ELECTRICITY METERS
& METER TESTING

A SERIES OF MONOGRAPHS
ON
ELECTRICAL ENGINEERING

Under the Editorship of
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VOLUME VI

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BY

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EDITORIAL PREFACE

The advances made within the realm of electrical engineering during the twentieth century have been phenomenal.

In the field of electric power supply the National Electricity Scheme has been conceived, designed and put into commission, thus increasing the reliability and availability of the service. Meanwhile the allied sciences of electrical communications and high frequency technique have moved forward with gigantic steps ; indeed, it is hardly too much to say that, due to the advent of the electronic tube and its application in both the power and communications branches, certain aspects of electrical engineering practice have been completely revolutionised.

In view of these epoch-making advances, it is not surprising that the electrical engineer whose college days are some years behind him finds it necessary to do a considerable amount of reading in order to keep abreast of modern developments, while the student who is reading for a University degree is nowadays expected to possess at least an outline of the field of electrical physics in addition to a comprehensive knowledge of electrical technology.

While, however, the literature dealing with these advances is both comprehensive and voluminous, it is also scattered and thus available only by extensive literary research. The aim of each monograph published in this series is to give a modern orientation of a particular subject within the confines of a small book, thus obviating the necessity for searching through the transactions of innumerable learned societies. At the same time, for those wishing further to extend their knowledge, each monograph contains references to the more important publications relating thereto.

The satisfactory accomplishment of this object necessarily implies that the author must fulfil the dual function of collator and interpreter ; for this reason, one of the editor's most important tasks has been to induce acknowledged authorities to write for this series. In this respect, he has been fortunate beyond his expectation, and each monograph has been written by an author who is eminent in his chosen field and thus writes with the authority which is the result of intimate knowledge.

It is hoped that the Monographs will succeed in filling a gap which has hitherto existed in scientific literature.

H. P. YOUNG.

AUTHOR'S PREFACE

THE passing of the Electricity Supply (Meters) Act, 1936, created an entirely new situation in the supply industry. Previously a meter was deemed to be correct unless the consumer disputed its accuracy. The Act now places on the electricity supply industry the onus of maintaining within stipulated limits of error all consumers' meters which register kWh ; and also of submitting these meters for certification by a Meter Examiner appointed by the Electricity Commissioners. In addition, official memoranda supplementary to the Act have standardised the testing procedure preliminary to the certification of meters, and have laid down specifications for the equipment of Approved Testing Stations. The consequence of this is that the engineer in charge of the work of testing consumers' meters now enjoys that definite status to which he has long been entitled, but which he has often in the past been denied.

It is considered that the new situation arising from the passing of the 1936 Act justifies the appearance of a new book dealing with the important subject of Meters and Meter Testing in a manner which will appeal to practical testing engineers. It is hoped that the present work will be found acceptable by those engineers engaged in this field who are now called upon to carry increased responsibilities. The general scheme of treatment followed in the book is of a practical nature, and, accordingly, mathematical symbolism is avoided as far as possible in the chapters dealing with the theory and construction of meters. Throughout the book the author has endeavoured to elucidate points of theory by verbal explanations from first principles.

The official specifications supplementary to the 1936 Act deal at present only with the testing of meters registering kWh. This testing procedure, together with the official requirements for the equipment of Approved Testing Stations, is fully summarised and explained in the present volume. In addition, considerable space is devoted to explanations of the operation of and to practical details for testing demand indicators, reactive and kVAh meters, which are not regulated by the 1936 Act. The preliminary chapter of the book deals with the elementary principles of tariff design and explains to the student the necessity for the various classes of meters used in the measurement of consumers' supplies.

The subject of meter testing has, in the past, been somewhat neglected in courses of instruction for the training of electrical engineers, but there are now welcome signs that lectures on this important subject will be regularly given in the technical colleges. It is therefore hoped that this book will be found useful by lecturers and students, as well as by those actually engaged in meter testing work.

G. W. S.

PREFACE TO SECOND EDITION

IN this edition several errors have been corrected, minor revisions have been made where necessary, and some additional matter relating to maximum demand measurement, test-house equipment and to meter testing technique has been included. The author takes this opportunity of thanking those correspondents who kindly pointed out mistakes in the original edition and who made some useful suggestions.

G. W. S.

CONTENTS

CHAPTER I

CHAPTER II

CHAPTER III

CHAPTER IV

CHAPTER I

THE MEASUREMENT OF CONSUMERS' SUPPLIES

The Cost of Electricity Supply. In the business of public electricity supply the necessity for electrical measurements arises in two distinct fields: the first is that of testing and checking the condition and efficiency of the machinery and apparatus used in generating the supply and distributing it to consumers' terminals; the second is that of measuring those electrical quantities upon which the charges for the supplies are based. Consumers or, rather, utilisers of public electricity supply require electrical energy for conversion into energy of other forms—optical, thermal, or mechanical, as the case may be—and the most straightforward method of charging a consumer for his use of the supply would be in direct proportion to the number of units of energy, or kWh, he absorbs in his consuming apparatus. This method of charging would be in line with those used in almost all other branches of retail trading, including generally the sale of gas. There are, however, two vital circumstances which make this method of charging for an electricity supply impracticable, or, at all events, inexpedient. In the first place, the cost of providing a kWh of electrical energy at the consumers' service terminals depends to a large extent upon the time during which this energy is used. Secondly, the money value to the consumer of a kWh of energy depends to a large extent upon the manner in which this energy is used.

The circumstances which affect the cost of supplying a kWh of electrical energy are the load factor, and the power factor when this supply is on the A.C. system. This first circumstance and its effect on the cost of supply are best explained by the theory that the total expenditure involved in generating electrical energy and distributing it to consumers' terminals can be divided into two components: the one called the standing cost represents the expenditure involved in readiness to supply any power demand up to the rated capacity of the generating plant, and the other, called the running cost, represents the additional expenditure involved when the generating machinery is actually operating and delivering energy to consumers. The first component, which includes not only what are called capital costs, but also the larger fraction of the salary and wages and a smaller fraction of the fuel bill, is roughly proportional to the power rating of the generating plant in kW. The second component, which includes the larger fraction of the fuel bill and a smaller fraction of the wages bill, is roughly proportional to the output of the generating plant in kWh.

If the whole of the output of a supply plant were used by a single consumer, then, in order adequately to remunerate the supply undertaking for the costs involved in giving the supply, the consumer would have to pay a sum made up of two components, the one proportional to the kW rating of the plant required to supply him, and the other to his energy consumption in kWh. Algebraically, the amount payable by the consumer would be represented by the formula $a + bn$, where a is the standing cost, b the running cost expressed as a rate per kWh, and n the actual consumption in kWh. The overall rate paid by the consumer per kWh would therefore be expressed by the formula $a/n + b$, and this rate would diminish till n had increased to the value which corresponds to the total output of the plant working continuously at full load. If this maximum kWh output be denoted by N, then the ratio $n/\mathrm{N} = \lambda$ is called the load factor of the supply. It is also, in this case, the load factor of the consumers' use of the supply. Since $n = \mathrm{N}\lambda$, the overall rate paid per kWh by the consumer will be given by the formula $\dfrac{a}{\mathrm{N}\lambda} + b$, and this rate will increase as the load factor falls.

In actual practice the total output of a power supply works is used by a large number of consumers, and the manner in which these consumers use the supply varies greatly. In other words, the load factors of the various consumers will not be the same. It would be possible to charge all the consumers for their supplies at a flat or uniform rate per kWh, which is based upon the load factor of the generating plant. It has long been found to be impossible to carry on the business of electricity supply on this principle, and the more complicated method of charging by differential rates per kWh is now firmly established, the actual rates depending upon the manner in which the supply is used, and being so adjusted that the total revenue they bring the undertaking is sufficient to remunerate it for the total cost of giving the whole supply.

Theoretical Bases of Tariff Design. These differential rates of charging for electricity supply may be justified by two theories. The first is that the whole charge for the supply to a consumer should be approximately equal to the total cost of producing this supply and delivering it to his service terminals. The second principle is that differential rates should be so adjusted that, whilst bringing an adequate total revenue to the undertaking, the use of electricity in all fields is stimulated by making these rates correspond to the value of the supply to the consumer. According to the first, or so-called 'scientific theory,' lighting rates per kWh are generally higher than power rates, because a kWh used for lighting costs more to produce and supply than one used for power. According to the second theory, the lighting rate is higher because the consumer is willing to pay at a higher rate for a supply used for lighting.

The two principles which operate in the fixing of rates of charging for electricity supplies—the one quasi-scientific and the other based upon pure expediency—are only partially consistent, but all methods of charging in use are supposed to be justifiable to some extent, at any rate, by the scientific theory.

Systems of Charging for Electricity Supply. According to the scientific theory of charging for electricity supply, consumers should share the total running costs in proportion to their kWh consumptions, and the total standing costs in proportion to the extent to which they contribute to the maximum power demand on the generating plant of the supply undertaking. The application of the first part of this principle is simple, because running costs can be expressed as a rate per kWh. The application of the second part of this principle is, however, not so straightforward. As a rough approximation the contribution of a consumer to the total maximum demand on the generating plant may be assumed to be represented by the maximum rate in kW at which he uses the supply. As this maximum rate can be measured, two quantities—the consumer's consumption in kWh and his maximum rate of using the supply in kW—would appear to furnish a quantitative basis for the application of the scientific system of charging. The matter is, however, not quite so simple as this. The actual times of the maximum uses of the supply by consumers will not, as a rule, coincide, so that the sum of these maxima is greater than the actual effective demand they impose on the generating plant. This circumstance, whereby the sum of a number of individual maximum demands is greater than the actual resultant maximum demand they impose upon the supply plant, is called diversity, and diversity is a characteristic of the manner in which the supply is used. It is evident, for instance, that the individual maximum demands of lighting loads will tend to closer coincidence in time than those of cooking loads, because the use of electricity supply for lighting depends to a great extent upon the natural phenomenon of darkness, while the use of the supply for cooking can hardly be said to be completely regulated by convention or custom.

Diversity is defined quantitatively, with respect to a group of consumers or to a particular use of the supply, as the ratio of the sum of the individual maximum demands to the total effective resultant demand on the generating plant of the undertaking. A nearly equivalent definition of diversity is the ratio of the load factor of a particular kind of supply as a whole to the average of the load factors of all the component consumptions of this supply. It is quite easy to formulate a quantitative definition of the diversity of a particular type of use of electricity supply, but, as this definition is based upon physical quantities which are generally incapable of measurement, such a definition is not of great interest to the engineer who is concerned with exact measurements. Diversity is a conception of too abstract a nature to be of practical use, although, as will be seen hereafter, it is often employed in arguments intended to justify systems of charging, really based upon expediency, by the scientific theory.

We have seen that a first approximation to the scientific theory of charging could be realised by charging consumers at a fixed rate per kWh of their consumption and at a rate per kW of their maximum demand corresponding to the standing cost per kW. We have also seen that, because of diversity, the rate per kW of maximum demand will be too high. If, however, the diversity of a class of consumers can be numerically estimated, then the consumers in this class will, according to the scientific theory, pay equitably for their supply in two components, the one proportional to their kWh consumptions at the rate of the running costs per kWh, and the other proportional to their kW maximum demands at a rate equal to the standing costs per kW divided by the diversity of the class. This application of the scientific theory requires that the diversity of the class be known, either by measurement or by estimation. Alternatively, of course, the rate per kW of maximum demand can be fixed by considerations of expediency, and this rate can be scientifically justified by making a suitable estimate of diversity This method of charging for electricity supply on the basis of kWh consumption and kW maximum demand is now a standard one.

A consumer's maximum demand is equal to his average demand divided by the load factor of his use of the supply. Now average demand is proportional to kWh consumption. If therefore the load factor and the diversity applicable to a particular use of the supply can be estimated, the demand component of the total charge for the supply can be calculated as a rate per kWh, and can so be merged into the second component which corresponds to the running costs. Thus all energy used in a particular way can be charged for at a flat rate per kWh, which depends, according to the scientific theory, upon the standing and running costs of the supply and upon the load factor and diversity corresponding to this use of the supply. This is the scientific justification for the differential flat rate system of charging for electrical energy, under which the consumer pays for his supply on the basis of kWh consumption, the rate of charging varying according to the use which is made of the supply. From the point of view of electrical measurement, this system only requires the determination of energy consumptions, but, as a separate measuring instrument and wiring circuit are required for each kind of use of the supply, it is, from this point of view, less desirable than the two-part maximum demand system in which the charge for the supply is based on two measurements in a single circuit.

These two methods of charging for electricity form the basis of all other methods. Thus the two-part domestic tariffs, in which a fixed quarterly payment is based upon some such quantity as floor area, number of rooms, rateable value, or connected demand, and which only require the measurement of kWh consumption, may be supposed to be modifications of the two-part maximum demand system of charging, the maximum demand being assumed to be approximately proportional to the quantity upon which the fixed quarterly component of the charge is based. Similarly, the block rate tariff, in which a fixed number of units, as with the two-part tariff, are charged for at a high rate, all consumption in excess being charged for at a low rate, may be considered also to be a modification of the maximum demand system, the portion of the charge corresponding to the assumed maximum demand being merged into the high rate charged for the first block of kWh consumed, the charge for all in excess being at a rate corresponding to the running charges. The scientific justification for these domestic tariffs involves so many doubtful assumptions that it is really of little value, and these tariffs are best justified by the principle of expediency in that they have been remarkably successful in extending the use of electrical energy in the domestic field and in increasing the prosperity of the electricity supply industry. This justification is sufficient for all practical purposes. From the point of view of the technique of electrical measurement, domestic tariffs are interesting in that one measuring instrument only is required for each consumer.

Power Factor Tariffs. When, as is usual, an electricity supply is given on the A.C. system, there is a circumstance additional to load factor which affects the cost of this supply. This additional circumstance is the power factor of the consumers' electrical load. Electrical plant is rated for duty according to the maximum current it can handle, and hence this rating is not in terms of power or kW, but of kVA. The amount of electrical plant required to supply a kW of A.C. power to a customer therefore varies inversely as the power factor of this load. A low power factor supply, since it requires more current per kW than one of the maximum power factor of unity, also involves increased heating losses in all electrical machinery, and hence the running costs of supplying a kWh of energy at the consumers' terminals are also increased. This effect of power factor on the cost of electricity supply is of most importance when the supply is used for industrial power purposes, as the induction motor, the simplest and most robust of all A.C. motors, operates with a power factor which is necessarily less than unity and which diminishes as the load falls.

The two-part maximum demand system of charging can be modified to allow for the effect of power factor on the cost of supply by basing the demand charge upon kVA instead of kW. This method of charging for A.C. power supplies is in close conformity with the requirements of the scientific theory, but it has the technical objection that it involves a measurement of alternating kVA.

The extra cost of supply of electrical energy can be passed on to the consumer in an approximate way by a system of charging whereby the rates are made to vary with a measured power or energy factor. The adjustment of the rates is usually in accordance with a schedule. Under a system of this kind the rates increase as the measured average power factor falls. This used to be called 'penalising low power factor'—a misleading expression, as it implies that low power factor is caused by an improper use of the supply. Low power factor actually is caused by the reactive VA necessarily required for the alternating magnetic fields upon which the operation of transformers and induction motors fundamentally depends. If the consumer uses motors of this type he must use the kVAr necessary for their working. This kVAr can either be drawn from the supply, in which case it causes low power factor, or it can be generated by plant such as static condensers, provided at the consumers' expense. A system of charging in which the rates are graded according to measured power or energy factor is best considered as one which reimburses the supply undertaking for the increased cost of supply caused by low power factor or which provides financial inducement for the consumer to generate the kVAr which his motors require instead of drawing this kVAr from the supply mains. From the point of view of electrical measurement, this system of charging according to graded rates is one which requires the measurement of some average power factor or energy factor value on which the rate applicable can be based.

There is a third system of charging for A.C. power supplies which depends upon a theory of charging formulated by Arno. According to this theory, the amount paid for the supply should be made up of two components, the one proportional to kWh paid for at a rate corresponding to the cost of supply at unity power factor, and the other at a lower rate proportional to the excess of the apparent energy or kVAh over the actual energy or kW. The total charge for this supply will thus be represented by a formula such as

$$a \times \text{kWh} + b \times (\text{kVAh} - \text{kWh}),$$

where a and b are the rates fixed to cover the cost of production. This formula is equivalent to

$$c \times \text{kWh} + d \times \text{kVAh},$$

where c and d are new fixed rates.

This formula as it stands involves the quantity kVAh, which, on technical grounds, is very difficult to measure. The formula can, however, be readily converted to another, approximately equivalent, which is more suitable on technical grounds. If $\cos\phi$ is the energy factor, or the ratio of kWh to kVAh, the last formula can be written

$$\text{kVAh}\,(c\cos\phi + d).$$

Now if an average or normal energy factor $\cos\theta$ can be assigned, then provided $\cos\phi$ does not depart greatly from $\cos\theta$ this formula can be written

$$\text{kVA}\{c\cos\phi + d\cos(\phi - \theta)\}$$

because in the conditions assumed $\cos(\phi - \theta)$ will not be more than $2\frac{1}{2}$ per cent. less than 1 provided the angles ϕ and θ do not differ by more than 13 degrees. By expanding the term $\cos(\phi - \theta)$ by elementary trigonometry the approximate formula can be transformed to

$$\text{kVA}\,(p\cos\phi + q\sin\phi)$$

or to

$$p \times \text{kWh} + q \times \text{kVArh},$$

where p and q depend upon a, b and $\cos\theta$. Thus the Arno theory of charging for A.C. supplies can be made to justify a tariff under which the consumer pays for his supply in two components, the one proportional to his energy consumption and the other proportional to his consumption of reactive kilovolt ampere hours. This method of charging is technically of great interest because, of all the measurements involved in the various methods in use, that of kVArh is, after energy measurements, the most simple and the most accurate.

Other Circumstances which affect the Cost of Supply. So far we have considered in detail the two main circumstances peculiar to the consumer which affect the cost of supplying him with a kWh of electrical energy. These are load factor and power factor, and each of these factors depends upon the manner in which the supply is used. These, of course, are not the only circumstances on which the cost per kWh of the supply depends. Thus, a consumer situated at a considerable distance from a generating centre will cost more to supply than one who is not so remote, because of the greater length of distribution conductors required and also because of the greater losses involved. It has, however, always been customary, and probably always will be, to ignore this circumstance in the electricity supply business, and to charge consumers in the same area of supply who use electrical energy in the same manner at the same rate per kWh, irrespective of any difference in the cost of the distribution conductors required to supply them.

There are, however, two circumstances connected with the manner in which a consumer uses an A.C. supply which have some bearing upon the cost of this supply per kWh, and in respect of which proposals have been made already or may be made in the future for differential rates. These two circumstances are: first, the unbalance of supplies taken from 3-phase mains; and secondly, the current wave distortion caused by the use of apparatus of variable impedance.

The effect of the unbalance of a 3-phase load is easily understood. With a balanced load of unity power factor and a line voltage of V, the power which can be transmitted per ampere of line capacity is $\sqrt{3}V$ watts. In the extreme condition of unbalance of the load drawn from a 3-phase supply the whole of the power is carried by two lines, and in this case the maximum power which can be transmitted per ampere of line capacity at unity power factor is V watts. Thus the capacity of the 3-phase electrical plant required to supply a kW of power to a consumer increases as the unbalance of the load increases. It is possible to specify the unbalance of a 3-phase load quantitatively in terms of the ratio of the symmetrical components of the load, and as these symmetrical positive and negative phase sequence components can be measured, it would also be possible to grade the rates of the supply of 3-phase power according to an unbalance factor in the same manner as they can be graded according to power factor. There is, however, an important difference between power factor and unbalance factor. Low power factor caused by normal use of an A.C. supply for power purposes is nearly always caused by lagging reactive currents, and as lagging reactive currents are all in phase they add arithmetically and the combined effect of several consumers of low power factor on the total supply is cumulative. Unbalance of 3-phase supplies, which may, on the symmetrical component theory, be considered to be caused by negative phase sequence currents, is not of this nature because the phase of the negative sequence currents which cause the unbalance of any particular load is indefinite, and these currents may be wholly or partially cancelled by negative sequence currents of the unbalanced load of a consumer adjacent. Because of this nature of unbalance, its actual effect on the cost of a consumer's supply cannot be considered to be altogether dependent upon any quantitative magnitude which can be assigned to it. Although proposals have from time to time been made to grade rates of charging for electricity supply according to measured unbalance, these proposals have not so far been practically adopted.

The effect of current wave distortion on the cost of giving a supply of electrical energy is, likewise, easily explained. When the wave forms of the current and voltage in an A.C. circuit are dissimilar, the power factor of the load as defined by the ratio, watts/volt-amperes, is always less than unity; and, moreover, this true power factor cannot be determined by the usual methods which are based upon the determination of kW and kVAr. In other words, an A.C. circuit in which the wave form of the current is distorted may, according to the registration of a meter measuring kVArh, take no reactive current from the mains, and yet require more than 1 kVA per kW. Wave distortion thus causes a lowering of the power factor additional to that set up by reactive currents. The effect of this wave distortion is, however, more serious than that of ordinary reactive kVA. Distorted waves of current set up pressure drops which lead to distortion of the waves of voltage supplying other consumers, and this voltage distortion adversely affects the efficiency of motors. The ultimate effect of the use of apparatus giving rise to distorted current waves is thus further-reaching than that of ordinary reactive currents which cause low power factor.

Current wave distortion takes place in any A.C. circuit the impedance of which varies with the instantaneous value of the current in it. Well-known examples of such circuits are those containing reactors having iron cores of variable permeability, and those containing an arc or vacuum discharge apparatus. Vacuum discharge lighting is rapidly increasing in use, and as this type of load becomes relatively larger the effect of the wave distortion of alternating currents which it causes will be progressively more serious. As with the unbalance of 3-phase loads, proposals have been made for methods of charging under which the kWh rate would be increased by reason of wave distortion. Such systems of charging would involve the assessment of maximum demand on the basis of R.M.S. amperes, and would therefore require the actual measurement of this demand by instruments of a particular type.

Although it is without the scope of this work to discuss in any detail the desirability or expediency of the practical use of these suggestions for grading kWh rates according to 3-phase unbalance or wave distortion, it may be pointed out that, whilst the technical problems involved are not inordinately difficult or unattractive to the technical expert, the extra complication of the tariff and the introduction into it of a complex electrical conception hardly to be comprehended by the consumer is very undesirable. Although, according to the scientific theory of charging, tariff rates should be so graded that the amount paid by a consumer is equal to the total cost of supplying him, this principle is incapable of general application, and some differential costs, such as those of distribution, must be averaged. Differential costs which arise from unbalance and distortion of current wave form are difficult to determine with precision, and are essentially those which are suitable for averaging. From the point of view of the principle of expediency, it is of high importance that tariff charges for electricity supply should be as simple as possible, and that they should be based upon physical quantities which are as simple as possible in character, capable of exact definition and accurate measurement, and as far as possible capable of comprehension by the consumer. On this ground differential charges based upon 3-phase unbalance and wave distortion would be ill-advised.

Physical Quantities involved in the Measurement of Consumers' Supplies. From the foregoing review of the methods of charging for consumers' supplies, it appears that the fundamental electrical quantity involved in assessing the amount which a consumer has to pay for his supply is energy. When the system of charging is a two-part one, the second component of the charge may be based upon maximum demand in kW, or some such electrical quantity which is determined once for all. When energy is supplied on the A.C. system and the rates depend upon power factor, the electrical measurement involved may be that of kVA maximum demand, average power factor, or reactive kVAh, in addition to the normal determination of energy consumption. We shall now discuss the physical nature of these electrical quantities and examine some ambiguities in the way they are usually defined.

Electrical Energy. Of all the physical quantities referred to in the preceding sections, electrical energy is the simplest. It is exactly equivalent to definite amounts of energy in other forms ; thus 1 kWh is equal to 2,656,400 foot-lbs., to 3,600,000 joules, and to 3,440 British heat units. Electrical energy can, fortunately, be measured with a very satisfactory accuracy by commercial instruments. An instrument which registers amounts of energy is conventionally called a meter. Contrary to what might be thought, *a priori*, meters for measuring energy in A.C. circuits are cheaper, simpler and more accurate than those for D.C. circuits. Energy or watt-hour meters for D.C. supplies are susceptible to greater errors than are A.C. meters, and for small currents D.C. energy meters are inordinately costly. For this reason it is legally permissible to take, as the energy consumption of a consumer, the quantity of electricity in his supply in ampere hours multiplied by one-thousandth of the declared or nominal value of the supply voltage. For the measurement of small D.C. supplies it is customary to use quantity or ampere-hour meters, the registers being arranged to read in fictitious kWh calculated according to this rule. The conception of electrical quantity is quite a definite one considered in relation to D.C. circuits. With A.C. supplies it is ambiguous, and an A.C. quantity meter might be one which registered quantity irrespective of direction, or one which registered a time integral of R.M.S. amperes. Actually, whilst it is easy to manufacture a cheap and accurate energy meter for A.C. circuits, it is very difficult to make an accurate A.C. quantity meter for any but small currents. Thus, although small D.C. supplies are measured on the basis of electrical quantity, A.C. ampere-hour meters are rarely used, excepting in one type of demand indication which will be described later.

The fictitious energy consumption registered by a D.C. quantity meter may, because of variations of the supply voltage, differ considerably from the true energy. Thus if the voltage at a consumer's terminals is always 5 per cent. low, his fictitious energy consumption as registered by a D.C. quantity meter will be about 5 per cent. higher than the actual energy he has used. As supply voltages are most likely to be low when the supply is most used, D.C. consumers with quantity meters will generally have to pay for more than the actual amount of energy they consume. It should be noted that there is no question of an error of measurement arising here, because the fictitious kWh, calculated on the basis of a constant nominal voltage, is the legal and agreed unit of supply. The amount of money a consumer pays for his supply depends upon more factors than his measured consumptions. It depends also upon the nature of the units of supply and upon the tariff rates. In the technique of the measurement of consumers' supplies it is only the accuracy of the determination of the supply in conventional units that is in question. The nature of these units and the accuracy of the rates are questions of a totally different kind. If, for instance, a municipally owned supply undertaking makes large profits, which are used in some part to subsidise the rates, there are considerable errors in the tariff rates, but this is not the concern of the engineer responsible for accuracy of measurement of the consumers' supplies.

Maximum Demand. The quantity maximum demand which, after energy, is next in importance in the measurement of consumers' supplies, is much more ambiguous in its nature than is electrical energy. The natural unit of demand is the kW of power, which is a rate at which energy is used. The simplest meaning of maximum demand would be the maximum value of instantaneous power in a D.C. circuit, or the maximum average cycle of oscillating power in an A.C. current. It is impracticable to base charges for a supply upon such instantaneous or nearly instantaneous values, and maximum demands are always assessed in terms of some kind of average power over a period. There is, however, no formal or even generally agreed definition as to how this average is to be obtained.

There are two methods in use of measuring maximum demands of consumers' supplies, which differ in fundamental principle. In the first method, what is called a Merz type of indicator is used, and this instrument records in average kW the maximum consumption of energy in any one of equal periods, defined by clock times, and which may vary from 15 to 60 minutes. The length of the integration periods is not authoritatively specified, but is settled by agreement of the supply authority and the consumer. As the averaging periods are defined by clock times, it follows that the effect of a short transient peak load on the reading of the indicator will depend, not only upon the extent of the integration period, but also upon the clock time of its incidence. To illustrate this point by a numerical example, suppose that a consumer normally takes a load of 100 kW and that an abnormal peak load of 120 kW lasting for 5 minutes occurs. Assume first that this peak falls entirely within one of the integration periods. If this period is 15 minutes, the consumption will be 25 kWh for the constant load and 10 kWh for the peak, giving a total of 35 kWh for 15 minutes, and this will be recorded as maximum average demand of 140 kW. If, however, the period is 30 minutes, the consumption in the period will be 50 kWh for the constant load and 10 for the peak, and the total of 60 kWh will be recorded as a maximum demand of 120 kW.

Again, assuming that the integration period is 15 minutes, consider that the 5-minute peak demand falls for 2½ minutes within one averaging period and 2½ minutes within the next. The total consumption in each of these two periods will be 30 kWh, which will be recorded as a maximum demand of 120 kW. Thus the record of a maximum peak of perfectly definite electrical character depends upon the integration period and the clock times to which these periods correspond.

An alternative method of measuring maximum demand is by means of an instrument responsive to the temperature of an element which is heated by the current in the load, the instrument being so designed that a steady current will produce its maximum temperature rise in a stipulated period which, like that of the averaging period of a Merz type of instrument, may be from 15 to 30 minutes. An instrument of this thermal type will be responsive to the demand in amperes, but it can be scaled to indicate kW at a declared voltage. As the rise of temperature of the heating element caused by a temporary peak lasting for a smaller period than the rated time lag will only be partially indicated by a thermal demand indicator, this instrument can be said, like the Merz indicator, to give some kind of maximum average demand. The nature of this demand is, however, not so definite as is the average of a Merz instrument because the rise of temperature of the element heated by a constant current is not at a constant rate, but it approaches the limiting rise exponentially. Thus when a constant current passes into a thermal demand indicator the advance of the pointer is relatively quickest at first, and gradually becomes slower as the reading corresponding to this current is approached. It therefore follows that a high peak of a duration which is small as compared with the nominal time lag period will be indicated as having a greater value than one comprising the same kWh consumption which is of longer duration. The thermal indicator does not give a true maximum average demand as does the Merz type of instrument. On the other hand, it is evident that the amount of demand indicated in respect of a peak load is independent of the clock time of its incidence.

The fundamental differences of these two types of demand indicators may be summarised as follows :—

Merz Type. Gives a true maximum average demand in kW.
Indication depends for short peaks on integration period.
Indication depends for short peaks on clock time of incidence.

Thermal Type. Does not give a true maximum average demand.
Indication depends upon current and not power in A.C. circuits.
Indication depends for short peaks on nominal time lag.
Indication is independent of clock time of incidence of peak.

It has been pointed out in the preceding section that a unit of measure used in a method of charging for consumers' supplies need not be exact in the scientific sense. It is, however, highly desirable that any such unit should be exactly defined and that it should mean the same thing in all places. This cannot now be said of the kW unit of maximum demand, as neither the method of measurement nor the averaging period is authoritatively specified, but is subject to the discretion or caprice of the supply authority.

It may also be pointed out that the effect of a single abnormal peak demand on the amount paid by a consumer for his supply depends upon a further factor, that of the accounting period. If demand indicators are read and re-set monthly the effect of such a peak on a year's power bill will evidently be less than if the period between successive re-settings is a quarter. Accounting periods, like averaging periods, should therefore be standardised, but this is a matter lying without the subject of the measurement of consumers' supplies.

A critical study of the possible effects of timing and length of integration period variations on the indication of a peak load by a Merz type of maximum demand indicator has been recently put forward by D. J. Bolton in an I.E.E. paper to which reference has been made in the Appendix to this book. After developing and enlarging upon the basic arguments set out above, Bolton deals with six load curves, obtained either by graphic instruments or by 1–2 minute readings of an energy meter, referring to supplies of the bulk, commercial, rural, domestic and industrial classes. He shows by actual calculation that variation of the clock time of incidence of the integrating periods, each of 30 minutes, can cause variations of the demand indicated by a Merz indicator ranging from 5 to 30 per cent. of the maximum possible indicated value. Although clock times of the now recommended standard integrating period of 30 minutes are stipulated by the Electricity Commissioners to start at the hour or half-hour, this calculation shows how much the effect of a short peak on recorded demand can be affected by the clock time of the peak. Bolton also investigates the effect of varying the integrating period between the limits of 5 and 30 minutes, he finds that such variations may affect the recorded demand by percentages of the same order as those given by variation of clock times, and he concludes that, to put the Merz method of demand measurement on a sure and certain basis, and to remove the possibilities of the serious anomalies that can now occur, a new type of instrument must be evolved which will integrate with a 'floating' integrating period, and so designed that each

1-minute increase of the record cancels the consumption during a similar period 30 minutes earlier in time. An instrument designed on these lines would admittedly be costly, but, considering the large sums of money that depend upon demand indicator readings for bulk supplies, it is considered that a costly instrument would be justified for the removal of the uncertainties and possible anomalies of the present method of measurement by successive integration and averaging.

The second part of Bolton's paper consists of the examination of the experimental results obtained by measuring a fluctuating load by several indicators of the thermal as well as the Merz type, together with measurements of the copper and oil temperatures of a transformer carrying this load. These results show that the amplitude of the temperature variations are much less than those of the demand indicator readings, although the rating of the transformer was relatively small, 75 kVA. It was concluded that these experiments show that an increase of integrating periods is justified, as demand charges are supposed to be based upon plant loading. This would be inherently desirable because the longer the period the less are the possible effects of variation of clock time of short peaks on the recorded maximum demand. Where indicators of the thermal type are used, Bolton recommends that the demand charge should be assessed upon the average of a number of readings taken at regular intervals.

Kilovoltamperes. We have examined in some detail the ambiguity of the expression 'a kW of maximum demand.' When A.C. power is supplied under a tariff in which demand is assessed in kVA instead of kW, there will be a further ambiguity in the maximum demand unit because the quantity kVA is not a physical quantity of a definite character. When 1 R.M.S. ampere is associated with 1 R.M.S. volt in an A.C. circuit the apparent power is said to be 1 VA, but the magnitude VA is a product of two average values. The instantaneous VA in an A.C. circuit is the same thing as instantaneous power, and the average VA is what is called A.C. power.

In a single-phase circuit the quantity VA can be defined as the product of R.M.S. volts and amperes, and VA demand can evidently be measured approximately by a thermal demand indicator, responsive to current, which is scaled to indicate kVA at a nominal declared voltage. The matter is not so simple with 3-phase circuits. Although electrically a 3-phase 3-wire circuit may be considered as a combination of three single-phase circuits with a suppressed common return, a 3-phase service line is considered to give one and not three supplies. It is not, therefore, legitimate to measure 3-phase demand by three separate single-phase instruments, because it is quite possible that the maxima in the three phases might occur at different times, whence the sum of the single-phase maxima would not represent the maximum of the actual 3-phase demand. Accordingly, 3-phase demand in kVA must be defined in terms of the characteristics of a 3-phase load.

The usual definition of 3-phase VA is the square root of the sum of the squares of the total watts and the total VAr. The total watts in a 3-phase load is a perfectly definite physical quantity. The total VAr is the algebraic sum of the separate single-phase VAr, that is, leading VAr in one phase are considered to cancel lagging VAr in another. The advantage of this definition, which of course is a convention, is that the 3-phase VAr so defined can be easily and accurately measured. According to this definition it is evident that the total equivalent VA can be obtained by compounding the single-phase VA as vectors in the same way that the VA in parallel single-phase circuits are combined to obtain a resultant value. For this reason the total equivalent VA in a 3-phase circuit is sometimes called the vector sum of the single-phase VA, but the possibility of the vector summation, it should be borne in mind, depends upon the convention that the lagging VA in one phase are cancelled by leading VA in another. Physically, of course, this is not the case, so that total equivalent VA is not really a vector sum, and, as a matter of fact it is possible to devise a 3-phase load, consisting of a capacitance and a reactor of equal ohmic values, and connected between two different pairs of lines of the supply, which absorbs negligible watts and negligible VAr as conventionally defined, and yet which takes from the supply considerable and equal currents in all three lines.

A second possible and rational definition of 3-phase VA is the arithmetical sum of the single-phase VA. As the arithmetical sum of the currents in parallel circuits is always greater in magnitude than their resultant, excepting when the power factors of these circuits are all the same, so the arithmetical sum of the single-phase VA is always greater than the total equivalent VA, excepting when the three single-phase power factors are all equal. With a 3-phase 3-wire load this condition only obtains when the load is balanced.

A third rational method of defining 3-phase VA is as $\sqrt{3}$ times the square root of the sum of the squares of the single-phase VA. When these single-phase VA are all equal the 3-phase VA, according to this definition, is equal to the arithmetical sum, and hence to the total equivalent VA value. Otherwise it is easy to show that the root-mean-square definition leads to a value which is greater than the other two. For, let the single-phase VA be represented by the quantities a, $a + m$, and $a - n$, then the square of the 3-phase VA, according to the root-mean-square definition, will be

$$9a^2 + 6a(m - n) + 3(m^2 + n^2).$$

The square of the arithmetical sum of the single-phase VA will be the square of $(3a + m - n)$ or

$$9a^2 + 6a(m - n) + m^2 + n^2 - 2mn,$$

which, as m and n are essentially positive, is less than the value according to the root-mean-square definition excepting when m and n are zero.

Thus the total equivalent VA, the arithmetical sum VA and the root-mean-square VA in an unbalanced 3-phase load are in ascending order of magnitude.

The differences between the three values of 3-phase VA obtained in accordance with these definitions can be illustrated by considering a simple numerical example. Suppose a single-phase load of 1 A is supplied from a 3-phase system with a line pressure of $\sqrt{3}$ at a power factor of unity. As the VAr in the two phases supplying the load will be equal, the one leading and the other lagging, the 3-phase VAr will be zero and the total equivalent VA will be $\sqrt{3} = 1{\cdot}73$. Each phase voltage is 1, so the arithmetical sum VA will be 2. The R.M.S.

value of the 3-phase VA will be $\sqrt{3} \times 2 = 2{\cdot}45$. In this extreme condition of unbalance the numerical values are very discordant. Whilst it is true that, with small unbalance, the differences of the three values are negligible, it seems desirable on general grounds that the definition of so important a quantity as a kVA should be authoritative and not left to the discretion of the supply authority.

The total equivalent VA and the root-mean-square VA are those in actual use for the measurement of 3-phase demands. Whilst the first conventional value has considerable scientific justification, it may be noted that the second might be considered to have some advantage in that it is greater than the first by an amount depending upon the unbalance. In the language of symmetrical component theory it is easy to show that if the 3-phase voltages are approximately balanced the total equivalent VA corresponds to the positive sequence component of the line currents, while the root-mean-square VA corresponds to the square root of the sum of the squares of the positive and negative sequence components.

Average Power Factor. The quantity average power factor which, as we have seen, is used in some methods of charging for A.C. power, is one which, like 3-phase VA, is uncertain in its meaning. The most obvious of the possible meanings is a time average such as would be given by a recording instrument. Such a value, although precise in its significance, would be very inconvenient as a basis for charging for an electricity supply, and an alternative value based upon the registrations of integrating instruments is evidently more desirable. A definition of average power factor alternative to that of a true average is by the definition kWh/kVAh. This ratio would be more correctly designated energy factor than average power factor. It is fairly definite in its significance, but it is practically inconvenient, because it involves the quantity kVAh which, as will be seen hereafter, is difficult to measure. A third definition is that given by the ratio of kWh to the square root of the sum of the squares of kWh and kVArh. This definition has the advantage that it involves physical quantities which, in 3-phase circuits at any rate, can be measured fairly easily and with considerable accuracy. On the other hand, the quantity as defined has no very clear physical significance.

These three definitions of average power factor only lead to the same numerical result when load and actual power factor are constant. Otherwise the values may be considerably discordant. This can be illustrated by a simple numerical illustration in which the three average values are calculated by a load and power factor variation represented by the figures in the following table :—

Time in Hours.	Power Factor.	kWh Consumption.	kVArh Consumption.	kVAh Consumption.
1	0·5	1	1·73	2
2	0·8	8	6	10
1	1·0	4	0	4

Total consumptions 13 7·73 16
Time average power factor = $\frac{1}{4}(0{\cdot}5 + 1{\cdot}6 + 1{\cdot}0)$ = 0·775
Energy factor = kWh/kVAh = 13/16 = 0·815
$\text{kWh}/\sqrt{\{(\text{kWh})^2 + (\text{kVArh})^2\}}$ = 13/15·1 = 0·862

The numerical differences are here sufficient to show that average power is in itself a term of uncertain significance without further definition. Owing to the relative ease with which the quantity kVArh can be measured in 3-phase circuits, the definition which involves this quantity and kWh is generally used in a system of charging under which the rate per kWh is graded according to average power factor.

Reactive Kilovoltampere Hours. This is the third electrical quantity upon which the charges for an A.C. power supply are directly based. Its fundamental nature has already been sufficiently explained in reference both to single- and 3-phase circuits.

Prepayment for Electricity Supply. So far we have considered, in reference to the question of electrical measurement, methods of charging for electricity supply under which the measurement is made in order to give the information necessary for assessing the charge. In another method of supply, largely used in this country for small consumers, payment is made before the supply is given, and the function of the measuring device is gradually to use up or exhaust the prepaid sum, and when this exhaustion is complete to cut off the supply. Meters which perform this function are called prepayment meters, and the simplest type of such meters is that which is based upon the flat rate method of charging, and which will pass a definite amount of energy for each unit of amount prepaid.

The prepayment method of supply can also be adapted to conform to those modifications of the two-part system of maximum demand system of charging which have already been mentioned as being in common use for domestic supplies. Thus the prepayment method can be made to conform to the two-part domestic tariff under which a fixed quarterly charge is payable by the use of a meter whereby the sum prepaid is exhausted, not only by the use of the supply, but also at a constant rate corresponding to a fixed quarterly charge. In such a meter there will be two moving elements, the one having a speed proportional to the power consumption, the speed of the second element being constant.

The block rate tariff for domestic supplies can be used with the prepayment method. In this case the number of kWh allowed for each unit of money prepaid is automatically increased when the consumption of energy in an accounting period reaches a stipulated amount.

There is a further method of prepayment supply which corresponds to no method of charging for energy which has been previously consumed. This is known as the load-rate method, and the prepayment meter used with this method is designed automatically to increase the number of kWh allowed for each unit of prepayment when the power consumption in kW exceeds a stipulated value. The adjustment to the meter mechanism is made by the agency of a relay responsive to the current in the meter. This method of prepayment supply appears to be in direct conflict with the scientific theory of charging for electrical energy in that a reduction of the load factor by a peak demand reduces the average rate per kWh which is prepaid for the supply. As this method of prepayment supply is said to be profitable to the supply undertaking and advantageous to the small domestic consumer, it seems to be an excellent illustration of the importance of the principle of expediency on the economics of electricity supply.

The Electricity Supply (Meters) Act, 1936. We have now briefly reviewed the various methods in use for charging for electricity supplies and the electrical measurements which are required thereby. The total sum which a consumer is required to pay for his supply depends upon the method of charging, the tariff rates and the measured electrical quantities which are relevant thereto. Tariff rates and methods of charging are, within wide limits, at the discretion of the supply authority and unregulated by statutory authority, while, on the other hand, a consumer can usually exercise some option as to the system of supply under which he will be charged, and so choose the one which gives for him the lowest overall rate per kWh. The accuracy of the measurement of the electrical quantities involved in assessing charges for electricity supply, so far as the determination of real or fictitious kWh is concerned, is regulated by statutory requirement, and limits of error for consumers' meters are explicitly defined.

Till the passing into law of the Electricity Supply (Meters) Act, 1936, these statutory requirements were, however, somewhat ill-defined. Powers and duties connected with the certification of consumers' meters were exercised by Electric Inspectors, who were virtually appointed by, but who were independent of, the supply authorities. As there appeared to be no obligation for an authority to appoint an Inspector, this statutory method of regulation was virtually a dead letter, although the general standard of the accuracy of consumers' meters was maintained well within the statutory limits of error by the efficient testing organisations of the supply authorities. Under the provisions of the Electricity Supply (Meters) Act, 1936, not only are consumers' meters required to be of approved design and construction as heretofore, but they must all be certified as being so approved and being capable of ascertaining the magnitude of the supply within the statutory limits of error by a Meter Examiner appointed by the Electricity Commissioners. The actual testing of consumers' meters for certification is still carried out by the supply authorities, subject to the provision that the testing equipment conforms to a specification issued by the Electricity Commissioners supplementary to the Act. The actual testing, moreover, must be carried out according to prescribed methods. Approval of testing equipment is given by a Meter Examiner, and a place containing such approved apparatus is known as a Testing Station. Testing work is, moreover, under the general supervision of a Meter Examiner. Thus, under the provisions of the 1936 Act, no meter can be used for the measurement of real kWh or of fictitious kWh based upon quantity unless it has been certified by a Meter Examiner. At the time of writing these requirements apply only to meters registering kWh or units. They do not apply to any other types of instruments used for the measurement of kW or kVA demand or of kVAhr.

The present limits of error for consumers' meters as prescribed in Supplementary Notes to the Act are 2½ per cent. plus or 3½ per cent. minus at any load at which the meter may be operating. This prescription will be considered in detail hereafter.

CHAPTER II

SUPPLY METERS AND METHODS OF MEASUREMENT

Instruments Used in Measuring Consumers' Supplies. In the preceding chapter we have seen that the assessment of the charges for consumers' supplies involves the measurement of two kinds of electrical quantities, energy consumption and power demand ; of the quantities used in power factor tariffs, kVArh is of the same physical nature as kWh, and kVA as kW. The difference in the physical nature of kWh and kW involves an important difference in the instruments used for their measurement. kWh is a time integral in its nature, and a meter registering kWh has an indicating register which advances at a rate proportional to the rate of energy consumption. Energy consumptions are cumulative and the consumption during any period is obtained as the difference of the readings of the register at the end and the beginning of the period. An error in the reading of a kWh meter will cause an error in the derived consumption for the period preceding, but this error will be compensated by an equal error of opposite sign, for the next consumption period. An instrument which indicates maximum demand, on the other hand, gives a reading which represents a quantity of the nature of an instantaneous rate of consumption. After the indicating pointer is read, it is set to zero. A mistake in the reading of an indicator of maximum demand is not therefore corrected by a compensating mistake for a subsequent period, and, as the record is obliterated after reading, this reading cannot subsequently be checked, as can the reading of the register of a kWh meter.

Fundamental Principles of Electricity Meters. Meters, or instruments for registering real or fictitious kWh, are, as regards the underlying principle of their action, of three main types. These are, electrolytic, motor and clock meters.

An electrolytic meter is one which is based upon Faraday's law of electrolysis according to which the quantity of an ion liberated from a solution by electrolysis is exactly proportional to the quantity of electricity which has passed. In an electrolytic meter the amount of the liberated ion is measured volumetrically by a device calibrated in kWh at a declared voltage, so that, by inspection of the meter scale, the fictitious kWh consumption during a period can be obtained as the difference of two readings.

The essential constituent of a motor meter is an element which rotates at a speed which is proportional to the power or current in the circuit containing the meter. The time integral of rotational speed or amount of turning of the rotating element will be proportional to the corresponding time integral of the power or the current, that is, to the energy or quantity. Each revolution of the moving element of a motor corresponds therefore to a definite amount of actual energy or fictitious kWh at a declared voltage. The constant of proportionality of revolutions of turning to kWh is generally expressed as revolutions per unit. The turning element is mechanically geared to the register, and the gearing ratio is such that the velocity ratio of the turning element to the spindle for the pointer indicating kWh is ten times the fundamental revolutions per unit constant if the meter is to register correctly. If this velocity ratio and the constant of the meter element are not in correspondence, the constant, or the speed of the element with 1 kW passing, can be adjusted to suit the gearing ratio, or the gearing ratio can be adjusted to suit the meter constant. It is usual to adopt a round figure for the gearing constant and to adjust the characteristic or constant of the meter element to suit it.

The ability of a motor meter to register energy correctly irrespective of variations of power in the circuit containing it, depends upon exact proportionality of speed to power. This condition is satisfied if the torque producing rotation is proportional to the power and if the rotation is impeded only by a braking torque proportional to the speed. For if the driving torque is aW, W being the power, and the braking torque $b\omega$, ω being the speed, then, as there are no other torques affecting rotation, $a\mathrm{W} = b\omega$ when the speed is steady, so that $\omega = a\mathrm{W}/b$.

A braking torque of the required characteristic is provided by a conducting disc attached to the rotating element and placed in the field of a permanent magnet. The eddy currents set up by relative motion of the braking disc and the permanent magnet field, and hence the braking torque, will be exactly proportional to the speed. Thus an eddy current brake is a secondary essential component of all motor meters.

Summarising the foregoing argument, the accuracy of a motor meter with an eddy current brake depends upon proportionality of driving torque to power or current, freedom of movement of the rotating element from all constraint excepting that exercised by the eddy current brake, and correspondence of the 'constant' in revolutions per kWh of the meter element with the gearing constant of the meter. We shall examine in a subsequent chapter how far these requirements can be practically realised.

The clock meter is the third type used for energy measurement. A body moving with constant speed will, according to Newton's first law, cover spaces proportional to times. Thus, with uniform speed, distance is a measure of time with translational motion and angle is a measure of time with rotational motion. This principle is utilised in a clock, for the movement of the hands indicates time intervals. If the rate of a clock can be increased or retarded by an amount proportional to electric power, then the gain or loss of this clock over a period will be a measure of the energy which has been the cause, because this gain or loss will be a time integral of difference between the actual and normal rates. It is easy, by means of a differential gear, to indicate the difference in the time readings of two clocks, one of which is modified in its action by electrical agency. This is the principle of the clock meter, in which the registration depends inherently upon the difference of the advances of two clocks controlled electrically.

It is necessary, in addition to accuracy in principle, that an electricity meter should absorb or waste but a very small fraction of the power it is intended to measure. The losses in a meter are resistance losses in the windings, and the power required in a motor meter to drive the moving element. A quantity meter has one circuit only, which carries the current, and the loss in this circuit is expressed as a voltage drop with the maximum or full load current passing. An energy meter has a voltage circuit connected to both supply mains, and the loss in this circuit is additional to that in the current circuit, and is nearly constant at all times. The loss caused by the power required to drive the moving element is generally negligible in comparison with the other losses.

Excepting in the case of meters for small currents, the whole of the current of the main circuit does not, as a rule, pass into the current coil. Meters for D.C. are shunted and A.C. meters have their current coils supplied by current transformers which supply a secondary current in a nearly constant ratio and of nearly the same phase as the main current which flows in their primary windings.

Alternative to the classification of meters according to their basic principle of operation, they may be classified according to the kind of supply they are capable of measuring. Electrolytic meters, which inherently integrate current, are generally suitable only for D.C. supplies. Motor meters of the quantity type, in which the torque corresponds in magnitude and direction to the current, are inherently suitable for D.C. only. An energy meter suitable for D.C. circuits will, if its accuracy is maintained over its whole voltage range, be a true integrating wattmeter, and will therefore register energy in A.C. circuits, although there may be practical objections to its use for this purpose. A clock meter responsive to power over its whole range of current and voltage will likewise register in either D.C. or A.C. circuits. A meter which depends for its working on any kind of transformer action will be suitable for A.C. only, and will not, of course, register in a D.C. circuit.

Electrolytic Meters. Although the electrolytic method was the earliest used for the measurement of consumers' supplies, only two meters of this type have survived. In the first of these, quantity of electricity or kWh at a declared voltage is measured by the electrolysis of a dilute solution of caustic soda in a graduated vessel, using nickel electrodes. The loss of water by electrolysis can be observed from the change in the level of the liquid, and the scale is graduated in kWh. When the level sinks to, or nearly to, the extent of the scale, the cell must be refilled by adding water. To prevent evaporation a little paraffin is poured on to the electrolyte. The electrolytic cell has a back e.m.f. of polarisation of about 1·7 volt. This means that its resistance to the flow of a current in it as given by the ratio of voltage drop to current is variable. The cell cannot therefore be shunted, as the proportion of the total current passing through the cell would not be constant, but would increase as the main current increased. Hence this kind of meter, being essentially of the total current kind, is only suitable for measuring very small supplies. The meter is simple and cheap, but it has several disadvantages. The voltage drop is high, varying from 1·7 to about 2½; it has to be refilled with water at frequent intervals, when the record is lost. The position of the level of the liquid is difficult to read after the meter has been in use for some time. The electrochemical 'constant' of the meter is given by the data that 100 ampere hours will decompose 34·66 c.c. of water.

In the second kind of electrolytic meter a saturated solution of mercury and potassium iodides completely fills a hermetically sealed glass vessel. The current enters the solution by an electrode consisting of a quantity of mercury and leaves it by an electrode of iridium. When current passes through the cell, mercury is dissolved at entry of the current and deposited at the iridium electrode, from which it falls into a tube graduated to read in kWh. When this tube fills, the mercury in it falls, by syphonic action, into a second tube of larger area graduated in 100 kWh divisions. As the electrolytic action is to make mercury go in and out of solution, there is practically no back e.m.f. of polarisation. The resistance of the cell is therefore constant, and it can be, and invariably is, shunted. The cell, however, has a negative temperature coefficient of resistance and a compensating resistance of iron is connected in series with it, so that changes of temperature have a negligible effect on the total resistance of the circuit connected in parallel with the shunt. This meter is reset, when the whole of the store of mercury is transferred from the measuring tubes, by inverting the container, whereby the mercury runs by gravity to its original position. The meter is simple and fairly easy to read, but unless it is fixed on a firm and rigid support mercury is liable to be shaken into the measuring tube by vibration.

Motor Meters for Measuring Quantity. Meters of this kind are either what are called commutator or mercury motor ampere-hour meters. The electrical circuits of the commutator quantity meter are shown in Fig. 1. A light pivoted armature with commutator and brushes is free to rotate in the field of a permanent magnet. The armature coils are generally contained in an aluminium casing, in which eddy currents are induced by rotation to form as dynamic brake. The brushes of the armature are connected to a shunt. If the main current is I and the armature current I_1, the shunt current will be $(I - I_1)$. The shunt resistance being R_2, the drop in the shunt will be $R_2(I - I_1)$. This drop will be equal to the sum of the resistance drop R_1I_1 in the armature circuit and the back e.m.f. caused by rotation of the armature in the permanent magnet field, which is proportional to the rotational speed ω of the armature. Thus

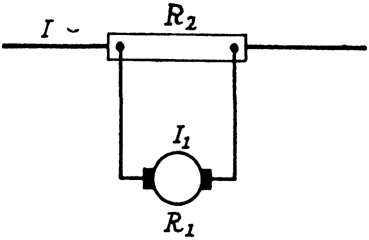


FIG. 1.—Commutator quantity meter.

$$R_2(I - I_1) = K\omega + R_1I_1,$$

where K is a constant.

Thus $$I_1 = \frac{R_2I - K\omega}{R_1 + R_2}.$$

The driving torque on the armature is proportional to the current in it, the number of conductors and the permanent magnet flux, and if the movement is impeded only by eddy current braking, the opposing torque will be proportional to ω.

Thus $$\frac{R_2I - K\omega}{R_1 + R_2} = K_1\omega,$$

where K_1 is another constant depending upon the braking. So

$$\omega = I \times \frac{R_2}{K + K_1(R_1 + R_2)}$$

and the speed of rotation is proportional to I.

This equation shows that a commutator quantity will work without an eddy current brake. For if the braking constant K_1 is zero, we shall have $\omega = I \times R_2/K$, a linear proportionality between ω and I. In this condition, moreover, ω is independent of R_1, the resistance of the armature circuit, and hence of the brush contact resistance. Actually the speed of an unbraked commutator meter would be too high for this property to be utilised.

The mercury motor ampere-hour meter is illustrated diagrammatically in Fig. 2. A pivoted disc of pure copper is completely immersed in a bath of mercury placed in the field of a permanent magnet. The current of the supply enters the centre of the circular bath and leaves it at a terminal on its periphery. A constant fraction of this current traverses the disc in a direction which is perpendicular to the permanent magnet field, and a driving torque proportional to the disc current and to the total flux causes rotation of the disc. The torque per ampere is increased by slotting the disc radially, whereby the current is constrained to follow more or less definite paths, but the meter will operate with a plain disc, although, as in this condition the disc current is not associated physically with a moving conductor, it is by no means self-evident why it does. As the disc rotates, eddy currents, superposed upon the circuit current, are produced and these give rise to a braking torque. Sometimes the braking torque is increased by providing a second permanent magnet, which gives a braking flux in the portion of the disc which does not carry the circuit current. This second magnet is shown in the figure. There is a back e.m.f. of rotation set up in the mercury meter as there is in the commutator meter, but as in the mercury meter there is only one equivalent armature conductor, the e.m.f. is so small as to be negligible.

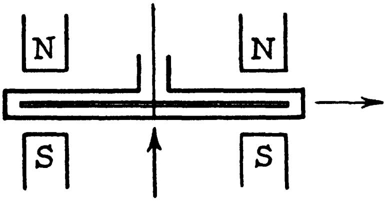


FIG. 2.—Mercury motor quantity meter.

Motor Meters for Measuring Energy in D.C. Circuits. These meters are of two kinds, known generally as the commutator or dynamometer watt-hour meter and the mercury motor watt-hour meter. The commutator meter is illustrated diagrammatically in Fig. 3. An armature connected in series with a resistance is connected to the supply mains, and this armature is free to rotate, subject to the constraint of an eddy current brake in the field of two coils which carry the circuit current. As the field is proportional to the current and the armature current to the voltage, the torque is proportional to the power and the total rotation of the armature is a measure of the energy in kWh. If the meter carries alternating current and the alternating magnetic flux is exactly in phase with the current producing it, and the armature current is in phase with the voltage, the instantaneous torque will correspond to the instantaneous VA and the average torque over a cycle to the A.C. power, so that the registration will be a true measure of the A.C. energy. The same result will be obtained if the phase difference of flux and current is exactly the same as that of armature current and voltage. Although the commutator watt-hour meter is generally used for D.C. circuits, it is quite possible to make the necessary adjustments whereby it may be an accurate instrument for measuring A.C. energy. We shall, however, shortly see that it is possible to measure A.C. energy by a different kind of instrument which, although similar in principle to the commutator or dynamometer energy meter, is much simpler in construction.

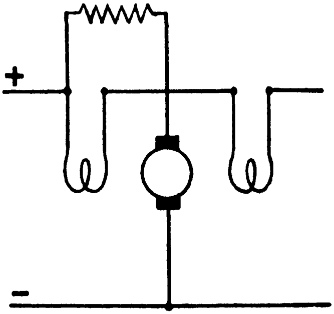


Fig. 3.—Commutator energy meter.

The mercury motor watt-hour meter is in a large measure similar to the corresponding quantity meter, and its construction is illustrated in Fig. 4. Here the pivoted disc immersed in mercury is free to rotate in the field of an electromagnet, which is excited by a winding connected to the supply mains. Eddy current braking is supplied by a separate disc moving in the field of a permanent magnet. If the flux of the electromagnet is proportional to the exciting current, and hence to the voltage, the torque will be proportional to the power in watts. This condition is, however, far from being satisfied, as the magnetic circuit for the flux contains iron. Because of hysteresis, the torque with constant current will not diminish proportionally with the voltage, but will be too high for voltages under the normal value and too low for voltages over this value. For this reason the mercury motor watt-hour meter is not a true energy meter, and it can only be regarded as a quantity meter which is very approximately compensated for such voltage variations as are likely to occur in D.C. circuits. As the instantaneous voltage in an A.C. circuit varies in a half cycle from a positive maximum through zero to a negative maximum, a mercury motor meter would be incompetent to follow these variations in strict proportion and in exact phase correspondence. This meter, therefore, is inherently unsuitable for A.C. circuits.

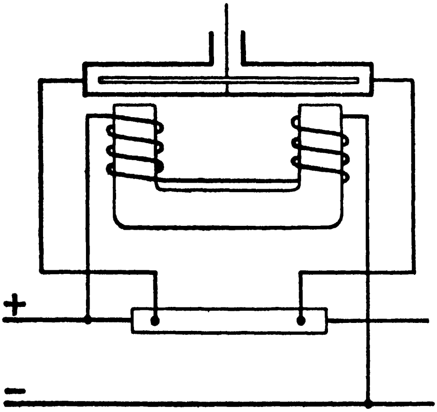


Fig. 4.—Mercury motor energy meter.

Motor Meters for Measuring Energy in A.C. Circuits. These meters are almost invariably what are called induction meters. The general construction of this meter is as shown in Fig. 5. Here a plain pivoted disc is free to rotate between two electromagnets, the upper of which has a winding connected to the supply voltage, and the lower of which is excited by the circuit current. The leakage alternating flux from the centre

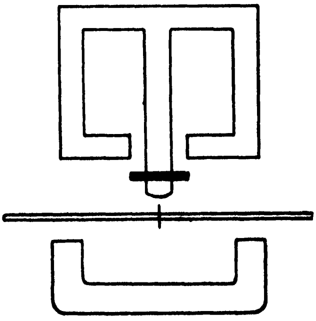


Fig. 5.—Induction meter.

pole of the voltage magnet induces currents in the disc, and these, being displaced in physical position with respect to the flux of the current magnet, give rise to a driving torque. The meter is thus similar to the commutator meter, excepting that the currents in the moving element which correspond to the voltage are induced by transformer action, instead of being conveyed into it by conduction through brushes. If the voltage flux lags exactly 90 degrees in phase on the voltage, and the eddy currents induced by this flux circulate in disc paths of negligible inductance, then these eddy currents will correspond in phase with the voltage, for there is a 90-degree phase difference between the e.m.f. causing them and the voltage flux which produces this e.m.f. These conditions being satisfied, the meter will correctly measure A.C. energy, for the field of the current coils, and the disc currents produced by the voltage flux are each in phase respectively with the circuit current and voltage. Actually these conditions are not satisfied without a compensating device, for the lag of the voltage flux is necessarily less than 90 degrees. The most usual compensating device is a copper ring placed on the salient pole of the voltage magnet as shown in the diagram. The current induced in this ring lags considerably in phase on the flux producing them, and the magnetic effect of this current is to produce a small additional lag in the phase of the leakage flux cut by the disc. The effect of the ring is adjustable by varying its position on the pole, so that the condition requisite for accurate A.C. energy measurement can be obtained exactly. This condition can be checked by observing that there is no torque when the circuit current and voltage are exactly 90 degrees out of phase. The induction meter movement is braked in the usual way by means of a permanent magnet placed so that the one disc rotates in its field.

Clock Meters. The principle and main features of the clock meter are illustrated diagrammatically by Fig. 6. Here AA are two short pendulums, the bobs of which are coils of fine wire connected in series with a resistance to the circuit voltage. These pendulums oscillate over fixed coils BB, which carry the circuit current. The directions of the windings are such that the magnetic effects of currents in the two sets of coils are equal and opposite. The action of the meter can be explained simply in the following way. The number of complete swings of a pendulum, N, in a unit of time depends upon its dimensions and the gravitational acceleration, g. Algebraically

$$N = K\sqrt{g},$$

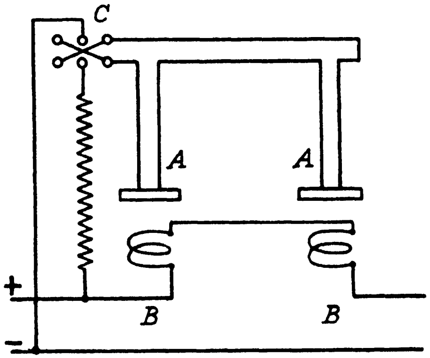


FIG. 6.—Clock meter.

where K is a constant of the pendulum. If a pendulum coil of the clock meter and the associated fixed coil each carry current, the effect of these currents will be to alter the restoring force per unit displacement of the bob from its position of rest, and this dynamically will be the same as an increase or decrease of the gravitational acceleration g. Further, the magnitude of this increase will be proportional to the product of the currents in the two coils, and hence to the instantaneous watts, W. Thus one of the pendulums will swing at a rate represented by

$$N_1 = K_1\sqrt{g + a_1W},$$

where a is a constant depending upon the physical characteristics of the coils; the second pendulum will swing at a rate represented by

$$N_2 = K_2\sqrt{g - a_2W}.$$

The difference of the two rates is therefore

$$N_1 - N_2 = K_1\sqrt{g + a_1W} - K_2\sqrt{g - a_2W},$$

and if a_1W and a_2W are both small in relation to g, this relation is very approximately the same as

$$N_1 - N_2 = \sqrt{g}(K_1 - K_2) + \frac{W}{2g}(a_1 + a_2) \quad . \quad . \quad \text{(i)}$$

The difference in the rates is thus a constant quantity depending upon the physical dimensions of the pendulums plus a quantity proportional to W. This total quantity represents the advance of pendulum 1 on pendulum 2. If now the direction of the currents on the pendulum coils be reversed, pendulum 1 will swing at a rate less than that of pendulum 2, and the difference in the ratio $N_2 - N_1$ will be

$$N_2 - N_1 = -\sqrt{g}(K_1 - K_2) + \frac{W}{2g}(a_1 + a_2) \quad . \quad . \quad \text{(ii)}$$

Now the pendulum meter contains a differential gear, whereby the difference in the rates of swing of the two pendulums is caused to advance a register. Periodically the direction of the current in the pendulum coils is made to reverse by means of the commutator C, and simultaneously the direction of the advance of the register is reversed mechanically. Thus, in the first period, if pendulum 1 is fast the advance of the register will correspond to equation (i) above. At the expiration of this period the current in the pendulum coils is reversed and the register will measure the excess of the rate of pendulum 2 on pendulum 1. During this second period the advance of the register will correspond to equation (ii). The total advance of the register in the two equal time periods will thus be

$$\frac{W}{g}(a_1 + a_2)$$

and proportional to the power W, the difference of the natural rates of the pendulums represented by the difference of the constants K_1 and K_2 being compensated by the double reversal.

The clock meter thus depends, for its inherent accuracy of principle, on the accelerative effect of the currents in the fixed and pendulum coils being small in relation to g and upon equality of the periods between successive simultaneous reversals of the pendulum coil current and the register gear. The usual period between these reversals is about 10 minutes, and the actual mechanism of them is controlled by the pendulums. The register is advanced by an electrically wound spring controlled by the pendulum escapements, so that the clock meter, unlike the motor meter, does not obtain its driving power electrically.

Shunted D.C. Meters. Reference has already been made to the use of shunts with direct current meters. Commutator quantity meters are necessarily shunted, while mercury motor meters are shunted in all but those of small current rating. As long as the circuit current remains constant the proportion of this current which passes into a shunted meter depends only upon the resistances of the meter and shunt branches, so that the quantity passing the meter is a fixed fraction of the total quantity. If the circuit current varies rapidly and a meter circuit having appreciable inductance is in parallel with a non-inductive shunt, this fixed relation of meter current to circuit current will not hold. Referring to Fig. 7, we have, equating e.m.f.'s in the two parallel branches, $R_2I - R_2I_1 = R_1I_1 + L\dot{I}_1$, $\dot{I}_1$ denoting the rate of change of I_1 and L the inductance of the meter branch. Thus

$$(R_1 + R_2)I_1 = R_2I - L\dot{I}_1.$$

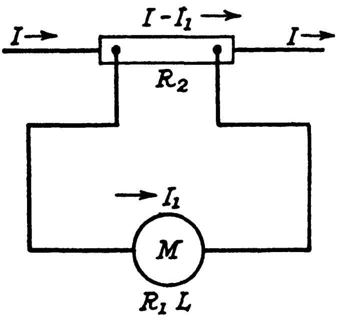


FIG. 7.—Shunted meter.

The time integral of the left-hand term is $(R_1 + R_2)$ times the quantity passing the meter. The time integral of R_2I is R_2 times the total quantity, while the time integral of $\dot{I}_1$, the rate of change of I_1 over a period commencing and ending with zero, is evidently the overall change in I_1, and is zero. Thus the quantity passing the meter is a fraction of the total quantity corresponding only to the resistance values. Similarly, the instantaneous driving torque with varying current I_1 will be aI_1, and this will equal the braking torque $a\omega$ plus the accelerating torque $J\dot{\omega}$ proportional to the acceleration $\dot{\omega}$ Thus

$$aI_1 = b\omega + J\dot{\omega},$$

and the time integral of I_1, the quantity passing the meter, is b/a times the time integral of ω, or the total rotation, since if ω is zero at the beginning and end of the period of integration the time integral of the acceleration or total change of velocity is zero. The relation between registration and the total quantity in the main circuit with a shunted meter is the same as the relation between speed and constant current irrespective of inductance of the meter branch.

This rule applies also to an energy meter if the voltage is constant, but not otherwise. An A.C. meter current coil cannot be shunted without error if the time constants of the two branches differ.

Three-wire D.C. and Single-phase A.C. Meters. Meters with two current coils, such as those shown in Figs. 3 and 5, can be connected with one coil in each 'outer' main of a D.C. or single-phase A.C. supply, and the rate of registration will then correspond to the sum of the 'outer' currents and the voltage between 'outers.' If the outer-to-neutral voltages are equal, and in phase with an A.C. supply, this rate will correspond to the total power, but not otherwise. This kind of meter is not approved by the Electricity Commissioners for the measurement of 3-wire supplies ; for this duty two separate meter elements must be used.

If a clock meter has its two current coils connected each in one main of a 3-wire supply and the corresponding pendulum coils, energised by the corresponding outer-to-neutral voltages, the acceleration and retardation of the pendulums will correspond to the power supplied through the outer main corresponding. The difference between the pendulum rates which cause the registration will thus be a measure of the actual power in the 3-wire supply, irrespective of voltage balance, and the meter will register the true energy in a 3-wire supply.

Three-phase A.C. Meters. Any kind of loading of a 3-phase 3-wire supply can be obtained by connecting two suitable impedances one between each of two pairs of lines. With the connection of the load, one of the lines is a common return for the two single-phase load currents ; and the energy consumed in each impedance can be measured in the normal way by a single-phase kWh meter. It therefore follows that the load in a 3-phase 3-wire supply can be measured by two wattmeters connected as if this load were absorbed by two impedances each supplied from a pair of lines. A 3-phase meter comprises two single-phase meter elements, each of which produces a driving torque in a single rotating member which is geared to a register giving 3-phase energy.

Similarly, as any kind of loading of a 3-phase 4-wire circuit can be produced by suitable impedances connected between any three pairs of lines, the total energy in a 4-wire supply can be measured by three single-phase meter elements. It is usual to connect a combination of meter elements for 4-wire measurements in such a way that the neutral of the supply is treated as the common return, in which case each element carries a line current and is energised by the corresponding phase voltage. This is not the only possible method of connection. Energy in 4-wire circuits can be accurately measured by a three-element combination with which one line is treated as the common return. In this case the elements carrying line currents will be energised by line voltages, while the third element carrying the neutral current will be energised by the phase voltage of the common return line.

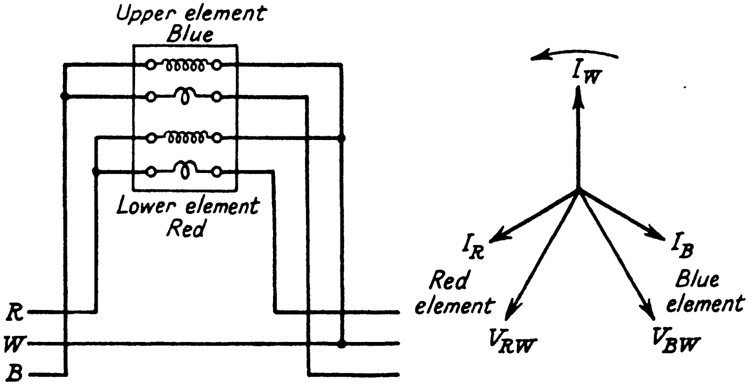


FIG. 8.—Three-phase 3-wire meter.

In a 3-phase meter the measuring elements are situated so close together that leakage flux from each is liable to affect the accuracy of the others. For this reason 3-phase meters should be so connected that the phase relation of the exciting voltages is the same as that for which the meter was calibrated. This condition is satisfied by the observation of a convention whereby the upper element carries the current in the blue-phase and the lower element the current in the red-phase of the supply. The standard connections of a 3-phase 3-wire meter and the vector diagram for balanced load, unity power factor, are shown in Fig. 8.

Approximate Methods of Measuring 3-phase Energy. There are various approximate methods of measuring 3-phase energy by fewer meter elements than are required according to the fundamental principle. These all depend upon some condition of the loading or of the voltages being satisfied. Thus if a 3-phase load is balanced, both as regards currents and voltages, that is, if the vector systems of currents and voltages are perfectly symmetrical, the total energy consumed by the load can be determined as one-third the energy consumed in one phase of the supply. This latter quantity can evidently be measured by a single-phase meter carrying a line current and having its voltage circuit connected between this line and the neutral of the supply or a point at the same potential as the neutral.

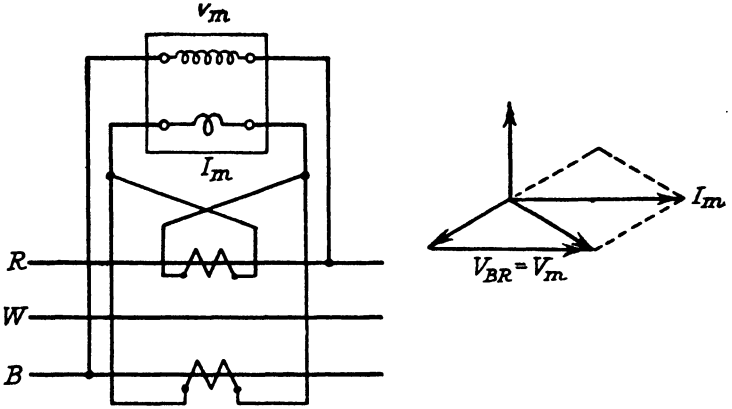


FIG. 9.—Three-phase balanced load meter.

Again a meter which has two current coils each carrying a line current and having its voltage coil connected between these lines will, if correctly connected, measure the total energy in a 3-phase supply. Instead of a double-coil meter, one of ordinary construction may be used in connection with two current transformers joined as shown in Fig. 9. This connection is such that the go and return currents of a single-phase load connected between the two lines containing the meter would flow in the same direction in the meter coil. It is clear from the vector diagram that the resultant current in the meter is $\sqrt{3}$ times a line current and that the meter power factor is the same as the power factor of the 3-phase balanced load. Hence the meter registers the energy supplied to the balanced load.

The two kinds of balanced load meters just described both depend for their accuracy of principle upon the condition of balance being accurately satisfied. Quite small departures from the balanced condition will give rise to errors which are too great to be negligible. These balanced load meters are not approved by the Electricity Commissioners for the measurement of consumers' supplies, but they are useful and often sufficiently accurate as sub-meters in large power installations for determining the allocation of total power charges among the several departments of the works.

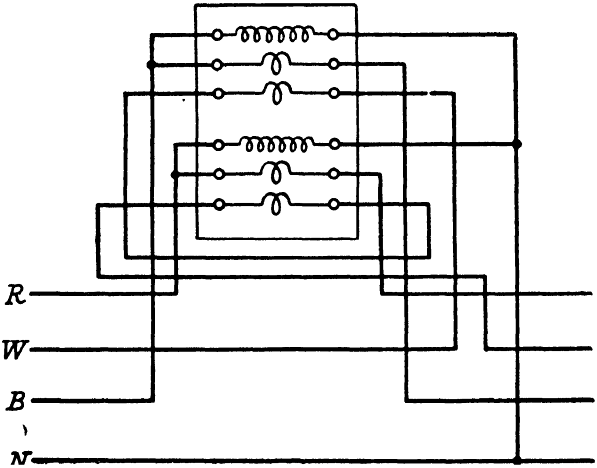


FIG. 10.—Three-phase 4-wire meter.

In certain circumstances the energy consumed in a 3-phase 4-wire meter can be measured by a two-element double-coil meter. One coil of each element carries a line current, and the voltage circuit is energised by the corresponding phase voltage.

The third line current flows in the reverse direction in the second coil of each element. The schematic diagram of connections is shown in Fig. 10. The association of red-phase current and voltage will produce a torque corresponding to the red-phase power. Similarly for the blue-phase current and voltage. The association of reversed white-phase current with red- and blue-phase voltages in the two elements will produce a torque which will correspond with the association of the white-phase current with the reversed resultant of red- and blue-phase voltages, and if this resultant is identical with white-phase voltage, the circulation of the white-phase current in the two windings will produce a torque corresponding to white-phase power. Thus the total torque will correspond to the total power and the two-element meter will register the total energy subject to the condition that the instantaneous sum of the phase voltages is always zero, or that their vectors form a closed triangle. The magnetic effect of two currents in separate windings can be obtained if suitable currents from secondary delta of a set of current transformers flow in the coils of an ordinary two-element meter, since the resultant current arising from the delta connection is of two line currents taken in opposite senses. In this case the full load current of the meter will be $\sqrt{3}$ times the full load secondary current of the transformers. This type of meter for the measurement of 4-wire supplies is approved by the Electricity Commissioners for existing meters, but not as regards future manufacture, presumably because, although the condition for accuracy is in practice very approximately satisfied, there is, nevertheless, an inherent defect in the underlying principle of the meter. It should be noted that it is not necessary for the phase voltages to be balanced when using this approximate method of 4-wire energy measurement; it is only requisite that the vectors of these voltages form a closed triangle.

Measurement of Maximum Power Demand. Reference has been made in the preceding chapter to two inherently different methods of measuring maximum demand. In the averaging or Merz method the consumptions of energy during equal successive clock periods are communicated by gearing to an auxiliary kW indicating register. The indicator gear is spring-loaded, and an arm actuated by the gearing pushes over a scale an indicating pointer, friction-tight on its spindle, by an amount corresponding to the energy consumption within the fixed clock period. The advance of the pointer from the zero of the scale is therefore representative of the average power consumption within the period. At the expiration of each clock period the indicator mechanism is thrown out of gear with the main register for a short time by means of an electrical device actuated by a clock, and the arm of the mechanism returns to the initial position. The friction-tight indicating pointer, however, remains at the position which it reached at the end of the period. If during the next or any subsequent period the consumption is less than that corresponding to the scale reading, the arm of the mechanism will fail to reach the pointer, which will be unaffected. If, however, any subsequent consumption during a clock period is greater than that corresponding to the existing indication, the pointer will be advanced further to correspond. Thus the scale reading of the pointer gives the maximum average consumption within a clock period during the whole accounting period of reading and resetting the indicator. The actual resetting of the pointer is done by pushing it back to zero by hand.

This type of demand indicator, whereby the indication is based on continuous integration, is evidently suitable for any kind of meter of the motor or clock types. When used in conjunction with a clock meter, the temporary disengagement of the indicator gear is controlled by the clock in the same way as the reversal of the main register and pendulum circuit connections. In this case the integration period for demand indication will correspond to a complete cycle of the main register reversal. A Merz indicator fitted to a motor meter registering kVAh will give a measure of the maximum apparent demand in kVA; these meters will be dealt with hereafter.

In the thermal type of demand indicator the indication corresponds to the maximum rise of temperature of a component of the instrument which is heated by the circuit current or by a constant fraction thereof. If a constant current is passed through the instrument, then, when a steady temperature rise is attained, the rate of loss of heat in the heated component proportional to θ, the temperature rise, will be equal to the rate of gain of heat, proportional to the square of the current, I^2. Thus the steady current indication will correspond to the square root of the temperature, and with a uniform temperature scale the current scale will be contracted near zero. Between the time of the incidence of a constant heating current and the reaching of a steady temperature rise the actual relation between temperature and time will be of an exponential character, and the advance of the pointer during this period will be as shown in the graph of Fig. 11. The advance of a Merz indicator with constant loading will be at a uniform rate, as is shown by the diagonal straight line. If a steady current flows only for a fraction of the time required to produce the maximum temperature rise, the thermal indicator reading may have such a value as OB ; the corresponding reading of a Merz type of indicator giving a true rate of consumption would be OA.

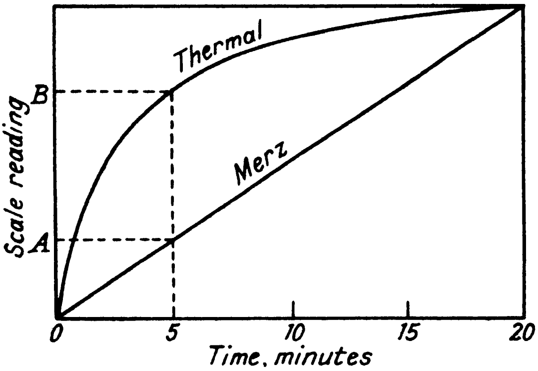


FIG. 11.—Demand indicator characteristics.

There are two kinds of thermal demand indicators. In one of these the temperature rise is measured by a differential air thermometer, and is indicated by the quantity of liquid which, by reason of the relative expansion of air in one bulb, is made to overflow into a measuring tube which is graduated in amperes, or kW or kVA at a declared voltage. In the second kind the indication is given by the relative movement of the free ends of two bi-metallic strips, one of which is heated by the current. This relative movement depends upon the heating of the current and is independent of variations of the ambient temperature. The strips are connected by gearing which pushes a pointer forward over a scale graduated in amperes or kW, the pointer, like that of a Merz type of indicator, being friction-tight on its spindle, so that it can be reset to zero after being read.

It has been stated already that, although thermal demand indicators are suitable for the measurement of kVA at a declared voltage in A.C. circuits, they cannot be used for 3-phase measurements of this kind.

Measurement of Reactive VAh. We have seen in Chapter I that two systems of charging for power supplies depend upon the measurement of kVArh, either as a direct basis for charging or for use in calculating an average power factor according to which the tariff rates are graded. Some of the methods of measuring kVAh in 3-phase circuits in commercial use also depend upon the simultaneous integration of kW and kVAr.

It may be taken as almost self-evident and without formal demonstration that if the voltage flux in a single-phase A.C. energy meter can be caused to lag exactly 90 degrees on the phase required for energy measurement, the registration of the meter, so modified, will give lagging kVArh. It is not so self-evident that if two such modified meters be connected in a 3-phase circuit in the same way as two ordinary meters would be connected for energy registration, the sum of the registrations of the modified meters would give the total 3-phase lagging kVAvh as defined in Chapter I. This proportion can easily be demonstrated by analysis, but its truth can be perceived without mathematics by reflecting that any leading kVAr in one phase of the 3-phase supply will tend to produce a reverse torque in the meter which measures it, and so to cancel the forward driving torque produced by lagging kVAr in other phases.

The required phase modification of the voltage flux of a single-phase meter for kVArh registration can be made by the use of a double voltage winding, one connected directly to the supply mains and the other being in series with a resistance. The directions of the two windings are in the opposite sense. Referring to the vector diagram (Fig. 12), the vector of the flux ϕ_1 produced by the directly connected winding, lags about 90 degrees on the voltage vector V. The flux produced by the second winding in series with a resistance will lag by less than 90 degrees, and as this winding is in opposition to the first, the vector of this second flux ϕ_2 will be as shown. The resultant of the two fluxes, ϕ, can, by adjustment of the series resistance, and hence of the lag of ϕ_2 on V, be made to be sensibly in phase opposition to V, that is, 90 degrees differing from the phase required for energy registration. By passing current in the correct direction through the current coils, such a meter will register lagging kVArh.

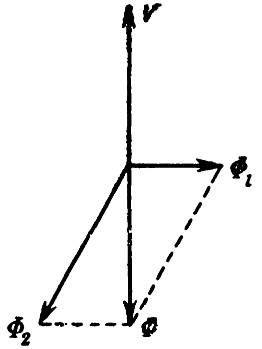


FIG. 12.—Vector diagram of single-phase reactive meter.

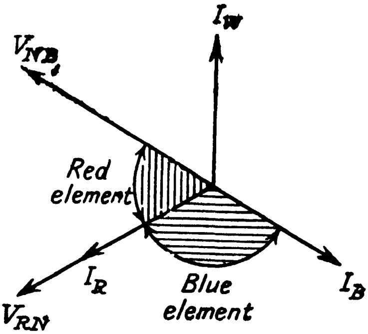


FIG. 13.—Vector diagram of 3-phase reactive meter.

As single-phase distribution is generally limited to domestic and small power supplies, the measurement of single-phase kVAr is rarely required. For 3-phase reactive measurements it is customary not to use single-phase reactive meter elements, but to excite an ordinary 3-phase energy meter by artificial voltages, derived from the supply, which lag 90 degrees on the phases of the voltages required for energy measurement. These artificial voltages, bearing a known ratio to the normal voltages and lagging them 90 degrees in phase, can readily be obtained if the voltages of the 3-phase supply can be assumed always to be balanced.

There are three methods whereby a 3-phase energy meter may be made to register lagging 3-phase kVArh by the use of artificial exciting, each lagging 90 degrees on the normal phase. The first of these is illustrated in the vector diagram (Fig. 13), which shows the vectors of the meter currents for balanced load, unity power factor, the vectors of the artificial meter voltages, and their respective association of current and voltage in each meter element. Here it is seen that the red-phase element is energised by the neutral to blue-phase voltage, whilst the blue-phase element voltage is the red to neutral voltage. These artificial voltages are each 90 degrees lagging on the normal voltages and are in magnitude $1/\sqrt{3}$ of these voltages, provided the vectors of the phase voltages of the supply are symmetrical. This method of measuring kVArh is only suitable if the neutral of the supply is accessible.

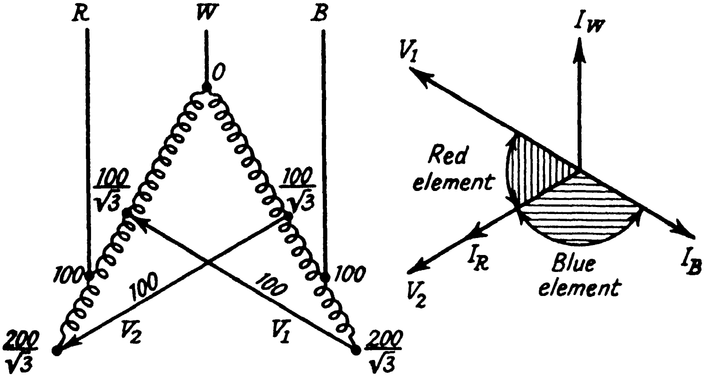


FIG. 14.—Three-phase reactive meter.

A second method of obtaining the required artificial voltages is shown in Fig. 14. Here two auto-transformers are used, connected in open delta to the supply. The line connections are taken to terminals corresponding to two 100 per cent. taps and the common terminal, and each meter voltage is obtained from a $200/\sqrt{3}$ tap on one transformer and a $100/\sqrt{3}$ tap on the other. The voltages between these pairs of taps are evidently in phase quadrature with the line voltages used for energy registration, and equal to the line voltages in magnitude. The association of currents and voltages in the reactive meter is shown in the vector diagram. A 3-phase energy meter connected in this way will register kVArh directly.

The vector diagram of a third method of connecting a 3-phase energy meter for kVArh measurements is shown in Fig. 15. Here the meter currents, derived from delta-connected current transformers, correspond to those required for 4-wire energy registration; the meter current in one element I_1 is the resultant of the red-phase current and the white-phase current reversed, and I_2 is likewise the resultant of blue-phase current and white-phase current reversed. The voltages on the meter for energy registration would be respectively red and blue star voltages. If, as shown in the diagram, I_1 is associated with white to blue, and I_2 with red to white line voltages, these meter voltages will be respectively 90 degrees lagging in phase on those of an energy meter and $\sqrt{3}$ times greater than these latter voltages in magnitude. A meter so connected will measure kVAr in a 3-phase 4-wire circuit, but in this case the phase voltages must be symmetrical.

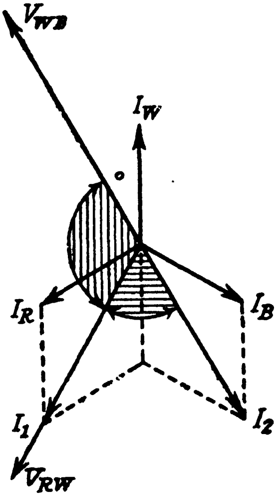


FIG. 15.—Vector diagram of 3-phase 4-wire reactive meter.

The foregoing methods of kVAr measurement in 3-phase circuits are all accurate in principle with balanced voltages, irrespective of current unbalance. There are three further methods, less accurate in principle, which depend upon balance of line currents as well as voltages. The first is explained in detail in all text-books on A.C. theory, and depends upon the well-known fact that $\sqrt{3}$ times the difference of the registrations of 2 watt-hour meters measuring the energy in a balanced load will be equal to the kVArh. A 3-phase meter can therefore be made to register $1/\sqrt{3}$ of the lagging kVArh if the voltage circuit connections to the red element are reversed. Again, a single-phase meter carrying a current in one line and energised by the voltage between the other two will, with a circuit power factor of unity, have its current and voltage in phase quadrature, and will, with lagging power factors, register $1/\sqrt{3}$ of the kVArh in a balanced load. It therefore follows that if the connections of a 3-phase meter are so modified that each element is energised by a voltage lagging 90 degrees on its current with a circuit power factor of unity, the meter will register $2/\sqrt{3}$ times the kVArh in a balanced 3-phase load. This is often known as the ‘ crossed-phase ’ method of kVArh measurement, and it is manifest that, for balanced loads, it is no more advantageous than a single-phase meter.

Measurement of kVAh and kVA Maximum Demand. We have already seen that maximum demand in fictitious kVA at a declared voltage will be given by a demand indicator of the thermal type connected in a single-phase A.C. supply circuit. Single-phase kVAh can be measured by an electrolytic meter of the mercury deposition kind which is supplied from an A.C. circuit by a current transformer and a full-wave metal oxide rectifier network. The amount of mercury deposited will correspond to the quantity in the rectified current passing into the meter, and this will bear a ratio to the time integral of R.M.S. amperes in the A.C. circuit which depends, not only upon the errors of the transformer and rectifier, but also upon the wave form. Apart from these errors, the reading of the meter with a suitably calibrated scale will give kVAh consumption provided the wave form of the circuit current is the same as that to which the calibration corresponds.

Three-phase kVA or kVAh may be defined in either of the three ways explained in Chapter I. Total equivalent VA, as defined by the quantity $\sqrt{\{(W)^2 + (VAr)^2\}}$, is a quasi-vector quantity, and if the whole current system of a 3-phase load has its phase continually varied with respect to the whole voltage system without change of the magnitudes or mutual phase differences of the individual currents, it can be shown that the total equivalent VA corresponds to the maximum value of the watts obtained by such variation. Denoting the total equivalent VA by M, the watts W are given by $W = M \cos \phi_1$, where ϕ_1 is the amount of phase shift from the phase giving maximum watts. ϕ_1 is also equal to arc tan VAr/W. These propositions may be considered self-evident, but those who do not find them so may be referred to books on A.C. theory for formal demonstrations.

It follows from the foregoing that if the phase of either the voltages applied to the elements of a 3-phase meter, or of the voltage fluxes in the elements, is modified from the natural value by an angle θ, the rate of registration will correspond not to $W = M \cos \phi_1$, but to $M \cos (\phi_1 \pm \theta)$. If in practice the angle ϕ_1, which determines the 3-phase power factor, does not vary very much, and the angle θ of phase variation of the meter voltages or voltage fluxes is made approximately equal in magnitude to the average value of ϕ_1, then the rate of registration of the meter can be made to correspond to $M \cos (\phi_1 - \theta)$, which will not differ from M by more than 4 per cent., provided $\phi_1 - \theta$ does not exceed arc cos $0{\cdot}96 = \pm$ 16 degrees.

Three-phase energy meters with a fixed phase compensation of this kind have been used for the measurement of maximum demand and kVA, it being assumed that the 3-phase power factor, $\cos \phi_1$, will not vary greatly from an estimated average value $\cos \theta$, to which the phase compensation corresponds. This assumption is reasonable, but it does not apply to conditions when the load is considerably less than the maximum, as then the actual power factor is likely to be less than the estimated average corresponding to peak loads. A meter of this kind, with fixed phase compensation, is not therefore for the continuous integration of kVA.

In order that such a meter may give kVA demand at estimated average lagging power factors, the angle of compensation θ corresponds to an additional phase lag of the voltage flux. The amount of compensation can be obtained to correspond to any angle between θ and 60 degrees, by exciting the meter by voltages lagging 60 degrees on those applying to ordinary energy measurements, and by diminishing the external compensation to the required value by connecting resistance in the voltage circuit. It is fairly evident that the initial compensation of 60 degrees can be obtained by choosing line voltages of suitable phase and polarity.

This method of approximate measurement of kVA demand has been developed to be suitable for continuous kVA integration by an automatic device whereby the phase compensation angle θ is always maintained within 5 degrees of the angle ϕ_1, which corresponds to the actual 3-phase power factor of the load.

This method of automatic compensation will be understood from the following description and from the diagram (Fig. 16). AB, BC and CA represent three potentiometer resistances, connected between the lines of the 3-phase supply. The vector of the voltage between any two tapping points, such as PQ, will be represented by the line PQ in magnitude and phase, if the triangle ABC represents the vectors of line voltages and the lengths p and q represent the ohmic resistances between the tapping points and the terminal C. Suppose that the vector BD, the median of the triangle, represents a datum of voltage, then to obtain an equal voltage displaced from this voltage by a stipulated phase angle α the tapping points P and Q must be so chosen that the line PQ is equal in length to BD and is inclined to it by an angle α. If AB = 1, BD = PQ = $\sqrt{3}/2$, and the angles C, P and Q of the triangle CPQ are respectively 60 degrees, (α + 30) degrees and (90 − α) degrees. Thus the lengths p and q, which give the correct tapping points, can be found from the following equations :—

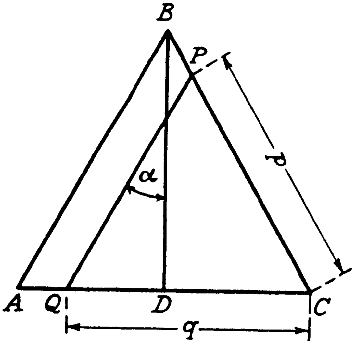


FIG. 16.—Automatic phase compensation for kVAh meter.

$$\frac{2\sin 60^\circ}{\sqrt{3}} = \frac{\sin(90^\circ - \alpha)}{p} = \frac{\sin(30^\circ + \alpha)}{q}.$$

If the three resistances are provided with tapping points arranged so that α can be varied in 10-degree steps, it will readily be understood that two pairs of moving contacts can be provided, and that voltages constant in value, with a mutual phase separation of 60 degrees for exciting a 3-phase meter, can be adjusted to correspond in phase to the angle ϕ_1 of a 3-phase load within a limit which need not exceed 5 degrees. This adjustment having been made by the choice of suitable taps on the resistances, the rate of registration of the meter will never be less than M cos 5 degrees $= 0{\cdot}9962 \times$ M, so that the meter will register kVAh with an error caused by defective phase compensation which is less than ½ per cent.

The actual selection of the correct taps is made by a Ferraris motor controlled by a power factor relay. The power factor relay is excited by the same voltages as the kVA meter, and the action of the relay is to cause the selecting contacts to rotate till the apparent power factor in the relay is unity. The rate of registration of the meter then corresponds to $\mathrm{M} \cos (\phi_1 - \theta)$, where, as already explained, the angle $(\phi_1 - \theta)$ does not exceed 5 degrees.

A rate of integration of a register which is approximately proportional to kVA can be obtained in a manner quite different from the foregoing. Conceive that a register is made to advance at a rate represented by $p\mathrm{W} + q\mathrm{X}$, where W represents 3-phase power, X 3-phase VAr and p and q are proper fractions. This rate will be $\mathrm{M}\ (p \cos \phi_1 + q \sin \phi_1)$, where $\cos \phi_1$ is the 3-phase power factor. If the fractions satisfy the two conditions, $p^2 + q^2 = 1$ and $q/p = \text{arc} \tan \theta$, then the rate is $\mathrm{M} \cos (\phi_1 - \theta)$, and it will be approximately the same as M, provided ϕ_1 does not differ much from θ which is fixed by p and q. A rate of advance corresponding to $p\mathrm{W} + q\mathrm{X}$ will be obtained by the use of a register which is driven through a differential gear by an energy and a reactive meter, the fractions p and q being adjustable to any desired values by suitable design of the gears between the meter's elements and the differential. Thus if the advance of the register corresponds to 0·8 of the kWh plus 0·6 of the kVArh, the register will give kVAh if the angle ϕ_1 of the load is arc tan 0·75, or about 37 degrees. Otherwise the rate of registration will correspond to $\mathrm{M} \cos (\phi_1 - 37 \text{ degrees})$.

The foregoing theory may be illustrated graphically. In Fig. 17 OM is the vector of total equivalent kVA at a 3-phase power factor of $\cos\theta$ and the length OW represents the 3-phase power. Drawing WB perpendicular to OM, $OB = W\cos\theta = pW$ and $BM = X\sin\theta = qX$, whence $q/p = \tan\theta$ and $pW + qX = M$. If the power factor $\cos\phi_1$ differs from $\cos\theta$ by a small angle as shown, the quantity pW will be represented by OB and qX by $CM_1 = BL$, where M_1L is the perpendicular from the end of the new OM_1 vector on OM. The sum of pW and qX is OL, and it is evident from the geometry of the diagram that, provided $(\phi_1 - \theta)$ is small, this is very approximately equal to the VA, OM.

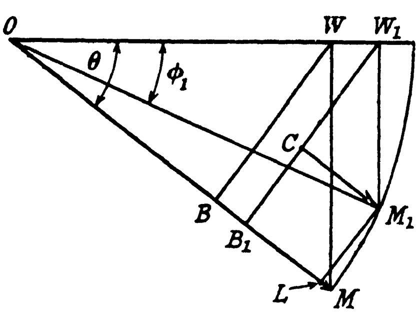


FIG. 17.—Approximate mechanical integration of kVA.

This method of measuring kVAh is developed in the 'Trivector' meter by the use of an ingenious mechanism, whereby the fractions p and q can have five sets of values corresponding to $\theta = 0$, $22\frac{1}{2}$, 45, $67\frac{1}{2}$ and 90 degrees. There will be five aggregate speeds corresponding to these five sets of values, and the rate of advance of the kVAh register corresponds to the greatest of these aggregates. This rate of advance cannot vary between wider limits than M and $M\cos 11\frac{1}{4}$ degrees $= 0{\cdot}98$ M, so that a very approximate measure of kVAh is obtained over the whole range of lagging power factors. The 'Trivector' meter is furnished with kWh and kVArh registers as well as a kVA register and maximum kVA demand indicator, so that accurate values of energy factor or average power factor over an extended period can be obtained.

The methods of kVAh measurement so far considered have all been approximate in fundamental principle. In the method now to be described the principle is exact, and fundamentally the kVAh register advances at a rate which is at all times exactly proportional to the total equivalent VA. This result is achieved by the use of an ingenious friction gearing, whereby the kVAh register is responsive to the rotations of kWh and kVArh meter elements. Referring to Fig. 18, which illustrates the principle of this mechanism, two friction wheels of identical size, driven respectively by the kWh and kVArh elements, make contact with an aluminium sphere of such a size that the contact points subtend a right angle at the centre. The third friction wheel which drives the kVAh register is attached to a framework free to turn about an axis through the centre of the sphere at right angles to the plane of the paper. The sphere takes up a position such that the rotations of the kWh and the kVArh spindles produce in it the same angular speed about an axis parallel to that of the kVAh wheel axis. When this is the case it is manifest that, as the peripheral speeds of the driving friction wheels are respectively proportional to W and X, the angular speed of the sphere, and hence of the kVAh friction wheel, will be proportional to $\frac{W}{\cos\alpha} = \frac{X}{\sin\alpha}$, where α is the shift of the kVAh wheel axis from parallelism with that of the kWh and kVArh wheels. It therefore follows that $\tan\alpha = \frac{X}{W} = \tan\phi_1$, where $\cos\phi_1$ is the 3-phase power factor. Thus, as $\alpha = \phi_1$, the angular speed of the kVAh wheel is $\frac{W}{\cos\phi_1}$ = the total equivalent voltamperes in the circuit containing the kWh and kVArh meter elements. The exactness of fundamental principle in this meter is obtained at the expense of the complication of a friction gear, which has not only to transmit the rotations of two driving elements, but also automatically to adjust its position to correspond to the 3-phase power factor.

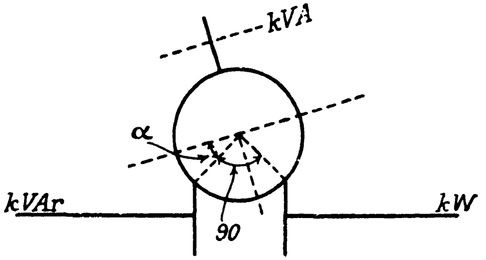


FIG. 18.—Exact mechanical integration of kVA.

In the last kind of 3-phase kVA meter to be considered the driving torque is produced, not by kWh and kVArh meter elements as in the foregoing kinds of meters, but by three shaded pole electromagnets having windings carrying the 3-line currents of the circuit in which the meter is connected. The shaded pole electromagnets are similar to those used in an induction ammeter, and they produce a torque on a freely rotating disc placed in the air gap, which is proportional to the product of the magnetic fluxes in the two portions of the shaded pole and to the sine of the angle of phase difference of these fluxes. This angle of phase difference depends upon the design of the magnet and on the frequency, so that the torque produced by a magnet is proportional to the square of the current in its windings. The movement of the discs which are subjected to this driving torque is restrained by two braking torques, the one produced by the field of a permanent magnet, and the other which is caused by the rotation of the discs in the alternating field of the driving magnets. This so-called self-braking torque is, like the permanent magnet braking torque, proportional to the speed of rotation. It is also proportional to the square of the total flux cutting the disc, and hence to the square of the alternating currents in the magnet windings, because the greater the flux the greater proportionally will be the eddy currents induced by rotation which interact with the flux to produce braking. It follows from this argument that the total driving torque produced by three electromagnets excited by the line currents of a 3-phase circuit will, if the winding characteristics of the magnets are identical, be proportional to the sum of the squares of these line currents. Denoting the root-mean-square or quadratic mean of these line currents by I_Q, the driving torque will be proportional to I_Q^2 and the total braking torque to $\omega(B^2 + kI_Q^2)$, where B corresponds to the permanent magnet flux and kI_Q^2 to the alternating flux, which produces the self-braking torque. The relation between I_Q and the rotational speed ω is therefore given by the equation $\omega = \dfrac{aI_Q^2}{B^2 + kI_Q^2}$. This relation is represented graphically in Fig. 19, and it is manifest from the algebraic formula that, when I_Q is small, the representative graph approximates to the parabola $\omega = aI_Q^2/B^2$, and that when I_Q^2 is very large, the graph is asymptotic to the straight line $\omega = a/k$. It is also manifest from the shape of the curve that there is a point on it at which the tangent passes through the origin of co-ordinates, and in the neighbourhood of this point the curve approximates to the straight line of proportionality of I_Q and ω. The meter is designed so that this neighbourhood represents the working range. As this proportionality of speed to I_Q is only approximate, the instrument is not suitable for the continuous integration of kVA, but used as a demand indicator to register maximum average kVA over restricted averaging clock it gives a closely approximate measure of the maximum 3-phase current demand in terms of the quantity I_Q, provided that during the averaging period of this maximum demand I_Q does not vary greatly and has a value in the neighbourhood of the approximately linear portion of the characteristic curve. The scale of the demand indicator is marked to correspond to 3-phase kVA at a declared voltage, and the markings are determined by the pointer advances produced by constant values of the nominal kVA corresponding. Demand indicators of this kind can be supplied with a voltage-compensating device whereby the driving torque per ampere in each element is increased or diminished by variations of the supply voltages. This compensating device consists of a

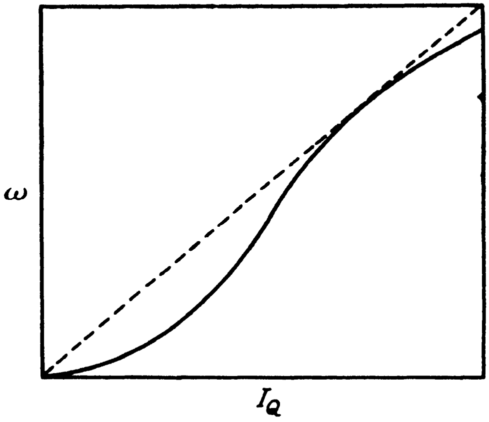


FIG. 19.—Characteristic of shaded-pole kVA meter.

movable shading piece in the air gap of the driving magnet, the position of which is controlled by an electromagnet excited by the corresponding phase voltage. A demand indicator so compensated will give an indication corresponding to the third definition of 3-phase kVA explained in Chapter I, that is, to the root-mean-square of the single-phase kVA in the three phases of the supply. This value of the kVA is, as already explained, greater than the value derived for kWh or kVArh meters, and also greater than the arithmetical sum of the single-phase kVA, excepting when the 3-phase load is balanced.

Shunts. The use of shunts in connection with D.C. meters has already been mentioned. When the energy in heavy current D.C. circuits is measured, care has to be exercised in the design and installation of the shunt. To prevent the errors caused by thermo-e.m.f.'s set up at the junction of the resistance alloy of the shunt and its terminal blocks, the alloy should be such that this thermo-e.m.f. per degree of temperature difference is as small as possible, and the shunt should preferably be mounted horizontally, so that the two terminal blocks are approximately at the same temperature.

For very heavy current D.C. circuits two or more similar shunts can be used in parallel, and, provided certain simple precautions are taken in connecting the meter to the shunts, the accuracy of the arrangement does not depend upon equality of the division of the main current between the several shunts. The underlying theory of shunts used in parallel is very simple. Consider a meter of resistance R_1 connected to a single shunt of resistance R by two leads, each of resistance R_2. Then, according to elementary principles, the current through the meter will be $I \times \frac{R}{R + R_1 + 2R_2}$, or if R is very small, $I \times \frac{R}{R_1 + 2R_2}$ very nearly, where I is the current in the main circuit. Consider now the arrangement shown in Fig. 20. Here a meter of resistance R_3 is connected to two shunts, each of resistance R, by leads each of resistance R_4, each lead being connected to a meter terminal and a shunt terminal. The symbolic values of the currents in the branches of the circuit are as shown in the diagram. Equating to zero the algebraic sum of the e.m.f. in the loop composed of the upper shunt, the leads and the meter, we have

$$R(I_1 - I_3) = 2R_4I_3 + R_2(I_3 + I_4).$$

Similarly for the corresponding lower loop,

$$R(I_2 - I_4) = 2R_4I_4 + R_2(I_3 + I_4).$$

Adding these equations, putting I for $I_1 + I_2$ and I_m for $(I_3 + I_4)$, the meter current, we find

$$RI = I_m(R + 2R_2 + 2R_4),$$

or, neglecting R as being very small in comparison with R_2 and R_4, we have

$$I_m = I \times \frac{R}{2R_2 + 2R_4}.$$

Comparing this expression with the one giving I_m for a single shunt of resistance R, we see that if the resistance of the leads R_4 with the double shunt combination is made twice the value correct for the single shunt, the current in the meter—and hence its rate of registration—will be halved by using two similar shunts in parallel, and that this relation is quite independent of the manner in which the total current divides between the two shunts. Hence a meter can be used in conjunction with two similar shunts sharing the total current of a circuit. Otherwise one D.C. meter can be used to measure the

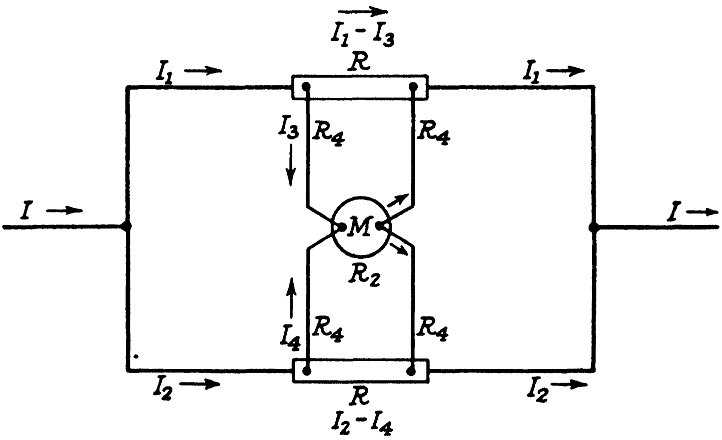


FIG. 20.—Shunts in parallel.

total energy in two circuits supplied at the same voltage if this meter is properly connected to two similar shunts, one in each circuit. The accuracy of the registration of this meter will not be affected if one of these circuits causes no current, provided that the shunt resistance is very small in comparison with that of the leads and the meter. It is easy to see that, generalising the foregoing argument, *n* similar shunts can be used in parallel if each connecting lead has a resistance of *n* times that required with a single shunt.

Instrument Transformers. The function of a current transformer in A.C. electrical measurements is similar to that of a shunt. Its underlying principle is, however, different, and it has the great practical advantage as compared with a shunt that the secondary circuit containing the meter is electrically insulated from the main circuit carrying the energy which is measured. The action of a current transformer depends upon the principle that the primary ampere-turns of the transformer are equal to the resultant of the secondary ampere-turns and the ampere-turns required to magnetise the core for the production of the secondary e.m.f. If the secondary e.m.f. is kept low by reducing the secondary impedance as much as possible, and if the core of the transformer is made of a high permeability alloy, the primary ampere-turns required for magnetisation are sensibly equal to the secondary ampere-turns, so that the secondary current bears a constant ratio to and is very nearly in phase with the primary current. Actually the small component of the primary ampere-turns gives rise to errors of transformation, whereby the magnitude of the secondary current is less than its nominal value and its phase is made to be slightly different from that of the primary current. This effect will be considered in greater detail when the subject of errors of measurement is discussed.

For measurements in high-pressure A.C. circuits the meter is excited by the secondary of a voltage transformer. A voltage transformer is exactly similar in principle to the power transformer, which is dealt with in detail in books on A.C. theory. The regulation of a voltage transformer gives rise to differences of the secondary voltage in magnitude and phase from the nominal correct value.

As the secondary circuit of a current transformer is electrically insulated from its primary circuit, it follows that the secondary circuits of two or more currents can be joined together to form a common circuit in the current coil of a meter. If the several main circuits carry power at the same voltage, as regards magnitude, phase and frequency, and one ampere in each secondary winding corresponds to the same primary current, a meter so connected as to carry the resultant of a number of secondary currents can measure the total energy in the corresponding main primary circuits. The condition enunciated is equivalent to saying that if two or more current transformers have each a part of their secondary circuits in common in the current coil of a meter, the ratio of the transformers must all be the same. This condition therefore corresponds to that applying to the use of shunts in parallel. When the total energy in two or more circuits is measured by connecting a single meter to two or more current transformers, there is no need for any particular precaution regarding the resistances of the connecting leads, since, to a high degree of approximation, the secondary current of the transformer is not affected by small changes in the secondary impedance. The meter current coil, however, must carry a current which is the resultant of all the secondary currents. Thus if *n* current transformers of normal type supply the current coil of a meter, the coil of the meter may have to carry $5n$ amperes.

Measuring the Total Energy in two or more Circuits. This process is known by practical engineers as ‘ summation metering,’ an ugly term which is so firmly established in electrical jargon that its use cannot be avoided. We have, in the preceding paragraphs, explained how under certain conditions a single meter can measure the total energy in two or more circuits by connecting it to shunts or current transformers included in these circuits. The practical use of this idea is very restricted in D.C. supplies, but is more used in A.C. circuits. The restriction of this method to circuits in which current transformers are of the same ratio can be avoided by the use of an auxiliary current transformer, which is generally known as a ‘ summation ’ transformer. This auxiliary transformer also enables the maximum current in the meter to be reduced to the standard value of 5 amperes.

A summation transformer is so designed that an ampere in any one of the main circuits produces the same current in the meter coil. The transformer has a number of primary windings which corresponds to the number of main circuits associated with the meter and a single secondary winding. Each primary winding of the summation transformer is connected to the secondary winding of one of the current transformers in a main circuit. The turns ratios of the primary windings of the summation transformer to its secondary winding are so designed that, with full load currents of the same phase in the main circuits, the meter current has a standard value of 5 amperes, and that an ampere in either of the main circuits produces the same current in the meter circuit.

This requirement can be illustrated by a simple numerical example. Suppose that it is required to measure the total energy in three circuits in which are current transformers of ratio 600/5, 300/5 and 100/5. The sum of the full load currents is 1,000 amperes, and this must produce 5 amperes in the meter coil. Of this 5 amperes, 3, 1·5 and 0·5 amperes must be contributed respectively by the three main circuits at full load. The secondary currents delivered by the current transformers to the primary windings of the summation transformer will in this condition each be 5 amperes. The ratios of three primary windings with respect to the secondary winding of this summation transformer will therefore be respectively 5/3, 5/1·5 and 5/0·5, and if this condition is satisfied, it is manifest that the overall ratio of main current to meter current in any circuit is 200/1.

The foregoing methods of summation measurement depend upon a single meter being supplied with a current proportional to, and in phase with, the resultant of the currents in the several main circuits. The voltage circuit of the meter is excited by the voltage common to all the main circuits. If the voltages of the several circuits are different, these methods are inapplicable.

Energy in a restricted number of circuits, of any voltage or frequency, can be measured by a single instrument containing a sufficient number of meter elements, all of which communicate driving torque to a common rotating system. A 3-phase 2-element meter, for instance, may, in certain conditions of 3-phase loading, measure the total energy consumed in two single-phase circuits, and such a meter could evidently be used to give the total energy in any pair of single-phase circuits irrespective of full load current, voltage and frequency, by suitable design of the windings or by the use of suitable instrument transformers. Summation metering is, however, in practice only required for 3-phase circuits, and in this field the application of a special type of instrument is limited to a 4-element meter, of which the two pairs of elements are connected in two 3-phase 3-wire circuits. For the measurement of the energy consumption in more than two 3-phase circuits other methods are in use.

In these methods a separate energy meter is installed in each circuit, and each meter is fitted with a special device whereby an electrical impulse is communicated to a summation register for each revolution of the moving elements. When an impulse is received by the summation register it advances by an amount corresponding to the kWh per revolution of the meter which originated the impulse. Thus the total advance of the summation register over a period corresponds to the number of advances originated by all the circuit meters, and hence to the total energy passing all these meters in the period. The electrical connection between the circuit meters and the summation register is by pilot wires, and the impulse currents which may be D.C. or rectified A.C. can readily be transmitted over considerable distances, although if the pilot wires are too long trouble may be experienced owing to their capacitance. The impulses may be originated in the circuit meters by means of a commutator fitted to the rotating element with which contacts are made by brushes. With this method the impulse current is usually interrupted by an auxiliary switch operated by the solenoid which advances the summation register, so that the office of the commutator is merely to make contact. Another method of originating the impulse is by means of a switch operated by a cam on the meter spindle, the office of which is, like the commutator, merely to establish the contact.

A third method of originating the impulses is by means of a weight, which is gradually raised by the action of the meter and which falls by gravity to its lowest position once per revolution. In the act of falling the weight is made to make and break a switch, whereby the impulse is sent through the pilot wires to the summation register. It will be evident from the foregoing brief description that the operation of the summation register is merely one of counting, so that this register is incompetent to distinguish between forward and backward registration of the circuit meters. As backward rotation would advance the summation register, the circuit meters are often fitted with ratchet devices to prevent this.

CHAPTER III

ERRORS OF MEASUREMENT

Necessity for Accuracy of Consumers' Meters. The necessity for the accuracy of consumers' meters is generally taken by electricity supply engineers as being almost self-evident on the ground that errors, 'slow' or 'minus,' will result in a loss of revenue to the undertaking. Thus it is considered that if all consumers' meters were registering 5 per cent. 'slow' the undertaking would only receive 95 per cent. of the revenue to which it was entitled, and that consequently financial loss would result. Hence efficient meter maintenance, which prevents such slow errors, is said to be financially justified by the loss of revenue which it avoids. This view is hardly correct. A loss of revenue caused by meter errors is of quite a different nature from the loss which occurs in electrical conductors. This latter is a real loss of energy; the loss which arises from meter errors is the consumer's gain. This, however, is not all. Tariff rates are adjusted empirically so as to bring an adequate revenue to the undertaking. If therefore all consumers' meters were 5 per cent. slow, but supposed to be correct, the tariff rates would be adjusted to yield the correct revenue, so that the consumers would all pay for their supplies the same as they would were the meters correct. The kWh basis of charging for electrical energy is one which has a scientific sanction, and this sanction is endorsed by legal obligation. From the purely commercial point of view, the kWh unit is inconveniently small, since small fractional adjustments in the kWh rate cannot be made in terms of monetary units. Thus the same inaccuracy of all consumers' meters really means that the supply is charged for on a basis of other than kWh, and to this there is no purely commercial or ethical objection, so long as the non-standard unit is the same for all consumers. The fundamental requirement in the measurement of consumers' supplies is that all meters shall give the same registration for the same amount of energy. This requirement is most rationally and simply met by making the meters register kWh with as high an accuracy as is practicable and economic. Meters are maintained in this condition of accuracy in order that the same energy consumption shall always result in the same meter registration. The fundamental reason for the maintenance of accuracy of consumers' meters is therefore based on considerations of equity rather than of avoiding financial loss by the supply undertaking.

There is an important distinction to be noted between the error of registration of a consumer's meter and an error in the sum of money he is required to pay for his supply. Under a flat rate or block rate tariff the charge for the supply is based upon kWh rates and meter registrations, so that an error of 1 per cent. of meter registration will make an alteration of 1 per cent. in the amount charged. Under the two-part tariffs, comprising a fixed charge and a kWh charge, the effect of meter errors on the consumer's bill is manifestly less. Thus if the standing charge is about equal to the kWh charge, a 1 per cent. meter error will affect the amount charged by only about ½ per cent. If kWh rates under two-part tariffs tend to decrease and standing charges to increase, the effect of meter errors on consumers' bills will tend to diminish. The limiting case of a two-part tariff would be one with which the kWh rate is zero. This, of course, would be a contract method of charging similar to that used for domestic water supplies. Under such a tariff meters would not be required.

When the registrations of electricity meters are required, not for assessing the charges for electricity supplies, but for measuring the efficiency of electrical machinery, the necessity for accuracy is of a different kind. In this case the ratio of two kinds of energy—the one the input and the other the output—is required. Accuracy of electrical energy measurement now depends upon the nature of the final result required. In acceptance tests on turbo-alternators, small variations of the measured efficiency make large money differences to the scheduled contract prices payable for the plant, and the highest possible accuracy is necessary in electrical energy measurements. Similar considerations apply to normal routine measurements of the output of turbo-alternators. The efficiency of electricity generating plants is kept under close observation by operating engineers, and efficiency values are customarily stated to three or four significant figures. In this connection it is worth noting that, taking the energy output of a generator at its *prima facie* value given by the advance of a meter register, this value may be in error to the extent of at least 1 part in 200, so that the use of four significant figures in a statement of plant efficiency based upon this advance is not justified, apart altogether from the matter of the possible errors of the other quantity involved in the efficiency ratio. To obtain a measure of electrical energy output correct to within 1 part in 1,000, the greatest error with which the use of four significant figures can be justified, it would be essential to correct the register advance for inherent errors of the meter and of its associated instrument transformers.

Nature of Meter Errors. The object of the designer of an electricity motor meter is to make an instrument whereof the angular speed of the rotating element is exactly proportional to the power or current in the circuit containing the meter. If the ratio of speed ω to watts W is constant for all values of W, then the number of revolutions of the moving element executed per kWh will also be independent of W, and if the constant 'revolutions per kWh' is made numerically equal to the velocity ratio of the register gear train between the meter rotor and the spindle of the register denoting tenths of a unit the meter will register kWh correctly for all values of W. If the gearing ratio does not exactly correspond to the 'revolutions per kWh' characteristic the registration will be inaccurate by a constant factor. An error of this kind, which is the same for all values of W, or at all loads, is known as one of calibration. It can be wholly or partly eliminated by adjustment either of the speed characteristic ω/W of the meter, or of the gearing constant.

There is a further class of errors which arise in the registration of an electricity meter and which are owing to the fact that the speed characteristic ω/W is not exactly constant but depends upon W. Because of this fundamental defect the gearing constant can only correspond to one value of the characteristic which corresponds to one or more values of W. At other loads the registration will necessarily be inaccurate. All meters have this kind of fundamental defect, although it can be reduced to some extent by auxiliary compensating devices. The object, in the final adjustment of the meter, is to bring the speed characteristic of the meter into such correspondence with the gearing constant that the overall error of the registration of the meter in actual service will be as small as possible.

A third class of errors arise when an electricity meter is operated in conditions slightly different from those for which it is rated. A typical example of this class of error would be that of a quantity meter in a circuit whereof the actual voltage differed from that for which the register was designed, although, as we have already pointed out, this circumstance would only give rise to an error from the scientific point of view, because quantity is a legally recognised basis for charging. Variations of voltage from the normal or rated value cause small errors in watt-hour meters, which, according to basic principles, should register accurately irrespective of these variations. Similarly, the speed characteristic ratio of a meter is affected by temperature changes, and, in the case of an A.C. instrument, by frequency changes. These effects we shall shortly consider in detail.

The actual error of registration of a meter is the resultant of the error of calibration and the inherent errors caused by defects in the fundamental characteristic. This error is expressed as a percentage ratio of the difference between the actual registration and the true energy corresponding to the true energy. If the registration exceeds the true energy the error is fast or plus, otherwise it is slow or minus. Otherwise the error can be expressed as the difference between the actual ω/W ratio and that corresponding to the gear ratio, stated as a percentage of the latter ratio.

Legal Requirements regarding Accuracy of Consumers' Meters. The legally prescribed limits of error allowed by the Electricity Commissioners for consumers' meters have recently been reviewed, and the official requirements are set out in Supplementary Notes to the 1936 Act. According to these Supplementary Notes, the Electricity Commissioners have brought under review the permissible limits of error for the various kinds of D.C. and A.C. meters—both old and new—including prepayment and polyphase meters; and for the varying conditions of temperature and possible loading in excess of the marked current under which they may operate when in actual service. The new prescribed limits, namely, an error not exceeding 2½ per cent. plus, or 3½ per cent. minus, at any load at which the meter may be operating, have been fixed with due regard to these matters, and are applicable to all meters submitted for certification without requiring any compensating allowances for varying conditions.

It will be noted that the limits of error specified as the maximum values allowable are those which may not be exceeded in the most adverse conditions in which the meter may be working. Thus the maximum error of a meter, which will decide whether it is eligible for certification, will include not only errors of calibration and those inherent errors caused by a speed/watts ratio varying with the load, but also those further errors which may be caused by variations of temperature from the value for which the meter is designed to give its highest accuracy.

Theory of Inherent Meter Errors. In this section we consider the theory of those errors of meters which arise because the speed/watts ratio varies with the load in the circuit containing the meter. The most important of these errors arise from the following causes :—

(*a*) Bearing and commutator friction.
(*b*) Fluid friction in mercury meters.
(*c*) Self-braking effect in A.C. induction meters.
(*d*) Variable permeability of iron in A.C. induction meters.
(*e*) Defective phase compensation in A.C. meters.

The effect of bearing friction is to set up a torque opposing rotation of the moving element which, to a first approximation, is independent of the speed. Indicating this torque, assumed constant, by f, we have when the speed is steady with constant W, the driving torque aW is equal to the sum of the permanent magnet braking torque $b\omega$ and the friction torque f. Thus

$$aW = b\omega + f.$$

If f is the same when the rotor is at rest as when it is moving, and if W_1 is the value of the watts which will just produce rotation, then ω being zero we have $aW_1 = f$.

Thus $$aW = b\omega + aW_1$$

and $$\omega = \frac{aW}{b} - \frac{aW_1}{b}.$$

Without friction the speed of the meter ω^1 would be $\frac{aW}{b}$. Thus the difference between the actual speed w and the speed ω^1, without the disturbing effect of friction, expressed as a fraction of ω^1 is $-\frac{W_1}{W}$. This fraction, generally expressed as a percentage, is the error caused by friction.

Considering ordinary fractional values, the error of a meter in which bearing friction is the only cause of variation of the speed/watts ratio will be given by the expression

$$\eta_a = \eta - \frac{W_1}{W},$$

where η is the error of calibration, that is, the fractional difference between the speed/watts ratio without friction and that corresponding to the gearing ratio. This relation between η_a and W is shown graphically in Fig. 21.

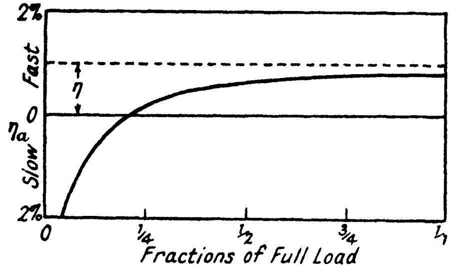


FIG. 21.—Error curve of meter with bearing friction.

The error caused by bearing friction can be corrected in energy meters by a device incorporated in the meter whereby a sensibly constant driving torque is superposed on the main driving torque proportional to the watts. This torque is derived from the voltage which energises the meter. In commutator meters an auxiliary coil, placed with its axis in line with those of the main current coils, connected in series with the armature, and, carrying the practically constant current of the voltage circuit, will give rise to a small driving torque, independent of W, which will tend to annul the effect of the bearing friction torque. In one kind of mercury motor energy meter a thermo-junction is continuously heated by a coil in the voltage circuit, and a small and practically constant current from the thermo-couple passes into the mercury bath, and, being superposed upon the main current therein, produces a small constant torque additional to the principal driving torque. In A.C. induction motors, a copper ring is placed so that some of the leakage flux passes through it, and in an unsymmetrical position with respect to the pole of the voltage magnet. This copper ring behaves in the well-known way as a ‘ shaded pole ’ to the voltage magnet, and causes a small and sensibly constant driving torque additional to that produced by the circuit current.

If the compensating torque produced by either of the foregoing methods exceeds the bearing friction torque, there will be a resultant driving torque when no current is passing through the meter, and the rotor will tend to move or to ‘ creep ’ with no load. As bearing friction tends to increase with length of service of the meter, it is customary to make the compensating torque such as to tend to produce rotation with no load and to prevent creep by some auxiliary device. This may be a piece of iron attached to the moving system of the meter, which, being feebly attracted by a magnetic pole, will prevent continuous rotation or creep with a feeble driving torque. Otherwise holes may be drilled in the disc of the rotating system, so that for certain positions of the same the resistance of the eddy current paths is distorted, and hence the torque set up by the compensating device is weakened or even cancelled.

The curve showing the relation of error to load with a meter which is over-compensated for bearing friction is shown in Fig. 22. Here the calibration error η is supposed to be negative so as to be opposite in sign to the plus or fast error caused by the compensation.

The second and third classes of inherent errors, caused respectively by fluid friction and self-braking, are somewhat similar in their relation to the load in the circuit containing the meter. Fluid friction torque is approximately proportional to the square of the speed, and as to a first approximation the speed is proportional to the watts, this fluid friction torque is nearly proportional to the square of the load. Self-braking errors are caused by the eddy currents which are set up by relative motion of the disc of an induction meter and the field of the current magnets. This torque is proportional to the speed and to the square of the current, and as speed is to a first approximation proportional to the load, we have that, at constant power factor, the self-braking torque is nearly proportional to the cube of the load.

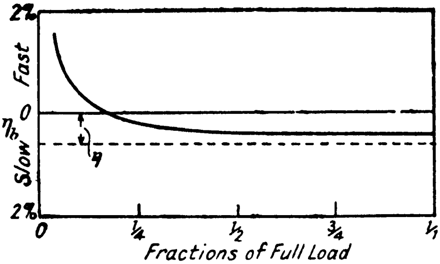


FIG. 22.—Error curve of over-compensated meter.

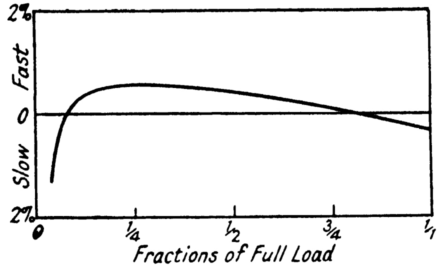


FIG. 23.—Error curve of meter with bearing and fluid friction.

The torque equation of a meter with fluid and bearing friction will thus be

$$aW = b\omega + aW_1 + kW^2,$$

so that
$$\omega = \frac{aW}{b} - \frac{W_1}{W} - \frac{kW}{a}$$

the relation of error to load will be given by

$$\eta_b = \eta - \frac{W_1}{W} - \frac{kW}{a}.$$

This relation is shown graphically in Fig. 23, and is the kind of error curve applicable to mercury motors in which fluid friction is considerable.

The torque equation of a meter with a self-braking error and bearing friction will be, at unity power factor when I is a measure of W,

$$aW = b\omega + aW_1 + k_1W^3,$$

so that the relation between the actual error η_c and W will be

$$\eta_c = \eta - \frac{W_1}{W} - \frac{k_1W^2}{a}.$$

This relation is shown graphically in Fig. 24.

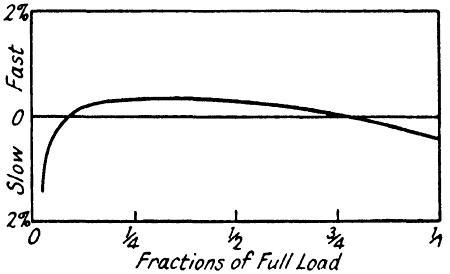


FIG. 24.—Error curve of A.C. meter with self-braking.

At power factors less than unity the current which causes the self-braking effect will be proportional to W/cos ϕ, so that the self-braking error will be represented by $\frac{k_1}{a}\frac{W^2}{\cos^2\phi}$ in the last equation. The effect of self-braking is thus increased as the power factor falls.

If a meter with self-braking is over-compensated for bearing friction, then it will be readily understood that the error curve will assume the form shown in Fig. 25.

The effect of fluid friction in mercury quantity meters can be compensated with fair approximation by superposing on the braking flux a reverse flux proportional to the current in the meter. This compensating flux may be provided by an auxiliary electromagnet excited by the meter current, or a reverse compensating flux may be directly superposed on the permanent magnet flux by a current winding encircling the magnetic circuit. When a separate magnet is provided for braking, the current coil can be arranged to strengthen the magnet which provides the flux for the driving torque and to weaken the magnet which is used only for braking. The effect of the reverse compensating flux depends upon the principle as previously explained, that the braking torque is proportional to the square of the flux. With correct compensation the effect of fluid friction in reducing the effective driving torque is negatived by a corresponding reduction in the braking torque by reason of the diminution of the braking flux.

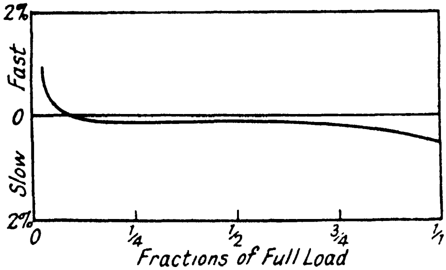


FIG. 25.—Error curve of over-compensated A.C. meter.

Mercury motor energy meters can be approximately compensated for fluid friction by providing a current winding on the electromagnet which provides the flux for the driving torque. The effect of this current winding is to strengthen the driving torque by an amount proportional to the square of the current to overcome the fluid friction torque which, as we have already seen, is approximately proportional to the same quantity.

It would appear to be feasible to compensate an A.C. induction meter for the effect of the self-braking torque by a shaded pole device incorporated in the current magnet which would produce a compensating torque proportional to the square of the current. Although the self-braking torque is approximately proportional to the cube of the current, depending upon ωI^2, such a compensating torque would greatly diminish the error of self-braking. The disadvantages of such a method of compensation would be two-fold. First, the meter would tend to register at considerable loads with the voltage circuit unexcited, and secondly, the torque compensation, depending upon current and not upon power, would be excessive at low power factors. This idea, therefore, has not found practical application, and A.C. induction meters can by suitable design be so made that the self-braking error is not of great practical moment.

The fourth source of error, that of variable permeability of the core of the current magnet of an induction meter, is not of great practical importance, but it is of interest because it may cause the error curve to assume a peculiar shape. It is well-known that at low flux densities the permeability of iron rises with the flux density. Consequently the flux per ampere in the current magnet of an induction meter will increase slightly as the load increases. An additional error will be caused by the variation of permeability, and the resultant error curve may be as represented in Fig. 26.

We have already seen that, because the driving torque of an A.C. induction meter is proportional to watts and the self-braking torque to the square of current, the error caused by this torque will increase as the power factor falls. This is a case of one inherent error being modified by power factor changes. Another kind of inherent error in A.C. meters is dependent only on power factor, and is caused by incorrect phase relation of the currents and alternating fluxes in the meter with the main circuit current and voltage from which they arise.

The correct registration of an induction meter depends upon the phase difference between the voltage flux and the disc eddy currents induced by the current flux being exactly the same as that between the voltage and current in the circuit containing the meter. For if the flux and eddy currents are proportional in magnitude to the voltage and current causing them, the average torque on the moving element will be

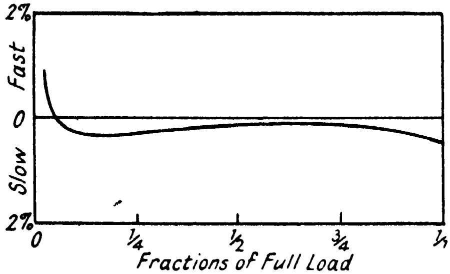


FIG. 26.—Error curve of A.C. meter with variable permeability.

proportional to IV cos ϕ. If, however, the phase compensation of the voltage flux is inaccurate, by an angle α, the average torque will correspond to IV cos $(\phi \pm \alpha)$ or to

$$IV (\cos\phi \cos\alpha \mp \sin\phi \sin\alpha).$$

The phase angle α being in practice very small, we may put $\cos\alpha = 1$ and $\sin\alpha = \alpha$, if α is expressed in radian measure. The torque therefore corresponds to

$$IV \cos\phi \mp \alpha IV \sin\phi$$

or to $W \mp \alpha X$, where X represents the VAr.

The difference between this value of the torque and that with no phase error, when $\alpha = 0$, expressed as a fraction of this latter torque is evidently $\mp \alpha \tan\phi$, and this gives the fractional error of registration caused by the phase error α. This error is independent of W, and for any assigned value of ϕ it is like an error of calibration. If the phase angle α arises because the voltage flux lags insufficiently on the voltage, α will be positive and the error will be negative or slow for lagging and positive or fast for leading power factors. If α arises from over-compensation there will be a fast error with lagging power factors, and *vice versa*.

An interesting effect of phase error on accuracy of registration may occur with 3-phase 2-element meters in which the phase errors of the two elements are unequal. Consider such a meter with phase errors of α and β in radian measure in the red and blue elements respectively. Then, with a balanced load, the meter torque will correspond to

$$\mathrm{IV}\left\{\cos\left(\phi+\frac{\pi}{6}-\alpha\right)+\cos\left(\phi-\frac{\pi}{6}-\beta\right)\right\}$$

and the ' torque error ' will be, assuming α and β to be small,

$$\mathrm{IV}\left\{\alpha\sin\left(\phi+\frac{\pi}{6}\right)+\beta\sin\left(\phi-\frac{\pi}{6}\right)\right\}$$

By expanding the sine terms in this last equation it can be shown that, as the correct torque corresponds to $\sqrt{3}\mathrm{IV}\cos\phi$, the fractional torque error will be

$$\tan\phi\left(\frac{\alpha+\beta}{2}\right)+\frac{\alpha-\beta}{2\sqrt{3}}$$

so that part of this error is independent of the power factor and is a constant error of calibration. In the extreme case in which α and β are equal in magnitude, but of opposite sign, the term $\tan\phi\,(\alpha+\beta)/2$ in the above expression would be zero, so that the 3-phase meter would be free from resultant phase error with balanced loads; notwithstanding both elements considered as single-phase wattmeters would be inaccurate on low power factors.

Errors caused by Temperature Changes. The accuracy of nearly all meters is to some extent affected by changes of temperature. Unshunted electrolytic meters may be considered for all practical purposes to be unaffected by temperature variations. The use of a compensating resistance to reduce temperature errors in shunted meters of this type has been mentioned on p. 28. The effect of temperature variations on the accuracy of clock meters of the unshunted kind can be made very small by designing the voltage circuit so that its overall temperature coefficient of resistance is negligible. If, however, a clock meter is shunted and its current circuit has an appreciable temperature coefficient of resistance, as it will if the current winding is of copper, the effect of temperature variation on the accuracy of the meter will not be negligible. If the resistance of the meter current circuit at normal temperature is R_1, and that of the shunt R_2, the meter current per ampere of main current will be $\frac{R_2}{R_1 + R_2}$. Suppose the temperature changes from the normal value by an amount θ. R_2 will remain sensibly unchanged, but the resistance of the meter circuit will alter from R_1 to $R_1(1 \pm k\theta)$, and the meter current per ampere of main current will be

$$\frac{R_2}{R_2 + R_1(1 \pm k\theta)} = \frac{R_2}{R_1 + R_2}\left(1 \mp \frac{R_1 k\theta}{R_1 + R_2}\right) \text{ approximately.}$$

The fractional alteration of the rate of registration caused by the temperature change will thus be $\mp \frac{R_1 k\theta}{R_1 + R_2}$, being negative or slow for a temperature rise, as k is positive for copper. The error caused by temperature change is thus nearly proportional to θ. The amount of this error per degree depends upon R_1, R_2 and k. If R_2 is small in comparison with R_1, as will be the case with heavy current meters, the error per degree of temperature change approximates to k, the temperature coefficient of resistance of the meter circuit. If this circuit is entirely composed of copper conductors, the error will thus approximate to 0·4 per cent. per degree C., a prohibitively high value. By the use of ‘swamping’ resistance of negligible temperature coefficient k can be reduced to 0·2 per cent., and this is the temperature error per degree C. of temperature change to which heavy current shunted clock meters are liable.

The effect of temperature changes on the accuracy of motor meters is, in the main, caused by changes in the resistivity of the material of the disc in which the braking eddy currents are induced. The magnitude of these currents—and hence the braking torque—per unit of speed and with constant permanent magnet flux will be inversely proportional to this resistivity.

This effect is most important in mercury motor quantity meters, in which a copper disc causes the circuit current and the eddy currents for the braking torque. Assuming that in an unshunted mercury motor meter the whole of the circuit current passes through the disc, the driving torque per ampere will be independent of temperature, but the braking torque per unit of speed will vary 0·4 per cent. per degree C. of temperature change. It therefore follows that, on this assumption, the error of the meter would be 0·4 per cent. per degree C., being fast for a temperature rise because of the reduction of braking torque. This assumption that the whole of the meter current traverses the disc is erroneous. Actually the disc is virtually shunted by the mercury.

Still making this assumption, consider the effect of shunting the meter. We have seen on p. 75 that the effect of a shunt having negligible temperature coefficient of resistance is to reduce the current per ampere in the meter by an amount which differs from the correct or nominal value by a fractional amount $\frac{R_1 k\theta}{R_1 + R_2}$, where the symbols have the meanings previously assigned. The effect of a rise of temperature is to reduce the meter current per ampere, so that shunting tends to negative the error caused by the reduction of braking torque. Shunting a mercury motor meter therefore reduces the error caused by temperature changes.

We have already stated that the submerged disc of a mercury meter is shunted by the mercury. The temperature coefficient of resistance of mercury is much less than that of copper, being of the order of 0·07 per cent. per degree C. It follows therefore that the actual current in the disc of a mercury motor meter per ampere of current entering the bath falls as the temperature rises. The driving torque per ampere of meter current is therefore reduced as well as the braking torque per unit of speed. Because of this shunting effect of the mercury the change of accuracy of an unshunted mercury meter is only about ⅛ per cent. per degree C.; a rise of temperature causes a fast error.

If a mercury motor quantity meter is shunted by a circuit of negligible temperature coefficient of resistance, the error per degree of temperature rise will be reduced, as explained on p. 78. As the compensating error caused by shunting tends to be equal to the temperature coefficient of resistance of the meter circuit as the resistance of the shunt diminishes, it follows that for a certain value of the shunt resistance, determined by the rated full load current in the main circuit, the two errors, one caused by shunting and the other by alteration of braking torque, would cancel. As quantity meters are only used in circuits of small current rating, this possibility is not of practical interest. Although from the point of view of reduction of the effect of temperature on meter accuracy, it would be advantageous to shunt small mercury motor meters, the gain would be outweighed by the loss of torque per ampere of current in the meter circuit. The torque per ampere is practically independent of the meter current for which the instrument is rated, and is consequently feeble in meters for low current circuits. Any loss of this torque by shunting would evidently increase the main current required to initiate movement of the disc, upon which, as we have seen, the fractional errors caused by bearing friction depend.

The foregoing are the principal causes of the temperature errors in mercury motor quantity meters, which errors are greater than those of most other kinds. A further source of error arises from the fact that the flux of a permanent magnet weakens as the temperature rises, to an extent which may be of the order of 0·03 per cent. The driving torque of a mercury motor being proportional to this flux, and the braking torque being proportional to its square, it follows that the weakening of the braking torque resulting from a reduction of the permanent magnet flux will predominate, and the overall effect of this cause will be to make the meter fast by about 0·03 per cent. per degree C. of temperature rise.

In commutator quantity meters the effect of temperature change on accuracy is the resultant of several component effects which have been considered in detail in the foregoing paragraphs. As the resistance of the shunt is very small in comparison with that of the meter, the reduction in meter current with a temperature rise is very nearly compensated by the decrease of the braking torque. Actually the errors of these meters caused by temperature changes is for all practical purposes negligibly small.

Mercury motor energy meters will be subject to the same causes producing temperature errors as are quantity meters of this kind. The general tendency of errors, caused by shunting and variations in the resistivity of the material of the brake disc, will, as has been seen, be to make the meter over-register with a rise of temperature, in meters of small or moderate current ratings. There is a further source of error in energy meters which arises from the changes in the voltage circuit resistance caused by temperature variations. This error will be opposite in tendency to those caused by shunting and variations of braking torque, since a temperature rise will generally tend to increase the voltage circuit resistance, and so to cause under-registration. If the whole of the supply pressure is not applied to the excitation coils of the voltage magnet, it is sometimes possible, by adjustment of the temperature coefficient of the series resistance, to make the error caused by the effect of temperature on the voltage circuit current partially or completely cancel those which are inherent in quantity meters.

The commutator energy meter will have its accuracy affected by temperature on account of the alteration of the resistivity of the material of the brake disc and the flux of the permanent magnet. These two effects which, as already explained, cause fast errors with a rise of temperature will be to some extent offset by the effect of temperature on the resistance of the voltage circuit, but as the greater part of this resistance is usually of negligible temperature coefficient, this offset is small. The principal effects of temperature variation on accuracy will also be reduced by shunting the meter. The overall temperature coefficient of error for this kind of meter therefore depends, as it does with the mercury motor energy meter, on a combination of circumstances, some of which in turn depend upon the current and voltage for which the meter is rated.

The effects of temperature variation on the accuracy of A.C. induction meters is inherently small, because the chief cause of temperature errors, alteration of the resistance of the rotating disc, is absent with the induction meter, as the eddy currents producing driving torque and those producing braking torque are each affected to the same extent by such resistance change. The effect of temperature variation on the flux of the braking magnets, already noticed, will cause an induction meter to register slightly fast with a temperature rise. Temperature variation will also affect the resistance of the voltage circuit of the meter, but, as the impedance of this circuit is mainly determined by its reactance, the magnitude of the flux per volt will remain practically unaffected. Temperature change will, however, affect the phase of the voltage flux because of the alteration of the resistance, and hence the resistance/reactance ratio of the voltage circuit.

A temperature rise will cause the phase of the voltage flux to shift in a leading direction on this account, and, further, by increasing the resistance of the copper ring used for phase compensation, will decrease this compensation, and so cause an additional leading phase shift. The effect of temperature change on the performance of an induction meter is thus twofold; a temperature rise reduces the braking torque and tends to cause a fast error; it also shifts the phase of the voltage flux in a leading direction by an amount approximately proportional to this rise, and, by increasing the angle of phase difference between voltage flux and disc currents induced by the current flux with lagging power factors, tends to cause a slow error. If the power factor is leading the temperature phase shift will cause a fast error. As the error of registration caused by a leading phase error of the voltage flux is $-\alpha \tan\phi$, the overall error per degree of temperature rise will be given by a formula like $K_1 - K_2 \tan\phi$, where ϕ is a lagging circuit phase angle. For a certain value of ϕ the total phase error will therefore be zero. Representative curves showing the effect of temperature and power factor on the errors of an induction meter are given in Fig. 27. These fit a formula $(0{\cdot}08 - 0{\cdot}045 \tan\phi)$ per cent. per degree C. of temperature rise.

In the foregoing discussion we have considered the effect of changes of the temperature of the actual components of a meter on its accuracy. These changes can occur from two separate causes: first, by alteration in the temperature of the ambient air in the position where the meter is installed; and, secondly, by the heating effect of currents in the meter circuits. As the case of a meter is made to be dust-tight, a small rate of production of heat within the case will result in a considerable internal rise of temperature, irrespective of any alteration of the temperature of the ambient air. This effect is usually known as self-heating, and its seriousness increases

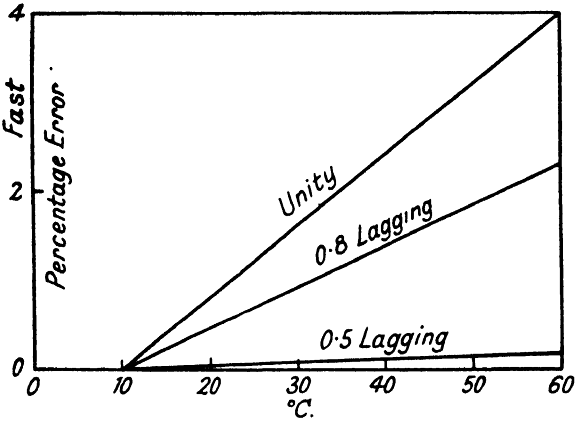


FIG. 27.—Temperature errors of induction meter.

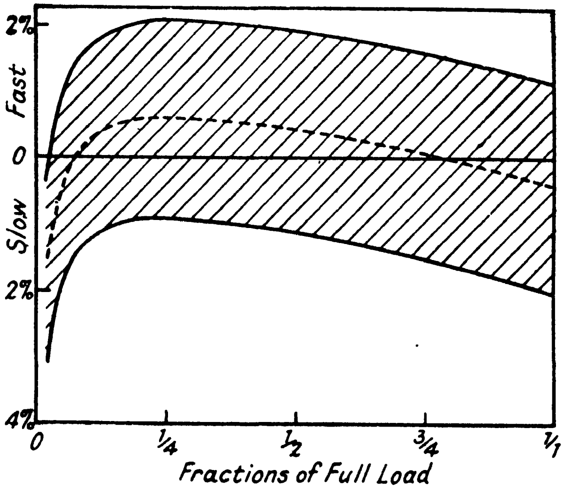


FIG. 28.—Performance of unshunted mercury motor with variable temperature.

with the internal losses in the meter and with the inherent temperature coefficient of error. The practical effect of self-heating is that the temperature of the component parts of a meter, and hence its error, will vary according to the load it carries and also according to the time the load has been carried. Additional to the temperature effects which have been discussed, a rise of temperature sometimes appears to increase bearing friction, and so to alter the shape of the error 'curve.'

It has been stated on p. 69 that the official limits of error for consumers' meters, as recently fixed by the Electricity Commissioners, have been fixed with due regard to the varying conditions of temperature under which these meters may operate when in actual service. It therefore follows that the performance of a meter, as regards actual accuracy of registration, is determined, not by a single curve giving the relation between error and load at a standard temperature, but rather by a band, bounded by the error curves corresponding to the greatest and least temperatures which may reasonably be anticipated in actual service. Thus, assigning the moderate values of 20 degrees and 10 degrees C. for these temperatures. the performance of an unshunted mercury motor quantity meter giving its best overall accuracy at 15 degrees C. will be as shown in Fig. 28, in which the error 'curve' becomes a wide band. The width of the band may, in unfavourable circumstances, be materially increased by self-heating. Thus, in view of the explicit stipulation by the Electricity Commissioners that the limits of error have been fixed with due regard to temperature effects, it appears that a temperature coefficient of error exceeding, say, 0·1 per cent. per degree C., will materially impair the eligibility of a meter for certification.

Errors caused by Voltage and Frequency Changes. We have previously noted that as the registration of a quantity meter gives fictitious kWh, each of which merely represents so many ampere-hours, changes of the supply voltage have no effect on the nominal accuracy of such meters. Energy meters are supposed, nominally, to be accurate at all currents and voltages up to those for which they are rated, and this property depends inherently upon the factors producing driving torque being exactly proportional to the circuit quantities from which they are derived. Thus, if the driving torque of a meter can be considered to be produced by the interaction of the circuit current and a magnetic flux derived from the circuit voltage, the accuracy of registration with varying voltages will depend upon exact proportionality of the voltage flux and the voltage from which it is derived. Meters are rarely used with voltages of a different order from those for which they are rated, and the only class of meter in which errors arising from this cause are of any moment is the mercury motor energy class. In these meters the magnetic circuit for the voltage flux lies largely in iron, and, as the excitation is by direct current, the effects of saturation and hysteresis will not only be to make the meter fast at low voltages, and *vice versa*, but also to cause the voltage flux at normal voltage to differ from its normal value after a fall and subsequent rise in the applied pressure. This defect of the mercury motor energy meter is not serious for the pressure variations likely to arise at a consumer's service terminals, but it makes the meter unsuitable for conditions in which voltage changes are considerable.

There is another class of effects of voltage variation in which the errors are of a secondary nature. The compensation for bearing friction in energy meters is always such that an auxiliary driving torque, derived from the circuit voltage and hence proportional to the square of its magnitude, opposes the nominally constant bearing friction torque. A given fractional change in voltage will thus produce twice this fractional change in compensating torque, so that the compensation will only be correct at normal pressure.

Another secondary effect of voltage variations arises in A.C. induction meters. In these meters rotation of the disc in the voltage flux produces a self-braking torque proportional to the square of the voltage, which, so long as the pressure is normal, is merely like additional permanent magnet braking. A fall in the voltage will cause a reduction of this self-braking torque, giving a fast error, and *vice versa*.

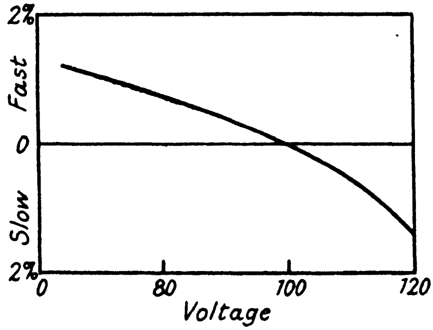


FIG. 29.—Errors of induction meter caused by voltage changes.

For small variations of voltage the error will be approximately proportional to the change, but a considerable rise of pressure will produce a larger slow error than the fast error arising from a similar fall of pressure. A representative curve showing the errors of an induction meter caused by voltage changes is shown in Fig. 29. It will be noticed that this curve is parabolic in form.

The effect of frequency changes on the accuracy of A.C. meters is a matter of small practical importance now that controlled frequency is so usual. Consideration of this effect may be limited to induction meters. This effect may be ascribed to three main causes. First, as the voltage flux varies inversely as the frequency, a rise of frequency will decrease the voltage self-braking torque, and *vice versa*. Secondly, frequency variations will affect the reactance of the eddy current paths in the meter disc. Thirdly, the phase compensation given by the copper ring on the pole of the voltage magnet will increase with a frequency rise, and *vice versa.*

At a circuit power factor of unity, changes in phase compensation will have a negligible effect on accuracy, and the effect of frequency variations will be limited to those caused by alterations of self-braking torque and of the impedance of eddy current paths. The first of these causes being proportional to the square of the frequency change will predominate with a fall in frequency. With a rise of frequency from the normal value, the second effect usually predominates, so that with unity power factor and constant watts an induction meter is fastest at the frequency for which it is rated.

With low circuit power factors, the change in the phase compensation is the predominating effect, and as a rise of frequency increases the lag of the voltage flux, it will cause a fast error with lagging power factors, and *vice versa.*

Phase Sequence. It has already been explained on p. 40 that, because of the effect of leakage fluxes on the driving torques of the elements of polyphase meters, the meters must be connected in circuit, so that the phase relation of the voltages of each pair of elements corresponds to those for which the meter was calibrated. If this condition is not satisfied, errors may arise to an extent depending upon the closeness of the proximity of the several elements. There is, however, no difficulty in connecting polyphase meters in circuit with the correct phase sequence, so that this source of error should be of no practical importance.

Meter Registration Errors with Varying Loads. The investigations of the theoretical forms of the 'error curves' of meters of various kinds given in the earlier part of this chapter lead to the establishment of relations between fractional speed errors and watts. These relations will also apply to fractional errors of energy registration over a period if the watts are constant, but not otherwise. The question of error of registration of a meter over a period during which the load varies will now be briefly discussed.

We will consider first the case in which bearing friction is the only cause of error. Referring to the equation for the errors of a meter of this kind on p. 70, it will readily be understood that we can restate this equation in the following form :—

$$W = W_m + \eta W_m - W_1,$$

where W_m is the rate at which the meter is registering kWh. Over a time period t we shall evidently have

$$E = E_m + \eta E_m - W_1 t,$$

where E is the energy and E_m the meter registration corresponding. The fractional error of registration is easily seen to be, since E_m is nearly equal to E,

$$\eta - \frac{W_1 t}{E} = \eta - \frac{W_1}{W_a},$$

where W_a is the average value of the watts.

The error of registration over the period is therefore equal to the speed error corresponding to the average watts. The calibration speed error η gives rise, as might be expected, to an approximately equal registration error.

Now consider a meter in which fluid friction is an additional source of error. We shall now have

$$W = W_m + \eta W_m - kW^2 - W_1,$$

so that over a time period t

$$E = E_m + \eta E_m - kW_Q{}^2 t - W_1 t,$$

where W_Q is the root-mean-square value of the varying watts. The fractional error of registration is thus

$$\eta - \frac{W_Q^2}{W_a} - \frac{W_1}{W_a}$$

and, as W_Q is necessarily greater than W_a, this error will be that corresponding to the speed error at a load somewhat greater than W_a.

A similar investigation applicable to a meter in which the self-braking torque of the current flux is a source of error will show that the registration error corresponding depends upon the ratio of the cube root of the mean cube of the power to its average value, so that with such a meter the error of registration will also correspond to the speed error at a load greater than the average.

Errors caused by Instrument Transformers. If a meter is energised by currents supplied by instrument transformers, its registration errors will depend, not only on its own inherent errors, but also upon those of the instrument transformers. These errors are of two kinds : the first arises from a difference between the actual secondary magnitude and its nominal value based upon the ratio of transformation. This error, expressed as a fraction or percentage, is known as the ratio error. The second kind of error is that of the phase difference between the primary and the secondary magnitude. It is called a phase error and is stated in minutes of angle, or preferably in percentages of a radian (centi-radians). The errors of an instrument transformer depend upon the characteristics of the impedance of its external secondary circuit. These characteristics are usually stated in terms of VA and power factor of the secondary output at the rated voltage or rated full load current of the transformer. The VA value of this output is called the burden.

The errors of a voltage transformer have a much less serious effect on meter accuracy than those of a current transformer, because, as the primary pressure of the transformer is practically constant, the errors are the same at all loads. The relation between the errors of a voltage transformer and its output or burden depends on considerations very similar to those which determine the regulation of a power transformer, excepting that, as the exciting current of a voltage transformer is usually relatively greater than that of a power transformer, a voltage transformer cannot accurately be considered to have a constant fictitious equivalent impedance. As with a power transformer, the phase difference of primary and secondary voltages will fall as the secondary power factor changes from unity to a lagging value. When the secondary burden of a voltage transformer consists wholly or in part of the voltage coil of an induction meter, the power factor will have a low lagging value and the phase error will tend to be very small. Ratio error for a specific burden can be accurately corrected by making the turns ratio differ slightly from the nominal ratio.

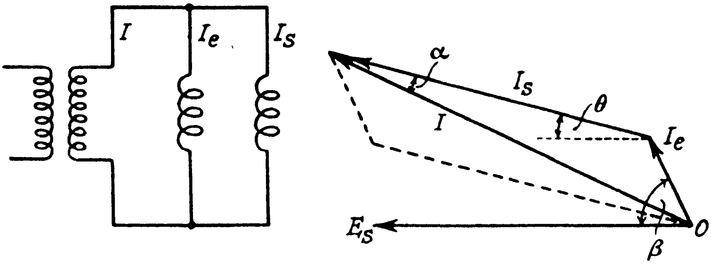


Fig. 30.—Theory of current transformer.

The errors of a current transformer are more important in their effect on meter accuracy because these errors are not constant, but vary with the primary current, and hence with the load. The elementary theory of the current transformer is based upon the idea that a component of the primary current is used up in magnetising the core, the remainder being transformed into secondary current without error. The magnetising or exciting component depends upon the secondary e.m.f. required to drive the secondary current against secondary impedance. It therefore follows that the performance of an actual current transformer will be the same as if it were free from error, but had its secondary circuit shunted by an impedance which always diverted from the burden a current corresponding to the exciting component. The equivalent circuit of a current transformer is thus as shown in Fig. 30, where I represents the nominal secondary current, I_s the actual current, and I_e the exciting component referred to the secondary side. The vectors of these currents are shown also in Fig. 30, in which E_s is the vector of secondary e.m.f. and θ the phase lag of I_s depending upon the secondary impedance. As θ varies, the impedance remaining constant, the extremity of I vector will rotate about O, so that for a certain value of θ the phase difference of I and I_s will become zero. If the angle of lag of I_e on E_s is β, then the fractional ratio error is approximately

$$\frac{I_e}{I}\cos(\theta - \beta)$$

and the phase error in radians is

$$\frac{I_e}{I}\sin(\theta - \beta),$$

I_s leading I. When θ is greater than β, $(\theta - \beta)$ is negative and the phase error is a lagging angle.

The magnitude of I_e will depend upon E_s, and hence on the value of the primary current and of the secondary impedance. The errors of a current transformer are therefore reduced by keeping the external impedance as low as possible. Since E_s depends upon the total impedance which includes the internal impedance, the errors cannot be reduced indefinitely, even if the external impedance is negligibly small.

If I_e were proportional to E_s, and hence to I_s, with constant secondary impedance, the errors would be constant for all values of I. The relation between I_e and I_s, however, corresponds to the magnetisation curve of the material of the core, and for low flux densities this curve is convex to the exciting ampere-turns axis. The exciting current per ampere of secondary current therefore increases as the secondary current falls. The errors of a current transformer are therefore considerable with primary currents of a small fractional value of the rated full load current, but fall to a small value as full load current is reached. The errors of current transformers can be made almost negligible, not exceeding 0·3 per cent. in ratio or phase for currents over one-tenth of full load, by the use of a high permeability nickel-iron alloy for the core.

The ratio errors of a current transformer can often be partly compensated by winding the secondary circuit with one turn less than corresponds to the nominal ratio. Thus, if the nominal number of secondary turns is 200 and actually 199 only are employed, the secondary current will, because of this, always be 0·5 per cent. higher than it would be with the nominal turns ratio. This method of compensation is coarse, and is only suitable when inherent ratio errors are high. There is no simple method of compensating for phase errors.

If the circuits of a single-phase meter are supplied from instrument transformers, the resultant ratio error will have a direct effect on the accuracy of registration. The phase errors of the transformers will cause the meter current and voltage to differ by an angle $(\phi - \alpha)$ with lagging power factors, where α is the resultant phase error, secondary magnitudes leading. If η_t is the overall ratio error, the rate of registration will correspond to

$$(1 - \eta_t)\,\mathrm{IV}\cos(\phi - \alpha)$$

or to

$$(1 - \eta_t)\,(\mathrm{W}\cos\alpha + \mathrm{X}\sin\alpha),$$

so that the fractional error of registration is very approximately

$$-\eta_t + \alpha\tan\phi,$$

α being in radian measure.

As the inherent phase error of a meter is possible of adjustment, it follows that it should be possible to make this error equal and opposite to the average phase errors of the associated instrument transformers. Accuracy of energy measurement is, however, best obtained by high inherent accuracy of the meter and of the associated transformers rather than by such compensation.

If a 3-phase meter is supplied by sets of instrument transformers having identical errors, the effect of the transformer errors will evidently be the same as with a single-phase meter, and as given above if currents and voltages are balanced. The effect of these errors is not so clearly perceived if the load is unbalanced, and is generally complex. If, however, we assume that the voltage is balanced, and that for the differing line currents the current transformer errors are sensibly identical, the effect of the errors can be explained simply in terms of symmetrical component theory, because the whole of the power is produced by the association of the balanced voltage system with the positive sequence symmetrical component of the current system. The effect of the phase error of the instrument transformers is thus to alter the phase difference of these two symmetrical systems from ϕ_1 to $(\phi_1 - \alpha)$, so that the fractional error of registration so caused is $\alpha \tan \phi_1$. In this case, however, $\tan \phi_1$ is defined as the ratio total VAr/watts.

Errors of Demand Indicators. A maximum demand indicator of the Merz type will be subject to the same errors as those of the integrating meter by which it is operated. Further, the accuracy of the kW indication will also depend upon the accuracy of the time intervals during which the demand indicator is in gear with the associated meter. The modern tendency is to control the throwing in and out of gear of the indicator wheel train by a synchronous motor clock. The accuracy of such a clock depends upon the frequency of the voltage supplying it being maintained constant at the declared value. The accuracy of a demand indicator controlled by such a clock will therefore be affected by such temporary frequency variations as might occur in conditions of emergency. If the supply to the clock should fail, the indication of the demand indicator will be spoiled. The accuracy with which a demand indicator can be read depends upon the size and design of the scale and pointer.

The indications of a demand indicator of the thermal type depend upon a difference of the temperatures of two component parts of the instrument, so that it should be independent of variations of the temperature of the surrounding air. The accuracy of such an indicator can only be assessed in terms of the accuracy with which the scale is marked to correspond with currents or fictitious kW of a constant value and which persist for a time period longer than the nominal averaging period. For currents persisting for shorter periods, or varying in magnitude, the nature of the indication is so indefinite that it is hardly possible to assign any measure to the accuracy of the indication.

Errors of Reactive Meters. A reactive meter registering kVArh is a modified A.C. energy meter, and it will accordingly be subject to the same tendencies to error which have already been discussed for A.C. energy meters. A single-phase reactive meter with two voltage windings depends for its accuracy on the correct phase of the current in the partly reactive circuit. This phase will vary with frequency, and a meter of this kind will only be correct at the rated frequency.

Three-phase reactive meters are usually standard energy meters excited by artificial voltages in lagging phase quadrature with respect to those required by a kWh meter. In one type of 3-phase reactive meter the voltages are 60 degrees leading on those applied to an energy meter, and the additional 30-degree phase shift of the voltage flux is obtained by added resistance in the voltage circuit. Internal compensation of this kind will evidently only be accurate at one frequency. As controlled frequency is now so common, errors caused by frequency variations are of small practical importance.

In most 3-phase reactive meters the voltage compensation is completely external to a standard energy meter, and is obtained by one of the methods explained on pp. 47–49. The accuracy of this compensation depends upon exact equality of the line voltages of the supply. If the voltages are unbalanced, the reactive meter voltages will be incorrect, both as regards magnitude and phase.

The errors of 3-phase externally compensated reactive meters can be expressed quantitatively in terms of the symmetrical components of the unbalanced supply voltage system. Consider the method of obtaining the artificial voltages illustrated in Fig. 14. It can be shown by elementary symmetrical component theory that the meter voltages derived from this arrangement of auto-transformers are each the resultant of the positive and negative sequence components of the voltage system displaced 90 degrees in phase, but with the sense of the quadrature phase shifts opposite. The torque set up in the meter can only arise from the association of symmetrical component currents and voltages of like phase sequence. As the connections of the meter are such as to cause forward rotation with positive sequence kVAr, the torque caused by negative sequence kVAr will, owing to the opposite sense of the quadrature phase shift of the negative sequence voltages, be a reverse one. The error of the meter will therefore be twice the negative sequence kVArh. It should be noted that this quantity depends on negative sequence currents and voltages. If both voltages and currents are approximately balanced, these components will each be fractionally small, so that their product—and hence the error of registration—will be negligibly small. This error, it must also be noted, depends on a quantity θ, expressive of the phase relation of negative sequence current and voltage. Over a period of registration the quantity, which is determined by conditions of unbalance, may vary over wide limits from a leading to a lagging value. The effect of this phase variation will be to reverse the sign of the error corresponding to negative sequence kVAr, so that the overall error of registration of actual kVAr will, by reason of this effect, be reduced.

It is evident from elementary principles that the voltages for a reactive meter obtained as shown in Fig. 13 have the same phase as, and are always in a constant ratio to, those given by the auto-transformer arrangement of Fig. 14. A reactive meter energised by phase to neutral voltages will therefore be liable to the same errors with voltage unbalance as one connected according to the method of Fig. 14. It can also be shown that the errors of a reactive meter using line voltages and composite currents, according to the method indicated in Fig. 15, will also register with a slow error corresponding to twice the negative sequence lagging kVAr.

The methods of approximate kVArh measurement described on p. 49 all depend for their accuracy on balance of both currents and voltages of the 3-phase supply. . Although it is legitimate to assume closely approximate balance of the voltages, this does not apply to the currents of a 3-phase supply, and these methods are inherently liable to considerable errors in practical use. As, however, the approximate method of kVArh measurement depending upon obtaining the difference of the registrations of two single-phase energy meters is still used as a basis for obtaining a value of average power factor, a short discussion of the errors of these approximate methods will be of practical use. We shall assume that the 3-phase voltages are balanced, and only consider errors caused by disymmetry of line currents.

Consider first the crossed-phase method of adapting a 3-phase energy meter referred to on p. 50. If the currents are balanced, that is, contain a positive sequence component only, the meter will with balanced voltages give $2/\sqrt{3}$ of the 3-phase kVArh. If there is a negative sequence component, this will tend in association with the meter voltages to produce a driving torque, for, because of the crossing of the phases, the meter voltages are of reversed, that is, of negative, phase sequence. But the blue-phase voltage is reversed in polarity so that, with respect to the negative sequence currents and the artificial negative sequence voltages, the meter behaves as an energy meter with one torque reversed. The torque corresponding to the negative sequence currents thus corresponds to $I_n V \sin \theta$, where I_n represents a negative sequence current and θ is the phase angle of the lead of this current. As the multiplier of a crossed-phase meter is $\sqrt{3}/2$, the error caused by current unbalance in the rate of registration is $\frac{\sqrt{3}}{2} I_n V \sin \theta$, positive when θ is leading.

Now consider the case of a 3-phase meter with the voltage connections to the red element reversed. With respect to negative sequence currents, the instrument will behave as a crossed-phase reactive meter, and as red-element voltage is reversed, forward torque will be produced when the negative sequence currents are leading. This torque will correspond to $2I_n V \sin \theta$, and as the multiplier of a reversed-phase meter is $\sqrt{3}$, the error in the rate of registration will be $2\sqrt{3}I_n \sin \theta$, positive when θ is leading. This error is of the same sign, but four times the magnitude of the error of a crossed-phase reactive meter.

Finally, consider a single-phase meter connected in a 3-phase circuit with its current coil in one line and its voltage coil connected between the other two. Both positive and negative sequence component currents in the meter coil will produce torque, and that arising from the negative sequence component will be $I_nV \sin \theta$, where θ is a lagging angle, since the connections are arranged for forward rotation with lagging power factors. As the multiplier of this meter is $\sqrt{3}$, the error caused by current unbalance will be $\sqrt{3}I_nV \sin \theta$, negative when θ is leading. This error is opposite in sign and one-half in magnitude the error of the reactive meter, which is an ordinary 3-phase energy meter with reverse torque in the red element.

It follows from this last result that if to the difference of two single-phase meters used for energy measurement in a 3-phase circuit is added twice the registration of a reactive meter with its current coil connected in the third phase, the result will give $\sqrt{3}$ times the kVArh in the circuit, and the errors caused by current unbalance in the two sets of meters will be eliminated.

Errors of kVAh Meters. kVAh meters which integrate the equivalent VA in a 3-phase circuit consist of a combination of energy and reactive meters, and generally errors of the component meters will affect the accuracy of kVAh registration correspondingly. Phase errors which are equal in magnitude and sign in the component meters will, however, have no effect on the accuracy of the kVAh register, since such a phase error will in effect merely modify the quantity ϕ_1 in the expression $\cos \phi_1$ for the 3-phase power factor, and the kVA is independent of this quantity.

When the so-called vector summation of kW and kVAr in a kVA meter is approximate and only correct at certain values of the power factor, there will be inherent errors of a fractional value $(1 - \cos \beta)$, where β is deviation of ϕ_1 from the values corresponding to those for which the summation is correct. This fractional error can nominally be halved by adjusting the calibration of the meter, so that with constant VA it is as fast at the power factors for which summation is correct as it is slow for those intermediate power factors for which the inherent error is greatest. This possibility is difficult to realise completely, and the possible errors of a kVAh meter are best considered as the resultant of the slow fractional error $(1 - \cos \beta)$ and those inherent in the component meters and their instrument transformers.

The accuracy of the shaded-pole type of kVA demand indicator will depend upon that of the scale marking for constant values of the currents in the three lines of a 3-phase circuit. As the scale is non-uniform, by reason of the non-proportionality of the speed of the movement to kVA, the indication may be different from that corresponding to kVAh in the averaging period when the kVA value varies within this period, although this difference is likely to be small. The inherent difference between the indications of this instrument and that of one indicating total equivalent 3-phase kVA has already been pointed out. Thus, if total equivalent VA be regarded as the correct value of 3-phase VA, the shaded-pole instrument will register incorrectly with unbalanced currents. It is easy to show by symmetrical component theory that, with balanced voltages, the total equivalent VA corresponds to the positive sequence component of an unbalanced current system, and the VA reading of a shaded-pole demand indicator corresponds to the square root of the sum of the squares of the positive and negative sequence components.

The accuracy of a shaded-pole type of demand indicator will depend upon the frequency being maintained at the rated value.

Errors caused by Wave Distortion. The final cause of errors of registration of A.C. energy and reactive meters which requires consideration is distortion of the waves of current and voltage in the supply measured by the meter. This cause of error is of increasing practical importance because the use of consuming and converting apparatus which take a distorted wave of current from A.C. supply systems is becoming more and more common. If the impedance of a circuit is not constant, but varies with the current it carries, the circuit will take a non-sinusoidal wave of current when connected to a source of sinusoidal alternating voltage. This distorted wave of current will set up a similarly distorted wave of voltage drop in the conductors between the circuit and the source of supply which will give rise to a non-sinusoidal resultant voltage at the terminals of the circuit.

A non-sinusoidal alternating quantity can, according to Fourier's theorem, be regarded as the equivalent of the resultant of a number of sinusoidal components having frequencies which are integral multiples of the main frequency. Further, the average power in a supply consisting of non-sinusoidal alternating current and voltage only arises from the association of harmonic components having the same frequency. It therefore follows that an energy meter measuring a supply with distorted current and voltage waves really has to integrate simultaneously a number of power components of different frequencies. If with a distorted current wave the associated voltage wave is very approximately sinusoidal, the whole of the power will be carried by the first harmonic component or fundamental of the current wave, and this power will be of normal frequency. The operation of the meter is therefore normal, excepting that, as we shall explain in detail hereafter, the current per watt at normal voltage and maximum power factor will be increased by wave distortion, so that the self-braking torque of an induction meter will also increase.

If both current and voltage waves are non-sinusoidal, then, as the total power is the sum of a number of components of various frequencies, the accuracy of the meter will depend upon the accuracy with which it can integrate power at the various harmonic frequencies. We have already briefly discussed the effect of small frequency variations on the accuracy of an induction meter. When the frequency is very much larger than that for which the meter is rated, the errors become considerable, both in calibration and phase. Calibration errors arise because of the increase of the impedance of the eddy current paths in the disc, and phase errors are caused by the phase shift of these currents with respect to the meter currents from which they arise, so that the meter registers slower on lagging than on unity power factor. At a frequency of 300 an induction meter rated for a standard frequency of 50 may register only about one-half of the total energy passing it at unity power factor, and may have a phase error of the order of 10 degrees. The integration of the harmonic components of the power will thus be very inaccurate. If, however, the voltage distortion is small, these harmonic components will also be relatively small, so that the actual meter error will not be serious. The behaviour of induction meters when measuring the supply to mercury arc rectifiers has been investigated in detail by Dr. C. Dannatt, and the results of this investigation are given in a paper mentioned in the appendix. It appears that the error of an induction meter caused by wave distortion when measuring the input to a 3-phase rectifier is not likely to exceed 1 per cent.

The calibration errors of an unshunted commutator energy meter when measuring A.C. power at frequencies higher than the rated value are much less than those of an induction meter. Thus the error of such a meter at a frequency of 500 is only of the order of 5 per cent. The phase error caused by increase of the reactance of the voltage current is, however, considerable, being of the order of 10 degrees and making the meter fast on lagging power factors. The behaviour of an unshunted clock meter in a circuit of high frequency may be expected to be similar to that of a commutator meter. If either of these kinds of meters is shunted large errors would be caused at high frequencies by the increase of the reactance of the meter circuit.

The superiority of the general performance of the induction meter is likely to ensure its use in the measurement of distorted wave supplies in which, as is now usual, the harmonic components of the voltage wave are relatively small. In special circumstances of serious voltage wave distortion the use of an unshunted commutator or clock meter might be desirable.

The measurement of reactive VA and average power factor in circuits with distorted current and voltage waves involves difficulties of a fundamental nature. A distorted wave of current will have an R.M.S. value of $\sqrt{(I_1^2 + I_3^2 \ . \ . \ .)}$, where I_1, I_3, etc., are the R.M.S. values of the harmonic constituents. This R.M.S. value is independent of the phases of the harmonics in respect of the fundamental. If a distorted wave of current is associated with a sinusoidal voltage in a reactive meter the rate of registration will correspond to $VI_1 \sin \phi_1$, as the meter will ignore the current harmonics having a different frequency from that of the voltage. The VA value derived from the registrations of an energy and a reactive meter will therefore correspond to I_1V, whereas the actual value of the VA is $\sqrt{(V^2I_1^2 + V^2I_3^2 \ . \ . \ .)}$. In other words, the square of the VA in a supply having distorted current and voltage waves is the sum of three components, W^2, X^2 and $X_D{}^2$, where W is the actual power, X the total reactive VA corresponding to phase displacements of harmonic constituents of current and voltage of like frequencies, and X_D is a kind of distortion VA, which arises from dissimilarity of current and voltage waves. Of these three components, only W and X can be detected and integrated by meters. Thus a value of power factor derived from meter readings will correspond to $W/\sqrt{(W^2 + X^2)}$, and this will be greater than the true power factor or W/(VA), which is equal to $W/\sqrt{(W^2 + X^2 + X_D{}^2)}$. As X_D is independent of phase displacements, it cannot be reduced by any phase-shifting devices, so that the maximum power factor, when $X = 0$, will be $W/\sqrt{(W^2 + X_D{}^2)}$.

It appears, therefore, that a reactive meter can only integrate the quantity X in a circuit with distorted current and voltage. The accuracy of the reactive meter will, as with an energy meter, depend upon its inherent error characteristic with large frequency changes, and, if the reactive meter is internally compensated for phase shift, there will be a source of error at high frequencies additional to those causing errors in energy measurement, because the increase in frequency will affect the phase compensation for reactive measurements. A full discussion of the behaviour of reactive meters in circuits with distorted currents and voltages will be found in Dr. Dannatt's paper, to which reference has been made.

It should be noted, in conclusion, that as the indication of a thermal demand indicator carrying a constant current corresponds to the true R.M.S. value of this current, the accuracy of the instrument is not affected by wave distortion.

CHAPTER IV

STANDARD AND SUBSTANDARD INSTRUMENTS USED FOR METER TESTING

Electrical Units. The unit of energy or the kWh is of fundamental importance in the business of electricity supply, because primarily a consumer buys electrical energy in order that it may be converted to other forms of immediate use to him. This economic utility of electrical energy gives it an objective reality which is not possessed by other electrical quantities, and as electrical energy has physical dimensions which can be stated in terms of the basic dynamical units of mass, length and time, there are scientific grounds for considering the kWh as the fundamental electrical unit. From the point of view of practical measurement, however, it is much more convenient to regard electrical energy as a complex electrical magnitude arising from the association of current, voltage and time, although actually current and voltage are complex in their physical nature, in that their dimensions cannot be expressed in terms of the basic dynamical units. The convenience of regarding energy as a complex electrical unit arises from the circumstance that practical units of current and voltage can be constructed without great difficulty. The ampere can be defined in terms of the rate of electrolytic deposition of silver and the volt in terms of the e.m.f. of a mercury-cadmium cell constructed according to a standard specification. Thus, having such precise means of measuring current and voltage by practical standards of this kind, electrical power in D.C. circuits can be measured, whence, by the use of a time-measuring device, energy at constant power can also be measured.

A further simplification in the measurement of electrical power is made possible by that physical property of conducting substances, expressed by Ohm's law, in virtue of which the voltage drop in a conducting circuit is exactly proportional to the current it carries, provided the temperature remains constant. The voltage drop for 1 ampere, or the resistance of a conductor, being known, the current in the conductor can be determined from this resistance if the voltage drop in it can be measured, and the measurement of power can be made to depend upon the measurement of voltages only, if resistances of known and suitable values are available.

Hence the measurement of power in D.C. circuits in terms of the fundamental electrical units only requires one standard e.m.f., the value of which is known in terms of the volt, that is, a standard cell, together with suitable standard resistances of known value, since resistance, being a physical character of a material conductor, may be presumed to remain constant so long as other physical characteristics do not vary. Apparatus components of this kind which enable current or voltage to be measured directly in terms of the electrical units are called standards, and instruments which are verified by means of such standards are called substandards.

Meter Testing. The object of the regulations for meter testing issued supplementary to the Electricity Supply (Meters) Act, 1936, is that the errors of all meters submitted for certification shall be determined by substandard instruments which are themselves periodically tested, either directly or indirectly, by ultimate standards of e.m.f. and resistance. In other words, the performance of a meter must be determined by a series of tests, which, starting with the meter, can be traced by definite steps to the standard cell and standards of resistance. The chain of connection between the meter and the standards may be of various orders of complexity. A quantity meter may be tested by an ammeter, which itself is tested by direct reference to the standards, and a timing device which is checked by a standard clock. Here the chain of connection is simple. A heavy current A.C. energy meter, on the other hand, may be tested by means of a substandard energy meter used with standard current transformers, this substandard being tested by a substandard indicating wattmeter and timing device, the wattmeter finally being tested with direct current by the ultimate standards. In this case not only is the chain of connection between the supply meter and the standards complex, but there is one link at least of doubtful integrity. The indicating wattmeter is tested by direct current and used to measure A.C. power. An instrument tested with one kind of current and used with another kind is called a transfer instrument, and is evidently necessary in referring an A.C. meter to the ultimate standards of e.m.f. and resistance. The transfer instrument is in practice a wattmeter, but it is possible to use a chain of tests in which the transfer instrument is an ammeter. The essential property of any transfer instrument is that it must give the same reading when carrying the same power or current, whether this power or current be steady or alternating. It may be said that if an ammeter is used as a transfer instrument it is only necessary that this identity of reading under differing conditions obtains for one stipulated value of the current. The accuracy of meter testing thus depends upon the accuracy of all substandards used in the chain of connection between the meter and the ultimate standards and upon the accuracy of transfer of the transfer instruments used with A.C. meters. The possible error of a chain of tests is the sum of the possible errors involved in each link of the chain, hence the inherent accuracy of the test of a meter in terms of the ultimate standards becomes necessarily less as the chain lengthens.

Apparatus Required in Approved Testing Stations. The apparatus which must be provided in any Testing Station in which supply meters are tested before being submitted for certification includes the following components :—

Standard Apparatus. Direct-current potentiometer, standard cells, voltage dividers and resistance standards for current measurements.
  Ship's chronometer or pendulum clock.

Substandard Apparatus. Indicating wattmeters, ammeters and voltmeters.
  Rotating meters.
  Electrolytic meters.
  Stopwatches or other timing devices.

Current, voltage and phase control apparatus.

This apparatus must conform to the Specifications laid down by the Electricity Commissioners and issued in publications supplementary to the 1936 Act.

Direct-current Potentiometers. The D.C. potentiometer comprises a resistance so arranged that contacts can be established at tappings on it between which any required decimal fraction of the whole of the resistance is included. The resistance carries a constant current, the value of which is so adjusted that the fall of potential between any two points on the resistance is equal in volts to the fractional part of the whole resistance included between these points.

The principle and use of the D.C. potentiometer are illustrated by the elementary diagram of connections (Fig. 31). R is a resistance divided by tapping points into, say, exactly ten equal parts, S is a uniform slide-wire having a resistance equal to one-tenth of R and fixed over a scale which gives exact fractional subdivisions of its whole length. A rheostat is connected in series with R and S whereby the current from an accumulator cell may be adjusted to the required value. Contact with this circuit can be established at two points—a tapping on the resistance R and an adjustable point on the slide-wire S. The current in the circuit is adjusted by connecting a standard cell SC of known e.m.f. in series with a galvanometer and a key switch to these two points so set that the fraction of R included between them is equal to the e.m.f. of the standard cell in volts. Thus, if this e.m.f. is 1·0183 volts, the two contacts will be set to include between them the whole of R and a fraction 0·183 of S. By adjustment of the rheostat the fall of potential in the potentiometer circuit can

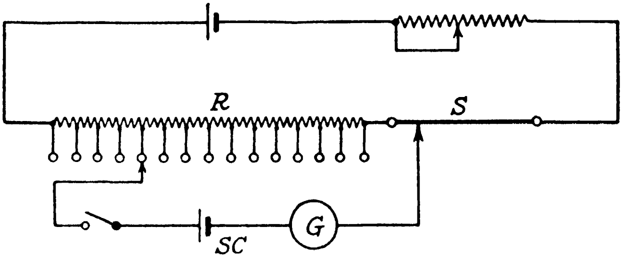


FIG. 31.—D.C. potentiometer.

be made exactly equal to the e.m.f. of the standard cell, this equality being shown by a null reading of the galvanometer when the key is pressed. When this condition is obtained the fall of potential in R is exactly 1 volt and that in S is exactly 0·1 volt, so long as the current from the accumulator does not vary. By adjustment of the two tapping points any fall of potential between them from 0 to 1·1 volts can be obtained, and the magnitude of this fall of potential in volts is given by the number of subdivisions of R and the length of S in which it takes place. An unknown voltage up to 1·1 volts can thus be measured by connecting its source in series with the galvanometer and key, and adjusting the positions of the two contact points till the fall of potential between them is equal to the unknown voltage, as shown by a null indication of the galvanometer. The value of the unknown voltage is then given by the positions of the tapping points.

The resistance R is generally divided into 15 equal parts. In this case the current is adjusted to such a value that the drop in each part is exactly 0·1 volt. With the arrangement the potentiometer will measure voltages between 0 and 1·6.

The foregoing is a description of the essential parts of a potentiometer. Modern instruments have a number of refinements designed to give convenience in use. The rheostat generally comprises two components—one for fine and the other for coarse adjustments. A multi-way double-pole switch is provided, so that two or more external circuits or the standard cell can be connected at pleasure in series with the contact key to the variable tapping points. In some potentiometers a special laminated contact key is provided, resistances being connected between the laminations. A light pressure on the key will establish contact between two laminations only, and a resistance will be included in circuit with the galvanometer of sufficient value to avoid a violent swing if the setting of the position of the tapping points does not approximately correspond to the voltage being measured. An increase of pressure on the key brings a second pair of laminations into contact, short-circuits the safety resistance, and gives the maximum galvanometer sensitivity for a final adjustment of the tapping contacts.

The disadvantage of the potentiometer, as has been described, lies in the fact that it is not suitable for the measurement of small voltages under 0·1, as for such measurements the fraction of the total resistance of the potentiometer is given by a short length of the slide-wire, and considerable errors may occur because of the difficulty of accurate observation of the length of the wire corresponding to the unknown resistance and to possible irregularities in the uniformity of its cross-section. For this reason modern potentiometers are fitted with a range-changing device, whereby the voltage drop in the potentiometer can be reduced to exactly one-tenth of its normal value. The range-changing device is illustrated in Fig. 32, and it comprises two extra resistance components, R_2 and R_3, together with a double-throw switch or plug device. With the normal range R_2 is short-circuited and R_3 is idle. To change the range, R_3 is connected in parallel with the potentiometer and R_2 is put in series with the parallel combination. R_3 is exactly one-ninth of the potentiometer resistance, so that the resistance of the combination is one-tenth of this resistance. The value of R_2 is exactly nine-tenths of the potentiometer resistance. With the connections for the lower range the total resistance of R_2 and the parallel combination is therefore exactly equal to the potentiometer resistance, so that the accumulator current is unaltered by changing the range. The current in the potentiometer branch of the parallel combination is, however, only one-tenth of its normal value, so that the fall of potential in it—and hence the fall of potential in any fractional part of it—

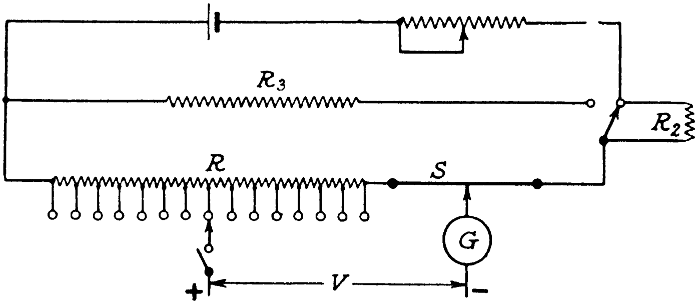


FIG. 32.—D.C. potentiometer with range-changing device.

is reduced in the same proportion. On the low range therefore the whole length of the slide-wire corresponds to 0·01 volt, and the sensitivity and accuracy of the measurement of very low voltages is very much improved. D.C. potentiometers for use in Approved Testing Stations must, according to the Electricity Commissioners' Specification, be fitted with a range-changing device, so that the maximum fall of potential with the low range is one-tenth of its normal value.

The subdivisions of the principal fractions of the potentiometer resistance may be made primarily by a subsidiary resistance with uniform tappings arranged in accordance with the Kelvin-Varley principle. The rudimentary arrangement of such a potentiometer is as shown in Fig. 33. Here there are eleven main components of the principal circuit and a pair of tapping points between which two such sections are bridged by the auxiliary resistance, which itself is provided with eleven points dividing it into ten equal parts. The total value of the auxiliary resistance is exactly equal to two fractional components of the main potentiometer circuit, so that the combined resistance between two of the taps in the main circuit bridged by the auxiliary resistance is exactly equal to one section of the principal fraction. Thus each primary voltage drop of 0·1

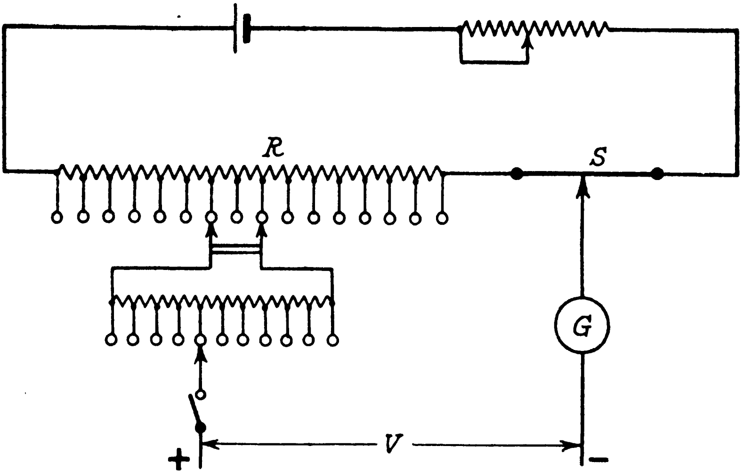


Fig. 33.—D.C. potentiometer with Kelvin-Varley dial.

in the potentiometer circuit is subdivided into secondary fractions of 0·01 volt by the taps on the auxiliary resistance, and by adjustment of the position of the pair of coupled contacts engaging with the principal taps and a second movable contact engaging with the auxiliary taps, the potential drop, which is compared with an unknown voltage, can be altered in steps of 0·01 volt. Finer adjustment is made by adjustment of the contact touching the slide-wire, and in the normal range the potential drop in this slide-wire need be only 0·01 volt. The Kelvin-Varley principle can be extended by using a second auxiliary resistance, whereby the potential drop in the first auxiliary resistance is further subdivided.

In the Vernier potentiometer an auxiliary resistance connected on the Kelvin-Varley principle is subdivided into 100 parts, and a resistance section, in place of the slide-wire, equivalent to one of these fractional parts is itself subdivided into 100 parts. The minimum fraction of potential drop obtained by adjustment of the contact position on this last resistance is thus $\frac{1}{10} \times \frac{1}{100} \times \frac{1}{100} = 0{\cdot}00001$ volt. The equating of the fraction of the potential drop in the potentiometer circuit to an unknown voltage is in this instrument entirely carried out by 'dial' switches, by which suitable tapping points are chosen.

The switches for the selection of tapping points of the resistance of a potentiometer are usually marked with the potential drops to which they correspond on the normal range, and these markings are correct only when the current in the main circuit has its correct value, or, what is the same thing, when the e.m.f. of the standard cell as measured by the potentiometer is correct. Constancy of the potentiometer current is best checked by observing that this last condition is satisfied, and some instruments are provided with a refinement whereby this check can be carried out without adjusting the potentiometer contacts. This arrangement is illustrated in Fig. 34, from which it will be seen that included in the main circuit is a resistance R_4, across which is a drop of 1·0177 volts when the current has its correct value. An additional resistance section S_1, which either consists of a slide-wire or is adjustable in not less than six steps, is provided so that the total drop can be increased to 1·0187 to allow for variations of the e.m.f. of the standard cell caused by temperature changes. The variable portion of this resistance is set to correspond to the temperature of the cell, and the accuracy of the potentiometer current can be checked at any time by comparing the e.m.f. of the standard cell with the potential drop in this component of the circuit, a double-throw switch being provided whereby the galvanometer may be put in circuit, either with the standard cell or with the source of unknown voltage being measured. This means of checking the potentiometer current by an independent resistance section is recommended in the Specification of the Electricity Commissioners.

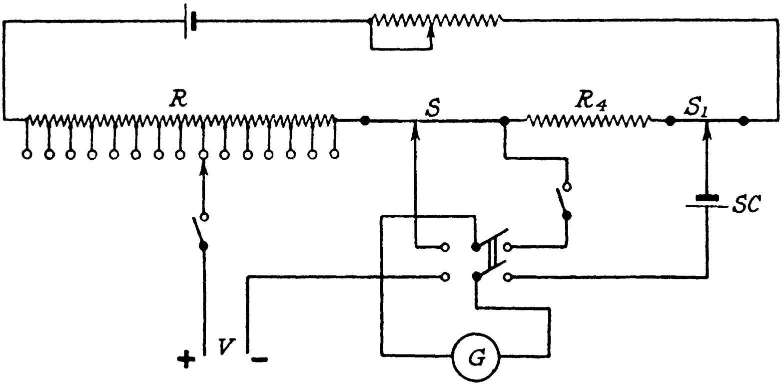


FIG. 34.—D.C. potentiometer with independent standard-cell section.

Constancy of the current in a potentiometer circuit can be very nearly obtained by using an accumulator of not too small rating which is in good condition. The accumulator should be partially discharged before use with a potentiometer, and prior to actual testing the potentiometer should be connected to the accumulator for about half an hour. When a D.C. potentiometer is in frequent use the connection of the accumulator may be permanent.

The following are the additional requirements of the Electricity Commissioners for D.C. potentiometers in Approved Testing Stations. The normal range shall be at least 1·5 volts, and a range-changing device is required. The minimum reading on the normal range is not to be greater than 0·0002. In a slide-wire instrument this reading corresponds to one division of the slide-wire, which must not be less than 1 millimetre in length. The resistance of the potentiometer circuit must be not greater than 200 ohms nor less than 10 ohms per volt. The error caused by departure of the resistance of any section of the potentiometer circuit shall not exceed 0·0002 volt on the normal range, and such error shall not be altered by more than 0·02 per cent. when the instrument is used on the lower range. Two adjusting rheostats are to be provided in the potentiometer circuit and a selector switch, giving choice of not less than three external circuits, excepting when a separate section of the potentiometer circuit is used for checking the value of the current, in which case provision for the choice of one of two circuits shall suffice. Exposed insulating material shall not be such as is affected by light. Ordinary black ebonite does not satisfy this requirement.

Standard Cells. The standard cells used in connection with a D.C. potentiometer are stipulated to have an e.m.f. at 20 degrees C. which does not vary beyond the limits of 1·01815 and 1·01835 volts. These cells are of the well-known mercury-cadmium or Weston type. Not less than two cells must be provided in an Approved Testing Station, and the integrity of these cells as standards of e.m.f. is inferred from the results of tests in which the e.m.f. of one, measured by another as a standard of reference, is found to be correct. The certainty with which the integrity of standard cells can be verified is much greater with three than with two, because a set of three gives three pairs for mutual comparison.

Voltage Dividers. A voltage divider is a resistance provided with suitable taps so arranged that when all or part of this voltage is connected to a source of voltage of unknown value, a known fractional part can be obtained, which is under 1·5 volts, for measurement by means of a potentiometer. When, in measuring this fraction, a final balance is obtained, as shown by the null reading of the galvanometer, no current passes from the tap to the potentiometer. The potential differences in the whole part of the resistance to which the measured voltage is connected, and in that part connected to the potentiometer, will be in direct proportion to the ohmic value of these parts. Voltage-dividing resistances used in Approved Testing Stations must have values lying between the limits 50 and 100 ohms per volt. The resistance per volt for each range must be constant. The error in the ratio of voltage division shall not exceed 0·02 per cent.

Fig. 35 illustrates the use of a voltage divider for measuring a voltage of 460 by a potentiometer. The whole of the dividing resistance is rated for a voltage of 500 and has a value of 100 ohms per volt. The source of voltage to be measured is connected to the terminals of this resistance, and the potentiometer is connected to a section having a resistance of 150 ohms. When the potentiometer is adjusted for correct balance no current passes out of the dividing resistance, and the unknown voltage is exactly 500/1·5 times the voltage measured by the potentiometer.

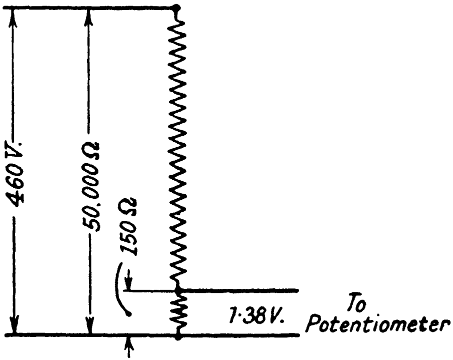


Fig. 35.—Voltage divider.

Standard Resistances for Measuring Current. A standard resistance used to measure currents by means of a potentiometer is like an ordinary D.C. ammeter shunt, excepting that, when the potentiometer adjustment is correct, the whole of the measured current passes through the resistances. Standard resistances are designed for a voltage drop of exactly 1·5 or 1·0 when the rated current is passing through them. To measure a current passing through a standard resistance, the voltage drop in it is measured by a potentiometer, and the unknown current is in the same ratio to the rated current of the resistance as the measured voltage is to the rated voltage drop.

The power wastage in standard resistances for potentiometer measurements is considerably greater than in ordinary D.C. ammeter shunts because of the much higher voltage drop.

Thus a resistance rated for 100 amperes and giving a drop of 1 volt will waste 100 watts, and this will cause a rise of temperature unless the resistance is correctly designed. The dissipation of the heat generated in standard resistances is assisted by enclosing them in a bath of oil.

Temperature rise in standard resistances tends to set up thermal e.m.f.'s if the temperatures of the two terminal blocks of the shunt are different. These e.m.f.'s are small in well-designed shunts, and their presence when measuring currents can be detected by interrupting the current in the resistance, setting the potentiometer reading to zero, that is, short-circuiting the leads from the resistance in the potentiometer, immediately after the voltage drop in it is measured and observing whether the galvanometer deflects from zero when the key switch is closed. Any deflection in this condition is caused by a thermal e.m.f., and the value of this e.m.f. can be estimated by observing the galvanometer deflection as quickly as possible, and then observing the deflection produced by setting the potentiometer to its minimum reading. From these two galvanometer readings the magnitude of the thermal e.m.f. can be calculated. If the galvanometer reading is reduced when the potentiometer is set to its minimum reading, the thermal e.m.f. is in the same direction as the voltage drop set up by the current in the resistance, and its value must be subtracted from the measured voltage drop to obtain its true value. If altering the potentiometer reading for zero to its minimum value causes the deflection of the galvanometer to increase, the value of the thermal e.m.f. must be added to the observed value of the voltage drop.

Standard resistances for current measurement used in Approved Testing Stations must be of manganin, or similar material, hard-soldered to the terminal blocks, or to copper pieces which may be soft-soldered to the terminal blocks. For current ratings up to 100 amperes the error in the resistance may not exceed 0·03 per cent., for current ratings over 100 amperes the limit of error is 0·05 per cent. The thermal e.m.f., after the resistance has been carrying its rated current for 30 minutes, must not exceed 0·01 per cent. of the nominal voltage drop.

Galvanometer. The galvanometer used in a potentiometer equipment is a null instrument, and its function is merely to indicate absence of current. The sensitivity of the galvanometer should be sufficient to enable it to give a reasonable deflection with the potentiometer out of balance to the extent of the minimum scale reading. The galvanometer may be provided with a multi-range shunt, so that its sensitivity can be reduced during preliminary adjustments, but this reduction in sensitivity is best obtained by a resistance in series with the galvanometer, otherwise a very faulty adjustment, with the galvanometer practically short-circuited, may allow of the flow of dangerously high currents in the potentiometer circuit. The galvanometer constants should be such that it is critically damped, for the best and most rapid work, and the instrument should preferably be provided with a magnetic shunt so that the field strength can be adjusted to obtain this result. The zero position of the galvanometer movement should be stable, and for this reason the sensitivity of the instrument should not be greatly in excess of that actually required. If there is vibration of the building in which the galvanometer is used, it will be necessary to support it by some anti-vibration device.

Precautions Necessary in Potentiometer Measurements. The sensitivity of the galvanometer used in potentiometer equipments makes it very liable to give false indications by reason of currents leaking between parts of the test circuit which are at different potentials. For this reason the greatest care should be taken thoroughly to insulate all component apparatus of the circuit, and to ensure that external circuits brought to the potentiometer terminals are all at the same potential. The accumulator supplying the potentiometer current and the standard cell should be efficiently insulated, and this can be done by standing them on blocks of paraffin wax. External circuits should be earthed through lamps at the potentiometer terminals. When a galvanometer of high sensitivity is used, errors caused by electrostatic effects can be avoided by thoroughly insulating the instrument and connecting one terminal to the case. If a potentiometer test circuit is properly set up, no change in the zero of the galvanometer should occur by manipulation of the selector switch or any switch in an external circuit connected to the potentiometer.

Voltage Standardisers. The normal duty of a potentiometer is to measure a voltage of unknown value. The instrument can also be used for a converse duty—that of indicating that a voltage is maintained constant at a desired value. For this latter duty the reading of the potentiometer would be set to this desired value of the voltage, and the actual value of this voltage would be adjusted so that the reading of the galvanometer was maintained at zero. As will be seen later, the testing of wattmeters is carried out with varying currents and with the voltage constant at the one or more values for which the instrument is rated, and for the maintenance of the voltage at the desired value instruments called voltage standardisers have recently been developed, which are virtually combinations of potentiometer and voltage divider.

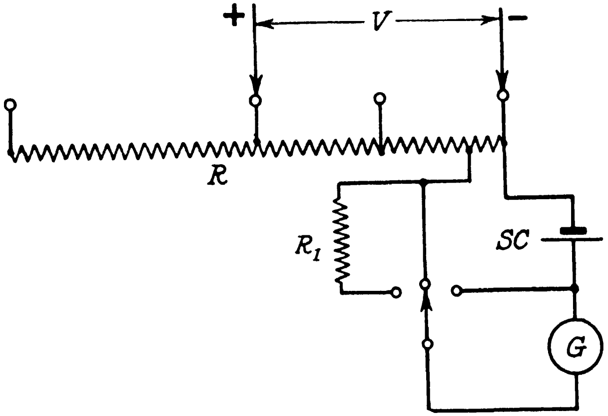


Fig. 36.—Constant-current voltage standardiser.

The constant-current type of voltage standardiser is illustrated schematically in Fig. 36. The resistance R is provided with tapping points corresponding to a number of standard voltages, and when the actual voltage of a supply connected to a section of R is exactly equal to the nominal value associated with this section, the fall of potential in the last section is equal to the e.m.f. of the standard cell, SC, connected thereto.

This equality is indicated by a null reading of the galvanometer, G. The standard cell and galvanometer circuit includes a 3-way selector switch, whereby contact with the divider resistance may be established either directly or through a resistance R_1. The third position of the selector switch short-circuits the galvanometer. The voltage applied to the dividing resistance is adjusted till the required null reading of the galvanometer is obtained. For most purposes sufficient accuracy is obtained in the adjustment if the resistance R_1 is in circuit and the galvanometer is used with low sensitivity. The value of R_1 can be so adjusted that its deflection indicates the fractional difference between the voltage applied to R and its nominal correct value. In the instrument, as shown in Fig. 36, the values of the components of R are adjusted to suit the e.m.f. of the cadmium standard cell at an average temperature. If desired, an adjustable tap for the standard cell circuit can be provided to correct for variations of the cell temperature.

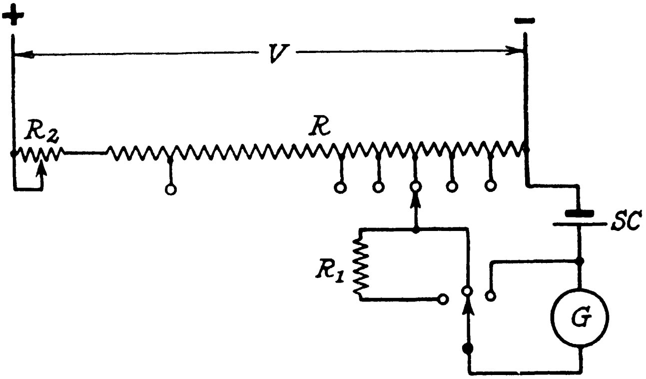


Fig. 37.—Constant-resistance voltage standardiser.

The constant-resistance type of voltage standardiser is shown in Fig. 37. The voltage to be standardised is applied to the whole of the dividing resistance and the connection of the standard cell is variable to suit the required standardised value of the applied voltage. Here the whole dividing resistance is shown with an adjustable section R_2, which is provided to compensate for variations of the standard cell e.m.f. caused by temperature changes.

Voltage standardisers used in Approved Testing Stations must conform to the Specification of the Electricity Commissioners. In the constant-current type the resistance per volt must not be less than 50 ohms per volt or more than 100 ohms per volt. In the constant-resistance type the current in the resistance taken from the source of standardised voltage must not be less than 1 milliampere or greater than 20 milliamperes. A standard cell additional to those specified for the D.C. potentiometer must be provided for the voltage standardiser, and if there is no adjusting device for compensation for variations of the temperature of this cell, its e.m.f. shall be assumed to be 1·01825 volts. The error in the ratio of voltage division must not exceed 0·02 per cent. on any range.

Substandard D.C. Potentiometers. D.C. potentiometer equipments which do not conform to the requirements for standard instruments may be used for a duty similar to that of a voltage standardiser, that is, for maintaining a testing voltage at a desired value, provided certain conditions are satisfied. The range of the instrument must be 0–1·5 volts, and the actual voltage applied to it must not be less than 0·75 volt. If the instrument is of the slide-wire type, one division of the wire must correspond to not more than 0·001 volt. Otherwise the lowest reading must not be more than 0·0005 volt. The maximum allowable error at a temperature of 20 degrees C. is 0·001 volt.

Standard Clocks. Ships' chronometers used as time standards in Approved Testing Stations must be provided with a seconds hand, the tip of which advances in steps of at least 0·5 millimetre. The same requirement applies to pendulum clocks unless the pendulum is visible. The error of a standard timing device must not exceed 30 seconds in 24 hours at any temperature between 10 degrees and 30 degrees C., and these standards must be checked periodically against the Post Office Speaking Clock or the Greenwich Time Signal. Synchronous motor clocks are not approved as time standards.

Substandard Ammeters and Voltmeters. The direct-current potentiometer is the first link in the chain of apparatus used to refer the performance of an electricity meter to the standard cell and standards of resistance. The only indicating instrument in a potentiometer equipment is a galvanometer, the office of which is merely to indicate that no current is flowing through it. The accuracy of measurements made by the direct-current potentiometer should, therefore, be of the same order as that of the resistance elements of which it is composed.

The next link in this chain of apparatus is formed by the group of indicating instruments which, being periodically checked for accuracy by means of the potentiometer, are used, either for the actual testing of supply meters or for the checking of integrating substandards. The simplest substandard instruments are direct-current ammeters and voltmeters and A.C. wattmeters. It may here be observed that the exact measurement of voltage in A.C. circuits is a matter of secondary importance only. Measurements of alternating current need only to be approximate.

Direct-current substandard ammeters and voltmeters are usually, but not necessarily, of the well-known moving-coil permanent-magnet type. Those used in Approved Testing Stations must have scales at least 5 inches long, with knife-edge pointers, and must satisfy the requirements of B.S.S. No. 89—1929 for substandard permanent-magnet moving-coil instruments. Additional requirements have been stipulated by the Electricity Commissioners. These relate to changes of indication caused by change of level, which may not exceed 0·15 per cent. for a variation of 1 degree ; stray field, which may not exceed 0·5 per cent. for a field of 1 C.G.S. unit ; self-heating, which may not exceed 0·2 per cent. for 30 minutes' application of full current or voltage ; change of polarity, which may not exceed 0·1 per cent. ; instability of indication, which may not exceed 0·1 per cent. All percentages are in terms of the full-scale reading.

Alternating-current substandard voltmeters must have scales at least 5 inches long. Used with direct current, they must not show a change of reading at the full-scale point, caused by reversal of polarity, which exceeds 0·5 per cent. The difference between the indications of a unidirectional voltage and an alternating voltage of the same value must not exceed 0·3 per cent. The former indication shall be taken as the mean of two values with opposite polarities.

When a dynamometer type voltmeter is used for the checking of a substandard wattmeter in the manner to be described hereafter the effect of reversal of polarity at the full-scale point must not exceed 0·1 per cent. and the scale length must be 12 inches.

Substandard indicating instruments must be tested by a standard D.C. potentiometer at intervals not exceeding six months.

Of the foregoing types of instruments, D.C. ammeters and voltmeters are tested for accuracy in conditions identical with those in which they are used to test D.C. meters. The A.C. voltmeter, on the other hand, is a transfer instrument; it is tested in a D.C. circuit and used to measure alternating voltages. Substandard A.C. voltmeters are usually of the dynamometer type, in which the torque deflecting the pointer depends upon the mechanical force on a pivoted coil placed in the field of a fixed coil. The two coils carry the same current, and, when the instrument is used in a D.C. circuit, the value of this current per volt of pressure applied to the instrument depends upon the total resistance only. The torque, for any given position of the moving coil, is therefore proportional to the square of the applied pressure, since the field of the moving coil is proportional to the current it carries. When the instrument is used in an A.C. circuit, then, with the coil in the same position, the average torque during a cycle per R.M.S. volt of applied pressure will be the same as before if the following conditions are satisfied: the current per volt in the instrument is the same for A.C. as for D.C., the field per ampere of instrument current is the same for A.C. as for D.C., and the field and moving coil current are in phase. These last two conditions can be satisfied almost exactly in instruments for commercial frequencies by avoiding using any metal to support the fixed coils. The first condition, that of equality of impedance at rated frequency to resistance, depends for its satisfaction upon the reactance being small in relation to the resistance. This condition can also be very approximately satisfied in dynamometer instruments for power voltages.

Substandard Wattmeters. The substandard wattmeter is the most important transfer instrument used in equipments for meter testing, as it is by this instrument that the accuracy of all A.C. meters is finally referred to the standards of resistance and e.m.f. Substandard wattmeters are of the dynamometer type, and when measuring the power in a circuit, the current of this circuit passes through fixed coils in the instrument which produce the magnetic field. A pivoted coil in series with a suitable resistance is connected to the circuit voltage. The value of the watts in a D.C. circuit is IV, in an A.C. circuit the average value of the watts is also IV, if I and V represent virtual or R.M.S. values and current and voltage are exactly in phase. If, therefore, a wattmeter is an accurate transfer instrument, then, for any given position of the moving coil, the steady torque per VA in a D.C. circuit must be exactly the same as the average torque per VA at unity power factor in an A.C. circuit. There is a further condition which must be satisfied by the instrument, which, although implicitly contained in the foregoing, should be explicitly stated. When the instrument is connected in an A.C. circuit the average torque will be proportional to the average of the instantaneous products of the magnetic field set up by the current coil and the current in the moving coil. This average torque, if the field Φ and moving-coil current I_1 are sinusoidal, will be proportional to $\Phi I_1 \cos \theta$, where θ is the angle of phase difference between Φ and I_1. For this quantity to represent the power $IV \cos \phi$ in the A.C. circuit containing the wattmeter, not only must Φ and I_1 have the same value per ampere and volt respectively in the A.C. as in a D.C. circuit, but the angle θ must be exactly equal to ϕ.

It therefore follows that errors of a dynamometer wattmeter as a transfer instrument, that is to say, the differences in readings given by the same values of alternating and constant power, may arise from four causes :—

(*a*) Difference of the moving-coil current per volt in the two conditions.

(*b*) Difference of the fixed coil magnetic flux per ampere in the two conditions.

(*c*) Phase difference of the alternating voltage and the moving-coil current it produces.

(*d*) Phase difference of the alternating flux and the fixed-coil current producing it.

The first of these causes, which is virtually the same as difference between impedance and resistance, is mainly owing to reactance in the moving-coil circuit. With high-voltage instruments self-capacitance between sections of the series resistance may cause a tendency for the current to be higher with A.C. than with D.C.

The second cause, that of a smaller flux per ampere with A.C. than with D.C., will itself be the result of eddy currents set up in masses of metal near the current coils, or in wattmeters for heavy currents in the metal of the coil itself. The two causes of transfer error will tend to make the wattmeter read lower with A.C. than with D.C. power.

The third and fourth causes of error are connected respectively with the first two. Reactance of the moving-coil circuit will cause the current in this circuit to lag in phase on the applied pressure, whilst self-capacitance in this circuit will cause a lead in phase of the current. Eddy currents set up by the instrument current will cause a lead in the phase of the alternating magnetic field.

The effect of these sources of transfer error is to cause a wattmeter which, when tested with D.C., is accurate, to give a reading with A.C. having a value $k\text{IV}\cos(\phi - \alpha)$, where k is a factor slightly different from unity representing the causes (*a*) and (*b*) above, and α is the difference between ϕ and the phase difference θ of the flux set up by the fixed coils and the current in the moving coils. This expression for the erroneous wattmeter indication can be transformed to

$$(1 - n)\text{IV}(\cos\phi\cos\alpha + \sin\phi\sin\alpha),$$

where $k = (1 - n)$. If α is so small that $\cos\alpha$ can be considered to be equal to 1, and $\sin\alpha$ is equal to the radian measure of α, then if n also is small, the indication corresponds to

$$\text{IV}\cos\phi - n\text{IV}\cos\phi + \alpha\text{IV}\sin\phi.$$

The transfer error expressed as a fraction of the true value of the power IV cos ϕ is therefore

$$-n + \alpha \tan \phi.$$

This fractional error therefore tends to increase indefinitely as cos ϕ tends to zero. When the circuit power factor is zero, and IV cos $\phi = 0$, sin $\phi = 1$, and the instrument indication will be αIV. Thus the quantity α can be determined if the instrument is connected in an A.C. circuit carrying known volt amperes at zero power factor; α in radians is the ratio of the instrument reading to the VA.

There is little difficulty in reducing the errors caused by reactance and self-capacitance in the voltage circuit of a dynamometer to negligible values if the instrument is rated for power voltages and frequencies, as a very large fraction of the impedance of the voltage circuit is made up of series resistance which can be constructed so as to be practically non-inductive. It is, however, more difficult to avoid the effects of eddy currents set up by the current if the instrument is rated for large currents. The most important effect of eddy currents is that of a phase lead of the alternating field of the instrument, whereby the angle θ is less than ϕ on lagging power factors. The recent improvement in the performance of current transformers has made it possible to fix standard current ratings of the order of 5 amperes and under for substandard dynamometer wattmeters. The power in A.C. circuits carrying currents differing greatly from the rated current of the wattmeter is measured by passing into the wattmeter the secondary current of a transformer bearing a definite and suitable ratio to and being almost exactly in phase with the current in the main circuit.

As regards construction, substandard dynamometer wattmeters may be of the pointer or torsion-head kind. In the pointer kind of instrument the moving coil is deflected till the control torque set up by the spring is equal to the electrodynamic torque caused by the power. It follows that the scale of a pointer dynamometer will only be uniform if the magnetic field threading the moving coil is independent of its position. In the torsion-head kind of wattmeter the moving coil is restored to a fixed reference position by turning the end of the spring by which the coil is suspended. When this adjustment is made the flux per ampere, through the moving coil, must always be the same, so that the electrodynamic torque, which is measured by the torsion of the spring which restores the coil to its reference position, is exactly proportional to the angle through which the spring has been turned. As this angle may be 360 degrees for the maximum rated power, the scale of a torsion-head instrument is not only inherently uniform, but can be made longer than that of a pointer instrument. On the other hand, the indication of a torsion-head wattmeter has to be found by adjustment, and not by mere inspection, so that in actual practice the pointer instrument is much easier to use.

Substandard wattmeters used in Approved Testing Stations must conform to the requirements of B.S.S. No. 89—1929 for substandard grade dynamometers, excepting that there is no limitation to the number or rating of the current ranges. The errors caused by changes of level, self-heating and instability, as defined for ammeters and voltmeters, are limited to 0·15, 0·2 and 0·1 per cent. respectively. When the instrument is carrying rated current at rated voltage the zero indication obtained by breaking the voltage circuit at either terminal must not differ from that obtained by breaking the current circuit by more than 0·1 per cent. of full-scale reading. The stray field error must not exceed 0·75 per cent. with an alternating field of 5 C.G.S. units in phase with the instrument current. The indications of the instrument with the same values of steady power and alternating power of rated frequency and unity power factor shall be identical within the accuracy of reading. The reading of the instrument with rated current and pressure at zero power factor lagging shall not exceed 0·3 per cent. of the VA.

It will be seen from a consideration of the last portion of this specification that the accuracy of a substandard wattmeter regarded as a transfer instrument in the reference of the performance of a supply meter to the ultimate standards of e.m.f. and resistance is deemed to be fixed by compliance with two requirements: identity of readings of the instrument with steady power and alternating power of the same value at unity power factor, and a reading at zero power factor which does not exceed a certain fraction of the VA. We have seen that this fractional reading at zero power factor can be identified with the radian measure α of a phase error which we have tacitly assumed to be a character of the instrument depending only on its physical properties and independent of the load it is measuring. The stipulated limit of reading at zero power factor is equivalent on this assumption to a phase error of 0·3 centiradian. This limit of phase error will give rise to a transfer error, the fractional value of which is $0{\cdot}3 \times \tan\phi$. Thus the transfer error at a power factor of 0·5 when $\tan\phi = 1{\cdot}73$ will be 0·52 per cent. of the instrument reading. The assumption that the phase error is a true characteristic of the instrument and independent of the magnitude and power factor of the load conditions seems to be almost self-evident from *a priori* considerations, but the rigid truth of this assumption is sometimes considered to be open to question.

The two characteristics of a substandard wattmeter indicative of its accuracy of transfer are not such as can be checked by the instruments available in an ordinary Testing Station. The verification of these characteristics evidently can only be obtained by the use of higher methods of testing, or of a special standard wattmeter, the transfer accuracy of which can be inferred with certainty from fundamental considerations. It may, however, be taken as reasonably certain that, once the transfer errors of a substandard wattmeter have been determined, these errors will remain constant.

Testing Substandard Wattmeters. The accuracy of transfer of a substandard wattmeter having been determined once for all, the actual accuracy of the instrument when used in A.C. circuits can be inferred from the results of testing with direct current. With such tests the nominal or true value of the reading of the instrument is the product of the value of the current in the one circuit and the voltage simultaneously applied to the other circuit. The wattmeter current and voltage need not be associated in a single circuit; the current coil can be supplied from one source and the voltage coil from another, as will be explained in greater detail in the next chapter. As the substandard wattmeter is, as far as A.C. measurements are concerned, the ultimate standard of reference, it is essential that in a D.C. test the components of the apparent power in the instrument are measured with the highest accuracy possible. The straightforward measurement of simultaneous current and voltage measurements to this degree of precision would require two standard potentiometers. In practice substandard wattmeters are used with voltages very near to the rated value, and are tested by applying this rated voltage to the moving-coil circuit and observing the reading of the instrument with several values of the current. This procedure simplifies considerably the equipment required for the accurate testing of a wattmeter, since it is merely necessary to hold the testing voltage at a definite value. This condition can be satisfied by three methods of testing, which are described and specified in Explanatory Notes supplementary to the 1936 Act.

The first of these methods is to use a calibrated D.C. voltmeter as an indicator of the required voltage. The voltmeter is first calibrated by means of a standard potentiometer by setting the potentiometer reading to correspond exactly to the rated voltage of the wattmeter to be tested, adjusting the voltage applied to the potentiometer equipment till a balance is obtained and observing the reading produced in the voltmeter by this voltage. The wattmeter can now be tested by using the calibrated voltmeter as an indicator. The voltage applied to the wattmeter and voltmeter is adjusted to and maintained so as to give the reading on the voltmeter corresponding to the nominal value, and, whilst this constant voltage is continuously maintained, currents of various values are passed through the current circuit of the wattmeter and measured by the standard potentiometer, the wattmeter readings being observed simultaneously with these current measurements.

This method of using a voltmeter as a calibrated indicator, which is specified by the Electricity Commissioners, may be considered to be open to criticism, in that, whilst the wattmeter is being tested, the reading of the voltmeter must be held by adjustment of the testing voltage at a value which does not necessarily correspond to a scale mark of the voltmeter. It would be more convenient during the wattmeter test to hold the testing voltage at such a value as would correspond to a scale mark, the true value of the voltage corresponding to this voltage being determined by the standard potentiometer.

The second method of wattmeter testing is to use a substandard potentiometer as an indicator of the voltage applied to the wattmeter. The reading of the substandard potentiometer which corresponds to the rated voltage of the wattmeter is first obtained by a test with a standard potentiometer. The standard instrument is set to the nominal value of the voltage, and the actual pressure applied to the two potentiometer equipments is adjusted till the standard is in balance. The reading of the substandard instrument corresponding is then observed. The substandard potentiometer equipment is then connected in parallel with the voltage circuit of the wattmeter during test, and, with the potentiometer reading corresponding to the required pressure, the pressure is continuously held at such a value that the substandard potentiometer galvanometer gives a null reading.

In the third method of testing substandard wattmeters a voltage standardiser is connected in parallel with the voltage coil of the instrument. With the voltage standardiser set to the required value of the testing voltage, the actual value of the voltage is continuously adjusted to maintain a null reading of the standardiser galvanometer.

Substandard Polyphase Wattmeters. These instruments comprise two dynamometer movements, with two voltage and two current circuits, and are intended to indicate the total power in a 3-phase circuit by single pointer and scale. Polyphase wattmeters are recognised as legitimate substandards in the Electricity Commissioners' Specifications supplementary to the 1936 Act, and they must conform to the requirements for ordinary single-element wattmeters as far as these requirements are applicable. No special directions are issued for testing polyphase wattmeters with direct current. The straightforward method of carrying out such tests would be to apply the rated pressure to both voltage circuits in parallel and to pass the same current through both current circuits in series. The indication of the instrument, if it is correct, should be twice the product of these current and voltage values.

Substandard Integrating Meters. These instruments, generally designated concisely as substandard meters, are used for the direct checking of the registration of supply meters. The substandard meters are tested by means of calibrated substandard indicating instruments, and the testing of a supply meter is carried out by comparing its registration with that of a substandard meter for the same energy or quantity. The registration of the supply and substandard meters may be observed either as dial readings or as angular travel of the rotating elements.

Substandard Electrolytic Meters. These substandard meters are used for testing supply meters of the electrolytic type. Substandard electrolytic meters in Approved Testing Stations must be of the mercury deposition kind. The maximum scale reading must be obtained in not less than 250 hours at rated current. The error at any scale point in the upper half must not exceed 0·5 per cent. for any current between a quarter and full value of the rated current. The scale must be readable within 0·3 per cent. of the maximum reading. The temperature coefficient of the meter must not exceed 0·1 per cent. per degree C.

Substandard Motor Meters. These meters are of two kinds. The first kind is generally similar to an ordinary supply meter, but provision may be made for the accurate observation of fractions of a kWh of consumption. This kind of substandard meter is often provided with two or more current ranges, and is used to test the accuracy of the actual dial registration of supply meters over considerable time periods.

The second kind of substandard motor meters is provided with a special dial which indicates revolutions and fractions of a revolution of the moving element of the meter. This kind of substandard is also often provided with a number of current ranges. The method of testing supply meters with this kind of substandard will be dealt with in detail hereafter, but briefly it consists in observing the total angular travel of the movement of the substandard, whilst the moving element of the supply meter under test executes an exact stipulated number of revolutions. This kind of substandard meter is usually of the A.C. energy type, and the rotation of the moving element can be started or stopped by means of a switch connected in the voltage circuit of the meter.

Substandard rotating meters used in Approved Testing Stations must generally conform to the requirements of B.S.S. No. 37—1930. The limits of error for substandard A.C. meters at unity power factor at 15 degrees C. are ± 0·5 per cent. from $1\frac{1}{4}$ to $\frac{1}{5}$ of rated current, ± 1 per cent. at $\frac{1}{10}$ rated current, and ± 1·5 per cent. at $\frac{1}{20}$ of rated current. At 0·5 power factor lagging the first two limits are doubled. A tolerance of 0·5 per cent. is allowed in one direction only, so that the range of error must not exceed that specified. These limits apply to all voltage ranges. A voltage variation of 10 per cent. must not change the error by more than 1 per cent. The temperature coefficient of error must not exceed 0·1 per cent. per degree C. at any power factor between unity and 0·5 lagging. The change of error between full and one-quarter load, unity power factor, shall not exceed 1·5 per cent. when the meter is placed in the centre of a coil 1 metre in diameter, giving a field corresponding to 100 ampere turns in phase with or 60 degrees leading on the meter current. The plane of the coil is to be perpendicular to the back of the case of the meter and to be vertical. The change in error caused by self-heating by the meter current with rated VA must not exceed 0·3 per cent. at unity power factor, or 0·5 per cent. at 0·5 power factor, nor shall the final error so caused exceed the specified limits. The change in error caused by self-heating by the voltage circuit current must not exceed 0·5 per cent. at any load between $1\frac{1}{4}$ and $\frac{1}{4}$ at unity power factor. A 5 per cent. variation of frequency must not change the error by more than 0·5 per cent. at unity power factor or 1·0 per cent. at 0·5 power factor lagging. The meter must not register with full rated current when the voltage circuit is open. No meter must show an instability of error exceeding 0·2 per cent. between $1\frac{1}{4}$ and $\frac{1}{4}$ of full load when loading conditions are maintained constant.

Substandard motor meters used in Approved Testing Stations must be tested periodically at intervals not exceeding one month by substandard indicating instruments. If two such substandard meters are always used in series, then, provided the corrected corresponding registrations do not exceed 0·25 per cent., the testing period may be extended to three months. The testing of substandard motor meters will be considered in detail hereafter.

Current Transformers. Current transformers used for extending the ranges of substandard wattmeters or meters in Approved Testing Stations are required to satisfy the requirements of B.S.S. No. 81—1936 for Class A.L. transformers. The limits of error for such transformers with a secondary burden of 7·5 VA, unity power factor, are ± 0·15 per cent. in ratio from $1\frac{1}{5}$ to $\frac{1}{10}$ of rated current and ± 3 minutes in phase from $1\frac{1}{5}$ to $\frac{3}{5}$ of rated current, ± 4 minutes from $\frac{3}{5}$ to $\frac{1}{5}$ of rated current, and ± 6 minutes from $\frac{1}{5}$ to $\frac{1}{10}$ of rated current. Current transformers whose errors do not exceed these limits with a burden of 3 VA may be approved.

There is no provision in the specifications supplementary to the 1936 Act for periodically checking the accuracy of current transformers used as auxiliaries to substandard instruments, and this lack of provision seems to imply that the accuracy of a current transformer, like that of a standard resistance, is a fixed quantity which will not change. It is doubtful, however, whether this implication is true. The errors of a current transformer may be sensibly changed if the secondary circuit is opened while the primary winding is carrying a considerable current. If the accidental open-circuiting is noticed, the accuracy of the transformer can be restored by demagnetising the core by an alternating current which is gradually reduced to zero. It is, however, quite possible for the open-circuiting to be accidental and unnoticed, and as the result of this the errors of a very accurate transformer might be considerably increased. It seems, therefore, that periodical checks of the accuracy of substandard current transformers are desirable.

To obtain a check of this kind it is not necessary to carry out a complete test on the transformer if at the time the actual errors are determined the secondary exciting current at a definite value of the secondary voltage is measured. This value may conveniently correspond to the condition when the primary is carrying rated current and the secondary is delivering rated burden. A brief study of Fig. 30 will make it clear that the exciting current in any condition, expressed as a percentage of the secondary current, is equal to the square root of the sum of the squares of the percentage ratio error and the phase error in centiradians. Thus, so long as the magnitude of the exciting current at a stipulated secondary voltage remains unchanged, the errors at this voltage—and, indeed, at any voltage representative of operating conditions—may with fair certainty be presumed not to have altered.

To consider this point in a more concrete way we see that the secondary voltage at rated current of a current transformer with a burden of 7½ VA is 1·5 volts. If the ratio error is 0·15 per cent. and the phase error 3 minutes = 0·09 centiradian, the exciting current at 1·5 volts will be $\sqrt{0{\cdot}15^2 + 0{\cdot}09^2}$ = 0·175 per cent. of 5 amperes, or 0·087 ampere. If this value is found by measurement when the transformer is originally tested, and is confirmed by subsequent measurement, this confirmation will be a very satisfactory check that the errors have not increased.

From the point of view of the technique of measurement, the exact determination of a small value of exciting current at a voltage of the order of 1·5 is difficult. Exactness of measurement is, however, not necessary; any approximate method will suffice provided that the test conditions are exactly specified and that these conditions are exactly repeated in subsequent measurements.

Substandard current transformers for meter testing are generally provided with some device whereby the interconnection of sections of the primary winding may be varied to furnish a number of ratios of transformation. This refinement is very necessary because, in the Approved Methods of Meter Testing issued by the Electricity Commissioners, it is stipulated that in an actual test the reading of an indicating instrument must not be less than two-fifths of the full scale value. By the use of a suitable multi-range current transformer single-phase power measurements in circuits carrying currents down to 2½ amperes can be made in accordance with this stipulation by a substandard wattmeter rated for 5 amperes. When the testing current is as low as 0·25 ampere a wattmeter with a 0·5 ampere range is normally required. The necessity for a second wattmeter can, however, be avoided by using a substandard current transformer of a kind now available giving a fractional ratio of 1/5. With this ratio the power in a circuit carrying 0·25 ampere at unity power factor can be measured by a wattmeter with a double current range of 5/2½ amperes. Multi-ratio substandard current transformers of this kind usually include a 5/5 ratio, so that over the whole range of testing currents the wattmeter is permanently connected to the secondary terminals of the transformer, changes in the effective range of the wattmeter being made by variation of the current transformer ratio.

Alternating-current Potentiometers. The dynamometer wattmeter is not the only instrument whereby alternating power may be measured in reference to ultimate standards of e.m.f. and resistance. It is possible to measure an alternating voltage and the active and reactive components of an alternating current with respect to this voltage by means of an A.C. potentiometer. The product of the values of the voltage and the active component of the current gives the alternating power if the voltage wave is sinusoidal. The transfer instrument used in an A.C. potentiometer equipment is a dynamometer ammeter, which is presumed to carry currents of the same value, either direct or alternating, when its movement deflects to a fixed reference mark. The A.C. potentiometer is first set with direct current by adjusting the current in it to the correct value to balance the standard cell. The dynamometer ammeter is set to deflect to the reference mark when this adjustment is made. Alternating voltages are measured in the usual way, with alternating current in the potentiometer of such a value as to deflect the ammeter exactly to the reference mark. The standard of phase quadrature is given by a mutual inductance which is used with infinite secondary impedance. The manipulative skill required in the use of the A.C. potentiometer is far higher than in the use of the A.C. instrument, and, although it is capable of yielding accurate results, the A.C. potentiometer is not a convenient instrument for alternating power measurements. Presumably for this reason, the A.C. potentiometer is not recognised by the Electricity Commissioners as a standard instrument for A.C. measurements in Approved Testing Stations, although such an instrument, if it complies with the Commissioners' Specification, may be approved as a standard direct-current potentiometer. A full account of the applications of the A.C. potentiometer will be found in the book on this subject by D. C. Gall, to which reference is made in the Appendix.

Time Substandards. The normal substandard instruments for the measurement of time intervals in meter testing are stopwatches, in which the movement of the time-indicating pointer is controllable by the observer. Substandard stopwatches for use in Approved Testing Stations must have dials readable to 0·05 second ; the smallest dial division may be 0·01 second, provided the length of a division is not less than 1 millimetre and a suitably thin pointer is provided. The error of time indication must not exceed 0·15 per cent. at any temperature from 10 to 30 degrees C., with the watch in a horizontal position and the main spring at least half wound. The combined error in starting and stopping the watch, caused by its own imperfections and independent of the personal error of the observer, must not exceed 0·05 second.

The foregoing specification implicitly requires that substandard stopwatches shall beat to 0·05 second. Stopwatches are available which will beat and record to 0·01 second. The overall advantage of using watches with a beat frequency exceeding 20 per second appears to be questionable, as the allowable errors may introduce an inherent accuracy of time measurement of 0·2 per cent., which is 0·12 second in a time duration of 1 minute. The total error in the measurement of a time interval includes the personal error of the observer, which cannot be reduced by increasing the precision of the reading of the stopwatch used. The fractional or percentage effect of personal errors can only be made negligible by adopting testing periods of sufficient length, which length has been proved by experience to be certainly more than 60 seconds.

As a stopwatch beating to $\frac{1}{20}$ second will nominally indicate a period of this duration to 1 part in 1,200, the occasion for a more complicated and delicate device with a greater beat frequency hardly arises in normal meter testing work.

The specification for substandard time-measuring instruments in Approved Testing Stations explicitly excludes any device depending upon a mains-driven synchronous motor, but methods of time measurement other than by stopwatches may be submitted for approval.

Substandard time-measuring devices in Approved Testing Stations must be checked against the time standard before each day's use.

CHAPTER V

TEST HOUSE EQUIPMENT

Basic Requirements for Meter Testing Equipment. The complete equipment of a testing station for supply meters comprises not only the standard and substandard instruments, whereby the performance of the meters may be referred to the ultimate standards of e.m.f., resistance and time, but also adequate and suitable sources of energy which can easily and conveniently be regulated as regards current and pressure over the requisite ranges. In testing supply meters of a rated current up to 10 amperes the meter current is usually supplied directly from the same source as the meter voltage. For rated currents exceeding 10 amperes the two circuits of an energy meter are generally supplied from separate sources, the source of the meter current supply being one of low voltage. In this condition the value of the power to which the meter responds is less than the actual rate of dissipation of energy in the testing load. The meter power is generally known as a phantom load.

A meter testing equipment must include a source, or sources, of testing current, alternating or direct, or both, which current can be varied in magnitude continuously from a value corresponding to the starting current of the lowest rated meter to be tested to a value corresponding to the full load current of meters of the highest rating. The requirements for the source or sources of testing voltage are not so stringent, since supply meters are tested for accuracy at their rated voltages. Supplies of testing voltage therefore need only correspond in value to the rated voltages of the energy meters which have to be tested, but provision must be made for varying these voltages by 10 per cent. on each side of each nominal value.

When large power consumers are supplied from a high-voltage network their supplies may be measured either on the primary or secondary sides of the transformers in the consumers' substation. If the supply is measured on the high-voltage side, or if the consumer uses motors supplied at high voltage, the meters will be used with voltage transformers, and these transformers, like current transformers and shunts, must be considered as essential components of the meters with which they are associated. When consumers' supplies are measured at high voltages the meter testing equipment should therefore include some provision whereby a meter rated for the high voltage may be tested as a complete unit with its voltage and current transformers. In actual meter testing practice high-voltage meters are often tested with their associated current transformers only, that is to say, as low-voltage instruments, and the results of the test in this condition are corrected by compounding with the meter errors so determined the constant errors of the voltage transformer, which transformer errors may be either determined by an actual test, or inferred from the result of a type test.

In testing A.C. supply meters there is a requirement of the testing equipment to be satisfied other than those of variation and current. All A.C. meters must be tested at a power factor of 0·5, and it is very desirable that arrangements are made whereby the behaviour of an A.C. meter may be observed with a phantom load having zero power factor. An A.C. meter can be tested with a real load of 0·5 power factor by using a reactor as well as a resistance to limit the load current. This method is, however, very inconvenient, and is now entirely superseded by the phantom load method of A.C. testing, in which the voltage applied to the meter is derived from a source of supply so arranged that the phase of the testing voltage can be regulated entirely independently of the testing current. Any appliance whereby the phase of a testing voltage may be varied without alteration of its value is called a phase-shifting device, and some such device which provides either for continuous phase variation or for phase shifts of definite steps is an essential component of a testing equipment for dealing with A.C. meters.

The Electricity Commissioners have laid down no definite specification for the apparatus which provides and controls the supplies in Approved Testing Stations, but have merely indicated that it is expected that this apparatus will be such as to give suitable regulation for the methods of testing adopted. One important stipulation is made in regard to the equipment for testing A.C. meters, and this stipulation is that the circuit control apparatus shall be such that the peak factor of the voltage waveform of the main supply is not to be varied by more than 3 per cent.

The peak factor of an alternating quantity is defined as the ratio of its maximum instantaneous value to its R.M.S. value. The foregoing stipulation means, therefore, that with constant R.M.S. value of the voltage of the main supply the peak value of this voltage must not be varied by more than 3 per cent. by any operation of the control apparatus. It is to be noted that the stipulation applies explicitly to the main supply voltage and not to the voltage actually applied to the meters, which may either be different in value from the main supply voltage or isolated from it by regulating or phase-shifting transformers.

We shall return later to this matter of voltage wave form and to means of verifying that wave distortion is avoided.

As we have already seen, and as will be dealt with in detail in the next chapter, there are two methods of testing motor meters, the one which consists inherently in measuring the angular speed of the moving elements for various constant values of the meter power, and the other which consists in comparing the registrations of the meter under test and a substandard meter measuring the same energy or quantity. For the first method of testing it is highly desirable that the pressures of the sources of testing current and voltage are both invariable once the required conditions of testing have been set by regulation. If these supply voltages are unsteady, exact constancy of the meter power during a speed test can only be obtained by hand regulation of the meter current, so that an instrument pointer is maintained in a fixed position relative to a scale. The necessity for this hand regulation will introduce an additional source of error in the method of testing. When meters are tested by the second method, that of comparison of registrations, exact constancy of the meter power is not important.

For this second method of meter testing, the testing current and voltage can be taken from the main supply. For the testing of meters by speed measurements and for the checking and calibration of substandards, it is highly desirable that a separate source of alternating current is available which will furnish a constant supply voltage. For direct-current meter testing by speed measurements, and for the calibration of substandard instruments by the standard D.C. potentiometer, separate sources of direct current at practically steady voltages are almost essential. Steady voltage direct-current supplies are in practice almost always obtained from batteries of accumulators.

Current Supply for D.C. Testing. The rated capacity of a battery used to supply current for D.C. testing will depend upon the full load current of the instrument of the highest rating which is likely to be handled. In D.C. testing stations it is the maximum meter rating which will fix the advisable capacity of the testing battery. In A.C. meter testing stations batteries will be required for the testing of substandard wattmeters and voltmeters by the standard potentiometer, and in this case the battery capacity for the current supply must be sufficient for the maximum wattmeter current rating. When D.C. meters of various ratings have to be tested in quantity it is advisable to provide two sources of battery supply, one for the highest current which will be required, and the other for testing currents up to some such value as 20 amperes. The voltage of batteries for direct-current supply must be such that the greater part of this voltage is absorbed in loading resistance of a negligible temperature coefficient, otherwise the unavoidable heating of copper connecting conductors will, with heavy currents, give rise to alteration of the total resistance in the test circuit, and hence to the value of the testing current. A convenient battery voltage is of the order of 20, and any economy obtained by providing less than 10 cells is ill-advised. It was formerly the custom to obtain an increased battery capacity for the highest testing currents by providing some arrangement whereby the two halves of the battery were connected in parallel. This arrangement is also ill-advised. In the first place, two sections of a battery will almost certainly not share an external output exactly equally, so that with the parallel connection and high output, one-half of the battery is discharged more than the other. Secondly, the parallel arrangement reduces the available voltage in the test circuit in a condition when, to avoid current instability caused by the heating of copper conductors, it should be the maximum possible. Nothing is more conducive to high accuracy of measurements with the standard D.C. potentiometer than a reliably steady source of testing current, and for steadiness of the output of a battery into a testing circuit an adequate voltage is very desirable.

A battery equipment for the supply of direct current must include suitable charging plant. The duty of a battery at high output can be relieved by supplying part of this output by the charging plant, but it is essential that there is a net discharge from the battery, otherwise its voltage will rise and the value of the testing current will be unsteady. Small variations of the charging current will only affect the testing current slightly, and this method of obtaining test currents of the order of the maximum nominal output of the battery, or even exceeding this value, is quite suitable for direct-current testing by substandard instruments. The battery should preferably be used alone for measurements with the standard potentiometer.

Particular care should be taken in the charging and maintenance of testing batteries, and this work should be placed under the responsible supervision of one member of the staff of the test department. Every testing battery should be provided with ampere-hour meters in the charge and discharge circuits, and these meters should be read daily and the readings recorded in a permanent log. Charging should be carried out according to a definite routine. Generally it may be said that the maintenance of a battery for supplying testing current should be as careful and conscientious as that of a battery for power supplies or for the control of large switchgear equipments.

If the duty of a battery for the supply of low currents is practically continuous during working hours, it is very advisable that a duplicate battery be provided for this purpose, so that the two batteries can be discharged and charged alternately under proper supervision.

A possible alternative to a high capacity battery for the supply of direct currents is a low-voltage generator connected to the test circuit in series with a ballast resistance, and separately excited from a constant voltage source. Variation of the testing current is obtained by field regulation, for which a potentiometer-connected rheostat is most desirable. This method of supply is, of course, very inferior to a battery of ample capacity and voltage, and it is never worth while providing a D.C. generator specially for the purpose. Occasionally it happens that a disused motor generator set is available and capable of easy conversion for giving a variable supply of testing current in the manner indicated. If care is taken that the brushes and commutator of the machine are maintained in excellent condition a fairly steady supply of heavy direct current can be obtained if the generator is driven by an A.C. motor from a source of steady frequency.

Voltage Supply for Direct-current Testing. A source of steady voltage must be provided for the voltage circuits of D.C. energy meters and substandard wattmeters, and also for substandard A.C. voltmeters. The minimum voltage of the battery for this supply must be at least 10 per cent. in excess of the maximum voltage rating of any instrument which will have to be tested. The actual capacity of this battery will generally only need to be small, as direct-current energy meters rarely have to be tested in quantity. If an A.C. supply is available the voltage supply battery can be maintained in a condition ready for use by trickle charging from a metal rectifier equipment.

A point of particular importance to be observed in the installation of a battery for voltage supplies is that of insulation. Double, or even triple, insulation is highly desirable, and the final insulators of the framework supporting the battery should be of ample size, quite plain in contour, so that they can easily be kept scrupulously clean. Defective insulation of a high-voltage battery is one of the most fruitful sources of vexation and error in measurements by a standard D.C. potentiometer.

Current and Voltage Control in Direct-current Testing. When small D.C. quantity meters are tested by methods which do not require exact constancy of the testing current, the supply for the testing can be taken from the direct-current mains. In this case there will be little need for exact current adjustment, and the values of the current required for testing can be obtained by fixed loading resistance units of suitable values. The switches controlling the loading circuits can be marked with the corresponding nominal current values, and these values can be checked by an ammeter included with the meters in the test circuit.

The output from a battery giving a supply for direct-current testing should be variable over its whole range, and provision must be made for the fine adjustment of this output current to any desired value. There are many ways in which these objects may be attained. One method, illustrated by the diagram of connections (Fig. 38), is by the use of two or three rheostats in conjunction with a fixed ballast resistance, each giving ten steps of variation of the output current, the whole range of the second rheostat corresponding to one step of the first. The diagram shows an arrangement for a battery with a maximum current output of about 200 amperes. It is seen that, by regulation of the rheostats, the current can be set at any desired value in the range 0–200 amperes to within at the most 2 amperes. Finer regulation can be obtained by a rheostat giving continuous resistance variation connected as shown in Fig. 39. The rheostats for the coarse control of the current may be of the controller type.

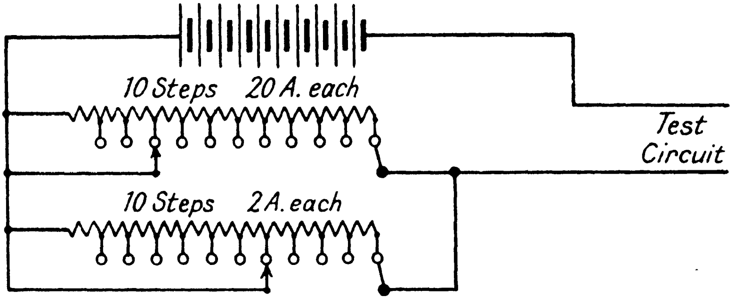


FIG. 38.—Control of D.C. test supply.

An alternative and simpler arrangement for the control of considerable direct currents is shown in the diagram of connections (Fig. 39). Here a limited number of loading resistance units give such values of the testing current that any total load can be built up by suitably combining them. With 8 units passing currents of 1, 2, 2, 5, 20, 20, 50 and 100 amperes any current can be set in the range 0–200 amperes to within 1 ampere. The loading resistance units can be controlled by knife switches marked with the value of the current corresponding. As these switches will be in frequent use, they should be of robust character, and the economy by reducing the current rating of those switches controlling any but the largest step of current is not worth while. Fine regulation of the testing current is obtained by an additional circuit containing a ballast resistance and a continuously variable slider resistance located on the actual test bench, the ballast resistance being designed to pass a current, with the rheostat short-circuited, which is slightly in excess of the smallest current of the main loading units.

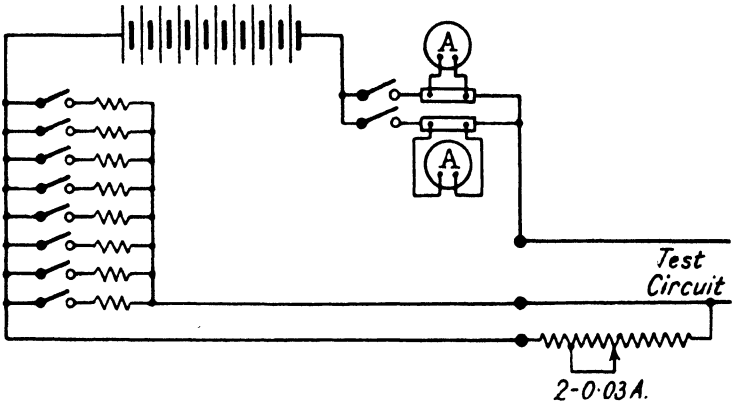


Fig. 39.—Control of D.C. test supply.

Either of the foregoing arrangements will be suitable for a battery with a low current output of the order of 20 amperes, but as such a battery will be likely to be in more frequent use than one for heavy currents it is perhaps preferable to provide current regulating gear of the rheostatic type shown in Fig. 38.

A method of fine current control, alternative to the use of a continuously variable loading resistance, is shown in Fig. 40. Here an auxiliary battery of four or five cells is used, and the current from this battery is superposed upon the main testing current in the meters and instruments in the test circuit. The maximum value of this auxiliary current is limited by the ballast resistance R_2, and its actual value can be varied from this value continuously down to zero by means of the potentiometer rheostat R_1. The double-pole double-throw switch enables the direction of the auxiliary current to be reversed at will, so that this current is added to or subtracted from the main current in the instruments in which it flows. The total range of current variation by this method is thus twice the maximum current delivered from the auxiliary battery.

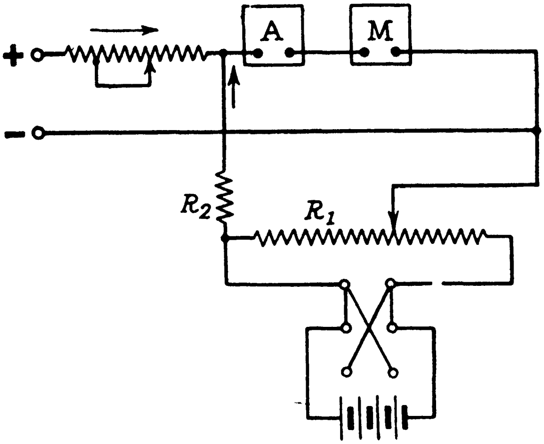


FIG. 40.—Fine control of D.C. test supply.

Control of the voltages obtained from a high-voltage battery is generally only necessary in the neighbourhood of a few fixed values. The arrangement shown in the diagram of connections (Fig. 41) is one suitable for most purposes. The position of the tapping point of the positive feed to the test circuit can be varied by means of plug sockets, any one of which may be made to close the circuit by the insertion therein of a short-circuited plug. Taps should be provided for the nominal testing voltages required, and also for voltages 10 per cent. in excess of these values. Fine adjustment of the testing voltage can be obtained in steps of 2 volts by means of a cell-regulating switch at the negative end of the battery. Finer regulation than this is best obtained by a potentiometer rheostat located on the test bench, and an auxiliary battery which can be charged independently of the main high-voltage battery. It is not advisable to connect a potentiometer rheostat to a section of the main battery, as the current taken by the rheostat will cause the cells in this section to be discharged more than those in the remainder of the battery.

Supplies for A.C. Testing. Power for testing small single-phase A.C. meters in quantity by methods which do not require absolutely steady loads can conveniently be obtained from the supply mains by means of permanently graduated loading resistances which give the nominal values of the testing power required. Testing voltages 10 per cent. in excess of the rated voltages can be obtained by suitable auto-transformers, as will be explained in greater detail hereafter.

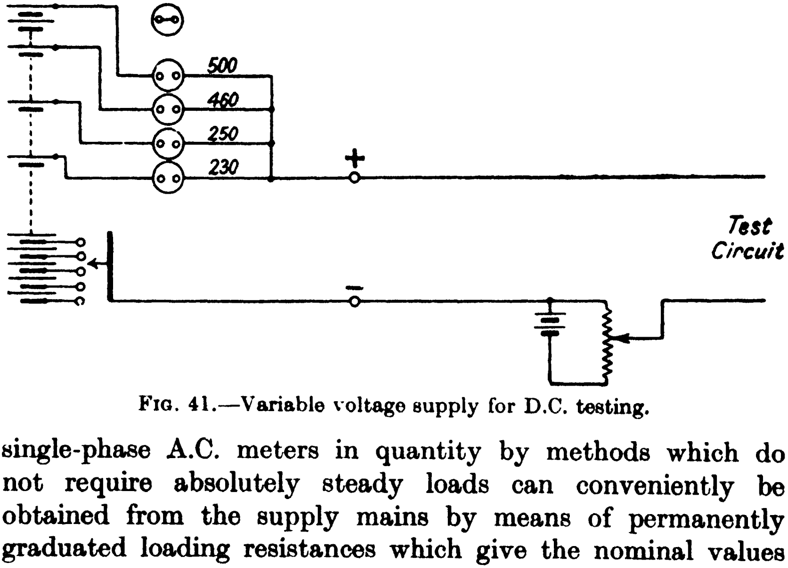


FIG. 41.—Variable voltage supply for D.C. testing.

Power for the testing of A.C. meters by speed measurements in which steady loads are required, or for the checking of substandard A.C. instruments, is best obtained from a source of steadier voltage than is given by supply mains. The ideal source of supply for A.C. testing is a motor generator set driven from a battery supply. This arrangement is costly, as the capacity of the battery must be sufficient to supply the motor generator over long testing periods, and the voltage of the battery should not be below 100. The duty of a battery supplying a motor generator can be reduced by supplying part of the input to the machine from the charging source, sufficient discharge being taken from the battery to maintain a steady voltage. The speed of a D.C. motor will, unless it is differentially compounded, vary with the load on the A.C. generator it drives. To maintain constant frequency of the A.C. output fine speed control must be provided for the D.C. motor. On the other hand, the frequency of an A.C. generator driven by a D.C. motor is under control, so supplies of a frequency above or below the standard value can be obtained.

An alternative to a battery-driven motor generator is one comprising a synchronous motor driven from the A.C. supply mains. The speed of a synchronous motor is not affected by variations of the voltage supplying it, and is only responsive to frequency changes. Commercial frequencies are generally so steady that the speed of the motor, and hence the voltage and frequency of an A.C. generator which it drives, will be practically constant. A testing generator driven by a motor with a constant speed characteristic will give a supply at one frequency only. Frequency variations of the testing supply with an A.C. motor drive can only be obtained by using a variable speed motor.

The A.C. generator for testing supplies will be of the 3-phase type, and it must be so designed as to give a nearly sinusoidal voltage wave over its whole range of load variation in accordance with the requirements of B.S.S. No. 225. The rated voltage of the generator may be the standard 3-phase distributing voltage, *i.e.*, 400/230, or it may be low, of the order of 20, so that real testing loads can be obtained with small power wastage. Current and voltage regulation in A.C. phantom testing loads is, however, so easy that this last advantage is of small moment in comparison with the advantage of being able to change over a testing circuit from the supply mains directly to the motor generator supply circuit at will.

Motor generator sets are now available for meter testing, which, with supply pressure, voltage variations of plus or minus 5 per cent., will deliver a testing pressure which is balanced and constant in value within the limits ± 0·05 per cent. These sets comprise a synchronous motor, a synchronous generator, a D.C. generator for exciting the field circuits of the two A.C. machines, together with the automatic voltage regulating gear. The speed of regulation is only 0·3 second, and this is obtained by the use of thermionic valve amplifiers in the voltage control circuit.

At one time it was not unusual to derive A.C. testing supplies from a double generator set, of which one generator was rated for low voltages to give the current supply for phantom loads, and the other generator was designed to give the voltage supply for all the testing pressures required by field regulation. The second generator was, moreover, provided with a movable stator for phase adjustment. It is now customary to derive A.C. testing supplies from a single generator and to obtain voltages of varying values and phases relative to associated testing currents by auxiliary transformers and voltage regulators, as will be explained hereafter. Phase shifting of a testing voltage by variation of the physical position of an alternator stator, although simple in theory, is inconvenient in practice unless some expensive device is provided for the remote control of the stator position from the test bench, and for the remote indication of this position to the operator.

Current Regulation for A.C. Testing. Heavy currents for A.C. testing are most conveniently obtained and regulated by the use of step-down transformers. Thus from a 230-volt supply a current of 200 amperes for a phantom testing load can be obtained from the secondary of a transformer with a voltage ratio of 20/1, with a current of only 10 amperes from the supply. This current can be obtained and its value may be adjusted in one of two ways. The primary winding may be directly connected to the supply mains, and the testing circuit with suitable limiting resistance connected to the transformer secondary. The current in the test circuit can be controlled in the usual way by variation of the amount of resistance in series with it. Otherwise the test circuit, comprising the current coils or current transformer primaries of the instruments under test, can be connected directly to the secondary winding of the step-down transformer, and the current in this test circuit may be regulated by a variable resistance connected between the transformer secondary and the supply. In the first condition, the arrangement is one of constant voltage step-down transformation ; the secondary voltage of the step-down transformer, and hence its secondary current, will, over the whole range of current variation, have a sinusoidal waveform, provided the supply voltage is sinusoidal. In the second condition, the transformer secondary is closed through a low impedance, and the current in its primary winding is determined to a large extent by the limiting resistance between this winding and the supply. The transformer can, therefore, be considered to be operating as a step-up current transformer.

Conceive that, in this condition, the transformer is delivering its rated secondary current through a low impedance, and the input current is limited to the correct value by a resistance in the primary circuit. The primary current waveform will correspond very approximately to that of the voltage, and a sinusoidal voltage will give a sinusoidal current. The waveform of the secondary current will not, however, correspond exactly to that of the primary, because the primary current must be considered to be composed, not only of the counterpart of the secondary current, but also of the current required to excite the transformer core. This latter component will be relatively small, so that the component which is transformed will be nearly sinusoidal, but as the exciting component waveform is distorted, the component which is transformed—and hence the secondary current—will not have a waveform corresponding exactly to that of the primary current.

The difference in the waveforms will, however, be practically inappreciable so long as the exciting component of the primary current is but a small fraction of its total value, as will be the case provided the transformer primary is rated for the supply voltage. Thus the secondary output of a transformer satisfying this condition can be regulated by a variable resistance in the primary circuit, and the waveform of the secondary current will correspond closely to that of the supply voltage. This, however, would not necessarily apply to a transformer rated for a voltage considerably less than the supply voltage, as in certain conditions of adjustment the actual voltage drop on the primary terminals might exceed this rated value. The exciting component of the total primary current would then, because of magnetic saturation, become relatively large, and the waveform of the secondary current would tend to become distorted. To avoid the possibility of wave distortion of heavy testing currents obtained by means of step-down transformers, these transformers should be rated for the voltage of the supply source.

Fig. 42 shows a typical circuit for the regulation of a current from 200 amperes downwards. The primary current is regulated by means of graded resistances giving nominal values of 5, 2 and 2 amperes, and these currents are increased twentyfold by the transformer having a voltage ratio of 20/1 and a current ratio of 1/20. Adjustments of the primary current in intervals less than 2 amperes is obtained by a continuously variable resistance in the usual way. When the value of the testing current required is below 10 amperes, this current is taken direct from the supply by the operation of the double-throw switch. In this condition the primary winding of the transformer is short-circuited, so that very little impedance is offered to the passage of the small value of the testing current through the secondary winding.

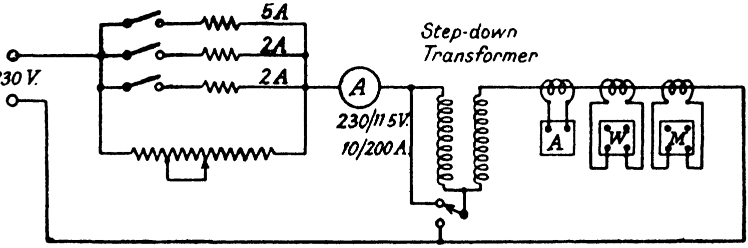


FIG. 42.—Control of A.C. test supply.

It should be noted that in a circuit containing a limiting resistance and a step-up current transformer, the total impedance consists not only of the resistance, but of the equivalent impedance of the transformer. This impedance, with a low external secondary impedance, will be partly reactive, so that, notwithstanding the limiting resistance is non-inductive, the current drawn from the supply, and hence the secondary output current in the test circuit, will lag in phase on the supply voltage.

Balanced 3-phase Testing Currents. The methods of meter testing approved by the Electricity Commissioners stipulate that, normally, polyphase meters are to be tested as such on a circuit of the type for which the meters were designed. This stipulation standardises the hitherto usual practice of testing 3-phase meters with balanced 3-phase current and voltage supplies. The currents for testing such meters must therefore be variable in magnitude, whilst being kept in balance both as regards magnitude and phase.

The straightforward method of obtaining a small real balanced 3-phase 4-wire test load is shown by the diagram of connections (Fig. 43). This diagram is almost self-explanatory. Each line current of the load is derived from a phase-to-neutral voltage, so that when the three currents are shown to be equal by equal ammeter readings, and these currents flow in non-inductive resistances, they will constitute a balanced load. For 3-wire 2-element meter testing the neutral connection should be open.

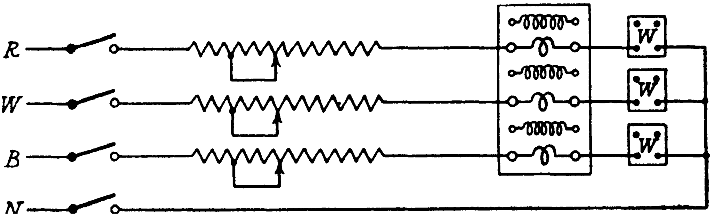


FIG. 43.—Three-phase 4-wire test supply.

Heavy currents for a phantom 3-phase test load at low voltage can evidently be regulated by means of three such circuits as are shown in Fig. 42. The feeds to the limiting resistances of these circuits will be taken to the line terminals, and the three return feeds from the double-throw switches will all be joined together and connected to the neutral for a 4-wire load.

An alternative method of obtaining a real balanced 3-phase 3-wire test load of low currents is shown in Fig. 44. Here two variable resistances and two ammeters only are required. One resistance, R_2, is connected between white line and the centre point tap of an auto-transformer joining red and blue lines. The second resistance is connected in parallel with the auto-transformer. To set a balanced load of required magnitude R_2 is first adjusted till the ammeter A_2, in series with it, gives the correct indication. R_1 is then adjusted till the reading of A_1 is equal to that of A_2. Adjustment of R_1 does not affect the current in the white line. Further, because of the symmetry of the connection of the auto-transformer, the currents in red and blue lines are necessarily equal. A possible disadvantage of this method is that the two current circuits of the meter under test are, unlike those in the arrangement of Fig. 43, at different potentials. This condition, it may be noted, however, corresponds to that of actual service.

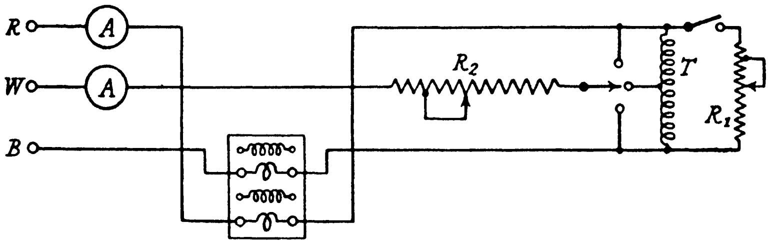


FIG. 44.—Three-phase 3-wire test circuit.

The selector switch in the connection between R_2 and the auto-transformer can be used for testing a 3-phase meter, in accordance with the official requirements, with current in each of the elements alone. By connecting the resistance to red or blue lines and by disconnecting R_1, meter current will only flow in the element corresponding.

A modification of the foregoing method of obtaining a 3-phase 3-wire balanced load for meter testing is shown in Fig. 45. Here a transformer is used with two separate winding sections, instead of a mid-point tap, and the meter current coils are included in the connection which joins these two sections together. The two current circuits of the meter, and those of the substandard instruments used for the testing of the same, are now at the same potential. For heavy current loads two step-down transformer primary windings are connected in the branches of the circuit which are shown in Fig. 45 to be containing the meter coils.

The advantages of the methods of loading shown in Figs. 44 and 45 are, of course, confined to the testing of 3-wire meters. The straightforward arrangement of Fig. 43 is the best for 4-wire meters, as with the neutral connection made, the current in each line can be adjusted independently of the others, and any slight unbalance which gives rise to a residual current does not affect the accuracy of the testing provided that three single-phase substandard wattmeters are used. When, however, 3-wire meters are tested or a 3-wire set of substandard wattmeter elements is used to test 4-wire meters, it is vital to the accuracy of the testing that there is no residual current in the load. This condition is easily satisfied with the circuit of Fig. 43 by breaking the neutral connection after the balanced load has been set. The arrangements of Figs. 44 and 45 enable a balanced 3-wire load to be set by two current adjustments only.

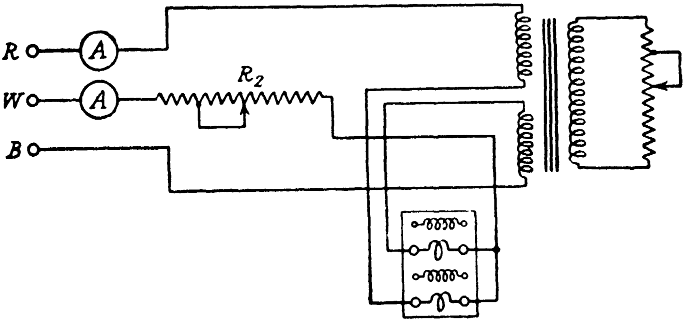


FIG. 45.—Three-phase 3-wire test circuit.

Voltage Supply for A.C. Testing. Voltage supplies of any required nominal value for A.C. meter testing can be obtained very readily from a constant pressure source by means of a variable ratio transformer. A 3-phase voltage supply should be derived from a double-wound transformer, which is delta-connected on the primary side and star-connected on the secondary side. This will ensure that the phase voltages used in testing 2-element 4-wire meters are free from residue, and that these phase voltages correspond in waveform to the line voltages of the supply. The phase-to-neutral voltages obtained in this way will, of course, be displaced 30 degrees in phase from those of the actual supply. As will be explained shortly, this is a matter of no importance in 3-phase meter testing, as the voltage supply for the meters is generally derived from a delta-star-connected phase-shifting transformer, whereby the phase of the 3-phase testing voltage is under complete control. The application of the variable-ratio transformer method of obtaining various values of a testing voltage will be illustrated later in diagrams showing typical meter testing circuits.

Artificial Power Factors for A.C. Testing. As all A.C. meters must be tested at a power factor of 0·5 lagging, an essential component of a meter testing equipment is some means of obtaining this condition of loading. We have already referred to the inconvenience of obtaining real loads of lagging power factor for meter testing by using inductive impedances for current limitation, and to the superiority of the use of phantom loads of artificial lagging power factor, of which the phase of the voltage component is under control. This kind of power factor control can be readily obtained when, as is usual, the testing supply is 3-phase. There is, however, one interesting method whereby the phase of a testing current may be varied over the whole range of 90 degrees, which is useful in circumstances where a 3-phase supply is not available and ordinary methods of power factor control cannot be used. This method, which was developed by the Metropolitan-

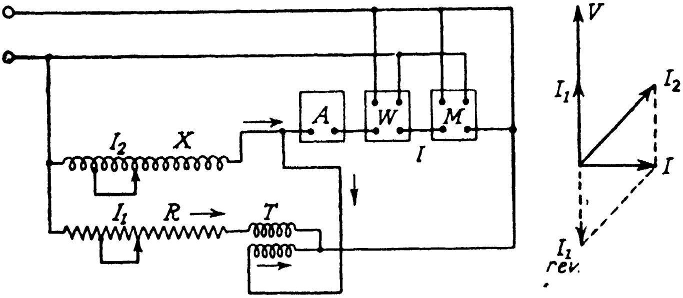


Fig. 46.—Artificial power factors from single-phase supply.

Vickers Electrical Co. Ltd., is illustrated in Fig. 46, which shows a single-phase testing circuit comprising an ammeter, a substandard wattmeter and an energy meter. X is an adjustable inductive impedance and R is an ordinary adjustable non-inductive resistance. The instrument current coils carry the current I_2 of X, and the reversed current I_1 of R, the reversal of I_1 being obtained by a double-wound transformer of unity ratio. The vector of the resultant current I in the instrument is therefore the difference of the I_1 and I_2 vectors, and it will readily be seen from the vector diagram that, by adjustment of I_1 and I_2, the phase of I can be made to lag by any angle between 90 degrees and that of I_2. If the connections of the instruments to the secondary of the transformer are reversed, then the vector of I will be the sum of the vectors of I_1 and I_2, so that, by adjustment of the magnitude of these component currents, the phase of I can be adjusted to any value between that of I_2 and that of I_1. By this means, therefore, artificial single-phase loads of any magnitude and power factor can be obtained from a single-phase supply. The method is not, however, very convenient in use because adjustment of either I_1 or I_2 affects both the magnitude and phase of I, but the method is useful in circumstances when the other and more

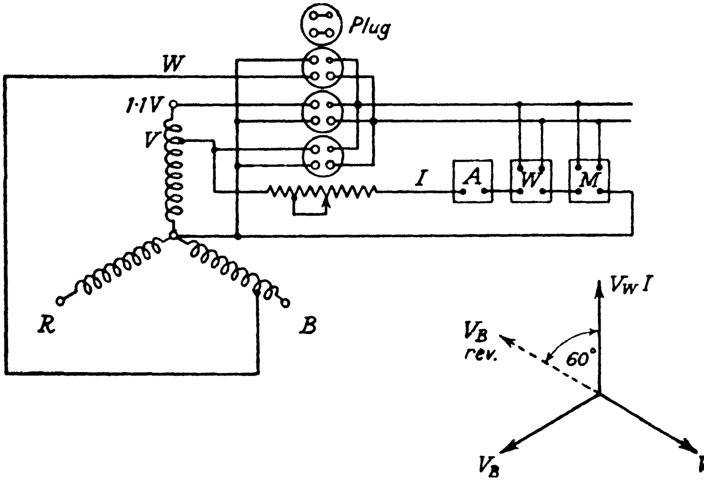


FIG. 47.—Artificial power factors from 3-phase supply.

convenient methods of obtaining power factor control cannot be used.

A 3-phase 4-wire supply gives voltages of six different phases, so that, by exciting the voltage coil of a meter by a voltage different from that from which the current is derived, artificial power factors can readily be obtained. Fig. 47 shows a simple single-phase testing circuit containing an ammeter, a substandard wattmeter and an energy meter, with an arrangement whereby the voltage circuits of the meter and wattmeter may be excited by normal voltage in phase with the testing current, so that the load is of unity power factor, a voltage 10 per cent. in excess of normal also in phase with the testing current, and a voltage of normal value leading 60 degrees on the phase of the testing current and giving an artificial power factor of 0·5 lagging. The selection of the testing voltage corresponding to the required condition is obtained by the use of three 4-pin sockets and a plug with pins of non-uniform size to ensure correct register. Two pairs of pins are internally joined together to establish the required connection. With the plug in the lowest socket, the voltage circuits of the instruments are excited by the same voltage as supplies the current. Placing the plug in the middle socket increases this voltage by 10 per cent. With the plug in the uppermost socket the instruments have their voltage circuits excited by a voltage lagging 120 degrees on that corresponding to the unity power factor condition, but reversed in polarity. This reversal gives a voltage on the instruments which leads 60 degrees in phase on that corresponding to the unity power factor condition. The artificial power factor in the phantom load is thus 0·5 lagging. This method of obtaining an artificial power factor is sometimes called cross-phasing. As the methods of testing approved by the Electricity Commissioners allow a tolerance of 10 per cent. plus or minus in the power factor of a nominal value of 0·5, which tolerance is equivalent to a phase tolerance of plus or minus 3 degrees, the cross-phasing method of obtaining artificial power factors is sufficiently accurate with a commercially balanced voltage supply and a non-inductive test load.

When heavy-current phantom test loads are obtained by means of a step-down transformer the phase of the testing current relative to the supply voltage will, as has already been pointed out, be governed not only by the current-limiting resistance in circuit with the transformer primary, but also by the equivalent impedance of this transformer. With a non-inductive limiting resistance, the lag of the secondary testing current on the supply pressure may exceed 3 degrees at full load, so that if an artificial lagging power factor is obtained by the cross-phasing method, this will be lower than 0·5 by an amount exceeding the allowed tolerance. Fig. 48 shows how the voltage applied to the meter for the low power factor test can be made to lag on its uncorrected phase by an amount which corresponds to the lag of the current caused by the transformer reactance. It will be seen from this diagram that a small voltage derived from the blue and white line pressure by a double-wound transformer is combined with the blue-phase voltage used for the 0·5 power factor test. This additional small voltage is in phase quadrature with blue line-to-neutral voltage, and the effect of this quadrature boost is to retard the phase of the test voltage without sensible alteration of its magnitude. The vector diagram illustrates this: OI is the vector of test current lagging on OV_W the supply voltage. OV_B is the reversed blue-to-neutral voltage, V_1 is the vector

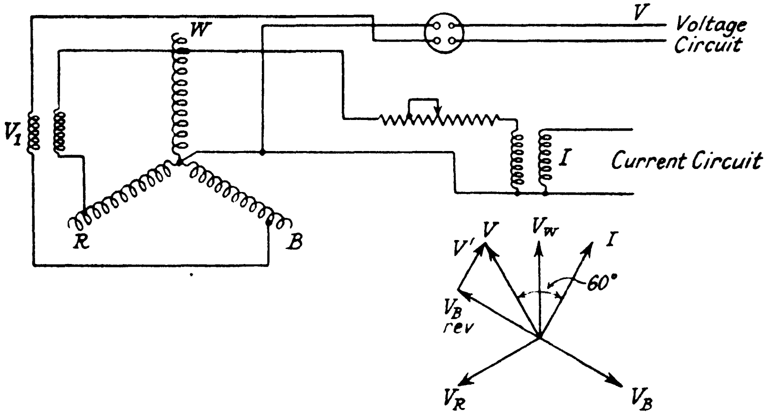


FIG. 48.—Artificial power factors from 3-phase supply for heavy-current circuit.

of the boosting voltage in phase with the red-to-white line pressure. OV is the resultant test voltage which can be made to lead 60 degrees on OI if the ratio of the magnitude of V_1 and OV_B is made equal to the radian measure of the lag of OI on the supply voltage. As meter tests with 0·5 power factor are only required to be made with rated full load current, the amount of the phase correction boost can be adjusted to suit this condition with considerable exactness. This phase correction will, of course, only be accurate for the load for which it is adjusted.

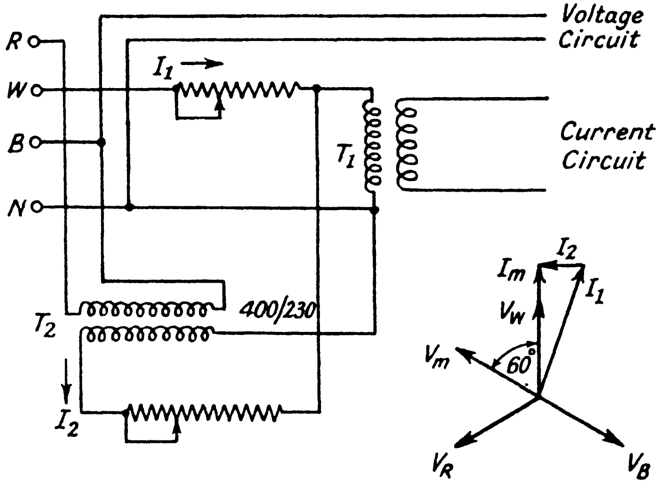


FIG. 48A.—Method of phase compensation.

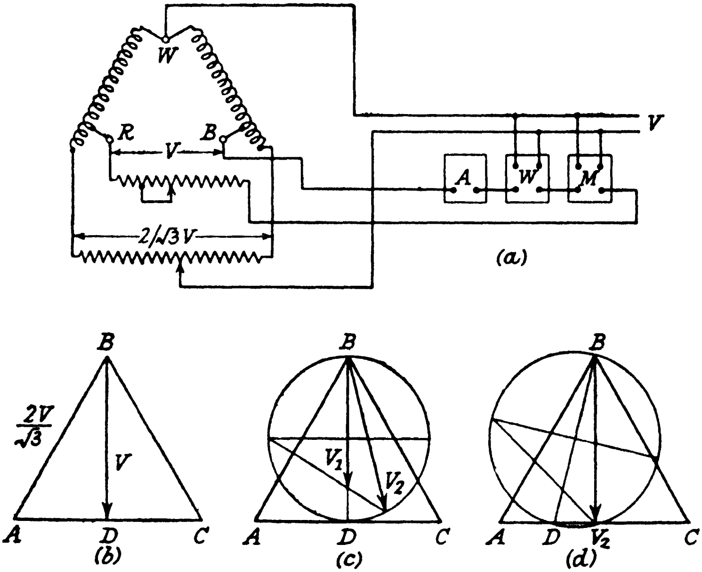
Another method of phase compensation for obtaining exactly 0·5 power factor is illustrated in Fig. 48A. Here a small secondary current derived from the secondary of a step-down transformer T_2 supplied from red and blue lines is superposed, either directly on the meter current, or, as shown in the figure, on the primary current of the step-up current transformer that supplies the meter test circuit. The current from T_2 is approximately in phase quadrature with the white star voltage from which the test current is derived, and, as shown by the vector diagram, a suitable phase shift of this current to obtain exactly 0·5 power factor lagging can be made by adjustment of the compensating current I_2.

A very convenient method of indicating a lagging power factor of 0·5 is by the use of a 3-phase balanced-load type, centre zero, induction wattmeter connected in the meter circuit. An instrument of this type has a 30-degree lead in the voltage flux so that no torque is produced in it when the lag of its current on the voltage exciting it is 60 degrees. 0·5 power factor can therefore be rapidly and accurately obtained with a variable compensation circuit by setting the compensation so that the wattmeter pointer stands exactly over the zero mark.

The lag of the phase of the secondary current of a step-down transformer on the supply voltage of a meter testing circuit gives rise to a power factor error in unity power factor tests. This, however, is not usually considered to be of material importance, as the effect of inherent phase errors of meters and wattmeters upon accuracy of registration and indication is very small at high power factors. When, therefore, the voltage circuit of a heavy-current meter is energised by the same pressure as the primary circuit of the step-down transformer, the condition of loading is generally deemed to approximate sufficiently closely to that of a power factor of unity.

It is evident from elementary considerations that other artificial power factors for meter testing than 0·5 can be obtained from a 3-phase 4-wire supply. Thus, if the meter current is derived from white phase, a voltage derived from the red-to-blue line pressure will be approximately in phase quadrature with this current, so that a power factor in the neighbourhood of zero will be obtained. This approximation to zero power factor is not of great practical use because when meters are checked with an artificial power factor of zero this condition must be exactly obtained. That is to say, the phase difference of meter current and voltage must be exactly 90 degrees. For this reason an artificial power factor of zero is almost always obtained by a device known as a phase-shifting transformer, whereby the phase of a testing voltage may be varied continuously and by any desired amount, without altering its magnitude.

Before passing on to a consideration of the theory and application of the phase-shifting transformer, reference may be made to a method of obtaining an artificial zero power factor for meter testing which is a particular application of the method of phase control described on p. 52. Fig. 49 (*a*) shows the rudimentary diagram of the connections of a meter test circuit with a zero power factor load obtained by this method. It will be seen that the meter current is derived from red and blue phases of a 3-phase supply, and the meter voltage is derived from white phase and the mid-point tapping of a potentiometer rheostat supplied at $2/\sqrt{3}$ times the nominal voltage from red and blue phases. The vector diagram (*b*) shows that, with balanced supply voltages, the meter voltage will be in phase quadrature with the non-reactive meter current, and of the correct value, provided the impedance of the meter and instrument voltage circuit is considerable in comparison with that of the potentiometer rheostat. Further, small variations of the phase of the test voltage can be obtained without sensibly altering its magnitude by small variations of the position of the tap of the rheostat. If, however, the impedance of the voltage circuit is of the same order as that of the rheostat,



(*a*)
(*b*) (*c*) (*d*)

FIG. 49.—Adjustable power factor by rheostat.

the magnitude and phase of the testing voltage will not generally be correct. The vector diagram (Fig. 49 (*c*)) shows the conditions with the potentiometer tap central for a voltage circuit impedance equal to that of the potentiometer. The vector BV_1 is that of the test voltage when the meter voltage circuit and the potentiometer resistance are both inductive. This voltage is correct in phase, but only 0·8 of the correct magnitude. If the meter voltage circuit is entirely reactive the vector of the test voltage is BV_2. This is seen to be nearly correct in magnitude, but to be wrong in phase. This phase error can be corrected by adjustment of the position of the tapping point, so that the vector diagram becomes as Fig. 49 (*c*). It appears, therefore, that with a reactive meter voltage circuit the quadrature voltage obtained by the potentiometer will correspond very closely to its nominal value, provided the impedance of this circuit is somewhat greater than that of the potentiometer rheostat, and, further, that phase adjustments in the neighbourhood of the quadrature position can be made without sensible variations of the magnitude of this voltage. The method, however, is not to be recommended for testing induction meters, as although the voltage circuits of these meters are highly reactive, the waveform of the current they take by normal excitation may, because of magnetic saturation, be distorted. If this exciting current passes through a phase-regulating resistance its wave form may be constrained to approximate to the sinusoidal form, so that the wave of voltage flux in the meters will become flat-topped. It therefore appears that if an induction meter is excited by a test voltage the magnitude or phase of which is controlled by a resistance, the conditions of the test will not correspond to those of actual service. This may not be of great importance in the zero power factor testing of meters, but it is, on general grounds, very desirable to avoid any possible sources of error in meter testing caused by faulty testing conditions; and for this reason, as well as for that of convenience, phase control of a test voltage is best obtained by means of a phase-shifting transformer.

Phase-shifting Transformers. A phase-shifting transformer is somewhat similar to a wound rotor induction motor.

The stator contains a 2-pole primary 3-phase winding so designed as to give a sinusoidal spatial flux distribution. The secondary winding is contained in a rotor the angular position of which, relative to the stator, can be controlled by means of a worm gear and indicated by a pointer and scale of degrees and cosines. If the stator winding is connected to a source of balanced 3-phase voltages, the rotating field set up will be constant in angular velocity, and sinusoidal in space form, and a sinusoidal e.m.f. will be induced in the rotor winding which will be constant for all positions of the rotor. The phase of the secondary e.m.f. relative to the primary voltage will, however, depend upon the relative physical positions of the two windings, and because of the uniform speed of the rotating field, a given angular shift of the rotor will produce an exactly corresponding angular shift of the phase of the secondary e.m.f., this phase shift being leading if the physical shift of the rotor is in the opposite direction to that of the rotating field, and *vice versa.* Thus, with a balanced voltage supply of sinusoidal waveform a secondary e.m.f. can be obtained from a phase-shifting transformer, the phase of which is continuously variable over a complete range of 360° or over by movement of the rotor. The phase of the secondary voltage is indicated by a pointer and degree scale. A second pointer is usually provided which is variable in position relative to the rotor. This second pointer can be set to the zero of the degree scale for any position of the rotor, so that phase shifts of the secondary e.m.f. from any desired datum can be conveniently observed.

If the voltage of the supply to the primary of a phase-shifting transformer is not accurately balanced, the value of the secondary e.m.f. will not be constant for all positions of the rotor, nor will the phase shift of this e.m.f. correspond accurately to the physical angular shift of the rotor. This is best explained by the theory of symmetrical components, according to which an unbalanced 3-phase system of voltages is considered to be the resultant of a balanced system of normal phase sequence and a second balanced system of negative or reversed phase sequence. Each of these component balanced voltage systems may be considered to give rise to a rotating field of uniform angular velocity, but the field caused by the negative sequence component voltage system will be in the reverse direction to that of the main component field. Assume that the normal sequence voltage component of the supply is of unity magnitude, that the negative sequence component has a fractional magnitude represented by n, and that the ratio of transformation is unity. Then, considering one winding of the secondary, the component rotating fields will induce in this winding e.m.f.'s of magnitude 1 and n. Suppose the rotor is in such a position that these e.m.f.'s are in phase. The resultant e.m.f. will be $(1 + n)$. Now suppose that the rotor is turned through an angle θ to produce a lag in the phase of the secondary e.m.f. This angular shift will be in the direction of the principal component field, but against that of the reverse component field set up by the negative sequence component of the supply voltage. The resultant secondary voltage with the rotor in its new position will therefore be that of two component voltages, the one of value unity, lagging on the first phase position by an angle θ, and the other of value n, leading on this position by the same angle. The vector diagram for this condition is given in Fig. 50, from which it is easily seen that, if n is small, as in practice it will be, the magnitude of the vector V of the resultant voltage is very nearly equal to $1 + n \cos 2\theta$, and the difference α between the phase of this resultant voltage and its nominal value corresponding to the angle θ is equal in radian measure to $n \sin 2\theta$. When $\theta =$ 90 degrees the angle α is $n \sin 180$ degrees and is equal to zero, and the magnitude of the component voltage is $1 + n \cos 180° = 1 - n$. As the rotor is turned through 360 degrees from the initial position specified, the magnitude of the e.m.f. in one secondary phase winding therefore varies and assumes the

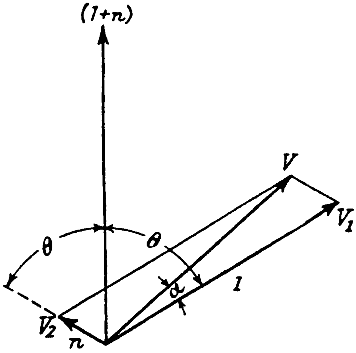


FIG. 50.—Vector diagram of phase-shifting transformer.

values $(1 + n)$, $(1 - n)$, $(1 + n)$ and $(1 - n)$. The phase shift of this e.m.f. corresponds to the angular shift of the rotor at positions displaced 90, 180 and 270 degrees from this initial position. The error of the phase shift has its greatest radian measure of n at positions displaced 45, 135, 225 and 315 degrees from the initial position. If the rotor is shifted from the 45 to the 135 degrees physical position, that is, through 90 degrees, the phase shift of the secondary e.m.f. will be $(90 + 2n \times 57{\cdot}3)$ degrees. Thus if the negative sequence component of the supply voltages has as small a value as 2½ per cent., the error in a 90-degree phase shift may be as much as $0{\cdot}05 \times 57{\cdot}3$, or nearly 3 degrees.

The balance of the supply voltages can be checked with a properly designed phase-shifting transformer by observing whether the secondary e.m.f. is independent of the rotor position. If this condition is satisfied, the phase shift of the secondary e.m.f. can be taken to correspond accurately to the corresponding angular shift of the rotor. If the secondary e.m.f. varies as the rotor is turned, then the maximum variation expressed as a fraction of the average value is evidently, from the preceding argument, equal to $2n$, or twice the fractional value of the negative sequence component of the supply voltage. This experimentally determined value of n will enable the maximum error of the phase of the secondary e.m.f. as indicated on the degree scale of the transformer to be estimated, and the position of the rotor which gives a maximum value to the secondary e.m.f. will be that from which the angle θ is reckoned in the foregoing argument. It is therefore possible by observing this position, and the value of the fraction n, to correct the phase readings of a phase-shifting transformer, with an unbalanced voltage. This refinement will, however, rarely be called for in meter testing, but the argument which has been given above will illustrate the desirability that the voltages supplying a phase-shifting transformer should be as nearly balanced as possible. For this reason the primary of a phase-shifting transformer should be connected directly to the source of supply, and any apparatus for regulating the value of the test voltage to single- or 3-phase circuits should be included in the secondary circuit.

As the magnetic circuit of a phase-shifting transformer contains an air gap, its voltage regulation will be very poor. The available secondary voltage will therefore vary considerably with the secondary output, and the nominal voltage ratio will only be obtained when this output is very small. For this reason it is desirable to provide, as an auxiliary to a phase-shifting transformer, some device whereby continuous variation of the testing voltage derived therefrom may be readily obtained over the whole range of the drop in the transformer terminal voltage which will arise in working conditions, and also to give the 10 per cent. over pressure required in A.C. meter testing. When the testing supply is obtained from a motor generator, regulation of the meter test pressure can, of course, be obtained by field control of the alternator, but this will affect the testing current. The most convenient method of obtaining continuous voltage regulation is by means of a moving-coil voltage regulator connected between the phase-shifting transformer secondary terminals and the step-up or step-down transformers which supply the voltage circuits of the meters which are tested.

Although the specification for test room equipment approved by the Electricity Commissioners only stipulates stability of the waveform of the supply voltage, it is desirable that the waveform of the secondary voltage of a phase-shifting transformer is determined for all conditions in which it is used. The writer has found that the waveform of the secondary voltage of a phase-shifting transformer of standard design is affected by the condition of secondary loading. Fig. 51 shows the voltage waveform of a modern phase-shifting transformer first with the secondary on open circuit, and then with the secondary supplying a balanced load of 200 VA non-inductive. The rated output of the transformer is 750 VA. The source of supply was an inverted rotary converter giving a voltage of very approximately sinusoidal waveform, and the actual waveforms of the phase-shifting transformer voltage were determined by the point-by-point method of measuring the average value of the rectified current in a condenser, the rectification being by a split-ring commutator on the shaft of the rotary converter supplying the transformer. These results seem to indicate that to obtain stability and correctness of the waveform of the voltage of a phase-shifting transformer it is desirable permanently to connect to the secondary terminals a balanced non-inductive load of the order of one-quarter of its rated output.

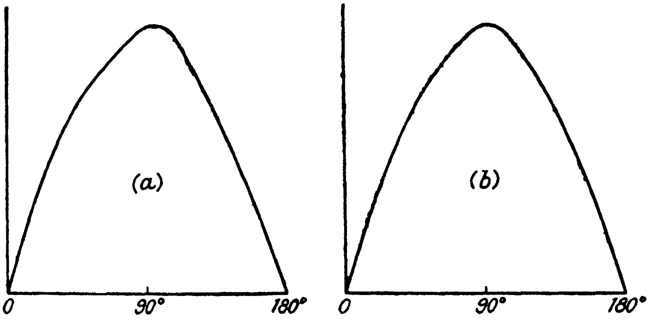


FIG. 51.—Waveforms of secondary voltage of phase-shifting transformer.

When the voltage supply for a 3-phase circuit for meter testing is obtained from a phase-shifting transformer with a voltage regulator for control of the testing pressure, the schematic arrangement of the connections is relatively simple. A typical arrangement of this kind is shown in Fig. 52. Here the nominal secondary voltage of the delta-star-connected phase-shifting transformer is 110, or 63 from line to neutral. Three auto-transformers, star-connected, are provided, so that a 3-phase voltage supply of either 63/110 or 230/400 can be obtained for meter testing. The required pressure is selected

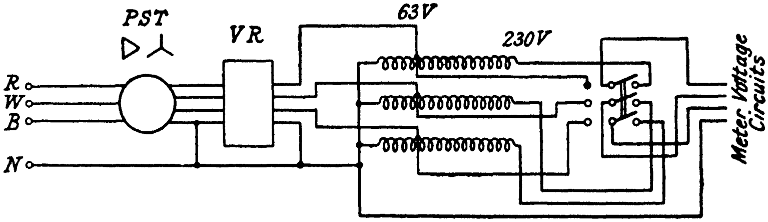


FIG. 52.—Three-phase meter voltage supply.

by means of a 3-pole double-throw switch. The actual voltages applied to the meters can be adjusted to the nominal value, or to 10 per cent. in excess of this value, by means of the voltage regulator, VR, interposed between the phase-shifting transformer and the step-up auto-transformers. The phase of the testing voltage relative to the testing current is controlled by the phase-shifting transformer. Since, in the actual test circuits, there is a phase shift of the testing voltage because of the delta-star connection of the auto-transformer, and a possible phase lag of the testing current when this is derived from step-down transformers, it is advisable to obtain by testing a reference datum for the regulation of the phase of the testing voltage. This can conveniently be done by setting the rotor of the phase-shifting transformer to such a position that zero power factor is obtained, this condition being indicated by a zero reading of the polyphase substandard wattmeter in the test circuit, or of equal and opposite readings of two single-phase substandard wattmeters. When the zero power factor condition is obtained the adjustable pointer of the phase-shifting transformer is set to 90 degrees on the degree scale, and for the particular load current setting, the reading of this pointer will correctly indicate the phase difference of the current and voltage systems of the test load for all positions of the rotor, provided, of course, that the supply voltages are balanced. Thus unity power factor is obtained by turning the rotor through exactly 90 degrees from the zero power factor position, and in such a direction as to give forward indications of the substandard wattmeters. In this condition the pointer of the phase-shifting transformer may read 180 degrees. If this is the case the pointer should be turned through half a revolution to give zero reading on the degree scale. Any other power factor can be set by turning the rotor of the phase-shifting transformer so that this power factor is indicated by the pointer on the cosine scale. As, with a heavy-current testing load, the phase of the current relative to the supply voltage will vary with load changes, the reference datum of the phase-shifting transformer should be checked for each current setting by the zero power factor test.

The sense of the phase shift of the secondary voltages of a phase-shifting transformer is marked either lead or lag on the degree scale. This indication will, of course, only be correct if the phase sequence of the voltages between the primary supply terminals is correct. When connecting a phase-shifting transformer it is therefore advisable to check the phase sequence of the supply by one of the well-known tests. A good and satisfying confirmatory test is to connect a wattmeter so that its current coil is in series with an inductive load and its voltage coil is supplied from the phase-shifting transformer. The rotor of the transformer is then turned to give the maximum wattmeter reading. A non-inductive load is then substituted for the inductive load, when, if the phase sequence of the supply voltages is correct, the rotor of the phase-shifting transformer should have to be turned in the leading direction to obtain a maximum reading of the wattmeter.

Wave form of Testing Voltages. Reference has already been made to the stipulation of the Electricity Commissioners that the control of alternating-current test loads must not vary the peak factor of the voltage waveform by more than 3 per cent. It is to be regretted that some method of testing peak factor was not also stipulated so that compliance with the Commissioners' requirement could be verified with certainty. A change of peak factor of the order of 3 per cent. is not very easy to detect by means of an oscillograph record. One simple method of detecting such a change is to measure the average current in a condenser supplied from the voltage under investigation. The instantaneous charge in a condenser is proportional to its capacitance and to the instantaneous voltage applied to it. The change in the charge during ½ cycle will therefore be proportional to the maximum value of the alternating charging voltage, and the average change in the charge will be the average value of the rectified current. A possible method of detecting changes in the peak factor of an alternating supply voltage is to measure the current in a condenser connected to the supply by means of a rectifier milliammeter, in all conditions of test loading. Although, because of the high impedance of a rectifier instrument, the average value indicated would not be of a very high order of accuracy, it might reasonably be inferred that if the rectifier current per R.M.S. volt did not change by more than 3 per cent. in all conditions of loading the stipulation of the Electricity Commissioners is satisfied. A 1-μF condenser will take an average current of about 130 milliamperes from a 460-volt supply.

An instrument has recently been brought out by Messrs. Elliott Bros. which is designed for the comparison of the peak factors of two voltages. The instrument comprises a potentiometer rheostat in series with a milliammeter, and a neon lamp connected to the rheostat. Constant current is maintained in the rheostat, and the position of the potentiometer tap giving the striking voltage of the neon lamp is noted whilst the circuit is connected in turn to the two voltages whose peak factors are to be compared. Any variation of the position of the tap corresponds to the difference of peak factors which can be read directly as a percentage on a dial switch.

The conscientious testing engineer will, however, not be satisfied with a mere compliance with the permanence of the waveform of the supply voltage ; he will wish to assure himself that no wave distortion occurs in the transformation of this voltage to that actually applied to the terminals of the meters which are tested. Facility for observing the waveform of testing currents is also desirable, although, provided integrity of the waveform of testing voltages is assured, this is not a matter of quite so much importance. The most convenient way of checking the goodness of a waveform is by means of an oscillograph record or indication, and as an oscillograph should certainly form part of the equipment of a testing department, the activities of which are not rigidly confined to the checking of supply meters, this instrument will, if available, give a convenient means for verifying that no sensible distortion of the waves of testing current or voltage is caused by the methods of control which are in use. An oscillograph record is not, however, a very convenient means for obtaining exact data regarding the waveform of steady voltages. A better way to obtain such data is to obtain points on the wave by actual measurement. A very convenient method of plotting waves in this way is by means of a synchronous rectifying commutator with brushes, the position of which is adjustable by a micrometer device and accurately observable over a range of 180 electrical degrees. An alternating waveform can readily be plotted by measuring the average value of the rectified current in a condenser by means of a moving-coil milliammeter for various phases of the rectification obtained by shifting the position of the brushes of the mechanical rectifier. The diagram of connections for this test is shown in Fig. 53. The alternating voltage, maintained at an absolutely steady value as shown by the constancy of the indication of the voltmeter V, is connected to a condenser C and a synchronous rectifier. The rectified current is measured by the milliammeter A. The zero point of the voltage wave is obtained by setting the brushes so that A reads zero. The readings of A for other brush positions of shift θ from this position are proportional to the instantaneous values of the alternating voltage of phase corresponding to θ. Thus, by varying θ from zero to 180 electrical degrees, any convenient number of readings, proportional to the instantaneous voltages corresponding, can be observed, and used, either to plot the waveform or for direct calculation of its characteristic constants.

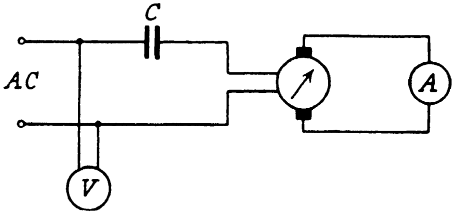


FIG. 53.—Delineation of waveform.

The theory of this method of wave delineation is very simple. Suppose that a voltage of distorted waveform is represented by the expression

$$V_m \sin pt + V'_m \sin (3pt + \theta),$$

this expression will represent the instantaneous charge per unit of capacitance of a condenser connected to the supply with this voltage. If the rectifier brushes are so set that rectification starts at a time t_1, the change in the charge corresponding to the fundamental component of the voltage during the half cycle that the milliammeter is connected to the A.C. circuit will be from $V_m \sin pt_1$ to $- V_m \sin pt_1$, so that the average value of the current, as measured by the instrument, will be proportional to $2V_m \sin pt_1$, the instantaneous value of the fundamental component of the voltage at time t_1. The charge corresponding to the harmonic will change in the half cycle from $V'_m \sin (3pt_1 + \theta)$ to a similar negative value, since a time corresponding to a half period of the fundamental will correspond to one and a half periods of the harmonic. The average current corresponding to this change, in the milliammeter circuit, will thus be proportional to $2V'_m \sin (3pt_1 + \theta)$. The instrument reading, being proportional to the sum of these components of average current, that is, to

$$2\{V_m \sin pt_1 + V'_m \sin (3pt_1 + \theta)\},$$

will represent the instantaneous value of the alternating voltage at time t_1. A similar argument applies to any harmonic higher than the third.

The waveform of an alternating current can similarly be obtained by passing this current through the primary winding of a mutual inductance and measuring the values of the rectified secondary e.m.f. for various positions of the brushes of the synchronous rectifier.

Protection of Meter Testing Circuits. A matter of some practical importance in the arrangement of a meter testing circuit is that of the efficient protection of the substandard instruments from the harmful effect of inadvertent short-circuit or overload currents. Fuses in the main supply circuit are hardly adequate for this duty, and other and more sensitive protective devices, additional to fuses, should always be provided. One fairly efficient method of protection for A.C. instrument circuits is obtained by small electromagnetic circuit breakers such as used in domestic installations. These devices are cheap, and practically instantaneous in operation; and are quite efficient in the interruption of excessive currents. As circuit breakers are not adapted for rapid alterations of the setting, it is best to provide two or three adjusted to settings corresponding to the current ranges of the substandard instruments, the required range being obtained by selecting switches arranged so that any desired breaker can be put in circuit. In a 3-phase testing circuit one set of breakers will be required in each line. A more efficient method of protection is by means of sensitive instantaneous over-current relays, which on operation will open a shunt-trip type of circuit breaker controlling the whole testing supply. One of these relays is connected in the secondary circuit of each of the current transformers used to supply the substandard wattmeters, or, in low-current circuits, in series with the standard wattmeter current coils. Relays of this type can be supplied with a wide range of current settings, which can be varied by alteration of a plug position, so that the setting appropriate to the instrument range can be readily made.

The Lay-out of a Meter Testing Station. The lay-out of a meter testing station should be designed with two main objects: the first is that of obtaining the highest possible standard of the testing work, and the second is the reduction in the total cost of meter testing and maintenance. In regard to the first of these objects a few principles of general application can be laid down. One of the most important of these is the desirability of a separate laboratory section of the testing station, which is devoted principally to the verification of substandards and which is maintained quite separate from the main test room, in which the routine testing of consumers' meters is carried out. The work of verifying the accuracy of substandards, whether indicating or integrating, is of a higher order than that of routine testing, and in order that the best possible accuracy is achieved in this work it is highly desirable that it is organised as belonging to a distinct section of the whole department and carried out in a special section of the testing station. The actual work of the laboratory section should be recognised as carrying special responsibility, and proper accommodation should be provided in this section for the careful calculation and orderly recording of test results. The complete segregation of substandard verification would involve the provision in the laboratory of a separate equipment for supplying and controlling 3-phase test loads for the verification of 3-phase substandard motor meters. It may not be possible to carry the segregation to this point, excepting in the larger testing stations, but all that can be done to separate substandard checking from routine meter testing should be done.

In conclusion, mention may be made of the obvious desirability of installing testing batteries in a spacious and well-ventilated battery room and of installing a motor generator for testing supplies in such a position that instruments in the laboratory are not subjected to vibration.

CHAPTER VI

METER TESTING TECHNIQUE

Scope of Meter Testing. The primary ultimate object of a meter testing station is the determination of the errors of supply meters under approved conditions and by approved methods with a view to submitting these meters for certification. This includes not only the actual testing of the meters, but the verification of the substandards used in connection therewith. The work of a testing station is not, however, confined within these limits. Most supply undertakings have in operation systems of charging which require the measurement of electrical quantities other than kWh, and the instruments used for these measurements—demand indicators, kVArh and kVA meters—although not at present eligible for certification, must be checked for accuracy. A further component of the work of a testing station which lies outside the scope of the Electricity Supply (Meters) Act of 1936 is that of checking the integrity of the mechanism of prepayment meters which regulates the energy allowed to pass the meter for each unit of prepayment.

Verification of Indicating Substandards. The verification of a substandard indicating meter consists essentially in comparing the true with the indicated values of the quantity purported to be measured by the instrument approximately at stipulated points of the scale. There are two ways in which this comparison may be made. First, the quantity to be measured may be adjusted to such a value that the pointer of the instrument deflects exactly to a position on the scale representing the stipulated nominal value, its true value then being measured by the standard potentiometer. Otherwise the ·value of the quantity measured may be adjusted to be exactly equal to the nominal value required by means of the potentiometer, and the actual instrument reading in this condition observed.

Consider, for instance, the verification of a substandard voltmeter. The connections of the test circuit will be as shown in Fig. 54. Suppose the range of the voltmeter is 0–250 volts and that it is to be checked at the 230-volt point. The range of the voltage dividing resistance used with the potentiometer will be 300, so that the ratio of voltage subdivision is 1/200. A pressure of exactly 230 volts on the dividing resistance will therefore correspond to a potentiometer reading of 1·1500. With the first method of comparison the voltage on the instrument and dividing resistance in parallel is adjusted so that the instrument pointer reads exactly 230. This adjustment can be made with considerable precision with an instrument fitted with a scale mirror and knife-edge pointer. With the testing voltage maintained at this value, the potentiometer reading is adjusted till the balanced condition is obtained. In this condition the reading may be found to be 1·1534, whence the actual voltage applied to the instrument is 230·68. The error of 0·68 volt at the 230 scale point is measured with an accuracy which practically depends only upon the precision with which the pointer is set over the 230 scale mark. In the second method of comparison the potentiometer would be set to read 1·1500, the pressure on the voltmeter and voltage dividing resistance adjusted to give a null galvanometer indication and the reading of the voltmeter observed in this condition. This reading would be 229·32, but in this case the accuracy with which the error of 0·68 volt could be observed would depend

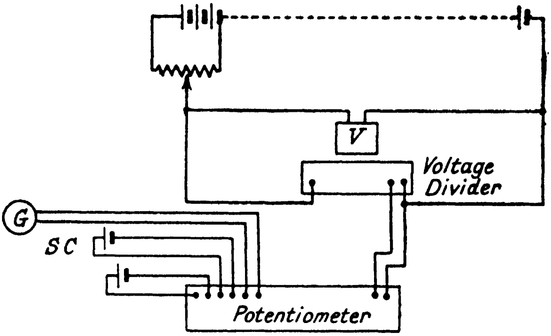


FIG. 54.—Checking substandard voltmeter.

upon the precision with which the angular movement of the pointer corresponding could be read. This accuracy of determination of the error would be much inferior to that of the first method, because whereas a pointer can be set in accurate correspondence with a scale marking, a relatively small deviation of its position from this mark can only be approximately estimated. The first method of verification thus appears to be the better one. Attention has, however, already been drawn to the fact that in the specification of the method of testing substandard wattmeters by means of a calibrated voltmeter, which has been approved by the Electricity Commissioners, the method of calibrating the voltmeter is by the second method, that is, its reading is observed when the pressure applied to it is adjusted exactly to the nominal value. The second method of verification may therefore be taken to be the correct one in accordance with the implication of this specification.

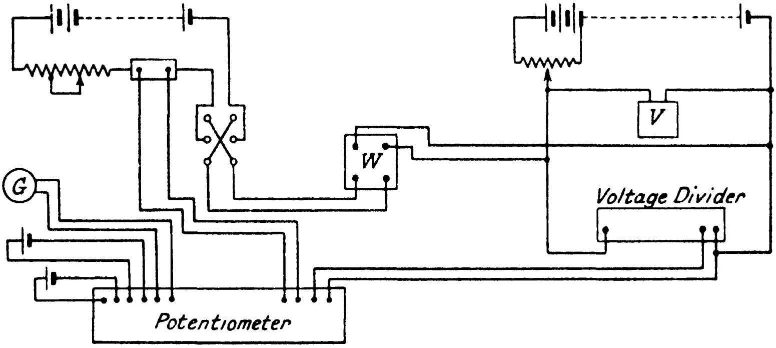


FIG. 55.—Checking substandard wattmeter.

The diagram of connections for the verification of a substandard wattmeter will be as shown in Fig. 55. This shows a calibrated voltmeter for maintenance of the correct pressure on the wattmeter under test, but, as explained on pp. 114–115, a voltage standardiser or a substandard potentiometer may be used for this purpose. The wattmeter is tested at various points of the scale by maintaining the applied pressure constant at the correct value, setting the current to the required values by means of the standard potentiometer and observing the corresponding wattmeter readings. Thus, consider the verification of a wattmeter rated for 5 amperes and 250 volts with a phantom D.C. load of 5 amperes at 230 volts, giving 1,150 watts. The standard resistance used in conjunction with the potentiometer may be rated for a voltage drop of 1·5 with a current of 15 amperes. The potentiometer reading corresponding to 5 amperes will be 0·5 volt. The current is set to such a value that with this potentiometer reading the galvanometer shows a balance, and the pressure is set to give a voltmeter reading which has been found by a calibration test to correspond to 230 volts. When these conditions are obtained, the reading of the wattmeter is observed and this reading corresponds to a phantom load of 1,150 watts. The test is repeated with the phantom load of exactly the same value, but with the direction of the currents in both circuits of the wattmeter reversed. The true indication corresponding to the load of 1,150 watts will be taken to be the average of these two readings.

The verification of an A.C. voltmeter will be generally similar to that of a direct-current instrument, excepting that two readings of the instrument will be taken with the current through the instrument in both directions.

According to the approved directions for meter testing, an indicating substandard instrument may not be used under 40 per cent. of its full scale value. For the purpose of meter testing, therefore, the verification of such instruments will be required only at such points as those at which it will be used in the upper three-fifths of its scale. For general purpose testing it will generally be considered advisable to check the accuracy of the instruments in the lower part of the scale. If an instrument has been completely verified, its reading will have been determined for every value of the measured quantity which will be required in meter testing.

Verification of Time Substandards. Substandard stopwatches can be verified for accuracy very easily by a standard instrument with a seconds dial. A comparison of the stopwatch reading with the advance of the standard chronometer or clock over a period of 5 minutes will reveal any error of the order of the maximum of 0·15 per cent. permissible with certainty. In practice it is best to reject for immediate use and to put aside for examination and regulation any stopwatch which shows an error exceeding 0·1 per cent., that is, an actual error of 0·3 second in 5 minutes. The indicated error of a stopwatch will include the 'personal' error of starting and stopping, and this will vary with the observer. It is best for the checking of stopwatches to be always carried out by the one test assistant who has been proved by actual experience to have the smallest 'personal' error, as shown by the greatest consistency in two or more successive test results in the same watch.

If the standard clock is not fitted with a seconds hand, stopwatches must be tested by the counting of pendulum swings. This is a very tedious process, and it can be made much more convenient—and hence less liable to error—by fitting the standard clock with an optical device whereby a shutter attached to the pendulum bob allows a light flash to pass from a lamp, through the aperture in the shutter, every time it reaches one extreme limit of its swing. By means of a suitable mirror the indicating flash can be made visible at a test bench. If the standard clock has a seconds pendulum and a 2-second period of oscillation, the flash will be given with a period of 2 seconds, so that the observer, seated at the test bench, can count the flashes and keep the watch under observation in comfort. The position of the lamp will have to be adjusted with nicety by trial, so that the flash is given at the extreme point of the travel; any error in this position will result either in no flash being obtained or in the flash being double. Once the adjustment has been made, the integrity of the flash will be a very good indication that the swing of the pendulum of the standard clock is correct.

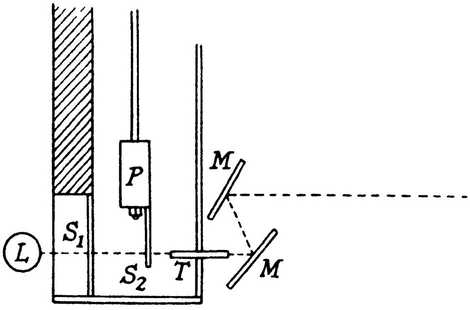


FIG. 56.—Optical device for checking stopwatches.

Fig. 56 is a diagrammatic representation of a device of this kind, which was described by 'A. H. H.' in the *Electrical Times* of September 15th, 1938. Here L is the lamp, which is located behind the screen S_1, in which is a $\frac{3}{32}$-inch hole. S_2 is a light aluminium shutter attached to the bob B of the pendulum, which is also drilled to give a second $\frac{3}{32}$-inch hole to register with that in S_1. T is a length of $\frac{1}{4}$-inch tube, through which the indicating flash passed by the holes in S_1 and S_2 proceeds to the adjustable mirrors MM, from whence the flash is reflected to any convenient position in the laboratory. The lateral positions of the screen S_1 and the tube T are adjustable so that they can be brought in line with the hole in S_2 when the pendulum is at the extreme limit of its swing.

Verifying Substandard Integrating Meters. As a substandard energy or quantity meter forms a link of connection between supply meters and substandard indicating meters, it follows that this kind of meter must itself be verified by indicating substandards. Substandard meters for D.C. testing differ in no way from ordinary supply meters, and, apart from the greater care and precision which is desirable, the testing of these substandards is similar to that of ordinary supply meters using indicating instruments and stopwatches. This process will be considered fully in a subsequent section.

Alternating-current substandard motor meters are, as explained on p. 126, usually special types of instruments furnished with a register whereby fractions of a revolution of the movement of the rotating element are indicated. These meters can be tested by means of indicating substandards in the manner applicable to supply meters. A special technique for the checking of A.C. substandard meters has been proposed whereby the time during which a load of measured power passes the meter is regulated automatically by a standard clock, the object of this technique being to obtain greater accuracy of verification by eliminating the actual measurement of a time interval by the observer. The automatic time control is obtained either by special contacts fitted to the 'scape' wheel shaft of the clock or by light flashes received by a photo-electric cell and transmitted electrically to an impulse relay. The voltage circuit of the substandard meter under test is closed and opened by a contactor relay controlled indirectly by the clock, so that the voltage circuit current of the meter flows during a time interval which is exactly equal to a prescribed number of seconds. The operation of testing the meter then consists in comparing the advance of the register in revolutions and fractions of a revolution of the moving element with the energy corresponding to the measured constant test power flowing for the time period controlled by the standard clock. This method of testing avoids two sources of error : first, the inherent error of a stopwatch ; and secondly, the observer's error in either starting and stopping the watch with a speed test, or in starting and stopping the meter in a dial advance test. This method of time control is largely used in America, and a standard clock and relay equipment has recently been developed in this country and, after testing for accuracy of the timing periods at the National Physical Laboratory has been approved by the Electricity Commissioners. A typical equipment of this type has been described by L. B. S. Golds.* The general adoption of this method in this country would materially improve the overall accuracy of the chain of testing whereby the performance of an A.C. supply meter is referred to the ultimate practical standards of e.m.f. and resistance.

Methods of Meter Testing. We have already in Chapter II referred briefly to the methods whereby supply meters are tested for accuracy. These are three in number. In the first the actual ratio of speed to watts is compared with that for which the gearing constant is suited. In the second the energy required for a definite integral number of revolutions of the meter rotor is measured. In the third the advance of the meter register corresponding to a stipulated quantity of energy is observed.

In the first method of testing the value of the power in the test circuit, actual, or nominal at a declared pressure, is measured by substandard indicating instruments, and the accuracy of the performance of the meter is assessed by the measurement of the time interval required for a stipulated number of revolutions of the rotor of the meter. According to the Methods of Meter Testing prescribed by the Electricity

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* Testing Substandard Meters, *Electrical Times*, 27th July, 1940, p. 111.

Commissioners, the measured time interval must correspond to three revolutions of the meter rotor or be not less than 100 seconds, whichever be the longer period. For dealing with supply meters in quantity, this timing method has several disadvantages. The accuracy practically obtainable depends upon the testing load being maintained constant at its stipulated value during the test, and this calls for ceaseless vigilance, and, if necessary, careful regulation of this load throughout the test period by an observer additional to the one who actually measures the time interval. Further, the test merely checks the accuracy of the instrument as a wattmeter; it does not verify that the actual gearing constant is the same as the nominal value.

In the second method of testing, which is used for A.C. meters, the energy corresponding to a stipulated integral number of revolutions of the rotor of the meter under test is measured by means of a substandard meter which registers revolutions and fractions of a revolution of its moving element. Constancy of the test load is not essential with this method of testing, so that one observer only is required. Apart from the inherent errors of the substandard, the accuracy of the test depends only upon the precision with which the operation of this instrument is regulated in reference to the rotation of the meter under test. In the Methods of Meter Testing approved by the Electricity Commissioners it is stipulated that with this method of testing the number of revolutions of the meter under test must bear the same ratio to 40 as the actual testing load bears to the rated full load of the meter, provided that this number of revolutions must not be less than 5 and need not be greater than 25. This method of testing, like the first, does not verify the correctness of the actual value of the gearing constant, and consequently does not check the accuracy of the meter as an integrating instrument.

In the third method of testing the actual advance of the register of the meter under test is observed for the passage of a measured amount of energy through it. This energy may be measured by a substandard meter or by measuring the value of the power, which must be maintained constant, and the time corresponding. A substandard meter is used for testing supply meters, but for the verification of substandards the energy must be measured in terms of power and time. This method of testing is one which completely verifies the accuracy of a meter as an integrating instrument, and it is very convenient for the checking of supply meters in quantity. According to the Approved Methods of Meter Testing, the duration of tests on supply meters by this method must correspond to not less than ten revolutions of the pointer of the last dial. The test will therefore take an inconveniently long time with a testing load which is only a small fraction of the rated full load of the meter, and for this reason this method of testing is usually carried out at full load.

The Approved Methods of Meter Testing stipulate that all supply meters submitted for certification must be tested in three conditions :—

(*a*) At 5 per cent. of its marked current or, in the case of a D.C. meter with a marked current of less than 10 amperes, at 10 per cent. of its marked current.

(*b*) At a current intermediate between this fraction and the full value of the marked current, and

(*c*) At the actual marked current for a D.C. meter, or at this current or one exceeding it by one-fifth for an A.C. meter.

The value of the current in the condition (*b*), although not explicitly specified, should be that at which the ratio speed/watts is likely to leave its maximum value. It is generally taken as one-half of full load.

It is also stipulated that, whatever actual method of testing be employed, one test must be carried out by comparing the advance of the dial register with a measured amount of energy. This must be a test addition to three tests carried out by other methods. These other methods will be either by substandard motor meters or indicating instruments for A.C. supply meters, but by indicating instruments for D.C. supply meters. The load on any substandard A.C. meter used for measuring the energy corresponding to a stipulated number of revolutions of the rotor of a test meter must not be less than one-quarter nor greater than one and one-quarter times its full load. The load on any substandard indicating meter must not give a reading of less than two-fifths of its full scale reading. This stipulation applies irrespective of the scale length of the instrument.

In testing watt-hour meters the voltage circuits of the meters and of all substandard instruments used for testing must be energised for a period of 1 hour prior to actual testing.

Meter Testing by Time Measurement. In the practical application of this method the time actually required for the execution of a definite exact integral number of revolutions of the meter rotor with a stipulated load is compared with the nominal time which corresponds to the gearing constant. This gearing constant may be given in many forms, and it is marked either on the nameplate of the meter or a label fixed inside the meter case. The gearing constant of D.C. quantity meters is generally expressed as ampere-seconds per revolution of the meter rotor, and the method of calculating the times required for stipulated numbers of revolutions in various loading conditions can best be illustrated by a numerical example.

Suppose the meter to be tested has a marked current of 5 amperes and a gearing constant of 5·32 ampere-seconds per revolution. The meter will be tested at loads of 5, 2½ and 0·5 amperes. The correct time required in any condition is given by the formula

$$\text{seconds} = \frac{5{\cdot}32 \times \text{revs.}}{\text{amps.}}$$

so that for a load of 5 amperes 100 revolutions should correspond to a time of 106·4 seconds, a test period which conforms to the requirement of a minimum of 100 seconds. At the loads of 2½ and 0·5 amperes this correct period of 106·4 seconds will correspond respectively to 50 and 10 revolutions. The test is therefore carried out by maintaining the meter current at steady values of 5, 2½ and 0·5 amperes and observing the times required for exactly 100, 50 and 10 revolutions of the meter rotor. If the meter is correct these times will all be exactly 106·4 seconds.

If the observed time differs from this calculated correct time the fractional error is the difference between the two times expressed as a fraction of the observed time, this error being plus or fast if the observed time is the smaller, and *vice* *versa.* For, if the correct time is T_1 and the observed time T for N revolutions, the actual rotor speed is N/T and the correct speed is N/T_1. The difference between these speeds is

$$N\left(\frac{1}{T} - \frac{1}{T_1}\right) = N\left(\frac{T_1 - T}{T_1 T}\right)$$

and this difference, expressed as a fraction of the correct speed N/T_1, is

$$\frac{T_1 - T}{T}$$

Thus, if in the test at 5 amperes the time was found to be 104 seconds, the percentage error would be

$$100 \times \frac{106{\cdot}4 - 104}{104} = \frac{240}{104} = 2{\cdot}3 \text{ per cent. fast.}$$

If the errors determined in this way are the same at the three test loads, these errors are merely of calibration, and either the inherent speed/load characteristic of the meter may be altered to suit the gearing constant or the gearing constant may be altered to suit the actual characteristic. In practice, the three errors or the three time periods actually observed will never be exactly equal. If these periods do not deviate from the correct calculated period by more than 2 per cent. under or 3 per cent. above, the meter may be considered as sufficiently accurate to be submitted for certification. Otherwise some adjustment must be made to satisfy this condition. It may happen that the observed times differ among themselves by more than 5 per cent. In this case it is evident that no adjustment either of the speed/load characteristic or of the gearing constant can make the meter accurate within the limits specified, and the meter is either inherently defective or requires overhaul.

The adjustment of a meter so that its errors at the three loads at which it is tested do not exceed the stipulated limits is usually made by variation of the braking flux, which cuts the disc in which the eddy currents are induced. This is done either by alteration of the position of the brake magnets relative to the disc, or by alteration of the position of a magnetic shunt relative to the magnet which provides the braking flux. This adjustment has to be made by trial, and it can be considered to be most successful when the maximum final errors of the meter, fast and slow, are in the ratio of 2 to 3.

The adjustment of an inaccurate meter may be also made by altering the gearing constant so that the errors, as calculated by the new constant, satisfy the condition mentioned in the last paragraph. When this method of adjustment is used, the register train contains what is called a ‘change wheel,’ and a table is supplied giving the ‘ampere-seconds per revolution’ gearing constants corresponding to change wheels having various numbers of teeth. The three test results are expressed in the ampere-seconds per revolution form, and a change wheel is selected which gives a constant corresponding to the values calculated from the test results. Suppose, for instance, that the observed times in tests on the meter already considered are 104, 102·5 and 105·5 seconds respectively at 5, 2½ and 0·5 amperes. The ampere-seconds per revolution corresponding will be 5·2, 5·12 and 5·27. The values of the gearing constants obtained by change wheels may be found from table to be, say, 5·12, 5·2 and 5·29, and in this case the wheel giving the 5·2 constant will be selected. When the gearing constant is changed in this way, its new value must be marked in the appropriate position in or on the meter case or meter dial plate.

The number of teeth in a change wheel to suit a set of test results can generally be calculated directly without much difficulty. Thus, to take a numerical example, suppose that the gear train of the 5-ampere meter is made up as follows: an 8-tooth pinion on the rotor spindle gearing with a change wheel of n teeth on a single worm spindle, a 40-tooth worm wheel with an 8-tooth pinion gearing with an 80-tooth wheel on the spindle carrying the pointer registering one-tenths of a kWh. The gearing constant will evidently be

$$\frac{8}{n} \times \frac{1}{40} \times \frac{8}{80} = \frac{1}{50n} \text{ kWh per revolution.}$$

Now, taking the representative value of the actual ampere-seconds per revolution characteristic to be 5·2, this corresponds for a 230-volt meter to

$$\frac{5{\cdot}2 \times 230}{3{,}600 \times 1{,}000} \text{ kWh per revolution.}$$

So that, if the gearing constant is to suit the meter characteristic, we must have

$$\frac{1}{50n} = \frac{5{\cdot}2 \times 230}{3{,}600 \times 1{,}000}$$

whence $$n = \frac{3{,}600 \times 2}{5{\cdot}2 \times 23} = 60 \text{ almost exactly,}$$

so that a 60-tooth change wheel will give a gearing constant corresponding to an ampere-seconds per revolution constant of 5·2.

The advantage of the change wheel method of adjustment is that it is made with certainty. The disadvantages are twofold: in the first place, the adjustment is not continuous, but in steps which, when the number of change wheel teeth is of the order of 60, are of about $1\frac{1}{2}$ per cent.; and secondly, the adjustment involves interference with the gear train, and it involves not only the selection of the correct wheel, but also considerable care that the depth with which this wheel gears with the associated pinion is correct.

The gearing constant of a D.C. watt-hour meter may be given as the kW-seconds per revolution. In this case the correct time for a convenient number of revolutions in all test conditions is easily calculated. To give a numerical example, we consider a 50-ampere 460-volt meter with a gearing constant of 30 kW-seconds per revolution. The full load power will be 23 kW, so that in this condition we have

$$\text{time} = \frac{30 \times \text{revs.}}{23}$$

Thus 80 revolutions with 50 amperes at rated voltage should take 104·3 seconds. This time should also be taken for 40 revolutions at 25 amperes and for 4 revolutions at $2\frac{1}{2}$ amperes.

In most A.C. meters the gearing constant is given as revolutions per kWh. It is evident at once that kW-seconds per revolution is given by the formula 3,600/revolutions per kWh, whence the correct times for a convenient number of revolutions at various test loads can easily be calculated. Thus a 5-ampere 230-volt A.C. meter having a gearing constant of 2,000 revolutions per kWh should execute 1 revolution for 3,600/2,000 = 1·8 kW-seconds, and with the full load unity power factor of 1·15 kW the correct time will be given by the formula

$$\frac{1{\cdot}8 \times \text{revs.}}{1{\cdot}15}$$

whence 80 revolutions should take 125 seconds.

Substandard A.C. meters will be tested throughout their range by the method of timing rotor revolutions, a supplementary dial register test being made as with supply meters. These tests ought to be of a more searching character than those of supply meters, as the magnitude of the errors is required to be known precisely. Each test should occupy a period of the order of 3 minutes, and it should be checked by repetition.

Throughout the foregoing description it has been tacitly assumed that the substandard indicating instruments are free from error. This in practice is not likely to be the case, so that, in testing substandard meters at any rate, allowance must be made for the errors of indicating instruments. This allowance can be made in two ways. First, in testing, the value of the measured quantity can be adjusted to give that indication of the standard which has been found, by potentiometer tests, to represent the required correct nominal value. The meter test results obtained by this method will then be based on the true values of the integrated quantity and will be authentic. Practically, this method suffers from the disadvantage that it is not easy to maintain a testing load in continual adjustment to a scale reading which does not correspond exactly to a scale marking. The second method of allowance is to correct the final test results by subtracting from the observed meter errors the corresponding errors of the substandard errors. This is quite simple if a correct convention regarding algebraic signs of errors is observed. Thus a fast error of a meter and a high reading of a substandard will both have positive signs. The subtraction of the substandard errors from the observed meter errors will then be carried out in accordance with the rules of algebra.

Short-time Meter Tests with Substandard Motor Meters. In this method of testing the number of revolutions of the rotor of the meter under test is stipulated, and when the substandard meter registers in rotor revolutions and fractions thereof, the advance of the substandard corresponding to the stipulated revolutions of the test meter must be calculated. This calculation is very easy. If the revolutions per kWh constant of the two meters is the same, then, if both meters are correct, the rotors thereof will rotate synchronously. Otherwise the ratio of the revolutions of the test meter to those of the substandard will be in the ratio of these gearing constants so defined. To take a numerical example, suppose that a 5-ampere 230-volt A.C. meter with a 2,000 revolutions per kWh gearing constant is being tested by a substandard meter of which the gearing constant is 1,800. Then for the full-load test 40 revolutions of the test meter will correspond to $\frac{40 \times 1{,}800}{2{,}000} = 36$ revolutions of the substandard. Similarly for the half-load test 20 revolutions of the test meter will correspond to 18 revolutions of the substandard. The revolutions per kWh constant of the substandard meter will depend upon its rated full load current, and this will be affected by changes of its range, either by internal rearrangement of the interconnection of sections of the current winding or by change of the ratio of an associated substandard current transformer or by change of its voltage range. The revolutions per kWh constants of substandard meters should be clearly tabulated for all ranges, both of current and voltage.

The error of a meter tested in this way is found from the circumstance that the calculated value of the substandard revolutions corresponds to the registration of the test meter, whilst the observed value corresponds, apart from substandard errors, to the correct registration; the percentage error is thus

$$\frac{\text{calculated revs.} - \text{observed revs.}}{\text{observed revs.}} \times 100.$$

The error so determined will be corrected for substandard error as previously explained.

Dial Testing. Long period dial tests on supply meters are very simple in character. Approximately full load is passed through the meters till the required register advance is obtained, and these actual advances are compared with that of a substandard meter. The dial pointers should be set to zero before the test, and the quantity of energy may conveniently be such as to give an advance on the substandard which, allowing for its error, corresponds to an exact decimal multiple of a kWh. Thus, supposing the nominal energy to give the required advance is 1 kWh and the substandard is 0·4 per cent. fast, then, if the load is interrupted when the substandard advance is 1·04 kWh, the dial readings of the test meters will give percentage errors directly because the true energy will be 1 kW. There seems no reason why in good testing work the result of a dial test should differ by more than 0·5 per cent. from a test by any other method at the same load.

Dial testing of substandard meters must be carried out by maintaining a constant load throughout the test and by obtaining the true value of the energy from the readings of substandard indicating and time measuring instruments. With substandard D.C. meters of ordinary pattern the test is tedious, and it is questionable whether, once having been made, it is necessary to repeat it so frequently as the verifying timing tests. Long time measurements are best made by the standard clock if this is fitted with a seconds hand. The testing current can be maintained for such a time interval, either as will give an exact decimal multiple of a kWh or as gives an exact decimal multiple advance of a kWh as the meter dials.

Alternating-current substandard meters registering in rotor revolutions may be tested by the method of timing revolutions, or preferably by observation of the registration corresponding to measured power flowing for measured time. The revolutions per kW-second of a meter rotor will, if the instrument is correct, be given by the formula

$$\frac{\text{revolutions per kWh}}{3{,}600}$$

whence the correct registration for any given loading conditions can be at once obtained. If the revolution registration exceeds this calculated amount the meter is fast, and *vice versa*, and the percentage error, uncorrected for substandard error, will be

$$\frac{\text{observed registration} - \text{calculated registration}}{\text{calculated registration}} \times 100.$$

The regulation of the duration of the test load to an exact number of seconds is achieved by the operation of a switch connected by means of a flexible cord in the voltage circuit of the meter. A flashing device actuated by a standard clock is better for this purpose than a stopwatch. Best of all is an automatic control device such as was described on p. 174.

Stroboscopic Methods of Meter Testing. This is a method of checking the speed of a meter disc which is analogous to null methods of electrical measurement. It consists in viewing the disc, the face of which is provided with regularly spaced markings, by the light of a lamp which is caused to flash with a frequency controlled by the speed of rotation of the disc of a special substandard meter. This control is obtained by regularly spaced perforations in the substandard disc upon which is focussed a beam of light. As the disc rotates, the periodic flashes through the perforations are received by a light-sensitive cell, which, by the agency of an amplifying circuit, controls the supply to a neon lamp, used to view the disc of the meter under test. If the speed of this meter corresponds to that of the standard used to check it, the marks on the disc will appear to be stationary. If the meter speed is too high, the apparent motion of the disc will be forward at a rate corresponding to the meter error relative to the substandard, and *vice versa.* The adjustment of the meter under test therefore consists in setting the position of its brake magnets so that the disc, viewed stroboscopically, appears to be stationary. Measurements of time intervals and personal errors incident thereto are avoided, and the setting of the brake magnet is made with certainty.

Stroboscopic methods are inherently suitable only for test loads not less than half the rated full load of the meter. At lower loads the disc speeds are so small that the stroboscopic method becomes unreliable or fails entirely to give its characteristic phantom differential speed effect. One manufacturer has adapted the method to low-load testing by temporarily substituting, for the ordinary brake magnets of the substandard and test meters, special low-flux magnets which increase the meter speeds tenfold. By this means it is stated that meters can be tested stroboscopically at one-twentieth of full load.

The stroboscopic method of testing has been refined in America for the determination of actual meter errors by direct indication. The substandard which controls the flash frequency is provided with a specially calibrated brake magnet adjusting device, geared to a dial indicating deviations from correct speed in percentages fast and slow. The speed of the substandard is adjusted to exact correspondence with that of the test meter, and the error of this meter is read off directly on the special dial.

The stroboscopic method of testing is most useful in circumstances where large batches of meters of similar rating have to be handled and adjusted rapidly. It is therefore inherently one for the manufacturer rather than for the ordinary user of meters. The technique of stroboscopic testing differs considerably from that of ordinary methods, but, once it has been acquired by the operator, there seems little doubt that it effects considerable economies in the adjustment of large batches of meters to good commercial accuracy.

Testing Electrolytic Meters. Electrolytic meters must be tested by what is the equivalent of a long period dial test, and in the Approved Methods of Meter Testing it is stipulated that meters of this type are to be tested by a substandard electrolytic meter at a load of approximately three-quarters of the full load value. The duration of the test must not be less than 100 hours. The error after about 50 hours and the error at the time of syphoning are also to be recorded.

The substandard electrolytic meter used for these tests must, according to the Electricity Commissioners' stipulations, be checked for accuracy at periods of not less than twelve months, and these checks will most conveniently be made by a substandard D.C. motor meter. This latter meter must be itself tested for accuracy immediately before and immediately after being used to test the electrolytic substandard and at the exact load used for this test. The measured error of the electrolytic substandard will be corrected for the error of the motor substandard, this latter error being presumably as the mean of the results of the two check tests.

Clock meters must necessarily be tested by long period dial tests, and the duration of each test must correspond exactly to a complete number of forward and reverse directions of the current in the voltage circuits. The testing can be by substandard indicating instruments or by a substandard motor integrating meter, which will generally be of the direct-current type, as clock meters are rarely used in A.C. circuits. The latter method of testing would be similar to that prescribed for electrolytic meters, and presumably it would be necessary to check the accuracy of the substandard meter before and after testing the clock meter.

Possible Overall Accuracy in Meter Testing. A question of some practical importance is that of the overall accuracy obtained in meter testing. To put this question in a concrete form, suppose that the error of a 5-ampere A.C. supply meter at full load unity power factor is found to be 0·2 per cent. fast. This means that if the meter is placed in a circuit in which 1,000 kWh are consumed its registration will be 1,002. How near is this statement likely to be to the actual truth ? Could this be determined ? It is very difficult to answer a question of this kind satisfactorily. The meter in question would have been tested by a motor substandard, which itself would have been tested by a substandard wattmeter, and probably a standard clock. The wattmeter would have been tested with direct current by the standard potentiometer and by a verified substandard voltage indicator. The possible errors of the potentiometer and its adjuncts, which constitute the ultimate standards of the testing station, are very small and cannot amount in cumulation to more than a fraction of 0·1 per cent. If the errors of instruments in the chain of tests were correctly measured and allowed for in the subsequent link in the chain, it might appear that an overall error of 0·1 per cent. in the result of a test on a supply meter would not be exceeded. An estimate of this kind is, however, very optimistic. At each stage in the chain of tests which ultimately purports to give the error of a supply meter there are possibilities of errors of observation by reason of the personal limitations of the observers. In this connection it is to be noted that the Electricity Commissioners state that it is not expected that any meter shall be submitted for certification if its errors are within 0·5 per cent. of the stipulated limits of 2½ per cent. plus and 3½ per cent. minus.

The interpretation of this statement as bearing on the question of overall accuracy of testing seems to be that possible errors of considerably more than 0·1 per cent. may be expected in meter testing, and it is questionable if the possible overall error in the most careful meter testing will be less than one-quarter of 1 per cent. Much light would be thrown on a question of this kind if a meter, tested in the normal way in a Testing Station, were re-tested at the National Physical Laboratory, and a check of this kind would be well worth the expense. Certainly the overall accuracy of meter testing is not such as to warrant the recording of percentage errors to more than one place of decimals. If a test result is recorded as, say, 0·42 per cent., the figure 2 in this record makes an implied claim that the result is correct to the nearest one-hundredth of 1 per cent.—a claim which would not be actually made by a standardising laboratory of the highest character.

Over-pressure Tests. Every watt-hour meter submitted for certification must be tested for ‘ creep ’ with its current circuit open and with a voltage one-tenth greater than the rated voltage applied to its voltage circuit. We have explained in the preceding chapter how an A.C. meter testing circuit is arranged so that this test can be conveniently carried out. The friction compensating device of the meter should cause incipient movement of the rotor in certain positions, but this should be checked by the ‘ anti-creep ’ device, so that a complete revolution cannot be completed. If it is necessary to adjust the compensations to prevent continuous rotation, this adjustment will tend to alter the error at one-twentieth of full load, and the meter should be re-tested in this condition.

Low Power Factor Tests. Alternating-current meters submitted for certification must be tested at the full rated VA load at 0·5 power factor lagging. A tolerance of 10 per cent. is allowed in the power factor for this test. The arrangement of a meter test circuit whereby this test may be conveniently carried out has been already explained in Chapter V.

The difference between the errors of an A.C. meter at full rated VA at 0·5 power factor and the same watt load at unity power factor may arise from two causes—the first being an inherent phase error of the meter and the second being the relatively greater ratio of self-braking torque to driving torque in the 0·5 power factor condition. Phase error can be detected without a quantitative test by energising the meter with rated VA at zero power factor, when the rotor should not turn if this error is practically negligible. This zero power factor test is conveniently carried out before the quantitative test at 0·5 power factor. If the substandard wattmeter is itself free from phase error, the zero power factor condition is easily obtained by adjustment of the rotor of the phase-shifting transformer so that, with full load meter current, the wattmeter reads zero. If in this condition the meter shows a tendency to register, this can be corrected by adjustment of the phase-compensating ring on the voltage magnet. After adjustment of this ring it is advisable to interrupt the current and to observe that there is no tendency to creep with voltage alone. Should the wattmeter have a small phase error and give a reading with zero power factor, then if the ratio of this reading to the VA is known, the phase of the voltage can easily be set in quadrature with the meter current. Let this ratio, which is the radian measure of the substandard wattmeter phase error, be called α. Then, first observing the maximum reading of the wattmeter which can be obtained by phase adjustment, the phase of the voltage is advanced till the wattmeter reading is α times this amount and in the correct direction. A wattmeter with phase error will usually give a forward deflection with zero power factor lagging.

Having established zero power factor and checked the meter for lack of creep in this position, the 0·5 power factor condition can be established by retarding the phase of the voltage by 30 degrees, by adjustment of the phase-shifter or by changing over the voltage circuit of the meter to a special supply in correct phase relation to the current, as shown in Fig. 48.

The Methods of Testing approved by the Electricity Commissioners have been criticised on the ground that a single test with full load VA is not sufficient at 0·5 power factor. This criticism does not seem to be well founded. The effect of inaccurate phase compensation on the accuracy of a meter at a given power factor would seem, on *a priori* grounds, to be independent of the meter current, and to be completely detected by a single test. The effect of the relative increase in the self-braking torque of the current flux will be greatest at full load VA. Thus, if the performance of the meter at 0·5 power factor is satisfactory at the rated VA, there is no reason why it should be worse at any other load. A more useful check on the 0·5 power factor test than one at a load less than that specified would be a second test at 0·5 power factor leading, as the difference between the errors in the two power factor conditions will be double the change of accuracy caused by phase error at the 0·5 power factor, and this difference, if more than it should be, can readily be detected.

Testing 3-wire D.C. and Single-phase A.C. Meters. Meters of this kind, with the possible exception of the clock type, must comprise two separate elements communicating driving torque to a common spindle. Three-wire meters may be tested for certification in a 2-wire circuit, the voltage circuits being in parallel and the current circuits in series. This test is, however, not conclusive of the accuracy of the meter when integrating an unbalanced load, because an accurate result with the 2-wire test will be obtained if one element is fast and the other is slow by a corresponding amount. Additional tests are therefore necessary in which the extreme condition of unbalance is simulated ; that is to say, with both voltage circuits excited and with current in one circuit only. If the balance of the elements is correct, or the torque per watt set up by each is the same, the tests with full load current in each element only should give results within the limits of error. Clock meters arranged for 3-wire circuits will be tested similarly.

Testing 3-phase Meters. We have already recorded the stipulation that every 3-phase meter submitted for certification must be tested in a 3-phase circuit. Three-phase 4-wire meters may be tested in 3-wire circuits which supply the testing currents, in which case the total power in the phantom test load can be measured either by two wattmeters or by a 2-element polyphase wattmeter ; alternatively, tests can be made by substandard 2-element meters. With this method of testing 4-wire meters the common terminal of the three voltage circuits will be connected to the neutral of the voltage supply. The rudimentary connections for testing a 4-wire supply meter by a 3-wire substandard are shown in Fig. 57.

When a 3-phase meter is tested by a substandard integrating meter, by either the short- or long-period method, the technique differs in no way from that of the testing of single-phase meters other than that of setting the test loads to the required magnitude and to the balanced condition. This matter has been sufficiently considered in the preceding chapter. When, however, a 3-phase supply substandard meter is tested by means of indicating substandards the maintenance of a steady load during the actual testing period may not be so straightforward. If the power in the phantom 3-phase test load is measured by a single polyphase substandard wattmeter, small adjustments can be made to one of the three variable loading resistances to maintain the required indication of the wattmeter, as these small adjustments will not cause sensible departure from the balanced condition of the load which will have been obtained before the commencement of the test. If, however, the total power in the phantom load is measured by two substandard wattmeters it is by no means easy to maintain an absolutely constant value of this total power by adjustment of a loading resistance, if the supply voltage is not quite

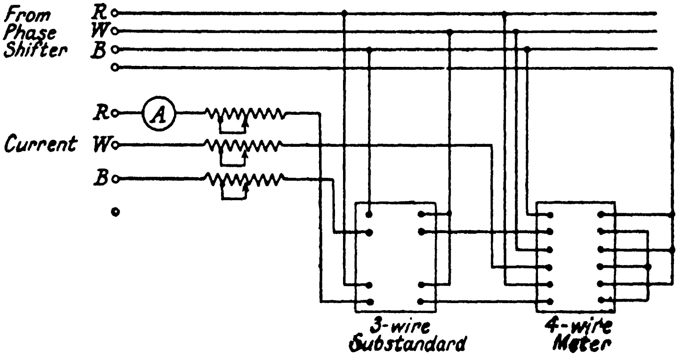

Fig. 57.—Testing 3-phase 4-wire meters.

steady, because, with a star-connected load, such adjustment will alter the power indicated by each wattmeter. It may be taken for granted that it is practically impossible to maintain a 3-phase test load, supplied by a varying voltage, at an absolutely constant value by adjustment if the total power is indicated by two wattmeters, because one observer cannot keep the indications of two wattmeters under continual observation. For the highest class of 3-phase testing by two single-phase substandard wattmeters it is almost essential to have a source of supply at steady pressure. This, of course, does not apply to the testing of 3-phase supply meters by substandard motor meters. In this method of testing small variations of the value of the testing load are immaterial.

Every polyphase meter submitted for certification must be tested for balance of the elements in addition to the usual tests with a balanced load. The test for balance is carried out by exciting both voltage circuits in the normal way and passing full load current through each element in turn. These tests are directed to be carried out at power factors of unity and 0·5 lagging, the latter power factor being subject to a tolerance of 10 per cent. It will be observed that, in the balance test, unity and 0·5 power factors refer to the single-phase load actually producing torque in the meter. Thus, whereas with the 3-phase test at unity power factor the red element current is lagging 30 degrees on the red element voltage, with the balance test when the red element only is producing driving torque, the current and voltage will be in phase. Similarly with the 0·5 power factor test, the meter current will lag 60 degrees on the meter voltage, not 90 degrees, as in the condition of balanced load. When a star-connected test load is used, the balance tests are conveniently carried out with one line supply to the load interrupted, as shown in Fig. 58. The necessity for testing 3-phase meters for balance at a power factor of 0·5 will be manifest from the investigation on p. 77, where it is shown that the difference between the phase errors of the two elements leads to an error in balanced load conditions which cannot be detected by testing at low power factor, and which is only revealed when the actual difference in phase errors is revealed by two single-phase tests.

As 3-phase meters used with current transformers have to be tested with these transformers and in a 3-phase test circuit, it follows that the equipment for checking this class of meters must be suitable for the instrument of the highest rating which will have to be handled. As this requirement may in many cases be onerous, it may be of interest to enquire as to whether there are any methods of testing heavy-current 3-phase meters without using a complete double 3-phase testing circuit, which would with sufficient practical accuracy yield results indicative of the performance of the meter in working conditions. At one time it was not uncommon to test 2-element 3-phase meters as single-phase instruments, that is, with the voltage circuits in parallel and the current circuits in series. If a meter were connected in a single-phase circuit in this way so as to give forward rotation, then by reversing the polarity of the current and voltage connections another condition of test also giving forward rotation would be established. The results of two single-phase tests in these two conditions are rarely the same, and to a first approximation it was assumed that the mean of the results would represent the performance of the meter in normal conditions of balanced 3-phase loading. The difference between the two conditions, 3-phase and single-phase, is, however, often too great for this kind of test to be considered satisfactory. In the first place, as the torque per watt of each element is to a slight extent affected by leakage flux from the voltage magnet of the other, a test with the two voltage fluxes in phase is made in a condition differing considerably from that in which these fluxes are

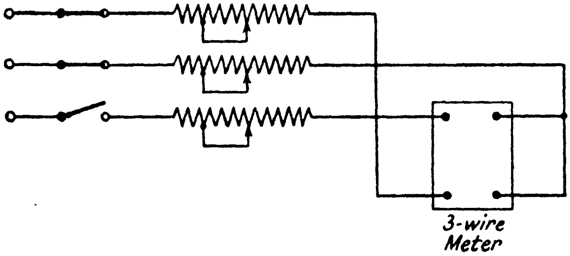

FIG. 58.—Balance test for 3-phase meter.

60 degrees out of phase. Secondly, with the single-phase test with full load current and unity power factor, the driving torque is greater than that in the corresponding test by the factor $2/\sqrt{3}$. Thirdly, the single-phase test will not be subject to the constant error of calibration caused by different phase errors which affects the performance of the meter with a 3-phase test. Although a perfect 3-phase meter should be accurate, whether tested in a 3-phase or a single-phase test circuit, the imperfections of commercial meters are at the present time sufficient to make it essential that they are tested in conditions

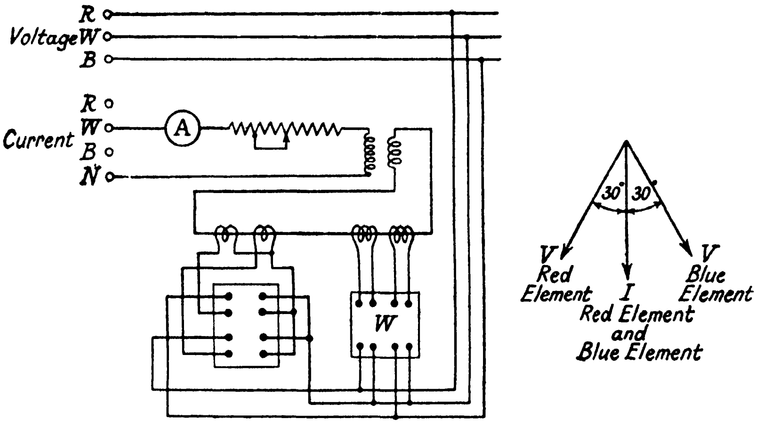

FIG. 59.—Modified circuit for testing heavy-current 3-phase meters.

which approximate more closely to those of actual service than those of the single-phase test.

A possible simple method of testing 3-phase meters, which reproduces actual working conditions more closely than the single-phase test, is illustrated by the circuit of Fig. 59. Here a heavy-current 3-phase meter used with two current transformers is excited in the usual way by voltages between red and white and blue and white lines, but the current in each element is derived from a single-phase circuit connected to blue and neutral terminals of the supply. The power in the phantom load is measured by a polyphase substandard wattmeter. The vector diagram shows that at unity power factor the current in each element is 30 degrees out of phase with the associated voltage, as with a 3-phase load, but this phase difference is leading in the red and lagging in the blue element. With this exception, the conditions in the meter correspond to those of actual service, and, in view of the simplicity of the circuit for the supply of the heavy testing current, and of the ease whereby the phantom load can be controlled, this method of testing 3-phase meters of abnormal current rating may give results which for all practical purposes are as accurate as those obtained by the use of more complicated testing circuits. It must, however, be emphasised that the foregoing scheme is merely suggestive in character, as regards the testing of supply meters for certification, as the type of testing circuit is not strictly that for which 3-phase meters are designed.

No simplification in testing, comparable with that outlined in the preceding paragraph, appears to be possible for 3-phase 4-element meters. The only alternative to testing in a 3-phase double circuit is a single-phase test with all voltage circuits in parallel and all current circuits in series, and this, for similar reasons to those set out above, is inadmissible. The only possible simplification allowed in the Approved Methods of Meter Testing is that of the use of a 3-wire test circuit for the current supply. The balance test with 3-element meters must be carried out at unity and 0·5 power factors with current alone in each of the three elements, all voltage circuits being excited normally. The method of carrying out this series of tests is obvious, as in a 3-element meter the meter and circuit power factors are the same with a balanced load.

Meters used with Instrument Transformers. The stipulation in the Approved Methods of Meter Testing that a meter must be tested with the instrument transformers with which it is to be used is qualified by a statement in an Explanatory Memorandum that the Electricity Commissioners may in appropriate cases approve other methods and tests, such as the separate testing of meters and their transformers. This qualification would probably apply to the testing of meters for high-voltage supplies, as a rigid application of the normal method of testing would require a 3-phase high-voltage test circuit. In this case, doubtless the accuracy of the combined meter equipment could be inferred from the results of tests of the meter with current transformers and a separate test of the voltage transformer. In this case the testing voltage would actually be 110, but, for purposes of calculation, the testing load would be multiplied by the nominal ratio of the voltage transformer.

The qualification referred to might also be applicable to the testing of heavy-current 3-phase meters, and it is possible that either some such alternative test might be approved as was suggested in the preceding paragraph, or that these meters might be tested as 5-ampere instruments, and the results of these tests corrected for the measured errors of the current transformers.

The ratio error of a current transformer is reckoned positive if its secondary current is greater than its nominal value ; the phase error is reckoned positive if the secondary current leads in phase on the primary, since such an error causes a positive error of registration in an associated meter with lagging power factor. It therefore follows that, according to this convention, if a meter is tested as a 5-ampere instrument, the error at unity power factor of the combination of meter and instrument transformers will be the algebraic sum of the meter error and the transformer ratio error at the current corresponding. As the effect of the phase error of a transformer on the performance of a meter is to cause a registration of $\alpha \tan \phi$, the actual error of the combination at a power factor of 0·5 lagging will be equal to the algebraic sum of the measured meter error, the transformer ratio error, and $\sqrt{3}$ times the radian measure of the phase error of the associated current transformers in the conditions corresponding. This rule tacitly assumes that the errors of the two or three current transformers used with a polyphase meter will be equal in the same conditions. In practice, differences of measured errors will be slight, as the current transformers used with a polyphase meter should be similar in design and VA rating. If the differences are more than negligible, an average value can be taken for purposes of correction. It is extremely unlikely that differences of current transformer ratio error will ever be sufficient to give rise to the constant calibration error referred to on p. 77.

The subject of testing instrument transformers for ratio and phase error is so wide, and the available methods of testing are so numerous and diverse, that adequate treatment of this branch of electrical measurements is not possible in the present work. Exhaustive and critical information on instrument transformer testing will be found in the books by B. Hague and D. C. Gall, of which mention is made in the Appendix. We may remark here that, when a current transformer is used with a polyphase supply meter, its burden is small and partly inductive, so that the method of testing the transformer for error should be such that, in the ratio and phase error test circuit, the burden is identical with that corresponding to actual service. For this reason, so-called absolute methods of testing current transformers, in which the ratio and phase difference of primary and secondary currents are actually compared, are not good for the purpose of correcting meter errors, because in these absolute methods a considerable non-inductive secondary burden is essential for the test, so that it is impossible to obtain the errors with a burden corresponding to that of actual service. The better method of testing is the comparison method, in which the errors of the test transformer are determined in relation to those of a substandard current transformer. The comparison test should be such that the burden of the test transformer is not increased by connecting in its secondary circuit any instrument coil whatsoever other than the current coil of the meter with which it will be used, or one similar. Further, the method of testing should be a null one, and not one in which the difference between two secondary currents passes into a wattmeter, as with this latter method of testing the accuracy obtained is vitiated by the unavoidable wattmeter impedance. Most of the conventional methods of current transformer testing are defective, from the point of view of correcting meter errors, because they do not satisfy these requirements. There is urgent need for standardisation and specification of a few good methods of testing the errors of meter current transformers, when the meter with these transformers cannot be tested as a single unit and the alternative method of indirect testing is approved by the Electricity Commissioners. As this approval will presumably only be given in exceptional circumstances, these current transformer tests will be infrequent, so that approved methods of testing should be such as only call for ordinary laboratory instruments and substandard transformers. Testing the errors of a current transformer for the purpose of correcting the errors of meters measured in a 5-ampere test circuit is a very different process from a test to verify that these errors do not exceed those stipulated for its class in the relevant B.S. Specification.

The foregoing remarks relating to the testing of current transformers will apply equally to the testing of voltage transformers used with polyphase meters for high-voltage supplies.

Testing Reactive Meters. As the only meters requiring certification are those which register kWh, there is no officially approved procedure for testing the accuracy of a meter which either registers kVArh or which is a component of a meter registering kVAh. If substandard instruments indicating 3-phase VAr were available, the testing of reactive meters would be a process similar to that of testing 3-phase energy meters. A polyphase substandard wattmeter or Wh meter can be made to indicate 3-phase VAr by supplying its voltage circuits from a bank of voltage transformers arranged as explained on p. 48. With balanced testing voltages the error of the VAr indication will depend upon the inherent errors of the wattmeter and upon the ratio and phase errors of the substandard quadrature voltage transformer. A suitable quadrature transformer of high accuracy for supply voltages should not be an expensive item, and when reactive meters have frequently to be tested the provision of such a substandard transformer whereby VAr can be indicated by a wattmeter or integrated by a substandard motor meter is very advisable, as the technique of testing reactive meters will be the same as that of testing energy meters. The normal tests of a reactive meter will be carried out at zero power factor when the VAr are equal to the VA. In addition, the behaviour of the meter with rated VA unity power factor and zero VAr will be observed, and a test will be made with an intermediate lagging power factor. The test for balance of the elements of a polyphase reactive meter is made with normal voltage excitation and with current in each element singly. The meter torque in this test will correspond to the single-phase power in the active meter element if the compensation by reactive measurements is entirely external, and this power can be measured by a wattmeter in the usual way. If a reactive meter is internally compensated partially, that is to say, if each element is neither a true wattmeter nor a true reactive meter, a balance test will be difficult. The normal tests of a reactive meter may be either by timing or by substandard energy meter arranged to integrate VAr.

If there are no means available for the measurement of VAr or VArh directly by substandard instruments, indirect methods must be used. A straightforward indirect method of obtaining 3-phase VAr is to obtain the watts by actual measurement and the VAr from ammeter and voltmeter readings. This method is cumbersome and undesirable, as it involves simultaneous readings of three ammeters, three voltmeters and one or two wattmeters, together with an awkward calculation. Moreover, this method is only fundamentally accurate with a balanced load, but the error with small unbalance is practically negligible.

Another indirect method of obtaining the VAr measure of a balanced 3-phase test load is from the readings of two single-phase wattmeters used to measure the power. $\sqrt{3}$ times the difference of these readings will give the 3-phase VAr if both currents and voltages are accurately balanced. The disadvantage of this method is that the accuracy of the VAr measure so obtained depends so much upon the balance of the currents and voltages. The percentage error of VAr measurement may be of the same order as the percentage difference between any two line currents or voltages of the test load. It therefore follows, as errors of over 1 per cent. may easily arise from this cause, the method is inherently inaccurate.

A third method of VAr measurement is by using a phase-shifting transformer having an accurate degree scale. In this method the VAr value is obtained indirectly from the reading of a substandard wattmeter. The reactive meter to be tested is connected in an ordinary 3-phase double circuit, of which the voltage supply is derived from the secondary circuit of a phase-shifting transformer. A polyphase substandard meter is included in the circuit. To carry out the tests at zero power factor, maximum VAr, the testing currents are first set to the required value, and the phase of the testing voltage is then set so that the wattmeter reading corresponds to zero power. The rotor of the phase-shifting transformer is then turned through exactly 90 degrees. Throughout this movement the value of the testing voltage should be constant if the angular movement of the rotor corresponds accurately to the phase shift. Provided this condition is satisfied, the wattmeter reading will be the maximum possible with the given current and voltage values, and this reading will give the 3-phase VA, and hence the VAr in the zero power factor condition. The wattmeter reading is noted and the phase of the test voltage is again adjusted to the zero power factor position. The time required for a suitable number of revolutions of the rotor of the reactive meter is then observed in the usual way, and this is compared with the true time corresponding to the observed VAr value. During this test constancy of the load can be checked by observation of ammeter readings, and after the test the actual value of the VAr can be checked by shifting the phase of the testing voltage again through 90 degrees and taking a second wattmeter reading. In this last condition of unity power factor the rotor of the reactive meter can be observed to verify that it is stationary, or practically so, as it should be. Tests at other loads are carried out similarly. A test at a power factor intermediate between unity and zero can be made by setting the phase of the testing voltage to a value corresponding to the required power factor. The value of the VAr in this test can be obtained as the square root of the difference of the squares of the wattmeter condition in the unity power factor and the actual test conditions. Otherwise this value can be obtained by multiplying the value of the VA by the sine of the angle of phase shift of the testing voltage from the unity power factor position.

Neither of these three indirect methods can be considered as anything more than a makeshift. The only satisfactory way to test a reactive meter is by reactive substandards, and the most satisfactory available substandards for this purpose are ordinary substandard wattmeters or integrating meters used with substandard voltage compensators. The accuracy of such substandards will, of course, depend upon exactness of balance of the supply voltages, and if unbalance errors of the substandards are similar in magnitude and phase to those caused by voltage unbalance in the reactive meter, the test will indicate the accuracy of the meter in balanced voltage conditions. No test of a reactive meter can be considered conclusive till single-phase internally compensated reactive substandards are available of an accuracy at standard frequency, comparable with that of a substandard wattmeter.

Testing Kilowatt Demand Indicators. A Merz demand indicator is merely an additional dial registering average kW. The accuracy of this indication, provided the associated meter is free from error, will depend upon the accuracy of the gearing ratio controlling the advancement of the pointer and the accuracy of the averaging periods during which the demand indicator gearing engages with that of the meter register. When the averaging periods are controlled by a time switch, external to the meter, this switch can be checked for accuracy separately, and the accuracy of the demand indicator gearing can be tested by a procedure similar to a long-period dial test. The engagement of the demand indicator mechanism is controlled manually, so that the averaging periods correspond to the nominal value and the reading of the indicator is observed for three or four averaging periods. The corresponding advances of the substandard meter will give the true values of the average power corresponding. The tests should be taken at full load and about one-quarter of full load, and at the commencement of each testing period the pointer should be set to a reading slightly less than that which is anticipated, so that actual advancement of the pointer over the scale only takes place at the end of the period. When the averaging periods are controlled by a self-contained clock, the substandard meter reading must be observed at the commencement of each averaging period and the time durations of these periods simultaneously measured. The true average kW in this case will be obtained from the substandard advances and the observed times.

Thermal demand indicators are tested by passing steady currents of suitable values through them and comparing the final pointer indication with the true value of the steady nominal power. The reading of the pointer should be observed at the ends of time periods of one, two, three, four and five quarters of the nominal averaging period. The steady reading should be given at the end of the fourth quarter. The checking of a thermal demand indicator is in principle similar to that of checking an ammeter.

Testing 3-phase kVAh Meters and kVA Demand Indicators. These instruments, as already explained in Chapter II, are of four classes : kWh and kVArh combinations with a separate kVAh mechanism, phase-compensated kWh meters, current-operated instruments with voltage compensation and kWh meters with fixed phase compensation which are accurate at one power factor only.

The first steps in testing an instrument of the first class are to test the kWh and the kVArh components separately by the methods which have already been described. The kVAh register and the kVA demand indicator should then be tested by methods corresponding to a long-period dial test and to an ordinary kW demand indicator test at kVA loads of various power factors. The method of checking the instrument will, provided the kVA load be known, follow the standard procedure, but it is in practice difficult to measure the kVA value of a 3-phase test load over a range of power factors. One method of doing this is illustrated by the diagram of connections (Fig. 60). The kVA instrument is tested without its current transformers, as a 5-ampere instrument. This meter, together with a substandard 3-phase watt-hour meter and a substandard polyphase wattmeter, carries the current of a non-inductive test load. The substandard watt-hour meter voltage circuits are in parallel with this load, those of the kVAh meter and the polyphase wattmeter are energised from the secondary of a phase-shifting transformer. The magnitude of the voltages applied to the kVAh meter is adjusted by the voltage regulator to be exactly equal to those applied to the substandard watt-hour meter, this equality being indicated by the voltmeters V_1 and V_2. In this circuit the phantom volt-amperes in the kVAh instrument and in the polyphase wattmeter are the same as the actual volt-amperes in the substandard watt-hour meter, and, as the load of the circuit containing this latter meter is non-inductive, the rate of advance of the substandard meter will correspond to the kVA in the phantom load for all values of the power factor of this load. Thus,

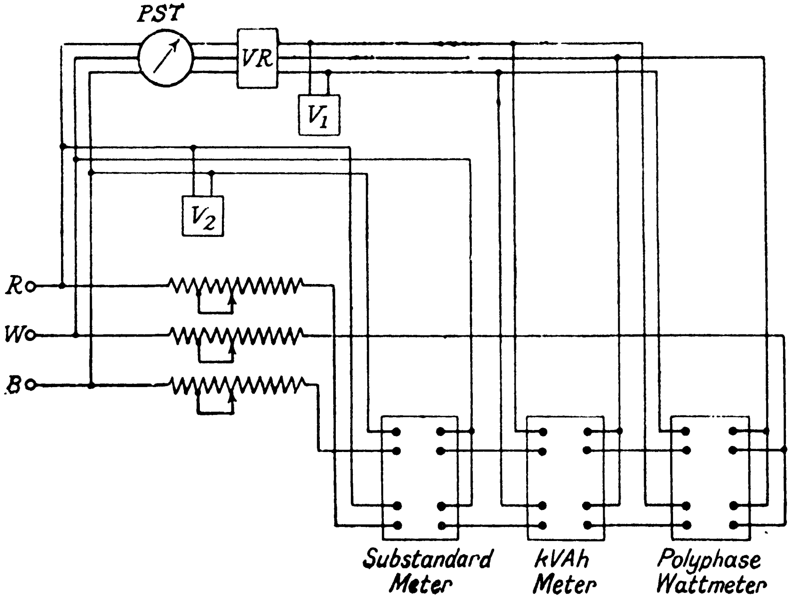

Fig. 60.—Testing kVAh meters.

setting the rotor of the phase-shifting transformer to give desired values of the power factor of the phantom load in the kVAh meter, the advance of the kVAh register or of the kVA indicator should correspond to the advance of the substandard watt-hour meter. The polyphase substandard wattmeter is used to obtain the position of the phase-shifter rotor corresponding to zero power factor. When this position has been obtained, as indicated by zero power reading of the wattmeter, the movable pointer of the phase-shifting transformer is set to 90 degrees in the usual way. The power factor of the phantom load in the kVAh meter will thereafter be directly indicated on the power factor scale of the transformer. kVAh meters should be tested at zero and unity power factors and two or three power factors intermediate. These intermediate power factors should not, preferably, correspond to phase angles which are an exact simple submultiple of 90 degrees.

The foregoing method of testing is defective, in that it does not follow the standard and most desirable practice of testing supply meters and associated current transformers as a single measuring unit. The convenience of obtaining a direct measure of the kVAh in the phantom test load by means of a 3-phase substandard meter is so great, as compared with the indirect methods, that this convenience seems to outweigh the slight defect to which reference has been made. The only possible adjustment which has to be made during tests of the kVAh register and the kVA indicator is that of maintaining the kVAh meter and substandard watt-hour meter voltages exactly equal.

The foregoing method of testing will be immediately applicable to kVAh meters with automatic phase compensation, the instrument first being tested as a standard 3-phase energy meter in the usual way without its auxiliary relay and voltage dividing resistances, and at its nominal voltage.

kVA demand indicators, which consist of a watt-hour meter with fixed phase compensation, can also be tested by the method just described at the nominal power factor for which they are correct. This power factor can be set by the phase-shifting transformer, and it can be checked by comparing it with the ratio of the wattmeter reading to the maximum reading which can be obtained by phase adjustment of the voltage. The meter should then be tested with the phase of the voltage shifted 18 degrees on each side of the position corresponding to the average power factor to verify that, in each of these positions, a slow error of about 5 per cent. is obtained.

Three-phase kVA demand indicators containing three current-operated electromagnetic elements should be tested for balance at two or three loads, the voltage-compensating circuits being excited, and current, of a phase corresponding to balanced load conditions, being passed through each element in turn. The speed of the rotor should be the same for the same current in each element, but there is no definite relation between speed and current. Thereafter the instrument will be tested as a 3-phase kVA demand indicator at selected points of the scale at constant 3-phase loads corresponding. The test at rated full load may be carried out at three or four values of the power factor and at unity power factor with values of the applied voltage 10 per cent. above and below the rated value to check the accuracy of the voltage compensation. In this latter test it will be legitimate to assume that the accuracy of the substandard watt-hour meter is, for all practical purposes, unaffected by the voltage changes.

Testing Prepayment Meters. The testing of the measuring component of a prepayment meter will be carried out in the usual way, as these meters have to be submitted for certification of accuracy of measurement. In addition to these tests, the integrity of the prepayment mechanism must be checked and the performance of the meter when the switch is tripped with one-twentieth of full load should be observed. The extra retarding torque in this condition will tend to impair the meter accuracy, but it should not do so unduly, and there should be no tendency for the meter to fail to register.

The correctness of the rate of charging per kWh must be checked by observation of the actual registration of the energy required to trip the switch after three or four coins have been inserted.

In prepayment meters for a two-part tariff the operation of the switch is controlled not only by the meter, but also by a continuously rotating motor energised by the supply voltage. The correctness of the rate at which credit is continually extinguished in this way must also be checked. As this rate is normally of the order of one or two shillings a week, this test is necessarily a lengthy one for meters in which the shilling is the coin prepaid. When a number of prepayment meters are checked in one circuit a practical difficulty arises because, without a special arrangement of the circuit, the opening of the switch of one of the meters will not be self-registering, as the operation of the continuously rotating motor depends upon voltage and not upon load current. Fig. 61 shows the connections of a circuit for testing the accuracy of the fixed charge collecting component of a number of prepayment meters. HM is a time meter which registers the actual time during which pressure has been maintained on the voltage circuits of the meters under test. This voltage supply is controlled by all the meter switches in series. When credit is extinguished in any meter, the voltage supply is interrupted for the whole circuit, and the reading of the time meter shows the number of hours allowed by this particular meter for the amount prepaid. This reading having been made, the current leads of the meter which has opened the circuit are removed and joined together, and the test is resumed till the next meter switch operates, and so on. By this method the test can proceed continuously, and it is immaterial if a meter switch opens when the test house is unattended.

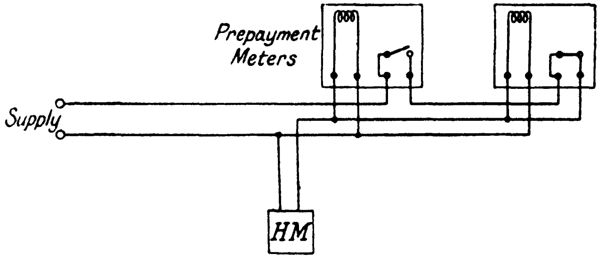

Fig. 61.—Testing prepayment meters.

Off-circuit Tests. So far we have considered the testing of supply meters, either for certification or before being put into actual service. When meters are removed from consumers' premises, either by reason of being changed for another meter of higher rating, because of some defect, or by reason of the expiry of the period of useful service, a series of tests of the accuracy of the meter is often taken immediately it is received in the test house. These tests are not so searching as those preliminary to certification, and are usually taken at full load and one-twentieth of full load only. The utility of these so-called 'off-circuit tests' is not generally agreed, and it is unquestionable that, because of possible disturbance consequent upon the actual handling and transport of the meter, they do not necessarily represent the accuracy with which it was actually registering prior to removal. On the other hand, off-circuit tests seem to be the only means whereby a comparison may be made of the long-service performance of meters made by different manufacturers, and, although not conclusive but only indicative in character, they are for this purpose of some value. The technique of off-circuit testing calls for no comment. The interpretation of the results of off-circuit tests as a measure of quality will be considered in the following chapter.

CHAPTER VII

RECORDING AND ANALYSIS OF TEST RESULTS

Records and Data. The results of tests of consumers' meters are generally and best entered on separate test sheets which, when filed, form a permanent record.

References to these sheets, by a serial number or otherwise, can be made on the record cards, on which the complete history of all meters is chronicled.

The results of tests on substandard instruments might be recorded in greater detail, and there is something to be said for a system of loose leaf books, one for each instrument or group of instruments, in which the results of tests are entered. These books will form a compact and complete technical record of all the substandard instruments. Each substandard instrument should be provided with some method of fixing for a detachable label or chart, on which the results of the latest standardisation tests are recorded for the use of observers.

Great care should be taken in the calculation of the results of tests on substandard instruments. It is questionable whether an ordinary 10-inch slide rule is good enough for this purpose. A slide rule of the Fuller type should be provided for the laboratory section of the testing station, and the use of this rule should be insisted upon for all calculations made therein. The records of tests on substandard instruments should all be checked and passed by a senior assistant.

Clear diagrams of connections should be made of all permanent testing circuits, and appropriate white prints should be framed and fixed in a position convenient for the observers. Permanent drawings should also be made of the connections of circuits for special tests for which there are no permanent arrangements. When it is desired to repeat such a test, a print of this diagram can then be handed to the tester responsible. Diagrams of connections of test circuits should, preferably, be a standard size, stored in box files and properly indexed.

Every effort should be made to make the routine testing of consumers' meters as automatic as possible and to avoid the necessity for any preliminary calculations by the observer. For testing meters by the method of timing revolutions, the appropriate number of revolutions and the corresponding 'correct' times in seconds should be calculated and tabulated for all values of the meter rating and of the gearing constants which will be met with. It is best for these tabulated calculations to be written by hand on tracing linen by a draughtsman, so that prints can be supplied, mounted on stiff cardboard, for the use of testing observers. Similarly, when meters are tested by substandard motor meters, the correct number of revolutions and the corresponding advances of substandard meters for all meter ratings and testing constants should be clearly tabulated. The ideal system of tabulation will be one such that a new meter tester can readily and rapidly grasp.

Records for Certification of Supply Meters. When meters are submitted for certification the test results must be recorded on duplicate forms of a pattern prescribed by the Electricity Commissioners. The information on these forms will be taken from the meter test sheets. The meters on the official forms must be distinguished by the makers' serial numbers. After scrutiny and approval by the Meter Examiner, one of the forms is returned to the Testing Station for filing.

Off-circuit Tests as an Indication of Quality. Ordinary tests of consumers' meters are made primarily with a view to verifying that they register correctly within stipulated limits of error, and the results of these tests actually give the magnitude of the error in various conditions of loading to within a possible inaccuracy between ¼ and ½ per cent. This degree of accuracy to which the meter has been adjusted will not as a rule be maintained indefinitely after it is placed in service to measure a consumer's supply. The maximum period of service is generally limited by a programme of periodical examination and re-calibration. A meter will not necessarily survive this maximum period. It may fail to register whilst in service and be reported as defective by the meter reader, or it may be exchanged for a meter of larger rating and be returned to the testing station, in which case it will be examined and re-tested before further use. The programme of periodical examination is so fixed that the number of meter failures is relatively small, so that the majority of supply meters remain in service for the maximum period determined by this programme. During this period there are two causes of a change in the registration errors as determined by the original tests. In the first place, mechanical deterioration will tend to increase friction, and so to cause a slow error, the percentage value of which is inversely proportional to the load. Secondly, a tendency to weakening of the brake magnets of a motor meter will give rise to a tendency to a fast error which is independent of the load. These considerations apply only, of course, to motor meters. There seems to be no reason why, excepting from serious defects, the accuracy of an electrolytic meter should deteriorate in service, and the only cause of a change of error of a clock meter would appear to be serious mechanical defects. These considerations may not be confirmed by actual experience, but changes in the accuracy of electrolytic and clock meters, if they occur, are generally the result of causes which are somewhat difficult to discover.

Resuming the consideration of the behaviour of motor meters in service, we understand that, if after the prescribed maximum period an off-circuit test is carried out at full load and one-twentieth of full load on such a meter which is returned to the testing station, the effect of increased friction will be much the greater on the low load accuracy. It is possible to segregate the two causes of change of accuracy quantitatively from the results of an off-circuit test. Suppose that the change of percentage error at full load is η_{100}, that is to say, that the full load ‘ off-circuit ’ test gives a result which is η_{100} per cent. faster than the original test before the period of service, and suppose that the change of percentage error at one-twentieth of full load is η_5. The effect of a weakening of the brake magnets, considered alone, will be to cause a change of error in the fast direction. Call this change, in percentage value, p. The effect of friction, considered alone, will be to cause a change of error in the slow direction, which is inversely proportional to the load. Let this in percentage value be q at one-twentieth load. At full load it will be $\frac{q}{20}$. We therefore have

$$\eta_{100} = p - \frac{q}{20}$$

and

$$\eta_5 = p - q,$$

whence $q = \frac{19}{20}(\eta_5 - \eta_{100})$ or $(\eta_5 - \eta_{100})$ very nearly.

and $p = \eta_{100} + \frac{q}{20}$.

Thus from the changes of accuracy at full load and one-twentieth of full load, as determined from the result of an off-circuit test, it is possible to calculate two characteristic values, p and q, which are respectively representative of the change in calibration caused by variation of the permanent magnet braking flux and the effect of increased friction. It may be noted that when the actual change of the errors, η_5 and η_{100}, is determined, p and q can be easily calculated mentally by inspection. Thus suppose that an off-circuit test shows that a meter is 4 per cent. slower at one-twentieth of full load and 1·2 per cent. faster at full load. The characteristic q is equal approximately to 5·2 per cent. and p is $1{\cdot}2 + \frac{1}{20} \times 5{\cdot}2$, or 1·5 per cent. nearly. A brief and easy analysis of this kind therefore gives useful information regarding the change in the fundamental physical characteristics of a meter which has taken place during its period of service, and this information can be regarded as an authentic quantitative indication of quality.

Statistical Analysis of Meter Test Results. If the magnitudes characteristic of quality are obtained for a number of meters all of the same rating, type and manufacture, and after the same period of service, these data will contain implicitly the quality characteristics of this class of meters. A study of this mass of data will enable some sort of judgment to be made of the quality of the class, but, even with so small a number of meters as 100, a quantitative judgment will be difficult, and liable to bias if made by mere inspection. If similar data for another set of meters of the same type and rating, and after the same period of service, but of a different manufacture, are collected, then, although the two collections implicitly contain all the information requisite for the comparison of quality, it will be practically impossible to make this comparison fairly by mere study and inspection. Meters of different manufacture can only be compared as regards quality on the basis of off-circuit test results which are digested into a few representative magnitudes which are characteristic, not of any particular meter, but of a whole class. Such representative magnitudes are called statistical averages.

There are three averages used by statisticians, each of which is a single number representative of a group. These averages are the mean, the median, and the mode. The mean of a collection of magnitudes is the ordinary average value which is instinctively used by engineers. It has the characteristic property that of all the magnitudes in the collection it represents, the sum of those exceeding it is equal to the sum of those it exceeds. The median is found by setting out or ‘ arraying ’ the collection in order of magnitude, when the median is the middle value of the array. This average has the characteristic property that there are as many values exceeding it as those which it exceeds. The third statistical average, the mode, can be roughly defined as the value to which the greatest number of magnitudes in the collection tend to approximate. It has the characteristic property that it corresponds to the value for which the frequency distribution is the greatest.

The mean, or arithmetical average, of a collection of test results, or of characteristic magnitudes derived therefrom in the manner described in the preceding section, is the average value which would generally be calculated to represent the whole collection. It is not, however, of itself the best for this purpose. This point can be illustrated by considering a collection of 100 test results, of which the great majority were good, but a few were very bad, having large errors of the same sign. These bad results would affect the mean value to an extent which would mask the uniform goodness of the greater number. For this reason the modal average has been suggested as one which is better representative of a collection of meter test results. If, for instance, of a group of 100 test results fifty were between 0 and 1 per cent. of error, the modal error would lie between these two figures, and it would be unaffected by the circumstance that there might be a few errors of abnormal magnitude. The disadvantage of the mode, as a representative average of a collection of test results, is that it is difficult to calculate. It can be estimated by plotting what is called a cumulative frequency diagram. Such a diagram is shown in Fig. 62. The abscissæ represent percentage errors, p, obtained

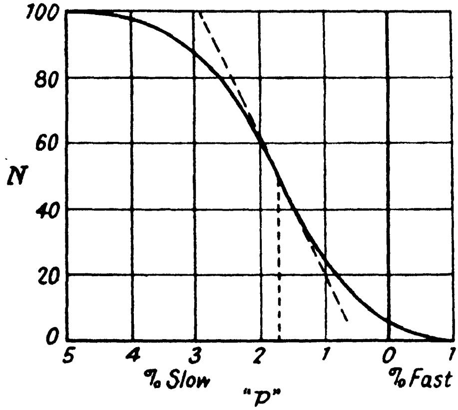

FIG. 62.—Cumulative frequency diagram.

as described. The ordinates represent actual numbers of meters which have 'p' values not more than the abscissæ corresponding. Thus, for the curve shown, of a collection of test results for 100 meters, 88 have 'p' values of 3 per cent. slow and less. The modal error is defined by the point of maximum slope of this curve, and this mode is seen to be about 1·7 per cent. slow. The position of the inflecting point of maximum slope cannot be determined very accurately, but this graphical method of estimating a modal average is quicker than the arithmetical process. The disadvantage of the mode, as a representative average, is that a cumulative frequency curve, constructed in the manner described, may inflect at three points, giving two points of maximum slope. Although the greater value of these maxima will be taken as the mode, the significance of the average is indefinite.

The median average has not been proposed as a representative value of a collection of test results. This average has the advantage that it is easily determined, and that, like the mode, it is not affected by a few abnormally great errors.

No single statistical average of a collection of meter test results can, however, be considered to be satisfactory as a measure of quality. Much more definite information can be conveyed by two representative average numbers, the one being the mean and the other the square root of the average sum of the squares of all the differences from this mean. This latter representative number is known by statisticians as the standard deviation, and it is evidently a measure of the 'spread' with respect to the arithmetical average. As the standard deviation is based on an average square value, it is independent of the direction of the differences from the mean. The significance of a 'standard deviation' average in the interpretation of a mass of numerical data is manifest. In regard to experimental data, such as meter test results representative of performance, standard deviation is a measure of uniformity. Thus if the characteristic changes of the permanent magnet braking flux and the frictional torque of meters, denoted by p and q in the preceding section, are averaged for a class, and the mean and standard deviation are calculated, these two statistical characteristics of the class will furnish a good measure of quality, or, rather, poorness of quality.

The calculation of standard deviation can be expedited by a simple artifice which depends upon the fact that its square is equal to the mean square of the magnitudes averaged less the square of their mean. This is easily proved. Denoting standard deviation by the usual symbol, σ, and the mean of a series of N magnitudes x_1, x_2, etc., by $\bar{x}$, we have by definition

$$\begin{aligned} \mathrm{N}\sigma^2 &= \Sigma(x - \bar{x})^2 \\ &= \Sigma x^2 + \mathrm{N}(\bar{x})^2 - 2\bar{x}(\Sigma x) \\ &= \Sigma x^2 + \mathrm{N}(\bar{x})^2 - 2\bar{x} \times \mathrm{N}\bar{x}. \end{aligned}$$

Whence
$$\sigma^2 = \frac{\Sigma x^2}{\mathrm{N}} - (\bar{x})^2.$$

By this artifice the value of σ is easily calculated at the same time as the mean, by obtaining the sum of the squares of the magnitudes to be averaged.

A method of statistical analysis of the changes of meter accuracy after prescribed service carried out by one of the foregoing methods gives a rational basis for the comparison of classes of meters and a justification for conscientious 'off-circuit' testing. In no other way can this comparison be satisfactorily made. Statistical analysis of this kind will be assisted if the results of all tests at full load and one-twentieth of full load are entered on the cards on which the histories of the meters are recorded. Whilst it must be admitted that an 'off-circuit' test may possibly not represent with fidelity the actual performance of the meter before removal, this test gives the most approximate indication of this performance that in practice can be obtained.

APPENDIX

Recent Books and Papers dealing with Meters and Meter Testing

BOOKS

‘ Electrical Measuring Instruments,’ by C. V. Drysdale and A. C. Jolley. Part II.

An exhaustive and authoritative treatise dealing very fully with the theory and construction of electricity meters.

‘ Electrical Measurements and Measuring Instruments,’ by E. W. Golding.

A comprehensive text-book which forms a very useful work of reference for testing engineers.

‘ Meter Engineering,’ by J. L. Ferns.

A very useful and comprehensive handbook which contains a wealth of practical information relating to the running of a supply authority’s meter testing and repair department.

‘ Electric Power Metering,’ by A. E. Knowlton.

An excellent book by an American author on the theory and practice of the measurement of consumers’ supplies.

‘ Instrument Transformers,’ by B. Hague.

An authoritative treatise in which all methods of testing instrument transformers for ratio and phase errors are fully and critically described.

‘ Direct and Alternating Potentiometer Measurements,’ by D. C. Gall (Vol. 4 of this series).

An exhaustive treatise in which the application of the A.C. potentiometer to instrument transformer testing is fully described.

PAPERS

‘ Grid Metering,’ by J. Henderson. *Journal I.E.E.*, 1934, Vol. 75, p. 185.

This paper contains a critical review of the various methods of summation measurements of energy and VAr.

'A Study of the Induction Watthour Meter,' by T. **Havekin.** *Journal I.E.E.*, 1935, Vol. 77, p. 355.

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A critical discussion based on both theoretical reasoning and experimental data.

INDEX