 'ω' is negative for Motor.

(A)Voltage balance equations of the machine (assuming ω as positive):

$$\begin{bmatrix} u_d \\ u_f \\ u_q \\ u_g \end{bmatrix} = \begin{bmatrix} R_a+L_d\,p & L_{df}\,p & \omega\,L_{rq} & \omega\,L_{rg} \\ L_{df}p & R_f+L_{ff}p & 0 & 0 \\ -\ \omega\,L_{rd} & -\ \omega\,L_{rf} & L_{qg}p & R_q+L_qp \\ 0 & 0 & L_{gq}\,p & R_g+L_{gg}p \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_q \\ i_g \end{bmatrix} \tag{2.26}$$

The symbols used in matrix equations (2.26) have the meanings as listed below:

u_d, u_f, u_q, u_g are the impressed voltage across 'D', 'F', 'Q' and 'G' coils respectively.

i_d,i_f,i_q,i_g are the currents through 'D', 'F', 'Q' and 'G' coils respectively.

Ra is the armature winding resistance of generalized four coil primitive machine model.

Rg is the resistance of 'G' coil.

L_d, L_q,L_{ff} and L_{gg} are the self-Inductances of 'D' ,'Q','F', and 'G' coils respectively.

L_{df} is the mutual inductance between 'D' and 'F' windings.

ω is the speed in rad/sec

L_{rq} is the fictitious inductance of 'D' coil due to rotational effect produced by q-axis current.

L_{rg} is the fictitious inductance of 'D' coil due to rotational effect produced by q-axis current.

L_{rd} is the fictitious inductance of 'Q' coil due to rotational effect produced by d-axis current.

L_{rf} is the fictitious inductance of 'Q' coil due to rotational effect produced by q-axis current.

(B) Power Balance Equation:

Equation 2.26, after suitable mathematical manipulation and with reference to Figure.2.5the total input power ‘P’ to all the coils can be expressed as:

$$P = \sum i^2 R + \frac{d}{dt}(\frac{1}{2} L_d i_d^{\,2}) + L_{fd}(i_d p i_f + i_f p i_d) + \frac{d}{dt}(\frac{1}{2} L_{ff} i_f^{\,2}) + \frac{d}{dt}(\frac{1}{2} L_q i_q^{\,2}) +$$

$$L_{qg}(i_q p i_g + i_g p i_q) + \omega[id\varphi q - iq\varphi d] + \frac{d}{dt}(\frac{1}{2} L_{gg} i_g^{\,2}) \tag{2.27}$$

When two coils are bypassed (F and G Coils)

Input power can be renamed as P_{new} where P_{new} can be expressed as:

$$P_{new} = \sum i_d{}^2 Ra + \sum i_q{}^2 Ra + \frac{d}{dt}\left(\frac{1}{2} L_d i_d{}^2\right) + \frac{d}{dt}\left(\frac{1}{2} L_q i_q{}^2\right) + \omega[i_d\varphi_q - i_q\varphi_d]$$

(2.28)

where

$$\varphi_d = L_{rd} i_d$$
$$\varphi_q = L_{rq} i_q \qquad (2.29)$$

With reference to equation (2.28), we can pick up separately the third term in the right hand side of the equation and it can be further simplified to draw a local conclusion about the phenomenon of electromagnetic torque. This term symbolized as P_3can be expressed as follows:

$$P_3 = \omega[i_d\varphi_q - i_q\varphi_d] \qquad (2.30)$$

Therefore, the electromagnetic torque ($T_{saliency}$) after bypassing the ‘F’ and ‘G’ coils comes out to be:

$$T_{saliency} = \frac{P_3}{\omega} = [i_d\varphi_q - i_q\varphi_d]$$
$$= -i_d i_q (L_{rd} - L_{rq}) \qquad (2.31)$$

The negative sign can be included for motor torque. Hence

$$T_{saliency} = i_d i_q (L_d - L_q) \qquad (2.32)$$

Here $(L_d - L_q)$ depends on reluctance variation.

The torque component $T_{saliency} = i_d i_q (L_d - L_q)$ is due to saliency only.

Hence Total Torque (T_{total}) =Torque due to saliency ($T_{saliency}$) + Torque due to permanent magnet contribution. This particular statement can be considered as a local conclusion.

The paper entitled “Analysis of current regulated voltage source inverters for permanent magnet synchronous motor drives in normal and extended speed ranges” [13], deals with the analysis of Voltage Source Inverter(VSI) fed permanent magnet synchronous motor drive system. The inverter used is a current regulated VSI and it appears that the effect of such inverters is near to that of a CSI. The important aspect of the analysis is that the contribution of permanent flux vector has been represented in stator reference frame through the application of a position sensor or a position encoder. Another interesting aspect of this paper is that as the permanent magnet machine does not involve any separate field winding, the interaction of rotor speed and the permanent magnet flux contribution can be treated as a back emf vector as it is usually done in the case of simulation of DC machines. In the case of synchronous motor drive system such vector will be e_{fa}, e_{fb}, e_{fc} as expressed in the paper and they are position dependent. Here, 'e', indicates equivalent back e.m.f and the suffixes fa,fb,fc indicates the phase windings in phase a, b and c respectively. Once the ac model is finalized, Park’s transformation matrix has been successfully applied to obtain the axis currents. Finally the obtained torque speed characteristics can be used further as an explicit functional form or equation by the method of polynomial fitting. Such model can be easily used for analysis of stability aspects.

Steady state stability criteria of alternating current(AC) drives play a dominant role for making the drive system practically successful. Generally such analysis is done using small perturbation model. But the complexity of the mathematical formulation depends on the nature of the drive system used. There is a companion paper titled “Analysis of steady state stability of a CSI fed synchronous motor drive system with damper windings included” [14], in this direction. This paper presents a detailed analysis of steady state stability criterion based on small perturbation model of a current source inverter fed synchronous motor drive system taking d-axis and q-axis damper winding into account using generalized theory of electrical machines. The modeling also clearly shows that even at no load the system satisfies steady state stability criterion. Routh-Herwitz criterion is used to finalize the result. The methodology of the proposed research work can be stated as follows: The synchronous motor has been treated as a five coil primitive machine model using the concept of generalized theory of electrical machines. Using the concept of Park's transformation the armature current in d-q model has been represented by suitable equations as a function of armature current magnitude in phase model (I_s) and the field angle (β).As the system under consideration is basically a current source inverter fed system, I_s has been considered as a constant and as a consequence the field angle (β) finally appears as a control variable. Furthermore voltage balance equations of the five coils have been expressed in time domain and those equations has been transformed accordingly after applying Laplace transform technique and then small perturbation technique is applied on the transformed equations. Finally the transfer function $\Delta\beta(s)/\Delta T_L(s)$ have been formulated; where $\Delta\beta(s)$ and $\Delta T_L(s)$ represent small change in transformed field angle and load torque respectively. The analysis concludes that the absence of damper winding leads to instability of the machine system.

The paper entitled:"Toward Condition Monitoring of Damper Windings in Synchronous Motors via EMD" [15], is basically devoted to the Empirical Mode Decomposition(EMD) analysis of breakage in damper bars in synchronous motor. The EMD Analysis was basically applied by Hound and colleagues [16],and the theory is famous as Hilbert – Huang transform (HHT). HHT is basically a signal processing tool and it is applied efficiently to the area of synchronous motor in context to the problem of breakage of damper bars. As a whole, such problem can be looked upon as a conditioning monitoring problem and now a days this particular area is drawing more attention by the practicing engineers. The concepts presented by J A Antonio et al is basically old concept but the presentation appears to be very nice. Superposition theorem has been efficiently used to bring up the fact that " Asynchronous machine with one broken bar = A healthy machine + A synchronous machine with current source with value , -I, in the bar that breaks". It may be noted that it is not exactly the superposition theorem for electrical networks but presently it is a superposition of two different configurations. Furthermore this fault current due to brakeage in the bar must flow through the short circuit rings or end rings and rest of the cage bar, originating a magnetic field in the air gap. This field must be a function of space and time and can be decomposed as the sum of spatial harmonics. It is a matter of interest to note that such spatial harmonics can be formulated clearly only when time is considered as a frozen parameters(or in other words, time 't' is fixed). The continued analysis lead to the conclusion that the fault field will be a pulsating field. From our classical concept it is well known that a pulsating magnetic field has its axis stationary in space (space means here position with respect to rotor), but the peak pulses with respect to time. Hence such function can be easily represented as equivalent to " Cosθ Cosωt where 'θ' and 't' indicates space and time respectively. Now Cosθ and Cosωt can be represented as a linear combination of cosine of (θ+ωt) and cosine of (θ-ωt). Hence physically with reference to particular F*ault Field* every harmonic pulsating field will lead to two series of rotating components with constant amplitude and speed obtained: a series with the same rotating direction as the rotor(+) and another series with opposite direction (-).

As a consequence several current harmonics in stator windings are induced. The detection of these harmonics in the Fourier spectrum of the steady -state current is considered as the foundation of the classical method for fault diagnosis involving bar breakage in induction motors [17], [18].

Even though the present paper [15], refers to the synchronous motor as a machine component, the theory of induction motor can very easily applied here in context to the bar breakage problem. The reasoning is straight forward because a practical synchronous motor with damper winding can be treated as partly induction motor and partly synchronous motor. Based on this philosophical discussion, the mathematical analysis is continued to reach the stage of generation of side band frequencies having lower side band and upper side band extremities. At this juncture it may be noted that the original problem of condition monitoring in synchronous motor has merged to become a signal processing problem. Furthermore those side band frequencies must be function of 'base band frequency' and 'slip'. The term 'slip' appears here due to the known fact that the role of damper winding as an induction motor has been brought into the analysis, [19],[20].

"A current controller design for current source inverter-fed permanent magnet synchronous motor drive system" [21], is a particular research work that has concentrated on the ‘s’ domain analysis and ‘z’ domain analysis of a current source inverter fed AC machine drive system. The author of the present thesis has been very much impressed by this paper, mainly due to three reasons:

1. CSI is involved as the same situation in the present Thesis also(refer to figure.3.1.in chapter 3 of this thesis)
2. The AC machine involved in this paper is a generalized term. As a particularization permanent magnet synchronous motor or synchronous reluctance motor are considered for the analysis. In the present thesis also the main machine component is a three phase salient pole type synchronous motor which has also variable air gap saliency as similar to the case in reluctance machine. Therefore the authors' inclination to such paper can be considered as a very natural fact.
3. Similar to the analytical method in this paper the author of the present thesis also has extensively used the method of Laplace Transform to analyze the research problem stated in the thesis. However the only difference lies in the fact that the author of this thesis has not converted the continuous time domain model of the present drive problem into a discrete model (using ‘z’ transform), due to lack of time for exploring this.

This paper basically analyses the whole CSI fed ac machine drive system using synchronous reference frame and ‘d-q’ decomposition. Another special feature of this paper is the inclusion of a virtual resistor in the mathematical model, which is also known as the active damping method. Such inclusion has been necessitated to avoid the instability of the designed current controller due to the parameter error or some other disturbances like digital delay and non-linearity of the inverter.

"Quantized-Input Control Lyapunov Approach for Permanent Magnet Synchronous Motor Drives"[22], is a paper that basically analyzes the stability aspects of PMSM drive system using quantized input Lyapunov approach. The d –q frame model of a PMSM have been expressed in the structure of the state variable model and the stabilization of the electrical and mechanical sub-systems have been analyzed separately by choosing suitable energy functions. The energy function chosen for mechanical sub-system is constructed by choosing rotor speed and rotor position as two independent variables and the energy functions for electrical subsystem has been developed by choosing d axis and q axis armature currents and also by including the energy function for mechanical subsystem. It is quite obvious that the inclusion of the energy function for mechanical subsystem into the model of energy function for electrical subsystem basically signifies that mechanical function and electrical function of any motor cannot be decoupled.

"Permanent magnet flux linkage adaptive observer for permanent magnet synchronous motor" [23], is an interesting paper that deals with the development of an adaptive observer for PMSM drive system. The author of the present thesis has been motivated by one important aspect of this paper which is the parameter sensitivity analysis. From this analysis presented in this paper it is clearly understood that to reduce the sensitivity of q-axis inductance (Lq) an adaptive observer combined with Lq is proposed. However development of any adaptive observer or adaptive control scheme is totally based on the selection of the adaptive control law. Such control law is also selected in this paper based on the d-q decomposition model of the whole synchronous motor.

2.4. Review Work Based on Text Books

The earlier discussions on literature survey basically include the published research papers in different journals and conference proceedings. Though these types of survey are a must to track the modern trends in research, it will not be customary to forget that any research area has some fundamental concepts and those aspects should always be referred for preparation of a research manuscript. The author of the present thesis while looking into such directions firstly reminds a renowned literature which is a text book titled ,"The performance and design of alternating current machines"[24]. In this reference, dynamics of synchronous motor has been dealt in detail leading to the concept of steady state stability and transient stability. Furthermore the special aspect of what will happen to the dynamics of synchronous motor if a cyclic distributive torque is associated also has been lucidly dealt with.

The important point in reference to the text book titled "Electric Machinery" [25], is that the chapter titled "Electro-Mechanical Energy Conversion principles" may be treated as a foundation to carry out any research work on any type of rotating electrical machine. In this chapter, the concept of field energy and co-energy are nicely explained and research exposure in the field of steady state and transient stability of synchronous motor can be smoothly carried on using the concept of field energy and co-energy as explained in the book.

Most of the literature review indicate that generalized theory of electrical machines may be considered as a foundation stone to carry out the modeling and stability analysis work of synchronous motor .In this context the book titled, "General theory of alternating current machines" [12], is found to be very useful. Adkins *et.al* has started systematically from the concept of primitive machine model and finally they have reached the method of performance analysis of synchronous machine in both generator and motor mode through the stages of analysis involving primitive impedance matrix, generalized torque equations and Park’s transformation, expressions for operational impedances and state variable model.

An interesting text book entitled, “Electric machines analysis and design applying MATLAB” [26], draws the attention of the author because the MATLAB simulation of synchronous machines direction of view point make the foundation program strong for further researchers in the concerned area. The textbook entitled “Power System Stability, Volume III Synchronous Machines” [25], though concerns the power system stability problem, its detailed modeling of synchronous machine in ‘d’, ‘q’ frame may also help a researcher involved in the analysis of synchronous motor drive system.

2.5. Conclusions

- The detailed survey in the area of stability analysis of Synchronous Motor clearly indicates that the attention by researchers are mainly directed to Permanent Magnet Synchronous Motor as compared to the same problems of normal Synchronous Motor.
- Most of the models used in the literature are ‘d-q’ models. The possible reason behind this trend is that the coils fixed on ‘d’ and ‘q’ axis will see fixed permeances and as a result Inductance parameters will no longer be functions of space angle. Such philosophy leads to a model convenient for computer simulations.
- Most of the researchers target the stability problems (either Steady state or Transient Stability) associated with Permanent Magnet Synchronous Motor. But generally a Synchronous Motor starts as an Induction Motor with the help of

damper winding and pools into synchronism under certain conditions .Hence it is required to analyze in more detail regarding what is happening during the transition from induction motor state to Synchronous Motor stage. A flow chart presented in this direction in the paper may draw the attention of the researchers in the concerned area.

- So far as the trend in industry application is concerned, it has been observed that to run a synchronous motor being excited from a current source inverter in place of being excited directly from bus obviously have some specific advantage. The fact is that armature reaction phenomenon becomes a mathematical function of a single variable. i.e. the field angle(β), even though the joint contribution of magnitude of armature current and field angle constitute a physical model of armature reaction. The important point is that for controlling the cost, if the synchronous motor is run on being excited directly from bus then the analysis in the above said direction will not hold good. Such situation will make the analysis complicated, but the complexity is a must to clearly understand the stability criteria. The author of the present thesis predicts that an analysis involving partial differentiation may serve the purpose.

References

[1] Sergelen, B., "Mathematical model of salient pole synchronous motors supplied by a frequency converter," *International Forum on Strategic Technology, IFOST 2007.* ,3-6October, Ulaanbaatar,pp-390 - 393. doi: 10.1109/IFOST.2007.4798612.

[2] Najafi, S.; Kar, N.C., "Effect of Short-Circuit Voltage Profile on the Transient Performance of Saturated Permanent Magnet Synchronous Motors," *Power Engineering Society General Meeting, 2007. IEEE* , vol., no., pp.1,6, 24-28 June 2007 doi: 10.1109/PES.2007.386288

[3] Kondo, K.; Matsuoka, K., "Stability analysis of permanent magnet synchronous motors for railway vehicle traction in a sudden line voltage change," The 27th Annual Conference of the IEEE Industrial Electronics Society, 2001. IECON '01., vol.2, no., pp.1445,1450 vol.2, 2001. doi: 10.1109/IECON.2001.975996.

[4] Shimmin, D.W.; Wang, J.; Bennett, N.; Binns, K.J., "Modelling and stability analysis of a permanent-magnet synchronous machine taking into account the effect of cage bars,", IEE Proceedings - Electric Power Applications, vol.142, no.2, pp.137,144, Mar 1995. doi: 10.1049/ip-epa:19951719.

[5] Magne, P.; Marx, D.; Nahid-Mobarakeh, B.; Pierfederici, S., "Large-Signal Stabilization of a DC-Link Supplying a Constant Power Load Using a Virtual Capacitor: Impact on the Domain of Attraction," Industry Applications, IEEE Transactions on , vol.48, no.3, pp.878,887, May-June 2012 doi: 10.1109/TIA.2012.2191250.

[6] Wijenayake, Ajith H., and Peter B. Schmidt. "Modeling and analysis of permanent magnet synchronous motor by taking saturation and core loss into account." Power Electronics and Drive Systems, 1997. Proceedings., 1997 International Conference on. Vol. 2. IEEE, 1997..

[7] Cheng-Ting Hsu, "Transient stability study of the large synchronous motors starting and operating for the isolated integrated steel-making facility," Industry Applications, IEEE Transactions on , vol.39, no.5, pp.1436,1441, Sept.-Oct. 2003 doi: 10.1109/TIA.2003.816534.

[8] Al-Taee, M.A.; AlZu'bi, H.S.; Al-Din, M.S., "Stable and highly efficient operation of open-loop controlled PM synchronous motor drive," Systems, Signals and Devices, 2008. IEEE SSD 2008. 5th International Multi-Conference on , vol., no., pp.1,6, 20-22 July 2008. doi: 10.1109/SSD.2008.4632845.

[9] Mellor, P.H.; Al-Taee, M.A.; Binns, K.J., "Open loop stability characteristics of synchronous drive incorporating high field permanent magnet motor," Electric Power Applications, IEE Proceedings B , vol.138, no.4, pp.175,184, Jul 1991.

[10] Lixin Tang; Gui-Jia Su, "Boost mode test of a current-source-inverter-fed permanent magnet synchronous motor drive for automotive applications," Control and Modeling for Power Electronics (COMPEL), 2010 IEEE 12th Workshop on , vol., no., pp.1,8, 28-30 June 2010 doi: 10.1109/COMPEL.2010.5562430.

[11] Abdelfattah, M.Y.; Ahmed, M.M., "New digital simulation technique for a current-source inverter fed synchronous motor. I," Industrial Electronics Society, 1990. IECON '90., 16th Annual Conference of IEEE , vol., no., pp.1007,1012 vol.2, 27-30 Nov 1990. doi: 10.1109/IECON.1990.149276.

[12] B.Adkins.; R.G.Harley., ”General Theory of Alternating Current Machines” , Chapman and Hall Publications Limited, London, 1954, pp – 28-41.

[13] Dhaouadi, R.; Mohan, N., "Analysis of current-regulated voltage-source inverters for permanent magnet synchronous motor drives in normal and extended speed ranges," Energy Conversion, IEEE Transactions on , vol.5, no.1, pp.137,144, Mar 1990. doi: 10.1109/60.50825.

[14] Chattopadhyay, A.B., Sunil Thomas., Ruchira Chatterjee., "Analysis of Steady State Stability of a Csi Fed Synchronous Motor Drive System with Damper Windings Included.", Trends in Applied Sciences Research, 2011, vol.,no., 6, pp: 992-1005.doi: 10.3923/tasr.2011.992.1005.

[15] Antonino-Daviu, J.A.; Riera-Guasp, M.; Pons-Llinares, J.; Roger-Folch, J.; Perez, R.B.; Charlton-Perez, C., "Toward Condition Monitoring of Damper Windings in Synchronous Motors via EMD Analysis," Energy Conversion, IEEE Transactions on , vol.27, no.2, pp.432,439, June 2012. doi: 10.1109/TEC.2012.2190292.

[16] Kizhner, S.; Huang, N.E.; Blank, K.; Conwell, E., "On the Hilbert-Huang transform data processing system development," Aerospace Conference, 2004. Proceedings. 2004 IEEE , vol.3, no., pp.,1979 Vol.3, 6-13 March 2004 doi: 10.1109/AERO.2004.1367979.

[17] Kliman, G.B.; Koegl, R.A.; Stein, J.; Endicott, R.D.; Madden, M.W., "Noninvasive detection of broken rotor bars in operating induction motors," Energy Conversion, IEEE Transactions on , vol.3, no.4, pp.873,879, Dec 1988.doi: 10.1109/60.9364.

[18] Elkasabgy, N.M.; Eastham, A.R.; Dawson, G.E., "The detection of broken bars in the cage rotor of an induction machine," Industry Applications Society Annual Meeting, 1988., Conference Record of the 1988 IEEE , vol., no., pp.181,187 vol.1, 2-7 Oct. 1988.doi: 10.1109/IAS.1988.25060.

[19] El Hachemi Benbouzid, M., "A review of induction motors signature analysis as a medium for faults detection," Industrial Electronics, IEEE Transactions on , vol.47, no.5, pp.984,993, Oct 2000. doi: 10.1109/41.873206

[20] Filippetti, F.; Franceschini, G.; Tassoni, C.; Vas, P., "Recent developments of induction motor drives fault diagnosis using AI techniques," Industrial Electronics, IEEE Transactions on , vol.47, no.5, pp.994,1004, Oct 2000 doi: 10.1109/41.873207.

[21] Hak-Jun Lee; Sungho Jung; Seung-Ki Sul, "A current controller design for current source inverter-fed PMSM drive system," Power Electronics and ECCE Asia (ICPE & ECCE), 2011 IEEE 8th International Conference on , vol., no., pp.1364,1370, May 30 2011-June 3 2011. doi: 10.1109/ICPE.2011.5944414.

[22] Prior, G.; Krstic, M., "Quantized-Input Control Lyapunov Approach for Permanent Magnet Synchronous Motor Drives," Control Systems Technology, IEEE Transactions on , vol.21, no.5, pp.1784,1794, Sept. 2013. doi: 10.1109/TCST.2012.2212246

[23] Qiu Tengfei; Wen Xuhui; Zhao Feng; Wang Yongxing, "Permanent magnet flux linkage adaptive observer for permanent magnet synchronous motor," Transportation Electrification Asia-Pacific (ITEC Asia-Pacific), 2014 IEEE Conference and Expo , vol.,no.,pp.1,4,Aug.31,2014-Sept.32014. doi: 10.1109/ITEC-AP.2014.6940981.

[24] Say.M.G, " The Performance and Design of Alternating Current Machines", CBS Publishers and Distributors, New-Delhi,2002, pp – 465-517.

[25] A.E. Fitzgerald, C. Kingsley, Jr., (S.D. Umans), Electric Machinery, McGraw-Hill, 1st ed. 1952, 6th ed. 2002.

[26] J.J. Cathey, Electric Machines: Analysis and Design Applying MATLAB, McGraw-Hill, 2000.

[27] Edward Wilson Kimbark, Power System Stability, Volumes I, II, III: (An IEEE Press Classic Reissue, Paper Edition, 3 Volume Set), IEEE Press, New Jersey, pp- 78-118.1995.

Chapter 3: Mathematical Treatment on Steady State Stability Aspects of a Current Source Inverter fed Synchronous Motor Drive System taking the Effect of Damper Windings into account.

3.1. Introduction

Steady state stability criteria of alternating current(AC) drives play a dominant role for making the drive system practically successful. Generally such analysis is done using small perturbation model. But the complexity of the mathematical formulation depends on the nature of the drive system used. This chapter presents a detailed analysis of steady state stability criterion based on small perturbation model of a current source inverter fed synchronous motor drive system taking d-axis and q-axis damper windings into account using generalized theory of electrical machines. The modeling also clearly shows that even at no load the system satisfies steady state stability criterion. Routh-Herwitz criterion is used to finalize the result. The methodology of the proposed research work can be stated as follows: The synchronous motor has been treated as a five coil primitive machine model using the concept of generalized theory of electrical machines. Using the concept of Park's transformation, the armature current in d-q model has been represented by suitable equations as a function of armature current magnitude in phase model (I_s) and the field angle (β).As the system under consideration is basically a current source inverter fed system, I_s has been considered as a constant and as a consequence the field angle (β) finally appears as a control variable. The explanation on relation between field angle (β) and load angle (δ) has been already presented in Chapter-1 of this thesis. Furthermore voltage balance equations of the five coils have been expressed in time domain and those equations has been transformed accordingly after applying Laplace Transform technique. As the analysis is focusing on the steady state stability criteria, the author feel that small perturbation technique will be appropriate to apply on the transformed equations. Finally the transfer function $\Delta\beta(s)/\Delta T_L(s)$ have been formulated; where $\Delta\beta(s)$ and $\Delta T_L(s)$ represent small change in transformed field angle load torque respectively. The analysis concludes that the absence of damper winding leads to instability of the machine system.

For achieving flexibility in operation, the traditional method of feeding direct supply to the major components of an alternating current drive system i.e. synchronous machine or induction machine are avoided and the void is filled up by the use of power electronic converters. In this context, alternating current motor drives using inverter-fed synchronous motors are used in some specific application areas which have certain features that make them preferable to induction motor drives[1]. One of those specific examples is the accurate simultaneous speed control of a number of motors by using synchronous motors. There are companion papers in the direction of power electronic control of synchronous motor drive systems. The research paper [2-3] basically deals with the analysis of control mechanism of a synchronous motor drive system with the help of a cyclo-convertor: direct torque control of Permanent Magnet Synchronous Motor(PMSM) . The research manuscript [4], explain the direct torque control of PMSM.

The research paper by *Chan,* [5] was on a flux-observer method to estimate the rotor speed of a Permanent Magnet Synchronous Motor. Another interesting work [6], has been reported in 2008. This paper deals with fuzzy logic control of inverter fed synchronous motor based on simple mathematical model, algorithmic investigations of stability criteria.

Using fuzzy logic algorithm, similar work on Permanent Magnet Synchronous motor [7], has been reported and the controller was found to be robust for high-speed applications. Though most of the inverters used in alternating current drive are voltage source inverters[8], current source inverters(CSI) are also being recognized due to simplicity, greater controllability and ease of protection. This chapter examines steady state stability aspects of a CSI fed synchronous motor drive system considering the presence of damper winding on both direct and quadrature axis. There is also a necessity of providing a damper winding to be placed on the q-axis to assure the steady-state stability at no-load for the motor only with d-axis damper winding. In this chapter of the present thesis, it also confirms the above said statement by presenting a rigorous analysis based on generalized theory of electrical machines [9]. Furthermore, there exists an interesting research work [10], in connection to the dynamics of a three phase synchronous motor being fed from a single phase supply, using state space model. Korshunov has published an interesting paper [11], in the area of stability analysis of a synchronous motor with permanent magnets by using small perturbation model. Even though the author of the present thesis have used the axis model of synchronous motor for analysis of steady state stability, the reference [11], has drawn the attention because such work relates with the state variable model.

From the research paper [12], we can observe that the paper deals with the mathematical model of a synchronous motor energized through a power electronic converter and ultimately develops simulation results of time response of phase currents/phase voltages and motor speed. These results are not in line with the results submitted in the present chapter of this thesis because the objectives are different. However, a very good similarity is observed involving the fact that the paper uses *'u-i'* (voltage-current) relations, and generalized torque equations in the primitive machine model and the same type of equations have also been used as a part of problem formulation in this thesis work.

Another comparison of the present work has been carried out in the research manuscript [13].In this research work, the main similarity lies in the method of modeling using voltage balance equations in the *'d-q'* model. As such there is no similarity in the results presented in the paper[13], “Effect of Short-Circuit Voltage Profile on the Transient Performance of Saturated Permanent Magnet Synchronous Motors”, and the present chapter, an observation on the literature review behind that paper clearly indicates that the transient response of the performance figure taking torque angle, phase currents etc. into account, can lead to a specific inference on stability criteria. Though the present chapter of this thesis deals with the steady state stability analysis of a three phase synchronous motor, the said similarity appears to be important as it falls in the broad category of stability studies of synchronous motors.

The results presented in the paper [14], uses block diagram approach for stability analysis of permanent magnet synchronous motors for railway vehicle traction in a sudden line voltage change, and the formulation presented in the present chapter can easily lead to block diagram modeling in *‘s’* domain. Secondly in both the works, researchers are ultimately interested in the stability analysis, but they have some dissimilarity. This is due to the fact that the motor used in their work is a permanent magnet synchronous motor and the chapter presented here uses synchronous motor consisting of physical field winding system. As a third aspect it is observed that, the paper uses the concept of real part of the eigen values to be negative for the assessment of the stability and the work presented here uses the same concept for stability assessment but in a slightly different form, that is on the relative stability criteria based on Routh-Herwitz array.

It is a well-known fact that the literature review part of any research work is generally considered to be complete only when the literature review is made from the view point of both that aspects which are in favor and in contrast of the study. The following paragraph is developed on the basis of the literature review in contrast of the proposed study.

The well-known research work [15], clearly indicates that the small perturbation model can be applied to a synchronous motor drive with current source inverter (the motor being without damper winding) for determination of steady state stability criteria. This particular paper is in contrast with the proposed study because the author of the present thesis have considered a synchronous motor with damper windings.

The next important work [16], has drawn the attention to the author of the present thesis. This work involves the stability improvement of a V/f controlled PWM inverter fed induction motor using a dynamic current compensator. This particular paper has been considered in contrast with the proposed study because induction motor can be treated as a singly excited magnetic field system where as synchronous motor considered by the author in the present thesis can be treated as a multiply excited magnetic field system. Hence the difference in the views of the research philosophy is clearly observed.

Another important work [17], as a part of the literature review in contrast of the study is related to induction motor as a component of the power system and transient modeling for such motor leading to the assessment of transient stability has been considered as the objective of the research work presented in that paper. This literature review is in contrast of the study carried by the author of the present thesis due to the fact that the paper [17], does not involve any analysis related to steady state stability criteria but it is clear that the model associated with the dynamics of induction motor can be easily modified to be used in determination of the steady state stability criteria of the system.

The research work [18], does not involve synchronous motor as a topic of research but the main similarity lies in the fact that this paper also uses Laplace Transform Technique as a tool for mathematical modeling using state variable approach applied to solar array power system and another work [19], is basically a research work related to the fault location in a power system. This paper uses neural network technique for the problem formulation but it can be linked up with the proposed research work involving synchronous motor in the sense that the same power system can consist of the synchronous motor drive system as a load. An interesting research work [20], basically deals with the simulation of a power system consisting of a turbo generator using matlab. The points of similarity with the research work presented in this thesis are:

(i) It is also a stability problem even though different techniques like Lyapunov Technique has been used.

(ii) Laplace Transform has been used to carry out the block diagram approach.

(iii) Torque angle and power balance equations under dynamics have been used. It may be noted that the term torque angle [20], corresponds to the term field angle referred in the present chapter of the thesis.

The research work [21],basically deals with the distance relay protection scheme implemented on a transmission line in presence of Unified Power Flow Controller (UPFC).The UPFCis a member of the FACTS family with attractive features. An interesting paper [22], analyze the effect of electrical parameters of an induction generator on the transient voltage stability of a variable speed wind turbine system. The similarity of this paper with the present thesis lies in the fact that the induction generator also has been modeled in d-q frame through voltage balance and flux linkage equations. Furthermore this paper uses the torque balance equations in the phase model which can be converted to *'d-q'* model using the well known torque balance equation i.e., $T_e = \Psi_d i_q - i_d \Psi_q$.

There are many representative form of transfer function in association with the steady state stability analysis of a current source inverter fed synchronous motor drive system. Taking the practical aspect into account, the current chapter targets to derive an expression in a suitable form for transfer function, which is the ratio of the Laplace Transform of the small signal version of the change in field angle (β) between the field (rotor) m.m.f. axis and armature (stator) m.m.f. axis to the Laplace transform of the small signal version of change in load torque (T_L). The mathematical analysis in the said direction is presented in the subsequent section. The objective of this study is to diagnose the fact whether the synchronous motor with damper winding and fed through a current source inverter can sustain small perturbation in load torque or not, from the view point of the concept of steady state stability criteria of an electrical drive system.

3.2. Problem Formulation

The basic block diagram of the proposed scheme is shown in figure.3.1

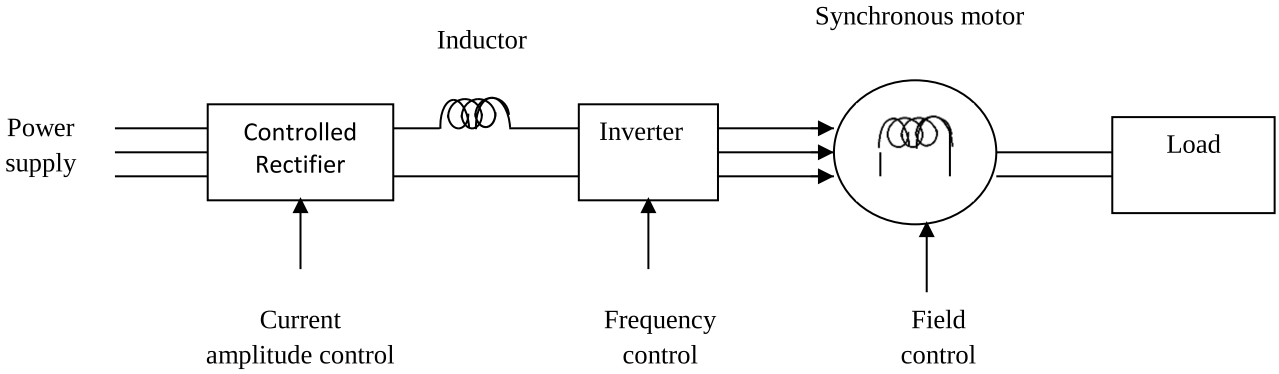


Figure.3.1 Drive configuration for open-loop current-fed synchronous motor control

To have a better feeling of the method of analysis, the primitive machine model of the synchronous motor is drawn and it is shown in figure 3.2. To have a better feeling on the stage wise development of the whole problem formulation, the following subsections are created. The first subsection is devoted to the expression for small perturbation for d- axis and q-axis components of armature currents in time and Laplace domain(s-domain), respectively.

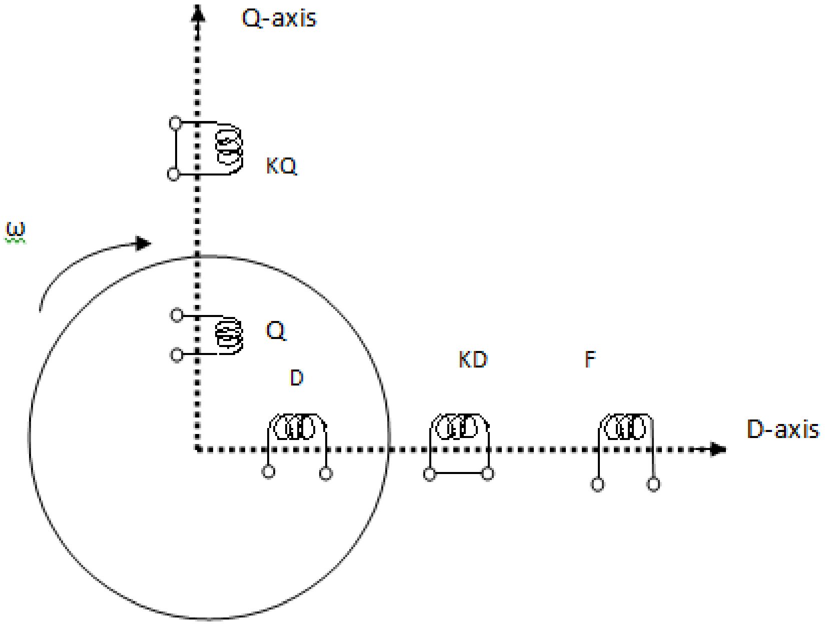


Figure.3.2. Primitive machine model of a synchronous motor

3.2.1. Derivations for axis components of armature currents as a function of field angle

In the following analysis, magnetic saturation is ignored but provision is made for inclusion of saliency and one number of damper winding on each axis. Following Park's transform, a constant stator current of value at a field angle 'β' can be represented by direct and quadrature axis currents as,

$$i_d = i_s \cos\beta \tag{3.1}$$

$$i_q = i_s \sin\beta \tag{3.2}$$

Designating steady state value by the subscript '0' and small perturbation by Δ, the perturbed equations of the machines are

$$\Delta i_d = -i_s \sin\beta_0 \Delta\beta \tag{3.3}$$

$$\Delta i_q = i_s \cos\beta_0 \Delta\beta \tag{3.4}$$

Applying Laplace Transform to the equations (3.3) and (3.4), it yields,

$$\Delta I_d(s) = -i_s \sin\beta_0 \Delta\beta(s) \tag{3.5}$$

$$\Delta I_q(s) = i_s \cos\beta_0 \Delta\beta(s) \tag{3.6}$$

Once the expressions for the resolved armature currents in both time domain and 's' domain are obtained, the next stage work remains to express the voltage balance equations of all the members(windings) of the primitive machine model (Figure 3.2) and subsequently to develop the torque balance equations in a systematic form. All the equations are presented in time domain in absolute form and in small perturbation form separately. Laplace Transform is applied only to the equations involving perturbed variables.

3.2.2. Derivations for Voltage balance and Torque balance equations

The generalized expression for electromagnetic torque of a primitive machine model is an established one and it is expressed as,

$$
\begin{aligned}
T_e &= \Psi_d i_q - i_d \Psi_q \\
&= \left(L_d i_d + L_{md} i_f + L_{md} i_{kd}\right) i_q - \left(L_q i_q + L_{mq} i_{kq}\right) i_d \\
&= \left(L_d - L_q\right) i_d i_q + L_{md} i_q i_f + L_{md} i_{kd} i_q - L_{mq} i_{kq} i_d \qquad (3.7)
\end{aligned}
$$

Small signal version of torque equation in time domain is expressed as

$$
\begin{aligned}
\Delta T_e = {} & \left(L_d - L_q\right)\Delta i_d i_{q0} + \left(L_d - L_q\right)\Delta i_q i_{d0} + L_{md}\Delta i_q i_{f0} + L_{md}\Delta i_f i_{q0} \\
& + L_{md}\Delta i_{kd} i_{q0} + L_{md} i_{kd0}\Delta i_q - L_{mq}\Delta i_{kq} i_{d0} - L_{mq} i_{kq0}\Delta i_d \qquad (3.8)
\end{aligned}
$$

Equation (3.8) after being transformed takes the shape as given by

$$
\Delta T_e(s) = \left(L_d - L_q\right) i_{q0}\Delta I_d(s) + \left(L_d - L_q\right) i_{d0}\Delta I_q(s) + L_{md} i_{f0}\Delta I_q(s) + L_{md} i_{q0}\Delta I_f(s)
$$

$$
\begin{aligned}
& + L_{md} i_{q0}\Delta I_{kd}(s) + L_{md} i_{kd0}\Delta I_q(s) - L_{mq} i_{d0}\Delta I_{kq}(s) - L_{mq} i_{kq0}\Delta I_d(s) \\
& = [\left(L_d - L_q\right) i_{q0} - L_{mq} i_{kq0}]\Delta I_d(s) + \left[\left(L_d - L_q\right) i_{d0} + L_{md} i_{kd0} + L_{md} i_{f0}\right]\Delta I_q(s) \\
& \quad + L_{md} i_{q0}\Delta I_f(s) + L_{md} i_{q0}\Delta I_{kd}(s) - L_{mq} i_{d0}\Delta I_{kq}(s) \qquad (3.9)
\end{aligned}
$$

To tackle equation (3.9) in an easier form, it is expressed as

$$
\Delta T_e(s) = c_1\Delta I_d(s) + c_2\Delta I_q(s) + c_3\Delta I_f(s) + c_4\Delta I_{kd}(s) + c_5\Delta I_{kq}(s)
$$
(3.10)

where,

$$c_1 = \left[\left(L_d - L_q\right) i_{q0} - L_{mq} i_{kq0}\right] \qquad (3.10a)$$

$$c_2 = \left[\left(L_d - L_q\right) i_{d0} + L_{md} i_{kd0} + L_{md} i_{f0}\right] \qquad (3.10b)$$

$$c_3 = L_{md} i_{q0} \qquad (3.10c)$$

$$c_4 = L_{md} i_{q0} \qquad (3.10d)$$

$$c_5 = -L_{mq} i_{d0} \qquad (3.10e)$$

The small perturbation model of the transformed voltage balance equations of F-coil, KD-coil and KQ are expressed as,

$$0=\Delta U_f(s)= R_f\Delta I_f(s) + sL_{ff}\Delta I_f(s) + sL_{md}\Delta I_d(s) + sL_{md}\Delta I_{kd}(s) \tag{3.11}$$

$$0=\Delta U_{kd}(s)=R_{kd}\Delta I_{kd}(s) + sL_{kkd}\Delta I_{kd}(s) + sL_{md}\Delta I_d(s) + sL_{md}\Delta I_f(s) \tag{3.12}$$

$$0=\Delta U_{kq}(s)=R_{kq}\Delta I_{kq}(s) + sL_{kkq}\Delta I_{kq}(s) + sL_{mq}\Delta I_q(s) \tag{3.13}$$

As the damper winding on d-axis and q-axis are short-circuited within themselves, ΔU_{kd}=0 andΔU_{kq}=0. So in transformed version ΔU_{kd}(s)=0 and ΔU_{kq}(s)=0 as shown in eqns. (3.12) and (3.13). Furthermore in general, the voltage fed to the field winding is fixed. That is why ΔU_f(s)=0 as shown in equation (3.11).

From equations (3.11) and (3.12) it yields,

$$\Delta I_f(s)=\left(\frac{s^2(L_{md}{}^2-L_{md}L_{kkd})-sL_{md}R_{kd}}{s^2(L_{kkd}L_{ff}-L_{md}{}^2)+s(L_{kkd}R_f+R_{kd}L_{ff})+R_{kd}R_f}\right)\Delta I_d(s) \tag{3.14}$$

$$=\left(\frac{a_1s^2+a_2s}{D(s)}\right)\Delta I_d(s) \tag{3.15}$$

where,

$$a_1=L_{md}{}^2-L_{md}L_{kkd} \tag{3.15a}$$

$$a_2=-L_{md}R_{kd} \tag{3.15b}$$

$$D(s)=b_1s^2+b_2s+b_3 \tag{3.15c}$$

$$b_1=L_{kkd}L_{ff}-L_{md}{}^2 \tag{3.15d}$$

$$b_2=L_{kkd}R_f+R_{kd}L_{ff} \tag{3.15e}$$

$$b_3=R_{kd}R_f \tag{3.15f}$$

Similarly equations. (3.11) and (3.12) yields,

$$\Delta I_{kd}(s)=\left(\frac{s^2(L_{md}{}^2-L_{md}L_{ff})-sL_{md}R_f}{s^2(L_{kkd}L_{ff}-L_{md}{}^2)+s(L_{kkd}R_f+R_{kd}L_{ff})+R_{kd}R_f}\right)\Delta I_d(s) \tag{3.16}$$

$$= \left(\frac{d_1 s^2 + d_2 s}{D(s)} \right) \Delta I_d(s) \tag{3.17}$$

where

$$d_1 = \left(L_{md}{}^2 - L_{md} L_{ff} \right) \tag{3.17a}$$

$$d_2 = -L_{md} R_f \tag{3.17b}$$

From equation (3.13) it is obtained,

$$\Delta I_{kq}(s) = \left(\frac{-sL_{mq}}{R_{kq} + sL_{kkq}} \right) \Delta I_q(s) \tag{3.19}$$

$$= \left(\frac{e_1 s}{Q(s)} \right) \Delta I_q(s) \tag{3.20}$$

where,

$$e_1 = -L_{mq} \tag{3.20a}$$

$$Q(s) = f_1 s + f_2 \tag{3.20b}$$

$$f_1 = L_{kkq} \tag{3.20c}$$

$$f_2 = R_{kq} \tag{3.20d}$$

Substituting eqns. (3.14), (3.16), (3.19) in eqn. (3.10), it yields

$$\Delta T_e(s) = c_1 \Delta I_d(s) + c_2 \Delta I_q(s) + c_3 \left[\frac{a_1 s^2 + a_2 s}{D(s)} \right] \Delta I_d(s) + c_4 \left[\frac{d_1 s^2 + d_2 s}{D(s)} \right] \Delta I_d(s) + c_5 \left[\frac{e_1 s}{Q(s)} \right] \Delta I_q(s) \tag{3.21}$$

$$= \left[c_1 + \frac{c_3 a_1 s^2 + c_3 a_2 s + c_4 d_1 s^2 + c_4 d_2 s}{D(s)} \right] \Delta I_d(s) + \left[c_2 + \frac{c_5 e_1 s}{Q(s)} \right] \Delta I_q(s) \tag{3.22}$$

Equation (3.21) can be re expressed as,

$$\Delta T_e(s) = \left[\frac{(m_1 s^3 + m_2 s^2 + m_3 s + m_4) \Delta I_d(s) + (n_1 s^3 + n_2 s^2 + n_3 s + n_4) \Delta I_q(s)}{l_1 s^3 + l_2 s^2 + l_3 s + l_4} \right] \tag{3.23}$$

where,

$$m_1 = f_1c_1b_1 + c_3a_1f_1 + c_4d_1f_1 \tag{3.23a}$$
$$m_2 = c_1f_1b_2 + c_3a_2f_1 + c_4d_2f_1 + c_1b_1f_2 + c_3a_1f_2 + c_4d_1f_2 \tag{3.23b}$$
$$m_3 = c_1b_2f_2 + c_3a_2f_2 + c_4d_2f_2 + c_1b_3f_1 \tag{3.23c}$$
$$m_4 = c_1b_3f_2 \tag{3.23d}$$
$$n_1 = f_1c_2b_1 + c_5e_1b_1 \tag{3.23e}$$
$$n_2 = c_2f_2b_1 + c_2f_1b_2 + c_5e_1b_2 \tag{3.23f}$$
$$n_3 = c_2f_2b_2 + b_3c_2f_1 + b_3c_5e_1 \tag{3.23g}$$
$$n_4 = c_2f_2b_3 \tag{3.23h}$$
$$l_1 = b_1f_1 \tag{3.23i}$$
$$l_2 = b_2f_1 + b_1f_2 \tag{3.23j}$$
$$l_3 = b_3f_1 + b_2f_2 \tag{3.23k}$$
$$l_4 = b_3f_2 \tag{3.23l}$$

Substituting the expressions for $\Delta I_d(s)$ and $\Delta I_q(s)$ from equations. (3.3) and (3.4) in equation. (3.23), we have

$$\Delta T_e(s) = \left[\frac{(m_1s^3 + m_2s^2 + m_3s + m_4)(-i_s \sin\beta_0(s)) + (n_1s^3 + n_2s^2 + n_3s + n_4)(i_s \cos\beta_0(s))}{l_1s^3 + l_2s^2 + l_3s + l_4} \right] \Delta\beta(s) \tag{3.24}$$

Equation (3.24) can be re-expressed as,

$$\Delta T_e(s) = \left[\frac{x_1s^3 + x_2s^2 + x_3s + x_4}{l_1s^3 + l_2s^2 + l_3s + l_4} \right] \Delta\beta(s) \tag{3.25}$$

where,

$$x_1 = n_1i_s\cos\beta_0 - m_1i_s\sin\beta_0 \tag{3.25a}$$
$$x_2 = n_2i_s\cos\beta_0 - m_2i_s\sin\beta_0 \tag{3.25b}$$
$$x_3 = n_3i_s\cos\beta_0 - m_3i_s\sin\beta_0 \tag{3.25c}$$
$$x_4 = n_4i_s\cos\beta_0 - m_4i_s\sin\beta_0 \tag{3.25d}$$

The above said equations basically give the mathematical structure of the portion (of the synchronous motor), which is responsible for producing the electromagnetic torque

Hence as a logical consequence, the next stage of the problem formulation should be responsible for development of the transfer function. As the transfer function for a drive system , in most of the cases, involves transformed small change in load torque as a input signal, the interaction of load torque with the electromagnetic torque must come into picture. Such interaction is expressed through suitable mathematical equations and it is presented in the following final subsection.

3.2.3. Development of Transfer Function of the said drive system.

The torque dynamic equation of a synchronous motor can be written as,

$$T_e - T_L = J\frac{d\omega}{dt} \tag{3.26}$$

where,

ω = Motor speed in mechanical rad./sec.

J = Polar moment of inertia of motor and load (combined) kg-m^2

The small change in speed 'ω' equal to Δω can be related to small change in field angle, Δβ as given by,

$$\Delta\omega = -\frac{d(\Delta\beta)}{dt} \tag{3.27}$$

The negative sign in equation physically indicates a drop in speed (ω) due to increase in field angle (β).

Based on equation (3.27), the following expression can be written,

$$J\frac{d(\Delta\omega)}{dt} = J\frac{d}{dt}\left[-\frac{d}{dt}(\Delta\beta)\right] = -J\frac{d^2}{dt^2}(\Delta\beta) \tag{3.28}$$

The small-perturbation model of equation (3.28) can be written as,

$$\Delta\mathrm{T_e} - \Delta\mathrm{T_L} = J\frac{d(\Delta\omega)}{dt} \tag{3.29}$$

.

Combining equations (3.28) and (3.29), it yields

$$\Delta\mathrm{T_e} - \Delta\mathrm{T_L} = -J\,\frac{d^2}{dt^2}(\Delta\beta) \tag{3.30}$$

The transformed version of eqn. (3.30), with initial condition relaxed, comes out to be

$$\mathrm{T_e}(\mathrm{s}) - \mathrm{T_L}(\mathrm{s}) = -\mathrm{Js}^2\Delta\beta(\mathrm{s}) \tag{3.31}$$

Substituting the expression for ΔTe(s) from eqn. (3.25) in eqn. (3.31), we have

$$\left[\frac{x_1 s^3 + x_2 s^2 + x_3 s + x_4}{l_1 s^3 + l_2 s^2 + l_3 s + l_4}\right]\Delta\beta(s) + Js^2\Delta\beta(s) = \Delta T_L(s) \tag{3.32}$$

Equation (3.32) gives, after manipulation, a Transfer Function, T(s) expressed as,

$$\mathrm{T}(\mathrm{s}) = \frac{\Delta\beta(s)}{\Delta T_L(s)} = \frac{l_1 s^3 + l_2 s^2 + l_3 s + l_4}{K_1 s^5 + K_2 s^4 + K_3 s^3 + K_4 s^2 + K_5 s + K_6} \tag{3.33}$$

where,

$$\mathrm{K_1} = \mathrm{J}l_1 \tag{3.33a}$$

$$\mathrm{K_2} = \mathrm{J}l_2 \tag{3.33b}$$

$$\mathrm{K_3} = (\mathrm{J}l_3 + \mathrm{x_1}) \tag{3.33c}$$

$$\mathrm{K_4} = (\mathrm{J}l_4 + \mathrm{x_2}) \tag{3.33d}$$

$$\mathrm{K_5} = \mathrm{x_3} \tag{3.33e}$$

$$\mathrm{K_6} = \mathrm{x_4} \tag{3.33f}$$

The expression for $\frac{\Delta\beta(s)}{\Delta T_L(s)}$ in equation (3.33) gives a light in the direction of analysis of steady state stability criterion. In fact the denominator polynomial of right-hand side of equation (3.33), set to zero, becomes the characteristic equation. A Routh-Herwitz(R-H) analysis of the characteristic equation will ultimately lead to the status of steady state stability. The detailed R-H analysis, considering a suitable example and the associated interpretation of the results are presented in the next section.

3.3. Results and Discussions

The denominator polynomial of the right hand side of equation (3.33) being set to zero, takes the form as given by

$$K_1s^5 + K_2s^4 + K_3s^3 + K_4s^2 + K_5s + K_6 = 0 \qquad (3.34)$$

The Routh-Herwitz table for equation (3.34) is developed taking suitable values of the polynomial co-efficient (K_1 to K_6). The machine data based on which the K_1 to K_6 are calculated are presented in Appendix. The calculated values of $K_1$to K_6 are as follows.

$$K_1 = 2.2871; \quad K_2 = 0.3585; \quad K_3 = 0.5577; \quad K_4 = 0.0776; \quad K_5 = 0.0023; \quad K_6 = 2.419\times10^{-6}$$

The coefficients of the first column of the R-H table do not show any change of sign. Therefore the steady state stability is assured. Generally it is observed that at no-load, a synchronous motor without damper winding does not satisfy steady state stability criteria. At the no load condition with β_0=0, the transfer function expressed in the equation,(3.33) for a C.S.I. fed synchronous motor with damper winding takes the shape for the motor without damper winding, as given by,

$$\frac{\Delta\beta(s)}{\Delta T_L(s)} = \frac{1}{s^2J + [L_{md}i_{f0}i_s + (L_{md} - L_{mq})i_s{}^2]} \qquad (3.35)$$

The denominator of the right hand side of the equation (3.35), being set to zero leads to a characteristic equation expressed as

$$Js^2 + C = 0 \tag{3.36}$$

where,

$$C = L_{md} i_{f0} i_s + \left(L_{md} - L_{mq}\right) i_s^2 \tag{3.37}$$

Equation (3.37) clearly shows that the pair of roots of the characteristic equation are pure imaginary and complex conjugate. Such roots lead to a marginally stable system, which does not clearly indicate the system is completely stable.

However with damper winding considered and at no load (β_0=0), the denominator polynomial of the right hand side of the equation (3.33) being set to zero is made subjected to R.H. analysis. With the R-H table formed the 1st column coefficient of that table do not show any change in sign. This assures that at no load also, the motor satisfies the steady state stability criterion. The values of the coefficient K1 to K6 for no load condition are calculated based on the machine data, which are also present in the Appendix. The calculated values of K1 to K6 for no load condition are as follows.

$$K_1 = 2.2871; \quad K_2 = 0.3585; \quad K_3 = 0.0421; \quad K_4 = 0.0045; \quad K_5 = 1.567 \times 10^{-4}; \quad K_6 = 1.7 \times 10^{-7}$$

The reason behind using βo=0, in the concerned equation, for no-load condition of the motor lies in the fact explained as follows:

The left hand term of the equation (3.35) is a transfer function expressed not as the ratio of two absolute transformed quantities rather as the ratio of two transformed perturbed quantities. That is why the denominator appears in the left hand side as $\Delta T_L(s)$, not as

$T_L(s)$. Now the interesting matter to be noted is that to simulate the effect of no-load condition, $\Delta T_L=0$ has not been used , rather βo=0 has been implemented in the concerned voltage-balance and torque-balance equations. The reason behind such analytical aptitude lies in the fact that $\Delta T_L=0$, does not signify a no-load condition of the motor (because even though the load torque is constant, the change in load torque is zero), whereas βo=0 (or very small value) indicates a physical no-load condition of the motor.

3.4. Conclusions

This chapter highlights the following salient concluding points

1) The transfer function expressed in equation (3.33) has been developed in more general form such that the transfer function for no load condition can be derived from equation (3.33) directly just by substituting suitable marginal condition.
2) Statements similar to point no.1 are generally valid for the case of derivation of transfer function for a C.S.I. fed synchronous motor without damper winding.

References

[1] Marx, D.; Pierfederici, S.; Davat, B., "Nonlinear control of an inverter motor drive system with input filter — large signal analysis of the DC-link voltage stability," *Power Electronics Specialists Conference, 2008. PESC 2008. IEEE* , vol., no., pp.498,503, 15-19 June 2008. doi: 10.1109/PESC.2008.4591978.

[2] Das, S.P.; Chattopadhyay, A.K., "Analysis, simulation and test results for a cycloconverter-fed AC commutator-less motor drive with a modified machine model," *Electric Power Applications, IEE Proceedings -* , vol.151, no.5, pp.622,627, 9 Sept. 2004. doi: 10.1049/ip-epa:20040634.

[3] Yan, Y.; Zhu, J.G.; Guo, Y.G., "Initial rotor position estimation and sensorless direct torque control of surface-mounted permanent magnet synchronous motors considering saturation saliency," *Electric Power Applications, IET* , vol.2, no.1, pp.42,48, Jan. 2008. doi: 10.1049/iet-epa:20070083.

[4] Sayeef, S.; Foo, G.; Rahman, M.F., "Rotor position and speed estimation of a variable structure direct torque controlled IPM synchronous motor drive at very low speeds," *Power Engineering Conference, 2008. AUPEC '08*. Australasian Universities , vol., no., pp.1,6, 14-17 Dec. 2008.

[5] Chan, T.F.; Wang, W.; Borsje, P.; Wong, Y.K.; Ho, S.L., "Sensorless permanent-magnet synchronous motor drive using a reduced-order rotor flux observer," *Electric Power Applications, IET* , vol.2, no.2, pp.88,98, March 2008 doi: 10.1049/iet-epa:20070234.

[6] Fabijanski, P.; Lagoda, R., "A very simple fuzzy control system for inverter fed synchronous motor", *13th Power Electronics and Motion Control Conference,*. EPE-PEMC,2008.vol.,no.,pp.1040,1043,1-3Sept.2008. doi:10.1109/EPEPEMC.2008.4635404.

[7] Uddin, M.N.; Azizur Rahman, M., "High-Speed Control of IPMSM Drives Using Improved Fuzzy Logic Algorithms," *Industrial Electronics, IEEE Transactions on* , vol.54, no.1, pp.190,199, Feb. 2007. doi: 10.1109/TIE.2006.888781.

[8] Uddin, M.N.; Radwan, T.S.; Azizur Rahman, M., "Performance of interior permanent magnet motor drive over wide speed range," *Energy Conversion, IEEE Transactions on* , vol.17, no.1, pp.79,84, Mar 2002. doi: 10.1109/60.986441.

[9] B.Adkins.;R.G.Harley., ”General Theory of Alternating Current Machines” , *Chapman and Hall Publications Limited*, London, 1954, pp – 28-41.

[10] Al-Ohaly, A.A.; Hamouda, R.M.; Badr, M.A., "Torsional oscillations associated with the starting of three phase synchronous motors from a single phase supply," *Energy Conversion, IEEE Transactions on* , vol.12, no.1, pp.10,16, Mar 1997
doi: 10.1109/60.577274.

[11] Korshunov, A.I .," Analysis of the static stability of a synchronous motor with permanentmagnets by the classical method", *Russ.Electr.Eng.*,80:81-85,2009.
doi: 10.3103/S1068371209020060.

[12] Sergelen, B., "Mathematical model of salient pole synchronous motors supplied by a frequency converter," *International Forum on Strategic Technology,. IFOST* 2007. vol., no., pp.390,393, 3-6 Oct. 2007. doi: 10.1109/IFOST.2007.4798612.

[13] Najafi, S.; Kar, N.C., "Effect of Short-Circuit Voltage Profile on the Transient Performance of Saturated Permanent Magnet Synchronous Motors," *Power Engineering Society General Meeting, 2007. IEEE* , vol., no., pp.1,6, 24-28 June 2007. doi: 10.1109/PES.2007.386288.

[14] Kondo, K.; Matsuoka, K., "Stability analysis of permanent magnet synchronous motors for railway vehicle traction in a sudden line voltage change," *Industrial Electronics Society, 2001. IECON '01. The 27th Annual Conference of the IEEE* , vol.2, no., pp.1445,1450 vol.2, 2001. doi: 10.1109/IECON.2001.975996.

[15] Slemon, G.R.; Dewan, S.B.; Wilson, James W.A., "Synchronous Motor Drive with Current-Source Inverter," *IEEE Transactions on Industry Applications,* vol.IA-10, no.3, pp.412,416, May 1974. doi: 10.1109/TIA.1974.349170.

[16] Jee-Hoon Jung; Gang-Youl Jeong; Bong-Hwan Kwon, "Stability improvement of V/f-controlled induction motor drive systems by a dynamic current compensator," *IEEE Transactions on Industrial Electronics*, vol.51, no.4, pp.930,933, Aug. 2004.doi: 10.1109/TIE.2004.831766.

[17] Tanneeru, S.; Mitra, J.; Patil, Y.J.; Ranade, S.J., "Effect of Large Induction Motors on the Transient Stability of Power Systems*," 39th North American Power Symposium, 2007. NAPS '07*. vol., no., pp.223,228, Sept. 30 2007-Oct. 2 2007. doi: 10.1109/NAPS.2007.4402314.

[18] Chattopadhyay, A.B.; A. Choudhury .;A. Nargund., "State variable model of a solar power system" *Trends Applied Sci. Res.,2011* vol., no 6: pp.563-579. doi: 10.3923/tasr.2011.563.579.

[19] H. Zayandehroodi.; A. Mohamed.; H. Shareef.; M. Mohammadjafari., "Automated Fault Location in a Power System with Distributed Generations using Radial basis Function Neural Networks", *Journal of Applied Sciences,2010*. vol.10, issue:23, pp:3032-3041. doi: 10.3923/jas.2010.3032.3041.

[20] Ghorab, N.; O. Ghorab N.E.; Debbache., "Real time dynamic simulation of retrofitted turbo generator in matlab/simulink environment", 2007,*J. Applied Sci*., 7: 3119-3122. doi 10.3923/jas.2007.3119.3122.

[21] Jazaeri, M.; M. Gholamzadeh.; M. Pazoki., "Analysis of over/under-reaching of distance relay on transmission line in presence of UPFC*", Trends Applied Sci. Res*., 2011 vol. no.6: pp.580-594. doi: DOI:10.3923/tasr.2011.580.594.

[22] Babainejad, S.;R. Keypour., "Analysis of transient voltage stability of a variable speed wind turbine with doubly fed induction generator affected by different electrical parameters of induction generator". *Trends Applied Sci. Res.,*2010 vol.no. 5: pp.251-278. doi: 10.3923/tasr.2010.267.278.

Chapter 4: Modeling and Simulation of a Synchronous Motor Drive System: Transition Zone in Complex Frequency Domain

4.1. Introduction

Modeling of synchronous motor plays a dominant role in designing complicated drive system for different applications, especially large blower fans etc. for steel industries. As synchronous motor has no inherent starting torque generally it is started as an induction motor with the help of damper windings and it pulls into synchronism under certain conditions. The present chapter exactly concentrates on this particular zone of transition from induction motor to synchronous motor mode for a current source inverter fed synchronous motor drive system. Due to complexity of synchronous motor in terms of number of windings and finite amount of air gap saliency, direct modeling of such transition zone in time domain becomes cumbersome at the first instance of modeling. That is why the modeling in complex frequency domain (s-domain) has been taken up using small perturbation model. Such a model clearly shows the role of induction motor as noise function or disturbance function with respect to the open loop block diagram of synchronous motor. Such finding can lead to important results and that is done in the present chapter of this thesis, such that the results can help the designer for the successful design of a synchronous motor drive system. Many constant speed applications such as fans, fuel pump and compressors comprising a considerable amount of total electrical appliances [1], basically need a 3 phase synchronous motor. Even though permanent magnet synchronous motors are widely used in such applications, current source inverter fed normal synchronous motors can also be applied in many constant speed applications [2-3]. The steady state stability study of a current source inverter fed synchronous motor was basically initiated in 1974 [4], and after this as an extension on this work by Chattopadhyay et.al.[5], presented a detailed analysis of a current source inverter fed synchronous motor drive system taking damper windings into account in 2011. So far the research accuracy of the paper [5], is covered, it is not clear that exactly what is happening in the transition zone when the machine is jumping from induction motor mode to synchronous motor mode. Again the concentration on such detailed aspect is a matter of long discussion and in this context many researchers have tried to put sufficient light on the matter. The research paper [6], explains the analysis of magnetic fields and temperature fields for a salient pole synchronous motor in the process of steady state analysis. They have used the *'d-q'* model of the synchronous motor but the role of field winding in transition from induction motor to synchronous motor has not been reflected in the mathematical model.

Similar observations are valid on the other works [7-8], and it represents a good state variable model and its mathematic simulations in time domain. The research papers[9-10], carries important works on the mathematical modeling of a salient pole synchronous motor supplied by a frequency converter and also the effect of short circuit voltage profile on the transient performance of permanent magnet synchronous motors .An important work on non-linear control of an inverter motor drive system with input filter [11-13], draws attention and the author has presented a detailed signal analysis of the DC-link voltage stability.

Another interesting research manuscripts [14-15], clearly portrays the critical aspects of starting a large synchronous motor. Even though this particular work does not involve much mathematical analysis but the range of the slip presented in this paper with reference to pull in torque of a synchronous motor may help a designer to select a particular synchronous motor for any specific application. There exists a good number of literature [16-19] on the starting of synchronous motors which throw light on different aspects to be considered during the starting of synchronous motors.

Based on the above said literature review, to the best of the authors understanding it reveals that researchers have not put sufficient light on the fact that exactly what happens to the mathematical model of a synchronous motor in time domain or complex frequency domain during the period when field winding is disconnected from the external resistance (generally 6-7 times of main field winding resistance to avoid the effects due to George’s phenomenon) and immediately thrown to the dc source. In the authors' opinion, investigation in such direction becomes very much crucial because before connecting the field winding to the DC source, characterization of the machine can be done as three phase induction motor. Therefore it will be very logical to get the semi induction semi synchronous machine character to be reflected in the resultant mathematical model. This particular work has been presented in this chapter of the present thesis. In the next section the mathematical methods will be explained in detail under the section, "Problem Formulation".

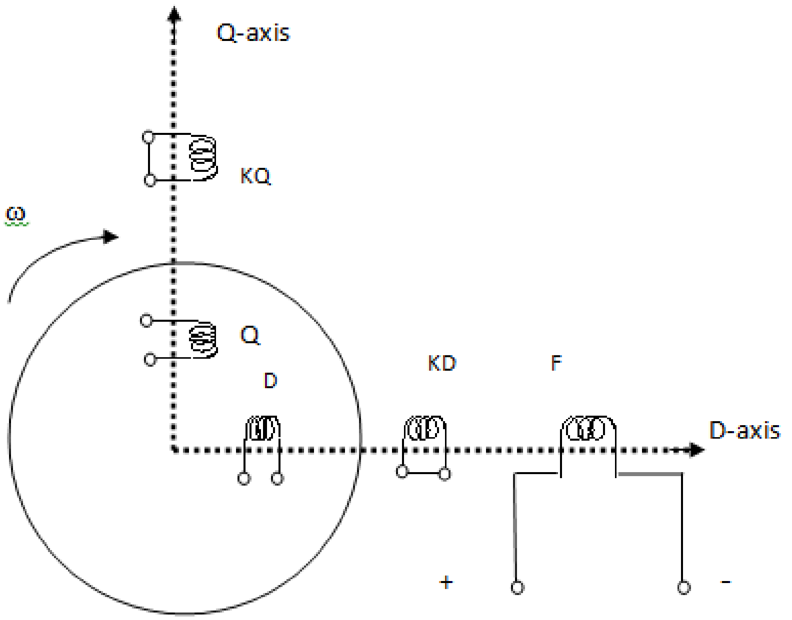
4.2. Problem Formulation

The philosophy of problem formulation in the present chapter is based on the observation what happens to change in the impressed voltage across the field winding when a synchronous motor starts as an induction motor with the help of damper winding. However for the sake of the development of the complete model under the present operating condition all the voltage balance and torque balance equations in convenient forms must be developed. This total work is represented in the following subsections:

4.2.1. Development of voltage balance equations (small perturbed and transformed)

It may be reminded at this stage that the primitive machine model structure of the synchronous motor remains same as discussed and presented in chapter 3 (refer section 3.2). As a consequence, the same figure, Figure 3.1and Figure 3.2 and the concerned subsequent analytical expressions in chapter 3(equation 3.1 - 3.11e)can be exactly followed in the present section. But in the present chapter, the configuration of synchronous motor has a little change because initially (at the time of start), the field winding should not be directly supplied from dc source. Such fine tuning comes out from the fact when the synchronous motor has started as an induction motor with the help of damper winding, field winding should be closed through an external resistance to minimize the effect of George’s phenomenon. However the complete discussion about George’s phenomenon will be presented in the next subsection.

The above said physical concept about the status of the field winding during the transition from induction motor mode to synchronous motor mode needs a special attention and such attention leads to a schematic view of the modified primitive machine model drawn in figure.4.1(modified form of figure. 3.2).



Field winding should be closed through an external resistance (generally 6-7 times of main field winding resistance) to minimize the effect of George's phenomenon

Figure.4.1.Modified form of Primitive machine model of synchronous motor during the transition from induction motor mode to synchronous motor mode.

Based on figure 4.1, Δu_f (small change in impressed voltage across the field winding), can be expressed as

$$\Delta u_f = c + s_1 k \,, \tag{4.1}$$

where

c =220 Volts, S_1= slip and k = (N_f/Na)*(415/√3)

N_f= Number of turns in field winding

Na= Number of turns in armature winding.

Following the nature of analysis in chapter 3, finally in the present chapter the small perturbation model of the transformed voltage balance equations of F-coil, KD-coil and KQ are expressed as,

$$\frac{c}{s}+s_1\frac{k}{s}=\Delta \mathrm{U_f}\left(\mathrm{s}\right)= \ \mathrm{R_f}\Delta \mathrm{I_f}\left(\mathrm{s}\right)\ +\mathrm{sL_{ff}}\Delta \mathrm{I_f}\left(\mathrm{s}\right)+\ \mathrm{sL_{md}}\Delta \mathrm{I_d}\left(\mathrm{s}\right)\ +\mathrm{sL_{md}}\Delta \mathrm{I_{kd}}\left(\mathrm{s}\right) \tag{4.2}$$

$$0=\Delta \mathrm{U_{kd}}\left(\mathrm{s}\right)=\mathrm{R_{kd}}\Delta \mathrm{I_{kd}}\left(\mathrm{s}\right)\ +\mathrm{sL_{kkd}}\Delta \mathrm{I_{kd}}\left(\mathrm{s}\right)+\ \mathrm{sL_{md}}\Delta \mathrm{I_d}\left(\mathrm{s}\right)\ +\mathrm{sL_{md}}\Delta \mathrm{I_f}\left(\mathrm{s}\right) \tag{4.3}$$

$$0=\Delta \mathrm{U_{kq}}\left(\mathrm{s}\right)=\mathrm{R_{kq}}\Delta \mathrm{I_{kq}}\left(\mathrm{s}\right)\ +\mathrm{sL_{kkq}}\Delta \mathrm{I_{kq}}\left(\mathrm{s}\right)+\ \mathrm{sL_{mq}}\Delta \mathrm{I_q}\left(\mathrm{s}\right) \tag{4.4}$$

As the damper winding on d-axis and q-axis are short-circuited within themselves, $\Delta U_{kd}=0$ and $\Delta U_{kq}=0$. So in transformed version $\Delta U_{kd}(s)=0$ and $\Delta U_{kq}(s)=0$ have been used in eqns. (4.3) and (4.4). Furthermore in general, the voltage fed to the field winding is fixed. It is a well-known fact that a synchronous motor cannot start from itself and the easiest way to start a synchronous motor is to start it as an induction motor with the help of damper windings. But the problem is that we have to investigate what will be status of field winding of the synchronous motor when the damper winding is in action. As already the winding was physically embedded (existing), and during the running of the machine one cannot take it out. In other words when damper winding is in action, field winding effect has to be inactivated. Such inactivation may be done by the following methods:

a) Field winding completely open circuit

b) Field winding short circuit in itself

The status of field winding in (a) can be looked upon as a transformer whose primary winding constitutes of 3 phase armature winding supplied from 415V (L-L) ac and whose secondary winding is the field winding being open circuited. As generally in a normal synchronous machine of normal design N_f/N_a>> 1, (where N_f is number of field windings and N_a is number of armature windings), the induced voltage in the open field terminal will be large and it may lead to hazardous conduction so far as operator safety is concerned. Hence this case is rejected.

Status of the field winding in (b): The induced e.m.f in field winding due to transformer action will produce a single phase alternating current and in turn will produce a pulsating field in field winding. It is well known that a pulsating field m.m.f can be resolved as a combination of forward rotating and backward rotating m.m.f magnetic fields (strengths of each resolved component is half of the original pulsating m.m.f). The effect of backward rotating magnetic field will produce a torque opposite to the (asynchronous/induction) motor torque and it will dominate at some value of slip. Hence a situation may arise and motor may stall due to the negative effect of backward component. This phenomenon is known as George's phenomenon.

Hence such case cannot be completely accepted. However there is some remedial method. The field winding may be closed through an external resistance which is about 6-7 times of original field resistance; such that the magnitude of short circuit current diminishes and as a result effect of resolved backward component will be less or reduced.

Even though a lot of physical significance about the status of the field windings have been discussed in the above said paragraphs, still a finite amount of academic gap remains about the fact what happens to change on the voltage impressed on the field winding. This particular direction of thinking leads to the necessity of representation in a separate subsection, as follows.

4.2.2 What happens to change in voltage impressed on field winding (Δu_f)

In the current research problem 'Δu_f' cannot be equal to zero because originally it was an induction motor with the field winding short circuited in it or closed through an external resistance of large value and at a later stage it was pulled into synchronism when dc supply is fed to the winding.

Quantitatively, 'Δuf' should depend on a particular property of induction motor and that property must be 'SLIP'. Here the technique of mathematical modeling appears as a novel approach and this approach forms the foundation of the proposed analysis. The proposed modeling considers that field winding is closed within itself. In other words the presence of external large resistance has not been considered in modeling to make the mathematical treatment comparatively easy. However it does not affect the accuracy of the system as the external resistance can be lumped or clubbed with the field winding.

At this stage , after a considerable amount of discussions on the physical significance of the whole process during the transition from induction motor mode to synchronous motor mode, one naturally will demand a further mathematical picture involving the small perturbation form of the transformed electromagnetic torque. The following subsection serves this purpose.

4.2.3 Convenient form of expressions for electromagnetic torque (as a small change in variable and then transformed)

From equations (4.2) and (4.3) it yields,

$$\Delta I_f(s) = \left(\frac{s^2(L_{md}{}^2 - L_{md}L_{kkd}) - sL_{md}R_{kd}}{s^2(L_{kkd}L_{ff}{}_{-}L_{md}{}^2) + s(L_{kkd}R_f + R_{kd}L_{ff}) + R_{kd}R_f} \right) \Delta I_d(s) - \mathrm{F}_{32}(\mathrm{s}) \qquad (4.5)$$

Where $\mathrm{F}_{32}(\mathrm{s}) = (R_{kd} + sL_{kkd})[\frac{c + s_1 k}{s^2 L_{md}}]$ (4.5a)

Similarly equations. (4.2) and (4.3) yields,

$$\Delta I_{kd}(s) = F_{31}(s) - \left(\frac{s^2(L_{md}{}^2 - L_{md}L_{ff}) - sL_{md}R_f}{s^2(L_{kkd}L_{ff}{}_{-}L_{md}{}^2) + s(L_{kkd}R_f + R_{kd}L_{ff}) + R_{kd}R_f} \right) \Delta I_d(s) \qquad (4.6)$$

$$= F_{31}(s) - \left(\frac{d_1 s^2 + d_2 s}{D(s)} \right) \Delta I_d(s) \qquad (4.7)$$

where

$$F_{31}(s) = \frac{\mathrm{c} + \mathrm{s}_1 \mathrm{k}}{s^2} \left[1 - \left\{ \left(\frac{R_f}{SL_{md}} + \frac{L_{ff}}{L_{md}} \right) \left(R_{kd} + sL_{kkd} \right) \right\} \right] \qquad (4.8)$$

From equation (4.5) it is obtained,

$$\Delta I_{kq}(s) = \left(\frac{-sL_{mq}}{R_{kq} + sL_{kkq}} \right) \Delta I_q(s) \tag{4.9}$$

$$= \left(\frac{e_1 s}{Q(s)} \right) \Delta I_q(s) \tag{4.10}$$

where,

$$\mathrm{e}_1 = -\mathrm{L}_{\mathrm{mq}} \tag{4.10a}$$

$$\mathrm{Q}(\mathrm{s}) = \mathrm{f}_1\mathrm{s} + \mathrm{f}_2 \tag{4.10b}$$

$$\mathrm{f}_1 = \mathrm{L}_{\mathrm{kkq}} \tag{4.10c}$$

$$\mathrm{f}_2 = \mathrm{R}_{\mathrm{kq}} \tag{4.10d}$$

Substituting eqns. (4.5), (4.6), (4.9) in eqn. (3.10), it yields

$$\Delta T_e(s) = c_1 \Delta I_d(s) + c_2 \Delta I_q(s) + c_3 \left[\frac{a_1 s^2 + a_2 s}{D(s)} \right] \Delta I_d(s) + c_4 \left[\frac{d_1 s^2 + d_2 s}{D(s)} \right] \Delta I_d(s) + c_5 \left[\frac{e_1 s}{Q(s)} \right] \Delta I_q(s)$$

$$= \left[c_1 + \frac{c_3 a_1 s^2 + c_3 a_2 s + c_4 d_1 s^2 + c_4 d_2 s}{D(s)} \right] \Delta I_d(s) + \left[c_2 + \frac{c_5 e_1 s}{Q(s)} \right] \Delta I_q(s) + \mathrm{F}_3(\mathrm{s}) \tag{4.11}$$

where, $\mathrm{F}_3(\mathrm{s}) = \mathrm{F}_{31}(\mathrm{s}) - \mathrm{F}_{32}(\mathrm{s})$ (4.12)

Equation (4.11) can be re expressed as,

$$\Delta T_e(s) = \left[\frac{(m_1 s^3 + m_2 s^2 + m_3 s + m_4)\Delta I_d(s) + (n_1 s^3 + n_2 s^2 + n_3 s + n_4)\Delta I_q(s)}{l_1 s^3 + l_2 s^2 + l_3 s + l_4} \right] + \mathrm{F}_3(\mathrm{s}) \tag{4.13}$$

where,

$$\mathrm{m}_1 = \mathrm{f}_1\mathrm{c}_1\mathrm{b}_1 + \mathrm{c}_3\mathrm{a}_1\mathrm{f}_1 + \mathrm{c}_4\mathrm{d}_1\mathrm{f}_1 \tag{4.13a}$$

$$\mathrm{m}_2 = \mathrm{c}_1\mathrm{f}_1\mathrm{b}_2 + \mathrm{c}_3\mathrm{a}_2\mathrm{f}_1 + \mathrm{c}_4\mathrm{d}_2\mathrm{f}_1 + \mathrm{c}_1\mathrm{b}_1\mathrm{f}_2 + \mathrm{c}_3\mathrm{a}_1\mathrm{f}_2 + \mathrm{c}_4\mathrm{d}_1\mathrm{f}_2 \tag{4.13b}$$

$$\mathrm{m}_3 = \mathrm{c}_1\mathrm{b}_2\mathrm{f}_2 + \mathrm{c}_3\mathrm{a}_2\mathrm{f}_2 + \mathrm{c}_4\mathrm{d}_2\mathrm{f}_2 + \mathrm{c}_1\mathrm{b}_3\mathrm{f}_1 \tag{4.13c}$$

$$\mathrm{m}_4 = \mathrm{c}_1\mathrm{b}_3\mathrm{f}_2 \tag{4.13d}$$

$$\mathrm{n}_1 = \mathrm{f}_1\mathrm{c}_2\mathrm{b}_1 + \mathrm{c}_5\mathrm{e}_1\mathrm{b}_1 \tag{4.13e}$$

$$\mathrm{n}_2 = \mathrm{c}_2\mathrm{f}_2\mathrm{b}_1 + \mathrm{c}_2\mathrm{f}_1\mathrm{b}_2 + \mathrm{c}_5\mathrm{e}_1\mathrm{b}_2 \tag{4.13f}$$

$$n_3 = c_2f_2b_2 + b_3c_2f_1 + b_3c_5e_1 \quad (4.13g)$$
$$n_4 = c_2f_2b_3 \quad (4.13h)$$
$$l_1 = b_1f_1 \quad (4.13i)$$
$$l_2 = b_2f_1 + b_1f_2 \quad (4.13j)$$
$$l_3 = b_3f_1 + b_2f_2 \quad (4.13k)$$
$$l_4 = b_3f_2 \quad (4.13l)$$

Substituting the expressions for $\Delta I_d(s)$ and $\Delta I_q(s)$ from equations.(3.3) and (3.4) in equation. (4.13), we have

$$\Delta T_e(s) = \left[\frac{(m_1s^3 + m_2s^2 + m_3s + m_4)(-i_s \sin\beta_0(s)) + (n_1s^3 + n_2s^2 + n_3s + n_4)(i_s\cos\beta_0(s))}{l_1s^3 + l_2s^2 + l_3s + l_4}\right]\Delta\beta(s)$$
$$+F_3(s) \quad (4.14)$$

Equation (4.14) can be re-expressed as,

$$\Delta T_e(s) = \left[\frac{x_1s^3 + x_2s^2 + x_3s + x_4}{l_1s^3 + l_2s^2 + l_3s + l_4}\right]\Delta\beta(s) + F_3(s) \quad (4.15)$$

$$\Delta T_e(s) = T_1(s)\Delta\beta(s) + F_3(s) \quad (4.15a)$$

where,

$$T_1(s) = \left[\frac{x_1s^3 + x_2s^2 + x_3s + x_4}{l_1s^3 + l_2s^2 + l_3s + l_4}\right] \quad (4.16)$$

$$x_1 = n_1i_s\cos\beta_0 - m_1i_s\sin\beta_0 \quad (4.16a)$$
$$x_2 = n_2i_s\cos\beta_0 - m_2i_s\sin\beta_0 \quad (4.16b)$$
$$x_3 = n_3i_s\cos\beta_0 - m_3i_s\sin\beta_0 \quad (4.16c)$$
$$x_4 = n_4i_s\cos\beta_0 - m_4i_s\sin\beta_0 \quad (4.16d)$$

The above set of equations only represents the transition zone in the form of an equivalent electromagnetic torque in the frame of small perturbation and Laplace domain. But this picture will be finally tuned only when the load interaction is brought into the modeling. This particular job is mathematically accomplished in the following subsections:

4.2.4. Mathematical model of the transition zone after the load torque interaction

The torque dynamic equation of a synchronous motor can be written as,

$$T_e - T_L = J\frac{d\omega}{dt} \tag{4.17}$$

where,

ω=motor speed in mechanical rad./sec.

J=polar moment of inertia of motor and load (combined)

The small change in speed 'ω' equal to Δω can be related to small change in field angle, Δβ as given by,

$$\Delta\omega = -\frac{d(\Delta\beta)}{dt} \tag{4.18}$$

The negative sign in equation physically indicates a drop in speed (ω) due to increase in field angle (β).Based on equation (4.18), the following expression can be written,

$$J\frac{d(\Delta\omega)}{dt} = J\frac{d}{dt}\left[-\frac{d}{dt}(\Delta\beta)\right] = -J\frac{d^2}{dt^2}(\Delta\beta) \tag{4.19}$$

The small-perturbation model of equation (4.17) can be written as,

$$\Delta\mathrm{T_e} - \Delta\mathrm{T_L} = J\frac{d(\Delta\omega)}{dt} \tag{4.20}$$

Combining equations (4.19) and (4.20), it yields

$$\Delta\mathrm{T_e} - \Delta\mathrm{T_L} = -J\frac{d^2}{dt^2}(\Delta\beta) \tag{4.21}$$

The transformed version of eqn. (4.21), with initial condition relaxed, comes out to be

$$\mathrm{T_e}(\mathrm{s}) - \mathrm{T_L}(\mathrm{s}) = -\mathrm{J}\mathrm{s}^2\Delta\beta(\mathrm{s}) \tag{4.22}$$

Substituting the expression for ΔTe(s) from eqn. (4.15) in eqn. (4.22), we have

$$T_1(s)\Delta\beta(s) + Js^2\Delta\beta(s) + \mathrm{F_3(s)} = \Delta T_L(s) \tag{4.23}$$

$$\frac{1}{\left[T_1(s) + Js^2\right]}\Delta T_L(s) - \frac{1}{\left[T_1(s) + Js^2\right]}\mathrm{F_3(s)} = \Delta\beta(s) \tag{4.24}$$

The block diagram representation of the system obtained from the above equation is shown in figure.4.2

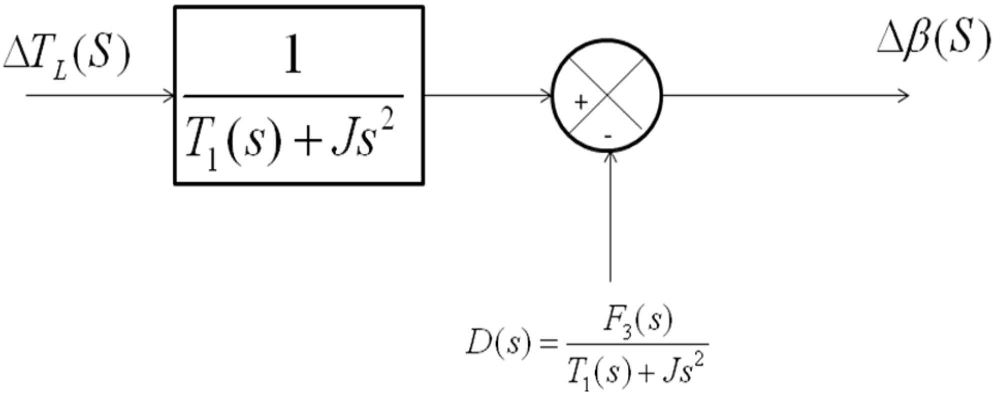


Figure 4.2. Block diagram representation of the system with disturbance function D(s)

The disturbance function can be taken separately and a detailed analysis in complex frequency domain is carried out as follows

$$F_3(\mathrm{s}) = \mathrm{F}_{31}(\mathrm{s}) - \mathrm{F}_{32}(\mathrm{s}) \tag{4.25}$$

Where, $$F_{32}(\mathrm{s}) = (\mathrm{R}_{kd} + sL_{kd})(\frac{c+s_1k}{s^2L_{md}}) \tag{4.26}$$

And $$F_{31}(\mathrm{s}) = \frac{(\mathrm{c+s_1k})}{s^2}\left[1-\left(\frac{R_f}{sL_{md}}+\frac{L_{ff}}{L_{md}}\right)\left(R_{kd}+sL_{kd}\right)\right] \tag{4.27}$$

Let $k_1 = c + s_1k$

$$F_{31}(\mathrm{s}) = \frac{k_1}{s^2}\left[\frac{\left\{sL_{md} - R_fR_{kd} - s(\mathrm{L}_{kkd}\mathrm{R}_f + \mathrm{R}_{kd}\mathrm{L}_{ff}) - \mathrm{s}^2 L_{ff}L_{kkd}\right\}}{sL_{md}}\right] \tag{4.28}$$

$$F_{31}(\mathrm{s}) = \frac{k_1}{s^3L_{md}}\left\{-\mathrm{s}^2 L_{ff}L_{kkd} + s(L_{md} - \mathrm{L}_{kkd}\mathrm{R}_f - \mathrm{R}_{kd}\mathrm{L}_{ff}) - R_fR_{kd}\right\} \tag{4.29}$$

$$F_{31}(\mathrm{s}) = \left(-\frac{k_1L_{ff}L_{kkd}}{L_{md}}\right)\left(\frac{1}{s}\right) + \frac{k_1\left\{L_{md} - \mathrm{L}_{kkd}\mathrm{R}_f - \mathrm{R}_{kd}\mathrm{L}_{ff}\right\}}{s^2L_{md}} - \left(\frac{R_fR_{kd}k_1}{L_{md}}\right)\frac{1}{s^3} \tag{4.30}$$

Let $$c_1 = \left(\frac{k_1 L_{ff} L_{kkd}}{L_{md}} \right) \tag{4.31}$$

$$c_2 = \frac{k_1 \left\{ L_{md} - \mathrm{L}_{kkd} \mathrm{R}_f - \mathrm{R}_{kd} \mathrm{L}_{ff} \right\}}{L_{md}} \tag{4.32}$$

$$c_3 = \left(\frac{R_f R_{kd} k_1}{L_{md}} \right) \tag{4.33}$$

Hence

$$F_{31}(\mathrm{s}) = -c_1 \left(\frac{1}{s} \right) + c_2 \left(\frac{1}{s^2} \right) - c_3 \left(\frac{1}{s^3} \right) \tag{4.34}$$

Similarly F_{32} (s) can be expanded as a polynomial in ‘s-domain’ and the disturbance function D(s) can also be expressed in convenient form. However at this stage any researcher can demand that the behavior of the disturbance function as an equivalent analog filter will be a good portrait of the function of the machine in the transition zone. But generally in the case of a development of an analog filter use of complex frequency is avoided because practically(or experimentally) one cannot quantize the effect of damping which is mainly responsible for producing the real part of the complex frequency ‘s’. That is why to substitute $s=j\omega$, in the transfer function of any standard filter comes out to be a natural job for analyzing the behavior of the filter. In other words, in the context of the current research problem the frequency response of the disturbance function must be studied at this stage. Such necessity demands for a separate representation of D(jω) (after substituting $s=j\omega$ in the expression for D(s)). The formulation for D(jω) along with the subsequent results and discussions for this whole work can be put jointly in a separate section under the heading "Results and Discussions". This section is presented below.

4.3. Results and Discussions

Substituting s=jω in equation 4.34, it yields

$$F_{31}(\mathrm{j}\omega) = -c_1\left(\frac{1}{j\omega}\right) + c_2\left(\frac{1}{-\omega^2}\right) - c_3\left(\frac{1}{-j\omega^3}\right) \tag{4.35}$$

Since $\frac{1}{-j} = j$

$$F_{31}(\mathrm{j}\omega) = jc_1\left(\frac{1}{\omega}\right) - c_2\left(\frac{1}{\omega^2}\right) - jc_3\left(\frac{1}{\omega^3}\right)$$

$$= \left\{-\frac{c_2}{\omega^2}\right\} + j\left\{\frac{c_1}{\omega} - \frac{c_3}{\omega^3}\right\} \tag{4.36}$$

$$= A_{31} + jB_{31}$$

where

$$A_{31} = -\frac{c_2}{\omega^2} \text{ and } B_{31} = \frac{c_1}{\omega} - \frac{c_3}{\omega^3} \tag{4.37}$$

$$F_{32}(\mathrm{s}) = \left(R_{kd} + sL_{kkd}\right)\left(\frac{k_1}{s^2 L_{md}}\right)$$

$$= \left(\frac{k_1}{L_{md}} R_{kd}\right)\frac{1}{s^2} + \left(\frac{k_1 L_{kkd}}{L_{md}}\right)\frac{1}{s} \tag{4.38}$$

Let, $c_4 = \frac{k_1 R_{kd}}{L_{md}}$ and $c_5 = \frac{k_1 L_{kkd}}{L_{md}}$ (4.39)

$$F_{32}(\mathrm{s}) = \frac{c_4}{s^2} + \frac{c_5}{s}$$

(4.40)

substituting s = jω

$$F_{32}(\mathrm{j}\omega) = -\frac{c_4}{\omega^2} - j\frac{c_5}{\omega} \tag{4.41}$$

$$= A_{32} + jB_{32} \tag{4.42}$$

where

$$A_{32} = -\frac{c_4}{\omega^2} \,\&\, \mathrm{B}_{32} = \frac{c_5}{\omega} \tag{4.43}$$

$$\begin{aligned} F_3(\mathrm{j}\omega) &= \mathrm{F}_{31}(\mathrm{j}\omega) - \mathrm{F}_{32}(\mathrm{j}\omega) \\ F_3(\mathrm{j}\omega) &= A_3 + jB_3 \end{aligned} \tag{4.44}$$

$$\begin{aligned} A_3 &= A_{31} - A_{32} \\ &= -\frac{c_2}{\omega^2} + \frac{c_4}{\omega^2} = \frac{c_4 - c_2}{\omega^2} \\ B_3 &= \frac{c_1}{\omega} - \frac{c_3}{\omega^3} + \frac{c_5}{\omega} = \frac{c_1 + c_5}{\omega} - \frac{c_3}{\omega^3} \end{aligned} \tag{4.45}$$

$$\begin{aligned} & T_1(\mathrm{s}) + \mathrm{Js}^2 \\ &= \left\{ \frac{X_1 s^3 + X_2 s^2 + X_3 s + X_4}{l_1 s^3 + l_2 s^2 + l_3 s + l_4} \right\} + Js^2 \end{aligned} \tag{4.46}$$

Put s = jω

$$\begin{aligned} & T_1(\mathrm{s}) + \mathrm{Js}^2 \,|_{s=j\omega} \\ &= \left\{ \frac{-jX_1\omega^3 - X_2\omega^2 + jX_3\omega + X_4}{-jl_1\omega^3 - l_2\omega^2 + jl_3\omega + l_4} \right\} - J\omega^2 \\ &= \left\{ \frac{-jX_1\omega^3 - X_2\omega^2 + jX_3\omega + X_4}{-jl_1\omega^3 - l_2\omega^2 + jl_3\omega + l_4} \right\} \\ &= \frac{j\left\{ l_1\omega^5 J - \omega^3 X_1 - \omega^3 l_3 J + \omega X_3 \right\} + \left\{ \omega^4 l_2 J - l_4\omega^2 J - \omega^2 X_2 + X_4 \right\}}{-jl_1\omega^3 - l_2\omega^2 + jl_3\omega + l_4} \\ &= \frac{N_1(\mathrm{j}\omega)}{D_1(\mathrm{j}\omega)} \end{aligned} \tag{4.47}$$

Where

$$\begin{aligned} N_1(\mathrm{j}\omega) &= \mathrm{M}_1 + jN_1 \\ D_1(\mathrm{j}\omega) &= \mathrm{P}_1 + jQ_1 \end{aligned} \tag{4.48}$$

$$\{T_1(\mathrm{s}) + \mathrm{Js}^2\}\,|_{s=j\omega} = \left\{ \frac{M_1 + jN_1}{P_1 + jQ_1} \right\} \tag{4.49}$$

where

$$\begin{aligned} M_1 &= \omega^4 l_2 J - l_4 \omega^2 J - \omega^2 X_2 + X_4 \\ N_1 &= l_1 \omega^5 J - \omega^3 X_1 - \omega^3 l_3 J + \omega X_3 \\ P_1 &= l_4 - l_2 \omega^2 \\ Q_1 &= \omega l_3 - \omega^3 l_1 \end{aligned} \tag{4.50}$$

$$D(s) = \frac{F_3(\mathrm{s})}{T_1(\mathrm{s}) + \mathrm{J}\, s^2} \tag{4.51}$$

$$D(\mathrm{j}\,\omega) = \frac{(\mathrm{A_3P_1} - B_3\,\mathrm{Q_1}) + \mathrm{j}(\mathrm{B_3P_1} + \mathrm{A_3Q_1})}{M_1 + jN_1} \tag{4.52}$$

$$|\,\mathrm{D(j}\,\omega)\,| = \frac{\sqrt{(\mathrm{A_3P_1} - B_3\,\mathrm{Q_1})^2 + (\mathrm{B_3P_1} + \mathrm{A_3Q_1})^2}}{\sqrt{M_1^{\;2} + N_1^{\;2}}} \tag{4.53}$$

$$\angle D(\mathrm{j}\,\omega) = \tan^{-1}\left(\frac{\mathrm{B_3P_1} + \mathrm{A_3Q_1}}{\mathrm{A_3P_1} - B_3\,\mathrm{Q_1}} \right) - \tan^{-1}\left(\frac{N_1}{M_1} \right) \tag{4.54}$$

The overall mathematical treatment basically interprets the role of induction motor action just before switching to synchronous motor action in a convenient mathematical from in the complex frequency domain. The motivation to formulate this problem in **s** domain (complex frequency domain) is not intentional rather it is a natural tendency because this chapter may be treated as an extension of the previous chapter, i.e. Chapter 3 of this thesis. In that chapter the whole intention of the author of this thesis was to investigate the steady state stability aspects using small perturbation model and that is why the formulation was finalized in ‘s’ domain. As an extension of such motivation it reveals that investigations can be made on the same machine just before switching to synchronous motor action (running as an induction motor). The overall model in complex frequency domain in the form of small perturbation model taking the role of induction motor into account does not lead to stability assessment model. However the nature of the function D(s) can be investigated in the marginal condition after substituting $s=j\omega$. After such mathematical substitution the function D(jω) involves magnitude and phase angle plot against 'ω' and such behavior are similar to analog filter characteristics. Hence the role of induction motor action for a physical synchronous motor may be looked upon as an analog filter behavior in the pure frequency domain in the concerned mathematical model. Those magnitude and phase angle plots of D(jω) are shown in figures. 4.3 and 4.4 respectively. As the role of D(s) can be looked upon as a disturbance function in relation to the model given by equation $D(s)=\frac{F_3(\mathrm{s})}{T_1(\mathrm{s})+\mathrm{J}\,s^2}$. In the equation (4.24), one part T(s) = $\frac{1}{T_1(\mathrm{s})+\mathrm{J}\,s^2}$ can be treated as transfer function of the pure synchronous motor behavior of the same physical synchronous motor. Similar to D(jω) formulation, T(s) can be converted to T(jω) after substitution of $s = j\omega$ in the said function. The magnitude and phase angle plot of T(jω) against 'ω' are shown in figures 4.5 and 4.6 respectively. Even though the plots in Figures.4.3, 4.4, 4.5, 4.6 are based on the same equation (4.24), they need some physical explanation and the corresponding interpretations are presented in the next section.

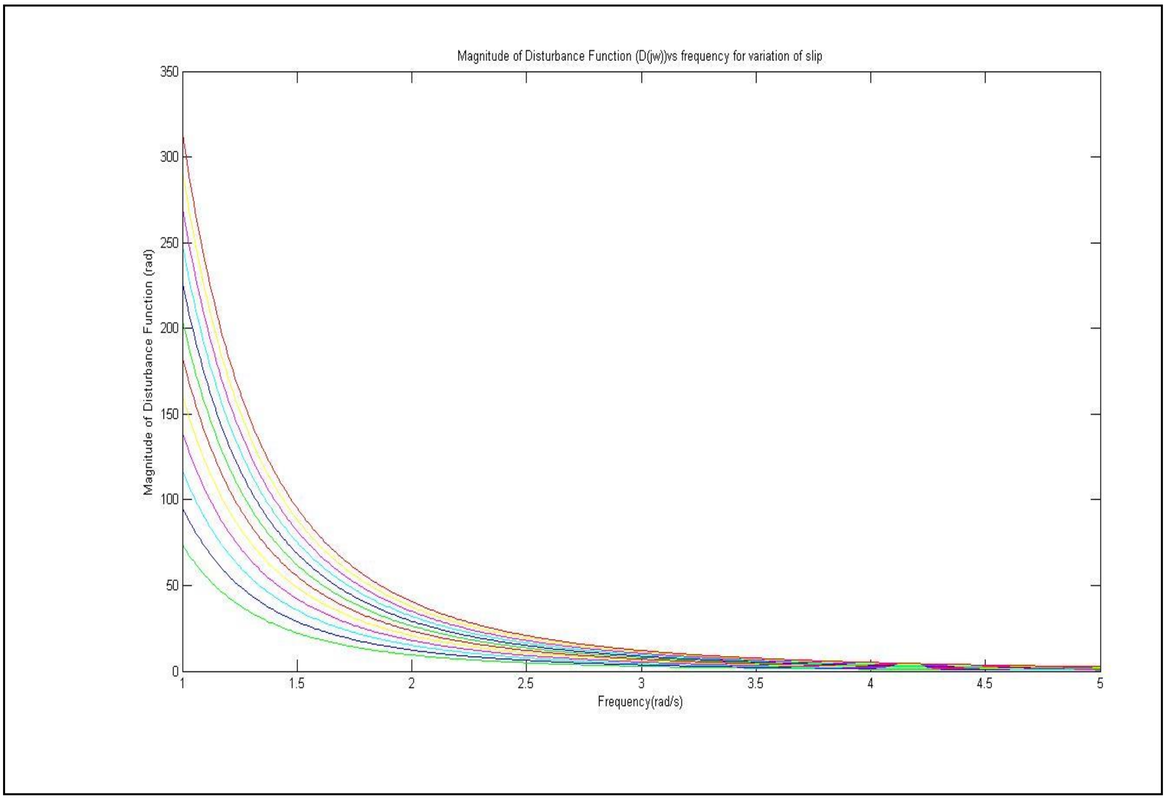


Figure.4.3: Magnitude of the |D(jω)| plotted against ω

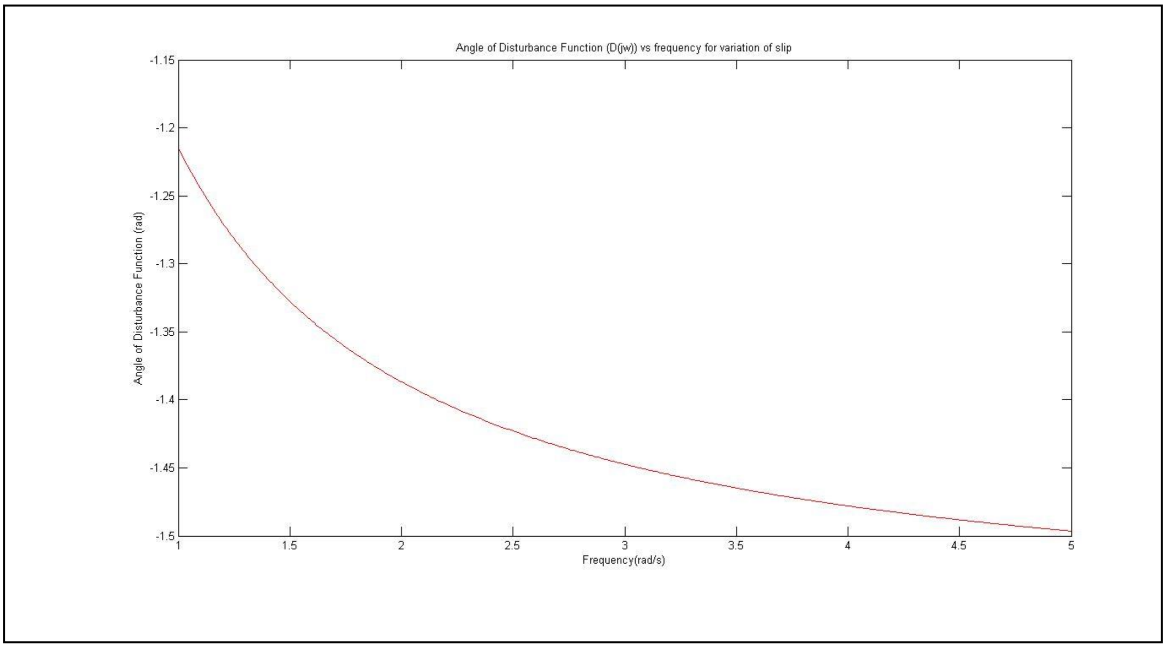


Figure.4.4:Angle of the |D(jω)| plotted against ω

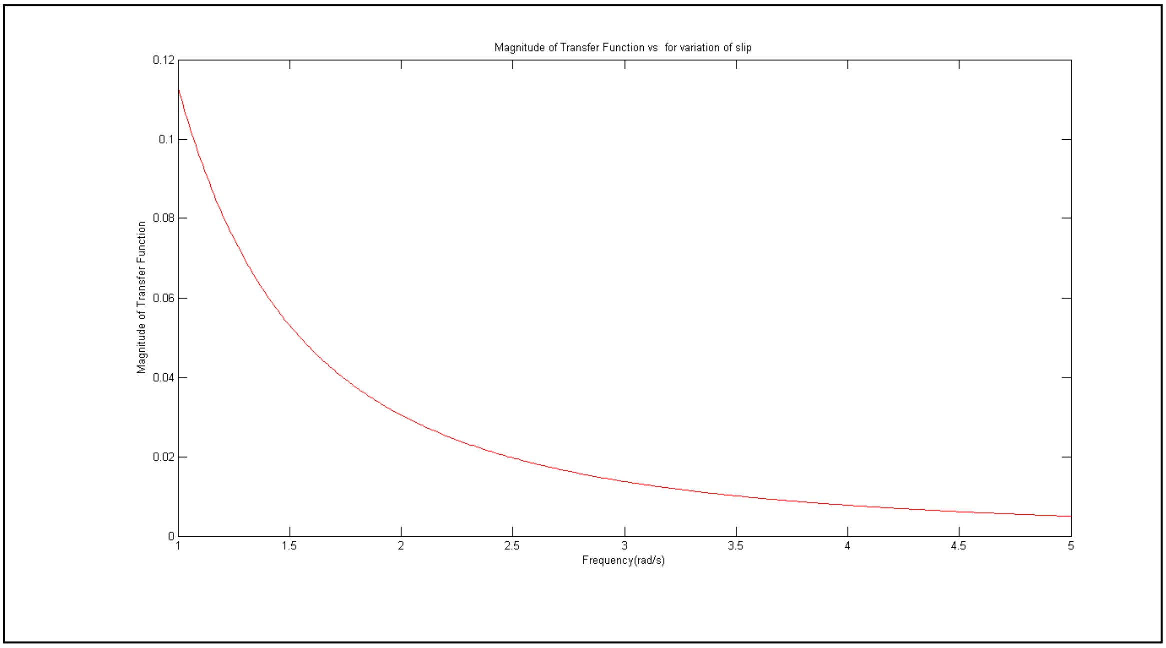


Figure.4.5: Magnitude of the transfer function plotted against ω

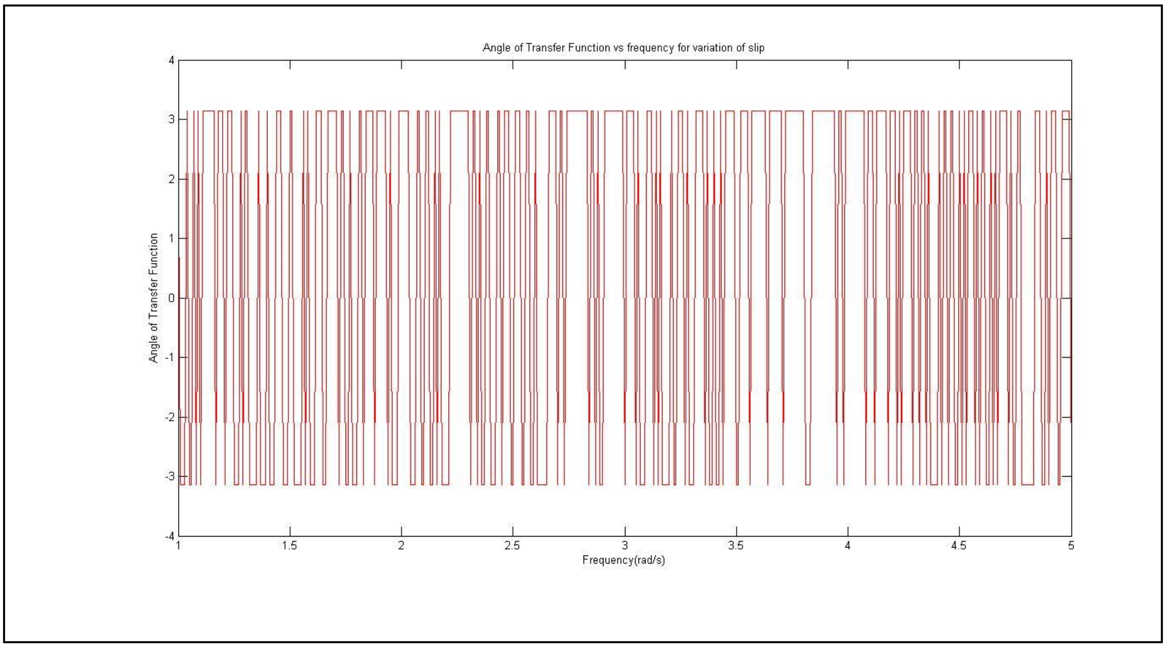


Figure.4.6: Angle of the transfer function plotted against ω

Magnitude of the $|D(j\omega)|$ plotted against 'ω' in figure 4.3, shows the nature of inverse function and also a family of plot is obtained by varying slip. It is already known that

$$D(\mathrm{j}\omega) = \frac{F_3(\mathrm{j}\omega)}{T_1(\mathrm{j}\omega) + \mathrm{J}(\mathrm{j}\omega)^2} = \frac{F_3(\mathrm{j}\omega)}{T_1(\mathrm{j}\omega) - \mathrm{J}\,\omega^2}$$

It may be recalled that during the period of induction motor action $\left[T_1(s) + Js^2\right]\Delta\beta(s) + \mathrm{F}_3(\mathrm{s}) = \Delta T_L(s)$, where each function has been expressed in the equations. Hence $F_3(s)$ can be treated as an equivalent to additional transformed load torque. This additional function has appearance due to the role of the induction motor when synchronous motor has started as an induction motor with the help of damper windings. We know that roughly induction motor torque is an increasing function of slip in the steady state zone. The transformations from time domain has been obtained using Laplace Transform operator and ultimately the formulation is obtained in the complex frequency domain (in terms of kernel 's'). Hence the physical transformation from induction motor to synchronous motor, during the starting period should not be confused with the mathematical transformations from time domain to Laplace domain and hence the role of slip remains same in Laplace domain as in time domain. The function $\left[T_1(s) + Js^2\right]$ in Laplace domain appears due to the induced dynamics of the physical synchronous machine when it is exactly running as synchronous machine. Therefore the parameter slip from the view point of engineering conception should not affect this function and furthermore it is well known that slip is the indication of an asynchronous machine.

Even though the major results have been interpreted but still insights on figures 4.3 and 4.4 are necessary. Such thinking leads to the representation in a separate subsection as follows

4.3.1. Explanation of the magnitude plot of D(jω) vs ω at a particular value of slip.

Again we shall recall the function $\frac{1}{\left[T_1(s)+Js^2\right]}$.It has been observed that the function, $\left[T_1(s)+Js^2\right]$ for $s=j\omega$, has a numerator polynomial containing highest power ω^3 and has a denominator polynomial containing highest power, ω^5. Hence such functions should show dominance near zero frequency. Now let us separately concentrate on the function $F_3(s)$ which has a real part containing highest power ω^2 and negative imaginary parts containing highest power ω^3. Hence D(jω) which is product of $F_3(j\omega)$ and $\frac{1}{\left[T_1(s)+Js^2\right]}$ must be dictated by profile of $\frac{1}{\left[T_1(s)+Js^2\right]}$ (for $s=j\omega$) , when ω is varied. That is why in the plot of |D(jω)| vs. ω shows an inverse function nature at a particular value of slip with high functional value around zero frequency.

Figure 4.5 represents the plot of magnitude of D(jω) against 'ω' based on equation (4.54). It may be reminded that in equation (4.54) the angle of $F_3(s)$ at $s = j\omega$ can be estimated very roughly as $\tan^{-1}\left[\frac{\left(\frac{1}{\omega^3}-\frac{1}{\omega}\right)}{-\frac{1}{\omega^2}}\right]$. Such rough expression will help the designer to predict the nature of the variation. The above said variation has a decaying nature with respect to ω. However the other part of the D(jω) function which is T(jω) = $\frac{1}{\left[T_1(s)+Js^2\right]}$ at $s = j\omega$ has an oscillatory behavior of its phase angle with respect to ω. This oscillatory behavior of angle of T(jω) vs. ω can be easily interpreted because this function can be treated roughly as $\tan^{-1}(\omega)$ and that is why it is bounded between some specific values of angle (value= π). This behavior is plotted in figure 4.6. The plot of phase angle of D (jω) vs. 'ω' involves the behavior of phase angle of T(jω) vs. 'ω' but the dominance of phase angle of F_3 (jω) is more, that is why the overall plot of phase angle of D(jω) vs. 'ω' shows a dragging nature and it is plotted in figure 4.6. With reference to equation $\Delta\beta(j\omega) = T(j\omega)\Delta T_L(j\omega) - D(j\omega)$, at this stage only the interpretation of the plot of T(jω) remains as pending because the other 3 plots (figures 4.3, 4.4 and 4.5) have been interpreted. The pending interpretation for figure 4.6 is as follows:

$$T(s) = \frac{1}{\left[T_1(s) + Js^2\right]} \text{ ,at } s = j\omega \text{ , } T(j\omega) = \frac{1}{\left[T_1(\mathrm{j}\omega) - J\omega^2\right]}$$

$\frac{1}{\left[T_1(\mathrm{j}\omega) - J\omega^2\right]}$ has a denominator polynomial in ω, having highest power '5'. Hence at the high frequency region, |T(jω)| decreases very fast. This plot is shown in figure 4.6

4.4. Conclusions

Role of Induction motor in Laplace domain appears as an equivalent "system" consisting of multi-frequencies.

- Filtering of the equivalent "NOISE" may not be needed due to natural attenuation.
- To avoid or minimize George's phenomenon during the transition zone, inclusion of external resistance in the field circuit basically leads to a complicated mathematical formulation, but clubbing this resistance along with the original field winding resistance develops the problem formulation in a more straight forward manner.

- With reference to Figure.4.3, the time-frequency contour of the function (in s-domain), D(s) can be developed using STOCKWELL-Transform(S-Transform)[20], and such contour will be able to provide more design information to the designer.

References

[1] Isfahani, A.H.; Vaez-Zadeh, S., "Effects of Magnetizing Inductance on Start-Up and Synchronization of Line-Start Permanent-Magnet Synchronous Motors," *IEEE Transactions on Magnetics,* vol.47, no.4, pp.823,829, April 2011. doi: 10.1109/TMAG.2010.2091651.

[2] Knight, A.M.; McClay, C.I., "The design of high-efficiency line-start motors," Industry Applications Conference, *Thirty-Fourth IAS Annual Meeting Conference Record of the 1999 IEEE* , vol.1, no., pp.516,522 vol.1, 1999. doi: 10.1109/IAS.1999.800002.

[3] Weifu Lu; Yingli Luo; Haisen Zhao, "Influences of Rotor Bar Design on the Starting Performance of Line-Start Permanent Magnet Synchronous Motor," *Sixth International Conference on Electromagnetic Field Problems and Applications (ICEF),* 2012, vol., no., pp.1,4, 19-21 June 2012.doi: 10.1109/ICEF.2012.6310360.

[4] Slemon, G.R.; Dewan, S.B.; Wilson, James W.A., "Synchronous Motor Drive with Current-Source Inverter," *IEEE Transactions on Industry Applications* , vol.IA-10, no.3, pp.412,416, May 1974. doi: 10.1109/TIA.1974.349170.

[5] Chattopadhyay, A.B.; Sunil Thomas.; Ruchira Chatterjee., "Analysis of Steady State Stability of a Csi Fed Synchronous Motor Drive System with Damper Windings Included.", *Trends in Applied Sciences Research,* 2011, vol.,no., 6, pp: 992-1005.doi: 10.3923/tasr.2011.992.1005.

[6] Ma Xianhao; Ding Shuye; Li Weili, "Calculation and Analysis of Magnetic Fields and Temperature Fields for Salient Pole Synchronous Motors in the Process of

Starting,". *International Conference on Power System Technology,* 2006. Power Con 2006, vol., no., pp.1,6, 22-26 Oct. 2006. doi: 10.1109/ICPST.2006.321414.

[7] Wang Xiaoyuan; Ren Na, "Simulation of asynchronous starting process of synchronous motors based on Matlab/Simulink,". *Sixth International Conference on Electrical Machines and Systems, ICEMS* 2003 , vol.2, no., pp.684,687 vol.2, 9-11 Nov. 2003.

[8] Das, J.C.; Casey, J., "Characteristics and analysis of starting of large synchronous motors," *Industrial & Commercial Power Systems Technical Conference, 1999 IEEE.* , vol., no., pp.10 pp.,, Aug 1999. doi: 10.1109/ICPS.1999.787222.

[9] Sergelen, B., "Mathematical model of salient pole synchronous motors supplied by a frequency converter,". *International Forum on Strategic Technology, IFOST* 2007 , vol., no., pp.390,393, 3-6 Oct. 2007. doi: 10.1109/IFOST.2007.4798612.

[10] Najafi, S.; Kar, N.C., "Effect of Short-Circuit Voltage Profile on the Transient Performance of Saturated Permanent Magnet Synchronous Motors," *Power Engineering Society General Meeting, 2007. IEEE* , vol., no., pp.1,6, 24-28 June 2007. doi: 10.1109/PES.2007.386288.

[11] Marx, D.; Pierfederici, S.; Davat, B., "Nonlinear control of an inverter motor drive system with input filter — large signal analysis of the DC-link voltage stability," *Power Electronics Specialists Conference, 2008. PESC 2008. IEEE* , vol., no., pp.498,503, 15-19 June 2008. doi: 10.1109/PESC.2008.459197.

[12] Harnefors, Lennart; Pietilainen, K., "Inverter DC-link stabilizing control with improved voltage sag ride-through capability," *Power Electronics and Applications, 2005 European Conference on* , vol., no., pp.10 pp.,P.10, 0-0 0 doi: 10.1109/EPE.2005.219553.

[13] Mosskull, H., "Some issues on stabilization of an induction machine drive," *Decision and Control, 2004. CDC. 43rd IEEE Conference on* , vol.4, no., pp.4441,4446 Vol.4, 14-17 Dec. 2004. doi: 10.1109/CDC.2004.1429450.

[14] Das, J.C.; Casey, J., "Characteristics and analysis of starting of large synchronous motors," *Industrial & Commercial Power Systems Technical Conference, 1999 IEEE.* , vol., no., pp.10 pp.,, Aug 1999. doi: 10.1109/ICPS.1999.787222.

[15] Al-Ohaly, A.A.; Hamouda, R.M.; Badr, M.A., "Torsional oscillations associated with the starting of three phase synchronous motors from a single phase supply," *Energy Conversion, IEEE Transactions on* , vol.12, no.1, pp.10,16, Mar 1997. doi: 10.1109/60.577274

[16] Modran, L.N., "Digital simulation of induction and permanent magnet synchronous motor starting," *11th International Conference on Optimization of Electrical and Electronic Equipment, OPTIM* 2008. , vol., no., pp.139,146, 22-24 May 2008. doi: 10.1109/OPTIM.2008.4602513.

[17] Ugale, R.T.; Bhanuji, A.; Chaudhari, B.K., "A novel line start permanent magnet synchronous motor using two-part rotor," TENCON 2009 - 2009 IEEE Region 10 Conference, vol., no., pp.1,5, 23-26 Jan. 2009.
doi: 10.1109/TENCON.2009.5395857.

[18] Moussa, M.F.; Dessouky, Y.G., "High starting performance synchronous motor: Topic number: T2," *International Symposium on Power Electronics Electrical Drives Automation and Motion (SPEEDAM),* 2010 , vol., no., pp.293,297, 14-16 June 2010. doi: 10.1109/SPEEDAM.2010.5545097.

[19] A. M. Knight and C. I. McClay, “The design of high efficiency line start motors,” *IEEE Transactions on Industry applications*, vol.36, No. 6,Nov. / Dec. 2000, pp. 1552-156.doi: 10.1109/IAS.1999.800002.

[20] Media, M.; Martin, F.; Craciunescu, A.; Ciumbulea, G.; Rodriguez, A., "Induction motor's broken bars detection by using Stockwell transform," *Power Electronics, Electrical Drives, Automation and Motion (SPEEDAM), 2012 International Symposium on* , vol., no., pp.1023,1026, 20-22 June 2012. doi: 10.1109/SPEEDAM.2012.6264459

Chapter 5: Modeling and Simulation of a Synchronous Motor Drive System: Transition Zone in Time Domain

5.1 Introduction

As the work in this chapter is an extension of the work carried out in chapter 4 of this thesis, the literature review part[1-13], remains mostly the same. It is true that the said literature review does not show any clear path of approach to the modeling of the drive system during the transition zone from induction motor mode to synchronous motor mode, including the effect of Georges phenomenon.

Basically the formulation will be done in complex frequency domain involving two transformed quantities: (i) small perturbation in load torque (ii) small perturbation in field angle. This is a traditional technique of formulation to put some light on the overall modeling of synchronous motor and also on the steady state aspects in general and transient state analysis in particular. But in the present problem as the machine's transition from induction motor mode to synchronous motor mode is of prior importance, a disturbance function in complex frequency domain is expected to appear in the resultant formulation and it is expected to disturb the linkup between the above said two transformed quantities (field angle and load torque).It is quite natural that the modeling of the disturbance function in complex frequency domain will give a lot of information to the design of a synchronous motor applied to various drive system, but still the necessity of getting the time domain response of that disturbance function remain within the scope of the research. Hence to get the time domain behavior of that disturbance function becomes the main objective of the author of the present thesis; because it will give more detailed information regarding the convergence and divergence nature of the disturbing function such that a designer can pre-assume what should be the time instant for synchronizing the motor. It reveals from the industrial experience that typical types of load for synchronous motor in some industries(e.g. steel industries) may be of discontinuous types. In such case, modeling of synchronous motor taking the effect of load discontinuity into account makes the modeling more complicated. In such case of application, to choose the exact method of starting of synchronous motor also becomes a major engineering decision. The easy way to tackle such problem is to choose a synchronous motor which has damper windings as additional constructional feature. But such decision leading to the ease in starting process will positively lead to the complexity in analysis or modeling because synchronous motor will start as an asynchronous motor(induction motor). The asynchronous motor appears because the concept of damper winding or coil closed in itself under the proximity of a rotating magnetic field created by the three phase stator windings. This idea is nothing but the behavior of a three phase asynchronous (induction) motor. But another problem is that a separately constructed induction motor will have a uniform radial air-gap length throughout the stator periphery. But this is not the situation in case of a three phase salient pole synchronous motor which has a well-defined air-gap saliency over the complete armature periphery. Hence the individual theory or circuit theory of induction motor cannot be principally applied , while a complete modeling of transition zone of induction motor mode to synchronous motor mode is to be mathematically explored. In other words, the principle of induction within a machine environment under the proximity of non-uniform air-gap has to be mathematically modeled accurately and it appears as a challenge before the author of the present thesis.

The present chapter exactly concentrates on the analytical aspects of time domain behavior of the disturbance function being influential during the particular zone of transition from induction motor to synchronous motor mode for a current source inverter fed synchronous motor drive system. It is well known that the complexity of synchronous motor in terms of number of windings and finite amount of air gap saliency, forces a researcher to face challenge in modeling of such transition zone in time domain directly, without applying Laplace Transform at the first instance of modeling. That is why firstly the modeling is presented in complex frequency domain(Chapter 4) and then the time domain modeling is obtained by applying inverse Laplace Transform technique. Furthermore the time domain response of the disturbance function may help a designer to fix up the time instant when the pull in phenomenon will be imposed by throwing the field winding to a dc supply.

5.2 Problem Formulation

The block diagram representation of the system obtained from the equation(4.34) is shown in figure.5.1, (For convenience, Figure.4.1, is reproduced as 5.1).

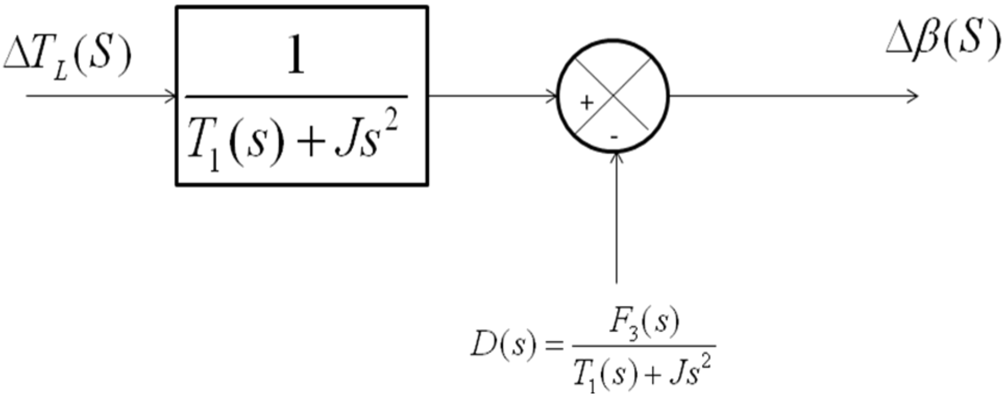


Figure 5.1.Block diagram representation of the system with disturbance function D(s)

The disturbance function can be taken separately and a detailed analysis in complex frequency domain is carried out as follows

$$F_3(\mathrm{s}) = \mathrm{F}_{31}(\mathrm{s}) - \mathrm{F}_{32}(\mathrm{s}) \tag{5.1}$$

where, $$F_{32}(\mathrm{s}) = (\mathrm{R}_{kd} + sL_{kd})(\frac{c + s_1 k}{s^2 L_{md}}) \tag{5.1a}$$

$$F_{31}(\mathrm{s}) = \frac{(\mathrm{c+s_1k})}{s^2}\left[1 - \left(\frac{R_f}{sL_{md}} + \frac{L_{ff}}{L_{md}}\right)\left(R_{kd} + sL_{kd}\right)\right] \tag{5.1b}$$

Let $k_1 = c + s_1 k$ (5.2)

$$F_{31}(\mathrm{s}) = \frac{k_1}{s^2}\left[\frac{\left\{sL_{md} - R_f R_{kd} - s(\mathrm{L}_{kkd}\mathrm{R}_f + \mathrm{R}_{kd}\mathrm{L}_{ff}) - \mathrm{s}^2 L_{ff} L_{kkd}\right\}}{sL_{md}}\right] \tag{5.3}$$

$$F_{31}(\mathrm{s}) = \frac{k_1}{s^3 L_{md}}\left\{-\mathrm{s}^2 L_{ff} L_{kkd} + s(L_{md} - \mathrm{L}_{kkd}\mathrm{R}_f - \mathrm{R}_{kd}\mathrm{L}_{ff}) - R_f R_{kd}\right\} \tag{5.4}$$

$$F_{31}(\mathrm{s}) = \left(-\frac{k_1 L_{ff} L_{kkd}}{L_{md}}\right)\left(\frac{1}{s}\right) + \frac{k_1\left\{L_{md} - \mathrm{L}_{kkd}\mathrm{R}_f - \mathrm{R}_{kd}\mathrm{L}_{ff}\right\}}{s^2 L_{md}} - \left(\frac{R_f R_{kd} k_1}{L_{md}}\right)\frac{1}{s^3} \tag{5.5}$$

Let $$c_1 = \left(\frac{k_1 L_{ff} L_{kkd}}{L_{md}}\right) \tag{5.6}$$

$$c_2 = \frac{k_1\left\{L_{md} - \mathrm{L}_{kkd}\mathrm{R}_f - \mathrm{R}_{kd}\mathrm{L}_{ff}\right\}}{L_{md}} \tag{5.7}$$

$$c_3 = \left(\frac{R_f R_{kd} k_1}{L_{md}}\right) \tag{5.8}$$

Hence

$$F_{31}(\mathrm{s}) = -c_1\left(\frac{1}{s}\right) + c_2\left(\frac{1}{s^2}\right) - c_3\left(\frac{1}{s^3}\right) \tag{5.9}$$

The disturbance function can be taken separately and a detailed analysis in time domain is carried out as follows:

$$D(\mathrm{s}) = \frac{F_3(\mathrm{s})}{T_1(s) + Js^2} \tag{5.10}$$

$$F_3(s) = \frac{-c_1}{s} + \frac{c_2}{s^2} - \frac{c_3}{s^3} - \frac{c_4}{s^2} - \frac{c_5}{s} \tag{5.11}$$

$$= \frac{-(c_1 + c_5)}{s} + \frac{(c_2 - c_4)}{s^2} - \frac{c_3}{s^3} \tag{5.12}$$

$$F_3(\mathrm{t}) = -(c_1 + c_5)u(\mathrm{t}) + (\mathrm{c}_2 - \mathrm{c}_4)\,\mathrm{t} - \frac{\mathrm{c}_3}{2}t^2 \tag{5.13}$$

$$= -1.4588*10^3 + 543.0379t - 0.0127t^2 \tag{5.14}$$

$Denominator = Q(s)$

$$\frac{1}{T_1(\mathrm{s}) + \mathrm{Js}^2} = \frac{l_1 s^3 + l_2 s^2 + l_3 s + l_4}{(\mathrm{s} - \mathrm{j}\,0.3899)(\mathrm{s} + \mathrm{j}\,0.3899)(\mathrm{s} + 0.1018)(\mathrm{s} + 0.0538)(\mathrm{s} + 0.0011)} \tag{5.15}$$

$$= \frac{A}{s - s_1} + \frac{B}{s - s_2} + \frac{C}{s - s_3} + \frac{D}{s - s_4} + \frac{E}{s - s_5} \tag{5.16}$$

$$\begin{aligned} A &= -j0.3666 \\ B &= j0.3666 \\ C &= -2.9253*10^{-16} \\ D &= 6.754*10^{-17} \\ E &= -7.8382*10^{-18} \end{aligned} \tag{5.17}$$

$$\frac{1}{T_1(\mathrm{s}) + \mathrm{Js}^2} = -j0.3666e^{j0.3899t} + j0.3666e^{-j0.3899} + (2.9253*10^{-16})\,\mathrm{e}^{-0.1018t} + (6.754*10^{-17})\,\mathrm{e}^{-0.0538t}$$
$$+(-1.8382*10^{-18})\,\mathrm{e}^{-0.011t} \tag{5.18}$$

$$D(\mathrm{t}) = \mathrm{F}_3(t) * L^{-1}\left(\frac{1}{T_1(s) + Js^2}\right) \tag{5.19}$$

$$= \int (-1.45\times 10^3 + 543\tau - 0.0127\tau^2)\{\mathrm{j}\,0.3666(\mathrm{e}^{-j0.3899(\mathrm{t}-\tau)} - \mathrm{e}^{j0.3899(t-\tau)})\}\,\mathrm{d}\,\tau \tag{5.20}$$

$$I_1 = \int_0^t \sin(0.3899(\mathrm{t}-\tau)) * (-1.45 * 10^3)\,d\tau \tag{5.21}$$

$$= \frac{1.45\times 10^3}{0.3899}\cos(t) - \frac{1.45\times 10^3}{0.3899} \tag{5.22}$$

$$I_2 = \int_0^t \sin(0.3899(\mathrm{t}-\tau))543\tau\,\mathrm{d}\,\tau \tag{5.23}$$

$$= 543\left(\frac{\mathrm{t}}{0.3899}\right) - \frac{\sin(0.3899t)}{0.3899^2} \tag{5.24}$$

$$I_3 = \int_0^t \sin(0.3899(\mathrm{t}-\tau))(-0.0127\tau^2)\,\mathrm{d}\,\tau \tag{5.25}$$

$$I_3 = -0.0127\left[\frac{t^2}{0.3899} + \frac{2}{0.3899^3} + \frac{\cos(0.3899(\mathrm{t}-\tau))}{0.3899^2}\right] \tag{5.26}$$

$$D(\mathrm{t}) = 0.7332\left[\frac{1.45\times 10^3}{0.3899}\{\cos(0.3899t) - 1\} + 543\left(\frac{t}{0.3899} - \frac{\sin(0.3899\,\mathrm{t})}{0.3899}\right)\right.$$
$$\left. -0.0127\left\{\frac{\mathrm{t}^2}{0.3899} + \frac{2}{0.3899}\left(\frac{\cos(0.3899t)}{0.3899^2} - \frac{1}{0.3899^2}\right)\right\}\right] \tag{5.27}$$

where * indicates convolution in time domain.

5.3. Results and Discussions

The expression for D(t) in equation(5.27), demands some numerical calculations such that the different aspects of D(t) contributing to overall action of synchronous motor cane be realized. This particular philosophy forces the authors to present the results in graphical forms in figures 5.2, 5.3 and 5.4. Furthermore the philosophy of calculation and extended results lead to the presentation in figure.5.5 and Table 5.1.

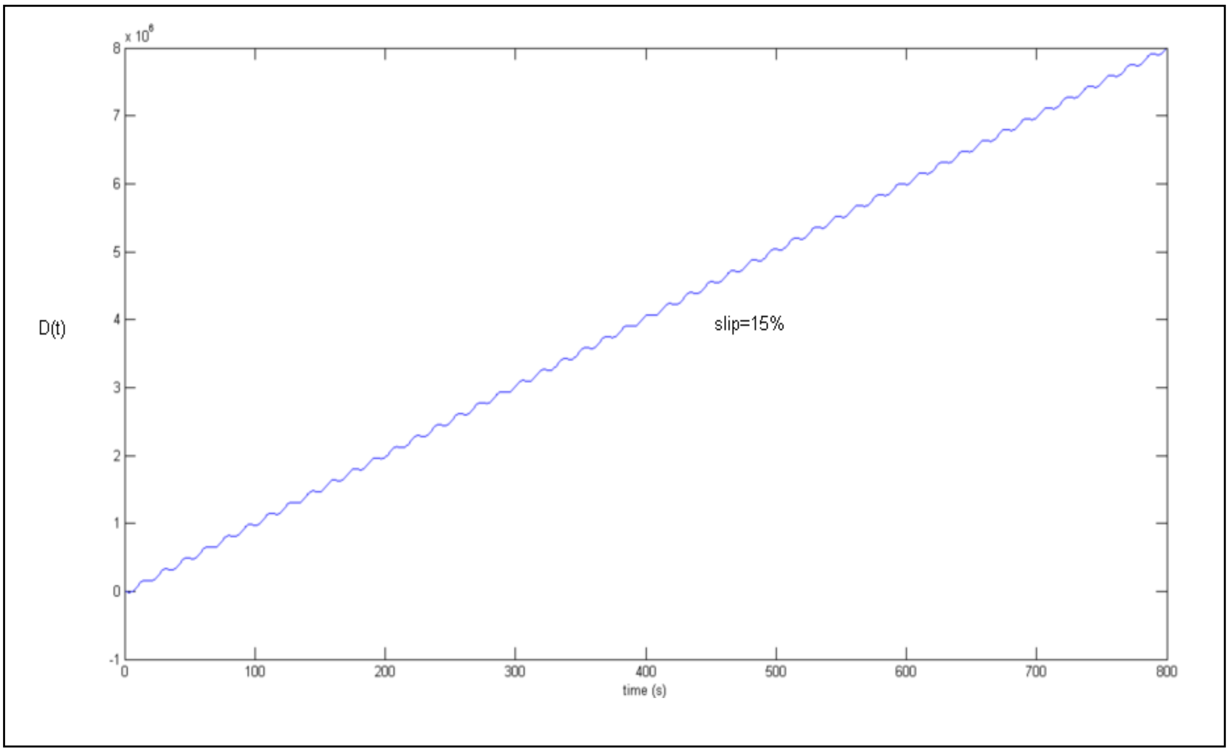


Figure.5.2: Disturbance function D(t) vs. time

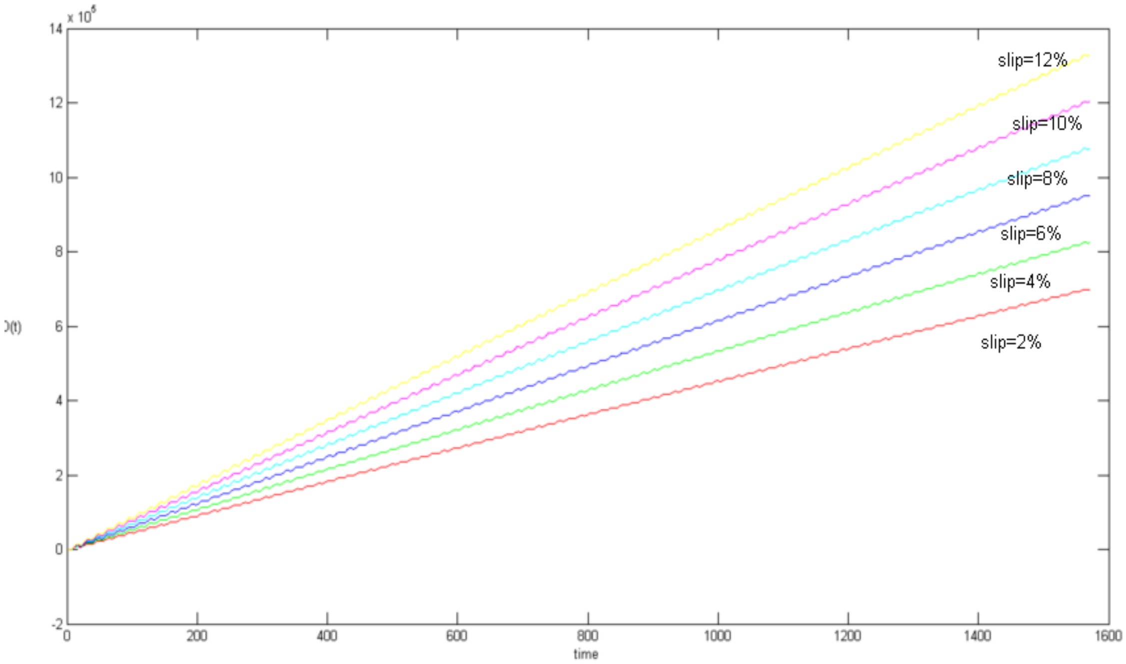


Figure.5.3: D(t) vs. time for various values of slip

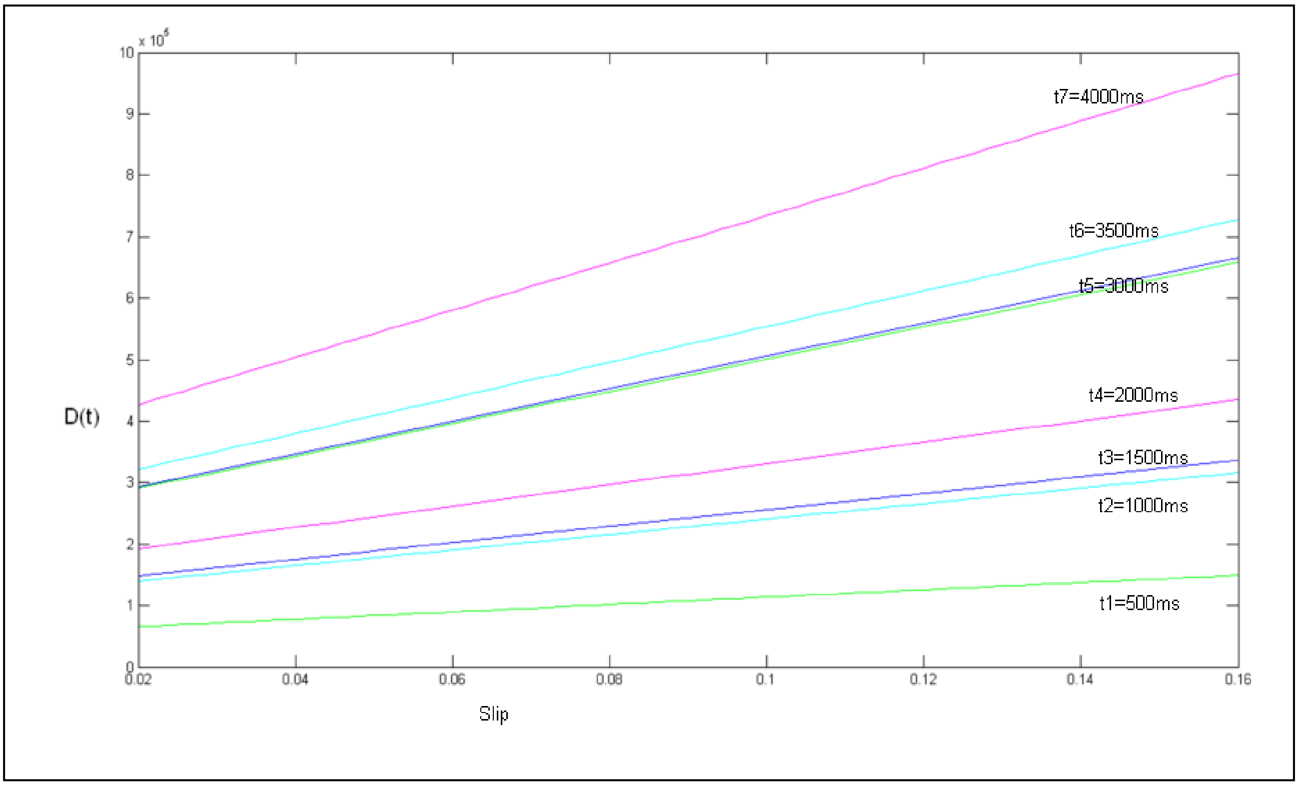


Figure.5.4: Variation of magnitude of D(t) against slip for a fixed time instant

Figure. 5.2 explains the variation of magnitude of the disturbance function D(t) against time at a particular value of slip of 0.15.From the nature of the problem formulation it is already well known that the asynchronous behavior of the synchronous machine in motor mode basically can be looked upon as a disturbance phenomenon, even though this should not be treated as a negative one because it indicates basically the starting method of synchronous motor. However qualitative treatment always does not guarantee the quality of a research work. As a supporting point to this statement the variation of D(t) is plotted against time in figure.5.2.From this plot, it is very clear that ,out of all the terms, the coefficient associated with the term’t^2’is dominant. That is why the plot in figure 5.2, is observed as an increasing function. Furthermore the oscillations superimposed on the straight line(strictly speaking it is a parabola) in figure 5.2 basically indicates the involvement of sinusoidal function of time in the expression for D(t) and in reality exactly it is happening so far as the formulation is concerned. Figure5.3 represents a family of plots of D (t) against time for different values of slips.

Figure 5.4 shows the variation of disturbance function D (t) with respect to slip at different time instants. From the expressions for D (t) it is quite natural that oscillations will be superimposed on straight line nature. As seen from the expressions for involved co-efficient for D (t), it is clear that the term ‘slip’ (s_1) appears in the numerator of the concerned expressions, expressed as fractional terms. Hence the nature of the profile shown in figure5.4 resembles with the physical fact. One interesting point is to be noted that the effect of oscillations superimposed on the profile of straight line have been observed in the figures 5.2, 5.3 and 5.4 and accordingly all the physical conclusions or inferences have been drawn in the above said paragraphs with sufficient engineering explanations. However the authors reveal that the mathematical nature of D (t) can be looked upon as a function of multivariable's as ‘t’ (time instant) and ‘s_1’ (slip).Hence strictly speaking D(t) can be looked upon as the function $D(s_1,t)$.Now is the question arises how to look the oscillations contributed by ‘time’ and ‘slip’ variations. This whole philosophical concept can be expressed in the form of a self-explanatory flow chart given in figure 5.5.

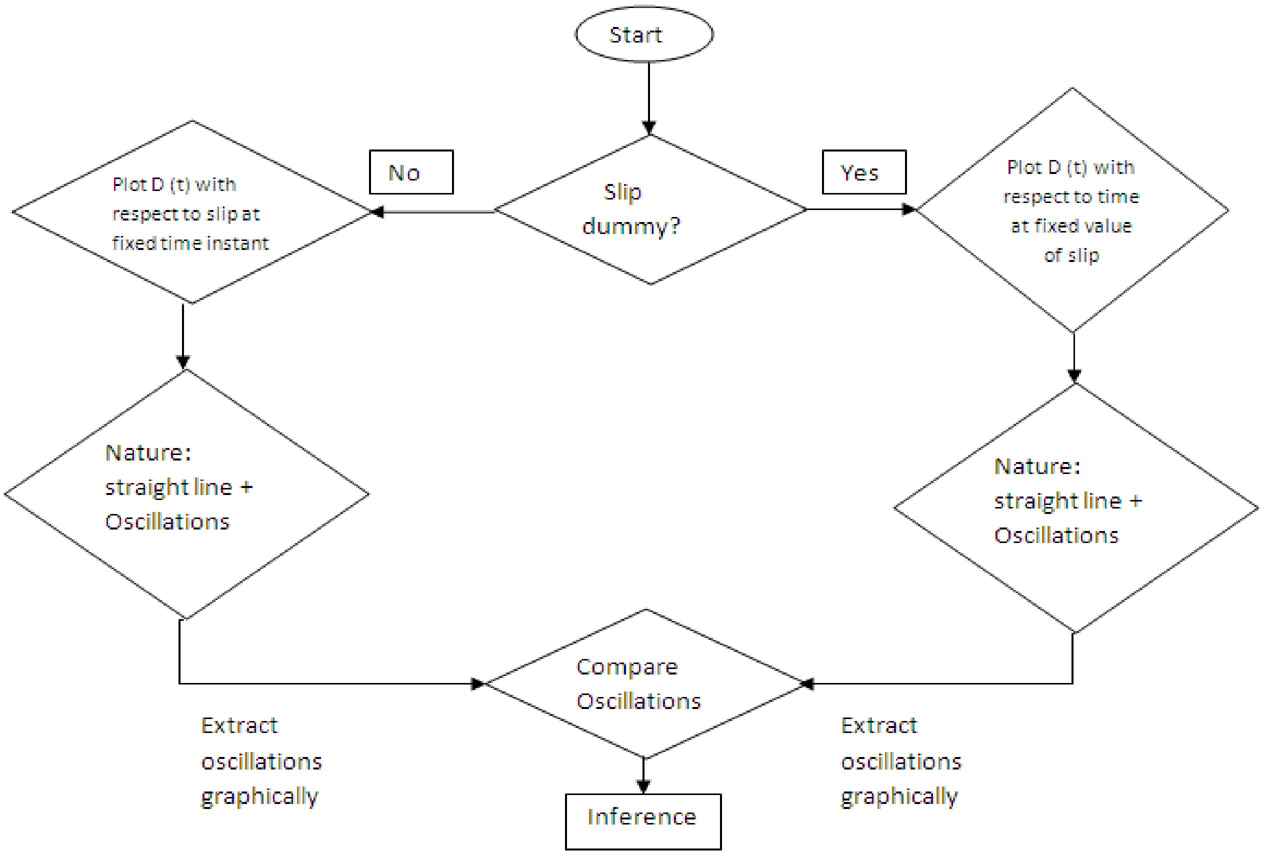


Figure.5.5: Flow chart pertaining to contributions of oscillations to the responses in Figures. 5.2, 5.3 and 5.4

5.3.1Additional interesting Facts about the results

With reference to figure .5.1, it is clearly observed that the role of D(t) can be looked upon as a transformed noise or disturbance function. For a control model of any system, it is very logical to observe the integrated value of the noise function in time domain for the sake of maintaining the health of the system. Based on this idea, Table 5.1 has been developed, which shows the numerical value of ∫D (t) dt over selected time spans, based on the expression for D(t) in equation(5.27).

Each cycle is 0.02 seconds	1st cycle	1st cycle+ 2nd cycle	1st cycle+2nd cycle+3rd cycle
∫D (t) dt	0.2042	0.8168	1.8377

Table.5.1:Numerical value of ∫D (t) dt over selected time spans based on equation(5.27).

The subsequent further explanations related to the table 5.1, are as follows: It is difficult to infer about the nature of D(t) vs. time because the dominance of algebraic terms over trigonometric term are not observable at the first glance. Furthermore, relative dominance within the algebraic terms are also not understood from the expressions for D(t).That is why the integral effect of D(t) over time are calculated and it shows the existence of dominance of specific algebraic term .i.e.' t^2'..The reason behind the calculation of integrated values of D(t) over time[first cycle, first and second cycle and first ,second and third cycles] is mainly to observe the integrated effect of noise during the sub transient period. Strictly speaking when the machine behave as an induction motor, the terminology, "sub transient period", has not much physical significance. But this terminology is used intentionally to emphasize the fact that it is not an isolated induction motor, rather it is a part of the whole synchronous motor which has a damper winding being mainly responsible for creating sub transient state.

5.4. Conclusions

During the transition zone the synchronous motor is started as an induction motor with the help of damper winding. During the transition, in order to avoid George's phenomenon the field winding of the synchronous motor is closed through an external resistance which is about 6-7 times that of the field winding resistance value. During the start of the above said process the change in voltage across the field winding is given by $220+s_1k$ where 's_1' is the slip of the induction motor and 'k' is a constant. At start slip = 1, hence $\Delta U_f = 220+s_1k$ is maximum. As the slip decreases towards 0, ΔU_f decreases towards 220 Volts. When this happens the field flux gets weakened as the slip movers from $s_1 = 1$ to $s_1 = 0$. This in turn decreases the electromagnetic torque developed in the machine for a fixed value of β (load angle). Hence T_e-T_L will also decrease during this process. This will in turn, reduce the rate of change of ω with respect to time in the motor because every machine dynamics will have to obey the law; $T_e - T_L = J\frac{d\omega}{dt}$. To adjust the decreased 'ω', the field angle (β), of the machine will have to increase accordingly to stabilize the system. Hence during the transition period Δβ will have to increase as slip goes from 1 to 0.

References

[1] Isfahani, A.H.; Vaez-Zadeh, S., "Effects of Magnetizing Inductance on Start-Up and Synchronization of Line-Start Permanent-Magnet Synchronous Motors," *IEEE Transactions on Magnetics*, vol.47, no.4, pp.823,829, April 2011. doi: 10.1109/TMAG.2010.2091651.

[2] Knight, A.M.; McClay, C.I., "The design of high-efficiency line-start motors," Industry Applications Conference, *Thirty-Fourth IAS Annual Meeting Conference Record of the 1999 IEEE* , vol.1, no., pp.516,522 vol.1, 1999. doi: 10.1109/IAS.1999.800002.

[3] Weifu Lu; Yingli Luo; Haisen Zhao, "Influences of Rotor Bar Design on the Starting Performance of Line-Start Permanent Magnet Synchronous Motor," *Sixth International Conference on Electromagnetic Field Problems and Applications (ICEF),* 2012, vol., no., pp.1,4, 19-21 June 2012.doi: 10.1109/ICEF.2012.6310360.

[4] Slemon, G.R.; Dewan, S.B.; Wilson, James W.A., "Synchronous Motor Drive with Current-Source Inverter," *IEEE Transactions on Industry Applications* , vol.IA-10, no.3, pp.412,416, May 1974. doi: 10.1109/TIA.1974.349170.

[5] Chattopadhyay, A.B., Sunil Thomas., Ruchira Chatterjee., "Analysis of Steady State Stability of a Csi Fed Synchronous Motor Drive System with Damper Windings Included.", *Trends in Applied Sciences Research*, 2011, vol.,no., 6, pp: 992-1005.doi: 10.3923/tasr.2011.992.1005.

[6] Ma Xianhao; Ding Shuye; Li Weili, "Calculation and Analysis of Magnetic Fields and Temperature Fields for Salient Pole Synchronous Motors in the Process of Starting,". *International Conference on Power System Technology,* 2006. Power Con 2006, vol., no., pp.1,6, 22-26 Oct. 2006. doi: 10.1109/ICPST.2006.321414.

[7] Wang Xiaoyuan; Ren Na, "Simulation of asynchronous starting process of synchronous motors based on Matlab/Simulink,". *Sixth International Conference on Electrical Machines and Systems, ICEMS* 2003 , vol.2, no., pp.684,687 vol.2, 9-11 Nov. 2003.

[8] Das, J.C.; Casey, J., "Characteristics and analysis of starting of large synchronous motors," *Industrial & Commercial Power Systems Technical Conference, 1999 IEEE.* , vol., no., pp.10 pp.,, Aug 1999. doi: 10.1109/ICPS.1999.787222.

[9] Sergelen, B., "Mathematical model of salient pole synchronous motors supplied by a frequency converter,". *International Forum on Strategic Technology, IFOST* 2007 , vol., no., pp.390,393, 3-6 Oct. 2007. doi: 10.1109/IFOST.2007.4798612.

[10] Najafi, S.; Kar, N.C., "Effect of Short-Circuit Voltage Profile on the Transient Performance of Saturated Permanent Magnet Synchronous Motors," *Power Engineering Society General Meeting, 2007. IEEE* , vol., no., pp.1,6, 24-28 June 2007. doi: 10.1109/PES.2007.386288.

[11] Marx, D.; Pierfederici, S.; Davat, B., "Nonlinear control of an inverter motor drive system with input filter — large signal analysis of the DC-link voltage stability," *Power Electronics Specialists Conference, 2008. PESC 2008. IEEE* , vol., no., pp.498,503, 15-19 June 2008. doi: 10.1109/PESC.2008.459197.

[12] Harnefors, Lennart; Pietilainen, K., "Inverter DC-link stabilizing control with improved voltage sag ride-through capability," *Power Electronics and Applications, 2005 European Conference on* , vol., no., pp.10 pp.,P.10, 0-0 0 doi: 10.1109/EPE.2005.219553.

[13] Mosskull, H., "Some issues on stabilization of an induction machine drive," *Decision and Control, 2004. CDC. 43rd IEEE Conference on* , vol.4, no., pp.4441,4446 Vol.4, 14-17 Dec. 2004. doi: 10.1109/CDC.2004.1429450.

Chapter 6: Sensitivity Analysis of Transfer Function of a Current Source Inverter fed Synchronous Motor Drive System as an Extension to Steady State Stability Analysis

6.1. Introduction

It is well known that the transfer function plays a dominant role in the behavior of a three phase synchronous motor drive system from the view point of steady state stability analysis. Simultaneously it is also true that the transfer function with respect to those machine design parameters of the synchronous motor should be designed in a way such that the magnitude of the sensitivity of the transfer function may lie within a tolerable range. Such view point of the fabrication process can lead to the development of a new prototype of synchronous motor to become successful in the open market competition. In the present chapter, those machine design parameters have been selected from the view point of experience and practical applications and accordingly the sensitivity analysis have been brought into a particular shape for the ease of fabrication process.

It is well known that now a days the “Sensitivity Analysis" is being portrayed by the current researchers as a powerful tool in the areas of torque ripple reduction [1], parameter identification [2], control behavior of permanent magnet machines [3-5],robust design of permanent magnet machines[6]. All the above said references are related to three phase synchronous motor drive systems. However the authors believe that even though the sensitivity analysis have wide applications in the periphery of synchronous and induction motors, in the case of a synchronous generator as a component of power system, sensitivity analysis can be applied in an effective manner to meet certain objectives like "Tuning the voltage regulator parameters" [7] and area of “Machine insulation co-ordination through high voltage/surge test” [8].

From the mathematical treatment in the above said literature, one point becomes very clear that the researcher will have to be expert in identifying the system parameters or machine design parameter, tuning by which the sensitivity of the selected performance figure or the objective function will come out as a compact numerical data. This particular paragraph defines the basic responsibility of the designer involved in designing partly or fully a synchronous motor drive system or a synchronous generator as a part of the power system. In context to the above said discussion it can be indicated that the present chapter of this thesis looks upon the transfer function of a current source inverter (CSI) fed synchronous motor drive system from the view point of the sensitivity analysis.

At this stage it looks awkward if the researcher directly brings the methodology of sensitivity analysis of the transfer function of a current source inverter (CSI) fed synchronous motor drive system (where the transfer function has been developed to access the steady state stability analysis criteria [9]). To avoid such awkwardness it looks better to give an outline concept of sensitivity and then to bring the actual methodology into picture. The subsequent paragraph is presented in this particular direction.

If 'u' becomes a function of any engineering parameter 'y', then if the parameter is tuned in most of the practical engineering problems it is needed to observe what is the corresponding change in 'u'. If a very small change in 'y' leads to a very large change in 'u', then the system is said to be very sensitive with respect to that particular parameter. Such exercise becomes a necessity in designing an engineering problem. This general philosophy on sensitivity can also be applied to the current research problem on synchronous motor drive system. With reference to the steady state stability problem of a current source inverter (CSI) fed synchronous motor, the transfer function developed in Laplace domain [10] generally becomes a complex frequency dependent function. For specific cases like determination of frequency responses etc, $s='j\omega'$, can be substituted in the transfer function and different observations on the frequency responses can be obtained. But to treat this current research work as a machine design problem it is necessary to examine the effect of machine parameters on the particular dependant variable, either in complex frequency domain or in time domain as per necessity. In the present problem the assessment of steady state stability has been done using the Routh-Hurwitz criteria which only needs the characteristic equation in Laplace domain. For a given set of values for machine parameters, the above criterion is applied and necessary inferences were made on the stability aspect. But it has not investigated that how the co-efficient of the characteristic equation dominate the pole placements or in other words how the transfer function is sensitive to the machine design parameter. Hence, at this stage to perform the sensitivity analysis of the transfer function with respect to different machine design parameters is needed and the detailed mathematical process is presented in the next section.

As synchronous motor has no inherent starting torque, it is generally started with the help of damper winding as an induction motor. Hence the damper winding resistances along *'d'* and *'q'* axis (i.e. R_{kd}, R_{kq}) must play a dominant role in the performance of the motor. Hence the roles of resistances of damper windings placed on *'d'* and *'q'* axis (i.e. R_{kd}, R_{kq}) are crucial in the entire starting process of the three phase synchronous motor drive system. As a starting step of the analysis 'R_{kd}' is first taken up as the machine design parameter to observe the sensitivity.

6.2. Sensitivity analysis of damper windings with respect to the obtained transfer function.

As stated in the end of section 6.1, before performing sensitivity analysis, the particular performance index or transfer function should be formulated in a clear cut manner. Furthermore from the view point of the final formulation of the said function it should be very clear that which are the possible parameters or the independent variables, by tuning which the sensitivity of the dependent variable or the function can be found out. At the present stage of the analysis the above said responsibility is completed and therefore naturally the next section will be devoted for the sensitivity analysis of the concerned transfer function. The selected machine design parameters for carrying out the sensitivity analysis are *'d'* axis damper winding resistance and *'q'* axis damper winding resistance. The reasoning behind deciding the said machine design parameters is not heuristic and the possible explanation is presented as follows. Chapters 3,4 and 5 of this thesis basically can be looked upon as a modeling work of a synchronous motor (as a whole). Out of these three chapters, chapter 4 and 5, pertain to the modeling of asynchronous behavior of a synchronous machine. This asynchronous behavior, obviously, a contribution by the damper windings which becomes responsible mainly for creating the induction phenomenon. That is why R_{kd},and R_{kq} are chosen to be the tuning parameter for carrying out the proposed sensitivity analysis .

The sensitivity of magnitude of transfer function obtained in equation 3.33 (refer chapter 3), can be obtained by using the basic formula:

$$^{T}S_{Rkd} = \left(\frac{\partial T}{\partial R_{kd}}\right)\left(\frac{R_{kd}}{T}\right) \tag{6.1}$$

6.2.1 Calculation of sensitivity of T(s) with respect to R_{kd}

Step -1 (Substitute $s=j\omega$ in equation no.(6.1))

$$T(\mathrm{j}\omega) = \frac{-j\omega^3 l_1 - \omega^2 l_2 + j\omega l_3 + l_4}{j\omega^5 k_1 + \omega^4 k_2 - j\omega^3 k_3 - \omega^2 k_4 + j\omega k_5 + k_6} \tag{6.2}$$

$$= \frac{A_1 + jB_1}{C_1 + jD_1} \tag{6.3}$$

Where, $A_1 = l_4 - \omega^4 l_2$ (6.3a)

$$B_1 = \omega l_3 - \omega^3 l_1 \tag{6.3b}$$

$$C_1 = \omega^4 k_2 - \omega^2 k_4 + k_6 \tag{6.3c}$$

$$D_1 = \omega^5 k_1 + \omega k_5 - \omega^3 k_3 \tag{6.3d}$$

Step – 2

$$|\,\mathrm{T(j\omega)}\,| = \sqrt{\frac{A_1{}^2 + B_1{}^2}{C_1{}^2 + D_1{}^2}} \tag{6.4}$$

Step – 3

$$\frac{|\,\partial T(\mathrm{j}\omega)\,|}{\partial R_{kd}} = \frac{1}{\sqrt{C_1{}^2 + D_1{}^2}}\,\frac{\partial}{\partial R_{kd}}\left[\sqrt{A_1{}^2 + B_1{}^2}\right] + \left(\sqrt{A_1{}^2 + B_1{}^2}\right)\left(-\frac{1}{2}\right)(\mathrm{C}_1{}^2 + \mathrm{D}_1{}^2)^{-\frac{3}{2}}\,\frac{\partial}{\partial R_{kd}}\left[\mathrm{B}(\mathrm{C}_1{}^2 + \mathrm{D}_1{}^2)\right] \tag{6.5}$$

Step – 4

$$A_1{}^2 + B_1{}^2 = (\mathrm{l}_4 - \omega^2\,\mathrm{l}_2)^2 + (\omega\mathrm{l}_3 - \omega^3\,\mathrm{l}_1)^2 \tag{6.6}$$

$$= (\mathrm{l}_4)^2 + \omega^4 l_2{}^2 - 2\omega^2 l_4 l_2 + \omega^2 l_3{}^2 + \omega^6 l_1{}^2 - 2\omega^4 l_3 l_1 \tag{6.7}$$

$$\frac{\partial}{\partial R_{kd}}\left[A_1{}^2 + B_1{}^2\right] = 2l_4\left(\frac{\partial l_4}{\partial R_{kd}}\right) + 2l_2\omega^4\left(\frac{\partial l_2}{\partial R_{kd}}\right) - 2\omega^2 l_4\Big|_{R_{kd}=R_{kd\,0}}\left(\frac{\partial l_2}{\partial R_{kd}}\right)$$
$$-2\omega^2 l_2\left(\frac{\partial l_4}{\partial R_{kd}}\right) + \omega^2\,2l_3\left(\frac{\partial l_3}{\partial R_{kd}}\right) + 0 - 2\omega^4 l_1\left(\frac{\partial l_3}{\partial R_{kd}}\right) \tag{6.8}$$

Step – 6

$$\left.\frac{\partial l_4}{\partial R_{kd}}\right| = R_f\,R_{kq}(1) \tag{6.9}$$

$$\left.\frac{\partial l_2}{\partial R_{kd}}\right| = L_{kkq}L_{ff} \tag{6.10}$$

$$\left.\frac{\partial l_3}{\partial R_{kd}}\right|_{R_{kd}=R_{kd0}} = R_f L_{kkq} + R_{kq} L_{ff} \tag{6.11}$$

Step – 7

$$\frac{\partial}{\partial R_{kd}}\sqrt{A_1^2 + B_1^2} = \frac{1}{2}\left(A_1^2 + B_1^2\right)^{-\frac{1}{2}} \frac{\partial}{\partial R_{kd}}\left[A_1^2 + B_1^2\right] \tag{6.12}$$

$$= \frac{1}{2}\left(A_1^2 + B_1^2\right)^{-\frac{1}{2}}\left[R_f R_{kq}\left(2l_4 - 2\omega^2 l_2\right) + L_{kkq} L_{ff}(-2\omega^2 l_4 + 2 l_2 \omega^4) + \left(R_f L_{kkq} + R_{kq} L_{ff}\right)\left(2\omega^2 l_3 - 2\omega^4 l_1\right)\right] \tag{6.13}$$

$$= \frac{1}{2}\left(A_1^2 + B_1^2\right)^{-\frac{1}{2}}\left[P_1 + P_2 + P_3\right] \tag{6.14}$$

$$k_1 = 2.29 \tag{6.15}$$

$$k_2 = 0.45776 + 7.52 R_{kd} \tag{6.16}$$

$$\frac{\partial k_2}{\partial R_{kd}} = 7.52 \tag{6.17}$$

$$\frac{\partial k_4}{\partial R_{kd}} = J\,\frac{\partial l_4}{\partial R_{kd}} \tag{6.18}$$

$$\frac{\partial k_4}{\partial R_{kd}} = 0.012 + 0.022 = 0.034 \tag{6.19}$$

$$k_6 = 8.26 \times 10^{-5} R_{kd} \tag{6.20}$$

$$\frac{\partial k_6}{\partial R_{kd}} = 8.2 \times 10^{-5} \tag{6.21}$$

$$c_1 = \left(L_d - L_q\right) L'_{q0} - L_{md} L'_{kq0} \tag{6.22}$$

$$c_1 = -0.0726 \tag{6.23}$$

$$b_3 = 0.0015 R_{kd} \tag{6.24}$$

$$f_2 = 0.039 \tag{6.25}$$

$$m_4 = -4.25\times10^{-6} R_{kd} \tag{6.26}$$

$$\begin{aligned}C_1^{\,2} + D_1^{\,2} &= \left[0.45776 + 7.52R_{kd} + 8.20\times10^{-5} R_{kd}\right]^2 + (0.034)^2\\ &-2\left((0.45776 + 7.52R_{kd}) + 8.20\times10^{-5} R_{kd}\right)(0.034)\\ &+(9.177 + 0.061R_{kd})^2 + (0.4133)^2\\ &-2\times0.4133(9.177 + 0.061R_{kd})\end{aligned} \tag{6.27}$$

For $R_{kd} = 0.03$

$$\begin{aligned}C_1^{\,2} + D_1^{\,2} &= \left[0.683 + 2.46\times10^{-6}\right]^2 + (0.034)^2\\ &-2\left[0.683 + 2.46\times10^{-6}\right](0.034) + 84.25 + 0.1708\\ &-0.8266(9.1788)\end{aligned} \tag{6.28}$$

$$= 0.466 + 1.156\times10^{-3} - 0.046 + 84.25 + 0.1708 - 7.58$$

$$= 77.26$$

To Find $\sqrt{A_1^{\,2} + B_1^{\,2}}$

$$A_1 = l_4 - \omega^2 l_2$$

$$B_1 = \omega l_3 - \omega^3 l_1$$

$$l_1 = 1.055 \qquad l_2 = \left(1.22\times10^{-3} + 0.94R_{kd}\right)$$

$$l_3 = \left(0.052R_{kd} + 6.564\times10^{-5}\right)$$

$$l_4 = 5.85\times10^{-5} R_{kd} \tag{6.29}$$

For $\omega = 1$p.u and $R_{kd} = 0.03$

$$A_1^{\,2} = \left(l_4 - \omega^2 l_2\right)^2 = \left(l_4 - l_2\right)^2$$

$$= \left[5.85\times10^{-5}\times0.03 - \left(1.22\times10^{-3} + 0.94\times0.03\right)\right]^2$$

$$=\left[1.755\times10^{-6}-0.02942\right]^2$$

$$=8.654\times10^{-4} \tag{6.30}$$

$$B_1^{\ 2}=\left(l_3-l_1\right)^2$$

$$=\left(0.052\times0.03+6.654\times10^{-5}-1.055\right)^2=1.10 \tag{6.31}$$

$$A_1^{\ 2}+B_1^{\ 2}=8.654\times10^{-4}+1.10$$

$$=1.1$$

$$\sqrt{A_1^{\ 2}+B_1^{\ 2}}=\sqrt{1.1}=1.049 \tag{6.32}$$

$$C_1^{\ 2}+D_1^{\ 2}=\left(\omega^4k_2+k_6\right)^2+\omega^4\left(\mathrm{k}_4\right)^2-2\left(\omega^4k_2+k_6\right)\left(\omega^2\,\mathrm{k}_4\right)+\left(\omega^5k_1+\omega k_5\right)^2$$
$$+\omega^6k_3^{\ 2}-2\omega^3k_3\left(\omega^5k_1+\omega k_5\right) \tag{6.33}$$

$$\frac{\partial}{\partial R_{kd}}\left[C_1^{\ 2}+D_1^{\ 2}\right]=2C_1\frac{\partial C_1}{\partial R_{kd}}+2D_1\frac{\partial D_1}{\partial R_{kd}} \tag{6.34}$$

$$\frac{\partial C_1}{\partial R_{kd}}=\omega^4\frac{\partial k_2}{\partial R_{kd}}-\omega^2\frac{\partial k_4}{\partial R_{kd}}+\frac{\partial k_6}{\partial R_{kd}} \tag{6.35}$$

$$\frac{\partial D_1}{\partial R_{kd}}=\omega\frac{\partial k_5}{\partial R_{kd}}-\omega^3\frac{\partial k_3}{\partial R_{kd}} \tag{6.36}$$

$$k_3=Jl_3+x_1$$

$$\frac{\partial k_3}{\partial R_{kd}}=J\frac{\partial l_3}{\partial R_{kd}}+\frac{\partial x_1}{\partial R_{kd}}$$

$$\frac{\partial k_3}{\partial R_{kd}}=J\frac{\partial l_3}{\partial R_{kd}}+0 \tag{6.37}$$

$$k_4 = Jl_4 + x_2$$

$$\frac{\partial k_4}{\partial R_{kd}} = J\,\frac{\partial l_4}{\partial R_{kd}} + \frac{\partial x_2}{\partial R_{kd}}$$

$$\frac{\partial k_3}{\partial R_{kd}} = 0.4133$$

(6.38)

$$k_5 = 0.9848x_3 - 0.173m_3$$

$$= (0.9848)(0.073)R_{kd} - (0.173)(0.058)R_{kd}$$

$$= R_{kd}\left[0.071 - 0.010\right]$$

$$= 0.061R_{kd}$$

(6.39)

$$\frac{\partial D_1}{\partial R_{kd}} = (\omega)\frac{\partial k_5}{\partial R_{kd}} - \left(\omega^3\right)\left(\frac{\partial k_3}{\partial R_{kd}}\right)$$

(6.40)

$$= (1)(0.061) - (1)^3(0.4133)$$

$$= -0.352$$

$$\frac{\partial C_1}{\partial R_{kd}} = \omega^4\,\frac{\partial k_2}{\partial R_{kd}} + \frac{\partial k_6}{\partial R_{kd}} - \omega^2\,\frac{\partial k_4}{\partial R_{kd}}$$

$$\omega = 1\,\text{pu}$$

$$= 7.52 + 8.2\times10^{-5} - 0.034 = 7.48$$

(6.41)

$$\frac{\partial}{\partial R_{kd}}\left[C_1{}^2 + D_1{}^2\right] = 2C_1\,\frac{\partial C_1}{\partial R_{kd}} + 2D_1\,\frac{\partial D_1}{\partial R_{kd}}$$

$$= 2C_1(7.48) + 2D_1(-0.352)$$

$$C_1 = \left(\omega^2 k_2 + k_6\right) - \omega^2 k_4$$

$$D_1 = \left(\omega^5 k_1 + \omega k_5\right) - \omega^3 k_3$$

$$C_1 = \left(0.45776 + 7.52R_{kd} + 8.25\times10^{-5}R_{kd}\right) - \left(0.012R_{kd} + 0.022\right)$$

$$= 0.66$$

(6.42)

$$D_1 = (k_1 + k_5) - k_3$$

$$= (9.177 + 0.061R_{kd}) - 0.4133 = 8.76 \tag{6.43}$$

$$\frac{\partial}{\partial R_{kd}}\left[C_1^{\,2} + D_1^{\,2}\right] = 2 \times 0.66 \times 7.84 - 2 \times 8.76 \times (-0.354)$$

$$= 16.04 \tag{6.44}$$

$$P_1 = R_f R_{kq}\left(2l_4 - 2\omega^2 l_2\right) \tag{6.45}$$

$$P_1 = -1.0992 \times 10^{-4} R_{kd} - 1.427 \times 10^{-7}$$

$$P_2 = L_{kkq} L_{ff}\left(-2\omega^2 l_4 + 2l_2\omega^4\right) \tag{6.46}$$

$$P_2 = 1.767R_{kd} + 2.293 \times 10^{-3}$$

$$P_3 = \left(R_f L_{kkq} + R_{kq} L_{ff}\right)\left[\left(2\omega^2 l_3 - 2\omega^4 l_1\right)\right] \tag{6.47}$$

$$P_3 = 5.304 \times 10^{-3} R_{kd} - 0.1076$$

$$P_1 + P_2 + P_3 = 1.772R_{kd} - 0.1053$$

$$P_4 = \frac{1}{2}\left[A_1^{\,2} + B_1^{\,2}\right]^{-\frac{1}{2}} \times \left[P_1 + P_2 + P_3\right] \tag{6.48}$$

$$P_4 = \frac{1}{2}\left[0.8863R_{kd}^{\,2} - 0.10751R_{kd} + 1.117\right]^{-\frac{1}{2}} \times \left[1.712R_{kd} - 0.1053\right] \tag{6.49}$$

All these part calculations are substituted in equation (6.2) and the sensitivity magnitude is plotted against R$_{kd}$. The plot is shown (Figure .6.1) and discussed in section no.6.3.The next important calculation remains due, which is the sensitivity calculation of transfer function with respect to q-axis damper winding resistance (R$_{kq}$).This calculation process is presented in the next section.

6.2.2 Calculation of Sensitivity of T(S) with Respect to R$_{kq}$

$$K_1 = Jl_1 = b_1 f_1 = 2.289 \tag{6.50}$$

$$K_2 = Jl_2 = J(b_2 f_1 + b_1 f_2) = 0414 \tag{6.51}$$

$$K_3 = Jl_3 + x_1 = J(\mathrm{b_3 f_1 + b_2 f_2}) + \mathrm{n_1 i_s cos}\beta_0 - \mathrm{m_1 i_s sin}\beta_0$$

$$= 2.76 + 0.32R_{kq} \tag{6.52}$$

$$K_4 = Jl_4 + x_2 = J(\mathrm{b_3 f_2}\ ) + \mathrm{n_2 i_s cos}\beta_0 - \mathrm{m_2 i_s sin}\beta_0$$

$$= 0.543R_{kq} - 0.196 \tag{6.53}$$

$$K_5 = x_3 = \mathrm{n_3 i_s cos}\beta_0 - \mathrm{m_3 i_s sin}\beta_0$$

$$= 0.05R_{kq} + 6.155\times10^{-5} \tag{6.54}$$

$$K_6 = x_4 = \mathrm{n_4 i_s cos}\beta_0 - \mathrm{m_4 i_s sin}\beta_0$$

$$= 6.3\times10^{-5} R_{kq} \tag{6.55}$$

$$\begin{aligned} l_1 &= 0.28625 \\ l_2 &= 0.051 \\ l_3 &= 3.26\times10^{-5} + 0.04R_{kq} \\ l_4 &= 4.5\times10^{-5} R_{kq} \end{aligned} \tag{6.56}$$

$$\begin{aligned} &\frac{\partial}{\partial R_{kq}}[A_1{}^2 + B_1{}^2] = \frac{\partial l_4}{\partial R_{kq}}[2l_4 - 2\omega^2 l_2] + \frac{\partial l_2}{\partial R_{kq}}[-2\omega^2 l_4 + 2l_2\omega^4] + \frac{\partial l_3}{\partial R_{kq}}[2\omega l_3 - 2\omega^2 l_1] \\ &= 3.2\times10^{-3} R_{kq} - 0.02 \end{aligned} \tag{6.57}$$

$$\begin{aligned} &\frac{1}{2}[A_1{}^2 + B_1{}^2]^{-\frac{1}{2}} = \frac{1}{2}[(l_4{}^2 + \omega^4 l_2{}^2 - 2\omega^2 l_4 l_2 + \omega^2 l_3{}^2 + \omega^6 l_1{}^2 - 2\omega^4 l_3 l_1]^{-\frac{1}{2}} \\ &= \frac{1}{2}[2.025\times10^{-9} R_{kq}{}^2 + (3.26\times10^{-5} + 0.04R_{kq})^2 + 0.023R_{kq} + 0.08]^{-\frac{1}{2}} \end{aligned} \tag{6.58}$$

$$\begin{aligned} &\frac{\partial}{\partial R_{kq}}[\sqrt{A_1{}^2 + B_1{}^2}] = \frac{1}{2}(A_1{}^2 + B_1{}^2)^{-\frac{1}{2}} \frac{\partial}{\partial R_{kq}}(A_1{}^2 + B_1{}^2) \\ &= \frac{1}{2}[2.025\times10^{-9} R_{kq}{}^2 + (3.26\times10^{-5} + 0.04R_{kq})^2 + 0.023R_{kq} + 0.08]^{-\frac{1}{2}} \times (3.2\times10^{-3} R_{kq} - 0.02) \end{aligned} \tag{6.59}$$

$$\frac{\partial}{\partial R_{kq}}[C_1^{\,2}+D_1^{\,2}]=2C_1\frac{\partial C_1}{\partial R_{kq}}+2D_1\frac{\partial D_1}{\partial R_{kq}} \tag{6.60}$$

$$\frac{\partial C_1}{\partial R_{kq}}=-0.543 \tag{6.61}$$

$$\frac{\partial D_1}{\partial R_{kq}}=-0.27 \tag{6.62}$$

$$\begin{aligned}\frac{\partial K_4}{\partial R_{kq}}&=\frac{\partial}{\partial R_{kq}}[0.543R_{kq}-0.196]\\&=0.543\end{aligned} \tag{6.63}$$

$$\begin{aligned}\frac{\partial K_5}{\partial R_{kq}}&=\frac{\partial}{\partial R_{kq}}[6.3\times10^{-5}R_{kq}]\\&=6.3\times10^{-5}\end{aligned} \tag{6.64}$$

$$\begin{aligned}C_1&=\omega^4K_2-\omega^2K_4+K_6\\&=0.61-.543R_{kq}\end{aligned} \tag{6.65}$$

$$\begin{aligned}D_1&=\omega^5K_1-\omega K_5+\omega^3K_3\\&=0.47-0.27R_{kq}\end{aligned} \tag{6.66}$$

$$\begin{aligned}\frac{\partial}{\partial R_{kq}}[C_1^{\,2}+D_1^{\,2}]&=[2\times(0.61-.543R_{kq})\times-0.543)]+[2\times(0.47-0.27R_{kq})\times-0.27)]\\&=-0.407+0.735R_{kq}\end{aligned} \tag{6.67}$$

All these part calculations are substituted in equation no.(6.2) and the sensitivity magnitude is plotted against R_{kq}. The plot is shown in Figure .6.2. The simulated results and discussions on the obtained plots are explained in detail in the next section.

6.3. Results and Discussions

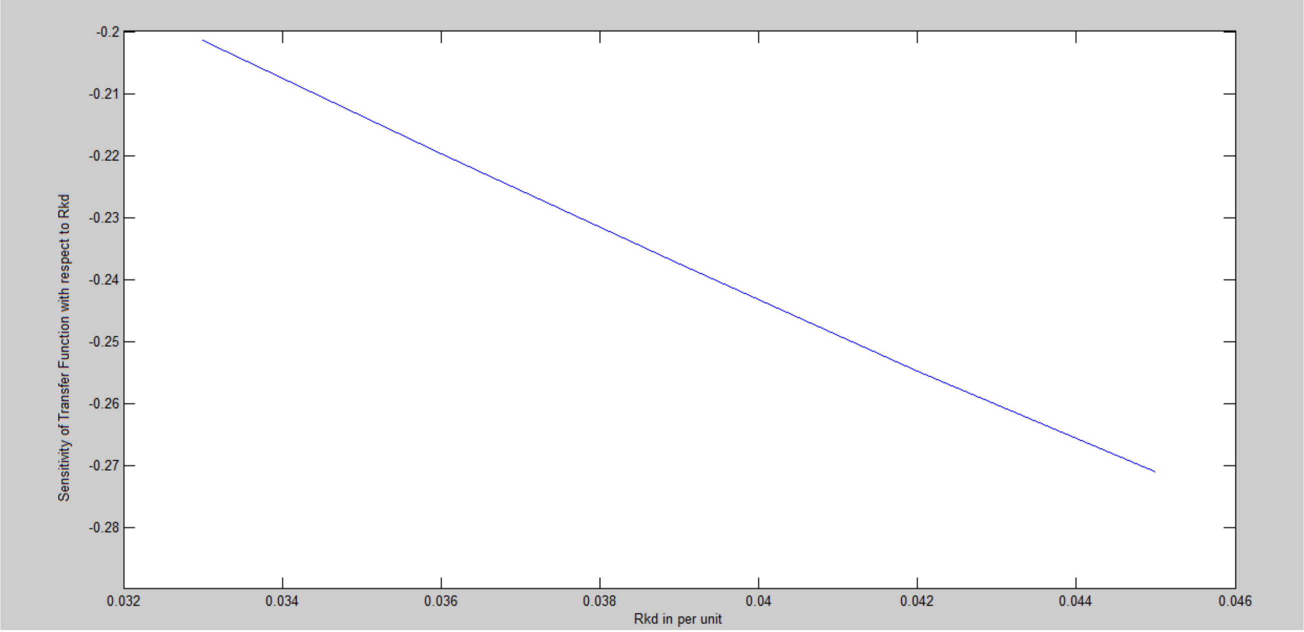


Figure.6.1: Sensitivity of transfer functions with respect to R_{kd}

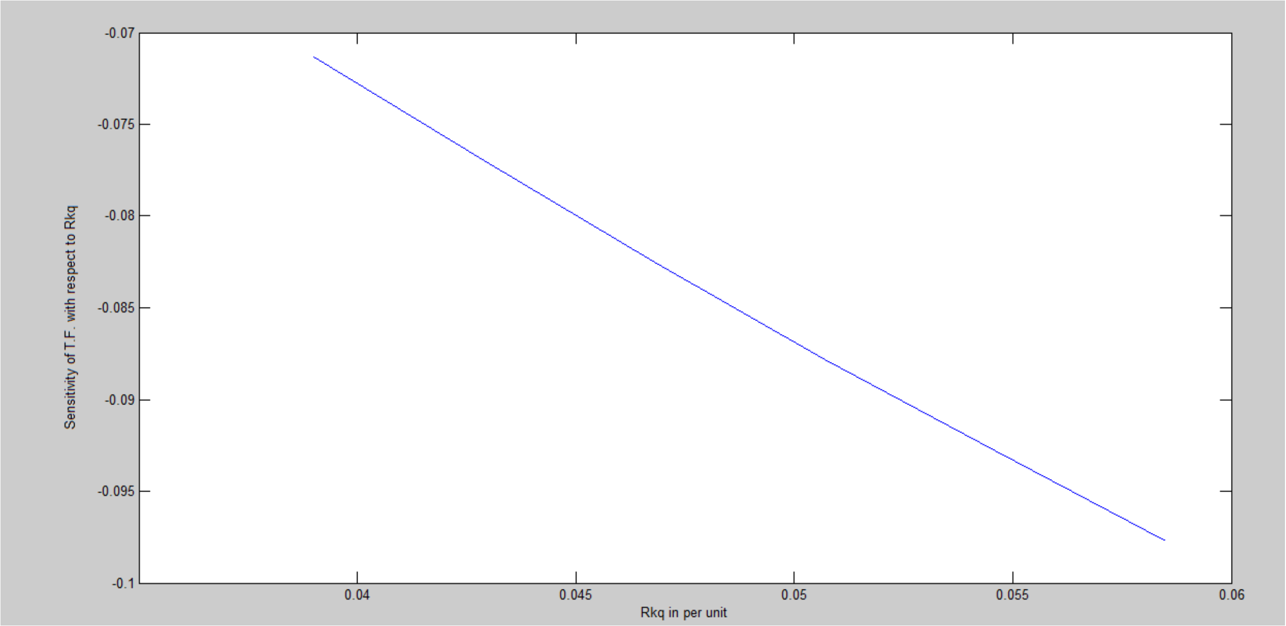


Figure.6.2: Sensitivity of transfer functions with respect to R_{kq}

i) The combination of R_{kd} and R_{kq} can be looked upon as a whole squirrel cage rotor system (comprising of aluminum or copper bars and end rings) of a three phase squirrel cage induction motor.

ii) Based on the first point it reveals that with the increase in values of R_{kd} and R_{kq} , the present machine under consideration(which is a three phase salient pole synchronous motor) will tend to become a three phase induction motor, which has very less significance of torque angle being dependent on load torque. Hence sensitivity of transfer function (which is $\frac{\Delta\beta(s)}{\Delta T_L(s)}$) of a synchronous motor can be explained from this particular view point.

iii) With reference to point no. (ii), the matter of interest, at present should not be the keen observation of sensitivity of transfer function with respect to damper winding resistances rather it should be the profile of variation of magnitude of the above said sensitivity with respect to the damper winding parameters.

iv) The motivation behind such direction of thinking stated in point no. (iii), comes out from the fact that the whole purpose of development of this study is to convert the analytical problem into a suitable machine design problem such that a suitable fabrication process can be planned subsequently.

v) Relative comparison of figures .6.1 and 6.2 also gives one information that the magnitude of sensitivity is more dependent on R_{kd} rather than on $R_{kq..}$Such observation is quite logical because the present machine under consideration is not a motor with dual field excitation rather a single field excitation on ‘d’- axis.

vi) A natural query may be raised within the mindset of any researcher, why such behavior mentioned in point no.(v), is not observed in the case of a three phase induction motor while its sensitivity aspects are analyzed. As an answer to this query, it may be noted that the induction motor is a uniform air gap machine and hence in most of the cases (except some special transient cases),'d-q' resolution is not needed. Hence the concept of R_{kd} and R_{kq} do not appear. Furthermore conceptually each of the R_{kd} and R_{kq} resembles to the rotor winding resistances referred to stator of a three phase induction motor. However, an induction motor under the transient condition, with a hypothetical variable air gap(saliency) can be looked upon as two induction motors, having uniform air-gap with different values of rotor winding resistances.

vii) Another important point in continuation with point no. (vi), is that why not such transfer function (which is $\frac{\Delta\beta(s)}{\Delta T_L(s)}$, where 's' stands for kernel of Laplace Transform) does not appear in the case of three phase induction motor even though conceptually a three phase induction motor is embedded in a three phase synchronous motor. The answer behind such question lies in the fact that in the case of a three phase induction motor, the parameter slip(s) plays the equivalent role of a torque angle (δ).It is a matter of interest to note that in a synchronous motor concept of slip is not applicable because the speed of the motor is fixed at synchronous value.

6.4. Conclusions

a) Figures 6.1 and 6.2 of section 6.3 will be very much helpful from the view point of designing a machine depending on the application of the particular synchronous motor coupled to a particular load. Primarily the designer will have to study the nature of the load and accordingly it is to be decided what should be the range of sensitivity of transfer function with respect to R_{kd} and R_{kq} separately. In the next stage those ranges can be selected from Figs. 6.1 and 6.2 and accordingly the tolerances of R_{kd} and R_{kq} can be used in the machine design process.

b) Even though this chapter of the thesis can be treated basically a research work on synchronous motor drive system, the present discussions around the zone of R_{kd} and R_{kq} immediately gives in the mindset of the designer, a mapping concept, from the view point of transformation from the constructional geometry of a synchronous motor to that of an induction motor.

Following the mathematical process in the same way (as presented), the sensitivity analysis of the same objective function with respect to other design parameters can be carried on. However the practical importance of those sensitivity analyses will depend on the particular designer's experience and view point.

References

[1] Hao Zhu; Xi Xiao; Yongdong Li, "Torque Ripple Reduction of the Torque Predictive Control Scheme for Permanent-Magnet Synchronous Motors," *Industrial Electronics, IEEE Transactions on* , vol.59, no.2, pp.871,877, Feb. 2012. doi: 10.1109/TIE.2011.2157278.

[2] Babel, A.S.; Cintron-Rivera, J.G.; Foster, S.N.; Strangas, E.G., "Evaluation of a Parameter Identification Method for Permanent Magnet AC Machines Through

Parametric Sensitivity Analysis," *Energy Conversion, IEEE Transactions on* , vol.29, no.1, pp.240,249, March 2014. doi: 10.1109/TEC.2013.2288235.

[3] Babel, A.S.; Foster, S.N.; Cintron-Rivera, J.G.; Strangas, E.G., "Parametric sensitivity in the analysis and control of permanent magnet synchronous machines," *Electrical Machines (ICEM), 2012 XXth International Conference on* , vol., no., pp.1034,1040, 2-5 Sept. 2012. doi: 10.1109/ICElMach.2012.6350003.

[4] Amoros, J.G.; Andrada, P., "Sensitivity Analysis of Geometrical Parameters on a Double-Sided Linear Switched Reluctance Motor," *Industrial Electronics, IEEE Transactions on* , vol.57, no.1, pp.311,319, Jan. 2010 doi: 10.1109/TIE.2009.2032208.

[5] Rahman, K.M.; Hiti, S., "Identification of machine parameters of a synchronous motor," *Industry Applications, IEEE Transactions on* , vol.41, no.2, pp.557,565, March-April 2005. doi: 10.1109/TIA.2005.844379.

[6] Zhiyong Wu; Hong Guo; Hao Qian; Dayu Wang, "Sensitive analysis of geometrical parameters on robust design for surface mounted permanent magnet synchronous motor," *Electrical Machines and Systems (ICEMS), 2013 International Conference on* , vol., no., pp.1169,1173, 26-29 Oct. 2013 doi: 10.1109/ICEMS.2013.6713352.

[7] Antonino-Daviu, J.A.; Riera-Guasp, M.; Pons-Llinares, J.; Roger-Folch, J.; Perez, R.B.; Charlton-Perez, C., "Toward Condition Monitoring of Damper Windings in Synchronous Motors via EMD Analysis," *Energy Conversion, IEEE Transactions on* , vol.27, no.2, pp.432,439, June 2012. doi: 10.1109/TEC.2012.2190292.

[8] Mink, F.; Kubasiak, N.; Ritter, B.; Binder, A., "Parametric model and identification of PMSM considering the influence of magnetic saturation," *Optimization of Electrical and Electronic Equipment (OPTIM), 2012 13th International Conference on* , vol., no., pp.444,452, 24-26 May 2012 doi: 10.1109/OPTIM.2012.6231768.

[9] Islam, M.S.; Mir, S.; Sebastian, T.; Underwood, S., "Design considerations of sinusoidally excited permanent magnet machines for low torque ripple applications," *Industry Applications Conference, 2004. 39th IAS Annual Meeting. Conference Record of the 2004 IEEE* , vol.3, no., pp.1723,1730 vol.3, 3-7 Oct. 2004. doi: 10.1109/IAS.2004.1348703.

[10] Grubic, S.; Restrepo, J.; Aller, J.M.; Bin Lu; Habetler, T.G., "A New Concept for Online Surge Testing for the Detection of Winding Insulation Deterioration in Low-Voltage Induction Machines," *Industry Applications, IEEE Transactions on* , vol.47, no.5, pp.2051,2058, Sept.-Oct. 2011. doi: 10.1109/TIA.2011.2161972.

[11] Chattopadhyay, A.B.; Sunil Thomas.; Ruchira Chatterjee., "Analysis of Steady State Stability of a Csi Fed Synchronous Motor Drive System with Damper Windings Included.", *Trends in Applied Sciences Research*, 2011, vol.,no., 6, pp: 992-1005.doi: 10.3923/tasr.2011.992.1005.

[12] Chattopadhyay, A.B.; Sunil Thomas., “Modeling and Simulation of Current Source Inverter Fed Synchronous Motor in Complex Frequency Domain Taking the Transition Zone From Induction Motor to Synchronous Motor Mode into Account”. *Research Journal of Applied Sciences, Engineering and Technology*, 2014. vol.,no.,7(8), pp: 1489-1499.

Chapter 7: Conclusions

7.1. Concluding points about the present research work

The following salient concluding points about the present research work completed are being hereby highlighted as follows:

i) Even though it is well known that due to sudden load disturbance, the produced oscillations in the speed and torque angle in a synchronous motor are subdued by the use of damper windings. But the concrete mathematical modeling presented in the chapter 3 clearly shows the quantitative role of the damper winding in assuring the steady state stability of a current source inverter(CSI) fed synchronous motor under the case, where the perturbation is not large.

ii) From the traditional literature it is well known that at the time of starting, a three phase synchronous motor works as an induction motor with the help of damper winding and pulls into synchronism under certain conditions. But just at the exact transition point, when the main field winding is decoupled from the external closing resistance and thrown to the DC supply, no mathematical model was available (to the best of the understanding of the author) to quantify the role of induction motor as a well-defined mathematical function associated with another mathematical function which should be the representative of functional aspect of the synchronous motor.

Starting from the fundamentals, to reach a time-domain modeling and to reflect the above said philosophical statement, it has been experienced that, direct formulation to have a time domain model is very cumbersome. However, firstly converting the basic time-domain equations (flux linkage-current equations or impressed voltage- current equations or torque balance equations) into complex frequency domain by using Laplace Transform technique and then secondly applying inverse Laplace Transform to the transformed equations meet the objective. Chapter 4 does the work of first responsibility i.e., complex frequency domain modeling during the transition from induction motor mode to synchronous motor mode. The important outcome of this modeling is that induction motor during this transition period can be looked upon as an analog filter. However the exact filter characterization and properties have not been investigated or modeled because presently such investigation does not come inside the scope of the defined research problem.

iii) In this context it can be reminded that the modeling presented in chapter 4, lead to the concept that during the transition zone (even though the time duration of the zone is less but from the view point of the principle of operation it behaves as a dominant zone), the induction motor plays the role of a disturbing element being reflected mathematically as a disturbance function. Even though complex frequency domain modeling is a very natural stage of the mid process of the whole modeling, ultimately time domain representation of any phenomenon speaks a lot about the operational and design aspects of the process or equipment or machine. The modeling presented in chapter 5 serves this purpose.

From the result of chapter 5 it can be concluded that knowing the time instant exactly when the Induction motor role should be truncated, can only be mathematically predicted after knowing the behavior of disturbance function with respect to time. Hence this result presented in that chapter works as a design data for the designer.

iv) Once the exact application of a synchronous motor is known, a designer may primarily fix-up the range of sensitivity of the transfer function (T(s) = $\frac{\Delta\beta(s)}{\Delta T_L(s)}$), by choosing the tuning parameters as R_{kd} and R_{kq}. In the next stage or finally, the results of the sensitivity analysis may be analyzed in detail to fix-up the design value of R_{kd} and R_{kq}. Furthermore the design of damper bars may be finalized based on the above said findings. Here lies the importance of analysis and simulation work presented in chapter 6 of this thesis.

7.2. Specific Contributions

i) To give inference on steady state stability aspects of a three phase current source inverter fed synchronous motor drive system taking the effect of damper winding and air gap saliency into account.

ii) To bring up the role of induction motor as a specific disturbance function when the synchronous motor is started as an induction motor with the help of damper windings. This modeling is done in complex frequency domain.

iii) To show the above said disturbance function in time domain also.

iv) Sensitivity analysis of the transfer function (used for steady state stability analysis), obtained for the above said drive system , indicating clearly the tuning effects of damper winding resistances on the transfer function. This work is basically a machine design aspect for fabrication purpose.

7.3. Further Scope

Following are the highlighted points in connection with the future research work aspects coming out from the view point of extension of the present research work.

i) In chapter 4 magnitude of the i_d and i_q were taken as constants because the motor was a CSI fed machine. But if due to restricting the cost aspect for certain application area, CSI can be avoided and then the steady state stability aspect phenomenon of a pure synchronous motor drive system (without CSI) will involve both armature current and torque angle as independent variables. Under such conditions it will be the responsibility of the future researcher to explore the complicated mathematical modeling and to draw the conclusions regarding the status of the steady state stability of the above said machine.

ii) In chapter 5,the role of induction motor in the whole starting phenomenon of synchronous motor, appears as a noise function ($F_3(s)$) in Laplace domain. This is due to the asynchronous behavior of a synchronous motor in starting zone. The matter of interest to be noted is that similar way to any circuit theory analysis under transient condition, this noise function of the complex frequency can be looked upon as an analog filter characteristics having a specific amplitude response and phase angle response, when '$s= j\omega$' is substituted in the expression for noise function. Exactly at this point the author of this thesis has truncated his work, because beyond this point the role of circuit theory or analog filter comes in. It is totally depending on the research objective of the researcher whether he should continue the work in the direction of filter analysis and performances or he should truncate at this point to divert his attention to the further analysis of the performance of the system as a machine element or electrical drive element.This can be carried out as a future research work but it needs a very good command on low frequency modeling of energy conversion device.

iii) The disturbing function in chapter 5 has been modeled as time domain function but still it remains in the category of steady state stability domain. It reveals that much hard work may not be needed to convert the small perturbation model into an equivalent suitable model for approximate calculations involving the transient stability assessment. The reason behind such statement lies in the fact that whenever the assumption of constant parameter during the perturbation period is nullified then only transient stability model succeeds. This work can be taken up as a future research work.

iv) The coefficients of the characteristic equation are machine dependant and hence machine with abnormalities and faults can be investigated as an extension to this work.

v) In connection with sensitivity analysis(Chapter 6) , it is needed to elaborate on the machine model, if damper winding resistances are increased. This analysis leading to machine design aspect can be taken up as a future scope of the present research work.

APPENDIX

The machine data used for the analysis are given as:

J=8 p.u.

L_d=1.17 p.u.

L_{md}=1.03 p.u.

L_q=0.75 p.u.

L_{mq}=0.61 p.u.

L_{kkd}=1.122 p.u.

L_{kkq}=0.725 p.u.

L_{ff}=1.297 p.u.

R_{kd}=0.03 p.u.

R_{kq}=0.039 p.u.

R_f=0.0015 p.u.

1) When machine is at load

i_s=1 p.u.

i_{f0}=0.97 p.u.

i_{kd0}=0 p.u.

i_{kq0}=0 p.u.

$\beta_0=10^0$

2) When machine is at no load

i_s=0.1 p.u.

i_{f0}=0.9 p.u.

i_{kd0}=0 p.u.

i_{kq0}=0 p.u.

$\beta_0=0^0$

LIST OF PUBLICATIONS

[1] Chattopadhyay, A.B.; Sunil Thomas.; Ruchira Chatterjee., "Analysis of Steady State Stability of a Csi Fed Synchronous Motor Drive System with Damper Windings Included.", *Trends in Applied Sciences Research,* 2011, vol.,no., 6, pp: 992-1005.doi: 10.3923/tasr.2011.992.1005.

[2] Sunil Thomas.;Chattopadhyay, A.B., "A 20-year survey on general modeling of synchronous motor and its stability studies. Part I: first literature review." *Advances in Modelling -Automatic Control (theory and applications),*2013,vol.,no.,68, Issue 2.

[3] Sunil Thomas.;Chattopadhyay, A.B., "A 20-year survey on general modeling of synchronous motor and its stability studies. Part I: first literature review." *Advances in Modelling -Automatic Control (theory and applications),*2013,vol.,no.,68,Issue 2.

[4] Chattopadhyay, A.B.; Sunil Thomas., "Modeling and Simulation of Current Source Inverter Fed Synchronous Motor in Complex Frequency Domain Taking the Transition Zone From Induction Motor to Synchronous Motor Mode into Account". *Research Journal of Applied Sciences, Engineering and Technology,* 2014. vol.,no.,7(8), pp: 1489-1499.

[5] Chattopadhyay, A.B.; Sunil Thomas., "An Insight into the Time Domain Phenomenon during the Transition Zone from Induction Motor to Synchronous Motor Mode for a Current Source Inverter Fed Synchronous Motor Drive System". *Research Journal of Applied Sciences, Engineering and Technology,*2014. vol.,no., 8(13), pp: 1510-1521.

[6] Chattopadhyay, A.B.; Sunil Thomas., "Sensitivity analysis of transfer function of a Current Source Inverter fed synchronous motor drive system as an extension to steady state stability analysis" – Communicated to *Journal of Electrical Engineering*, Springer Publications, Berlin Germany.

BRIEF BIOGRAPHY OF THE CANDIDATE

Mr. **Sunil Thomas** completed his Masters in Power Electronics and Drives from Vellore Institute of Technology, Vellore, Tamil Nadu, India in the year 2007. He started his teaching career as a Lecturer in the Department of Electrical and Electronics Engineering at the International Center of BIT, MESRA at Ras Al Khaimah, United Arab Emirates. In 2009, Mr. Thomas joined BITS Pilani Dubai Campus as a lecturer in the department of Electrical and Electronics Engineering. His research interests include Electrical machines, Power Electronics and Electrical Drives.

BRIEF BIOGRAPHY OF THE SUPERVISOR

Dr. **Adhir Baran Chattopadhyay** graduated as Bachelors and Masters in Electrical Engineering from Bengal Engg. College, University of Calcutta, India and National Institute of Technology, Jamshedpur in the year 1979 and 1990 respectively. He completed his PhD in Electrical Engineering from Indian Institute of Technology, Kharagpur, India in 1998. Dr. Chattopadhyay started his career as a graduate trainee in M/s Crompton Greaves Ltd, Ahmed Nagar, Maharashtra(India) in October 1980. In May 1985 he completed his assignment in the said company as a Quality Control Engineer in Machine Division-III. Later he joined as a Lecturer in the Department of Electrical Engineering, National Institute of Technology, Jamshedpur in May 1985. He completed his job at NIT Jamshedpur in August 2006 as an Assistant Professor. Since 2006, Dr Chattopadhyay is working as an Assistant Professor in the Department of Electrical and Electronics at BITS Pilani, Dubai Campus and he became Associate Professor in 2010. He is currently heading the Department of Electrical and Electronics Engineering at BITS Pilani, Dubai Campus. His research areas include Electrical Machines, Power Electronics, Electrical Drives, Power Systems, Control Engineering and Applied Mathematics