CHAPTER 3

PRICING, INVESTMENT AND ORDER QUANTITY DECISIONS IN COLLABORATION BETWEEN A MANUFACTURER AND A RETAILER

Supply chain models based on intra and inter-component collaboration between manufacturer and retailer are developed. The objective of this research is to obtain the model that provides the maximum profit for the manufacturer and the retailer in the supply chain. This chapter explains the analytical models and numerical results of pricing, investment and order quantity decisions between a manufacturer and retailer. Managerial implications based on the results obtained are also presented.

3.1 Analysis of Pricing, Investment and Order Quantity for a Manufacturer and Retailer

Joglekar et al., (2006) have indicated a number of possibilities for intra- and inter-organizational coordination for the pricing and order quantity decisions in a supply chain. Organizations can benefit from coordination between marketing department that makes the pricing decisions and the operations department that makes the order quantity decisions. Similarly, a supply chain can benefit from the coordination of pricing and order quantity decisions of the manufacturer and the retailer. Inter-organizational coordination for price and order quantity decisions comes from the marketing and the operations perspective. Joglekar et al., (2006) presented a set of eight models of coordination for pricing and order quantity decisions in a supply chain consisting of one manufacturer and one retailer of a product with price sensitive demand. Their models assume that price and order quantity affect the profits of individual channel members and the total channel as a function of product demand elasticity. Gurnani et al., (2007) highlighted that firms in supply chain may compete and co-operate with each other in order to maximize their profits. They considered the scenario where the manufacturer invests in the technology to improve quality, and the retailer invests in selling effort to develop the market for the product. Although a larger investment in technology improves the quality of the product which results in an increased demand potential for the product, the technology investment/quality-improvement costs are directly incurred by the manufacturer only. A retailer has the opportunity to influence final demand by choosing the appropriate selling/promotional efforts.

This research extends the work of Joglekar et al., (2006) to present relative merits of a set of eight different models based on intra and inter collaboration for price, investment and order quantity decisions. The case of a single retailer and single manufacturer with a single product are considered with customer demand as a function of price, retailer's selling effort and manufacturer's quality level. The retailer must determine the optimal retail price for the consumer and optimal order quantity. The demand placed on the manufacturer is same as that for retailer. The retailer sets a price and influences product demand by investing in demand enhancing efforts. The supplier sets a price and invests in quality improvement. The retailer incurs the cost of selling effort and the manufacturer incurs the cost of quality improvement. In all the models considered in the study, it is assumed that both manufacturer and retailer have explicit knowledge of all the relevant cost parameters.

The remainder of the section is organized as follows. Notations and assumptions made in the analysis of eight coordination models are described. Algebraic expressions of investment, price and order quantity for eight different models based on intra and inter collaboration are derived and solution procedure is provided. Optimized results for all eight models for demand as a function of price, selling effort, and product quality level and sensitivity analysis of the demand parameters are provided.

The notations used in the coordination models are:

A	constant, which is the annual demand at $P_1=0$
C_1	retailer's inventory costs
C_2	manufacturer's inventory costs
C	total inventory costs of retailer and manufacturer
D	annual demand for the product at price $P_1=(A-eP_1+\gamma\alpha+\lambda\theta)$
e	coefficient of product's demand elasticity
F_1	retailer's annual fixed costs
F_2	manufacturer's annual fixed costs
h_1	retailer's inventory holding cost per unit of inventory per year
h_2	manufacturer's inventory holding cost per unit of inventory per year
O_1	retailer's ordering cost per order

O_2 manufacturer's ordering processing cost per order of retailer

P_1 price charged by the retailer per unit

P_2 price charged by the manufacturer per unit

Q order quantity

R manufacturer's production capacity

S_2 manufacturer's production setup cost per setup

V_1 retailer's variable costs per unit

V_2 manufacturer's total variable production cost per unit

Z_1 retailer's net profit per unit

Z_2 manufacturer's net profit per unit

Z total net profit of the retailer and manufacturer per unit

α retailer's selling effort

γ influence of retailer's selling effort on demand

θ quality selected by manufacturer

λ impact of product quality on demand

Π_1 retailer's marketing profit per unit

Π_2 manufacturer's marketing profit per unit

Π gross profit of retailer and manufacturer per unit

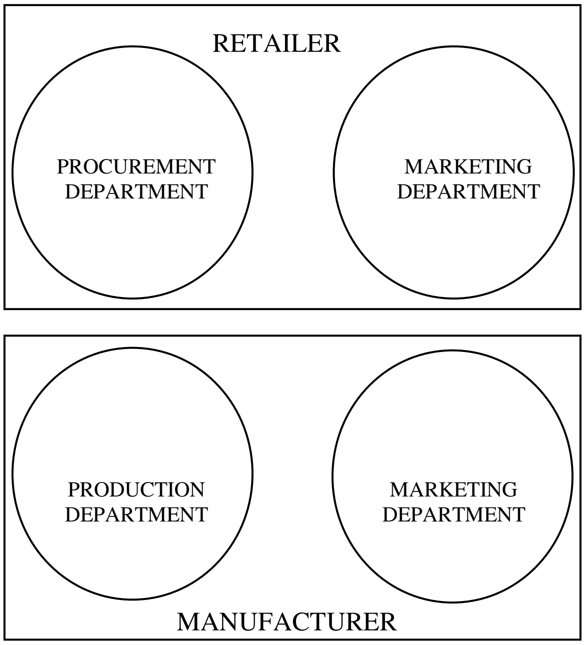
3.1.1 Coordination models

In the Model I, it is assumed that there is no coordination between manufacturer and retailer. In the Model II, on intra coordination at the manufacturer, the manufacturer's goal is to maximize net profit per unit, Z_2, while in the Model III, on the intra coordination at the retailer only, the retailer's goal is to maximize net profit per unit, Z_1. The Model IV discusses the intra organizational coordination at the manufacturer and at the retailer, wherein the manufacturer and retailer independently maximizes their net profits. In Model V, on inter organizational coordination for order quantity, total inventory costs of the manufacturer and retailer are minimized. When there is inter component coordination for investment and price decisions only, as in Model VI, the manufacturer and retailer seek to maximize the joint marketing profit Π. And in Model VI, the manufacturer and retailer seek to minimize the respective inventory related costs C_1 and C_2. When there is inter organizational coordination for order quantity & investment and pricing decisions only, as in Model VII, the manufacturer and retailer seek to minimize joint inventory costs *C*, while the manufacturer and retailer seek to maximize their joint marketing profit Π. Finally when there is total coordination in supply chain, the goal is to maximize total net profit of the retailer and manufacturer per unit, *Z*. Fig. 3.1 presents a schematic summary of eight models considered in this study (Joglekar et al., 2006).

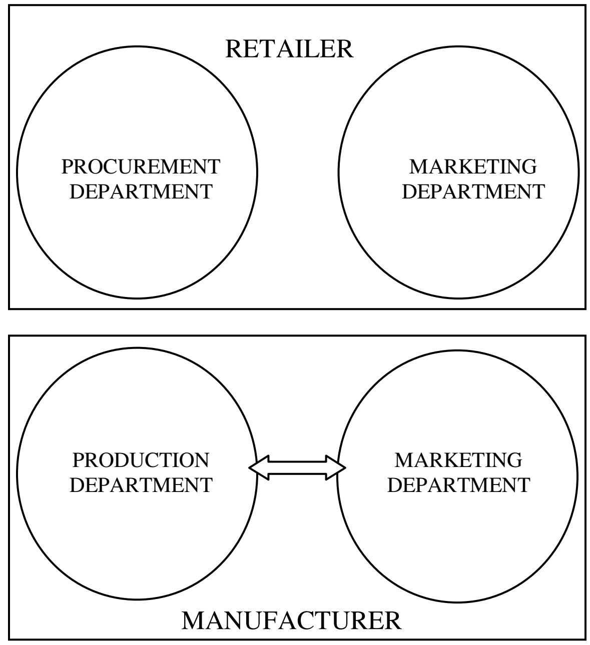
The assumptions made in the analysis are:

(1) The demand rate for the product on the retailer is a linear function of (i) selling price to customer, (ii) buyer's selling effort, and (iii) quality level selected by the supplier. The demand model considered in this work is similar to that used by Gurnani et al., (2007).

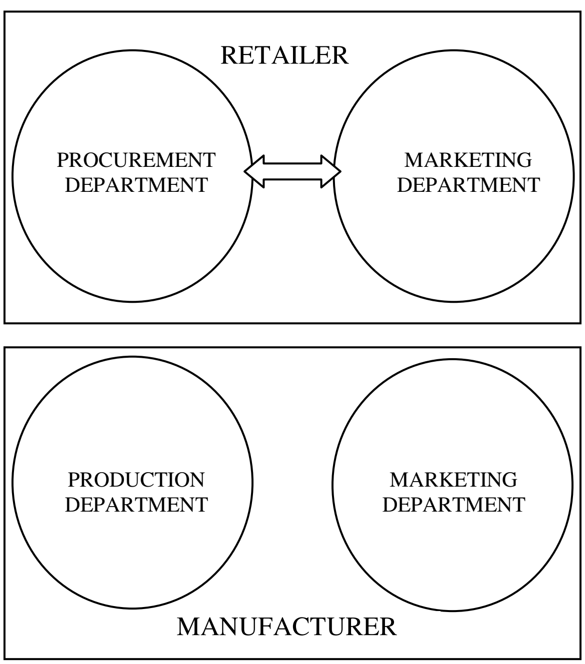
(2) Both the manufacturer and retailer seek to optimize their respective objectives and they agree to coordinate their decisions that represent their joint interests.



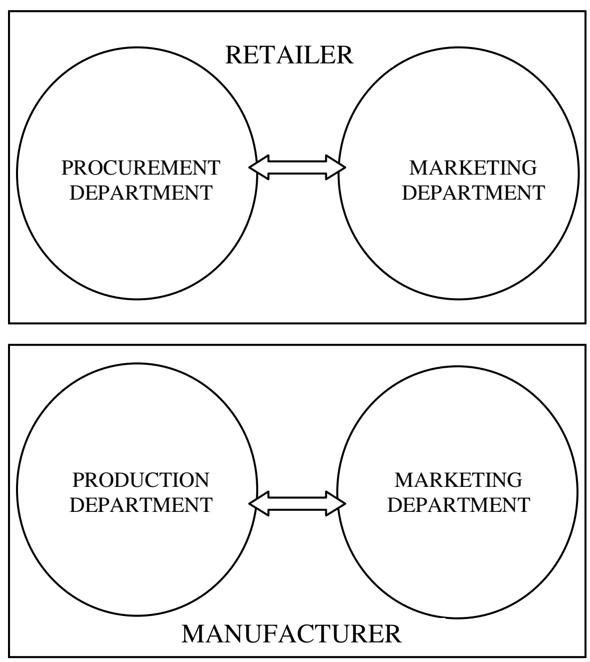
MODEL I - No Coordination
Manufacturer invests in product quality (θ) and sets the price (P_2) for the retailer.
Retailer determines the optimal selling effort (α) and sets the price (P_1)



MODEL II - Intra Coordination Of Manufacturer
Manufacturer optimizes net profit to determine product quality (θ) and price (P_2) for the retailer.
Retailer determines the optimal selling effort (α) and sets the price (P_1)

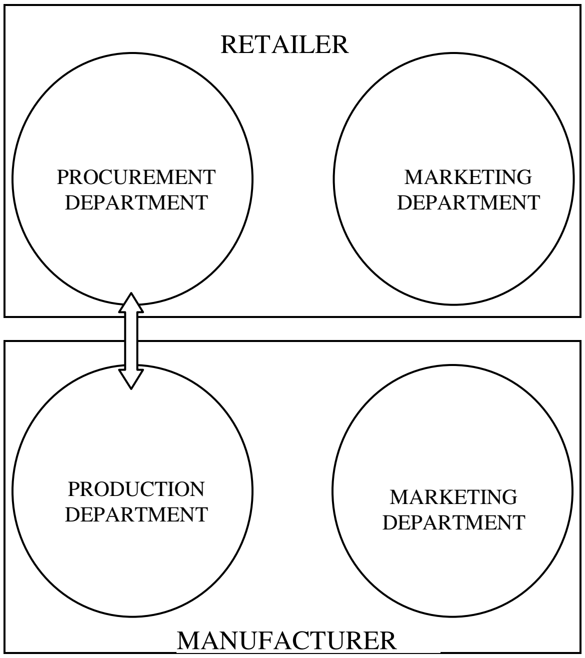


MODEL III - Intra Coordination Of Retailer
Manufacturer invests in product quality (θ) and sets the price (P_2) for the retailer.
Retailer optimizes the net profit to determine optimal selling effort (α) and price (P_1)

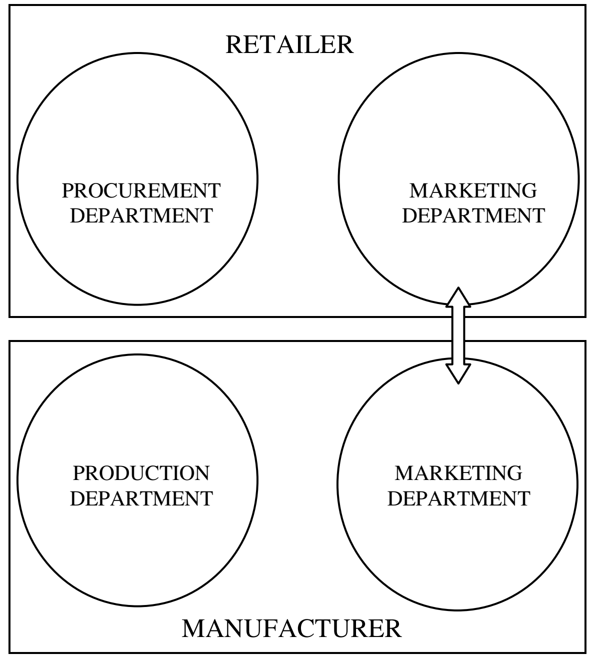


MODEL IV - Intra Coordination Of Manufacturer And Retailer
Manufacturer optimizes net profit to determine product quality (θ) and price (P_2) for the retailer.
Retailer optimizes the net profit to determine optimal selling effort (α) and price (P_1)

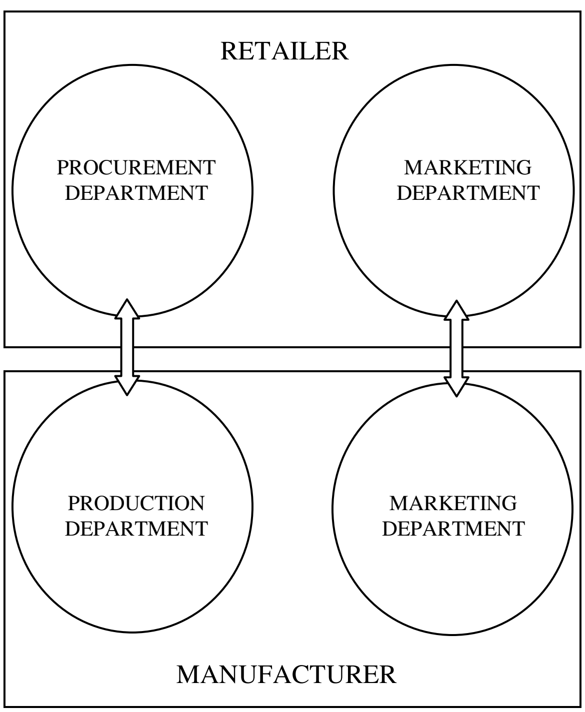
Fig. 3.1 Supply chain models for manufacturer and retailer



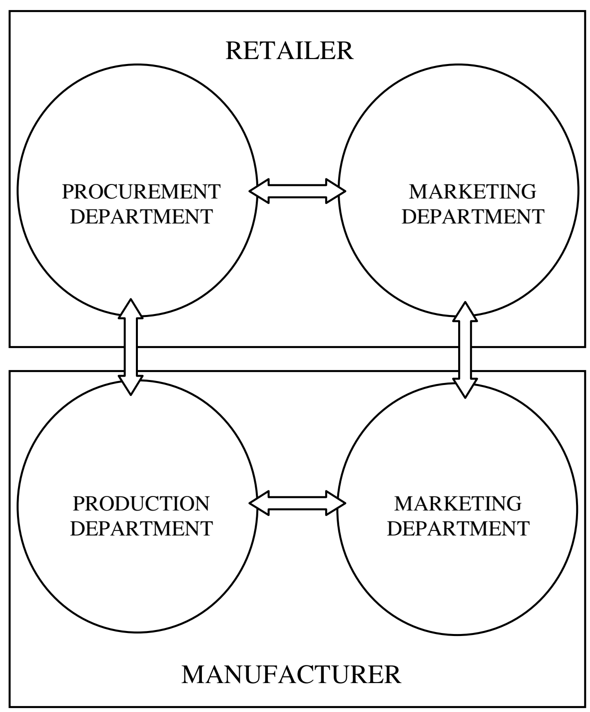
MODEL V - Inter Coordination For Order Quantity
Manufacturer invests in product quality (θ) and sets the price (P_2) for the retailer. Retailer determines the optimal selling effort (α) and sets the price (P_1). Both manufacturer and retailer coordinate their order quantity decisions.



MODEL VI - Inter Coordination For Investment And Pricing
Manufacturer and retailer coordinate their optimal investment and profit margin decisions, product quality (θ), selling effort (α) and price (P_1)



MODEL VII - Inter Coordination For Order Quantity & Investment And Pricing
Marketing and the operations departments of manufacturer and retailer collaborate independently of each other



MODEL VIII - Inter And Intra Coordination Of Manufacturer And Retailer
Marketing and the operations departments of manufacturer and retailer fully collaborate within as well as across the channel.

Fig. 3.1 Supply chain models for manufacturer and retailer…. contd

3.1.1.1 Model I No-coordination

In this model there is no channel collaboration. The manufacturer and the retailer acts independently of the other in selecting its profit margin. The manufacturer determines the investment in product quality and sets the price for the retailer. Subsequently retailer determines the optimal selling effort, order quantity and the price. Both retailer and manufacturer set prices in order to maximize their profits.

The retailer's net profit per unit is (Joglekar et al, 2006),

$$Z_1 = \Pi_1 - C_1 \tag{3.1}$$

where

$$\Pi_1 = P_1\left(A - eP_1 + \gamma\alpha + \lambda\theta\right) - \left(V_1 + P_2\right)\left(A - eP_1 + \gamma\alpha + \lambda\theta\right) - F_1 - \frac{\eta\alpha^2}{2} \tag{3.2}$$

$$C_1 = \frac{\left(A - eP_1 + \gamma\alpha + \lambda\theta\right)O_1}{Q} + \frac{Qh_1}{2} \tag{3.3}$$

The cost of selling effort is $\frac{\eta\alpha^2}{2}$ similar to that used by Gurnani et al., (2007). The condition for the maximization of the retailer's marketing profit is obtained by solving $\frac{\partial\Pi_1}{\partial P_1} = 0$. This results in an optimal value given by;

$$P^*_{1,I} = \frac{A + \gamma\alpha + \lambda\theta}{2e} + \frac{\left(V_1 + P_2\right)}{2} \tag{3.4}$$

Using the expression of P_I from Eq. (3.4), the marketing profit of the retailer can be written as:

$$\Pi_1 = \left[\frac{A + \gamma\alpha + \lambda\theta}{2e} - \frac{\left(V_1 + P_2\right)}{2}\right]\left[\frac{A + \gamma\alpha + \lambda\theta}{2} - \frac{e\left(V_1 + P_2\right)}{2}\right] - F_1 - \frac{\eta\alpha^2}{2} \tag{3.5}$$

Next, the optimal selling effort level that maximizes the retailer's marketing profit is found by solving $\frac{\partial\Pi_1}{\partial\alpha} = 0$, which yield,

$$\alpha^*_I = \frac{\gamma\left[A + \lambda\theta - e\left(V_1 + P_2\right)\right]}{2e\eta - \gamma^2} \tag{3.6}$$

In order to ensure concavity of P^*_I and α^* in Π_1, it is required that $2e\eta - \gamma^2 > 0$ as shown in Appendix A.1.

The retailer's operations department minimizes its inventory related costs by solving $\frac{\partial C_1}{\partial Q} = 0$ and the resulting optimal value is given by,

$$Q^* = \sqrt{\frac{2(A - eP_1 + \gamma\alpha + \lambda\theta)O_1}{h_1}} \tag{3.7}$$

The manufacturer invests in quality improvement efforts which improve the demand potential for the product. The investment in quality improvement increases fixed costs, $\frac{\zeta\theta^2}{2}$. It is considered that the quality level has also an impact on the variable costs, $V_2(1+\nu\theta)$, similar to that analyzed by Gurnani et al., (2007).

The manufacturer's net profit per unit is (Joglekar et al, 2006),

$$Z_2 = \Pi_2 - C_2 \tag{3.8}$$

where

$$\Pi_2 = P_2(A - eP_1 + \gamma\alpha + \lambda\theta) - V_2(1+\nu\theta)(A - eP_1 + \gamma\alpha + \lambda\theta) - F_2 - \frac{\zeta\theta^2}{2} \tag{3.9}$$

$$C_2 = \frac{1}{Q}(A - eP_1 + \gamma\alpha + \lambda\theta)(S_2 + O_2) + \frac{Qh_2}{2}\left[\frac{(A - eP_1 + \gamma\alpha + \lambda\theta)}{R}\right] \tag{3.10}$$

Using the expression of P^*_1 from Eq. (3.4) and α^* from Eq. (3.6), in Eq. (3.9), the marketing profit of the manufacturer is simplified as:

$$\Pi_2 = \frac{e\eta}{(2e\eta - \gamma^2)}[P_2 - V_2(1+\nu\theta)][A + \lambda\theta - e(V_1 + P_2)] - F_2 - \frac{\zeta\theta^2}{2} \tag{3.11}$$

In order to ensure concavity of P_2 and θ in Π_2, it is required that $(2e\eta - \gamma^2) > 0$ and $\left[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right] > 0$ as shown in Appendix A.1.

The manufacturer's marketing profit is maximized by solving $\frac{\partial \Pi_2}{\partial P_2} = 0$;

$$P^*_{2,I} = \frac{A + \lambda\theta - e[V_1 - V_2(1+\nu\theta)]}{2e} \tag{3.12}$$

Using the expression of P^*_2 from Eq. (3.12), the marketing profit of the manufacturer is written as follows:

$$\Pi_2 = \frac{\eta}{4(2e\eta - \gamma^2)}\{A + \lambda\theta - e[V_1 + V_2(1+\nu\theta)]\}^2 - F_2 - \frac{\zeta\theta^2}{2} \tag{3.13}$$

The optimal quality level selected by the manufacturer that maximizes the manufacturer's marketing profit is obtained by solving $\frac{\partial \Pi_2}{\partial \theta} = 0$.

$$\theta^*{}_I = \frac{\eta(\lambda - eV_2\nu)[A - e(V_1 + V_2)]}{[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2]} \tag{3.14}$$

Using the expression of θ^* from Eq. (3.14), the manufacturer's marketing profit is expressed as:

$$\Pi_2 = \frac{\zeta\eta[A - e(V_1 + V_2)]^2}{2[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2]} - F_2 \tag{3.15}$$

For $[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2] > 0$, and $(\lambda - eV_2\nu) > 0$, Π^*_1, Π^*_2, P^*_1, α^*, P^*_2 and $\theta^*$are all decreasing in retailer's cost of selling effort, η, as shown in Appendix A.2. If the retailer has a lower cost of selling effort, retailer provides more selling effort, and also manufacturer provides a higher quality product. Both retailer's price per unit and marketing profit are higher for lower retailer's selling effort. Also manufacturer's price per unit and marketing profit increase with decreasing retailer's selling effort.

Π^*_1, Π^*_2, P^*_1, P^*_2 *and* $\theta^*$are all decreasing in cost of quality, ζ, while α^* increases with cost of quality, ζ, for $[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2] > 0$, and $(\lambda - eV_2\nu) > 0$. If the manufacturer has a higher cost of quality, he is likely to provide lower quality product, while the retailer makes more selling effort. Both retailer's price per unit and manufacturer's price per unit are decreasing with increasing cost of quality. Manufacturer's and retailer's profits are decreasing in increasing cost of quality, ζ, as shown in the Appendix A.2.

3.1.1.2 Model II Intra-coordination of manufacturer

In this model the manufacturer coordinates the decisions on product quality and profit margins. The manufacturer determines the investment in product quality and sets the price for the retailer. Subsequently retailer optimizes selling effort, price and order quantity. In this scenario also, both retailer and manufacturer set prices in order to maximize their profits. Retailer's decisions in this model are similar to that analyzed in Model I.

The manufacturer's net profit, Z_2 per unit is given in Eq. (3.8). Substituting the expression of P^*_1 from Eq. (3.4) and α^* from Eq. (3.6), in Eq. (3.8), the net profit per unit of the manufacturer is given by:

$$Z_2 = \frac{e\eta}{(2e\eta-\gamma^2)}\left(P_2 - V_2(1+\nu\theta) - \frac{(S_2+O_2)}{Q} - \frac{Qh_2}{2R}\right)[A+\lambda\theta - e(V_1+P_2)] - F_2 - \frac{\zeta\theta^2}{2} \tag{3.16}$$

The net profit of the manufacturer per unit, Z_2 is optimized with respect to manufacturer's price per unit, P_2 by solving $\frac{\partial Z_2}{\partial P_2} = 0$;

$$P^*_{2,II} = \frac{A+\lambda\theta}{2e} - \frac{1}{2}\left(V_1 - V_2(1+\nu\theta) - \frac{(S_2+O_2)}{Q} - \frac{Qh_2}{2R}\right) \tag{3.17}$$

The net profit of the manufacturer per unit, Z_2 can be re-written using the expression of P^*_2 from Eq. (3.17), as:

$$Z_2 = \frac{\eta}{4(2e\eta-\gamma^2)}\left[A+\lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]^2 - F_2 - \frac{\zeta\theta^2}{2} \tag{3.18}$$

The net profit of the manufacturer per unit, Z_2 is optimized with respect to manufacturer's optimal quality, θ, by solving $\frac{\partial Z_2}{\partial \theta} = 0$;

$$\theta^*_{II} = \frac{\eta(\lambda - eV_2\nu)\left[A - e\left(V_1 + V_2 + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]}{\left[2\zeta(2e\eta-\gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} \tag{3.19}$$

Using the expression of θ^* from Eq. (3.19), the manufacturer's net profit per unit is expressed as:

$$Z_2 = \frac{\zeta\eta\left[A - e\left(V_1 + V_2 + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]^2}{2\left[2\zeta(2e\eta-\gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} - F_2 \tag{3.20}$$

Lemma 3.1. *When there is no-coordination between the retailer and manufacturer compared to intra-coordination of the manufacturer, the manufacturer invests more in product quality,* $\theta^*{}_I > \theta^*{}_{II}$, *the retailer invests more in selling effort,* $\alpha^*{}_I > \alpha^*{}_{II}$ if $\frac{\left[2\zeta\left(2e\eta-\gamma^2\right)+\eta\left(\lambda-eV_2\nu\right)^2\right]}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta\left(\lambda-eV_2\nu\right)^2\right]}>0$. *However, manufacturer's sets lower price per unit,* $P^*{}_{2,I} < P^*{}_{2,II}$ if $\left[\zeta\left(2e\eta-\gamma^2\right)-\eta\lambda\left(\lambda-eV_2\nu\right)\right]>0$, *while the price per unit set by the retailer is high,* $P^*{}_{1,I} > P^*{}_{1,II}$ if $\left(2e\eta-\gamma^2\right)\left[\lambda\eta\left(\lambda-eV_2\nu\right)-\zeta\left(2e\eta-\gamma^2\right)\right]>0$. Proof of Lemma 3.1 is shown in Appendix A.5.

The algorithm to obtain the solution to this model is as follows:

Step 1: Initialize $P^*{}_1$, α^*, $P^*{}_2$ and θ^*.

Step 2: Solve Eq. (3.4) to determine new value of $P^*{}_1$ using Newton - Raphson iterative procedure, Press et al, (1994).

Step 3: Using the $P^*{}_1$ found in Step2, determine α^*, $P^*{}_2$ and θ^*, from Eqs. (3.6), (3.17) and (3.19). The convergence criteria for algorithm is that the absolute of difference in the new and old values of α^*, $P^*{}_2$ and θ^* are within 0.001. Steps 2 and 3 are repeated until convergence is obtained.

3.1.1.3 Model III Intra-coordination of retailer

In this model, the selling effort, pricing and order quantity decisions of the retailer are coordinated. Both retailer and manufacturer set prices in order to maximize their profits. The manufacturer determines the investment in product quality and sets the price for the retailer. Further, retailer determines the optimal selling effort, price and order quantity.

The retailer's net profit per unit, Z_1 is given by Eq. (3.1). Solving $\frac{\partial Z_1}{\partial P_1}=0$;

$$P^*{}_{1,III} = \frac{A+\gamma\alpha+\lambda\theta}{2e}+\frac{1}{2}\left(V_1+P_2+\frac{O_1}{Q}\right) \tag{3.21}$$

Substituting the expression of retailer's price, $P^*{}_1$ from Eq. (3.21) in Eq. (3.1), the retailer's net profit per unit is expressed as:

$$Z_1 = \left[\frac{A+\gamma\alpha+\lambda\theta}{2e} - \frac{1}{2}\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right]\left[\frac{A+\gamma\alpha+\lambda\theta}{2} - \frac{e}{2}\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right] - F_1 - \frac{\eta\alpha^2}{2} - \frac{Qh_1}{2} \tag{3.22}$$

The condition for the maximization of the retailer's net profit is obtained by setting $\frac{\partial Z_1}{\partial \alpha} = 0$. This results in an optimal value of selling effort given by;

$$\alpha^*_{III} = \frac{\gamma}{2e\eta - \gamma^2}\left[A + \lambda\theta - e\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right] \tag{3.23}$$

Substituting retailer's optimal selling effort given in Eq. (3.23), in Eq. (3.22), the retailer's net profit per unit is further simplified to:

$$Z_1 = \frac{\eta}{2(2e\eta - \gamma^2)}\left[A + \lambda\theta - e\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right]^2 - F_1 - \frac{Qh_1}{2} \tag{3.24}$$

The manufacturer invests in quality improvement efforts which increases fixed costs, influences variable costs and improves the demand potential for the product. Using the expression of P^*_1 from Eq. (3.21) and α^* from Eq. (3.23), in Eq. (3.9), the gross profit of the manufacturer is expressed as:

$$\Pi_2 = \frac{e\eta}{(2e\eta - \gamma^2)}\left[P_2 - V_2(1+\nu\theta)\right]\left[A + \lambda\theta - e\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right] - F_2 - \frac{\zeta\theta^2}{2} \tag{3.25}$$

The manufacturer's gross profit is maximized by solving $\frac{\partial \Pi_2}{\partial P_2} = 0$, which yields,

$$P^*_{2,III} = \frac{A + \lambda\theta - e\left(V_1 - V_2(1+\nu\theta) + \frac{O_1}{Q}\right)}{2e} \tag{3.26}$$

The gross profit of the manufacturer per unit, Π_2 in Eq. (3.25) can be re-written using the expression of P^*_2 from Eq. (3.26), as:

$$\Pi_2 = \frac{\eta}{4(2e\eta - \gamma^2)}\left[A + \lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{O_1}{Q}\right)\right]^2 - F_2 - \frac{\zeta\theta^2}{2} \tag{3.27}$$

The gross profit of the manufacturer per unit, Π_2 is optimized by solving $\frac{\partial Z_2}{\partial \theta} = 0$;

$$\theta^*_{III} = \frac{\eta(\lambda - eV_2\nu)\left[A - e\left(V_1 + V_2 + \frac{O_1}{Q}\right)\right]}{\left[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} \tag{3.28}$$

Using the expression of θ^* from Eq. (3.28), the manufacturer's gross profit per unit is simplified as:

$$\Pi_2 = \frac{\zeta\eta\left[A - e\left(V_1 + V_2 + \frac{O_1}{Q}\right)\right]^2}{2\left[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} - F_2 \tag{3.29}$$

Lemma 3.2. *When there is no-coordination between the retailer and manufacturer compared to intra-coordination of the retailer, the manufacturer invests more in product quality,* $\theta^*_I > \theta^*_{III}$, *and sets higher price per unit,* $P^*_{2,I} > P^*_{2,III}$ if $\left[\zeta(2e\eta - \gamma^2) + \eta eV_2\nu(\lambda - eV_2\nu)\right] > 0$, *while the retailer invests more in selling effort,* $\alpha^*_I > \alpha^*_{III}$ if $\frac{\left[3\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]}{\left[2\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} > 0$, *and sets higher price per unit,* $P^*_{1,I} > P^*_{1,III}$ if $\left[\eta(\lambda - eV_2\nu)(3\lambda - eV_2\nu) - 2\zeta(3e\eta - 2\gamma^2)\right] > 0$. Proof of Lemma 3.2 is shown in Appendix A.6.

An iterative algorithm is used to obtain the solution to Eq. (3.21) to determine new value of P^*_1, from the initial values of P^*_1, α^*, P^*_2 and θ^*. Updated value of P^*_1 is used to determine α^*, P^*_2 and θ^*, from Eqs. (3.23), (3.26) and (3.28). The procedure to determine the current values of α^*, P^*_2 and θ^* is repeated till convergence is obtained. The convergence criteria for algorithm is that the absolute of difference in the new and old values of α^*, P^*_2 and θ^* are within 0.001.

3.1.1.4 Model IV Intra-coordination of manufacturer and retailer

In this model the selling effort, pricing and order quantity decisions of retailer are coordinated. Also in this model the manufacturer's product quality and profit margins are coordinated. Both retailer and manufacturer set prices independently in order to maximize their net profits. The manufacturer determines the investment in product quality and sets the price for the retailer. Subsequently retailer optimizes selling effort, price, and order quantity.

The manufacturer's net profit, Z_2 per unit is given in Eq. (3.8). Substituting the expression of P^*_1 from Eq. (3.21) and α^* from Eq. (3.23), in Eq. (3.8), the net profit per unit of the manufacturer is simplified as:

$$Z_2 = \frac{e\eta}{(2e\eta-\gamma^2)}\left(P_2 - V_2(1+\nu\theta) - \frac{(S_2+O_2)}{Q} - \frac{Qh_2}{2R}\right)\left[A+\lambda\theta - e\left(V_1 + P_2 + \frac{O_1}{Q}\right)\right] - F_2 - \frac{\zeta\theta^2}{2} \tag{3.30}$$

The net profit of the manufacturer per unit, Z_2 is optimized by solving $\frac{\partial Z_2}{\partial P_2} = 0$;

$$P^*_{2,IV} = \frac{A+\lambda\theta - e\left(V_1 + \frac{O_1}{Q} - V_2(1+\nu\theta) - \frac{(S_2+O_2)}{Q} - \frac{Qh_2}{2R}\right)}{2e} \tag{3.31}$$

The net profit of the manufacturer per unit, Z_2 is further simplified using the expression of P^*_2 from Eq. (3.31), as:

$$Z_2 = \frac{\eta}{4(2e\eta-\gamma^2)}\left[A+\lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{O_1}{Q} + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]^2 - F_2 - \frac{\zeta\theta^2}{2} \tag{3.32}$$

The net profit of the manufacturer per unit, Z_2 is optimized wrt θ, by solving $\frac{\partial Z_2}{\partial \theta} = 0$;

$$\theta^*_{IV} = \frac{\eta(\lambda - eV_2\nu)\left[A - e\left(V_1 + V_2 + \frac{O_1}{Q} + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]}{\left[2\zeta(2e\eta-\gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} \tag{3.33}$$

Using the expression of θ^* from Eq. (3.33), the manufacturer's net profit per unit is expressed as:

$$Z_2 = \frac{\zeta\eta\left[A - e\left(V_1 + V_2 + \frac{O_1}{Q} + \frac{(S_2+O_2)}{Q} + \frac{Qh_2}{2R}\right)\right]^2}{2\left[2\zeta(2e\eta-\gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} - F_2 \tag{3.34}$$

Lemma 3.3. *When there is no-coordination between the retailer and manufacturer compared to intra-coordination of manufacturer and retailer, the manufacturer invests more in product quality,* $\theta^*_I > \theta^*_{IV}$ *, and sets higher price per unit,* $P^*_{2,I} > P^*_{2,IV}$, if

$$\left(\frac{O_1}{Q}\right)\frac{\left[\zeta\left(2e\eta-\gamma^2\right)+\eta eV_2\nu(\lambda-eV_2\nu)\right]}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]}+\left(\frac{(S_2+O_2)}{Q}+\frac{Qh_2}{2R}\right)\frac{\left[\lambda\eta(\lambda-eV_2\nu)-\zeta\left(2e\eta-\gamma^2\right)\right]}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]}>0,$$

while the retailer invests more in selling effort, $\alpha^*_I > \alpha^*_{IV}$, if $\frac{\left(2e\eta-\gamma^2\right)}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]}>0$,

and sets higher price per unit, $P^*_{1,I} > P^*_{1,IV}$ if

$$\left(\frac{O_1}{Q}\right)\frac{\left[3\zeta\left(2e\eta-\gamma^2\right)+\eta eV_2\nu(\lambda-eV_2\nu)\right]}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]}+\left(\frac{(S_2+O_2)}{Q}+\frac{Qh_2}{2R}\right)\frac{\left[2\lambda\eta(\lambda-eV_2\nu)-\zeta\left(2e\eta-\gamma^2\right)\right]}{\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]}>0.$$

Proof of Lemma 3.3 is shown in Appendix A.7.

Similar to model III, an iterative algorithm is used to obtain the solution to Eq. (3.21) to determine new value of P^*_1, from the initial values of P^*_1, α^*, P^*_2 and θ^*. The new value of P^*_1 is used to determine α^*, P^*_2 and θ^*, from Eqs. (3.23), (3.31) and (3.33). The iterative procedure is repeated until convergence is obtained in the values of α^*, P^*_2 and θ^*. The convergence criteria for algorithm is that the absolute of difference in the new and old values of α^*, P^*_2 and θ^* are within 0.001.

3.1.1.5 Model V Inter-coordination of manufacturer and retailer for order quantity

In Model V, the manufacturer and retailer coordinate their order quantity decisions with each other to minimize the system's total inventory costs. However the manufacturer and retailer set their respective investments and profit margins independently of each other and independently of their operations departments. In this case, the focus is on minimizing the system's total inventory costs. Hence separate values of C_1 and C_2 are no longer relevant.

The total inventory related costs are given by

$$C_j=\frac{1}{Q}(A-eP_1+\gamma\alpha+\lambda\theta)(O_1+O_2+S_2)+Q\left(\frac{h_1}{2}+\frac{h_2}{2}\left[\frac{(A-eP_1+\gamma\alpha+\lambda\theta)}{R}\right]\right) \quad (3.35)$$

Solving $\frac{\partial C_{joint}}{\partial Q}=0$;

$$Q^*_j = \sqrt{\frac{(A - eP_1 + \gamma\alpha + \lambda\theta)[O_1 + O_2 + S_2]}{\frac{h_1}{2} + \frac{h_2}{2}\left[\frac{(A - eP_1 + \gamma\alpha + \lambda\theta)}{R}\right]}} \tag{3.36}$$

Substituting Eq. (3.36) in Eq. (3.35), results in

$$C_{Q^*_j} = 2\sqrt{\{(A - eP_1 + \gamma\alpha + \lambda\theta)[O_1 + O_2 + S_2]\}\left\{\frac{h_1}{2} + \frac{h_2}{2}\left[\frac{(A - eP_1 + \gamma\alpha + \lambda\theta)}{R}\right]\right\}} \tag{3.37}$$

3.1.1.6 Model VI Inter-coordination of manufacturer and retailer for investment and pricing

In model VI, the manufacturer and retailer coordinate their optimal investment and profit margin decisions to maximize their joint marketing profit. In the inter-coordination case, the decision variables are: product quality, selling effort and price. The manufacturer and retailer jointly optimize the investment in product quality, selling effort and price. But the retailer minimizes their inventory related costs independently of their marketing department.

The joint gross profit of the manufacturer and retailer is given by:

$$\Pi_j = P_1(A - eP_1 + \gamma\alpha + \lambda\theta) - [V_1 + V_2(1 + \nu\theta)](A - eP_1 + \gamma\alpha + \lambda\theta) - F_1 - F_2 - \frac{\eta\alpha^2}{2} - \frac{\zeta\theta^2}{2} \tag{3.38}$$

Solving $\frac{\partial \Pi_{joint}}{\partial P_1} = 0$ yields,

$$P^*_{1,VI} = \frac{A + \gamma\alpha + \lambda\theta}{2e} + \frac{[V_1 + V_2(1 + \nu\theta)]}{2} \tag{3.39}$$

Substituting the joint price offered by manufacturer-retailer from Eq. (3.39), the joint gross profit is re-written as

$$\Pi_j = \frac{1}{4e}\{A + \gamma\alpha + \lambda\theta - e[V_1 + V_2(1 + \nu\theta)]\}^2 - F_1 - F_2 - \frac{\eta\alpha^2}{2} - \frac{\zeta\theta^2}{2} \tag{3.40}$$

Next the optimal selling effort level that maximizes the joint manufacturer-retailer gross profit is found by solving $\frac{\partial \Pi_j}{\partial \alpha} = 0$, which yield,

$$\alpha^*_{VI} = \frac{\gamma\{A + \lambda\theta - e[V_1 + V_2(1 + \nu\theta)]\}}{2e\eta - \gamma^2} \tag{3.41}$$

Substituting joint optimal selling effort from Eq. (3.41) in the joint profit of Eq. (3.40), yields,

$$\Pi_j = \frac{\eta}{2(2e\eta-\gamma^2)}\{A+\lambda\theta-e[V_1+V_2(1+\nu\theta)]\}^2 - F_1 - F_2 - \frac{\zeta\theta^2}{2} \tag{3.42}$$

The optimal quality level selected under inter-coordination of manufacturer-retailer that maximizes the joint marketing profit is obtained by solving $\frac{\partial \Pi_j}{\partial \theta} = 0$.

$$\theta^*_{VI} = \frac{\eta(\lambda - eV_2\nu)[A-e(V_1+V_2)]}{\left[\zeta(2e\eta-\gamma^2)-\eta(\lambda-eV_2\nu)^2\right]} \tag{3.43}$$

Substituting the joint optimal quality level from Eq. (3.43) in Eq. (3.42), the joint marketing profit is,

$$\Pi_j = \frac{\zeta\eta[A-e(V_1+V_2)]^2}{2\left[\zeta(2e\eta-\gamma^2)-\eta(\lambda-eV_2\nu)^2\right]} - F_1 - F_2 \tag{3.44}$$

To ensure concavity of the joint gross profit in P^*_I, α^*, and θ^*, it is required that $(2e\eta-\gamma^2)>0$, $\left[2e\zeta-(\lambda-eV_2\nu)^2\right]>0$, $(\lambda-eV_2\nu)>0$, and $\left[\zeta(2e\eta-\gamma^2)-\eta(\lambda-eV_2\nu)^2\right]>0$ as shown in Appendix A.3.

For $(2e\eta-\gamma^2)>0$, $\left[2e\zeta-(\lambda-eV_2\nu)^2\right]>0$, $(\lambda-eV_2\nu)>0$ and $\left[\zeta(2e\eta-\gamma^2)-\eta(\lambda-eV_2\nu)^2\right]>0$, Π^*_j, P^*_I, α^* and θ^* are all decreasing in retailer's cost of selling effort, η, as shown in Appendix A.4. If the inter-coordinated manufacturer-retailer has a lower cost of selling effort, then they provide more selling effort and higher quality product. Both inter-coordinated manufacturer-retailer price per unit and marketing profit are higher for lower cost of retailer's selling effort.

Π^*_j, P^*_I, α^* and θ^* are all decreasing in cost of quality, ζ, for $(2e\eta-\gamma^2)>0$, $(e\eta-\gamma^2)>0$, $\left[2e\zeta-(\lambda-eV_2\nu)^2\right]>0$, $(\lambda-eV_2\nu)>0$ and $\left[\zeta(2e\eta-\gamma^2)-\eta(\lambda-eV_2\nu)^2\right]>0$. If the joint manufacturer-retailer has a higher cost of quality, they are likely to provide lower quality product and more selling effort. With increase in the cost of quality, joint price offered by the manufacturer-retailer is increased, while the inter-coordinated marketing profit is decreased, as shown in the Appendix A.4.

3.1.1.7 Model VII Inter-coordination of manufacturer and retailer for order quantity & investment and pricing

In this model, the marketing and the operations departments of manufacturer and retailer collaborate independently of each other. As in Model VI, in this model also, the marketing departments of manufacturer and retailer coordinate with each other in setting optimal selling effort, quality and price. Subsequently, the manufacturer and retailer coordinate their order quantity decisions with each other to minimize the system's total inventory costs.

3.1.1.8 Model VIII Inter and Intra-coordination of manufacturer and retailer

In model VIII, it is assumed that the two channel members fully collaborate within as well as across the channel. Their objective is to maximize the net profit of the system. The manufacturer and retailer jointly coordinate their optimal investment, profit margin and order quantity decisions to maximize their joint net profit. In this model, the decision variables are: product quality, selling effort, price and order quantity.

The joint (retailer-manufacturer) net profit per unit is

$$Z_j = \Pi_j - C_j \tag{3.45}$$

$$\Pi_j = P_1(A - eP_1 + \gamma\alpha + \lambda\theta) - [V_1 + V_2(1+\nu\theta)](A - eP_1 + \gamma\alpha + \lambda\theta) - F_1 - F_2 - \frac{\eta\alpha^2}{2} - \frac{\zeta\theta^2}{2} \tag{3.46}$$

$$C_j = \frac{1}{Q_j}(A - eP_1 + \gamma\alpha + \lambda\theta)(O_1 + O_2 + S_2) + Q_j\left(\frac{h_1}{2} + \frac{h_2}{2}\left[\frac{(A - eP_1 + \gamma\alpha + \lambda\theta)}{R}\right]\right) \tag{3.47}$$

Solving $\frac{\partial Z_j}{\partial P_1} = 0$;

$$P^*_{1,VIII} = \frac{A + \gamma\alpha + \lambda\theta}{2e} + \frac{1}{2}\left(V_1 + V_2(1+\nu\theta) + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right) \tag{3.48}$$

Substituting the joint price offered by manufacturer-retailer from Eq. (3.48), the joint net profit is expressed as,

$$Z_j = \frac{1}{4e}\left[A + \gamma\alpha + \lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right)\right]^2 - F_1 - F_2 - \frac{\eta\alpha^2}{2} - \frac{\zeta\theta^2}{2} - \frac{Q_j h_1}{2} \tag{3.49}$$

Next, the joint manufacturer-retailer net profit is maximized by solving $\frac{\partial Z_j}{\partial \alpha} = 0$,

$$\alpha^*_{VIII} = \frac{\gamma}{2e\eta - \gamma^2}\left[A + \lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right)\right] \tag{3.50}$$

Substituting joint optimal selling effort from Eq. (3.50) in the joint net profit of Eq. (3.49), yields,

$$Z_j = \frac{\eta}{2(2e\eta - \gamma^2)}\left[A + \lambda\theta - e\left(V_1 + V_2(1+\nu\theta) + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right)\right]^2 - F_1 - F_2 - \frac{\zeta\theta^2}{2} - \frac{Q_j h_1}{2} \tag{3.51}$$

The optimal quality level that maximizes the joint net profit is obtained by solving $\frac{\partial Z_j}{\partial \theta} = 0$.

$$\theta^*_{VIII} = \frac{\eta(\lambda - eV_2\nu)\left[A - e\left(V_1 + V_2 + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right)\right]}{\left[\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} \tag{3.52}$$

The joint net profit is re-written in Eq. (3.53), by substituting the joint optimal quality level from Eq. (3.52) in Eq. (3.51).

$$Z_j = \frac{\zeta\eta\left[A - e\left(V_1 + V_2 + \frac{(O_1 + O_2 + S_2)}{Q_j} + \frac{Q_j h_2}{2R}\right)\right]^2}{2\left[\zeta(2e\eta - \gamma^2) - \eta(\lambda - eV_2\nu)^2\right]} - F_1 - F_2 - \frac{Q_j h_1}{2} \tag{3.53}$$

Lemma 3.4. *When there is inter-coordination between the retailer and manufacturer compared to inter and intra-coordination of manufacturer and retailer, the inter coordinated manufacturer and retailer invests more in product quality,* $\theta^*_{VI} > \theta^*_{VIII}$, *invests more in selling effort,*

$\alpha^*_{VI} > \alpha^*_{VIII}$ if $\frac{\left(2e\eta-\gamma^2\right)}{\left[\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]} > 0$, *and sets higher price per unit,* $P^*_{1,VI} > P^*_{1,VIII}$ if $\frac{\left[(\gamma-1)\zeta\left(2e\eta-\gamma^2\right)+2\lambda(\lambda-eV_2\nu)\right]}{\left[\zeta\left(2e\eta-\gamma^2\right)-\eta(\lambda-eV_2\nu)^2\right]} > 0$. Proof of Lemma 3.4 is shown in Appendix A.8.

The algorithm to obtain the solution to this model is as follows:
Step 1: Initialize P^*_1, α^* and θ^*.
Step 2: Solve Eq. (3.48) to determine new value of P^*_1 using iterative procedure.
Step 3: Using the P^*_1 found in Step2, determine α^* and θ^*, from Eqs. (3.50) and (3.52). Steps 2 and 3 are repeated until convergence is obtained. The convergence criteria for algorithm is that the absolute of difference in the new and old values of α^* and θ^* are within 0.001.

3.1.2 Numerical investigations

Tables 3.1 and 3.2 outline the assumptions common to all eight models (Joglekar et al, 2006). Table 3.1 shows the results obtained for demand as a function of price, selling effort and product quality (Gurnani et al., 2007) given by the formula: $D = 50{,}000 - 4{,}000P_1 + 500\alpha + 15{,}000\theta$. The example described in Table 3.1 is referred as the base case. With reference to Table 3.1 which refers to the results for investment in selling effort and product quality, in no coordination situation (Model I), the net profit margin show minor increase from the net profit margin of Models II, III, IV. In Model IV, the supply chain's total net profit Z is smaller that what is in either Model II or Model III and smaller yet compared to the profit in Model I. It is also interesting to note that when both channel members coordinate their internal decision making, each member makes only a smaller net profit compared to the situation of no coordination. In Models II to IV, when a supply chain's total net profit Z decreases, the retail price P_1, selling effort α, product quality level θ, product demand D, order quantity Q, total inventory costs C, and total gross profits Π in each of these models are slightly lower than those in Model I. Thus when there is no collaboration across the channel, the customer as well as the supply chain yield lower net profits as compared to a no coordination situation.

In Model V, the supply chain attains small improvement in the net profit (Table 3.1). Hence, collaboration across the channel for the minimization of inventory related costs only is not promising.

Models VI, VII, and VIII result in an increase in the price that the consumer has to pay P_1 due to investments by manufacturer and retailer (Table 3.1). As Table 3.1 shows, the supply chain's total net profit Z in each of the Models VI, VII, VIII is significantly larger than the net profit in Model I.

As shown in Table 3.1, Model VIII, with total collaboration across components of supply chain, produces the best profit for the supply chain, followed by Model VII, which assumes collaboration across components of supply chain in marketing (selling effort, product quality, pricing) and operations departments (order quantity decisions). However, due to the costs of coordination in the implementation of Models VII and VIII, Model VI may be the most desirable.

The results in Table 3.2 are obtained for demand as function of price (Joglekar et al, 2006) given by: $D = 50{,}000 - 4{,}000P_1$. When the retailer invests in selling effort and the manufacturer invests in product quality level, analysis of results in Models I to IV indicates (Table 3.1) that the total net profits, total gross profits, total inventory costs, demand, and order quantity are higher in comparison to the case when there is no investment (Table 3.2). However, when there is no investment, the retail prices P_1 in each of the Models II to IV are slightly higher than those in Model I (for no coordination), while the retail prices P_1 in each of the Models VI to VIII are lower than those in Model I (Table 3.2). Thus component collaboration is highly desirable from the customer's perspective when there is no investment by the manufacturer and retailer.

In the base case example considered in Table 3.1, the parameter values were selected rather arbitrarily, and hence a sensitivity analysis of the parameters A, e, γ and λ are provided in Tables 3.3 to 3.6. Any one of the particular parameter (A, e, γ and λ) is increased by 10% and keeping all other values at the base case level.

Table 3.3 shows optimized decisions for various models as a result of 10% increase in *A*, which is the potential demand at P_1=0. As *A* increases, it also induces a higher demand for the product (*D*) and larger stocking quantity (Q^*). An increase in the stocking quantity increases the total inventory costs of retailer and manufacturer (*C*). Consequently the quality level selected by the manufacturer (θ^*) and the retailer's selling effort (α^*) are at a higher level. With an increase in demand the price charged by the retailer (P^*_1) to the customer is also higher. The total gross profit (Π) of the retailer and manufacturer is higher with a higher demand and higher price set for the product. Also the total net profit (*Z*) of the retailer and manufacturer are high with an increase in *A*.

Table 3.1 Optimized results for various models for demand as a function of price, selling effort, and product quality level

Assumptions common to all models (base case) A=50,000 units/year, e=4,000 units/dollar, F_1=\$10,000 /year, V_1=\$1.25 /unit, O_1=\$250 /order, h_1=\$1 /unit/year, R=30,000 units/year, F_2=\$20,000 /year, V_2=\$1.5 /unit, O_2=\$100 /order, S_2=\$600 /setup, h_2=\$0.5 /unit/year, γ=500, η=1,000, λ=15,000, ζ=10,000, υ=2								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
Θ^*	0.801	0.775	0.792	0.765	0.801	1.708	1.708	1.670
P^*_2	\$9.080	\$9.152	\$8.995	\$9.067	\$9.080	NA	NA	NA
A^*	1.336	1.291	1.321	1.275	1.336	2.847	2.847	2.784
P^*_1	\$13.001	\$12.984	\$12.995	\$12.979	\$13.001	\$13.568	\$13.568	\$13.542
D	10,685	10,326	10,566	10,203	10,685	22,774	22,774	22,274
Q^*	2311	2272	2299	2259	4151	3374	5601	5555
C_1	\$2,311.4	\$2,272.3	NA	NA	NA	\$3,374.4	NA	NA
C_2	\$3,441.7	NA	\$3,420.2	NA	NA	\$5,364.6	NA	NA
C	\$5,753	\$5,649	\$5,718	\$5,612	\$4,890	\$8,739	\$7,726	NA
Π_1	\$17,650	\$15,827	NA	NA	\$17,650	NA	NA	NA
Π_2	\$32,089	NA	\$30,935	NA	\$32,089	NA	NA	NA
Π	\$49,739	\$47,856	\$49,120	\$47,193	\$49,739	\$81,022	\$81,022	NA
Z_1	\$15,339	\$13,554	\$15,887	\$14,085	NA	NA	NA	NA
Z_2	\$28,647	\$28,653	\$27,514	\$27,496	NA	NA	NA	NA
Z	\$43,986	\$42,207	\$43,402	\$41,581	\$44,849	\$72,283	\$73,296	\$73,351

Table 3.4 shows optimized decisions for various models as a consequence of 10% increase in *e*, which is the coefficient of product's demand elasticity. An increase in *e*, results in decrease in the product demand (*D*) and hence order quantity. Lower order quantity (Q^*) results in lower total inventory cost (*C*) for the retailer and manufacturer. With decrease in the product's demand,

the manufacturer chooses lower quality level (θ*) and buyer opts for lower selling effort (α*). An increase in the value of *e* implies that customers are becoming more price-sensitive and hence price charged to customer ($P*_1$) need to be reduced. Due to decrease in demand and the price charged by the retailer ($P*_1$) to customer, the total gross profit (Π) of the retailer and manufacturer is decreased. The total net profit (Z) of the retailer and manufacturer also decreases.

Table 3.2 Optimized results for various models for demand as a function of price

Assumptions common to all models A=50,000 units/year, e=4000 units/dollar, F_1=$10,000 /year, V_1=$1.25 /unit, O_1=$250 /order, h_1=$1 /unit/year, R=30,000 units/year, F_2=$20,000 /year, V_2=$1.5 /unit, O_2=$100 /order, S_2=$600 /setup, h_2=$0.5 /unit/year								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
$P*_1$	$10.063	$10.109	$10.119	$10.124	$10.063	$7.625	$7.625	$7.738
$P*_2$	$6.375	$6.468	$6.375	$6.383	$6.375	NA	NA	NA
$Q*$	2208	2187	2182	2180	3992	3123	5288	5241
D	9,750	9,563	9,521	9,505	9,750	19,500	19,500	19,050
C_1	$2,207.9	$2,186.6	NA	NA	NA	$3,122.5	NA	NA
C_2	$3,270.5	NA	$3,227.7	NA	NA	$4,878.9	NA	NA
C	$5,478	$5,423	$5,410	$5,405	$4,641	$8,002	$7,007	NA
Π_1	$13,766	$12,861	NA	NA	$13,766	NA	NA	NA
Π_2	$27,531	NA	$26,414	NA	$27,531	NA	NA	NA
Π	$41,297	$40,375	$40,167	$40,087	$41,297	$65,063	$65,063	NA
Z_1	$11,558	$10,674	$11,571	$11,495	NA	NA	NA	NA
Z_2	$24,261	$24,278	$23,186	$23,187	NA	NA	NA	NA
Z	$35,819	$34,952	$34,757	$34,682	$36,656	$57,061	$58,056	$58,106

Tables 3.5 and 3.6 shows optimized decisions for various models as a consequence of 10% increase in γ (influence of buyer's selling effort on demand) and λ (impact of product's quality on demand) respectively. With increase in either γ or λ by 10% and keeping all other values at the base case level, it is found that it results in increase in product demand (D) and hence order quantity (Q*). Consequently, both the optimal value of buyer's selling effort (α*) and the quality level selected by manufacturer (θ*) increase with increase in either γ or λ. Also the retailer charges a higher price ($P*_1$) to the customer and hence an increase in both total gross profit (Π) and total net profit (Z) of the retailer and manufacturer.

Table 3.3 Optimized results for various models for demand as a function of price, selling effort, and product quality level as a result of 10% increase in A

Assumptions common to all models same as in base case except A=55,000 units/year								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
θ^*	0.904	0.879	0.896	0.870	0.904	1.927	1.927	1.890
P^*_2	$10.051	$10.120	$9.972	$10.040	$10.051	NA	NA	NA
α^*	1.507	1.465	1.493	1.450	1.507	3.212	3.212	3.150
P^*_1	$14.315	$14.299	$14.310	$14.294	$14.315	$14.954	$14.954	$14.929
D	12,055	11,716	11,943	11,600	12,055	25,693	25,693	25,206
Q^*	2455	2420	2444	2408	4367	3584	5846	5807
C_1	$2,455.1	$2,420.3	NA	NA	NA	$3,584.2	NA	NA
C_2	$3,683.7	NA	$3,664.3	NA	NA	$5,785.4	NA	NA
C	$6,139	$6,045	$6,108	$6,013	$5,245	$9,370	$8,350	NA
Π_1	$25,194	$23,242	NA	NA	$25,194	NA	NA	NA
Π_2	$46,301	NA	$45,075	NA	$46,301	NA	NA	NA
Π	$71,496	$69,490	$70,839	$68,796	$71,496	$111,314	$111,314	NA
Z_1	$22,739	$20,822	$23,321	$21,387	NA	NA	NA	NA
Z_2	$42,618	$42,623	$41,411	$41,396	NA	NA	NA	NA
Z	$65,357	$63,445	$64,731	$62,783	$66,251	$101,944	$102,964	$103,016

Table 3.4 Optimized results for various models for demand as a function of price, selling effort, and product quality level as a result of 10% increase in e

Assumptions common to all models same as in base case except e=4,400 units/dollar								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
θ^*	0.407	0.391	0.401	0.385	0.407	0.829	0.830	0.808
P^*_2	$7.110	$7.228	$7.036	$7.154	$7.110	NA	NA	NA
α^*	1.130	1.085	1.115	1.070	1.130	2.304	2.304	2.244
P^*_1	$10.619	$10.648	$10.629	$10.659	$10.619	$9.845	$9.845	$9.885
D	9,940	9,550	9,810	9,414	9,940	20,272	20,272	19,748
Q^*	2229	2185	2215	2170	4025	3184	5366	5313
C_1	$2,229.4	$2,185.2	NA	NA	NA	$3,183.7	NA	NA
C_2	$3,305.8	NA	$3,281.7	NA	NA	$4,995.1	NA	NA
C	$5,536	$5,418	$5,496	$5,378	$4,692	$8,179	$7,178	NA
Π_1	$11,819	$10,138	NA	NA	$11,819	NA	NA	NA
Π_2	$22,811	NA	$21,698	NA	$22,811	NA	NA	NA
Π	$34,631	$32,885	$34,056	$32,261	$34,631	$57,309	$57,309	NA
Z_1	$9,589	$7,953	$10,144	$8,485	NA	NA	NA	NA
Z_2	$19,506	$19,514	$18,416	$18,398	NA	NA	NA	NA
Z	$29,095	$27,467	$28,560	$26,883	$29,939	$49,130	$50,131	$50,189

Table 3.5 Optimized results for various models for demand as a function of price, selling effort, and product quality level as a result of 10% increase in γ

Assumptions common to all models same as in base case except γ=550								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
θ^*	0.807	0.780	0.798	0.771	0.807	1.721	1.721	1.683
P^*_2	$9.099	$9.171	$9.015	$9.086	$9.099	NA	NA	NA
α^*	1.480	1.430	1.463	1.413	1.480	3.156	3.156	3.085
P^*_1	$13.040	$13.022	$13.034	$13.016	$13.040	$13.651	$13.651	$13.626
D	10,762	10,402	10,643	10,278	10,762	22,950	22,950	22,439
Q^*	2320	2281	2307	2267	4164	3387	5616	5570
C_1	$2,319.7	$2,280.6	NA	NA	NA	$3,387.4	NA	NA
C_2	$3,455.7	NA	$3,434.2	NA	NA	$5,390.3	NA	NA
C	$5,775	$5,671	$5,741	$5,635	$4,910	$8,777	$7,764	NA
Π_1	$17,862	$16,030	NA	NA	$17,862	NA	NA	NA
Π_2	$32,466	NA	$31,308	NA	$32,466	NA	NA	NA
Π	$50,328	$48,437	$49,707	$47,772	$50,328	$81,879	$81,879	NA
Z_1	$15,542	$13,750	$16,093	$14,282	NA	NA	NA	NA
Z_2	$29,011	$29,016	$27,874	$27,855	NA	NA	NA	NA
Z	$44,553	$42,766	$43,966	$42,137	$45,418	$73,102	$74,115	$74,170

Table 3.6 Optimized results for various models for demand as a function of price, selling effort, and product quality level as a result of 10% increase in λ

Assumptions common to all models same as in base case except λ=16,500								
	Model I	Model II	Model III	Model IV	Model V	Model VI	Model VII	Model VIII
θ^*	1.302	1.260	1.289	1.246	1.302	3.066	3.066	3.000
P^*_2	$11.015	$11.023	$10.913	$10.919	$11.015	NA	NA	NA
α^*	1.447	1.400	1.432	1.384	1.447	3.406	3.406	3.333
P^*_1	$15.159	$15.073	$15.131	$15.044	$15.159	$18.759	$18.759	$18.624
D	11,577	11,201	11,453	11,073	11,577	27,249	27,249	26,663
Q^*	2406	2367	2393	2353	4294	3691	5967	5922
C_1	$2,405.9	$2,366.6	NA	NA	NA	$3,691.1	NA	NA
C_2	$3,600.4	NA	$3,578.6	NA	NA	$6,005.7	NA	NA
C	$6,006	$5,900	$5,972	$5,864	$5,122	$9,696	$8,676	NA
Π_1	$22,460	$20,393	NA	NA	$22,460	NA	NA	NA
Π_2	$36,438	NA	$35,236	NA	$36,438	NA	NA	NA
Π	$58,897	$56,766	$58,201	$56,025	$58,897	$102,838	$102,838	NA
Z_1	$20,054	$18,026	$20,571	$18,526	NA	NA	NA	NA
Z_2	$32,837	$32,839	$31,658	$31,635	NA	NA	NA	NA
Z	$52,891	$50,866	$52,229	$50,161	$53,775	$93,142	$94,162	$94,223

It is found that increase any one of the particular parameter (A, e, γ and λ) by 10% resulted in approximately the same trend in percentage change in the optimized results (θ^*, α^*, P^*_1, D, Q^*, C, Π, Z) in each one of our models. Thus our results about the relative merits of different forms of supply chain coordination are valid over a wider range of the parameters (A, e, γ and λ).

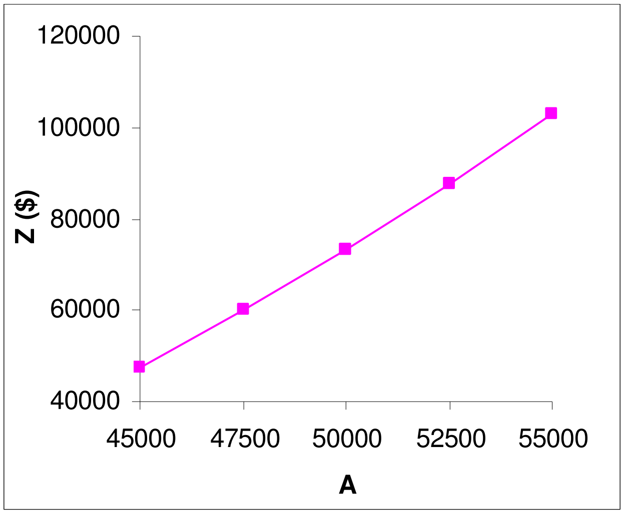


Fig. 3.2 Variation of Z with A

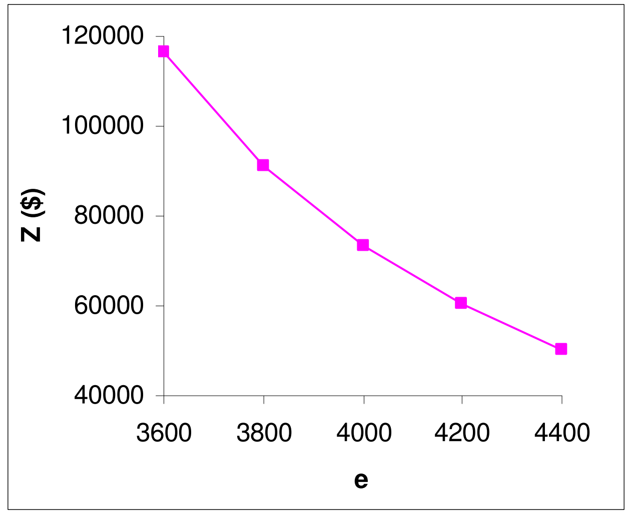


Fig. 3.3 Variation of Z with e

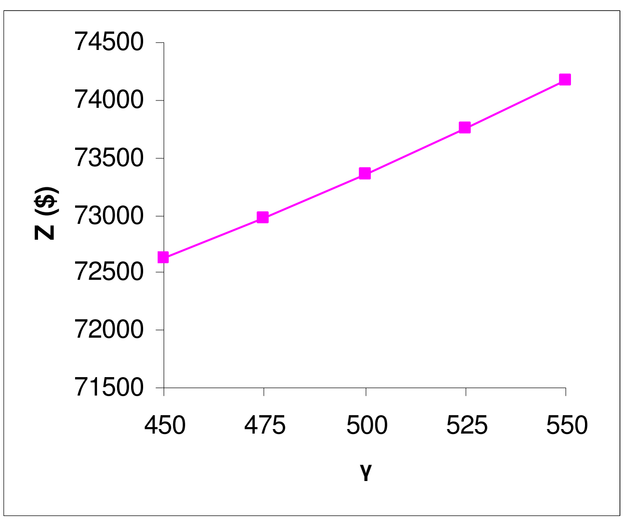


Fig. 3.4 Variation of Z with γ

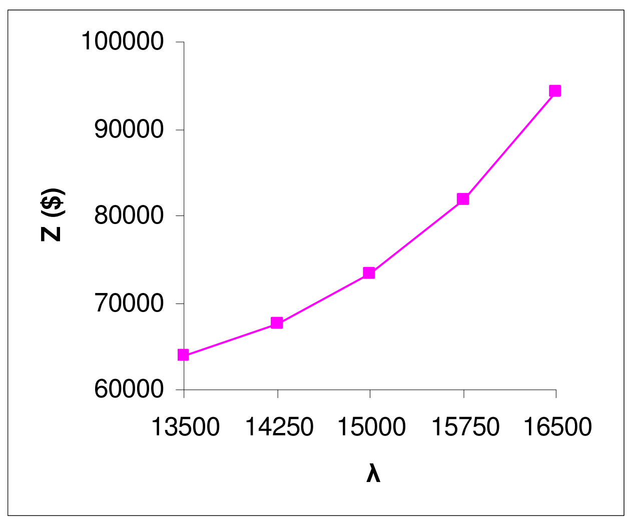


Fig. 3.5 Variation of Z with λ

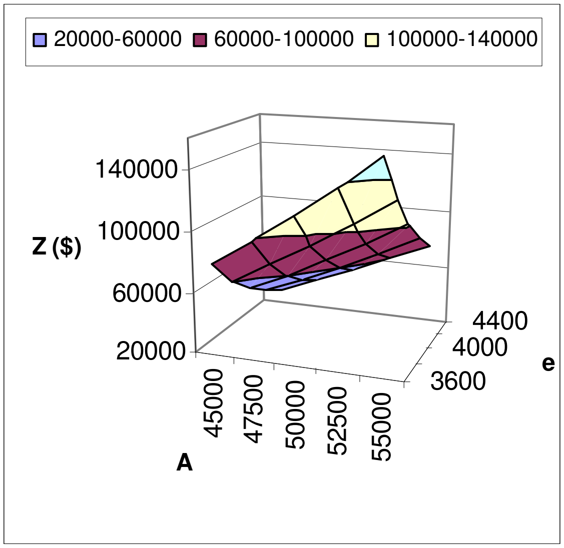


Fig. 3.6 Variation of Z with A and e

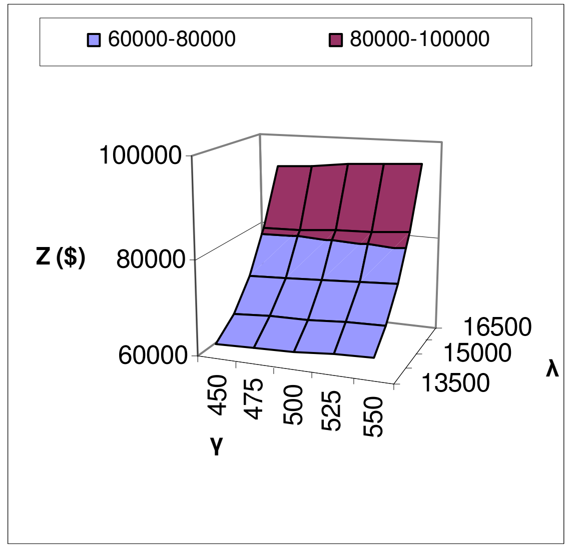


Fig. 3.7 Variation of Z with γ and λ

Figures 3.2 – 3.7 show the variation of joint retailer-manufacturer net profit per unit (Z) for the inter and intra-coordination of manufacturer and retailer. Figures 3.2 – 3.5 show the variation of Z with A (constant), e (coefficient of product's demand elasticity), γ (influence of retailer's selling effort on demand) and λ (impact of product quality on demand) respectively. As shown in Figs. 3.2 – 3.5, the joint net profit (Z) increases with increase in A, γ and λ, while the joint net profit (Z) decreases with increase in e. Figure 3.6 shows the variation of the joint net profit (Z) with A and e. As shown in the Fig. 3.6, the joint net profit (Z) is higher for higher values of A and lower values of e. Figure 3.7 shows that joint net profit (Z) is higher for higher values of γ and λ.

3.2 Managerial Implications

The managerial implications are as follows:

- When there is no coordination between manufacturer and retailer, for a lower cost of retailer's selling effort, retailer provides higher selling effort, higher price and higher marketing profit. While for a lower cost of retailer's selling effort, the manufacturer provides a higher quality product, higher price and higher marketing profit. Under no coordination, for a manufacturer with higher cost of quality, retailer makes more selling effort and manufacturer provides lower quality product. Retailer's and manufacturer's price as well as their marketing profits are decreasing in increasing cost of quality. As shown in Appendix A.2, for $\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta\left(\lambda-eV_2\nu\right)^2\right]>0$, and $\left(\lambda-eV_2\nu\right)>0$, Π^*_1, Π^*_2, P^*_1, α^*, P^*_2 and θ^* are all decreasing in retailer's cost of selling effort, η. For $\left[2\zeta\left(2e\eta-\gamma^2\right)-\eta\left(\lambda-eV_2\nu\right)^2\right]>0$, and $\left(\lambda-eV_2\nu\right)>0$, Π^*_1, Π^*_2, P^*_1, P^*_2 *and* θ^* are all decreasing in cost of quality, ζ, while α^* increases with cost of quality, ζ,.

- It is found that the intra collaboration at any one or both components of the channel actually reduces the total net profit of supply chain compared to the base case with no collaboration. For investment in selling effort and product quality with reference to Table 3.1, in no coordination situation (Model I), the net profit per unit is $43,986, while the net profit margin of Model II (intra-coordination of manufacturer), Model III (intra-coordination of retailer), and Model IV (intra-coordination of manufacturer and retailer) are $42,207, $43,402 and $41,581 respectively.

- When the inter-coordinated manufacturer-retailer has a lower cost of selling effort, then they provide more selling effort and higher quality product. When the inter-coordinated manufacturer-retailer has a higher cost of quality, then they are likely to provide lower quality product and more selling effort. With decrease in the cost of retailer's selling effort inter-coordinated manufacturer-retailer price per unit and marketing profit are increased. With increase in the cost of quality, joint price offered by the manufacturer-retailer is increased, while the inter-coordinated marketing profit is decreased. As shown in Appendix A.4, for $\left(2e\eta-\gamma^2\right)>0$, $\left[2e\zeta-\left(\lambda-eV_2\nu\right)^2\right]>0$, $\left(\lambda-eV_2\nu\right)>0$ and $\left[\zeta\left(2e\eta-\gamma^2\right)-\eta\left(\lambda-eV_2\nu\right)^2\right]>0$, Π^*_j, P^*_1, α^* and θ^* are all decreasing in retailer's cost of selling effort, η. For $\left(2e\eta-\gamma^2\right)>0$, $\left(e\eta-\gamma^2\right)>0$, $\left[2e\zeta-\left(\lambda-eV_2\nu\right)^2\right]>0$, $\left(\lambda-eV_2\nu\right)>0$ and $\left[\zeta\left(2e\eta-\gamma^2\right)-\eta\left(\lambda-eV_2\nu\right)^2\right]>0$, Π^*_j, P^*_1, α^* and θ^* are all decreasing in cost of quality, ζ.
- Complete inter component collaboration produces the highest net profit for the supply chain. Inter component collaboration for only investment and price determination produces nearly same net profit for the supply chain as that of complete collaboration. As shown in Table 3.1, the net profit in complete inter component collaboration is $ 73,351

while the net profit in inter component collaboration for only investment and price determination is \$ 72,283.

- Results of sensitivity analysis in Tables 3.3, 3.4, 3.5 and 3.6 shows that the optimized results (θ^*, α^*, P^*_I, D, Q^*, C, Π, Z) in each one of the models increase with increase in the parameters: A (annual demand at P_I=0), γ (influence of buyer's selling effort on demand) and λ (impact of product quality on demand), while the optimized results decrease with increase in e (product's demand elasticity).