ELECTRICAL MEASUREMENTS IN THEORY AND APPLICATION

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ELECTRICAL MEASUREMENTS IN THEORY AND APPLICATION

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BY

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PREFACE

This book is a revision and enlargement of the *Principles of Electrical Measurements*, and is written for those who have passed one year of college physics and desire further knowledge regarding electrical and magnetic matters. It can be used as a guide in laboratory work, but it is more than a laboratory manual. The principles involved are amply treated, and the reader is led to reason out the relations between the various quantities rather than to memorize a formula. The student is urged to learn the facts from his own observations, and direct information is often replaced by a suggestion as to how the knowledge can be obtained.

The time has certainly come when the electron theory of electrical phenomena should be presented to all students of physics and electrical engineering. Regarding the electron tubes used in radio communication, for instance, there is no doubt that the stream of electrons through the tube continues as an electron current through the connecting wires. This point of view has been taken throughout the book and electrical currents are considered as the flow of electrons along the circuit. This is in accordance with the ideas of modern Physics, and it adds a concreteness to the subject that is especially helpful in the class room.

The book is arranged on the progressive system. The simpler and more fundamental parts of the subject are taken up in the first chapters, and in the first part of each chapter, while the more difficult measurements and the methods involving more extended knowledge are reserved until the student has attained greater proficiency. For example, Chapter I shows how to measure current, resistance, electromotive force, and power, by ammeter and voltmeter methods. For an elementary course in Electrical Measurements nothing could be better than this series of simple experiments, well understood. They bring out the fundamental relations with a minimum of apparatus to confuse the mind; and they are not out of place at the beginning of a more extended course that contemplates using the entire book.

The strict definitions that establish the magnitudes of the various electrical units are the condensed statements of a very intimate knowledge of the subject. They are introduced, therefore, after the reader has obtained a considerable acquaintance with the various electrical quantities through use and measurement.

Graphical methods have been introduced freely for the solution of inductance and capacitance bridges. They are used to supplement rather than replace the analytical methods, and they give a clear insight into the relations of the currents and electromotive forces in the various circuits. By placing these in a separate chapter they can be studied either before or after the other methods.

The Leeds & Northrup Co., the Weston Electrical Instrument Co., and the General Radio Co., have generously furnished material for some of the illustrations.

ARTHUR WHITMORE SMITH.

UNIVERSITY OF MICHIGAN
July, 1924.

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ELECTRICAL MEASUREMENTS IN THEORY AND APPLICATION

INTRODUCTION

UNITS AND DEFINITIONS

1. Electrons.—The atoms of all substances are complicated structures, consisting of electrons and protons. The hydrogen atom consists of one electron, which is the smallest electrical charge (negative) that has ever been found, and a nucleus of one proton, which likewise is the smallest positive charge that has been isolated. The atoms of other elements contain a larger number of electrons and protons. In certain kinds of atoms some of the electrons can circulate from atom to atom, while in other atoms each electron is confined rather closely to its own atom. A substance composed of atoms of the former kind is an electric conductor. If the atoms are of the latter kind, the substance is an insulator.

The electrons in a conductor are moving in every direction. When, in addition to this motion, there is a drift of the electrons along the conductor, this is called an "electron current."

2. Electronic Charge.—When a piece of glass is rubbed with silk some of the electrons are removed from the glass and are added to the silk. Just how this is done is not yet fully understood, but that does not alter the fact. As water poured upon the ground tends to return to the ocean, so the electrons tend to return to their former state of level equilibrium. The reason they do not do so at once is because it is almost impossible for them to move through silk or glass, whereas they can move readily through copper or other metals.

It has long been the custom to call this condition of the glass "positive" electrification, and the complementary condition of the silk "negative" electrification. In accordance with this established nomenclature, electrons are "negative."

Since an electron is negative, the addition of an electron to a neutral body will give it a negative charge. The addition of more electrons would increase this negative charge. The natural unit in which to express the magnitude of any charge would be the electron, but this is too small. The practical unit is defined in another way (see Art. 8).

The protons are not readily transferred from one body to another, and the usual method of obtaining a positive charge on a body is to remove some of its electrons. The body from which the electrons are taken will be left with a positive charge.

3. Electron Current.—Bodies in which the electrons can freely move from one part to another are called "electrical conductors." If electrons are added or removed at one point of a conductor there is a movement of the electrons in the conductor to reestablish equilibrium. This flow of the electrons constitutes an electron current.

When a plate of zinc is placed in a solution of zinc sulphate there is a strong tendency for zinc ions to go into solution, leaving electrons on the zinc plate. On the other hand, when a copper plate is placed in a copper sulphate solution there is a great tendency for electrons in the plate to join the copper ions to form atoms of copper. The direction of these effects is noted here without stopping to consider why they are so. When these two arrangements are brought together, as in the Daniell cell, and the two plates are connected by a wire, the electrons on the zinc plate flow along the wire to the copper plate, where they meet the copper that is being deposited. In the solution positive and negative ions travel between the electrodes. Thus there is a circulation of electrons through the circuit.

The number of electrons in a copper wire is many times greater than the number of atoms of copper. These electrons are in motion within the atoms, and a comparatively small drifting of a few of these electrons along the copper wire is sufficient to make an electron current. The wire becomes heated, and the space surrounding the wire becomes a magnetic field.

4. Electric Current.—The appearance of a magnetic field around a wire has been known for over a century (since Oersted's experiment, 1821) and has been ascribed to an electric current in the wire. The nature of this current was unknown but a given direction was assigned to it in accordance with the magnetic field surrounding it. The current was said to flow in the direction that a righthanded screw advances when turned in the positive direction of the magnetic field.

It is now known that a current is a drifting of electrons along the wire, and, since the electrons are negative in sign, they move in the opposite direction to that formerly assigned to the positive electric current.

In order to understand the language used in the older books, it is still useful to retain some of the old terms and to speak sometimes of the positive direction of an electric current. If the motion of the electrons is to be indicated, we can speak of the electron current without any confusion of terms. Since electrons are negative, either way of speaking results in the same meaning.

5. Unit of Current.—In practical measurements the electron current is measured in *international amperes.* It is not possible to preserve a current for a standard, so the unit is defined in terms of the amount of silver deposited by such a current under carefully specified conditions.

One international ampere is the amount of current that, flowing steadily through a solution of silver nitrate in water, will deposit silver at the rate of 0.00111800 *of a gram per second.*

This unit of current is used in electrical measurements, and ammeters are calibrated to read international amperes. It is the legal unit of current in the United States and many other countries. It is intended to be equal to the true ampere[1] defined from the C.G.S. unit of current, and it differs from the latter by only a few parts in 100,000. In all except the most precise measurements, the two units can be considered identical.[2]

6. Unit of Resistance.—The amount of current that flows from a given battery is determined by the resistance of the electric circuit. The unit of resistance that is used in electrical measurements is the *international ohm.* Standard ohms can be made and preserved in the form of mercury columns held in glass tubes.

An international ohm is the resistance of a uniform column of mercury at the temperature of melting ice, 106.300 *cm. in length, and weighing* 14.4521 *grams.*

Coils of wire which have been compared with such mercury ohms are very useful as secondary standards.

The international ohm is the legal standard of resistance in the United States and many other countries. It is as near to

[1] See Art. 230.

[2] One international ampere = 0.9999_7 (10^{-1} C.G.S.) ampere. See GLAZEBROOK, "Dictionary of Physics," vol. 2, p. 950.

the value of the ohm[1] derived from the C.G.S. unit of resistance as could be determined at the time of the London Conference in 1908. It is now known to be slightly larger,[2] but the difference does not affect ordinary measurements.

7. Unit of Electromotive Force.—Using Ohm's law, *an international volt is equal to the electric difference of potential between the two terminals of an international ohm when it is carrying one international ampere.*

Comparing this with the E.M.F. of a Weston standard cell, the latter is found to be 1.0183 international volts at 20° C. Knowing thus the value of its E.M.F., it is often more convenient to use a Weston cell and a known resistance for the measurement of a current in international amperes than to set up a silver voltameter.[3]

8. International Coulomb.—Unit quantity of electrons, for practical measurements, is the quantity flowing across a given cross section of an electric circuit each second when the current is constant and equal to one international ampere. This quantity is called one international coulomb.

9. International Watt.—The unit of power is the rate of expenditure of energy represented by one international ampere flowing through a difference of potential of one international volt. This amount of power is called one international watt.

10. International Joule.—The unit of energy is the international joule, which is the energy expended each second when the power is one international watt.

11. Common Usage.—Inasmuch as these international units are commonly used in most electrical measurements, it is not necessary to repeat the word "international" each time, and when the terms "ampere," "volt," "ohm," etc. are used, it is to be understood that the name refers to the value defined above, unless there is a distinct statement to the contrary.

[1] See Art. 235.

[2] One international ohm = 1.0005_2 (10^9 C.G.S.) ohms. See GLAZEBROOK, "Dictionary of Physics," vol. 2, p. 950.

[3] Because the international ohm is slightly too large, the international volt is also too large. One international volt = 1.0004_9 (10^8 C.G.S.) volts.

CHAPTER I

AMMETER AND VOLTMETER METHODS

12. Hot Wire Ammeter.—When an electron current flows through a wire, one of the most noticeable effects is that the wire becomes warmed. The direct result of this is an increase in the length of the wire. It thus becomes possible to measure the value of the current in terms of the change in length of such a wire, and some ammeters are constructed on this principle.

13. The Weston Ammeter.—In addition to heating the wire, the current produces a magnetic effect in the surrounding space. This is manifest by its action upon a magnetic needle near it, or by the force which the wire itself experiences when in the magnetic field of a magnet or another current. A current is not attracted by a magnet, but the wire carrying the current is urged *sidewise* across the magnetic flux (lines of force) near it.[1] From this it follows that a *loop* of wire carrying a current will tend to *turn* in a definite direction when it is in a magnetic field.

In the Weston ammeter the current passes through a coil of many turns of fine wire wound on a rectangular form. This coil is mounted on jeweled bearings and can turn in the magnetic field between the poles of a strong permanent magnet. It is held in a definite position by a spiral spring at either end. When a current is passed through the coil, the wires parallel to the axis are urged sidewise across the magnetic flux between the poles of the permanent magnet, and the coil turns until the torque of the springs is sufficient to balance the couple due to the forces between the current and the magnetic field. A pointer attached to the coil moves over the scale and indicates the angle turned through; the scale is not graduated in degrees, but in terms of the current required to produce the deflection. The scale is thus direct reading and gives the value of the current in amperes.

[1] This effect furnishes the basis for defining the value of unit current (see Art. 230), and the precise relation between the current and the force acting upon the wire is fully worked out in Chap. XI.

In ammeters for measuring large currents, a low-resistance shunt is placed in parallel with the moving coil to allow only a moderate current through the latter. The scale is then graduated to read the value of the large total current through both the coil and its shunt. This arrangement is entirely similar, in principle, to the voltmeter and shunt described in Art. 32, the moving coil system acting as a sensitive voltmeter.

14. The Weston Voltmeter.—The construction of a voltmeter is the same as an ammeter, except that, instead of having a shunt in parallel with the moving coil, there is a high resistance in series with the coil. The current which will then flow through the instrument depends upon the E.M.F. applied to the terminals of this resistance, and the numbers written on the scale are not the values of the currents in the coil, but are the corresponding values of the fall of potential over the resistance of the voltmeter. Therefore, it is sometimes said that a voltmeter is really only a sensitive ammeter measuring the current through a fixed high resistance; while an ammeter is really only a sensitive voltmeter measuring the fall of potential over a low resistance.

15. Laws of Electron Currents—Use of an Ammeter.—The Weston ammeter is a good and accurate instrument for the measurement of electron current. It is a very delicate and sensitive instrument and must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the instrument is not burned out thereby.

When it is desired to use the ammeter for the measurement of current it is connected in series with the rest of the circuit, and therefore the entire current passes through the instrument. Great care should always be exercised never to allow a larger current to flow through an ammeter than it is intended to carry. It is always best to have a key in the circuit and, while keeping the eye on the needle of the ammeter, tap the key gently, thus closing it for a fraction of a second only. If the needle does not move very far, the key can be held down for a longer time, and if it is then seen that the needle will remain on the scale, the key can be held down until the needle comes to rest. Back of the needle is a strip of mirror, and by placing the eye in such a position that the image of the needle is hidden by the needle itself, the error due to parallax in reading the scale can be avoided.

The scales of these instruments are graduated to read the current directly in amperes. Sometimes the pointer does not stand at the zero of the scale when no current is flowing. When this is the case, the position of rest should be carefully noted and the observed reading corrected accordingly.

For this exercise join a dry cell, a coil of several ohms resistance, a key, and the ammeter in series, that is, one after the other to form a single and continuous circuit.

The electrons flow from the negative, or zinc, pole of the cell out into the external circuit. In order to read properly, the ammeter should be connected into the circuit so that the electrons

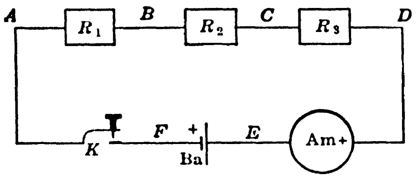


FIG. 1.—Resistances in series.

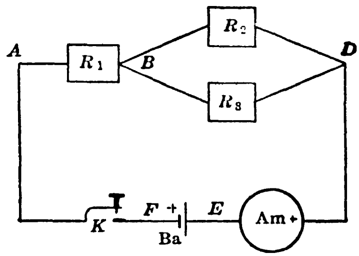


FIG. 2.—Resistances R_3 and R_2 in parallel.

will enter it at the post marked – and leave the ammeter at the post marked +. Measure and record the value of the current at different points along this circuit, to determine whether the current has the same value throughout its path or whether it is smaller after passing through the resistances. Next add the remaining coils to the circuit, keeping them all in series, and note the value of the current at the same points as before. State in your own words the effect of adding resistance to the circuit.

Remove one of the coils and connect it in parallel with one of those still remaining in the circuit, *i.e.*, so that the current in the main circuit will divide, a part going through each of the two coils in parallel. Measure the part of the current through each coil; also the main current. Could you have foretold the value of the latter without measuring it?

This exercise should show that the only thing in common to coils in *series* is the value of the *current* passing through them. For this reason an ammeter is always joined in series with the circuit in which the value of the current is desired. Record the readings of the ammeter in a neat tabular form similar to the following:

To Measure the Current along an Electric Circuit

Ammeter zero	Ammeter reading	Current	Point at which current is measured

16. Difference of Potential, E.M.F., Fall of Potential.—The difference of potential between two points is that difference in condition which produces an electron current from one point to the other as soon as they are connected by a conductor. Every battery or other electric generator possesses a certain power of maintaining a difference of potential between its terminals, and, therefore, a power of driving a continuous current. This difference of potential produced by a cell or other generator, and which may be considered as the cause of the current, is called "electromotive force." It must be remembered that this quantity is not a force at all, and, in order to avoid using the word "force," it is commonly called "E.M.F."

When a current flows through a conductor, there is a difference of potential between any two points on the conductor. This difference of potential is greater the further apart the points are taken, and as the change is gradual, it is usually called a "fall of potential." It might also be called a "rise of potential," since, when *following* the electron current through a resistance, one is lead to points that are more strongly + in potential. It can always be expressed by the formula RI, where R is the resistance of the conductor, or conductors, under consideration.

This apparent duplication of names may at first appear unnecessary, but the corresponding ideas are quite distinct and the correct use of the proper term will add conciseness to one's thinking and speaking. Thus we have the E.M.F. of a battery; the fall of potential along a conductor; and the more general and broader term, difference of potential, which includes both of the above as well as some others for which no special names are used.

17. Fall of Potential in an Electrical Circuit. *Use of a Voltmeter.* The Weston voltmeter is a good and accurate instrument for the measurement of a fall of potential. It is a very delicate and sensitive instrument and must always be handled with care. Mechanical shocks or jars will injure the jeweled bearings, and too large a current through it will wrench the movable coil and bend the delicate pointer, even if the E.M.F. is not much larger than that intended to be measured by the instrument.

Since a voltmeter is always used to measure the difference of potential between two points, it is not put into the circuit like an ammeter, but the two binding posts of the voltmeter are connected directly to the two points whose difference of potential is desired. The voltmeter thus forms a shunt circuit between the two points. There is a current through the voltmeter proportional to the difference of potential between the two points to which it is joined. This current passing through the movable coil of the instrument deflects the pointer over the scale, but the latter is graduated to read, not the current, but the number of volts between the two binding posts of the voltmeter. In some instruments there is a strip of mirror placed below the needle. When the eye is so placed that the image of the needle is hidden by the needle itself, the reading can be taken without the error due to parallax. Sometimes the pointer does not stand at the zero of the scale when there is no current through the instrument. When this is the case, the position of rest should be carefully noted and the observed reading corrected accordingly.

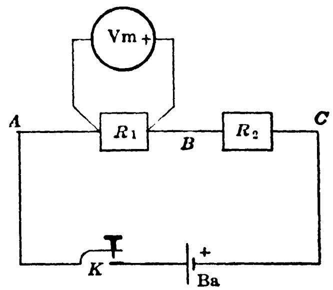


FIG. 3.—Connections for a voltmeter.

For this exercise join a cell, two coils of several ohms resistance, and a key, in series. With the voltmeter measure the fall of potential over each coil, also over both together. Add a third coil and measure the fall of potential over each; also over all. Note where these last readings are the same as before and where they are different. Add a second cell and repeat the above readings. Note changes.

Join two of the coils in parallel, thus forming a divided circuit and allowing a part of the current to flow through each branch (see Fig. 2). Measure the fall of potential over each branch. Add a third coil in parallel with the other two and again measure the fall of potential over each.

This exercise should show, especially in connection with the preceding one, that the only thing common to several circuits in *parallel* is that each one has the same *fall of potential.* Hence a voltmeter is always joined in parallel with the coil, the fall of potential over which is desired. The voltmeter indicates the fall of potential over itself; and if it forms one of the parallel circuits, the fall of potential over each one is the same as that indicated by the voltmeter.

Record the voltmeter readings as below:

FALL OF POTENTIAL IN AN ELECTRIC CIRCUIT

Voltmeter		Fall of potential	Position of voltmeter
Zero	Reading		

18. Ohm's Law.—The electron current that flows through any conductor is directly proportional to the potential difference between its terminals. This statement was first formulated in 1827 by Dr. Ohm, as the result of many experiments and measurements, and it is known as Ohm's law. It is usually written

$$V = RI \qquad \text{or} \qquad I = \frac{V}{R},$$

where V denotes the potential difference over the circuit through which is flowing the current, I. The factor R is called the resistance of the conductor, and its value depends only upon the dimensions and material of the wire and its temperature. It is entirely independent of V and I.

This relation holds equally well whether the entire circuit is considered or whether only a portion of such circuit is taken. In the former case the law states that the current through the circuit is equal to the total E.M.F. in the circuit divided by the resistance of the entire circuit, including that of the battery and the connecting wires. When applied to a single conductor, *AB*, the law states that the current flowing through the conductor is equal to the fall of potential between *A* and *B* divided by the resistance, *AB*.

To determine the resistance of a conductor, it is then only necessary to measure with an ammeter the current flowing through it, and with a voltmeter measure the difference of potential between its terminals. In case the current is at all variable, the two instruments must be read at the same time, for Ohm's law applies only to simultaneous values of the current and voltage.

19. Measurement of Resistance by Ammeter and Voltmeter. *First Method.*—Join the conductor whose resistance, *R*, is to be measured, in series with an ammeter, *Am*, a key, a battery, and sufficient auxiliary resistance to keep the current from being too large. The electron current should leave the ammeter at the post marked +. Keeping the eye fixed on the needle of the ammeter, close the key for a fraction of a second. If the deflection is in the right direction, and is not too large, the key can be closed again and the value of the current read from the scale of the ammeter. Should the current be too large, the auxiliary resistance can be increased until the current is reduced to the desired value.

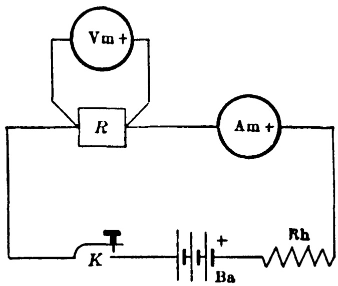


FIG. 4.—To measure a small resistance, *R*.

To measure the fall of potential over the conductor, its two terminals are joined to the binding posts of the voltmeter, by means of additional wires. That terminal of the resistance at which the electron current leaves should be joined to the voltmeter post marked +. Close the key for an instant as before, keeping the eye on the voltmeter needle, and if the deflection is in the right direction and not too large, the reading of the voltmeter can be taken.

Now close the key and record simultaneous readings of the ammeter and the voltmeter. Do this several times, changing the current slightly by means of the auxiliary resistance before each set of readings. Compute the resistance of *R* from each set of readings by means of the formula

$$R = \frac{V}{I},$$

where V and I are the voltmeter and ammeter readings, corrected for the zero readings. The mean of these results will be the approximate value of the resistance.

A more exact value of R can be obtained by correcting the current as measured by the ammeter for the small current, i, which flows through the voltmeter. The current through R is, strictly, not I, but $I - i$. This gives then,

$$R = \frac{V}{I - i}.$$

The value of the current, i, through the voltmeter can be computed. If the resistance of the voltmeter is S ohms,

$$i = \frac{V}{S}.$$

20. Measurement of Resistance by Ammeter and Voltmeter. *Second Method.*—This method differs from the first method by the position of the voltmeter. In the first method the ammeter measured both the current through R and the small current through the voltmeter, and therefore its readings were somewhat too high.

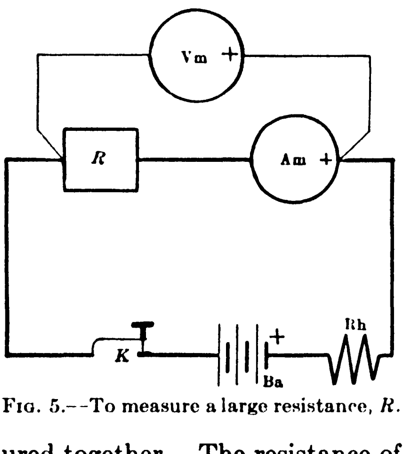


Fig. 5.—To measure a large resistance, R.

If the connections are made as shown in the figure, this error is avoided, as the current now passing through R is strictly the same as that measured by the ammeter. The voltmeter, however, now measures the fall of potential over both R and the ammeter, and therefore the resistances of both are measured together. The resistance of R is then found by subtracting the resistance of the ammeter from the measured amount. The formula then becomes

$$R = \frac{V}{I} - A,$$

where A is the resistance of the ammeter.

This correction is easily made and therefore this is the preferable method, except for very small resistances.

Measure the resistance of two coils and check results by also measuring them in series and in parallel. The measured resistances should be compared with the values computed from the formula—for series, $R = R_1 + R_2$, and for parallel,

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Record the readings as follows:

To Measure the Resistance of..........

Coil measured	Ammeter reading	I	Voltmeter reading	V	Resistance	
					$\frac{V}{I}$	Corrected

21. Graphical Solution for Resistances in Parallel.—The equivalent resistance of two resistances in parallel is easily determined by a simple geometrical construction. Let two lines, a and b, be drawn at right angles to a base line, PQ, and at a convenient distance, m, apart. Let the lengths of these lines represent the two resistances on a convenient scale. Join the top of each line to the bottom of the other, as shown by the dotted lines. From the intersection of these diagonals draw the line c, also perpendicular to the base line. The length of c gives the equivalent resistance of the parallel circuits on the same scale that was used in drawing a and b.

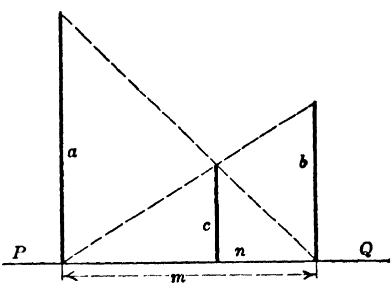


Fig. 6.—The resistance of a and b in parallel is shown by c.

This is easily shown. From similar triangles,

$$\frac{c}{a} = \frac{n}{m}$$

and also

$$\frac{c}{b} = \frac{m - n}{m} = 1 - \frac{n}{m} = 1 - \frac{c}{a},$$

from which

$$c = \frac{ab}{a + b}.$$

This diagram will thus show quickly the effect of connecting any two resistances in parallel.

22. Comparison of the Two Methods.—In each of the preceding methods for measuring resistance by means of an ammeter and a voltmeter, it is necessary to apply a correction to the observed readings in order to obtain the true value of the resistance being measured. It is the object of this section to inquire under what conditions these corrections are a minimum. In order to compare the two correction terms with each other, they will both be expressed in the form of factors by which the observed values are multiplied to obtain the true values.

In the first method the correction is applied to the ammeter reading, the true current through R being $I' - i$. The true value of the resistance is, then,

$$R = \frac{V'}{I' - i} = \frac{V'}{I' - \frac{V'}{S}} = \frac{V'}{I'\left(1 - \frac{V'}{I'S}\right)} = \frac{R'}{1 - \frac{R'}{S}},$$

where S is the resistance of the voltmeter, and R' is written for $\frac{V'}{I'}$, the uncorrected value of R.

In the second method the value of R is given by

$$R = \frac{V''}{I''} - A = R'' - A = R''\left(1 - \frac{A}{R''}\right),$$

where A is the resistance of the ammeter, and R'' is written for the uncorrected value of R in this method.

In the first method the correction factor is nearly unity when the resistance being measured is small. Other things being equal, a voltmeter having a high resistance, S, is better than a low-resistance instrument. In the second method the correction factor is near unity when the resistance being measured is large. The smaller the resistance of the ammeter the better. Either method, thus corrected, should give the correct resistance of R.

23. To Find the Best Arrangement for Measuring Resistance with an Ammeter and a Voltmeter.—The division point between large and small resistances is not evident. In fact, it depends upon the resistances of the ammeter and the voltmeter that are being used. The corrections differ from unity by $\frac{R'}{S}$ and $\frac{A}{R''}$, respectively, and at this division point neither of the corrections is very large, and it is nearly true to write

$$R' = R'' = R$$

and

$$\frac{R}{S} = \frac{A}{R}.$$

The division point is, then, at

$$R = \sqrt{AS}.$$

For resistances smaller than this, the first method has the smaller correction. For larger resistances, the second method

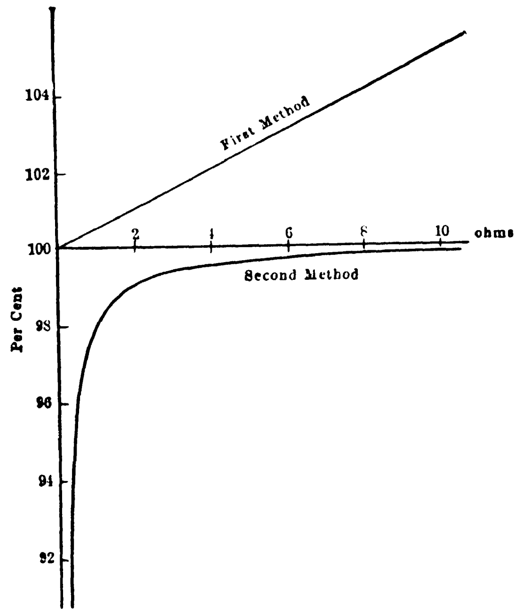


FIG. 7.—Showing the errors in the uncorrected values given by the two methods of measuring resistance.

should be used. Whether the correction is applied or not, it should always be made as small as possible. If the proper method is selected, the correction will almost never be as large as 1 per cent.

The curves in Fig. 7 show the factor by which R', or R'', must be multiplied to give the true resistance when using a

200-ohm voltmeter and a 0.02-ohm ammeter. It is seen that with a resistance of 2 ohms the corrections are equally large for each method, one giving uncorrected results that are 1 per cent too high, and the uncorrected results of the other being 1 per cent too low.

In measurements of this kind a larger error may be introduced by neglecting the temperature of the wire that is being measured. The resistance of a copper wire increases about 2 per cent for each 5° C. rise in temperature, and, unless the temperature is known more closely than 2°, there is not much use in measuring the resistance closer than 1 per cent.

24. Internal Resistance of a Battery.—The internal resistance of a battery can be measured by the ammeter and voltmeter method as readily as any other resistance. Let r denote the resistance within the cell, and R the resistance of the external circuit. The current, I, that will flow through this circuit, when the key is closed, is

$$I = \frac{E}{R + r},$$

where E is the E.M.F. of the battery.

Solving this for r gives

$$r = \frac{E - IR}{I} = \frac{E - E'}{I}, \tag{A}$$

where E' is written for RI, the fall of potential over the external circuit.

If the value of R is known, the ammeter can be omitted, and the value of the current determined from the relation $I = \frac{E'}{R}$. Equation (A) then becomes

$$r = R\frac{E - E'}{E'}. \tag{B}$$

E' can be measured with a voltmeter. The determination of E is not so simple, but an approximate value can be obtained when the voltmeter is connected directly to the cell, and no other current is flowing through the battery. This reading of the voltmeter will not be much less than E if r is smaller than 1 ohm. When r is larger than this, the following method is better.

25. Measurement of a Resistance Containing an E.M.F.—The resistance of a circuit in which there is also an E.M.F. can be measured without knowing the value of this E.M.F. This problem might be the measurement of the internal resistance of a battery, as in the preceding article, or it might be the determination of the resistance of a metallic circuit containing one or more E.M.F's. If sufficient current can be drawn from the circuit, no other battery is required.

The cell, or resistance to be measured, is joined in series with a variable resistance, a key, and an ammeter, as shown in Fig. 8. A voltmeter is joined in parallel with the resistance and key. When the key is closed, the current drawn from the cell is

$$I = \frac{E}{R' + r'},$$

where R' is the combined resistance of R and the voltmeter in parallel, and r' includes not only the resistance of the battery but also that of the ammeter. Clearing of fractions,

$$E = R'I + r'I.$$

The voltmeter measures $R'I$, and, denoting this by E',

$$E = E' + r'I. \qquad \text{(A)}$$

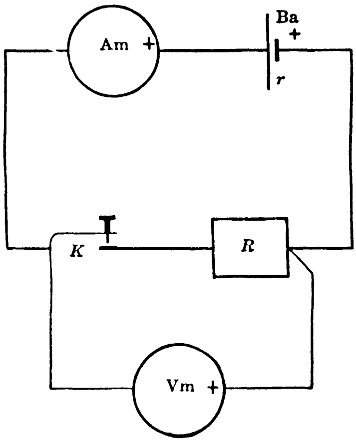


FIG. 8.—To measure the internal resistance of the cell, *Ba*.

When the key is open, there is a small current flowing through the ammeter and the voltmeter. The value of this current is

$$i = \frac{E}{S + r'},$$

where S denotes the resistance of the voltmeter. This relation may be written

$$E = Si + r'i.$$

The voltmeter measures the part Si, which is the fall of potential over its own resistance. The part $r'i$ is the fall of potential in the cell and the ammeter, and is negligible only when r' is very small. If V is this voltmeter reading

$$E = V + r'i. \qquad \text{(B)}$$

Equating the two values of E in Eqs. (A) and (B) gives

$$V + r'i = E' + r'I,$$

from which

$$r' = \frac{V - E'}{I - i}. \quad \text{(C)}$$

Thus it is not necessary to measure the E.M.F. of the battery. E' and I are simultaneous readings of the voltmeter and the ammeter with the key closed. V and i are the corresponding readings taken as soon as possible after the key is opened. It is not necessary to correct these readings for the zero error of the instruments, since only differences are used in Eq. (C). Stated in words, this result is

$$Resistance = \frac{Change\ in\ Potential\ Difference}{Change\ in\ Current},$$

which is another form for Ohm's law.

Since the resistance, A, of the ammeter was included in r', it must now be subtracted from the computed result to obtain the resistance, r, of the battery alone. Thus,

$$r = r' - A.$$

The observations can be recorded as follows:

INTERNAL RESISTANCE OF................CELL

Am. zero = Vm. zero = Am. resistance =

Kind of cell	R	Key closed		Key open			
		E'	I	V	i	r'	r

26. Relation between Available E.M.F. and Current.—Ohm's law, when applied to a complete circuit, gives the relation

$$I = \frac{E}{R + r},$$

or,

$$E = RI + Ir = E' + Ir. \quad \text{(A)}$$

The term Ir is the fall of potential over the internal resistance of the cell. RI is the fall of potential over the entire external part of the circuit, and is often denoted by the single symbol E'. Various names have been applied to this term E', such as "terminal E.M.F.," "terminal potential difference," "pole potential," "available E.M.F.," etc. It is that part of the total E.M.F. of the cell that is available for doing useful work. Its value, from Eq. (A), is

$$E' = E - Ir, \tag{B}$$

and from this it appears that the available E.M.F. is less as the current becomes larger.

The battery, or E.M.F. to be examined, is joined to a variable resistance, R, and an ammeter in series. By using various values of R the current can be varied throughout its possible range, the values being read from the ammeter. A voltmeter joined to the poles of the battery gives the corresponding values of E'. The key should be kept closed as little as possible to avoid unnecessary polarization of the battery.

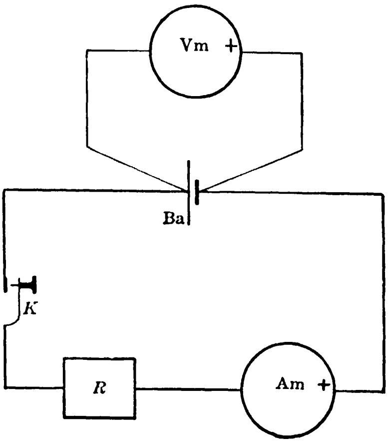


FIG. 9.—To measure the available E.M.F. corresponding to the current drawn from the battery, *Ba*.

The observations can be recorded as follows:

RELATION BETWEEN AVAILABLE E.M.F. AND CURRENT FOR A . . . . CELL

R	Ammeter		I	Voltmeter		E'
	Zero	Reading		Zero	Reading	

From the values of E' and I plot a curve, as in Fig. 10. The currents should have been chosen so that the plotted points will be well distributed along this curve. The curve shows that the available E.M.F. continually decreases as more current is drawn from the source, and it will decrease to zero if the current is sufficiently increased. This is true whether the current is supplied by a battery, a dynamo, or any other source, but it is not always safe to allow this current to flow if its value would be too large.

For the sake of comparison, curves should be drawn for different types of cells using at least one cell of low internal resistance. Determine from the curves the maximum current which each cell can furnish. Find one reason why an ammeter should not be joined to the poles of a cell, as is done with a voltmeter.

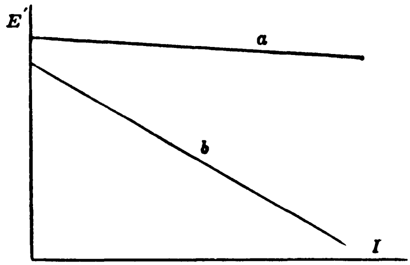


FIG. 10.—Relation between the available E.M.F and the current drawn from *a*, a low-resistance battery, and *b*, a high-resistance battery.

27. Maximum Current from a Battery.—The current from a given battery depends upon the resistance in the circuit, and by Ohm's law is

$$I = \frac{E}{R + r}.$$

The smaller the resistance of the circuit, the larger the current. The limit comes when the external resistance R has been reduced to zero, and then

$$I = \frac{E}{r}$$

is the maximum current that can be drawn from the battery.

With a storage battery, or a dynamo, where r is very small, this maximum current may be dangerously large. No attempt should ever be made to measure this current without first knowing approximately what its value may be, or else starting with considerable resistance, R, in the external circuit and cautiously reducing it while the ammeter is watched as the current increases. A single storage cell with internal resistance $r = 0.001$ ohm will supply over 1,000 amperes on short circuit. If the ammeter is designed to measure as large a current as this, it will not be injured. But it would be rather hard on the cell, and ruinous to an ammeter of smaller range.

28. Useful Power from a Cell.—Electrical power is measured by the product EI, where I denotes the current and E the fall of potential over the circuit in which the power is being expended. The unit of power is the watt, one watt being the product of one volt by one ampere.

When a battery is furnishing a current, the total power expended is supplied by the chemical reactions within the cell. Part of this power is expended in the external circuit where it may be used in running motors or doing other useful work. The remainder is spent within the cell and only goes to warming the contents of the battery. In some cases the greater part of the energy is thus wasted within the cell.

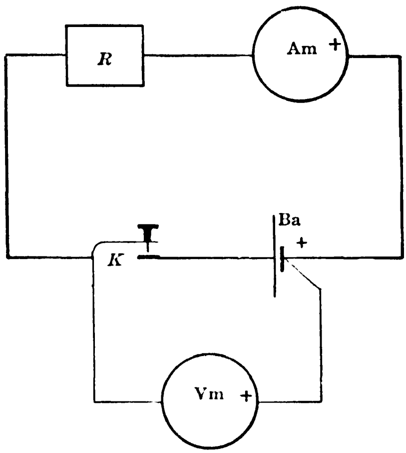


FIG. 11. Measurement of power.

The object of this experiment is to measure the power in the external circuit when various currents are flowing. The cell is joined in series with an ammeter and a resistance which will carry the largest current that may be used. A voltmeter measures the fall of potential. Probably there will be found a point beyond which the useful power decreases even though the current is made larger. Since the current is proportional to the amount of chemicals used up in the cell, it will be well to express the results as a function of the current. This is best done by means of a curve, using values of the current for abscissæ and the corresponding values of power for the ordinates.

Plot also the curve showing the total power, EI, supplied by the cell. The ratio of the ordinates of these two curves, that is, the ratio of the useful power to the total power, gives the efficiency of the cell.

The efficiency can also be plotted as a curve on the same sheet with the other two curves.

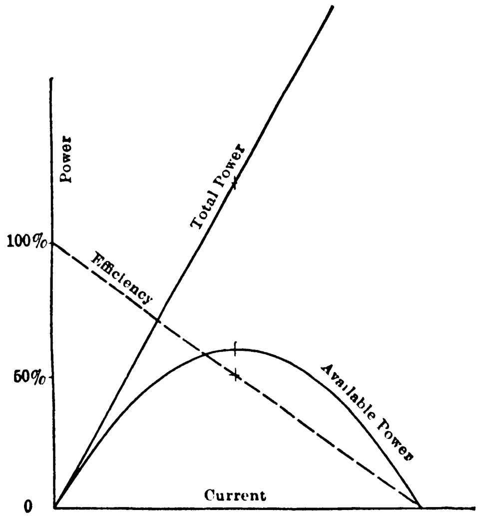


Fig. 12.—Power from a battery.

Record the observations as follows:

Useful Power from a . . . . Cell

R	Ammeter		I	Voltmeter		E'	W = E'I
	Zero	Reading		Zero	Reading		

Problem.—How much power is expended in an ammeter of 0.01-ohm resistance if it is connected to a storage battery of 10 volts and 0.01-ohm internal resistance? Why is it best not to try the actual experiment in the laboratory?

29. Maximum Amount of Available Power.—The amount of available power corresponding to a given current, OC, is shown graphically in Fig. 13 by the area of the rectangle $OABC$, whose sides are corresponding values of E' and I. When I is small, the area of this rectangle is small, increasing as I increases. After a certain point, E' decreases faster than I increases, and the rectangle decreases to zero. Plotting the areas of the rectangles ($=W$) against the corresponding values of the current gives the curve of available power, Fig. 12.

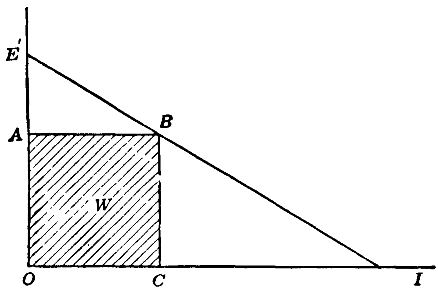


FIG. 13.—Showing the amount of available power W corresponding to a given current ($= OC$).

At the point where the power, W, is a maximum, the curve between W and I becomes parallel to the I-axis, and in the notation of the calculus,

$$\frac{dW}{dI} = 0.$$

The relation between W and I is

$$W = E'I,$$

where both E' and I are variables. Substituting for E' its value in terms of I from Eq. (B), Art. 26, gives

$$W = EI - I^2r,$$

and the derivative of this is

$$\frac{dW}{dI} = E - 2Ir.$$

For a maximum value of W, this expression is equal to zero, which is the case when the current is

$$I = \frac{E}{2r}.$$

When the current has this value, the battery is delivering more power than when the current is larger or smaller. Since, for this current, the resistance of the entire circuit is $2r$, and that of the battery is r, it follows that the resistance of the external circuit is also equal to r. Therefore the power *delivered* by the cell is a maximum when the external resistance is equal to the internal resistance. In the case of power from a distant source delivered over a long line, all of the resistance up to the point where the power is delivered should be counted as "internal" resistance.

30. Why a Voltmeter Does Not Measure the Full E.M.F. of a Battery.—When a cell, B, is joined to a voltmeter, the reading of the latter strictly does not give the E.M.F. of the cell. This discrepancy is small for cells of low internal resistance, but in many cases it cannot be neglected. If the resistance of the voltmeter is S, and that of the cell is r, then by Ohm's law the small current through the cell and voltmeter is

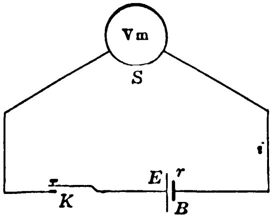


FIG. 14.—A voltmeter does not measure the full E.M.F. of a battery.

$$i = \frac{E}{S + r}.$$

From this the E.M.F. of the cell is

$$E = Si + ri,$$

where Si is the fall of potential over the external circuit, in this case the voltmeter. Since a voltmeter measures only the fall of potential over its own resistance, the reading of the voltmeter is not E, but Si, which is less than E by the amount ri. When r is small, this term can be neglected, but not otherwise, unless i can be made very small.

In case r is known, a correction may be added to the voltmeter reading to obtain the value of E. Writing the above equation in the form

$$E = Si\left(1 + \frac{r}{S}\right) = E' + \frac{r}{S}E'$$

it is seen that the last term can be added to the voltmeter reading, E', to obtain the full value of E.

31. How a Voltmeter Can Be Made to Read the Full E.M.F. of a Battery.—In this method no current is taken from the battery, and therefore its full E.M.F. can be measured. The battery, B, is connected to a voltmeter as in Fig. 15, and then a second circuit is formed, consisting of the voltmeter, another battery, Ba, larger than B, and a variable resistance, R.

Since there is no key in this second circuit, the voltmeter stands deflected and its reading, V, indicates the difference of potential between its terminals. When the key, K, is closed, the current through it may be either in one direction or in the opposite direction, or it may be zero, depending upon the value of V. If V is small, the current through K will be in the direction determined by the cell, B, and the voltmeter will carry the combined current, $I + i$. If the potential difference, V, is large, it will cause a current to flow through the cell, B, in opposition to its E.M.F., E. In this case the main current, I, from the battery, Ba, divides at the voltmeter and flows through the two branches in parallel, just as though B were not present, except that the current in the lower branch is smaller than it would be with B absent.

If i denotes the value of this reverse current through B, the voltmeter current is $I - i$. The change in the voltmeter current from I to $I + i$, or to $I - i$, when the key is closed, will cause a change in the voltmeter reading, and no other ammeter is needed to tell the direction of the current through B.

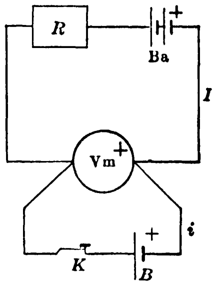


FIG. 15.—To measure the E.M.F. of B.

Since the current through B can be in either direction, there is one point when this current will be zero. This will be when the tendency for the current to flow through B and K, due to the potential difference, V, across the voltmeter, is just balanced by the tendency for the current to flow in the other direction due to E. For this case of no change in the voltmeter deflection when K is opened or closed, there is no current through B, and $E = V$. The value of E can now be read from the voltmeter.

Measure in this way the E.M.F. of several cells and compare the results obtained in each case with the readings of the voltmeter when used alone.

The readings may be recorded as follows:

Name of cell	Voltmeter readings		R
	Alone	With aux. battery	

32. Measurement of Current by a Voltmeter and Shunt.—When a current, I, flows through a resistance, R, the fall of potential over R is $E = IR$, which is in accordance with Ohm's law and the definition of the term "fall of potential." In Art. 19

both E and I were measured and the value of R was then computed. When R is known, the experiment can be reversed and by measuring E the value of I can be computed. Thus a voltmeter may be used to measure currents in place of an ammeter. The arrangement is shown in Fig. 16.

Usually the resistance that would be suitable for this purpose is less than an ohm. Therefore any variation in the position of the voltmeter connections at a and b would cause considerable change in the resistance between these points. It is best to have these connections soldered fast, and let the voltmeter connections be made at the auxiliary points a' and b', where a little resistance, more or less, in the voltmeter circuit will be inappreciable.

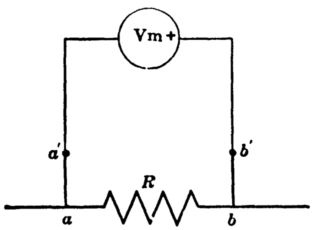


FIG. 16.—Measurement of current.

Such shunts are often made having a resistance of 0.1, 0.01, 0.001, or less, of an ohm. The current is then 10, 100, 1000, or more, times the voltmeter reading. This principle is also used in the construction of ammeters for measuring large currents. The greater part of the current is carried by a shunt of low resistance, while the delicate moving coil carries only a small current and thus in reality acts as a sensitive voltmeter. The numbers on the scale, however, instead of reading volts, are made to give the corresponding values of the currents passing through the instrument.

33. Potential Divider. It is often necessary to use only a part of the given E.M.F. This may be because a large E.M.F. is to be measured by a low-reading instrument, or it may be that a small fraction of a volt is required for use in some experiment. For this purpose, the source, E, is joined to a resistance, AB, which is large enough not to draw too much current from E, or to become overheated itself. C is a sliding contact that can be moved from

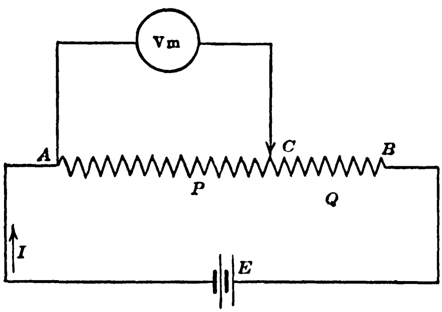


FIG. 17.—Showing the use of a potential divider, AB.

A to B, dividing the resistance in two parts, P and Q. The fall of potential between A and C is

$$V = PI = \frac{P}{P + Q} E.$$

The internal resistance of the battery is here considered negligible in comparison with the large resistance of $P + Q$. In case it is not negligible, it should be included as a part of Q, or else E should denote the fall of potential over AB.

A resistance, AB, which can be thus divided in various ratios is called a "potential divider." The E.M.F., E, is divided into two parts, PI and QI, and by moving C, PI can be made any desired fraction of E.

The same effect can be obtained by using two resistance boxes for P and Q. By changing the ratio of the resistances in P and Q, the fall of potential across AC can be varied.

34. To Use a Potential Divider.—Join the ends of a sliding rheostat to a constant E.M.F. Connect a voltmeter to the sliding contact and to one end of the rheostat. Observe how the reading of the voltmeter varies as the sliding contact is moved from one end to the other. Note that moving the sliding contact does not affect the amount of current that is drawn from the battery, as would be the case when the rheostat is used as a variable resistance.

In the same way the potential difference between A and C can be used wherever a fraction of the E.M.F. of E is desired.

35. Measurement of a High Resistance by a Voltmeter Alone.—This method is a modification of the ammeter and voltmeter method for the measurement of moderate resistances, and it is based on the fact that a voltmeter is really a very sensitive ammeter. It can be used as an ammeter whenever the high resistance in series with the moving coil will not interfere with the desired measurement of current.

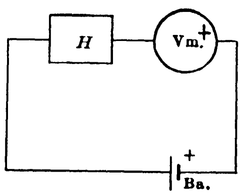


FIG. 18.—To measure the high resistance, H.

The voltmeter is joined in series with a battery and the high resistance, H, to be measured. In this way the voltmeter serves as an ammeter to measure the current through the circuit. The reading, V, of the voltmeter gives the fall of potential over its own resistance, S, or, in symbols,

$$V = Si.$$

Considering the entire circuit, the value of the current is given by the expression

$$i = \frac{E}{H + S}.$$

Eliminating i from these two equations and solving for H gives

$$H = S\frac{E - V}{V}.$$

The value of E, the E.M.F. of the battery, is easily measured by the same voltmeter by connecting it directly to the battery. A key arranged to short-circuit the high resistance will readily change the voltmeter from its position as an ammeter to that of a voltmeter.

When the E.M.F. of the source is too large to be measured by the voltmeter, or for any other reason cannot be determined, the resistance can still be measured. In the circuit shown in Fig. 18, the current is

$$i = \frac{V}{S} = \frac{E}{H + S}.$$

Now add a large known resistance, R, in series with H and the voltmeter. Then the current through the circuit will be

$$i' = \frac{V'}{S} = \frac{E}{H + S + R}.$$

Dividing one equation by the other gives

$$\frac{V}{V'} = \frac{H + S + R}{H + S},$$

and

$$H = \frac{V'(S + R) - VS}{V - V'}.$$

The observations can be recorded as follows:

TO MEASURE THE RESISTANCE OF..........

Vm. zero = Vm. resist. = ...ohms

Name of object measured	V	E	R

36. Time Test of a Cell.—The time test of a cell is designed to show how well the cell can maintain a current, how effective is the action of the depolarizer, and the rapidity and extent of the recovery of the cell when the current ceases. Such a test usually continues for an hour, and the results may best be shown graphically by means of curves. These curves should show the value of the E.M.F. of the cell at each instant during the test; the available E.M.F.; the current; and the internal resistance of the cell. After maintaining the current for an hour, the circuit is opened and the cell allowed to recover. The curves should show the rate and extent of this recovery.

Of course it is evident that such a continued test may prove rather severe for cells intended for only a few minutes' use at one time. On the other hand, cells which make a good showing under the test might not be the best for long-continued service. Nevertheless, such a test gives the most information regarding the behavior of the cell that can be obtained in the short space of 2 hours.

The set-up for making a time test is the same as that for measuring the internal resistance of the cell, save that the battery circuit remains closed all the time, except when it is opened for an instant to measure the E.M.F. of the cell. With the proper preparation beforehand it is not difficult to observe all the necessary data and record it in a neat and convenient form.

Readings should be taken once a minute Practice in doing this should be obtained by measuring the E.M.F. of the cell before any current is drawn from it. When ready to commence the test proper, the circuit is closed through 5 or 10 ohms, or about as much resistance as the internal resistance of the cell, and as soon thereafter as possible the first reading for the available E.M.F. is taken. One minute later the circuit is opened just long enough to take a reading of the E.M.F. of the cell. Thus every 2 minutes the available E.M.F. is recorded and on the intermediate minutes the value of the total E.M.F. is measured.

From these data two curves are plotted—one showing the variation in the E.M.F. of the cell during the test, and the other showing the same with respect to the available E.M.F. The internal resistance is computed at 5-minute intervals, the values of E and E' being taken from the curves. The current is computed from the values of R and the available E.M.F.

After the first hour of the test the battery key is changed so as to keep the circuit open, except for an instant each minute when it is closed long enough to read the voltmeter. Since the voltmeter draws some current from the cell, and as it is sometimes difficult to obtain simultaneous readings of the ammeter and voltmeter at the instant desired, more accurate results are possible when the measurements are made by the condenser and ballistic galvanometer methods described in the next chapter. The keys can be worked by hand and if proper care is exercised good results may be expected. Better results may be obtained by using a special battery testing key or a pendulum apparatus which will close and open the keys in precisely the same manner each time.

During the second hour the battery will recover from the effects of polarization, more or less completely, and at the end of the 2-hour test the value of E should be about the same as at the beginning. The recovery curve may be plotted backward across the sheet containing the other curves, thus showing very clearly the extent of the recovery.

The observations may be recorded as below.

Time Test of a........Cell

Time of day hour minute	E	E'	I	r

CHAPTER II

BALLISTIC GALVANOMETER AND CONDENSER METHODS

37. A Voltmeter Without Current.—The voltmeters discussed in Chap. I are essentially nothing but sensitive ammeters with a fixed resistance in series with the moving coil. Each one draws some current from the source whose E.M.F. is being measured; in most cases this is immaterial, but sometimes even a small current changes the E.M.F. that it is desired to measure.

In the present chapter we are to consider a voltmeter that draws no current, and therefore, is free from the last objection. In a sense, then, this will be more like a real voltmeter, but a more important consideration is that the condenser methods do not change the potential differences that are being measured. It will also be possible to measure values of E.M.F. that do not exist long enough to read on a voltmeter.

This chapter might appropriately be combined with Chap. X, but it is introduced here because the manipulation of these methods is very simple, and with a little precaution there is not much danger of injuring valuable apparatus through the inexperience of the investigator. Articles 188–194 should be read for a more complete understanding of the use of condensers

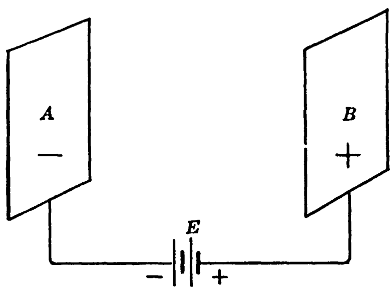


FIG. 19.—Electrons leave *B* and accumulate on *A* until the difference of potential has become as large as the E.M.F. of the battery.

38. Capacitance.—The use of this term has been growing in favor, and with this spelling it stands with resistance and inductance in expressing a property of a conductor. It means precisely the same as the older term "capacity," and it has the advantage that it does not suggest the misleading notion "all it can hold."

When two insulated conductors, A and B, Fig. 19, are connected together through a battery, a current of electrons will flow out of one conductor, through the battery, and into the other conductor until the difference of potential thus produced is sufficient to balance the E.M.F. of the battery. If the battery is increased, more electrons will flow from the first conductor to the other, and the only limit to the quantity of electrons that can thus be taken away from one conductor and added to another is the limit of the E.M.F. of the battery and the completeness of the insulation of the conductors.

When the conductors are near together, the electrons that are transferred from one conductor to the other are still so near to the place from which they started that, in effect, they partially neutralize the lack of electrons in the conductor they have left. This allows more electrons to be transferred before the E.M.F. of the battery is balanced.

An arrangement of conductors whereby a greater number of electrons can be transferred by a given E.M.F. than would be possible with the conductors separated is called a "condenser."

39. Condensers.—In order to make the capacitance as large as possible, condensers are constructed with broad sheets of tin foil placed as near as possible to other similar sheets. Actual contact is prevented by thin layers of mica, glass, or paraffined paper. Large capacitances are formed by building up alternate sheets of tin foil and dielectric, every other sheet of tin foil being connected to one terminal post, and the intermediate ones to the other terminal (Fig. 20).

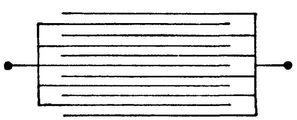


FIG. 20.—The tin foil plates of a condenser.

The best condensers and those intended for standards are made with thin sheets of mica as insulation between the sheets of tin foil. Very good condensers are made with paper insulation, the whole pressed firmly together and boiled in paraffine until all the air and moisture has been expelled, when the whole is allowed to solidify.

40. Meaning of a Charge "Q" in a Condenser.—A charge "Q" in a condenser means that there is a charge of $-Q$ coulombs on one set of plates, and an equal, though positive, charge of $+Q$ coulombs on the other set of plates. The discharge of the condenser consists of the electrons, $-Q$, flowing through the discharging circuit to the other set of plates and thus supplying the lack of electrons, $+Q$, thereon. The surplus of electrons on one conductor is sometimes called a "negative charge of electricity," and corresponding to this the other conductor, which has lost the electrons, is said to possess a "positive charge of electricity." As explained elsewhere, this flow of electrons is an electron current, and the direction of the electron flow is the negative direction of the electric current.

The positive charge, consisting of protons, is in the nuclei of the atoms and these cannot move without taking the atoms with them. There is no flow of atoms along a copper wire.

41. Unit Capacitance.—The larger the area of tin foil in a condenser and the nearer the sheets are to each other, the greater will be the number of electrons transferred by a given E.M.F., E. If the electrons are measured in coulombs,[1] the quantity, Q, transferred from one set of plates to the other is

$$Q = CE,$$

where C denotes the capacitance of the condenser expressed in units of such magnitude as will make this a true equation.

In the international units the unit of capacitance is the *international farad, which is the capacitance of a condenser that is charged to a potential difference of one international volt by one international coulomb.* The capacitance of a condenser, $C = \frac{Q}{E}$, is thus measured by the number of *coulombs per volt* required to charge it.

The farad is far too large a capacitance for ordinary use, and it is customary to express capacitances in terms of a smaller unit, the *microfarad*, which, as its name indicates, is one-millionth of a farad.

[1] The quantity of electricity represented by one electron is inconceivably small, yet its value has been accurately measured (Millikan, "The Electron," p. 119), and found to be

$$e = 15.92 \times 10^{-20} \text{ coulombs.}$$

If ordinary electrical charges were measured by counting the number of electrons that are thus collected together, the resulting numbers would be inconveniently large. If the electrons constituting a negative charge of one coulomb were placed in a row, 40,000 in each millimeter, the line would reach from the earth to the sun.

Therefore, instead of counting the individual electrons, we shall count 6.281×10^{18} of them at each clip and express the result in coulombs.

Problem.—A 3-microfarad paraffined paper condenser is about 1 ft. square and an inch in thickness. How large a pile of such condensers would have a capacitance of 1 farad? A 30 ft. cube?
How large a pile would it take to have a capacitance of 1 c.g.s. electromagnetic unit? A 6 mile cube?

Note.—The capacitance of a solid metal sphere of 1 cm. radius is one c.g.s. electrostatic unit, when it is far from all other conductors.

42. Ballistic Galvanometer.—A ballistic galvanometer is one in which the moving system, whether coil or magnet, is made comparatively heavy and massive so that it will swing slowly. Such a galvanometer is designed to measure, not steady currents like an ammeter, but transient currents which may exist for only a very small fraction of a second. Indeed, the duration is so short that it is customary not to speak of them at all as currents, but only to consider the total quantity of electrons that has passed.

Thus there are two reasons for having a slow-moving galvanometer. In the first place, it is most sensitive when in the position of rest and therefore should not turn appreciably from this position before all the electrons have been able to flow through the galvanometer and exert their full effect in turning the coil. In the second place, the coil gives one kick and then settles back to the position of rest again and the only thing that can be measured is the maximum deflection which it attains. Therefore it must move slowly enough to enable one to read the deflection at the end of its swing.

43. Damping of a Galvanometer. *Critical Damping.*—After the galvanometer has given its deflection, the very fact that the moving system is massive, and at the same time can move freely, which is essential for a good ballistic galvanometer, makes it very slow in coming to rest again. It will swing back and forth many times until its energy has been used up in friction against the air, and in other ways, when it will finally settle down at rest. That the swings decrease at all is due to the *damping* of the motion, as this effect of friction, etc., is called.

If the damping is increased, as would be the case if the coil were surrounded with oil, the swings would decrease more rapidly and the coil would quickly come to rest. Such damping might be so great that there would be no swings, and the coil would slowly creep back to the position of rest, possibly taking longer to do so than when it is allowed to swing freely.

Thus it is seen that there is some intermediate value of the damping that would allow the coil to swing back to rest not too slowly, and yet bring it to rest without its swinging to the other side. This value of the damping is called *critical damping*, and with this damping the coil is brought to rest in the minimum time.

The most convenient way to increase the damping of a ballistic galvanometer is to join its terminals by a key or a low resistance, as shown at S, Fig. 21. Due to the motion of the coil in a strong magnetic field, an induced current will flow through S, when closed, and the supply of energy in the coil is quickly dissipated as heat by the electric current in the wire. Often S is a resistance adjusted to such a value as to give critical damping. The galvanometer can then be used once a minute, or oftener if desired.

The effect of the shunt in reducing the deflections is discussed in the next chapter.

44. Use of Ballistic Galvanometer and Condenser.—When the poles of a battery are joined to the plates of a condenser, the latter becomes charged, as explained above. The amount of this charge depends upon the E.M.F., E, of the battery and the capacitance, C, of the condenser, being given by the relation

$$Q = CE$$

When the condenser is connected to a ballistic galvanometer, the charge, Q, in the condenser passes through the galvanometer and produces a deflection, d, the relation being,

$$Q = kd,$$

where k is the constant of the galvanometer.

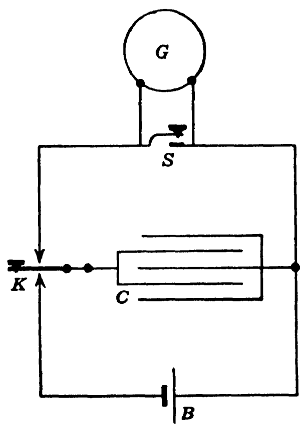


FIG. 21.—Use of a condenser.

If this operation has been carefully arranged so that there has been no leakage of the charge elsewhere, it is evident that the quantity that has been discharged through the galvanometer is the same quantity that was put into the condenser by the battery. That is,

$$CE = kd.$$

This is a very useful relation, since it can be used to compare the value of any one of the factors involved when the other three are known or can be measured.

The best arrangement for using a ballistic galvanometer with a condenser is shown in the figure, where G represents the galvanometer, C the condenser, with the battery at B. These are connected through the key as shown. *Note that the tongue of the key is connected to the condenser, and to the condenser only.* This precaution is necessary in order that by no possibility can the battery ever be joined directly to the galvanometer. Arranged as shown, when the key is depressed the condenser is joined to the battery and becomes charged. When the key is raised, the condenser is joined to the galvanometer and the charge passes through it, producing a deflection.

45. To Determine the Constant of a Ballistic Galvanometer.— For small deflections, the maximum swing is proportional to the quantity of electrons passed through the galvanometer, and the proportionality factor is called the constant of the galvanometer. It is determined by discharging through the galvanometer a known quantity and noting the resulting deflection. Then

$$Q = kd,$$

where k is the desired constant. Knowing k for a galvanometer, any other quantity of electrons can be measured by sending it through the galvanometer and noting the corresponding deflection.

The galvanometer, condenser, and battery are connected as shown in Fig. 21. The scale and telescope should be adjusted so that both the divisions and the numbers on the scale are distinctly seen in the telescope. The eyepiece must be focused on the cross-hair of the telescope, which should appear very clearly defined. When focusing the telescope upon the image of the scale as seen reflected from the mirror of the galvanometer, it must be remembered that the image is not on the surface of the mirror, but lies as far back of the mirror as the scale is in front of it.

After the set-up has been tried and found to work correctly with an old dry cell for B, a standard cell whose E.M.F. is much more exactly known may be substituted for it. Ten independent determinations of d should be made, and the mean value used in the computation for k. If the scale is movable, it will be necessary to measure the distance of the scale from the mirror of the galvanometer and compute the value of d for a standard distance of 1 meter.

The relation given in Art. 44 is

$$CE = kd.$$

In the present case the condenser used must be one of known capacitance. Likewise, E must be known as exactly as the value of k is desired. And d is measured. Hence the computed value of k is

$$k = \frac{CE}{d}.$$

If the value of C is given in microfarads, k will be expressed in *microcoulombs per scale division.* A microcoulomb is one-millionth of a coulomb and is the quantity of electrons that is represented by a current of one ampere flowing for one-millionth of a second.

The observations can be recorded as follows:

Galvanometer			Deflection for scale at 1 meter	C	E	k
Zero	Reading	Deflection				

46. Comparison of E.M.Fs. by the Condenser Method.—The arrangement of a condenser with a ballistic galvanometer may be conveniently used to measure the E.M.F. of a battery, or any other difference of potential. It thus serves as a voltmeter, and has an advantage over the ordinary voltmeter, in that it measures the total E.M.F. of the battery, no matter what the internal resistance of the latter may be.

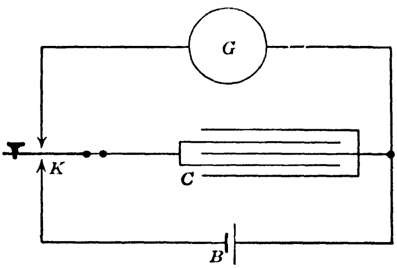


Fig. 22.—To measure the E.M.F. of B.

The set-up is arranged as shown in Fig. 22. When the key is depressed, the condenser is charged, and by raising the key it is discharged through the galvanometer. It is immaterial whether the key works this way or whether the condenser is charged when the key is up and is discharged by depressing the key. It is absolutely necessary, however, that the tongue of the key be joined to the condenser, as is shown in Fig. 22.

When the battery is connected to the condenser, a current flows until the difference of potential between the two sets of condenser plates is equal to the E.M.F. of the battery. When this current is flowing, the available E.M.F. (see Art. 26) is only

$$E' = E - Ir$$

but as the charge in the condenser increases, the current flowing through the battery becomes less and less, and when it reaches zero

$$E' = E.$$

The final difference of potential impressed on the condenser is equal, therefore, to the total E.M.F. of the battery, and thus this E.M.F. can be measured.

The final charge in the condenser is

$$Q = CE = kd.$$

Solving this for E gives

$$E = \frac{k}{C} d.$$

The factor $\frac{k}{C}$ can be determined by using a cell of known E.M.F., E_s, and observing the corresponding deflection, d_s. Then,

$$E_s = \frac{k}{C} d_s \qquad \text{or} \qquad \frac{k}{C} = \frac{E_s}{d_s},$$

so that, finally,

$$E = \frac{E_s}{d_s} d.$$

Having determined this coefficient of d once for all, the E.M.F. of any cell can be measured quickly and easily by observing the corresponding deflection of the galvanometer.

Inasmuch as the reading must be caught quickly at the end of the swing, it will be best to take several trials and use the mean deflection for computing the value of E. The readings may be recorded as follows:

Name of cell	Galvanometer			Mean deflection	$\frac{k}{C}$	E.M.F. of cell
	Zero	Reading	Deflection			

47. Comparison of Capacitances by Direct Deflection.—The same arrangement described above, and shown in Fig. 22, can be used equally well for the measurement of the capacitance of a condenser. It is only necessary to go through the experiment as before, and observe the galvanometer deflection with the first condenser, for which

$$CE = kd.$$

Now, replacing the condenser by another one, but using the same battery and everything else the same as before, the relation becomes

$$C'E = kd',$$

where d' is the galvanometer deflection when the condenser of capacitance C' is used. Dividing the second equation by the first gives

$$C' = C\frac{d'}{d}.$$

If C is a known capacitance then the value of C' can be determined as exactly as the flings d and d' can be measured. Each of these should be taken several times, and the mean values used in the computation.

48. Internal Resistance of a Battery by the Condenser Method. The condenser method offers a convenient and elegant means for determining the internal resistance of a cell, the principal advantage being the wide range of resistances that can be measured, and the short time that the cell must be in use. In the ammeter-voltmeter method a considerable current must oftentimes be drawn from the cell and for a period long enough to read both instruments. Such readings seldom can be repeated, for, owing to polarization, the cell does not return to its original condition.

The set-up for using the condenser method is shown in the figure. When K_2 is closed, a current flows through R and the cell, the value of which is

$$I = \frac{E}{R + r},$$

where r is the internal resistance of the battery. This gives

$$r = R\frac{E - E'}{E'}, \tag{A}$$

where E' is written for RI, the external fall of potential. If R is known, it only remains to measure E and E'.

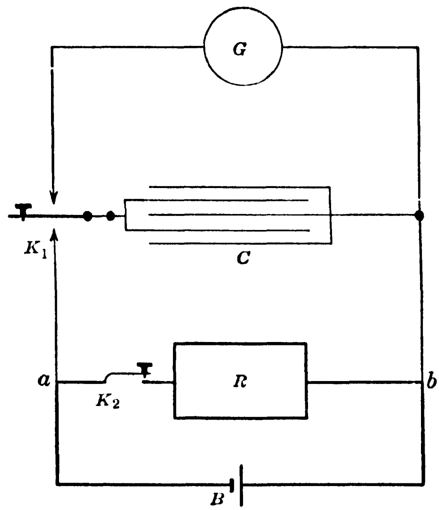


Fig. 23.—To measure the resistance of B.

When K_1 is depressed, the condenser is charged to the E.M.F., E, of the cell. This E.M.F. is measured by the deflection of the galvanometer when K_1 is raised, the relation being

$$E = \frac{k}{C}d.$$

When a current is drawn from the cell, the potential difference between a and b is

$$E' = E - Ir = RI.$$

If the condenser is now charged from this potential difference, the relation is

$$E' = \frac{k}{C}d'.$$

Substituting these expressions in Eq. (A) gives

$$r = R\frac{d - d'}{d'}. \tag{B}$$

It is best to use a value of R that will make d' about half as large as d; thus neither d' nor $d - d'$ need be too small for accurate measurement.

When using this method it is necessary to keep K_2 closed only long enough to depress and raise K_1. With skill, this interval can be reduced to less than a second when the keys are worked by hand. If conditions require that the current be drawn from the cell for a shorter time than this, a special testing key can be used.

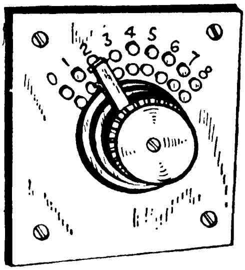


FIG. 24.

49. Special Battery Testing Switch.—A useful switch for making different connections quickly is shown in Fig. 24. In the diagram of Fig. 25 this switch is shown with the various contact points joined to the battery, galvanometer, condenser, and resistance. When thus connected it takes the place of K_1 and K_2 in the arrangement of Fig. 23. By turning the brushes over

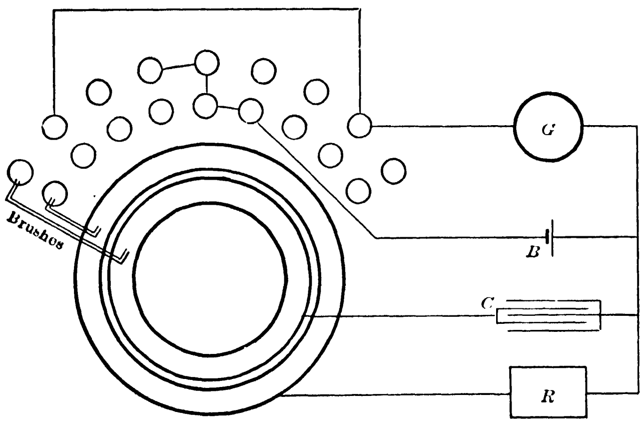


FIG. 25.—Diagram for the rotary switch and connections for measuring internal resistance.

the contacts in a clockwise direction, the galvanometer gives the deflection, d'. When the brushes move over the same contacts, but in the opposite direction, the galvanometer gives d. Current is drawn from the battery only while the brushes are moving over the central contacts, and measurements can be duplicated more easily than when separate keys are used. When many measurements are required, the ease and rapidity of obtaining the readings is a desirable feature of this arrangement.

50. Measurement of Insulation Resistance.—A ballistic galvanometer can be used to advantage in the measurement of extremely high resistances. Usually such resistances need not be determined with great accuracy, but the order of magnitude is required. Resistances up to a few megohms, for example, the insulation of the electric wiring of a building from the water pipes, can be measured by a voltmeter, as explained in Art. 35. Larger resistances, such as the insulation of the coils from the frame of an electric motor, can be measured by the ammeter-voltmeter method (Art. 20), using a sensitive galvanometer in place of the ammeter.

But the current through the highest resistances, such as the insulation of electric cables, or the resistance between the plates of a good condenser, is too small to be observed, even with a sensitive galvanometer. True, its value can be computed, but it is usually called a "leakage" current. It is then necessary to use one of the following methods.

51. Insulation Resistance by Leakage.—This method is used when the resistance to be measured is so large that the current which it is possible to pass through it is too small to be measured by a sensitive galvanometer. The method consists, in brief, in letting the current flow into a condenser for a sufficient time, and then discharging the accumulated quantity through a ballistic galvanometer.

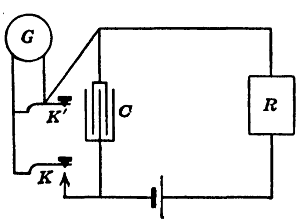


FIG. 26.—To measure a resistance of many megohms.

The set-up is arranged as shown in Fig. 26, where R denotes the large resistance to be measured. A battery of sufficient E.M.F., E, supplies the current which flows through R and gradually charges the condenser, C. When a sufficient charge is accumulated it is discharged through the galvanometer by closing the key, K. The other key, K', is for damping the swings of the galvanometer and bringing it to rest, and it should be kept closed all the time that the galvanometer is not being observed through the telescope.

At any instant during the time the current is flowing, the fall of potential over R is

$$RI = E - V, \tag{1}$$

where V is the difference of potential across the condenser.

In this method it is assumed that the condenser has considerable capacitance, and that the charging is discontinued before V has reached an appreciable part of E. If the experiment is not worked in this way, the following discussion does not apply.

At the start, and as long as V can be neglected in comparison with E, the current through R is, from (1),

$$I = \frac{E}{R}. \tag{2}$$

If this current flows into the condenser for t seconds, the accumulated charge is

$$Q = It, \tag{3}$$

and when the condenser is discharged through the galvanometer, there is a deflection, or fling, of d scale divisions, such that

$$Q = kd. \tag{4}$$

Thus the current is

$$I = \frac{kd}{t},$$

and, from (2),

$$R = \frac{E}{I} = \frac{tE}{kd}.$$

Determination of the Constant.—The "constant," k, of the galvanometer can be determined by the method described in Art. 45, giving

$$k = \frac{E'C'}{d'},$$

where E', C', and d' denote the particular values used in this determination of the constant.

If the same battery is used in finding the constant as in the experiment proper, $E = E'$, and the absolute value of the E.M.F. employed does not enter into the computation. Then

$$R = \frac{t}{C'}\frac{d'}{d}.$$

52. Comparison of High Resistances.—If the value of one high resistance is known, then other resistances of the same order of magnitude can be directly compared with it, and this can be done without knowing the constant of the galvanometer or the capacitance of any condenser.

Suppose another resistance, R_1, is substituted for R in the set-up shown by Fig. 26, and the current allowed to leak through it into the condenser as before.

In the first case.

$$R = \frac{tE}{kd}$$

and now

$$R_1 = \frac{t_1E}{kd_1},$$

where t_1 and d_1 denote the observations when the current was leaking through R_1.

Eliminating E and k by division gives

$$R_1 = R\,\frac{t_1d}{td_1},$$

where, as in the preceding article, it is supposed that V can be neglected in comparison with E. In the following article it appears that this approximate value of R_1 becomes more nearly correct as d and d_1 are more nearly equal to each other.

53. The More Exact Formula.—In Art. 51 an assumption was made which is not quite true. It was there supposed that the leakage current remained constant, as shown by curve I, Fig. 27.

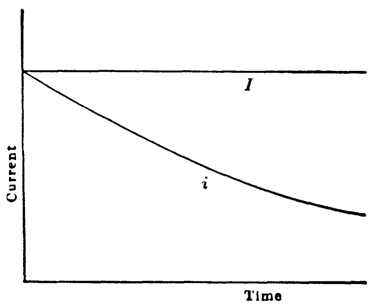


FIG. 27.—The current flowing into a condenser decreases with time, as shown by the curve i.

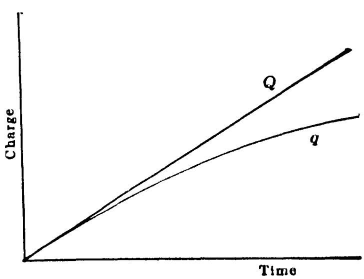


FIG. 28.—The charge in a condenser increases with time, but more and more slowly as shown by curve q.

From this assumption it followed that the charge collected in the condenser increased directly with the time the current is flowing or $Q = It$, as shown by curve Q, Fig. 28. As a matter of fact, the current becomes less and less as the condenser becomes charged, as shown by curve i, Fig. 27. The corresponding charge collected in the condenser is shown by curve q, Fig. 28. At the beginning there is not much difference between Q and q, and it is seen that the method above is approximately true when t is a small part of the time required to half charge the condenser.

In order to make a satisfactory measurement, it is necessary to collect a sufficient quantity in the condenser to give a measurable deflection when it is discharged through the galvanometer. Likewise, the time allowed for the collection of this quantity cannot be determined with accuracy if it is too short. Either, or both, of these conditions may make it desirable to allow the current to flow for a longer period, and thus render it no longer permissible to neglect the increasing value of V.

Taking these conditions into account leads to the following considerations. The current flowing into the condenser at any instant is

$$i = \frac{E - V}{R},$$

or, in terms of q, the charge in the condenser,

$$i = \frac{dq}{dt} = \frac{E - \frac{q}{C}}{R},$$

since $q = CV$.

Separating the variables for integration puts this in the form

$$\frac{-dq}{Q - q} = \frac{-dt}{RC},$$

where Q is written for EC. This Q is the charge in the condenser, C, when it is joined directly to E. Integrating this equation gives

$$\log (Q - q) = \frac{-t}{RC} + K,$$

where K is the constant of integration.

At the start, when both t and q are zero, this equation gives

$$\log Q = K.$$

After t' seconds, when q' has collected in the condenser,

$$\log (Q - q') = \frac{-t'}{RC} + K.$$

Subtracting the latter from the former gives the change during t' seconds.

$$\log \frac{Q}{Q - q'} = \frac{t'}{RC}.$$

Hence the value of R is

$$R = \frac{t'}{C \log \frac{Q}{Q - q'}} = \frac{t'}{2.303\, C \log_{10} \frac{d_o}{d_o - d'}},$$

where d_o and d' are the deflections when the quantities Q and q are discharged through the galvanometer.

54. Insulation Resistance of a Cable.—Let a measured length of the cable be coiled up and placed in a tank of water to give contact over the outside surface if there is not a lead covering.

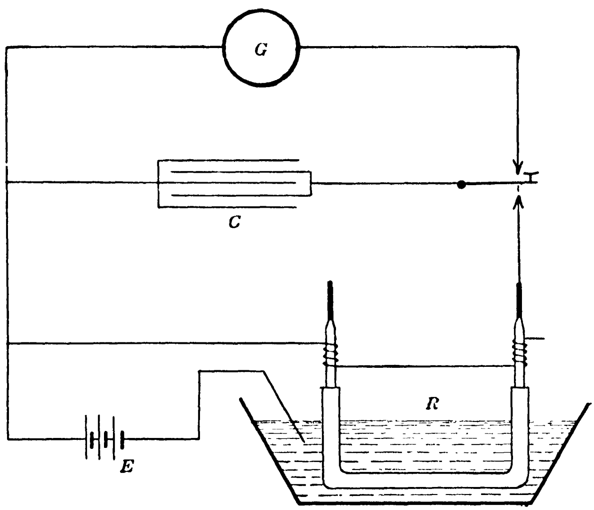


FIG. 29.—To measure the insulation resistance, R, of a length of a cable.

Each end of the cable should extend out of the water or sheath for some distance, and must be kept thoroughly dry. A few turns of bare copper wire are tightly wound around the cable near the middle of this dry portion of the insulation and connected to the common point between the condenser and the battery, as shown in Fig. 29. This guard wire serves to intercept any current that may be leaking along the surface of the insulation. Connection is made to the water by means of a wire of the same kind as that in the cable, to avoid voltaic effects. The wires connecting R to C and the key should be well insulated from the table and from all places where a charge can leak across. As far as possible, they should be stiff enough to stand in the air. Where necessary, they can be supported on blocks of sulphur.

The determination consists, then, in allowing the leakage current to flow into the condenser long enough (t' seconds) to give a fair charge, q', which, when discharged through the galvanometer, will give a deflection, d'. When R is short-circuited and the condenser charged directly from E, the charge is Q, giving a deflection, d. For best results q' should be about half of Q.

Owing to dielectric absorbtion of the charge, the first values of R will be too small. In reporting any value of R, the duration of the test should also be stated.

55. Insulation Resistance by Loss of Charge.—If the high resistance has also considerable capacitance, it will not be necessary to use a separate condenser. Thus, if it is required to measure the insulation of a condenser or a long cable, the arrangement will be as shown in Fig. 30, where CR represents the condenser of capacitance C and resistance R. Upon closing K, with K' also closed, the condenser is charged to the full potential difference of the battery. When the key is opened, the charge Q begins to leak away through the high resistance R. At the start, and before the charge in the condenser has been appreciably reduced by the leakage, the current through R is,

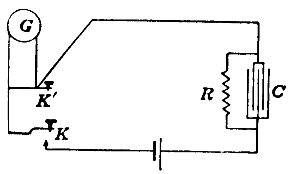


FIG. 30.—To measure the resistance of a condenser.

$$I = \frac{E}{R}.$$

This current will reduce the original charge in the condenser by the amount

$$q = It$$

in t seconds, where t is not too great to consider the current constant during this interval. This loss of charge can be determined by recharging the condenser through the galvanometer to the original difference of potential. Then, as before,

$$q = kd, \qquad \text{and} \qquad I = \frac{kd}{t},$$

from which

$$R = \frac{E}{I} = \frac{Et}{kd} = \frac{t}{C'}\frac{d'}{d},$$

where E cancels out if the same battery is used to determine k.

At the beginning of this test the values of R will usually be too low because of the effect of "absorption," by which a part of the charge disappears. This reduces the charge in the condenser just as though it had leaked out. The true value of the resistance will be obtained only after several hours, in some cases several days, but if a first test is being made, it is well to determine the value of R at intervals of a few minutes. A curve plotted with the time of day for abscissæ and the corresponding values of R for ordinates will show this variation and indicate the maximum value of the insulation resistance.

A resistance not having any capacitance can be measured by this method by adding a condenser in parallel with it. But in such a case the arrangement shown in Fig. 26 would be preferable.

56. A More Exact Formula for the Loss of Charge Method.—As in the previous case, the current leaking out of C can be considered constant only as an approximation for a short portion of the discharge. For more precise results it will be necessary to take account of the decreasing value of this current.

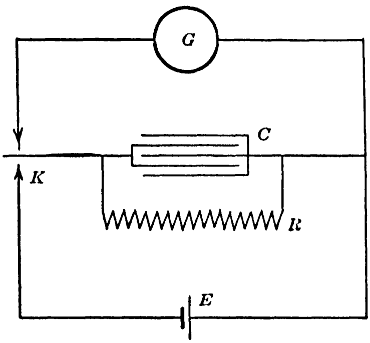


FIG. 31.—The charge in the condenser can leak away through the high resistance, R.

As the charge in the condenser leaks away, the potential difference across the condenser decreases and finally becomes zero. Let V denote the value of this potential difference at the time t. The current flowing through R at this instant is $i = \frac{V}{R}$. Since this current is supplied from the charge in the condenser, it is measured by the rate of *decrease* of this charge,

or
$$i = -\frac{dq}{dt} = \frac{V}{R} = \frac{q}{CR},$$

since $V = \frac{q}{C}$.

Separating the variables for integration,

$$\frac{dq}{q} = \frac{-dt}{CR},$$

and performing the integration gives

$$\log q = \frac{-t}{CR} + B, \qquad \text{(A)}$$

where q is the charge remaining in the condenser after the charge has been leaking out for t seconds.

At the start, when $t = 0$, $q = Q$, and putting these values in (A) gives for the value of the constant of integration, B,

$$\log Q = B.$$

If q' denotes the quantity that has leaked out of the condenser in t' seconds, then

$$q = Q - q'$$

and

$$\log (Q - q') = \frac{-t}{CR} + \log Q.$$

Solving this for R gives

$$R = \frac{t'}{C \log \frac{Q}{Q - q'}} = \frac{t'}{2.303\ C \log_{10} \frac{d_o}{d_o - d'}},$$

where d_o and d' are the deflections when Q and q' are discharged through the galvanometer.

57. To Reduce the Effect of Absorption.—One difficulty in measuring the leakage resistance of a cable or a condenser arises from the absorption of the charge by the dielectric. (See Art. 193.) This may be avoided in part by first charging the condenser and allowing it to leak for a period of t_1 seconds. The remaining charge, q_1, is then discharged through a ballistic galvanometer and the deflection, d_1, observed. Then recharge and allow the charge to leak away for a longer period, t_2, say twice as long. For the first period,

$$\log q_1 = \frac{-t_1}{CR} + B,$$

and for the second,

$$\log q_2 = \frac{-t_2}{CR} + B.$$

The difference is

$$\log \frac{q_1}{q_2} = \frac{t_2 - t_1}{CR},$$

and

$$R = \frac{t_2 - t_1}{C \log \frac{q_1}{q_2}} = \frac{t_2 - t_1}{2.303\ C \log_{10} \frac{d_1}{d_2}}. \qquad \text{(B)}$$

An electrometer can be used in place of the galvanometer, and it has the advantage that the loss of charge can be observed continuously. By taking readings at definite times, the curve of discharge can be plotted. The effect of dielectric absorption of the charge is shown by the variation of this curve from the one plotted from Eq. (A). Any two points of this curve can be used in Eq. (B) to obtain the value of R.

CHAPTER III

THE CURRENT GALVANOMETER

58. Two Types of Galvanometers.—A galvanometer is a delicate and sensitive instrument for the measurement of small electric currents. All galvanometers consist of two essential parts—a coil of wire through which can flow the current to be measured, and a permanent steel magnet. In some galvanometers the coil is comparatively large and is rigidly fixed to the frame of the instrument, while the magnet is a small piece of steel suspended lightly by a fiber of untwisted silk or of quartz. In other galvanometers the arrangement is reversed. The coil is made as light as possible and is suspended by a thin strip of phosphor bronze between the poles of a large and strong magnet which often forms the body of the instrument. In either style the movable portion is made to turn as easily as possible, the amount of turning being measured by the mirror, scale, and telescope method.

There are two ways of using a galvanometer. A transient current, like the discharge of a condenser, will produce a fling or kick of the galvanometer after which it will settle back to the original position. Evidently the only thing that can be measured in this case is the maximum fling. But if the current is steady, the galvanometer will settle down at a deflected position, and the deflection, as the distance of this position on the scale from the position of rest is called, measures the current.

59. Figure of Merit. (*a*) *By Direct Deflection.*—Most galvanometers are so constructed that, for small angles at least, the deflection is directly proportional to the current. That is,

$$I = Fd.$$

This proportionality factor, F, is called the "figure of merit" of the galvanometer, and it is defined as the *current per scale division* (1 mm.) that will deflect the galvanometer. The figure of merit of most galvanometers is smaller than one hundred-millionth of an ampere per millimeter.

Inasmuch as the deflection will vary with the distance of the scale from the mirror, this distance should be made 1 meter. If for any reason the scale is at a different distance, the observed deflection must be corrected to what it would have been had the scale been at the standard distance.

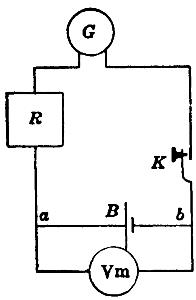


FIG. 32.—To determine the figure of merit of G.

In order to determine the figure of merit, it is necessary to send a small known current through the galvanometer and observe the *steady* deflection it produces. The method can be understood by reference to Fig. 32. The galvanometer is joined in series with a battery, a large resistance, and a key. When the key is closed, the current flowing through the circuit, and therefore through the galvanometer, is, by Ohm's law,

$$i = \frac{E}{g + r + R},$$

where E denotes the E.M.F. of the battery, and g, r, R, the resistances of the galvanometer, battery, and R, respectively. If this current produces a steady deflection of d scale divisions (mm.), the figure of merit is $F = \frac{i}{d}$.

By using a voltmeter to measure directly the fall of potential between a and b, the current, i, will be given by

$$i = \frac{V}{R + g},$$

where V is the voltmeter reading. Then

$$F = \frac{V}{(R + g)d}.$$

Thus, by the use of the voltmeter, the somewhat uncertain E.M.F. of the cell is replaced by the definite voltmeter reading, and the unknown resistance of the cell does not appear in the equations. Of course V and d must be simultaneous values, and it is understood that d is the *steady* deflection produced by the steady current, i.

60. Figure of Merit. (*b*) *By Potential Divider Method.*—With a sensitive galvanometer it is usually not possible to make R large enough to use the simple method. It is then most convenient to use a value for E which is only a small fraction of the E.M.F. of the cell. This can be done by the potential divider method shown in Fig. 33. The fall of potential between a and b is now PI instead of E, and both P and I can be made as small as necessary. For most galvanometers it is convenient to make $P + Q = 1{,}000$ ohms, and R about 100,000 ohms.

The current from the battery divides at a, one part, i, going through the galvanometer, another part, I, through P, and the third part flows through the voltmeter. The fall of potential from a to b across P is the same as through R and the galvanometer, or

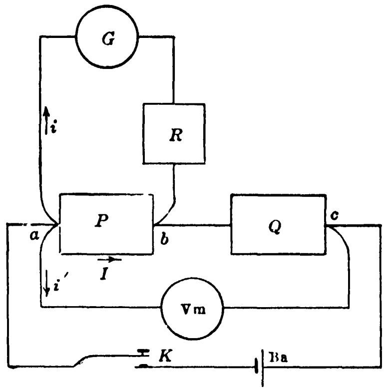


FIG. 33.—The potential divider, PQ, gives a small fraction of E across ab.

$$PI = (R + g)i, \qquad (1)$$

where g is the resistance of the galvanometer.

The current through Q is $I + i$, and the fall of potential from a to c, which is measured by the voltmeter, is

$$PI + Q(I + i) = V. \qquad (2)$$

Eliminating I between Eqs. (1) and (2), and solving for i,

$$i = \frac{PV}{(Q + P)(R + g) + PQ}.$$

The figure of merit is, then,

$$F = \frac{PV}{(P + Q)(R + g) + PQ} \cdot \frac{1}{d}.$$

Usually the term PQ can be omitted in comparison with QR, since P is much less than R, and by keeping P small, the error thus introduced may be made less than the error in reading the voltmeter or the galvanometer deflection. It is always a wise precaution not to connect the battery into the circuit until after the rest of the set-up is completed and has been carefully examined to make sure that no unintended connections have been made.

Record the observations as follows:

TO DETERMINE THE FIGURE OF MERIT OF GALVANOMETER NO.....

Zero	Reading	Deflection	R	V	P	Q	F

At the start, P should be set very much smaller than Q, perhaps $P = 1$, and $Q = 1{,}000$, so that the first deflection of the galvanometer will not be too large. When this has been tried, P can be increased enough to give a deflection of 100 or 200 mm.

61. Current Sensitivity of a Galvanometer.—The figure of merit expresses the sensitivity of a galvanometer in the old way. The smaller the figure of merit the more sensitive is the galvanometer. It is especially useful when the galvanometer is used to measure a small current, for then

$$i = Fd.$$

The more modern practice is to express the sensitivity as the deflection, D, that is (or would be) produced on a scale at one meter distance by a current of 1 microampere (= 0.000,001 ampere). This deflection is not observed directly, but is computed, like F, from the observations in the above experiment. This gives

$$D = \frac{1}{10^6 F} = \frac{(P + Q)(R + g) + PQ}{10^6 PV} a.$$

62. Megohm Sensitivity.—The sensitiveness of a galvanometer is often expressed in megohms (millions of ohms). This means the number of megohms that can be placed in series with the galvanometer to give a deflection of one scale division per volt of the applied E.M.F. Evidently this number is the same as D. In some respects this is a better way of expressing the sensitivity. A value of $D = 5{,}000$, for example, is the result of a computation and could not be actually observed, while it would be possible to place D megohms in series with the galvanometer and obtain a deflection of one millimeter when one volt is applied.

63. Voltage Sensitivity.—For some purposes it is necessary to know the sensitiveness of a galvanometer to differences of potential applied to its terminals. This will depend largely upon the resistance of the galvanometer, as a small resistance will allow a larger current to flow. The voltage sensitivity is expressed in volts per scale division, and is given by the product of the figure of merit and the resistance of the galvanometer.

64. The Best Galvanometer.—The best galvanometer to use for a particular purpose depends upon the special requirements. When the same current is passed through several galvanometers in series, the one that gives the largest deflection is the most sensitive for current measurements. Probably this one could have been picked out from the fact that it has the highest resistance because it is wound with many turns of fine wire. If the same galvanometers are joined in parallel and connected to a difference of potential of a few microvolts, probably a very different galvanometer will now show the largest deflection. This one will have a low resistance, and it will be sensitive for voltage measurements, provided that the source can supply the current required.

For many purposes a galvanometer may be "too good." If the current to be measured is not extremely minute, the deflection may be too large to measure on the scale. Extreme sensitiveness is often obtained by using a very fine suspension fiber. This makes the period long and the zero, or resting point, is not as definite as it is with a stiffer suspension.

For most purposes a galvanometer with D about 100 scale divisions per microampere, and g about 100 ohms, is satisfactory. The suspension should be strong enough to withstand moderate jars. The moving parts should have sufficient clearance to allow the use of the instrument without too tedious a process of leveling it. The interior should be easily accessible.

65. Factor of Merit.—When the sensitiveness of a galvanometer is expressed in terms of the deflection, D, at a distance of 1,000 scale divisions, due to a current of one microampere, this sensitiveness can be increased by using a weaker suspension wire or by winding more turns of wire on the coil, without making any real improvement in the design of the galvanometer. In order to make a fair comparison of different galvanometers, it is necessary to take these differences into account.

To make this comparison it is usual to reduce the observed deflection, D, to what it would be if the period of the galvanometer were 10 seconds and the resistance 1 ohm. It is found that D varies as T^2 and about as $g^{2/3}$, where T denotes the period of the galvanometer and g is its resistance.

The sensitiveness, thus modified, is called the "factor of merit" of the galvanometer. Thus,

$$\text{Factor of Merit} = \left(\frac{10}{T}\right)^2 \left(\frac{1}{g}\right)^{\frac{2}{3}} D.$$

The unit of this quantity is not named. The unit of a similar quantity in which the reference period is taken as one second has been called a "D'Arson."

66. The Common Shunt.—When the current to be measured by any instrument is larger than the range of the scale, the latter can be increased to almost any desired extent by placing a shunt in parallel with the instrument, as shown in Fig. 34. The shunt and instrument thus form two branches of a divided circuit and the current through one branch is directly measured. Knowing the current in this branch, the total current can be computed.

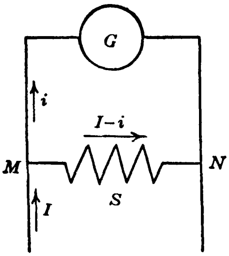


FIG. 34.—Use of a shunt.

Thus let G denote the galvanometer or ammeter, and S the shunt. The fall of potential through the galvanometer from M to N is equal to the fall of potential through the shunt between the same points, or

$$ig = (I - i)s,$$

where g denotes the resistance of the galvanometer and s that of the shunt. This gives for the value of the main current

$$I = i\,\frac{g + s}{s}.$$

67. The Multiplying Power of a Shunt.—The factor, $\frac{g + s}{s}$, by which the current measured by the galvanometer must be multiplied to give the total current in the main circuit is called the "multiplying power" of the shunt. In order that this factor may be expressed in convenient round numbers, 10, 100, 1,000, etc., it is necessary to have a series of shunts carefully adjusted to, $\frac{1}{9}$, $\frac{1}{99}$, $\frac{1}{999}$, etc., of the resistance of the galvanometer. Such shunts will not have the same multiplying power when used with a galvanometer of different resistance, and therefore can be used advantageously only with the galvanometer for which they were made. Placing a shunt in parallel with a galvanometer reduces the total resistance of the circuit, and therefore the current measured by the galvanometer times the multiplying power of the shunt does not give the value of the original current, but the value of the new main current. Sometimes an extra resistance of $0.9g$, $0.99g$, or $0.999g$, is inserted to keep the total resistance of the circuit constant.

When the galvanometer is used ballistically, these shunt ratios are not the same as for steady currents because of the varying amounts of damping produced.

68. The Universal Shunt.—The universal shunt is so called because it can be used with any galvanometer and its shunt

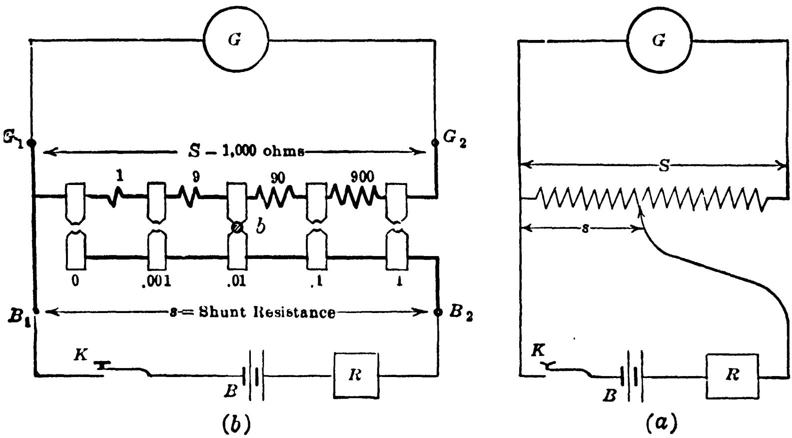


FIG. 35. Diagram illustrating the universal shunt.

ratios will be the same. This arrangement is shown in Fig. 35. The total resistance, shown in the figure as 1,000 ohms, is connected as a permanent shunt on the galvanometer. The current to be measured is passed through only a part of this shunt, shown as 10 ohms in the figure, the remaining 990 ohms acting merely as resistance in series with the galvanometer.

The expression for the multiplying power of a universal shunt may be derived as follows: Let g denote the resistance of the galvanometer, S the total resistance of the shunt, and s the portion of S corresponding to a common shunt and which carries that part of the current not flowing through the galvanometer. The fall of potential over the galvanometer and the part of S that is in series with the galvanometer is equal to the fall of potential over the part s. Let I denote the value of the main current, and i the current through the galvanometer. Then

$$i[g + (S - s)] = (I - i)\ s,$$

or

$$I = i\ \frac{g + S}{s}.$$

For this form of shunt, therefore, the multiplying power is inversely as s, since $g + S$ remains constant while s varies. For this reason, a shunt of this type can be used for different instruments, and the deflections of each will be proportional to the values of s for the same value of the main current. That is, the shunt will have the same series of multiplying powers for every galvanometer.

69. Arrangement of the Universal Shunt.—Any ordinary resistance box having a traveling plug for making a third connection at any intermediate point can be used as a universal shunt for any galvanometer. For all values the shunt ratios are very accurate, since all the coils are even ohms and can be adjusted much more precisely than in the case of common shunts. Differences in temperature between the galvanometer and shunt produce no error, but should remain constant while measurements are being made.

The change produced in the resistance of the total circuit is not as easily determined as for a common shunt. Indeed, the resistance is frequently greater with the shunt than when the galvanometer is used alone. In many kinds of work it is not essential that the resistance shall be constant or even known. Where it must be known, it can be determined for the galvanometer and its shunt combined as readily as for the galvanometer alone.

Shunts are marked with the numbers 0.1, 0.01, 0.001, implying the fractions of the current which they pass through the galvanometer. It is evident that when the universal shunt is used at the point marked 1, the galvanometer is not quite as sensitive as with no shunt connected. If S is several times g, this slight reduction in the sensitiveness is of small moment. The essential thing is that when the shunt is set at 0.1, 0.01, etc., the same total current will give deflections 0.1, 0.01, etc., as large as with the shunt set at 1. And this arrangement of the galvanometer shunts is especially useful because a given series of shunts will have the same relative multiplying powers when used with any galvanometer. Since the damping is constant, the shunt ratios remain the same when the galvanometer is used ballistically.

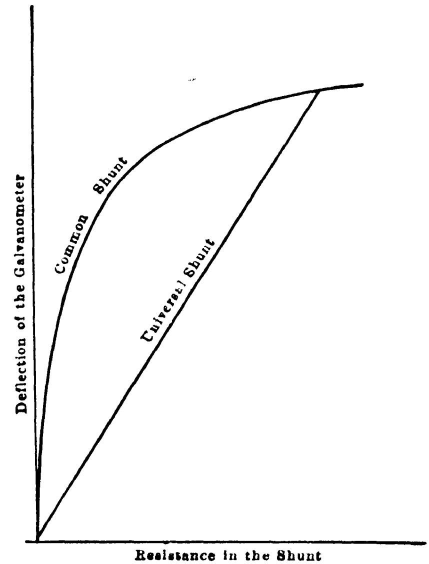


FIG. 36.—Shows the relation between the galvanometer deflection and the resistance in the shunt. With the same total current for each deflection, the straight-line curve for the universal shunt shows that the galvanometer deflection is proportional to the resistance in the shunt.

The curve for the common shunt shows the deflections of the same galvanometer with the same total current when common shunts of different resistances are used. For small values of the shunt, the deflections increase rapidly as the shunt is made larger. The larger deflections are not changed much by increasing the resistance of the shunt.

The advantage of the universal shunt is evident.

70. To Measure the Multiplying Power of a Shunt. *Test of a Shunt Box.*—The effect of a shunt in increasing the amount of current that can be measured by a galvanometer can be determined experimentally for the actual conditions of using the shunt.

Let us consider the case of a steady current, I, flowing through the circuit and dividing between the galvanometer and its shunt. The galvanometer will give a steady deflection, d, due to the current, i, through it. Then

$$i = Fd,$$

where F is the figure of merit of the galvanometer alone.

Without changing the deflection or the current in any way, let us, secondly, think of the combined galvanometer and shunt as a single instrument. The total current, I, passes through this instrument and the deflection is d. Then

$$I = F'd,$$

where F' is the figure of merit of the combined galvanometer and shunt.

The multiplying power, m, of a shunt is the ratio of the main current to the current through the galvanometer. This gives

$$m = \frac{I}{i} = \frac{F'd}{Fd} = \frac{F'}{F}.$$

Thus, by determining the values of F and F', the multiplying power of a shunt can be obtained. Or, if m is known, the value of F' can be computed $(=mF)$. This measured value of m should be compared with the computed value, $m = \frac{g + s}{s}$, when possible.

In the same manner, all of the shunts in the shunt box should be tested, and the multiplying power of each one determined. The results should be compared with the values stamped on the shunt box, and also with the computed values as determined by the relative resistances of the galvanometer and the shunt.

When using the shunts of lowest resistance, it may be better to determine the figure of merit by the direct deflection method, Art. 59. The scale deflection should be about the same for each shunt.

71. Resistance of Galvanometer by Half Deflection. (*a*) *Resistance in Series.*—In some of the foregoing exercises it is necessary to know the resistance of the galvanometer as it has been used, either alone or combined with a shunt. If this is unknown, it can be determined with a fair degree of accuracy by the method of half deflection.

Let the galvanometer be connected to a source of potential difference that is small enough to keep the deflection on the scale with no additional resistance in series with the galvanometer. Then add enough resistance, R, in series with the galvanometer to make the deflection exactly one-half of its former value. This means that the current has been reduced to half its former value and therefore the resistance, $R + g$, in the galvanometer circuit has now been made twice as much as when the galvanometer was used alone. That is,

$$R + g = 2g \qquad \text{or} \qquad g = R.$$

To avoid the errors arising from thermal currents, etc., it is best to reverse the battery and repeat the measurements, taking the mean of the two results as the correct value of g.

72. Resistance of a Galvanometer by Half Deflection. (*b*) *Resistance in Parallel.*—When the resistance of the galvanometer is low, or a small E.M.F. is not readily available, the galvanometer may be placed in series with a very large resistance and sufficient E.M.F. to give a fairly large deflection. Then let a resistance box be joined in parallel with the galvanometer, and the resistance varied until the deflection is just half of its former value. The current is now divided between the galvanometer and its shunt, half of the original current flowing through the galvanometer and the rest through the shunt. If the main current is unchanged, this means that the current is equally divided between the galvanometer and its shunt. From this it follows that the resistance of the galvanometer is equal to the resistance of the shunt.

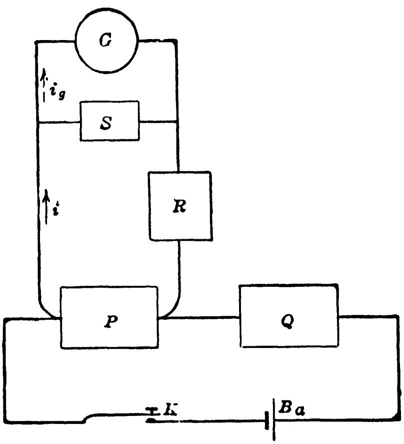


FIG. 37.—Resistance of a galvanometer. When $S = g$, $i_g = \frac{1}{2}i$.

It is true that the addition of the shunt has reduced the resistance of this portion of the circuit, but as this is only a small part of the total resistance in the battery circuit, the main current in the second case, and which is divided between the galvanometer and the shunt, will be larger than the current in the first case by less than can be read on the galvanometer scale. This point can be tested by placing the shunt resistance in series with the galvanometer and noting whether it produces an appreciable effect in reducing the deflection.

Problem.—Let the student give a mathematical proof that $g = S$.

73. Making an Ammeter.—An interesting way to use a galvanometer is to construct a direct-reading ammeter with the galvanometer as the moving element. The problem is to set up the galvanometer with a suitable shunt, and other resistances, so that when a current of 1 ampere (or other assigned amount) is passed through the arrangement the galvanometer will stand deflected 100 divisions.

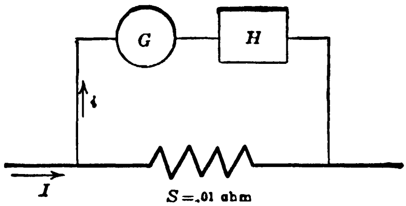


Fig. 58.—An ammeter built up with a galvanometer and shunt.

If there is available a known resistance of 0.01 ohm, it would make a useful shunt to carry most of the current that is to be measured. This low resistance should have permanent current and potential terminals, as shown in Fig. 91. If the galvanometer is connected to the potential terminals, the current will divide between the two parallel circuits, most of it flowing through the low resistance of the shunt and a small part of it, i, flowing through the galvanometer.

From the law of shunts,

$$I = \frac{g + s}{s} i = \frac{g + s}{s} F d',$$

where d' is the desired deflection corresponding to the main current, I.

Since F and s are already fixed, the galvanometer resistance should be

$$g = \frac{Is}{Fd'} - s,$$

where the last term, s, is probably insignificantly small. If the galvanometer resistance is less than this, it can be increased by adding in series sufficient resistance, H, to give the desired amount.

If F has been determined accurately, the galvanometer should show a deflection of d' divisions when the main current is I amperes. If this check is satisfactory, it gives confidence that by changing H and s the galvanometer can be set to measure any other desired range of current.

74. To Make a Voltmeter.—A very excellent voltmeter can be made with a galvanometer and a high resistance of 100,000 ohms or more. The idea of a voltmeter is an instrument that will measure E.M.F. without destroying or changing what it is trying to measure. Therefore, the less the current that is drawn from the battery, or other source, the better, since this will allow the available E.M.F. at the terminals to be nearer the value of the full E.M.F.

Suppose the problem is to make a voltmeter that will read 100 divisions per volt. The current through the galvanometer will be, then,

$$i = 100F.$$

But the current that will flow through the high resistance, H, and the galvanometer in series, when connected to an E.M.F. of 1 volt, is

$$i' = \frac{1}{H + g}.$$

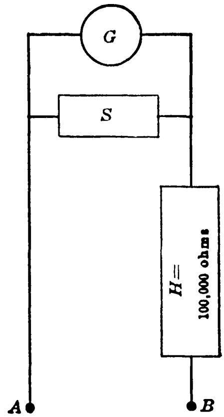


FIG. 39.—A voltmeter built up with a galvanometer and a high resistance.

Probably g can be neglected in comparison with H.

If i' is larger than i, and H cannot be increased to make $i' = i$, it will be necessary to use a shunt on the galvanometer whose multiplying power is

$$m = \frac{i'}{i} = \frac{g + s}{s}.$$

The resistance of a shunt having this multiplying power is

$$s = \frac{g}{m - 1}.$$

Of course s cannot be computed any closer than g is known. Hence the importance of knowing g is evident. Since the galvanometer coil is of copper wire, which changes with temperature, the value of g should be determined at the time it is used.

The voltmeter thus built can be compared with a Weston voltmeter. When both voltmeters are in parallel with each other they should read the same.

To show the advantage of a high-resistance voltmeter, it can be used to measure the E.M.FS. of several old dry cells. After this is done, the same cells can be measured with a Weston voltmeter. The latter will be found to give results that are too low.

75. Differential Galvanometer.—A differential galvanometer has two independent coils, as nearly alike as possible. A current passed through either one will produce a deflection, and if the current flows through both coils in series, the deflection is due to the effect of both coils acting together. If the effect of the current in one coil acting alone is to produce a deflection in one direction, and the effect of a current in the other coil is to produce a deflection in the other direction, the effect of both coils acting together will be the difference of the two, and the resulting deflection will be smaller than for either coil alone. If the coils have been carefully made, and adjusted so that the magnetic effect of each upon the needle is the same, then no deflection will be produced by equal currents flowing through the two coils in opposite directions, for the effect of one coil is just neutralized by the other. Usually this balance is not exact, and a final adjustment is required before using the galvanometer, and since it is impossible to have the coils exactly alike, the two currents will not be equal for a balance.

Calling the current through one coil I', and that through the other I'', the relation between them would be

$$I' = nI'', \tag{1}$$

where n is a constant whose value is about unity.

To compare two resistances, R and X, they are joined in parallel with each other, as shown in the figure. In series with each is one coil of the galvanometer. A resistance, P, is placed in series with the battery in order to keep the current from being excessive.

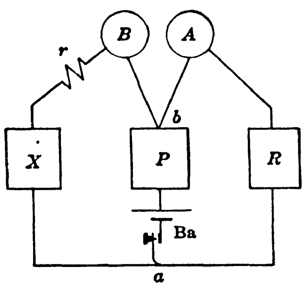


FIG. 40.—Use of the differential galvanometer, AB.

The galvanometer is adjusted as follows. R and X are shortcircuited by inserting all of the plugs, or otherwise. This is better than removing them from the circuit, as no change in connections will be required when they are to be used. On now closing the key, the current through the galvanometer coils will depend only upon their relative resistances, and probably these are not such as will give a balance. Let a resistance, r, be now added to the side having too large a current and adjusted to give a balance. Then, from (1),

$$\frac{V}{A} = n\frac{V}{B + r}, \tag{2}$$

where A and B are the resistances of the two coils, and V is the fall of potential between a and b.

Inserting R and X, and adjusting R till a balance is again obtained, gives, from (1),

$$\frac{V'}{A + R} = n\frac{V'}{(B + r) + X}, \tag{3}$$

where V' is the new value of V.

Exchanging R with X and readjusting R again to balance gives

$$\frac{V''}{A + X} = n\frac{V''}{(B + r) + R'}, \tag{4}$$

where R' and V'' are the slightly changed values of R and V. Dividing (2) by (3) gives

$$\frac{A + R}{A} = \frac{(B + r) + X}{(B + r)}.$$

Clearing of fractions and solving for X,

$$X = \frac{R(B + r)}{A}. \tag{5}$$

Similarly, from (2) and (4)

$$X = \frac{R'A}{B + r}. \tag{6}$$

Multiplying (5) and (6) and extracting the square root,

$$X = \sqrt{RR'}.$$

The value of X is thus a mean proportional between the values of R required to give the two balances. If these values are nearly equal, no appreciable error is made by taking X as the arithmetical mean. In case a change of 1 ohm in R produces a readable deflection, the tenths of ohms can be obtained by interpolation.

76. The Weston Ohmmeter.—The principle of the differential galvanometer is utilized in the Weston ohmmeter, shown in Figs. 41 (a) and 41(b). This instrument reads directly in ohms the value of the resistance that is connected between the binding posts marked XX. The moving coil of the ohmmeter has two windings, as indicated by A and B in the diagram of Fig. 41(a).

A current through one of these windings tends to deflect the pointer up the scale, while a current through the other winding tends to deflect the pointer down the scale. With two equal currents, the action of one is balanced by the effect of the other and the needle stands undeflected at zero on the scale. When resistance is added in series with the second coil the current is lessened and the pointer moves up the scale, due to the undiminished effect of the current in the first coil.

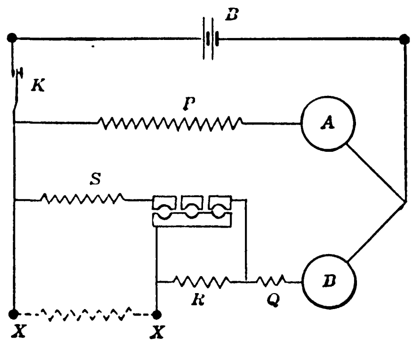


FIG. 41(*a*).—Connections of a Weston ohmmeter. *A* and *B* indicate the differential windings on the moving coil. The resistance to be measured is connected at *XX*.

Like any voltmeter, the reading depends upon the voltage of the battery that is used. The ohmmeter shown in Fig. 41(*b*) is designed to operate on two dry cells. Within the instrument is a standard resistance, *S*, equal to the amount measured by the full scale deflection. With the movable plug in the position shown in Fig. 41(*b*), this standard resistance is connected across *XX*, and when the key is depressed, the pointer should stand at the top of the scale. In case it does not, the instrument is quickly adjusted by changing the position of a magnetic shunt with respect to the pole pieces of the permanent magnet. In this way any variation in the voltage of the battery is compensated, and the scale readings are independent of the actual current through the instrument. The middle hole is merely a place to stand the plug. When placed in the hole "100," the plug short-circuits 50 ohms, and therefore 50 ohms must be connected across *XX* to give the zero reading.

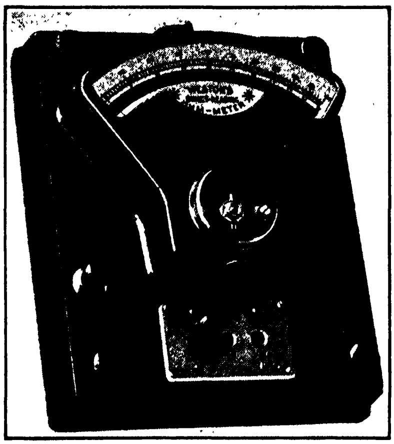


FIG. 41(*b*).—The Weston ohmmeter.

In this position of the plug the scale reads from 50 to 100 ohms. Other ranges measure up to 2,000 or 3,000 ohms.

77. Differential Galvanometer in Shunt.—The differential galvanometer is even more useful in the comparison of two low resistances. In this case each of the galvanometer coils is used as a sensitive voltmeter to measure the fall of potential over a resistance. If the fall of potential over each of the two resistances is the same when they are in series, the resistances must be equal.

The galvanometer coils should be of high resistance, say several thousand ohms. This means that they will have a great many turns of (copper) wire, which will make the galvanometer sensitive to small currents. Therefore only a little current will be shunted from the low resistances that are being compared.

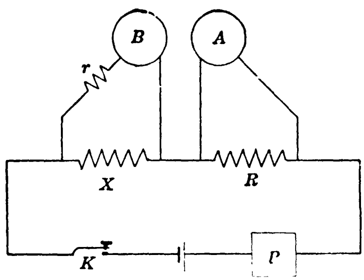


Fig. 42.—Differential galvanometer, AB, in shuntwith R and X.

For this comparison it is necessary to have a standard low resistance which can be varied by small steps. This is joined in series with the unknown resistance X, as shown in Fig. 42. Some additional resistance, P, is placed in the battery circuit to keep the current from being too large. The two coils of the galvanometer are connected as shown.

To adjust the galvanometer, the two coils are connected together in parallel and both shunted across the same low resistance. Each coil will now have the same fall of potential and the galvanometer should give no deflection. In case there is a deflection, the current through one of the coils must be reduced by adding some resistance, r, in series with this coil until the deflection is brought to zero. This will make the resistances of the two shunts unequal, but the deflection will be zero when each shunt has the same fall of potential.

When connected as shown in Fig. 42, a balance will be obtained when R has been adjusted to equal the value of X, provided that the current through each is the same. In general, this will not be the case, for, by introducing r, the shunt currents are not the same. Therefore a balance is obtained when the resistance of X with its shunt equals the resistance of R with its shunt. That is, when,

$$\frac{XB}{X + B} = \frac{RA}{R + A}.$$

A second balance is obtained by exchanging X and R, and readjusting the value of the latter to R' for zero deflection. Then,

$$\frac{R'B}{R' + B} = \frac{XA}{X + A}.$$

Dividing the first of these equations by the second gives

$$\frac{X(R' + B)}{R'(X + B)} = \frac{R(X + A)}{X(R + A)}$$

or

$$\frac{X}{R'}\frac{X}{R} = \frac{(X + A)\,(X + B)}{(R + A)\,(R' + B)} = 1, \text{ very nearly.}$$

Even in the extreme case of $A = 1{,}000$ and $B = 2{,}000$ with $X = 1$ ohm, this ratio differs from unity by only about 0.0002, and the difference is correspondingly less when A and B are more nearly equal.

Therefore,

$$X = \sqrt{RR'}.$$

CHAPTER IV

THE WHEATSTONE BRIDGE

78. The Wheatstone Bridge.—The Wheatstone bridge consists, essentially, of two circuits in parallel and through which an electric current can flow. Let these circuits be represented by ABD and ACD, Fig. 43, and let the currents through the two branches be denoted by I and I'. Since the fall of potential from A to D is the same whichever path is considered, there must be a

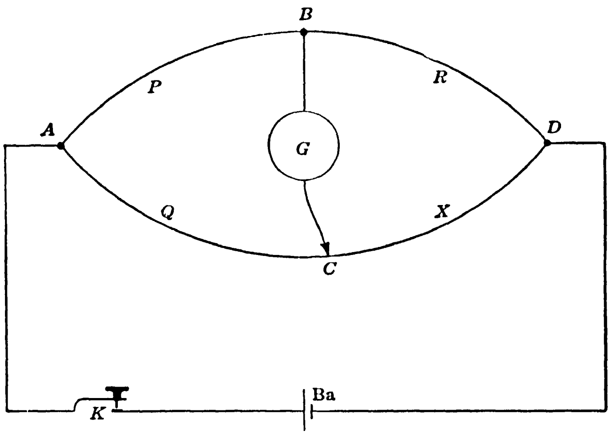


FIG. 43.—Principle of the Wheatstone Bridge.

point, C, on one circuit which has the same potential as any chosen point, B, on the other. If one terminal of a galvanometer is joined to B and the other terminal is moved along ACD, the galvanometer will indicate zero deflection when the point C has been found. Since B and C have the same potential, the fall of potential from A to B is the same as from A to C, or, in terms of the currents and resistances,

$$IP = I'Q,$$

where P and Q are the resistances of AB and AC, respectively. Similarly, for the other part of the circuits

$$IR = I'X.$$

Dividing one equation by the other eliminates the unknown currents and gives

$$\frac{P}{R} = \frac{Q}{X}$$

as the relationship of the resistances when the bridge is balanced. In the usual method of using the Wheatstone bridge three of these resistances are known and the value of the fourth is easily computed from the above relation as soon as a balance is obtained.

79. The Slide Wire Bridge—Simple Method.—The Wheatstone bridge principle is used in several forms of apparatus

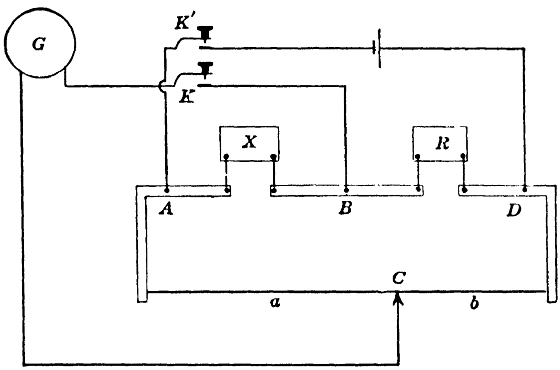


FIG. 44.—The simple slide wire bridge.

for the measurement of resistance. The simplest of these is the slide wire bridge as shown in Fig. 44. The unknown resistance which is to be measured is placed at X, while at R is the known resistance, usually a box of coils. The branch ACD consists of a single uniform wire, usually 1 meter in length, stretched alongside or over a graduated scale. This wire should be of a high-resistance alloy. The balance is obtained by moving the contact, C, along the wire until a point is found for which the deflection of the galvanometer is zero when K' and K are closed. This contact should not be scraped along the wire, but always raised, moved to the new point, and then gently but firmly pressed into contact with the wire. Neither should it be used for a key, as the continual tapping will dent the wire and destroy its uniformity. The two keys, K' and K, are both combined into a single successive contact key, often called a Wheatstone bridge key, in which one motion of the hand will first close the battery key, and then, after the currents have been established, will close the galvanometer circuit. The need of such a key is very evident when there is self-inductance in X.

When the point C has been located, we have, as in the preceding article,

$$xI' = apI,$$

where a is the length of the bridge wire from A to C, and p is the resistance per unit length of this wire. x denotes the value of the resistance of X. Similarly,

$$RI' = bpI,$$

where b is the length of the bridge wire from C to D. In this discussion the resistances of the heavy straps are neglected and A and D are considered as being at the ends of the bridge wire. Dividing the first equation by the second gives

$$\frac{x}{R} = \frac{a}{b}.$$

Thus the ratio of the unknown resistance to R is given by the ratio of the two lengths into which the bridge wire is divided by the balance point.

This relation can be expressed in terms of a single length, a, by writing

$$b = c - a,$$

where c denotes the total length of the bridge wire. Then

$$x = R\frac{a}{c - a} = R\frac{a}{1{,}000 - a},$$

if the total length of the bridge wire is 1,000 mm.

Measure in this way the resistances of two or more coils. Also, measure the same coils when joined in series and compare the result with the computed value,

$$R = R' + R''.$$

When two coils are joined in parallel, the measured resistance should fulfil the relation,

$$\frac{1}{R} = \frac{1}{R'} + \frac{1}{R''}.$$

Problems

1. Exchange the positions of the battery and the galvanometer and then deduce the formula for x, as above.

2. Prove that for three resistances in series

$$R = R_1 + R_2 + R_3$$

and in parallel

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}.$$

3. Deduce the corresponding expressions for five resistances.

80. Calibration of the Slide Wire Bridge.—In deducing the formula for the slide wire bridge it was assumed that the bridge wire was divided into 1,000 parts of equal resistance, and that the readings obtained from the scale corresponded to these divisions. To make sure that the scale readings do thus correspond to the bridge wire, it is necessary to calibrate the wire, that is, to determine experimentally what readings on the scale correspond to the 1,000 equiresistance points on the wire.

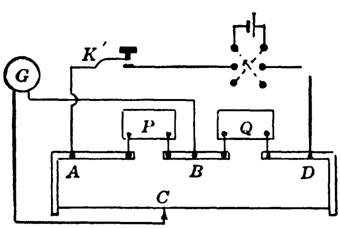


FIG. 45.—Calibration of the simple bridge.

Two well-adjusted resistance boxes are inserted in the two back openings of the bridge, and the battery and galvanometer connected in the usual manner. Only the battery key is needed in this calibration, as good resistance boxes are non-inductive. Closing the galvanometer key often gives a small deflection due to thermal E.M.FS., and this deflection interferes with the determination of the balance point. It is better, therefore, to keep the galvanometer circuit closed, and observe the deflection when the battery circuit is opened and closed. If now, for example, 500 ohms are put in each box, the balance point will be at the middle of the bridge wire and this should be at the point marked 500 on the scale. If it is not, but falls a distance, f, below 500, then f is the correction which must be added to the observed reading to obtain the true reading. Since there may be thermal currents in the galvanometer circuit, this value of f should be computed from the mean of two readings, one taken with the battery current direct and the other taken with the battery reversed.

In the same way the true location of the points 100, 200, 300, 400, 500, 600, 700, 800, and 900 can be found. It will be found convenient to keep the sum of P and Q always constant at 1,000 ohms.

Finally, a calibration curve is drawn, with the readings of the scale for abscissæ and the corresponding corrections for ordinates. The corrections for any point on the scale can then be read directly from the curve. The corrected readings are thus expressed in thousandths of the total bridge wire, including all the resistance of straps, connections, etc., between the two points where the battery is attached.

CALIBRATION OF BRIDGE NO. ——

P	Q	Scale readings			True reading	Cor.
		Battery direct	Battery reversed	Mean		

81. Double Method of Using the Slide Wire Bridge.—In the simple method above, only a single balance point was obtained, and the value of the unknown resistance was computed from the relation,

$$\frac{x}{R} = \frac{a}{1{,}000 - a},$$

where a denotes the reading on the scale at the point of balance, and is assumed to be the length of one portion of the bridge wire.

The measurement of resistance will be more precise if x and R are exchanged with each other, without, however, changing the value of either one, and a new balance point determined. This second balance point will be, say, at a' on the scale, and

$$\frac{x}{R} = \frac{1{,}000 - a'}{a'}.$$

Combining these two equations by the addition of proportions,

$$\frac{x}{R} = \frac{1{,}000 + (a - a')}{1{,}000 - (a - a')} = \frac{1{,}000 + d}{1{,}000 - d},$$

in which the actual values of a and a' do not appear, but only their difference. Thus all questions regarding the starting point of the scale or the wire are eliminated, and if d is small, any error made in its determination will have only a small effect upon the value of x as computed from this equation.

82. Advantages of the Double Method.—When it is desired to use the slide wire bridge with some degree of precision, several precautions are necessary in order to avoid the principal errors. Prominent among these are the effects due to thermal E.M.F. in the galvanometer circuit, to balance which it is necessary to set the sliding contact on the wire at a point somewhat to one side of the true balance point to obtain zero deflection of the galvanometer. If the scale is displaced endwise with respect to the wire, or if the index from which the readings are taken is not exactly in line with the point at which contact is made on the wire, the effect is much the same.

Let a' denote the observed reading on the scale, and $a' + f$ the true balance point, where f denotes the displacement of the reading due to the causes noted above. The actual value of f is unknown, but it is constant in amount and sign, at least while one set of readings is being taken. For a balance of the bridge with x and R in the positions shown in Fig. 44, we have

$$\frac{x}{R} = \frac{a' + f}{c - (a' + f)},$$

where c is the total length of the bridge wire expressed in the same units as a' and f—usually in millimeters.

Exchanging x and R gives a new balance at a'', and

$$\frac{x}{R} = \frac{c - (a'' + f)}{a'' + f}.$$

Combining these two expressions by the addition of proportions gives

$$\frac{x}{R} = \frac{c + (a' - a'')}{c - (a' - a'')} = \frac{c + d}{c - d}.$$

It is seen that f has been eliminated by this double method and the only measured quantity appearing in the final expression is d, the length of the wire between the two *observed* balance points. The value of c should be determined by the method given in Art. 99, as this may be greater than the meter of bridge wire because of the added resistance of the copper straps and the connections at each end of the wire. The "bridge wire" really includes all of the resistance from A to D. However, if d is small, a slight uncertainty in the value of c will produce a negligible error in the computed value of x. This means that R should be taken as near to the value of x as is convenient.

83. The Best Position of Balance. (*a*) *Simple Bridge.*—The formula deduced in Art. 79 for the value of a resistance measured by the simple slide wire bridge is

$$x = R\frac{a}{c - a}. \qquad \text{(A)}$$

Suppose that the value of a can be read to within $\pm h$ mm. of its true value. This uncertainty will be the same at one part of the scale as at any other, but its effect in the formula depends upon the value of a. Using a value of a that is uncertain by $\pm h$ gives

$$R\frac{a \pm h}{c - (a \pm h)} = x \pm f, \qquad \text{(B)}$$

where f denotes the corresponding error introduced into x. It is required to find the value of a that will make the effect of $\pm h$ as small as possible.

Subtracting (A) from (B) leaves

$$\pm f = R\left(\frac{a \pm h}{c - (a \pm h)} - \frac{a}{c - a}\right) = R\frac{\pm hc}{(c - a \mp h)(c - a)}. \qquad \text{(C)}$$

While the uncertainty h cannot be neglected in computing the value of x, in finding the small quantity f it is near enough to call $c \mp h = c$. Then,

$$f = R\frac{hc}{(c - a)^2}. \qquad \text{(D)}$$

In the notation of the calculus, the same result is obtained. When h is small, f corresponds to dx and h to da. Differentiating (A) gives

$$\frac{f}{h} = \frac{dx}{da} = R\frac{c}{(c - a)^2}$$

the same as before. But the actual error made in finding the value of x is not of as much importance as the relative error. It is evident that an error of 1 ohm in a total of 10 ohms is a very different thing from an error of 1 ohm in 1,000 ohms. The relative error is the ratio of the actual error to the total quantity measured. Thus from (A) and (D) the relative error, e, is

$$e = \frac{f}{x} = h\frac{c}{(c - a)a}.$$

From this it appears that even the relative error is not the same for the same error in reading, but it depends upon the value of a. Examining this expression for a minimum value of e,

$$\frac{de}{da} = \frac{-c(c - 2a)}{((c - a)a)^2} h = 0.$$

This is satisfied if $(c - 2a) = 0$.

Thus in reading the value of a, a given error (say 1 mm.) will produce the least effect on the computed value of x when the balance point comes at the middle of the bridge wire.

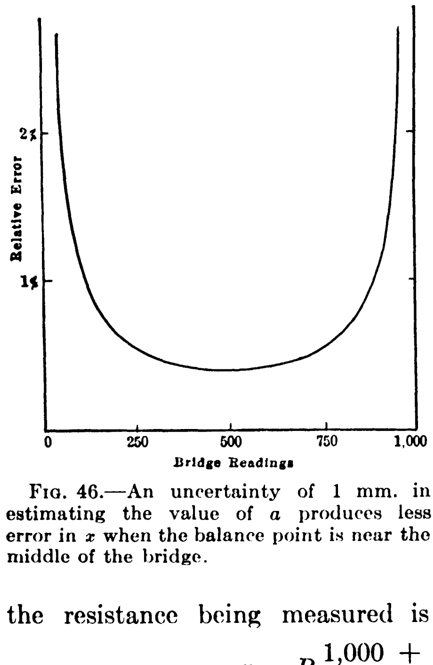


FIG. 46.—An uncertainty of 1 mm. in estimating the value of a produces less error in x when the balance point is near the middle of the bridge.

While this error is smallest at the middle of the bridge wire, it is not much greater when the balance falls at any point on the middle third of the scale. Figure 46 shows how rapidly this error increases when the balance point is near either end of the bridge wire.

(*b*) *Double Method.*—The above discussion applies to the simple slide wire bridge. When the double method is used, the formula for the resistance being measured is

$$x = R\frac{1{,}000 + (a' - a'')}{1{,}000 - (a' - a'')},$$

as deduced in Art. 80.

Writing this as

$$x = R\frac{c + z}{c - z},$$

it can be shown in the same way that a given error in measuring z will have the least effect upon the computed value of x when $z = 0$, that is, when both a' and a'' are at the middle of the bridge wire. In this case

$$\frac{f}{h_s} = \frac{dx}{dz} = R\frac{2c}{(c - z)^2}$$

and

$$e = h_z \frac{2c}{(c - z)(c + z)} = \frac{2c\, h_z}{c^2 - z^2},$$

which evidently is a minimum when $z = 0$.

84. Sources of Error in Using the Slide Wire Bridge.—These may be summarized as

1. Errors in setting, due to:
   a. Thermal currents.
   b. Contact maker not in line with index.
   c. Non-uniform wire or scale.
   d. Ends of wire and scale not coincident.
2. Errors in reading.
   e. The position of balance.
   f. True value of R, loose plugs, etc.

The effect of *a* and *b* can be eliminated by using the double method, as explained in Art. 82.

The only way to avoid the effect of *c* or *d* is by calibration of the bridge wire and correcting all readings.

The error in reading the position of the index after a balance has been found is often greater than the uncertainty of the setting. In the preceding section it was shown that this error, which is about the same for all parts of the scale, has the least effect on the computed value of x when the reading is near the middle of the bridge wire.

The error in the resistance coils of a good box is very small. However, the value of R read from the box and used in computations may be very different from the actual resistance of the experiment. If some of the plugs are loose, or make poor contact because of dirt or corrosion, the resistance may be considerably increased. Moreover, the resistance actually used in the bridge includes all the connections and lead wires used to join the box to the bridge. In the same way the resistance measured includes the lead wires and connections.

85. The Wheatstone Bridge Box.—In the slide wire form of the Wheatstone bridge the balance is obtained by locating a certain point on the wire, and the accuracy of the measurement depends upon the accuracy with which the lengths of the two portions of the wire can be measured. In the Wheatstone bridge box the wire is replaced by a few accurately adjusted resistance coils. Thus, while the number of ratios that can be employed is less than ten, the values of these few ratios are precise, even when the ratio is far from unity. The usual arrangement is to make P and Q (Fig. 43) the two ratio arms with the unknown resistance in X and obtain the balance of the bridge by adjusting the resistance of R. The value of the unknown is then given by the usual relation

$$X = R \frac{Q}{P}$$

and is known as accurately as are the values of P, Q, and R.

In a common form of the Wheatstone bridge box, P and Q each contain 1-, 10-, 100-, and 1,000-ohm coils, thus giving ratios

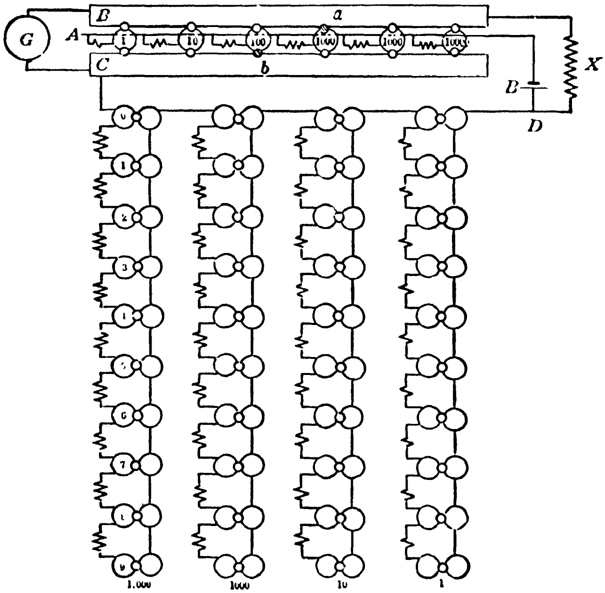


FIG. 47.—Diagram of a decade Wheatstone bridge box.

of 1,000, 100, 10, 1, 0.1, 0.01, 0.001. The rheostat arm, R, can be varied by 1-ohm steps from 0 to 11,110 ohms. This gives a range of measurement of unknown resistances from 0.001 to 11,110,000 ohms. In using such a bridge it is best first to set the ratio arms equal, say to 1,000 ohms each, and obtain an approximate value of the unknown resistance. Then change the ratio to such a value that R may be given in four figures. This will give the resistance of the unknown to four significant figures also.

A more convenient form is the decade bridge. The rheostat arm is arranged on the decade plan with one plug for each decade. The resistance in this arm is indicated by the position of the plugs, which always remain in the box. The ratio arms consist of a single series of coils of 1-, 10-, 100-, 1,000-, 1,000-, 10,000-ohms, but any coil can be used in either arm (but of course the same coil cannot be used in both arms at the same time). The connections not visible are clearly indicated by lines drawn on the top of the box. The different parts should be carefully compared with the diagram of Fig. 42, Art. 77, and the points A, B, C, D located before attempting to use the box. The resistance to be measured is joined to the posts marked X, the battery, and the galvanometer as shown, with a key in each circuit—preferably a successive contact key.

86. To Use a Wheatstone Bridge Box.—When starting to obtain a balance, the ratio arms are each set at 1,000 ohms and an approximate value of the resistance determined. This is done by shunting the galvanometer with the smallest shunt available, and with R set at 1 ohm the keys are quickly tapped and the direction of the deflection noted. The key should not be held down long enough to cause a large deflection, as the direction can be seen from a small one just as well and with less danger to the galvanometer. Next, R is set at 9,000 ohms and the key tapped. Usually this deflection will be in the opposite direction. If it is not, try zero and infinity. Knowing that the value of X lies between 1 and 9,000, say, this range is divided by next trying 100 ohms, and if X is less than this, try 10 ohms. Suppose X is between 10 and 100. This range is divided by trying 50, and so on until it is reduced to a single ohm. Let us suppose that X is found to be between 68 and 69 ohms. Then the ratio arms are changed so as to make R come 6,800 or 6,900. The exact value for a balance is determined by continuing the same process and is found, say, when R is 6,874 ohms. This example then gives $X = 68.74$ ohms.

If the best balance, obtained with no shunt on the galvanometer, still gives some deflection, the next figure for X can be obtained by interpolation, but this is not usually required. If greater accuracy is desired, it is necessary to make a second measurement with the battery current reversed through the bridge. This will reverse some of the errors, and especially the effect of thermal currents in the galvanometer branch. The mean of these two measurements will then be nearer the true value of X than either one alone.

Measure the resistance of several coils and check the results by measuring their resistance when joined in series and parallel. If some of these coils have an iron core, notice the effect of first closing the galvanometer key and then closing the battery key. Remember that the formula for this method was deduced on the assumption that all of the currents were steady, and that there was no current through the galvanometer.

The bridge balances can be recorded as follows:

Object measured	P	Q	R	X

87. The Per Cent Bridge.—A very useful and convenient form of the Wheatstone bridge is the arrangement shown in Fig. 48. When it is desired to compare coils that are nearly equal to each other, the ratio of the unknown resistance to the standard resistance is often more desired than the actual value of the unknown resistance. In the per cent bridge the coils are arranged so that the dial readings give directly the ratio of the unknown resistance to the resistance that it is being compared with.

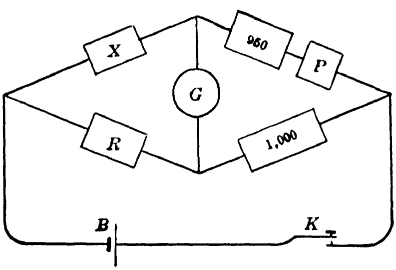


FIG. 48.—The direct-reading per cent bridge.

As shown in Fig. 48, one of the ratio arms of the bridge is a fixed resistance of 1,000 ohms. The other ratio arm consists of a fixed resistance of 950 ohms in series with P, which is a dial resistance box of tens, units, and tenths of ohms. When x and R are equal, it will require 50 ohms in P to give a balance. Mark this setting of P as 100 per cent. If x is 1 per cent larger than R, it will require another 10 ohms in P, and this setting can be marked 101 per cent; 70 ohms would correspond to 102 per cent, etc. If the setting of P

for a balance is 76.1 ohms, it indicates that x is 102.61 per cent of R. If the index on the first dial can be set to read "20" when it stands at 70 ohms, the last three figures, 261, can be read directly from the box. The range of this bridge is from 95 to 105 per cent. By increasing the ratio arms to 10,000 ohms each, the setting of P can be read to thousandths of 1 per cent. If lead wires are needed to connect x to the bridge, the lead wires for R should be made of the same resistance.

88. Location of Faults.—By a fault on a telephone, electric light, or other line, is meant any trouble by which the insulation of the line is impaired, or which interferes with the proper working of the line. The principal kinds of faults are named as follows:

A *ground* is an electrical connection more or less completed between the line and the earth. In the case of a cable any connection from one of the wires to the lead covering of the cable consitutes a ground.

A *cross* is an electrical connection between two wires.

An *open* is a break in the line.

In testing for a fault it is first necessary to determine in which of these classes the given trouble belongs. A testing circuit is made by connecting a battery in series with a voltmeter or other current indicator. The faulty line is then picked out from among the good ones by one of the methods outlined below.

Test for Grounds.—The test to find grounded wires can be made at any point along the line. The battery side of the test-

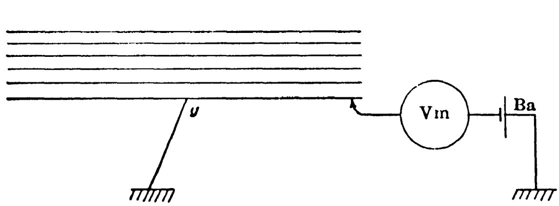


FIG. 49.—Picking out the grounded wire.

ing circuit just described is connected to the ground, and the wire from the voltmeter side is brought into contact successively with each wire to be examined. When the grounded wire is reached, the battery circuit is completed through the earth, and this will be indicated by the deflection of the voltmeter.

Tests of this kind made with alternating current are often unreliable because of the capacitance of the line.

Test for Opens.—Before testing for opens, the distant ends of the wires are joined together and grounded. The test is

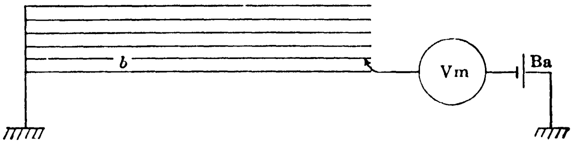


FIG. 50.—Picking out the broken line.

applied at the near end, using the testing circuit in the manner just described. As each wire is tried, the voltmeter will indicate the ground which has been placed on the other end, unless the line is open at some intermediate point. If the line is broken, the pointer of the voltmeter will remain at rest, showing that there is no electrical connection through that wire.

Test for Crosses.—In testing for crosses, the near ends of all the wires are connected together and to one end of the testing circuit. The wires are then disconnected one at a time, and the free end joined to the other end of the testing circuit.

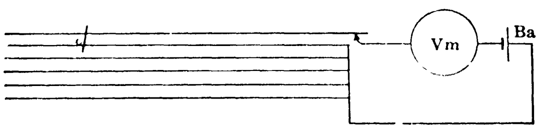


FIG. 51.—Picking out the crossed lines.

A movement of the voltmeter pointer shows that the wire being tested is crossed with one of the other wires. If there is no indication of a cross, the wire is laid at one side and the test continued with other wires until all the good wires have been removed, and only crossed wires remain.

In case there are several sets of crossed wires among those remaining, the near ends of all the crossed wires should be separated. Then connecting them, one at a time, to the battery side of the testing circuit, touch each of the other wires in succession with the wire from the voltmeter. A deflection indicates that the line touched is crossed with the one joined to the battery.

Of course, the wires must not be connected at the other end, or through any switchboard, as this would give the test for a cross.

89. Methods for Locating Faults.—The usual methods for locating faults in a telephone line or cable are based on the principles of the simple slide wire bridge. The following methods are illustrative examples of this kind of measurement and show the general mode of procedure. They are called loop tests, because the wire being tested is joined to a good wire, thus forming a long loop out on one wire and back on the other.

In case the line is open so it cannot be used as one arm of a Wheatstone bridge, the position of the break can be located by comparing the capacitance of the line out to the break with the capacitance of a similar line whose length is known.

90. The Murray Loop.—In the Murray loop method the grounded wire is joined to one end of a slide wire bridge, as

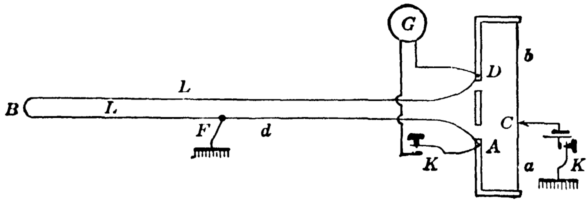


FIG. 52.—Locating the position of the ground, *F*.

shown at *A*, Fig. 52. A second wire of the same length and resistance and similar to the first, but free from faults, is joined at *D* to the other end of the bridge. These two wires are joined together at the far end of the line, thus forming the loop. This loop is divided into two portions by the fault at *F*, which in the figure is assumed to be a ground. These two parts form two arms of a Wheatstone bridge, the other two being formed by the bridge wire, *ACD*. It is usually best to connect the galvanometer and battery in the position indicated, the battery connection at *F* being made through the earth.

Let C indicate the balance point. Let p denote the resistance of 1 mm. of the bridge wire, and p' the resistance of unit length of the wire forming the loop. Then, by the principle of the Wheatstone bridge for a balance,

$$apI = bpI' = (c - a)\,pI',$$

where c $(= a + b)$ denotes the whole length of the bridge wire, and a is the reading on the bridge scale, measured from the end connected to the faulty wire. Similarly,

$$dp'I = (2L - d)\,p'I',$$

from which

$$d = 2L\frac{a}{c},$$

where d is the distance to the fault and L is the length of the faulty wire.

This determination can be checked by exchanging the good and faulty wires in the arms of the bridge. This change requires a slight modification in the formula used in solving for d.

91. Fisher's Method.—It sometimes happens that no good wire like the grounded one can be obtained. It is still possible to locate the fault, provided only that *two* good wires can be obtained, the only requisite being that they extend from the bridge to the far end of the faulty wire.

First make connections as in Fig. 52, using one of the good wires, H, Fig. 53, to complete the loop with the faulty wire. Then, as before,

$$\frac{a}{c} = \frac{dp'}{Lp' + Hp''}, \qquad \text{(A)}$$

where Hp'' is the resistance of the good wire.

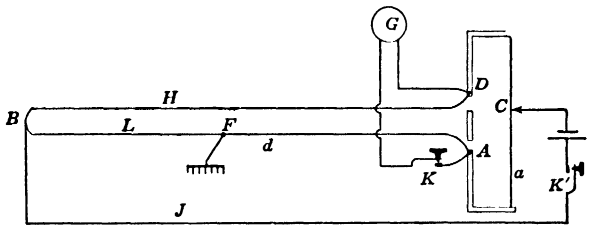


FIG. 53.—Finding the relative resistance of the two lines.

Next use the other good wire, J, to connect the battery to the far end of the line as shown in Fig. 53 and obtain a second balance. The ground at F will make no difference if there is not a second ground at some other point. In this case, if a' is the reading,

$$\frac{a'}{c} = \frac{Lp'}{Lp' + Hp''}. \qquad \text{(B)}$$

From (A) and (B),

$$\frac{a}{a'} = \frac{d}{L} \qquad \text{or} \qquad d = L\frac{a}{a'},$$

where, as before, d is the distance to the fault and L is the length of the faulty wire.

92. Location of a Cross.—The methods for locating a cross are similar to those just given for locating a ground. The only difference is that instead of connecting the battery to F by means of the earth, as shown in Fig. 52, the connection is now made by means of the wire which is crossed with the line AB.

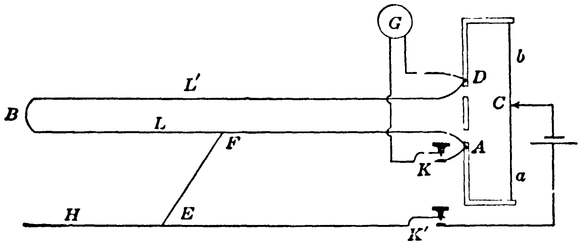


FIG. 54.—Locating the position of the cross, EF.

The balance point is found the same as before, and the distance to the fault computed by the formulas given above. Figure 54 shows the Murray loop for locating the cross EF between the lines L and H. Of course the location of the cross could have been made equally well by using line H in the bridge in the place of L.

93. Location of a Cross. *Wires Unlike.*—In case the lines L and L' are unlike, it will be necessary to use another line, J, as shown in Fig. 55 and obtain a second balance as in the preceding method, Art. 91. The distance to the cross is then given by the formula $d = L\frac{a}{a'}$, derived as before.

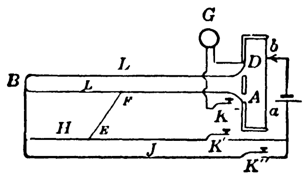


FIG. 55.—Locating the position of a cross when the wires are all unlike.

94. The Varley Loop Test.—It is often convenient to make the test for fault location by the use of a Wheatstone bridge box or a portable testing set, instead of using the slide wire bridge. In the Varley loop method the far end of the grounded wire is connected to a good wire, and the circuit or loop thus formed is connected into the Wheatstone bridge as the unknown arm. This arrangement is shown in Fig. 56. The usual balance of the bridge gives the resistance, S, of this complete loop.

If now the battery connection is changed from D to the ground, which, in effect, means that the battery is connected at F, and the bridge again balanced, the relation is

$$\frac{S - x}{R + x} = \frac{P}{Q} = A,$$

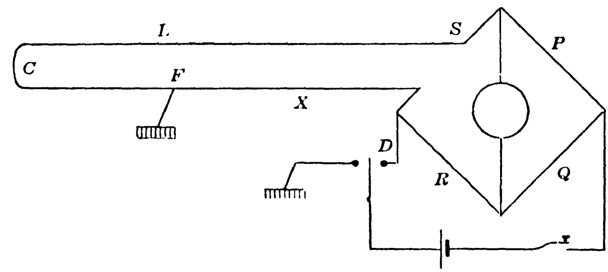


FIG. 56.—Varley loop method for locating the ground at F, using a Wheatstone bridge box (Fig. 57).

where x is the resistance of the line from the bridge out to the ground at F, and A is written for the ratio of the arms P and Q. Usually A is 0.1 or 0.01. This gives

$$x = \frac{S - AR}{1 + A}.$$

Knowing the number of ohms per meter or the ohms per mile for this wire, the distance out to F is readily computed (by division). In case both of the wires forming the loop are alike, the distance from the bridge end of the line to the ground at F is

$$d = \frac{x}{S} L,$$

where L is the total length of the loop. In practice this method is readily applied, as it requires only two balances of the bridge testing set, and a simple computation gives the distance, d.

In locating the position of a cross between two wires, the only difference in procedure is that the battery is connected to the line that is crossed with the one being tested, instead of being connected to the ground.

95. The Fault Finder.—Inasmuch as the meter slide wire bridge is awkward to use, except on a laboratory table, the essential parts are arranged in a compact box for field use.

One form of such a portable testing set is shown in Fig. 57. The ratio arms of the bridge are varied by turning the left hand dial, and the other four dials control the known resistance, R. The resistance to be measured is connected to the large binding posts at the front of the box. The battery and galvanometer are contained within the box.

This testing set is not only a portable Wheatstone bridge, but, by means of the knife switches at the left, the connections can be quickly changed for the Murray or Varley loop methods.

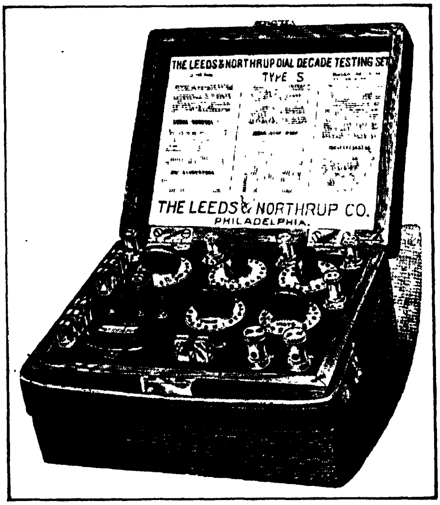


FIG. 57.—Portable testing set.

96. Location of Opens.—When one of the lines is broken, it will not be possible to use it for one arm of a Wheatstone bridge. If the line is one of a pair, it may have sufficient capacitance to be measured. The simplest way is to charge one piece of the broken wire and discharge it through a ballistic galvanometer. This deflection is compared with the deflection obtained when a known length of a similar wire is charged and discharged in the same way. Then

$$d = L\frac{d'}{d''}.$$

Usually a more exact determination can be made by the bridge method (Art. 195). Let *ac* and *ef* represent the two wires of a pair, of which *ac* is broken at the point *b*. A similar pair is shown by *mn* and *hj*. The line *mn* and the part *bc* of the broken line are joined to the non-inductive resistances R' and R'', as shown. The other wires of these pairs, and the remainder of the broken wire, are joined to the ground. An induction coil, *I*, or some other sources of alternating E.M.F., is used to charge the lines through the resistances. When the latter are adjusted to give a minimum sound in the telephone T,

$$d = L\frac{R''}{R'}.$$

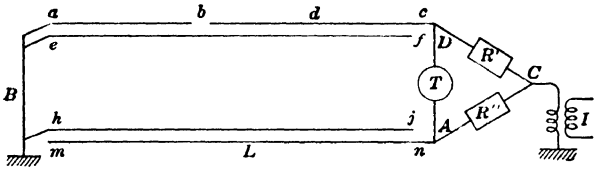


FIG. 58.—Finding the distance out to the break at b.

97. Resistance of Electrolytes.—When a current flows through an electrolyte it is accompanied by a separation of the substance in solution. The negative ions move in the same direction as the electron current, while the positive ions travel in the opposite direction, each being liberated at the electrodes. In general, this action causes polarization, which tends

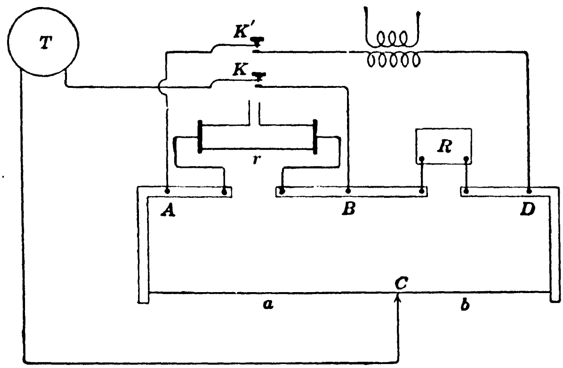


FIG. 59.—Resistance of an electrolyte.

to oppose the flow of current. In order, therefore, to measure the resistance of an electrolyte, it is necessary to employ an alternating current. This can be most readily obtained from a small induction coil.

The electrolyte is placed in a suitable cell, and made the fourth arm of a Wheatstone bridge, the induction coil being used in place of the usual battery. The resistance of the electrolyte can then be determined by the Wheatstone bridge method in the usual way, and when the bridge is balanced,

$$r = R\frac{a}{b}.$$

Since an alternating current is employed, this balance can be found by means of a telephone receiver connected in the usual place for a galvanometer. For purposes of instruction, the best form of cell for holding the electrolyte is a cylindrical tube with a circular electrode closing each end. The resistance measured by the bridge is then the resistance of the electrolyte between the two electrodes, and knowing the resistance of this column of the electrolyte, the resistivity, s, of the solution can be calculated the same as for metallic conductors, or,

$$s = r\frac{A}{L},$$

where A is the cross-section of the tube containing the solution and L is the distance between the electrodes.

The conductivity of the solution, c, is the reciprocal of this, or

$$c = \frac{1}{s} = \frac{L}{rA}.$$

Since the resistance of an electrolyte, or, more strictly, its conductivity, depends upon the amount of the substance in solution—that is, upon the number of ions per cubic centimeter—if we wish to compare the conductivities of different electrolytes it is necessary to express the concentrations in terms of the number of ions per cubic centimeter. This is usually stated in terms of the number of gram-molecules of substance that are dissolved in one liter of the solution. For the purpose of this experiment it is necessary to express the concentrations in terms of the number of gram-molecules in one cubic centimeter of the solution. The molecular conductivity, μ, of an electrolyte is then defined as the conductivity per gram-molecule of salt contained in each cubic centimeter of solution.

$$\mu = \frac{c}{m} = \frac{1}{ms} = \frac{L}{mrA} = \frac{bL}{amRA}$$

where m = number of gram-molecules in one cubic centimeter of the solution.

The most interesting application of the conductivity of solutions is the knowledge it gives regarding the degree of dissociation of the dissolved substance. The conductivity of an electrolyte is due entirely to the ions it contains and is directly proportional to the number of ions per cubic centimeter. Most salts are completely dissociated in very dilute solutions, and therefore the molecular conductivity of such solutions is not increased by further dilution. Call this value μ_0. Then if μ denotes the molecular conductivity of a more concentrated solution of the same salt, the relative dissociation in the solution is

$$\alpha = \frac{\mu}{\mu_0}.$$

Express results by means of a curve, using values of μ for ordinates and the corresponding values of $\frac{1}{m}$ (= number of cubic centimeters containing one gram-molecule) as abscissæ.

CHAPTER V

THE WHEATSTONE BRIDGE (*Continued*)

98. The Slide Wire Bridge with Extensions.—The measurement of resistances by the slide wire bridge can be made with more precision by using a longer bridge wire. The uncertainty in locating the balance points probably will be about the same, but since the distance, $a' - a''$, between the two balance points is increased, the percentage error will be less.

As it would be inconvenient to have the apparatus much over a meter in length, and as only the middle portion of the bridge wire is used in making careful measurements, the effective length of the bridge wire is increased by adding a resistance at each end. These extensions may consist of known lengths of wire similar to that used for the bridge wire—or any two equal resistances may be used and their equivalent lengths determined experimentally by the method shown below. The meter of wire provided with a scale then becomes only a short portion along the middle of the total length of the bridge wire. While this arrangement makes possible a greater precision of measurement, it also lessens the range of the bridge, as only those balances which fall on this limited section of the wire can be read.

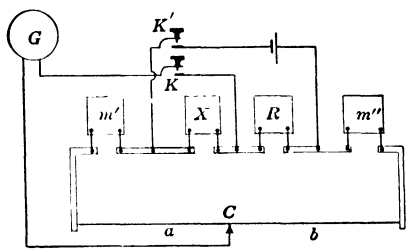


FIG. 60.—Bridge with extensions, m' and m''.

The extensions are placed in the outside openings on the back of the bridge, between the ends of the bridge wire and the battery connections. They should be nearly equal. Let m' and m'' denote the number of millimeters of bridge wire having the same resistances as each extension, respectively, and let L denote the total length of the bridge wire including both of its extensions.

With the resistance, x, to be measured, and the known resistance, R, in the middle openings of the bridge, as shown in Fig. 60, the first balance point is found. The reading on the scale at this point will be called a'. Then

$$\frac{x}{R} = \frac{m' + a'}{m'' + b'} = \frac{m' + a'}{L - (m' + a')}. \tag{1}$$

Exchanging the positions of x and R, and calling the scale reading at the new balance point a'',

$$\frac{x}{R} = \frac{L - (m' + a'')}{m' + a''} \tag{2}$$

And by addition of proportions,

$$\frac{x}{R} = \frac{L + (a' - a'')}{L - (a' - a'')} = \frac{L + d}{L - d}. \tag{3}$$

It is evident that this arrangement reduces the range of the bridge, for only those values of x can be measured which are near enough equal to R to give balance points on the scale. But what is lost in range is more than made up in the greater precision of measurement.

Dividing out the fraction in (3) gives

$$\frac{x}{R} = 1 + 2\frac{d}{L} + 2\frac{d^2}{L^2} + 2\frac{d^3}{L^3} + \cdot \tag{4}$$

All the terms after the second are negligible if d is small in comparison with L, so that

$$x = R + 2R\frac{d}{L}. \tag{5}$$

The only part of x, then, which is measured by the bridge is the second term, and a small error in it will only slightly affect the computed value of x.

99. To Find the Length of the Bridge Wire with Its Extensions. The total length of the bridge wire, including the extensions at each end, can be determined as follows. A good resistance box, P, is used in place of the unknown resistance, x, shown in the figure of the preceding section. Then with both extensions connected in the bridge, the values of P and R are adjusted to bring the balance point near one end of the scale. Let a' denote this scale reading, corrected if necessary by the calibration curve for this wire when used as a simple bridge. Then

$$\frac{P}{P + R} = \frac{m' + a'}{m' + c + m''} = \frac{m' + a'}{L},$$

where c denotes the original length and L the total length of the bridge wire.

Exchanging P and R, the balance falls near the other end of the wire and

$$\frac{R}{P + R} = \frac{m' + a''}{L}.$$

By subtraction,

$$\frac{P - R}{P + R} = \frac{a' - a''}{L},$$

whence

$$L = \frac{P + R}{P - R}(a' - a'').$$

This method may also be used to determine whether there is any extra resistance in the straps and connections at the ends of the usual meter of bridge wire.

Notice that the value of L can be determined only as accurately as the distance, $d = a' - a''$, can be measured. Therefore, this distance should be as long as can be conveniently measured on the scale, say 85 or 90 cm. The reason for making d small in Art. 98 does not apply here.

Problem.—In calibrating a bridge wire it was found that for $P = 100$, and $Q = 900$ ohms, the balance point fell at 95 on the scale; while for $P = 900$ and $Q = 100$ the balance point was at 903. What is the effective length of the bridge wire? *Ans.* $L = 1{,}010$ mm.

100. To Calibrate the Slide Wire Bridge with Extensions.—The formula deduced for this method in Eq. (5) above works very well as long as x and R are nearly equal; but several errors may occur in its use, the principal of which are:

1. Using a wrong value for L.
2. Neglecting all the terms containing d in powers higher than the first.
3. Errors in the determination of d, due to non-uniform bridge wire, scale errors, etc.

The method of calibration described below corrects for all of these errors at once by finding a correction to be added to the observed value of d, which will give to $\left(1 + 2\frac{d}{L}\right)$ the true value of $\frac{x}{R}$.

With the bridge set up as shown in Fig. 60, with the extensions in place, and two good resistance boxes, P and Q, in place of x and R, we have

$$P = Q\left(1 + \frac{2d'}{L}\right) \tag{A}$$

and solving for d' gives

$$d' = \frac{L}{2Q}(P - Q). \tag{B}$$

This is the value that d' must have in order that (A) shall give the correct values of the resistances being measured.

Starting with P and Q each 1,000 ohms, the value of d should be zero. Then increasing P by successive small steps, the corresponding observed values of d can be determined. These observed values of d will not agree with the values of d' computed from Eq. (B) above, and therefore if used in Eq. (A), will not give the correct values for P. This is because of the errors noted above. It is, therefore, necessary to add to the observed length of bridge wire, d, a certain amount, h, such that

$$d + h = d'$$

and this corrected value, d', should be used in Eq. (5) above.

In the present case where P and Q are known, the values of d' are computed from Eq. (B), while the corresponding values of d are observed on the bridge wire. The differences give the values of h, and a calibration curve can be drawn, as shown, which will give the correction to be used at each point. The correction increases rapidly with d, owing to the increasing importance of the second error noted above.

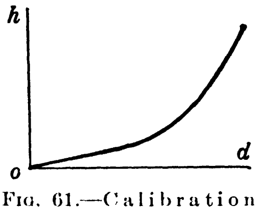


FIG. 61.—Calibration curve.

In case d is not zero when P and Q are nominally equal, it means that one is really a little larger than the other. Let d_o denote this value of d. This can be reduced to zero by adjusting the value of Q; or, if it is more convenient, d_o may be subtracted from each value of d throughout this calibration, as a sort of zero correction.

The readings may be recorded as below:

TO CALIBRATE BRIDGE NO. .... WITH EXTENSIONS NO. .... AND NO. ...

P	Q	a'	a''	d $=a' - a''$	d' Eq. (B)	h $=d' - d$

101. Measurement of Resistance by Carey Foster's Method.—One of the most exact methods for comparing two resistances is the one devised by Prof. Carey Foster of England. The Wheatstone bridge is arranged in the same manner as was used for the slide wire bridge with extensions, except that now the extensions become the resistances to be compared.

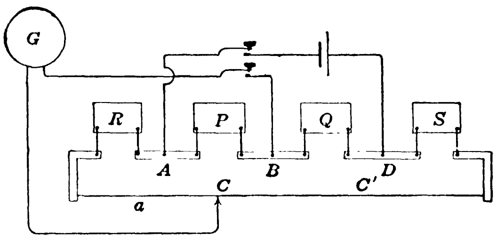


FIG. 62.—Carey Foster bridge, comparing R and S.

Thus, in the figure, let S be the resistance that is to be compared with R. These two are placed in the bridge as shown, being connected together by the bridge wire. The other arms of the bridge, P and Q, become merely ratio coils and may have any value, although they must be nearly equal to each other in order that the balance may fall on the bridge wire, and for the most sensitive arrangement *all the arms* of a Wheatstone bridge should be nearly equal. For the greatest accuracy, R and S should each be in a constant-temperature oil bath.

Let the balance point be found by moving the contact, C, along the wire until a point is reached for which there is no deflection of the galvanometer. Let a' be the scale reading at this point. It is immaterial whether this scale extends the entire length of the bridge wire or not.

Now let R and S be exchanged with each other. This will make no difference in the total length of the extended bridge wire, ACD. But if the resistance of S is less than that of R, the new balance point will not fall at the same point as before, for it will be necessary to add to S enough of the bridge wire to make the part AC the same as before. The resistance of AC in the first instance is

$$R + a'p = \frac{P}{P + Q} T.$$

After R and S have been exchanged, the resistance of AC' is

$$S + a''p = \frac{P}{P + Q} T,$$

where T denotes the total resistance of ACD. Equating these two expressions,

$$R + a'p = S + a''p,$$

and

$$S = R - (a'' - a')p,$$

where p is the resistance per unit length of the bridge wire.

102. Correction for Lead Wires or Other Connections.—If either S or R is connected to the bridge by wires or other connections, the resistance of these wires will be included in the measured values. If S and R denote the values of these resistances, exclusive of the lead wires, the last equation above should be written

$$S + s = R + r - (a'' - a')p,$$

where s and r denote the resistances of the connections.

If S and R can be removed or short-circuited, leaving only the connections, a second pair of balances will give

$$s = r - (a_2 - a_1)p.$$

Subtracting this from the first equation gives

$$\begin{aligned} S &= R - [(a'' - a') - (a_2 - a_1)]p \\ &= R - (d' - d_1)p \end{aligned}$$

where d_1 and d_2 are the differences in the balance points in the two cases.

103. The Carey Foster Bridge.—One form of the Carey Foster bridge is shown in Fig. 63. Three separate bridge wires are provided to give a choice in the value of p that can be used. The coil holder consists of a hard rubber base, upon which are mounted massive copper bars for holding the resistance under test and the comparison resistance, and for connecting these resistances with the other parts of the bridge. There is also a

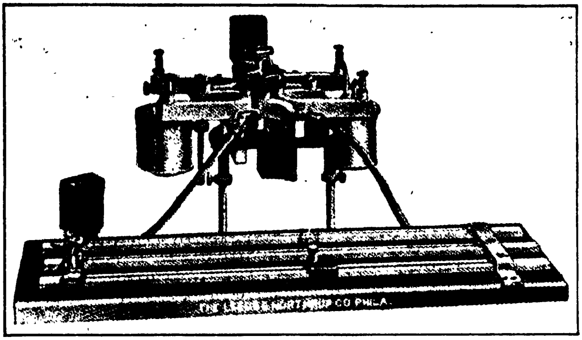


FIG. 63.—Carey Foster bridge, showing the coil-holder and commutator.

commutating device for interchanging the standard and the coil under comparison. Connection to the bridge wire is made by two large flexible copper cables.

104. To Determine the Value of p.—The resistance per unit length of bridge wire can be readily determined by the same arrangement as shown above. Let R and S be two resistances that are nominally equal. When the two balances are obtained,

$$S = R - (a'' - a')p,$$

as shown above.

Now let S be shunted by a rather large resistance, P, so as to give a small but definite change in this resistance. Then

$$\frac{SP}{S + P} = R - (a_2 - a_1)p,$$

where a_1 and a_2 are the new balance points. By subtraction,

$$S - \frac{SP}{S + P} = [(a_2 - a_1) - (a'' - a')]p$$

and

$$p = \frac{S^2}{(S + P)(d_1 - d')}$$

The resistances S and R should be well known, but extreme accuracy is not essential in order to obtain a fair degree of accuracy in the values of p.

105. Comparison of Two Nearly Equal Resistances by Substitution.—The Wheatstone bridge box, described in Art. 85, is a fairly accurate instrument for the measurement of resistance. Its accuracy is limited by the accuracy of its own coils and the uncertainty in the connections to the unknown resistance. It is possible, however, to use such a box for measurements that are more accurate than its own coils.

Thus, suppose that we wish to measure a resistance that is slightly less than 100 ohms, and there is available a standard 100-ohm coil whose resistance is accurately known. With the unknown resistance connected in the Wheatstone bridge, the latter is balanced as exactly as possible. If the best balance is not exact, the deflection of the galvanometer is carefully observed and recorded. Then, without changing the bridge or moving any of the plugs, the standard resistance, S, is substituted in place of the unknown resistance, X. If the key is now closed, the bridge will be found unbalanced, because, as we have supposed in this case, S is larger than X. When this is found to be the case, a high resistance box, R, is connected in parallel with S and the resistance adjusted until the galvanometer gives the same deflection as was observed for the best balance with X. Then the combined resistance of S and R must now be equal to X, or

$$X = \frac{SR}{S + R} = S\left(1 - \frac{S}{S + R}\right).$$

The actual values of the other arms of the bridge are thus immaterial.

An uncertainty in the resistance of the shunt, R, will only affect the value of the fraction $\frac{S}{(S + R)}$, which is small compared with 1. When X is nearly equal to S, thus requiring a large value of R to balance the bridge, the uncertainty due to R will be slight.

In case the unknown resistance is larger than the standard, the shunt should be applied to it. Then

$$S = \frac{XR}{X + R} \qquad \text{and} \qquad X = \frac{SR}{R - S}.$$

106. Precise Comparison of Two Resistances.—The last method leads directly to a method for the accurate comparison of two resistances that are nearly equal.

Let the two resistances to be compared form two arms, S and R, of a Wheatstone bridge. The other arms may be two resistances, A and B, of about the same nominal resistance as S. The more nearly $A = B = S$ the better, but the exact values do not enter in the measurement. The bridge is balanced by adjusting one or both of the high-resistance shunts, P and Q, that are in parallel with R and S respectively. When R is not closely equal to S, the balance may be obtained by the use of only one of the shunts. If R is only slightly larger than S, the value of Q necessary to give a balance would be very high, and possibly beyond the range of any available resistance box. In this case a moderate resistance can be used in P, making the resistance of this arm equal to

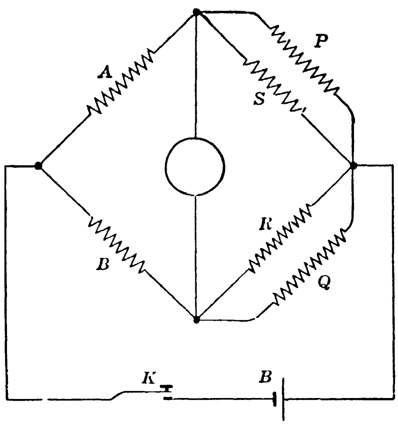


FIG. 64.—The balance is obtained by adjusting the shunts, P and Q.

$$S_1 = \frac{SP}{S + P}.$$

It will then be possible to obtain a balance by adjusting Q, giving for the resistance of R and Q the value

$$R_1 = \frac{RQ}{R + Q}.$$

When the bridge is thus balanced we have

$$\frac{A}{B} = \frac{S_1}{R_1},$$

where the ratio $\frac{A}{B}$ is a constant but unknown quantity nearly equal to unity.

For the next step the resistances A and B are interchanged with each other without altering the rest of the bridge. This, in general, will upset the balance of the bridge, but the latter can be restored by readjusting the shunts to new values, P' and Q'. We will now have

$$\frac{A}{B} = \frac{R_2}{S_2},$$

where

$$R_2 = \frac{RQ'}{R + Q'} \quad \text{and} \quad S_2 = \frac{SP'}{S + P'}.$$

Equating this to the former expression for $\frac{A}{B}$ eliminates this unknown ratio, giving

$$\frac{S_1}{R_1} = \frac{R_2}{S_2}$$

or

$$\frac{R}{S} = \sqrt{\frac{PP'}{QQ'}\frac{(R + Q)(R + Q')}{(S + P)(S + P')}}.$$

In computing the value of this factor it is permissible to use the nominal values of S and R, since these are added to the large resistances P and Q.

The advantages of this method are apparent when it is seen that all uncertainty in the ratio of the arms A and B is completely eliminated, and, since P and Q enter in both numerator and denominator, the uncertainties in their values will have only a slight effect on the result. Of course, the usual precautions against thermal E.M.FS. and changes in temperature should be observed.

In some cases it is better to use the shunts on the ratio arms of the bridge, thus varying the ratio to equal $\frac{R}{S}$.

107. Measurement of Low Resistance.—It is often necessary to measure a small resistance, either in the form of a short length of a metal rod or a coil of low resistance. The determination of the resistivity of a metal usually requires the accurate measurement of a bar of the material.

The usual forms of the Wheatstone bridge are not well adapted for the measurement of such very small resistances. This is because of the uncertain resistances of the contacts and connections by which the small resistance is joined into the bridge. If the measured resistance is small, it is evident that a small contact resistance may introduce a relatively large uncertainty in the computed result.

108. Millivoltmeter and Ammeter Method.—If a sensitive millivoltmeter is available, the resistance of a sample piece of wire, MN, Fig. 65, can be measured by the ammeter-voltmeter method. As shown in Fig. 65, the voltmeter measures the fall of potential between *m* and *n*. Since the main current does not pass through the contacts at *m* and *n*, the resistance of these contacts will not affect the fall of potential between *m* and *n*. These contact resistances are, in fact, a portion of the volt meter circuit and therefore, are negligible. The resistance measured by this method is, then, the resistance of that portion of the wire between *m* and *n*.

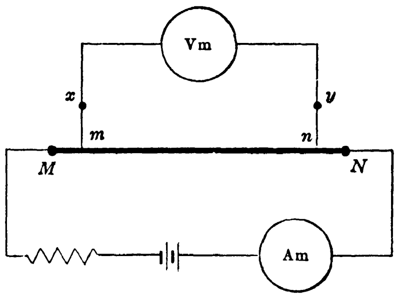


FIG. 65.—Measurement of *mn* by the ammeter-voltmeter method.

109. Four-terminal Resistances.—If it is desired to preserve this particular sample, *mn*, Fig. 65, as a fixed and constant resistance, the potential wires at *m* and *n* should be soldered to the main wire, *MN*, at *m* and *n*. Then each time it is employed the same resistance, *mn*, will be used. The voltmeter is disconnected at *x* and *y*, and the current circuit is

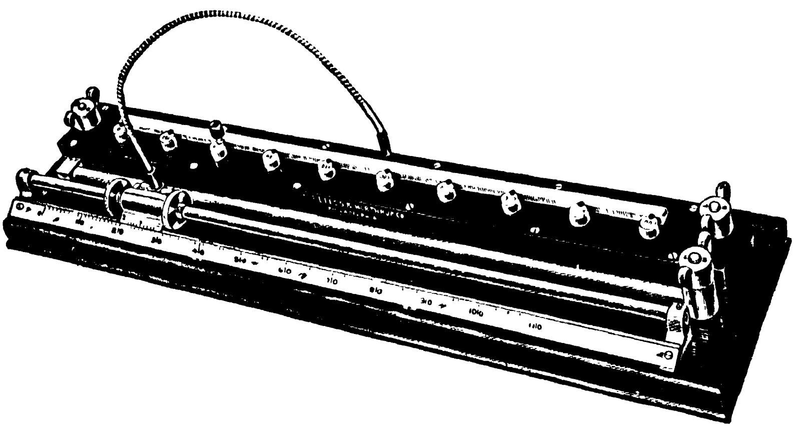


FIG. 66.—Variable low-resistance standard.

broken at *M* and *N*. All contact resistances in the measured portion of the wire are thus eliminated. Such fixed resistances are called "four-terminal resistances." Standards of low resistance are always thus provided with four terminals, two for current terminals and two for potential terminals.

110. Low-resistance Standard.—An external view of a standard of low resistance is shown in Fig. 66, and the diagram of connections is given in Fig. 67. *AB* is a heavy piece of resistance metal of uniform cross-section and uniform resistance per unit of length; *CD* is another piece of resistance metal of smaller cross-section, and the two are joined together by a heavy copper bar, *AC*, into which both are silver soldered; *LL* are the current terminals and *PP* are the potential terminals. The resistance of *AB* between the marks 0 and 100 on the scale, *S*, is 0.001 ohm.

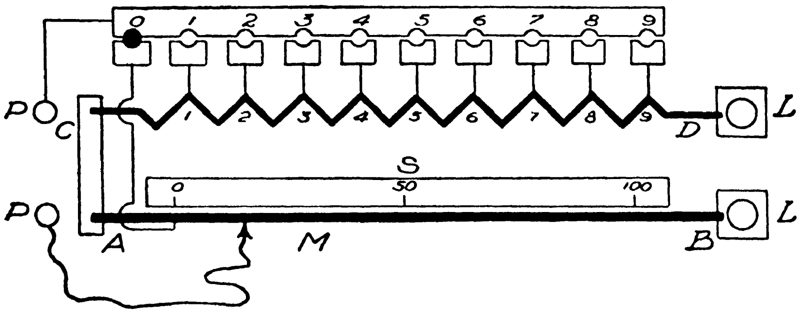


FIG. 67.—Diagram of a variable standard low resistance.

From the point 1 on the resistance, *CD*, to 0 on *AB* is also 0.001 ohm, from 2 to 0 is 0.002 and so on, and from 9 to 100 is the total resistance of 0.01 ohm. The slider, *M*, moves along the resistance, *AB*, and its position is read on the scale, *S*, which is subdivided into 100 equal parts and can be read by a vernier to thousandths. Subdivided in this way the resistance between the tap-off points, *PP*, may have any value from 0.000001 to 0.01 ohm by steps of 0.000001 ohm.

There is a binding post on the bar, *AC*, which makes it possible to use *AB* separately. On account of its large current-carrying capacity, this bar is a very satisfactory standard resistance for measuring current by the potentiometer method.

111. The Kelvin Double Bridge.—While low resistances can be measured by the use of a millivoltmeter and an ammeter, the accuracy of the determination is limited by the accuracy with which the deflections of the instruments can be read. The best and most precise method for the measurement of low resistances is that of the Kelvin double bridge. This is a modification of the Wheatstone bridge, and being a null method, the results do not depend upon the readings of any instrument.

The general arrangement is shown in Fig. 68. The resistance to be measured is shown by X, with current terminals at M and N, and potential terminals at x and y. R is the standard low resistance with current terminals, S and T, and potential terminals, v and z. R and X are joined at J with as good connection as practicable and, together with P and Q, constitute the arms of a Wheatstone bridge. In the ordinary Wheatstone bridge the

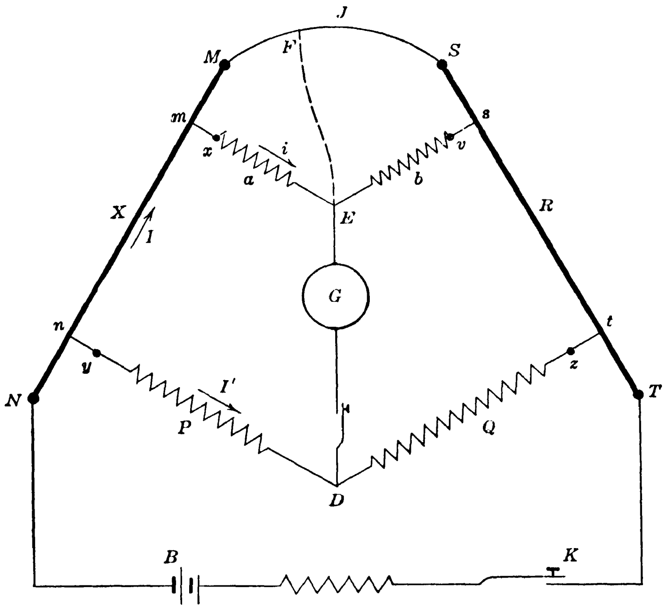


FIG. 68.—Diagram of the Kelvin double bridge.

galvanometer is connected between D and F, but in the present case the resistance of the connection at J cannot be neglected in comparison with X or R. If the galvanometer could be connected at a point, F, which divides the resistance, J, of the connections from m to s in the same ratio as X and R, then the resistance of J will not interfere with the comparison of X and R. This it is impossible to do, but the same thing, in effect, is accomplished by the use of the potential divider, ab. Since ab is in parallel with J, there is some point, E, having the same potential as F, and when the galvanometer is joined to E the effect is the same as though the connection were made at the corresponding point, F. By adjusting P and Q, a value can be found for which there is zero deflection of the galvanometer.

For this condition of balance, E and D will be at the same potential, and the fall of potential from n to D is equal to the fall from n to E. Writing out this equation gives

$$PI' = XI + ai.$$

Similarly, for the branches Dt and Et,

$$QI' = RI + bi.$$

By division,

$$\frac{P}{Q} = \frac{X + a\frac{i}{I}}{R + b\frac{i}{I}}.$$

Let the ratio, $\frac{i}{I}$, be denoted by r, for short. Then

$$\frac{i}{I} = \frac{J}{J + a + b} = r.$$

Evidently r will be a small number, and its smallness will depend upon the smallness of the joint resistance, J. Then

$$\frac{P}{Q} = \frac{X + ar}{R + br}$$

and

$$X = \frac{P}{Q}(R + br) - ar$$
$$= \frac{P}{Q}R + rb\left(\frac{P}{Q} - \frac{a}{b}\right).$$

If $\frac{P}{Q} = \frac{a}{b}$, this last term vanishes. In case the equality is not precise, the effect of this term is made less by making r as small as possible. This means that the connection MJS, Fig. 68, should have as low a resistance as possible.

112. Adjustment of the Ratios.—The ratio $\frac{a}{b}$ can be made equal to $\frac{P}{Q}$ by slightly adjusting the value of a. Opening the joint at J makes a Wheatstone bridge which will balance when

$$\frac{P}{Q} = \frac{X + a}{R + b}.$$

X and R are usually small in comparison with a and b, and also are in the same ratio, so that the balance expressed by this equation can be made perfect by adjusting the value of a or b. In this way $\frac{a}{b}$ can be made to equal $\frac{P}{Q}$, and then a, b, and J do not appear in the final value for X.

When $\frac{P}{Q}$ is set at 10^k the result is given by the direct reading of R with the decimal point moved k places.

113. Advantages of the Kelvin Bridge.—Low resistances can also be measured by the potentiometer method. (See Art. 144.) With the Kelvin bridge, however, an absolutely steady current is not essential, and the result is given by a simple computation from a single direct reading.

If, for any reason, it is necessary to use long wires to connect the bridge to the potential terminals of the low resistance, it will probably be better to use the potentiometer method.

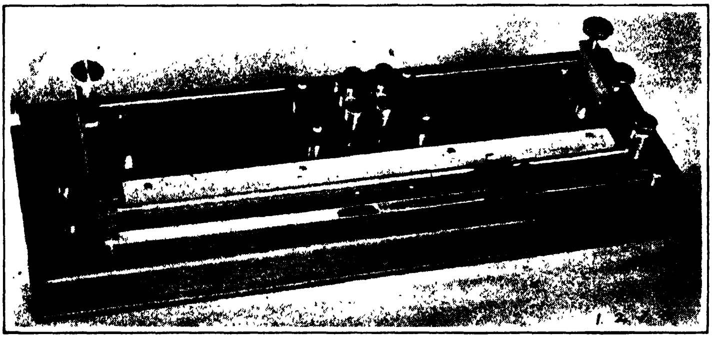


FIG. 69.—Student's Kelvin bridge.

114. Student's Kelvin Bridge.—The student's Kelvin bridge is a simplified instrument for making low-resistance measurements and for teaching the principles of these measurements. It consists of a 0.01-ohm calibrated variable standard of resistance, a set of double ratio coils with three different multiplying ratios, and a pair of current and potential contacts for the sample to be measured. It has, therefore, three ranges: 0 to 0.001 ohm, 0 to 0.01 ohm, and 0 to 0.1 ohm. It is designed for measuring samples about 18 in. long, and up to $\frac{3}{8}$ in. in diameter, but binding posts are provided for connecting to any four-terminal resistance.

As small a current should be used as will give the necessary sensitivity, and not more than 20 amperes should be passed through the standard resistance. Excessive current will heat the wire and its resistance will be increased. The zero reading of the galvanometer should be observed after the galvanometer has been connected to the bridge, to eliminate the effect of any thermal E.M.F. that may be present. Place both ratio plugs at the same value (marked 10, 1, or 0.1), close the battery circuit, and balance the galvanometer by moving the contact on the standard slide wire resistance. The maximum accuracy is obtained when

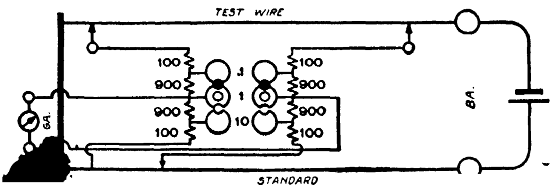


FIG. 70.—Diagram of connections.

the contact is at the upper end of the slide wire, and therefore a ratio value should be used that will bring the balance point as high as possible. The value of the unknown resistance, X, is obtained from the formula

$$X = R\frac{P}{Q},$$

where R is the reading on the scale, and the values of $\frac{P}{Q}$ are stamped between the ratio blocks.

115. Definite Relation between Resistance and Temperature. When the resistance of a wire is measured at different temperatures it is found to have different values. Usually the resistance increases as the temperature rises. At each definite temperature, however, there is a definite resistance for each wire, and if corresponding values of resistance and temperature are plotted, the result will be a smooth curve which is not far from a straight line.

In the case of alloys the change in resistance is very much less than it is for the pure metals, and in at least one case the resistance actually decreases with an increase of temperature.

Let $LMNP$, Fig. 71, represent the relation between resistance and temperature for a given coil. Suppose the portion MN has been experimentally determined and plotted. If this limited part of the curve is nearly a straight line, the relation between the resistance, R, and the temperature, T, is easily expressed, as is shown below.

116. The Indicated Temperature of Zero Resistance.—Let the straight line MN, Fig. 71, be extended back until it meets the axis of temperature at Z. The point Z gives the indicated temperature of zero resistance for this portion of the curve, and this point is useful in computing the value of the resistance at any

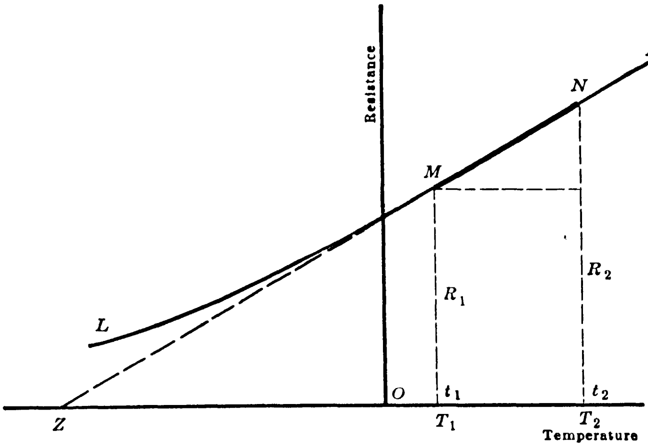


FIG. 71.—The relation between resistance and temperature. Over the observed range, MN, the resistance is proportional to the temperature above the point Z.

temperature *between* t_1 and t_2. For this purpose it is convenient to count the temperature from Z as zero. Thus, if t_1 degrees Centigrade is the temperature of M on the Centigrade scale, it will be $t_1 - T_z = T_1$ degrees above Z, where T_z denotes the position of Z on the same scale. Likewise, the temperature of N will be $t_2 - T_z = T_2$ degrees above Z.

Let R_1 and R_2 denote the resistances of a coil at T_1 and T_2 degrees above Z. Then, from the similar right angled triangles, ZM and MN,

$$\frac{R_1}{T_1} = \frac{R_2 - R_1}{T_2 - T_1} = s = \text{the slope of the line.}$$

Solving this for the value of T_1 gives

$$T_1 = \frac{R_1}{s}.$$

Thus it is not necessary actually to extend the line back to Z, since this point is readily determined by computation. Both R_1 and s are obtained from the known portion, MN, of the curve, and the indicated temperature of zero resistance is then

T_1 degrees below the temperature at which the resistance is R_1 ohms.

117. Determination of the Indicated Zero Point.—In this experiment several coils of different metals are arranged in an oil bath where the temperature can be raised as desired. A very convenient arrangement is to use an electric water heater to hold the oil bath. With a suitable resistance in series this can be easily warmed and maintained at the desired temperatures. Starting at room temperature, the resistance of each coil is carefully determined. A Wheatstone bridge box gives a convenient and accurate method for resistances having large temperature coefficients like copper and iron. For alloys like German silver and manganin it is better to use a more delicate method, such as the Carey Foster bridge.

After the resistances of the coils have been obtained at room temperature, the bath is warmed 10° or 15° and when things have become steady at the new temperature the resistances are again measured. In the same way the resistances are determined at five or six different temperatures, and the results plotted as a temperature-resistance curve for each coil.

The temperature at which this portion of the curve indicates that the resistance of a given coil would be zero is determined by calculation, as shown above. In this computation it is best to use corresponding values of resistance and temperature that are read from the curve, and which are, therefore, less liable to error than are single observations.

Thus if the chosen points on the curve correspond to temperatures of 20° and 90° C., the slope of the line is

$$s = \frac{R_{90} - R_{20}}{70}$$

and

$$T_{20} = \frac{R_{20}}{s}.$$

The indicated temperature of zero resistance is, then,

$$20^\circ - T_{20} = T_z$$

on the Centigrade scale.

These formulas take no account of the change in dimensions with change of temperature and therefore apply to the conductor as a whole.

INDICATED TEMPERATURES OF ZERO RESISTANCE IN THE RANGE 0 TO 100° C.

Metal	T_z (Centigrade)	Metal	T_z (Centigrade)
Aluminum	−236	Lead	−243
Brass	−556	Manganin	−40,000
Copper	−234	Platinum	−273
German silver	−3,200	Silver	−250
Gold	−272	Tungsten	−196
Iron (pure)	−160	Zinc	−249
Mercury	−1,100	Platinoid	−4,500

It is to be noted that these values of T_z imply nothing regarding the behavior of these metals at very low temperatures. This is merely a very simple way of expressing the slope of the temperature-resistance curve in the region where it has been studied.

118. Use of the Indicated Zero.—Knowing the resistance, R_1 of a conductor at a temperature T_1 degrees above the indicated temperature of zero resistance, it is a simple matter to find its resistance, R, at any other temperature, T. Since ZMN is a straight line,

$$\frac{R_1}{T_1} = \frac{R}{T}.$$

The proportion shown in this equation is very convenient and can be used in many ways. It is especially useful when the change in resistance is large.

Problems

1. What is the resistance at 100° C. of a coil of copper wire that is 13.46 ohms at 30° C.? The indicated zero for copper is −234° C.

Ans. $R_{100} = \frac{334}{264} \times 13.46 = 17.02$ ohms.

2. What is the resistance at 100° C. of a coil of copper wire that is 13.46 ohms at 30° C. if $\alpha_{20} = 0.00394$?

Solution.—First find the resistance at 20° C. Then, using this value, find the resistance at 100° C.

$$R_{100} = 13.46\,\frac{1 + 0.00394(100 - 20)}{1 + 0.00394(30 - 20)} = 17.02 \text{ ohms.}$$

3. How much resistance is there in a coil of copper wire that increases 1 ohm per 1° C.?

4. What is the resistance at 50° C. of a coil of platinoid wire that is 40 ohms at 10° C.? The indicated temperature of zero resistance for platinoid is −4,500° C.

Ans. 40.4 ohms.

5. What is the resistance at 100° C. of a coil of iron wire that is 10 ohms at 20° C.? *Ans.* 14.44 ohms.
6. A certain electrical machine should not be allowed to run hotter than 70° C. The resistance of a copper coil embedded in the machine can be measured as the temperature rises. How large can this resistance be allowed to become if it is 15 ohms at 20° C.? *Ans.* 17.95 ohms.

119. Resistance Thermometer.—Oftentimes it is impossible to use a mercurcy thermometer in some situation where the temperature is desired. This may be an inaccessible part of some machine or a distant place in a factory. If a coil of copper or platinum wire can be used and its resistance measured, the temperature can be determined from the relation above.

One of the best methods for measuring temperatures that are either too high or too low for the use of mercury is to measure the resistance of a fine platinum wire, and then read the temperature from the resistance-temperature curve. In fact, temperatures can be measured more exactly with a platinum-resistance thermometer than with a mercury thermometer, even at ordinary temperatures.

120. Temperature Coefficient of Resistance.—The change of resistance with temperature is often expressed in another way. From

$$\frac{R_1}{T_1} = \frac{R}{T}$$

it follows that

$$R = R_1 \frac{T}{T_1}.$$

Clearing of fractions by performing this division gives

$$\begin{aligned} R &= R_1\left[1 + \frac{1}{T_1}(T - T_1)\right] \\ &= R_1(1 + a_1 t) \end{aligned}$$

where t is written for the temperature difference, T-T_1, and a_1 is written for $\frac{1}{T_1}$. The coefficient, a_1, is called the temperature coefficient of resistance.

This formula is convenient to use in computing a small change in resistance corresponding to a small change, t, in temperature. Note that a_1 is the reciprocal of the temperature, T_1, above the indicated zero point, for which the resistance is R_1.

121. Second-degree Relation.—In case a straight line cannot represent the resistance of a coil with sufficient accuracy, a curve showing this relation can be used. In this case all first-degree equations are likewise insufficient and it is then necessary to use a second-degree equation to approximate more closely to the actual resistance-temperature curve. Usually this is written in the form

$$R = R_0(1 + at + bt^2),$$

where a and b are coefficients to be determined.

122. Temperature Coefficient of Copper.—The effect of a small amount of other elements alloyed with copper not only appears in the decreased conductivity of the metal, but the temperature coefficient is also decreased in the same ratio. Hard drawing the metal has a similar effect. The temperature coefficient of a sample of copper wire, expressed in terms of the resistance at 20° C., is given by multiplying the number expressing the per cent conductivity by 0.00394.[1]

The table below gives a few values for copper furnished for electrical purposes and for the temperature range 10 to 100° C.

Per cent conductivity	a_{20}	Indicated temperature of zero resistance (Centigrade)
101	0 00398	−231.3
100	0.00394	−233.8
99	0.00390	−236.4
98	0.00386	−239.1
97	0.00382	−241.8
96	0.00378	−244.6

[1] *Bull.* Bureau of Standards, vol. 7, p. 83.

CHAPTER VI

POTENTIOMETER AND STANDARD CELL METHODS

123. The Point Principle. *Kirchhoff's First Law.*—When a steady electron current is flowing in a circuit, the number of electrons arriving at any point on the conductor each second is the same as the number leaving the same point in the same time. This must be so since there is no accumulation of electrons at any place in the circuit when the potential is unchanging. When two currents unite and flow together in a single conductor, the resultant current is the sum of the two currents for the same reason.

The point principle can be extended to any number of currents meeting in a common junction. If all currents flowing to the point are considered positive, and all currents flowing from the point are considered negative, then when the currents have reached their steady values the algebraic sum of all the currents meeting at the point must be zero, since it is not possible for electrons to accumulate indefinitely at any point. The same is true when the currents are not steady, provided that there is no capacitance at the point considered.

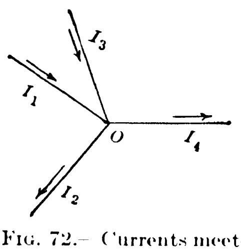


FIG. 72.—Currents meeting at O.

Thus, in Fig. 72.

$$+I_1 - I_2 + I_3 - I_4 = 0.$$

124. Potential Differences.—Any hill may be called up, or down, according to the direction in which one is going. If, after traversing several paths, one returns to the starting point, there will have been as much downhill as uphill in the entire journey.

If going uphill is called positive and downhill is called negative, the total height, up and down, that one has gone in the entire journey is zero when the journey ends at the starting point.

In this sense, then, let us notice that when we follow a stream downhill, that is, in the direction in which the water flows, our final position is lower than that at which we started, and our change in level is negative.

The word "level" is commonly used when speaking of points on the ground. Two points are at the same level when water will not flow from one to the other, when it is free to do so. They are still at the same level when the water is absent.

The word "potential" is used to express the same idea in electrical phenomena. Two points are at the same potential when electrons will not flow from one to the other, when they are free to do so. There is a difference of potential between two points when electrons will flow from one to the other when a conductor is provided. There will still be a difference of potential between these points if the conductor is absent.

125. The Circuit Principle. *Kirchhoff's Second Law.*—An electrical circuit is any path along which a current can flow. It requires but little extension of this idea to include any path whatever that can be traced. The circuit exists whether a current flows along it or not.

Let us consider a network of conductors, with cells of various E.M.FS. in the different branches of this network. Let us pick out any path we choose through the network. The currents may, or may not, follow the circuit that we have selected. If we imagine ourselves to start at any point on this path, and to follow the circuit through the conductors back to the starting point, we shall have found various ups and downs in potential, but the total change in potential for the entire circuit will be zero. This is so because the circuit began and ended at the same point, and therefore at the same potential.

Briefly, then, the circuit principle states that the *sum of all the potential differences in a complete circuit is zero.*

In making this sum each potential difference must be counted with its proper sign. There is no confusion here, for whether we think of the electron current or the electric current the ups and downs along the circuit are identical. In accordance with the usual meaning of the + and − signs, the positive pole of a battery is at a higher potential than the negative pole. Therefore, when we pass along the circuit each E.M.F. (of a battery or other source) that we encounter is counted as positive when we go up the potential difference, and negative when we pass down the potential difference, *i.e.*, in the direction in which the electrons are urged by the E.M.F.

The same rule applies when we are passing along a resistance. The only uncertainty is in finding which end of the resistance is at the higher potential. If we can follow an electron current, I, through a resistance, R, we know that we are going to points of higher potential, because the electrons always tend to rise to the points of highest potential. In this case our change in potential has been $+RI$. On the other hand, when we are going along a resistance, R, against the electron current, I, we are going down towards points of lower potential and our change in potential is $-RI$. It often simplifies the problem to determine first which end of each RI is positive, and mark it + as in Fig. 73, and mark the lower end −. Then, when the circuit is being traced, it is easy to see whether one is going towards points of higher or lower potential.

If we had chosen to speak in terms of the *electric* current considered as flowing downhill from points of higher to points of lower potential, we would have found the same ups and downs in potential. The same end of R would be marked +, and when we follow the electric current through a resistance to the other end the change of potential is $-RI$, the same as before.

126. Illustrations of the Circuit Principle. *For a Simple Circuit.*—Consider a circuit consisting of a cell, E, joined in series with a resistance, R. Starting at A, Fig. 73, and going around the circuit counterclockwise, we have a fall of potential RI in the part, AB. In passing from B on to the starting point, A, there is the further resistance, r, of the battery, and therefore a further fall of potential of rI. Between B and A there is also the E.M.F. of the battery, and since we are passing, in this case, from the negative side to the positive side, there is a rise of $+E$ volts.

The sum of the potential differences in the circuit is, then,

$$-RI - rI + E = 0.$$

For the Wheatstone Bridge.—When the bridge is balanced, no current flows through the galvanometer. Then writing down the potential differences for the circuit $ABCA$ gives

$$PI' - QI'' = 0,$$

since there is no E.M.F. in this circuit and no fall of potential in the galvanometer branch, BC.

Similarly, for the circuit $BDCB$

$$RI' - SI'' = 0.$$

Eliminating the I' and I'' gives the relation

$$PS = QR.$$

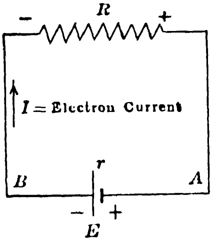


FIG. 73.

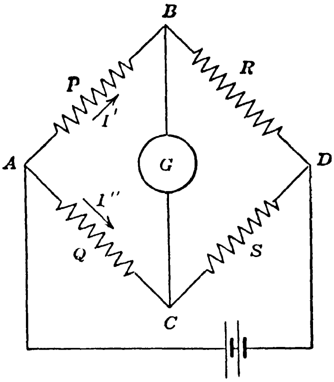


FIG. 74.

Typical circuits. The arrows indicate the direction of the electron currents.

127. Further Illustration of the Circuit Principle.—Let us consider the case shown in Fig. 75, which shows a single net, or circuit, from a network of conductors through which currents are flowing. We will assume that the electron currents flow as indicated by the arrows; should the value of any current be found negative in the final solution, it will mean simply that the actual electron current flows in the opposite direction.

Starting at any point we please, say D, and tracing out the circuit in either direction, say $D.F.A.B.C.D.$, we can express the differences of potential over each branch in terms of the resistances and E.M.FS. Since the electron current is taken as flowing from D to F, D must have the lower potential, and therefore as we trace out the path from D to F, we are moving to points of higher potential. Since D was the starting point, and has a potential indicated by V_4, the potential of F is given by

$$V_5 = V_4 + R_4 i_4.$$

In the next portion, FA, of the circuit that is being traced, there is a further rise of potential, $R_5 i_5$, and, in addition, there is the rise of potential, E_5, as we pass through the cell from the negative to the positive electrode. Therefore the potential of A is

$$V_1 = V_5 + R_5 i_5 + E_5 + r_5 i_5.$$

where r_5 denotes the internal resistance of the battery. In this resistance there is a fall of potential of the same sign and nature as though the resistance r_5 were outside of the battery and in series with it. (See Art. 25.) E_5 denotes the total E.M.F. of the battery as measured by the condenser method or by the potentiometer.

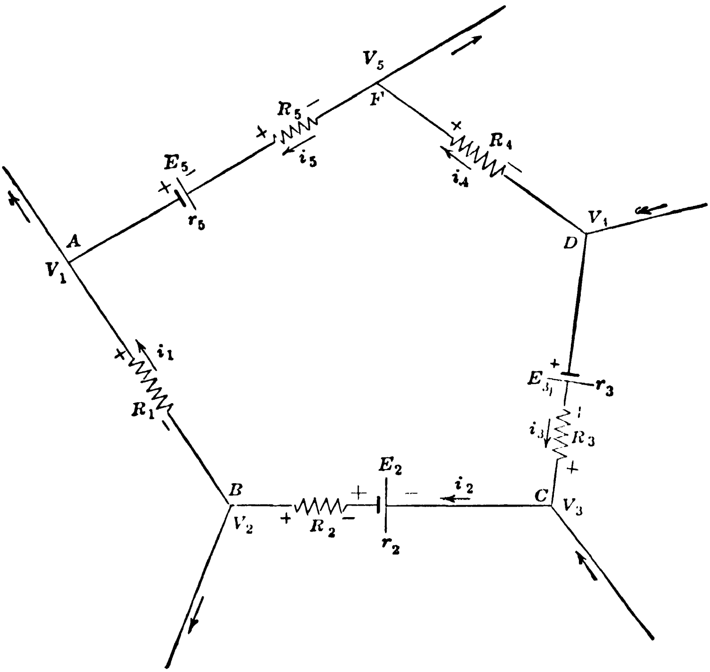


FIG. 75.—Showing the electron currents and the potential differences in one loop of a network of conductors.

In the next branch the electron current is flowing in the direction opposite to that in which we are tracing the circuit; therefore B must be at a lower potential than A, and

$$V_2 = V_1 - R_1 i_1.$$

In the same way the potential of C is still lower, and

$$V_3 = V_2 - R_2 i_2 - E_2 - r_2 i_2$$

because there is a drop in potential as we pass through E_2 in addition to the fall along the conductors.

As we return to the starting point along the branch CD we find both a fall and a rise of potential, giving for D

$$V_4 = V_3 - R_3 i_3 + E_3 - r_3 i_3,$$

since the cell in this branch is set so that we pass through it from the negative to the positive electrode. It is to be observed that the sign to be given E has nothing to do with the direction in which the current is supposed to be flowing through the cell. In such a network of conductors and cells it may often happen that the current will flow through some of the cells in the direction opposite to that in which it would flow if such cells were acting alone.

It is also to be noticed that the direction of the current has nothing to do with the direction in which we choose to follow a given path. In fact, it does not matter if there is no current whatever in some portions of the path that we trace out.

Substituting in the last equation the value of V_3 from the preceding equation, and the same for V_2, V_1, etc., gives

$$V_4 = V_4 + R_4i_4 + R_5i_5 + E_5 + r_5i_5 - R_1i_1 - R_2i_2 - E_2 - r_2i_2 - R_3i_3 + E_3 - r_3i_3$$

or,

$$R_4i_4 + R_5i_5 + E_5 + r_5i_5 - R_1i_1 - R_2i_2 - E_2 - r_2i_2 - R_3i_3 + E_3 - r_3i_3 = 0.$$

Thus if we write down the value of each rise and fall of potential as we come to it in following along any path whatsoever, the sum must be zero when we have again reached the point from which we started.

128. The Divided Circuit Principle.—This is a corollary, or modification, of the circuit principle. A divided circuit implies that there are two or more paths from one point, A, to another point, B. Since the paths start at the same point, and end at another point, the total difference of potential between A and B is the same along one path as along any other. Counting each potential difference with its proper sign as given above, the sum of all the potential differences along one path from A to B is equal to the sum along the second path from the same starting point to the same ending point.

129. The Potentiometer Method.—The most precise and elegant method for measuring an E.M.F. is that known as the potentiometer method. One great advantage of this method is that no current is drawn from the E.M.F. that is being measured. Being a null method, it is capable of greater sensitiveness and precision than any method that depends upon the reading of a deflection of a galvanometer or other instrument. The principle employed will be readily understood by referring to Fig. 76. AD represents a meter wire and scale similar to a slide wire bridge but of much higher resistance. The ends of this wire are connected to the battery, B, the E.M.F. of which is greater than that of any cell to be measured. C is the sliding contact, and any fraction of the fall of potential along the wire from zero to the full amount can be included between A and C. The points, A and C, are also connected through a parallel circuit containing the galvanometer, a key, and the cell whose E.M.F., E, is to be measured.

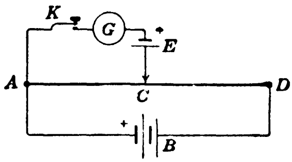


FIG. 76.—Slide wire potentiometer.

By moving C along AD a point can be found for which there is zero deflection of the galvanometer when K is closed. Let a' be the distance from A to this point; the fall of potential along this portion of the wire is $a'pi$, where p is the resistance of unit length of the wire and i is the current flowing through it from the battery at B.

Writing down the various potential differences in the galvanometer circuit, from C through E, G, K, to A, and back to C again, gives, for the case of a balance,

$$E' - a'pi = 0,$$

since, in this case, there is no current, and, therefore, no fall of potential, in the resistance of the galvanometer and the cell.

Now let another cell, whose E.M.F., E'', we may suppose is larger than E', be substituted for E. In order to obtain a balance and zero deflection, it will be necessary to move C nearer to D. Let a'' be the reading of this position. Then, as before,

$$E'' - a''pi = 0.$$

Eliminating pi from these equations gives

$$E' = a' \frac{E''}{a''}.$$

If E'' is known, E' can be computed from this proportion, being simply a constant $\left(\frac{E''}{a''}\right)$ times the scale reading, a'. By making the wire 2 meters in length and adjusting the current to give a fall of potential of just two volts between A and D, the millimeter scale becomes direct reading, 1 mm. corresponding to 1 millivolt. With this arrangement the E.M.F. of any cell can be read from the scale as soon as the galvanometer balance is obtained.

130. The Resistance Box Potentiometer.—In the resistance box potentiometer the wire AD of the preceding section is replaced by two similar and well-adjusted resistance boxes; and instead of actually moving C along the wire, resistance is transferred from AC to CD by changing the plugs while keeping the total resistance unchanged.

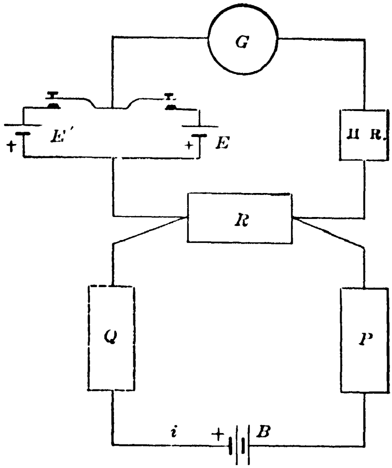


FIG. 77.—A potentiometer built up with resistance boxes.

The arrangement is shown in Fig. 77. The cell to be measured is placed at E, where it is in series with a sensitive galvanometer, a high resistance, HR, a key, and a resistance box R. Through the latter can be passed a small and constant current from the battery, B. In Art. 31 this R was the voltmeter, and the *current* through it was varied until the fall of potential, Ri, just balanced the E.M.F. of the cell. In the potentiometer method the *current is kept constant* and R is varied. Furthermore, the galvanometer is a more sensitive indicator of when the balance has been secured.

The rest of the set-up consists of a constant E.M.F. battery, B, for supplying the constant current, and a resistance box, P, which should be identical with R for convenience. The resistance used in both P and R should be kept equal to the total amount of one box; and if the boxes are alike it is very easy to keep the sum at this value.

Set up and test the arrangement, using first an old cell for E so as not to endanger a valuable standard by an accident or wrong connection. Keeping $R + P$ at the total amount in one box, a balance should be obtained without much difficulty. When it is certain that everything is working correctly the cells to be measured may be substituted for the cell at E.

For zero deflection of the galvanometer

$$E = Ri, \tag{1}$$

where i is the current from the auxiliary battery, B.

When the known E.M.F., E' of another cell has been balanced by the proper adjustment of R, this relation may be expressed in the same way as before,

$$E' = R'i. \tag{2}$$

Since both E' and R' are known, this equation determines the value of the current i as

$$i = \frac{E'}{R'}, \tag{3}$$

and if the current has been kept constant while R was varied, by also varying P so as to keep $(P + R)$ constant, and the battery, B, has not changed, then this value of i can be used for the value of the current in (1), giving

$$E = E' \frac{R}{R'}, \tag{4}$$

which expresses the E.M.F. of the cell to be measured in terms of known quantities.

131. A Direct-reading Resistance Box Potentiometer.— It is more satisfactory to read the value of the measured E.M.F. directly from the setting of the potentiometer than to make the computation indicated above. This can be done by adjusting the current through R and P to the proper value. When

$$i = \frac{E'}{R'} = 0.00010000 \quad \text{ampere}$$

the computation consists merely in moving the decimal point four places to the left. This can be done as follows:

Set $R = 10{,}000\ E'$, where E' is numerically equal to the E.M.F. of the standard cell at its present temperature. Set P at its complementary value in order that later it will be convenient to keep $P + R$ constant. This, in general, will not give a balance, but another resistance, Q, Fig. 77, may be introduced into the main circuit and adjusted to give the balance. Leaving Q at this value, the cell at E is replaced by one whose E.M.F. is desired and the balance obtained by adjusting R and P, keeping their sum constant as before.

The E.M.F. of this cell is

$$E = \frac{R}{10,000} \qquad \text{volts,}$$

where R is the resistance required for a balance.

The readings can be arranged as below:

POTENTIOMETER MEASUREMENTS

Name of cell	R	P	Q	E

132. Standard Cells.—In all measurements of E.M.F. with the potentiometer it is necessary to have a known E.M.F. which can be used as explained in Art. 130 (E', Eq. (4)), to standardize the value of the current through the potentiometer. Such a known E.M.F. is furnished by a standard cell, which is a primary battery set up in accordance with definite specifications so that it will possess a definite E.M.F. Such a cell is used as a standard of E.M.F., and is never expected to furnish a current. Since the E.M.F. of a cell depends upon the materials used in its construction, and not at all upon its size, standard cells are made small, both for economy of materials and for convenience in handling them.

133. The Weston Normal Cell.—The Weston normal cell was devised by Edward Weston. It has been the subject of much study and investigation, so that now it is possible for investigators in different parts of the world to set up such cells and know that they will have the same E.M.F. to within less than a ten-thousandth of a volt. In order to attain such accuracy, it is necessary that the cells be set up in strict accordance with the specifications, using only the purest materials.

The cell is usually set up in an H-shaped glass vessel having dimensions of a few centimeters. At the bottom of one leg is placed pure mercury to form the positive electrode of the cell. Connection to this is made by a fine platinum wire sealed into the glass, the inner end being completely covered by the mercury. At the bottom of the other leg there is placed, similarly, some cadmium amalgam which, when warm, can be poured in like the mercury and then hardens as it cools. A second platinum wire through the glass at the bottom of the leg makes electrical connection with this electrode. The electrolyte is a saturated solution of cadmium sulphate, containing crystals of cadmium sulphate in order to keep the solution saturated at all times. The mercury electrode is protected from contamination by the cadmium in this solution by a thick layer of a paste consisting mainly of mercurous sulphate. Cadmium ions from the solution coming through this paste form cadmium sulphate, and only mercury ions pass on and come into contact with the mercury electrode. This paste is thus an efficient depolarizer.

The E.M.F. of a Weston normal cell that has been set up in accordance with the specifications is

$$E_t = 1.01830 - 0.0000406\ (t - 20^\circ\text{ C.}) - 0.00000095\ (t - 20^\circ\text{ C.})^2 + 0.00000001\ (t - 20^\circ\text{ C.})^3.$$

This temperature formula was recommended by the London International Conference on Electrical Units in 1908, and the value, 1.01830 international volts, was found by the International Scientific Committee after an exhaustive series of measurements.

134. The Weston Unsaturated Standard Cell.—The Weston normal cell described above is seldom used in ordinary electrical measurements. When it is kept at a constant temperature its E.M.F. is constant and definitely known, but it is not always convenient to maintain a constant temperature chamber.

A considerable part of this temperature variation is due to the change in concentration as more or less of the crystals of cadmium sulphate dissolve. By omitting the crystals the electrolyte is unsaturated and remains at a constant concentration. Consequently, the E.M.F. changes very slightly with the temperature. The temperature coefficient is usually less than 0.000005 volt per degree centigrade and is almost invariably negative in sign. For most purposes the E.M.F. of the cell can be considered as constant for ordinary changes in room temperature. This modified form of cell is, therefore, more suitable for ordinary laboratory use, although it is necessary to determine the E.M.F. of each cell that is made.

The construction of this Weston standard cell is illustrated in Fig. 78. The materials forming the electrodes are held in place by porcelain retainers provided with cotton packing. The cell is sealed to prevent leakage and evaporation.

To preserve the constancy of a standard cell, it should not be subjected to temperatures below 4° or above 40° C. If by chance it is subjected to extreme temperatures, it should be set aside for a month at a practically constant temperature. It will probably then be nearly back to its original E.M.F.

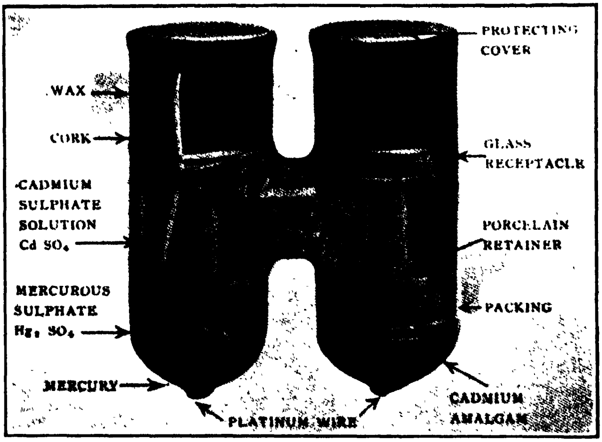


FIG. 78.—Construction of the Weston unsaturated type standard cell.

135. Use of a Standard Cell.—No current greater than 0.0001 ampere should ever be permitted to pass through a standard cell, and then only for a moment. A standard cell should never be connected to a voltmeter. The internal resistance is high, and therefore it would not be possible for it to furnish much current. In fact, any appreciable current drawn from the cell would polarize it somewhat, thereby decreasing its E.M.F. by an unknown amount and thus destroying the only value which the cell possesses. The depolarizer tends to restore the E.M.F. to its original value, but the time required depends upon the amount of polarization.

Standard cells may be used to charge condensers to a known difference of potential, for in this case there is no steady current drawn from the cell and the transient current is not sufficient to cause an appreciable polarization. When used in connection with a potentiometer, the cell should always be placed in series with a sensitive galvanometer. If it is necessary to reduce the deflection, this should be done by means of a high resistance in series with the cell, instead of a shunt on the galvanometer which would still allow the large current to flow through the standard cell. When using the galvanometer, the key should be lightly and quickly tapped so as to give a deflection of only a centimeter or two. This will indicate the direction of the current as clearly as a larger deflection and does not injure the standard cell.

136. The Student's Potentiometer.—For purposes of instruction the three-box potentiometer just described enables one to see clearly all parts of the set-up, and, as has been shown, it is convenient to use. But if one has many measurements of E.M.F.

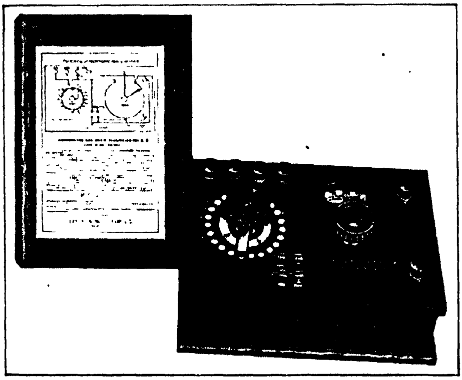


FIG. 79.—Student's potentiometer.

to make, the mere changing of the resistances to obtain the balances becomes a laborious task. It is then desirable to have a quicker and simpler way of varying the resistances.

In the student's potentiometer shown in Fig. 79 and the diagram Fig. 80, the resistances corresponding to the slide wire of Fig. 76 are arranged in two dials. Moving the sliding contacts over these dials corresponds to moving the ends, A and C, of the galvanometer circuit along the slide wire of Fig. 76. For accuracy and compactness, the greater part of the resistance consists of 22 resistance coils of 100 ohms each. These are adjusted to be like one another to within one-twenty-fifth of an ohm. In series with these coils is a circular slide wire which also has a resistance of 100 ohms. When the current is adjusted to 0.001000 ampere, by an auxiliary rheostat, the fall of potential over each coil is 0.1 volt. The galvanometer circuit, containing the standard cell or the E.M.F. to be measured, is connected to the two sliding contacts by means of the binding posts marked + and −, Fig. 80. The left-hand dial moves in steps of 0.1 volt. The final balance is obtained by turning the right-hand dial over the slide wire, which is graduated in thousandths of a volt.

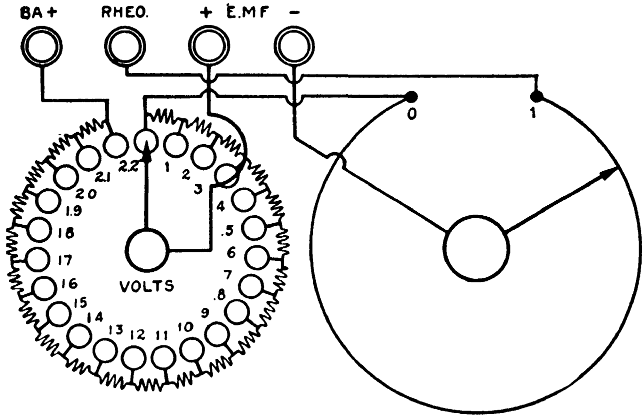


FIG. 80.—Diagram of the electrical circuits in the student's potentiometer.

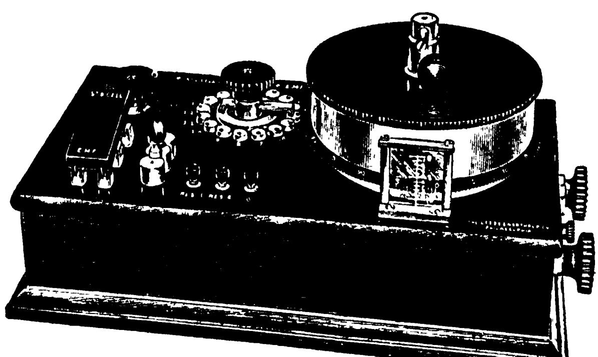


FIG. 81.—Type K potentiometer.

Although this reading is marked "volts," it is evident that this is so only when the proper current is flowing through the coils. This current can be supplied by two new dry cells, and after a balance has been obtained with a dry cell in the galvanometer circuit a standard cell can be used, to make sure that the arrangement is working properly, and the main current adjusted to the value that will make the potentiometer direct reading, as explained in Art. 131.

137. The Potentiometer.—A more elaborate form of potentiometer is shown in Fig. 81 and the electrical connections are shown in Fig. 82. The essential part of the instrument consists of 15 5-ohm coils, *AD*, adjusted to equality to a high degree of accuracy, connected in series, and having in series with them an extended wire, *DB*, the resistance of which from 0 to 1,000 on its scale (the entire scale reading from 0 to 1,110), is also 5 ohms. A contact point *M*, is arranged so that it can make contact

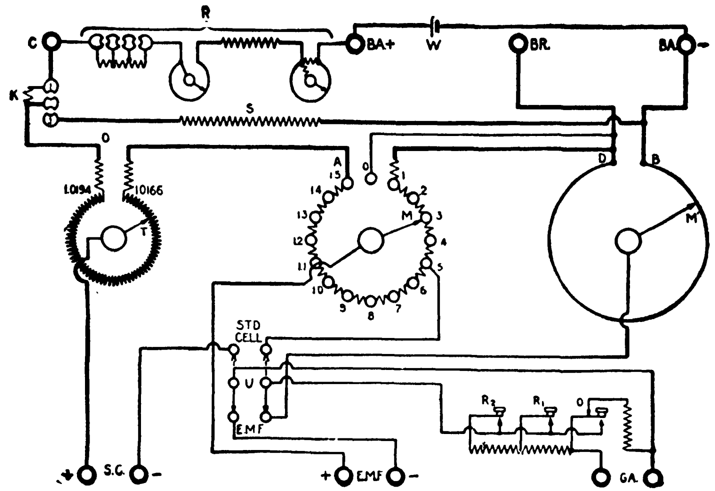


FIG. 82.—Diagram of the electrical circuits in the potentiometer shown in Fig. 81.

between any two of the 5-ohm coils, and a contact point, *M'*, so that it can make contact at any point on the extended wire, *DB*. Current from the battery, *W*, flows through these resistances and by means of the regulating rheostat, *R*, it is so adjusted as to be exactly one-fiftieth of an ampere. The fall of potential across any one of the coils, *AD*, is consequently one-tenth of a volt, and that across the extended wire, *DB*, is 0.11 volt. By placing the contact point, *M'*, at zero and by moving the contact, *M*, the fall of potential between *M* and *M'* may be varied by steps of one-tenth of a volt, from 0 up to 1.5 volts. By moving the contact point, *M'*, along the wire, the fall of potential between *M* and *M'* may be varied in infinitesimal steps. In making measurements, the unknown E.M.F. is introduced in series with a galvanometer between the points *M* and *M'* and in opposition to the fall of potential along *AB*. The contact points, *M* and *M'*, are then adjusted until the galvanometer shows that no current is flowing when the value of the unknown E.M.F. can be read from the position of the points, M and M'. In the diagram, Fig. 82, the unknown E.M.F. is introduced at the binding posts marked "E.M.F.", and the galvanometer at the point GA. When the key R_1 is depressed, the circuit is closed through a high resistance; when key R_2 is closed, the circuit is closed through a lower resistance, and when key R_0 is closed the circuit is closed through zero resistance. The purpose of keys R_1 and R_2 is to protect the galvanometer against excessive deflections when the opposing E.M.FS. are not approximately balanced.

138. Quick Method of Applying the Standard Cell.—The known E.M.F.—in practice, the standard cell—may be connected across *one* portion of AB, and the current adjusted, as described above; and the unknown potential difference may be balanced against the fall of potential in *another* portion of AB. The connections for accomplishing this with a single galvanometer are shown in Fig. 82. At the point 0.5 on the series of resistance, AD, a wire is permanently attached which leads to one point of the double-throw switch, U. Between A and O there is a series of 19 resistances with a sliding contact, T. The resistance between 0.5 and A is exactly that which corresponds with the E.M.F. of 1 volt, and that between A and 1.0166 a sufficient addition to make the resistance between 0.5 and this point correspond to an E.M.F. of 1.0166 volts. At this point there is connected in the circuit a small circular slide wire with a movable contact point, T, through which connection to the standard cell is made. The slide wire has a resistance value such that a practically continuous variation in the standard cell circuit of voltages from 1.0166 to 1.0194 can be obtained. This range corresponds with the variations in different cadmium cells and the circuits are so connected that the two points, T and 0.5, may be thrown in series with the galvanometer and the keys, R_1, R_2, and R_0. In this circuit are also the two binding posts marked "Standard Cell," to which the standard cell is to be connected. To adjust the current to one-fiftieth of an ampere, throw over the double-throw switch, U, to the position indicated by the dotted lines. Set the point T to correspond with the E.M.F. of the standard cell and regulate R until the galvanometer shows no deflection. The unknown E.M.F. may then be measured as before with the contact points, M and M', the double-throw switch having been placed in the position indicated by the full lines. After a balance has been obtained, the current may be checked by simply changing the position of U and touching the contact key, R_0. A galvanometer balance shows that no change has occurred. A slight deflection calls for a slight readjustment of R and a corresponding readjusment of M'.

139. Galvanometers for Use with a Potentiometer.—The function of a galvanometer in connection with a potentiometer is that of an indicator of absence of current, and, consequently, of absence of potential difference at its terminals. It is essential, therefore, that it should respond readily, by a deflection, when a slight potential difference exists.

When the total resistance of the circuit containing the galvanometer is relatively low, a small unbalanced potential difference will produce a larger current than the same potential difference when the total resistance is high. In the former case, which represents conditions in ordinary voltage and current measurements, and in thermocouple work, the instrument should have a good microvolt sensitivity; while in the latter, representing the conditions met with in many of the potential measurements of physical chemistry, an instrument of high current sensitivity is desirable. At the same time, the internal resistance and external critical damping resistance, in the first instance, should be low; in the second, these values should be correspondingly higher.

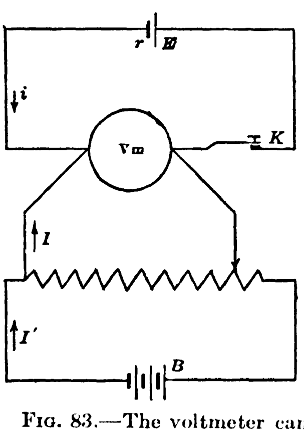


FIG. 83.—The voltmeter can be adjusted to read directly the E.M.F. of E.

140. A Direct-reading Potentiometer without a Standard Cell.—A direct-reading potentiometer which dispenses with both the fragile standard cell and the delicate galvanometer can be readily built up, using a voltmeter of suitable range to read the E.M.F. to be measured. The arrangement is very simple and the measurements are as accurate as the voltmeter (calibrated) can be read.

The voltmeter is connected to the slider of a wire rheostat or potential divider so that it can be made to read at any point of its scale. The cell or other E.M.F. to be measured is placed at E,

Fig. 83. When the key is closed, the potential differences in the upper circuit are

$$E - ir - V = 0,$$

where V is the voltage across the voltmeter.

When the key is open, the voltmeter stands deflected, due to the current, I, flowing through it from the working battery. When the key is closed, the current through the voltmeter is $I \pm i$, the sign depending upon the relative values of E and V. Due to this change in the voltmeter current, the pointer will move from V towards the value of E, and the potential divider should be varied to change V in the direction indicated by this motion of the pointer. By adjusting V until this motion is reduced to zero, and therefore $i = 0$, we have

$$E = V.$$

The voltmeter reading that remains unchanged by opening and closing the key gives, therefore, the value of the E.M.F. that is being measured. This voltmeter-potentiometer is especially useful in locations where laboratory facilities are not readily available.

141. Calibration of a Voltmeter by the Potentiometer Method. A low-reading voltmeter can be readily calibrated by the potentiometer method described in Art. 130 above. The voltmeter is connected to the sliding part of a potential divider so that the pointer of the voltmeter can be brought to any desired place on the scale. The resistance of the potential divider should be a few hundred ohms, or enough to draw only a small current from the battery so that the voltage will remain fairly constant. The voltmeter, while still connected with its battery, is also inserted in the galvanometer circuit of the potentiometer in place of the standard cell. There will thus be two electric circuits, each with its own battery; one battery, W, supplying the current through the potential divider and the voltmeter, while the potentiometer current through P and Q comes from the battery B. The galvanometer circuit connects these two circuits, and the current through it may be in either direction, or made zero by adjusting the resistance in P.

Writing the equation for the potential differences in the circuit through the galvanometer and the voltmeter, before a balance has been obtained, gives

$$S(I + i') - R(i - i') + Gi' = 0,$$

where S is the resistance of the voltmeter. For no current through the galvanometer this equation becomes

$$SI = R'i,$$

where R' is the value of R that gives the balance.

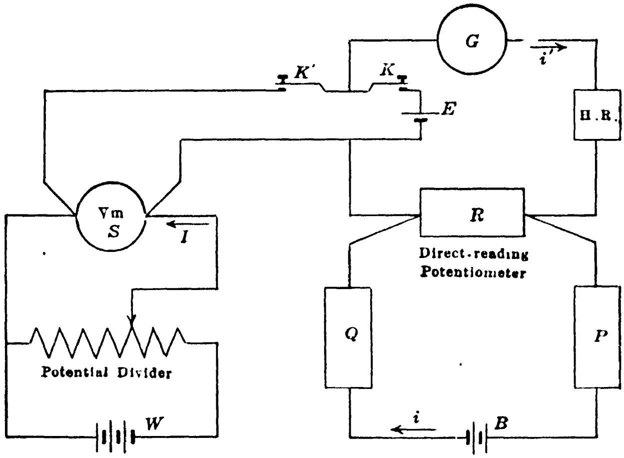


FIG. 84.—Calibration of the millivoltmeter Vm.

The value of the constant current, i, is determined by using a cell of known E.M.F. in the same manner as explained in Art. 130 (Eq. (3)). The voltmeter, with its battery, is removed from the galvanometer circuit by leaving K' open, and the standard cell is inserted in the same circuit by using the key K as shown in Fig. 84. The resistance in R is now adjusted to a new value R'' to give no current through the galvan meter. Then,

$$E = R''i,$$

since, by keeping $R + P$ constant, the current is the same as before. Therefore,

$$SI = E\frac{R'}{R''},$$

and this is what the voltmeter should read. If the voltmeter reading is V, the correction to be applied is

$$c = E\frac{R'}{R''} - V.$$

142. The Calibration Curve.—The correction curve should be drawn with the observed voltmeter readings for abscissæ and the corresponding corrections for ordinates.

In order to show the corrections clearly, they will be plotted on a magnified scale. This will mean that any uncertainty, say 0.01 volt, will affect the ordinates much more than the abscissæ. The points for the calibration curve should be plotted then, not

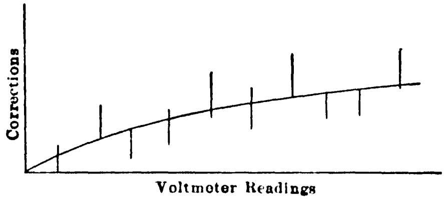


Fig. 85.—The uncertainties in the measured values of the corrections can be indicated by the lengths to which the observed values are drawn out by the magnified vertical scale.

as dots, but as short vertical lines whose lengths will show the magnified uncertainty in the corrections when they are plotted on this scale. A line that passes across each of these short lines gives the calibration curve. If the position of this curve is not sufficiently definite, additional points should be located.

143. Measurement of Voltages Greater Than about 2 Volts.— Inasmuch as most direct-reading potentiometers are designed

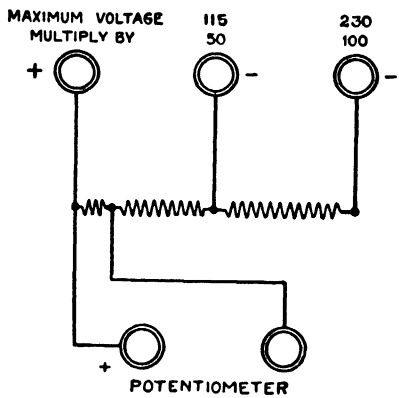


Fig. 86.—Diagram of a volt box.

to measure voltages up to 1.5 or 2 volts, it is necessary to use a volt box when higher voltages are measured. The volt box is a particular form of potential divider which is connected to the large voltage to be measured. A definite fraction of this voltage is then measured by the potentiometer.

The connections of a volt box are shown in Fig. 86. Any voltage up to 230 volts can be applied to the total resistance of 40,000 ohms. The fall of potential over 400 ohms of this resistance can be measured with a potentiometer, and 100 times this value gives the voltage that is applied to the box.

It is evident that a small current is drawn from the source, and therefore a volt box can be used to measure only those voltages that are not altered by this current.

With the aid of such a volt box, a high-reading voltmeter can be calibrated up to 230 volts with a potentiometer. By increasing the resistance and the insulation, the range of a volt box can be extended to measure several thousand volts.

144. Comparison of Resistances by the Potentiometer.—One of the accurate methods for comparing two resistances, particularly when these are not very large, is by means of a

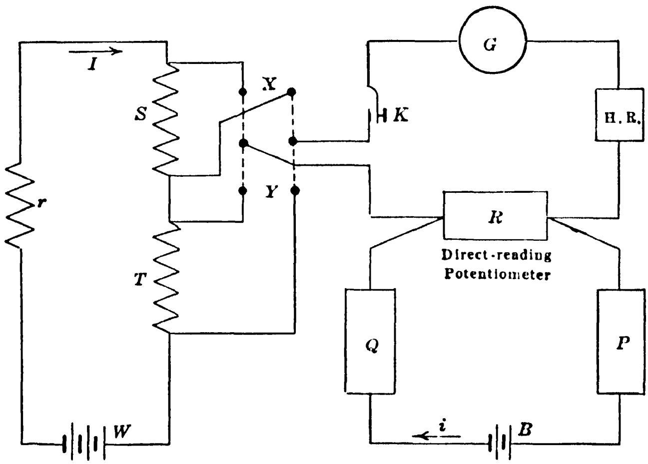


FIG. 87.—Comparison of the resistances, S and T.

potentiometer. The two resistances to be compared are joined together in series with a battery and sufficient other resistance to insure a steady current. This current should not be large enough to change the resistances by heating them. Other things being equal, it is desirable to have the fall of potential over each resistance about 1 volt. Let the two resistances be denoted by S and T; then the fall of potential over each will be SI and TI, respectively.

The actual measurements are very simple. With the potentiometer set up as used for the comparison of E.M.FS., the fall of potential over each resistance is measured. Let the readings on the potentiometer be R' and R'', respectively. Then from the conditions of balance,

$$TI = R'i \quad \text{and} \quad SI = R''i,$$

where i denotes the current through the potentiometer resistances.

From this it follows at once that

$$T = S\frac{R'}{R''}$$

and this relation can be determined as accurately as the potentiometer measurements can be made.

145. Direct Comparison of an Unknown Voltage with a Standard Cell.—It is not always necessary to have a potentiometer in order to use a standard cell for precise measurements. Often it is desirable to have a single balance give a direct comparison of the voltage measured and the E.M.F. of the standard cell. The two following methods illustrate this use of a standard cell.

146. Calibration of a Voltmeter. *Divided Potential Method.*—The voltmeter is joined to a storage battery or other source which will maintain the deflection steady at the point it is desired to calibrate. If the voltmeter is direct reading, it should read the difference of potential between its own binding posts. To calibrate the scale, then, it is only necessary to measure this same difference of potential by some precise method and compare this measured value with the reading of the voltmeter.

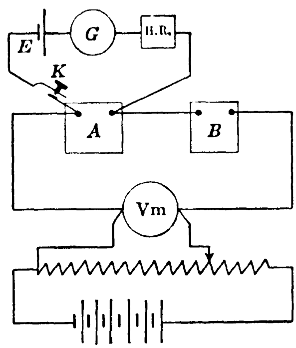


FIG. 88.—Calibration of the voltmeter, Vm.

In parallel with the voltmeter is joined a circuit consisting of two resistance boxes, A and B, and in parallel with A is a circuit containing the galvanometer, a standard cell, a high resistance, and a key. It is best to have about 1,000 ohms in A and B together for each volt read by the voltmeter. A preliminary calculation will give the resistance which should be placed in each A and B to give a fall of potential over A about equal to the E.M.F. of the standard cell. When these approximate resistances have been placed in A and B, the key, K, can be quickly and cautiously tapped and the direction of the deflection noted. In using a standard cell in this way, as little current as possible should be allowed to flow through it. Even a slight polarization will lower its E.M.F. by an unknown amount and then it can no longer be called a "standard" cell. Furthermore, nothing is gained by deflecting the galvanometer "off the scale," as a deflection of a centimeter or two takes less time and is fully as definite as the larger deflection. The high resistance is inserted for this very protection of the cell and therefore is better than a shunt on the galvanometer.

By adjusting the values of A and B the galvanometer deflection can be reduced to zero. The high resistance can be short-circuited for the final balance if the deflections are small. When thus balanced, the potential differences in the circuit through the voltmeter and A and B are

$$Ai + Bi - SI = 0,$$

where SI is the fall of potential over the voltmeter, and is what should be indicated on its scale.

Similarly, for the circuit through the standard cell

$$Ai = E.$$

Eliminating i by division,

$$SI = E\frac{A + B}{A}.$$

If V is the reading of the voltmeter, then

$$V + c = SI,$$

and the correction to be applied at this point is

$$c = SI - V.$$

The computations can be greatly reduced if A is kept at the value $A = 1{,}000E$, Thus if the E.M.F. of the standard cell is 1.018 volts at the temperature of the room, A would then be set at 1,018 ohms and all of the adjustment made by changing B. The values of SI are then given directly by the sum of A and B divided by 1,000.

Other points on the scale can be obtained by using a different number of cells in the main battery, or, if the voltmeter is low reading, by adding some resistance in series with the battery. If the battery cannot be divided into a sufficient number of steps, a useful method is to join across the battery terminals a high-resistance rheostat and connect the voltmeter to one terminal and to the sliding contact. Any desired voltage can then be obtained up to the maximum of the battery by simply sliding the movable contact along the rheostat. It is evident that this method cannot be used to calibrate points below the E.M.F. of the standard cell.

A calibration curve should be plotted, with voltmeter readings as abscissæ and the corresponding corrections as ordinates. If there is a "zero correction" because the needle does not indicate zero correctly, such correction should be made before computing the calibration corrections.

147. A More Convenient Method.—In case the voltmeter reading can be varied continuously, or by very small steps, either by means of a sliding rheostat as shown in Fig. 88, or by means of a resistance box in series with the battery, it will be found more expeditious to set *A* and *B* at the values corresponding to a given voltage, and then by varying the rheostat to bring the voltmeter to this voltage, the balance being indicated by the galvanometer the same as before. The voltmeter reading should give this same voltage; if it does not, the discrepancy is the correction which must be applied to the voltmeter reading at this point.

CHAPTER VII

MEASUREMENT OF CURRENT

148. Galvanometers.—The D'Arsonval type of galvanometer consists of a moving coil suspended between the poles of a permanent magnet something like the arrangement in an ammeter, but greater sensitiveness is secured by using a long, fine strip of phosphor bronze for the suspension in place of the jewel bearings. Such a galvanometer measures very small currents and is useful in measurements where the current is small or is made zero in the final adjustment. It can be used to measure larger currents by using a low-resistance shunt, as in the ammeter; and it will serve as a voltmeter when used in series with a high resistance.

Since the scale from which is read the deflection of the galvanometer usually is divided into millimeters, it will be necessary to calibrate it in order to read the value of the current. A set-up for determining the figure of merit can be used (see Fig. 33) and deflections corresponding to different values of the current can be observed. A curve can then be plotted between deflections and currents, and from this curve can be read the value of the current corresponding to any deflection.

149. The Tangent Galvanometer.—In the instruments heretofore described it has been supposed that the scale was graduated to read the value of the current directly in amperes, or that it could be calibrated so to read. But nothing has been said as to how the value of a current can be expressed in terms of the ampere as defined in Art. 230. The tangent galvanometer furnishes the means for establishing the value of a current in terms of the C.G.S. unit. The method is readily understood and formerly was the principal method for determining the absolute value of a current. Other methods (see Arts. 153–157) now offer more precise measurements, but the tangent galvanometer is as accurate as it ever was and it still holds sufficient historical interest to warrant its description.

The tangent galvanometer consists of a coil of relatively small cross-section and large diameter. At the center is a short magnetic needle, suspended by a fine silk or quartz fiber and carrying either a long pointer or a mirror for use with a telescope and scale. When hanging freely, the needle will point north and south in the magnetic meridian. The large coil should stand vertically in this same meridian. When a current is passed through the coil, the magnetic field at the center of the coil due to this current will be directed east and west. The resultant field due to the combination of this field with the original field, H, of the earth will be in a direction intermediate between the two and making an angle θ, say, with the direction of the latter. The needle, when free to turn, will take up this resultant direction; therefore the angle θ is determined by observing the change in the scale reading when the needle is deflected.

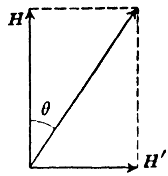


FIG. 89. — Resultant of two magnetic fields.

The intensity of the magnetic field at the center of a coil of n turns is, from Chap. XI (see Art. 231).

$$H' = \frac{2\pi I n}{r},$$

where r is the mean radius of the coil.

Since the coil of the tangent galvanometer is so placed that H' is at right angles to the earth's field, H, the angle which the resultant of these two makes with the latter is given by the expression,

$$\tan\theta = \frac{H'}{H} = \frac{2\pi I n}{rH},$$

whence,

$$I = \frac{Hr}{2\pi n}\tan\theta,$$

where I denotes the value of the current in c.g.s. units and θ is the corresponding deflection of the needle.

150. The Coulometer.—Another effect of the current is manifest when it passes through an electrolyte, such as a solution of copper sulphate in water. Copper is deposited upon the electrode at which electrons are supplied by the current through the metallic conductors—one atom of copper (divalent) being deposited for each two electrons in the current. The mass of copper deposited is thus a very precise measure of the quantity of electrons that have passed along the circuit. With proper precautions this gives a good method for measuring current, and, indeed, the value of the international ampere is fixed in terms of the amount of silver deposited per second in a silver nitrate solution.

When the copper coulometer is used, the electrodes should be of copper, the best form being obtained by winding a meter of large copper wire into a loose spiral that will just fit inside the glass jar containing the solution. Another spiral wound smaller and suspended within the first one serves as the cathode. All of the copper surface should be clean and never touched with the fingers. When ready to be used, the current should be passed through the coulometer for a sufficient time to deposit a uniform layer of new copper upon the smaller spiral, which is always used for the cathode. During this preliminary run, the current should be adjusted to the value it is desired to measure.

As soon as the current is stopped the spiral cathode is removed from the solution and thoroughly rinsed in distilled water, then dipped in alcohol to remove the water. The alcohol evaporates readily when the coil is gently swung in warm air. When completely dry, the cathode is carefully weighed.

When ready to begin the run, the cathode is placed in position in the solution, and at a given moment the circuit is closed, thus starting the current at a known time. The current should be maintained as constant as possible for an hour and then stopped at another known time. The cathode is removed, washed, dried, and weighed as before. The gain in weight gives the amount of copper deposited, from which the average value of the current can be computed. Since 1 coulomb will deposit 0.0003283 gram of copper, the current is given by the expression,

$$I = \frac{M}{0.0003283t},$$

where M denotes the mass of copper deposited in t seconds.

151. The Silver Voltameter.[1]—In accordance with the decisions of the London Conference, the silver voltameter is to be

[1] It would seem more appropriate to call this instrument the silver coulometer, since it actually measures a deposit of silver which is proportional to the number of coulombs passing through it. But the Electrical Congress at London in 1908 officially adopted the word "voltameter" because of its derivation from "volta-electro-meter" used by Faraday in 1834, and this name was used by the Congress in the specifications for the measurement of the international ampere. It has no connection with a voltmeter, or a voltammeter. For deposits of other metals the name "coulometer" is generally used.

used to measure the international ampere. In connection with the international ohm it may also be used to measure international volts.

The silver voltameter is not suited to ordinary measurements of current, but in the hands of a physicist who has specialized in its use very precise results can be obtained. Since it is not possible to preserve an electric current, the results of the voltameter measurements are usually expressed in terms of the E.M.F. of a Weston normal cell. The Weston cell can be used later with a standard resistance to establish a known current.

In the standard form of the silver voltameter the cathode is a bowl of platinum of from 125 to 400 $cm.^3$ capacity. The anode should be of pure silver, and it should be coated with electrolytic silver. It should be as large as can conveniently be used in the bowl. The electrolyte consists of a solution of pure silver nitrate in distilled water, having 15 to 20 grams of silver nitrate in each 100 $cm.^3$ of the solution.

The current during a deposit should be maintained constant at a value not over 1 ampere and for a period of an hour or longer. The deposit should be firm and crystalline, and free from striations. The slime that forms on the anode must be kept from the cathode. Porcelain porous pots of fine quality surrounding the anode are sometimes used for this purpose. Filter paper or silk should never be allowed to come in contact with the electrolyte.[1]

152. A Simple Voltameter.—A voltameter of small cost has been devised by the Bureau of Standards.[2] The cathode is a small sheet of platinum rolled into a cylindrical form. The weight of platinum is about 10 grams. This is placed within a larger cylinder of silver which serves as the anode. Below the cathode is a glass tray which can be removed from the solution with the cathode. The whole is arranged within a glass dish that holds the electrolyte. The smaller amount of platinum makes it less expensive than the standard form, but the deposit is concentrated on a smaller area and the accuracy is slightly less perhaps by a few hundredths of 1 per cent.

153. Measurement of Current by Means of a Standard Cell.—Although the silver voltameter is the legal standard for the measurement of current, its use is practically limited to reference

[1] For further description of the silver voltameter and its use see *Bull.* Bureau of Standards, vol. 13, p. 500, etc.

[2] See *Bull.* Bureau of Standards, vol. 10, pp. 529–530.

measurements of primary standardization. The potentiometer method is more convenient, and in the hands of the ordinary user it is more accurate, because it depends only on the measurements of a resistance and a difference of potential, both of which can be determined with a very high degree of precision. In this method the current, I, is passed through a four-terminal resistance, R, of an ohm or less, the value of which is accurately known. The resulting fall of potential, RI, is then measured with the potentiometer in terms of a standard cell, the E.M.F. of which is expressed in terms of the Weston normal cell, taken as 1.01830 international volts at 20° C. (see Art. 133). Since this value was determined from an exhaustive series of comparisons with the silver voltameter, the measurement of a current in terms of a standard cell amounts to a measurement in terms of the silver voltameter.

In the method described below the measurements are expressed in terms of a standard cell, either by the regular potentiometer method (Art. 137) or by a direct comparison similar to the method for calibrating a voltmeter (Art. 146).

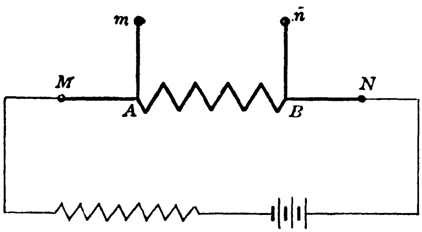


FIG. 90.—Diagram showing the four-terminal resistance, AB, connected into the circuit.

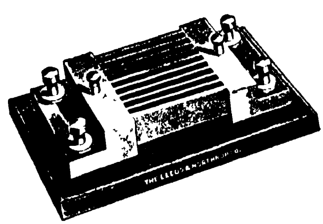


FIG. 91.—Current-carrying resistance; 0.00002 ohm for currents from 500 to 2,000 amperes.

154. Standard Resistances for Carrying Current.—When a low resistance is used, the question of contact resistance at the connections becomes important and such a resistance is usually provided with potential terminals. This arrangement is shown diagrammatically in Fig. 90. MN is a low resistance with potential terminals permanently attached at A and B. When a current I is flowing through this resistance, the fall of potential between m and n is unaffected by any contact resistance at M or N. And if very little or no current is taken through m and n, resistance at these points has little or no effect on the measured fall of potential.

It is evident, then, that the only useful resistance in this arrangement is that of the main circuit between A and B, where the potential terminals are connected, and it is the resistance of this portion that is marked on standard resistances of this type. The potential terminals are clearly seen in Fig. 91.

These four-terminal resistances cannot be added in series or in parallel. They can be measured and used with a potentiometer or with the Kelvin bridge.

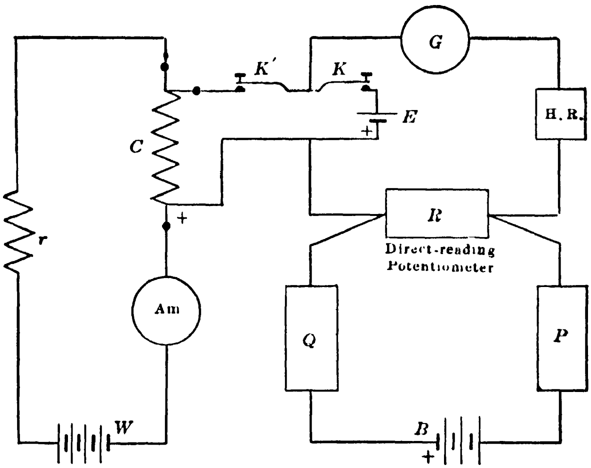


FIG. 92.—Calibration of the ammeter, Am.

155. Calibration of an Ammeter. *Potentiometer Method.*—For the calibration of an ammeter, the instrument is joined in series with a standard resistance. The same current must, therefore, pass through them both. Its amount is determined by measuring the fall of potential over the standard resistance and this value is compared with the reading of the ammeter. The difference is the ammeter correction. The arrangement is shown in Fig. 92.

The ammeter is connected in series with a storage battery, a variable rheostat, and the known low resistance, C, which should be a standard manganin resistance provided with permanent potential terminals. The fall of potential over the latter is measured by the potentiometer in the usual way. The best results are obtained when this is about 1 volt. By means of the keys, K' and K, either C or the standard cell, E, can be used in the galvanometer circuit. In the latter case, when a balance has been obtained by adjusting R until there is no deflection of the galvanometer,

$$E = R'i.$$

When the coil, C, carrying a current, I, has been substituted for the standard cell and a balance obtained by readjusting R to some new value, R'', we have,

$$CI = R''i.$$

From which

$$I = \frac{E}{C}\frac{R''}{R'}.$$

This value for the current is compared with the ammeter reading, I_A, and the corresponding correction is

$$c = I - I_A.$$

A calibration curve should be drawn, using the observed ammeter readings as abscissæ and the corresponding corrections for ordinates.

156. A Potentiometer Reading Directly in Amperes.—By adjusting the auxiliary resistance, Q, in the battery circuit so that when the potentiometer is balanced against the standard cell, the reading is $R' = 10{,}000\frac{E}{C}$, the potentiometer becomes direct reading in amperes, and the true value of the current flowing through the ammeter can be read directly from the setting of the potentiometer.

157. Calibration of an Ammeter. *Divided Potential Method.*—In this method the same arrangement that was used above to calibrate the voltmeter is now used to measure the fall of potential over the standard resistance that carries the current to be measured.

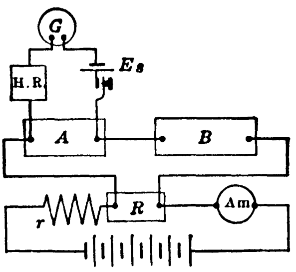


FIG. 93.—Calibration of the ammeter, Am.

The current from a storage battery flows in series through the ammeter, the standard resistance, R, and an adjustable resistance, r. In parallel with R is a circuit of much higher resistance and consisting of two well-adjusted resistance boxes. In parallel with one of these is the galvanometer with a standard cell and a high resistance of about 10,000 ohms to prevent too great a current through the standard cell.

Let the currents through the ammeter, standard resistance, B, and the galvanometer be denoted by I, I', i, and i' respectively. When the galvanometer shows zero deflection,

$$E_s = Ai.$$

At the same time the circuit ABR gives

$$RI' = (A + B)\,i.$$

Solving for the current in R,

$$I' = \frac{E_s}{R}\,\frac{A + B}{A}.$$

Since $I = I' + i$, we have, finally,

$$I = \frac{E_s}{R}\,\frac{A + B + R}{A}.$$

It is evident that if A is kept at some round number, say 5,000, and the adjustment made by varying B, the computations will be much simplified.

If the reading of the ammeter is I_A, then

$$I_A + c = I,$$

and the correction to be added to this reading is

$$c = I - I_A.$$

Calibration of Ammeter No. ....

Am. zero = E_s = at °C. R =

Ammeter		Standard cell		Resistances			
Reading	Corrected	Temp.	E.M.F.	A	B	I	Correction

158. The Kelvin Balance.—The Kelvin balance is an accurate semi-portable instrument for the measurement of current. There are six flat coils placed horizontally and through which the current passes in series. Two of these are carried on a balanced beam, one at either end, while above and below each of these movable coils is one of the fixed coils. The diagram shows the relative positions of the coils. The movable coil, *cd*, is shown in a vertical section through a diameter, while the dotted lines indicate the magnetic field in the same plane due to the fixed coils, *ab* and *ef*. It will be seen that at *c* the field is horizontal and directed to the right. If the electron current in the coil *cd* is flowing out of the paper at *c*, then this portion of the circuit will be urged downward. At the other side of the

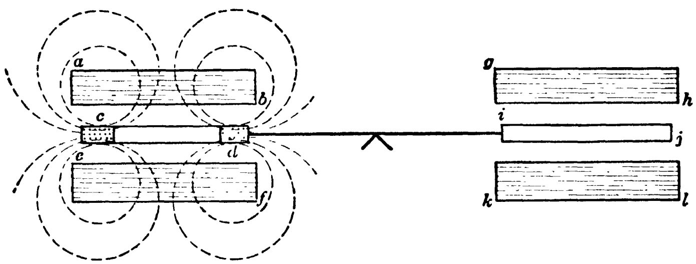


FIG. 94.—Coils of the Kelvin balance.

coil the direction of the field is to the left, and the electron current at *d* is flowing into the paper. Hence this side of the coil will also be urged downward. The same is true for all parts of the coil, *cd*, and therefore the coil as a whole is urged downward with a force proportional to the product of the current it carries and the density of the magnetic flux in which it moves (see Art. 231). The latter is proportional to the current in the fixed coils; therefore the downward pull is proportional to the square of the current through the coils. The action at the other end of the balance is the same, but the direction of the current through the coils is such that the movable coil, *ij*, is urged upward. Thus this effect is added to the other.

To restore the balance, a sliding weight is drawn along a graduated beam until the movable coils again stand in their original position midway between the fixed coils. This position is indicated by a short scale at the end of the beam, over which moves a pointer. The position of the weight on the beam is read from a scale of equal divisions, and, as shown above, this is proportional to the square of the current. To obtain the value of the current in amperes, the square root of this reading is multiplied by the constant corresponding to the particular weight used. There are four such weights and the constants are 0.5, 1, 2, and 4, respectively, for most balances. In the centi-ampere balance the result will then be given in centi-amperes.

One other matter must be noticed. When the sliding weight is placed on the beam at the zero end of the scale it is necessary to place an equal counterpoise in the pan at the other end. If this does not establish a complete balance, there is a small brass flag carried by the moving system which can be turned so as to throw more weight to one side or the other as may be required to restore the balance.

159. Calibration of a Kelvin Balance.—The weights that accompany a Kelvin balance are adjusted to give the correct value of the current when the corresponding constants are used. If for any reason it is desired to check the readings of the balance more closely, the scale can be calibrated by means of a potentiometer and a standard Weston cell. The arrangement is the same as that used for the calibration of an ammeter and shown in Fig. 92 above.

In case the corrections are nearly proportional to the readings on the uniform scale, it may be desirable to change the mass of the sliding weights.

160. The Electrodynamometer.—The e-lec″tro-dy″na-mom′-e-ter is an instrument for measuring currents. It consists essentially of two vertical coils, one fixed in place, and the other free to turn about the vertical axis common to both coils. Sometimes the movable coil is outside the other, as in the Siemens type; in other forms the movable coil is within the fixed coil. In either case, when a current flows through the movable coil it tends to turn in the same manner as the coil of a D'Arsonval galvanometer. But in the electrodynamometer the magnetic field is not due to a permanent steel magnet, but is produced by the current flowing in the fixed coil. Thus the deflection depends upon the current, I, in the fixed coil as well as upon the current, i, in the movable coil; and the resulting deflection is given by

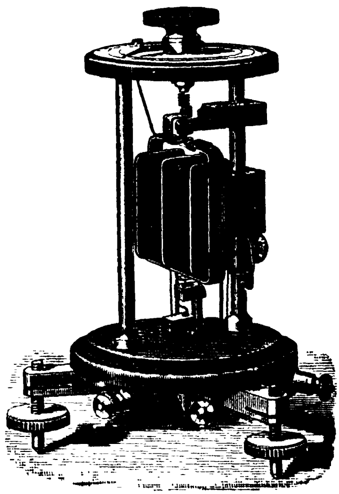


FIG. 95.—Siemens electrodynamometer.

$$iI = A^2D,$$

where A^2 is a constant including all the factors relating to the size and form of the coils, etc., and also including the restoring couple of the suspension.

If the same current flows through both coils in series,

$$I^2 = A^2D \quad \text{and} \quad I = A\sqrt{D}.$$

In the Siemens electrodynamometer the coil is brought back to its initial position by the torsion of a helical spring. D is the number of divisions of the scale which measures the amount of this torsion.

When a coil carrying a current is suspended in a magnetic field, *e.g.*, the earth's field, it tends to turn so as to add its magnetic field to the other. If the electrodynamometer is set in such a position that the earth's field is added to its own, evidently the deflection will be increased by a corresponding amount. If the two fields were opposed to each other, the deflection would be lessened. This effect can be eliminated by turning the instrument so that the plane of the movable coil is east and west.

161. Calibration of an Electrodynamometer.—In order to use an electrodynamometer for the measurement of a current, it is necessary to know the value of the constant, A, or, what is better, to have a calibration curve. Such a curve is obtained by joining the instrument in series with a calibrated ammeter or a Kelvin balance, and observing corresponding readings on the two instruments when carrying the same current. The curve is carefully plotted, using currents as ordinates and the corresponding deflections for abscissæ. This gives a horizontal parabola passing through the origin, and from this curve the value of the current corresponding to any deflection can be read. With such a curve, the dynamometer becomes a direct-reading ammeter. The deflection is independent of the direction of the current through the instrument, and therefore it can be used with alternating currents as well as with direct currents. Reversing the direction of the current through one coil only, however, will reverse the direction of the deflection.

CHAPTER VIII

MEASUREMENT OF POWER

162. The measurement of electrical power usually resolves itself into the simultaneous measurement of E.M.F. and current. As stated in Art. 9, the unit of power is the watt, and is the power expended by a current of one ampere under a potential difference of one volt. In Art. 28, there was given a simple method for measuring the power expended in a circuit by the current from a battery, using an ammeter and a voltmeter. A single instrument combining in itself the functions of both an ammeter and a voltmeter is called a wattmeter. With such an instrument the power may be read directly from a single scale, in the same way as the current is read from the scale of an ammeter.

163. The Use of an Electrodynamometer for the Measurement of Power.—An ammeter and a voltmeter connected as shown in Fig. 96 (a) for the measurement of a resistance will give at the same time the power expended in R. Let B denote the source of the current. The voltmeter, Vm, measures the fall of potential, E, between the terminals, while the ammeter, Am, gives the value of the current. The product, $EI = W$, gives the power in watts.

This result can be expressed in a different form. If in place of a direct-reading voltmeter there had been a large resistance of S ohms in series with a mil-ammeter for measuring the current, i, through it, then

$$E = Si, \quad \text{and} \quad W = SiI.$$

In this form it is seen that the measurement of power implies the product of two currents; and in Art. 160 it was seen that an electrodynamometer is an instrument for measuring the product of two currents. Therefore, an electrodynamometer can be used as a wattmeter if it is connected into the circuit in the proper manner for this purpose.

Let R, Fig. 96 (c), be the circuit in which the power is to be measured. The low-resistance coil, a, of the wattmeter, W, is connected in series with R as was the ammeter of Fig. 96 (a). The other coil, v, is joined in series with a resistance of several hundred ohms to form a shunt circuit of high resistance, and this is connected in the place of the voltmeter to measure the fall of potential over R and a. Let i denote the value of the current through this shunt circuit, and S its resistance. The

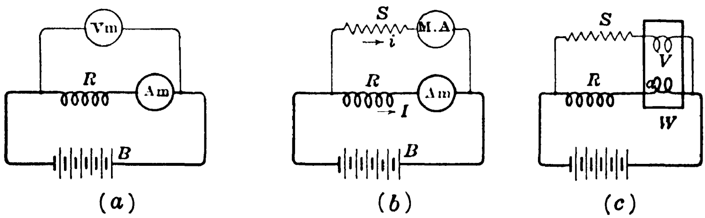


FIG. 96.—Measurement of power.

fall of potential is then Si, as in Fig. 96 (b). This current, i, through one coil of the instrument, together with the main current, I, through the other coil, will produce a deflection, D, proportional to the product of the two currents. From the equation of the electrodynamometer,

$$iI = A^2D,$$

where A is the same constant that was previously determined.

Since the power being expended in R is $W = SiI$, we now have

$$W = SiI = SA^2D.$$

If the constant of the instrument is known, then SA^2 becomes the factor for reducing the scale readings to watts. In case this factor is unity, as it can be made by adjusting the value of S, the wattmeter is said to be direct reading.

It may be that the value of A is not known, but instead there is a calibration curve for the instrument when used as an electrodynamometer. In this case the value of A^2D can be obtained directly from the curve, for it is the square of the current I' which would produce the same deflection, D.

Thus the power expended in R is

$$W = SI'^2,$$

where S is the resistance of the shunt circuit and I' is not any real current, but it is the current which gave the same deflection when the instrument was used as an electrodynamometer and the value of which can be obtained from the calibration curve.

164. The Weston Wattmeter.—The Weston wattmeter consists, essentially, of a moving coil electrodynamometer. The fixed coil is wound in two sections on a long cylindrical tube, within and between which is the movable coil. The latter is wound with fine wire upon a very short section of a cylindrical tube of somewhat smaller diameter than the fixed coil, and supported on pivots so that it can readily turn about its vertical diameter. Attached to the movable coil is a long light pointer which moves over a graduated scale.

In the position of rest the axis of the movable coil makes an angle of about 45 deg. with the axis of the fixed coil. When deflected so that the pointer is at the middle of the scale, the two coils are at right angles. At the extreme end of the scale the coil stands at 45 deg. on the other side of the symmetrical position. This gives a fairly uniform scale over its entire length. A spiral spring brings the coil to the zero position and provides the torque necessary to balance the electrodynamic couple due to the currents in the two coils.

In addition to the main fixed coil there is another fixed coil of fine wire and having the same number of turns as the other, so that a current sent through one coil and back through the other will produce no magnetic field at the place of the movable coil. It is then possible to compensate for the effect of the shunt current passing through the series coil, for the shunt current can be lead back through this second coil and thus be made to neutralize its action upon the movable coil. When it is desired not to use the compensation coil it is replaced by an equal resistance, this connection being brought out to a third binding post.

In the wattmeter reading up to 150 watts, the resistance of the series coil is 0.3 ohm, and that of the shunt circuit is 2,600 ohms. The compensation winding is about 3 ohms.

165. Comparison of a Wattmeter with an Ammeter and a Voltmeter.—The reading of a wattmeter can be compared with the power measured by an ammeter and a voltmeter, provided that the latter instruments are connected to measure precisely the same power as the wattmeter. This means, for the uncompensated wattmeter, that the current through the series coil of the wattmeter must be measured by the ammeter, and the voltmeter must be connected so as to measure the same fall of potential as the shunt coil of the wattmeter.

This is accomplished by the connections shown in Fig. 97. The power thus measured is not that expended in R alone, but it includes the power expended in the voltmeter and in the shunt circuit of the wattmeter. But since both instruments

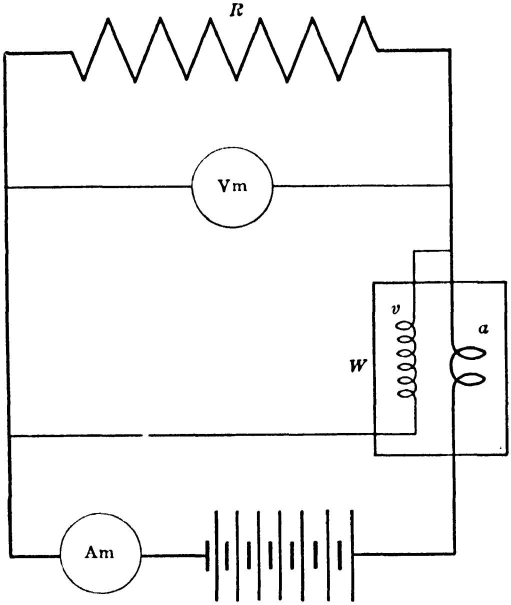


Fig. 97.—Comparison of a wattmeter, W, with an ammeter and a voltmeter.

measure this same power, the reading of the wattmeter should agree with the product of the readings from the ammeter and voltmeter.

If the wattmeter is compensated so that the power measured by it does not include the power expended in its own shunt circuit, then the ammeter must be connected so as not to measure this shunt current. But as it is not possible to connect the ammeter and voltmeter so as not to measure the power expended in one or the other of them, the best arrangement will be to join the voltmeter in parallel with R as shown in Fig. 98. The power measured by the ammeter and voltmeter will be that expended in both R and the voltmeter; that measured by the wattmeter will be greater than this by the amount of power expended in the ammeter. The latter can be computed from the

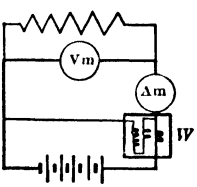


Fig. 98.

formula rI^2, and added to the product of the readings from the ammeter and voltmeter. With this slight correction the wattmeter reading should equal VI.

166. Power Expended in a Rheostat. (*a*) *When Carrying a Constant Current.*—The object of this exercise is to give the student some personal experience in the measurement of power and in the careful use of a variable rheostat. In this first part a variable resistance is joined in series with a large E.M.F. and considerable other resistance, so that the variations in r will not materially change the value of the current through the circuit. If desired, this variable resistance may include an ammeter, and the current may be kept always at the same value.

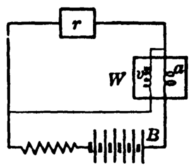


FIG. 99.—Measurement of the power expended in r.

The wattmeter is connected to r as shown in Fig. 99, and readings of the power expended in the rheostat are taken for the entire range of the resistance. If the values of the latter are not known, they can be measured by one of the methods previously given. A curve should be drawn, using the resistances in the rheostat as abscissæ and the corresponding amounts of power as ordinates. The report should contain a discussion explaining why this curve comes out with the form it has.

(*b*) *When under Constant Voltage.*—In this part of the exercise the arrangement is much the same as before, except that the high E.M.F. is replaced by a few cells of a storage battery, and r is now the only resistance in the circuit. Starting with the largest values of r, readings of the wattmeter are taken and plotted as before. It will not be safe to reduce r to zero, and readings should be continued only for current values that are not too large for the apparatus used. This portion of the report should give a discussion of what would probably happen if the rheostat were reduced to zero.

167. Efficiency of Electric Lamps.—An interesting exercise, and one that furnishes considerable information, is the determination of the efficiency of various types of electric lamps. By means of a wattmeter, the amount of electrical energy supplied to the lamp is readily measured. The light that the lamp furnishes can be measured by a photometer. The efficiency of the lamp is expressed as the ratio of the candle power of the lamp to the electrical power expended, and is given as so many "candles per watt." Thus the efficiency is not an abstract ratio, as in most cases, because it is not possible to measure light in watts. But this does not prevent a satisfactory comparison of different lamps.

Any good form of photometer can be used to measure the candle power of the lamp being examined. If none is at hand, a simple form can be made by standing some object in front of a white screen. The standard light and the one being measured will each cast a shadow of the object on the screen, and by varying the distance of one of the lights from the screen, the intensities of the two shadows can be made equal. The intensities of the two lights are then to each other as the squares of their respective distances from the screen.

If a number of different kinds of lamps are available, it is instructive to measure the efficiency of each one. Tungsten lamps can be compared with carbon lamps, and lamps that have been in use for a long time can be compared with new lamps of the same kind. It is also interesting to determine the efficiency of the same lamp when burned at different voltages, and it is well to plot a curve with efficiencies as ordinates with the corresponding voltages for abscissæ. Incidentally, the curve showing the variation of candle power with voltage can be plotted.

168. Measurement of Power in Terms of a Standard Cell.— In the preceding chapters there have been given methods for measuring either a current or a difference of potential in terms of the E.M.F. of a standard cell. By combining two of these methods it is possible to measure power in like manner, and this is especially useful when it is desired to know accurately the value of a given amount of power. For example, in some methods of calorimetry it is necessary to have supplied a known amount of heat. Often the actual amount is not essential, but, whatever it is, it must be known to a high degree of accuracy. In such cases it is convenient to generate the heat by an electric current flowing through a resistance coil, and then to measure the electrical power expended in terms of a standard cell of known E.M.F. This means the measurement of both the current and the fall of potential, as the resistance of the coil usually cannot be accurately determined under the conditions of actual use.

One convenient arrangement, which is capable of wide variation in the amount of power that can be measured, is shown in the figure. The heating coil in which the power is expended is denoted by H. In series with this is a coil, C, of sufficient current-carrying capacity not to be heated by the current through it. There is also a variable resistance, r, by which the current can be brought to any desired value. The current through C is measured by the method for calibrating an ammeter (see Art. 157) and the fall of potential over H is measured by the method for calibrating a voltmeter (see Art. 146).

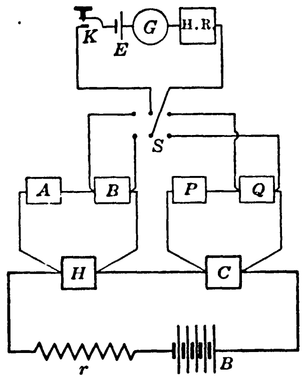


FIG. 100. — Measurement of power expended in H in terms of a standard cell, E.

The standard cell is joined in series with a sensitive galvanometer and a high resistance. The circuit thus formed is connected to the middle of a double-throw switch, S. One end of this switch is connected to B, which is a portion of a shunt around H. By adjusting A and B, the fall of potential over the latter can be made equal to the E.M.F. of the standard cell, as shown by no deflection of the galvanometer when the key, K, is closed. The total fall of potential over H is, then,

$$V = \frac{A + B}{B} E_s.$$

In the same way there is a shunt circuit, PQ, in parallel with C, and the part Q is joined to the other end of the double-throw switch. When the switch is thrown to this side, P and Q can be adjusted to give no deflection of the galvanometer when the key is closed. This means that the fall of potential over Q has been made the same as the E.M.F. of the standard cell, and the total fall over C is computed as shown above for H. This, divided by the resistance, C, gives the current through C as

$$I = \frac{E_s}{C} \frac{P + Q}{Q}.$$

A little consideration will show that the main current through the battery is larger than I by the amount of current that flows through the shunt circuit, PQ; and the current through H is smaller than the main current by the amount of current that flows through the shunt, AB. If Q is set equal to B, then, since the fall of potential over each is the same, the currents through these two shunts will be equal. Therefore the current through H will be equal to the current through C. The power expended in H is, then,

$$W = VI = \frac{E^2}{B^2}\frac{(A + B)(P + Q)}{C},$$

when the two shunt currents have thus been made equal.

169. Calibration of a Non-compensated Wattmeter.—The wattmeter in this case may be an electrodynamometer with two separate coils, or it may be the regular Weston wattmeter

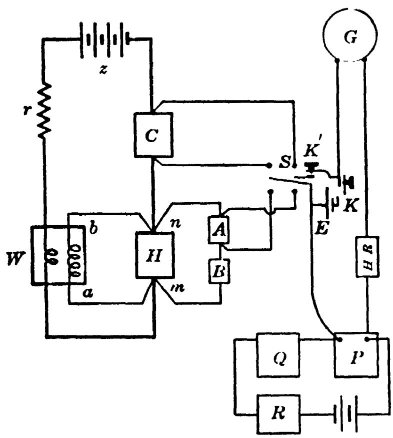


FIG. 101.—Calibration of the wattmeter, W.

used without the compensation coil. The series coil of the wattmeter is connected in series with a resistance, H, in which can be expended the power measured by the wattmeter. There is also in series a standard resistance, C, whose value is accurately known and which has sufficient current-carrying capacity not to be heated by the currents used in the calibration. A variable resistance, r, and a storage battery, z, complete the main circuit. The shunt coil, ab, of the wattmeter is connected in parallel with H. Two accurate resistances, A and B, are also connected in parallel with H. The power measured by the wattmeter is then the total power expended in H, the wattmeter shunt circuit, and the circuit consisting of A and B. This power is the product of the current through these three in parallel and the potential difference between m and n. Each of these quantities is determined by the potentiometer.

The potentiometer is represented by the three resistances, P, Q, and R. Across P is joined the galvanometer and standard cell in the usual way. When P has been adjusted for a balance,

$$E = iP, \qquad \text{or} \qquad i = \frac{E}{P},$$

where E is the E.M.F. of the standard cell and i is the steady constant current that is flowing through the main circuit of the potentiometer.

To determine the value of the current through the standard resistance, wires are brought from the terminals of C to the double-throw switch, S. When this switch is thrown up and K' is closed, C is connected into the galvanometer circuit in the place of the standard cell. Readjusting the potentiometer for a new balance, P', gives

$$IC = iP' = \frac{EP'}{P}$$

and, hence,

$$I = \frac{EP'}{CP}.$$

To determine the fall of potential over H, the double-throw switch is thrown down, thus connecting the resistance, A, into the galvanometer circuit. Let i' denote the value of the small shunt current flowing through A and B. Then, adjusting the potentiometer for a balance,

$$i'A = iP'' = \frac{EP''}{P},$$

where P'' is the new reading of the potentiometer. The fall of potential over both A and B is $\frac{A + B}{A}$ times as large, or,

$$V = \frac{EP''}{P} \frac{A + B}{A},$$

and this is the same as the fall of potential over H.

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2}{P^2} \frac{P'P''}{C} \frac{A + B}{A}.$$

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = W' - W.$$

Different readings of the wattmeter are secured by changing the current through H. A calibration curve can be plotted, using the readings of the wattmeter as abscissæ and the corresponding corrections for ordinates.

170. Calibration of a Compensated Wattmeter.—For this purpose a potentiometer may be used, as in the preceding method, but as this instrument is not always at hand, a method using resistance boxes is given here. The principal difference

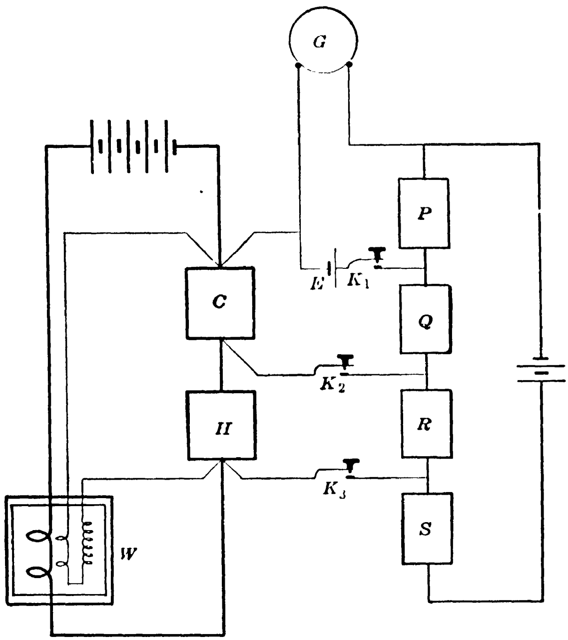


FIG. 102.—Calibration of the wattmeter, W.

between the compensated wattmeter and the uncompensated one is that the former measures only the power expended in the circuit to which it is attached, while the latter measures, in addition to this, the power expended in its own shunt circuit.

Thus let W, Fig. 102, represent the wattmeter connected in the circuit to measure the power expended in H and C together. C is a standard resistance for use in the measurement of the current, and H is sufficient other resistance to give the required amount of power. As the power expended in the shunt coil is not measured by the wattmeter, it should not be measured by the standard cell; therefore C is placed inside, next to H.

The calibration circuit consists of four resistance boxes, P, Q, R, and S, joined in series with a battery of a few volts sufficient to maintain a small constant current through the circuit. This circuit is joined through the galvanometer to the standard cell and to the wattmeter circuit in three places, as shown, each connection being provided with a key. When finally balanced, no current flows through any of these connections.

The measurements are made as follows: First P is set at some convenient value, say 1,000 ohms, and some of the remaining resistances are then changed until there is no deflection of the galvanometer upon closing the key, K_1. This fixes the total resistance of this circuit and it is thereafter kept constant at this amount. The fall of potential over C should be a little larger than the E.M.F. of the standard cell. It will then require a little more resistance added to P to give no deflection when the key, K_2, is used. This is added by varying Q and S, keeping their sum constant, until there is no deflection of the galvanometer upon closing the key, K_2. This balance measures the value of the current through C, for

$$IC = i(P + Q) = (P + Q)\frac{E}{P},$$

and, therefore,

$$I = \frac{E}{C}\frac{P + Q}{P}.$$

This is the effective current actuating the wattmeter.

The fall of potential over CH is measured in the same way. This will be greater than that over C alone, therefore for a balance it will require a resistance greater than $P + Q$. The resistance R is now varied, keeping $R + S$ constant, until there is no deflection of the galvanometer upon closing K_3. The fall of potential over CH is then the same as that over the three resistances, P, Q, and R. That is,

$$V = i(P + Q + R) = (P + Q + R)\frac{E}{P}.$$

The power measured by the wattmeter is, then,

$$W' = VI = \frac{E^2(P + Q)(P + Q + R)}{P^2C}.$$

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = W' - W.$$

In case V is too large to be measured directly as here shown, there may be placed a shunt, AB, around H as shown in the preceding method and the potential fall over A alone measured. The total fall of potential over H is then readily computed. The addition of this shunt will make no difference in the wattmeter, since H with its shunt now replaces H alone and the wattmeter measures whatever power is expended in either arrangement.

171. Calibration of a High-reading Wattmeter.—A high-reading wattmeter is one that measures large amounts of power. In calibrating such a wattmeter it is often impossible,

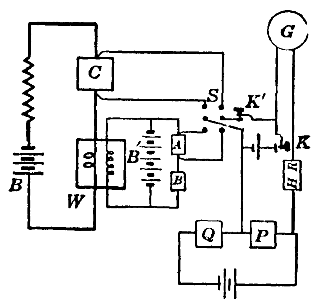


FIG. 103.—Calibration of the wattmeter, W.

and usually it is inconvenient, to expend sufficient power to bring the reading up to the high values indicated on the scale. But this is not necessary, for all that is required is that there shall be a large current through the series coil and a small current at high voltage through the shunt circuit. By using different batteries to supply these two currents, there need be no great expenditure of energy. As shown in the figure, the battery, B, consisting of one or two large storage battery cells, supplies a large current through the series coil of the wattmeter and the low standard resistance, C. The latter should be of such a value that the fall of potential over it will be a volt or less in order that this may be readily measured by the potentiometer. Since the resistance of the shunt circuit is large, it will require a large number of cells in the battery, B', but these cells can be small, as only a small current will be needed. In parallel with this circuit is placed another high-resistance circuit, AB, so divided that the fall of potential over the portion A shall be about 1 volt. The calibration is then the same as given above for the case of an uncompensated wattmeter, and the wattmeter should read the product of the current through C and the voltage across A and B.

If the reading of the wattmeter is W, the correction to be added to this reading is

$$c = VI - W.$$

In making this calibration it is best to have the currents through the wattmeter about the same as will be used when the instrument is measuring power.

CHAPTER IX

ELECTRON TUBES

172. Electron Emission.—It has been stated in earlier chapters that a current is a stream of electrons along the wire. Evidence of such a stream of electrons is afforded by the phenomena within an electron tube. When a hot filament is used in a vacuum tube with a cold electrode it is possible to pass an electron current through the vacuum space from the hot filament to the cold electrode by an E.M.F. that maintains the filament a few volts more negative than the cold electrode. When both electrodes are cold, the vacuum makes a very good insulator.

It has been shown by experimental investigation that the particles passing through the vacuum from the hot filament to the cold electrode have the same mass and carry the same charge as the electrons that are found in many other phenomena. That is, each one has a mass that is only $1/_{1,850}$ of the mass of an atom of hydrogen, while the charge carried by each one is equal to the charge carried by each atom of hydrogen in electrolysis. The supply of these electrons seems to be inexhaustible and is vastly greater than could have been supplied by the filament itself. The electrons, therefore, must have been supplied to the filament by flowing in through the colder metal connections, and as there is no accumulation of electrons on the cold electrode in the vacuum tube, they must have been able to flow away over the connecting wires. Tested with an ammeter, these wires are found to carry an electric current, the negative direction of which is the direction taken by the electrons as they pass along the wire on their way back from the cold electrode to the hot filament. If the battery between the cold electrode and the hot filament is reversed, no current passes through the circuit, since no electrons can leave the surface of the cold electrode.

173. Electron Tubes.—As usually seen at the present time, an electron tube contains three essential parts. The *filament* is similar to the filament of an ordinary electric light bulb, but shorter and of less resistance, so that it is lighted by a low-voltage battery. In some tubes the tungsten filament contains a small amount of thorium, and an abundant emission of electrons is obtained at a temperature below red heat.

Surrounding the filament, and a short distance from it, is a thin metal sheath called the *plate*. This is provided with an outside terminal, and, together with the filament, constitutes a two-electrode tube. In most tubes, however, there is a network of fine wires surrounding the filament and about midway between the filament and the plate. From its form this is called the *grid*. The grid is well insulated from both the filament and the plate, and it has its own outside terminal. Such a tube is called a three-electrode electron tube and has the familiar four prongs projecting from its base for the external connections. In general appearance an electron tube resembles an electric light bulb, and the pressure within the tube is reduced to a very good vacuum. Because of this fact these tubes are often called vacuum tubes.

174. Diagram of an Electron Tube.—In the diagrammatic representation of an electron tube the arrangement shown in Fig. 104 is used. The filament is shown by the narrow loop, *FF*, the + and − signs indicating the direction in which the heating current passes through the filament. The battery, of a few cells, that supplies this heating current is commonly called the "A" battery.

The vertical line, *P*, at the right-hand side of the figure represents the plate, taken as being at right angles to the plane of the paper. In reality, of course, this sheath surrounds the filament equally on all sides.

The wavy line, *G*, represents the grid of fine wires between the filament and the plate. In an actual tube the connections to the plate and to the grid are placed in the base near the filament connections. In the figure these are shown at the side and top of the diagram to avoid confusion in the drawing. The circle around the whole indicates that these elements are enclosed within the glass bulb of the electron tube.

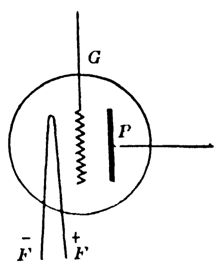


Fig. 104.

The potential of the plate is usually reckoned from the negative terminal of the filament, and the battery that supplies this voltage is commonly called the "B" battery. If a battery is used to give a definite potential to the grid, it is called the "C" battery.

175. Effect of Temperature upon Electron Emission.—As already suggested, electrons are more readily emitted from the filament when it is hot than when cold. This relation between the electron current and the temperature can be investigated with an arrangement as shown in Fig. 105.

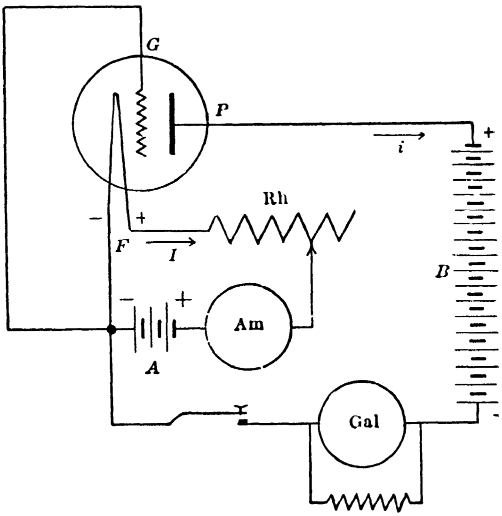


Fig. 105.—The plate current, *i*, is increased by increasing the temperature of the filament.

The filament is heated by a current from the "A" battery of a few storage cells. The current, *I*, can be varied by the rheostat, *Rh*, and measured by the ammeter, *Am*. The temperature of the filament can be taken as being proportional to I^2.

From the plate is a circuit through the "B" battery of 20 to 100 volts, a key, and the galvanometer, and leading to the negative end of the filament. If the "B" battery is connected so as to drive electrons from the plate to the filament, no current flows, even with the filament heated to incandescence. With the + end of "B" connected to the plate, a current is readily obtained, and it may be necessary to use a shunt with the galvanometer. In using 100 volts with a galvanometer, great care must be exercised to make sure that there will be no leakage current from any part of the battery to the galvanometer. The connection from the + terminal to the plate should be especially well insulated from the rest of the arrangement.

Keeping the voltage of the "B" battery constant, and starting with a small current through the filament from the "A" battery, observe the growth of the "plate current," i, that flows through the tube as the filament is heated more and more. Since this current flows across the space from the filament to the plate,

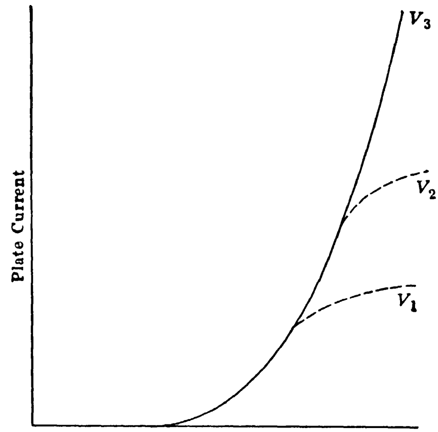


FIG. 106.—The plate current increases very rapidly as the filament-heating current is increased.

it is sometimes called the "space current." This relation can be shown by a curve between corresponding values of I^2 and i. (See Fig. 106.)

Similar curves can be determined for several different values of the "B" battery voltage. If i seems to reach a maximum value as I is increased, it is not because of any lack of electrons but because the potential gradient[1] near the filament is not sufficient to overbalance the effect of the space charge.

176. The Space Charge.—As pointed out above, current does not pass through the tube when the plate is connected to the negative end of the "B" battery. If the negative potential of the plate is due to a static charge of electrons upon it, the effect is the same, and any electrons in the neighborhood are urged back towards the filament. The negative charge need not even be on a metal plate in order to produce this modification of the potential

[1] The term "potential gradient" denotes the fall of potential per centimeter, and it is measured in volts per centimeter.

gradient between the plate and the filament, and every electron in the space around the filament, whether stationary or moving in the current from the filament to the plate, produces something of this effect.

When the effect of the negative charge of the electrons in the space around the filament is considered, the electrons concerned are referred to collectively as the space charge. The electron

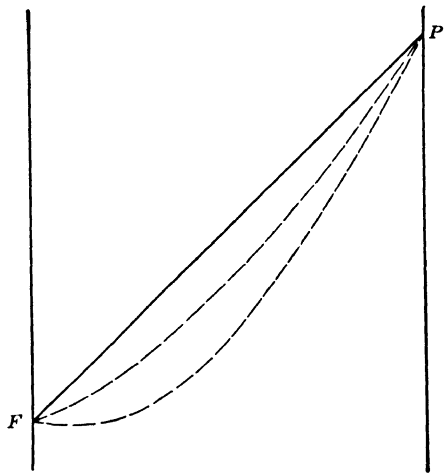


FIG. 107.—The potential gradient is uniform between two parallel plates. A space charge changes this, as is indicated by the dotted lines.

current is most concentrated near the filament and it is here that the effect of the space charge is most marked. When there is a large emission of electrons, the space charge around the filament may be sufficient to act like a negative potential on the plate and prevent a further increase of the electron current from the filament.

When a difference of potential is applied to two parallel plates, as *P* and *F*, Fig. 107, this can be represented as a uniform fall of potential across the space from one plate to the other. But a cloud of electrons in this space will lower the potential of this region so that the actual fall of potential is no longer uniform, but is more like the dotted line in Fig. 107. The larger the number of electrons the greater is this effect, until the condition is reached in which there is no rise in potential near *F*, and there would be no tendency for electrons to move from *F* towards *P*. This would prevent the continued increase of the current.

In the electron tube the case is similar to the parallel plates, but it is not as simple. The filament and sheath are more nearly like two concentric cylinders. The fall of potential across the space between two concentric cylinders is not uniform, but it is logarithmic, as shown in Fig. 108. Since the surface of the filament is very small, the density of the electrons near it is large

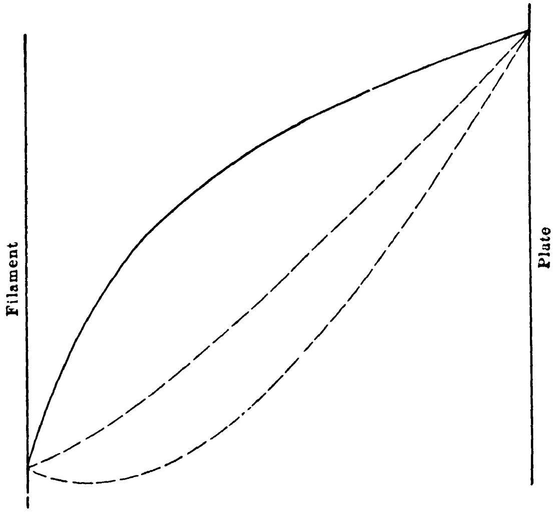


FIG. 108.—Most of the potential difference between a wire and a concentric cylinder is near the wire. But a space charge may greatly modify this distribution.

and the effect of this large negative charge near the filament reduces the potential gradient in this region, as explained above, in spite of the large gradient of the normal distribution of the potential, and it may even reverse the direction of the fall of potential, as shown in Fig. 108.

A larger voltage for the "B" battery will give a steeper potential gradient throughout the space between the filament and the plate, and this will give sufficient rise of potential around the filament to carry a larger stream of electrons. This is shown in Fig. 106 by the larger final values of the plate current.

177. Relation of the Plate Current to the Plate Voltage.—The current through the plate circuit depends to some extent upon the voltage of the "B" battery. As this voltage is increased the current becomes larger, until a point is reached where a further increase in voltage does not increase the current. The actual value of this saturation current depends upon the temperature of the filament. As the supply of electrons becomes more abundant, the approach of the saturation value of the current is less marked, but the final value of the current is larger. Figure 109 shows curves for several different temperatures of the filament. The normal current for this filament is 1.00 ampere, and with this heating the saturation value of the plate current is not reached with 80 volts. However, with a current of 0.80 ampere through the filament, the plate current reaches its saturation value at 40 volts, and does not increase as the voltage is further increased.

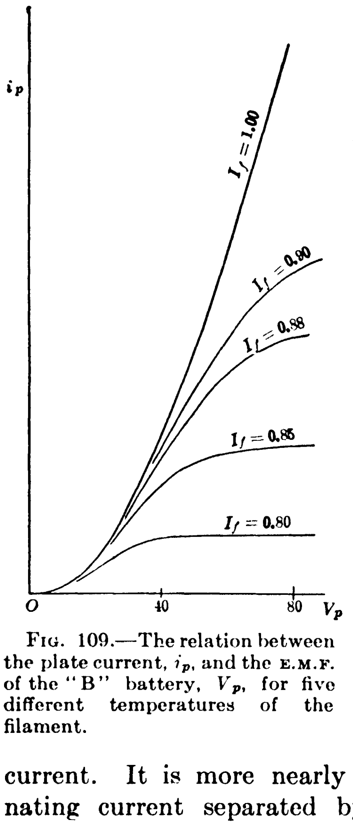


FIG. 109.—The relation between the plate current, i_p, and the E.M.F. of the "B" battery, V_p, for five different temperatures of the filament.

178. The Electron Tube as a Rectifier.—As indicated in Art. 172, the stream of electrons can pass through the tube in only one direction. When an alternating E.M.F. is used in the plate circuit in the place of the "B" battery there will be current through the circuit whenever the E.M.F. is in the proper direction, but there will be no current during the other half of the cycle. The current that flows, then, will be in one direction only, but it is not a steady current. It is more nearly like the positive loops of an alternating current separated by intervals of zero current. The English name of "valves" for these tubes indicates this rectifying property. This variable direct current is not desirable where a steady current is required, but it is suitable for some purposes and it is often useful when no other source of direct current is available. By rectifying a high-voltage alternating current, it is possible to obtain a high-voltage direct-current source. A modification known as the "tungar rectifier" is used for charging storage batteries.

179. The Grid.—The effect of the space charge around the filament in reducing the electron current has been noted in Art. 176. By placing a grid of fine wires near the filament, the potential of this region can be varied at will. When the grid is given a negative potential with respect to the filament, the effect of the space charge is increased, and this may be carried so far as practically to stop the electron current. On the other hand, by making the grid positive with respect to the filament, the effect of the space charge can be neutralized, or even overbalanced, thus giving an increased potential gradient near the filament and a greater acceleration to the electrons in this region.

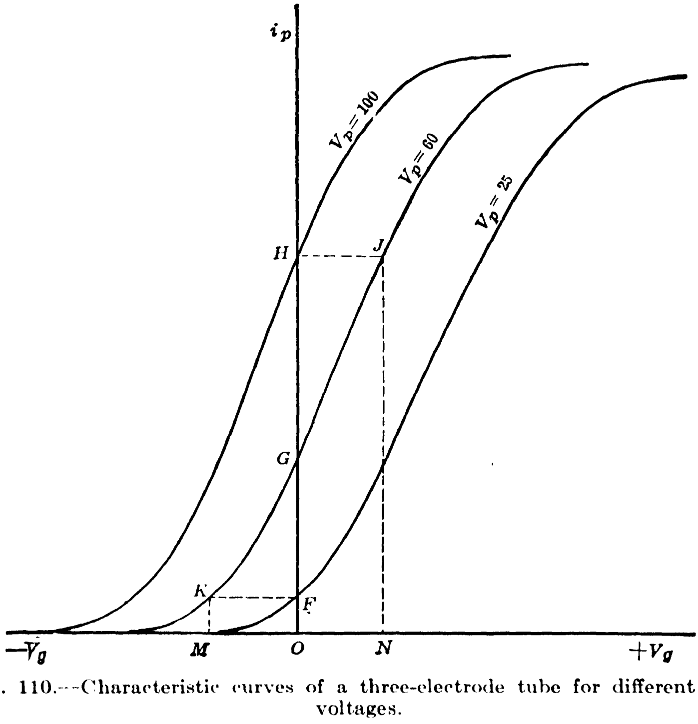


FIG. 110.—Characteristic curves of a three-electrode tube for different plate voltages.

The electron tube with such a grid serves as a very useful relay in an electric circuit. By merely applying a voltage to the grid, involving very little expenditure of power, the current through the plate circuit can be varied considerably. Since this plate current is driven by the high-voltage "B" battery, the variation in power is correspondingly greater. The response of the plate current is very prompt and the tube thus acts as an inertialess relay.

180. Static Characteristic of a Three-electrode Tube.—The variation that is produced in the plate current by changing the potential of the grid is shown by the curve *KGJ* in Fig. 110. The "B" battery is supposed to have maintained a constant voltage, V_o, in the plate circuit for all points on this curve. Such a curve is called the "static characteristic curve" for the electron tube and it shows the steady value of the plate current for any given set of steady values of the grid and plate voltages. While the tube is not ordinarily used in this manner, curves obtained in this way are useful in showing what effects may be expected in the tube when these voltage values are varied in any manner.

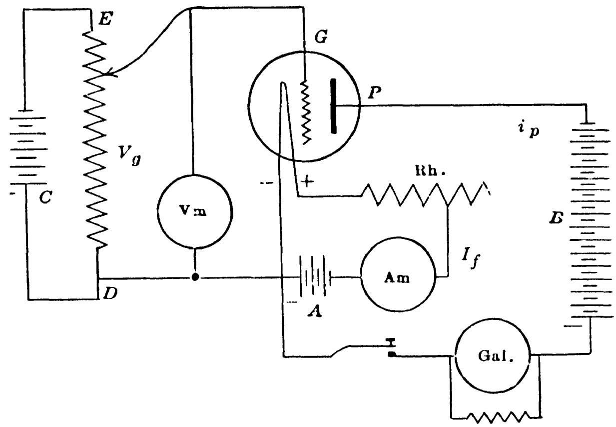


FIG. 111.—An arrangement for the determination of the characteristic curve of the electron tube.

The arrangement for the experimental determination of the static characteristic curve is shown in Fig. 111. The "A" and "B" batteries are connected in the usual way. With a sliding potential divider, *ED*, connected as shown in the figure, the grid can be maintained at any desired voltage. The potential of the grid is usually compared with the potential of the negative end of the filament, and this value is measured by the voltmeter, *Vm*. For negative grid voltages, the battery "C" is reversed. The current through the plate circuit is measured by the galvanometer as before. Plotting corresponding values of the grid voltage, V_g, and the plate current, i_p, gives one of the curves shown in Fig. 110. If the plate had been maintained at a different potential, by using a different value for the "B" battery voltage, V_p, a similar characteristic curve would have been obtained but shifted to the right or left, as shown by the curves of Fig. 110.

181. The Surface Characteristic.—The total characteristic of an electron tube involves more factors than can be represented by a single curve, or even by a series of such curves, although a person who is familiar with these curves can read much information from a set of curves like those shown in Fig. 110. It is not as easy, however, to see how the current increases through the values *OF*, *OG*, and *OH*, as V_p is increased as it is to refer to the i_p-V_p curve of Fig. 109 for $I_f = 1.00$ ampere. In fact, the latter curve can be considered as being drawn on a plane through *OH* and perpendicular to the plane of the paper in Fig. 110. If the entire series of both sets of mutually perpendicular curves were drawn, the result would be a surface. The curves shown in Figs. 109 and 110 are the intersections of this surface with different vertical planes.

Such a surface can be constructed in the form of a plaster model. A series of small holes are drilled in a smooth board 1 cm. apart each way, dividing the board into squares. Each row of holes in one direction is marked with a value of V_p from zero to the maximum that is used. The rows of holes in the cross-direction are marked with the values of V_g, letting the zero line be through the center of the board. Along one row of holes, set a series of vertical wires of lengths to represent the values of i_p. The tops of these wires will make a curve like one of those in Figs. 109 or 110. When all the rows are filled with vertical wires, their tops will indicate the characteristic surface corresponding to the filament temperature that was maintained while these currents were being measured. To make the surface more apparent, the whole group of wires can be filled up with plaster of Paris, smoothed off even with the tops of the wires. The whole series of characteristic curves can be drawn on this surface by connecting the ends of the wires in each row for which V_p has a constant value. Such a surface is useful in giving a complete survey of the action of the tube with a given filament temperature. A series of such surfaces will represent the characteristics for different temperatures of the filament.

It will be found that a certain limited part of these surfaces contains the most desirable characteristics for the principal uses of an electron tube. In drawing a single characteristic curve, a person familiar with the general form of these surfaces will chose such values of V_p, V_g, and I_f as will place the curve through this most favorable region.

182. Voltage Amplification.—Since the "B" battery maintains the plate at a constant potential, at first sight there would seem to be no possibility of voltage amplification. One meaning that may be given to this term is illustrated in the curves of Fig. 110. With a plate voltage of 60 volts, and the grid connected to the negative end of the filament, the plate current of a few milliamperes is represented by *OG*. This current can be increased to *OH* by increasing the plate potential to 100 volts, or, without changing V_p, the current can be increased to this same value by increasing the potential of the grid by the amount *ON* (= 5 or 6 volts), giving the value *JN* as the new plate current. In this case the application of a few volts on the grid has produced as great a change in the current as could be given by an additional 40 volts in the plate circuit.

In the same way the current could be reduced to *OF* by reducing the plate voltage to 25 volts. Or, without changing the plate voltage, the current can be reduced to *KM* by giving the grid a negative potential of a few volts (= *OM*).

183. Amplification Constant.—It was shown above that a change, V_g, in the grid potential is equivalent to a much larger change, V_p, in the plate potential. This is expressed by the relation

$$a\,\Delta V_g = \Delta V_p$$

or,

$$a = \frac{\Delta V_p}{\Delta V_g}$$

This ratio, a, is called the voltage amplification constant for the tube. Referring to the model of the surface characteristic, a is the ratio of the slope of the surface in the direction of V_g to the slope in the direction of V_p. The value of a is not constant for all values of V_g, but it is nearly constant for the voltages commonly employed.

184. Determination of the Amplification Constant.—While the value of the amplification constant, a, can be obtained from the curves of Fig. 110 when these are accurately drawn to scale, this only gives its value for steady conditions. Electron tubes are commonly used with rapid changes in voltage, and it is desirable to make a direct determination of a under conditions resembling those that obtain in the use of the tube. This can be done with the arrangement shown in Fig. 112, in which a small alternating voltage is introduced into the grid circuit and at the same time enough alternating voltage is added to the plate battery to keep the current from changing.

By means of the steady voltages from the batteries *A*, *B* and *C*, the electron tube can be set at any desired point on the characteristic curve, or surface. A small alternating current can be

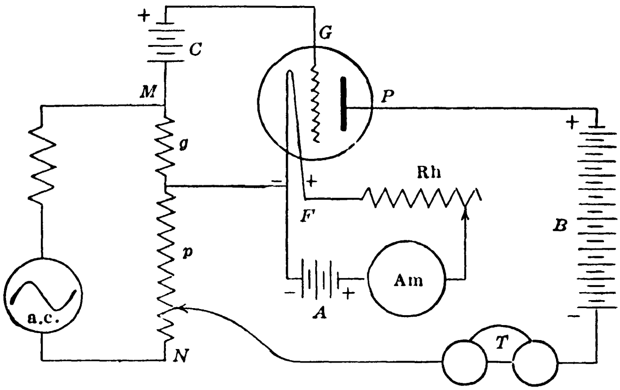


FIG. 112.—The arrangement for the direct determination of the voltage amplification constant.

passed through the non-inductive resistances, *g* and *p*. These resistances should not be large enough to increase materially the resistances of the grid and plate circuits, and for the same reason a low-resistance telephone receiver, *T*, should be used.

By tracing out the connections, it will be seen that when the alternating current, i', is flowing from *M* to *N*, the fall of potential gi', is added to the voltage of the "C" battery. This tends to increase the plate current. At the same time, the fall of potential, pi', is subtracted from the voltage of the "B" battery, and this tends to decrease the current. When these two effects are balanced, there is no change in the plate current, and no sound is heard in the telephone receiver, *T*. The amplification constant is, then,

$$a = \frac{pi'}{gi'} = \frac{p}{g}$$

for this point on the characteristic.

By changing the voltage of one of the batteries, the value of *a* for the new conditions can be determined in the same way.

185. The Electron Tube as a Relay.—One of the important uses of an electron tube is that of a relay, in which the plate current is made to reproduce all of the variations of the original current. Along the middle portion of the characteristic curve, the relation between V_g and i_p is very nearly linear. Therefore, when V_g is made to vary in any given manner, the plate current, i_p, will vary in the same manner and reproduce the same changes. For example, if a weak telephone current is transformed to a higher voltage (and a still smaller current), and this voltage applied to the grid of the electron tube, the plate current will respond with the same variations and much greater power. When the changes in V_g are large enough to extend beyond the linear part of the curve, the plate current is no longer proportional to the applied voltage, and the resulting wave form is distorted from the original.

186. Radio Communication.—Electron tubes are used extensively in radio communication, both as detectors and as amplifiers, but it is beyond the design of this book to discuss the various circuits that are used. To understand how the results are obtained requires at least an elementary knowledge of alternating currents and the effects of inductance and capacitance upon such currents. These subjects are treated in the following chapters, and after the last page has been read it will be easier to understand the following article on the action of an electron tube as a generator of alternating currents. This topic is introduced here because the device may be employed as a source of high-frequency alternating current for use in some of the measurements of capacitance and inductance that are described in the following chapters.

187. The Electron Tube as an Alternating-current Generator.—In studying the effects of inductance and capacitance, and in using alternating currents for bridge measurements, it is desirable to have a frequency of about 1,000 cycles per second. This is not readily obtained from alternating-current dynamos, but such a current can be obtained with the arrangement shown in Fig. 113. The grid of the electron tube is connected to the resonance circuit, *CL*, which contains the proper amounts of inductance and capacitance to resonate freely at the desired frequency. When the resonance current is oscillating through *C* and *L*, the potential of the grid is raised and lowered with every surge of the charge in and out of the condenser. This will impress the same variations on the current in the plate circuit, but this current is still a direct current. In this circuit is placed the primary of a telephone transformer, the secondary of which becomes a source of alternating current of the frequency of the variations in the plate current. In order to maintain the oscillations in the resonance circuit, *CL*, the plate current also passes through a coil, *Q*, which is inductively coupled with *L*, and in the sense that will tend to increase the oscillations in *CL*. For a 1,000-

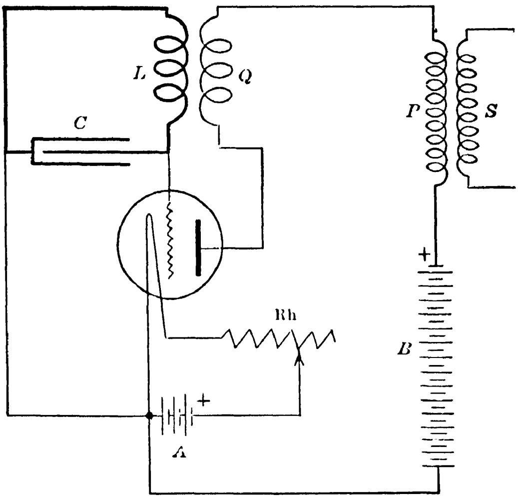


Fig. 113.—An electron tube generator for supplying audio frequency alternating current from the terminals of *S*.

cycle current it has been found suitable to use the coils of a small, bell-ringing transformer for *L* and *Q*, taking the 110-volt winding for *L* and the 14-volt winding for *Q*. A small "grid" condenser of 0.005 mf. is used for *C*. A 5-watt tube is satisfactory and 200 volts can be used for the plate battery. Such a generator will begin to oscillate as soon as the circuits are closed and will continue as long as the proper filamen current is maintained.

CHAPTER X

MEASUREMENT OF CAPACITANCE

188. Standard Condensers.—Mica condensers of a few tenths of a microfarad are often assembled in a box for convenient use. When the capacitances have been adjusted to the given values, such condensers serve as useful standards. Figure 114 shows

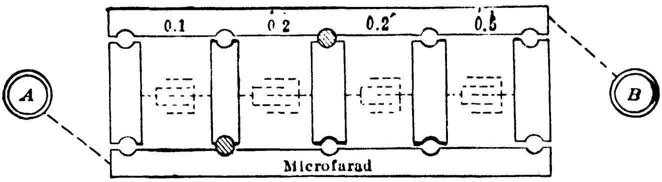


FIG. 114.

the arrangement of the brass blocks on the hard rubber top of a box containing four condensers. Each condenser is permanently connected to two of these metal blocks. The longer blocks at either side are connection pieces to the binding posts, A and B. By means of plugs, one or more of the condensers can be connected to these long blocks and thus to the binding posts. With two plugs inserted as shown in the figure, the condenser of 0.2

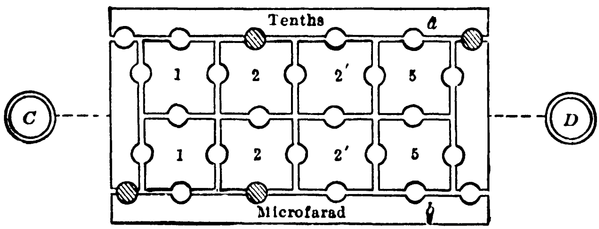


FIG. 115.

mf. is connected to A and B. When the four condensers are all connected in parallel, the capacitance is 1 mf. When they are all used in series, the combined capacitance is $\frac{1}{22}$ mf. In this box the different condensers are permanently connected together.

The top of another condenser box is shown in Fig. 115. The terminals of each condenser are connected to an independent pair of square blocks, each of which is marked with the capacitance of the condenser. Thus, the two blocks marked "1" are the terminals of a condenser having a capitance of 0.1 mf. The long bars are for connections. With four plugs inserted as shown in Fig. 115, the condenser having a capacitance of 0.2 mf. is connected to the binding posts, C and D. By means of plugs, the four condensers can be placed in parallel or in series as desired. Thus, two additional plugs placed at a and b will join the 0.5-mf. section in parallel with the 0.2 mf. already connected, giving a capacitance of 0.7 mf. connected to C and D.

189. Condensers in Parallel.—When two condensers are connected, individually, to the same E.M.F., each one, of course, becomes charged to this difference of potential. The total charge is

$$Q = Q_1 + Q_2 = C_1E + C_2E = CE,$$

where Q_1 and Q_2 are the charges in each condenser. This combination acts, then, like a single condenser whose capacitance is

$$C = C_1 + C_2.$$

Hence,

Law I.—The combined capacitance of several condensers in parallel is equal to the sum of the separate capacitances.

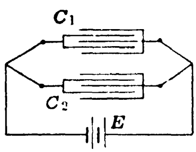


FIG. 116.—Condensers in parallel.

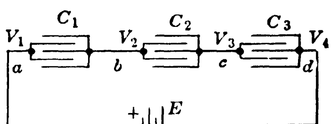


FIG. 117.—Condensers in series.

190. Condensers in Series.—If the condensers are joined in series, as shown in Fig. 117, it is evident that the difference of potential over each condenser is only a part of the total E.M.F. of the battery. Each of the condensers will have the same charge, for, being joined in series, all of the electrons that leave one set of plates in C_2 must go to the adjoining set of plates in C_1, if the intermediate parts are well insulated. This is shown more fully below.

When a series of condensers are connected to a battery of E.M.F. $= E$, as shown in Fig. 117, electrons are urged along the battery circuit from a, through the battery, to d. Whatever may be the number or the capacitances of the condensers in series, this flow of electrons will continue until the potential difference between a and d has reached the full value of E.

The intermediate portions of the condensers consist of a series of insulated sections of conductors. In each of these sections the electrons will distribute themselves so as to make the whole section at a uniform potential. The large accumulation of electrons at d tends to make everything in that neighborhood at a negative potential, including the C_3 end of the section C_3C_2. In the attempt to keep this section at a level potential, the electrons will flow from the C_3 end to the C_2 end. To do this and to neutralize the effect of the charge of q_3 electrons in C_3 will require that another group of q_3 electrons move out of the other set of plates of the condenser C_3, thus leaving them with a "positive" charge, and making the total charge on both sets of plates in this condenser equal to zero.

In the next insulated section the electrons will likewise move so as to keep all parts at the same potential level, and this will result in the same number as before moving from C_2 to C_1. Thus there has been the same number of electrons moving along each section of the circuit, and each condenser has the same charge.[1]

The capacitance between each condenser as a whole and the surrounding conductors and walls of the room is usually insignificant. In case it is not, the charges due to such capacitances are added to those discussed above.

191. Equivalent Capacitance of Condensers in Series.—The potential differences across each condenser, Fig. 117, respectively will be

$$V_1 - V_2 = \frac{Q}{C_1}$$

$$V_2 - V_3 = \frac{Q}{C_2}$$

$$V_3 - V_4 = \frac{Q}{C_3}.$$

[1] This condition is well illustrated by a water analogy, in which a long trough is divided into a number of sections by several transverse partitions. When the half-filled trough stands on the table, the water is at the same level throughout. But when one end of the trough is raised a few inches, the water must flow along each section of the trough in order to remain level. Evidently it cannot all be at the same level, and at each dam there is a difference of level. The sum of these differences is equal to the total difference in level between the two ends.

Adding these equations gives

$$V_1 - V_4 = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = \frac{Q}{C}.$$

Hence the equivalent capacitance, C, of the combination is given by the relation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

The resulting capacitance, C, is smaller, therefore, than the smallest capacitance in the series.

Law II.—When several condensers are connected in series, the joint capacitance is the reciprocal of the sum of the reciprocals of the several capacitances.

This is similar to the law for resistances in parallel.

192. Comparison of Capacitances by Direct Deflection.—When a condenser of capacitance C farads is charged to a potential difference of E volts, the quantity it contains is

$$Q = CE, \qquad \text{coulombs} \tag{1}$$

If this quantity is discharged through a ballistic galvanometer, giving a fling of d mm. as measured on the scale, we have

$$Q = kd. \tag{2}$$

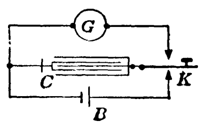


FIG. 118.

Combining (1) and (2) gives

$$C = \frac{k}{E}d. \tag{3}$$

Now if the same experiment is repeated, using the same battery and galvanometer, but with another condenser of capacitance C_1 we have

$$C_1 = \frac{k}{E}d_1 \tag{4}$$

and from (3)

$$C_1 = C\frac{d_1}{d}. \tag{5}$$

If C is a known capacitance, then the value of C_1 can be determined as exactly as the flings d and d_1 can be measured. Each of these should be taken several times and the mean values used in the computation.

193. Study of Residual Discharges.—When a condenser or a cable is charged for a long time and then discharged, it is nearly always found that the quantity of electricity obtained from the condenser on discharge is less than the total amount of the original charge. The remainder of the original charge is said to be "absorbed," meaning thereby that this charge remains in the condenser after the plates have been brought to the same potential, but not specifying the manner in which it is retained. After a short interval of time a portion of this absorbed charge is released and can be discharged by again joining the two plates of the condenser; and this is termed a "residual discharge." Several such residual discharges can be obtained from ordinary condensers and a great many from a poor condenser.

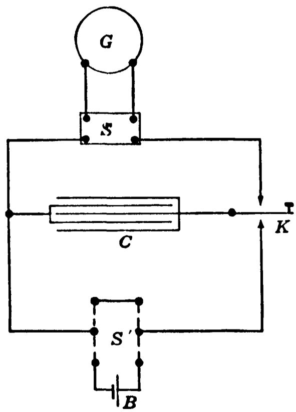


FIG. 119.

This phenomenon depends upon the material composing the dielectric between the plates of the condenser, and is the more marked the greater the heterogeneity of the dielectric. In case the dielectric is air, or a sheet cut from a crystal of quartz or iceland spar (*i.e.*, a homogeneous substance), there is no residual discharge. The charge which is thus absorbed and can be recovered later should not be confused with any leakage there may be through the condenser.

To make a short study of residual discharges proceed as follows: Charge a paraffined paper condenser from six or eight cells for 3 minutes. When ready to begin observations on residual discharges disconnect the battery from the condenser and thoroughly discharge the latter by throwing the switch, *S*, to the other side for 5 seconds and then leaving it open. Wait 1 minute and again discharge the condenser—this time through the galvanometer—and observe the deflection. The key, *K*, is supposed to remain closed on the lower contact shown in Fig. 119. Probably this would be the upper contact on the actual key. Pressing the key to the other contact joins the condenser to the galvanometer. Discharge the condenser through the galvanometer at minute intervals for half an hour. Express the results in the form of a curve using time for abscissæ and the sum of all previous discharges for ordinates. In some instances the sum of the residual discharges is greater than the original discharge. Repeat, charging the condenser for 15 minutes in order to observe the effect of the time of charging upon the amount of the residual charge.

194. Charging a Condenser through a Resistance.—When a condenser is connected in series with a resistance and a battery, a current flows until the potential difference across the condenser equals the E.M.F. of the battery. The initial value of this current, as well as the time required to charge the condenser, depends upon the amount of resistance in the circuit.

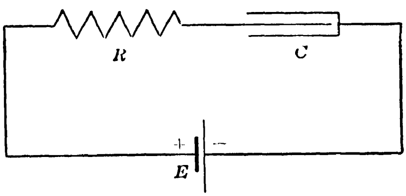


FIG. 120.—Charging a condenser through a resistance.

Figure 120 shows such a circuit, where R represents the total resistance, including that of the battery. Writing down the potential differences in this circuit gives

$$E - Ri - \frac{q}{C} = 0$$

or, expressing the current in terms of the charge,

$$E - R\frac{dq}{dt} - \frac{q}{C} = 0.$$

In order to separate the variables, q and t, this can be written in the form

$$\frac{-dq}{EC - q} = \frac{-dt}{RC}.$$

Integrating this gives

$$\log (EC - q) = -\frac{t}{RC} + B,$$

where B denotes the constant of integration. Rewriting this in the exponential form, we have

$$EC - q = e^{B} e^{-\frac{t}{RC}}.$$

If t is measured from the instant that the circuit is closed, when $q = 0$, then when $t = 0$, this equation becomes

$$EC = e^{B}$$

and, since this is a constant, it always has this value. We now have

$$q = Q - Qe^{-\frac{t}{RC}},$$

where Q is written for EC, this being the maximum value of q, and the final charge in the condenser.

FIG. 121.—Increase of the charge in a 1-mf. condenser when charged through 1,000 ohms.

Figure 121 shows the increase in the charge of a 1-mf. condenser when it is charged through 1,000 ohms. A larger E.M.F. will give a larger final charge, but the same time will be required to bring the charge to half of its final value.

195. Bridge Method for Comparing Two Capacitances.—This is a null method and therefore capable of more exact measurements than the preceding. The two condensers are placed in two arms of a Wheatstone bridge set-up, as shown in Fig. 122.

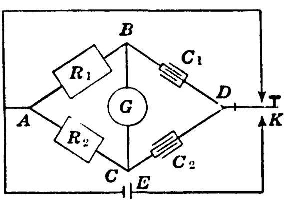


FIG. 122.—Bridge method.

When the key is depressed, both condensers become charged to the full potential difference of E, and the points A, B, and C all come to the same potential. If no charge passes through the galvanometer, then C_1 is charged through R_1 and C_2 through R_2. During the very short interval that is required to charge the condensers there will be transient currents through R_1 and R_2, and perhaps in the galvanometer also.

By working the charge and discharge key quickly, the deflection of the galvanometer may be checked even when R_1 and R_2 are far from a balance. The galvanometer should be thus protected.

Writing the equation for the potential differences in the circuit *BACB* at any instant while the condensers are being charged gives

$$R_1i_1 + L_1\frac{di_1}{dt} - R_2i_2 - L_2\frac{di_2}{dt} + Gi + L\frac{di}{dt} = 0 \qquad \text{(A)}$$

where $L_1 \dfrac{di_1}{dt}$, etc., are the E.M.FS. due to self-induction in the corresponding branches. The left-hand side of E is taken as positive. If i_1 denotes the electric current flowing from A to B, the point A is at the higher potential. If the electron current flowing from B to A is denoted by i_1, A is still at the higher potential, and Eq. (A) is written the same as before.

196. Integration of Eq. (A).—The quantities appearing in Eq. (A) are not directly measurable. The ballistic galvanometer, G, measures the total quantity of electrons passing through it, and this quantity is expressed as

$$Q = \int dq = \int idt,$$

where dq is the quantity transferred by the current, i, in the interval, dt. This suggests that Eq. (A) can be solved by integration, as follows:

$$\int R_1dq_1 + \int L_1di_1 - \int R_2dq_2 - \int L_2di_2 + \int Gdq + \int Ldi = 0,$$

where dq_1 is written for i_1dt, etc. Performing the integration thus indicated gives the general equation

$$R_1q_1 + L_1i_1 - R_2q_2 - L_2i_2 + Gq + Li = B.$$

At the beginning, before any of the currents have had time to start, i, i_1, and i_2, are each equal to zero. The quantities q, q_1, and q_2 likewise are each equal to zero at the start. Therefore B, the constant of integration, has the value zero.

At the end, after things have reached a steady condition, i, i_1, and i_2 are zero again. Therefore, when this steady state has been reached, the general equation above reduces to

$$R_1Q_1 - R_2Q_2 + GQ = 0,$$

where Q_1 is the value of q_1 at this time, and denotes the total quantity that has passed through R_1 since the key was closed.

As no currents are flowing, this value does not change further, however much longer the key may be held closed. In the same way, Q and Q_2 are the final values of q and q_2.

197. Conditions for a Balance.—It is to be noticed that no deflection of the galvanometer does not mean no current through it, as there may be currents through it in both directions before the condensers are fully charged. But zero deflection means that as many electrons have passed through the galvanometer in one direction as in the other, and that, taking account of signs, the total quantity through the galvanometer has been zero. Hence, when R_1 and R_2 are adjusted so that opening or closing K produces no deflection of the galvanometer, $Q = 0$.
Then

$$R_1Q_1 = R_2Q_2.$$

But, since no charge passed through the galvanometer,

$$Q_1 = C_1E \quad \text{and} \quad Q_2 = C_2E.$$

Substituting these values gives

$$C_1 = C_2 \frac{R_2}{R_1}.$$

The resistances, R_1 and R_2, should be large, 5,000 ohms or more, so that the fall of potential produced by the small charging currents may be appreciable. While self-inductance in these arms does not affect the result, as seen by the preceding integration, a large amount of inductance may render it more difficult to determine when a balance has been reached.

Make several determinations of each unknown capacitance by using various values for R_1 and finding the corresponding values of R_2. Check results by measuring the capacitance of the condensers when joined in series and in parallel.

The observations can be recorded in a form like the following:

To Measure the Capacitance of..... .....

R_1	R_2	C_2	C_1

198. Potential Changes in the Bridge.—To understand what is taking place in the bridge circuits during the short interval in which currents are flowing, let us consider the changes in potential occurring at the points A, B, C, and D, Fig. 122. To be definite, let us suppose that the key, K, is on its upper contact, connecting D and A, and that the negative (right-hand) side of E is grounded, thus maintaining it at the potential of the earth.

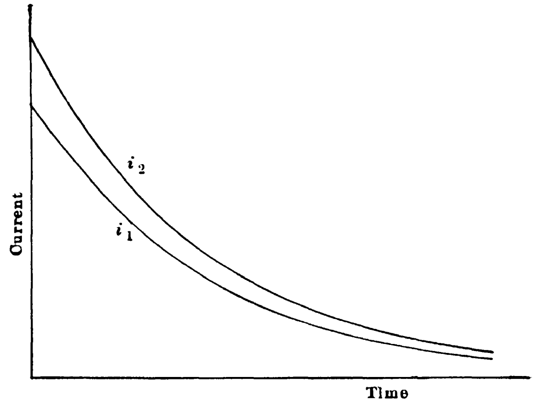


FIG. 123.—The currents through R_1 and R_2 decrease to zero in a short time.

Before K is depressed, the entire bridge is at a uniform potential equal to that of the positive terminal of the battery, E. When K is depressed, D drops at once to the potential of the earth (and that

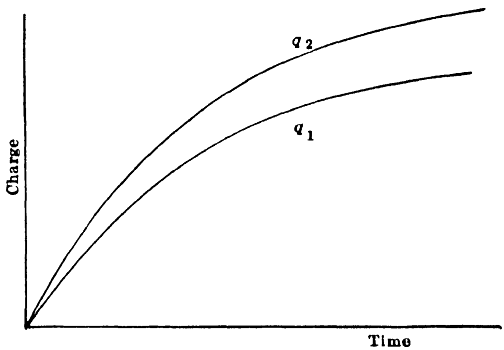


FIG. 124.—The charges in the condensers increase and finally reach constant values.

of the negative side of E), to which it is thus connected. At the first instant there are no charges in the condensers, and therefore no potential differences across them. Therefore the potentials of B and C will also drop with that of D, and the full E.M.F. of the battery is impressed upon R_1 and R_2. This determines the initial values of the currents through these resistances. As the condensers become charged, the currents fall to zero, as shown in Fig. 123. Since these currents flow into the condensers, the charges increase as shown in Fig. 124.

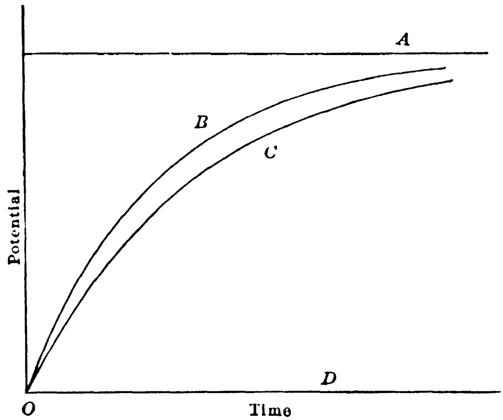


FIG. 125.—These curves show the change in potential of the points *A*, *B*, *C*, *D*, in the bridge of Fig. 122, after the key is depressed. For a balance, the curves for *B* and *C* must coincide.

The potentials of the points *B* and *C* rise as the condensers become charged, because the potential of *D* remains constant and the difference of potential across each condenser is proportional to its charge. (See Fig. 125.) Since the galvanometer is connected across *BC*, zero deflection indicates that *B* and *C* are at the same potential (on the average), and thus for a balance the two curves must coincide (if they are exponential).

Had any other point been selected as the grounded one, the corresponding curve would have been a straight line at zero potential, but the relative difference between the various curves would be the same as before.

199. Comparison of Capacitances by Gott's Method.—This is another bridge method and differs in arrangement from the preceding only by having the galvanometer and battery interchanged. In the bridge method the balance is obtained for the conditions which exist *during* the charging, or the discharging, of the condensers. In the present method the capacitances of the condensers are compared after everything has reached the steady and permanent condition.

The arrangement is shown in Fig. 126. When the battery key, K', is closed the two condensers in series are charged to the difference of potential between A and D. The point B, between the two condensers, has a potential intermediate between that of A and D. The measurement consists in adjusting R_1 and R_2 until C, on the upper circuit, has the same potential as B in precisely the same way as in the measurement of resistance by the Wheatstone bridge method. When this adjustment is correct, there will be no deflection of the galvanometer upon closing K''.

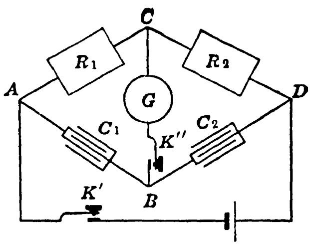


FIG. 126.—Gott's method.

Since the condensers are joined in series, each one must contain the same charge. The point B is insulated as long as K'' remains open, and therefore whatever charge appears on one condenser must have come from the other one. Moreover, for a balance, the closing of K'' produces no deflection of the galvanometer, *i.e.*, there is no change in the charges on the conductors joined at B.

Writing out the equations for the potential differences in the circuits $BACB$ and $DBCD$ gives

$$\frac{q}{C_1} - R_1 i = 0 \quad \text{and} \quad \frac{q}{C_2} - R_2 i = 0,$$

from which

$$C_1 = C_2 \frac{R_2}{R_1}.$$

It should be noted that closing K'' brings B to the potential of C, whether there is a balance or not. A second closing of K'' can produce no deflection. It is necessary, therefore, to discharge the condensers completely after each closing of K''. This can be most quickly done by opening K' before opening K''.

If one of the condensers has considerable absorption or leakage, it will seriously influence the results, for after some of the charge has leaked away it is no longer true to say that the charges in the two condensers are equal. The effect of this source of error is reduced by closing K'' as soon as possible after closing K'. When the resistances R_1 and R_2 are free from inductance and capacitance it is allowable to omit K'' from Fig. 126 and observe the deflections of the galvanometer when K' is closed or opened.

The observations can be recorded in the same form as used for the preceding experiment.

For this method Fig. 127 shows the way the potentials of different points in the bridge change after closing the key. It is seen that each point reaches its final potential at once and remains there till the key is opened.

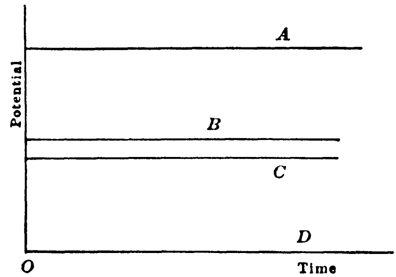


FIG. 127.—Showing the potentials of four points on the bridge of Fig. 126 (with D grounded) after closing the key. For a balance, the curves for B and C coincide.

200. Comparison of Capacitances by the Method of Mixtures.—This method was devised by Lord Kelvin to avoid some of the difficulties in the preceding methods. It is especially applicable to cases where the two capacitances are very dissimilar, *e.g.*, if the capacitance of a long cable is to be compared with that of a standard condenser. The method consists in charging the condensers to such potentials that each will contain the same quantity. They are then discharged, the one into the other, and the charges allowed to mix. If the charges are not equal, the difference

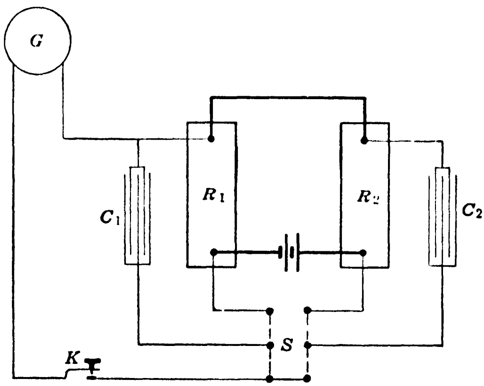


FIG. 128.—Method of mixtures.

will remain in the condensers and is later discharged through the galvanometer.

The arrangements and connections are shown in Fig. 128. Two moderately high resistance boxes, R_1 and R_2, are joined in series with a battery of a few cells. A small current, i, is allowed to flow continuously through R_1 and R_2, thus maintaining across R_1 a steady potential difference,

$$V_1 = R_1 i,$$

and across R_2 a potential difference,

$$V_2 = R_2 i.$$

The condenser to be measured is connected in parallel with R_1 through the double-pole, double-throw switch S. It is thus charged to the potential difference of $R_1 i$ volts, and therefore receives a charge,

$$Q_1 = C_1 R_1 i.$$

At the same time the known condenser is connected in parallel with R_2 by the other side of the switch. The charge it receives is

$$Q_2 = C_2 R_2 i.$$

By opening S, these condensers are disconnected from R_1 and R_2, but each condenser still retains its charge. When S is closed on the other side, each of the condensers discharges into the other, and the electrons constituting the negative charge of one condenser thus "mix" with the positive charge of the other condenser. If the charges in the two condensers are equal, one will just balance the other, and there will be no resultant charge in either condenser. In case the charge in one condenser is larger than that in the other, a part of this charge will be left after the "mixing," part of it being in each condenser.

By closing the tapping key, K, the condensers can be discharged through the ballistic galvanometer. If there is zero deflection, it shows that there was no charge remaining after the mixing, and therefore

$$Q_1 = Q_2$$

This condition can be obtained by varying R_1, and, therefore, Q_1, until zero deflection is obtained. Then,

$$C_1 R_1 i = C_2 R_2 i$$

or,

$$C_1 = C_2 \frac{R_2}{R_2}.$$

201. Absolute Capacitance of a Condenser.—The comparison of one capacitance with another, as shown in Art. 192, is a simple matter, but it is more difficult to measure accurately the capacitance of a condenser independently of any known capacitance. One method that has been used employs a tuning fork to operate the key, K, of Fig. 118. If the condenser can be fully charged and discharged n times each second, the rapid succession of charges through the galvanometer will produce a steady deflection equal to the deflection for an equivalent steady current of

$$i = nQ$$

and this can be measured as exactly as the deflection can be read. Then

$$Fd = nQ = nCE$$

and

$$C = \frac{Fd}{nE},$$

where E is the E.M.F. of the charging battery and F is the figure of merit of the galvanometer.

202. Absolute Capacitance by the Wheatstone Bridge Method. From the preceding method it appears that the condenser and its vibrating key are equivalent to a resistance of

$$N = \frac{E}{i} = \frac{1}{nC}.$$

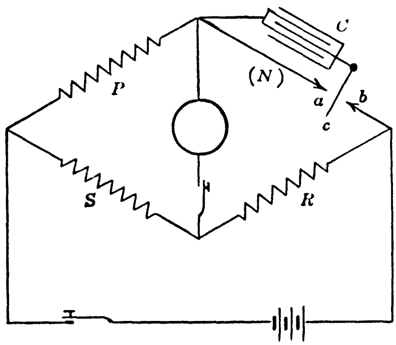


FIG. 129.—Absolute capacitance of a condenser. A motor-driven commutator connects c, first to a, and then to b, alternately, n times each second.

It is possible, therefore, to use this arrangement as one arm of a Wheatstone bridge, Fig. 129, and obtain a steady balance in the usual way. To a first approximation,

$$C = \frac{1}{nN} = \frac{S}{nPR}.$$

A closer examination will show that the zero deflection of the galvanometer is due to the resultant action of a steady current in one direction and the intermittent charges in the other direction. Taking into account the entire effect, the complete formula is

$$C = \frac{S}{nPR}\left\{\frac{1 - \dfrac{S^2}{(S+R+r)(S+P+g)}}{\left(1 + \dfrac{Sr}{P(S+R+r)}\right)\left(1 + \dfrac{Sg}{R(S+P+g)}\right)}\right\}.$$

The proper choice of the resistances that are used is important. The use of storage batteries will make r small, and by setting $\frac{S}{R} = \frac{100}{100{,}000}$ or less, the factor in brackets can be made nearly equal to unity.

It is also essential that the condenser shall be fully charged each time the contact is closed, and this requires a low resistance in the charging circuit. With $S = 100$, $P = 1{,}000$, $g = 20$, $r = 6$, and $R = 100{,}000$, the condenser will reach its full charge in less than 0.001 second.[1]

After obtaining a balance of the bridge, the condenser should be disconnected and a second balance obtained to determine the capacitance of the connections and other metal parts in this arm of the bridge. The change in capacitance is the capacitance of the condenser.

203. Constant Speed.—The value of n that enters in the formula above can be accurately determined from the speed of the commutator that makes the condenser connections. In the method as used at the Bureau of Standards, this commutator was driven by an electric motor that was hand-controlled by a carbon rheostat in the armature circuit. The slightest variation in speed would deflect the galvanometer one way or the other. By controlling the speed of the driving motor so that the galvanometer was kept at or close to zero, and measuring on a chronograph the total number of revolutions for several minutes, the value of n can be found as accurately as may be required.

This combination of a Wheatstone bridge and condenser can be used as an auxiliary apparatus for holding the speed of a motor steady in other situations where a uniform angular velocity is required.

[1] For the complete discussion of this method and the description of the special commutator used to charge and discharge the condenser, see *Bulletin* of the Bureau of Standards, vol. 1, p. 153.

CHAPTER XI

FUNDAMENTAL MAGNETIC AND ELECTRICAL UNITS

204. C.G.S. Units.—Magnetic measurements are made in c.g.s. (centimeter, gram, second) units. It is appropriate, therefore, to consider now the basis and development of this system of magnetic and electrical units. The set of electrical units that is based upon the magnetic units is called the c.g.s. electromagnetic system of units, and all of the common units, such as ohm, ampere, volt, etc., are based upon these units.

There is another set of electrical units that is used in electrostatic measurements. In this system the fundamental unit is determined by the repulsion of two similar charges of electricity. These units constitute the c.g.s. electrostatic system of units. In the present treatment, these units are not further considered.

205. The Magnetic Circuit.—The idea of the magnetic circuit was early introduced in the study of magnetism. If the region around a wire carrying an electric current is investigated with the aid of a magnetic needle, it will be found that, although the needle is strongly acted upon, neither pole is attracted towards the wire carrying the current. A little investigation, however, will show that a magnetic pole is urged along a path encircling the current and returning to the starting point.

If the wire is wound into a helix, forming a solenoid, the pole is urged along the inside of the solenoid from one end to the other, and then back to the starting point along a continuation of its path through the surrounding space. It is natural, therefore, to associate the idea of a magnetic circuit with the phenomena of magnetism.

Similar paths can be traced through the region around a permanent magnet, but in this case the test pole cannot be carried through the magnet itself, and other methods of measurement are necessary to show that the paths are continuous through the magnetized steel. There is no limit to the number of such paths that can be traced with a magnetic pole, and therefore this number has no special significance. The whole sheaf of paths that can be traced in any given case maps out the magnetic circuit for that case.

206. Magnetic Flux.—The bundle of lines showing the various paths that can be traced around a magnet or current indicates a real physical condition of the surrounding medium. This is made evident when this region is investigated with a wire whose ends are connected to a ballistic galvanometer. When such a wire is moved across these lines, it causes a deflection of the galvanometer. The quickness with which the wire is moved makes no difference, but if the wire is moved across a greater area, the galvanometer deflection is correspondingly increased.

Quantities which are thus measured with reference to an area are called "fluxes," and in this case the wire is said to cut across the "magnetic flux."

If the test wire is moved along the direction of these lines of flux, there is no deflection of the galvanometer, and nothing in this direction is discovered.

Magnetic flux is detected, then, by the deflection of the galvanometer, and the magnitude of this deflection may be taken as an indication of the total flux cut by the wire. It does not matter whether the wire is moved across the flux, or the flux is moved across the wire—the effect on the galvanometer is the same in either case.

The presence of magnetic flux can be shown in another way. If instead of using a simple wire connected to a galvanometer, we were to use a wire carrying an electric current, the wire will be found to experience a force urging it sidewise across the magnetic flux. Because of this force on a wire carrying a current, it will require work to push the wire across the flux, and the amount of this work will depend upon how much flux the wire is pushed across. There is no tendency for the wire to move along the lines of flux, and such a wire is not attracted by a magnetic pole.

The magnetic flux is always continuous and forms a closed circuit with itself, like the electric current. At one part it may be spread out over a large cross-section and at another part of the circuit be confined within narrow limits, but the total amount of magnetic flux is the same for each cross-section of the circuit.

The unit of magnetic flux is called a "maxwell." This name was adopted by the International Electrical Congress at Paris in 1900.

It will be shown later that whenever magnetic flux is cut by a wire an E.M.F. is produced in the wire. The international volt being the unit of E.M.F. that is used in practical measurements, the corresponding unit of magnetic flux is an international maxwell having such a value that when 100,000,000 international maxwells are cut each second at a uniform rate the induced voltage is one international volt.

The relation between these units and the fundamental C.G.S. units is shown in this chapter.

207. Representation of Magnetic Flux.—Since the magnetic flux is continuous throughout the magnetic circuit, such a

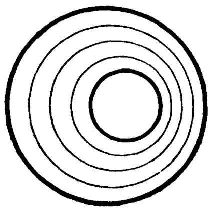


FIG. 130.

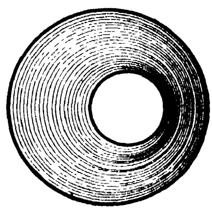


FIG. 131.

FIG. 130.—Magnetic flux in a magnetic circuit of wood or air. The three lines represent a magnetic flux of 3 maxwells due to a M.M.F. of 12 gilberts. (See Fig. 132.) The flux has a greater density where the circuit is narrow.

FIG. 131.—Magnetic flux in a circuit of iron. The 20 lines represent a magnetic flux of 20 maxwells due to a M.M.F. of 12 gilberts. (See Fig. 132.) In comparison with Fig. 130, this should show 2,000 maxwells or more, but it is not possible actually to draw all of the lines in this small figure.

circuit may be thought of as divided lengthwise into a number of parallel circuits so that there will be 1 maxwell of magnetic flux in each strand of this divided circuit. Such strands have been called "tubes of induction" or "lines of force." Perhaps it is better to speak of the "magnetic flux," and to think of these lines as merely a pictorial representation of the flux. Such representation is useful in showing how the flux passes continuously from one medium into another with no change in amount at the boundary. The total flux in a given circuit can be shown by the total number of these strands or lines.

Figures 130 and 131 represent the magnetic flux in two circuits of different material, but having the same size and subjected to the same magnetizing effect of the electric current. The total flux as well as the flux density is much greater in the iron ring. If the wire connected to a ballistic galvanometer could be made to cut across this flux, there would be a much greater deflection for the iron ring than for the wooden one. Since the wire cannot be moved through the iron, the same effect is obtained by reversing the direction of the flux while the wire remains stationary. Half of the galvanometer deflection then measures the flux.

208. Arrows of Magnetomotive Force. *The Downhillness of a Magnetic Circuit.*—If a magnetic pole had been used in the investigation of the magnetic field instead of the wire carrying a current, a very different condition would have been found. It would have taken no work to move the pole across the magnetic flux. The pole will be urged *along* the lines of flux, and it will require work to push it in the opposite direction. This means that there is a magnetic gradient in the direction of these lines. Suppose that a unit magnetic pole is pushed up this magnetic

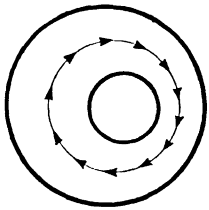


FIG. 132.—The arrows show the amount and distribution of the M.M.F. for a circuit of either iron or wood. The M.M.F. depends solely upon the number of ampere-turns of the magnetizing current, and is independent of the material of the magnetic circuit.

gradient for a distance sufficient to require the expenditure of 1 erg of work. Mark the end of this distance by an arrowhead on the line that is being followed. Let another erg be expended in carrying forward the unit pole, and another arrowhead placed on the line to indicate the position that has been reached. Let this process be continued for the complete circuit, marking the expenditure of each erg with an arrowhead. The total work to carry the pole around the complete circuit is given by the total number of these arrowheads. The magnetic quantity that is thus measured by the work per unit pole is called the "magnetomotive force." This name is often abbreviated to the initials M.M.F. The C.G.S. unit of magnetomotive force is called a "gilbert." The arrowheads are spaced, then, 1 gilbert apart and the *distribution* of this M.M.F. is vividly shown by the distribution of the arrowheads along the circuit. The total M.M.F. is given by the number of these arrows along the circuit.

209. M.M.F. in a Divided Circuit.—The use of arrows to show the distribution of M.M.F. along the magnetic circuit presupposes that it is possible to carry a magnetic pole along the lines of flux. In case the flux passes through solid materials, like wood or iron, it will not be possible to follow it with an actual steel magnet, and the amount of M.M.F. distributed over this portion of the circuit must be determined by an indirect method.

Let us consider two parallel branches of a magnetic circuit, dividing at the point A, Fig. 133, and joining again at the point B. Let G denote the number of units of M.M.F. between A and B. This

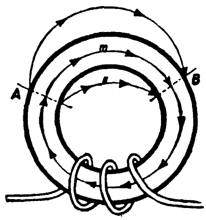


FIG. 133.—A ring of iron magnetized by three turns of current. The arrows indicate a M.M.F. of 9 gilberts, and show how this is distributed. Between A and B the M.M.F. along one path is the same as along any other path.

will be the same for each path because A (and likewise B) has a definite magnetic potential, or level, and the difference between these two potentials can have only a single value, provided that no electric current is included between these different paths. If this difference is G gilberts, there will be G arrows along the path AmB, and likewise G arrows along the path AnB. This consideration determines how the flux will divide between the two branches.

If one of these paths, say AnB, Fig. 133, is in air, it will be possible to carry a magnetic pole along it, and thus determine the value of G. This, then, will also be the M.M.F. along AmB.

210. Reluctance.—The amount of magnetic flux due to a given M.M.F. in a circuit is determined by the magnetic resistance, or reluctance, as it is called, of the circuit. The reluctance of a given circuit depends upon the dimensions of the circuit in much the same way, *e.g.*, as electrical resistance. That is, a portion of a circuit L cm. in length and having a constant cross-section of A sq. cm. has a reluctance $\frac{L}{A}$ times as large as a portion 1 cm. long and 1 sq. cm. in cross-section. A circuit that does not have a constant cross-section can be considered as a series of circuits having lengths dL_1, dL_2, etc., and cross-sections A_1,

A_2, etc. The total reluctance is then the sum of the reluctances of these various portions of the circuit.

The reluctance of a circuit also depends upon the material of which it is composed. Iron in its various forms is one of the best conductors of magnetic flux. Most other substances are rather poor conductors, but there are no insulators of magnetic flux.[1]

Even when a magnetic circuit is made largely of iron, the magnetic flux does not follow the iron exclusively, but there is some flux which finds a path through the air. It is extremely difficult to build a circuit, even of soft iron, in which all of the flux will follow the iron path. If two paths are open to the magnetic flux it will divide just as an electric current would do between two wires in parallel. The total reluctance of a circuit includes the effect of the reluctance of such parallel leakage paths.

In the case of iron and similar magnetic substances, there is a difference, too, between electric resistance and magnetic reluctance inasmuch as the former is constant for all ranges of current, while the value of the reluctance of a magnetic circuit depends upon the amount of magnetic flux through it, being greater for very small or very large values of the magnetic flux than it is for moderate values.

The C.G.S. unit of reluctance is called an "oersted."

211. Law of the Magnetic Circuit.—The relation between the amount of magnetic flux, and the applied M.M.F. can be expressed in a form quite similar to Ohm's law for an electric circuit. Thus the law of the magnetic circuit can be written

$$\textit{Magnetic Flux} = \frac{\textit{Magnetomotive Force}}{\textit{Reluctance}}.$$

This law can be applied to a complete circuit or to a limited portion of a magnetic circuit; and it holds true for circuits of any material, with one exception. For substances that show magnetic hysteresis, such as iron, steel, nickel, etc., this law is applicable only when the substance is being magnetized by a M.M.F. which is greater than any M.M.F. that has been used since the substance was unmagnetized.

[1] In somewhat the same way there are no insulators of the electric flux between the oppositely charged plates, *e.g.*, of a condenser.

MAGNETIC UNITS

Thus far we have been speaking familiarly of the various magnetic units, and now we can understand the definitions that fix the magnitude of each unit.

212. Action between Magnets.—Everyone who handles magnets discovers that they will attract or repel other magnets and that this effect is connected with certain parts, called "poles." These poles are not located at points, but occupy considerable regions. However, a long steel wire, with an iron ball at each end, makes a magnet which acts as though its poles were at the centers of the balls. The repulsive force between two such north-seeking, ball-shaped poles, m and m', is given by Coulomb's experiment as

$$F = k\frac{mm'}{r^2},$$

where the distance, r, from center to center, is a large distance compared with the dimensions of either pole.

213. Unit Pole.—Without understanding just why one pole repels another, this relation can be used to define the value of a unit pole. For simplicity, the poles m and m' should be measured in units of such magnitude that the proportionality factor, k, is made unity.

Such a unit pole is defined as follows:

Definition.—A unit magnetic pole is one of such strength that when placed one centimeter from an equal pole in vacuum it will be repelled with a force of one dyne.

Of course, this definition supposes that these poles are at points 1 cm. from each other. When this cannot be done, it is necessary to consider the integrated effect of each part of one pole upon every part of the other. The definition, however, serves as well in either case.

214. Magnetomotive Force. G.—Magnetomotive force is measured by the work per unit pole required to carry a magnetic pole along the magnetic circuit. The unit of M.M.F. is called a "gilbert," and is defined as follows:

Definition.—A gilbert is the M.M.F. *between two points in a magnetic circuit when one erg per unit pole is required to carry a magnetic pole from one point to the other.*

It is not always necessary actually to take the unit pole around the circuit, for if we know enough about the circuit it will be possible to compute the number of ergs of work that would be required to carry the pole along the given path, and thus to determine the value of the M.M.F.

215. Magnetic Intensity. *H*.—The gradient of M.M.F. at any point, or the M.M.F. per centimeter, is called the "magnetic intensity" at that place.

From the relation between force and work, it follows that the magnetic intensity is measured by the *force per unit pole* that would be experienced by a magnetic pole.

The magnetic intensity is measured, then, in gilberts per centimeter, and the letter *H* is commonly used to denote this quantity.

This is expressed by the equation

$$H = \frac{dG}{dL} \qquad \text{gilberts per centimeter,}$$

where dL is the distance over which the M.M.F. is dG.

The name "gauss" for unit intensity was adopted by the International Electric Congress at Paris in 1900, one gauss being a magnetic intensity of one gilbert per centimeter.[1]

Definition.—A gauss is the intensity of a magnetic field in which a magnetic pole experiences a force of one dyne per unit pole.

216. Arrows per Centimeter.—Where the arrows are close together in the representation of M.M.F., there the magnetic intensity, *H*, is great; where they are far apart, *H* is less. The value of *H* at any point of the circuit is given by the number of arrows per centimeter at that point. *H* thus measures the downhillness, or the magnetic gradient, of the circuit. This value may change abruptly where the circuit passes from one medium into another, as from iron into air, although a wire and ballistic galvanometer would show that the magnetic flux is the same in each medium.

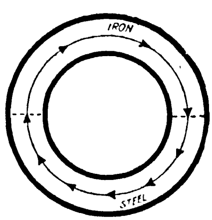


FIG. 134.—This shows the distribution of M.M.F. in a circuit half of steel and half of the more permeable iron.

It is thus seen that a unit magnetic pole can be used to investigate certain properties of a magnetic field. The *work* required to carry it along a given path measures

[1] The name "gauss" is sometimes used to denote a flux-density of one maxwell per square centimeter, but this does not seem to have been the intention of the Paris Conference.

the M.M.F. along that path, and the *force* that it experiences at any point measures the magnetic intensity, H, at that point.

217. Reluctance. *R.*—The unit of reluctance is conveniently taken as the remaining quantity to be independently defined. Therefore the reluctance of a centimeter cube of vacuum is chosen as a standard, and this is taken as unity. This unit of reluctance is called an "oersted."

Definition.—An oersted is a reluctance equal to the reluctance of a centimeter cube of vacuum, measured from one face to the face opposite. A piece of vacuum dL cm. long and dA cm.² in cross-section will have a reluctance of $\frac{dL}{dA}$ *oersteds.*

218. Magnetic Flux. ϕ.—The value of a maxwell follows from the law of the magnetic circuit:

Definition.—A maxwell is the flux that exists in a magnetic circuit where the M.M.F. *is one gilbert per oersted.*

219. Flux Density. *B.*—The term "flux density" denotes the amount of flux per square centimeter. Its value is given by the equation

$$B = \frac{d\phi}{dA} \qquad \text{maxwells per square centimeter}$$

where dA is the area over which the flux $d\phi$ is distributed. There is no name for the unit of flux density. From the meaning of the term we have the following definition:

Definition.—A uniformly distributed flux of one maxwell per square centimeter normal to the direction of the flux is a flux density of unity.

220. Permeability.—The reluctance of a magnetic circuit depends upon the nature of the material of which the circuit is made, in the same way that the resistance of an electric circuit depends on the conductivity of the metal of which it is made. The amount of reluctance in a given magnetic circuit is directly proportional to the length of this circuit, and it is inversely proportional to the cross-section of the circuit. If the medium is other than vacuum, the effect of the material is expressed by an appropriate factor which measures the ability of the substance to carry magnetic flux. This ability is called its "permeability," and its value is denoted by μ. The unit of permeability has no name. A magnetic circuit of large permeability will have a correspondingly small reluctance.

Expressed in symbols, then, the reluctance of a circuit is

$$R = \frac{L}{\mu A} \qquad \text{oersteds,}$$

where L denotes the length of the circuit having a cross-section of A sq. cm.[1] The factor μ corresponds to the conductivity in the expression for electrical resistance.

The law of the magnetic circuit can now be written

$$\phi = \frac{\text{M.M.F.}}{\dfrac{L}{\mu A}}.$$

221. Relation between B and H in a Medium.—For a portion of the magnetic circuit, over which the values of B and H are sensibly constant, the magnetic flux is $\phi = Ba$, and the M.M.F. over this portion is Hl, where l is the length and a is the cross-section of the portion considered. Therefore the law of the magnetic circuit can be written for this portion as

$$Ba = \frac{Hl}{\dfrac{l}{\mu a}},$$

which simplifies at once to

$$B = \mu H.$$

The last relation is very generally used in expressing the magnetic properties of different substances, and it gives a ready means for determining the value of μ when both B and H can be measured.

222. Permeability of Vacuum.—As a consequence of the definition of unit reluctance, μ has the value of unity for vacuum. In this case the flux density (B lines per square centimeter) is numerically equal to the magnetic intensity (H arrows per centimeter), but this does not mean that they are quantities of the same nature in a vacuum any more than the current is the same thing as the voltage in an electric circuit of unit resistance.

223. Relative Permeability.—Another way in which the term "permeability" is sometimes used is to compare the amount of magnetic flux in two media. For example, if the flux density

[1] If the circuit is not uniform in cross-section or material, it can be considered as a series of uniform sections, and

$$R = \frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} + \frac{L_3}{\mu_3 A_3} + \text{etc.}$$

is B_i in a certain circuit consisting of iron, and if the flux density is B_a for the same circuit when the iron is replaced by air, then,

$$\frac{B_i}{B_a} = \mu',$$

where μ' is the relative permeability of iron as compared with air.

The relative permeability, being the ratio of two similar quantities, is an abstract number. This is not the case with the permeability, μ, which is the ratio of quantities, B and H, that are dissimilar in nature.

224. Intrinsic Magnetization.—The magnetic flux through a magnetized circuit may be considered as the sum of two superimposed parts—one portion due to the current in the magnetizing solenoid, and a second part due to the electronic M.M.F. within the substance.

If the magnetizing solenoid were in vacuum, there would be some flux through it, and

$$B_0 = u_0 H, \tag{1}$$

where the subscripts refer to the values in vacuum. When a magnetic substance is within the solenoid, the total flux density is this B_0, *plus* the flux density, B_I that is contributed by the substance. Thus

$$B = B_0 + B_I. \tag{2}$$

When H is made larger and larger the value of B continues to increase due to the part B_0. The part, B_I, seems to reach a definite limit for each substance, which is reached when all of the movable electronic orbits, or whatever parts are free to turn, are oriented as nearly as possible into alignment with the applied M.M.F.

225. Intensity of Magnetization.—In the older theory of magnetism more emphasis was laid on magnets and poles, and the intrinsic magnetization was expressed in terms of magnetic poles. If we consider a long bar uniformly magnetized lengthwise, with all of the flux entering and leaving the bar at its ends only, the ends of the bar will be magnetic poles with one unit pole for each 4π maxwells of flux. The magnetic pole strength over each end of the bar is then

$$I = \frac{B_I}{4\pi} \tag{3}$$

units poles per square centimeter measured normal to the direction of the flux. If the bar is cut across at any place, the same value of the pole strength will appear. This quantity, I, is called the "intensity of magnetization." It should be noted that this is not magnetic intensity, H.

A more generalized conception of this quantity is obtained from the following considerations. If a bar of uniformly magnetized steel of length, dL, and cross-section, dA, with pole strength, m, is placed in a magnetic field of intensity, H, the maximum couple exerted by the field is

$$T = mHdL = IdA\ dL \cdot H = I \times \text{Volume} \times H$$

or,

$$I = \frac{T}{H \times \text{Volume}}. \tag{4}$$

Hence the intensity of magnetization, I, for a given piece of material is given by the maximum couple per cubic centimeter that would be exerted by a unit magnetic field.

226. Susceptibility.—The susceptibility, κ, of a substance is the ratio of the intensity of magnetization, I, to the value of H necessary to produce it, or

$$\kappa = \frac{I}{H} \tag{5}$$

There is a simple relation between κ and μ for a substance in a given magnetic condition. Substituting in Eq. (2) Art. 224, the values of the different flux densities with the aid of Eqs. (1), (3), and (5), gives

$$\mu H = \mu_o H + 4\pi\kappa H$$

or

$$\mu = \mu_o + 4\pi\kappa. \tag{6}$$

227. Effect of the Ends of a Bar.—When a closed iron ring is magnetized, the circuit is uniform and each part offers equal reluctance. This uniformity is disturbed if an air gap is cut across the ring, and this disturbance is described in two very different ways. In the language of the magnetic circuit it is said that the reluctance of this part of the circuit has been greatly increased by the introduction of the air gap in place of the iron. In the language of magnetic poles it is said that the ends of the iron have become magnetic poles, and these poles introduce a new set of magnetic intensities, H', opposed to H and tending to demagnetize the iron. The effect of magnetic flux passing through the surface of iron and air, and thus producing poles on the surface, is the same as though these poles were absent and equal poles on the ends of external steel magnets were brought up to the same position. This shows the difficulty of studying the magnetic properties of straight bars, or other forms of open magnetic circuits.

228. Electronic M.M.F.—In the usual consideration of the subject of the magnetic flux through a medium, such as iron, it has been supposed that the large amount of flux through the circuit is due to the high permeability of the iron. The modern conception of atomic structure indicates that even the interior of atoms is as empty as the solar system, with a few electrons circulating in orbits through this space. There is little, then, even in solid iron, that can be considered as having the nature of a substance uniformly distributed and completely filling the space it occupies. And this leads to a modification of the former notion regarding the reason for the large flux through iron.

If we are to think of a magnetic circuit, even when made of solid iron, as being practically as empty of any "medium" as vacuum itself, then the reluctance of such a circuit must be practically the same as the reluctance of a circuit of vacuum of the same length and cross-section as the actual circuit.

Rewriting the law of the magnetic circuit as

$$\phi = \frac{\mu' G}{\mu' R},$$

the denominator indicates this larger reluctance. The μ' cancels the numerical part of μ in R, but does not change the "dimensions," so $\mu' R \left(= \frac{\mu' L}{\mu A} \right)$ is a reluctance, expressed in oersteds.

The numerator, $\mu' G$ is a correspondingly increased M.M.F. due to the equivalent ampere-turns that are added by electrons moving in their orbits.

The effect of the atoms and electrons in a magnetic substance may then be looked upon as increasing the effective M.M.F. to μ' times the M.M.F. that is applied by the external electron current. This would be the case if some of the electronic orbits were turned so that the electrons traversed them in the same sense that the electrons in the magnetizing current flow along the electric circuit. Professor Ewing has recently made an atomic model in which an inner group of electrons can turn into alignment with the electron current in the magnetizing coil, and thus increase the resultant M.M.F. by many fold.

This conception of the effect of iron in increasing the magnetic flux does not involve any change in the relation between B and H as given above. B will still express the flux density in maxwells per square centimeter, and H, expressed in gilberts per centimeter, will continue to be the magnetic gradient, or intensity, along the circuit due to the applied M.M.F. The electrons in their orbits add their effect, giving a total magnetic intensity whose value is

$$H_T = \mu' H,$$

where μ' is the relative permeability of the substance compared with vacuum.

While the atoms of all elements have electrons circulating in orbits, in only a few substances is it possible for any of the orbits to change into alignment with the external current and produce the marked effect shown by iron, nickel, etc.

ELECTRICAL UNITS

229. Work to Move an Electron Current across Magnetic Flux.—Experiment shows that it requires work to move a wire carrying a current so as to cut across magnetic flux, or to move magnetic flux across an electric current. The amount of this work is proportional to the number of maxwells of magnetic flux that cut across the conductor, and it does not matter whether this flux is spread out or concentrated. The work also is proportional to the amount of current flowing in the wire.

These facts are simply expressed by the relation

$$W = k\phi I,$$

where k is a proportionality factor that depends upon the unit in which the current is measured.

230. Unit Current.—The electromagnetic units of electrical quantities are so called because they are based on the magnetic units. The unit of electron current is a current of such amount that k will be unity in the expression, $W = k\phi I$. This leads to the following:

Definition.—Unit current is flowing in a conductor when it requires one erg to move the conductor across one maxwell of magnetic flux.

This C.G.S. unit of current has no name, although it is sometimes called an ab-ampere. When the current is measured with this unit we have

$$W = \phi I.$$

The ampere is one-tenth of this current.

231. Force on an Electron Current.—The relation between force and work enables us to find the force acting on a conductor when carrying an electron current in a magnetic field. Let us consider a straight wire, L cm. long and carrying a constant current of I C.G.S. units, or ab-amperes. When this wire is moved sidewise a short distance, ds, across a flux of $d\phi$, the work required is

$$Fds = dW = Id\phi$$

where F is the force on the wire.

From this

$$F = I\frac{d\phi}{ds} = IL\frac{d\phi}{Lds} = BIL,$$

where B denotes the flux density normal to L and F.

This force is used in many practical ways for the measurement of current in ammeters, galvanometers, and similar instruments, and for the turning of electric motors, etc.

When the flux density is uniform, there is little difficulty in determining the meaning of B. In more complicated cases the value for B is found by considering a slight movement of the conductor. If in moving over a small area, Lds, the conductor cuts $d\phi$ maxwells, then the value of B is

$$B = \frac{d\phi}{Lds}.$$

232. Unit Quantity.—The definition of unit quantity of electrons follows at once from the definition of unit current:

Definition.—Unit quantity of electrons is the quantity transferred by unit current in one second.

This unit quantity is 3×10^{10} times larger than the electrostatic unit of quantity.

233. Resistance.—When an electron current is flowing through a circuit, it requires a continual expenditure of energy to maintain the current. This energy appears in the form of heat in the conductor carrying the current. The amount of heat thus generated depends upon the amount of current flowing through the conductor, the time that it flows, and upon a property of the conductor called its resistance. By using different currents and different conductors it has been found that the amount of heat produced in a given conductor is proportional to the square of the current, so that, if W denotes this amount of heat energy measured in ergs,

$$W = RI^2t \qquad \text{ergs},$$

where R denotes the resistance of the conductor and t the time that the current, I, is flowing. Of course, the resistance must be measured in units of such magnitude as to make this expression a true equation. This leads at once to the following definition:

Definition.—Unit resistance is that resistance in which one erg of heat is produced each second by the passage of unit current.

234. Unit Potential Difference.—Having now defined the unit of current and the unit of resistance, the value to be chosen for the unit difference of potential follows from Ohm's law:[1]

Definition.—Unit potential difference is the difference of potential over unit resistance when carrying unit current.

Since $E = RI$, and $Q = It$, from the preceding definitions, the amount of energy expended in a conductor when carrying a current may be expressed as

$$W = RI^2t = EQ,$$

where Q denotes the total quantity of electrons that has passed through the conductor. This gives another way of stating the value of unit potential difference, *viz.*,

COROLLARY.—Unit difference of potential is that difference of potential through which 1 erg can raise a unit quantity of electrons.

235. The Practical Units. *The Ohm.*—The unit of resistance just defined is inconveniently small (because an erg is so small) and, therefore, for practical use a multiple unit, 1,000,000,000 *times as large,* is used. This larger unit is called an "ohm."

The Ampere.—In the same way, the unit of current defined above turns out to be too large for convenience in practical measurements, and, therefore, the "ampere" is defined as being *one-tenth of the* C.G.S. *unit.*

The Volt.—Having now defined the ampere and the ohm, it follows that the practical unit for difference of potential is *the difference of potential that, steadily applied to a conductor whose*

[1] See Art. 18.

resistance is one ohm, will produce a current of one ampere. This unit is called a "volt." Evidently it is 100,000,000 C.G.S. units.

The Coulomb.—Having the ampere for the practical unit of current, it follows that the corresponding unit of quantity is the quantity transferred by one ampere in one second. This unit is called a "coulomb."

The Watt.—The power expended in maintaining one ampere under a potential difference of one volt is called a "watt."

The Joule.—The work done each second by one watt is called a "joule." This is the work that will transfer one coulomb through a potential difference of one volt.

236. Concrete Examples of These Units.—These definitions of the electrical units do not furnish, directly, any standards for use in making actual measurements. Therefore the most careful investigations have been made to determine the tangible values of these units as defined above, in order to express them in terms of definite concrete quantities. From the very nature of the case, such determinations never can express the exact values of the units, but they have been determined much closer than is ever required in ordinary measurements. At a conference of scientific delegates, which met in London, Oct. 12, 1908, the following resolutions were adopted. These resolutions stated the concrete values of the fundamental units in accordance with the best and latest opinion of the scientific men of the world.

237. The Conference on Electrical Units and Standards. London, 1908.

RESOLUTIONS

I. The Conference agrees that as heretofore the magnitudes of the fundamental electric units shall be determined on the electromagnetic system of measurement with reference to the centimeter as the unit of length, the gram as the unit of mass, and the second as the unit of time.

These fundamental units are (1) the Ohm, the unit of electric resistance which has the value of 1,000,000,000 in terms of the centimeter and second; (2) the Ampere, the unit of electric current which has the value one-tenth (0.1) in terms of the centimeter, gram, and second; (3) the Volt, the unit of electromotive force which has the value 100,000,000 in terms of the centimeter, the gram, and the second; (4) the Watt, the unit of power, which has the value 10,000,000 in terms of the centimeter, the gram, and the second.

II. As a system of units representing the above and sufficiently near to them to be adopted for the purpose of electrical measurements and as a basis for legislation, the Conference recommends the adoption of the International Ohm, the International Ampere, and the International Volt, defined according to the following definitions.

III. The Ohm is the first Primary Unit.

IV. The International Ohm is defined as the resistance of a specified column of mercury.

V. The International Ohm is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm.

To determine the resistance of a column of mercury in terms of the International Ohm, the procedure to be followed shall be that set out in specification I, attached[1] to these resolutions.

VI. The Ampere is the second Primary Unit.

VII. The International Ampere is the unvarying electric current which, when passed through a solution of nitrate of silver in water, in accordance with the specification II, attached[1] to these resolutions, deposits silver at the rate of 0.00111800 of a gram per second.

VIII. The International Volt is the electrical pressure which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere.

IX. The International Watt is the energy expended per second by an unvarying electric current of one International Ampere under an electric pressure of one International Volt.

The Conference recommends the use of the Weston Normal Cell as a convenient method of measuring both electromotive force and current, and when set up under the conditions specified in schedule C may be taken, provisionally, as having, at a temperature of 20° C., an E.M.F. of 1.0184 volts. (1.0183 since Jan. 1, 1911.)

In cases in which it is not desired to set up the Standards provided in the resolutions above, the Conference recommends the following as working methods for the realization of the International Ohm, the Ampere, and the Volt.

1. *For the International Ohm.*

The use of copies, constructed of suitable material and of suitable form and verified from time to time, of the International Ohm, its multiples and submultiples.

2. *For the International Ampere.*

(*a*) The measurement of current by the aid of a current balance standardized by comparison with a silver voltameter.

[1] *Elec. Review*, vol. 63, p. 738, 1908.

(*b*) The use of a Weston Normal Cell whose electromotive force has been determined in terms of the International Ohm and International Ampere, and of a resistance of known value in International Ohms.

3. *For the International Volt.*

(*a*) A comparison with the difference of electrical potential between the ends of a coil of resistance of known value in International Ohms, when carrying a current of known value in International Amperes.

(*b*) The use of a Weston Normal Cell whose electromotive force has been determined in terms of the International Ohm and the International Ampere.

237(a). C.G.S. and Practical Units.—The following table shows the relative values of the electromagnetic c.g.s. units and the electrostatic c.g.s. units in terms of the practical units.

RELATIONS BETWEEN DIFFERENT SYSTEMS OF UNITS

Practical unit	Number of C.G.S. units in one practical unit		Number of electrostatic units in one electromagnetic unit
	Electromagnetic	Electrostatic	
Ampere	10^{-1}	3×10^{9}	v
Ohm	10^{9}	$\frac{1}{9} \times 10^{-11}$	$\frac{1}{v^2}$
Volt	10^{8}	$\frac{1}{3} \times 10^{-2}$	$\frac{1}{v}$
Farad	10^{-9}	9×10^{11}	v^2
Watt	10^{7} (ergs)	10^{7} (ergs)	1
Henry	10^{9}	$\frac{1}{9} \times 10^{-11}$	$\frac{1}{v^2}$

$v = 2.9986 \times 10^{10}$ centimeters per second

CHAPTER XII

MEASUREMENT OF MAGNETIC FLUX AND M.M.F.

238. Induced E.M.F.—When work or energy is measured in electrical units the general expression is

$$dW = EIdt \qquad \text{ergs,}$$

and this relation furnishes the means for defining the unit of E.M.F. This unit must be of such a magnitude that this equation will be a true equality. One way of doing this was given in Art. 234. Another way of *saying the same thing* follows from the discussion of the work to move a current across a magnetic field.

Consider a current, I, flowing in a wire which is allowed to move across a magnetic flux, doing work thereby. For simplicity, let us suppose that the current is maintained constant by varying the E.M.F. of the source, if necessary. Then,

$$\begin{aligned} dW &= EIdt = (e' + \epsilon)Idt \\ &= RI^2dt + \epsilon Idt, \end{aligned}$$

where the first term is the energy used in merely keeping the current flowing through the circuit, whether the wire moves or not. If any other work is done, it is included in the part

$$dW = \epsilon Idt,$$

where ϵ is the extra E.M.F. that is required to keep the current constant while the wire is moving.

It was shown in Art. 230 that the work done by a current moving across a magnetic flux is

$$dW = Id\phi.$$

Equating these two expressions for the same amount of work gives

$$\epsilon\, dt = d\phi$$

from which

$$\epsilon = \frac{d\phi}{dt}.$$

This ϵ is the extra E.M.F. that is required to keep the current *constant* when the wire is moving. Therefore the motion of the wire must have resulted in the production of an E.M.F. equal to ϵ in the wire, because it is necessary to add ϵ in order to keep constant the total E.M.F. in the circuit.

The E.M.F. induced in the wire is, therefore, equal and opposite to ϵ, *viz.*,

$$e = -\epsilon = -\frac{d\phi}{dt} \qquad \text{C.G.S. units}$$

$$= -10^{-8}\frac{d\phi}{dt} \qquad \text{volts.}$$

Since this expression is independent of the value of the current in the wire, e will have the same value when I is zero. Therefore, when a wire carrying no current is moved across a magnetic flux there is induced in it an E.M.F. of this amount, and if the circuit is closed, this will cause a current to flow. The direction of this current will be opposite to that of the constant current, I, considered above.

Definition.—One C.G.S. *unit of* E.M.F. *is the* E.M.F. *induced in a conductor when it is cutting across magnetic flux at the rate of one maxwell per second.*

This is not a second definition, because both here and in Art. 234 the unit of potential difference is defined in terms of the erg. Here the erg is expended in pushing a current across a magnetic flux. There it was expended in heating the conductor. Both ways of saying it lead to the same unit of E.M.F. Which one is the more convenient depends upon the use that is required of it.

239. Measurement of Magnetic Flux.—Having seen above that an E.M.F. is induced in a wire when the latter cuts across a magnetic flux, it is at once evident that this offers a ready means for the measurement of magnetic flux. As this E.M.F exists only while the flux is being cut, it gives rise to a transient current, the total quantity in which can be measured by a ballistic galvanometer. If the galvanometer is connected to the wire when it is moved across a magnetic flux, the quantity that passes through the circuit during a short interval, dt, is

$$dQ = idt = \frac{e}{R}dt,$$

where e is the E.M.F. acting in the circuit at the instant under consideration. This consists of two parts—the induced E.M.F.,

$\frac{d\phi}{dt}$, due to the rate of cutting the magnetic flux that is being measured, and the E.M.F., $-L\frac{di}{dt}$, due to the self-inductance, L, of the circuit (see Art. 299). Therefore,

$$e = \frac{d\phi}{dt} - L\frac{di}{dt}$$

and

$$dQ = \frac{1}{R}\frac{d\phi}{dt}dt - \frac{L}{R}\frac{di}{dt}dt.$$

The total quantity, Q, passing through the galvanometer is found by integrating this expression from a time, t, before any flux is cut, until the time, t', when the induced current has ceased to flow through the galvanometer.

$$Q = \frac{1}{R}\int d\phi - \frac{L}{R}\int di = \frac{\phi}{R} - \frac{Li}{R} + h.$$

At the start, and before any flux has been cut, Q, ϕ, and i are each zero. Hence the constant of integration, h, is zero also. At the end, after the action has ceased, $Q = Q'$, $\phi = \phi'$, $i = 0$, and

$$Q' = \frac{\phi'}{R},$$

where Q' denotes the quantity of electrons that passed through the galvanometer when the flux ϕ', was cut.

240. The Ballistic Galvanometer.—A ballistic galvanometer is one designed to measure transient currents, or quantities of electrons, like the induced current in a secondary coil or the discharge of a condenser. The duration of the current is very brief as compared with the period of the needle of the galvanometer, and thus the coil remains practically at rest until the entire quantity has passed through it. It is impossible, therefore, to obtain a steady deflection, but the galvanometer coil gives a sudden throw and then settles down to the position of rest. The extent of this throw gives a measure of the quantity that has passed through the galvanometer; the greater the quantity the greater the throw, the exact relation depending upon the galvanometer employed.

These galvanometers are of two general types: (1) The magnet may be fixed, and the coil suspended and movable; or, (2) the magnet may be suspended and movable and the coil fixed.

241. Correction for Damping.—Let θ_1, θ_2, θ_3, etc., be the successive deflections of the galvanometer to the right and left when it is allowed to swing freely after the discharge of the condenser through it. It is observed that each deflection is less than the one before it by a certain constant ratio, so that

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \text{etc.} = f, \text{ say.}$$

If these deflections are laid off as ordinates, each one being erected at that point on the axis of abscissæ corresponding

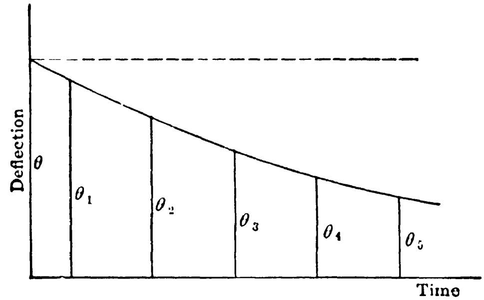


FIG. 135.—Damping of a galvanometer.

to the time at which it was observed, they would appear as in Fig. 135. Time is reckoned from the discharge of the condenser and when the galvanometer coil begins to move. After half of a single period the first throw, θ_1, is observed. The succeeding deflections follow at equal intervals of a whole single period. Knowing the value of f, any given deflection, say θ_1, can be computed from the later readings, since

$$\theta_1 = \theta_2 f = \theta_3 f^2 = \theta_4 f^3 = \text{etc.}$$

where the exponent of f in each case is equal to the number of single periods between the desired and observed deflections.

Had there been no damping, all of these deflections would have been larger, and each equal to θ, the value of which is

$$\theta = \theta_1 f^{\frac{1}{2}},$$

where the exponent of f is ½ because the interval between θ and θ_1 is half of a single period.

Since

$$f = \frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_1 + \theta_2}{\theta_2 + \theta_3} = \frac{s_1}{s_2},$$

the value of f is seen to be also given by the ratio of two consecutive swings—a swing being the full amplitude from the turning point at the right to the turning point at the left.

Using this value of f gives

$$\theta = \theta_1\left(\frac{s_1}{s_2}\right)^{\frac{1}{4}} = j\theta_1.$$

242. Closed Circuit Constant.—It thus appears from the above discussion that a given impulse may not always produce the same deflection, θ_1, but if the damping is made greater, the observed deflection is less. If k is the constant of the galvanometer without damping, the equation should be written

$$Q = kjd,$$

where jd is the deflection of the galvanometer, corrected for damping. In the present case where the galvanometer is connected to a closed circuit of comparatively low resistance, R, the damping is very great, often so great that the damping factor cannot be determined in the usual way. But even so it is found that the observed deflection is proportional to the quantity of electrons that passed through the galvanometer, and kj may be called the closed circuit constant.

243. Magnetic Ballistic Constant.—The flux cut by the galvanometer circuit is, then, from Art. 239,

$$\phi' = RQ' = Rkjd = cd,$$

where c is the combined constant, Rkj. This is called the "magnetic ballistic constant," and its value depends upon the resistance of the galvanometer circuit as much as upon k or j.

244. Flux-turns.—If the wire cutting the flux is wound up into a coil of n turns, and the flux, ϕ, is cut by each of these turns, the galvanometer deflection will be n times as large, and we have

$$\phi n = cd.$$

The product, ϕn, is called "flux-turns." If the entire flux does not pass through all of the turns, the "flux-turns" are the summation of the amounts of flux cut by each turn of the coil. This summation can be denoted by (φn).

245. Flux from a Permanent Magnet.—The following exercises will illustrate the manner in which magnetic flux is measured. They may also serve to make clear some points that perhaps have not been fully understood from merely reading the theory of the subject. In regard to the second one, it is important that the facts be impartially observed and that a full and sufficient reason therefor be thoroughly understood.

1. Place a coil of one turn about the middle of a bar magnet. Note the deflection of the ballistic galvanometer when the magnet is quickly withdrawn. Repeat, using two, three, five, and ten turns.

2. Does it make any difference whether the coil is drawn off the north or the south end of the magnet?

3. Place the coil just off one end of the magnet and suddenly withdraw the latter. Note the deflection. Repeat, moving the coil instead of the magnet. Repeat again, moving the coil at right angles to the magnet so as to "cut" across the magnetic flux. Is it true to say that the deflection depends only upon the total change in the flux threading the coil, and not at all upon the manner in which that change is produced?

246. To Find the Constant, *c*.—The constant, c, is readily determined if a standard magnet is at hand. This is a steel magnet the total flux through which is known. Call this Φ maxwells. A coil of n' turns is placed around the middle of the magnet and connected to the galvanometer. When the magnet is withdrawn, the flux, Φ, is cut n' times, and as before,

$$\Phi n' = cd',$$

from which,

$$c = \frac{\Phi n'}{d'}.$$

247. To Determine *c*, **Using a Known Mutual Inductance.**—The magnetic ballistic constant, c, of the combined galvanometer and its circuit, is most satisfactorily determined by means of a known mutual inductance. This consists of two coils, either wound upon the same spool or fixed in a definite relation to each other. One of these coils, called the secondary, is placed in the galvanometer circuit and the other, called the primary, may be connected to a battery when it is to be used.

Let I_1 denote the current through the primary when the circuit is closed. Then the flux-turns through the secondary are MI_1 (see Art. 298), where M is the mutual inductance (C.G.S.) of the given pair of coils, and is numerically equal to the flux threading the secondary when unit (C.G.S.) current flows in the primary. Each time the primary current, I_1, is made or broken, the secondary is cut by this flux, causing a fling d_1 of the galvanometer, if the circuit of the latter is closed. The change in the current should be such as to give a suitable deflection. This gives the relation,

$$MI_1 = cd_1 \qquad \text{or,} \qquad c = \frac{MI_1}{d_1}.$$

The value of c can be changed by changing the resistance in the galvanometer circuit. Therefore, when the desired value has been obtained, this resistance should not be altered. This may require leaving the secondary coil of M connected in the galvanometer circuit while the measurements of magnetic flux are being made.

248. To Compute the Value of M.—In case it is not possible to obtain a known mutual inductance, or to measure M by one of the methods of Chap. XVII, it is possible to compute the value of M when the coils are wound in a simple form. Thus, let the primary be a single layer of p turns per cm. wound in a cylindrical tube that is long in comparison with its diameter. Let the primary current be I C.G.S. units. The magnetic intensity, H, near the middle of this long solenoid is a little less than

$$H = 4\pi pI,$$

which is the value for a very long solenoid. The correction term is found by considering the solid angles, ω_1 and ω_2, subtended by the ends of the solenoid from a point near the middle.

Thus, imagine a unit pole at this place. Of its total flux of 4π maxwells, the greater part will pass through the sides of the solenoid, but $(\omega_1 + \omega_2)$ maxwells will pass out at the ends. Consider the work $(= \phi I)$ to move the unit pole 1 cm. along the axis of the solenoid, which is

$$W = HL = (4\pi - \omega_1 - \omega_2)pIL,$$

giving

$$H = (4\pi - \omega_1 - \omega_2)pI = 4\pi\left(1 - \frac{d^2}{2h^2}\right)pI,$$

where $\frac{d}{h}$ is the ratio of the diameter to the length of the solenoid.

Let the secondary coil of n turns be wound on a short cylinder within and coaxial with the long solenoid. The flux-turns through this coil are

$$\phi n = BAn = \mu HAn$$

and

$$M = \frac{\phi n}{I} = 4\pi\mu\left(1 - \frac{d^2}{2h^2}\right)pAn,$$

where A is the cross-sectional area of the coil. The product An can be considered as the total area enclosed by the secondary coil.

249. M.M.F. Due to a Current.—The most convenient way to obtain a definite M.M.F. in a circuit is to link the circuit with a number of turns of wire carrying an electric current, as indicated in Fig. 133. Since the unit current has been defined in terms of magnetic flux, and the gilbert is defined in terms of a unit magnetic pole, it follows that there is an intimate connection between them. A unit magnetic pole is never made or used in actual measurements, but the idea of such a unit pole enables us to investigate certain problems in a simple manner. The following articles show the steps by which we can find the relation between the M.M.F. and the current in a solenoid.

250. Flux from a Unit Pole.—A unit pole is the origin of a certain amount of magnetic flux. This amount is independent of the surrounding medium and can be determined from the following considerations.

When a unit test pole is placed r cm. from the unit pole in question, in vacuum, the force of repulsion is

$$F = \frac{mm^1}{r^2} = \frac{1}{r^2} \qquad \text{dynes.}$$

This means that the magnetic intensity at this point is

$$H = \frac{F}{m} = \frac{1}{r^2} \qquad \text{gilberts per centimeter.}$$

The flux density is then

$$B = \mu H = \frac{1}{r^2} \qquad \text{maxwells per square centimeter,}$$

since $\mu = 1$, for vacuum. From symmetry, the density would be the same at each point over the surface of a sphere of radius r. The area of such a sphere is

$$A = 4nr^2 \qquad \text{square centimeters.}$$

The total flux through this area is, then,

$$\phi = BA = \frac{1}{r^2} \times 4\pi r^2 = 4\pi \qquad \text{maxwells,}$$

and this flux all comes from the unit pole at the center of this sphere. The flux is the same amount when the pole is in any other medium.[1]

251. Work to Carry a Unit Pole around a Current.—When a magnetic pole is moved along any path that returns to the starting point it will, in general, move *against* magnetic forces in a part of the path, and *with* such forces in other portions of the path. If, at the end, the pole is at rest in the same position that it was at the beginning, the total work done is, in general, zero. But if the path is so chosen that the pole is carried around a current, the work is not zero for the complete path.

Every current flows in a closed circuit. Imagine a unit pole with its 4π maxwells of flux radiating out to infinity. As this pole is carried along a path that is linked with the circuit of the current, and continues on to the starting point, each one of these 4π maxwells must cut across the current at some point. Therefore, the work to carry a unit pole around a current I is

$$W = 4\pi I \qquad \text{ergs}$$

when I is expressed in ab-amperes. (Art. 230.)

[1] FLUX FROM A MAGNETIC POLE.—That the flux from a magnetic pole is independent of the medium surrounding the pole is readily shown by a consideration of the work done when a magnetic pole is carried through a long solenoid. Suppose, in the first instance, that the total flux from the pole is ϕ maxwells and that the pole is carried through N turns of wire carrying I units of current. The work done will be

$$W = \phi NI \qquad \text{ergs.}$$

If the pole is brought back along the same path, the whole amount of this work is recovered. This is the case for air.

Suppose, again, that the solenoid is filled with a ferric chloride solution and the same operation is repeated. Suppose that in this medium the amount of flux from the same pole becomes Φ maxwells. The work now required to carry the pole through the solenoid will be

$$W' = \Phi NI \qquad \text{ergs,}$$

and this work is recovered when the pole is brought back.

Now suppose that one side of the solenoid is filled with air and the other side is filled with the ferric chloride solution, and let the pole be carried through the air and brought back through the solution. The work expended is ϕNI and the work recovered is ΦNI, and the total work for the whole trip is zero because the pole has not been carried around any electric current (see Art. 229).

Therefore $$\Phi NI - \phi NI = 0.$$
and
$$\phi = \Phi.$$

The flux from a pole is not changed, then, by altering the surrounding medium.

252. M.M.F. Due to a Solenoid.—Consider a current flowing in a long, uniform helix or solenoid of many turns ($=N$) of wire, and let a unit pole be carried down inside the solenoid and brought back outside. The work done for the entire path is

$$W = 4\pi NI \qquad \text{ergs,}$$

and therefore the M.M.F. in this circuit is

$$\text{M.M.F.} = 4\pi NI \qquad \text{gilberts.}$$

The greater part of this M.M.F. is distributed along the portion of the magnetic circuit that is within the solenoid.

When the current is measured in amperes, this value must be divided by 10, giving

$$\text{M.M.F.} = 0.4\pi NI' = 1.257NI' \qquad \text{gilberts.}$$

The product, NI', is called "ampere-turns" and differs from M.M.F. by the constant factor of $\frac{4\pi}{10} = 1.257$. It is evident that a small current in a coil of many turns will produce the same M.M.F. as a large current in a coil of few turns.

253. Magnetic Intensity within a Solenoid.—Since the magnetic intensity, H, is the gradient of the M.M.F. along the circuit, we have

$$H = \frac{\text{M.M.F.}}{L} = \frac{4\pi NI}{L} = 4\pi pI \qquad \text{gilberts per centimeter,}$$

where L is the length of the solenoid, and p is the number of turns per centimeter.

This gives the value of H to the same degree of accuracy that is implied by saying that the total M.M.F. is uniformly distributed over the length, L. At the end of the solenoid, H has one-half of this value, while near the middle of a long solenoid the approximation is very close.

When the value of the current is expressed in amperes we have

$$H = 0.4\pi pI' = 1.257pI' \qquad \text{gilberts per centimeter.}$$

254. The Magnetic Intensity within a Ring Solenoid.—The ring solenoid is a uniform spiral or helix of many turns of wire, bent into a circle to make an endless spiral, as in the case of a ring uniformly wound with N turns of wire. There is thus no end effect. The magnetic flux is all within the solenoid and in the direction of the length of the ring.

In making the complete circuit along the ring, the path of a magnetic pole will be linked with a total current of NI. The M.M.F. for this circuit is, then,

$$\text{M.M.F.} = 4\pi NI.$$

If the path is a symmetrical circle of radius, r, the magnetic intensity will have the same value, H, at each point of the path. The length of this path is $L = 2\pi r$, and, therefore,

$$H = \frac{4\pi NI}{2\pi r} = \frac{NI'}{5r},$$

where I' is the value of the current in amperes. This shows that the magnetic intensity is not constant across the section of the ring, but varies inversely as r, being greater on the inner side of the ring.

255. Measurement of M.M.F.—When a M.M.F. is due to electric currents, it is very easy to compute its value from the relation, M.M.F. $= 4\pi NI$, and direct measurement is not necessary. But it is often desirable to measure M.M.FS. which are not thus

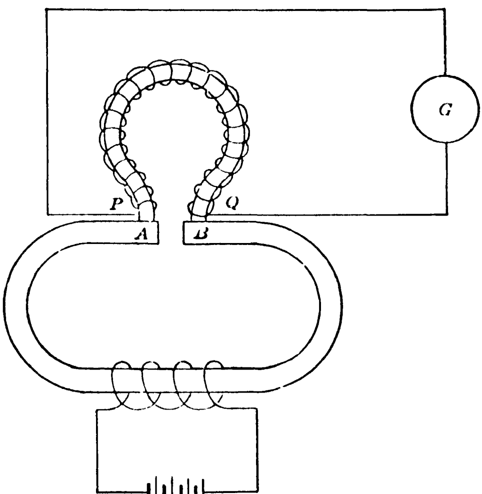


FIG. 136.—The flexible solenoid, PQ, measures the M.M.F. between A and B.

easily computed; for example, an M.M.F. due to steel magnets, or the part of a known M.M.F. that is distributed over a certain portion of the circuit.

This direct measurement can be made[1] with a "magnetic potentiometer" consisting of a ballistic galvanometer and a flexible

[1] CHATTOCK, *Phil. Mag.*, vol. 24, p. 94, 1887.

tube wound over its entire length with a uniform solenoid that serves as the test coil. Thus, if it is desired to measure the M.M.F. between A and B, Fig. 136, the ends of this flexible solenoid are placed at A and B. There will be some flux through some of the turns of the test solenoid, and these flux-turns can be measured by reading the galvanometer deflection when the flux is reversed, or when the ends of the solenoid are removed from AB and then quickly brought together.

If the solenoid has a constant cross-section, a, and is uniformly wound with p turns per centimeter, the flux-turns, $d(\varphi n)$, for a short length, ds, of the solenoid are

$$d(\varphi n) = B'apds,$$

where B' denotes the component of the flux density parallel to the axis of the solenoid at this place. The flux along the solenoid is Ba, and this flux passes through pds turns.

Let H' denote the magnetic gradient (magnetic intensity) along the solenoid. Then $B' = \mu H'$, and,

$$d(\varphi n) = ap\mu H'ds = ap\mu dG.$$

The product $H'ds$ is dG, the M.M.F. over ds, and the integral of this expression along the solenoid gives the M.M.F., G, between its ends. Thus, for the entire solenoid

$$(\varphi n) = ap\mu G.$$

By bringing the ends of the solenoid quickly together, G is reduced to zero and the corresponding change in the flux-turns is measured by the deflection of the galvanometer, giving

$$(\varphi n) = cd.$$

Therefore

$$G = \frac{c}{ap\mu} d = gd.$$

The galvanometer deflection is thus proportional to the change in M.M.F. between the ends of the solenoid.

256. To Determine the Constant, g.—To determine the value of g, which is a constant for a given solenoid with its galvanometer, let the solenoid encircle a known current, I_1, with the two ends of the solenoid placed together, thus forming an endless ring solenoid. Then

$$G = 4\pi I_1 = gd_1$$

and

$$g = \frac{4\pi I_1}{d_1},$$

where I_1 is in C.G.S. units and d, is the galvanometer deflection when I_1 is reduced to zero.

257. Units Used in Laboratory Magnetic Measurements.—The units used in magnetic measurements are the gilbert, maxwell, oersted, etc., and the relations of these C.G.S. units to the ampere and volt have been traced in the preceding chapters. Inasmuch as the international ampere and the international volt are the values actually used in laboratory practice, it follows that the magnetic measurements made in the laboratory will be in terms of the international units. These are, therefore, stated below.

258. International Magnetic Units. *International Gilbert.*—In Art. 252 it is shown that one ampere-turn is equivalent to a M.M.F. of 0.4π gilbert. Therefore, one international gilbert is equal to the M.M.F. produced by $\frac{10}{4\pi}$ international ampere-turns.

International Maxwell.—When magnetic flux cuts across a wire at a uniform rate, there is produced in the wire a steady and constant E.M.F. The motion is relative and it is immaterial whether the flux or the wire is in motion. One international maxwell is the amount of magnetic flux cutting across a conductor each second when the induced E.M.F. is 10^{-8} international volts.

International Oersted.—An international oersted is the reluctance in which one international gilbert is required to produce a flux of one international maxwell.[1]

[1] In terms of these units, the permeability of vacuum is not exactly unity, but is about 0.9994_8 (see footnotes on pp. 3 and 4).

CHAPTER XIII

MAGNETIC TESTS OF IRON AND STEEL

259. Study of a Magnetic Circuit—Bar and Yoke.—The bar and yoke apparatus consists of a heavy rectangle of iron, through the center of which is the bar to be studied. This bar is divided in the middle and the two ends surfaced to fit well together. One half of the bar is fixed rigidly in place, while the other half can be withdrawn by the handle, *H*. Around the bar near the middle is a small bobbin on which is wound

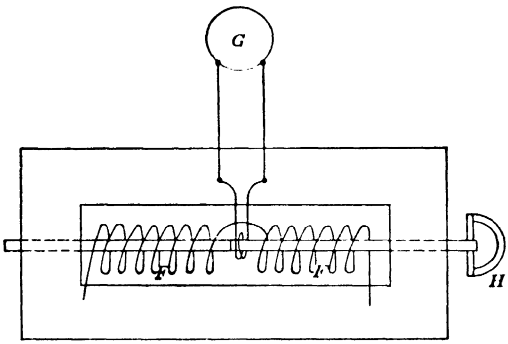


FIG. 137.—Bar and yoke.

the coil which is joined to the ballistic galvanometer. When the movable part of the bar is withdrawn, this bobbin flies out to one side, thus carrying the coil into a region of no, or very small, magnetic flux. Thus the deflection of the galvanometer shows the total flux through the bar before it was parted.

The magnetizing current is passed through the coils, *FF*, which extend the entire length of the bar, save for the short portion occupied by the bobbin at the middle. The bar is thus subjected to a uniform magnetizing force throughout its entire length. No magnetic poles are produced, since the flux through the bar completes its circuit through the yoke, the reluctance of which is small in comparison with that of the bar. Thus the length of the magnetic circuit for all practical purposes is equivalent to the length of the bar alone.

Starting with a small current through the magnetizing coils, the corresponding deflection of the galvanometer is determined. Then the current is increased by small steps and the corresponding deflections are noted, till the iron is fully magnetized. If the bar has been previously magnetized, it will be necessary to demagnetize it before using it. This can be done by starting with the largest current previously used and decreasing it by steps, while at the same time it is slowly reversed in direction eight or ten times at each step.

260. Computation for M.M.F.—The M.M.F. is readily computed from the value of the current by the formula,

$$\text{M.M.F.} = 0.4\pi NI',$$

where I' is the value of the current expressed in amperes.

261. Measurement of Magnetic Flux.—The magnetic flux is measured by the deflection of the galvanometer, the latter being directly proportional to the amount of flux cut by the wires on the bobbin when it flies out from its position around the bar. If ϕ denotes the total flux through the bar, and n the number of turns of wire on the bobbin, the effect on the galvanometer is the same as cutting a flux of ϕn with a single wire. Therefore

$$\phi n = cd,$$

where d is the deflection in scale divisions, and c is the *magnetic ballistic constant* of the galvanometer expressed in flux-turns per scale division. The actual flux through the bar is, then,

$$\phi = \frac{c}{n} d.$$

262. Plotting the Results.—The results obtained from this study of a magnetic circuit can best be shown by means of curves. The first one may be plotted taking values of the M.M.F. as the abscissæ, and the corresponding values of magnetic flux for the ordinates.

From the values of the flux and the M.M.F. compute the reluctance of the bar for different values of the flux. Express the results by means of a second curve, this time taking the flux for abscissæ and using the corresponding values of the reluctance for ordinates.

263. Flux Reversal.—The bar and yoke method is very useful for purposes of illustration and instruction, since it is readily understood that the wire on the little bobbin cuts across the magnetic flux when it is released by the withdrawal of the bar. But the air gaps at the end of the bar and at the side where it slides in the yoke introduce considerable reluctance into the magnetic circuit, and therefore the magnetic flux is smaller than it would be for a similar circuit containing no air gaps. Such closed circuits are used in the methods described below, and while it is not possible for the wire to cut across the flux, yet when the flux is changed from one direction to the reverse it is evident that the change in the flux through the coil has been twice the original amount. In fact, if the bobbin in the bar and yoke had remained around the bar while the magnetizing current was reversed, the deflection of the galvanometer would have been twice the amount that it was when the bar was withdrawn.

264. The Ring Method.—While this is one of the older methods, it still remains a standard method for determining the magnetic properties of iron or other magnetic material, and other methods are checked by comparing their results with those obtained by the ring method. This is due to the simplicity and uniformity of the magnetic circuit. The ring has no ends and there are no joints across which the flux must pass. When a ring is wound with a uniform solenoid of p turns per centimeter, the M.M.F. is uniformly distributed along the circuit, and if the material is homogeneous, all of the flux follows the iron path.

From what has been said thus far it will be seen that the magnetic flux cannot be measured directly. It is only the *change* in the *flux turns* that affects the galvanometer; and the value of the total flux must be inferred from such measurements. The most usual change of flux is that produced by reversing the magnetizing current. It is then assumed that the flux is also reversed, and therefore the change produced is twice the total flux.

The experimental arrangement is shown in Fig. 138. The iron ring is shown at Z, with the primary winding connected to the reversing switch, S. The ammeter is placed on the battery side of this switch so the current through it is not reversed. When S is thrown over, the current through the primary winding is reversed, thus changing the flux in the iron from ϕ in one direction to ϕ in the other direction—a change of 2ϕ. If the galvanometer is connected to the test coil (key closed) when this change in flux occurs, there is a deflection, d, and, as shown before (Art. 243).

$$\textit{Change in Flux} = 2\phi n = 2BAn = cd,$$

where A is the cross-sectional area of the iron, and n is the number of turns in the test coil. This coil should be wound closely

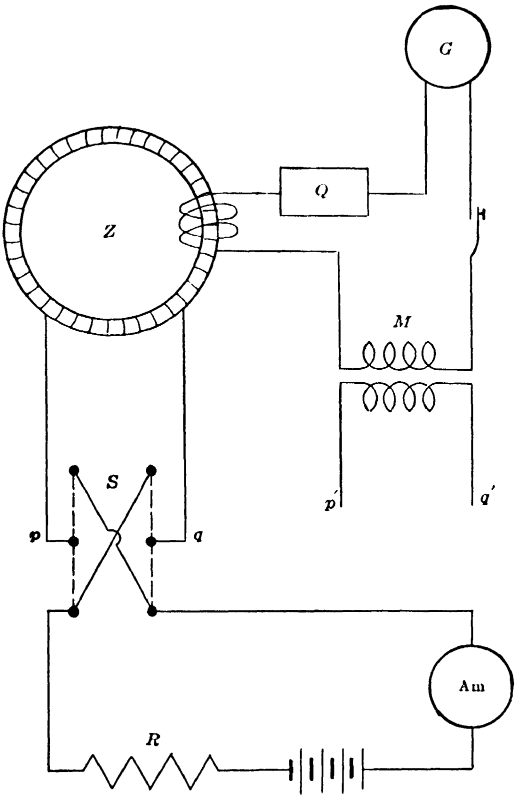


FIG. 138.—Arrangement for determining the normal magnetization curve for the ring, Z.

about the iron so as to include a minimum amount of the flux through the air.

The Flux Density.—In studying a given sample of iron or steel we are usually not so much interested in the properties of the particular piece under investigation as in the specific properties of that grade of iron. The actual total magnetic flux, $\phi = BA$, depends as much upon the dimensions of the sample studied as upon the quality of the iron, and therefore the flux density, $B = \frac{\phi}{A}$, is of greater importance than ϕ.

From the relation above, the value of the flux density is

$$B = \frac{cd}{2An} = Jd.$$

All of these factors can be found and the numerical value of J computed once for all. If it is possible to adjust the resistance of the galvanometer circuit, it simplifies the computation to make $J = 100$. The values of B can then be obtained as fast as the deflections can be read and multiplied by J.

The Magnetic Intensity, H.—The value of the magnetic intensity due to the current in the N turns of the primary winding is

$$H = \frac{NI'}{5r},$$

where I' is in amperes, and r is the mean radius of the iron ring.

265. The Iron Ring.—Since the primary turns are closer together on the inside of the ring than on the outside, the magnetic intensity, H, will not be uniform across the iron, but will be greater near the inner side. This is also seen from the relation, $H = \frac{NI}{5r}\cdot$ For this reason it is best to use a wide, thin ring, shaped like a wagon tire, with a radial thickness less than one-tenth (preferably one-twentieth) of the radius of the ring.[1]

If this ring is tested as a sample of a larger piece, the material should be altered as little as possible in making the ring. It is better to cut it out of a thick plate, but if it must be bent and welded, it should be worked as little as possible. If welded or cast, the outside scale should be taken off all around to about one-eighth of an inch deep, so that the measured cross-section shall represent homogeneous material. As working in the lathe hardens the iron near the cut surfaces, the ring should be carefully annealed in a furnace from which air is excluded, by packing the ring in aluminum oxide.

If the ring is a new one that has not been magnetized since it was annealed, the smallest currents may be used first to determine the virgin B-H curve. In this case great care must be observed that at no time does the current exceed, even for an instant, the value being used at the time. If the iron has been previously magnetized it must be thoroughly demagnetized before it is used.[2]

[1] *Bull.* Bureau of Standards, vol. 5, p. 442.

[2] See Art. 288.

266. Correction for the Flux through the Air.—The flux through the test coil consists mainly of the flux through the ring, but there is always some flux through the air space surrounding the iron and included within the test coil. Let the cross-sectional area of the iron be denoted by A cm.^{2} and that of the surrounding air space by a cm.^{2} The area of the test coil is then $(A + a)$ cm.^{2} and the flux through this coil is

$$\phi = BA + B_0 a,$$

where B_0 denotes the value of the flux density in the air around the iron. The flux density in the iron is

$$B = \frac{\phi - B_0 a}{A}.$$

The value of B_0 can be computed, since $B_0 = \mu_0 H$, and μ_0 is very nearly unity for air. Usually this correction is negligible, but when high values of H are used, this correction should be subtracted from the measured values of ϕ as shown above.

In case there is considerable air space between the iron and the test coil, it is sometimes found convenient to place a small auxiliary test coil in this space. By making

$$B_0 a' n' = B_0 a n,$$

where a' and n' refer to the auxiliary coil, the flux-turns through this coil can be made to neutralize the above correction. When the two test coils are in series, the resultant measured by the galvanometer is

$$\phi n = BAn + B_0 an - B_0 a' n' = BAn,$$

and

$$B = \frac{\phi}{A}.$$

267. Determination of the Magnetic Ballistic Constant.—The value of c is determined by the method shown above, using a known mutual inductance. Since $c = Rk$, this value depends upon the resistance in the galvanometer circuit. Therefore, the secondary of this mutual inductance coil should be included in the galvanometer circuit when the readings for B are taken. It is then ready to use for the determination of the constant.

By disconnecting the primary of the ring from S, Fig. 138 and connecting in its place the primary of the known inductance, M, the same battery and ammeter can be used again. When a current, I_1, is reversed through $p'q'$, giving a deflection, d_1, the constant is

$$c = \frac{2MI_1}{d_1},$$

as shown in Art. 247.

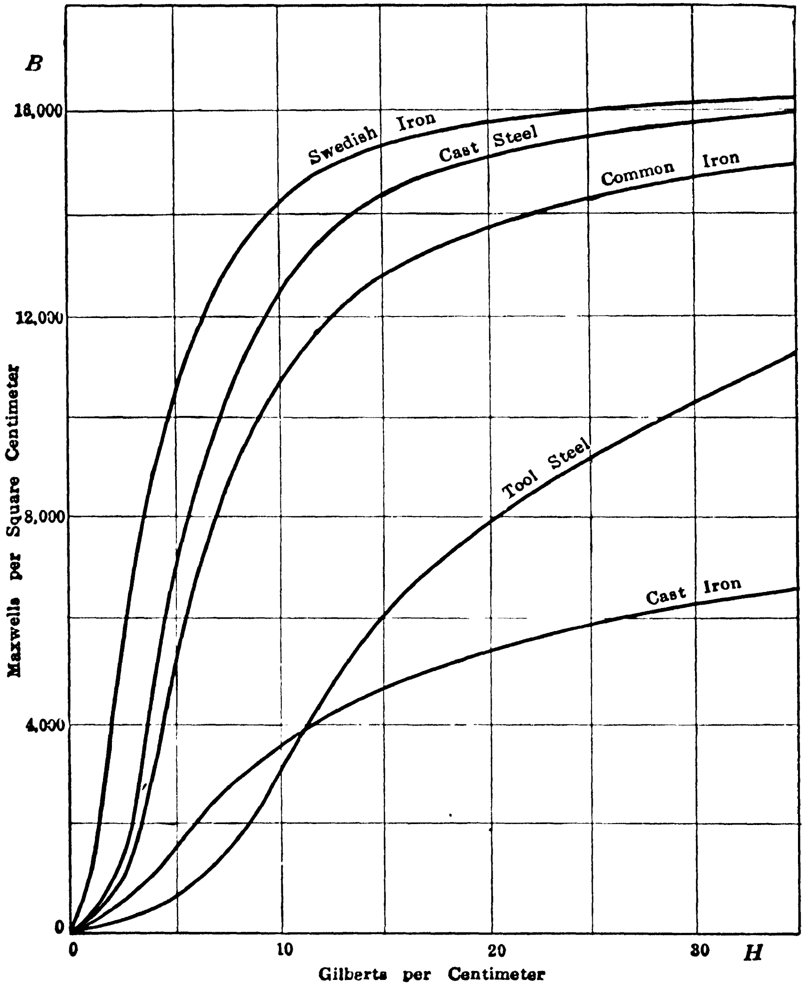


FIG. 139(*a*).—Normal magnetization curves.

268. Normal Magnetization Curve.—The relation between the corresponding values of the flux density, B, and the magnetic gradient, or intensity, H, computed from the above measurements are best shown by a normal magnetization curve. This curve will be similar to one of those shown in Fig. 139(a). It is seen that the relation is not constant, but the flux density rises quickly at first and then approaches a limiting value as H is increased.

The observations for this curve can be taken in connection with the process of demagnetization (see Art. 288). In this case the largest values are measured first, and the points on the curve are traced successively back towards the origin. Note that this curve is the locus of the upper tips of a series of hysteresis loops (see Art. 278).

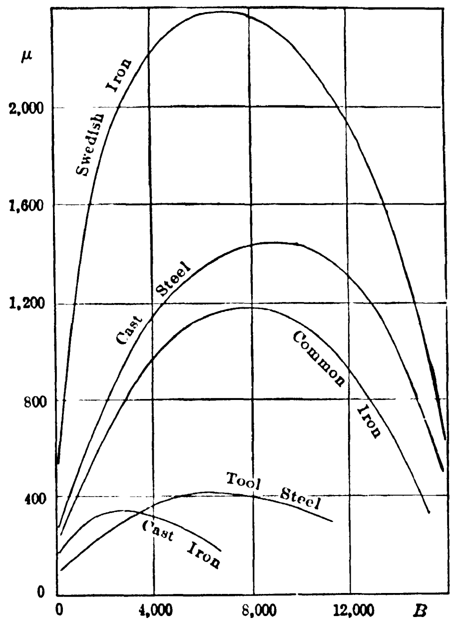


FIG. 139(b).—Permeability curves.

It will be noticed that this curve is very similar to the curve showing the relation between ϕ and M.M.F. in a magnetic circuit (see Art. 262). There is this difference, however, that while the latter applies to a particular *circuit*, the normal magnetization curve (B-H) applies to any circuit of this particular *grade of iron.*

269. Permeability Curve.—The qualities of different grades of iron are well shown by the permeability curves, in which values of the permeability,

$$\mu = \frac{B}{H}$$

are plotted as ordinates, with the corresponding values of B for abscissæ. Figure 139(b) shows the permeability curves corresponding to the magnetization curves of Fig. 139(a). It will be seen that the maxima of these curves correspond to the points where lines through the origin are tangent to the magnetization curves.

The premeability starts at a value of about 50 or 100, and, after passing the maximum, decreases rapidly and then slowly to a limiting value of unity. For the largest values of B that have been measured, μ is about 2.

270. Double Bar and Yoke.—Because of the difficulty of making rings and the labor of carefully winding each one, many other arrangements have been used in which the coils are wound on permanent forms that can be slipped over a bar or rod of iron. In most cases the magnetic circuit is made as nearly as practicable like that in the ring method.

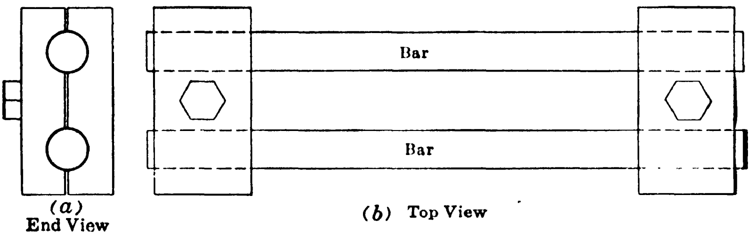


Fig. 140.—Double bar and yoke, without the magnetizing coils.

In the double bar and yoke method, the magnetic circuit consists of two massive yokes of Swedish iron, fitted to carry two round bars of the iron or steel to be tested. When clamped together the bars and yokes form a rectangular circuit, the bars forming the longer sides. Over the entire length of each bar is a brass spool on which are wound 300 turns of wire to carry the current used to magnetize the iron. The joints where the bars pass through the yokes are carefully fitted to avoid any unnecessary reluctance at these places. Thus the magnetic circuit consists of the two bars and very little else, since the reluctance of the yokes is small in comparison.

This method differs from Hopkinson's bar and yoke in that the bar cannot be pulled open to allow the test coil to fly out and cut the flux it is desired to measure. Therefore the test coil is wound around the middle of the bars, and the flux through the circuit is reversed by reversing the magnetizing current. This gives twice the change of flux through the coil and therefore twice the deflection that would be obtained by simply cutting the flux once.

In making a test by this method the procedure is the same as in the ring method just described. Of course, some error is introduced by neglecting the reluctance of the yokes. This is minimized by making the distance between the bars as short as possible and still leaving room for the primary windings, and also by placing the test coil near the middle of the bar. It would improve matters to supply an appropriate M.M.F. at each yoke, but this would increase by many times the labor of making the test (see Art. 273).

271. Burrows' Double Bar and Yoke.—In the double bar and yoke method as used at the Bureau of Standards the effect of the extra reluctance in the joints and yokes is balanced by

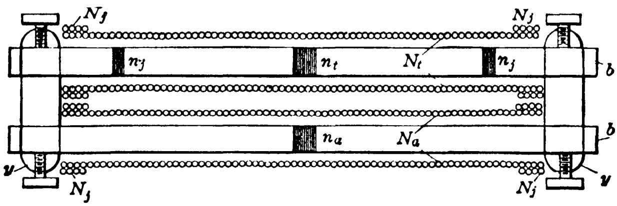


FIG. 141.—Burrows' double-yoke method.

extra current windings around the bars close to the yokes. In case only one bar for testing is available, a second bar of similar material is used. There are then three magnetizing circuits, and the currents through these are adjusted to such values as will give a uniform magnetic flux throughout the length of the test bar.

The condition of uniformity along the test bar is determined by the use of two test coils. One of these is wound around the middle of the bar, under the primary winding, and each half of the other coil is similarly placed about an inch from either end of the bar. When these coils are connected in series with the ballistic galvanometer and opposing each other, there will be no deflection if all the flux passing through the middle section of the bar continues in the bar to the ends. The current through the extra end windings is adjusted until the flux is thus uniform along the bar.

In case the auxiliary bar is of different material, the magnetizing current around it is adjusted until the flux through it is the same as that through the bar being tested. This equality is shown by a third test coil placed around the middle of the auxiliary bar and having the same number of turns and used in opposition to the main test coil around the bar being tested. When the flux has been made uniform throughout the circuit, its value is measured by the galvanometer deflection as in the ring method.

272. The Fahy Permeameter.—This is an arrangement of a yoke and the test bar in which all of the M.M.F. is supplied by a magnetizing coil around the yoke. This M.M.F. is used in part over the yoke portion of the magnetic circuit and in part over the bar to be tested. The distinctive feature of this method is the measurement of the M.M.F. acting on the bar by a magnetic potential solenoid (see Art. 255) arranged in parallel with the bar. The deflection of a ballistic galvanometer connected to this solenoid measures the M.M.F. when the latter is reversed.

Because of the uneven distribution of the magnetizing coils, the flux does not have the same value along the circuit. The average value of the flux through the bar is measured by the deflection of a ballistic galvanometer as in the method of reversals, using for the test coil a solenoid wound uniformly over the full length of the bar. This method has the same simplicity as the ring method in not requiring any adjustment or compensation of the magnetizing current, and it has the further advantage that different test bars can be used without winding new coils.

273. Single Bar and Yoke.—Inasmuch as there are no magnetic insulators by means of which the magnetic flux can be kept within prescribed paths, there is more or less leakage of flux from a magnetic circuit when the M.M.F. is applied over only a part of the circuit. The very great advantage of the ring method lies in the fact that the M.M.F. is uniformly distributed along the entire circuit, *i.e.*, it is distributed in the same way as the magnetic reluctance.

This principle has been used[1] in the single bar and yoke method for testing short bars of steel. Each end of the steel bar is clamped into the C-shaped yoke of iron that completes the circuit. A uniform winding of p turns per centimeter is carried by a brass spool surrounding the bar. Another winding

[1] A. W. Smith, *Phys. Rev.*, vol. 19, p. 424, 1922.
Smith, Campbell, and Fink, *Phys. Rev.*, vol. 23, p. 377, 1924.

covers the entire length of the yoke, with extra turns placed over each of the joints. The current through each of these windings can be adjusted independently so as to supply to each portion of the circuit a M.M.F. proportional to the reluctance of that part. The magnetic intensity is then given by

$$H = 1.257pI_a$$

and the corresponding value of the flux is measured by the galvanometer deflection when both currents are simultaneously reversed.

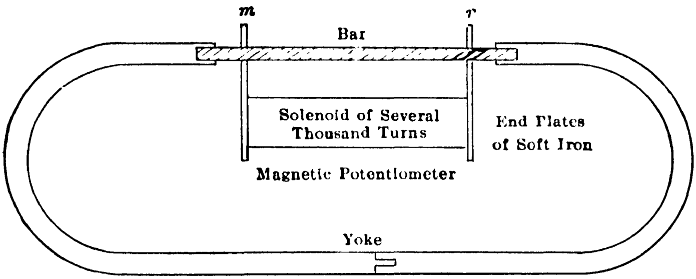


FIG. 142.—Bar and yoke, giving a closed magnetic circuit. The magnetic potentiometer is shown, connected to the bar.

The condition of uniform distribution of M.M.F. is determined by a magnetic potential solenoid in parallel with the greater portion of the bar, as shown in Fig. 142. (See also Art. 255.) This solenoid consists of many layers of fine wire wound on a spool having a non-magnetic core and with ends, *m*, *n*, of thick plates of soft iron. These end plates extend far enough on one side to be tightly clamped against the bar. The plates thus make contact with the bar at points near its ends, and there is the same difference of magnetic potential between them as there is along the bar between these points. This difference of potential is measured by the flux through the solenoid. The turns of wire in the magnetizing coil that are displaced by these end plates of the potential solenoid reaching through to the bar are wound as near as possible to their proper place to keep the turns per centimeter the same as for the rest of the bar. This winding is also continued over the rest of the bar up to the beginning of the yoke.

When the M.M.F. is applied bit by bit along the circuit, with just enough at each place to carry the flux across that portion, there is no resultant difference of potential between different points on the circuit. There will be, therefore, no tendency for flux to pass through the parallel path of the magnetic potential solenoid, and no deflection of the galvanometer upon reversal. By adjusting the yoke current until the potential solenoid shows no difference of magnetic potential over the bar, the flux is made uniform along the bar and the M.M.F. necessary to maintain this flux through each centimeter of the bar is just equal to the M.M.F. supplied by the ampere-turns around that centimeter length. This adjustment of the yoke current is made during the process of demagnetization (see Art. 288) of the bar and yoke from the larger current down to the new value to be used.

274. Fluxmeter-galvanometer.—In the measurements of magnetic flux it is necessary to read the deflection of a galvanometer, and with the usual ballistic galvanometer it requires considerable expertness to read the scale just at the end of the deflection. A galvanometer having a high critical damping (Art. 43) resistance (70,000 ohms) is very greatly overdamped on a low-resistance circuit, and the free motion of the coil may be extremely slow. When magnetic flux is cutting the closed galvanometer circuit, the deflection is quick and responsive, and the galvanometer coil turns so as to cut out of the galvanometer circuit as many flux-turns as are being cut into the circuit by the external flux through the test coil. When the flux ceases to change, the galvanometer coil stops moving and stands practically at rest. If the ending point of the deflection is near the natural resting point of the galvanometer, there is very little tendency for the scale reading to change after the flux has reached its steady value, and the observation is taken under stationary conditions.

275. Fluxmeters.—Portable instruments are made that will measure the change in magnetic flux by the deflection of a pointer moving over a scale graduated in maxwells. In appearance, one resembles a sensitive voltmeter, but in principle it is simply an overdamped ballistic galvanometer with the moving element suspended by an almost torsionless suspension so that the pointer stands at any place on the scale. When flux is measured, the pointer moves quickly from one position to another.

276. The Step-by-step Method.—This method is much like the method of reversals, the only difference being that the magnetizing current is not reversed to give the galvanometer reading. Instead of this, the current is constantly maintained in one direction. When the current is first turned on, the iron is magnetized to the corresponding amount, and if the galvanometer circuit is closed, there will be a proportional deflection. When the galvanometer has returned to its zero position the current is suddenly increased to its next value. The corresponding deflection of the galvanometer measures not the actual magnetism of the ring, but the *increase* over the former amount. In the same way the current is increased by steps until its maximum value is reached, while each of the corresponding deflections are carefully noted. The actual magnetism of the ring at any stage is measured by the *sum* of all the deflections up to that point. If this sum is denoted by Σd, the magnetic induction is

$$B = \frac{c\Sigma d}{nA}.$$

Since the ammeter in the primary circuit gives the total current at any point, the expression for the magnetic force will be the same as before,

$$H = \frac{4\pi NI}{10L}.$$

The *H*-*B* curve plotted from these values should be the same, very closely, as that obtained by the ring method using reversals of the current.

277. Hysteresis—Step-by-step Method.—In the step-by-step method just described, great care was observed never to have a larger current in the primary coil than that being used at the time, and the magnetization curve was obtained by always using increasing values of the current.

Suppose that after reaching the maximum, the current should be *decreased* by steps, and the corresponding deflection noted. Would the magnetization curve be retraced, or would a new curve be obtained? As the current is slowly removed, it would be found that the magnetization of the iron does not decrease to its former values, and when the current is reduced to zero there still remains a large amount of "residual" magnetism. This return curve can be traced perfectly well by the step-by-step method, and it is shown by the curve, *AD*, Fig. 143.

It will even require the application of a reversed magnetizing force, equal to *CG*, to reduce the magnetization to zero. This value of *H* is called the coercive force of the iron. It is large for hard iron, and steel, but small for soft iron and silicon-iron alloys. If the reversed field is increased to a value, *CF*, equal to *CE*, the iron will be magnetized as strongly as before, but in the opposite direction, and it will hold this magnetization just as persistently as the other. If *H* is reduced to zero and again increased to *CE*, the magnetization follows as shown by the curve *PJA*. This lagging of the values of *B* behind the corresponding changes in *H* is called hysteresis, from a Greek word meaning "to lag behind." The complete curve as thus drawn between *B* and *H* is called a hysteresis curve.

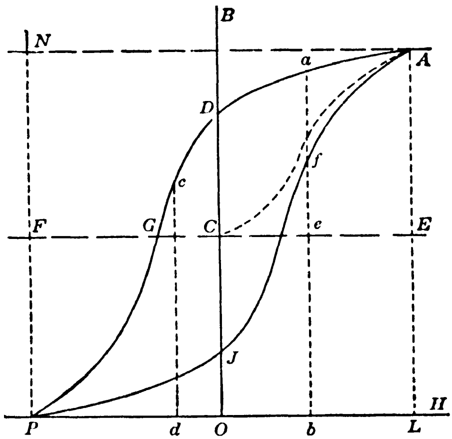


FIG. 143.—Hysteresis curve for tool steel.

For the determination of a hysteresis curve by this method, the set-up and manipulation would be the same as in the preceding experiment. And the results, plotted on a *B-H* diagram, will give a curve similar to the one shown in Fig. 143.

278. Hysteresis by Direct Deflection.—Referring to Fig. 143, suppose that the iron has been carried around the cycle, *ADPJA*, several times and left in the condition represented by the point *a*. If, then, it is carried from *a* to *P* by a single step, the deflection of the galvanometer will measure the corresponding change in the magnetic induction, represented by the ordinate, *ab*. By carrying the iron around the cycle to the point *a* again, this reading can be repeated as many times as desired, or, by stopping at some other point near *a*, the corresponding ordinate can be determined. Thus the curve, *ADP*, can be traced.

Let the set-up be made as shown in Fig. 144, where Z is the ring of iron to be studied. This iron is magnetized by a current from a few cells of the storage battery. The resistance, R, is set at the value which gives the maximum current that is desired. S is an ordinary double-throw, two-pole switch which is made into a reversing switch by the addition of two diagonal connections. As shown in the figure, one of these diagonal con-

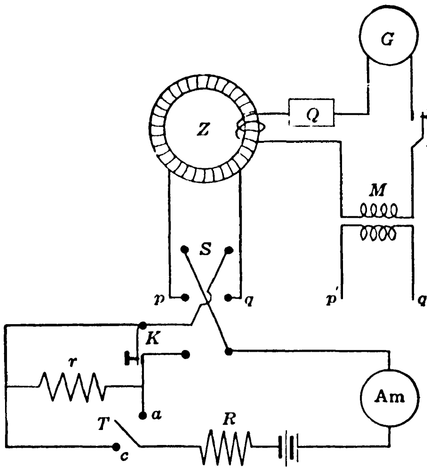


FIG. 144.—To determine the hysteresis curve for the ring, Z.

nections of S is formed by the adjustable rheostat, r, which can be short-circuited by closing K. The rheostat R can be connected to either end of the rheostat r by closing the switch, T, on either point a or point c. This introduces r into the battery circuit when S is up, or down, respectively.

With T Closed on a.—When the switch, S, is thrown down, thus connecting the primary of Z directly to the battery circuit, the magnetic state of the iron is represented by the point, P, Fig. 143. When it is thrown up, the current through Z is reversed, but the current will be smaller than before because the extra resistance, r, is now in the circuit, and the iron will be brought to some point as f on the curve JA. By closing K, the iron is brought to A, and when K is opened the iron comes to some point as a, depending upon the setting of r. The resistance in r must have been previously adjusted to the proper value in order that the iron may be brought to the desired point when K is opened. (In case r is adjusted after K is opened, the iron must be taken around the entire cycle several times before it is certain that the point a is reached.)

The galvanometer is joined in series with the secondary coil of the standard mutual inductance, M, a resistance box, Q, and the few turns of wire forming the secondary winding on the ring. It is often convenient to use a shunt on the galvanometer, adjusted to give critical damping. If S is now thrown down, changing the value of H from $+Ce$ to $-CF$, the galvanometer deflection will measure the corresponding change in B, indicated by ab, Fig. 143. In order to get back to the top of the curve for another determination, S is thrown up, bringing the iron to f on the lower curve. Then K is closed, thus carrying the iron on to A. Opening K brings the iron to the point on AD corresponding to the setting of r.

The cycle of operations is thus: (1) Throw S down, (2) throw S up, (3) close K, (4) open K. The galvanometer is read during the first operation.

With T Closed on c.—To locate points on the curve between D and P requires a slight change in the relative position of r. This is effected by changing the connection at T from a to c. Then when S is thrown up, the iron is always at A, Fig. 143, and when thrown down, the current will be reversed but less than its maximum value, because r is now in the circuit. By setting r at the proper value, the iron can be brought to any desired point between D and P. When K is closed, the iron is carried on to P and the corresponding deflection of the galvanometer gives the ordinate cd.

The cycle of operations is now: (1) Throw S up, (2) throw S down, (3) close K, (4) open K. The galvanometer is read at the time of the third operation.

In all of these measurements the iron is carried to P, and thus the final magnetizing force is the maximum that has been used. This is better than ending with a weak value of H, because under weak fields the iron will not come to its final magnetization as promptly as under stronger fields.

The remaining portion of the curve, PJA, can be determined in precisely the same way. With K and S set so that the iron is at P, let the connections from S to the ring, Z, be interchanged. This will carry the iron to A. By turning Fig. 143 bottom side up, it will be seen that the curve PJA corresponds exactly with the former curve ADP; and it can be traced by repeating the observations previously made, but the ordinates must now be plotted from the line AN. Or, since the curves are the same, the former set of ordinates can be used again for the second portion of the curve.

The actual plotting of the curve AJP downward from the line AN is often awkward, as usually AN does not coincide with one of the main divisions of the cross-section paper. An easier way is to lay a second piece of paper under the curve ADP and fasten both together by two pins through A and P. Several points along the path of the curve are pricked through both papers with a needle point. The lower paper is then placed over the curve, with the points A and P interchanged, and fastened on the two pins. The intermediate points are then pricked through onto the other sheet, thus outlining the curve PJA.

279. Determination of the Values of B.—In this experiment the change in the magnetic induction is not a reversal, but a change from ea, Fig. 143, in one direction to eb in the other. This gives a total change of ab, which may be denoted by the symbol ΔB. The corresponding change in flux-turns in the secondary circuit is ΔBAn, and this is measured by the galvanometer deflection, the relation being

$$\Delta BAn = cd. \tag{1}$$

The change in the magnetic induction is then

$$\Delta B = \frac{c}{An}d = Jd, \tag{2}$$

and this is plotted as the ordinate ab, Fig. 143, the values being laid off from the axis, OH. After the curve is drawn, the true axis, FE, is drawn through the middle of the figure.

280. The magnetic ballistic constant, c, may be determined by means of a known mutual inductance, as in the previous methods. From Art. 247,

$$MI' = cd',$$

where I' is the change in the current through the primary of the inductance M, measured in C.G.S. units. Since $c = JAn$, from (2), the deflection of the galvanometer for any desired value of J is

$$d' = \frac{MI'}{JAn}.$$

Usually it is convenient to choose $J = 100$, and then adjust the resistance in series with the galvanometer until the deflection is d' for a change of I' C.G.S. units of current in the primary of the calibration coil.[1]

The secondary of this mutual induction must remain a part of the galvanometer circuit, as shown at M, Fig. 144, since any change of the resistance of this circuit will change the value of the constant.

281. The values of the magnetic intensity, H, are determined by the current, which may be read by an ammeter. Values of H may then be computed as shown in Arts. 254 or 264. These values are plotted along the axis FCE, with C as the origin.

282. Energy Loss through Hysteresis.—Since it requires a reversed current to bring the magnetization of a ring or bar to zero, there is always a considerable loss of energy when a piece of iron is carried through a cycle of magnetic changes. This energy is represented by the area of the hysteresis loop, which is narrow for wrought iron, while for cast iron it is large and broad.

The amount of energy thus transformed into heat can be determined as follows: The work required to magnetize the iron is partly lost as heat in the magnetizing solenoid. This is the regular Ri^2 loss. Another part is used in maintaining the current against the induced E.M.F. due to the newly formed magnetic field. This induced E.M.F. is

$$e = -N\frac{d\phi}{dt} = -\frac{ANdB}{dt},$$

where A is the cross-section of the iron and N is the number of turns in the magnetizing solenoid.

Since e is negative, *i.e.*, opposed to the current, the positive work required to maintain the current is

$$W = \int eidt = \int\frac{ANidB}{dt}dt = \int\frac{AHl}{4\pi}dB = \int\frac{\text{Volume}}{4\pi}\int HdB,$$

since $Hl = 4\pi Ni$.

[1] Inasmuch as no current is used for the ring, Z, while the constant of the galvanometer is being determined, the same battery, ammeter, and rheostats can be used to furnish the current, I'. To make this change it is only necessary to exchange the connections, pq, for $p'q'$, Fig. 144.

Since the zero position of a sensitive D'Arsonval galvanometer depends upon the direction in which it was last deflected, all deflections should be in one direction only, and therefore the constant must be determined for deflections in this direction also. If greater accuracy is desired, the scale should be calibrated by determining the value of J for deflections throughout the range that will be used.

For one complete cycle, $\int HdB$ is the area of the hysteresis curve, measured in the units of H and B. This area, then, gives the energy expended per cycle in each 12.57 cc. of iron.

283. Permanent Magnets.—Referring to the hystersis curve for the bar and yoke, it is seen that the bar is still magnetized after the current is stopped. The magnetic flux through the bar completes its circuit through the heavy yoke, and outside the iron there is little to indicate that it is magnetized. When the bar is withdrawn from the yoke, it is in the same condition as before, but now the flux must complete its circuit through the air and there is plenty of evidence that the bar is magnetized. The bar itself is a permanent magnet, and the ends where the flux enters and leaves the iron are its poles.

284. Hysteresis Coefficient.—When the iron is carried through a hysteresis cycle between larger values of B, the hysteresis curve is higher and a little broader, but the area does not increase quite as fast as B^2. Experiment shows that over the usual range of the flux density in transformers and electrical apparatus, *i.e.*, from $B = 1{,}000$ to $B = 12{,}000$, the energy loss due to hysteresis is proportional to about the 1.6 power of B. For most practical purposes the empirical relation,

$$W = \eta B^{1.6} \qquad \text{ergs per cycle per cubic centimeter,}$$

gives the hysteresis loss over this range with sufficient accuracy, but above $B = 15{,}000$ the formula may be in error by 40 per cent.

The coefficient, η, is called the "coefficient of hysteresis" and its value indicates the magnetic qualities of the steel. The smaller this coefficient the smaller is the heat due to hysteresis and the more suitable is the material for use in transformers, etc.

The value of the exponent of B varies between 1.4 and 1.8 for different materials and for different ranges of the flux density. If a different value of the exponent is used in the equation for W above, it will require a different value for η; and since no value of the exponent will give the actual loss for any material over the entire range of B, the usual value of 1.6 is always taken in practice. The values of η are then comparable and serve as an index to the characteristics of the different materials. The range of values for this coefficient is shown in the table.

HYSTERESIS COEFFICIENTS FOR VARIOUS STEELS

Material	η	Material	η
Best silicon steel..........	0.0006	Soft machine steel.......	0.009
Silicon steel sheet.........	0.0010	Cast steel.	0.012
Soft iron wire.............	0.0020	Hard cast steel	0.025
Ordinary sheet iron.......	0.0040	Hard tungsten steel.......	0.058

285. Elementary Magnets.—Over thirty years ago Ewing explained the effects of magnetization by supposing the molecules of iron were little magnets that could be arranged in a definite direction by a sufficiently strong magnetic field. In a rotating field the direction of the field changes gradually throughout the cycle instead of decreasing to zero and then increasing in the opposite direction. In a strong field, where the elementary magnets are held in alignment with the field and all are turned gradually and orderly, the hysteresis loss per cycle for a rotating field is found to be only a small fraction of the hysteresis loss per cycle due to the reversal of the magnetic field. This indicates that the elementary magnet is free to be oriented in any direction.

Iron, while subjected to a magnetic field, has been examined[1] by x-rays and it seems probable that the molecules or atoms of iron are not turned around. The elementary magnet is more probably a single electron or a small group of electrons that can turn together within the structure of the atom. Ewing has a new model illustrating this.[2]

286. Effect of Temperature.—When iron is warmed, it is usually more easily magnetized by small values of H, but it does not reach as high values of B for larger values of H. Above a red heat the values of B are not much larger than for other substances.

In general, the hysteresis loss of energy per cycle diminishes when the temperature increases.

287. Eddy Current Loss.—There is another cause of energy loss in iron besides that due to magnetic hysteresis. This is the heating by the electric currents that are induced in the solid metal by the changing magnetic flux. This is reduced by using thin sheets of iron or steel, but it cannot be wholly eliminated.

[1] COMPTON and ROGNLEY, *Phys. Rev.*, vol. 16, p. 464, 1920.

[2] *Proc. Roy. Soc., Edinburgh.*, vol. 42, p. 97, 1922.

The energy loss due to the eddy currents is proportional to the square of the frequency of reversals, and for high frequencies may exceed the loss due to hysteresis.

The total loss can be expressed as

$$W = af + bf^2,$$

where f denotes the frequency. If W can be measured (by wattmeter or calorimeter) for two different frequencies, the constants, a and b, can be determined. The hysteresis loss, af, can then be compared with the loss, bf^2, due to the eddy currents.

288. Demagnetization of Iron.—When a ring of iron has been magnetized by a current and the current discontinued, the iron still retains a large part of its magnetism, as shown by the point D, Fig. 143. To wipe out the effect of this magnetization requires a process of demagnetization.

Simply removing the current has little effect. Reversing the current leaves the iron as strongly magnetized in the other direction. If the current is reduced slightly (not over 10 per cent), and the iron carried around the hysteresis cycle eight or ten times with this reduced current, the final hysteresis loop will be similar to the original loop, but a little smaller. The current can then be reduced a little more and the iron carried around a still smaller loop, and so on.

Many reversals are desirable but, having regard for the labor and time involved, ten are approximately sufficient, especially if they are slow[1] enough to allow the eddy currents to die away between each reversal. This requires smaller steps and slower reversals on the steep part of the B-H normal magnetization curve. The allowable speed of reversals can be determined as follows: Let t be the number of seconds between one reversal and the next. Then with the arrangement as in Fig. 137, let the current be reversed, and t seconds later close the galvanometer key, K'. If there is any sign of a deflection from the changing flux, t is too short, and the reversals should be made more slowly. Alternating currents are useless except for fine wires or very thin sheets of iron or steel.

289. Normal Magnetization Determination During Demagnetization—In this way the magnetization of the iron can be decreased a little at a time until it has been reduced as much as desired. At any stage in the process, after reversals with

[1] SMITH, *Demagnetization of iron*, *Phys. Rev.*, vol. 10, p. 284, 1917.

a current I, the iron is magnetized as strongly as though the demagnetization had been completed and then the current, I, applied and reversed a few times. It is, therefore, possible to take the readings for a curve of normal magnetization during the process of demagnetization, and with the saving of considerable time and effort.

290. Magnetic "Poles."—Inasmuch as magnetic "poles" appear only at surfaces where magnetic flux passes from one substance to another of different permeability, it should not be necessary to use the idea of poles at all. Of course there are many problems where the mathematical calculations are very greatly simplified by considering the action of poles, but every case of a piece of material turning or pointing in a certain direction is merely the effect of electronic currents tending to move across the magnetic flux in their neighborhood in the same way that the coil of a galvanometer turns because there is a force of

$$dF = BIdL$$

exerted on each element of the circuit, dL, tending to make it move at right angles both to the direction of B and to its own length.

The idea of a magnetic pole is simple and it has long been used. The fundamental units of magnetism were defined in terms of a "unit magnetic pole" and the idea may still be useful in discussing the relations of magnetic quantities. The unit pole stands for a bundle of 4π maxwells radiating from a point, and the expression "unit pole" is a short way of describing such a bundle of magnetic flux.

CHAPTER XIV

INDUCED ELECTROMOTIVE FORCE

291. Electromagnetic Induction.—When the current flowing through a circuit is started or stopped, or changed in any manner whatever, it is observed that other currents are set up in all of the other closed circuits which are near the first one. If some of these circuits are not closed, the tendency to produce a current is present just the same, but, being open circuits, no current results. In other words, an E.M.F. is induced in every conductor near a circuit in which the current is varying If a current in the second circuit is varying, there will be a corresponding E.M.F. induced in the first circuit. This action is called mutual induction.

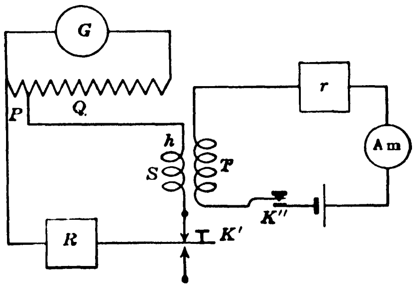


FIG. 145.—Mutual induction.

292. Laws of Mutual Induction.—A very satisfactory study of mutual induction can be made with a pair of coils and a sensitive ballistic galvanometer, connected as shown in Fig. 145. One of the coils is joined in series with a battery, an ammeter, a key, and a rheostat, r, for varying the current over a wide range. This is called the "primary circuit," and whichever coil is used in this circuit is called the "primary coil." The other coil, called the "secondary," is connected in series with the galvanometer and a resistance box. In order that the current may be varied without affecting the galvanometer when a reading is not desired, a switch or key, K', will be required in the secondary circuit.

It is also desirable to open the secondary circuit *before* making the reverse change in the primary current, in order to avoid deflecting the galvanometer to the other side of its resting point. There may be a shift of the zero, or resting, point of the galvanometer after it has been deflected in the opposite direction. In this case the first two or three readings in either direction should be discarded. In taking a series of readings it is best to keep the deflections in the same direction and never allow the galvanometer to swing across zero to the other side. If this precaution is taken the zero position should be quite constant.

293. The Effect of Varying the Primary Current.—The first part of this experiment is to study the relation between the change in the primary current and the resulting deflection of the galvanometer. The primary current is adjusted to such a value that closing K'', with K' closed, of course, will give a fairly large deflection. Note the effect of opening K''; also the effect of closing K'' first, and then closing K'.

The actual value of the current has no effect upon the deflection, as this is the same when the current is changed from zero to 1 ampere as when the change is from 4 to 5 amperes. If a primary current of 1 ampere is reversed, the change in the current is evidently 2 amperes, and the deflection will be twice as great as for either making or breaking the circuit.

Law I.—The quantity of electrons flowing through the secondary circuit depends directly upon the change produced in the primary current.

To investigate this relation, the rheostat, r, is adjusted to give the largest current that is to be used, and R and P are set so as to make the corresponding fling of the galvanometer as large as can be conveniently measured. Starting with this current, the deflection is read when the circuit is closed. The reverse kick of the galvanometer is avoided by opening K' as soon as the reading is obtained, and then opening K''. This reading should be repeated to make sure of consistent results. The current should be kept flowing as little as possible to avoid heating the coils—especially changing the resistance of the secondary coil by warming it. The current is now reduced about 10 per cent and another set of readings obtained, and so on until the current is reduced to zero.

If the scale readings are not proportional to the quantities of electrons discharged through the galvanometer, it may be necessary to correct them by the use of a calibration curve. The corrected readings are then plotted as ordinates against the corresponding changes in the primary current. This should be a straight line passing through the origin, an represented by the equation,

$$d = aI, \qquad (1)$$

where a is the slope of the curve. The value of a depends upon the number of turns of wire in the primary and secondary coils, and it also contains as one factor the reciprocal of the resistance of the secondary circuit.

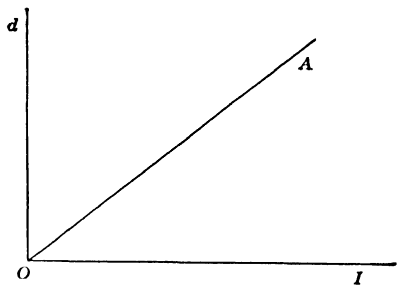


FIG. 146.—d shows the quantity of electrons through a constant-resistance secondary circuit when a primary current of I amperes is started or stopped.

294. The Effect of Varying the Secondary Resistance.—The second part of the experiment deals with the effect of changing the secondary circuit. Keeping the primary circuit constant, so that there will be the same change in the current each time the key is closed, a series of deflections are obtained by using different resistances in the secondary circuit. At each step two or three readings are taken in the same way as before. All of the resistance in this circuit should be measured under the conditions of the experiment, especially if the coil has been warmed appreciably by the current in the adjacent primary.

The galvanometer should not be joined directly in series with the rest of the circuit, for when the resistance of this circuit is varied, it will alter the damping factor of the galvanometer, and therefore its deflections will not be proportional to the quantities of electrons that are discharged through it. This trouble can be avoided by using a constant damping shunt consisting of two resistances, P and Q. P should be a few ohms, as many as necessary, and $P + Q$ sufficiently large to give critical damping to the galvanometer, that is, as large as possible and still keep the swing of the mirror aperiodic. A universal shunt of the proper resistance may be used for $P + Q$.

The total resistance of the secondary circuit is

$$R' = R + h + p,$$

where h denotes the resistance of the secondary coil, p the combined resistance of the galvanometer and shunt, and R is the additional resistance in the box, R.

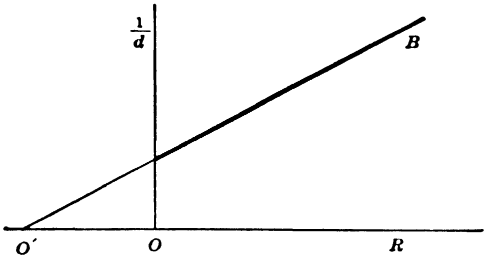


FIG. 147.—When a given primary current, I_0, is started or stopped, the quantity of electrons through the secondary circuit is inversely as the secondary resistance.

It will be found that the deflections become larger as R' is made smaller by decreasing the resistance in R; and plotting the corrected deflections against $\frac{1}{R'}$, will give a straight line passing through the origin, as in the first part. Inasmuch, however, as R is the only observed part of R', and $h + p$ may be unknown, it is more satisfactory to plot $\frac{1}{d}$ as ordinates against the corresponding values of R. This will give a straight line as before, but not through the origin. Extending this back to the axis of R, it will intersect it at a point $h + p$ ohms behind the origin of R; hence this point is the origin of R'.

The relation between d and R' is given, then, by the expression,

$$\frac{1}{d} = bR', \qquad \text{or} \qquad d = \frac{1}{bR'} \tag{2}$$

where R' is the total resistance in the secondary circuit and b is the slope of the line, $O'B$. The value of b depends upon the number of turns in each of the coils, and it also contains as one factor the constant value of the current which was made and broken in the primary circuit when the deflections were observed.

Law II.—The quantity of electrons flowing through the secondary circuit varies inversely as the resistance of the circuit.

From these two relations it is seen that the complete expression for the relation between the deflection, the change in the primary current, and the resistance of the secondary circuit is

$$d = f\frac{I}{R'}, \tag{3}$$

where f is the proportionality factor. Evidently, $\frac{f}{R'}$ is the a of Eq. (1), if R' is the constant resistance of the secondary circuit when curve A was determined. Likewise, fI is the $\frac{1}{b}$ of Eq. (2), if I denotes the change that was made in the primary current when curve B was determined.

The ballistic galvanometer indicates the passage of a quantity of electricity through it, and from the nature of the circuit to which it is connected it is seen that this quantity does not come from the discharge of a condenser, but represents the passage of a transient current. And, furthermore, if there is a current in the secondary circuit, there must be an E.M.F. causing this current.

In Art. 238 it was shown that an E.M.F. is induced in a wire which cuts across a magnetic flux. In the present case, as well as in the experiments of the preceding chapter, the wire is stationary while the flux cuts across it. When this wire is wound into a coil of n turns, the flux cuts the wire in each turn, thus inducing an E.M.F. in each turn.

The total E.M.F. induced in the coil is, then,

$$e = n\frac{d\phi}{dt}, \tag{4}$$

where ϕ denotes the flux linked with the n turns of the coil. This is the E.M.F. causing the current through the galvanometer and the secondary circuit. In case the same flux does not pass through each turn of the coil, the two factors, ϕ and n, of the flux-turns cannot be separated, but this quantity, ϕn, must then be considered as a summation extending to all the turns of the coil.

295. Meaning of Mutual Inductance.—The two coils, T and S, Fig. 145, are similar to the primary and secondary windings on the iron ring Z, Fig. 144, except that now the magnetic circuit is wholly of air. The M.M.F. due to the current in T is $4\pi Ni'$, and this causes a flux,

$$\Phi = \frac{4\pi Ni'}{Reluctance},$$

through this coil and the surrounding air. The portion, ϕ, of this flux passing through the n turns of coil S depends upon the relative position of the two coils. Let p denote this fraction. Then $\phi = p\Phi$, and the E.M.F. induced in this coil by the change in the primary current is

$$e = \frac{d(p\Phi n)}{dt} = pn\frac{d}{dt}\left(\frac{4\pi Ni'}{Rel.}\right) = \frac{4\pi pNn}{Reluctance}\frac{di'}{dt}. \qquad (5)$$

Writing a single symbol for these various constants,

$$e = M\frac{di'}{dt}, \qquad (6)$$

where the coefficient, M, is called the coefficient of mutual induction, or simply the *mutual inductance* of the pair of coils. This equation indicates the units in which the mutual inductance, M, should be measured.

Definition.—The C.G.S. *unit of mutual inductance is the inductance between two circuits so arranged that an* E.M.F. *of one* C.G.S. *unit is produced in one of them when the inducing current in the other changes at the rate of one* C.G.S. *unit of current per second.*

296. Relation between the C.G.S. Unit and the Henry.—When e is expressed in volts and i in amperes, Eq. (6) is still true if M is expressed in terms of the proper unit. This unit is called a henry[1] and is defined as follows.

Definition.—One henry of mutual inductance is the inductance between two circuits when an E.M.F. *of one volt is produced in one of them when the inducing current in the other changes at the rate of one ampere per second.*

Since a volt is 10^8 C.G.S. units and an ampere is 10^{-1} C.G.S. units, a henry becomes equal to 10^9 C.G.S. units of inductance.

297. Quantity Passing through the Secondary Circuit.—When the current is started or stopped in the primary coil of a mutual inductance, the E.M.F. in the secondary rises quickly to a maximum value and then subsides to zero. Even the maximum value is not constant for the same change in the primary current, but depends upon how quickly the current is made to change. Likewise, the secondary current is not definite either in amount or duration. The one thing that is definite is the quantity of electrons, Q, that passes along the secondary circuit when a given change, I, is made in the primary current. This is the same whether the current is changed quickly or slowly.

[1] Joseph Henry, American physicist, 1797–1878.

Let e denote the instantaneous value of the E.M.F. induced in the secondary coil at the time, t. Then, as shown above, the E.M.F. induced in the secondary coil is

$$e = M\frac{di'}{dt}.$$

Expressed in terms of the current, i, that flows in the secondary circuit, this is

$$e = Ri + L\frac{di}{dt}.$$

Equating these two values of e gives

$$M\frac{di'}{dt} = Ri + L\frac{di}{dt}.$$

Integrating this equation with respect to the time,

$$\int_t^{t'} M\frac{di'}{dt} = \int_t^{t'} Ridt + \int_t^{t'} Ldi = R\int_{q=0}^{q=Q} dq + L\int_{i=0}^{i=0} di = RQ$$

where the integration is extended from the time, t, before the primary current begins to flow, till the time, t', when the primary current has reached its steady value, I, and the secondary current has died away to zero.

Therefore,

$$MI = RQ, \tag{7}$$

or, solving for the value of Q,

$$Q = \frac{MI}{R} \tag{8}$$

in C.G.S. units.

If I is expressed in amperes, R in ohms, and M in henrys, then Q will be given in coulombs.

298. Secondary Definition of Mutual Inductance.—The induced E.M.F. has been expressed above in two different ways, *viz.*,

$$-e = n\frac{d\phi}{dt} \qquad \text{and} \qquad -e = M\frac{di}{dt}.$$

Equating these two values and integrating gives

$$\phi n = MI,$$

where ϕn denotes the flux-turns in the secondary due to a steady current, I, in the primary.

Solving for M gives

$$M = \frac{\phi n}{I}.$$

This equation leads to a secondary definition of M that is often useful, and it gives another view of mutual inductance.

The mutual inductance of a pair of coils is measured by the flux-turns through the secondary per unit of current in the primary. This gives M in C.G.S. units.

299. Meaning of Self-inductance.—It is but a step from considering the action of a current on an adjacent circuit to the case of action upon the same circuit. If the second circuit above were included as a part of the primary circuit, then the induced E.M.F. would be in the same circuit as the inducing current, and in a direction opposed to it. Furthermore, this same kind of action would appear in every turn of the primary coil, the current in each turn inducing an E.M.F. in each of the other turns. The total E.M.F. induced in the circuit by the inducing current is

$$e = -L\frac{di}{dt},$$

where the $-$ sign means the same as before and L is a constant depending upon the number of turns and the dimensions of the circuit. It is called the "*self-inductance*" of the circuit and is measured in henrys.

Definition.—One henry of self-inductance is the inductance in a circuit when an E.M.F. *of one volt is produced by the current changing at the rate of one ampere per second.*

There is also the C.G.S. unit of self-inductance. The henry is 10^9 times the C.G.S. unit.

300. Secondary Definition of Self-inductance.—The induced E.M.F. of self-induction is expressed by two different relations, *viz.*,

$$e = -n\frac{d\phi}{dt} \qquad \text{and} \qquad e = -L\frac{di}{dt}. \tag{1}$$

Equating and integrating these expressions gives

$$LI = n\phi, \tag{2}$$

from which

$$L = \frac{n\phi}{I}. \tag{3}$$

This shows that the self-inductance of a coil is measured by the number of flux-turns through it when carrying a current of one C.G.S. unit.

For one henry there would be a billion maxwell turns when an ampere flows in the wire.

301. Effect of Iron in the Magnetic Circuit.—These definitions of inductance presuppose that μ is constant, and that ϕ is proportional to the primary current. This is the case in air. When iron is near the coils, the inductance is no longer constant, but is different for each value of the current. From Eqs. (1) above,

$$L\frac{di}{dt} = n\frac{d\phi}{dt} = n\frac{d\phi}{di}\frac{di}{dt}$$

or

$$L = n\frac{d\phi}{di},$$

which might be called the instantaneous value of the inductance. When ϕ is proportional to i, this reduces to the second definition given above, but this is not the case with iron in the magnetic circuit. By referring to the magnetization curves for iron it is seen that $\frac{\phi}{I}$ and $\frac{d\phi}{dI}$ are not the same. In this case the value of L varies from point to point as the current changes, and the two forms of definitions give different values for the inductance for the same value of the current, which, moreover, are not the same for a decreasing current as for an increasing current. The choice of the proper definition for any particular problem is decided by a consideration of the attending circumstances.[1]

302. Starting and Stopping a Current.—When a constant E.M.F., E, is applied to an inductive circuit, Fig. 148, the current starts at the value zero and increases to its final steady value. While the current is thus increasing, there is induced in the same wire an E.M.F., the value of which is

$$e = -L\frac{di}{dt}. \qquad (1)$$

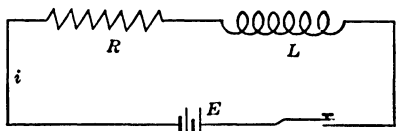


Fig. 148.—A circuit containing resistance and inductance.

This E.M.F. is in the direction that will oppose the change in the current. For an increasing current, it is directed in opposition to the E.M.F. that is causing the current to flow. When the current is decreasing, this induced E.M.F. helps to maintain the current. It is then added to E, but it is still written as $-L\frac{di}{dt}$ because di is now negative and this makes the whole expression positive.

[1] Starling, "*Electricity and Magnetism*," p. 302.

Let R denote the resistance and L the self-inductance of a circuit to which E is applied. The sum of all the potential differences in the circuit is

$$E - Ri - L\frac{di}{dt} = 0. \tag{2}$$

This equation is often written in the form

$$E = Ri + L\frac{di}{dt}, \tag{3}$$

which shows that while a part, Ri, of the applied E.M.F. is effective in maintaining the current, i, another part, $+L\frac{di}{dt}$, is required to make the current increase. This equation is very important in all discussions of varying currents.

303. Value of the Current.—Solving Eq. (2) for the value of the current at any instant after the steady E.M.F. has been applied gives

$$i = \frac{E - L\frac{di}{dt}}{R}, \tag{4}$$

which is fully analogous to Ohm's law for a steady current.

At the start, when the current is changing most rapidly, the second term in the numerator is nearly equal to E. As the current approaches its final value, it changes more slowly, the induced E.M.F. becomes less and less, and, finally, the steady value of the current is $i = \frac{E}{R}$.

304. Dying Away of a Current.—Suppose that a steady current has been flowing in a circuit when, suddenly, the battery, or other source of E.M.F., is removed without breaking the circuit or in any way changing its resistance or inductance. Equation (3) now becomes

$$0 = Ri + L\frac{di}{dt} \tag{5}$$

and the current thus left to itself dies away to zero, as is shown by the following relations:

Rewriting (5) to separate the variables,

$$\frac{di}{i} = -\frac{R}{L}dt. \tag{6}$$

The integral of this is

$$\log i = -\frac{Rt}{L} + c \tag{7}$$

or, in the exponential form,

$$i = C\epsilon^{-\frac{Rt}{L}}. \tag{8}$$

The value of the constant of integration, C, is given by the fact that at the start, when $t = 0$, the current had the value I. Putting these values in (8) gives $I = C$. Therefore (8) becomes

$$i = I\epsilon^{-\frac{Rt}{L}} \tag{9}$$

From this equation it appears that it is the self-inductance that prevents the current from falling to zero immediately, and the greater the self-inductance the more slowly will the current die away. If it is desired to state how rapidly the current decreases, it is necessary to state how long it takes for the current to fall to half value—or to some other definite fraction of its original amount—since the value of i from (9) will reach zero only after an infinite time.

When t has the value $\frac{L}{R}$, the current equals $\frac{I}{\epsilon}$, or $\frac{I}{2.72}$, and this interval in which the current falls to 0.368 of its original value is called the *time constant* of the circuit.

305. Beginning of a Current.—When a circuit containing an E.M.F. is closed, the current rises from zero to its final value at a rate depending upon the self-inductance in the circuit. This rate of increase can be found from the equation

$$E = Ri + L\frac{di}{dt}.$$

Separating the variables gives this in the form

$$\frac{-di}{\frac{E}{R} - i} = -\frac{R}{L}dt,$$

the integral of which is

$$\log\left(\frac{E}{R} - i\right) = -\frac{Rt}{L} + c,$$

or,

$$i = \frac{E}{R} - C\epsilon^{-\frac{Rt}{L}}.$$

The constant of integration is determined by the condition that when $t = 0$, $i = 0$. Hence, $C = \frac{E}{R} = I$, where I is the final value of the current.

Therefore,

$$i = I - I\epsilon^{-\frac{Rt}{L}}.$$

Here also it is seen that it is the self-inductance that keeps the current from rising suddenly to its full value as soon as the circuit is closed. The greater the self-inductance in comparison with the resistance, the more slowly will the current rise, but it never quite reaches its maximum value. Therefore in comparing different circuits it is necessary to compare the periods taken for the currents to rise to half value—or to some other definite fraction of the final steady values. When $t = \frac{L}{R}$ the current lacks $\frac{I}{\epsilon}$ of its final value, and this period in which the current rises to 0.632 of its maximum value is called the *time constant* of the circuit.

Problems

1. Given a circuit in which the resistance is 10 ohms, and the self-inductance is 0.01 henry, draw a curve showing the rise of the current for the first 0.004 of a second after applying a steady E.M.F. of 100 volts.
2. The above circuit is placed in parallel with a non-inductive resistance of 10 ohms, when the current in the main line is 20 amperes. Draw a curve showing how the current dies away in the parallel circuits when the main line switch is suddenly opened.

The logarithms which appear in the equations through the process of integration are, of course, not the common logarithms with the base 10, but are the natural logarithms with the base $\epsilon = 2.718+$. The following table will be useful in finding the time at which the current will have a given fraction of its maximum value.

Example.—Suppose it is desired to find the time at which the current in Problem 1 has risen to three-tenths of its maximum value. In this case, then, $\epsilon^{-\frac{Rt}{L}} = 0.3$, and this value is found in the first column of the table. In the last column is given the value of the exponent, $-\frac{Rt}{L} = -1.204$, which is the natural logarithm of 0.3. Therefore,

$$t = 1.204\frac{L}{R} = 0.001204 \textit{ second.}$$

This procedure is simpler than the computation to find the value of the current at an arbitrarily chosen time.

SHORT TABLE OF NATURAL LOGARITHMS

Number $\epsilon^{-\frac{Rt}{L}}$	Natural logarithms	
	Tabular value	Numerical value $-\frac{Rt}{L}$
0.90	9.895-10	-0.105
0.80	9.777-10	-0.223
0.70	9.643-10	-0.357
0.60	9.489-10	-0.511
0.50	9.307-10	-0.693
0.40	9.084-10	-0.916
0.30	8.796-10	-1.204
0.20	8.391-10	-1.609
0.10	7.697-10	-2.303
0.07	7.341-10	-2.659
0.04	6.781-10	-3.219
0.01	5.395-10	-4.605

CHAPTER XV

MEASUREMENT OF SELF- AND MUTUAL INDUCTANCE

306. Effects of Variable Currents.—The inductance of a circuit is not in evidence when steady currents are flowing. It is only when the current is started or stopped, or made to vary in some other way, that the effect of inductance is made manifest and it is through the measurement of such effects that the value of an inductance can be determined. In Chap. XVII several methods are given for measuring inductance when the current varies as a pure sine wave and the conditions for a balance are determined by graphical methods. The conditions for a balance can also be found by analytic methods, and for currents that vary in any manner whatsoever. Even though similar bridge arrangements may be considered in both methods, it will be simpler to study the analytic methods together in this chapter, and bring the graphical methods together in Chap. XVII.

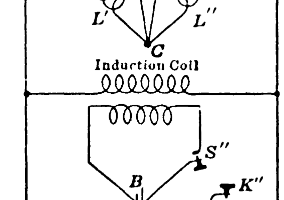


FIG. 149.—Comparison of two self inductances.

307. Comparison of Two Self Inductances.—The self-inductance of a coil can be measured by the bridge method, using variable currents. A Wheatstone bridge arrangement is set up as shown in Fig. 149, with the coil to be measured as one arm of the bridge and a variable standard of self-inductance in the corresponding arm. The other two arms consist of two noninductive resistance boxes which can be adjusted to balance the bridge. A galvanometer, G, and a battery, B, are connected in the usual way with the double key, K', K''.

In order to use variable currents, the secondary of an induction coil, or other source of alternating current, is connected to the bridge at the same points as the battery, without disturbing the latter connections. The primary of this induction coil is connected to the battery through the switch, S''. The same battery can serve in both places, since it will not be needed in one circuit while the other is being used. As an ordinary galvanometer is not deflected by an alternating current, a telephone receiver is connected between B and C by the switch, S'. The two switches, S' and S'', may well be the two blades of a double-pole switch which will close both circuits by a single motion.

The induction coil should be enclosed in a well-padded box to reduce the noise as much as possible. By listening at the telephone receiver while the inductance of the standard is varied, the position for a minimum sound is readily determined. Before recording the readings, the direct-current balance should again be tried to make sure that the resistances have not changed while the second balance was being made. The contacts in the variable standard are not always constant, and perhaps other changes may occur. A Wheatstone bridge can be balanced for varying currents only when *both* the resistances and the inductances are adjusted to the proper values.

The relation between the inductances and resistances which will give this double balance may be found as follows: Writing the instantaneous values of the potential differences for the circuit $ABCA$ at any instant when the currents are i and i' gives

$$Pi - R'i' - L'\frac{di'}{dt} = 0. \tag{1}$$

Similarly, for $BDCB$

$$Qi - R''i' - L''\frac{di'}{dt} = 0, \tag{2}$$

since there is no current from B to C, as is indicated by no sound in the telephone.

Transposing and dividing (1) by (2)

$$\frac{L'}{L''} = \frac{Pi - R'i'}{Qi - R''i'} = \frac{P\left(i - \frac{R'i'}{P}\right)}{Q\left(i - \frac{R''i'}{Q}\right)} = \frac{P}{Q}$$

since the resistances are adjusted for a direct-current balance and therefore $\frac{R'}{P} = \frac{R''}{Q}$.

Thus for a balance with varying currents, the triple equation

$$\frac{L'}{L''} = \frac{R'}{R''} = \frac{P}{Q}$$

must be satisfied.

This means that each of the inductive arms of the bridge must be *equally inductive*, or, in other words, the henrys per ohm must be the same for each arm in order to give a balance with variable currents. All of this is on the supposition that L' is a constant, *i.e.*, contains no iron or other magnetic substance. It is seen that the balance is independent of the particular way in which the currents are made to vary.

In case no balance can be obtained within the range of the standard, some non-inductive resistance may be added to the more inductive of the two inductive arms. And by varying this resistance several readings may be obtained on different parts of the scale.

308. Grounding the Telephone.—Each time the current changes in the inductance bridge, Fig. 149, there is a corresponding change in the potentials of the various parts, and if these parts have any capacitance there is also a change in their electrostatic charges. The currents due to these varying charges are usually too small to produce a noticeable effect in the arms of the bridge, but if they pass through the telephone it may cause an audible hum and make it impossible to secure silence when the bridge is balanced. It is desirable, therefore, to have the telephone receivers remain at the potential of the earth, and this can be accomplished without actually connecting the telephone to the ground.

Let a potential divider be connected from A to D, Fig. 149, in parallel with the bridge and let the point b divide it in two parts, p and q, having the ratio $\frac{P}{Q}$. Then if b is grounded, B and the telephone will also be at the potential of the earth. By connecting the telephone temporarily between b and B, the potential divider can be set so that $\frac{p}{q} = \frac{P}{Q}$. If the arms, Q and R', of the bridge were interchanged it would require an inductance in one part of the potential divider to make it similar to the path ABD. This grounding device is shown in Fig. 184 for a capacitance bridge.

309. Potential Changes in the Bridge.—The potential changes of the points A, B, C, and D on the bridge of Fig. 150 immediately after the closing or the opening of the battery key, K'', can be studied in the same way as was done in the case of the capacitance bridge of Fig. 122. Such a study helps to give a clear idea of what is going on in the bridge when the currents are changing, and it is well worth while to think through the various changes in potential when one point, A, B, or D, is grounded.

310. An Inductance Bridge.—An inductance bridge box is made by Leeds and Northrup, and in general appearance

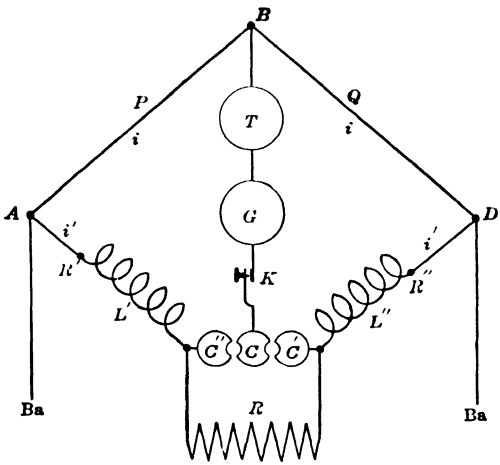


FIG. 150.—Diagram of the inductance bridge

resembles the Wheatstone bridge box made by the same firm. The ratio arms are similar in the two boxes and each has a rheostat arm, but as the actual resistance in this arm is immaterial in the inductance bridge, the fine adjustment is made by two small circular rheostats, each having many steps. The ratio arms correspond to P and Q of Fig. 149, while the rheostat arm of variable resistance is connected between the two inductance coils as shown in Fig. 150. By means of a plug key, the galvanometer connection, C, may be joined to either C' or C'', thus putting the resistance, R, in series with either R' or R'' as desired. The resistance balance is easily made by varying R, after which the inductance balance is obtained by varying L'. The value of the unknown inductance is, then,

$$L'' = \frac{Q}{P}L'$$

where $\frac{Q}{P}$ is an integer power of 10.

311. Comparison of a Mutual Inductance with a Self Inductance.—The preceding method gives also a very satisfactory way of measuring the mutual inductance between two coils. Call the self-inductances of the coils L_m and L_n, and their mutual inductance M. The E.M.F. induced in these coils when connected in series and carrying a current i, is

$$L_m\frac{di}{dt} + M\frac{di}{dt} + L_n\frac{di}{dt} + M\frac{di}{dt} = (L_m + L_n + 2M)\frac{di}{dt}.$$

But the coefficient of $\frac{di}{dt}$ is, by definition, the inductance of the circuit, and if the two coils thus joined together in series were made one arm of the bridge shown in Fig. 149, the inductance measured by that method would be

$$L_s = L_m + L_n + 2M. \qquad (1)$$

If one of the coils were reversed, the E.M.F. induced in each coil by the mutual inductance of the other would be reversed also, and therefore the inductance measured by the bridge in this case would be

$$L_r = L_m + L_n - 2M. \qquad (2)$$

Subtracting (2) from (1) gives

$$L_s - L_r = 4M.$$

Hence to determine the mutual inductance of a pair of coils it is only necessary to measure the self-inductance of both together when joined in series—first with the coils direct, and again with one of the coils reversed. One-fourth of the difference between the two inductances gives the value of the mutual inductance.

312. Measurement of a Self Inductance by Means of a Capacitance. Maxwell's Method.—In this method the self-inductance to be measured is placed in one arm of a Wheatstone bridge, the other arms of which should be as free from inductance as possible. By closing the keys in the usual order, the bridge can be balanced for steady currents giving the relation

$$PS = QR. \tag{1}$$

But if the galvanometer key is closed first, there will be a large deflection upon closing the battery key even though the bridge is balanced for steady currents. This is because the current through the inductive branch, *ABD*, Fig. 151, does not reach its full value as quickly as the current through the non-inductive branch, *AED*, and, therefore, just at the start the fall of potential over *S* is not as great as that over *R*. This effect can be balanced by a suitable inductance in *P*, as was done in Art. 307 or by placing a condenser in parallel with *R*, as shown in Fig. 151. With the condenser so placed, a large part of the current through *P*, just at the start, will flow into the condenser and therefore the fall of potential over *R* will not be as large as it would be without the condenser. If the capacitance of the condenser is of the proper amount, this latter effect will just equal the effect of the coil in reducing the current through *S*, and therefore the potentials of *B* and *E* will rise together. In this case there will be no deflection of the galvanometer, even though its key is closed first, and a telephone or other current detector will indicate no current through the branch, *BE*.

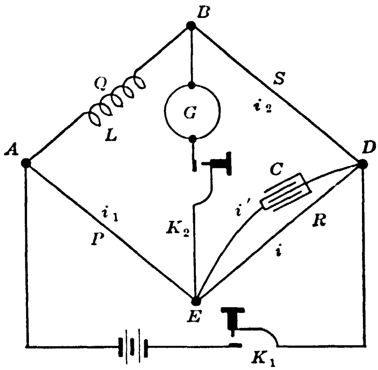


FIG. 151.—Comparison of capacitance and self inductance.

When such a balance has been obtained we can write for the circuit *BAEB* at any one instant

$$Qi_2 + L\frac{di_2}{dt} - Pi_1 = 0 \tag{2}$$

and for DEBD

$$Ri - Si_2 = 0 \tag{3}$$

where

$$i_1 = i + i' = i + \frac{dq}{dt} = i + \frac{d}{dt}(CRi) = i + CR\frac{di}{dt}, \tag{4}$$

since both the current, i, through *R*, and i' flowing into the condenser, are added together to make the current, i_1, through *P*.

Substituting in (2) the value of i_2 from (3) and i_1 from (4) gives

$$Q\frac{Ri}{S} + L\frac{R}{S}\frac{di}{dt} - P\left(i + CR\frac{di}{dt}\right) = 0$$

or

$$(PS - QR)i + PCRS\frac{di}{dt} = LR\frac{di}{dt}. \tag{5}$$

But because of the direct-current balance (1) this reduces to

$$L = PSC, \tag{6}$$

and the bridge is balanced when S has the value required by (6) in addition to the requirement of (1). Since $PS = QR$, $\frac{L}{Q} = RC$. But $\frac{L}{Q}$ is the "time constant" of the branch AB, and RC is the "time constant of the branch ED. The bridge is balanced for varying currents when these two time constants are equal.

In practice, then, the bridge is first balanced for steady currents. Then, with K_2 closed, the balance is tried by closing and opening the battery key, K_1. If the value of C is not just right to give a balance with varying currents, and if it cannot be varied, its effect can be varied by changing R. This will necessitate a corresponding change in S and another trial with varying currents until this double balance has been obtained. A telephone can be used in place of the galvanometer for the second balance if alternating currents are used. Or if the apparatus is at hand, it is convenient to use the alternating-current generator with a commutator on the same shaft for rectifying the galvanometer current. The 140 volts generated by the machine is entirely too much to use directly on good resistance boxes. By means of a transformer, the voltage can be reduced to 12 volts and this can be used instead of the battery shown in Fig. 151.

PRACTICAL APPLICATIONS OF MAXWELL'S METHOD

313. Case A. Self Inductance Varied. *Calibration of a Variable Standard of Self-inductance.*—The double balance required in the previous experiment makes the method long and tedious. But if Q is a variable self-inductance, some value of L can be found for each steady current balance. The method is therefore very useful in the calibration of a variable self-inductance.

The operation of the calibration is simple. A direct-current balance is obtained with values of P, S, and C which will give a product equal to the value of L it is desired to calibrate. Then, turning to alternating currents, the inductance is varied until a balance is obtained. If L' is the reading at this point, the correction to be applied is

$$c = PSC - L'.$$

A calibration curve should be plotted between L' as abscissæ and the corresponding values of c as ordinates.

314. Case B. Resistance Varied. *Inductance of a Single Coil.*—In case the self-inductance to be measured is not variable, but is a coil having a fixed and constant value for L, the balance can readily be obtained as follows: Let R and S, Fig. 151, be two similar decade resistance boxes so that the resistances of these two arms of the bridge can easily be kept equal to each other. Start with some convenient value, such as $R = S =$ 1,000 ohms.

In order to obtain the direct-current balance, it will now be necessary to make $P = Q$. Usually Q is not very large, and if P is a resistance box the balance can be obtained only to the nearest ohm. The final balance can be made by adjusting a low-resistance rheostat in series with the coil and forming a part of the arm, Q. The amount of resistance used from this rheostat, as well as the resistance of the coil itself, need not be known, but it is essential that the rheostat be non-inductive.

The battery and galvanometer are now replaced by an alternating E.M.F. and a current detector. By varying R and S together, thus keeping them equal, the alternating-current balance is readily found and the value of S giving this balance is the one required in Eq. (6). If R and S are two similar decade boxes, as specified above, they can be kept equal and the balance found as easily as though a single resistance was varied. If several independent determinations of L are required, a small resistance may be added to Q each time and the measurements repeated.

If it is desired to measure L when the coil is carrying a certain constant current, P and R can be set equal to each other and the direct-current balance obtained by adjusting S and Q. By maintaining a constant alternating-current voltage at AD, the current through Q will remain constant while P and R are being adjusted to give the alternating-current balance. This arrangement is especially desirable when the coil has an iron core, or whenever the value of L depends upon the current through the coil.

315. Case C. Capacitance Varied.—If the capacitance of the condenser C can be changed by several small steps, it is often convenient to obtain the alternating-current balance by varying the amount of capacitance in parallel with the resistance, R. Changing the capacitance of C will have no effect on the resistances of the bridge, and therefore if once in balance it will not be disturbed by making the alternating-current balance.

If the steps are too large to give an exact balance with varying currents, the two values nearest the balance can be tried and the deflections noted. Then the value of the capacitance which would give the exact balance can be determined by interpolation. This value of C is to be used in Eq. (6) to compute the value of L.

316. Case D. Effect of Capacitance Varied.—Sometimes it is necessary to measure the self-inductance of a coil with a con-

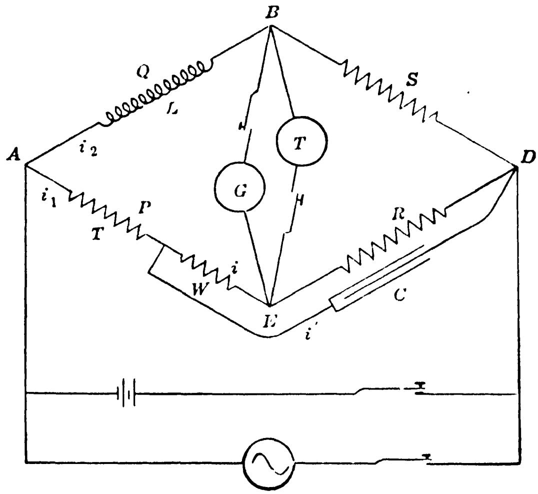


FIG. 152.—Arrangement for varying the effect of a condenser of fixed capacitance.

denser of fixed capacitance. In this case the effect of the capacitance can be varied by putting the condenser in parallel with R plus a part of P. Let P consist of two resistance boxes, T and W, so arranged that, while the resistance in either may be varied, yet the total resistance in both shall always be P. Let the condenser be placed in parallel with R and W as shown in Fig. 152. When the balance has been obtained and no current flows through the telephone, there will be the same current through R

and W, and the condenser will be charged to the potential difference $(R + W)i$.

Writing the instantaneous potential differences for the circuit, EAB, gives

$$Wi + Ti_1 - Qi_2 - L\frac{di_2}{dt} = 0, \tag{1}$$

where

$$i_1 = i + i' = i + \frac{dq}{dt} = i + C(R + W)\frac{di}{dt}. \tag{2}$$

Similarly for the circuit EDB,

$$Si_2 = Ri. \tag{3}$$

Rewriting (1) in terms of a single current, i, gives

$$Wi + Ti + TC(R + W)\frac{di}{dt} - Q\frac{R}{S}i - L\frac{R}{S}\frac{di}{dt} = 0.$$

At first sight this differential equation looks complicated, but a particular solution is readily obtained. In reality there are only two terms in the equation, one containing i and the other containing $\frac{di}{dt}$. If the coefficients of these two terms were zero, the sum of the two terms would always be zero and the equation would be satisfied whatever the values of i and $\frac{di}{dt}$ might be. Setting these coefficients equal to zero gives

$$(W + T) - Q\frac{R}{S} = 0$$

and

$$TC(R + W) - L\frac{R}{S} = 0$$

as the conditions that must be fulfilled to give a balance. The first of these is the usual Wheatstone bridge balance for resistances, and the second condition is met when

$$L = CTS\frac{R + W}{R}.$$

For the case when $W = 0$, this reduces to the formula in Maxwell's method, $L = PSC$.

An inspection of Fig. 152 shows that only the current through T is modified by the condenser, and as T becomes smaller, the effect of the condenser in balancing the inductance becomes less. When this method is used, the first adjustment should be such that the inductance of the coil is overbalanced by the condenser, that is, PSC should be larger than L. Then by varying T and W, keeping their sum equal to $\frac{QR}{S}$, a point will be found for which there is no current through the telephone.

317. Anderson's Method for Comparing a Self Inductance with a Capacitance.—In Anderson's modification of Maxwell's method, the effect of the condenser is varied without disturbing

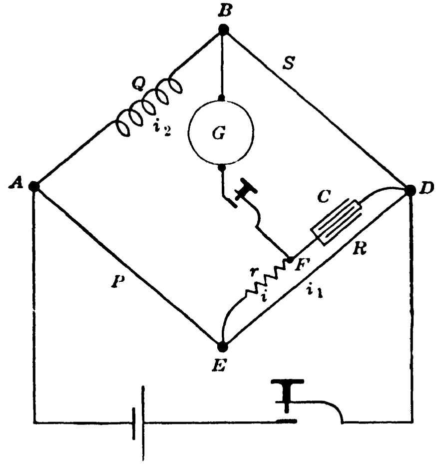


FIG. 153.—Comparison of capacitance and self inductance.

the direct-current balance. The set-up differs from Maxwell's method by having a resistance, r, in series with the condenser, and then connecting the galvanometer at F, Fig. 153, instead of at E. This makes no difference with the steady current balance, but with varying currents the effect of the condenser can be varied by changing the value of r. The alternating-current generator with its commutator for the galvanometer circuit can be used to advantage.

When r has been adjusted to give no current through the galvanometer branch, the potentials of B and F must always remain equal. Then

$$P(i_1 + i) + ri = Qi_2 + L\frac{di_2}{dt}, \qquad (1)$$

where the currents are as shown in the figure. Similarly, for the other branches

$$\frac{q}{C} = Si_2 \qquad (2)$$

and

$$Ri_1 = ri + \frac{q}{C}. \tag{3}$$

Substituting in (1) the values of i_1 and i_2 from (2) and (3) gives

$$\frac{P}{R}\left(ri + \frac{q}{C}\right) + (P + r)i = \frac{Q}{S}\frac{q}{C} + \frac{L}{SC}\frac{dq}{dt}$$

or

$$\left(\frac{P}{R} - \frac{Q}{S}\right)\frac{q}{C} + \left(\frac{Pr}{R} + P + r\right)i = \frac{L}{SC}i, \tag{4}$$

since

$$\frac{dq}{dt} = i.$$

Making use of the fact that $PS = QR$ from the steady current balance, (4) becomes

$$L = PSC + rC(S + Q), \tag{5}$$

which reduces to Maxwell's value when $r = 0$.

318. Direct Comparison of Two Mutual Inductances.—Mutual inductances are readily measured by comparing the E.M.FS. induced in their secondaries. If the current through the primary coils is direct, the induced E.M.F. will appear at the make and break of the circuit; while if alternating current is used, there will be an alternating E.M.F. in the secondary.

In the direct comparison of two mutual inductances the two primaries are joined in series and connected to the source of current, preferably a low-voltage alternating circuit. The two secondaries are also joined in series, with the two induced E.M.FS. opposed to each other. The coils should be placed far enough apart to avoid any induction between one primary and the other secondary. This method requires a variable standard of mutual inductance and a telephone, or a galvanometer and a rotating commutator, to indicate zero current in the secondary circuit. By turning the movable coil of the standard until the E.M.F. induced in it is just equal and opposite to that induced in the secondary of the other pair of coils as shown by zero deflection of the galvanometer, the value of the mutual inductance can be read directly from the standard. This is evident from the following considerations.

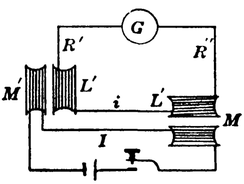


FIG. 154.—Comparison of two equal mutual inductances.

Writing the instantaneous values of the potential differences for the complete secondary circuit when not in balance gives, at any instant, t,

$$M'\frac{dI}{dt} - R'i - L'\frac{di}{dt} - Gi - L_g\frac{di}{dt} - M''\frac{dI}{dt} - R''i - L''\frac{di}{dt} = 0,$$

where G and L_g refer to the galvanometer.

For a balance $i = 0$, and is constant.

Therefore,

$$M' = M''.$$

It thus appears that only mutual inductances of the same value as the standard, that is, up to 10 millihenrys, can be measured by this method. However, the range can be extended by adding to the variable standard another mutual inductance of fixed value. This is done by joining the primaries in series and also the secondaries, thus adding together the E.M.FS. induced in the two secondaries.

319. Comparison of Two Unequal Mutual Inductances.—Let M be a pair of coils whose mutual inductance is known and M' another pair whose mutual inductance is desired. The primaries of the two coils are joined in series with a battery and key, the coils themselves being placed as far apart as possible and at right angles to each other so that each scondary will be influenced only by its own primary. The two secondaries are also joined in series with two resistance boxes, and a galvanometer is connected across from B to C. This is not a Wheatstone bridge arrangement, for the two E.M.FS. in the two secondaries act together and the current flows through the four arms in series.

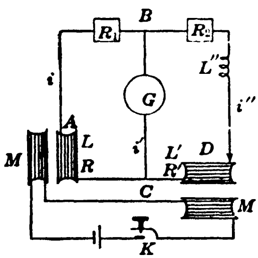


FIG. 155.—Comparison of two mutual inductances.

By properly adjusting R_1 and R_2, the galvanometer will give no deflection when K is opened or closed. Writing the instantaneous values of the potential differences for the circuit ABC, gives, at the instant t,

$$M\frac{dI}{dt} - Ri - L\frac{di}{dt} - R_1 i - Gi' - L_g\frac{di'}{dt} = 0$$

and for BDC

$$M'\frac{dI}{dt} - R'i'' - L'\frac{di''}{dt} + Gi' + L_g\frac{di'}{dt} - R_2 i'' - L''\frac{di''}{dt} = 0,$$

where L_g denotes the inductance of the galvanometer circuit.

Integrating each of these equations from the time when the key is first closed and $I = 0$, to the time when the primary current has reached its steady value, $I = I_0$, gives

$$(R + R_1)q = MI_0, \quad \text{and} \quad (R' + R_2)q'' = M'I_0,$$

if q', the integrated current through the galvanometer, is zero. Dividing one equation by the other gives

$$\frac{M}{M'} = \frac{R + R_1}{R' + R_2},$$

since $q = q''$, when $q' = 0$.

In this method the galvanometer current may be zero, but, in general, it is the sum of two transient currents each of which has an effect upon the galvanometer even when following one another in a short interval of time. This produces an unsteadiness of the galvanometer and renders an exact setting difficult, if not impossible. In order that no current should pass through the galvanometer, it is necessary that the potential difference between B and C shall remain zero for each instant while the primary current is changing, and this requires that the self-inductance of each branch shall be proportional to the E.M.F. induced in that part of the circuit.

Usually this is not the case, but it is not difficult to add some self-inductance, L'', in the part of the circuit that is deficient and thus fulfil this condition. The galvanometer will then indicate a much closer balance, or it may be replaced by a telephone, and an alternating current used in the primaries.

If the apparatus is at hand, the telephone may well be replaced by a galvanometer and rotating commutator, which will reverse the galvanometer terminals as often as the alternating current is reversed. This is more sensitive than the telephone, but it can be used only where the instantaneous galvanometer current is zero for a balance. In other words, this arrangement indicates zero current. It could not be used in the first arrangement where a balance was indicated by zero value of the integrated current.

320. Measurement of a Large Mutual Inductance.—A mutual inductance that is larger than the standard can be measured by the arrangement shown in Fig. 156. When variable inductometers of suitable range are available, the method is not so laborious as might appear, because each of the three balances is made independently.

The primary coils of the standard and of the inductance to be measured are connected in two arms of the bridge as shown. A variable self-inductance, L'', and a resistance, r, are included with the primary of the larger mutual inductance. The non-

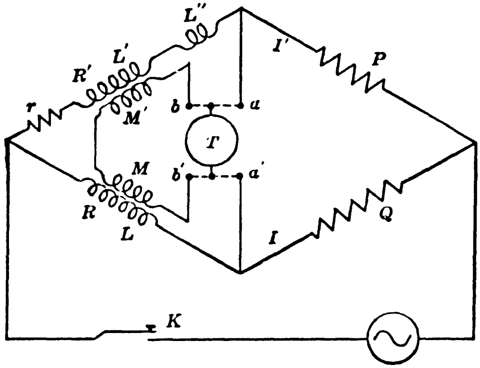


FIG. 156.—Measurement of a large mutual inductance, M' with a small variable standard, M.

inductive arms, P and Q, are set at the ratio approximately estimated for $\frac{M'}{M}$.

The first balance is made with direct currents by varying r until

$$\frac{R'}{R} = \frac{P}{Q},$$

where R and R' denote the total resistances in their respective arms of the bridge.

The second balance is made with 1,000-cycle alternating current by varying L'' until there is no current through the telephone. Then

$$\frac{L' + L''}{L} = \frac{P}{Q}.$$

(With sufficient skill and practice on the part of the operator both of these balances can be obtained with the alternating current, by adjusting r and L'' alternately until complete silence is obtained.)

When thus balanced, the current will divide in the ratio

$$\frac{I'}{I} = \frac{Q}{P}.$$

The two secondaries are now connected in series with the telephone and in opposition in each other, and the standard mutual inductance varied to give a balance. Then

$$MI = M'I'$$

and

$$M' = M\frac{P}{Q}.$$

321. Comparison of Mutual Inductances with Direct Currents. When variable inductances and alternating currents are not available, mutual inductances can be compared by the direct-

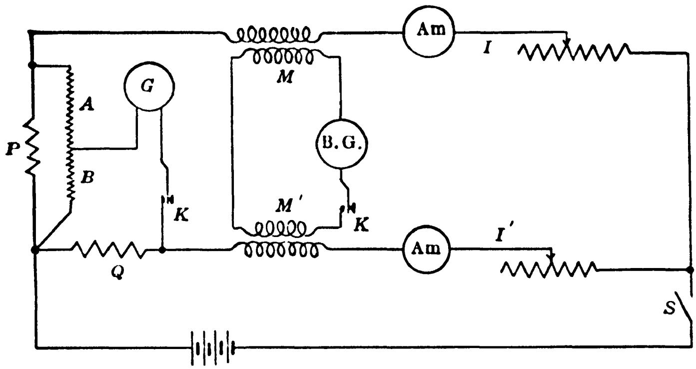


FIG. 157.—Comparison of two mutual inductances using direct currents.

current method shown in Fig. 157. Each primary coil is connected in a separate circuit with a rheostat for changing the current and an ammeter for reading the value of the current. The two secondaries are connected in opposition to each other and in series with a ballistic galvanometer or a fluxmeter. The currents, I and I', are adjusted to values inversely proportional to the estimated values of the inductances, and the deflection of the galvanometer is observed when the switch, S, is opened. For zero deflection,

$$MI = M'I'$$

when these values of the currents are read from the ammeters just before the switch is opened.

For a more exact comparison of the currents, their ratio can be determined by the potential divider, AB. P and Q are two accurate resistances of an ohm or two and capable of carrying the currents. In parallel with the larger fall of potential is placed the shunt circuit, AB, which may consist of two good resistance boxes. For zero deflection of G, the fall of potential over B will be equal to that over Q. Then

$$P(I - i) = (A + B)i$$

and

$$QI' = Bi,$$

which give

$$\frac{M}{M'} = \frac{I'}{I} = \frac{P}{Q}\frac{B}{A + B + P}.$$

322. Measurement of a Mutual Inductance in Terms of a Known Capacitance. Carey Foster's Method.—In this arrangement, shown in Fig. 158, one pair of coils is replaced by a condenser, and the transient current from the secondary coil, S, is balanced against the current that is charging the condenser, C.

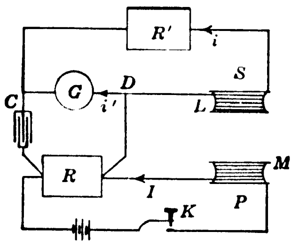


FIG. 158.—Comparison of a capacitance and a mutual inductance.

If the condenser were removed, there would be a current from S through R' and the galvanometer each time the key in the primary circuit is closed, and a current in the opposite direction when the key is opened. With the condenser in place, this current arrives at C just in time to charge the condenser, or to help charge it. The final charge in the condenser is

$$Q = C \times V = CRI, \tag{1}$$

where I is the final steady current through R. The amount of this charge is the same whether there is a current through the galvanometer or not. By adjusting the resistance in R', the current through it can be made too small, too large, or just sufficient to supply the charge in the condenser, the rest of the charge coming directly through the galvanometer. When there is no deflection of the galvanometer, the adjustment is complete, and the arrangement is said to be balanced.

Writing out the sum of the various potential differences in the circuit through $SR'G$ gives

$$M\frac{dI}{dt} - L\frac{di}{dt} - (S + R')i + Gi' + L_g\frac{di'}{dt} = 0. \quad (2)$$

Integrating this between the limits of time, t', just before K is closed, to t'', when the primary current has reached its steady value I, gives

$$MI - (S + R')\,Q' + Gq = 0, \quad (3)$$

where Q' is the total quantity that has passed through R', and q is the quantity through the galvanometer. Then, $Q' + q = Q$, the total charge in the condenser. But if the galvanometer deflection is zero, then $q = 0$, and

$$MI = (S + R')Q = (S + R')CRI$$

from (1). Hence

$$M = (S + R')CR.$$

It is to be noted that the galvanometer current is not required to be zero for a balance, and, in fact, it usually is not zero for each instant from t' to t''. Zero deflection merely indicates that the algebraic sum of the *quantities* passing through the galvanometer is zero. Nevertheless, the more nearly the galvanometer currents are equal to zero at each instant the steadier will be the zero deflection at the balance.

323. Modification of the Carey Foster Method.—In Art. 367 and Figs. 195 and 196 it is shown that there can be a balance at each instant, with zero current through the galvanometer. This is accomplished by adding a resistance, N, in series with the condenser. Then, for the condenser circuit, with the notation of Fig. 158,

$$Ni + \frac{q}{C} - R(I - i) = 0$$

or, by differentiation,

$$N\frac{di}{dt} + \frac{i}{C} - R\frac{dI}{dt} + R\frac{di}{dt} = 0. \quad (A)$$

For the circuit through S,

$$(R' + S)i + L\frac{di}{dt} - M\frac{dI}{dt} = 0. \quad (B)$$

Eliminating $\frac{dI}{dt}$ from (A) and (B) leaves

$$\left(\frac{N+R}{R} - \frac{L}{M}\right)\frac{di}{dt} + \left(\frac{1}{RC} - \frac{R'+S}{M}\right)i = 0.$$

This equation is satisfied for all values of i and $\frac{di}{dt}$ when

$$\frac{N+R}{R} = \frac{L}{M}$$

and

$$\frac{1}{RC} = \frac{R'+S}{M}$$

or when

$$M = (R'+S)CR$$

and

$$N = R\frac{L-M}{M} = \frac{L}{(R'+S)C} - R.$$

The first of these conditions is the same as was found in the preceding article. The second condition gives the value of the resistance that must be placed in series with the condenser to give zero current through the galvanometer at all times. It is convenient to obtain the final balance by varying the value of the mutual inductance, M.

CHAPTER XVI

ALTERNATING CURRENTS

324. An alternating current is the same as any other electric current, except that it flows in one direction for only a very short time; it then reverses and flows in the other direction for an equally short time. In ordinary lighting circuits there are from 100 to 300 such reversals each second. In some other cases there may be many millions of reversals each second. While the current is flowing in one direction it is the same as any other current of the same number of amperes. The only peculiarity of an alternating current is that it is continually being made to change. And just as a material body cannot change its velocity from one direction to the opposite without first slowing down to zero and then starting up in the other direction, so the current cannot instantly change from its full value in one direction to the full value in the other, but it requires some time to die down to zero and then to build up in the opposite direction. It does not have time to build up very far before it must begin to decrease again, so there is never a time when the current is not changing in amount. In fact, the value of the current as it changes from one direction to the other and back again goes through the same variations as the velocity of a pendulum bob when swinging to and fro.

325. Tracing Alternating Current and E.M.F. Curves.—In Chap. VII some methods were given for measuring the current flowing through a circuit. The same arrangements can be used to measure the value of an alternating current. Since the balance point would vary rapidly up and down the slide wire of the potentiometer, it will be necessary to add some mechanical device that will close the galvanometer circuit for only an instant at the particular time when the current has the value that it is desired to measure. As the current will have the same value 60 (say) times in each second, the galvanometer circuit can be closed 60 times each second, and this is often enough to produce a steady deflection of the galvanometer when the potentiometer is not balanced. Thus the actual setting is made as easily as when the current is steady.

Let *AD* represent the slide wire potentiometer, *S* the resistance through which the alternating current is flowing, and *M* the instantaneous contact maker which closes the galvanometer circuit for a very short time once in each cycle. Let *i* denote the value of the alternating current at the instant the galva-

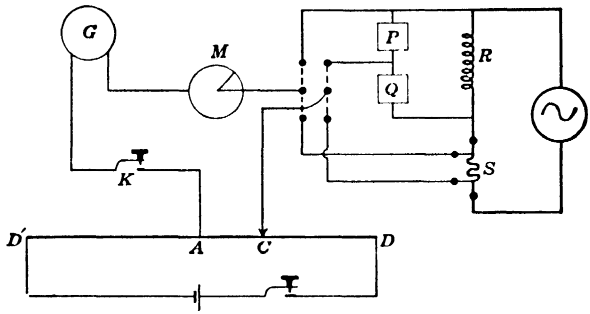


FIG. 159. Potentiometer for tracing alternating current curves.

nometer circuit is closed by *M*. The fall of potential over *S* is then *Si*, and if *C* is moved so that the fall of potential over *AC* is the same as *Si*, there will be no deflection of the galvanometer. Therefore the distance, *AC*, is proportional to the current, *i*.

Now let the contact maker, *M*, be turned a few degrees so as to close the circuit just a little later than before. The value of the current at this point will be somewhat different, and its value can be found by moving *C* along till a balance is again obtained. In this way the values of the current for the complete cycle can be determined, and a curve plotted showing how the current varies with the time.

When the current reverses and the fall of potential over *S* is in the other direction, *C* must be moved to the other side of *A* to find a balance. Therefore *AD′* is merely a second slide wire potentiometer on which all negative values of the current can be measured. In this manner the values of the current at various instants corresponding to the different settings of *M* can be measured. These should extend far enough to complete at least one cycle, that is, until the readings begin to repeat themselves. If the dynamo that is generating the current has two poles, there will be one cycle for each revolution of the armature. If there are more poles, there will be as many cycles in each revolution as there are pairs of poles.

It is absolutely necessary that the instantaneous contact maker, M, closes the galvanometer circuit at precisely the same point in each cycle. The contact maker is, therefore, placed on the dynamo shaft, but there is no electrical connection between the two; but this insures that the contact maker will keep rigid step with the current.

The curve for the alternating E.M.F. can be traced in the same manner. Usually the full E.M.F. is too large to be measured directly by the potentiometer, but by the fall of potential method, as used in the method for calibrating a high-reading voltmeter, any small portion of this E.M.F. can be obtained. It is then only a matter of increasing the scale to get the curve of the full E.M.F. When the current and E.M.F. curves are both drawn on the same sheet, the phase relation between them is clearly shown.

After tracing the curves for a current flowing through a non-inductive resistance, it will be interesting to do the same for a coil having a considerable inductance. This should show the effect of inductance in making the current "lag behind the E.M.F." If desired, the experiment can be further varied by connecting a condenser in the circuit and finding the effect it has upon the current and E.M.F. curves.

326. Instantaneous Values of the Current.—When an alternating current flows through a circuit, there are induced other E.M.FS. besides that impressed by the dynamo; and these extra E.M.FS. often have considerable influence in determining what the resulting current shall be. As we have seen, Art. 299, because of the fact that the current is changing at the rate of $\frac{di}{dt}$ amperes per second, there will be induced in the circuit an E.M.F. of $-L\frac{di}{dt}$ volts. In order to maintain the current, i, the dynamo must furnish not only the E.M.F., Ri, required by Ohm's law, but, in addition, must supply an E.M.F. sufficient to counterbalance, at each instant, this induced E.M.F. That is, the dynamo must supply an E.M.F. whose value at any instant is

$$e = Ri + L\frac{di}{dt}.$$

This is the general equation of any current varying in any manner whatsoever. Solving this equation for the current gives

$$i = \frac{e - L\dfrac{di}{dt}}{R},$$

which shows that the instantaneous value of the current is given by the usual form for Ohm's law—taking into account all of the E.M.FS. in the circuit at that instant.

327. Measurement of an Alternating Current.—Instruments, are seldom made to give the instantaneous value of the current, but they record some kind of an average value. Evidently, the arithmetical average of the values of a current which is negative as much as it is positive would be zero and, therefore, such an instrument as a galvanometer, or an ordinary ammeter, would be useless for the measurement of an alternating current. But an electrodynamometer, or the Kelvin balance, measures the current equally well whichever way the current flows through it.

328. Relation of the Instrument Reading to the Instantaneous Values of the Current.—When the current through an ammeter has a definite steady value, the reading of the instrument depends upon this value, but when the current is continually changing, the effect on the moving system is some kind of an average. As pointed out above, the average of an alternating current may be zero, but even for a rectified current, which flows always in a single direction, however much its value may vary, an ordinary ammeter will never measure the maximum value, but it will indicate an average value. In the case of an electrodynamometer this average value is not the arithmetical average of the different instantaneous values of the current because the effect on the instrument depends upon the *square* of the current. The reading of the electrodynamometer will therefore indicate the average, or the mean value, of the squares of the instantaneous values of the current.

The heating and power effects of an electric current are proportional to the square of the current, and therefore the mean square value measures the rate of heat production or the power developed by a current. Therefore, the average square of the current is the value usually required, and instruments giving these values find the greatest use. If the average value or the maximum value is required. either of these can be computed from the mean square value if the wave form is known. For a sine wave these relations are simple and can be derived as shown below.

329. Average Value of a Sine Current.—The usual alternating current follows closely the sine law, and its value at any instant is given by the equation,

$$i = I \sin \omega t,$$

where I and ω are constants.[1] To find the average ordinate of the sine curve, it is only necessary to determine the area included between the curve and the axis of abscissæ, and divide this by the length of the base. Then

$$\text{Average ordinate} = \frac{\text{area}}{\text{base}} = \int_0^{\pi} \frac{y\,dx}{\pi}.$$

But for a sine curve,

$$y = a \sin x.$$

Hence,

$$\text{Average ordinate} = \frac{a}{\pi}\int_0^{\pi} \sin x\,dx = -\frac{a}{\pi}\Big[\cos x\Big]_0^{\pi} = \frac{2a}{\pi}$$

$$= 0.6369 \text{ of maximum ordinate.}$$

330. Effective, or Root Mean Square, Value of a Sine Current. Inasmuch as the heating and power effects of an electron current, as well as the dynamometer readings, depend upon the square of the values of the current, it is more useful to know the average value of the square of the current than merely its average value. This can be found in the same way as before. Since the squares of negative quantities are positive, we are not limited to half a period, but can extend the integration over the whole period.

$$\text{Mean square of } y = \int_0^{2\pi} \frac{y^2\,dx}{2\pi} = \frac{a^2}{2\pi}\int_0^{2\pi} \sin^2 x\,dx.$$

But, $2 \sin^2 x = 1 - \cos 2x$, and

$$\int_0^{2\pi} \sin^2 x\,dx = \left[\left(\frac{x}{2} - \frac{\sin 2x}{4}\right)\right]_0^{2\pi} = \pi.$$

Hence, mean square of $y = \frac{a^2}{2}$, and the square root of the mean square value of the current is 0.707 of the maximum value.

[1] In case the current cannot be represented by a single term, it can always be expressed by a series of such terms.

331. Definition of an Ampere of Alternating Current.—Since an alternating current has no steady value, it is meaningless to speak of an ampere of alternating current without having some definite convention or definition. Naturally, an ampere of alternating current should be able to produce as much heating or other form of energy as an ampere of direct current. Inasmuch as the power expended in an electric circuit is proportional to the square of the current, the average value of i^2 is of more practical importance than the average value of i. The root mean square value of the current furnishes, then, the basis for defining the value of an alternating current.

If an electrodynamometer has been calibrated for direct current it may be used to measure alternating current, and, by definition, *that amount of current which will give the same deflection as one ampere of direct current is called one ampere of alternating current.*

332. Alternating-current Instruments.—Alternating-current ammeters and voltmeters are calibrated to read the effective, or R.M.S., value of the current because the power expended in a circuit depends upon the square of the current. The heat produced in a hot wire ammeter is evidently proportional to the average square of the current, and the deflection of an electrodynamometer (Art. 160) likewise depends upon the average square of the current. Therefore, when such instruments are calibrated to measure direct currents, if they are used to measure alternating current they will indicate the square root of the average square of the instantaneous values of the current. This value is called the "root mean square" value. It is also called the "effective" value.

333. To Find What E.M.F. Is Required to Maintain a Given Current in an Inductive Circuit.—Let an alternating current flowing in a circuit containing both resistance and self-inductance be given by the equation

$$i = I \sin \omega t. \tag{1}$$

It is required to find a similar expression for the E.M.F. which will maintain this current.

The general equation is

$$e = Ri + L\frac{di}{dt},[1]$$

[1] See Art. 302.

where Ri is the E.M.F. required to maintain the current through the ohmic resistance, and $L\frac{di}{dt}$ is the E.M.F. to balance the induced E.M.F.

Substituting the above value of the current gives

$$e = RI \sin \omega t + L\omega I \cos \omega t. \tag{2}$$

A person who remembers a little trigonometry will recall that a sine and a cosine term can be added, giving a result that has the general form of

$$e = k \sin (\omega t + a), \tag{3}$$

where k and a are constants depending on the values of R, L, and ω.

The values of k and a can be determined in terms of R, L, and ω by expanding $\sin (\omega t + a)$ as follows:

$$\sin (\omega t + a) = \cos a \sin \omega t + \sin a \cos \omega t \tag{4}$$

or, more generally,

$$k \sin (\omega t + a) = k \cos a \sin \omega t + k \sin a \cos \omega t, \tag{5}$$

where k is any common multiplier.

The right-hand side of this equation is seen to be similar to the right-hand side of (2), and it is identical with it when k and a have such values that

$$k \cos a = RI, \quad \text{and} \quad k \sin a = L\omega I, \tag{6}$$

that is, when

$$k = I\sqrt{R^2 + L^2\omega^2}, \quad \text{and} \quad a = \tan^{-1}\frac{L\omega}{R}. \tag{7}$$

If the right-hand members of (2) and (5) are identical, the left-hand members must also be equal when k and a have these values. That is

$$\begin{aligned} e &= k \sin (\omega t + a) \\ &= I\sqrt{R^2 + L^2\omega^2} \sin (\omega t + a). \end{aligned} \tag{8}$$

From this relation it appears that the *maximum* value of the impressed E.M.F., e, is

$$E = I\sqrt{R^2 + L^2\omega^2}, \tag{9}$$

where I is the maximum value of the (sine wave) current.

Problem

Draw the curve representing the current as given in (1) for at least two cycles. On the same axis draw the two components of the E.M.F. as given by (2) and by addition obtain the curve for e.

The following constants may be used, $R = 2$ ohms, $I = 1$ ampere, $\omega = 400$, $L = 0.01$ henry.

334. Graphical Solutions.—The term, $RI \sin \omega t$, in (2) may be represented by a line OA, Fig. 160. If this line is considered as rotating counterclockwise with the angular velocity, ω, its vertical projection at any instant, t, gives a length, $RI \sin \omega t$.

In the same way the term, $L\omega I \cos \omega t$, may be represented by a line OB, which must be drawn 90 deg. ahead of OA, since $\cos \omega t = \sin (\omega t + 90 \text{ deg.})$. The value of e at any instant will be given by the sum of the vertical projections of these two lines or, what is the same thing, by the vertical projection of OC, which is the geometrical sum of OA and OB.

From the figure it is evident that the length of OC is

$$I\sqrt{R^2 + L^2\omega^2}$$

and it is ahead of OA by the angle $a = \tan^{-1}\frac{L\omega}{R}$.

335. Components of E.M.F.—Figure 160 shows the parallelogram of E.M.FS. The side OA represents the E.M.F. required to keep the current flowing through the resistance, R. This is

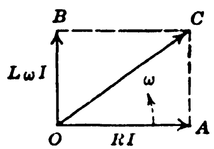


FIG. 160.—Addition of two harmonic quantities.

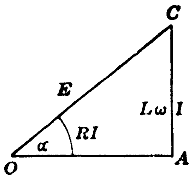


FIG. 161.—E.M.F. triangle.

called the active, or power, component of the E.M.F. The side OB represents the E.M.F. required to balance the induced E.M.F. This is called the reactive component of the E.M.F. The geometrical sum of these two gives the total E.M.F. that must be supplied to maintain the current, and it is called the impressed E.M.F. Since $OB = AC$, the same values are shown by the E.M.F. triangle, Fig. 161.

The angle of lag of the current behind the impressed E.M.F. is shown by the angle a. This may have any value from 0 to 90 deg. depending upon the amount of inductance in the circuit.

The Impedance Triangle.—If each side of Fig. 161 is divided by the current, I, the result will be the triangle of resistance and impedance as shown in Fig. 162.

The side R denotes the resistance, but this is not necessarily the same as the direct-current resistance. It is the factor which, multiplied by the square of the current, will give the amount of heat produced in the circuit.

The other side of the triangle, $L\omega$, is called the "reactance." It is also measured in ohms, but there is no loss of energy or production of heat because of it. The hypotenuse is called the "impedance."

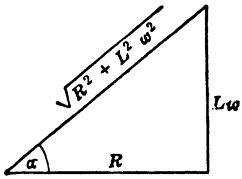


FIG. 162.—Impedance triangle

336. Components of Current.—In what has been said above the current has been the basis for reference, and the different components of E.M.F. have been considered in their relation to the current in the circuit. The same relations would have been obtained by starting from the impressed E.M.F. The current then would be represented by two components; the active component, I_a, being in phase with the E.M.F. impressed on the entire circuit, and the reactive component, I_r, being in quadrature with this E.M.F. The resultant of these two currents is I, Fig. 163, and is behind E by the phase angle, α, which is the same as shown in Fig. 161. The fact that these figures are printed on the page at different angles does not affect the relation between E and I.

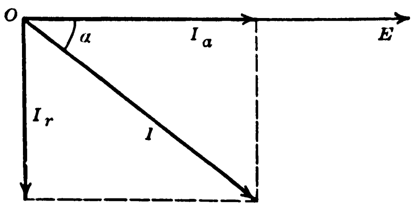


FIG. 163.—Relation between the E.M.F. and the components of the current.

The reactive component, I_r, of the current is sometimes called the "wattless component," because on the average over the whole cycle no power is required to maintain it. The power that is expended to make the current increase during a part of the cycle is received back again when the current decreases in a later part of the cycle.

337. Effective Values.—In all these diagrams, I denotes the maximum value of the current and when so used the result obtained for E will be the maximum value of the E.M.F. If the ammeter reading is used for I, the same construction can be used and the result will be the voltmeter reading value of E. But in this case the diagram no longer has the significance that was given it in Fig. 160, and it becomes merely a graphical construction to reach a desired result.

338. Measurement of Impedance by Ammeter and Voltmeter. From what has just been said it will be seen that the impedance of a circuit is merely the ratio of the impressed E.M.F. to the resulting current. To measure it, therefore, it is only necessary to measure the voltage and current in precisely the same way as the direct-current resistance is measured by an ammeter and a voltmeter. The ratio gives the impedance or,

$$\frac{E}{I} = \text{Impedance} = \sqrt{R^2 + L^2\omega^2}.$$

339. Self Inductance by the Impedance Method.—If the values of R and ω are known, this method gives a means for computing the self-inductance, L, of the circuit. If the current passes through n cycles per second, then $\omega = 2\pi n$.

The direct-current value may be used for R if the wire is not too large, and if there are no closed circuits, or masses of metal or iron in the vicinity.

In case heat is produced, or energy is otherwise expended, outside of R, it will be necessary to determine one other quantity before the impedance triangle can be drawn. This may take the form either of finding the angle of lag of the current behind the impressed E.M.F., or of determining the equivalent resistance of the circuit—that is, the non-inductive resistance in which the same amount of energy would be expended.

340. Impedance and Angle of Lag by the Three-voltmeter Method.—In this method a non-inductive resistance capable

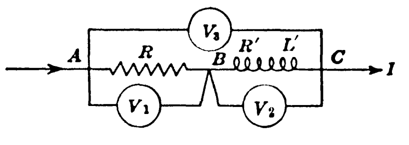


FIG. 164.—Three voltmeter method.

of carrying the current is placed in series with the impedance to be measured. Three voltmeter readings are taken, as nearly simultaneously as possible, to measure the voltage across the resistance and the impedance and across both together. It is best to use three voltmeters, as shown in Fig. 164, but if only one is available it may be transferred quickly from one position to another. If the voltmeter shunts an appreciable current from the main circuit, two equivalent resistances should occupy the places of the missing voltmeters.

The voltmeter is readily transferred to the various positions by means of two double-throw switches, S and T, as shown in Fig. 165, where Vm denotes the voltmeter, and U and W are resistances, each equal to the resistance of the voltmeter. With both switches thrown to the right, the voltmeter is placed across

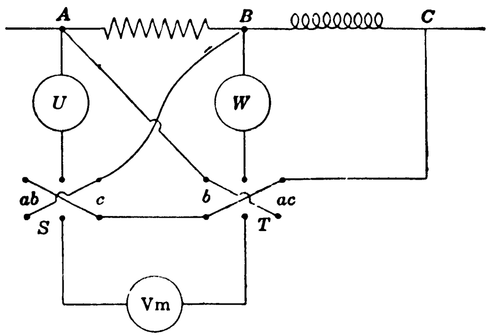


FIG. 165.—Switches for putting the voltmeter in three places.

AC, and measures the total voltage over both the impedance and the non-inductive resistance. When both switches are thrown to the left, the voltmeter is across BC, and measures the voltage over the impedance alone. When the switch, S, is thrown to the left and the switch, T, is to the right, the voltmeter

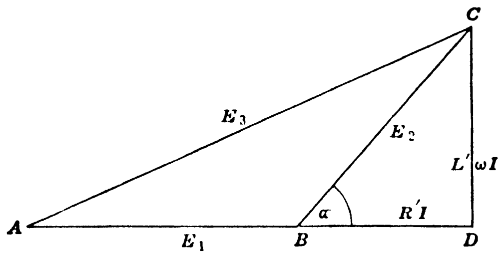


FIG. 166.—Determination of the angle of lag.

is across AB, and measures the voltage over the resistance, R, only. The resistances, V and W, are simultaneously transferred to the positions not occupied by the voltmeter.

Referring to Fig. 161 it will be seen that E_2, the reading of the voltmeter across BC, is the E.M.F. impressed upon the coil, and it would be the hypotenuse of the corresponding E.M.F. triangle if the other elements were known. In the same way, E_3 is the hypotenuse of the E.M.F. triangle for the entire circuit, AC, while for the part AB in which there is no inductance the hypotenuse coincides with the base of the triangle, and is measured by E_1.

Combining these three E.M.FS. gives the triangle, ABC, Fig. 166. Extending the side AB until it meets the perpendicular from C gives the complete E.M.F. triangle for the entire circuit, AC. The angle, DBC, gives the lag of the current behind E_2, and BDC is the E.M.F. triangle for the coil.

If the current is also known, either by direct measurement or by computation if R is known, the impedance of the coil is given by the relation

$$\text{Impedance} = \frac{E_2}{I}.$$

341. Determination of Equivalent Resistance.—Knowing the value of one side of the impedance triangle and the angle, a, the other sides are readily constructed. The base of this triangle, R, gives the value of the *equivalent resistance* of the coil.

In a coil of fine wire, with no other conductors near, this resistance will be the same as that measured by a Wheatstone bridge using direct current. In case there is a thick piece of metal or other closed circuit in which the varying magnetic flux from the coil can induce an electric current, it will result in a larger value for R'. When the coil consists of thick wire, there will be some eddy current circulating in this metal, thus increasing the heating effect. This is equivalent to the current, I, flowing through an increased resistance, which appears in the increased value of R' in the impedance triangle.

342. Wattmeter Determination of Equivalent Resistance.—The equivalent resistance can also be determined by the use of a wattmeter. From the definition of resistance as the property of a circuit by which the electrical energy is transformed into heat, we have

$$W = RI^2.$$

The electrical power, W, can be measured with a wattmeter and the current, I, with an alternating-current ammeter. Then,

$$R = \frac{W}{I^2}.$$

The effect of a solid iron core is greatly to increase this equivalent resistance over the ohmic resistance of the coil as measured by direct-current methods. The inductance is likewise increased.

If the core consists of a bundle of fine iron wire, the increase of the resistance is less, while the inductance is greater than with the solid core.

343. Inductive Circuits in Series.—When two inductive circuits are joined in series, the same current must, of course, flow through them both. But, in general, the E.M.F. over one will not be in phase with that over the other, and therefore the

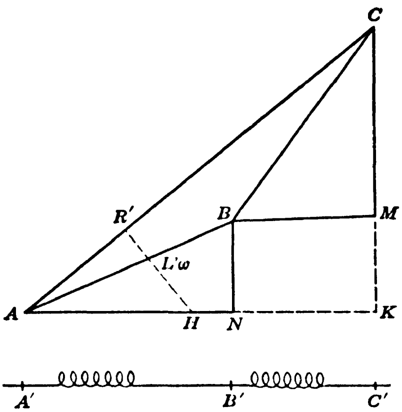


Fig. 167.—Two inductive circuits in series.

total E.M.F. required to maintain the current will be less than the sum of the two parts. This is readily seen from the figure. $A'B'C'$ represents the two inductive circuits in series, and the diagram ABC shows the E.M.F. triangles for each part, and for the whole circuit. The triangle ANB is the E.M.F. triangle for the portion AB, and corresponds to Fig. 161 for the first part of the circuit.

Similarly, the E.M.F. triangle for the part $B'C'$ is shown by BMC, which is drawn in the position shown because the point B in each triangle represents the one point, B', between the two parts of the circuit, and therefore it should occupy only one position on the diagram. The total E.M.F. over the entire circuit is given then by AC, while the E.M.F. triangle for the entire circuit is found by completing the triangle AKC.

From this construction it is seen that the total resistance in the circuit is the sum of the resistances of each part; and the total inductance is the sum of the separate inductances. Of course, the two parts are supposed to be far enough apart to avoid mutual induction between them. The angle of lag of the current behind the impressed E.M.F. is intermediate between the angles of lag in each part of the circuit considered separately, and is given by

$$a = \tan^{-1}\frac{L'\omega}{R'}.$$

Problems

1. Given two circuits in series, with $R_1 = 18$ ohms, $L_1 = 0.01$ henry, $R_2 = 4$ ohms, $L_2 = 0.02$ henry, $\omega = 400$. Find the E.M.F. necessary to maintain 10 amperes through the circuit; also the E.M.F. over each part.

   Solution.—Draw to scale a figure similar to Fig. 167. Then use the same scale to measure AC, AB, and BC.

2. Given the same circuit as above. What is the value of the current when the impressed E.M.F. is 100 volts?

   Solution.—Draw the line AC to represent the value of E. At A construct the angle $CAH = \tan^{-1}\frac{L'\omega}{R'}$, as shown by the dotted lines, Fig. 167. Extend AH to meet the perpendicular from C at K. Then $I = \frac{AK}{R'}$, where $R' = R_1 + R_2$.

   Solution.—Second method. Assume a value, I', for the current and find the corresponding value, E', for the impressed E.M.F. as above. Then $E' : 100 :: I' : I$.

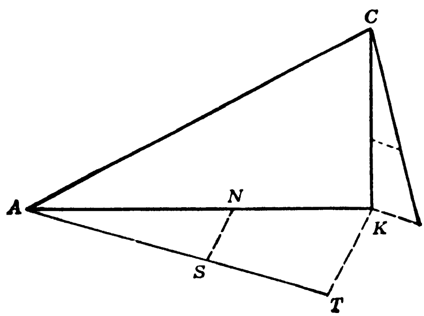


FIG. 168.—The line NS, drawn parallel to KT, divides AK in the same ratio as AT.

3. When the impressed E.M.F. is 200 volts, what is the E.M.F. over each part of the above circuit?

   Solution.—Draw the triangle KAC as before. Divide the side AK in the ratio of the two resistances by laying off AS and ST to scale and then drawing SN parallel to TK. In the same way the side KC can be divided in the ratio of the two inductances. Through the points M and N draw lines parallel to these sides; their intersection locates the position of B, Fig. 167. The values of AB and BC can then be measured.

344. Inductive Circuits in Parallel.—In the case of a divided circuit having two or more inductances in parallel, it is much more difficult to calculate what part of the current will pass through each branch, and graphical methods become more useful. The following example for two inductive circuits in parallel can, of course, be extended to as many parallel circuits as desired.

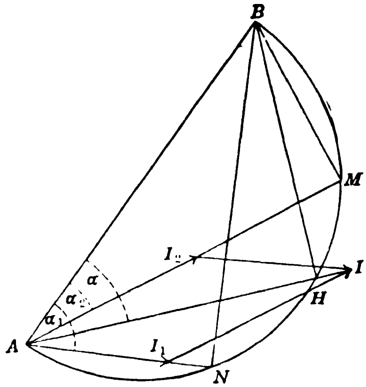


FIG. 169.—Inductive circuits in parallel.

Since each branch will have the same impressed E.M.F., the hypotenuse of each E.M.F. triangle will be identical. Let this be laid off to scale as shown by AB, Fig. 169. Since each triangle is right-angled, it will be inscribed within a semi-circle drawn on AB as a diameter. From the constants of the circuit the angle of lag in each branch can be determined. The base, AN, of the first triangle can then be laid off, making this angle with AB. The intersection of AN with the semicircle locates the other corner of this triangle at N, and the line NB completes the other side. The value of the current is $I_1 = \frac{AN}{R_1}$, and is laid off in the direction AN.

Similarly, the triangle AMB is laid out for the other branch, and the value of the current determined. The resultant current is the geometrical sum, AI, of these two components, and it lags behind the impressed E.M.F. by the angle HAB.

The point H, where the line AI cuts the semicircle, is the right-angled corner of a new triangle, AHB, which represents the resultant or equivalent effect of a single circuit which could replace the two parallel circuits. The E.M.F. AH is $R'I$, and HB is $L'\omega I$, where R' and L' denote the values of the resistance and the inductance of this equivalent circuit. These values can be taken from the figure as accurately as the lines can be measured.

Problems

1. Given a coil with $R_1 = 22$ ohms and $L_1 = 0.03$ henry, in parallel with a second coil with $R_2 = 8$ ohms and $L_2 = 0.03$ henry. $E = 100$ volts, and

$\omega = 400$. Find the values of the currents through each branch and in the main circuit; also the equivalent resistance and the equivalent inductance of the two coils in parallel, and the angle of lag of each current behind the impressed E.M.F.

2. Same as Problem 1, but with $L_2 = 0$.

345. The E.M.F. Required to Maintain a Current in a Circuit Having Capacitance and Resistance.—In a circuit having a condenser in series with a resistance and an alternating E.M.F., the relation at any instant is

$$e = Ri + \frac{q}{C}, \tag{1}$$

where $\frac{q}{C}$ is the difference of potential across the condenser of capacitance C farads, when its charge is q coulombs.

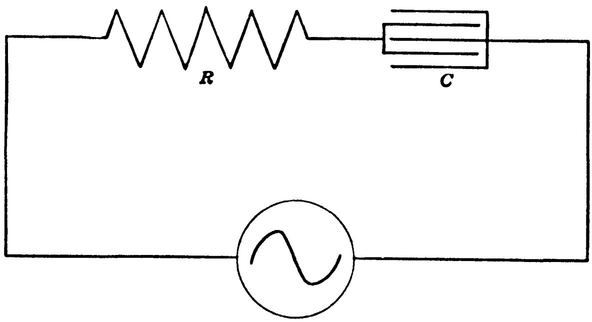


FIG. 170.—Capacitance and resistance in series.

If the current flowing through R, Fig. 170, and into the condenser is following the sine law, its value at any instant is

$$i = I \sin \omega t. \tag{2}$$

Then

$$q = \int idt = \int I \sin \omega t \, dt = -\frac{I}{\omega} \cos \omega t. \tag{3}$$

Putting this value in (1) gives

$$e = RI \sin \omega t - \frac{I}{C\omega} \cos \omega t \tag{4}$$

$$= RI \sin \omega t + \frac{I}{C\omega} \sin (\omega t - 90 \text{ deg.}),$$

Since

$$-\cos \omega t = +\sin (\omega t - 90 \text{ deg}).$$

This shows that the last term in (4) represents an E.M.F. that is 90 deg. behind the current.

Adding the two terms of (4) gives

$$e = k \sin (\omega t - a), \tag{5}$$

where k and a are new constants depending on the values of C, R, and ω.

Expanding $\sin (\omega t - a)$ gives

$$e = k \cos a \sin \omega t - k \sin a \cos \omega t. \tag{6}$$

Comparing this with (4) it is seen that

$$k = I\sqrt{R^2 + \frac{1}{C^2\omega^2}} \quad \text{and} \quad a = \tan^{-1}\frac{1}{RC\omega} \tag{7}$$

Hence, from (5),

$$e = I\sqrt{R^2 + \frac{1}{C^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{1}{RC\omega}\right).$$

The maximum value of e is

$$E = I\sqrt{R^2 + \frac{1}{C^2\omega^2}}.$$

If the resistance of the circuit is zero, the current flowing into the condenser is

$$I = EC\omega,$$

where I and E may denote either the maximum values or the effective values of the current and E.M.F.

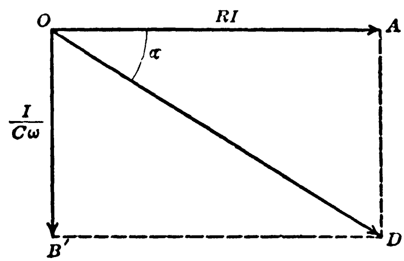


FIG. 171.—*OD* represents the resultant E.M.F. in a capacitance circuit.

346. Graphical Representation of the E.M.Fs. in a Circuit Having Resistance and Capacitance.—The two terms of equation (4) can be represented by two lines drawn at right angles, as was done in Art. 334. The result will be different, however, for after laying off OA, Fig. 171, equal to RI, the other side, OB, must be drawn *downward* in order to represent the negative term in (4). The geometrical sum of these two is

$$OD = E = I\sqrt{R^2 + \frac{1}{C^2\omega^2}}$$

and this is behind OA by the angle

$$a = \tan^{-1}\frac{1}{RC\omega}.$$

The current is thus ahead of the impressed E.M.F. by the angle a.

The Triangle of E.M.FS.—The triangle OAD shows the relation of these E.M.FS., and usually this part of Fig. 171 is all that need

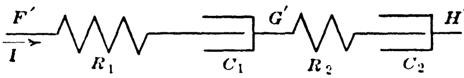


FIG. 172.—Two capacitance circuits in series.

be drawn. This triangle of E.M.FS. corresponds to the similar relation shown in Fig. 161. OA is called the active component of the E.M.F., and AD is the reactive component. The resultant of these, OD, is the impressed E.M.F.

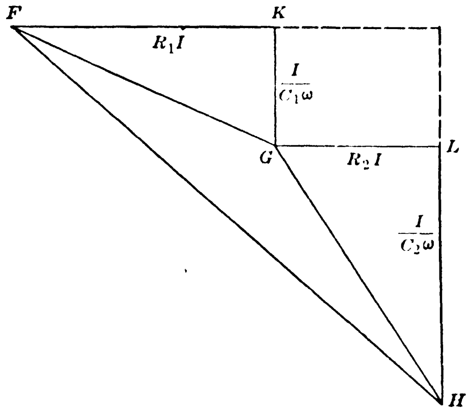


FIG. 173.—E.M.F. diagram for two capacitance circuits in series.

Impedance Triangle.—Dividing each side of the E.M.F. triangle by the value of the current, I, gives the corresponding impedance triangle.

347. Two Condenser Circuits in Series.—The graphical construction for two condenser circuits in series follows directly from the construction of Fig. 171. The triangle FKG, Fig. 173, represents the E.M.FS. in the first part, $F'G'$, of the circuit. The corresponding triangle for $G'H'$ will be drawn as shown by GLH. The total E.M.F. over the whole circuit is represented by the resultant, FH. The two circuits thus act like a single circuit of resistance, $R = R_1 + R_2$, and capacitance C, where

$$\frac{1}{C\omega} = \frac{1}{C_1\omega} + \frac{1}{C_2\omega}$$

Problems

1. Given the values of R_1, R_2, C_1, C_2, and ω, find by a graphical construction the E.M.F. required to maintain a current of I amperes. Also the E.M.F. across each part of the circuit.
2. Given the values of C, I, ω, and E, what resistance is in the circuit?

   Solution.—Lay off E as a diameter of a circle. Inscribe $\frac{I}{C\omega}$ and measure the value of RI.
3. Given C_1, R_1, R_2, E_1, E_2, E, and ω. Find C_2.
4. Given C_1, R_1, C_2, R_2, E, and ω. Find the E.M.F. over each part of the circuit.
5. Same as Problem 4, but with $R_1 = 0$.
6. Same as Problem 4, but with C_2 replaced with $R = 0$. Is this equivalent to $C_2 = 0$, or $C_2 = \infty$?

348. Capacitance Circuits in Parallel.—The graphical solution for two capacitance circuits in parallel is readily constructed. Since the two circuits are in parallel, each one will be subjected

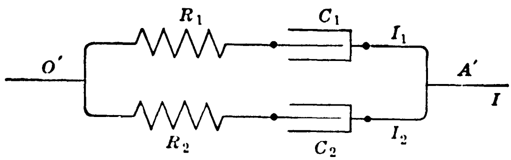


FIG. 174.—Two capacitance circuits in parallel.

to the same impressed E.M.F., E. Let E be laid off to scale along a straight line, as OA, Fig. 175, and at O construct the angle AOB by laying off OK and KT proportional to R_1 and $\frac{1}{C_1\omega}$, respectively. Draw the straight line OT and extend it until it intersects at B the circle drawn on OA as a diameter. OBA is then the right-angled triangle representing the E.M.FS. in the first circuit, R_1, C_1, Fig. 174. The current, I_1, in this branch will be in phase with OB, the active component of the E.M.F., and it can be shown on this diagram as I_1, laid off to an appropriate scale along OB.

In the same way the E.M.F. triangle for the other circuit, R_2C_2, can be constructed within the same semicircle, as shown by ODA. The current in this branch of the circuit is shown by I_2.

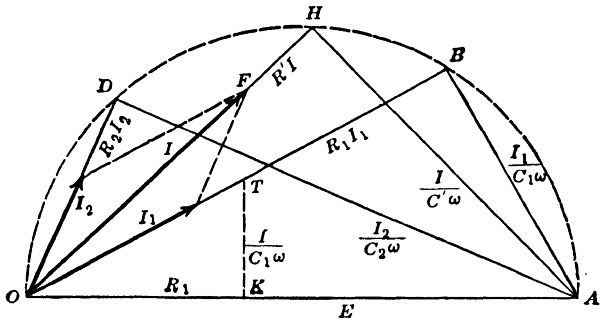


FIG. 175.—E.M.F. and current diagram for two capacitance circuits in parallel.

349. Equivalent Capacitance and Resistance.—The single circuit that would be equivalent to the two parallel circuits of Fig. 174 is easily determined. The current through such an equivalent circuit is I, the resultant of I_1 and I_2, and this is shown by the diagonal of the parallelogram, OF. Extending OF to the circle gives OH, which is the active component of the E.M.F. in the equivalent circuit. The value of this E.M.F. is obtained by measuring OH with the same scale that was used in laying off OA. In the same way, the line HA gives the value of the reactive component of the E.M.F.

Dividing the sides of this E.M.F. triangle, OHA, by the value of the resultant current, I, will give the corresponding impedance triangle, the sides of which are the resistance and the reactance of a single circuit that is equivalent to the two circuits in parallel.

Problems

1. Given the values of C_1, R_1, C_2, and R_2, for two circuits in parallel, draw the graphical construction and find the values of the equivalent resistance and capacitance.
2. Same as Problem 1, but with one of the circuits consisting of resistance only.
3. Same as Problem 1, but with one of the circuits consisting of a condenser only.

   Note.—Draw the E.M.F. triangle, OAB, with successively smaller values assigned to R_1, and observe the progressive changes as R_1 approaches zero.
4. A condenser of capacitance C is connected in parallel with a resistance R. Find the equivalent resistance and capacitance of the combination.

350. Circuits Having Resistance, Inductance, and Capacitance.—In a circuit having a condenser and a coil in series, there will be three parts in the total E.M.F. The instantaneous value of the impressed E.M.F. is

$$e = Ri + L\frac{di}{dt} + \frac{q}{C}$$

$$= RI \sin \omega t + L\omega I \cos \omega t - \frac{I}{C\omega} \cos \omega t.$$

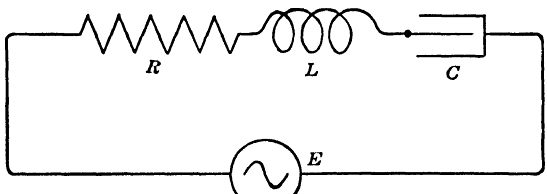


FIG. 176.—Resistance, self-inductance, and capacitance in series.

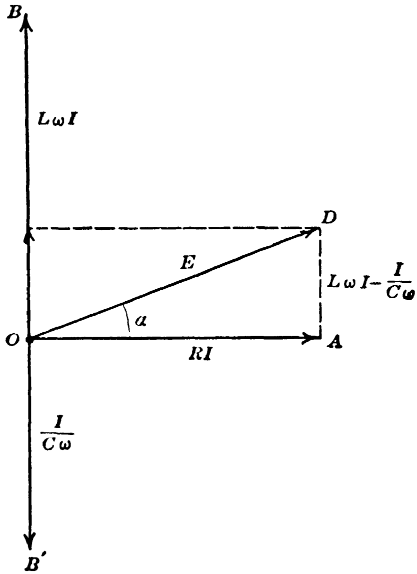


FIG. 177.—E.M.F. diagram for a circuit containing resistance, self-inductance, and capacitance in series.

There will thus be three components to be laid off in the vector diagram, *viz.*, RI as in Fig. 160 or Fig. 171, with $L\omega I$ drawn 90 deg. ahead of RI, as shown in Fig. 160, and $\frac{I}{C\omega}$ drawn 90 deg. behind RI as was done in Fig. 171. This is shown in Fig. 177. These three vectors can be combined by obtaining the resultant of any two, and then combining this resultant with the third one. In Fig. 177 the $L\omega I$ and $\frac{I}{C\omega}$ are first combined, giving a resultant, $L\omega I - \frac{I}{C\omega}$, which is combined with RI giving the final resultant, OD. This is the maximum value of the impressed E.M.F. and its value is given from Fig. 177 as

$$E = OD = I\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}.$$

In the case illustrated by Fig. 177, this resultant E.M.F. is ahead of the current by the angle AOD, and the circuit behaves like a slightly inductive circuit. Evidently, there is a wide range of possibilities in such a circuit, and the current may be ahead or behind the impressed E.M.F. according as $\frac{1}{C\omega}$ or $L\omega$ has the larger value.

351. Series Resonance in an Electrical Circuit.—In a circuit containing a condenser in series with a low-resistance coil of considerable inductance, the diagram of Fig. 177 will be modified by having OA very short. For large values of ω, the resultant, OD, will be nearly equal to $L\omega I$, while for small values of ω, this resultant E.M.F. is nearly the same as $\frac{I}{C\omega}$. At some intermediate value of ω we will have

$$L\omega I = \frac{I}{C\omega}$$

and for this condition the resultant E.M.F. is only

$$E = RI.$$

Thus when R is small, a small E.M.F. will maintain a large current through the coil and condenser, with a correspondingly large E.M.F. across each. This does not imply the expenditure of much power, because the current is so far out of phase with the E.M.F. that the power factor (cos θ, see Art. 372) is nearly zero. The circuit is said to be in resonance for this particular frequency, $n, \left(= \frac{\omega}{2\pi}\right)$ of the impressed E.M.F.

352. Parallel Resonance.—Another arrangement that shows electrical resonance consists of a condenser in parallel with a self-inductance as shown in Fig. 178. The E.M.F. diagram for this portion of the circuit is shown in Fig. 179. Let OA represent to scale the value of the impressed E.M.F. between O' and A'. This, then, is the hypotenuse of the E.M.F. triangle for the coil, the other sides of which are constructed with the aid of the dotted circle. The current is in phase with R_1I_1 and is represented by the line OI_1.

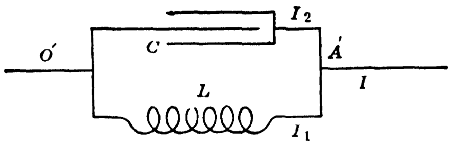


FIG. 178.—Parallel resonance. Alternating current for which $CL\omega^2 = 1$, cannot pass through this sifter.

The same E.M.F., E, is also impressed on the condenser, and the E.M.F. triangle for this part of the circuit is constructed in the upper half of the dotted circle. A small resistance, R_2, is

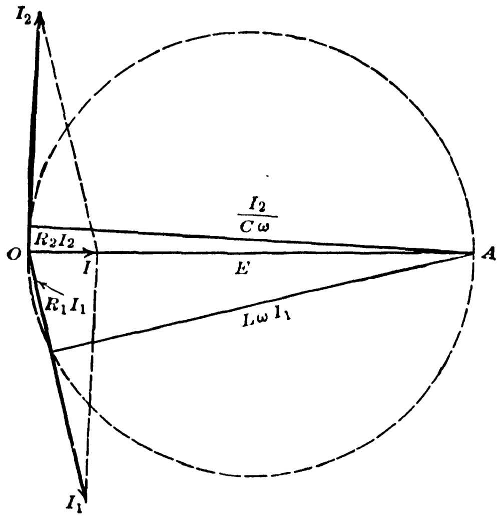


FIG. 179.—Parallel resonance with a capacitance C and an inductance L.

indicated in series with the condenser, and the condenser current is in phase with R_2I_2. When $R_2 = 0$, this current is 90 deg. ahead of E. The resultant current in the main line is shown by I.

When the frequency of the impressed E.M.F. is low, most of the current will flow through the coil. As the frequency increases, the condenser current increases. At the same time I_1 decreases, and this has a double effect. In the first place the resultant, I, comes nearer to OA; and in the second place, as R_1I_1 becomes less, it is more nearly in line with R_2I_2, which results in a very small value for I. Thus for a certain frequency the main current, I, is very small, while the current in the coil and condenser is large. This is called *parallel resonance.* Since only a small current is in phase with the impressed E.M.F., only a small amount of power is expended.

This combination of inductance and capacitance can be looked upon as an arrangement whereby a small main current is greatly multiplied by resonance to give a large current in the coil and condenser; or it may be considered as a *sifter* that will nearly stop current of this particular frequency while letting currents of all other frequencies pass through.

353. Frequency Sifters or Filters.—In making measurements of inductances and capacitances it is often essential that the alternating-current source shall supply a pure sine wave current. In other words, the frequency of the current must have only a single value, that of the fundamental, with all of the harmonics absent. Even when the frequency does not enter in the measurement as an explicit factor, a definite value is better, since the inductance or the capacitance that is being measured may not have the same value for different frequencies.

By means of a wave form filter or sifter, it is possible to pick out or greatly amplify a single frequency while suppressing the other frequencies that may be present. This is done by resonating circuits similar to Fig. 178, in which the desired frequency is selected and passed on to the bridge, usually by an air-cored transformer, while presenting a high impedance to the other frequencies.

CHAPTER XVII

ALTERNATING-CURRENT MEASUREMENTS

354. Sources of Alternating Current.—The alternating current used for power and lighting has a frequency of about 60 cycles per second (in some localities perhaps twice this). In alternating-current measurements requiring a current of several amperes, the most practical source is such a power supply. For more refined methods a higher frequency is desirable, for two principal reasons. Inasmuch as the effects of inductance and capacitance are due to the terms $L\omega$ and $C\omega$, it is seen that these quantities are more prominent when ω has a large value. And if a telephone receiver is used to determine the balance of a bridge, the combination of ear and telephone is more sensitive to a frequency of about 1,000 cycles per second ($\omega = 6,000$). For frequencies of 1,000,000 per second, the effects of inductance and capacitance are greatly increased, but this frequency is inaudible.

In some instances specially designed dynamos have been built with a large number of poles to give a frequency of 1,000 or 2,000 cycles.

355. Microphone Hummer.—A convenient, and not very expensive generator is shown in Fig. 180. This consists of an electrically driven tuning fork, which maintains the constant frequency of 1,000 cycles. The interrupted direct current passes through the primary of a transformer, the secondary of which is part of a circuit with sufficient capacitance to resonate freely at the frequency of the tuning fork. This resonance circuit is not joined directly to external connections, as this might interfere with the resonance, but by a second transformer the power can be taken for use with a

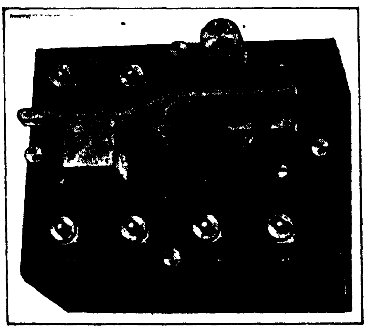


FIG. 180.—Microphone hummer.

bridge or other apparatus. The output is about 0.06 watt (0.012 ampere at 5 volts), which is sufficient for many measurements. When the bridge and telephone are used in a quiet room, very satisfactory balances can be obtained.

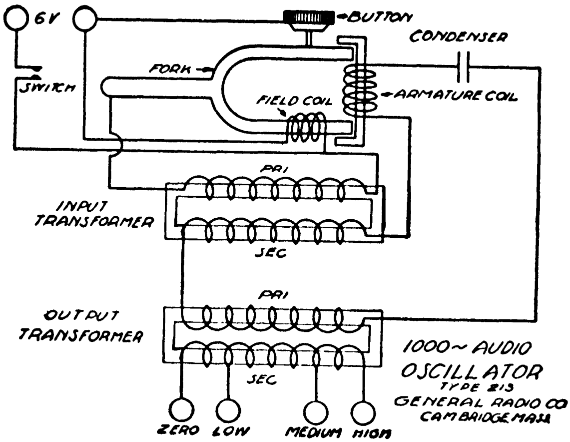


FIG. 180(*a*).—Diagram of the electrical circuits in the 1,000 cycle oscillator of Fig. 180.

An ordinary small induction coil with an automatic vibrator to interrupt the primary current can be used in some cases. The wave form of the secondary current probably will contain several harmonics, and it is more difficult to obtain complete silence in the telephone at the balance point.

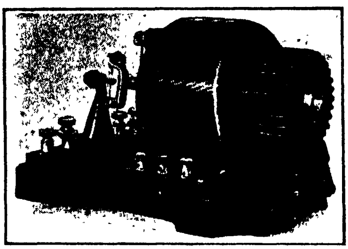


FIG. 180(*b*).—Constant speed high frequency generator.

355(*a*) High Frequency Generator.—This generator is driven by a universal motor which can be operated on 110 volts alternating or direct current. Its speed is maintained constant to ½ per cent by means of a governor which was developed in connection with various kinds of constant speed devices. The wave form of this generator is very symmetrical, although in a magnetic circuit containing iron it is impossible entirely to eliminate harmonics. By means of a suitable filter, a relatively pure sine wave can be obtained. The governor is adjusted to hold the frequency at 1,000 cycles, and small adjustments in this frequency are possible by changing the tension in the governor spring.

355(*b*) Electron Tube Generator.—An electron tube can be made to set up oscillating currents having a frequency that is determined by the inductance and capacitance of the oscillating circuit. The frequency can thus be adjusted to almost any desired value, and such an arrangement has the advantage of being entirely silent in operation (see Fig. 113), Art. 187

356. Alternating-current Detectors. *Telephone Receivers.*—The ordinary galvanometer cannot be used to indicate the balance of an alternating-current bridge, and most alternating-current galvanometers lack sufficient sensitiveness for this purpose. A telephone receiver is a very satisfactory instrument, although it does not indicate which way the bridge should be changed to approach nearer to the balanced condition. With a headset of high-grade Baldwin or Brandes receivers a very small alternating current can be detected.

Electron Tube Amplifiers.—In Art. 182 it was shown that an electron tube can be arranged to amplify the variations in a current. By using such an arrangement between the bridge and the telephone the balance can be set much closer. It may even be desirable to use two amplifying tubes in series if the balance point gives silence in the receivers when used alone. With the two-stage amplifier used with the telephone receivers a very definite setting of the bridge is possible. In making such a setting the variable resistance, capacitance, or inductance should be continuously variable and the setting varied back and forth from one side of the balance point to the other until the point of minimum sound is located. This cannot be done with the plug form of resistance or condenser.[1]

357. Comparison of Two Self Inductances. *Coils in Series.*—When two coils are placed in two adjacent arms of a Wheatstone bridge it is possible so to adjust the values of the resistances and the inductances that no current will flow through the galvanometer or telephone branch when an alternating E.M.F. is applied to the circuit. It is easy to see what is happening in each branch of the bridge when alternating currents are flowing through them. First let us consider the case where the two inductances to be compared are connected in series, as indicated in Fig. 181.

[1] See further, TERRY, "Advanced Laboratory Practice in Electricity and Magnetism."

The graphical representation of this part of the circuit is then shown by the two E.M.F. triangles, AEC and CFD, Fig. 181(a).

The corresponding diagram for the non-inductive circuit $A'B'D'$ reduces to a straight line which begins at A and ends at D, since this circuit is subjected to the same E.M.F. as $A'C'D'$.

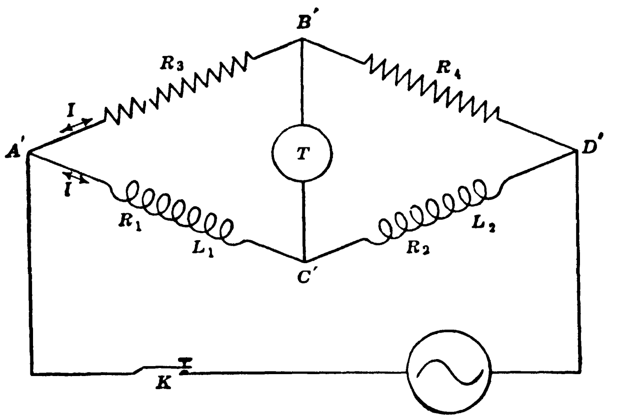


FIG. 181.—Inductance bridge with L_1 and L_2 in series.

This is shown by ABD. The E.M.F. impressed on the telephone is shown by BC, provided the telephone current is negligible compared with the currents through the other parts of the bridge.

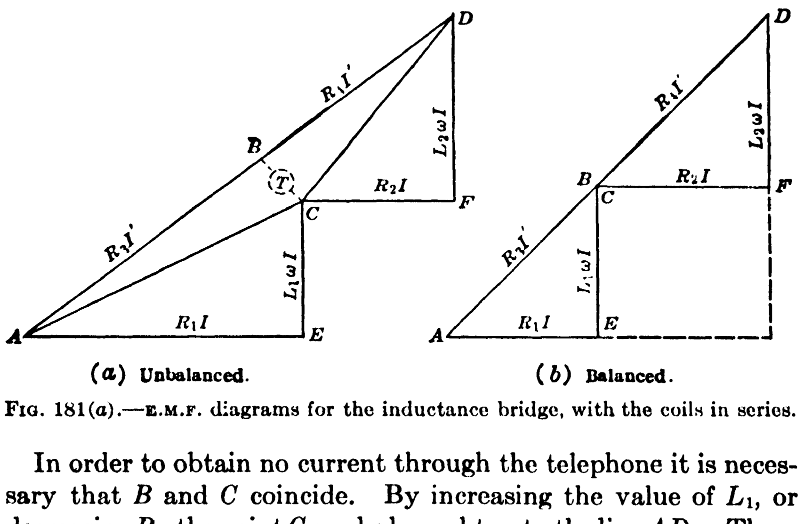


FIG. 181(a).—E.M.F. diagrams for the inductance bridge, with the coils in series.

In order to obtain no current through the telephone it is necessary that B and C coincide. By increasing the value of L_1, or decreasing R_1, the point C can be brought onto the line AD. Then, by varying R_3, the point B can be moved along AD until it coincides with C, as shown in Fig. 181(a) b. Then from similar triangles

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

If L_1 is variable, it is often convenient to set $R_1:R_2 = R_3:R_4$ by a direct-current balance, or otherwise, and then with alternating currents it is only necessary to vary L_1 to obtain a balance. But this is not necessary. By varying R_3 with alternating currents, B can be moved along AD until it is somewhere nearly opposite to C, Fig. 181(a) a. Then, by varying L_1, C is brought nearer to B. A further change of R_1 brings B nearer to C, and after a few such adjustments, B and C are brought together.

From the construction of the figure it is seen that the effect of a 1 per cent change in L_1 upon the position of C is greatest when C is midway between A and D. Therefore, for the greatest sensitiveness of the bridge the impedances of the arms should be as nearly equal as practicable.

358. Comparison of Two Self-inductances. *Coils in Parallel.* When the two inductances are placed in adjacent and parallel

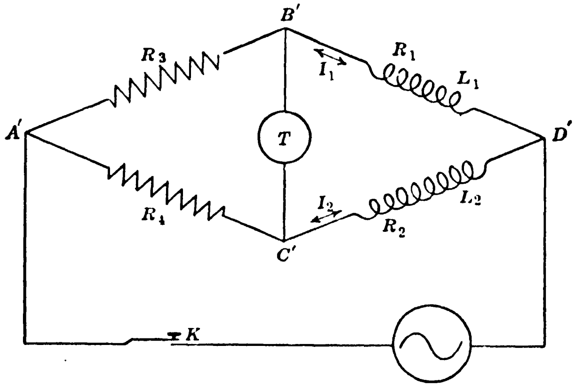


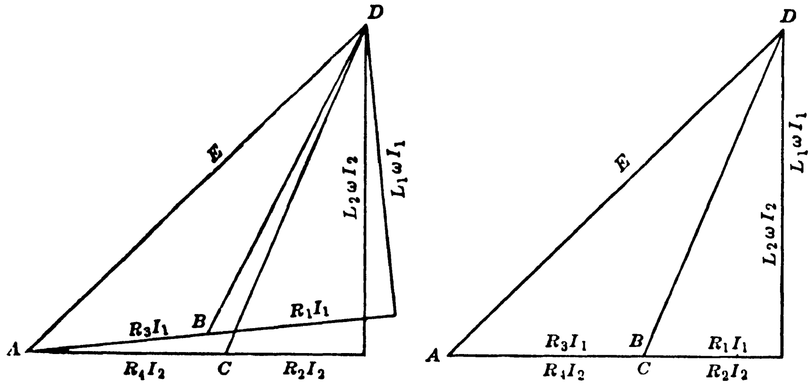
FIG. 182.—Inductance bridge with L_1 and L_2 in parallel.

arms of the bridge, as shown in Fig. 182, there is a non-inductive resistance in series with each coil. The diagram for the circuit $A'C'D'$ is shown by ACD, Fig. 183(a). The corresponding diagram for $A'B'D'$ is shown by the diagram ABD. For no E.M.F. on the telephone, B must be brought to coincide with C. This can be done as in the previous case, by adjusting, first R_4, and then L_2, until the minimum sound is given by the telephone.

When B is brought to C, the two diagrams coincide throughout, Fig. 183(b), and

$$\frac{L_1}{L_2} = \frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

It is often necessary to add some resistance to R_1 or R_2 in order that the ratio of these resistances shall be the same as the ratio



(a) Unbalanced. (b) Balanced.

FIG. 183.—E.M.F. diagrams for the inductance bridge, with the coils in parallel.

of the inductances. In other words, so that the E.M.F. diagrams for the two inductances shall be similar triangles.

The most sensitive arrangement is, as before, when AC and CD are nearly equal.

359. Comparison of Two Capacitances by Bridge Method.—When the condensers are placed in two arms of the bridge with a non-inductive resistance in series with each, as shown in Fig. 184, the arrangement amounts to two circuits in parallel. The E.M.F. triangles for these circuits are shown in Fig. 185, where E represents the E.M.F. applied to the bridge. It is supposed that there are no resistances in the condenser arms of the bridge. In this case the points B and F represent the points B' and F' where the telephone is connected. Since B and F are some distance apart in Fig. 185, it means that a corresponding voltage is applied to the telephone circuit. But by increasing R_2, the point F will be moved along the circle towards B, and, for the proper resistances, F and B can be made to coincide. There will then be no current through the telephone, and the bridge is said to be balanced.

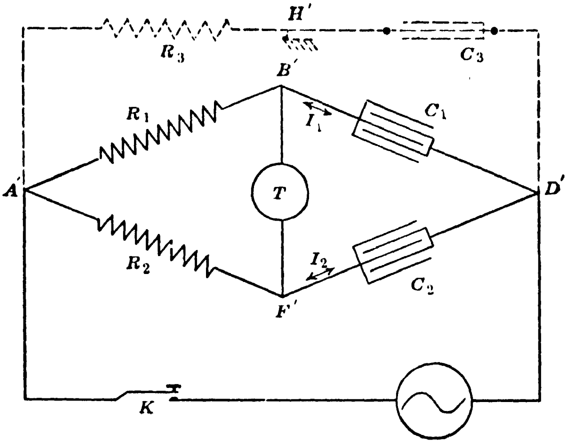


FIG. 184.—The capacitance bridge.

In this balanced condition the triangles ABD and AFD have been made similar, so that

$$R_1I_1 : R_2I_2 = \frac{I_1}{C_1\omega} : \frac{I_2}{C_2\omega}$$

or,

$$C_1 = C_2\frac{R_1}{R_2}.$$

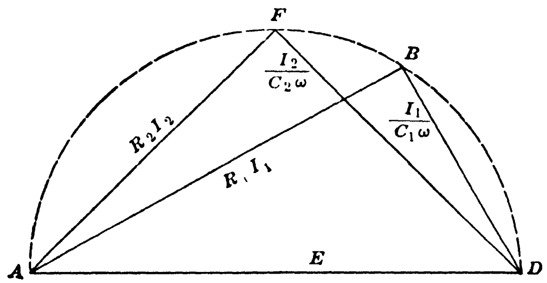


FIG. 185.—E.M.F. diagram for the capacitance bridge. By decreasing R_1, B is brought nearer to F and for a balance the two triangles coincide.

From the figure it appears that a 1 per cent change in R_2 will change the position of F by a greater amount when AF and FD are equal than when they are widely different. Therefore, for greatest sensitiveness, R_2 should be about equal to $\frac{1}{C_2\omega}$.

360. Grounding the Bridge.—When a current flows through a bridge the various parts of the apparatus are brought to different potentials, as shown in Art. 198. If these parts have any electrostatic capacitance between themselves and the surroundings, it will require the addition of corresponding charges thus to change the potentials. If any of these charges flow in through the telephone receivers, perhaps to charge the receivers themselves, there will not be silence when the bridge is balanced. Since the observer is usually connected to the earth, the points of the bridge connected to the telephones should be at zero potential also. If F' were grounded, the potential of the telephone would remain at zero, due to the addition of sufficient charges at this point, but such additional charges are not desirable in the bridge.

The same effect can be obtained by joining another circuit, $A'H'D'$, in parallel with the bridge, as shown by the dotted part of Fig. 184. This circuit is made similar to the bridge by setting $R_3 = \frac{R_1C_1}{C_3}$ with the aid of the telephone receiver connected temporarily between B' and H' after the bridge is approximately

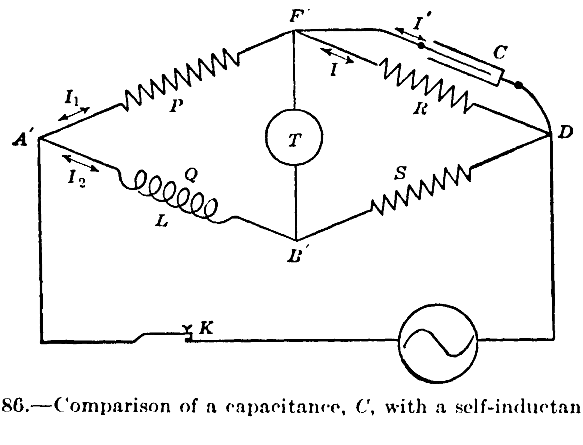


Fig. 186.—Comparison of a capacitance, C, with a self-inductance, L.

balanced. The point H' is then grounded, and this keeps B' and F' at zero potential also, without the addition of extra charges at these points.

Other bridges can be grounded in this manner by using a circuit of resistance, inductance, and capacitance similar to the bridge itself. In case it is desired to keep some other point of the bridge at zero potential, the corresponding point on the auxiliary circuit is grounded.

361. Comparison of an Inductance with a Capacitance.—It is often very desirable to compare an inductance with a standard condenser, and this is readily done with an alternating-current bridge. Inasmuch as the balance of the capacitance bridge, as well as the balance of the inductance bridge, is obtained by adjusting a resistance until B, Fig. 183(a) and Fig 185, is made to coincide with C or F it is possible to arrange a bridge in which a capacitance on one side is balanced by an inductance on the other. In Maxwell's method this is done by placing a resistance in parallel with the condenser and using this combination as one arm of the bridge. This makes it possible for currents of any frequency to flow through the bridge.

In drawing the E.M.F. diagram it will be simplest to solve first the parallel circuits between F' and D', in Fig. 186 (see Art. 349). R' and C' denote the equivalent resistance and capacitance of this arm of the bridge.

The currents I and I' through R and C are shown in Fig. 187 in the usual way, the impressed E.M.F. over the branch $F'D'$ being represented by FD. The resultant current, I_1, through this branch is ahead of the E.M.F. impressed on this part of the circuit by the angle α, where

$$\tan \alpha = \frac{I'}{I} = \frac{EC\omega}{\frac{E}{R}} = RC\omega \tag{1}$$

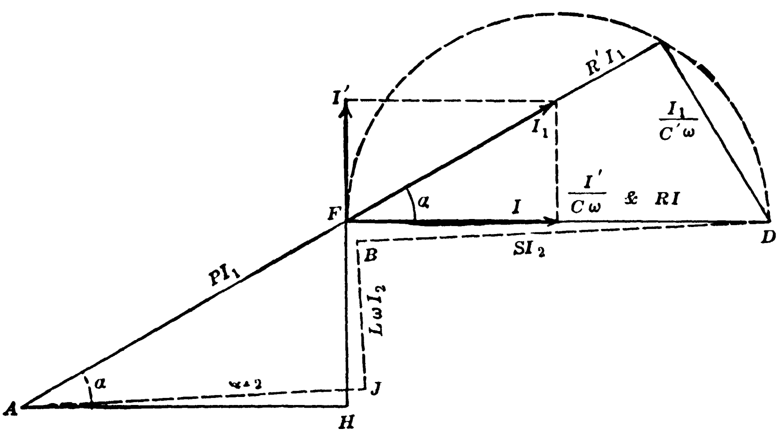


FIG. 187.—E.M.F. diagram for the comparison of C and L. The dashed lines show the condition when the bridge is unbalanced.

Since the non-inductive arm $A'F'$ carries this resultant current, I_1, the E.M.F. over this arm is in phase with I_1, as shown by AF.

The diagram for the inductive branch $A'B'D'$ presents no difficulties. In general, it would be as shown by the dashed lines ABD. For a balance of the bridge, L and Q must be varied to bring B into coincidence with F. This also means that J falls at H and the diagram is shown by the full lines $AHFD$. The phase angle HAF is equal to α and is given by

$$\tan HAF = \frac{L\omega}{Q} \tag{2}$$

when L and Q are the values that give the balance.

Comparing (1) and (2),

$$RC\omega = \frac{L\omega}{Q}$$

or

$$L = RQC \tag{3}$$

Equating equal parts from the two diagrams gives

$$QI_2 = PI_1 \cos\alpha \tag{4}$$

and

$$SI_2 = RI. \tag{5}$$

Since $I = I_1 \cos\alpha$, dividing one equation by the other gives

$$\frac{Q}{S} = \frac{P}{R} \tag{6}$$

as a second relation that exists when the bridge is balanced.

If Q is not accurately known, its value is given by (6), and then (3) becomes

$$L = PSC. \tag{7}$$

This method is conveniently used in the calibration of a variable self-inductance. With the bridge set for the direct-current balance (6), the inductance can be varied until the alternating balance is obtained. This value of L is then given by (7).

362. Anderson's Method for Comparing a Fixed Inductance with a Standard Capacitance.—In Maxwell's method, given above, the final balance was obtained by varying the self-inductance of the bridge arm, $A'B'$, which does not disturb the resistance ratios. When L is fixed, it is not easy to obtain a balance, and usually it is necessary first to obtain the direct-current balance, then try the alternating current, and repeat this process until a value of PS is found that will give a balance for both the direct and alternating current.

Anderson's modification of this arrangement makes it possible to adjust the conditions for a balance with alternating current without disturbing the resistance arms of the bridge. This is accomplished by adding a resistance, r, (Fig. 188), in series with the condenser, and then changing the telephone connection from F' to H'. The resistance balance is then left undisturbed while the effect of the condenser is varied by changing r.

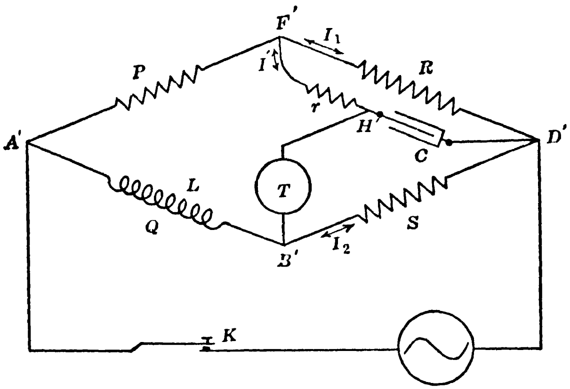


FIG. 188.—Anderson's method for comparing fixed values of C and L.

The conditions in the unbalanced state are shown in Fig. 189 (a). The parallel branches between D' and F' give the E.M.F. triangles DHF and DF, with the currents I' and I_1 as shown. Both of these currents flow through P, giving rise to PI' and PI_1, the vector sum of which is AF. $AJBD$ represents the E.M.FS. along $A'B'D'$. By varying r, the line DH can be brought to coincide with DB. By varying S, BJ can be made to coincide with HE. When H and B are together, there is silence in the telephone.

When the bridge is balanced (Fig. 189(b)),

$$SI_2 = \frac{I'}{C\omega} \tag{1}$$

and

$$L\omega I_2 = rI' + PI' + (EJ) \tag{2}$$

From similar triangles,

$$(EJ) = QI_2\frac{rI'}{I'/C\omega} = QI_2rC\omega \tag{3}$$

and rewriting (2) with the help of (1) and (3) gives

$$L\omega I_2 = rSC\omega I_2 + PSC\omega I_2 + QrC\omega I_2$$

from which

$$L = PSC + rC(S + Q).$$

When $r = 0$, this arrangement becomes the same as for Maxwell's method and B falls on F. For larger values of L

the addition of r makes it possible to bring H coincident with B. It is desirable, therefore, to set PSC a little below the expected value of L and then adjust r to give the final balance.

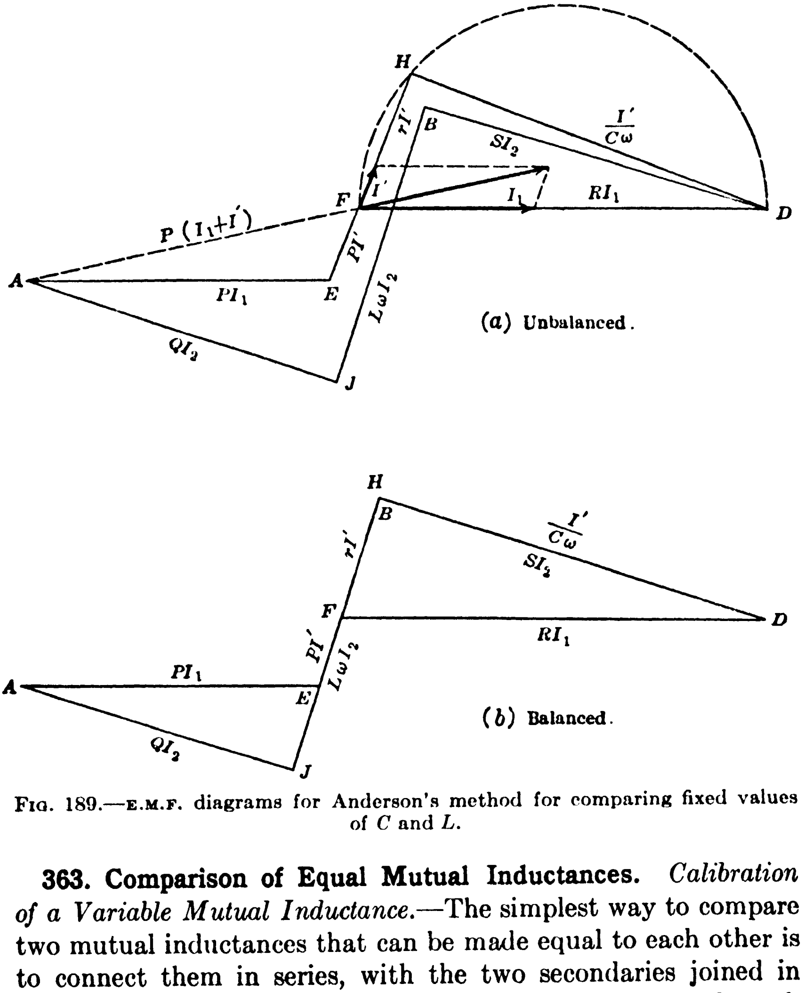


FIG. 189.—E.M.F. diagrams for Anderson's method for comparing fixed values of C and L.

363. Comparison of Equal Mutual Inductances. *Calibration of a Variable Mutual Inductance.*—The simplest way to compare two mutual inductances that can be made equal to each other is to connect them in series, with the two secondaries joined in opposition to each other. When the same current flows through the primary coils, the E.M.FS. induced in the secondaries are proportional to the mutual inductances of the coils. If the two secondaries are joined in series with a telephone, or other current detector, the secondary current is

$$i = \frac{MI - M'I}{R_s},$$

where R_s is the total impedance of the secondary circuit. If, now, the mutual inductance of one of the coils can be varied until the telephone shows by its silence that $i = 0$, we have,

$$M = M'.$$

A variable mutual inductance of unknown values can be quickly and easily calibrated by this direct comparison with a variable standard of similar range.

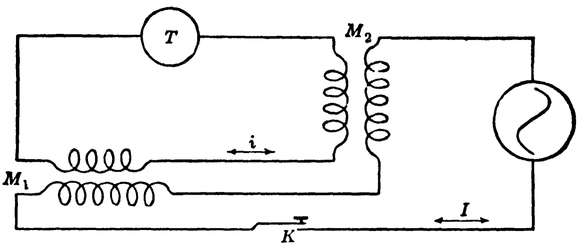


FIG. 190.—Comparison of two mutual inductances that can be adjusted to equality.

Because of the directness and simplicity of this method, it is less liable to errors than the more complicated arrangements. Care must be exercised that there is no mutual inductance between the primary of one pair of coils and the secondary of the other pair, by keeping them far apart and at right angles to each other.

When the two mutual inductances are not equal, it is often possible to add one or more mutual inductances of known values and thus bring the standard to the desired value. When the difference is considerable, one of the following methods can be used.

364. Comparison of Two Unequal Mutual Inductances.—When it is not possible to balance one mutual inductance against the other, as in the preceding method, they can be compared by the arrangement shown in Fig. 191. The two secondaries are joined in series, helping each other, and connected to an external circuit consisting of two resistance boxes in series. The current through this circuit is due to the combined E.M.FS. of both coils. This can also be thought of as one coil supplying the E.M.F. needed to keep the current flowing through its part of the circuit and the other E.M.F. being used to keep the current flowing through the rest of the circuit. If these portions of the circuit can be measured, the relative values of the E.M.FS. are determined.

Since the two primaries are in series and carry the same current, the E.M.FS. induced in their secondaries will be in phase with each other. When K is open, there is only a single current through the secondary circuit. Hence, R_1i and R_2i are in phase with each other, and $L_1\omega i$ with $L_2\omega i$ will be 90 deg. ahead of R_1i.

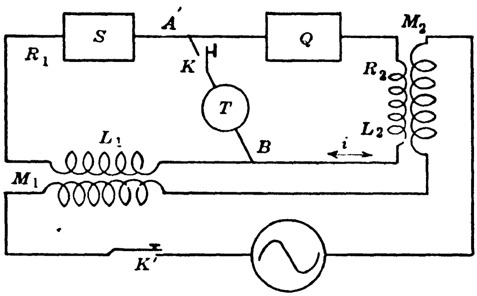


FIG. 191.—Comparison of two unequal mutual inductances. R_1 is the resistance in both S and L_1. R_2 denotes the total resistance in Q and L_2.

The E.M.F. diagram for the secondary circuit is then as shown in Fig. 192(a). Starting at A, the vector R_2i is laid off horizontally to the right, where R_2 denotes the total resistance from

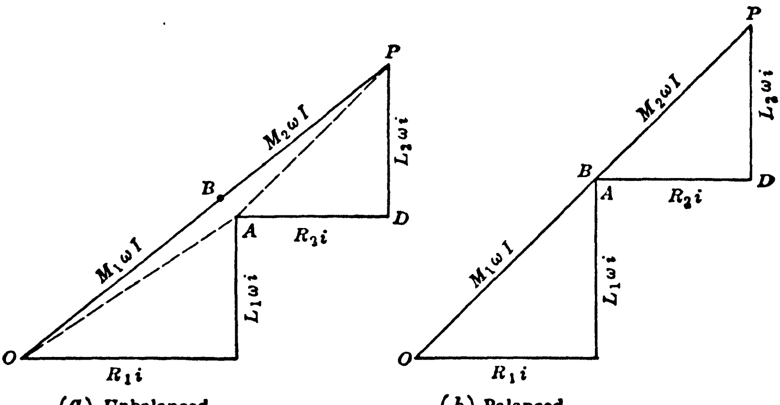


FIG. 192.—E.M.F. diagrams for two mutual inductances in series.

A' to B'. $L_2\omega i$ is 90 deg. ahead of R_2i and, therefore, is drawn as DP. The E.M.F. required to keep i flowing through this resistance and inductance is shown by AP.

In general, the E.M.F., $M_2\omega I$, induced in this coil is not AP, but differs in both phase and amount, as shown by BP. The difference between A and B shows the E.M.F. across the telephone circuit (with K open), and if this is not zero, the telephone will sound when K is closed.

Whether balanced or not, the E.M.F., $M_1\omega I$, induced in the other coil is in phase with $M_2\omega I$ and may well be represented by BO, which is a continuation of PB. From O the vector $R_1 i$ is laid off parallel to $R_2 i$, and $L_1\omega i$ is drawn parallel to $L_2\omega i$, ending at A for the complete circuit.

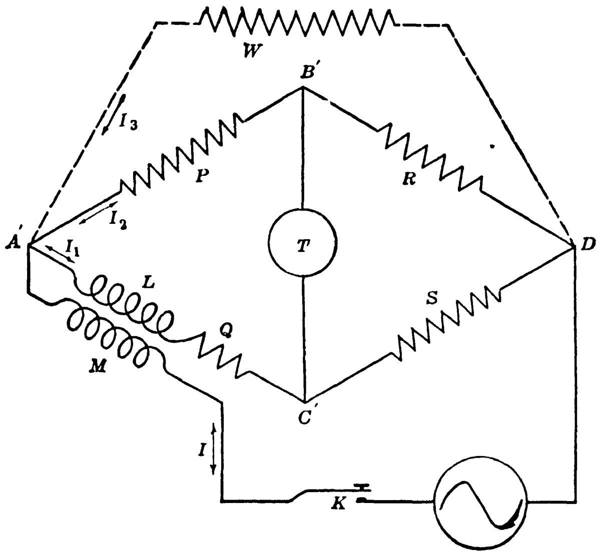


FIG. 193.—Comparison of the self inductance, L, with the mutual inductance, M

By varying R_1 and L_1, the point A can be brought to coincide with B. For this case no current will flow through a telephone joined from A to B, and therefore this diagram, Fig. 192(b), corresponds to the arrangement when there is silence in the telephone. The two triangles are similar and give the relation

$$\frac{R_1}{R_2} = \frac{M_1}{M_2} = \frac{L_1}{L_2}.$$

If R_1 and R_2 are very large resistances, a good balance can be obtained even when L_1 and L_2 are neglected, but in arranging the circuit it is best to fulfil this relation as nearly as possible. For the best balance a variable self-inductance should be included in the secondary circuit and adjusted as may be found necessary to reduce the sound from the telephone.

365. Comparison of the Mutual Inductance between Two Coils with the Self Inductance of the Secondary.—A convenient method for measuring mutual inductance in terms of a self-inductance is shown in Fig. 193. The self-inductance is placed in one arm of the Wheatstone bridge, which ordinarily would make it impossible to obtain a balance because of the E.M.F., $L\omega I$, in this coil. In the present arrangement the main current passes through the other coil, and this induces an E.M.F. of $M\omega I$ in the first coil, $A'C'$. In case $M\omega I$ is just sufficient to balance $L\omega I_1$, the effect of the latter is neutralized and the coil behaves as a non-inductive resistance. The bridge can then be balanced without difficulty.

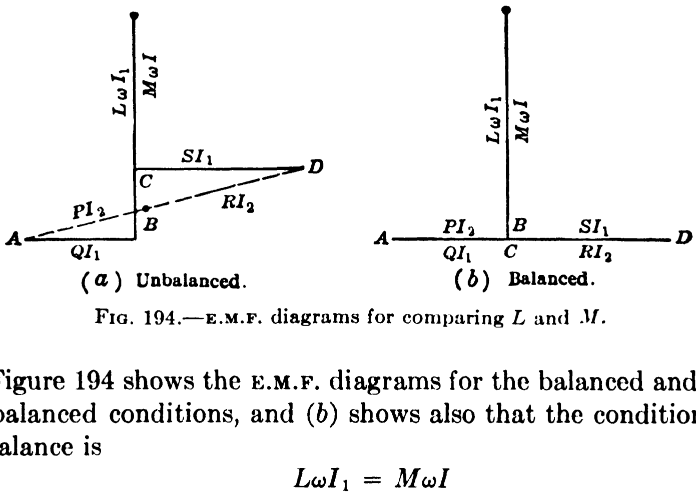


FIG. 194.—E.M.F. diagrams for comparing L and M.

Figure 194 shows the E.M.F. diagrams for the balanced and the unbalanced conditions, and (b) shows also that the condition for a balance is

$$L\omega I_1 = M\omega I$$

in addition to the usual relation

$$\frac{P}{Q} = \frac{R}{S}.$$

When M can be varied continuously by moving one coil with respect to the other, the balance is easily obtained. The value of M giving this balance is, then,

$$M = L\frac{I_1}{I} = L\frac{R}{R + S}.$$

In case M cannot be varied, it is still possible to obtain a balance by changing the current, I, through the primary coil. Ordinarily, this current is the total bridge current, $I_1 + I_2$, but by using another resistance, W, in parallel with the bridge the current, I, can be increased without unbalancing the bridge. Then

$$M = L\frac{I_1}{I} = L\frac{1}{1 + \frac{S}{R} + \frac{Q + S}{W}}$$

from the laws of parallel circuits.

366. Direct Measurement of a Small Self Inductance.— In the measurement of the self-inductance of a coil by the bridge method, the inductances and capacitances of the various parts of the bridge and connections enter as unknown factors. When L is small, this uncertainty may introduce considerable error. By using the method given above (Art. 365), this error can be reduced. When the balance has been obtained by varying M, let the small inductance, L', be added to L and placed so that there will be no mutual inductance between it and either of the original coils. A corresponding resistance can be added to P to maintain the direct-current balance. It will now require a larger value of M to give the alternating-current balance, and the computed value of the self-inductance will be larger than the former value by the amount L'.

367. Comparison of a Mutual Inductance with a Capacitance. One of the most satisfactory methods for comparing a mutual inductance with a standard condenser is a modification of the Carey Foster method so that the arrangement remains balanced at each instant. In this case alternating currents and a telephone receiver can be used. The principal change is in the addition of a resistance, N, in series with the condenser, as shown in Fig. 195.

In this method the induced E.M.F. in the secondary of a pair of coils is made to supply the varying charge in the condenser. The main current divides at A', the part I_2 passing through R, and the other part, I_1, flowing through the condenser, C, the resistances, N and S, and the self-inductance, L. The resistance of the secondary coil is included in the value of S, and N includes whatever equivalent resistance there may be in the condenser. At F' the currents unite and pass through P, the primary of the mutual inductance, M.

The conditions in this circuit are shown by drawing the diagram for the various E.M.FS, taking these in order along the path $A'B'SF'A'$. Starting with N, and laying off the voltage NI_1 as shown in Fig. 196, the voltage over the condenser would be drawn as HB. The voltage over S, which includes the fall of potential in the secondary coil, is drawn parallel to NI_1, since these are in phase, and $L\omega I_1$, is 90 deg. ahead, as shown by QD. There is also in this secondary coil the E.M.F. induced by the primary current. This E.M.F. is in two parts, which are not in phase with each other. The part of the current, I_1, that came through S and L continues on through the primary coil and induces in the secondary an E.M.F. of the amount.

$$e = M\frac{di_1}{dt} = M\omega I_1 \cos \omega t.$$

The maximum value of this E.M.F. is, then, $M\omega I_1$, and since it is a cosine term, it is either in the same phase as $L\omega I_1$, or directly opposite to $L\omega I_1$, according to the direction that I_1 flows through the primary coil.

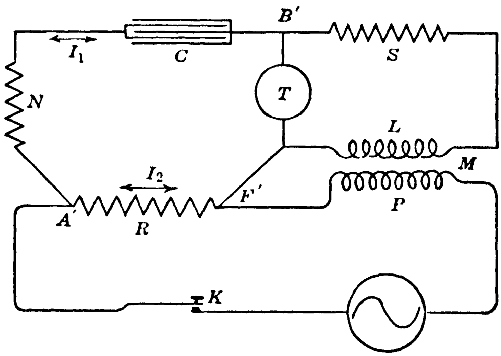


FIG. 195.—Comparison of a capacitance with a mutual inductance.

The other component of the current, I_2, also flows through the primary coil and gives rise to an E.M.F. of $M\omega I_2$ in the secondary, which differs by 90 deg. from RI_2. Since I_2 is behind I_1 by the angle α, $M\omega I_2$ is behind $M\omega I_1$ by the same angle.

Inasmuch as it is desired to obtain a balance, with no current through the telephone circuit, it will be necessary to connect the primary coil into the circuit in the direction that gives $M\omega I_1$ opposed to $L\omega I_1$, and $M\omega I_2$ an angle α behind, as shown in the figure by DJF. The current through R will be behind the condenser current I_1, and therefore RI_2 is represented by AF. As shown above, AFJ is a right angle. For zero E.M.F. across the telephone, it is seen that F must coincide with B. As N is varied, the right-angled lines JFA swing about J as a pivot and

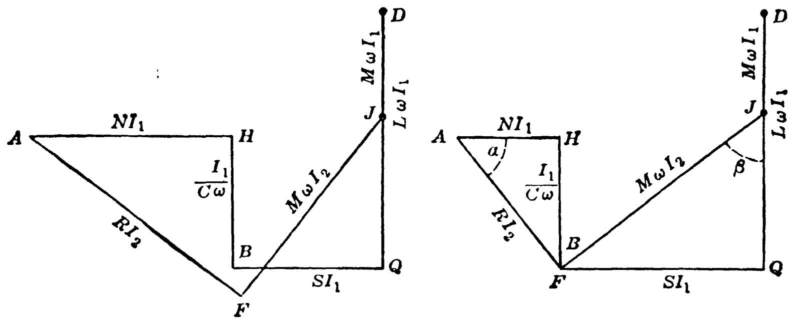
F can be brought nearer to B. As S is varied, B is moved along BQ and can be brought nearer to F. By adjusting first one and then the other, B can be made to coincide with F and the telephone shows this balance by silence.

When balanced, the angle α is equal to the angle β and, therefore, from these similar triangles,

$$\frac{M\omega I_2}{RI_2} = \frac{SI_1}{I_1/C\omega} = SC\omega = \frac{M\omega}{R}$$

or

$$M = SRC$$



(*a*) Unbalanced. (*b*) Balanced.

FIG. 196.—E.M.F. diagrams for the comparison of C and M.

The value of N that is necessary for this balance can be determined as follows: From the diagram

$$\frac{NI_1}{RI_2} = \frac{L\omega I_1 - M\omega I_1}{M\omega I_2}$$

and

$$N = R\frac{L - M}{M}.$$

If the rest of the circuit is well insulated it may improve the balance to have the point F' grounded.

368. Measurement of Frequency.—In most of the arrangements for measuring inductance and capacitance the balance does not depend upon the frequency of the alternating current except in so far as the quantity being measured varies with the frequency. There are a number of bridge methods, however, in which the frequency is one of the quantities contributing to the balance. When the other factors are known, these methods can be used for the measurement of the frequency.

Thus in Maxwell's method an inductance in one arm is balanced by a capacitance in the opposite arm of the bridge. The balance could have been effected with the condenser in the same arm as

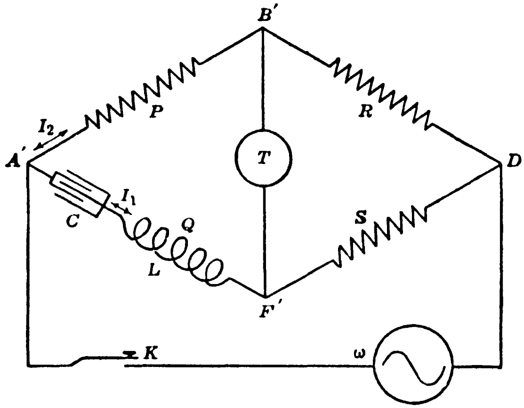


FIG. 197.—Frequency bridge.

the coil, as shown in Fig. 197, and with the other arms of non-inductive resistances. This means that the inductance and the capacitance must be adjusted to make the resultant effect of the

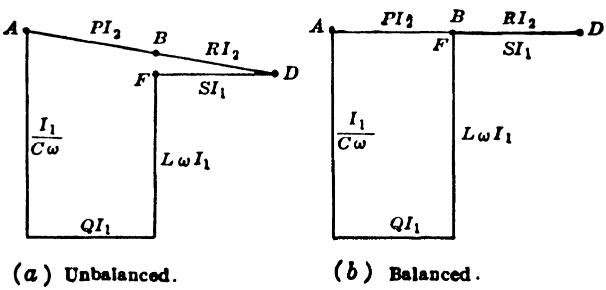


FIG. 198.—E.M.F. diagrams for the frequency bridge.

first arm non-inductive also. The diagrams for the unbalanced and the balanced conditions are shown in Fig. 198, and are self-explanatory.

For the balance

$$L\omega I_1 = \frac{I_1}{C\omega},$$

from which

$$\omega = \frac{1}{\sqrt{LC}},$$

and the bridge is not balanced for other frequencies. Therefore, with a complex wave form, the harmonics will still be heard in the telephone.

Since the frequency, n, is given by the relation,

$$\omega = 2\pi n,$$

its value is

$$n = \frac{1}{2\pi\sqrt{LC}}.$$

A frequency bridge can thus be made with fixed resistances and a variable self-inductance. In addition to the regular scale, the inductance can be calibrated to read directly the value of the frequency. By using a few fixed values of C, the use of the bridge can be extended to a wide range of frequencies.

CHAPTER XVIII

CALIBRATION OF ALTERNATING-CURRENT INSTRUMENTS

369. Alternating-current Ammeters.—In the usual direct-current ammeter the pointer moves up the scale to indicate the value of the current only when the current flows through the instrument in the proper direction. When the current is reversed, the moving coil tends to turn in the opposite direction, and in some instruments the scale is extended on each side of zero so that current in either direction can be measured. When an alternating current is passed through such an instrument the pointer tends to deflect first to one side and then to the other, but not being able to follow the reversals of the current, the resultant effect is merely a vibration of the pointer in some position near zero.

In other forms of instruments, such as the Kelvin balance and the electrodynamometer, the reading is independent of the direction in which the current flows through the instrument and an alternating current will give a definite and steady reading. Of course, this reading will not be the maximum value of the current, nor will it be the arithmetical mean of the instantaneous values. Since the deflection of an electrodynamometer depends upon I^2 (see Art. 160), the resultant deflection will be proportional to the mean square value of the current (see Art. 330). As pointed out above, this value represents the effect of an alternating current for heating, lighting, or power purposes, and an ampere of alternating current is defined (Art. 331) as that amount of alternating current that will deflect an electrodynamometer to the same reading as is given by one ampere of direct current.

Precision ammeters constructed on the electrodynamometer principle are made by the Weston Electrical Instrument Company (see Fig. 199). These ammeters work perfectly well with direct current and they can be calibrated by means of a standard cell and potentiometer (see Art. 155) to read the value of the current in international amperes. When used with alternating currents, the reading will indicate the effective, or root mean square, value of the current to the same degree of accuracy.

Alternating-current ammeters operating on the movable iron principle do not read alike on alternating and direct currents. These ammeters are calibrated by a direct comparison with an

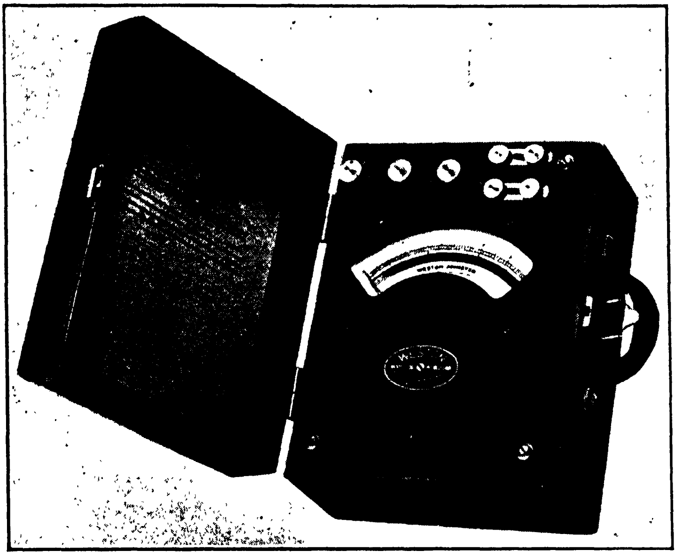


Fig. 199.—Direct-current and alternating-current ammeter.

electrodynamometer standard ammeter that has been checked against a standard cell with the potentiometer. The arrangement for this comparison is shown in Fig. 200.

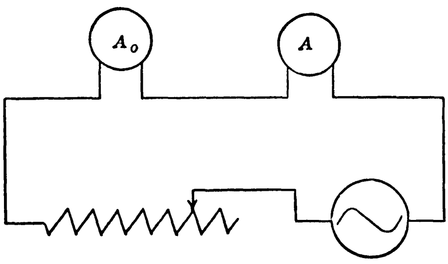


Fig. 200.—Comparison of the ammeter, A with the standard ammeter, A_0.

370. Alternating-current Voltmeters.—Having established the value of an alternating-current ampere, the value of the alternating-current volt follows directly from Ohm's law. Voltmeters are made on the electrodynamometer principle similarly to the ammeters, and these can be calibrated with direct-current voltage by the potentiometer and standard cell method (see Arts. 141 and 143). They will then measure alternating-current voltage (root mean square values) in international volts.

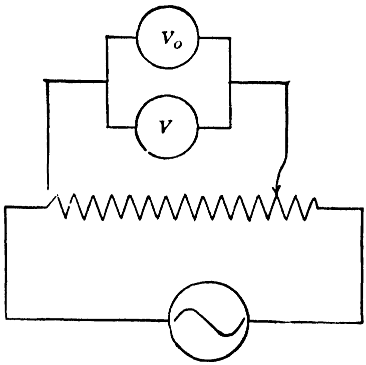


FIG. 201.—Comparison of the voltmeter, V with the standard voltmeter, V_o.

Alternating-current voltmeters operating on the movable iron principle are calibrated by a direct comparison with a calibrated voltmeter of the electrodynamometer type. This comparison is made by connecting the two voltmeters in parallel to the same potential divider, as shown in Fig. 201, and observing simultaneous readings of the two instruments.

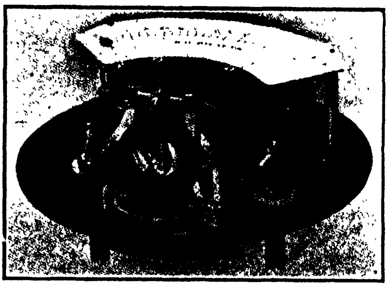


FIG. 202.—Interior of a wattmeter. Current through the fixed coils produces magnetic flux (perpendicular to the paper). Current in the movable coil tends to move across this flux, and the motion of the coil is shown by the attached pointer.

371. Alternating-current Wattmeters.—As shown in Art. 163, an electrodynamometer can be used for the measurement of power. If the high-resistance coil is used as a voltmeter to measure e, while the other coil carries the current, i, the turning effect will be proportional to ei. Since the moving system has quite a bit of inertia, it will average all the turning effects impressed upon it and take up a position corresponding to the average effect.

372. Alternating-current Power.—The power expended in an alternating-current circuit at any instant is

$$W = ei \text{ watts,}$$

where e and i are the values of the E.M.F. and current at this instant. The average expenditure of power is, therefore, the average value of this product. The relation of this average power to the effective values of the current and voltage is readily obtained for the case of sine wave currents. For this case

$$W = ei = [E_o \sin(\omega t + \alpha)] (I_o \sin \omega t),$$

where E_o and I_o denote the maximum values of e and i.

Expanding this expression gives

$$W = E_o I_o (\sin^2 \omega t \cos \alpha + \sin \omega t \cos \omega t \sin \alpha).$$

For a complete cycle, there is a negative product, $\sin \omega t \cos \omega t$, to cancel each positive product, thus giving the last term an average value of zero. The average value of $\sin^2 \omega t$ for one cycle (see Art. 330) is

$$\frac{1}{2\pi}\int_0^{2\pi} \sin^2 \omega t \, d(\omega t) = \frac{\pi}{2\pi} = 0.5$$

and therefore the average power is

$$\begin{aligned} W &= 0.5 \, E_o I_o \cos \alpha \\ &= 0.707 \, E_o \times 0.707 \, I_o \cos \alpha \\ &= EI \cos \alpha \end{aligned}$$

where E and I denote the effective values that would be indicated by a voltmeter and an ammeter. The factor $\cos \alpha$ is called the power factor of the circuit.

373. Comparison of a Wattmeter with an Ammeter and a Voltmeter.—When power is expended in a non-inductive circuit, the value of $\cos \alpha$ is unity and therefore

$$W = EI.$$

In this case the wattmeter can be compared directly with the ammeter and voltmeter as was done in Art. 165 and Fig. 97.

If the resistance in which the power is expended can be accurately determined, the voltmeter can be omitted and the power computed by the relation

$$W = RI^2.$$

374. Calibration of a Wattmeter on an Inductive Circuit.—It is sometimes desirable to check the readings of a wattmeter when it measures the power in an inductive circuit and therefore when the voltage impressed on the shunt circuit of the wattmeter is not in phase with the current through the main circuit.

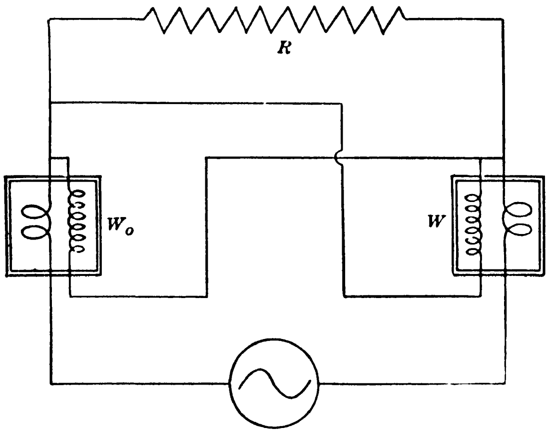


FIG. 203.—Comparison of the wattmeter, W with the standard wattmeter, W_o.

The power expended in an inductive circuit is, as shown above,

$$W = EI \cos \alpha$$

and therefore it cannot be measured with an ammeter and a voltmeter. If the inductive circuit has an alternating-current resistance, however, (see Art. 341), that is unchanged from its direct-current value, the power can be computed from

$$W = RI^2$$

and only the ammeter reading will be required. If the wattmeter is compensated so as not to measure the power expended in its shunt circuit, the ammeter should be placed where it will not include this shunt current in the measured value of I.

375. Comparison of a Wattmeter with a Standard Electrodynamometer Wattmeter.—A wattmeter that is carefully and accurately constructed on the electrodynamometer principle can be calibrated by one of the direct-current methods given in Chap. VIII. It will then measure alternating-current power also and can be used as a standard wattmeter. Other wattmeters can then be directly compared with this standard by connecting both instruments so that each will measure the same power. This arrangement is shown in Fig. 203.

376. Calibration of a Watt-hour Meter.—A watt-hour meter measures the total energy that has passed through it, and this requires a combination of a wattmeter and a time keeper. The latter usually means some kind of running mechanism, the speed of which at each instant is made proportional to the power being expended. A watt-hour meter can be calibrated by connecting it with a standard wattmeter so that the same current and voltage affects each, as shown in the arrangement of Fig. 201. By maintaining the power constant for a measured period of time, the total energy is given by the product of watts times hours, and this product should be recorded on the dials of the watt-hour meter. The time can be measured with a standard clock or a good watch. Usually it will not be necessary to continue the test long enough to observe the change in the dial readings, as the rotations of the faster moving disk can be counted for a period sufficient to determine the rate at which the meter is running.

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