Chapter 3

Steady Analysis Of Switching and Reboot Delay

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Chapter 3

Steady Analysis of Switching and Reboot Delay

"Engineers like to solve problems. If there are no problems handily available, they will create their own problems".

Scott Adams

3.1 Introduction

Modern technologies heavily depend on the proper functioning of automated machining systems, which can be observed in many industries like power, communication, security, manufacturing, etc. Any system or unit is subject to unpredictable failure due to arbitrary causes like wear and tear, overload, natural or man-made external causes, etc. and causing loss of efficiency. To control this situation, these systems require to analyze as a machine repair problem (MRP) via the queueing theoretic approach and seek to improve its efficiency. The machine repair problem has been a keen area of interest for several theoreticians (*cf.* Sztrik and Bunday [165]; Haque and Armstrong [59]; Shekhar et al. [159]), so it is worthwhile to give an in-depth overview on such significant works. In this chapter, the reliability characteristics of a machine repair problem with operating units prone to random failure, and a provision of mixed spare redundancy have been analyzed to increase the system efficiency and availability.

Redundancy is the duplication of critical components and functions of a system with the intent of high reliability of the system, in the form of a backup or fail-safe. The three types of backup or spare units are hot, warm, and cold which depend on their failure characteristics. The failure rate of the hot spare unit in standby mode, i.e. when it is not in operation, is the same as that of an operating unit, whereas the failure rate of a cold spare unit in standby mode is zero, i.e. it cannot fail until it is in the standby state. The warm spare unit is an intermediate case between hot and cold spare units. The failure rate of a warm spare unit thus lies between that of an operating unit and zero in standby state. The MRP with spare units provisioning has been studied by many researchers for different variants (*cf.* Wang and Sivazlian [171]; Ke and Wang [99]; Jain et al. [82]). Recently, Kumar et al. [108] investigated MRP with spare units provisioning under controlled admission policy and unreliable repairmen.

K-out-of-$M : F$ or a $(M-K)$-out-of-$M : G$ machining system is an arrangement with M operating units, where at any time, a minimum of $M-K$ units in working must be available. If more than K operating units fail, that is if less than $M-K$ units are working in the system, then the system shuts down completely. Failure of the system and its units can be caused by a number of reasons, like wear and tear of machinery, etc. An operating unit may fail during its running time, and a spare unit may fail in its standby time. In the short mode, the load is distributed over less than the required number of units. Load sharing by lesser operating units is the overloading case, where the time-to-failure of the unit is state-dependent, and the failure rate increases with a decreasing number of operating units due to their failure. Failed units in the system are repairable, with different repair rates for operating and spare units following a certain policy. In past, many research papers concerning the reliability characteristics of k-out-of-n warm spare units provisioning machining systems have been published (*cf.* Zhang et al. [204]; Kamalja [89]; Zhang [206]). Recently, Sutar and Naik-Nimbalkar [163] proposed an inference procedure for the dependence parameter associated with load sharing effect in k-out-of-n systems with non-identical components. Eryilmaz [40] obtained an exact and approximate expression for the survival function of the $(k_1, k_2, ..., k_m)$-out-of-n system having multi-type components. Ling et al. [123] compared two policies by mean of majorization order for optimal component allocation in k-out-of-n machining system.

A realistic phenomenon due to which the system is prone to absolute failure is common cause failure, where the system may fail completely at any state, due to unavoidable external causes or natural calamities, and is also of interest in current investigation. A special and fast repair facility is required to recover from a complete system failure state. Chao [23] derived a product-form solution for queue size distribution for the network of queues with a possibility of catastrophes at each station. The redundancy allocation problem was formulated with the objective of maximizing system reliability in the presence of common cause failures by Ramirez-Marquez and Coit [146]. In recent, the Bayesian approach was used for statistical analysis of common cause failure events in safety assessment (*cf.* Qiu et al. [143]; Nguyen and Gouno [134]).

On failure of the operating unit, the available spare unit switches to the operating state with some switching time and the substantial probability of successful switching. The switching probability (coverage factor) includes the chances of successful detection, extraction of the location of a failure. To overcome the switching failure, the system undergoes the prompt reboot process to extract/clear the failed unit. The concept of imperfect coverage in k-out-of-n machining system was introduced by Moustafa [131]. Many researchers studied the effect of imperfect coverage on failure in different point of view with different variants (*cf.* Trivedi [167]; Wang and Chiu [178]; Ke et al. [95]; Ke and Liu [96]; Shekhar et al. [155]) Yen and Wang [198] compared three different configurations with imperfect coverage and spare switching failures based on system reliability and availability. Kuo and Ke [110] used a supplementary variable technique to derive steady-state availability of the machining systems and compared cost/benefit ratio for different repair and service time distribution.

The optimal repair times have wide applications for the management of the fault-tolerant machining systems. The Newton-quasi method is suitable for the optimal analysis in the stochastic environment. The basic idea is to estimate subgradient to yield a search direction. It is proved that the resulting stochastic Newton-quasi algorithm is able to generate a sequence that converges to the optimal point, under certain conditions. Many theorists used Newton-quasi method for the optimization problem in repairable machining systems (*cf.* Kao and Chen [90]; Wang et al. [176]; Ke et al. [93]). Li et al. [121] extended the spectral conjugate gradient method with Newton-quasi directions and equations for the unconstrained optimization problem.

From the depth literature survey, the following research gap is clearly identified that no study has been done in the past on the effect of switching delay and reboot delay in the redundant repairable fault-tolerant Markovian machining system environment with probabilistic common cause failure. The novelty involved in the present study is the state-of-the-art mathematical modeling of the machining system with mixed spare units, common cause failure, switching delay, reboot delay, load sharing, and state-dependent failure rate. The purpose of the present learning is threefold: (i) The first objective is to develop the stochastic model of the machining system with mixed spare units provisioning. (ii) The second aim is to analyze the effect of various types of failures and delays on the performance of the fault-tolerant K-out-of-M system. (iii) The third goal is to frame the optimal strategy for maintaining the redundant repairable machining system for the seek of high quality of service (QoS) or performance.

A machining system may exist in different modes, namely normal, reboot, short, and failed. The machining system is in the normal mode when there are M operating units functioning in the system. If an operating unit fails during the normal mode, a spare unit is used to replace it, returning the system to normal functioning. If switching of the spare unit fails, the system enters the reboot mode. In reboot mode, the system is physically available but is not functioning. The system then tries to restore normalcy by rebooting at a fast rate and attempts another switching of a spare unit with the failed operating unit or by repairing it to full functionality. The attempts for switching continues till all the spare units are exhausted. When less than M units are functioning in the system, the system is said to be in the short mode. Now, the load is distributed over the remaining units, causing them to be overloaded. Overloading is an alarming condition wherein the failure rate of the remaining units increases sharply, with a decreasing number of operating units. If the number of failed units increases to more than K units, the system shuts down completely and refers as the failure state of the system. The system may enter the failure mode due to common cause failure from any state, as well.

A real-life example of the application of this machine repair problem is an electricity generator grid, supplying power to an area in the city. The generators and the spare generators are analogous to the operating units and spare units described above, respectively. These generators may fail due to many reasons like the fatigue of machinery, high voltage, overloading, etc. Upon failure of a generator, the load is shared over the remaining units. Then, a spare generator is activated to replace the failed operating generator. This switching is probabilistic, taking a considerable amount of time. In case of imperfect switching, the entire grid reboots to attempt another switching or to detect, extract, and repair of the failed generator. Overloading of the grid, when less than a certain number of generators are functioning, is equivalent to the short mode in the failure-prone system.

In this section, a depth literature review has been done and ascertain the uniqueness of the model under consideration. The rest contents of this chapter are presented in the following structure. A detailed description of the system is described in section (3.2) along with the governing Chapman-Kolmogorov differential-difference equations and the state probabilities computation. Using steady-state probabilities, various performance measures are computed in section (3.3). In section (3.4), the basics of the Newton-Quasi method for optimal analysis are presented. The comparative and optimal analysis is done for the variability of performance to varying parameters of the system in section (3.5). Finally, conclusions are drawn, and the future scope is discussed in the last sections (3.6).

3.2 Model Description and State Probabilities

In order to investigate the Markovian analysis of the fault-tolerant redundant repairable machining system via queueing-theoretic approach, the finite Markov model is developed using birth-death process.

3.2.1 Assumptions and Notations

In this section, a redundant repairable system of the form K-out-of-$M : F$ machining system considering some assumptions and notations is formulated. The detailed description of the machining system is as follows:

- The system consists of M operating units working simultaneously in parallel and independently with the provision of Y cold spare units and S warm spare units. For the standard functioning of the system, M operating units are required.
- The lifetimes of operating units and warm spares are exponentially distributed with rate λ and ν respectively.
- Whenever the operating unit fails, it is automatically replaced by the available spare unit. There is also a possibility that a spare unit may fail before it is used in operation and hence, it is continuously monitored with the automatic system. It is assumed that all the warm spare units are used exhaustively before the cold spare units.
- After the failure of the operating unit, the switching of spare unit to the operating state may be probabilistic in nature with the probability of successful switching ρ. The time-to-switching follows exponential distribution with parameter τ. Hence, if the switching succeeds then the failed operating unit is replaced by the available spare unit in time which is exponentially distributed with the rate $\frac{\rho}{\tau}$.
- In case of the switching failure of the spare unit, the system opts to reboot with time which is also exponentially distributed with the rate of $\frac{1-\rho}{\tau}$. In reboot state, the system is physically available but not in the working state (normal mode). The time-to-reboot follows an exponential distribution with rate σ which is to be very high so that no other event can take place during system reboot period.

- The load is shared with M operating units in normal mode and continues until there are at least $M-K$ units in the system in short mode. The lesser number of operating units also increases the chances of overloading making the remaining operating units more vulnerable to the failure. Hence, The system is operable until the failure of K operating units in the short mode. In short mode, operating unit fails with exponentially distributed time-to-failure with state-dependent mean rate Λ_n, where n denotes the number of failed operating units in short mode.

- The system not only fails due to failure of units but it may also fail due to some external unavoidable reasons collectively termed as common cause failure. The time-to-failure of the system due to common cause failure is exponentially distributed with a mean rate of λ_C.

- Henceforth, there is always a pre-arrangement of repair facility consisting of two repairmen. Following an exponential distributed repair times, repairman 1 repairs the failed operating unit with the rate of μ_1 while the repairman 2 repairs failed spare unit with the rate of μ_2.

- When the system is in shut down state, the special repair is facilitated and a repair time follows an exponential distribution with rate parameter μ_C.

All events are independent to state of the others. Fig. (3.1) depicts the state transition diagram with inflow and outflow rates for studied machine repair problem.

3.2.2 Chapman-Kolmogorov Equation

For the Markovian modeling, the notations adopted to describe the different states of the machine repair model at any instant t are as follows.

$$\begin{aligned}
\mathscr{I}(t) &\equiv \text{Number of operating units in the system at time } t \\
\mathscr{J}(t) &\equiv \text{Number of cold spare units in the system at time } t \\
\mathscr{K}(t) &\equiv \text{Number of warm spare units in the system at time } t \\
F(t) &\equiv \text{The system is in failed state at time } t
\end{aligned}$$

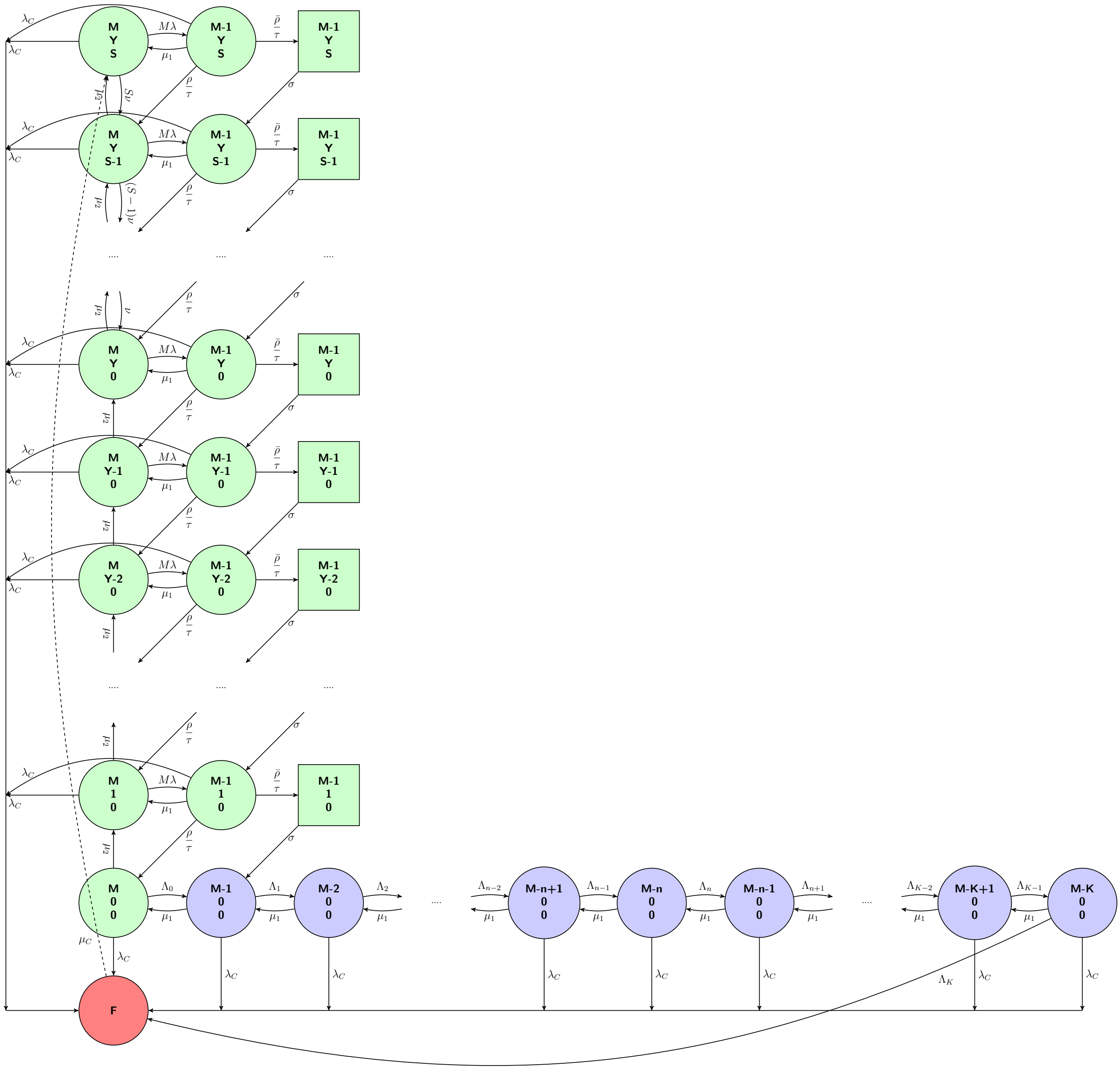

Figure 3.1: The state transition diagram

$\{\mathscr{I}(t), \mathscr{J}(t), \mathscr{K}(t) \cup F(t)\}$ is a continuous-time Markov chain. At any instant t, the state probabilities are defined as

$$P_{i,j,k}(t) = \text{Prob}\left(\mathscr{I}(t) = i, \mathscr{J}(t) = j, \mathscr{K}(t) = k \,\text{and system is in normal or short state}\right)$$
$$Q_{i,j,k}(t) = \text{Prob}\left(\mathscr{I}(t) = i, \mathscr{J}(t) = j, \mathscr{K}(t) = k \,\text{and system is in reboot state}\right)$$
$$P_F(t) = \text{Prob}\left(\text{system is in failed state}\right)$$

As $t \to \infty$, the system tends to stable condition and the governing Chapman-Kolmogorov forward difference equation in terms of inflow, outflow rates and state probabilities are as follows

Case 1: The warm spare units are utilized first on the failure of operating unit with perfect switching and random switching delay.

$$-\left[M\lambda + S\nu + \lambda_C\right] P_{M,Y,S} + \mu_1 P_{M-1,Y,S} + \mu_2 P_{M,Y,S-1} + \mu_C P_F = 0 \qquad (3.1)$$

$$\begin{aligned} &-\left[M\lambda + (S-k)\nu + \lambda_C + \mu_2\right] P_{M,Y,S-k} + (S-k+1)\nu P_{M,Y,S-k+1} \\ &+ \frac{\rho}{\tau} P_{M-1,Y,S-k+1} + \mu_1 P_{M-1,Y,S-k} + \mu_2 P_{M,Y,S-k-1} = 0; 1 \leq k \leq S-1 \end{aligned} \qquad (3.2)$$

Case 2: On exhaust of warm spare units, the cold spare units are utilized on the failure of operating unit with perfect switching and random switching delay.

$$-\left[M\lambda + \lambda_C + \mu_2\right] P_{M,Y,0} + \nu P_{M,Y,1} + \frac{\rho}{\tau} P_{M-1,Y,1} + \mu_1 P_{M-1,Y,0} + \mu_2 P_{M,Y-1,0} = 0 \qquad (3.3)$$

$$\begin{aligned} &-\left[M\lambda + \lambda_C + \mu_2\right] P_{M,Y-j,0} + \frac{\rho}{\tau} P_{M-1,Y-j+1,0} + \mu_1 P_{M-1,Y-j,0} \\ &+ \mu_2 P_{M,Y-j-1,0} = 0; \quad 1 \leq j \leq Y-1 \end{aligned} \qquad (3.4)$$

Case 3: On the exhaust of warm and cold spare units, the number of operating units are reducing on the failure of operating unit.

$$-\left[\lambda_C + \frac{1}{\tau} + \mu_1\right] P_{M-1,Y,S} + M\lambda P_{M,Y,S} = 0 \qquad (3.5)$$

Case 4: The warm spare units are utilized on the failure of operating unit with imperfect switching.

$$-\left[\lambda_C+\frac{1}{\tau}+\mu_1\right]P_{M-1,Y,S-k}+M\lambda P_{M,Y,S-k}+\sigma Q_{M-1,Y,S-k+1}=0;\ 1\le k\le S \quad (3.6)$$

Case 5: On exhaust of warm spare units, the cold spare units are utilized on the failure of operating unit with imperfect switching.

$$-\left[\lambda_C+\frac{1}{\tau}+\mu_1\right]P_{M-1,Y-j,0}+M\lambda P_{M,Y-j,0}+\sigma Q_{M-1,Y-j+1,0}=0;\ 1\le j\le Y-1 \quad (3.7)$$

Case 6: On imperfect switching, the system opts the reboot process.

$$-\sigma Q_{M-1,Y,S-k}+\frac{\bar{\rho}}{\tau}P_{M-1,Y,S-k}=0;\ 0\le k\le S \quad (3.8)$$

$$-\sigma Q_{M-1,Y-j,0}+\frac{\bar{\rho}}{\tau}P_{M-1,Y-j,0}=0;\ 1\le j\le Y-1 \quad (3.9)$$

Case 7: On exhaust of warm and cold spare units, the number of operating units are reducing on the failure of operating unit in safe mode and load sharing is distributed among remaining operating units.

$$-\left[\Lambda_0+\lambda_C+\mu_2\right]P_{M,0,0}+\frac{\rho}{\tau}P_{M-1,1,0}+\mu_1 P_{M-1,0,0}=0 \quad (3.10)$$

$$-\left[\Lambda_1+\lambda_C+\mu_1\right]P_{M-1,0,0}+\Lambda_0 P_{M,0,0}+\sigma Q_{M-1,1,0}+\mu_1 P_{M-2,0,0}=0 \quad (3.11)$$

$$-\left[\Lambda_i+\lambda_C+\mu_1\right]P_{M-i,0,0}+\Lambda_{i-1}P_{M-i+1,0,0}+\mu_1 P_{M-i-1,0,0}=0;\ 2\le i\le K-1 \quad (3.12)$$

$$-\left[\Lambda_K+\lambda_C+\mu_1\right]P_{M-K,0,0}+\Lambda_{K-1}P_{M-K+1,0,0}=0 \quad (3.13)$$

Case 8: The failed state of the system which may be due to less number of the required number of operating units or common cause failure.

$$
\begin{aligned}
-\mu_C P_F + \Lambda_K P_{M-K,0,0} + \lambda_C \left[\sum_{k=0}^{S} \left(P_{M,Y,S-k} + P_{M-1,Y,S-k}\right)\right] \\
+ \lambda_C \left[\sum_{j=0}^{Y-1} \left(P_{M,Y-j,0} + P_{M-1,Y-j,0}\right)\right] + \lambda_C \sum_{i=0}^{K} P_{M-i,0,0} = 0
\end{aligned} \tag{3.14}
$$

For above defined state probabilities, the normalizing condition is

$$
\sum_i \sum_j \sum_k P_{i,j,k} + \sum_i \sum_j \sum_k Q_{i,j,k} + P_F = 1 \forall i,j,k \tag{3.15}
$$

3.2.3 The Steady-State Solution

For the steady-state solution, the system of simultaneous linear equations Eq^n's (3.1)-(3.14) can be represented in matrix form as

$$
\mathbf{AP} = \mathbf{0} \tag{3.16}
$$

Here **A** is a square matrix of order $3Y + 3S + K + 2$ whose elements are the coefficient of state probabilities $P_{i,j,k}$ and $Q_{i,j,k}$, **P** is column vectors of unknown state probabilities and **0** be a null vector of suitable dimension. By imposing the normalizing condition in Eq^n. (3.15) in matrix form as

$$
\mathbf{Pe} = 1 \tag{3.17}
$$

where **e** is a vector of 1's, the above system of linear equations Eq^n. (3.16) can be expressed as

$$
\mathbf{CP} = \mathbf{B} \tag{3.18}
$$

where **C** is the same matrix as **A** except each element in the last row is replaced by 1 and **B** is a column vector having zero elements except the last element which is replaced by 1. The system of linear Eq^n. (3.18) given in matrix form has been solved to obtain the steady-state probabilities using the Gauss elimination extended numerical technique SOR method with over-relaxation parameter value 1.25 in MATLAB software.

In the next section, the performance indices for the machining system are established in terms of the state probabilities.

3.3 Performance Measures

The system physiognomies can be characterized by deriving the performance measures in terms of the steady-state probabilities. These measures play a substantial role in achieving the high performance and can be used as valuable tools by the system managers and industrial engineers for the enhancement of the grade of service (GoS) by predicting the preventive requirements and queueing indices of the concerned redundant machining system. Some performance measures are classified as follows

3.3.1 State Probabilities

- The probability that the system is in a normal state

$$P(N) = \sum_{k=0}^{S} \left(P_{M,Y,S-k} + P_{M-1,Y,S-k}\right) + \sum_{j=1}^{Y-1} \left(P_{M,Y-j,0} + P_{M-1,Y-j,0}\right) + P_{M,0,0} \tag{3.19}$$

- The probability that the system is in a down state

$$P(D) = \sum_{k=0}^{S} Q_{M-1,Y,S-k} + \sum_{j=1}^{Y-1} Q_{M-1,Y-j,0} \tag{3.20}$$

- The probability that all spare units are exhausted

$$P(S) = \sum_{i=1}^{K} P_{M-i,0,0} \tag{3.21}$$

3.3.2 Expectation

- Expected number of failed units in the machining system

$$\begin{aligned} E(N) = &\sum_{n=1}^{S} n\left(P_{M,Y,S-n} + P_{M-1,Y,S-n+1} + Q_{M-1,Y,S-n+1}\right) \\ &+ \sum_{n=S+1}^{Y+S} n\left(P_{M,Y+S-n,0} + P_{M-1,Y+S-n+1,0} + Q_{M-1,Y+S-n+1,0}\right) \\ &+ \sum_{n=Y+S+1}^{Y+S+K} n\left(P_{M+Y+S-n,0,0}\right) \end{aligned} \tag{3.22}$$

- Expected number of spare units in the machining system

$$E(S) = \sum_{n=0}^{S} \left(Y+S-n\right) \left(P_{M,Y,S-n} + P_{M-1,Y,S-n} + Q_{M-1,Y,S-n}\right) \\ + \sum_{n=S+1}^{Y+S-1} (Y+S-n) \left(P_{M,Y+S-n,0} + P_{M-1,Y+S-n,0} + Q_{M-1,Y+S-n,0}\right) \tag{3.23}$$

- Expected number of operating units in the machining system

$$E(O) = \sum_{n=0}^{S} M\left(P_{M,Y,S-n} + P_{M-1,Y,S-n} + Q_{M-1,Y,S-n}\right) \\ + \sum_{n=S+1}^{Y+S-1} M\left(P_{M,Y+S-n,0} + P_{M-1,Y+S-n,0} + Q_{M-1,Y+S-n,0}\right) \\ + \sum_{n=Y+S+1}^{Y+S+K} (M+Y+S-n) P_{M+Y+S-n} \tag{3.24}$$

- Expected delay time

$$E(D) = \frac{E(N)}{TT} \tag{3.25}$$

3.3.3 Queueing Measures

- The throughput of the machining system

$$Th = \mu_C P_{M,Y,S} + \mu_1 \left(\sum_{k=0}^{S} P_{M-1,Y,S-k} + \sum_{j=1}^{Y-1} P_{M-1,Y-j,0} + \sum_{i=1}^{K} P_{M-i,0,0} \right) \\ + \mu_2 \left(\sum_{k=0}^{S} P_{M,Y,S-K-1} + \sum_{i=1}^{K} P_{M-i,0,0} \right) \tag{3.26}$$

- Availability of the machining system

$$\mathrm{Av} = 1 - P_{M,Y,S} - \left(\sum_{k=0}^{S} Q_{M-1,Y,S-k} + \sum_{j=1}^{Y-1} Q_{M-1,Y-j,0} \right) \tag{3.27}$$

- Machine availability

$$MA = 1 - \frac{E(N)}{M+Y+S} \tag{3.28}$$

- Failure frequency of the machining system

$$FF = \Lambda_K P_{M-K,0,0} + \lambda_C \left[\sum_{k=0}^{S} \left(P_{M,Y,S-k} + P_{M-1,Y,S-k} \right) \right]$$
$$+ \lambda_C \left[\sum_{j=0}^{Y-1} \left(P_{M,Y-j,0} + P_{M-1,Y-j,0} \right) \right] + \lambda_C \sum_{i=0}^{K} P_{M-i,0,0} \qquad (3.29)$$

3.3.4 Expected Total Cost

The prime objective of the present learning is to determine the optimal design parameters in order to minimize the incurred value of the expected total cost. Now, the expected cost function is developed by considering the repair facility (rates) (μ_1, μ_2) as a decision variables and various cost elements involved in different activities. The cost elements associated with different activities are considered to be linear and defined as follows

C_H: Holding cost for each failed unit in the system

C_S: Cost incur for each spare unit in the system

C_O: Cost incur for each operating unit in the system

C_1: Cost involved in providing the service with rate μ_1

C_2: Cost involved in providing the service with rate μ_2

C_3: Cost involved in providing the service with rate μ_C

- Hence, the expected total cost is defined as

$$E(TC) = C_H E(N) + C_S E(S) + C_O E(O) + C_1 \mu_1 + C_2 \mu_2 + C_3 \mu_C \qquad (3.30)$$

3.4 Newton-Quasi Method

The Newton-quasi method is the procedure used to find local maxima and minima of the functions, as an alternative to Newton's method when Newton's method is not suitable due to the complexity of the function. The Newton-quasi method is used if the Hessian is unattainable or is too computationally complex at every iteration. Newton's method requires the Hessian for finding extrema. The search for an extremum of a scalar-valued function is nothing else than the quest for the zeroes of the gradient of that function, Newton-quasi method is readily suitable to find extrema of a function.

Newton-quasi methods are centred on Newton's method to explore the stationary point of a function, where the gradient is zero. Newton's method assumes that the function can be locally approximated as a quadratic in the region around the optimum, and uses the first and second derivatives to find the stationary point. In Newton-quasi methods, the Hessian matrix does not necessarily to be computed. The Hessian is updated by evaluating successive gradient vectors instead.

The Newton-quasi method is the variable-metric method used in optimization exploit following property. If g is the gradient of f, then searching for the zeroes of the vector-valued function g corresponds to the examination for the extrema of the scalar-valued function f; the Jacobian of g now becomes the Hessian of f. The main difference is that the Hessian matrix is a symmetric matrix, unlike the Jacobian when searching for zeroes.

In the chapter, the expected total cost is a uni-modal function but complex in nature. The first and second derivatives of expected total cost with respect to system parameters are not readily computable since it depends on system parameters intrinsically. On fixing the system parameters, the expected total cost is function of decision variables. The Newton-quasi method is employed to global search (μ_1, μ_2) until the minimum value of $E(TC(\mu_1, \mu_2))$, say $E(TC(\mu_1^*, \mu_2^*))$, is attained. The cost minimization problem can be illustrated mathematically as

$$TC(\mu_1^*, \mu_2^*) = \underset{\mu_1, \mu_2}{\text{minimize}}\, TC(\mu_1, \mu_2) \tag{3.31}$$

The essence of the Newton-quasi method is to find a search direction in each iteration. Different step length along this direction for a better solution is used until the tolerance is small enough. Define the vector $\vec{\Omega} = [\mu_1, \mu_2]^T$ and the respective gradient vector $\vec{\nabla} TC(\vec{\Omega})$ which consists of $\frac{\partial TC}{\partial \mu_1}$ and $\frac{\partial TC}{\partial \mu_2}$. Next, the Newton-quasi method is employed to find the global minimum expected cost and corresponding decision variables by using the following steps:

Step 1. Let $\vec{\Omega}_0 = [\mu_1, \mu_2]^T$

Step 2. Set the initial trial solution for $\vec{\Omega}_0$ and compute $TC(\vec{\Omega}_0)$

Step 3. Compute the cost gradient $\vec{\nabla} TC(\vec{\Omega}) = [\partial TC/\partial \mu_1, \partial TC/\partial \mu_2]^T \mid_{\vec{\Omega}_0}$ and the cost Hessian matrix

$$H(\vec{\Omega}) = \begin{bmatrix} \frac{\partial^2 TC}{\partial \mu_1^2} & \frac{\partial^2 TC}{\partial \mu_1 \mu_2} \\ \frac{\partial^2 TC}{\partial \mu_1 \mu_2} & \frac{\partial^2 TC}{\partial \mu_2^2} \end{bmatrix}$$

Step 4. Find the new trial solution

$$\vec{\Omega}_{n+1} = \vec{\Omega}_n - \left[H\left(\vec{\Omega}_n\right)\right]^{-1} \vec{\nabla} TC\left(\vec{\Omega}_n\right)$$

Step 5. Set $n = n+1$ and repeat steps $2-4$ until

$$\left|\frac{\partial TC}{\partial \mu_1}\right| < \varepsilon_1 \quad \text{and} \quad \left|\frac{\partial TC}{\partial \mu_2}\right| < \varepsilon_2$$

where $\varepsilon_1 = \varepsilon_2 = 10^{-7}$ are the tolerances.

Step 6. Find the global minimum value $TC\left(\vec{\Omega}_n^T\right) = TC(\mu_1, \mu_2)$

3.5 Numerical Result

The analytical results of the redundant repairable machining system performance measures are not sufficient to establish the worthiness of the developed model. To explore the practical applicability of the proposed repairable Markov model with switching and reboot delay, several numerical experiments are executed in MATLAB and the results are presented in Tables (3.1)-(3.5) and Figs. (3.2)-(3.8). For that purpose, numerical experiments with following default value of system parameters $M = 10$; $Y = 3$; $S = 4$; $K = 2$; $\lambda = 0.5$; $\lambda_C = 0.02$; $\nu = 0.3$; $\mu_1 = 2$; $\mu_2 = 5$; $\mu_C = 10$; $\rho = 0.5$; $\tau = 0.2$; $\sigma = 25$ are performed.

Fig. (3.2) presents the variability of the expected number of failed units in the machining system $E(N)$ with respect to system parameters. As the failure rates (λ, ν) of units increase, the values of $E(N)$ increases, which is an obvious result. The same trends are also noted for the increased number of units (M, S) in the system and the threshold K. To check the high value of $E(N)$, better corrective measures are recommended since it decreases for higher value repair rates (μ_1, μ_2), as well better preventive measures like the low probability of switching failure $(1-\rho)$ and low switching time (τ). Since the system fails with common cause failure, $E(N)$ is not much affected with λ_C and μ_C. The reboot process is also substantial in extracting the failed unit for repair.

The apparent results for the availability of the machining system Av are observed in Fig. (3.3) in which Av decreases for the higher rate of failure of units and common cause $(\lambda, \nu, \lambda_C)$ and increases with increased repair rate (μ_1, μ_2, μ_C). It is also inferred from Fig. (3.3) that suitable preventive measures are critical over the expensive arrangements of operating units and spare units. Prompt reboot, instant switching, and low switching failure probability are expected for better preventive measures.

Fig. (3.4) depicts the variation of the throughput of the machining system Th for the different values of system parameters. Throughput of the machining system Th is defined as the expected number of repaired units in the machining system. For the efficient corrective measures, higher throughput of the machining system is urged, which can be achieved with better service facilities like the higher value of repair rates (μ_1, μ_2, μ_C), prompt switching and reboot (ρ, τ, σ). The results of Fig. (3.4) support today's expectation and intention.

From the different sets of system parameters, the relation between machine availability MA and system parameters are established, and the results are depicted in Fig. (3.5). Fig. (3.6) displays the changes in the value of failure frequency FF for the different value of the system parameters. These performance measures are also critical in decision making for corrective, predictive and preventive maintenance and result in Fig. (3.5) & Fig. (3.6) will be helpful for system analysts.

Besides the assumed default system parameters value for Figs. (3.2)-(3.6), for cost analysis, the following unit cost elements $C_H = 80$; $C_O = 30$; $C_S = 15$; $C_1 = 15$; $C_2 = 5$; $C_3 = 50$ are also considered for computing the expected total cost. The expected total cost incurred in the machining system should be optimal with some constraints of availability of the system and system parameters. For that purpose, expected total cost function $E(TC)$ is defined in terms of various cost elements and states of the system. In Fig. (3.7), the surface plot of $E(TC)$ are presented for various combinations of system parameters. It reveals how the expected total cost is reduced with maintaining the expected grade of quality of service (QoS). It is obvious that expected total cost increases with number of units in the system and incurs more cost for better service which is also depicted in Fig. (3.7). Hence, there is necessity for finding the optimal repair facility, so the Newton-quasi method is employed for determining optimal decision parameters.

For the optimal analysis or employing the Newton-quasi method, it is necessary to check the nature of the expected total cost function $E(TC)$ with respect to decision variables and obviously it should be convex. For that purpose, in Fig. (3.8), contour plot for expected total cost with respect to decision parameters μ_1 and μ_2 is depicted for the following default parameters $M = 15$; $Y = 5$; $S = 8$; $K = 6$; $\lambda = 0.5$; $\lambda_C = 0.02$; $\nu = 0.3$; $\mu_1 = 2$; $\mu_2 = 10$; $\mu_C = 10$; $\rho = 0.5$; $\tau = 0.2$; $\sigma = 25$; $C_H = 80$; $C_O = 30$; $C_S = 15$; $C_1 = 15$; $C_2 = 5$; $C_3 = 50$ and infer that $E(TC)$ is convex in nature. For the illustration of convergent nature of the Newton-quasi method, two illustrative examples are taken into consideration with following default parameters $M = 15$; $Y = 5$; $S = 8$; $K = 6$; $\lambda = 0.5$; $\lambda_C = 0.02$; $\nu = 0.3$; $\mu_C = 10$; $\rho = 0.5$; $\tau = 0.2$; $\sigma = 25$; $C_H = 80$; $C_O = 30$; $C_S = 15$; $C_1 = 15$; $C_2 = 5$; $C_3 = 50$ and different initial guess value of decision parameters $\mu_1 = 2$; $\mu_2 = 10$ in Table (3.1) and $\mu_1 = 6$; $\mu_2 = 8$ in Table (3.2). Table (3.1) & Table (3.2) summarize the step value of decision variables, corresponding expected total cost, rate of change of cost with parameters.

The following stopping criterion of iterative procedure is assumed

$$Tol = \max\left(\left|\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_1}\right|, \left|\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_2}\right|\right)$$

with tolerance limit 10^{-7}. It is perceived from Tables (3.1 & 3.2) that the Newton-quasi method converges to same optimal decision values even it initiates with different initial guesses.

For Tables (3.3)-(3.5), the default parameters are set as follows $M = 15$; $Y = 5$; $S = 8$; $K = 6$; $\lambda = 0.5$; $\lambda_C = 0.02$; $\nu = 0.3$; $\mu_C = 10$; $\rho = 0.5$; $\tau = 0.2$; $\sigma = 25$; $C_H = 80$; $C_O = 30$; $C_S = 15$; $C_1 = 15$; $C_2 = 5$; $C_3 = 50$ with initial guess $\mu_1 = 2$; $\mu_2 = 8$ for iteration. Tables (3.3)-(3.5) comprise the results of optimal repair rates (μ_1^*, μ_2^*) and corresponding optimal expected total cost $E(TC(\mu_1^*, \mu_2^*))$ and other performance measures through the Newton-quasi method for different sets of system parameters. In Table (3.3), different combinations of (M,Y,S,K) are taken and find that higher value of repair rates is required if the system has more operating and spare units. For different set of system parameters $(\lambda, \lambda_C, \nu, \mu_C)$, from Table (3.4), it is observed that if units are more prone to failure with higher rate, better repair rate is required for achieving optimum service conditions. It is obvious results and support methodology used and analysis done herein. Similar kind of results are also tabulated in Table (3.5) for the system parameters (ρ, τ, σ). As ρ increases, optimal repair rate for operating/spare unit increases with decreased expected total cost. Decremental trends in the value of repair rates and expected total cost are observed for the increased value of τ. Negligible change is observed with reboot process since reboot is independent event to service.

From the in-depth comparative and optimal analysis, the following inferences are recommended.

- As preventive measures, too many spare units facility is not always beneficial. Prompt switching and reboot play a very vital role in maintaining a high grade of service.
- As corrective measures, the optimal repair rates must be maintained to avoid the idleness or extra cost incurred in service.
- Periodic maintenance is recommended to avoid the units/system failure, switching failure, which directly affects the performance of the machining system.

3.6 Conclusion

In the present model, the Markovian analysis has been done for the K-out-of-M : F redundant repairable machining system with the support of mixed spare units facing independent failure, switching failure, state-dependent failure, and common cause failure. Some real-time phenomena, like reboot delay, controlled parameter, etc. have also been incorporated. The governing Chapman-Kolmogorov equations have been formulated and solved by employing the SOR method to obtain the steady-state probabilities. Various performance characteristics for comparative and optimal analysis have been developed, and exhaustive experimental illustrations have carried out. This model can be used for application in various machining systems in different industries like power generation, communications, etc. The studied research problem takes into account many realistic phenomena and provides an informative outlook on the process of increasing the efficiency of automated machining systems. These systems have great importance in modern technology, and it is essential to optimize their performance for the advancement of the respective industries.

The present study may be extended with consideration of unreliable repairmen, additional repairmen, more realistic policies like a vacation, N, D, or T policy, setup time, etc. The impatience behavior of caretakers of failed units can also be incorporated with the present study to increase model insight for dealing with the more realistic problem. Researchers can look forward to some metaheuristics optimization techniques for complex systems.

Table 3.1: Illustrative example of the Newton-quasi method with initial guess $\mu_1 = 2, \mu_2 = 10$

μ_1	μ_2	$E(TC(\mu_1,\mu_2))$	$\left\lvert\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_1}\right\rvert$	$\left\lvert\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_2}\right\rvert$	Tol
2	10	1616.88	117.133775261	48.747081000	117.133775261
3.141312	14.43904	1434.55	24.336483598	8.230018143	24.336483598
3.987641	15.87876	1414.73	7.648146457	2.489152253	7.648146457
4.612385	16.68798	1410.86	1.571074445	0.486795543	1.571074445
4.82912	16.89946	1410.63	0.106397952	0.030448516	0.106397952
4.847014	16.91135	1410.63	0.000570863	0.000144221	0.000570863
4.847118	16.91138	1410.63	0.000000039	0.000000009	0.000000039

Table 3.2: Illustrative example of the Newton-quasi method with initial guess $\mu_1 = 6, \mu_2 = 8$

μ_1	μ_2	$E(TC(\mu_1,\mu_2))$	$\left\lvert\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_1}\right\rvert$	$\left\lvert\frac{\partial E(TC(\mu_1,\mu_2))}{\partial \mu_2}\right\rvert$	Tol
6	8	1487.46	19.726091639	34.621754774	34.621754774
6.160577	10.21133	1437.18	5.827643097	12.726419779	12.726419779
5.579954	12.89905	1417.68	2.090290318	4.614891336	4.614891336
5.060606	15.36998	1411.50	0.795980011	1.323134059	1.323134059
4.874426	16.65999	1410.65	0.143732241	0.192299092	0.192299092
4.847802	16.90419	1410.63	0.004479537	0.005450802	0.005450802
4.847119	16.91137	1410.63	0.000003970	0.000004684	0.000004684
4.847118	16.91138	1410.63	0.000000000	0.000000002	0.000000002

Table 3.3: The optimal repair rates and expected total cost for different system parameters

(M,Y,S,K)	μ_1^*	μ_2^*	$E(TC(\mu_1^*,\mu_2^*))$	Av	MA	FF	TH
(15,5,8,6)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281
(12,5,8,6)	4.3750	15.6801	1301.51	0.9537	0.9408	0.0191	5.2596
(14,5,8,6)	4.6979	16.5133	1374.56	0.9513	0.9430	0.0190	5.5782
(16,5,8,6)	4.9895	17.2986	1446.43	0.9491	0.9452	0.0190	5.8726
(15,4,8,6)	4.8403	16.9003	1395.61	0.9502	0.9420	0.0190	5.7260
(15,6,8,6)	4.8509	16.9171	1425.62	0.9502	0.9461	0.0190	5.7293
(15,8,8,6)	4.8542	16.9217	1455.58	0.9502	0.9496	0.0190	5.7303
(15,5,5,6)	4.9194	15.3628	1355.72	0.9504	0.9394	0.0190	5.3286
(15,5,7,6)	4.8738	16.4024	1392.37	0.9503	0.9427	0.0190	5.5959
(15,5,10,6)	4.7942	17.9138	1447.03	0.9500	0.9467	0.0190	5.9913
(15,5,8,5)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281
(15,5,8,8)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281
(15,5,8,10)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281

Table 3.4: The optimal repair rates and expected total cost for different system parameters

$(\lambda,\lambda_C,\nu,\mu_C)$	μ_1^*	μ_2^*	$E(TC(\mu_1^*,\mu_2^*))$	Av	MA	FF	TH
(0.5,0.02,0.3,10)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281
(0.4,0.02,0.3,10)	4.3746	15.6796	1391.34	0.9537	0.9471	0.0191	5.2595
(0.6,0.02,0.3,10)	5.2574	18.0442	1427.51	0.9473	0.9416	0.0189	6.1468
(1,0.02,0.3,10)	5.4371	4.5032	1922.16	0.9573	0.5886	0.0193	4.8190
(0.5,0.01,0.3,10)	4.8680	16.9565	1411.74	0.9511	0.9442	0.0095	5.7416
(0.5,0.03,0.3,10)	4.8262	16.8661	1409.51	0.9492	0.9441	0.0285	5.7146
(0.5,0.05,0.3,10)	4.7842	16.7752	1407.29	0.9474	0.9441	0.0474	5.6876
(0.5,0.02,0.1,10)	5.0773	14.9125	1396.48	0.9509	0.9482	0.0190	5.1080
(0.5,0.02,0.2,10)	4.9539	15.9386	1403.69	0.9505	0.9461	0.0190	5.4213
(0.5,0.02,0.4,10)	4.7533	17.8390	1417.31	0.9499	0.9423	0.0190	6.0291
(0.5,0.02,0.3,12)	4.8476	16.9125	1510.86	0.9505	0.9441	0.0190	5.7301
(0.5,0.02,0.3,16)	4.8483	16.9140	1711.16	0.9509	0.9441	0.0190	5.7326
(0.5,0.02,0.3,20)	4.8487	16.9149	1911.34	0.9511	0.9441	0.0190	5.7341

Table 3.5: The optimal repair rates and expected total cost for different system parameters

(ρ,τ,σ)	μ_1^*	μ_2^*	$E(TC(\mu_1^*,\mu_2^*))$	Av	MA	FF	TH
(0.5,0.2,25)	4.8471	16.9114	1410.63	0.9502	0.9441	0.0190	5.7281
(0.2,0.2,25)	6.9697	16.5545	1446.14	0.9264	0.9411	0.0185	6.3890
(0.4,0.2,25)	5.5678	16.7791	1422.64	0.9419	0.9431	0.0188	5.9613
(0.6,0.2,25)	4.1101	17.0620	1398.40	0.9588	0.9452	0.0192	5.4792
(0.5,0.1,25)	4.9743	21.3911	1446.68	0.9194	0.9372	0.0184	6.9915
(0.5,0.3,25)	4.4949	14.9285	1391.14	0.9629	0.9468	0.0193	5.0763
(0.5,0.5,25)	3.9476	12.9609	1369.60	0.9746	0.9491	0.0195	4.3416
(0.5,0.2,20)	4.8505	16.9069	1411.21	0.9389	0.9439	0.0188	5.6612
(0.5,0.2,30)	4.8447	16.9146	1410.23	0.9578	0.9443	0.0192	5.7736
(0.5,0.2,40)	4.8414	16.9189	1409.72	0.9675	0.9445	0.0194	5.8315

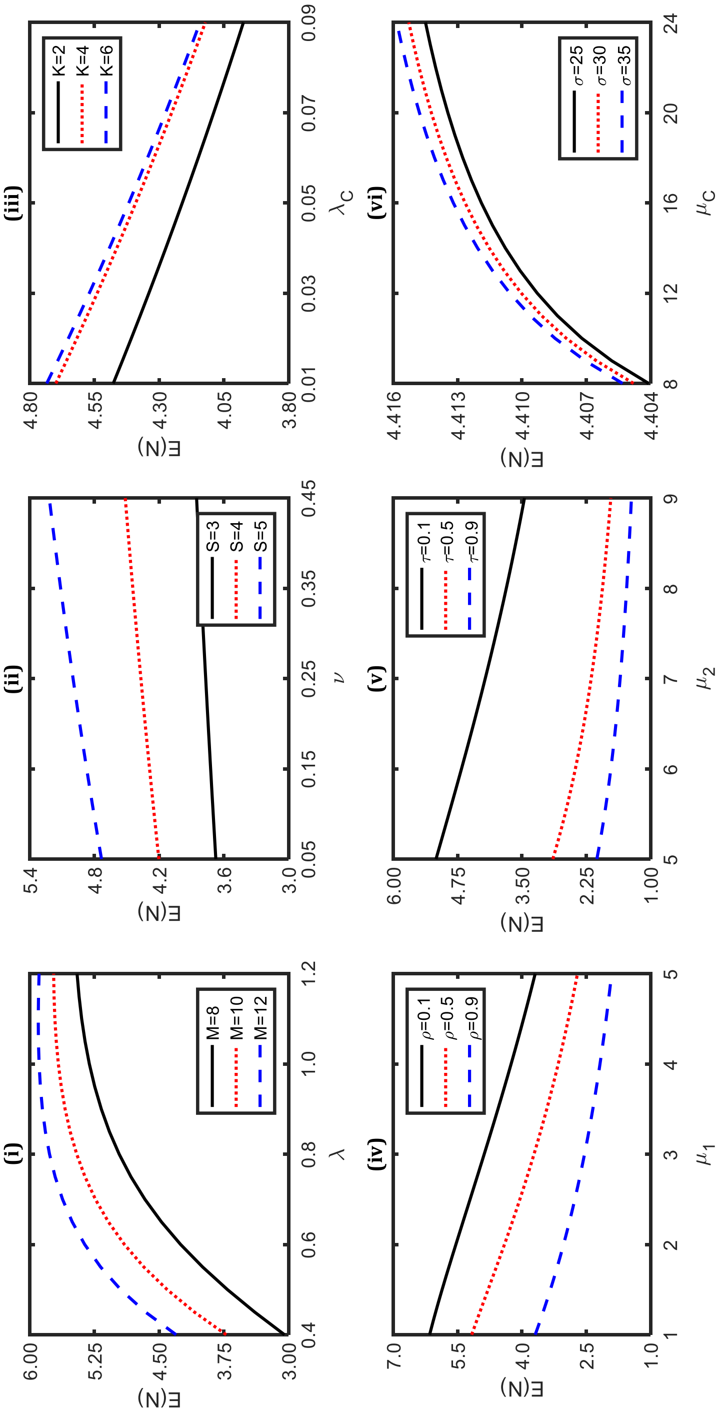

Figure 3.2: Expected number of failed units in the machining system

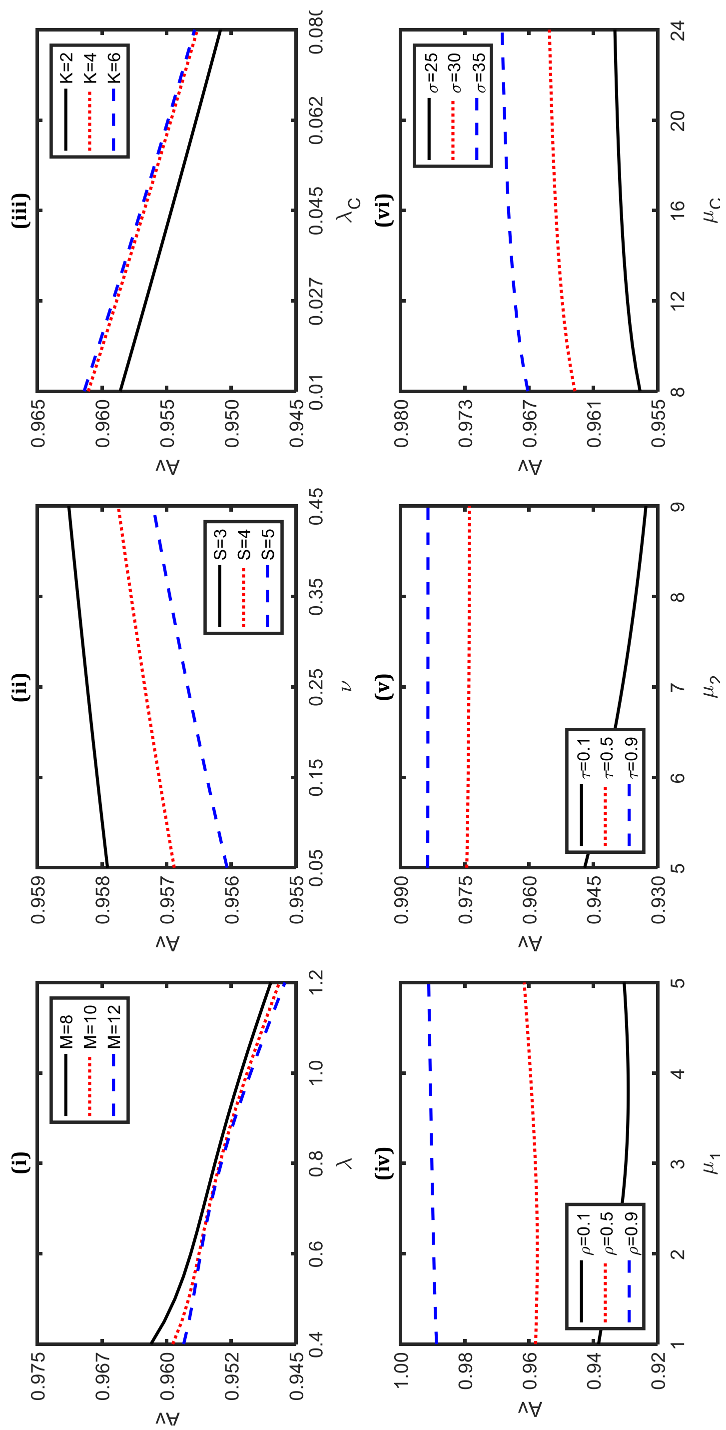

Figure 3.3: The availability of the machining system

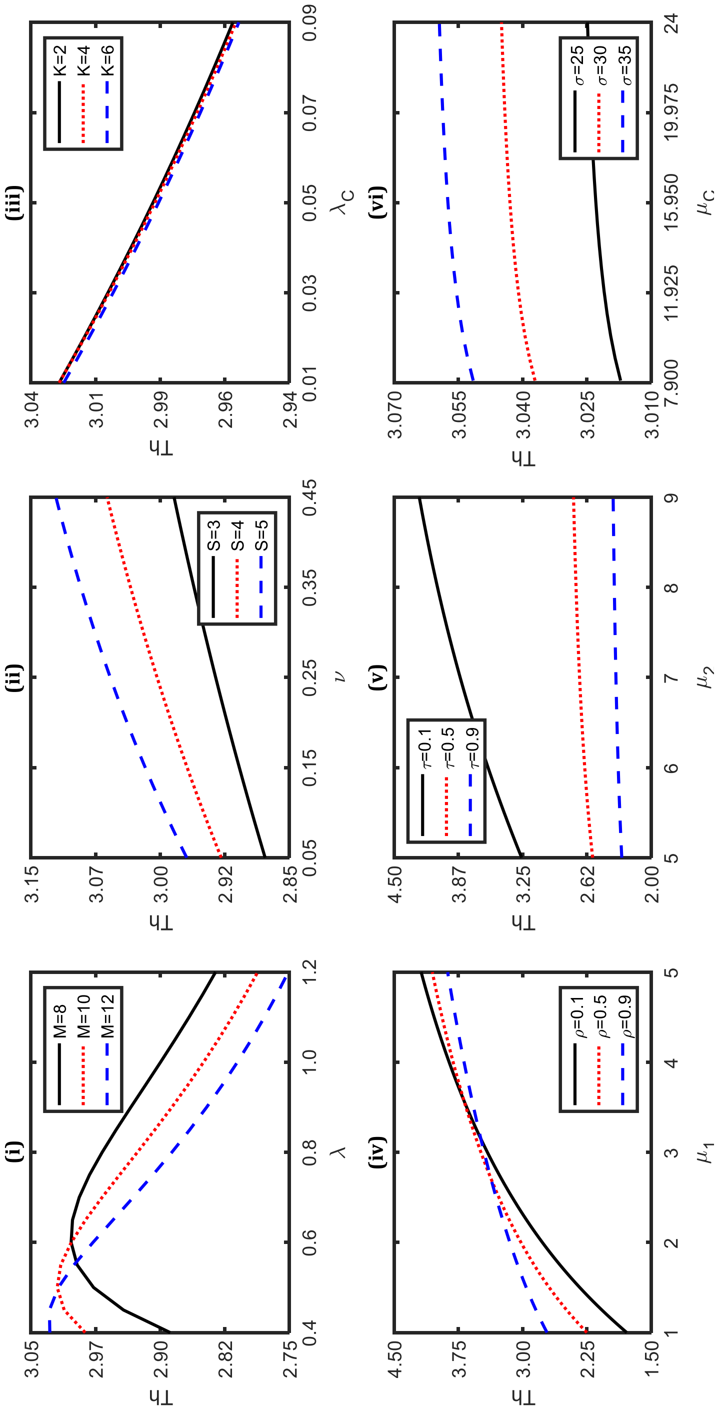

Figure 3.4: The throughput of the machining system

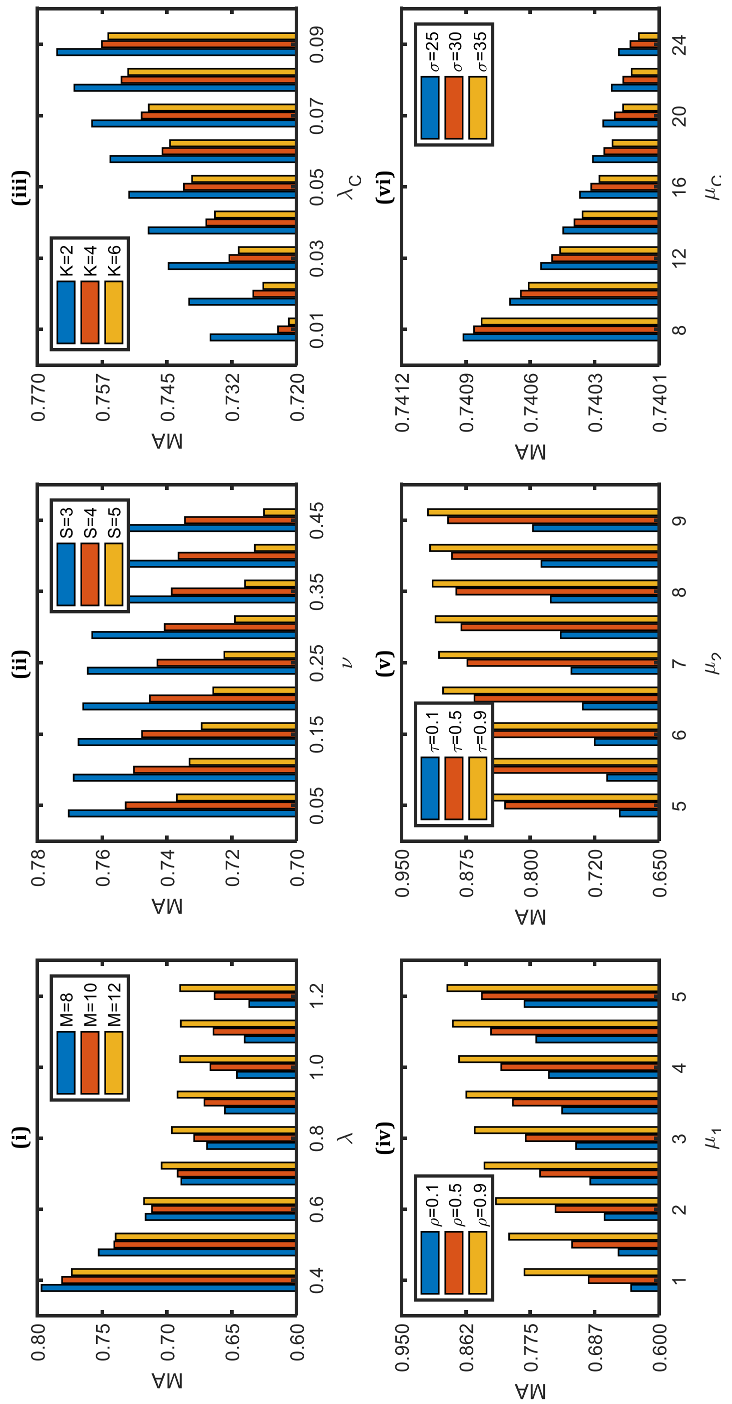

Figure 3.5: Machine availability

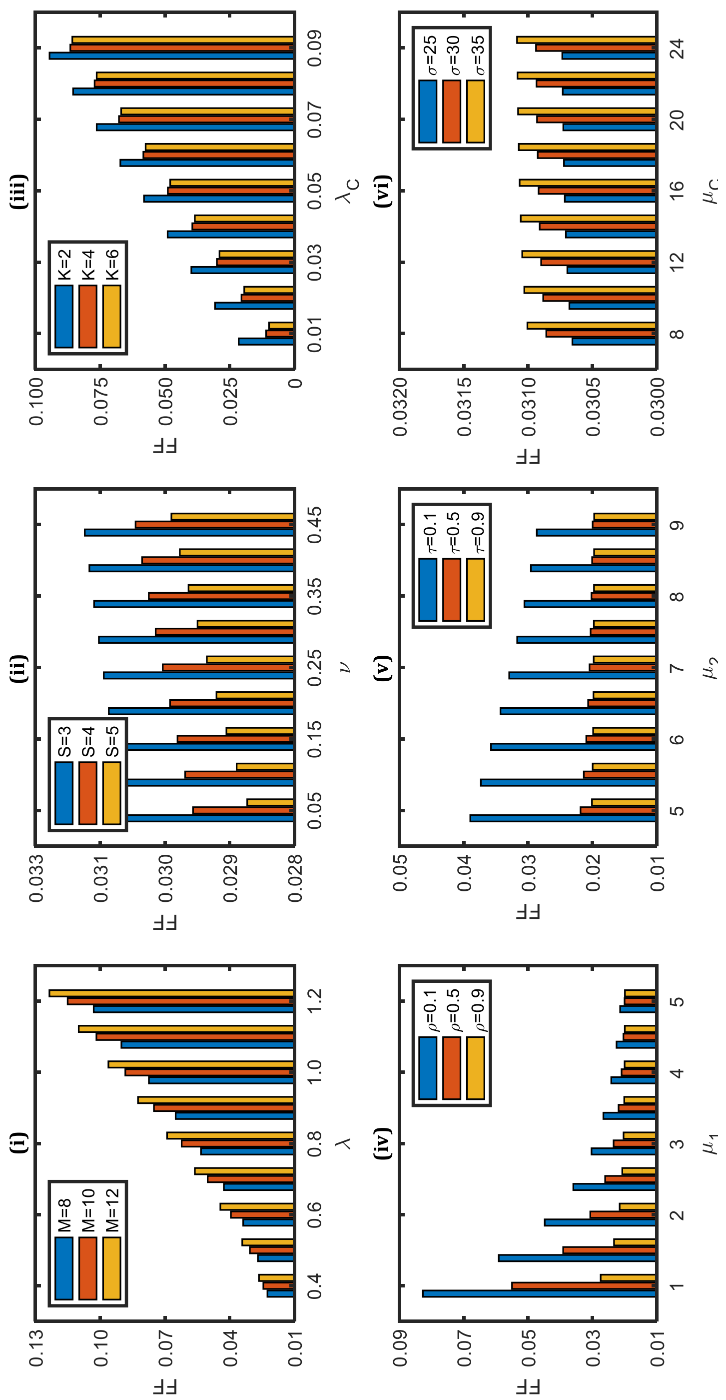

Figure 3.6: Failure frequency of the machining system

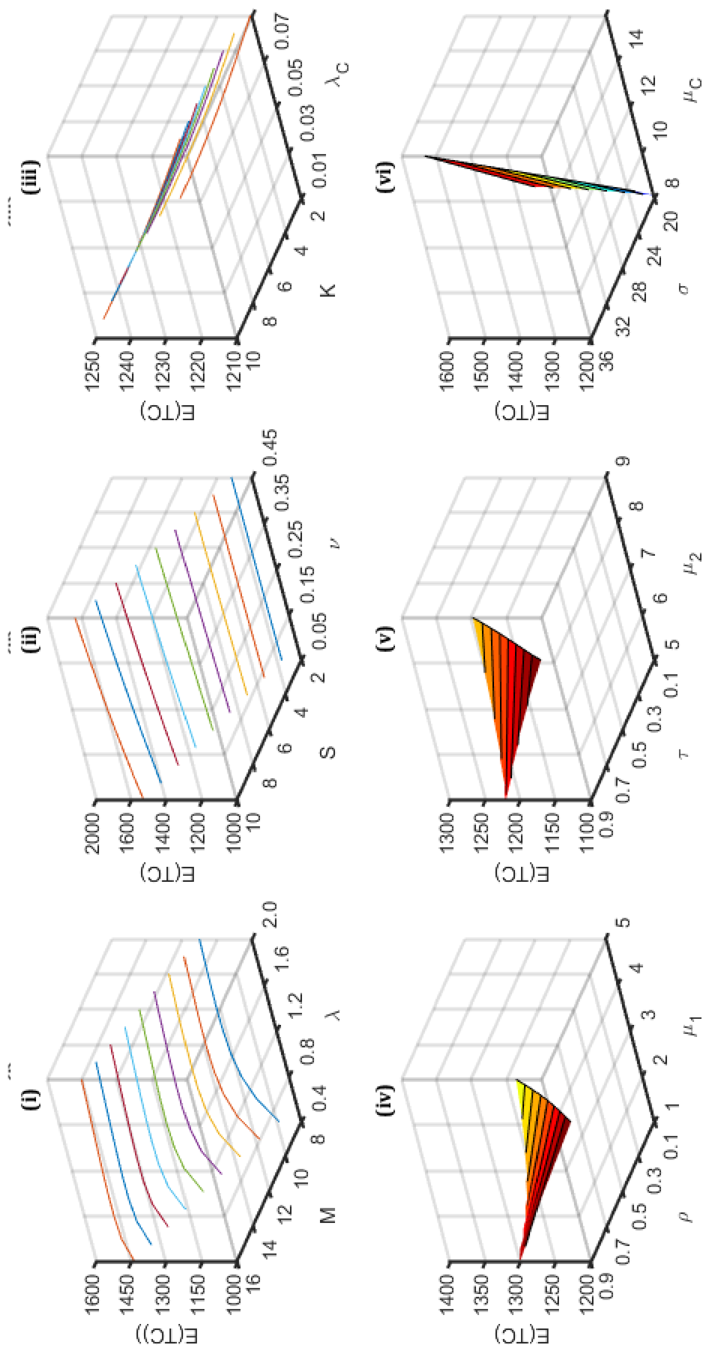

Figure 3.7: Expected total cost

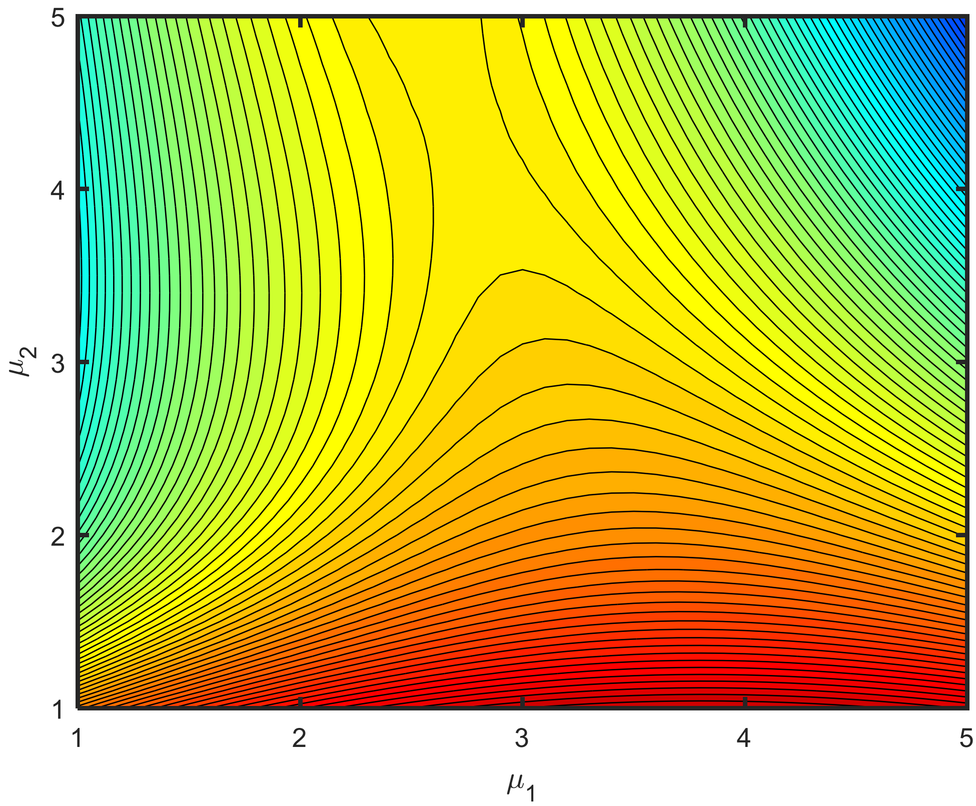

Figure 3.8: Contour of expected total cost wrt μ_1 and μ_2