Chapter 1

Introduction

Our Universe is textured with a variety of gravitationally bound structures on different scales: planets, stars, galaxies, clusters, and superclusters. Observations tell us that our Universe is expanding since its origin around 14 Gyr ago from a very dense and hot state (named as Big Bang), and recently its expansion has started taking place with acceleration, probably due to domination of some unknown form of energy called dark energy (DE). Cosmology is the study of the whole Universe with the aim to understand the origin, evolution and possible future of the Universe. For this purpose, relying on the Einstein's theory of general relativity, cosmologists have developed a mathematically tractable and observationally testable model of the Universe, the so-called ΛCDM (cosmological constant Λ + cold dark matter) model, also known as the standard cosmological model. This model is remarkably successful to explain the dynamics and observed features of the Universe such as the cosmic acceleration, large scale structure (LSS) formation and many more, and it finds an excellent fit to various observational data from different sources. On the other hand, it is plagued with a number of problems that need to be addressed. In this chapter, we briefly present the standard model of the Universe, both at the background and perturbation levels. Finally, the motivation for the research work carried out in this thesis is presented. Many comprehensive and excellent standard texts, describing the ΛCDM model, are there in the literature [1–5], which are followed in this background chapter.

1.1 Background Cosmology

This section provides basic information about background cosmology which includes major observational milestone, matter-energy ingredients of the Universe, background equations, and cosmological parameters.

1.1.1 Homogeneity and isotropy (cosmological principle)

The structure of the Universe on large scales possesses two important properties, homogeneity and isotropy, known as the cosmological principle. Homogeneity means that the Universe looks the same at each point, and isotropy means that the Universe looks the same in each direction. In other words, there are no preferred locations or directions in the Universe. So, there is no center of the Universe. Our Universe appears to become smooth (isotropic and homogeneous) at distance scales 100 Mega-parsec (Mpc) or more, the typical size of superclusters. Therefore, the structures larger than superclusters of galaxies are not observed. For, the large scale surveys such as the Two-degree Field (2dF) galaxy redshift survey and the Sloan Digital Sky Survey (SDSS) have not reported any structures bigger than the superclusters. Therefore, the cosmological principle is very well supported by the observations. It is the guiding principle for mathematical modeling of the Universe.

1.1.2 Expanding Universe: Hubble's law

The important observational evidence in modern cosmology is that almost all galaxies (observed in deep extragalactic space, 10 Mpc or more) in the Universe appear to be receding from us, and the farther away a galaxy is, the more rapid its departure appears to be. So the Universe is expanding! If a galaxy is moving away from us, the characteristic lines shift towards the red end of the spectrum and thus cause an effect called redshift (see Figure 1.1, taken from [6]). On the other hand, the light waves coming from a galaxy moving towards us, get crowded together and raise the frequency. This, in turn, causes blueshift since blue light corresponds to the high-frequency end of the spectrum. The

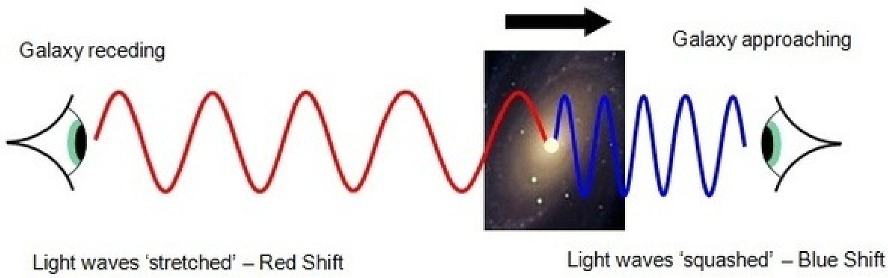

Figure 1.1: Redshift and blueshift.

redshift of a galaxy is denoted by dimensionless number z, defined as

$$z = \frac{\lambda_{\text{ob}} - \lambda_{\text{em}}}{\lambda_{\text{em}}},$$

where λ_{em} is the wavelength of the light at the time of emission from the remote galaxy while λ_{ob} is the wavelength of this light at the time of reception on Earth. In the case of blueshift, we have $z < 0$. However, almost all the galaxies are observed with $z > 0$.

Vesto Slipher, at the Lowell Observatory in 1912, first measured the wavelength shift of the light from M31 galaxy; which is one of the few galaxies exhibiting a blueshift. Slipher, by 1925, measured shifts of the spectral lines of around 40 galaxies and found that they were all nearly redshifted except some nearby galaxies within our Local Group. By 1929, there were enough galaxy redshifts measurements for the cosmologist Edwin Hubble to study whether a galaxy's redshift is related to its distance from Earth. A galaxy's redshift can be measured with high precision, but measuring its distance is not easy. For instance, Hubble knew the redshift z for around 50 galaxies, but estimated distances only for 20 galaxies. Nonetheless, he plotted redshift (z) versus distance (r) for the galaxies, shown in Figure 1.2 (extracted from Hubble's original paper [7]), and found the linear

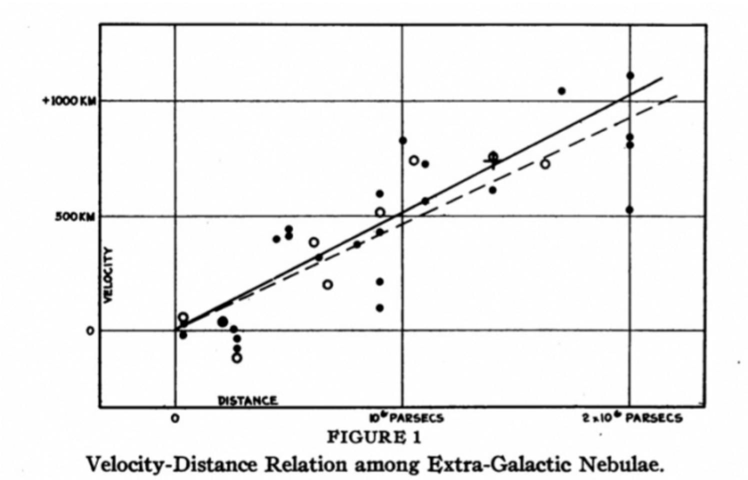

Figure 1.2: Hubble measurement of velocity and distance.

relationship between redshift and distance, now known as Hubble-Lemaître's Law[1]

$$z = \frac{H_0}{c} r,$$

where H_0 is a constant known as the Hubble constant and c is the speed of light. The Hubble's law is very well confirmed by modern observations. Hubble interpreted that the observed redshift of galaxies is due to their radial velocities away from the Earth (Doppler shift). In Hubble's analysis, the values of z were all small ($z < 0.04$), so he used the classical and non-relativistic relation, $z = v/c$, for the Doppler shift. Note that the relativistic relation is $z = \sqrt{(1+v/c)/(1-v/c)} - 1$, where v is the radial velocity of a galaxy or a light source. Hubble's law, interpreting the redshift as Doppler shift, takes the form

$$v = H_0 r. \tag{1.1}$$

The Hubble constant is measured in km s^{-1} Mpc^{-1}. The numerical value estimated by Hubble was $H_0 = 500$ km s^{-1} Mpc^{-1}. However, the best current estimate of the Hubble constant with the modern observations, considering the results from different observations, is much lower. For instance, a recently measured value of H_0 by the Baryon

[1]Two years before the publication of the Hubble's article, the astronomer and Belgian priest Georges Lemaître already published the research leading to the Hubble's Law.

Oscillation Spectroscopic Survey (BOSS) [8] reads as

$$H_0 = 67.6^{+0.7}_{-0.6}\ \text{km s}^{-1}\ \text{Mpc}^{-1}\ . \tag{1.2}$$

It may be noted that the expansion of the Universe is most significant over very large distance scales. For nearby galaxies such as the Andromeda galaxy in our local group, the gravity is strong enough to supersede the small effect caused by the expansion of the Universe. However, Hubble's law is consistent with all observations of the distant galaxies in their respective local groups. A galaxy within a cluster may have a velocity different from the Hubble flow due to the gravitational effects (local) within the cluster. Such a velocity is called the peculiar velocity of the galaxy. Thus, the total or effective velocity of a galaxy is given by

$$v_{\rm eff} = v_{\rm hubble} + v_{\rm peculiar}.$$

However, the peculiar velocities are negligible in comparison to the Hubble flow. Because of the cosmological principle, no location in the Universe is special. So observer in any galaxy will observe the other galaxies moving away from his/her own galaxy. It deserves to mention that the galaxies are not receding from each other in a pre-existing space rather the space between the galaxies evolves with the time that drags apart the galaxies from each other. The Universe is expanding. So extrapolating backward in time, all the stuff of the Universe can be imagined to crunch together. So it is believed that our Universe must have had a beginning from a very dense and hot state, called the hot Big Bang, around 14 billion years ago, and since then it is expanding continuously and growing in size. The structures in the Universe are formed during the course of the expansion of the Universe. The Hubble constant H_0 is usually parameterized by the dimensionless Hubble constant h via the relation: $H_0 = 100\,h$ km s^{-1} Mpc^{-1}. Furthermore, the Hubble constant H_0 stands for the present value of a time-varying parameter, known as the Hubble parameter:

$H = H(t)$. It is defined as

$$\dot{r}(t) = H(t)r(t), \tag{1.3}$$

where an overdot denotes the time derivative, and $r(t)$ is the physical radial distance. The physical distance, $r(t)$ is connected with the comoving distance, χ via the relation:

$$r(t) = \chi a(t), \tag{1.4}$$

where $a(t)$ is known as the scale factor of the Universe. Then, the Hubble parameter reads as:

$$H(t) = \frac{\dot{a}(t)}{a(t)}. \tag{1.5}$$

1.1.3 The story of hot Big Bang

Here are some of the historical developments that led to the theory of hot Big Bang regarding the origin and evolution of the Universe.

- The proposal (with a rigorous mathematical formulation) that the Universe may have had a definite beginning as a point of infinite density which set out to expand, was first made by the Belgian priest George Lemaître in 1927 (his mathematical formulation described a Universe which could either be expanding from a point or contracting into a point). But his original paper was published in a small Belgian journal and went largely unnoticed.

- Alexander Friedmann, a Russian mathematician and meteorologist, had already modeled such a Universe mathematically in 1922 using the general theory of relativity (of which Lemaître was unaware) and Einstein was well aware of his equations, but they both failed to give them any physical relevance, they just thought about them as mathematically consistent solutions to general relativity. Einstein was convinced that the Universe was static, and Friedmann died in 1925.

- Then in 1929, Hubble published his analysis on the redshift of distant galaxies and the proportionality between distance and redshift (Hubble-Lemaître's Law). But Hubble himself did not believe that expansion was the reason for the apparent redshift, he thought it must be just some apparent effect. Lemaître realized that Hubble's data fitted his model of an expanding Universe, and communicated with the already famous astronomer Arthur Eddington. Eddington did not believe Lemaître's proposal, he was also convinced that the Universe was static, but he nevertheless realized the importance of Lemaître's work and helped him to publish it in English in 1931, when the theory became widespread, already with a firm statement that the Universe was expanding and it must have started as a tiny point which he called "a primeval atom".

- The theory though received little support for many years, with most scientists favouring the steady-state Universe theory[2] to explain the apparent expansion. The name Big Bang was coined by Fred Hoyle in 1949 (who was a firm defender of the steady-state theory).

- George Gamow was the one who realized that the steady-state theory had many more conceptual problems than Lemaître's proposal and concluded that the Big Bang had to be the correct theory, and set out to develop it into a more rigorous and complete theory, developing many important aspects such as the details on how did the early Universe evolve, how did the mechanisms of primeval nucleosynthesis work, etc. Eventually, it was predicted that the Big Bang should have left a cosmic microwave background (CMB) radiation which should exist in the whole Universe today at a temperature 2.7255 ± 0.0005 K [9].

- The existence of the CMB radiation was first predicted by Ralph Alpher and his group in 1948 in connection with the research on Big Bang nucleosynthesis (BBN). It was first observed, after a sequence of coincidences, in 1965 by Arno Penzias and

[2]In steady-state theory, the Universe is eternal (no origin or beginning) with constant density. The major issue with the theory is that the continuous creation of matter out of nothing is proposed to maintain the constant density in the expanding Universe, violating the principle of conservation of energy.

Robert Wilson at the Bell Telephone Laboratories as a source of excess noise in a radio receiver. For this remarkable discovery, they shared the 1978 Nobel prize in physics. The experimental discovery of this CMB radiation finally put the hot Big Bang theory as the prevailing theory for the origin and evolution of the Universe. So far, it is the most accepted scientific theory regarding the origin and evolution of the Universe.

1.1.4 Accelerated expansion of the Universe

The Universe has been expanding after the Big Bang. So it is natural to expect that the gravitational attraction of the matter in the Universe would cause its expansion to be decelerating. But instead, the expansion of the Universe appears to be taking place at an increasing rate, so that the recession velocities of all distant galaxies are increasing continuously with time. Two independent projects, the Supernova Cosmology Project and the High-Z Supernova Search Team (see [10] and [11]) discovered the accelerated expansion of the Universe in 1998, where both the groups used distant type Ia supernovae (SNe Ia) to measure the accelerated expansion of the Universe. They considered these SNe Ia as standard candles, all having almost the same intrinsic brightness, and used the observed brightness of these SNe Ia to measure their distance from us. Comparing the distance with the cosmological redshift of the supernovae, they measured how fast these supernovae are moving away from us. They got the unexpected and surprising result that the Universe appears to be expanding with an accelerating rate. Three members of the two groups were subsequently awarded by the Nobel Prize of physics in 2011 for their remarkable and important discovery. Many complementary probes have confirmed the accelerated expansion of the Universe: CMB, baryon acoustic oscillations (BAO), and in analyses of the clustering of galaxies, are to name a few. The expansion of the Universe is believed to have been accelerating roughly 5 billion years ago. Further, it is believed that our Universe is witnessing an accelerating expansion due to the dominance of some exotic energy stuff, known as DE. The DE is mimicked by the cosmological constant Λ in the standard model of cosmology, as we will see later. There is one more point to ponder.

It is believed that Universe witnessed an accelerated expansion for a fraction of second just at the beginning, and inflated itself enormously. This idea is called cosmological inflation and was invented by Alen Guth in 1981. The original motivation of this ad-hoc idea was to explain the initial flatness and homogeneity of the Universe and the lack of certain relics that could have been produced in the very early Universe. The important property of inflation, which was discovered later, is that it provides a mechanism for generating the initial density fluctuations, the primordial perturbations, from which the structure such as stars and galaxies have grown. The cosmological inflation is a separate hot research topic nowadays in modern cosmology, and thus not a subject to discuss in detail here. For more details on inflation, one may refer to [12]. Figure 1.3 shows the schematic cosmic evolution (various epochs) of the Universe from the early Big Bang epoch until the present epoch of cosmic acceleration. This figure is taken from [13].

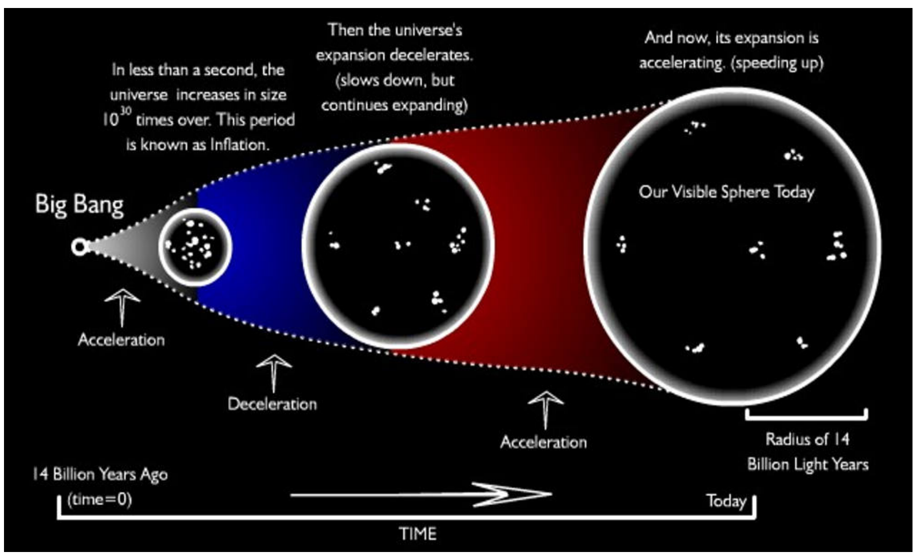

Figure 1.3: Cosmic evolution of the Universe from the time of Big Bang.

1.1.5 Energy components of the Universe

Fundamental particles are the constituents of everything in the Universe, and properties of these particles dictate the behaviour of the Universe as a whole. A non-relativistic particle moves with a speed much less than that of the light, and its rest mass-energy dominates the kinetic energy. On the other hand, a relativistic particle is a particle that moves with the speed of light or a substantial fraction of it. Its mass-energy does not dominate the kinetic energy. Thus, any particle with zero rest mass is always relativistic and moves with the speed of light. Thus, the total energy E of a particle is given by its mass-energy plus kinetic energy via the relation:

$$E^2{}_{\text{total}} = m^2c^4 + p^2c^2, \tag{1.6}$$

where m is the rest mass of the particle and p being the magnitude of momentum. In the relativistic limit, (i.e., for the particles having zero rest mass) we have

$$E = pc \tag{1.7}$$

In the non-relativistic limit (i.e., for the particles which have $v << c$), we have

$$E_{\text{total}} = mc^2 \left(1 + \frac{p^2}{m^2c^2}\right)^{1/2} \approx mc^2 + \frac{p^2}{2m}, \tag{1.8}$$

the first term in above equation is the rest mass-energy, and the second term is the classical kinetic energy of the particle. The momentum of the particle, $p = mv$ in the non-relativistic limit. The observable Universe comprises mainly five types of matter-energy components/particles: baryons, photons, neutrinos, dark matter (DM) and DE. In the following, we give a brief account of these components.

1. Baryons

A baryon is a particle that is defined to be made of three quarks. For instance, a proton contains two “up” quarks and a “down” quark; a neutron contains two “down” quarks and a “up” quark. So neutrons and protons both are examples of baryons. Indeed, the material objects that we see all around are made of the fundamental particles: neutrons, protons, and electrons. The simplest way to observe the baryons today is to see the galaxies. The greatest contribution to the density, though, comes not from stars in galaxies, but rather from gas in groups of galaxies. Various estimates place the limit on total baryonic matter roughly around 2-5% of the total density of the Universe. The estimation of total matter density in the Universe is much higher than the estimated baryonic matter, so this directly indicates the presence of a huge amount of non-baryonic matter in the Universe.

2. Photons

A photon (γ) is known to be a massless particle and travels with the speed of light. These are the constituent particles of the electromagnetic radiation. If its frequency is f or its wavelength is $\lambda = c/f$, then energy of the photon is, $E_\gamma = hf$, where $h = 2\pi\hbar$ stands for the Planck constant. Photons pervade the Universe with a wide range of energy, from radio to gamma rays. For cosmology, the CMB photons are the most relevant because they carry information of the very early Universe, and are present in a large abundance in the Universe such that the contribution of all other photons is negligible in comparison to the CMB photons.

3. Neutrinos

A neutrino (ν) is a lepton particle with three types, or "flavors": tau neutrinos, electron neutrinos, and muon neutrinos. Some or all may be massive or massless. It is not clear yet whether they are massive or massless, and intense research is in progress in this direction. Unlike photons and baryons, cosmic neutrinos have not been observed so far, so arguments about their presence and contribution to the energy density of the Universe are necessarily theoretical. However, these theoretical arguments are quite strong, based on very well-understood physics. A basic understanding of the interaction rates of neutrinos enables us to argue that neutrinos were once in equilibrium with the rest of the cosmic plasma, and decoupled from it at energy scale around 1 MeV before BBN. Neutrinos are extremely weakly interacting particles, and interact via the weak nuclear force with other particles. Neutrino cosmology is another active field of research and various experiments/surveys are currently in progress to detect them.

4. Dark Matter

BBN, the process of light elements production, provides a way to measure the baryon density of the Universe. From BBN prediction, it is found that ordinary matter (baryons) accounts for only a few percents (4-5%) to the total density of the Universe. But the direct estimates tell us that the total matter density is around 20-30% of the total density of the Universe. Thus, BBN provides a piece of evidence that there is a large amount of non-baryonic matter, which we call DM.

The history of DM is more than eighty years old. In 1933, Fritz Zwicky [14] provided the first evidence of DM on the basis of velocity dispersion of seven galaxies in the Coma cluster. He concluded that there is a large amount of non-ordinary matter within the Coma cluster of galaxies. Nowadays, there are several strong pieces of evidence of cosmological nature for the existence of DM. The strongest one is the observed anisotropies in the CMB. These present anisotropies are due to the presence of DM in the very early Universe. Other pieces of evidence include the strong and weak lensing effects, the brightness in X-rays of clusters of galaxies which depends on the potential well of the cluster [3], and the structure formation of the Universe which requires about at least 20% DM. In short, DM is a hypothetical type of matter which constitutes about 25% matter density of the Universe. DM has not been observed directly, its existence and properties are inferred from anisotropies in the CMB, galaxy rotation curves, weak and strong gravitational lensing, and formation of structures. The particle(s) which make this new form of matter is unknown so far. The most popular and simplest idea is that DM is made up of elementary particle(s) produced in the very early Universe.

There are three classes of DM particles based on their velocities, making up this matter: hot dark matter (HDM), warm dark matter (WDM), and cold dark matter (CDM). The HDM particles are those which decoupled when they were relativistic, i.e., they have relativistic velocities for at least some fraction of lifetime of Universe. So they form huge and rarer halos which are different in number from current observations. Thus, HDM particles are ruled out by several types of observations because they do not favor to form the observed structures in the Universe. A light neutrino, a very weakly interacting particle decoupled in very early Universe, might be an example of an HDM particle. CDM particles are those which have negligible velocity, hence they can cluster and form much smaller and denser halos. CDM particles favor the formation of the structure due to their mutual gravity and negligible pressure. Despite the ability of CDM particles in successfully explaining the observed structure of the Universe, there are some theoretical issues with these, as discussed in section 1.3.2. In the standard model of particle physics, a neutrino might be the most suitable candidate for DM particle if it has a small mass. The WDM is an intermediate case between HDM and CDM. Sterile neutrinos and gravitinos are the most common candidates for WDM particle. For more details about the pieces of evidence, candidates and methods of detection of DM, we refer the review articles [15–17]. For a recent status of direct and indirect searches of DM particles, see [18, 19].

5. Dark Energy

As mentioned above that the total matter density contributes only one third to the total density of the Universe. The remaining two-thirds of the density in the Universe comes from a hypothetical form of energy, known as DE. The important questions about DE such as, what is DE and in which form it is present in the Universe, still need answers? There are mainly three interpretations of DE: the cosmological constant, quintessence and phantom. The term cosmological constant, denoted by Λ, was first introduced by Albert Einstein in 1917, to make the Universe static. After the discovery of the expansion of the Universe by Edwin Hubble in 1929, Einstein dropped the cosmological constant from his field equations which describe the evolution of our Universe. For decades, most cosmologists assumed $\Lambda = 0$ in the cosmological models of the Universe and took it for granted that the expansion of the Universe is gradually slowing down due to the gravitational attraction of the matter content present in the Universe. After the discovery of the accelerated expansion of the Universe, the cosmological constant has turned out to be the most suitable candidate of DE explaining the observed acceleration of the Universe. The cosmological constant Λ is interpreted as the vacuum energy or energy of empty space. It was suggested by Einstein that empty space is not really empty. It is filled with pairs of particles and their anti-particles (virtual particles), appearing out of nowhere, which annihilates themselves and suddenly disappears, creating the vacuum. Quintessence is an alternative form of DE to the cosmological constant. There are many models for quintessence that arise from particle physics. The explanation for quintessence is that it is considered as a scalar field, defined as the strength of a field at each point of the space independent of the direction in the field. Therefore, one can treat cosmological constant as a particular case of a scalar field, in which field strength is the same at each point. Phantom is a hypothetical form of DE which has negative kinetic energy. The phantom form of DE may cause the expansion of the Universe to accelerate so quickly that it will lead to a Big Rip, a possible end of the Universe. A detailed description of various forms of DE can be seen in the review articles [20–22]. The precise nature of DE is still unknown and various cosmological probes are in progress in this direction on theoretical as well as observational grounds. Currently, the most popular candidate for DE is the cosmological constant and is a major part of the standard model since it is highly supported with most of the recent observational data. The individual contribution of various matter-energy components to the overall energy budget of the Universe are shown in Figure 1.4, which is taken from [23].

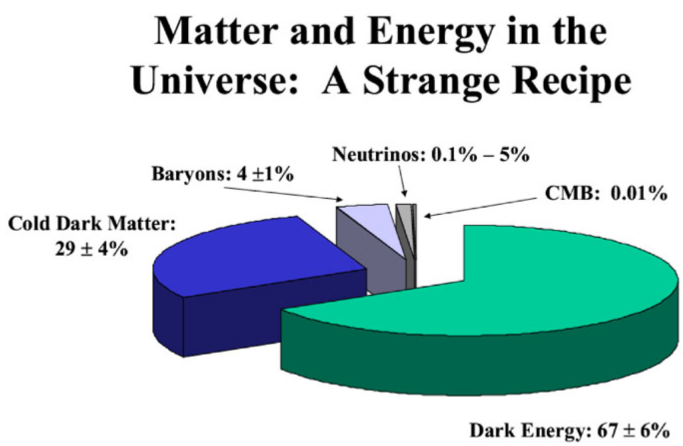

Figure 1.4: The individual contribution of components to the total energy budget of the Universe. It can be seen that the particles for around 95% energy stuff of the Universe are yet to be discovered.

1.1.6 Background equations and cosmological parameters

Here, we shall derive the background equations and cosmological parameters, governing the dynamics of the Universe. These equations and parameters allow us to describe the past and future evolution of the Universe.

1. Friedmann-Lemaître-Robertson-Walker metric

The metric that encapsulates the geometric properties of the homogeneous and isotropic Universe (i.e., respecting the cosmological principle) was derived by Alexender Friedmann in 1922. A few years later in 1927, George Lemaître independently arrived at the Friedmann metric. Finally, in 1935, Robertson and Walker rigorously proved that this is the only one metric representing homogeneous and isotropic spacetime. This metric is popularly known as Friedmann-Lemaître-Robertson-Walker (FLRW) metric [24–27]. In spherical coordinates (r, θ, ϕ), the metric of an isotropic and homogeneous three-dimensional space is given by

$$dl^2 = \left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{1.9}$$

where the constant k represents the spatial curvature. If $k < 0$, the 3D space is open or negatively curved; if $k = 0$, the 3D space is flat or Euclidean, and if $k > 0$, space is closed or positively curved. The most general metric of the four-dimensional spacetime of an isotropic, homogeneous and expanding Universe reads as

$$ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right], \tag{1.10}$$

where the function $a(t)$ is the scale factor of the Universe. It is the FLRW metric of the 4D spacetime.

2. Einstein's equation

Einstein's equation represents the relation between matter and geometry of the spacetime, given by [28], adpoting speed of light, $c = 1$.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}. \tag{1.11}$$

Here, $G_{\mu\nu}$ is the Einstein tensor that describes the geometry of the Universe; $R_{\mu\nu}$ is the Ricci tensor, which depends on the metric tensor $g_{\mu\nu}$ and its derivatives; R is the Ricci scalar, which is related to Ricci tensor via $R \equiv g^{\mu\nu}R_{\mu\nu}$; G is the Newton's gravitational constant and $T_{\mu\nu}$ is the stress-energy momentum tensor (a symmetric tensor) that describes the various matter and energy sources in the Universe. The Einstein tensor $G_{\mu\nu}$ determines the curvature of spacetime at a point in spacetime and is equated with the energy and momentum at the same point. It is diagonal ($G_{0i} = G_{i\neq j} = 0$) and isotropic ($G_{11} = G_{22} = G_{33}$).

The Ricci tensor is expressed in terms of the Christoffel symbols,

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\alpha,\nu} + \Gamma^{\alpha}_{\beta\alpha}\Gamma^{\beta}_{\mu\nu} - \Gamma^{\alpha}_{\beta\nu}\Gamma^{\beta}_{\mu\alpha}. \tag{1.12}$$

Here, the comma denotes derivative with respect to spatial coordinates. We notice that Ricci tensor involves Christoffel symbols, which are further expressed in terms of components of metric tensor $g_{\mu\nu}$:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left[\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\beta\nu}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\right] \tag{1.13}$$

In FLRW metric, $G_{\mu\nu}$ is computed using Christoffel symbols. The energy momentum tensor T^{μ}_{ν}, in case of a perfect isotropic fluid, is given by

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{1.14}$$

where ρ is the energy density, and p is the pressure of the isotropic cosmological fluid. The time-time or 00 component of the Einstein tensor reads as

$$G_{00} = 3\left[\frac{k}{a^2} + \left(\frac{\dot{a}}{a}\right)^2\right]. \tag{1.15}$$

In FLRW model, the 00 component, $G_{00} = 8\pi G T_{00}$, of the Einstein's equation gives

$$3\left[\frac{k}{a^2} + \left(\frac{\dot{a}}{a}\right)^2\right] = 8\pi G\rho \tag{1.16}$$

or,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{1.17}$$

This equation is called as the Friedmann equation. It gives the evolution of the Universe via the relation between, the energy density ρ, the scale factor $a(t)$ and the spatial curvature k.

3. Energy conservation

The Einstein's equation satisfies the first Bianchi identity $T^{\nu}_{\mu;\nu} = 0$, i.e., the conservation equation for the total energy-momentum tensor via the second Bianchi identity $G^{\nu}_{\mu;\nu} = 0$. In FLRW model, it gives the relation between the scale factor $a(t)$ and the energy density ρ:

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p). \tag{1.18}$$

This is known as the conservation law of energy in an expanding Universe. The dependency of energy density on the pressure p for different components of the Universe leads to the equation of state (EoS): $p \equiv p(\rho)$. For non-relativistic matter, such as baryons and DM, $p = 0$ due to the negligible kinetic energy. Therefore, (1.18) reduces to

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho \quad \Longrightarrow \quad \rho \propto a^{-3} \tag{1.19}$$

For relativistic matter such as the photons and massless neutrinos, $p = \rho/3$ because the particle velocity generates pressure. Therefore, (1.18) gives

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(1 + \frac{1}{3}\right)\rho = -4\frac{\dot{a}}{a}\rho \quad \Longrightarrow \quad \rho \propto a^{-4} \tag{1.20}$$

Thus, it is clear that in an expanding Universe, a relativistic fluid dilutes faster than the non-relativistic fluid.

4. Cosmological constant

In 1917, Einstein introduced the cosmological constant (Λ) to prove that the Universe is static. But after the Hubble's discovery of cosmic expansion in 1929, the cosmological constant was dropped. Later, it again came into the picture after the discovery of the accelerated expansion of the Universe in 1998. Thereafter, the cosmological constant is treated as the form of DE and was added to the left-hand side of the Einstein equation without violating the cosmological principle:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{1.21}$$

or

$$G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu} - \Lambda g^{\mu}_{\nu}. \tag{1.22}$$

Here, $g^{\mu}_{\nu} = g^{\mu\alpha} g_{\alpha\nu} = \delta^{\mu}_{\nu}$, and the stress-energy-momentum tensor of the homogeneous fluid in terms of cosmological constant is given by

$$\tilde{T}^{\mu}_{\nu} = -\frac{\Lambda}{8\pi G}\delta^{\mu}_{\nu} = \begin{pmatrix} -\frac{\Lambda}{8\pi G} & 0 & 0 & 0 \\ 0 & -\frac{\Lambda}{8\pi G} & 0 & 0 \\ 0 & 0 & -\frac{\Lambda}{8\pi G} & 0 \\ 0 & 0 & 0 & -\frac{\Lambda}{8\pi G} \end{pmatrix}. \tag{1.23}$$

Comparing (1.14) and (1.23), we see that the EoS of cosmological constant is $p = -\rho = \Lambda/8\pi G$. This shows that the cosmological constant does not vary with time ($\dot{\rho} = 0$).

5. Possible scenarios of the evolution history of the Universe

The total energy density of the Universe in terms of its contribution from different components of the Universe is written as

$$\rho = \rho_{\rm r} + \rho_{\rm m} + \rho_k + \rho_\Lambda, \tag{1.24}$$

where $\rho_{\rm r}$, $\rho_{\rm m}$, ρ_k and ρ_Λ are respectively, the energy densities of radiation (photons + neutrinos), matter (baryons + dark mater), spatial curvature, and the cosmological constant.

Therefore, we can rewrite the Friedmann equation (1.17) containing all components, as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\mathrm{r}} + \frac{8\pi G}{3}\rho_{\mathrm{m}} - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{1.25}$$

The energy densities of different components evolve with respect to the scale factor in the following manner: $\rho_{\mathrm{r}} \propto a^{-4}$, $\rho_{\mathrm{m}} \propto a^{-3}$, $\rho_k \propto a^{-2}$, and $\rho_\Lambda \propto a^0$. Therefore, with the expansion of the Universe they all dominate one after the other in the following order: radiation domination, matter domination, curvature domination, and cosmological constant domination. Let us understand the behaviour of scale factor, Hubble parameter and Hubble radius ($R_H = cH^{-1}$) during different dominating phases of the Universe.

I. Radiation domination:

$$\frac{\dot{a}^2}{a^2} \propto a^{-4}, \quad a(t) \propto t^{1/2}, \quad H(t) = \frac{1}{2t}, \quad R_H(t) = 2t. \tag{1.26}$$

II. Matter domination:

$$\frac{\dot{a}^2}{a^2} \propto a^{-3}, \quad a(t) \propto t^{2/3}, \quad H(t) = \frac{2}{3t}, \quad R_H(t) = \frac{3}{2}t. \tag{1.27}$$

III. Negative curvature domination:

$$\frac{\dot{a}^2}{a^2} \propto a^{-2}, \quad a(t) \propto t, \quad H(t) = \frac{1}{t}, \quad R_H(t) = t. \tag{1.28}$$

IV. Positive curvature domination: In the case of $k > 0$ and no cosmological constant, the RHS of Friedmann equation (1.25) becomes zero and the expansion of the Universe stops. When the scale factor decreases, H becomes negative but the RHS of (1.25) remains positive and hence the Universe recollapses.

V. Cosmological constant domination:

$$\frac{\dot{a}^2}{a^2} \propto \text{constant}, \quad a(t) \propto \exp(\Lambda t/3), \quad H(t) = 1/R_H = \sqrt{\Lambda/3}. \tag{1.29}$$

Thus, in the cosmolgical dominated phase, the Universe expands exponentially.

6. Cosmological parameters

The density required to make the Universe flat (i.e., curvature $k = 0$) is known as the critical density of the Universe. It is defined as

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}. \tag{1.30}$$

We now define the density parameter Ω_x as the ratio of the density of each species evaluated at present to the critical density of the Universe at present,

$$\Omega_x = \frac{\rho_{x0}}{\rho_{c0}}; \quad x \in \{\mathrm{r}, \mathrm{m}, \Lambda\}. \tag{1.31}$$

Also, the density parameter of the spatial curvature is given by

$$\Omega_k = -\frac{k}{a_0^2 H_0^2}, \tag{1.32}$$

Therefore, Friedmann equation reduces to

$$\Omega_\mathrm{r} + \Omega_\mathrm{m} + \Omega_\mathrm{k} + \Omega_\Lambda = 1. \tag{1.33}$$

This is usually called energy-budget equation of the Universe. The present physical density (ρ_{x0}) of a component x can be decomposed into standard units:

$$\begin{aligned}\rho_{x0} &= \Omega_x \frac{3H_0^2}{8\pi G} \\ &= \Omega_x \frac{3(100h\,\mathrm{km\,s^{-1}Mpc^{-1}})^2}{8\pi G} \\ &= \Omega_x h^2 \times 1.8788 \times 10^{-29}\mathrm{g\,cm^{-3}},\end{aligned} \tag{1.34}$$

where $\Omega_x h^2$ is the dimensionless number. This leads us to a new parameter called the physical density parameter, $\omega_x \equiv \Omega_x h^2$. In the measurement of CMB temperature, ω_γ is considered as the fixed-parameter:

$$\begin{aligned}\omega_\gamma &\equiv \Omega_\gamma h^2 \\ &= \frac{\rho_{\gamma 0}}{\rho_{c0}} h^2 \\ &= \left(\frac{\pi^2}{15} T_0^4\right)\left(\frac{8\pi G}{3H_0^2}\right) h^2 \\ &= \frac{8\pi^3 T_0^4}{45(H_0/h)^2 M_P^2}.\end{aligned} \tag{1.35}$$

At present CMB temperature, $T_0 = 2.7255 \pm 0.0005$ K at 68% confidence limit (CL) [29], we get $\omega_\gamma \equiv (2.472 \pm 0.002) \times 10^{-5}$. Moreover, neutrino abundance is relatively fixed to that of photons. Therefore, the total physical radiation density is given by:

$$\begin{aligned}\omega_{\rm r} &\equiv \omega_\gamma + \omega_\nu \\ &= \left[\frac{\pi^2}{15} T_0^4 + N_{\rm eff} \times \frac{7}{8} \times \frac{\pi^2}{15} T_{\nu 0}^4\right]\left(\frac{8\pi G}{3H_0^2}\right) h^2 \\ &= \left[1 + N_{\rm eff} \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right]\omega_\gamma\end{aligned} \tag{1.36}$$

In the above equation, $N_{\rm eff}$ stands for the effective number of neutrino species and can be viewed as a convenient parametrization of the relativistic energy density of the Universe beyond the energy density of the photons which is highly constrained by the measurement of the CMB temperature. By taking into account certain effects such as neutrino oscillations, non-instantaneous decoupling around electron-positron annihilation, etc., numerical calculations yield, $N_{\rm eff} = 3.046$ [30]. The factor $7/8$ apears due the fact that neutrinos are fermions, i.e., they follow Fermi- Dirac distribution with zero chemical potential. Now, substituting $N_{\rm eff} = 3.046$ and $T_0 = 2.7255 \pm 0.0005$ K in (1.36), we get the total physical radiation density as $\omega_{\rm r} = 4.183 \times 10^{-5}$. Thus, it is clear that the total radiation contribution can be ignored in the Friedmann equation, except in the early Universe.

1.2 Linear perturbation theory

It is well known that the present Universe is homogeneous and isotropic at sufficiently large scales about 100 Mpc or larger than that. Despite this, it has generated small inhomogeneities in the early stages of evolution, which have grown with time and formed the observed structures of the Universe such as a galaxy, galaxy clusters, superclusters, and voids. The observations of CMB tell us that the Universe was homogeneous and isotropic at the time of recombination, with very small inhomogeneities. The main physical mechanism responsible for the growth of these small initial inhomogeneities to be the large structures we observe today is the gravitational instability in the matter distribution. In order to understand how these small inhomogeneities have evolved from a very early stage of the Universe to the present CMB anisotropies and large-scale structure, it is necessary to study the evolution of the cosmological perturbations. For more details and related discussions, we refer to the classical paper [31]. In the following subsection, we provide a brief description of the linear perturbation theory, used in the thesis work assuming that the Universe is spatially flat.

1.2.1 Classification of perturbations

The metric and stress-energy tensors of the Universe are decomposed into spatial averages and linear perturbations as follows:

$$g_{\mu\nu}(t,\vec{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\vec{x}), \tag{1.37}$$

$$T_{\mu\nu}(t,\vec{x}) = \bar{T}_{\mu\nu}(t) + \delta T_{\mu\nu}(t,\vec{x}). \tag{1.38}$$

Here $\bar{g}_{\mu\nu}$ is the metric of the homogeneous and isotropic FLRW Universe. There are ten degrees of freedom in each of the two perturbed tensors, describing different aspects of gravity. It was shown by Bardeen in 1980 that these can be decomposed into scalars, vectors, and tensors via spatial rotations. These three sectors get decoupled at the first order in perturbation theory. Scalar and vector perturbations vanish in the vacuum, but tensor perturbations can propagate if these have been excited. The tensor perturbations account for gravitational waves. In the presence of matter, scalars provide the response of the metric to an irrotational distribution of matter, generalizing Newton's theory of gravitation. Vectors, on the other hand, represent the response of the metric to vorticity, and represent the phenomena with no equivalent in Newton's theory, called "gravitomagnetism". In the minimal cosmological models, the vorticity of the different matter components decays with time, and therefore vectors can be neglected. Tensors may have a a small role in CMB anisotropies, and these can be studied separately since they have decoupled from the scalar sector at first order in perturbations. In this thesis work, we consider only the scalar perturbations. The four scalar components of the metric as well as the stress-energy perturbed tensors are given as follows:

(i) The (00) time-time term:

In the perturbed metric $\delta g_{\mu\nu}$: it corresponds to the generalized gravitational potential ψ.

In $\delta T_{\mu\nu}$: it represents the energy density perturbation $\delta\rho$ (we will use the relative energy density perturbation $\delta = \frac{\delta\rho}{\bar{\rho}}$).

(ii) The trace (ii) space-space terms:

In $\delta g_{\mu\nu}$: the local distortion, given by ϕ, of the average scale factor $a(t)$ (the "local scale factor" is $(1-\phi)a$).

In $\delta T_{\mu\nu}$: the pressure perturbation δp.

(iii) The irrotational part of the $(0i)$ time-space vector:

In $\delta g_{\mu\nu}$: the potential b such that $\delta g_{0i} = \partial_i b$.

In $\delta T_{\mu\nu}$: the potential v of the irrotational component of the energy flux, $\delta T_i^0 = \partial_i v$.

(iv) the traceless longitudinal part of the (ij) space-space tensor:

In $\delta g_{\mu\nu}$: the potential μ of the metric shear: $\delta g_{ij} = (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\Delta)\mu$

In $\delta T_{\mu\nu}$: the potential s of the shear stress or anisotropic stress: $T_i^j = (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\Delta)s$.

In place of the velocity potential v, we use a variable θ (called the "velocity divergence") defined via

$$\partial_i \delta T_i^0 = (\bar{\rho} + \bar{p})\theta = \Delta v. \tag{1.39}$$

Similarly, one can use the function σ instead of s, with the definition

$$(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\Delta)\delta T^i_j = (\bar{\rho} + \bar{p})\sigma = \Delta(\Delta s). \tag{1.40}$$

The function σ is usually referred to as the anisotropic stress. To summarize, it is easy to see that one can manipulate four degrees of freedom representing the scalar perturbations of the matter fields: the density fluctuation, pressure perturbation, velocity divergence and anisotropic stress, that is, $\{\delta, \delta p, \theta, \sigma\}$.

1.2.2 Gauges

A gauge refers to the choice of time slicing. For the idealised FLRW Universe, there exists only one time slicing, which is compatible with the assumption of homogeneity. But, in a perturbed Univrese, there are infinitely many time slicings compatible with the perturbation theory, keeping the quantities close to their average values. The perturbation of any quantity at a given point is given by the difference between the true and the average values of the quantity at this point. For example, the perturbation of energy density ρ reads,

$$\delta\rho(t, \vec{x}) = \rho(t, \vec{x}) - \bar{\rho}(t). \tag{1.41}$$

Here $\rho(t, \vec{x})$ is a locally and unambiguously defined quantity, but $\rho(t)$ depends on the choice of equal-time hypersurface going through the point $(t, \vec{x})$. So with different choices, $\rho(t, \vec{x})$ would be compared to the averages performed on different sheets, leading to different values. Thus, $\delta\rho(t, \vec{x})$ depends on the choice of time slicing or gauge. In this thesis work, we use the Newtonian and synchronous gauges as described below.

1. Newtonian gauge

In Newtonian gauge (also called longitudinal gauge), it is imposed that the non-diagonal scalar perturbations of the metric vanish: $b = \mu = 0$. This prescription fixes a unique time slicing. In this gauge, the line element is given by

$$ds^2 = -(1 + 2\psi)dt^2 + (1 - 2\phi)a^2(t)(dx^2 + dy^2 + dz^2). \tag{1.42}$$

In terms of the conformal time η, defined by $d\eta = dt/a$, the line element (1.42) becomes

$$ds^2 = a^2(\eta)[-(1+2\psi)d\eta^2 + (1-2\phi)(dx^2 + dy^2 + dz^2)]. \tag{1.43}$$

The Newtonian gauge is with the advantage that the gauge-invariant Bardeen potentials $\{\Phi_A, \Phi_H\}$ reduce to the metric perturbations $\{\phi, \psi\}$. Note that for a homogeneous and spatially flat FLRW Universe with the metric

$$ds^2 = a^2(\eta)[-d\eta^2 + (dx^2 + dy^2 + dz^2)], \tag{1.44}$$

and with energy density $\bar{\rho}(\eta)$ and pressure $\bar{p}(\eta)$, the Einstein's equations $\delta G^\nu_\mu = 8\pi G \delta T^\nu_\mu$ give the following evolution equations for the expansion factor $a(\eta)$:

$$3\left(\frac{a'}{a}\right)^2 = 8\pi G a^2 \bar{\rho}, \tag{1.45}$$

$$3\frac{d}{d\eta}\left(\frac{a'}{a}\right) = -4\pi G a^2 (1+3w)\bar{\rho}. \tag{1.46}$$

where the primes denote derivatives with respect to conformal time η, and $w = \bar{p}/\bar{\rho}$ describes the EoS parameter of the cosmological fluid. The conservation of energy-momentum is a consequence of the Einstein's equations. Then the energy-momentum conservation equation

$$T^{\mu\nu}{}_{;\nu} = \partial_\mu T^{\mu\nu} + \Gamma^\nu_{\alpha\beta} T^{\alpha\beta} + \Gamma^\alpha_{\alpha\beta} T^{\nu\beta} = 0 \tag{1.47}$$

implies

$$\bar{\rho}' + 3\frac{a'}{a}(1+w)\bar{\rho} = 0. \tag{1.48}$$

For a given species with known w, one can solve (1.45) and (1.48) to find $\bar{\rho}$ and a in terms of η.

In the conformal Newtonian gauge and in Fourier space k, the first-order perturbed Einstein's equations $\delta G^{\nu}_{\mu} = 8\pi G \delta T^{\nu}_{\mu}$ give:

$$k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right) = -4\pi G a^2 \; \delta\rho \; , \tag{1.49}$$

$$k^2\left(\phi' + \frac{a'}{a}\psi\right) = 4\pi G a^2 \; (1+w)\bar{\rho}\theta \; , \tag{1.50}$$

$$3\phi'' + 3\frac{a'}{a}(\psi' + 2\phi') + 3\left(2\frac{a''}{a} - \frac{a'^2}{a^2}\right)\psi + k^2(\phi - \psi) = 4\pi G a^2 \; \delta p \; , \tag{1.51}$$

$$k^2(\phi - \psi) = 12\pi G a^2 \; (1+w)\bar{\rho}\sigma \; . \tag{1.52}$$

These equations are also valid for a single uncoupled fluid. They need to be modified for individual components if the components interact with each other. The conservation of energy-momentum is a consequence of the Einstein's equations. Then the perturbed part of energy-momentum conservation equation (1.47), in k-space, leads to the following scalar perturbation equations, namely the continuity equation

$$\delta' + 3\frac{a'}{a}\left(\frac{\delta p}{\delta \rho} - w\right)\delta + (1+w)(\theta - 3\phi') = 0 \; , \tag{1.53}$$

and the Euler equation

$$\theta' + \left[\frac{a'}{a}(1-3w) + \frac{w'}{1+w}\right]\theta - \frac{1}{1+w}\frac{\delta p}{\delta\rho}k^2\delta + k^2\sigma - k^2\psi = 0 \; . \tag{1.54}$$

We have $\bar{\rho}$ and a in terms of η from the background equations for a species with given w. So, for a given mode k, the above perturbation equations involve 6 variables: $\{\delta, \delta p, \theta, \sigma, \phi, \psi\}$, each to be determined in terms of η. But we have only four independent equations among the above six perturbation equations since the continuity and Euler equations are the consequences of the four Einstein's equations. In practice, we use the continuity and Euler equations along with two of the four Einstein's equations. Further, we need two more equations to close the system. For a perfect fluid, microscopic interactions impose local thermodynamical equilibrium, and the pressure is then isotropic with $\sigma = 0$. In addition, pressure perturbations obey $\delta p = c_s^2 \delta\rho$, where c_s is the sound speed of the fluid. In case of adiabatic perturbations, the adiabatic sound speed c_a is inferred from the EoS of the fluid via

$$c_a^2 = \frac{d\bar{p}}{d\bar{\rho}} = \frac{d}{d\bar{\rho}}(w\bar{\rho}) = w + \bar{\rho}\,\frac{dw}{d\bar{\rho}}\ . \tag{1.55}$$

The pressure perturbation for CDM can be neglected with respect to the density perturbation in the CDM case. Hence, CDM is equivalent to a perfect fluid, with no anisotropic stress and no pressure perturbation. So CDM has zero sound speed.

2. Synchronous gauge

In synchronous gauge, the line element reads

$$ds^2 = -a^2 d\tau^2 + a^2[(1-2\eta)\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j, \tag{1.56}$$

where $k^2E = -2/h - 3\eta$, h and η being the metric perturbation variables. The first-order perturbed Einstein's equations give

$$k^2\eta - \frac{1}{2}\frac{a'}{a}h' = 4\pi G a^2 \delta\rho\,, \tag{1.57}$$

$$k^2\eta' = 4\pi G a^2 (1+w)\bar{\rho}\theta\,, \tag{1.58}$$

$$h'' + 2\frac{a'}{a}h' - 2k^2\eta = -8\pi G a^2 \delta p\,, \tag{1.59}$$

$$h'' + 6\eta'' + 2\frac{a'}{a}\left(h' + 6\eta'\right) - 2k^2\eta = -24\pi G a^2 (1+w)\bar{\rho}\sigma)\,. \tag{1.60}$$

The continuity and Euler equations are, respectively, given by

$$\begin{aligned}
\delta' &= -(1+w)\left(\theta + \frac{h'}{2}\right) - 3\frac{a'}{a}\left(\frac{\delta p}{\delta\rho} - w\right)\delta\,, \\
\theta' &= -\frac{a'}{a}(1-3w)\theta - \frac{w'}{1+w}\theta + \frac{\delta p/\delta\rho}{1+w}\,k^2\delta - k^2\sigma\,.
\end{aligned} \tag{1.61}$$

1.3 Research work in the thesis

1.3.1 Origin of the research work

As mentioned earlier, from a sequence of remarkable and surprising discoveries in the 20th century, it is now known that our Universe is undergoing an accelerated expansion. The main goal of modern cosmology is to understand the origin, evolution, and future of our Universe. For this purpose, relying on Einstein's general theory of relativity, cosmologists have developed a mathematically tractable and observationally testable model of the Universe, the so-called the ΛCDM model, also known as the standard cosmological model. This model is remarkably successful to explain the dynamics and observed features of the Universe such as the cosmic acceleration, LSS formation and many more, and it finds an excellent fit for various observational data from different sources such as the CMB, LSS, BAO, SNe Ia, etc. Interestingly, the remarkably successful ΛCDM model is founded on two unknown and major energy ingredients, accounting for around 95% to the total energy budget of the Universe: (i) cosmological constant Λ, mimicking the simplest form of DE, responsible for cosmic acceleration (ii) CDM, the pressureless DM, responsible for structure formation. Unfortunately, scientists are yet to discover DE and DM particles in the laboratory or elsewhere, though the hunt for unfolding the mysteries of DM and DE is on via various ongoing and future experiments. Consequently, a large number of precise observational data related to CMB, BAO, SNe Ia, LSS and many other sources are pouring in. In near future, it is expected that these data would be able to pinpoint the precise nature/role of DM and DE in the dynamics and evolution of the Universe. Hence, the DM and DE model building and observational constraints acquire a special importance in cosmology. For instance, the Planck collaboration studies observational constraints using various data sets on the ΛCDM model by using some publicly available cosmology codes such as cosmic linear anisotropy solving system (CLASS) and MontePython in a high-performance computing (HPC) cluster. It deserves to mention that the ΛCDM model is plagued with a number of problems (such as fine-tuning, small scale, etc.), despite its great success in describing the Universe. So it is important and worthwhile to investigate the observational constraints on generalized dark sector models, and hence explore the possible physics beyond the standard ΛCDM model.

1.3.2 Issues with the standard model

The spatially-flat standard ΛCDM model is consistent with most of the observational data from different sources of different physical origin, and successfully explains the present accelerated expansion of the Universe [32]. This standard model of the Universe is filled with baryons, photons, neutrinos (sub-dominant at present epoch), and with the present major dominant components: DM as a cold specie and DE as a cosmological constant (Λ). The fundamental problem in the present-day cosmology is to understand the precise nature/behavior of both the dominant components: DM and DE [15, 16]. On the one hand, with the consideration of DM as a cold specie and DE as cosmological constant, the ΛCDM model has great successes on theoretical and observational grounds, providing us a substantial understanding of the dynamics and evolution of the Universe, and the formation of LSS. On the other hand, this model suffers from some serious theoretical issues:

- Fine-tuning problem
- The cosmic coincidence problem
- Small scale problem
- Tension in the measurements of some parameters

The fine-tuning problem is related to the cosmological constant, hence sometimes it is also called the cosmological constant problem. The fine-tuning problem can be stated as: the theoretically estimated value of vacuum energy is $\simeq 10^{76}$ GeV^4 [33] whereas the observed one is $\simeq 10^{-46}$ GeV^4 [32]. Thus, there is a huge discrepancy (about of order 123) between the theoretical and observed estimation of the cosmological constant. Another important issue with the cosmological constant is the cosmic coincidence [34]. This problem can be stated as why the DM and DE densities are approximately equal at present, although their cosmic evolution is completely different? The density of DM falls as a^{-3} with the expansion of the Universe whereas the density of the cosmological constant does not evolve, it stays constant all the time. Some issues are also with the CDM paradigm from observations on sub-galactic scales. These are called CDM small-scales anomalies and originated from the numerical simulations of the formation of structures. See [35] for a recent account. These anomalies include: core/cusp problem [36], missing satellite problem [37], and too big to fail problem [38]. There exist discrepancies (tensions) in the measurements of some parameters of the standard model. The well-known tensions are in the measurement of the present Hubble parameter H_0 and amplitude of present matter density fluctuations, characterized by σ_8. In the next two subsections, we discuss both the tensions in detail and some possible mechanisms to alleviate these tensions.

1.3.3 Tension on the Hubble constant

The Hubble constant, H_0 describes the present expansion rate of the Universe, and therefore an accurate measurement of this constant is important to understand the dynamics of the Universe. There are three methods, independent from each other, to measure the present value of Hubble parameter. The first method is based on CMB temperature anisotropy measurements in the framework of ΛCDM cosmology. Thus, it is a model-dependent measurement by using the Planck CMB data together with other complementary astrophysical probes. A second way of measuring H_0 is model independent and based on the cosmic distance ladder of astrophysical objects. It is called local measurement of H_0. The most recent and third method is also a model-independent: it uses recently detected gravitational waves (GWs) as standard sirens. The main advantage of H_0 measurement via standard siren is that it is not based on the cosmic distance ladder, but due to only a single data point, errors in the measurement are large. These large errors will be reduced by the more upcoming standard-siren measurements from future GWs sources like the Kamioka Gravitational Wave Detector (KAGRA) and Laser Interferom-

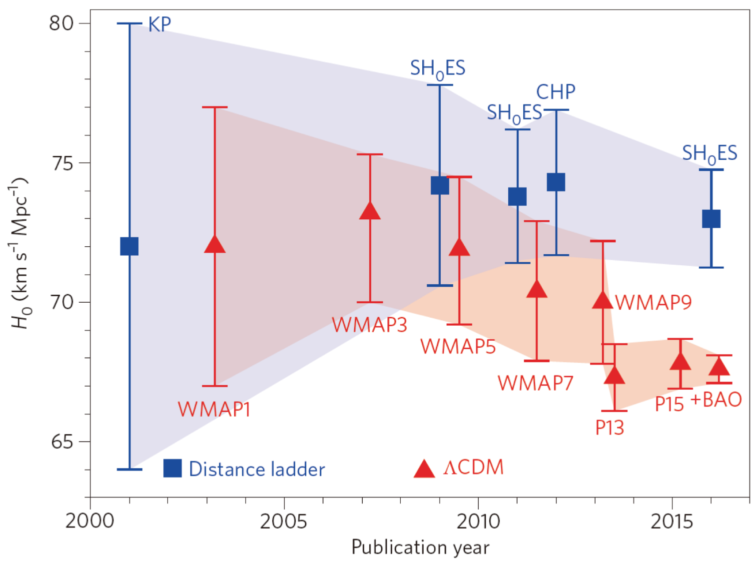

Figure 1.5: Measurements of H_0 from nearby Universe with cosmic distance ladder are shown in light blue color, and from Planck CMB within ΛCDM model are shown in light red color. Labels are showing different data sets and experiments in the determination of H_0. The shaded regions in blue and red color show the evolution of the uncertainties in these values. It can be seen that currently, these two methods are in big tension with each other.

eter Gravitational-Wave Observatory (LIGO)-India. Both the independent measurements of H_0 currently show a serious tension with the Planck CMB measurement. The latest Planck CMB (2018) measurement gives $H_0 = 67.36 \pm 0.54$ km s $^{-1}$ Mpc^{-1} [32] whereas from the local measurement it is $H_0 = 73.24 \pm 1.74$ Km s^{-1} Mpc^{-1} [39]. Using recently detected GWs as standard sirens, H_0 is constrained to be $H_0 = 70.0^{+12.0}_{-8.0}$ Km s^{-1} Mpc^{-1} [40]. The tension in the measurement of H_0 via distance ladder and from the Planck CMB can be seen in Figure 1.5, extracted from [41]. At present, there is no concrete evidence to believe which measurement gives the correct value of H_0.

1.3.4 Tension on matter density perturbation

The second important tension in cosmology arises in the measurement of root mean square fluctuation of present matter density perturbation in the sphere of radius 8 h^{-1}Mpc, characterized by σ_8. The value of σ_8 from the Planck CMB measurements and some LSS probes such as galaxy cluster counts [42, 43], weak lensing [44, 45] is in a serious disagreement within the framework of the ΛCDM model. The Planck CMB measurement gives a large value of σ_8 than the one predicted from the LSS measurements [46, 47]. The results from Planck CMB (2015) yields, $\sigma_8 = 0.831 \pm 0.013$ [32], which is about 2σ higher than $\sigma_8 = 0.75 \pm 0.03$ as given by the Sunyaev-Zel'dovich cluster abundances measurements [43]. This tension exists for a range of LSS indicators such as clusters, lensing, and redshift space distortions [48]. For instance, see Figure 1.6 extracted from [44].

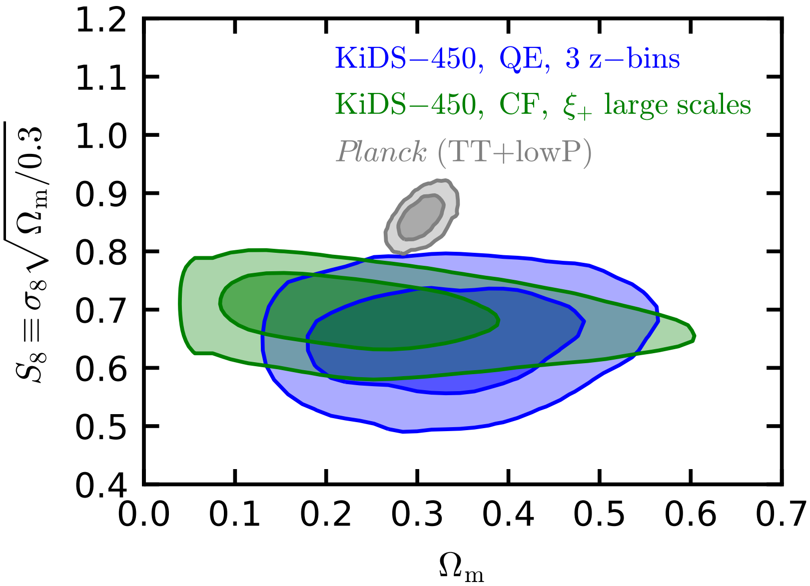

Figure 1.6: Confidence contours (68% and 95% CL) in $S_8 - \Omega_\mathrm{m}$ plane from kilo degree survey-450 (KiDS-450) with Quadratic Estimator, in 3 redshift bin, (denoted here by 'KiDS-450, QE, 3 z-bins'), the fiducial 'ξ_+ large scales' correlation-function analysis from KiDS-450, and Planck-2015 (TT + LowP).

These tensions in the parameters show that either there is systematics in data measurements or an indication of some new physics beyond the standard ΛCDM model. Under the pre-assumption that there is no systematics in the data measurements, several extended models of the minimal ΛCDM model have been constructed to reconcile these tensions. The possible extensions in this regard include the following: decaying DM, interaction in dark sector components, generalized DM than simple CDM, time-varying DE, other relativistic species (dark radiation), the inclusion of massive neutrinos of the standard model and beyond. Other possibilities also include modification in the law of gravity and/or consideration of curvature etc. For more detail in this regard, one may see the recent articles [49–52]. It may be noted that both the tensions have not been resolved simultaneously at a significant statistical level yet in the literature. Rather, by assuming neutrino properties, the parameters are correlated in such a way that lower values of σ_8 require higher values of total matter density and smaller values of H_0, which aggravates the tensions (e.g. [53]). In [54], it is argued that the presence of sterile neutrinos does not bring a new concordance, but possibly indicating systematic biases in the measurements. However, recently in [55], it has been argued that incorporation of the dissipative effects in the energy-momentum tensor can ameliorate both the tensions simultaneously. Likewise in [56], it is claimed that the presence of viscosity, shear or bulk or combination of both, can alleviate both the tensions simultaneously.

In this thesis, the main focus is to study observational constraints on the dark sector of the Universe to explore nature/features of DM and DE beyond the standard model by constructing physically relevant generalized dark sector models. We also look for the possible alleviation of well-known and widely discussed H_0 and σ_8 tensions while analyzing our models. We study observational constraints on the dark sector with the recent observational data sets that we describe in the next chapter.