Abstract

The primary interest of this thesis is to develop some efficient and higher order numerical techniques for solving singularly perturbed reaction-diffusion and convection-diffusion boundary value problems exhibiting boundary and/or interior layer(s). The differential equations in which the highest order derivative is multiplied by an arbitrary small parameter ε (say) which is known as singular perturbation parameter are called singular perturbation problems (SPPs). This leads the existence of the narrow region(s) in the domain where the gradient of the solution becomes steep as the perturbation parameter tends to zero. Because of this layer phenomena, it is challenging task to provide parameter-uniform numerical methods *i.e.,* the numerical methods in which the numerical solution converges to the exact solution independently with respect to ε.

This thesis contains 8 chapters. A brief note of basic introduction about singular perturbation problems and their applications in different fields, methodology and gaps in the present literatures has been discussed in Chapter 1. The explanation of different-different situations where the singular perturbation arises has also been discussed in the Introduction section. In Chapter 2, a uniformly convergent implicit numerical scheme on a fitted piecewise-uniform mesh condensing in the boundary layer region for a class of singularly perturbed parabolic partial differential equation with the time delay on a rectangular domain is proposed and analyzed which is applicable for delay which is either $o(\varepsilon)$ or $O(\varepsilon)$. It is shown that the discrete solution obtained by this scheme is first-order (up to a logarithmic factor) accurate. Then, the scheme is applied for the case when the delay of unit length is present in spatial direction in Chapter 3 which is more challenging because in addition to boundary layers an interior layer occurs due to the presence of delay term. The solution of these type of problems, in general, exhibits twin boundary layers (due to the presence of the perturbation parameter) and an interior layer (due to the presence of the large delay parameter in the reaction term).

In Chapter 4, we have developed the parameter uniform numerical scheme for singularly perturbed simple turning point problems and to resolve the boundary layer a fitted-mesh is constructed and the cubic B-spline basis functions on this mesh are used to discretize the equation. In Chapter 5, we have considered the case of multiple boundary turning point at left end point of the spatial direction. We have proposed a

numerical scheme comprising Crank-Nicolson scheme on a uniform mesh and finite difference scheme on a piecewise-uniform mesh (Shishkin-type mesh). The proposed scheme is proved to be parameter-uniform convergent and is of $\mathscr{O}((\Delta t)^2 + N^{-1}\ln N)$.

We have considered an important class of time dependent two parameter singularly perturbed boundary value problems in Chapter 6. The numerical method comprises the Crank-Nicolson scheme on an equidistant mesh in the temporal direction and the finite difference scheme on a predefined Shishkin mesh in the spatial direction is used. A parameter-uniform implicit scheme is developed for two different cases: Case I. $\varepsilon_1/\varepsilon_2^2 \to 0$ as $\varepsilon_2 \to 0$, and Case II. $\varepsilon_2^2/\varepsilon_1 \to 0$ as $\varepsilon_1 \to 0$. The order of accuracy in the Case I and Case II are shown $\mathscr{O}((\Delta t)^2 + N^{-1}(\ln N)^2)$ and $\mathscr{O}((\Delta t)^2 + N^{-2}(\ln N)^2)$ respectively. Then we have generalized this numerical scheme on the time dependent singularly perturbed two parameter boundary value problems having discontinuous convection coefficient and source term whose solution exhibits dual boundary layers and an interior layer for both the cases in Chapter 7. At the end of the whole work, we have summarized the specific contribution of the present work with novel results in Chapter 8 and proposed a scope of future work in this area.